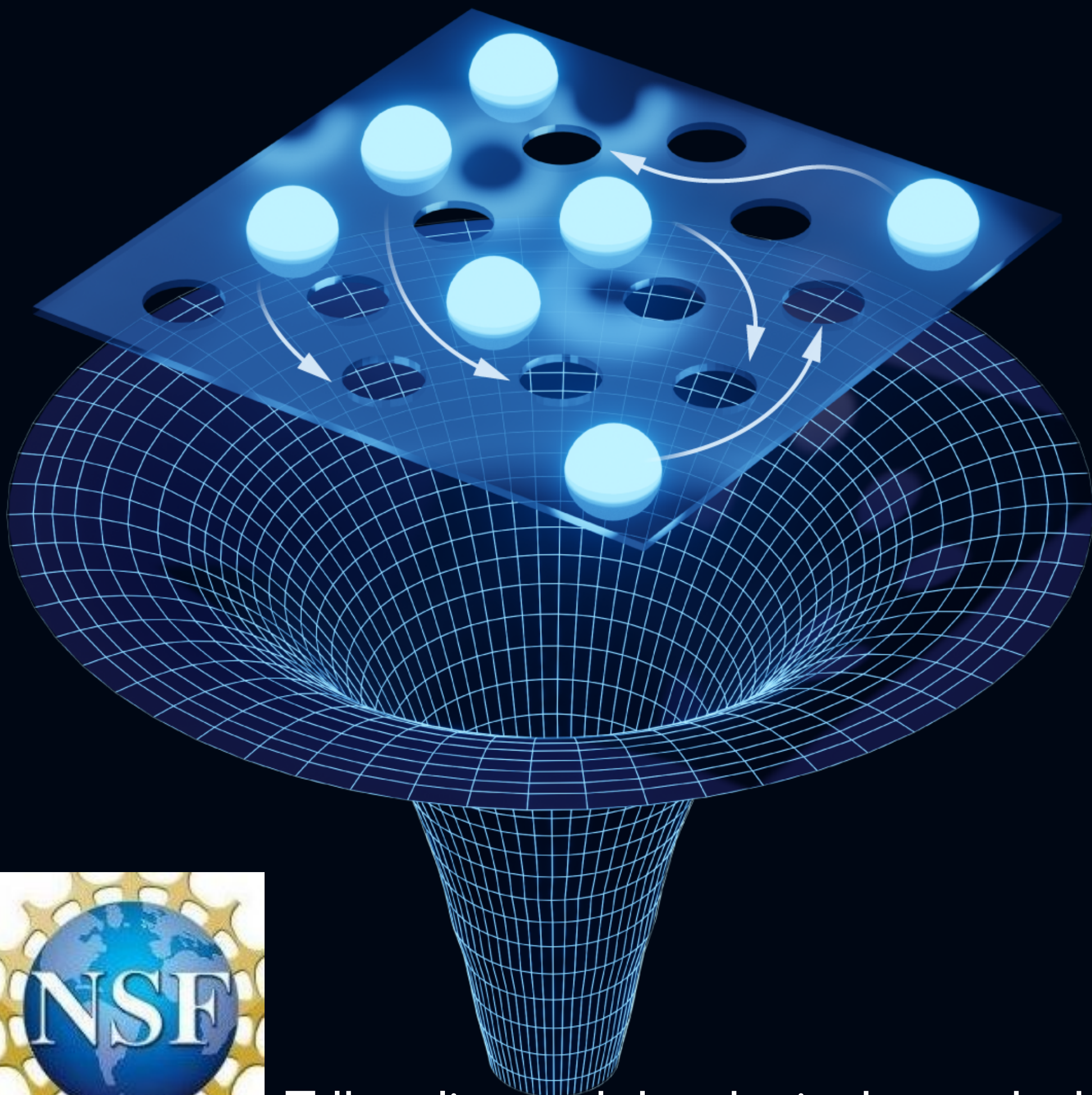


# When nature entangles millions of particles: from quantum materials to black holes



Hong Kong University,  
March 14, 2023

Subir Sachdev

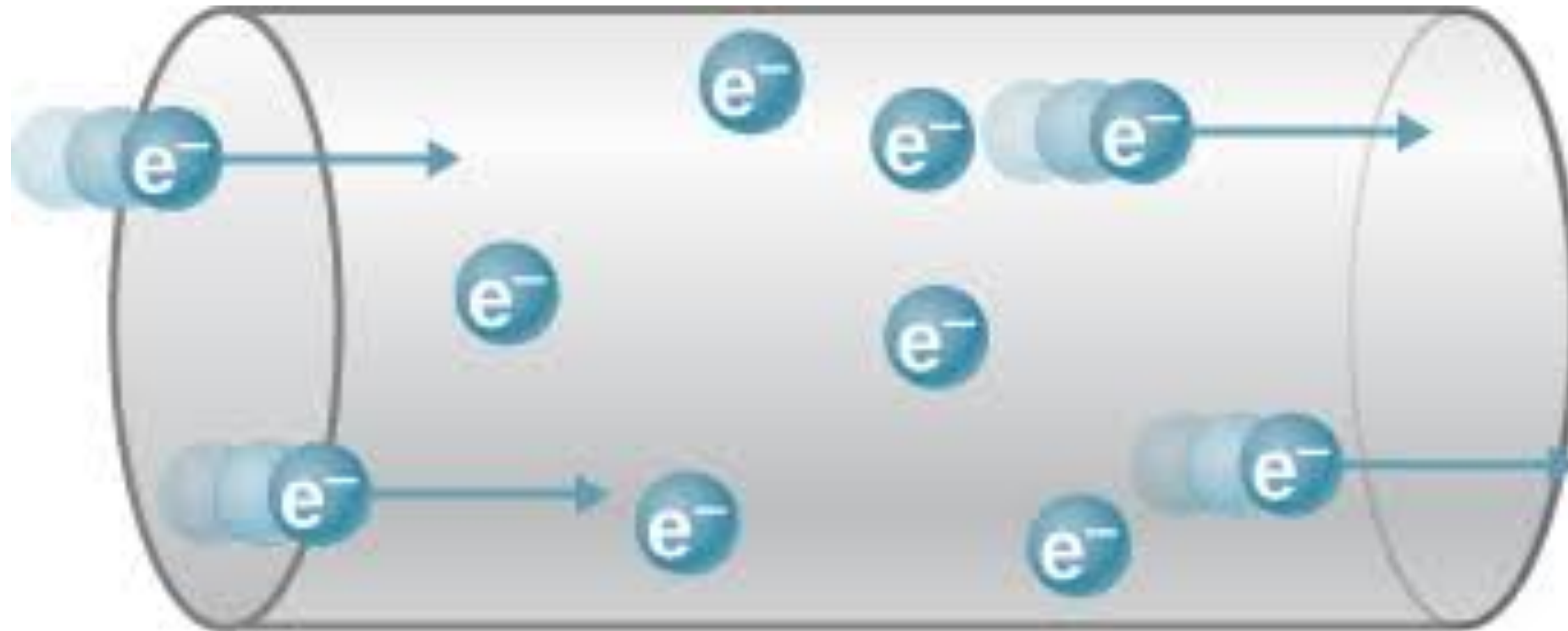


Talk online: [sachdev.physics.harvard.edu](https://sachdev.physics.harvard.edu)



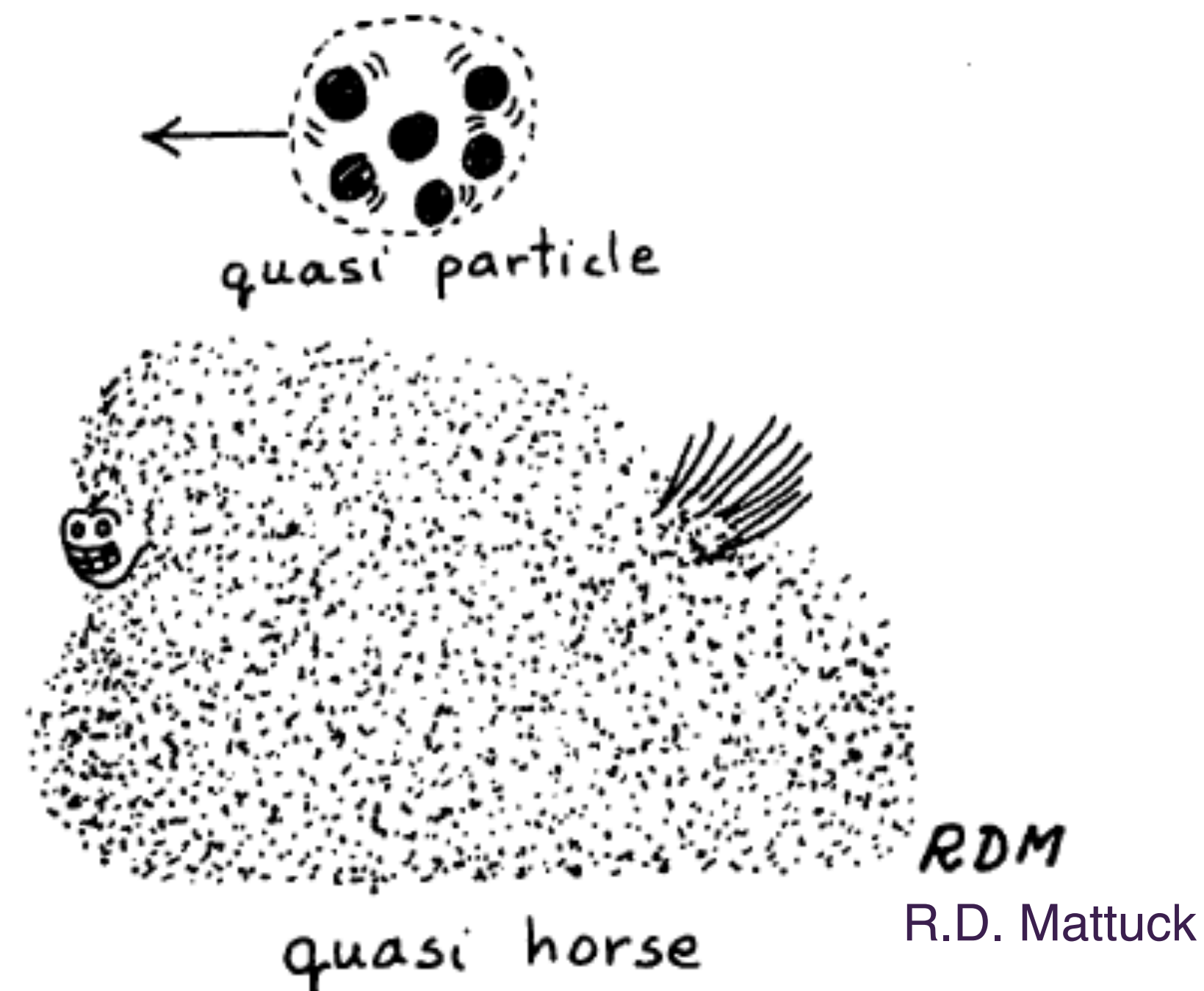
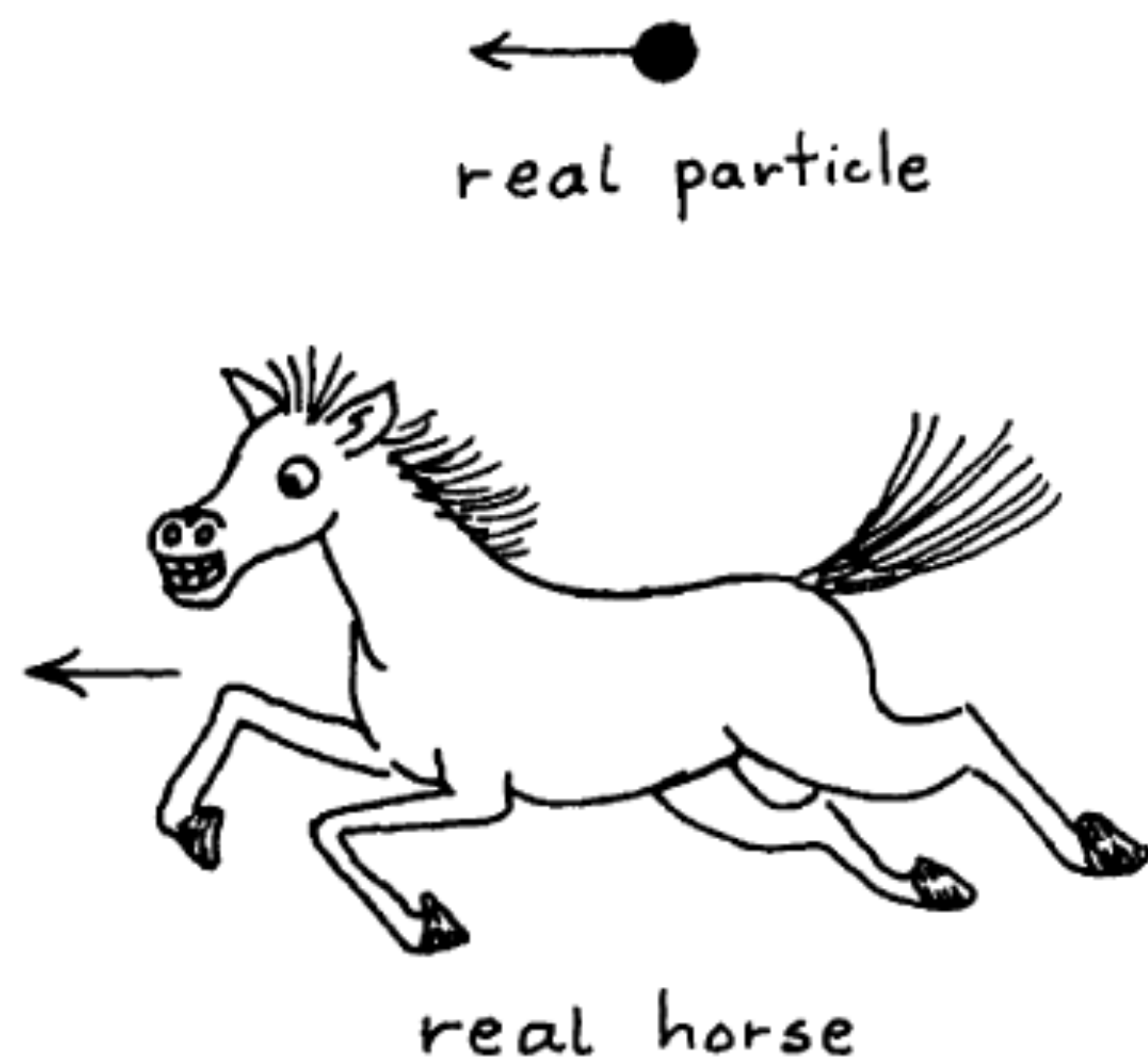
Quantum matter:  
ordinary metals,  
and  
strange metals

*Copper: ordinary metal quantum matter*



*Almost all many-electron systems are described by the quasiparticle concept:*

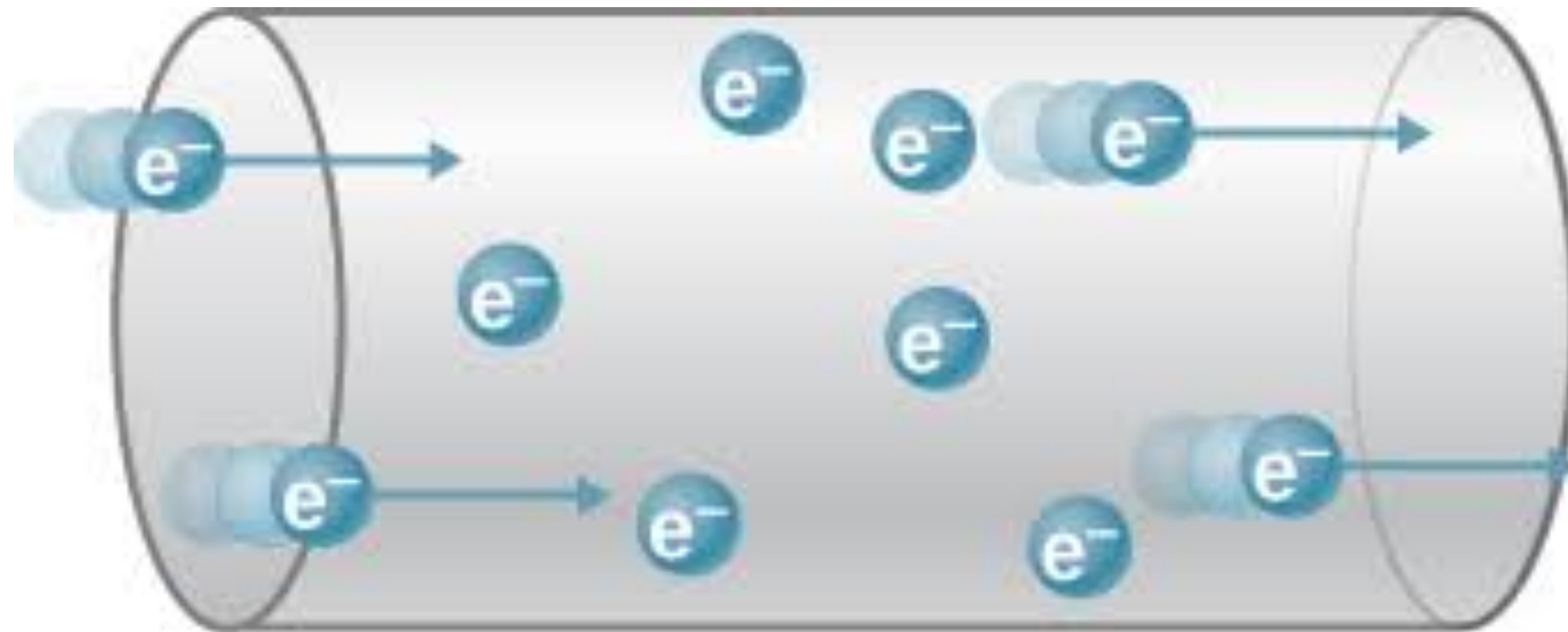
*a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.*



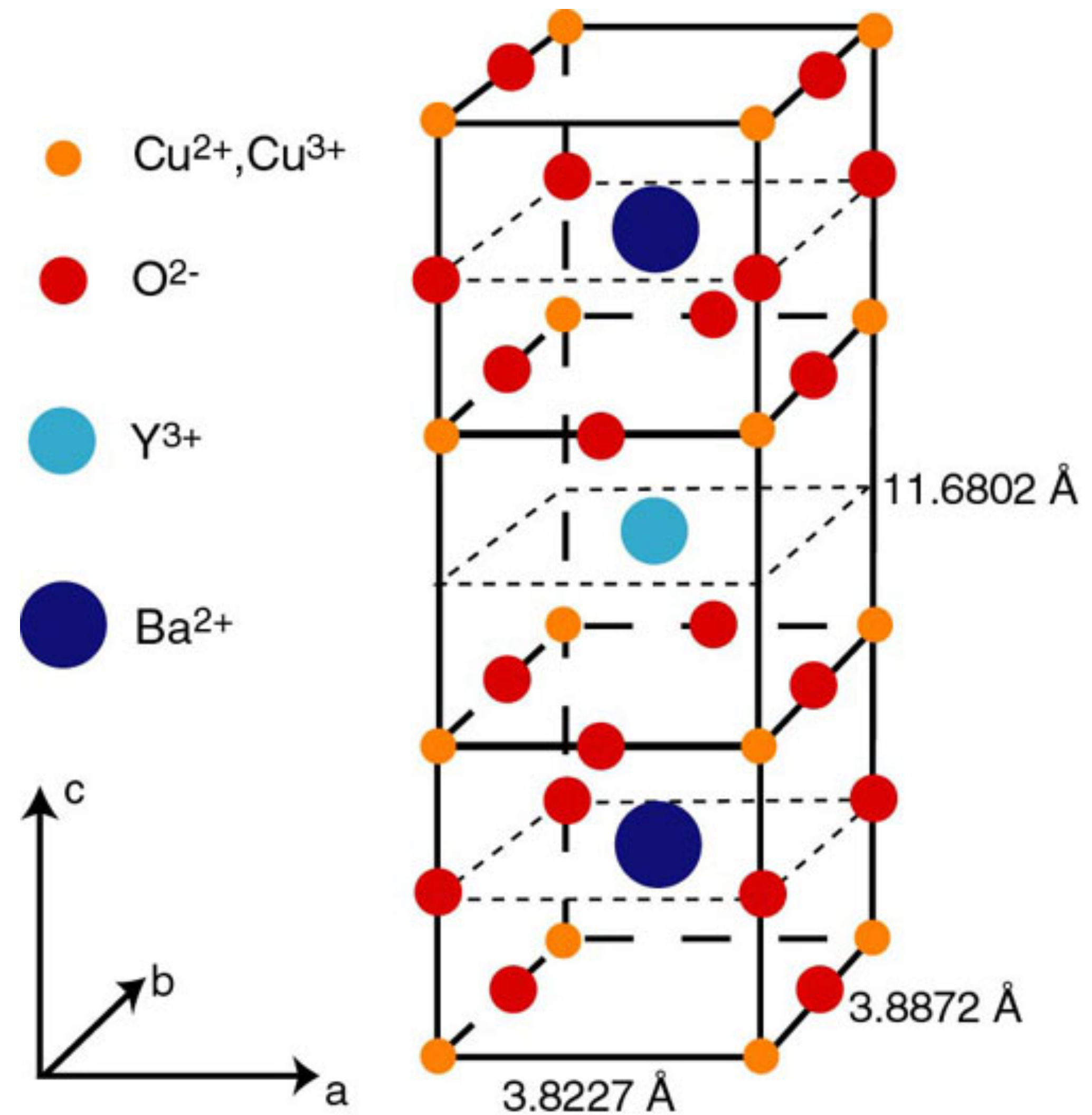
RDM

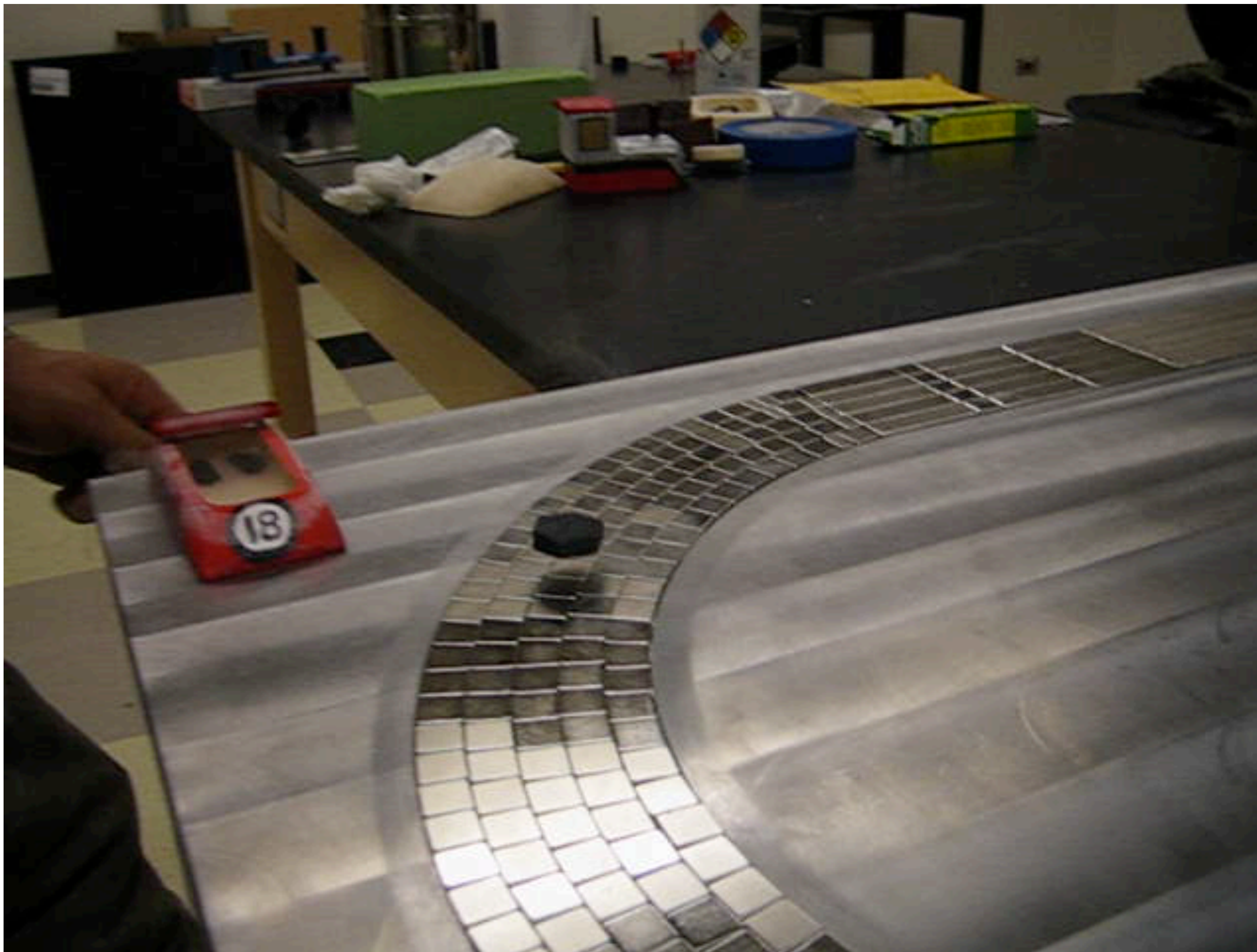
R.D. Mattuck

# *Current flow with quasiparticles in Copper*



# Cuprate high temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

# HTS Magnets: Enabling Technology

A new high temperature superconductor (HTS) recently reached industrial maturity: Rare Earth Barium Copper Oxide (REBCO). CFS is using HTS and has built its first-of-its-kind high-field large-bore superconducting magnet. HTS

magnets will allow for smaller, faster, and less expensive tokamaks using the science developed on Alcator C-Mod and other tokamaks.

## The surest path to limitless, clean, fusion energy

### ● Surest

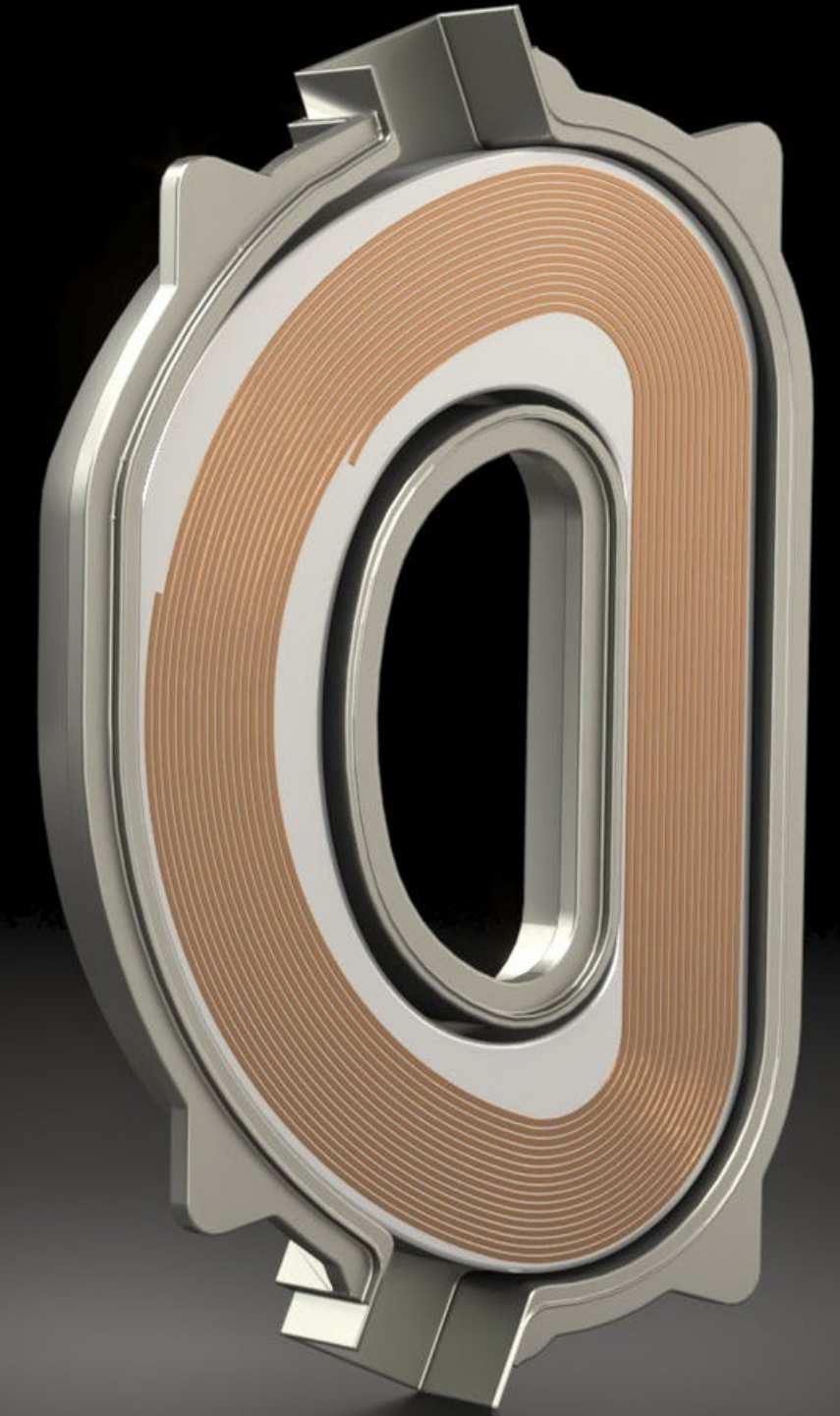
The fastest path to commercial fusion energy combining proven science with revolutionary magnet technology.

### ○ Limitless

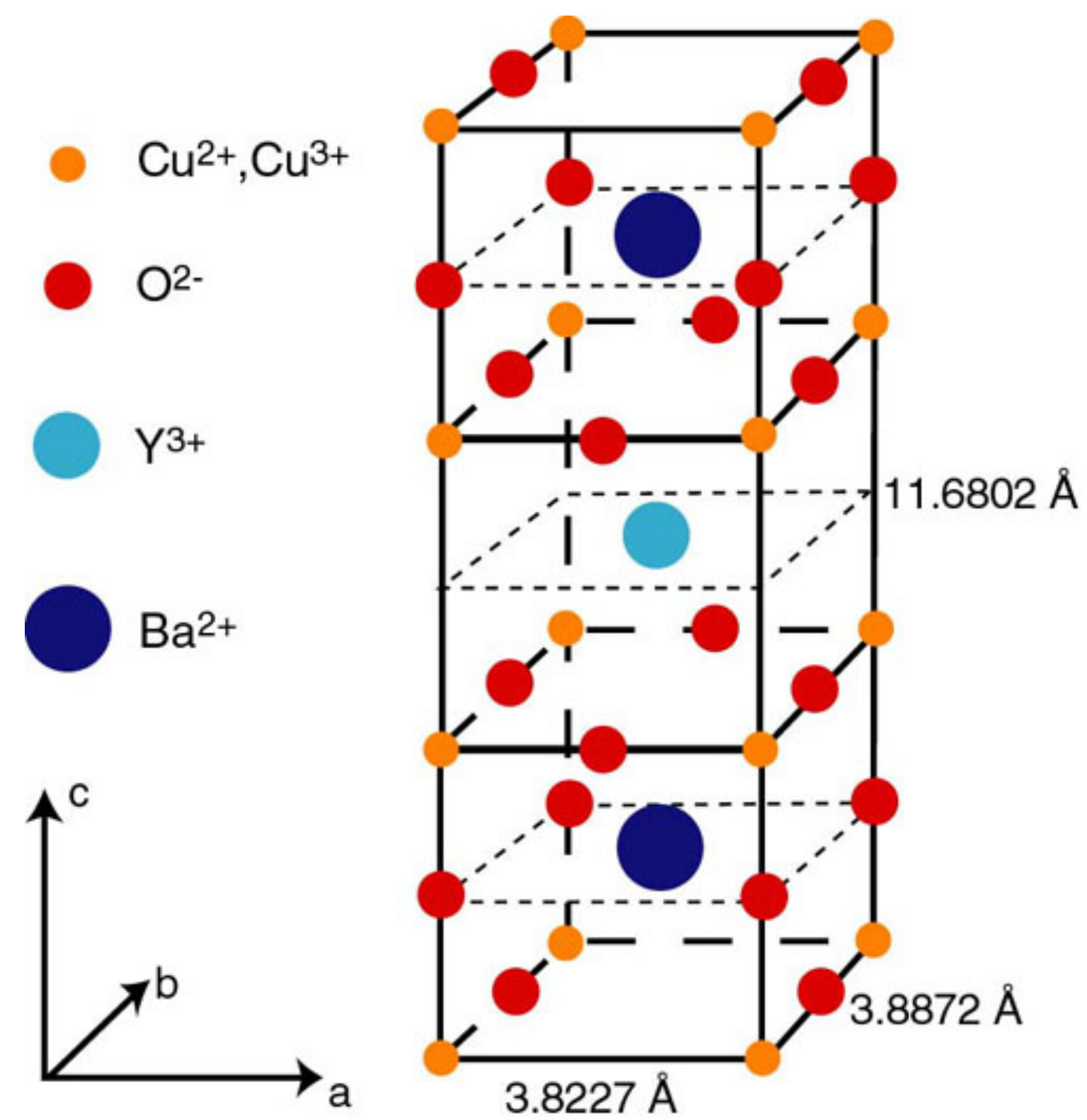
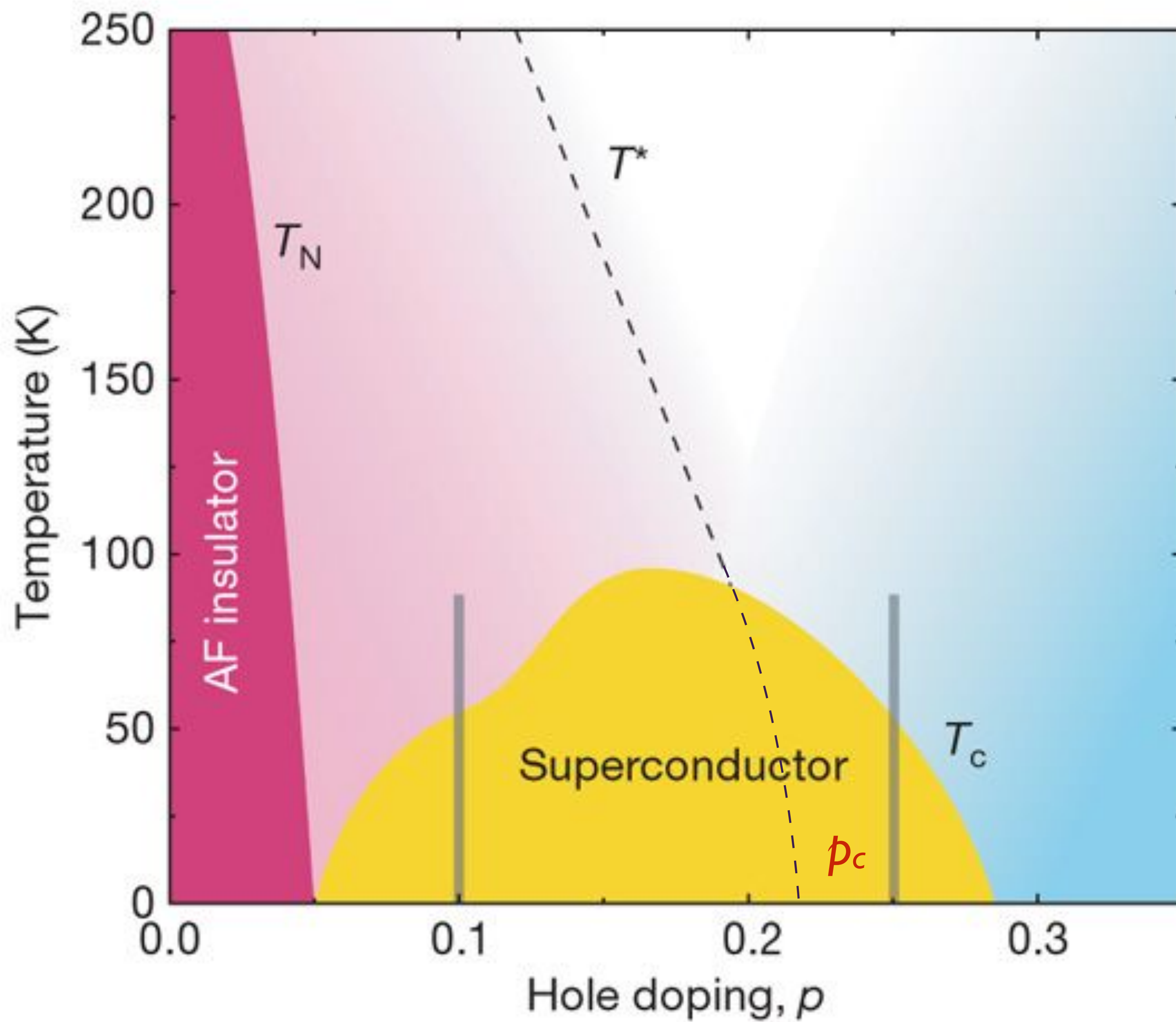
One glass of water will provide enough fusion fuel for one person's lifetime.

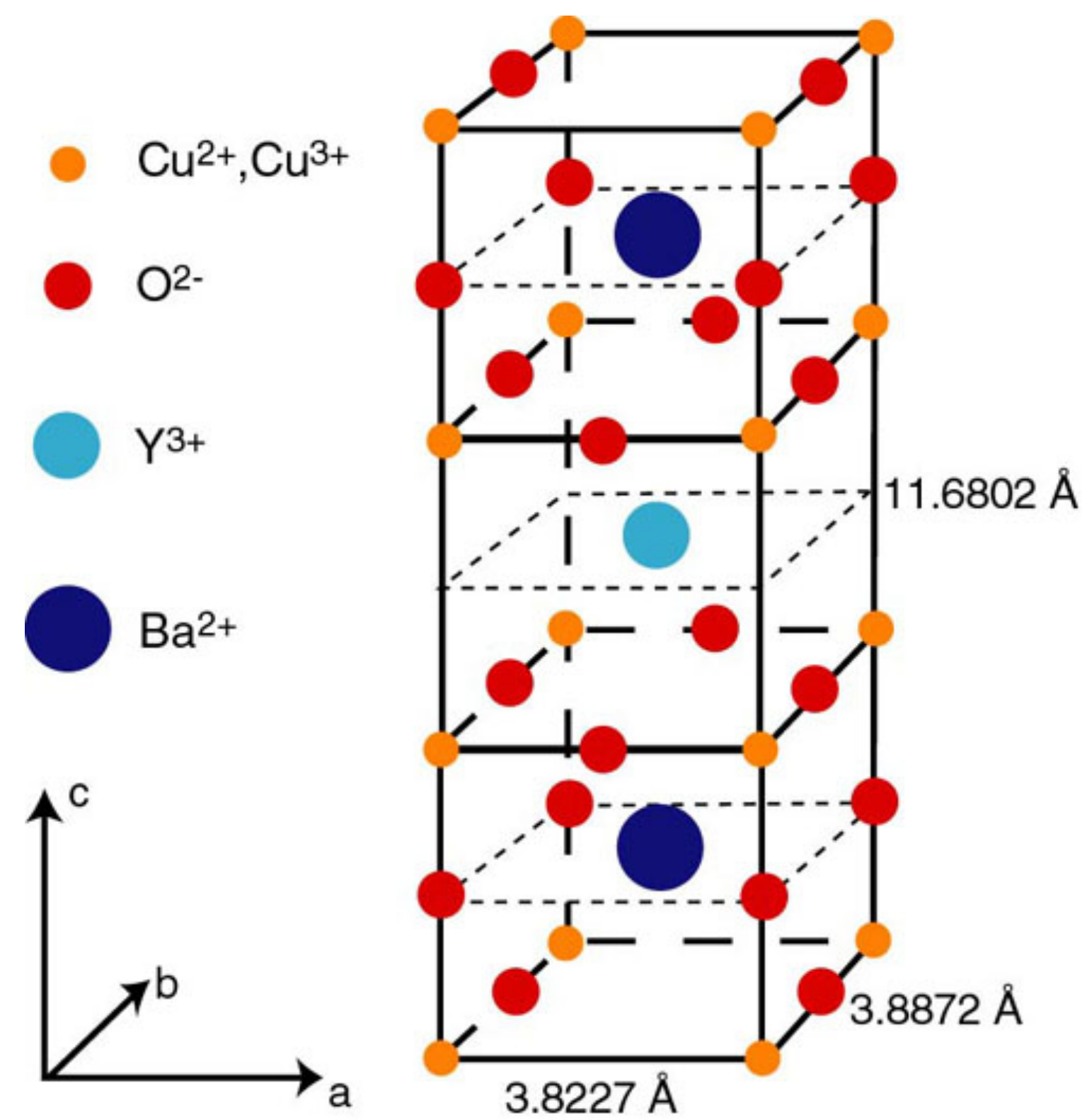
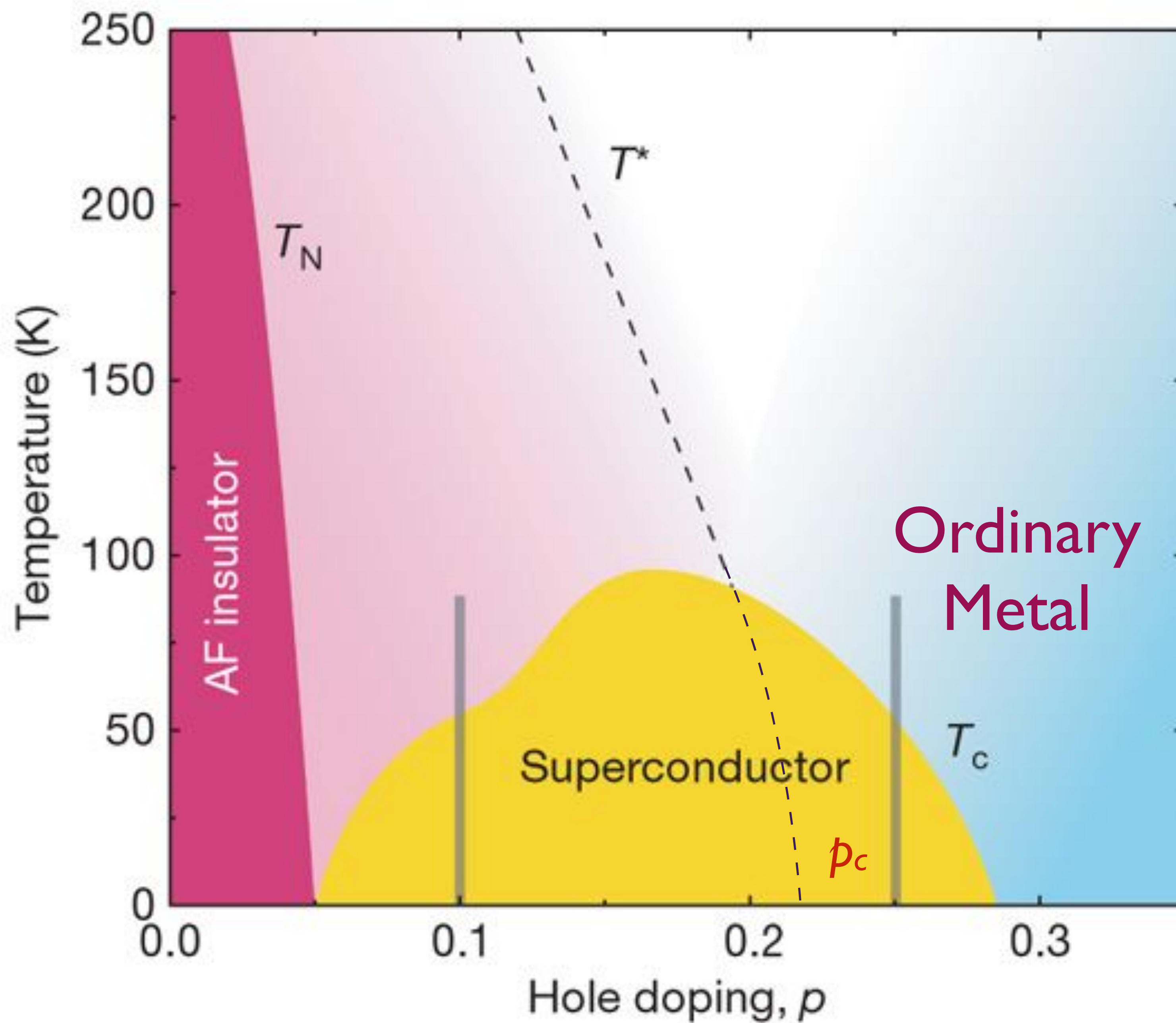
### ○ Clean

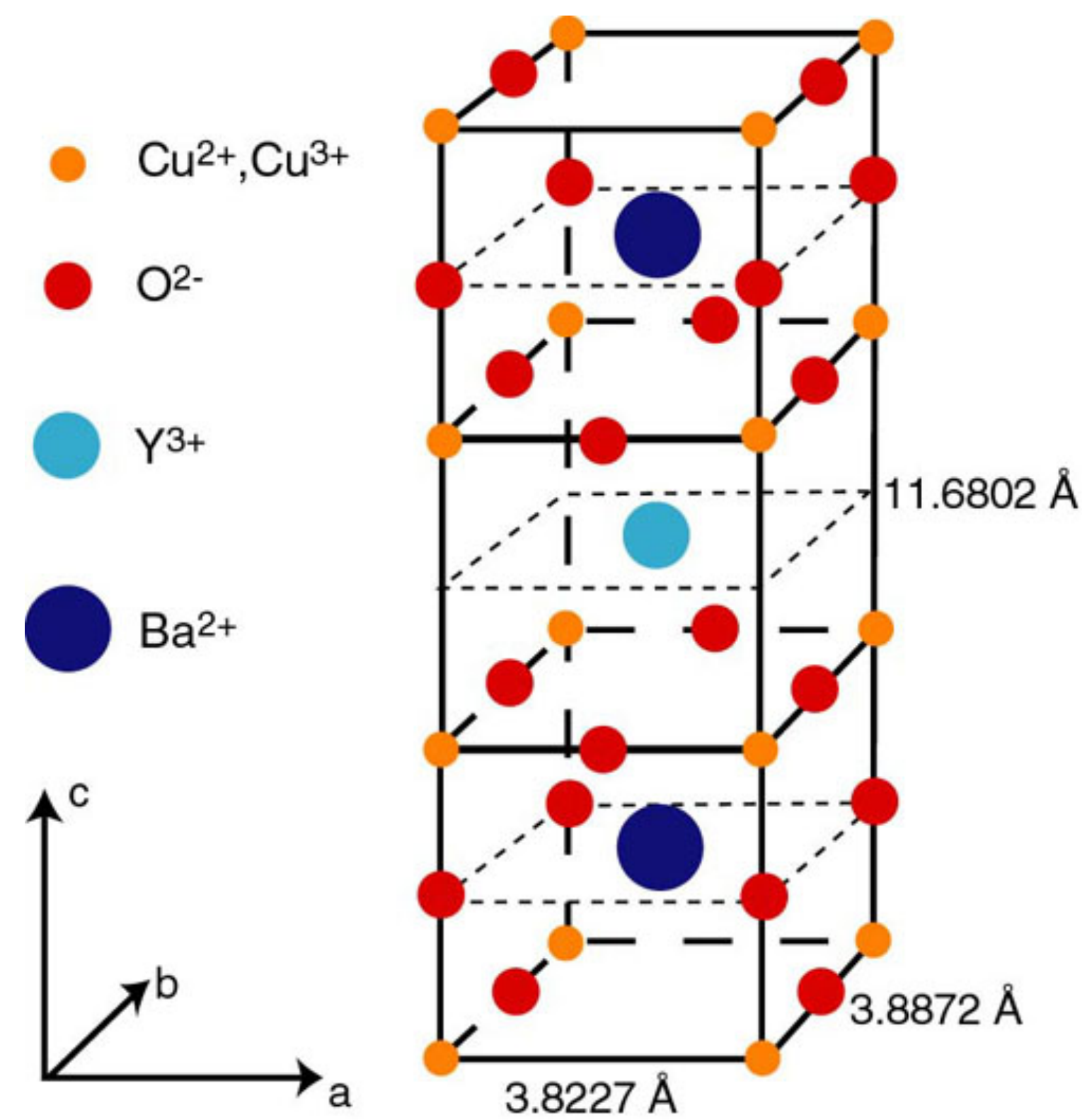
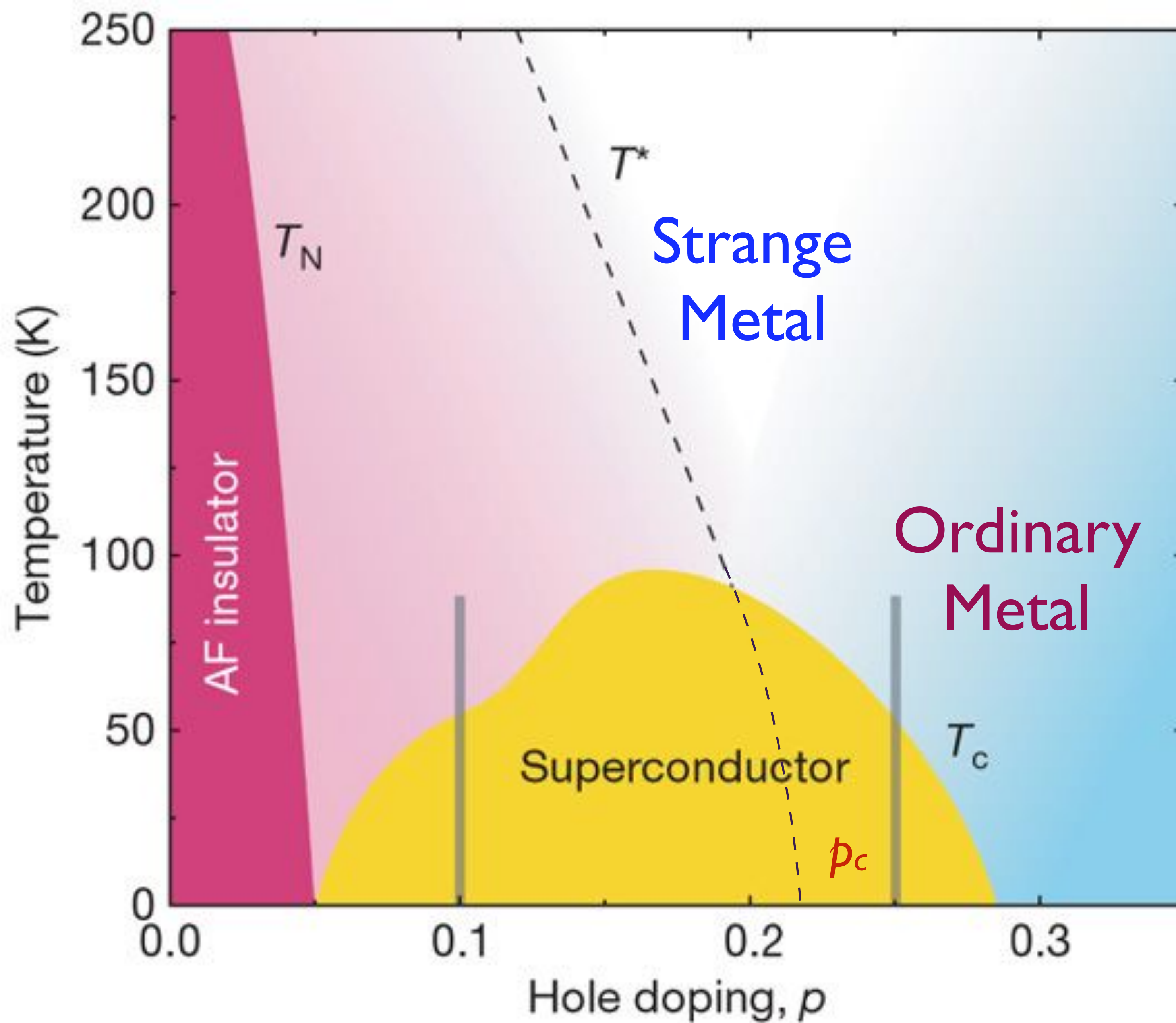
A new source of clean energy to meet our growing energy demands and combat climate change.

