

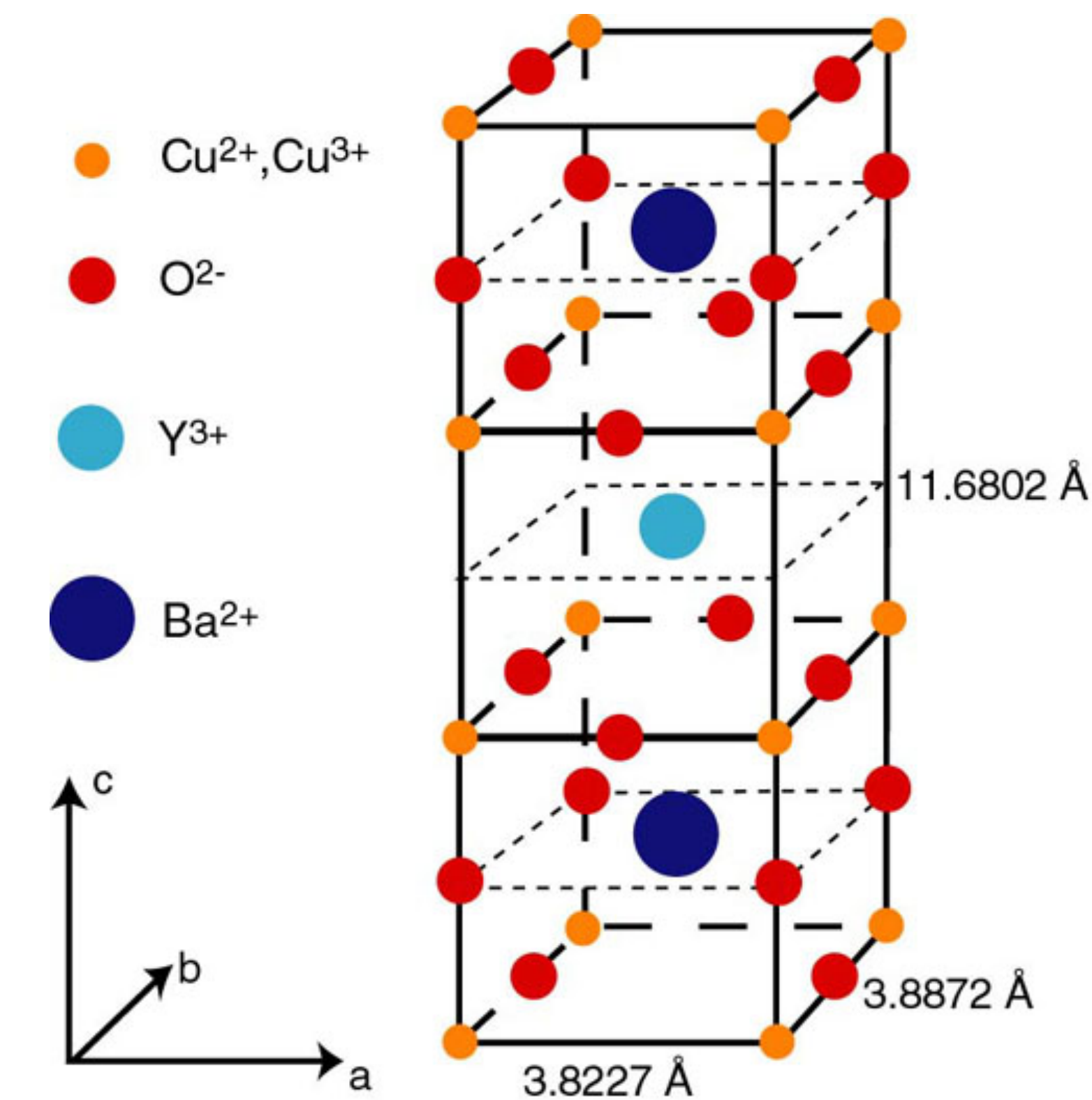
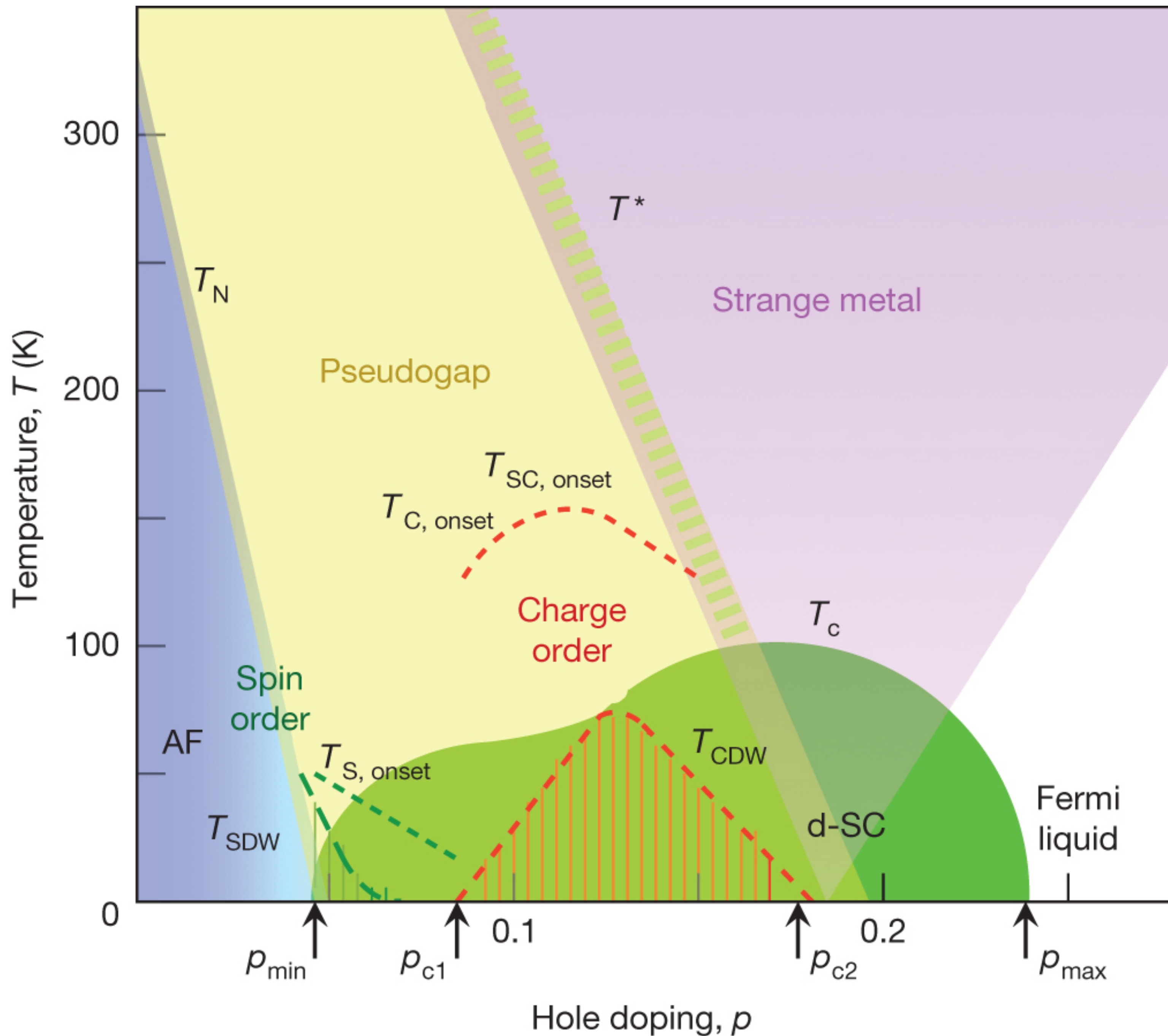
The foot, the fan, and the cuprate phase diagram: Fermi-volume-changing quantum phase transitions

Harvard University
February 7, 2025
Subir Sachdev

arXiv:2501.16417



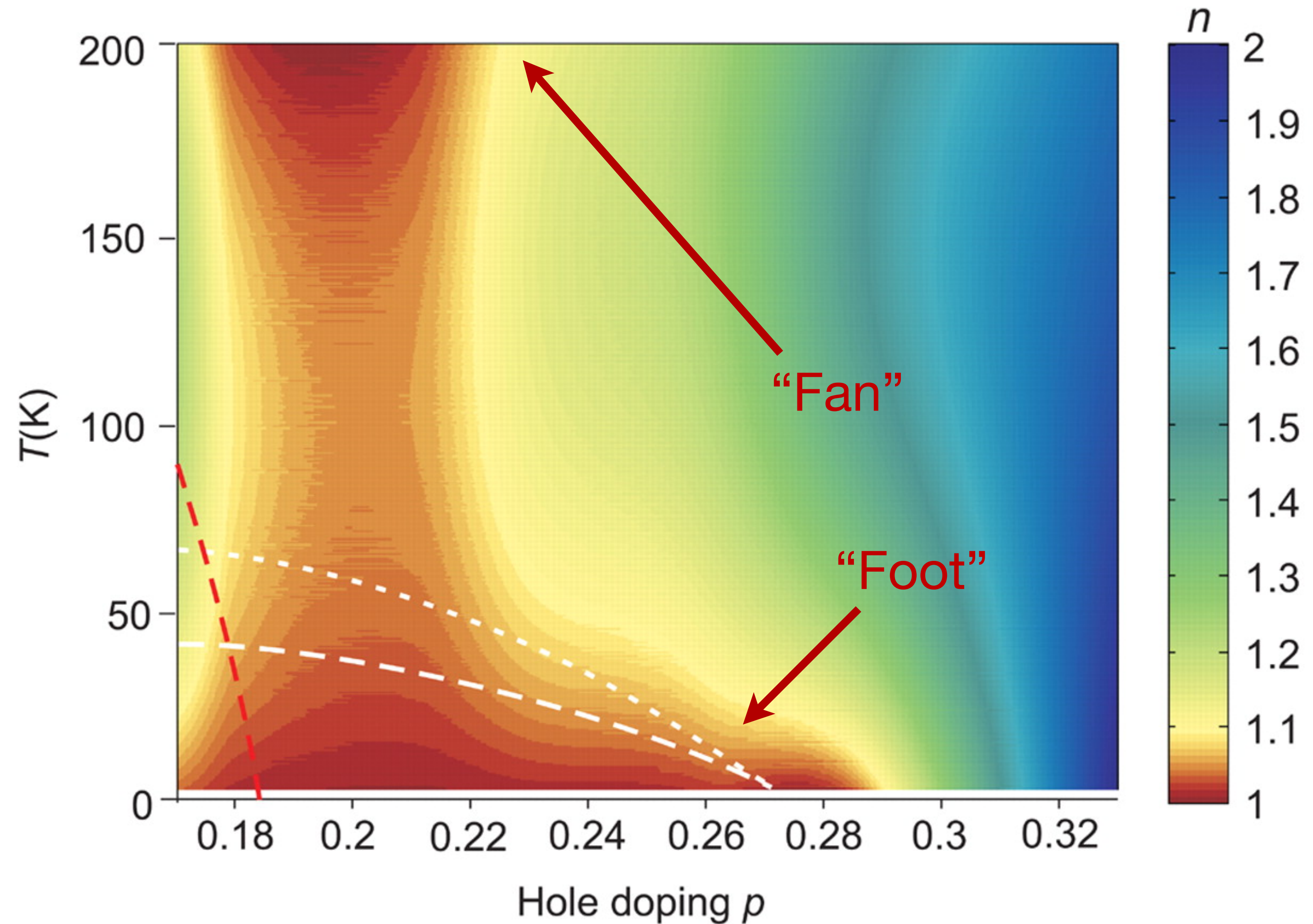
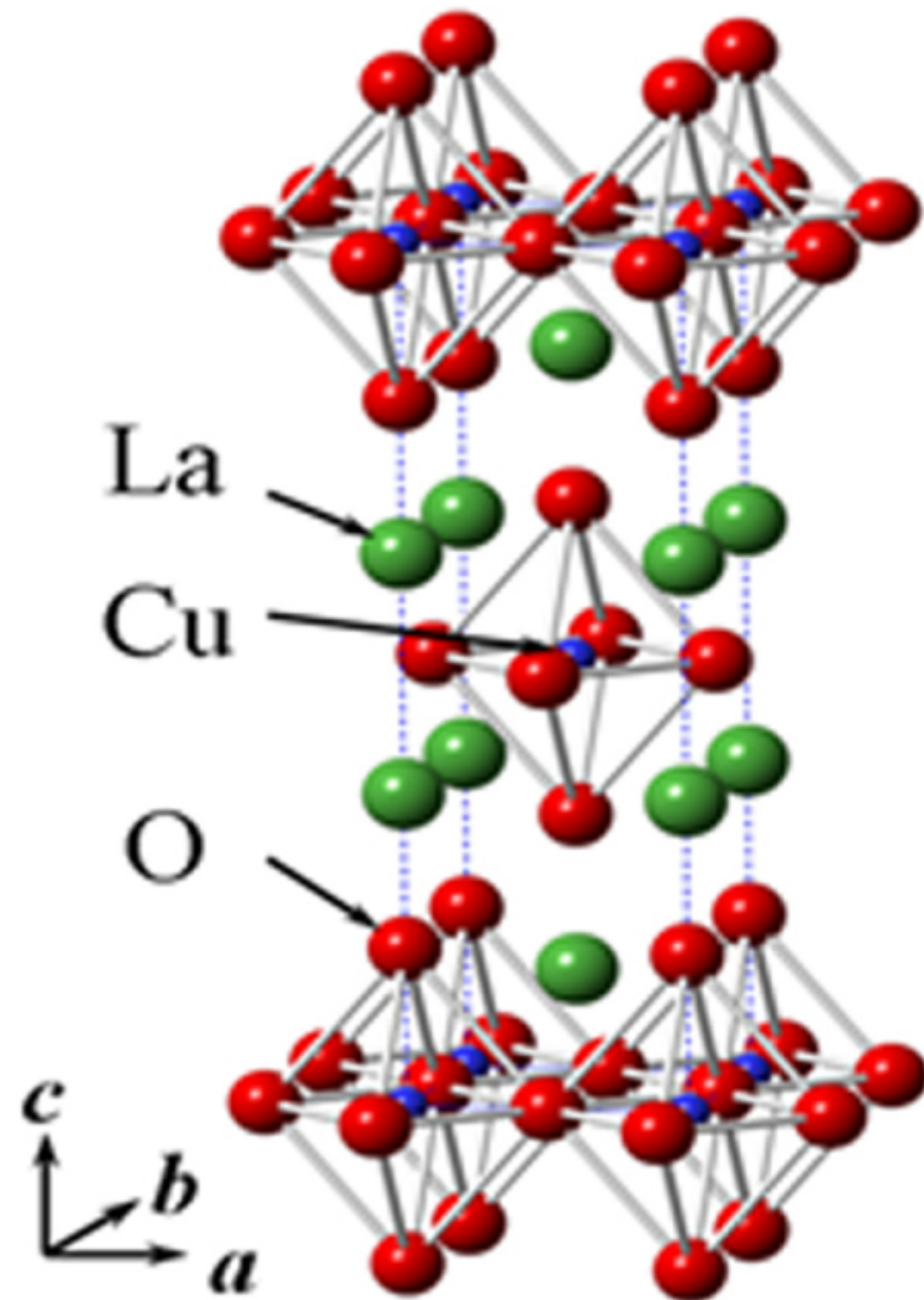
Talk online: sachdev.physics.harvard.edu

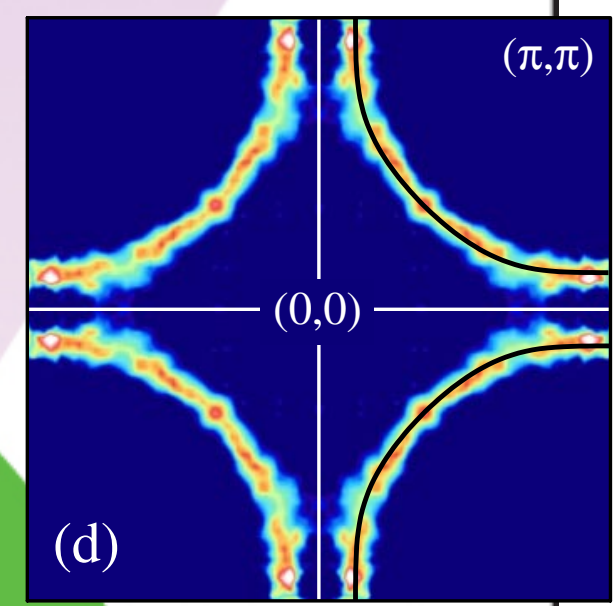
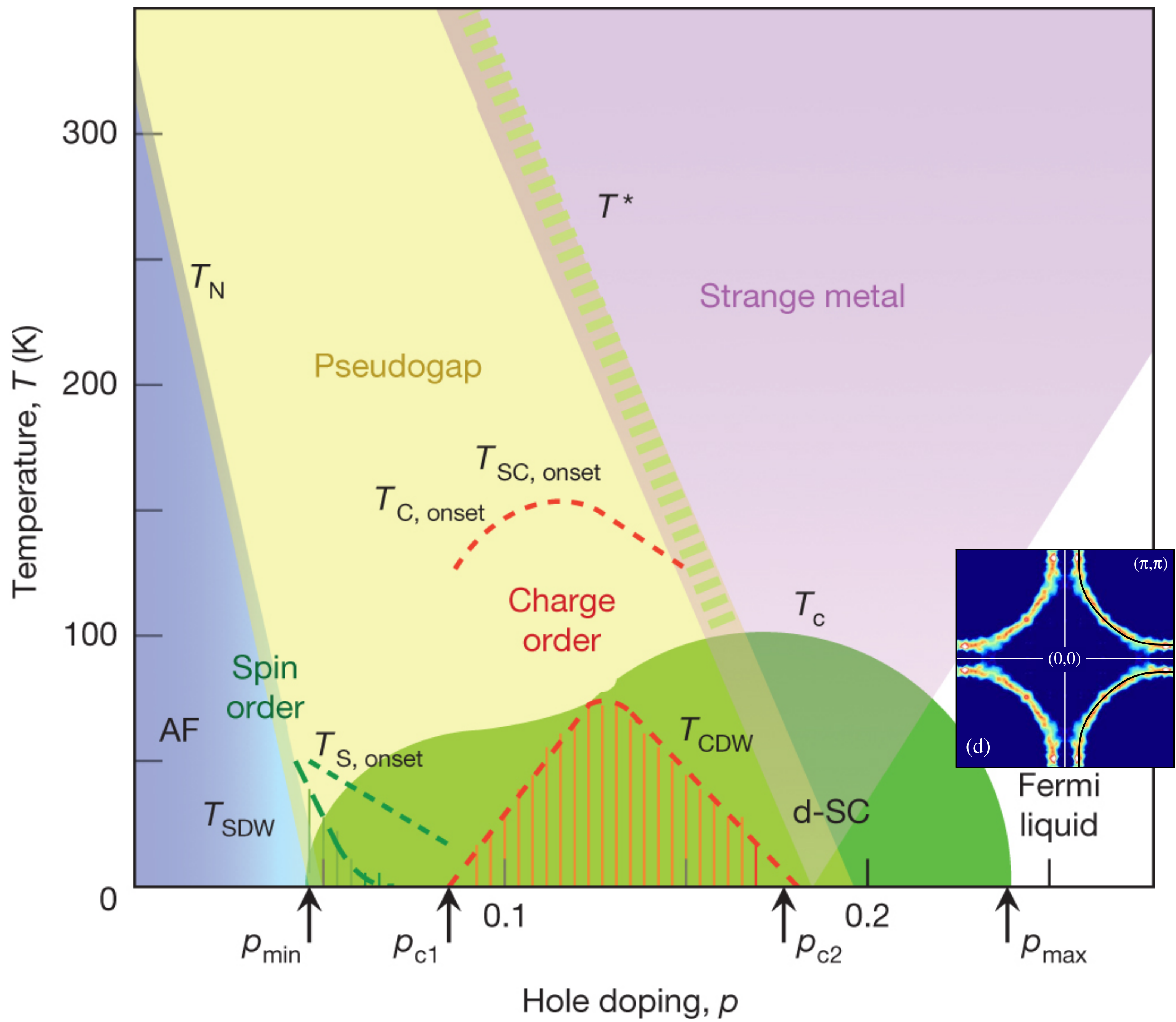


Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

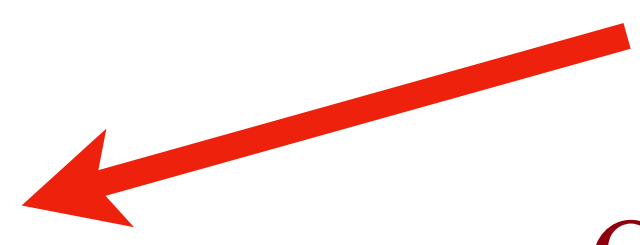
R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

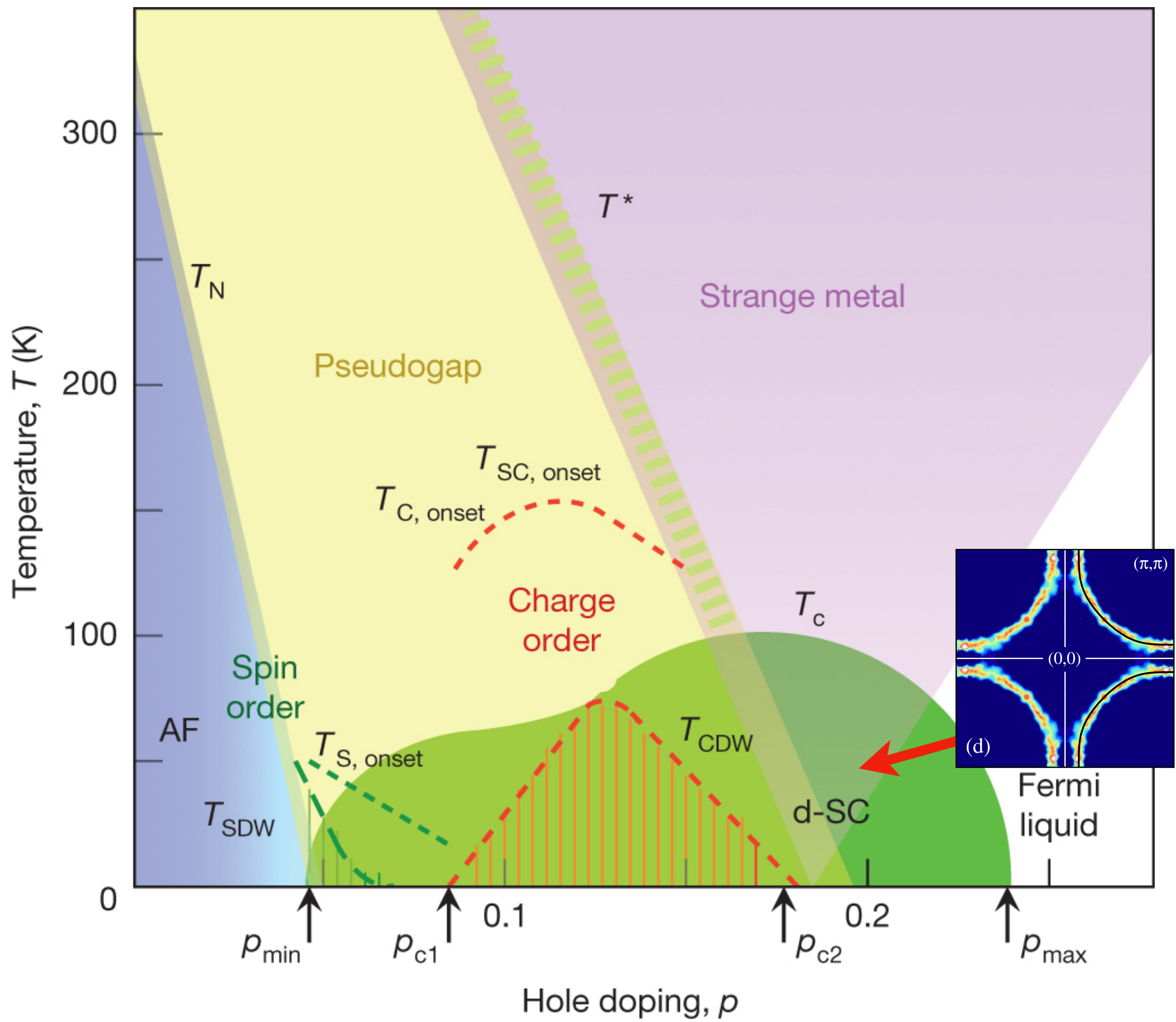
SCIENCE VOL 323 603 2009



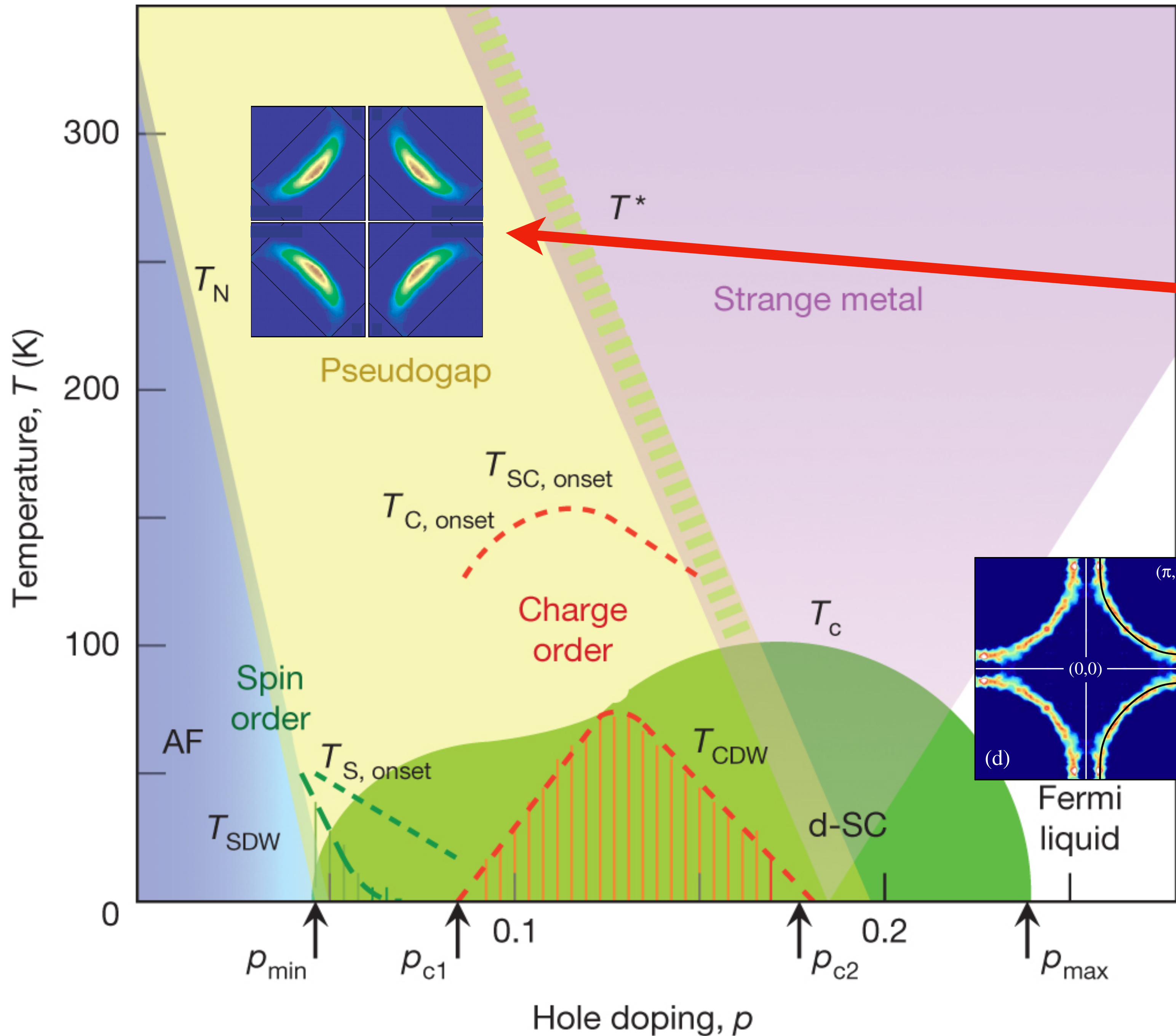


Fermi surface
as expected
in a model
of free electrons





BCS theory of *d*-wave superconductivity induced by antiferromagnetic spin fluctuations



“Pseudogap metal”
 Fermi surface
 modified by
 electron-electron
 interactions

Requires
 fractionalized
 spin liquid
 background

T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)
 R. K. Kaul, A. Kolezhuk, M. Levin, S. S., T. Senthil, PRB **75**, 235122 (2007)
 Y. Qi, S. S., PRB **81**, 115129 (2010)
 E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Anisotropic damping and wave vector dependent susceptibility of the spin fluctuations in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ studied by resonant inelastic x-ray scattering

H. C. Robarts, M. Barthélemy, K. Kummer, M. García-Fernández, J. Li, A. Nag, A. C. Walters, K. J. Zhou, and S. M. Hayden

PHYSICAL REVIEW B **100**, 214510 (2019)

- Difficult to have intense paramagnons from a small Fermi surface.
- Spin waves only present at low energies in the presence of antiferromagnetic order
- Most natural interpretation is a spinon continuum, similar to that observed on the triangular lattice in KYbSe_2

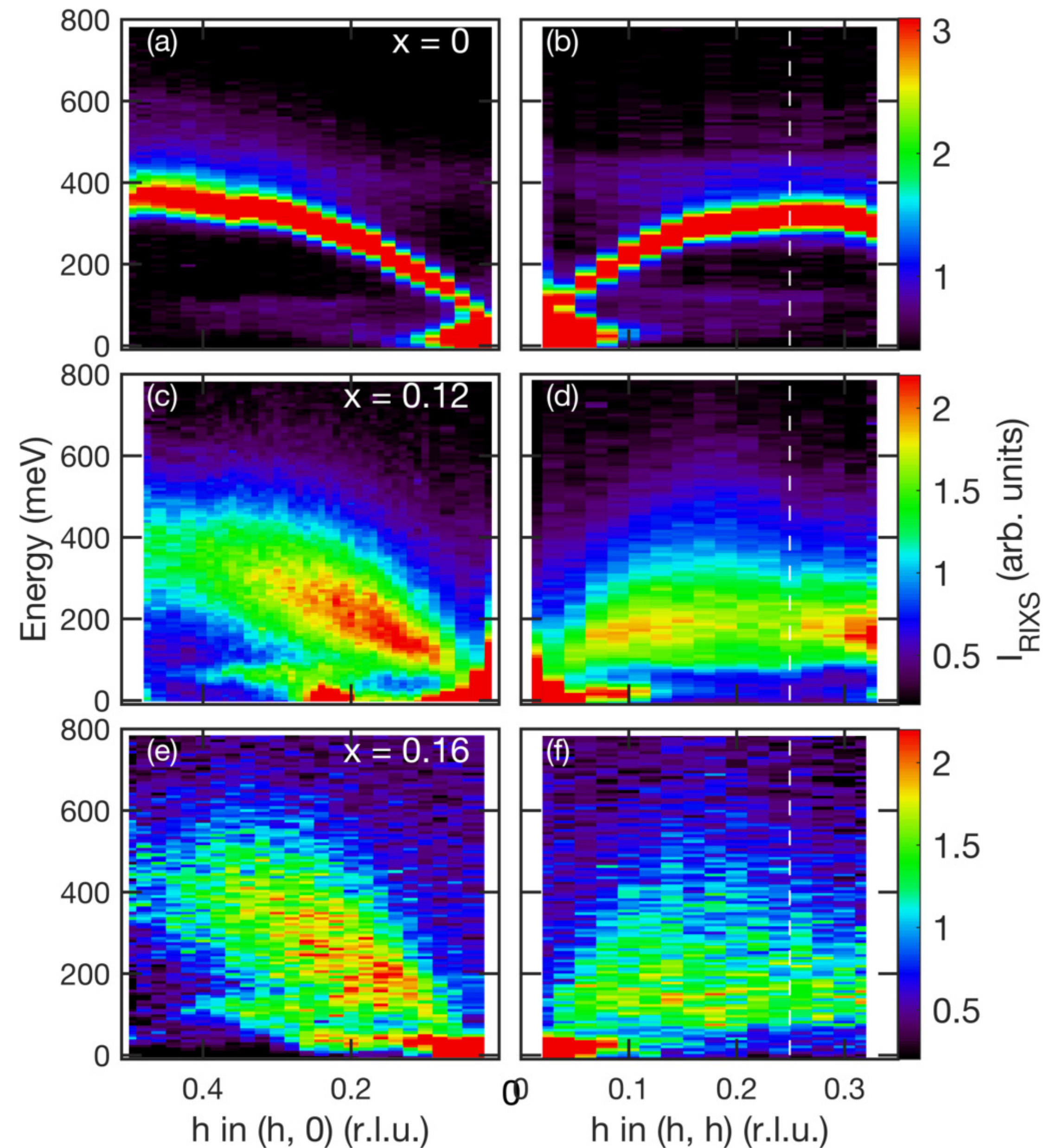
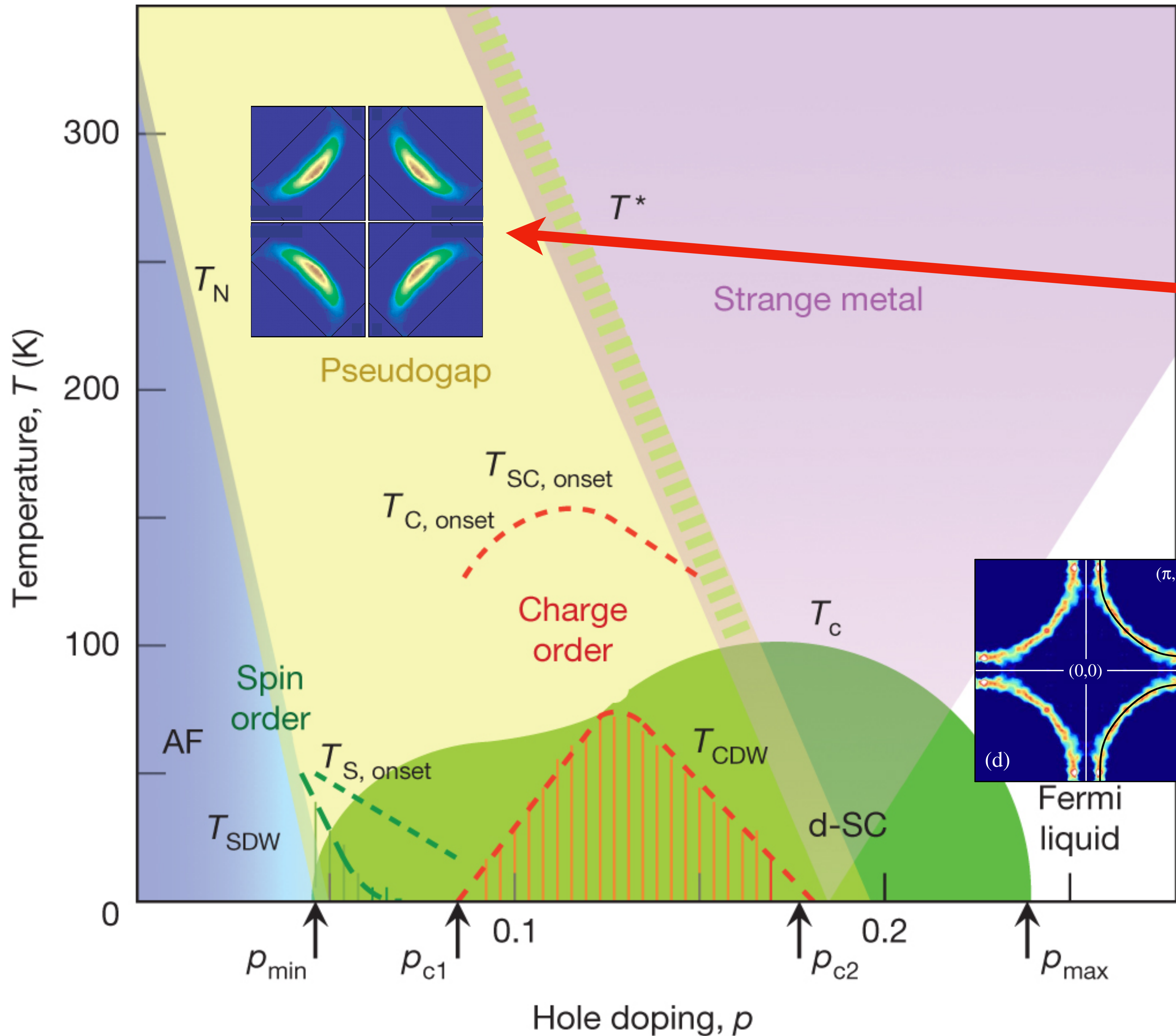


FIG. 2. I_{RIXS} intensity maps as a function of Q in LSCO $x = 0$ ($T \approx 20$ K), 0.12, and 0.16 ($T \approx 30$ K).



“Pseudogap metal”
 Fermi surface
 modified by
 electron-electron
 interactions

Requires
 fractionalized
 spin liquid
 background

T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)

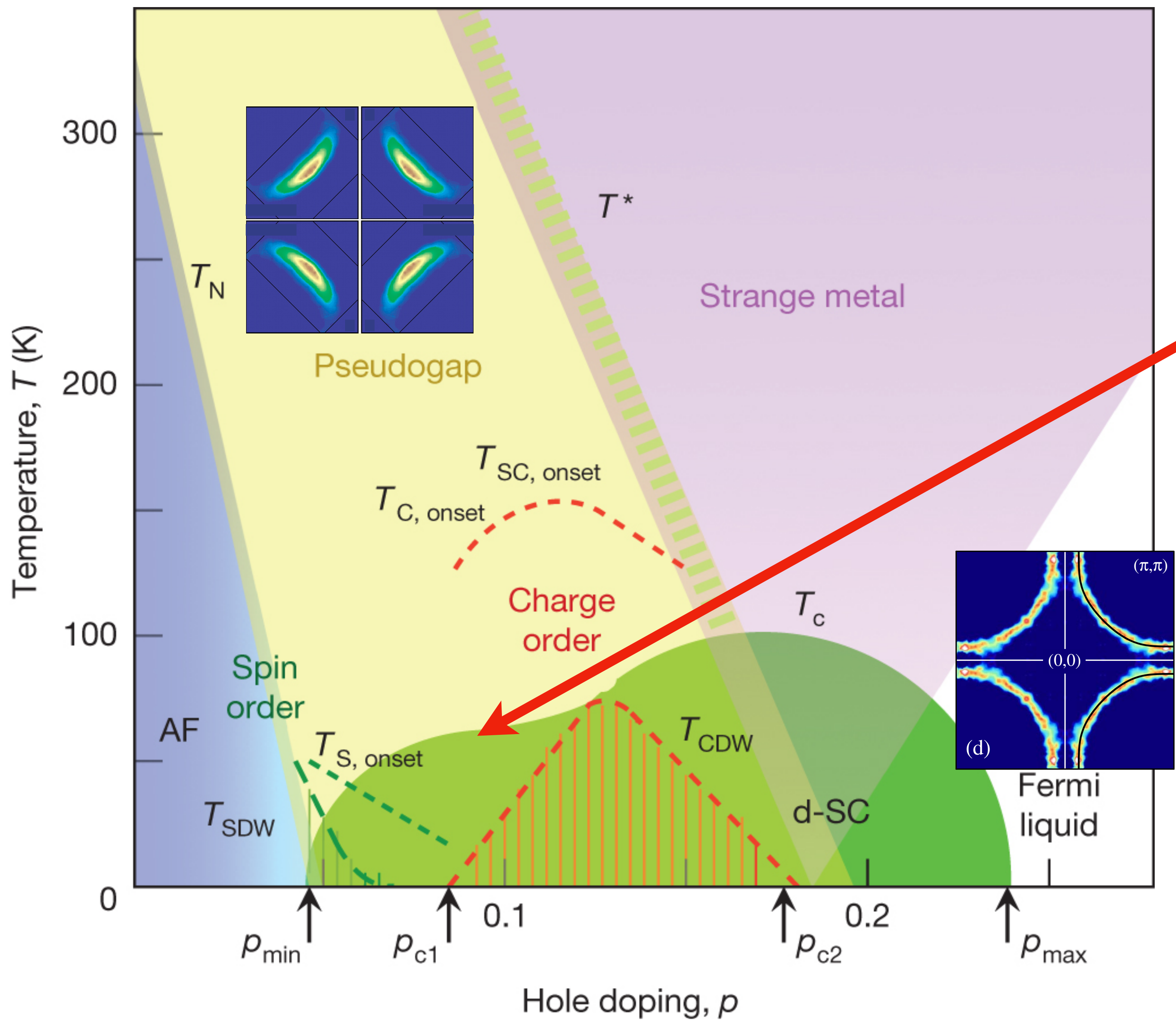
R. K. Kaul, A. Kolezhuk, M. Levin, S. S.,

T. Senthil, PRB **75**, 235122 (2007)

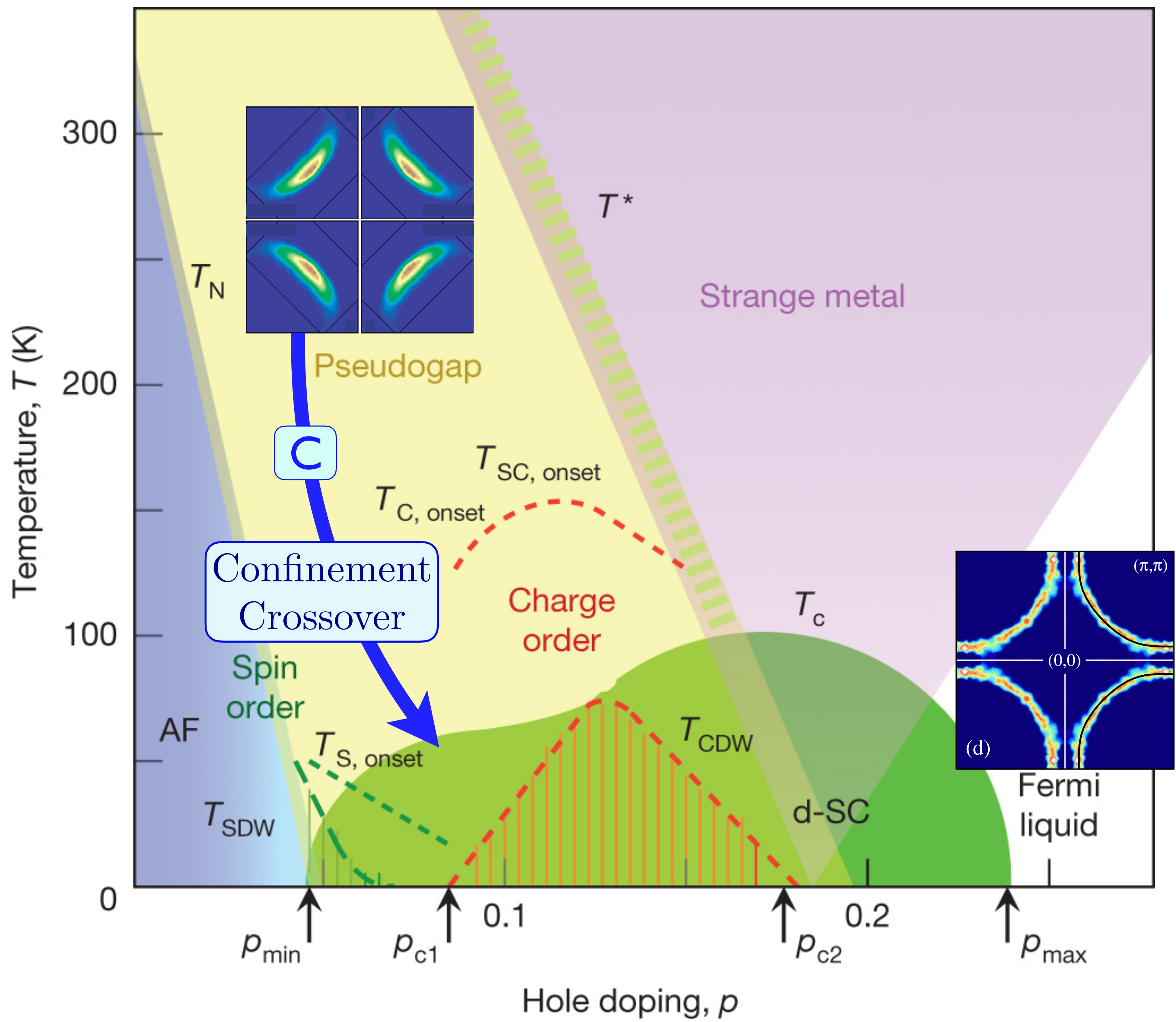
Y. Qi, S. S., PRB **81**, 115129 (2010)

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D. K. Morr, and S. S., PRB **105**, 075146 (2022)



