A model of the cuprates: from the pseudogap metal, to d-wave superconductivity, and charge order

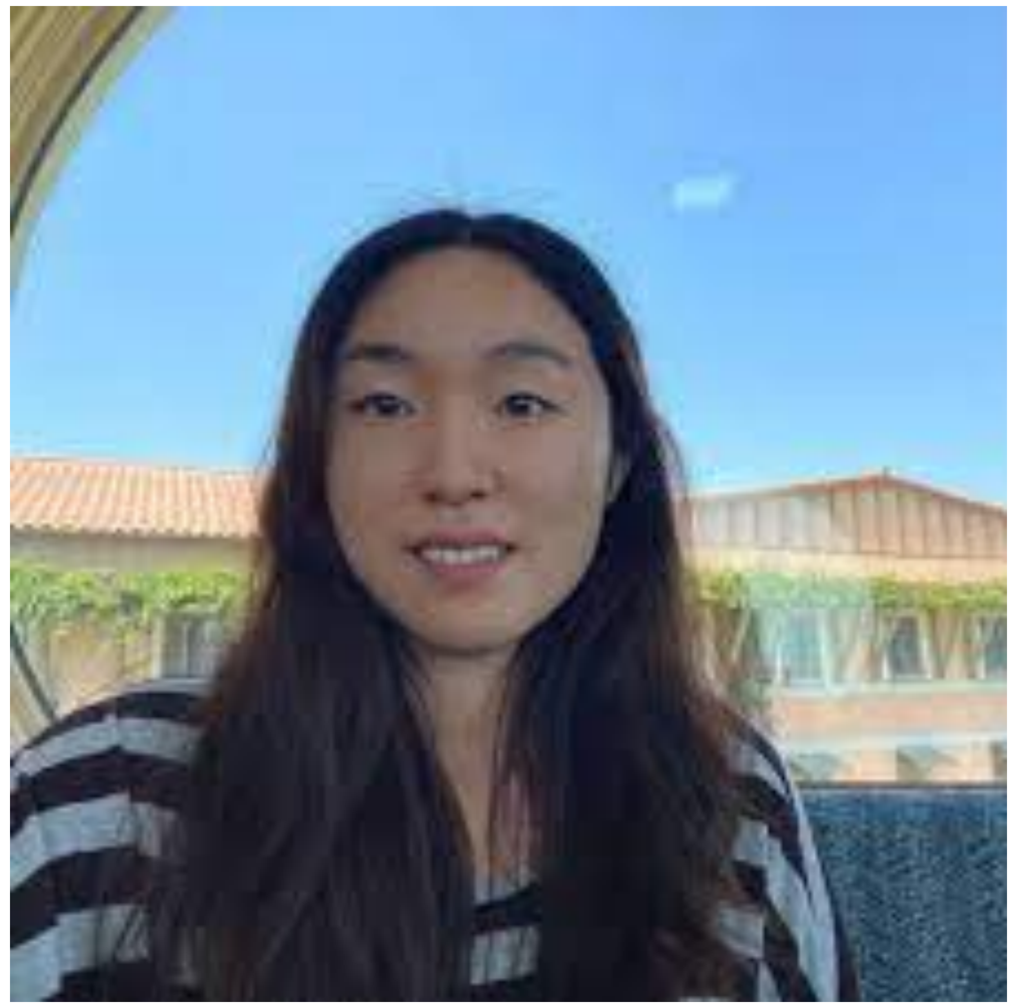
Quantum Matter Seminar
CMSA, Harvard
April 21, 2023
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang, Mathias Scheurer, and S. S., arXiv:2302.07885
Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi, and S.S., arXiv:2211.10452

Talk online: sachdev.physics.harvard.edu

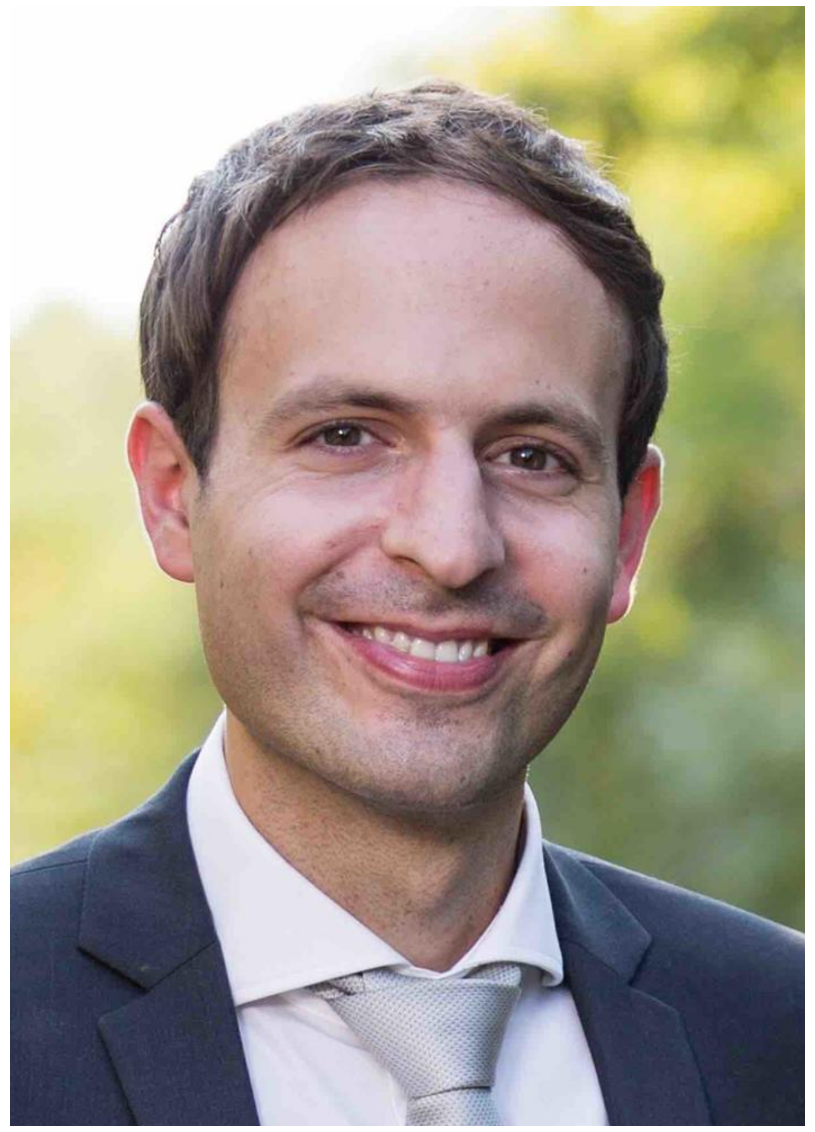
PHYSICS
VE RI TAS
HARVARD

Maine Christos

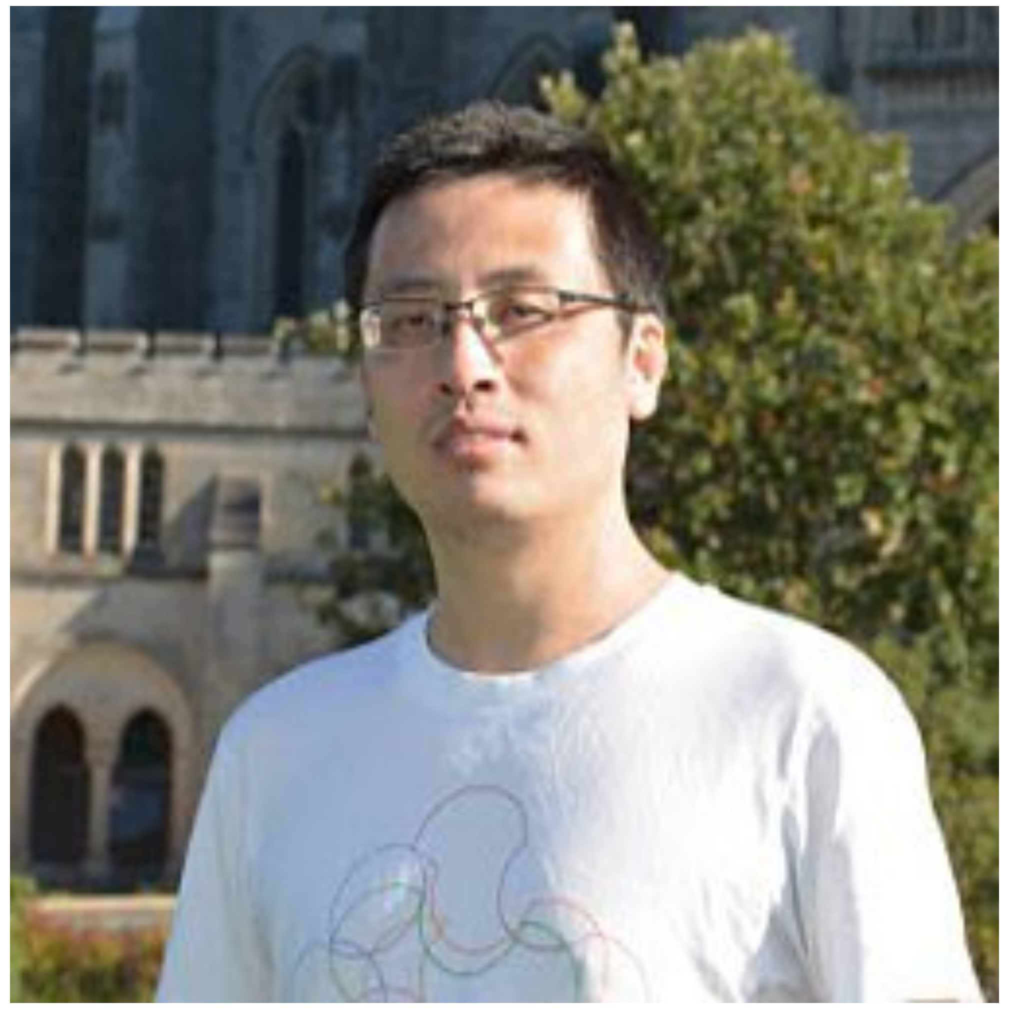


Zhu-Xi Luo

Henry Shackleton



Mathias Scheurer



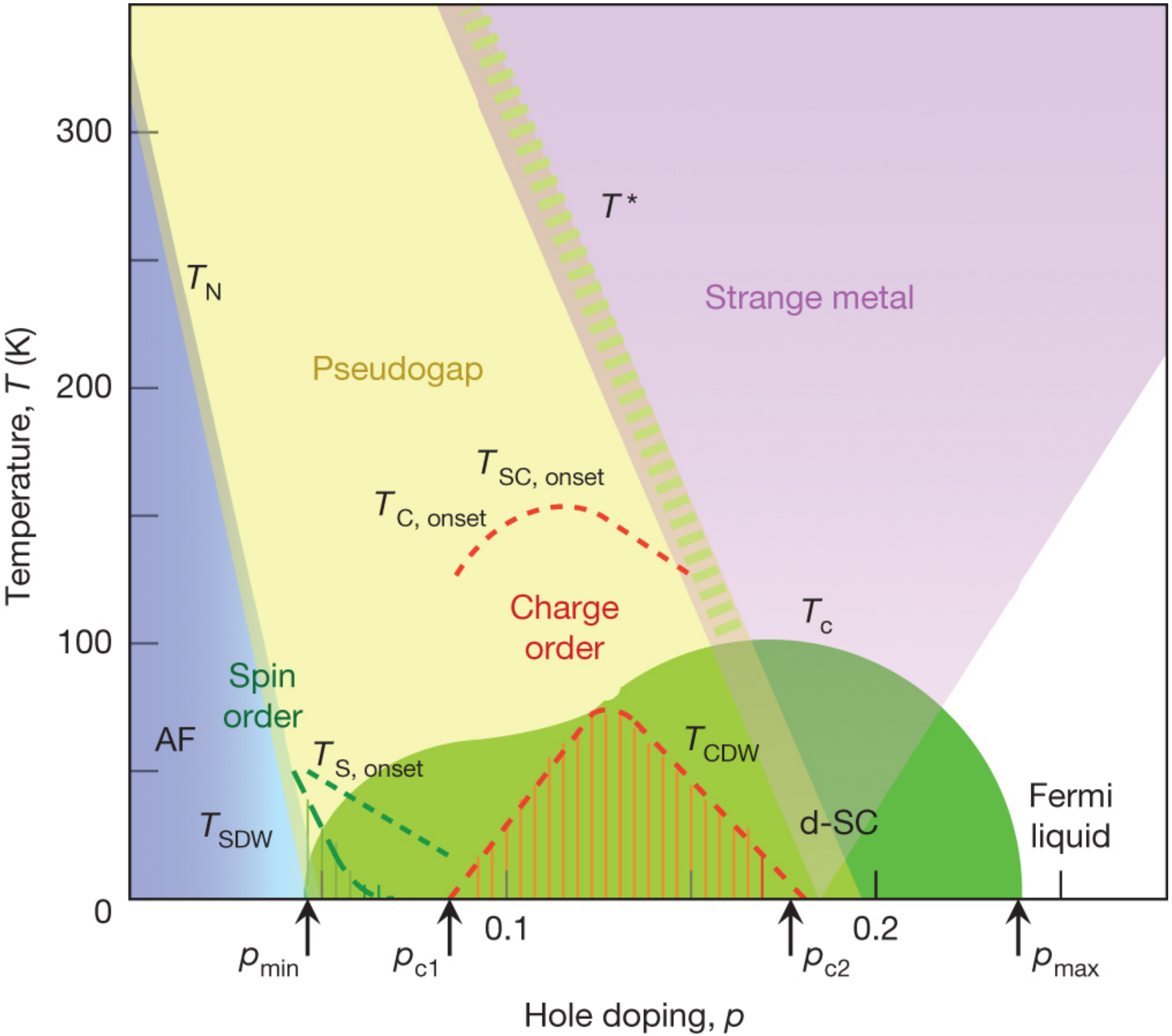
Ya-Hui Zhang

Alexander Nikolaenko

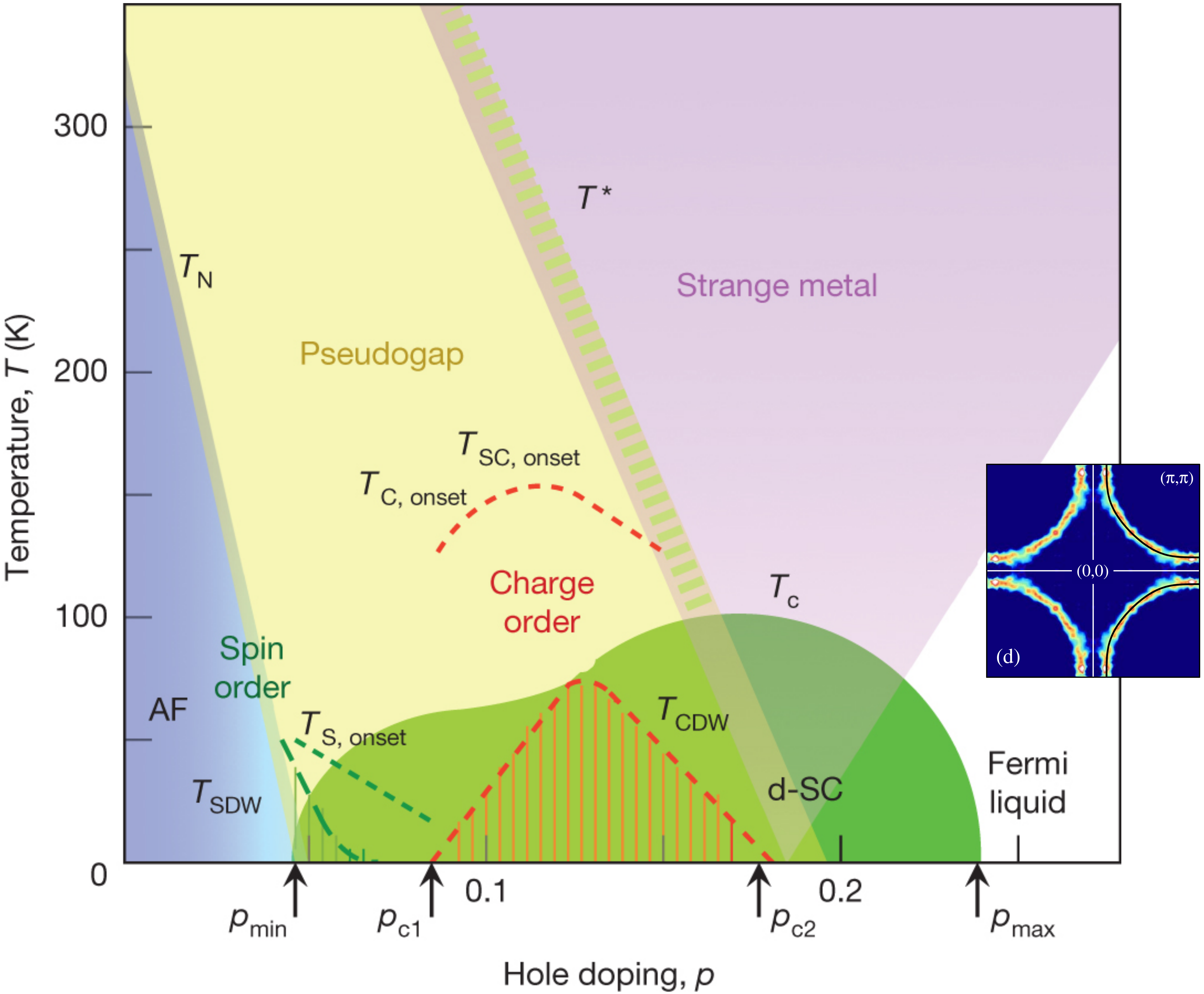
Darshan Joshi

Jonas von Milczewski

B Keimer *et al.* *Nature* **518**, 179-186 (2015)

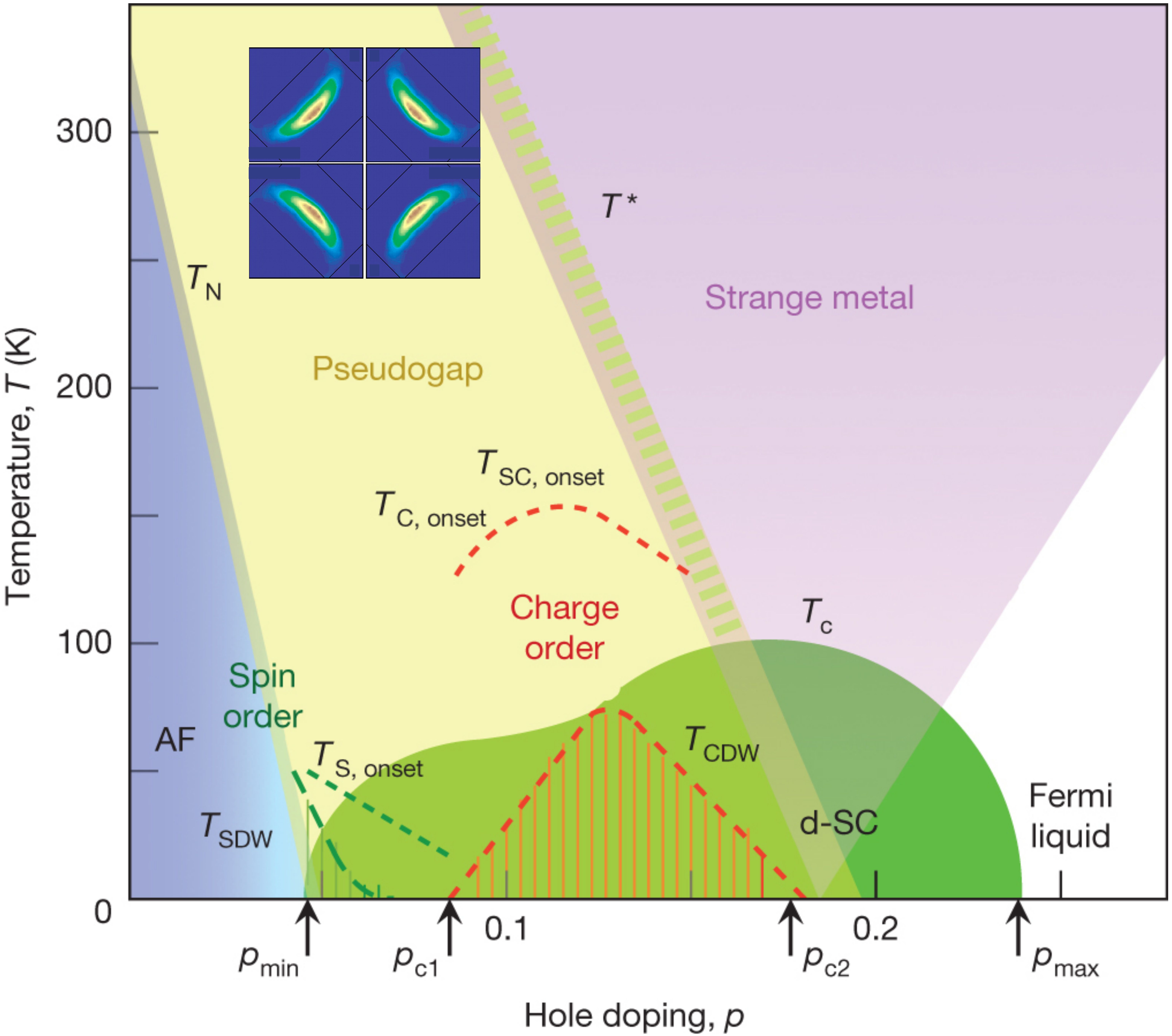


B Keimer *et al.* *Nature* **518**, 179-186 (2015)



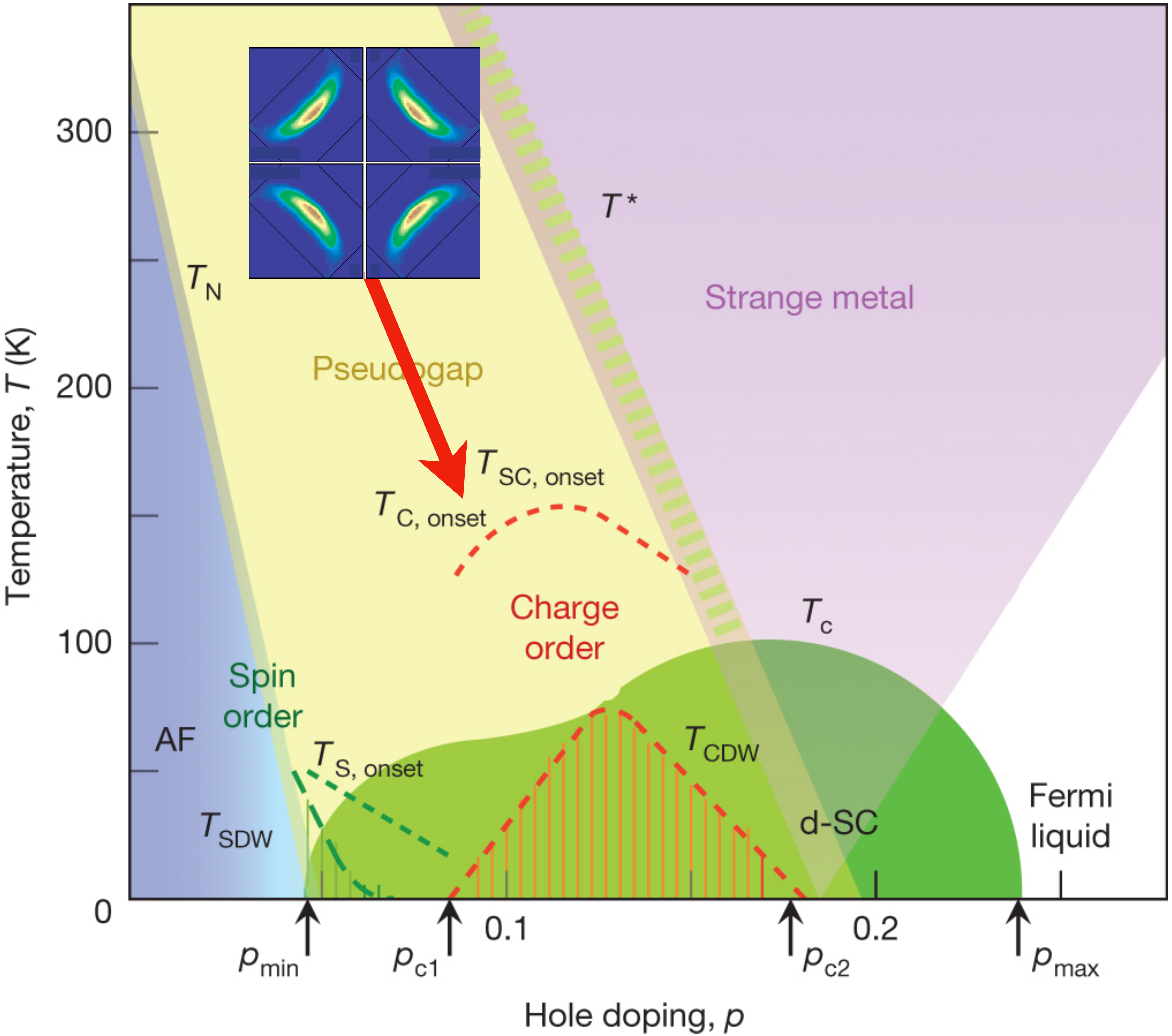
Fermi liquid
in the
overdoped metal

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



“Pseudogap metal” with “Fermi arcs”

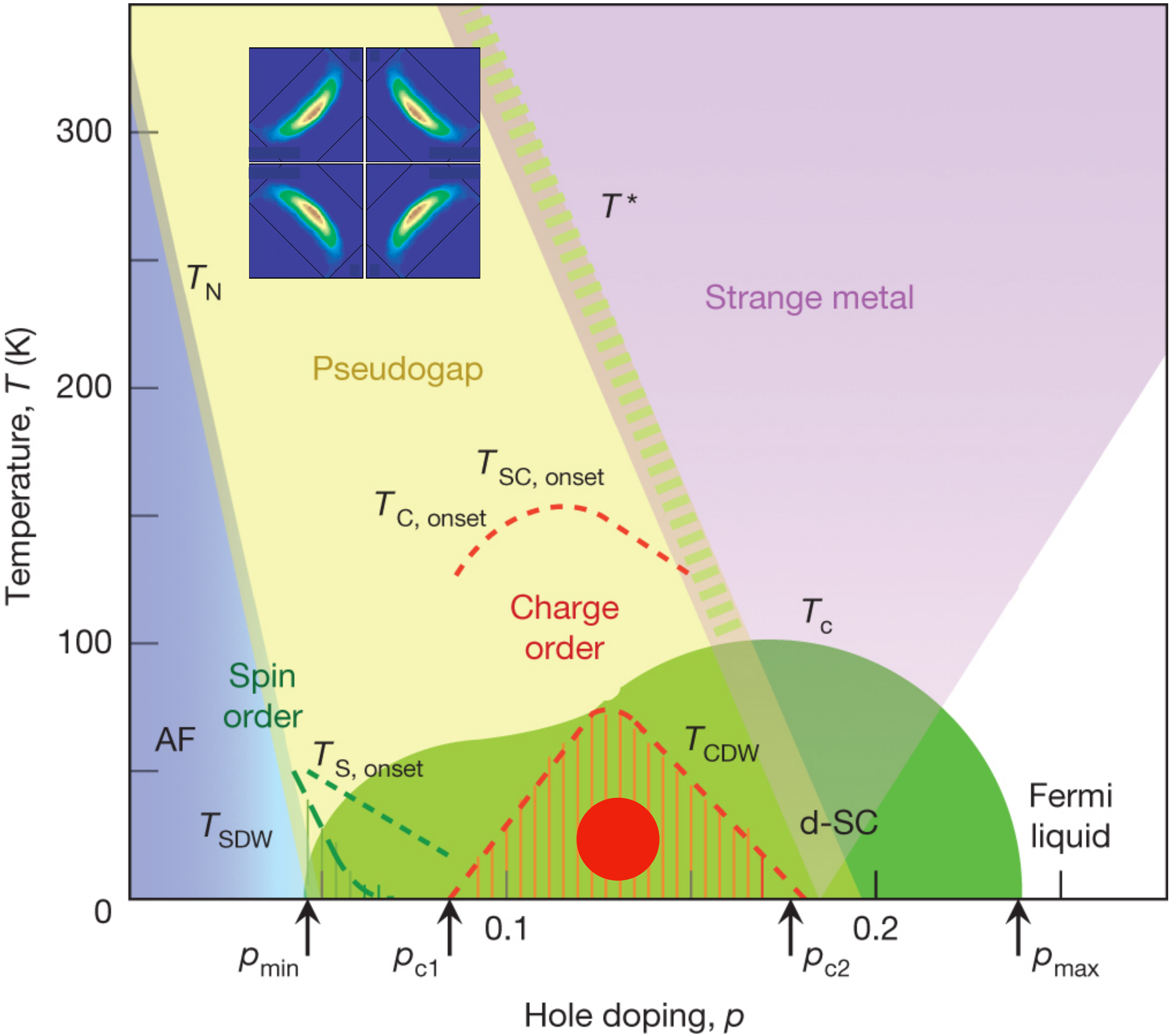
B Keimer *et al.* *Nature* **518**, 179-186 (2015)



Needed: a theory for the onset of charge order and d-wave superconductivity from the pseudogap metal.

Why are T_c and $T_{\rm CDW}$ about the same?

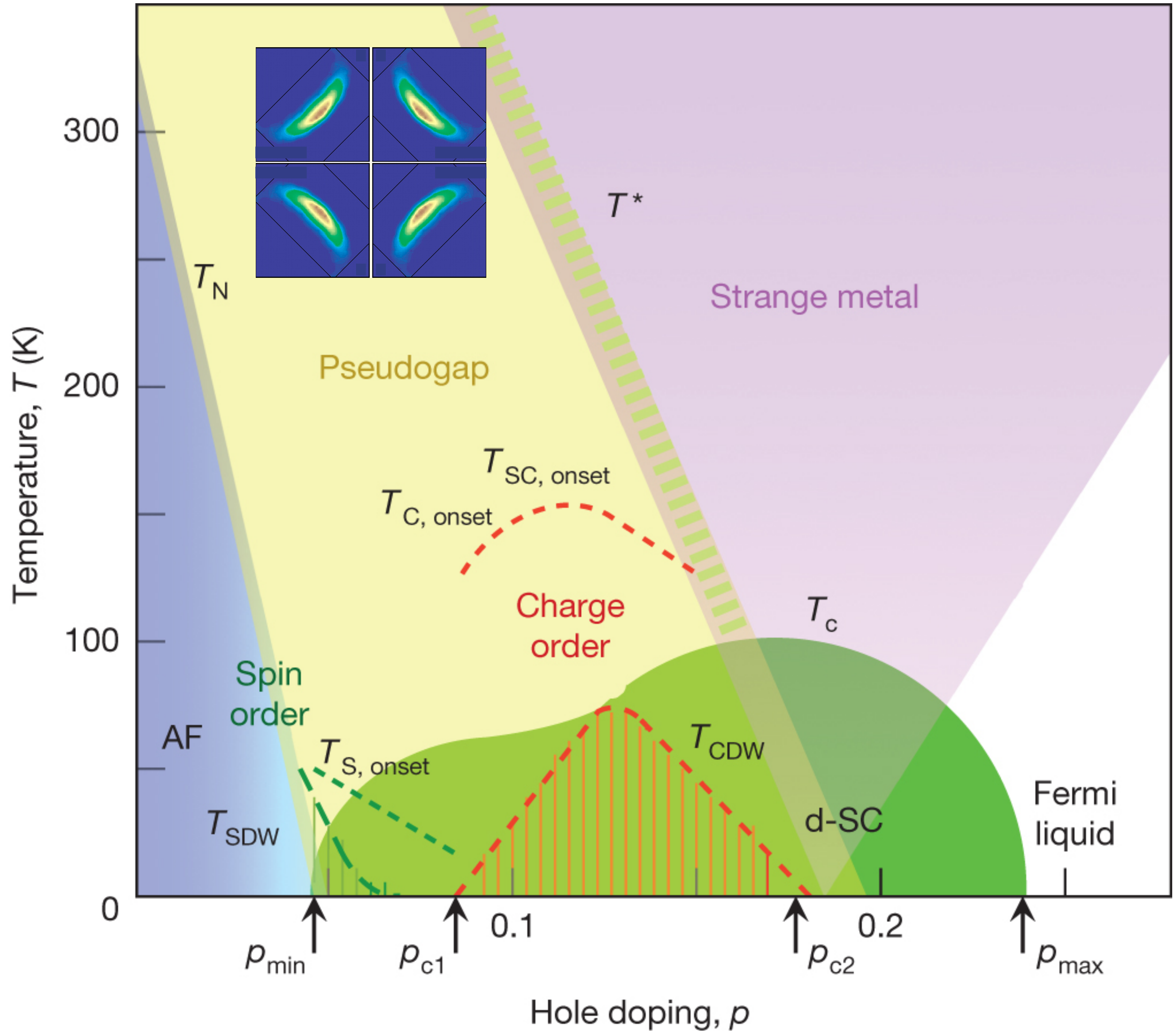
B Keimer *et al.* *Nature* **518**, 179-186 (2015)

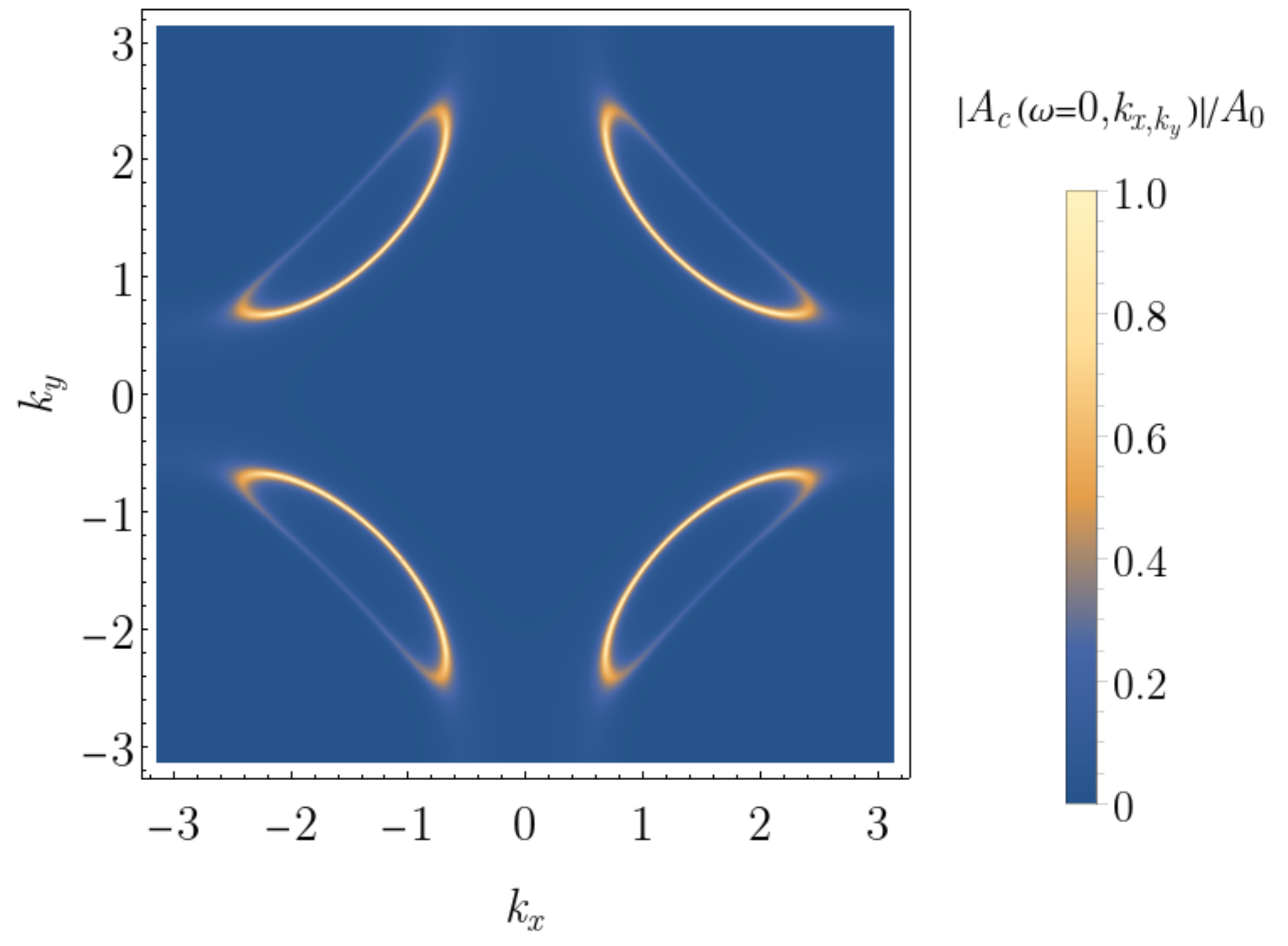


Quantum oscillations in the CDW phase at low T show only a single electron pocket.

This cannot be obtained in the theory of CDWs in a Fermi liquid.

B Keimer *et al.* *Nature* **518**, 179-186 (2015)





Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

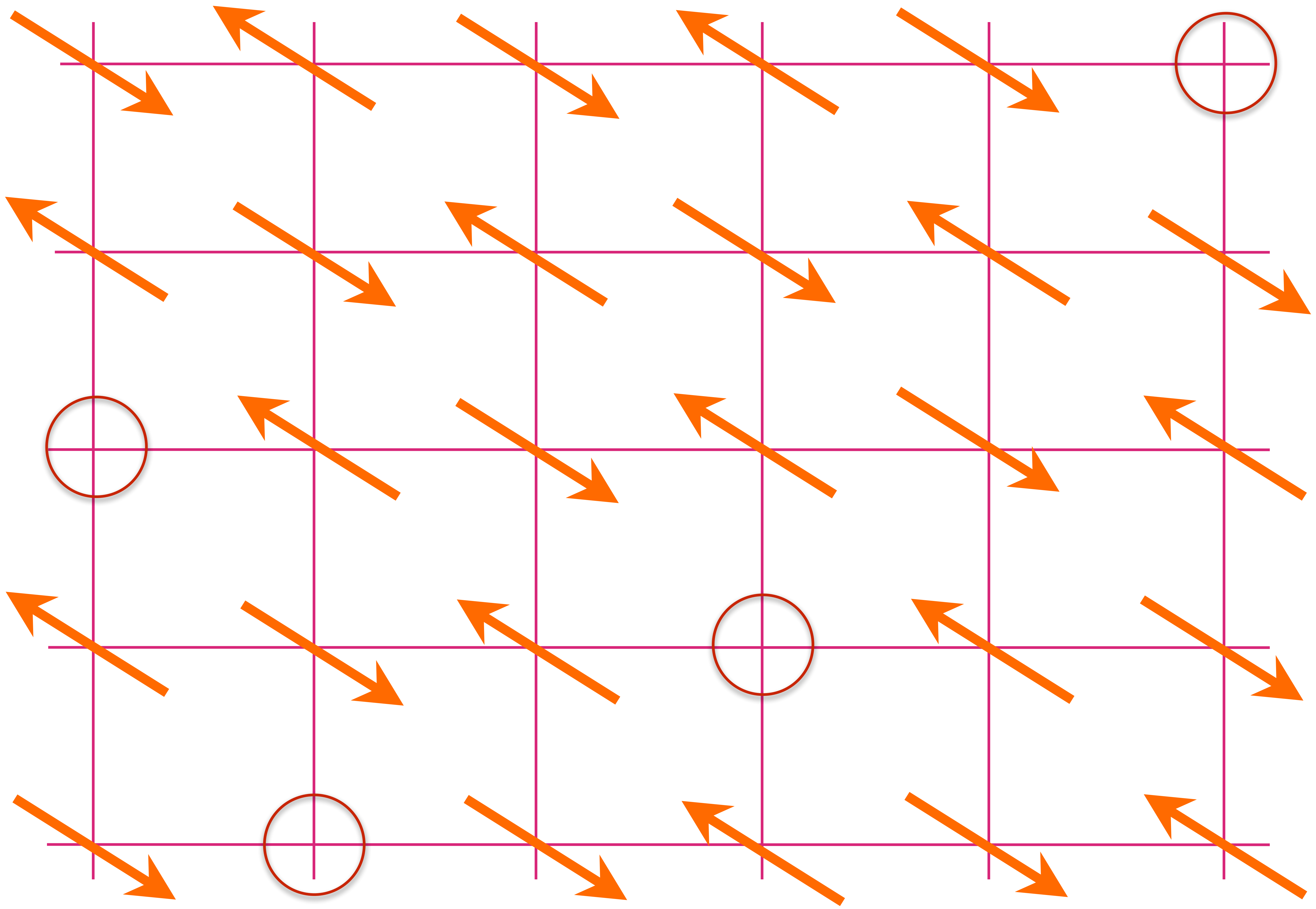
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. Sachdev, PRB **105**, 075146 (2022

Ancilla theory of the pseudogap metal

Hole pocket Fermi surfaces
of size p with
charge e, spin-1/2 quasiparticles
$+$
square lattice spin liquid.

FL*: Spin liquid is *required* because the Fermi surface does not enclose the Luttinger volume $(1 + p)$.