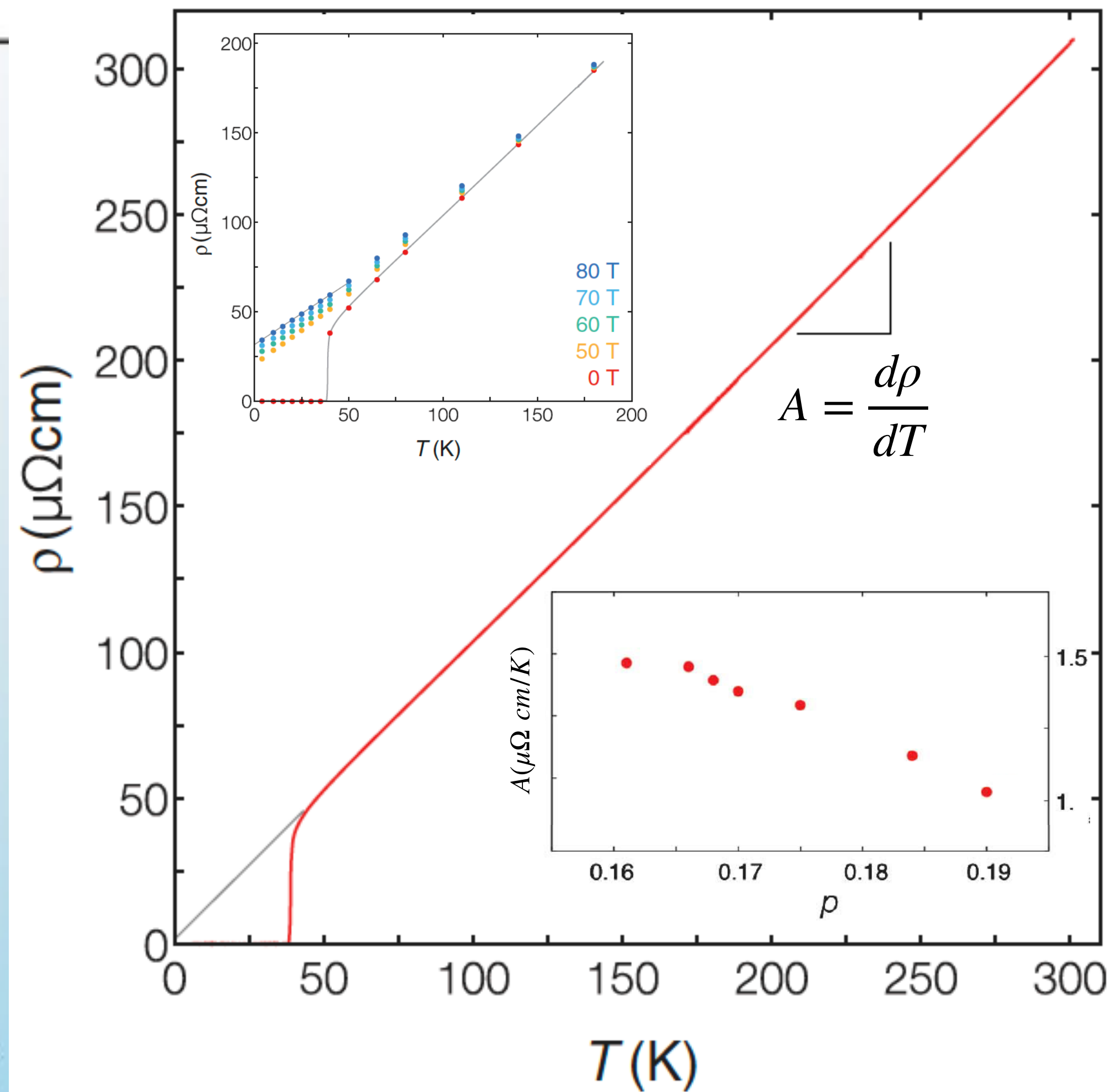
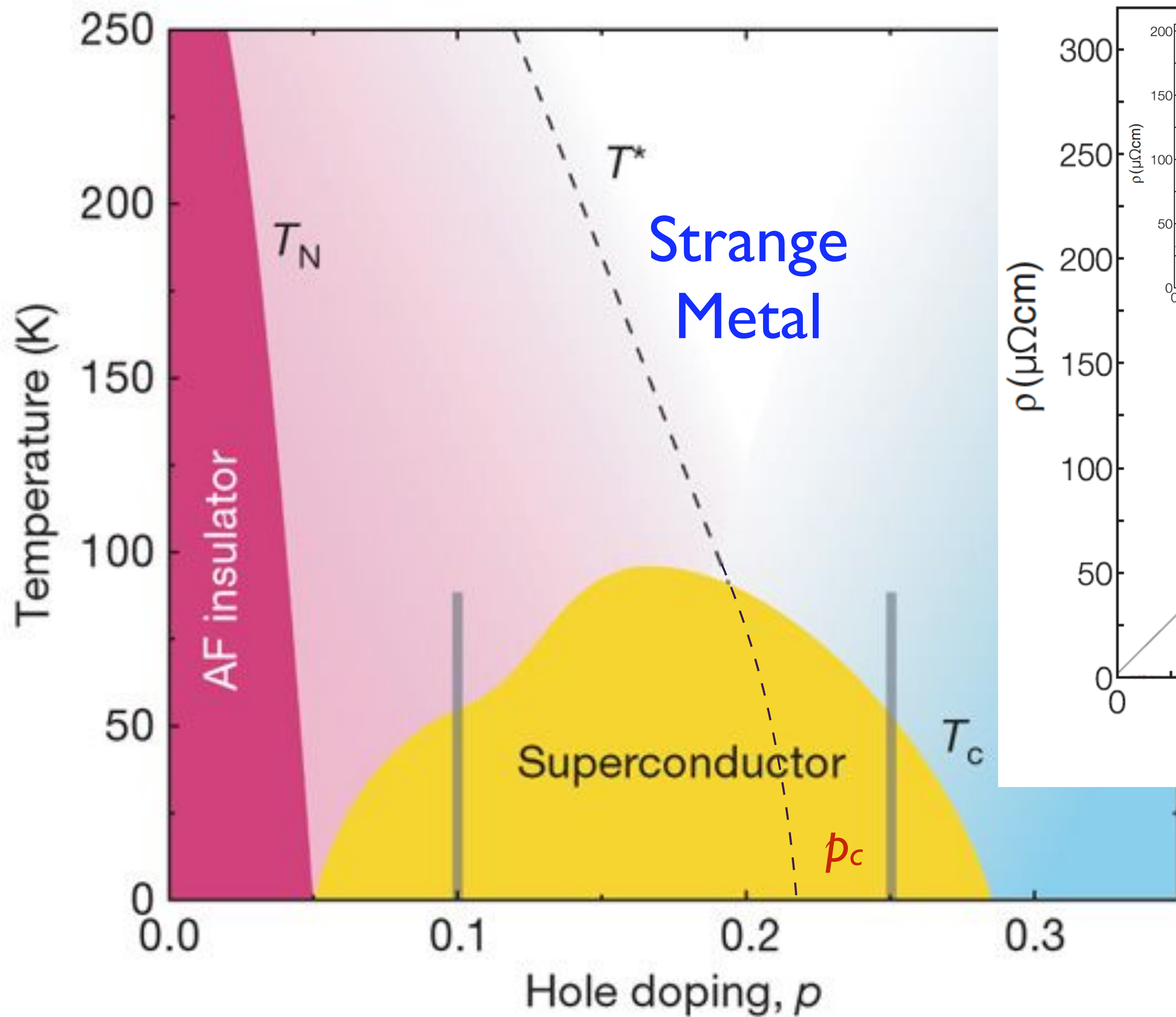


Commonwealth  
Fusion Systems

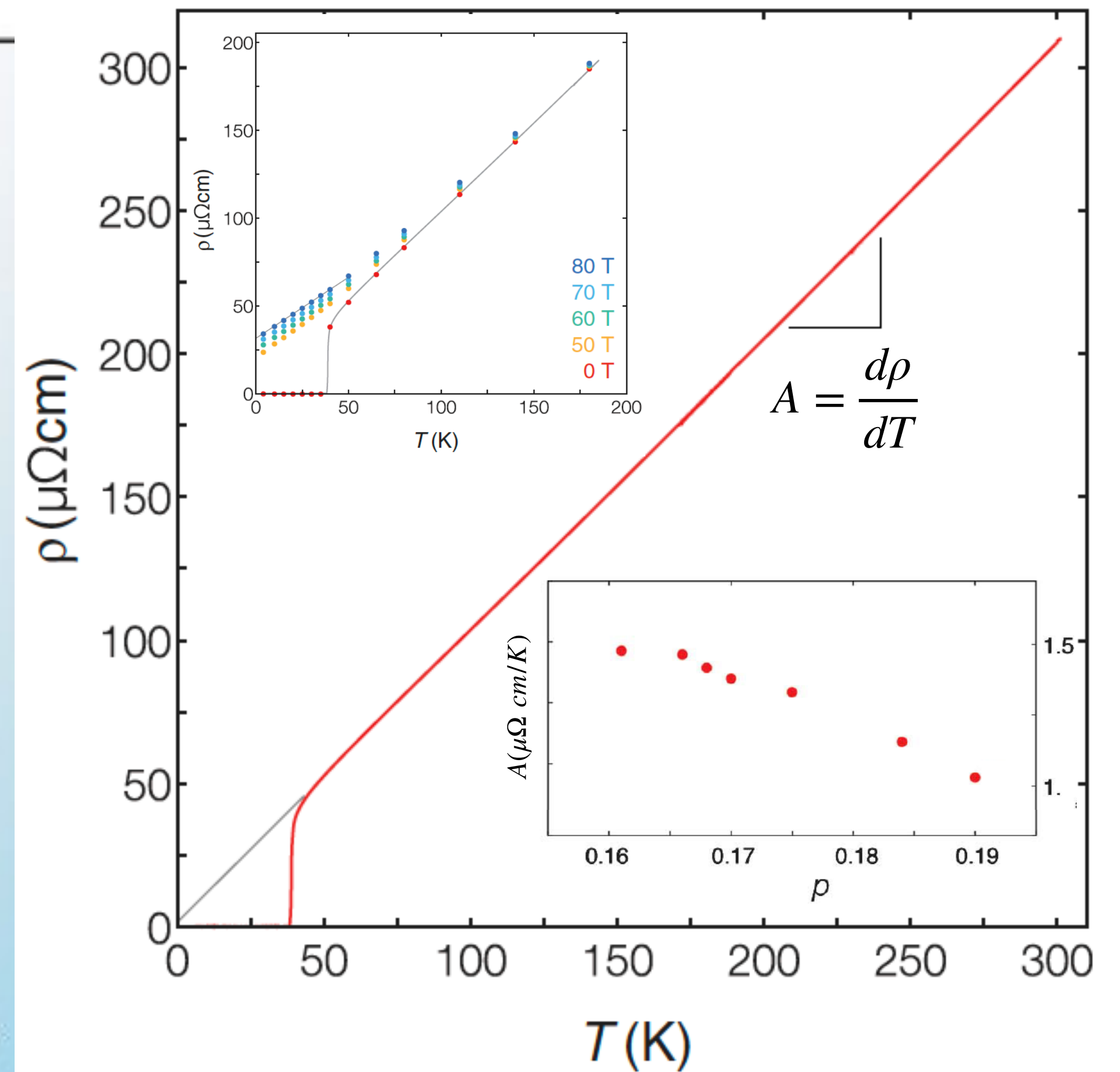
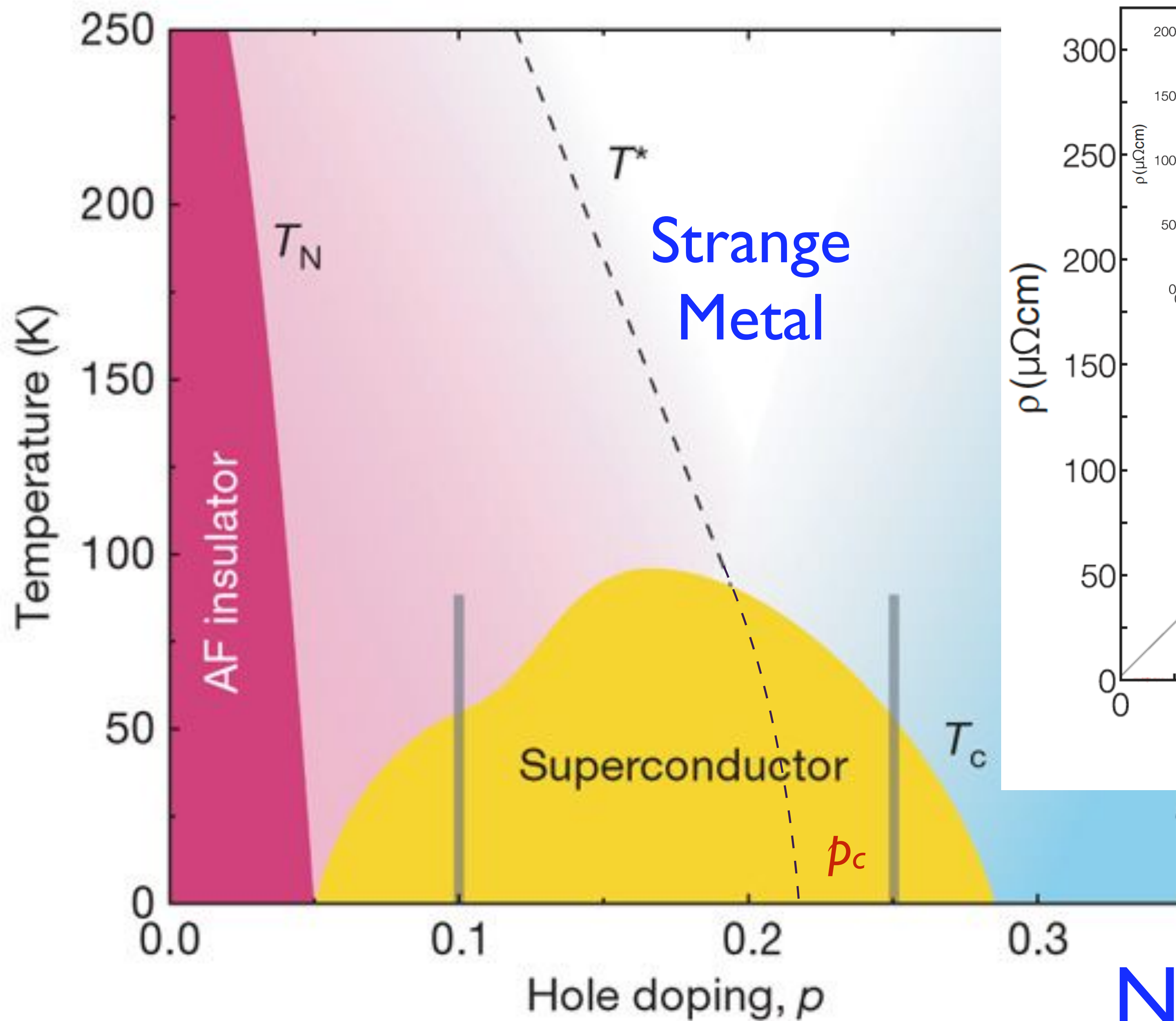








LSCO: Giraldo-Gallo et al. 2018



LSCO: Giraldo-Gallo et al. 2018

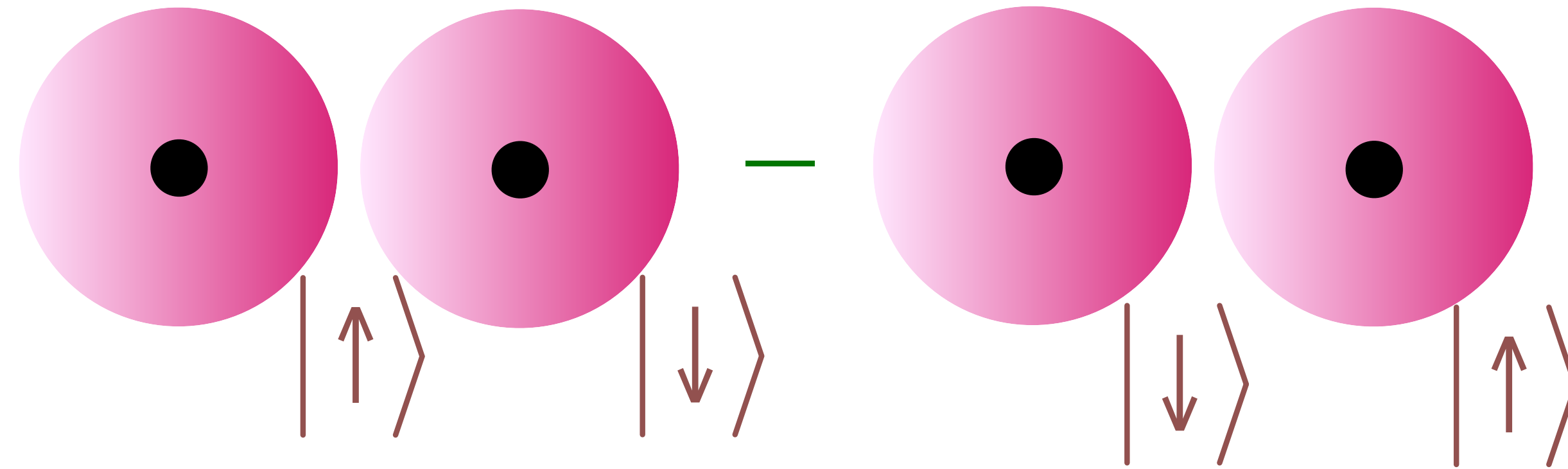
No quasiparticles ???

The Sachdev-Ye-Kitaev model  
of quantum matter without  
quasiparticles

The most remarkable new idea in the quantum theory is the  
*principle of superposition:*  
a physical system can be in a  
superposition of two (or more) distinct states.

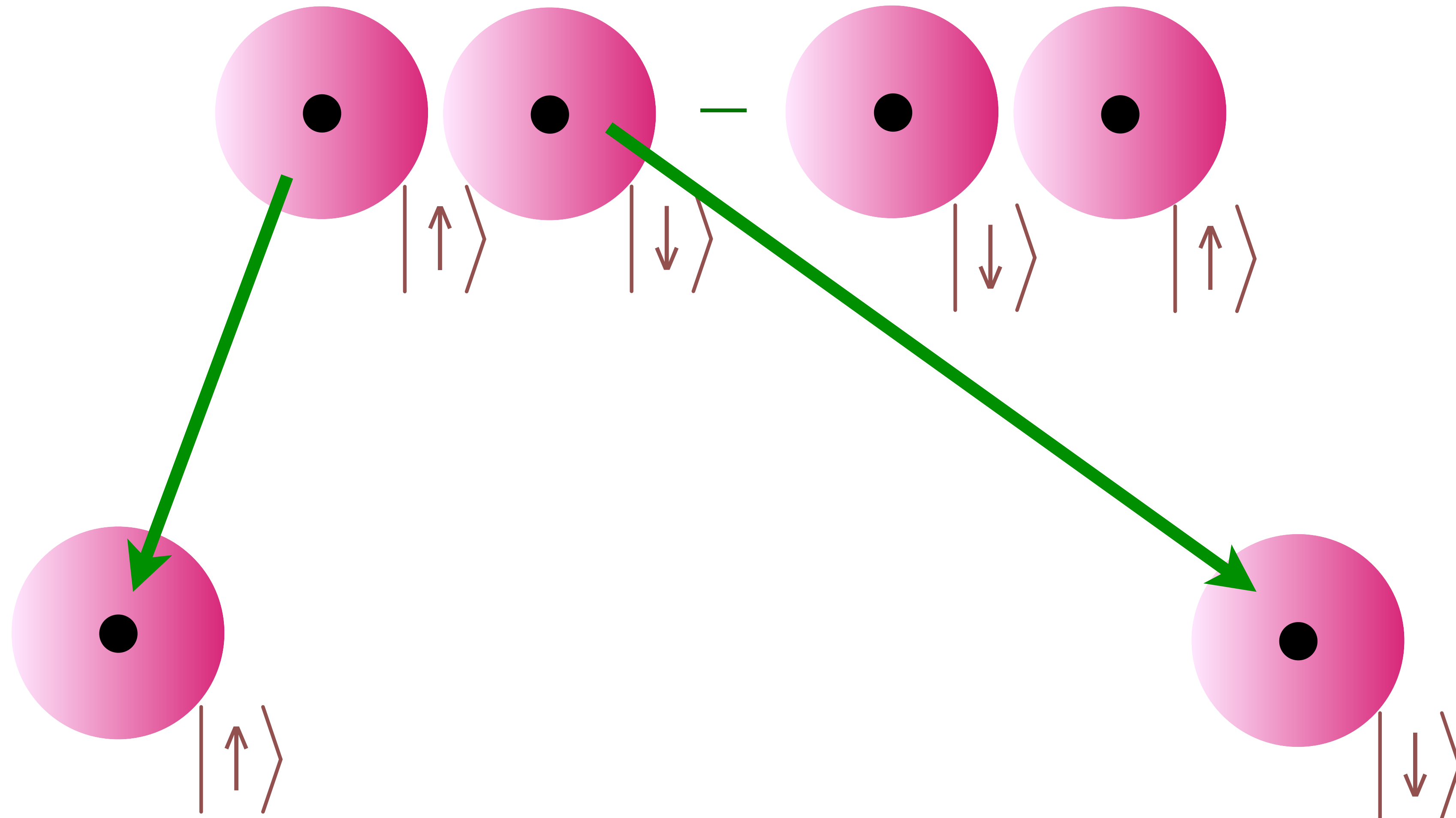
# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



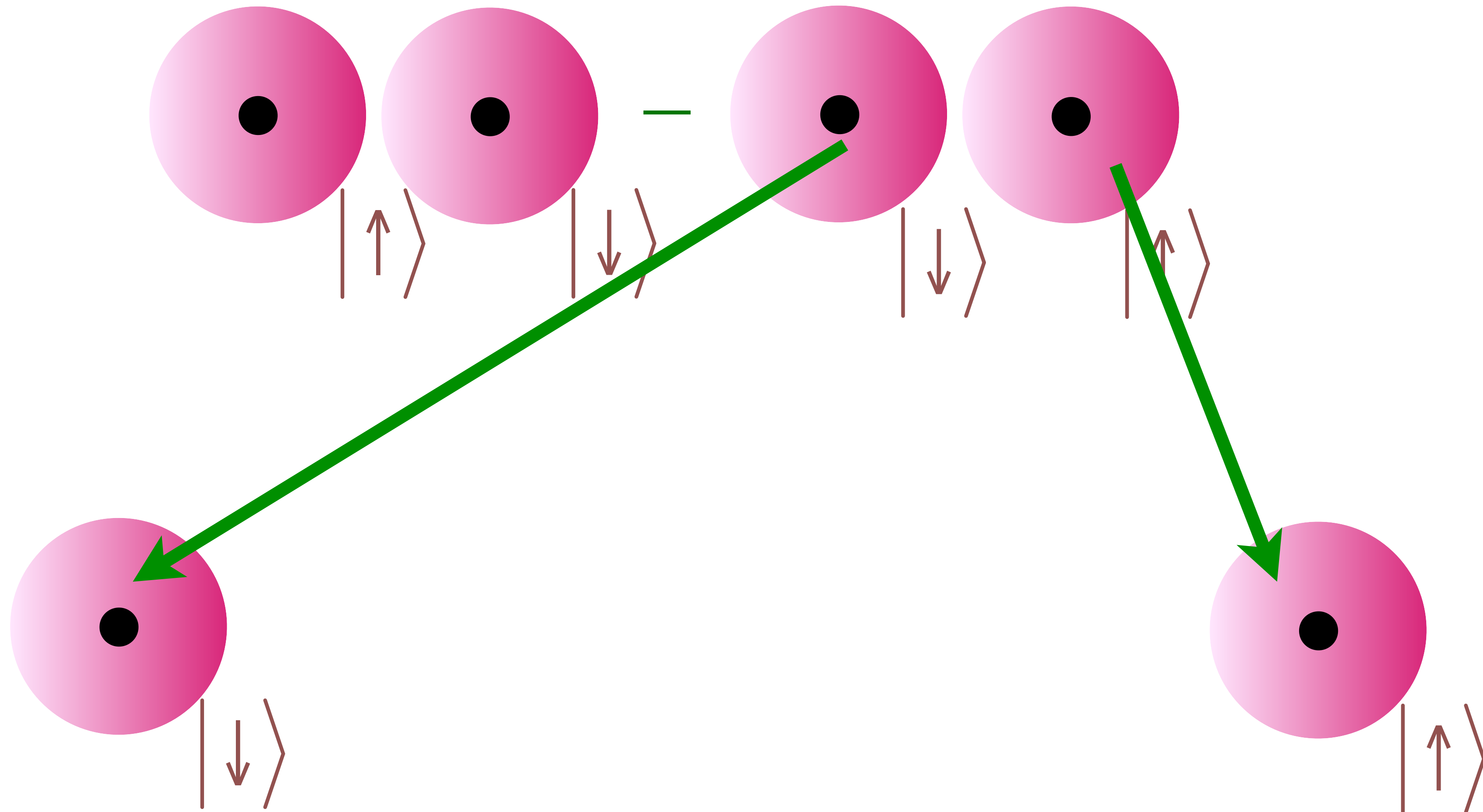
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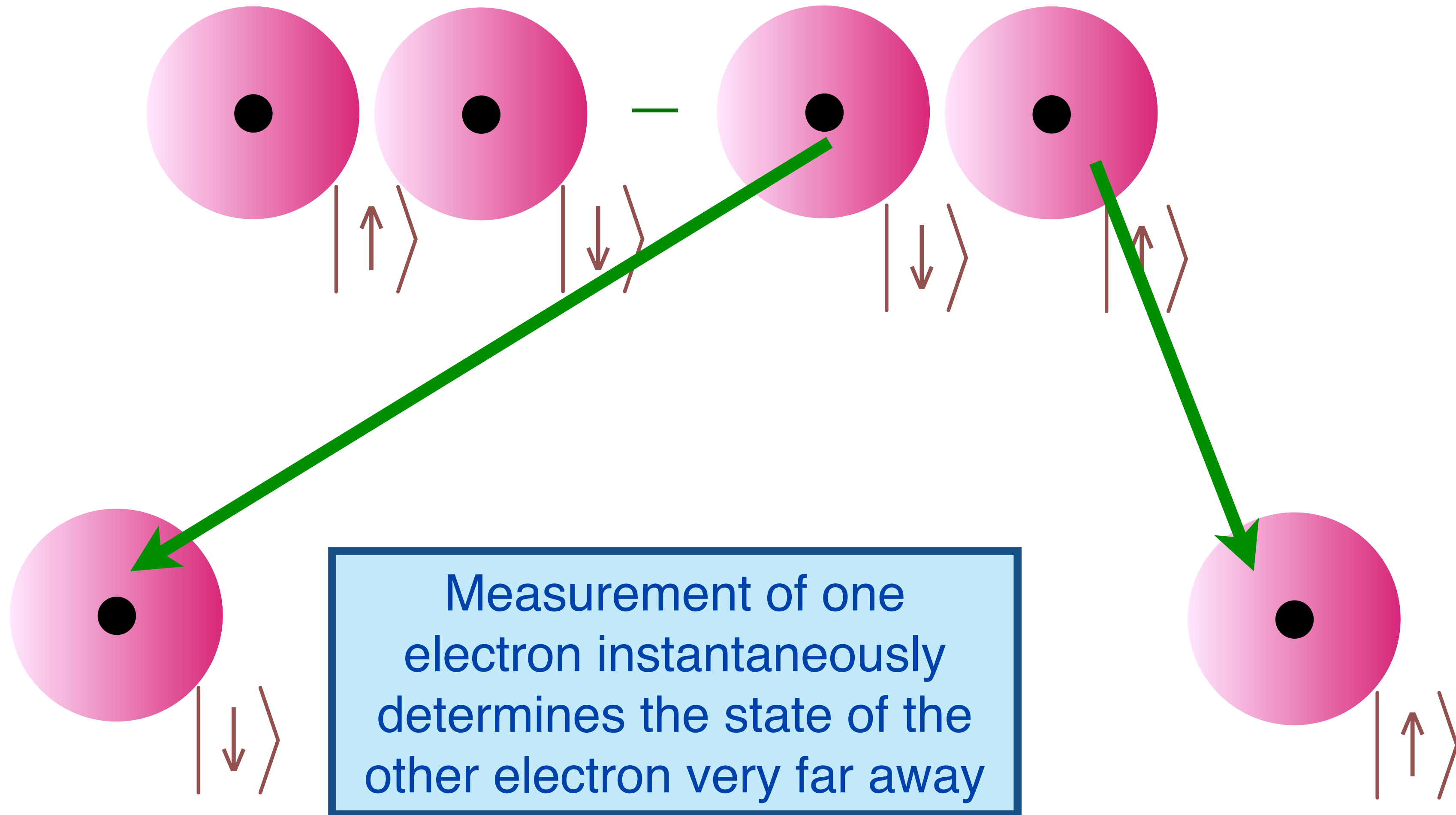
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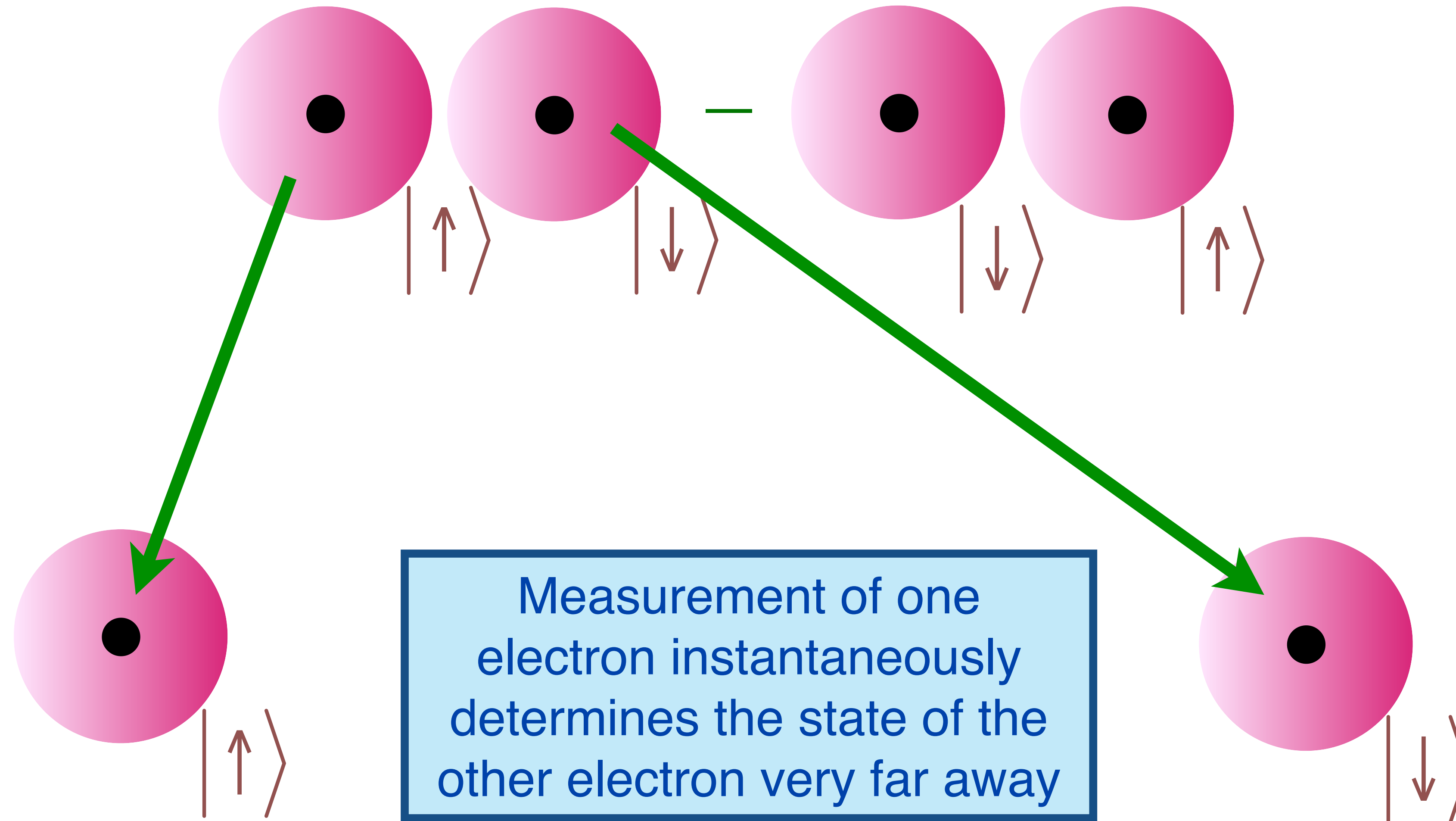
# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



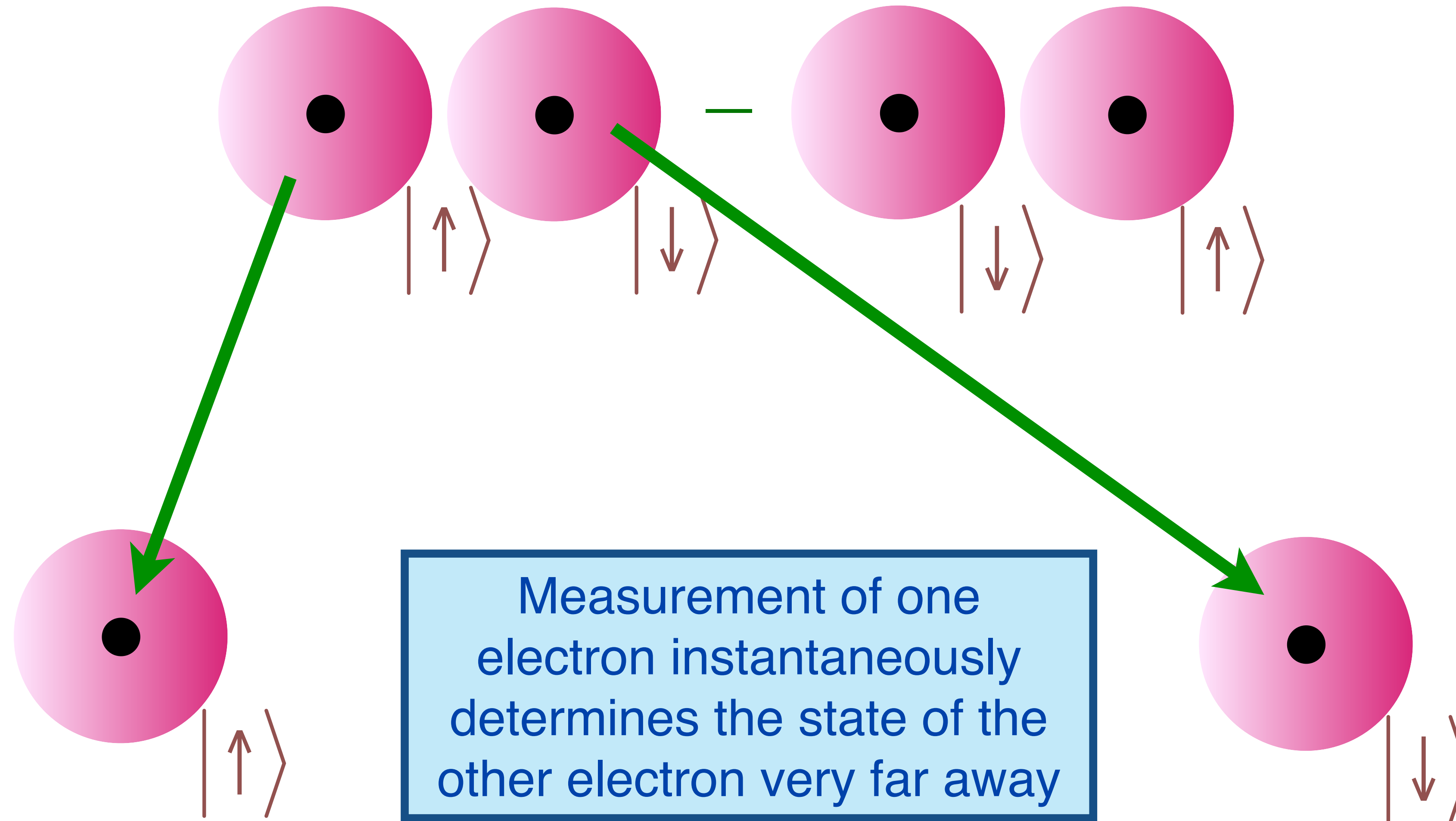
# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



# Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



**Spooky action at a distance !**

natürlicher  
deren Notwendigkeit im Raum  
mus ja zuerst von Dir klar erkannt wurde, einen Bedeutung  
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-  
an glauben, weil die Theorie mit dem Grundsatz unvereinbar  
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-  
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich  
überzeugt daß es wirklich mit der Theorie

amount of validity in the  
recognise clearly as necessary given the framework of  
malism. I cannot seriously believe in it because the theory cannot be rec-  
onciled with the idea that physics should represent a reality in time and  
space, free from spooky actions at a distance. I am, however, not yet  
convinced that it can really be achieved with a continuous field  
... this which so

I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

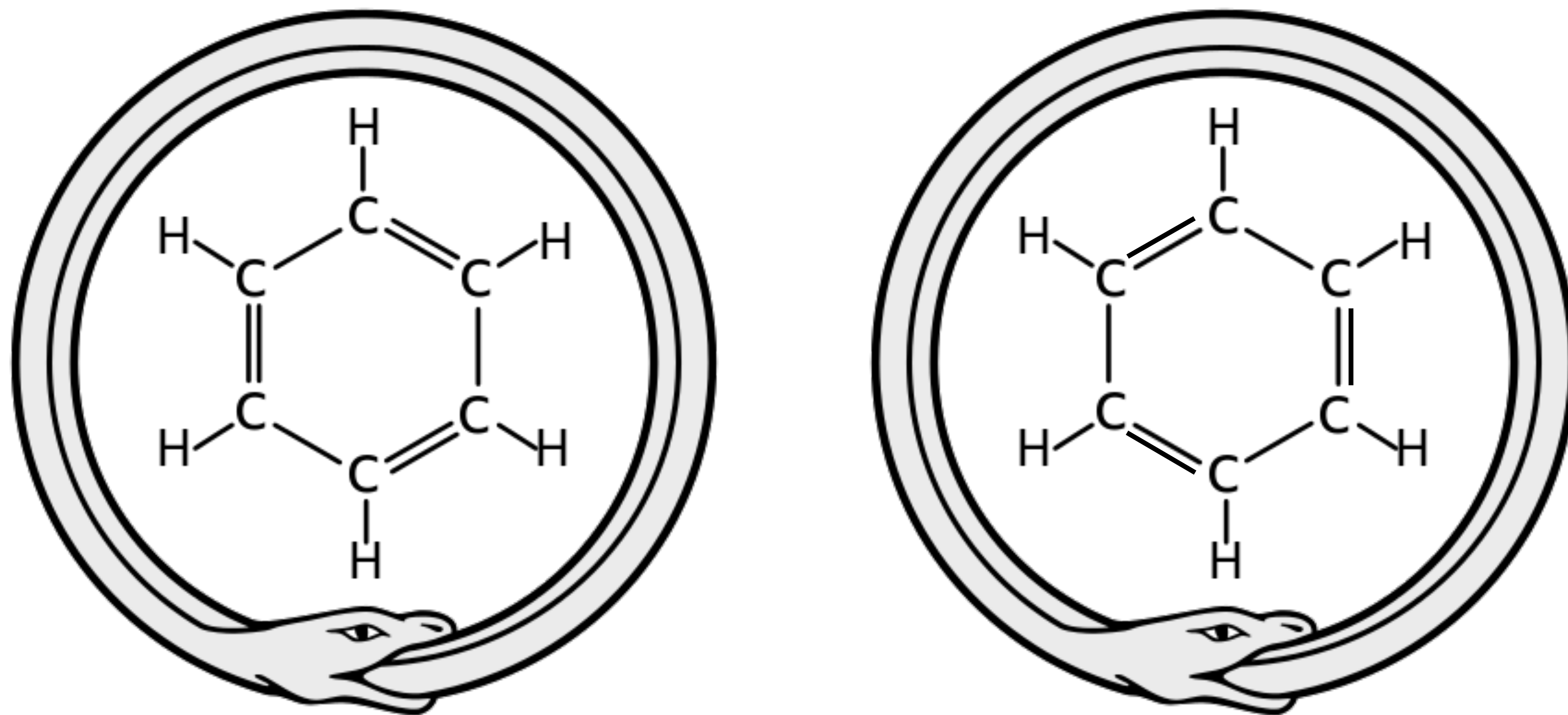
How about quantum entanglement  
of 3, 4, 5,  $\dots$   $\infty$  particles?



