Quantum entanglement and the phases of matter

Girdharilal Mehta Lecture
Harish-Chandra Research Institute, Allahabad
January 13, 2012

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PHYSICS
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Outline

1. Quantum critical points and string theory
   Entanglement and emergent dimensions

2. High temperature superconductors
   and strange metals
   Holography of compressible quantum phases

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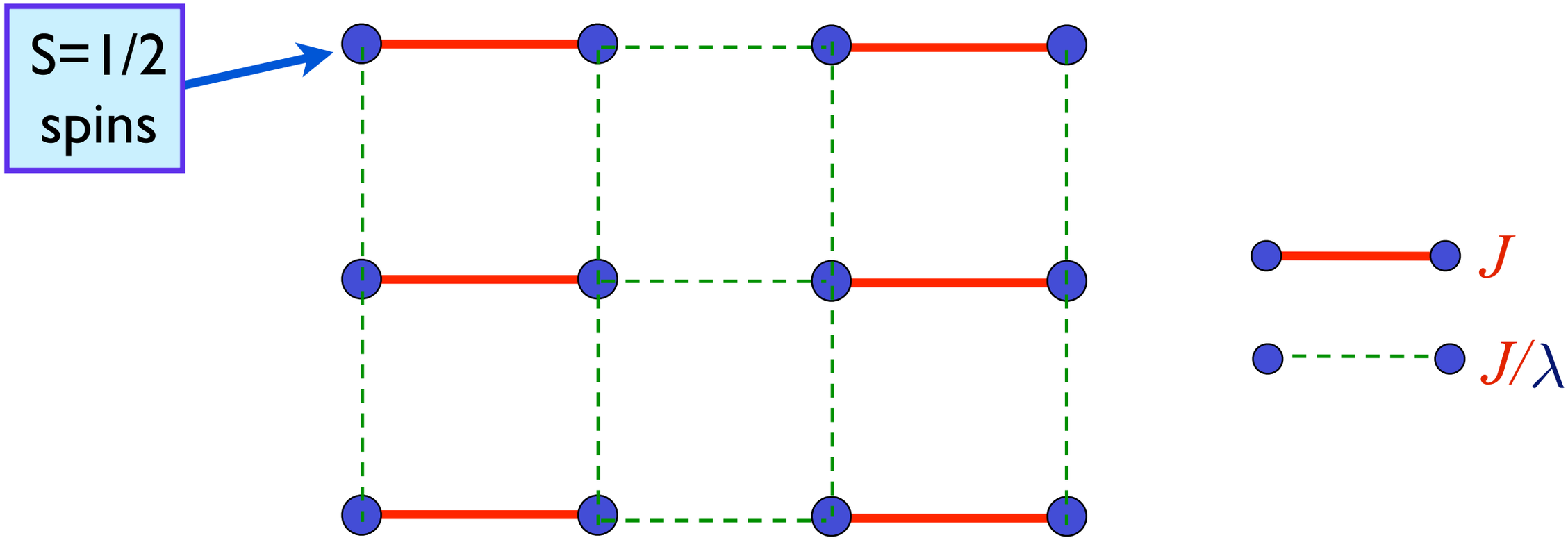
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

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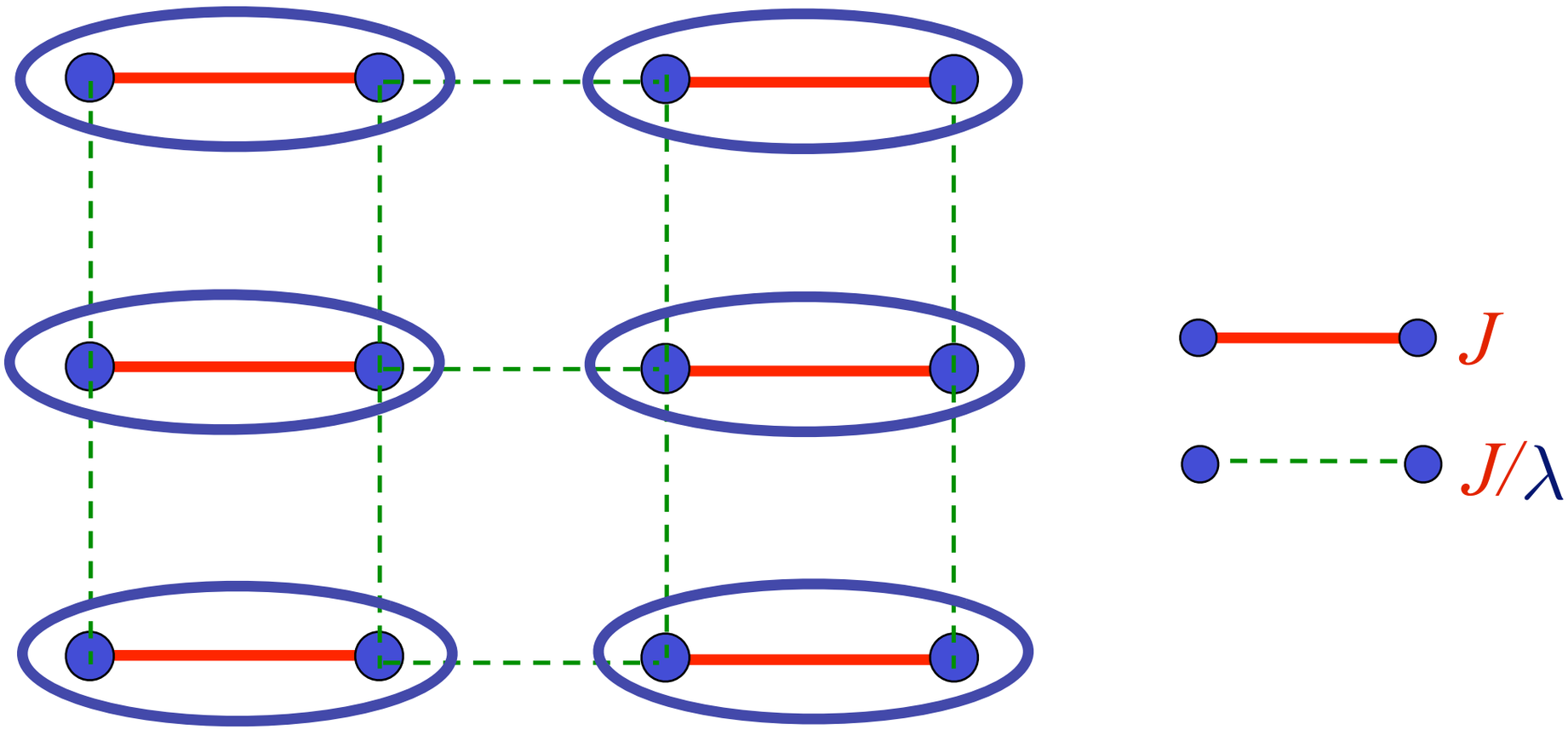

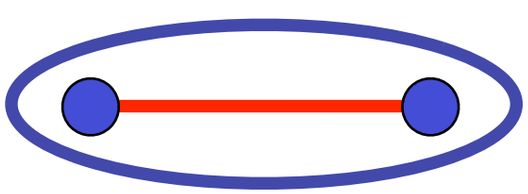

$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

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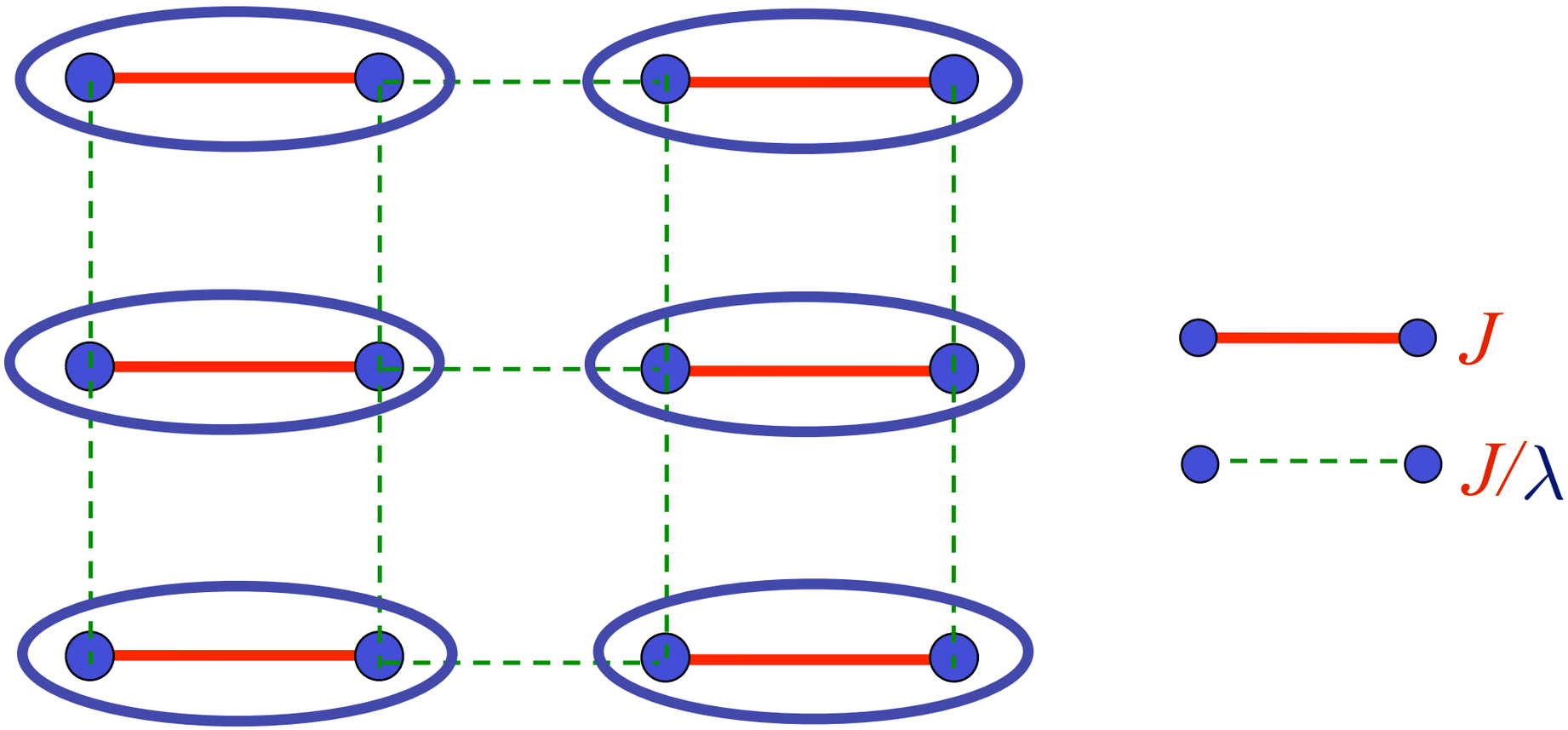

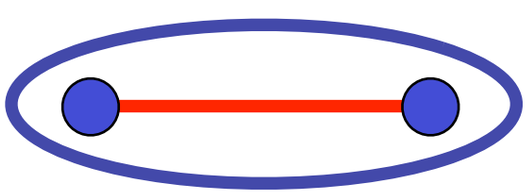

$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Square lattice antiferromagnet

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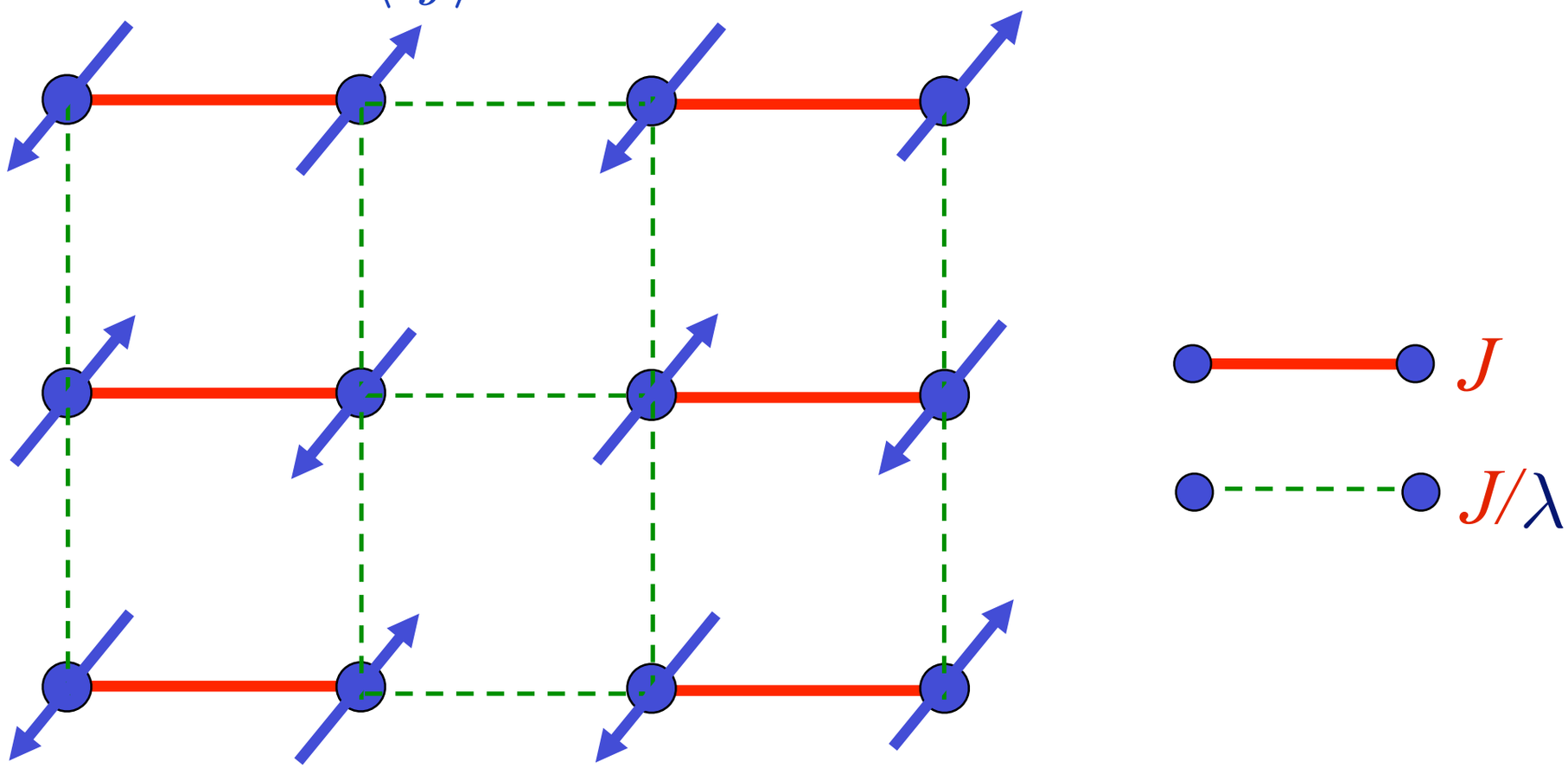

For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

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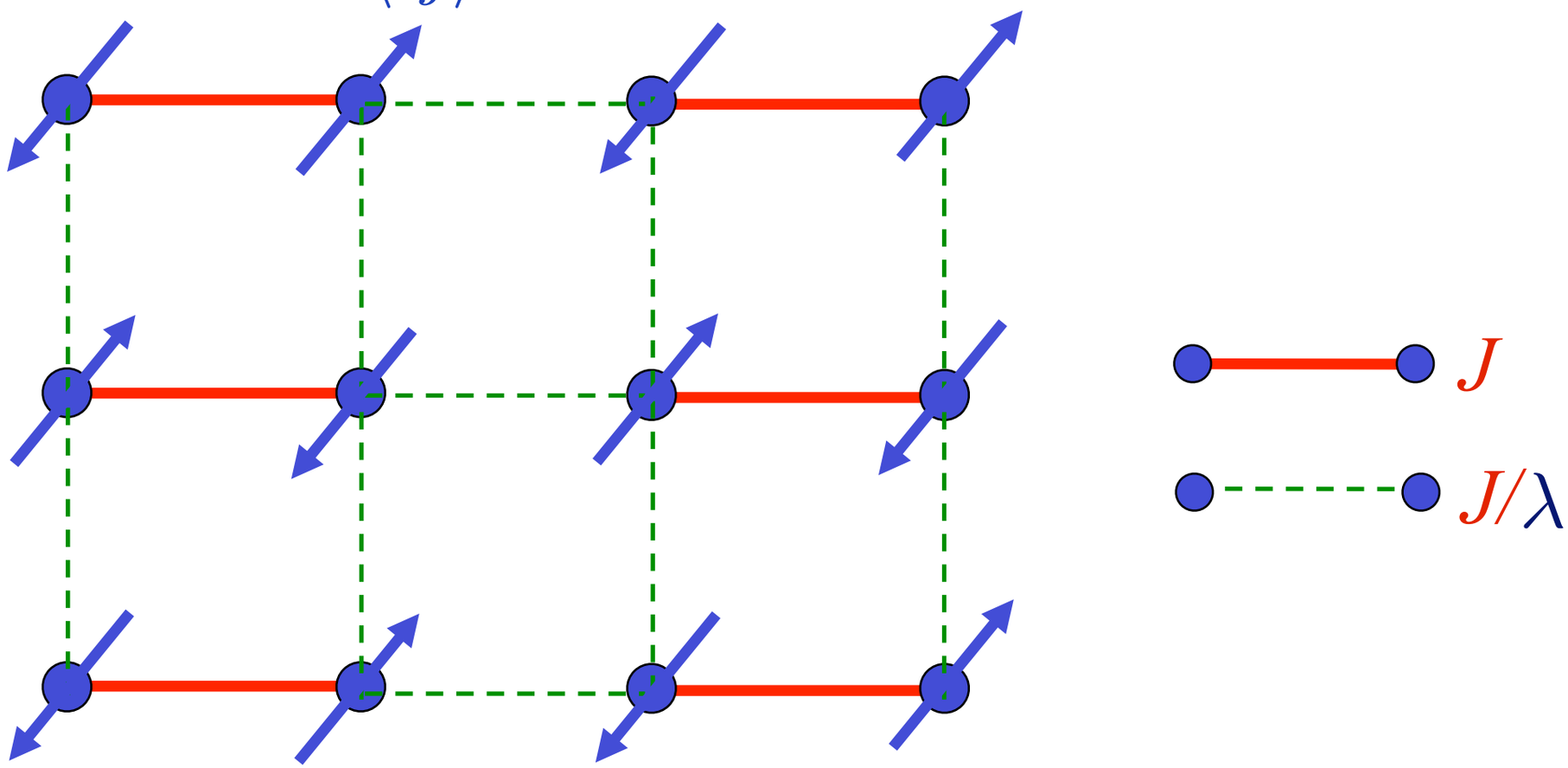

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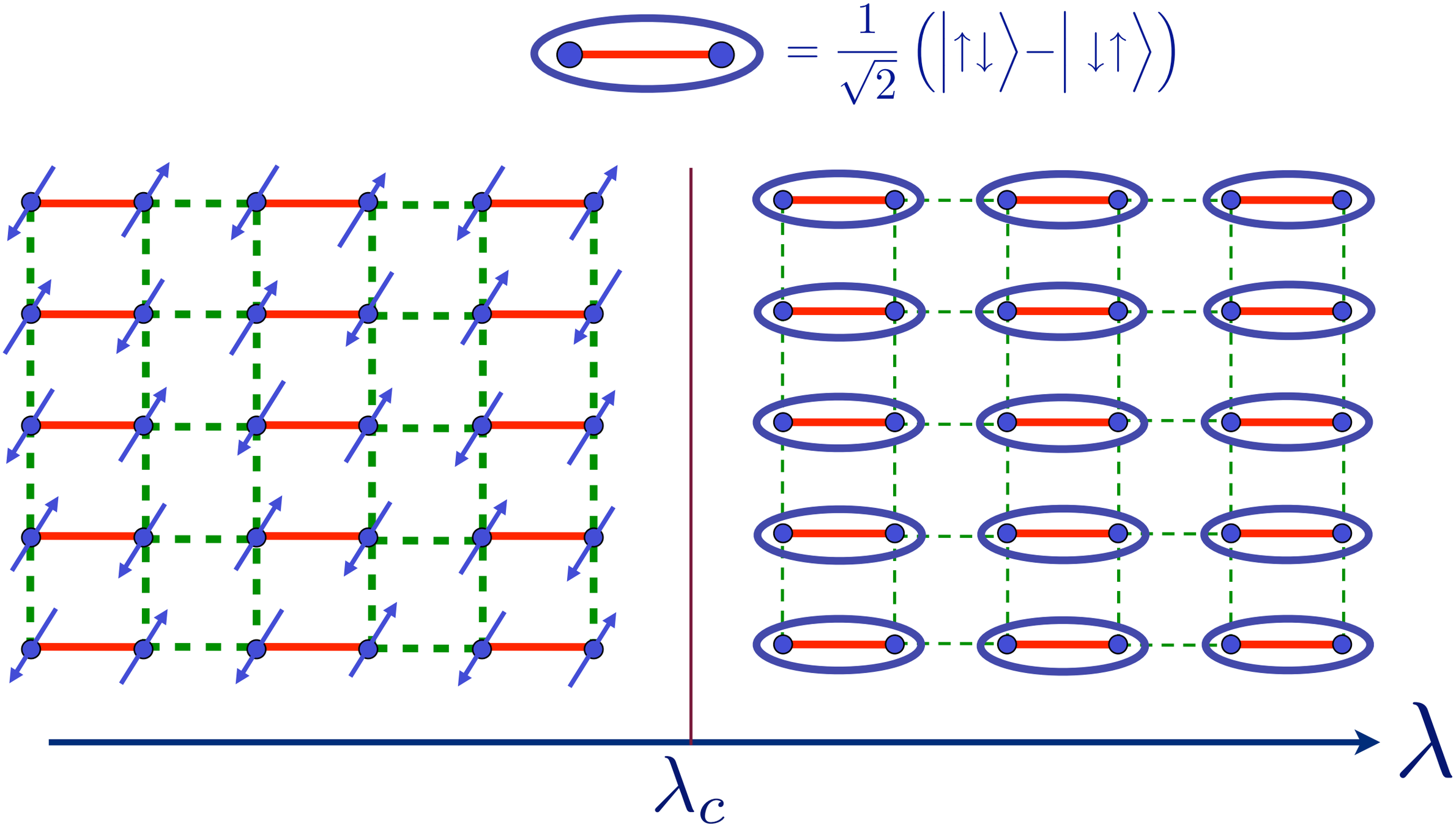
No EPR pairs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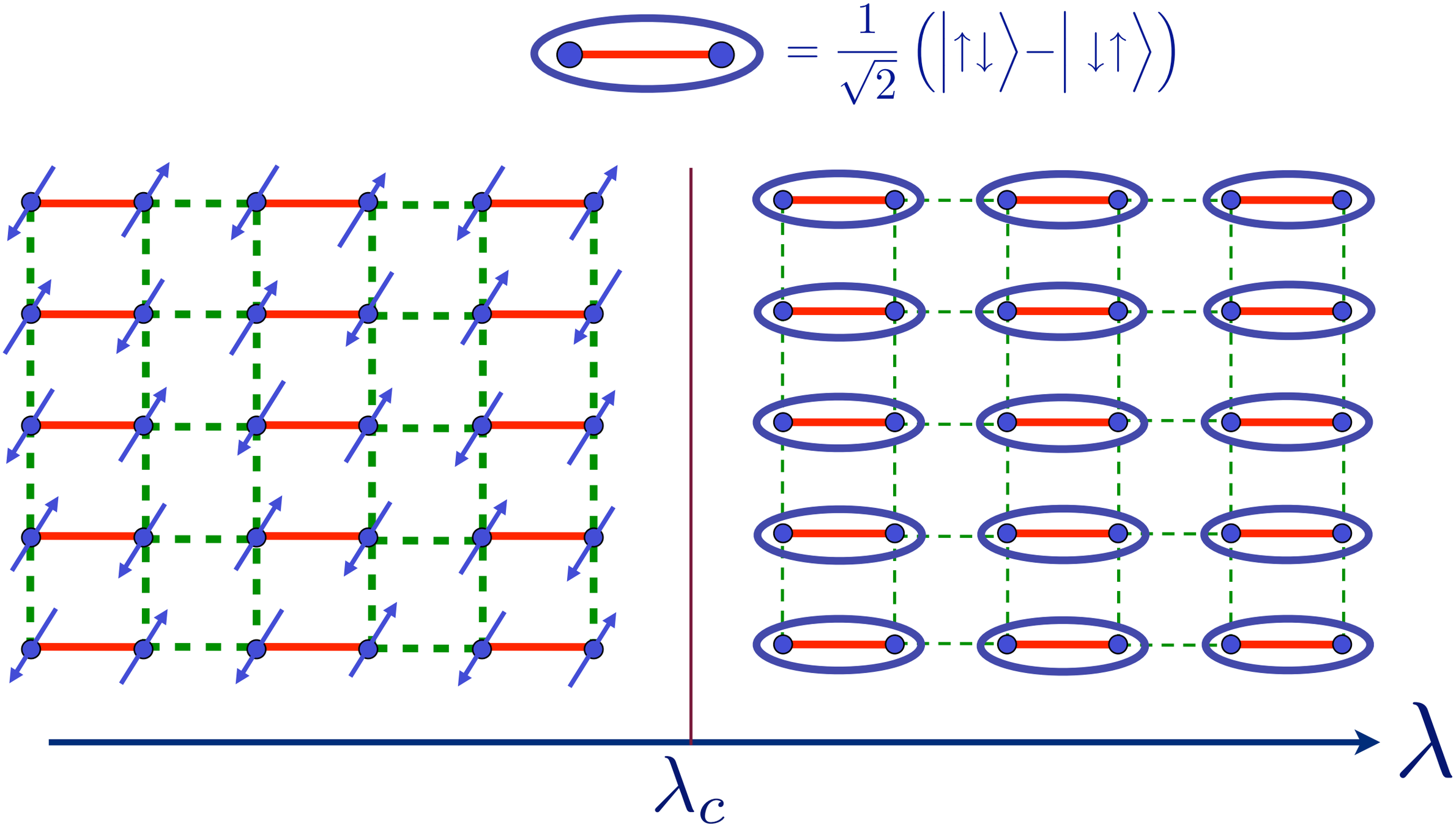
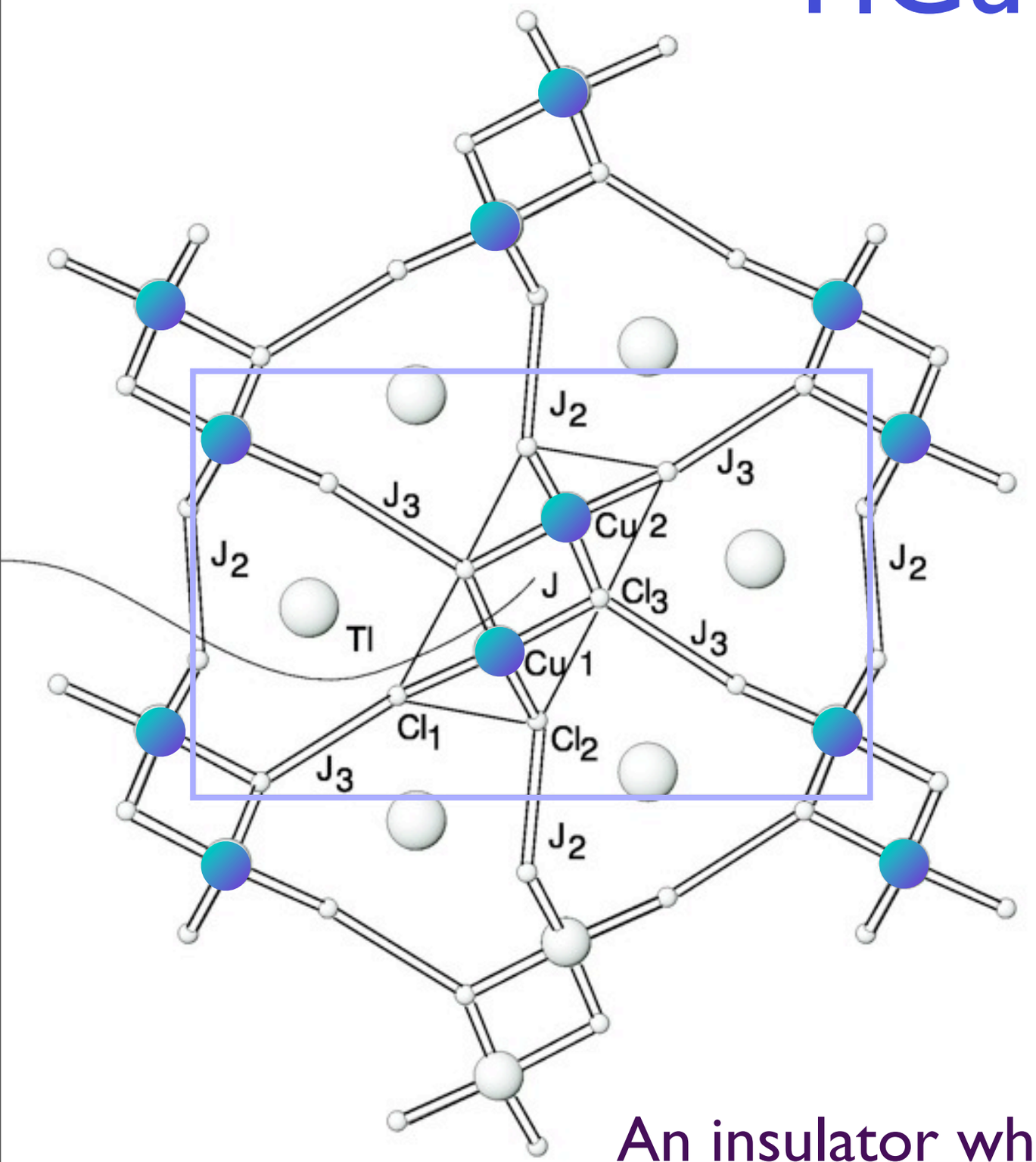
λ
λ_c

