Quantum matter and gauge-gravity duality

HRI, Allahabad
January 12, 2012

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PHYSICS
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HARVARD

1. The superfluid-insulator quantum phase transition

A. Field theory

B. Holography

2. Compressible quantum liquids

A. Field theory

B. Holography

1. The superfluid-insulator quantum phase transition

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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

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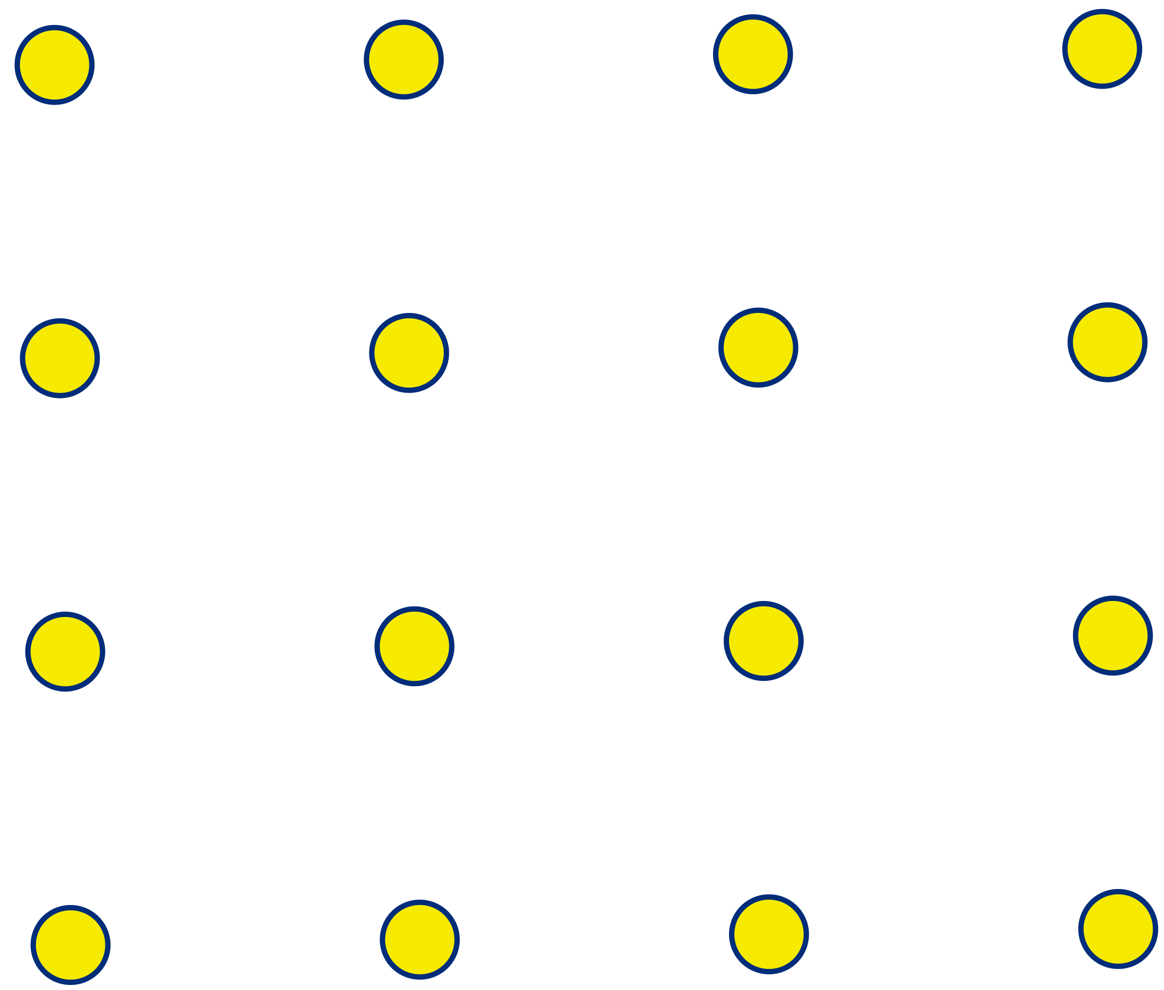
- "Relativistic" quantum critical systems are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

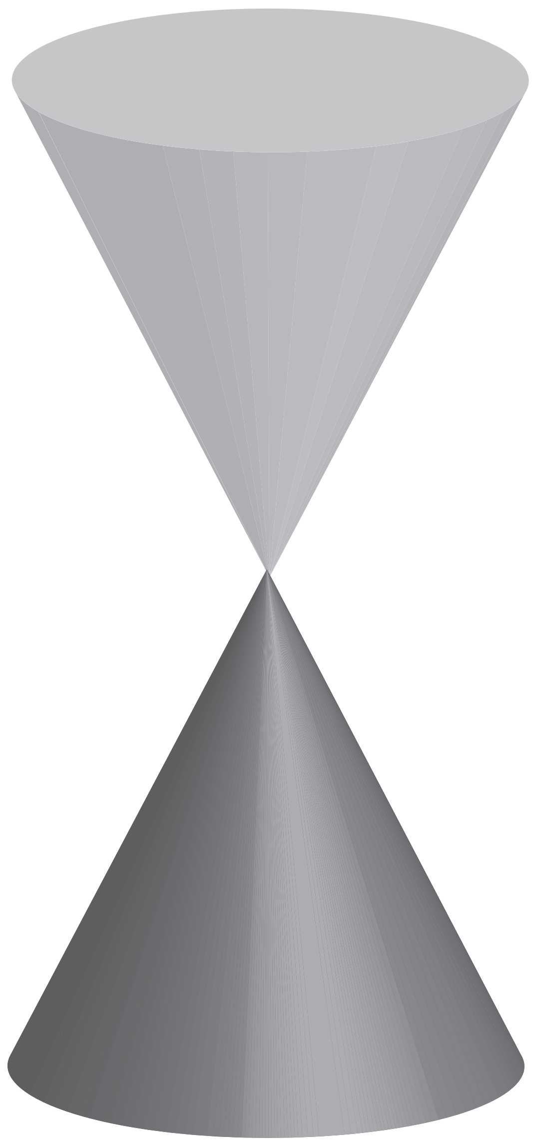
One compressible state is the **solid** (or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

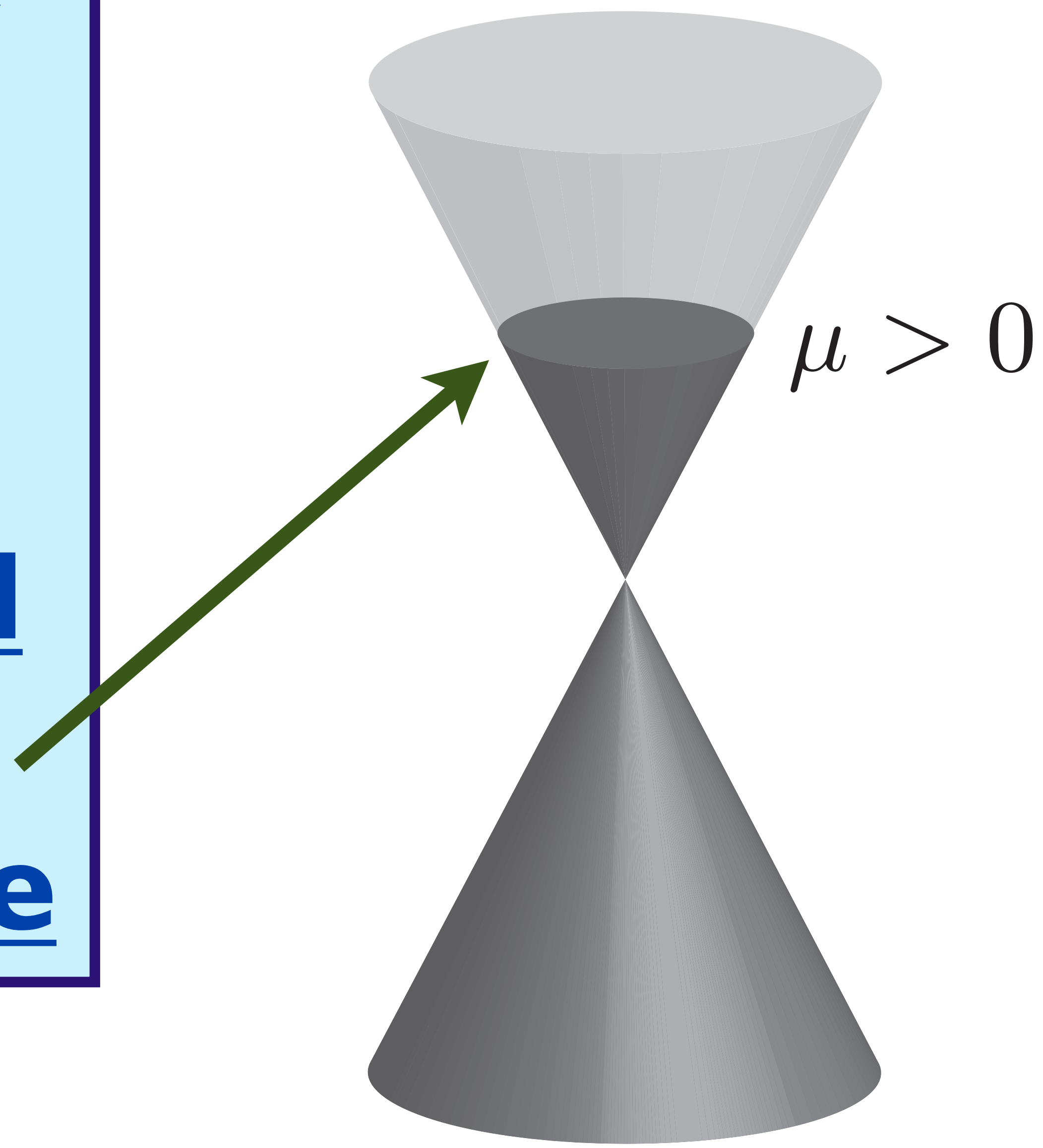
Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs



Graphene

The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**



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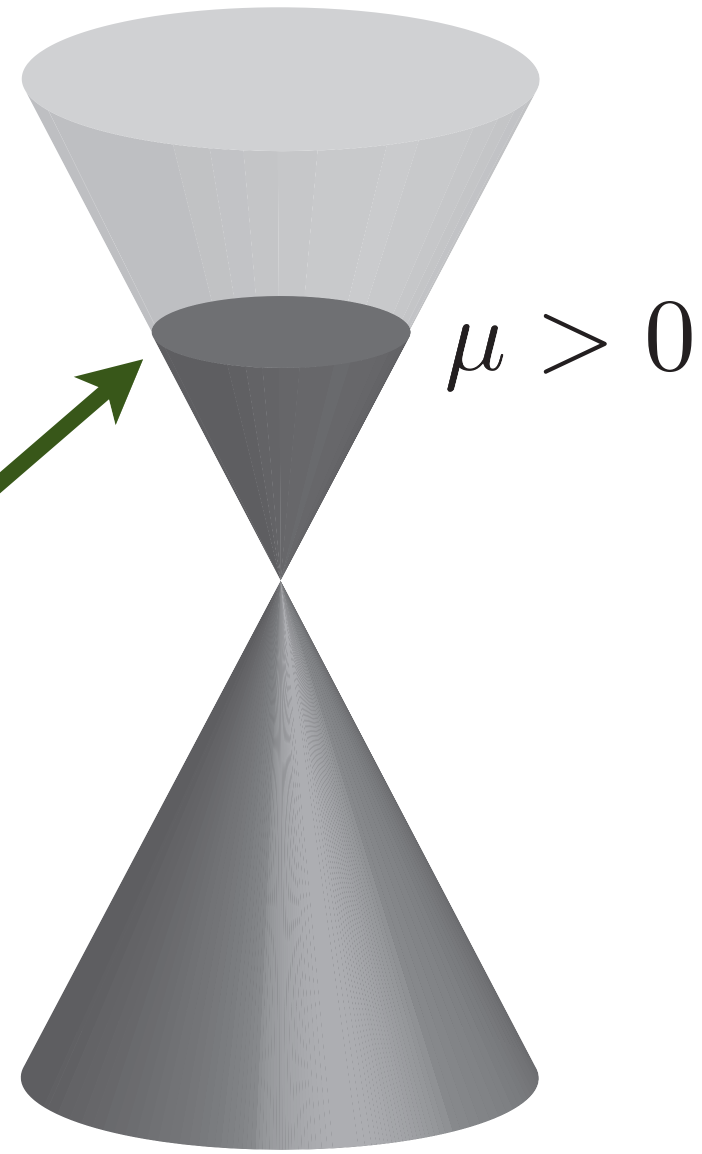

- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

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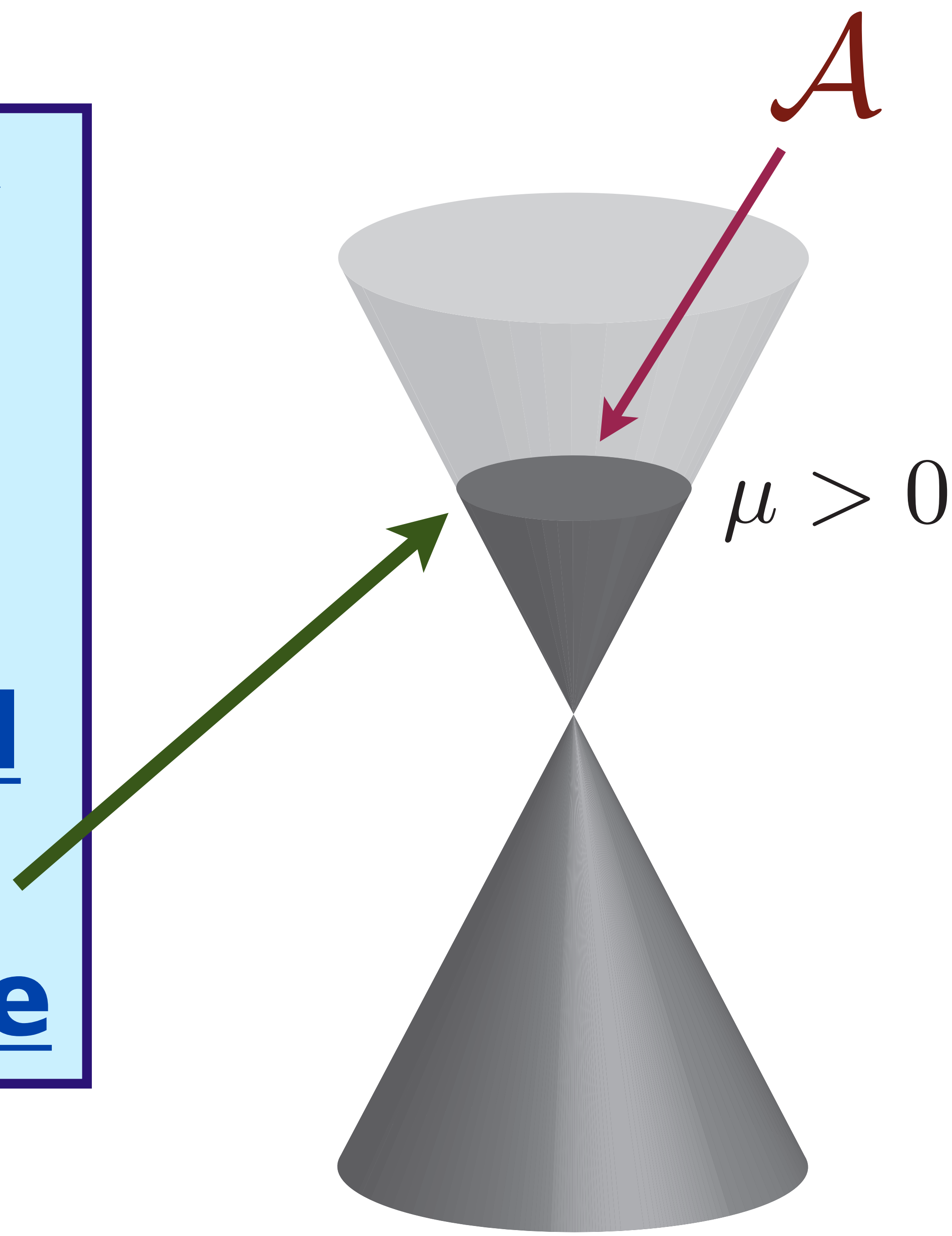

- **Luttinger relation:** The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$.

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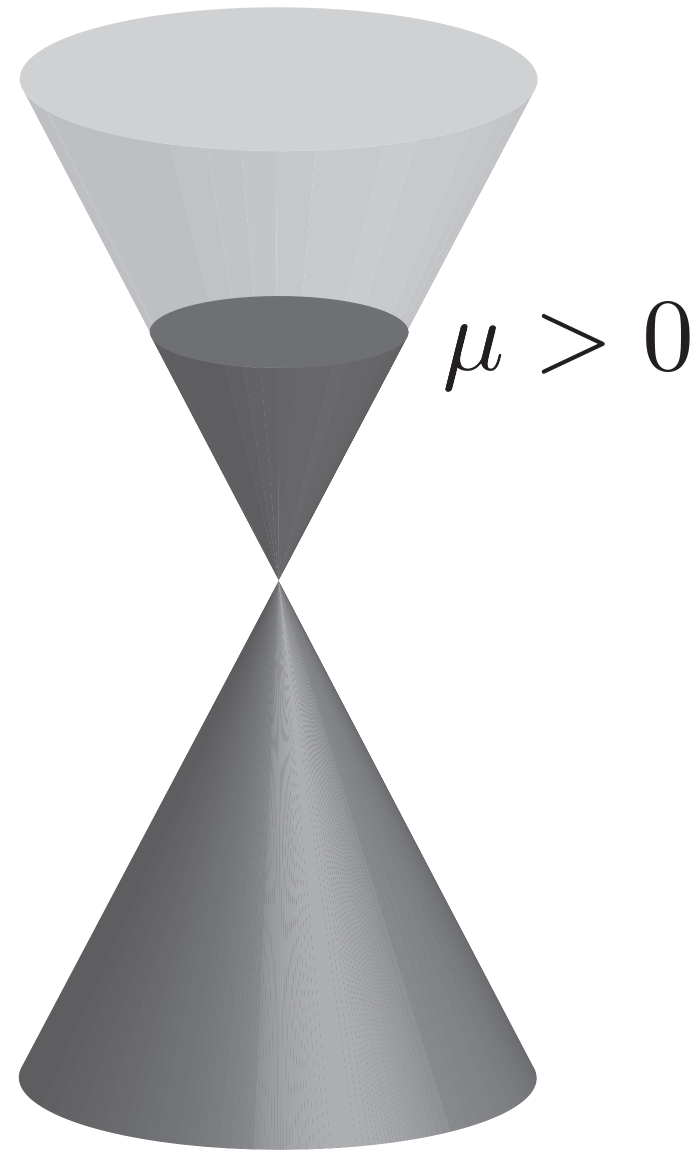
The Fermi Liquid (FL)



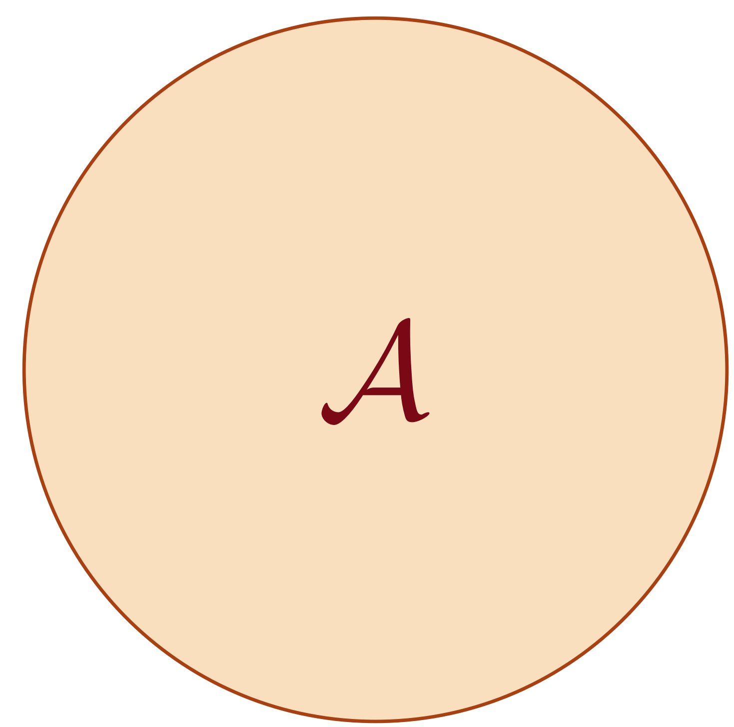
The Fermi Liquid (FL)

Most common example: electrons with short-range interactions (or screened long-range interactions), which are adiabatically connected to the non-interacting limit. The electron Green's function G_f has a pole which crosses zero energy at $k = k_F$, and the Fermi surface has the same area as the non-interacting case.