August Kekule, theory of the benzene molecule, 1865

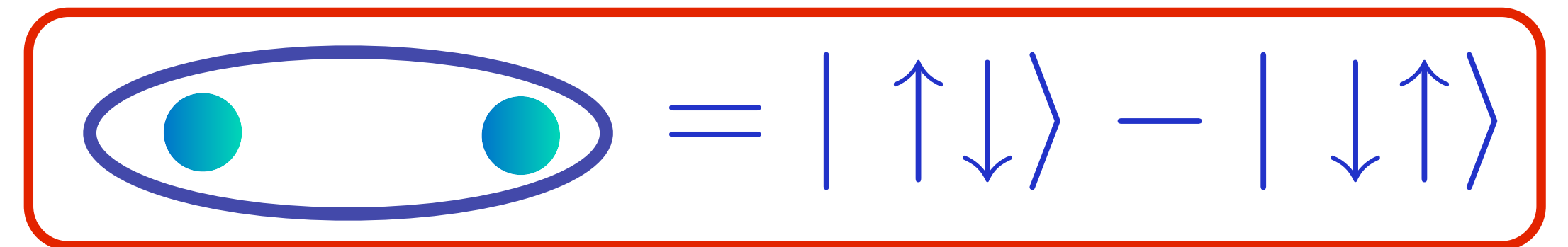
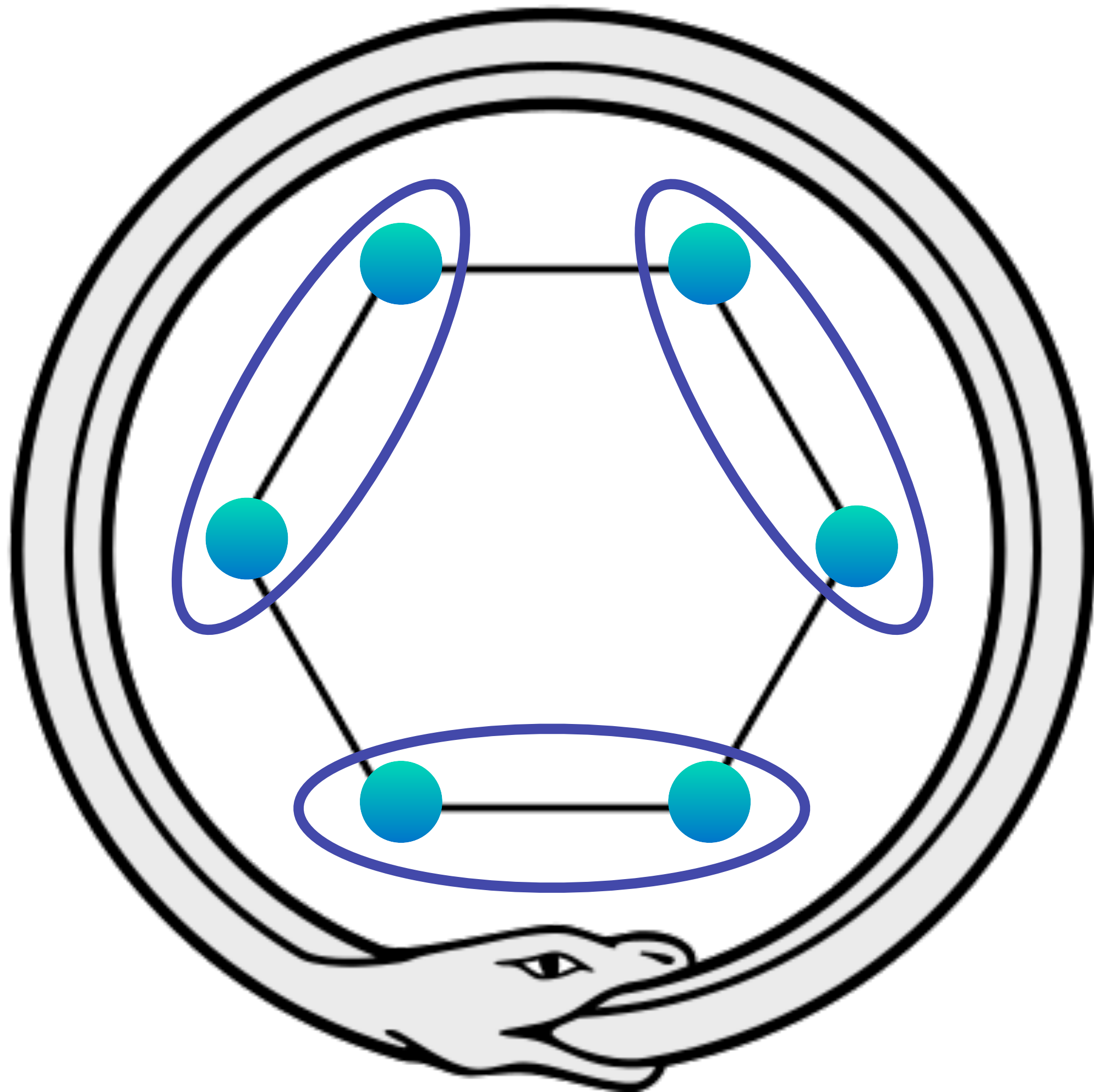
# Kekulé's spooky dream

Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail\*



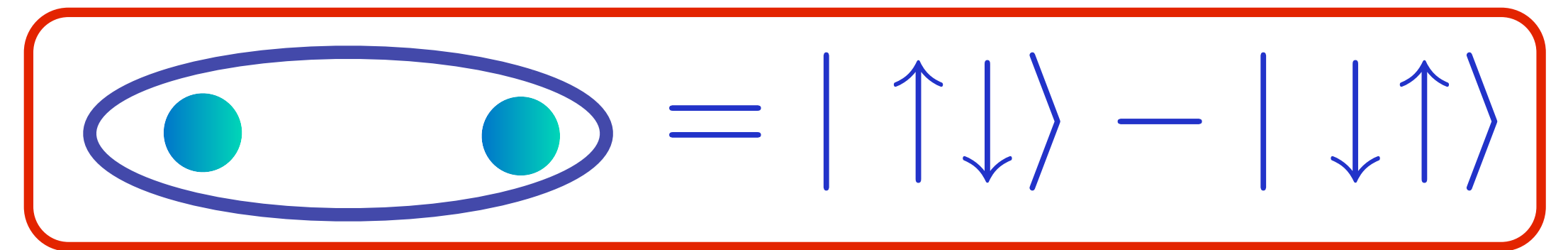
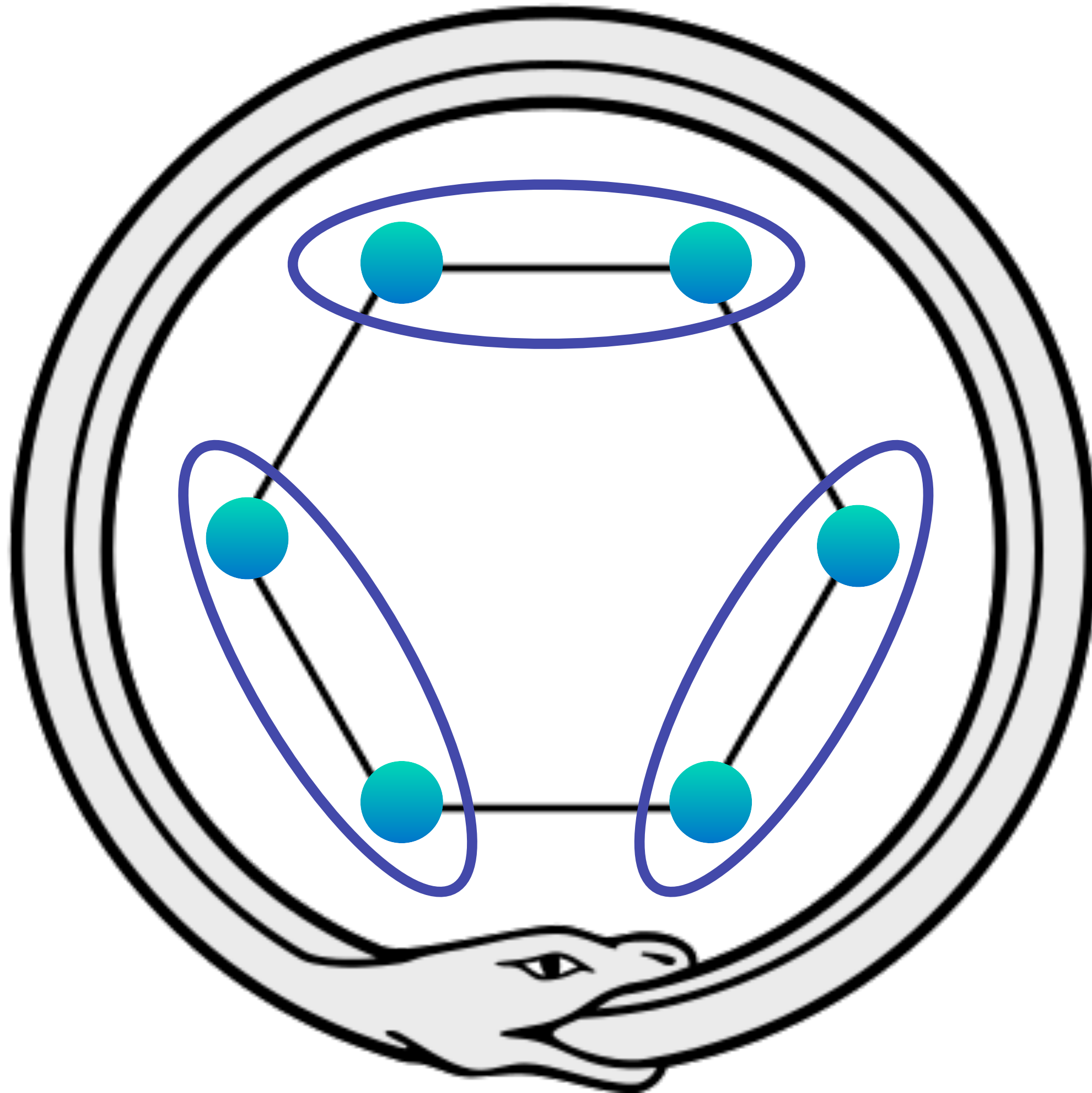
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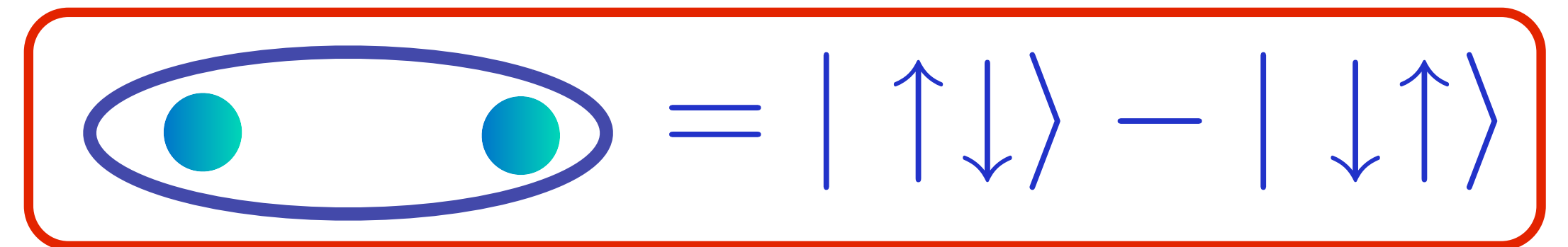
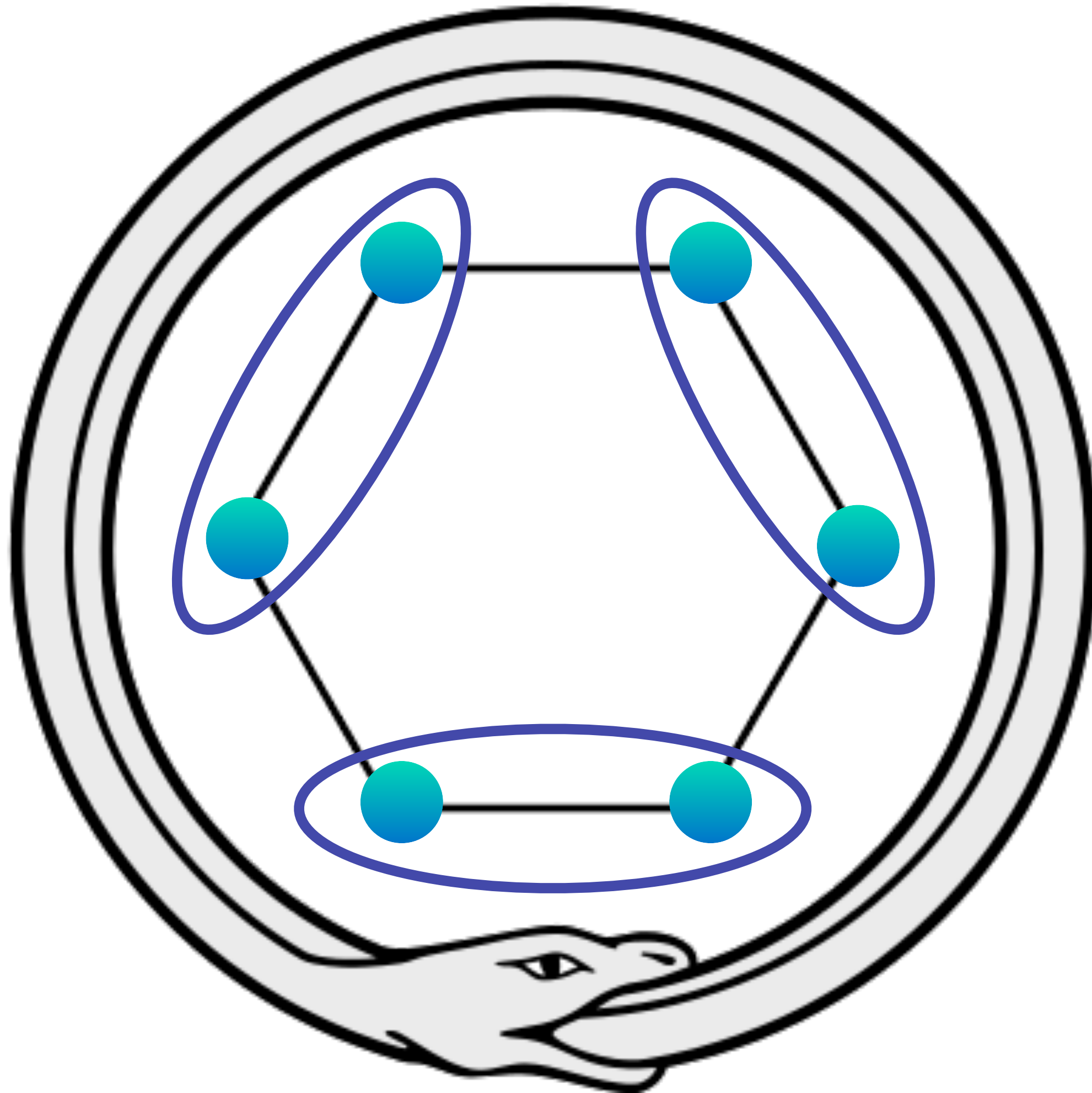
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# Kekule's spooky dream

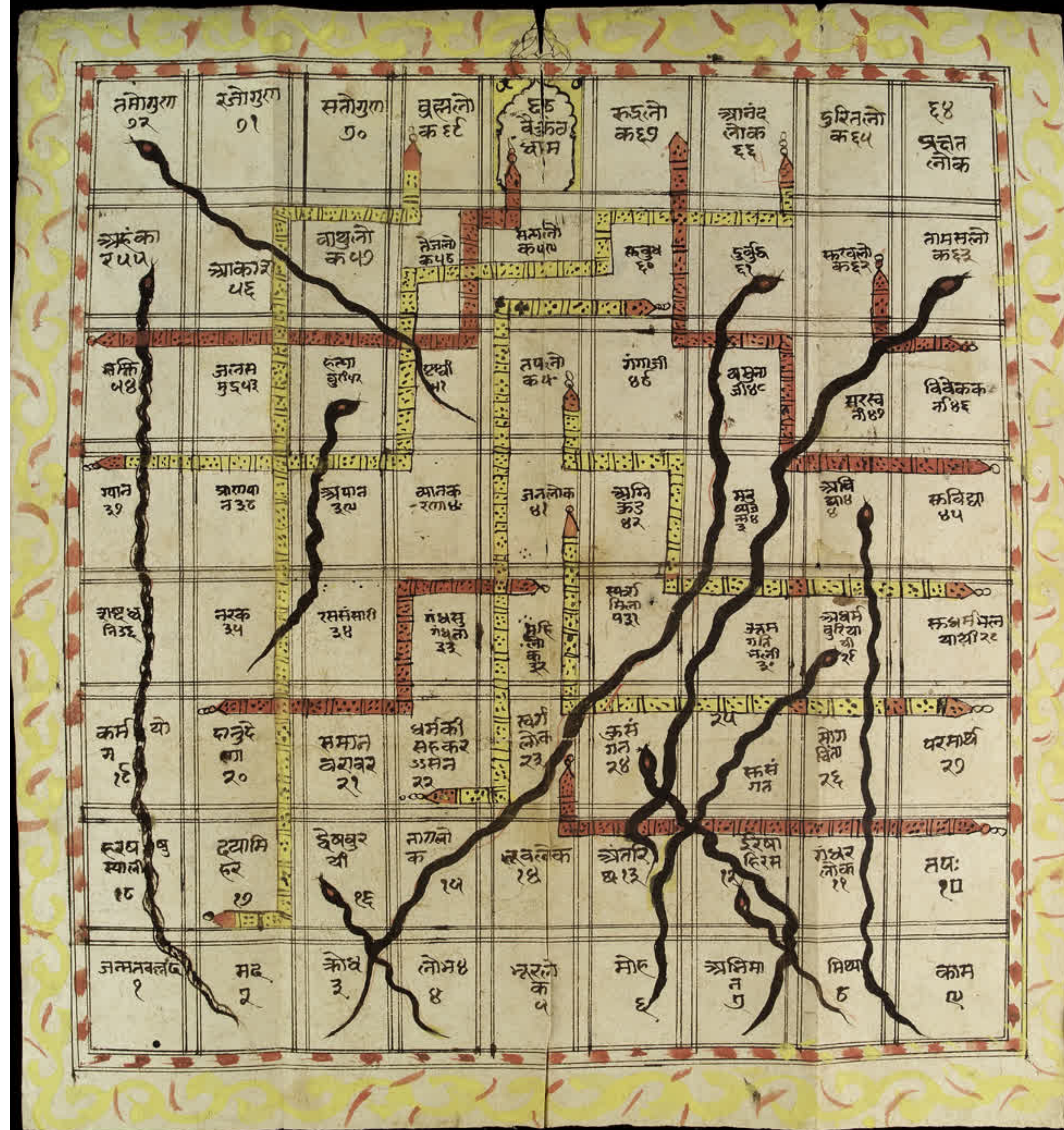
Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail\*



My  
spooky  
dream\*

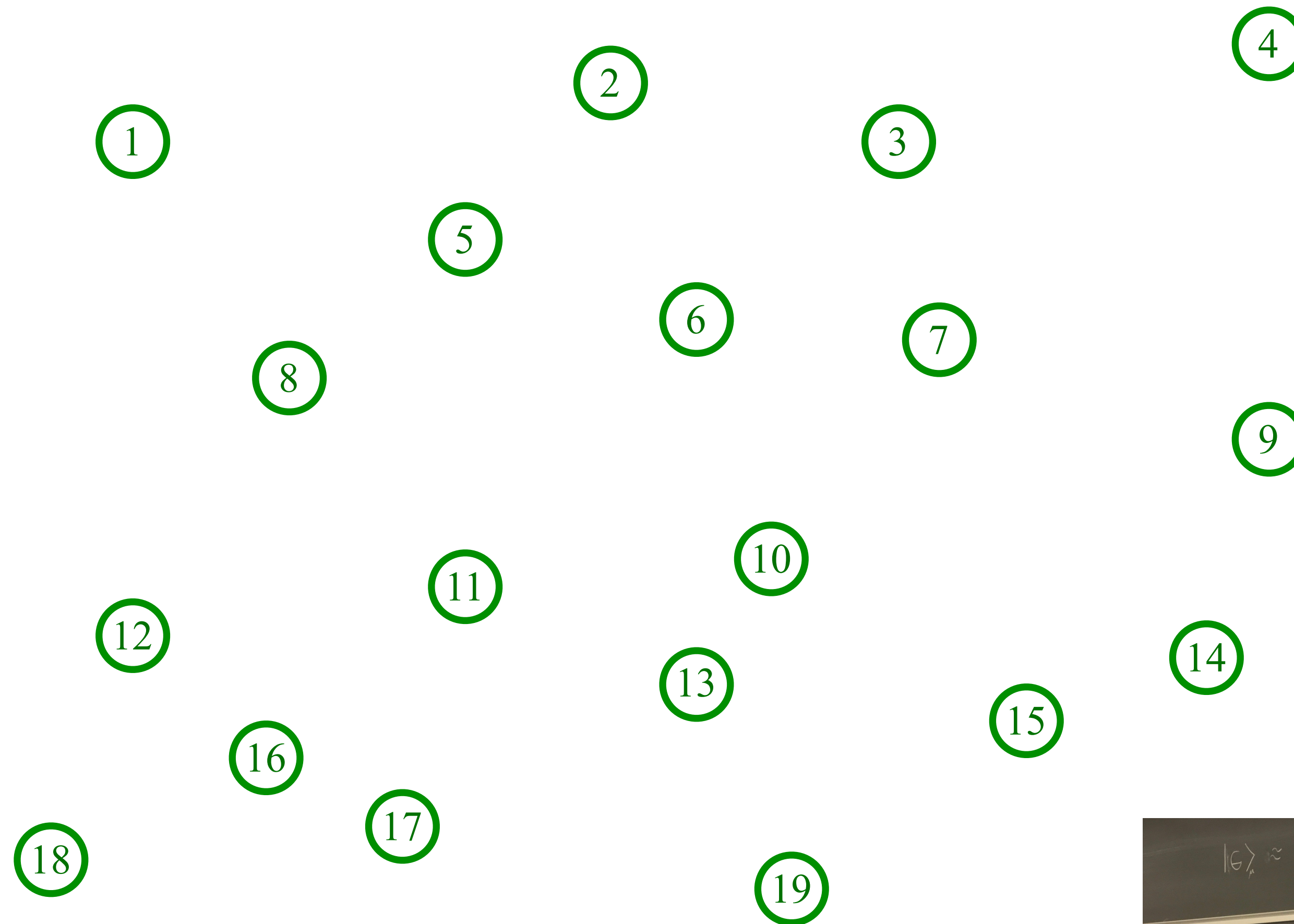
Ancient  
Indian  
game of  
Snakes  
and  
Ladders

\*Not true

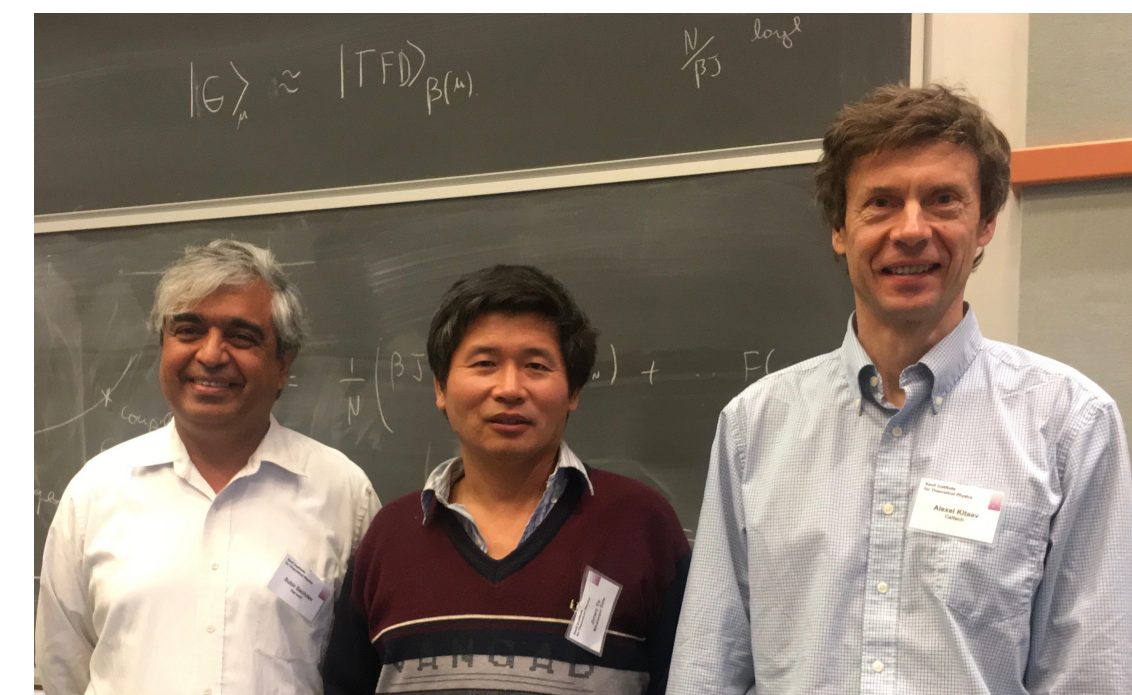


# The SYK model

Sachdev, Ye (1993); Kitaev (2015)

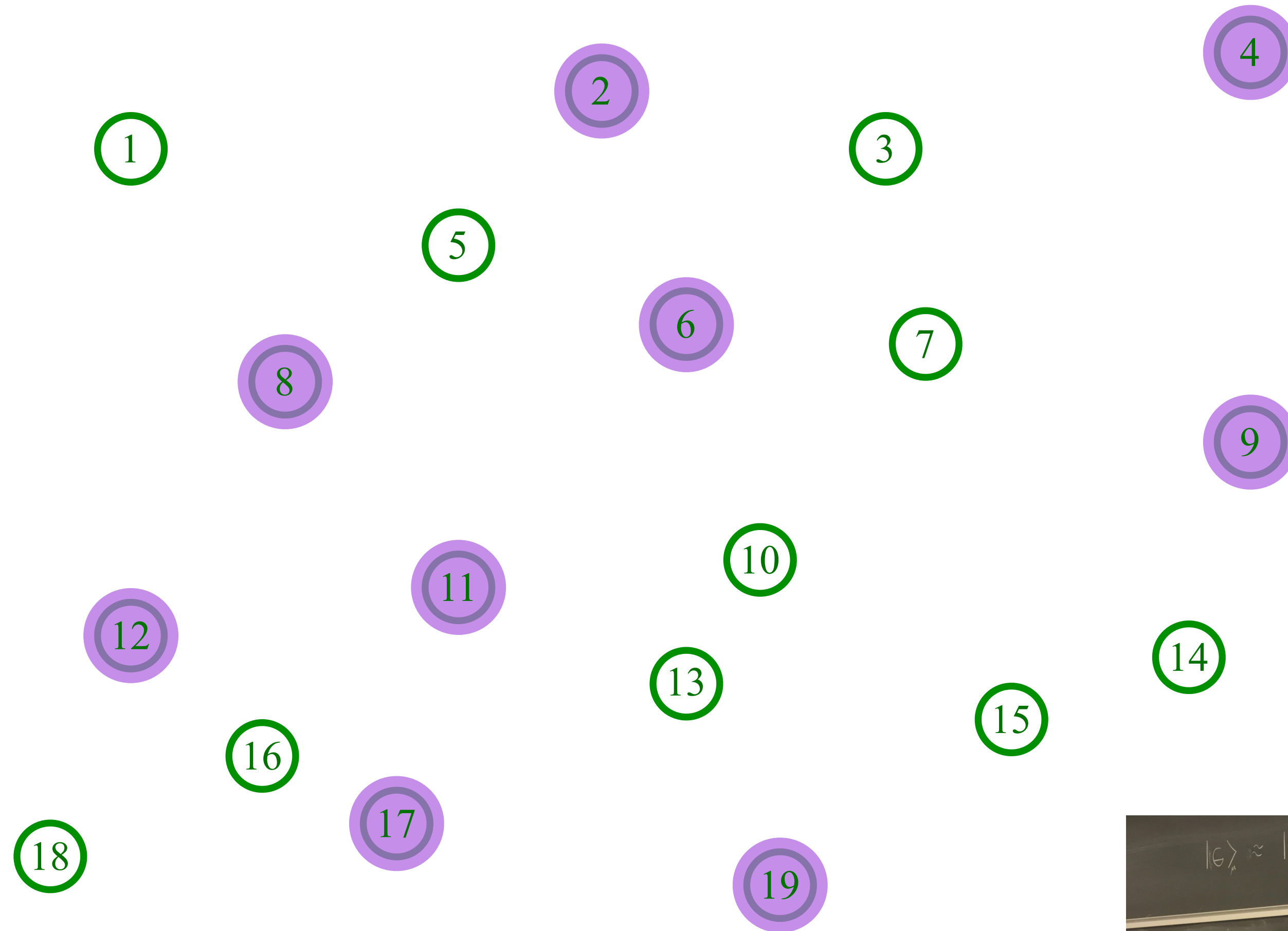


Pick a set of random positions

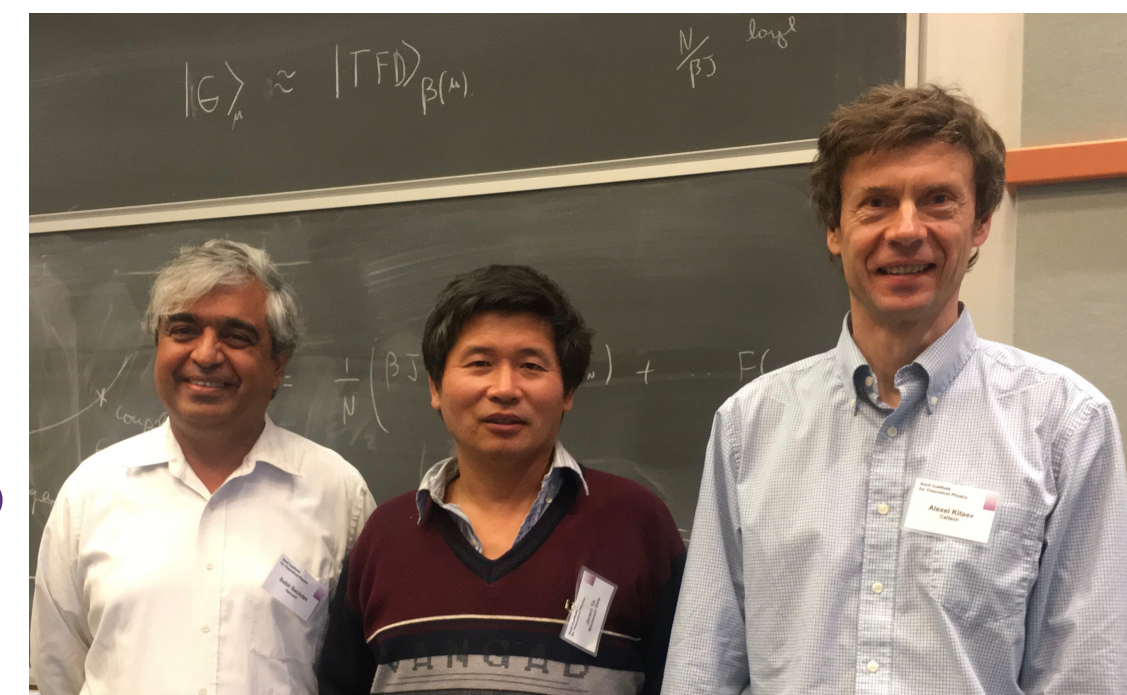


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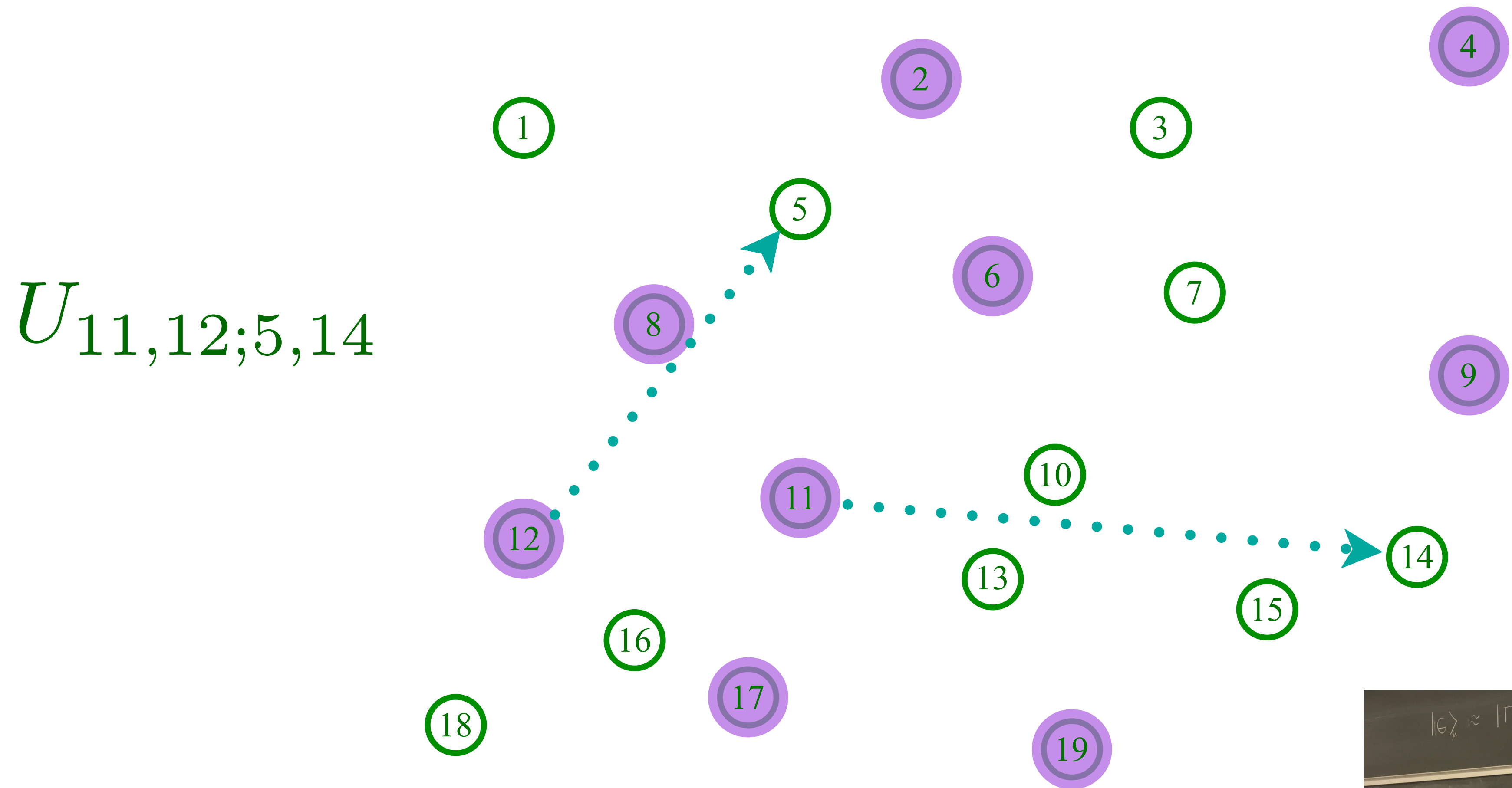


Place electrons randomly on some sites

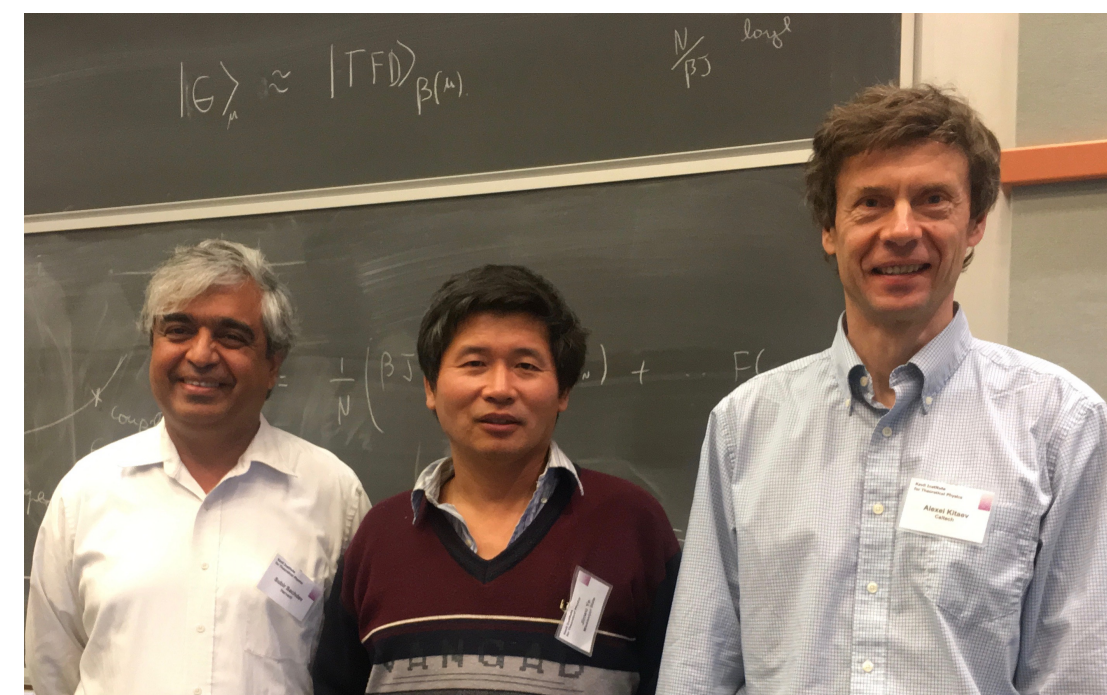


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Sachdev, Ye (1993); Kitaev (2015)



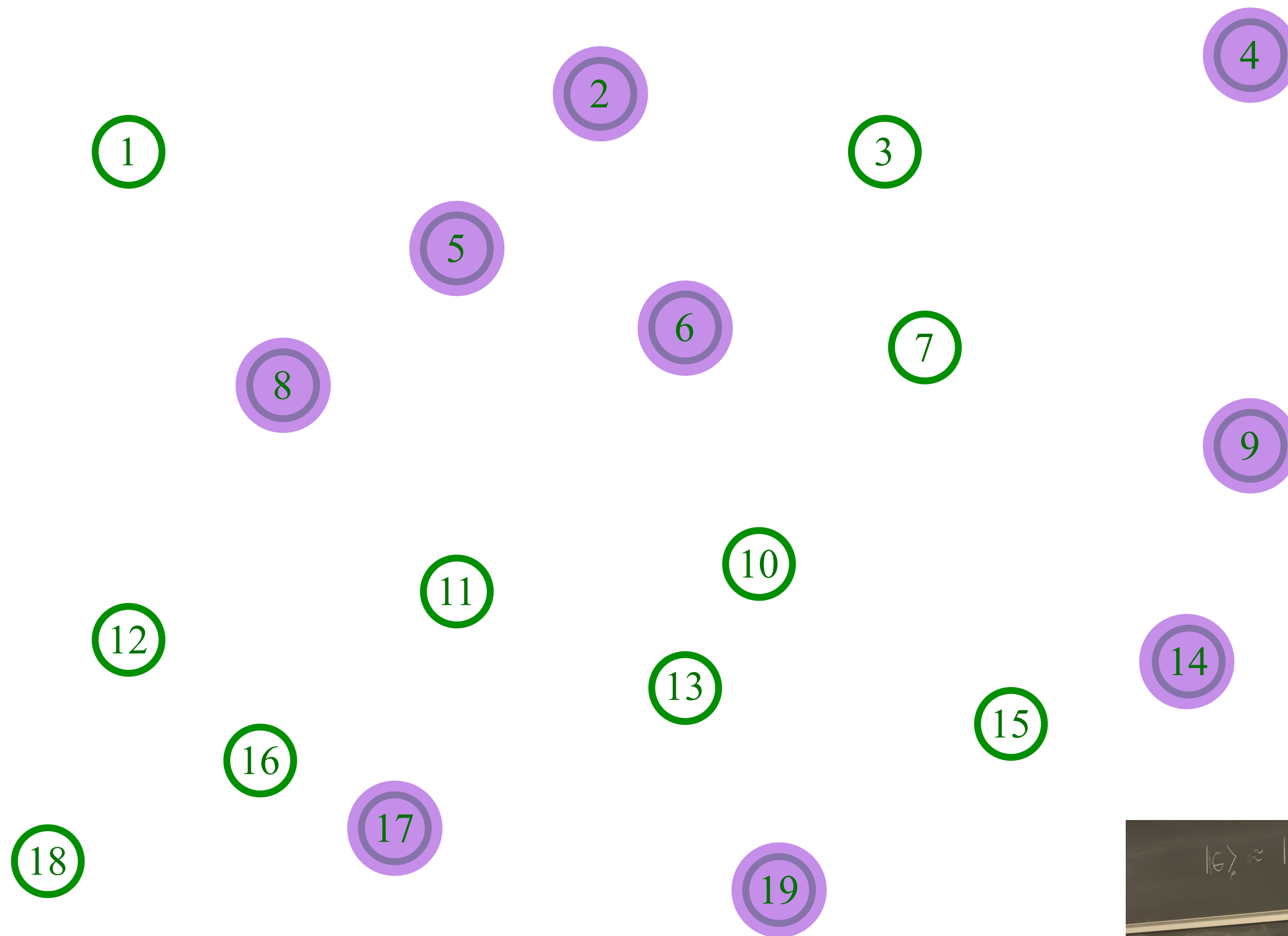
Place electrons randomly on some sites



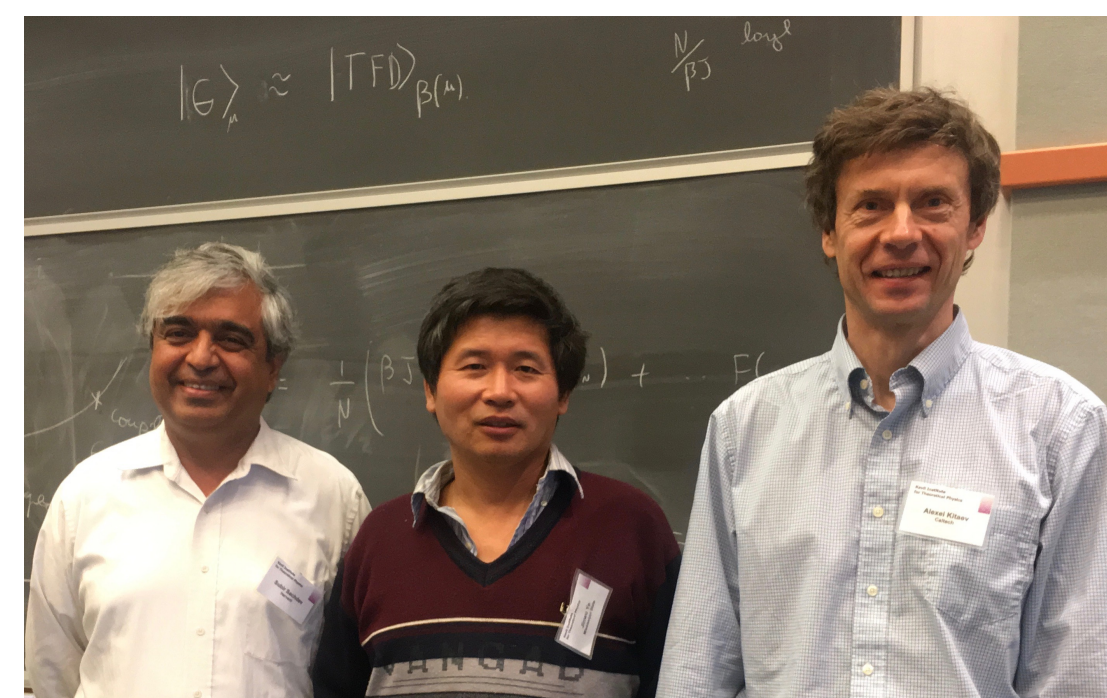
# The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



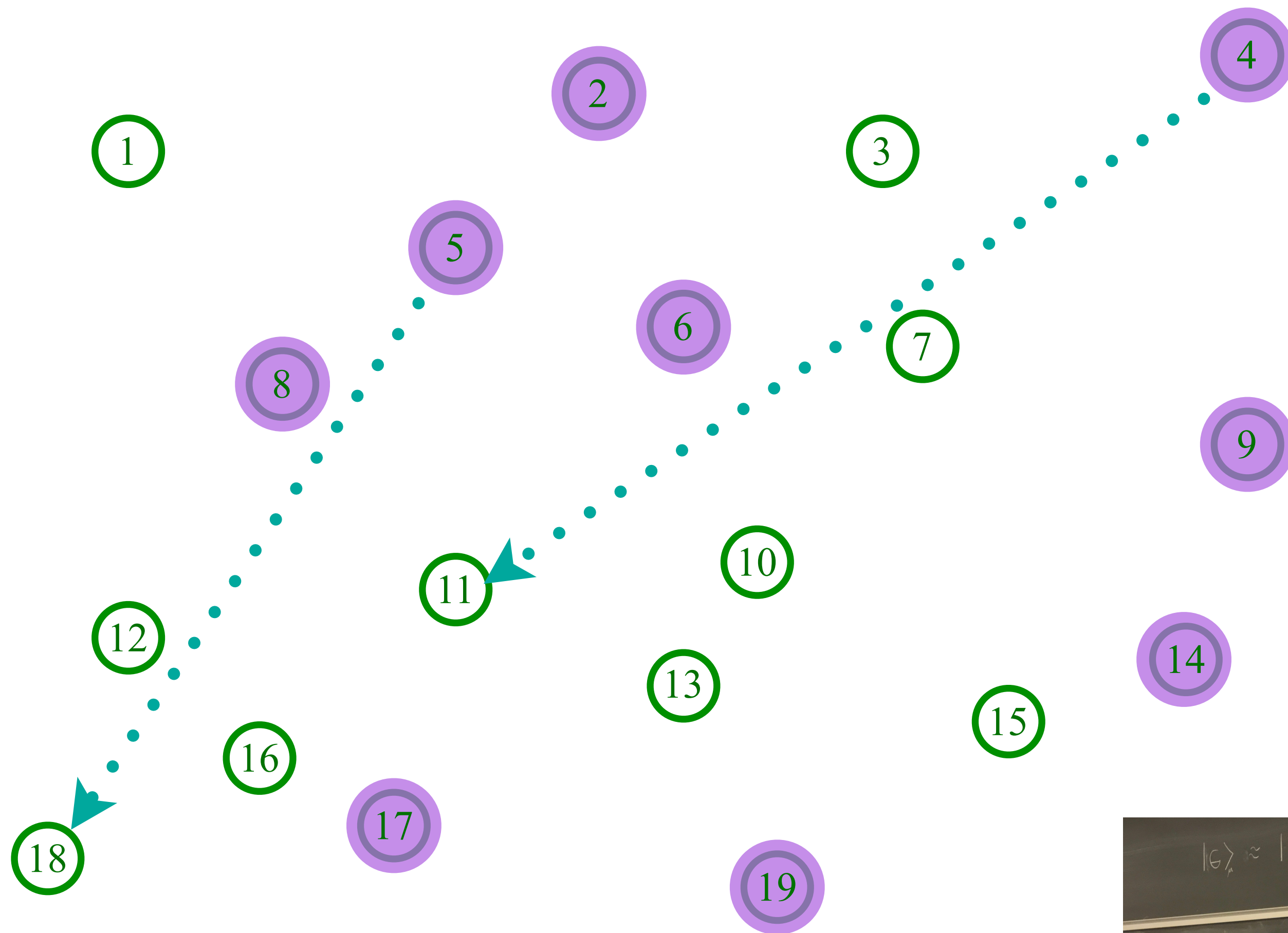
Entangle electrons pairwise randomly



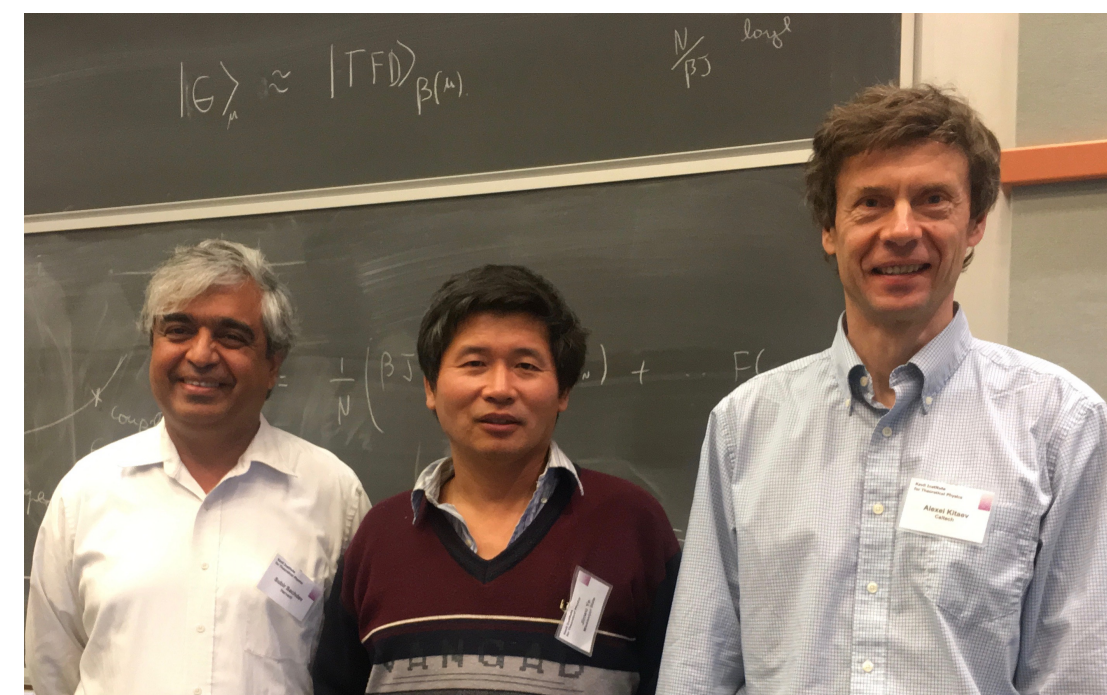
# The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



