Universal low temperature theory of SYK models of strange metals and charged black holes

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Higgs Centre for Theoretical Physics
University of Edinburgh
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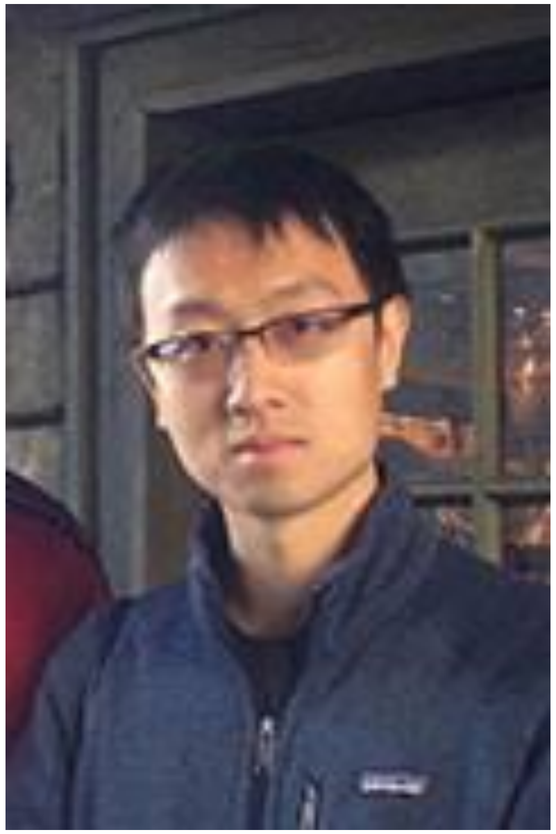
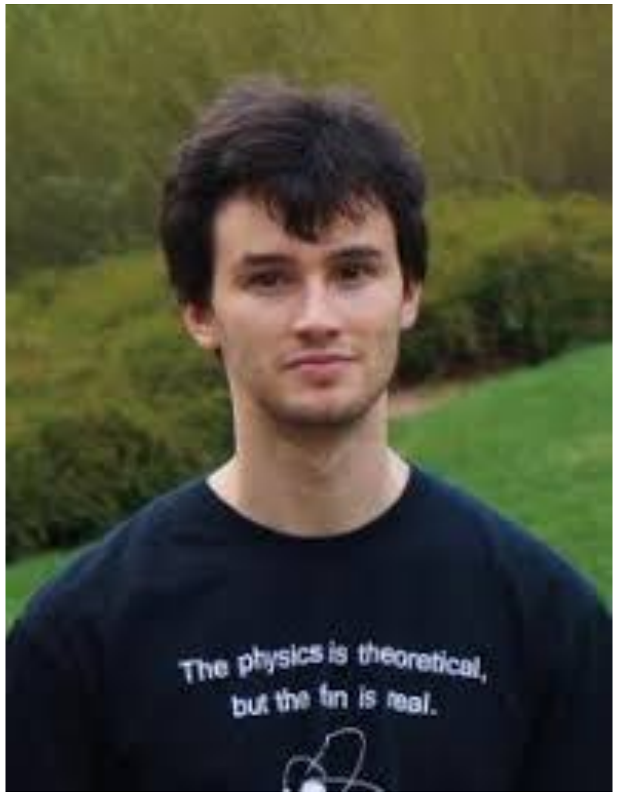
Wenbo Fu



Yingfei Gu



Grigory Tarnopolsky

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Main result

SYK model of fermions with random interactions of mean-square-value J, with total fermion number $\mathcal{Q}$, at temperatures $T \ll J$

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SYK model of fermions with random interactions of mean-square-value J, with total fermion number $\mathcal{Q}$, at temperatures $T \ll J$

and

Charged black holes in 3+1 dimensions of radius R_h, with total charge $\mathcal{Q}$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in $0+1$ dimensions

Main result

The common low T path integral is $\mathcal{Z} = \int \mathcal{D}f\, \mathcal{D}\phi\, e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left(\frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E} T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \mathrm{Sch}[\tan(\pi T f(\tau)), \tau] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with
$f(\tau + 1/T) = f(\tau) + 1/T$,
ϕ is a phase conjugate to the charge density with
$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n$, n integer,
$\mathrm{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$.
The couplings are related to the entropy $S(T, \mathcal{Q})$
and the chemical potential μ via

$$S(T \to 0, \mathcal{Q}) = s_0 + \gamma T, \quad K = \left(\frac{d\mathcal{Q}}{d\mu} \right)_{T \to 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{d\mathcal{Q}}$$

Main result

- Not the AdS/CFT correspondence, which involves only neutral black holes at $T > 0$.
- Unlike the AdS/CFT correspondence, *both* sides of the duality are solvable. This has enabled numerous recent studies of black holes quantum information.

Main result

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1. Normal and strange metals

(a) with quasiparticles (normal):

random matrix model

(b) without quasiparticles (strange):

the complex SYK model

2. Einstein-Maxwell theory of charged black holes in AdS space

3. Fluctuations, and the Schwarzian

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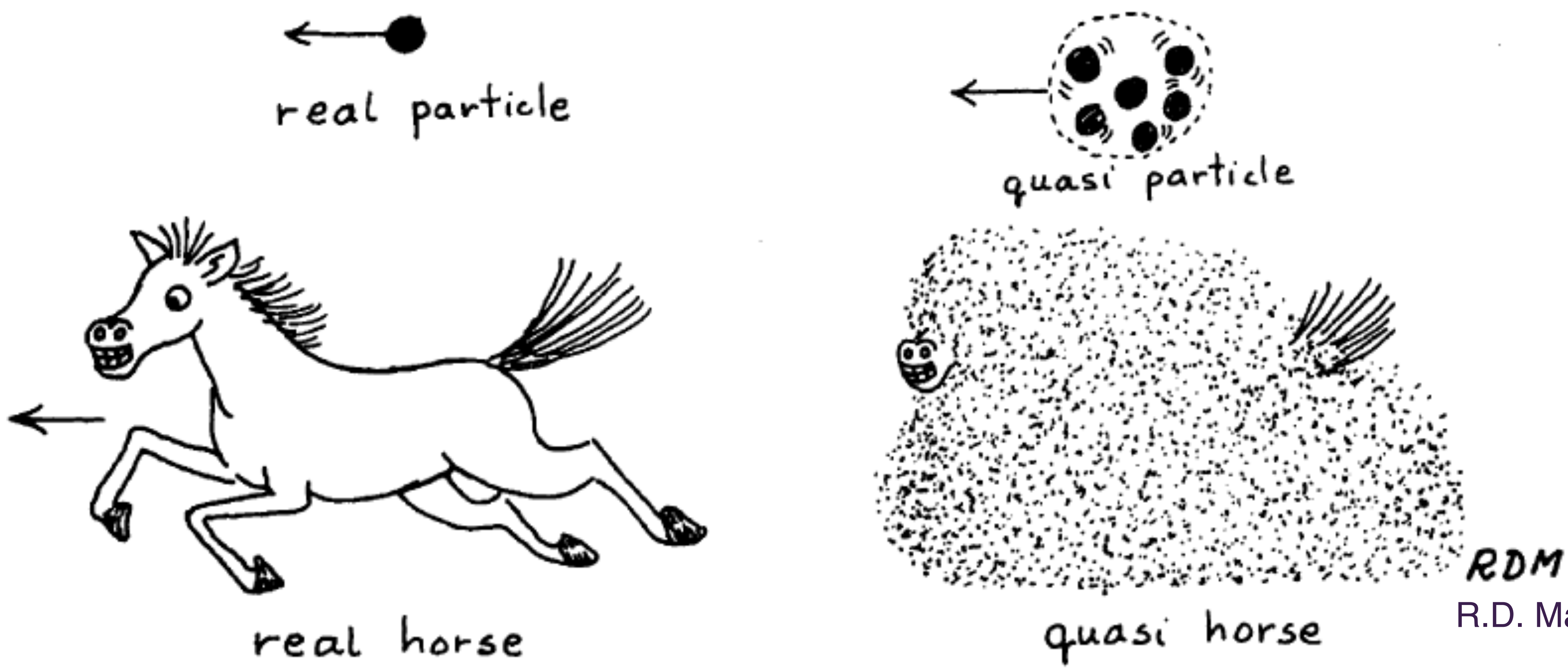
Ordinary metals

Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an "excited lump" in the many-electron state which responds just like an ordinary particle.



R.D. Mattuck

What are quasiparticles ?

- **Quasiparticles are additive excitations:** The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

What are quasiparticles ?

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where E_F is the Fermi energy.

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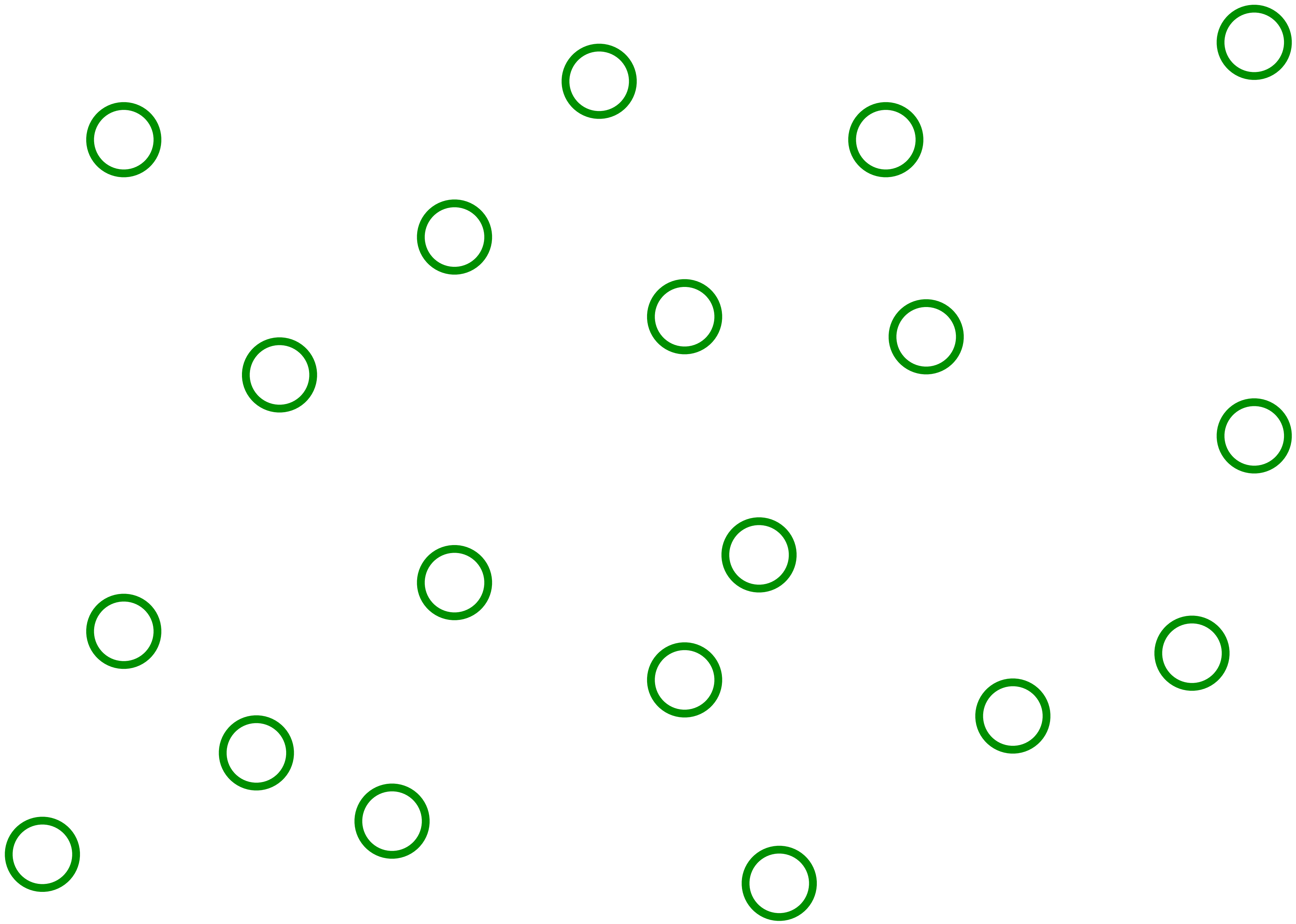
- This time is much longer than the 'Planckian time' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

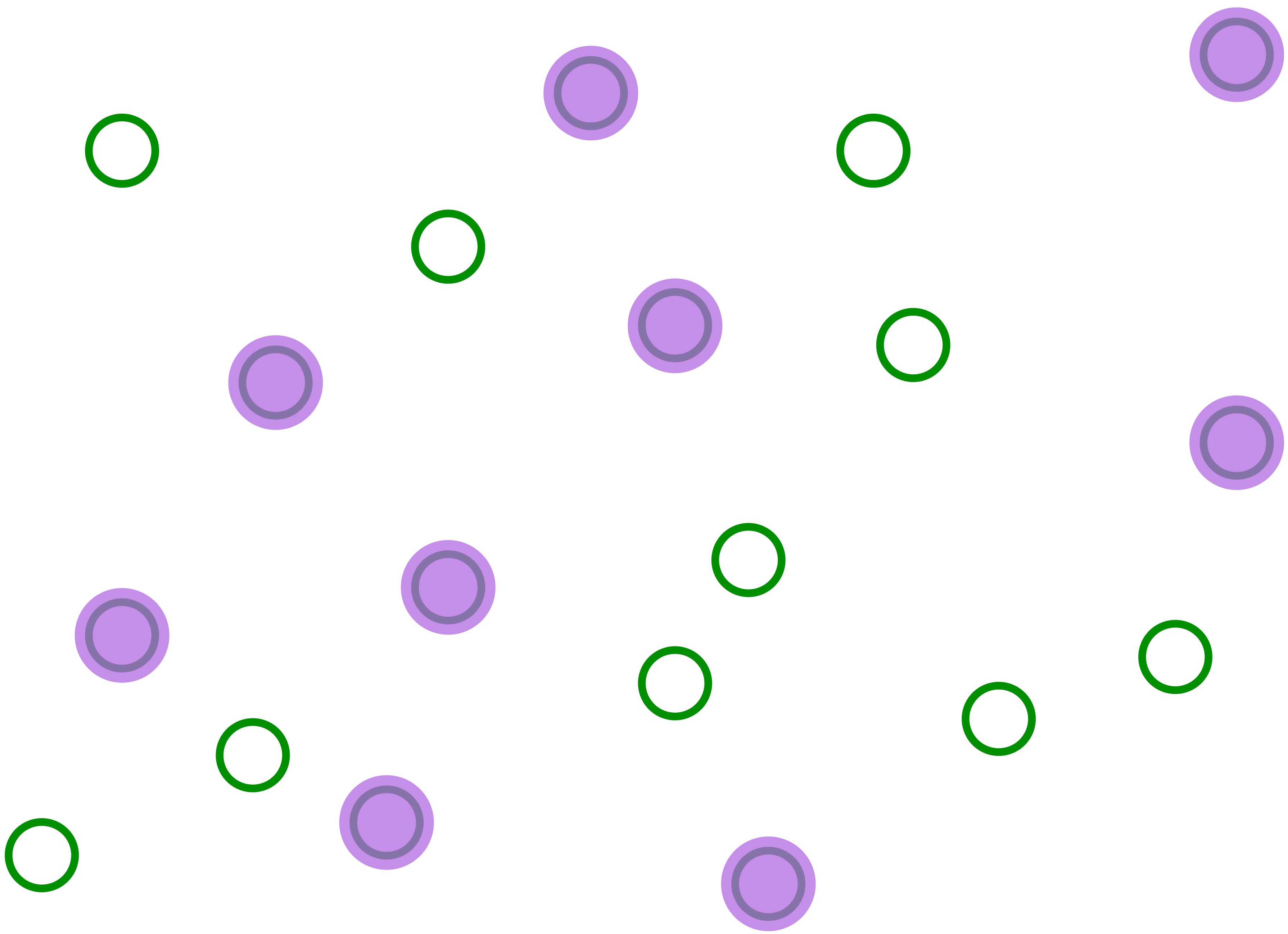
$$\tau_{\text{eq}} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

A simple model of a metal with quasiparticles



Pick a set of random positions

A simple model of a metal with quasiparticles



Place electrons randomly on some sites

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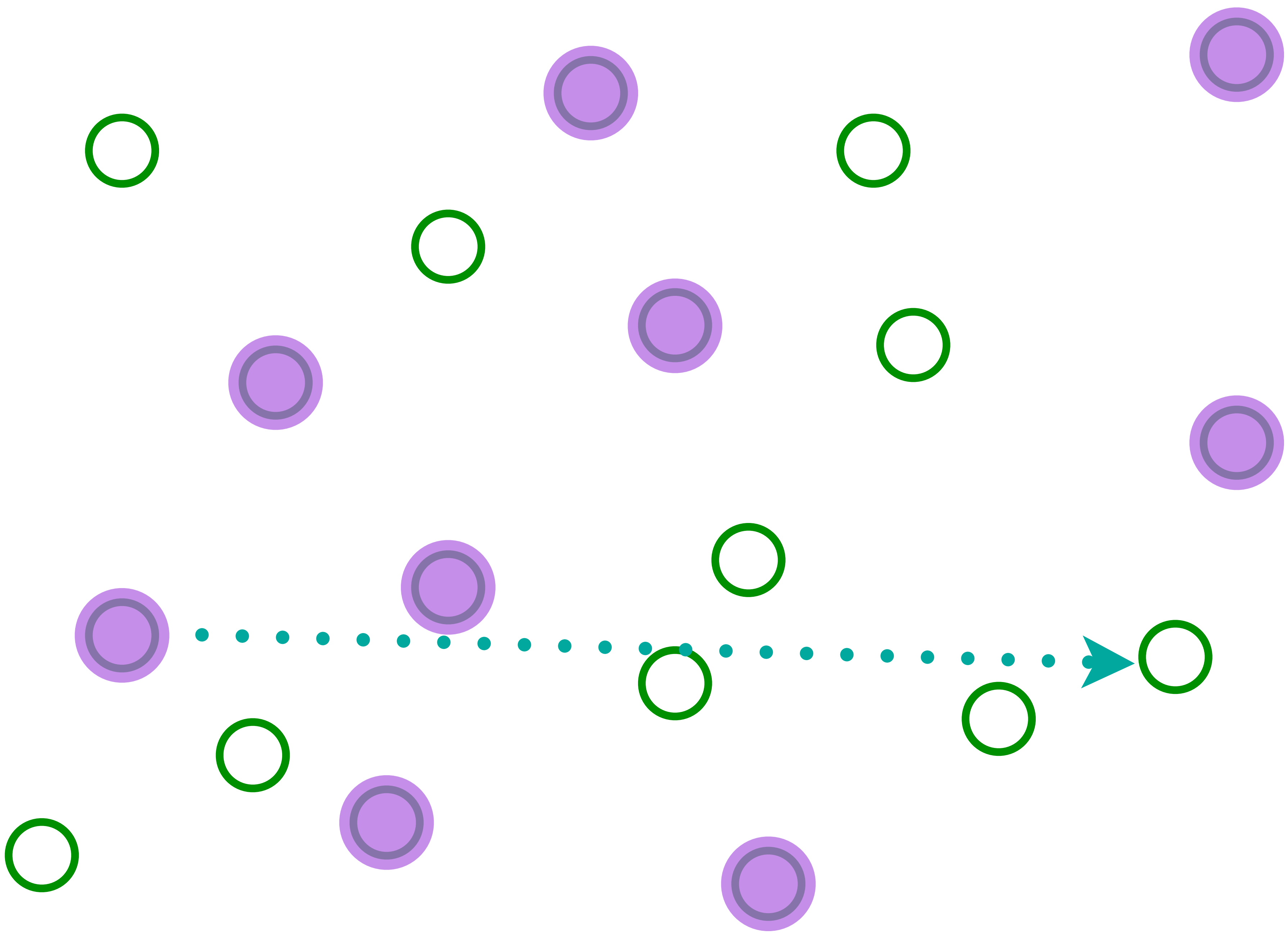

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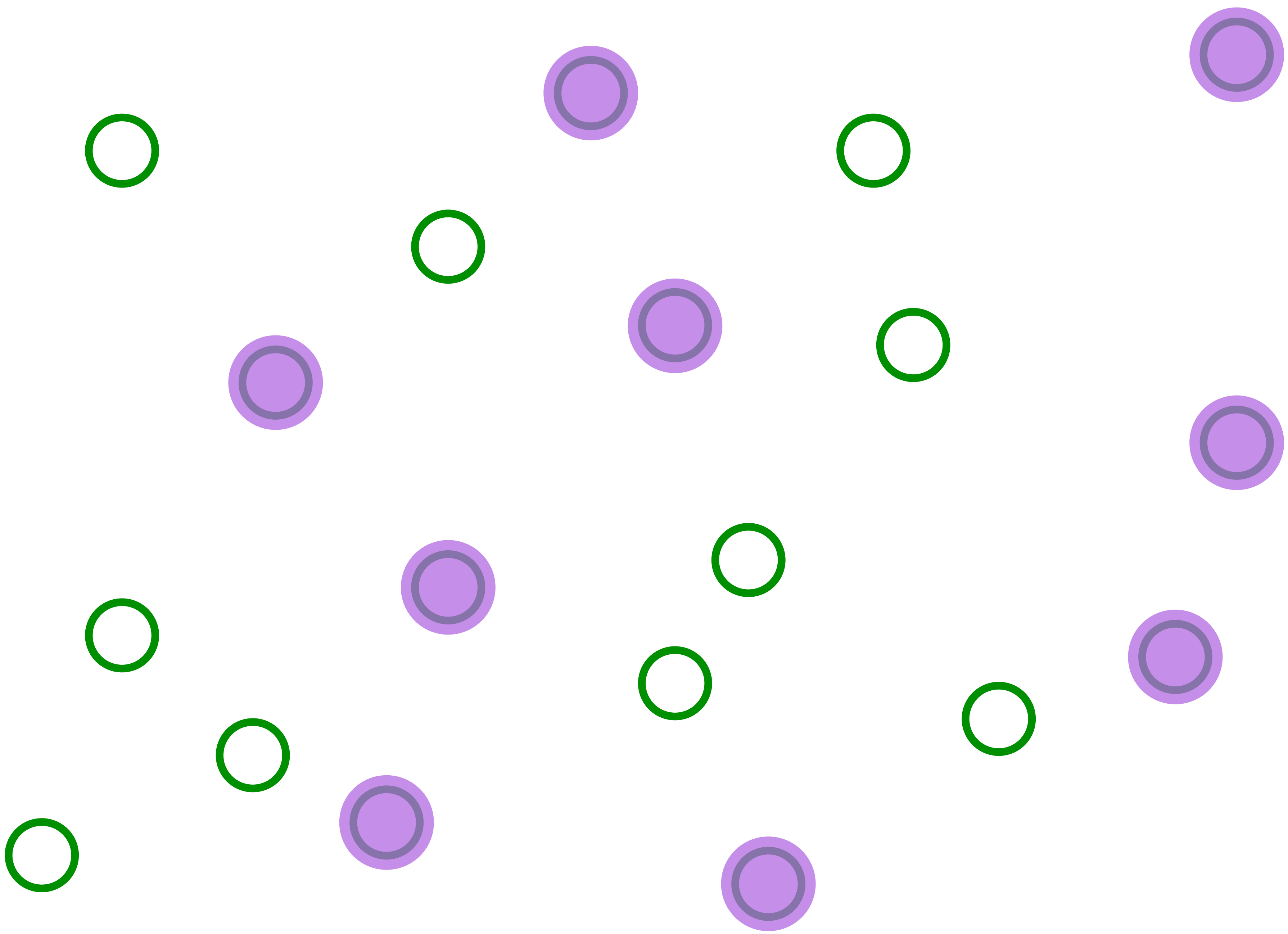

Electrons move one-by-one randomly

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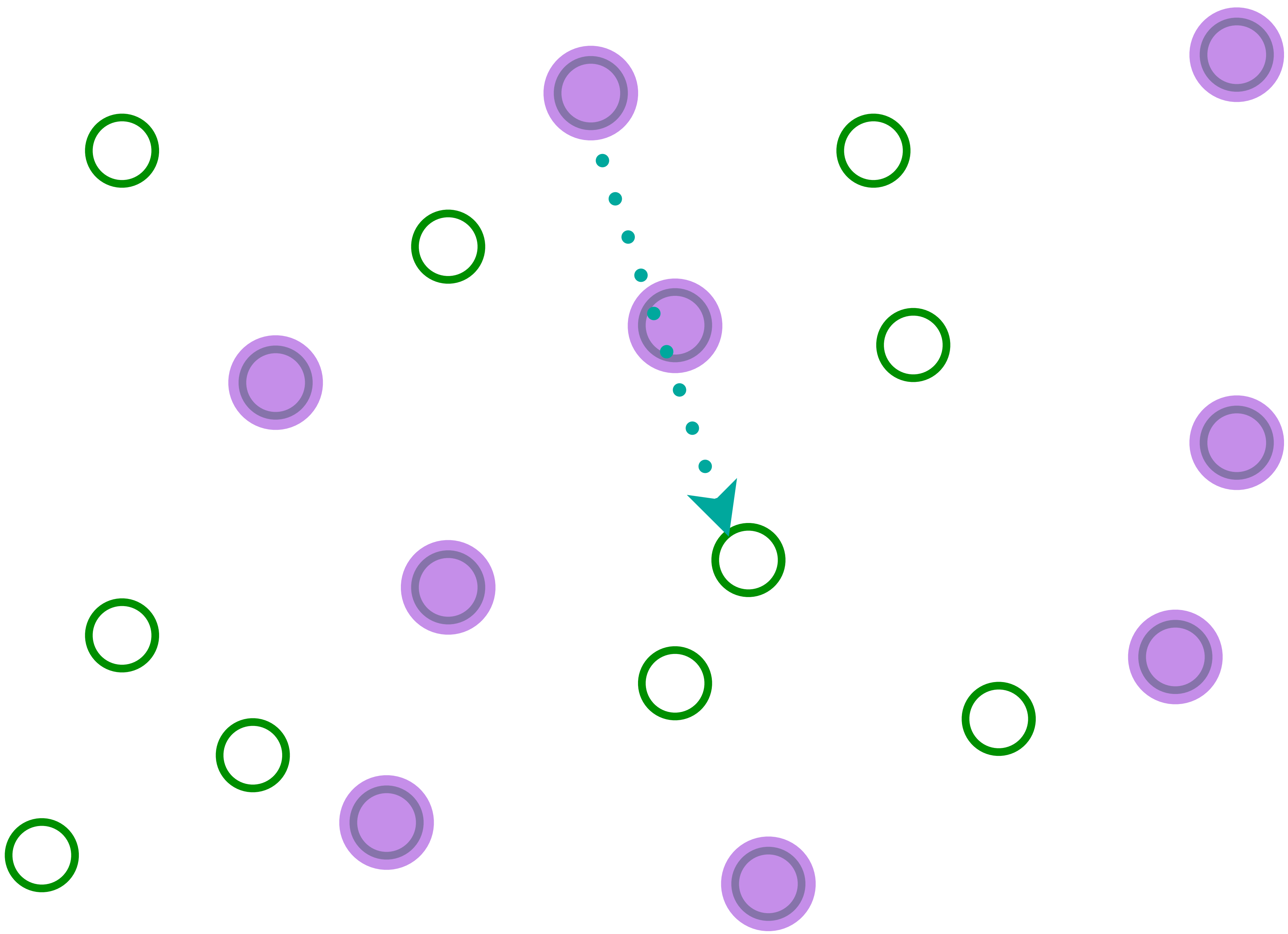

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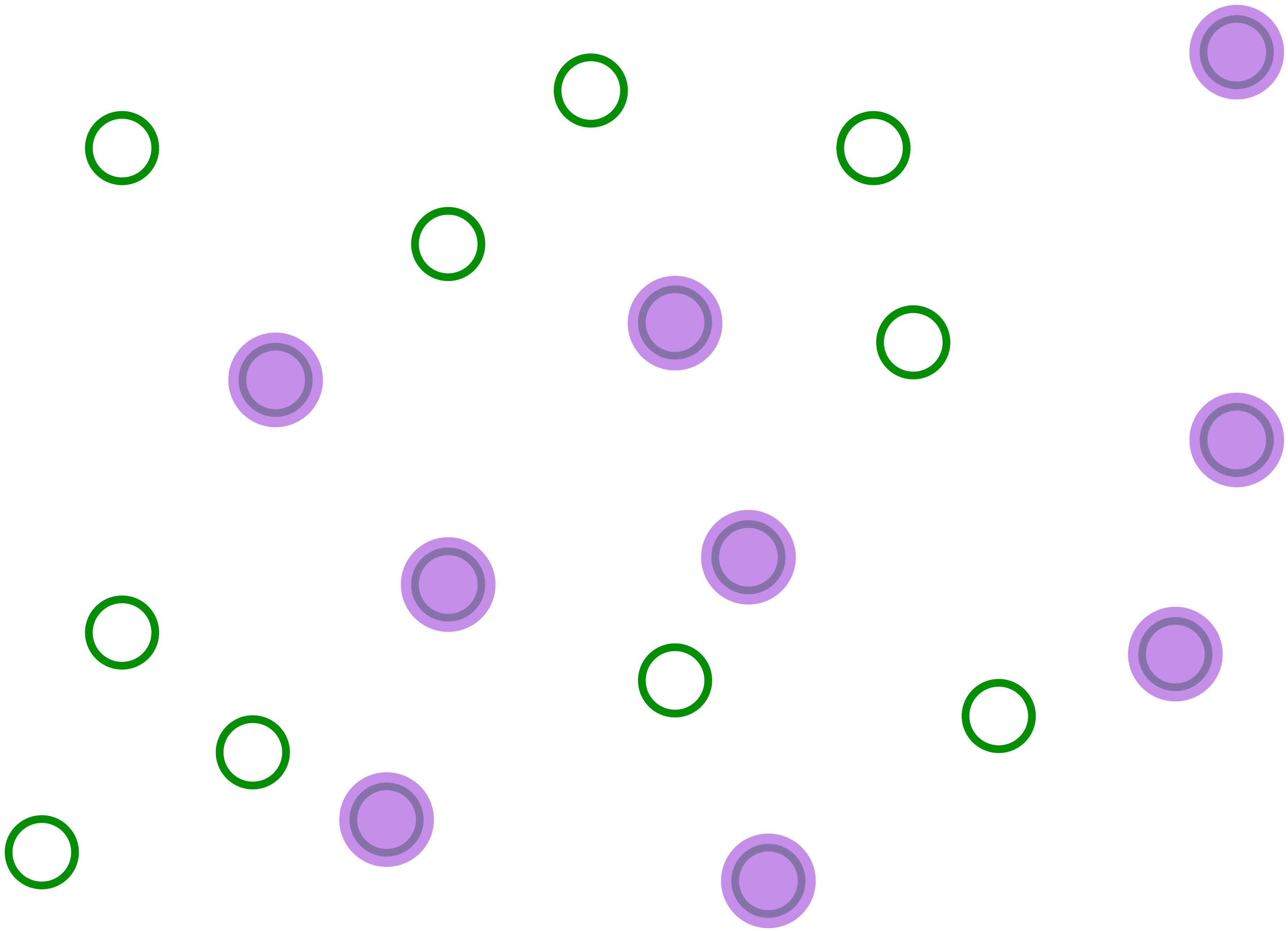

