Breakdown of the Landau-Ginzburg-Wilson paradigm at quantum phase transitions

Science **303**, 1490 (2004); *Physical Review* B **70**, 144407 (2004), **71**, 144508 and **71**, 144509 (2005), cond-mat/0502002

Leon Balents (UCSB)
Lorenz Bartosch (Harvard)
Anton Burkov (Harvard)
Matthew Fisher (UCSB)
Subir Sachdev (Harvard)
Krishnendu Sengupta (HRI, India)
T. Senthil (MIT and IISc)
Ashvin Vishwanath (Berkeley)

Talk online at http://sachdev.physics.harvard.edu

Outline

I. Statement of the problem
A. Antiferromagnets
B. Boson lattice models

II. Theory of defects: vortices near the superfluid-insulator transition
Berry phases imply that vortices carry "flavor"

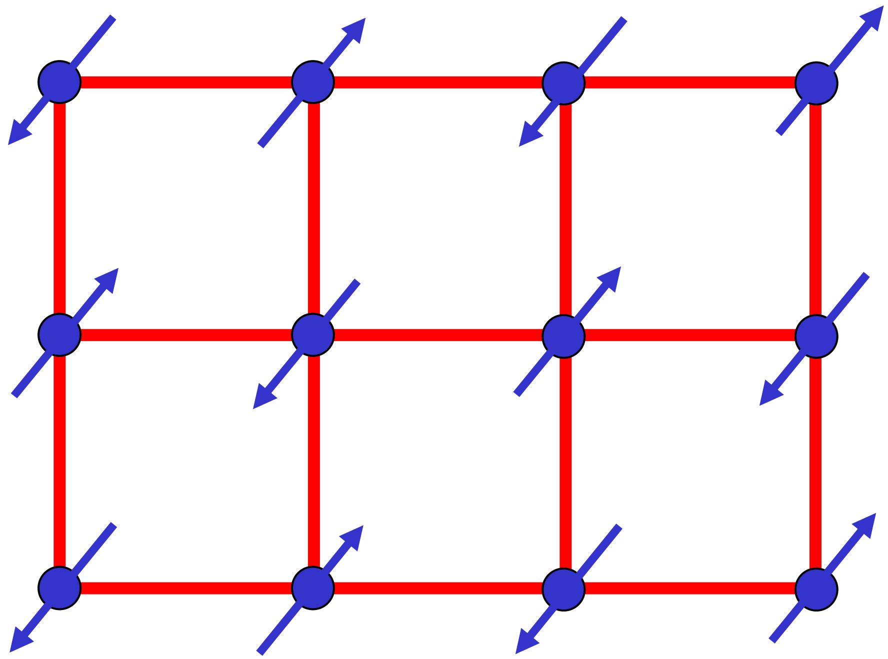
III. The cuprate superconductors
Detection of vortex flavors ?

IV. Defects in the antiferromagnet
Hedgehog Berry phases and VBS order

I.A Quantum phase transitions of $S=1/2$ antiferromagnets

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \quad ; \quad \vec{S}_i \Rightarrow \text{ spin operator with } S=1/2$$



Ground state has long-range Néel order

Order parameter $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$$\langle \vec{\varphi} \rangle \neq 0$$

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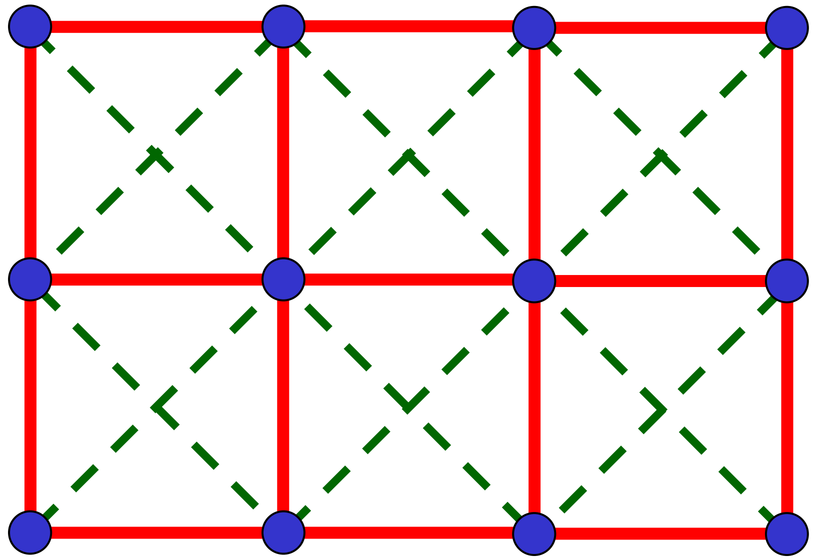

Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

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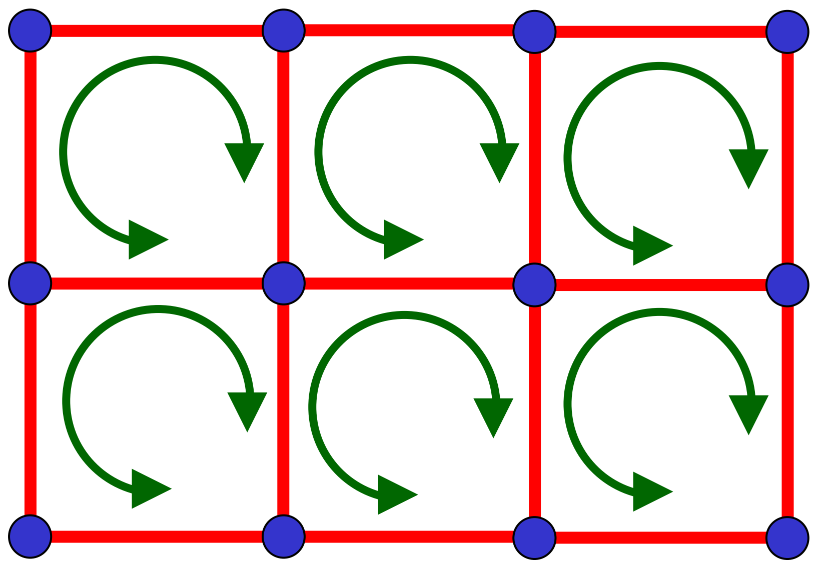

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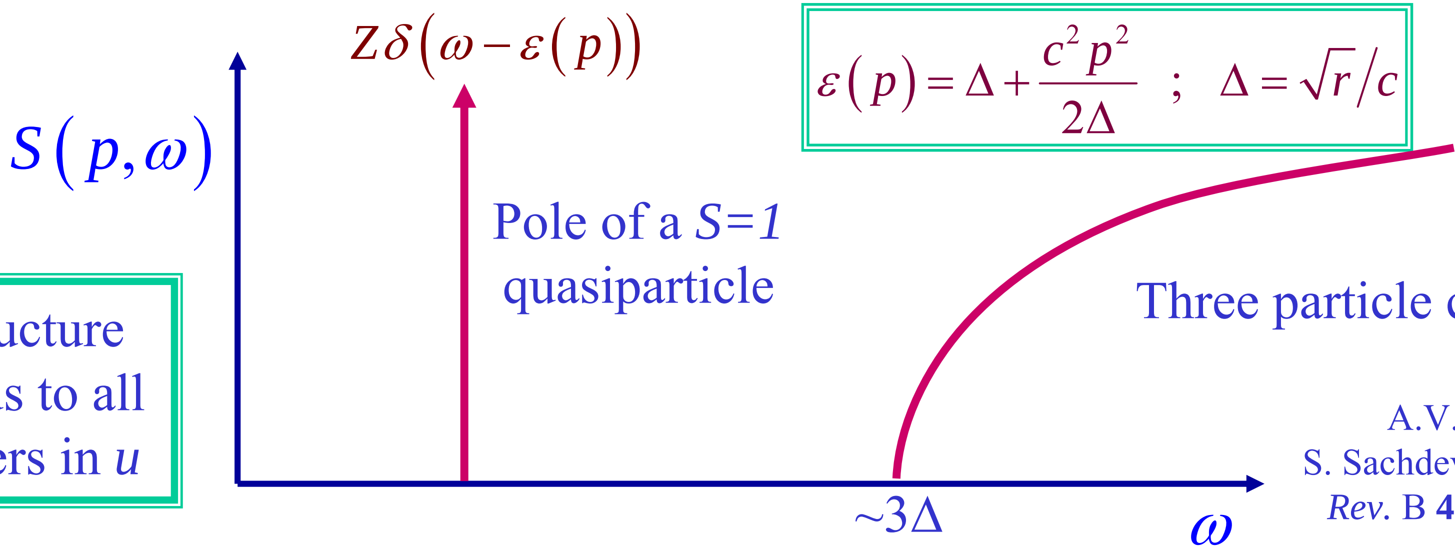
LGW theory for such a quantum transition

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_\varphi = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + \frac{1}{c^2}\left(\partial_\tau \vec{\varphi}\right)^2 + r\vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

For $r > 0$ oscillations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ lead to the following structure in the dynamic structure factor $S(p,\omega)$

$$\varepsilon(p) = \Delta + \frac{c^2 p^2}{2\Delta} \quad ; \quad \Delta = \sqrt{r}/c$$



Structure holds to all orders in u

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

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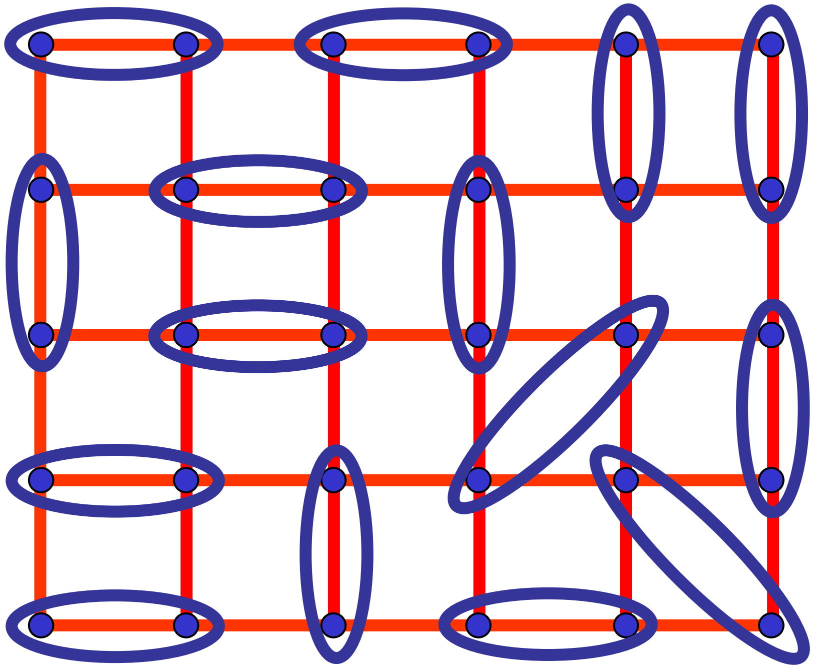

"Liquid" of valence bonds has fractionalized *S=1/2* excitations

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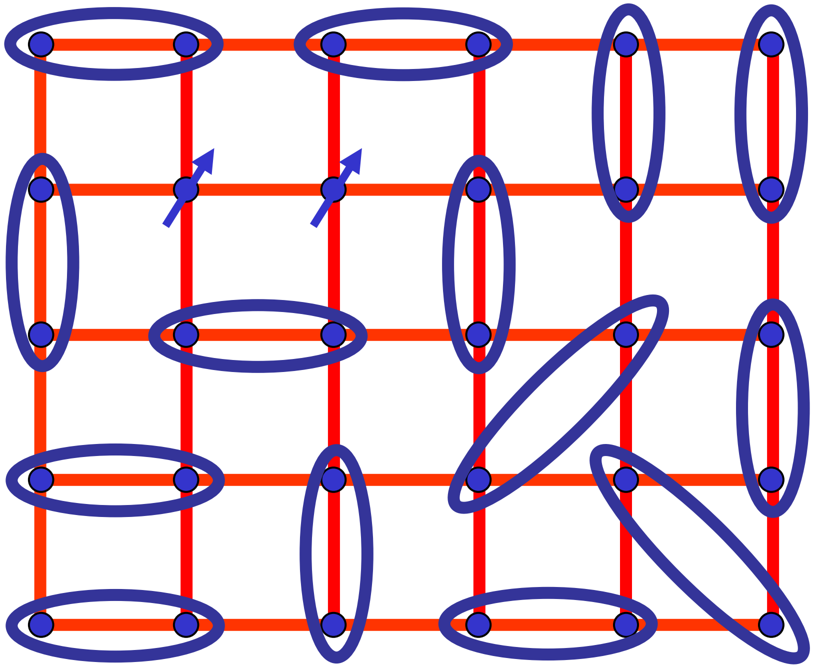

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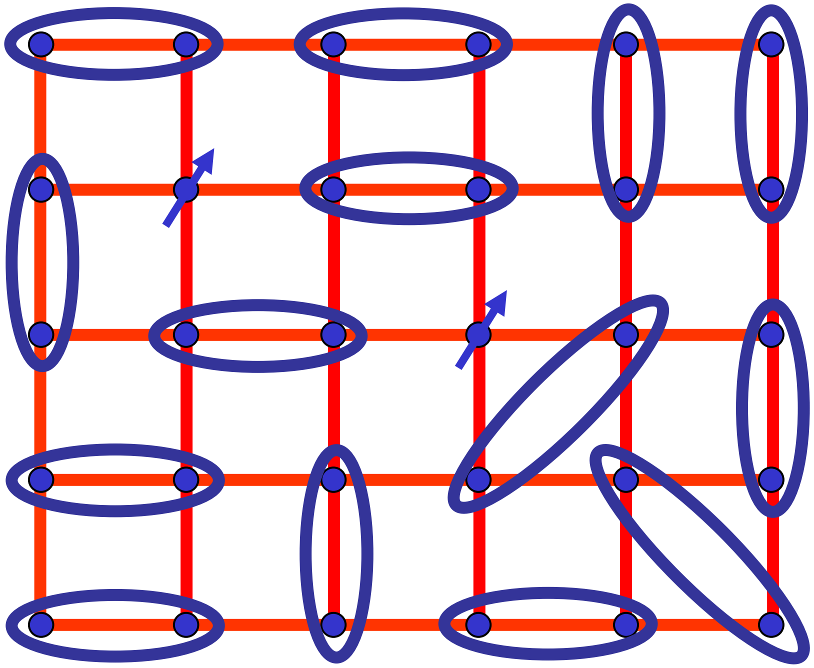

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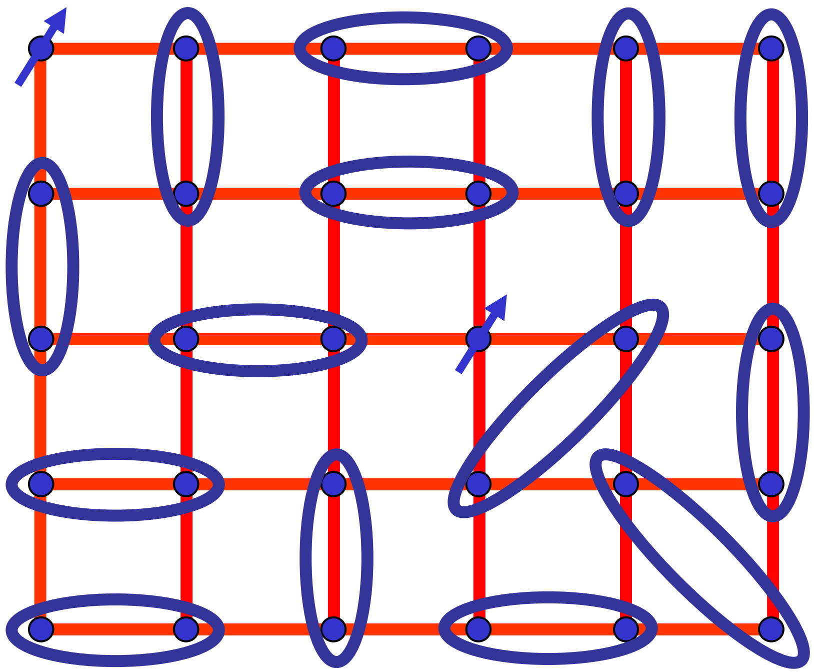

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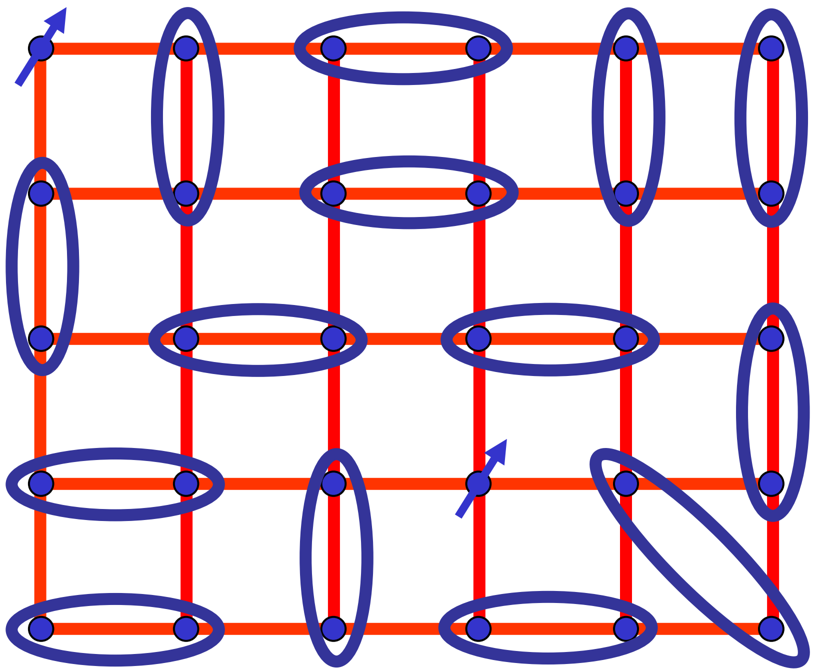