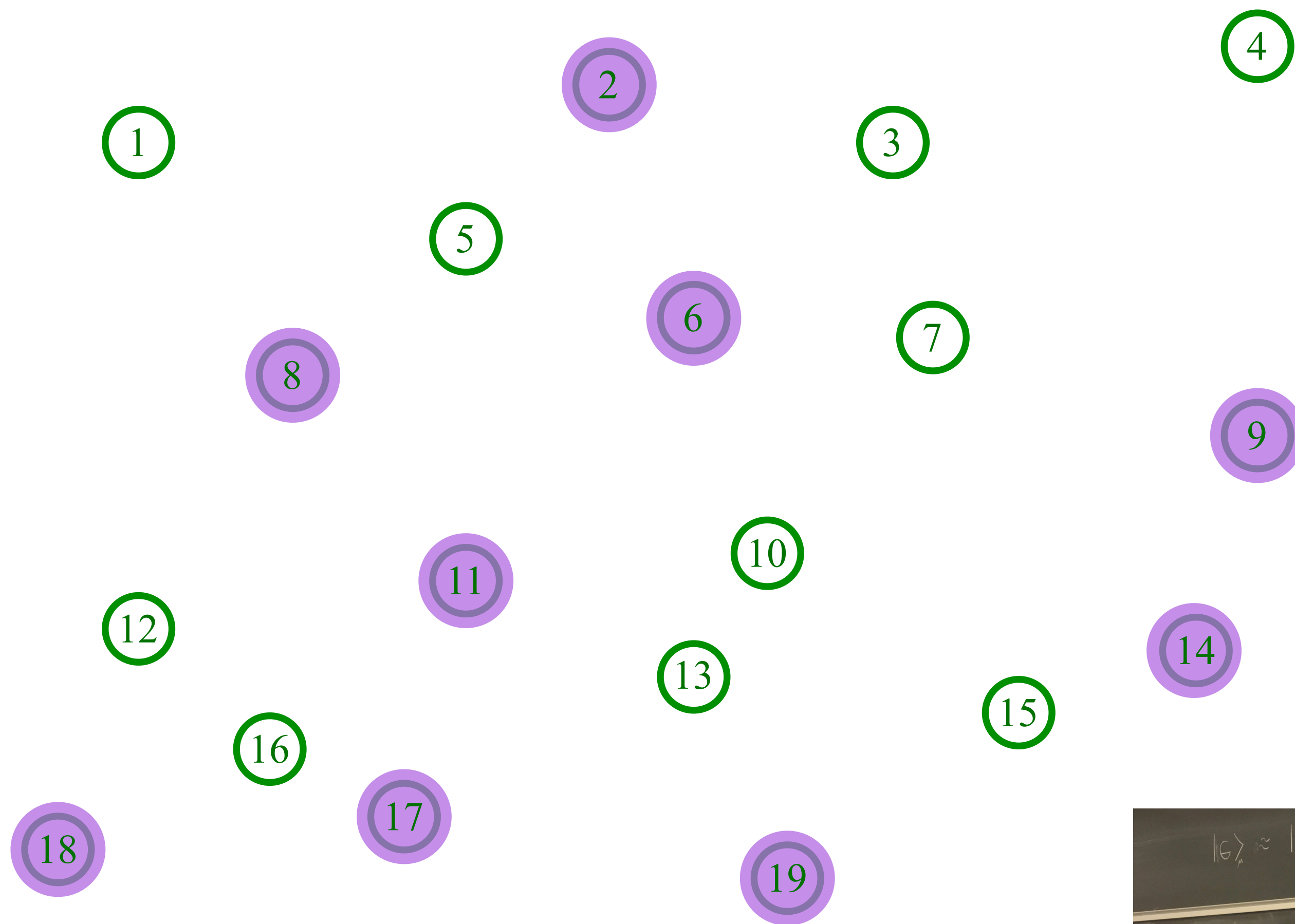
Entangle electrons pairwise randomly



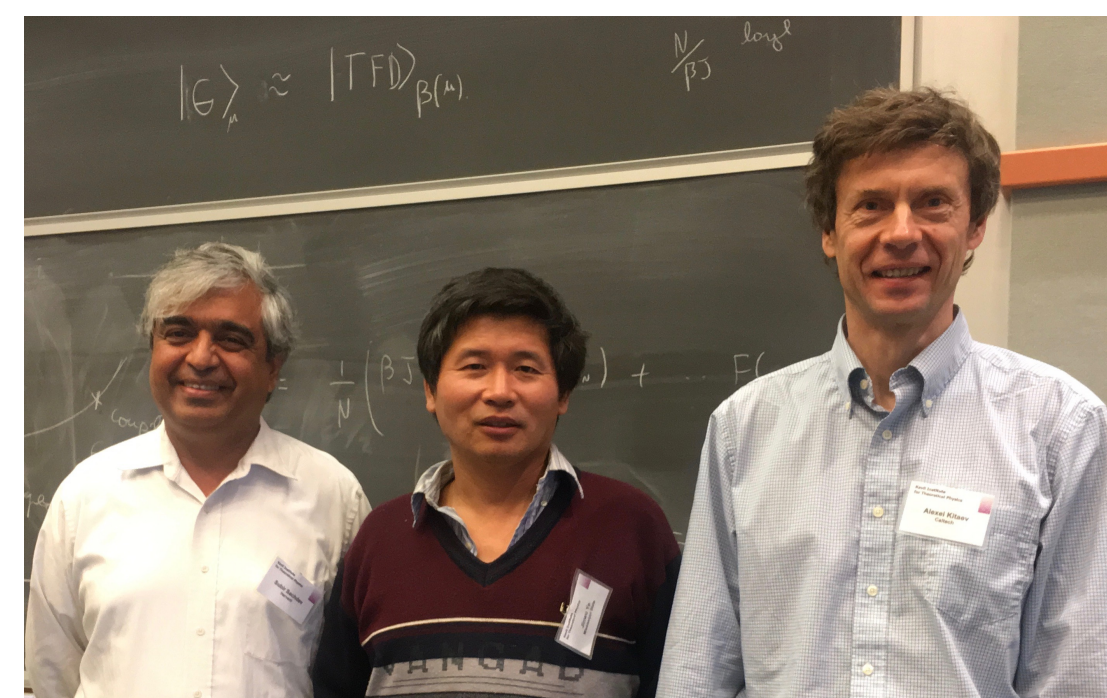
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Sachdev, Ye (1993); Kitaev (2015)

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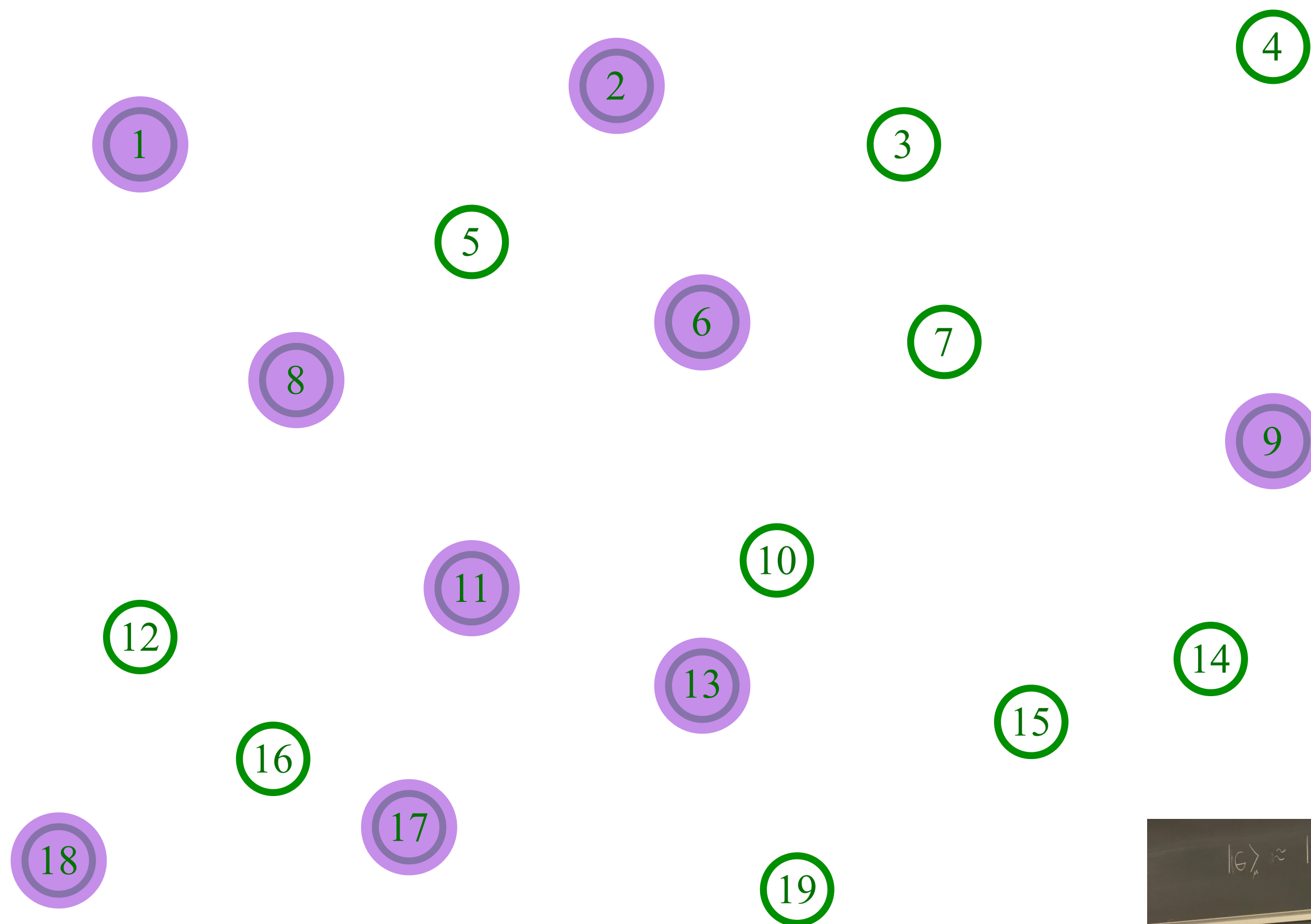




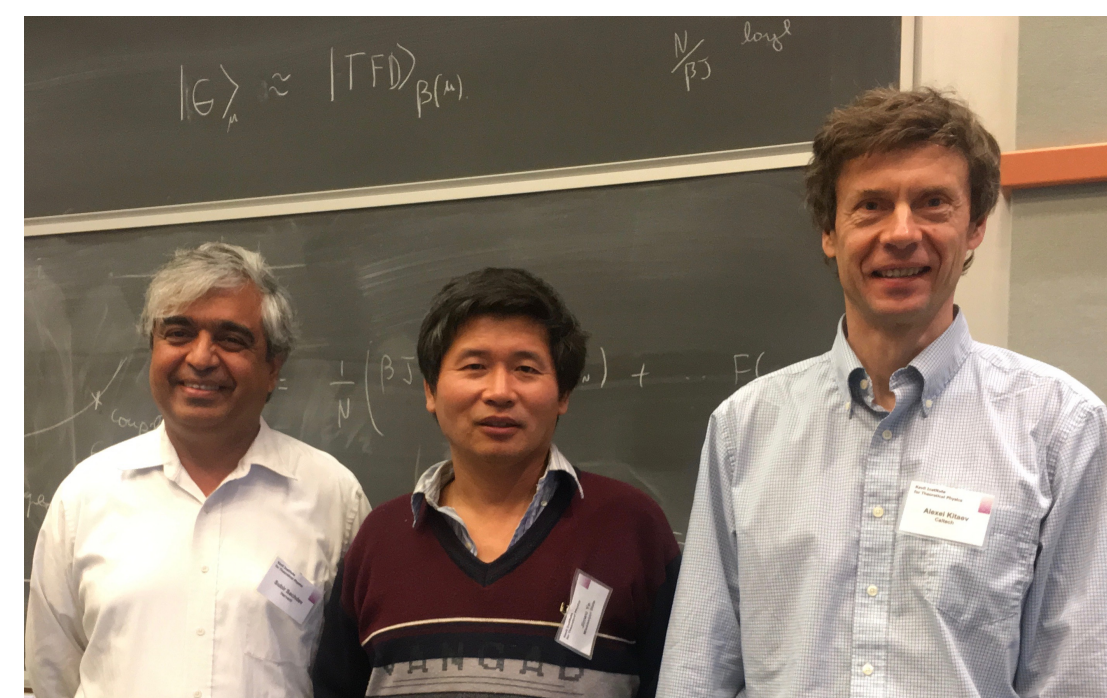
# The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



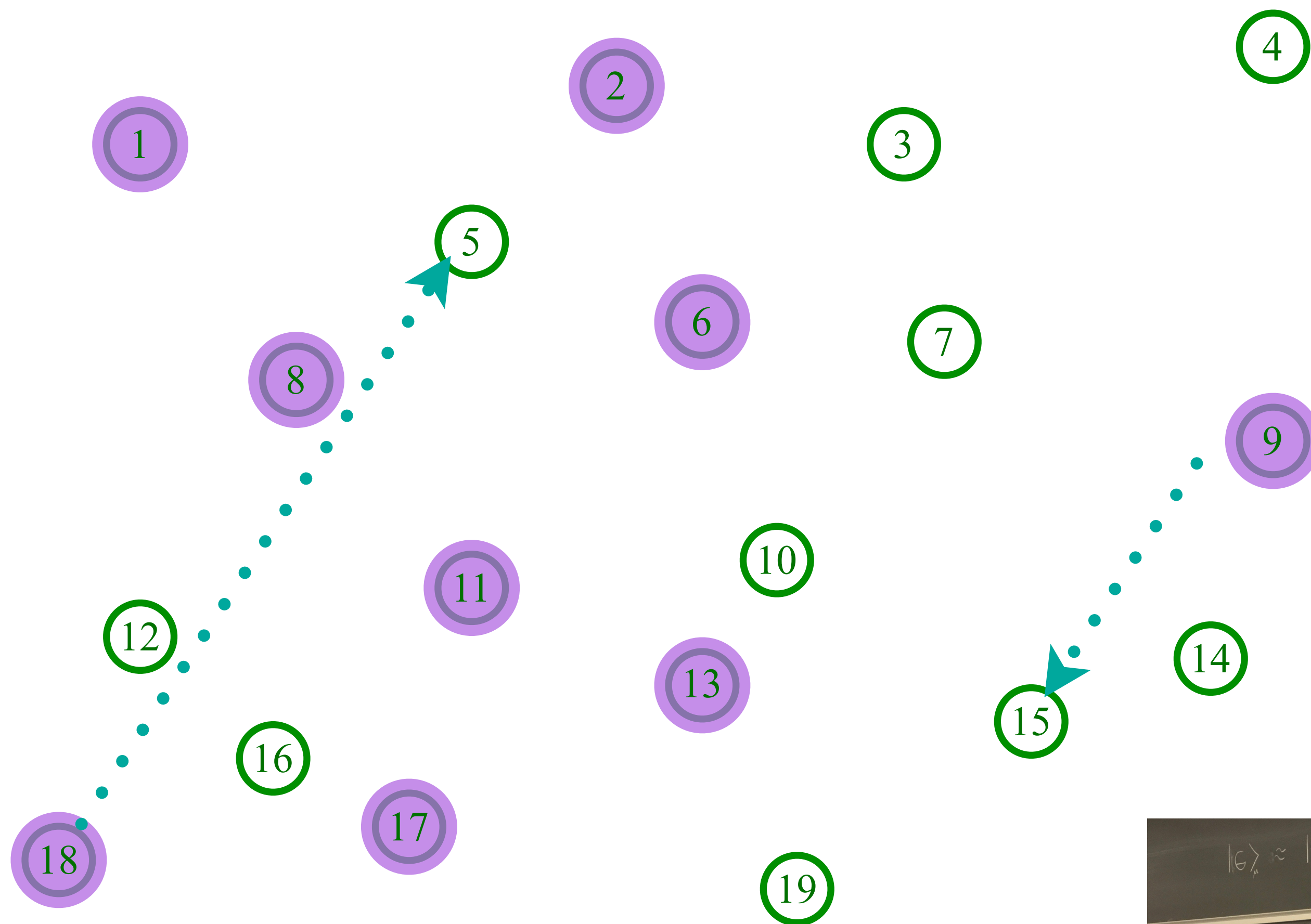
Entangle electrons pairwise randomly



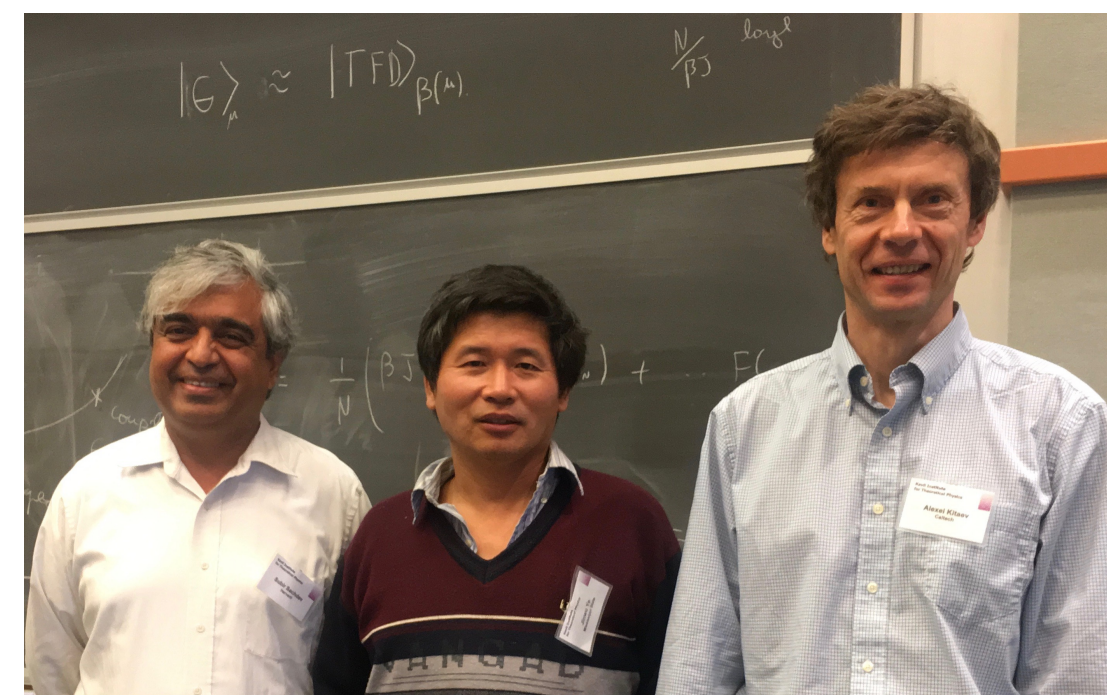
# The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



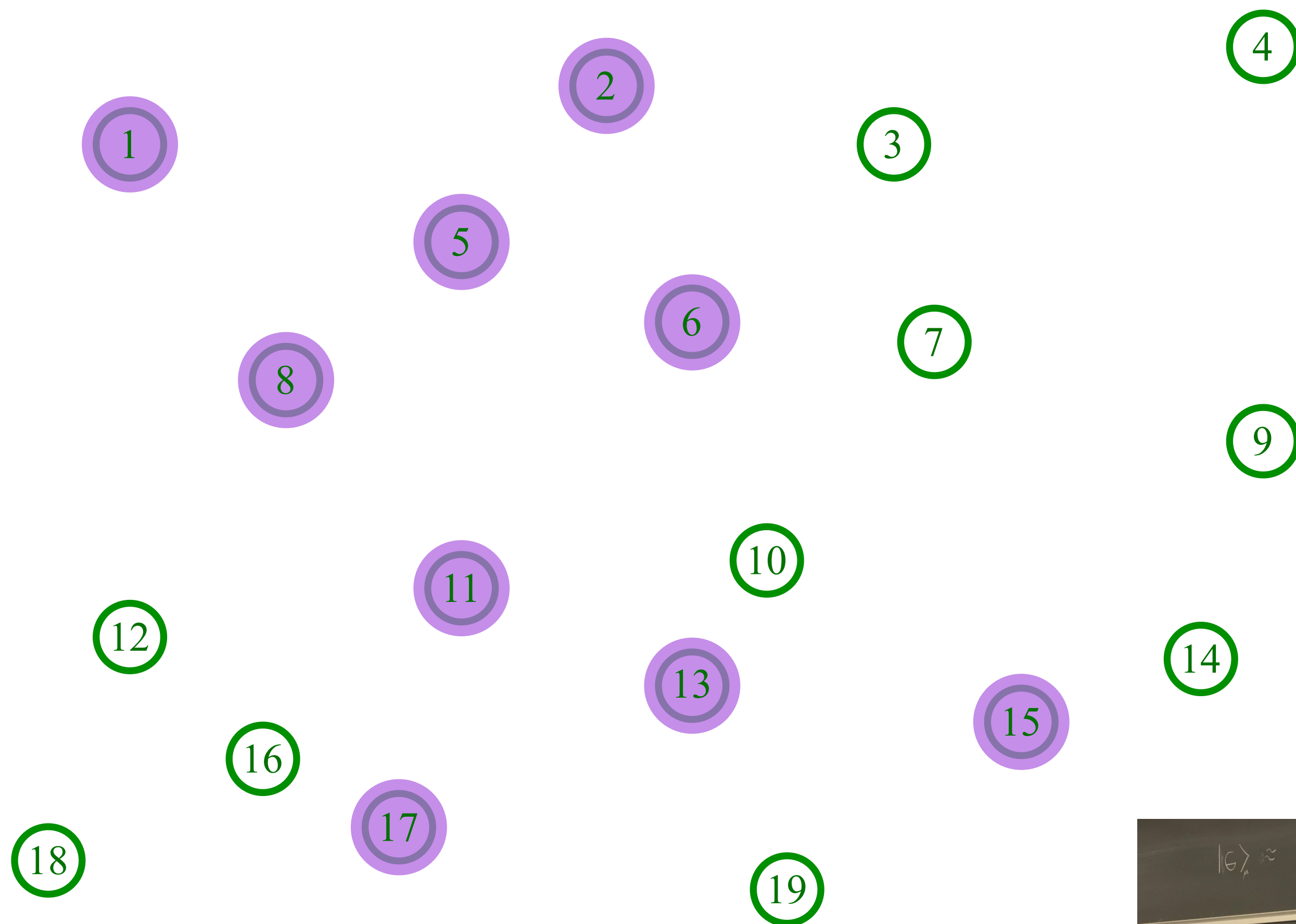
Entangle electrons pairwise randomly



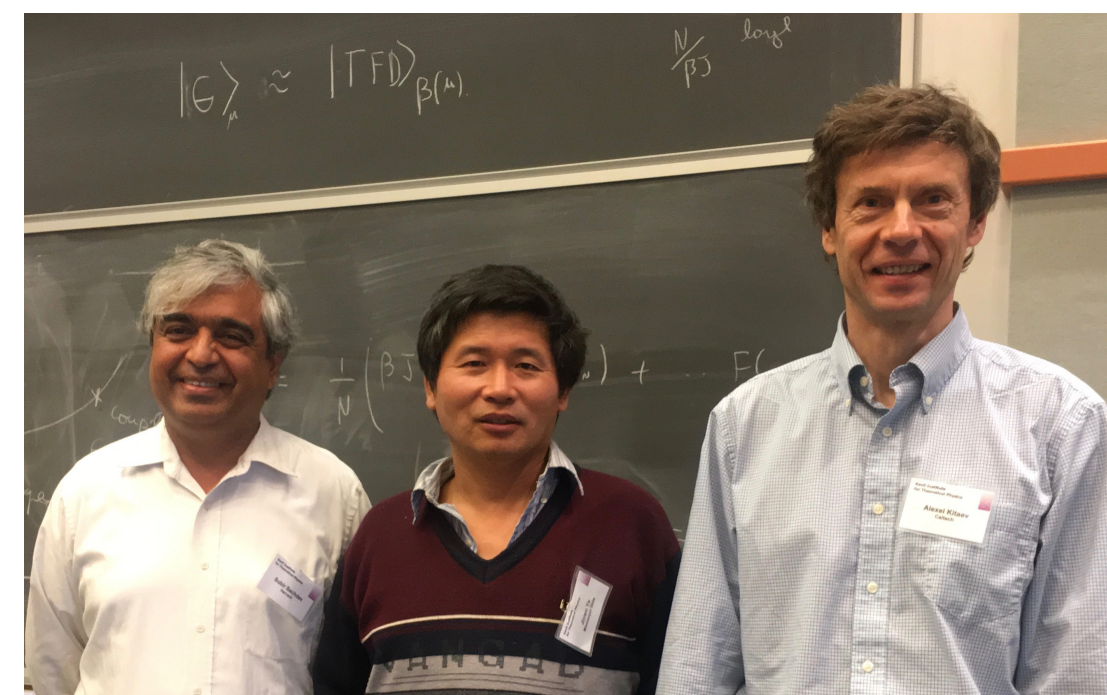
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Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



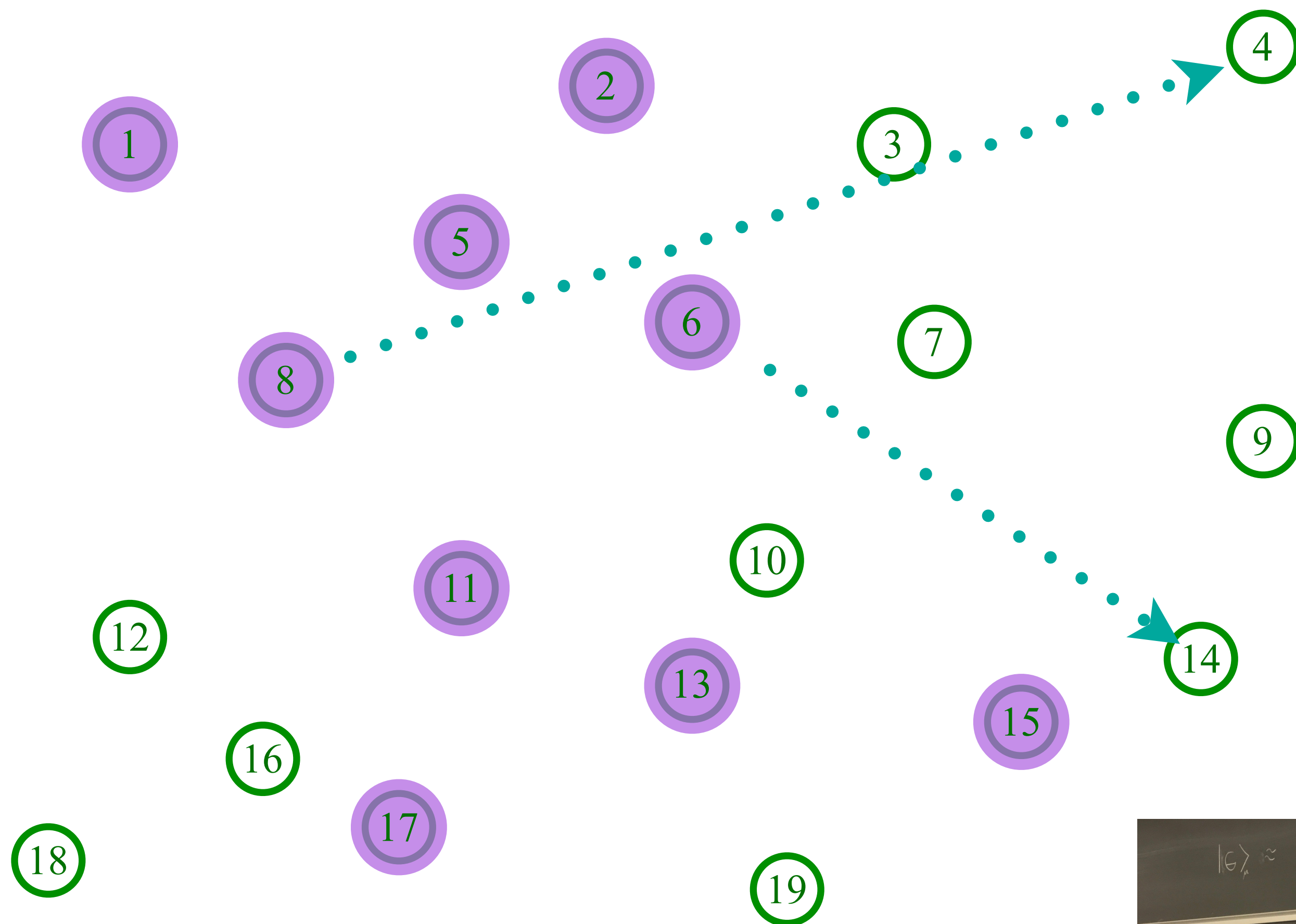
Entangle electrons pairwise randomly



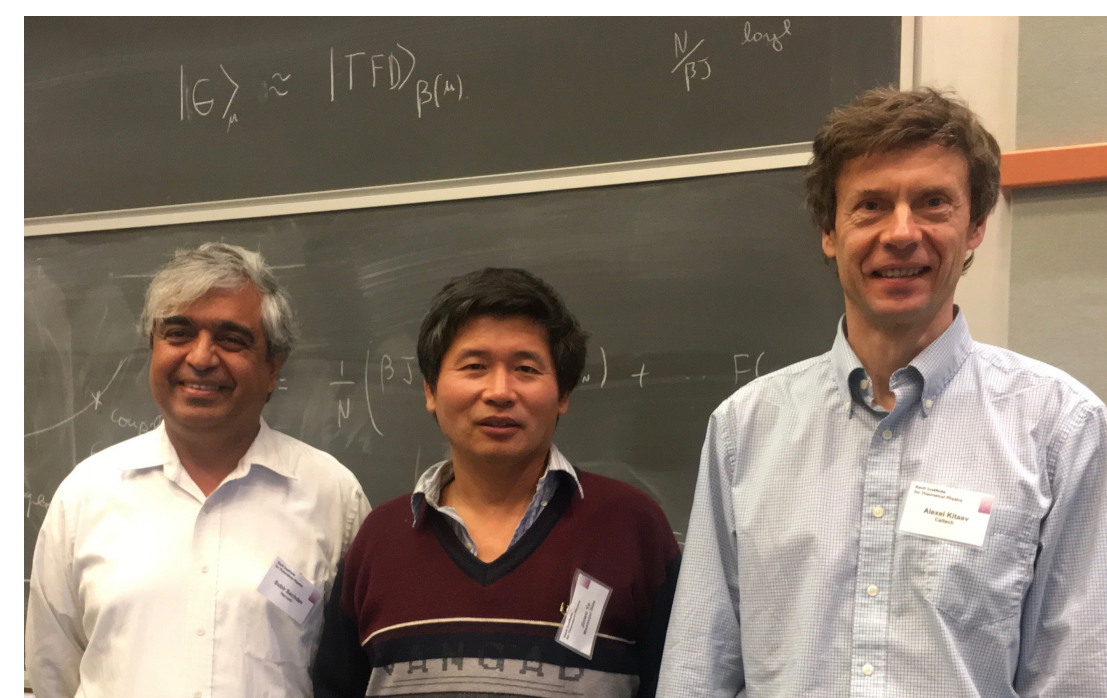
# The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



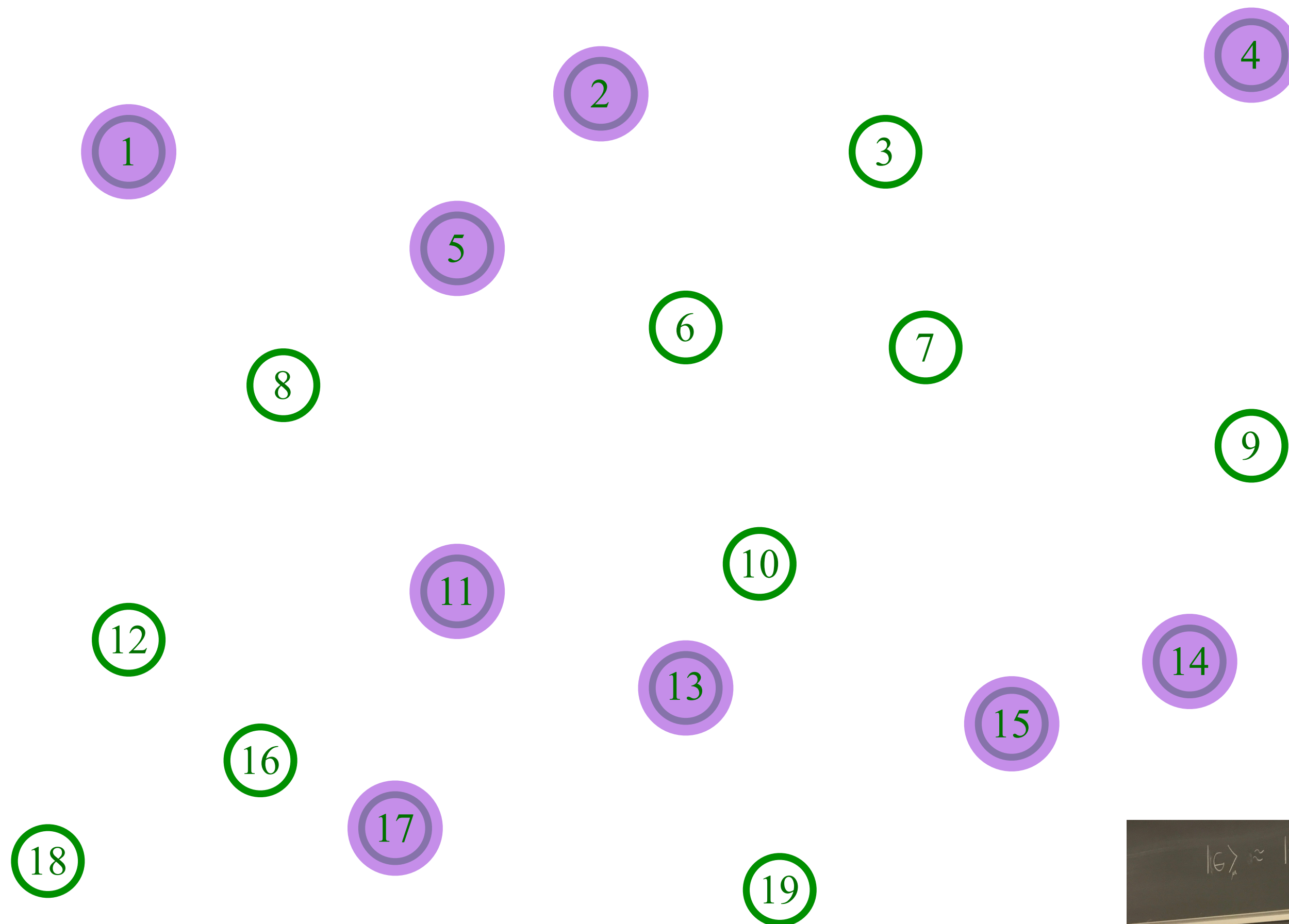
Entangle electrons pairwise randomly



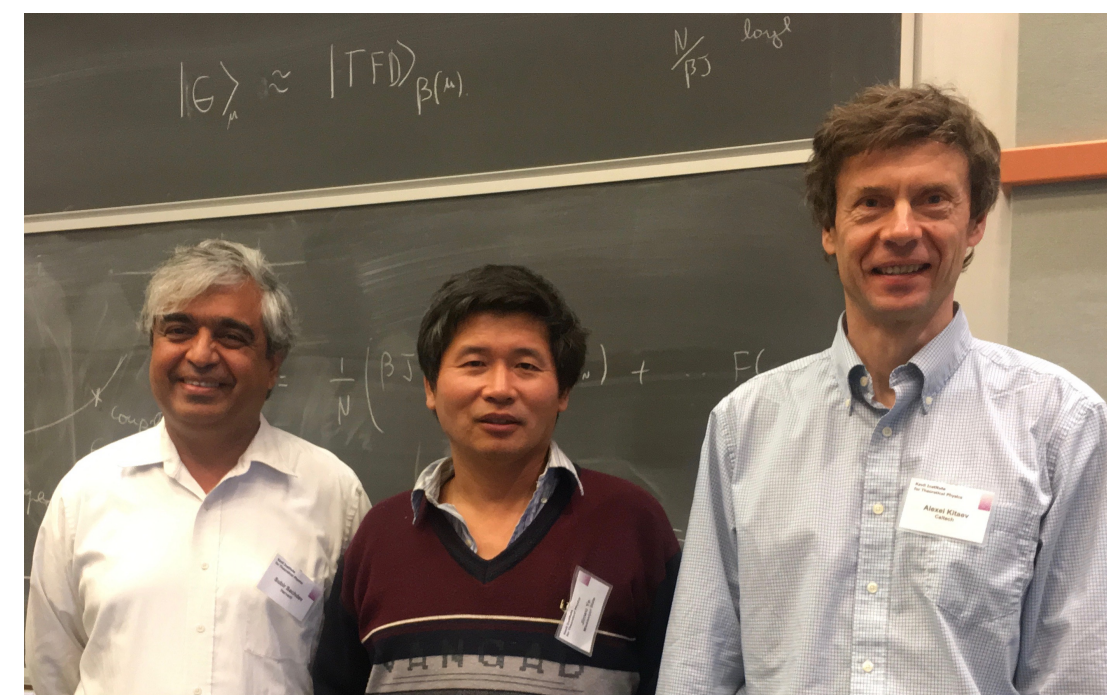
# The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



# The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large  $N$  limit;  
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

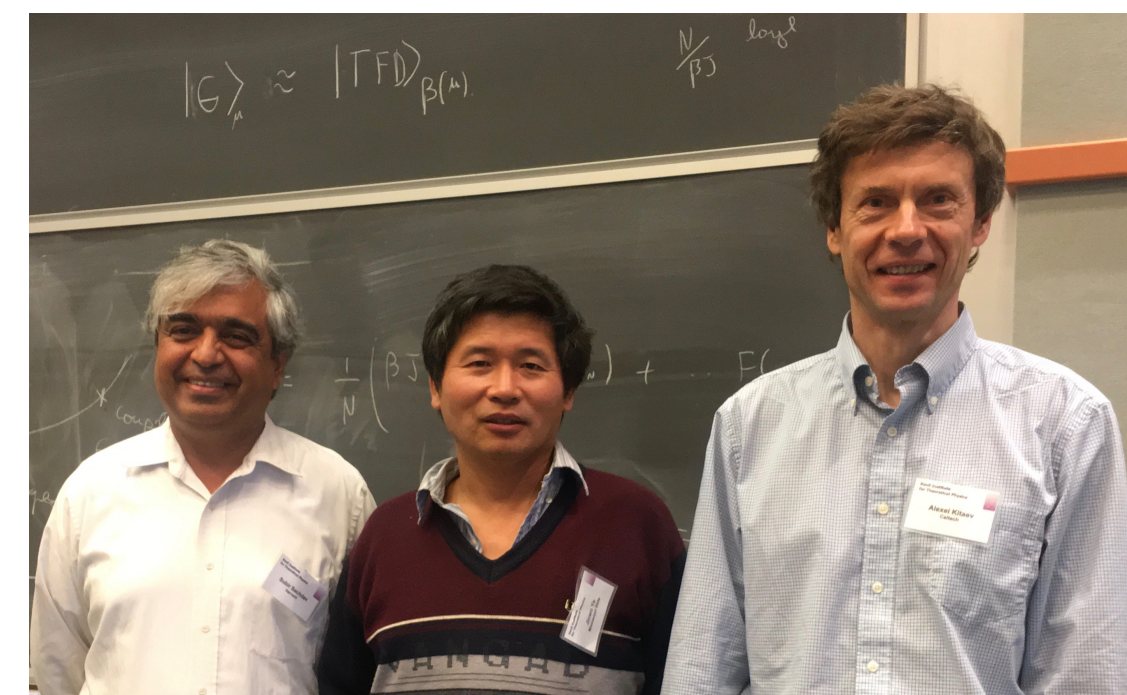
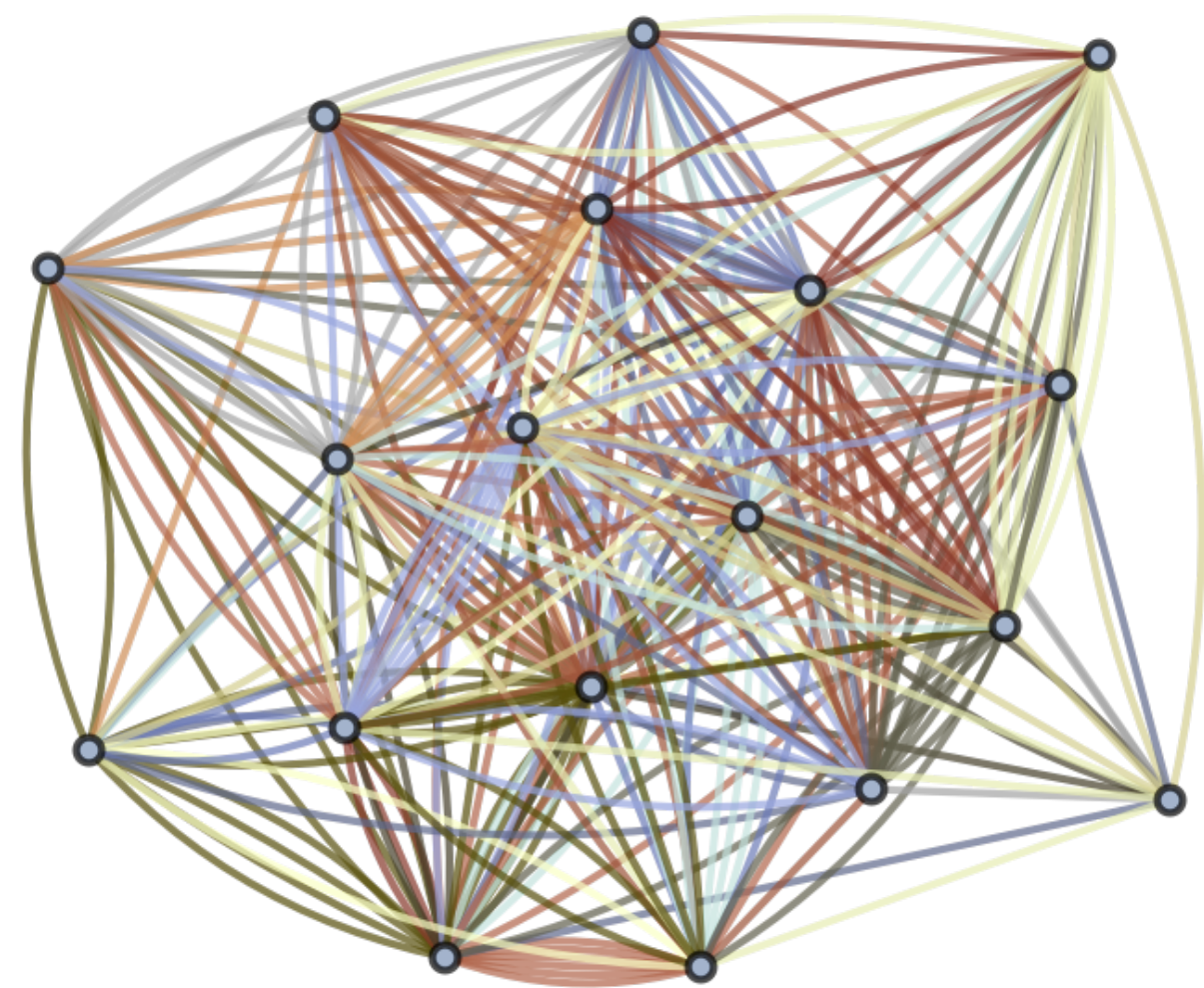
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}; \quad [\mathcal{H}, \mathcal{Q}] = 0; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$  are independent random variables with  $\overline{U_{\alpha\beta;\gamma\delta}} = 0$  and  $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$   
 $N \rightarrow \infty$  yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)



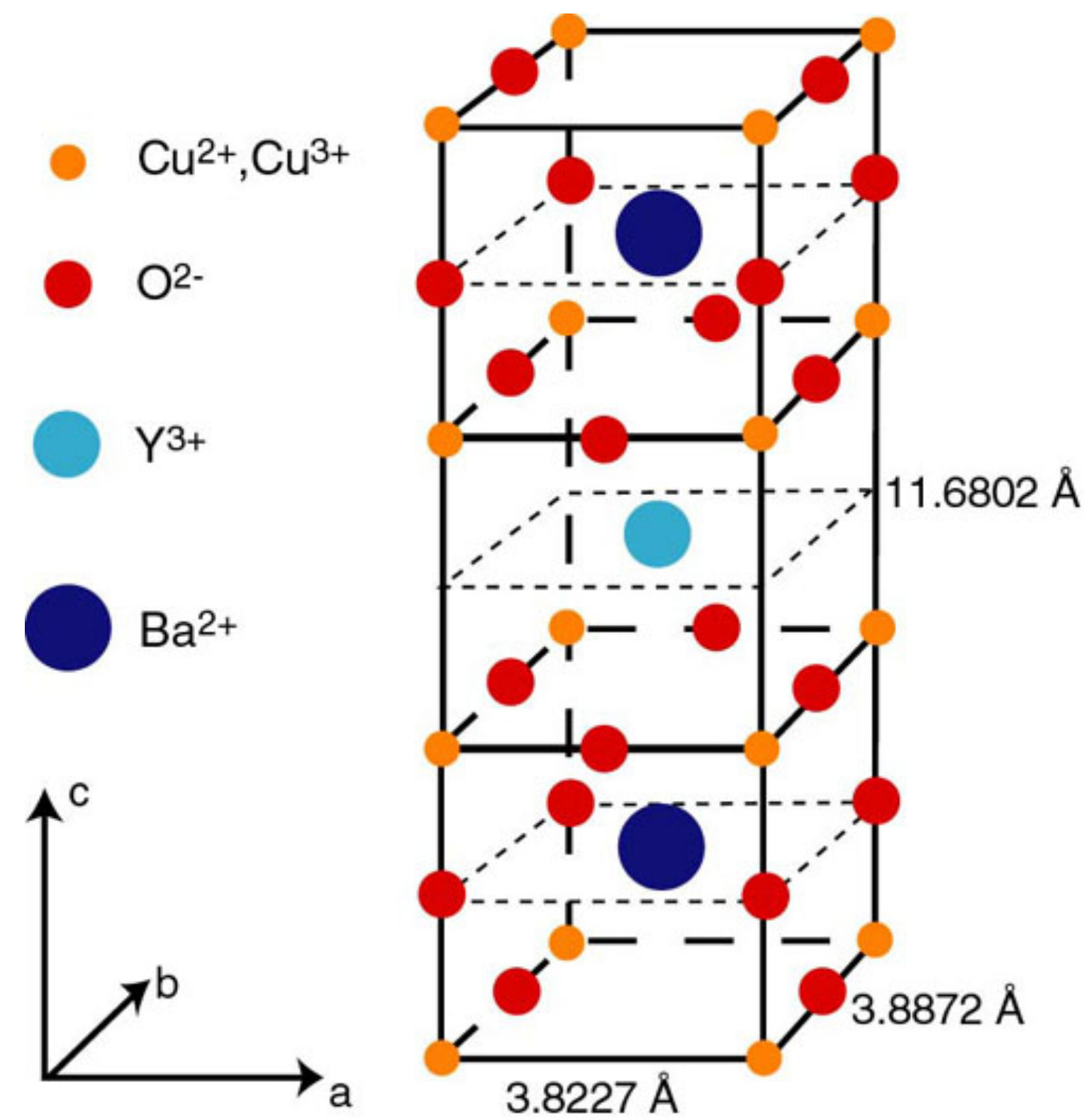
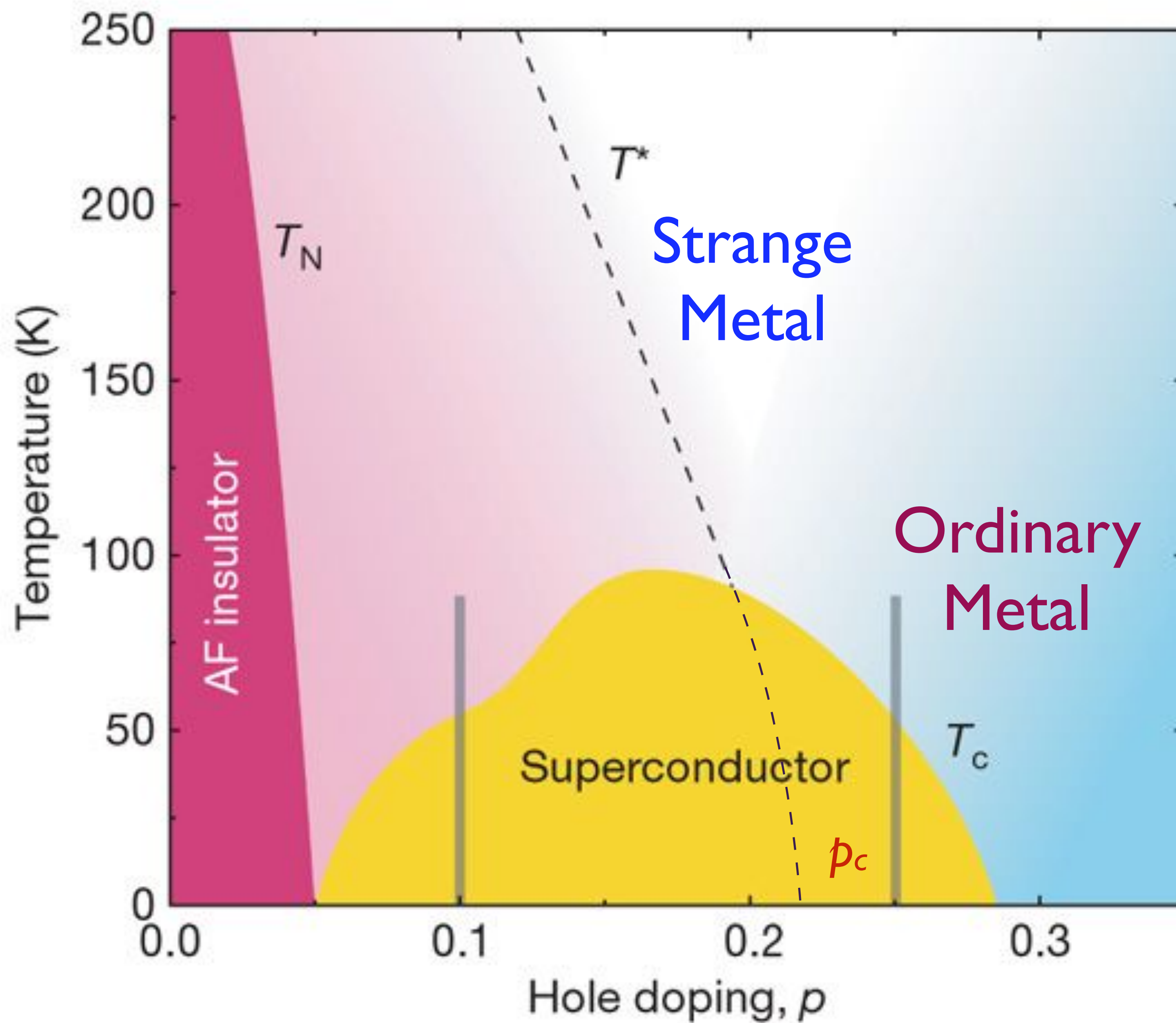
# The Sachdev-Ye-Kitaev (SYK) model