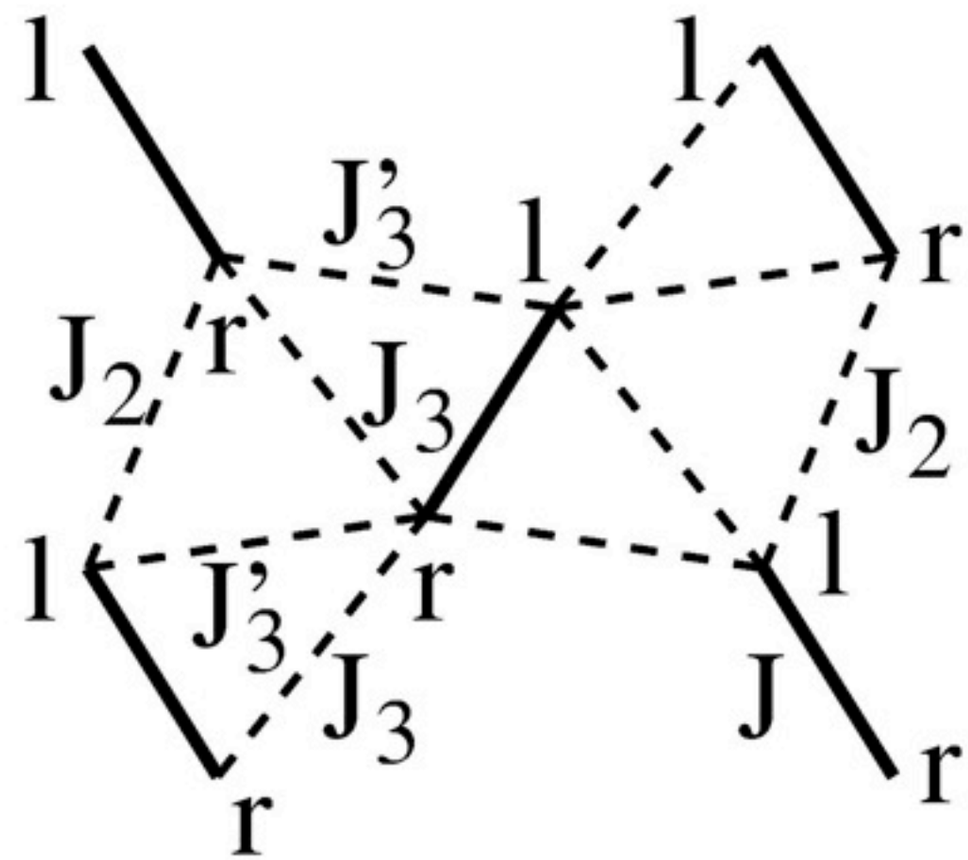


Pressure in $TlCuCl_3$

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

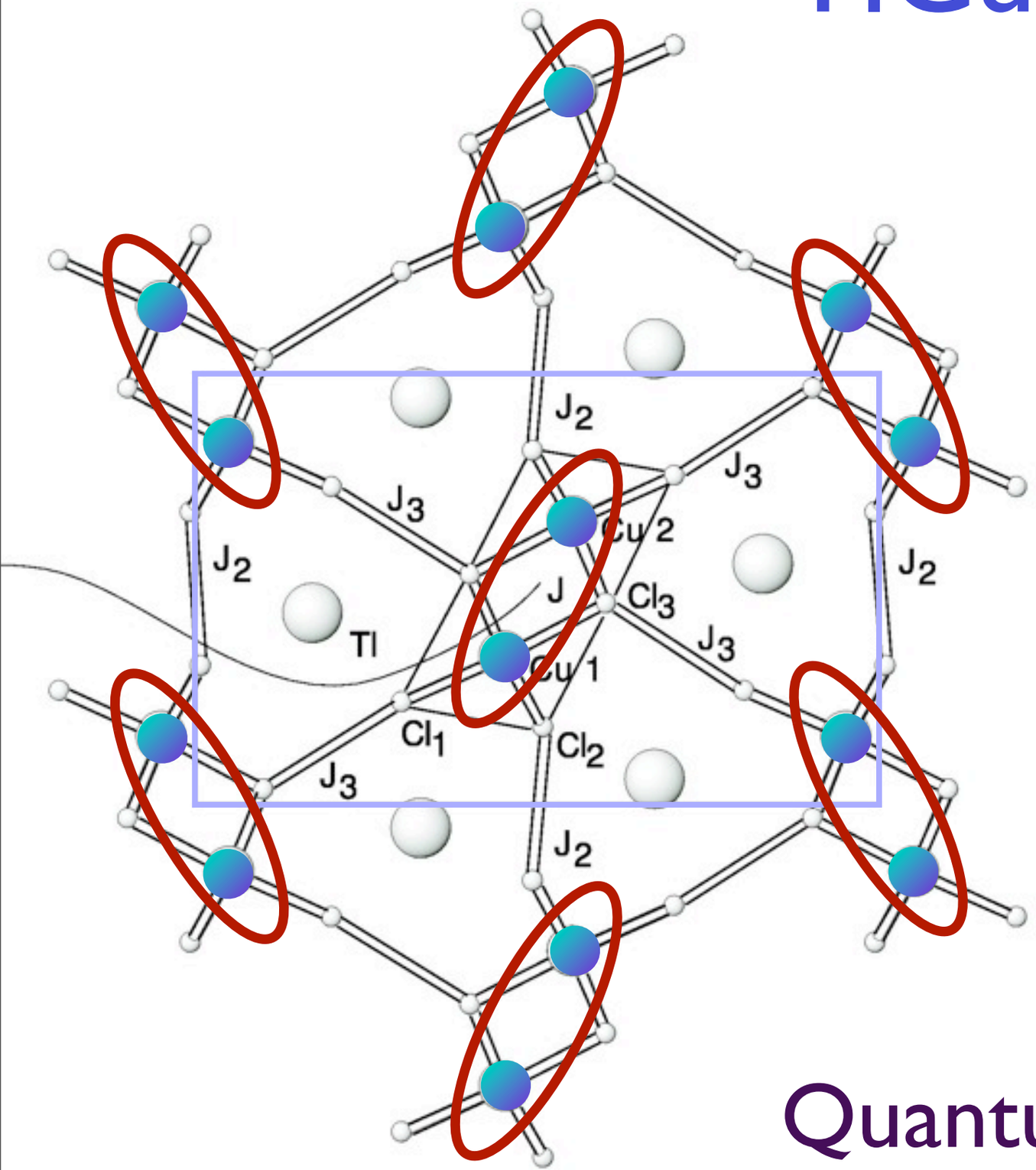
$TlCuCl_3$

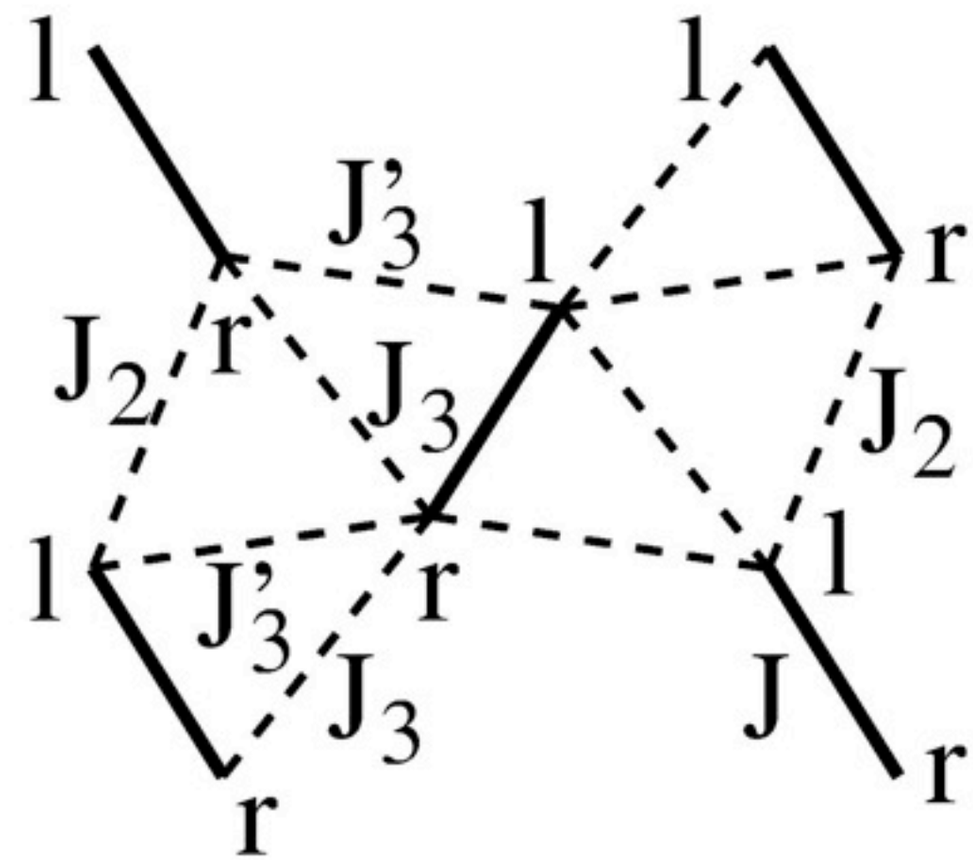




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

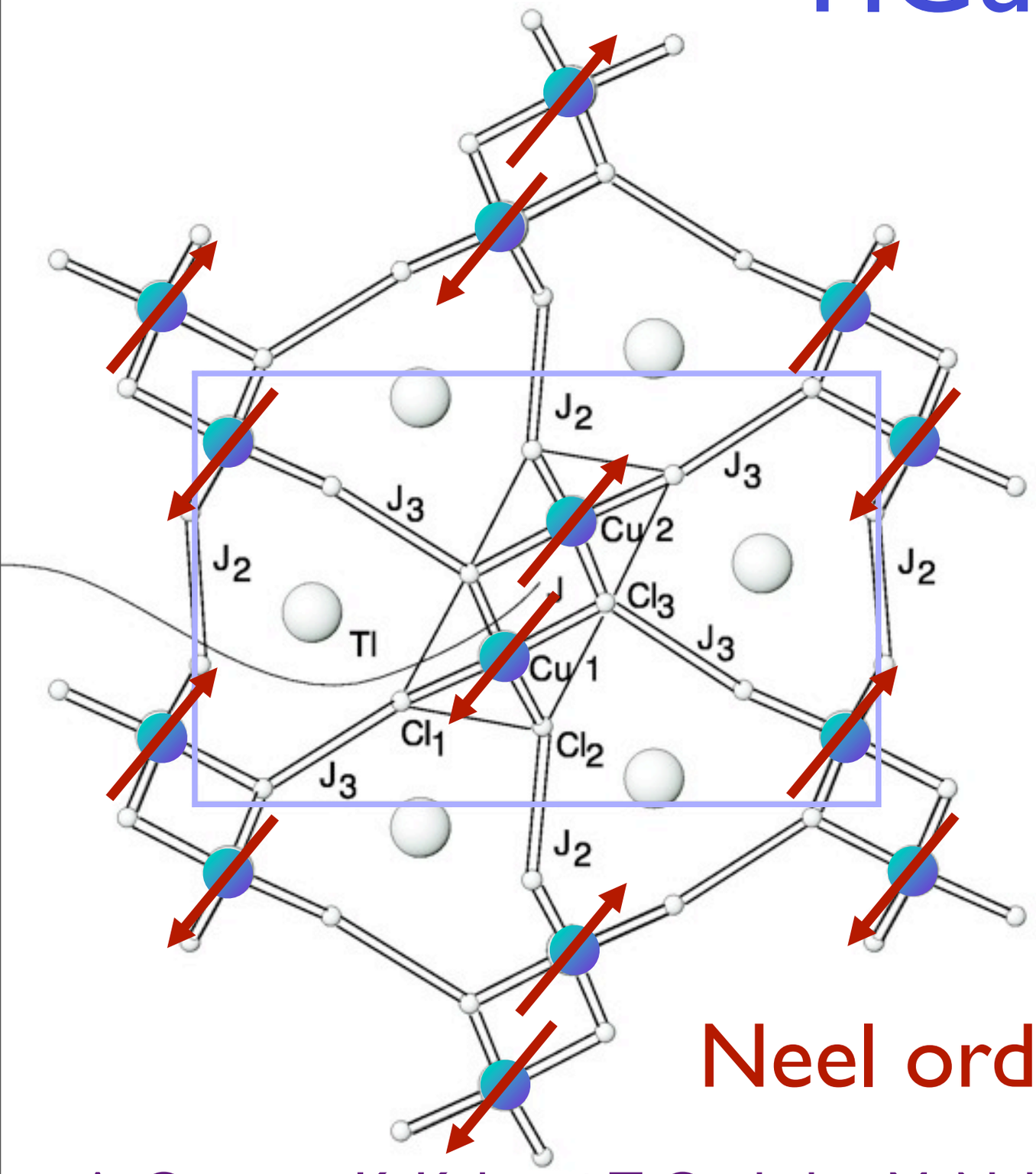
$TlCuCl_3$

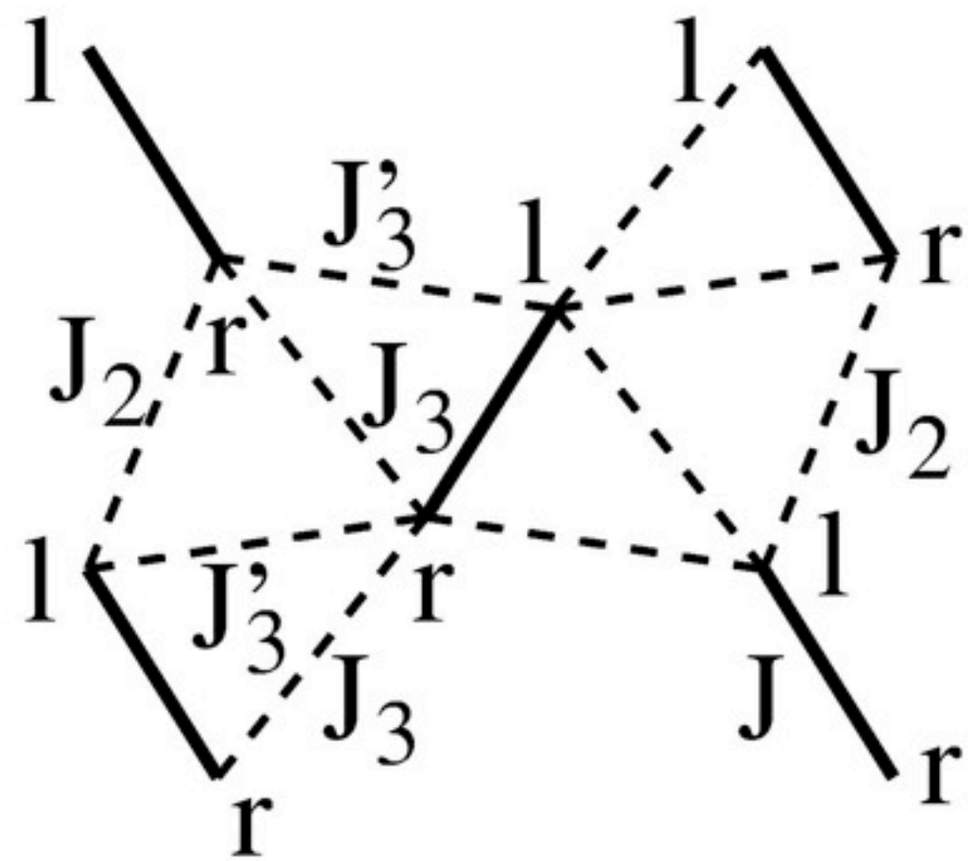




Quantum paramagnet at ambient pressure

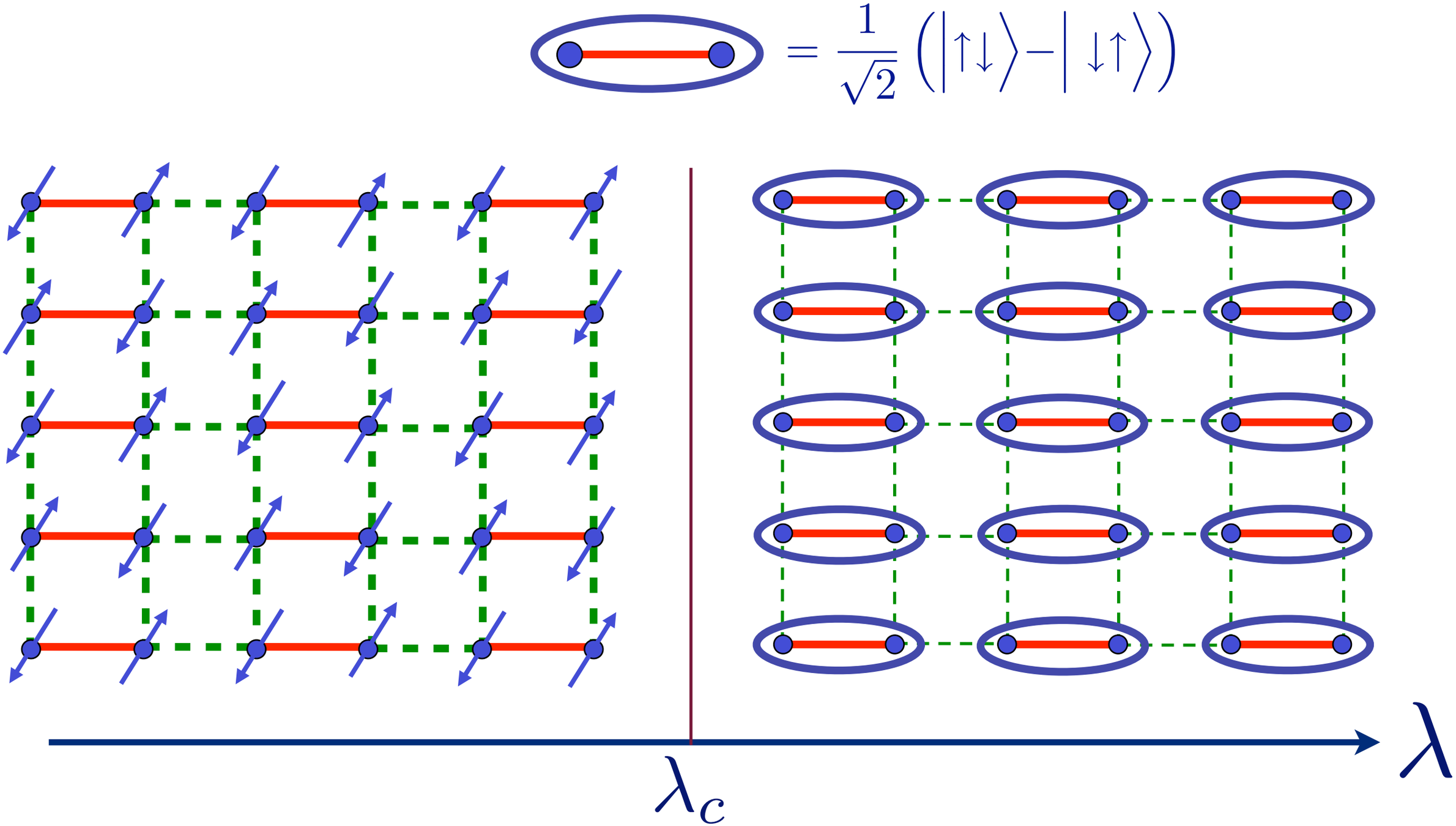
$TlCuCl_3$





Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).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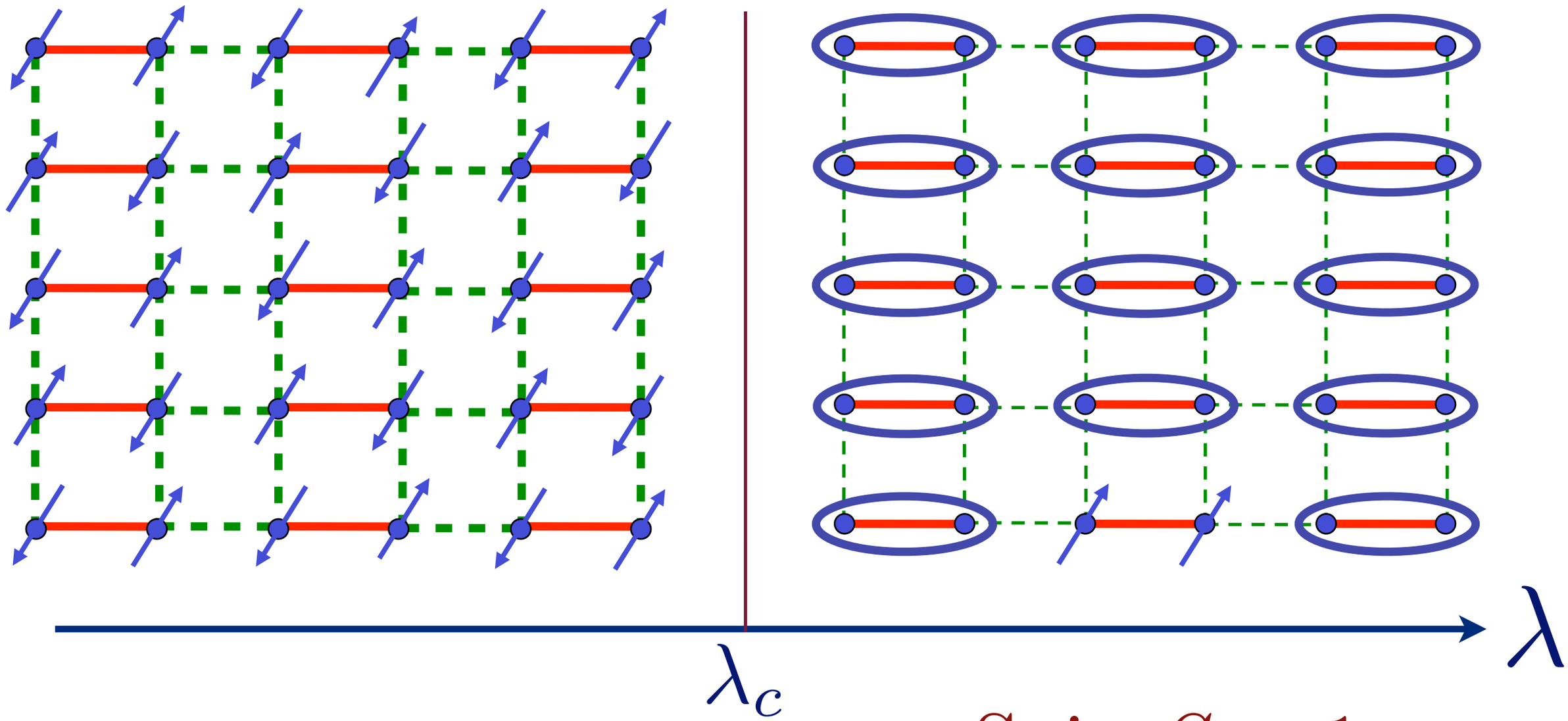
λ_c

Excitation spectrum in the paramagnetic phase



Spin $S = 1$
"triplon"

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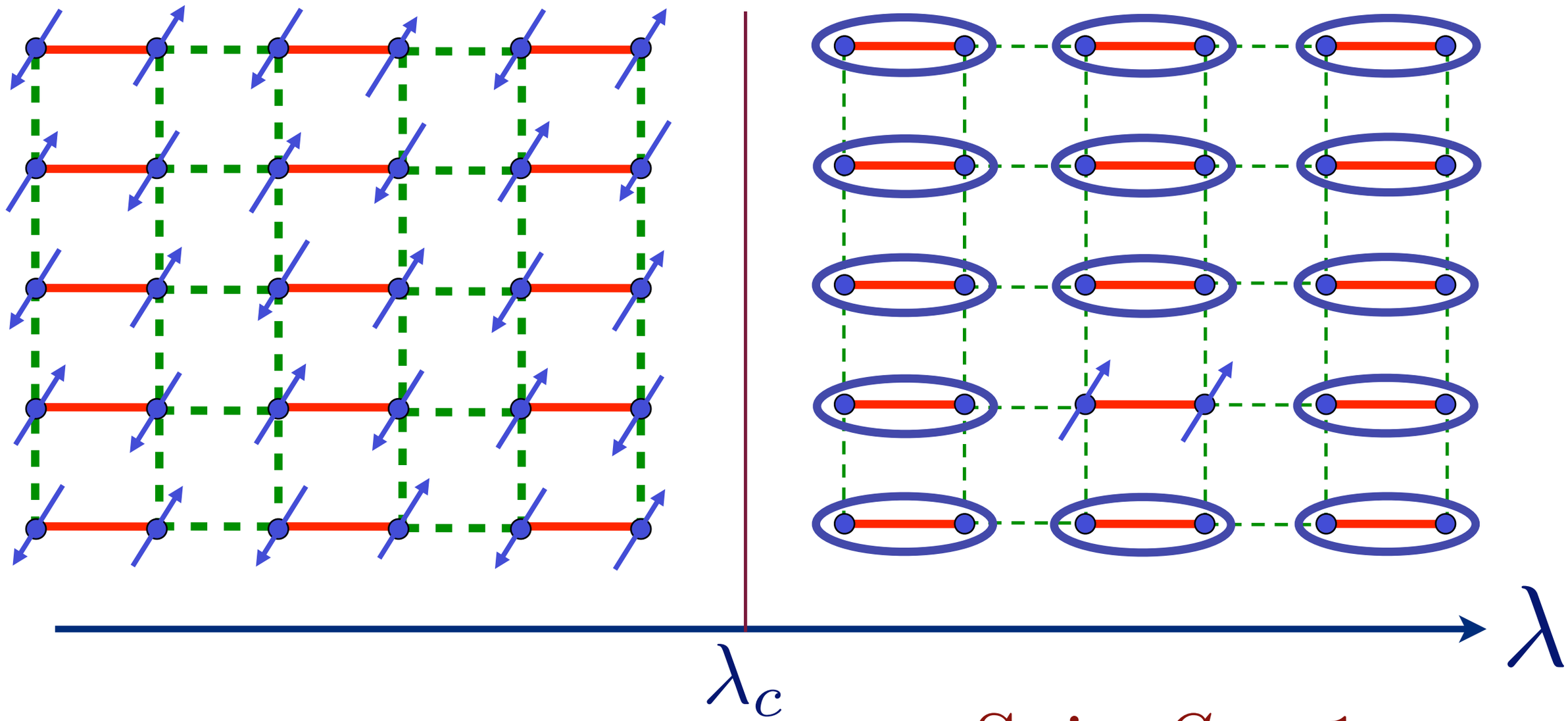

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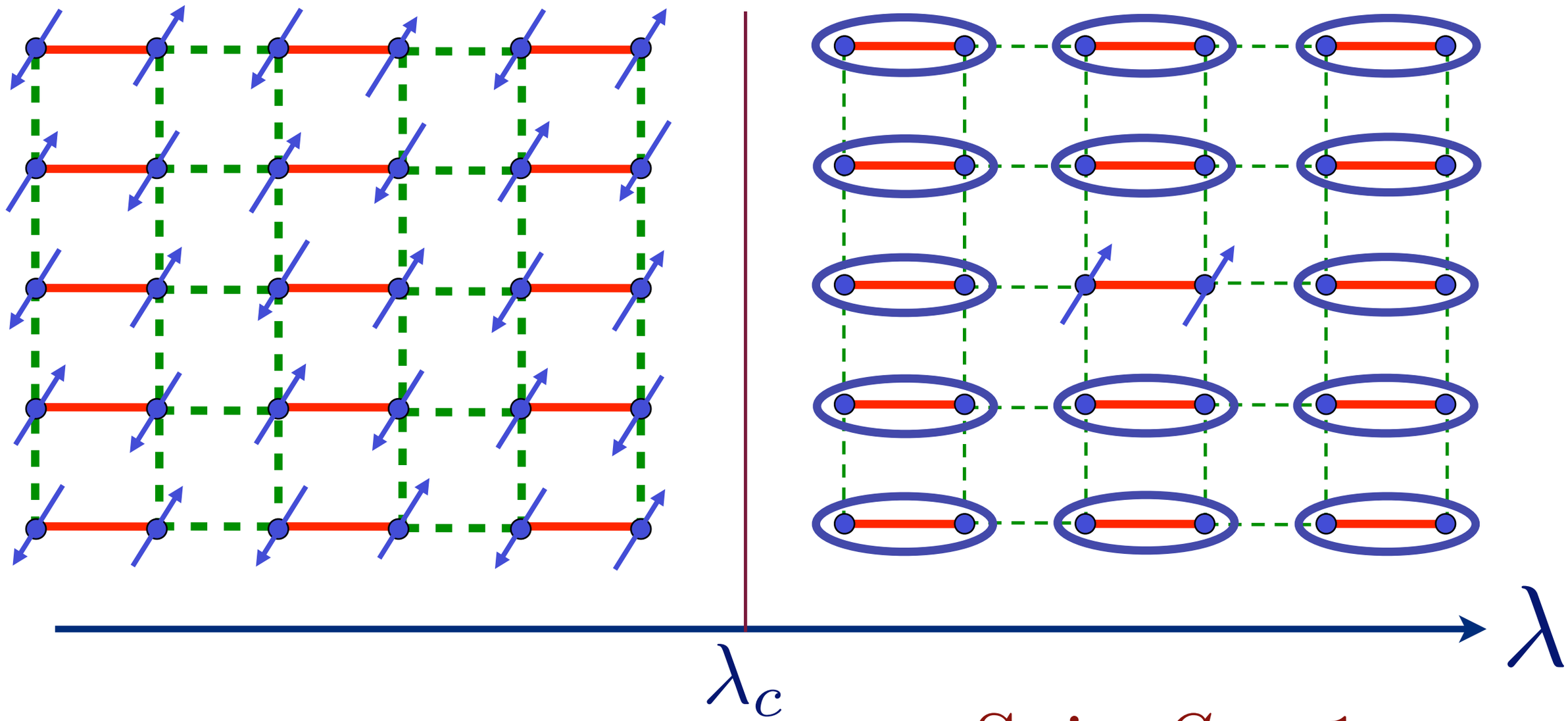

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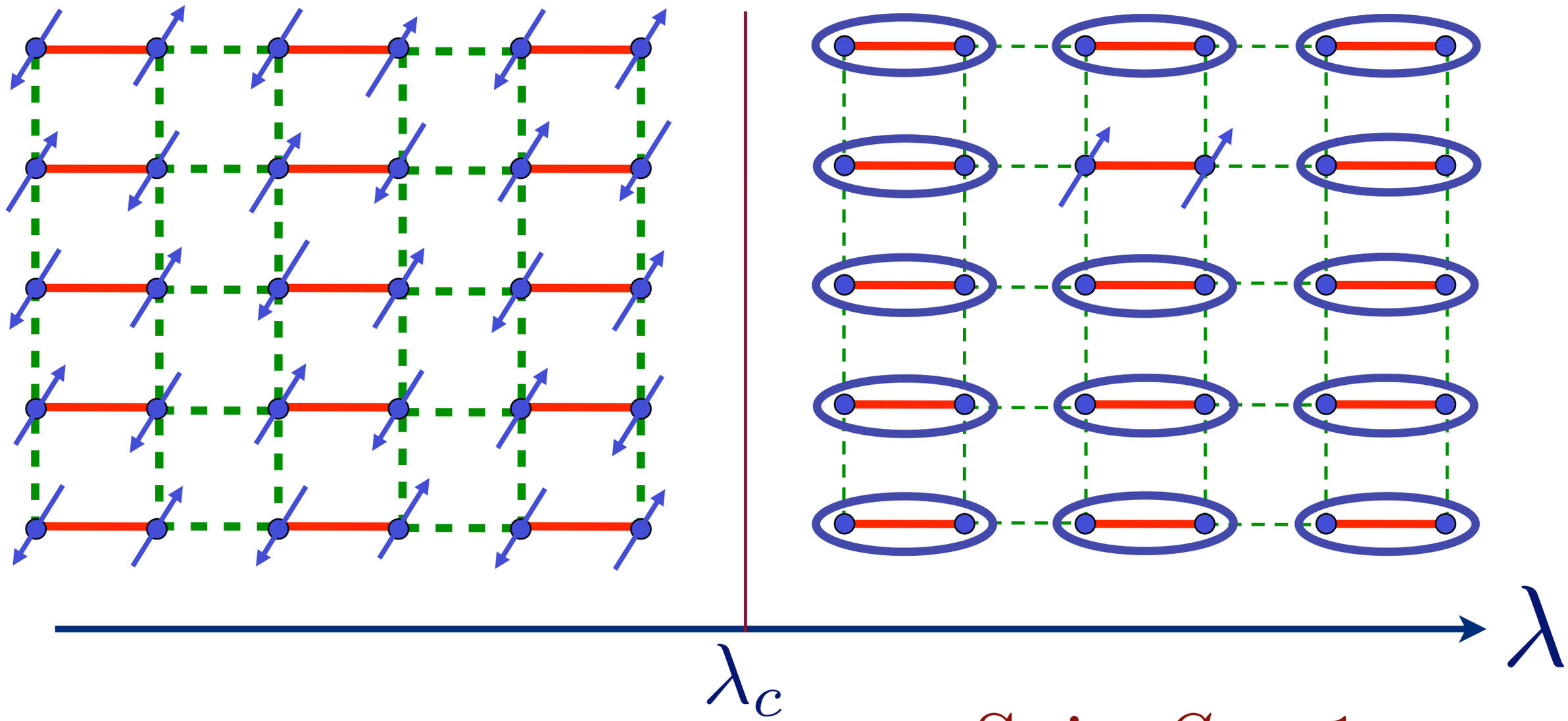

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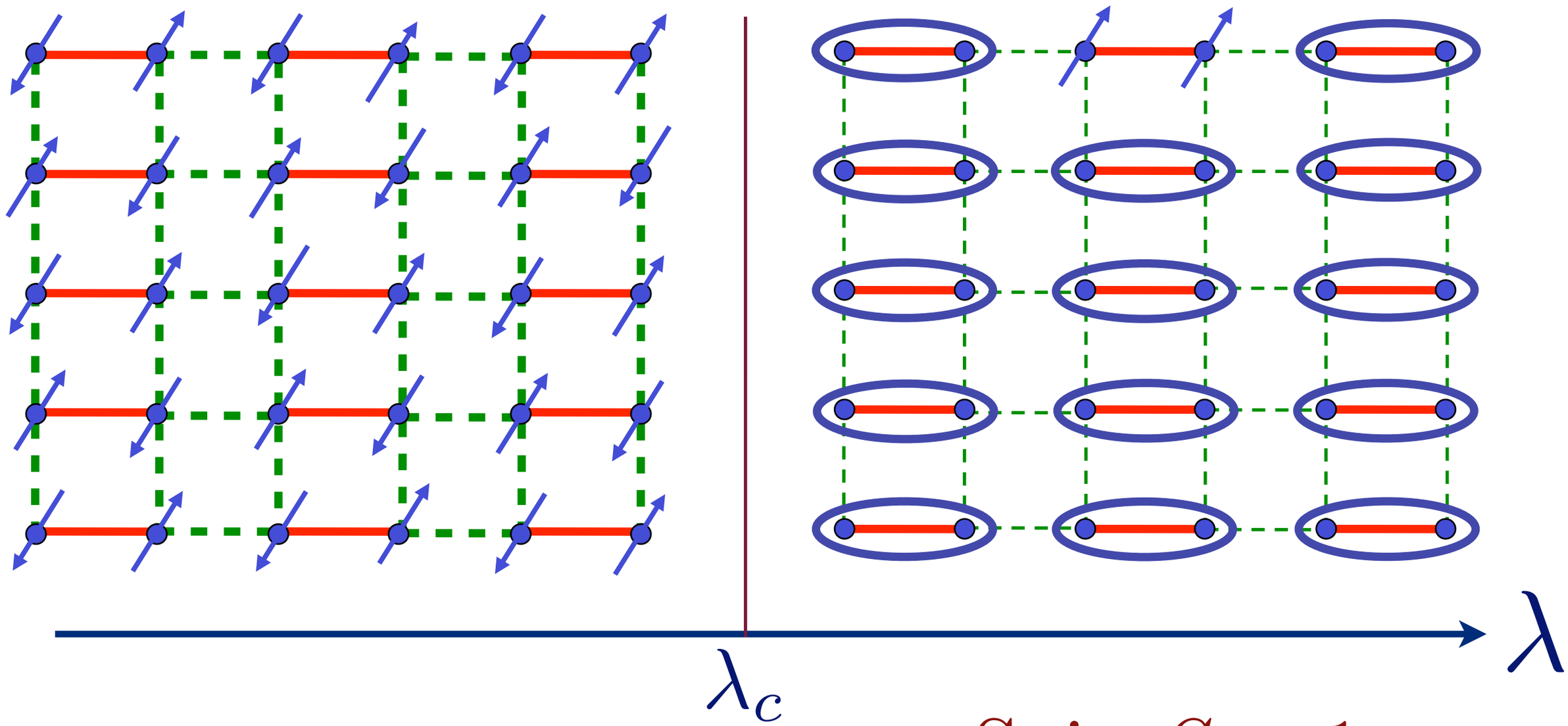

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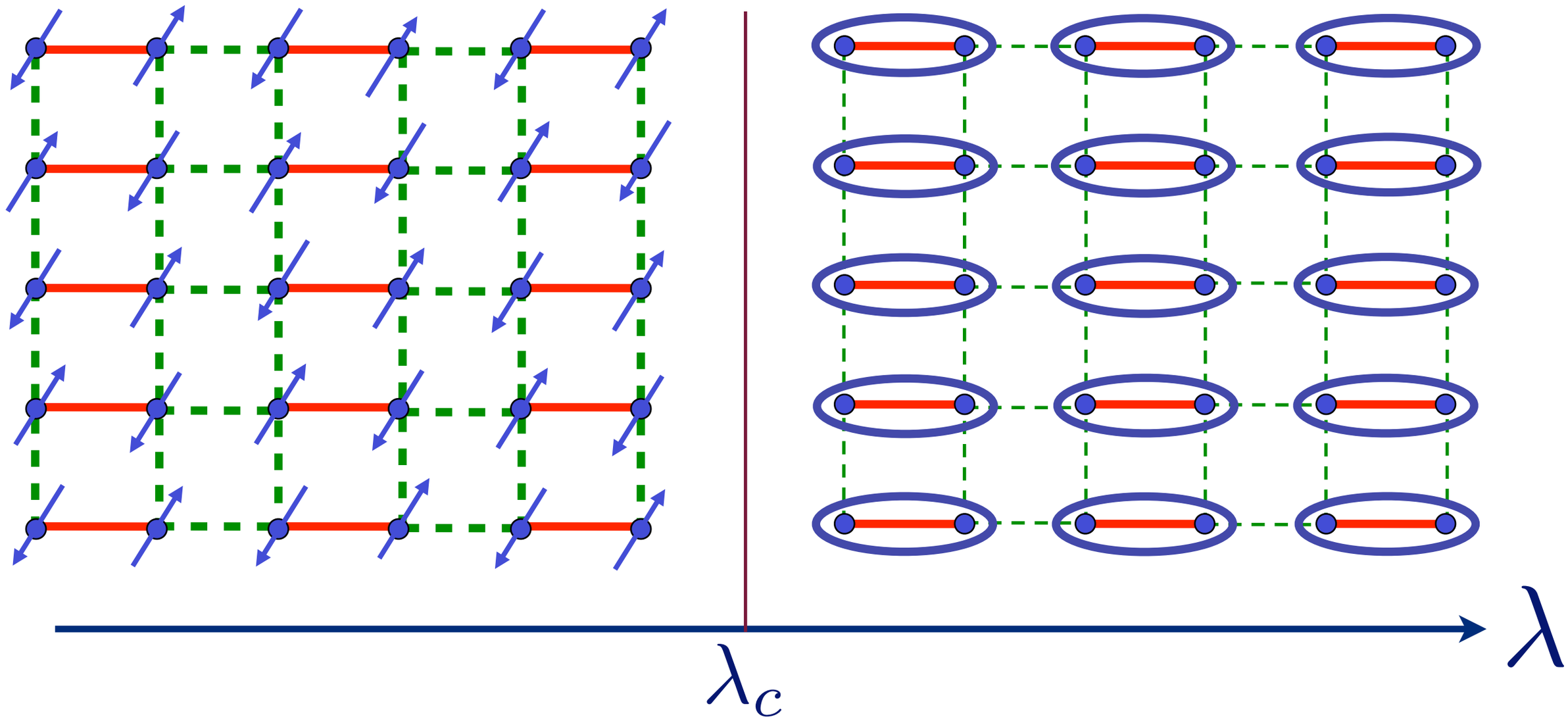
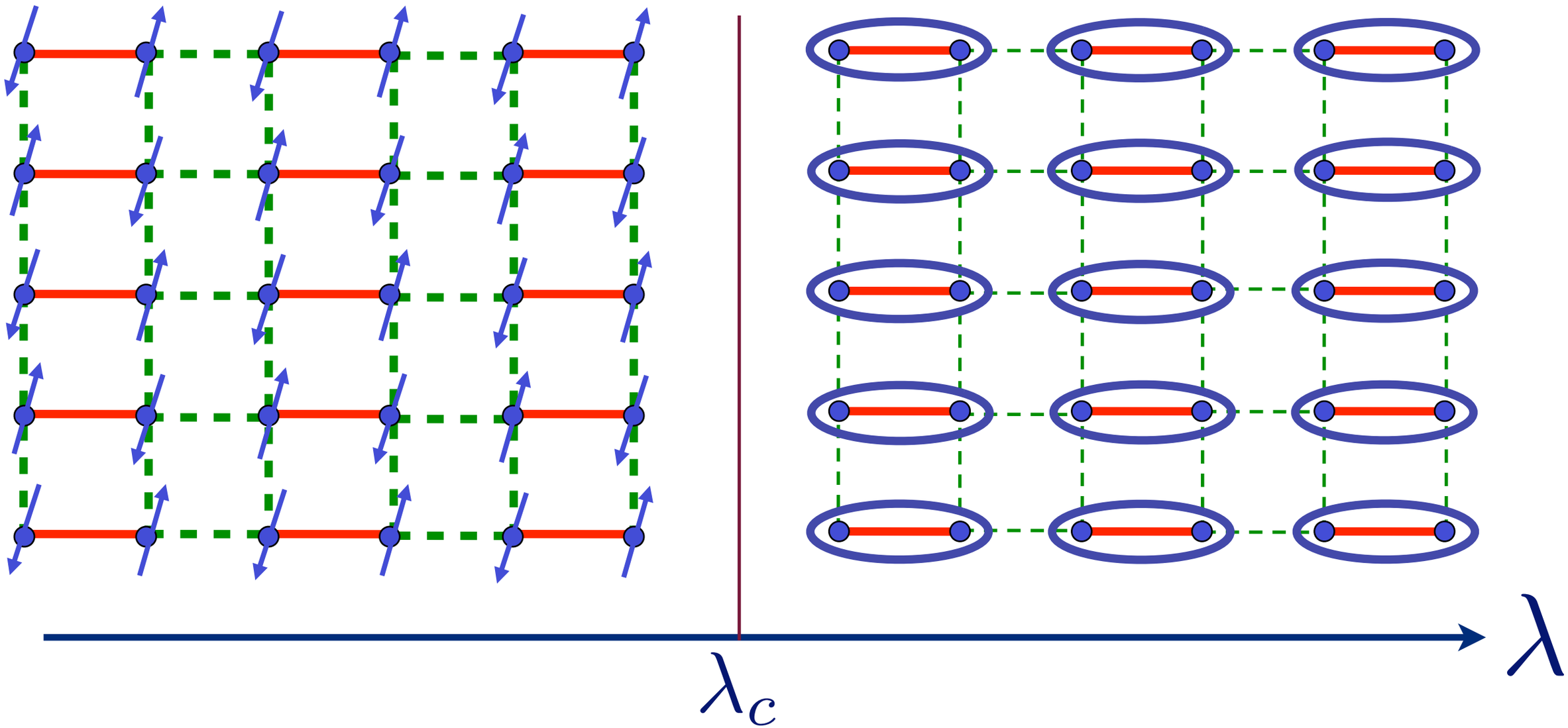
Excitation spectrum in the Néel phase



Spin waves

Excitation spectrum in the Néel phase



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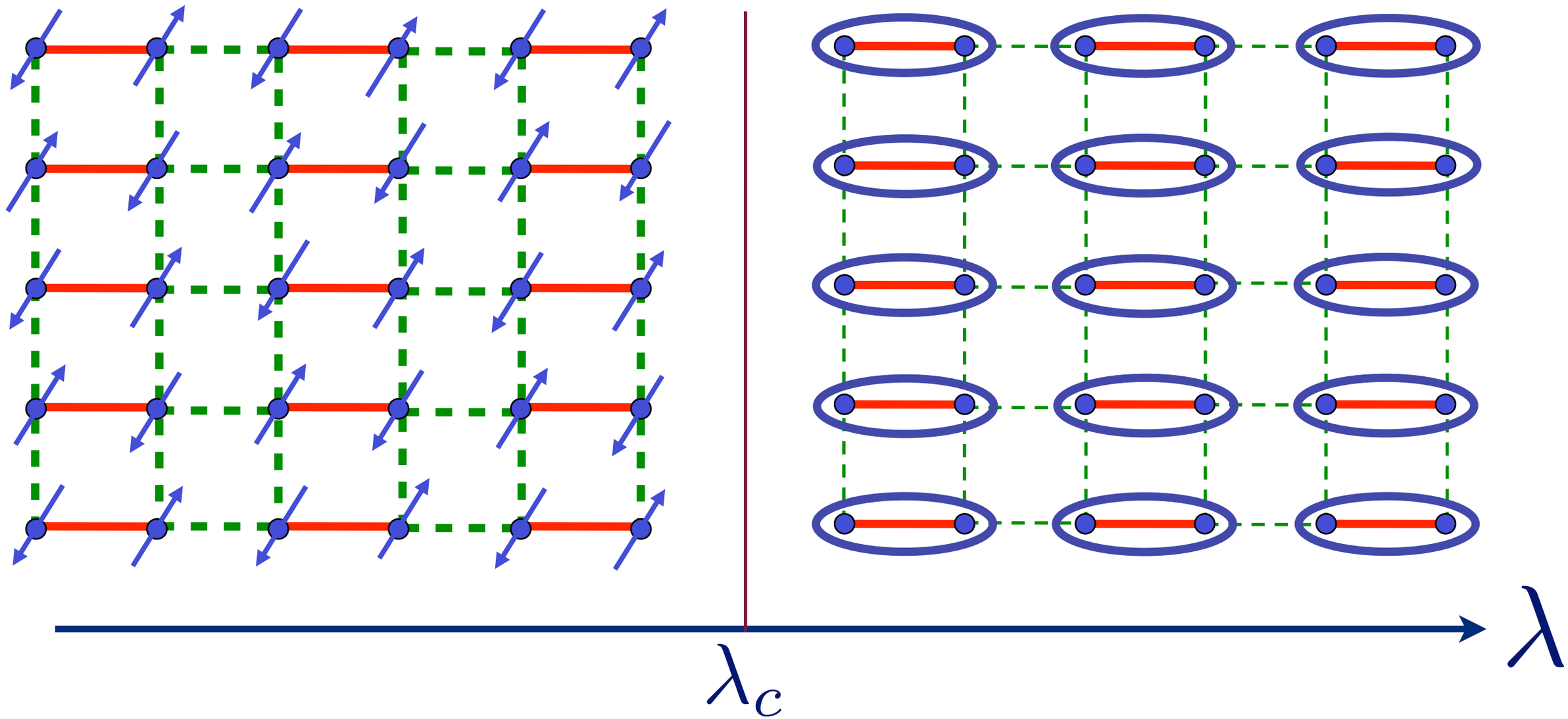
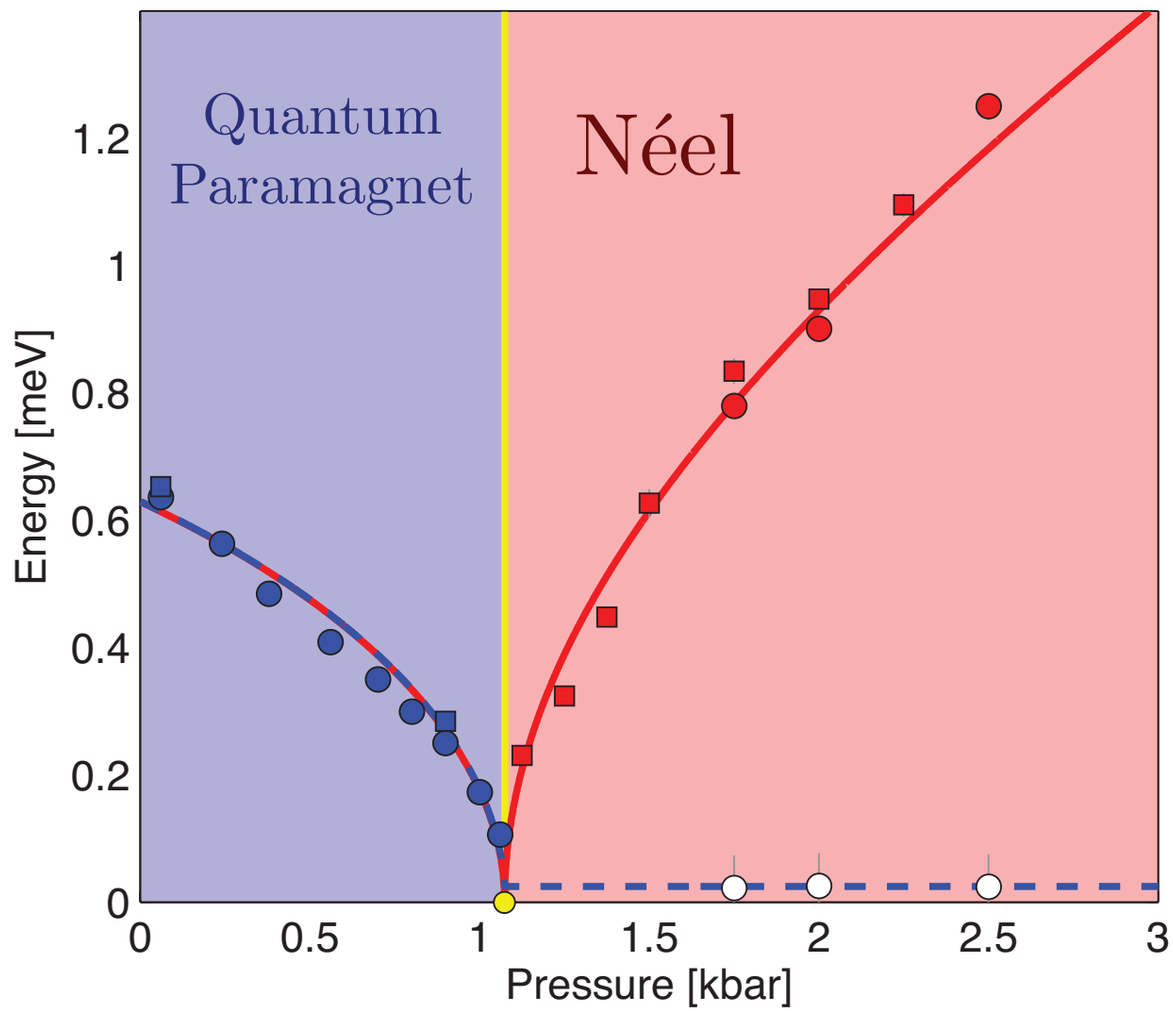