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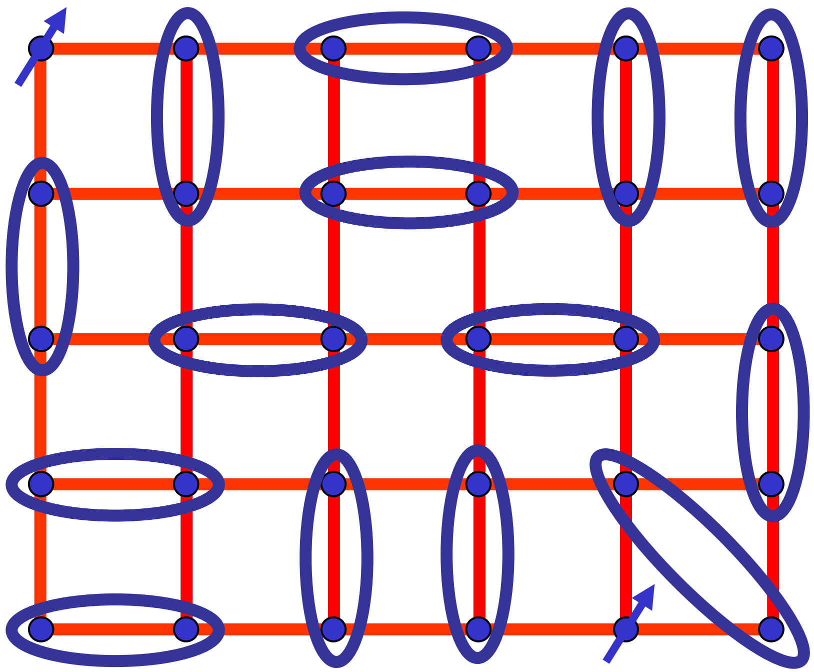
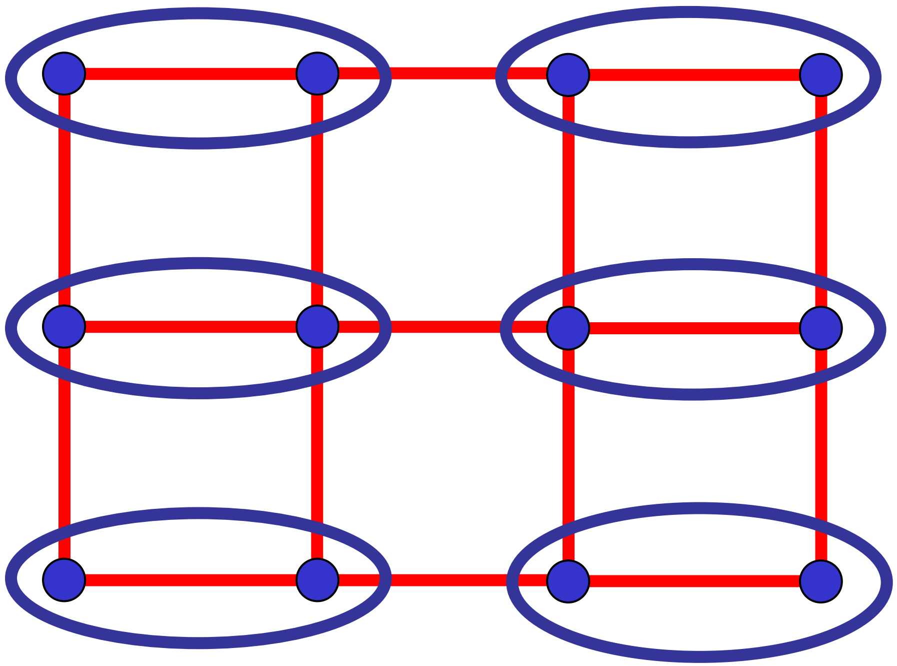


"Liquid" of valence bonds has fractionalized *S=1/2* excitations

Another possible state, with $\langle \vec{\varphi} \rangle = 0$, is the valence bond solid (VBS)



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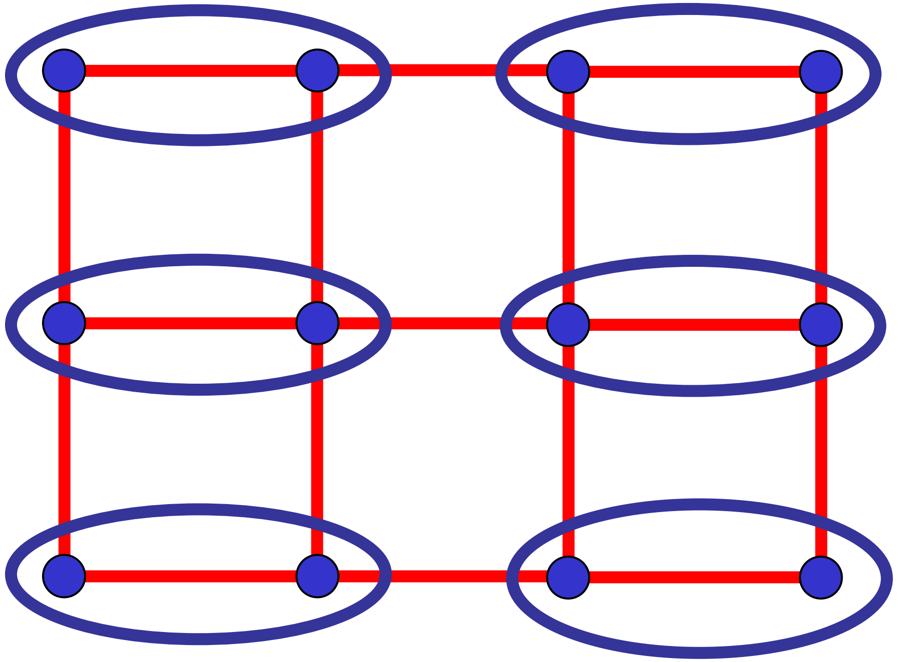

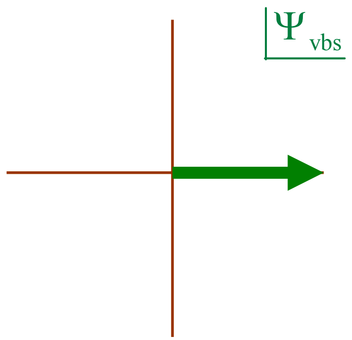

Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\left\langle \Psi_{\text{vbs}} \right\rangle \neq 0$, where Ψ_{vbs} is the ***VBS order parameter***

$$\Psi_{\text{vbs}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

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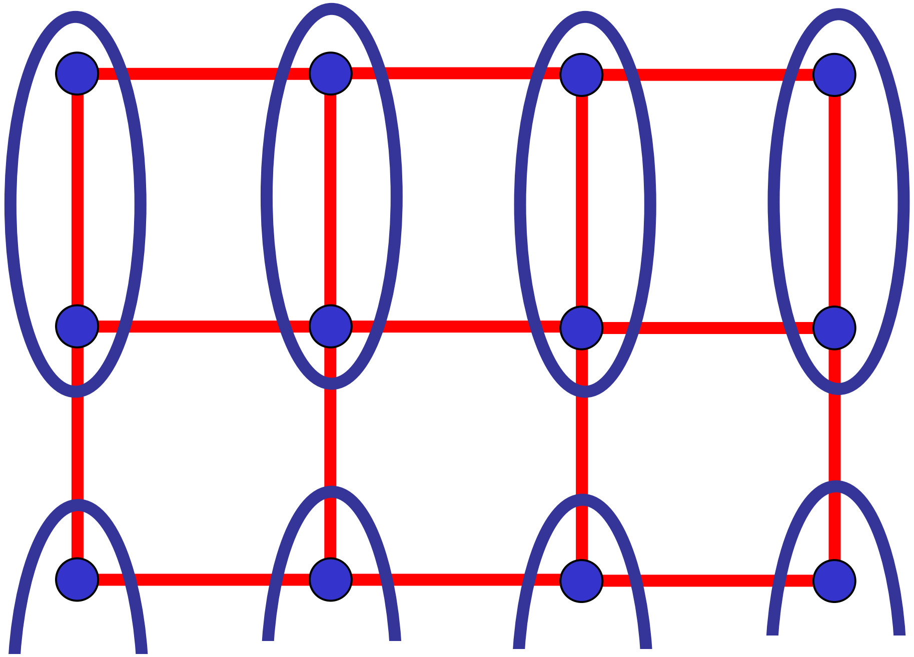

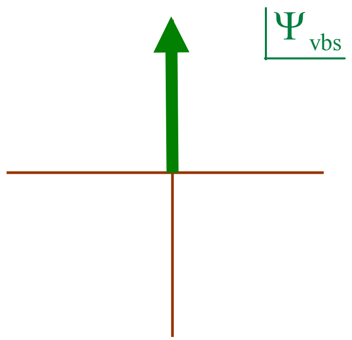

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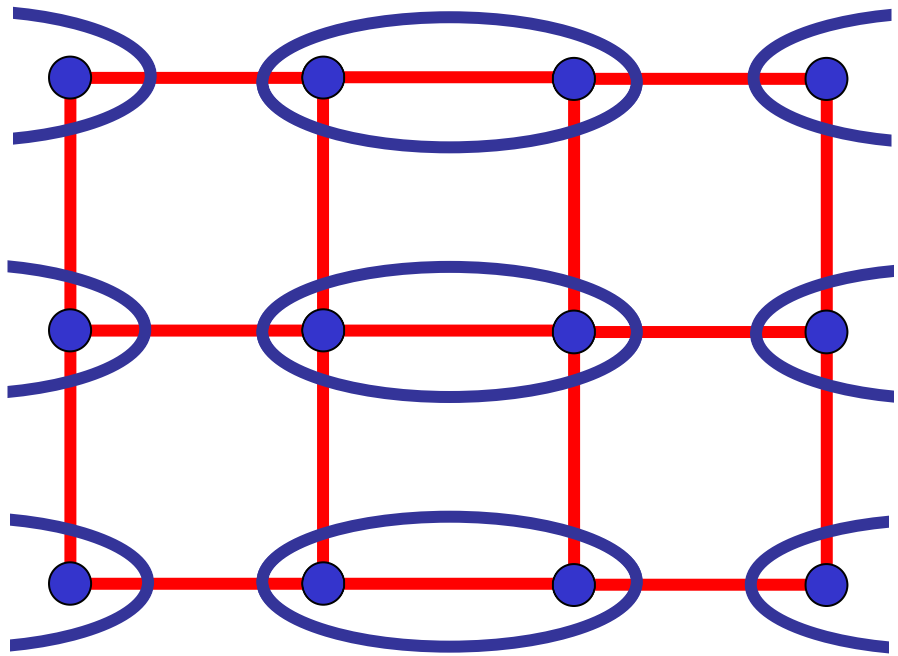

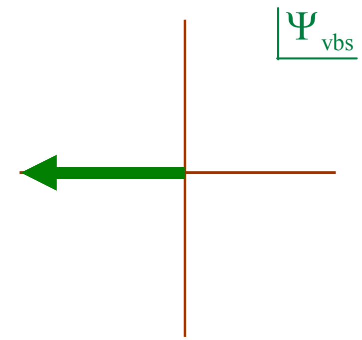

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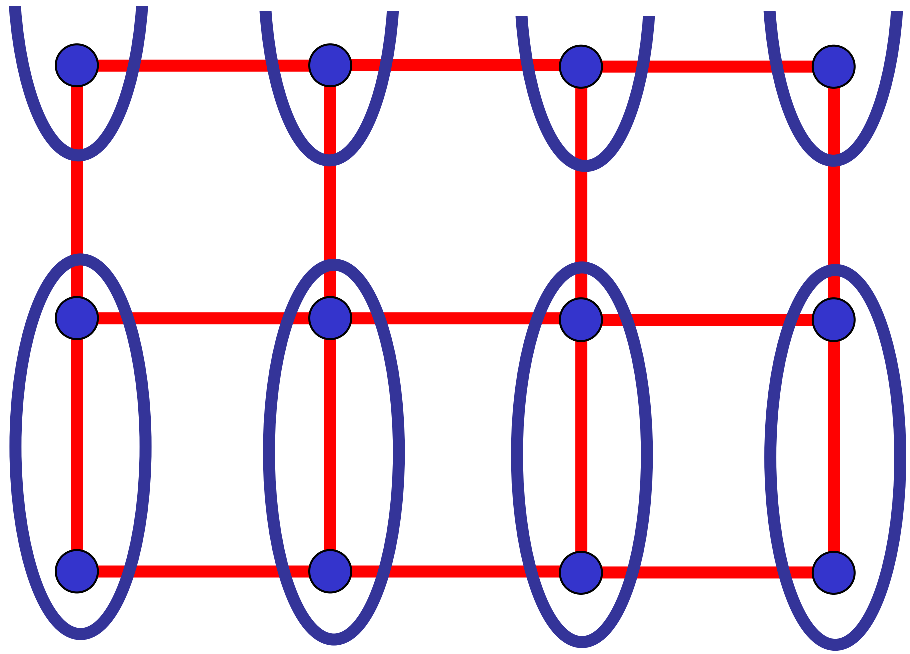

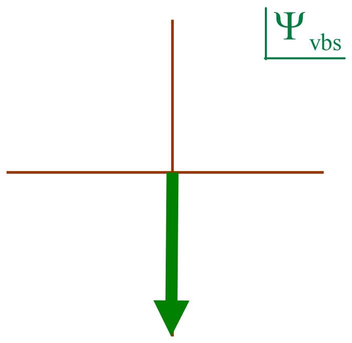

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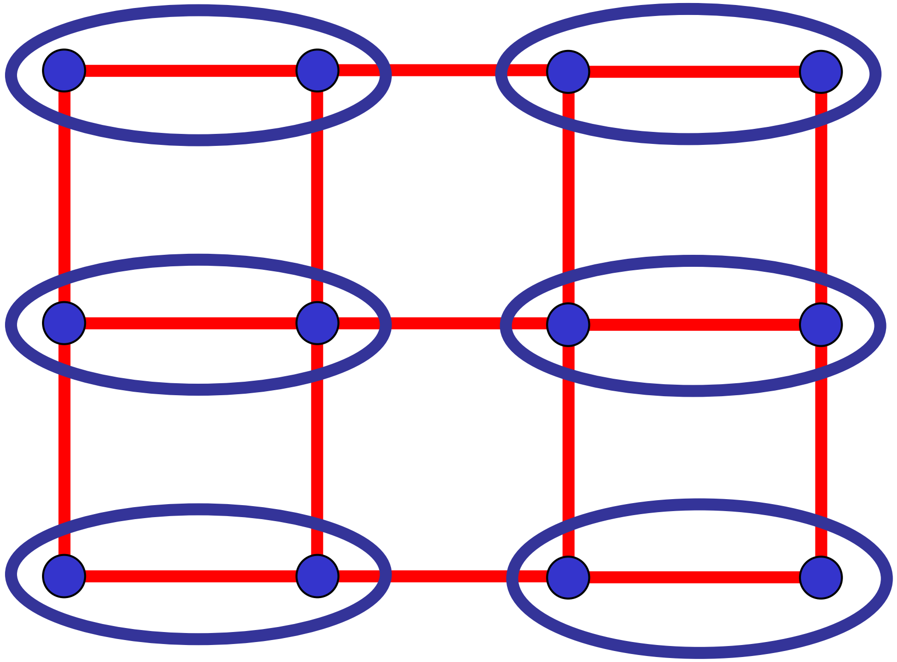

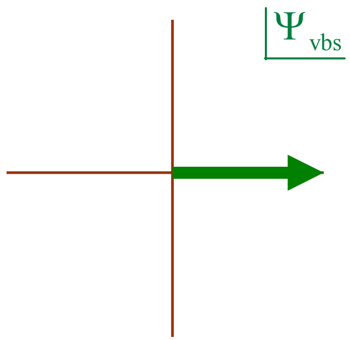

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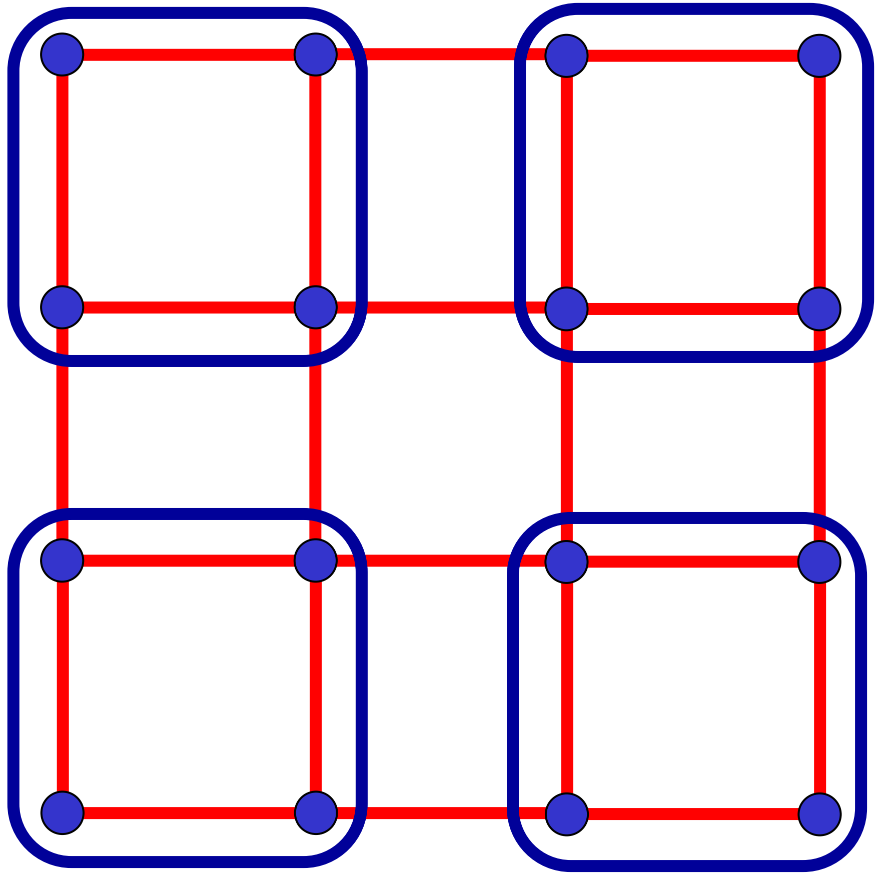