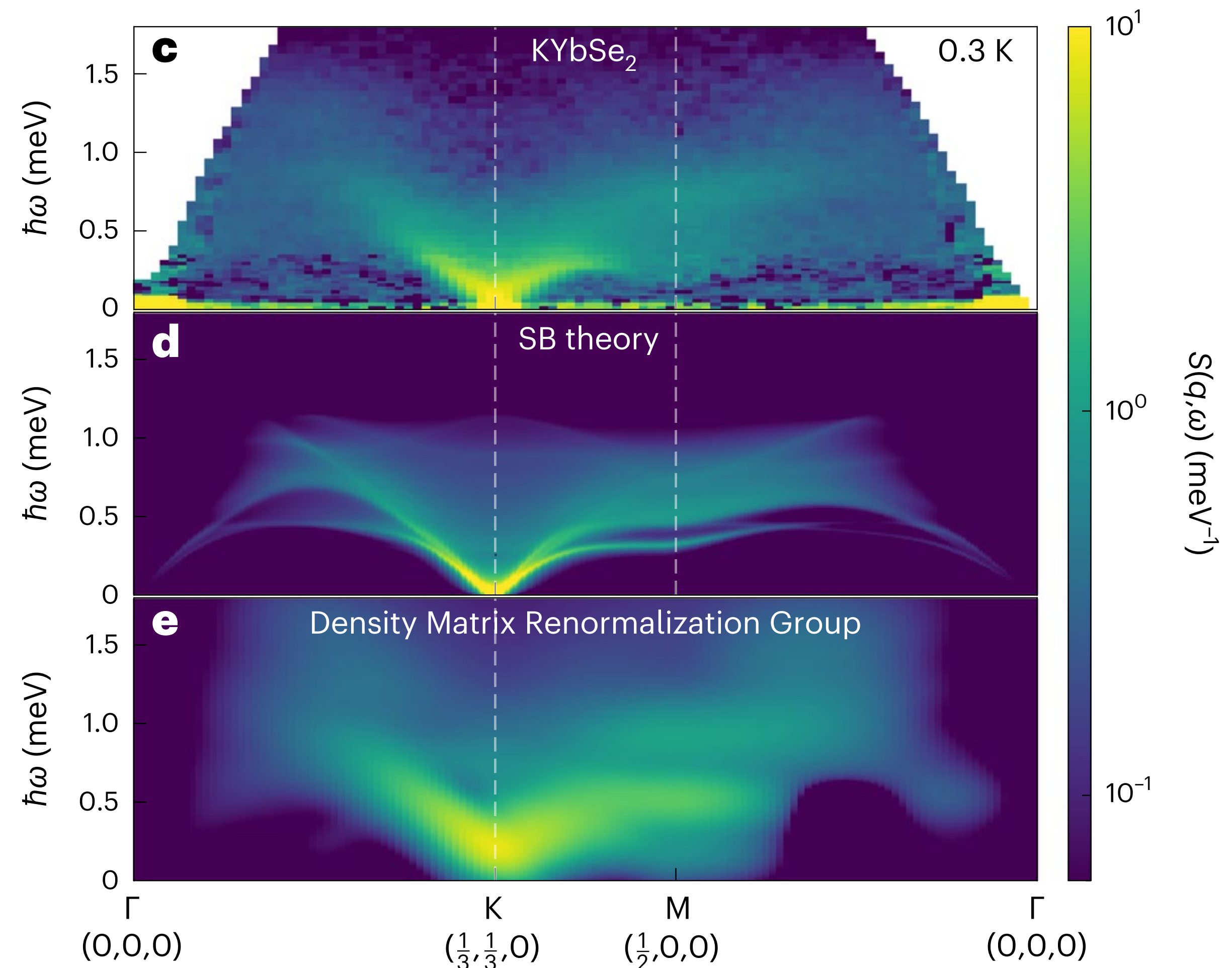
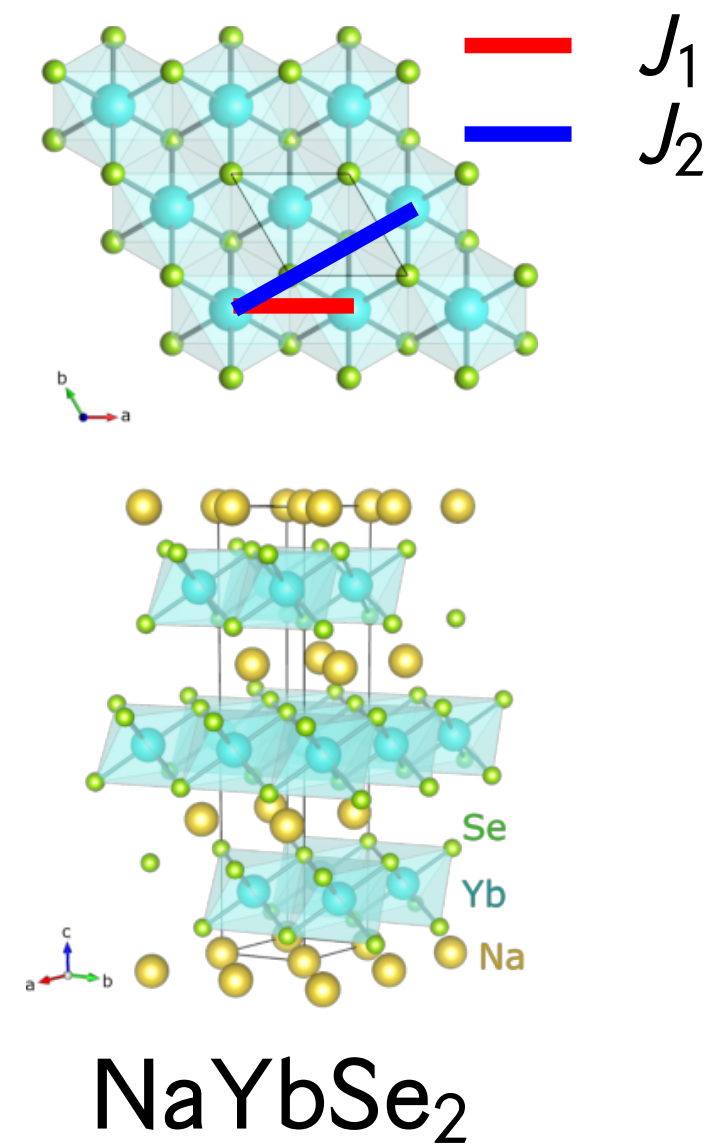
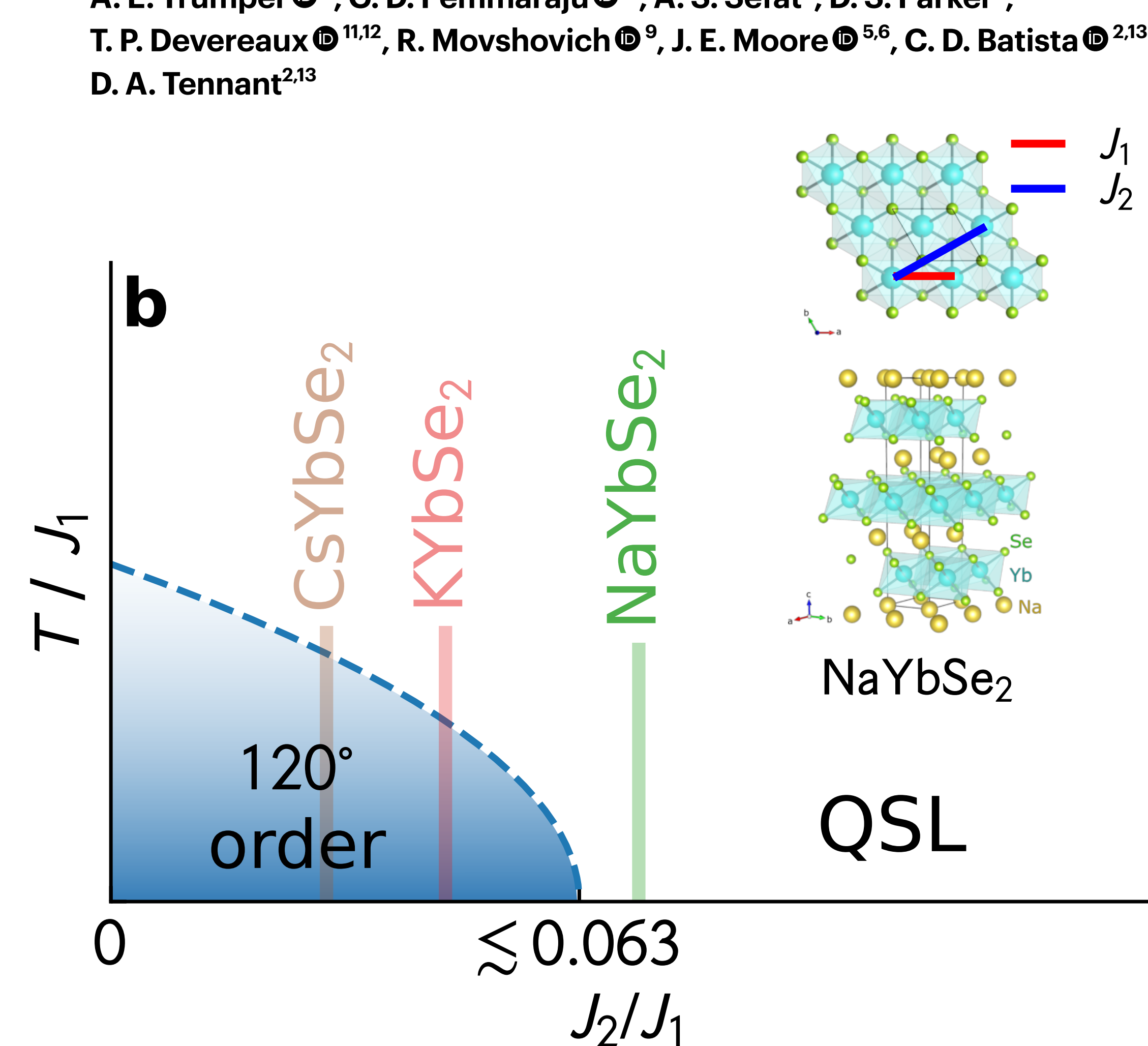
“Conventional” phases
with
symmetry breaking
and
without fractionalization



Proximate spin liquid and fractionalization in the triangular antiferromagnet KYbSe_2

A. O. Scheie¹✉, E. A. Ghioldi^{2,3}, J. Xing⁴, J. A. M. Paddison⁴, N. E. Sherman^{5,6}, M. Dupont^{5,6}, L. D. Sanjeewa^{7,8}, Sangyun Lee⁹, A. J. Woods⁹, D. Abernathy¹, D. M. Pajerowski¹, T. J. Williams¹, Shang-Shun Zhang¹⁰, L. O. Manuel³, A. E. Trumper³, C. D. Pemmaraju¹¹, A. S. Sefat⁴, D. S. Parker⁴, T. P. Devereaux^{11,12}, R. Movshovich⁹, J. E. Moore^{5,6}, C. D. Batista^{2,13}✉ & D. A. Tennant^{2,13}

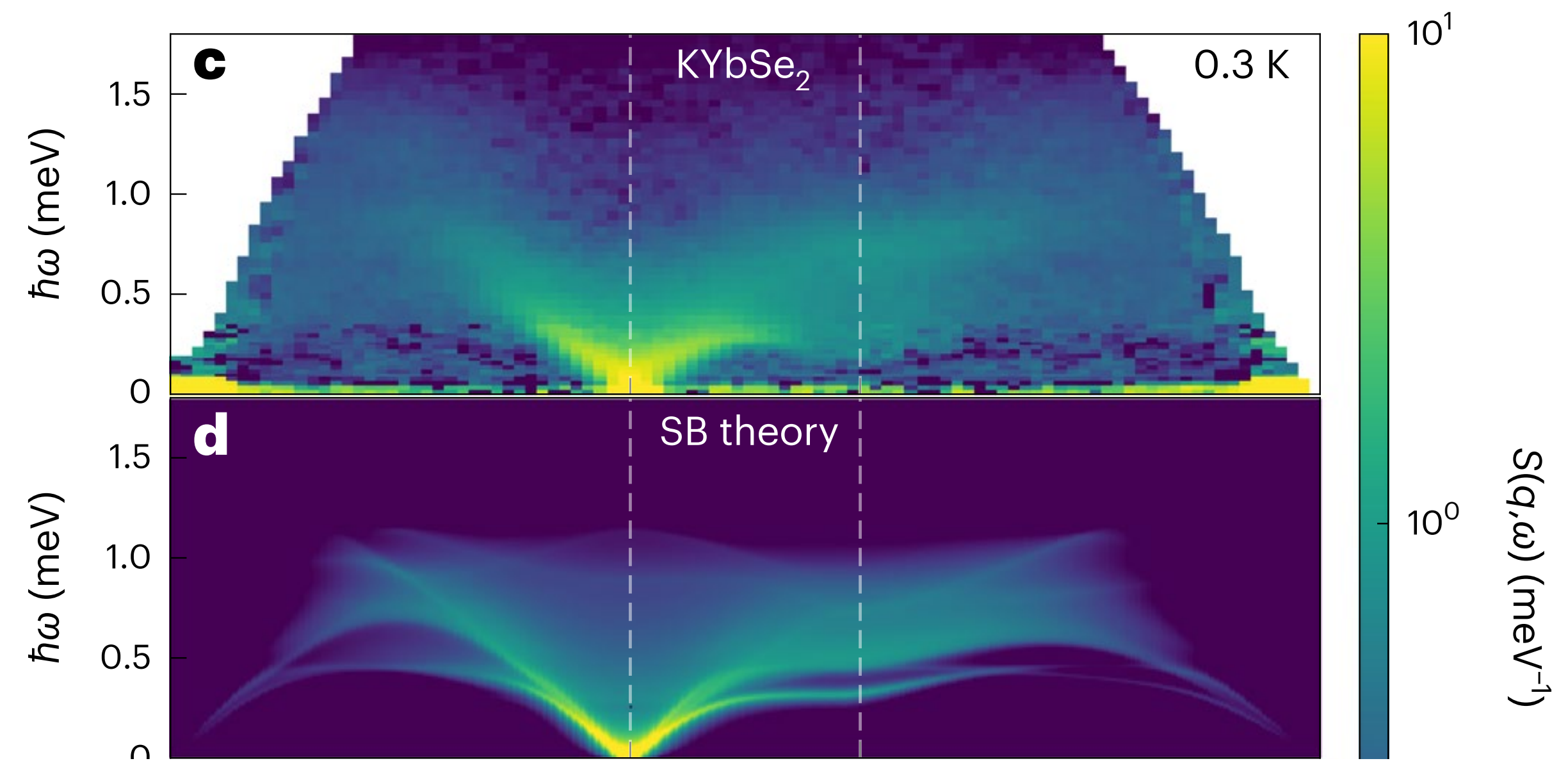
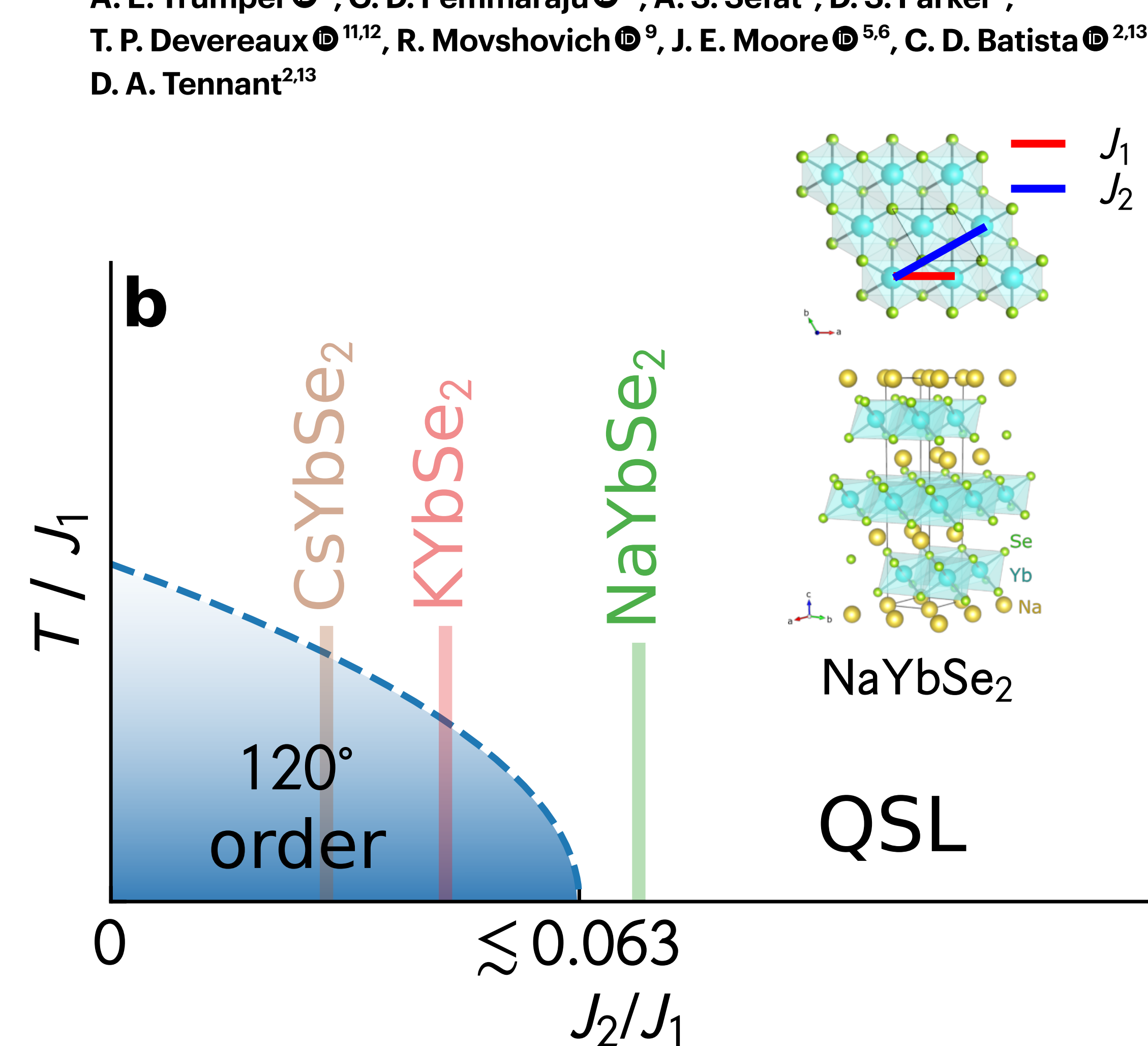
Nature Physics **20**, 74 (2024)



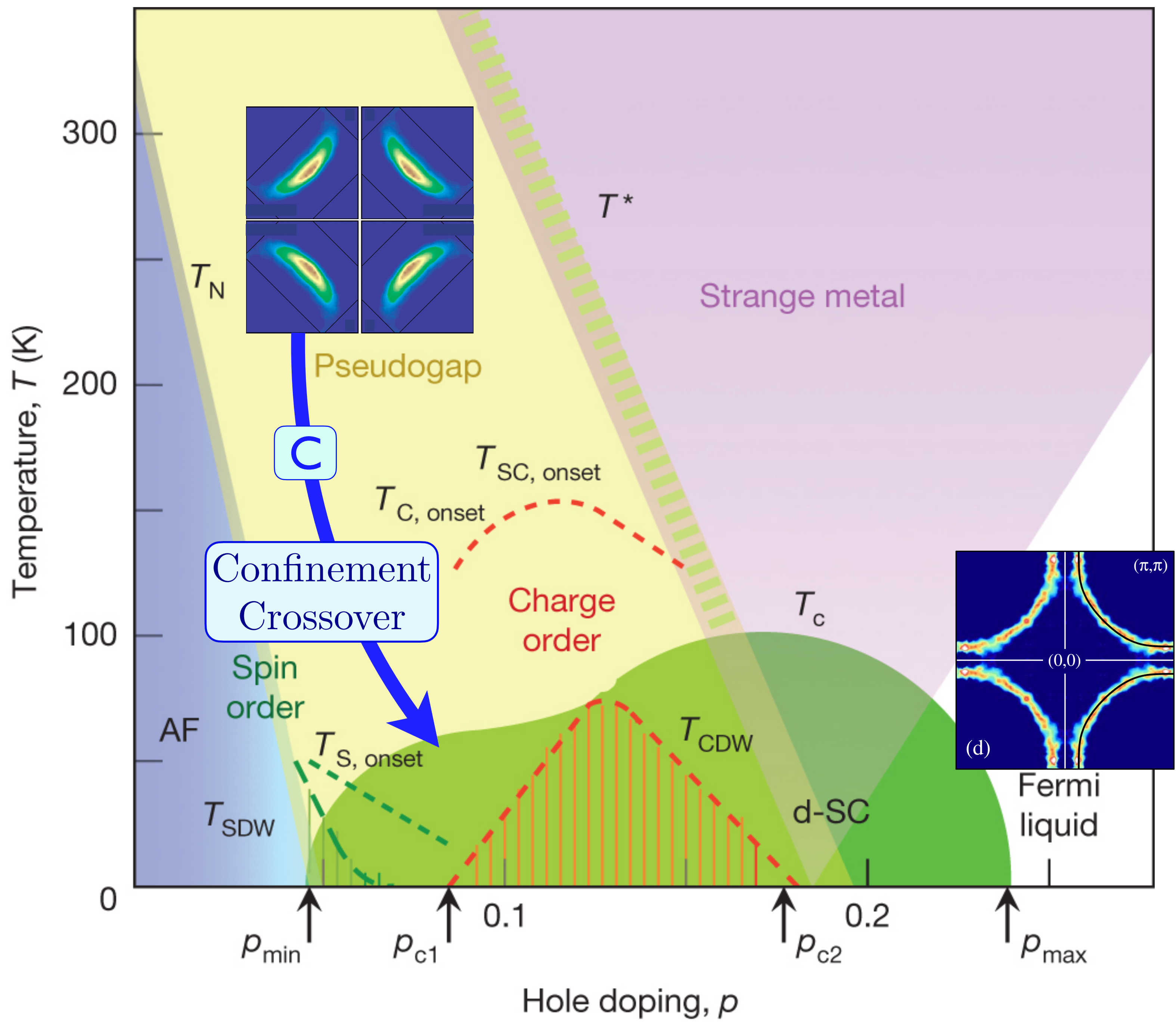
Proximate spin liquid and fractionalization in the triangular antiferromagnet KYbSe_2

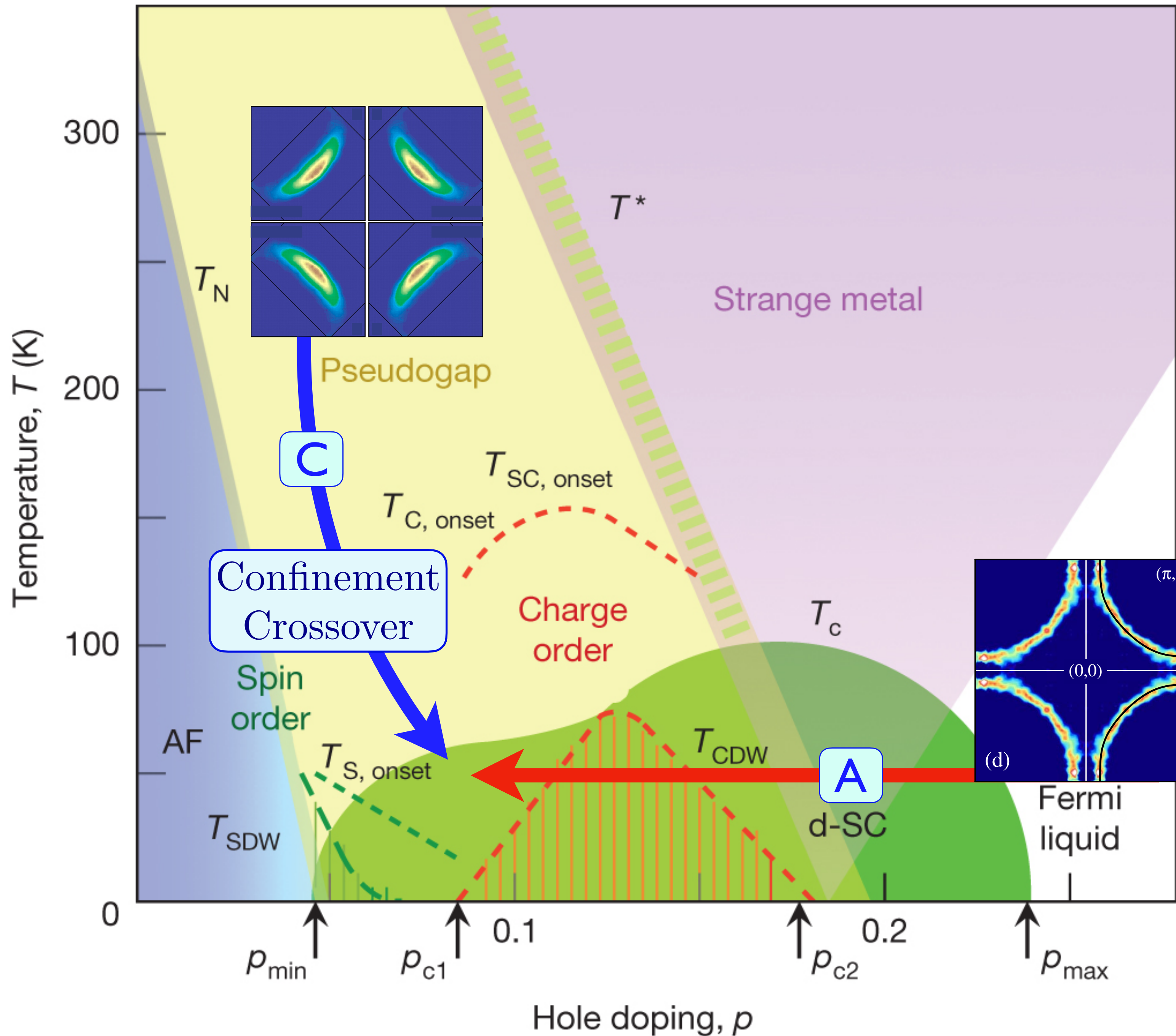
A. O. Scheie¹✉, E. A. Ghioldi^{2,3}, J. Xing⁴, J. A. M. Paddison⁴, N. E. Sherman^{5,6}, M. Dupont^{5,6}, L. D. Sanjeewa^{7,8}, Sangyun Lee⁹, A. J. Woods⁹, D. Abernathy¹, D. M. Pajerowski¹, T. J. Williams¹, Shang-Shun Zhang¹⁰, L. O. Manuel³, A. E. Trumper³, C. D. Pemmaraju¹¹, A. S. Sefat⁴, D. S. Parker⁴, T. P. Devereaux^{11,12}, R. Movshovich⁹, J. E. Moore^{5,6}, C. D. Batista^{2,13}✉ & D. A. Tennant^{2,13}

Nature Physics **20**, 74 (2024)



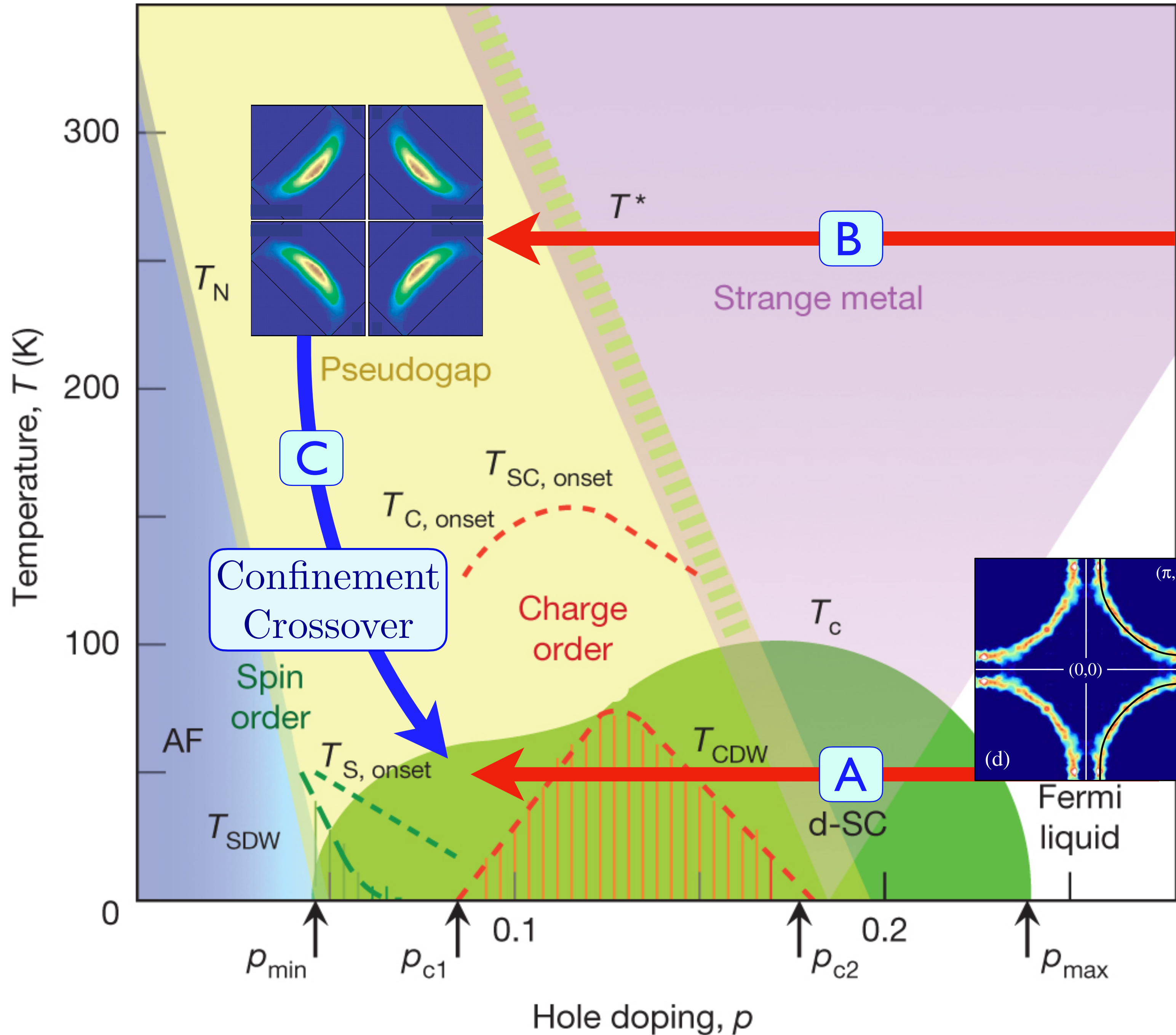
Neutron scattering observations “identify a diffuse continuum with a sharp lower bound within the measured spectra ... The key features of the data are reproduced by Schwinger boson theory” of fractionalized spinons.





Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT



Fermi-volume-changing QPT without symmetry breaking and with spatial disorder.

FL-FL* QPT
Requires fractionalization



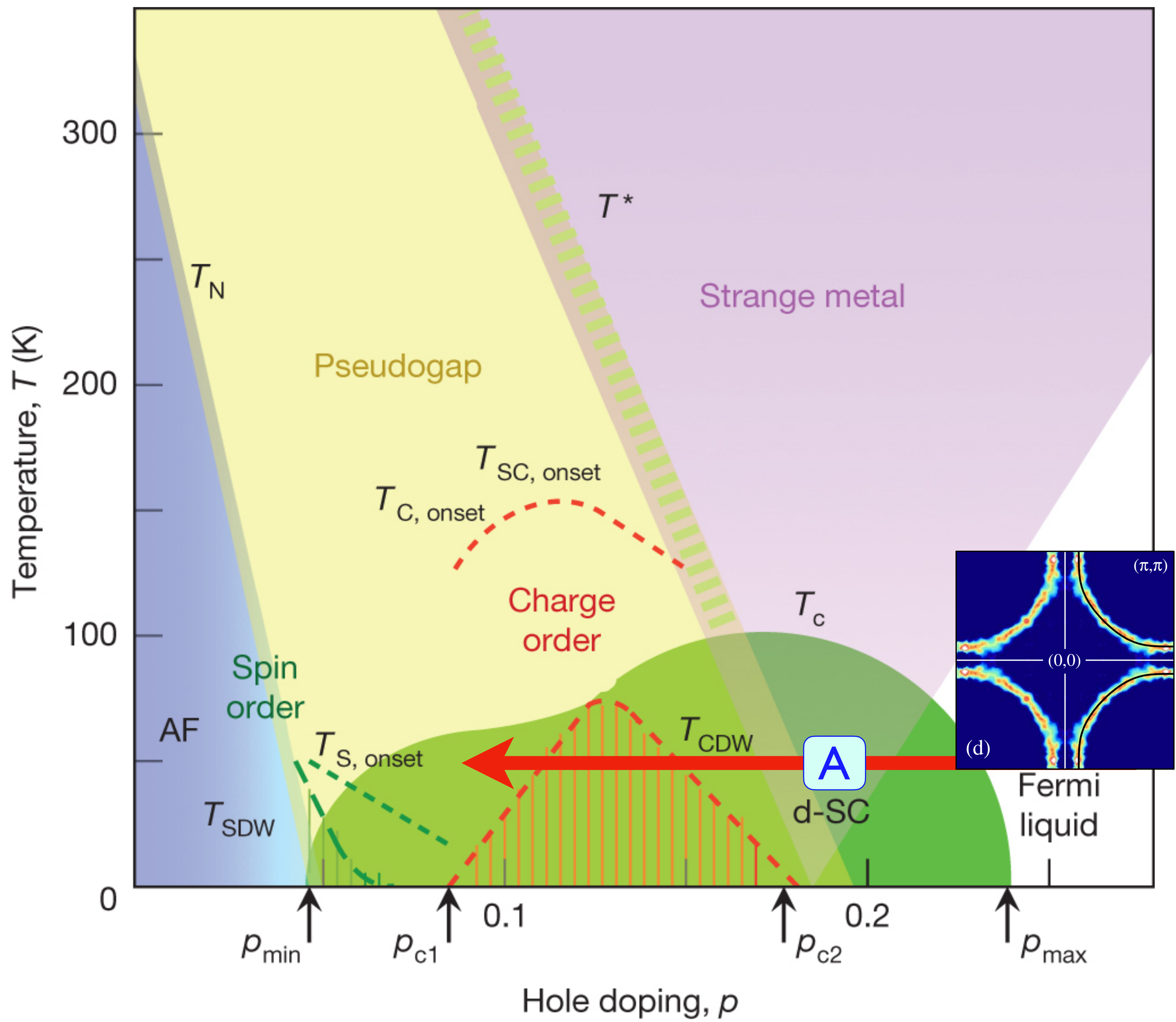
Fermi-volume-changing QPT with symmetry breaking and with spatial disorder.

FL-SDW QPT

A. FL-SDW QPT

B. FL-FL* QPT

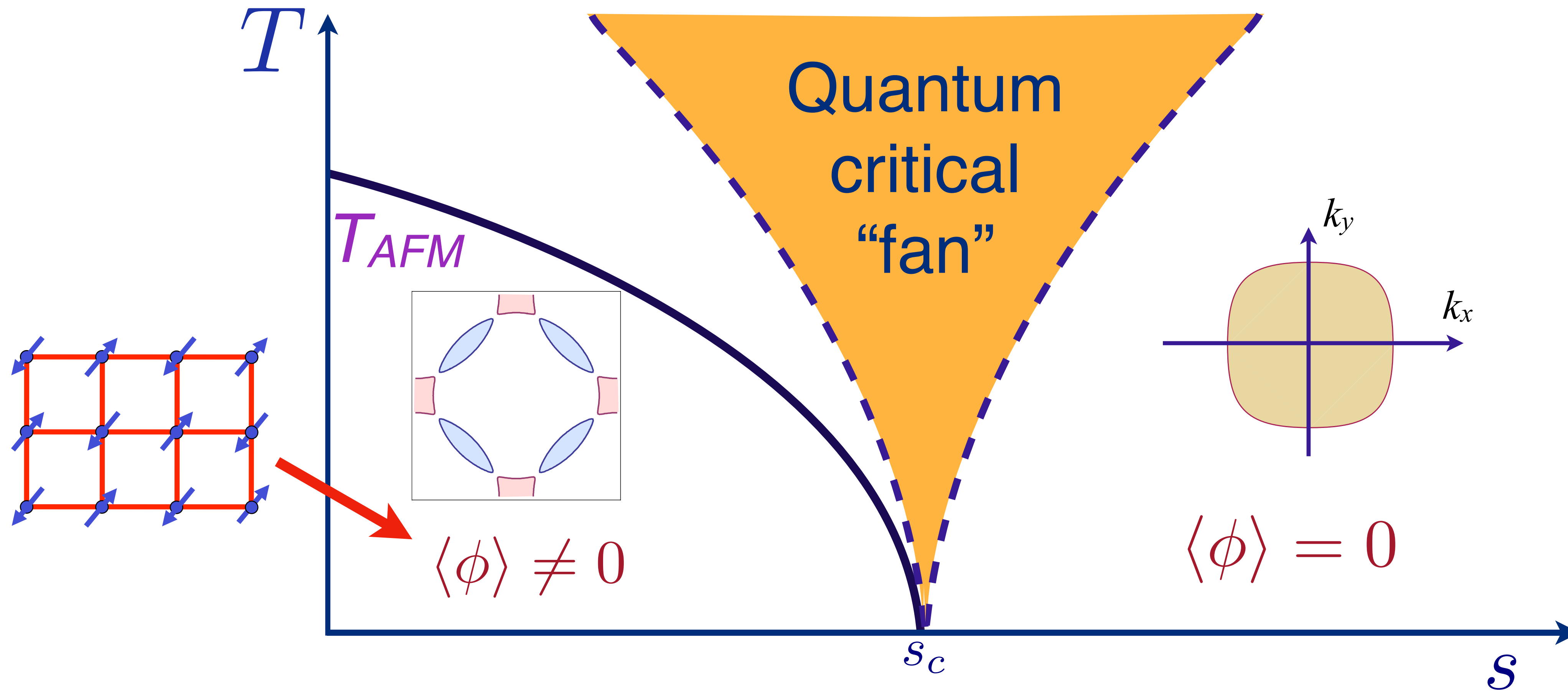
C. Confinement crossover to dSC



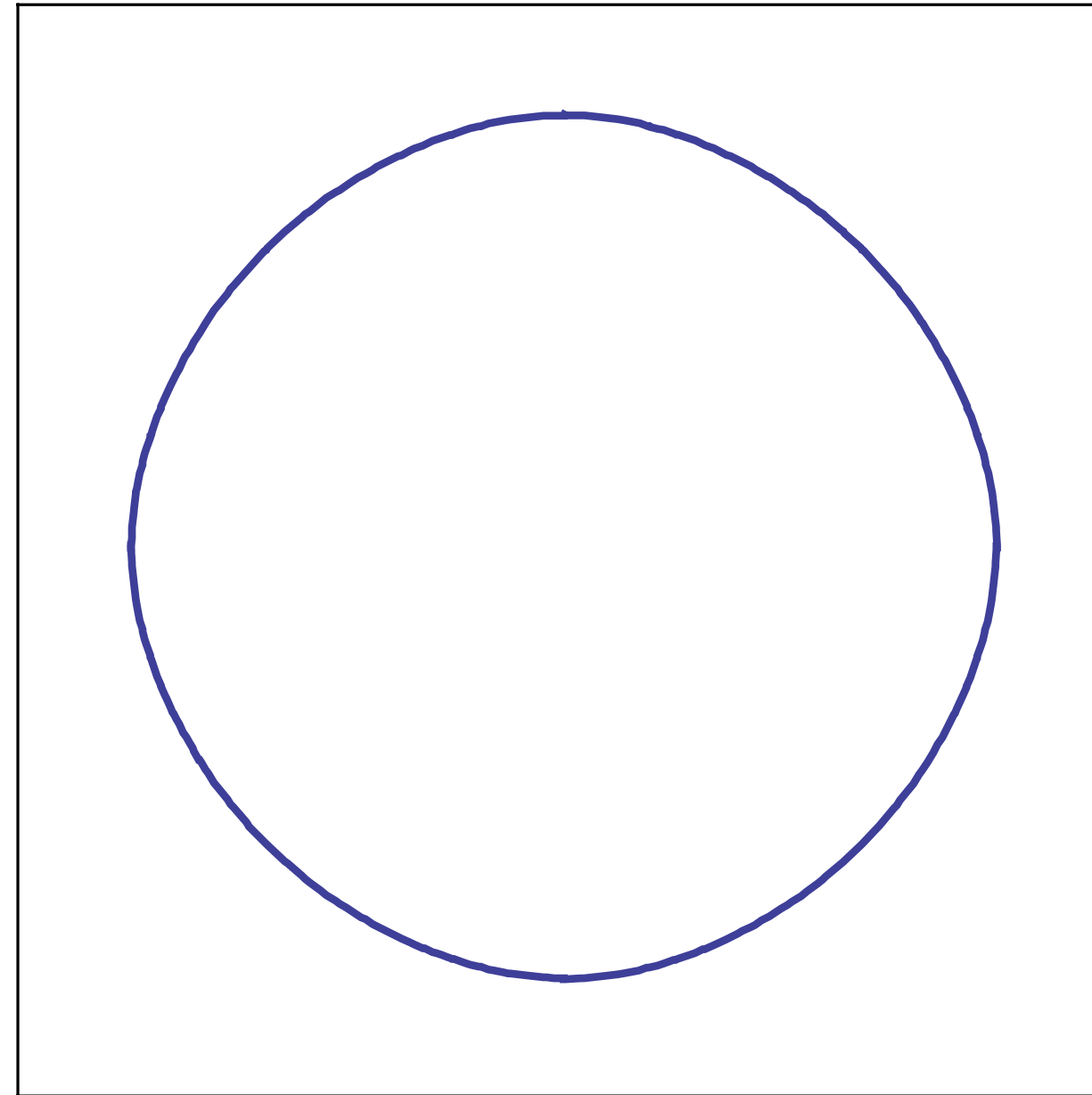
Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT

Fermi surface reconstruction from spin density wave (SDW) order

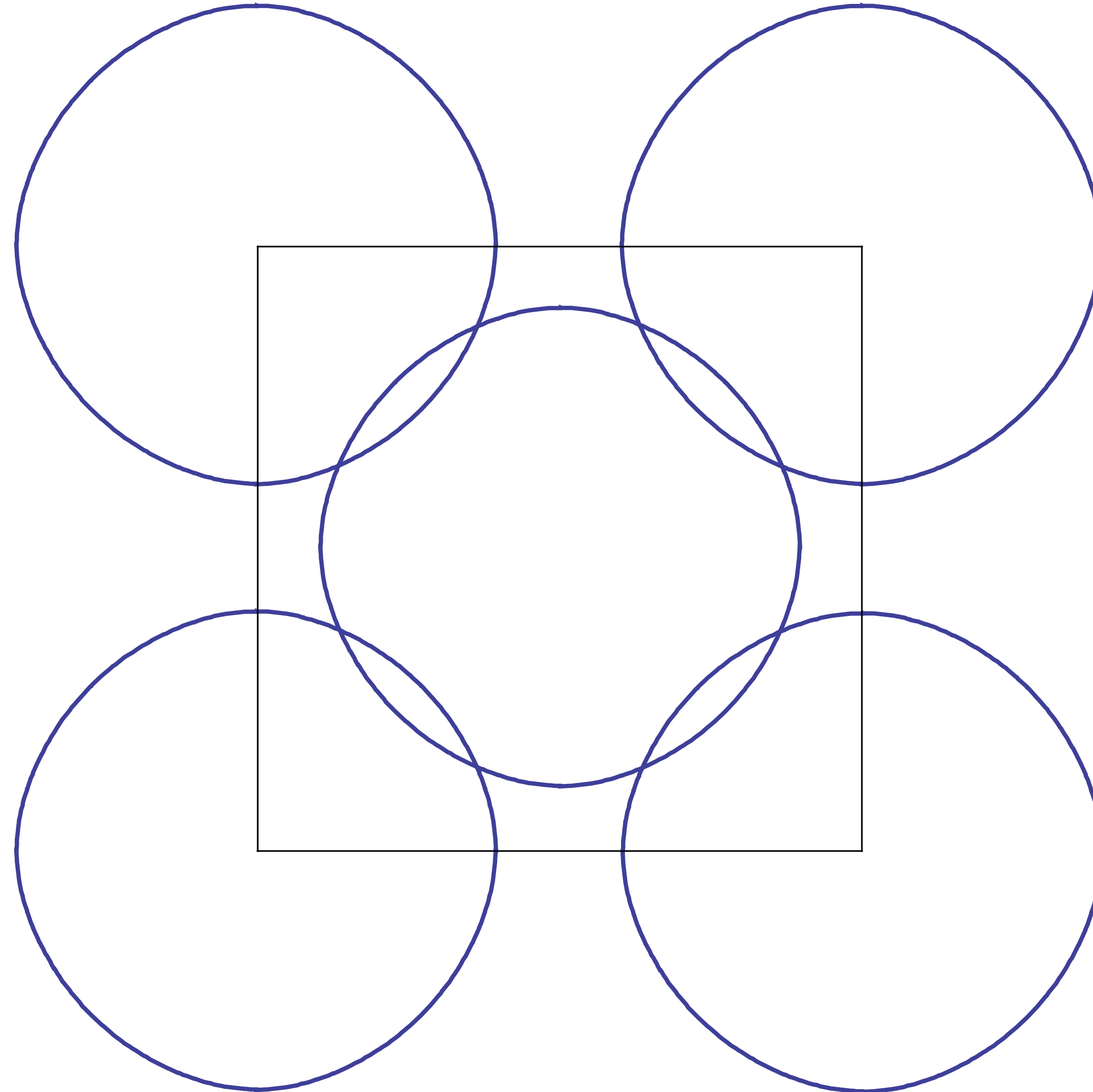


Fermi surface+antiferromagnetism



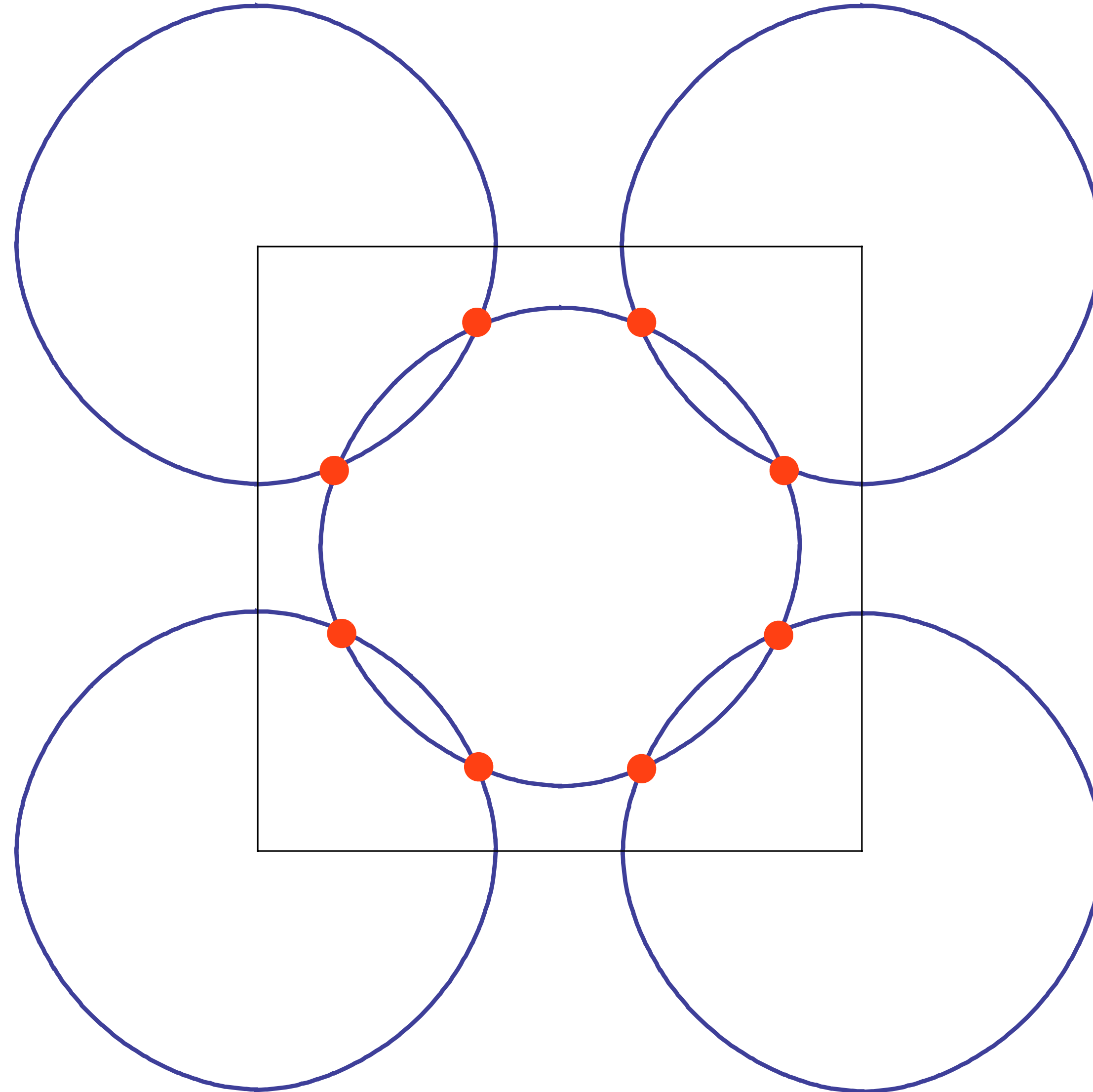
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