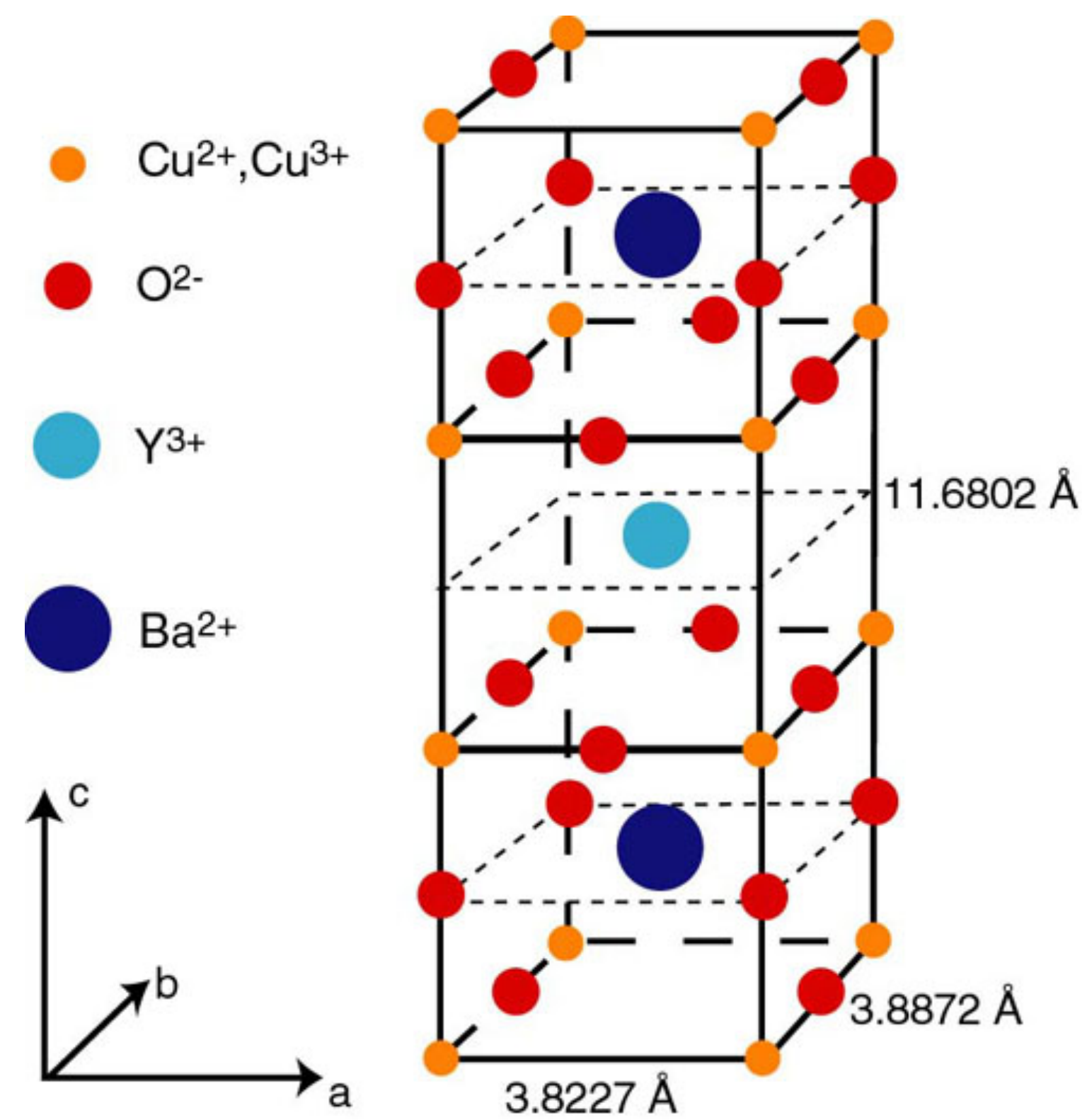
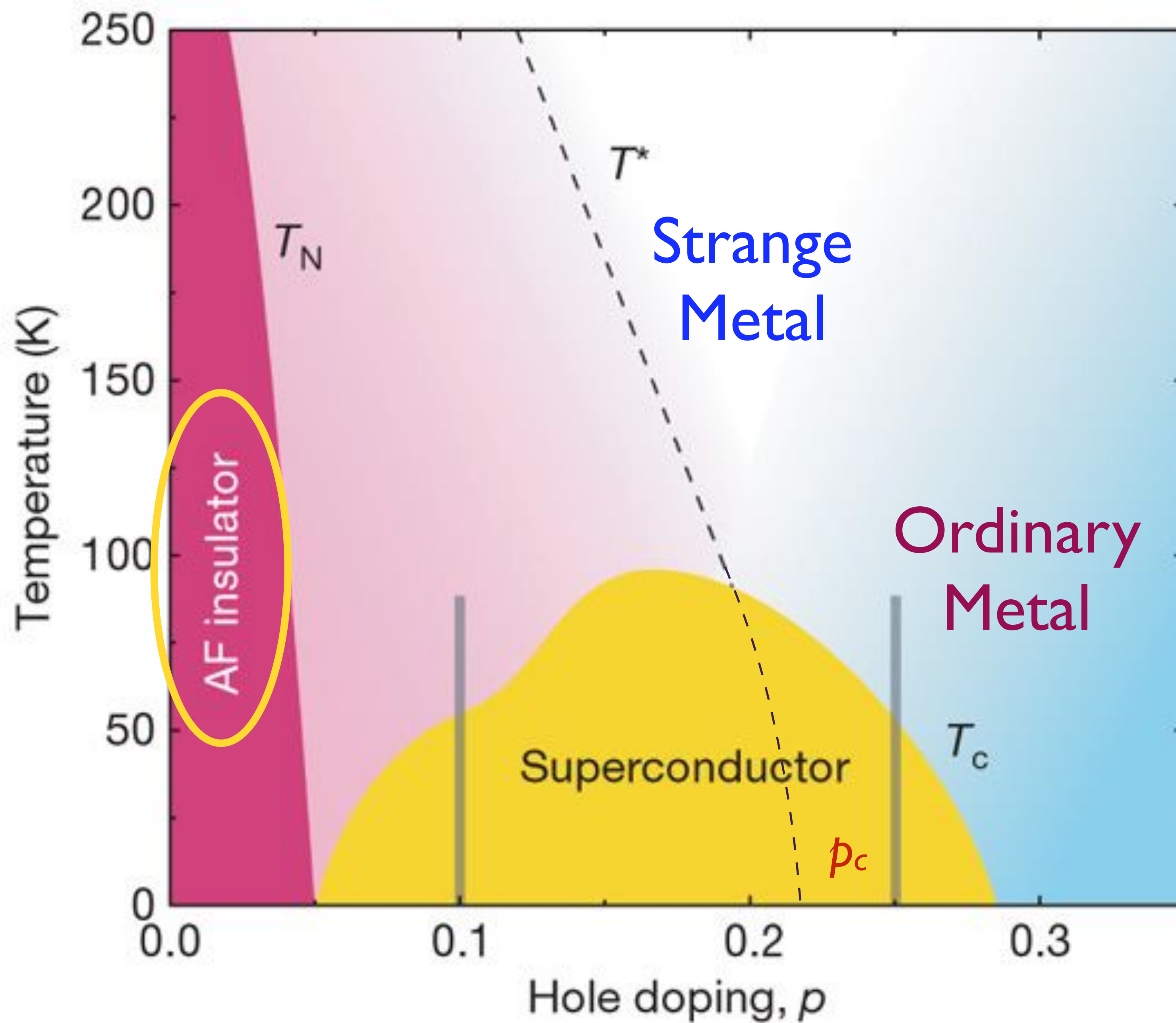
Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle  
quantum entanglement.

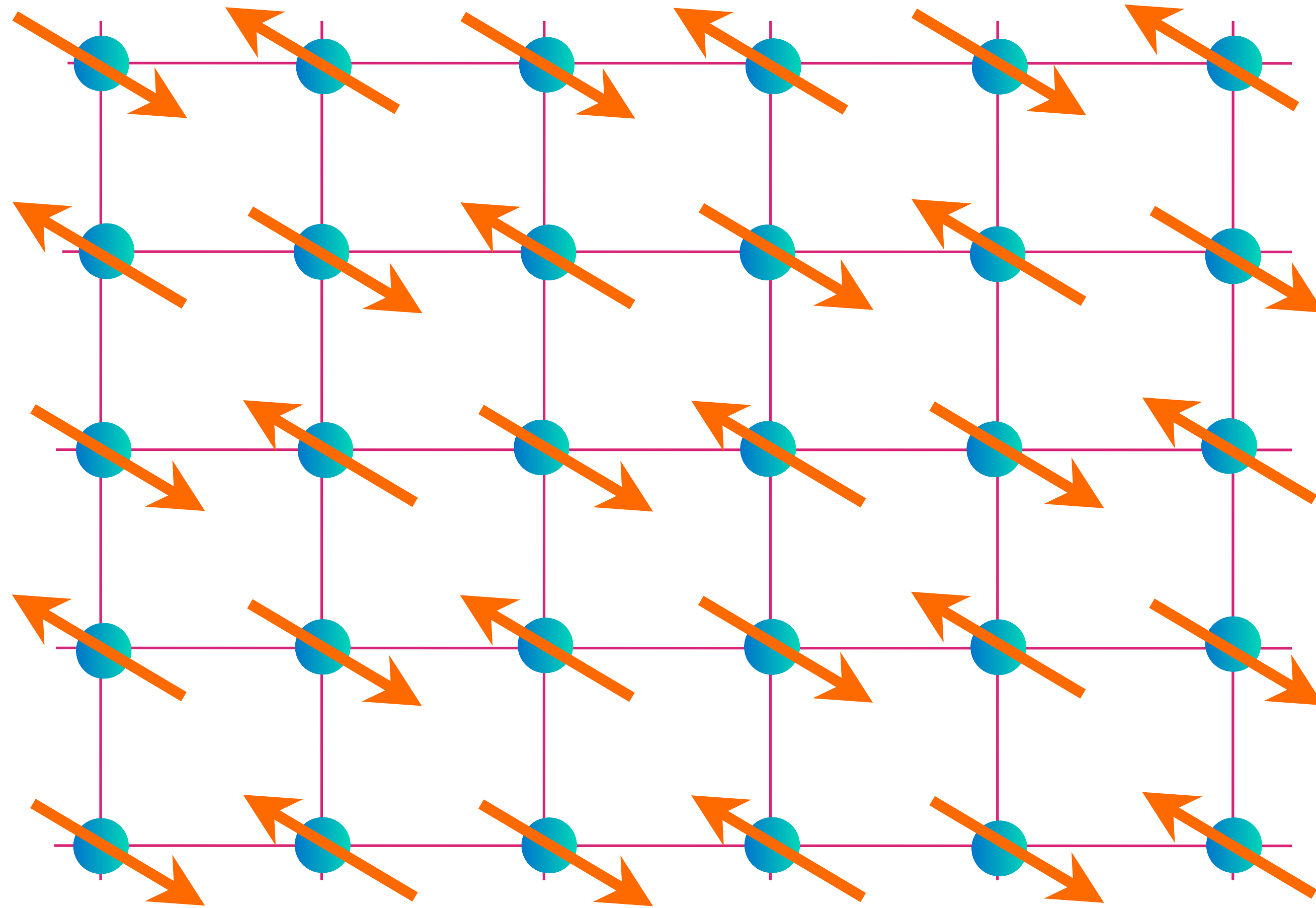
Yields a metal in which current is carried  
not by individual electrons,  
but by an entangled “quantum soup”

Spin liquids,  
fractionalization,  
and  
superconductivity





# The dance of electrons on Cu atoms in YBCO



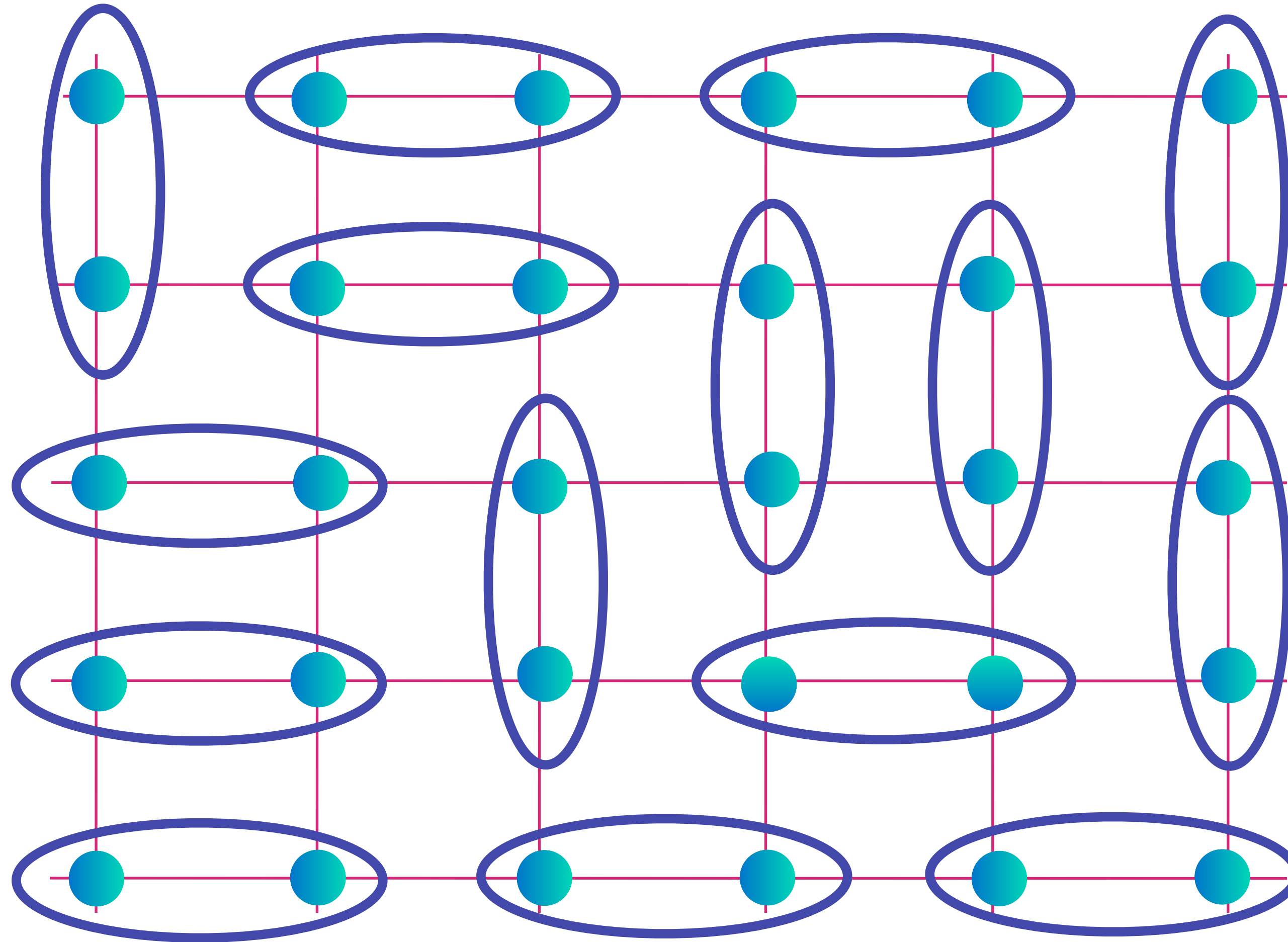
**Antiferromagnetism**

All nearest-neighbor pairs of electrons have opposite spins

# The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

**Spin liquid**



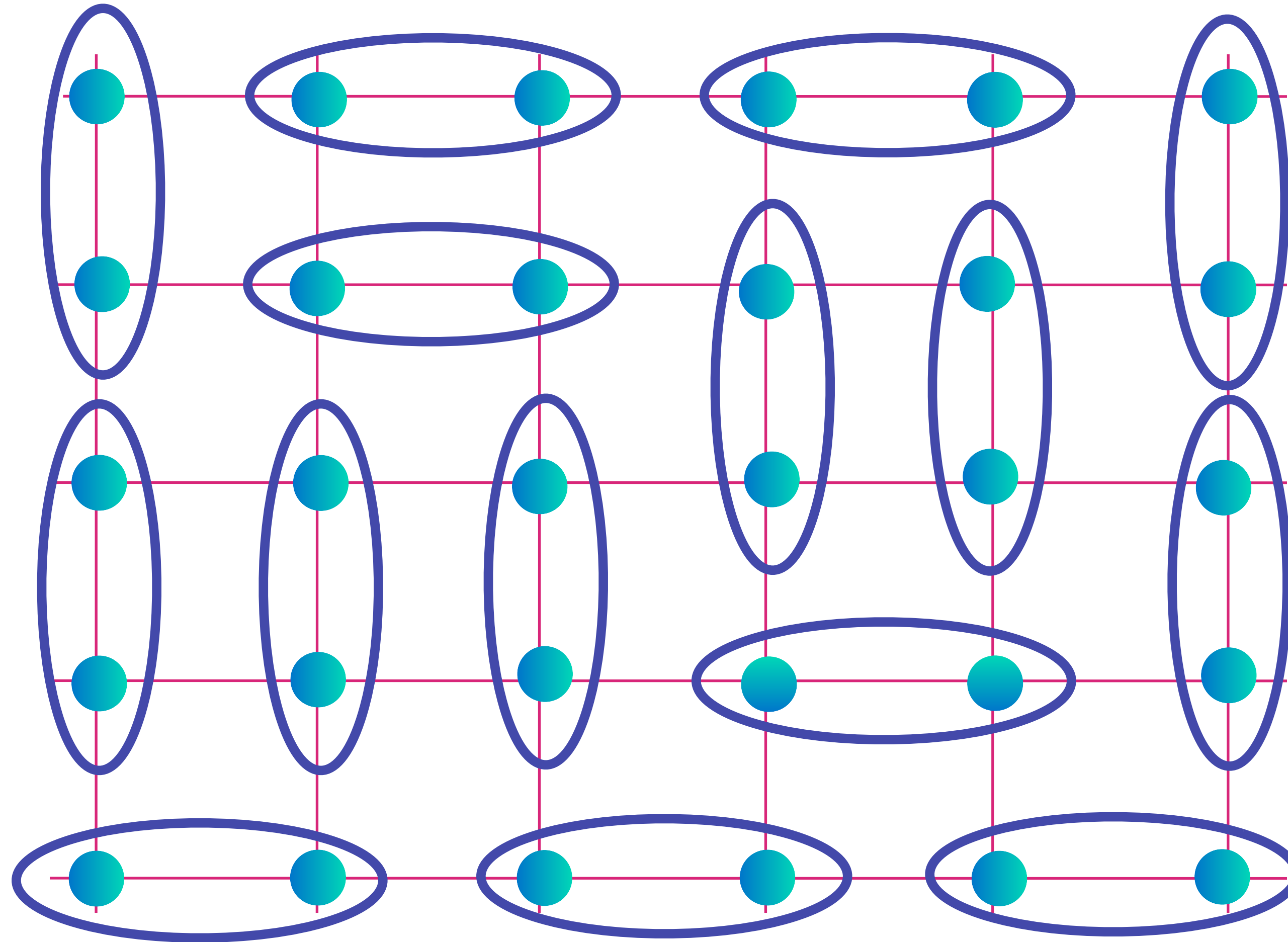
Electrons form entangled pairs, and the pairs entangle across the entire sample

$$\text{[Diagram of two teal dots in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

**Spin liquid**



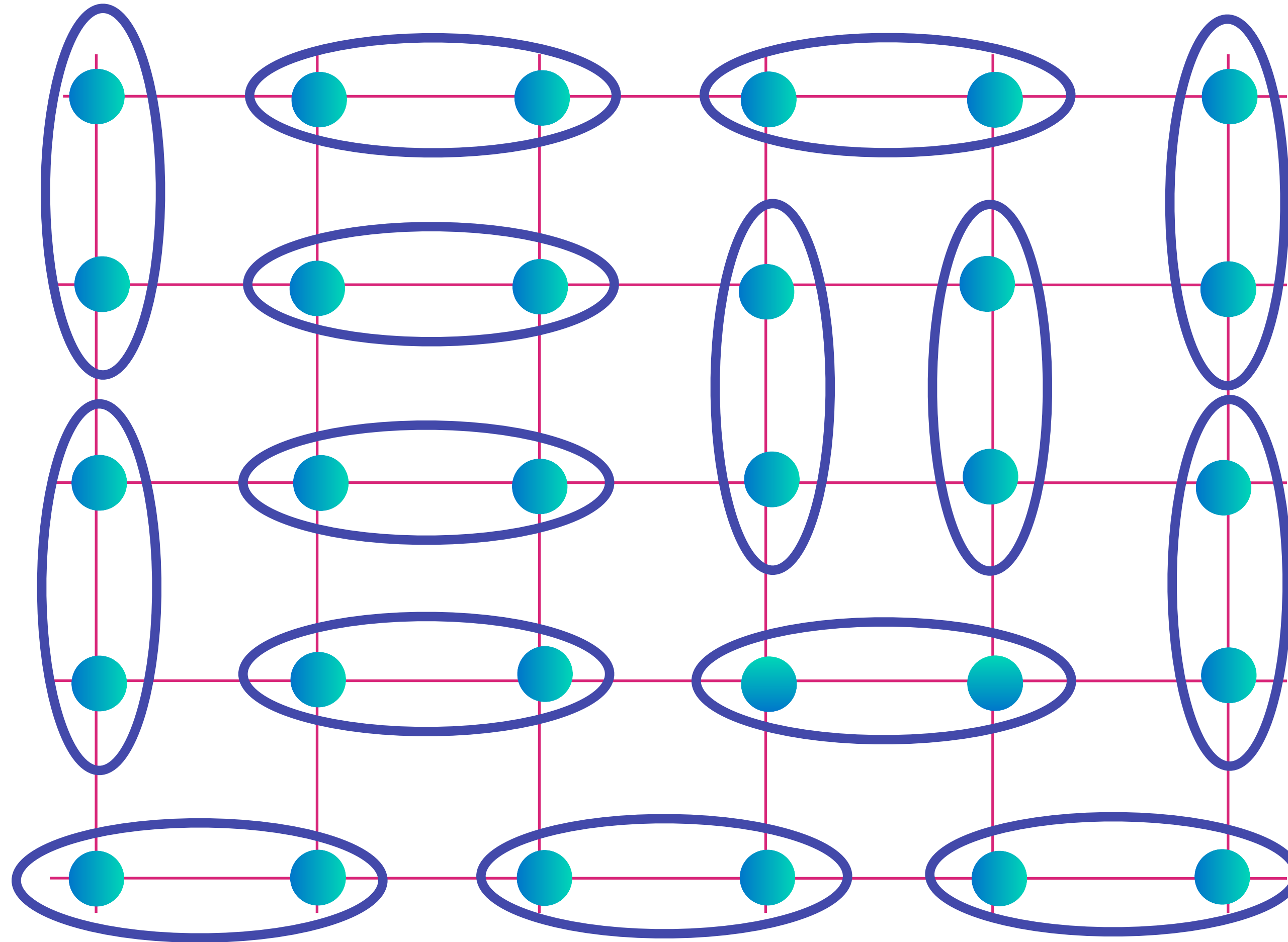
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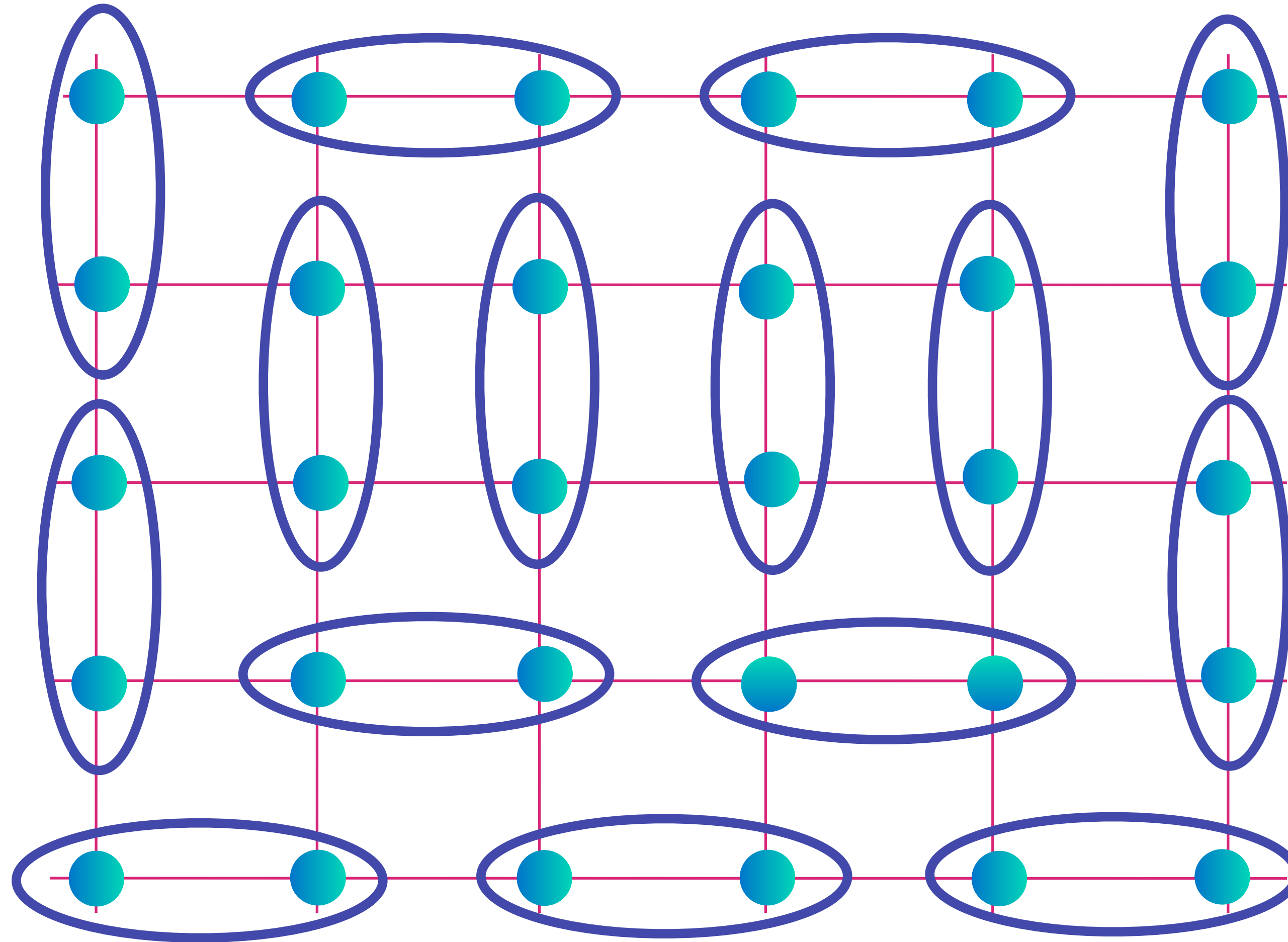
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P.W. Anderson (1973)

**Spin liquid**



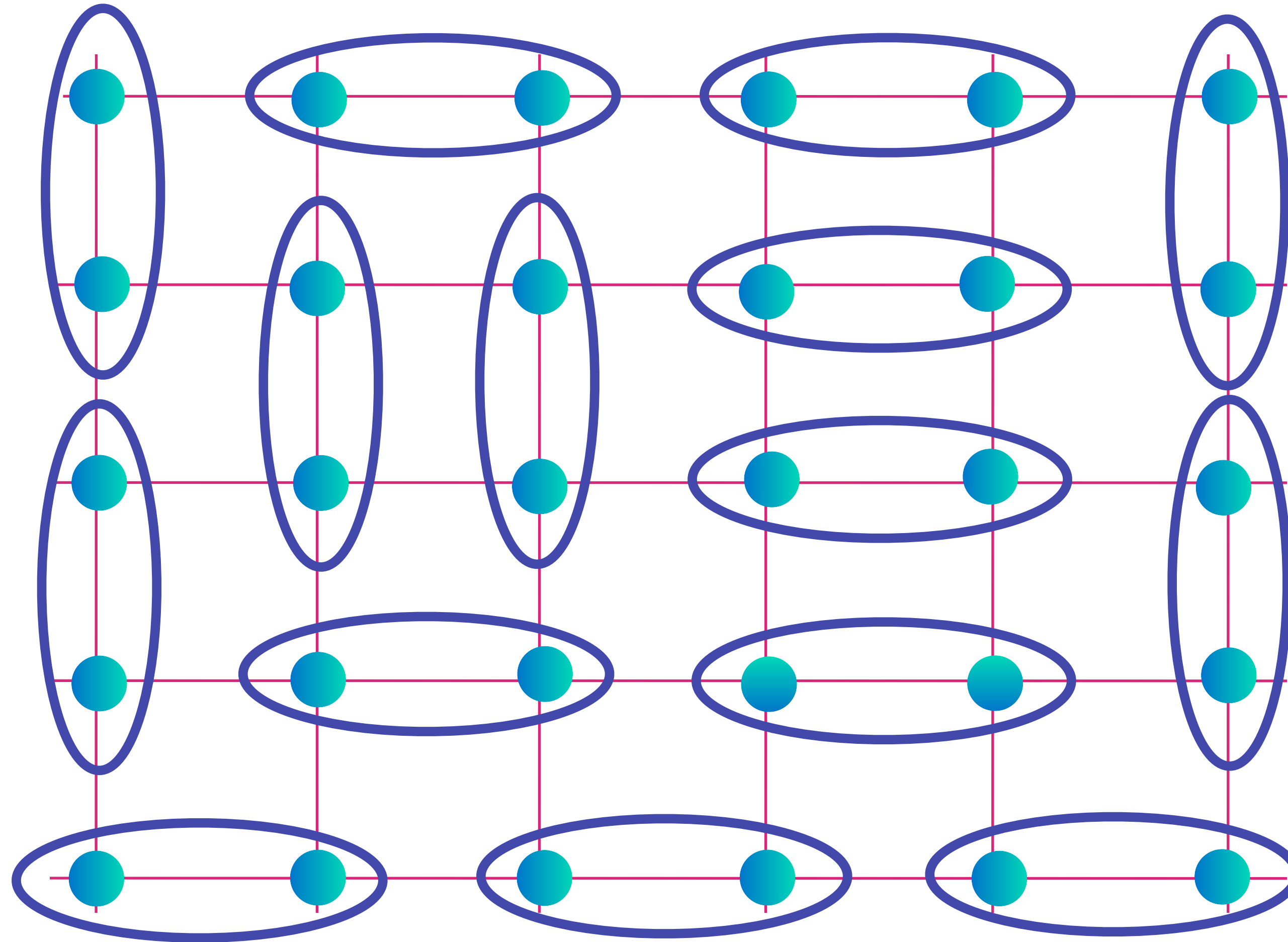
Electrons form entangled pairs, and the pairs entangle across the entire sample

$$\text{[Diagram of two teal dots in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

P.W. Anderson (1973)

**Spin liquid**



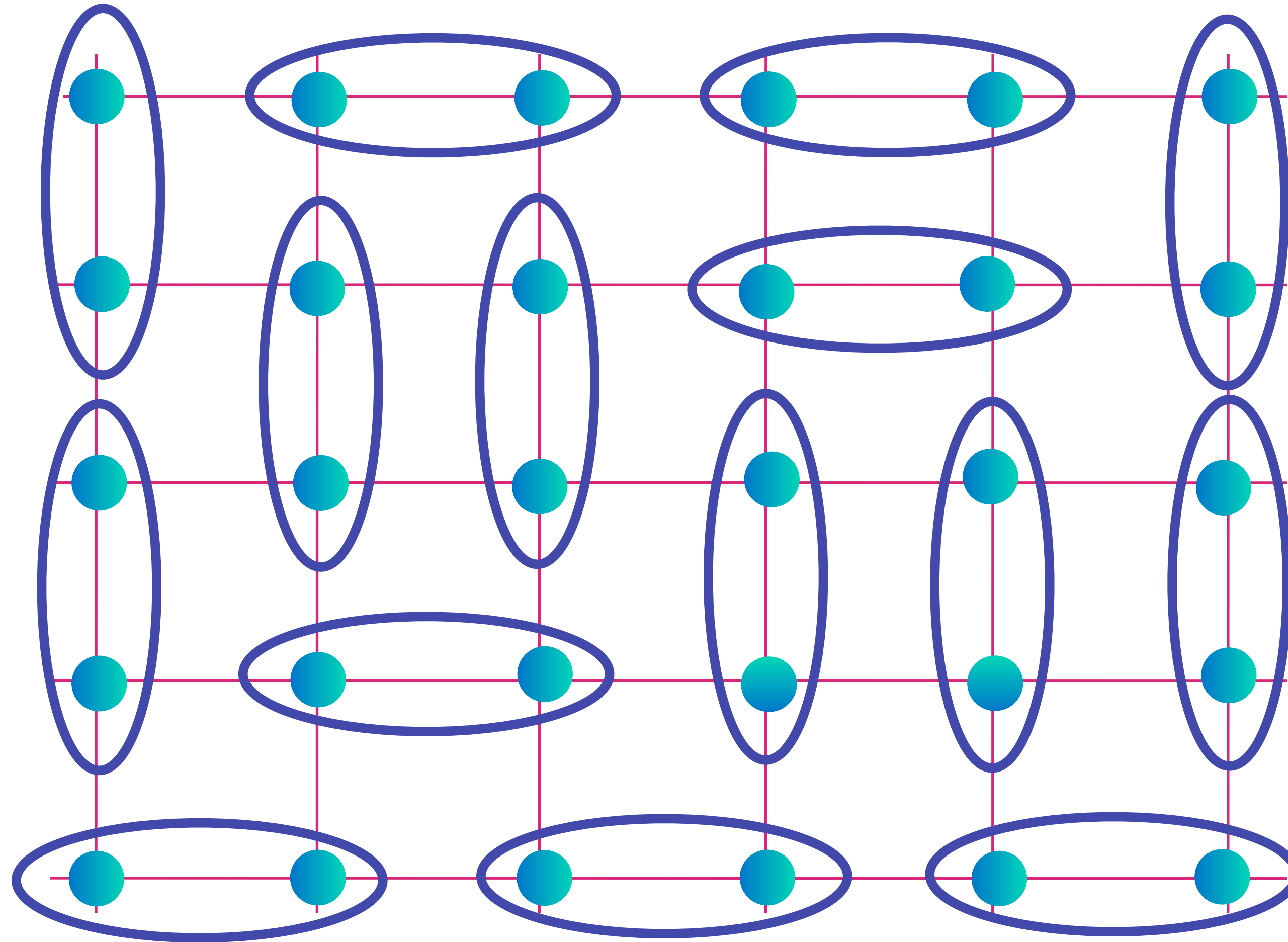
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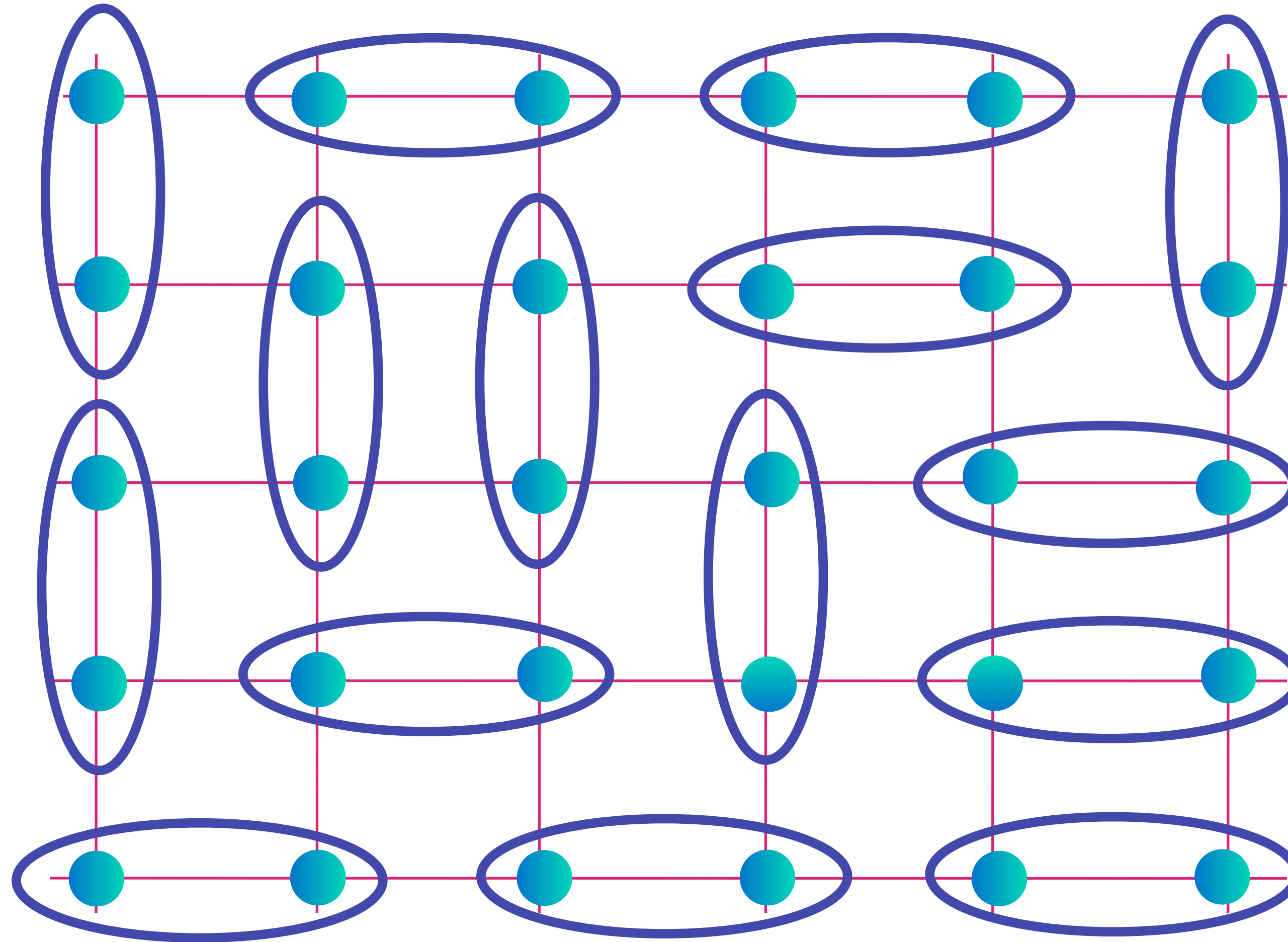
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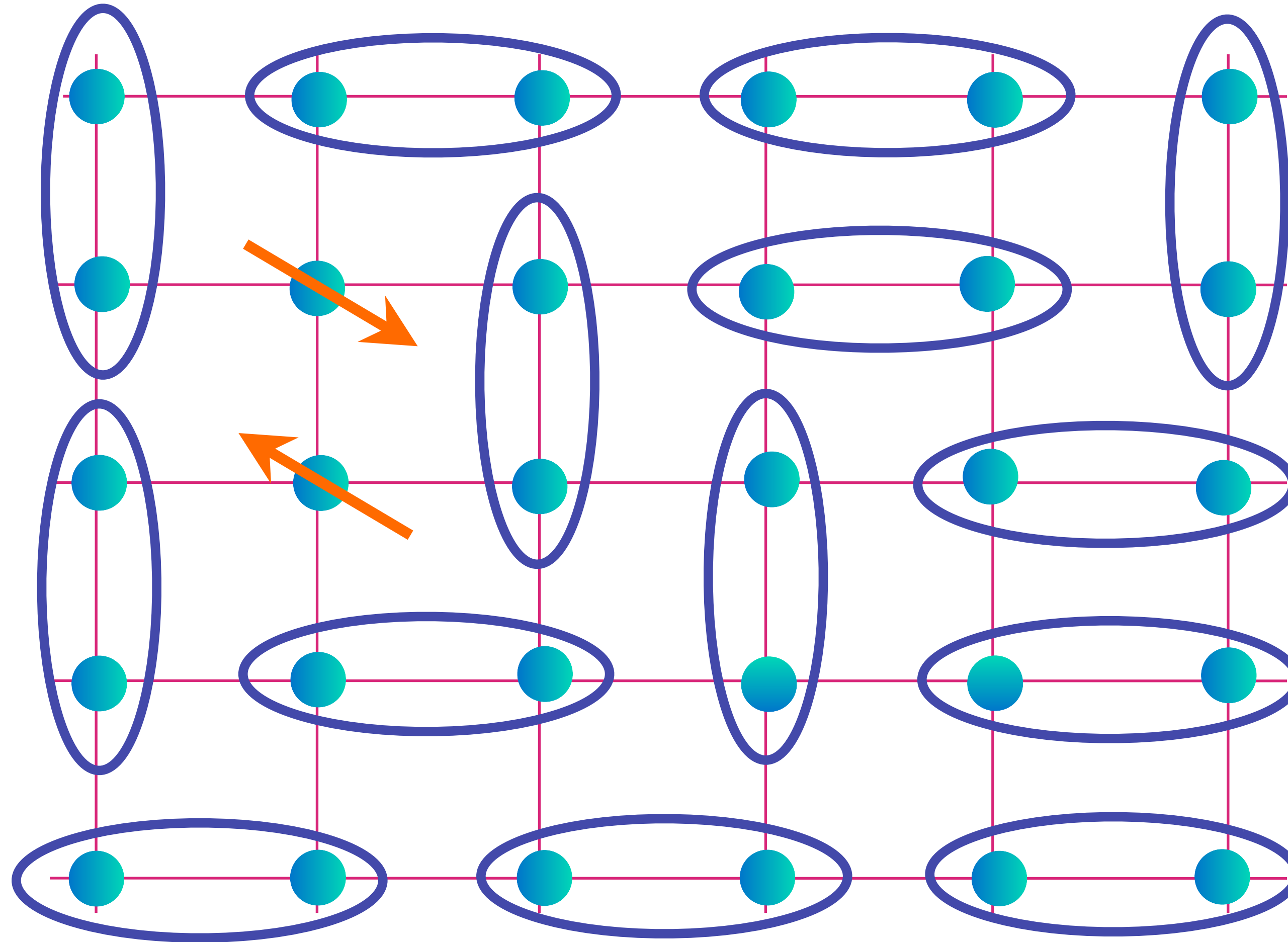
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# The dance of electrons on Cu atoms in YBCO

Kivelson (1987)

**Fractionalization**



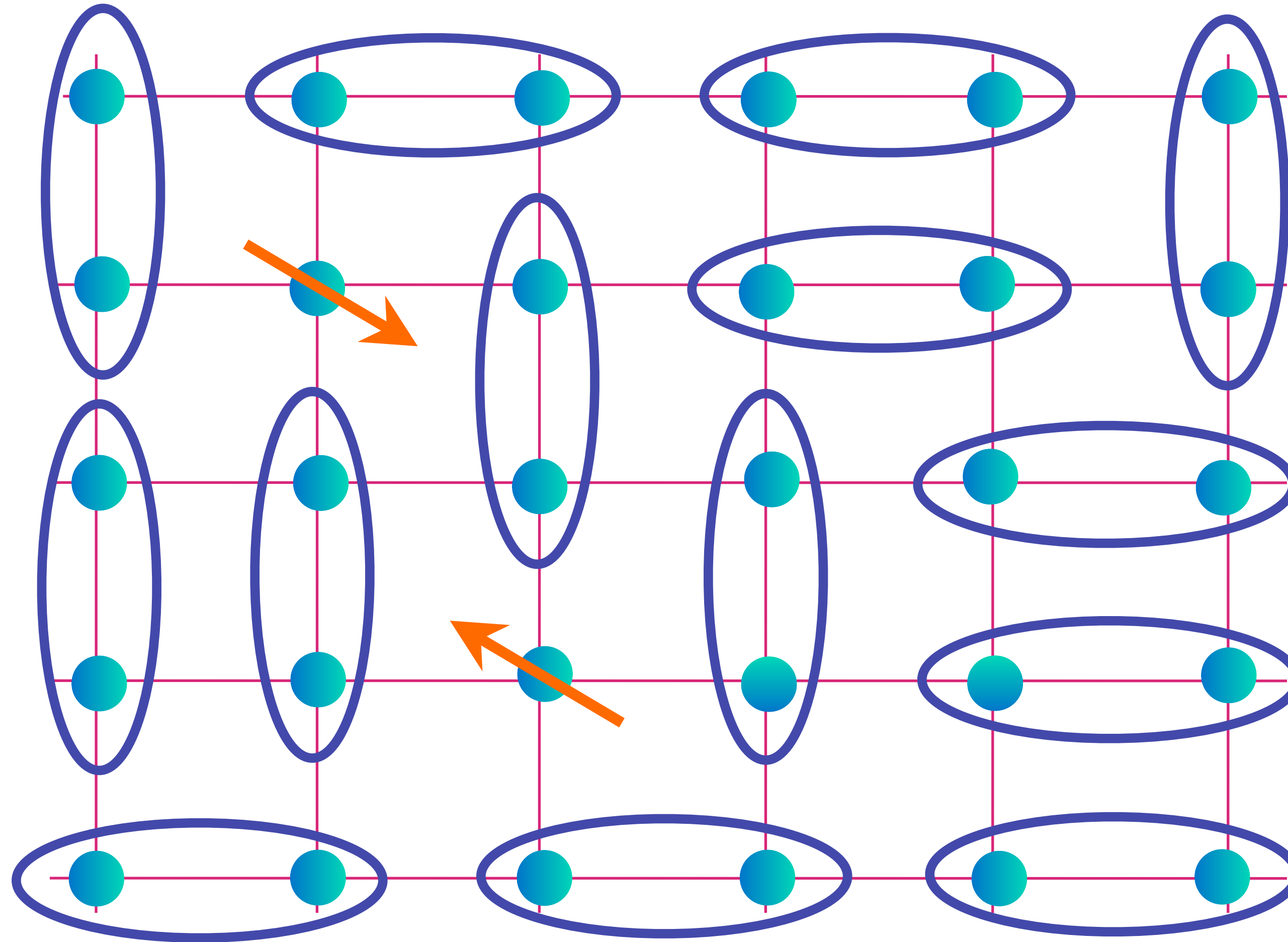
A spinon:  
an emergent  
particle which  
carries spin 1/2  
but no charge

$$\text{Spinon} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

Kivelson (1987)