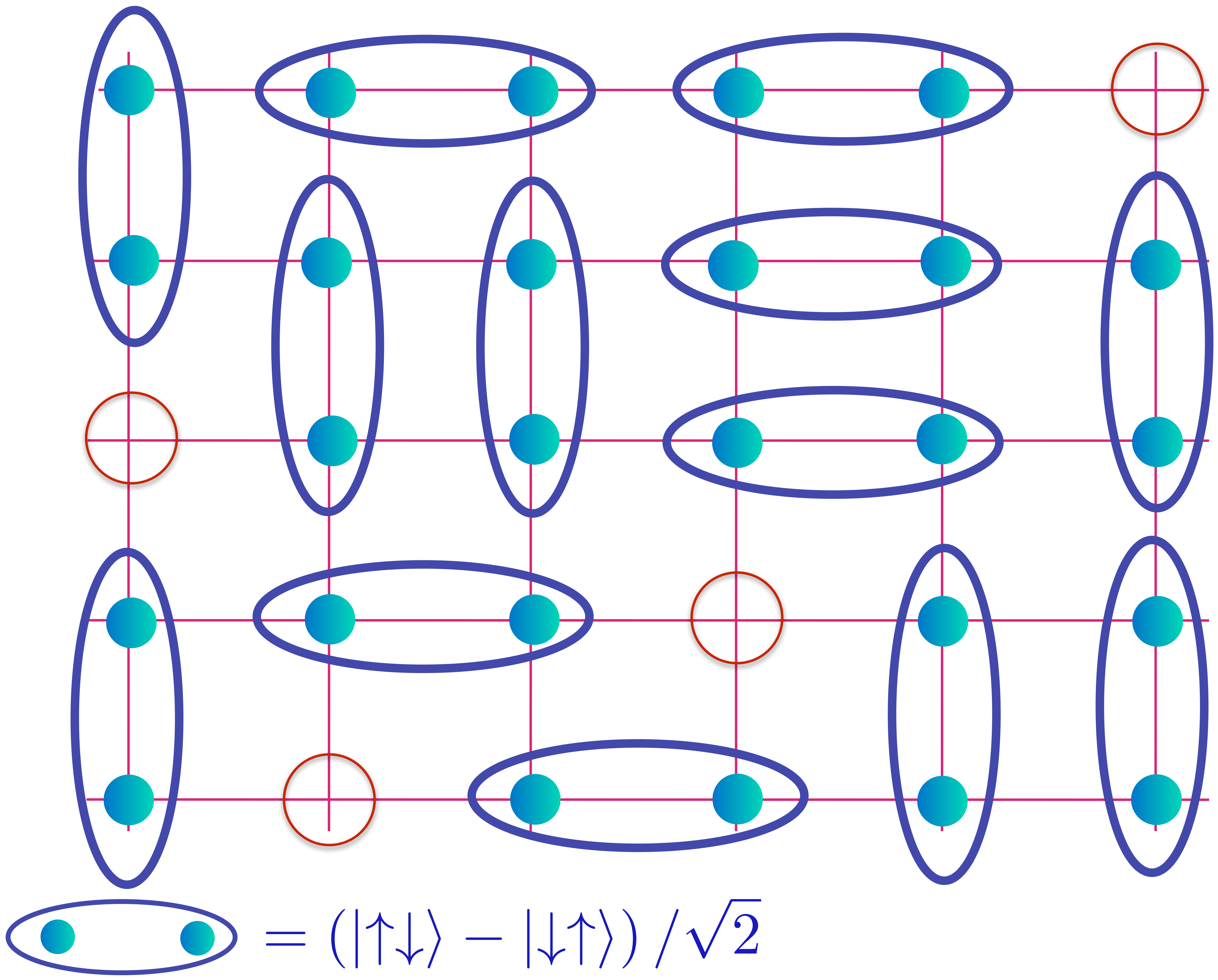
Earlier approach to FL* in a ***one-band*** model



Anti-ferromagnet with p holes per square

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

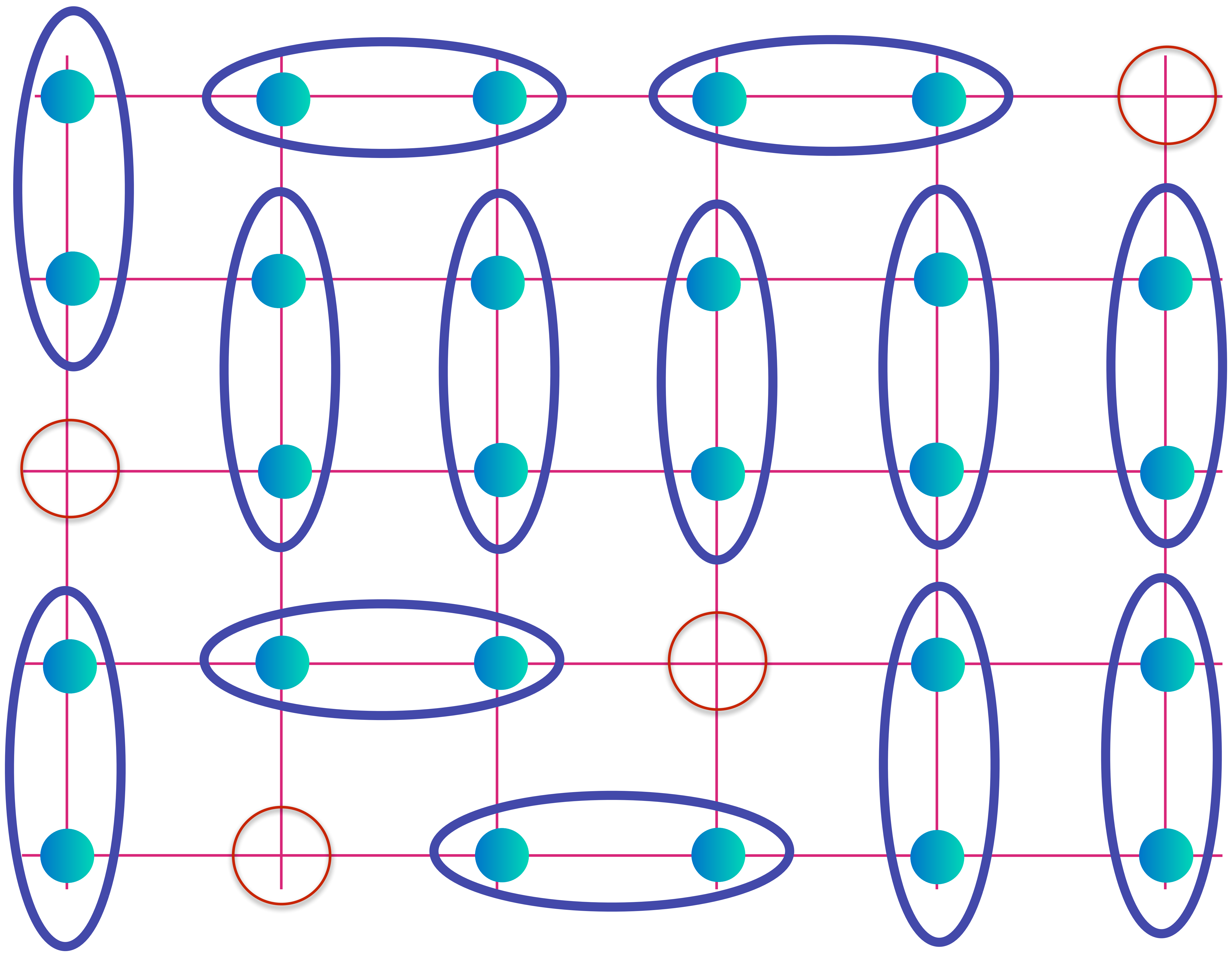


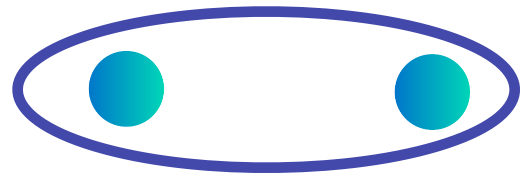
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid with density p of spinless, charge +e "holons".

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

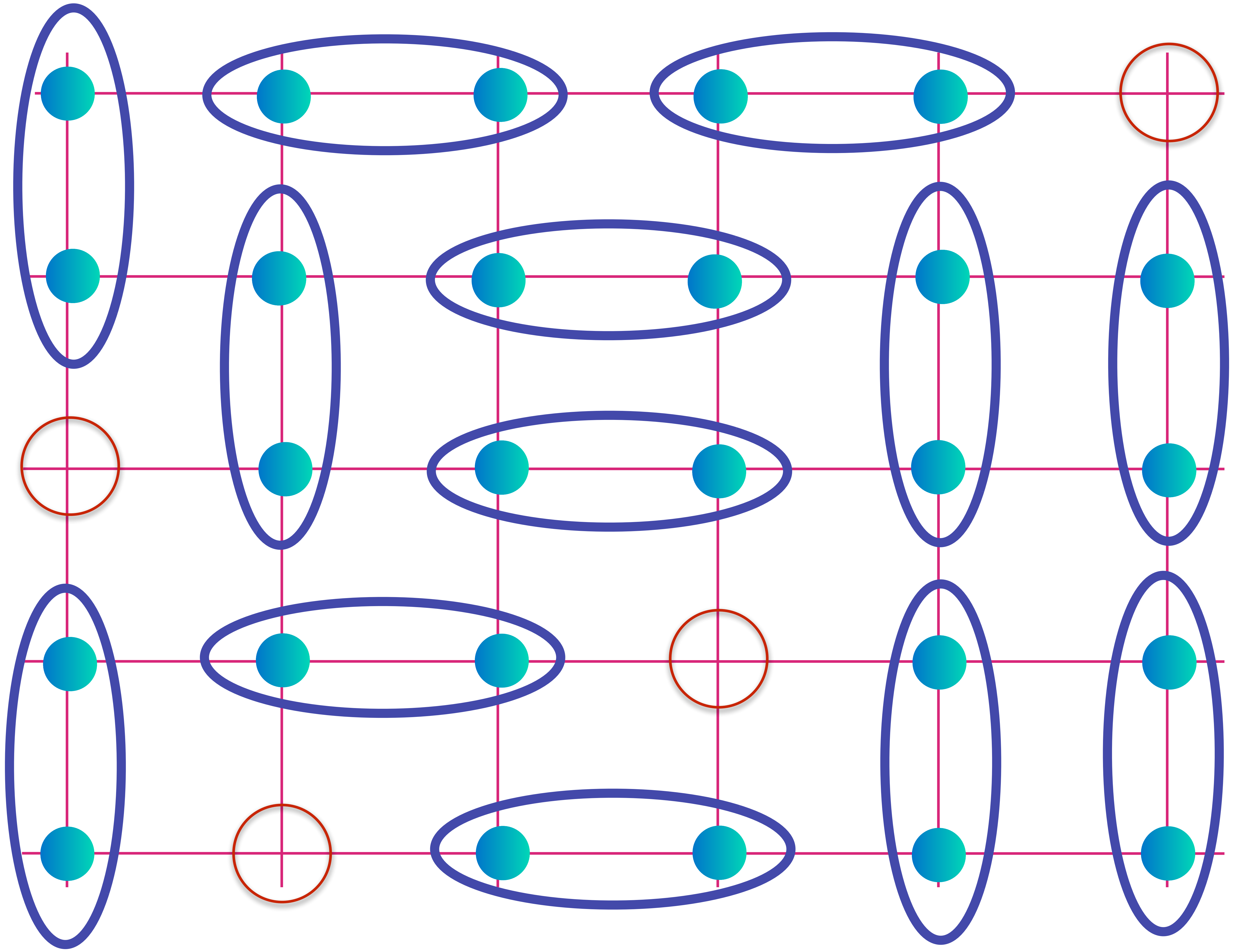


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

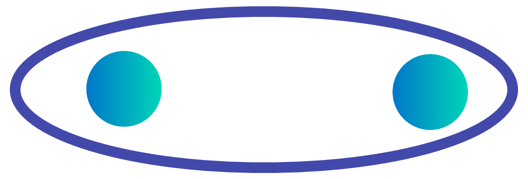
Spin liquid with density p of spinless, charge +e "holons".

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

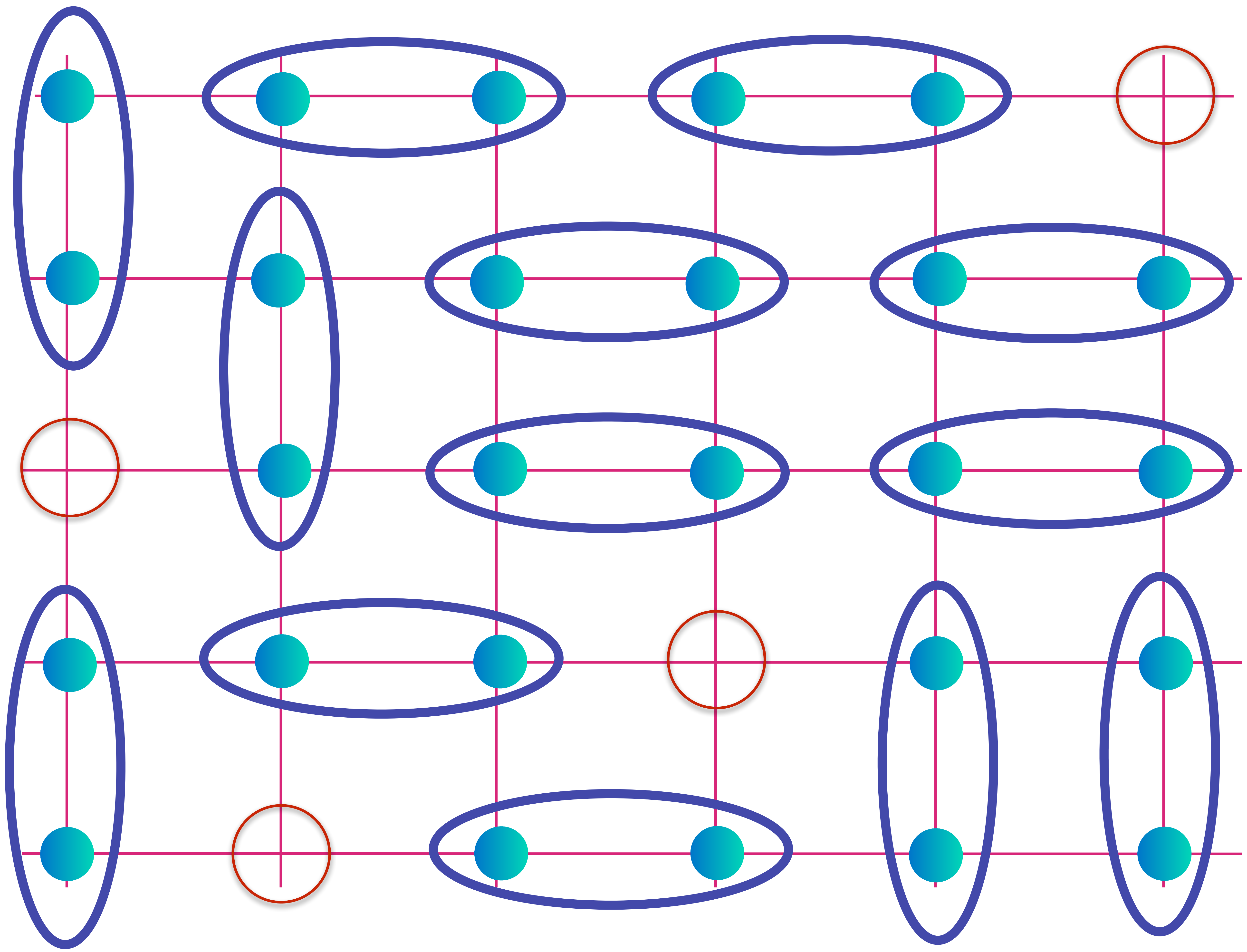


Spin liquid with density p of spinless, charge +e “holons”.

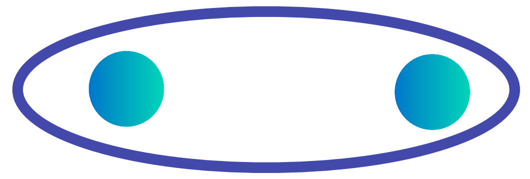
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



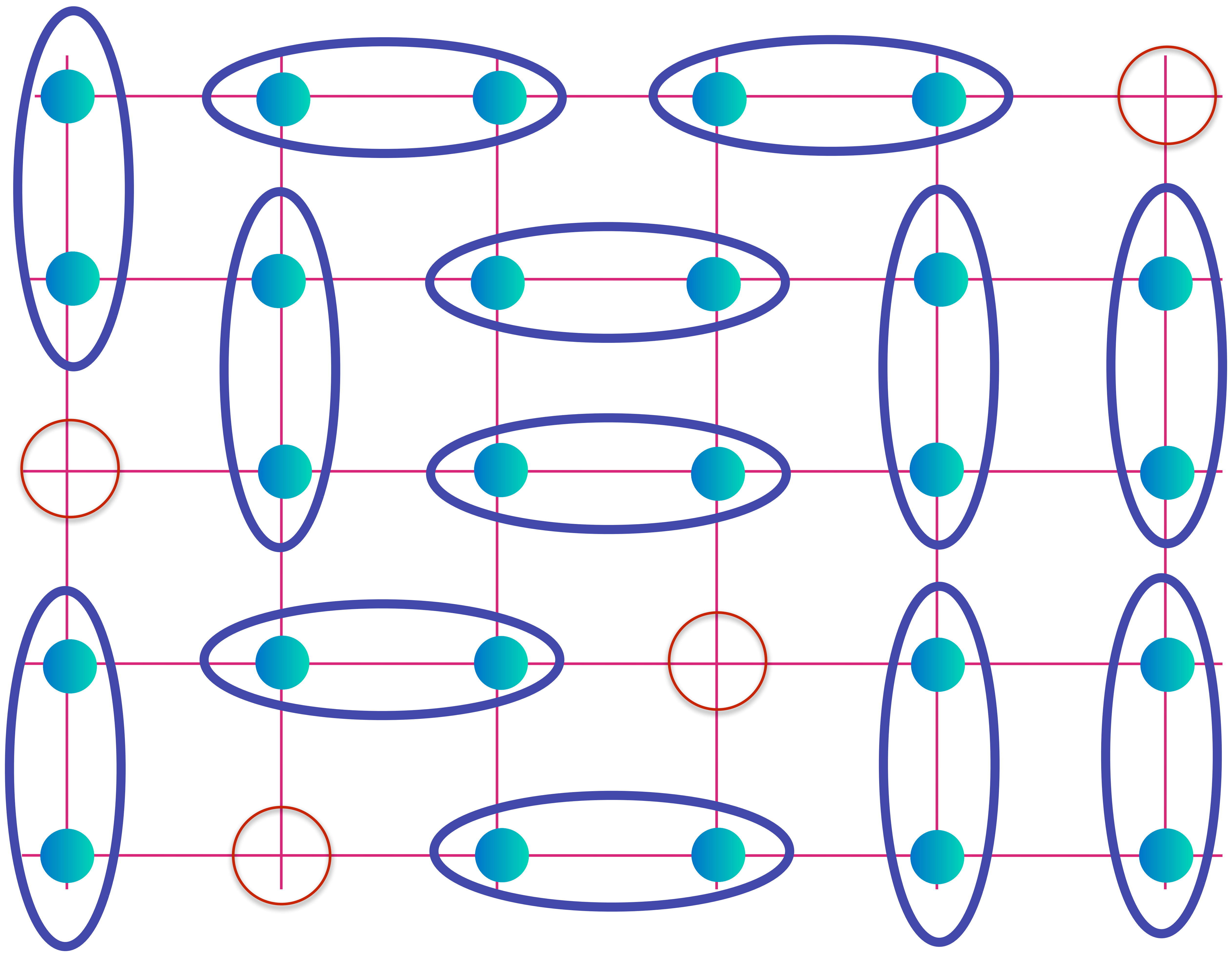
Spin liquid with density p of spinless, charge +e "holons".

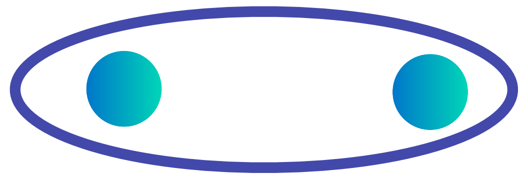


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

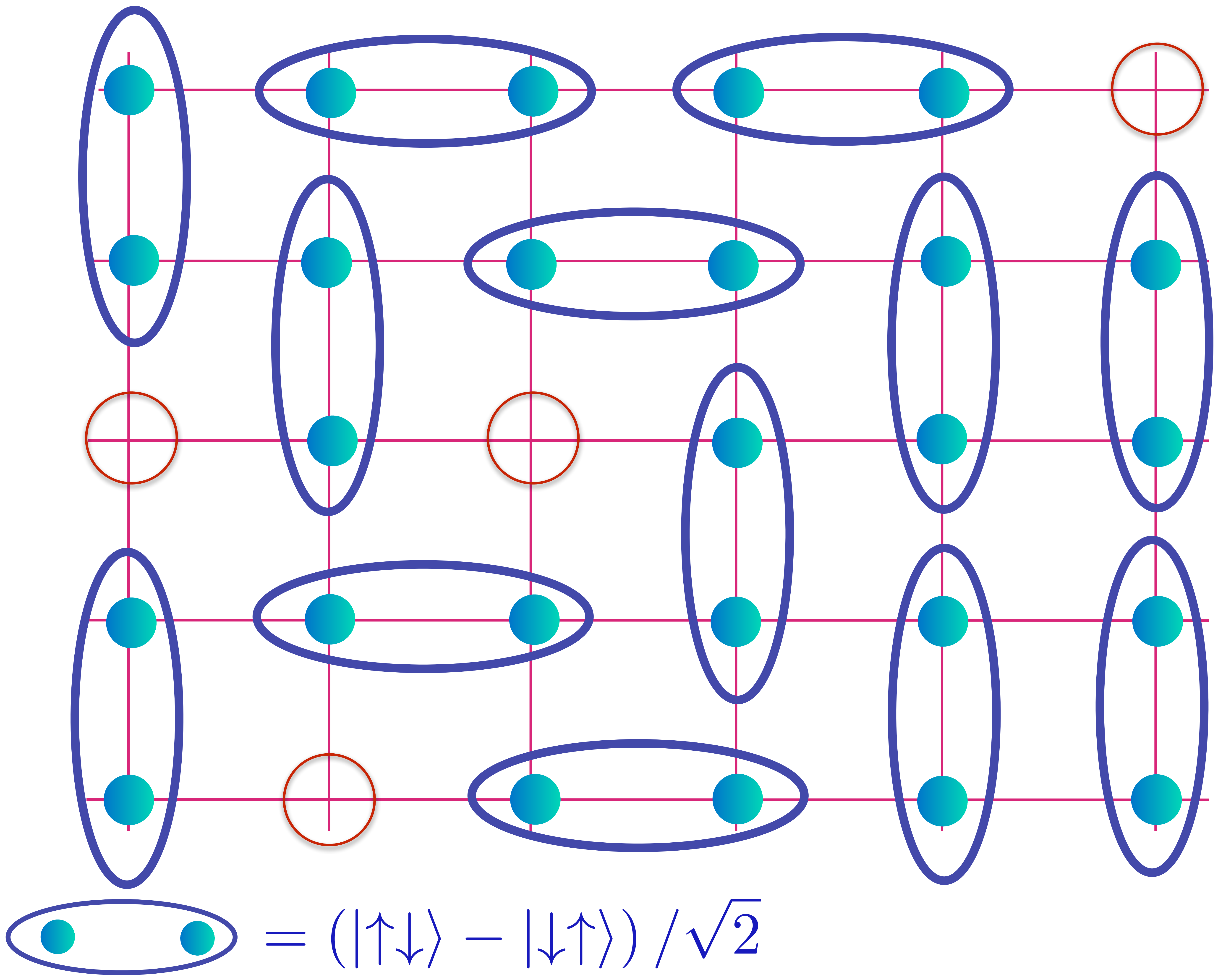


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Spin liquid with density p of spinless, charge +e "holons".

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

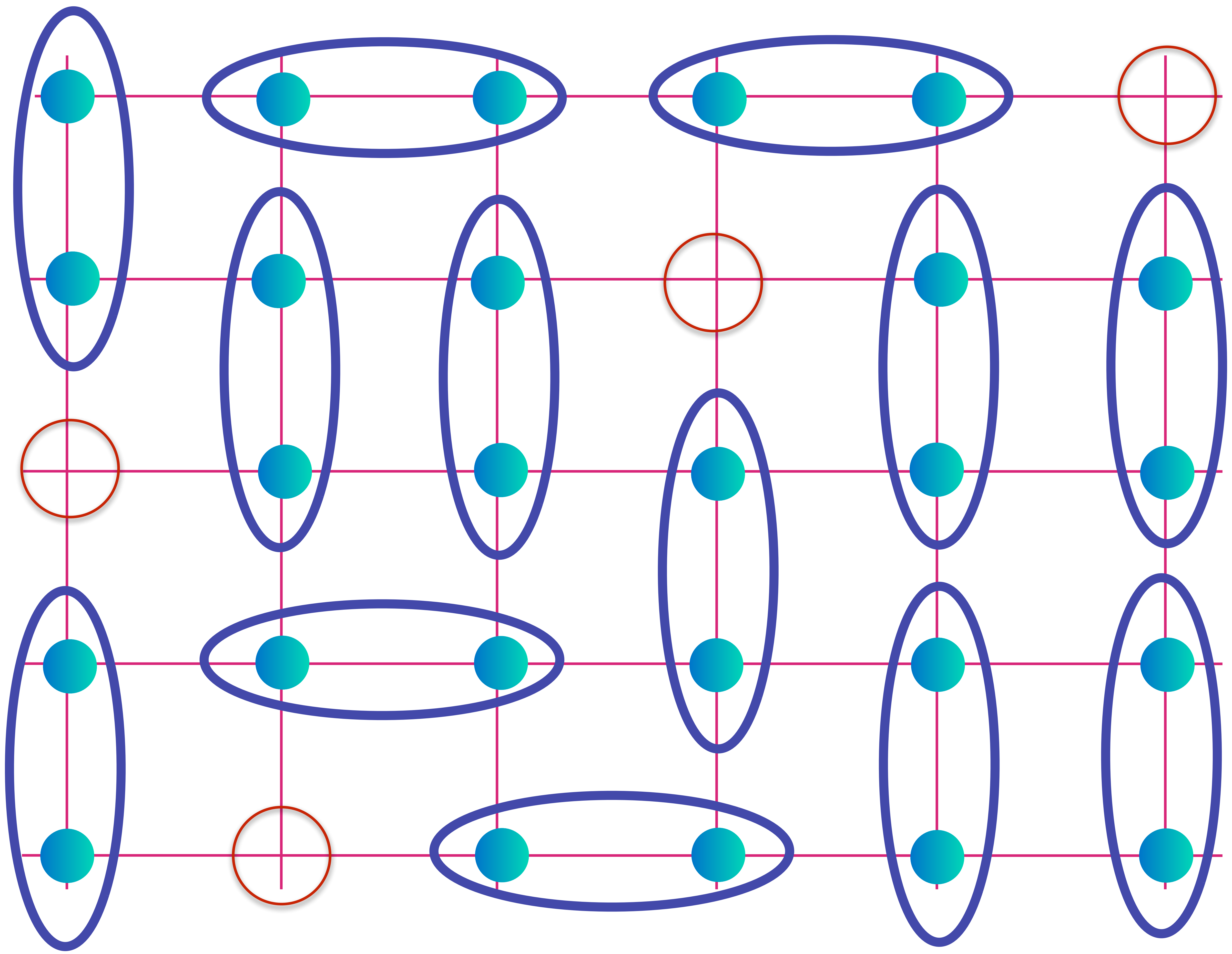


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

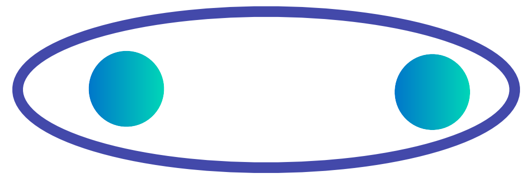
Spin liquid with density p of spinless, charge +e "holons".

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

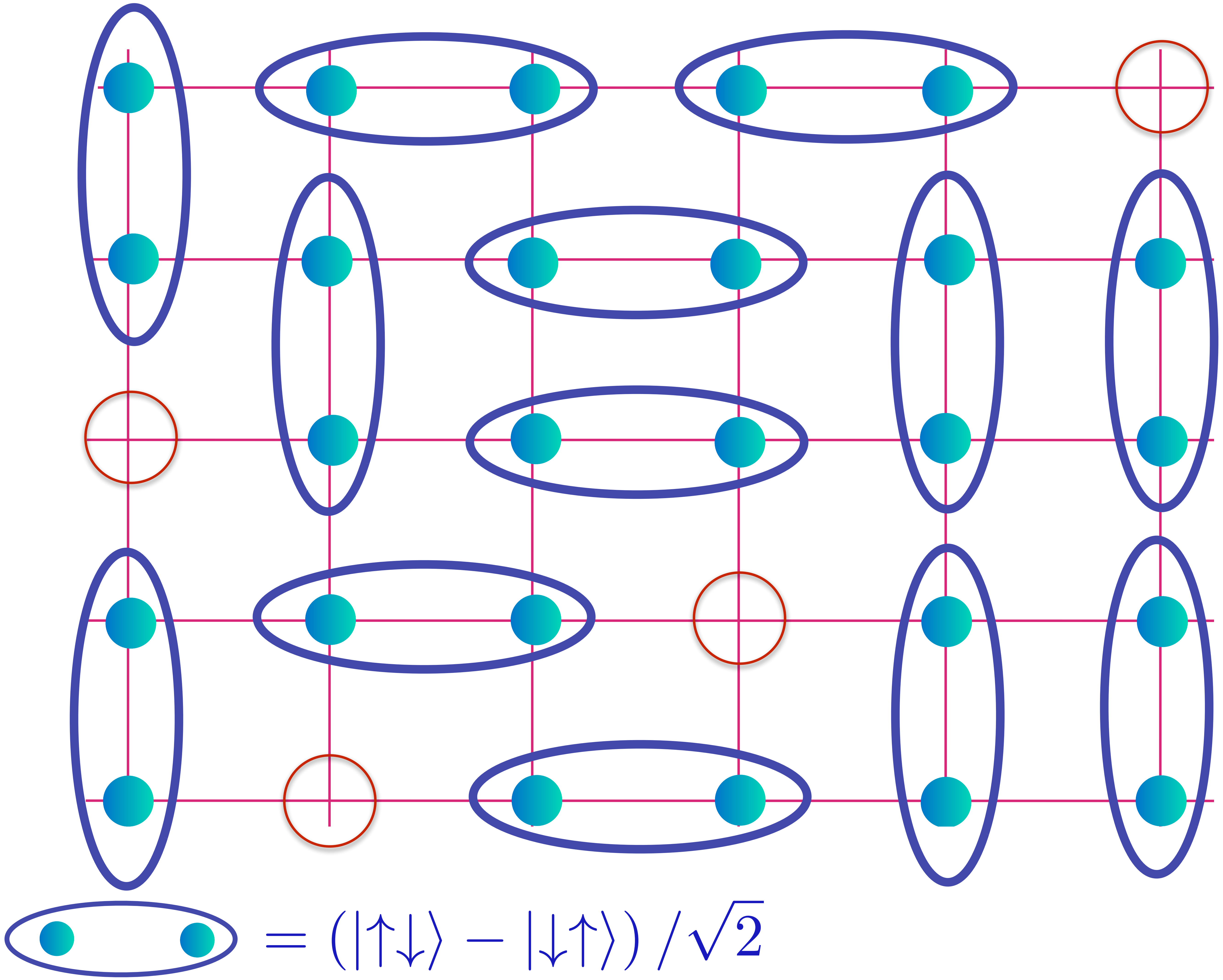


Spin liquid with density p of spinless, charge +e "holons".

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

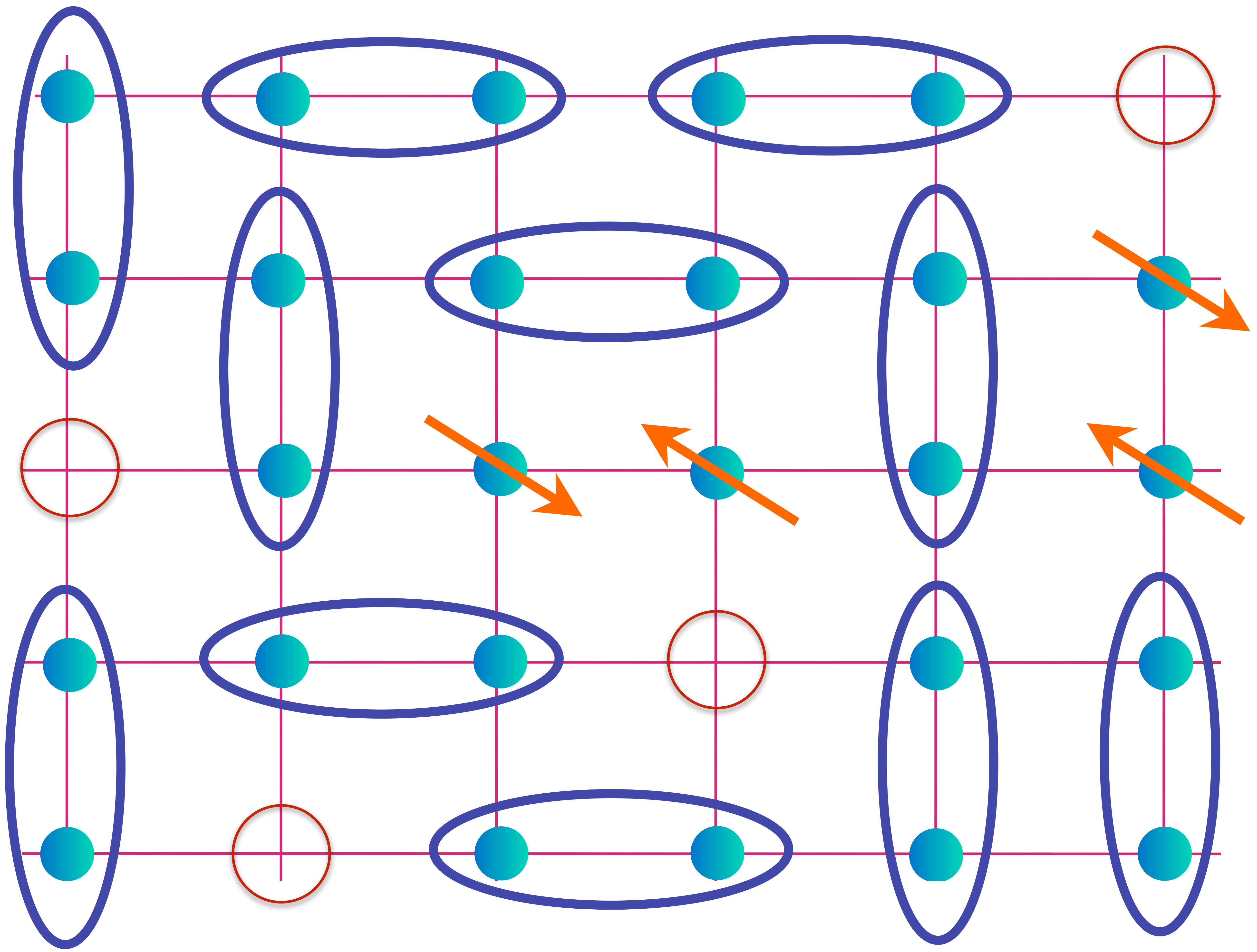


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

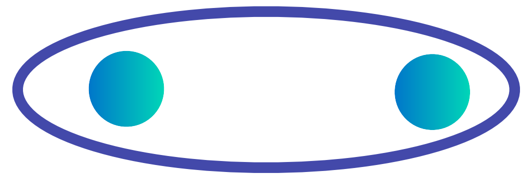
Spin liquid with density p of spinless, charge +e "holons".

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

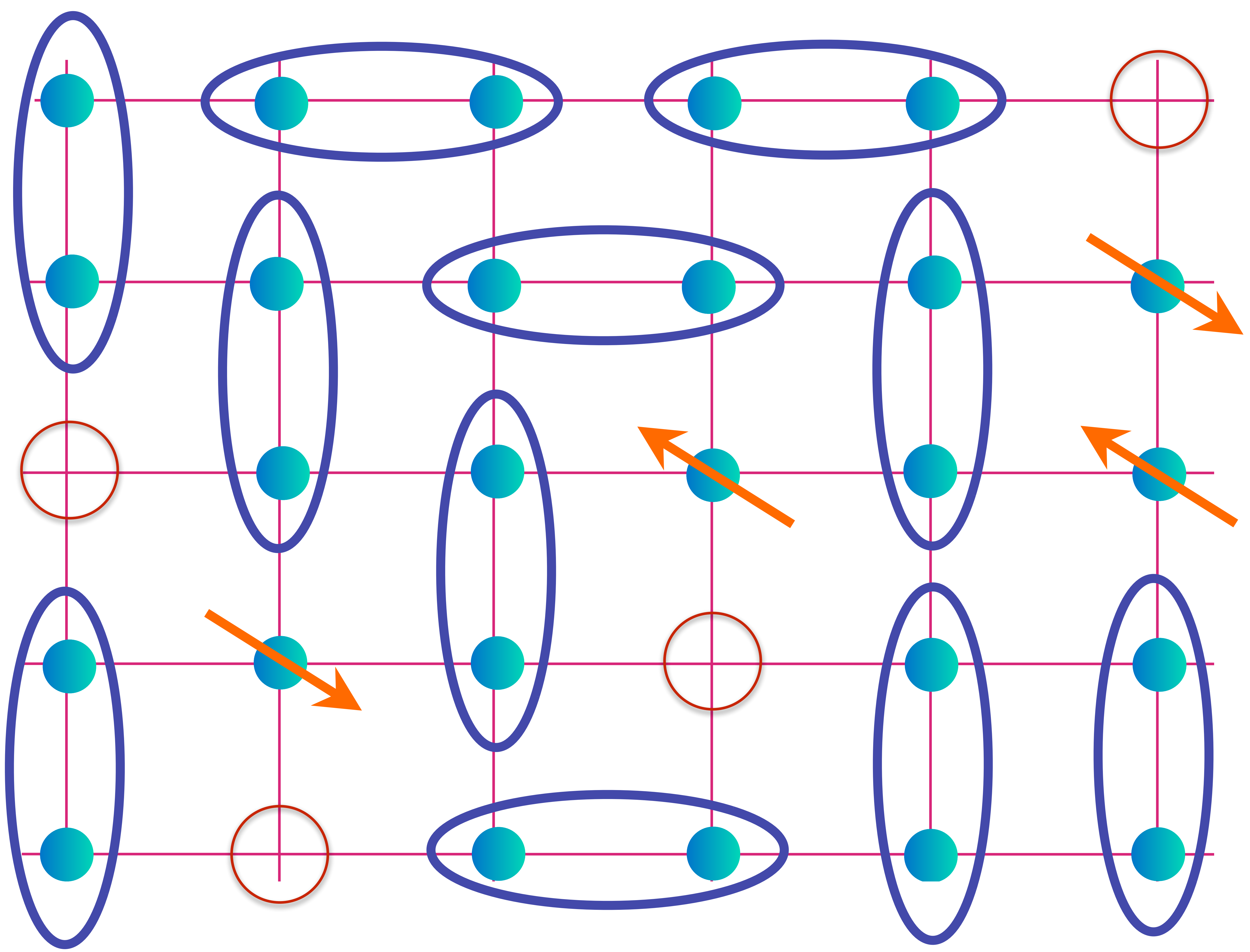


Spin liquid with density p of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".