Spin waves

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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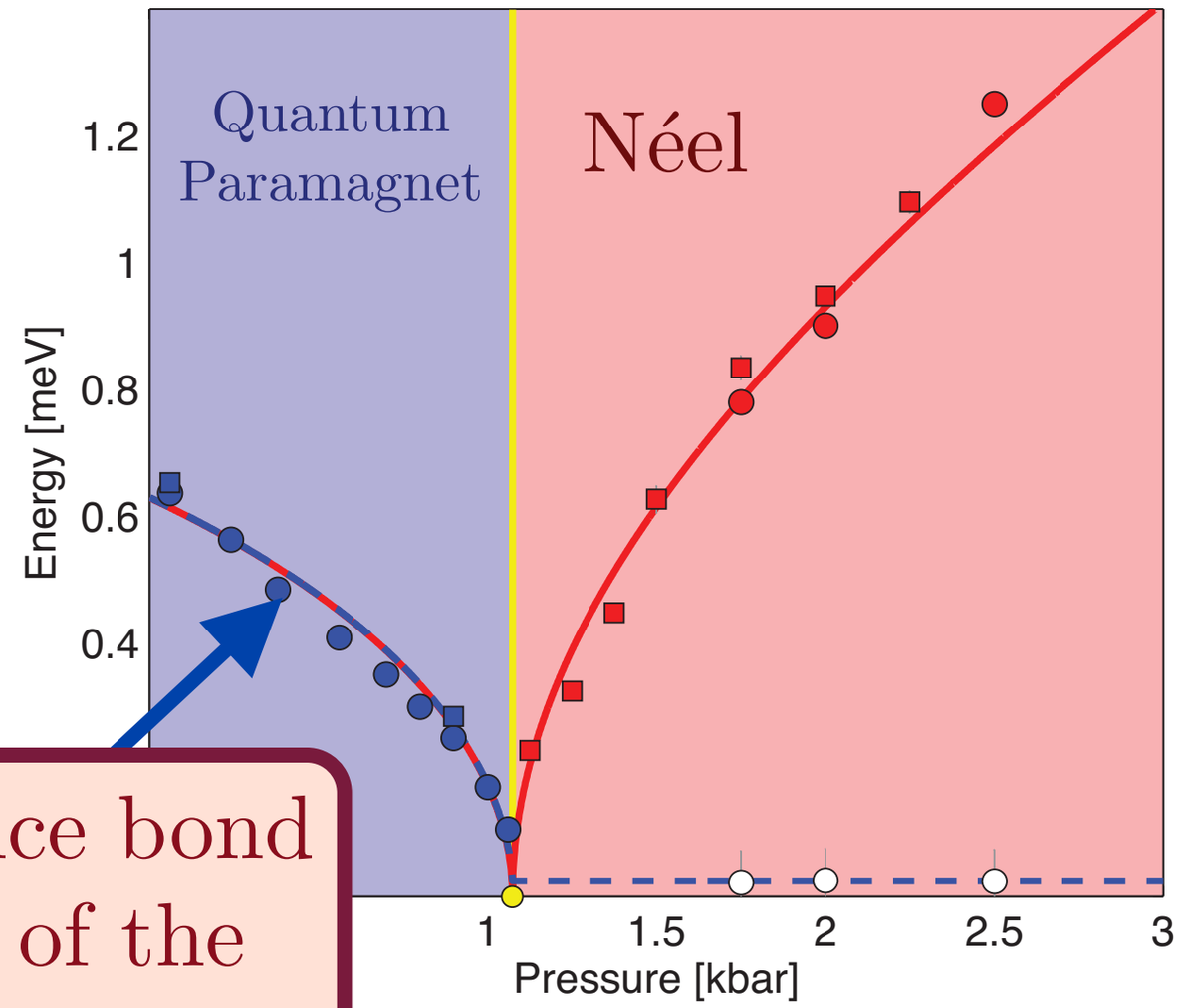

Broken valence bond excitations of the quantum paramagnet

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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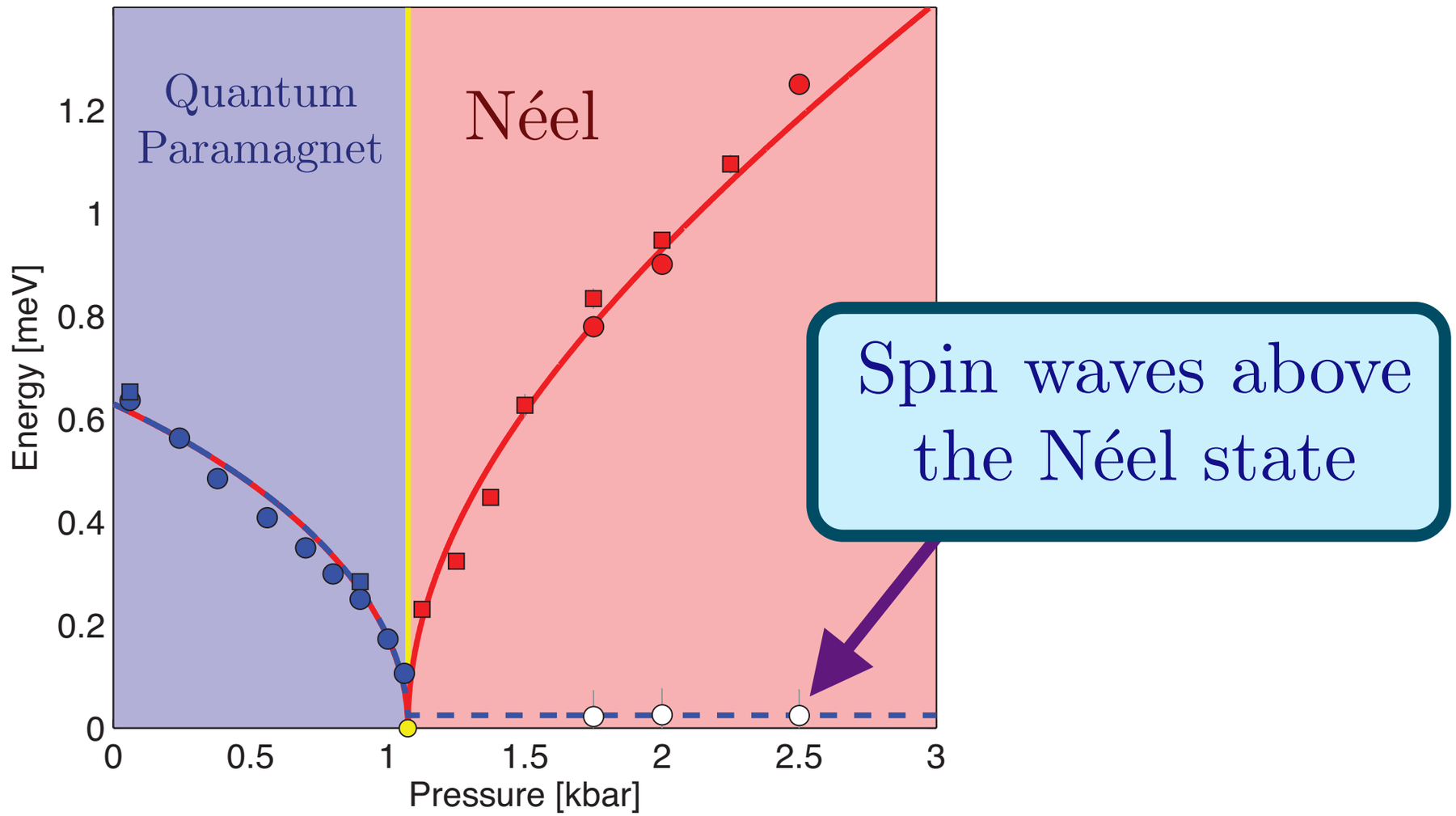

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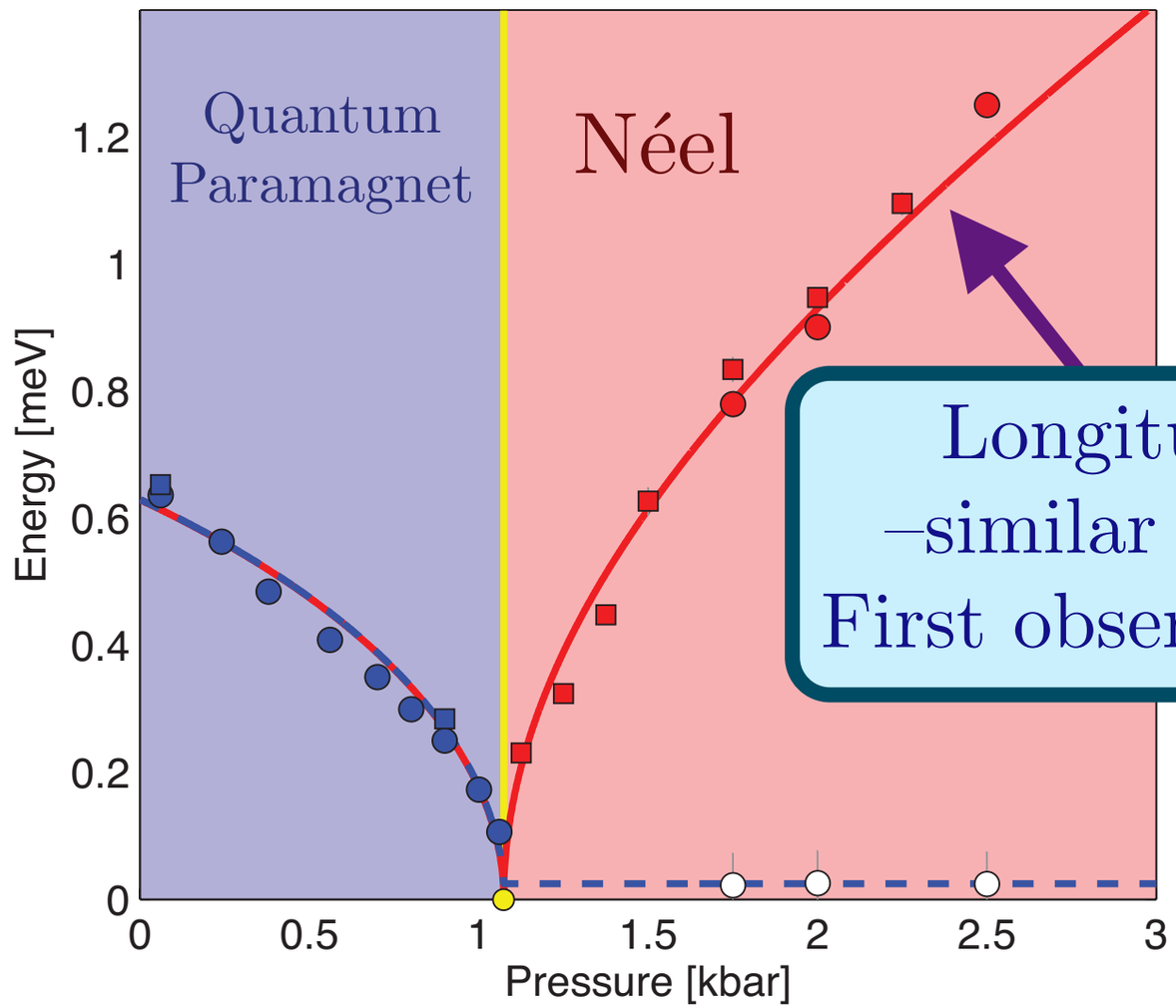

Longitudinal excitations
–similar to the Higgs boson
First observation of the Higgs !

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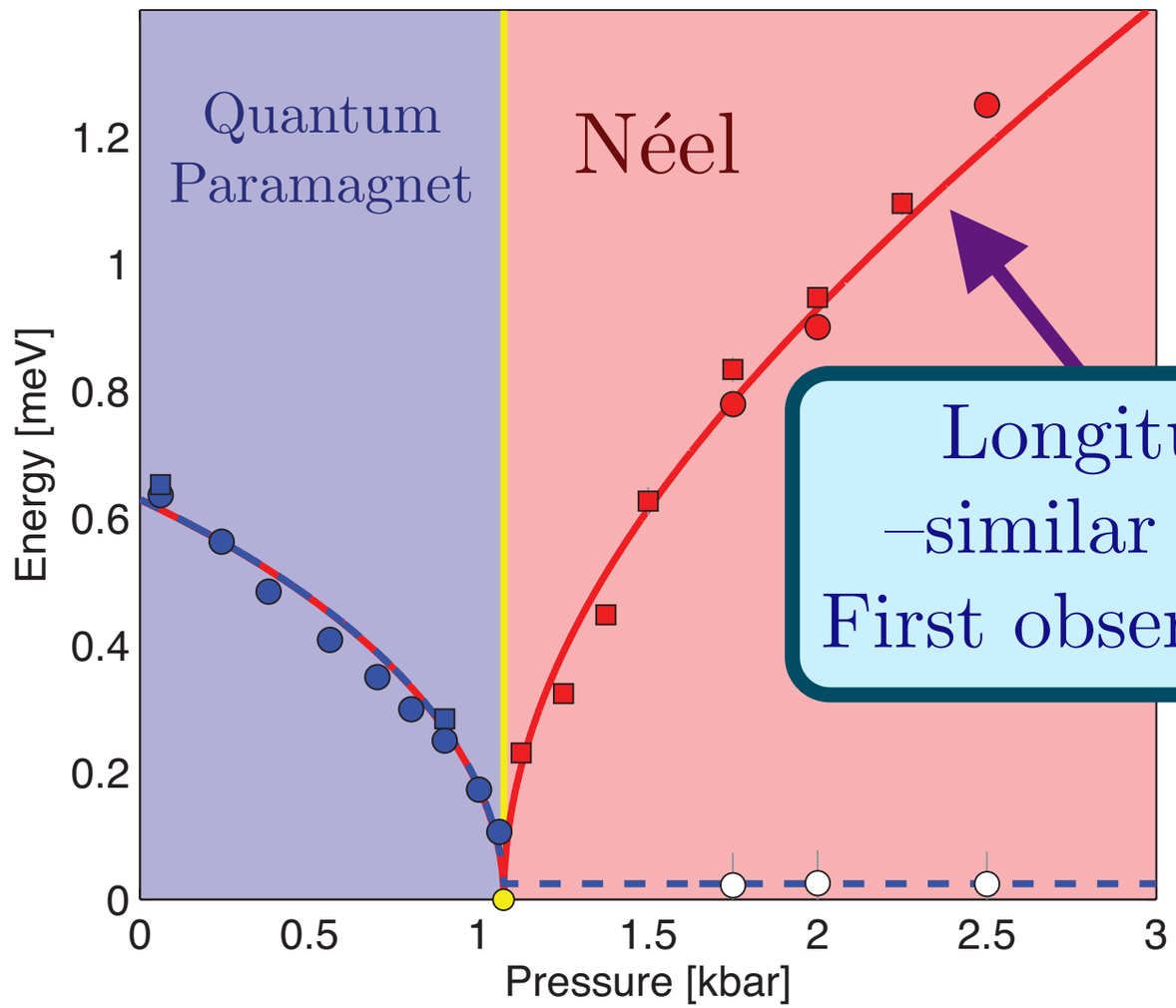

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"Higgs" particle appears at theoretically predicted energy

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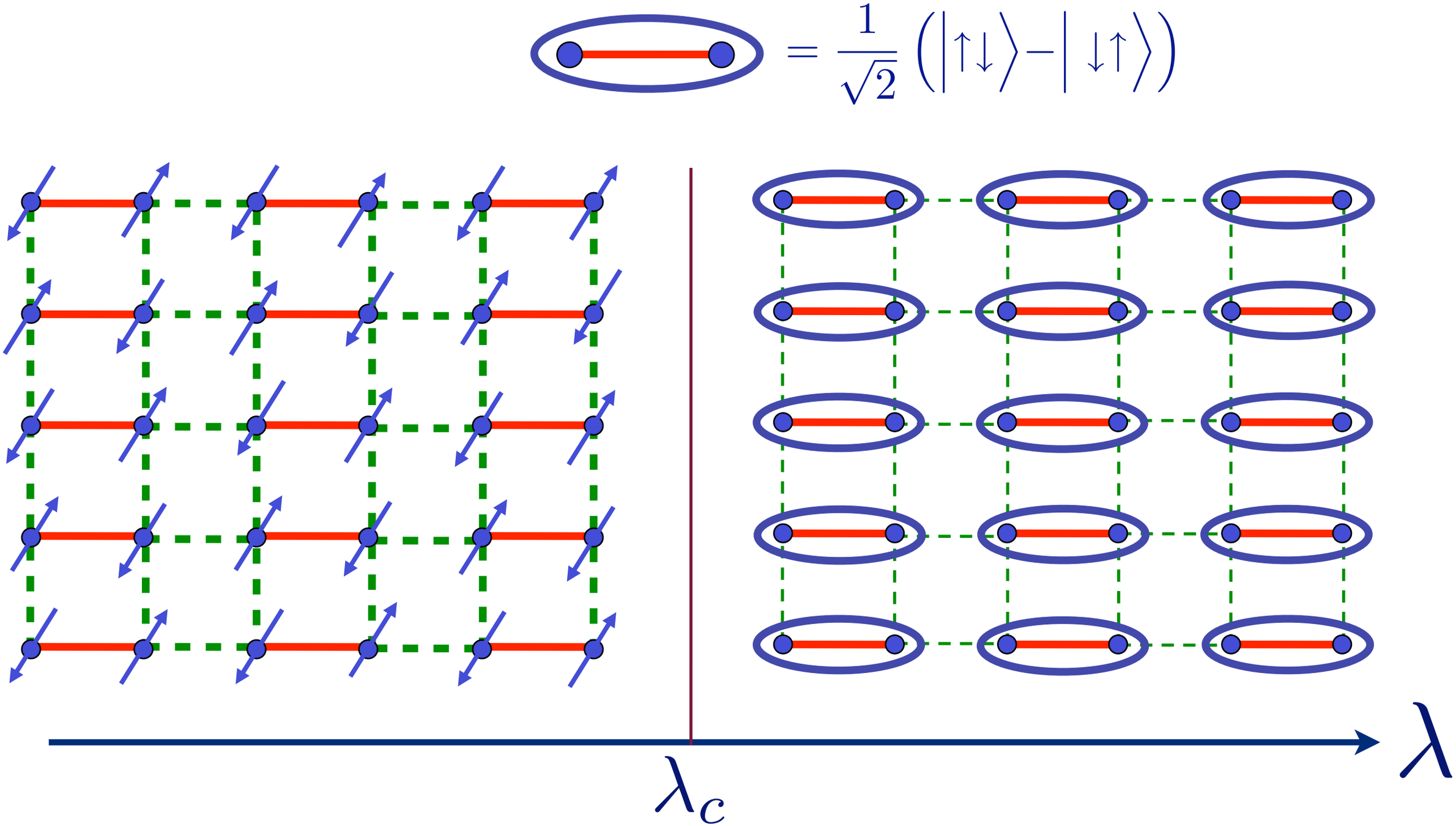
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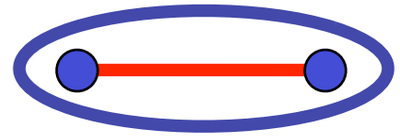

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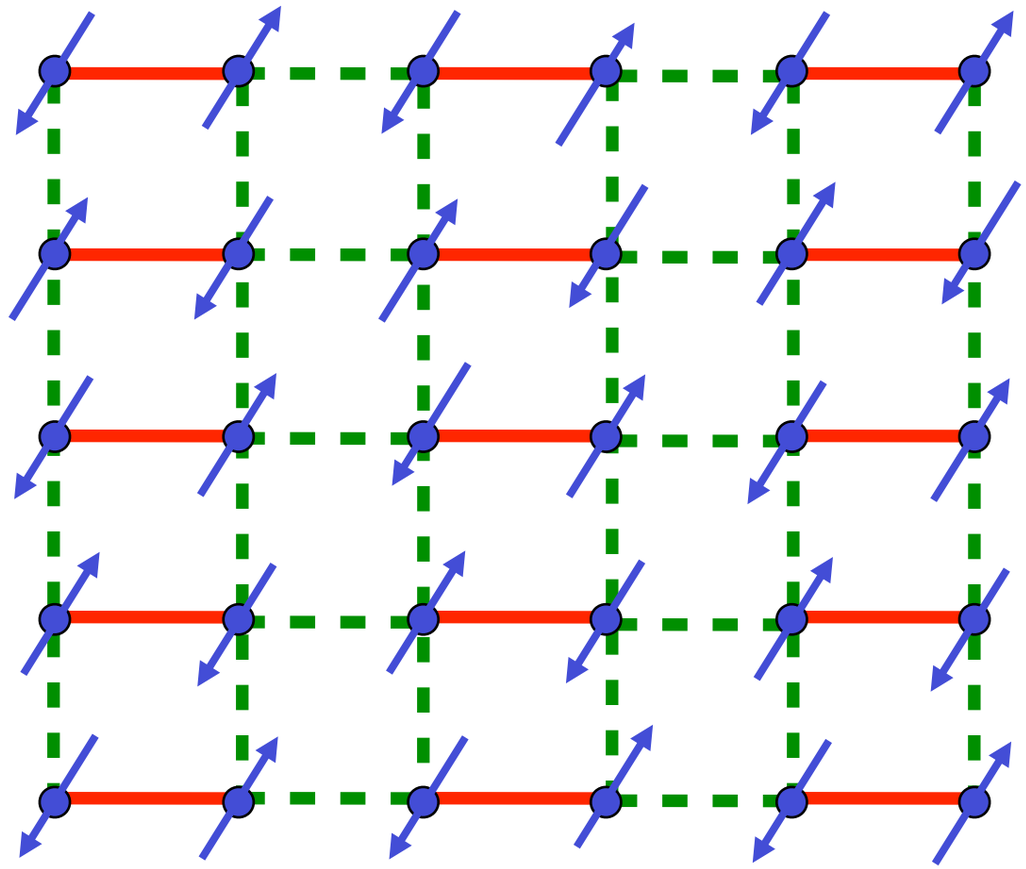
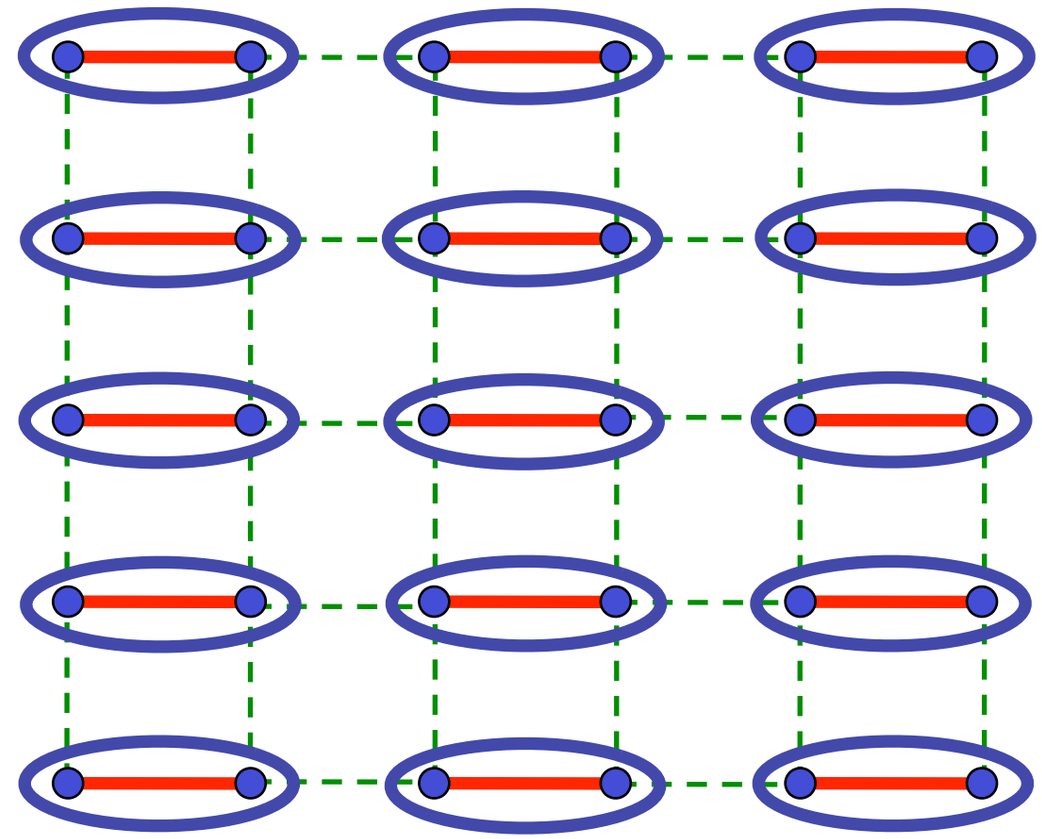


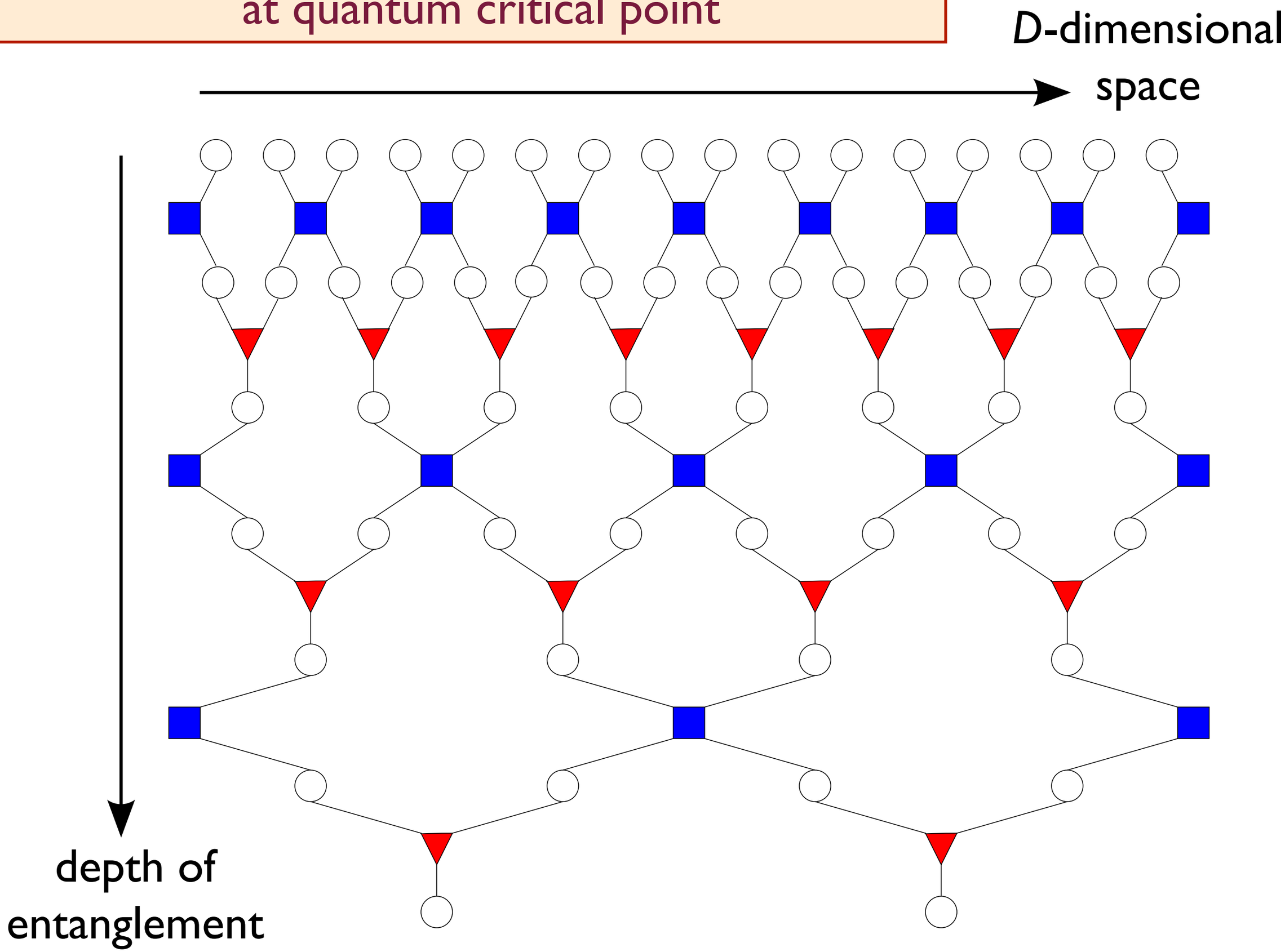


λ

λ_c

Quantum critical point with non-local entanglement in spin wavefunction

Tensor network representation of entanglement at quantum critical point



Characteristics of quantum critical point

- Long-range entanglement

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- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").

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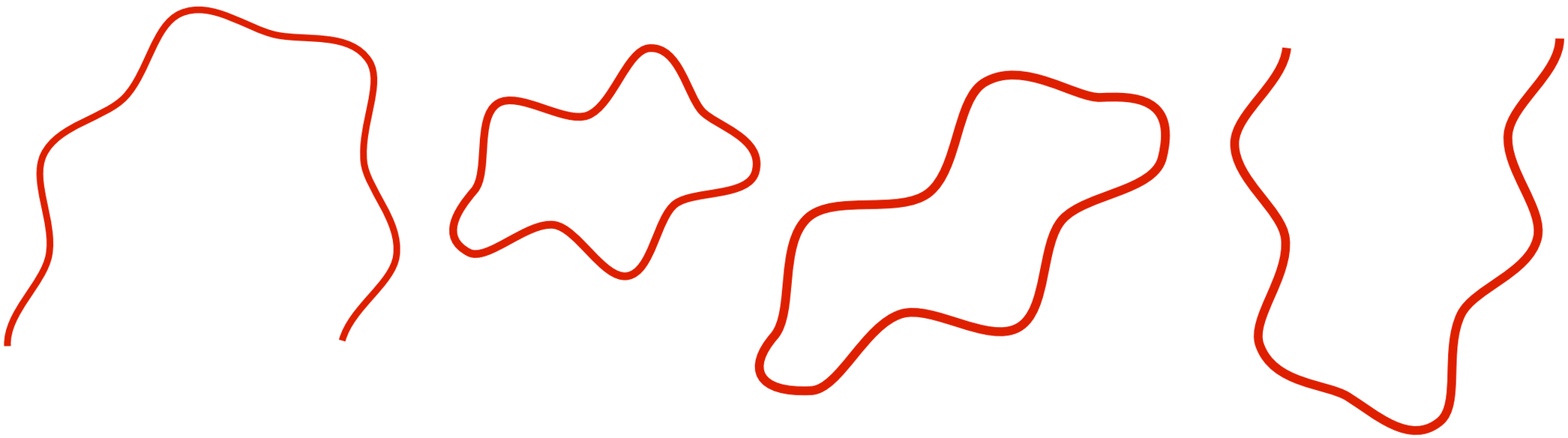
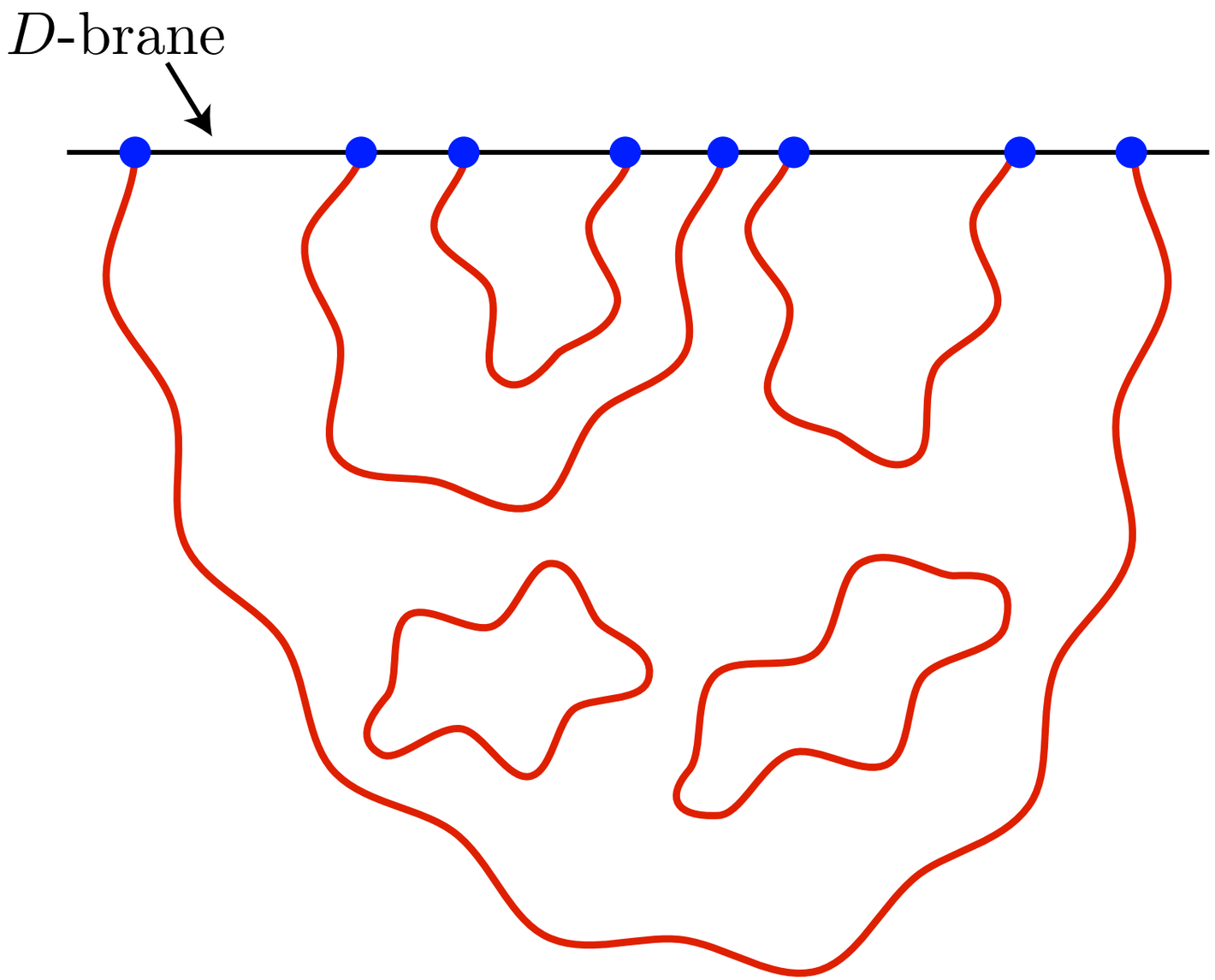
- Long-range entanglement
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").
- The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a **CFT3**

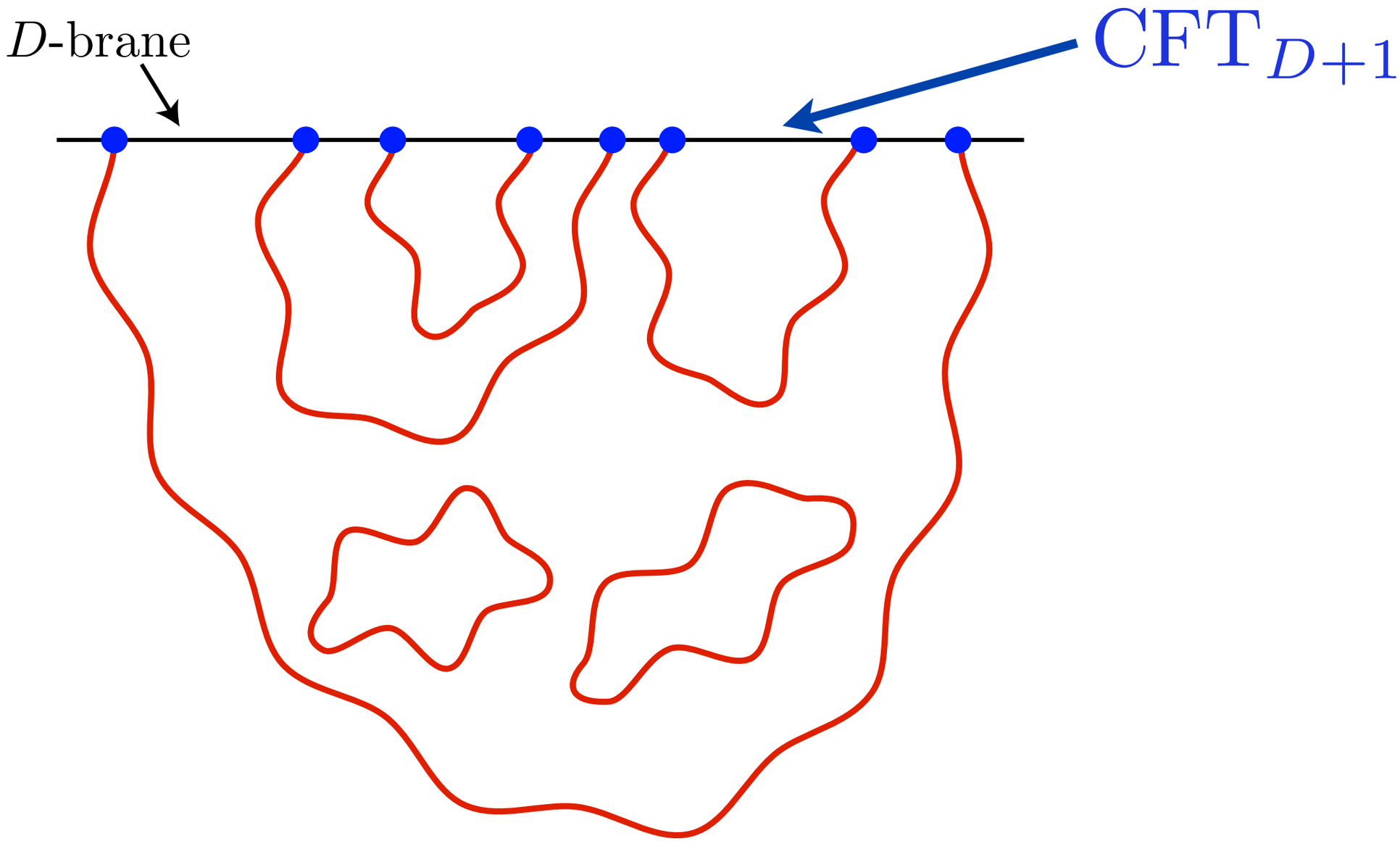
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.