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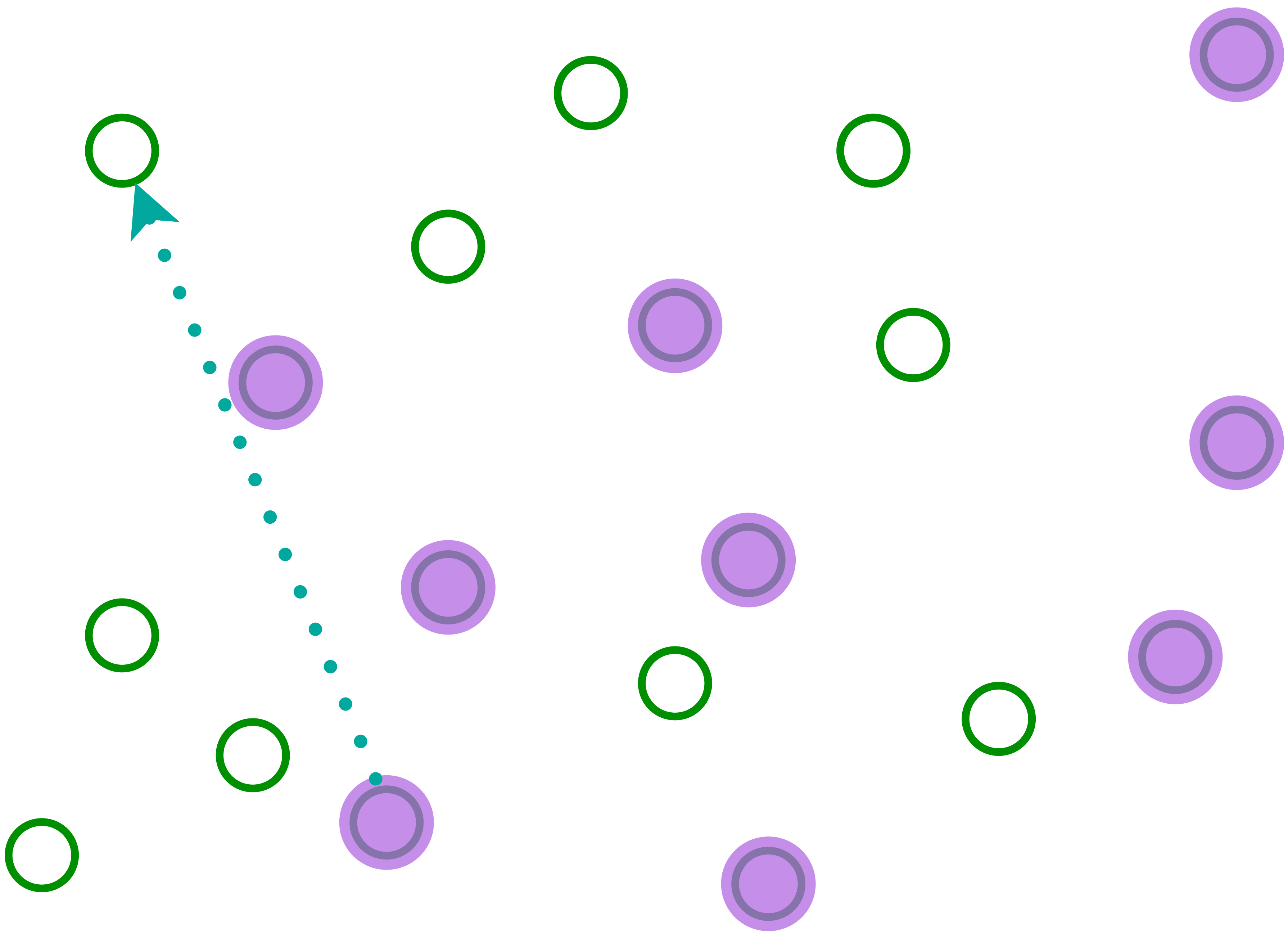

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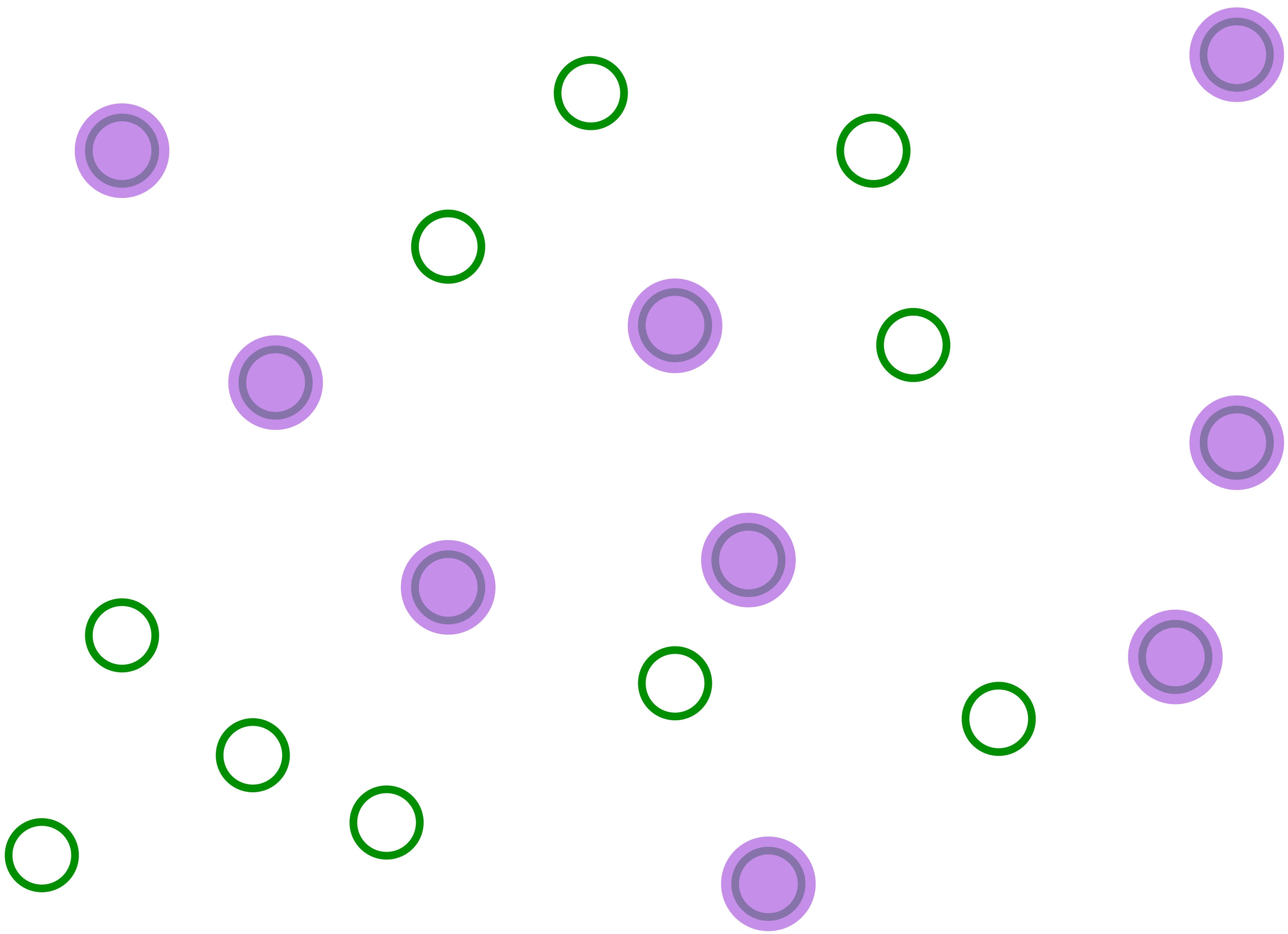

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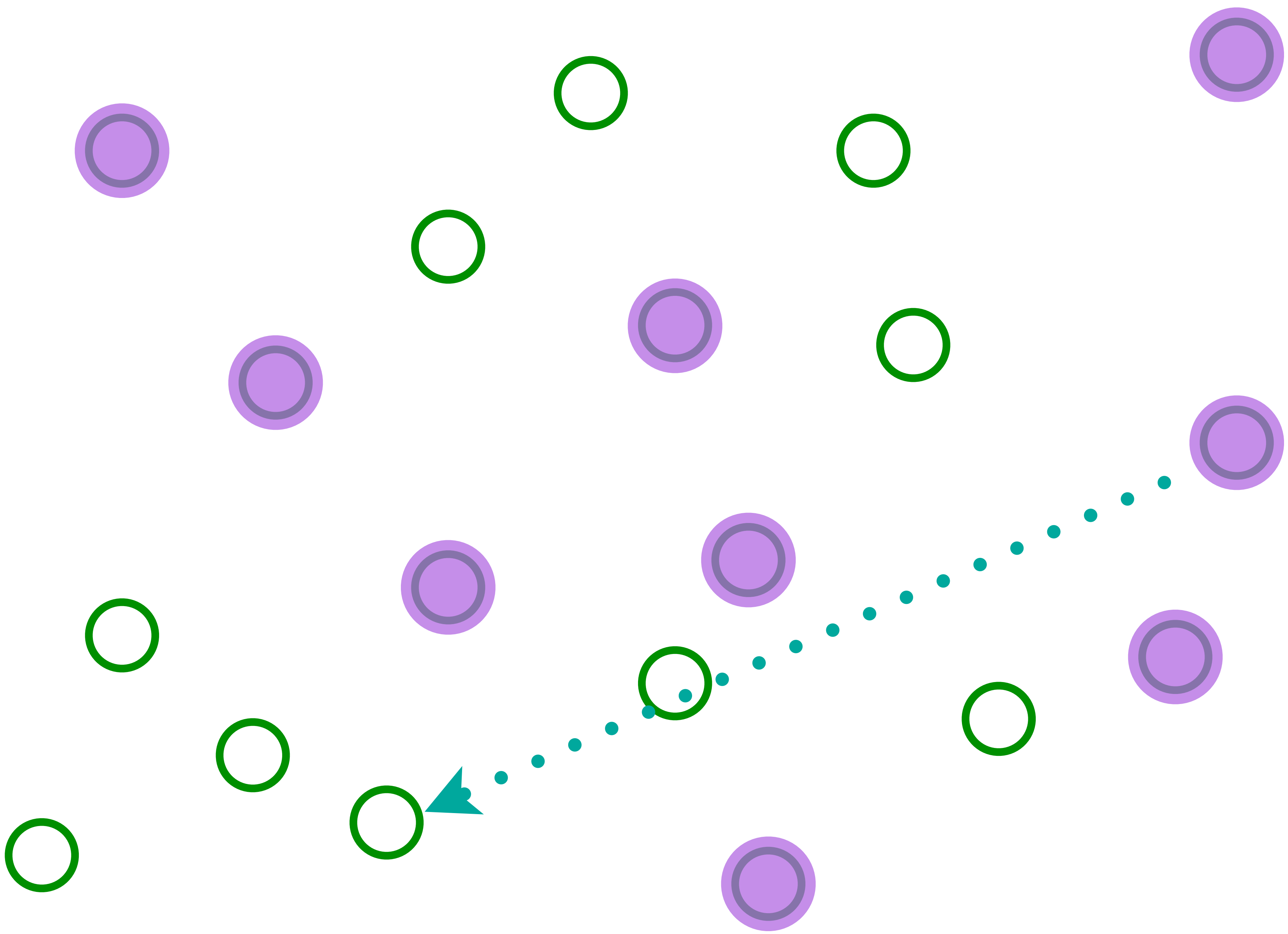

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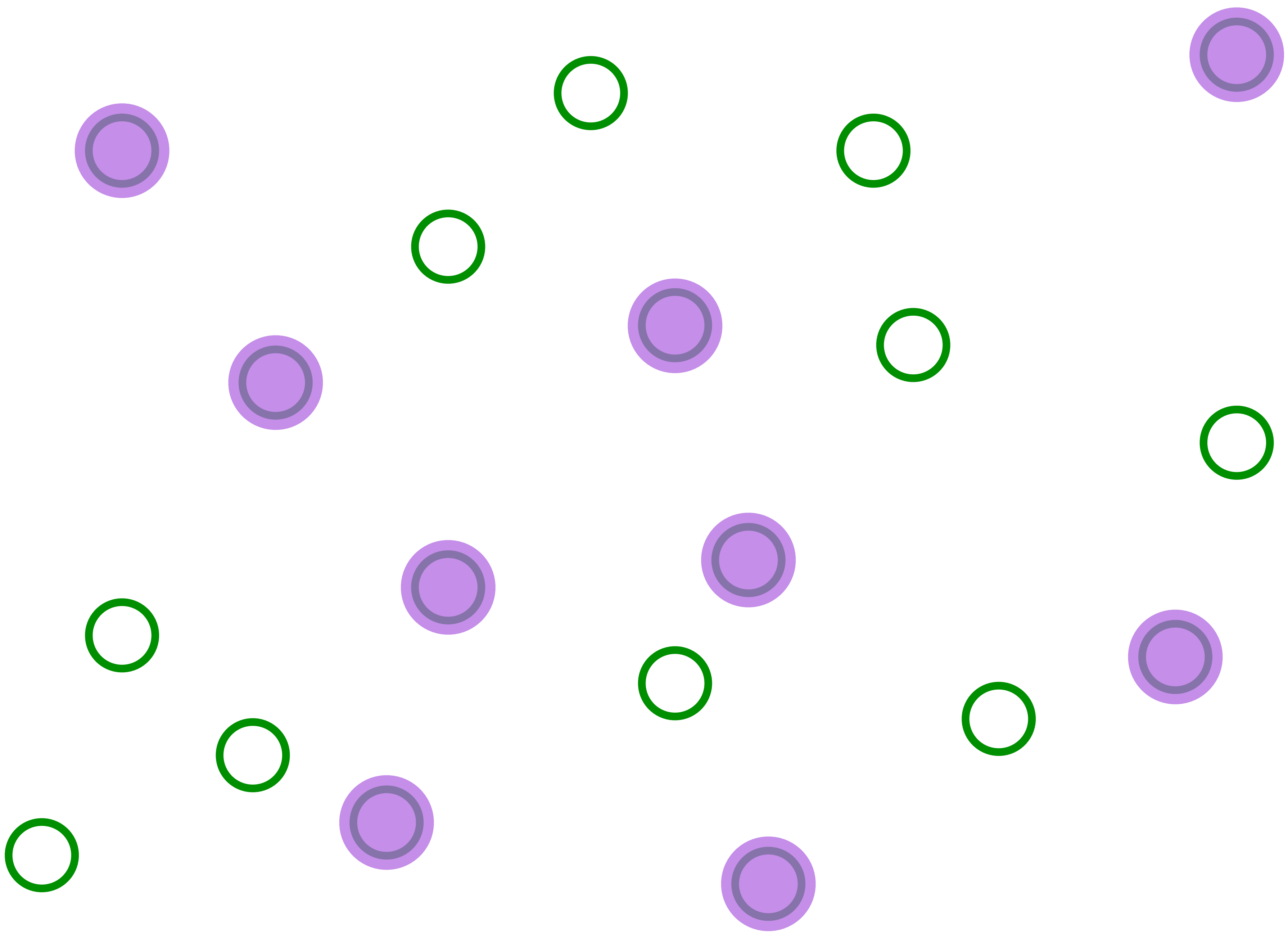

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A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

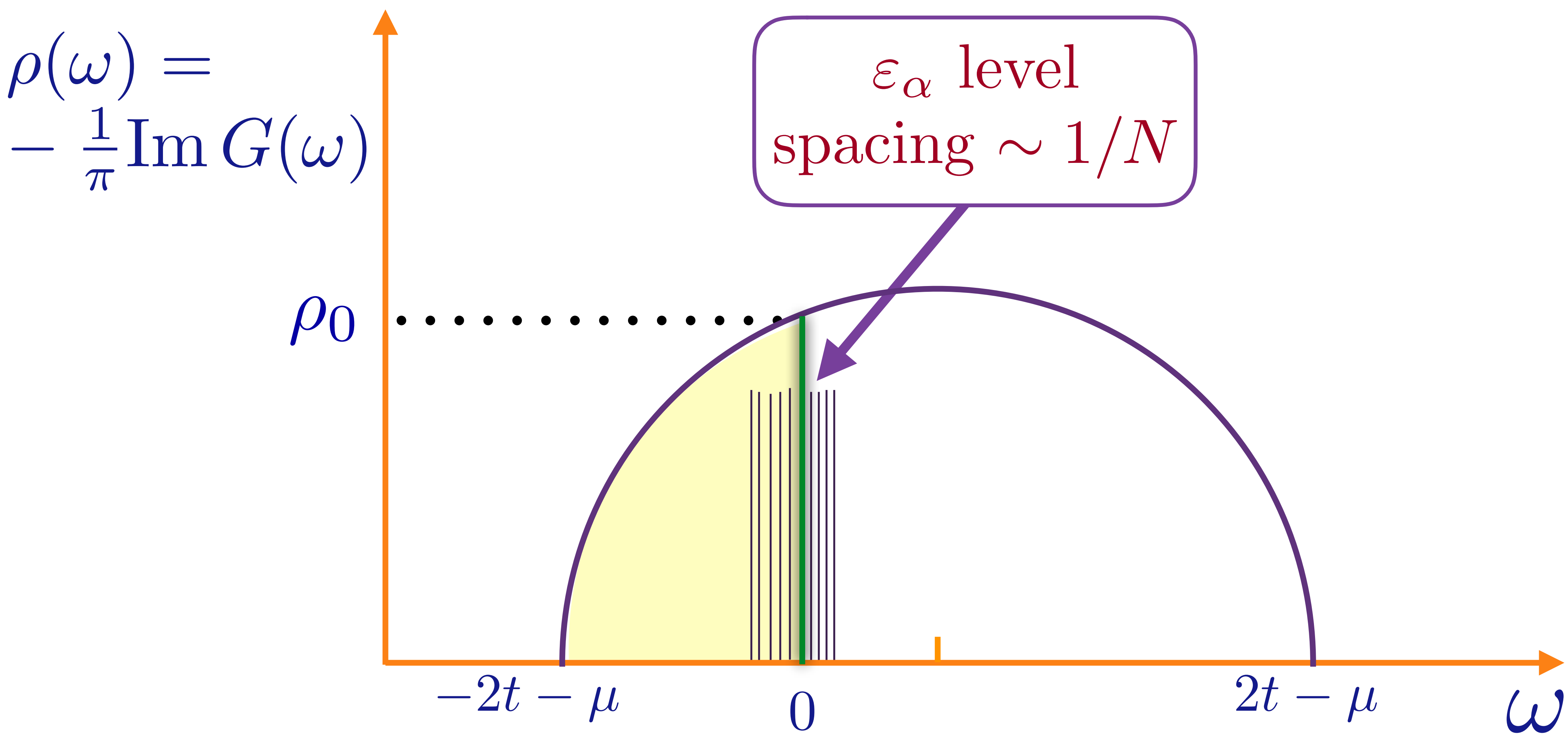
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a
N x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The single-particle density of states is
$\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$, and $\rho_0 \equiv \rho(\omega = 0)$.



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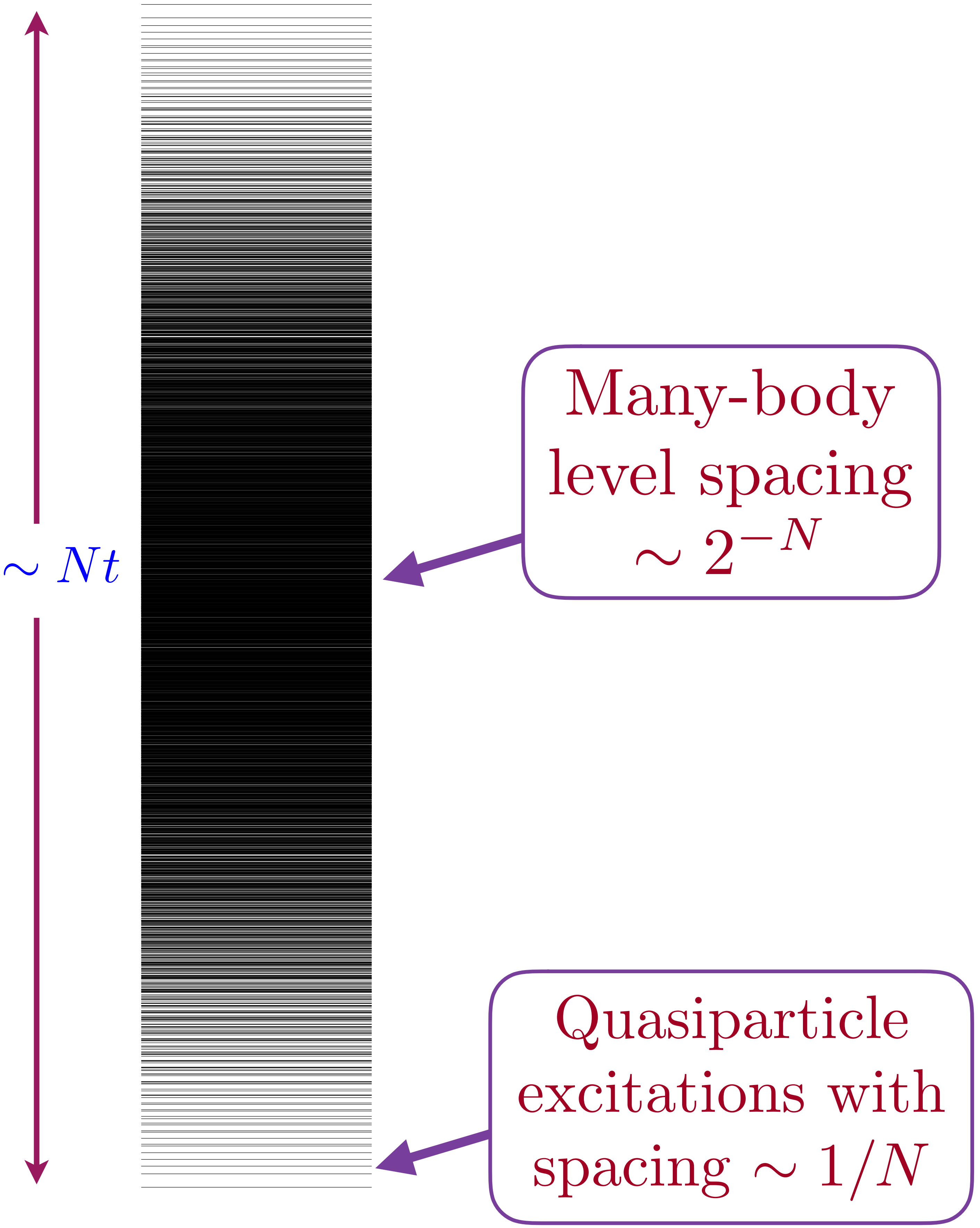

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

A simple model of a metal with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy ε_α. By Fermi's Golden rule, for ε_α at the Fermi energy

$$\begin{aligned}\frac{1}{\tau_\alpha} &= \pi U^2 \rho_0^2 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1 - f(\varepsilon_\gamma))(1 - f(\varepsilon_\delta))\delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta) \\ &= \frac{\pi^3 U^2 \rho_0^2}{4} T^2\end{aligned}$$

where ρ_0 is the density of states at the Fermi energy, and $f(\epsilon) = 1/(e^{\epsilon/T} + 1)$ is the Fermi function.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

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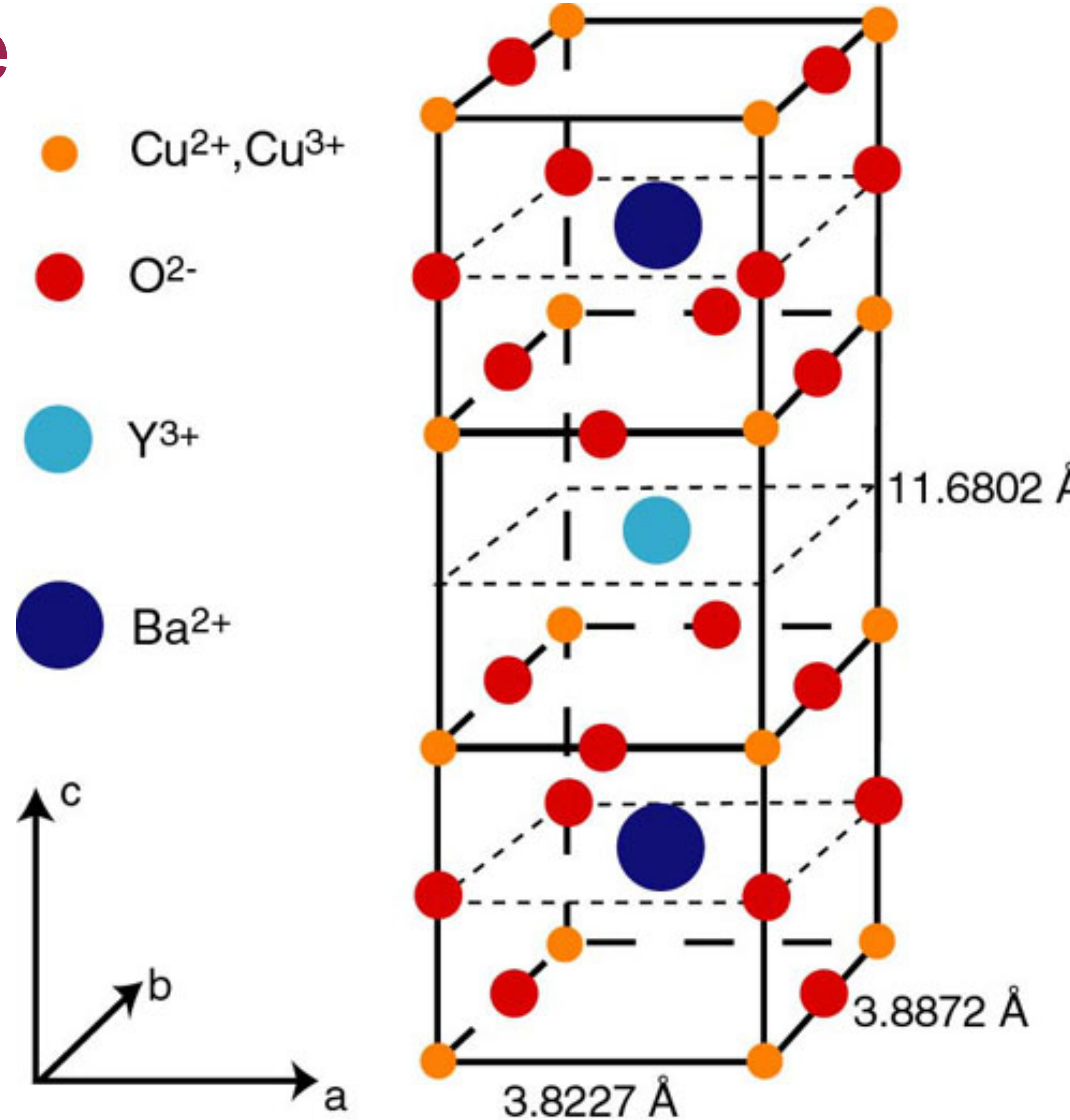
High temperature superconductors



$$YBa_2Cu_3O_{6+x}$$

"Strange",

"Bad",

or "Incoherent",

metal found ubiquitously at temperatures $T > T_c$ (the superconducting critical temperature) has a resistivity, ρ, which obeys

$$\rho \sim T,$$

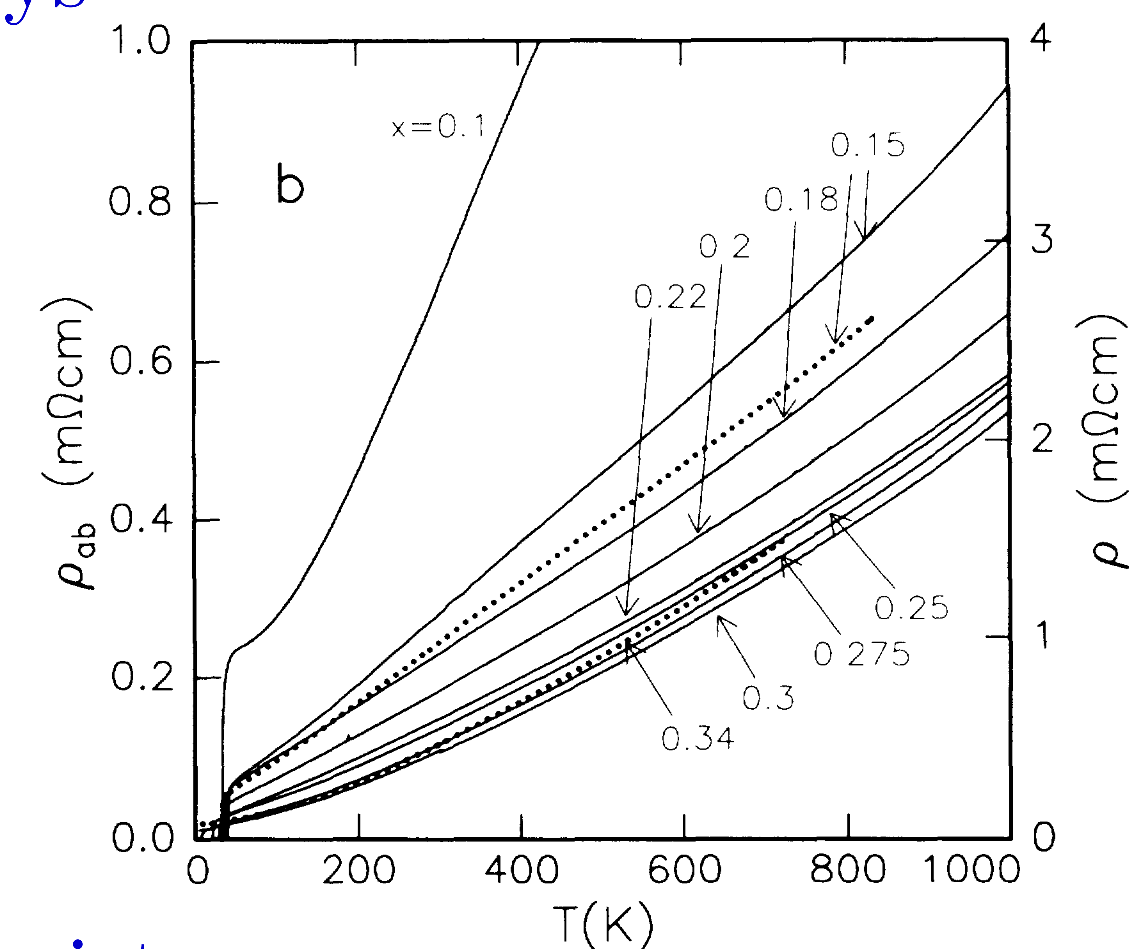
and

in some cases $\rho \gg h/e^2$

(in two dimensions),

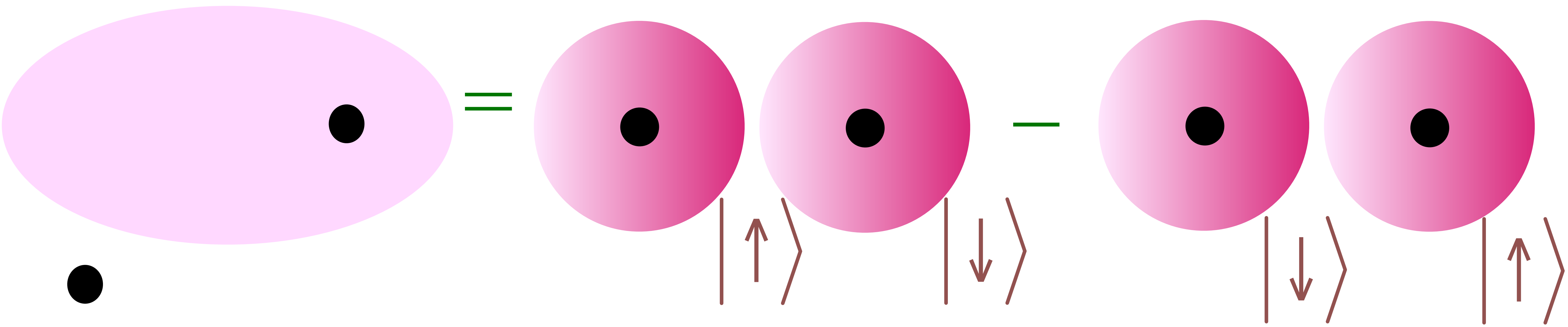
where h/e^2 is the quantum unit of resistance.

