Superconductors, Black Holes, and Quantum Matter

Friends of the Physics Department

Harvard University

May 3, 2026

Subir Sachdev

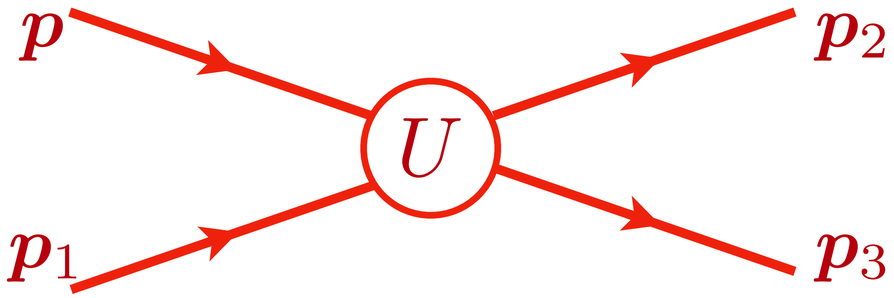
Boltzmann-Landau theory of ordinary metals: Cu, Ag

Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

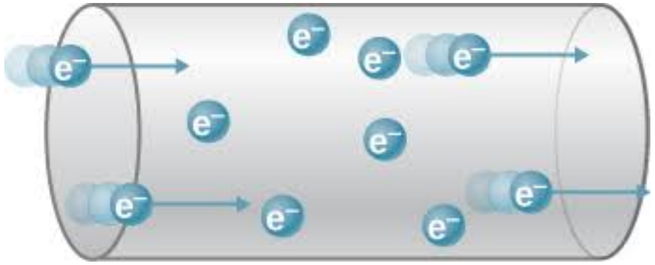
Quantum Boltzmann equation (Landau)
Dense gas of electrons

Collisions are also rare in a dense quantum gas at low temperatures because of the Pauli exclusion principle.

Neglect quantum interference (entanglement) between successive collisions



Current flow with electrons in ordinary metals



Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/(UT)^2$ (U is the strength of interactions),
resistivity $\rho(T) = \rho(0) + AT^2$

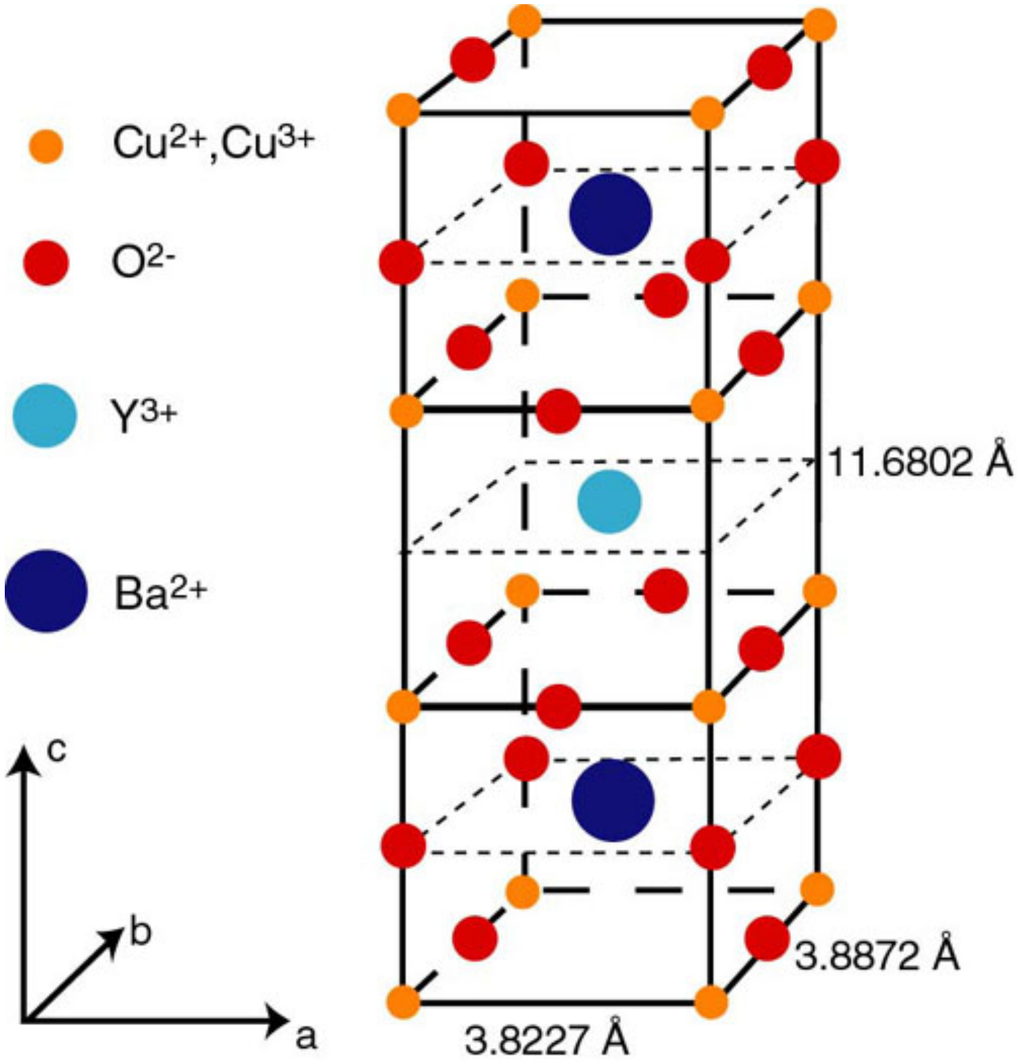
The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

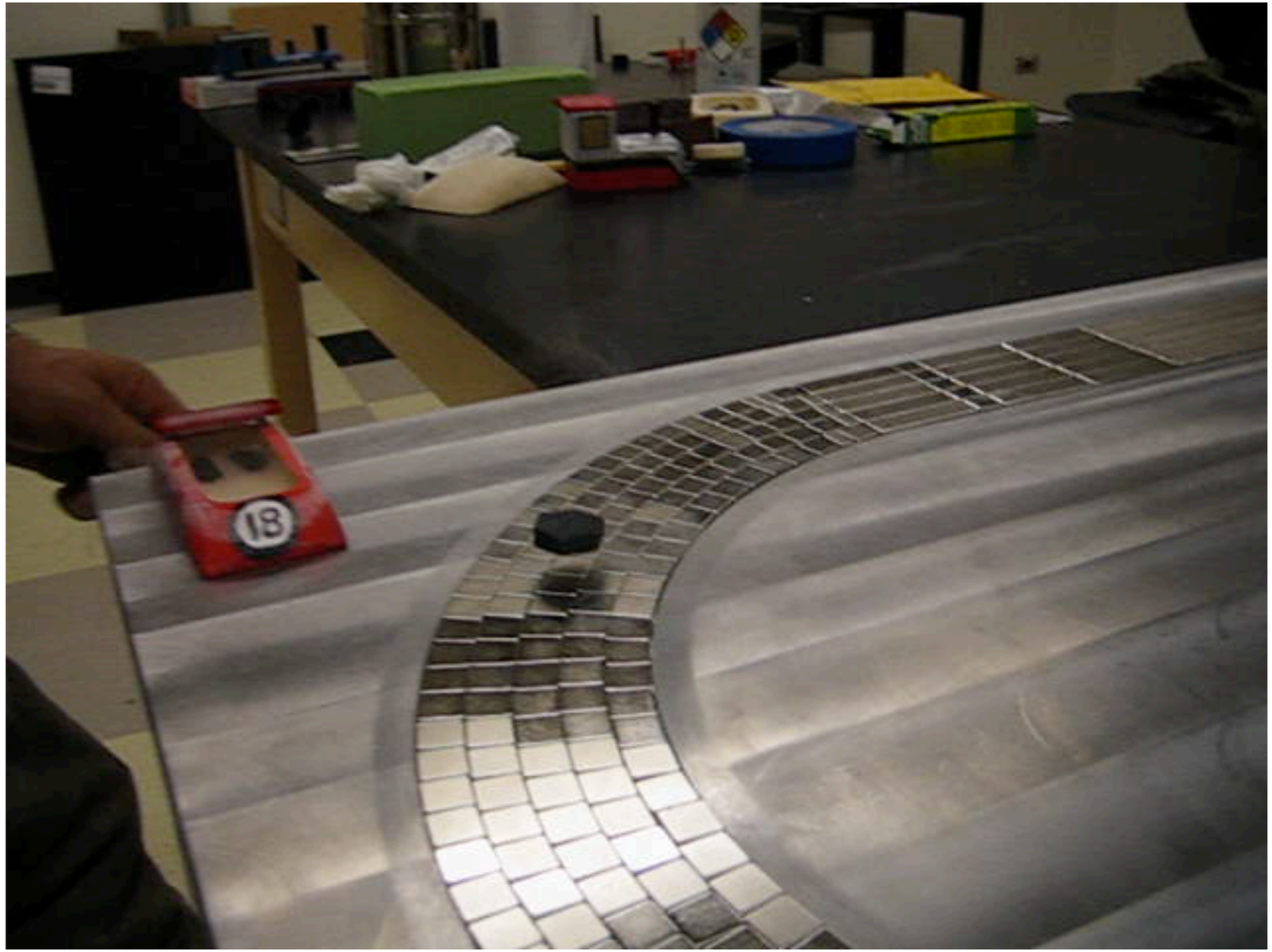
The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.

The cuprate high temperature superconductors

Cuprate high temperature superconductors



$YBa_2Cu_3O_{6+x}$



Nd-Fe-B magnets, YBaCuO superconductor

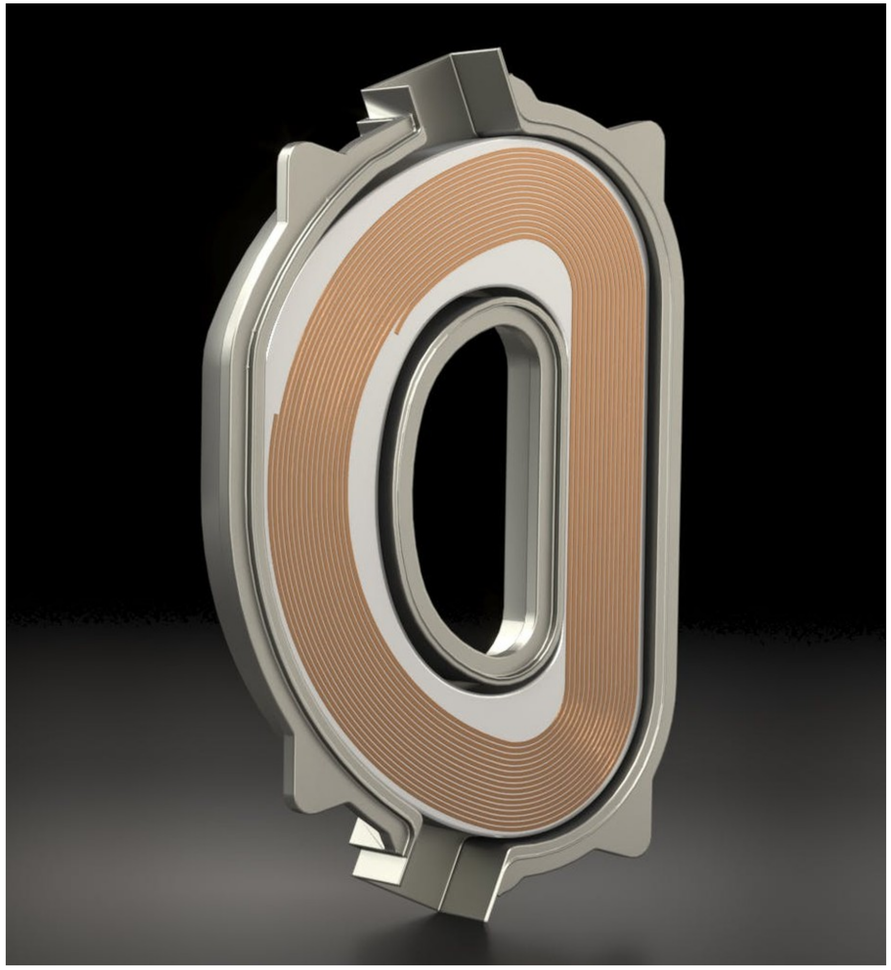
Julian Hetel and Nandini Trivedi, Ohio State University

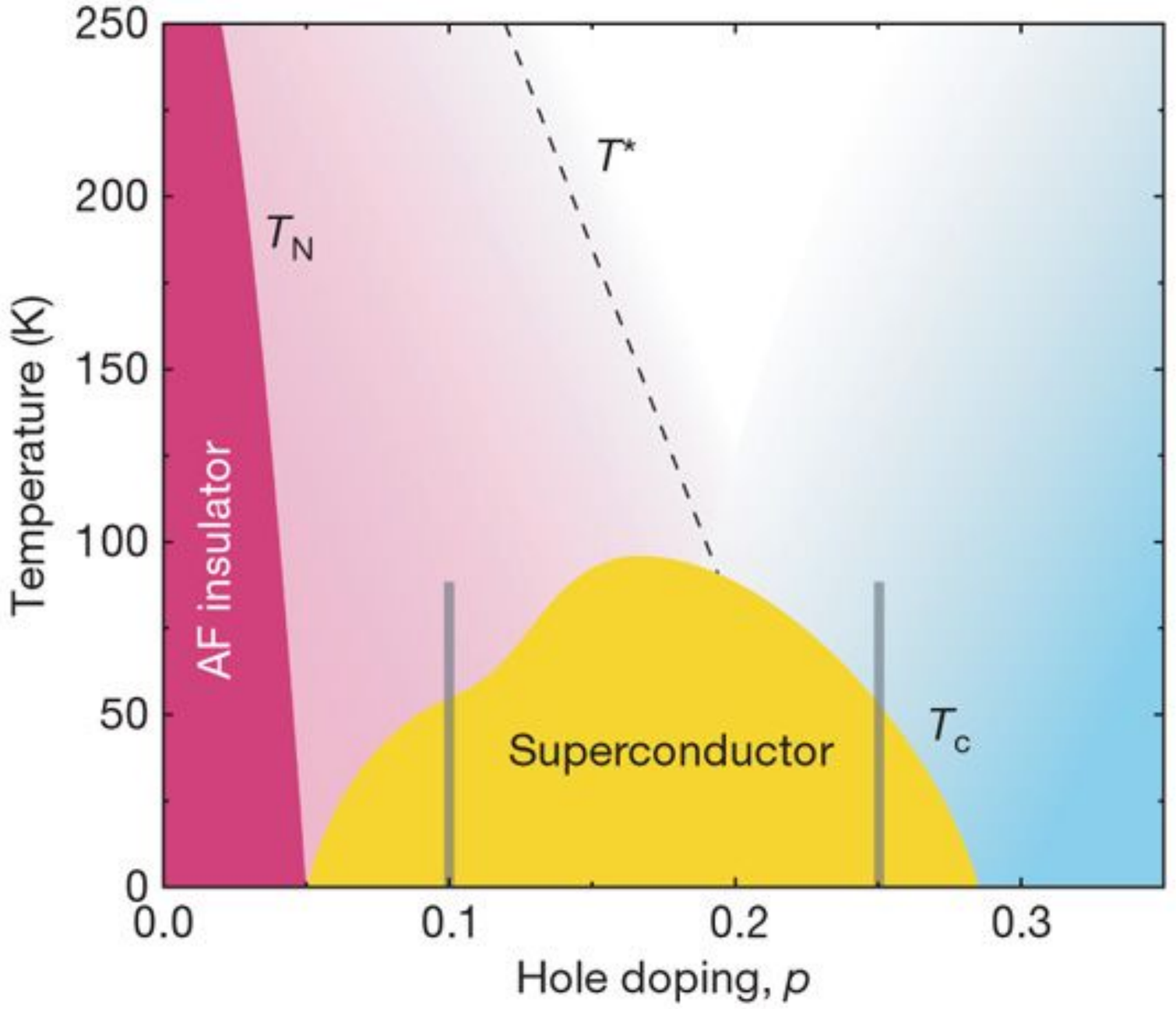
HTS Magnets: Enabling Technology

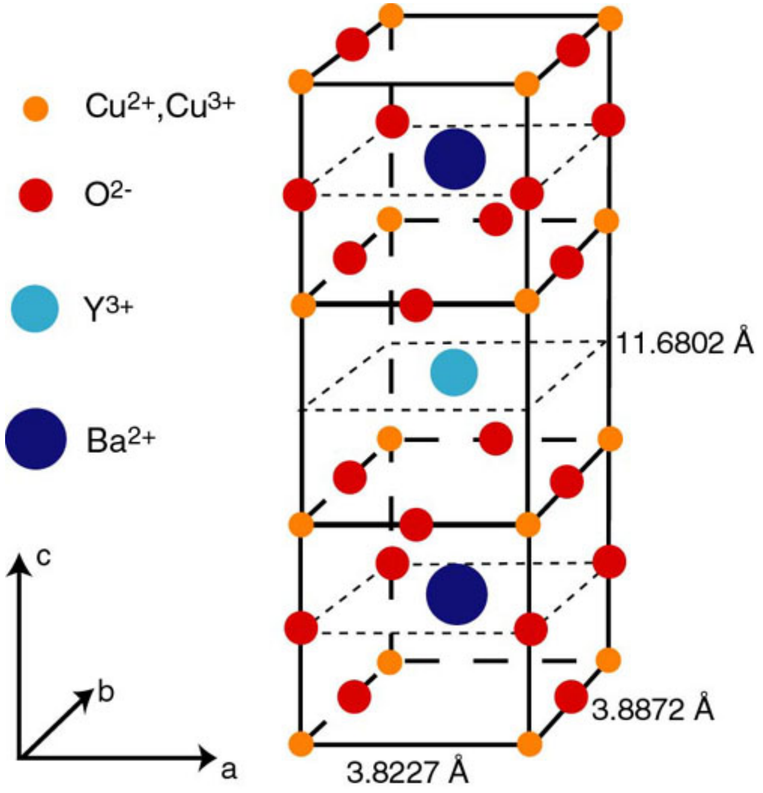
The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