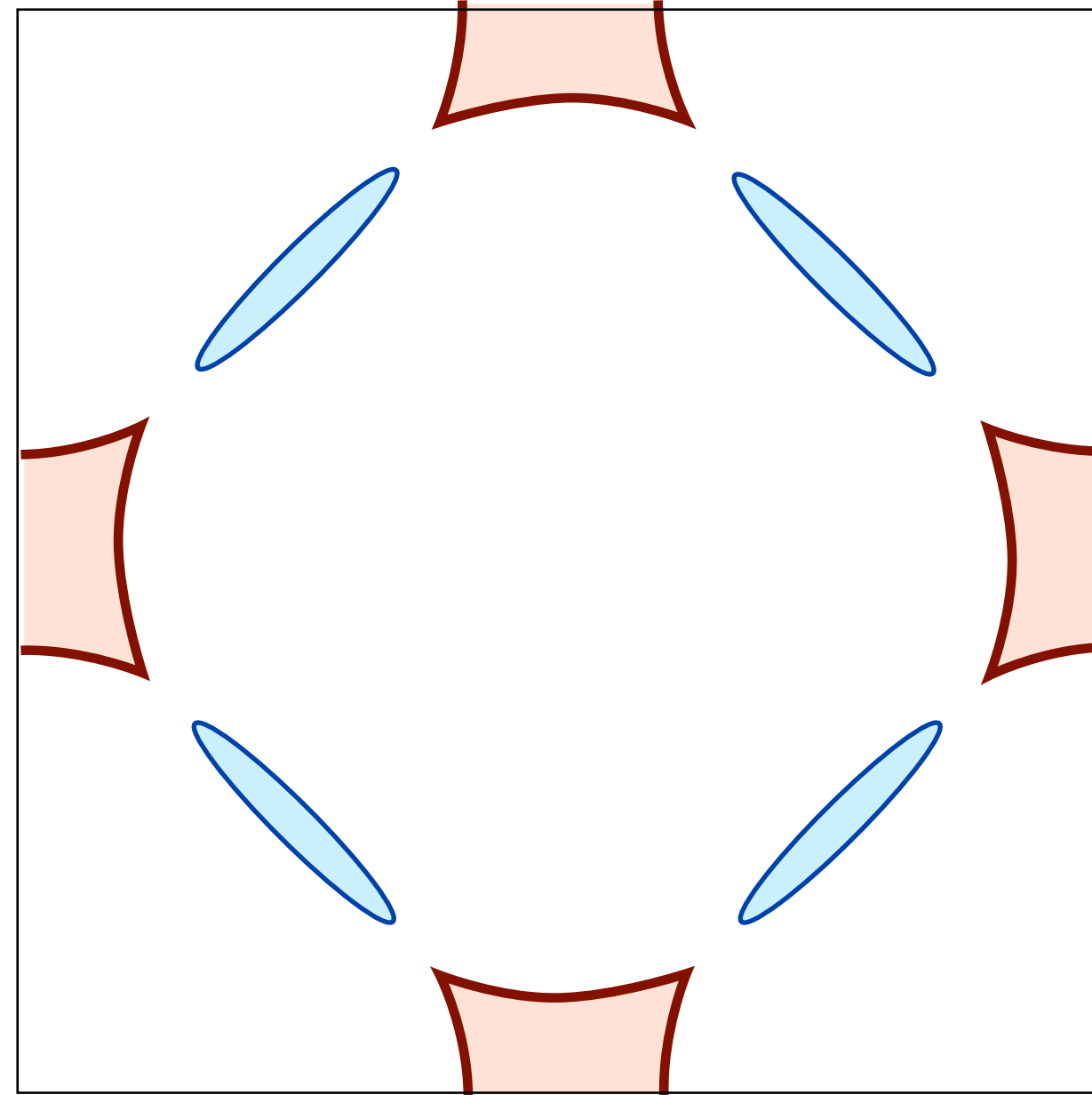
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



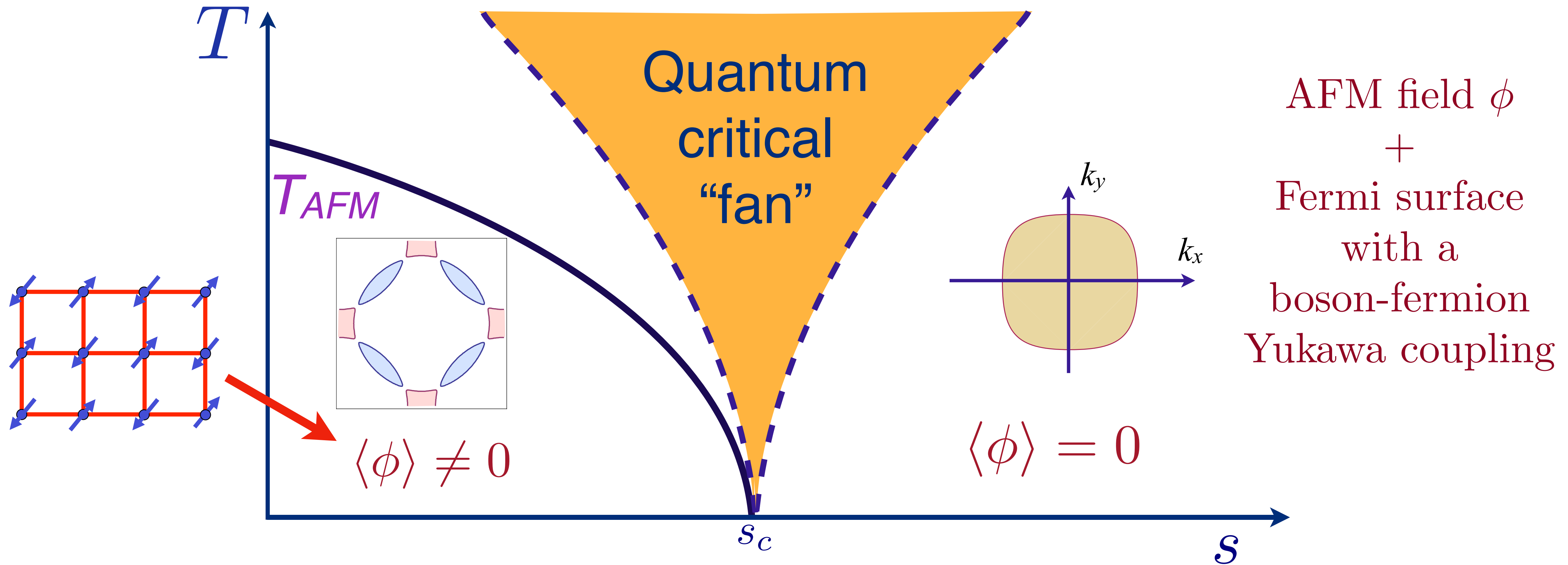
“Hot” spots

Fermi surface+antiferromagnetism



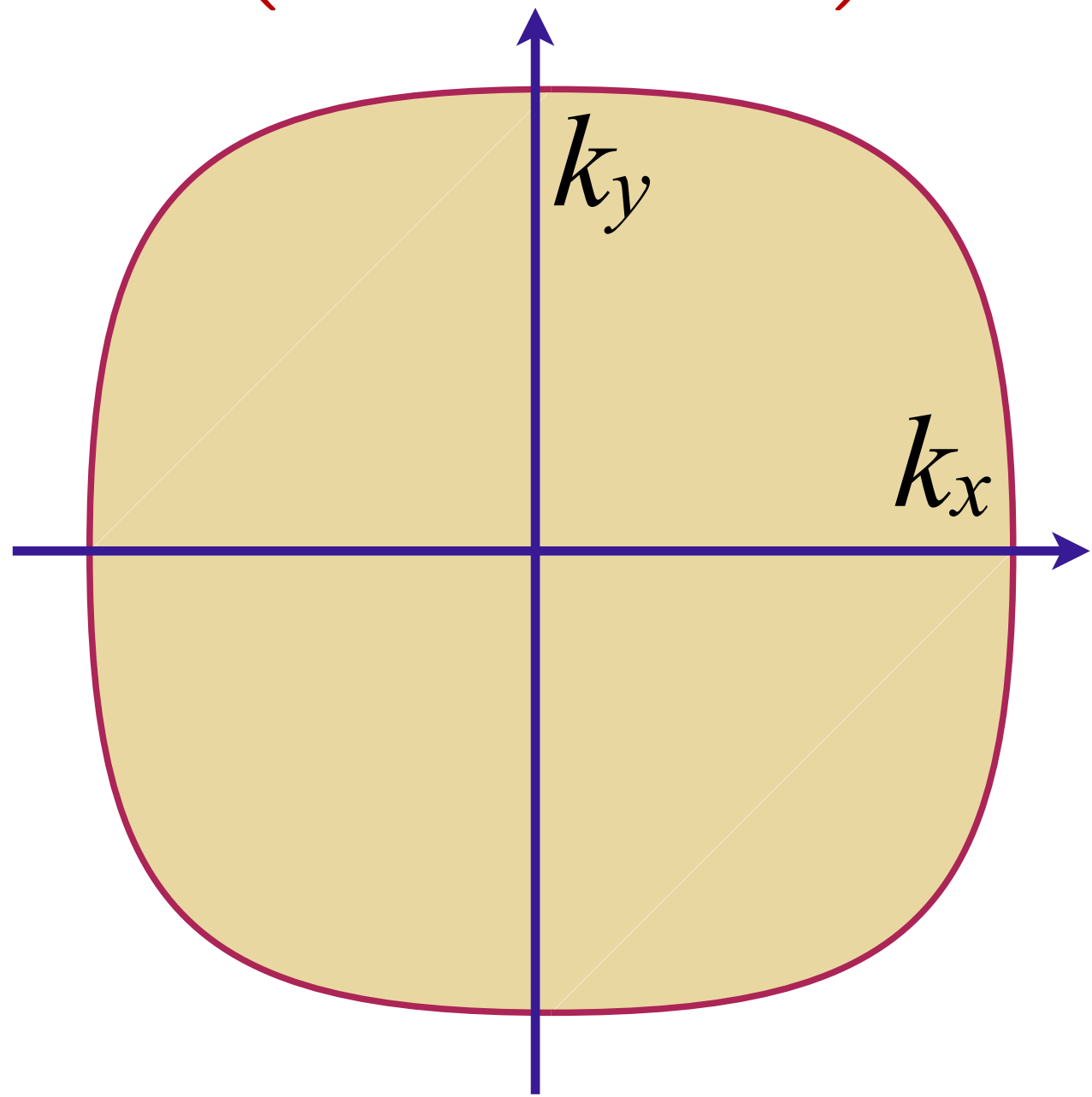
Electron and hole pockets in
antiferromagnetic phase with $\langle \phi \rangle \neq 0$

Fermi surface reconstruction from spin density wave (SDW) order



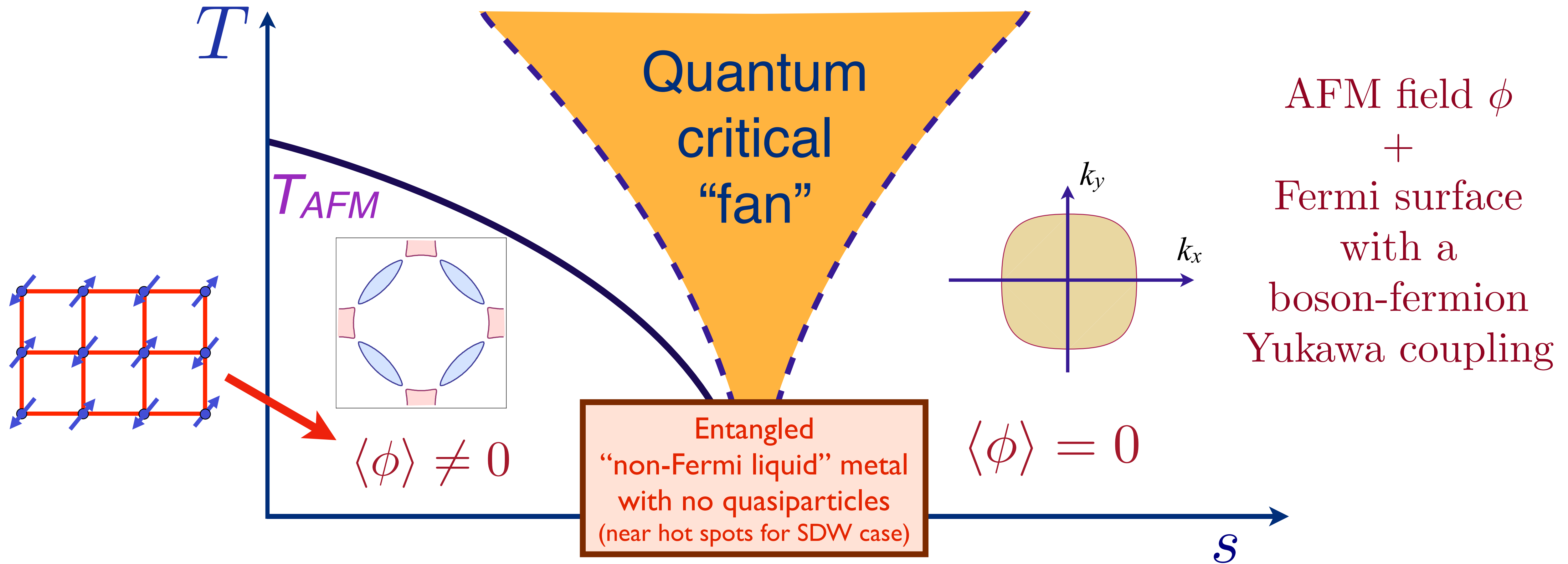
Fermi surface + critical boson with no spatial disorder

$$c_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) c_{\mathbf{k}\sigma}$$

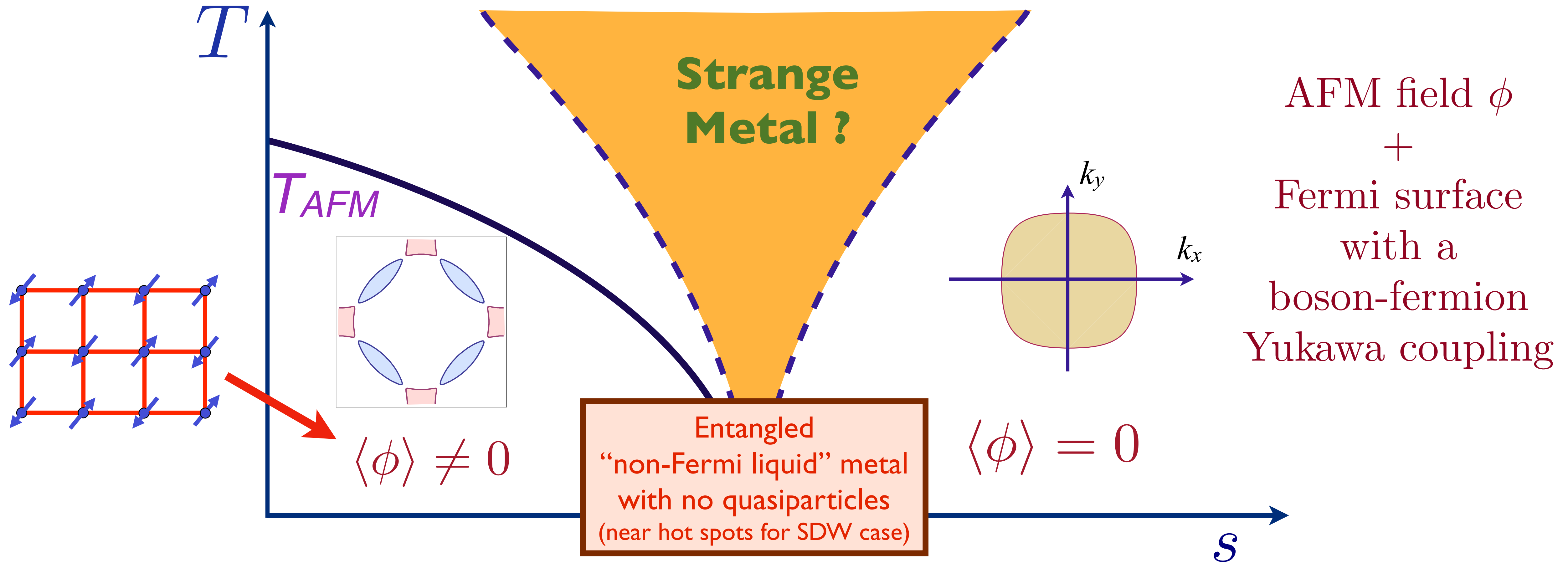


$$+s [\phi(\mathbf{r})]^2 \quad +g c_{\sigma}^\dagger(\mathbf{r}) \tau_{\sigma\sigma'}^a c_{\sigma'}(\mathbf{r}) \phi_a(\mathbf{r}) e^{i\mathbf{K}\cdot\mathbf{r}}$$
$$+K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4$$

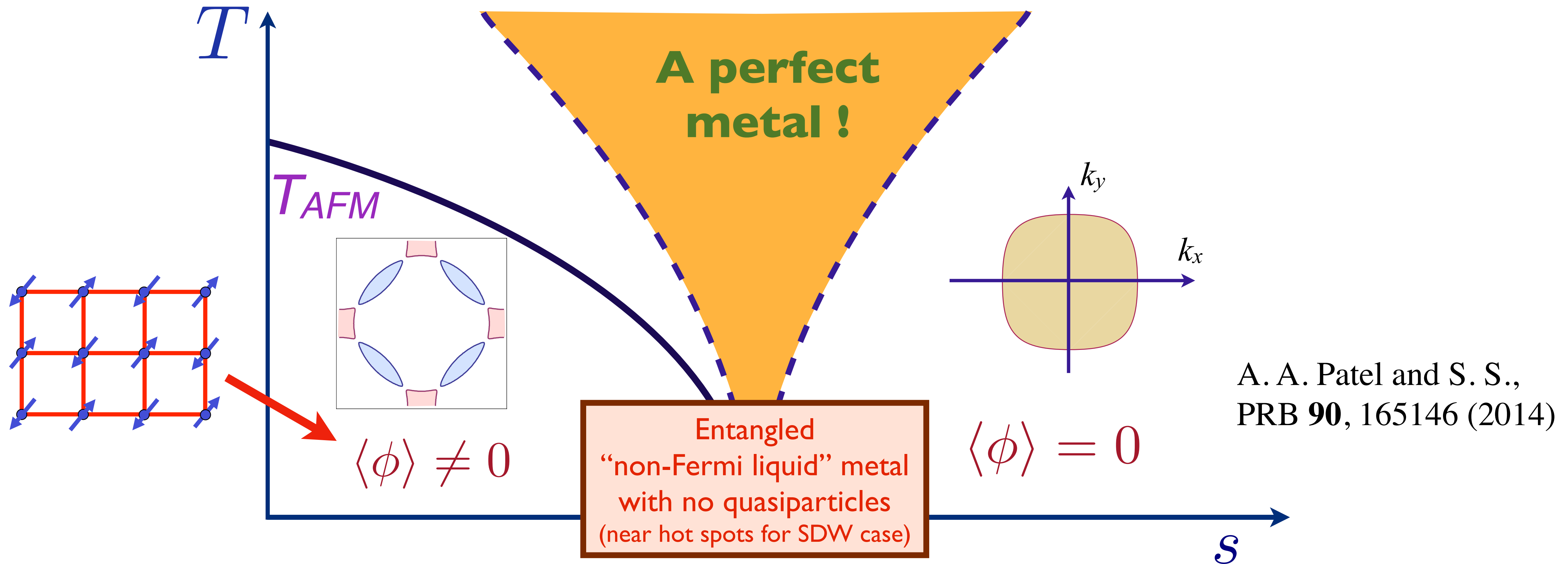
Fermi surface reconstruction from spin density wave (SDW) order



Fermi surface reconstruction from spin density wave (SDW) order



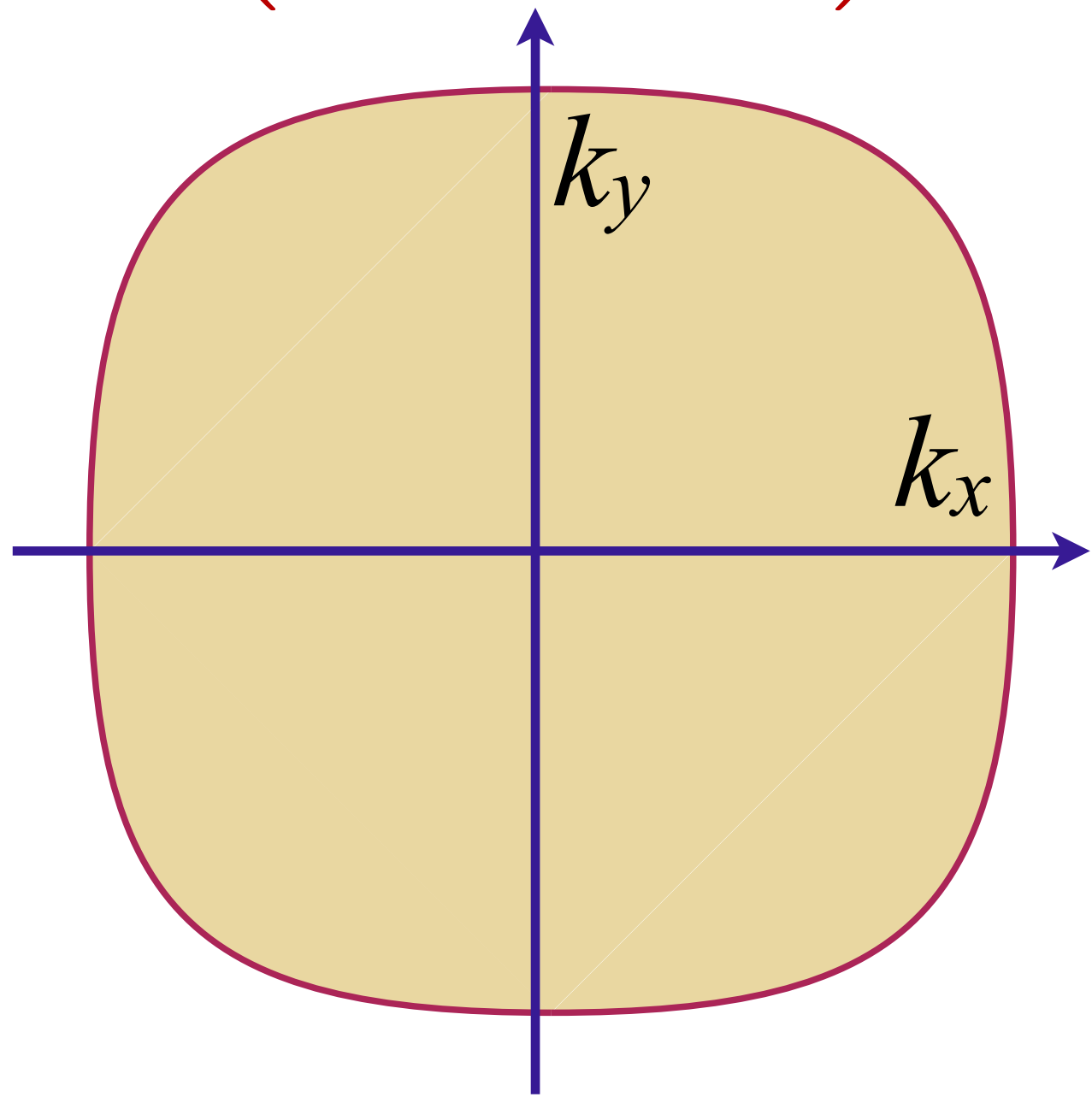
Fermi surface reconstruction from spin density wave (SDW) order



Extreme drag: the fermions c "drag" the bosons ϕ as they move, and so electrical current does not relax, even though strong c - ϕ scattering leads to absence of c quasiparticles.

Fermi surface + critical boson with no spatial disorder

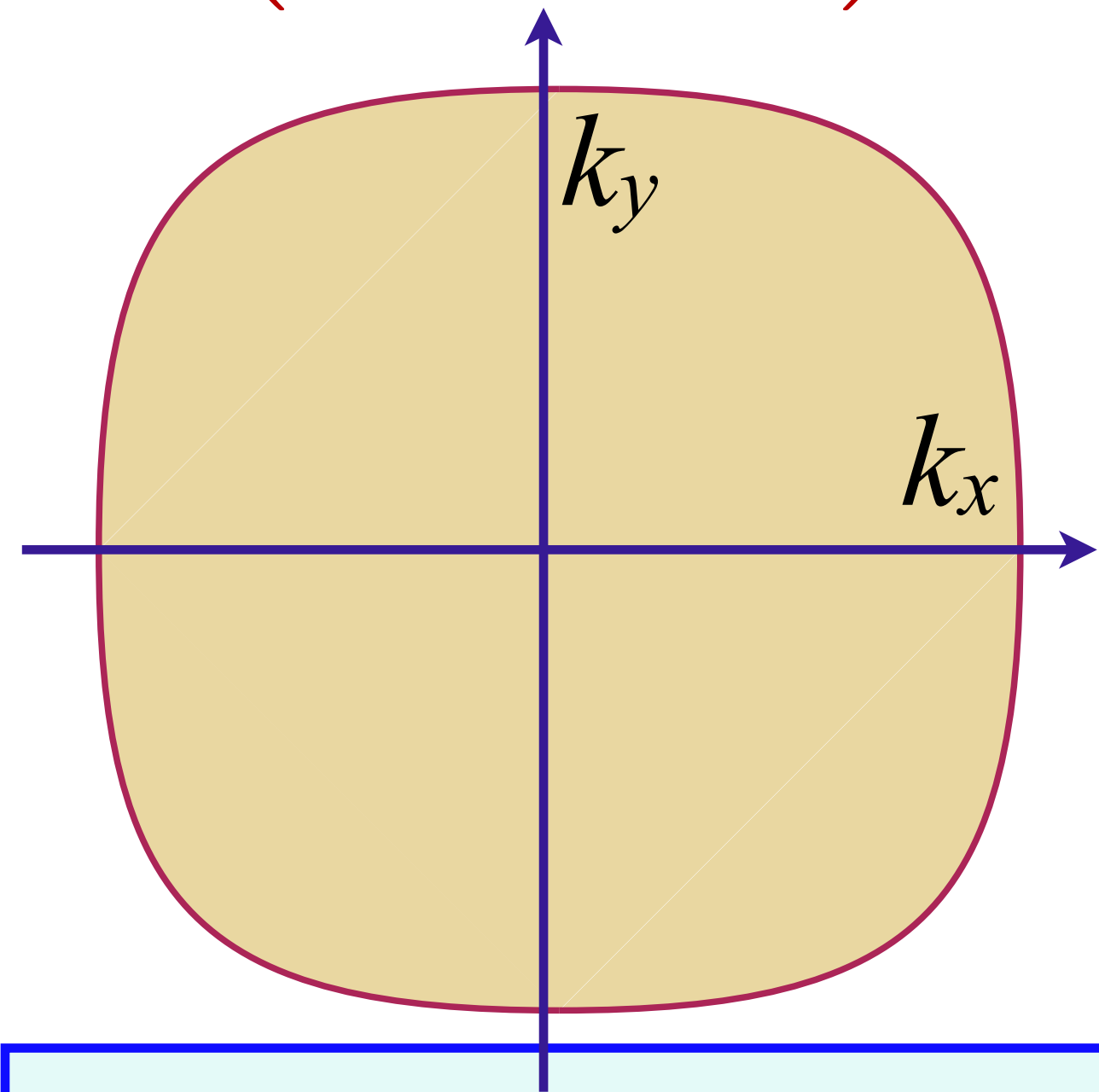
$$c_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) c_{\mathbf{k}\sigma}$$



$$+s [\phi(\mathbf{r})]^2 \quad +g c_\sigma^\dagger(\mathbf{r}) \tau_{\sigma\sigma'}^a c_{\sigma'}(\mathbf{r}) \phi_a(\mathbf{r}) e^{i\mathbf{K}\cdot\mathbf{r}}$$
$$+K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4$$

Fermi surface + critical boson with potential disorder

$$c_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) c_{\mathbf{k}\sigma}$$



$$+s [\phi(\mathbf{r})]^2$$

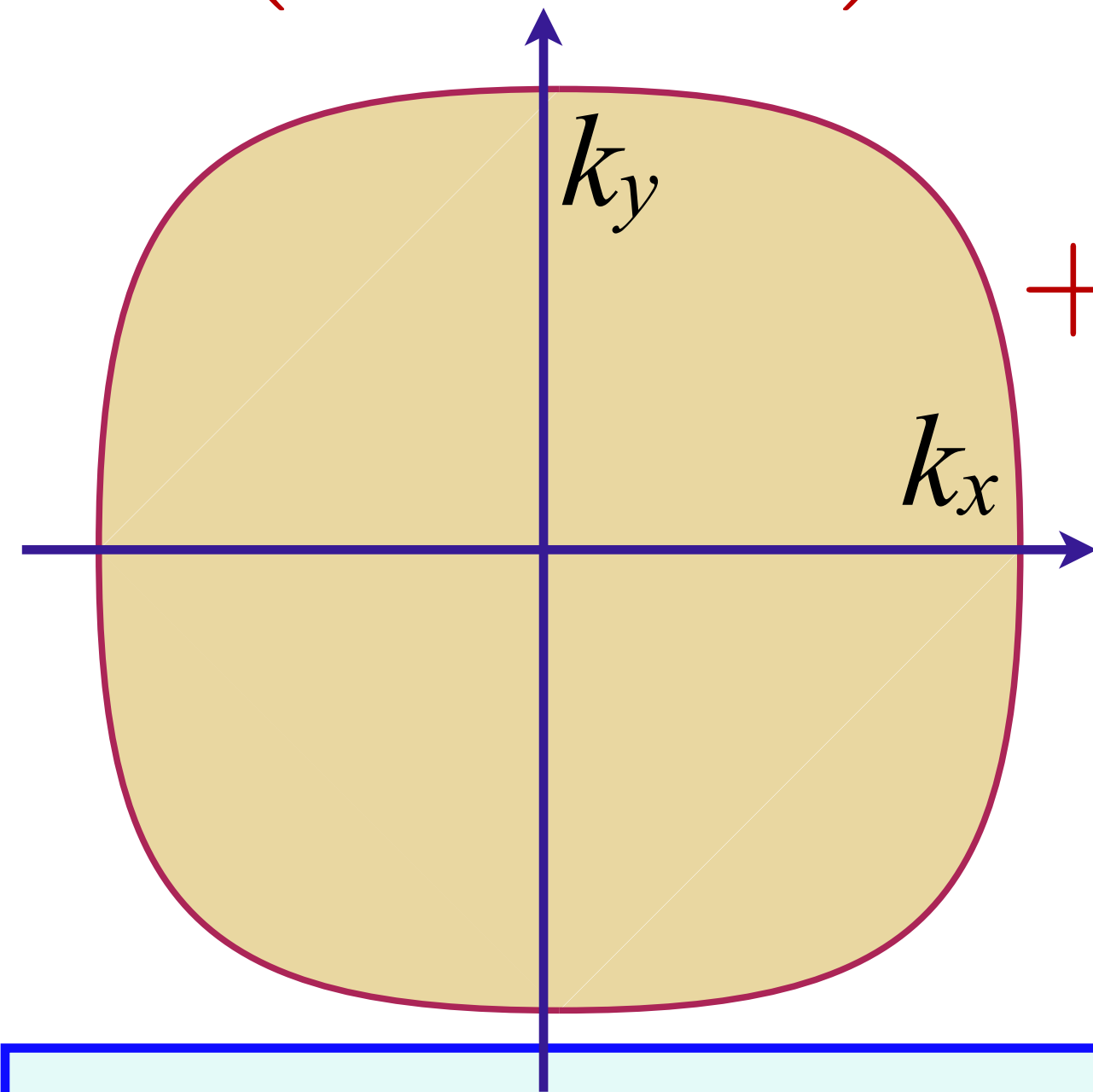
$$+g c_\sigma^\dagger(\mathbf{r}) \tau_{\sigma\sigma'}^a c_{\sigma'}(\mathbf{r}) \phi_a(\mathbf{r}) e^{i\mathbf{K}\cdot\mathbf{r}}$$

$$+K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) c_\sigma^\dagger(\mathbf{r}) c_\sigma(\mathbf{r})$$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Fermi surface + critical boson with potential and “mass” disorder

$$c_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) c_{\mathbf{k}\sigma}$$



$$+ [s + \delta s(\mathbf{r})] [\phi(\mathbf{r})]^2 + g c_\sigma^\dagger(\mathbf{r}) \tau_{\sigma\sigma'}^a c_{\sigma'}(\mathbf{r}) \phi_a(\mathbf{r}) e^{i\mathbf{K}\cdot\mathbf{r}}$$

$$+ K [\nabla_{\mathbf{r}} \phi(\mathbf{r})]^2 + u [\phi(\mathbf{r})]^4 + v(\mathbf{r}) c_\sigma^\dagger(\mathbf{r}) c_\sigma(\mathbf{r})$$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')}$ = $v^2\delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')}$ = $\delta s^2\delta(\mathbf{r} - \mathbf{r}')$

RG analysis (Harris criterion) shows that $\delta s(\mathbf{r})$ is most relevant disorder.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \left(\frac{s + \delta s_j}{2} \phi_{ja}^2 + \frac{u}{4M} (\phi_{ja}^2)^2 \right) \right]$$
$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j (\gamma |\Omega| + \Omega^2 / c^2) |\phi_{ja}(i\Omega)|^2,$$

where $a = 1 \dots M$ is a flavor index for an order parameter with $O(M)$ symmetry.

J. A. Hoyos, Chetan Kotabage, Thomas Vojta
Phys. Rev. Lett. **99**, 230601 (2007)

T. Vojta, J.A. Hoyos, Priyanka Mohan, Rajesh Narayanan,
J. Phys.: Condens. Matter **23**, 094206 (2011)
Mapping to RTFIM in strong disorder RG.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

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$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j (\gamma |\Omega| + \Omega^2 / c^2) |\phi_{ja}(i\Omega)|^2,$$

where $a = 1 \dots M$ is a flavor index for an order parameter with $O(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\bar{\mathcal{S}}_\phi = \int d\tau \left[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\widetilde{\delta s}_j}{2} \phi_{ja}^2 \right]$$

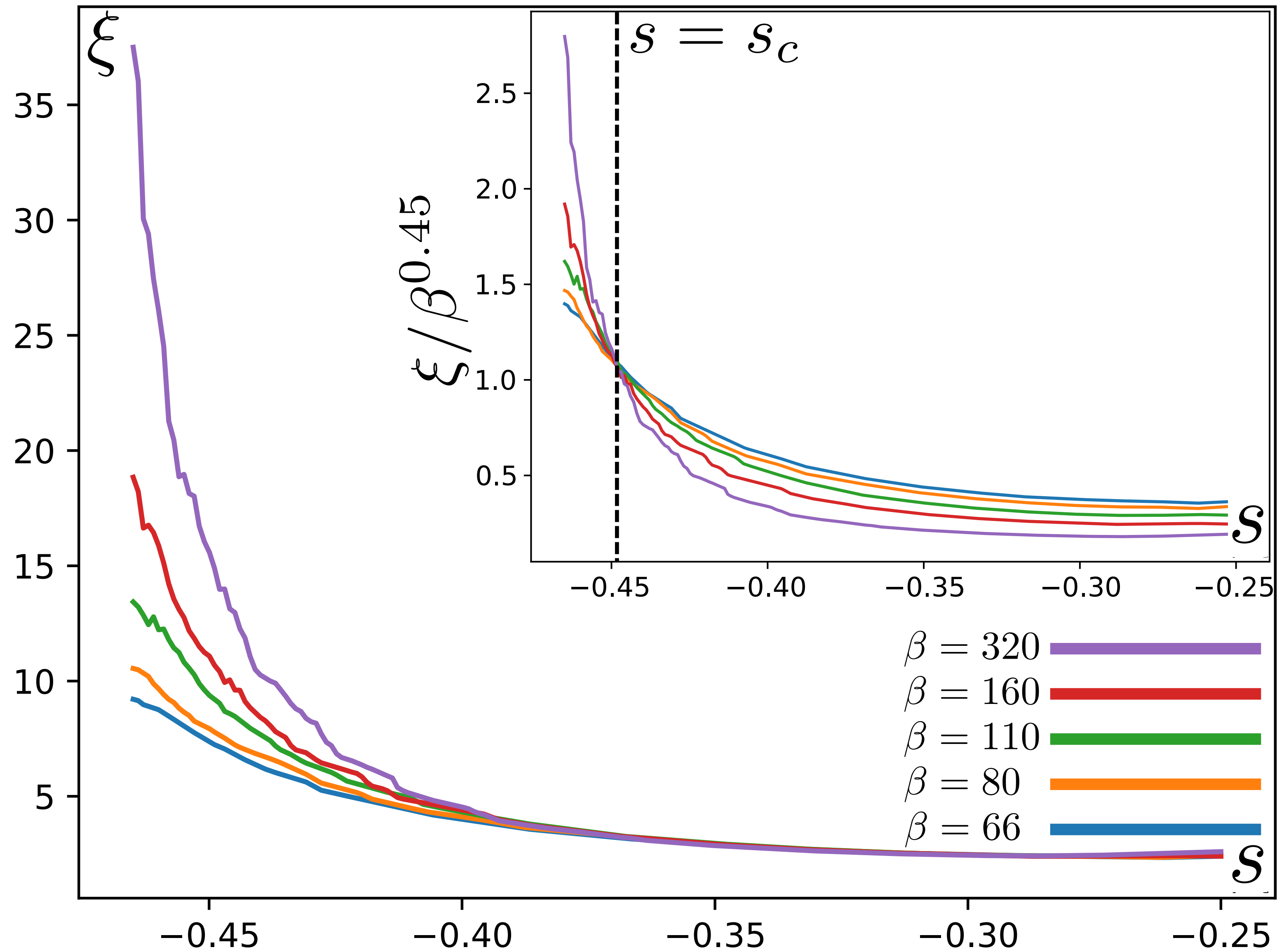
Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev,
Phys. Rev. Lett. **101**, 035701 (2008).

$$\widetilde{\delta s}_j = s + \delta s_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\bar{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + \delta s_j + uT \sum_{\Omega} \sum_{\alpha} \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma |\Omega| + \Omega^2 / c^2 + e_{\alpha}}$$

where e_{α} and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\bar{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \dots L^2$ for a $L \times L$ sample.