$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma + 4 \text{ Fermi terms}$$

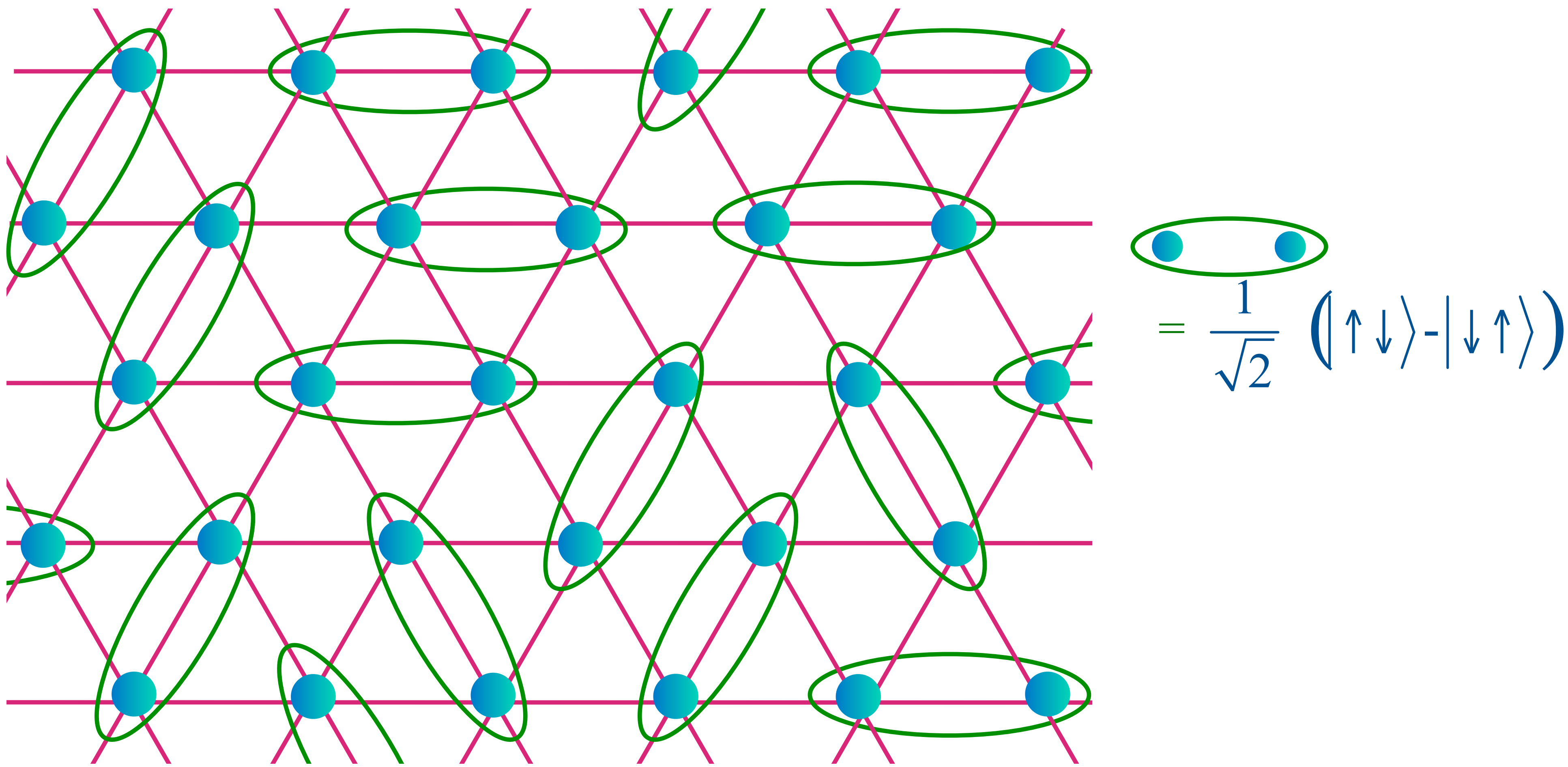
$$\mathcal{A} = \langle f_\sigma^\dagger f_\sigma \rangle = \langle \mathcal{Q}_\sigma \rangle$$

$$G_f = \frac{1}{\omega - v_F(k - k_F) + i\omega^2}$$



The Non-Fermi Liquid (NFL)

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field A_μ.



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

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- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field A_μ.

- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions*.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

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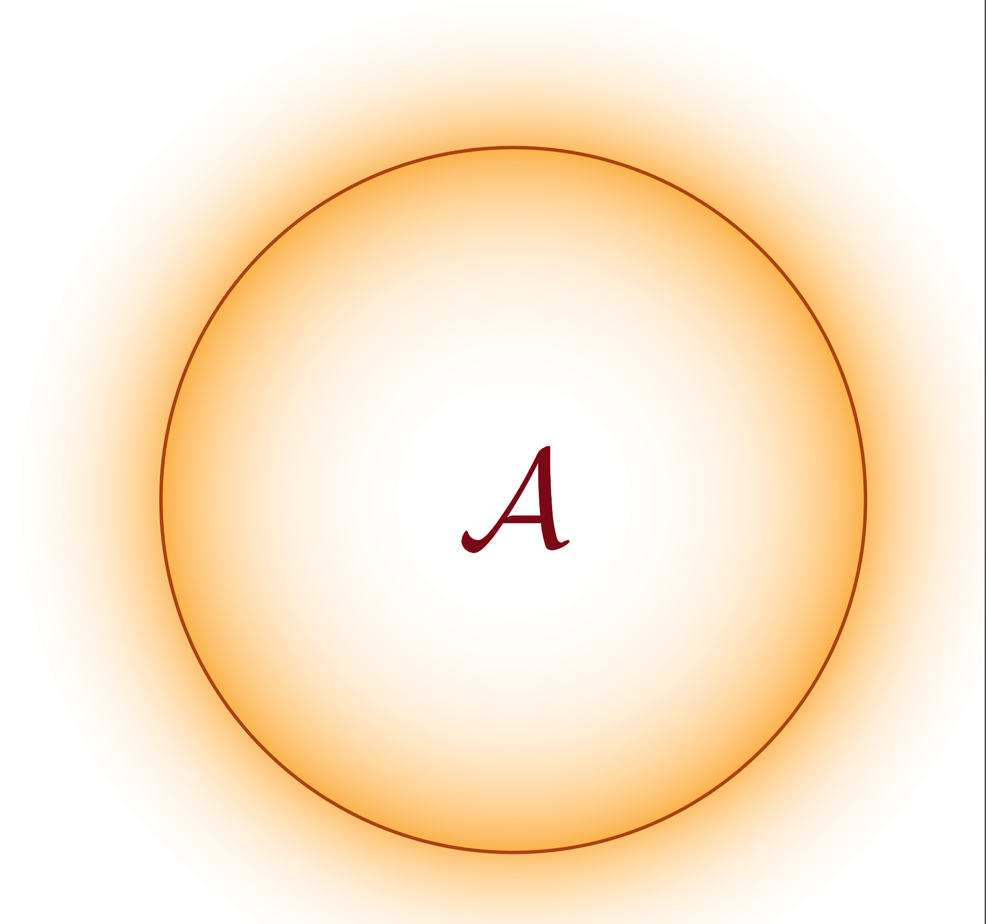
The Non-Fermi Liquid (NFL)

- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*



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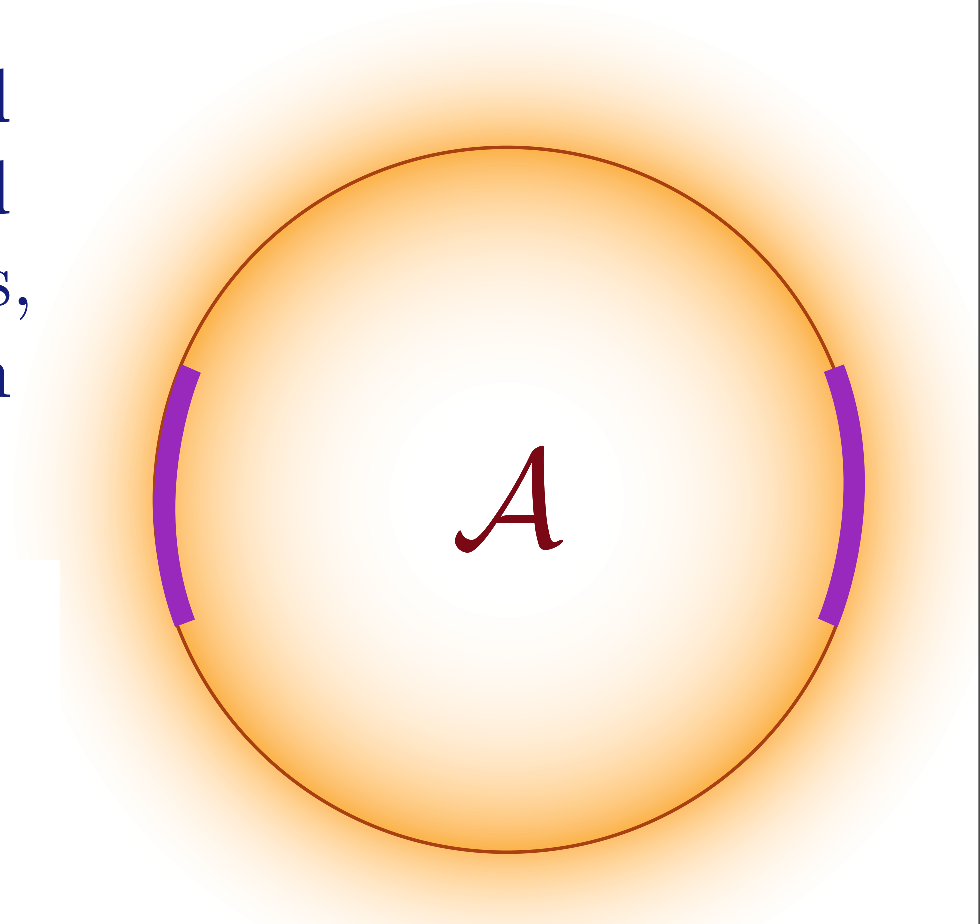
- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*
- Fluctuations near the Fermi surface are described by a strongly-coupled two-patch theory. Ward identities allow consistent matching of the patches, and patches along different directions decouple in the low energy limit.



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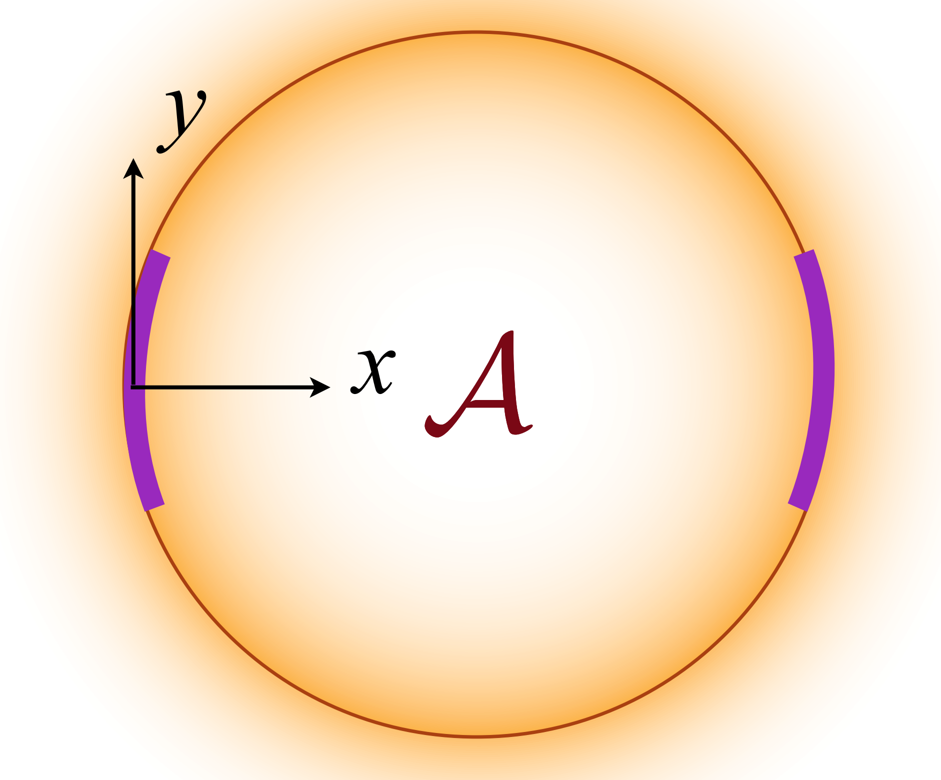
- Fluctuations near the Fermi surface are described by a strongly-coupled two-patch theory. Ward identities allow consistent matching of the patches, and patches along different directions decouple in the low energy limit.
- The singularity of the Green's function upon approaching the Fermi surface is described by the scaling form

$$G_f^{-1} = q^{1-\eta} F(\omega / q^z)$$

where $q_x = k_x - k_F$, $q_y = k_y$, and $q = q_x + q_y^2/(2k_F)$, and η and z are anomalous exponents. To three-loop order, we find $\eta \neq 0$ and $z = 3/2$.



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One-loop order: $G_f^{-1} \sim v_F q + i\omega^{2/3}$

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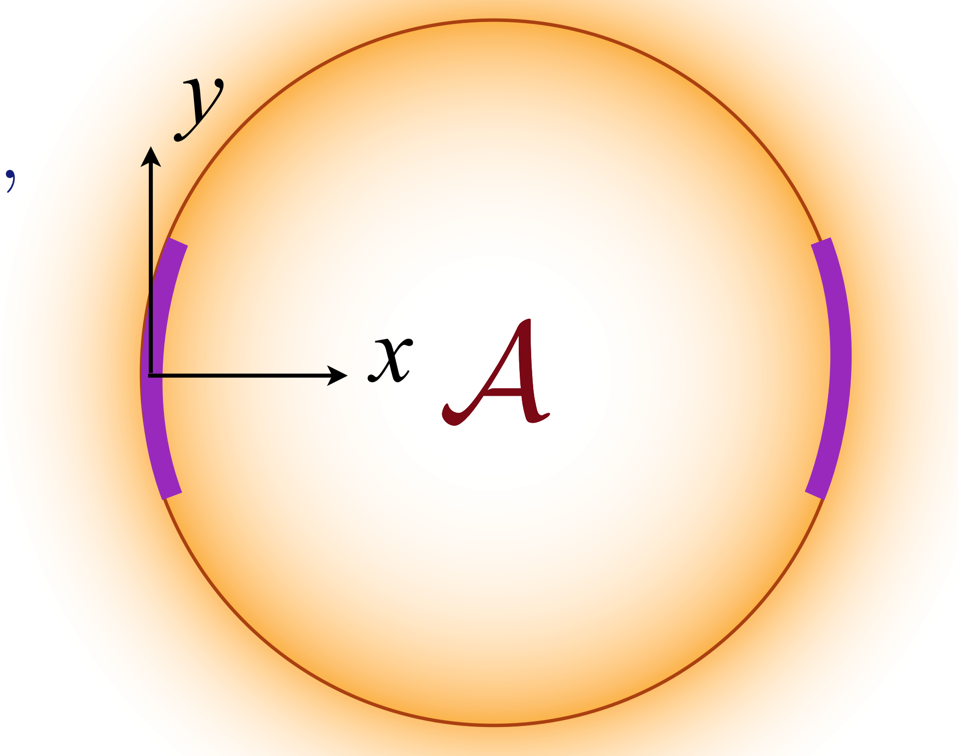

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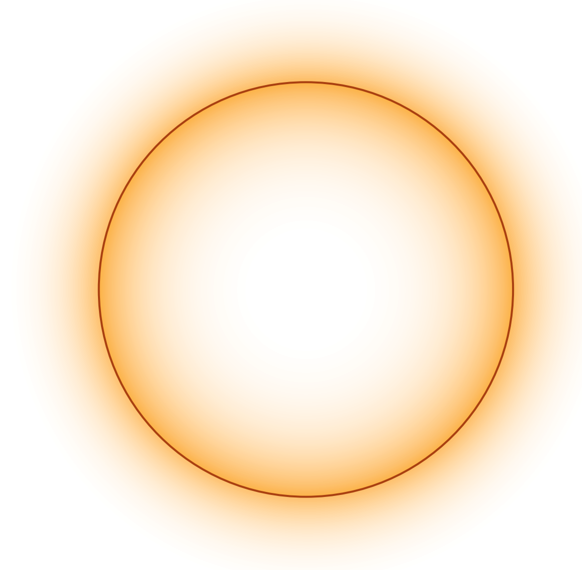
Key question:

How do we detect the
“hidden Fermi surfaces”
of fermions with gauge charges
in the non-Fermi liquid phases ?



These are not directly visible in the
gauge-invariant fermion correlations
computable via holography

One promising answer:

How do we detect the
"hidden Fermi surfaces"
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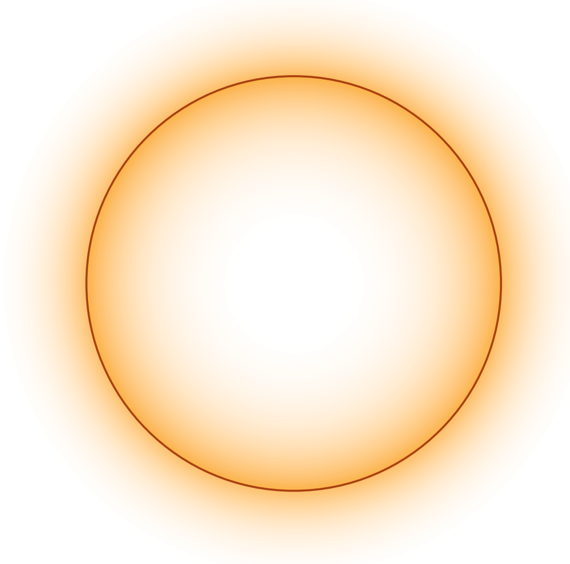
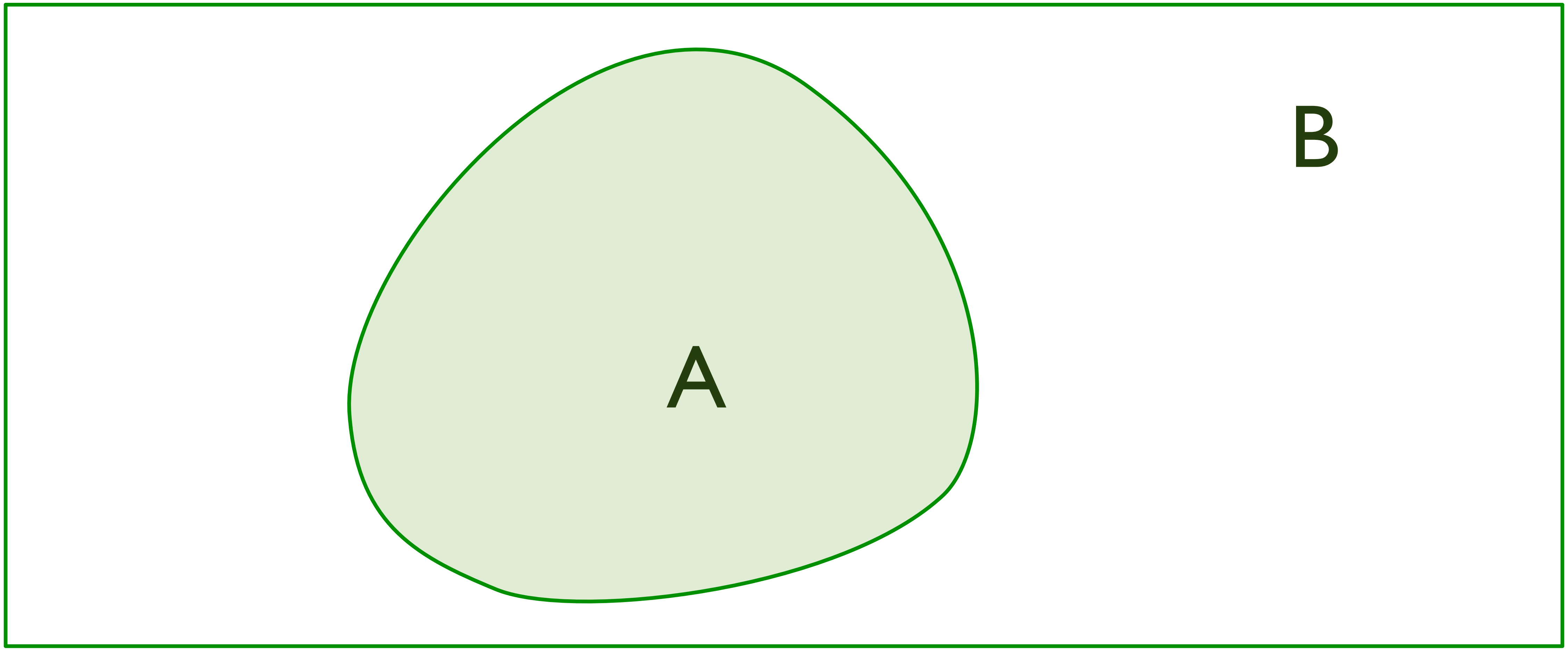


Compute
entanglement entropy

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, B. Swingle, and S. Sachdev arXiv:1112.0573

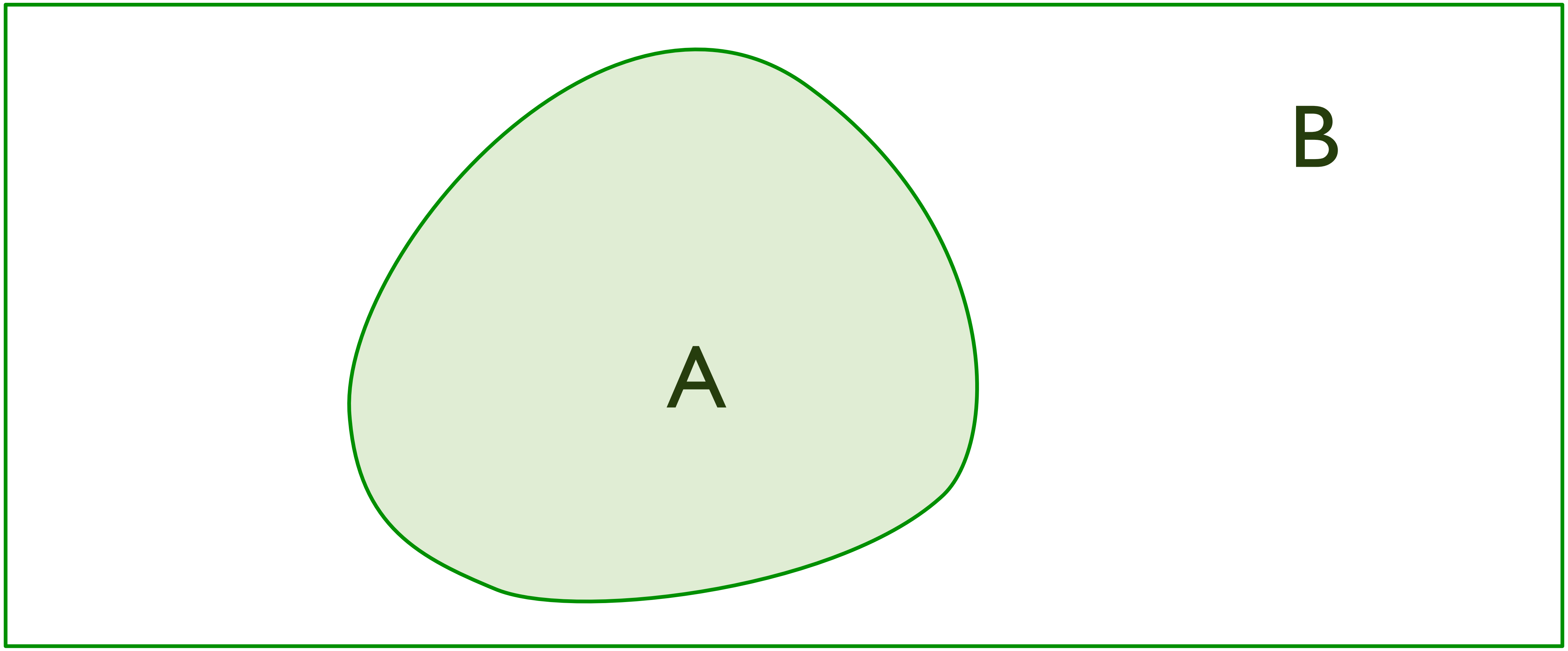
Entanglement entropy of Fermi surfaces



$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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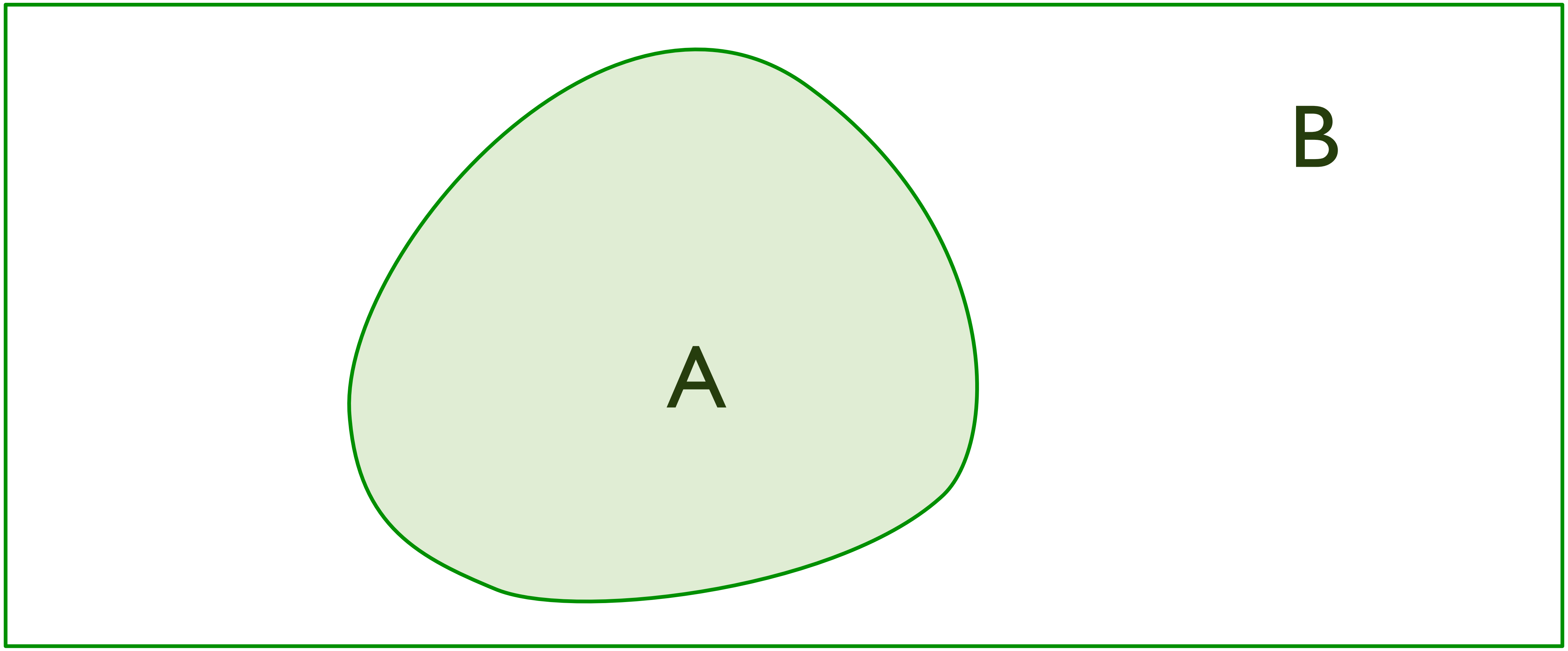

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Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