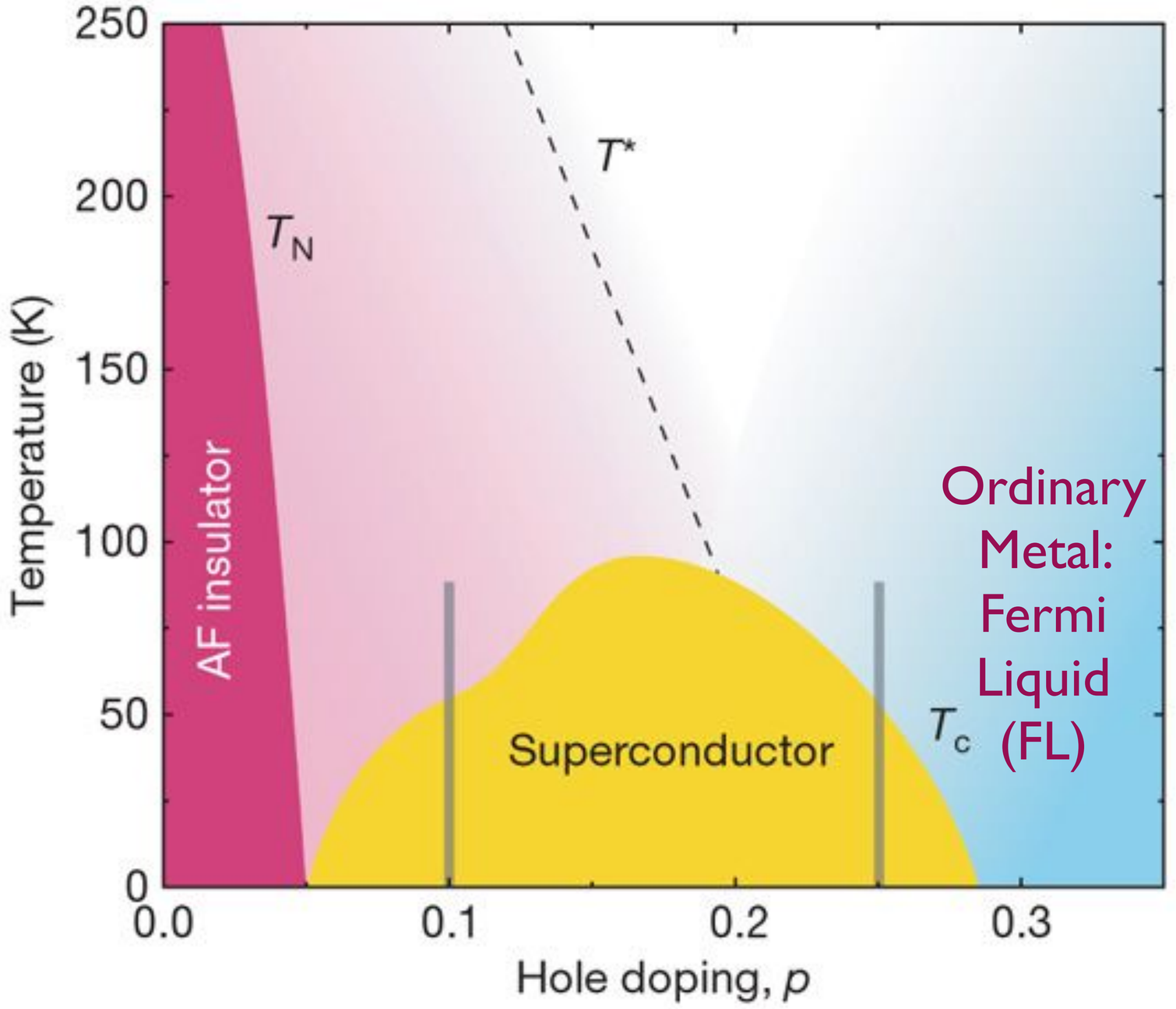
Commonwealth Fusion Systems

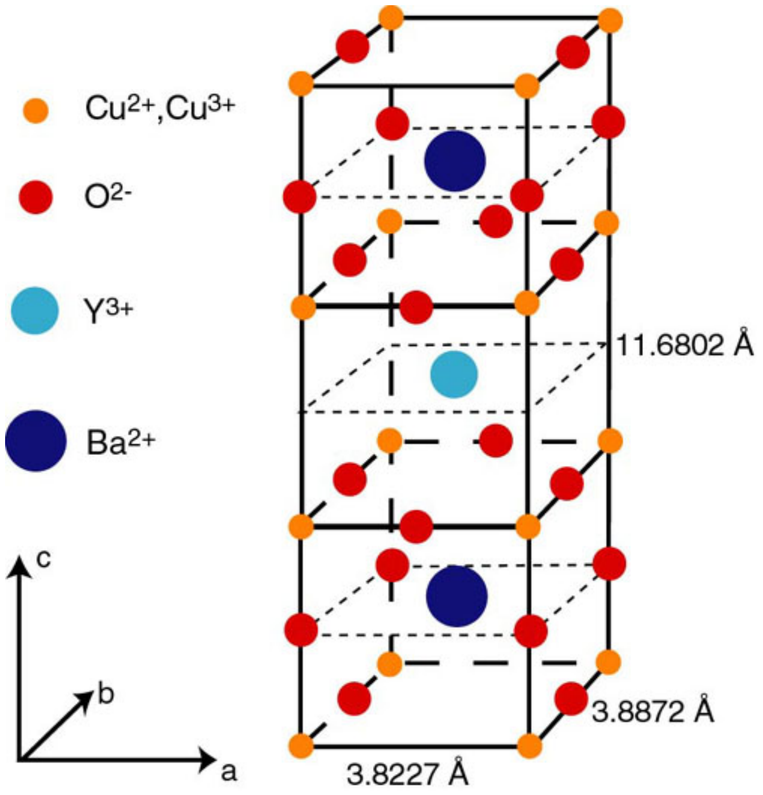



250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Superconductor
T_c

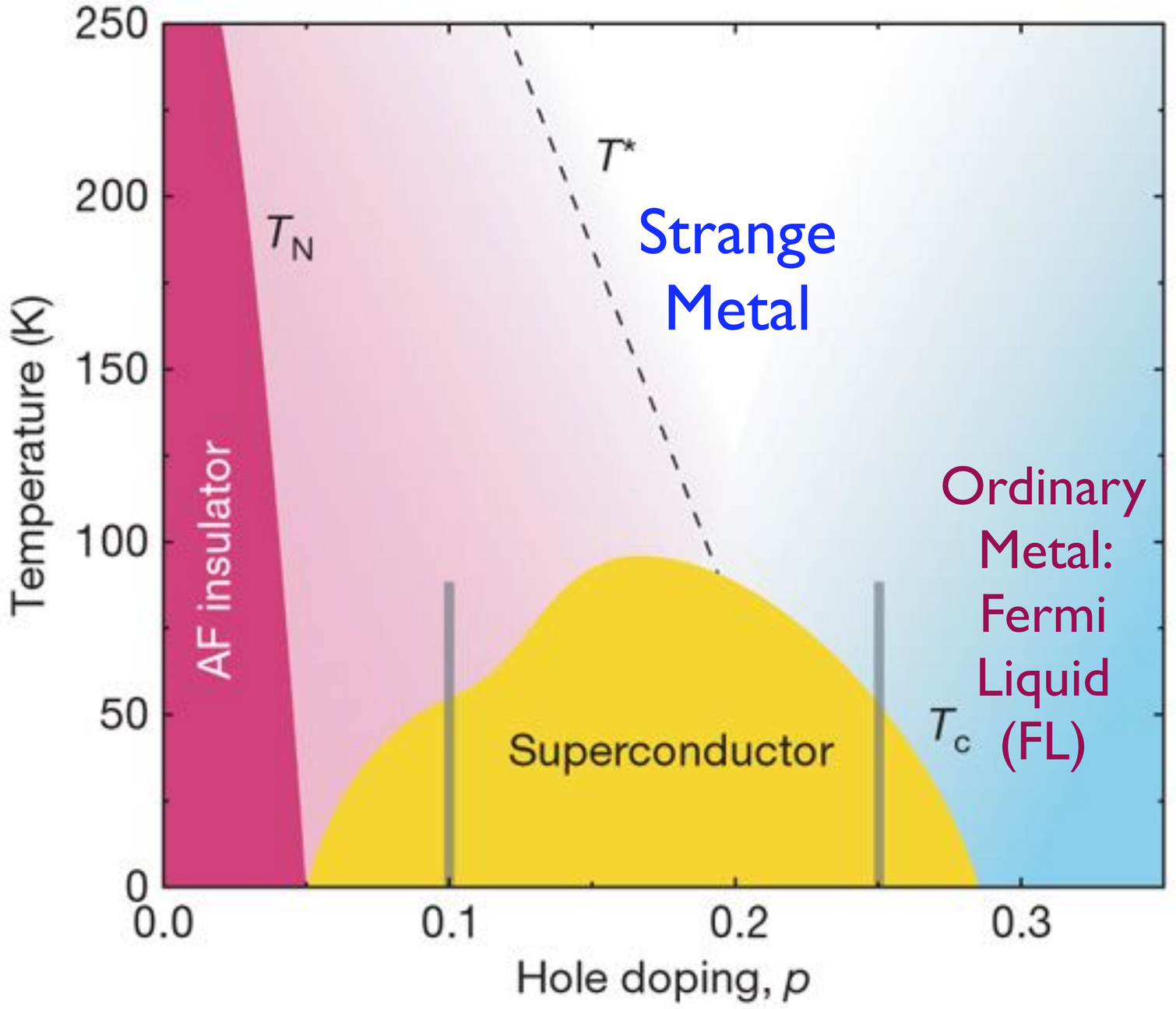

Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

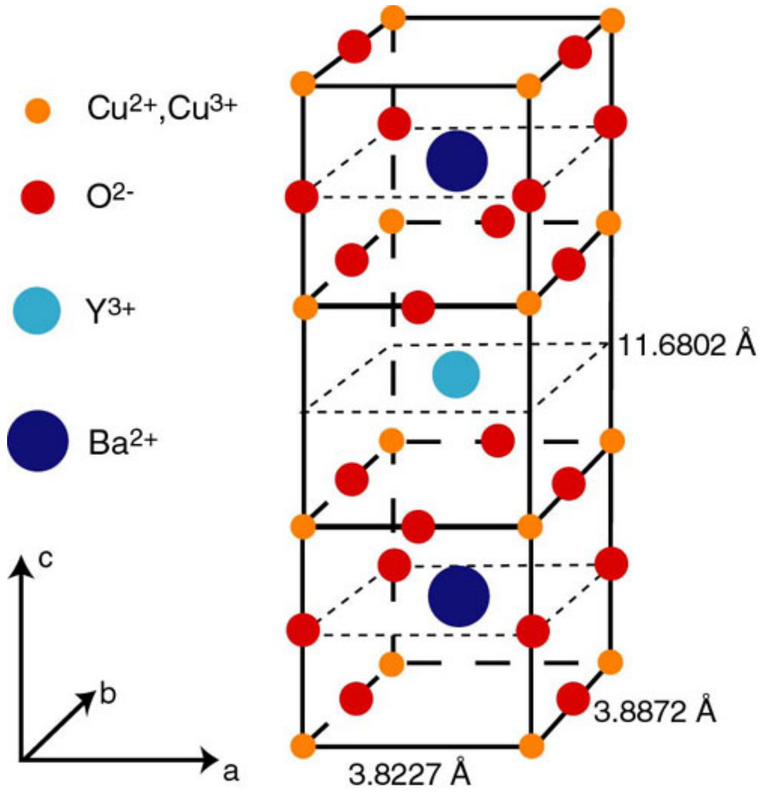
$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
T_N
T^*
AF insulator
Superconductor
T_c
Ordinary
Metal:
Fermi
Liquid
(FL)

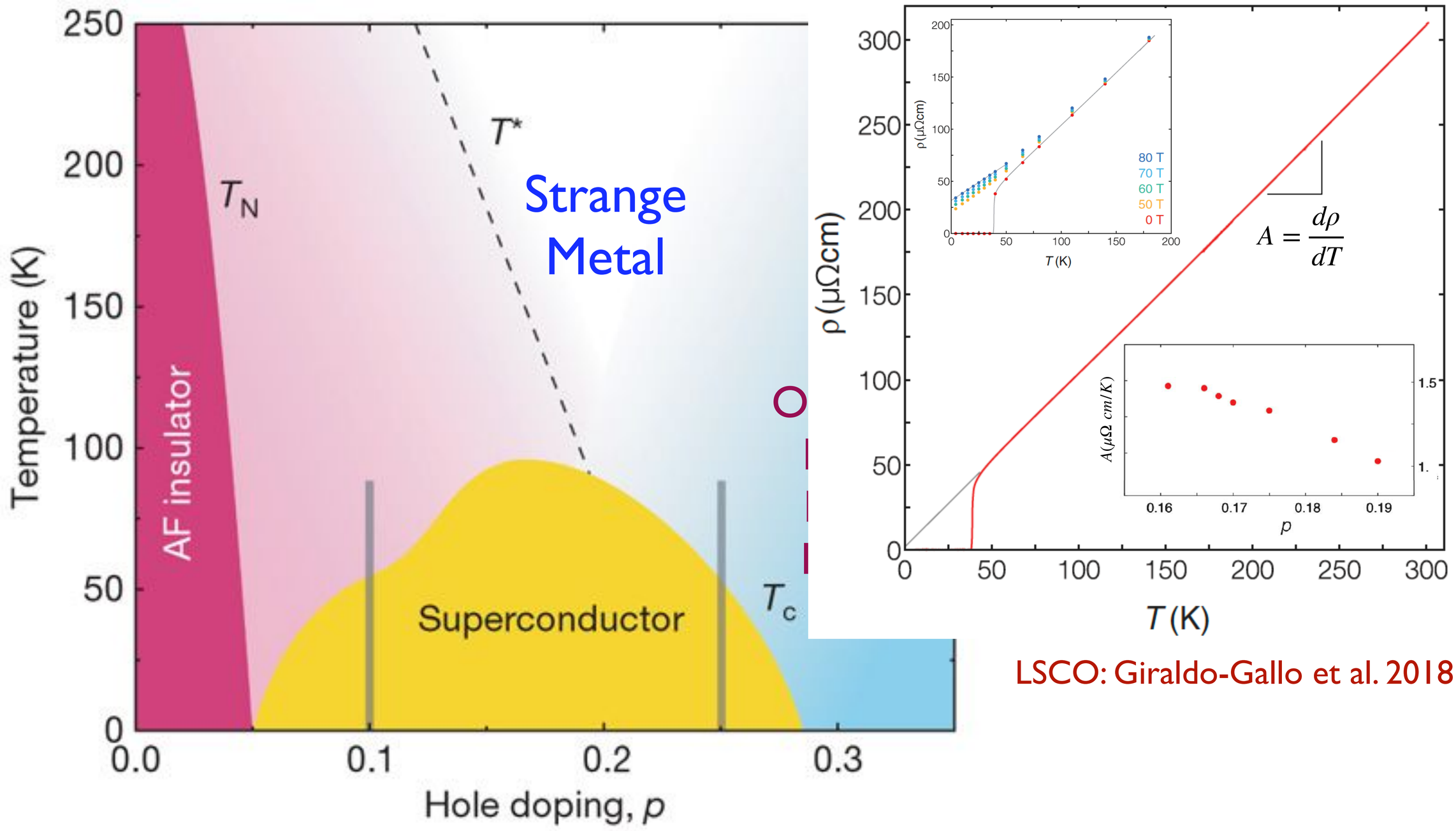

Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
T_N
T^*
AF insulator
Strange
Metal
Superconductor
T_c
Ordinary
Metal:
Fermi
Liquid
(FL)


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$



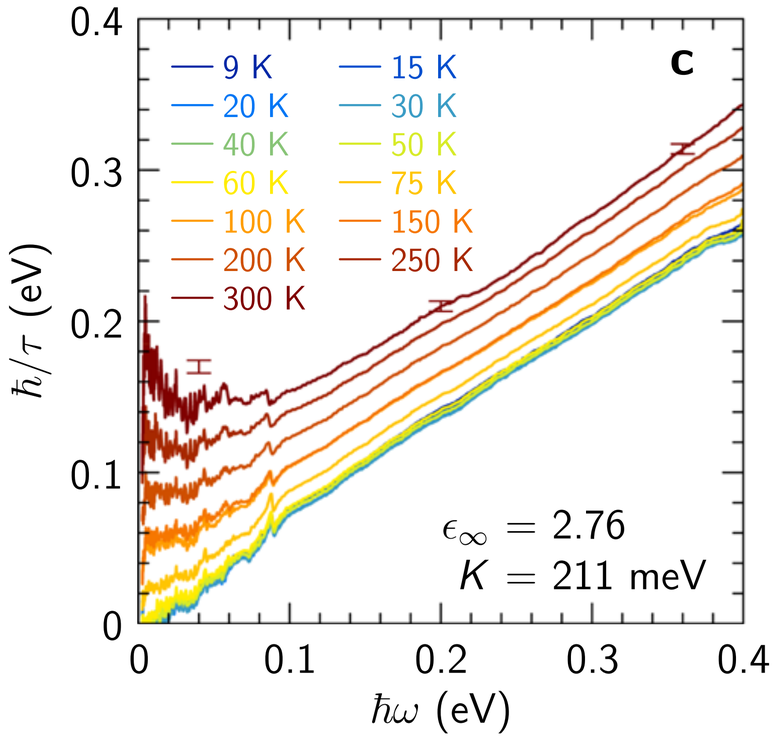
LSCO: Giraldo-Gallo et al. 2018

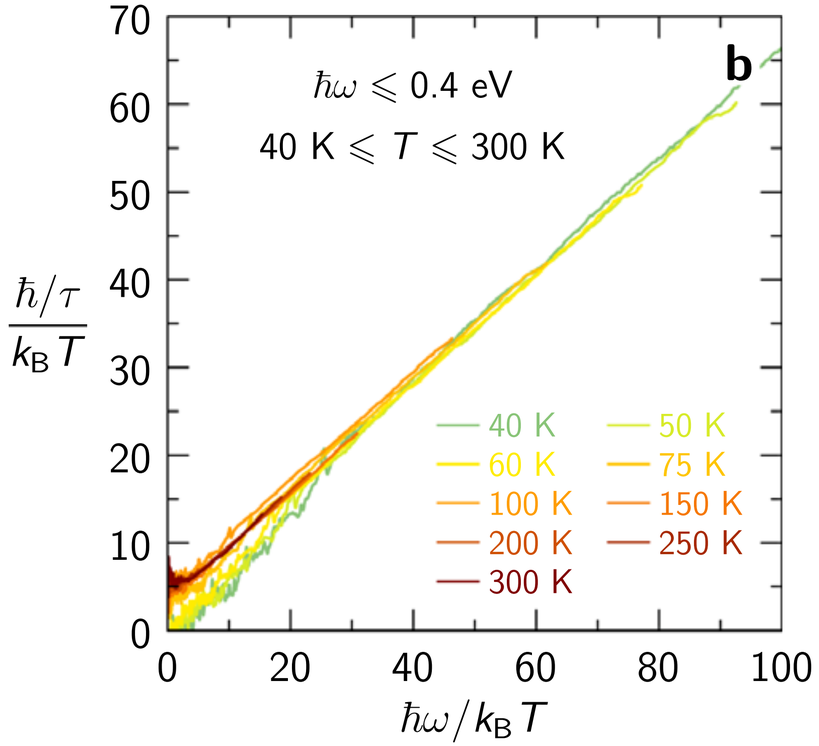
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

The time τ appears to be *independent* of interaction strength, contrary to Boltzmann.

Central questions:

What is the origin of the strange metal and why is it ubiquitous in correlated electron quantum materials?

Central questions:

What is the origin of the strange metal and why is it ubiquitous in correlated electron quantum materials?

What is special about the strange metal in the hole-doped cuprates that leads to the highest temperature superconductivity?

Central questions:

What is the origin of the strange metal and why is it ubiquitous in correlated electron quantum materials?

What is special about the strange metal in the hole-doped cuprates that leads to the highest temperature superconductivity?

Much progress has been made in addressing the first question in last 3 decades.

Central questions:

What is the origin of the strange metal and why is it ubiquitous in correlated electron quantum materials?

What is special about the strange metal in the hole-doped cuprates that leads to the highest temperature superconductivity?

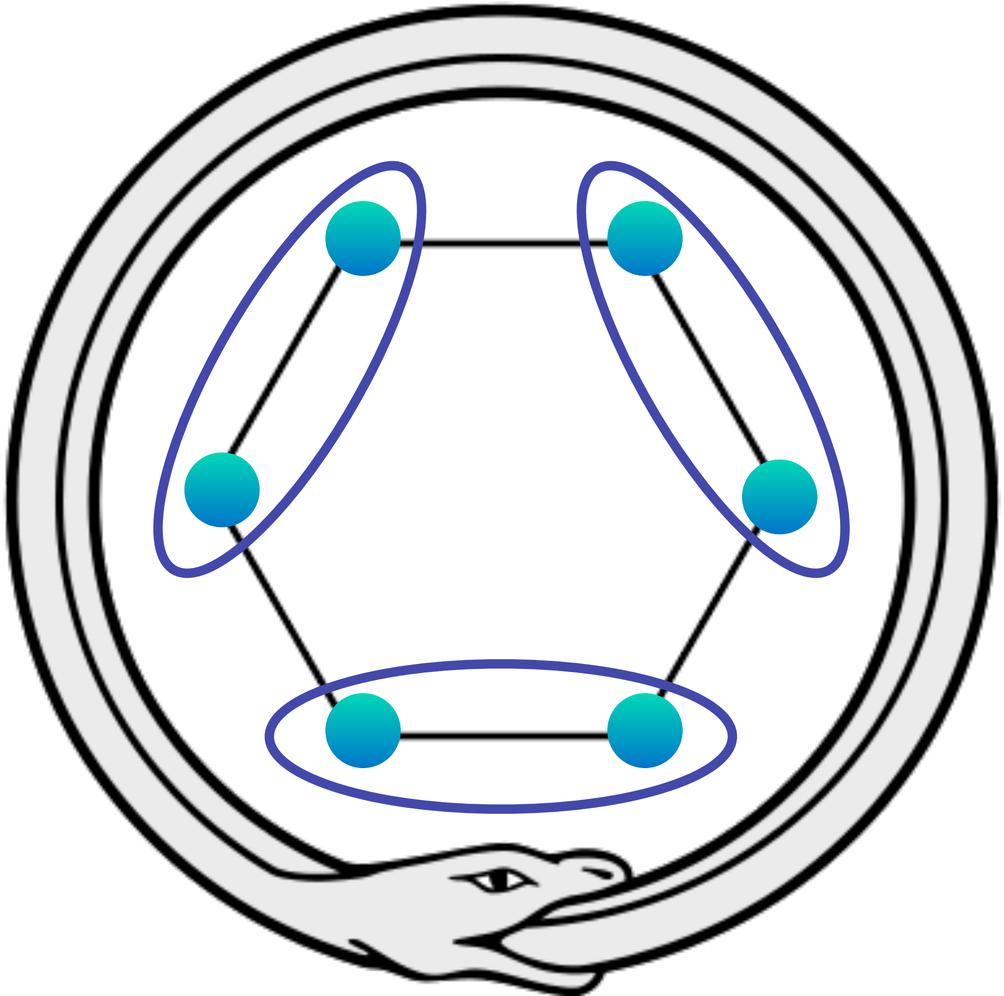
Much progress has been made in addressing the first question in last 3 decades.