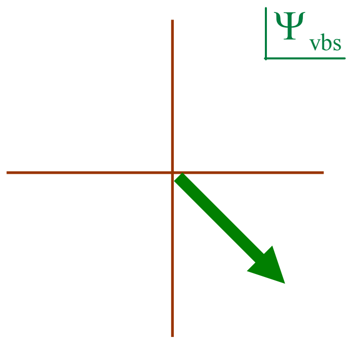

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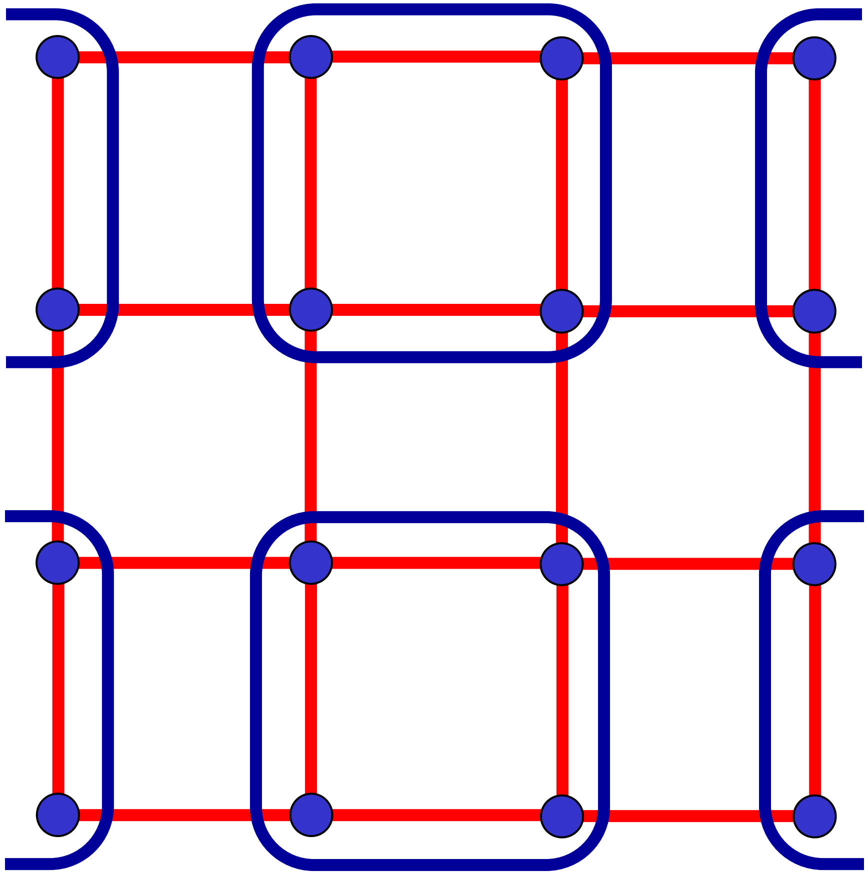

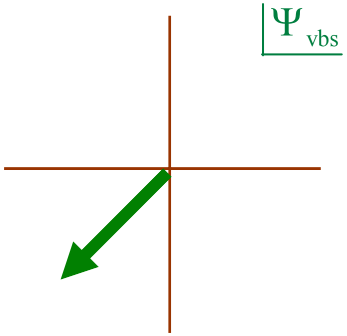

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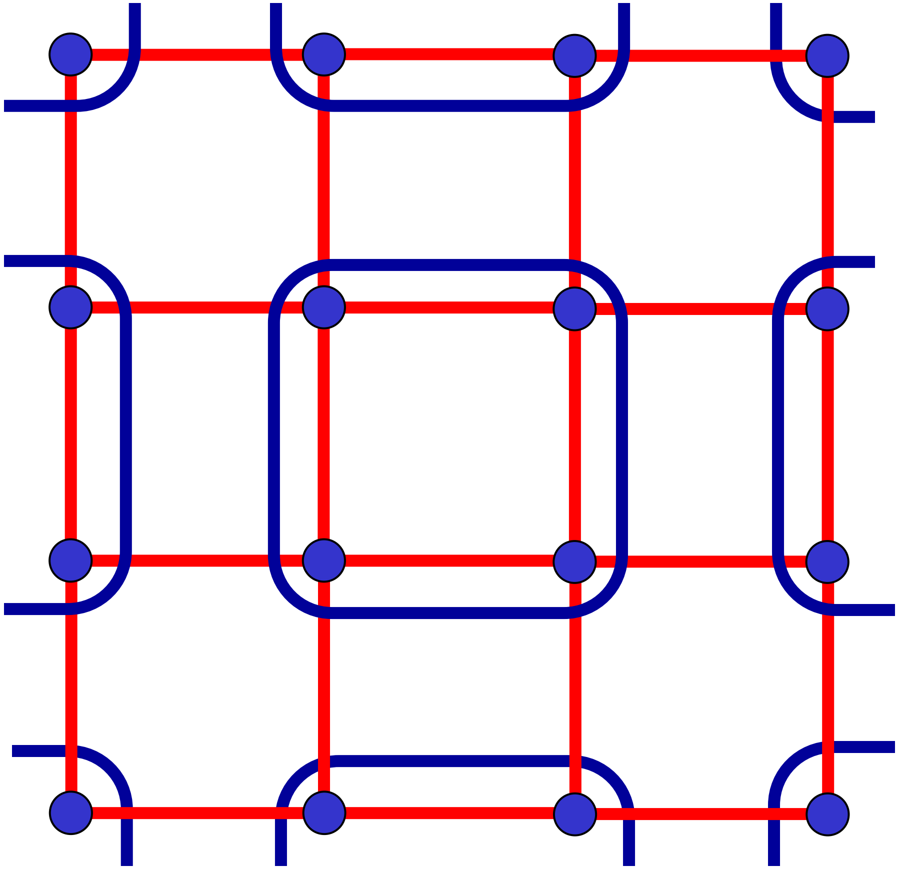

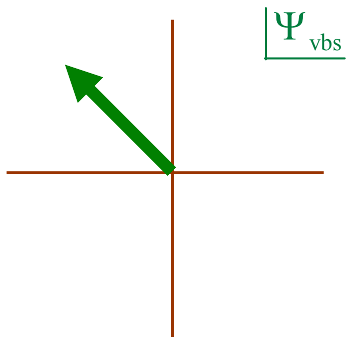

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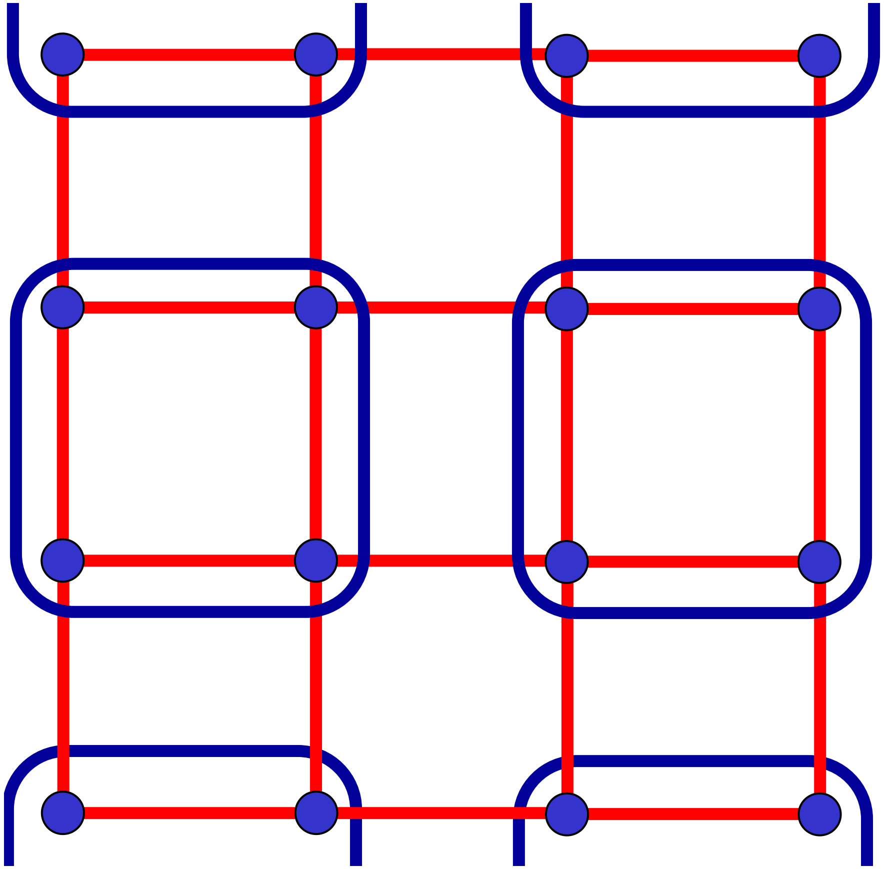

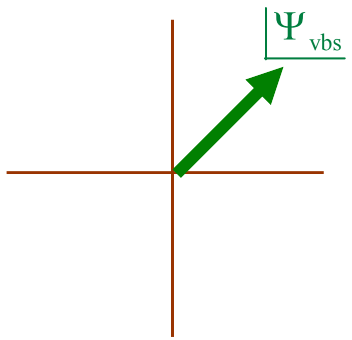

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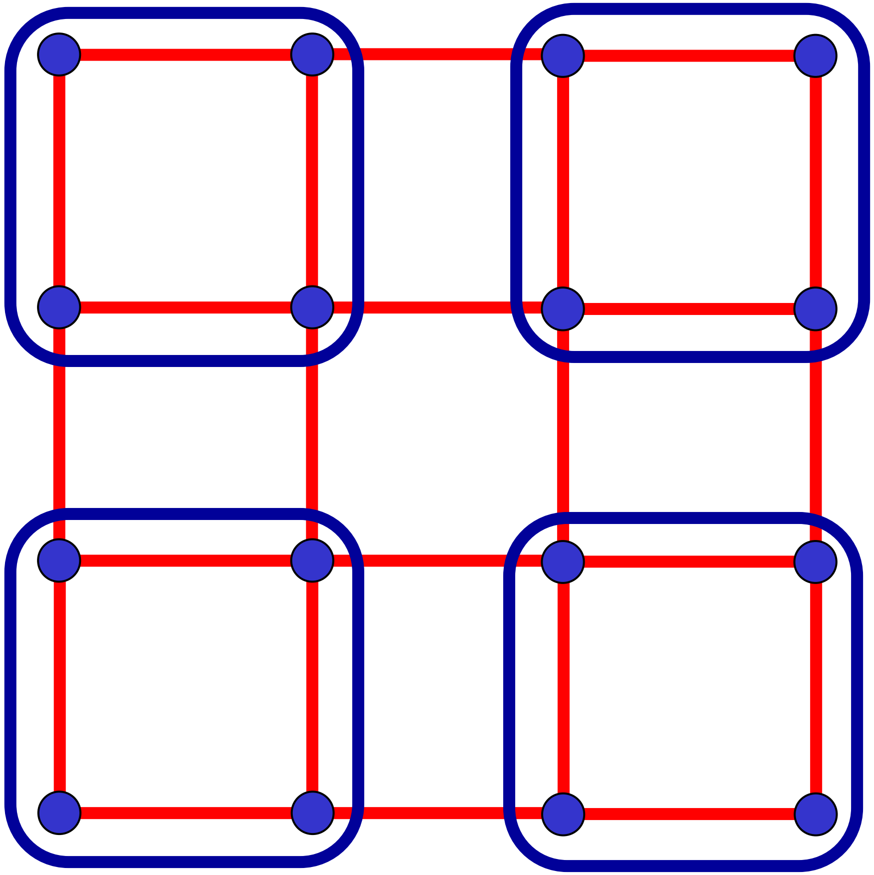

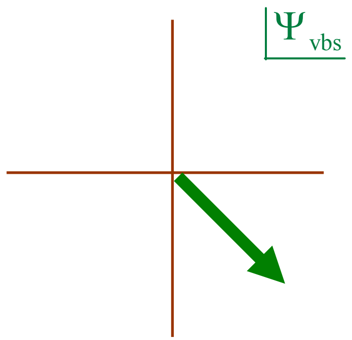

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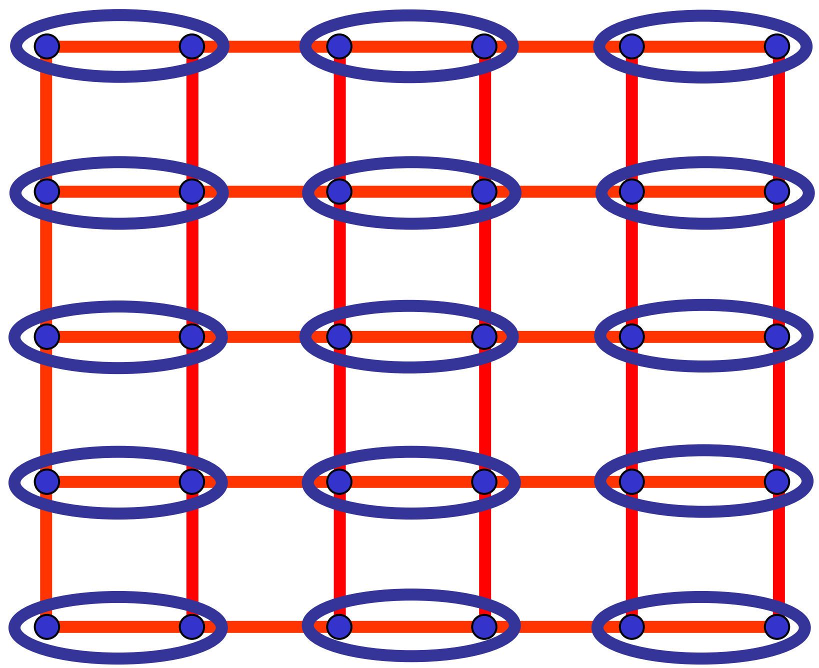

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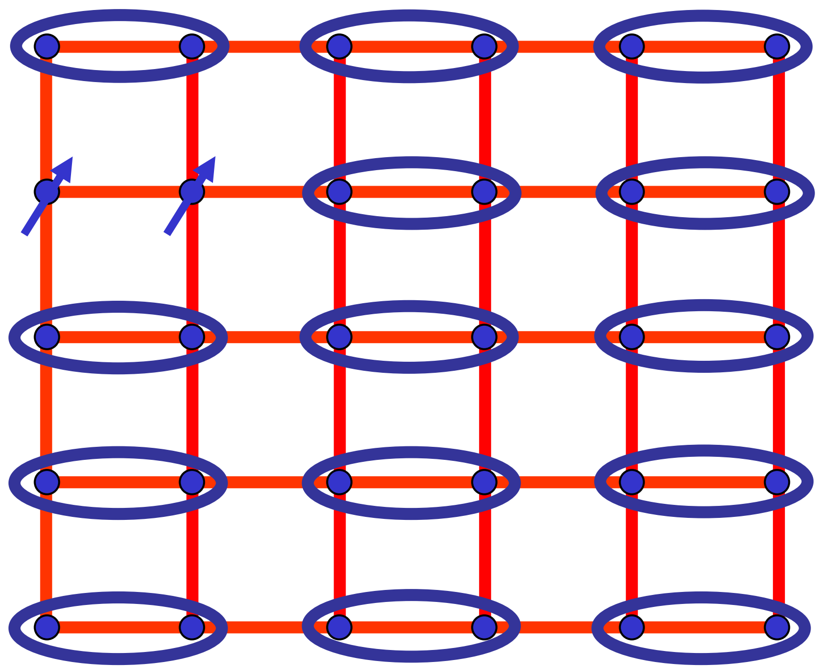

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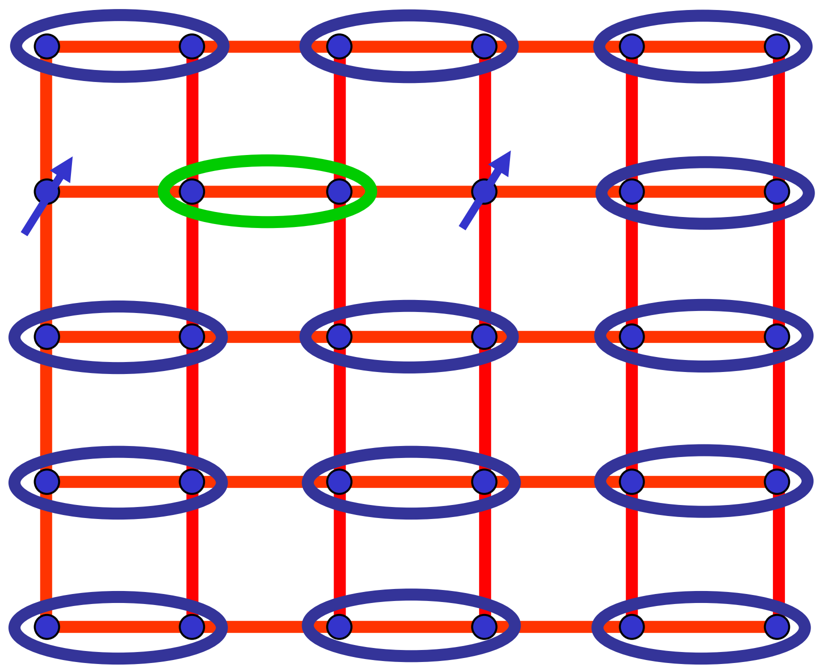

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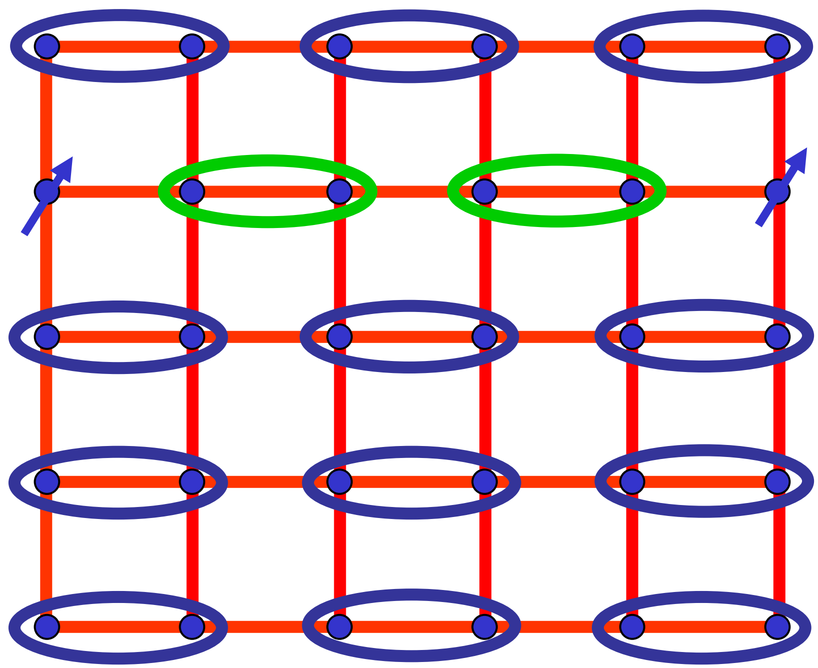

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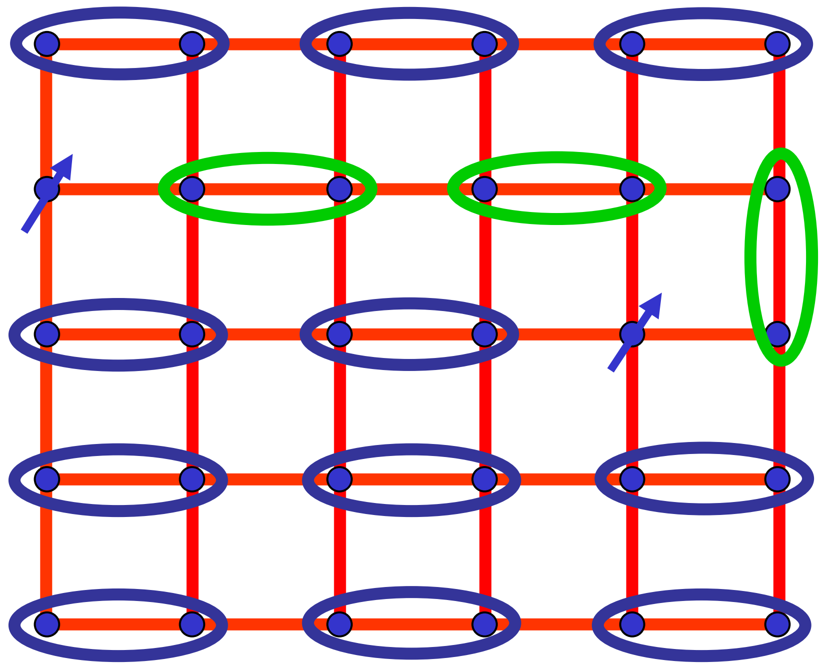