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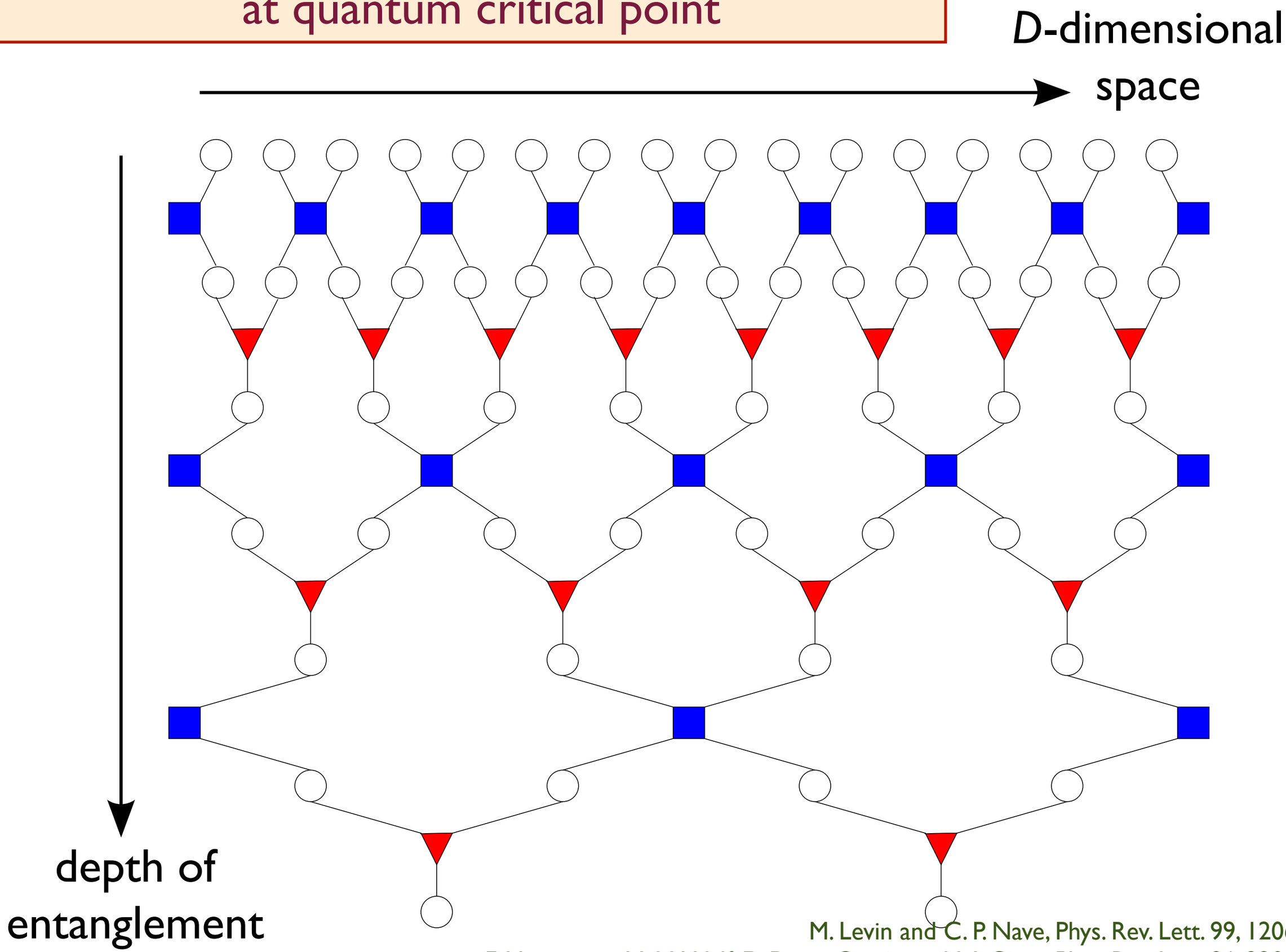
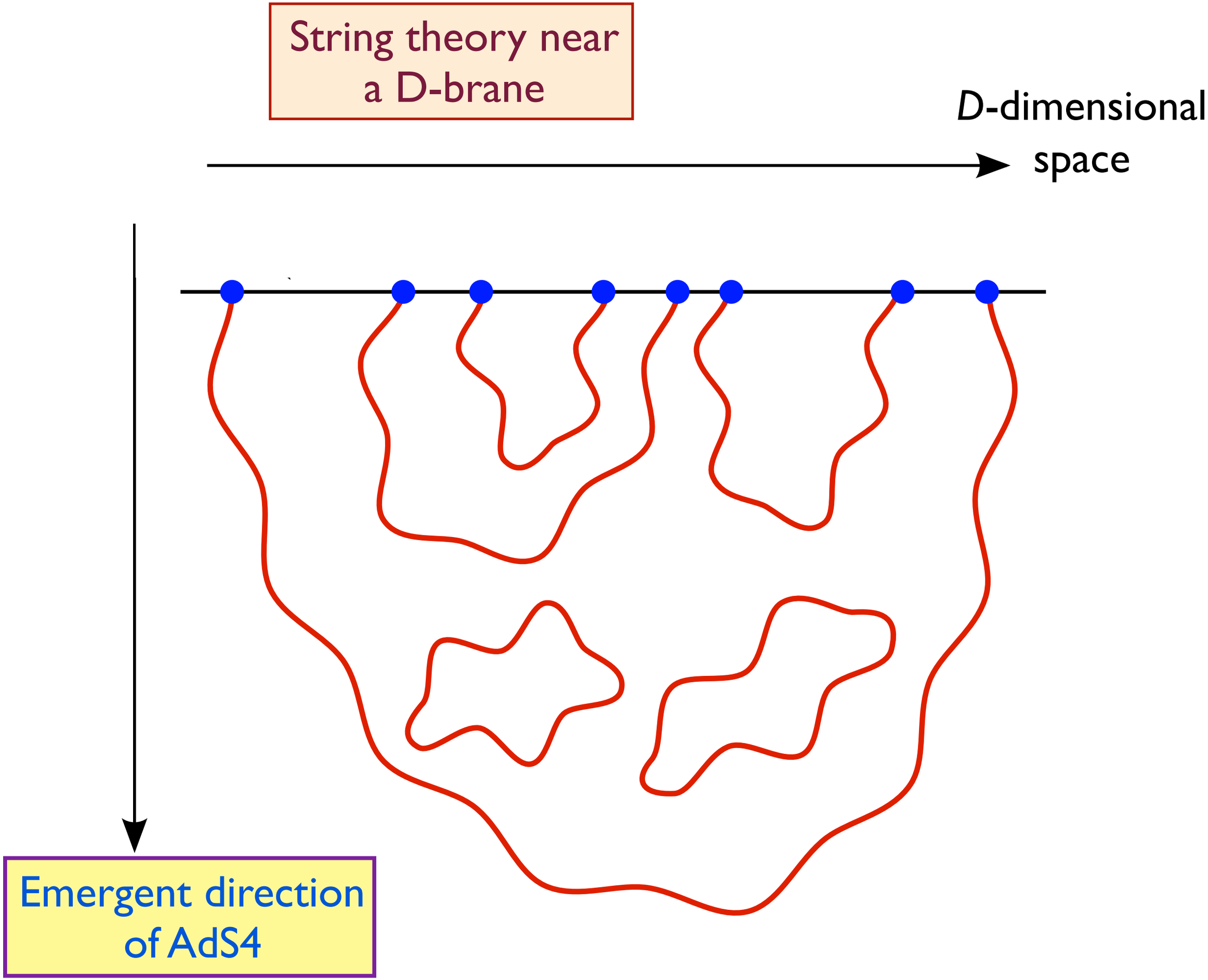
- In $D = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)


String theory near
a D-brane
D-dimensional
space
Emergent direction
of AdS4

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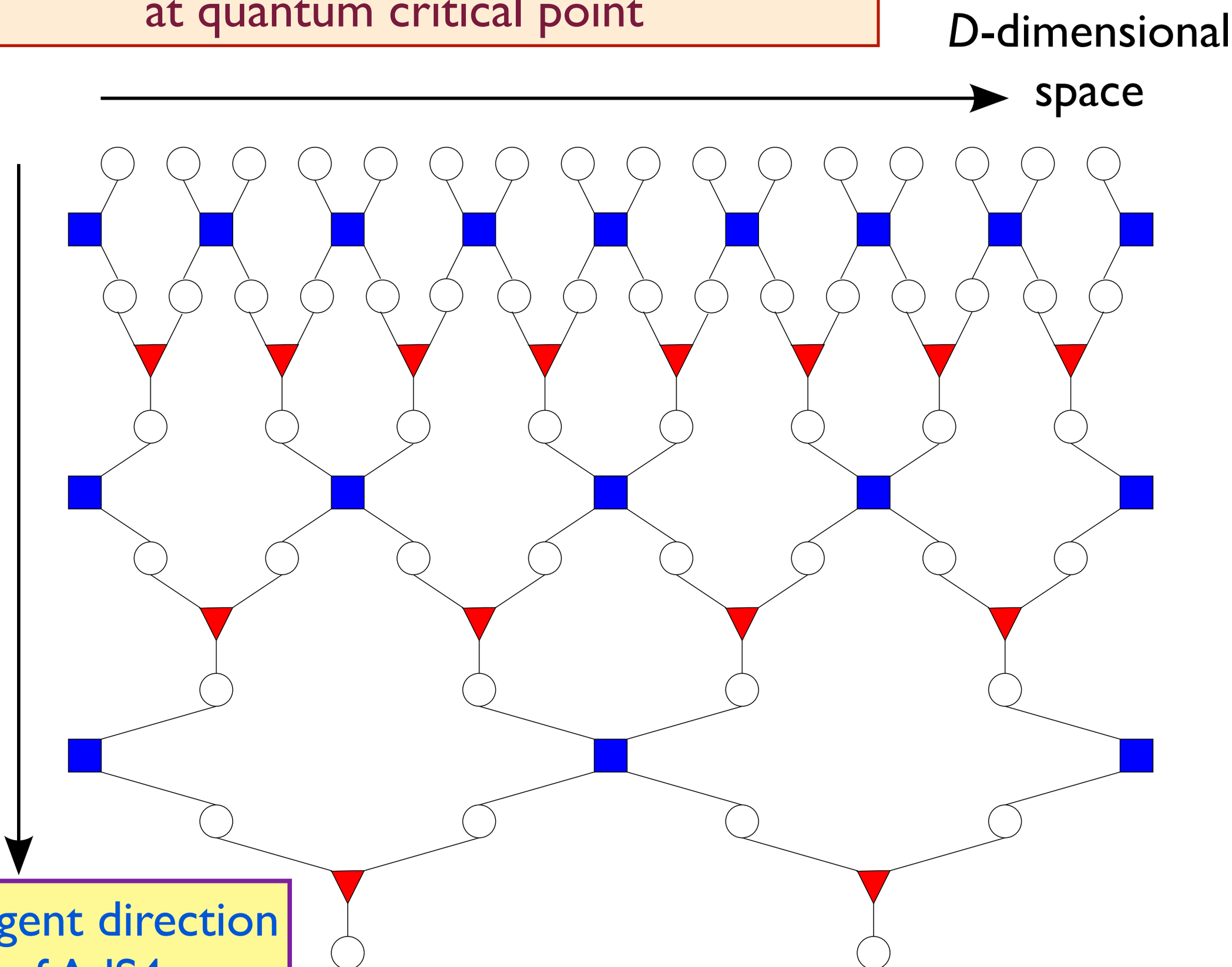
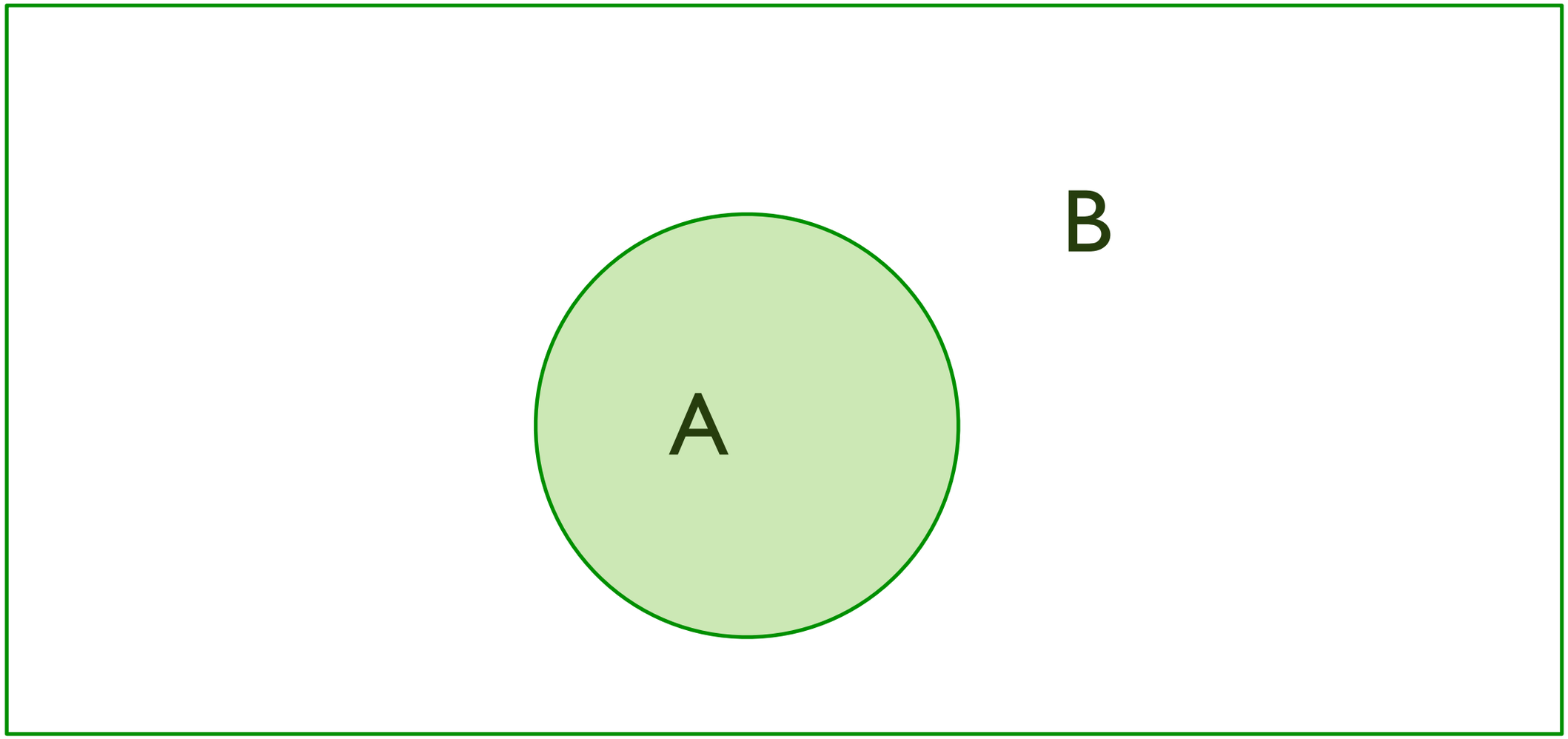


Brian Swingle, arXiv:0905.1317

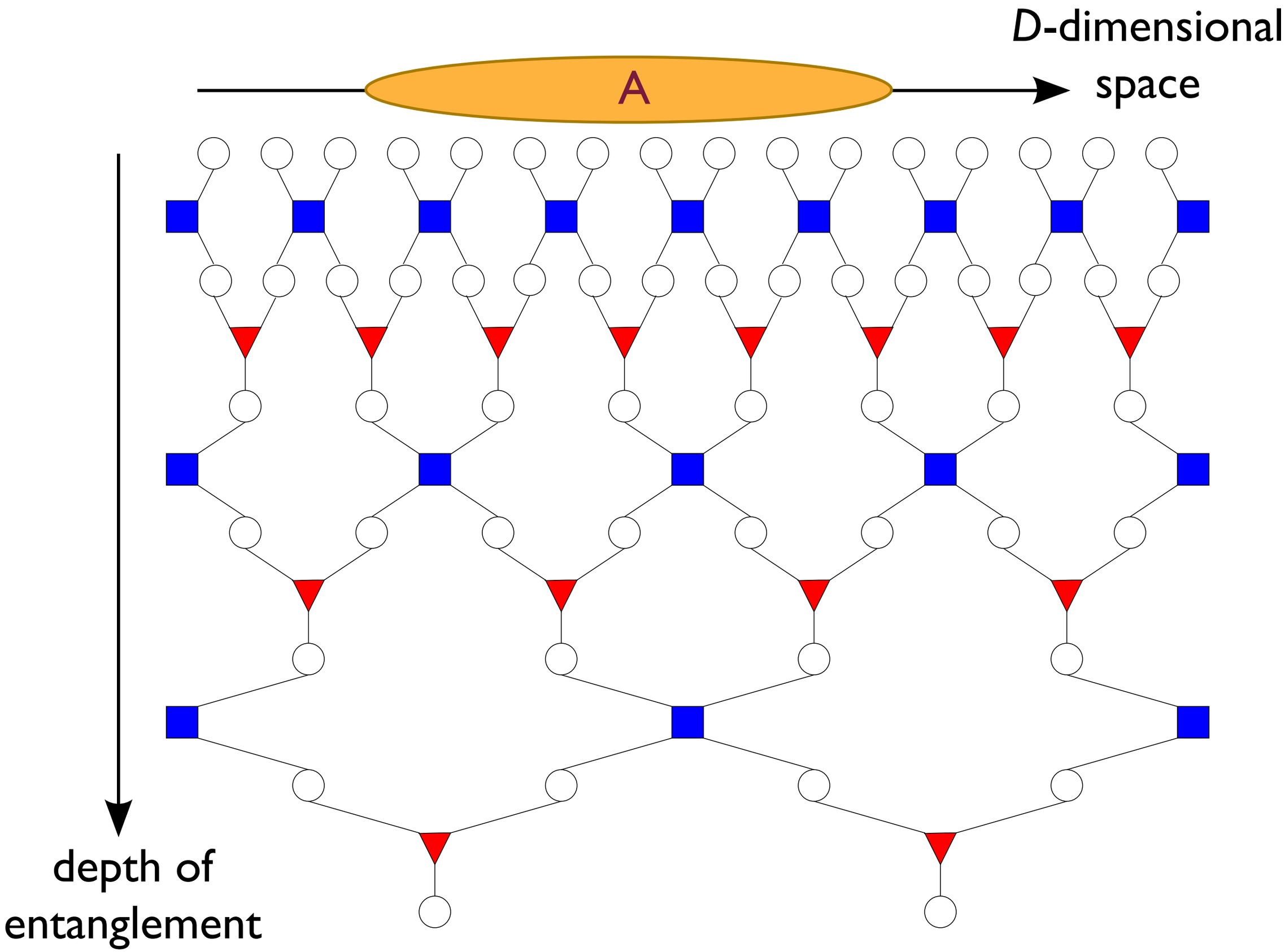
Entanglement entropy



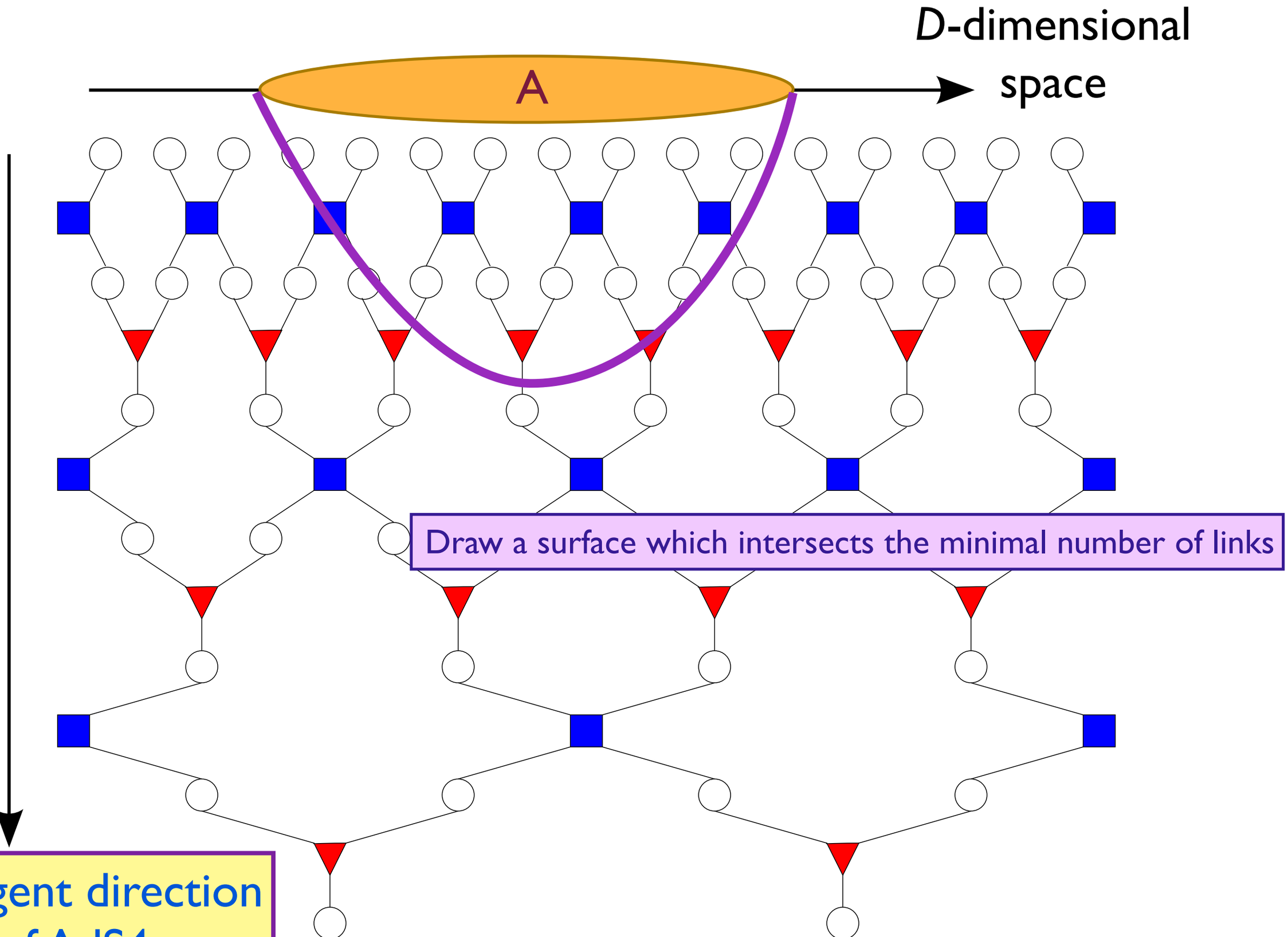
$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

Entanglement entropy $S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$

Entanglement entropy



Entanglement entropy



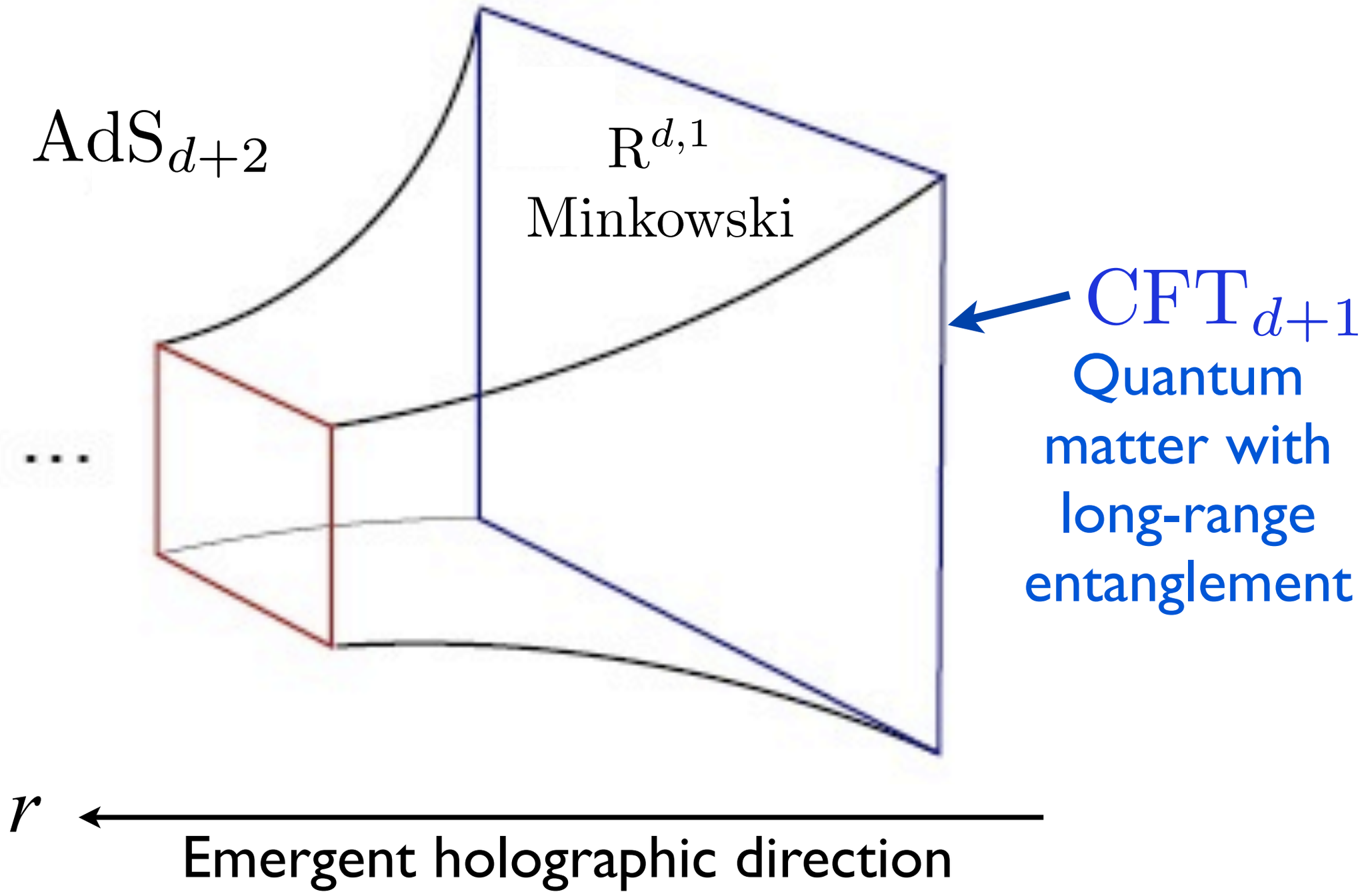
Emergent direction of AdS4

Entanglement entropy

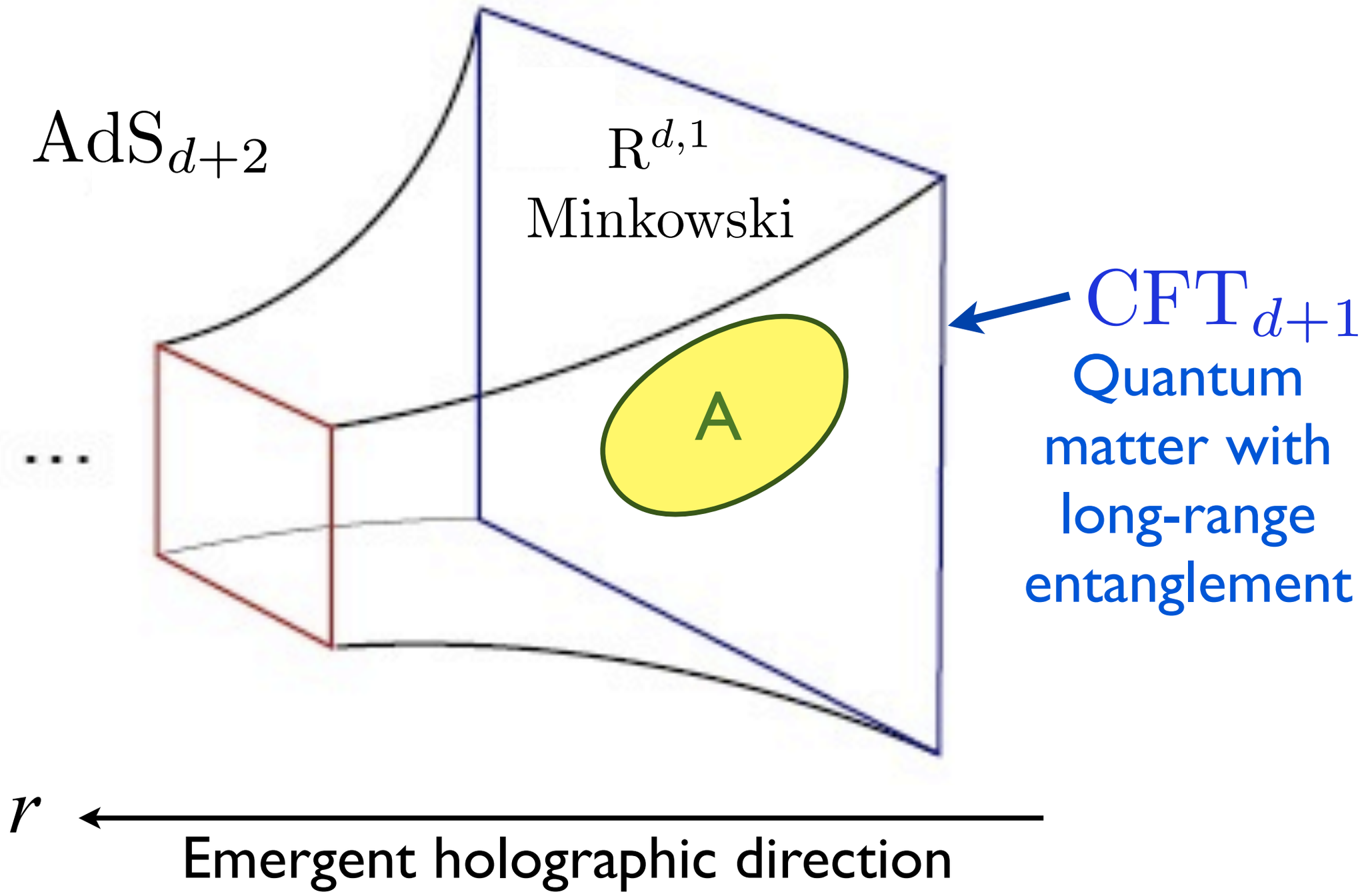
The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

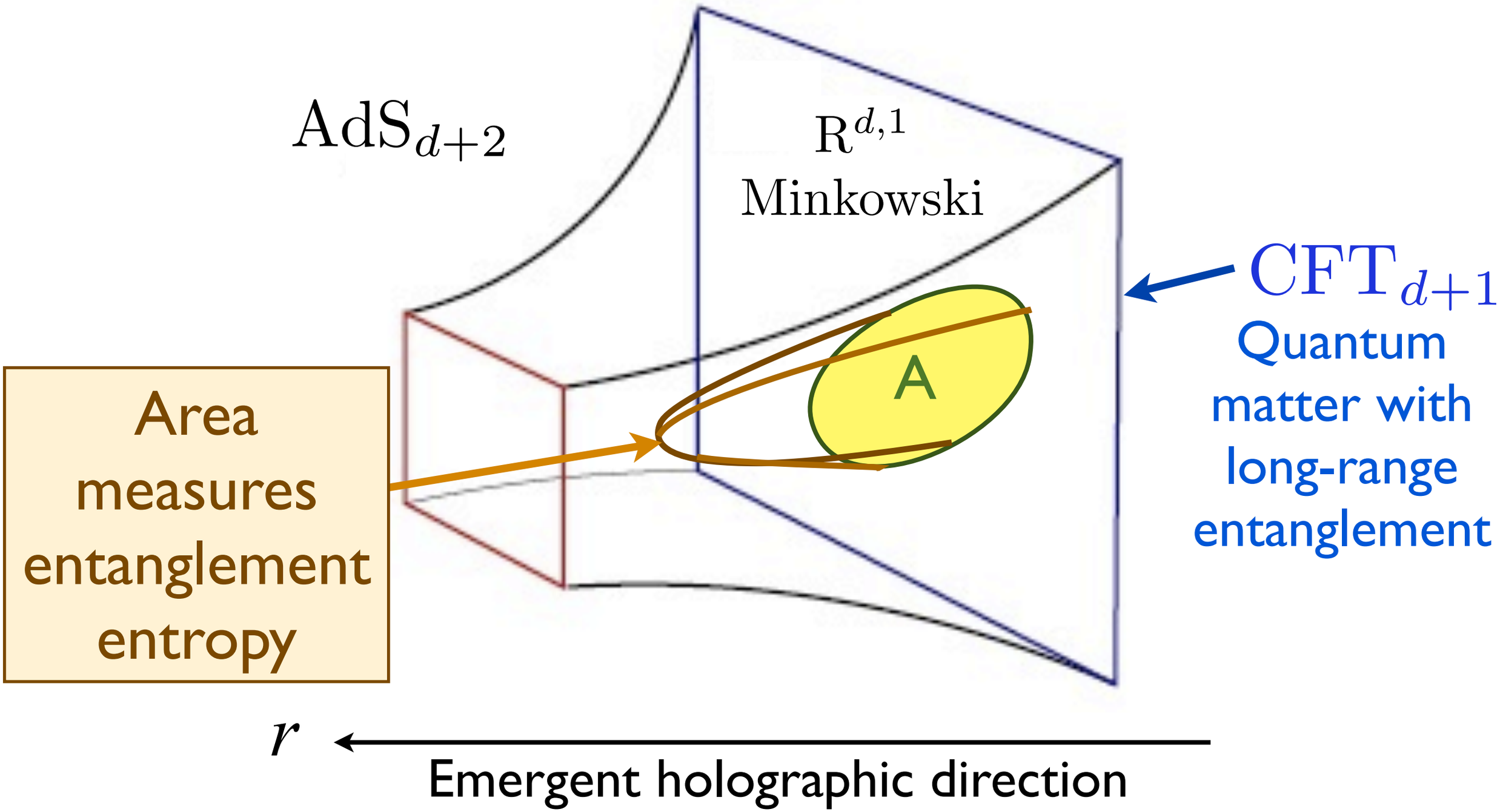
This can be seen both the string and tensor-network pictures

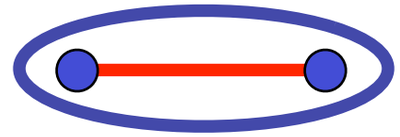
S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317



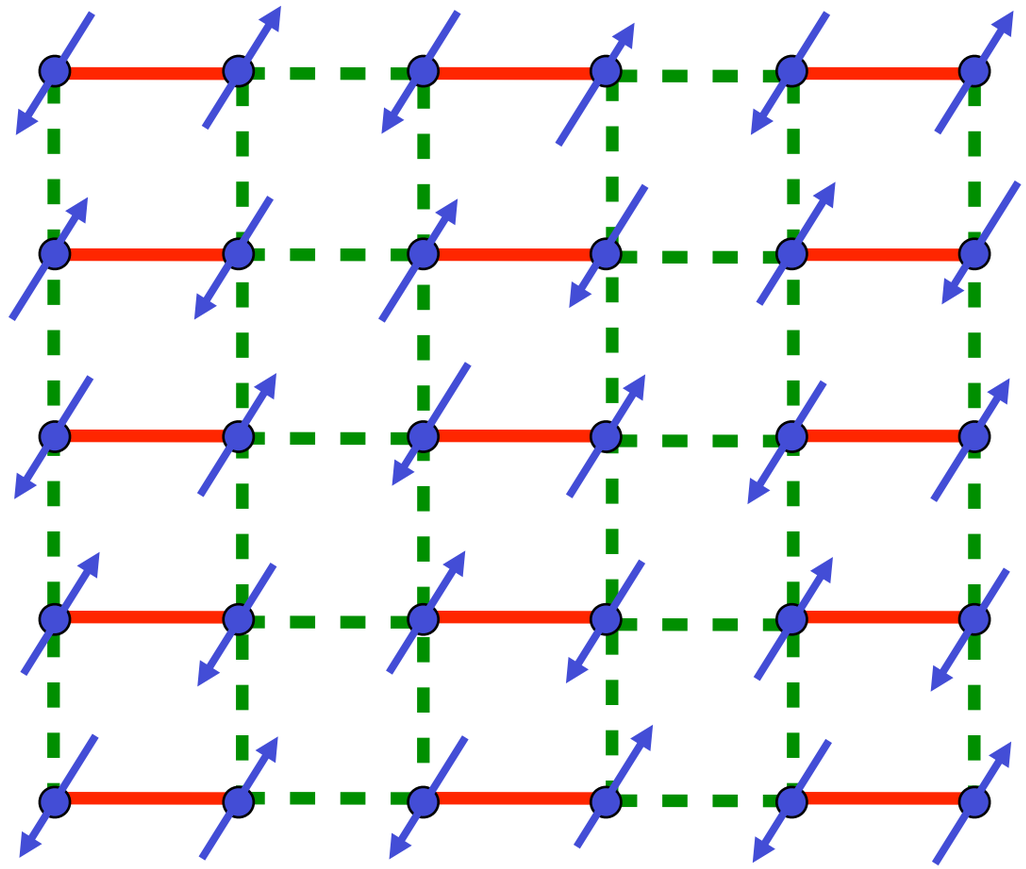
J. McGreevy, arXiv0909.0518

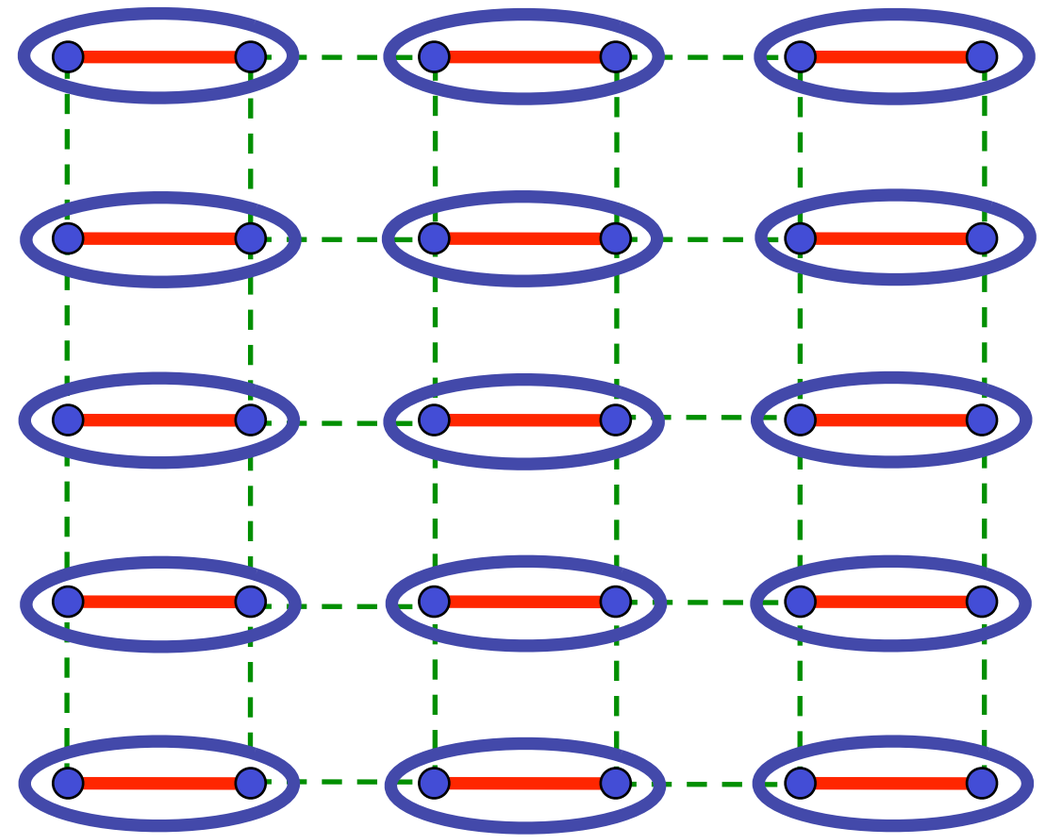

AdS_{d+2}
$\mathrm{R}^{d,1}$
Minkowski
A
...
CFT_{d+1}
Quantum matter with long-range entanglement
r
Emergent holographic direction


AdS_{d+2}
$\mathrm{R}^{d,1}$
Minkowski
A
CFT_{d+1}
Quantum matter with long-range entanglement
Area measures entanglement entropy
r
Emergent holographic direction