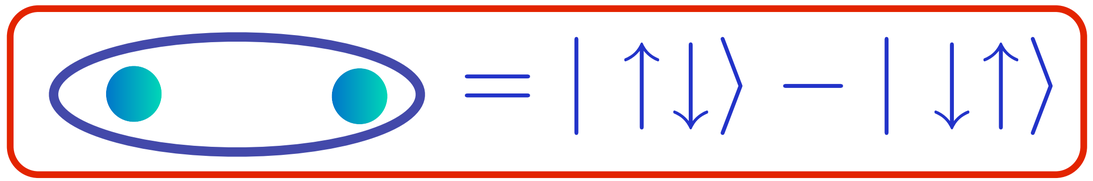
But an unexpected bi-product has been a much deeper understanding of the quantum theory of many particles, which has impacted numerous other fields of physics, including the quantum theory of black holes, and quantum error correction

The Sachdev-Ye-Kitaev model: solvable Planckian dynamics

Kekule's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail *



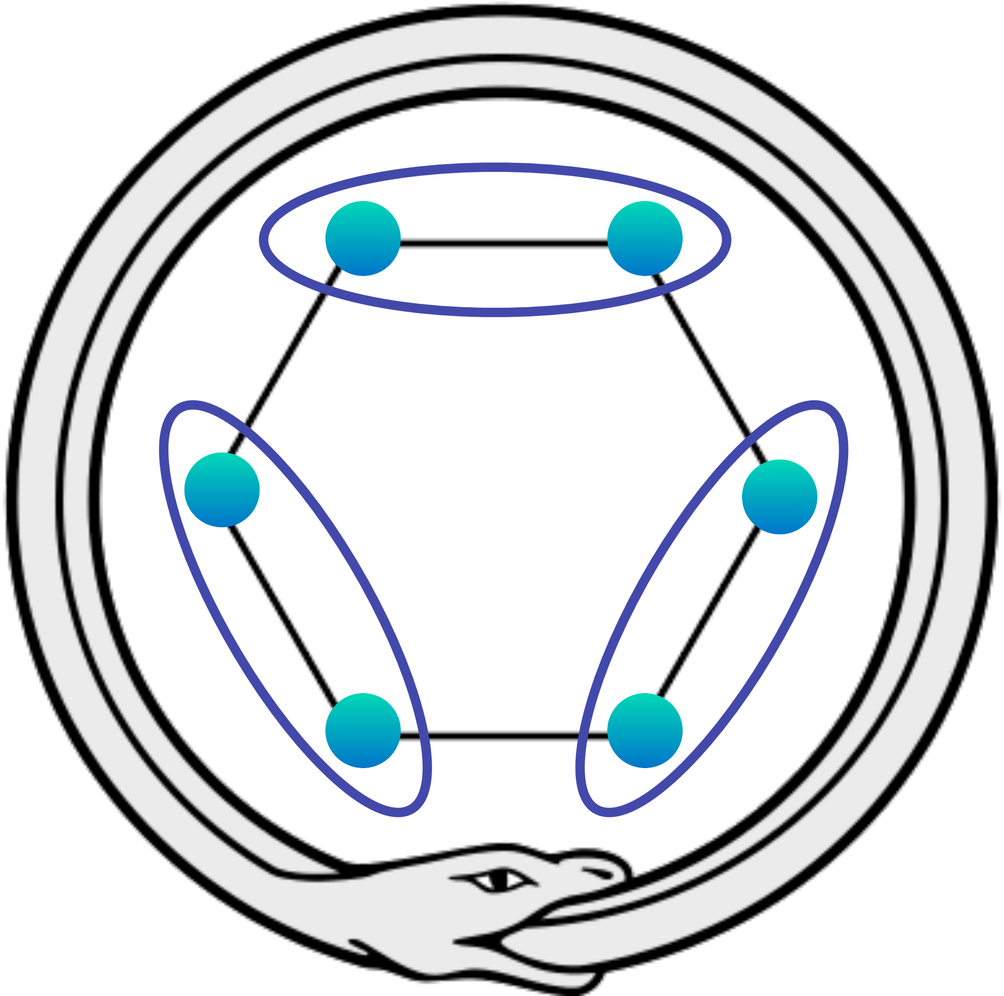


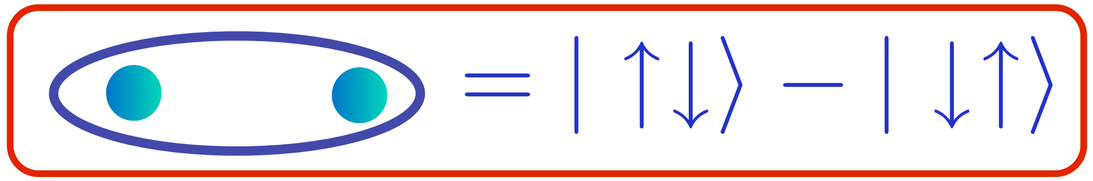
Benzene

*Wikipedia

Kekule's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail *



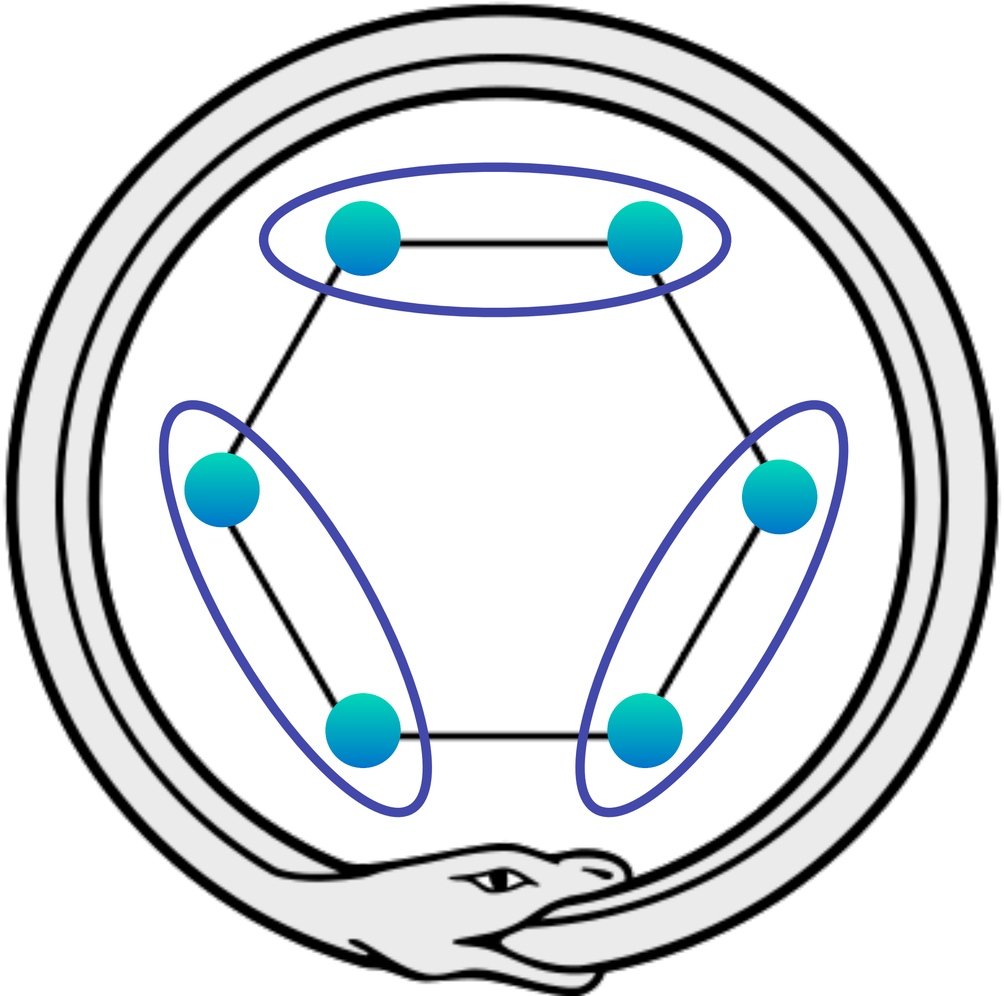


Benzene

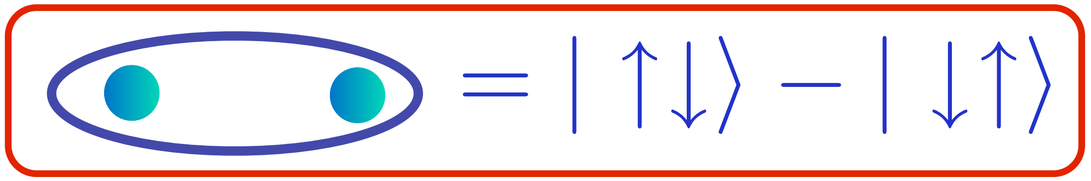
*Wikipedia

Kekule's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail *

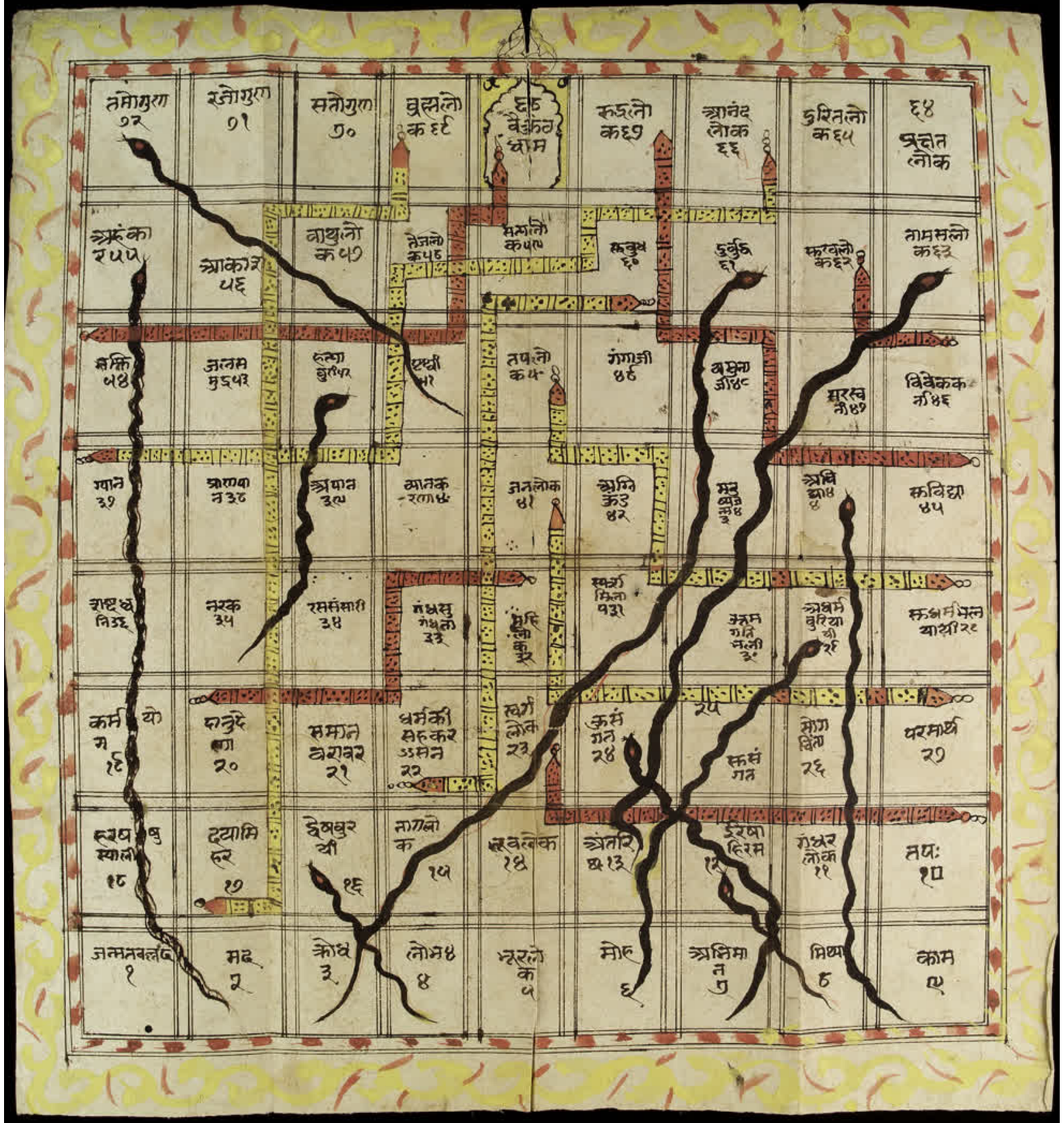


Quantum Entanglement of 6 electron spins!



Benzene

*Wikipedia



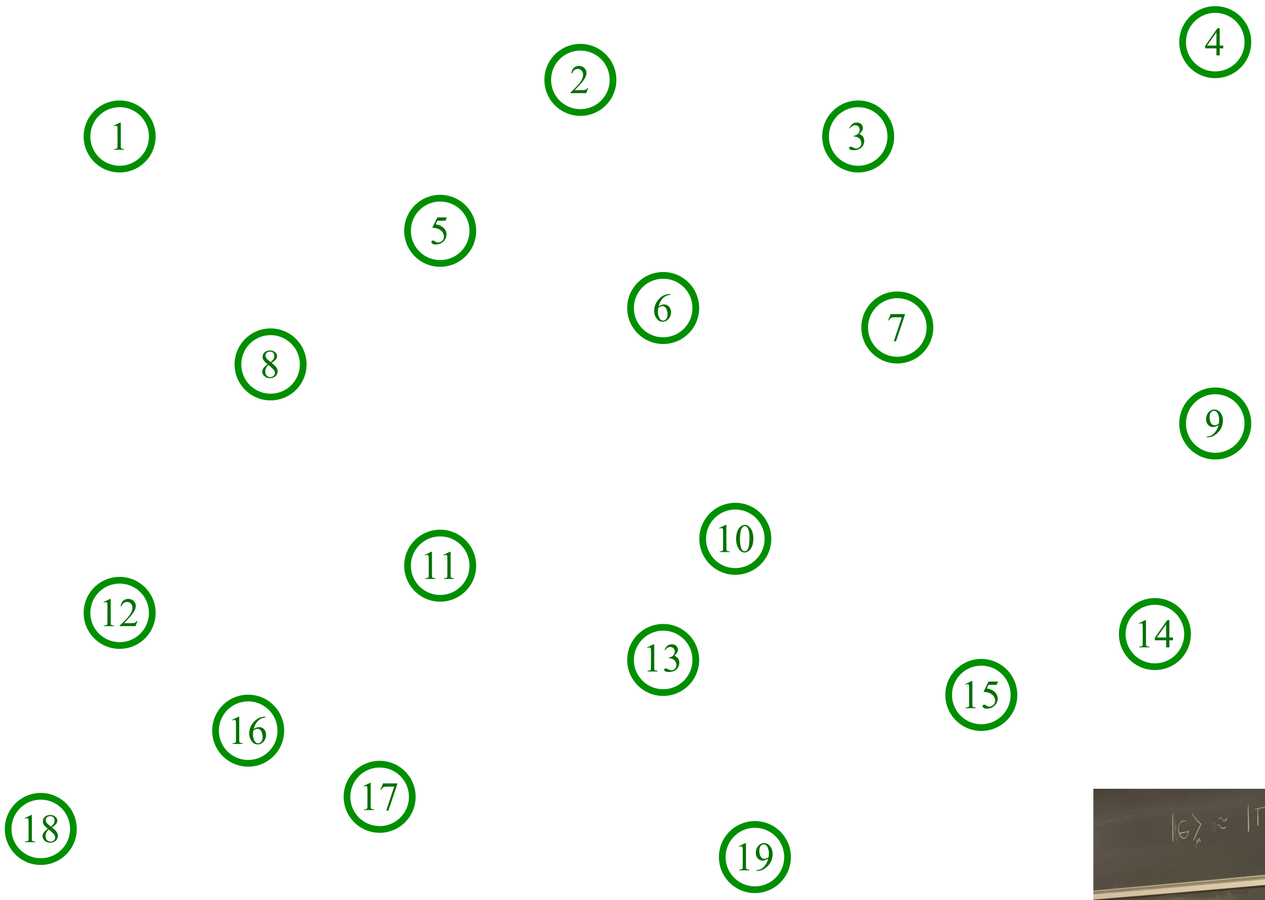
My spooky dream*

Ancient Indian game of Snakes and Ladders

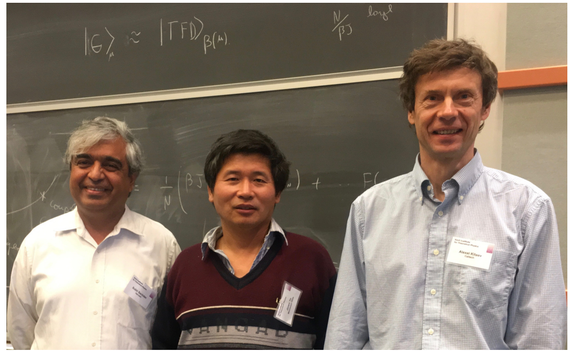
*Not true

The SYK model

Sachdev, Ye (1993); Kitaev (2015)