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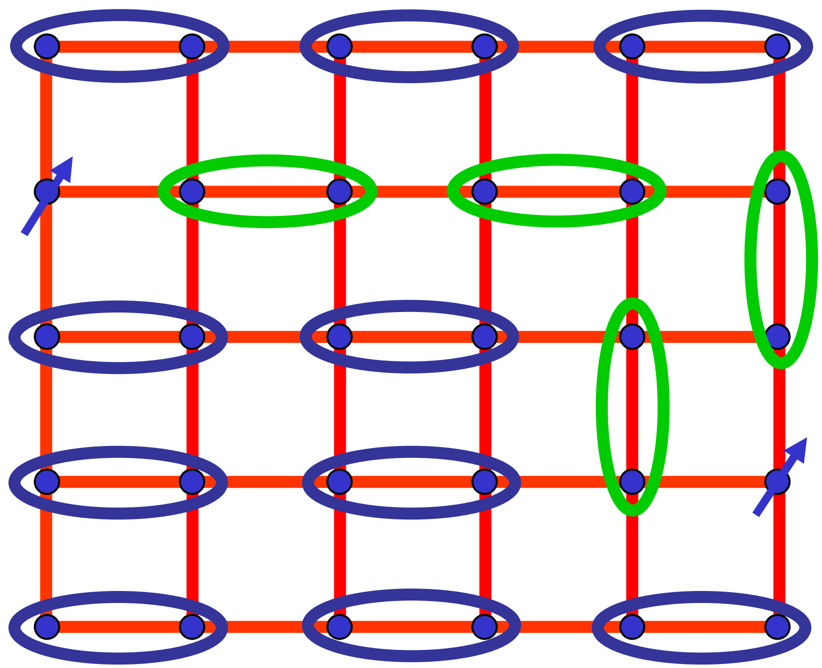


LGW theory of multiple order parameters

$$F = F_{\text{vbs}}\left[\Psi_{\text{vbs}}\right] + F_{\varphi}\left[\vec{\varphi}\right] + F_{\text{int}}$$

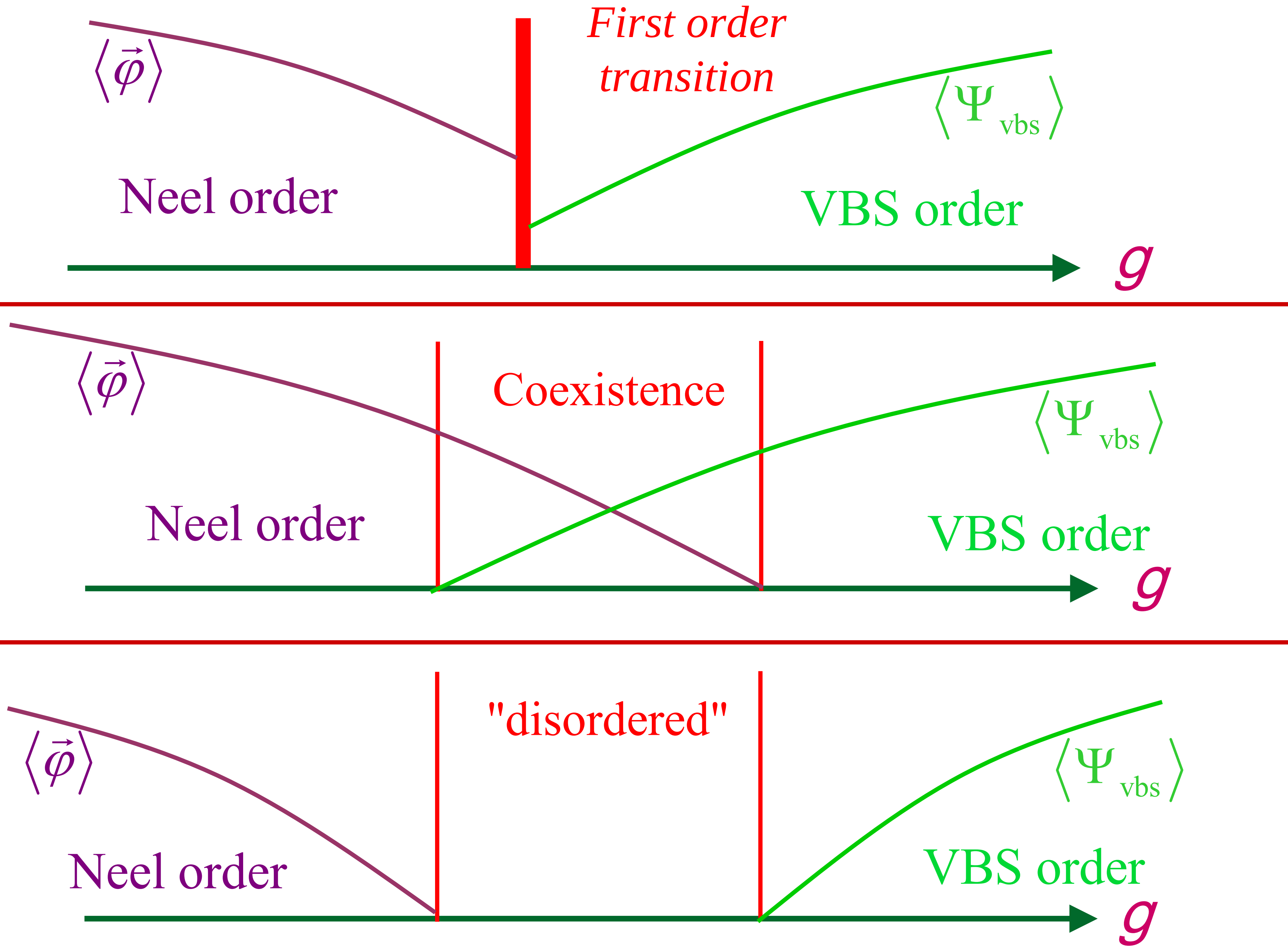
$$F_{\text{vbs}}\left[\Psi_{\text{vbs}}\right] = r_1\left|\Psi_{\text{vbs}}\right|^2 + u_1\left|\Psi_{\text{vbs}}\right|^4 + \cdots$$

$$F_{\varphi}\left[\vec{\varphi}\right] = r_2\left|\vec{\varphi}\right|^2 + u_2\left|\vec{\varphi}\right|^4 + \cdots$$

$$F_{\text{int}} = v\left|\Psi_{\text{vbs}}\right|^2\left|\vec{\varphi}\right|^2 + \cdots$$

Distinct symmetries of order parameters permit couplings only between their energy densities

LGW theory of multiple order parameters



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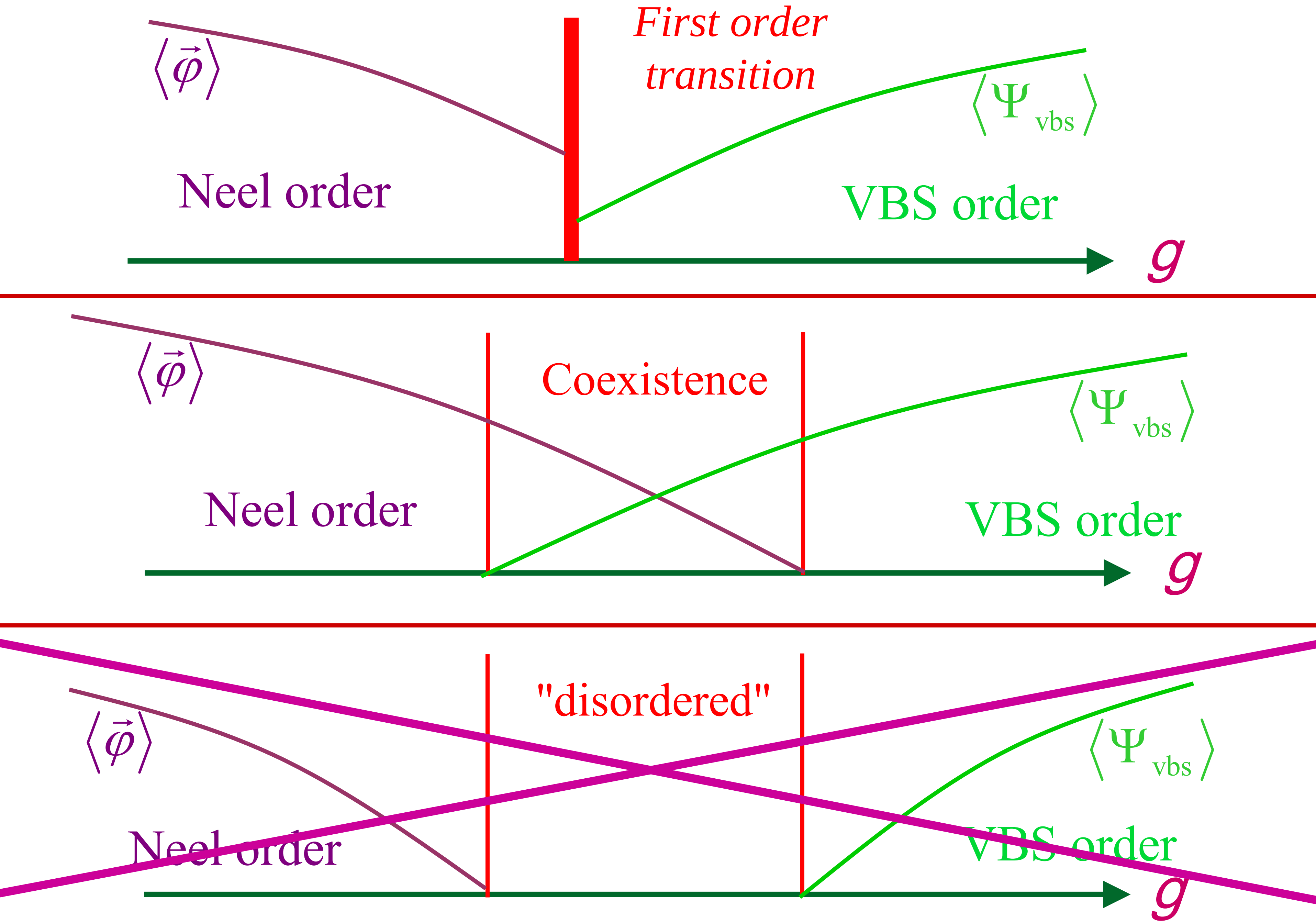

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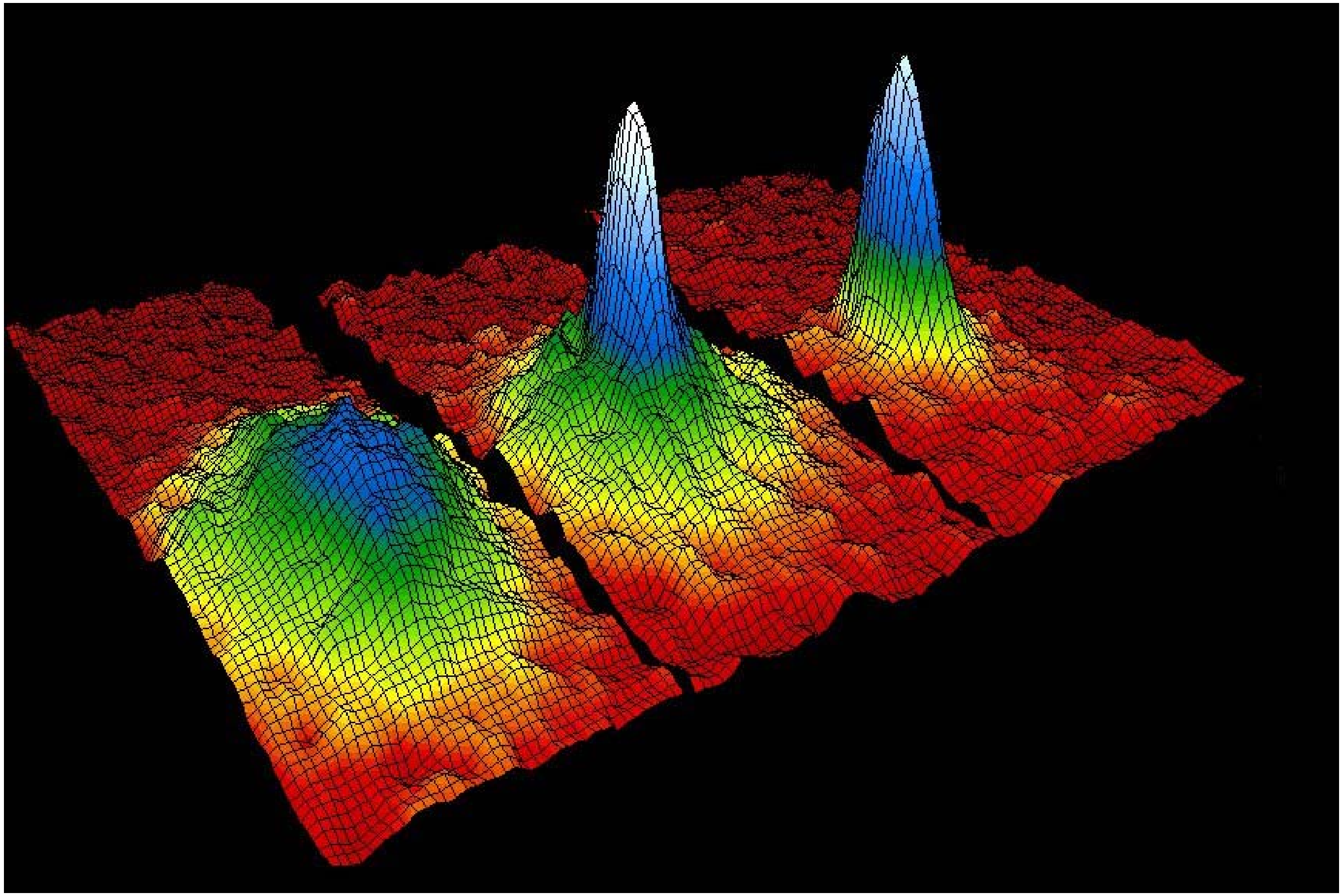
I.B Quantum phase transitions of boson lattice models