H. Takagi, B. Batlogg, H. L. Kao, J. Kwo, R. J. Cava, J. J. Krajewski, and W. F. Peck, Jr., Phys. Rev. Lett. **69**, 2975 (1992)



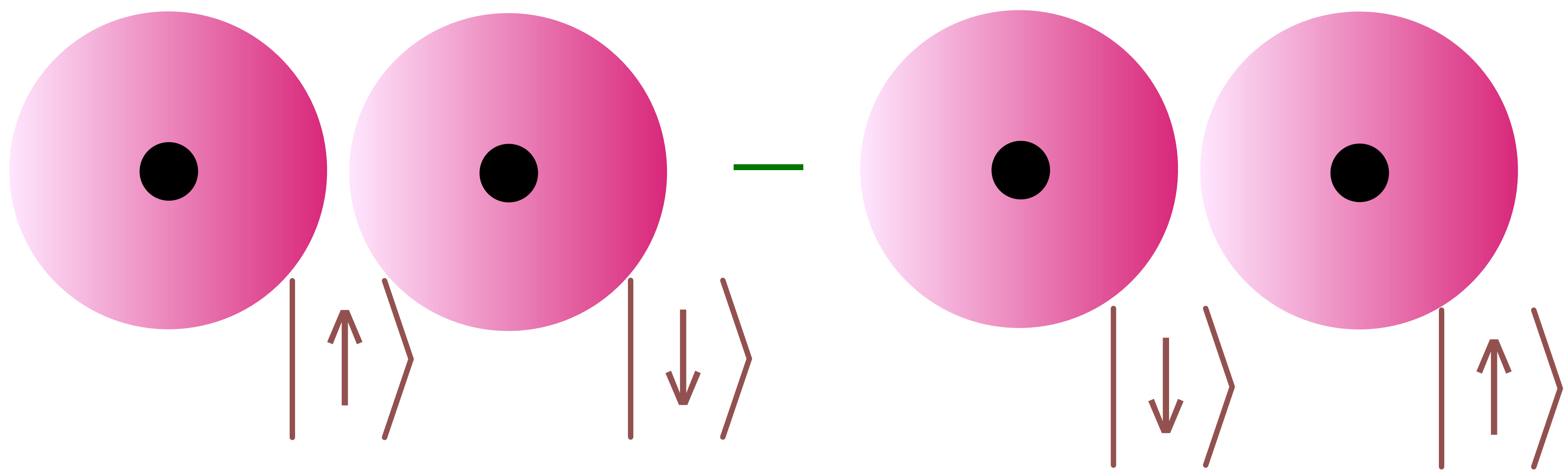
<u>Quantum Entanglement</u>: quantum superposition with more than one particle

Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Quantum Entanglement: quantum superposition with more than one particle



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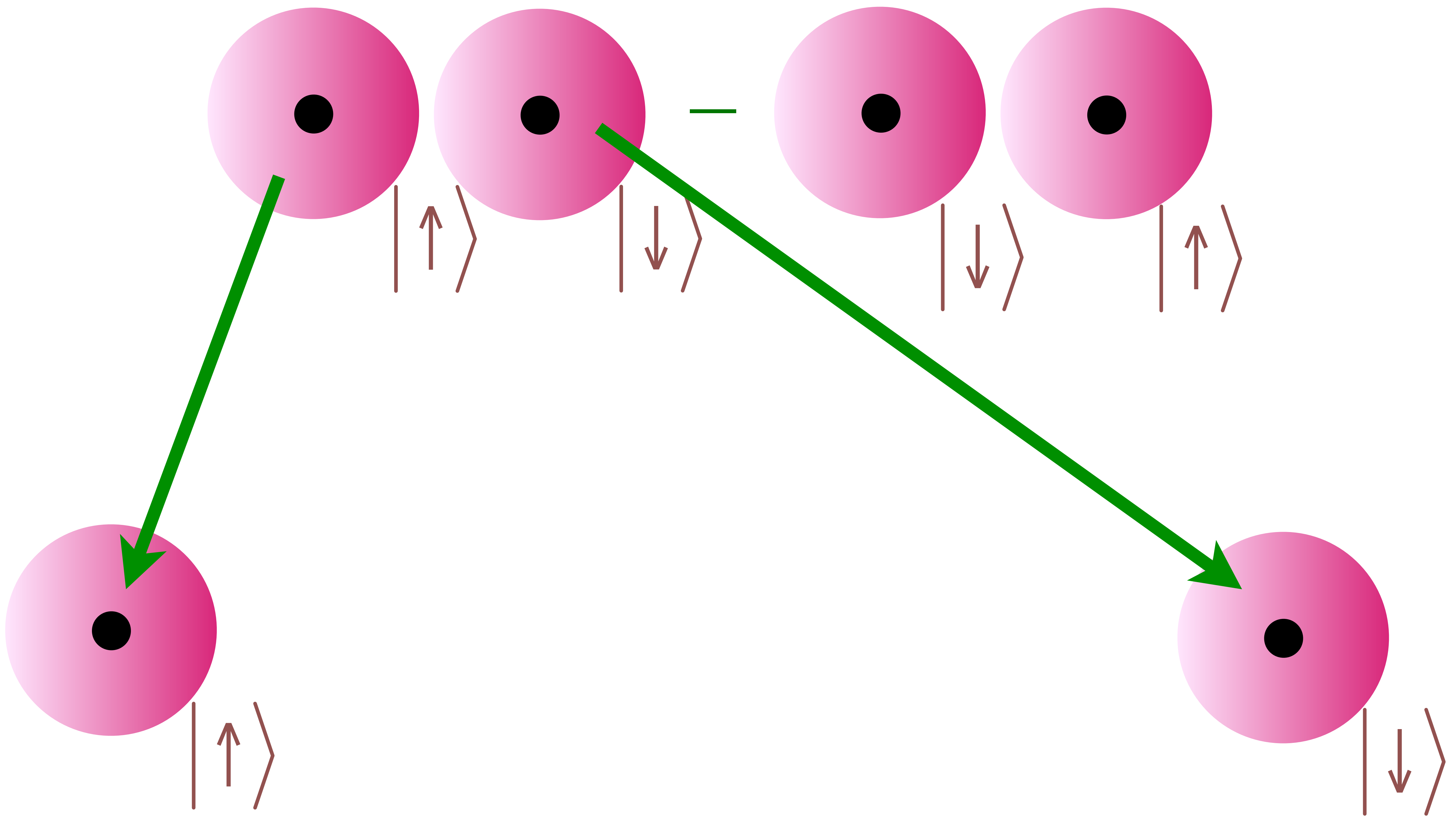

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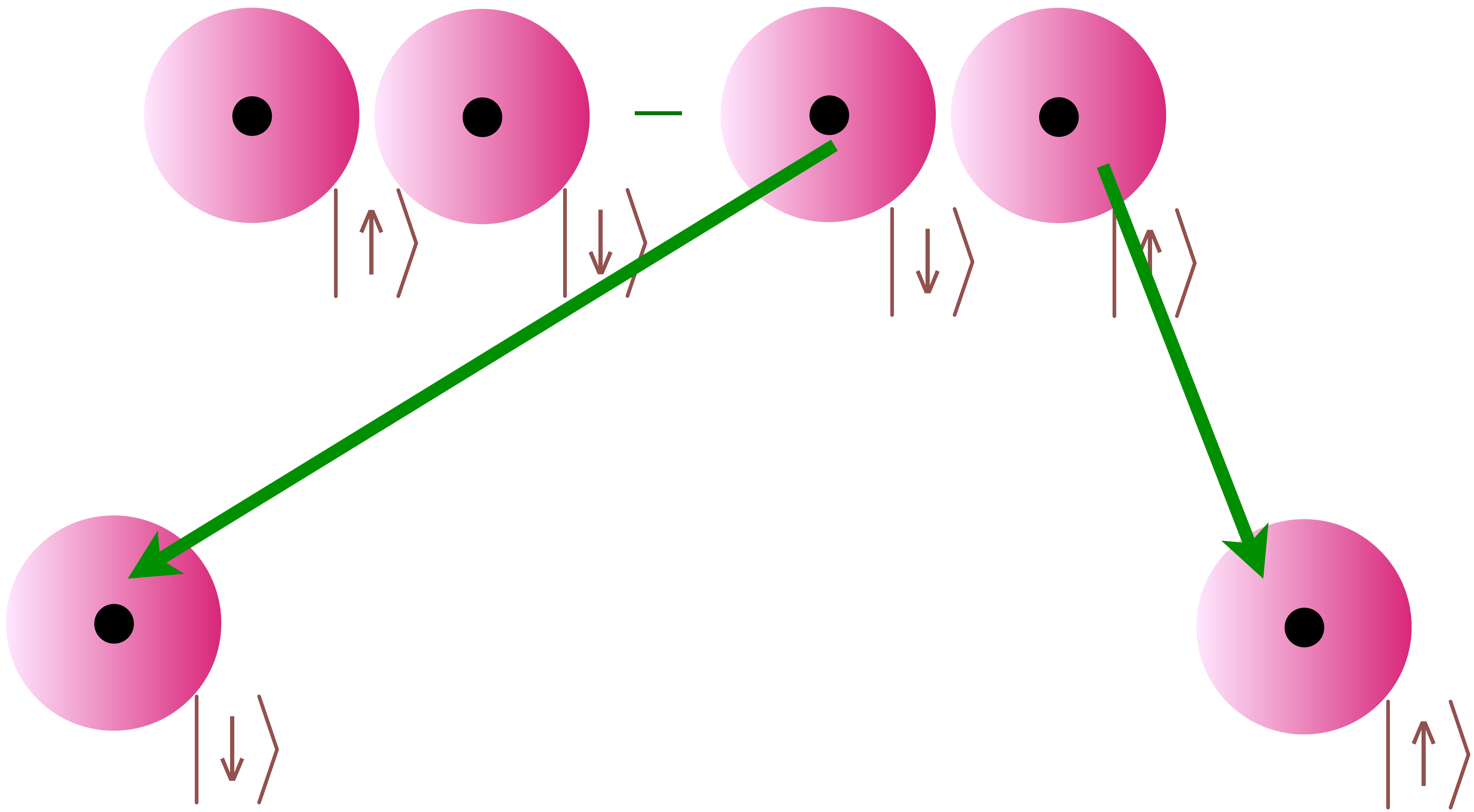
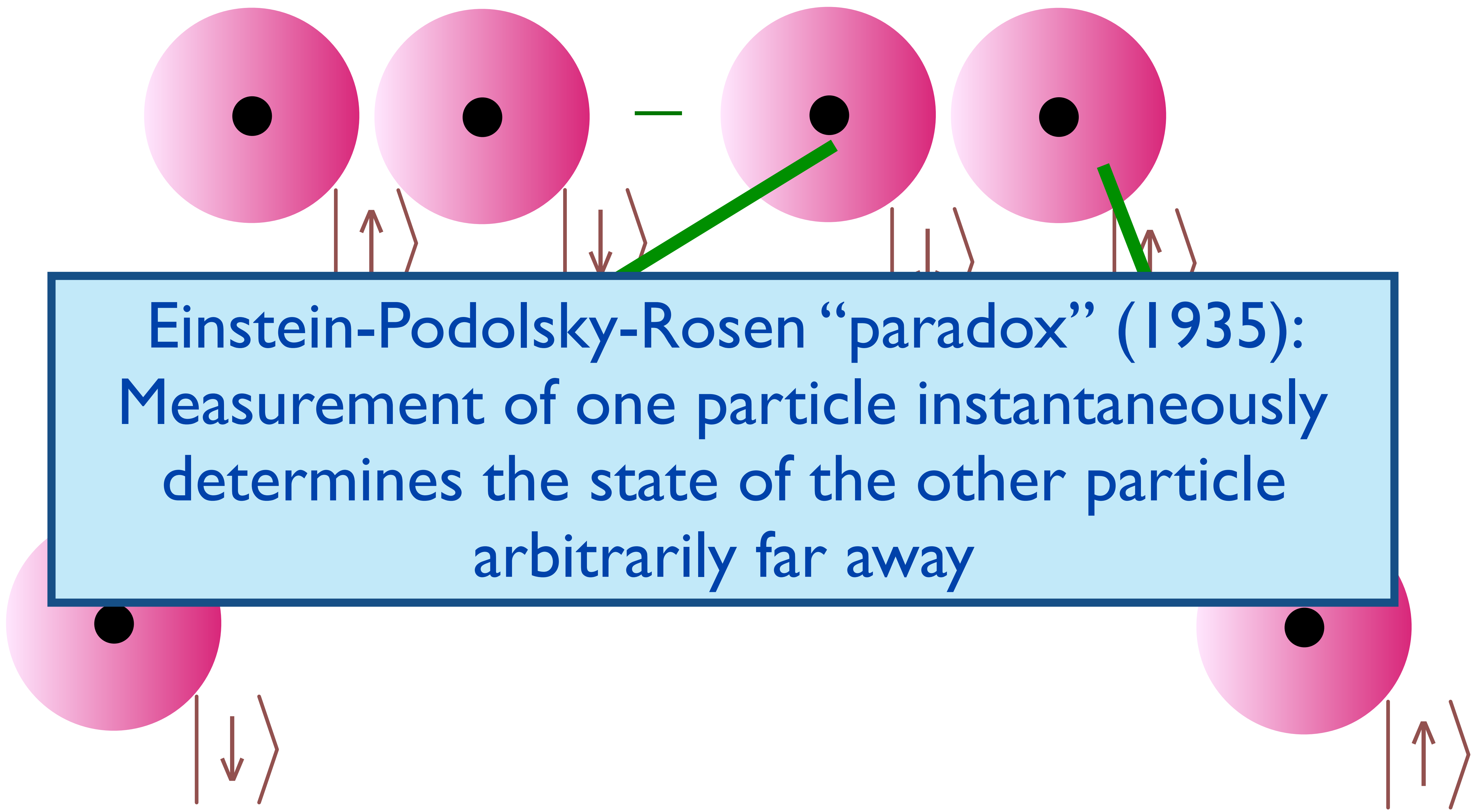
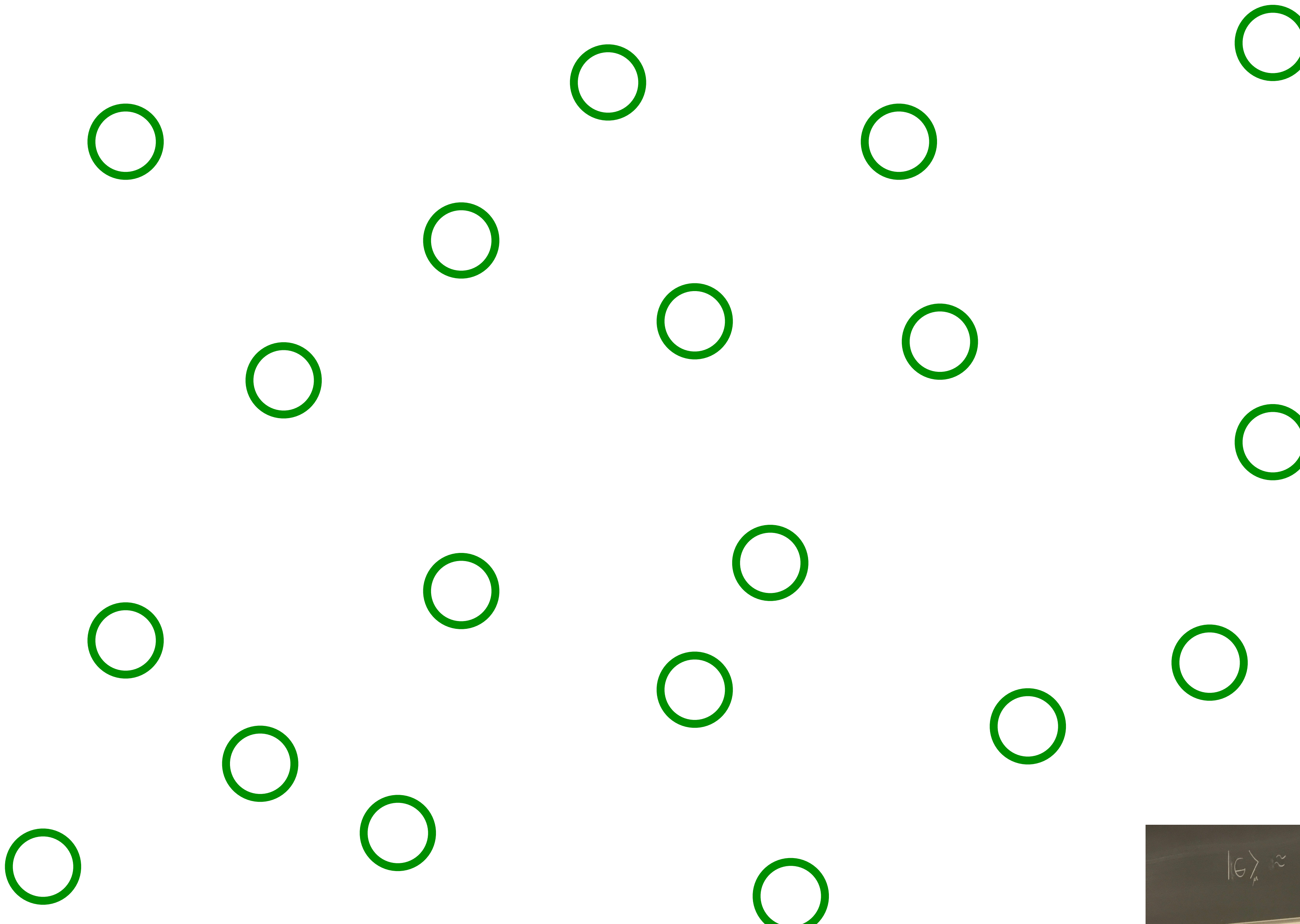


Quantum Entanglement: quantum superposition with more than one particle

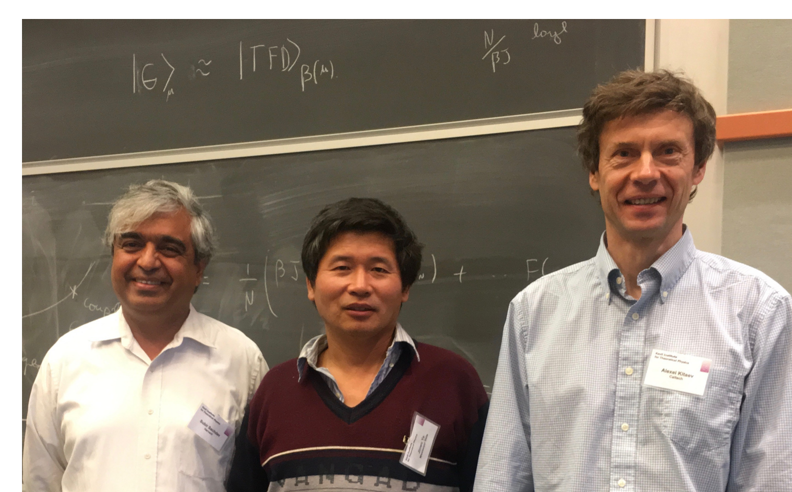


Einstein-Podolsky-Rosen "paradox" (1935):
Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away

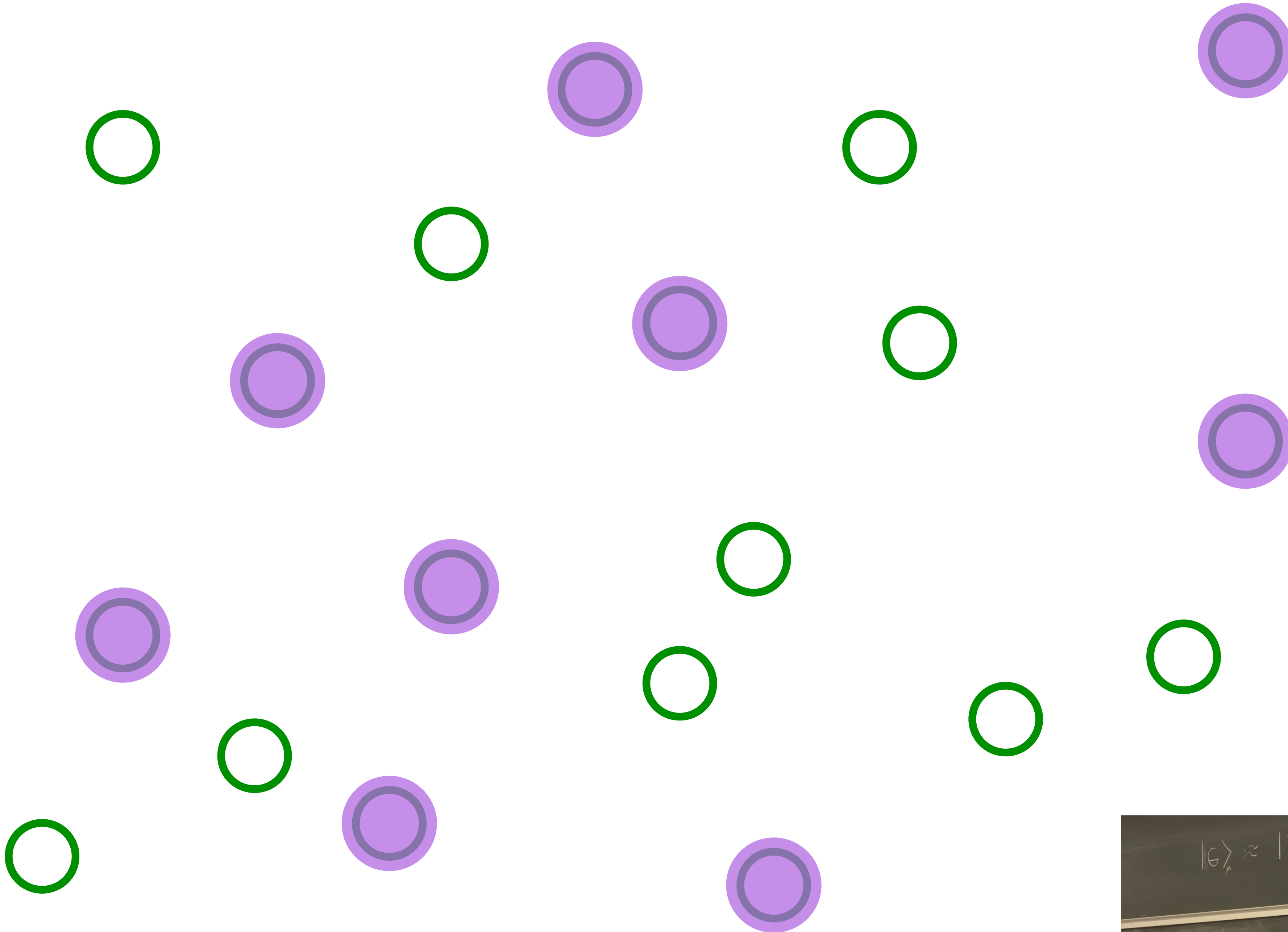
The Sachdev-Ye-Kitaev (SYK) model



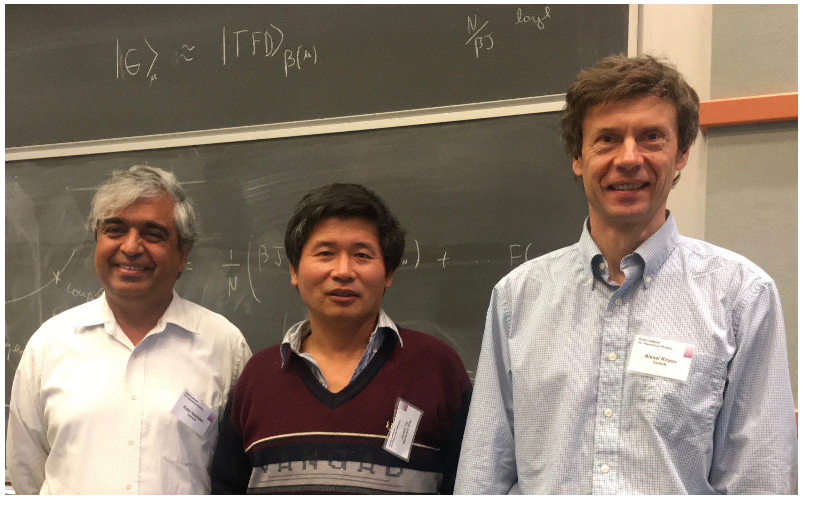
Pick a set of random positions



The SYK model



Place electrons randomly on some sites



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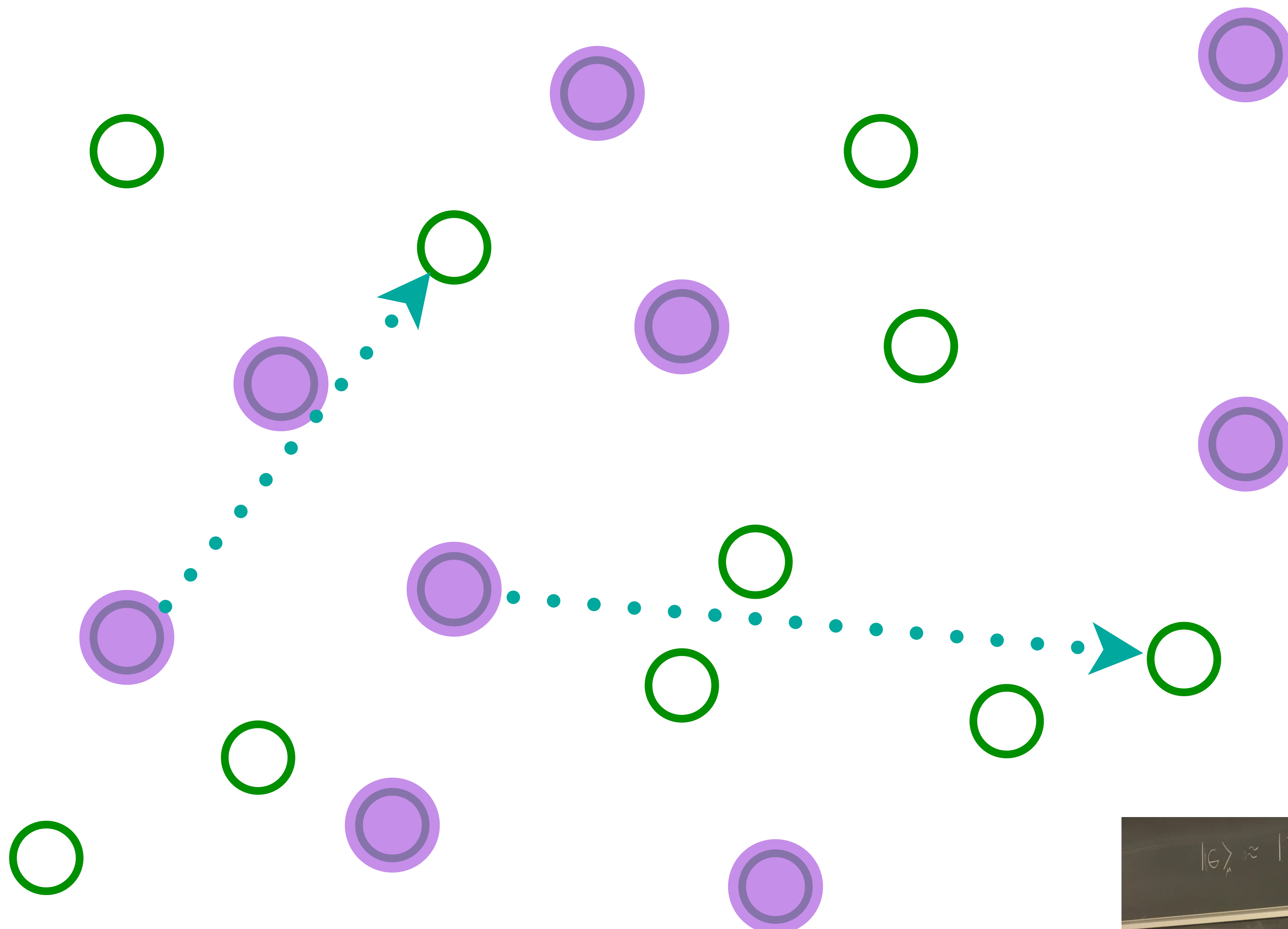

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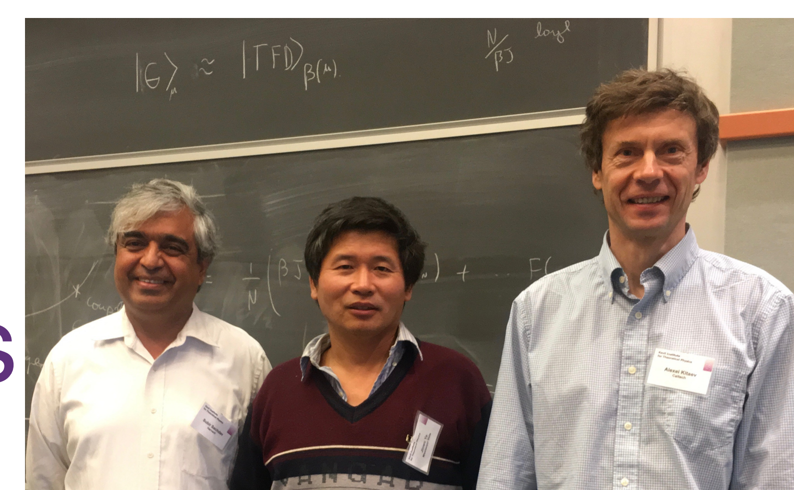

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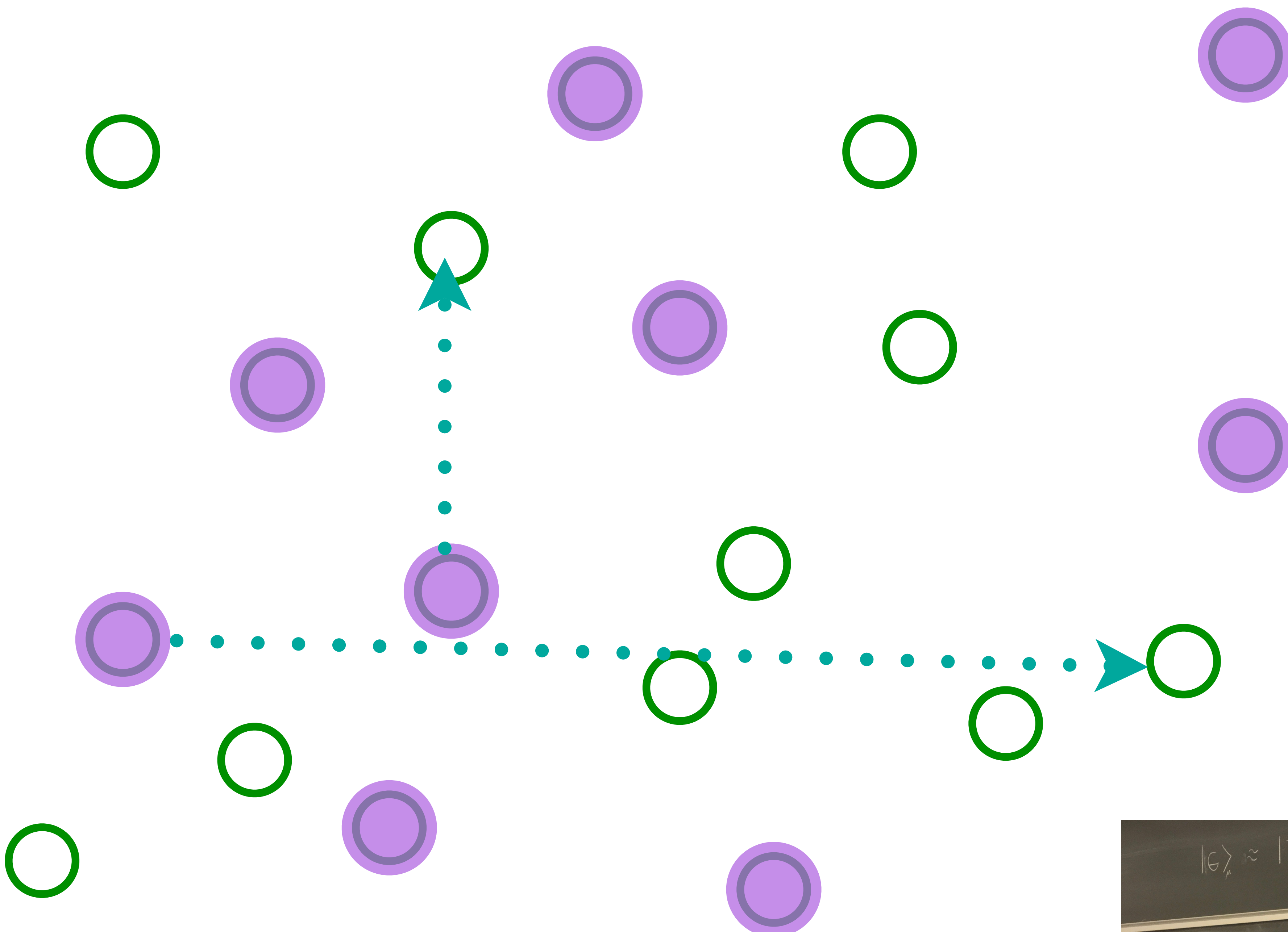