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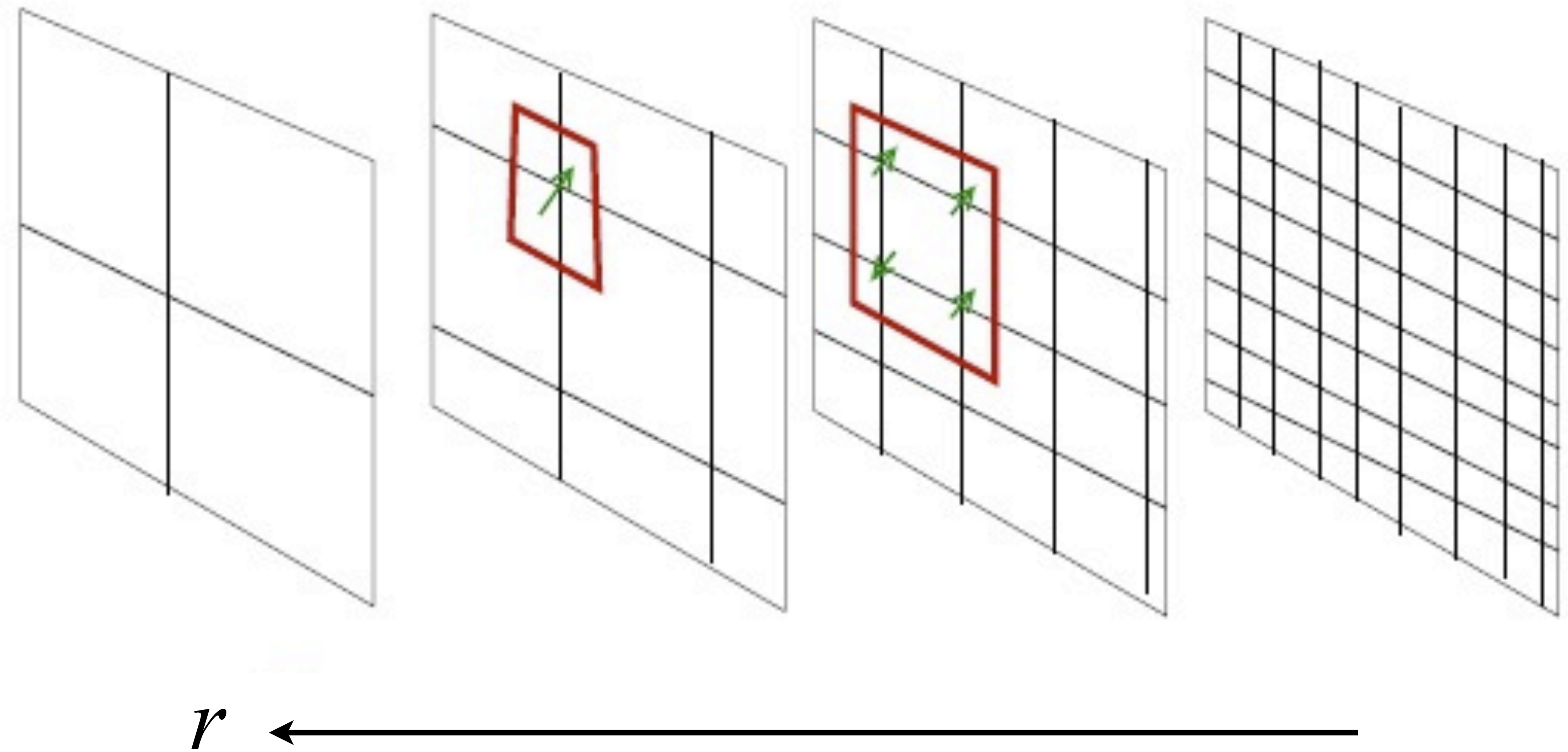

J. McGreevy, arXiv0909.0518

Consider the following (most) general metric for the holographic theory

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

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This metric transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

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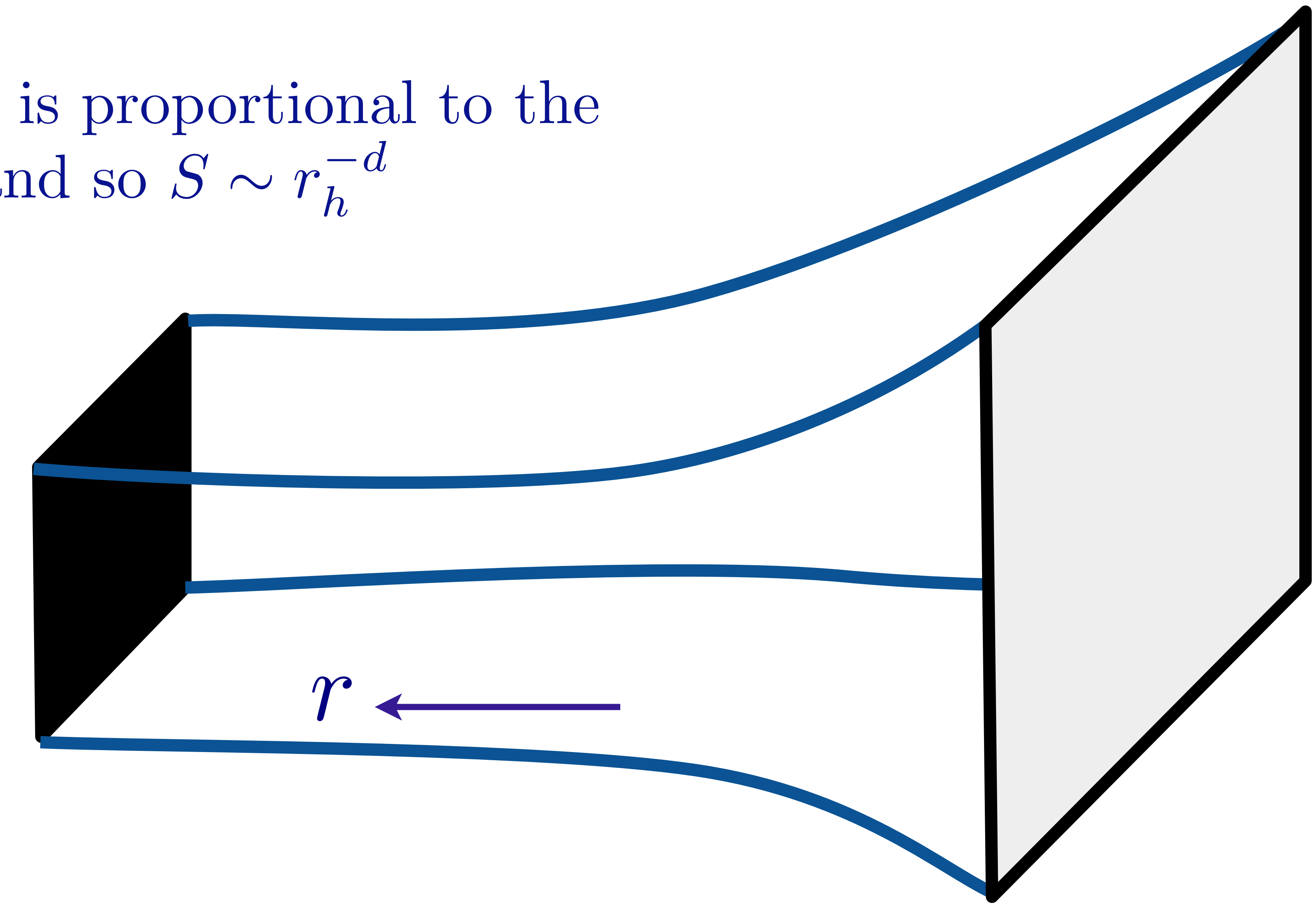
What is θ ? ($\theta = 0$ for "relativistic" quantum critical points).

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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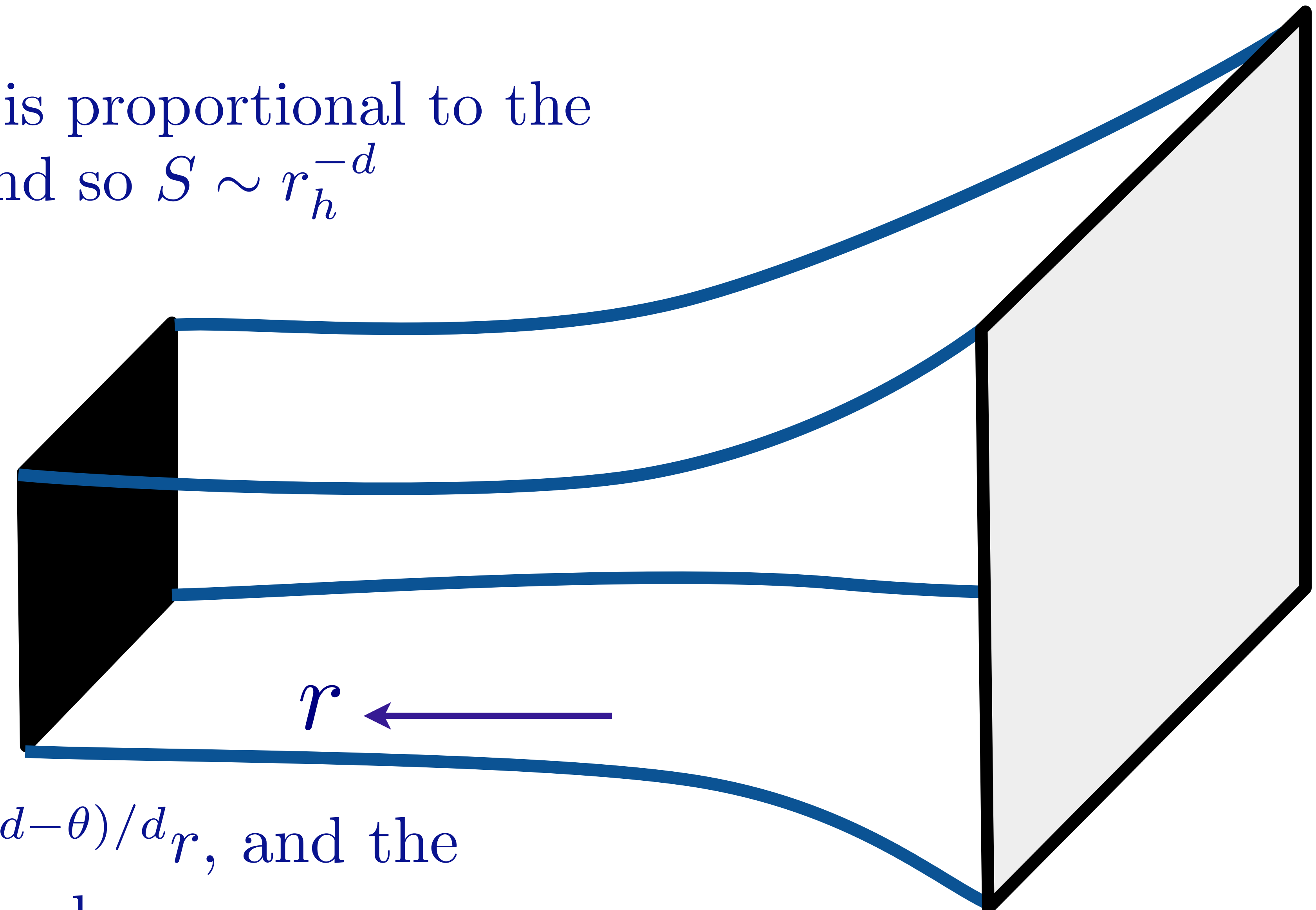

Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z}$$

So θ is the "violation of hyperscaling" exponent.