Bosonic eigenmodes in random mass Hertz theory

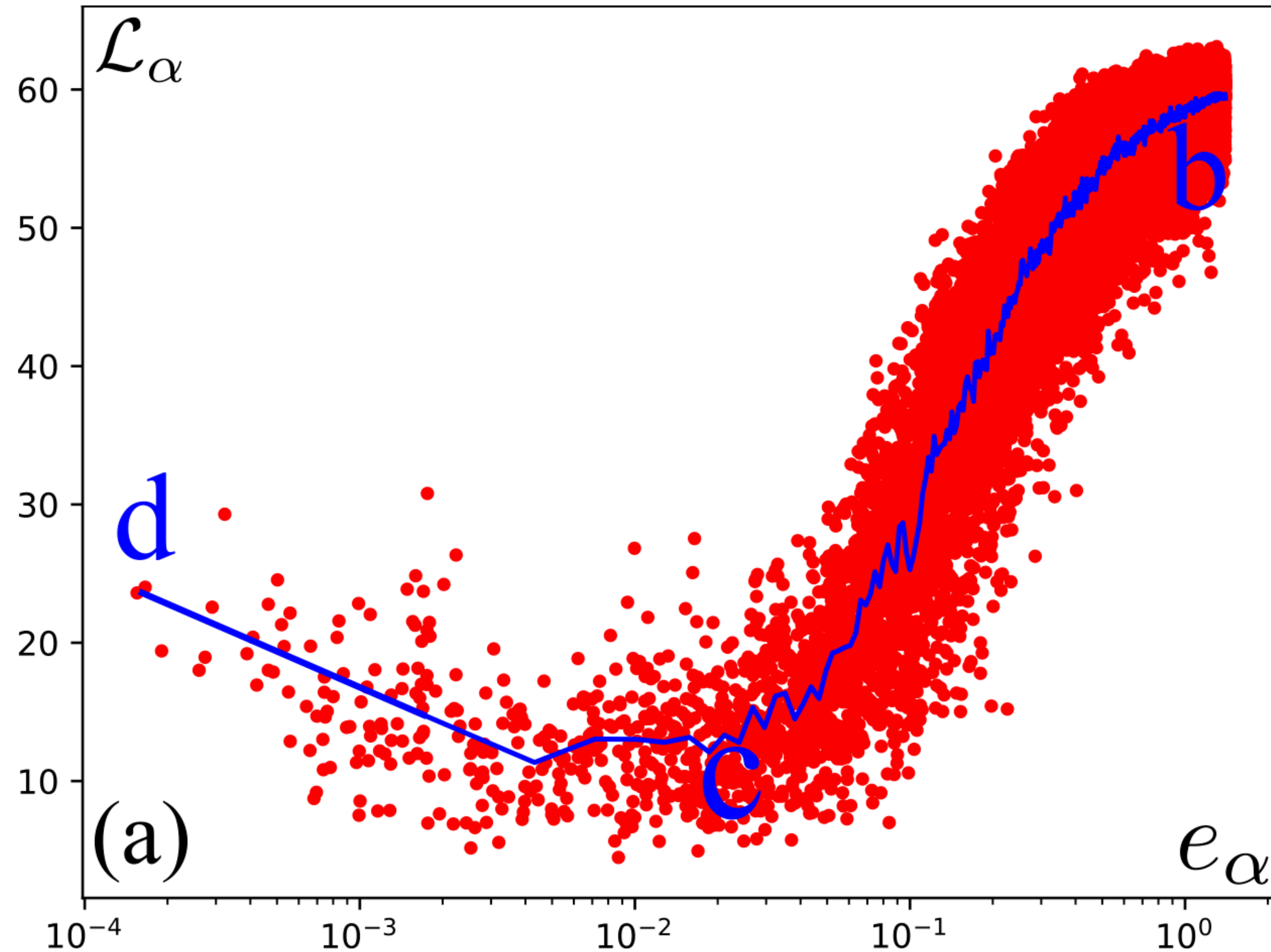
ϕ correlation length ξ



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α



QMC of random coupling model:
Aavishkar A. Patel,
Peter Lunts, and
Michael Albergo,
arXiv:2410.05365

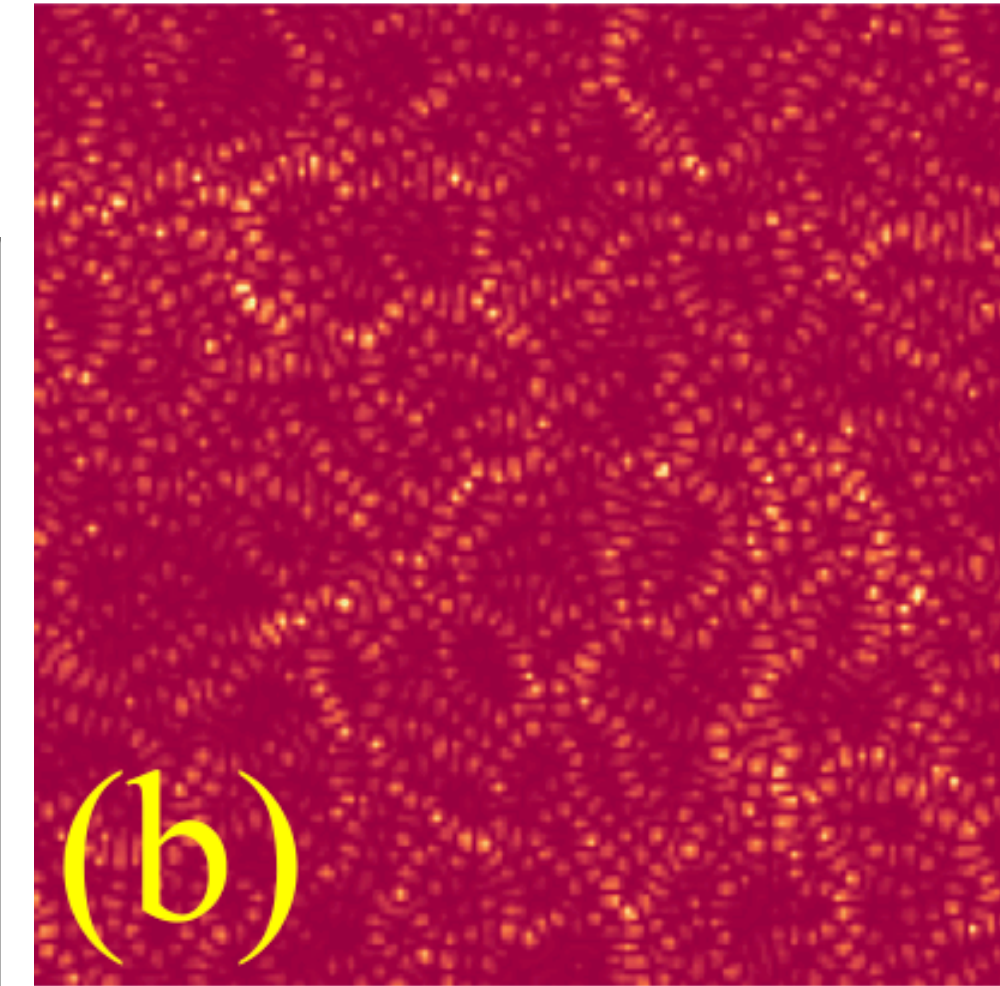
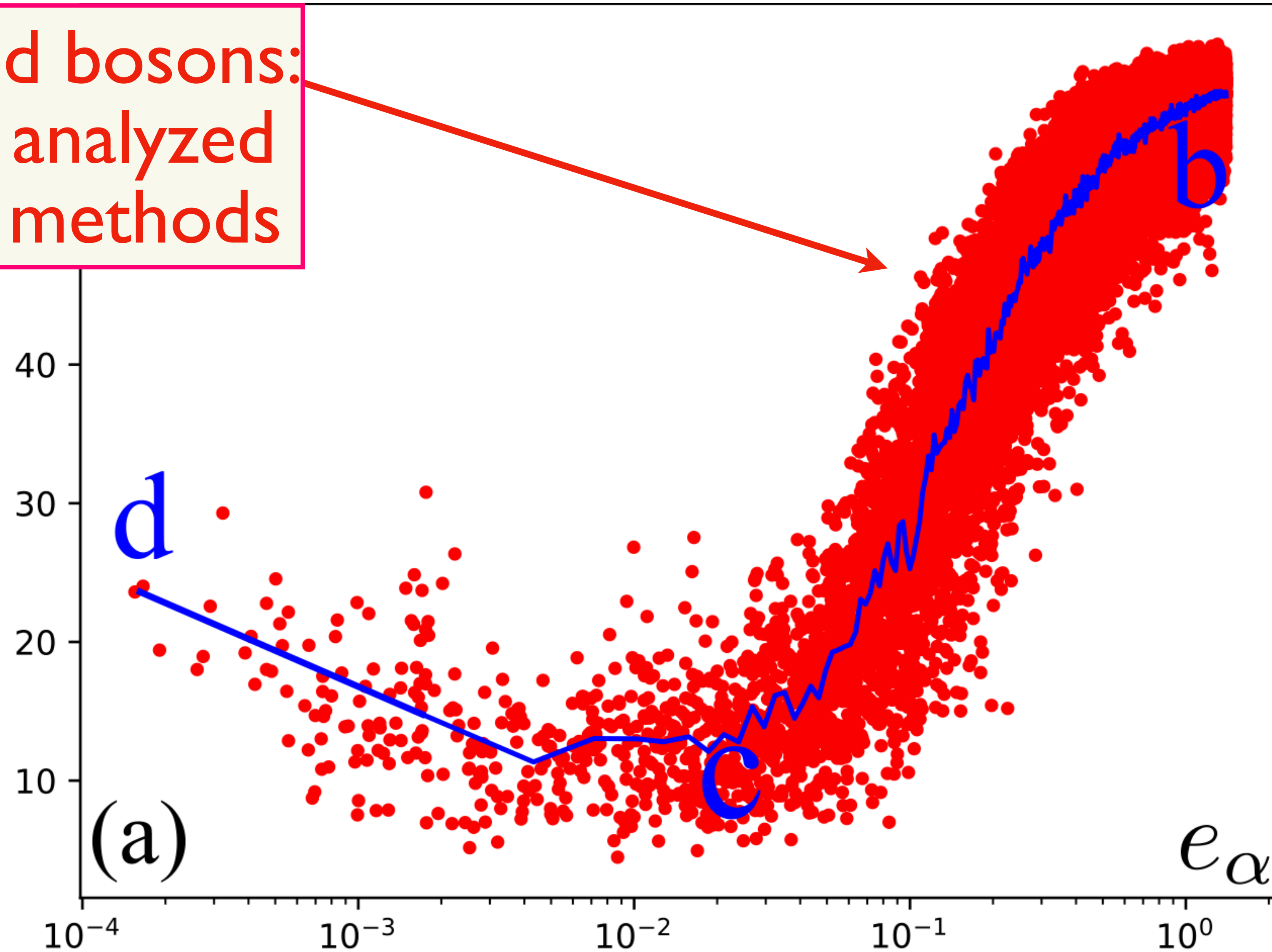


Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

Extended bosons:
Can be analyzed
by SYK methods



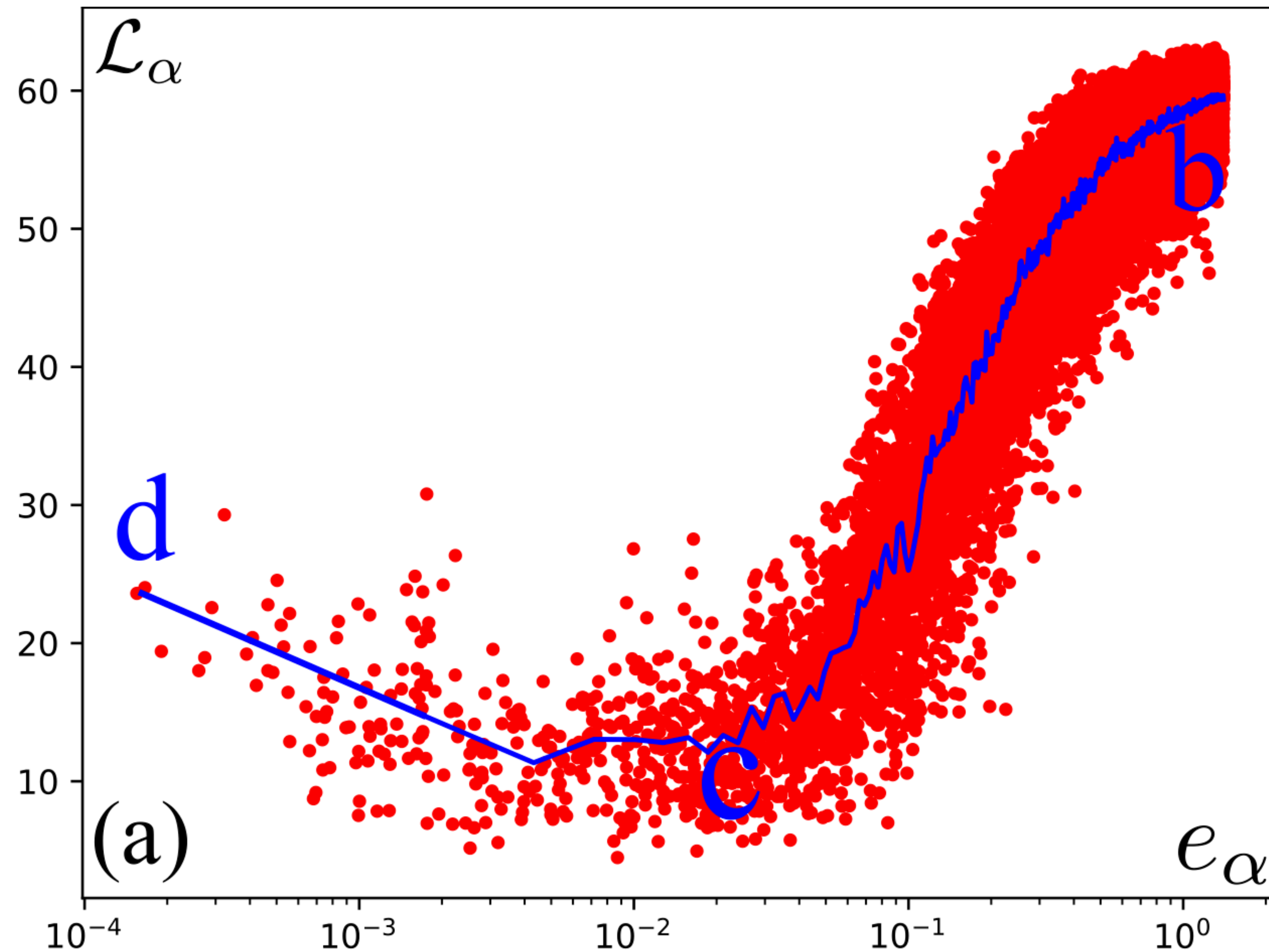
QMC of random coupling model:
Aavishkar A. Patel,
Peter Lunts, and
Michael Albergo,
arXiv:2410.05365



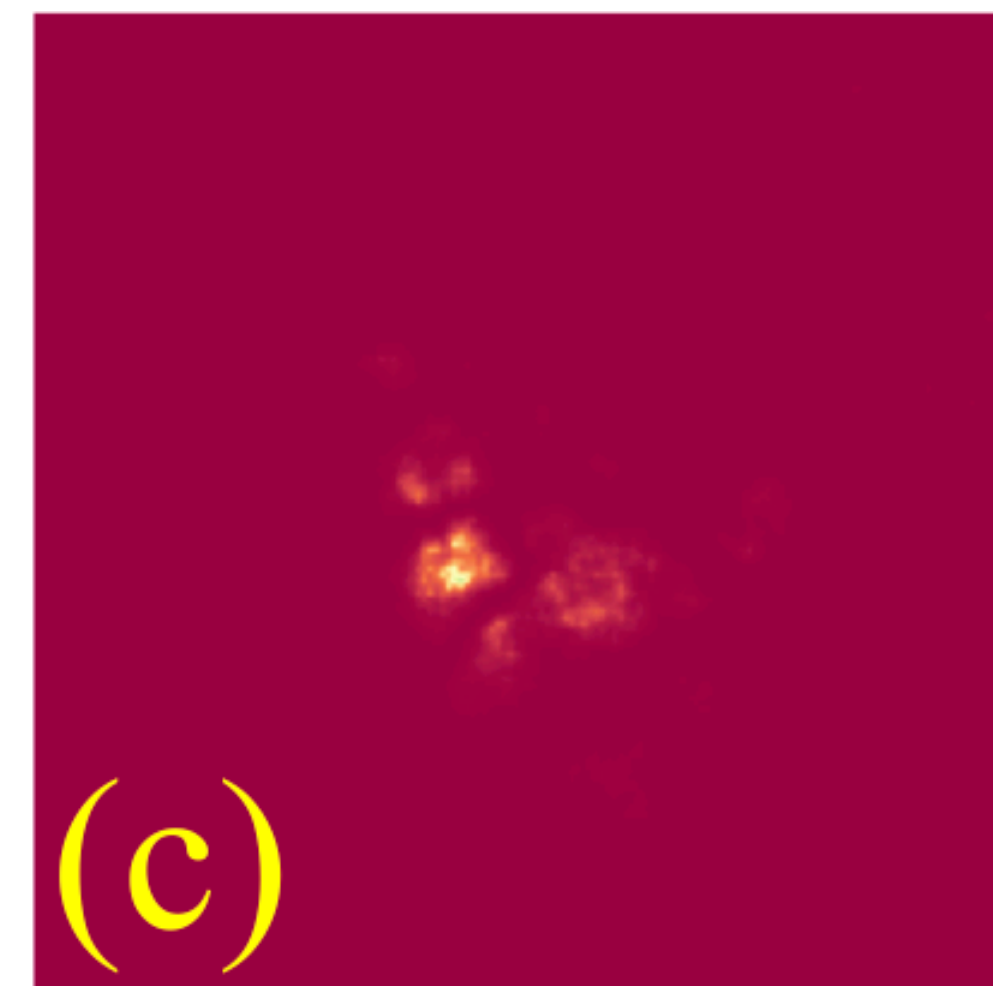
Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α



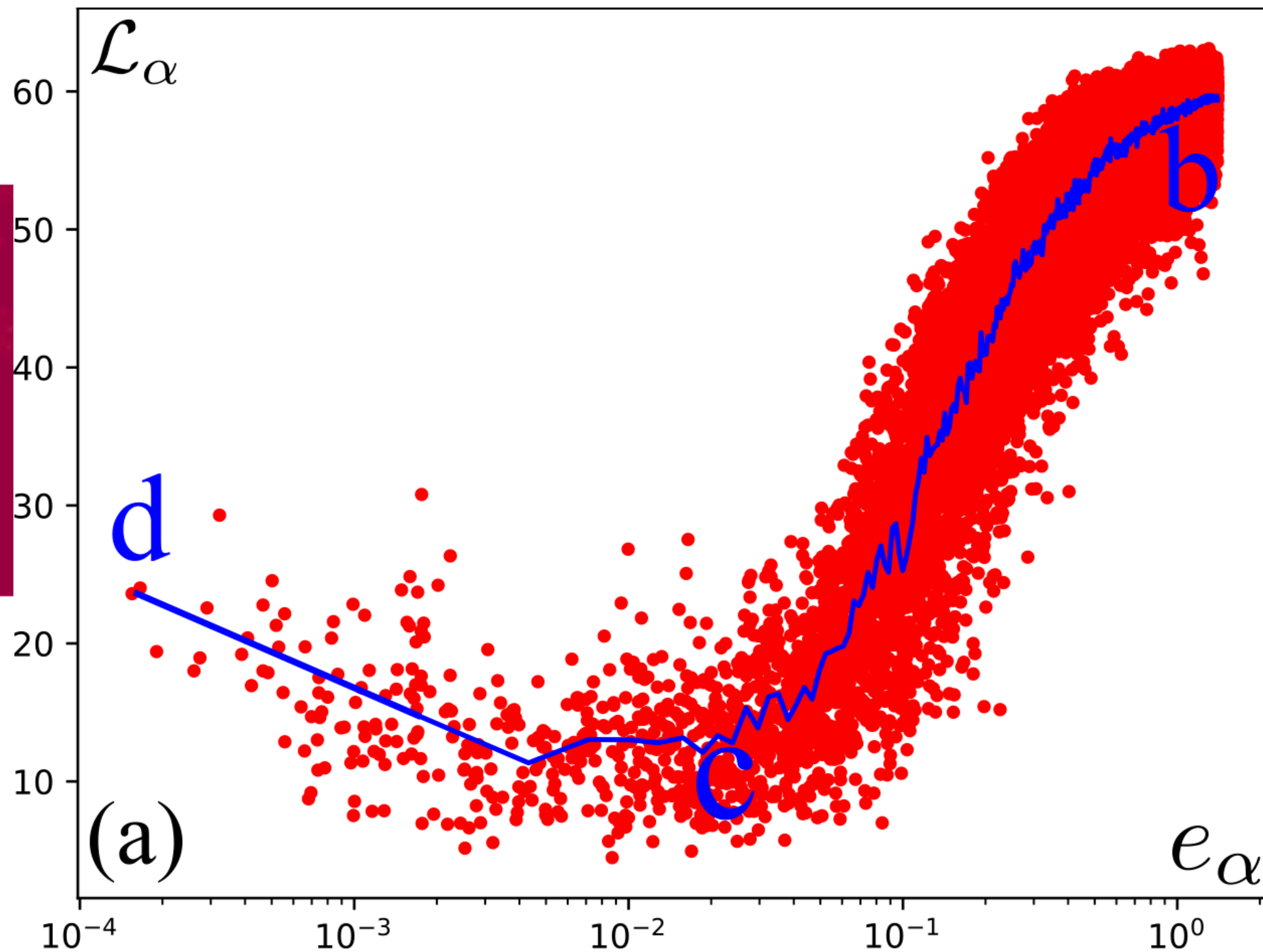
QMC of random coupling model:
Aavishkar A. Patel,
Peter Lunts, and
Michael Albergo,
arXiv:2410.05365



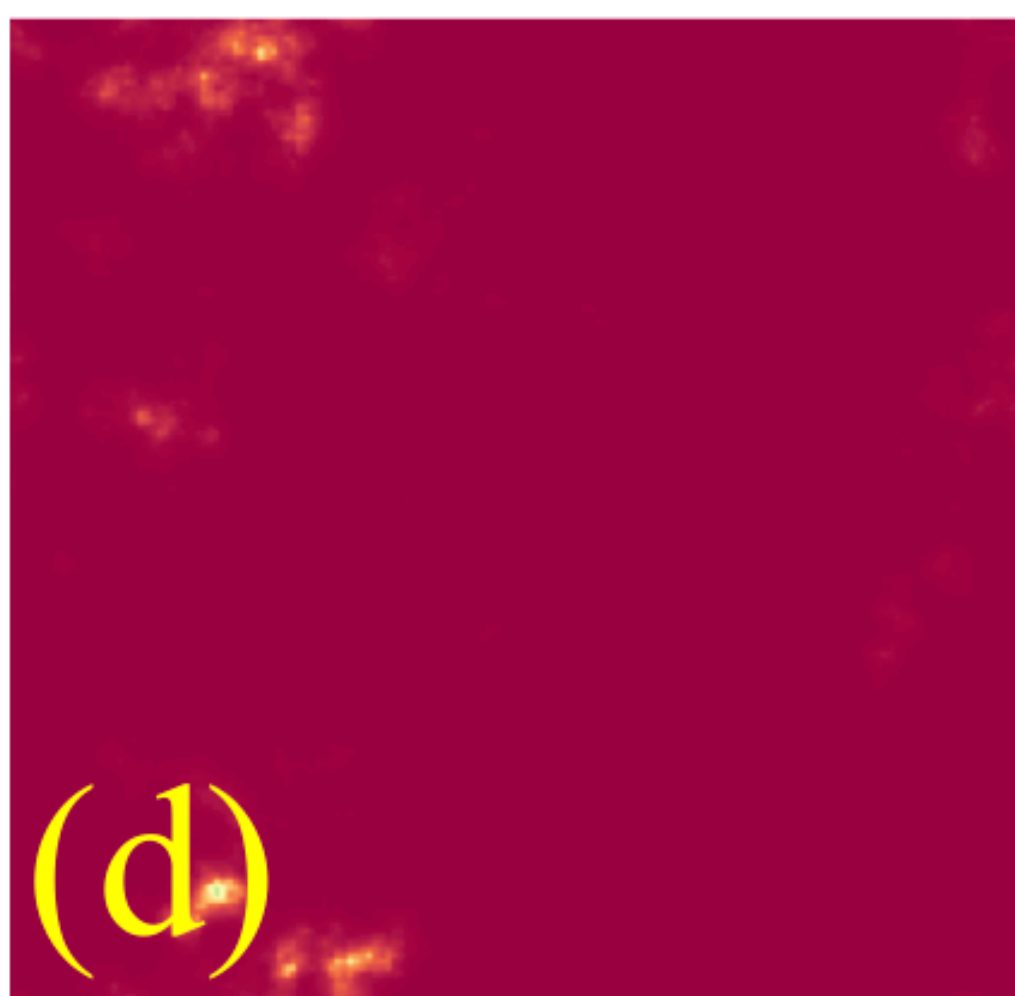
Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

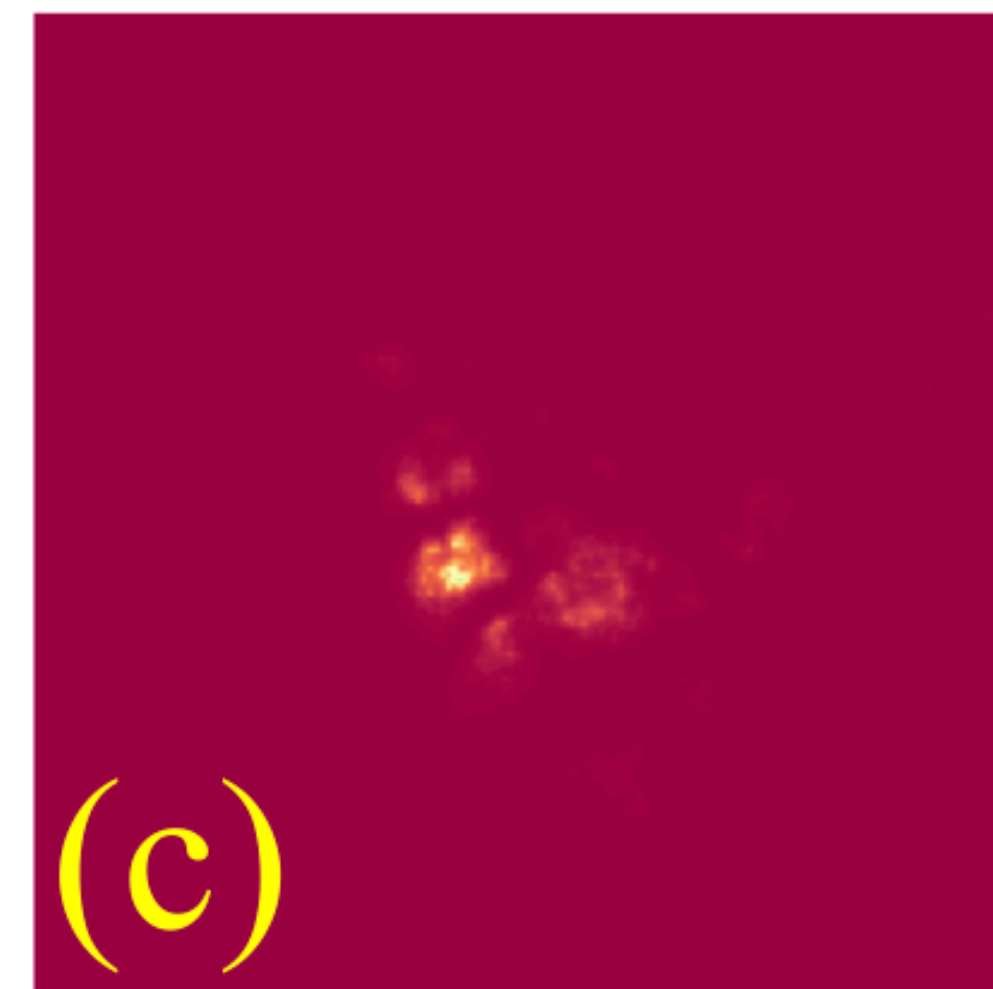
ϕ eigenmodes localization length \mathcal{L}_α



QMC of random coupling model:
Aavishkar A. Patel,
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arXiv:2410.05365



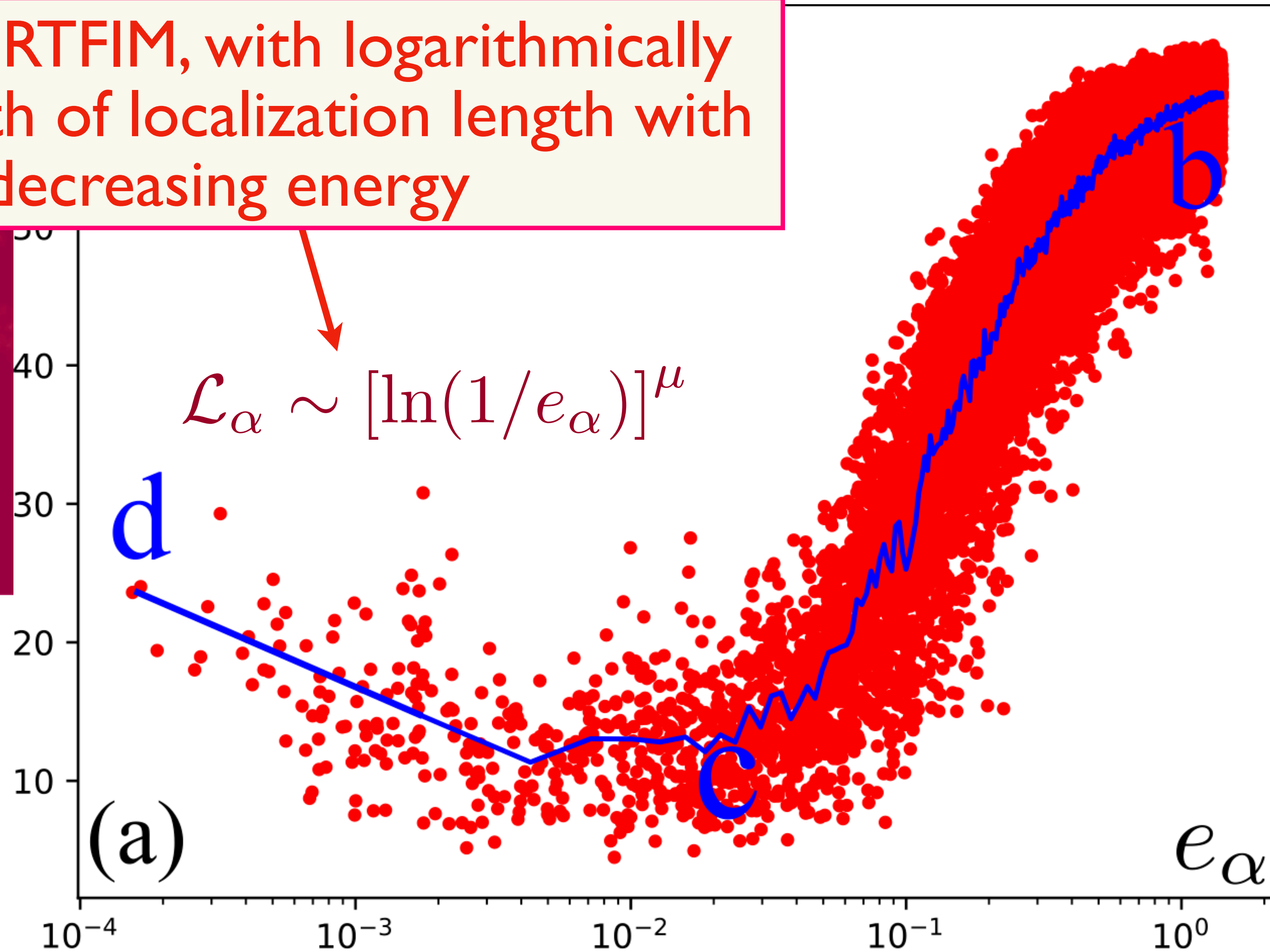
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Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)



Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length \mathcal{L}_α

Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy



J. A. Hoyos,
C. Kotabage,
T. Vojta
PRL **99**,
230601 (2007)

QMC of random
coupling model:
Aavishkar A. Patel,
Peter Lunts, and
Michael Albergo,
arXiv:2410.05365

(d)

(c)

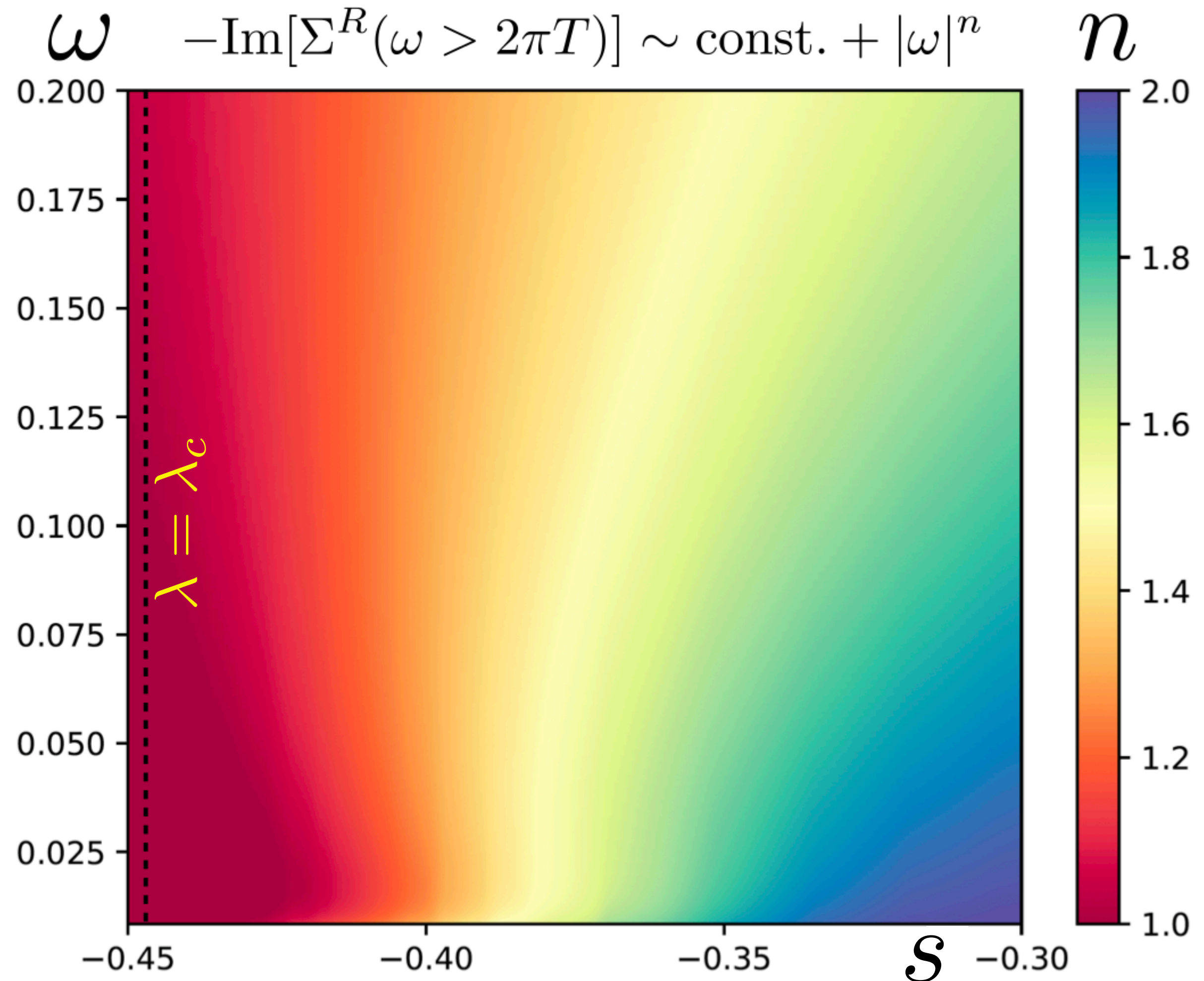


Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

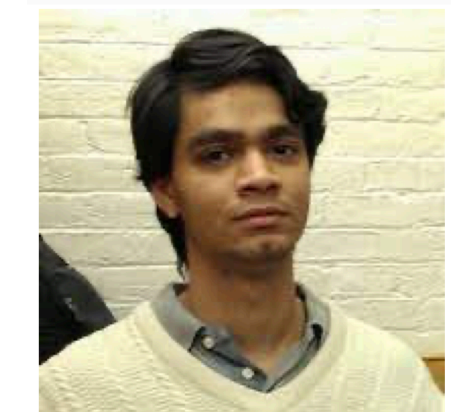
Transport scattering rate

$$\Sigma(i\omega) = -i\pi g'^2 \mathcal{N}_0 \frac{T}{L^2} \sum_{\alpha, \Omega} \frac{\text{sgn}(\omega + \Omega)}{\gamma|\Omega| + \Omega^2/c^2 + e_\alpha}.$$



$L = 160, \beta = 800, 10$ disorder samples

Extended region in λ with $n \approx 1$ - a strange metal *phase*



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

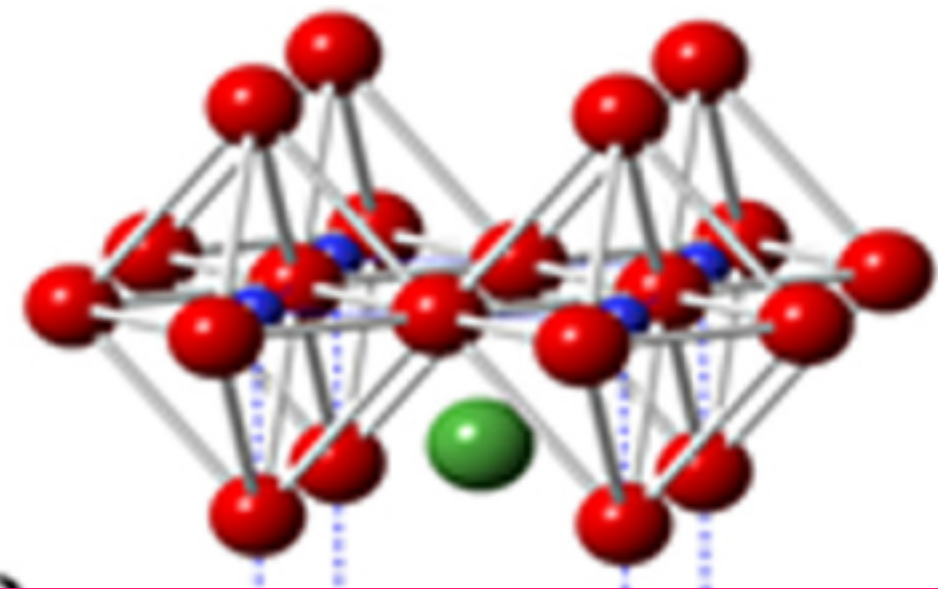
QMC of random
coupling model:
Aavishkar A. Patel,
Peter Lunts, and
Michael Albergo,
arXiv:2410.05365

Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

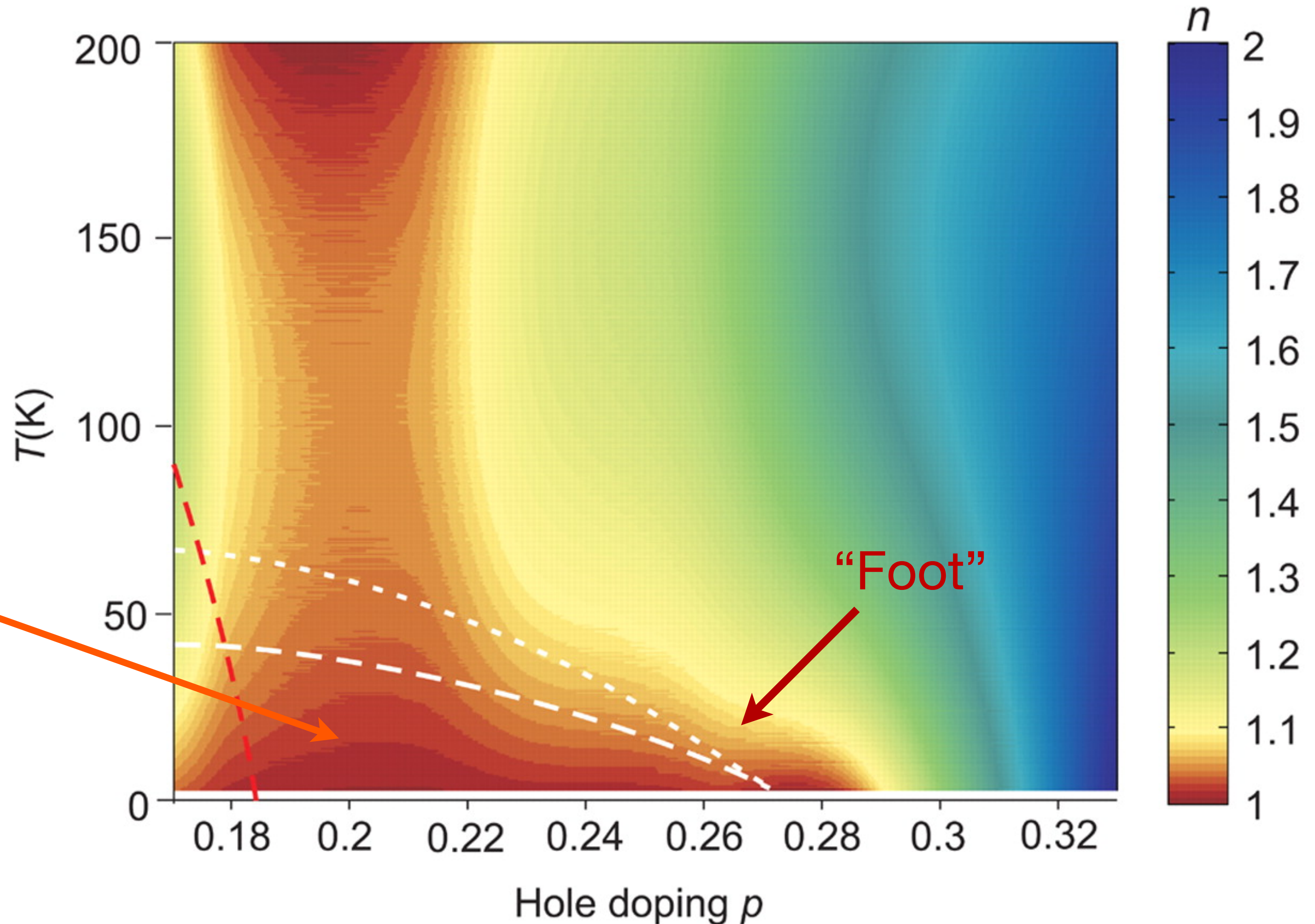
R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

FL-SDW QPT with Harris disorder provides a theory of the “foot”

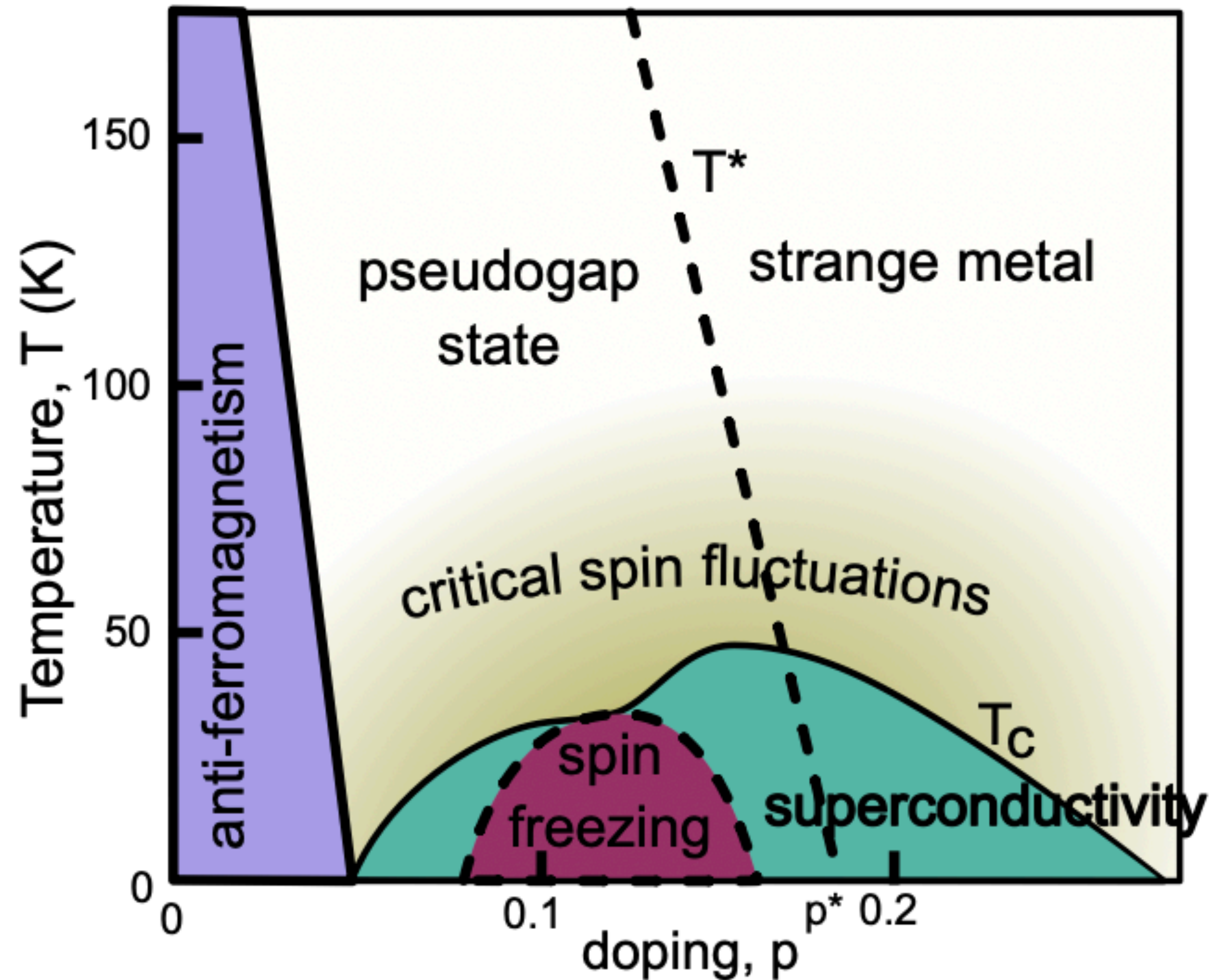
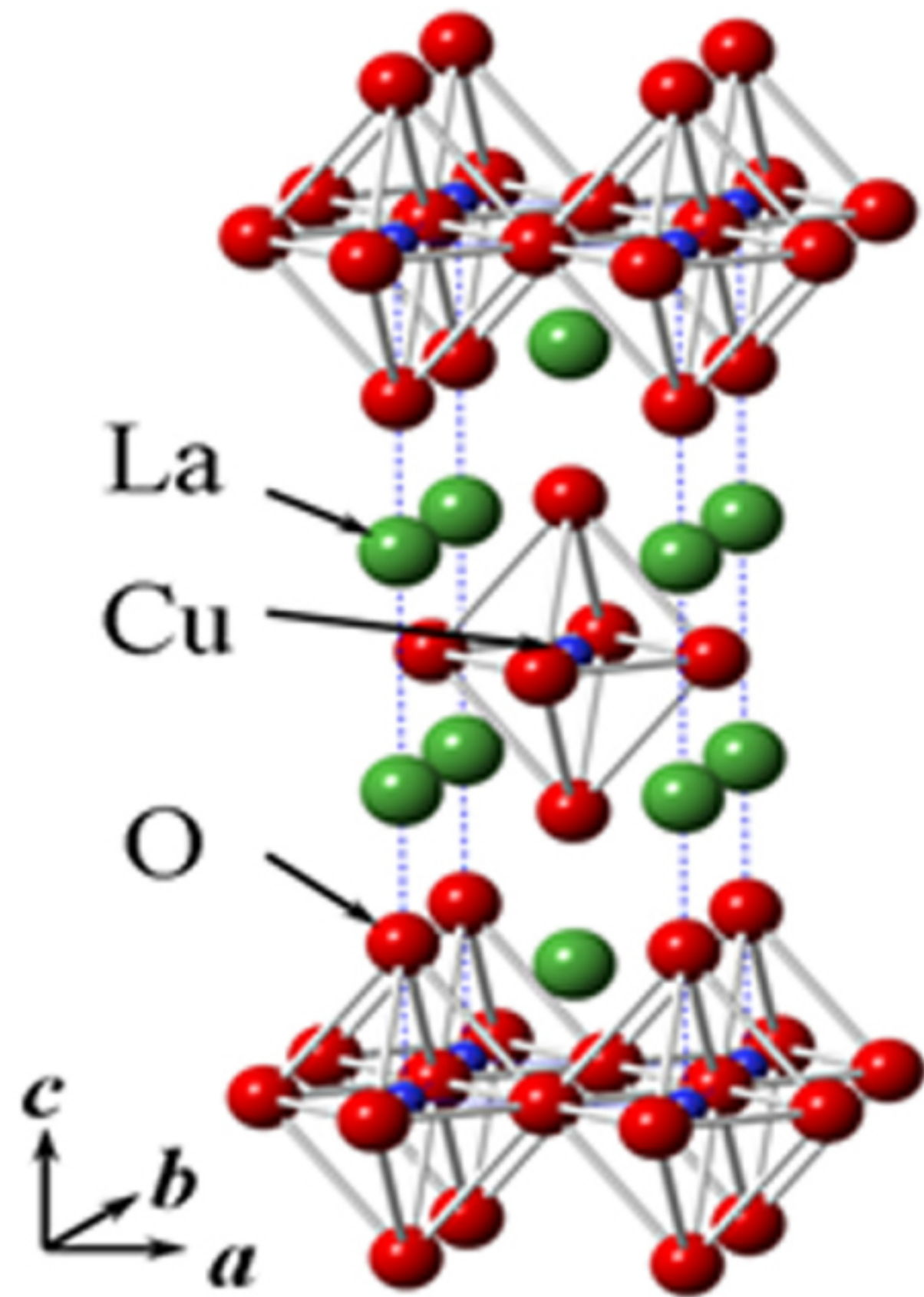
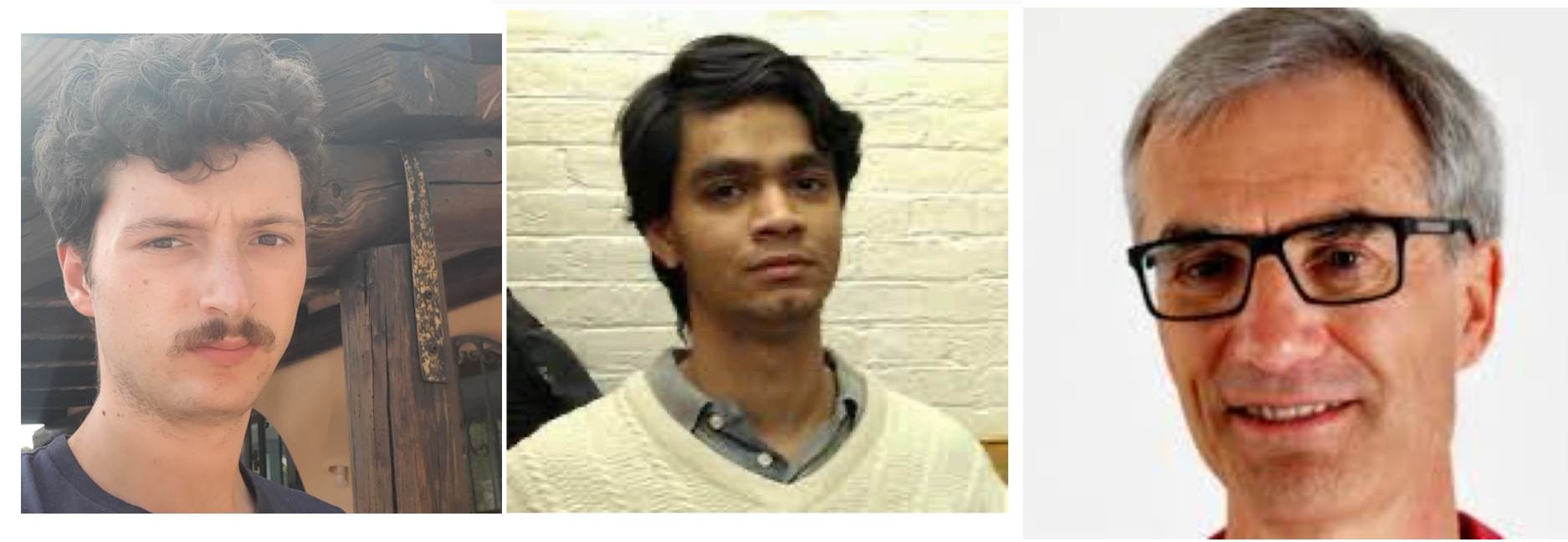