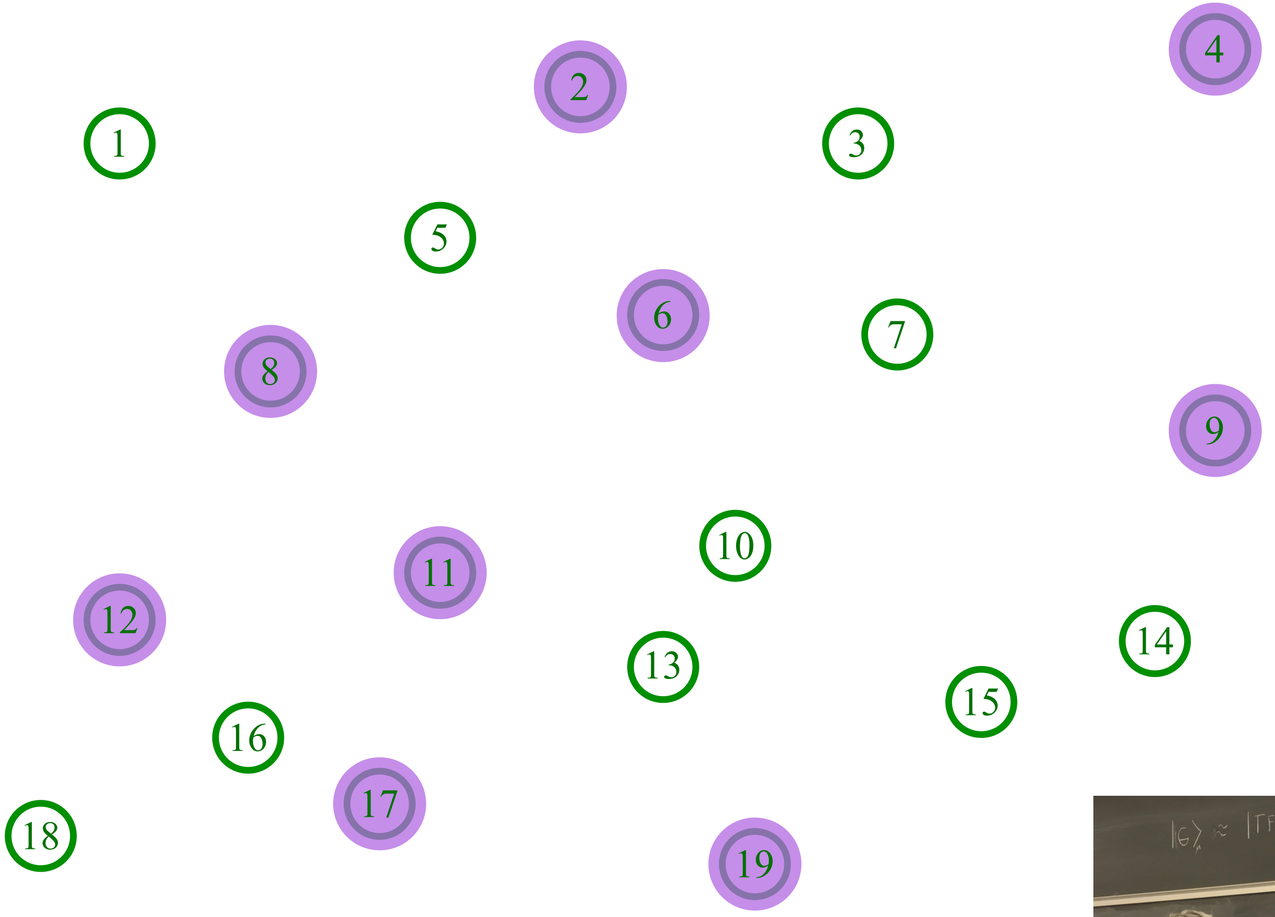


Pick a set of random positions

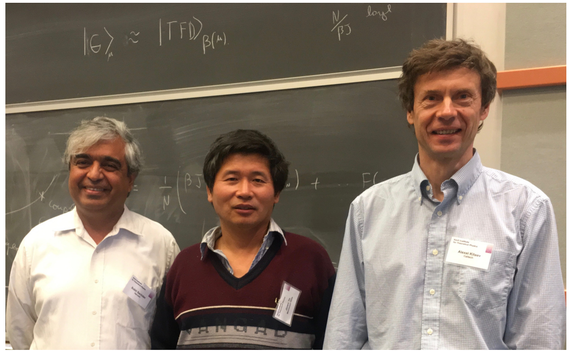


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



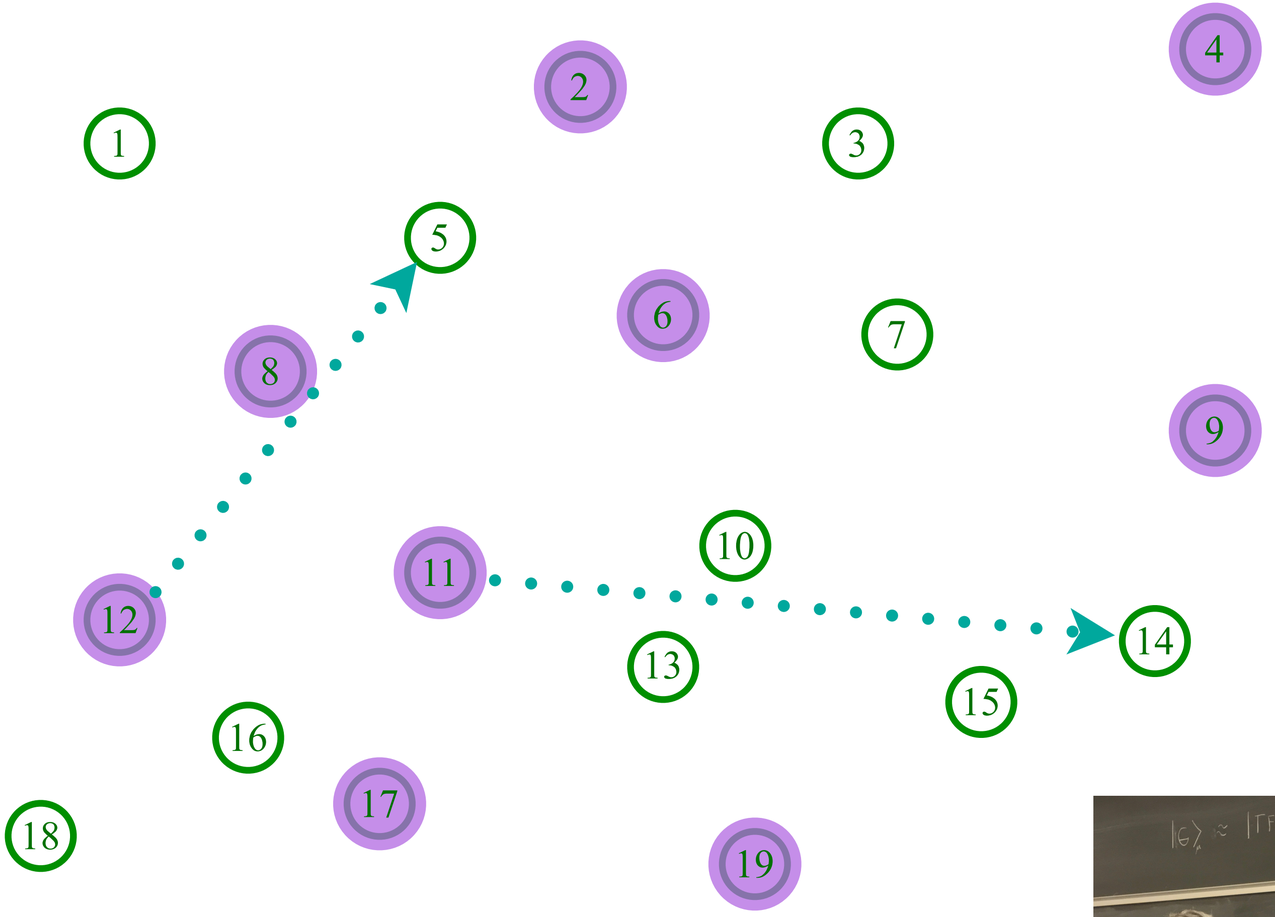
Place electrons randomly on some sites



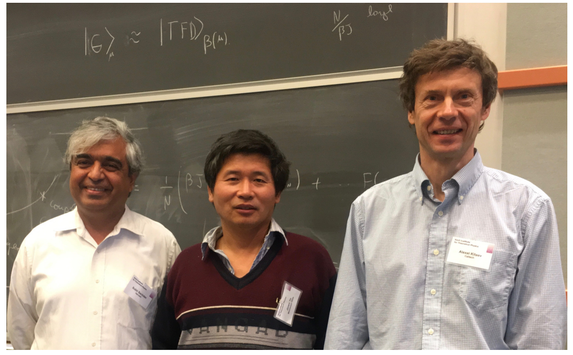
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



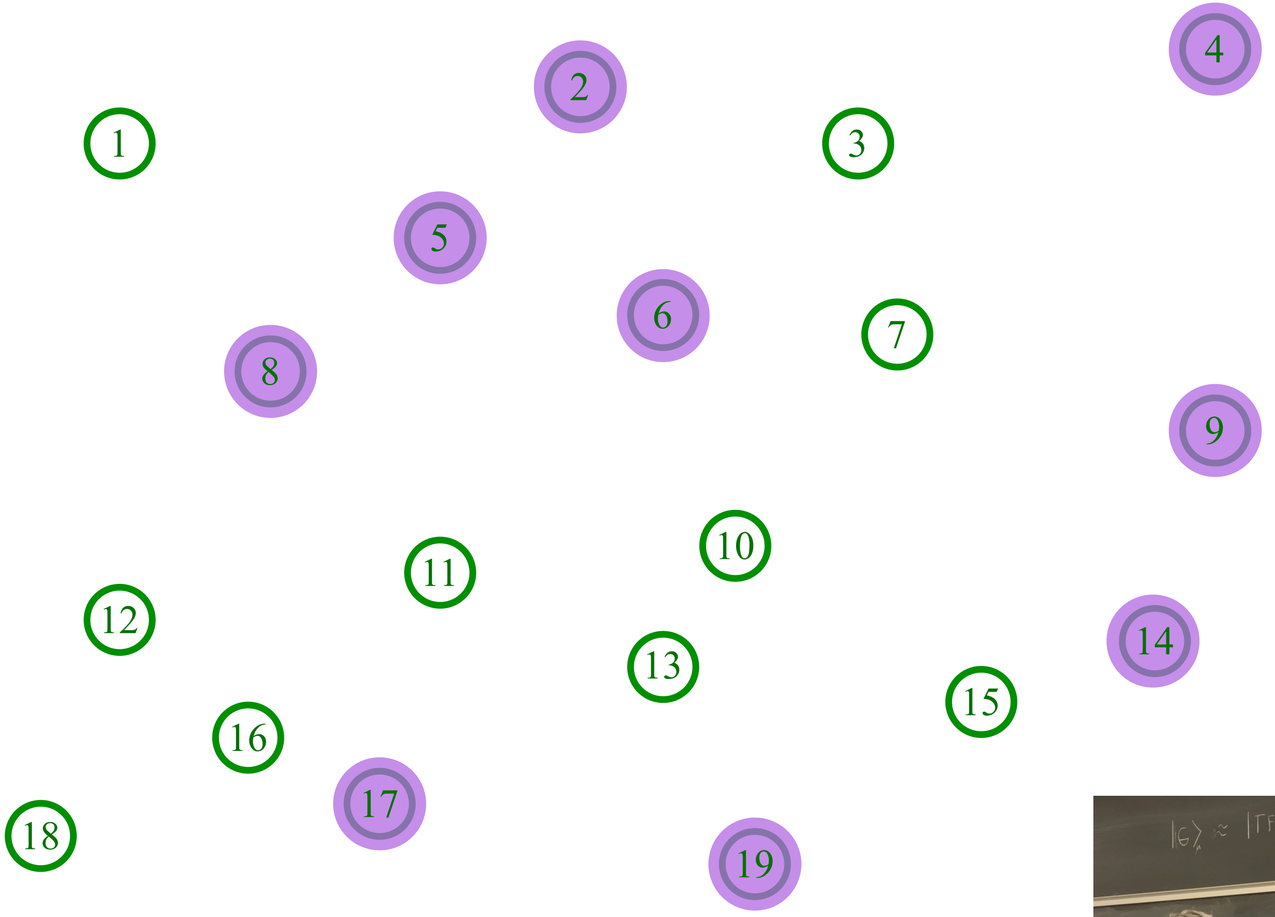
Place electrons randomly on some sites



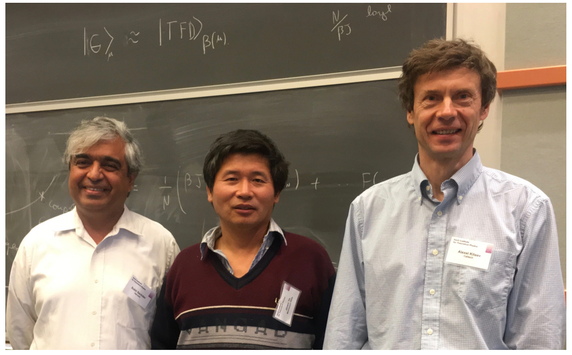
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{11,12;5,14}$



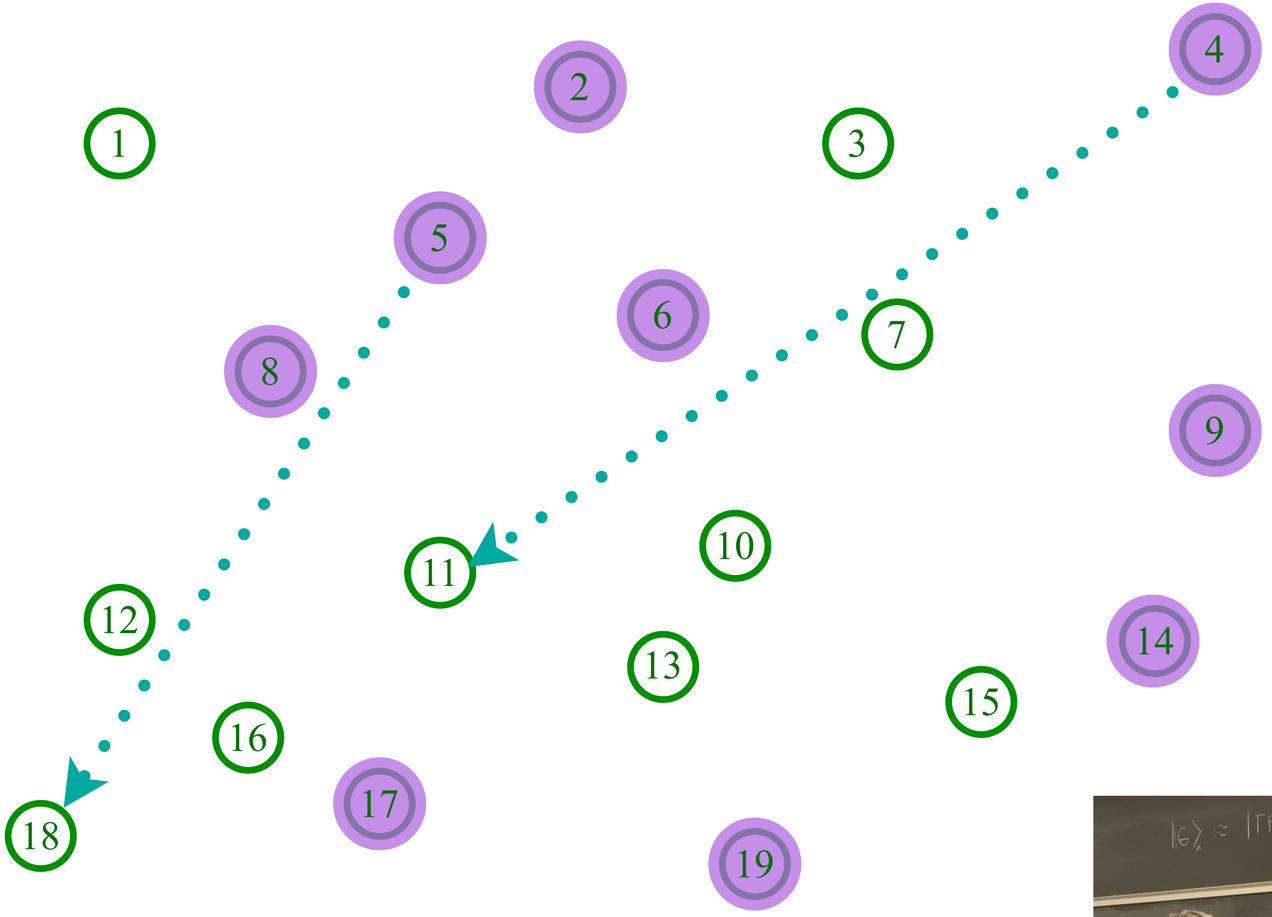
Entangle electrons pairwise randomly



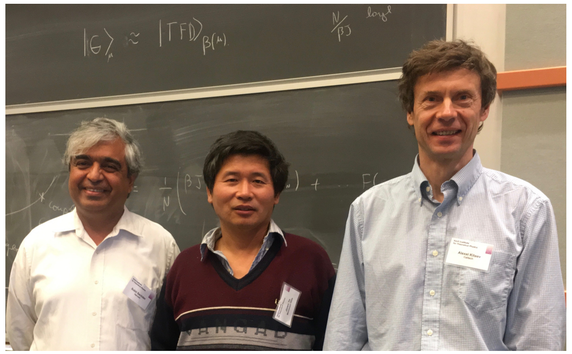
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



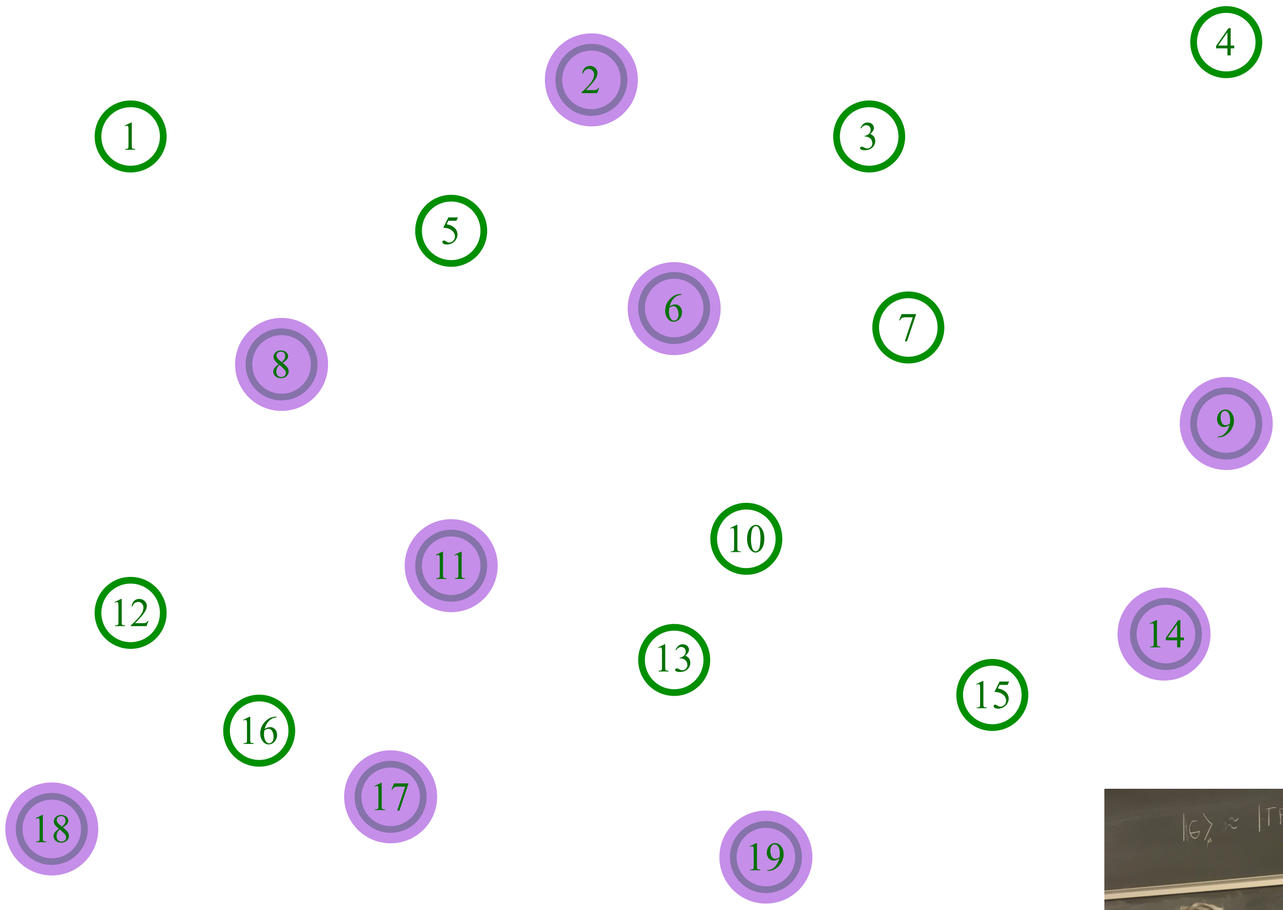
Entangle electrons pairwise randomly



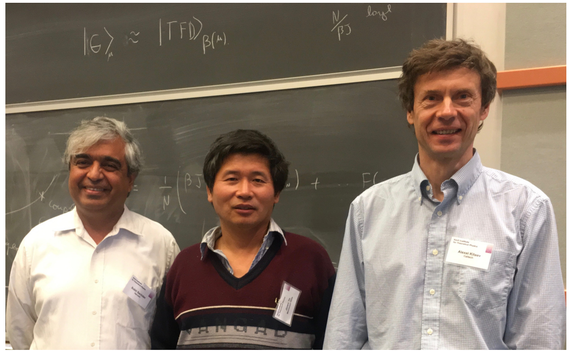
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