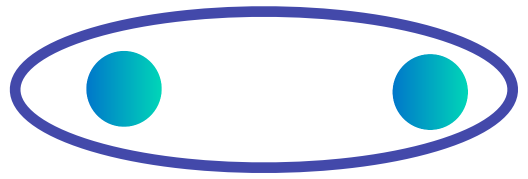
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



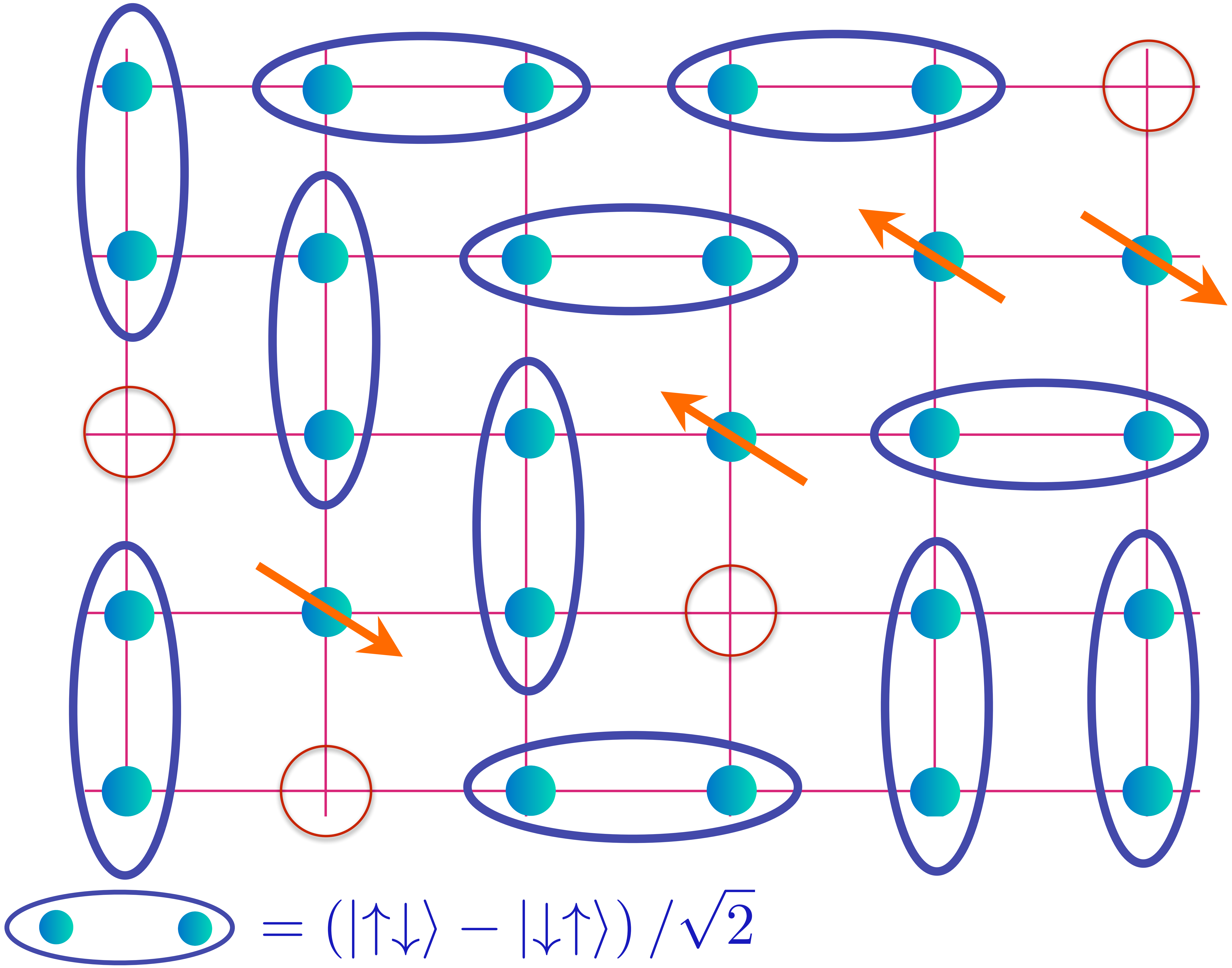
Spin liquid with density p of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

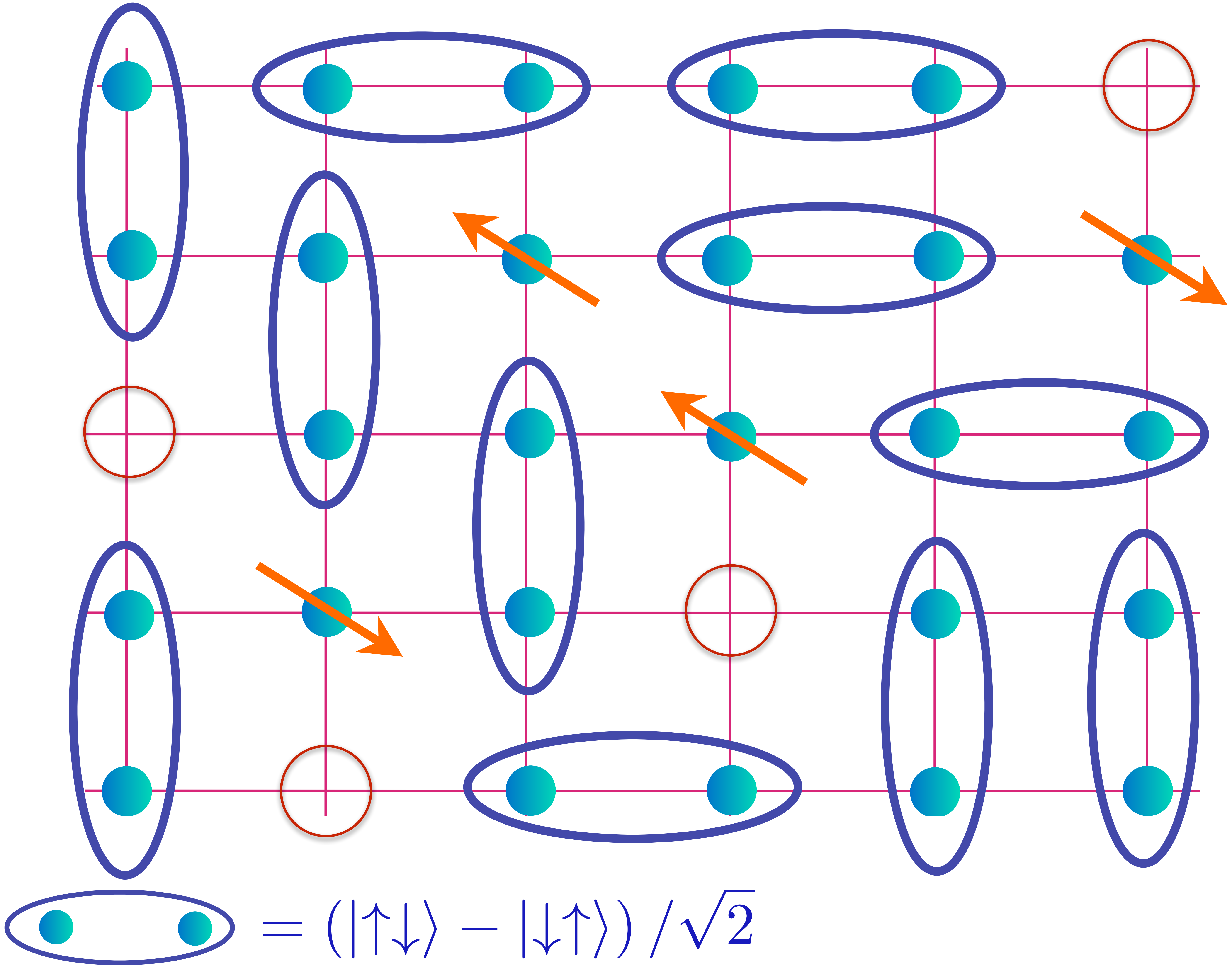


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Spin liquid with density p of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

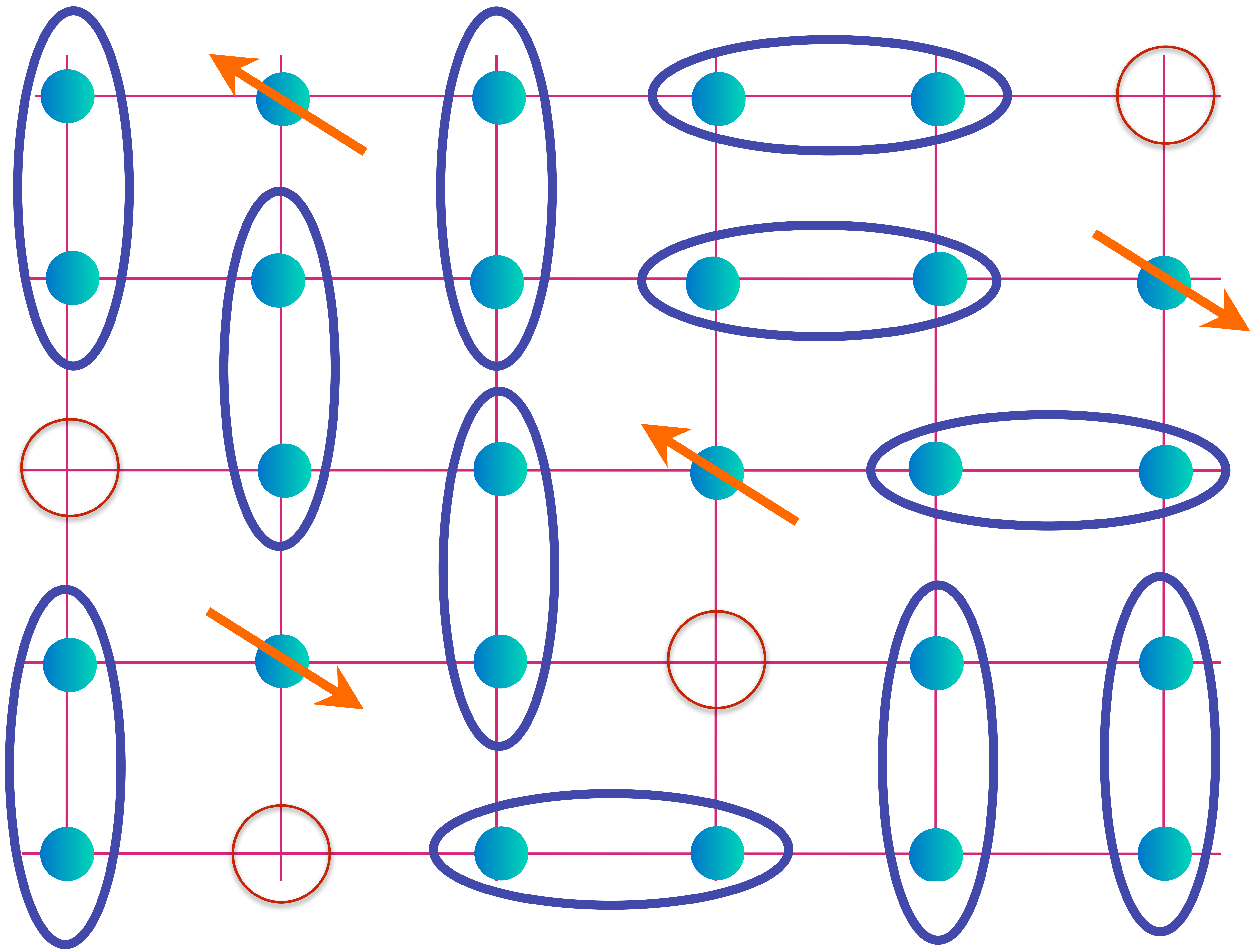


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

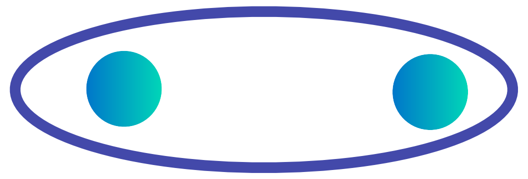
Spin liquid with density p of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



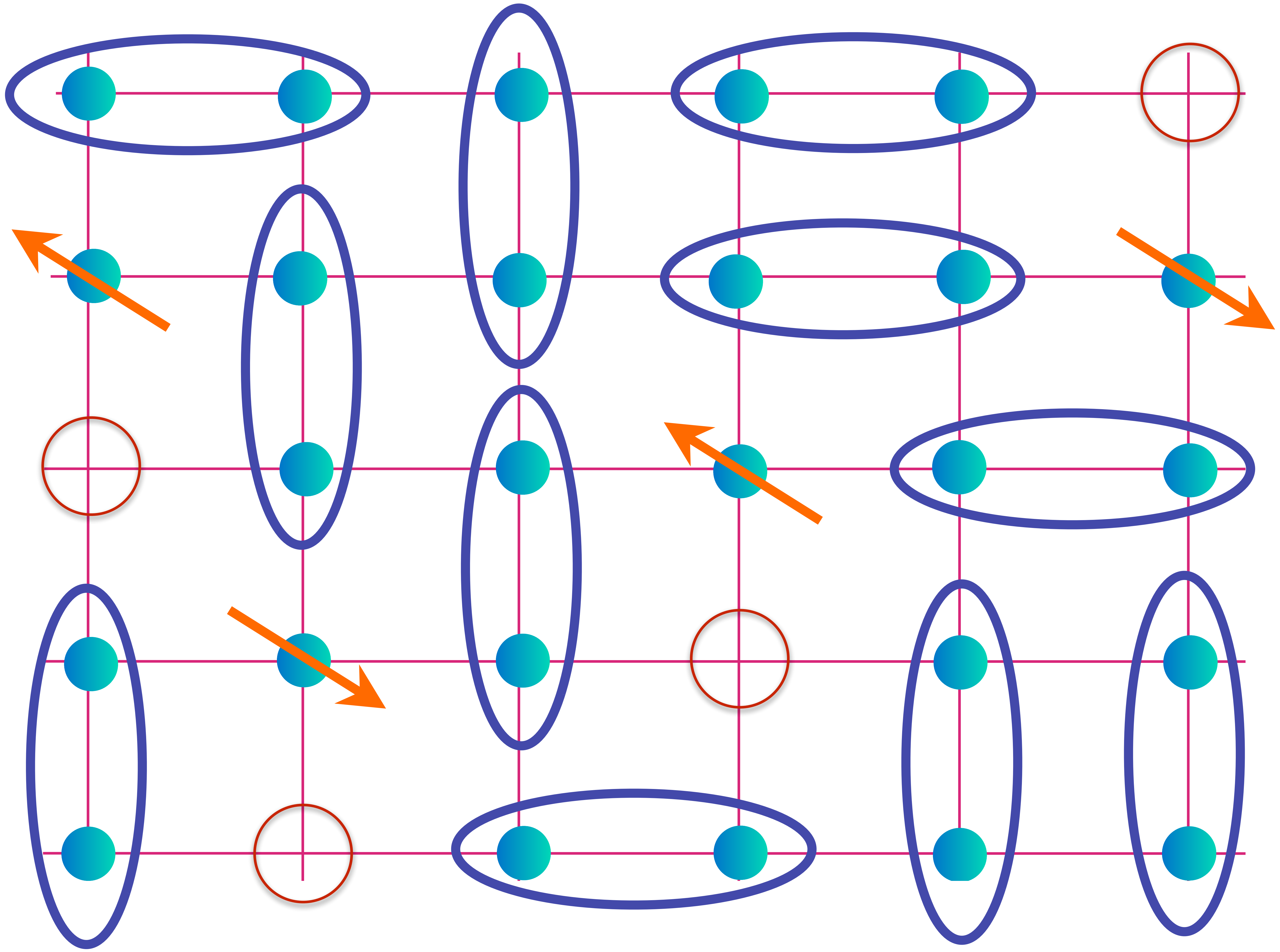
Spin liquid with density p of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".



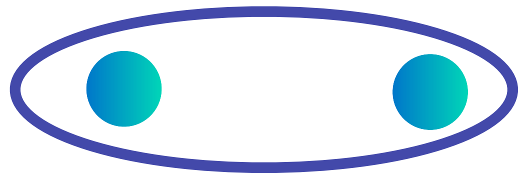
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$$

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



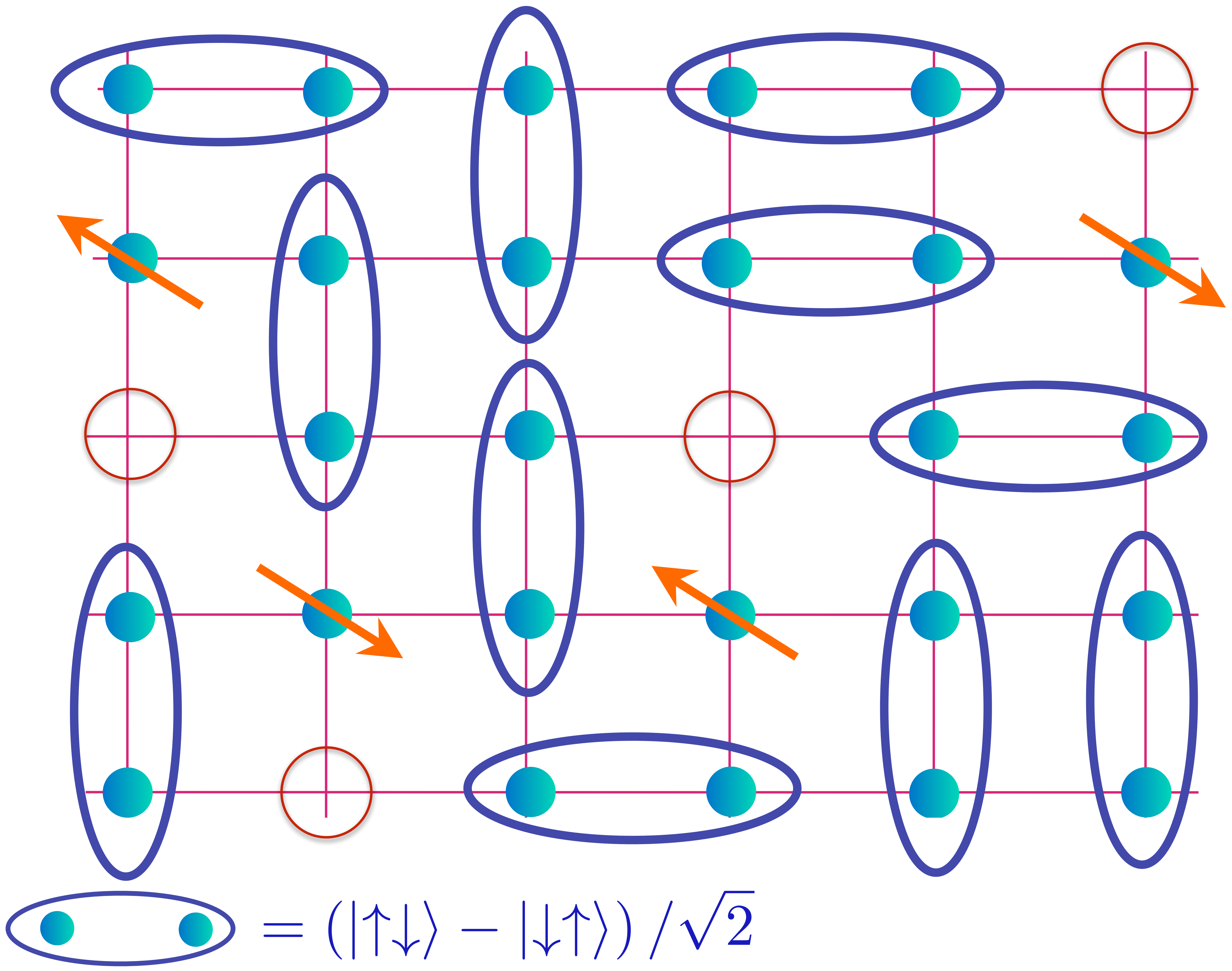
Spin liquid with density p of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

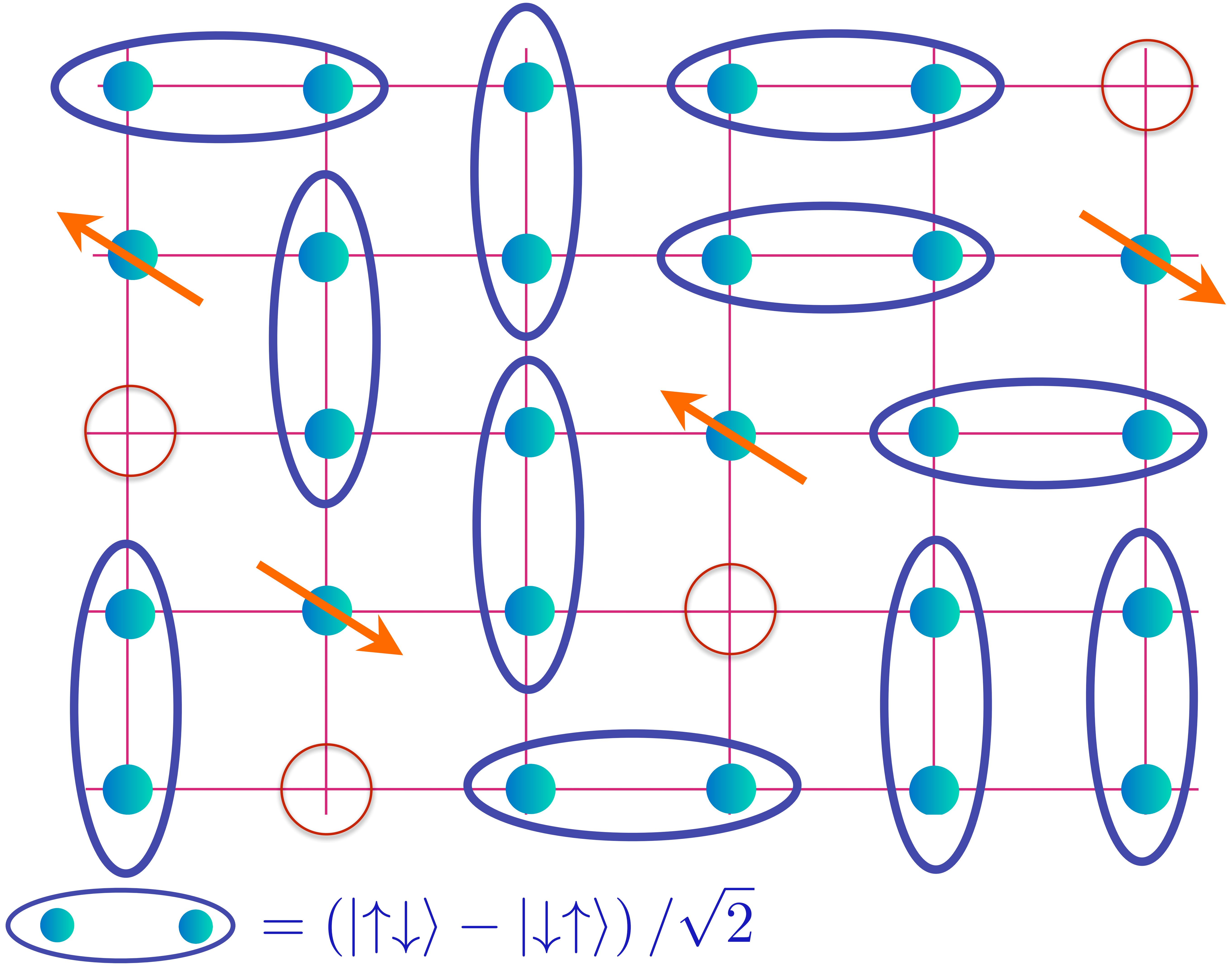


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid with density p of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)

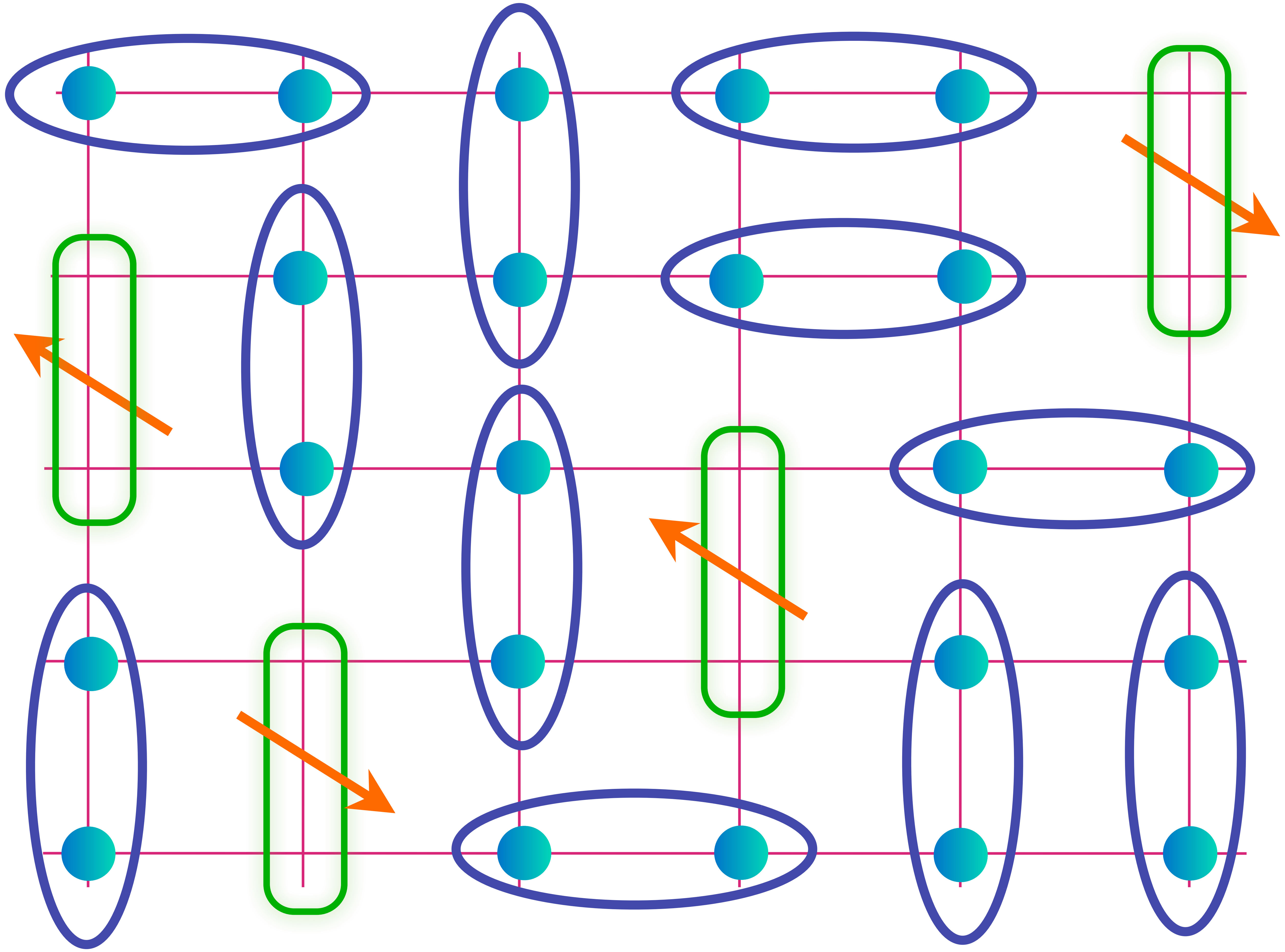


Spin liquid with density p of spinless, charge +e "holons" and charge 0, spin-1/2 "spinons".

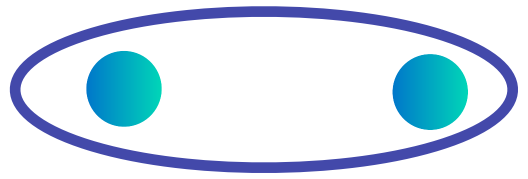
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

Earlier approach to FL* in a ***one-band*** model

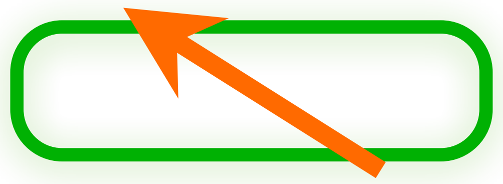
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin



$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$



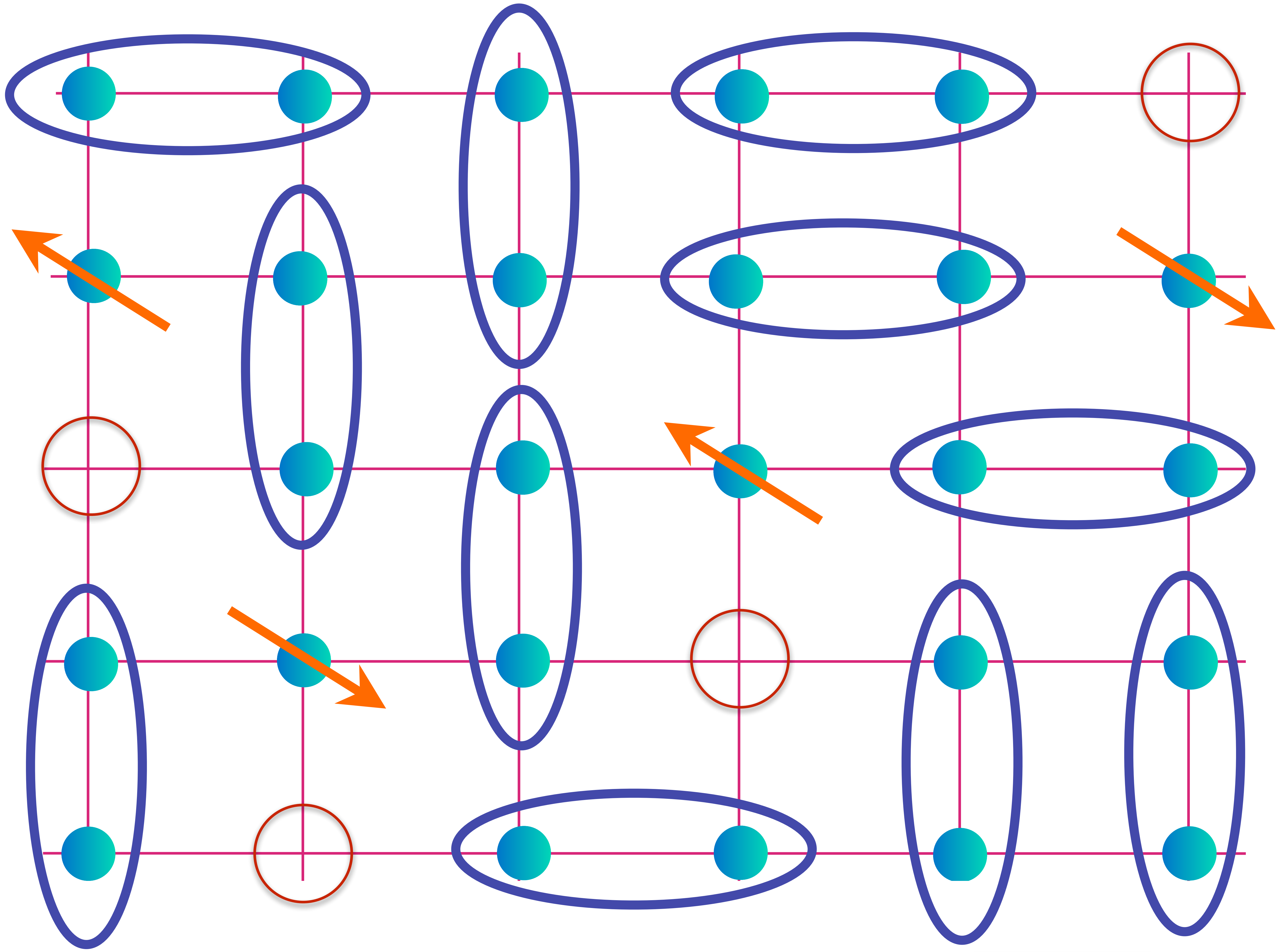
$$= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

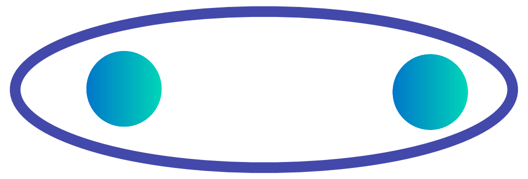
Earlier approach to FL* in a ***one-band*** model

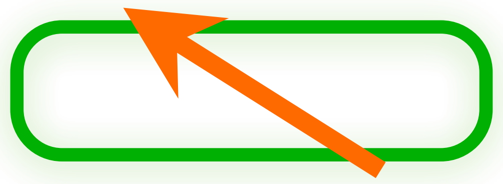
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

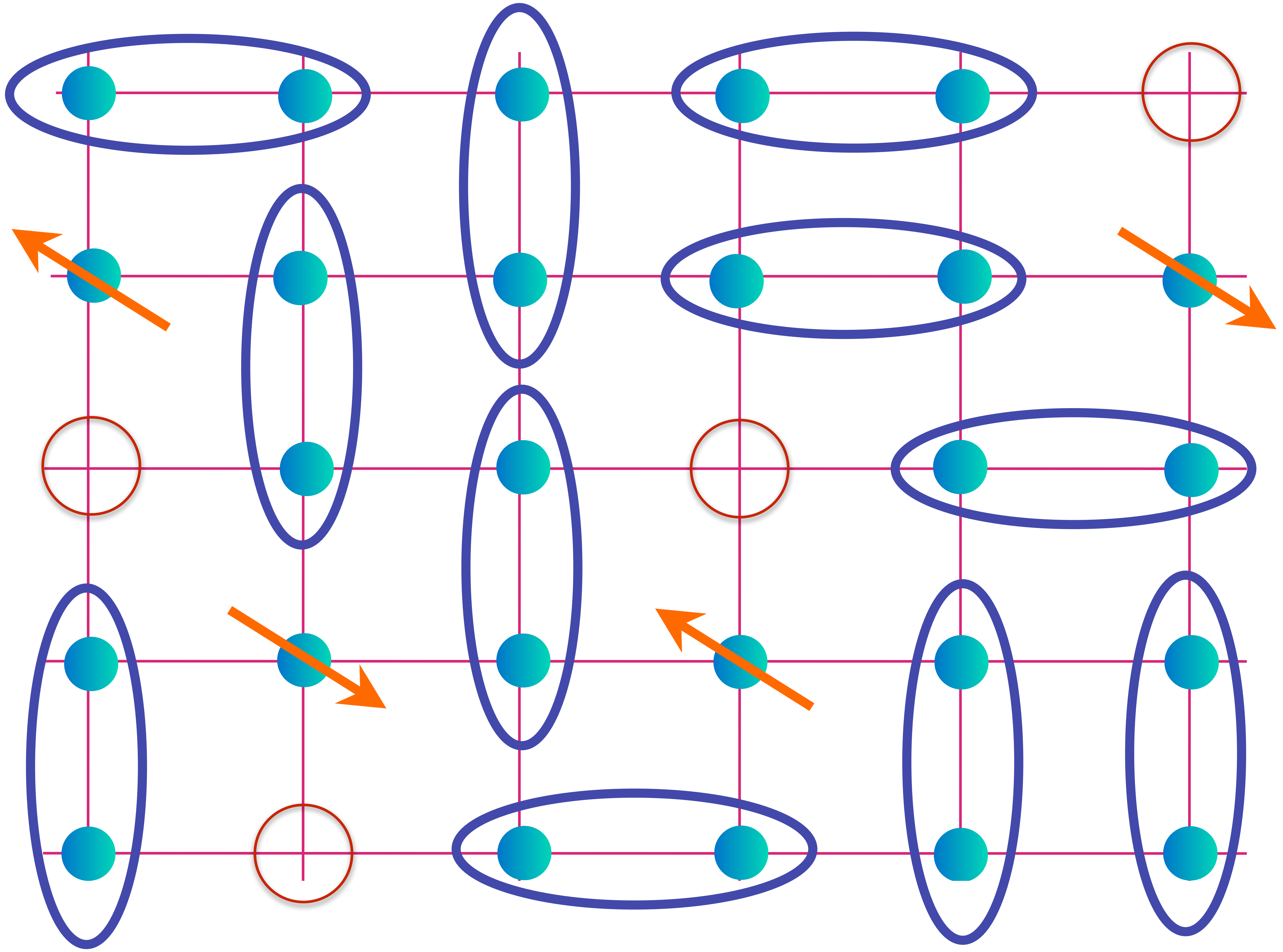
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

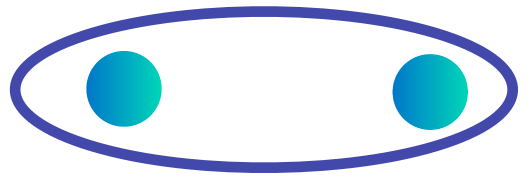
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

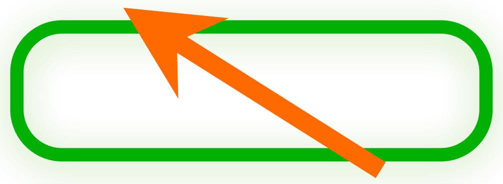
Earlier approach to FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

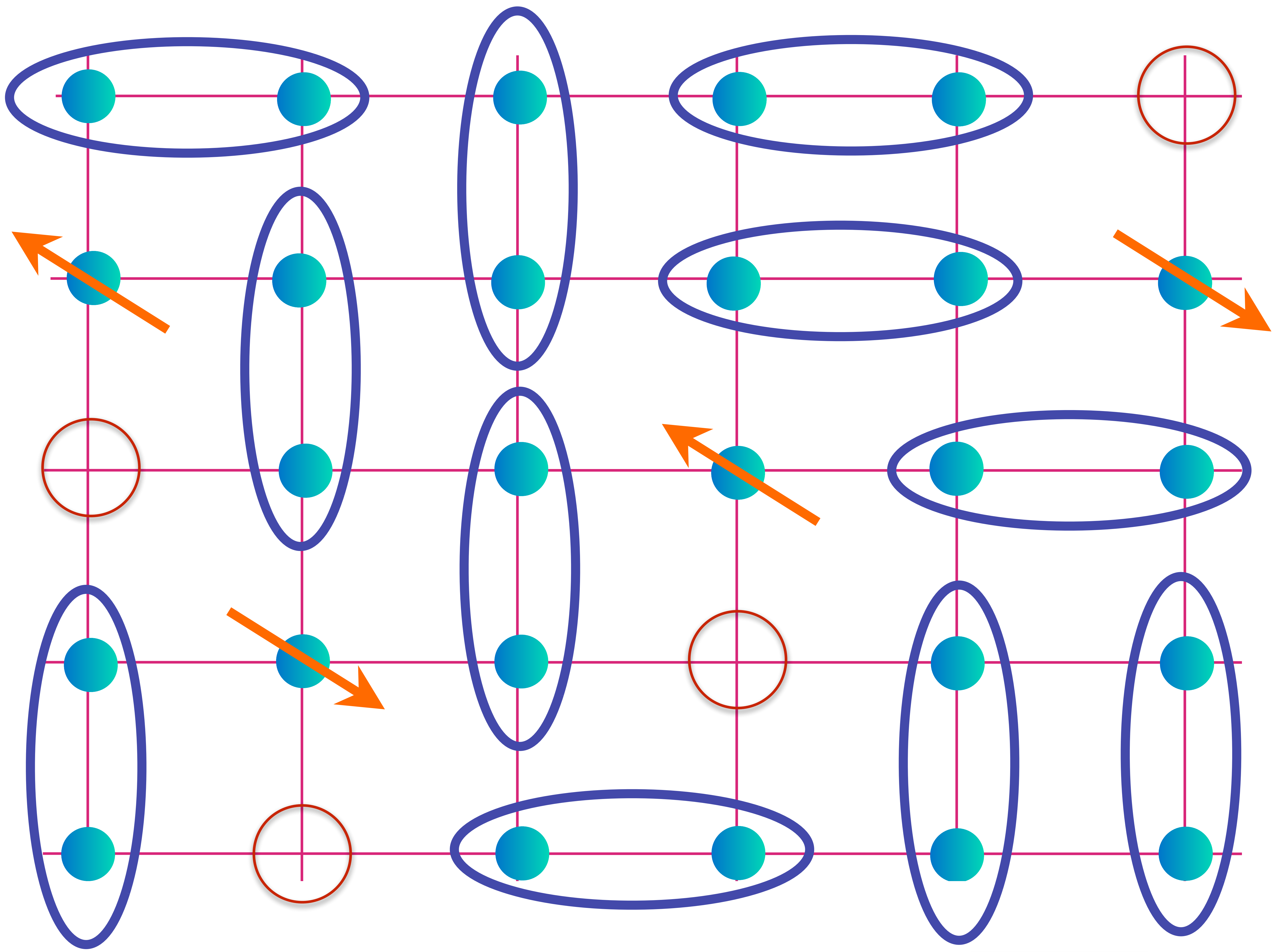
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

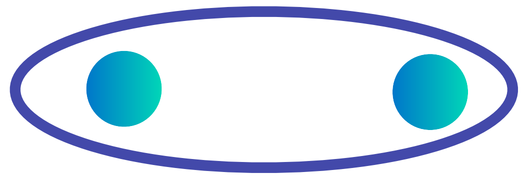
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Earlier approach to FL* in a ***one-band*** model

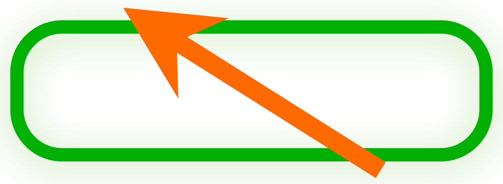
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

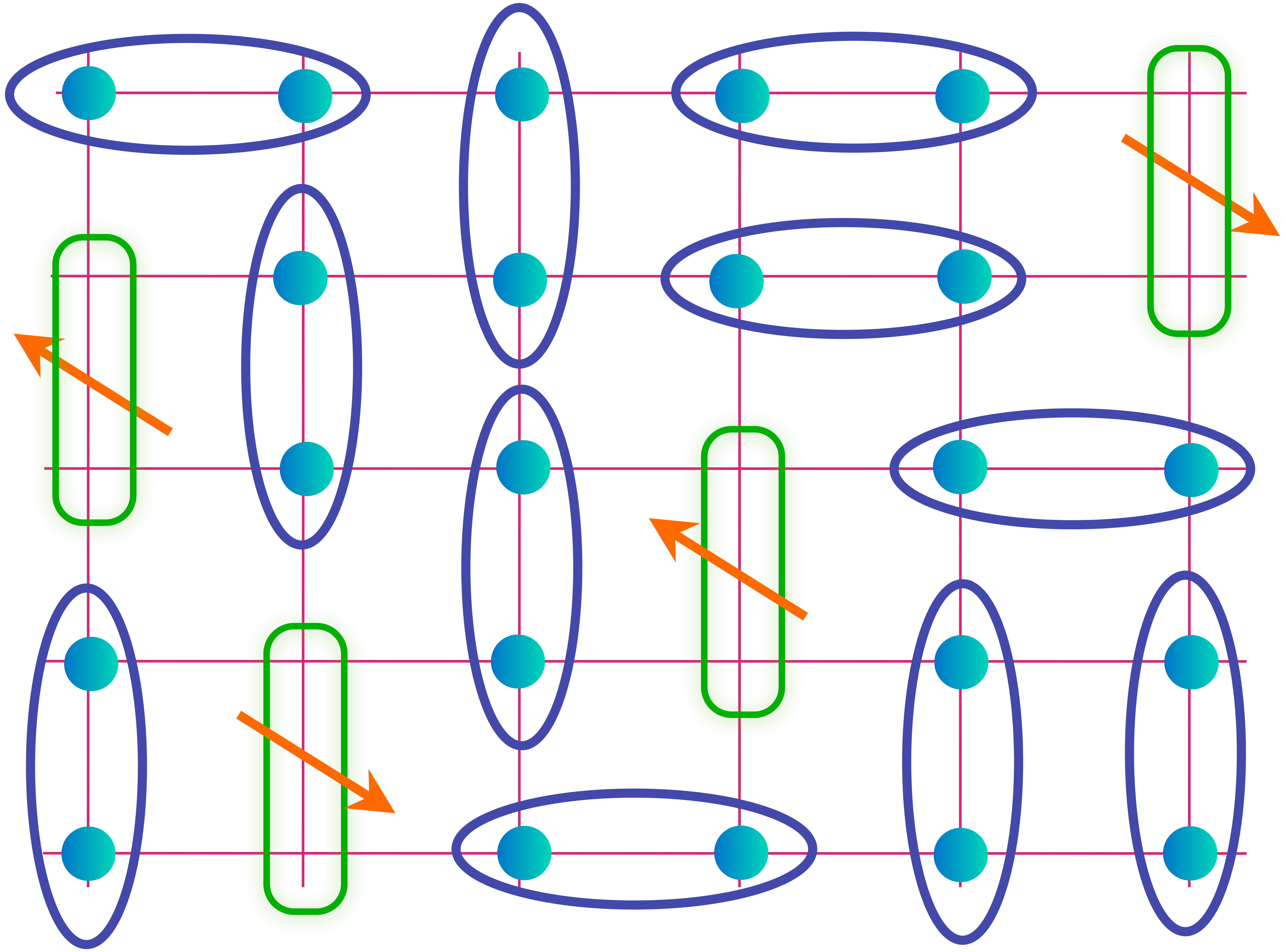


$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

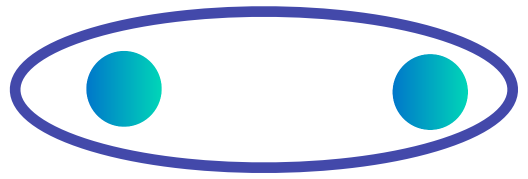
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Earlier approach to FL* in a ***one-band*** model

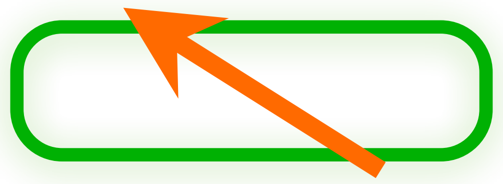
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

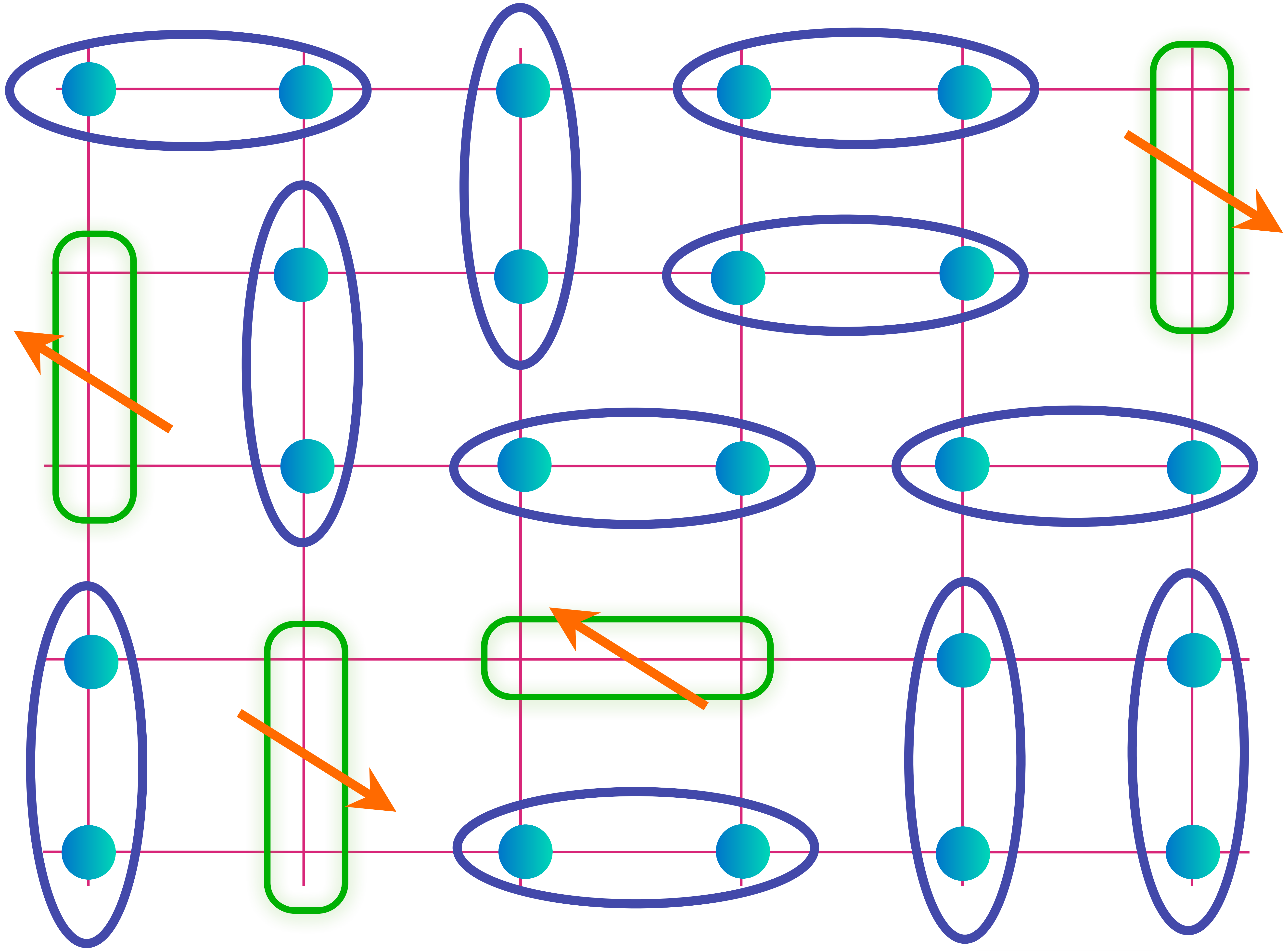


$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

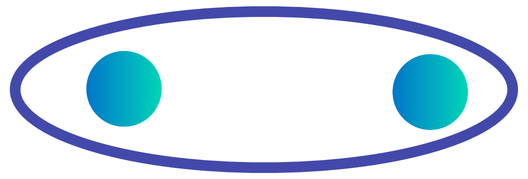
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