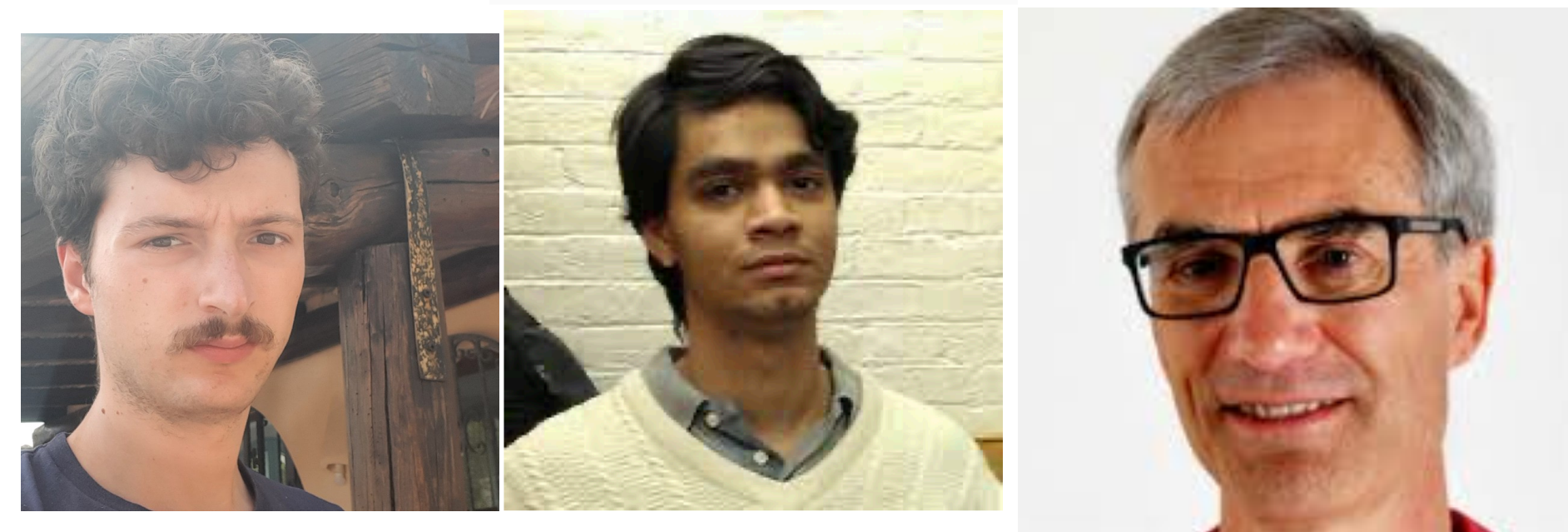
Localized overdamped SDW bosons, but extended fermions



Neutron scattering in LSCO

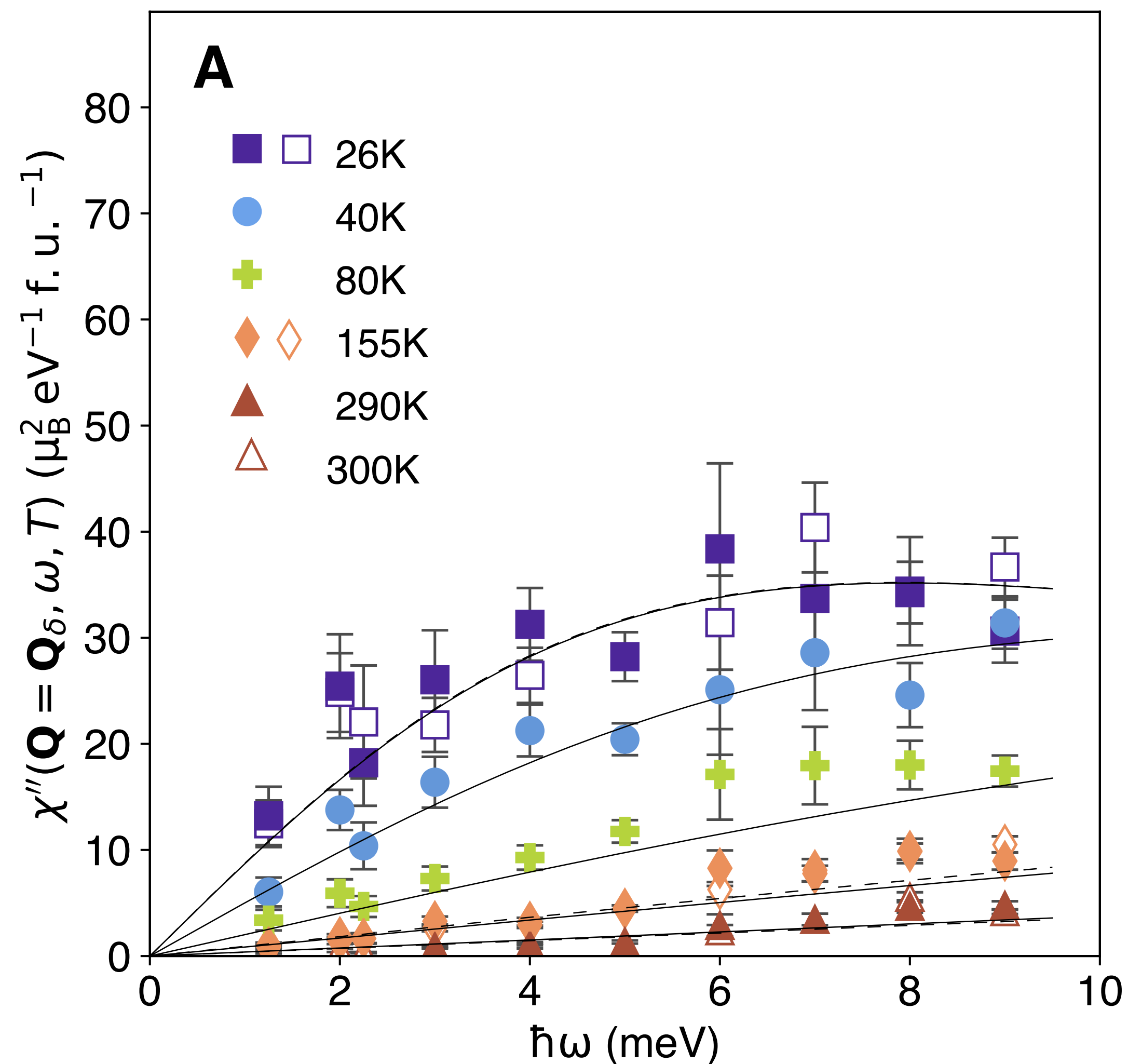
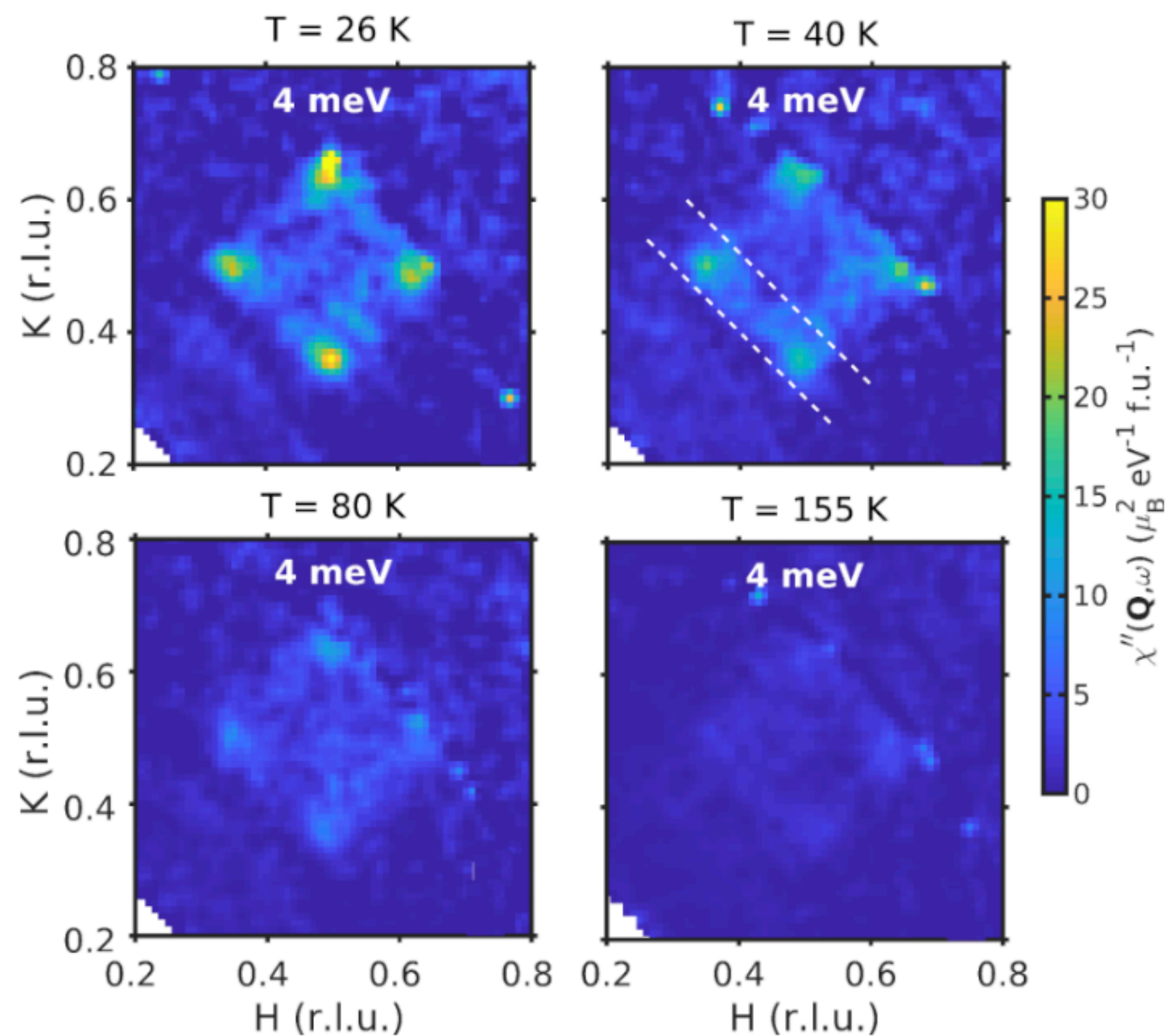
J. Radaelli, A.A. Patel, O.J. Lipscombe, R. Stewart, S. S. and S.M. Hayden,
Critical spin fluctuations across the superconducting dome in $La_{2-x}Sr_xCuO_4$,
to appear

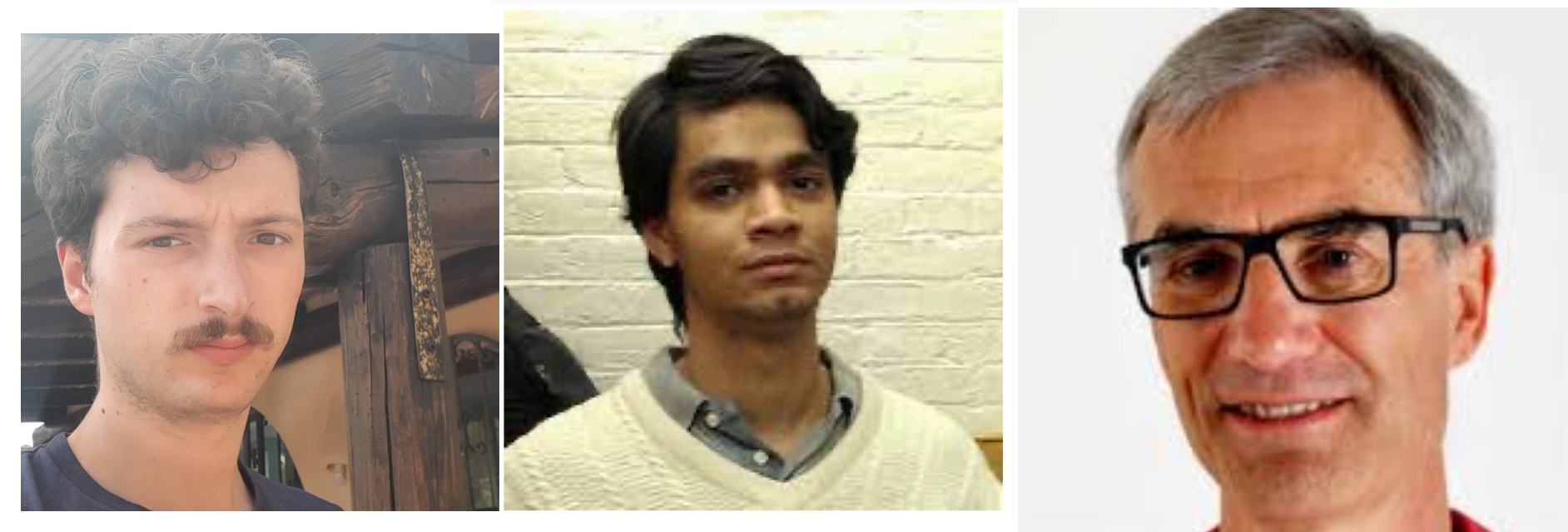




Neutron scattering in LSCO

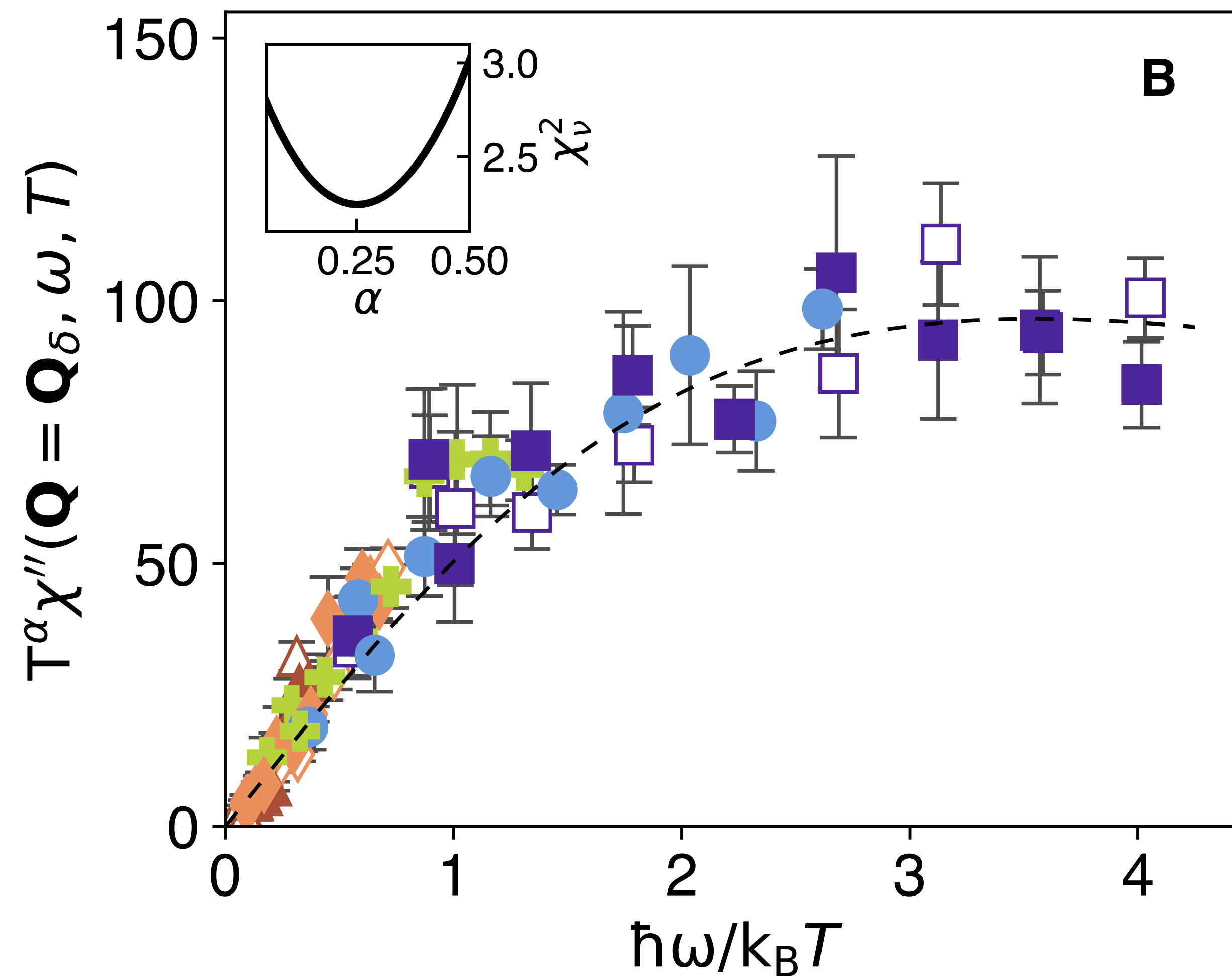
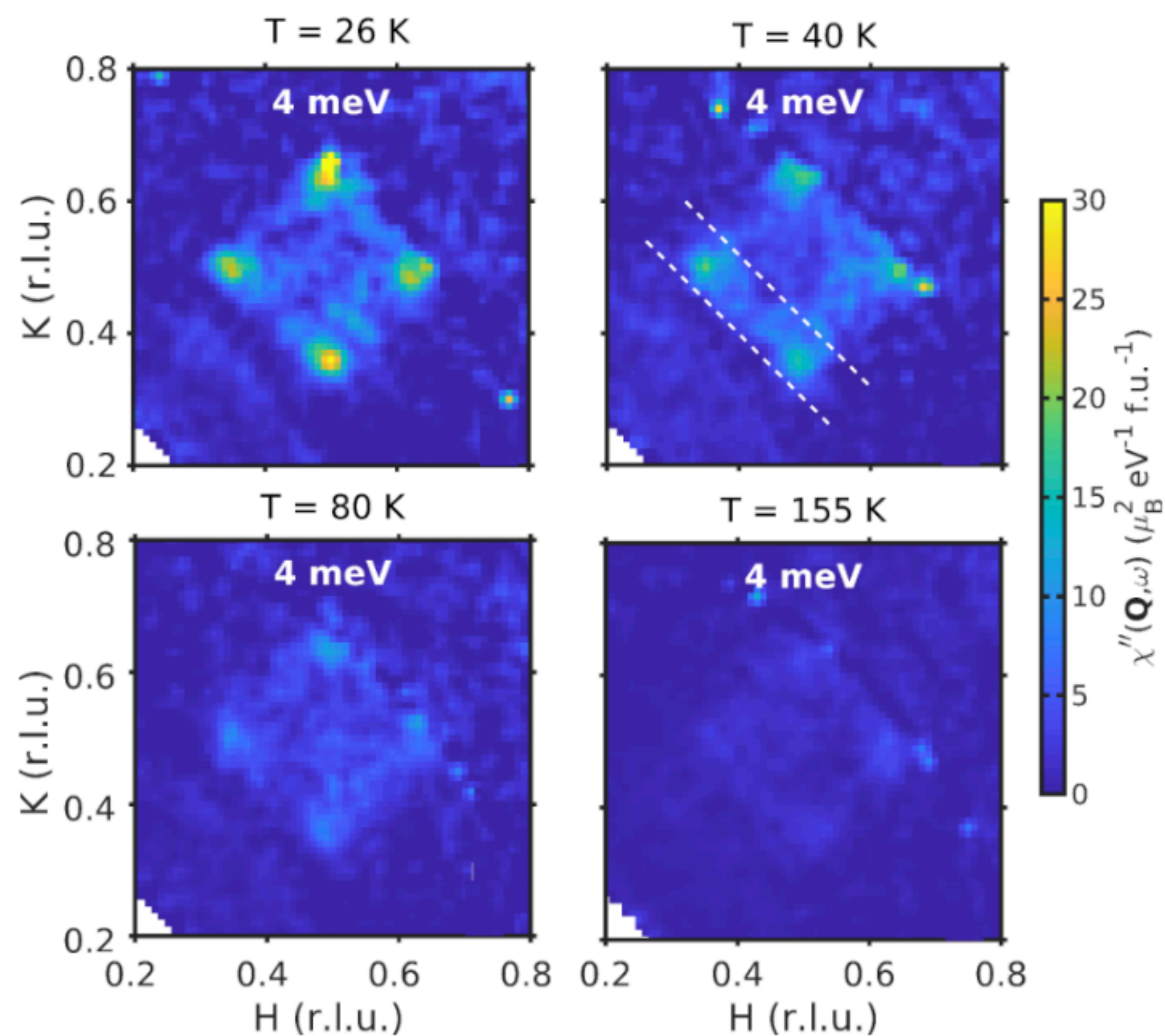
J. Radaelli, A.A. Patel, O.J. Lipscombe, R. Stewart, S. S. and S.M. Hayden,
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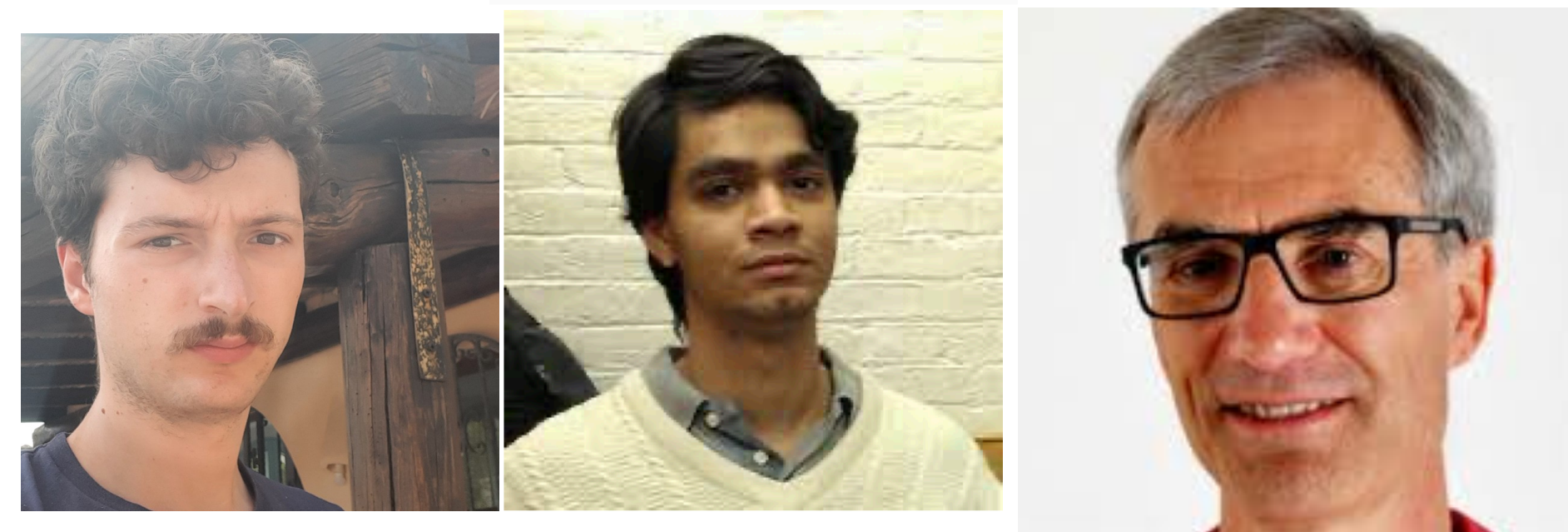




Neutron scattering in LSCO

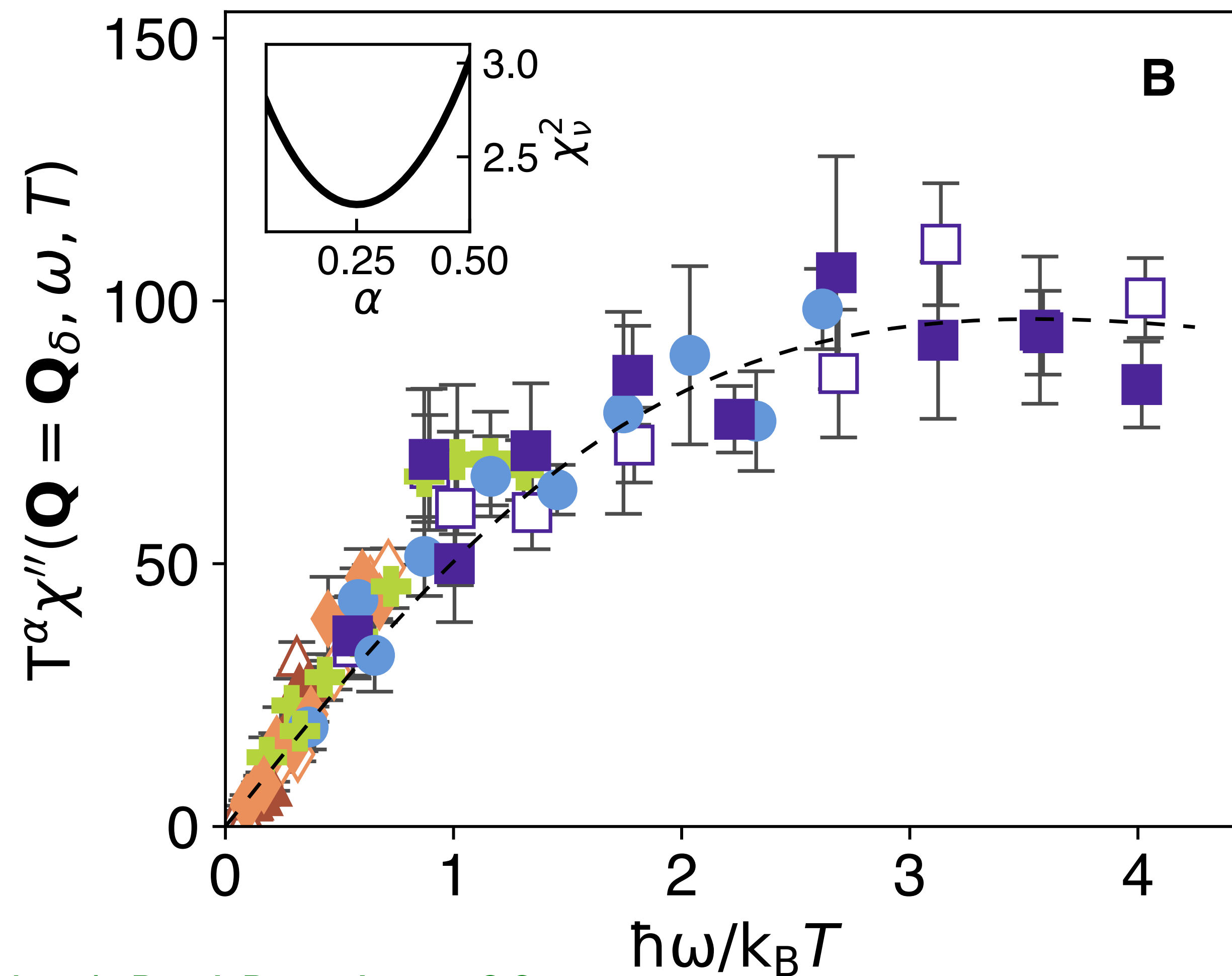
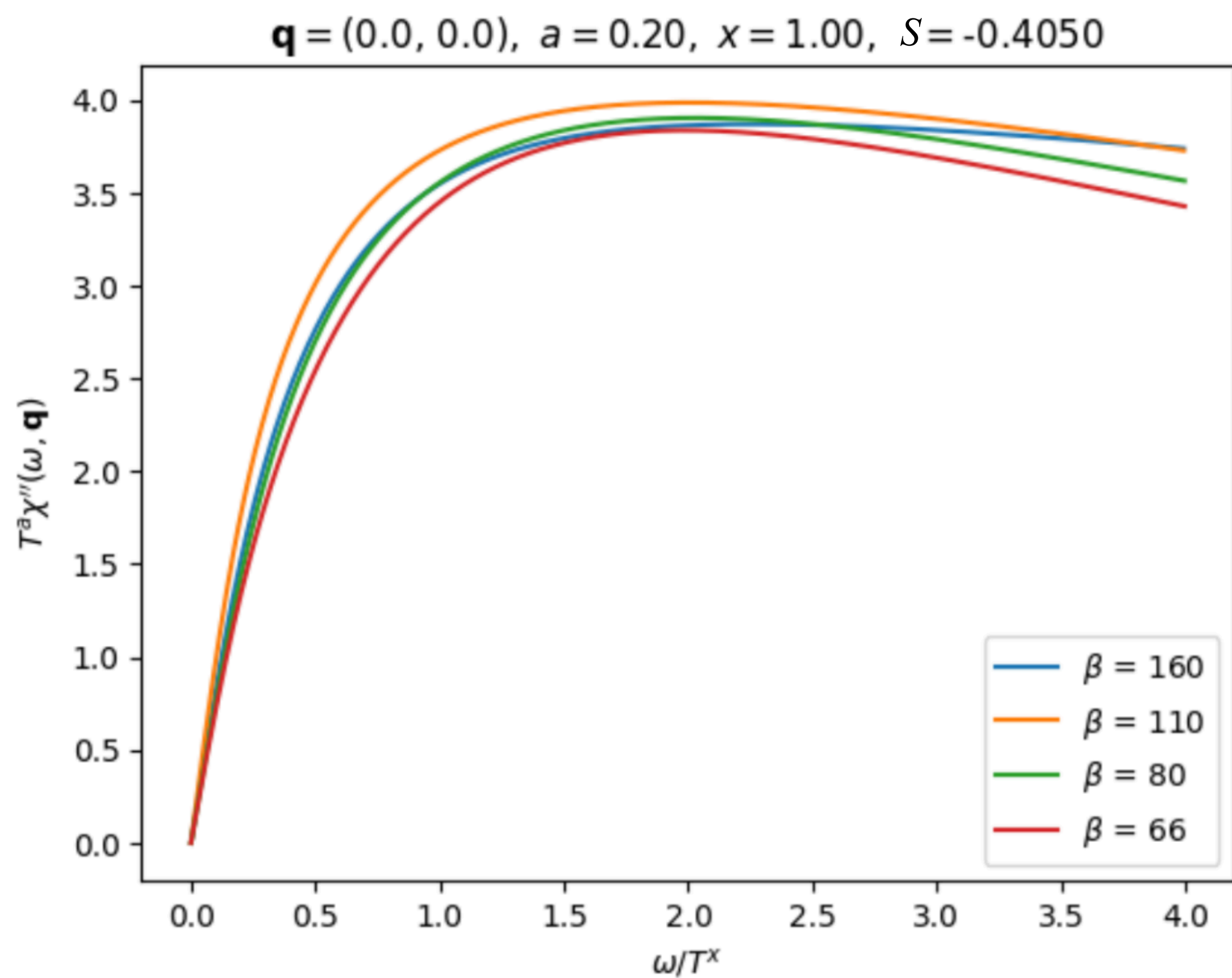
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Critical spin fluctuations across the superconducting dome in $La_{2-x}Sr_xCuO_4$,
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Neutron scattering in LSCO

J. Radaelli, A.A. Patel, O.J. Lipscombe, R. Stewart, S. S. and S.M. Hayden,
Critical spin fluctuations across the superconducting dome in $La_{2-x}Sr_xCuO_4$,
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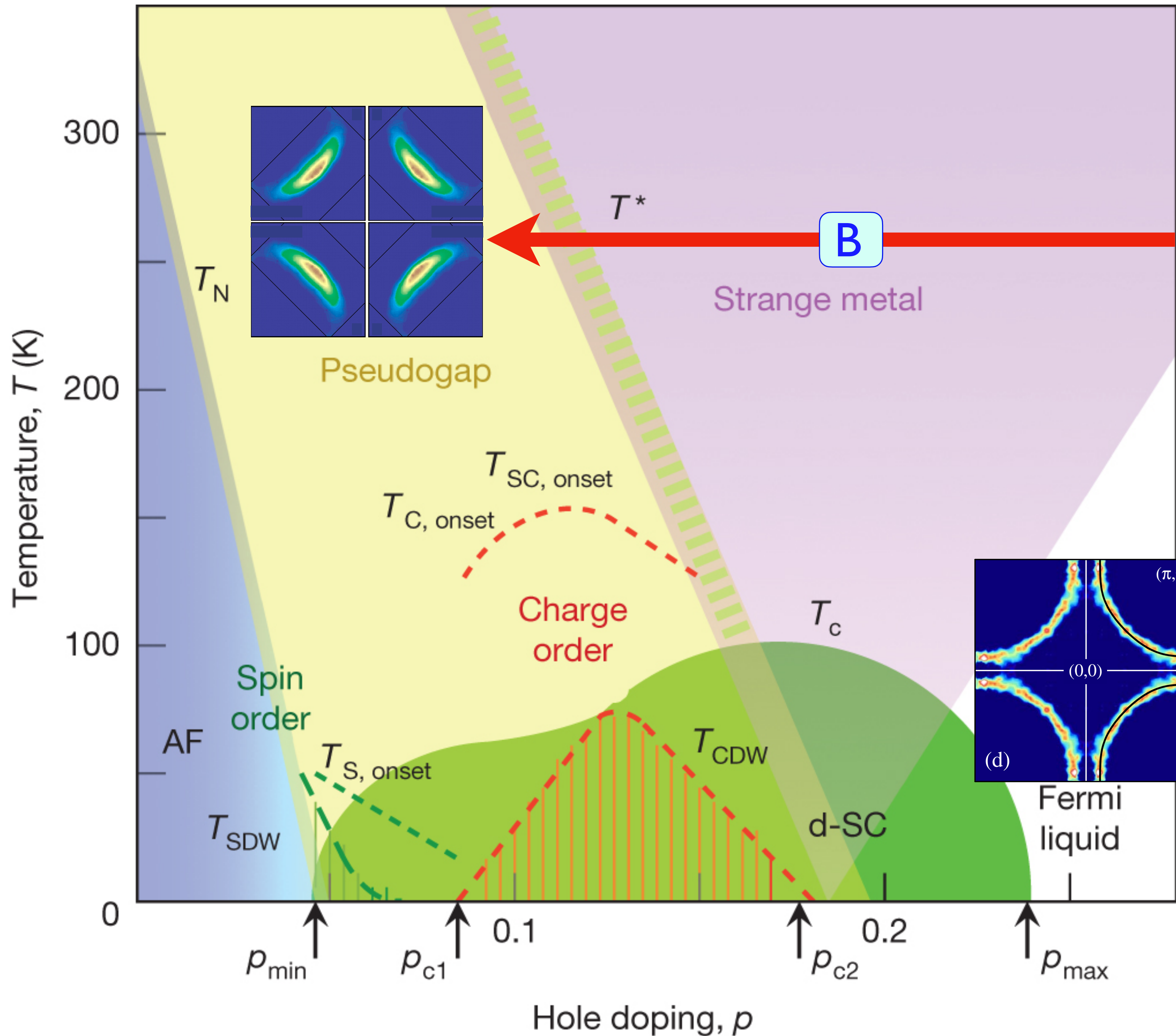


Aavishkar A. Patel, Peter Lunts, S.S.,
PNAS **121**, e2402052121 (2024)

A. FL-SDW QPT

B. FL-FL* QPT

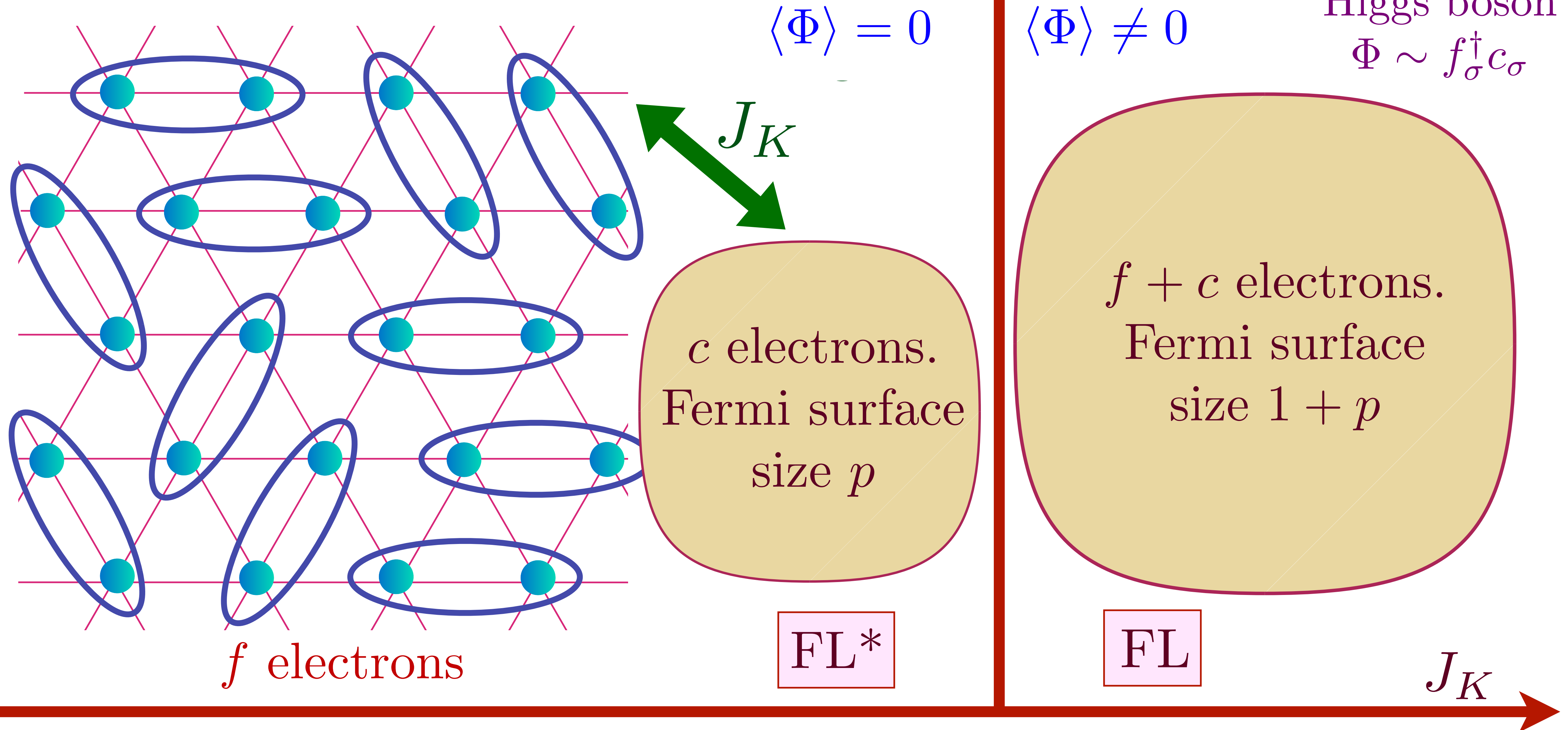
C. Confinement crossover to dSC



Fermi-volume-changing QPT
without symmetry breaking
and with spatial disorder.

FL-FL* QPT
Requires fractionalization

Fermi-volume-changing QPT in the Kondo lattice



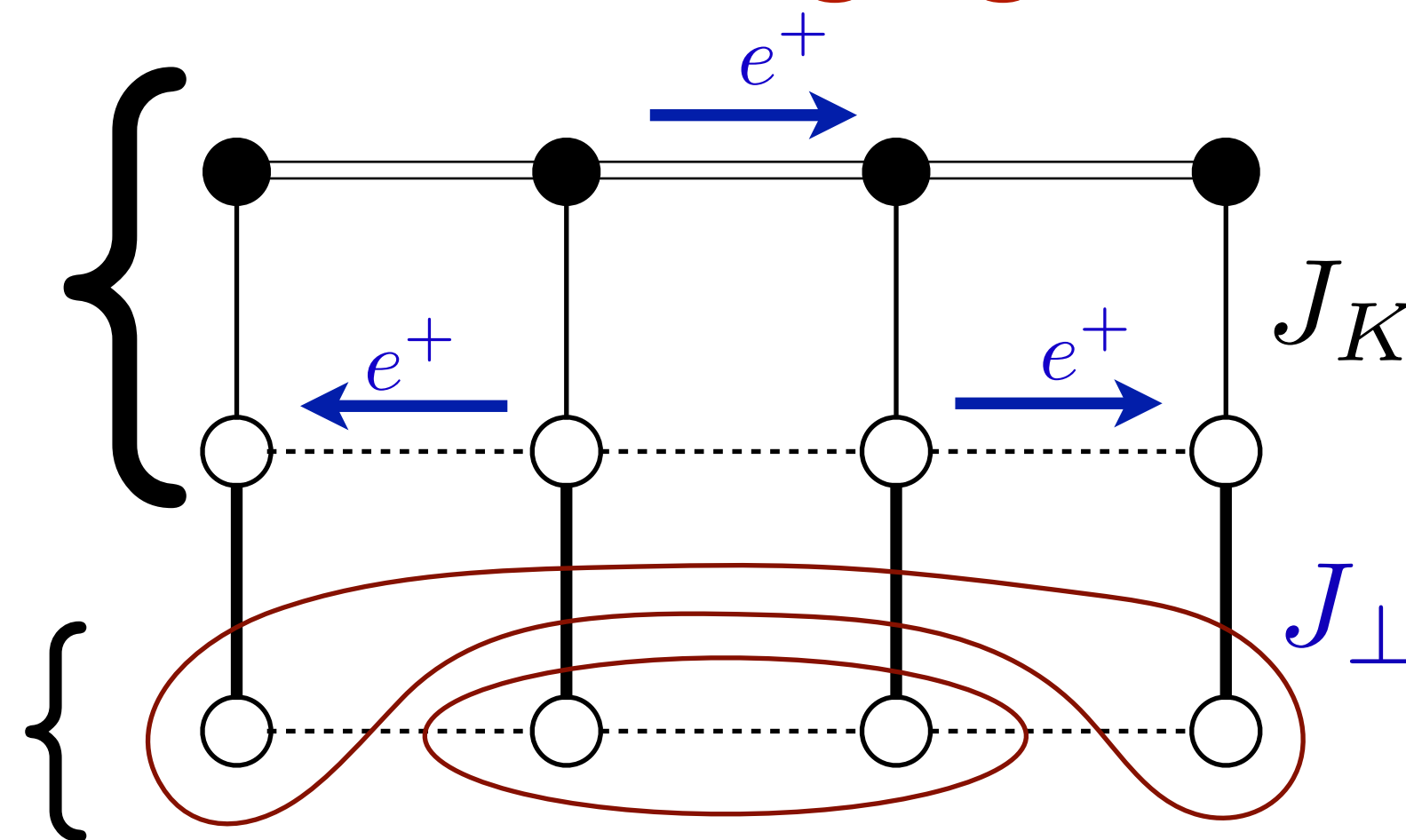
Fermi-volume-changing QPT in a one-band model

Ya-Hui Zhang and S. S.,
PRR 2, 023172 (2020)

Kondo lattice heavy
Fermi liquid.
Size $1 + p + 1$
 $= p \pmod{2}$.
Small Fermi surface!

$$\langle \Phi \rangle \neq 0$$

Spin liquid



Large
Fermi surface.
Size: $1 + p$

Trivial
insulator

J_K

FL*

$$\langle \Phi \rangle \neq 0$$

$$\langle \Phi \rangle = 0$$

FL

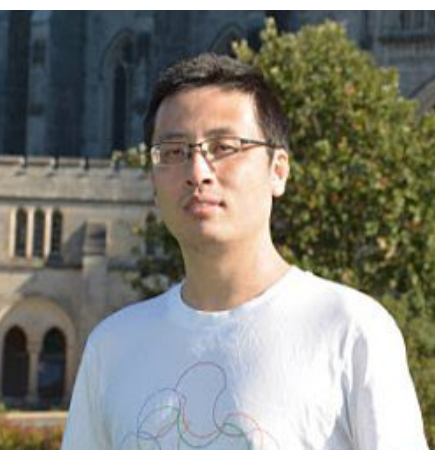
doping p

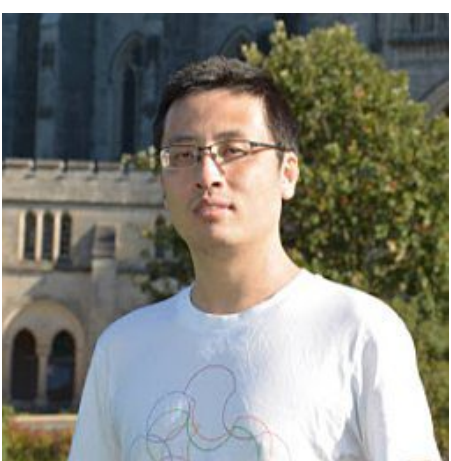
Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid
 \oplus
Spin Liquid

Fractionalized excitations of layer S_1 confined
by condensation of Higgs boson $\Phi \sim f_{1\sigma}^\dagger c_\sigma$.