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$



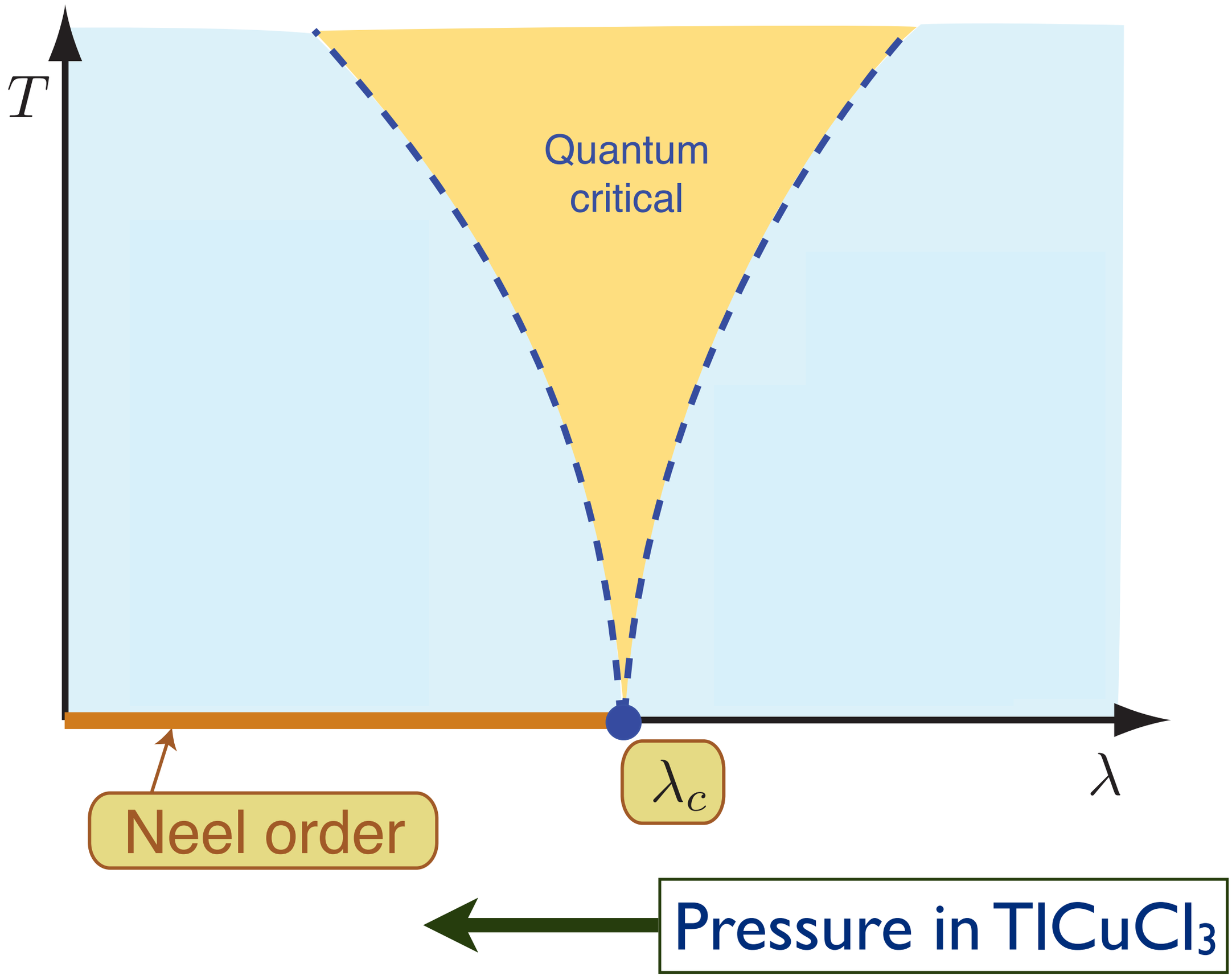


λ

λ_c

Quantum critical point with non-local entanglement in spin wavefunction

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



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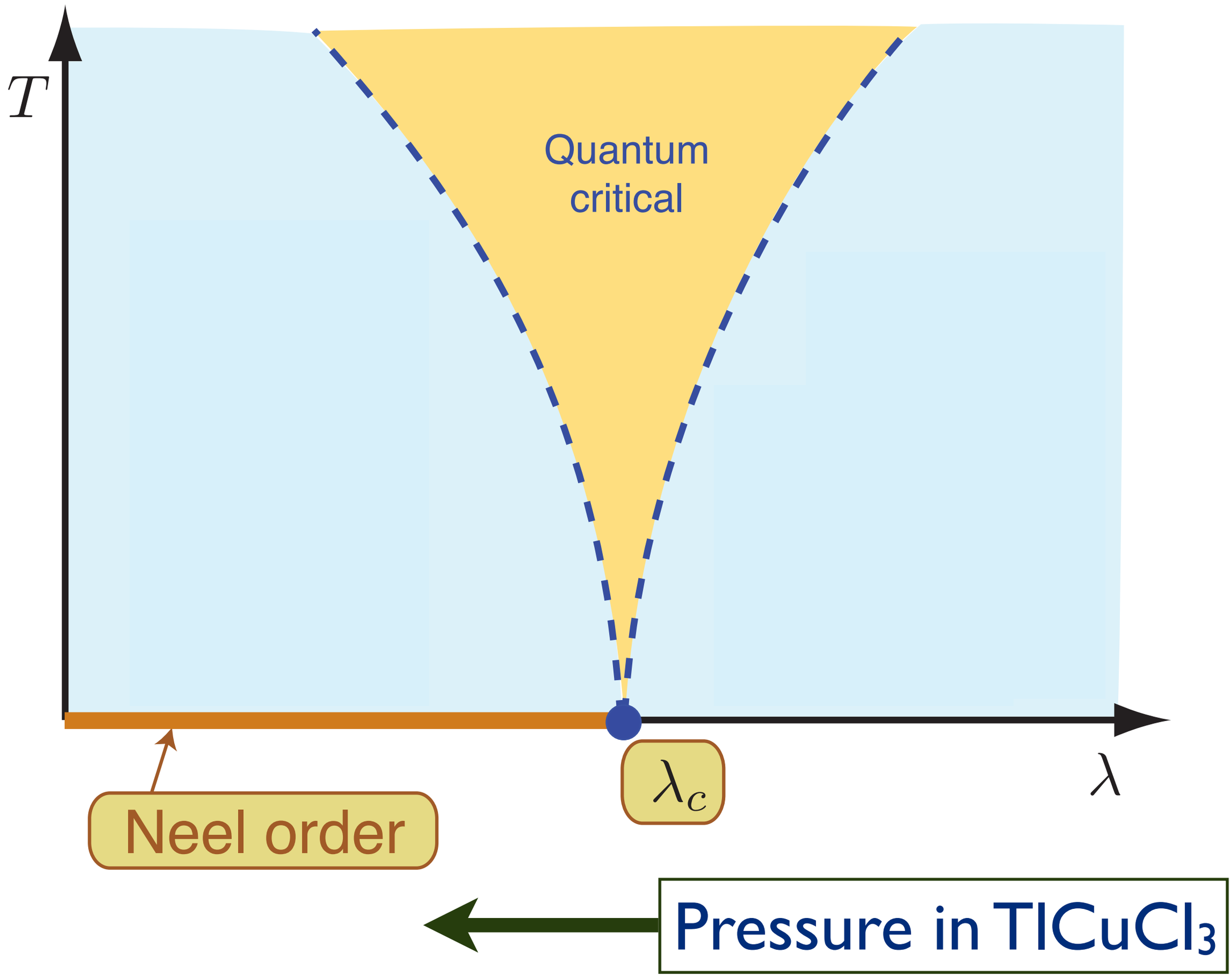

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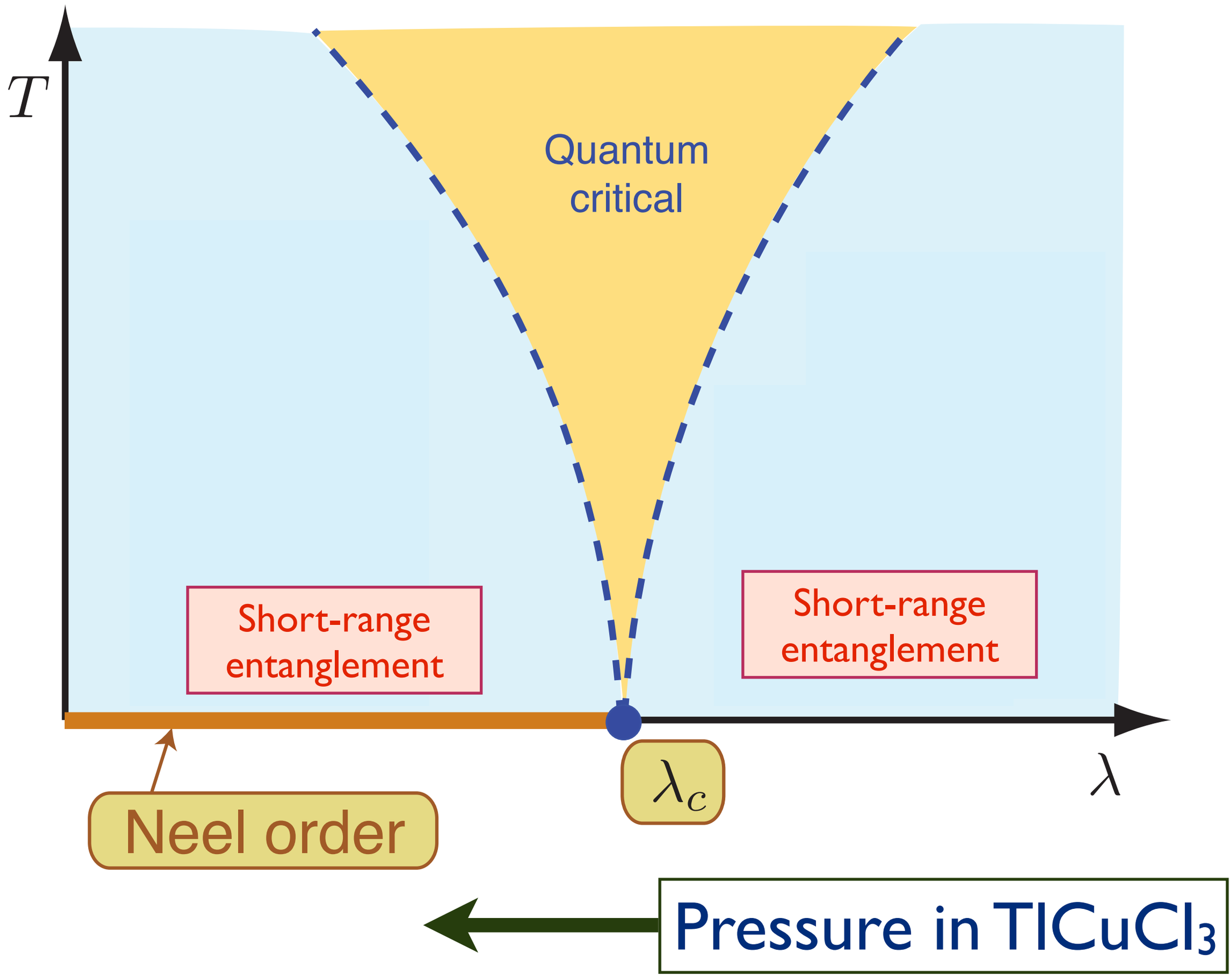

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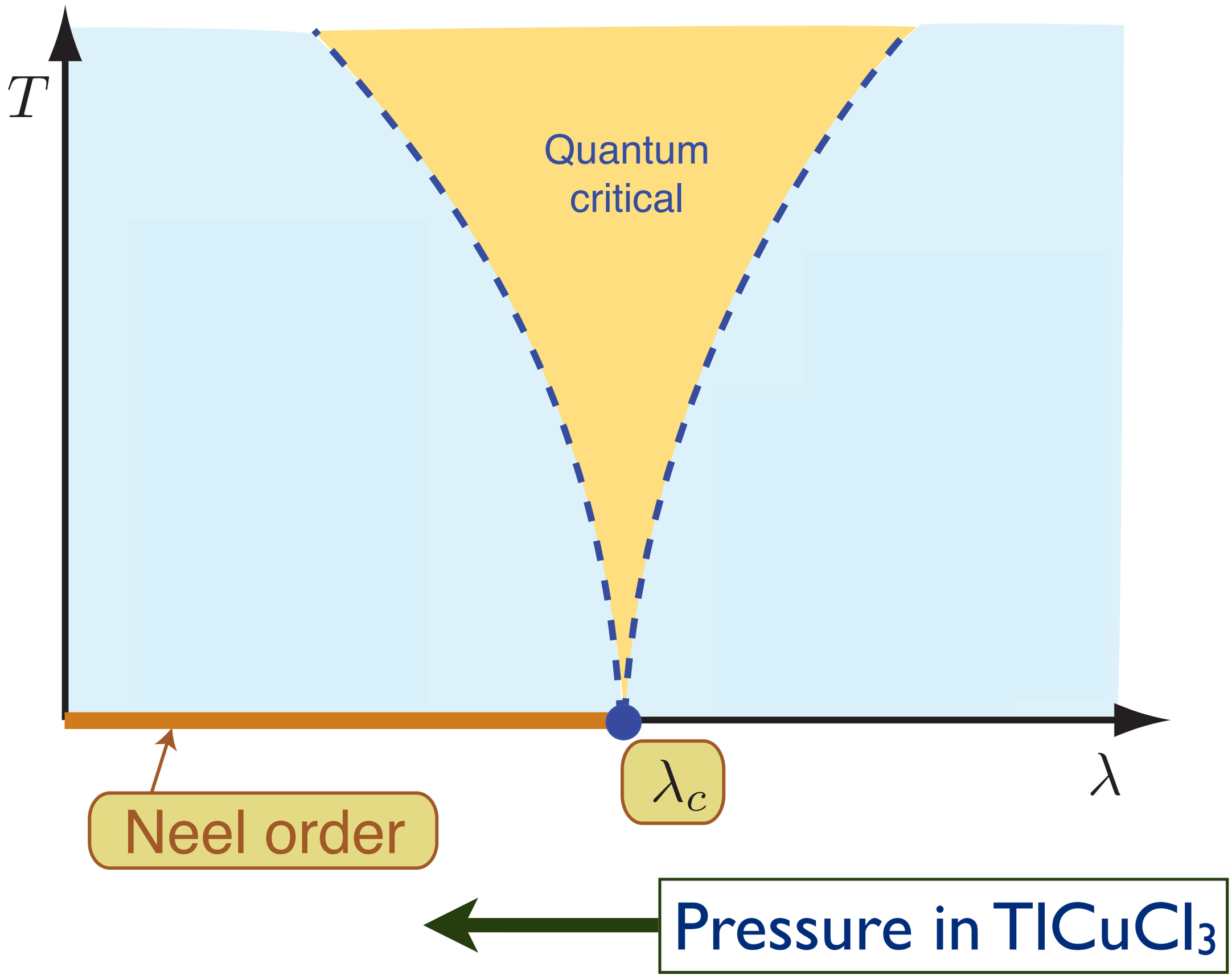

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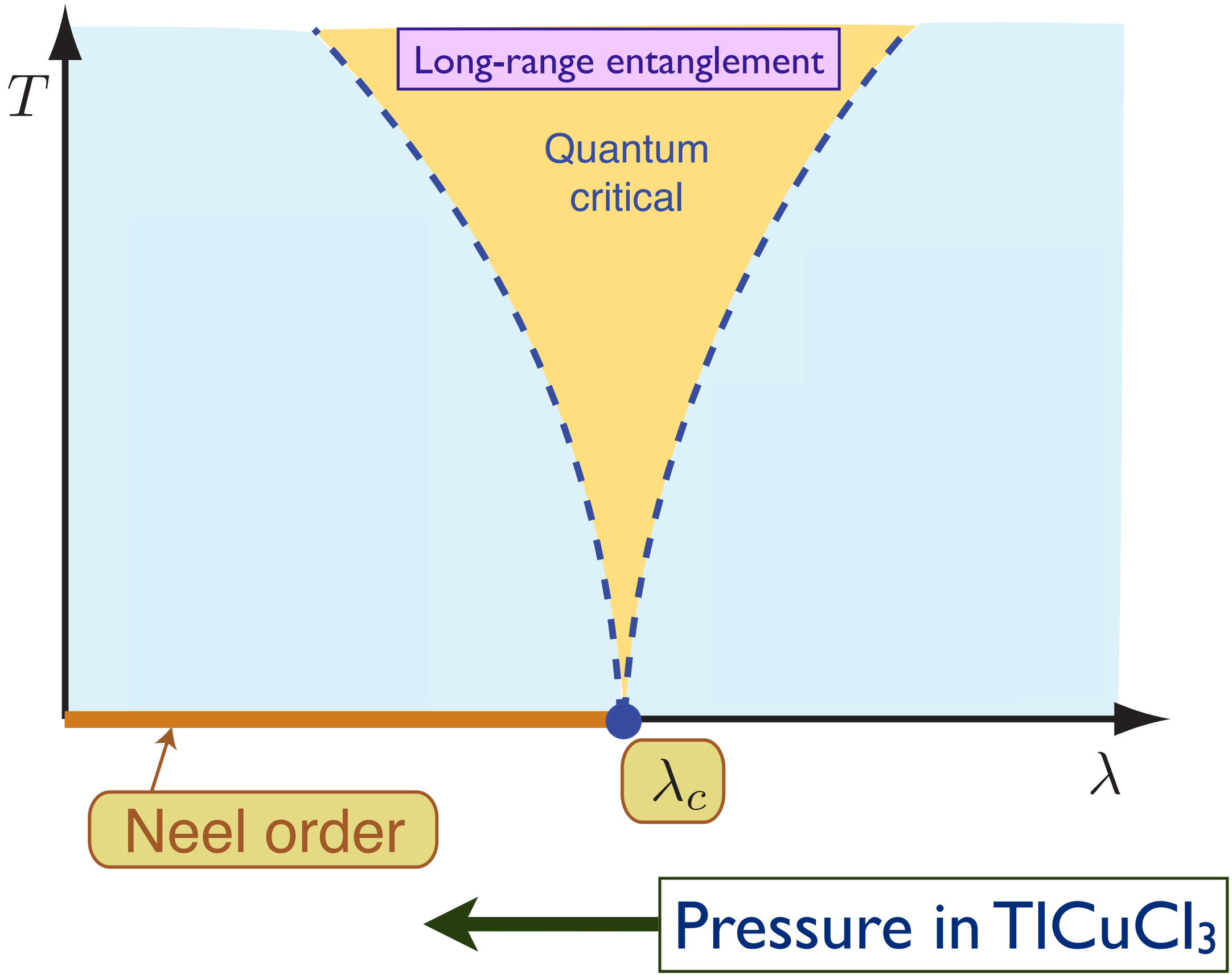
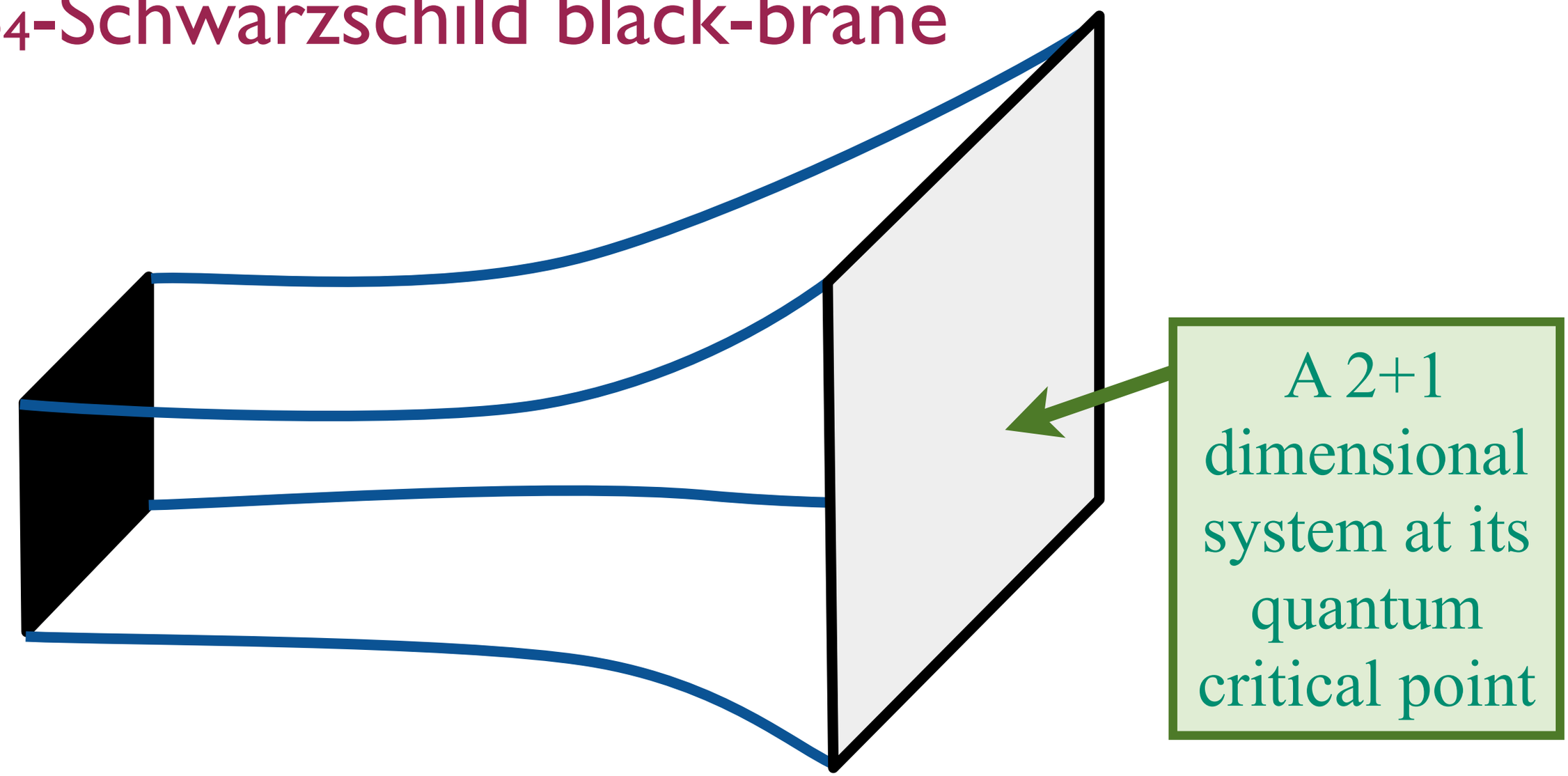


AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



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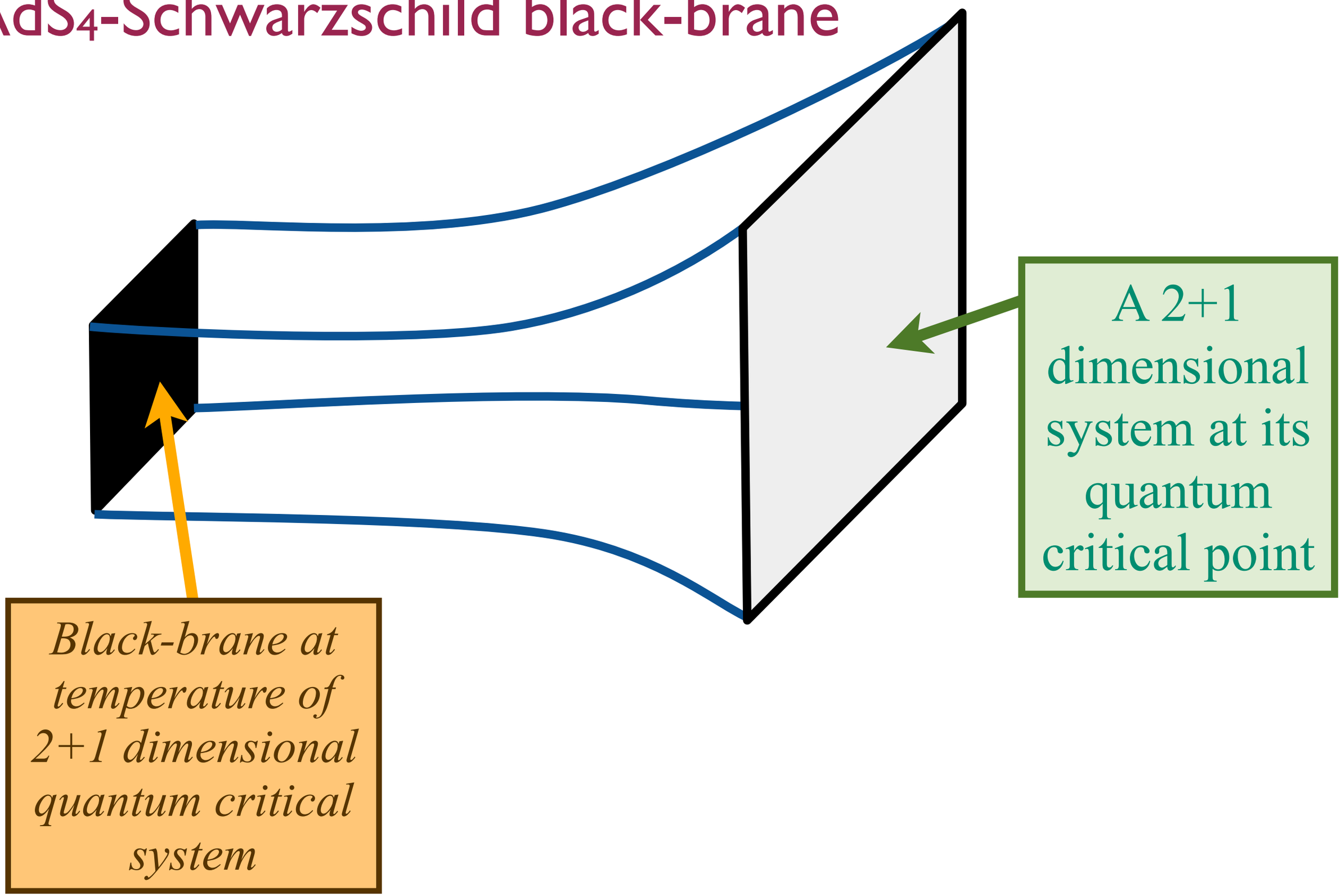

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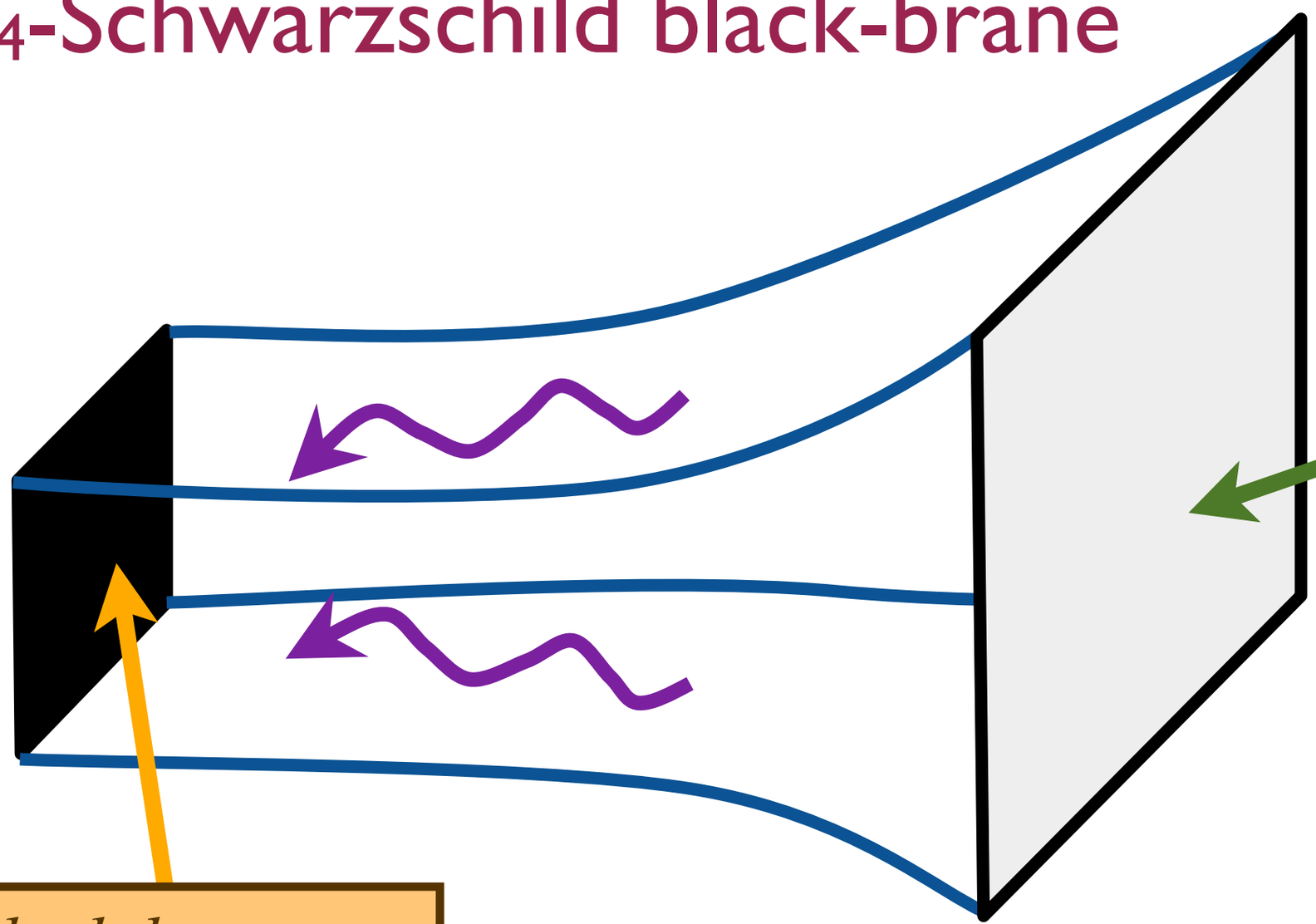


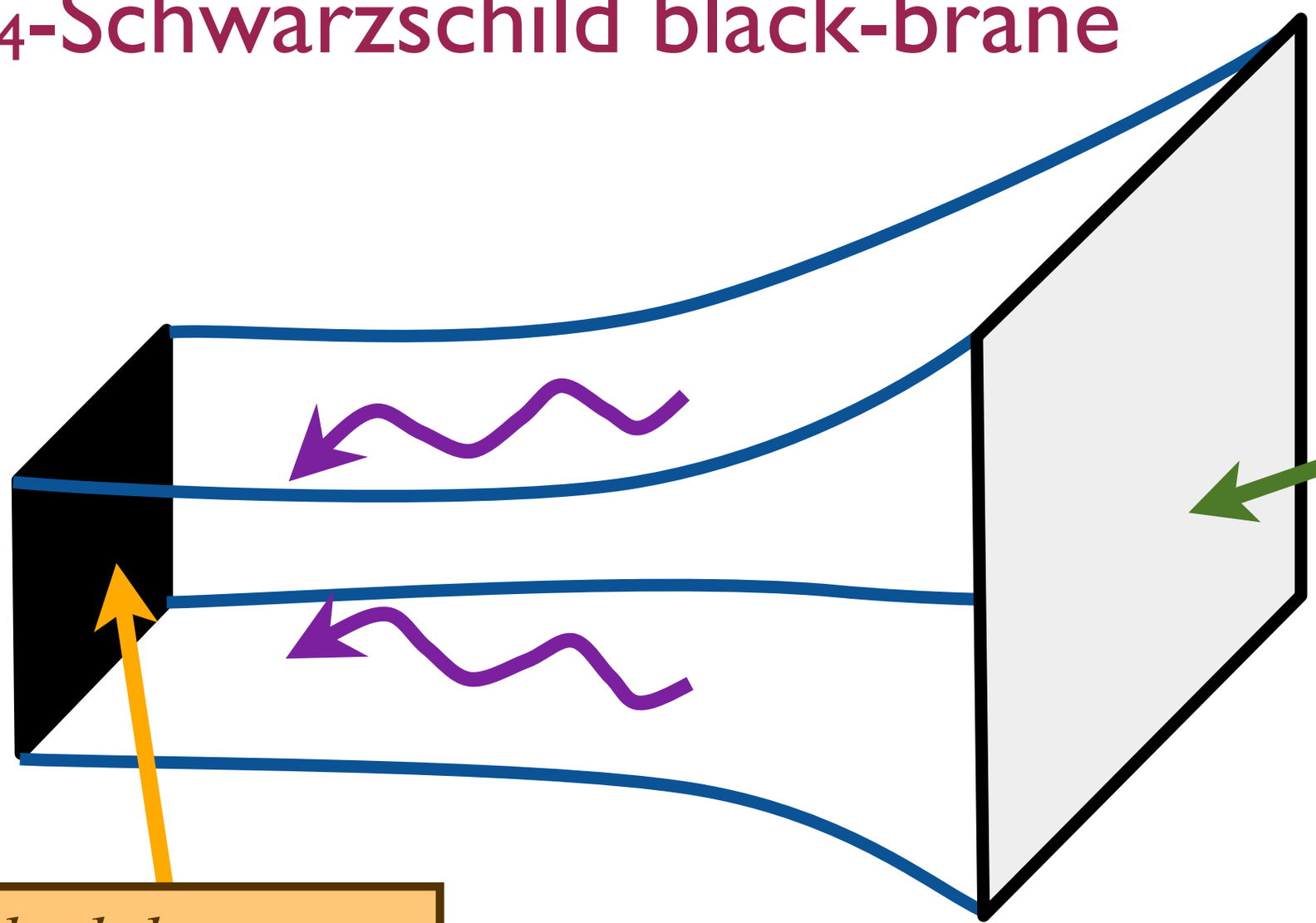
A 2+1 dimensional system at its quantum critical point

Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



A 2+1 dimensional system at its quantum critical point

Black-brane at temperature of 2+1 dimensional quantum critical system

Provides successful description of many properties of quantum critical points at non-zero temperatures

Outline

1. Quantum critical points and string theory
   Entanglement and emergent dimensions

2. High temperature superconductors
   and strange metals
   Holography of compressible quantum phases

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   Entanglement and emergent dimensions

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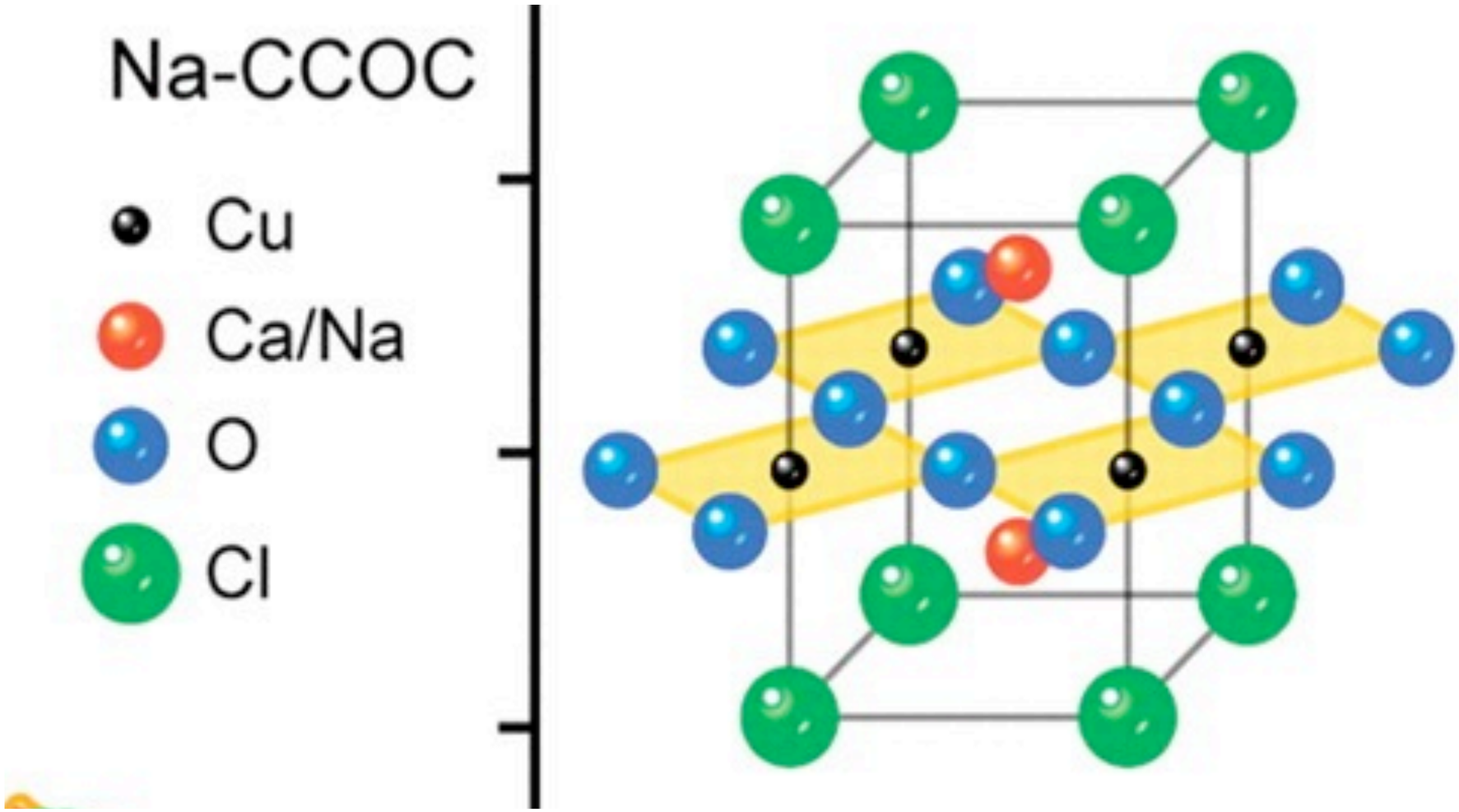
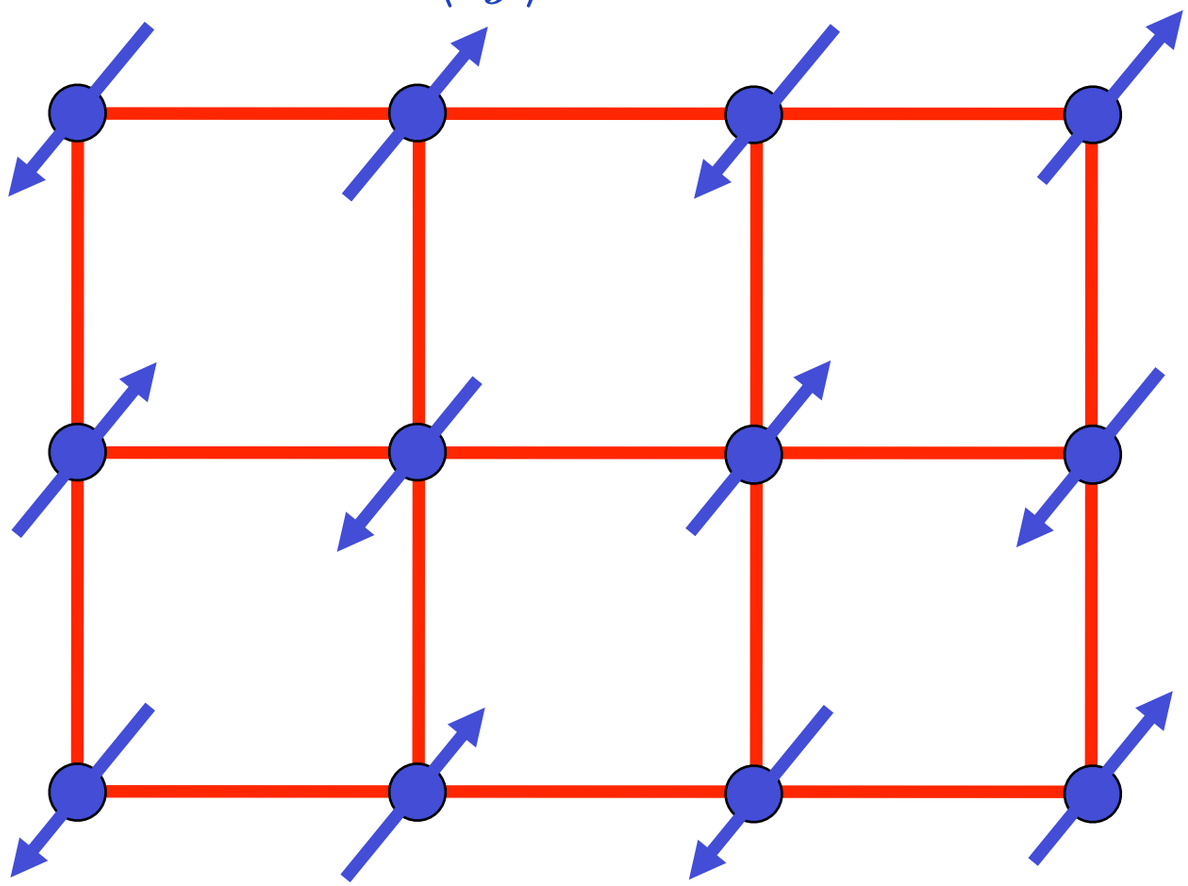
The cuprate superconductors