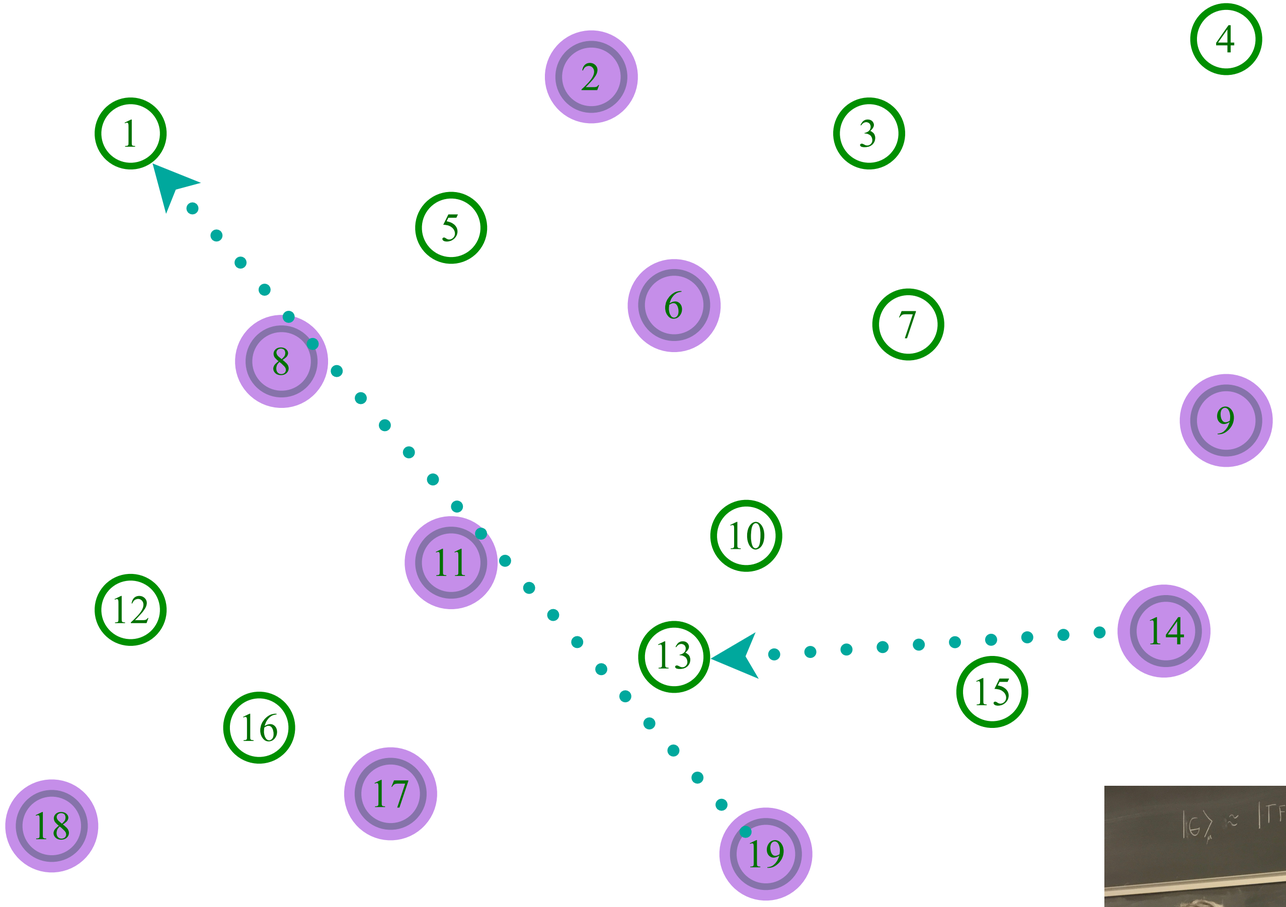
Entangle electrons pairwise randomly



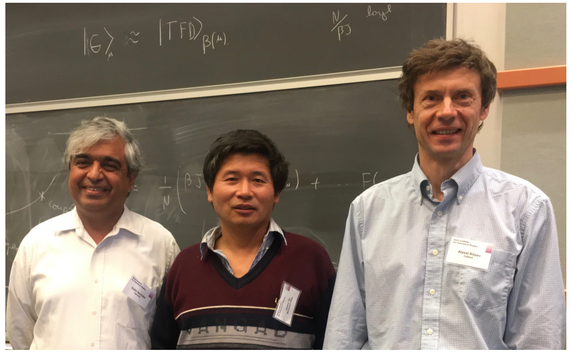
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



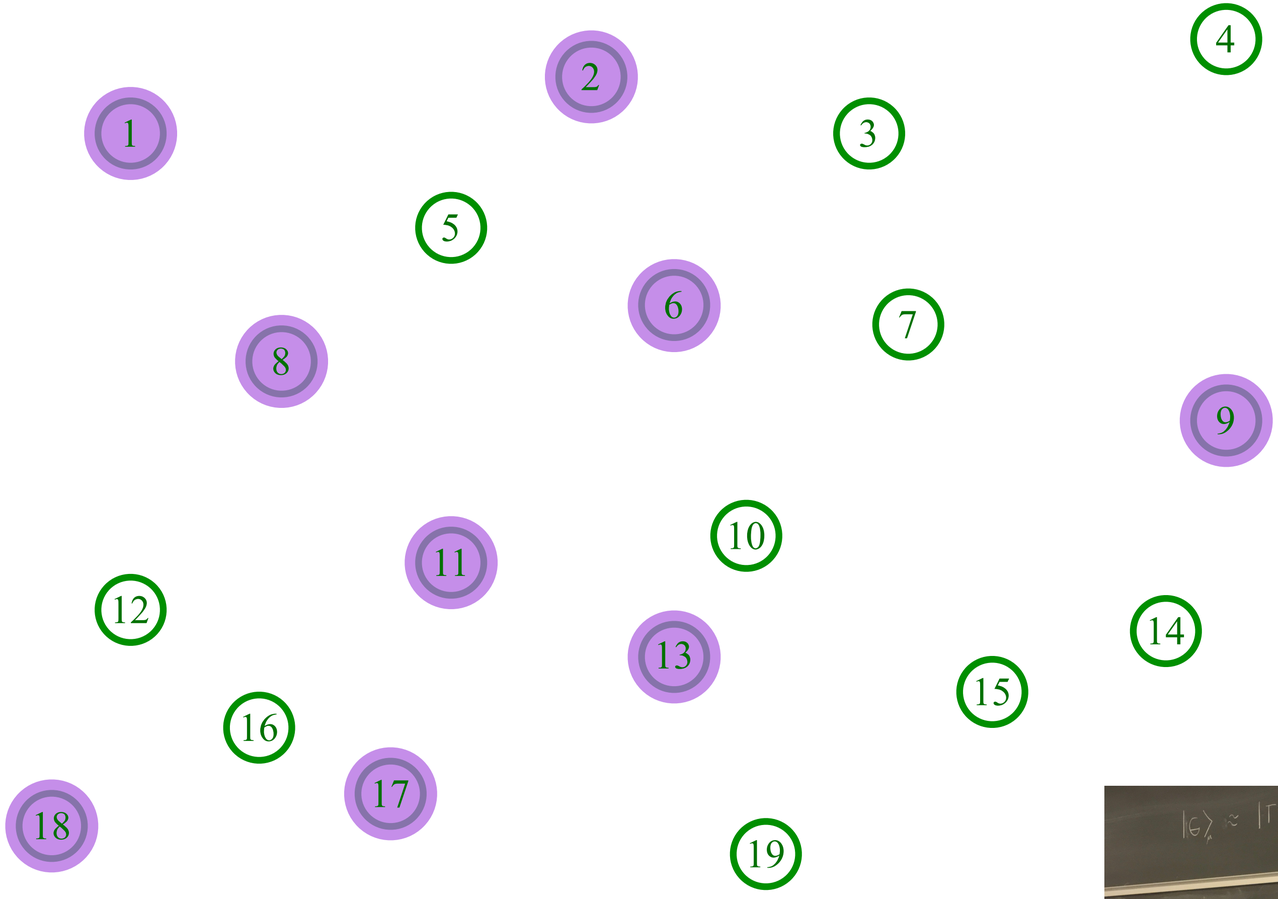
Entangle electrons pairwise randomly



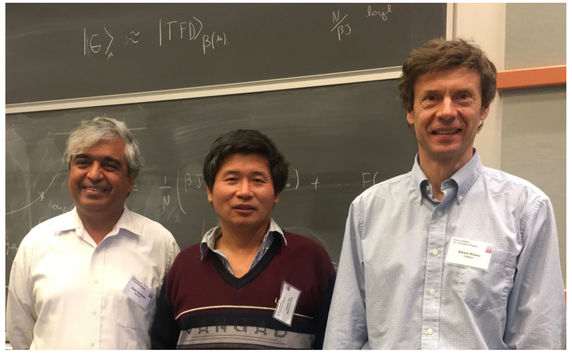
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



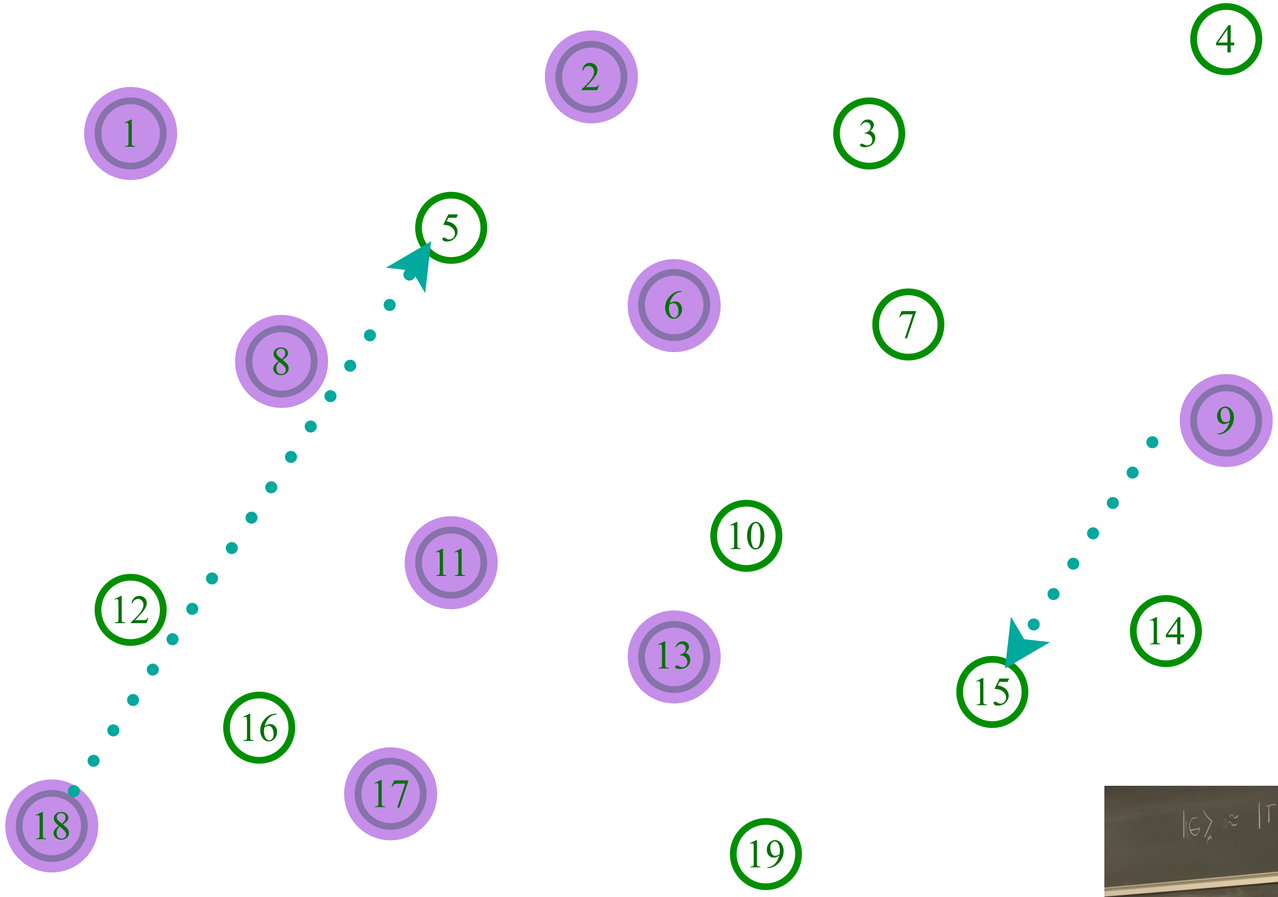
Entangle electrons pairwise randomly



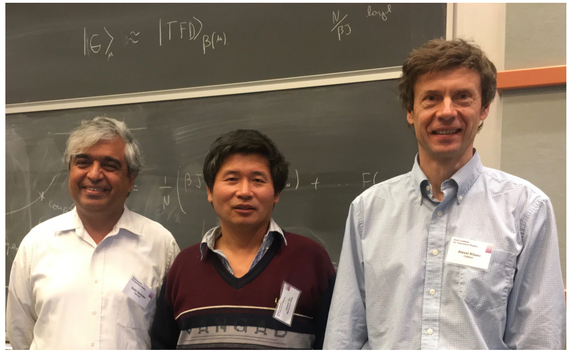
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{9,18;5,15}$



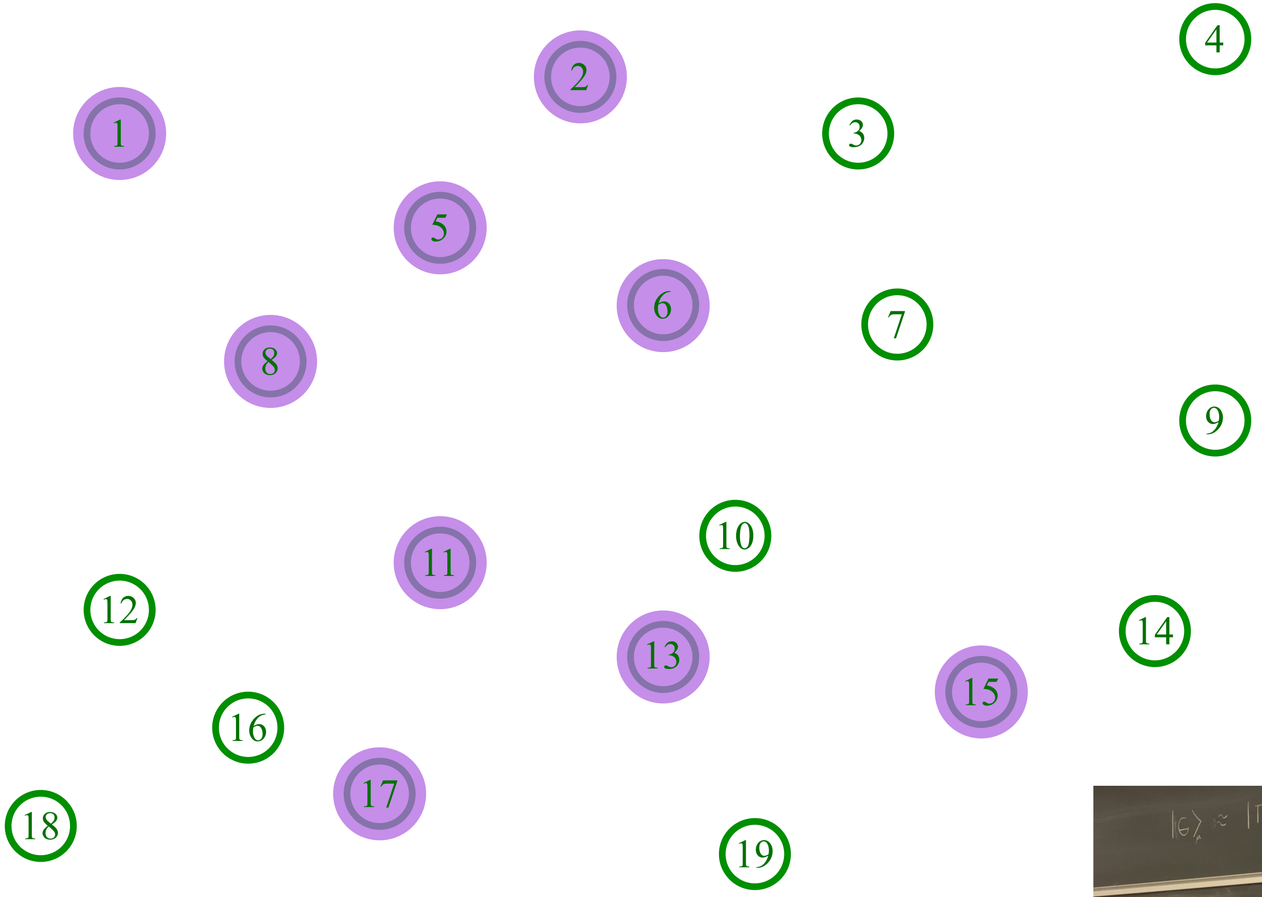
Entangle electrons pairwise randomly



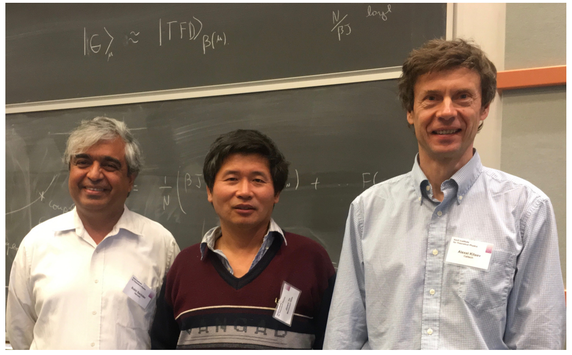
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{9,18;5,15}$



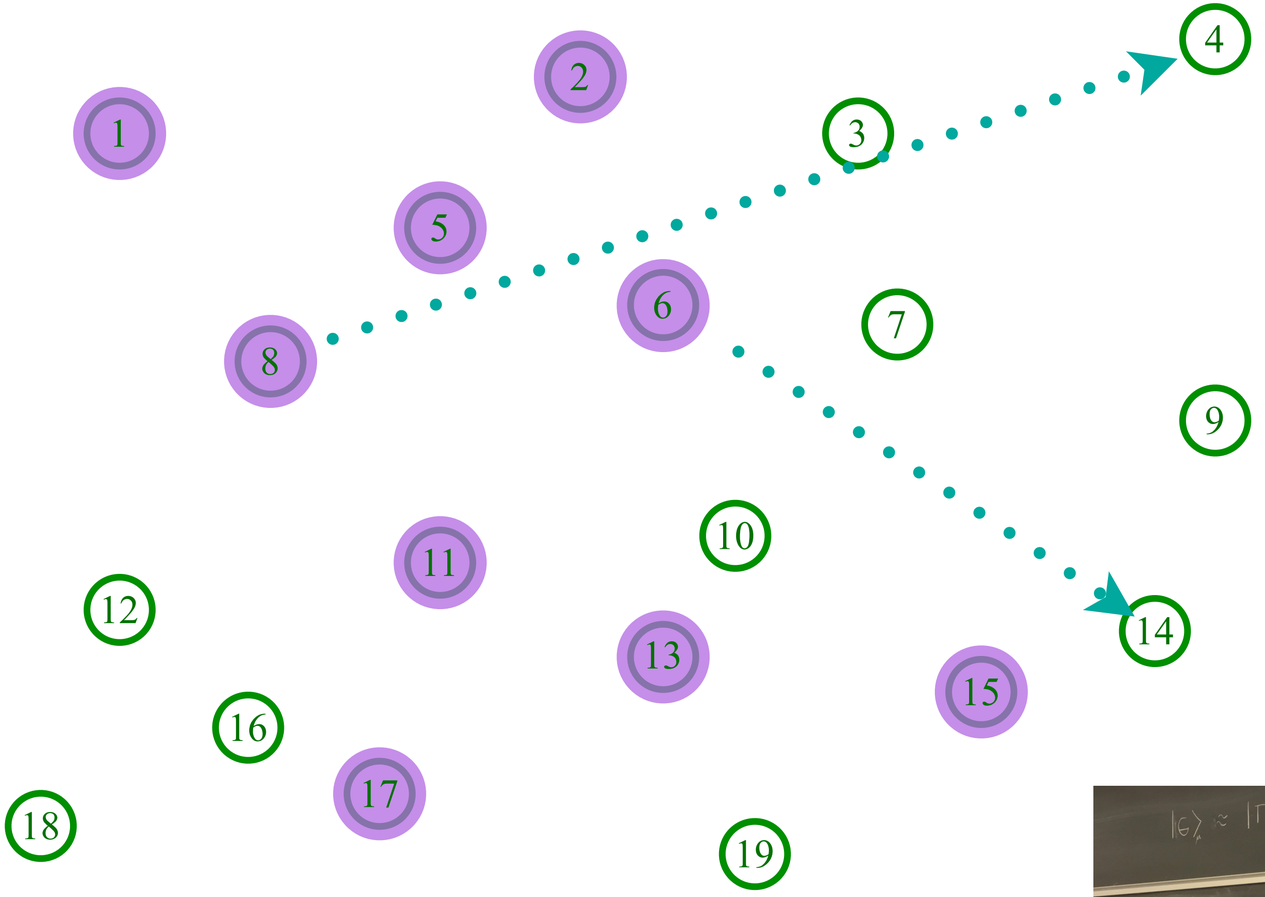
Entangle electrons pairwise randomly



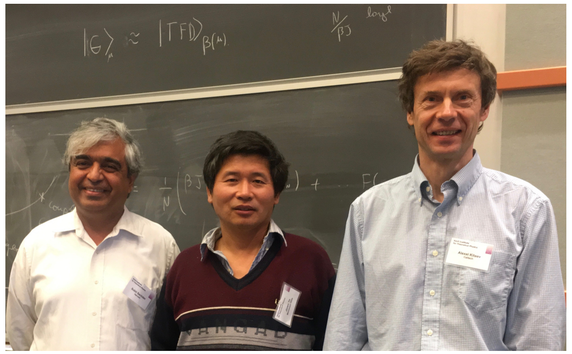
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{6,8;4,14}$