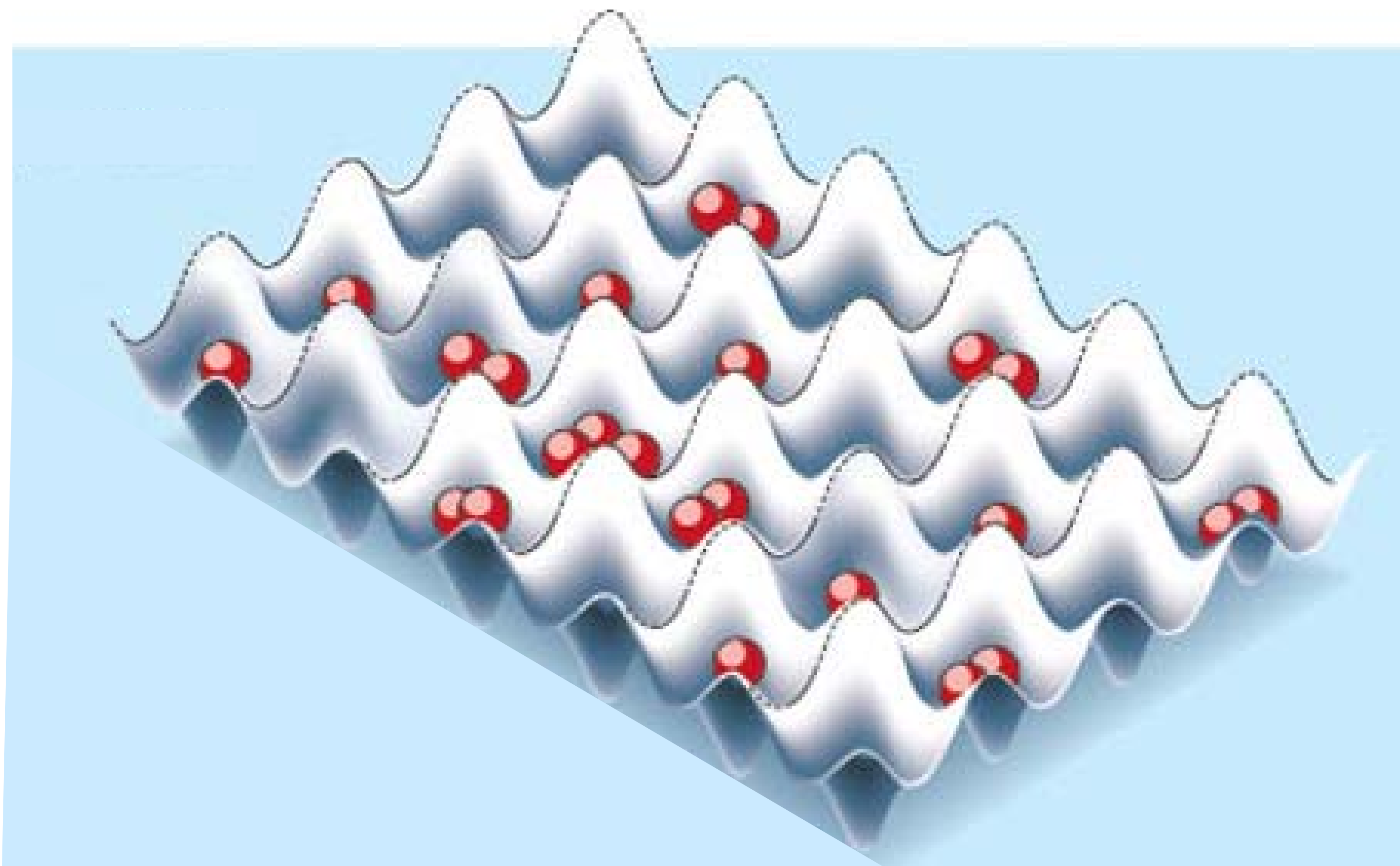
Bose condensation

Velocity distribution function of ultracold ^{87}Rb atoms



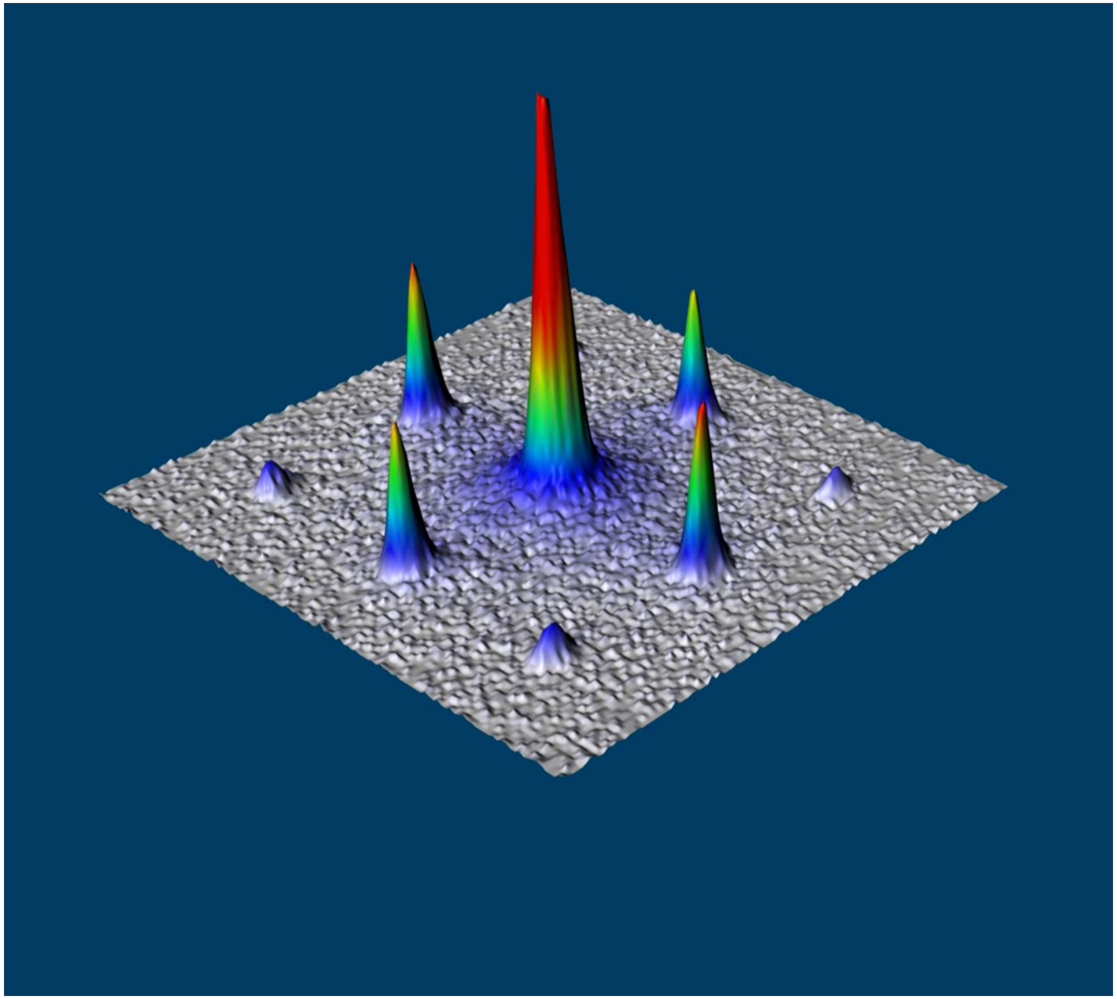
M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman and E. A. Cornell, *Science* **269**, 198 (1995)

Apply a periodic potential (standing laser beams) to trapped ultracold bosons (^{87}Rb)



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

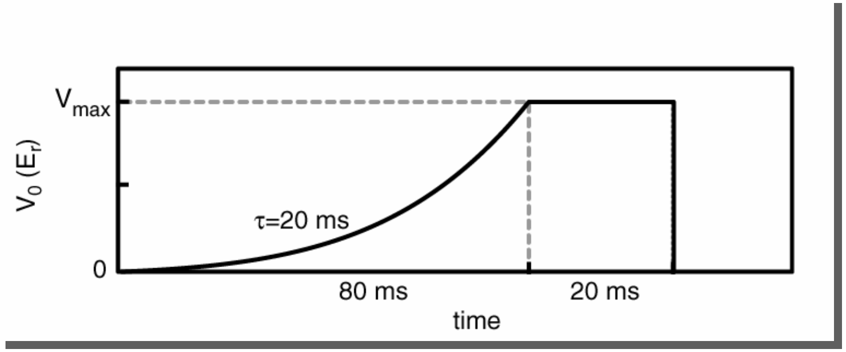
Momentum distribution function of bosons

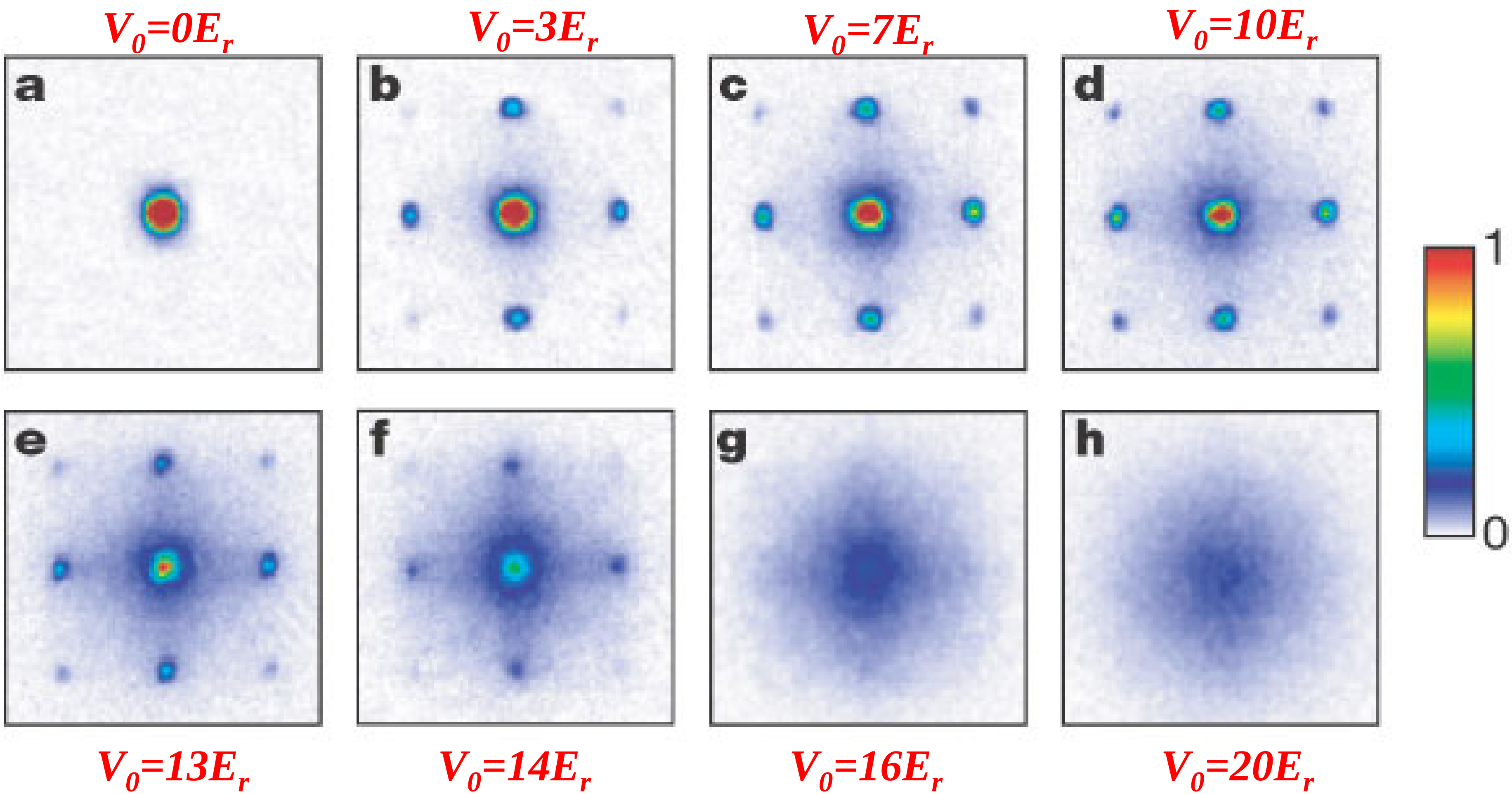


Bragg reflections of condensate at reciprocal lattice vectors

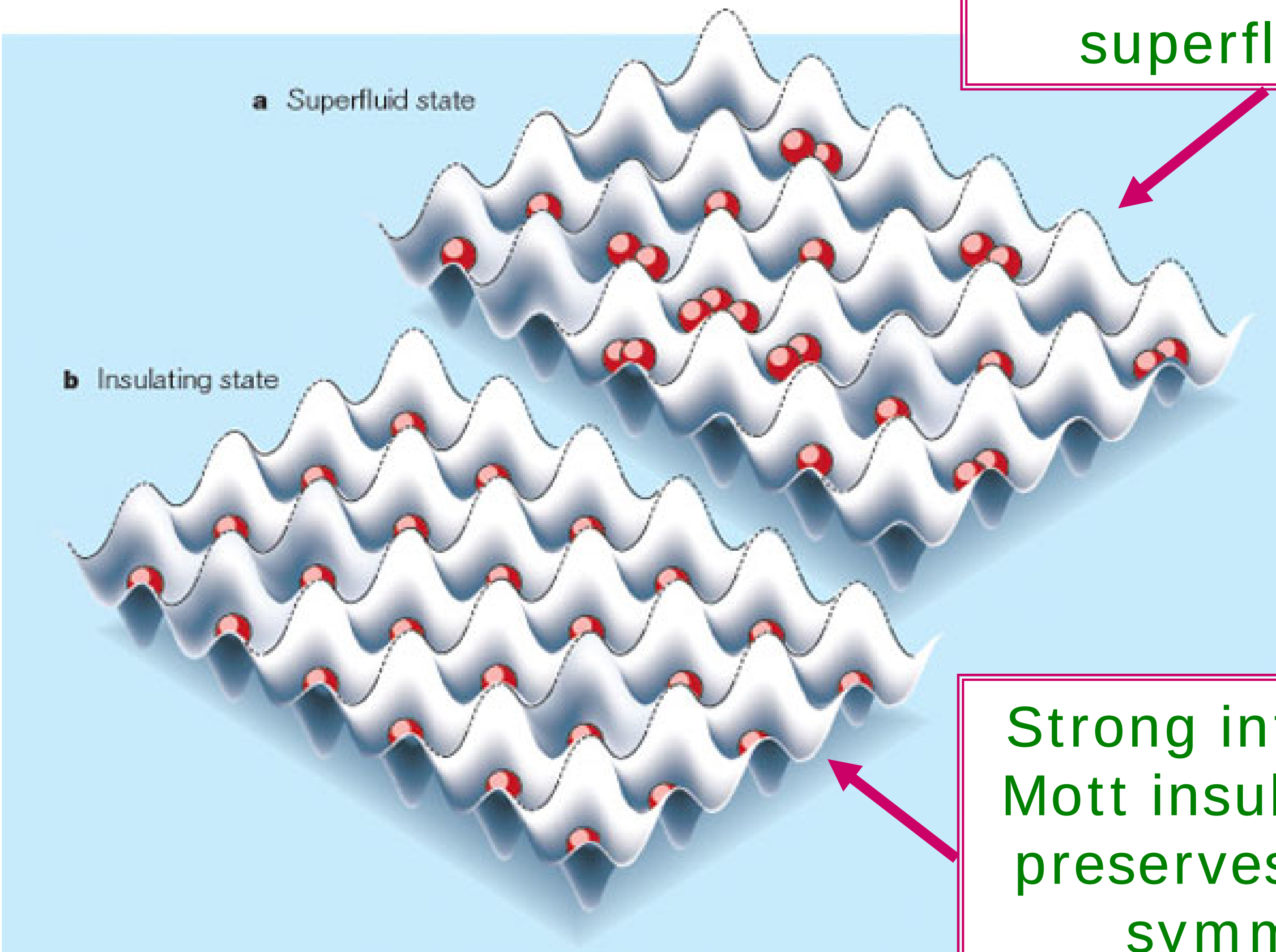
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Superfluid-insulator quantum phase transition at T=0





Bosons at filling fraction f = 1

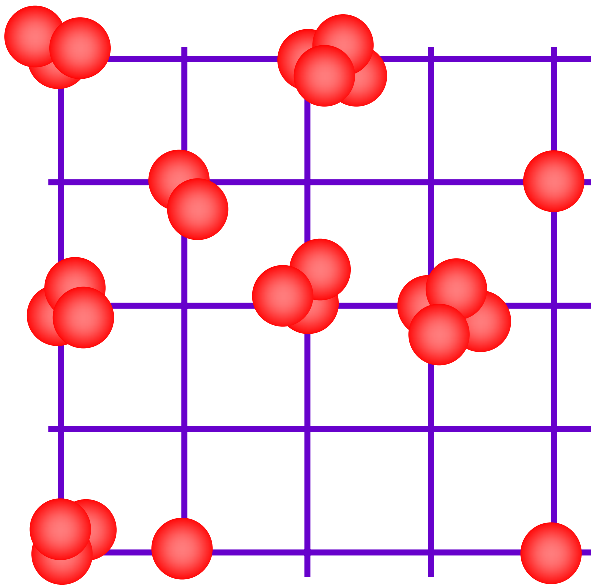


Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Bosons at filling fraction $f = 1$



$$\langle \Psi_{sf} \rangle \neq 0$$

Weak interactions: superfluidity

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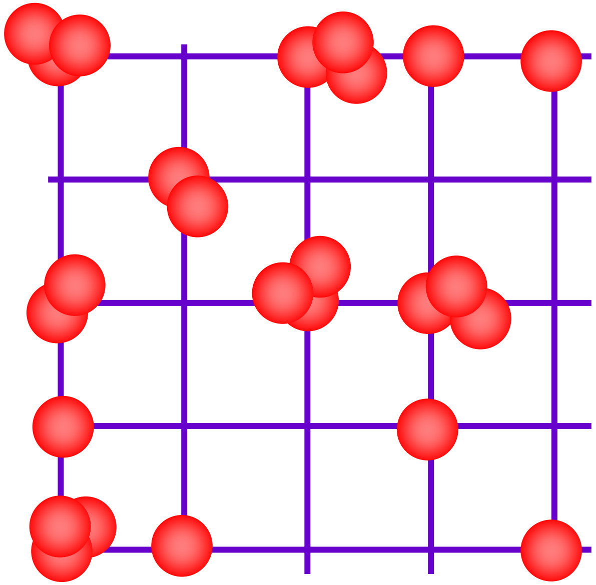

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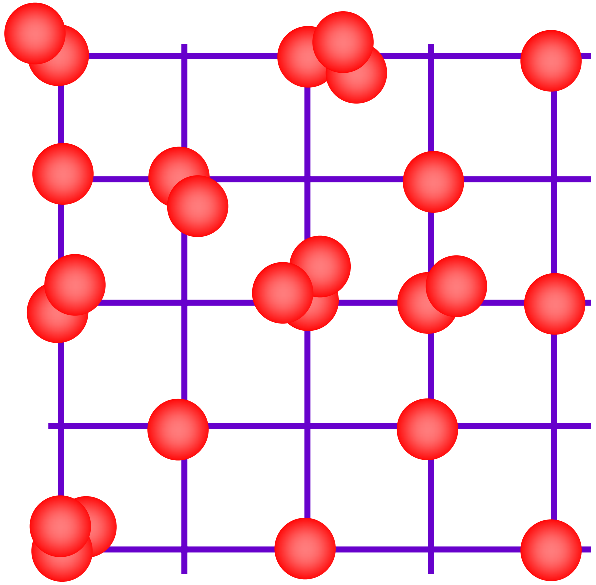

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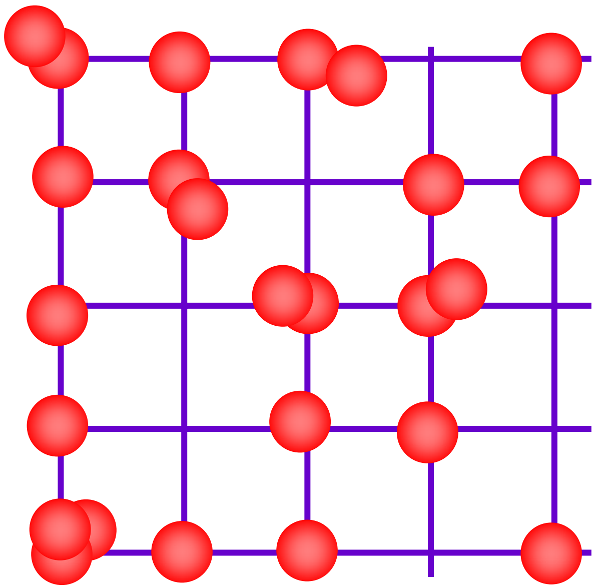

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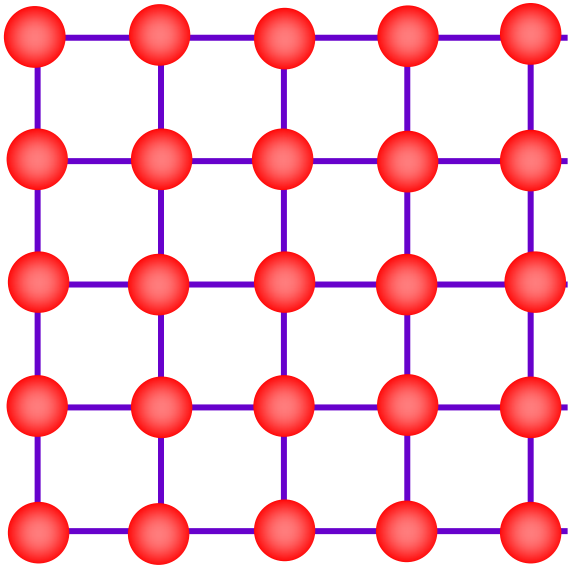

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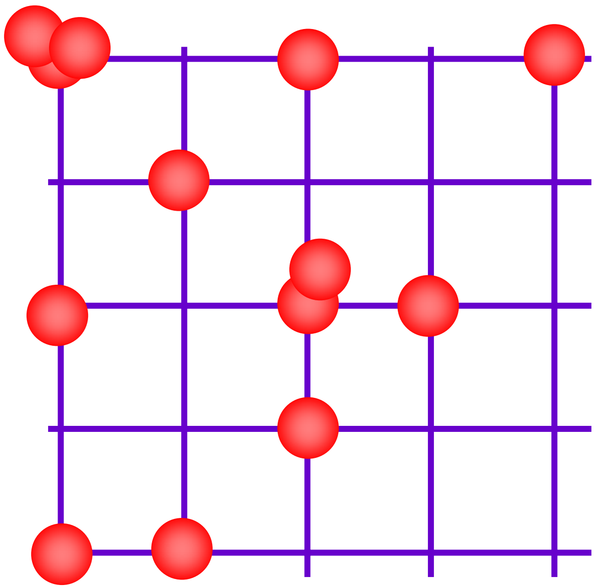
Strong interactions: insulator