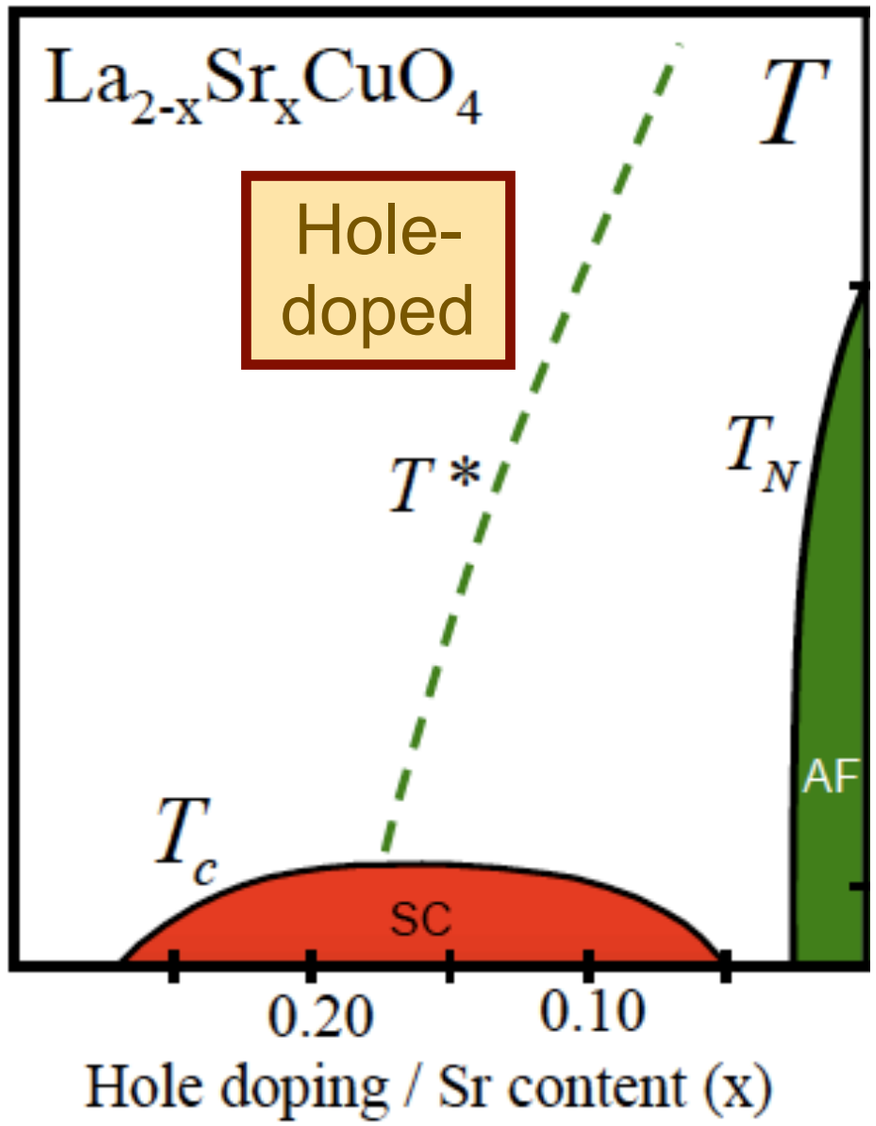
Davis

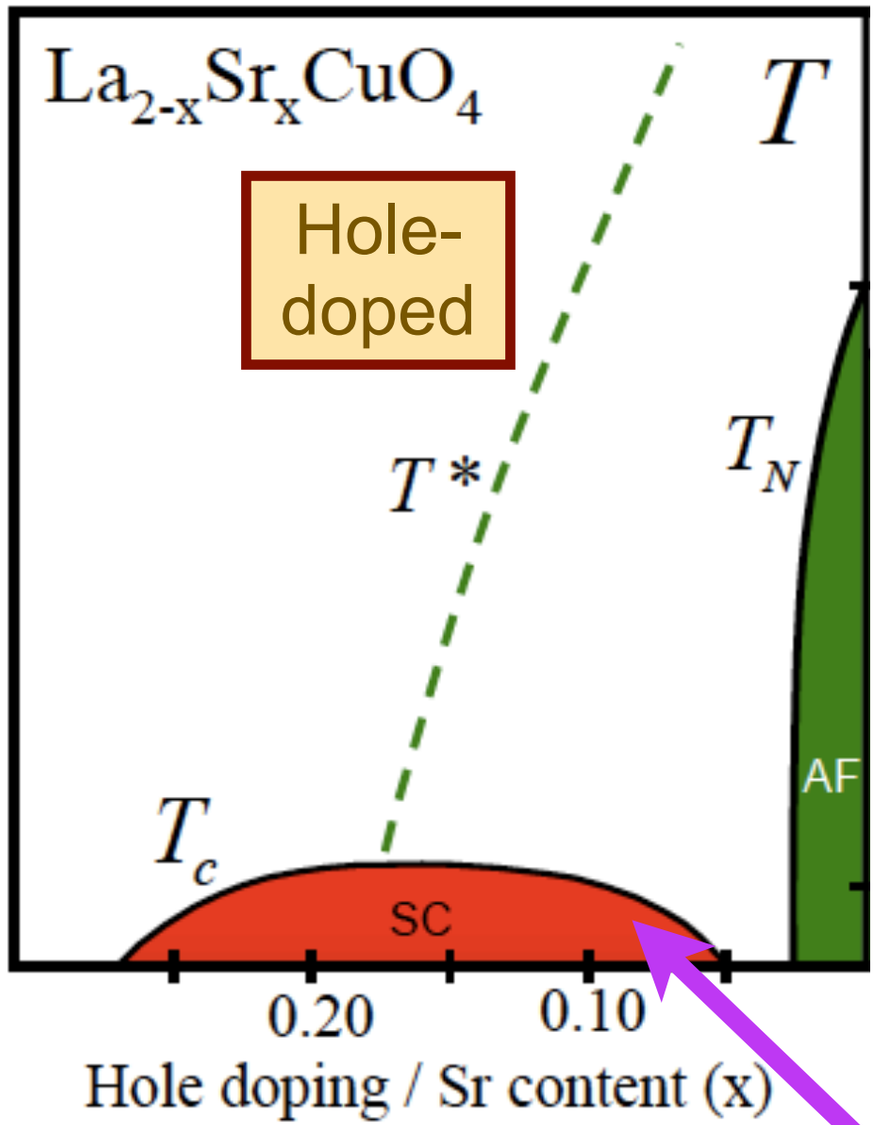
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



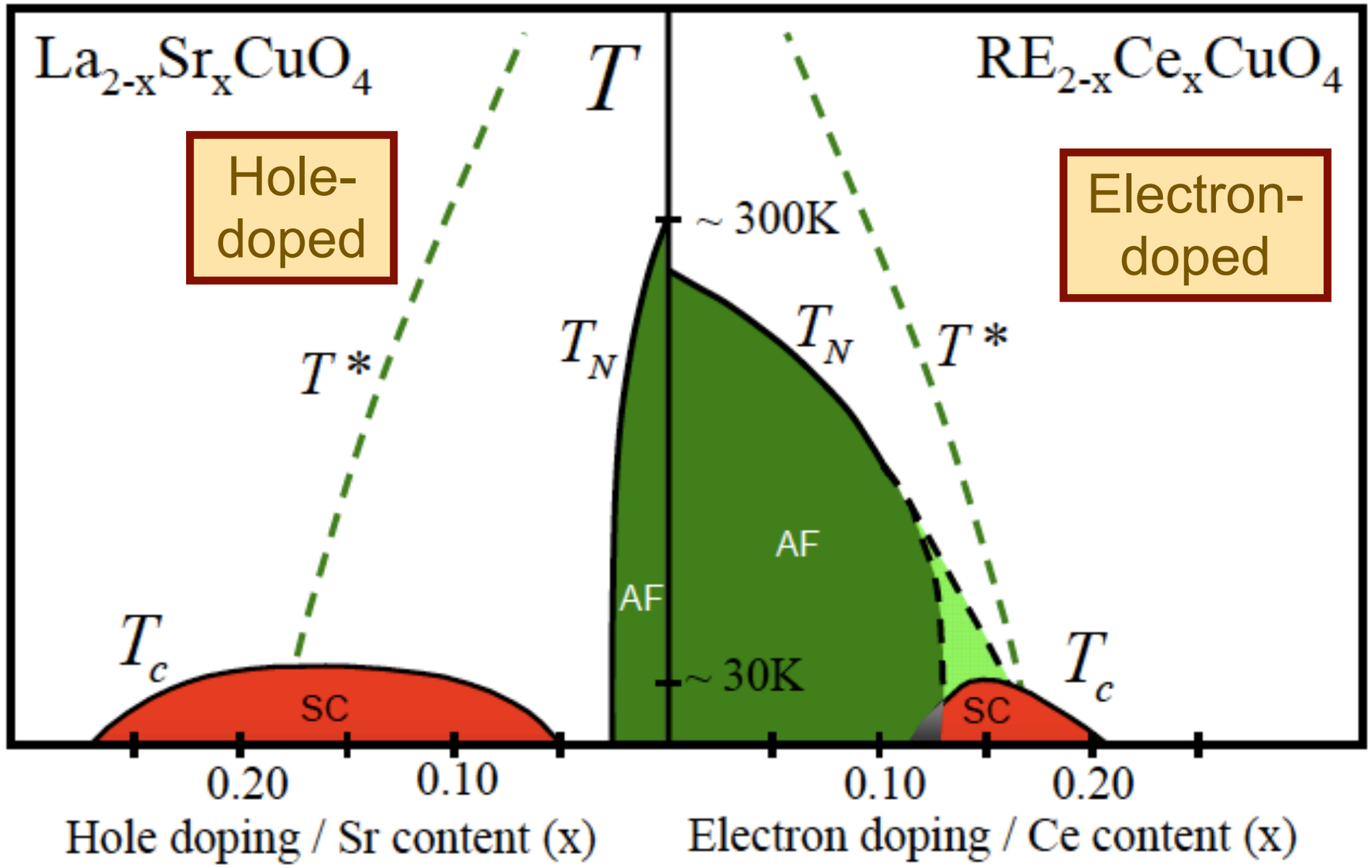
Ground state has long-range Néel order


$La_{2-x}Sr_xCuO_4$
T
Hole-doped
T^*
T_N
AF
T_c
SC
0.20
0.10
Hole doping / Sr content (x)



Superconductor
Bose condensate of pairs of electrons
Short-range entanglement

Electron-doped cuprate superconductors



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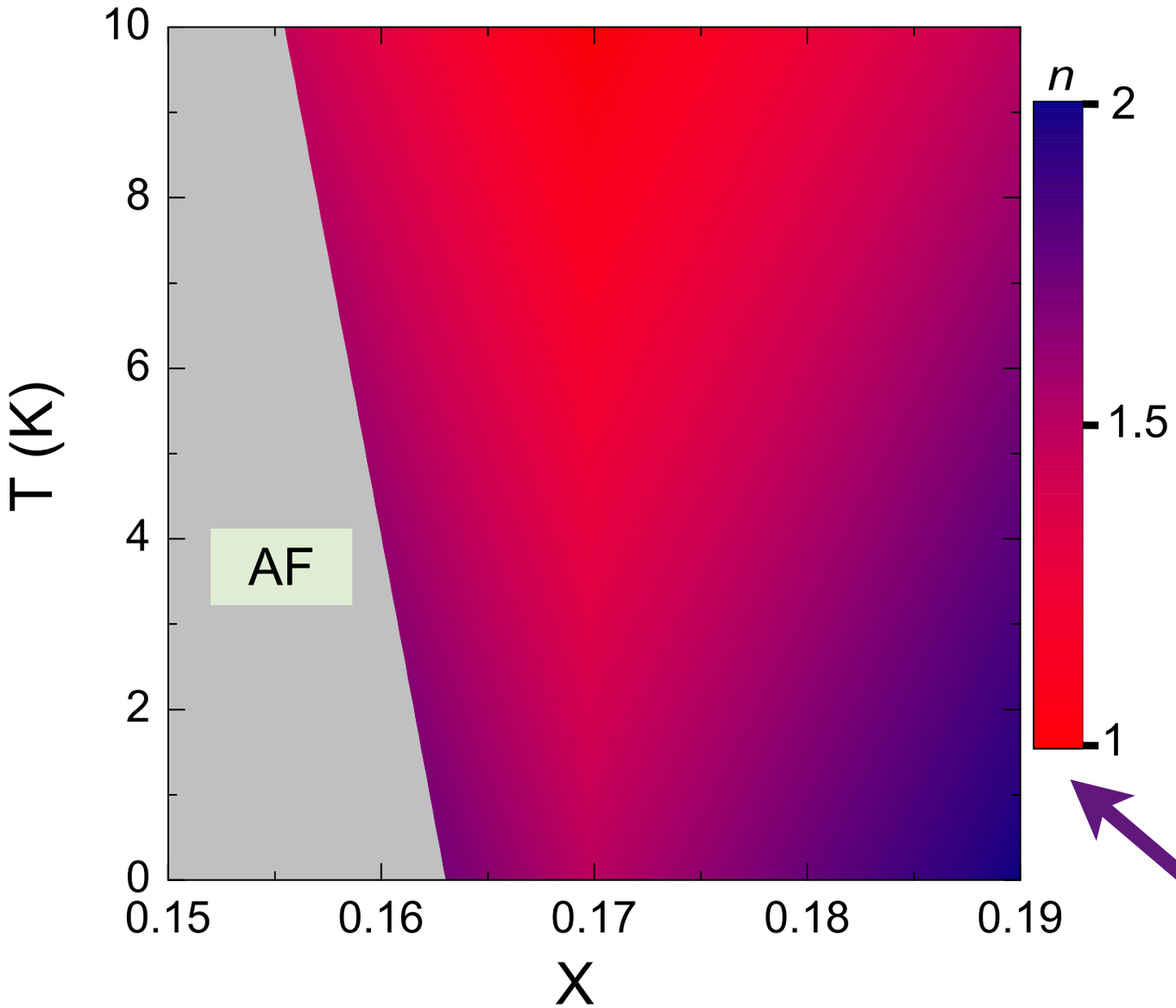


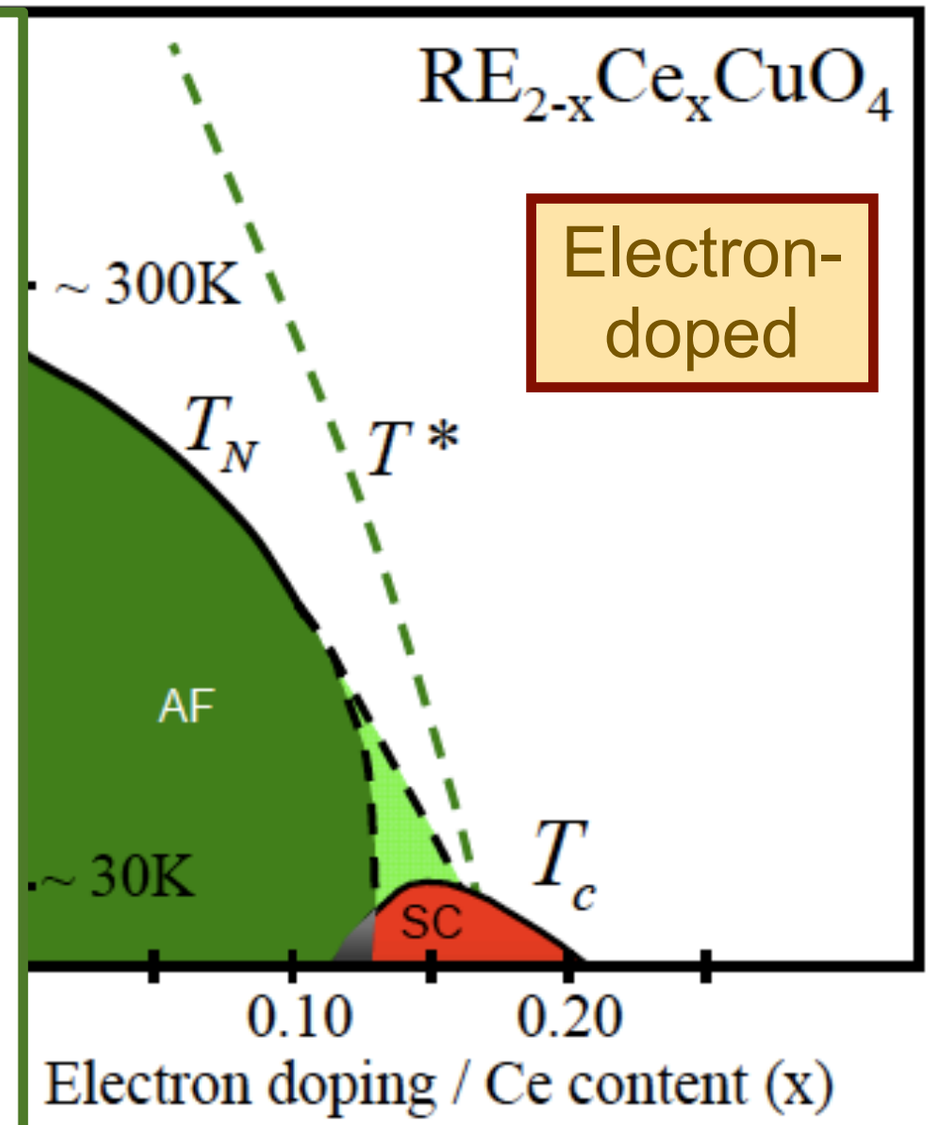
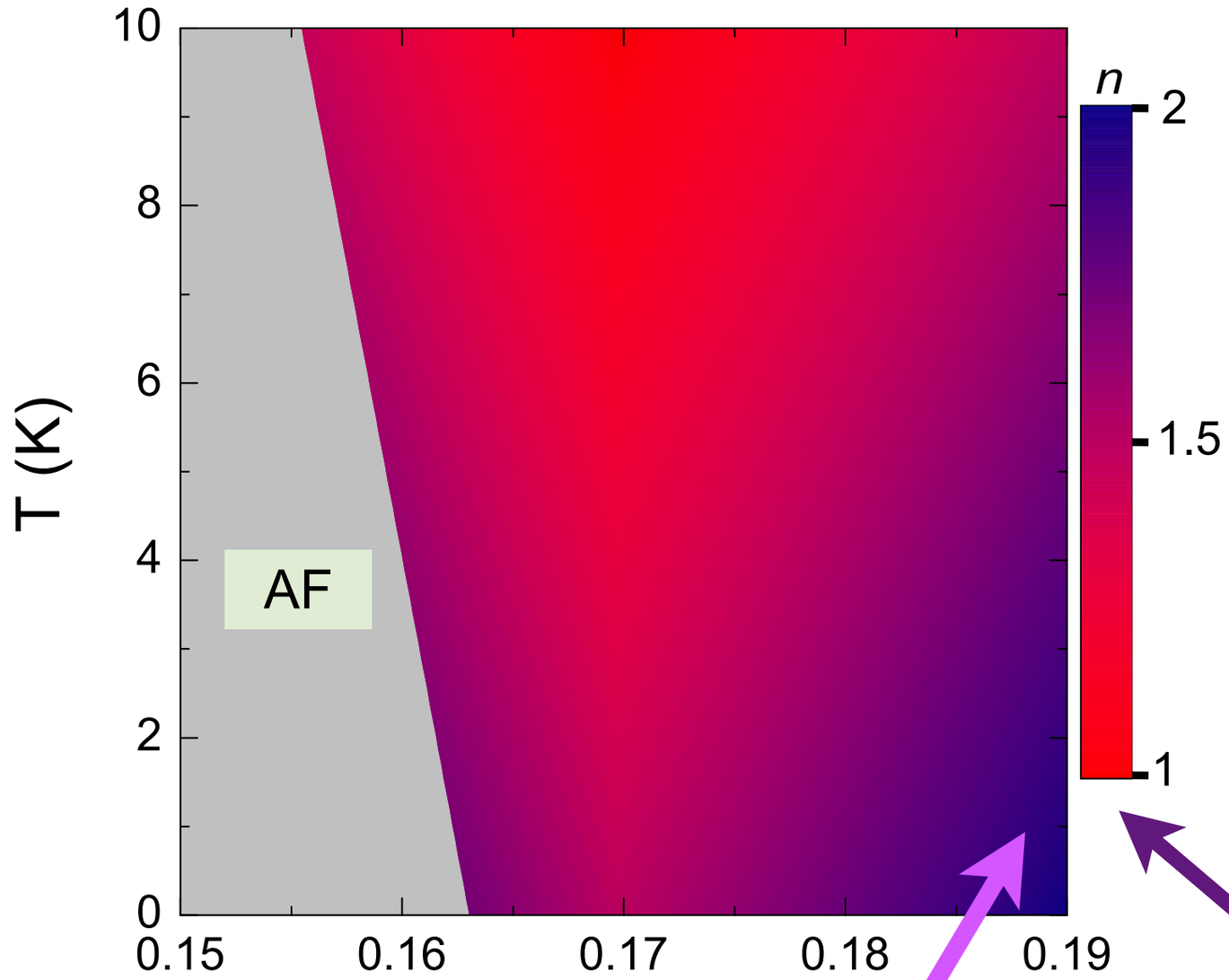
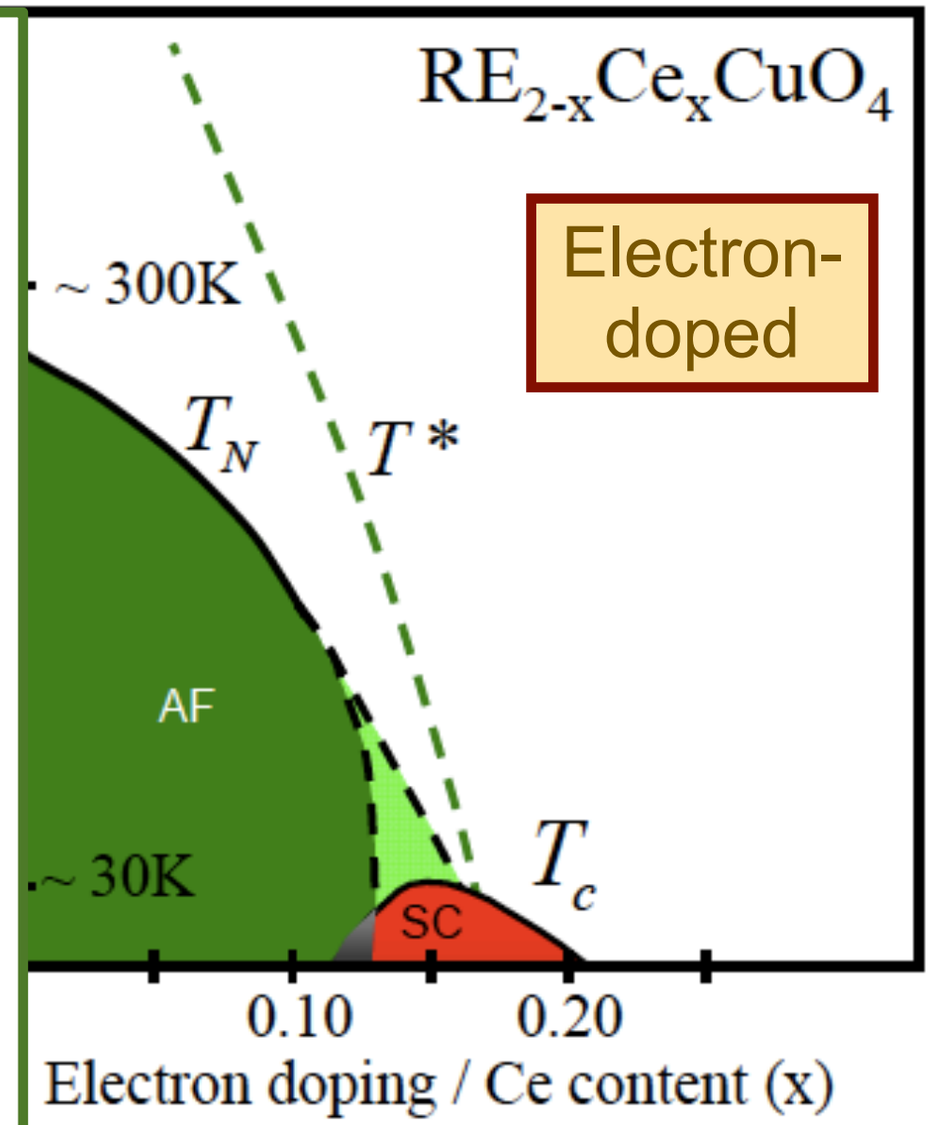


Figure prepared by K. Jin and and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. **82**, 2421 (2010).

Resistivity $\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors

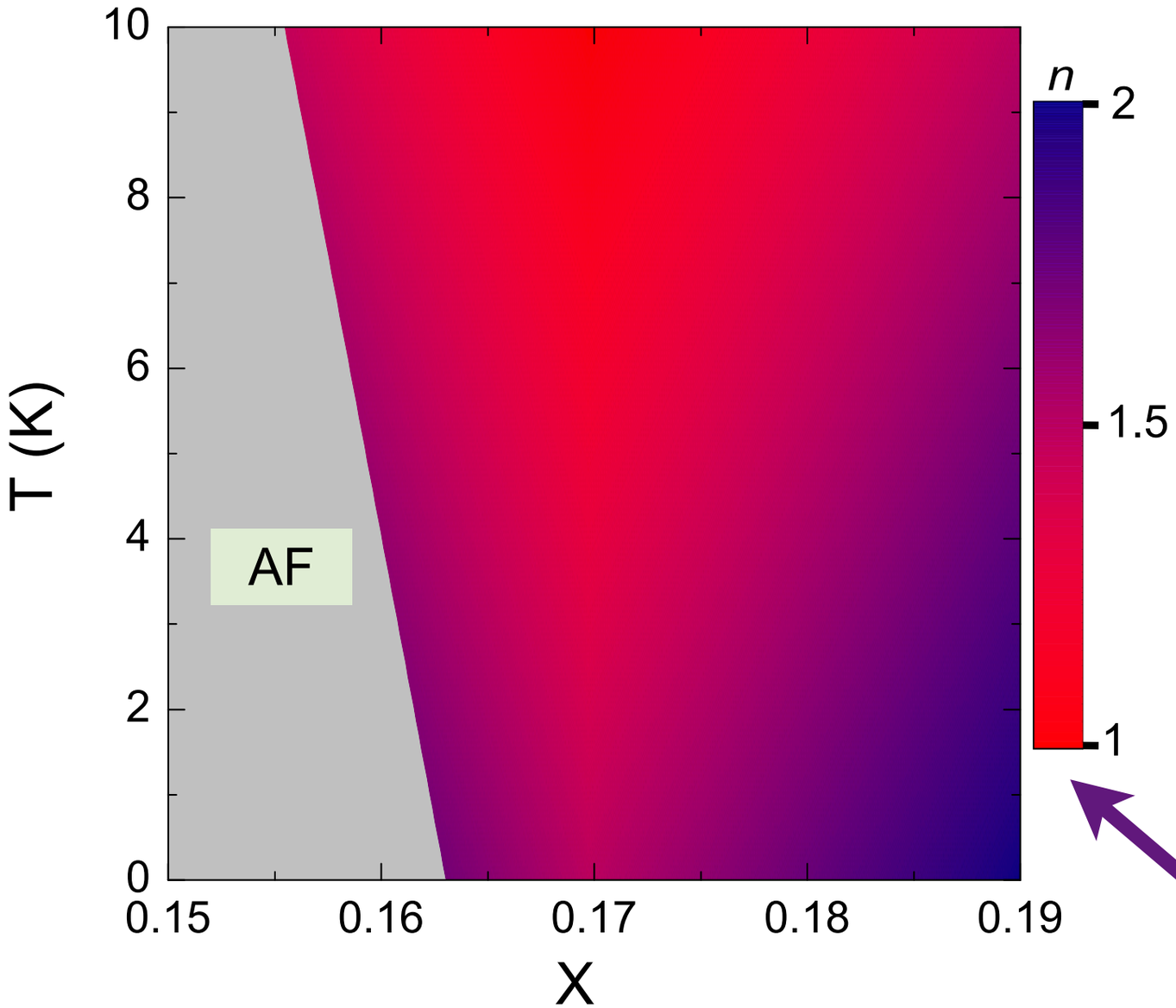


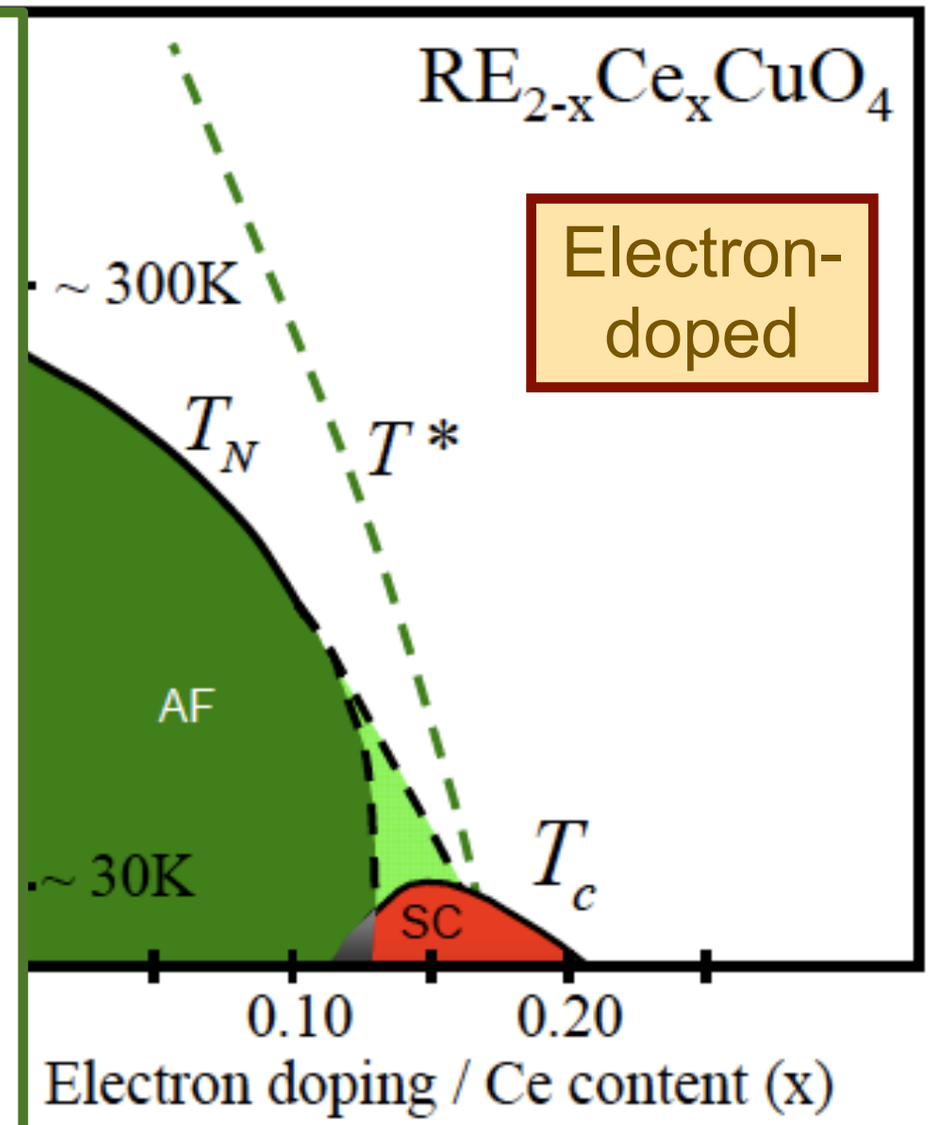


Ordinary metal
(Fermi liquid)
Short-range entanglement

Resistivity
$\sim \rho_0 + AT^n$

Electron-doped cuprate superconductors





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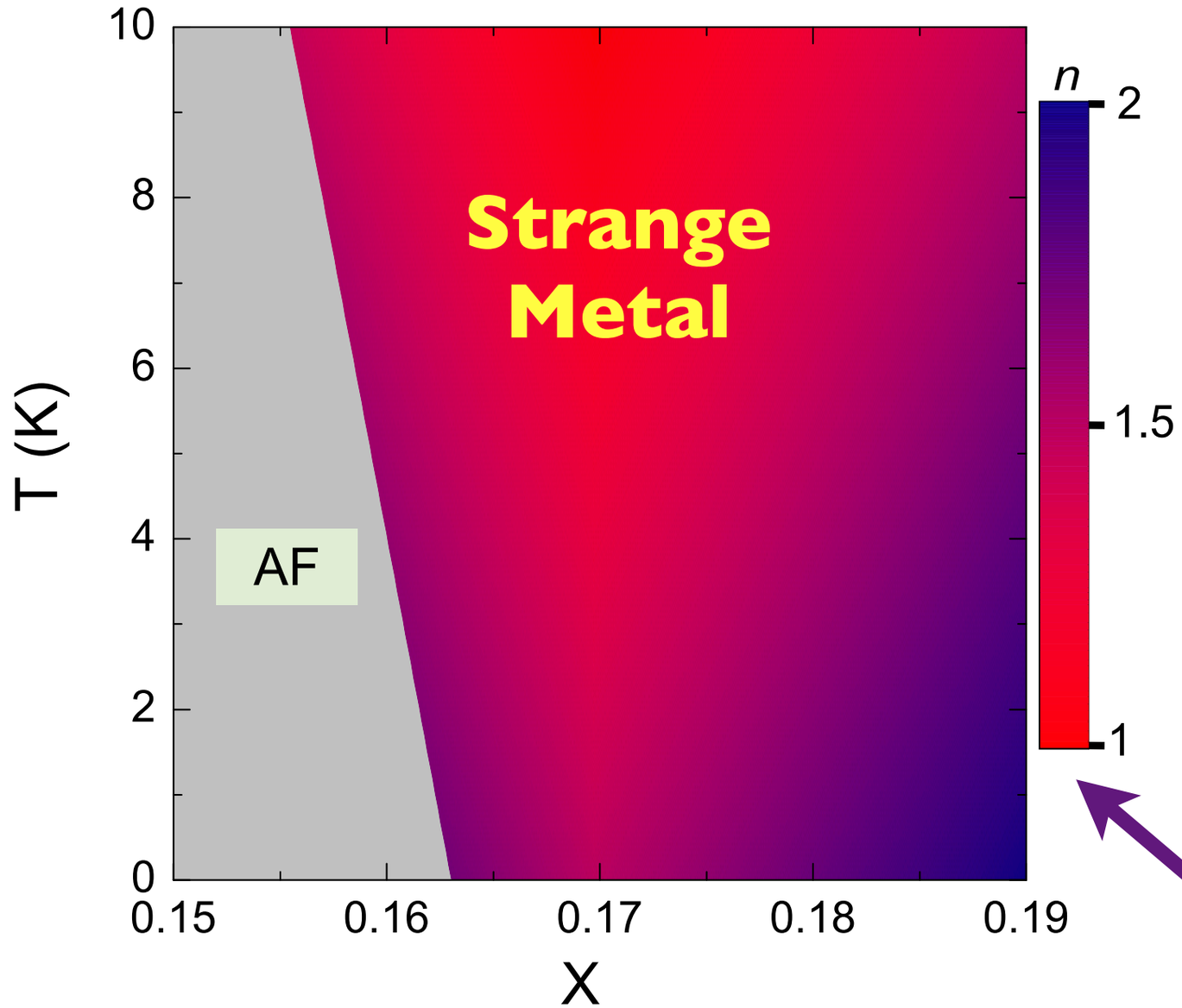

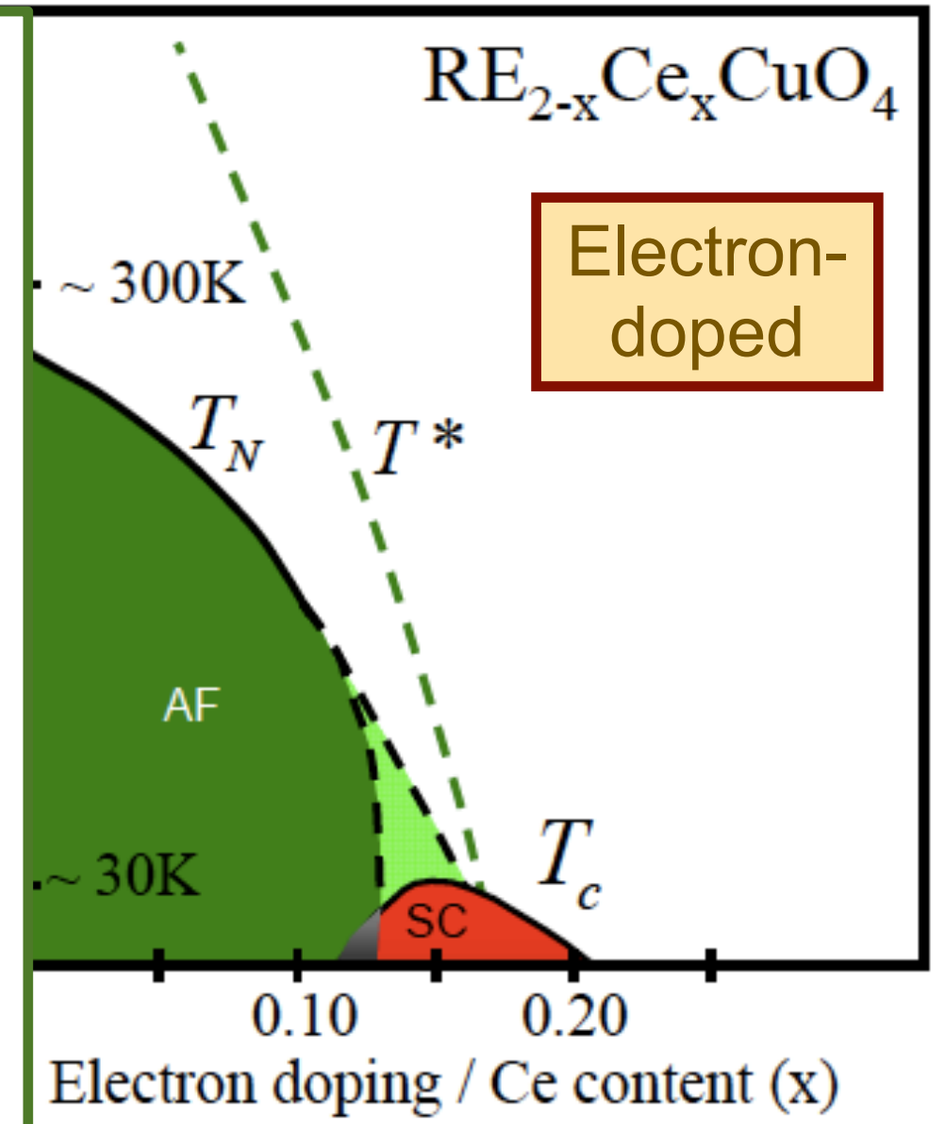

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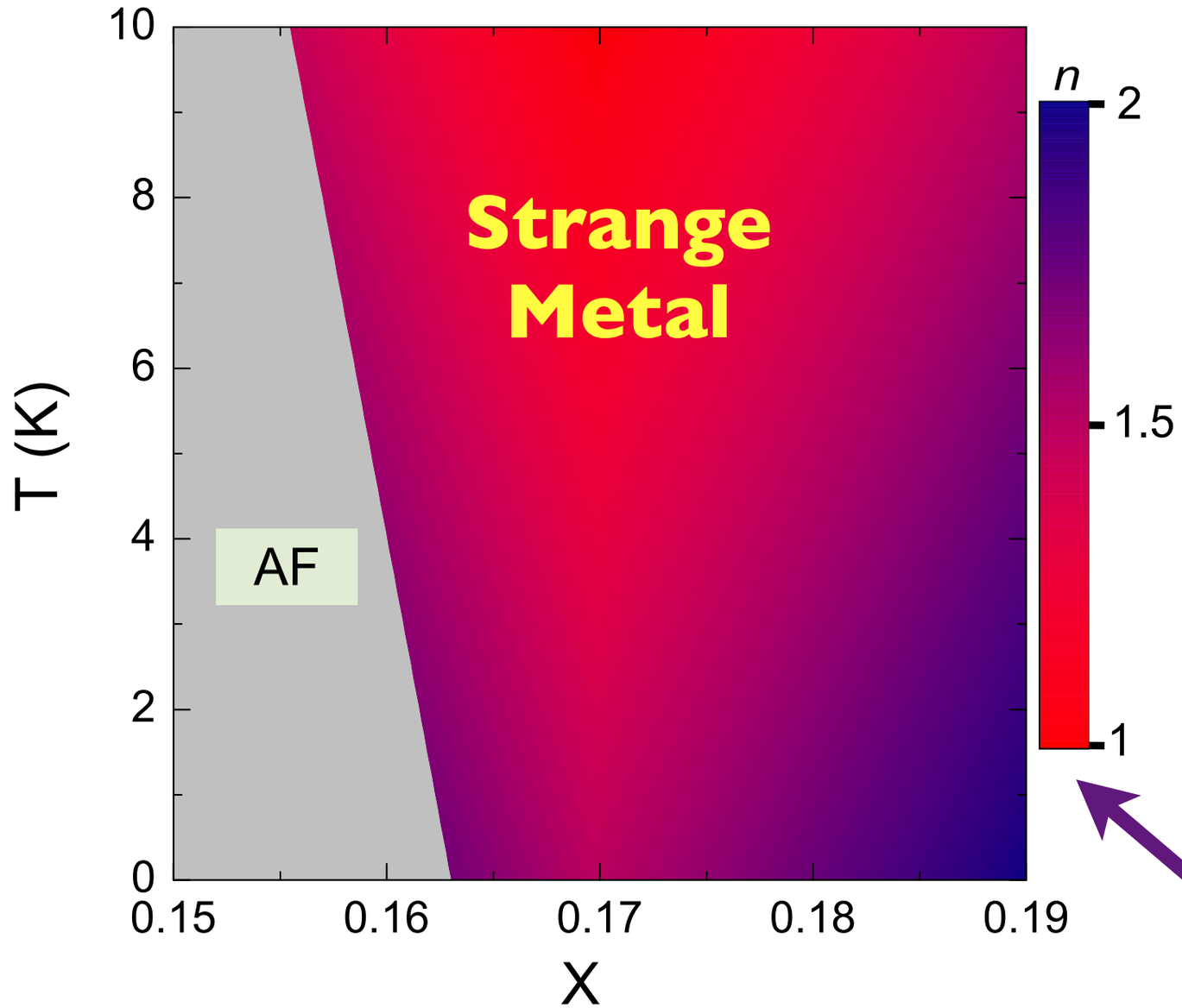