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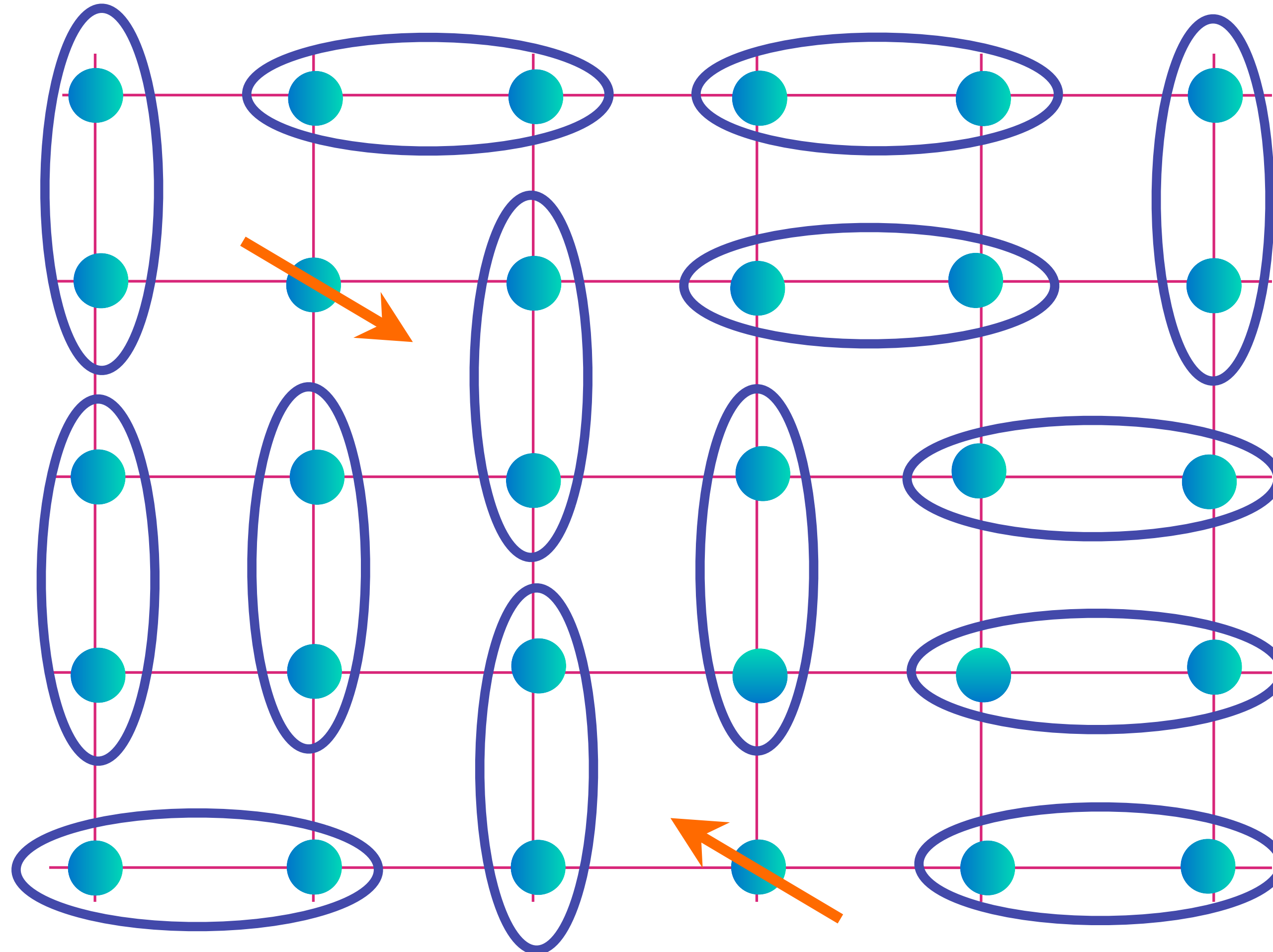
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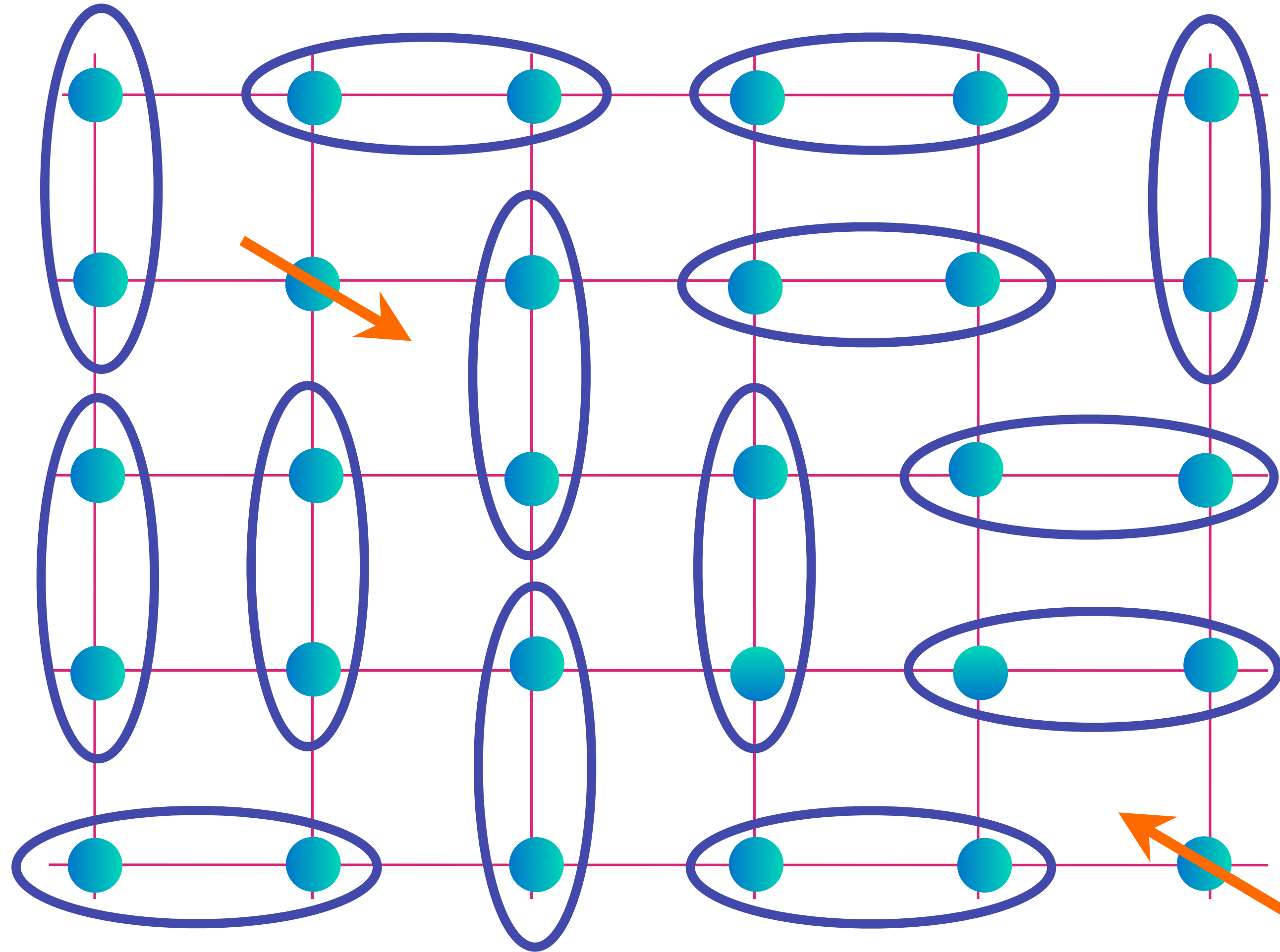
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$$\text{[Diagram of two electrons in a Cu atom]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

Kivelson (1987)

**Fractionalization**



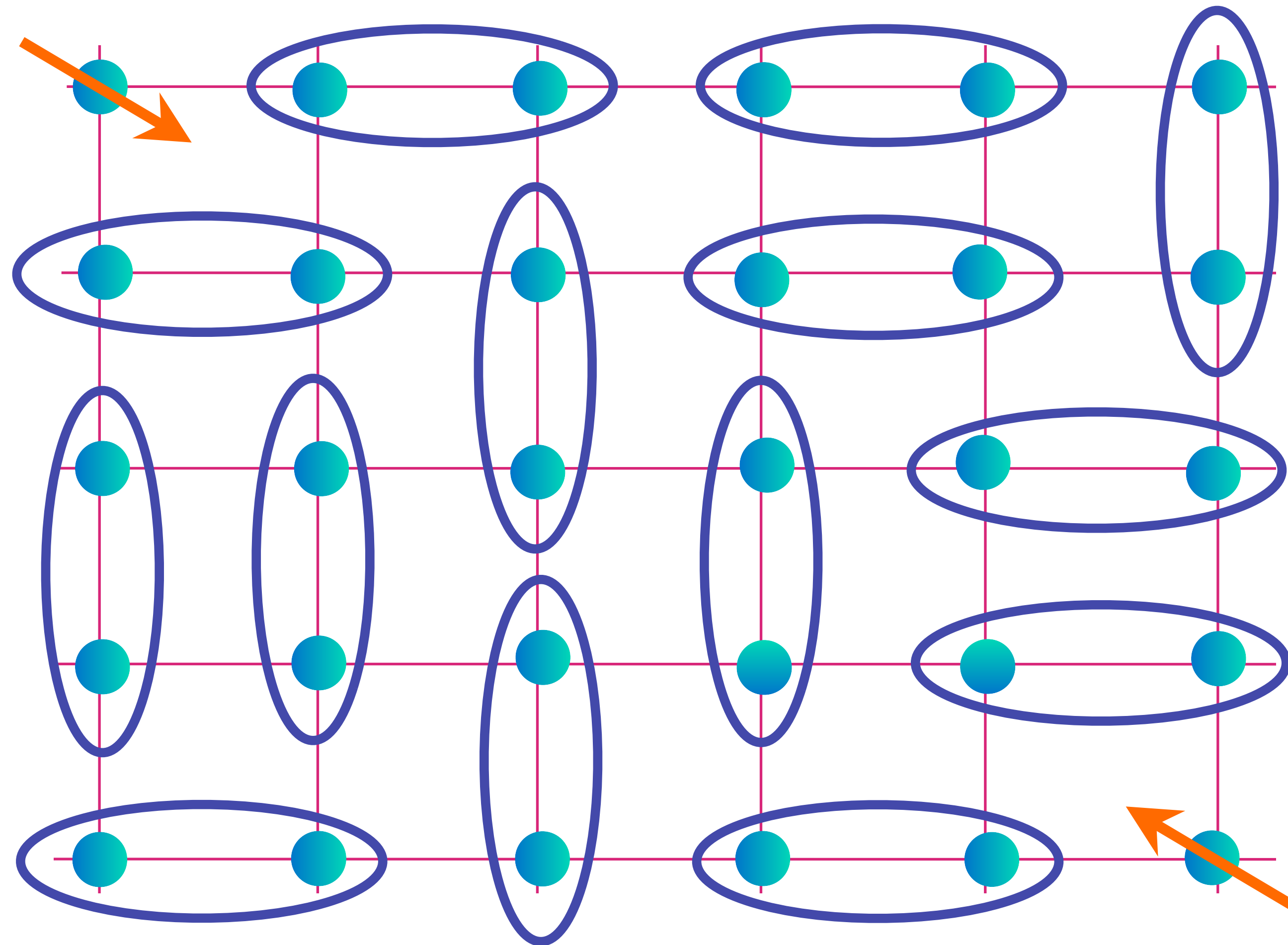
A spinon:  
an emergent  
particle which  
carries spin 1/2  
but no charge

$$\text{[Diagram of two teal circles in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

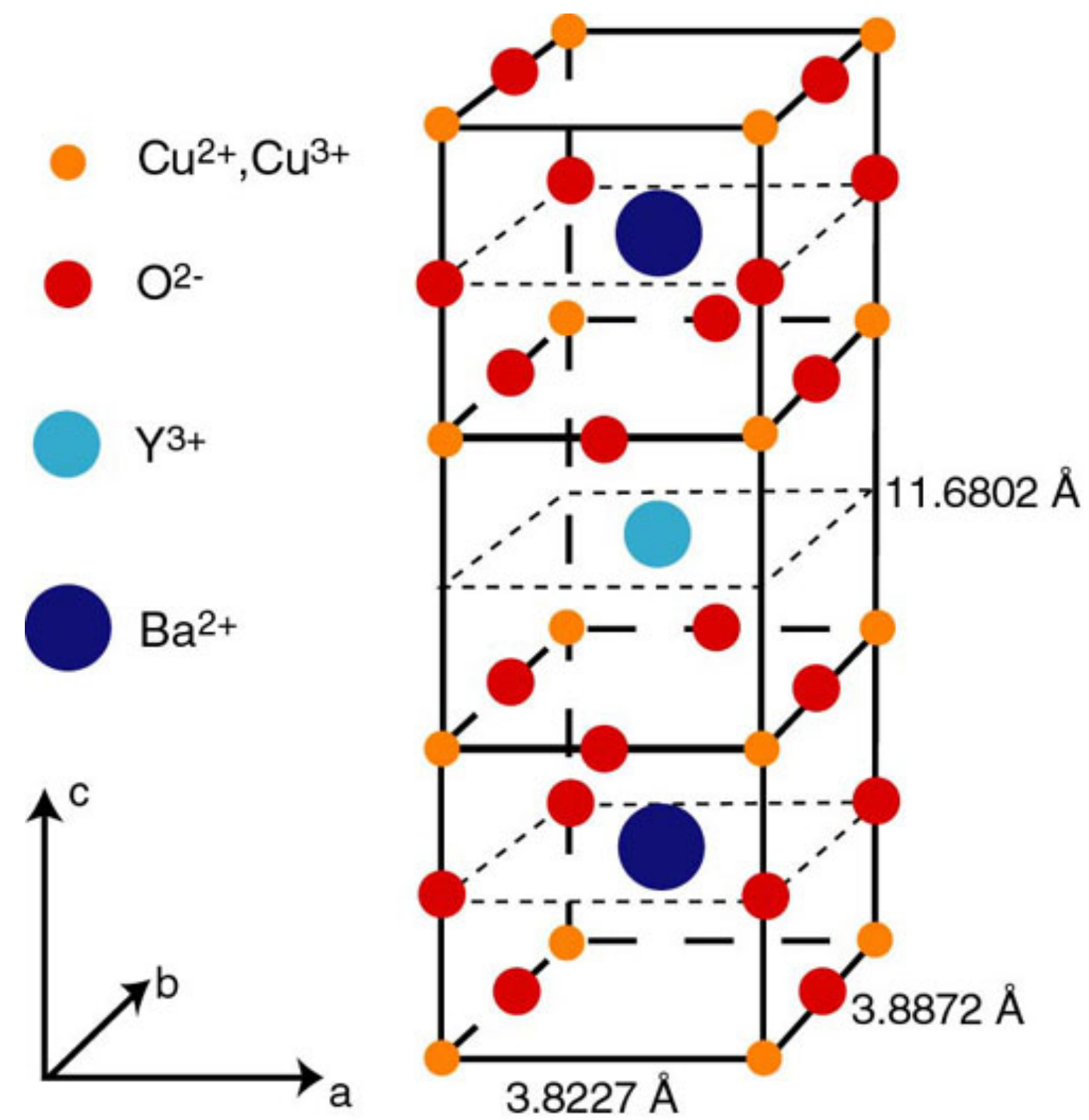
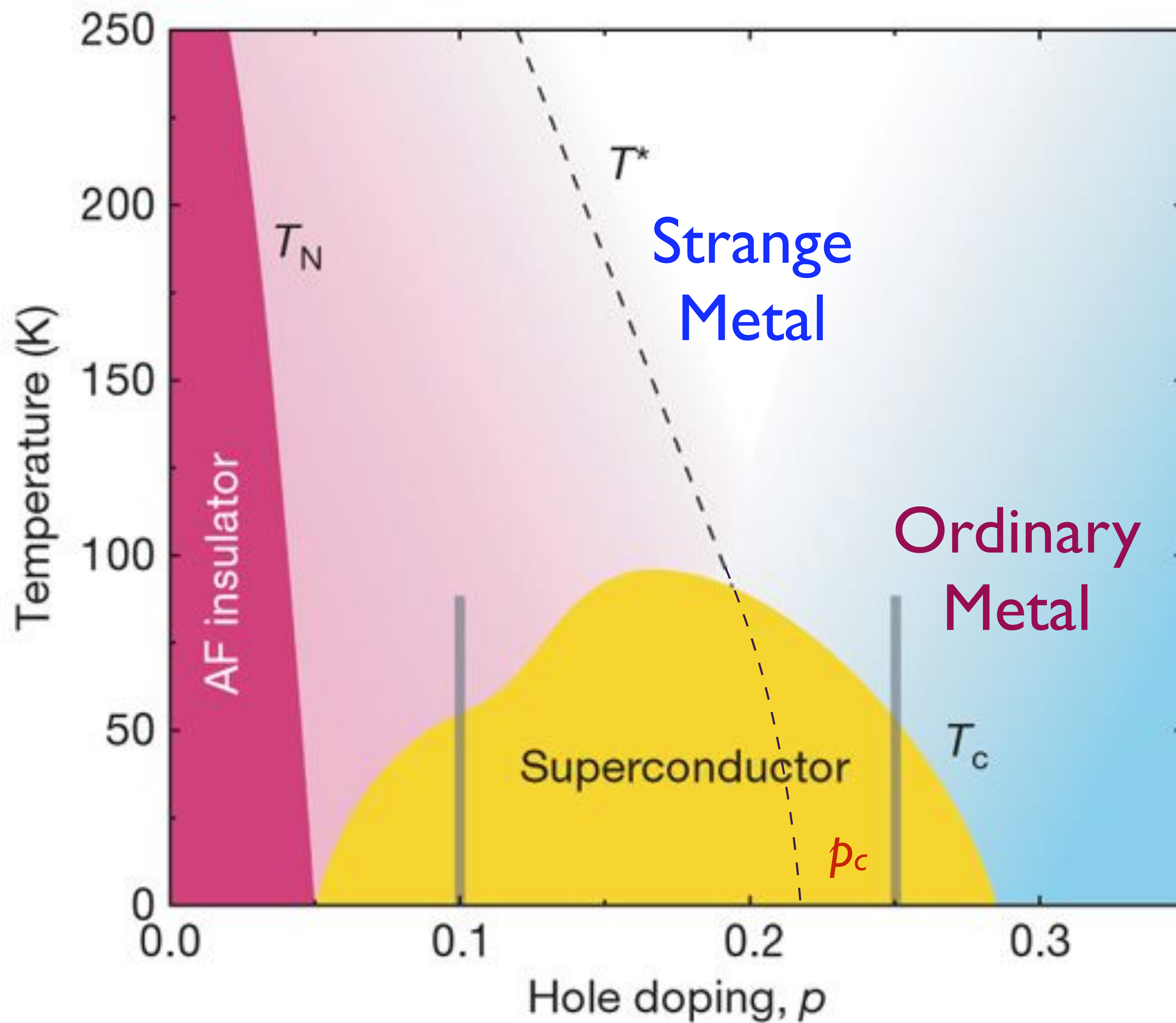
Kivelson (1987)

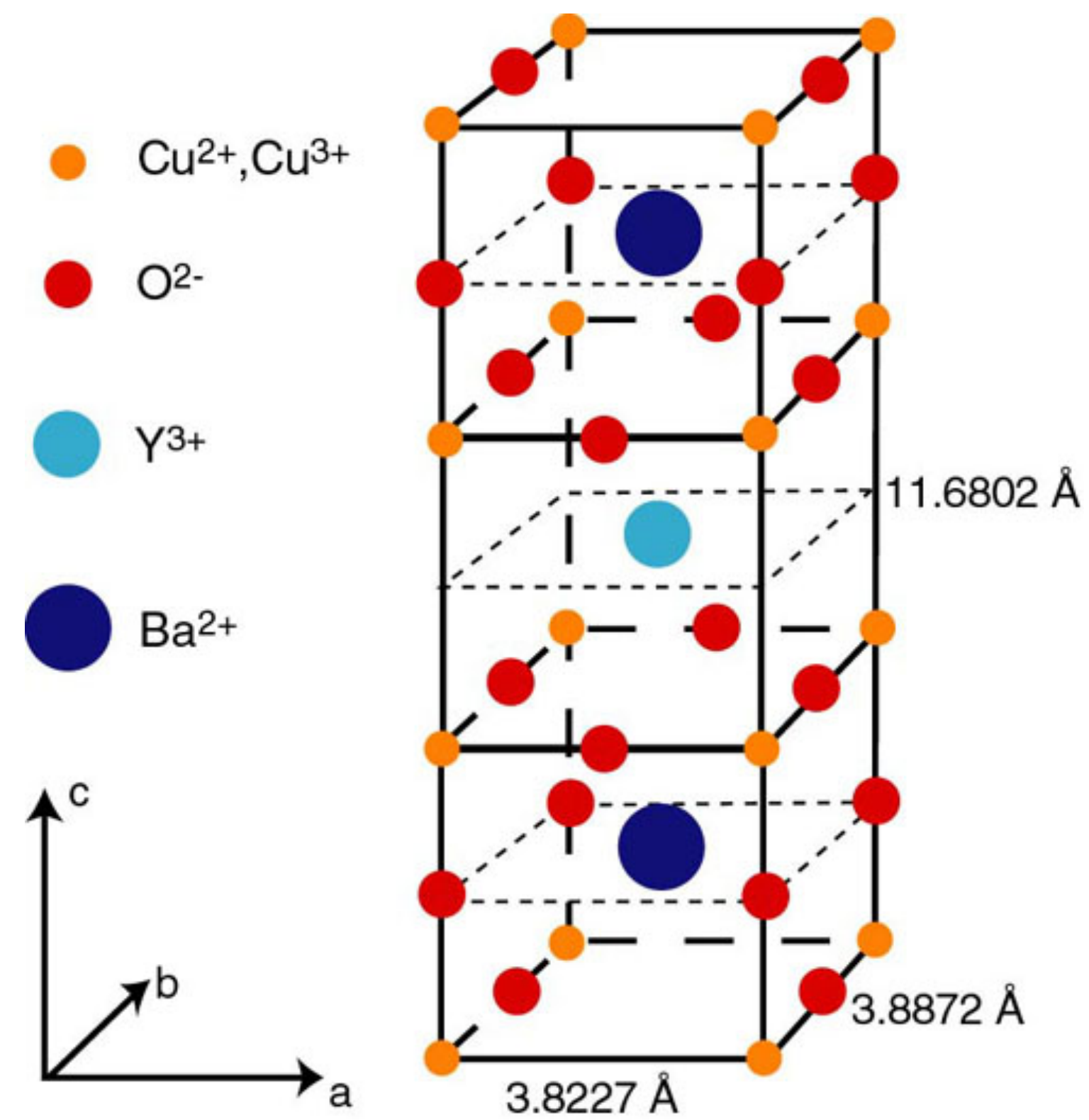
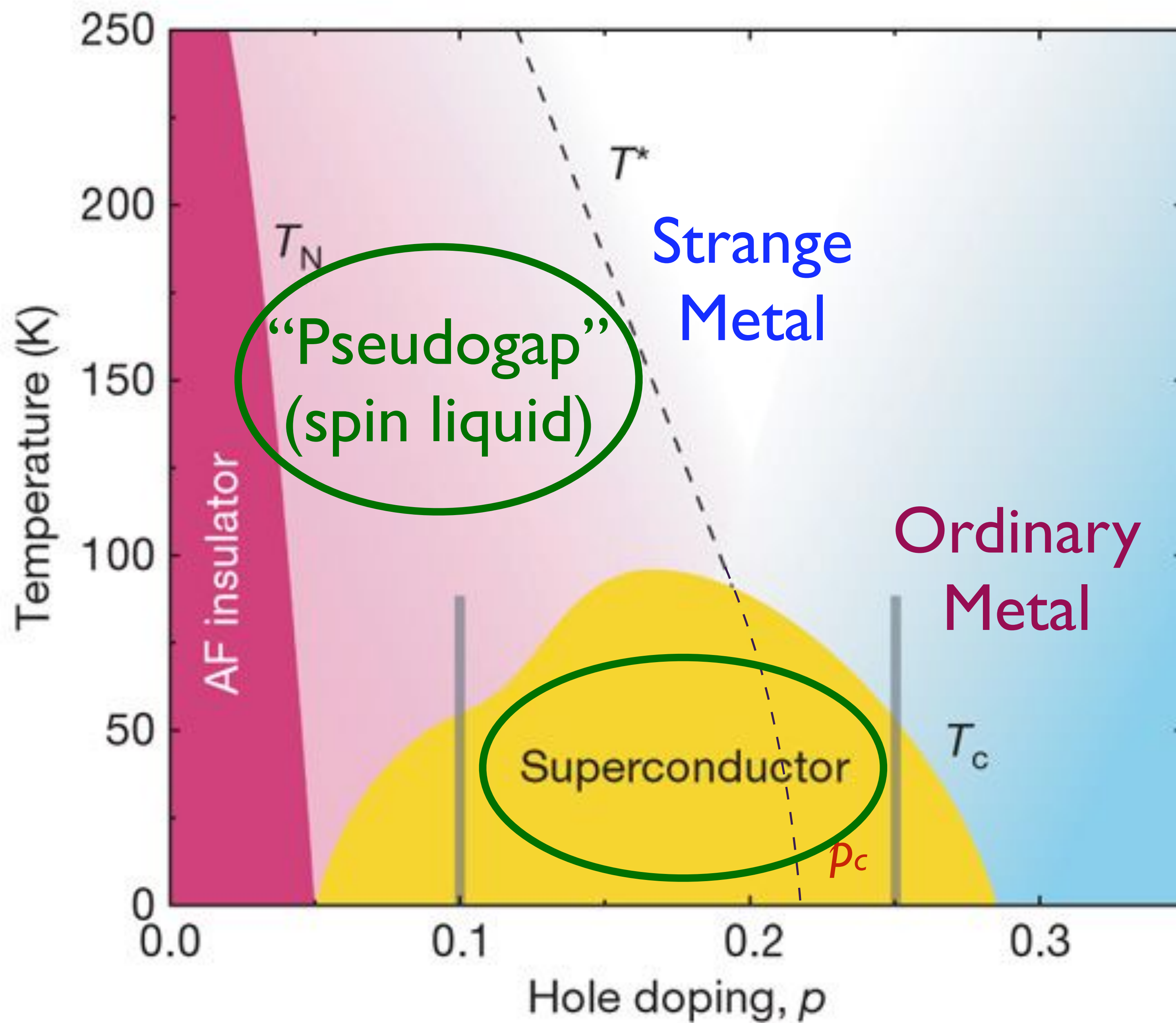
**Fractionalization**



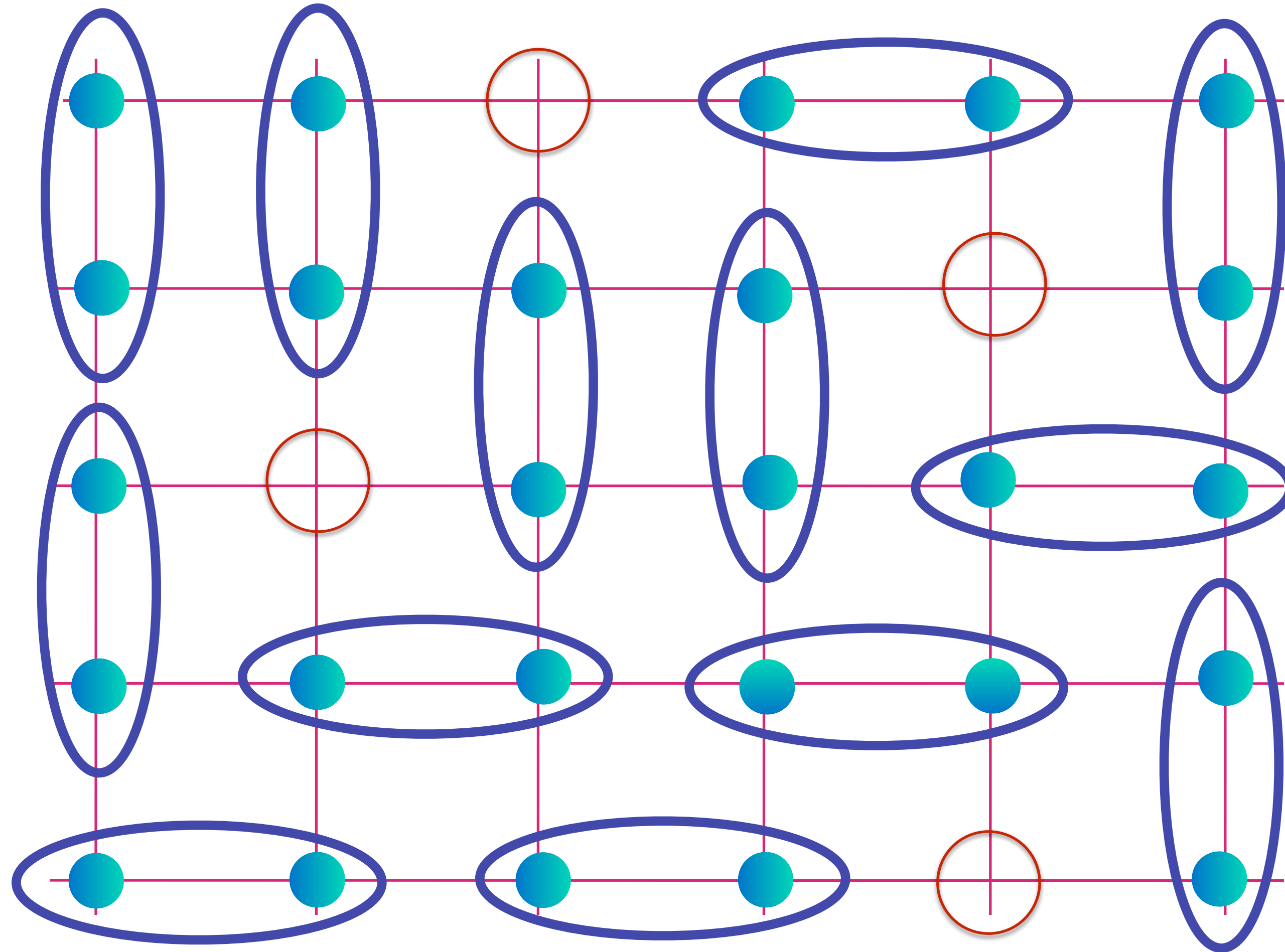
A spinon:  
an emergent  
particle which  
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but no charge

$$\text{[Diagram of two teal circles in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$





# The dance of electrons on Cu atoms in YBCO

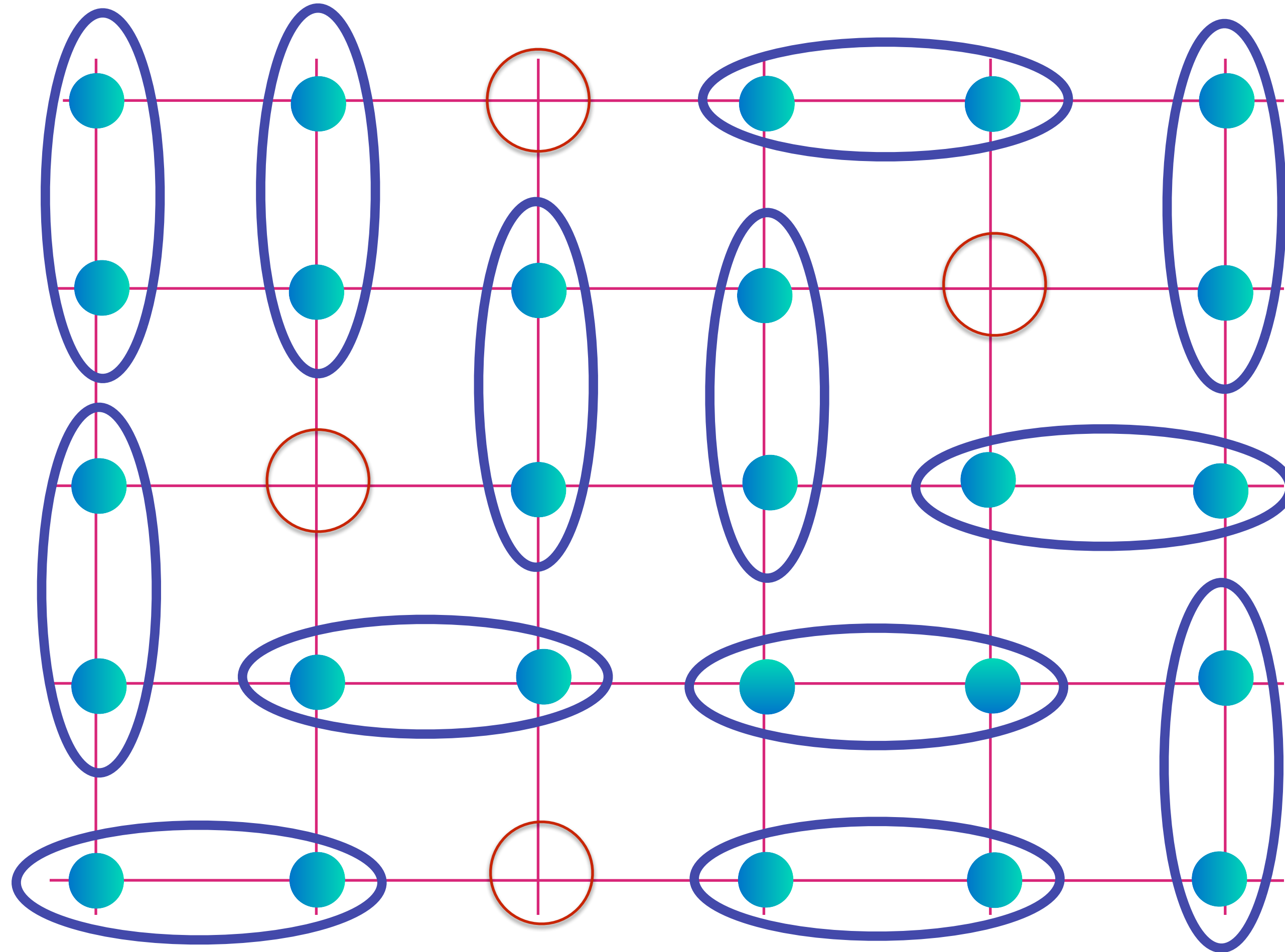


Pseudogap

Small density of mobile "holes" in a sea of entangled electron pairs

$$\text{[Blue Oval with 2 Teal Dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

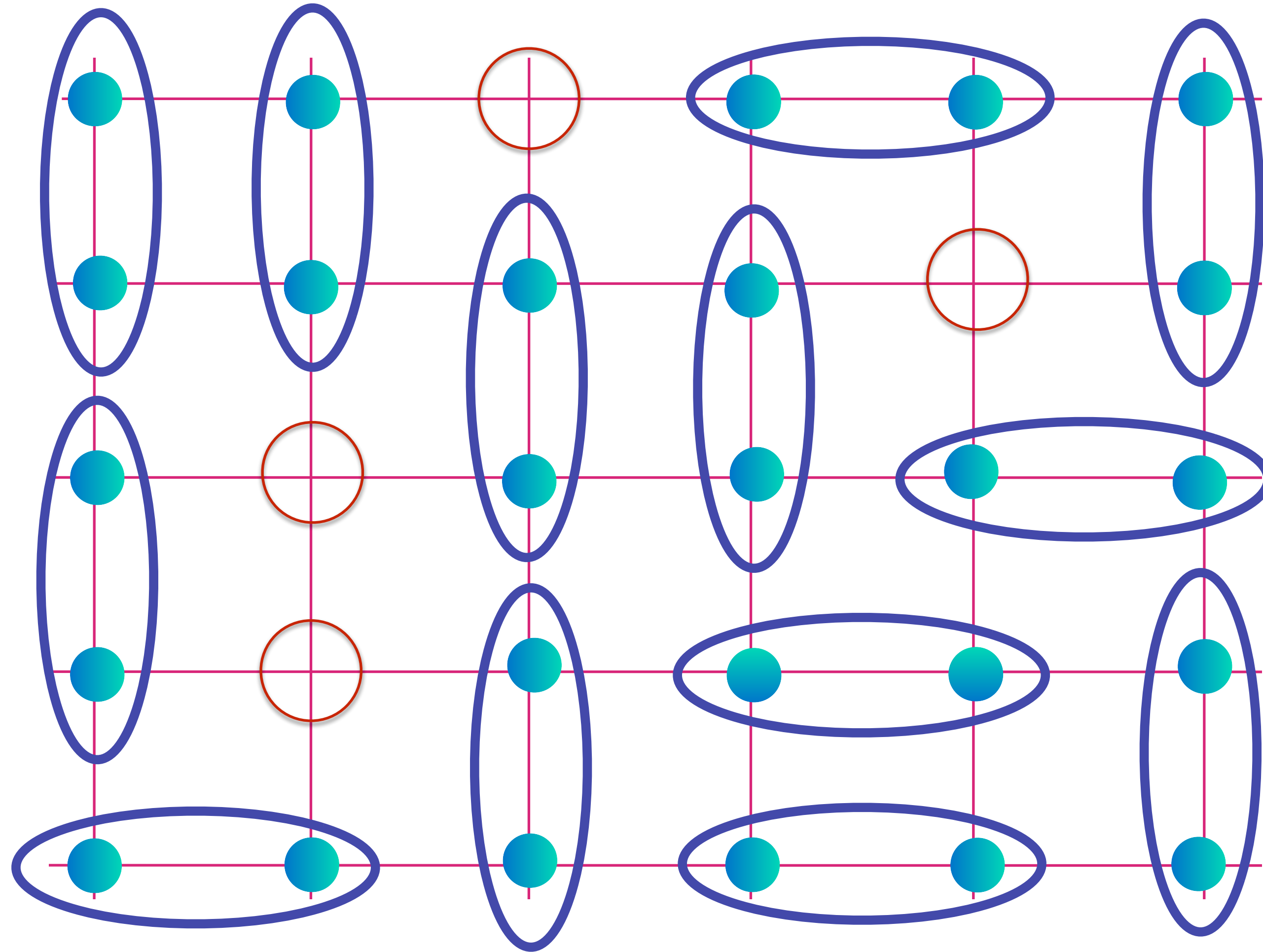


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Small density of mobile "holes" in a sea of entangled electron pairs

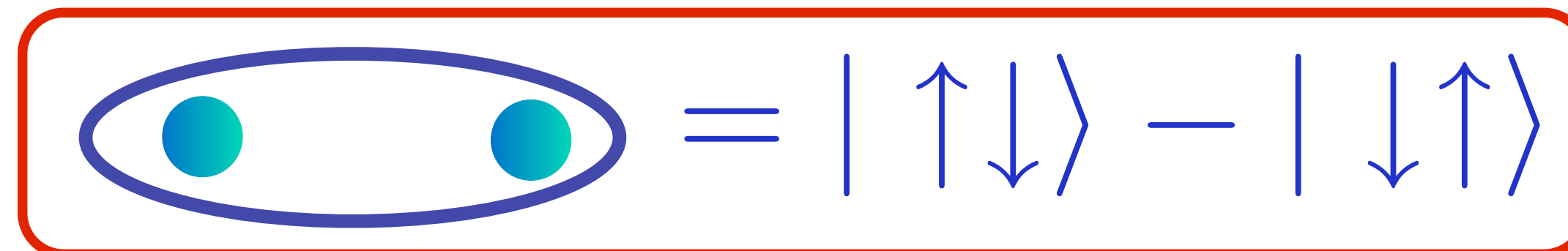
$$\text{[Blue oval with two cyan dots]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

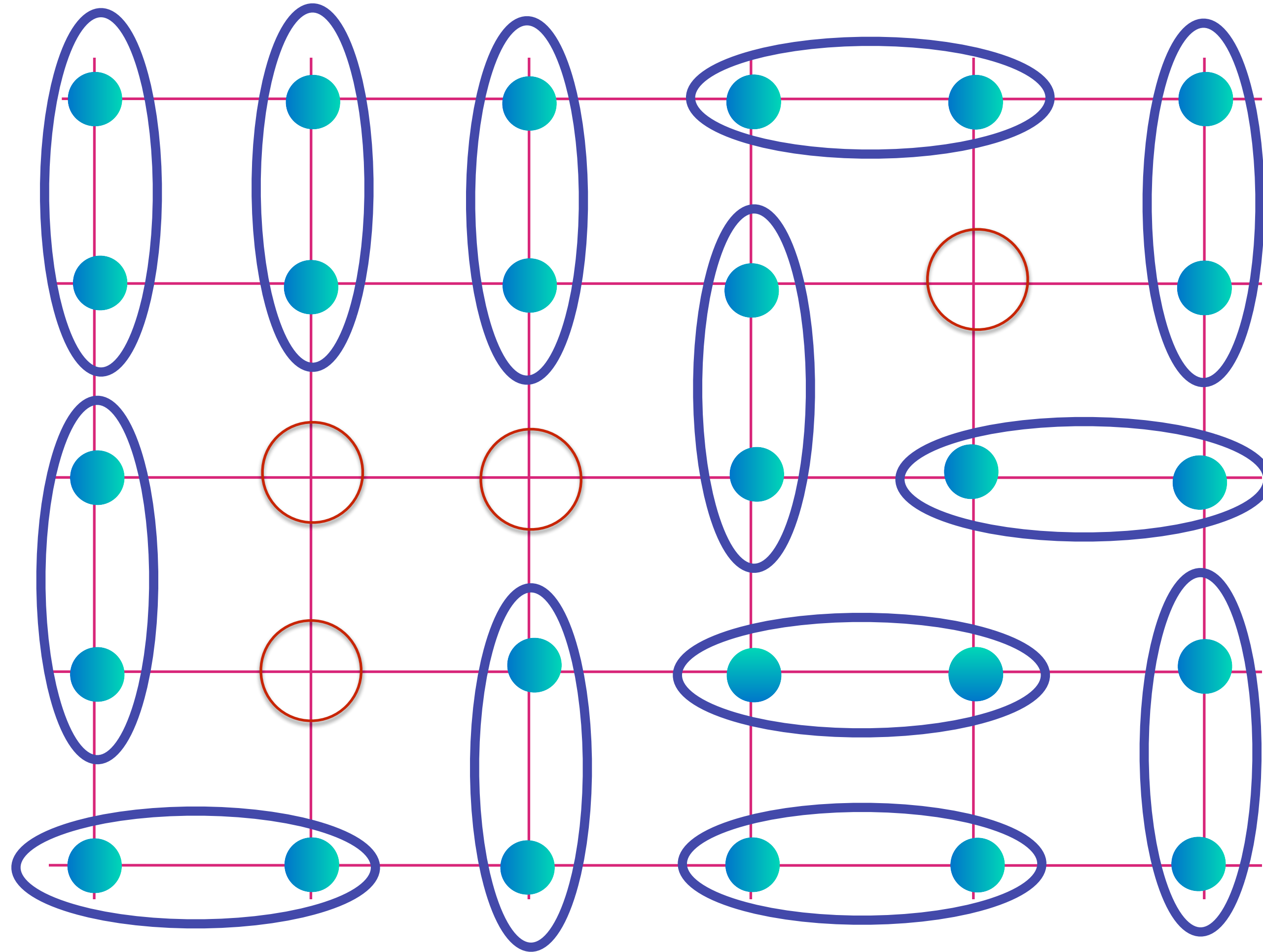


Pseudogap

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# The dance of electrons on Cu atoms in YBCO

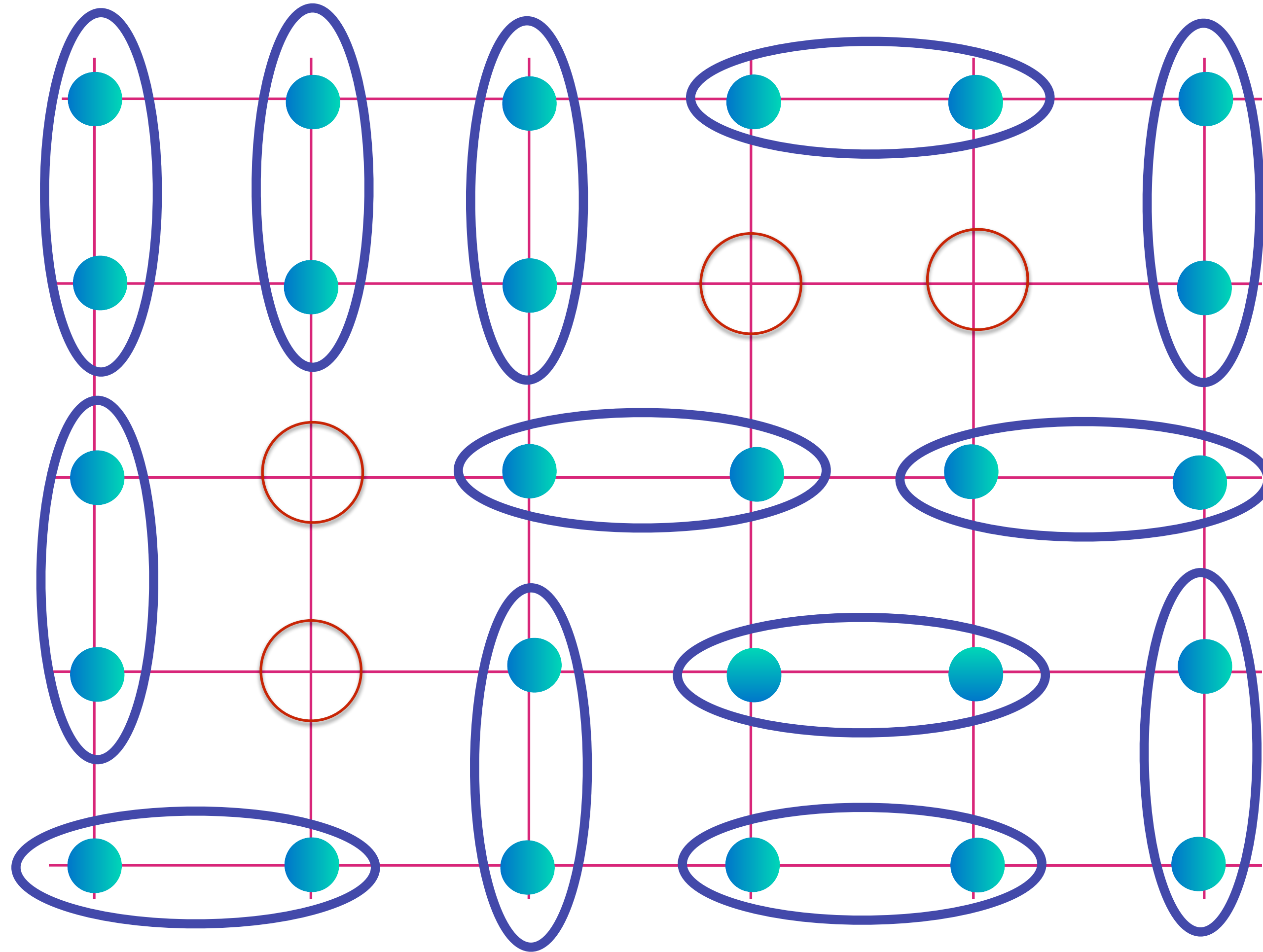


Pseudogap

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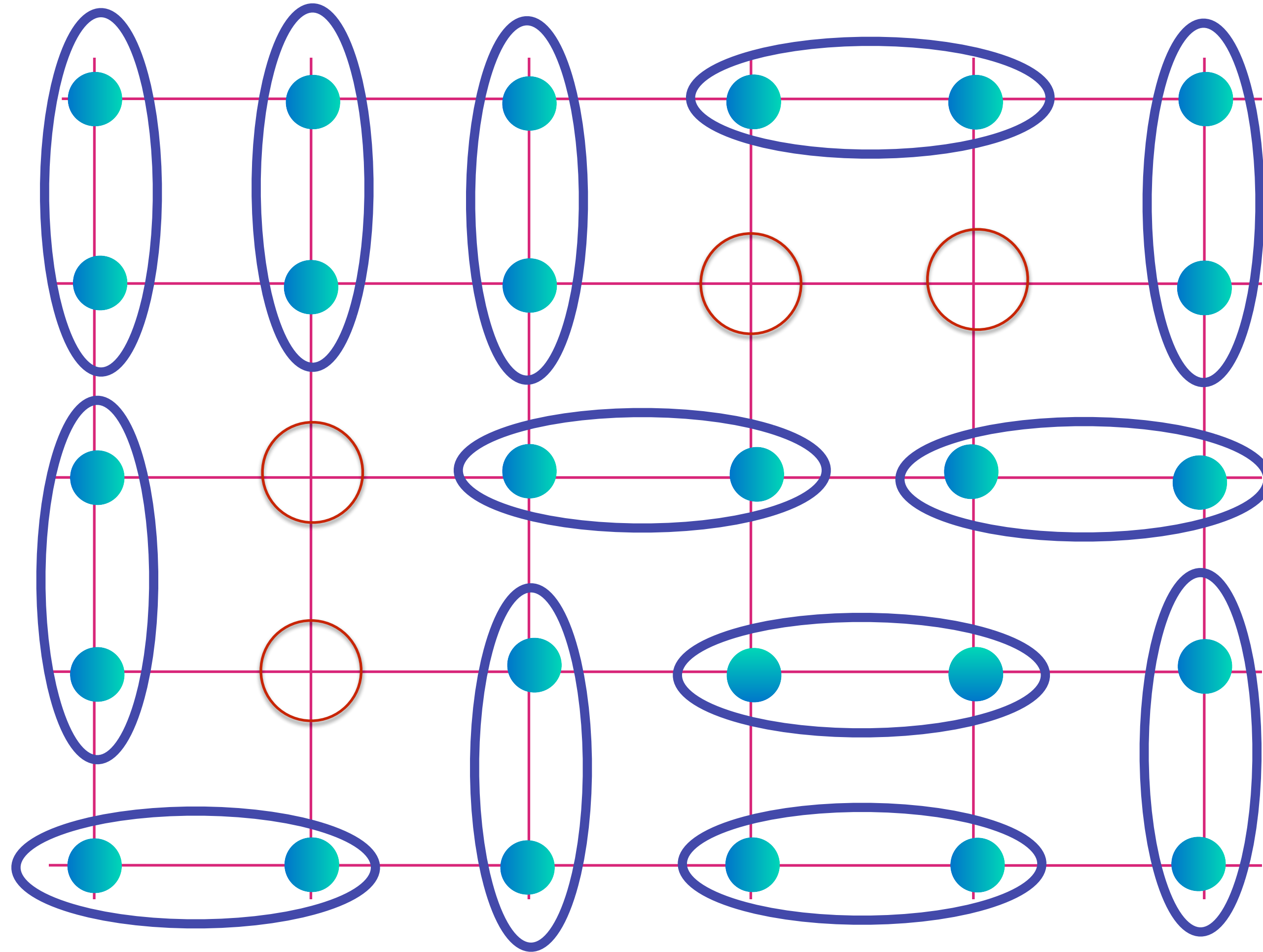


Pseudogap

Small density of mobile "holes" in a sea of entangled electron pairs

$$\text{[Diagram of two cyan dots in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