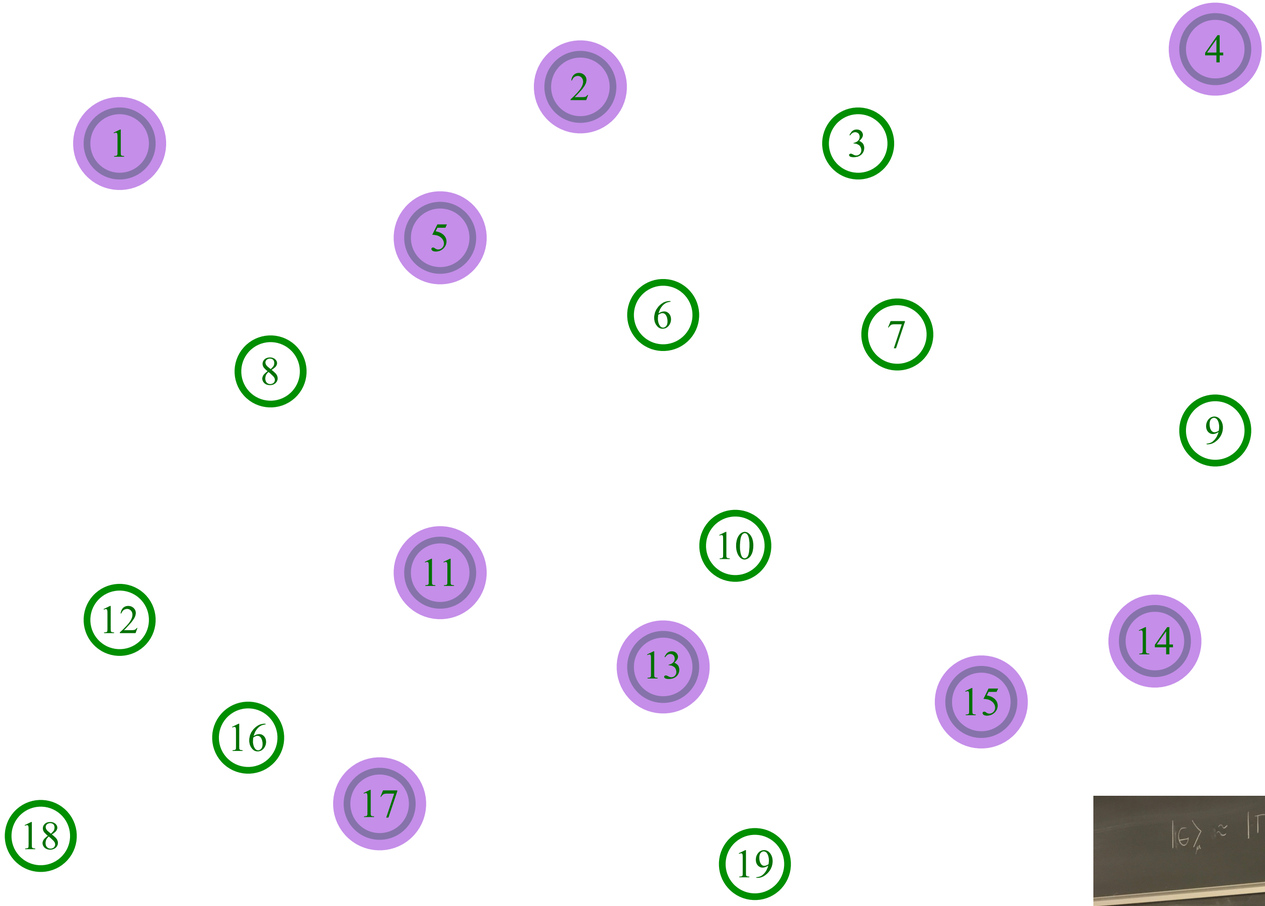
Entangle electrons pairwise randomly



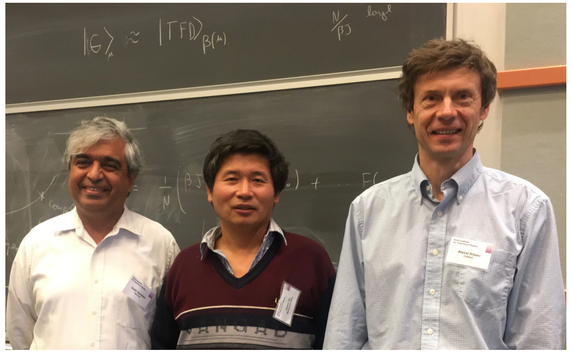
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{6,8;4,14}$



Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

Current is carried by an "entangled quantum soup"

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

Yields a quantum state whose excitations are not particle-like i.e. no bosons, fermions, anyons….

A key consequence of the absence of the particle-like excitations is *Universal Planckian Dissipation*.
The relaxation time, τ, when perturbed at a frequency ω is given by

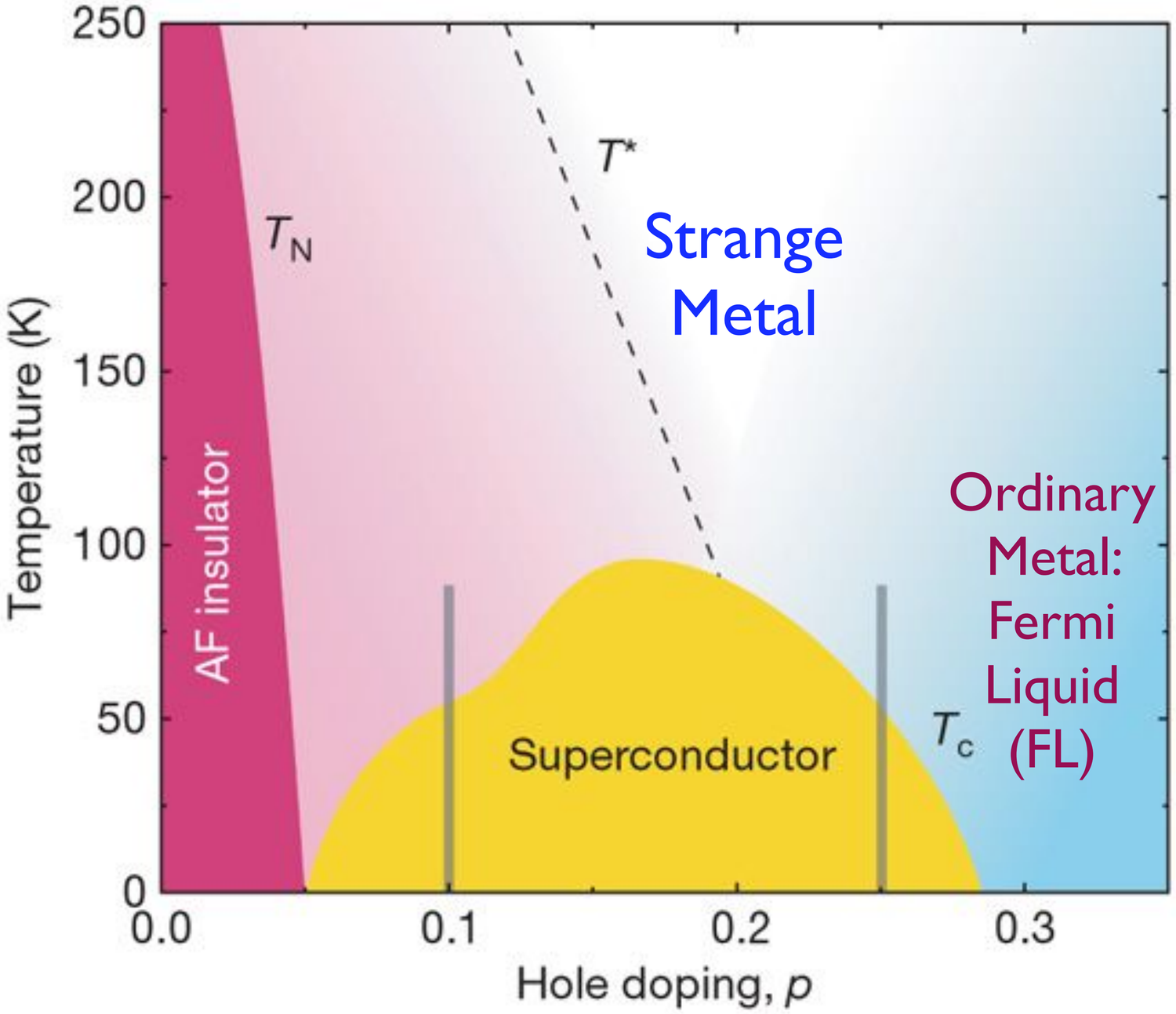
$$\tau = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

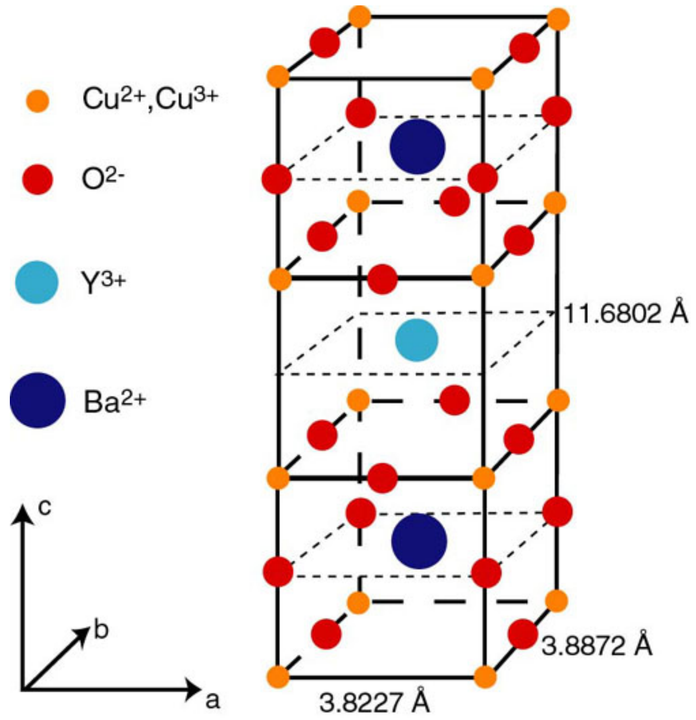
S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

where $\hbar$ is Planck's constant, T is temperature, and the function F is independent of the strength of interaction between the particles.

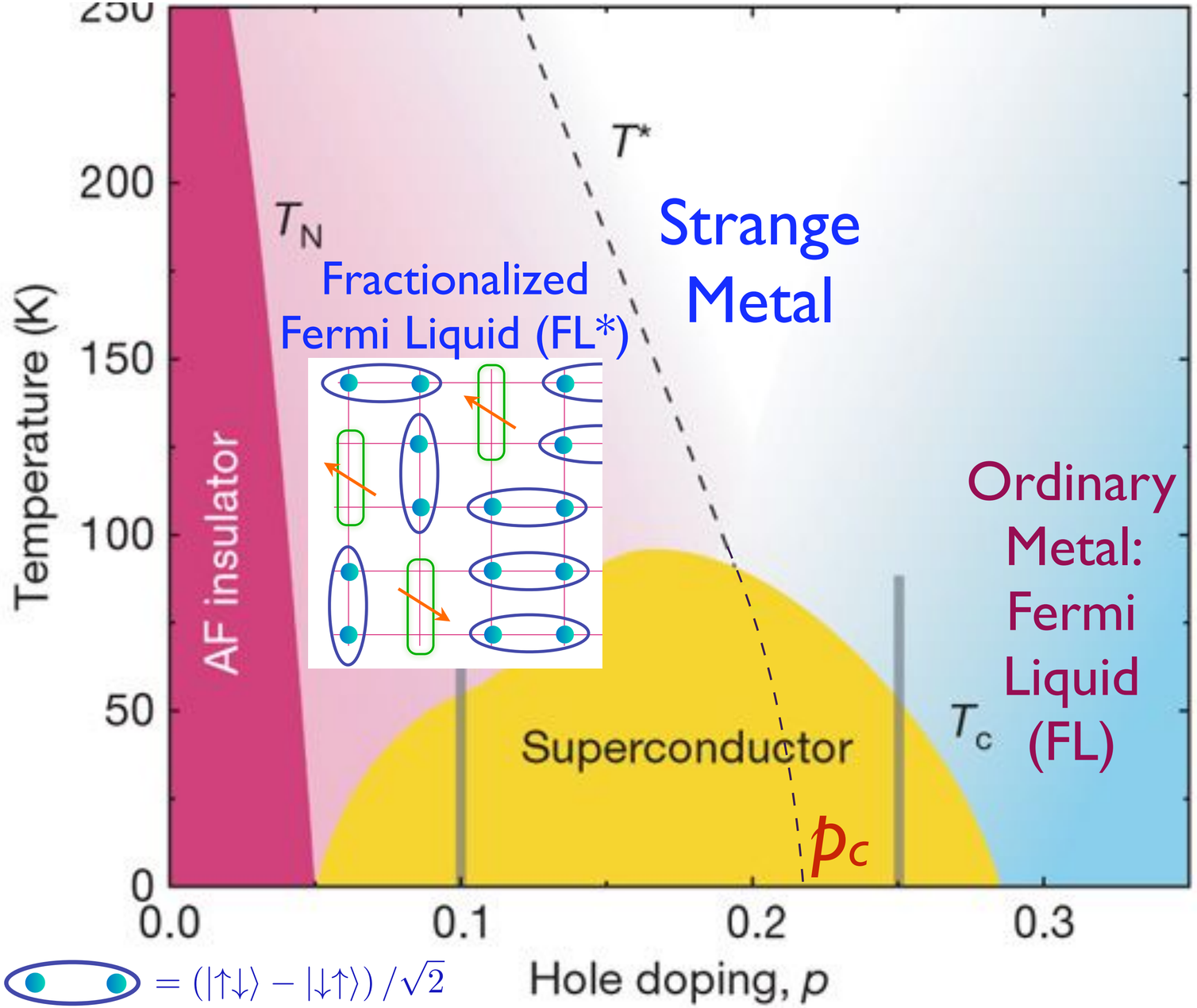
Universal theory of strange metals: two-dimensional Yukawa-SYK model


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
T_N
T^*
AF insulator
Strange Metal
Superconductor
T_c
Ordinary Metal: Fermi Liquid (FL)


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

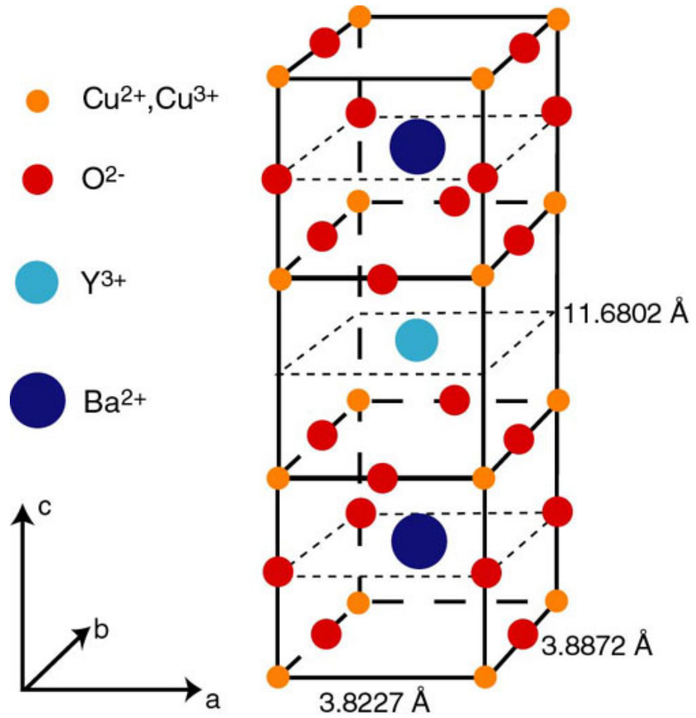
$YBa_2Cu_3O_{6+x}$

Quantum phase transition between two metals



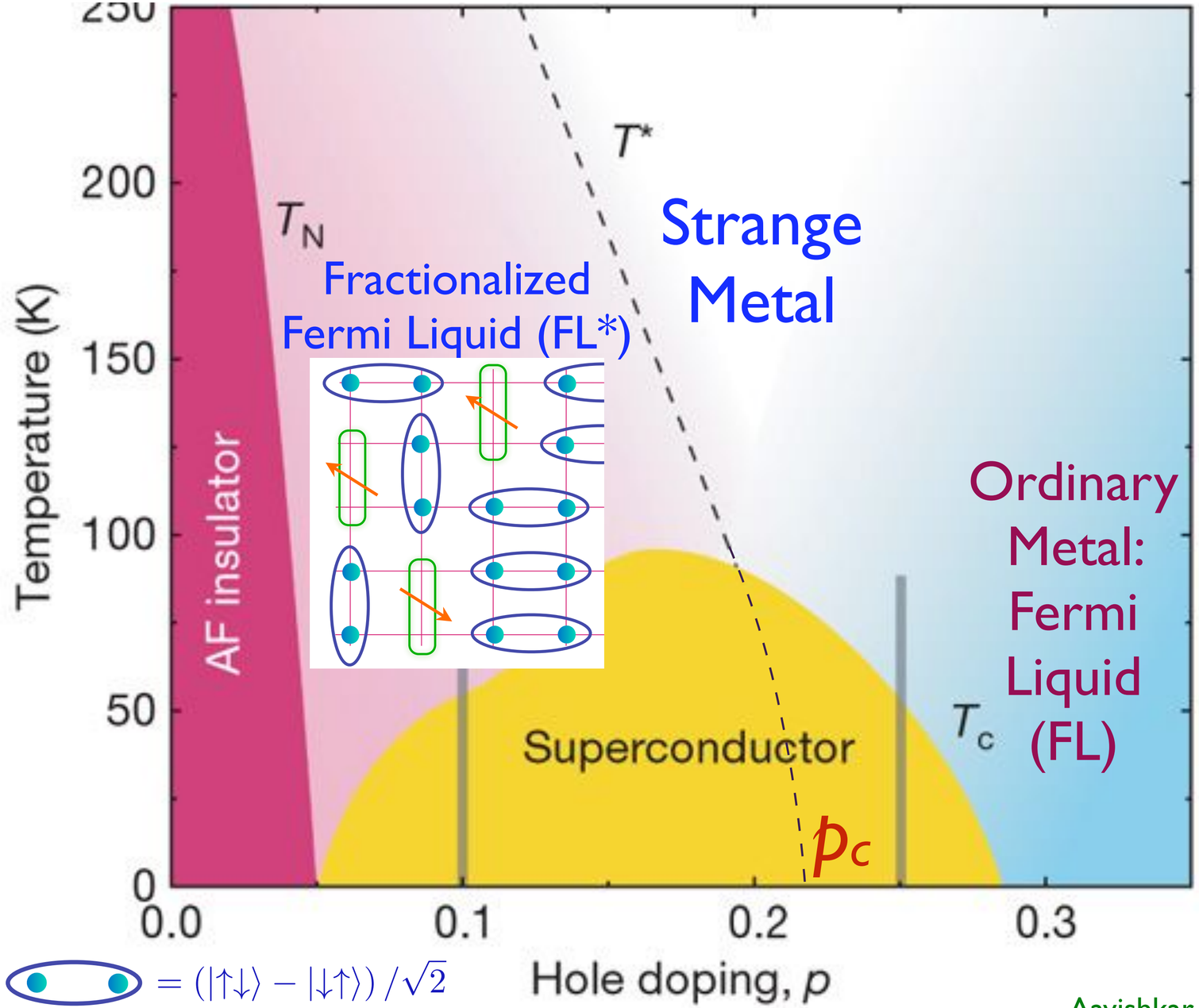
$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$



$YBa_2Cu_3O_{6+x}$

Quantum phase transition between two metals



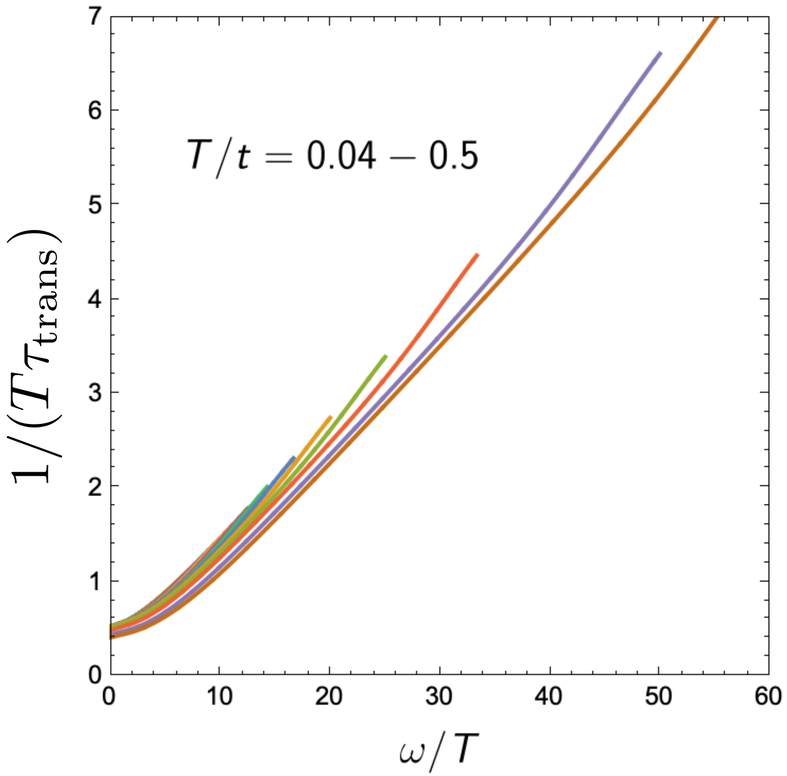
$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle)/\sqrt{2}$

Planckian dynamics from theory of FL*-FL transition: the 2D-YSYK model !