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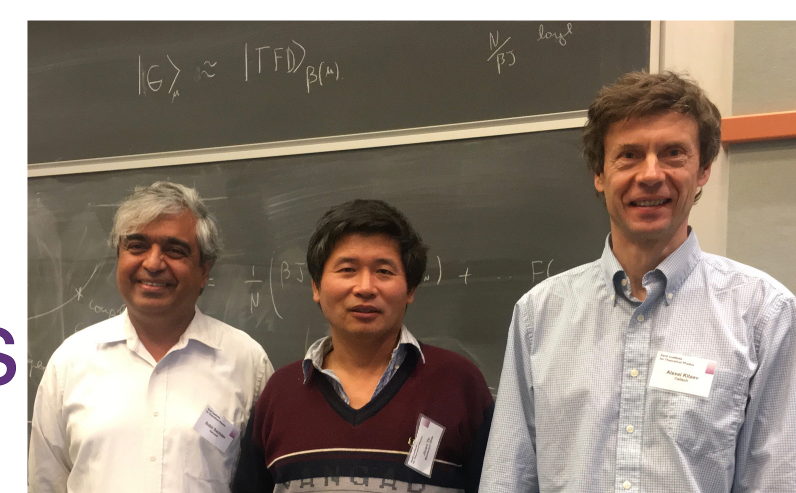

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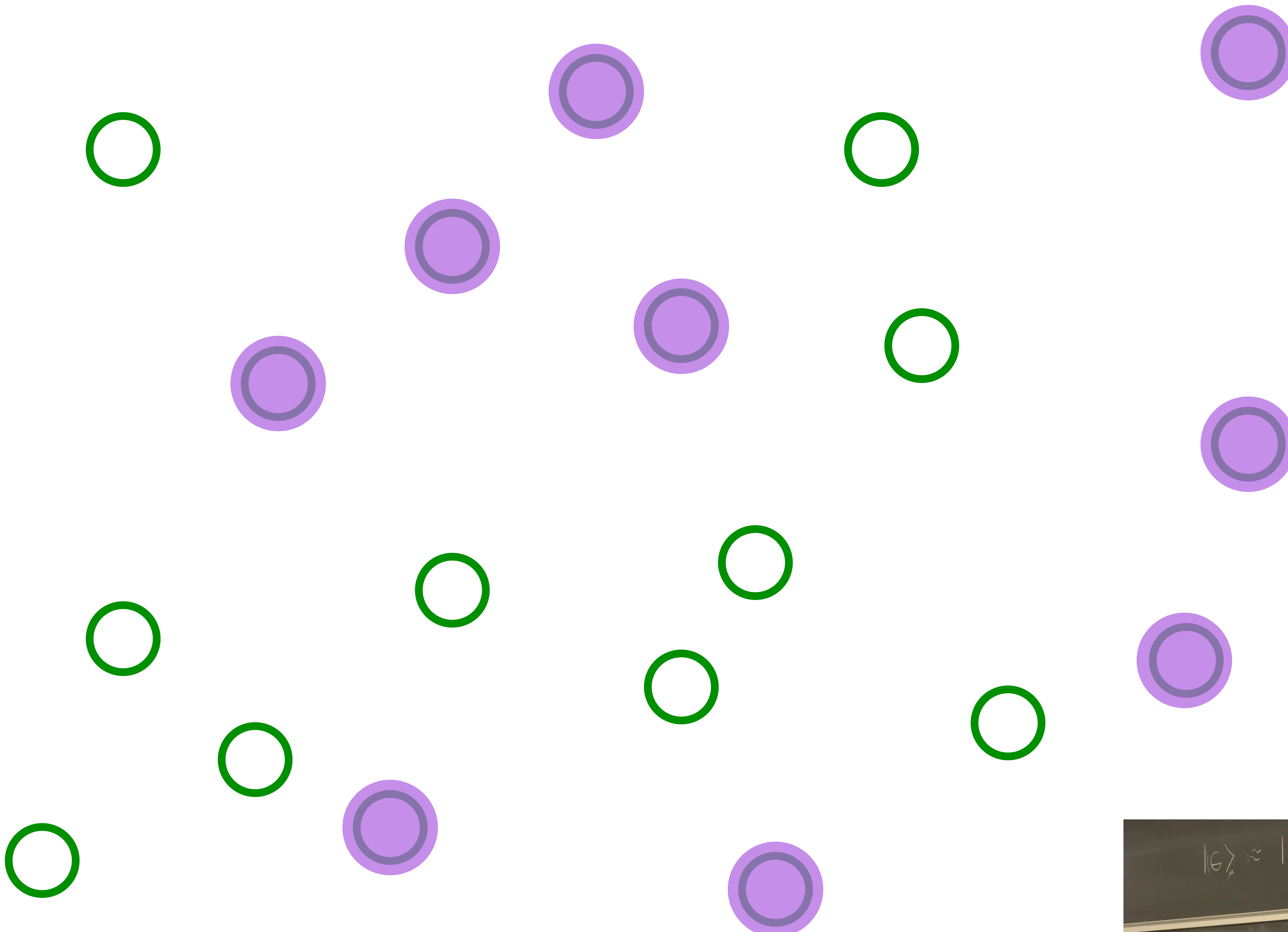
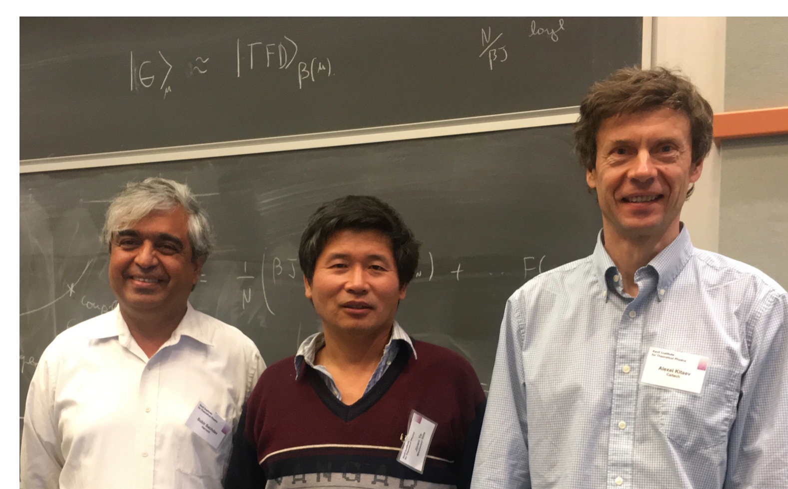


Entangle electrons pairwise randomly



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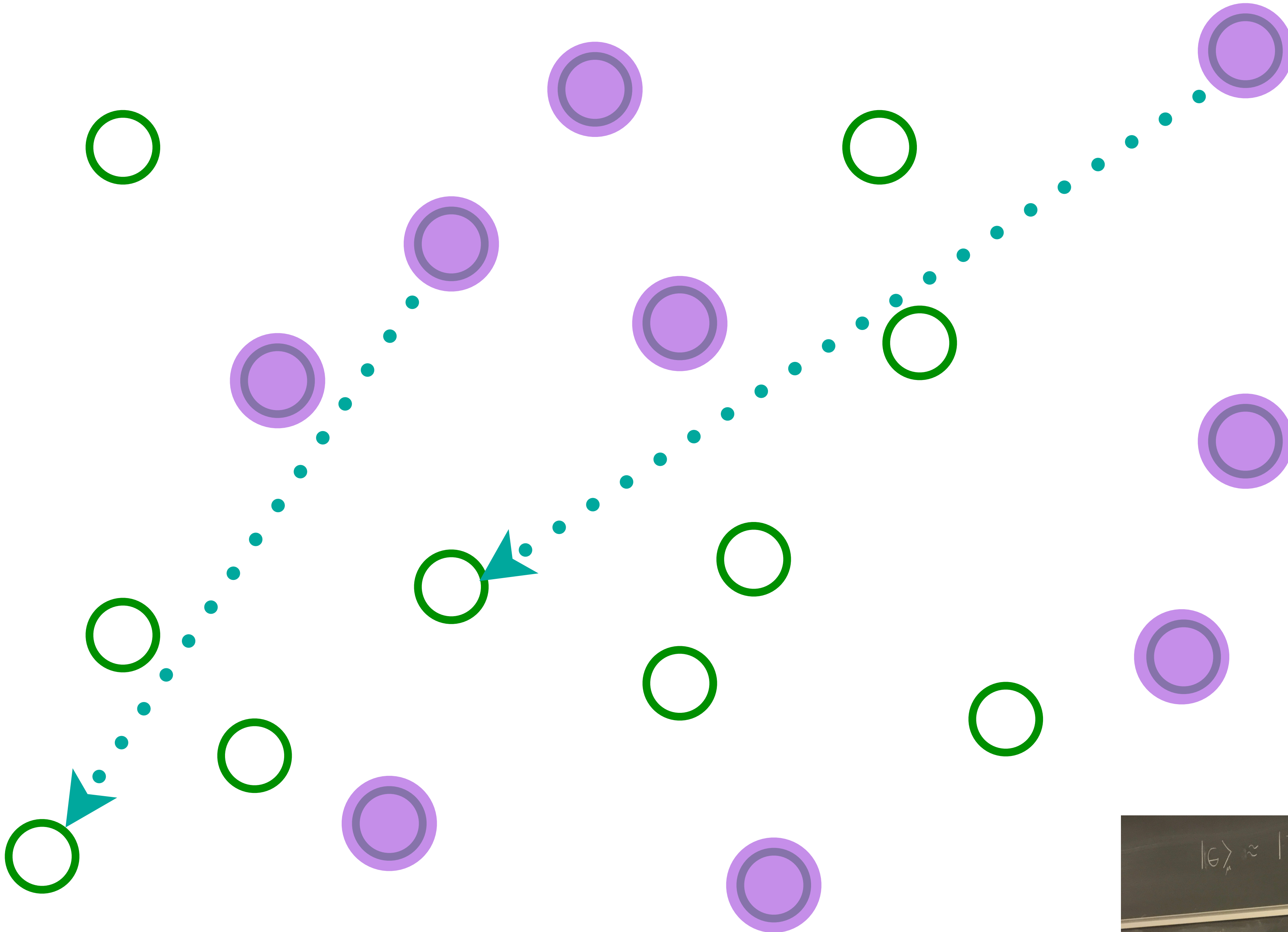

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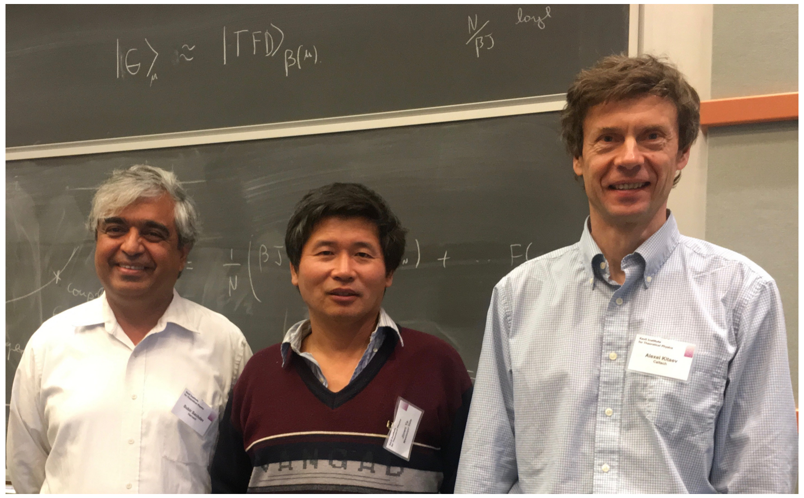

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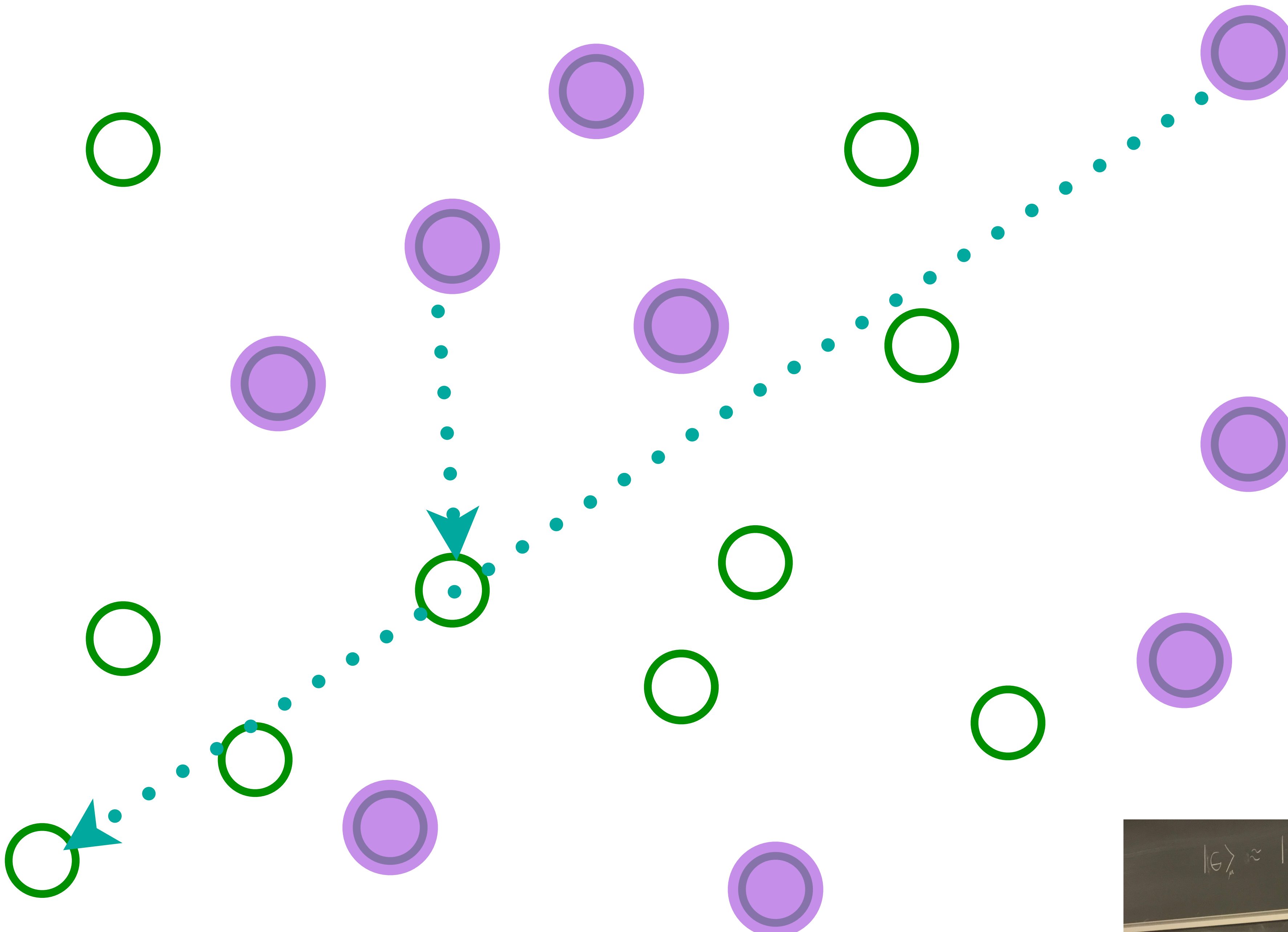

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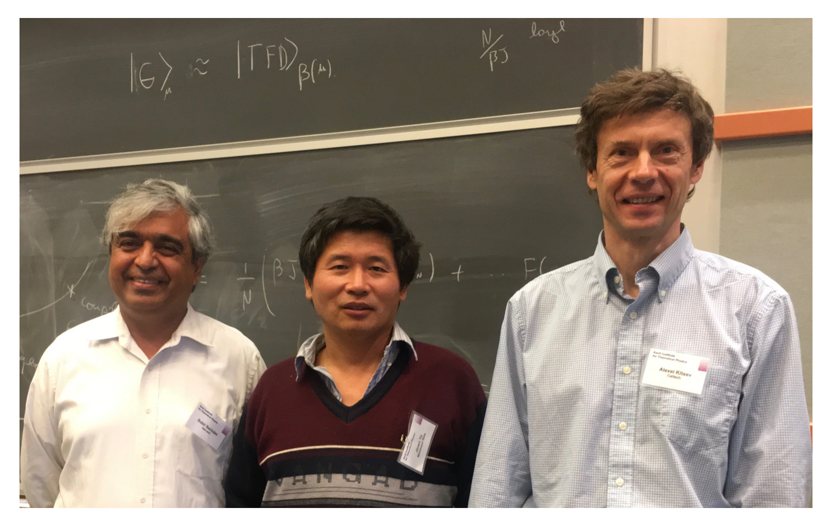

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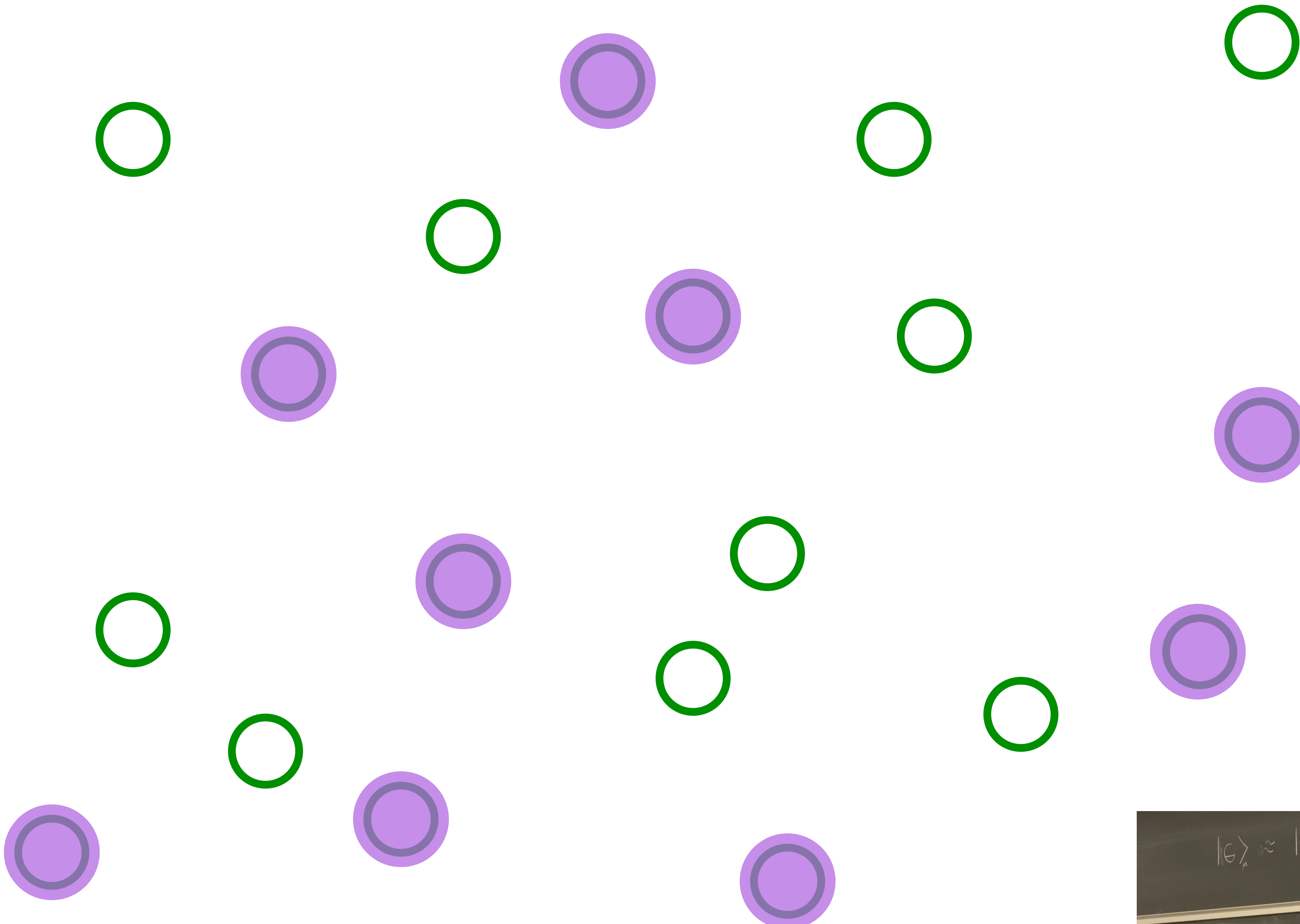

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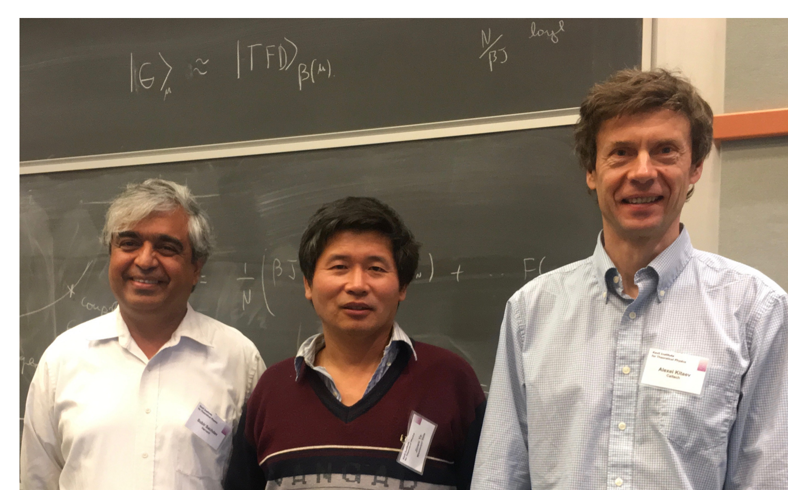

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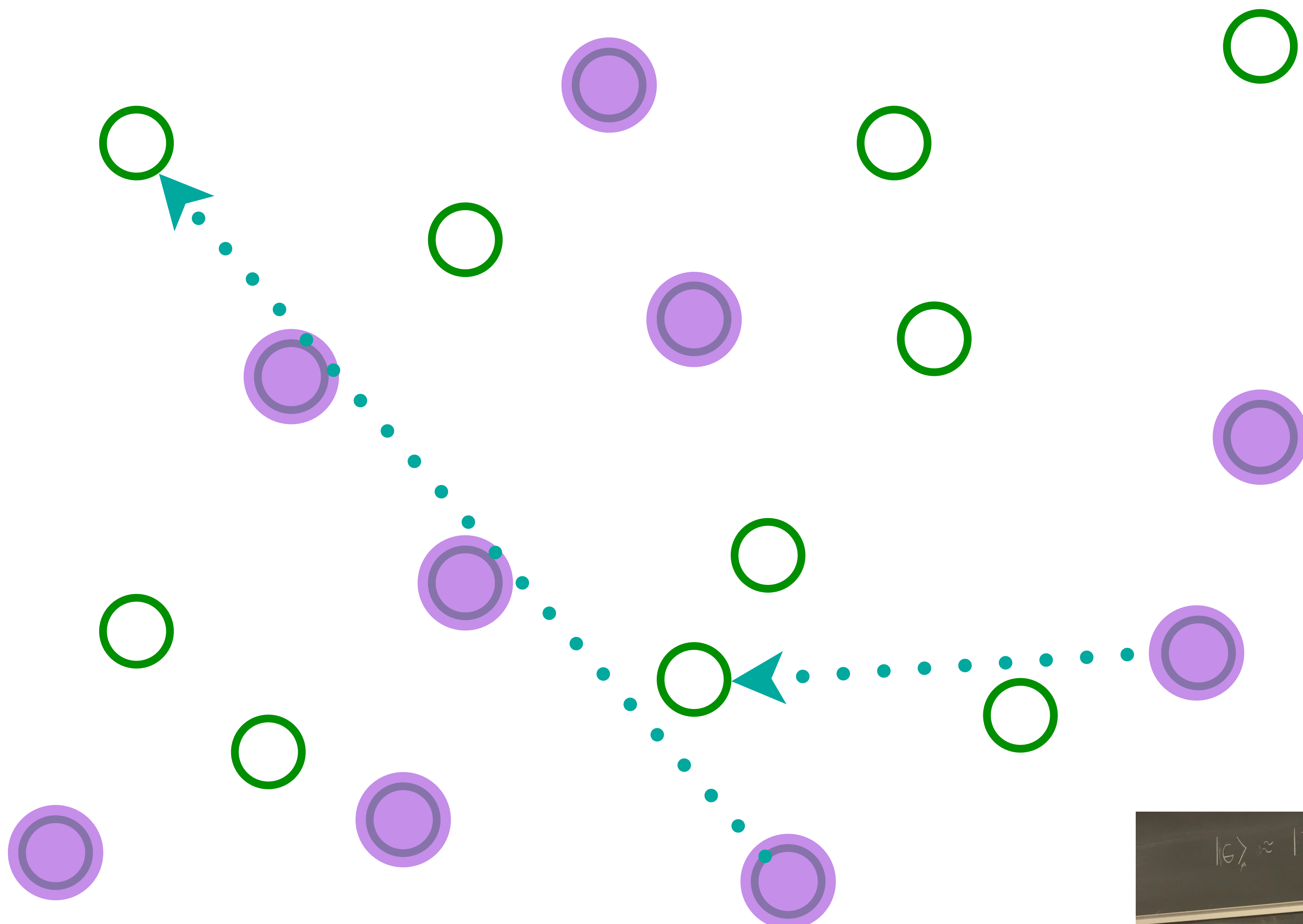

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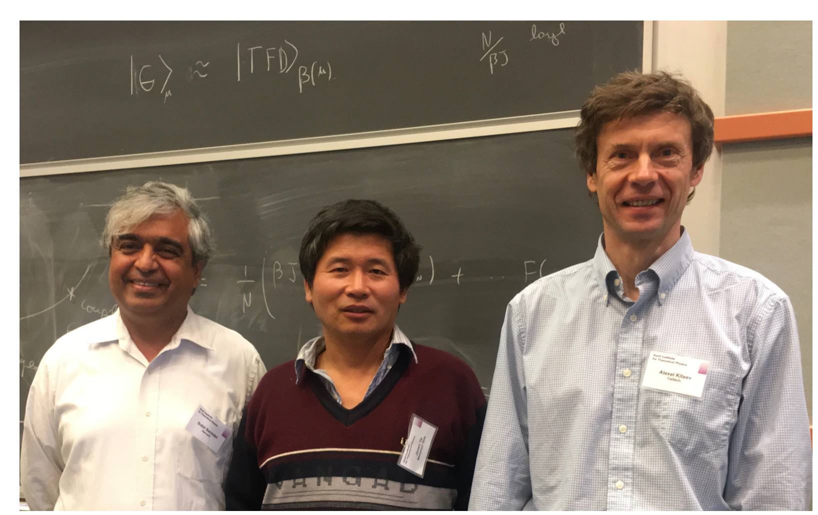

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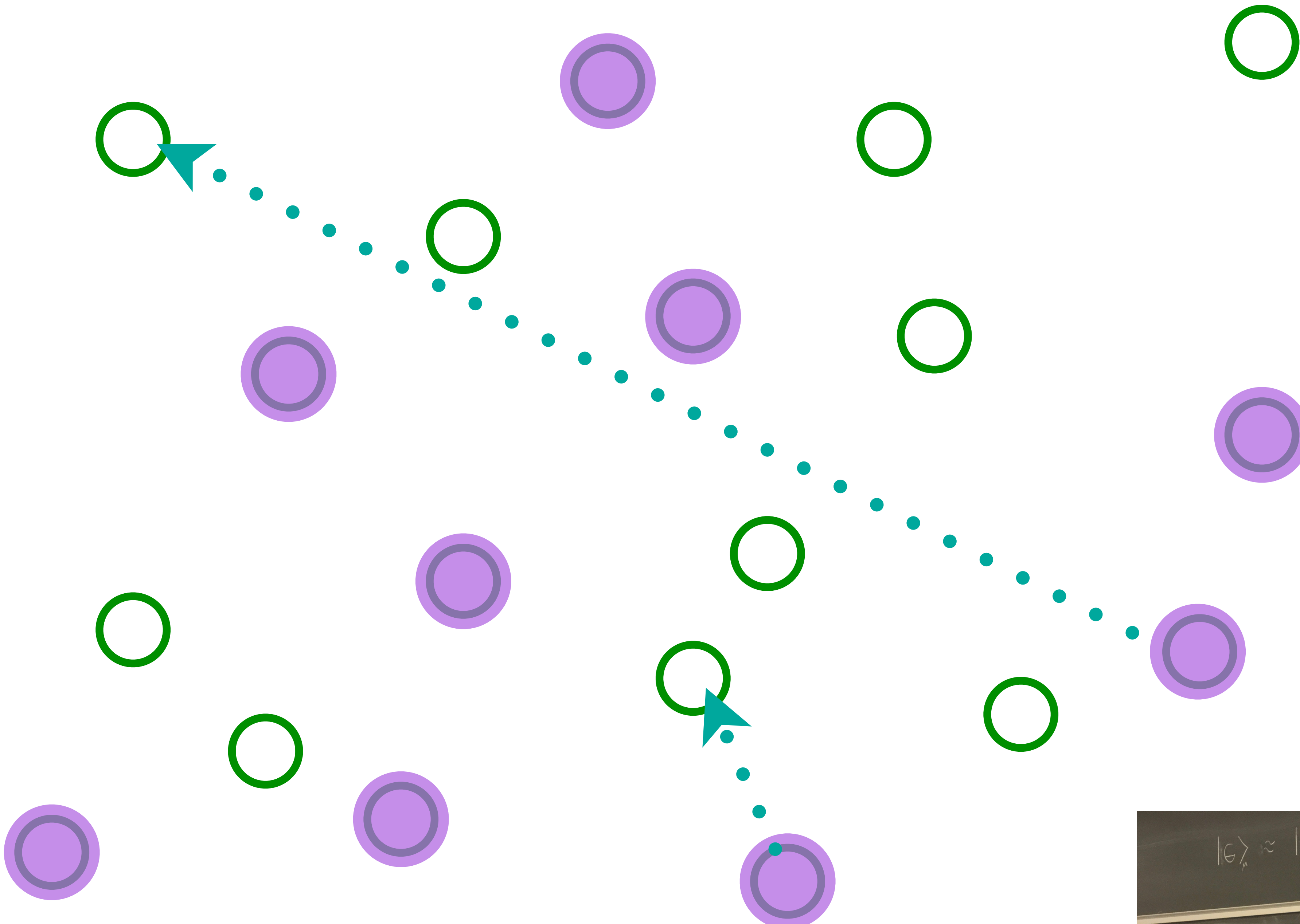