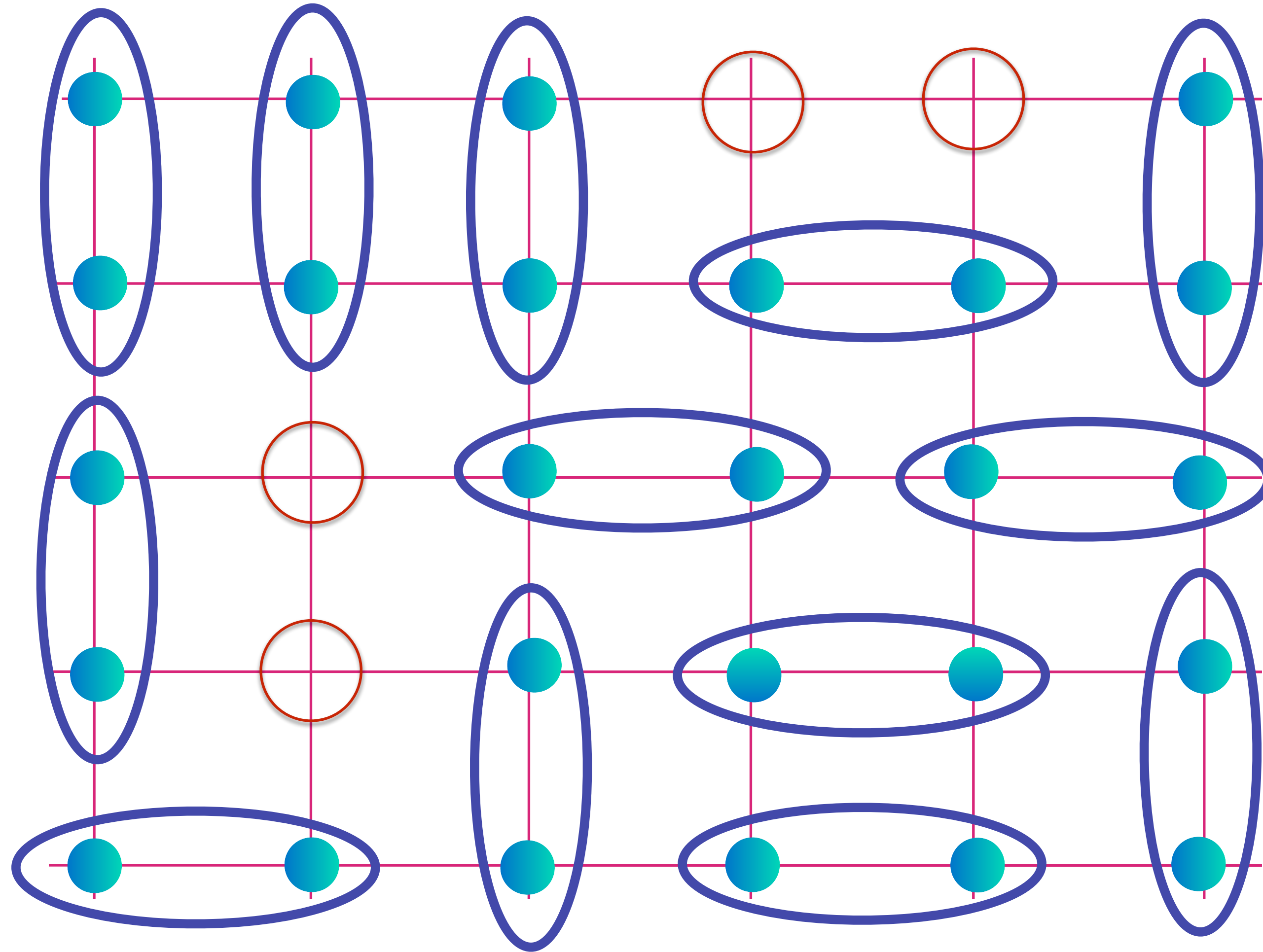


Superconductivity

Mobile  
entangled  
electron pairs

$$\text{[Diagram of two electrons in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

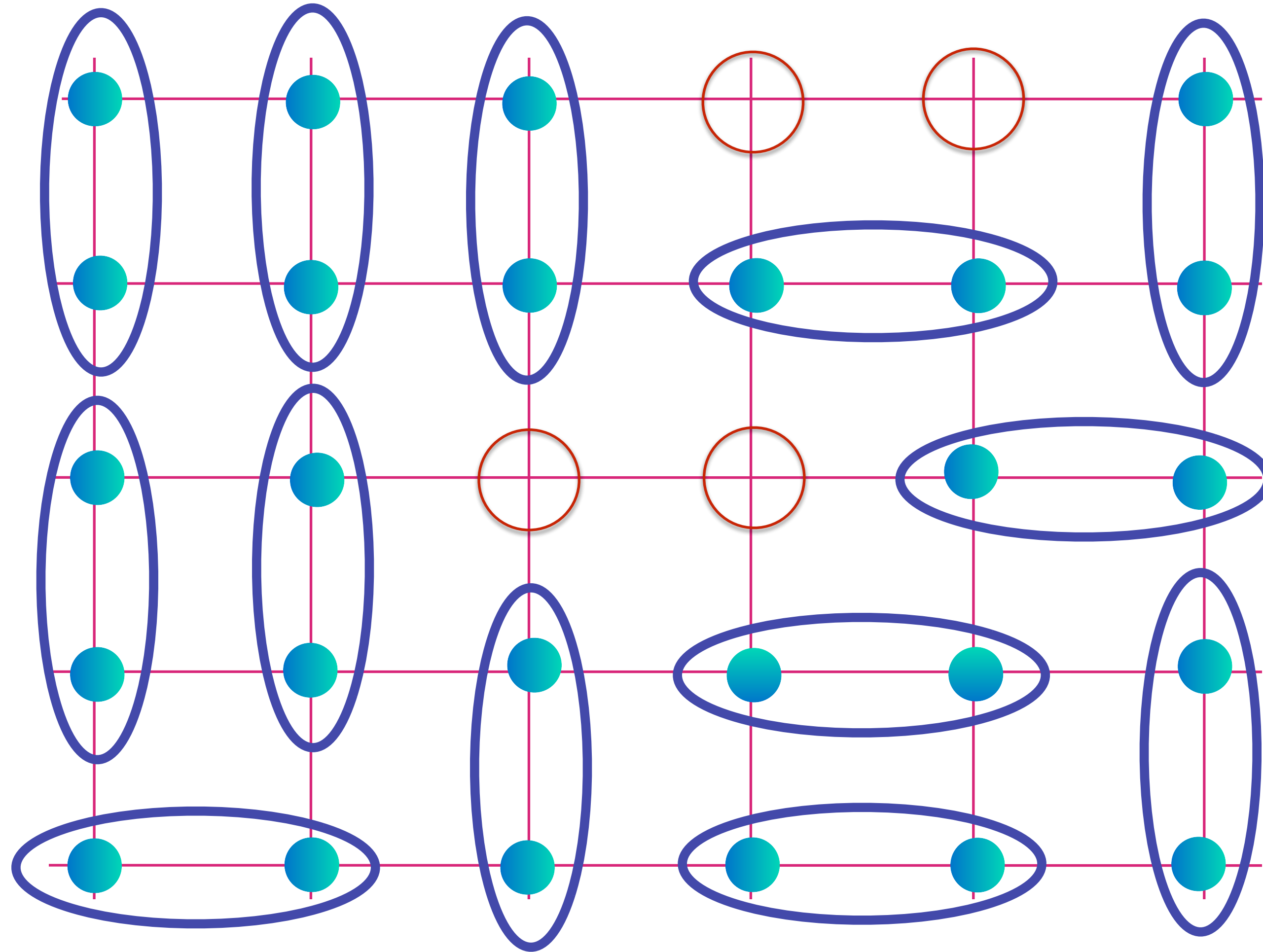


Superconductivity

Mobile  
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electron pairs

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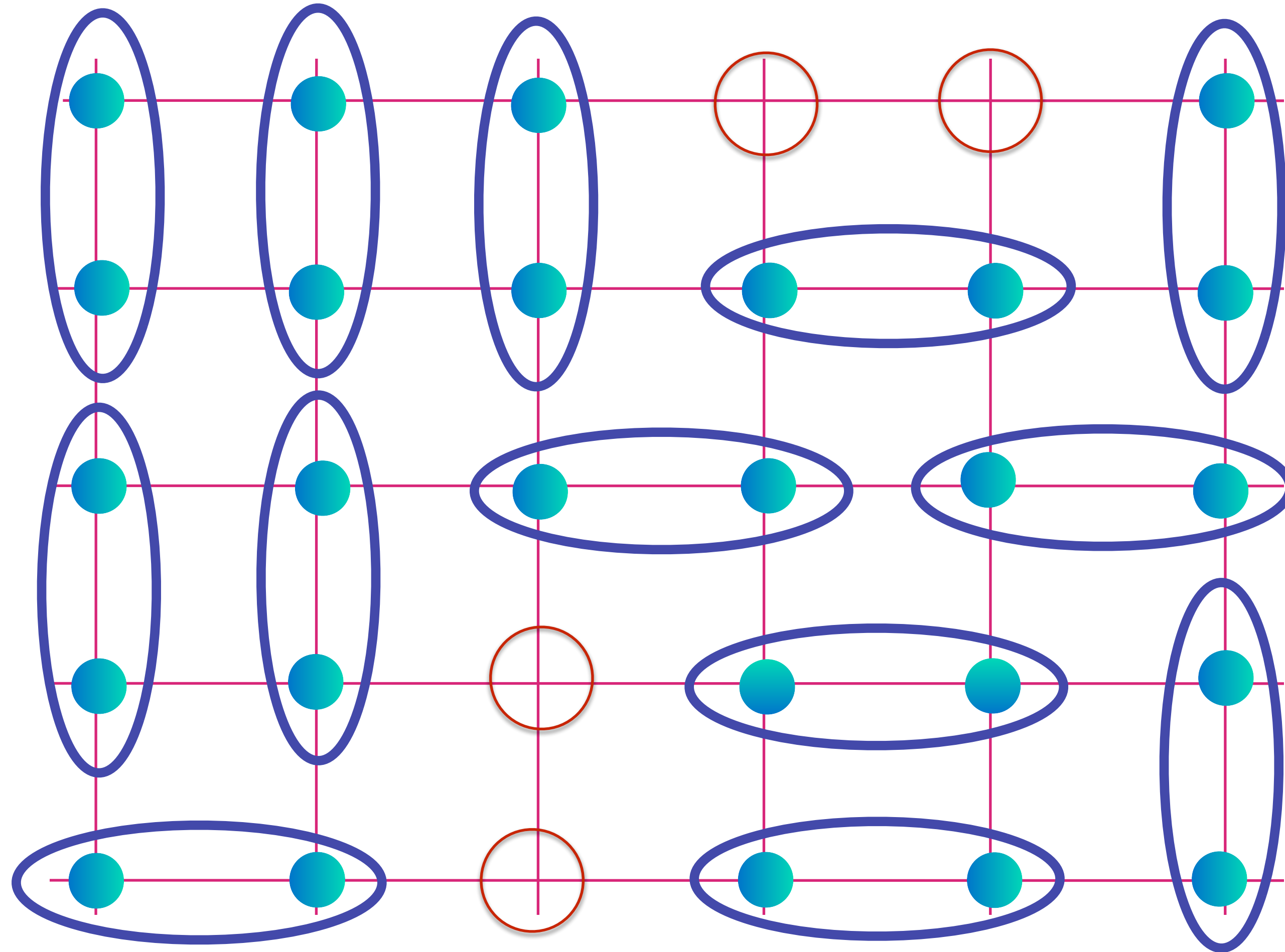


Superconductivity

Mobile  
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electron pairs

$$\text{[Diagram of two teal dots in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

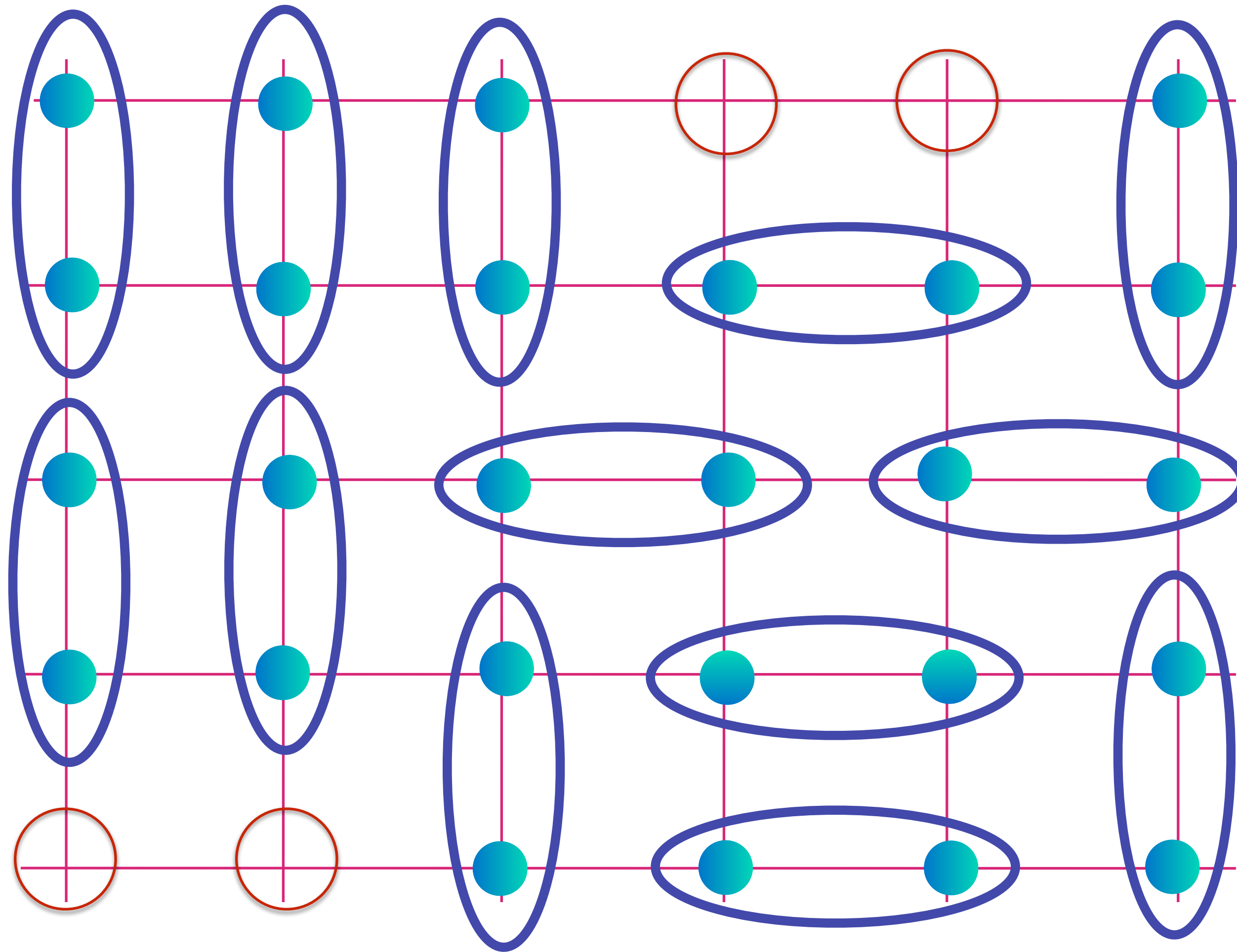


Superconductivity

Mobile  
entangled  
electron pairs

$$\text{[Diagram of a pair of electrons in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

# The dance of electrons on Cu atoms in YBCO

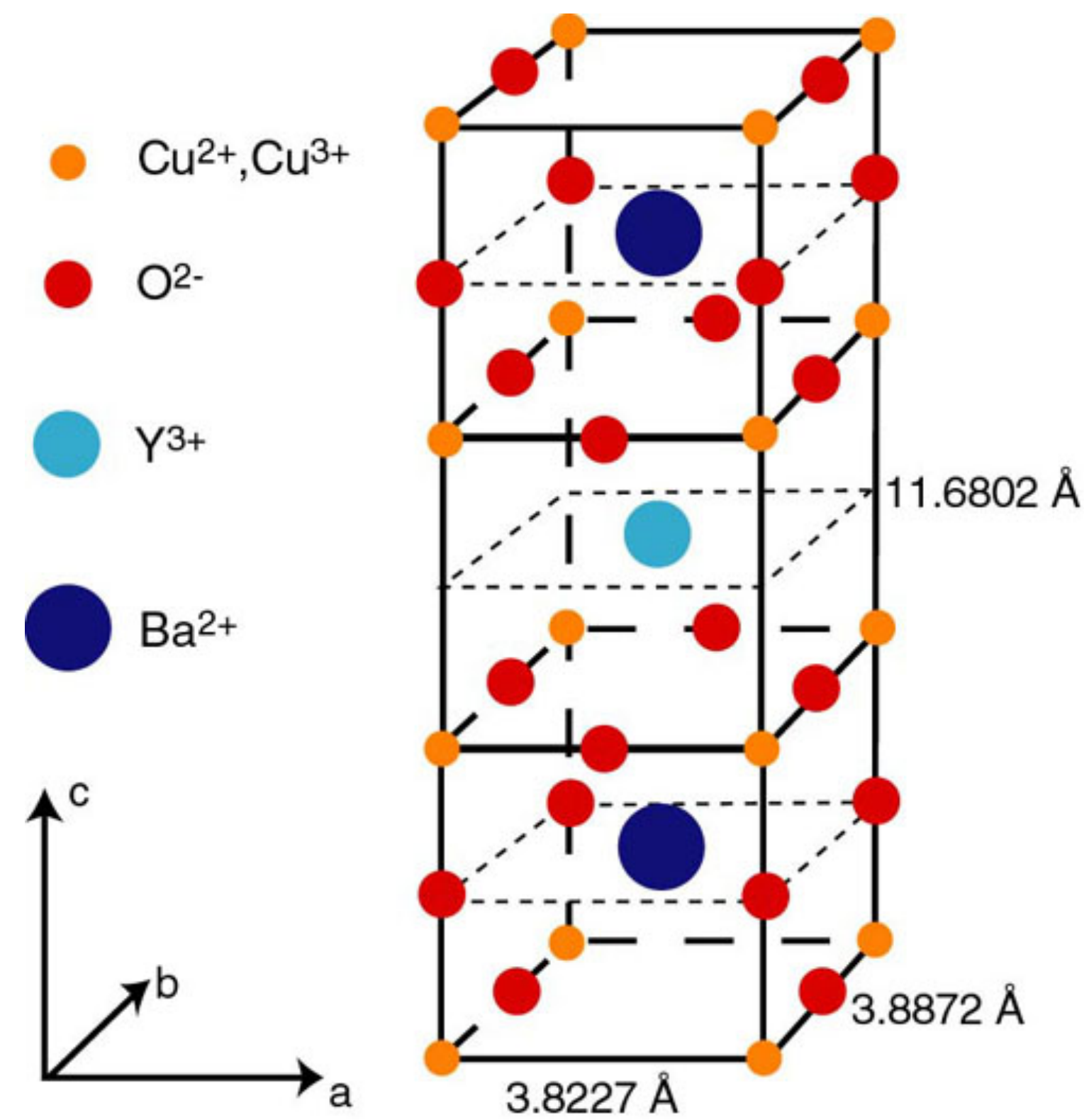
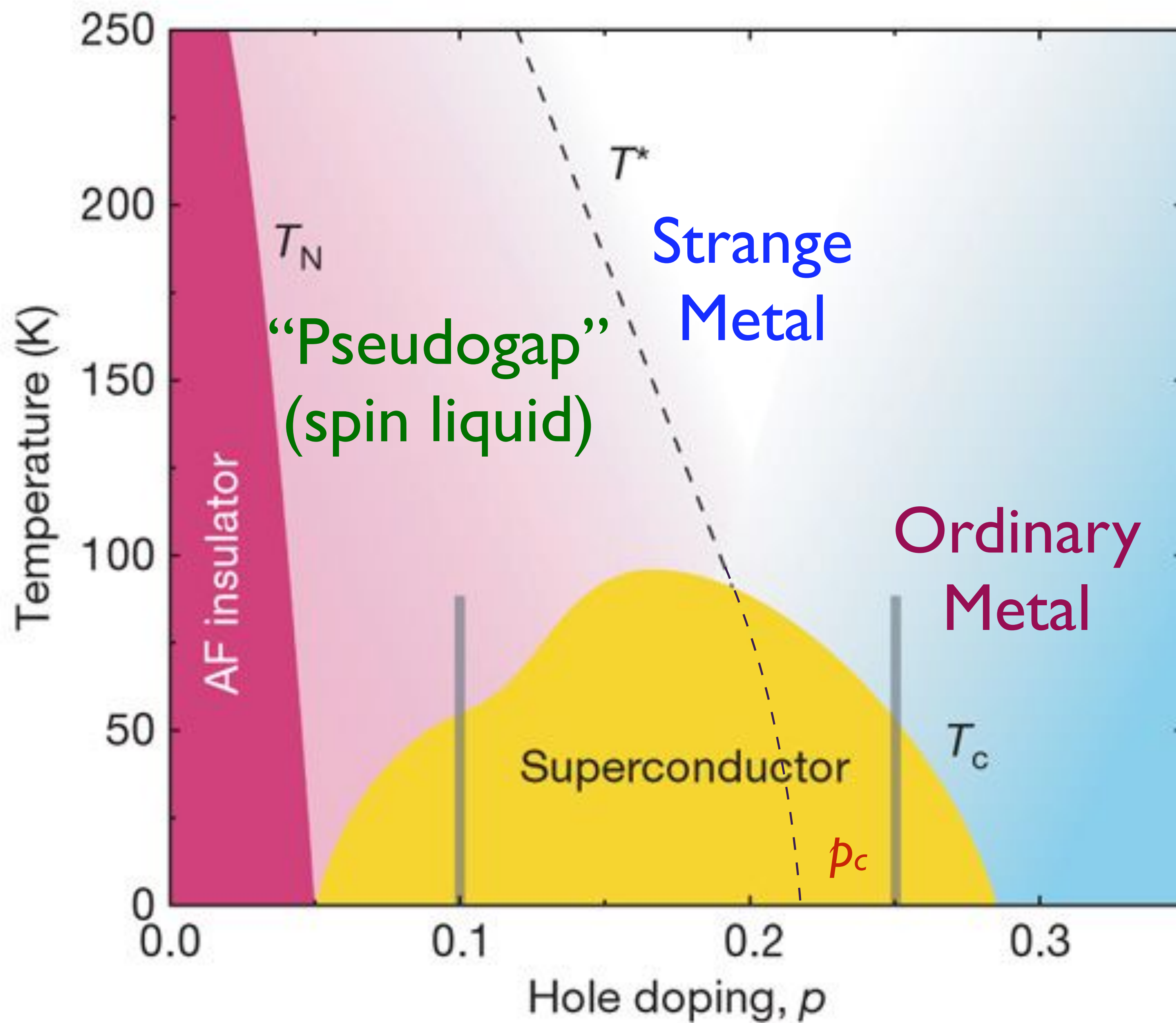


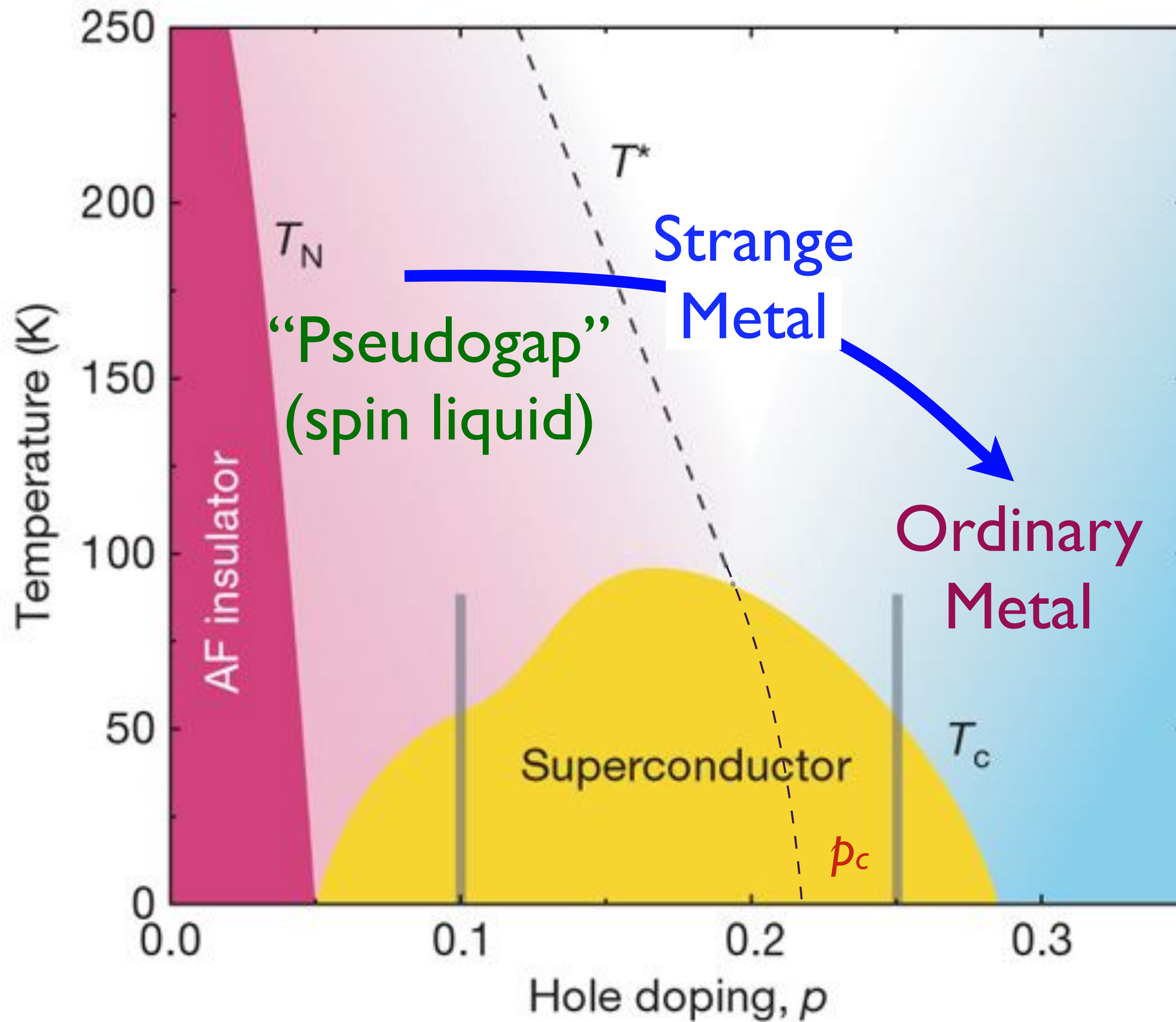
Superconductivity

Mobile  
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$$\text{[Diagram of a pair of electrons in a blue oval]} = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

From the SYK model  
to a theory of  
strange metals

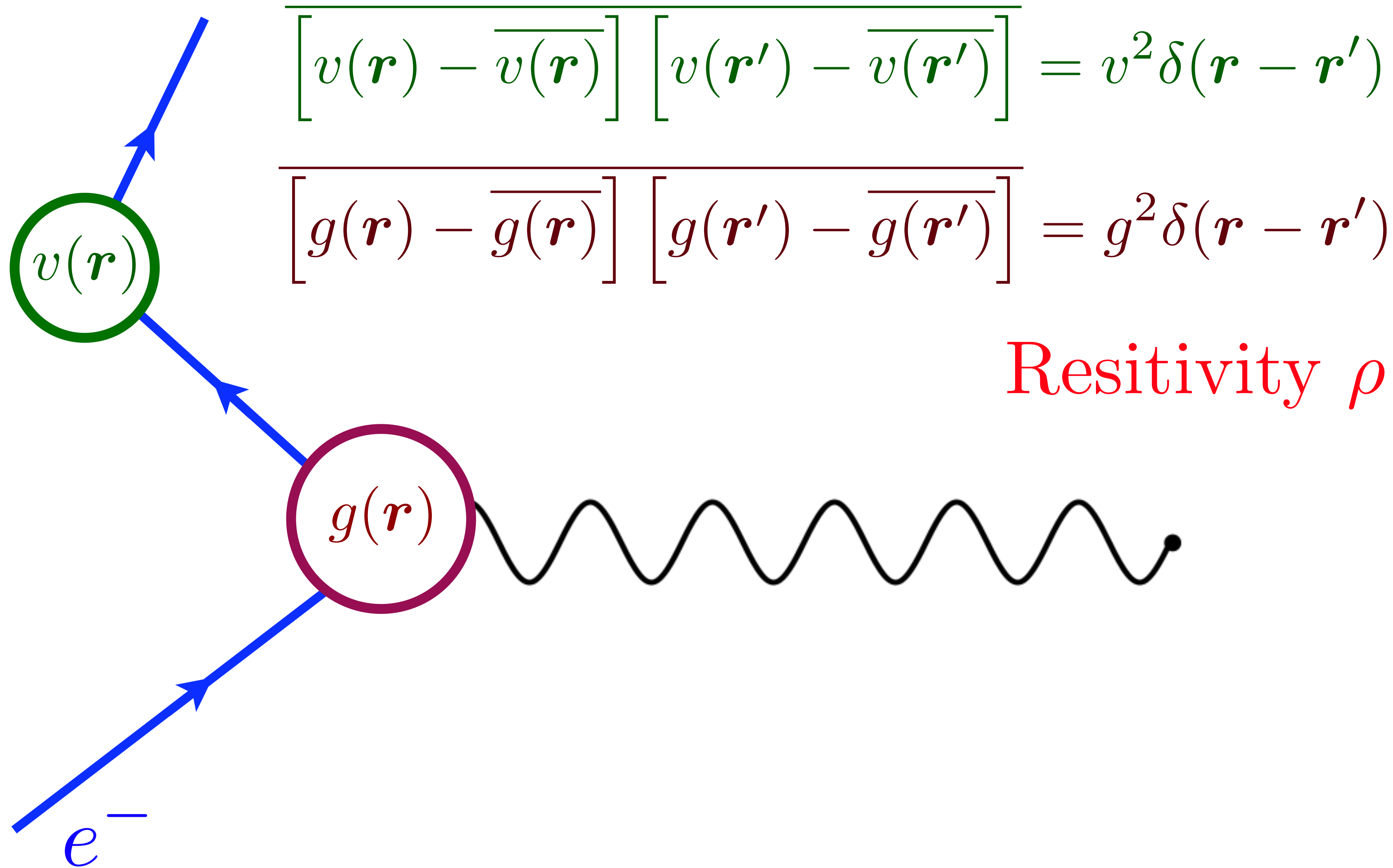




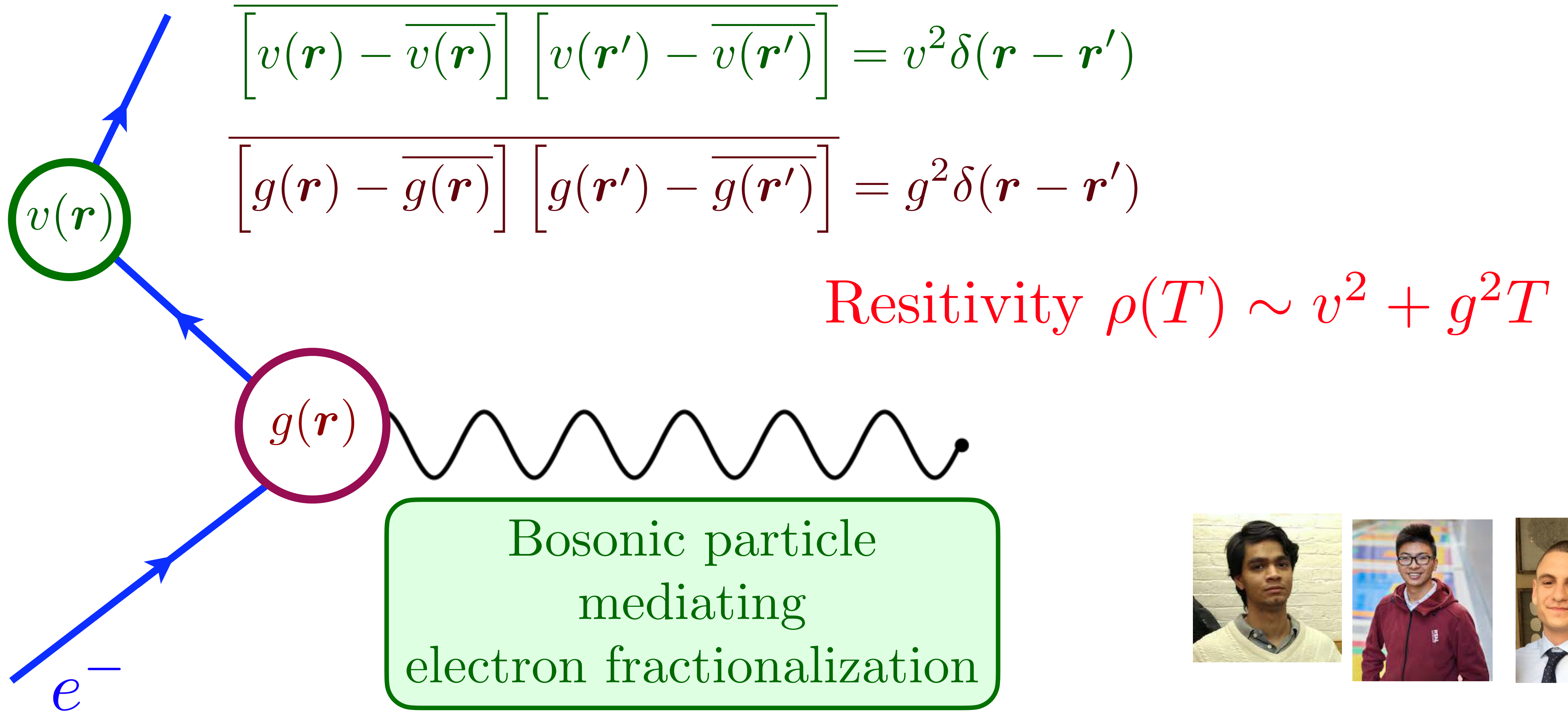
## Strange Metal

Arises from the evolution of quantum entanglement between the spin liquid and the ordinary metal

# The dance of electrons on Cu atoms in YBCO



# The dance of electrons on Cu atoms in YBCO



# The many faces of multi-particle entanglement

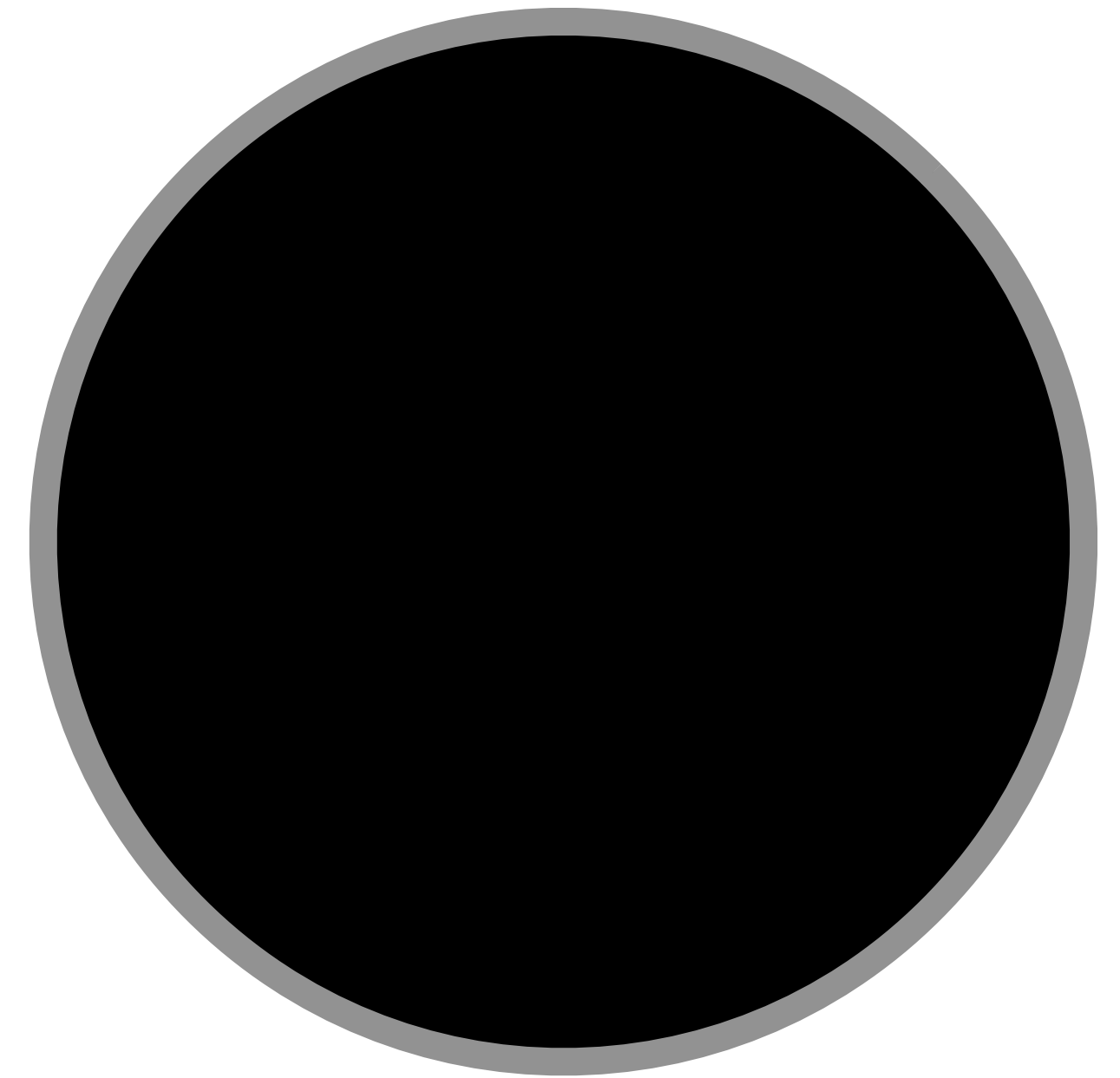
- Absence of quasiparticles, as in the SYK model and the strange metal
- Fractionalization and new emergent particles, as in spin liquids.
- Higher temperature superconductivity (?)

**Black  
holes**

# Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius  $R = \frac{2GM}{c^2}$



$G$  Newton's constant,  $c$  velocity of light,  $M$  mass of black hole  
For  $M = \text{earth's mass}$ ,  $R \approx 9 \text{ mm!}$

The supermassive black hole lurking at the heart of the Milky Way – Sagittarius A\* contains about 4.3 million solar masses, and, as it turns out, nearly all of the mass at the very center of the galaxy.

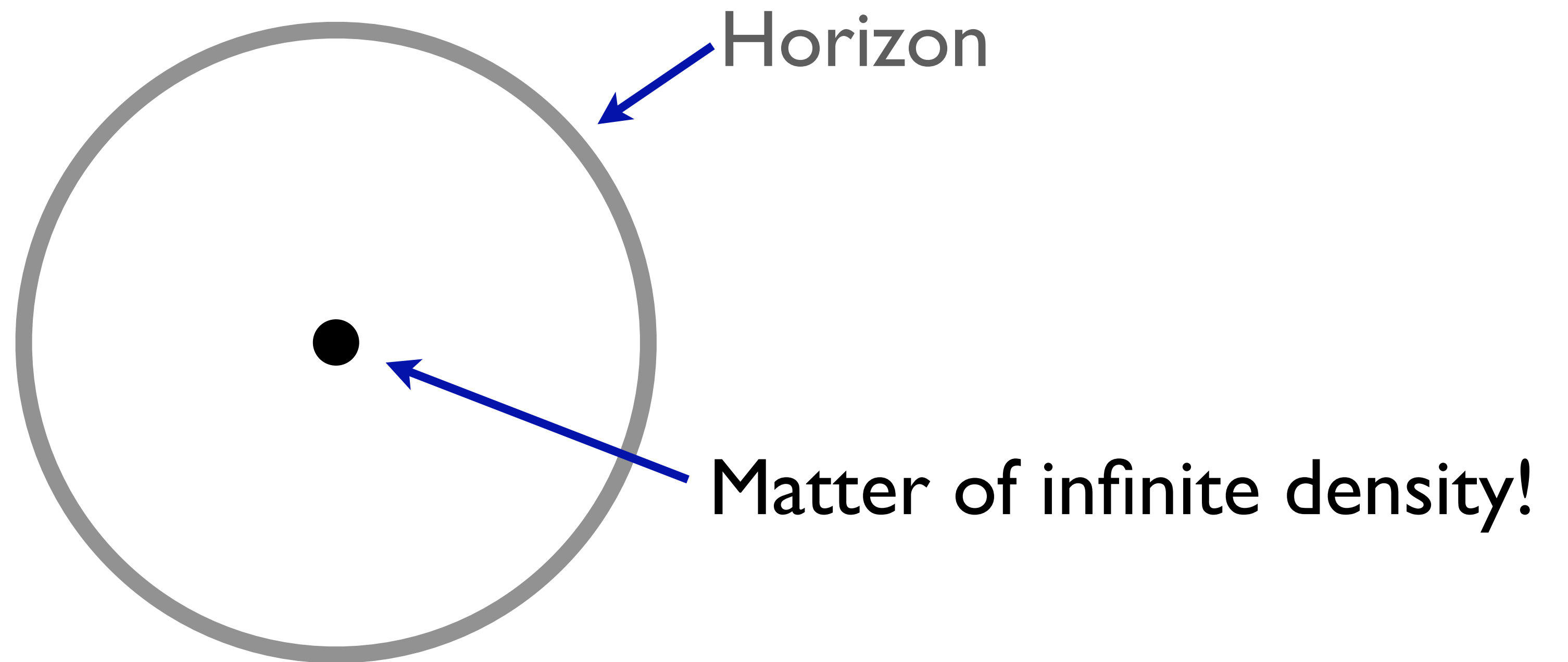
$$R = 1.3 \times 10^{11} \text{ m}$$

$\approx$  earth's orbit

An artist's impression of Sagittarius A\*, the supermassive black hole at the heart of the Milky Way. Image: International Gemini Observatory/NOIRLab/NSF/AURA/J. da Silva/(Spaceengine); M. Zamani (NSF's NOIRLab)

# What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



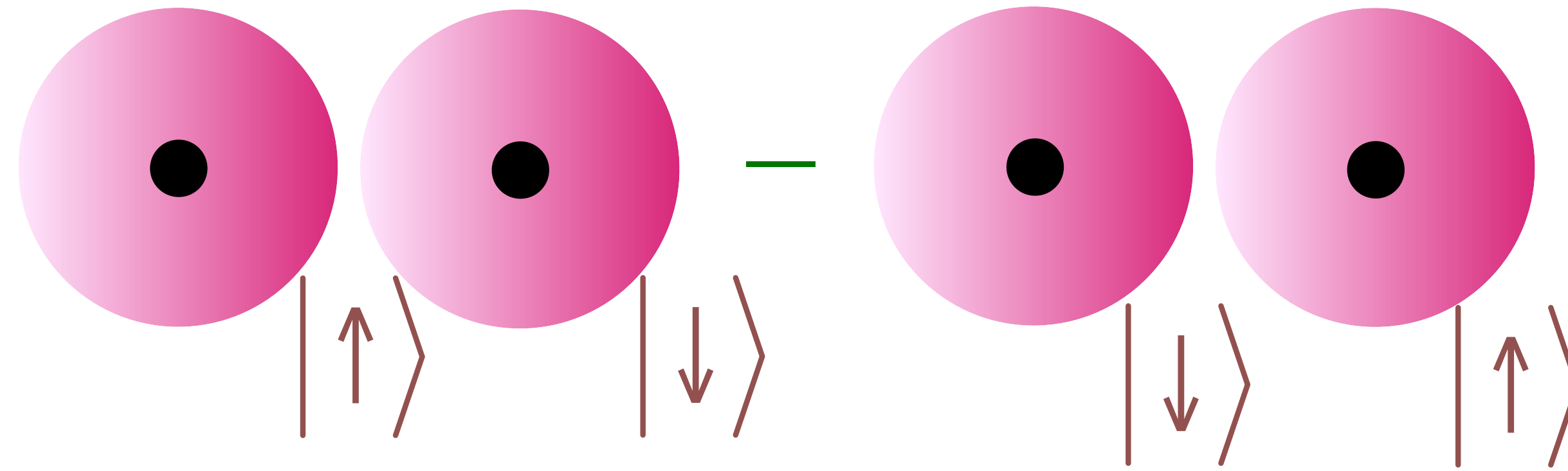
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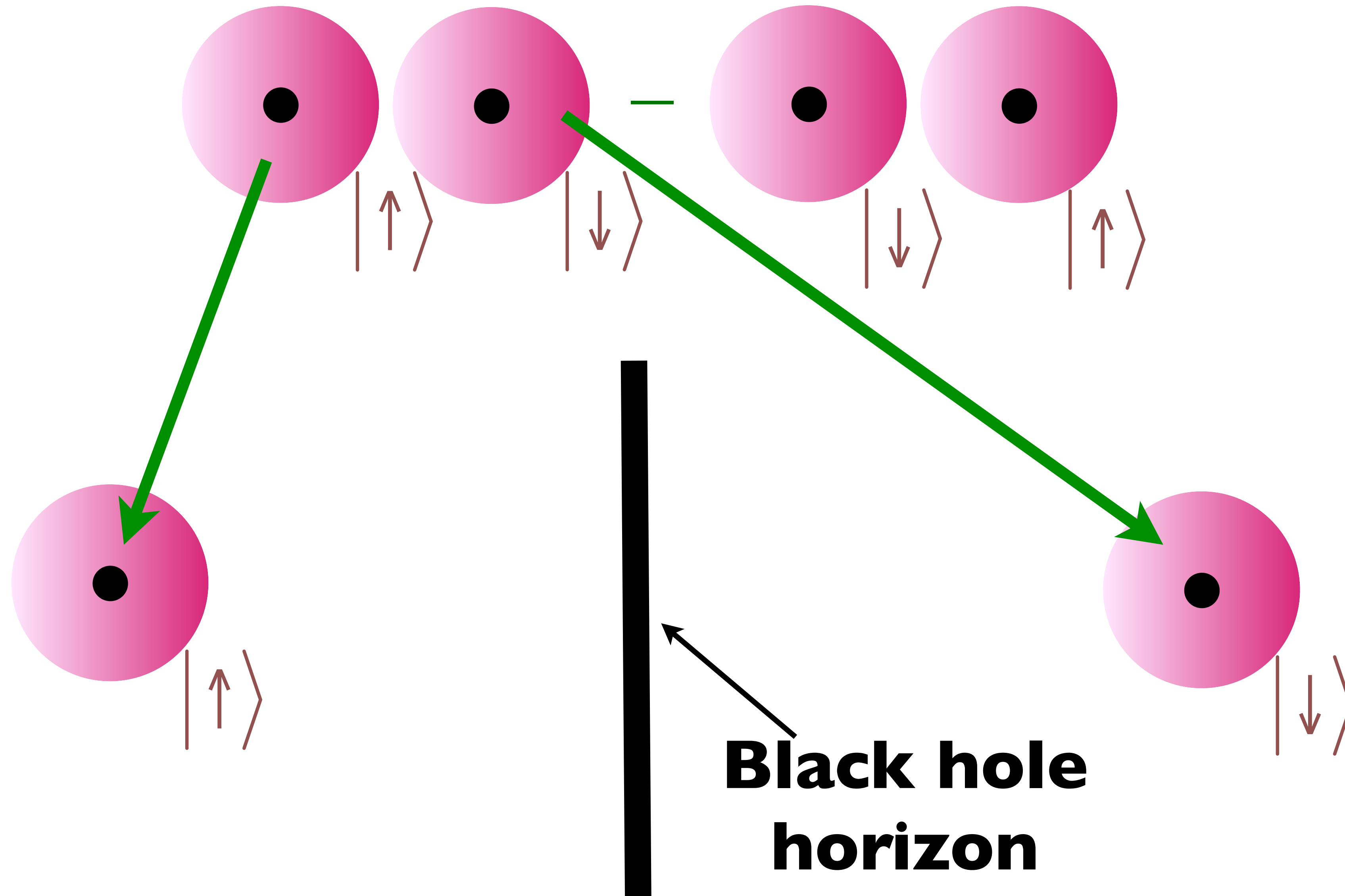
This singularity convinced many early on that black holes were unphysical solutions of Einstein's equations, and did not exist in our universe.