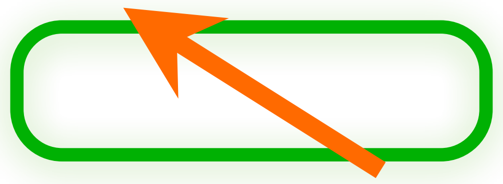
Earlier approach to FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

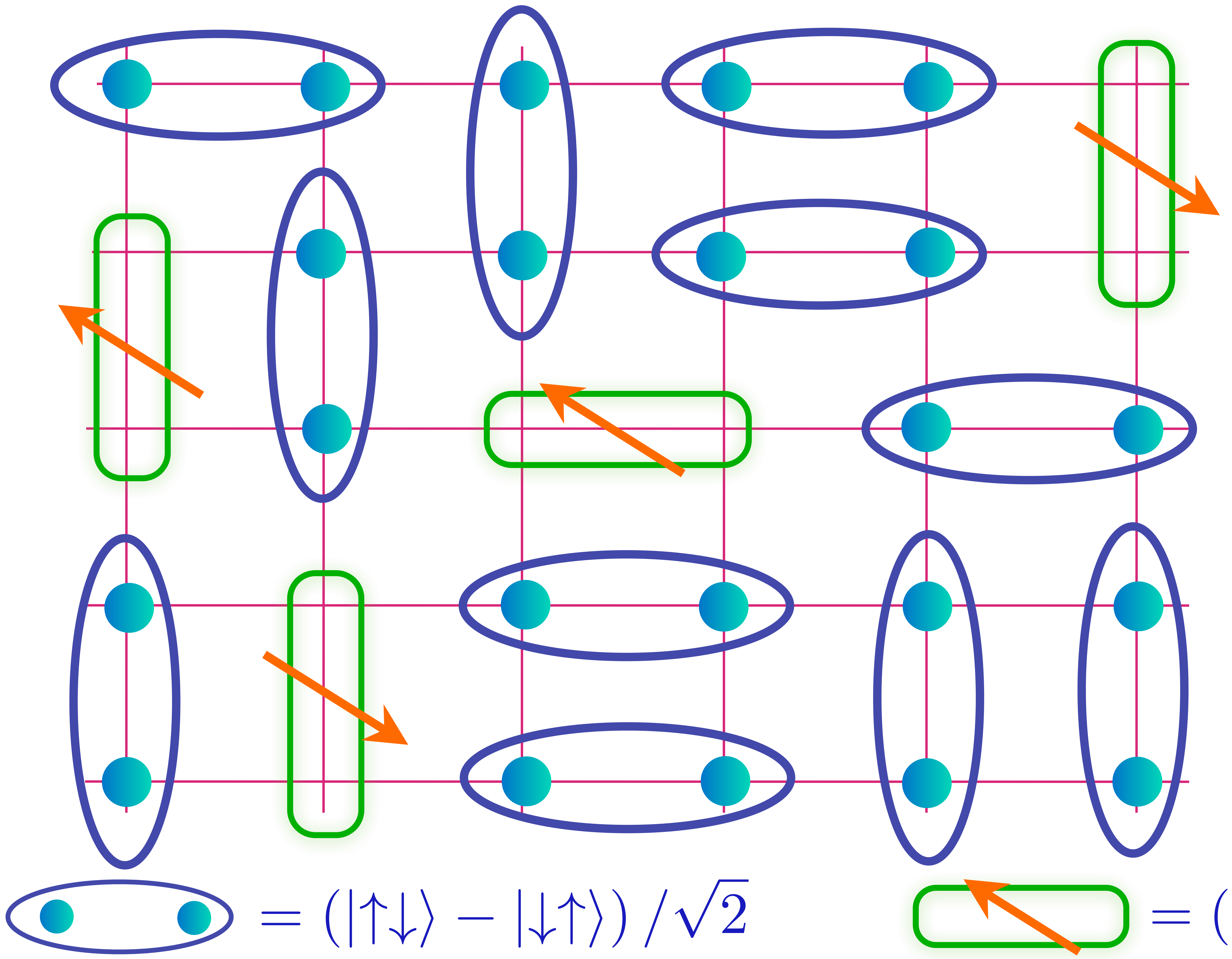
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Earlier approach to FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



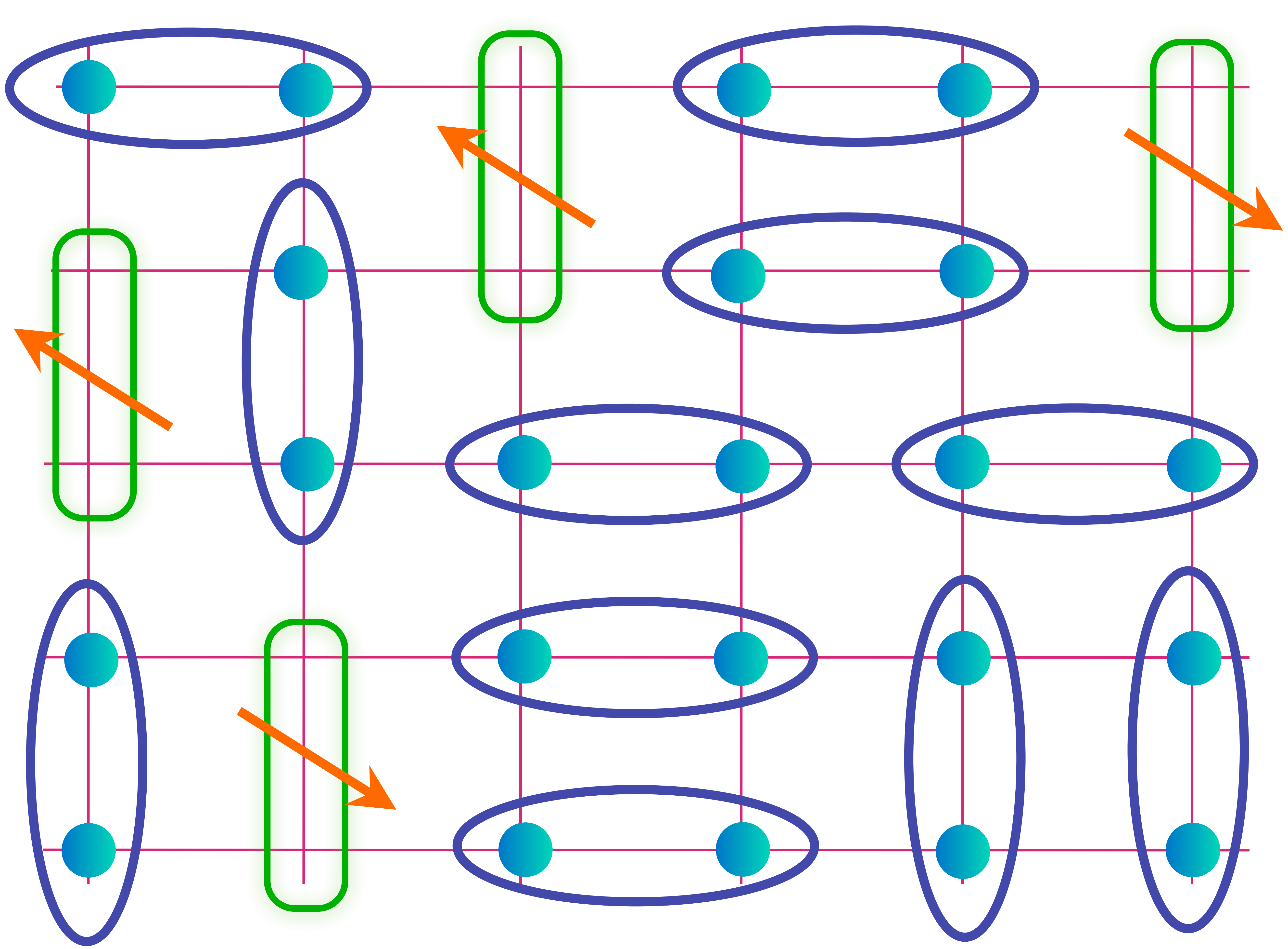
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$ $\quad$ $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

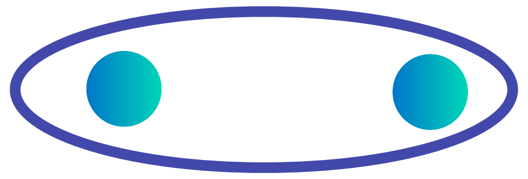
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

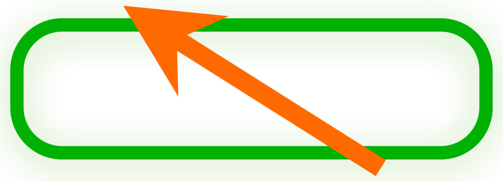
Earlier approach to FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields

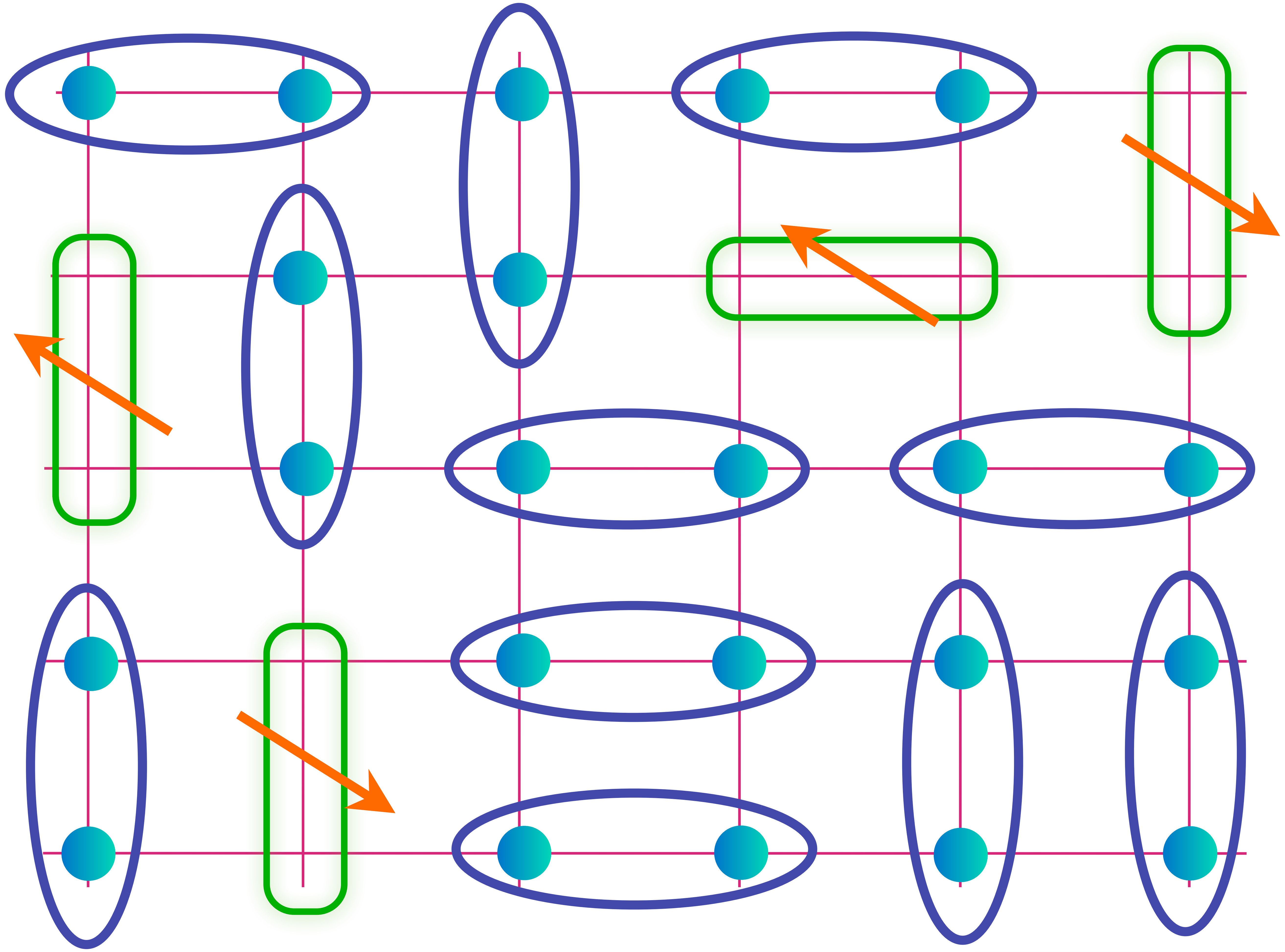
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

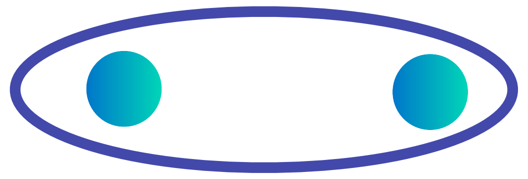
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

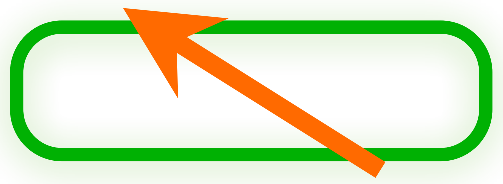
Earlier approach to FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

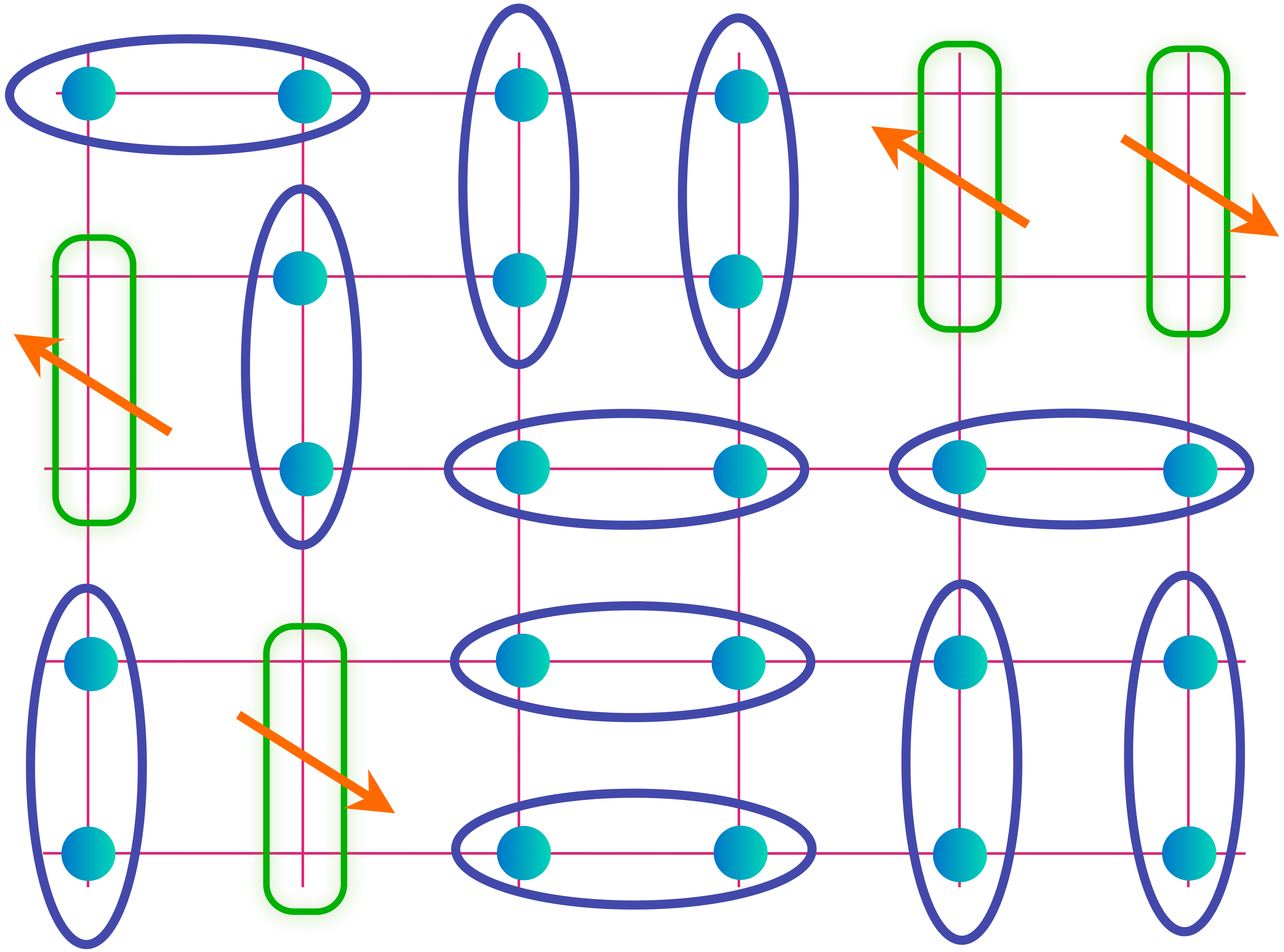
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

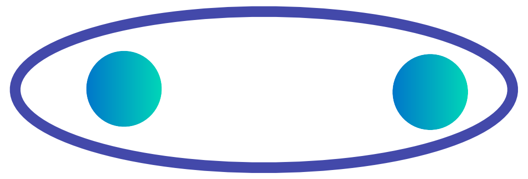
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

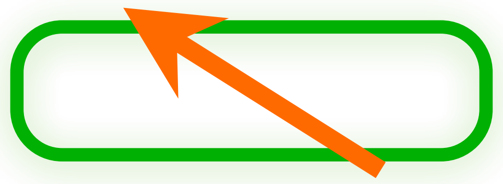
Earlier approach to FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

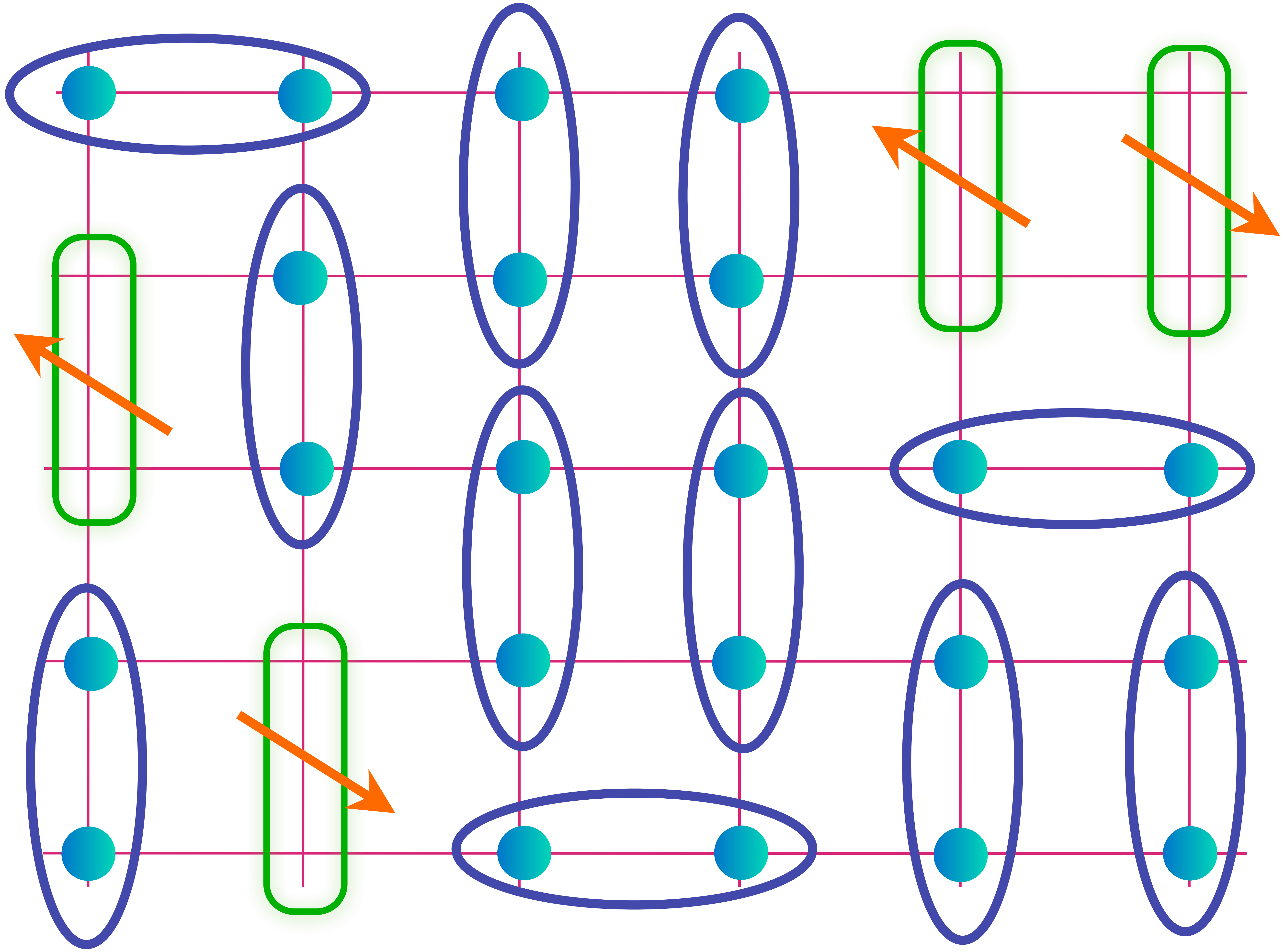
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

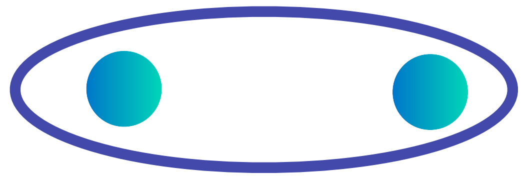
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Earlier approach to FL* in a ***one-band*** model

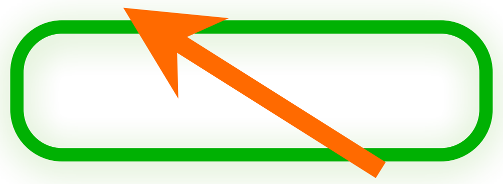
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

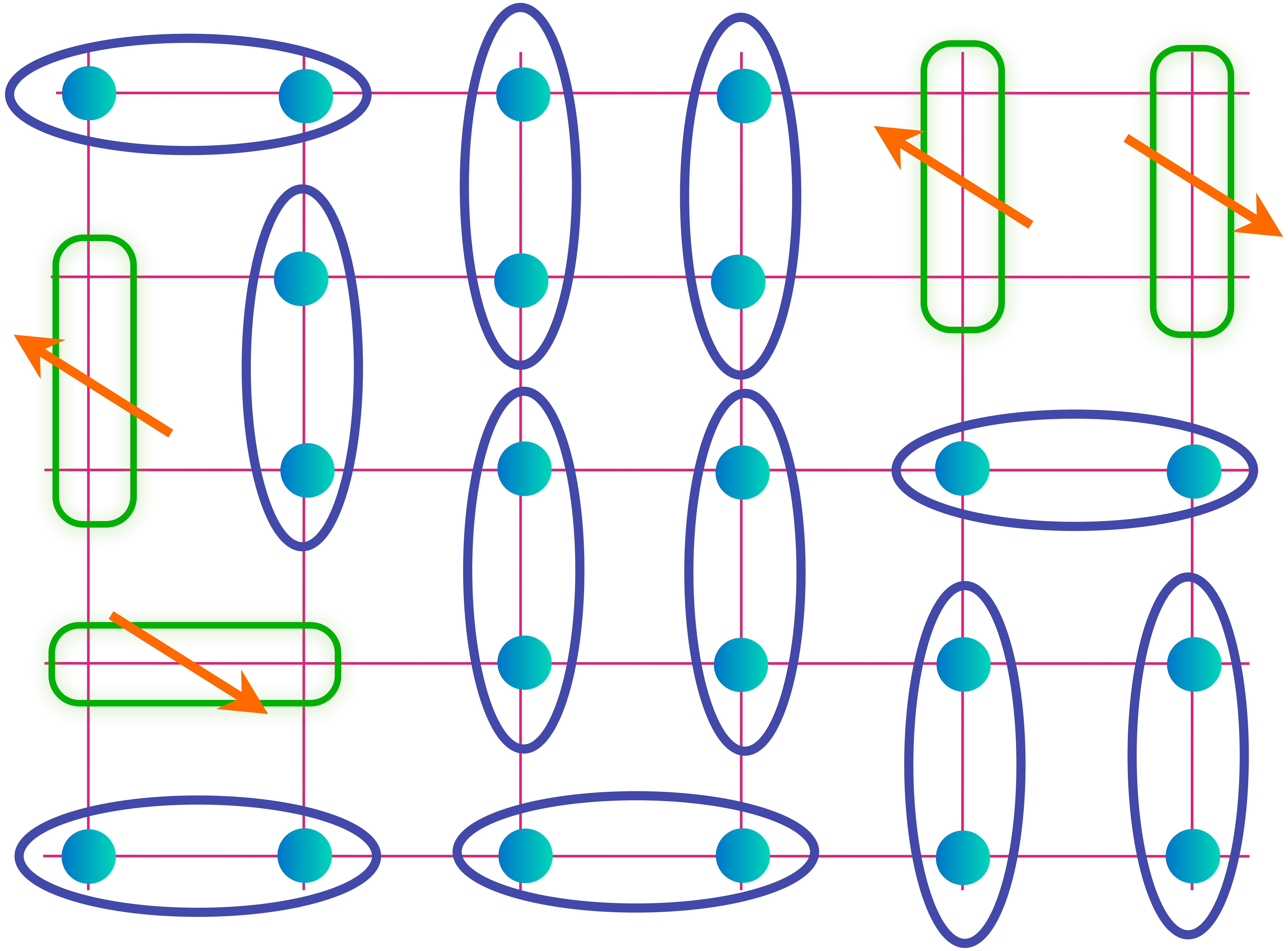


$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle) / \sqrt{2}$

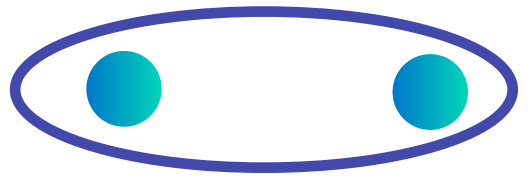
E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

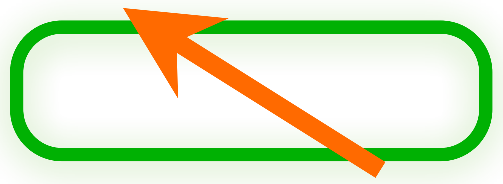
Earlier approach to FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



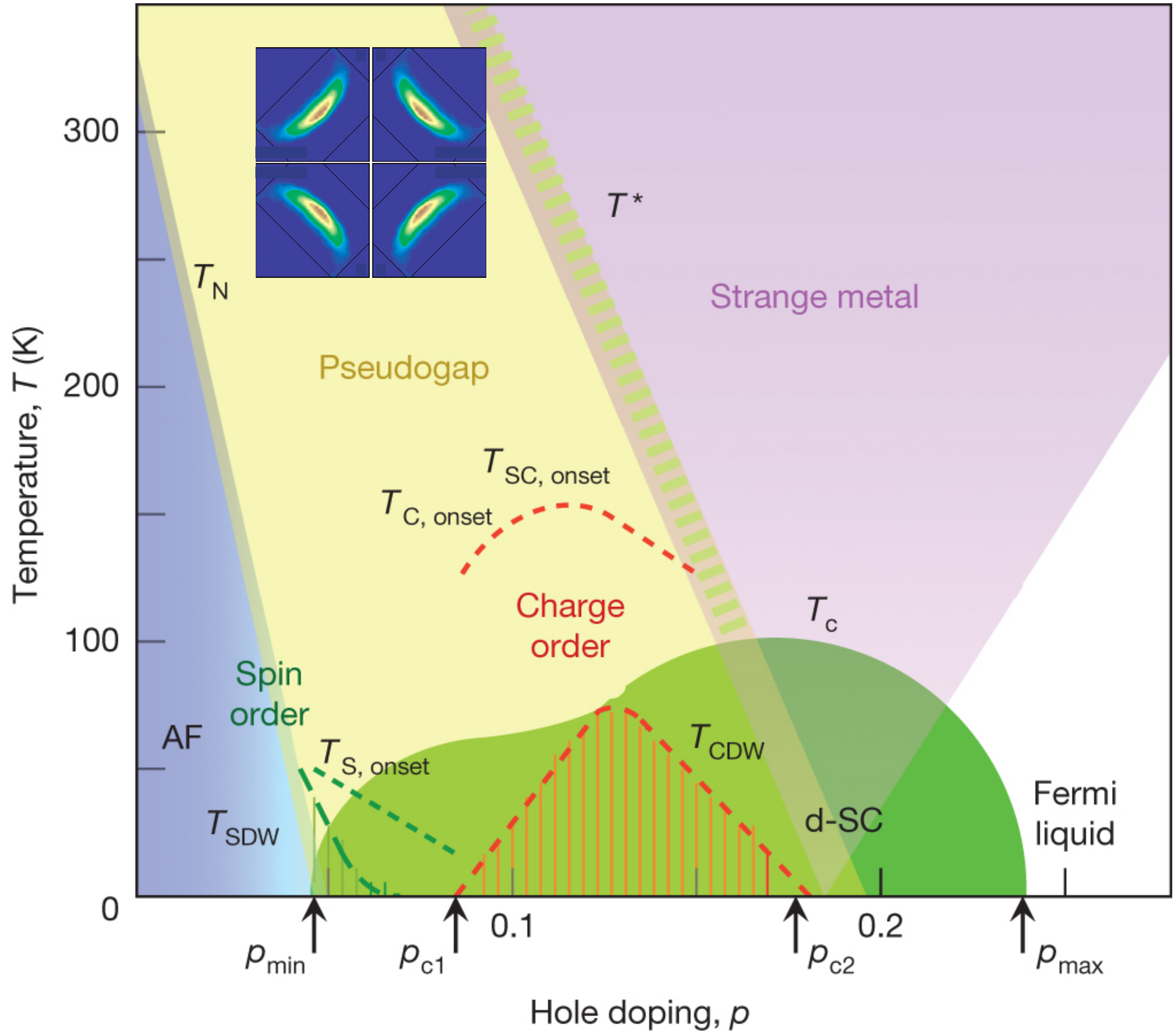
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

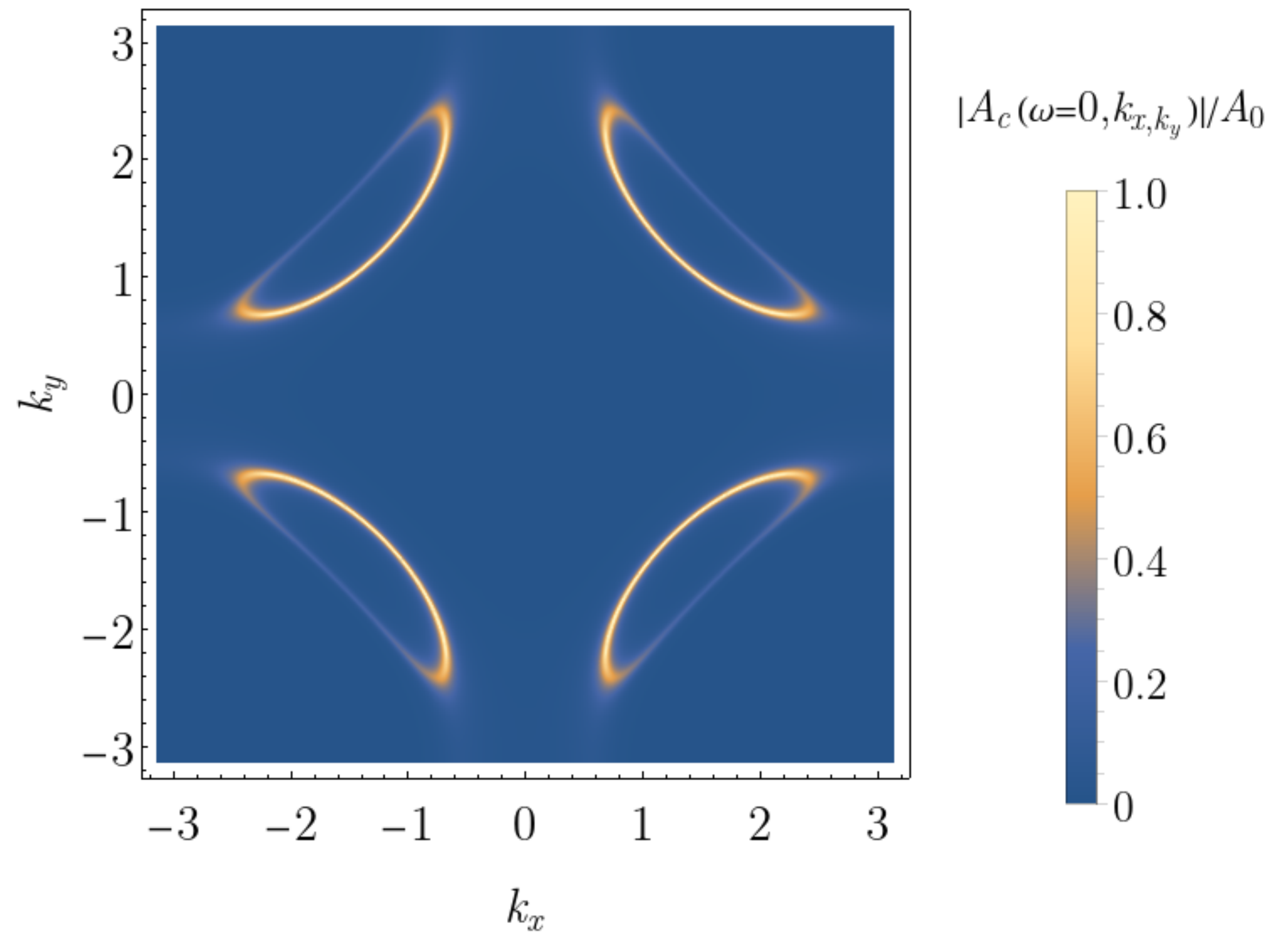
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

B Keimer *et al.* *Nature* **518**, 179-186 (2015)





Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

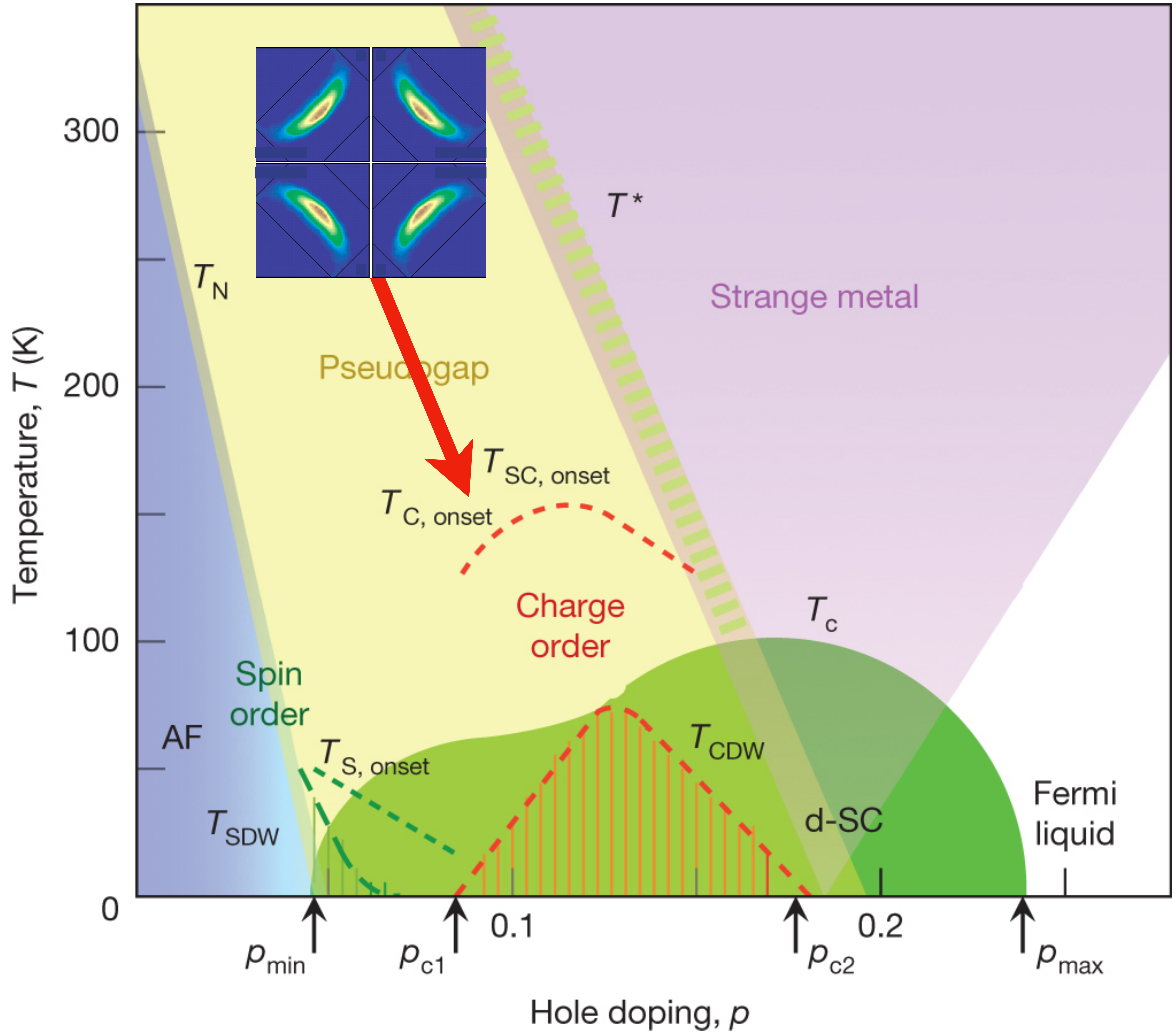
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. Sachdev, PRB **105**, 075146 (2022

Ancilla theory of the pseudogap metal

Hole pocket Fermi surfaces
of size p with
charge e, spin-1/2 quasiparticles
+
square lattice spin liquid.

FL*: Spin liquid is *required* because the Fermi surface does not enclose the Luttinger volume $(1 + p)$.

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



A theory for the confinement of fractionalized excitations in a *particular* square lattice spin liquid from electrically charged excitations.