Fractionalized excitations of layer S_2 in the π -flux spin liquid with
massless Dirac spinons and $SU(2)$ gauge field, dual to $\mathbb{C}P^1$ $U(1)$
gauge theory with critical bosonic spinons.

Ya-Hui
Zhang





Kondo lattice

FL*

$$\langle \Phi \rangle = 0$$

$$\langle \Phi \rangle \neq 0$$

FL

Small Fermi surface of size p

Large Fermi surface of size $1 + p$

0

J_K

One-band model

FL*

$$\langle \Phi \rangle \neq 0$$

$$\langle \Phi \rangle = 0$$

FL

Small Fermi surface of size p

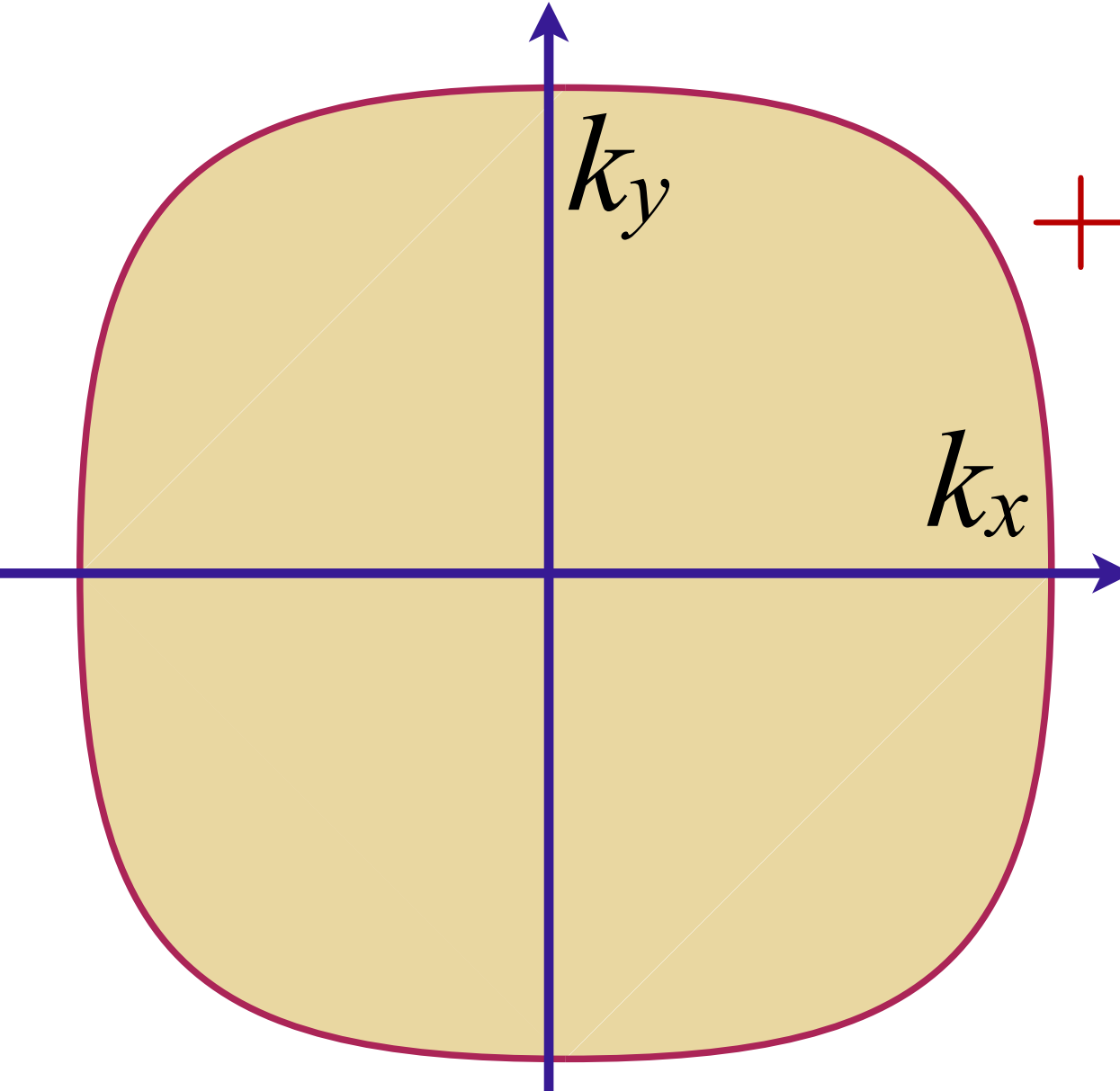
Large Fermi surface of size $1 + p$

J_K

0

One-band model has an ‘inverted’ Kondo lattice transition in a theory using a bilayer of ancilla qubits

Kondo lattice + critical boson with potential and “mass” disorder

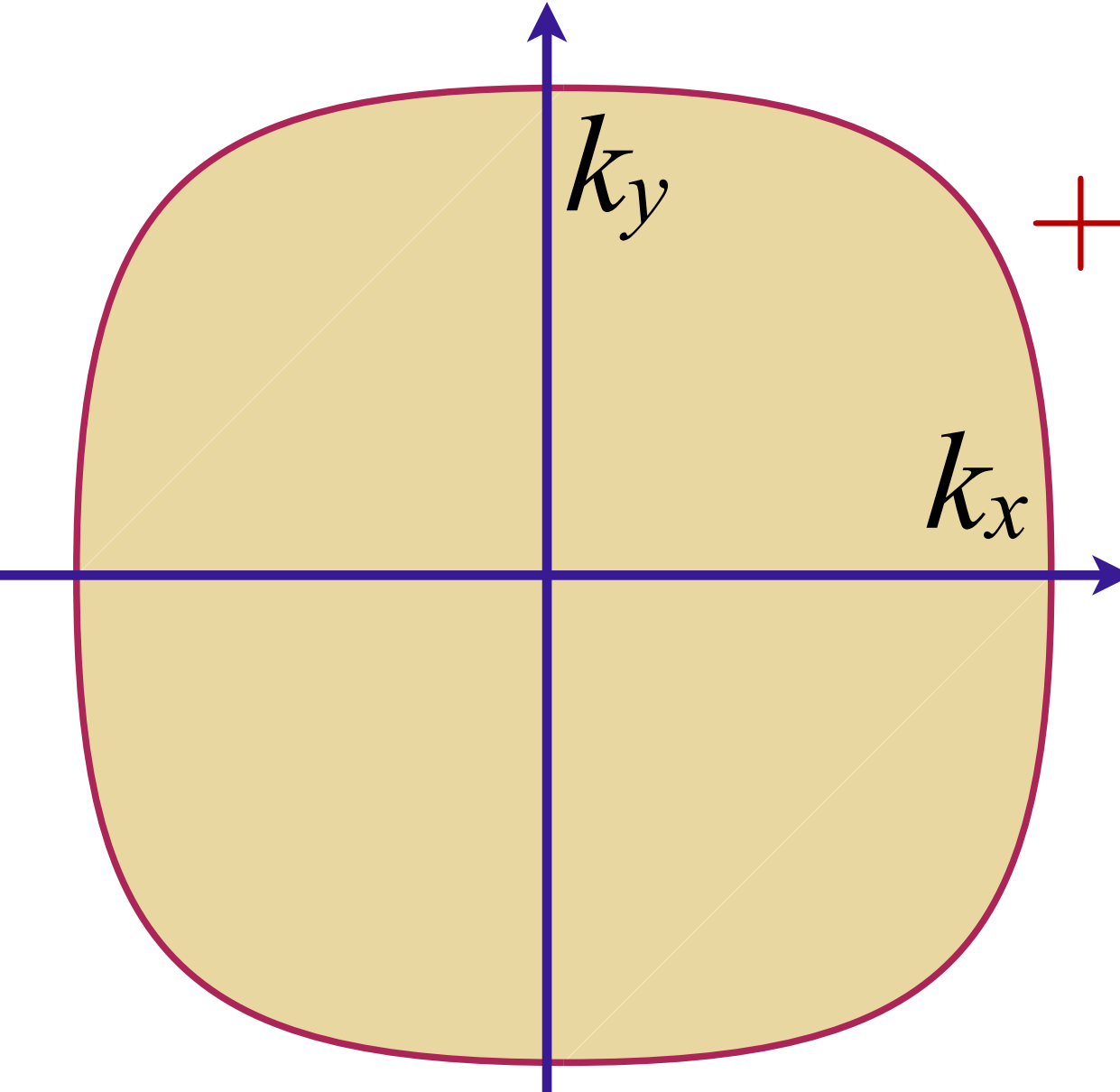
$$\begin{aligned}
 & c_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) c_{\mathbf{k}\sigma} + f_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon_1(\mathbf{k}) \right) f_{\mathbf{k}\sigma} \\
 & + [s + \delta s(\mathbf{r})] [\Phi(\mathbf{r})]^2 + g c_\sigma^\dagger(\mathbf{r}) f_\sigma(\mathbf{r}) \Phi(\mathbf{r}) + \text{H.c.} \\
 & + K [\nabla_{\mathbf{r}} \Phi(\mathbf{r})]^2 + u [\Phi(\mathbf{r})]^4 + v(\mathbf{r}) c_\sigma^\dagger(\mathbf{r}) c_\sigma(\mathbf{r})
 \end{aligned}$$


Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')}$ = $v^2\delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')}$ = $\delta s^2\delta(\mathbf{r} - \mathbf{r}')$

$v(\mathbf{r})$ leads to elastic scattering of c_σ and ‘Altshuler-Aronov’ corrections; localization of c_σ only at long length scales, not relevant for experiments

Kondo lattice + critical boson with potential and “mass” disorder

$$c_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) c_{\mathbf{k}\sigma} + f_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon_1(\mathbf{k}) \right) f_{\mathbf{k}\sigma}$$


$$+ [s + \delta s(\mathbf{r})] [\Phi(\mathbf{r})]^2 + g c_\sigma^\dagger(\mathbf{r}) f_\sigma(\mathbf{r}) \Phi(\mathbf{r}) + \text{H.c.}$$

Rescale Φ

$$+ K [\nabla_{\mathbf{r}} \Phi(\mathbf{r})]^2 + u [\Phi(\mathbf{r})]^4 + v(\mathbf{r}) c_\sigma^\dagger(\mathbf{r}) c_\sigma(\mathbf{r})$$

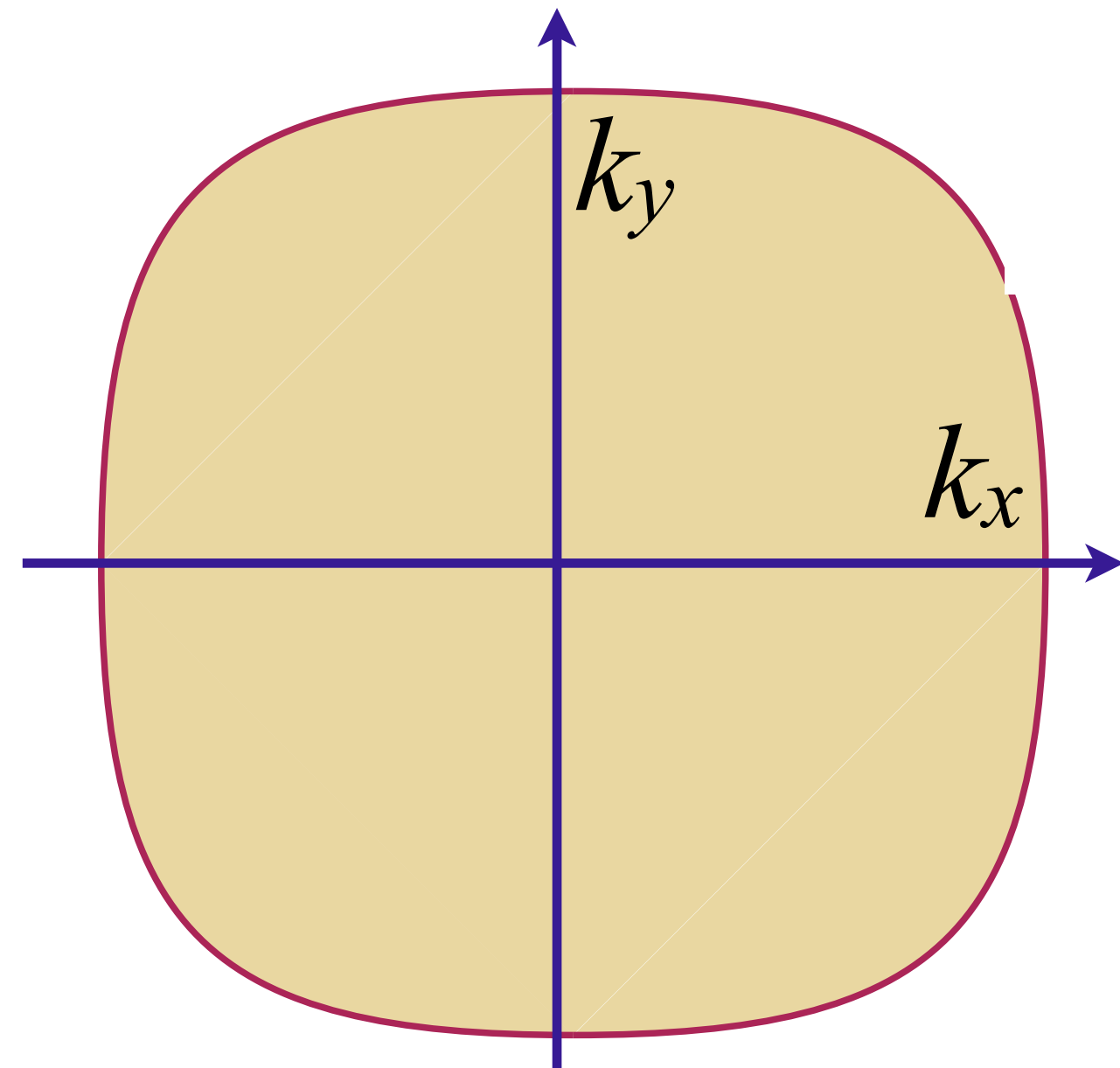
Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')}$ = $v^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random mass $\delta s(\mathbf{r})$ with $\overline{\delta s(\mathbf{r})} = 0$, $\overline{\delta s(\mathbf{r})\delta s(\mathbf{r}')}$ = $\delta s^2 \delta(\mathbf{r} - \mathbf{r}')$

$v(\mathbf{r})$ leads to elastic scattering of c_σ and ‘Altshuler-Aronov’ corrections; localization of c_σ only at long length scales, not relevant for experiments

Kondo lattice + critical boson with potential and interaction disorder

$$c_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) c_{\mathbf{k}\sigma} + f_{\mathbf{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon_1(\mathbf{k}) \right) f_{\mathbf{k}\sigma}$$



$$+ s [\Phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] c_\sigma^\dagger(\mathbf{r}) f_\sigma(\mathbf{r}) \Phi(\mathbf{r}) + \text{H.c.}$$

$$+ K [\nabla_{\mathbf{r}} \Phi(\mathbf{r})]^2 + u [\Phi(\mathbf{r})]^4 + v(\mathbf{r}) c_\sigma^\dagger(\mathbf{r}) c_\sigma(\mathbf{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

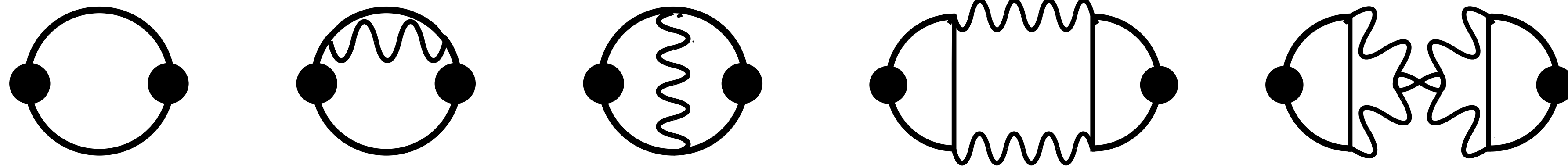
Analyze $2d$ -Yukawa-SYK model in a self-averaging manner as in the SYK model.
Should be applicable as long as eigenmodes of $\Phi(\mathbf{r})$ are extended.

2d-YSYK model: Fermi surface + critical boson with interaction disorder

$$\Sigma = \text{Diagram 1} \quad \Pi = \text{Diagram 2}$$

Diagram 1: A fermion self-energy diagram Σ consisting of a solid horizontal line with two white circular vertices. A wavy line labeled D connects the two vertices, with a dotted arc above it. A label G is placed below the right vertex.

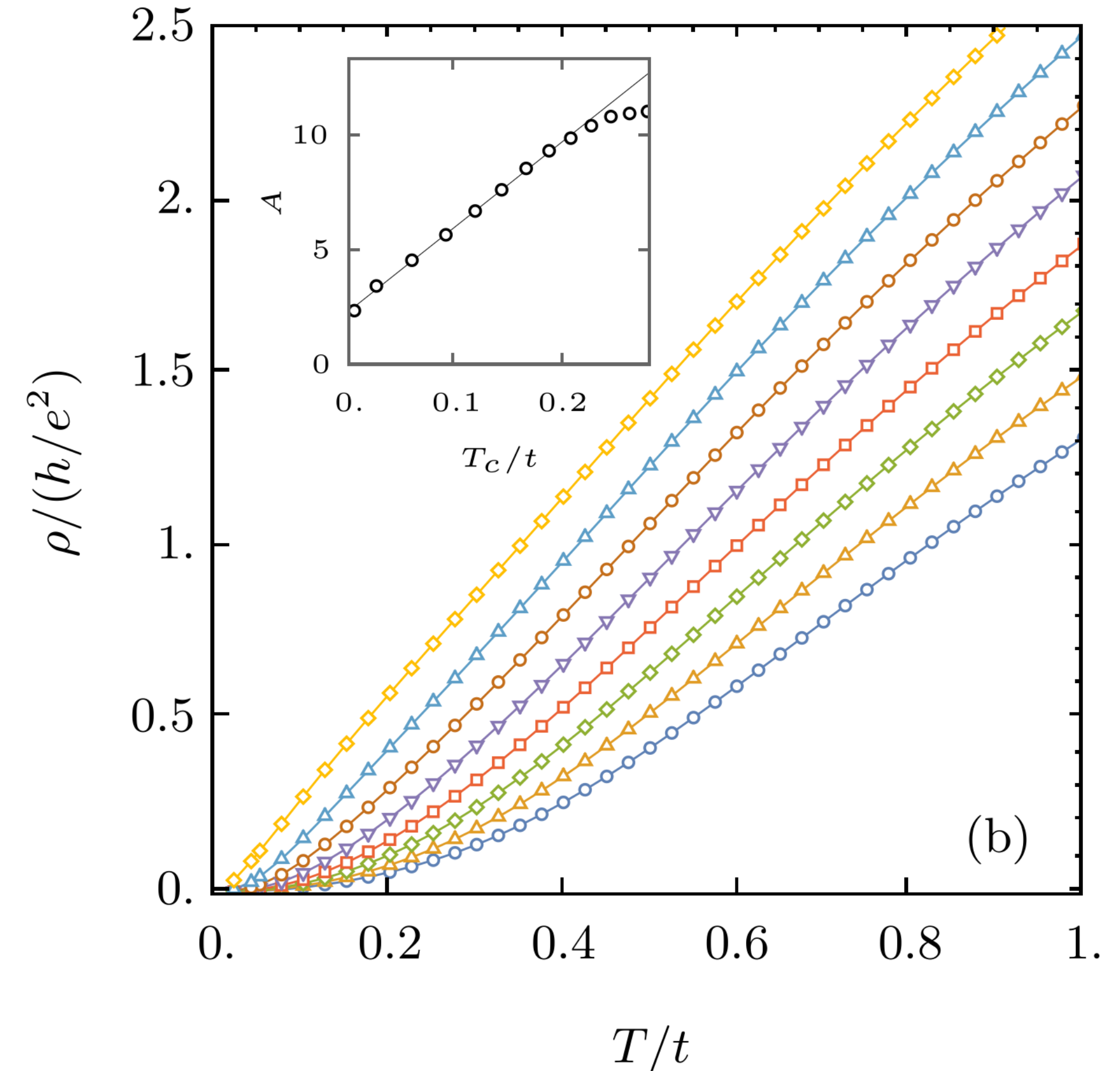
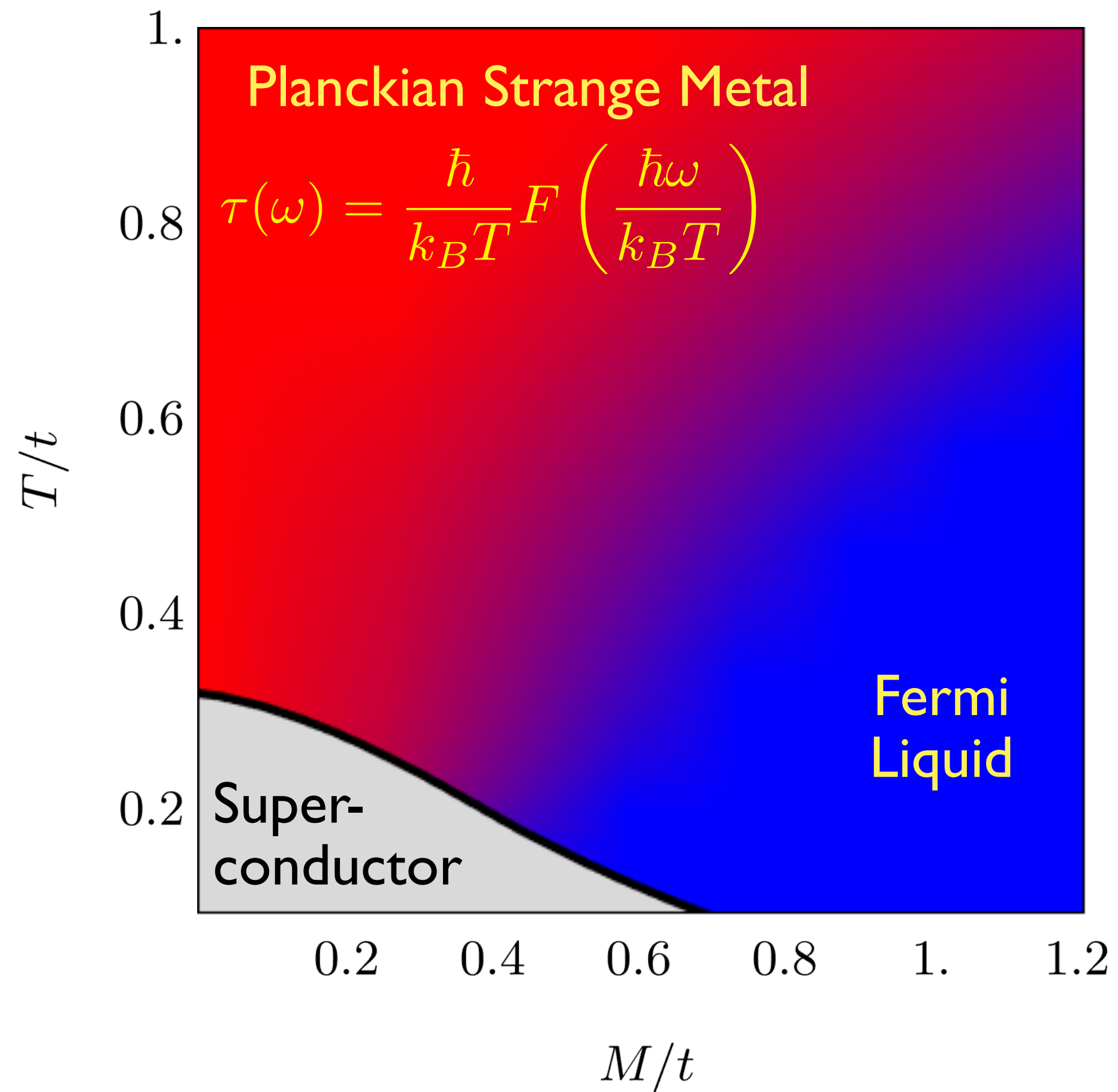
Diagram 2: A polarization diagram Π consisting of a solid circle with two white circular vertices on its left and right sides. Two wavy lines labeled G connect the vertices, one above and one below the circle. A dotted arc is above the circle.



Residual resistivity is determined by v^2
 Linear-in- T resistivity determined by g'^2
 Transport insensitive to g
 Marginal Fermi liquid self energy $\Sigma \sim \omega \ln \omega$
 $T \ln(1/T)$ specific heat

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentini, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

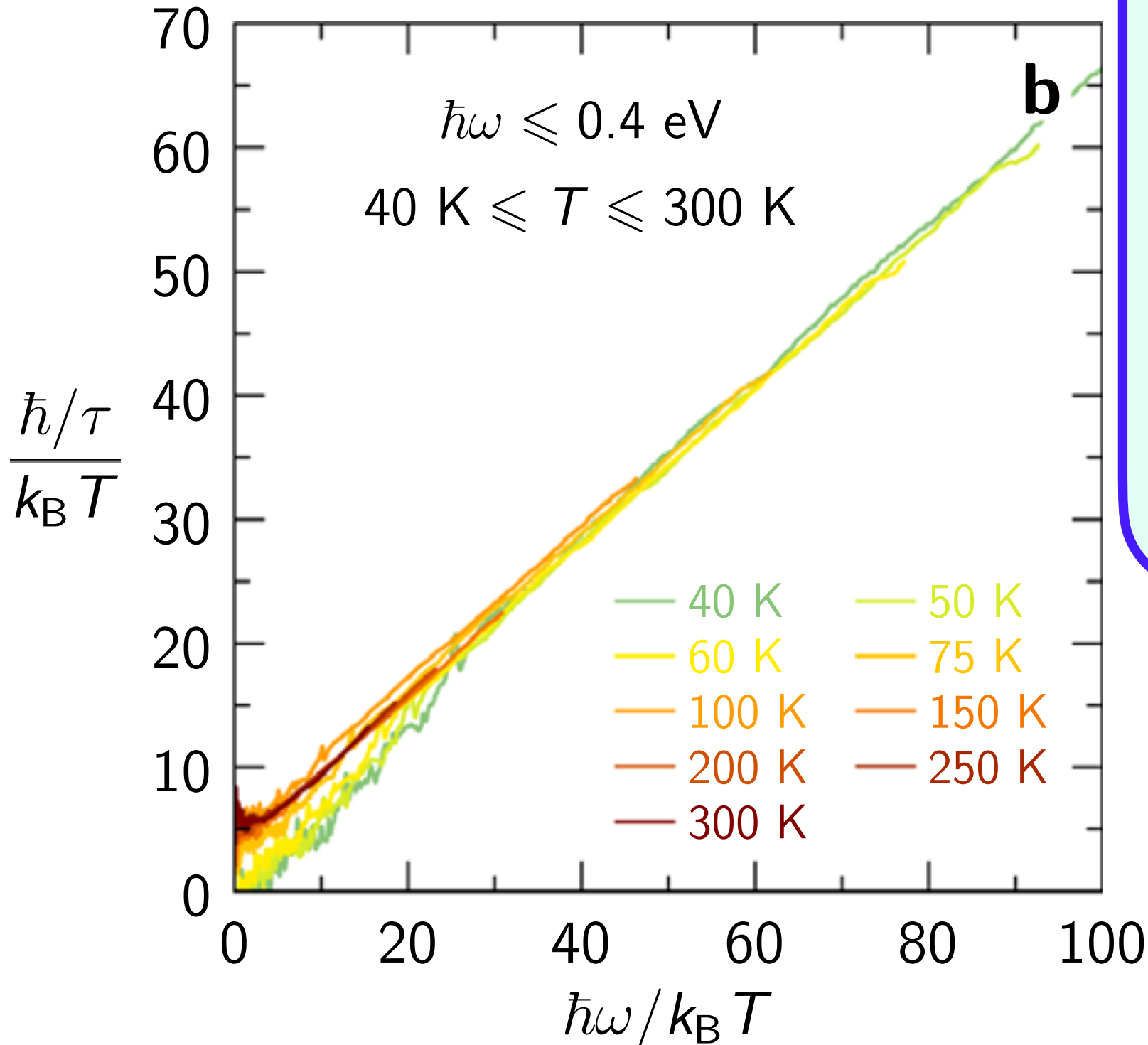
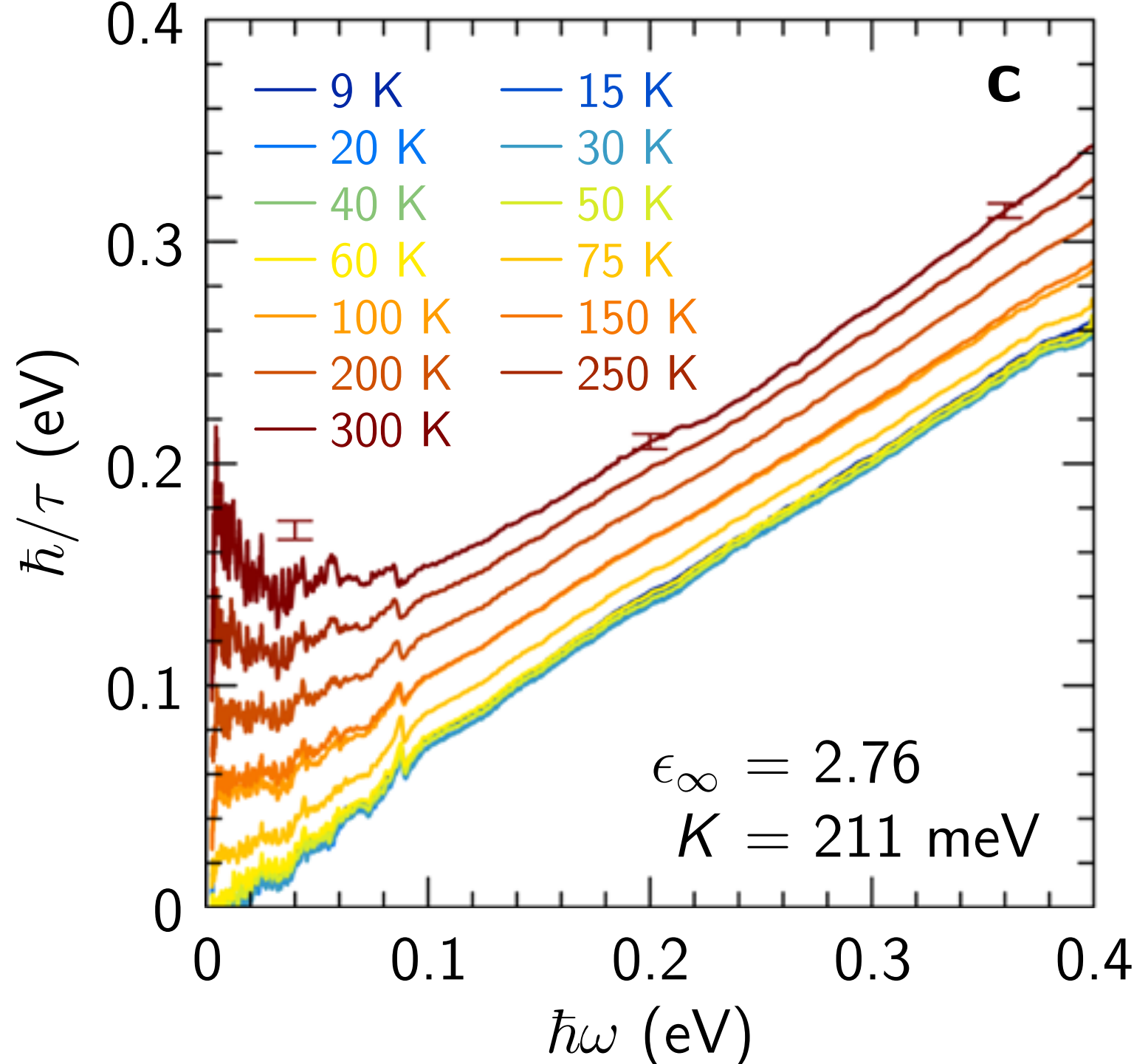


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$



Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

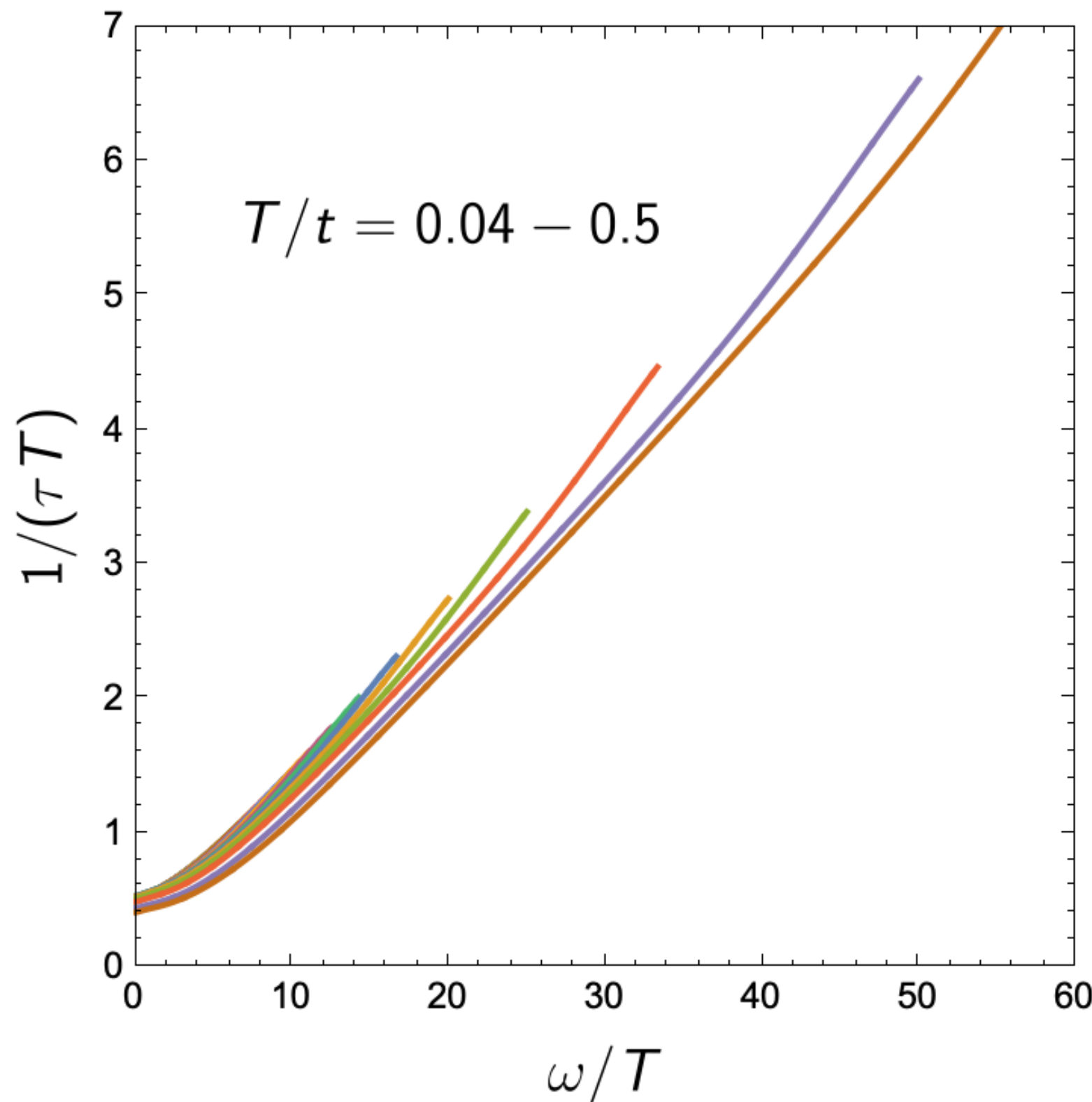
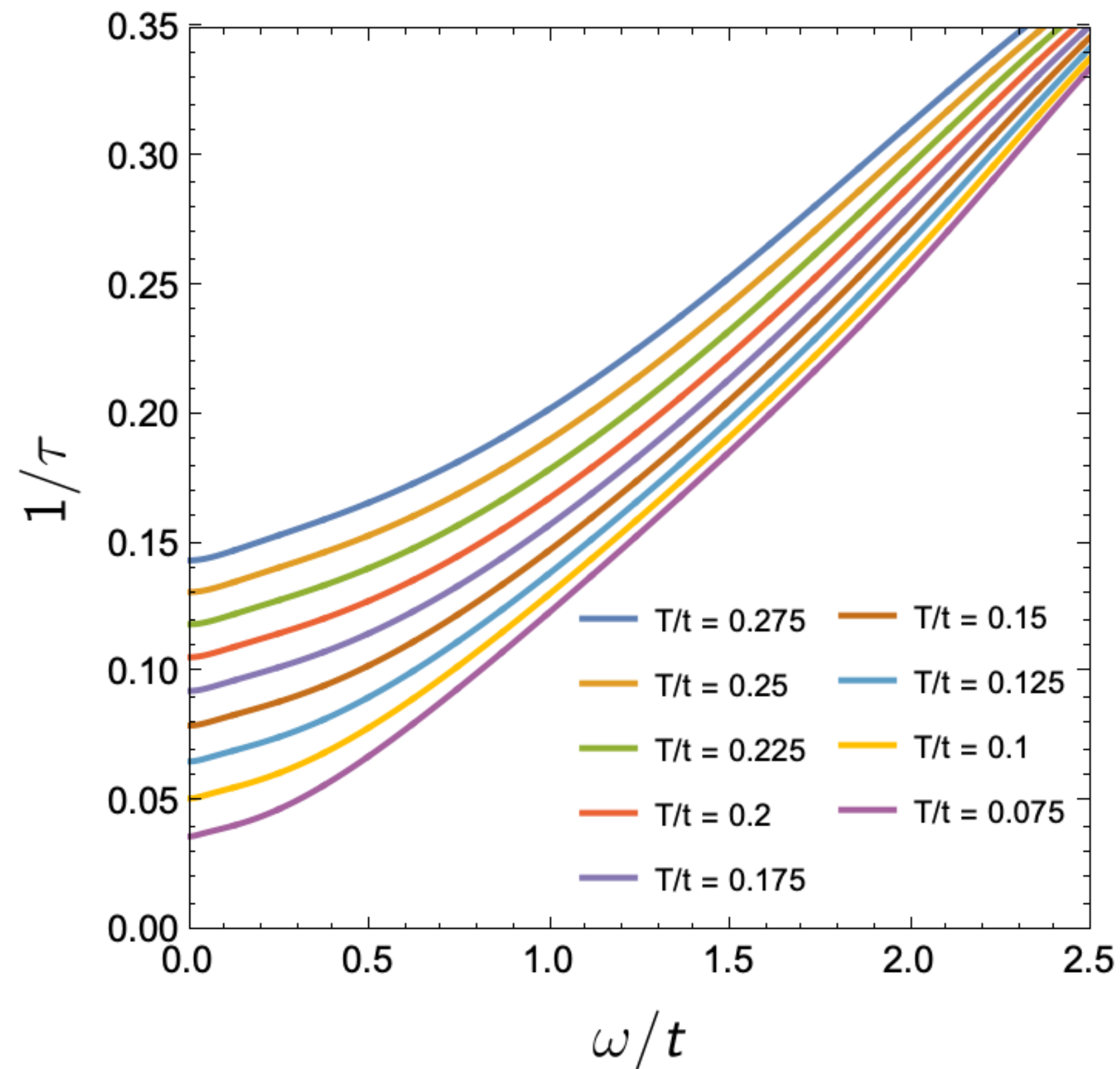
$$S(T \rightarrow 0) \sim T \ln(1/T).$$

$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$
 $p = 0.24$
 $T_c = 19$ K

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentini, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

$$\sigma(\omega) = i \frac{e^2 K / (\hbar d_c)}{\hbar \omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar \omega}{k_B T}\right)$$

and entropy



$S(T \rightarrow 0) \sim T \ln(1/T)$
in 2d-YSYK model
(unlike zero temperature entropy in SYK model).

Seebeck Coefficient in a Cuprate Superconductor: Particle-Hole Asymmetry in the Strange Metal Phase and Fermi Surface Transformation in the Pseudogap Phase

A. Gourgout,^{1,*} G. Grissonnanche,^{1,2,3,*,\dagger} F. Laliberté,¹ A. Ataei,¹ L. Chen¹ ,¹ S. Verret,¹ J.-S. Zhou⁴ ,⁴ J. Mravlje,⁵ A. Georges,^{6,7,8,9} N. Doiron-Leyraud,¹ and Louis Taillefer^{1,10,\ddagger}

PHYSICAL REVIEW X **12**, 011037 (2022)

Skewed non-Fermi liquids and the Seebeck effect

Antoine Georges ^{1,2,3,4} and Jernej Mravlje ⁵

PRR **3**, 043132 (2021)

The particle-hole asymmetry in the Φ propagator
(for the FL*-FL transition)

$$\sim 1/(-i\omega + q^2 + \gamma|\omega| + \alpha)$$

leads to a **skewed marginal Fermi liquid**.

P. Lunts,
A.A. Patel,
and S.S.,

arXiv:2412.15330

The ϕ propagator (for the SDW-FL transition)
does not have the $-i\omega$ term, and so is *not* skewed.