Bosons at filling fraction $f = 1/2$



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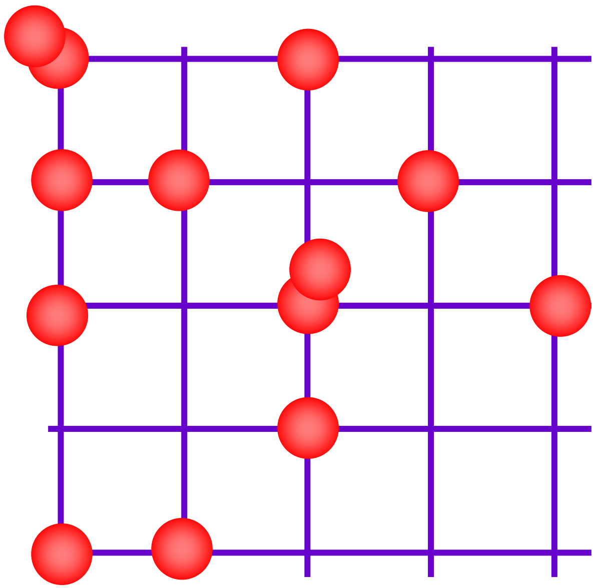
Weak interactions: superfluidity

Bosons at filling fraction f = 1/2



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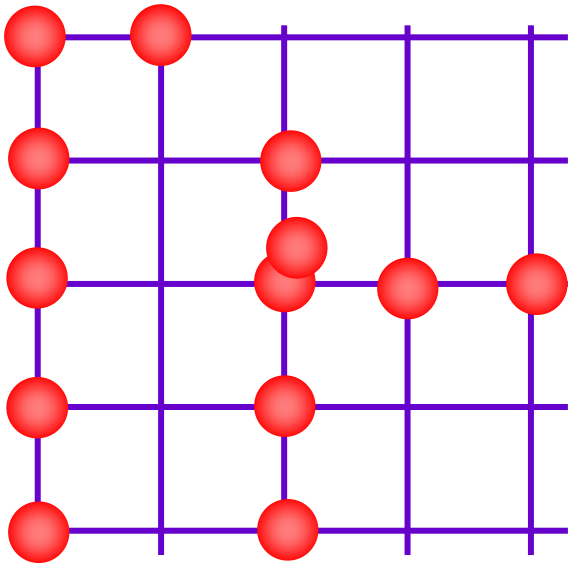

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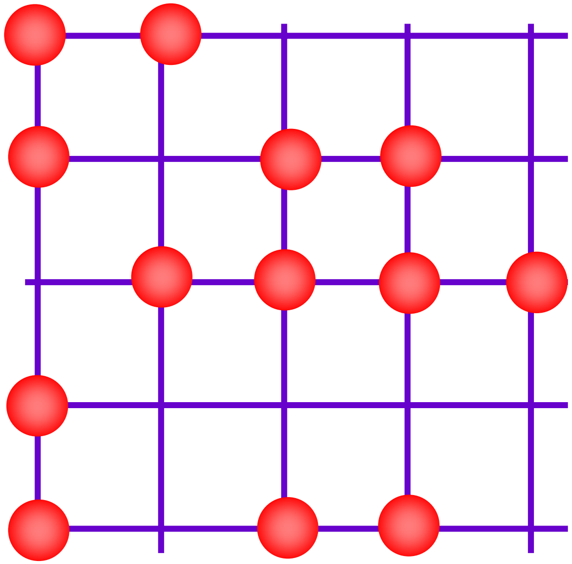

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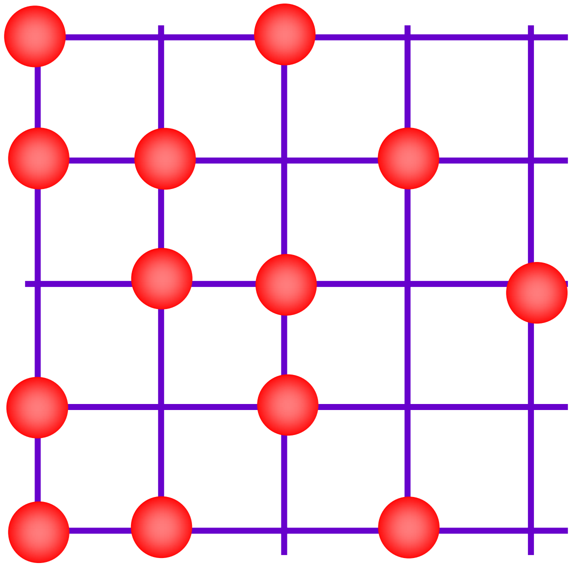

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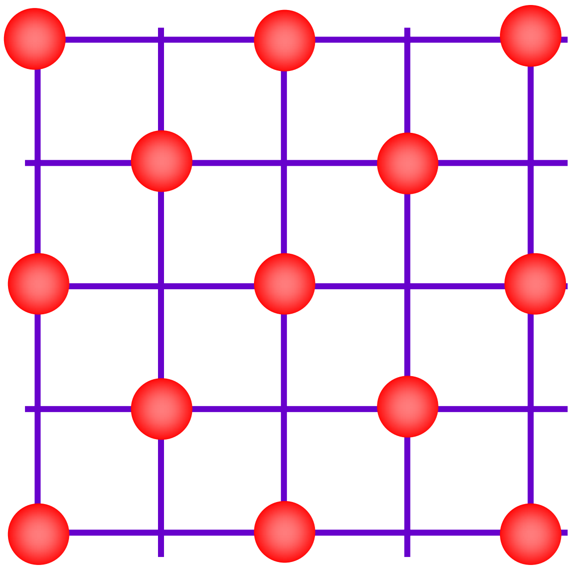

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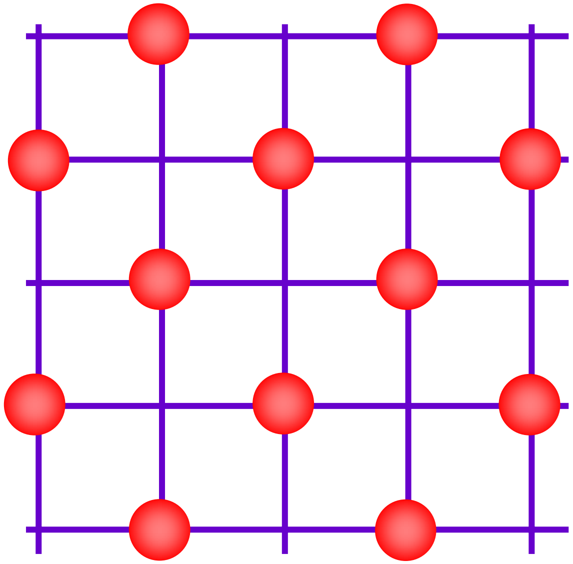

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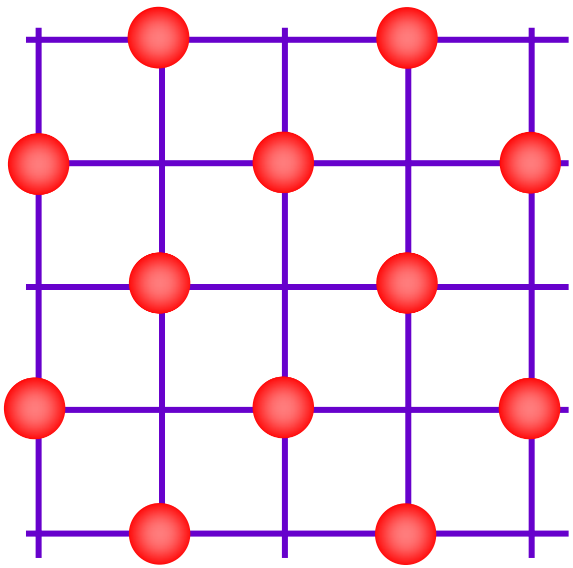

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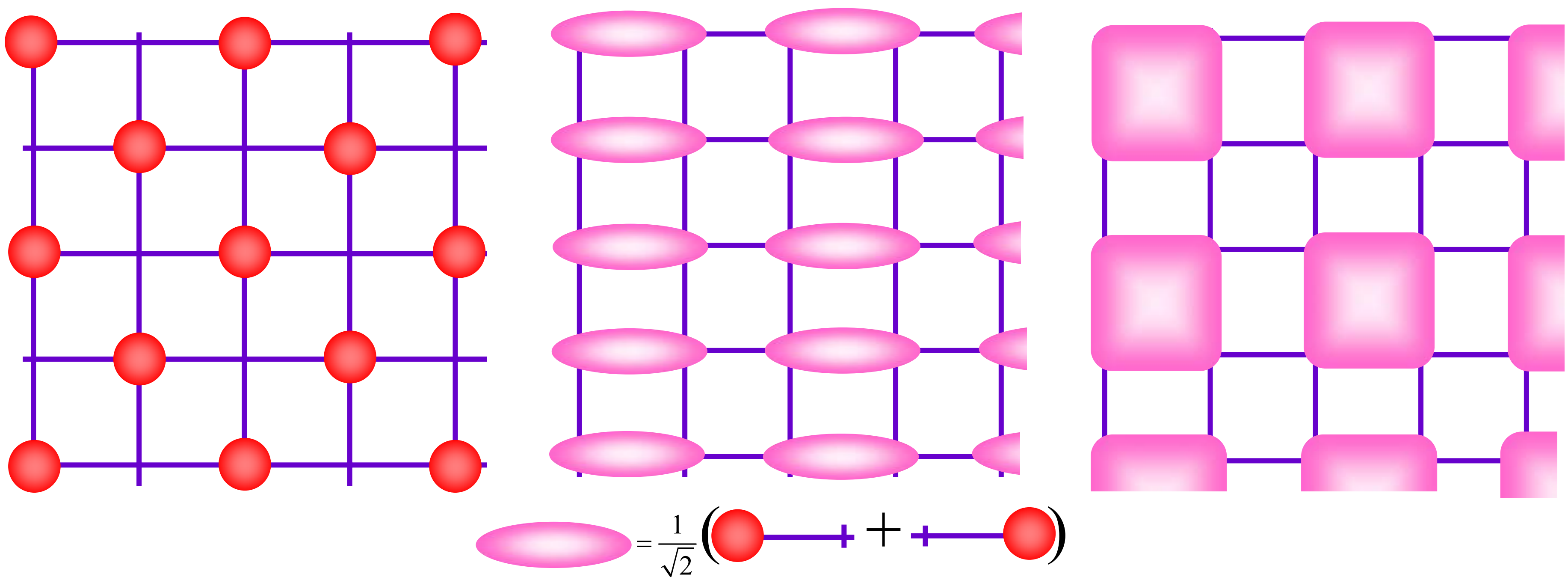
Strong interactions: insulator

Insulator has "density wave" order

Insulating phases of bosons at filling fraction $f = 1/2$



Charge density wave (CDW) order	Valence bond solid (VBS) order	Valence bond solid (VBS) order

Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi_{\text{sf}} \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\boldsymbol{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

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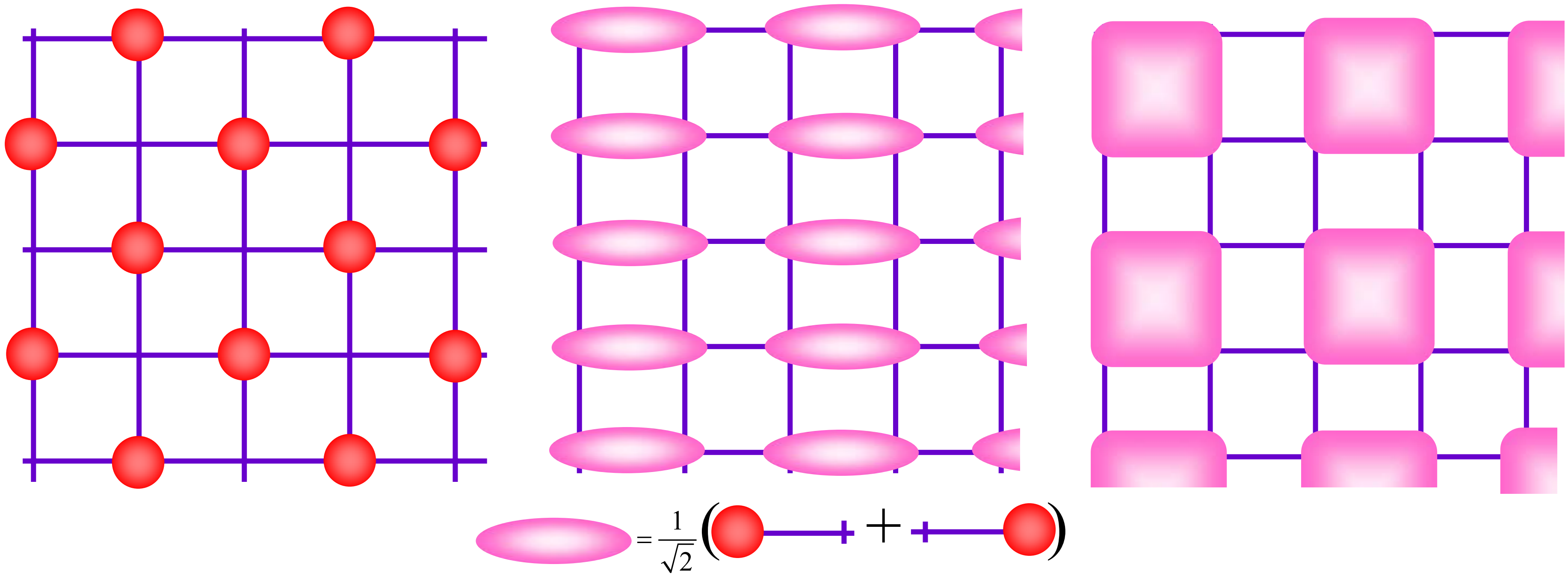

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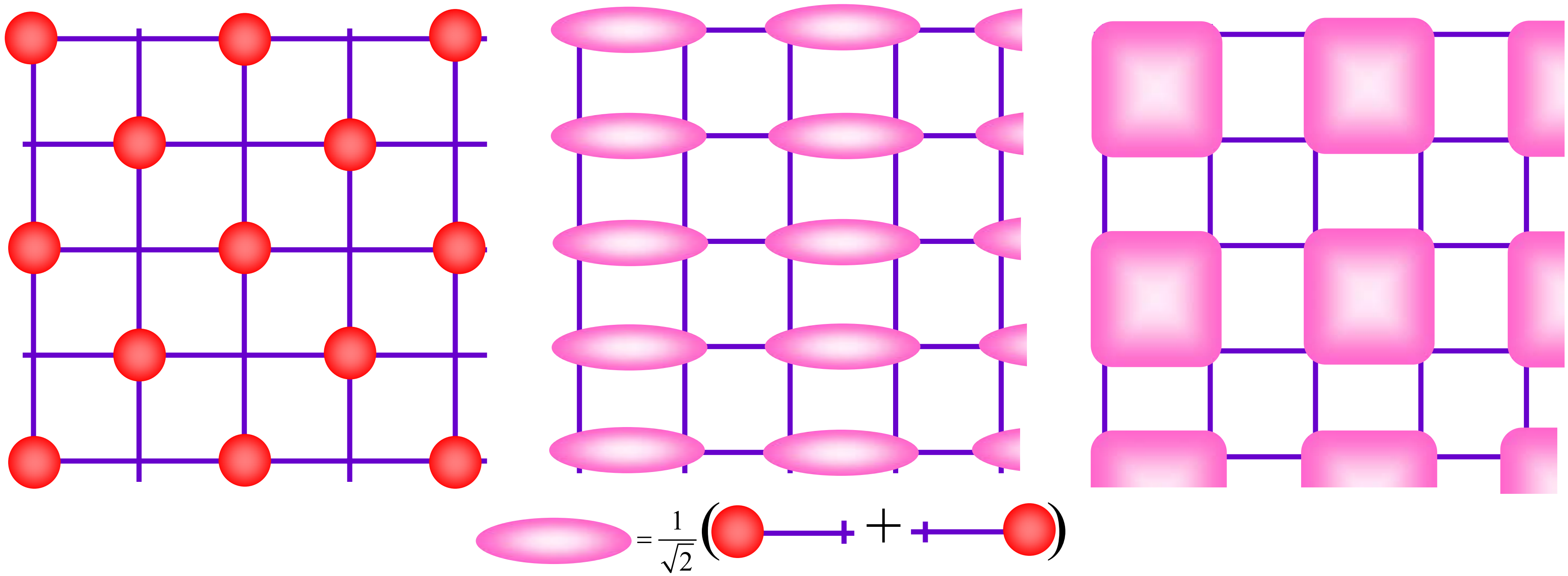