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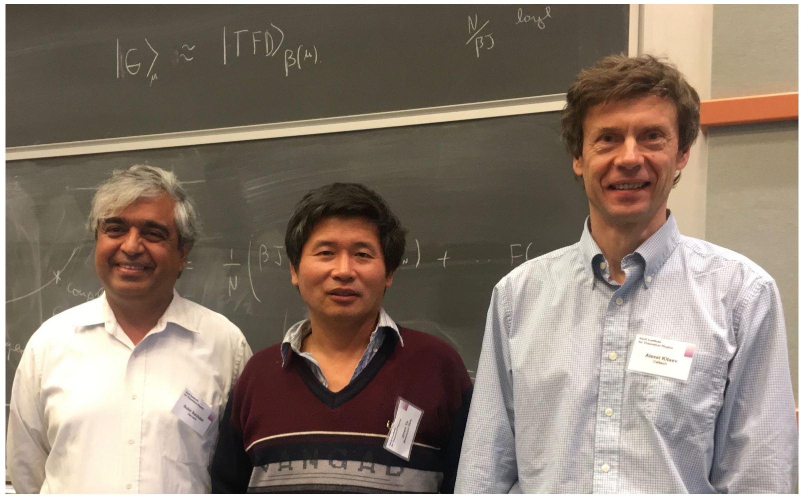

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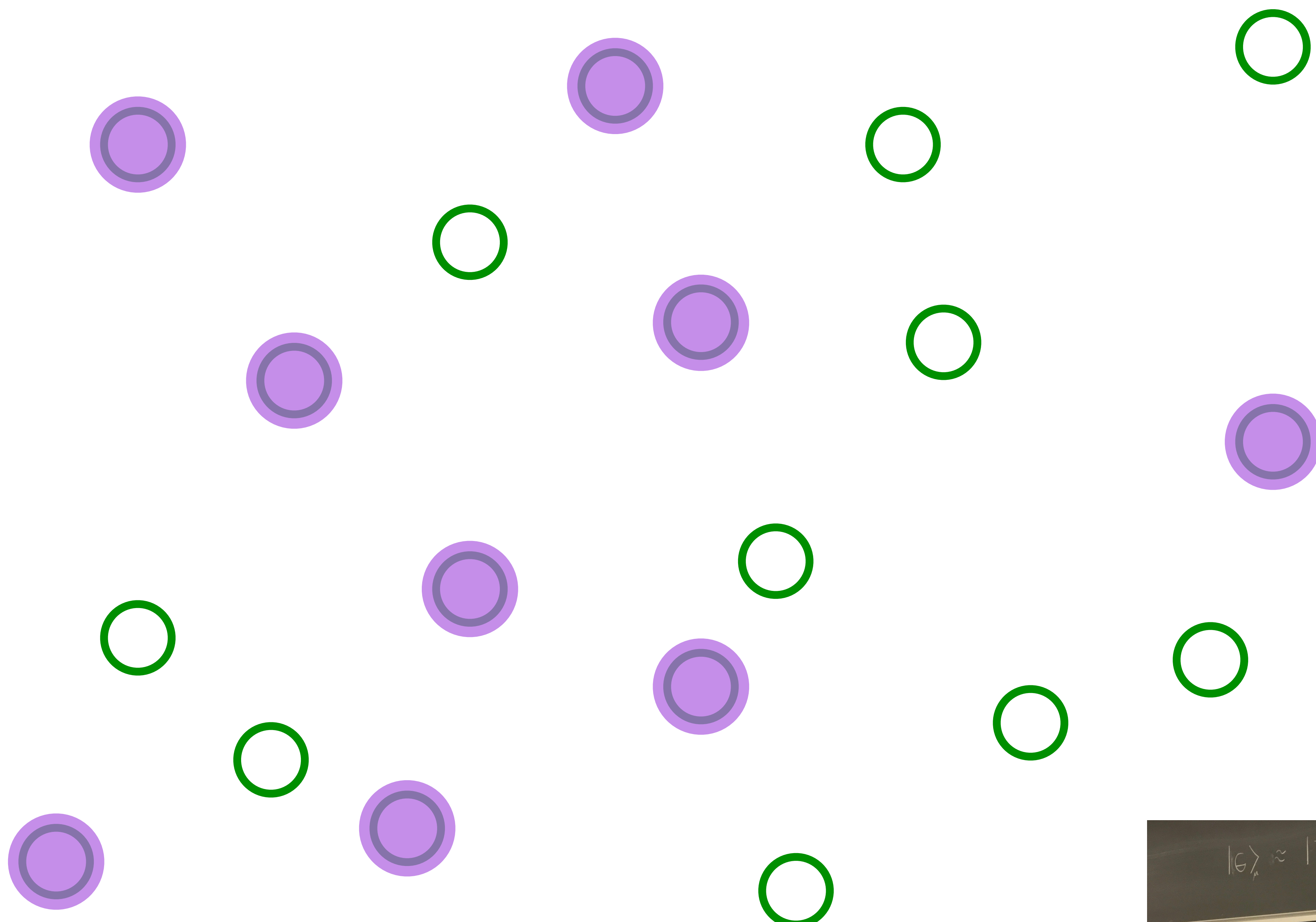

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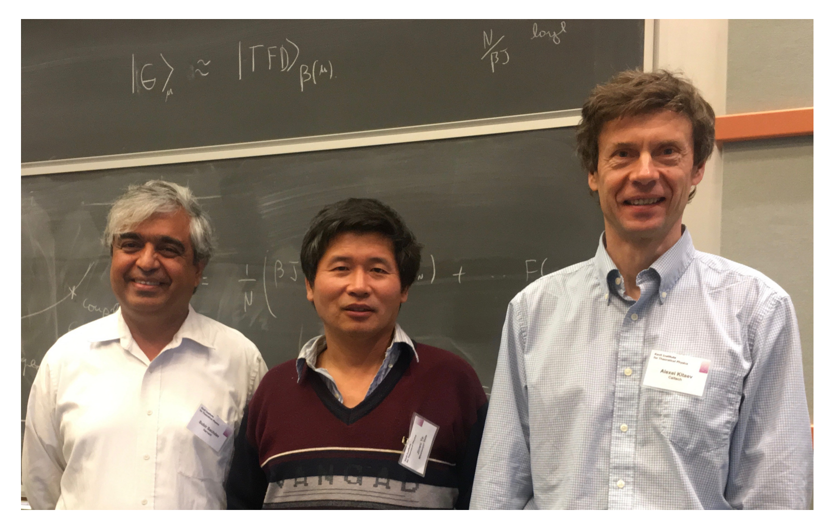

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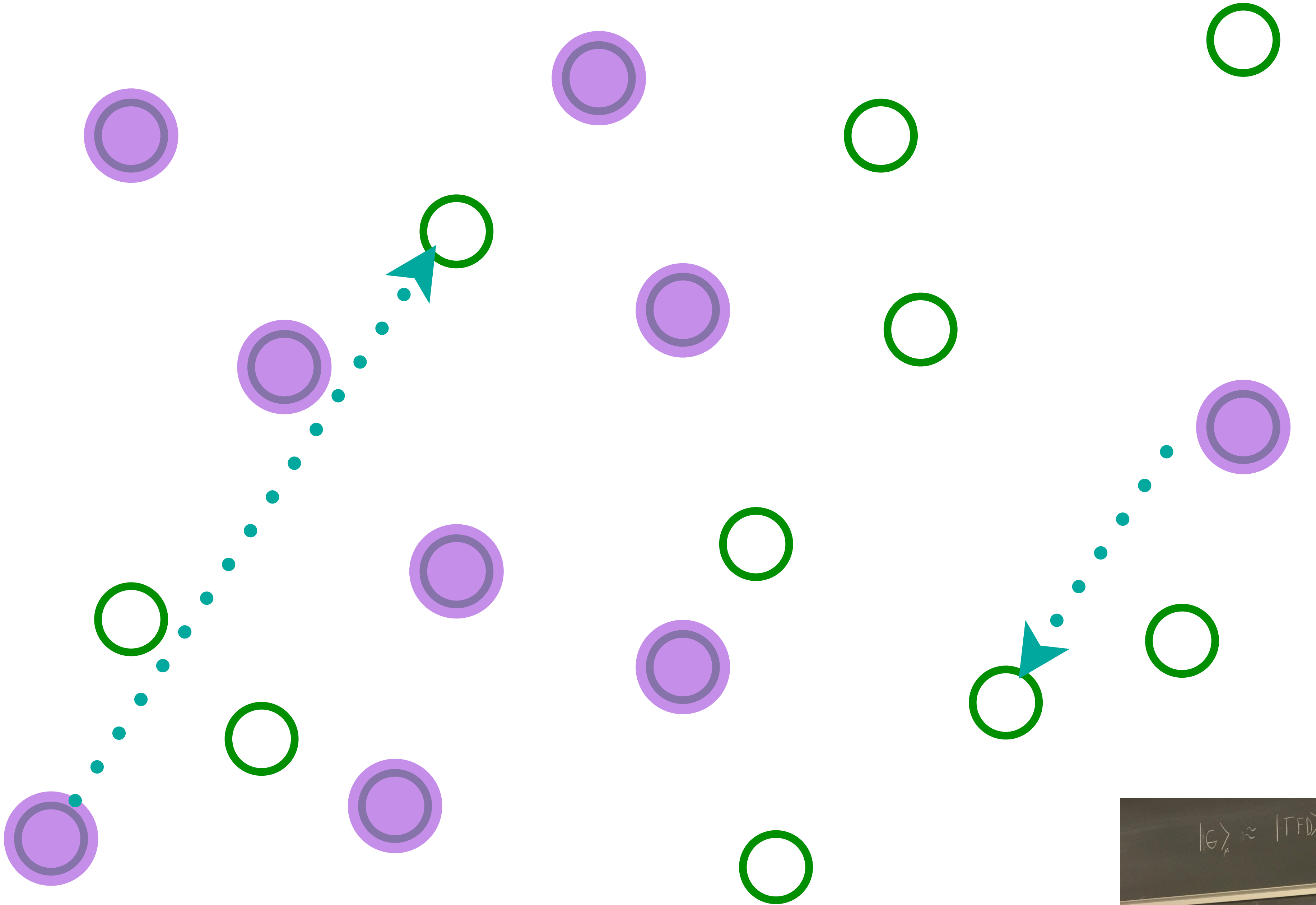

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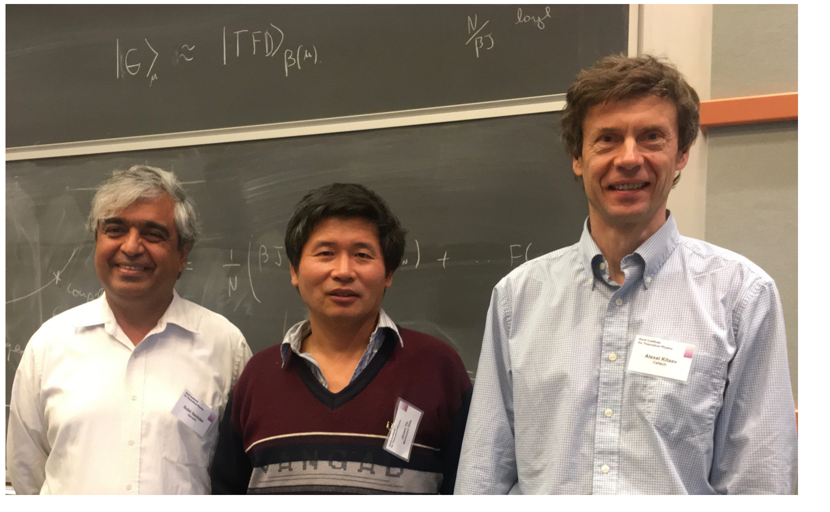

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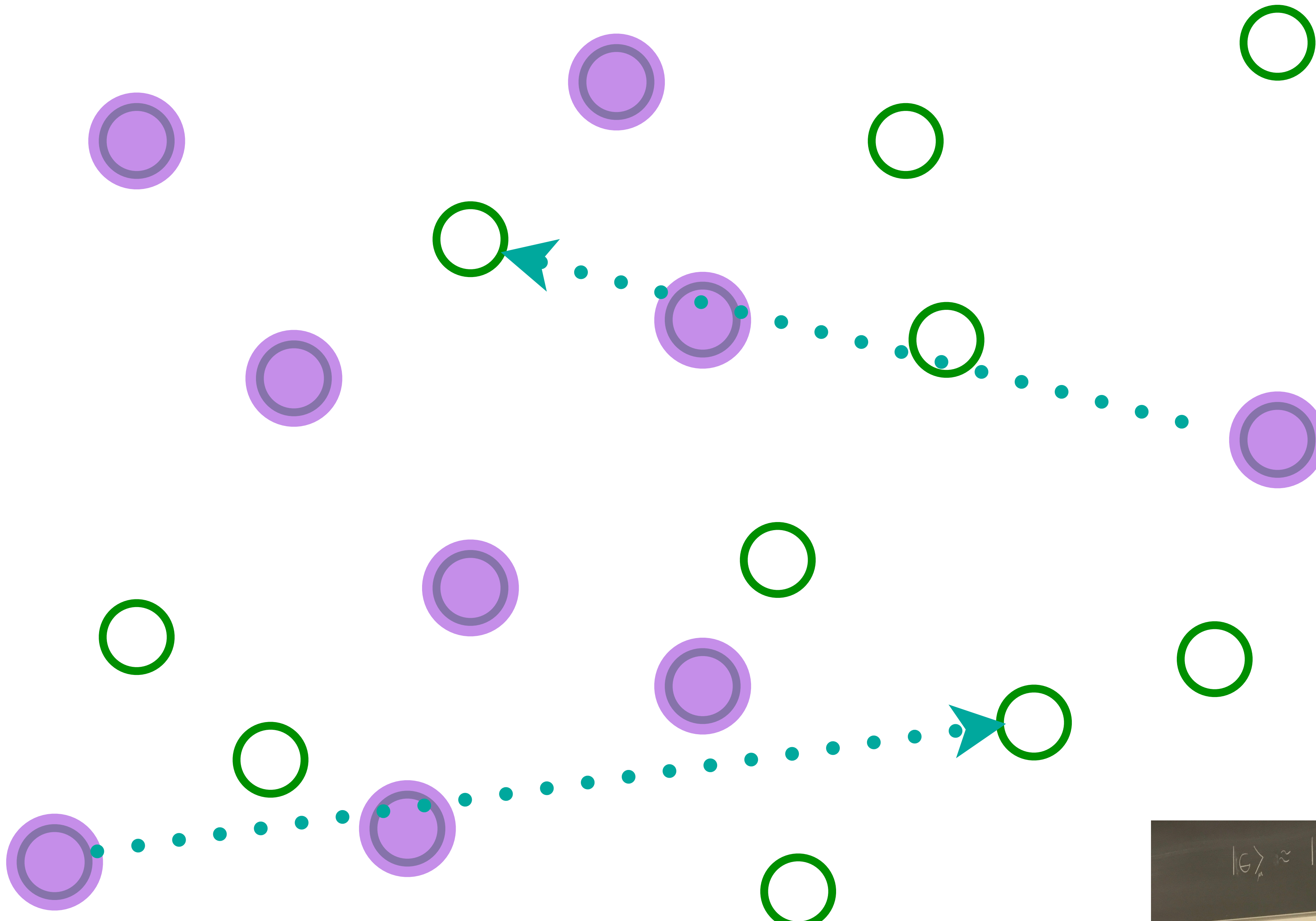

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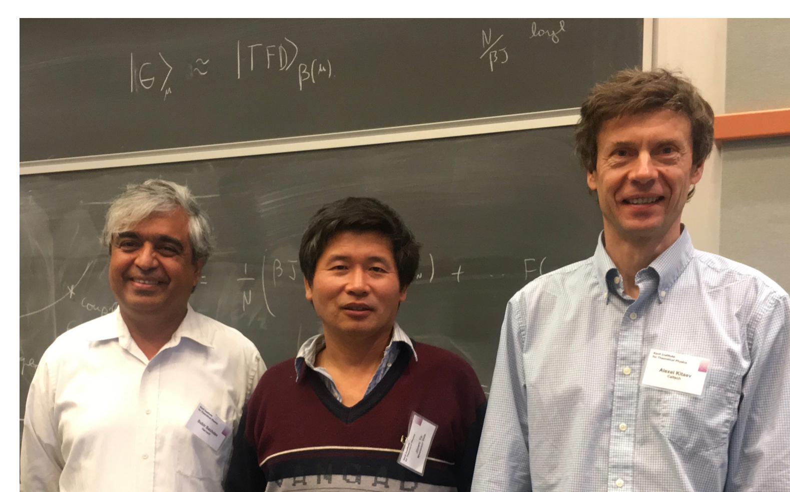
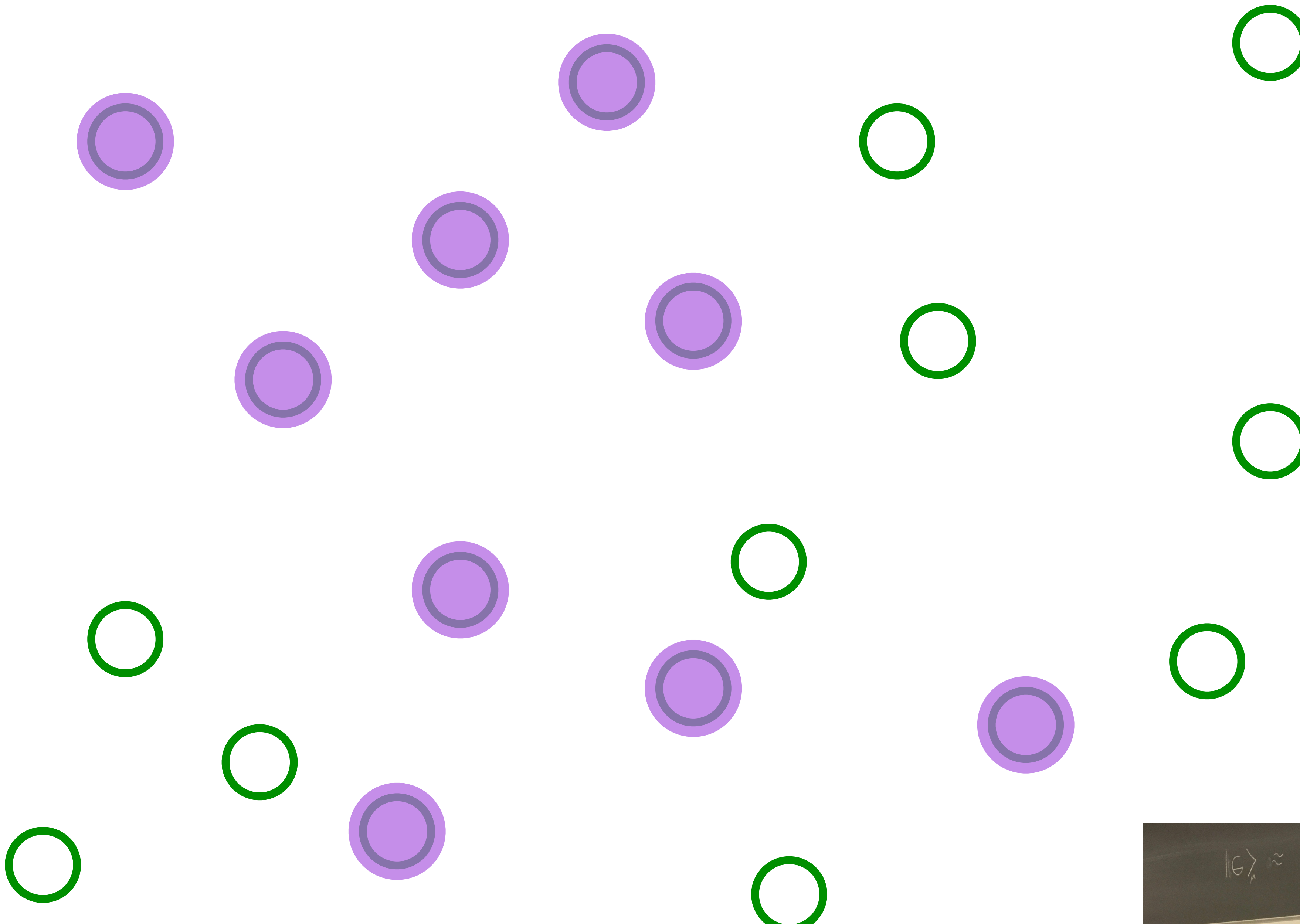


The SYK model



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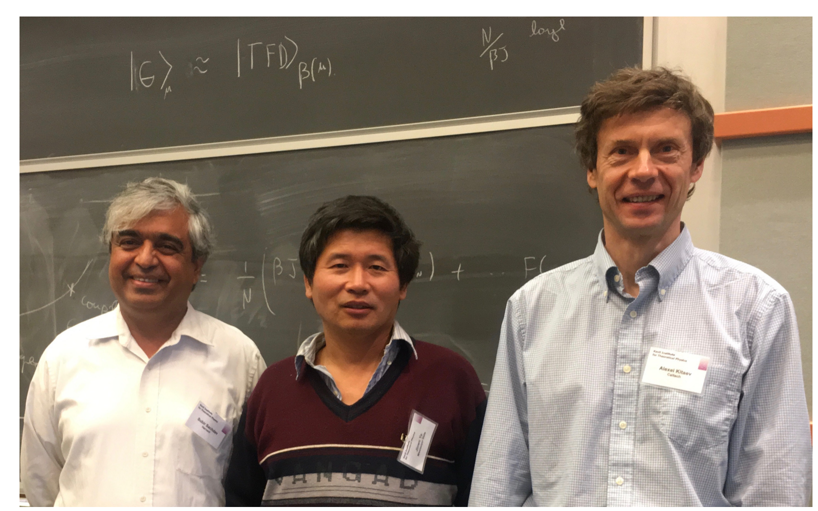
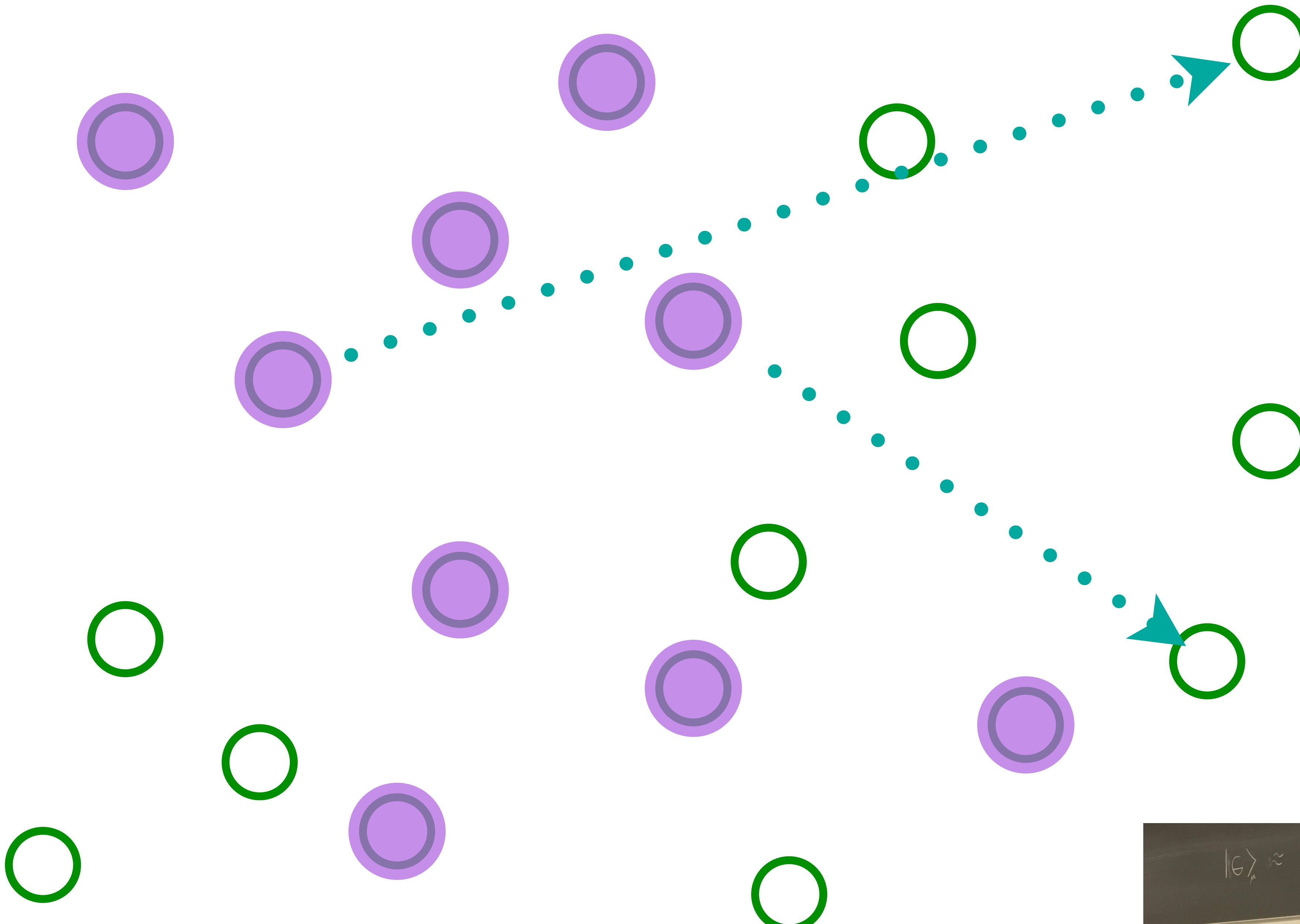


The SYK model



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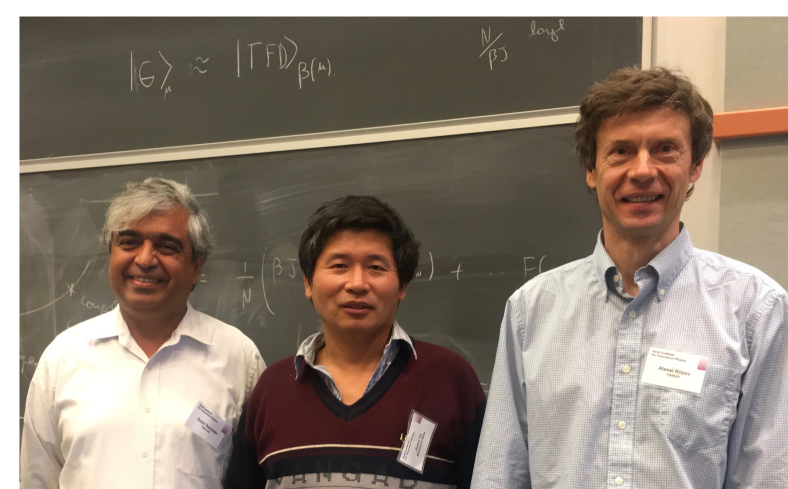
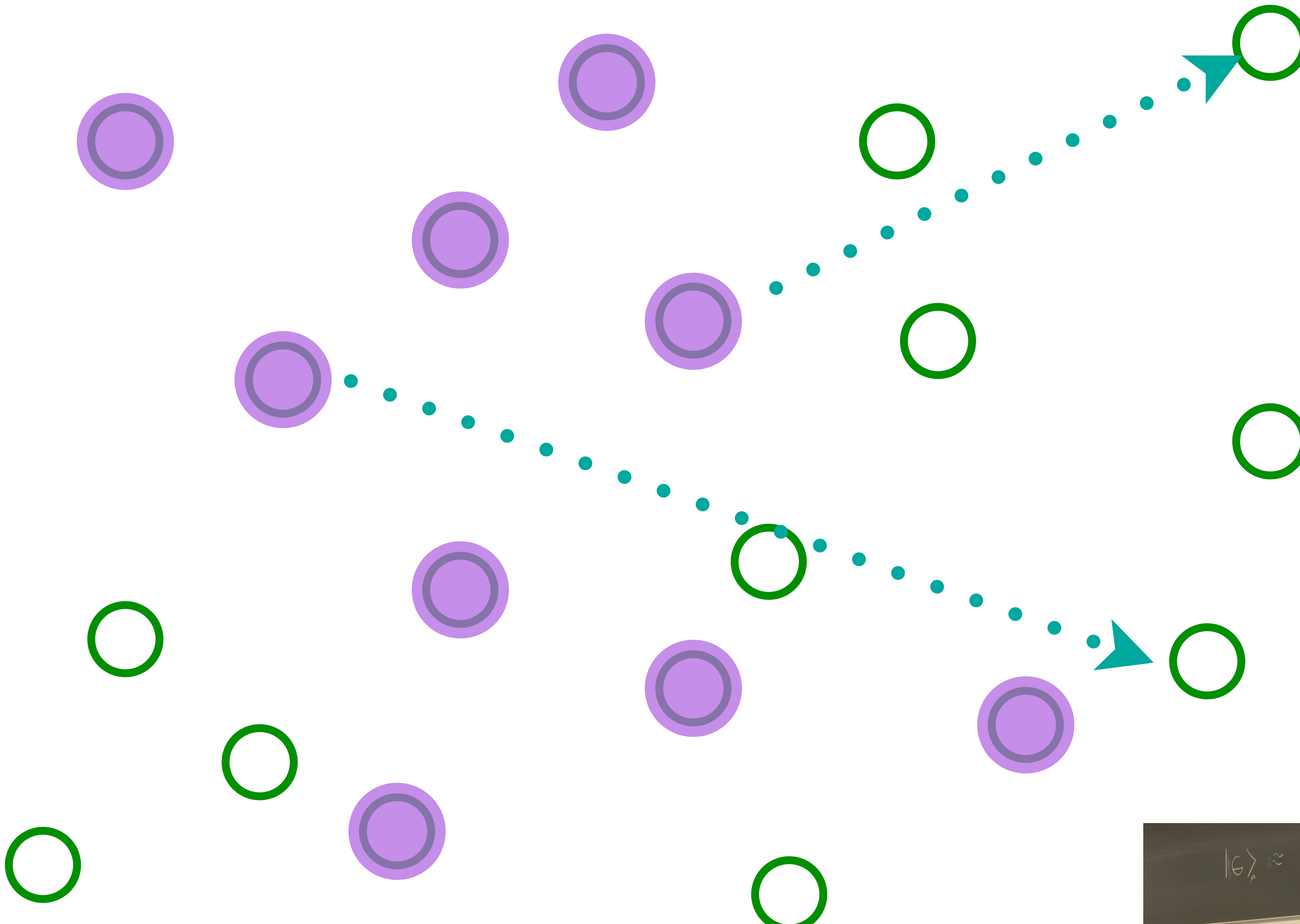