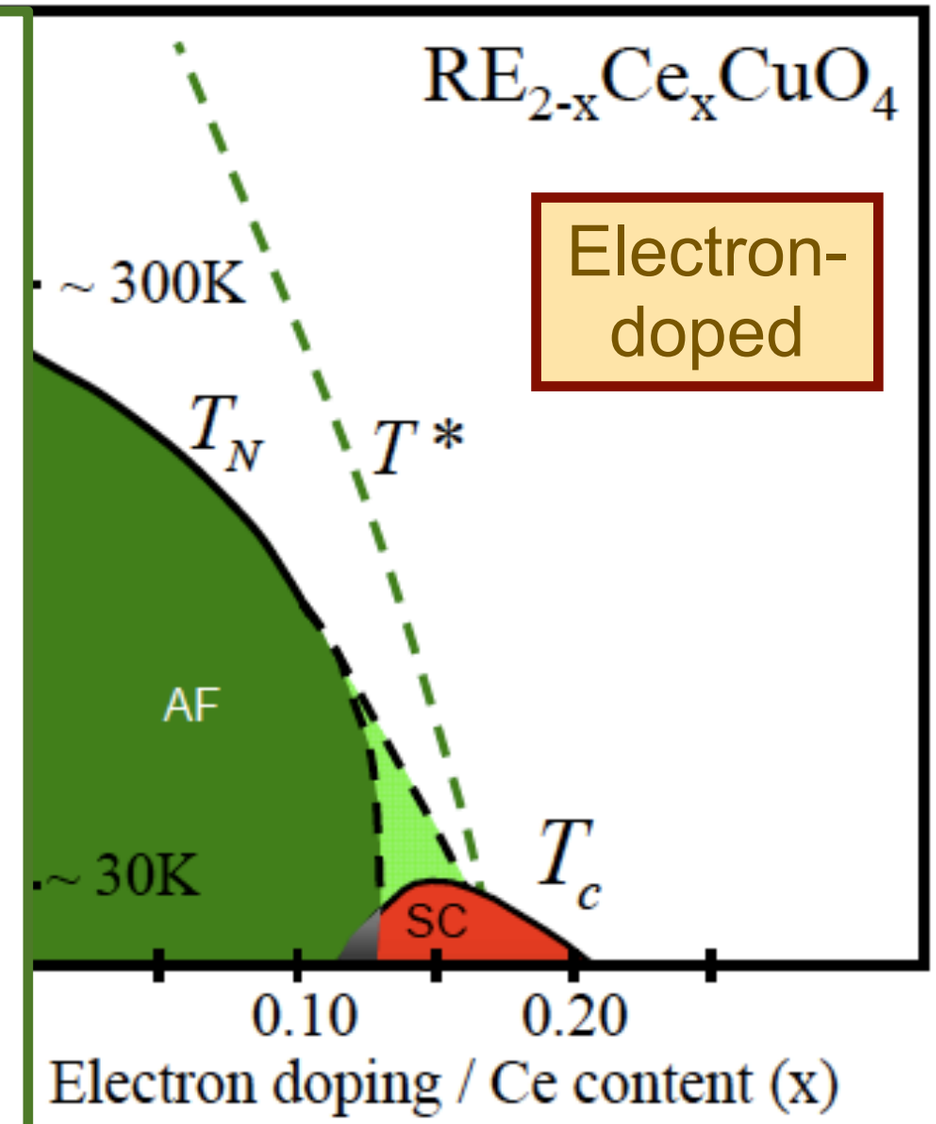

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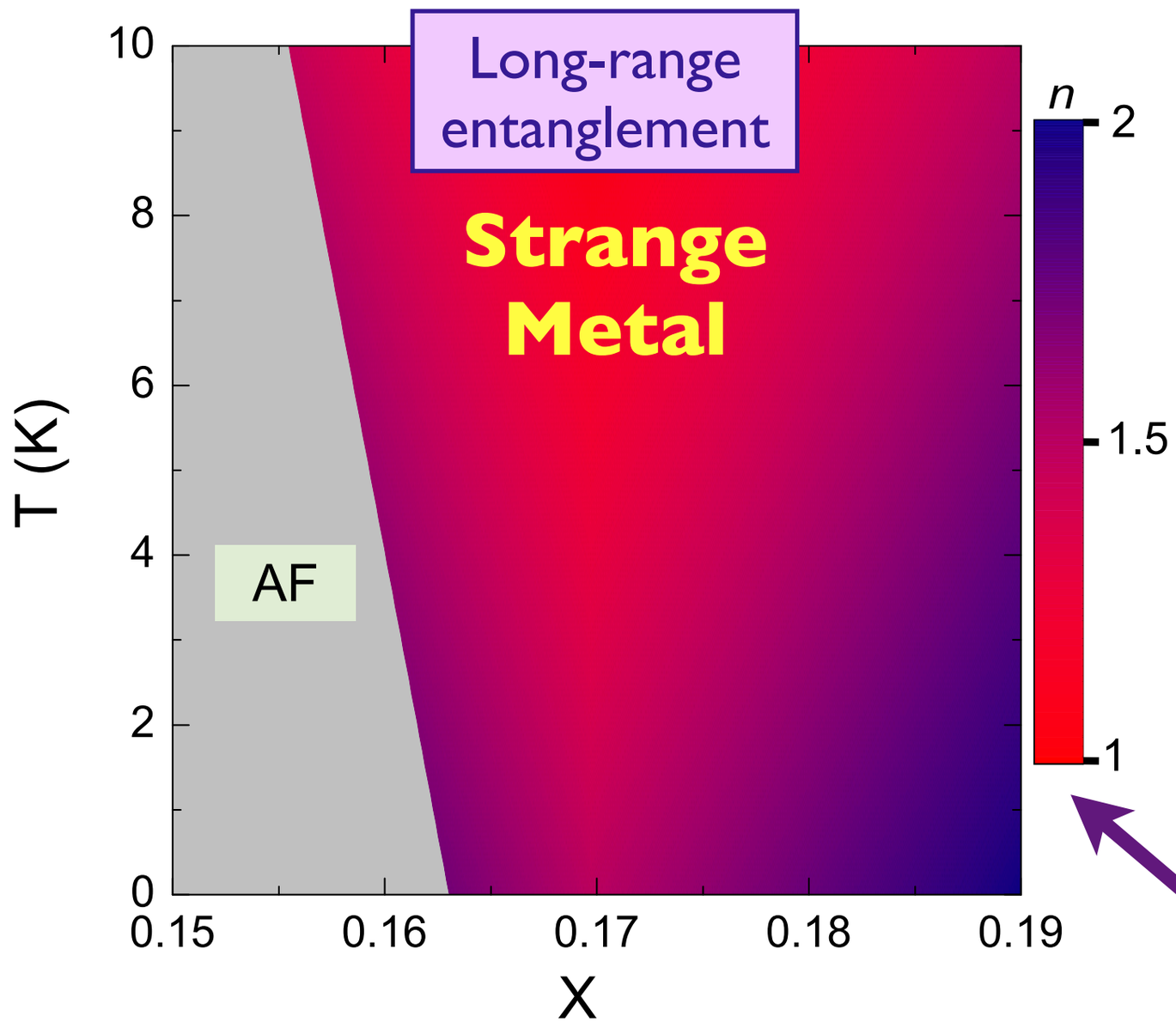

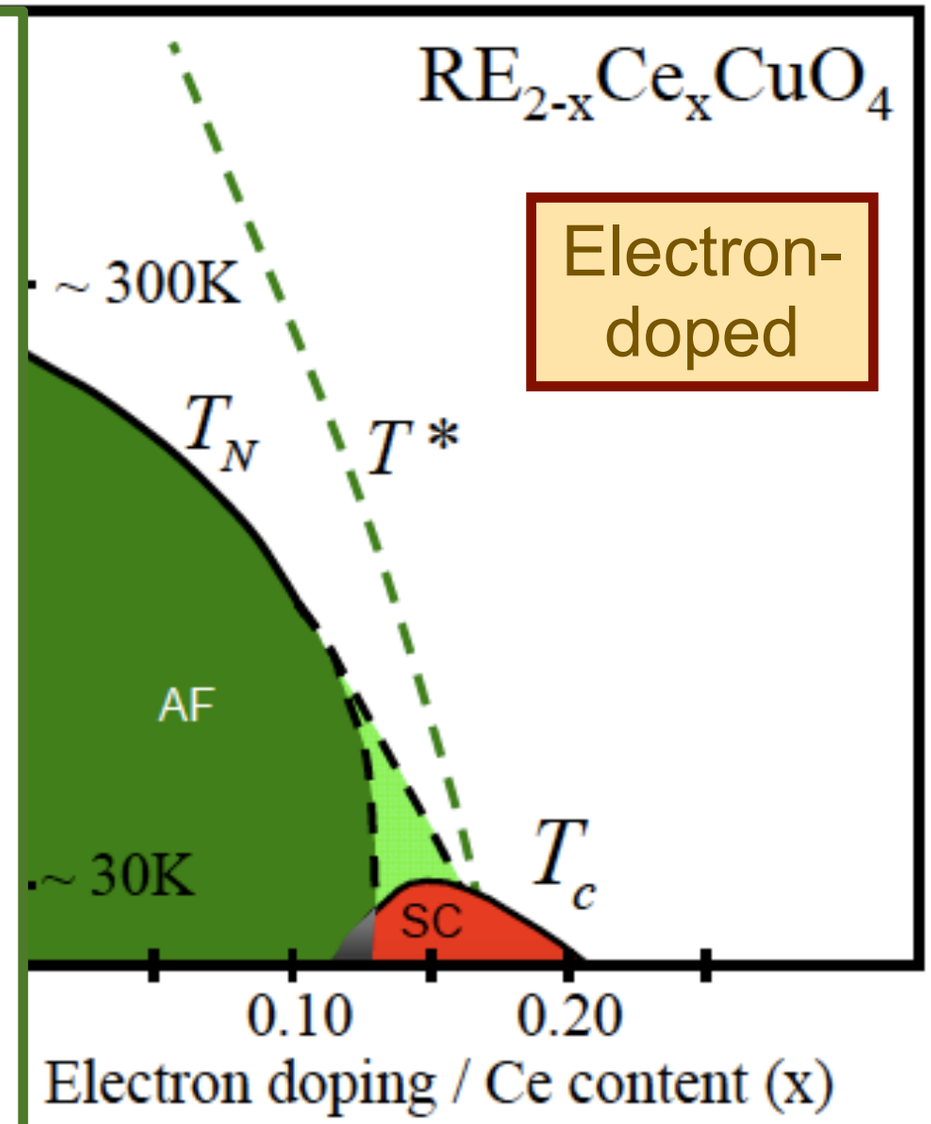

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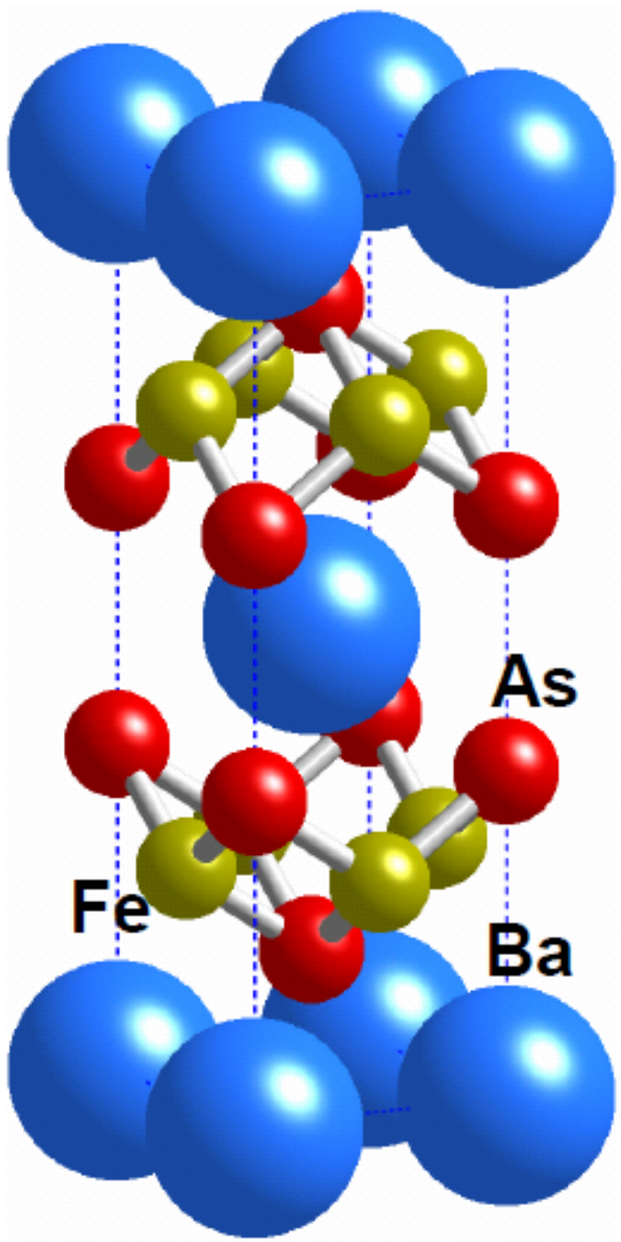
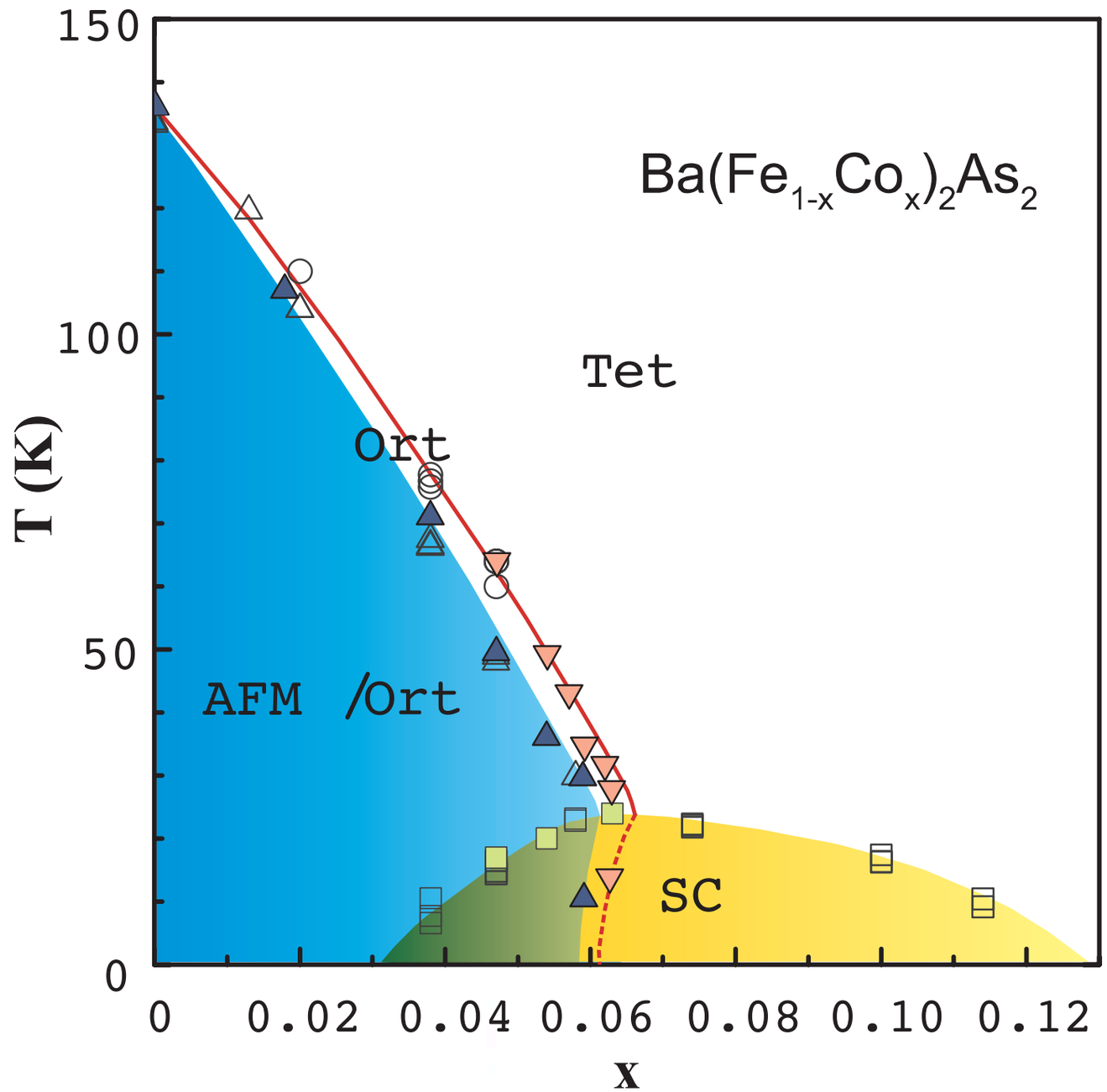
Resistivity $\sim \rho_0 + AT^n$

Iron pnictides:

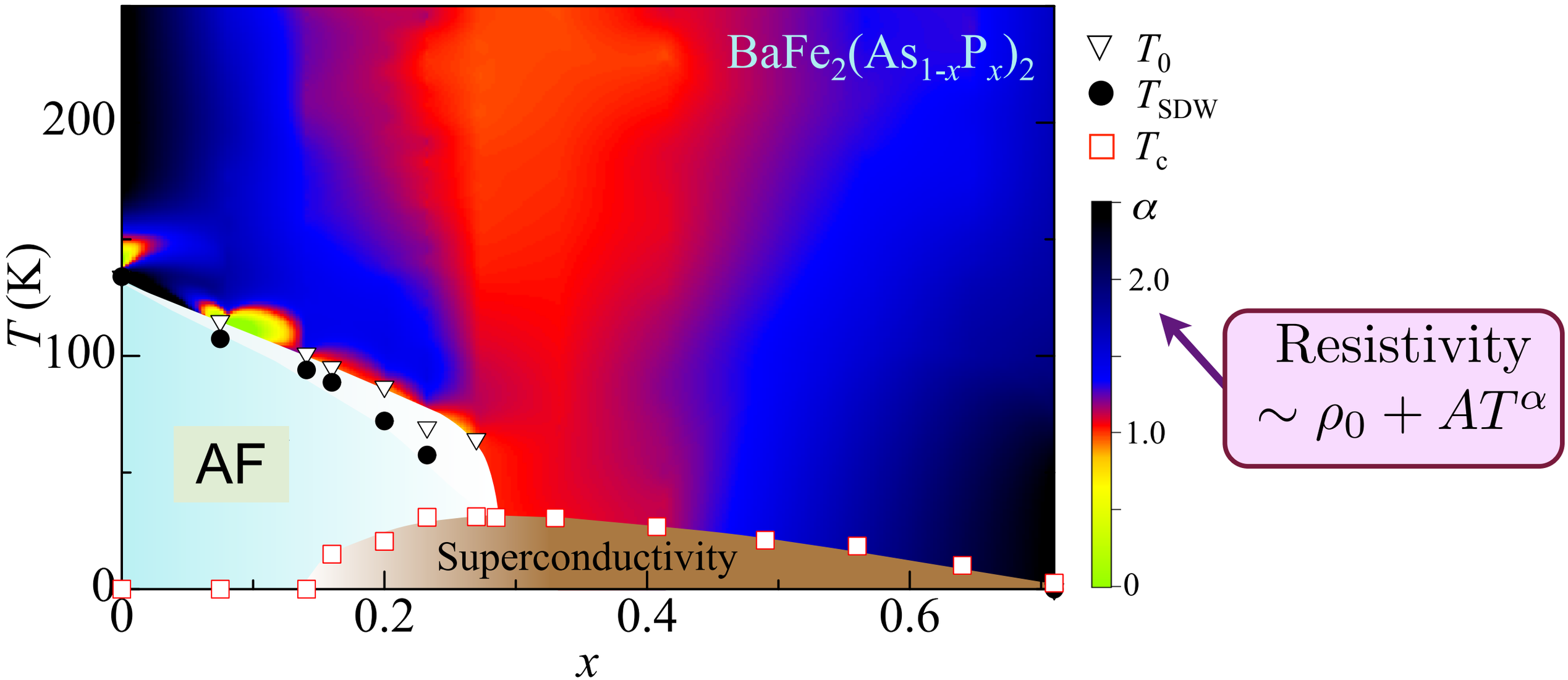
a new class of high temperature superconductors





S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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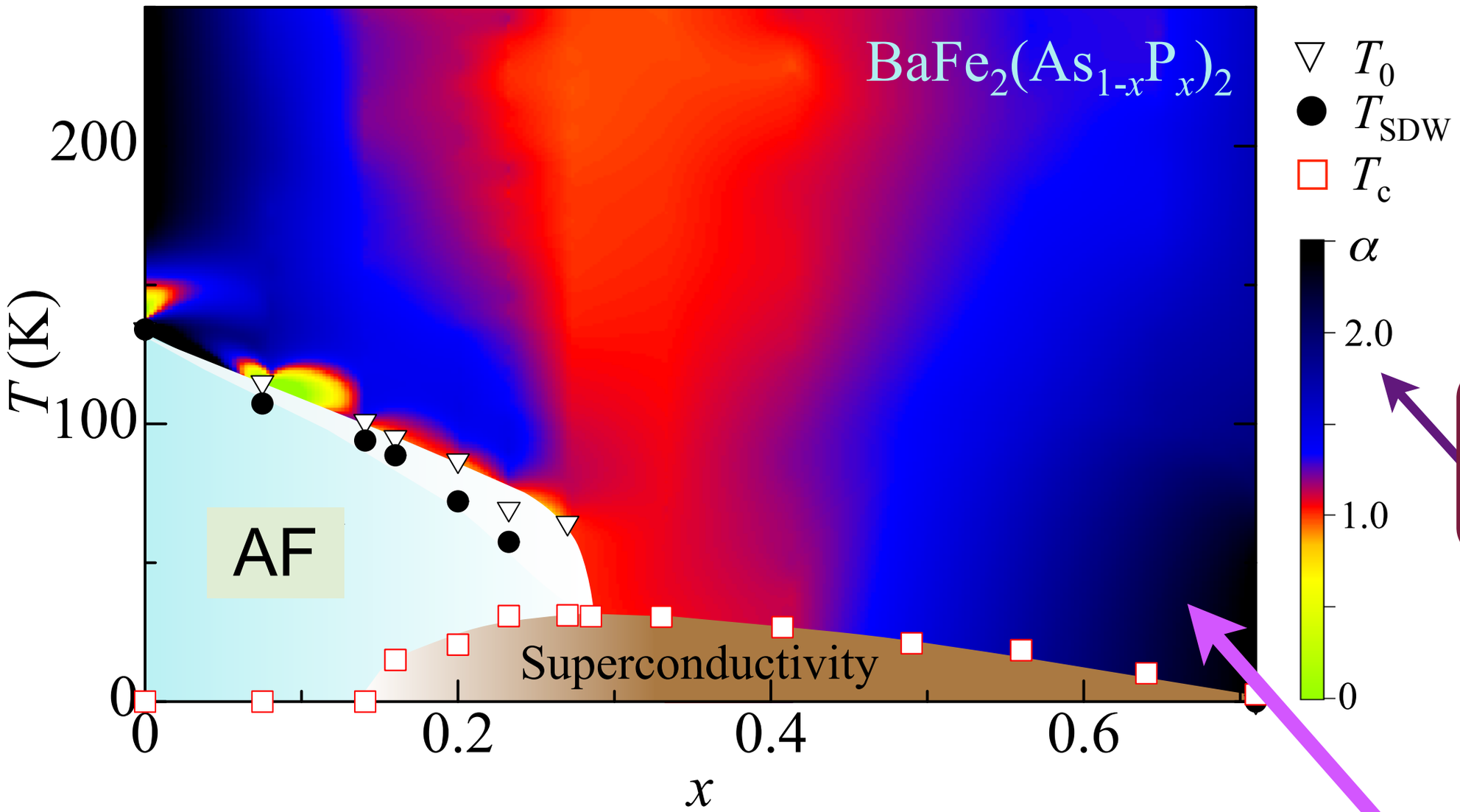

Resistivity
$\sim \rho_0 + AT^\alpha$

Ordinary metal
(Fermi liquid)
Short-range entanglement

S. Kasahara, T. Shibauchi, K. Hashi
H. Ikeda, H. Takeya,
Physical F

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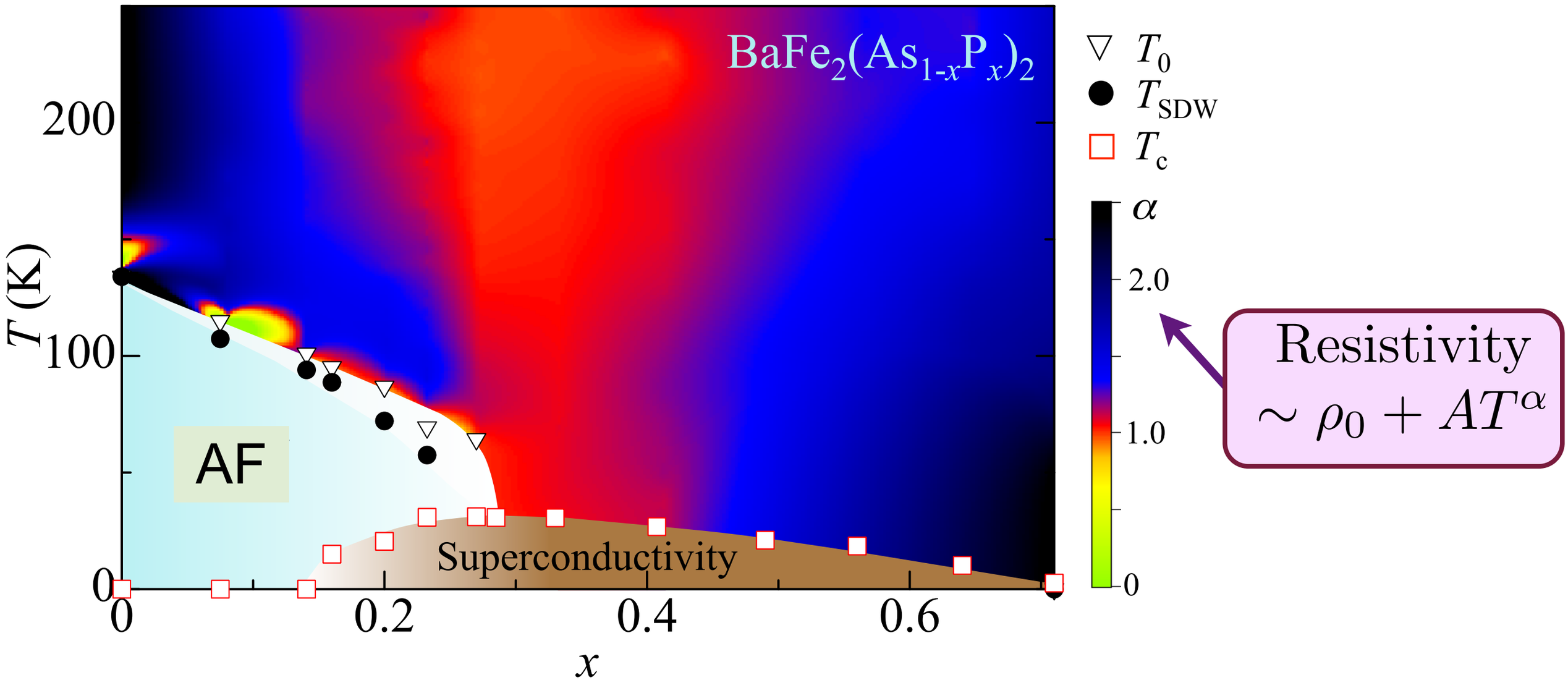

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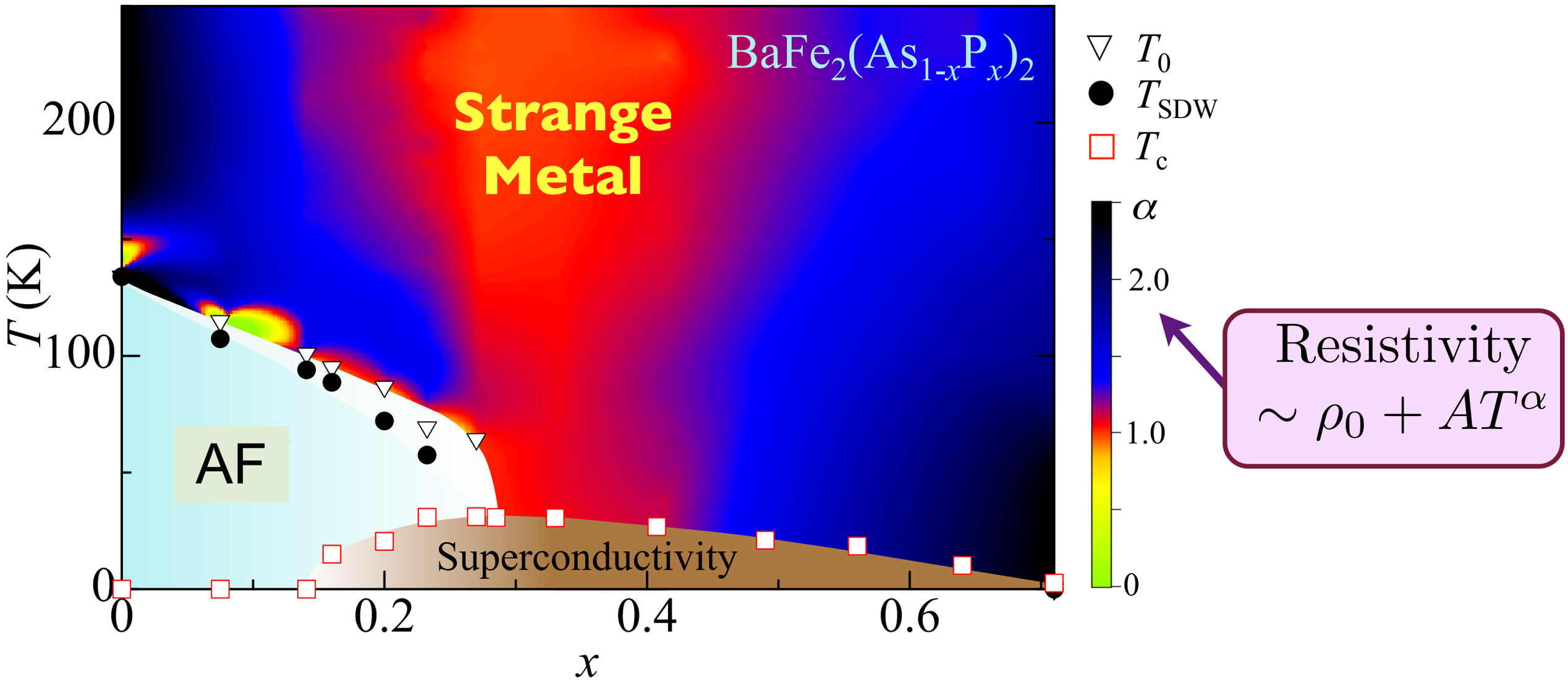

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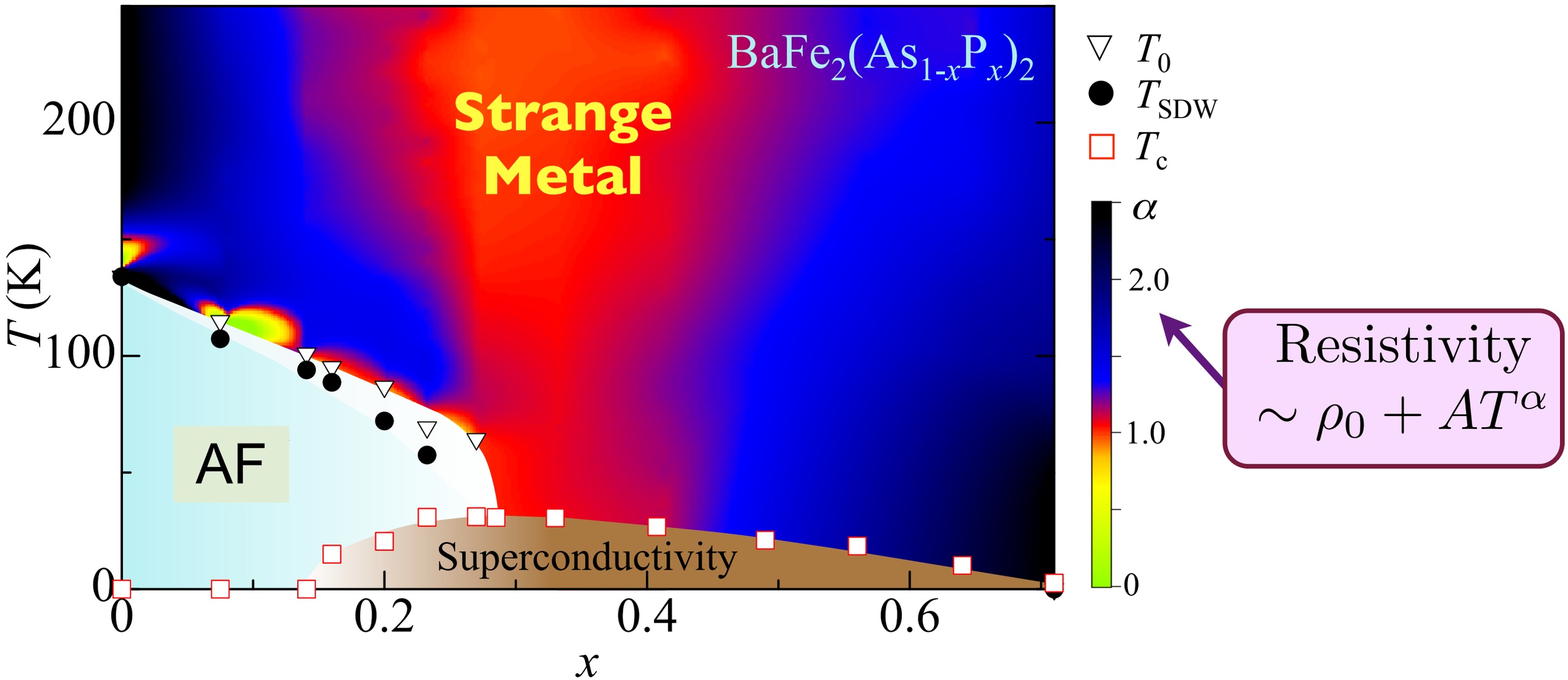