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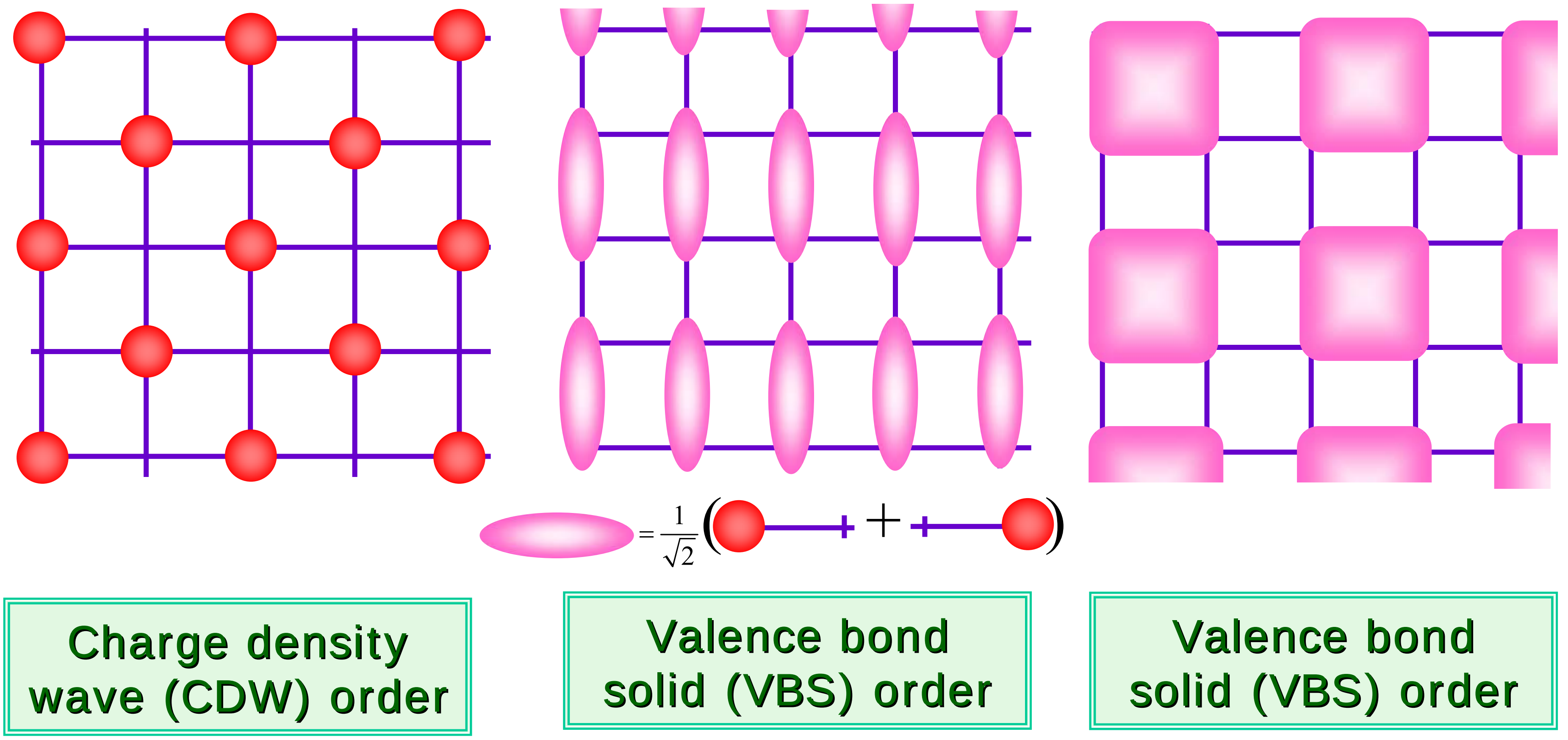

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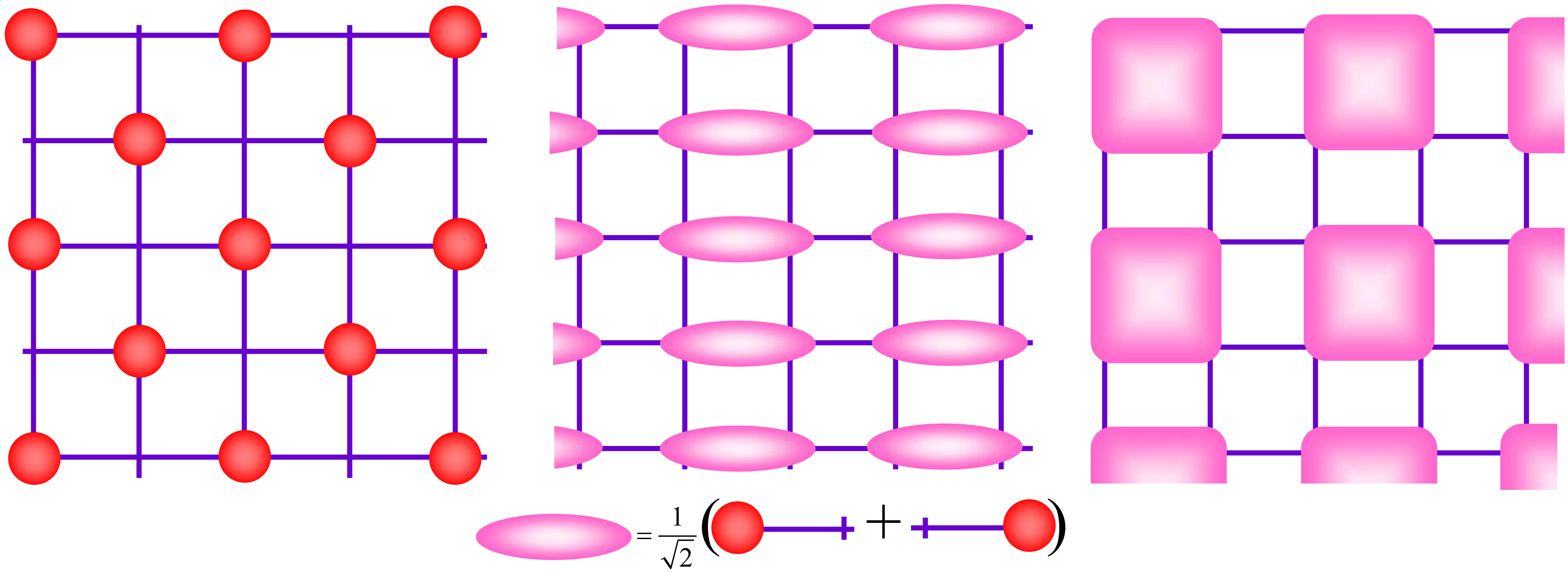

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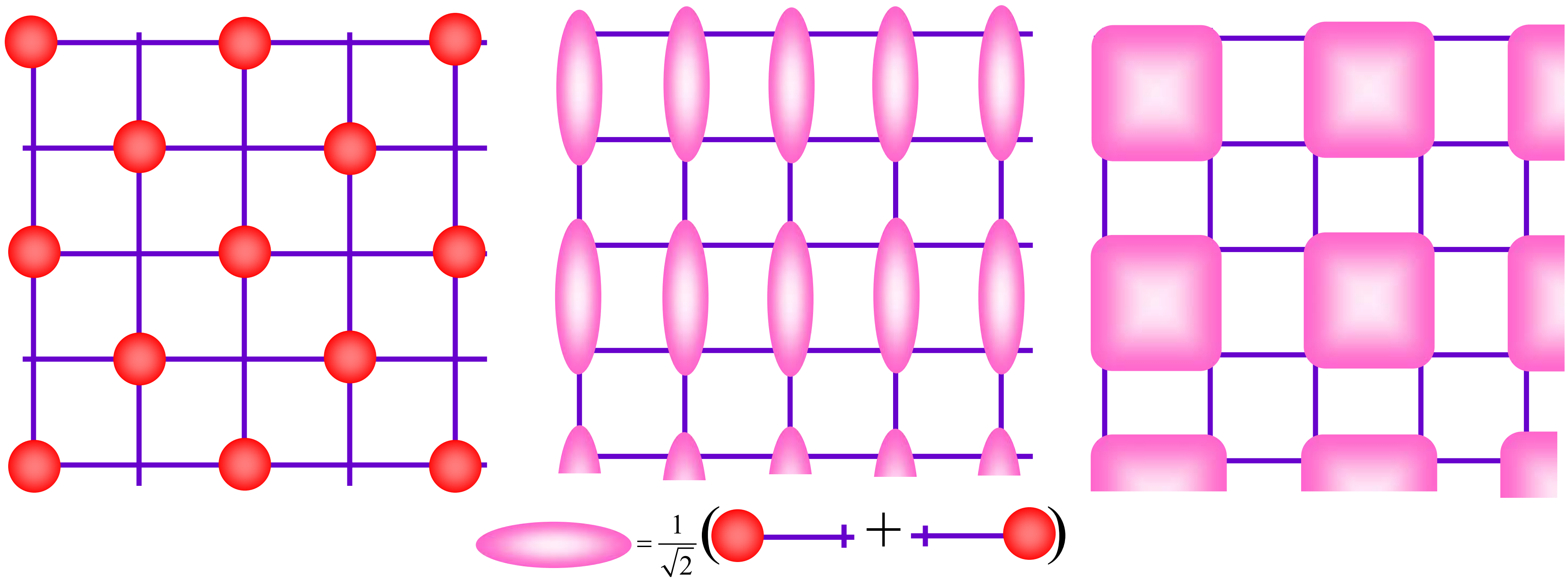

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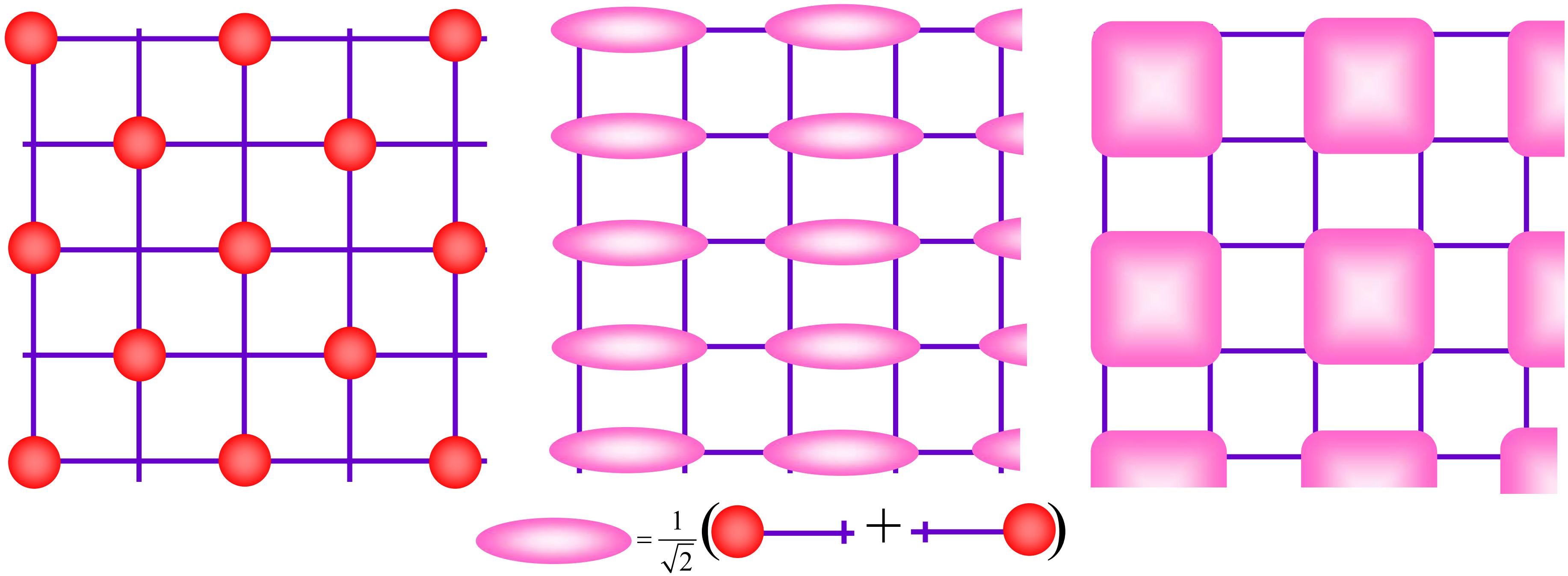

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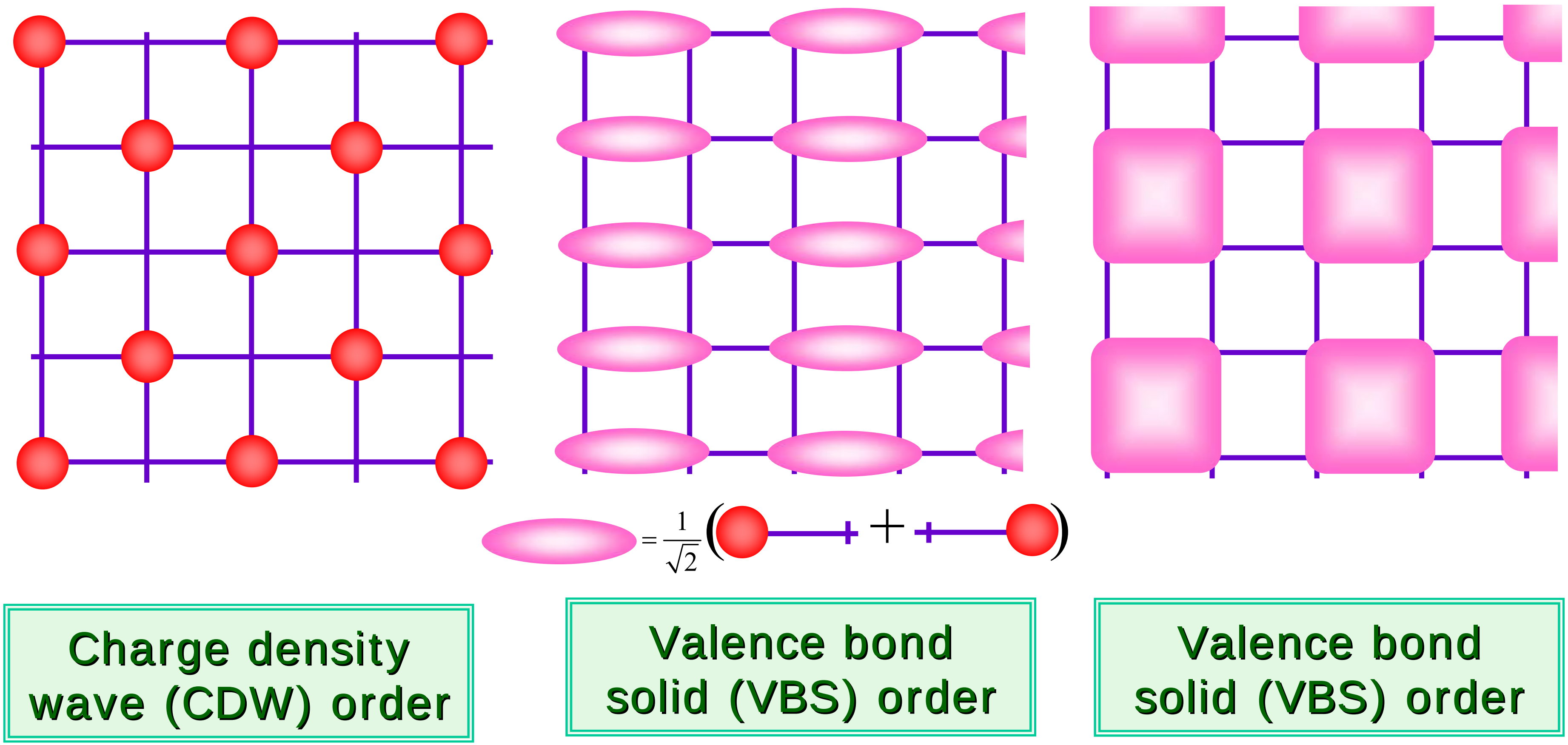

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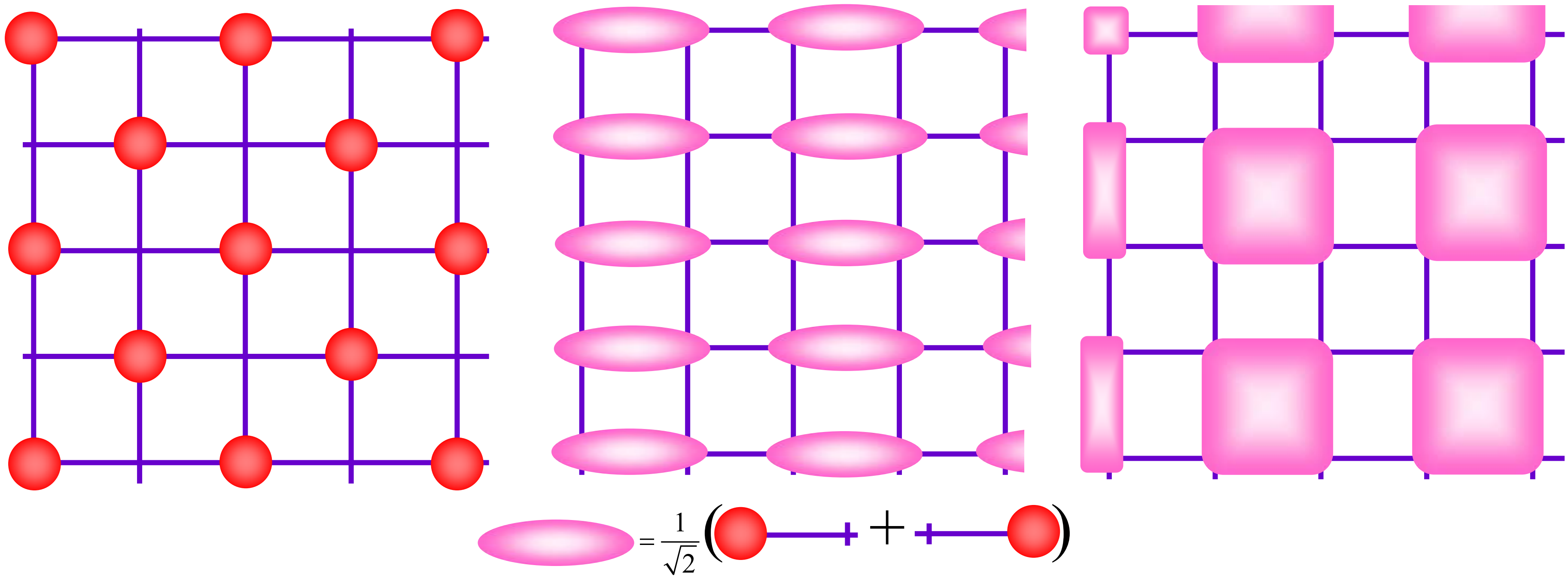

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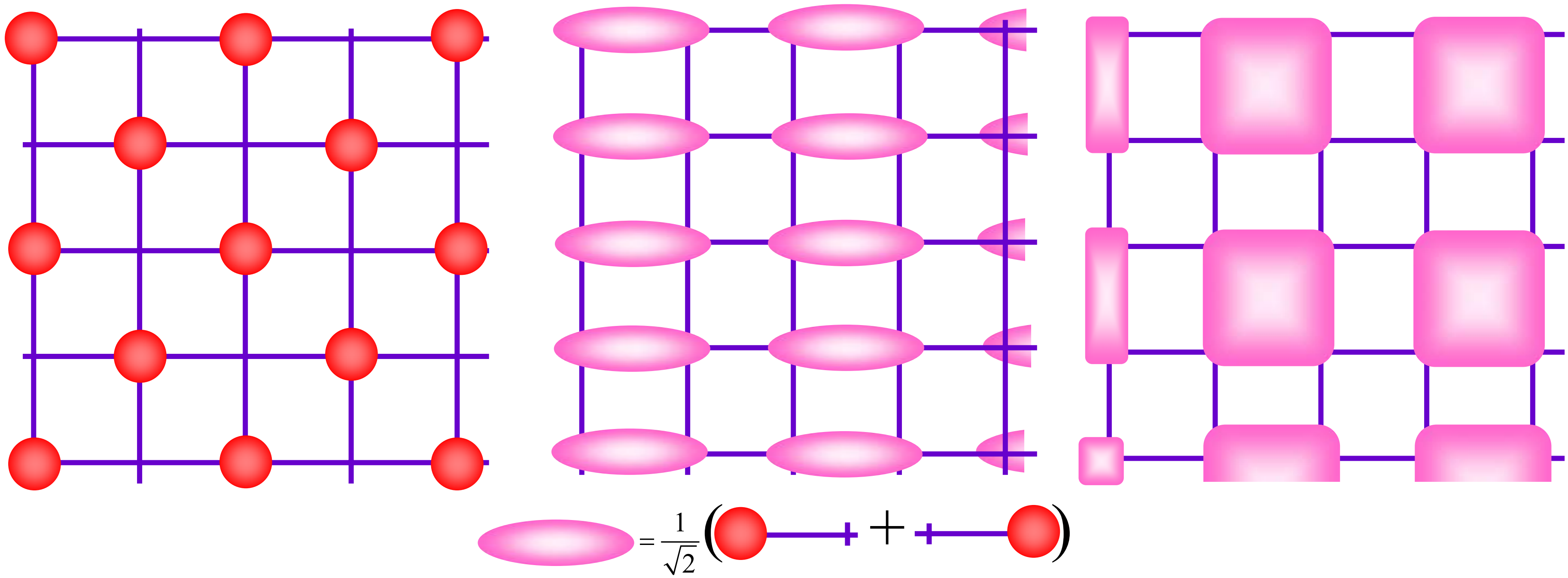

$$= \frac{1}{\sqrt{2}}\left(\text{(boson on left site)} + \text{(boson on right site)} \right)$$

