

Bosons,  $b_j$  hopping on the sites  $j$  of a square lattice with Hamiltonian

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_j n_j (n_j - 1)$$
$$n_j \equiv b_j^\dagger b_j$$

The boson operators obey the commutation relation

$$[b_j, b_k^\dagger] = \delta_{jk}$$

We restrict attention to the sector of the Fock space with

$$\sum_j n_j = \text{integer multiple of the number of sites}$$