Electron scattering time τ

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$



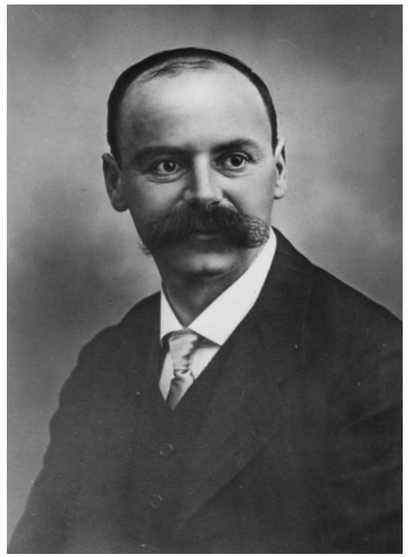
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

Planckian dynamics of black holes and the SYK model

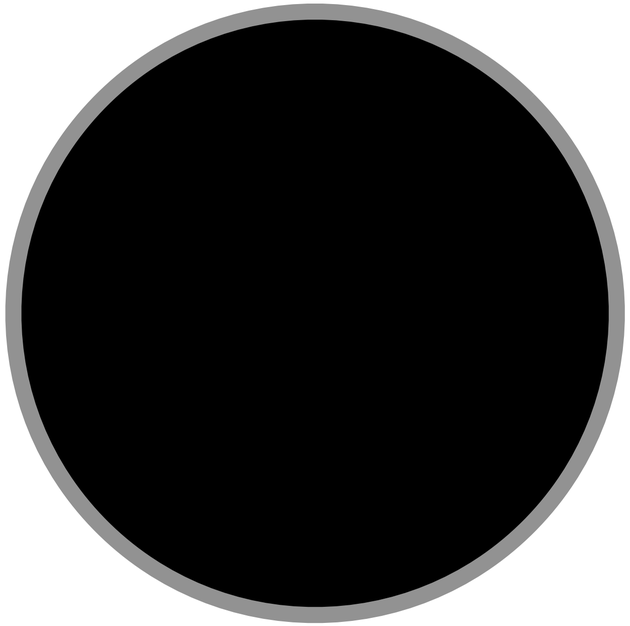
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

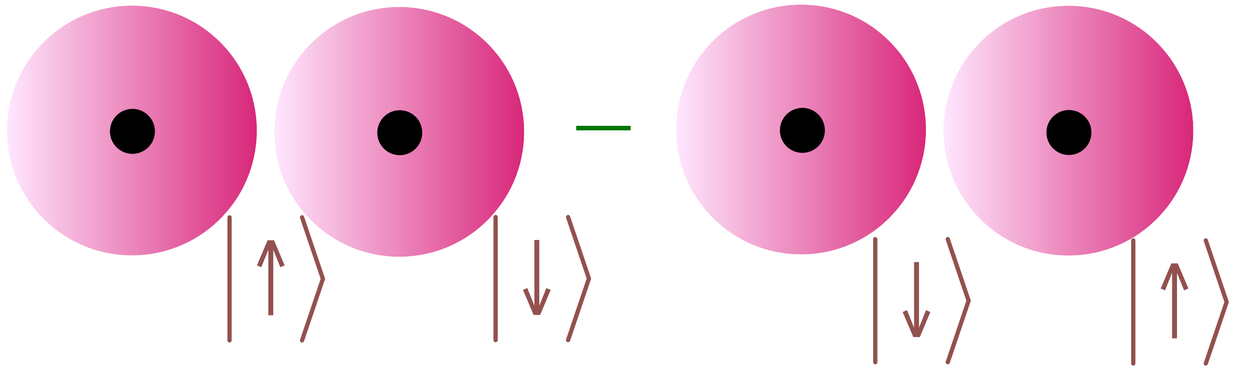
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



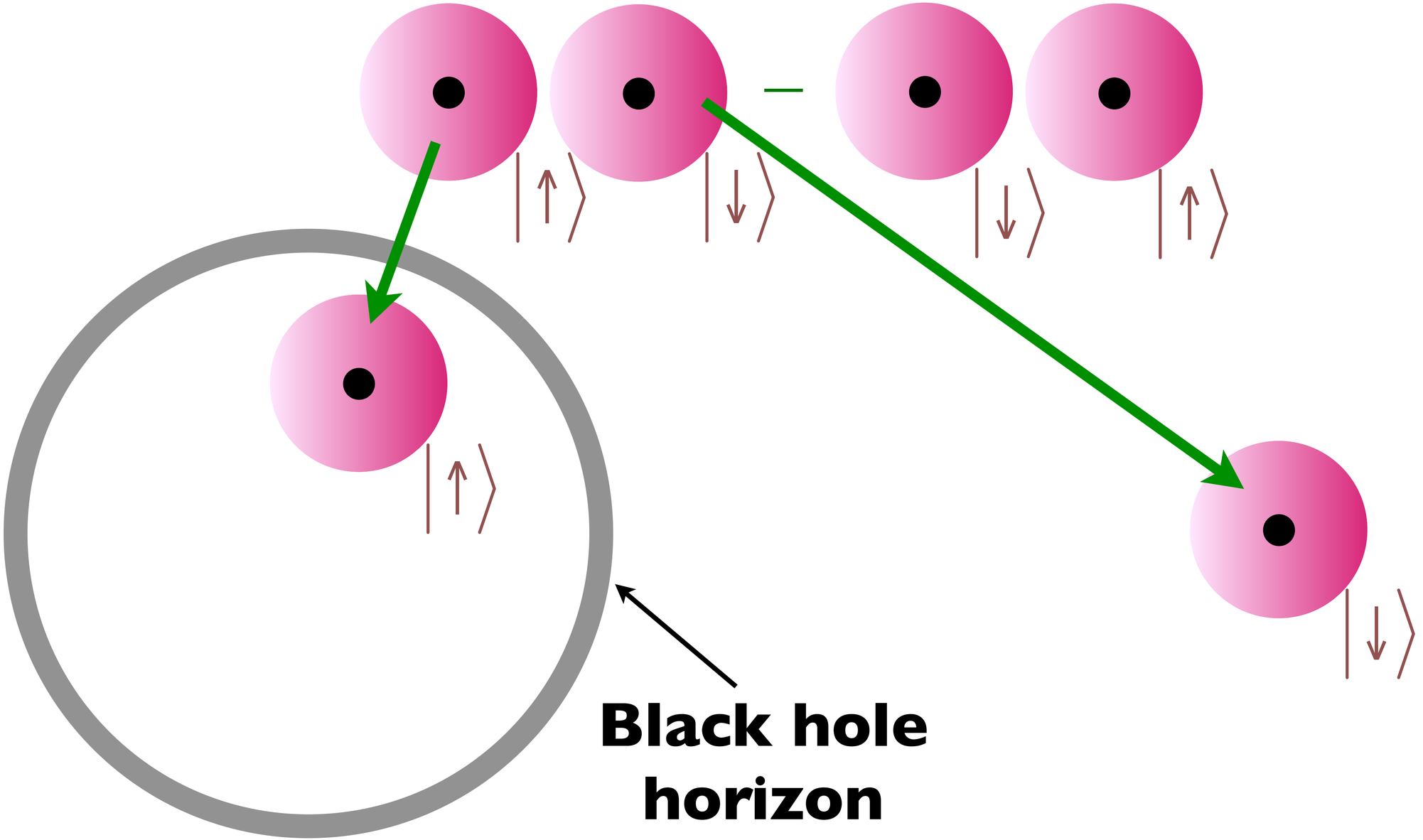
G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

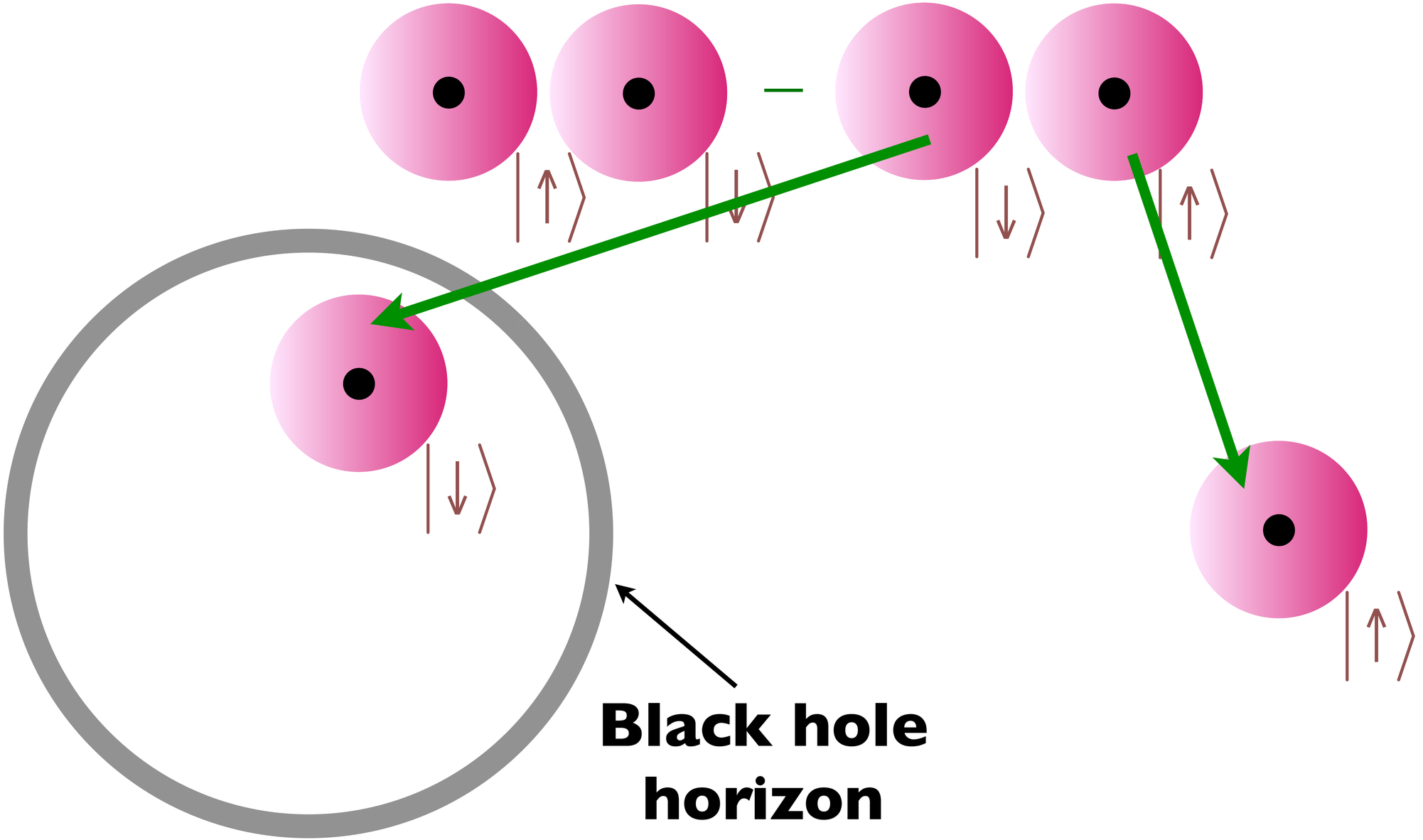
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

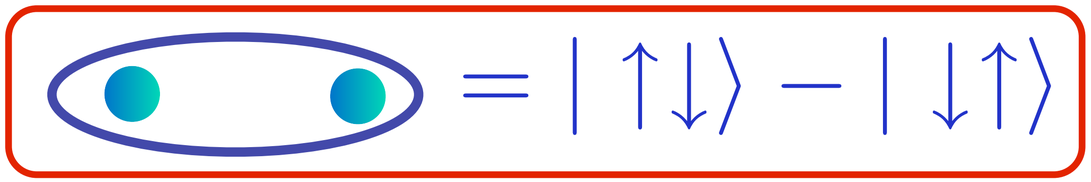


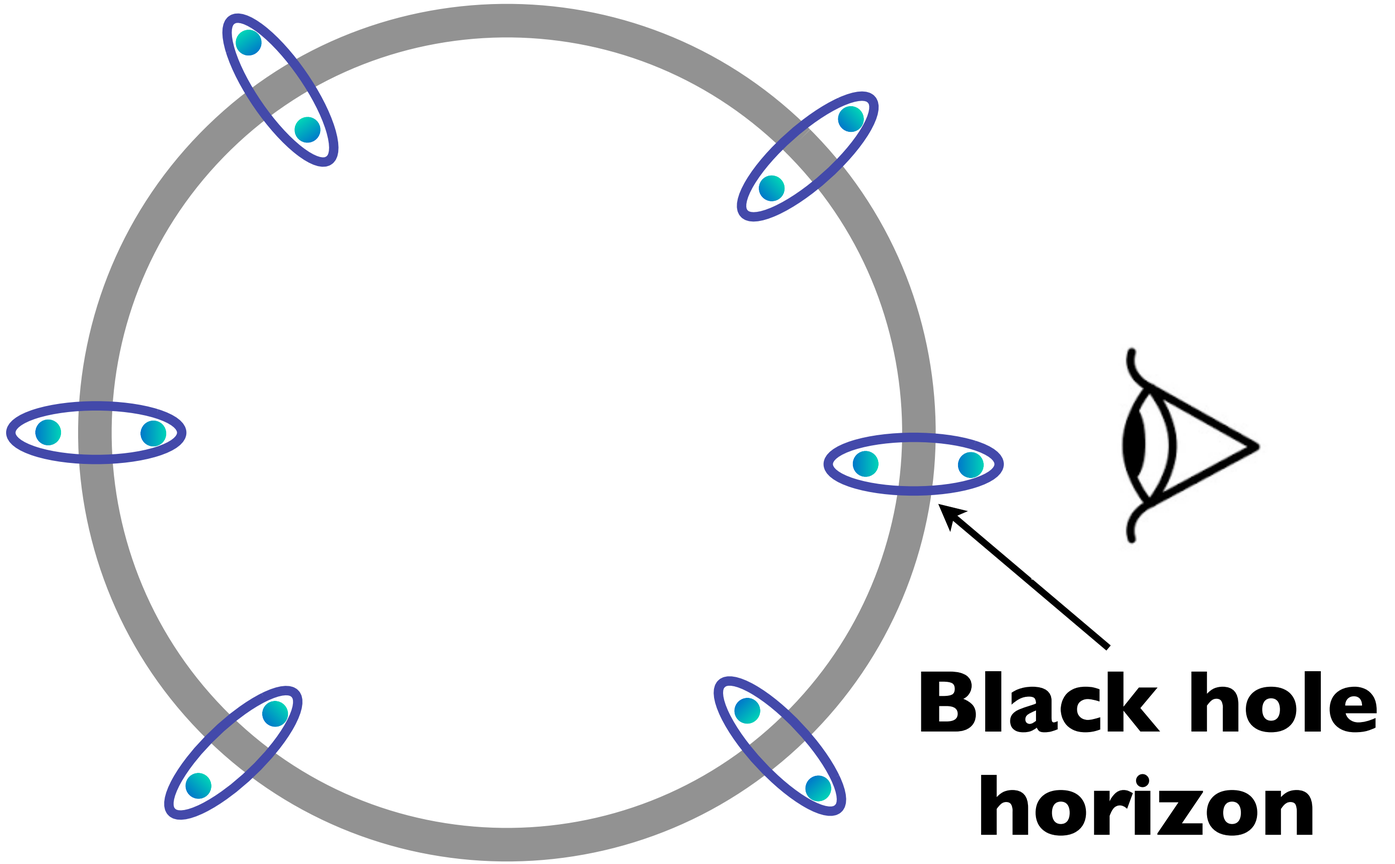
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface





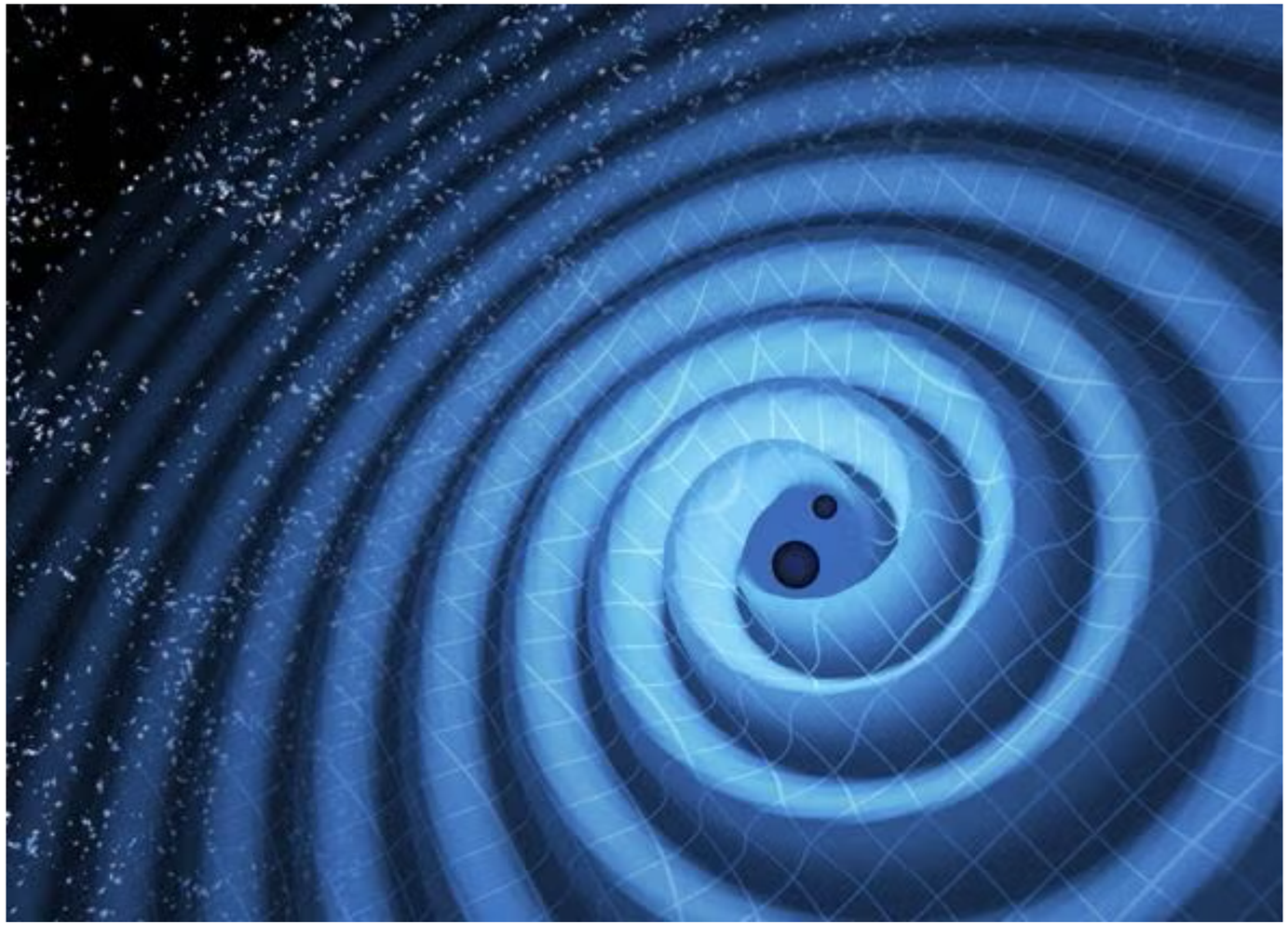
By computations *outside* the black hole, Hawking obtained the black hole entropy

$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Entanglement across a black hole horizon



Artwork depicting gravitational waves emanating from two black holes coalescing.
LIGO/T. Pyle

$$\tau_{\rm ring-down} \sim \frac{8\pi G M}{c^3}$$

C.V. Vishveshwara, Nature **227**, 936 (1970)

$$\text{Using } T_H = \frac{\hbar c^3}{8\pi G M k_B},$$

the Hawking temperature of the black hole,

$$\tau_{\rm ring-down} \sim \frac{\hbar}{k_B T_H}$$

Planckian dynamics of quasi-normal modes!