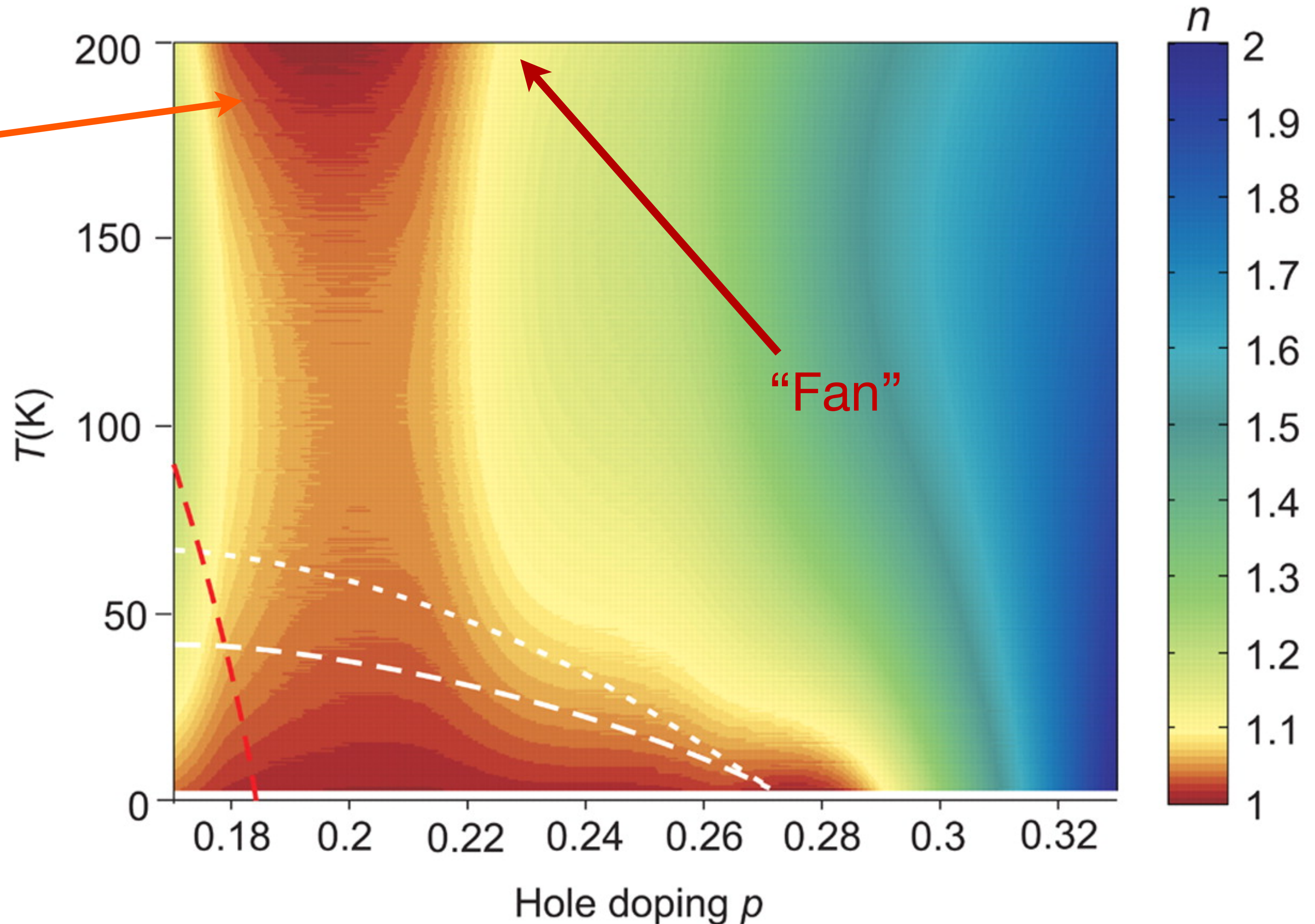
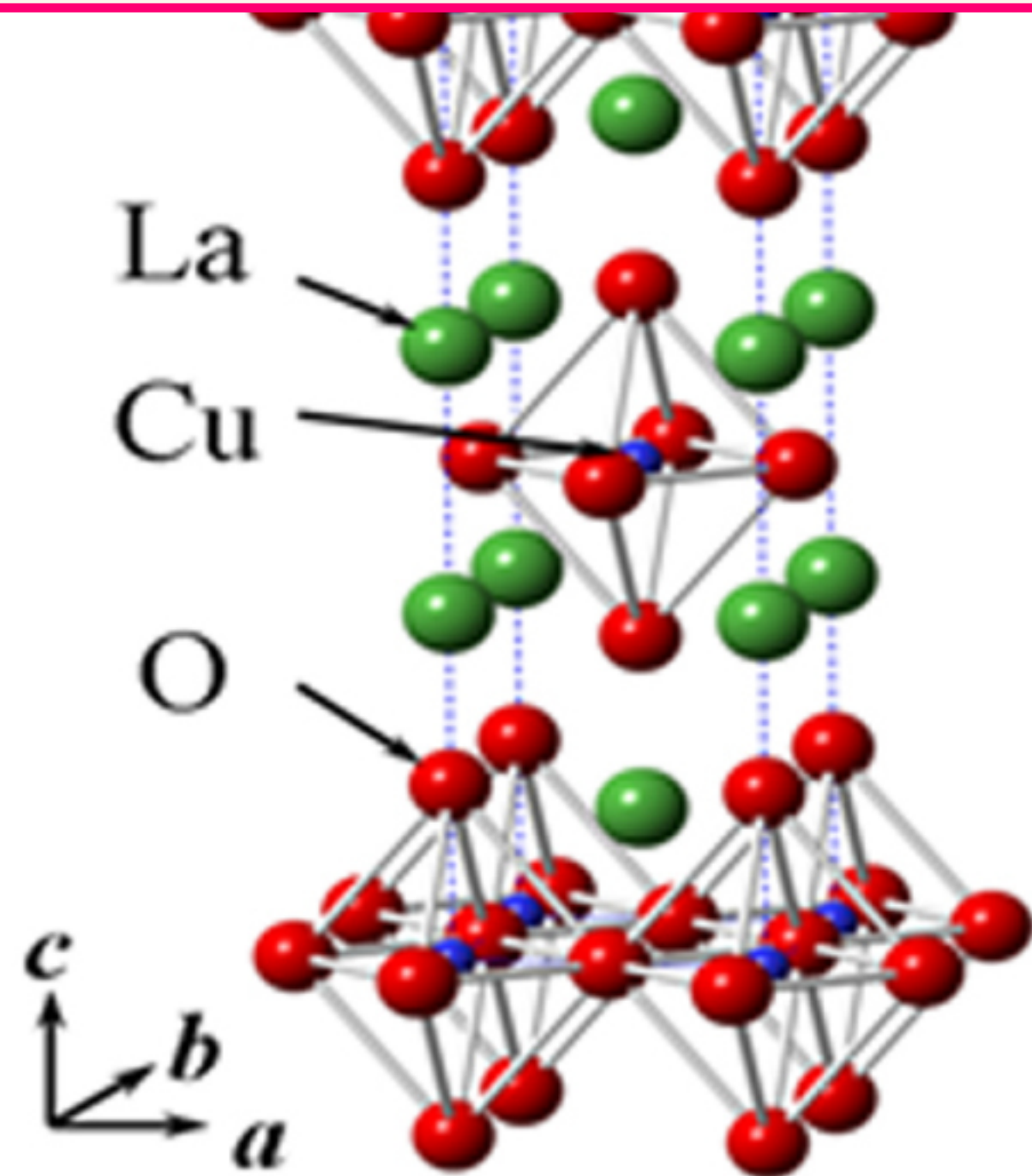
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SCIENCE VOL 323 603 2009

2dYSYK theory of FL-FL* QPT provides a theory of the “fan”

Extended fermions and Higgs bosons



Anomalous Criticality in the Electrical Resistivity of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

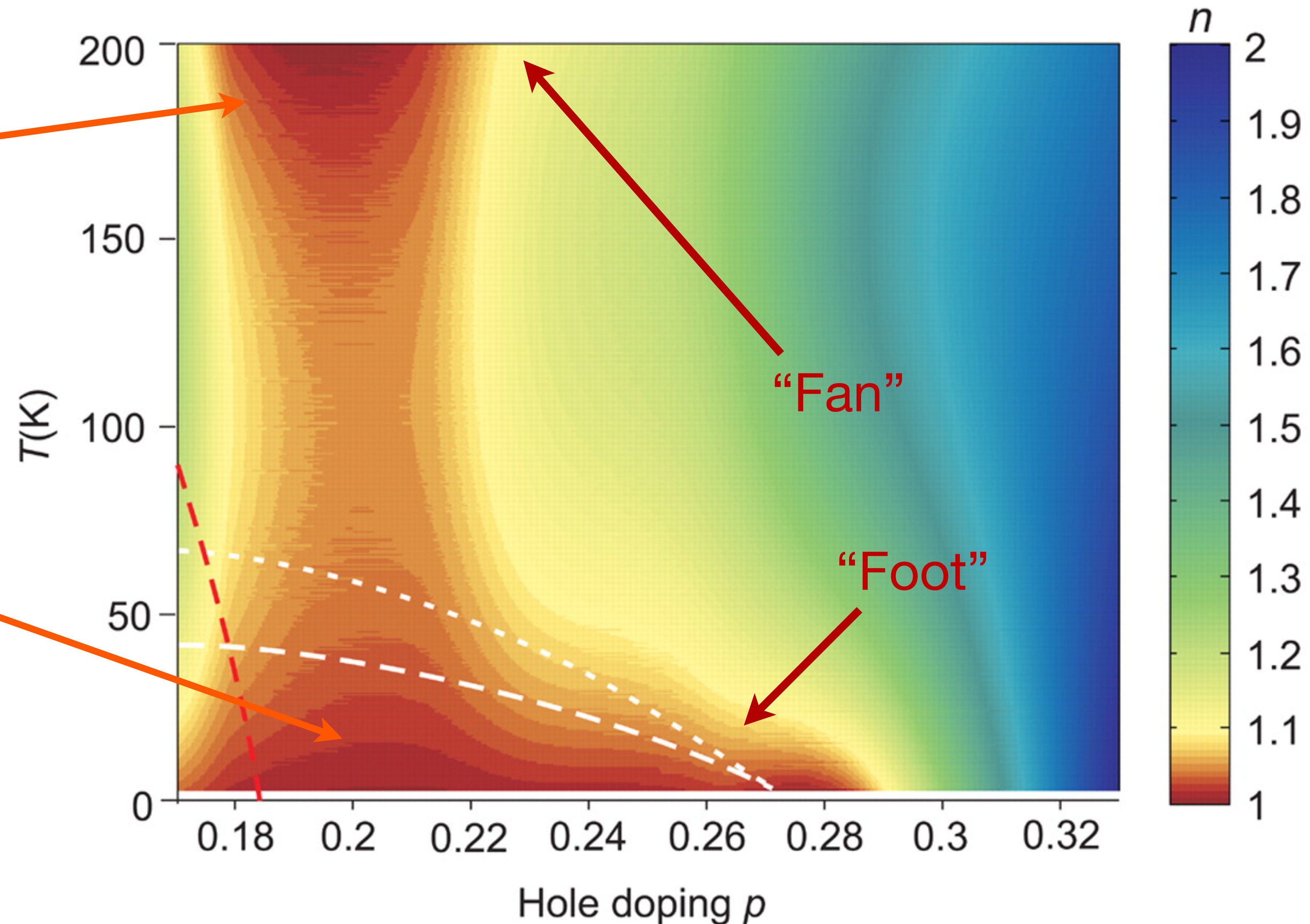
R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,^{4*} H. Takagi,⁴ Cyril Proust,² N. E. Hussey^{1†}

SCIENCE VOL 323 603 2009

FL-SDW QPT with Harris disorder provides a theory of the “foot”

Extended fermions and Higgs bosons

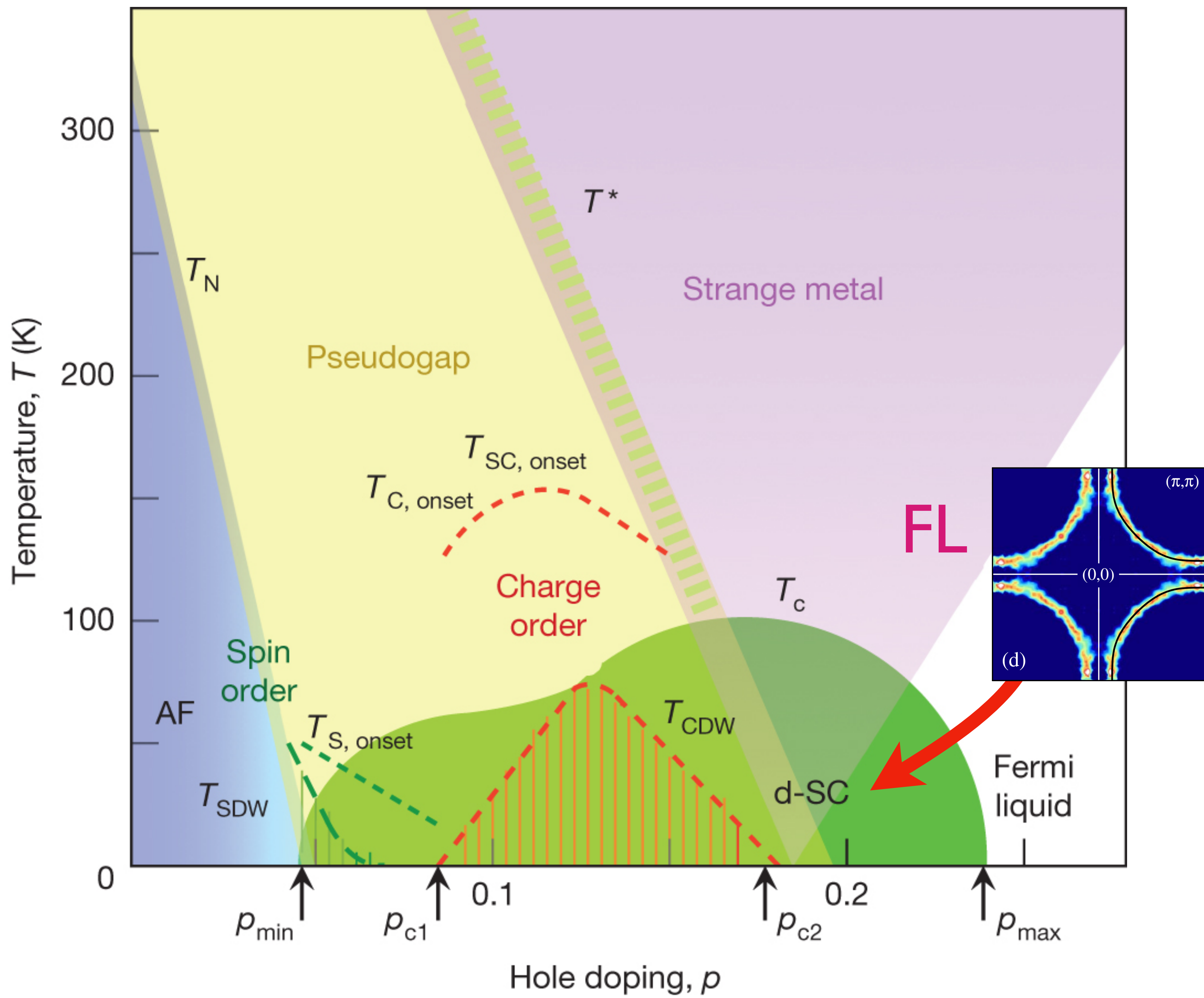
Localized overdamped SDW bosons, but extended fermions



A. FL-SDW QPT

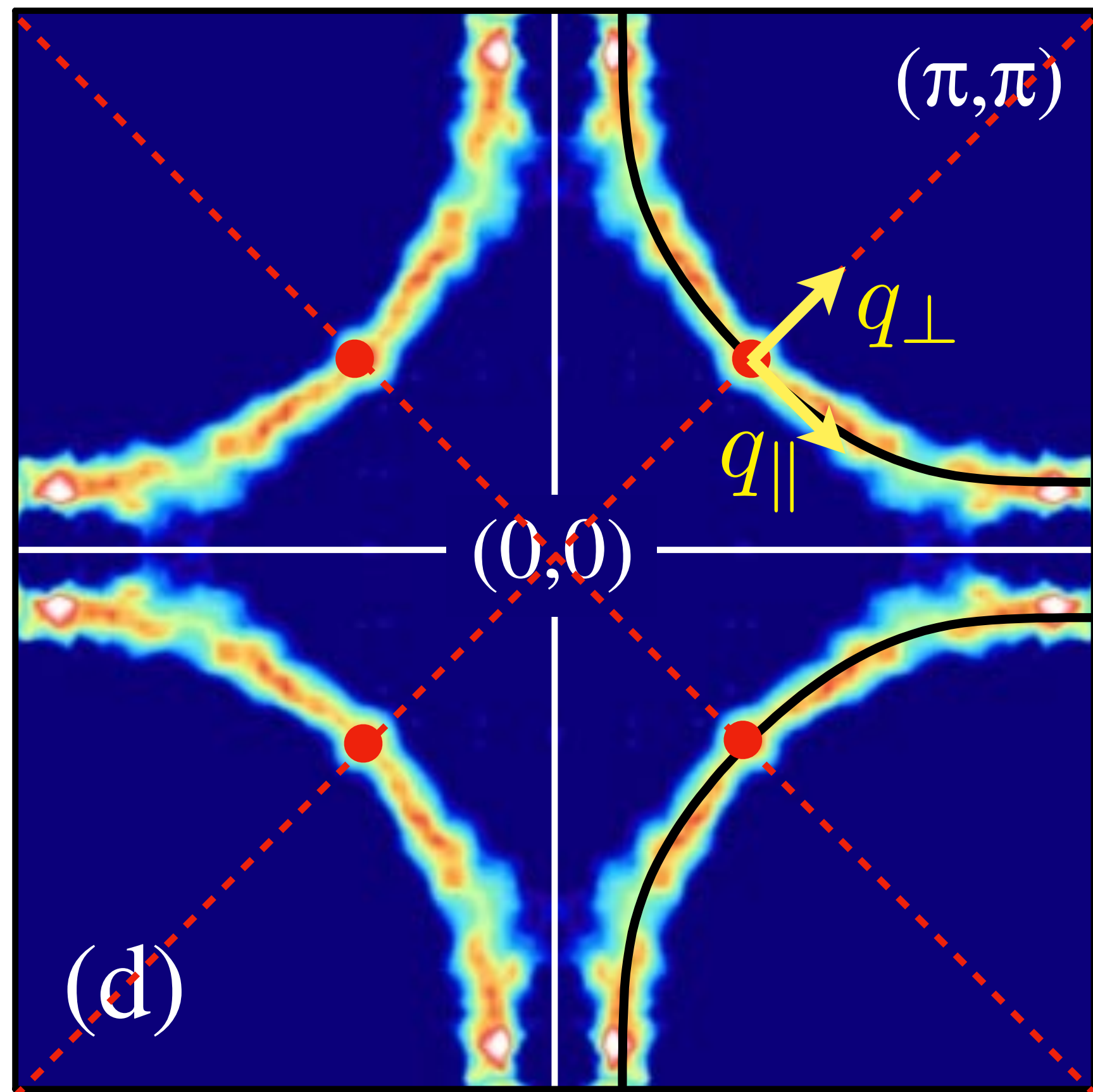
B. FL-FL* QPT

C. Confinement crossover to dSC



BCS-type theory of *d*-wave superconductivity (and charge order) induced by antiferromagnetic spin fluctuations.

FL → dSC



BCS/Bogoliubov quasiparticles
in a *d*-wave superconductor

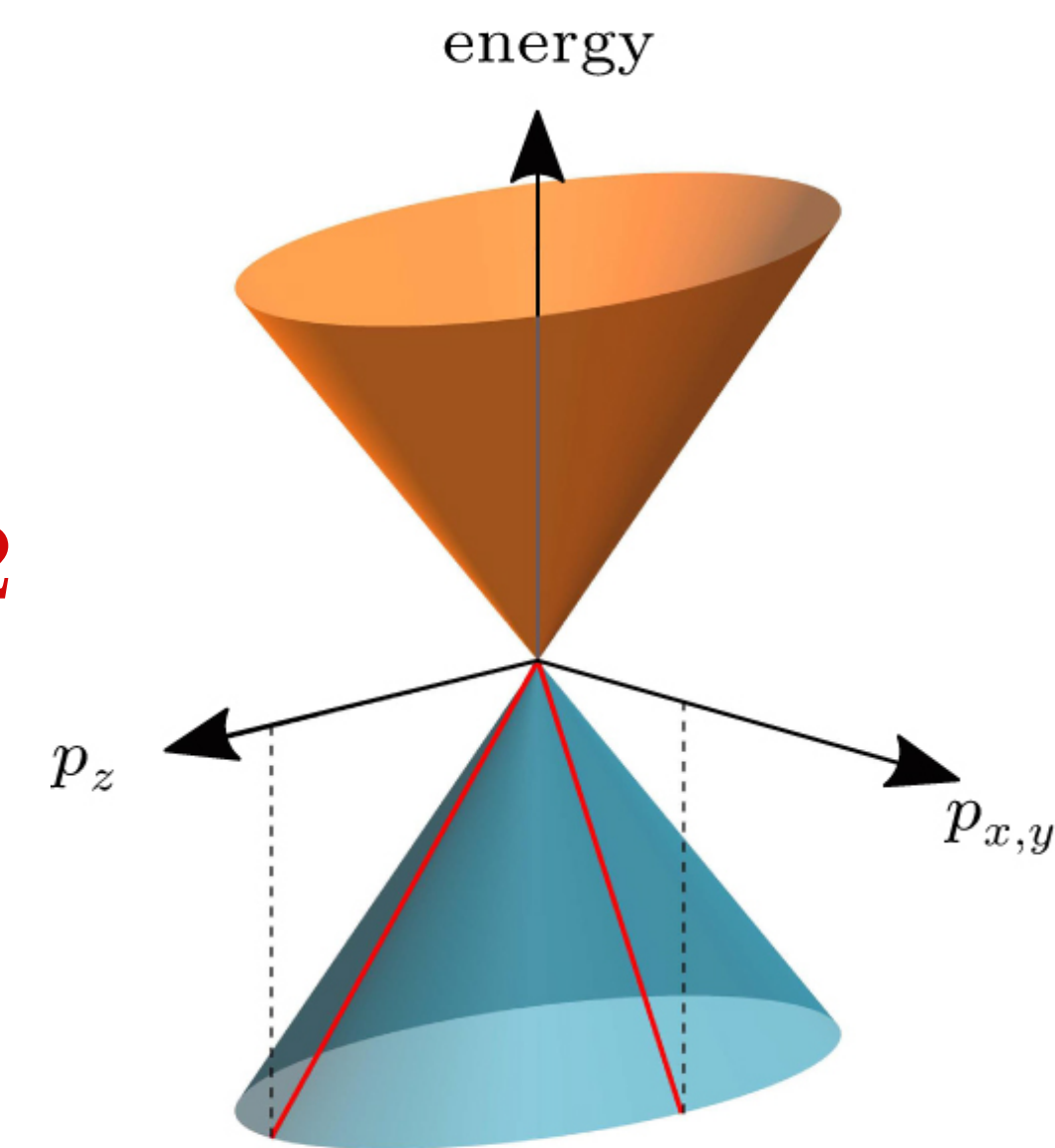
$$E_{\mathbf{k}} = \left(\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2 \right)^{1/2}$$

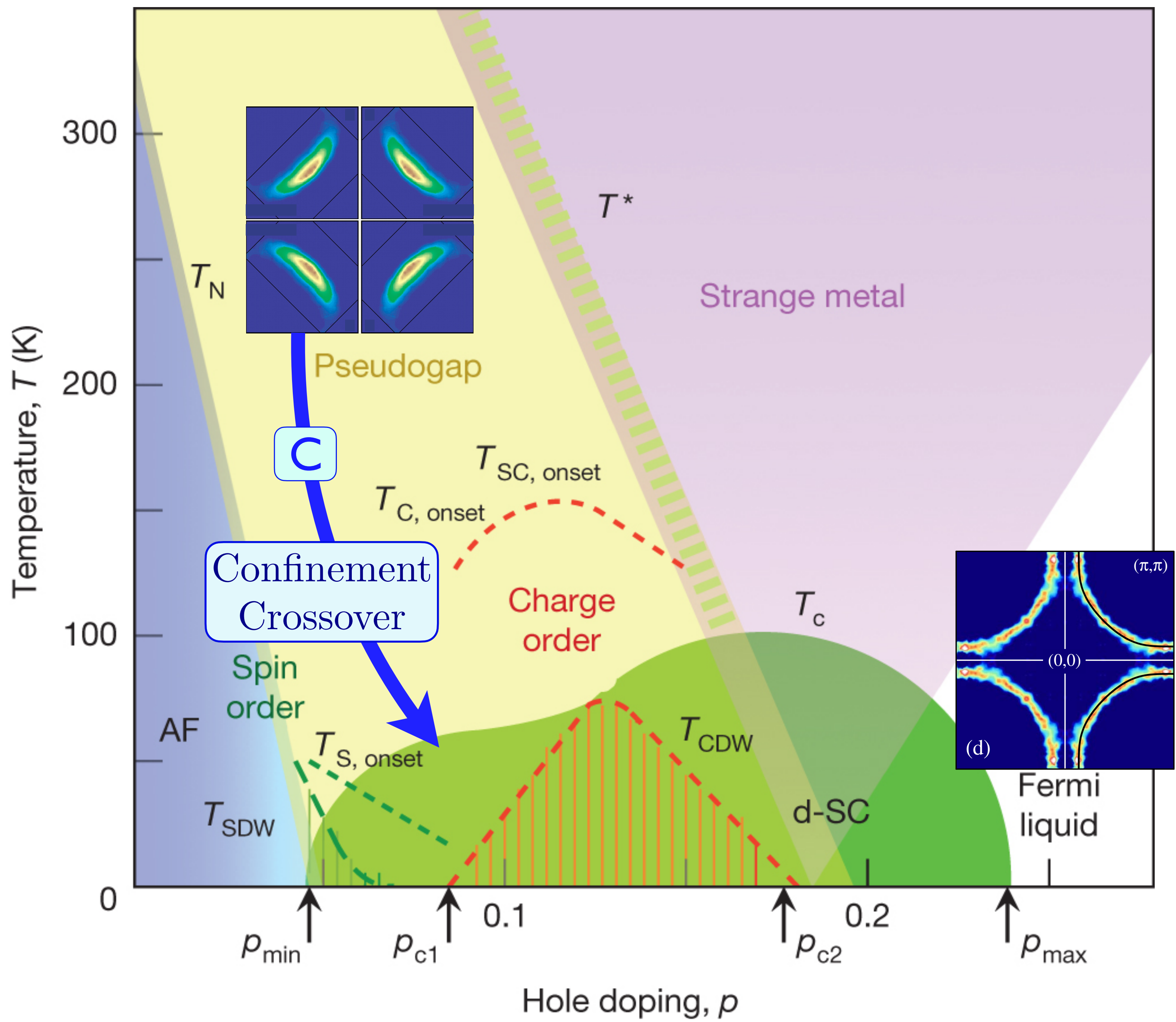
$$\Delta_{\mathbf{k}} = \Delta_0 (\cos k_x - \cos k_y)$$

4 nodal points where

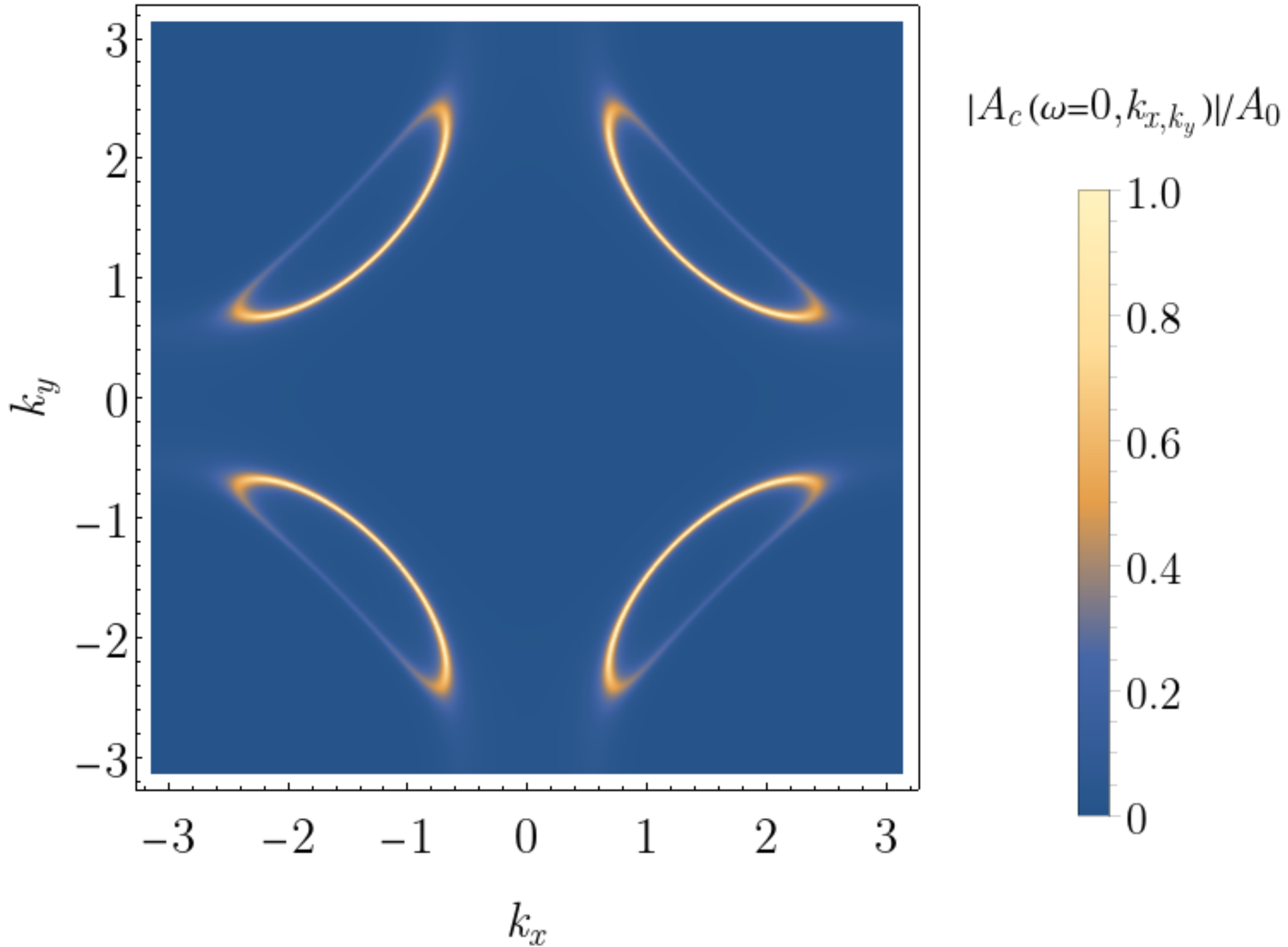
$$E_{\mathbf{k}_0 + \mathbf{q}} = \left(v_F^2 q_{\perp}^2 + v_{\Delta}^2 q_{\parallel}^2 \right)^{1/2}$$

with $v_F \gg v_{\Delta}$.



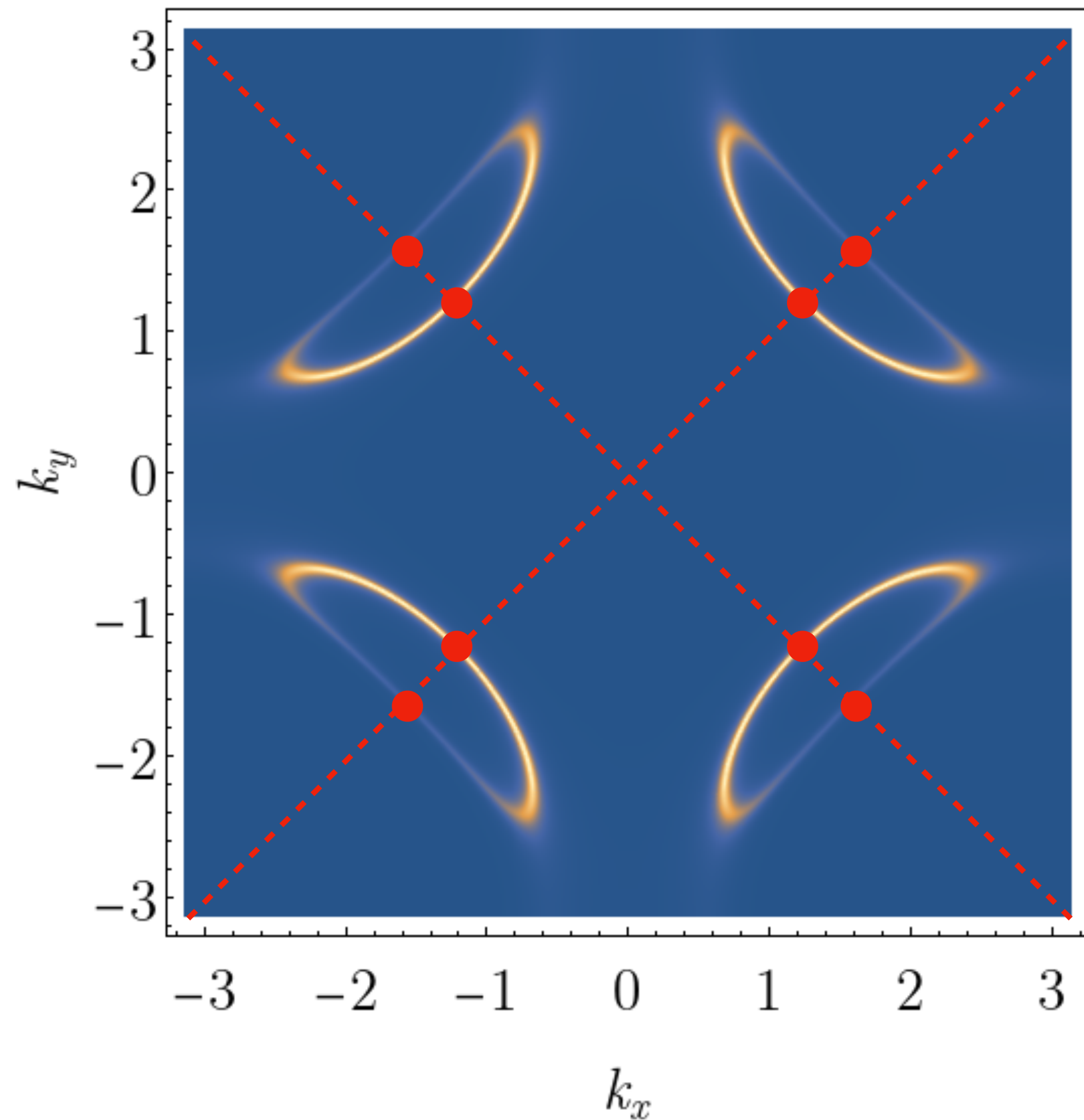


FL*

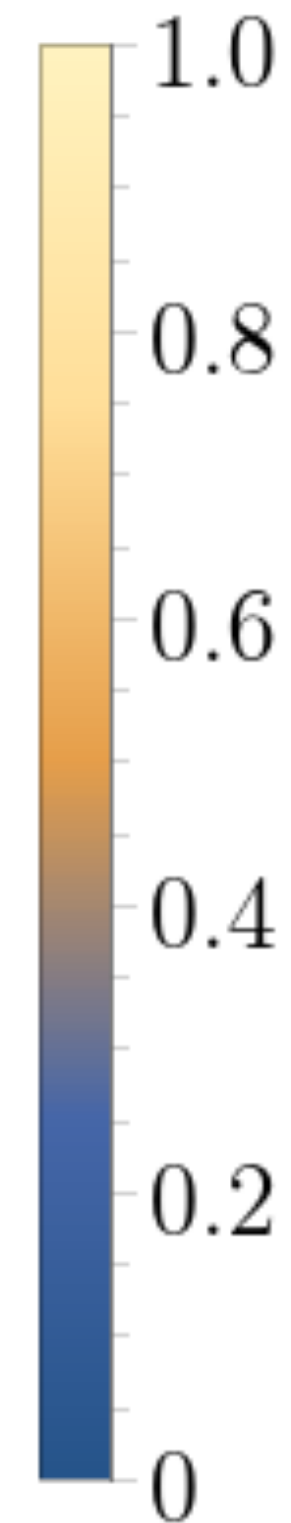


E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang,
D. K. Morr, and S. S., PRB **105**, 075146 (2022)

FL* → dSC*



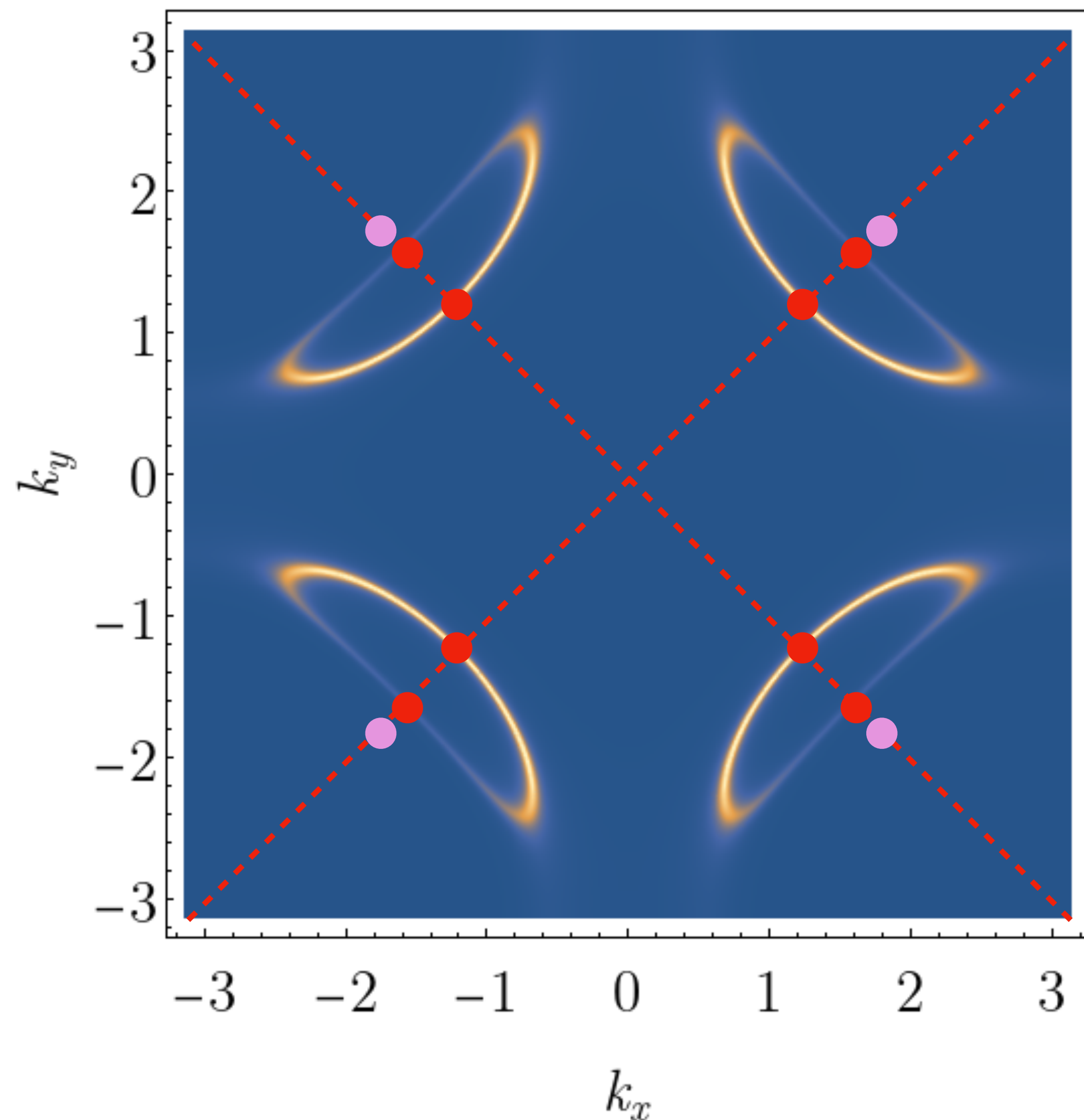
$$|A_c(\omega=0, k_x, k_y)|/A_0$$



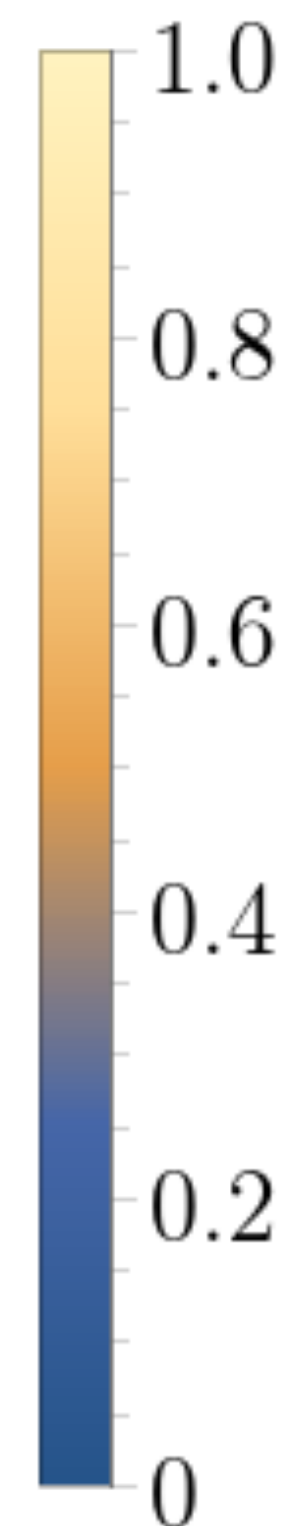
$$E_{\mathbf{k}} = (\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2)^{1/2}$$
$$\Delta_{\mathbf{k}} = \Delta_0 (\cos k_x - \cos k_y)$$

Adding *d*-wave pairing
to the hole pockets
leads to 8 nodal points???