A non-Fermi liquid has gapless fermionic excitations on the Fermi surface, which disperse in the single transverse direction with dynamic critical exponent z. So we expect compressible quantum states to have an effective dimension $d - \theta$ with

$$\theta = d - 1$$

Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Entanglement entropy of the metric

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The area law is obeyed for

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For $\theta = d-1$, the value expected for compressible quantum states, the entanglement entropy has log-violation of the area law

$$S_E = \Xi\, \mathcal{Q}^{(d-1)/d}\Sigma\, \ln\left(\mathcal{Q}^{(d-1)/d}\Sigma\right).$$

- Σ is the $(d-1)$-dimensional surface area of entangling region (in $d=2$, $\Sigma = P$ is the perimeter). Note S_E is otherwise independent of the shape of the entangling region, unlike other gapless systems. This is a characteristic property of a Fermi surface

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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- The dependence of the entanglement entropy on the boundary charge density, $\mathcal{Q}$, is computed by realizing the metric in an Einstein-Maxwell-dilaton theory.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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- Ξ is a dimensionless constant which is *independent* of $\mathcal{Q}$ and of any property of the entangling region.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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- The metric has a complicated dependence on $\mathcal{Q}$, but S_E is just proportional to $\mathcal{Q}^{(d-1)/d}$. Many UV details are irrelevant, and S_E flows to the universal $\mathcal{Q}$ dependence in the IR. By Luttinger's relation $\mathcal{Q} \sim k_F^d$, and so the prefactor is the area of the Fermi surface, as expected from field theory.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Inequalities

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The area law of entanglement entropy is obeyed for

$$\theta \leq d - 1.$$

The "null energy condition" of the gravity theory yields

$$z \geq 1 + \frac{\theta}{d}.$$

Remarkably, for $d = 2$, $\theta = d - 1$ and $z = 1 + \theta/d$, we obtain $z = 3/2$, the same value associated with the field theory.

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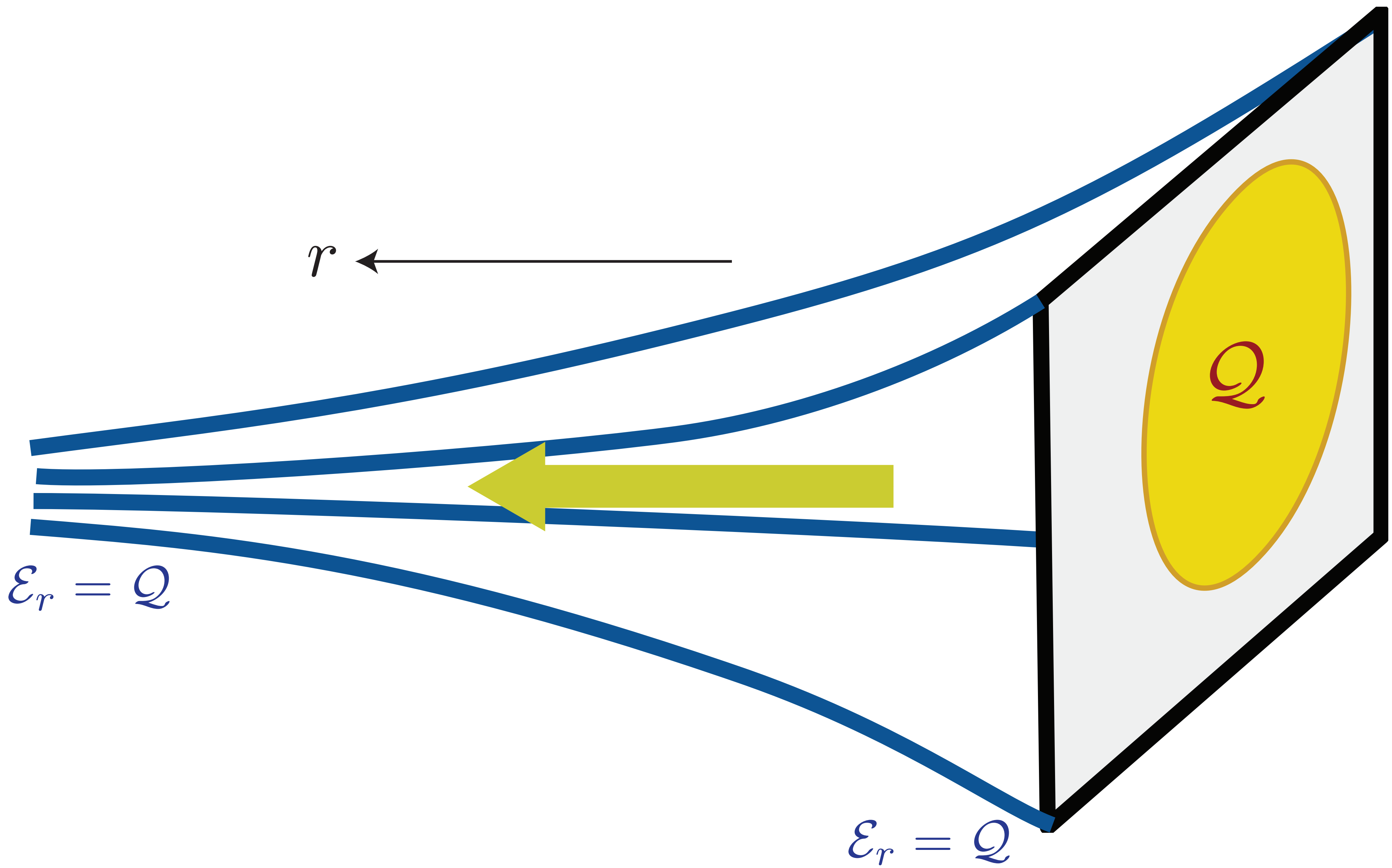
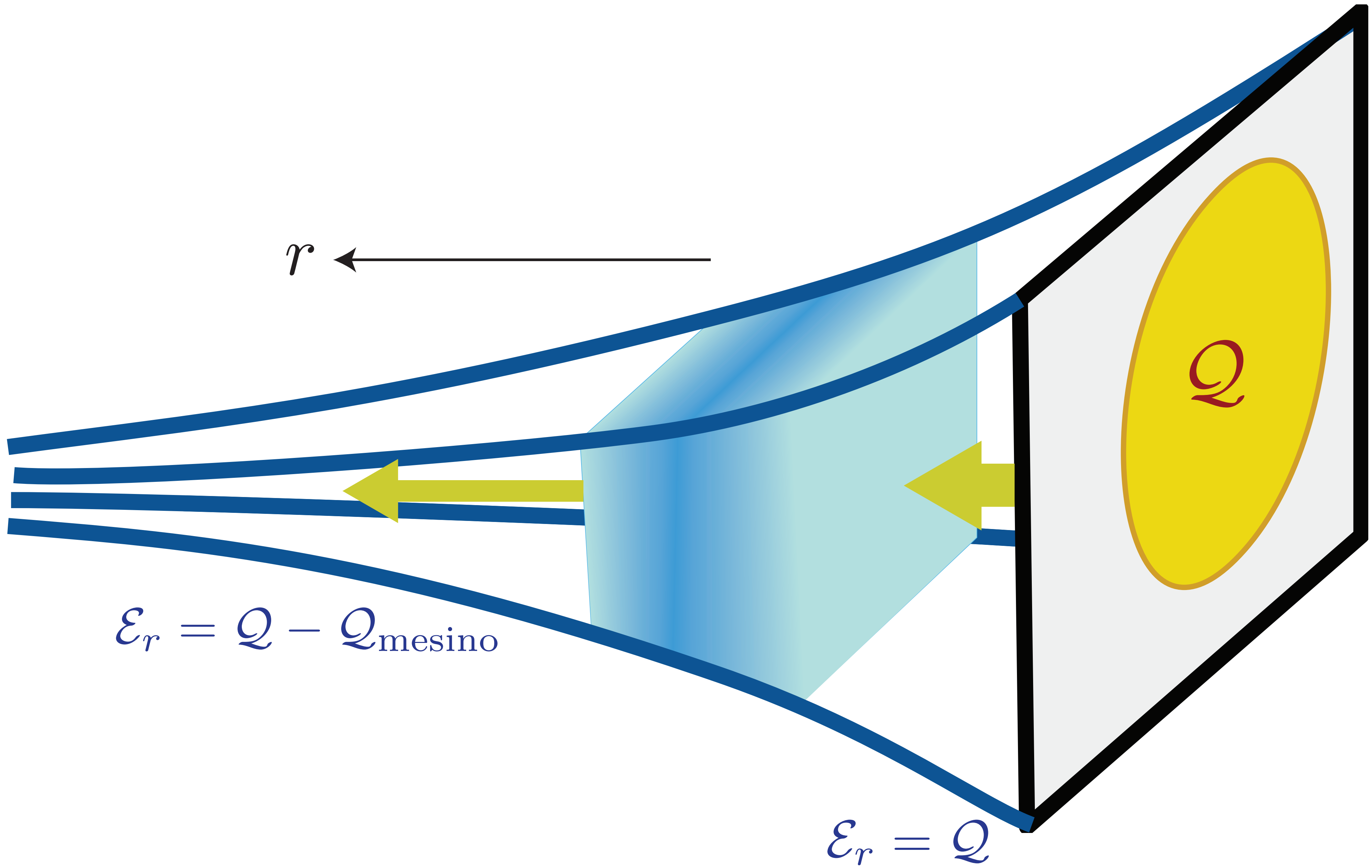
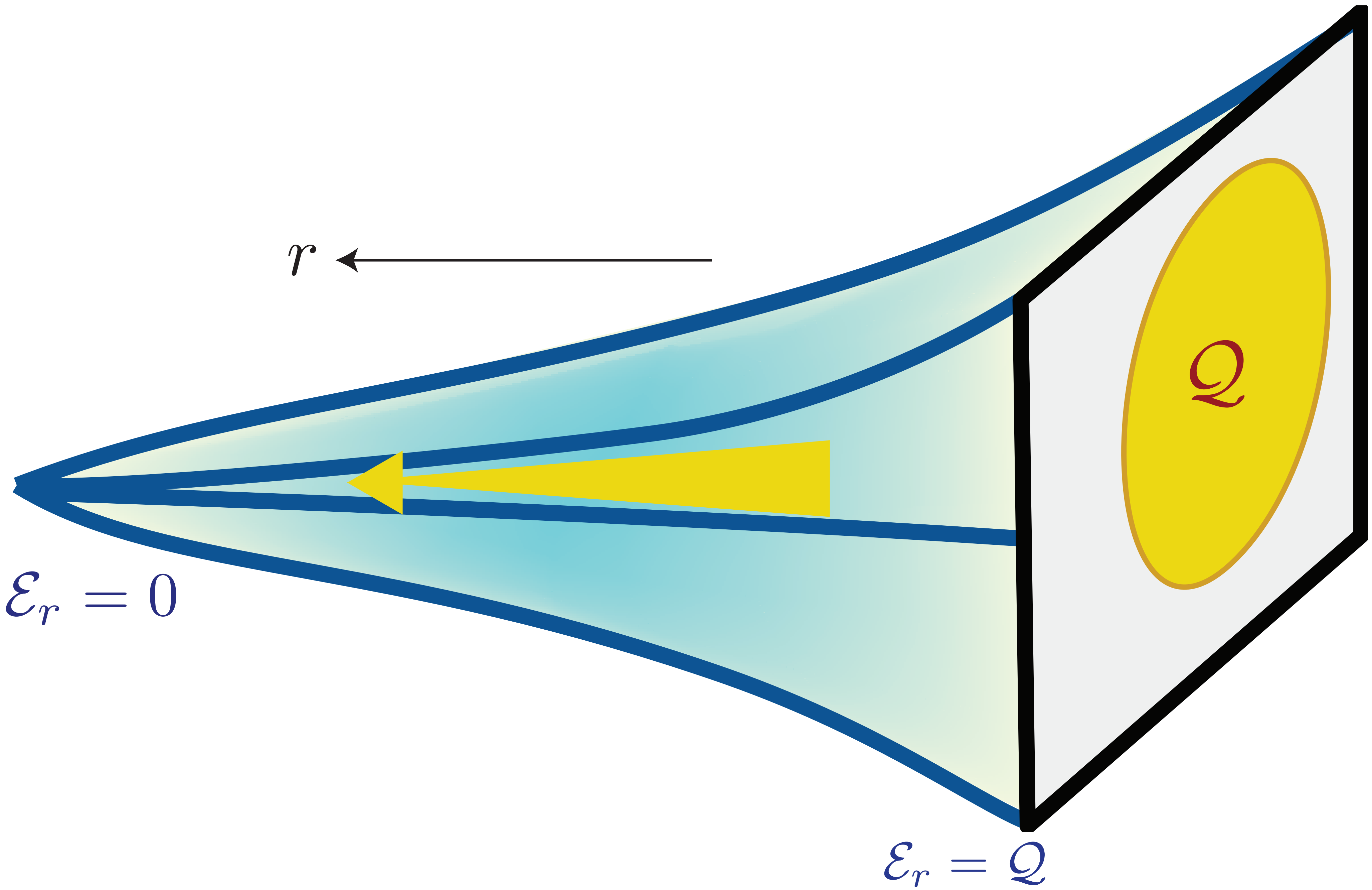
Holographic theory of a non-Fermi liquid (NFL)