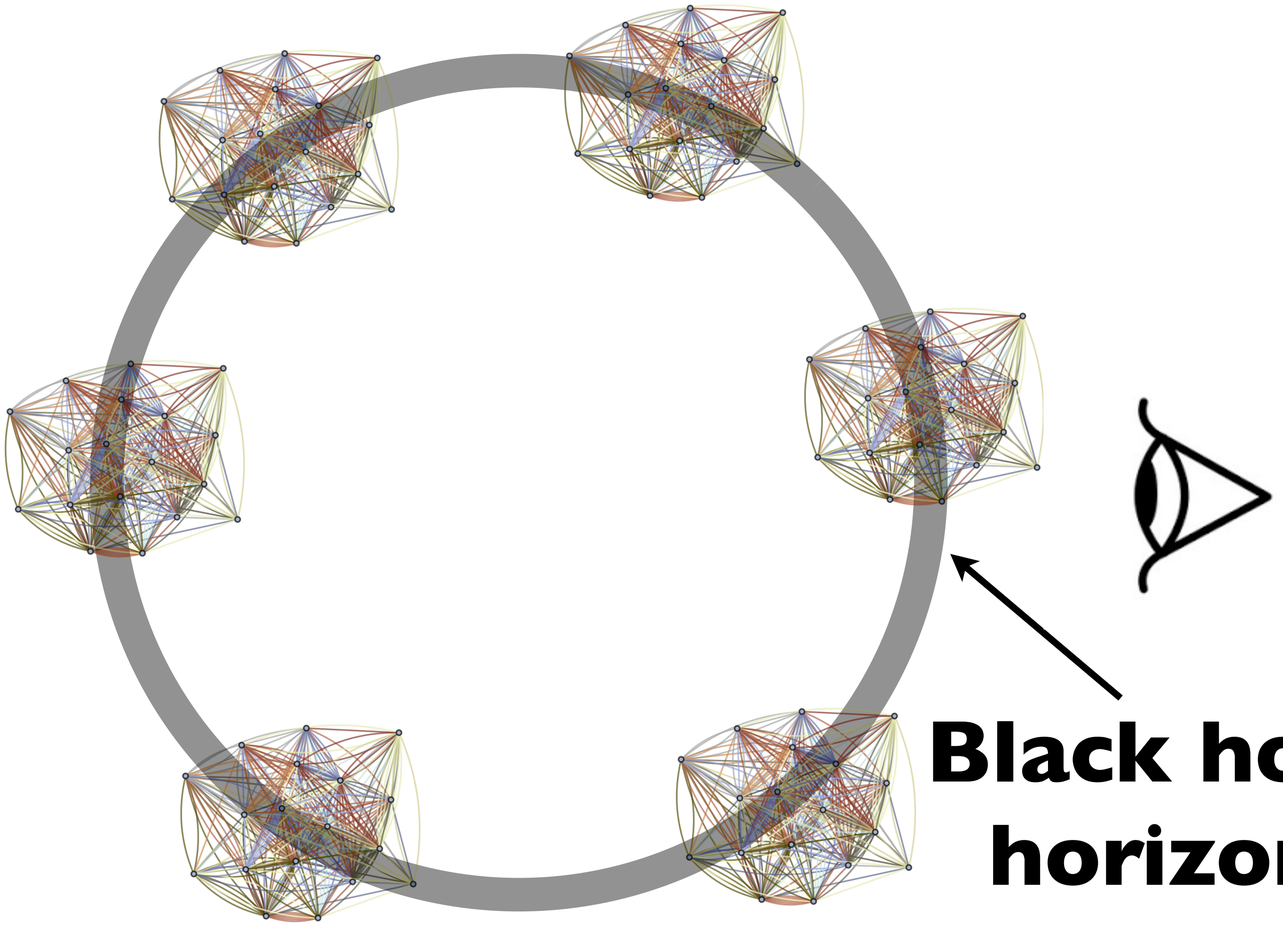
S. Sachdev, PRL **105**, 151602 (2010)

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

S. Sachdev, PRL **105**, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

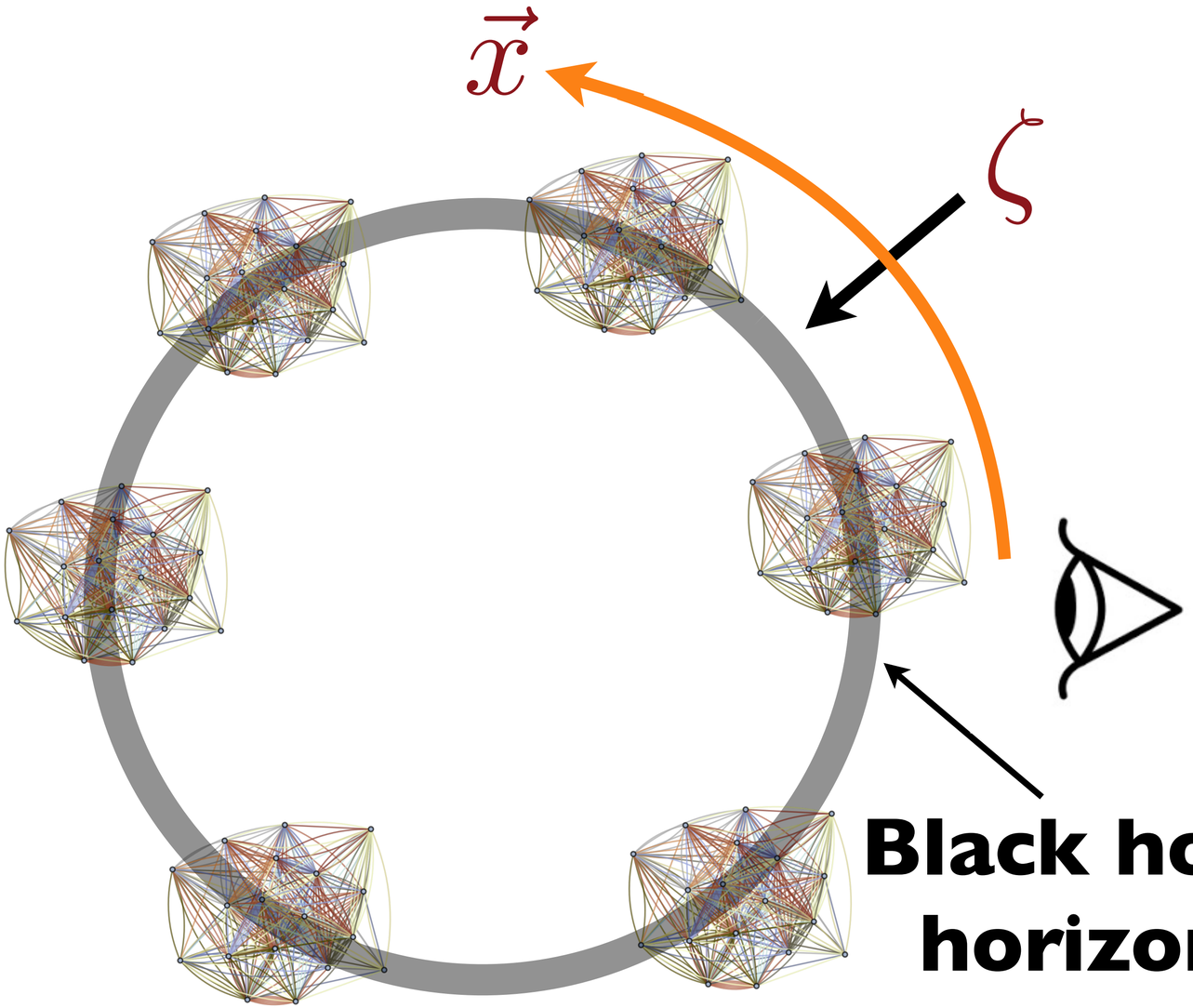


Black hole horizon

Subir Sachdev

"...This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R_2$ physics of Reissner- Nordström black holes."

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



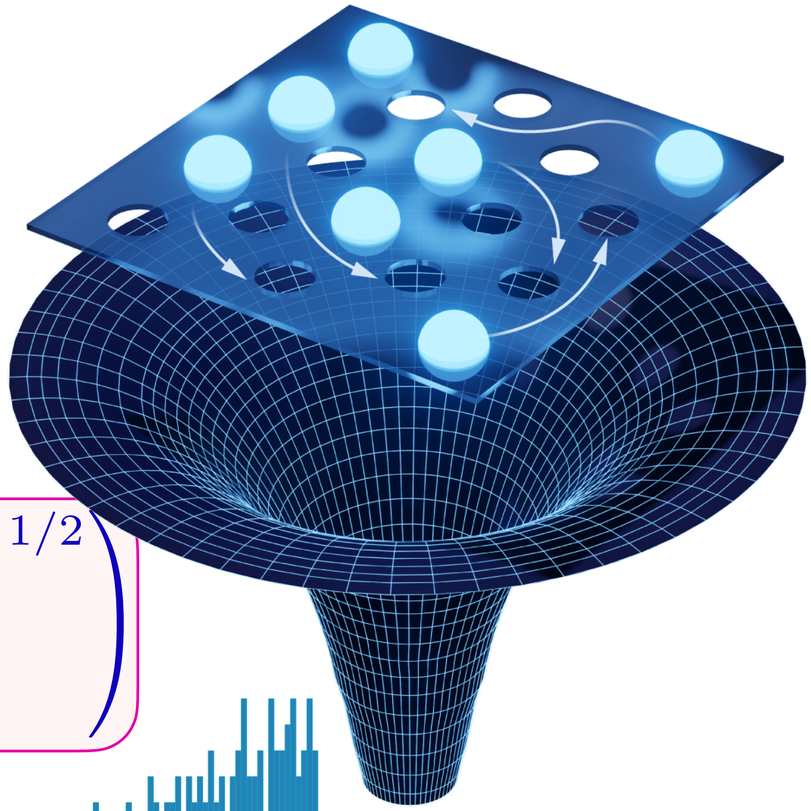
The quantum versions of Maxwell's and Einstein's equations in ζ space and time are also the equations describing electron entanglement in the SYK model!

Kitaev (2015), Maldacena Stanford (2015)

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

D(E) of charged black holes from the SYK model

- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

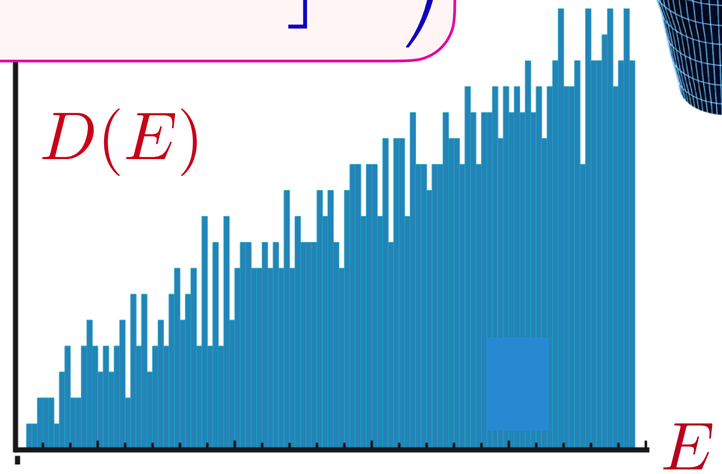


Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} {A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Iliesiu, Murthy, Turiaci (2022)

Developments from the SYK model



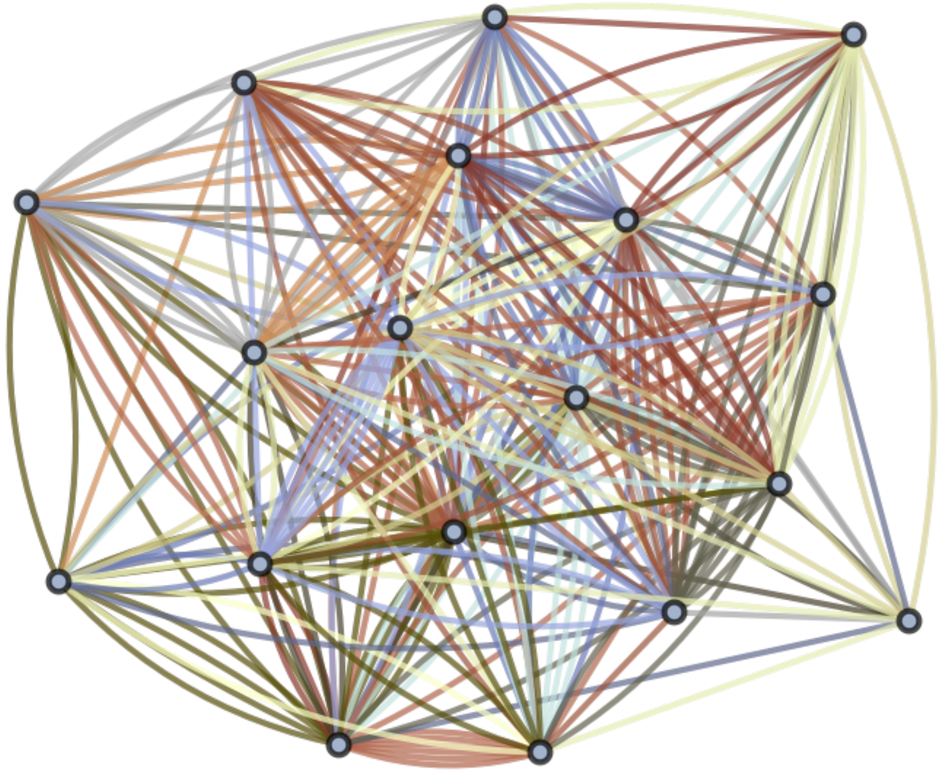
Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

Recap

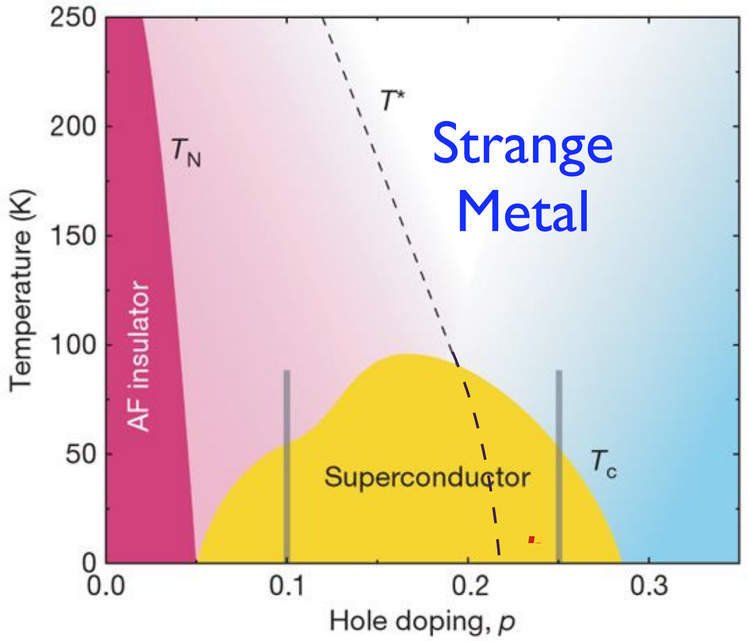
The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in Planckian dynamics and the loss of identity of the particles



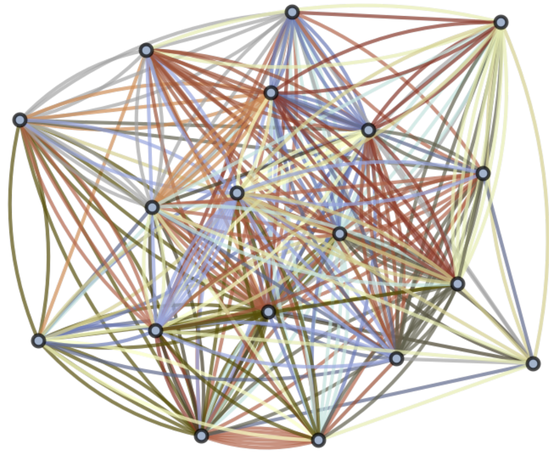
The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in Planckian dynamics and the loss of identity of the particles



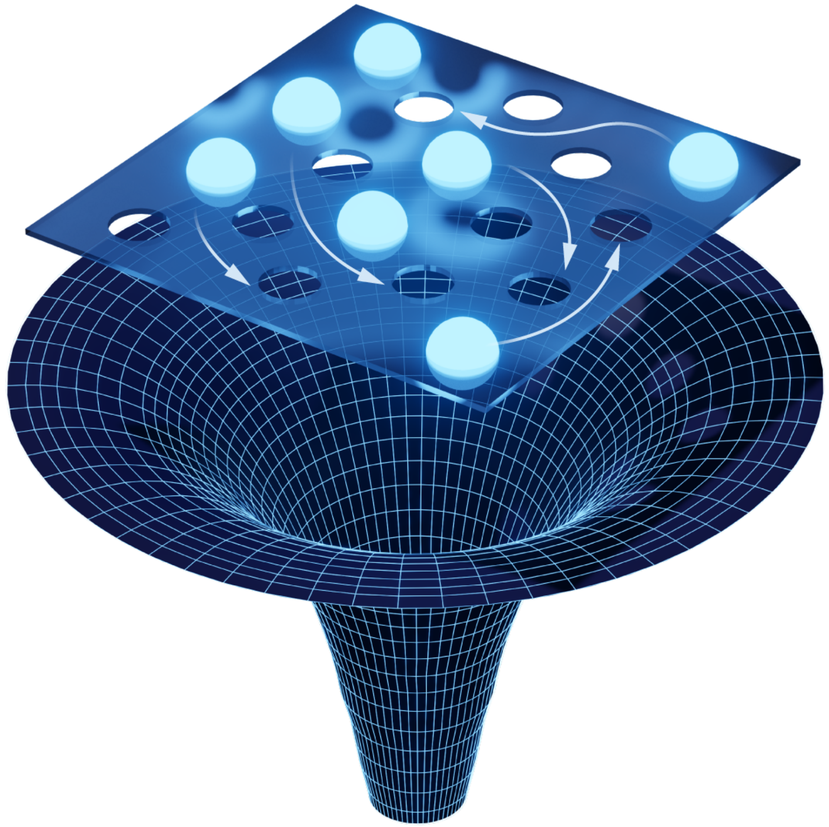
In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in Planckian dynamics and the loss of identity of the particles



In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)

In a *dual* set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)