In any case, it was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

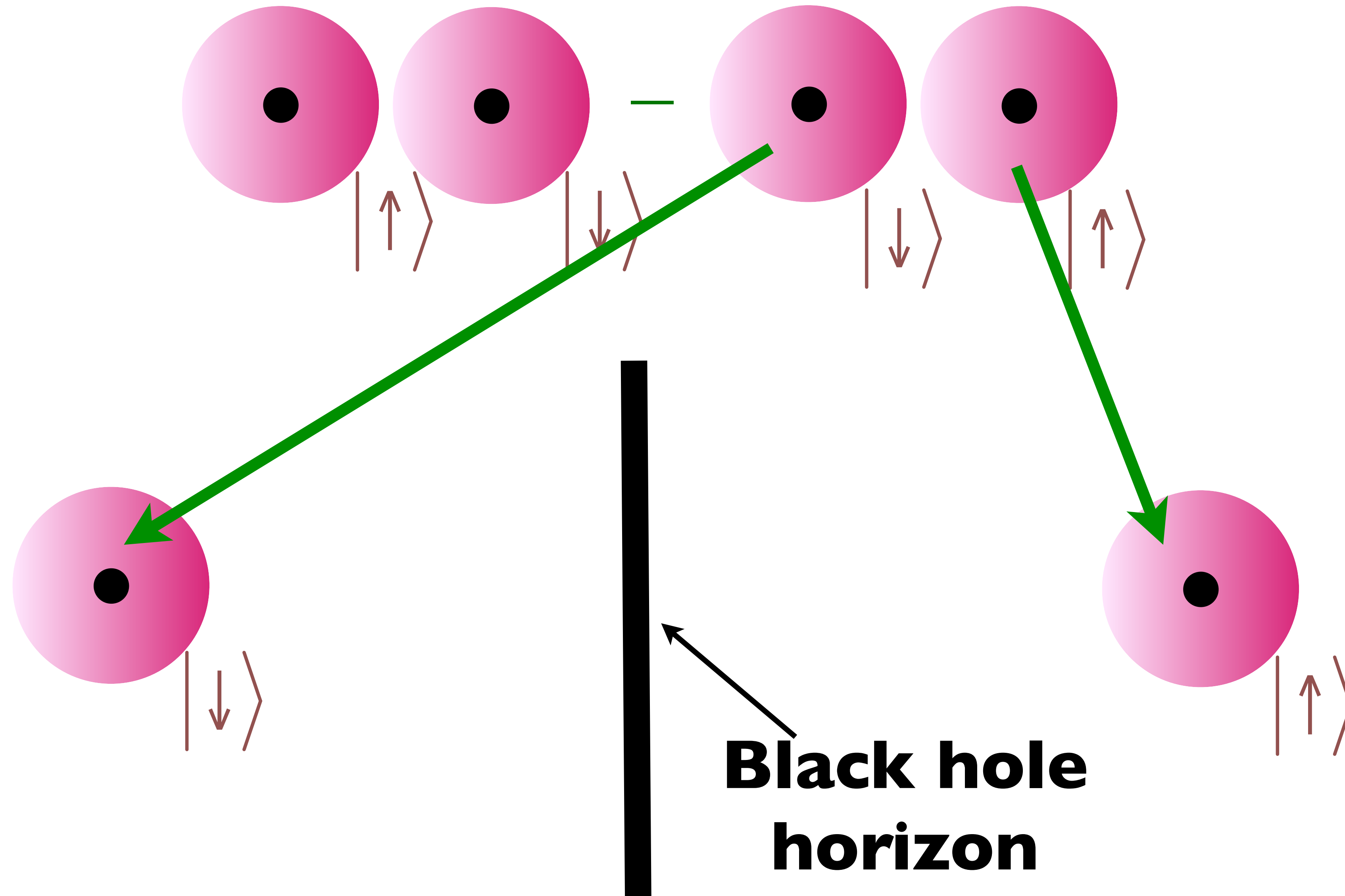
# Quantum Entanglement across a black hole horizon



# Quantum Entanglement across a black hole horizon

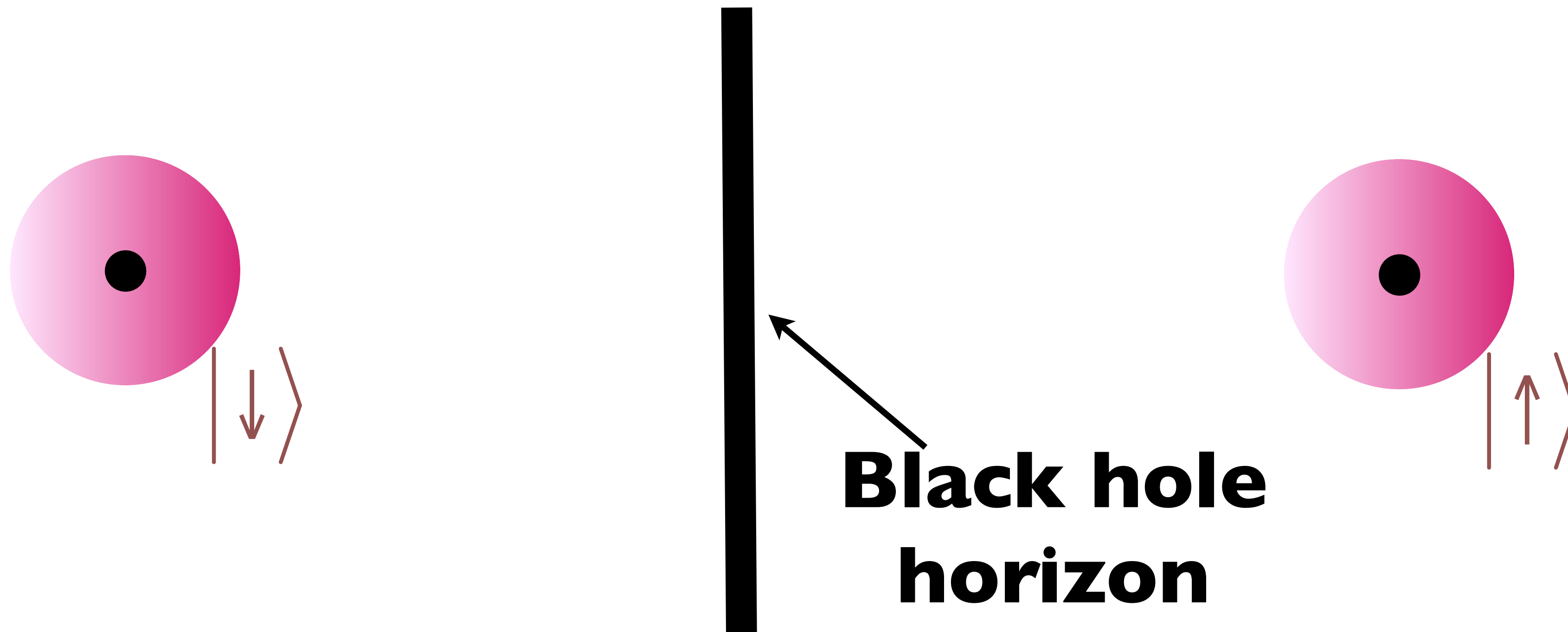


# Quantum Entanglement across a black hole horizon



# Quantum Entanglement across a black hole horizon

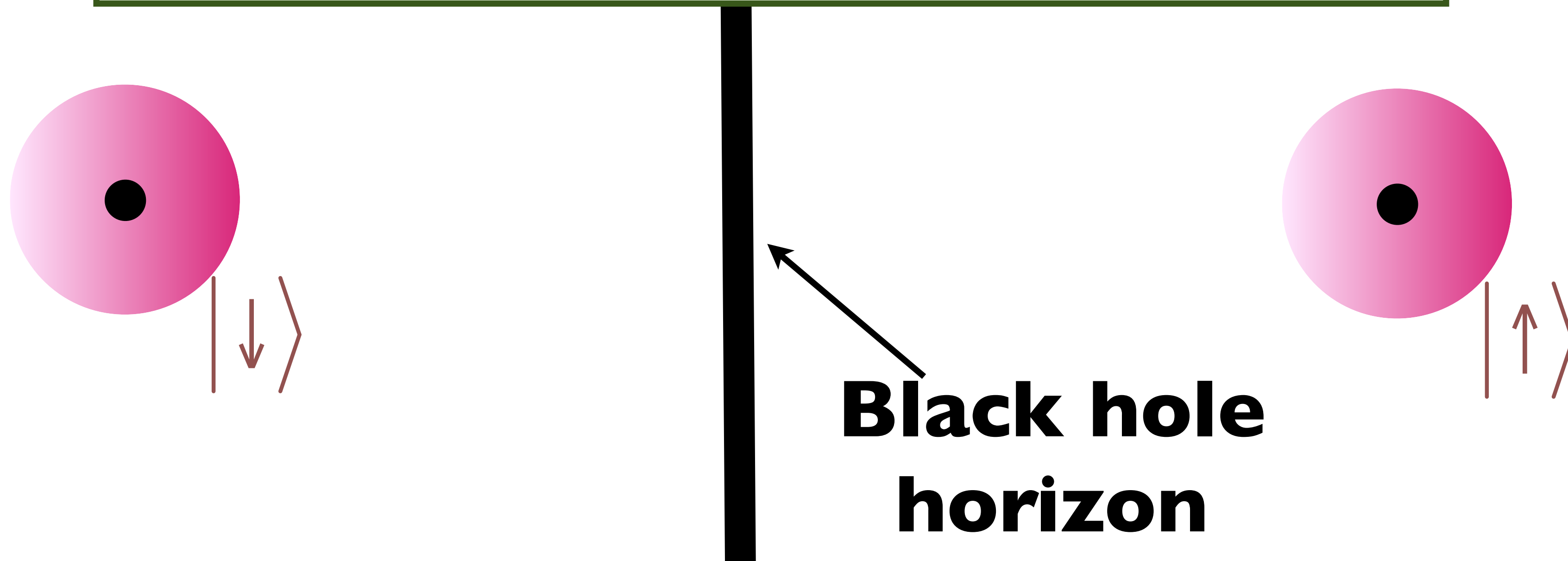
There is quantum entanglement between the inside and outside of a black hole



# Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature and an entropy

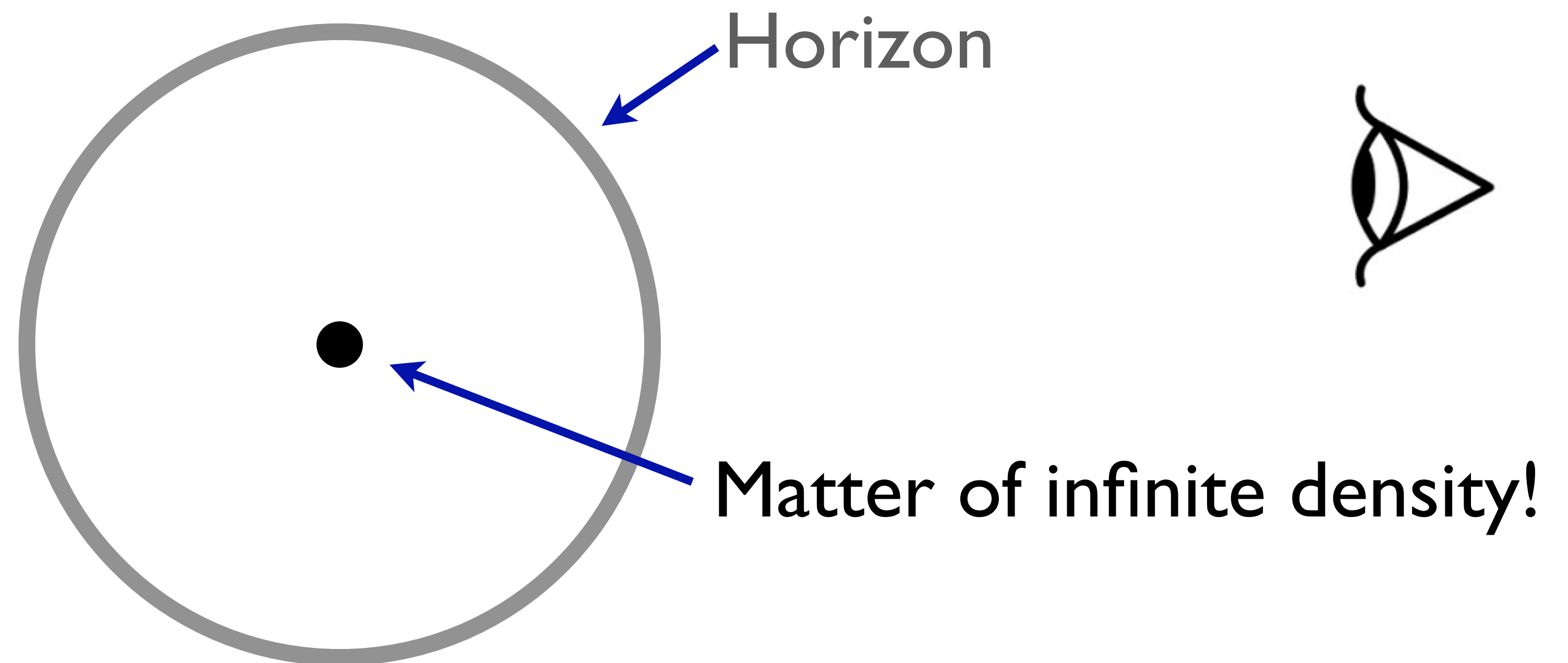
(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)



# Quantum Black Holes

Hawking obtained the black hole entropy by semiclassical computations for an observer outside the black hole horizon.

This allowed Hawking to avoid the contradictions associated with the singularity at the center of the black hole.



# Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states  $D(E) = \exp(S(E)/k_B)$



Ludwig Boltzmann

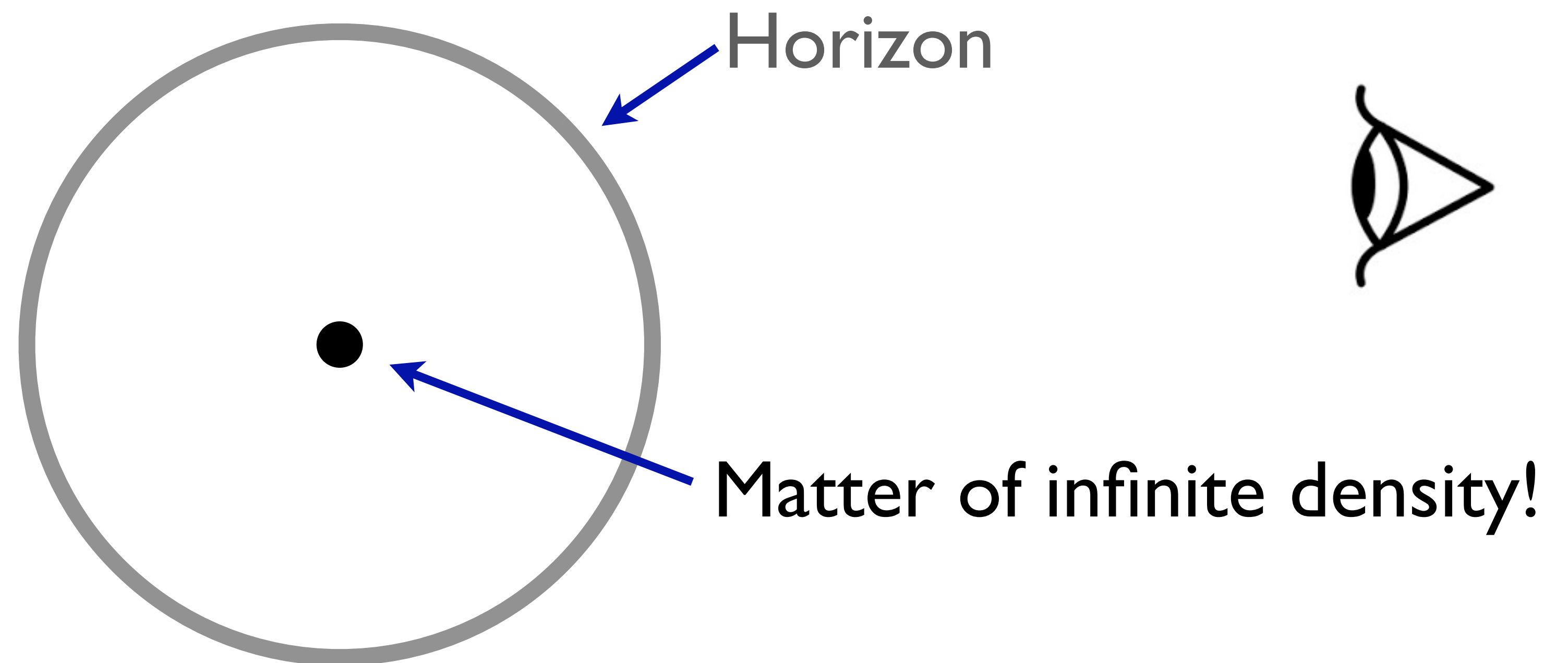
20 February 1844 - September 5, 1906

Vienna, Austria

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## Quantum Black Holes

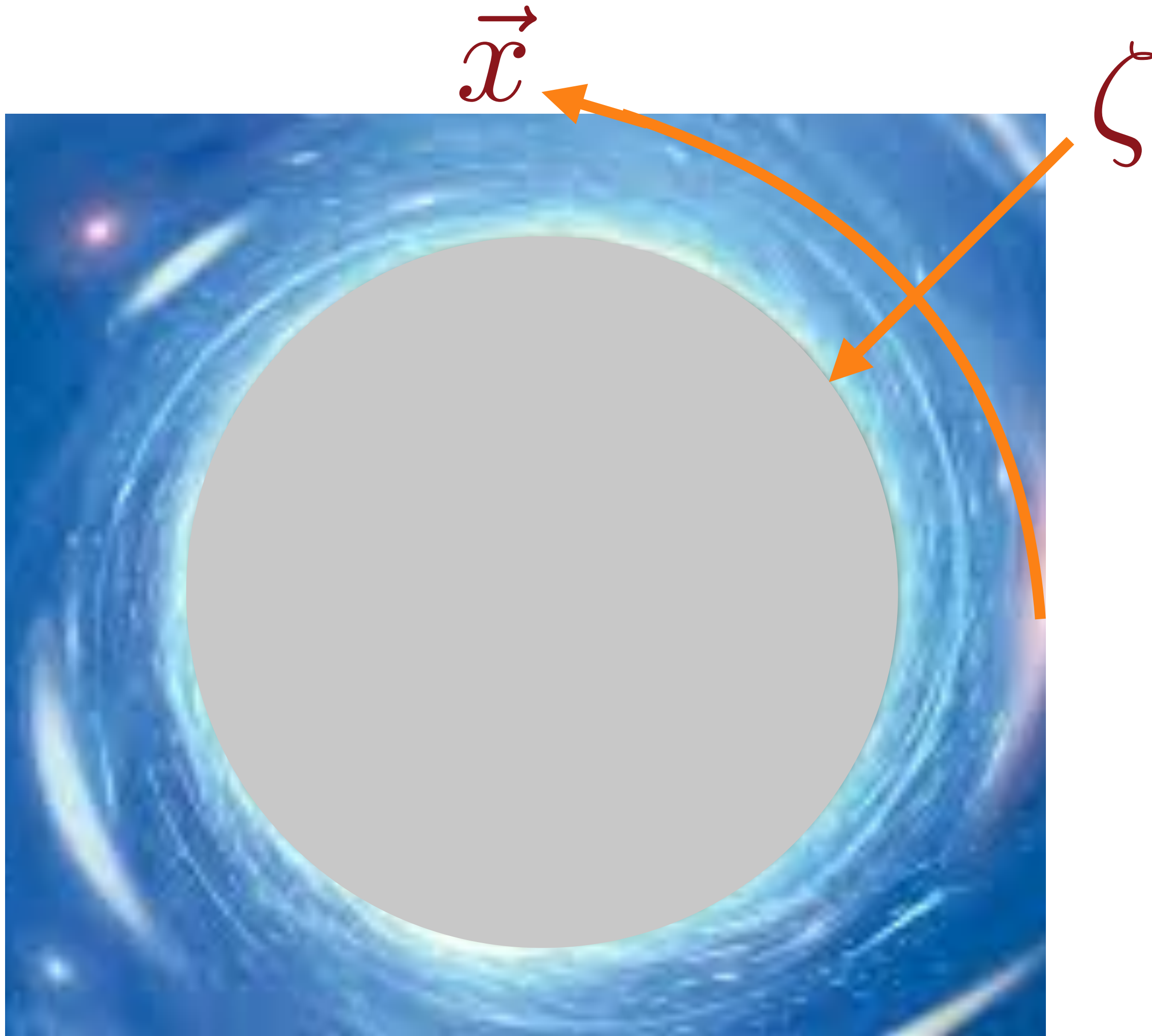
Hawking obtained the black hole entropy by semiclassical computations for an observer outside the black hole horizon.

Can we find a quantum theory for the collapsed matter at the center of the black hole, whose density of quantum states matches the Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics ?

From the SYK model  
to a quantum theory of  
charged black holes

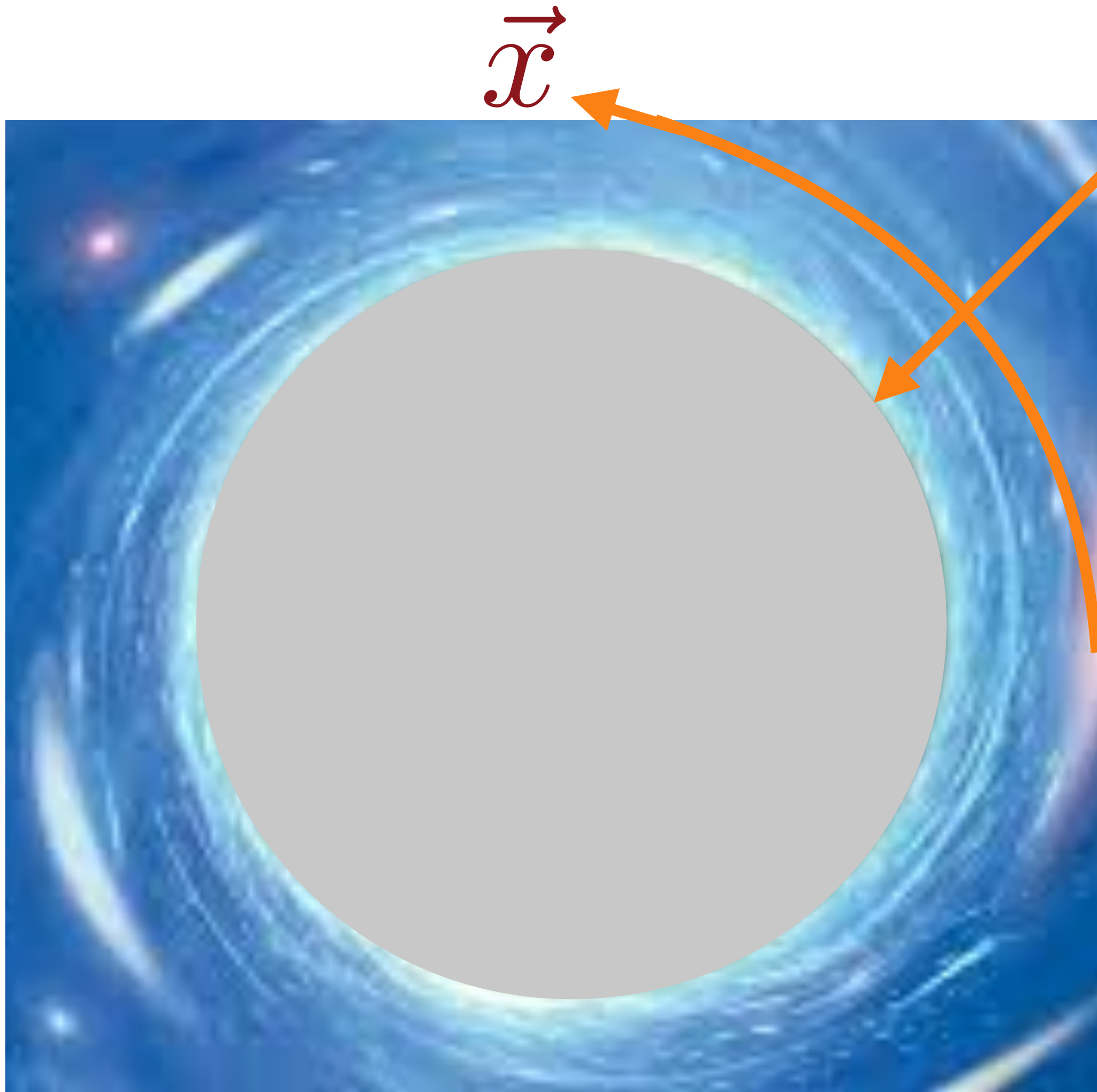


Maxwell's electromagnetism  
and Einstein's general relativity  
allow black hole solutions with a net charge





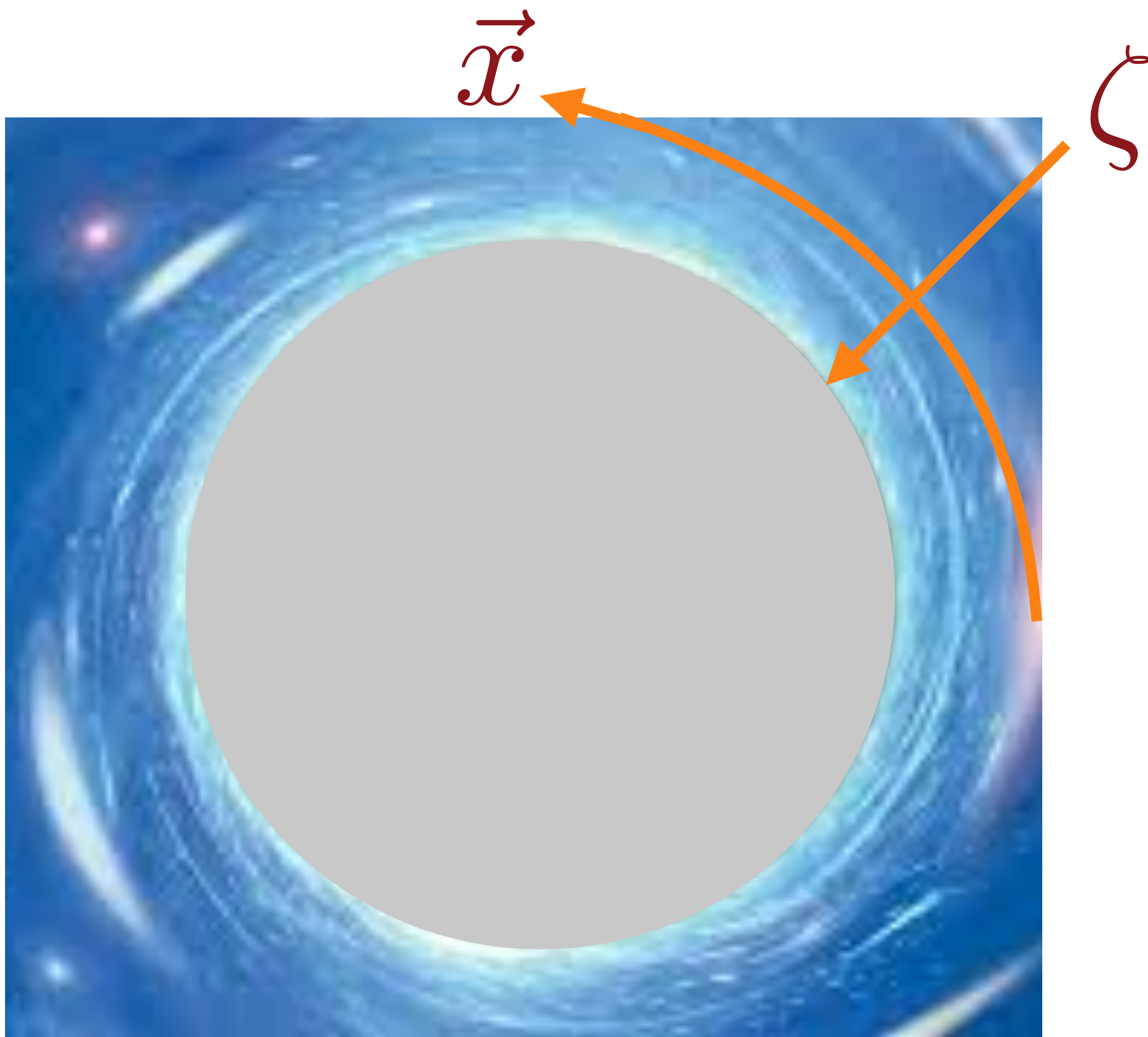
Maxwell's electromagnetism  
and Einstein's general relativity  
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Zooming into the near-  
horizon region of a charged  
black hole at low  
temperature, yields a  
quantum theory in one  
space ( $\zeta$ ) and one time  
dimension



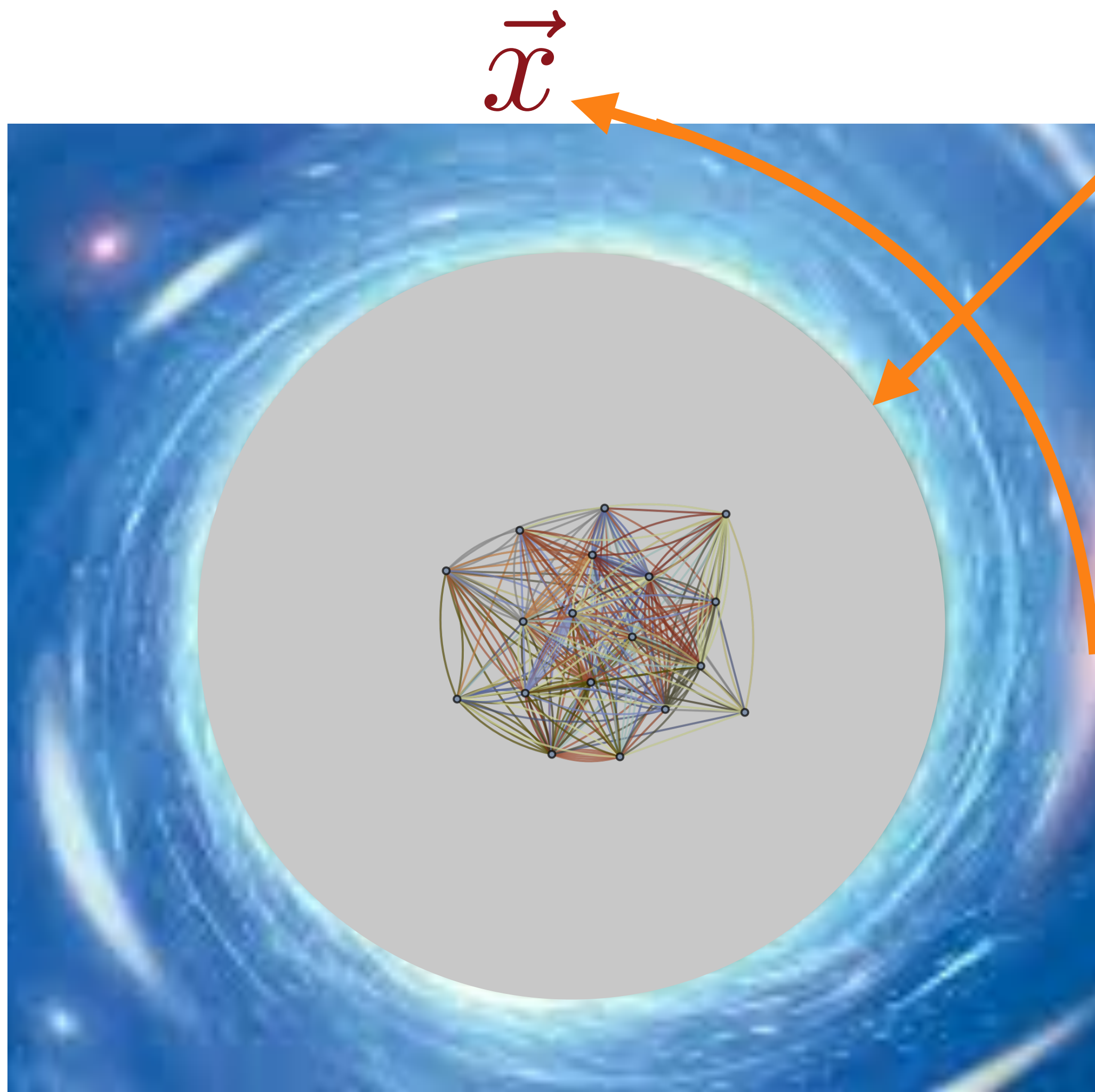
Maxwell's electromagnetism  
and Einstein's general relativity  
allow black hole solutions with a net charge



The quantum versions of  
Maxwell's and Einstein's  
equations in this  
two-dimensional spacetime are  
also the equations describing  
electron entanglement in the  
SYK model!



Maxwell's electromagnetism  
and Einstein's general relativity  
allow black hole solutions with a net charge

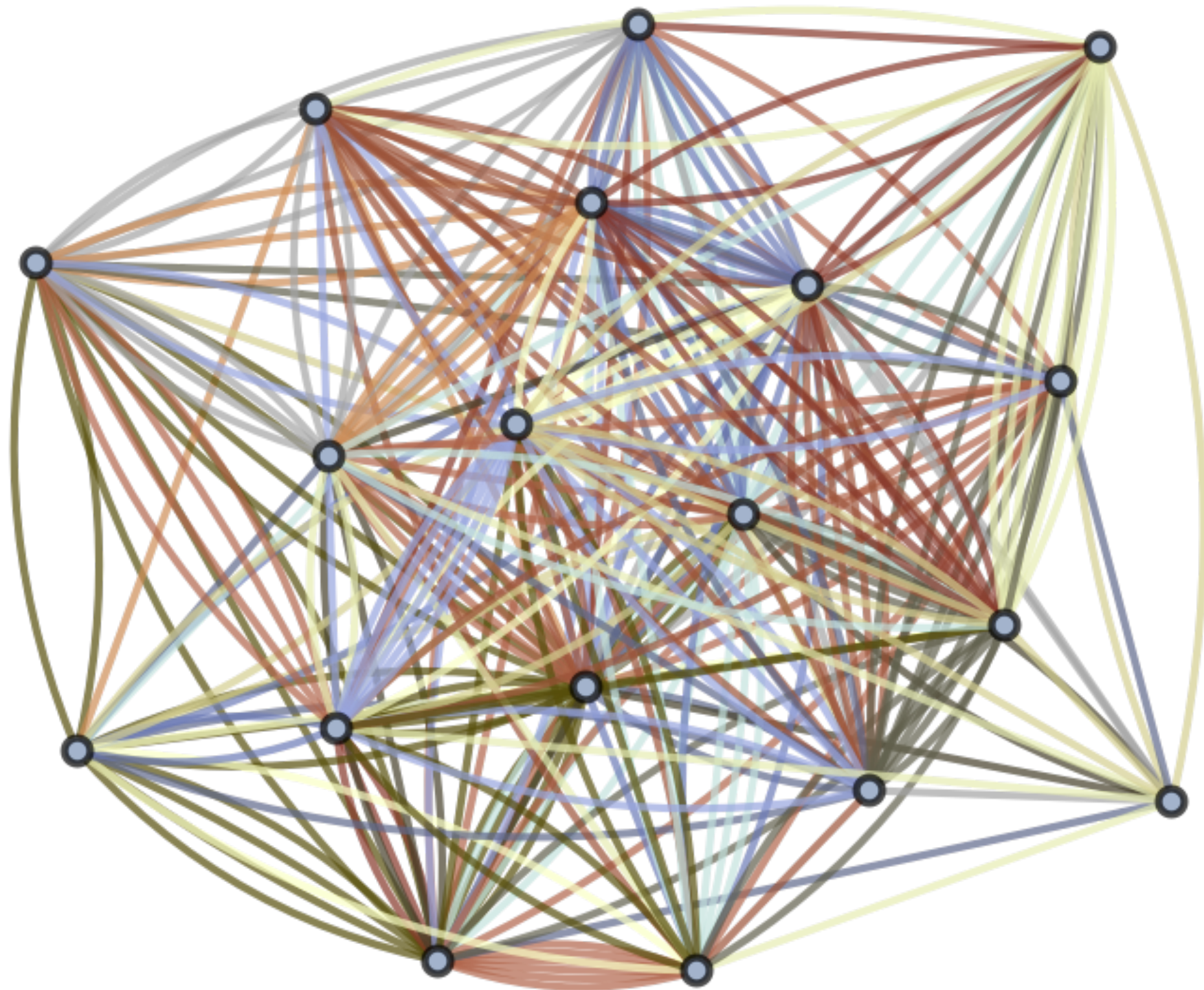


The SYK provides the  
needed realization of the  
black hole interior, and its  
density of quantum states  
matches gravitational  
entropy computations for  
charged black holes !

Recap

# The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

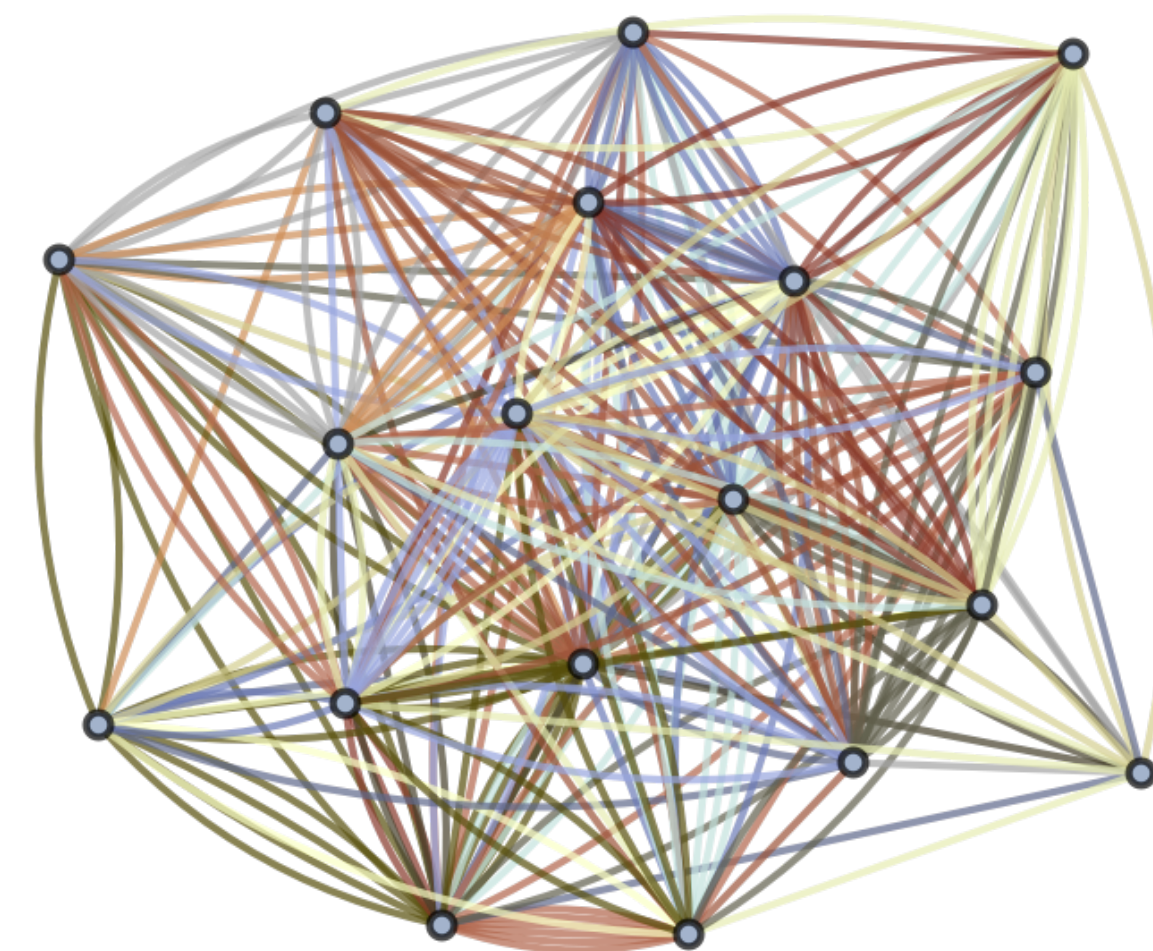
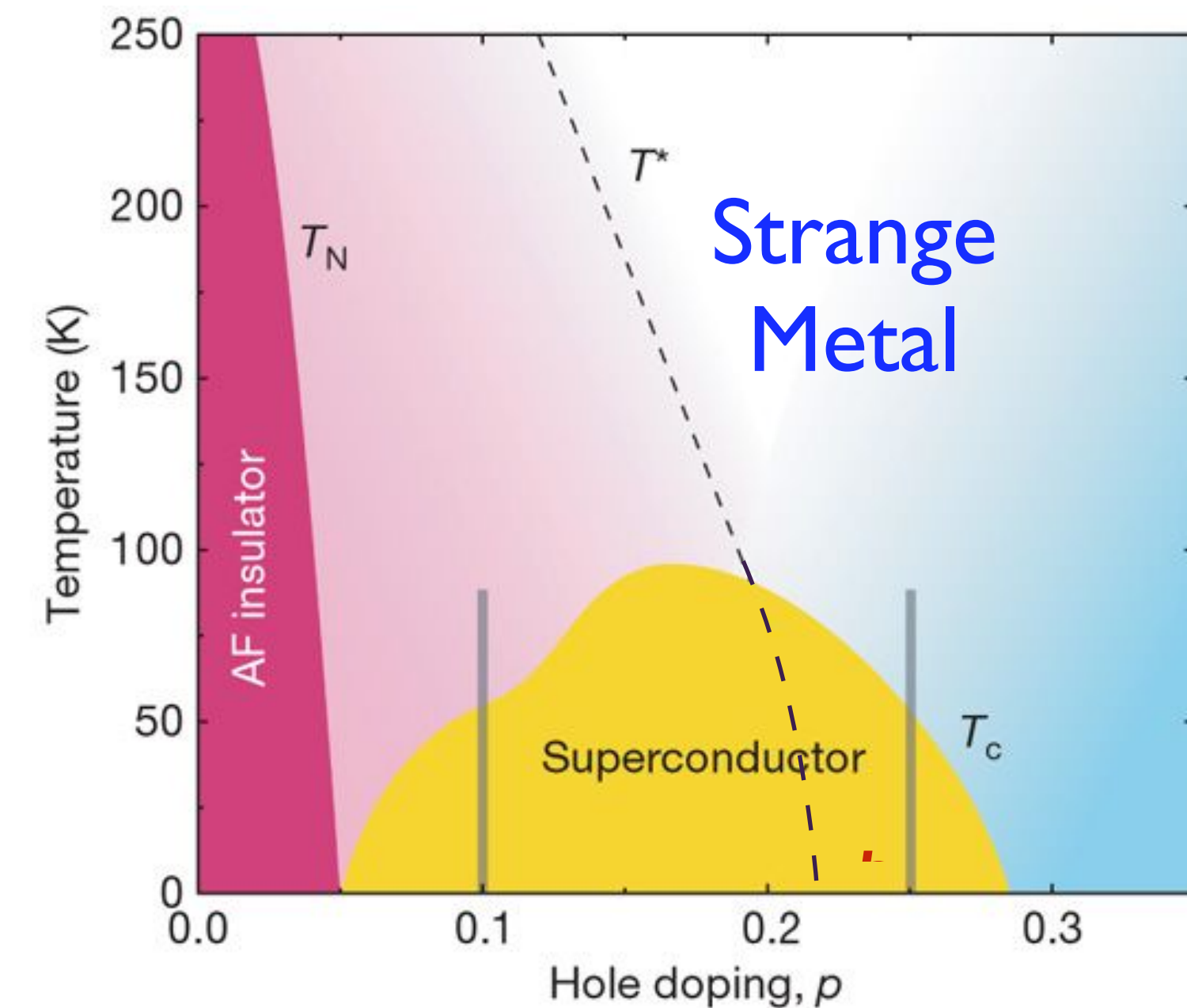


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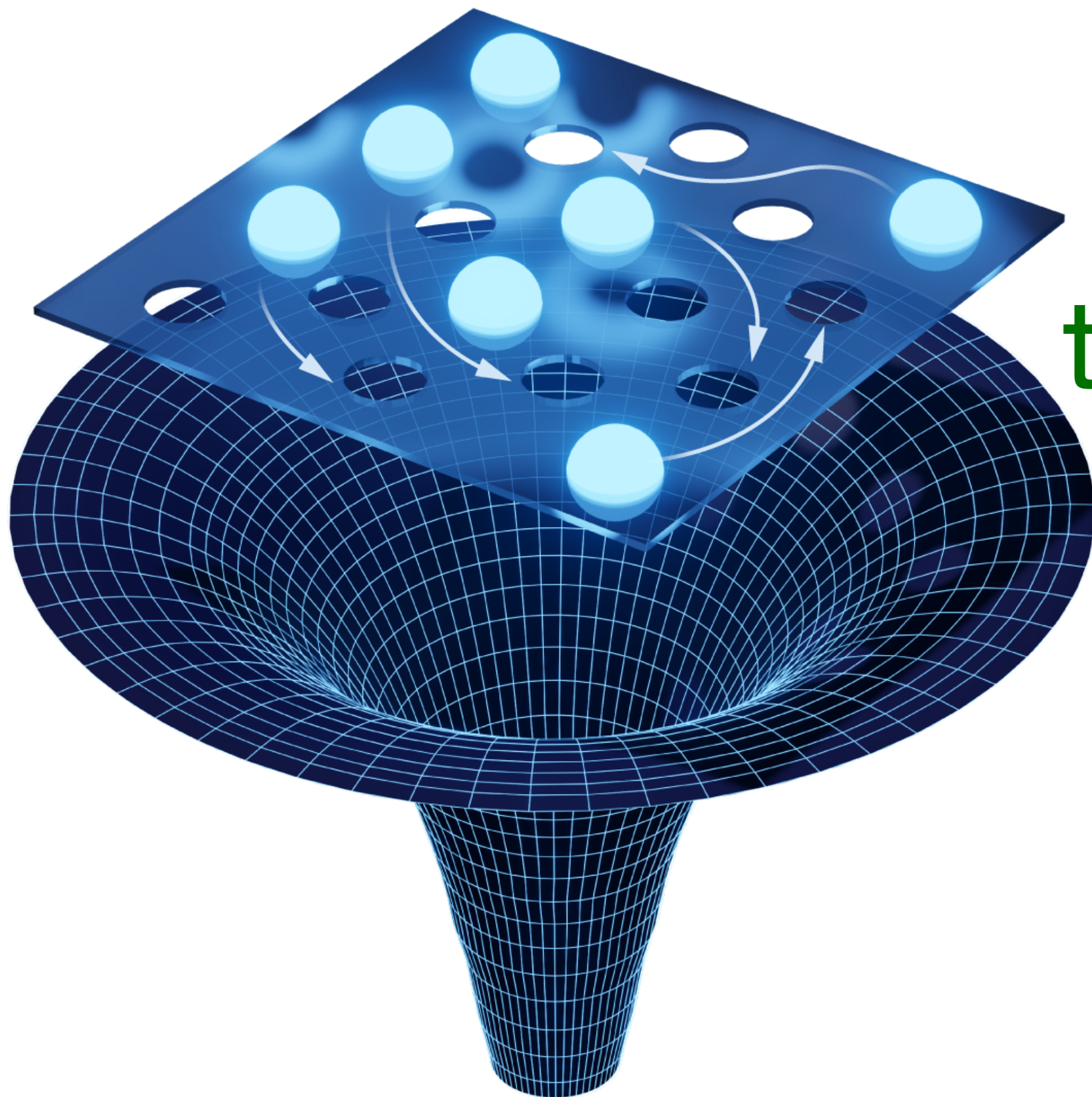
In one set of variables, it helps describe the *strange* electrical properties of YBCO

Sachdev, Ye (1993)



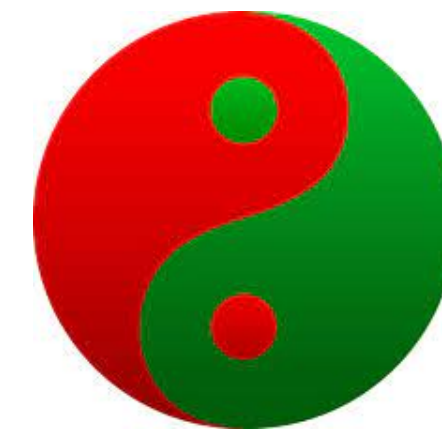
# The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



In a ***dual*** set of variables it describes ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

# The many faces of multi-particle entanglement

- Absence of quasiparticles, as in the SYK model and the strange metal
- Fractionalization and new emergent particles, as in spin liquids.
- Higher temperature superconductivity (?)

# The many faces of multi-particle entanglement

- Absence of quasiparticles, as in the SYK model and the strange metal
- Fractionalization and new emergent particles, as in spin liquids.
- Higher temperature superconductivity (?)
- A quantum theory of the interior of a black hole.