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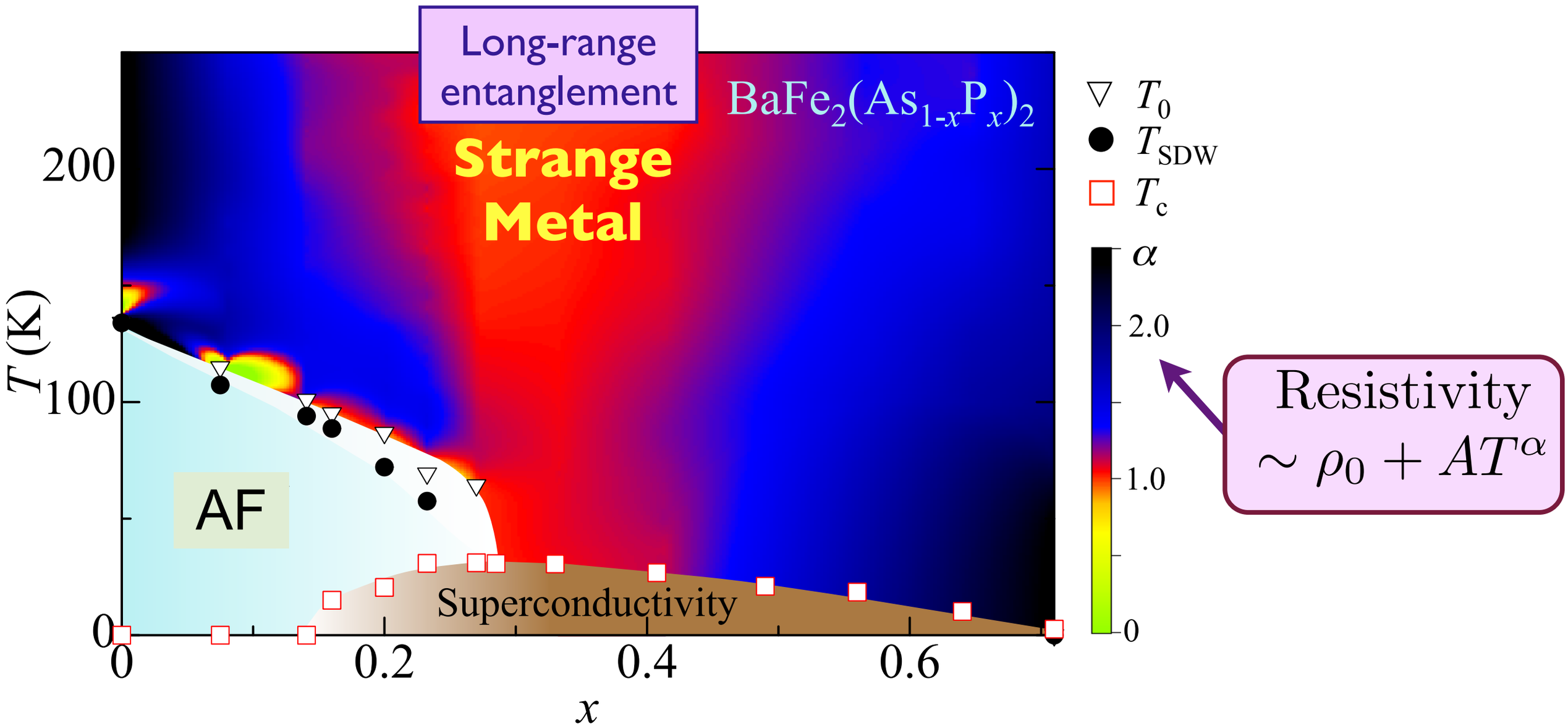
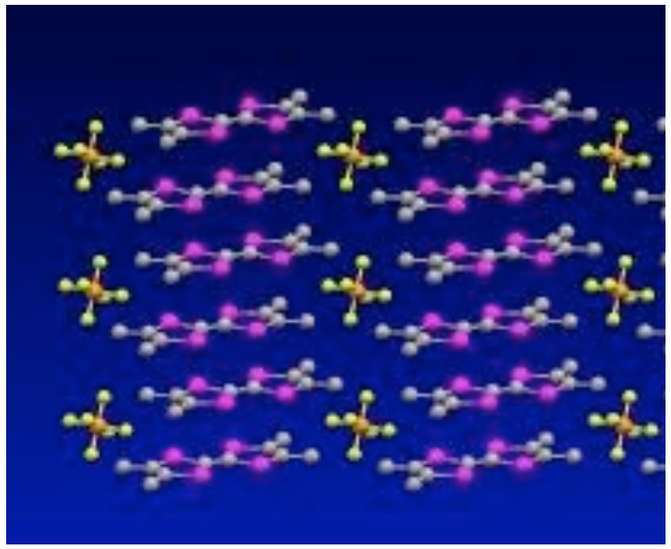
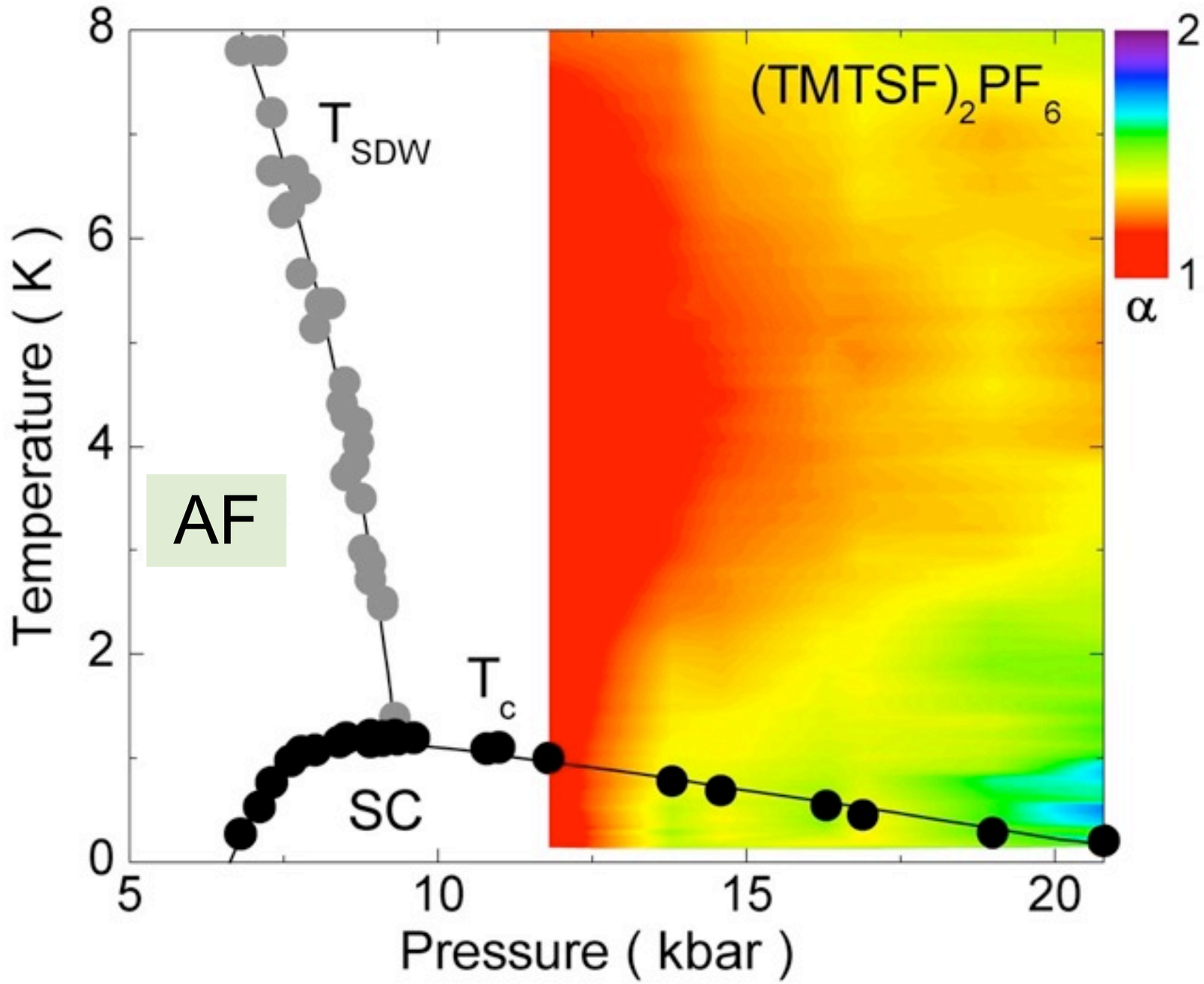


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

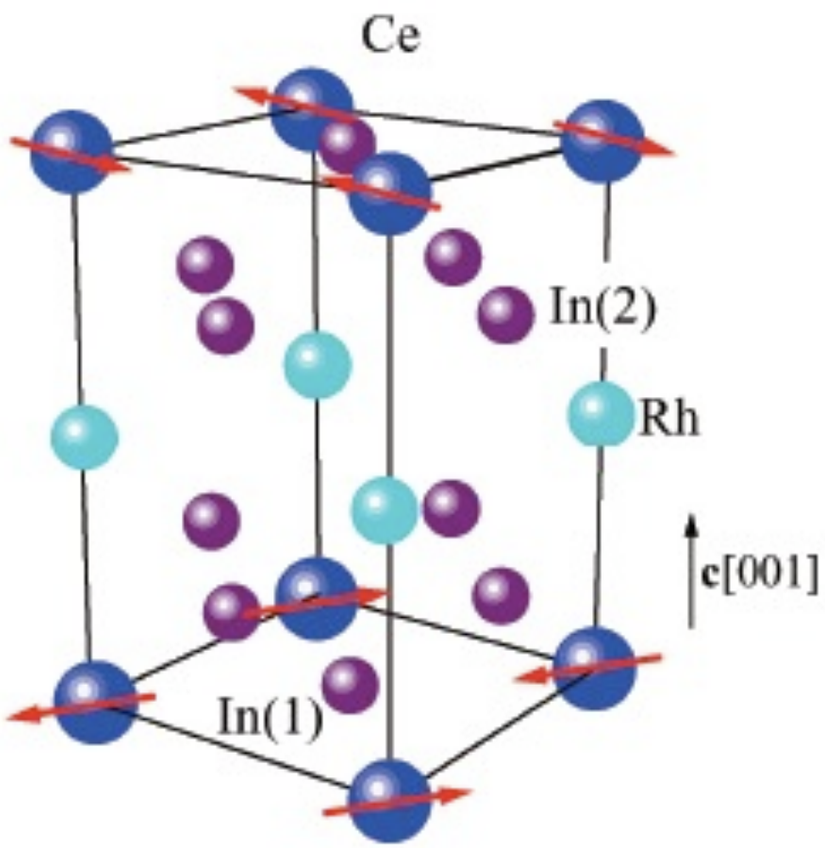
Temperature-pressure phase diagram of an organic superconductor

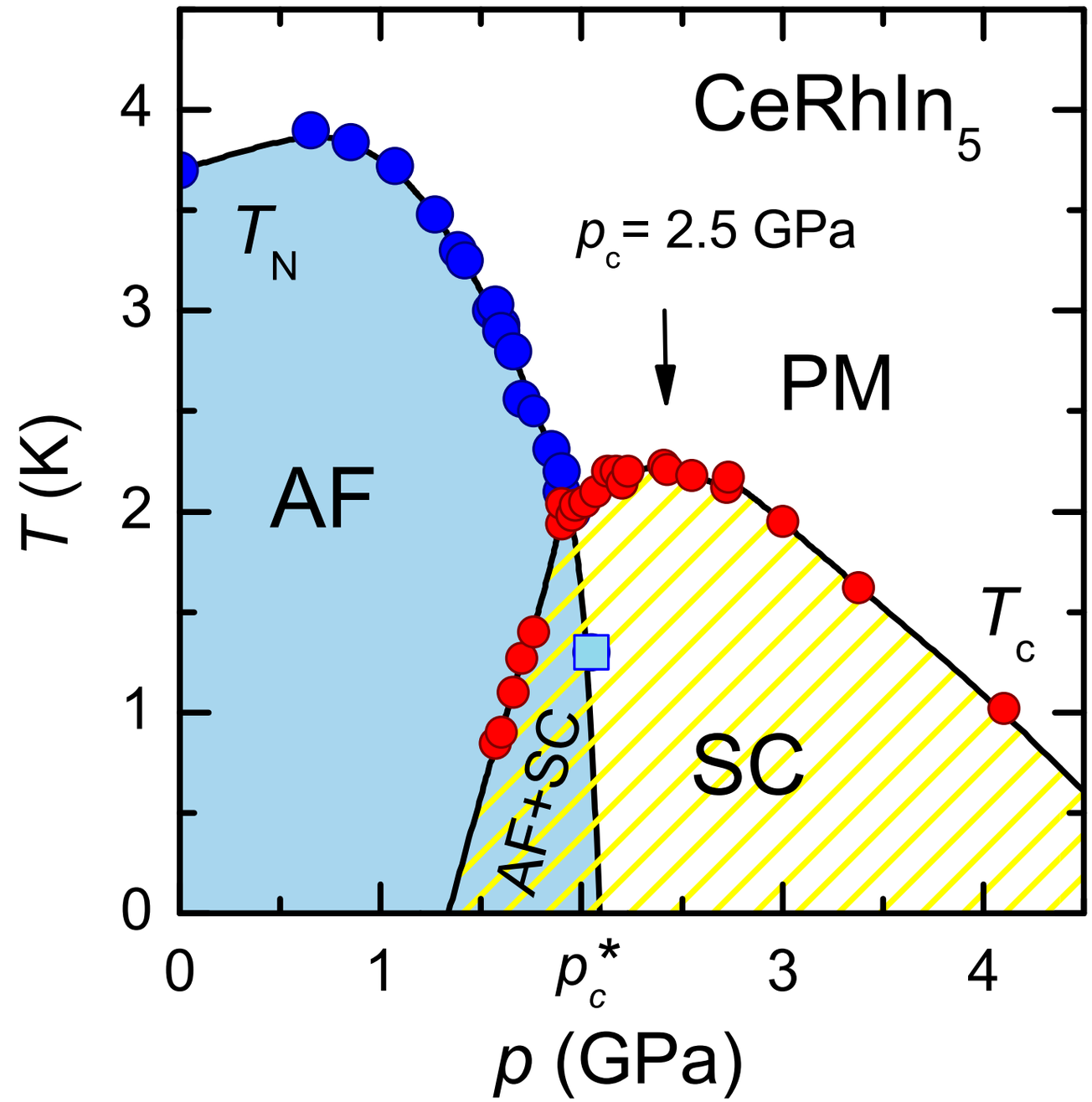




N. Doiron-Leyraud, P. Auban-Senzier, S. Rene de Cotret, A. Sedeki, C. Bourbonnais, D. Jerome, K. Bechgaard, and Louis Taillefer, Physical Review B 80, 214531 (2009)

Temperature-pressure phase diagram of an heavy-fermion superconductor



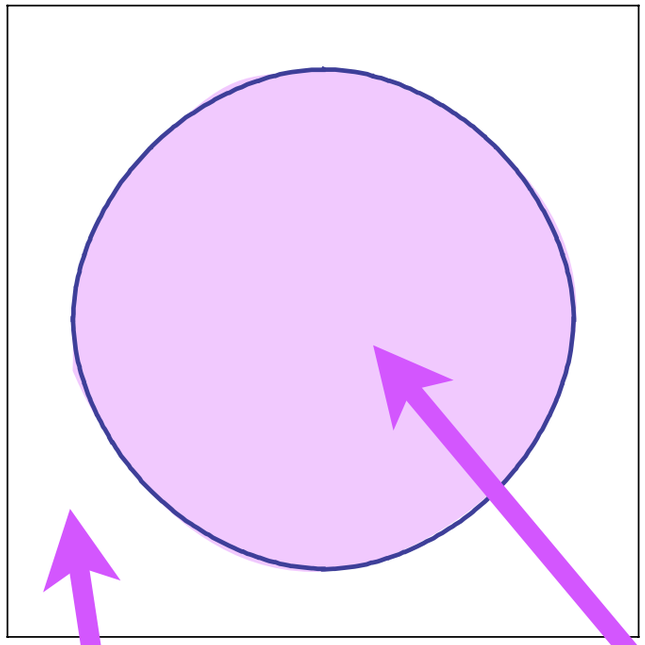


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223.

Tuson Park, F. Ronning, H. Q. Yuan, M. B. Salamon, R. Movshovich, J. L. Sarrao, and J. D. Thompson, *Nature* **440**, 65 (2006)

Ordinary metals (Fermi liquids)

Metal with "large" Fermi surface

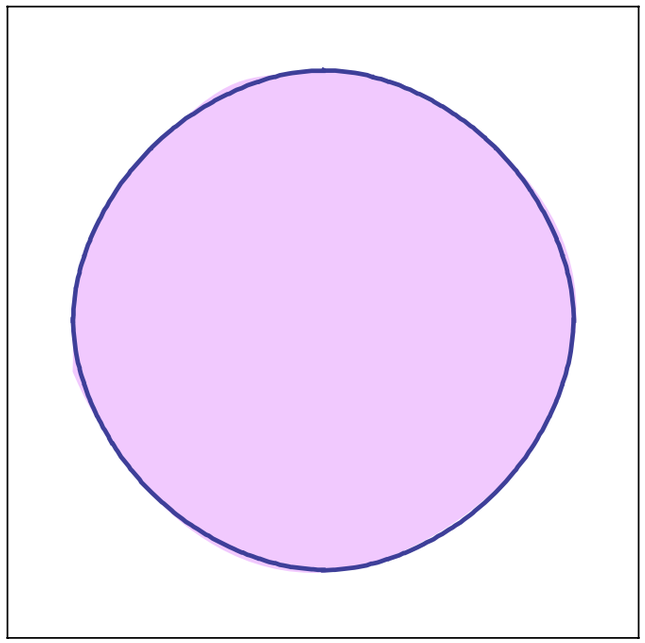


Momenta with electron states occupied

Momenta with electron states empty

Fermi surface+antiferromagnetism

Metal with "large"
Fermi surface



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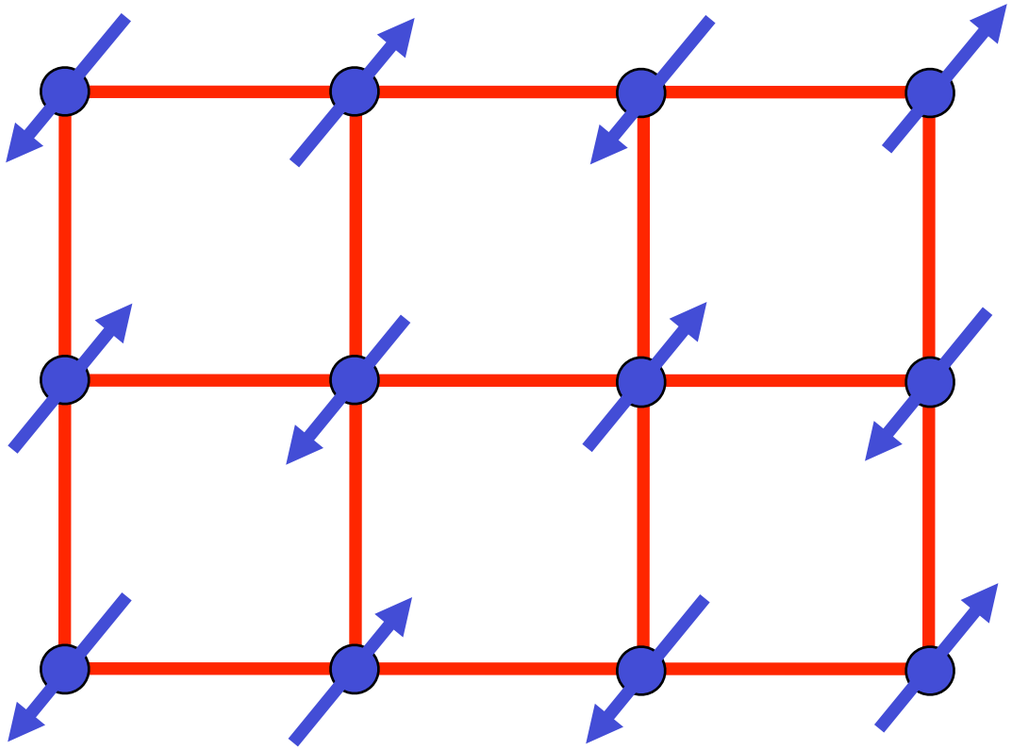
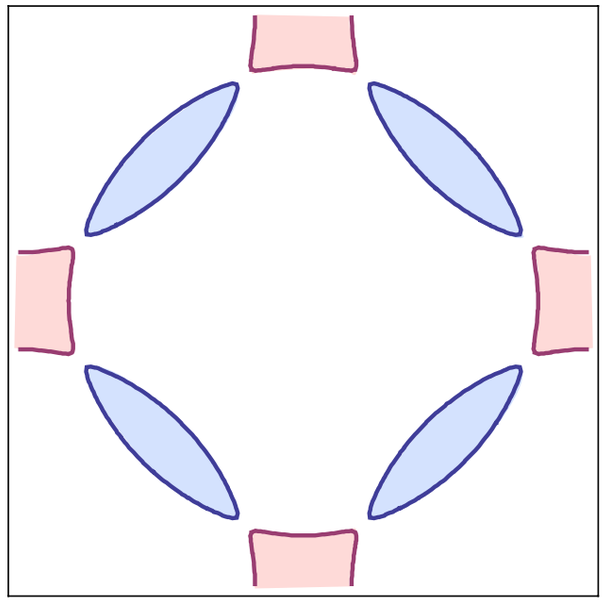


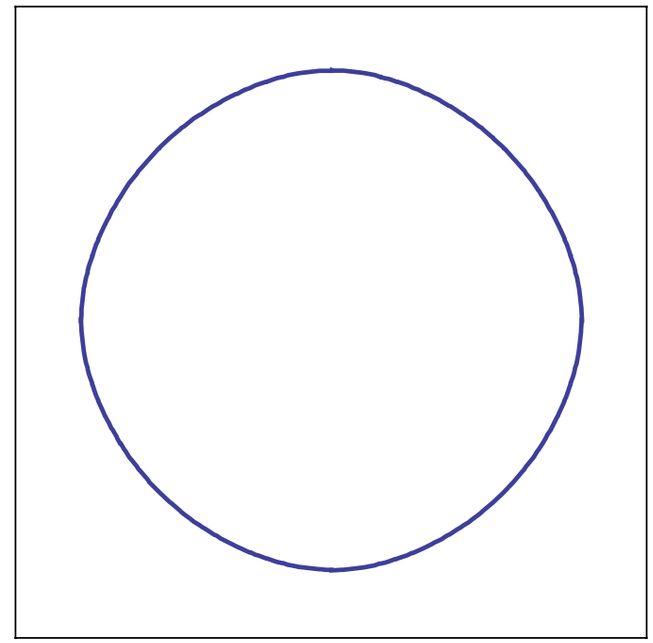
The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism





AF $\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

Metal with electron and hole pockets

Metal with "large" Fermi surface

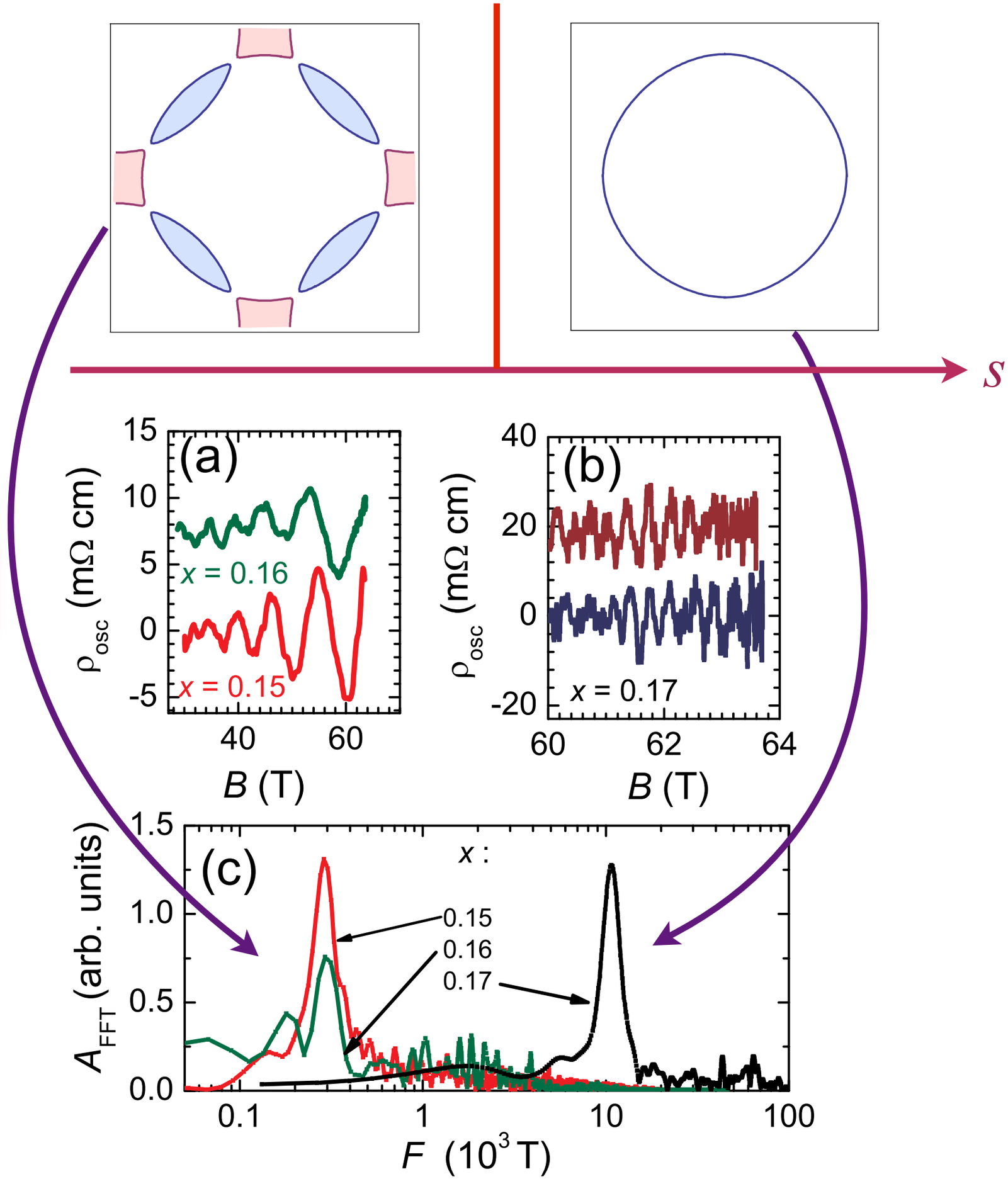
Increasing interaction

Fermi surface reconstruction and onset of antiferromagnetism

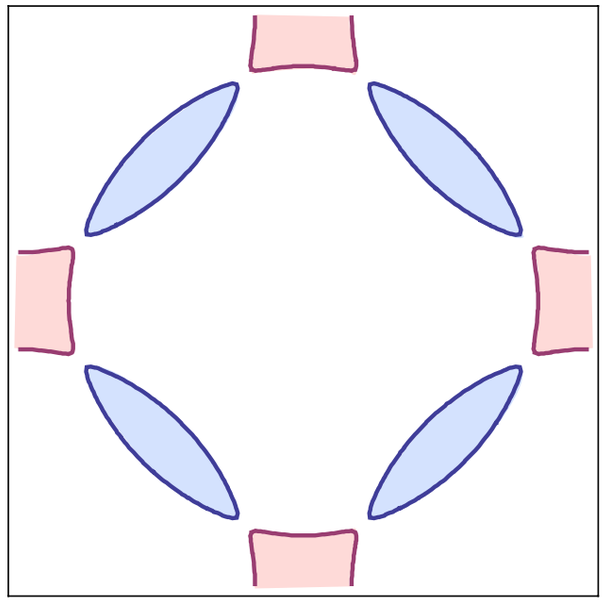
Quantum oscillations

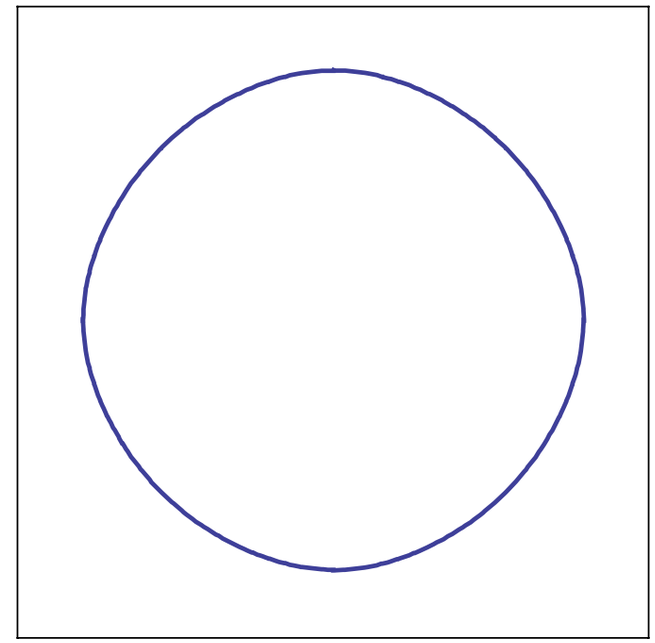
$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).



Fermi surface+antiferromagnetism





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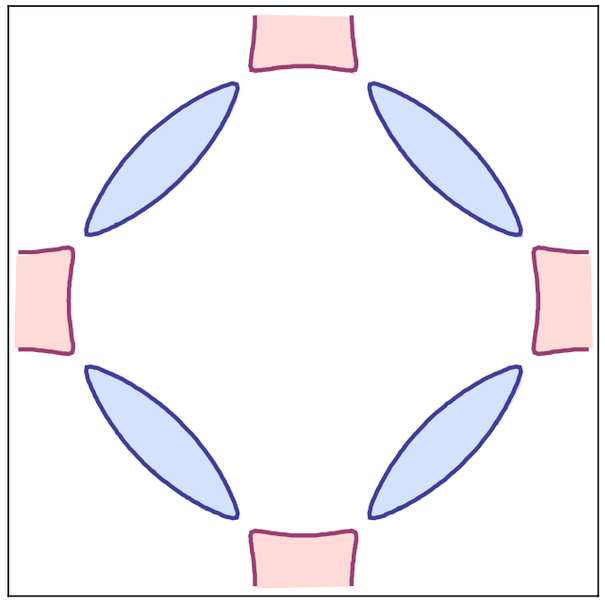
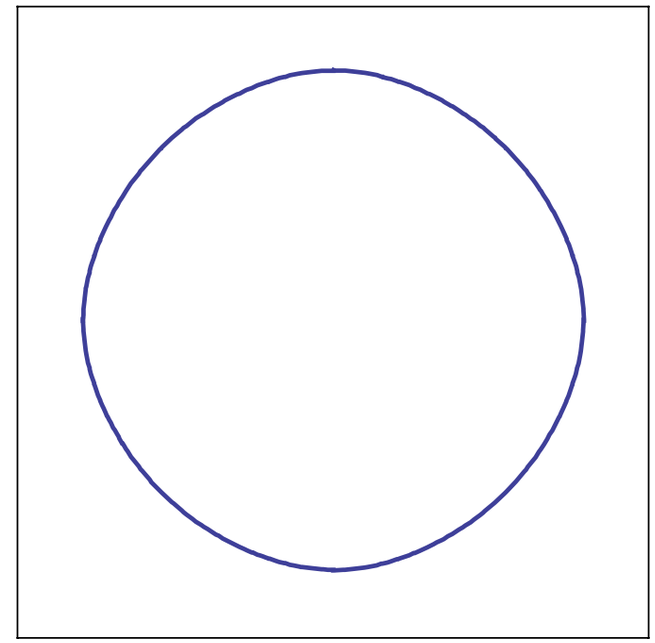
Metal with electron and hole pockets

Metal with "large" Fermi surface

s

Fermi surface+antiferromagnetism





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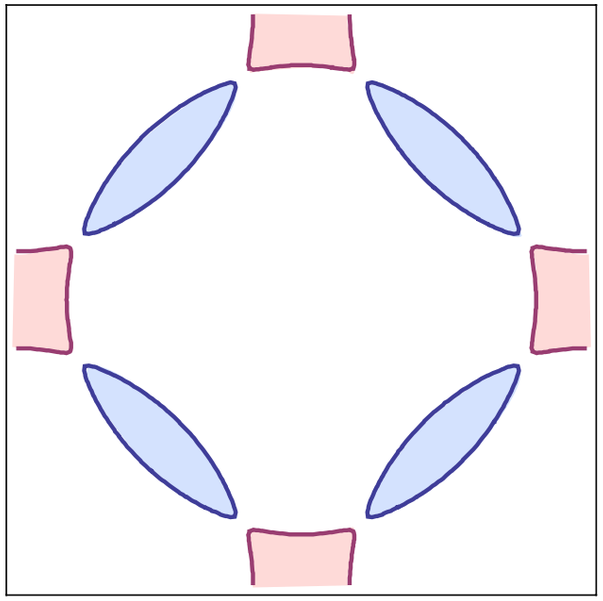
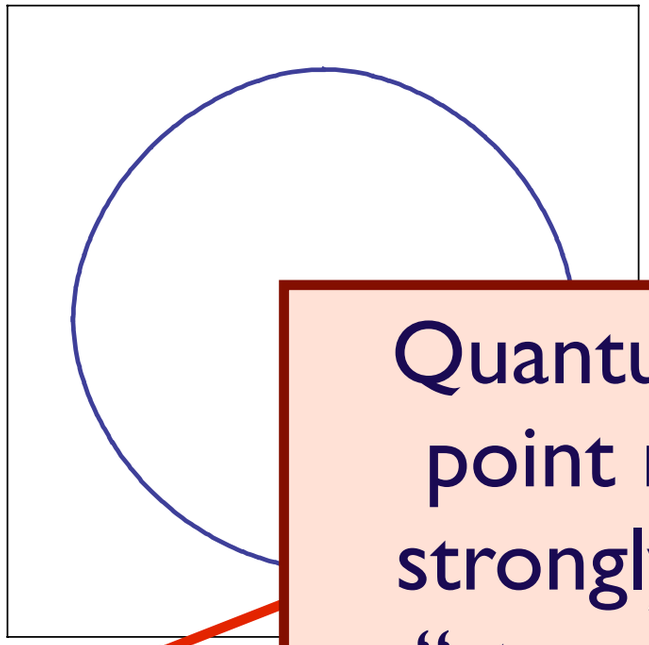




Quantum critical point realizes a strongly-coupled "strange metal" with long-range entanglement !

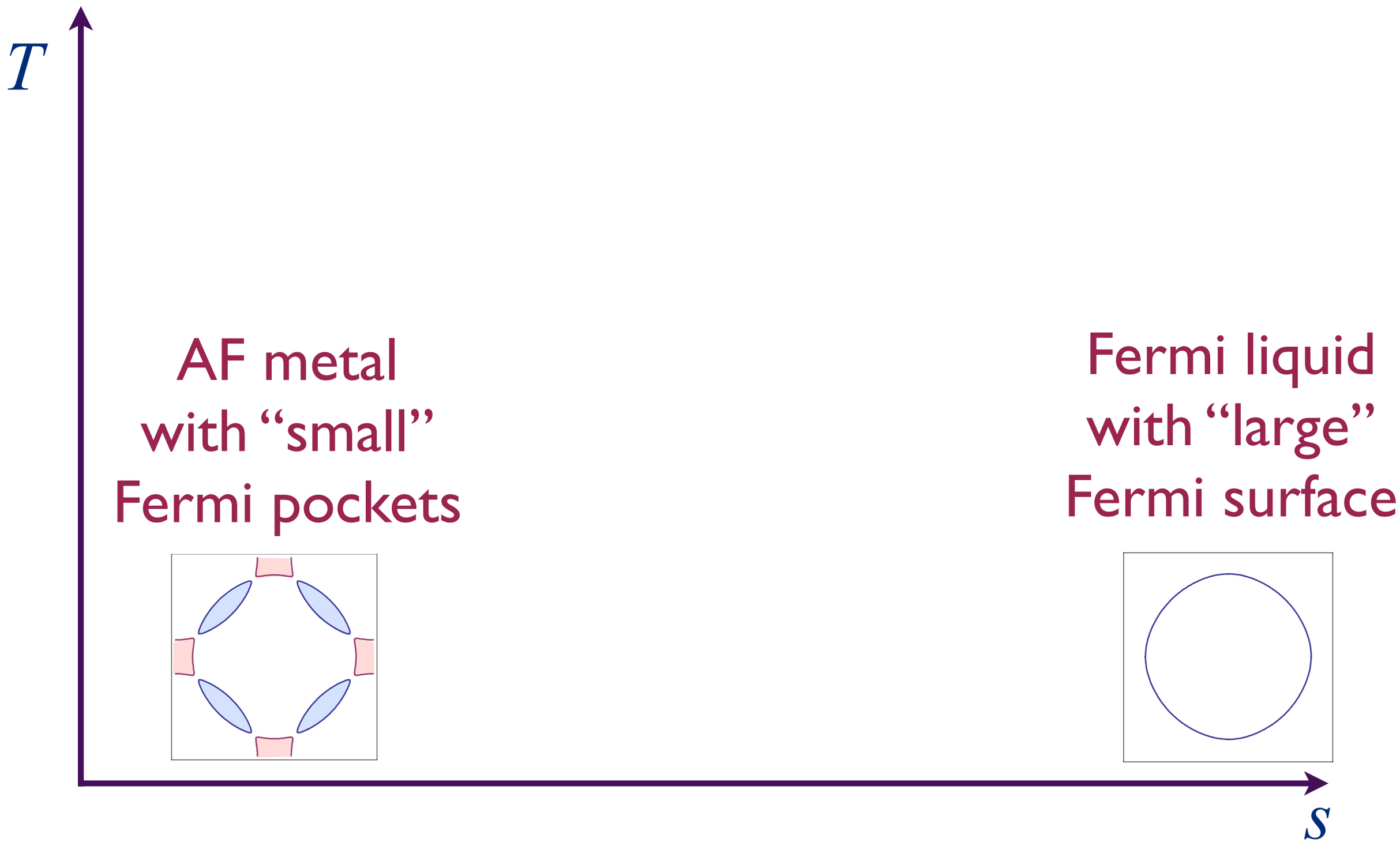
AF $\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi}$

Metal with electron and hole pockets

Metal with "large" Fermi surface

s


T
AF metal
with "small"
Fermi pockets
Fermi liquid
with "large"
Fermi surface
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D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
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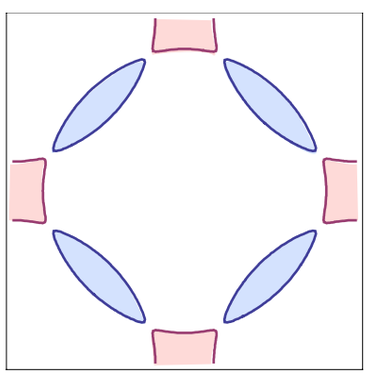
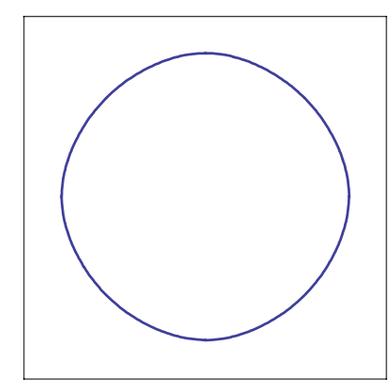


High temperature
Superconductor ✓

Fermi liquid
with "large"
Fermi surface



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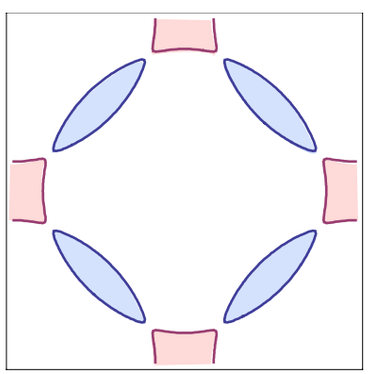

High temperature
Superconductor ✓

Fermi liquid
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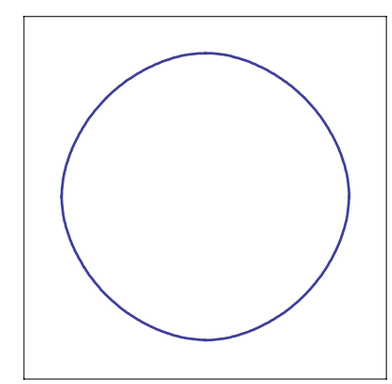

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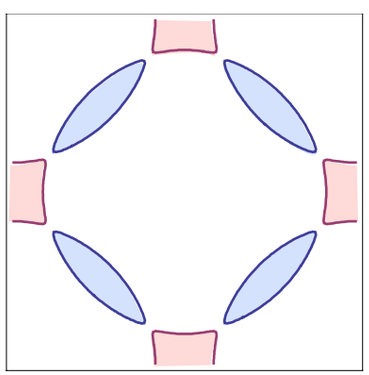
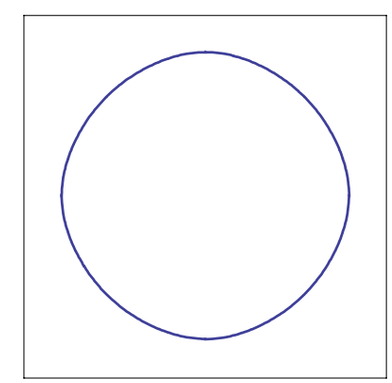
Strange Metal with long-range entanglement

AF metal with "small" Fermi pockets



High temperature Superconductor ✓

Fermi liquid with "large" Fermi surface



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Challenge to string theory:

Describe quantum critical points and phases of metallic systems

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Can we obtain holographic theories of superconductors and ordinary metals (Fermi liquids)?

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Can we obtain holographic theories of superconductors and ordinary metals (Fermi liquids)?

Yes

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Does holography yield metals other than ordinary metals ?

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Does holography yield metals other than ordinary metals ?

Yes, lots of them, with many "strange" properties !

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic "strange metals" have the correct type of long-range entanglement ?

Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic "strange metals" have the correct type of long-range entanglement ?

Yes, a very select subset has the proper logarithmic violation of the area law of entanglement entropy !!
These are now being studied intensively.....

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023; L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.

Conclusions

Simplest examples of long-range entanglement are at quantum-critical points of insulating antiferromagnets

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

Conclusions

String theory and holography offer a remarkable new approach to describing states with long-range quantum entanglement.

Conclusions

String theory and holography offer a remarkable new approach to describing states with long-range quantum entanglement.

Much recent progress offers hope of a holographic description of "strange metals"