The SYK model



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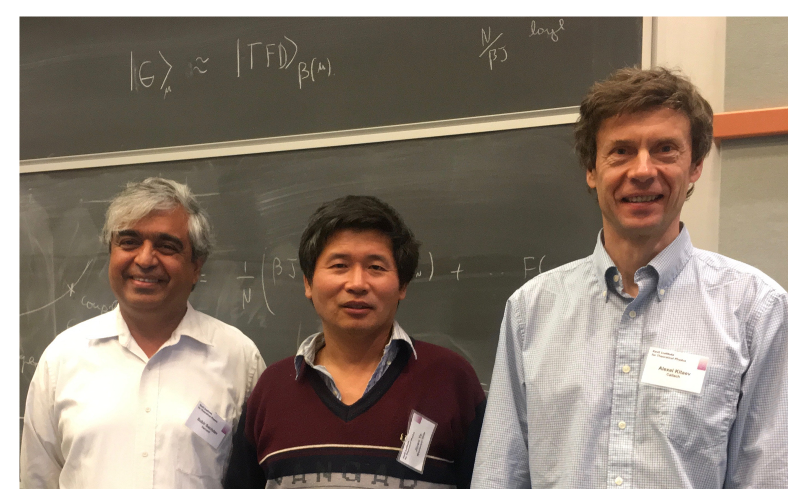

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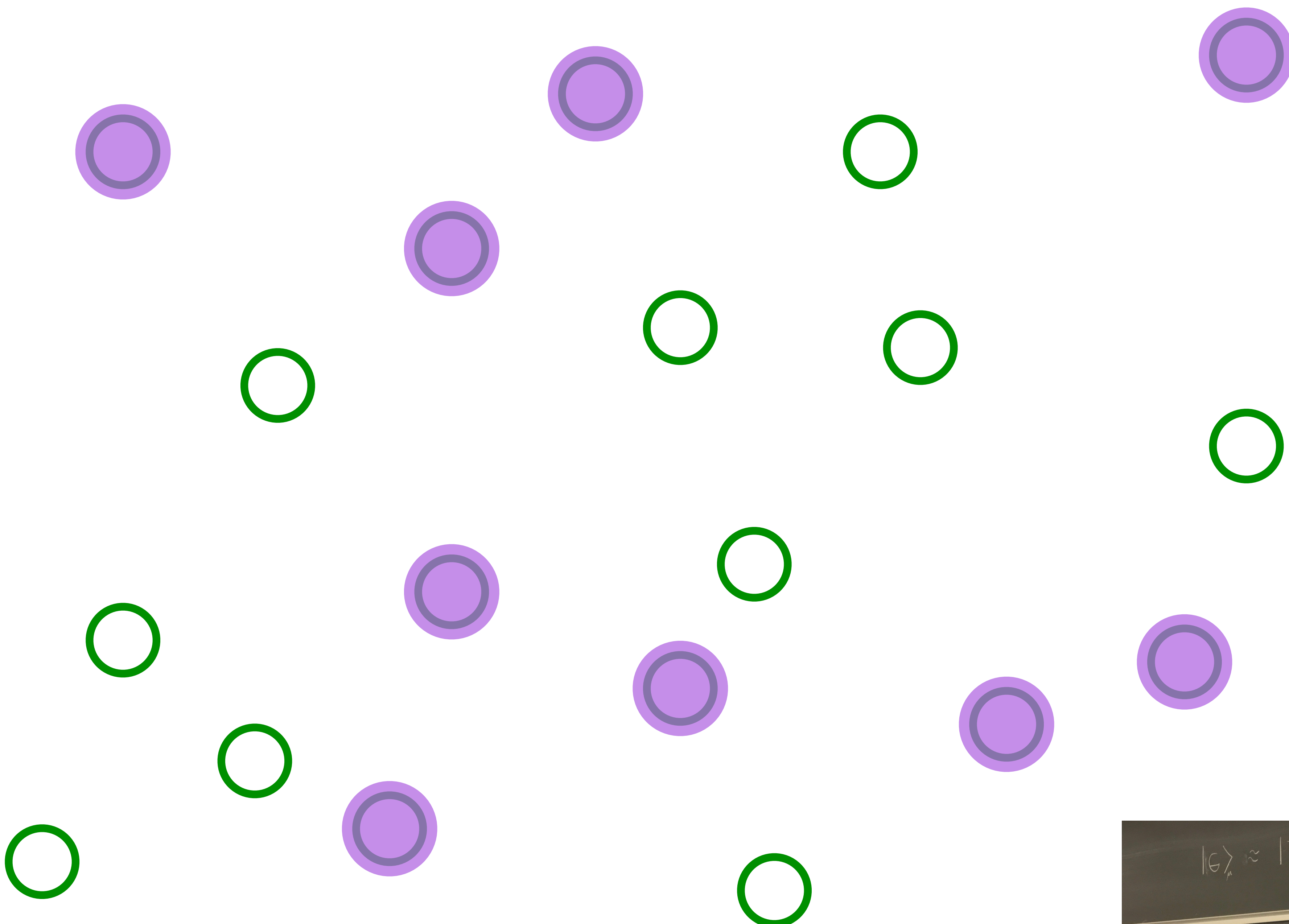

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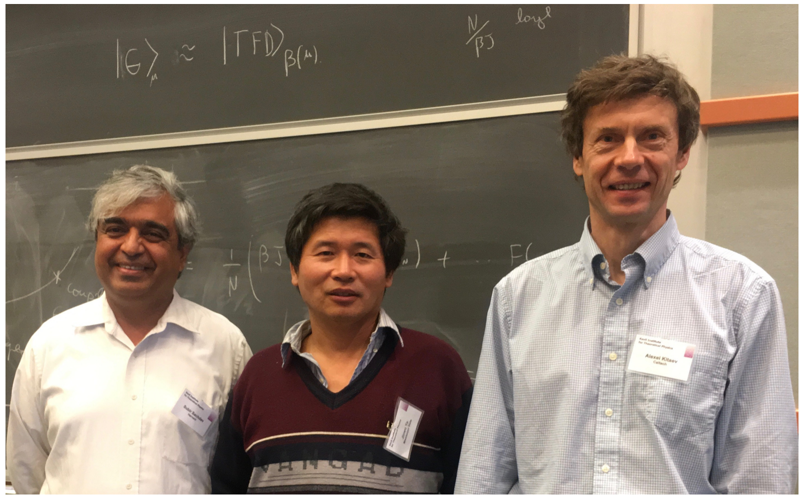

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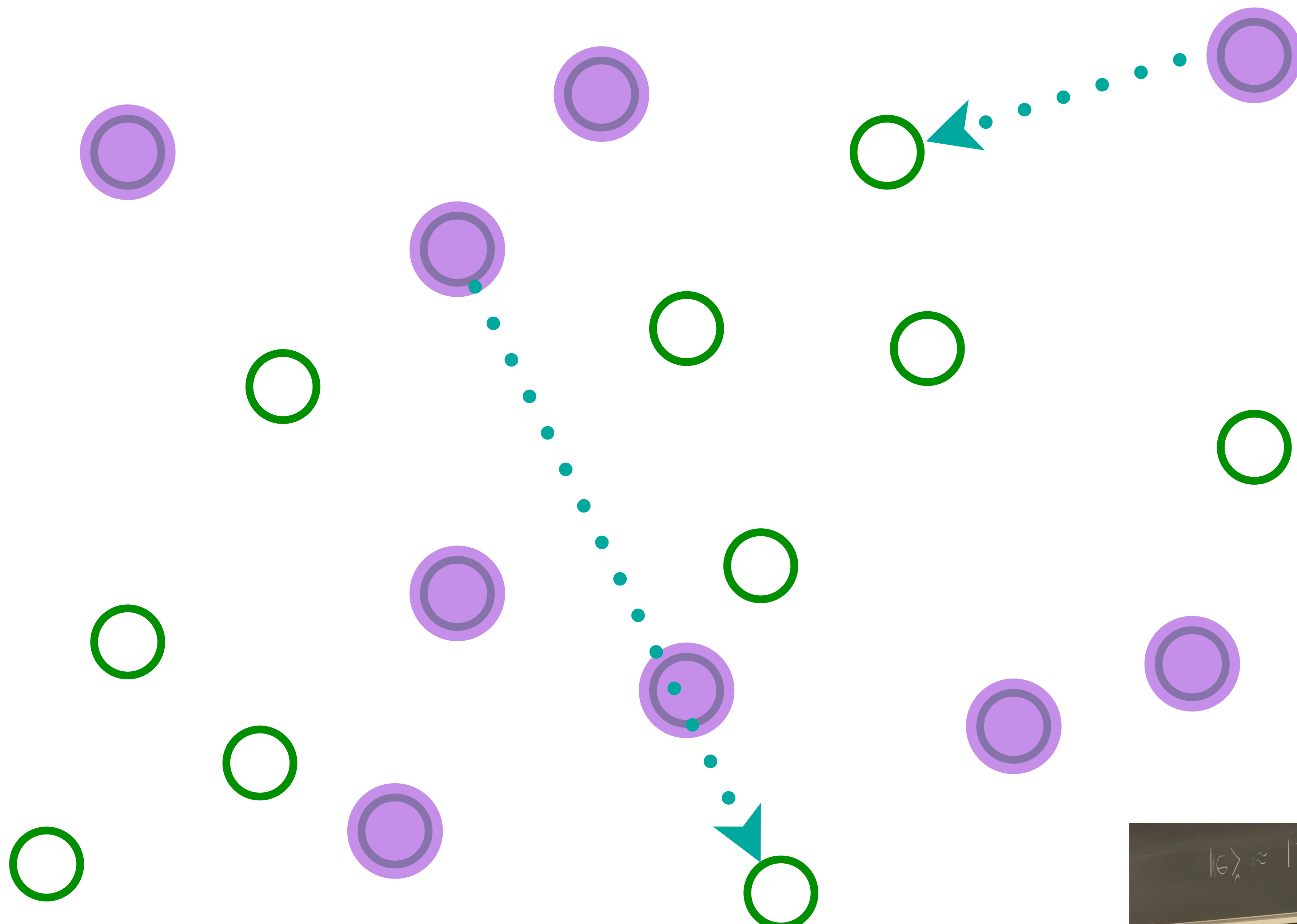

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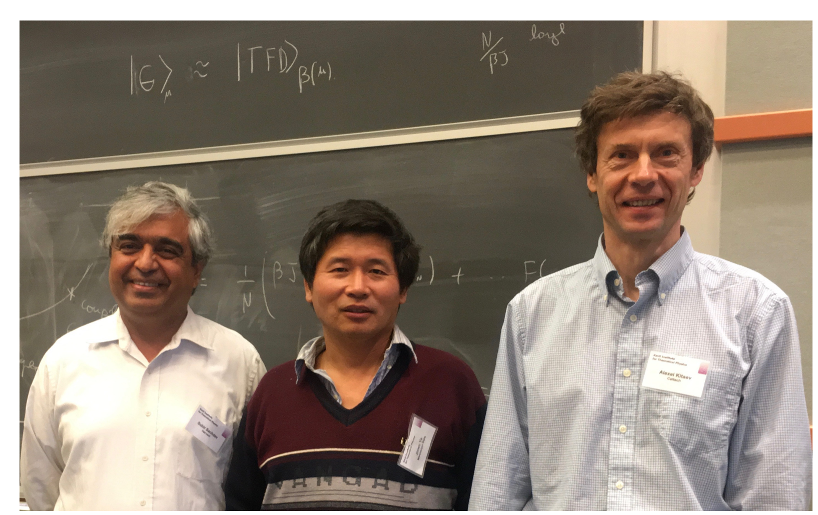

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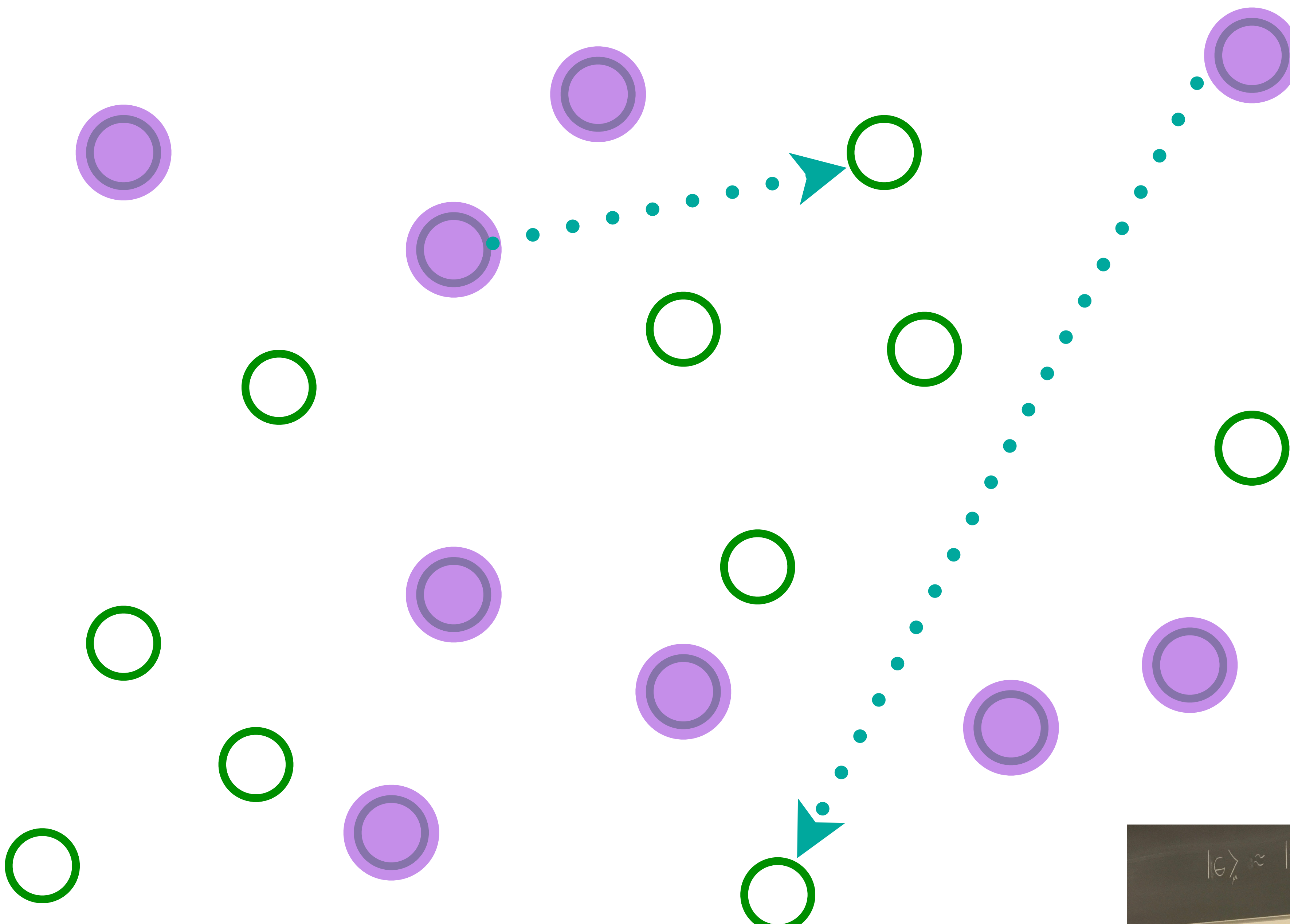

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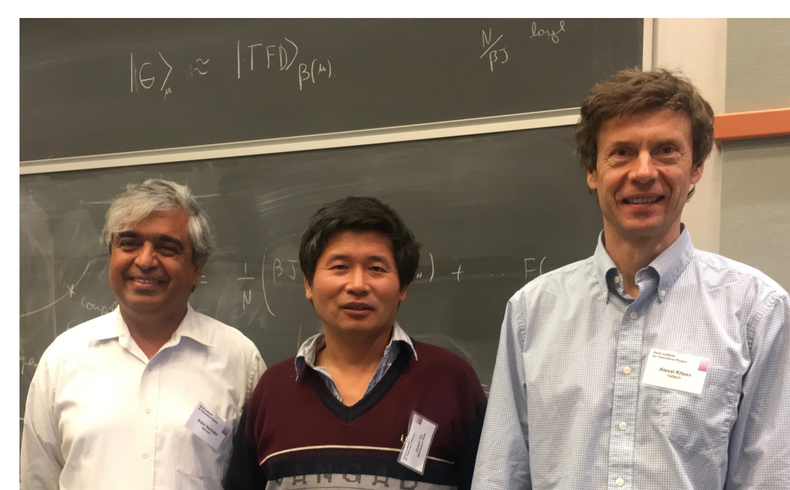
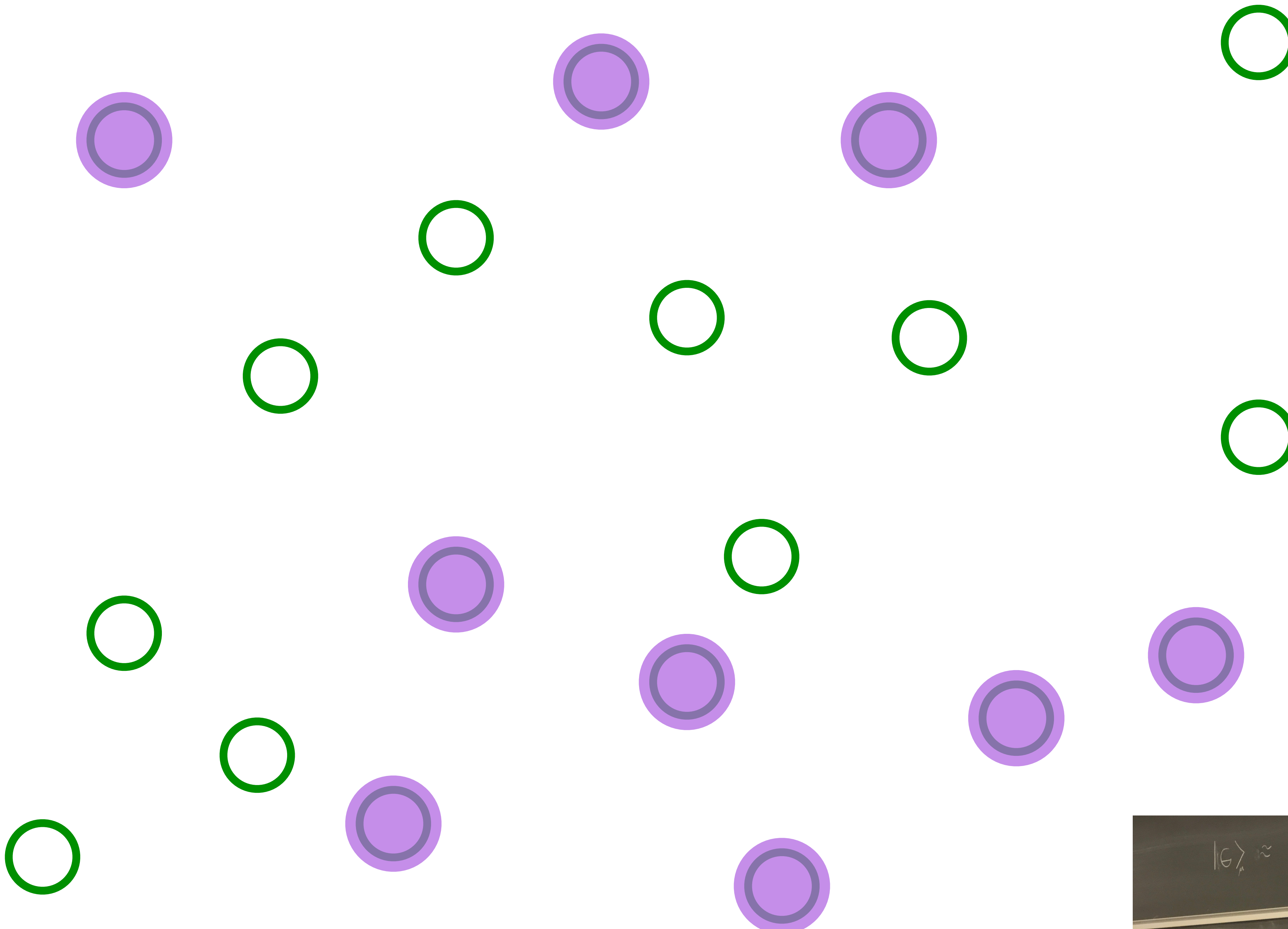


The SYK model



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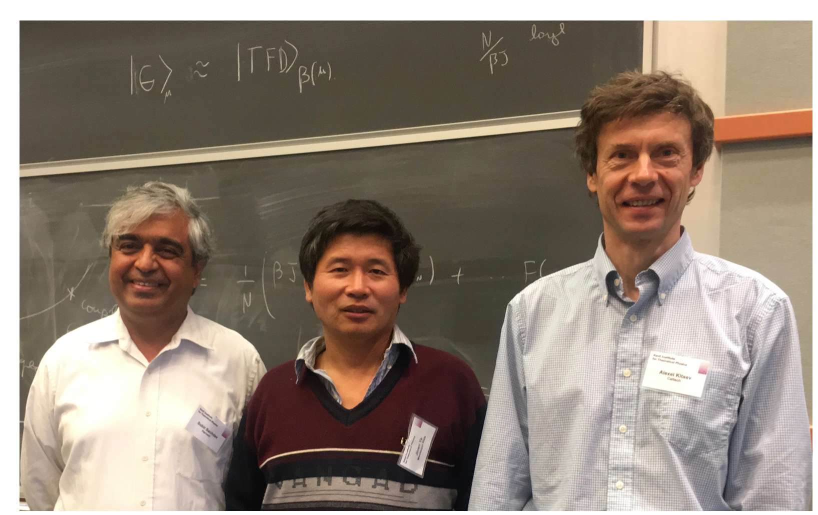

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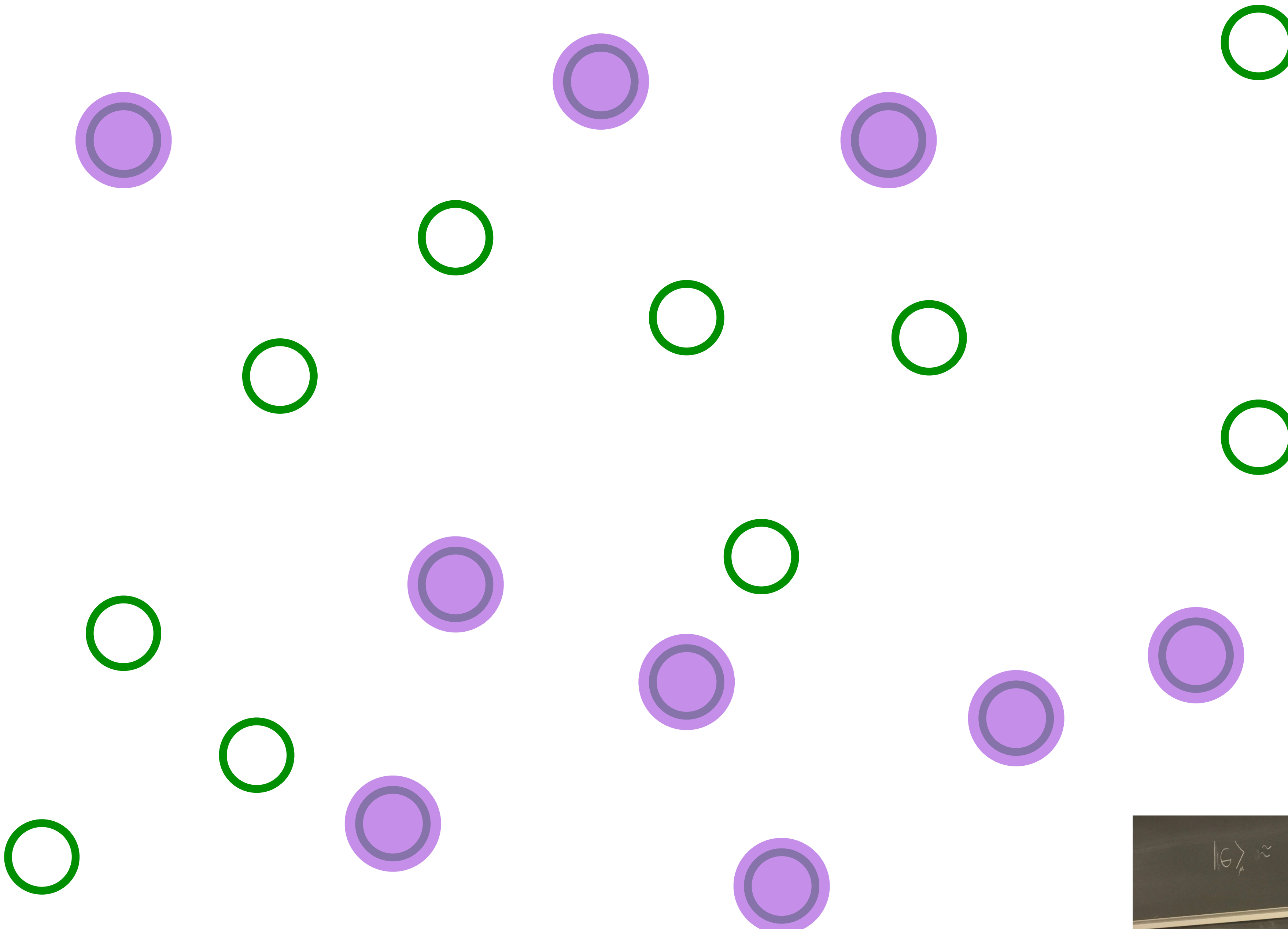
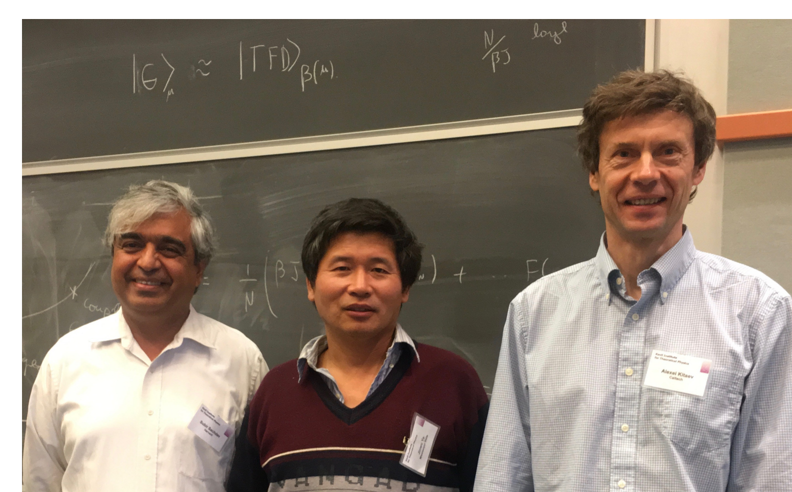


This describes both a strange metal and a black hole!



The SYK model

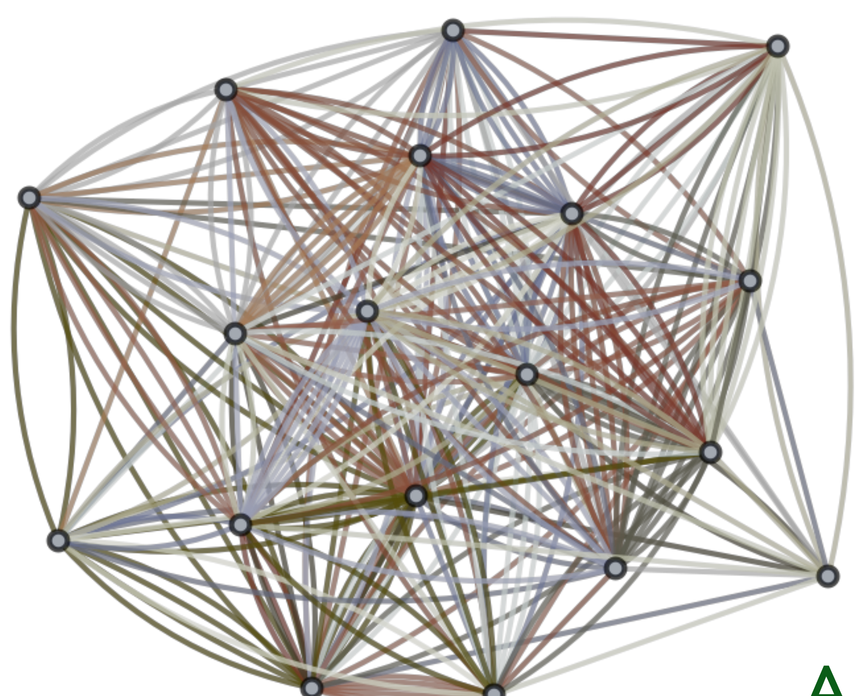
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

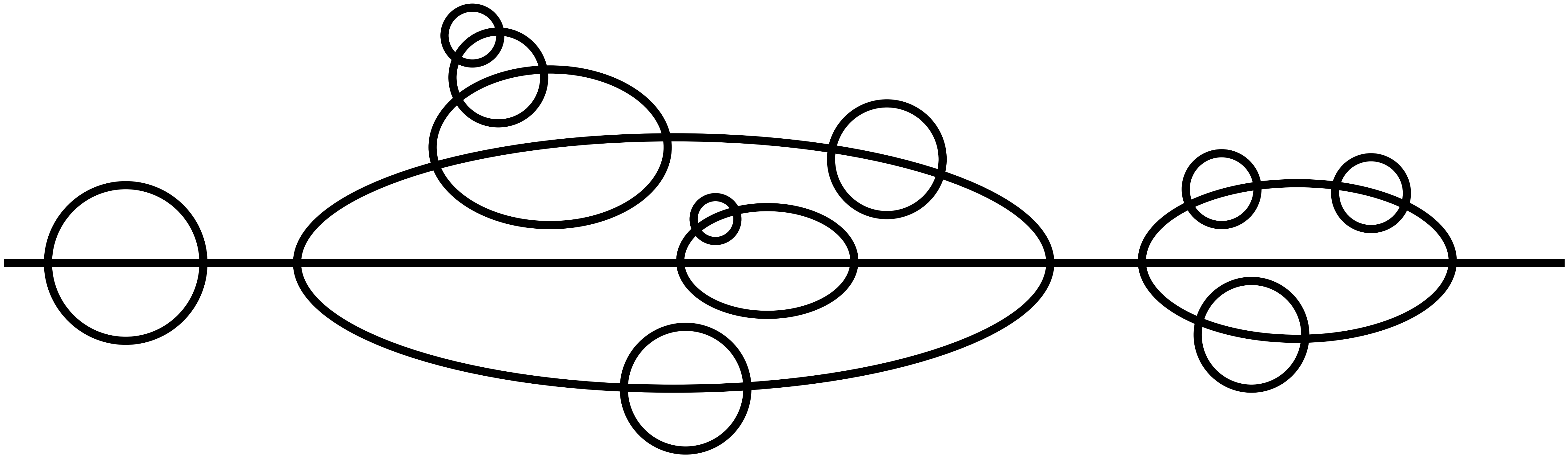


S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

The large N limit is given by the sum of "melon" Feynman graphs



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The SYK model

- Solution of the melon diagrams yields a Green's function at $T = 0$ of the form

$$G(\tau) \sim \begin{cases} -\tau^{-2\Delta} & \tau > 0 \\ e^{-2\pi\mathcal{E}}(-\tau)^{-2\Delta} & \tau < 0 \end{cases} \quad , \quad T = 0$$

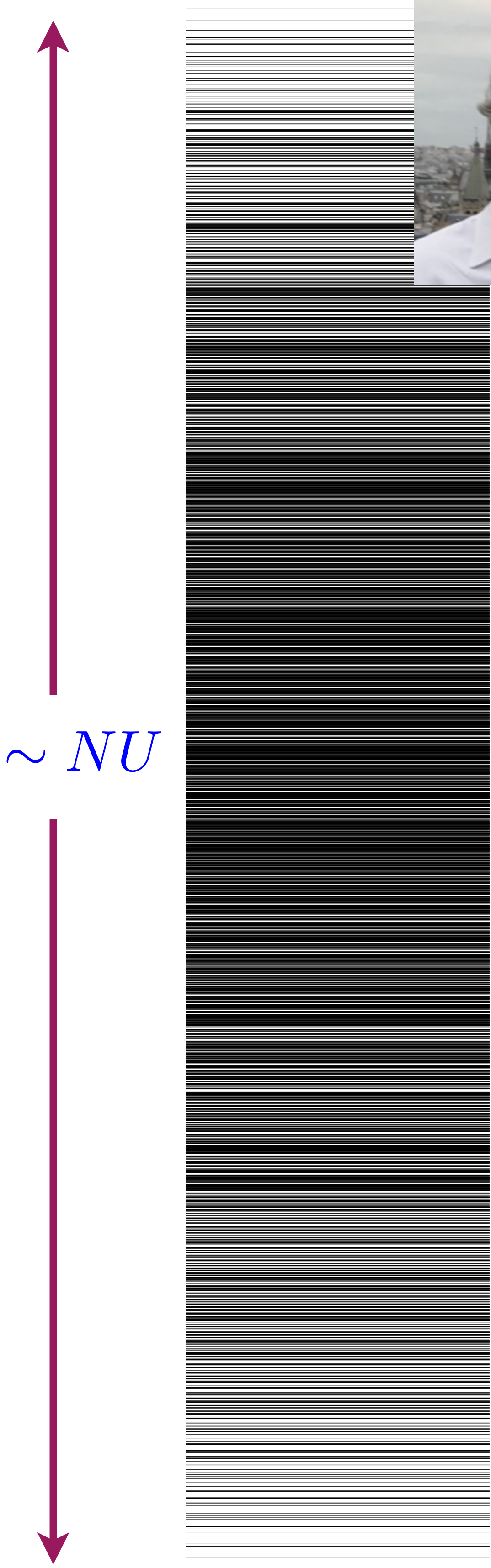
  where $\Delta = 1/4$ is the fermion scaling dimension. The *particle-hole asymmetry* is determined by the parameter $\mathcal{E}$ which universally depends only upon $\mathcal{Q}$.