1. Confinement of the π-flux spin liquid at half-filling

2. Ancilla theory of the pseudogap metal

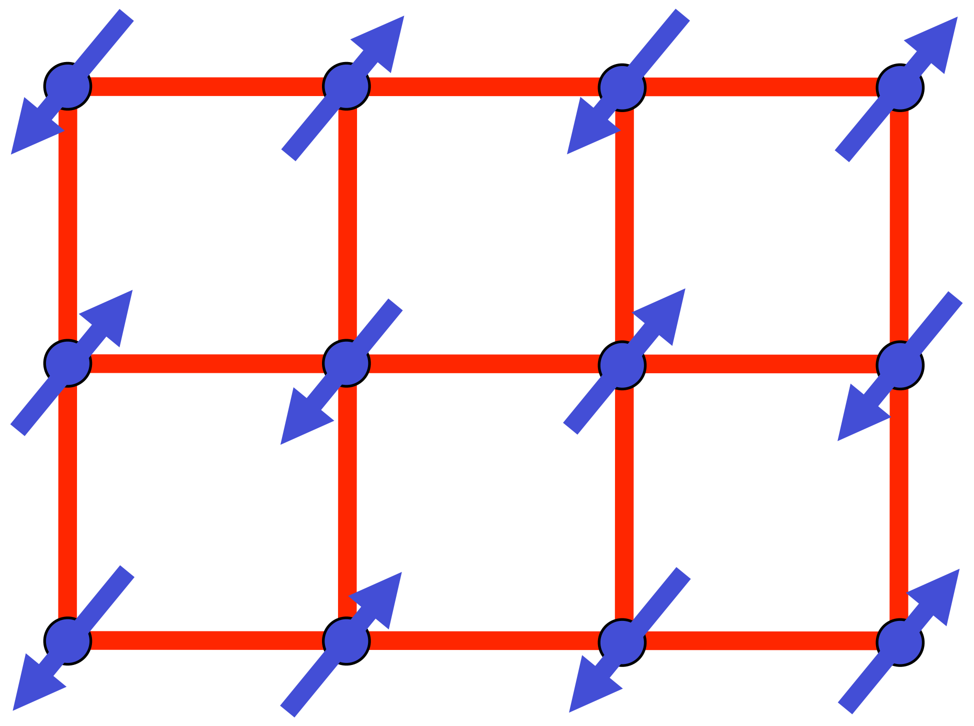
3. Confinement of the pseudogap metal at non-zero doping

1. Confinement of the π-flux spin liquid at half-filling

2. Ancilla theory of the pseudogap metal

3. Confinement of the pseudogap metal at non-zero doping

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

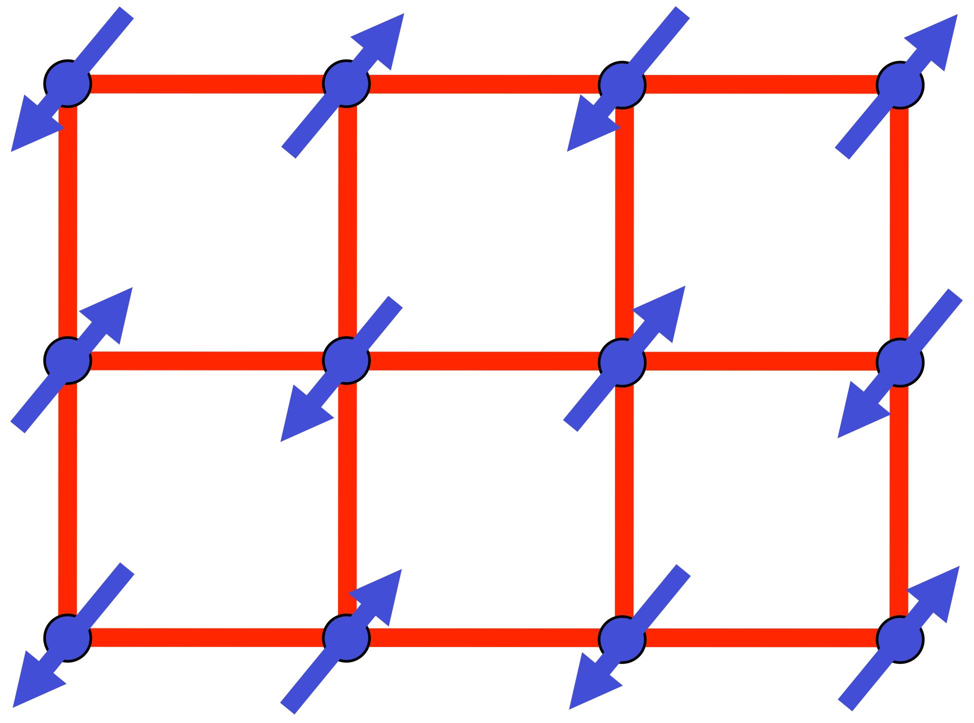
$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \qquad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Insulating S=1/2 antiferromagnet

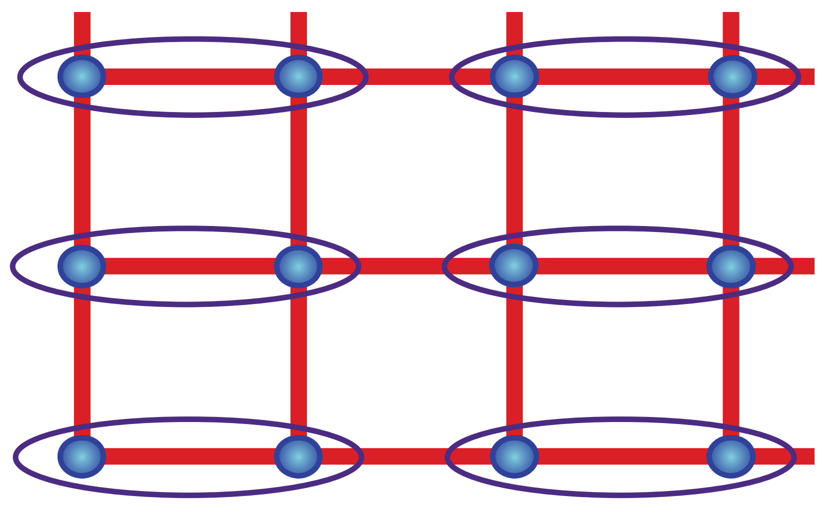
$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

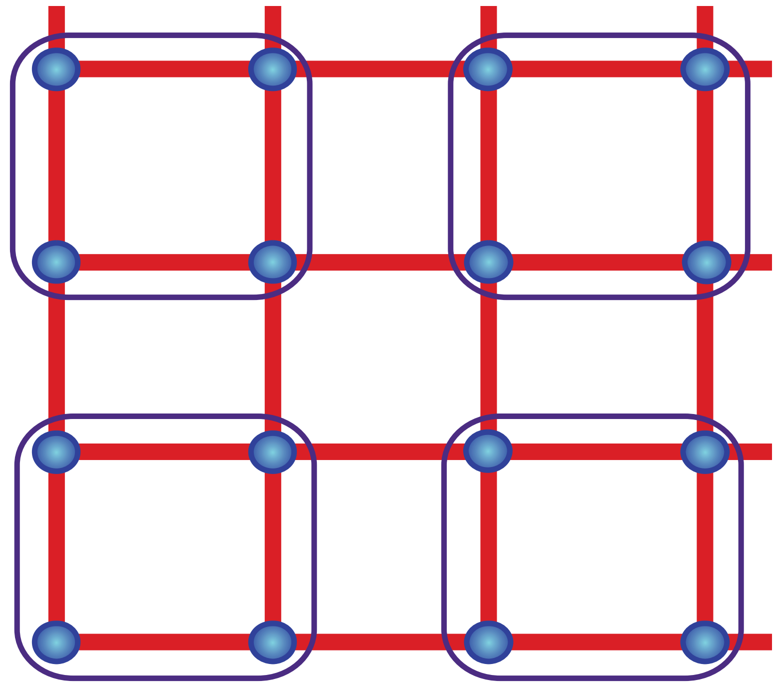
$$\boldsymbol{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} b^\dagger_{i\alpha} b_{i\alpha} = 1$$



Higgs phase, $\langle z_\alpha \rangle \neq 0$:
Néel order



or



Confining phase, $\langle z_\alpha \rangle = 0$:
VBS order

s

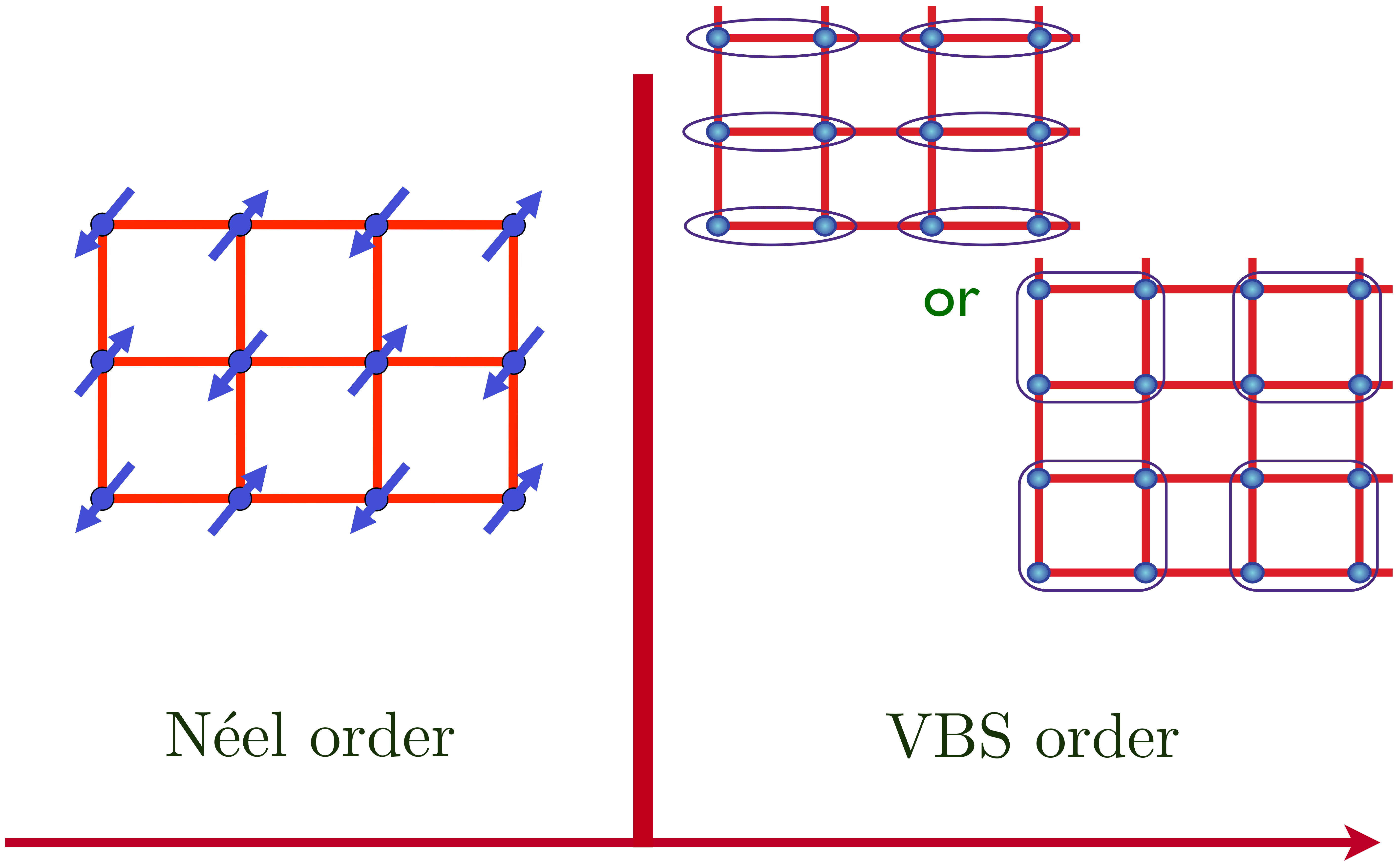
Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. Sachdev, Physical Review Letters **62**, 1694 (1989)

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

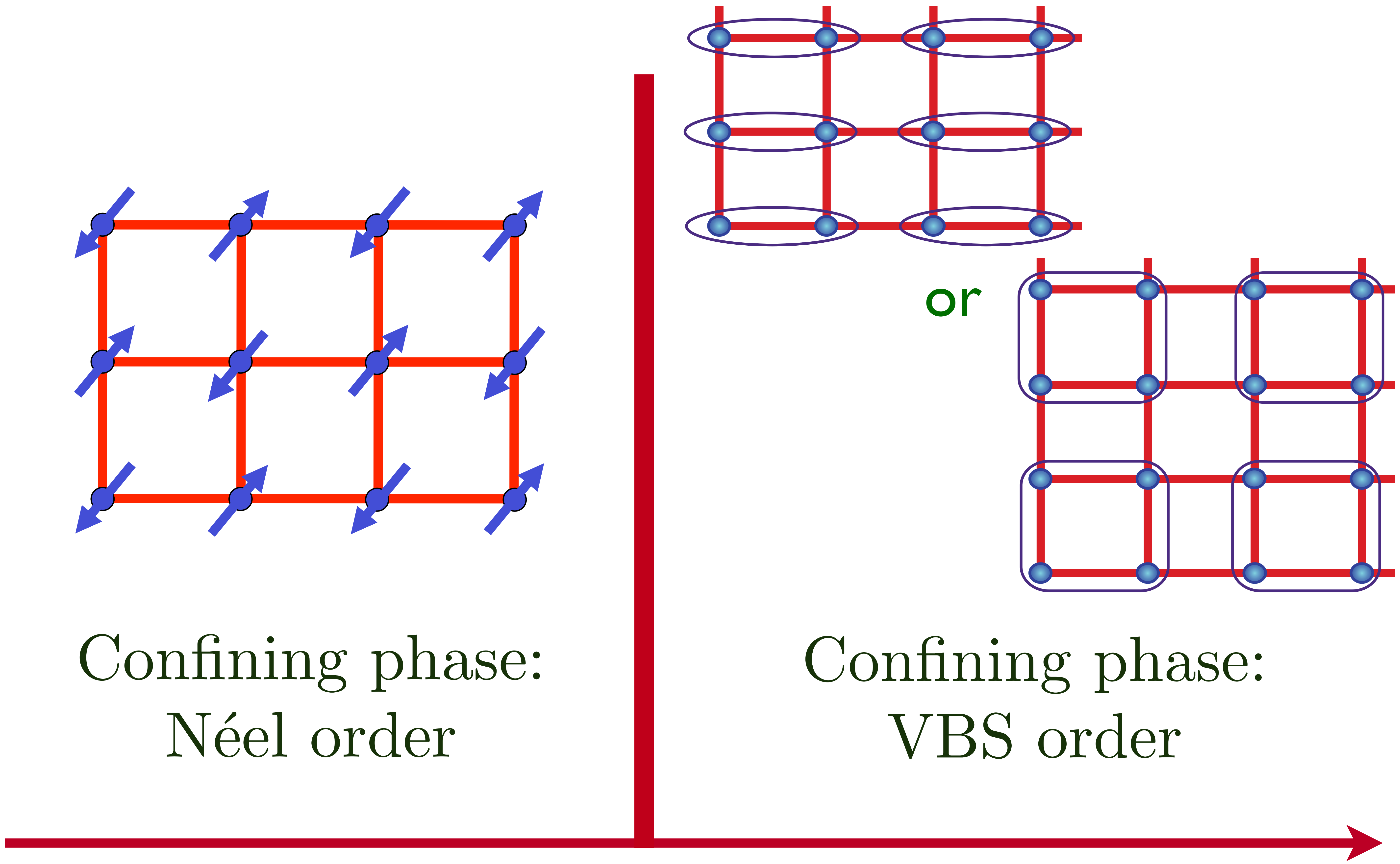
$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \, , \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

Insulating S=1/2 antiferromagnet

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^{\dagger} \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \,, \qquad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^{\dagger} f_{i\alpha} = 1$$

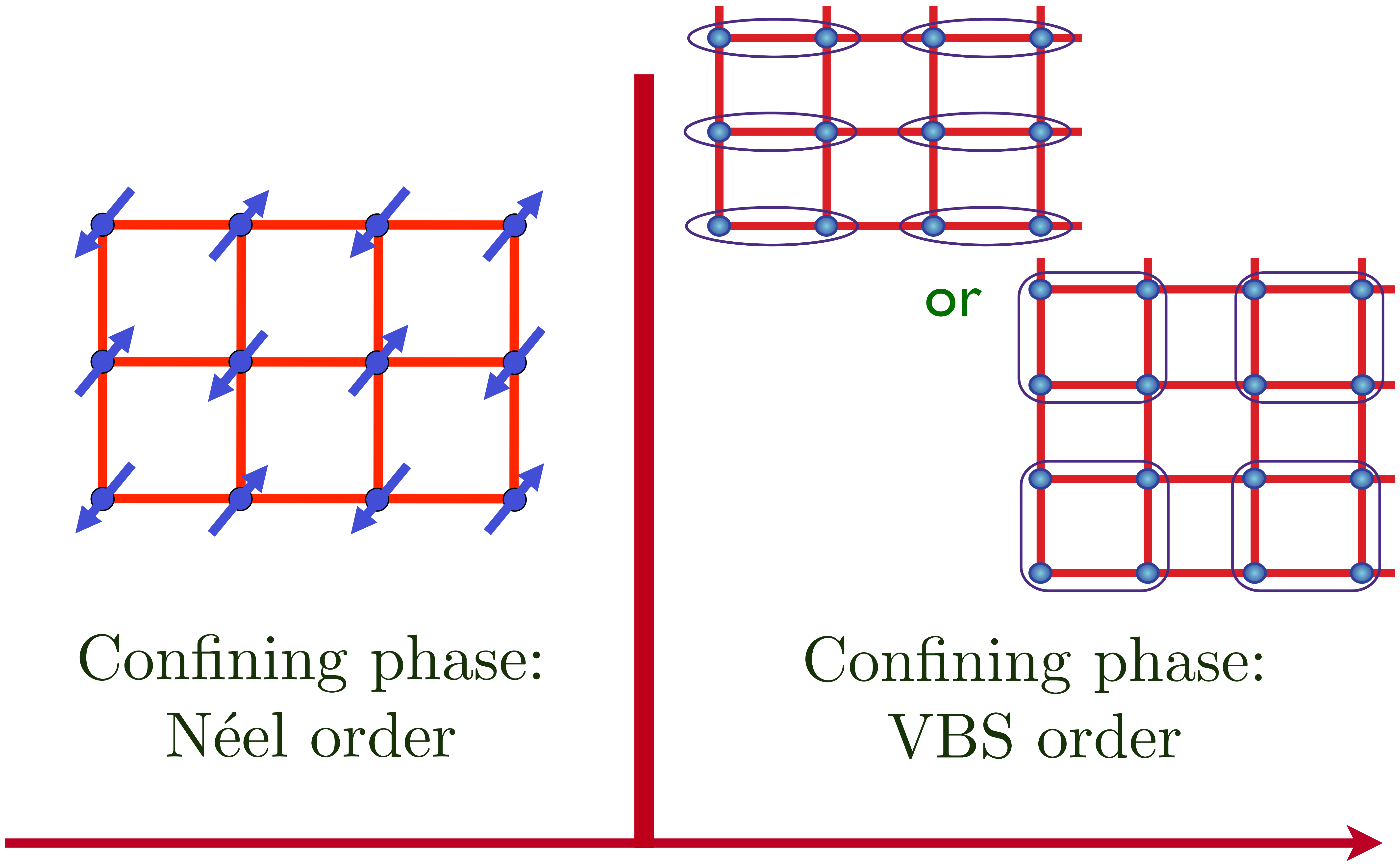


π-flux mean-field leads to a low energy theory of $N_f = 2$ Dirac fermions Ψ_s coupled to an emergent SU(2) gauge field. Confining order parameters are Néel and VBS states!

$$\mathcal{L} = i \overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + \ldots$$

Ying Ran and X.-G. Wen, cond-mat/0609620; Ying Ran, Ph.D. thesis, MIT (2007)

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π-flux mean-field leads to a low energy theory of $N_f = 2$ Dirac fermions Ψ_s coupled to an emergent SU(2) gauge field. Confining order parameters are Néel and VBS states!

$$\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + \ldots$$

Dual to $\mathbb{CP}^1$ U(1) gauge theory.

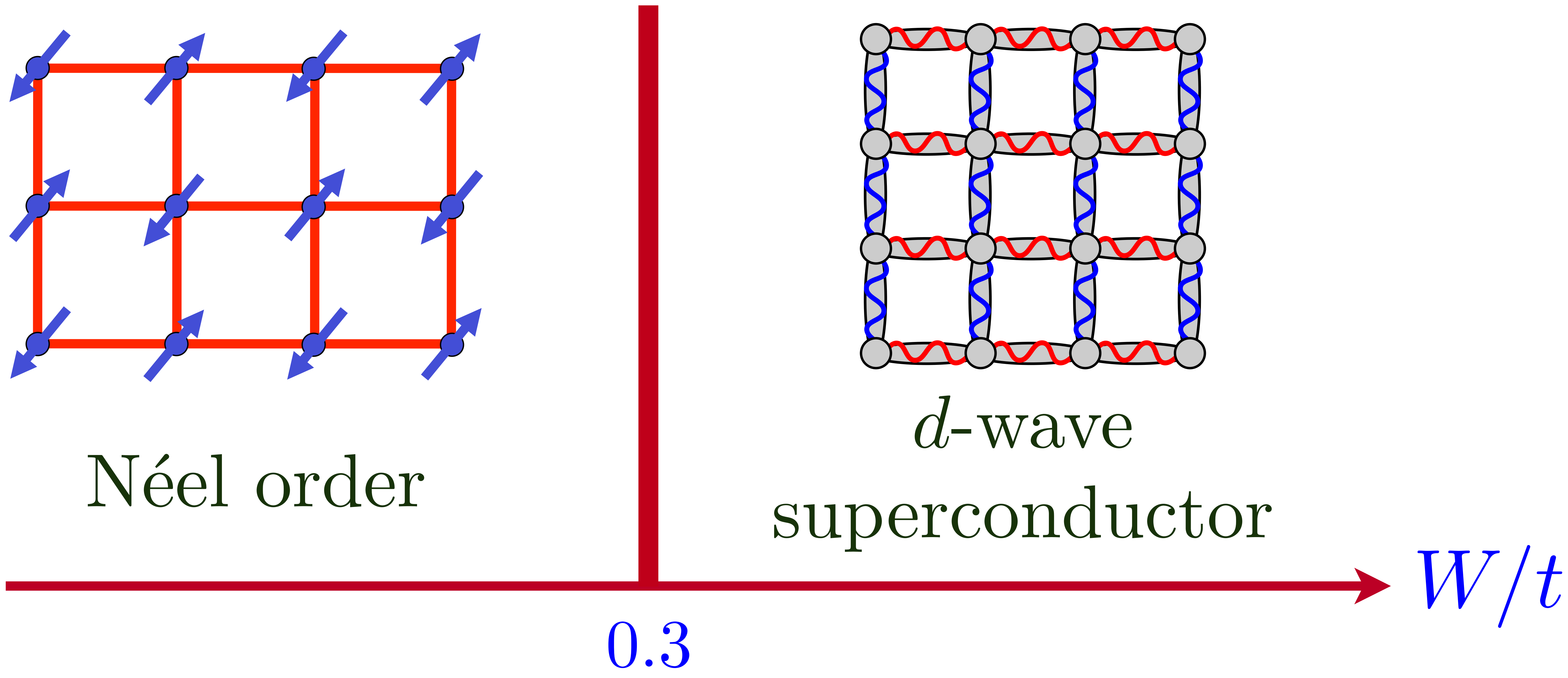
Ying Ran and X.-G. Wen, cond-mat/0609620; Ying Ran, Ph.D. thesis, MIT (2007)
C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, Physical Review X **7**, 031051 (2017)

Include charge fluctuations at half-filling: repulsive Hubbard model with pair-hopping

$$H = -\frac{t}{2}\sum_i K_i + U\sum_i (n_{i\uparrow} - \tfrac{1}{2})(n_{i\downarrow} - \tfrac{1}{2}) - W\sum_i K_i^2 \,,$$

$$K_i = \sum_{\hat{e}=\pm\hat{x},\pm\hat{y}} \left(c_{i\alpha}^\dagger c_{i+\hat{e},\alpha} + c_{i+\hat{e},\alpha}^\dagger c_{i\alpha} \right)$$

$U/t = 4$

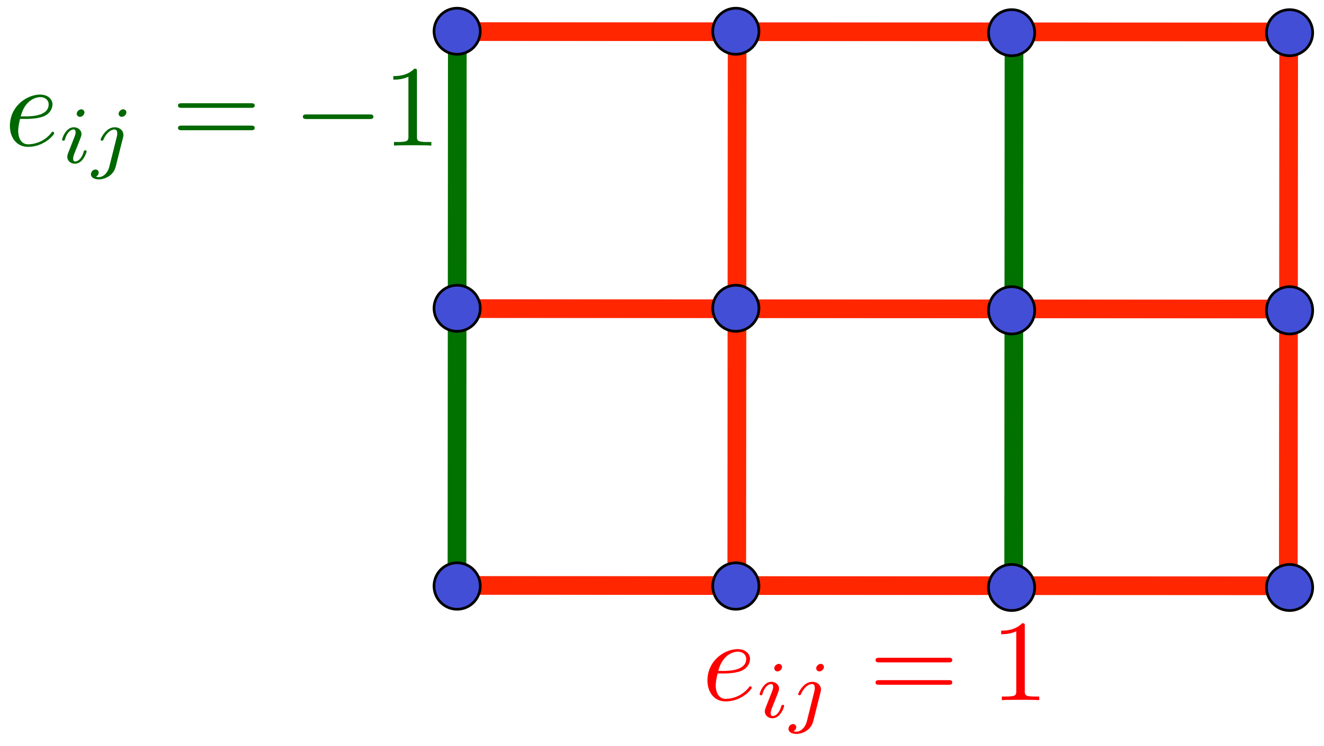


F. F. Assaad, M. Imada, and D. J. Scalapino, PRL **77**, 4592 (1996)
S. Raghu, S.A. Kivelson and D. J. Scalapino, PRB **81**, 224505 (2010)

Include charge fluctuations at half-filling: confinement of SU(2) gauge theory

- Begin with the π-flux spin liquid in the fermionic spinon description.

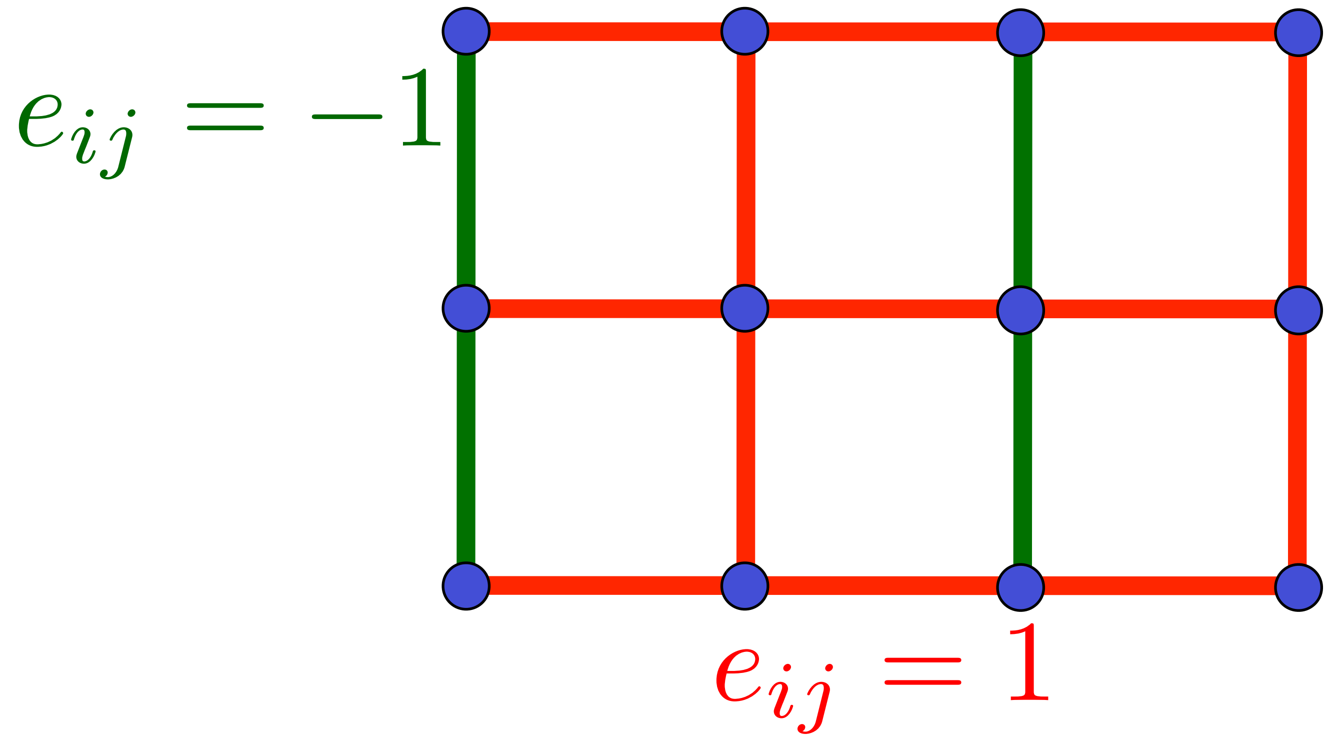
$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right)$$



Include charge fluctuations at half-filling: confinement of SU(2) gauge theory

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f^\dagger_{i\alpha} f_{j\alpha} - f^\dagger_{j\alpha} f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi^\dagger_i \Psi_j - \Psi^\dagger_j \Psi_i \right) ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f^\dagger_{i\downarrow} \end{pmatrix}$$

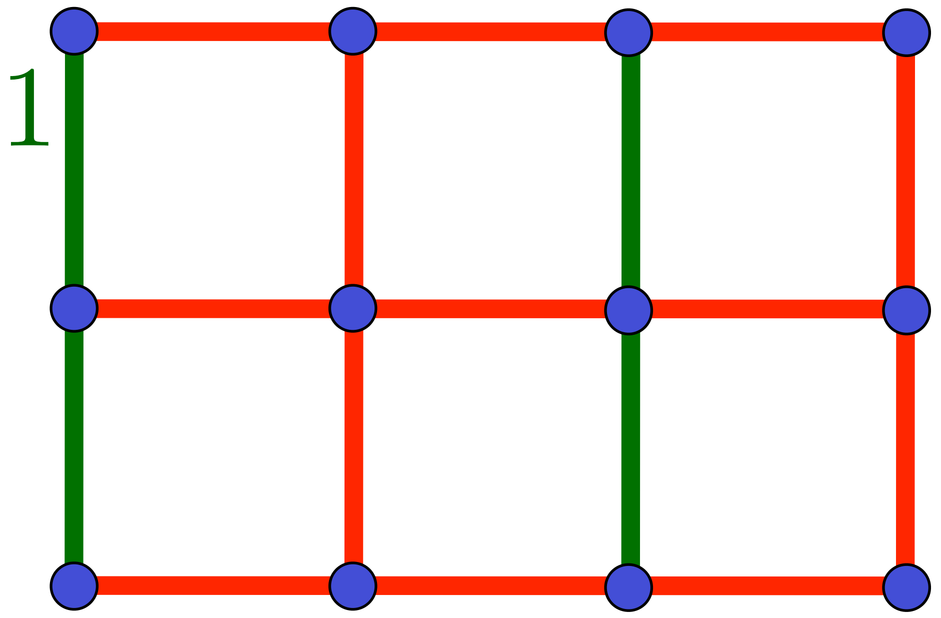


H_f is invariant under distinct SU(2) rotations in spin and Nambu space.

Include charge fluctuations at half-filling: confinement of SU(2) gauge theory

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger \Psi_j - \Psi_j^\dagger \Psi_i \right); \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$



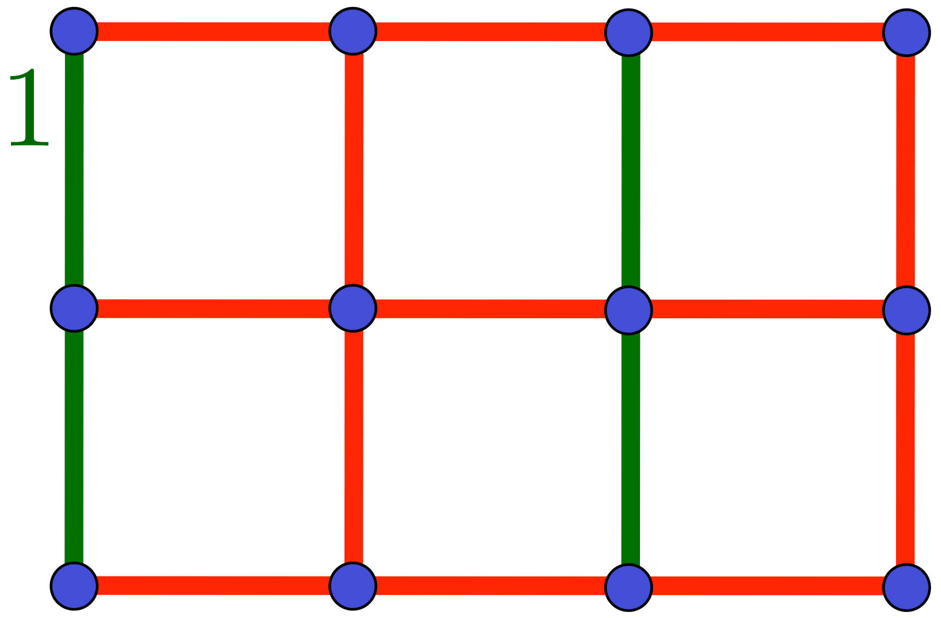
H_f is invariant under distinct SU(2) rotations in spin and Nambu space.

- We can fully confine the SU(2) gauge field by condensing a boson, B_i, which is a fundamental of gauge SU(2). To obtain superconductivity with charge $2e$ pairs in the confining phase, B_i should also carry electromagnetic charge e.

Include charge fluctuations at half-filling: confinement of SU(2) gauge theory

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger \Psi_j - \Psi_j^\dagger \Psi_i \right) ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$



H_f is invariant under distinct SU(2) rotations in spin and Nambu space.

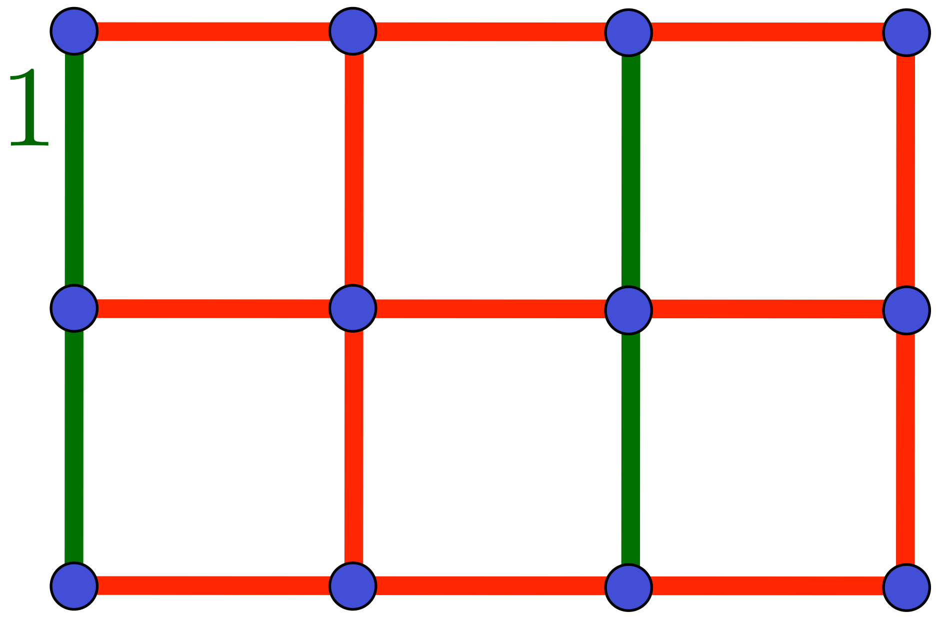
- We can fully confine the SU(2) gauge field by condensing a boson, B_i, which is a fundamental of gauge SU(2). To obtain superconductivity with charge $2e$ pairs in the confining phase, B_i should also carry electromagnetic charge e.

- This uniquely identifies B_i as the 'chargon' of X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996), related to the electrons $c_{i\sigma}$ by

$$B_i = \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix} \quad ; \quad \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix} \sim \begin{pmatrix} B_{1i}^* & B_{2i}^* \\ -B_{2i} & B_{1i} \end{pmatrix} \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

Include charge fluctuations at half-filling: confinement of SU(2) gauge theory

- Begin with the π-flux spin liquid in the fermionic spinon description.



$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger \Psi_j - \Psi_j^\dagger \Psi_i \right) ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

H_f is invariant under distinct SU(2) rotations in spin and Nambu space.

- We can fully confine the SU(2) gauge field by condensing a boson, B_i, which is a fundamental of gauge SU(2). To obtain superconductivity with charge $2e$ pairs in the confining phase, B_i should also carry electromagnetic charge e.

- This uniquely identifies B_i as the 'chargon' of X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996), related to the electrons $c_{i\sigma}$ by

$$B_i = \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix} \quad ; \quad \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix} \sim \begin{pmatrix} B_{1i}^* & B_{2i}^* \\ -B_{2i} & B_{1i} \end{pmatrix} \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

- Knowing the projective symmetry transformations of Ψ_i, we can deduce those of the B_i, and obtain the effective Hamiltonian for B_i

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger B_j - B_j^\dagger B_i \right) + \ldots$$

$$\mathcal{L}(B) = H_B + \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g \sum_{\langle \boldsymbol{ij} \rangle} \left|\Delta_{\boldsymbol{ij}}\right|^2 + J_1 \sum_{\langle \boldsymbol{ij} \rangle} Q_{\boldsymbol{ij}}^2 + K_1 \sum_{\langle \boldsymbol{ij} \rangle} J_{\boldsymbol{ij}}^2 .$$

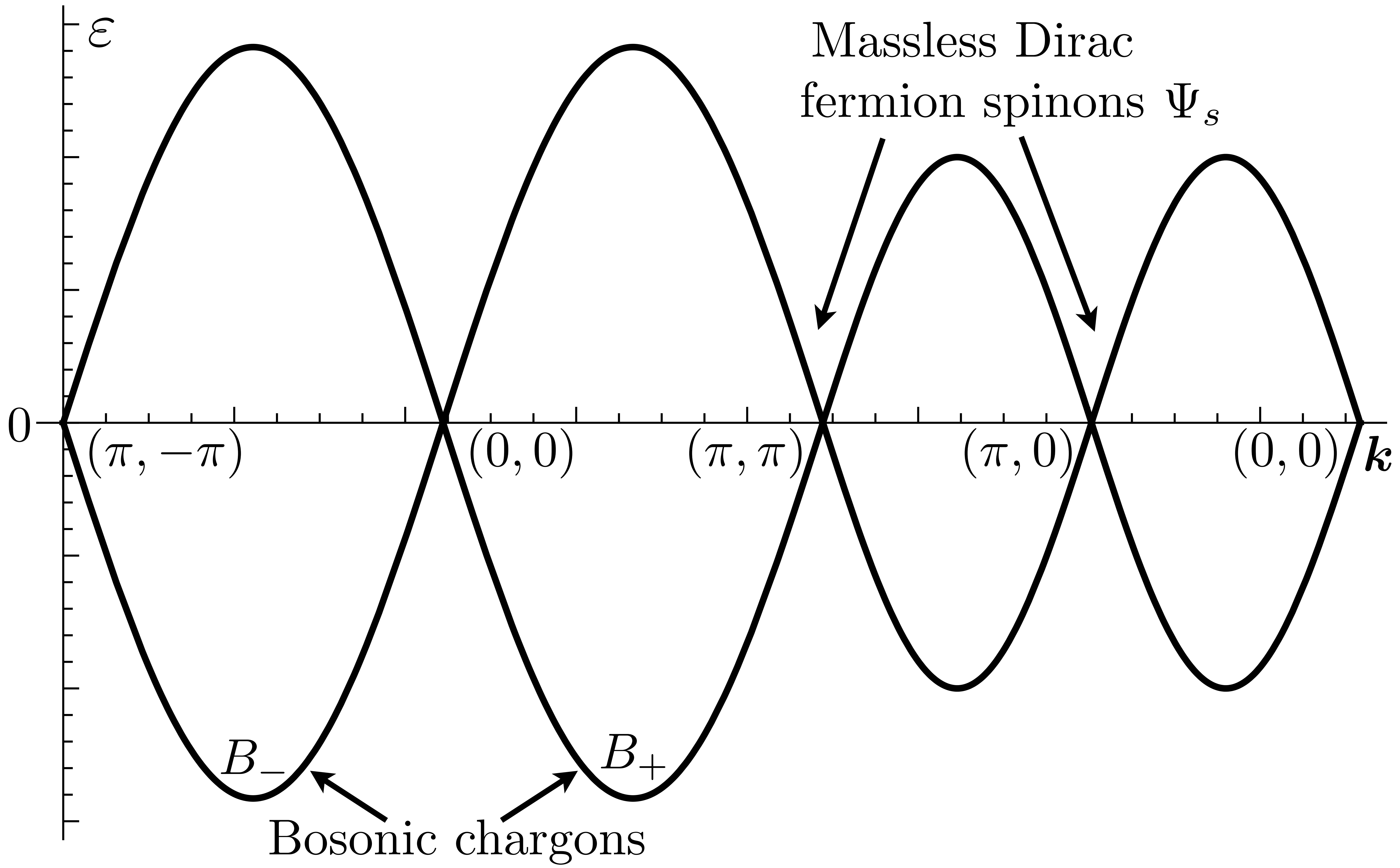
site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

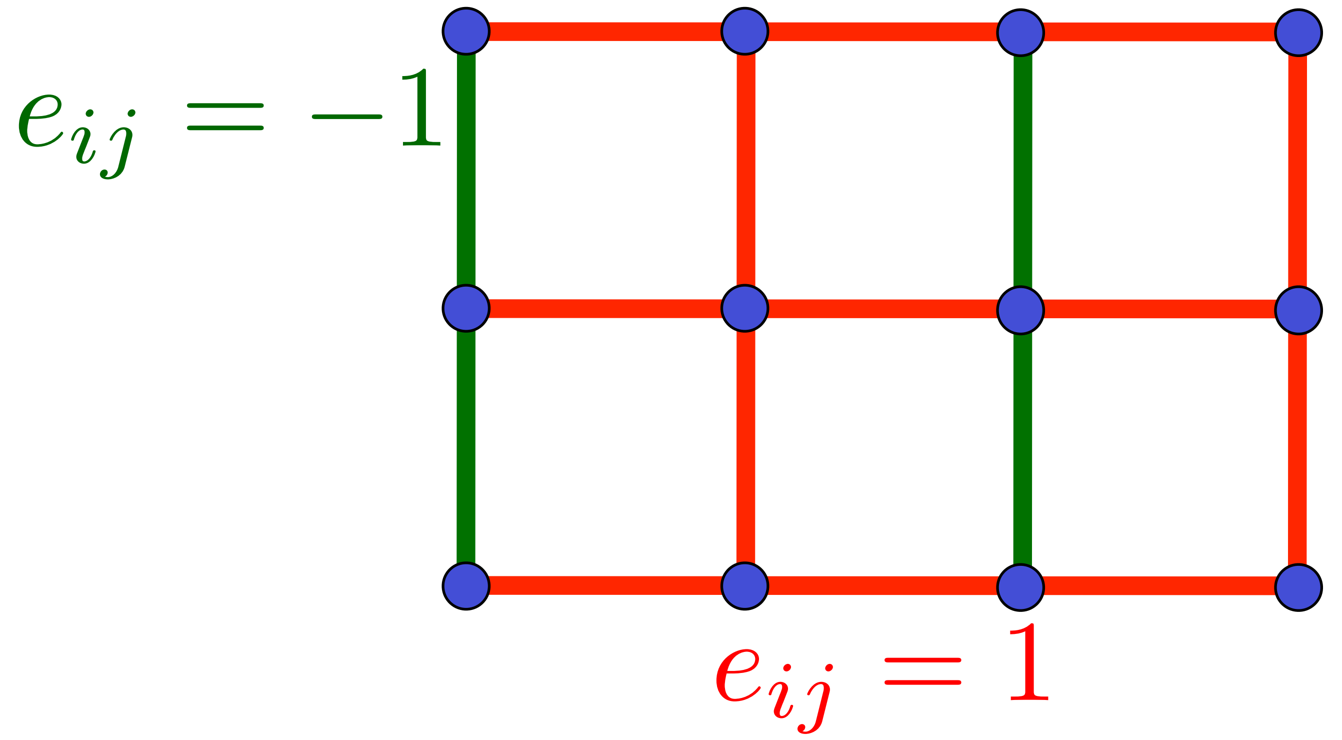
bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

bond current: $i \left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

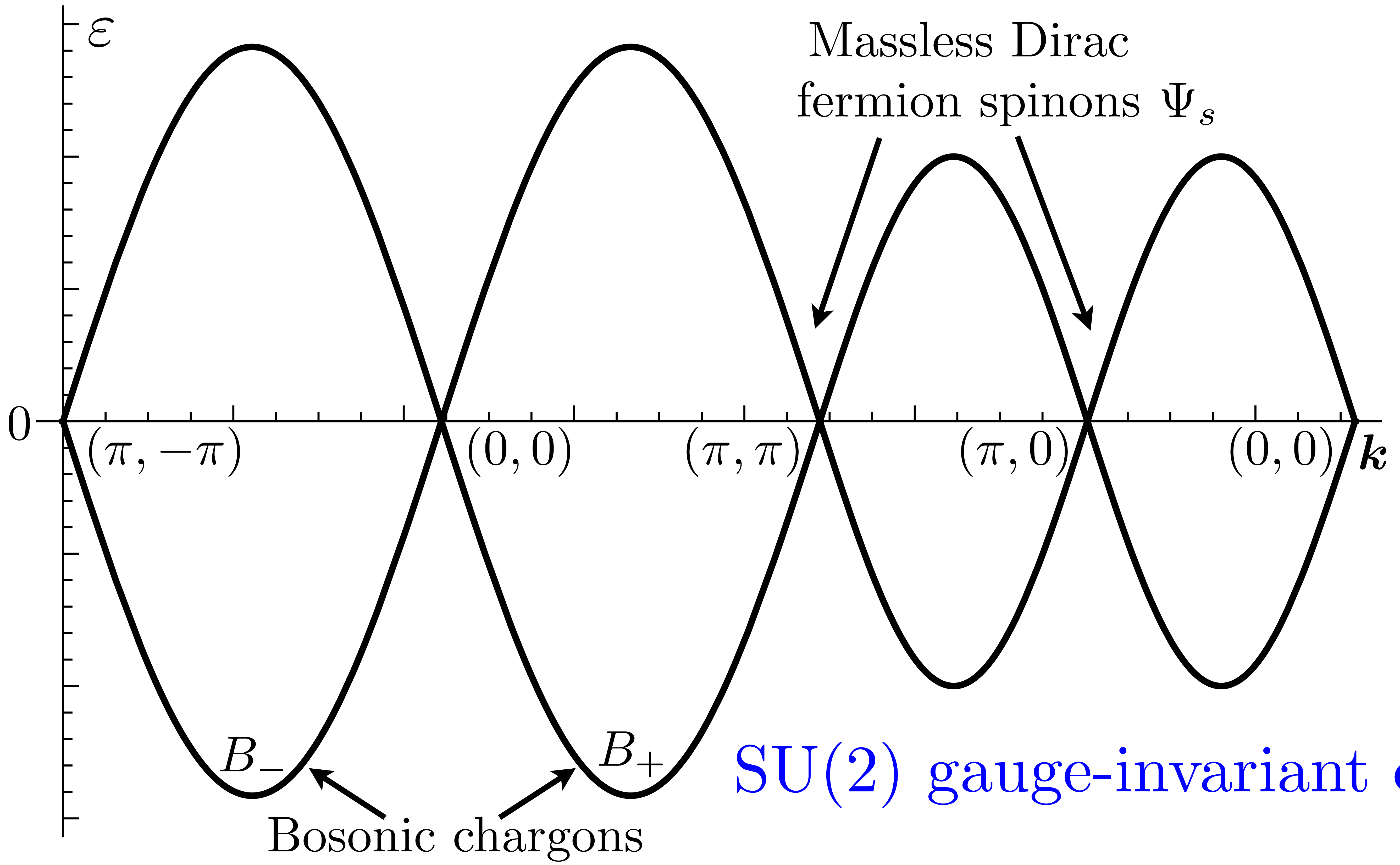
Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$.