Holographic theory of a fractionalized-Fermi liquid (FL*)



Holographic theory of a Fermi liquid (FL)



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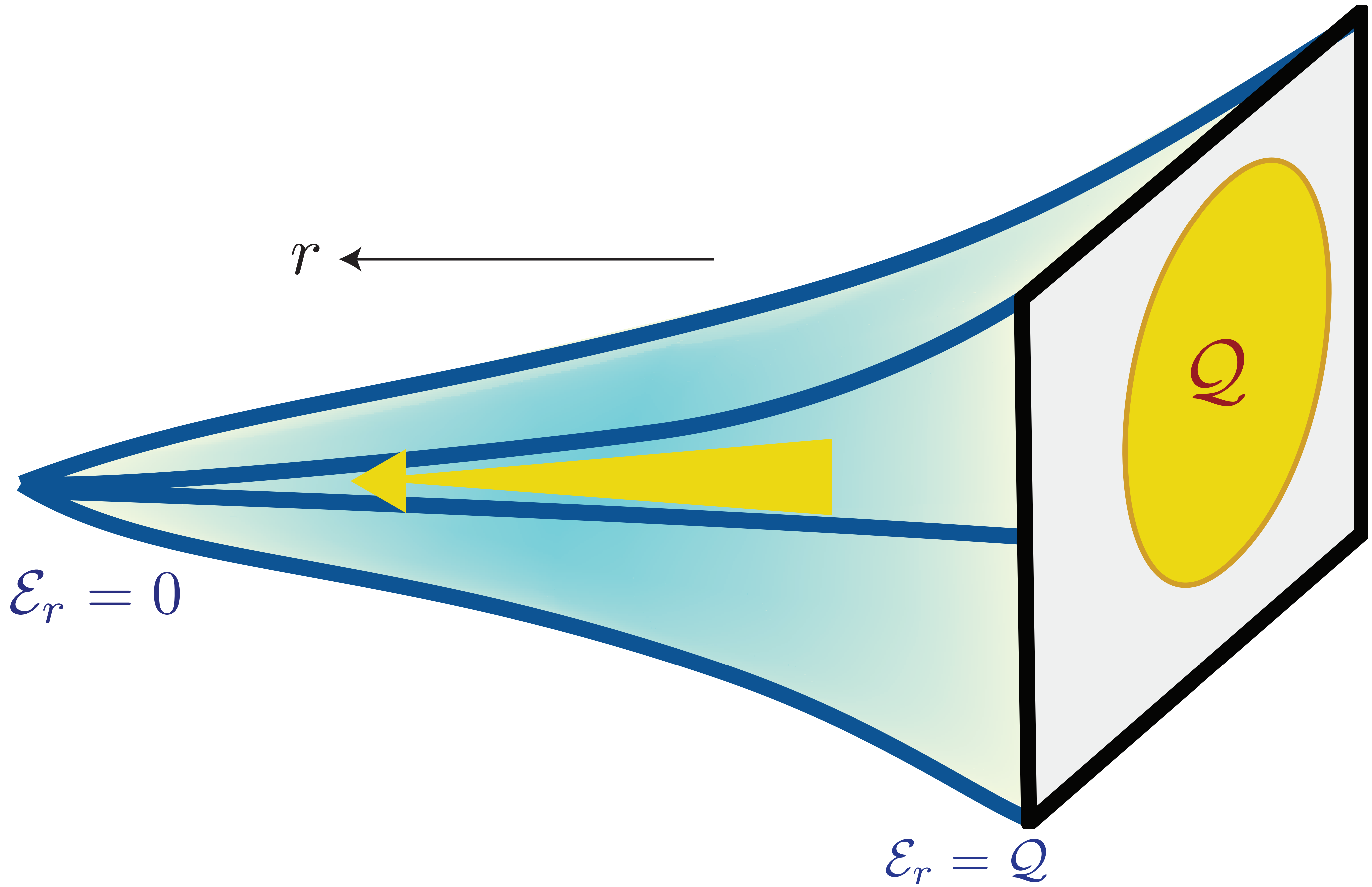

Gauss Law in the bulk

$\Leftrightarrow$ Luttinger theorem on the boundary

Theory of a non-Fermi liquid (NFL)

Field theory	Holography
A gauge-dependent Fermi surface of overdamped gapless fermions.	Fermi surface is hidden.

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Thermal entropy density $S \sim T^{1/z}$ in $d = 2$, where z is the dynamic critical exponent.	Thermal entropy density $S \sim T^{1/z}$ in all d for hyperscaling violation exponent $\theta = d - 1$, and z the dynamic critical exponent.

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Thermal entropy density $S \sim T^{1/z}$ in $d = 2$, where z is the dynamic critical exponent.	Thermal entropy density $S \sim T^{1/z}$ in all d for hyperscaling violation exponent $\theta = d - 1$, and z the dynamic critical exponent.
Logarithmic violation of area law of entanglement entropy, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.	Logarithmic violation of area law of entanglement entropy for $\theta = d - 1$, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.

Theory of a non-Fermi liquid (NFL)

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Gauge neutral 'mesinos' reduce the volume enclosed by Fermi surfaces of gauge-charged fermions to $\mathcal{Q} - \mathcal{Q}_{\rm mesino}$.	Gauge neutral 'mesinos' reduce the volume enclosed by hidden Fermi surfaces to $\mathcal{Q} - \mathcal{Q}_{\rm mesino}$.

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- "Visible" Fermi surfaces of gauge-neutral particles are realized are realized in a confining metric.