- At $T > 0$ this has the conformal form

$$G(\tau) = -A \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1 + e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T \tau)} \right)^{2\Delta} \quad , \quad 0 < \tau < 1/T \,.$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)

The SYK model



There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}\,.$$

At $\mathcal{Q} = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.

Many-body level spacing $\sim 2^{-N} = e^{-N\ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-Ns_0}$

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model

No quasiparticles

Julia Steinberg

- Rapid local thermal equilibration (of fermion correlators) in a 'Planckian' time

$$\tau_{\text{eq}} \sim \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

Established by solution of Schwinger-Keldysh equations for a quench.

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A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

Established by solution of Schwinger-Keldysh equations for a quench.

- Presence of quasiparticles should slow down thermalization, so *all* quantum systems obey

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

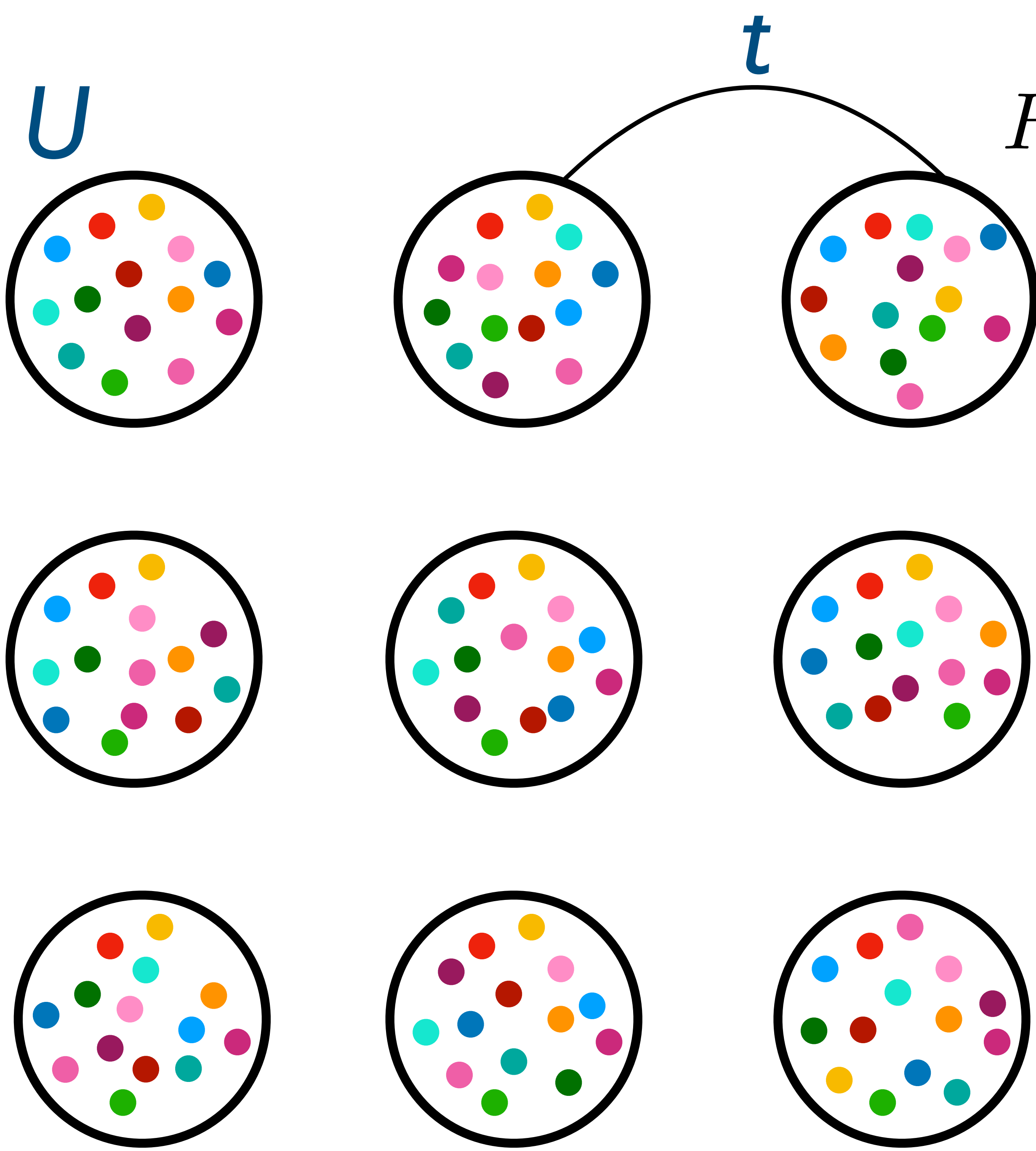
Absence of quasiparticles $\Leftrightarrow$ Fastest possible thermalization

Coupled SYK Islands

Xue-Yang Song

SYK quantum islands of electrons with random or regular hopping between them.

Aavishkar Patel



$$H = \sum_{x} \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx}$$

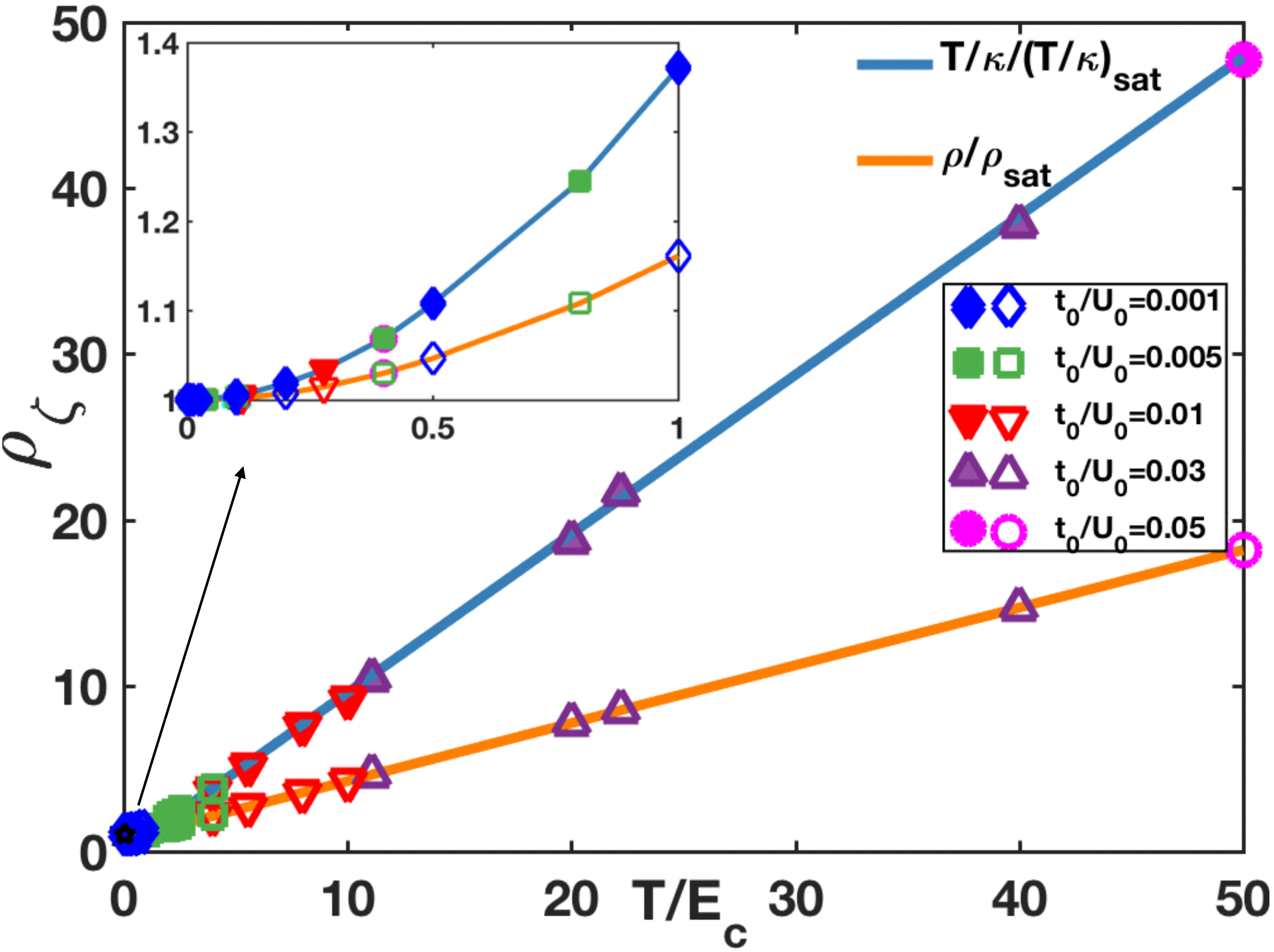
$$+ \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)
Pengfei Zhang, PRB **96**, 205138 (2017)
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 021049 (2018)
Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)
See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands

Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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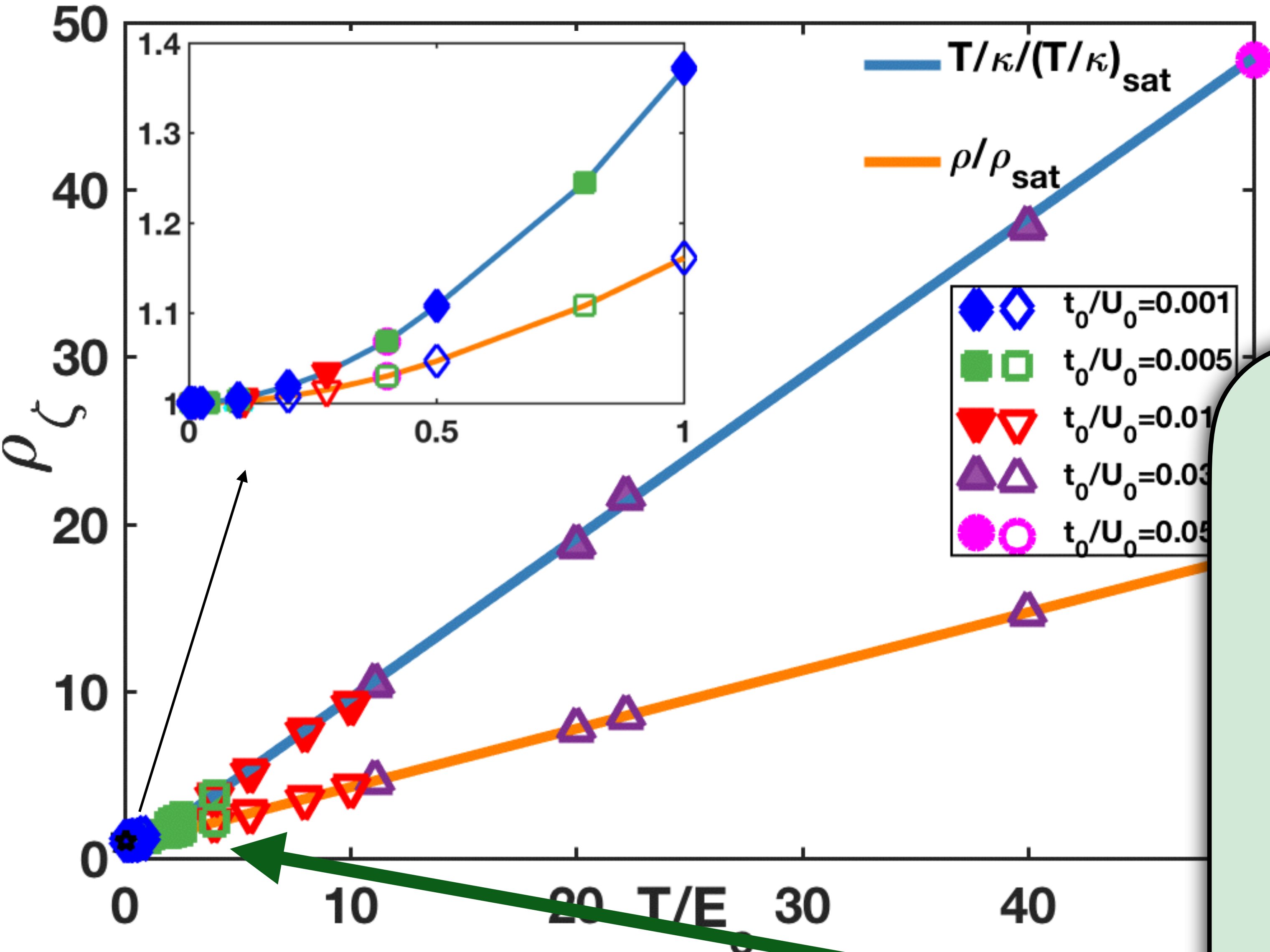

For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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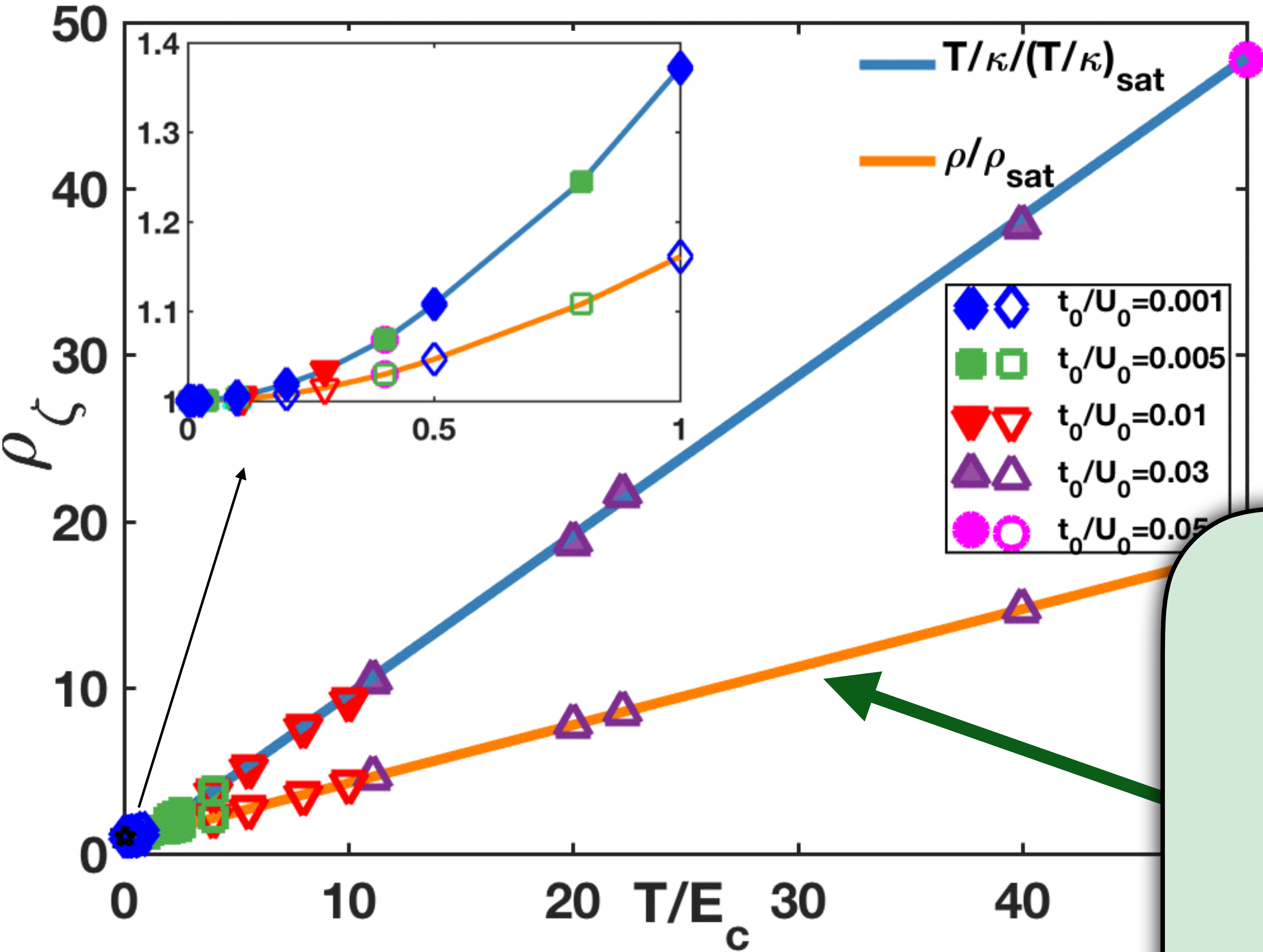

For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right) \quad , \qquad s = s_0$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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random matrix model

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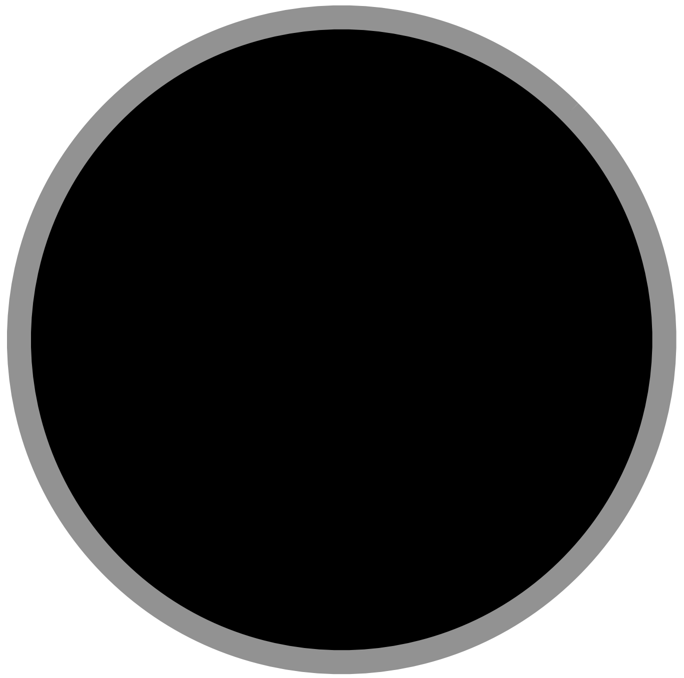
3. Fluctuations, and the Schwarzian

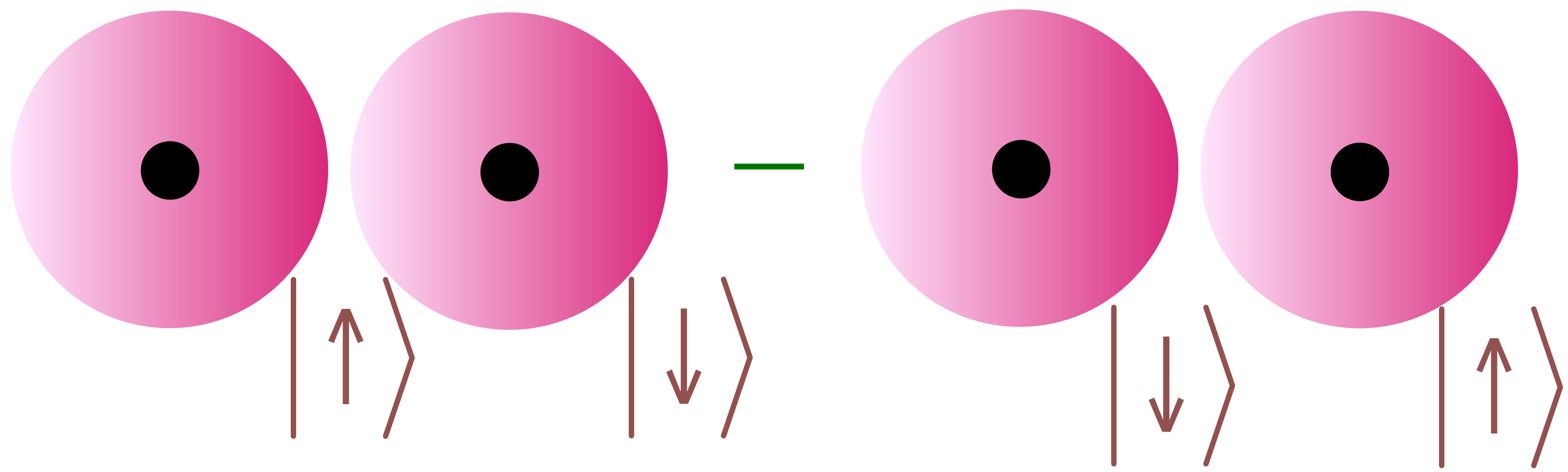
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.



Quantum Entanglement across a black hole horizon



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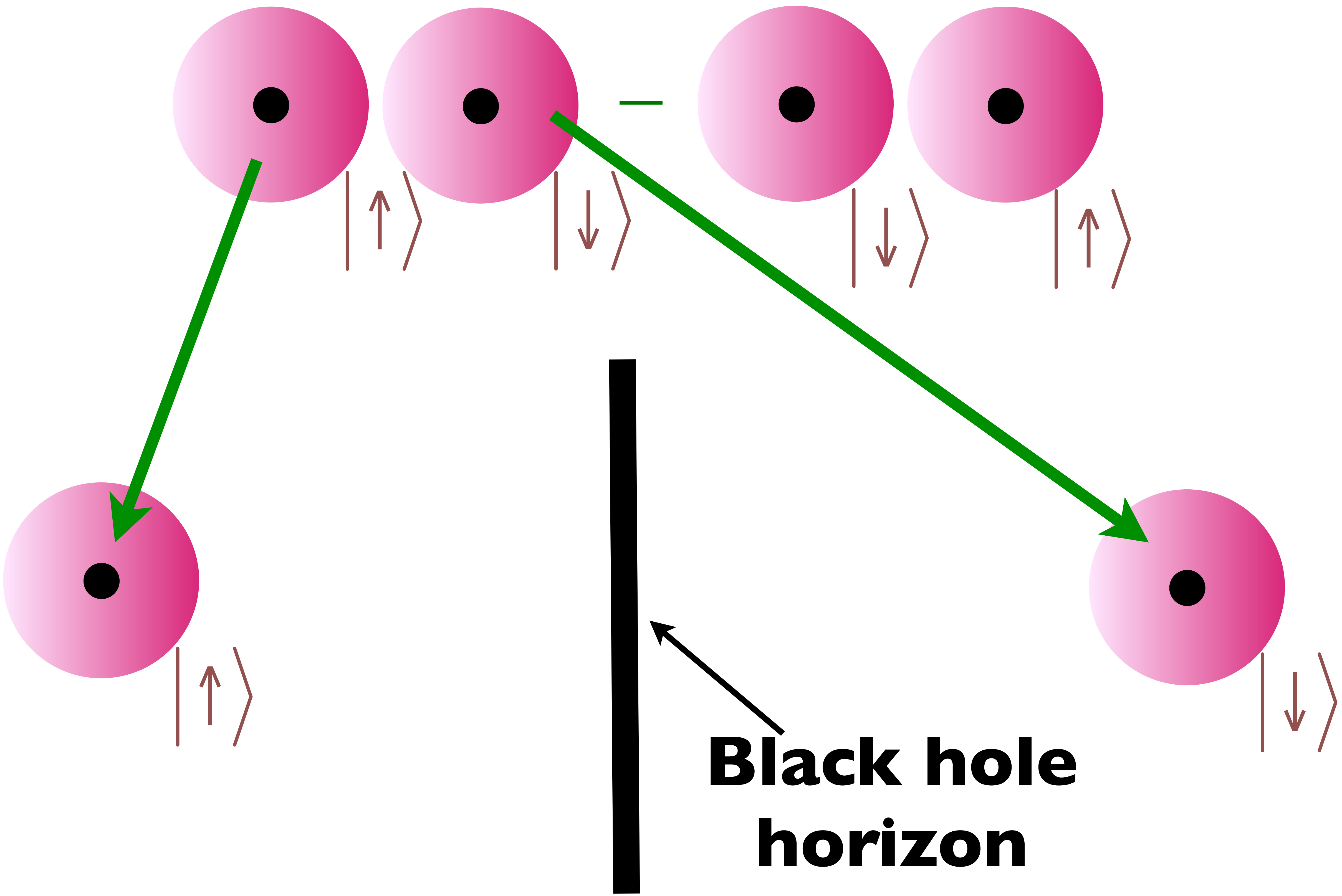

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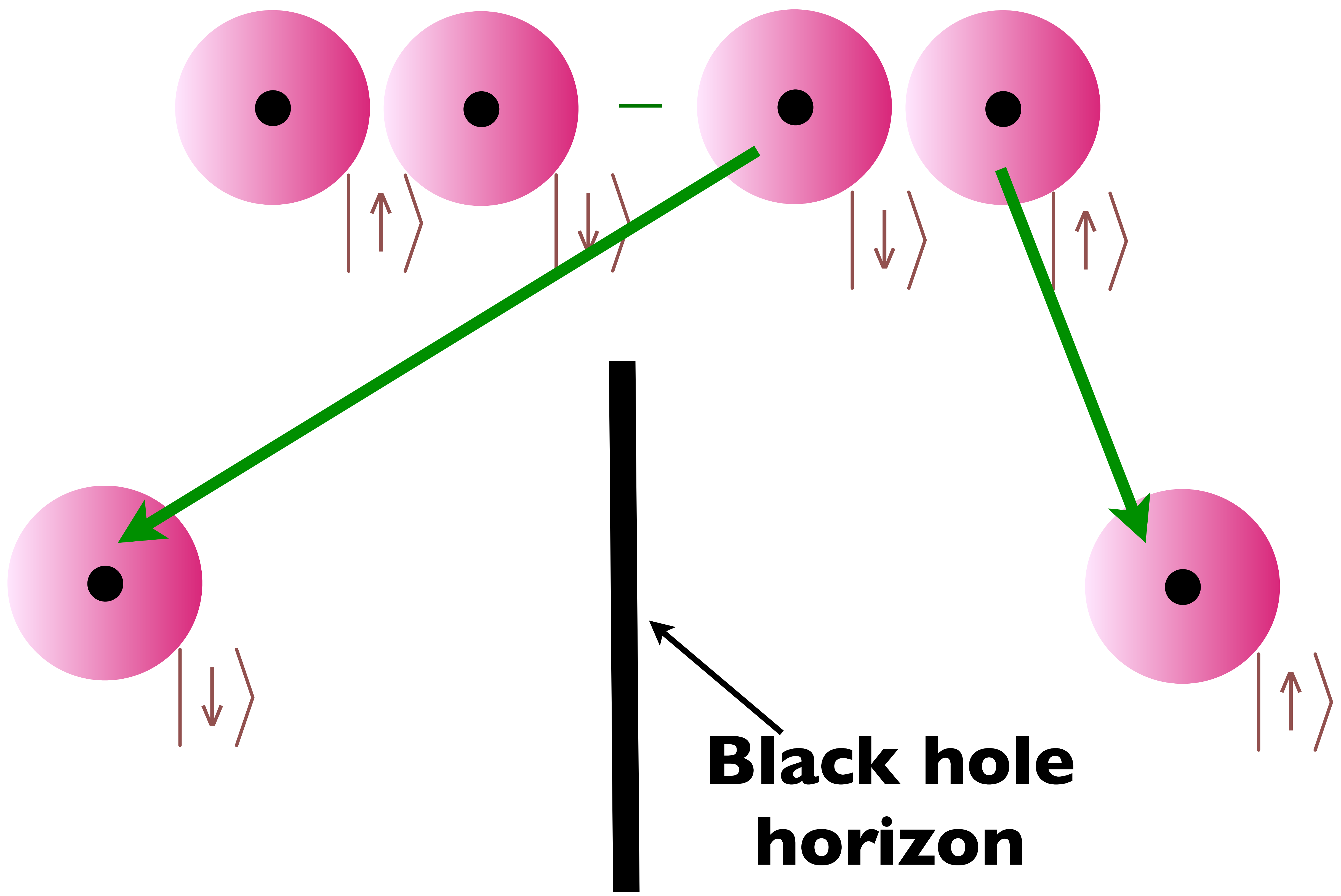
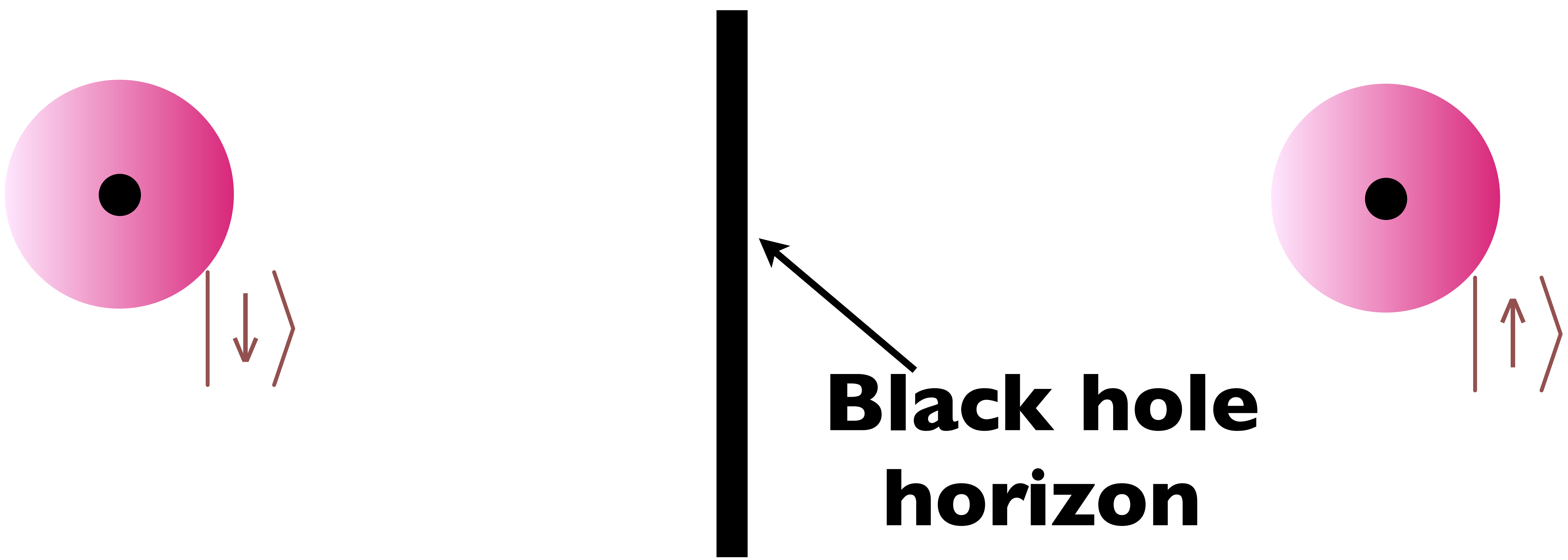