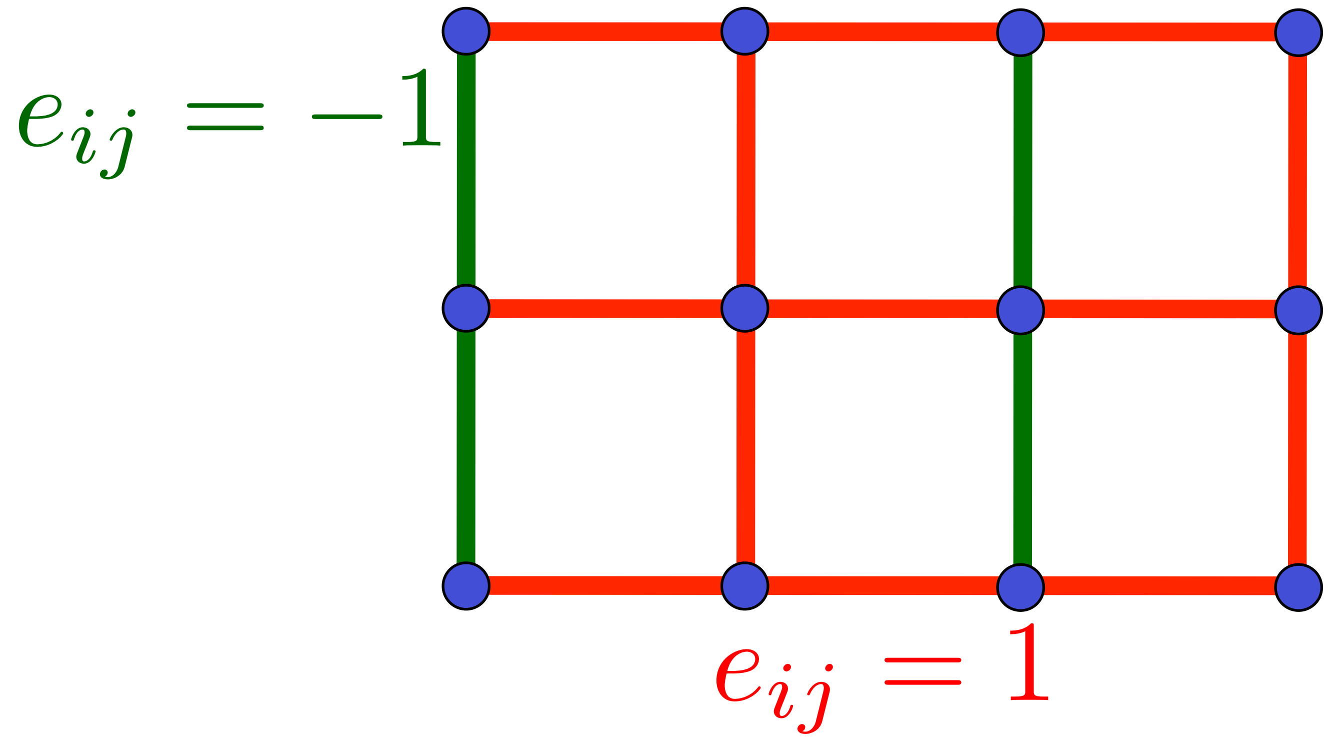
Include charge fluctuations at half-filling: confinement of SU(2) gauge theory





Include charge fluctuations at half-filling: confinement of SU(2) gauge theory

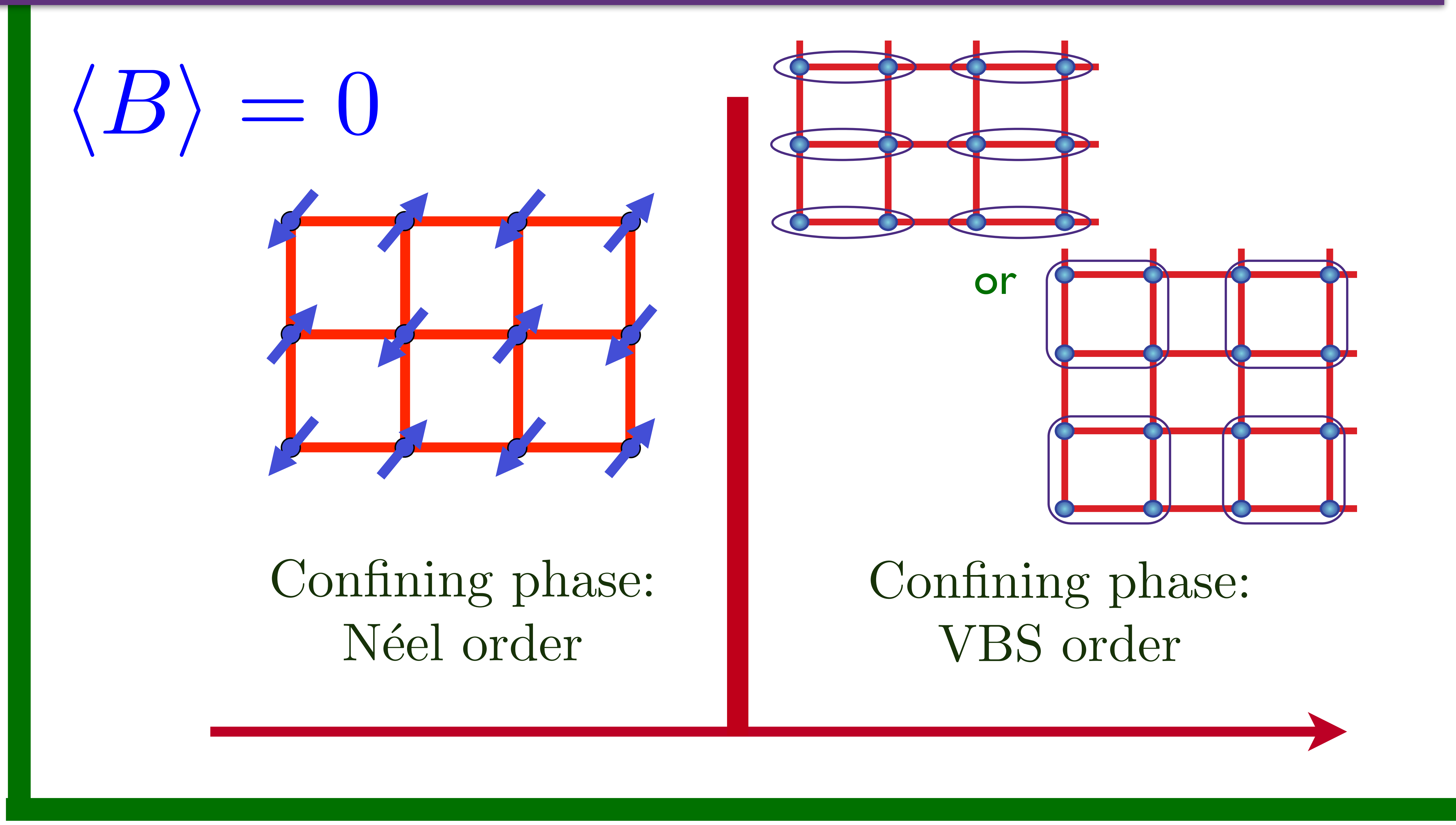




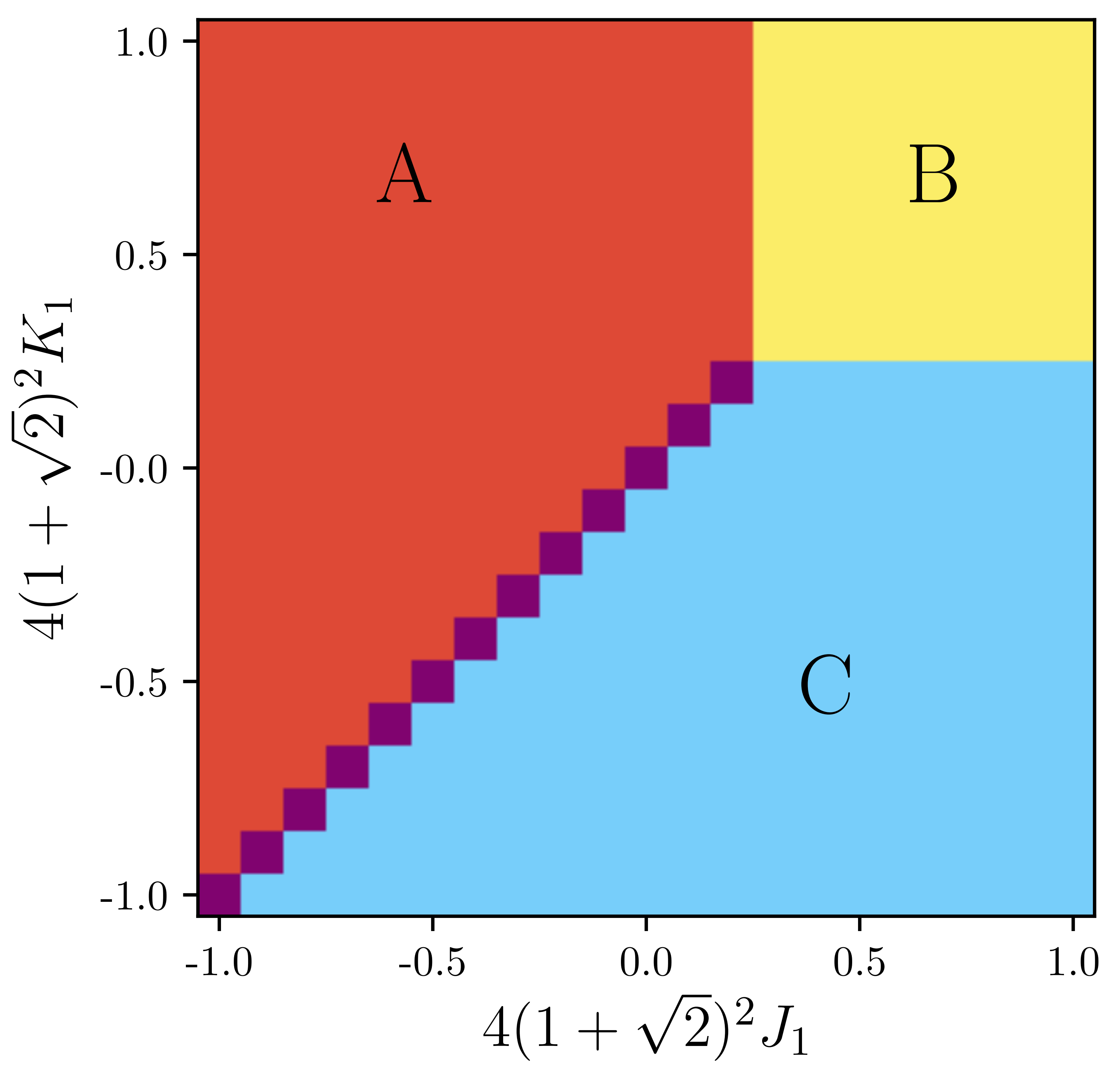
SU(2) gauge-invariant order parameters of Higgs phases:

$$
\begin{aligned}
x\text{-CDW}: &\quad \rho_{(\pi,0)} = B^*_{a+}B_{a+} - B^*_{a-}B_{a-} \\
y\text{-CDW}: &\quad \rho_{(0,\pi)} = B^*_{a+}B_{a-} + B^*_{a-}B_{a+} \\
d\text{-density wave}: &\quad D = i\left(B^*_{a+}B_{a-} - B^*_{a-}B_{a+}\right) \\
d\text{-wave superconductor}: &\quad \Delta = \varepsilon_{ab}B_{a+}B_{b-}
\end{aligned}
$$

Include charge fluctuations at half-filling: confinement of SU(2) gauge theory

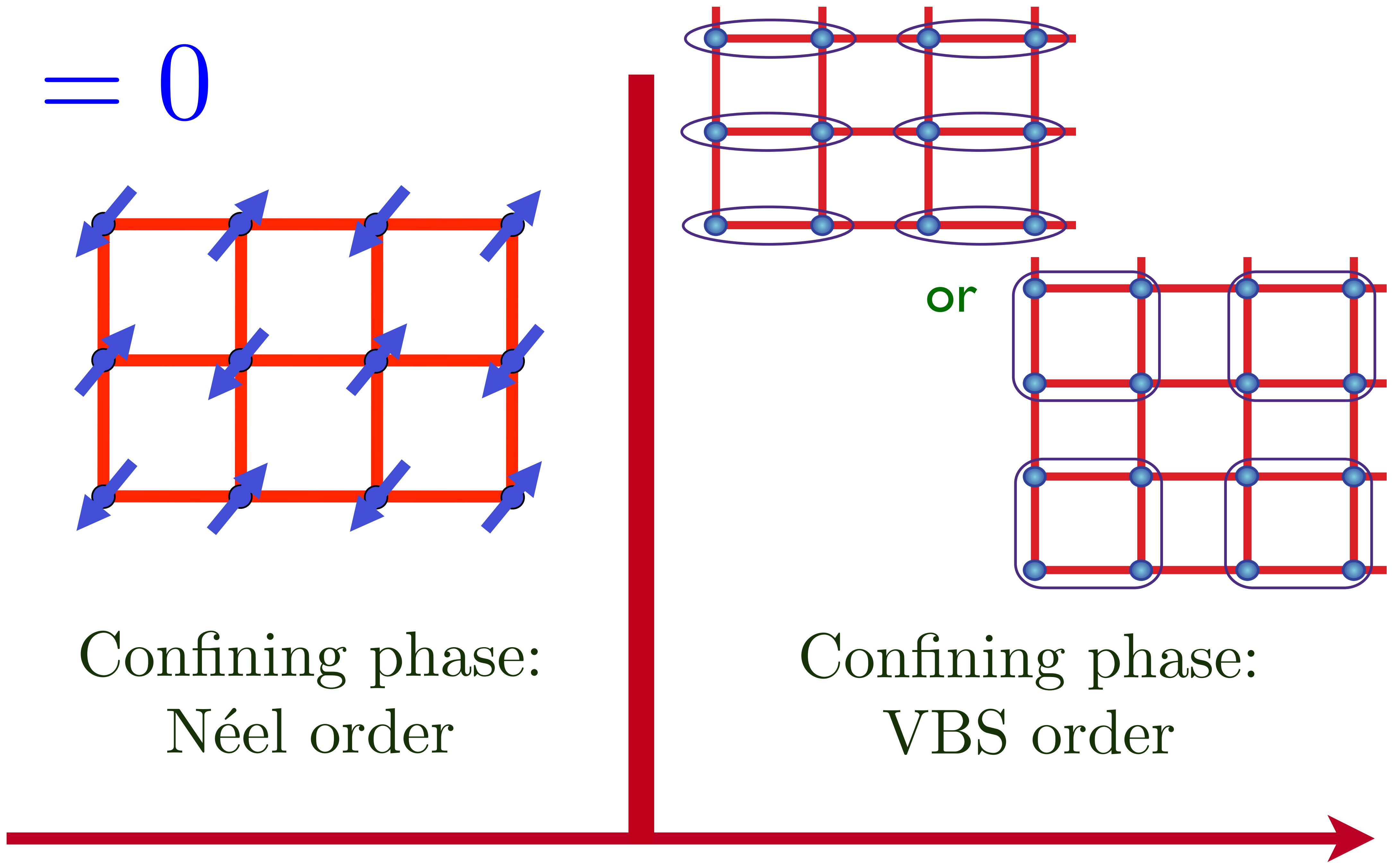


Include charge fluctuations at half-filling: confinement of SU(2) gauge theory



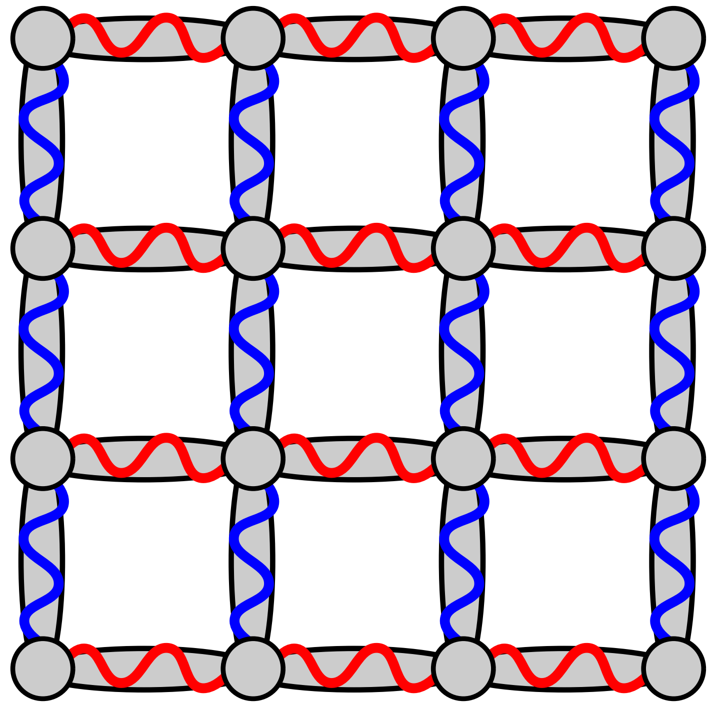
$\langle B \rangle \neq 0$

$\langle B \rangle = 0$



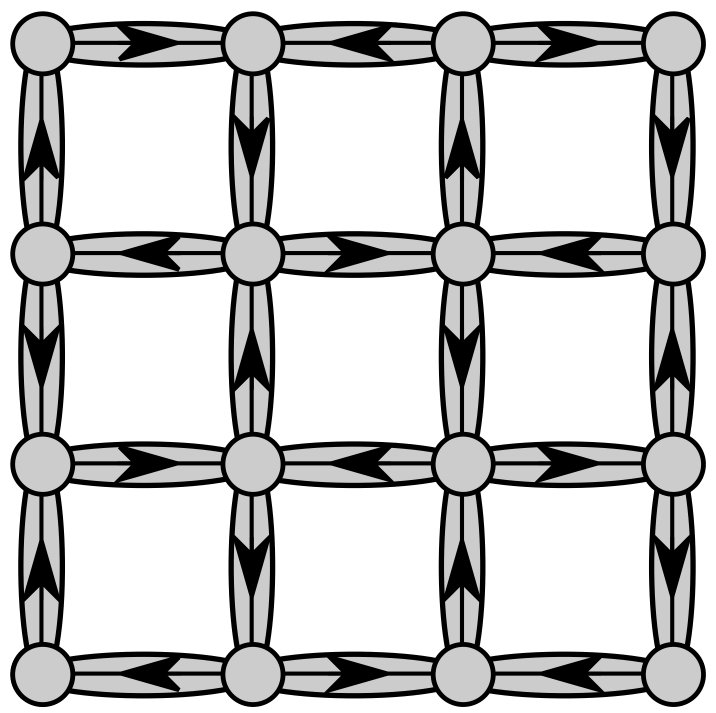
Phase B

d-wave SC



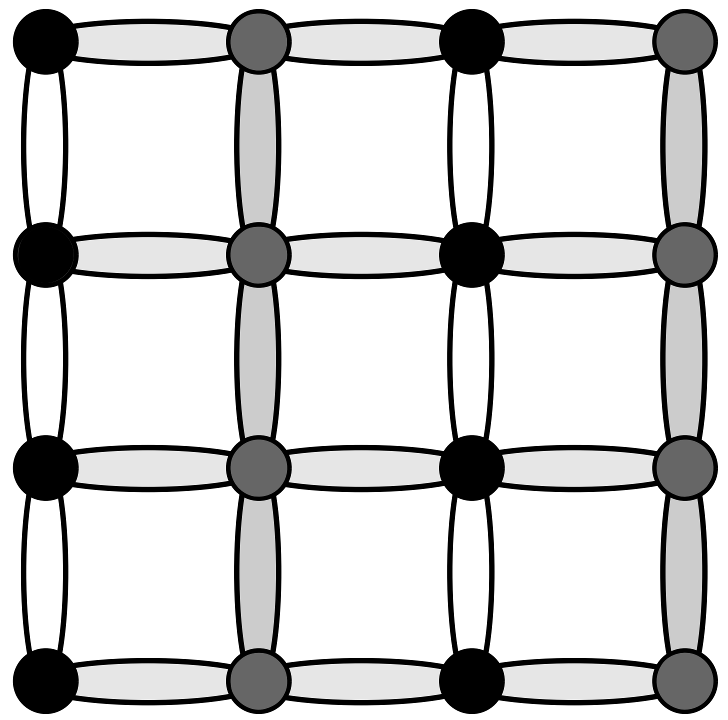
Phase C

d-density



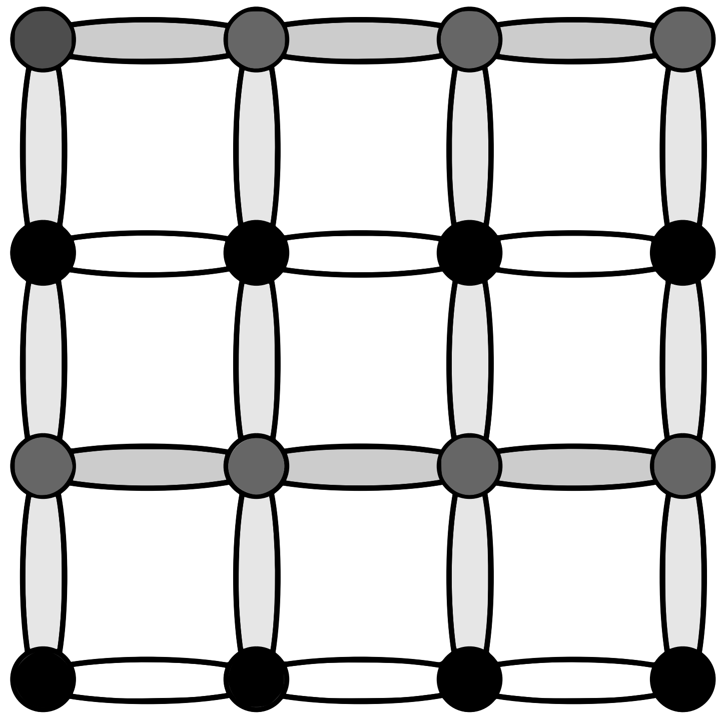
Phase A

(π,0) stripe

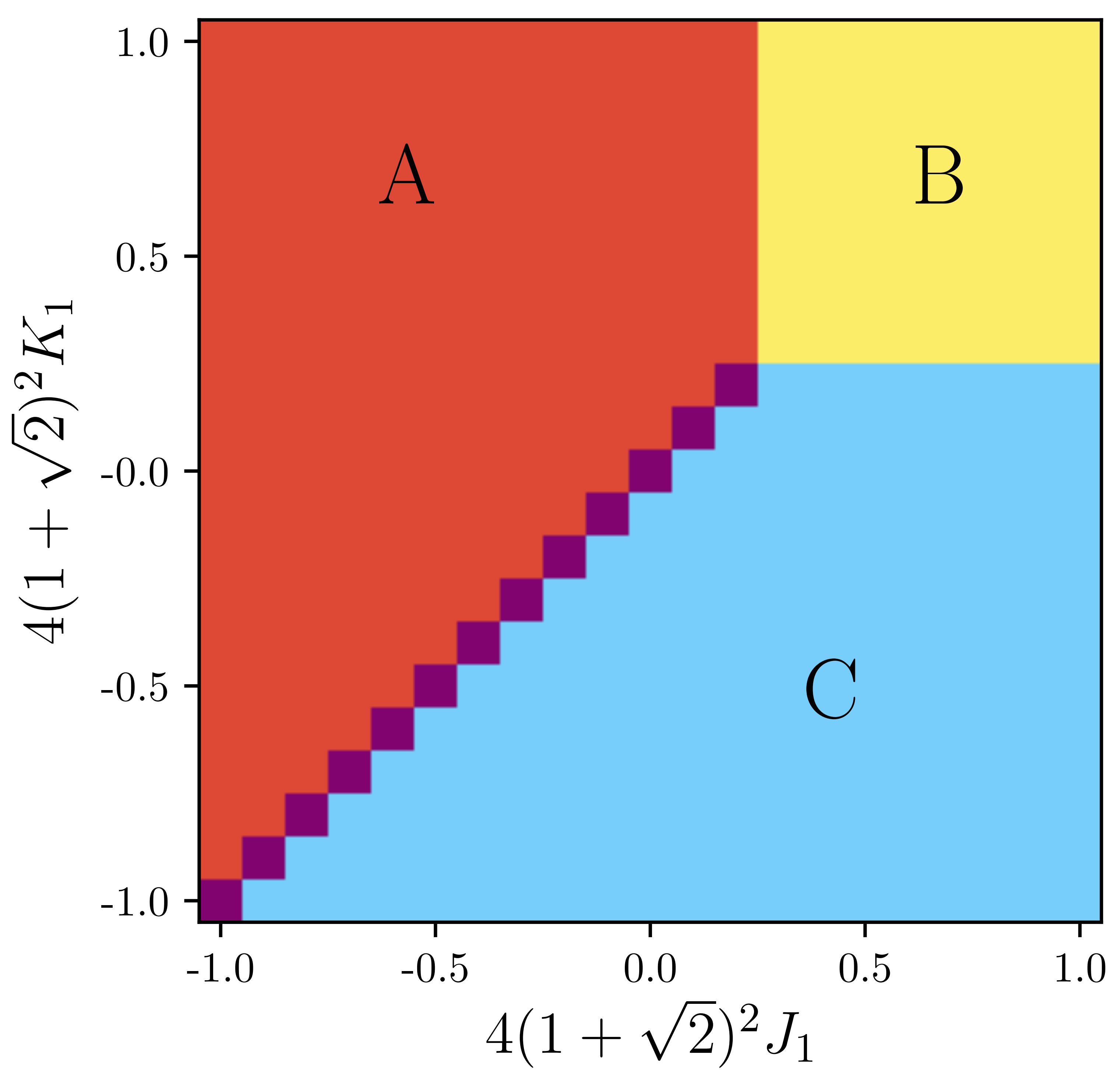


Phase A

(0,π) stripe



Include charge fluctuations at half-filling: confinement of SU(2) gauge theory



$\langle B \rangle = 0$

Confining phase.
$SO(5)_f$ broken.
Néel or
valence bond solid
order.

$\langle B \rangle \neq 0$

Higgs phase.
$SO(5)_b$ broken.
d-wave superconductivity or
period-2 stripes or
d-density wave order.

r $\qquad r_c$

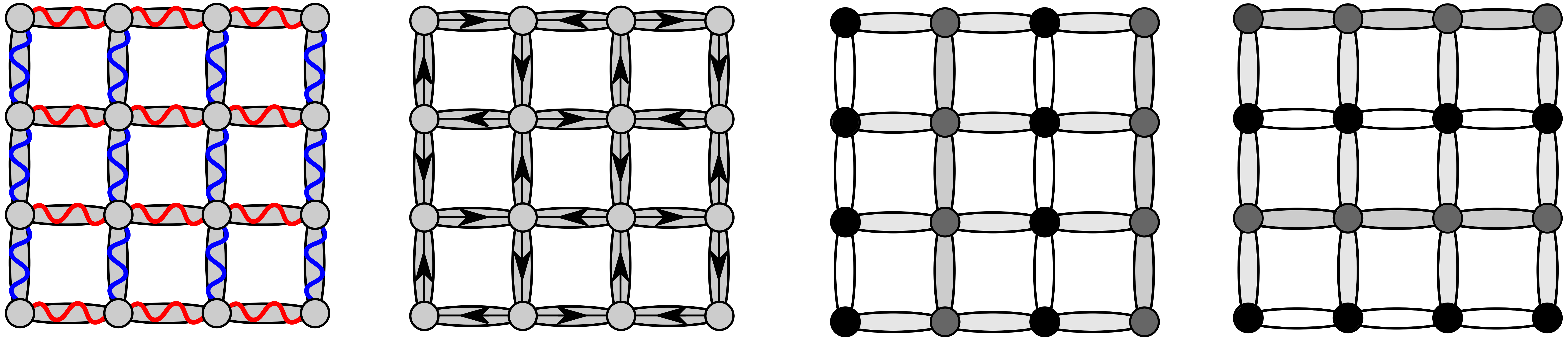
$$\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + |D_\mu B_s|^2 + r|B_s|^2 \ldots$$

Phase B d-wave SC

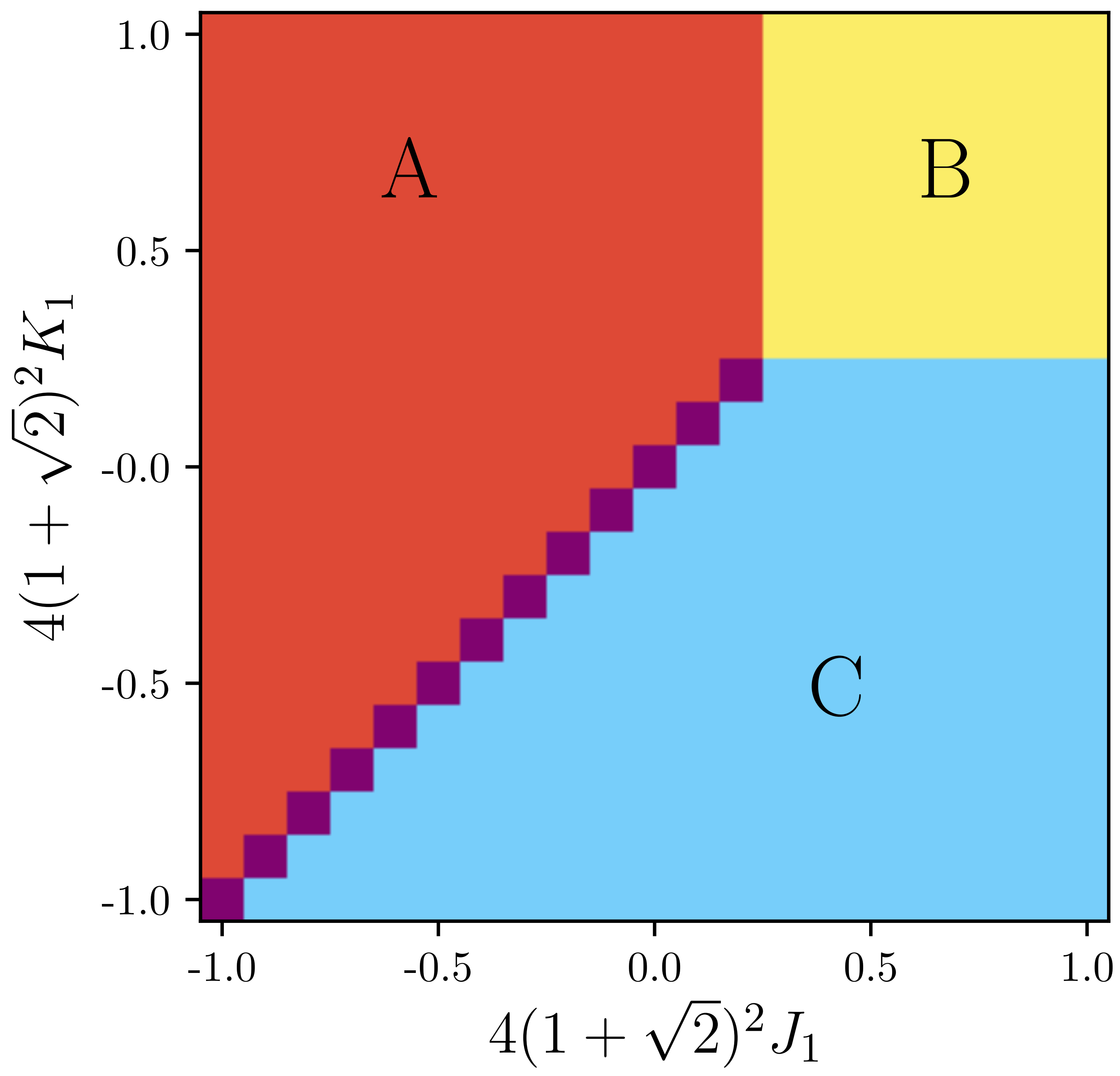
Phase C d-density

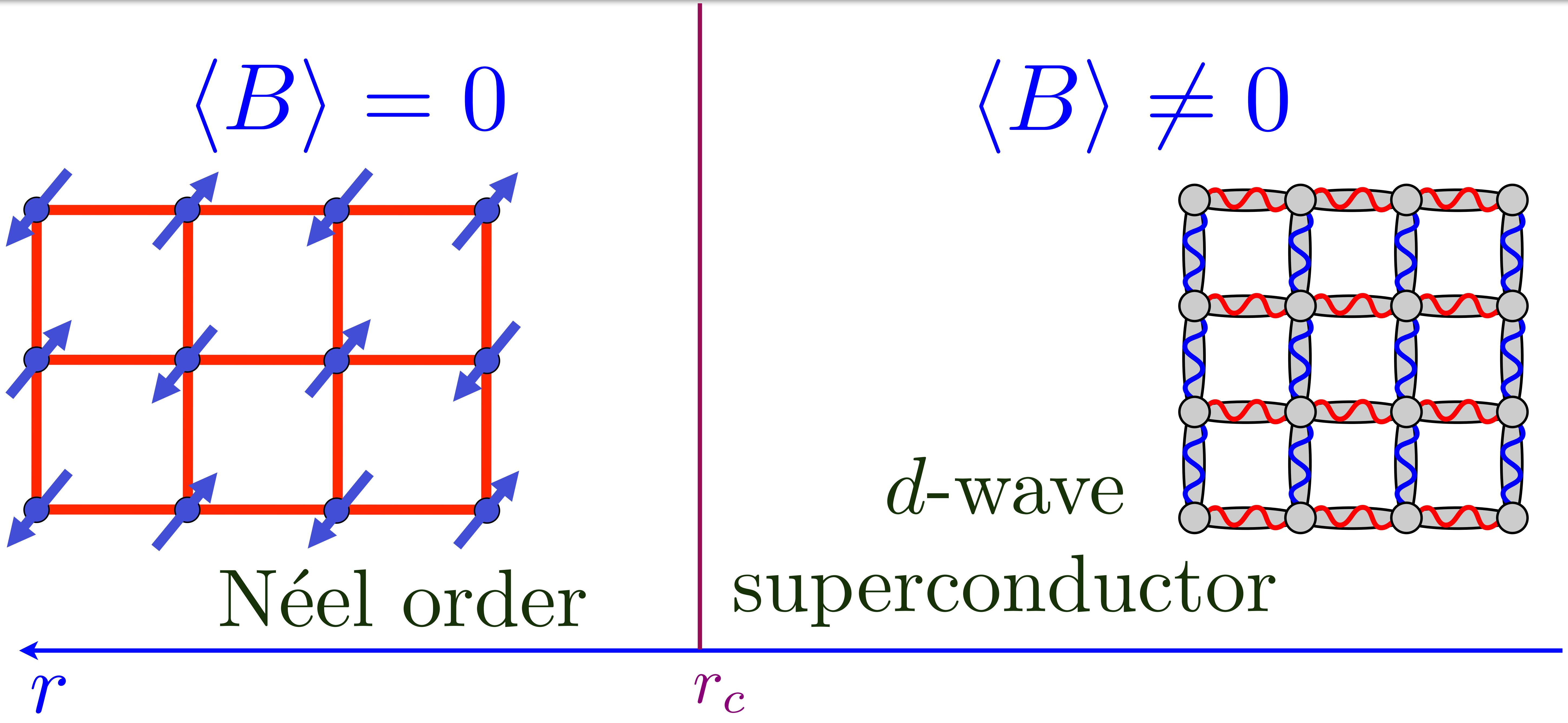
Phase A (π,0) stripe

Phase A (0,π) stripe

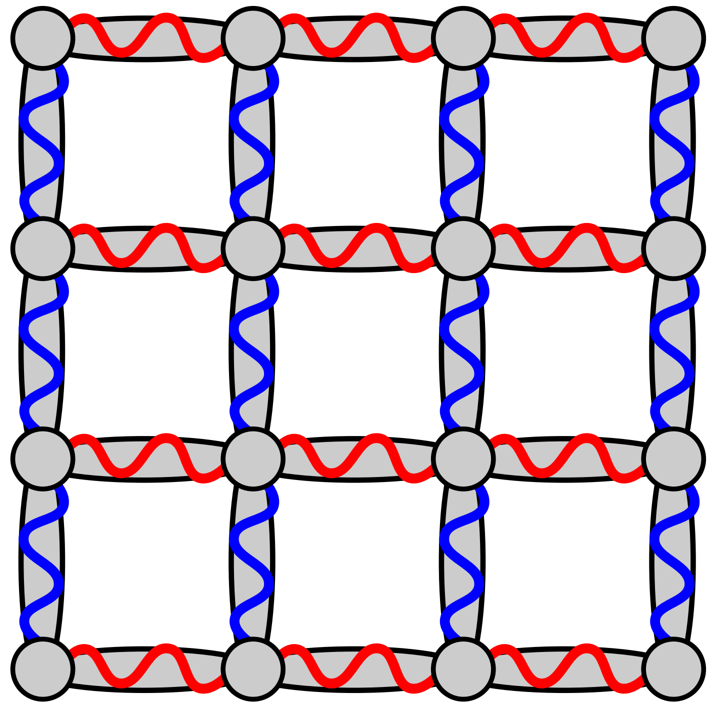


Include charge fluctuations at half-filling: confinement of SU(2) gauge theory

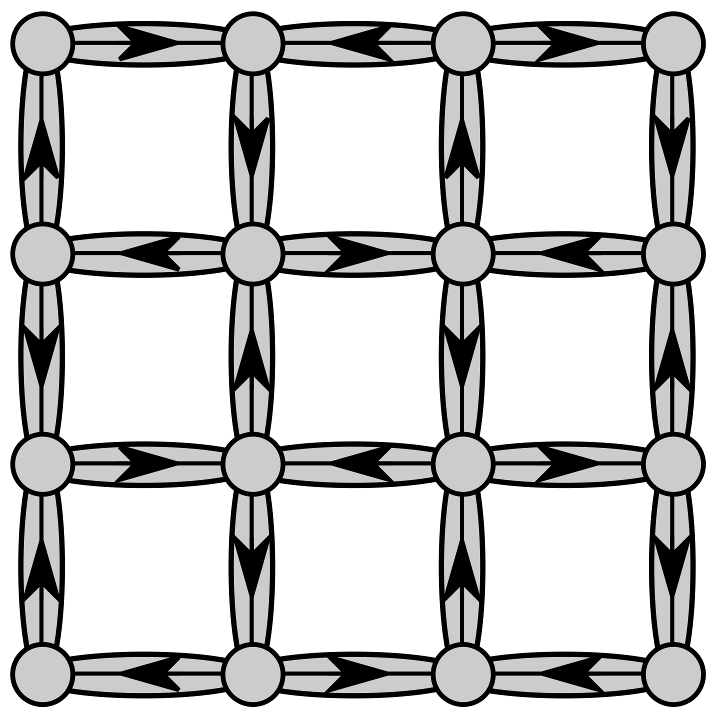




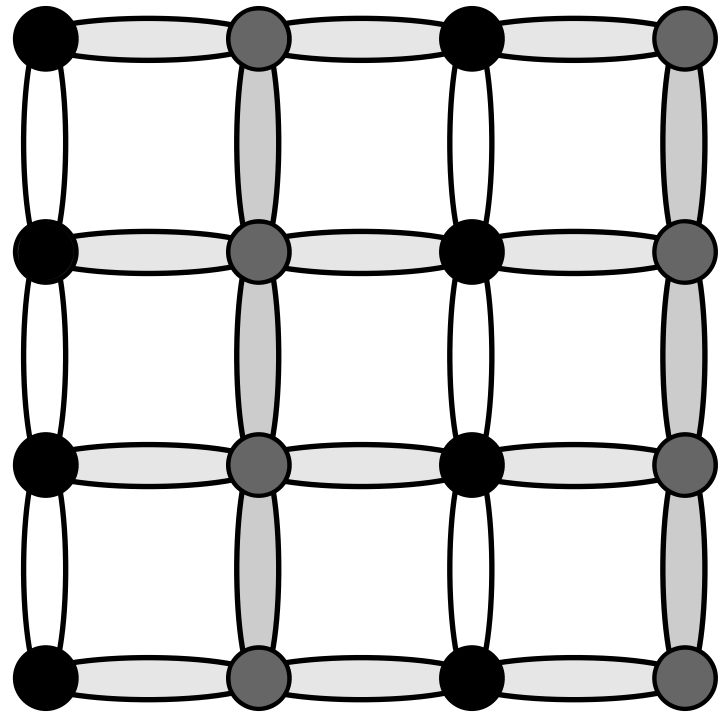
Phase B
d-wave SC



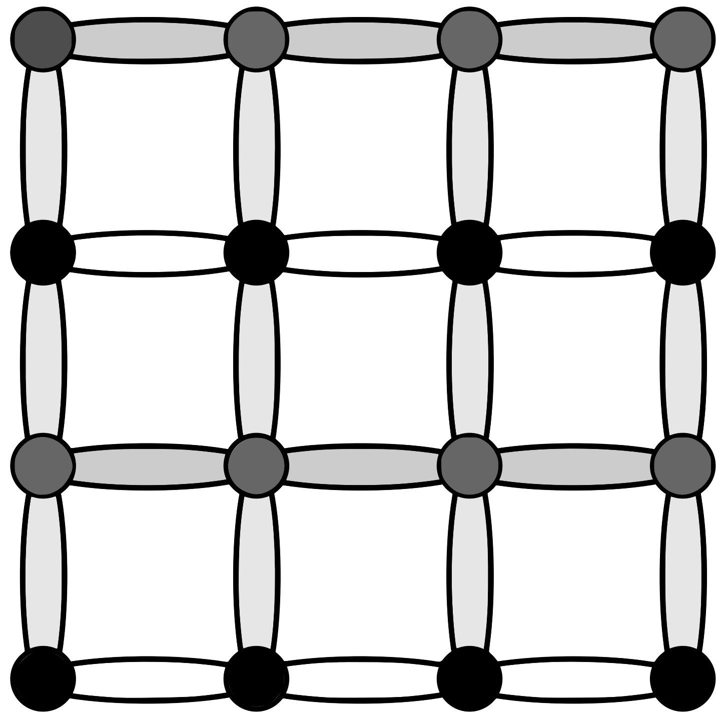
Phase C
d-density



Phase A
(π,0) stripe

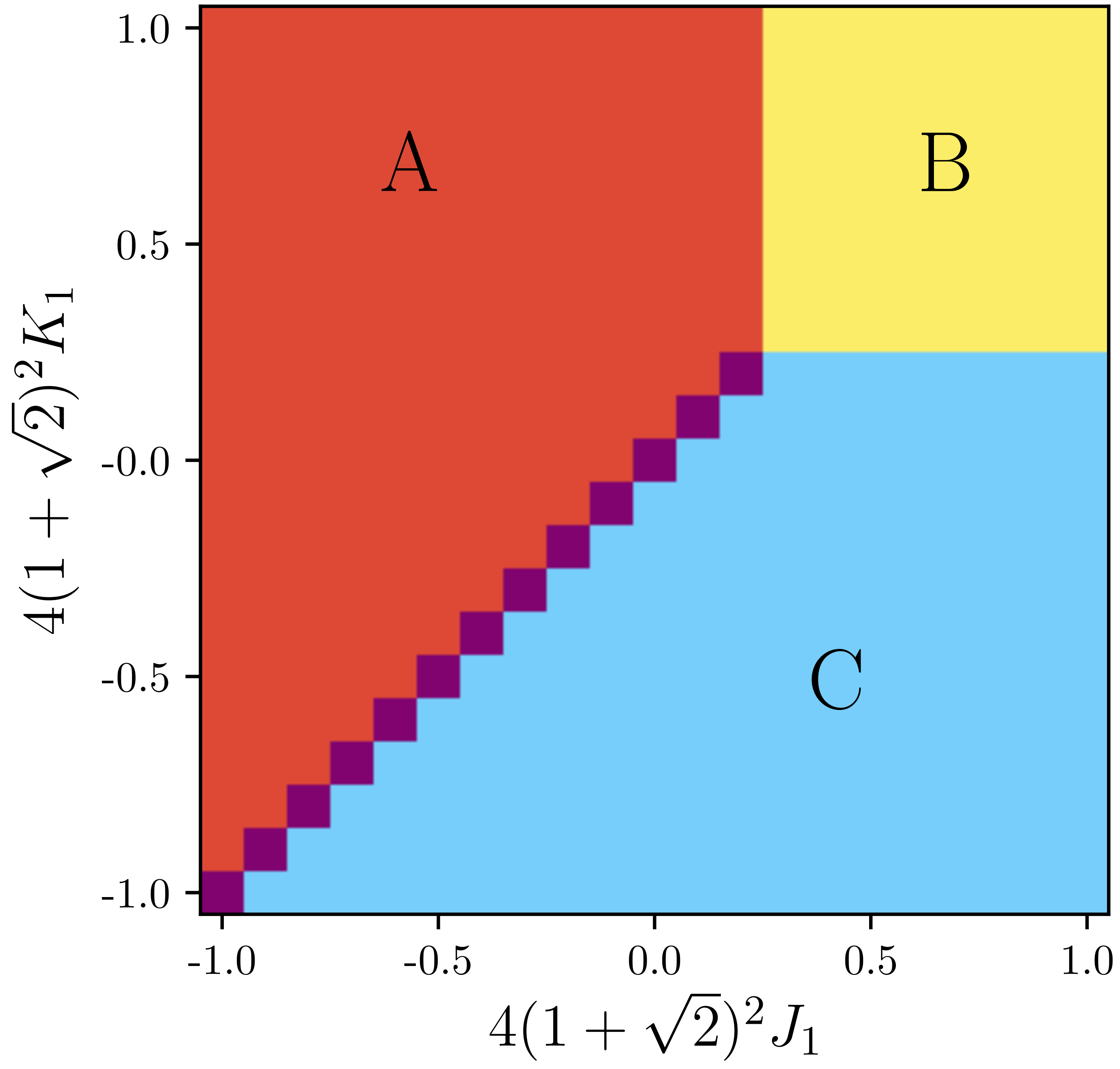


Phase A
(0,π) stripe



$$\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + |D_\mu B_s|^2 + r|B_s|^2 \ldots$$

Include charge fluctuations at half-filling: confinement of SU(2) gauge theory



$\langle B \rangle = 0$

Confining phase.
$SO(5)_f$ broken.
Néel or
valence bond solid
order.