$FL^* \rightarrow dSC^*$



$|A_c(\omega=0, k_x, k_y)|/A_0$



8 nodal points of
Bogoliubov quasiparticles
from the Fermi pockets

and

4 nodal points of
fermionic Dirac spinons from
the π -flux spin liquid

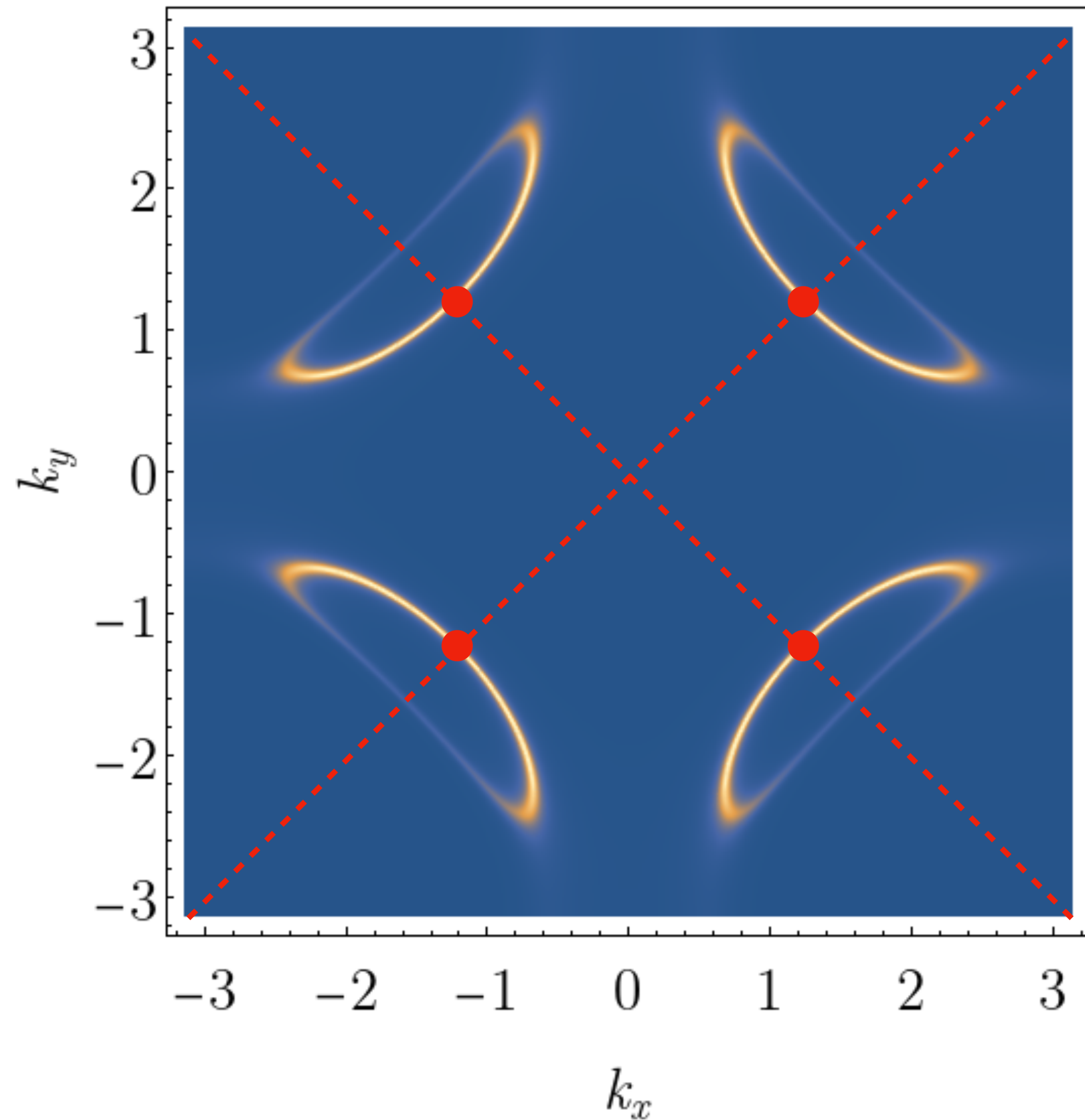
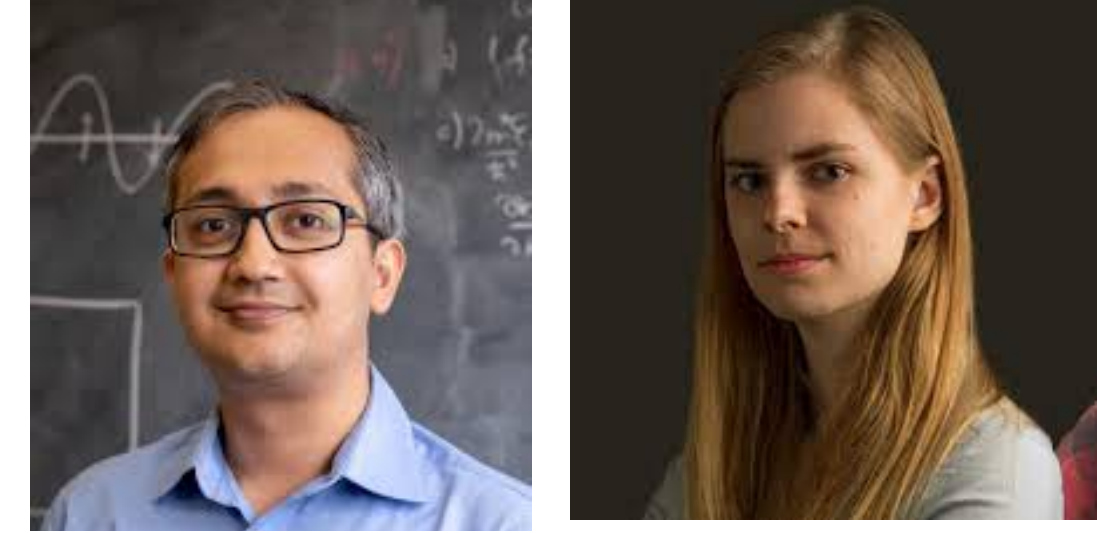
A spin liquid is required
to be present
in the background for FL^* :
realized by S_2 spins.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

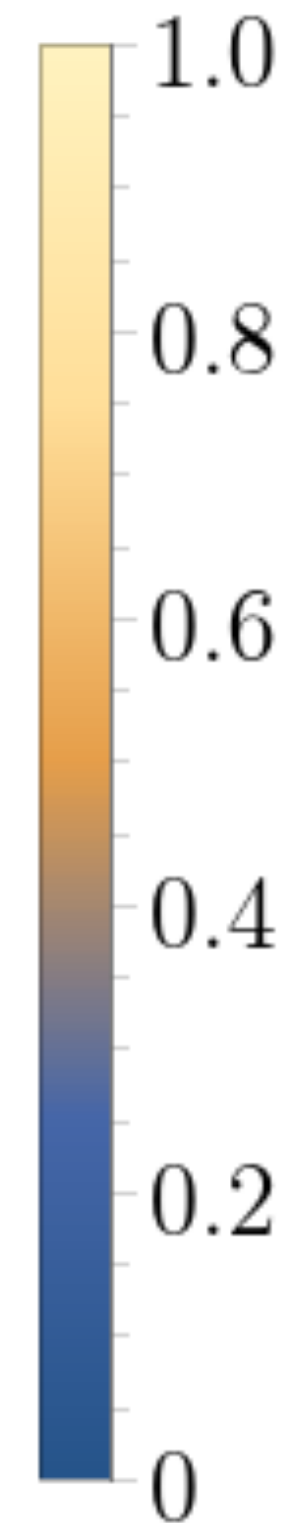


FL* → dSC

Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S. Sachdev,
npj Quantum Materials **9**, 4 (2024)



$$|A_c(\omega=0, k_x, k_y)|/A_0$$



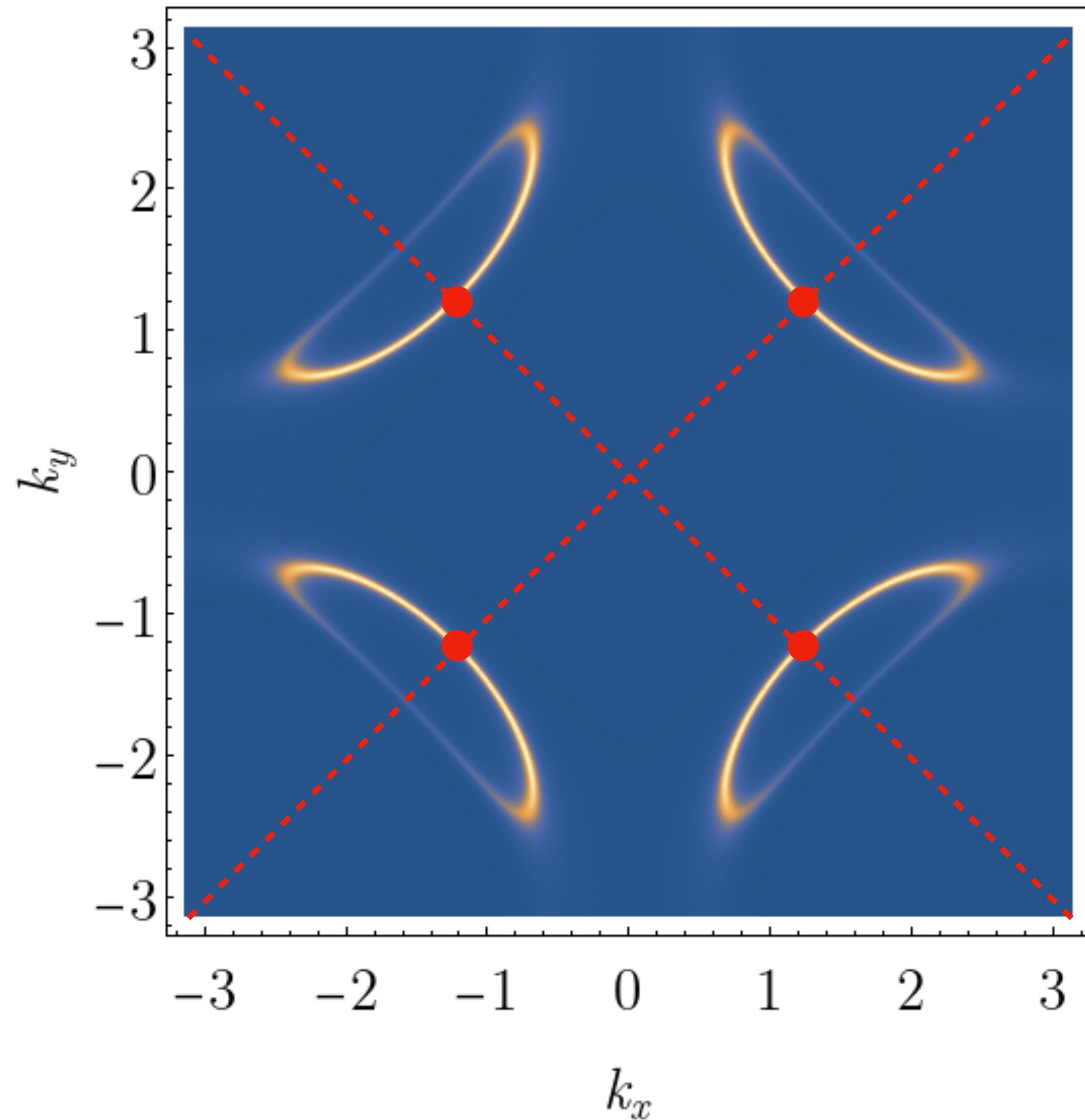
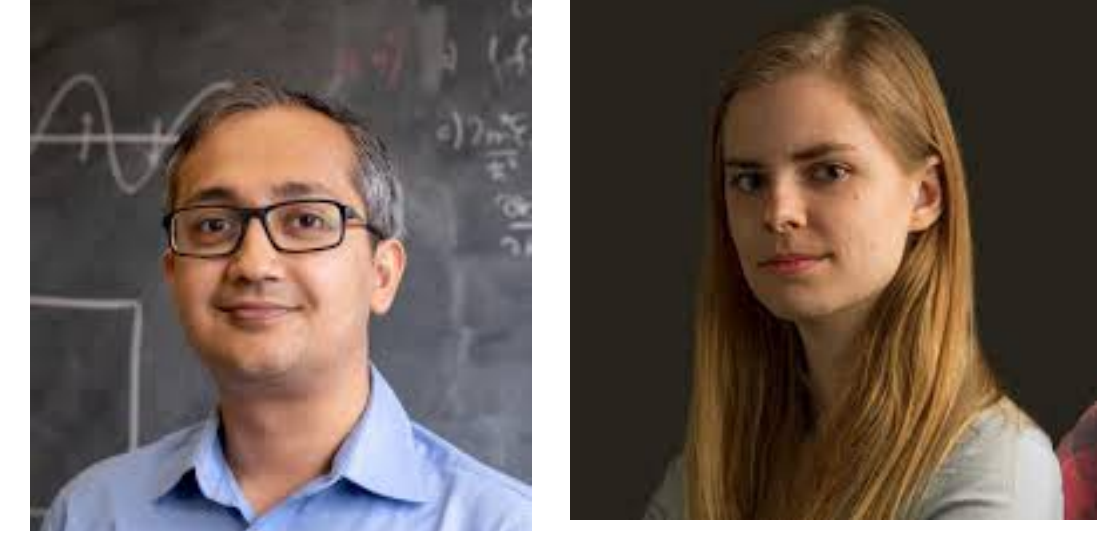
d-wave superconductor obtained
by condensing charge- e , $SU(2)$
fundamental boson B .

The B Higgs condensate allows
spinons and Bogoliubov
quasiparticles
to hybridize.

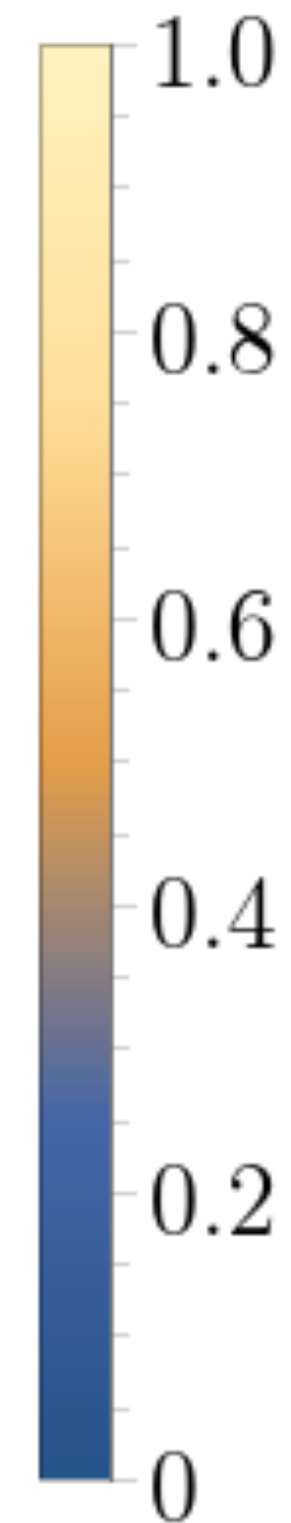
8 nodal points annihilate each
other, leaving 4 nodal points
with anisotropic velocities, just
as in a BCS *d*-wave state.

FL* → dSC

Shubhayu Chatterjee and S. Sachdev,
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$$|A_c(\omega=0, k_x, k_y)|/A_0$$



The spinons do *not* become the Bogoliubov quasiparticles, they *annihilate* the unwanted Bogoliubov quasiparticles. This leads to a *d*-wave superconductor with 4 nodal Bogoliubov quasiparticles, with $v_F \gg v_\Delta$, consistent with observations.

Onset of superconductivity in a SU(2) gauge theory with a fundamental Higgs field B

Complex bosons B_{ia} , $a = 1, 2$ is a SU(2) gauge index, hopping in π flux. U_{ij} are 2×2 SU(2) matrices on the links. Classical free energy in 2+0 dimensions $\mathcal{F}_2 + \mathcal{F}_4$:

$$\mathcal{F}_2 = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \kappa \sum_{\square} \left\{ 1 - \frac{1}{2} \text{ReTr} \prod_{ij \in \square} U_{ij} \right\}$$

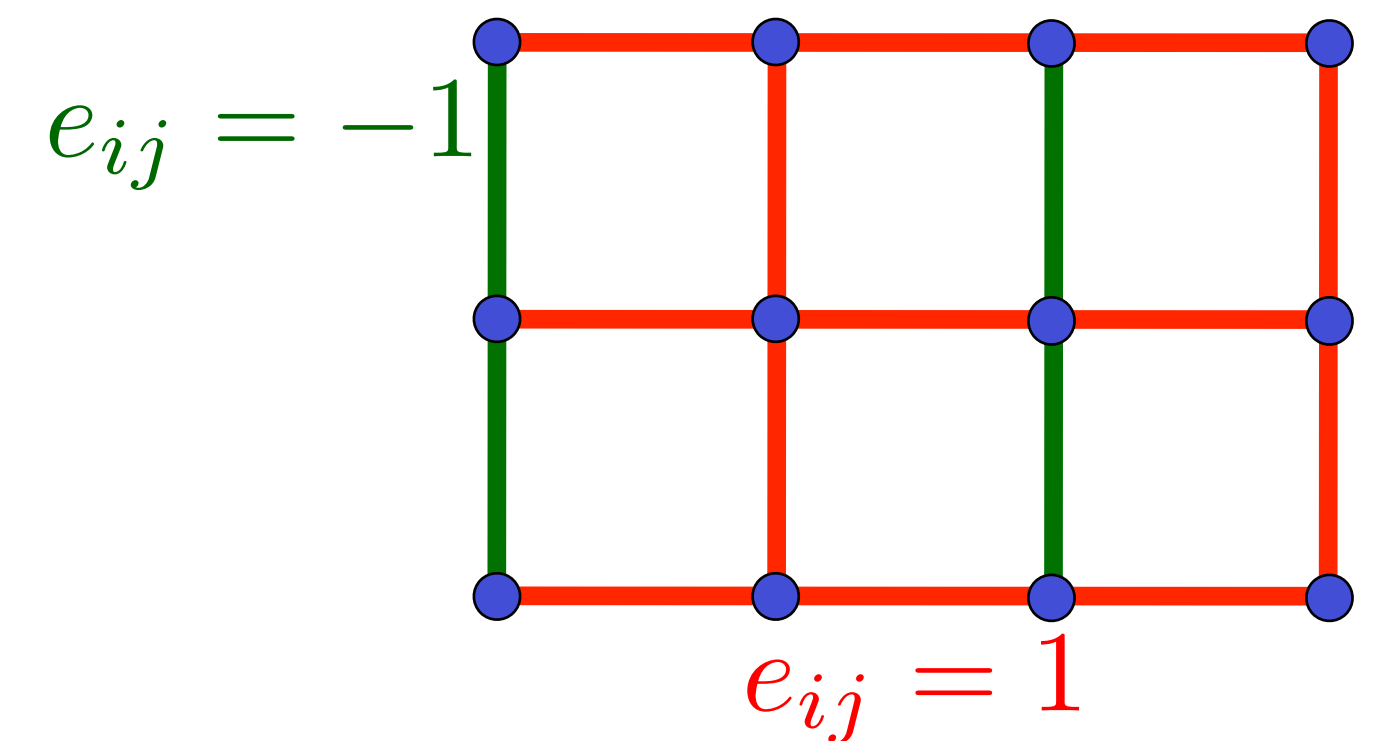
$$\mathcal{F}_4 = \frac{u}{2} \sum_i \rho_i^2 + V_1 \sum_i \rho_i (\rho_{i+\hat{x}} + \rho_{i+\hat{y}}) + g \sum_{\langle ij \rangle} |\Delta_{ij}|^2 + J_1 \sum_{\langle ij \rangle} Q_{ij}^2 + K_1 \sum_{\langle ij \rangle} J_{ij}^2$$

site charge density: $\langle c_{i\alpha}^\dagger c_{i\alpha} \rangle \sim \rho_i = B_i^\dagger B_i$

bond density: $\langle c_{i\alpha}^\dagger c_{j\alpha} + c_{j\alpha}^\dagger c_{i\alpha} \rangle \sim Q_{ij} = Q_{ji} = \text{Im} \left(B_i^\dagger e_{ij} U_{ij} B_j \right)$

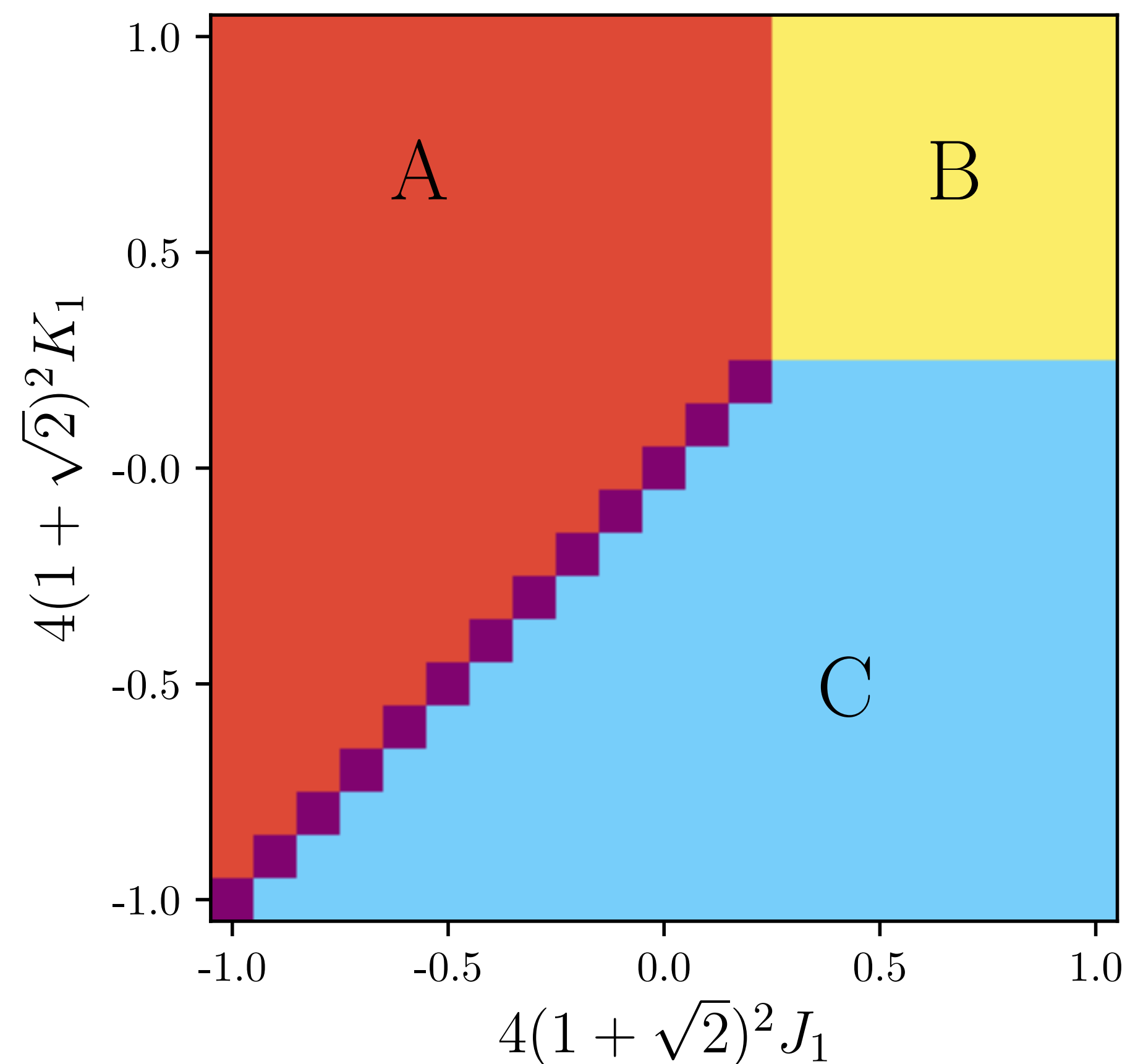
bond current: $i \langle c_{i\alpha}^\dagger c_{j\alpha} - c_{j\alpha}^\dagger c_{i\alpha} \rangle \sim J_{ij} = -J_{ji} = \text{Re} \left(B_i^\dagger e_{ij} U_{ij} B_j \right)$

Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{ij} = \Delta_{ji} = \varepsilon_{ab} B_{ai} e_{ij} U_{ij} B_{bj}$.

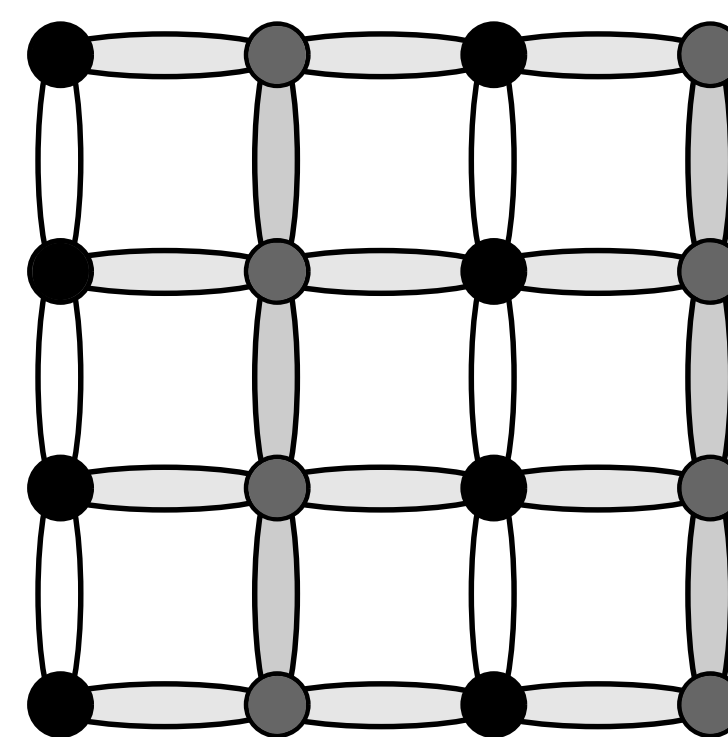


Onset of superconductivity in a $SU(2)$ gauge theory with a fundamental Higgs field B

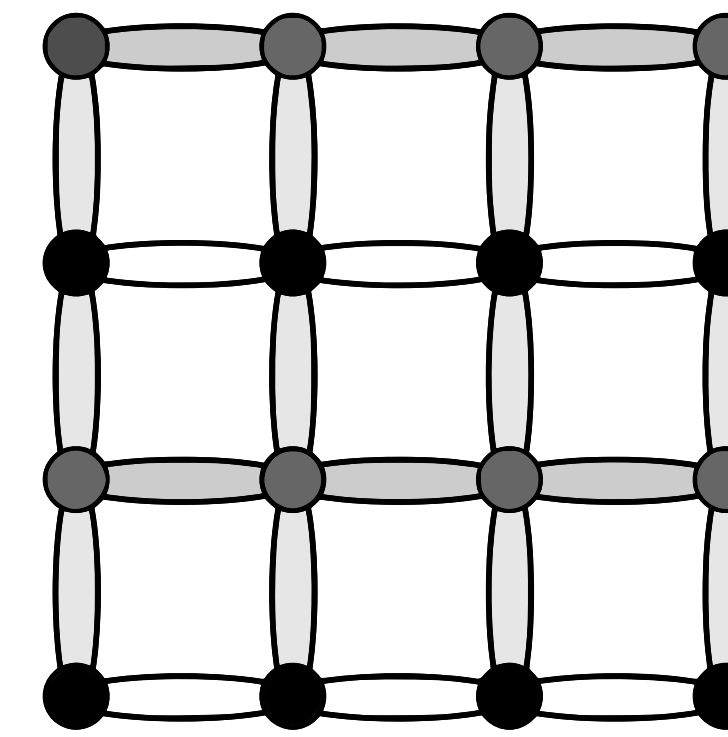
$T=0$ phase diagram



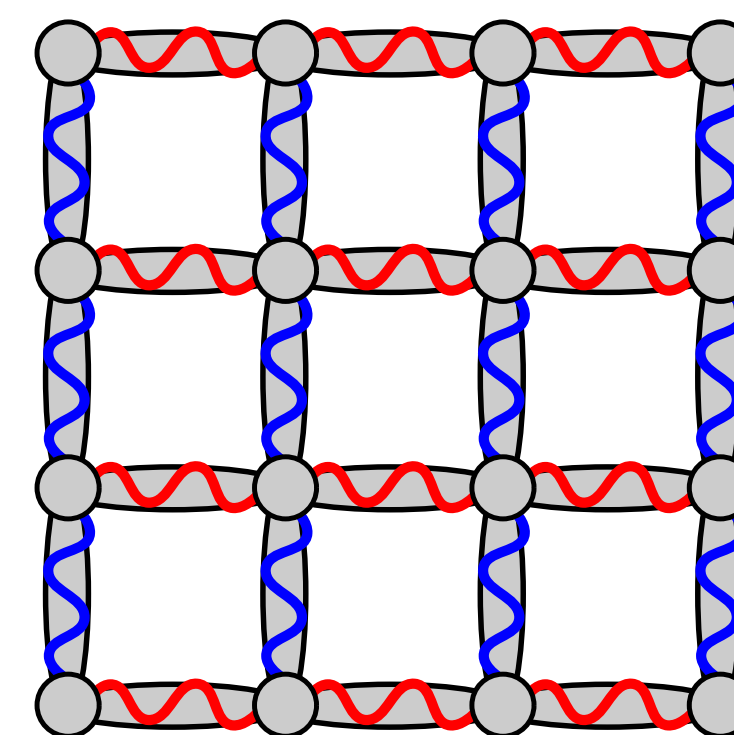
Phase A
 $(\pi, 0)$ stripe



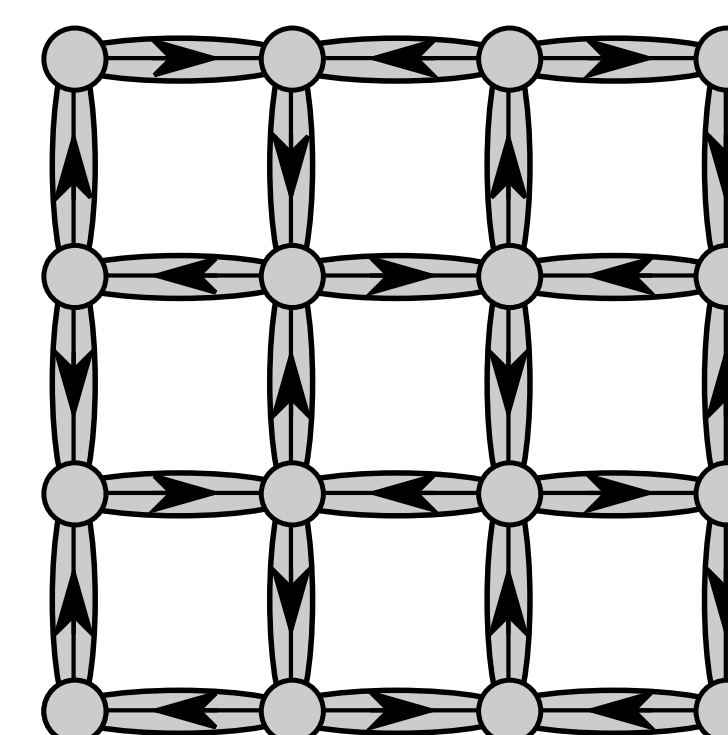
Phase A
 $(0, \pi)$ stripe



Phase B
 d -wave SC



Phase C
 d -density

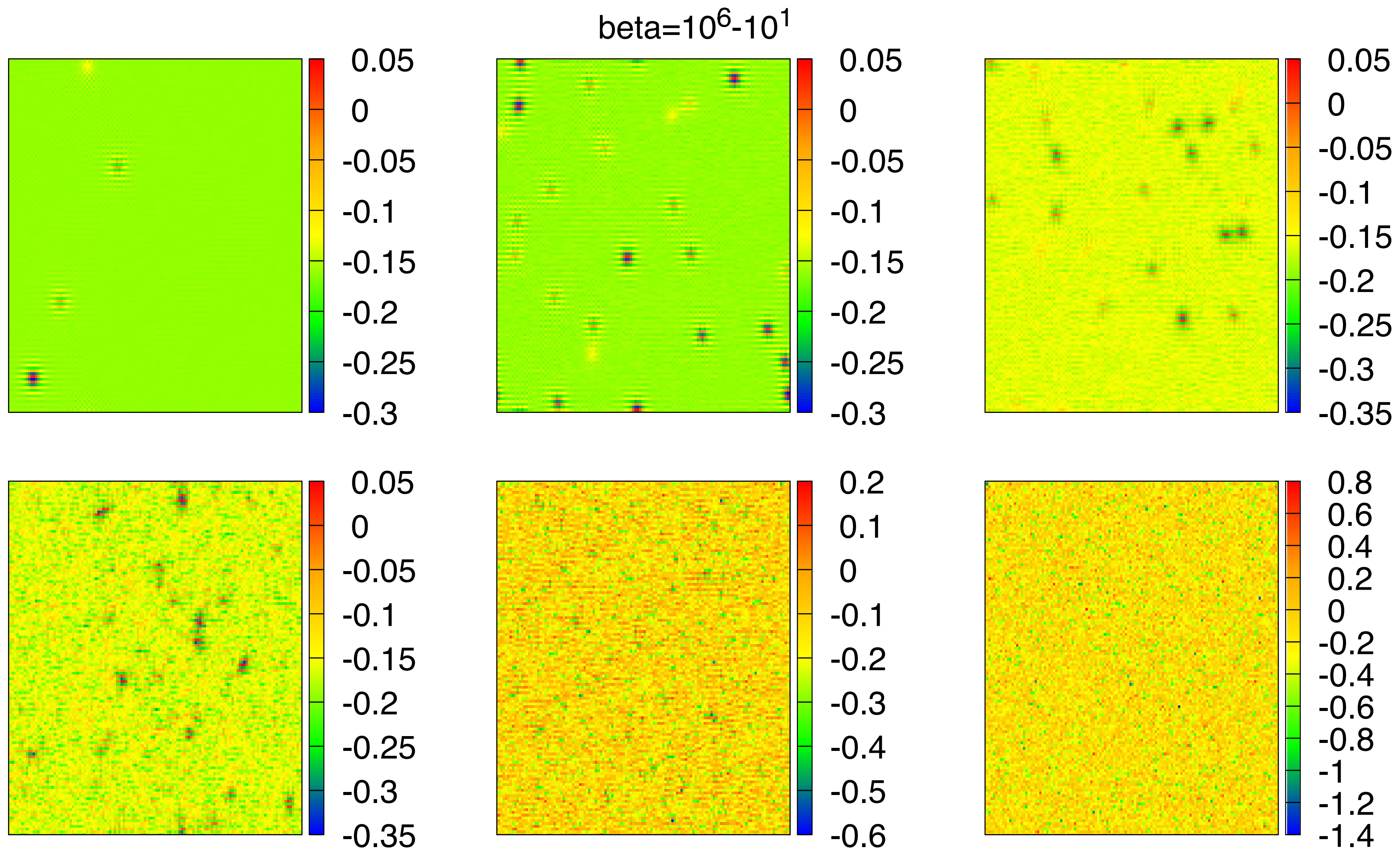


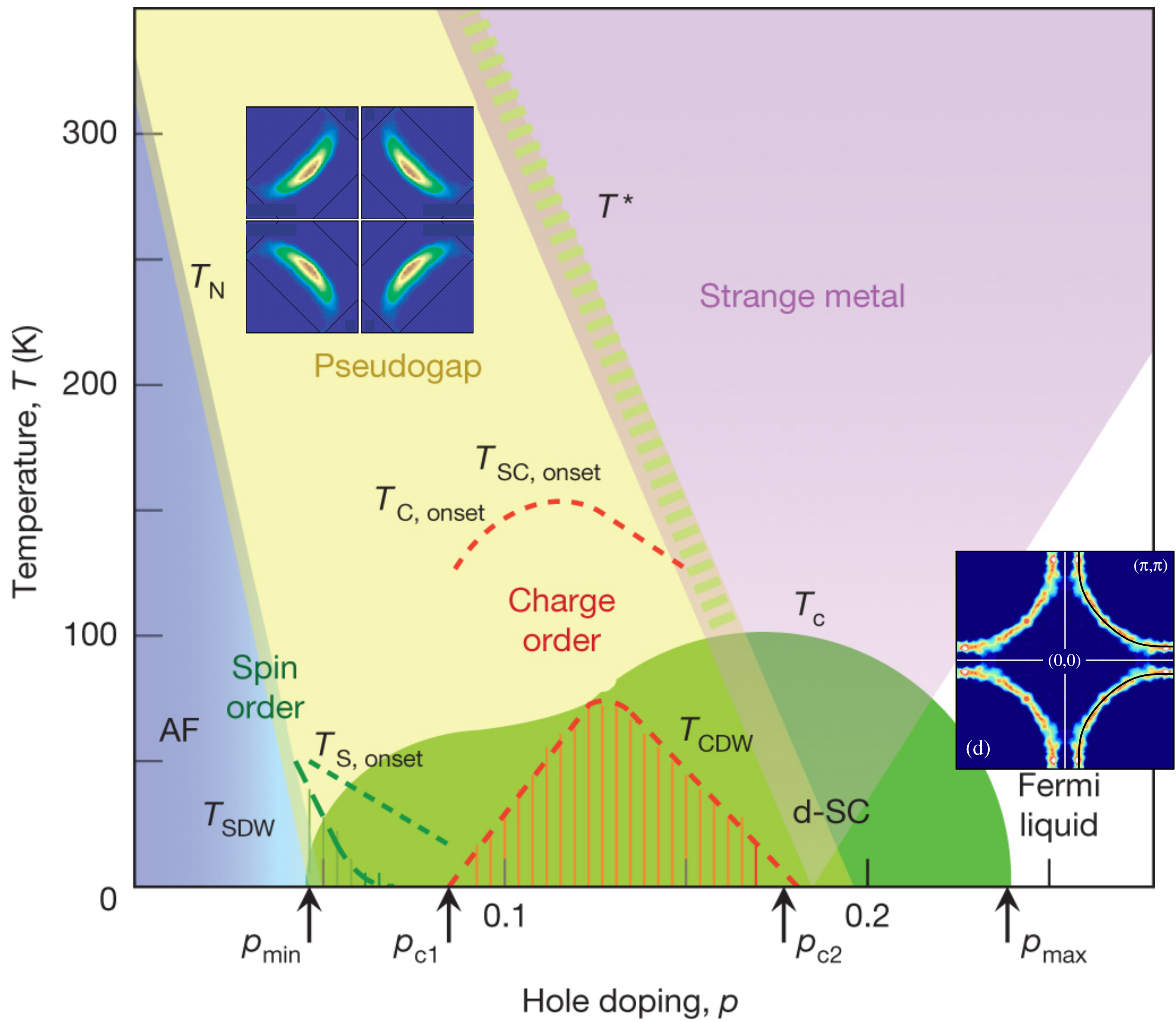
Onset of superconductivity in a SU(2) gauge theory with a fundamental Higgs field B

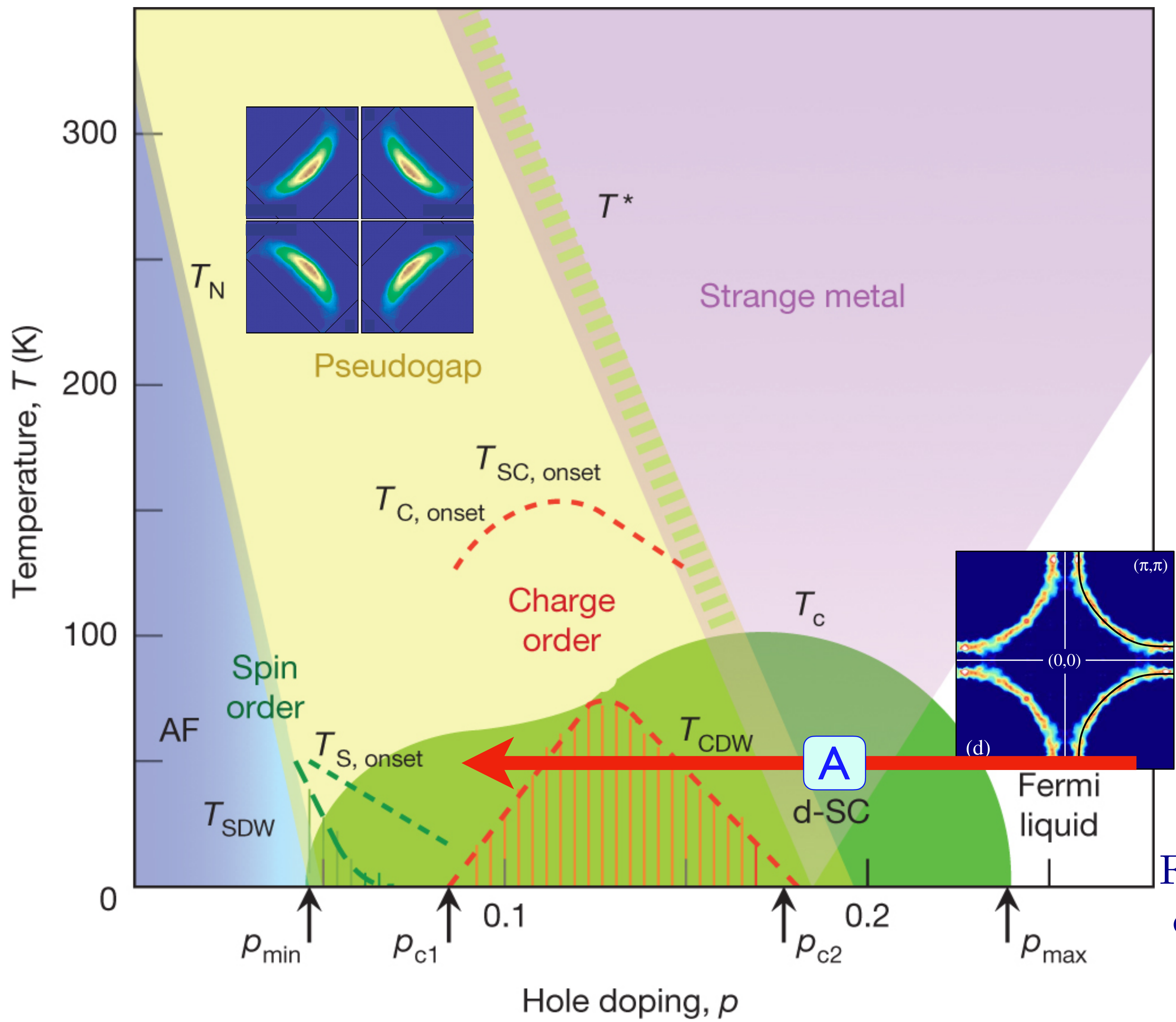
$T > 0$ Monte Carlo
192 x 192 square lattice

Plot of
bond density

Harshit Pandey,
Sayantan Sharma,
Maine Christos,
Pietro Bonetti....
to appear

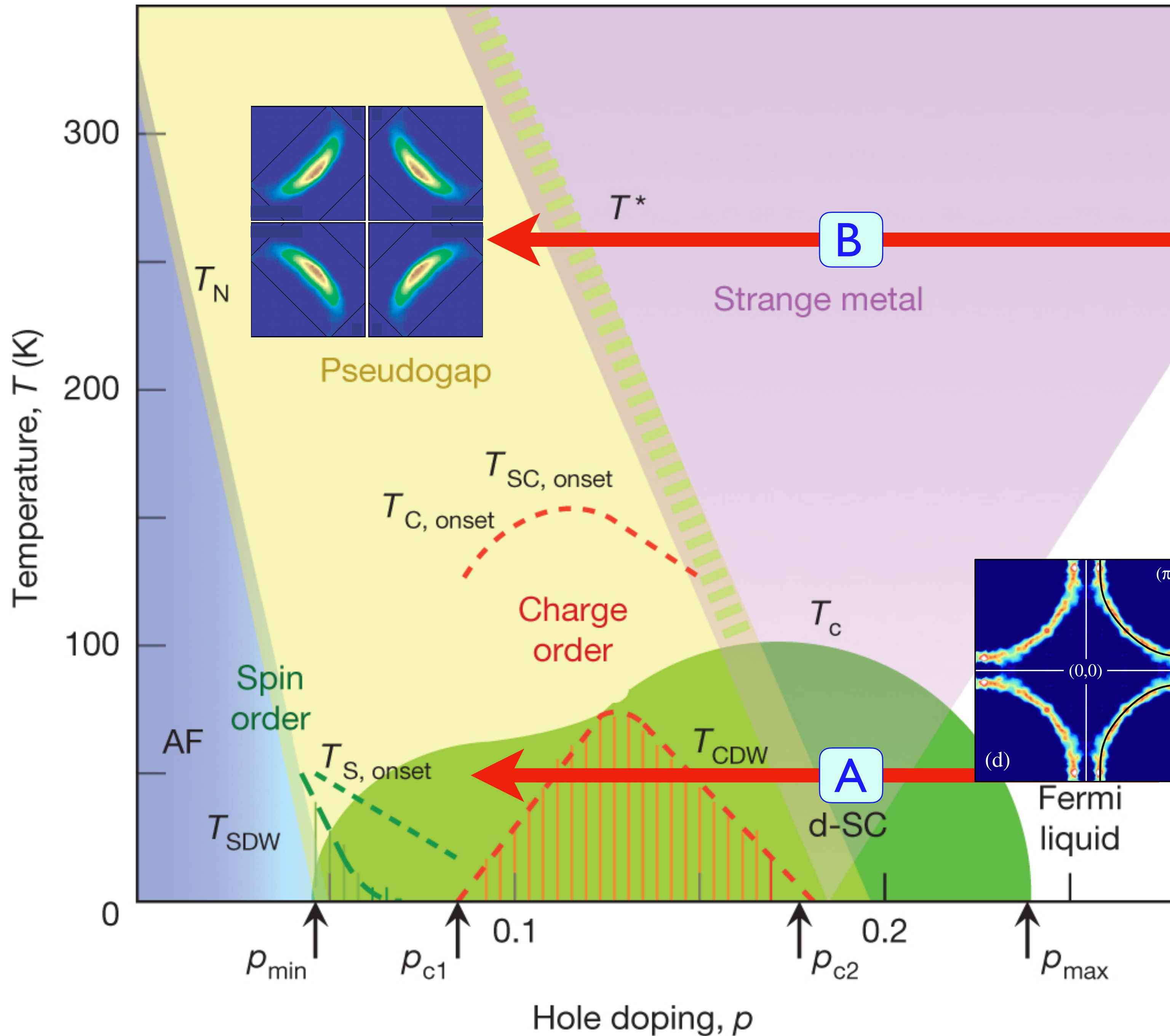






Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT with Harris disorder with
extended fermions and localized bosons
provides a theory of the “foot”



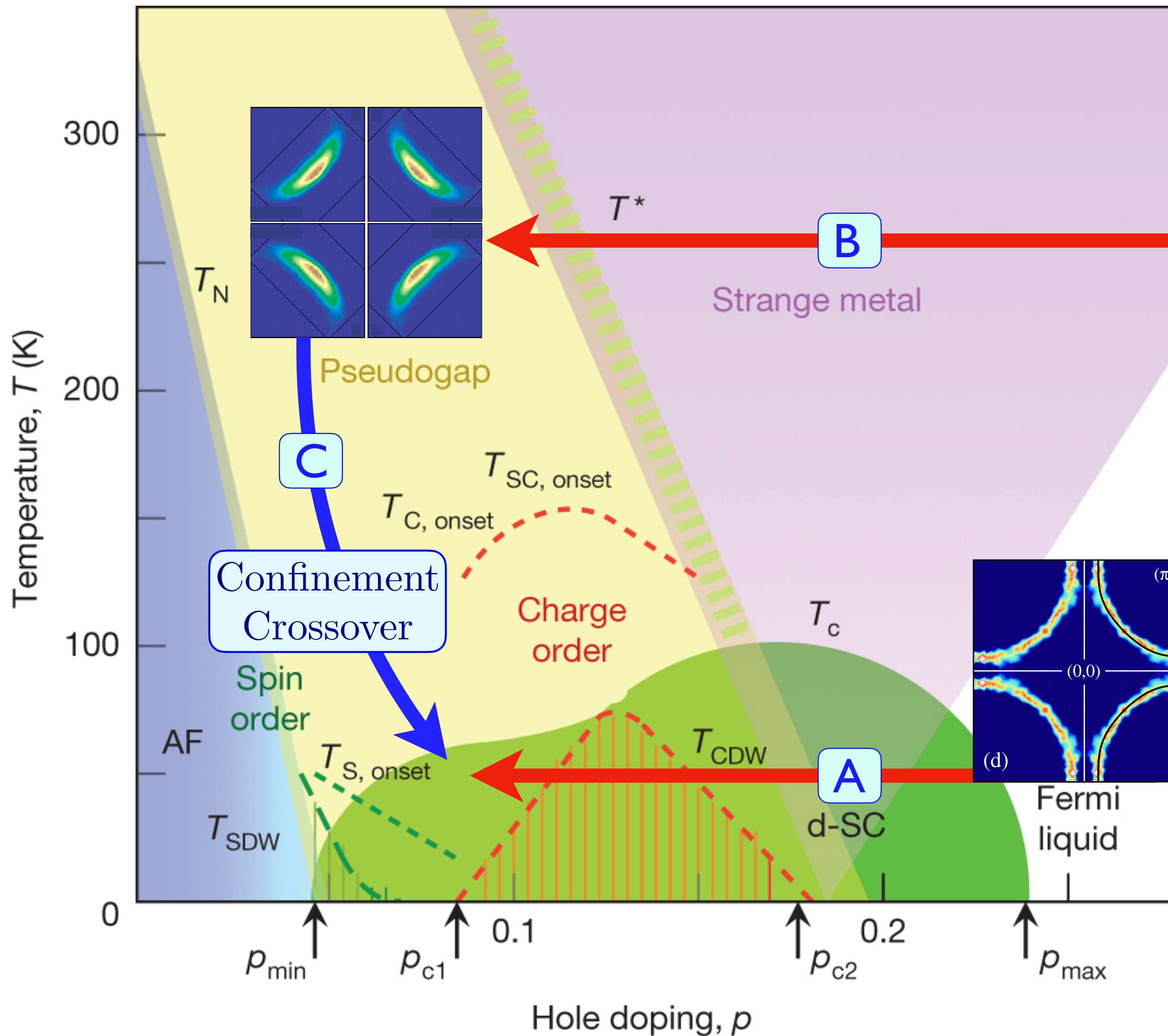
Fermi-volume-changing QPT without symmetry breaking and with spatial disorder.

2dYSYK theory of FL-FL* QPT with extended fermions and bosons provides a theory of the “fan”



Fermi-volume-changing QPT with symmetry breaking and with spatial disorder.

FL-SDW QPT with Harris disorder with extended fermions and localized bosons provides a theory of the “foot”



Fermi-volume-changing QPT without symmetry breaking and with spatial disorder.

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