Quantum Entanglement across a black hole horizon

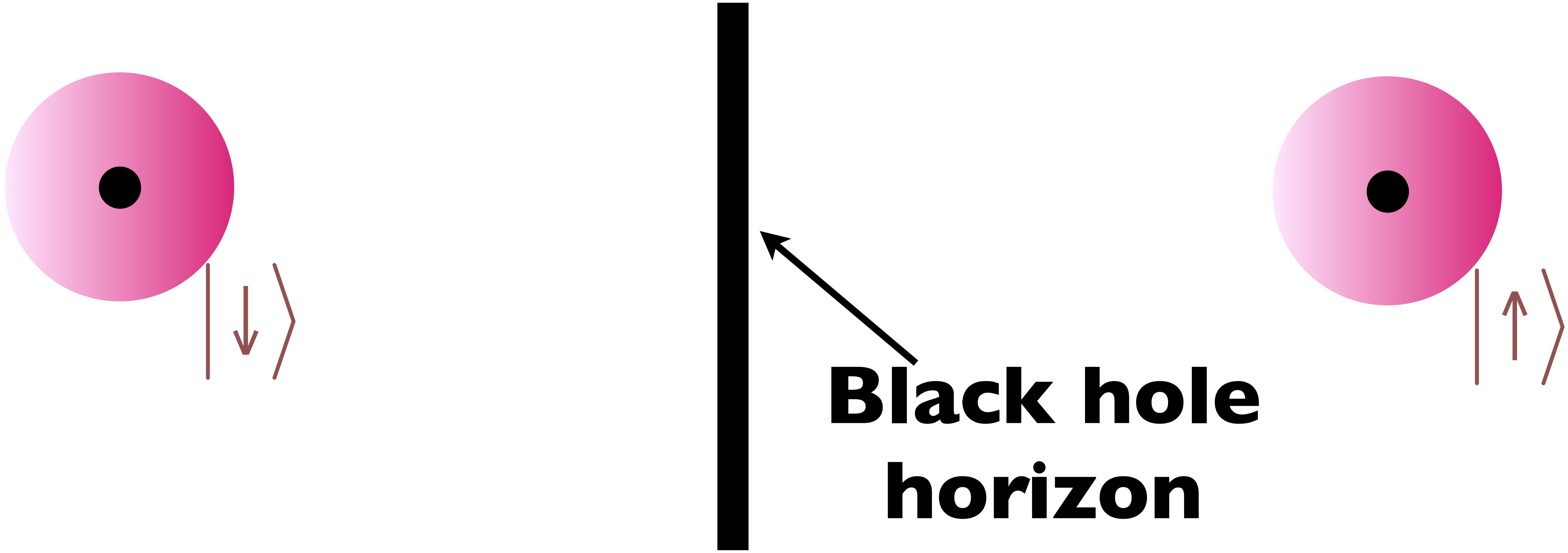
There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature

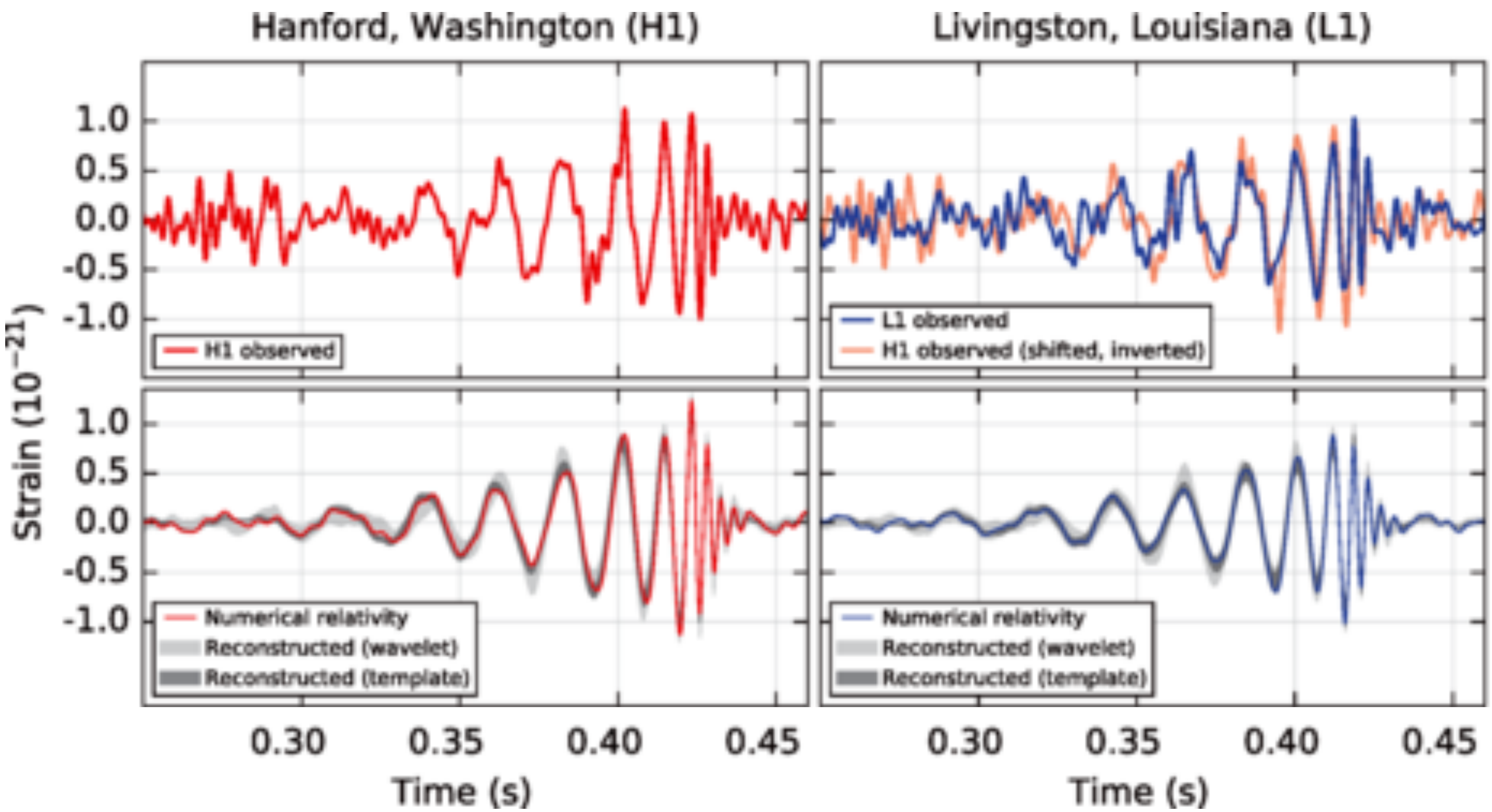
(because to an outside observer, the state of the electron inside the black hole is an unknown)

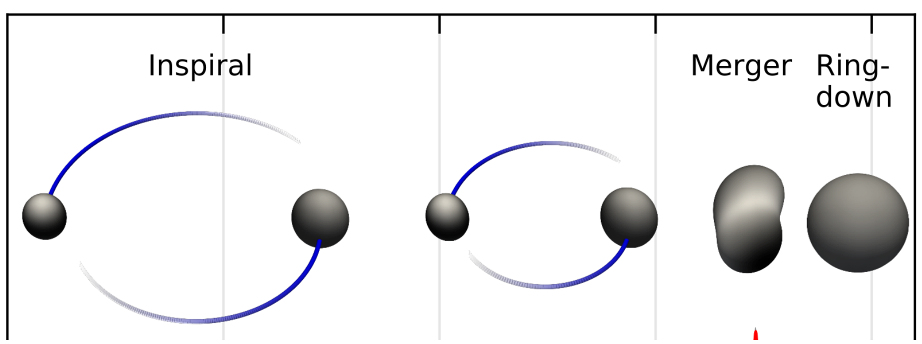


Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)



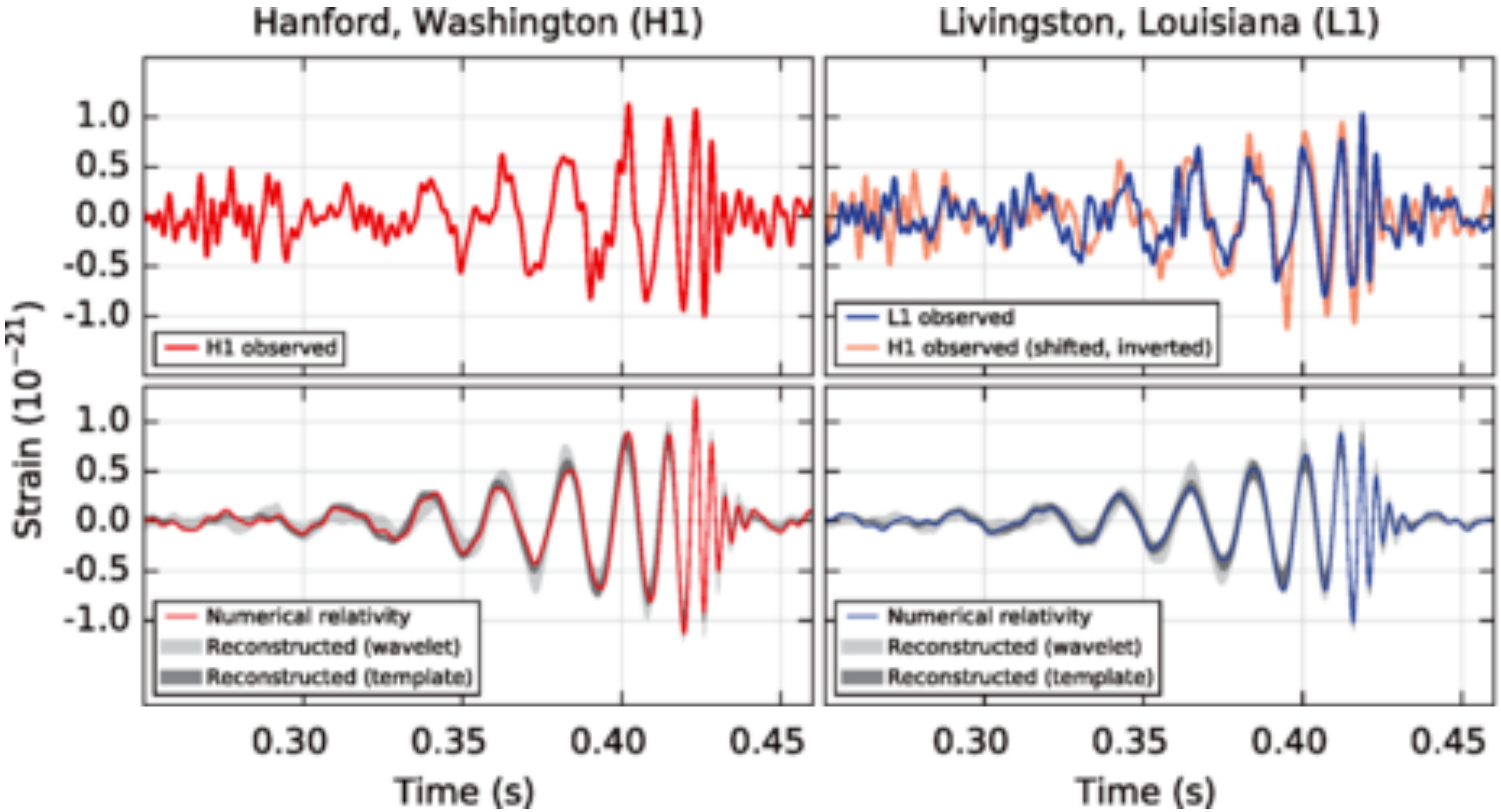


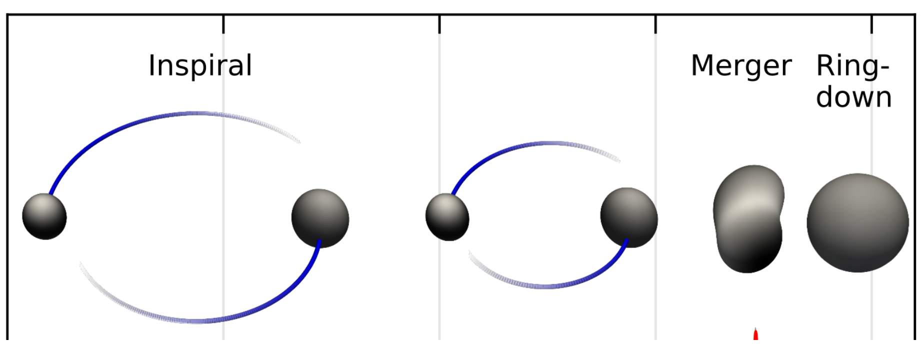
LIGO

September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds.

C. V. Vishveshwara, Nature **227**, 936 (1970)



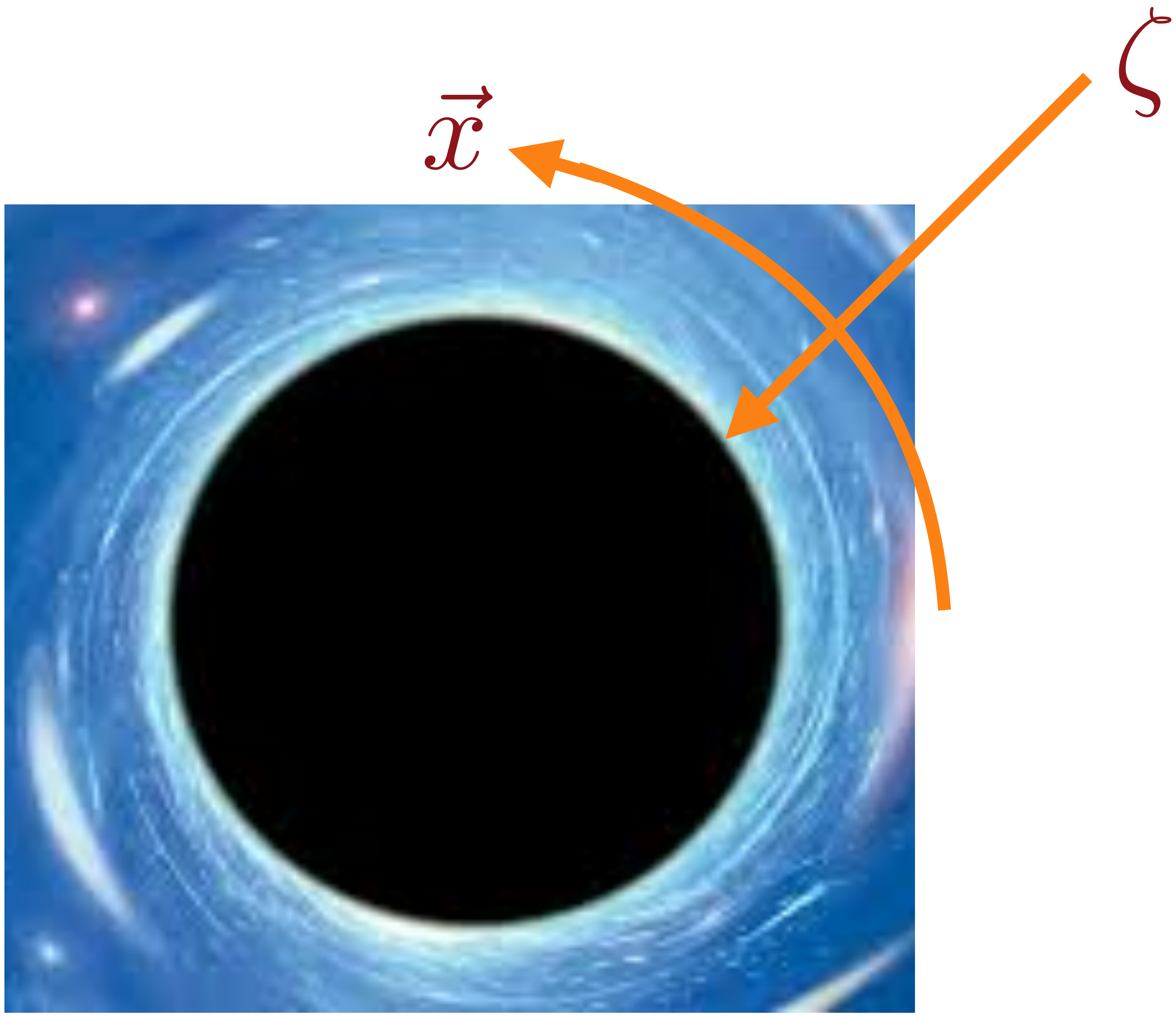


LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\frac{\hbar}{k_B T_H}$: so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

Black holes

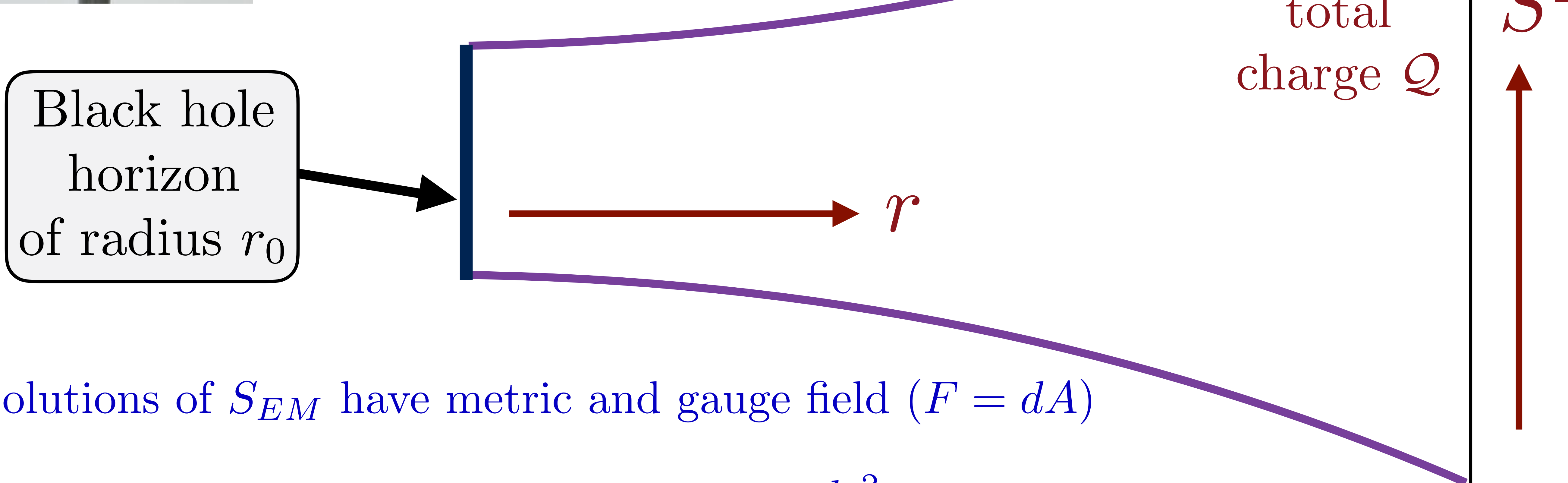
- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.



Zooming into the near-horizon region of a charged black hole, zooming at low temperature, yields a quantum theory in one space (ζ) and one time dimension

Charged black holes

$$S_{EM} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(\mathcal{R} + \frac{6}{L^2} - \frac{L^2}{g_F^2}F^2\right)$$



Solutions of S_{EM} have metric and gauge field ($F = dA$)

$$ds^2 = -V(r)dt^2 + r^2 d\Omega_2^2 + \frac{dr^2}{V(r)} \quad , \quad A = \mu\left(1 - \frac{r_0}{r}\right)dt$$

$$V(r) = 1 + \frac{r^2}{L^2} + \frac{\Theta^2}{r^2} - \frac{M}{r}.$$

where $d\Omega_2^2$ is the metric of the 2-sphere. All parameters of the solution, and the thermodynamics are determined in terms of the chemical potential μ, and the Hawking temperature of horizon, T.

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

Charged black holes

In the $T \to 0$ limit, at fixed μ, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of R_h

- In the near-horizon region, we change co-ordinates from r to ζ so that

$$r - R_h = \frac{R_2^2}{\zeta} \quad , \quad R_2 = \frac{L R_h}{\sqrt{6 R_h^2 + L^2}}.$$

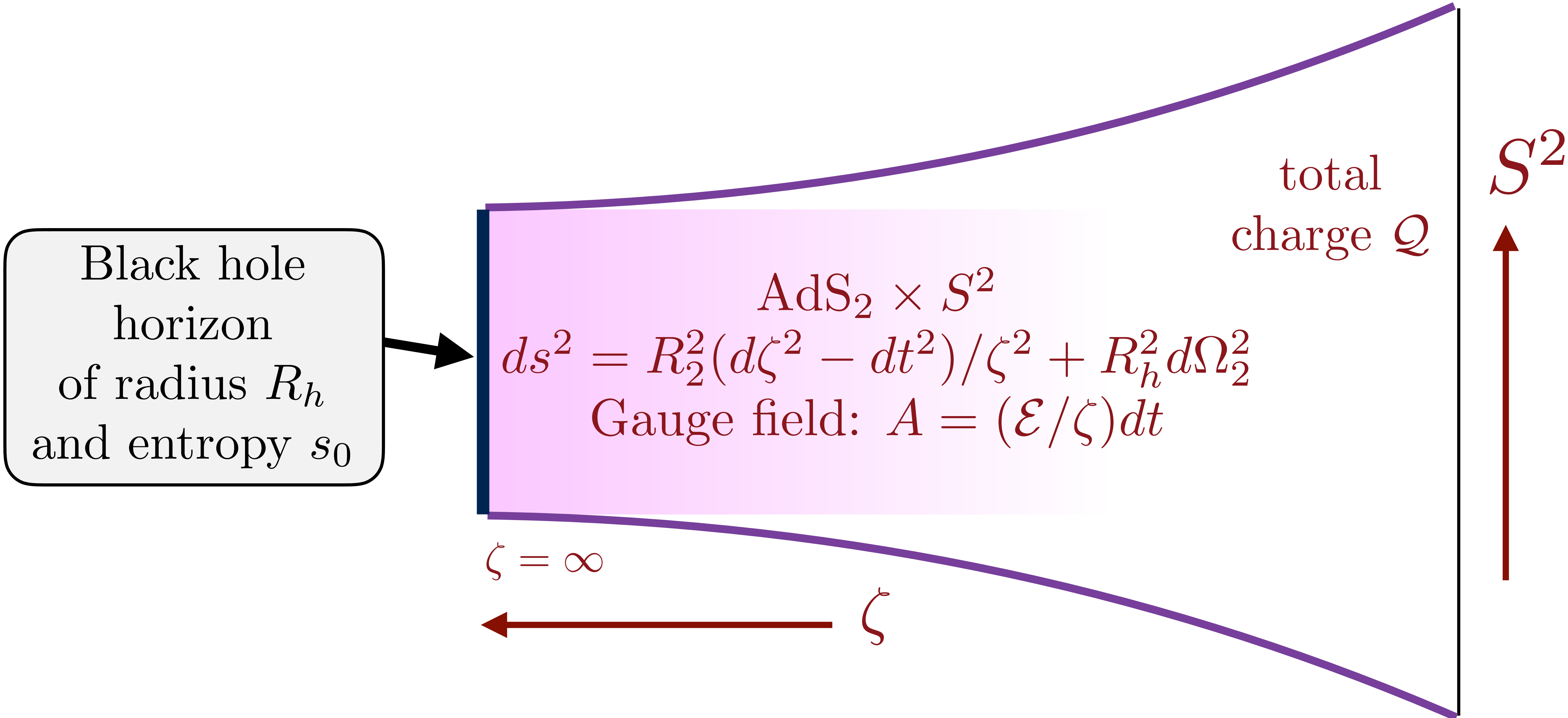
Then the near-horizon metric becomes $\mathrm{AdS}_2 \times \mathrm{S}_2$, with

$$ds^2 = R_2^2 \left[\frac{-dt^2 + d\zeta^2}{\zeta^2} \right] + R_h^2 d\Omega_2^2 \quad , \quad A = \frac{\mathcal{E}}{\zeta} dt\,.$$

where the dimensionless electric field $\mathcal{E}$ is

$$\mathcal{E} = \frac{g_F R_h \sqrt{3 R_h^2 + L^2}}{6 R_h^2 + L^2}.$$

Charged black holes



- The entropy s_0, the charge $\mathcal{Q}$, and the dimensionless electric field $\mathcal{E}$ obey

$$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen, JHEP **0509**, 038 (2005)

Charged black holes

Probe fermion in the AdS_2 near horizon

- A probe fermion has a near-horizon Green's function at $T=0$ of the form

$$G(\tau) \sim \begin{cases} -\tau^{-2\Delta} & \tau > 0 \\ e^{-2\pi\mathcal{E}}(-\tau)^{-2\Delta} & \tau < 0 \end{cases} \quad , \quad T = 0$$

where Δ is the fermion scaling dimension. The *particle-hole asymmetry* is determined by the dimensionless electric field $\mathcal{E}$ at the surface of the black hole.

- At $T > 0$ this has the conformal form

$$G(\tau) = -A \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1+e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T \tau)} \right)^{2\Delta} \quad , \quad 0 < \tau < 1/T \, .$$

T. Faulkner, Hong Liu, J. McGreevy, D. Vegh, PRD **83**, 125002 (2011)

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(a) with quasiparticles (normal):

random matrix model

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The SYK model

The theory of fluctuations about the large N saddle point are expressed in terms of a bilocal time Green's function, $G(\tau_1, \tau_2)$. This theory is invariant at low energies under a time reparameterization f, and an emergent gauge transformation g

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{G}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, 2015
S. Sachdev, PRX **5,** 041025 (2015)

The SYK model

The large N saddle point is

$$G_s(\tau_1 - \tau_2) \quad \sim \quad (\tau_1 - \tau_2)^{-1/2}.$$

The saddle point will be invariant under a reperamaterization $f(\tau)$ when choosing $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ leads to a transformed $\widetilde{G}(\sigma_1, \sigma_2) = G_s(\sigma_1 - \sigma_2)$ (and similarly for Σ). It turns out this is true only for the SL(2, R) transformations under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken down to SL(2, R) by the saddle point.

A. Kitaev

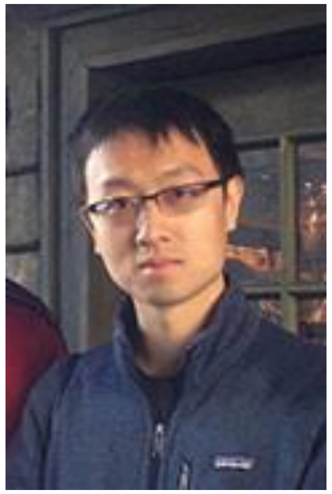
The Schwarzian theory of the SYK model

The effective theory of time reparameterizations $f(\tau)$ broken down to SL(2,R), and gauge transformations $g(\tau) = e^{i\phi(\tau)}$ is

$$S_{\rm eff}[f,\phi] = \frac{K}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau f)^2 - \frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \{\tan(\pi T f(\tau)), \tau\},$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$, the couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2 .$$



Yingfei Gu

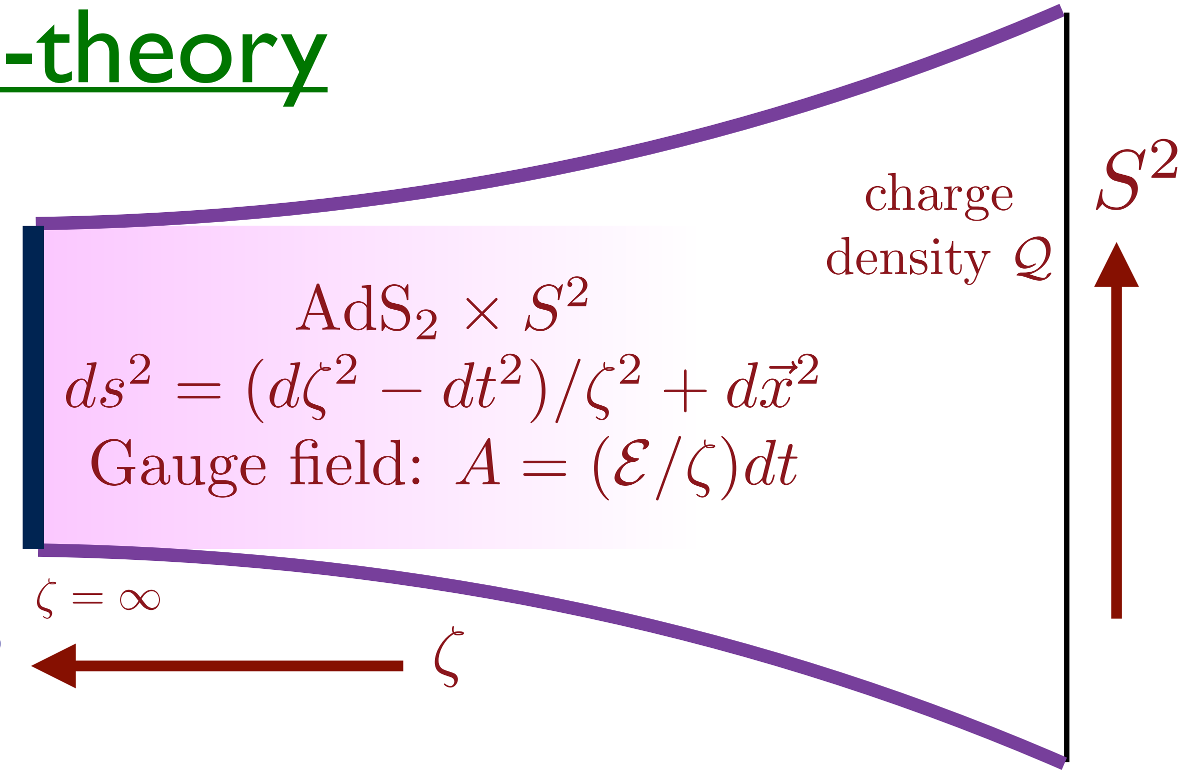
Specifically, an argument constraining the effective action at $T = 0$ is

$$S_{\rm eff}\left[f(\tau) = \frac{a\tau + b}{c\tau + d}, \phi(\tau) = 0\right] = 0,$$

and this is origin of the Schwarzian.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746;
Yingfei Gu and S. Sachdev, unpublished

Einstein-Maxwell-theory



$$S_{4D} = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

Near-extremal black hole of radius R and $T \ll 1/R$ from 4D Einstein-Maxwell

$\Longrightarrow$ 2D Einstein-Maxwell with a scalar field (fluctuations of black hole radius)

$\Longrightarrow$ Absence of 2D quantum gravity fluctuations: reduction to (0+1)D

$\Longrightarrow$ $(0+1)$D theory is *precisely the Schwarzian theory of the SYK model.*

$$S_{\text{eff}}[f, \phi] = \frac{K}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau f)^2 - \frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \, \{\tan(\pi T f(\tau)), \tau\},$$

J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438; P. Nayak, A. Shukla, R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547; U. Moitra, S.P. Trivedi, and V. Vishal, arXiv:1808.08239; P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062; A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746

SYK models and black holes

- Reparameterization and gauge invariance are defining properties of Einstein-Maxwell theory

- In imaginary time, AdS_2 is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \text{ with } ad - bc = 1.$$

A. Kitaev, 2015
J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857

Quantum matter without quasiparticles

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- Lattices of SYK islands have led to a partial understanding of strange metals.