$\langle B \rangle \neq 0$

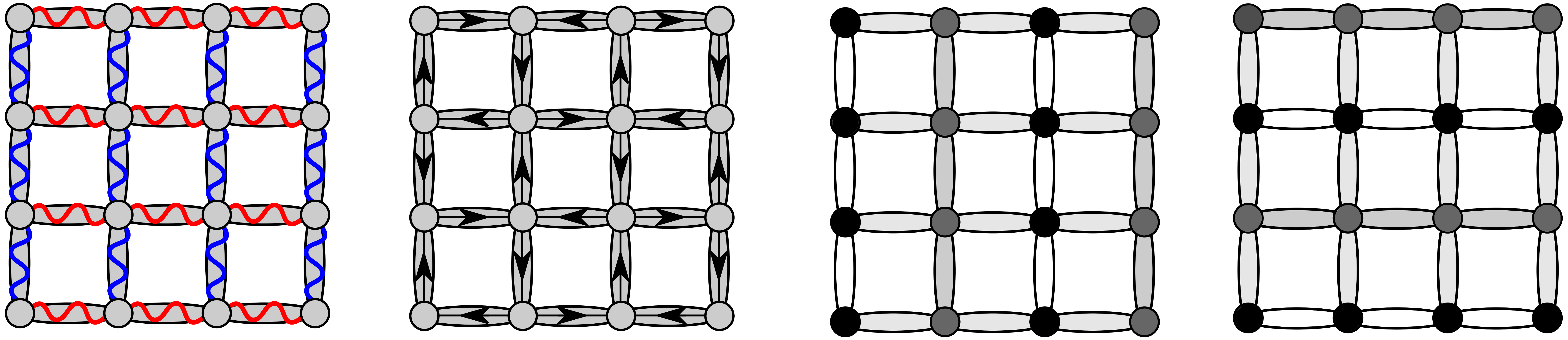
Higgs phase.
$SO(5)_b$ broken.
d-wave superconductivity or
period-2 stripes or
d-density wave order.

r $\quad$ r_c

$$\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + |D_\mu B_s|^2 + r|B_s|^2 \ldots$$

Possible CFT.
DQCP with
$SO(5)_f \times SO(5)_b$
symmetry.

Phase B d-wave SC | **Phase C** d-density | **Phase A** (π,0) stripe | **Phase A** (0,π) stripe



1. Confinement of the π-flux spin liquid at half-filling

2. Ancilla theory of the pseudogap metal

3. Confinement of the pseudogap metal at non-zero doping

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

We use the operator equation (valid on each site i):

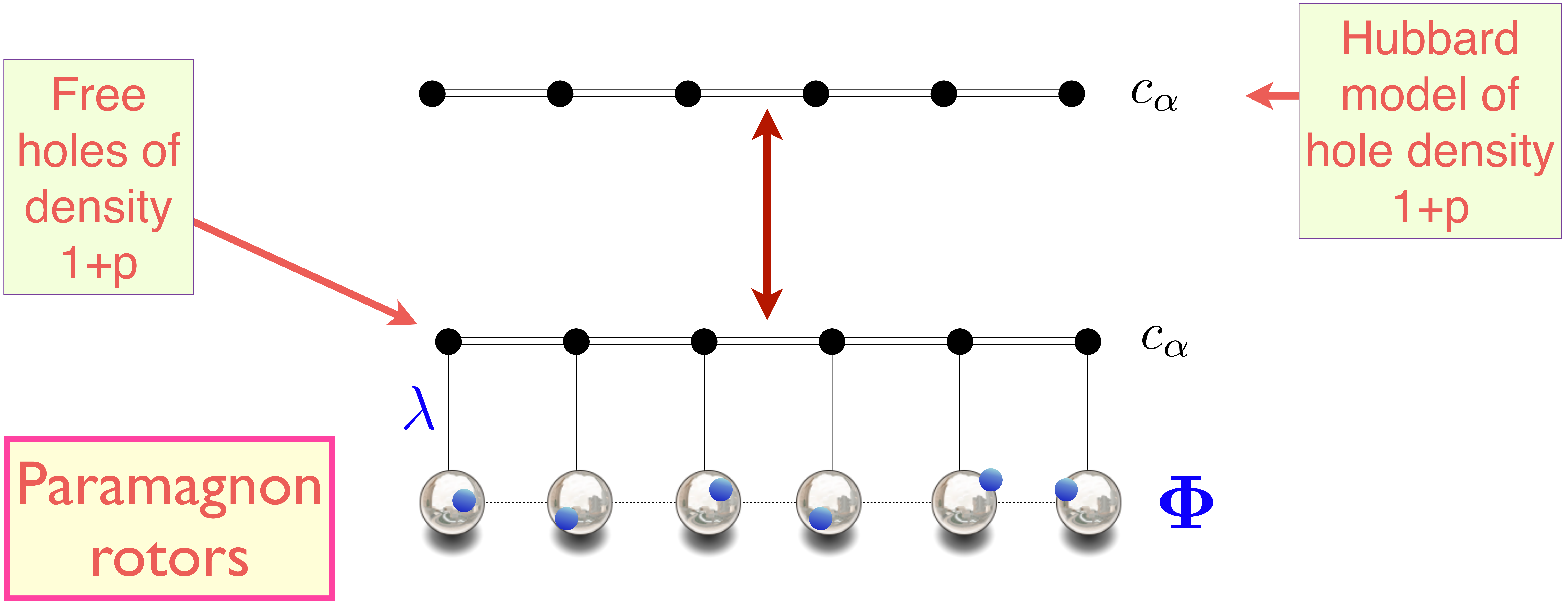
$$U\left(n_\uparrow - \frac{1}{2}\right)\left(n_\downarrow - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

Then we decouple the interaction via

$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\alpha}^\dagger \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'}\right]\right)$$

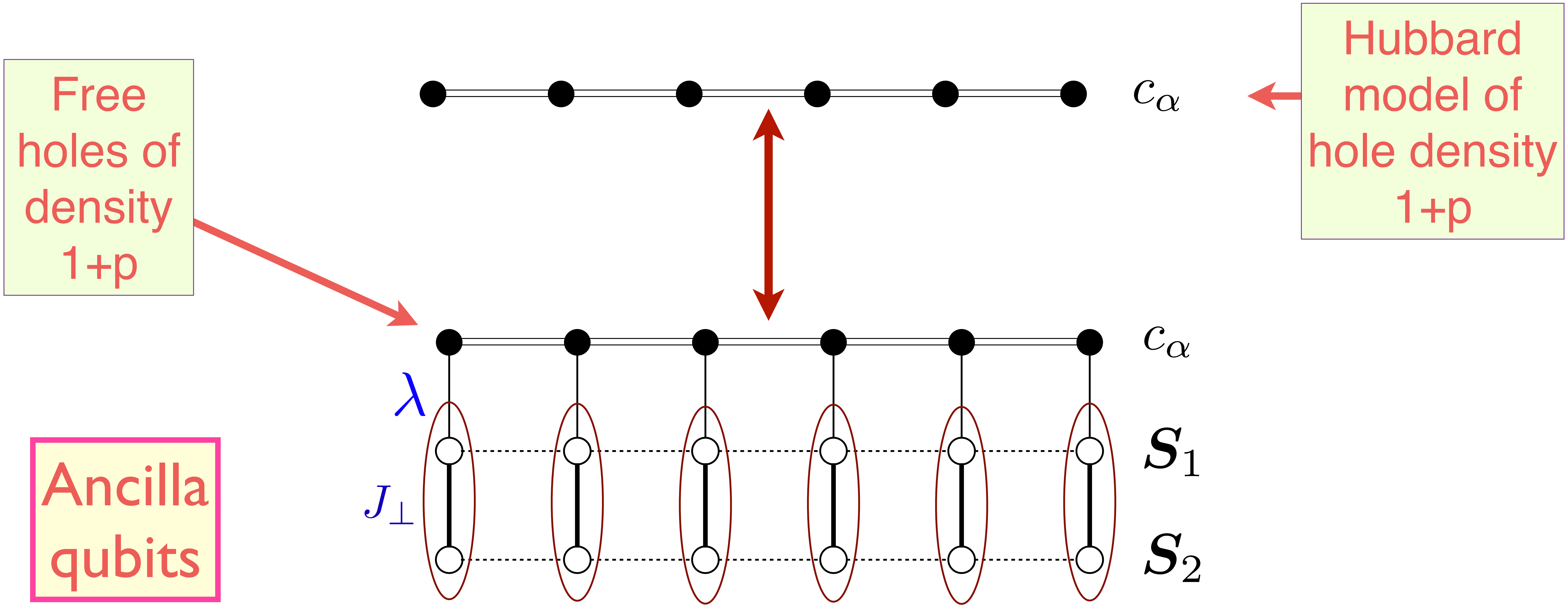
This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\alpha}$.

Paramagnon theory of the Hubbard model



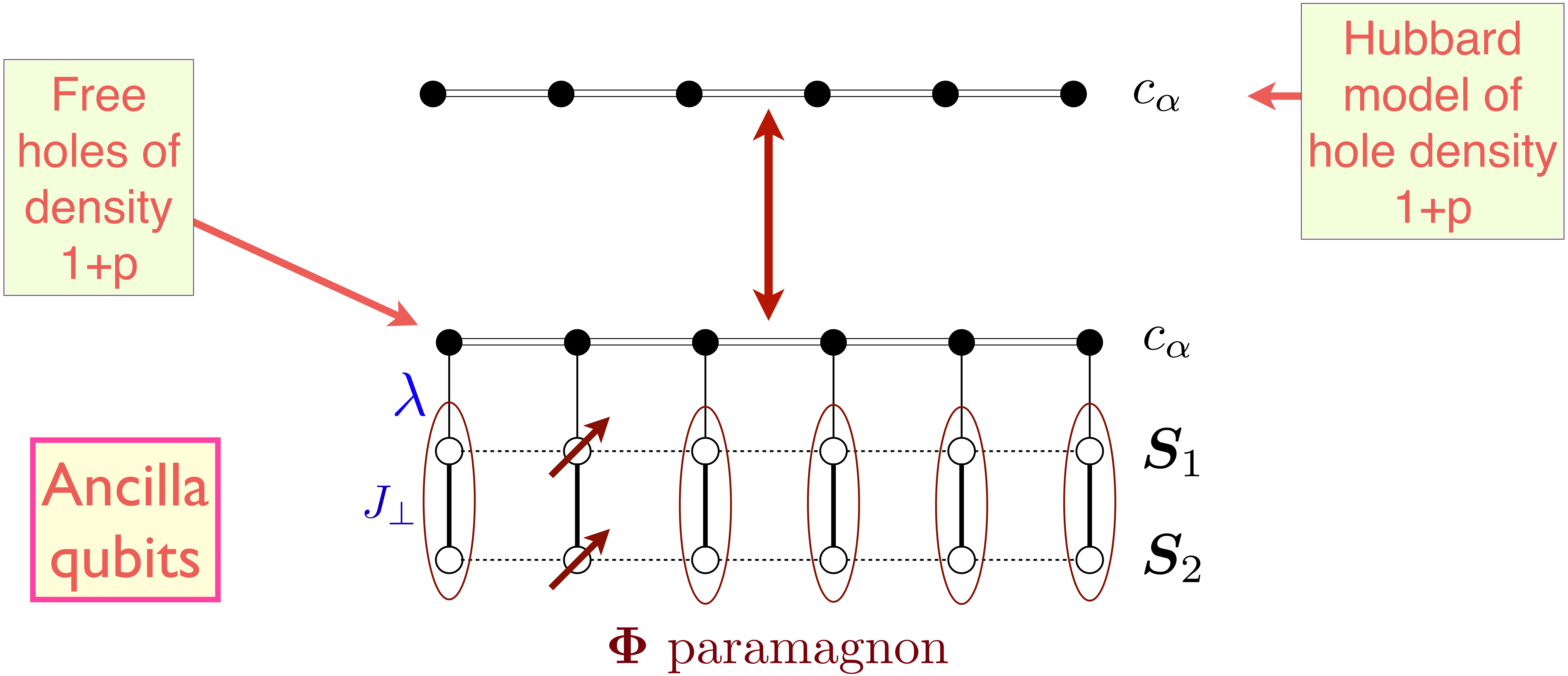
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} - \lambda \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \mathbf{\Phi}_i + \frac{J_\perp}{2} \sum_i \boldsymbol{P}_{\mathbf{\Phi}i}^2 + \sum_i V(\mathbf{\Phi}_i) + \ldots$$

Paramagnon theory of the Hubbard model



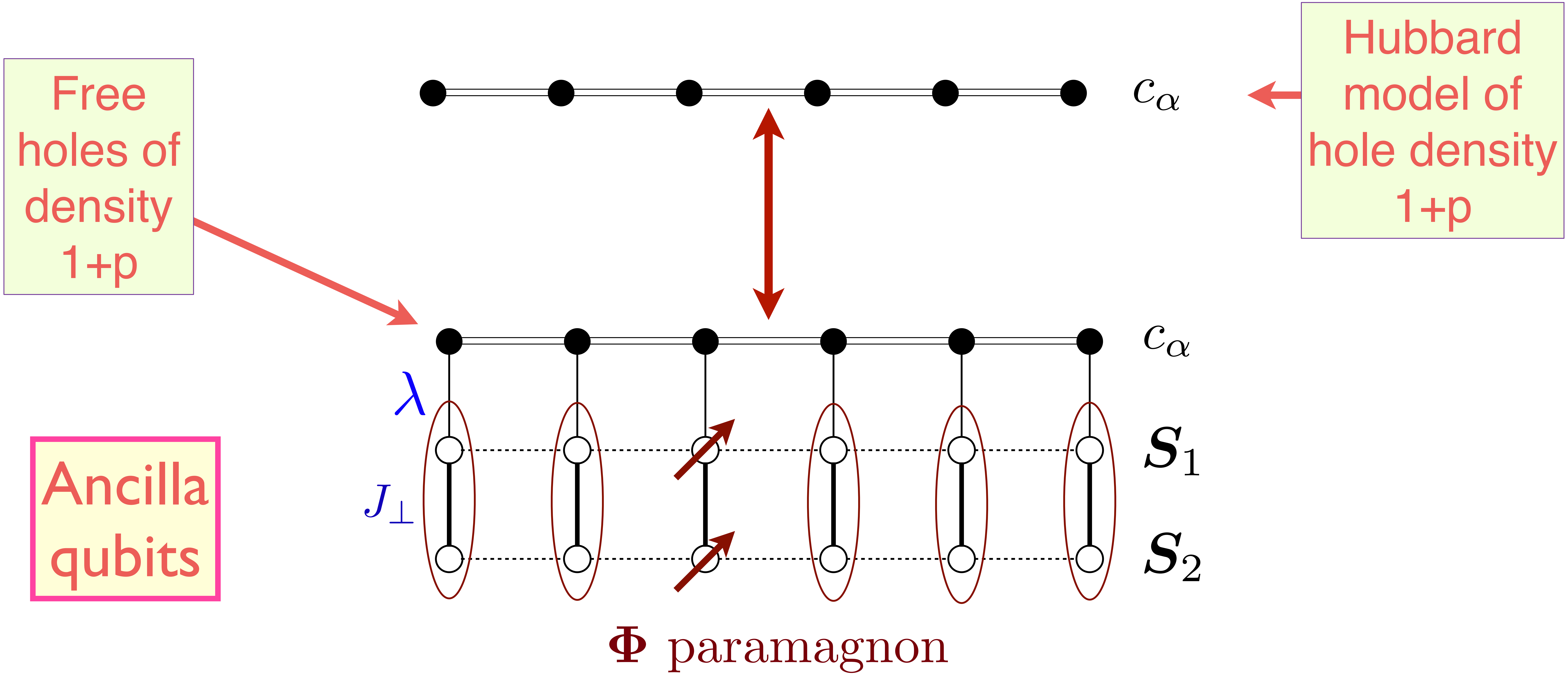
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} - \lambda \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{\Phi}_i + \frac{J_\perp}{2} \sum_i \boldsymbol{P}_{\boldsymbol{\Phi}i}^2 + \sum_i V(\boldsymbol{\Phi}_i) + \ldots$$

Paramagnon theory of the Hubbard model



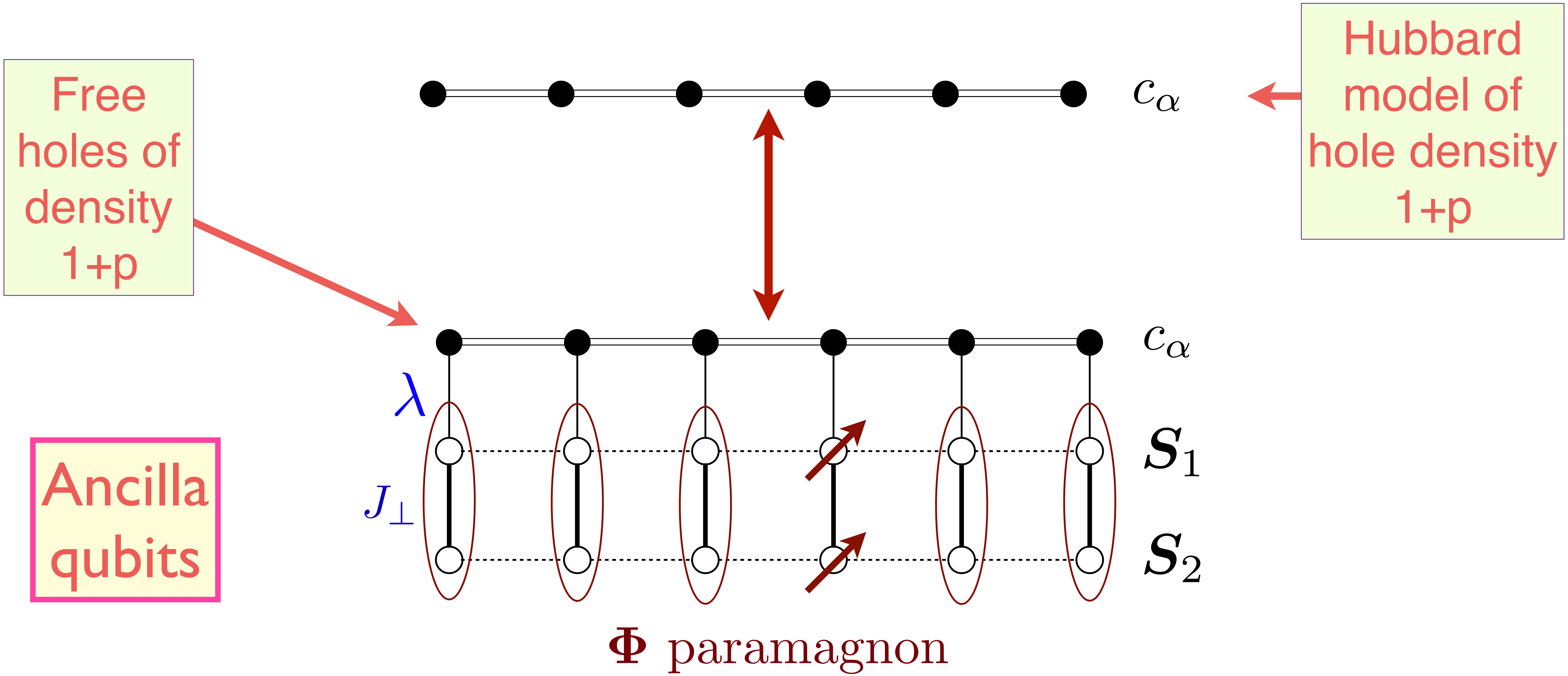
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} - \lambda \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{\Phi}_i + \frac{J_\perp}{2} \sum_i \boldsymbol{P}^2_{\boldsymbol{\Phi}i} + \sum_i V(\boldsymbol{\Phi}_i) + \ldots$$

Paramagnon theory of the Hubbard model



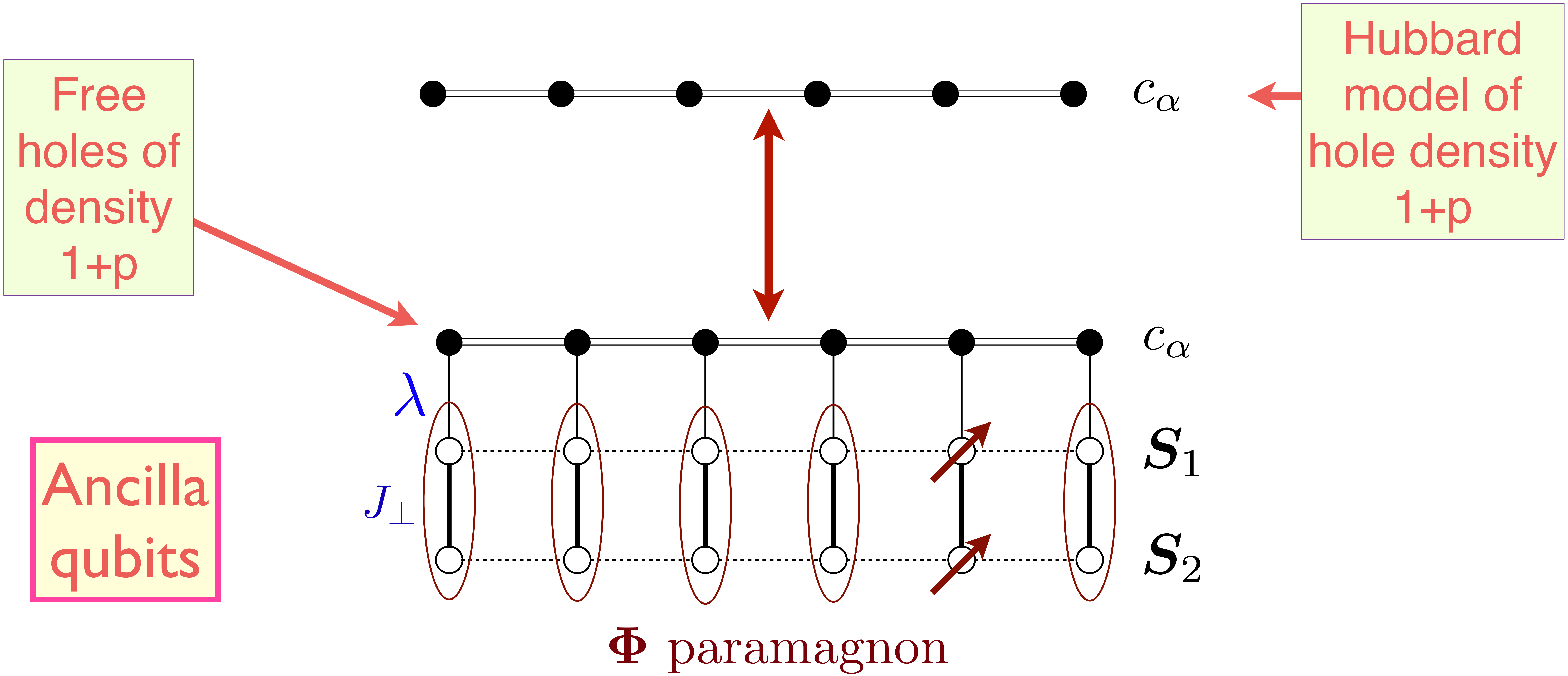
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} - \lambda \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{\Phi}_i + \frac{J_\perp}{2} \sum_i \boldsymbol{P}_{\boldsymbol{\Phi} i}^2 + \sum_i V(\boldsymbol{\Phi}_i) + \ldots$$

Paramagnon theory of the Hubbard model



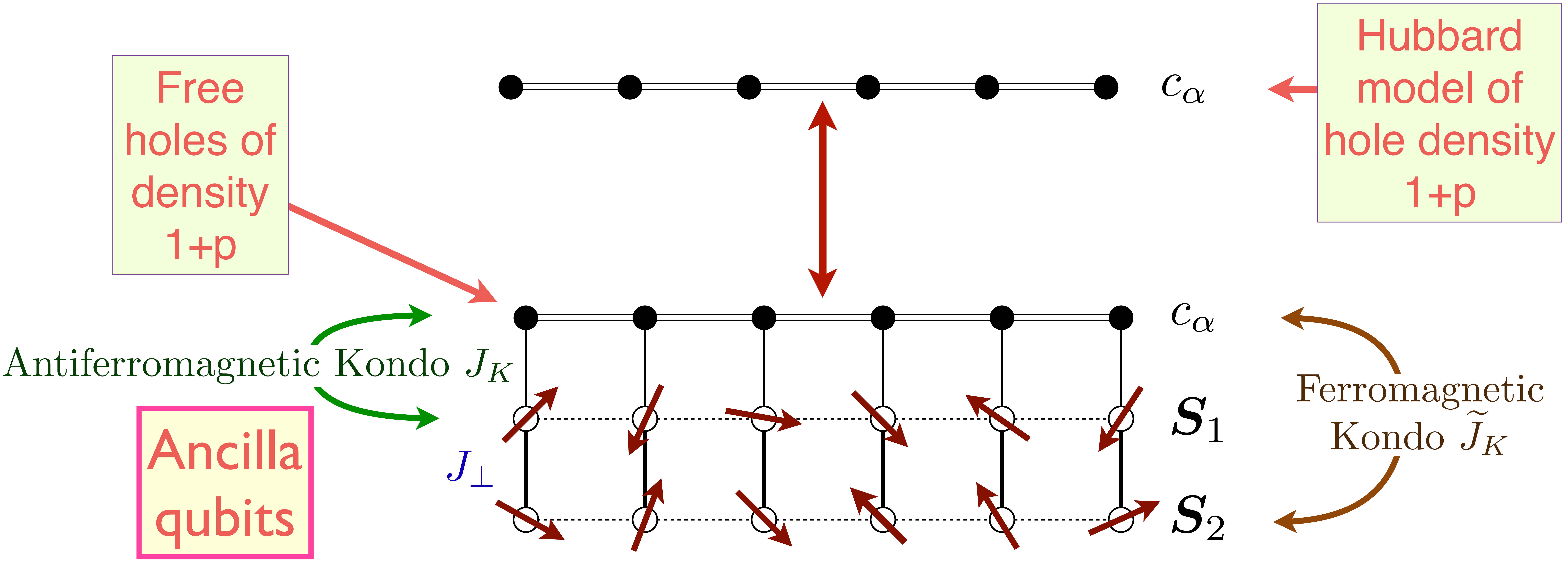
$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} - \lambda \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{\Phi}_i + \frac{J_\perp}{2} \sum_i \boldsymbol{P}^2_{\boldsymbol{\Phi}i} + \sum_i V(\boldsymbol{\Phi}_i) + \ldots$$

Paramagnon theory of the Hubbard model



$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} - \lambda \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{\Phi}_i + \frac{J_\perp}{2} \sum_i \boldsymbol{P}_{\boldsymbol{\Phi} i}^2 + \sum_i V(\boldsymbol{\Phi}_i) + \ldots$$

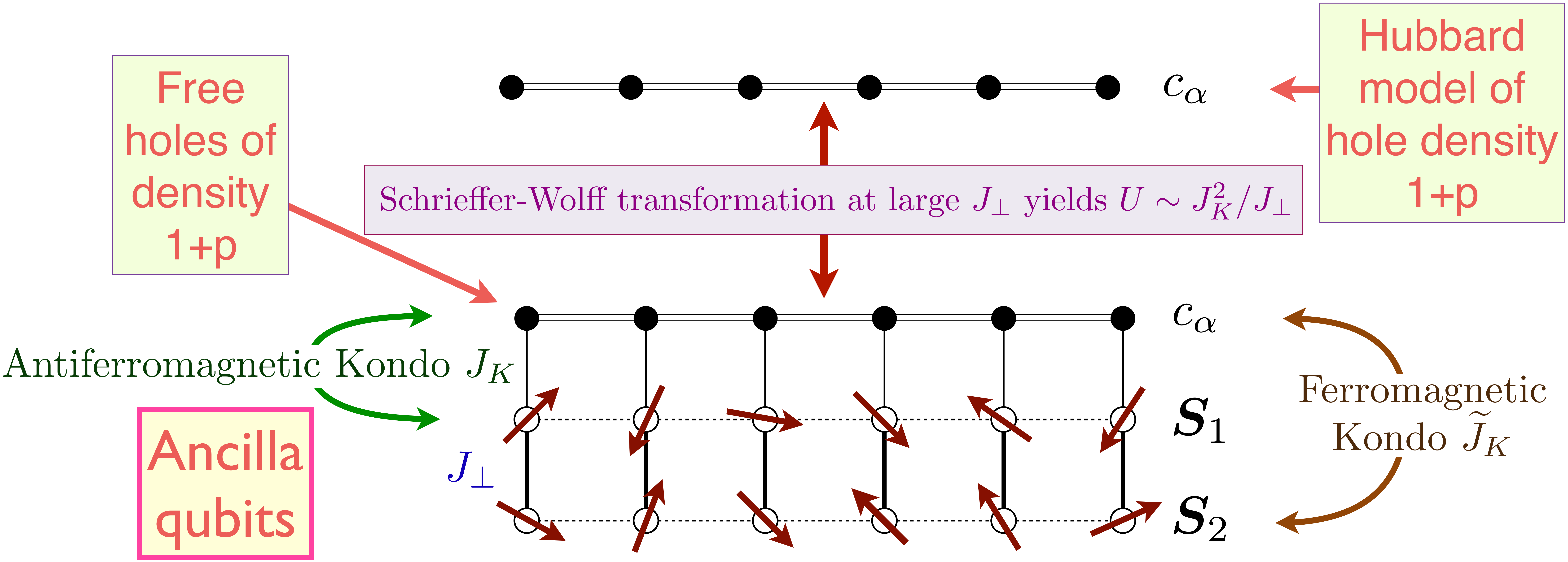
Paramagnon fractionalization theory of the Hubbard model



$$\boldsymbol{\Phi}_i = \frac{1}{\sqrt{3}}\left(\boldsymbol{S}_{2i} - \boldsymbol{S}_{1i}\right)$$

$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} - \widetilde{J}_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{2i} + \ldots$$

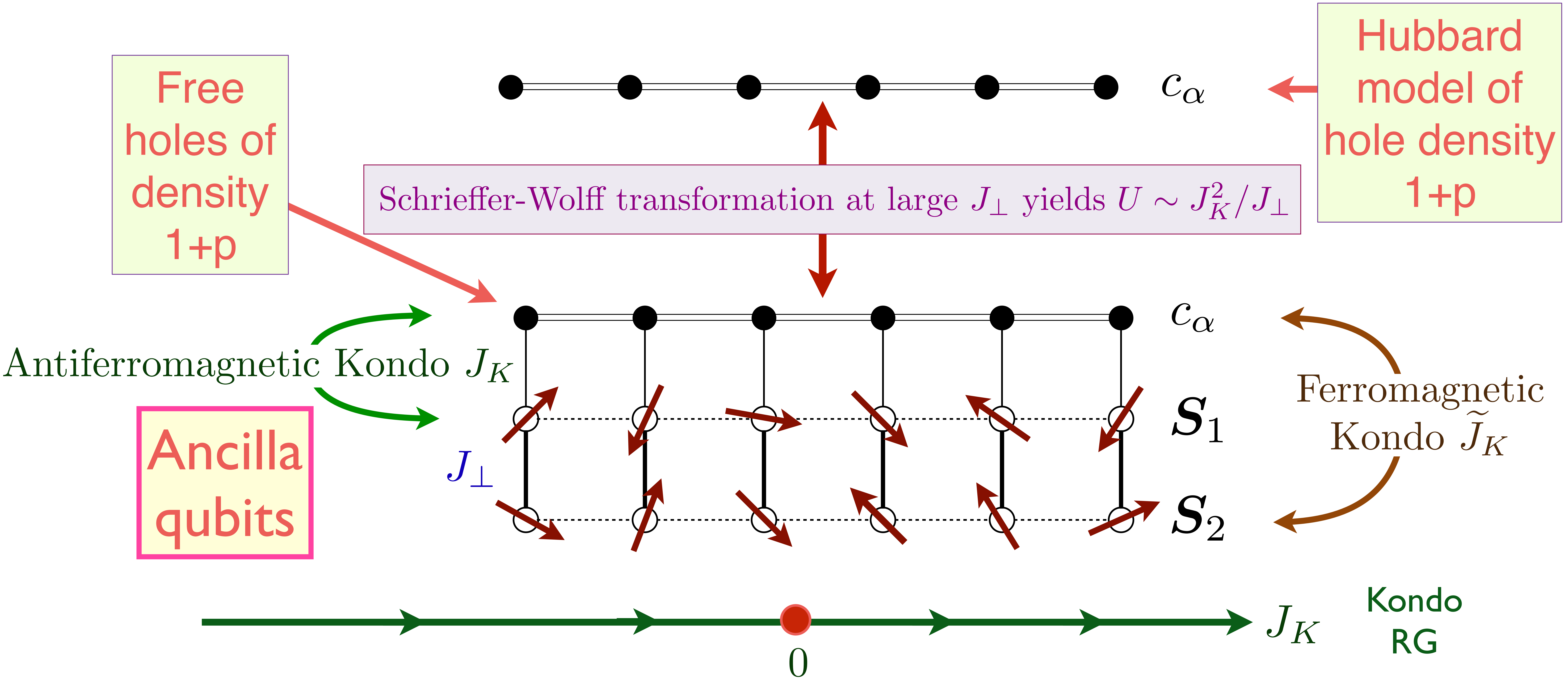
Paramagnon fractionalization theory of the Hubbard model



$$\boldsymbol{\Phi}_i = \frac{1}{\sqrt{3}}\left(\boldsymbol{S}_{2i} - \boldsymbol{S}_{1i}\right)$$

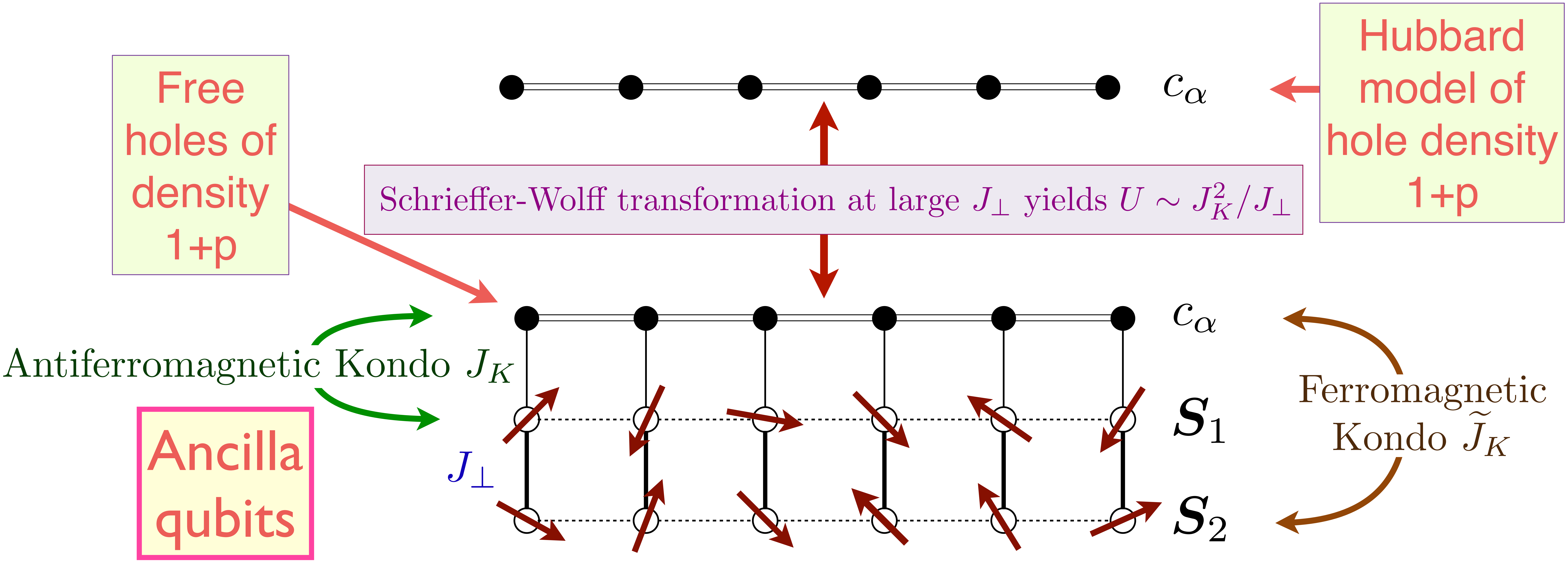
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} - \widetilde{J}_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{2i} + \ldots$$

Paramagnon fractionalization theory of the Hubbard model



$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + J_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} - \widetilde{J}_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\tau}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{2i} + \ldots$$

Paramagnon fractionalization theory of the Hubbard model



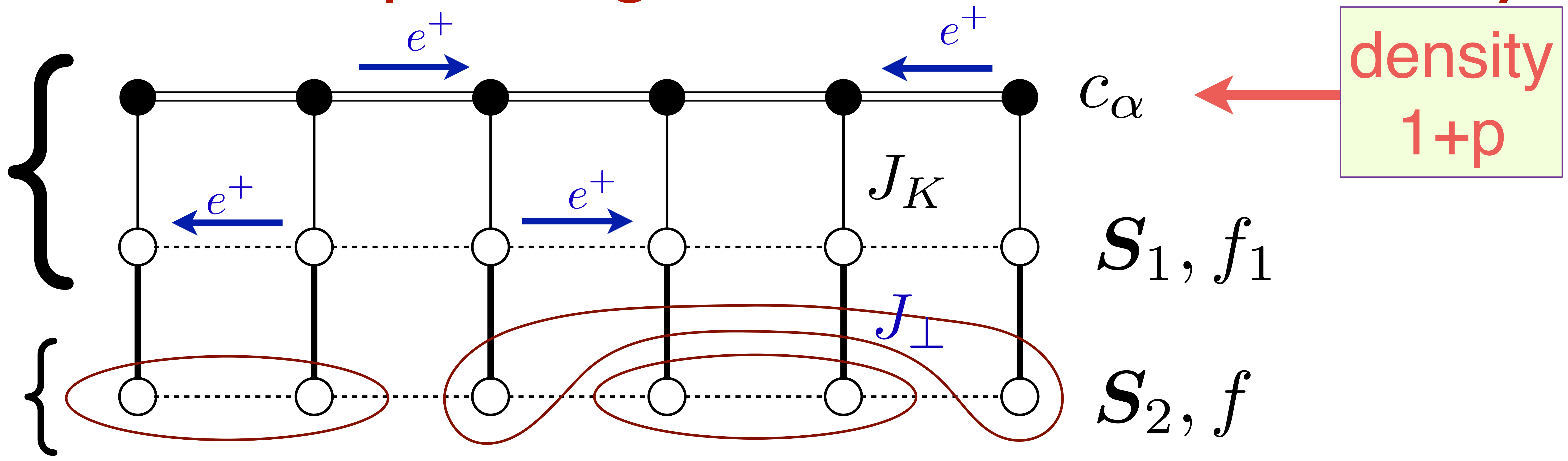
A FL* state is realized when the antiferromagnetic Kondo coupling dominates over $J_\perp$, and the c_σ and $\boldsymbol{S}_1$ form a heavy Fermi liquid state (as found in the heavy fermion compounds) of hole density $(1+p)+1 = 2+p = p$ mod 2!

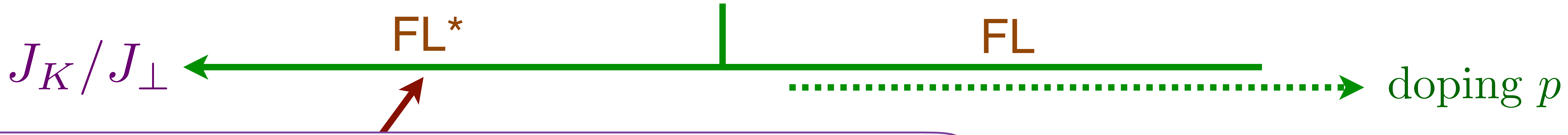
The $\boldsymbol{S}_2$ must form an 'odd' spin liquid which does not break translational symmetry, to obtain a metal with a non-Luttinger volume Fermi surface.

Trial wavefunctions in the paramagnon fractionalization theory

Small Fermi surface.
Size $2 + p \cong p$

Spin liquid

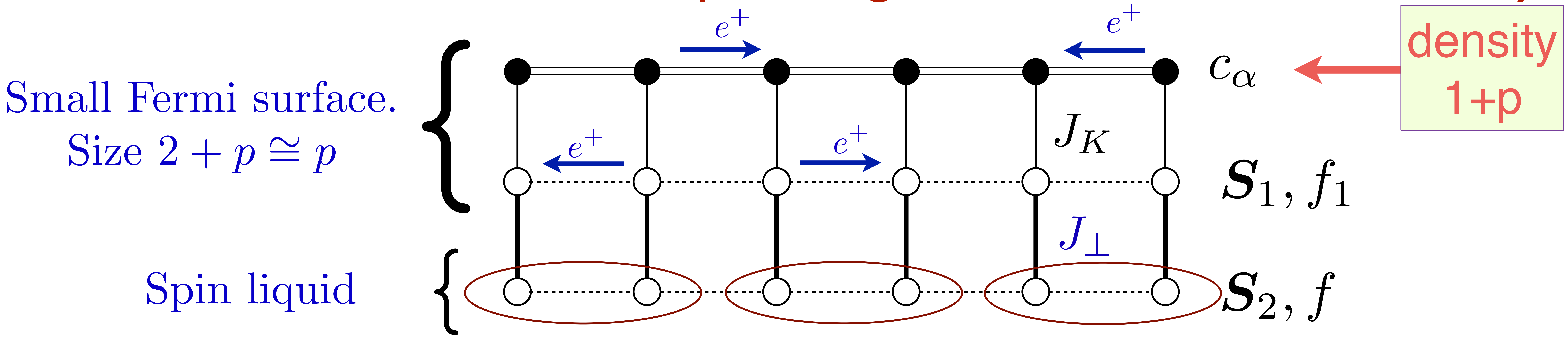


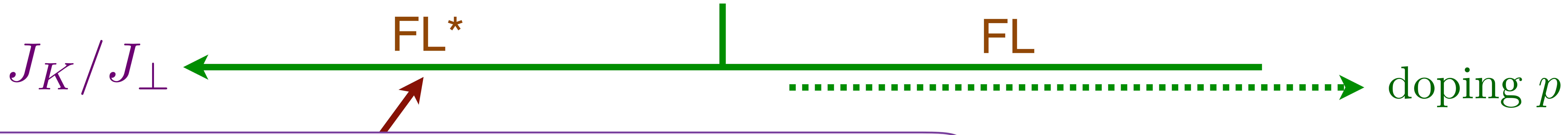


Small Fermi surface of size p

$$|\text{FL}*\rangle = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2]$$
$$\bowtie |\text{Slater determinant of } (c, f_1)\rangle$$
$$\otimes |\text{Spin liquid of } \boldsymbol{S}_2\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory

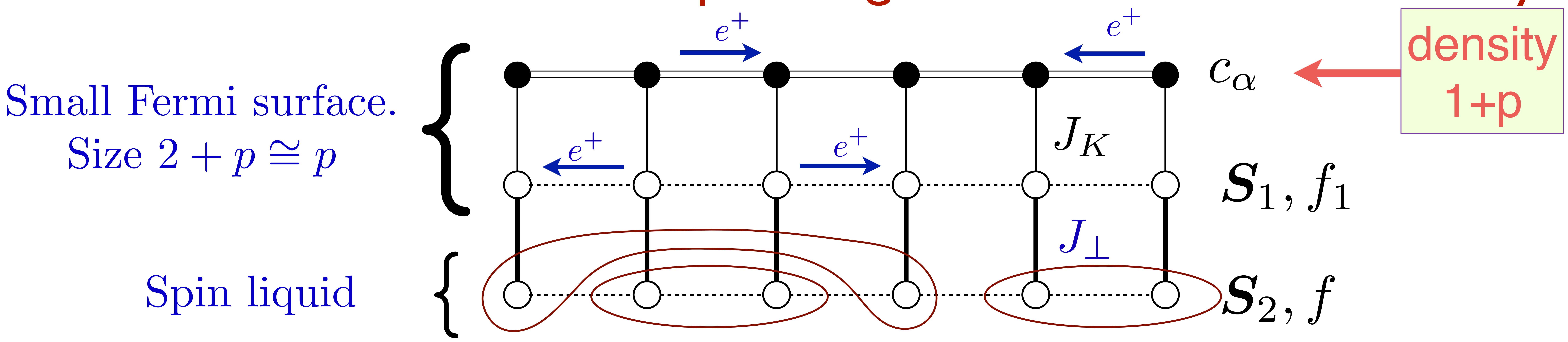


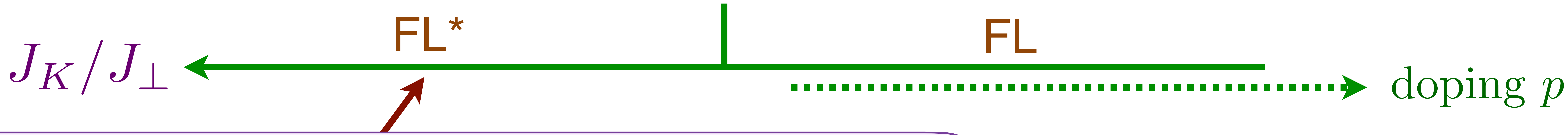


Small Fermi surface of size p

$$|\text{FL}*\rangle = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2]$$
$$\bowtie |\text{Slater determinant of } (c, f_1)\rangle$$
$$\otimes |\text{Spin liquid of } \boldsymbol{S}_2\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory





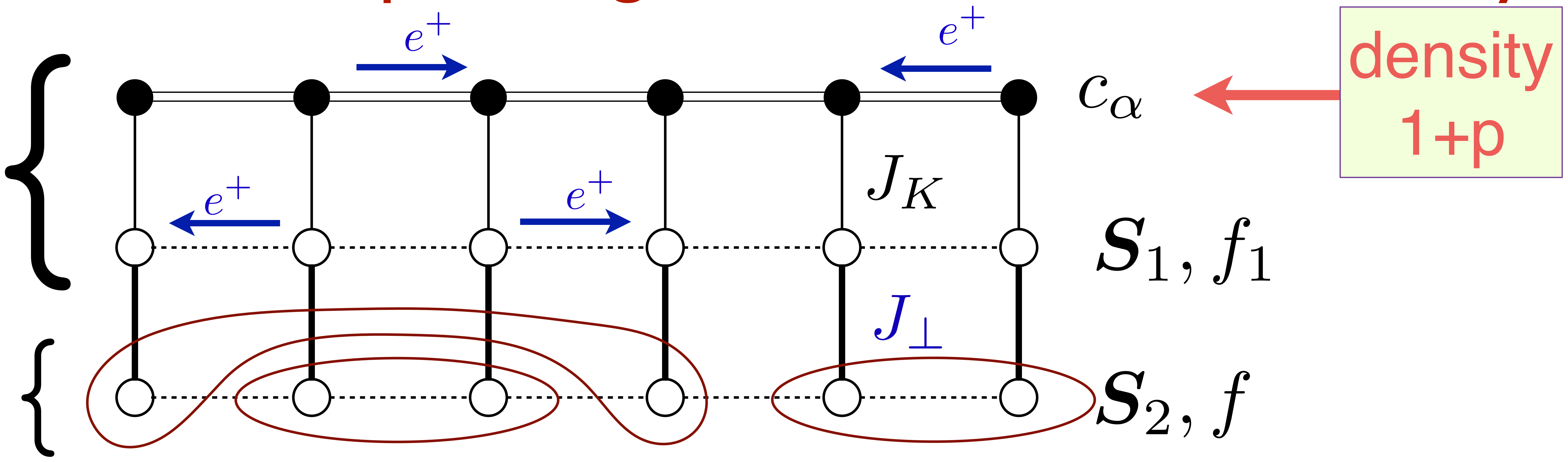
Small Fermi surface of size p

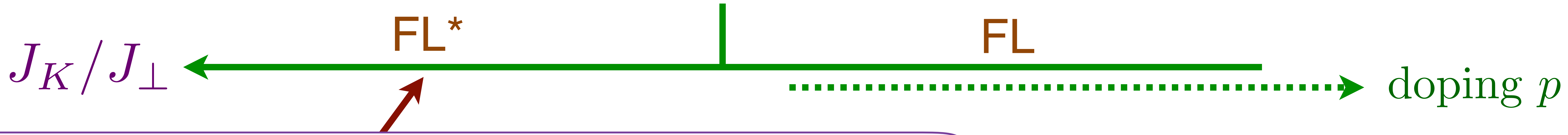
$$|\text{FL}*\rangle = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \\ \bowtie |\text{Slater determinant of } (c, f_1)\rangle \\ \otimes |\text{Spin liquid of } \boldsymbol{S}_2\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory

Small Fermi surface.
Size $2 + p \cong p$

Spin liquid



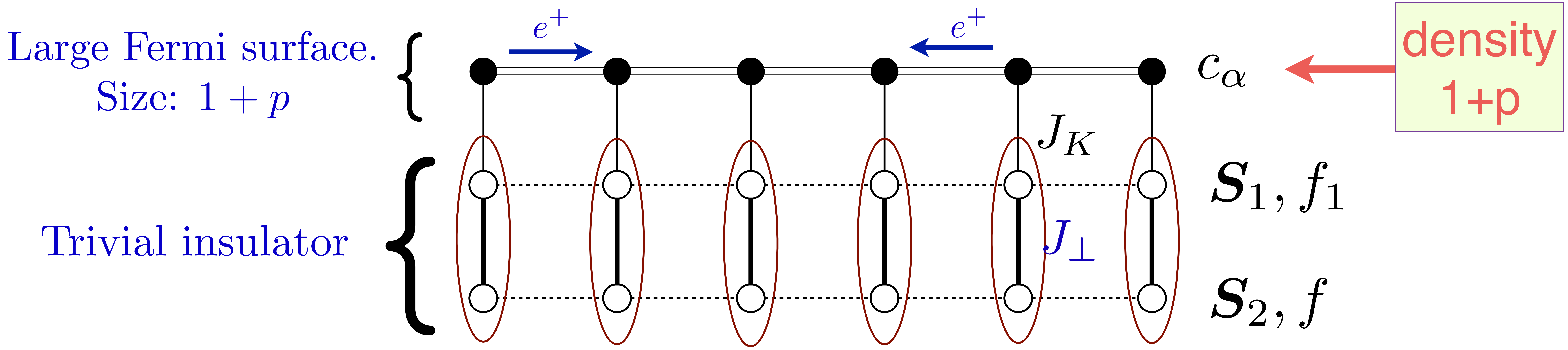


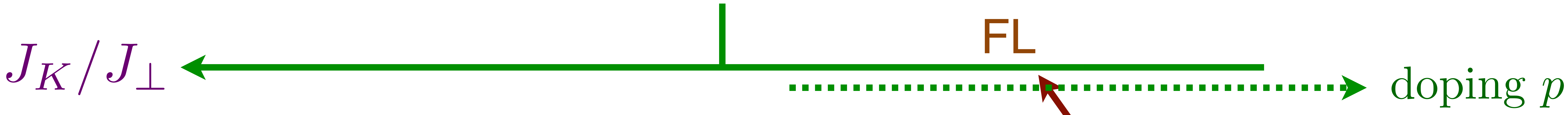
Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Spin liquid of } \boldsymbol{S}_2\rangle$$

Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid
$\oplus$
Spin Liquid

Trial wavefunctions in the paramagnon fractionalization theory

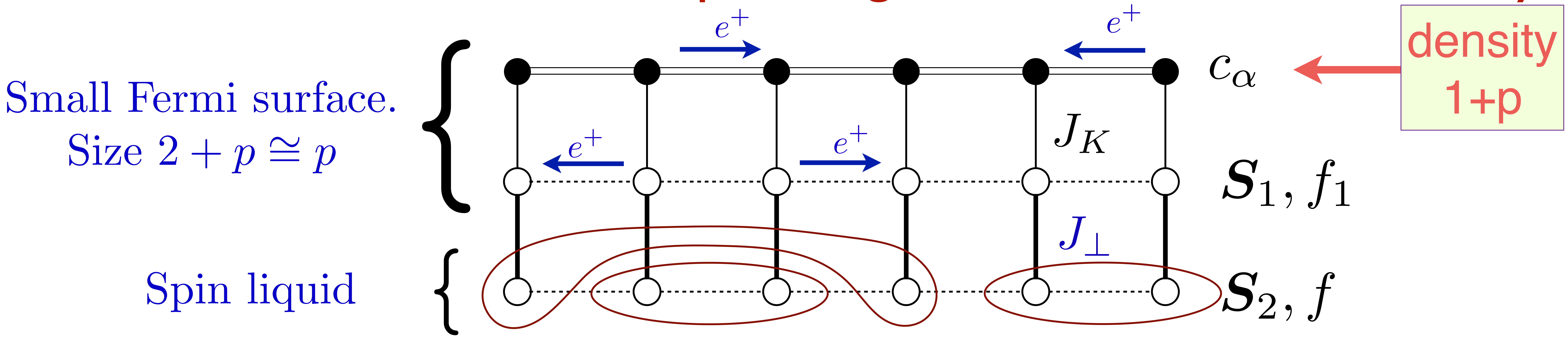


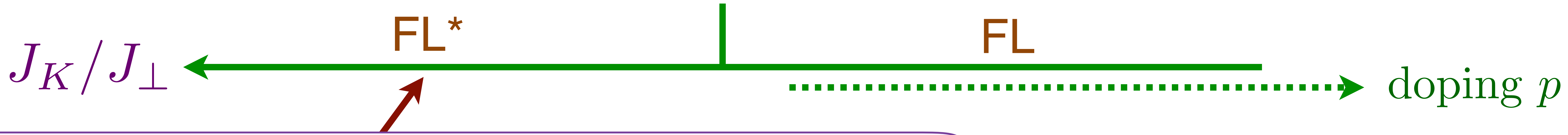


Large Fermi surface of size $1+p$

$$|\mathrm{FL}\rangle = |\text{Rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2\rangle \otimes |\text{Slater determinant of } c\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory



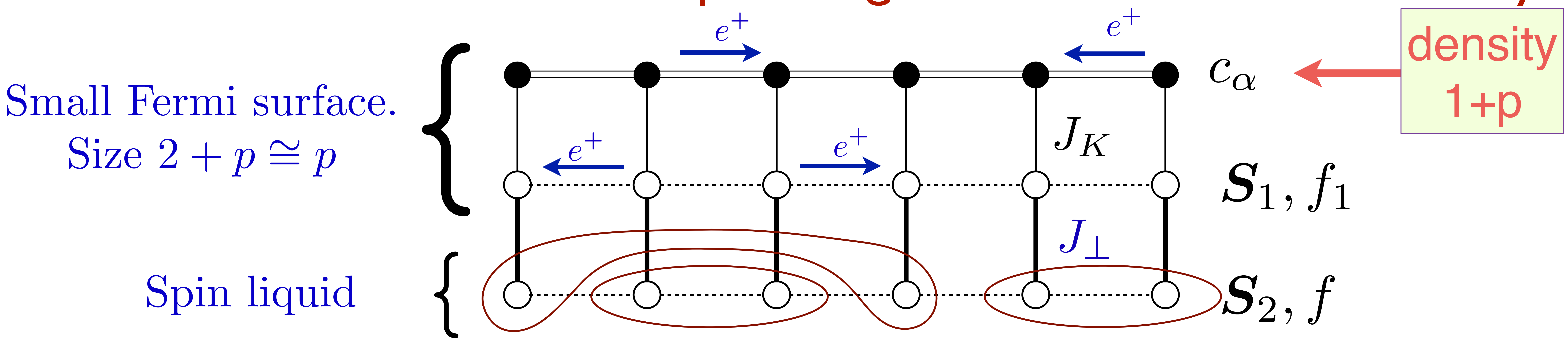


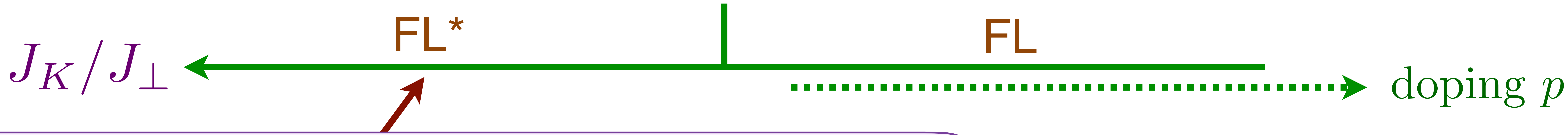
Small Fermi surface of size p

$|\text{FL}*\rangle = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2]$
$\bowtie |\text{Slater determinant of } (c, f_1)\rangle$
$\otimes |\text{Spin liquid of } \boldsymbol{S}_2\rangle$

Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid
$\oplus$
Spin Liquid

Trial wavefunctions in the paramagnon fractionalization theory





Pseudogap metal

f_α form π-flux spin liquid: $\langle c^\dagger_\alpha f_{1\alpha} \rangle \neq 0$

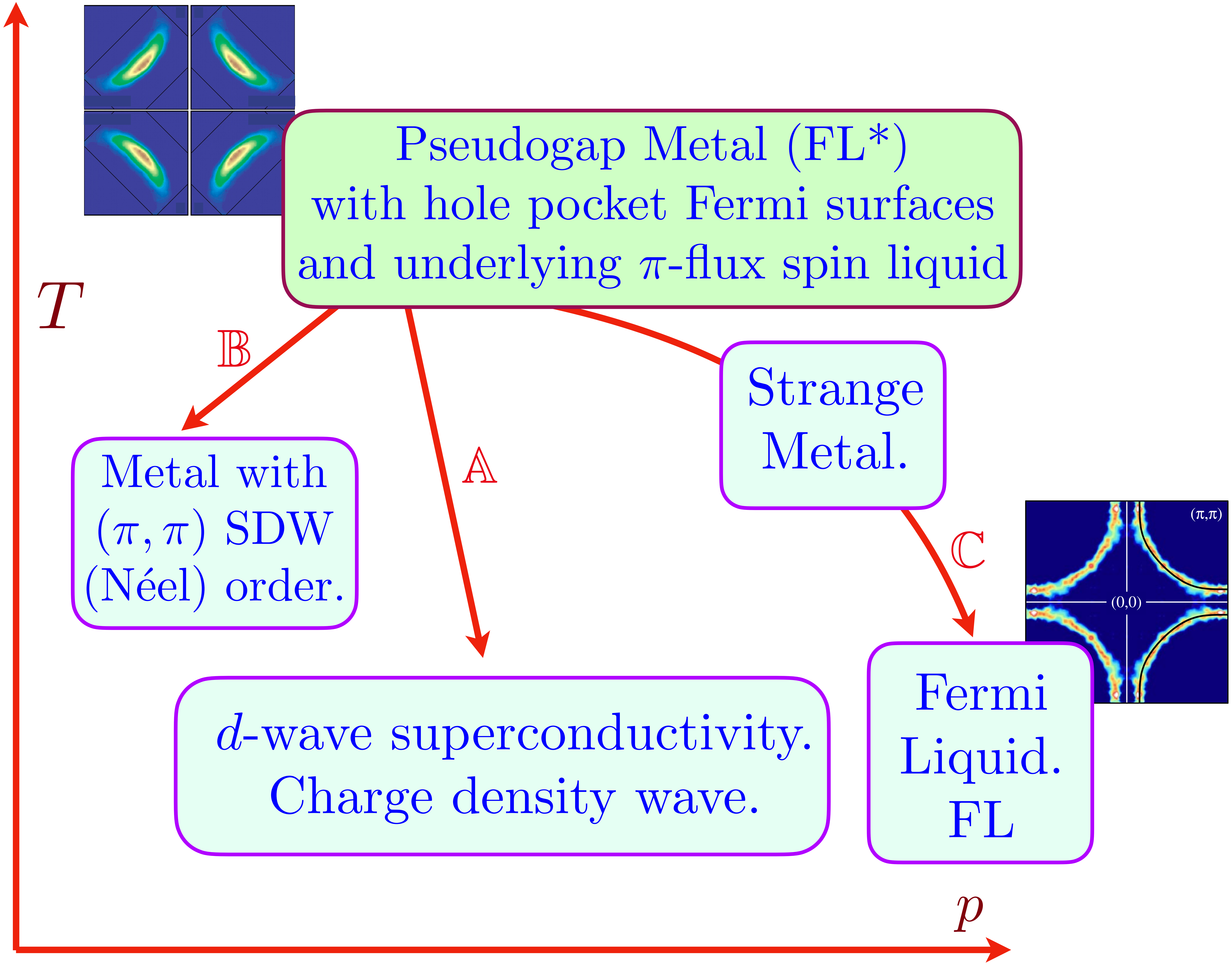
$$B \sim \begin{pmatrix} \left\langle f^\dagger_{1\alpha} f_\alpha \right\rangle \\ \left\langle \varepsilon_{\alpha\beta} f^\dagger_{1\alpha} f^\dagger_\beta \right\rangle \end{pmatrix}$$

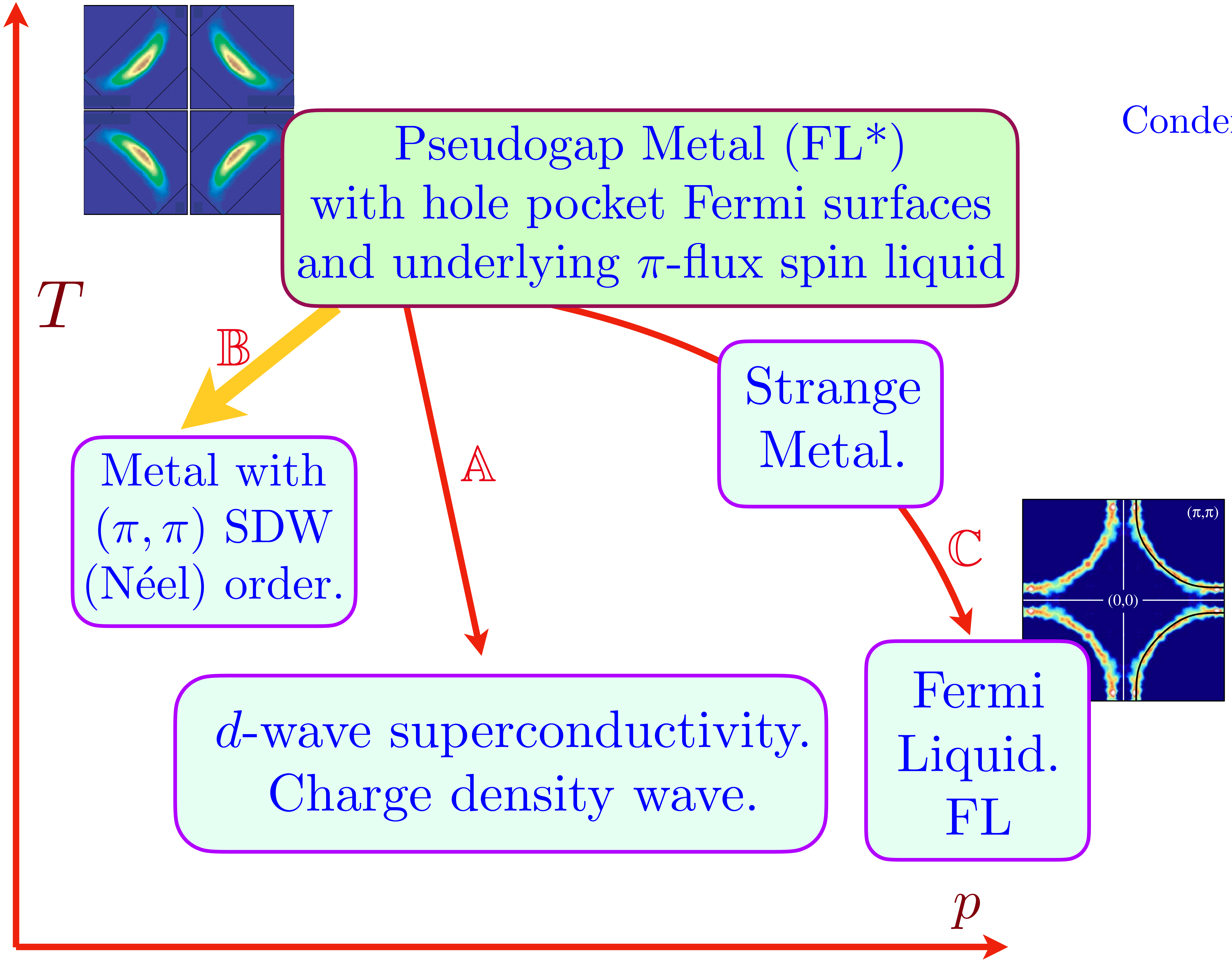
Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid
$\oplus$
Spin Liquid

1. Confinement of the π-flux spin liquid at half-filling

2. Ancilla theory of the pseudogap metal

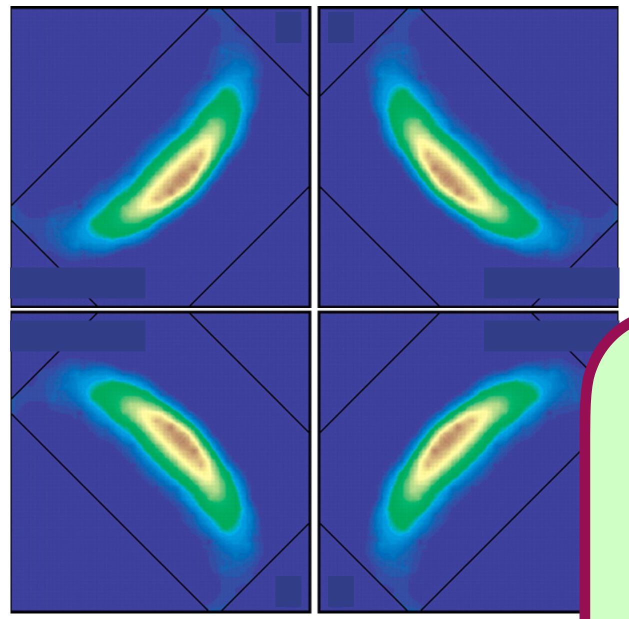
3. Confinement of the pseudogap metal at non-zero doping


T
p
Pseudogap Metal (FL*)
with hole pocket Fermi surfaces
and underlying π-flux spin liquid
𝔹
𝔸
ℂ
Metal with
(π, π) SDW
(Néel) order.
Strange
Metal.
d-wave superconductivity.
Charge density wave.
Fermi
Liquid.
FL
(π,π)
(0,0)



Arrow $\mathbb{B}$

Condensation of z_α in dual $\mathbb{CP}^1$ U(1) gauge theory.



Pseudogap Metal (FL*) with hole pocket Fermi surfaces and underlying π-flux spin liquid

T

$\mathbb{B}$

Metal with (π, π) SDW (Néel) order.

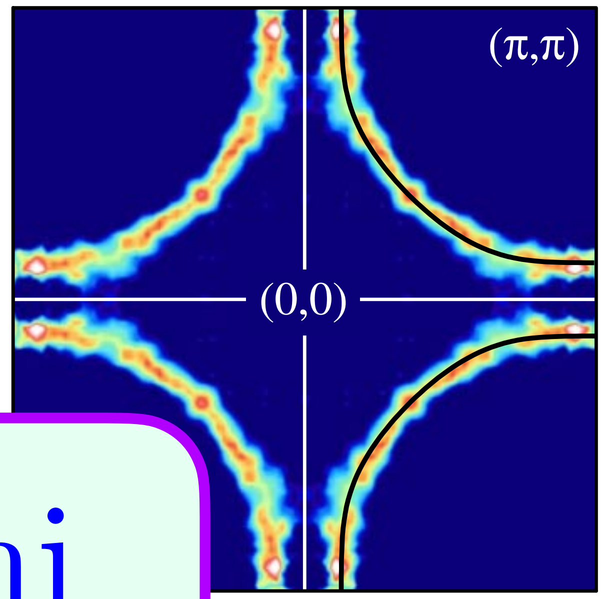
$\mathbb{A}$

d-wave superconductivity.
Charge density wave.

Strange Metal.

$\mathbb{C}$

Fermi Liquid. FL

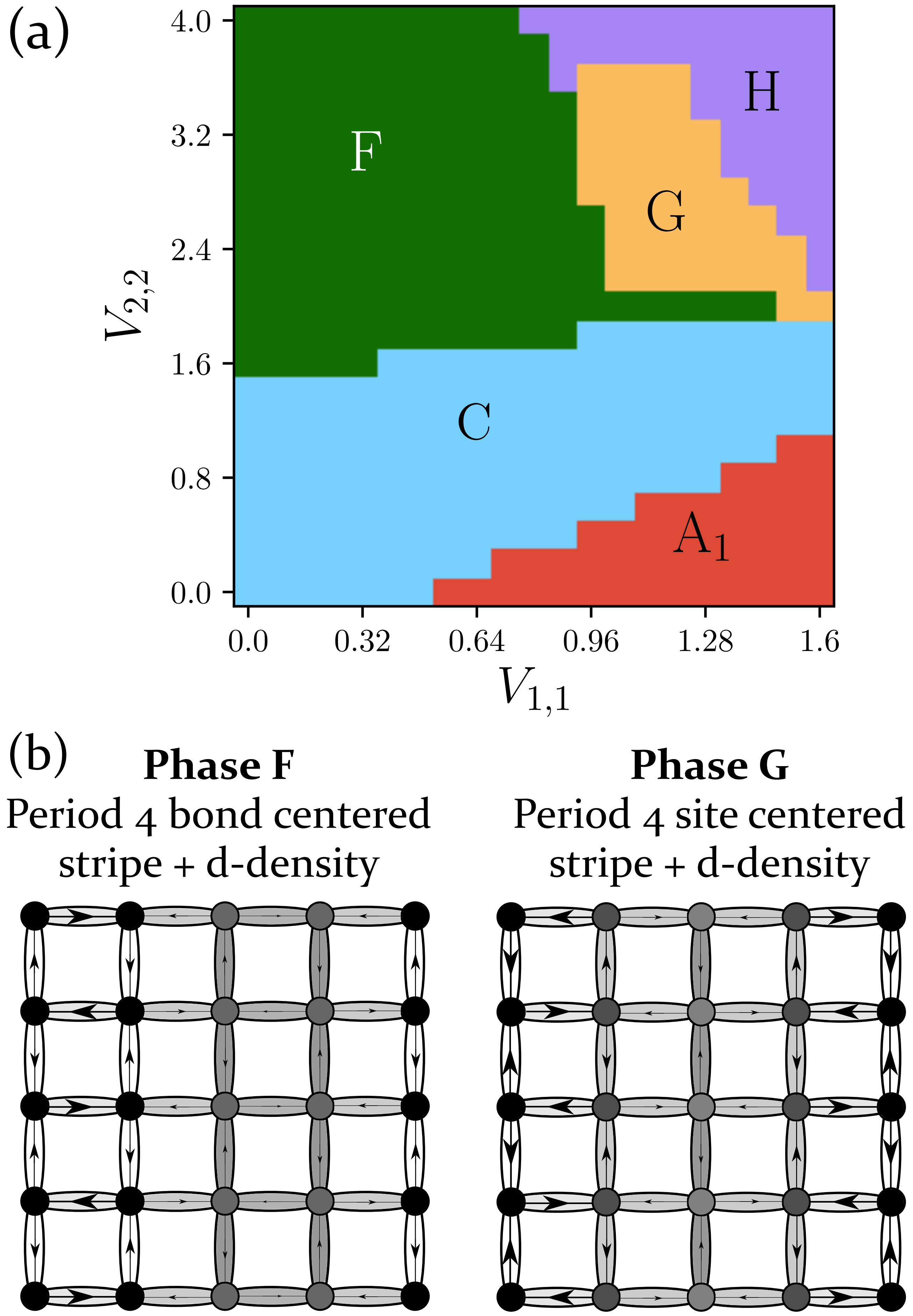


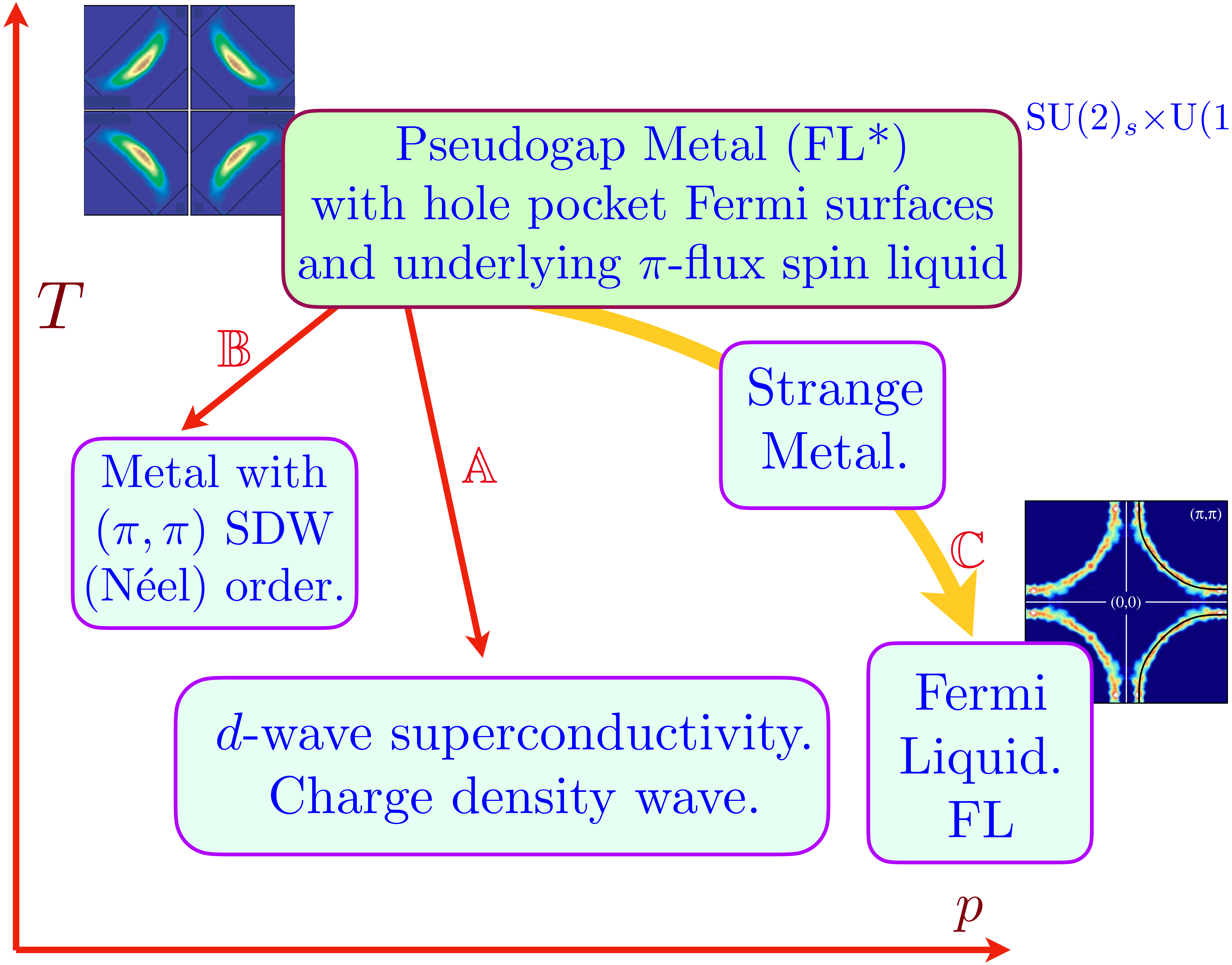
p

Arrow $\mathbb{A}$

Condensation of B in SU(2) gauge theory.

Longer-range couplings in H_B can lead to charge order with other periods

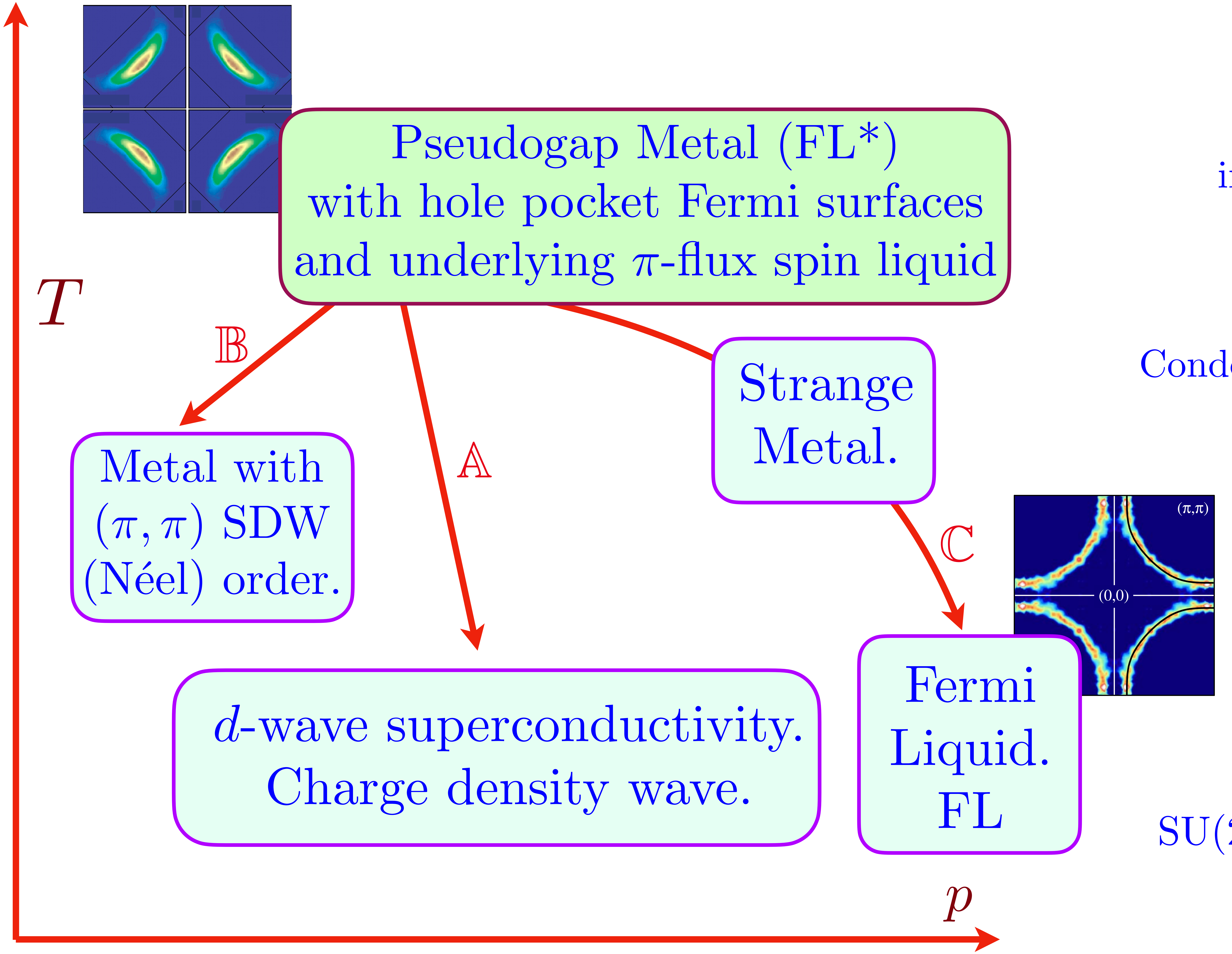




<u>Arrow ℂ</u>

$SU(2)_s \times U(1)_a$ gauge theory in ancilla model

Ya-Hui Zhang and S. Sachdev,
Physical Review Research **2**, 023172 (2020);
Physical Review B **102** , 155124 (2020)



<u>Arrow $\mathbb{A}$</u>

Condensation of B
in SU(2) gauge theory.

<u>Arrow $\mathbb{B}$</u>

Condensation of z_α in dual $\mathbb{CP}^1$
U(1) gauge theory.

<u>Arrow $\mathbb{C}$</u>

$\mathrm{SU}(2)_s \times \mathrm{U}(1)_a$ gauge theory
in ancilla model