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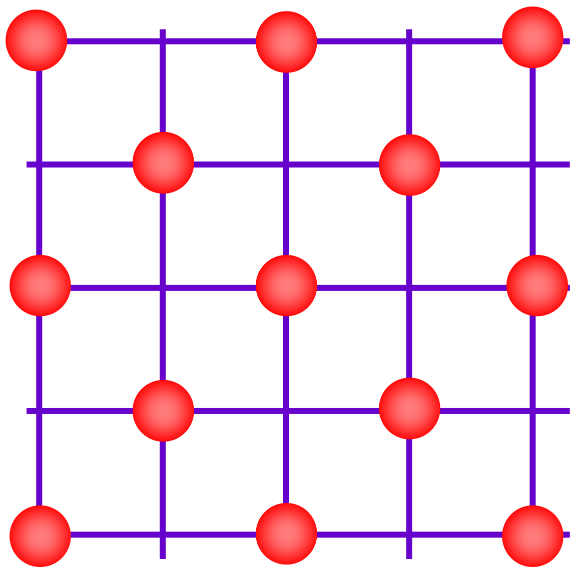

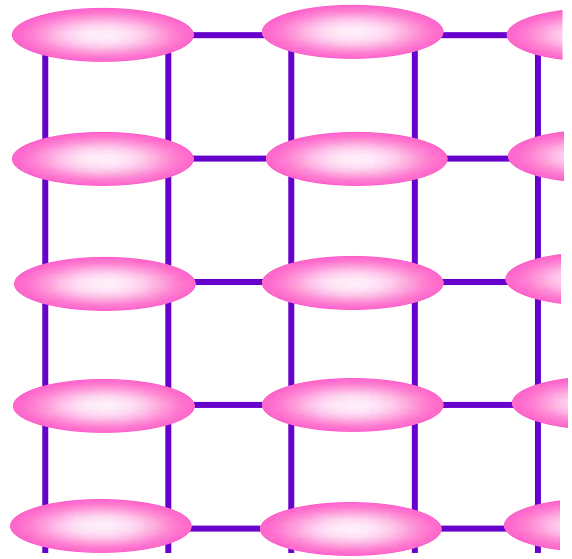

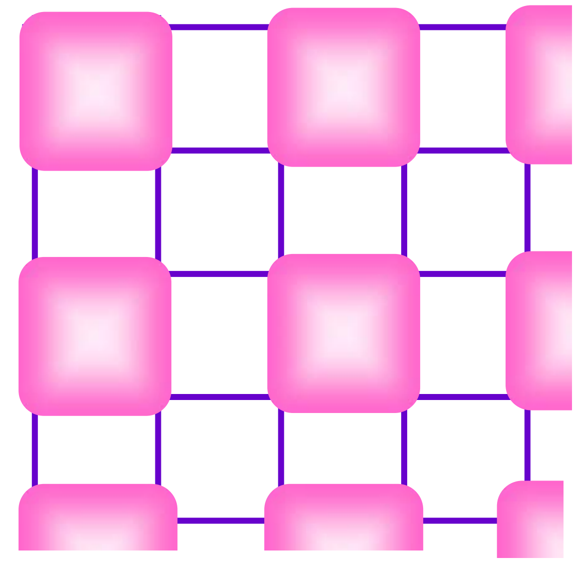

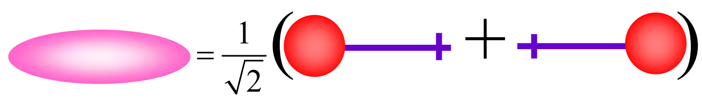

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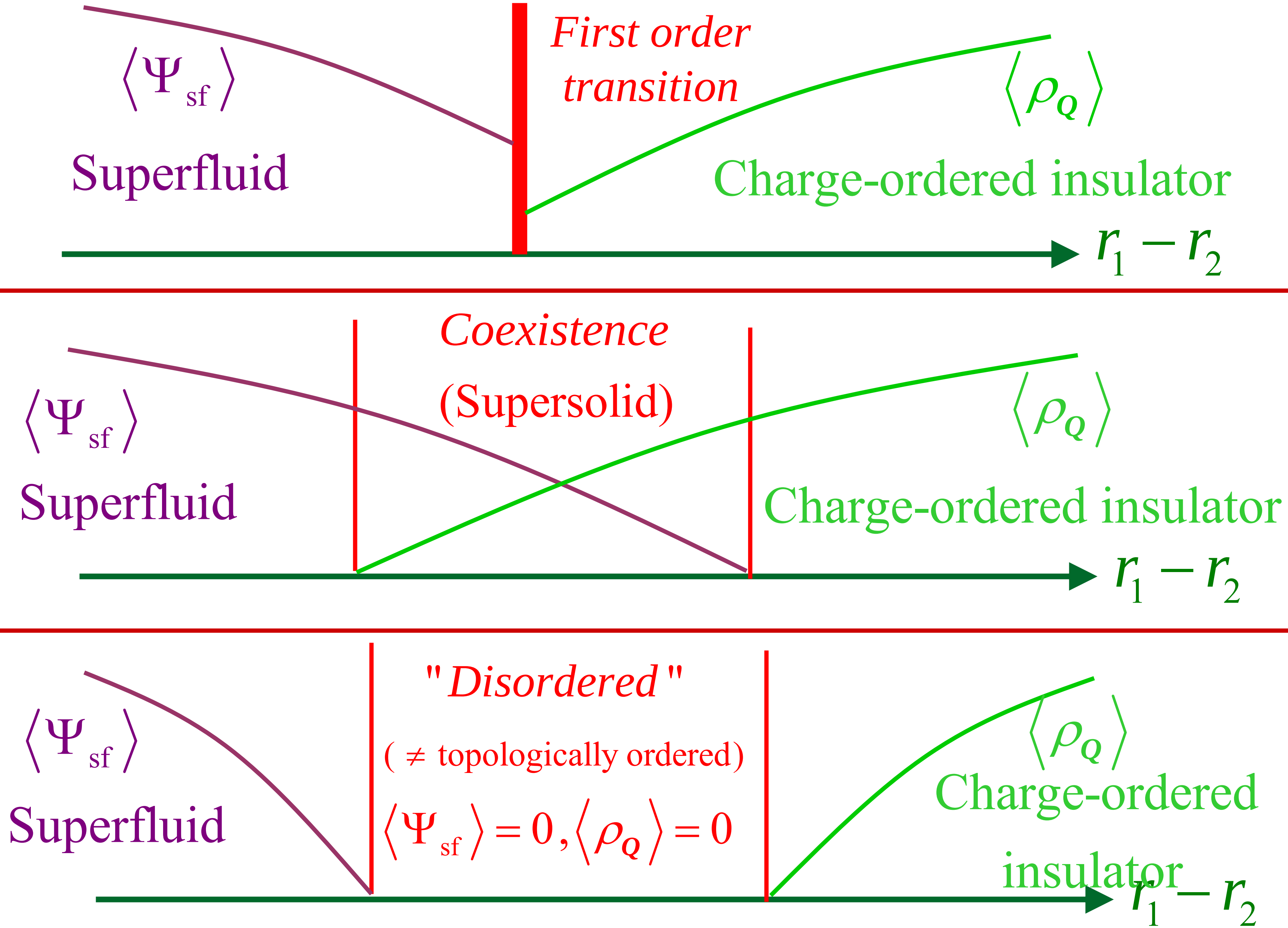
Predictions of LGW theory



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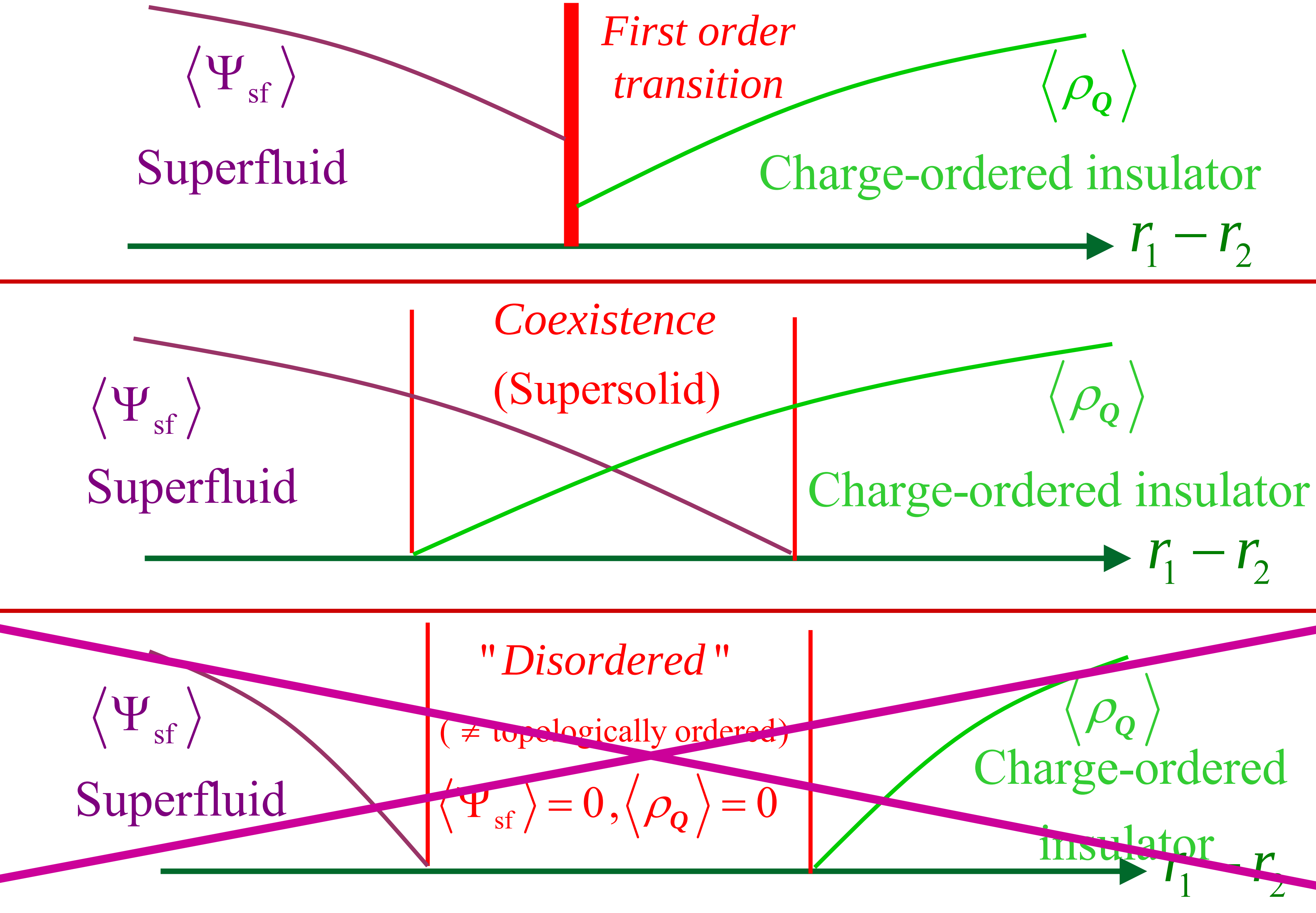


Outline

I. Statement of the problem

A. Antiferromagnets

B. Boson lattice models

II. Theory of defects: vortices near the superfluid-insulator transition

Berry phases imply that vortices carry "flavor"

III. The cuprate superconductors

Detection of vortex flavors ?

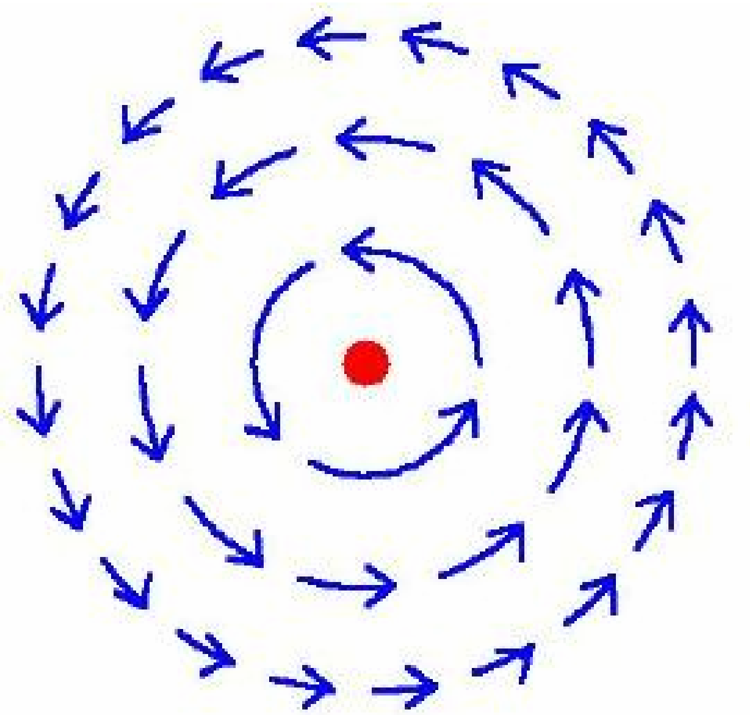
IV. Defects in the antiferromagnet

Hedgehog Berry phases and VBS order

II. Theory of defects: vortices near the superfluid-insulator transition

Berry phases imply that vortices carry "flavor"

Excitations of the superfluid: **Vortices**

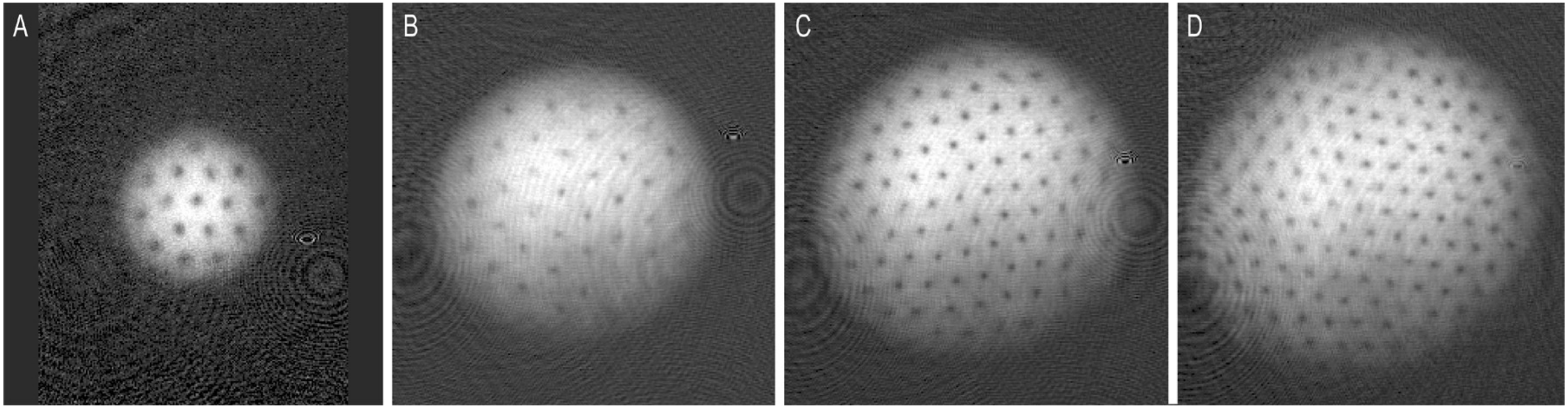


The circulation of a vortex is quantized:

$$\oint \mathbf{v}_s \cdot d\mathbf{r} = \frac{\hbar}{m} \oint \nabla\theta \cdot d\mathbf{r} = n\frac{h}{m}$$

where n is an integer.

Observation of quantized vortices in rotating ultracold Na



J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle,
Observation of Vortex Lattices in Bose-Einstein Condensates,
Science **292**, 476 (2001).

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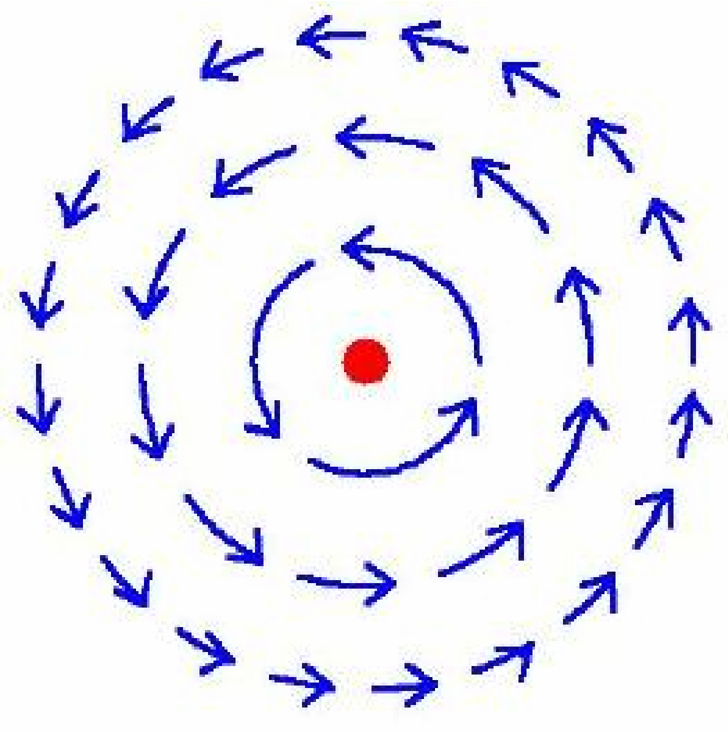
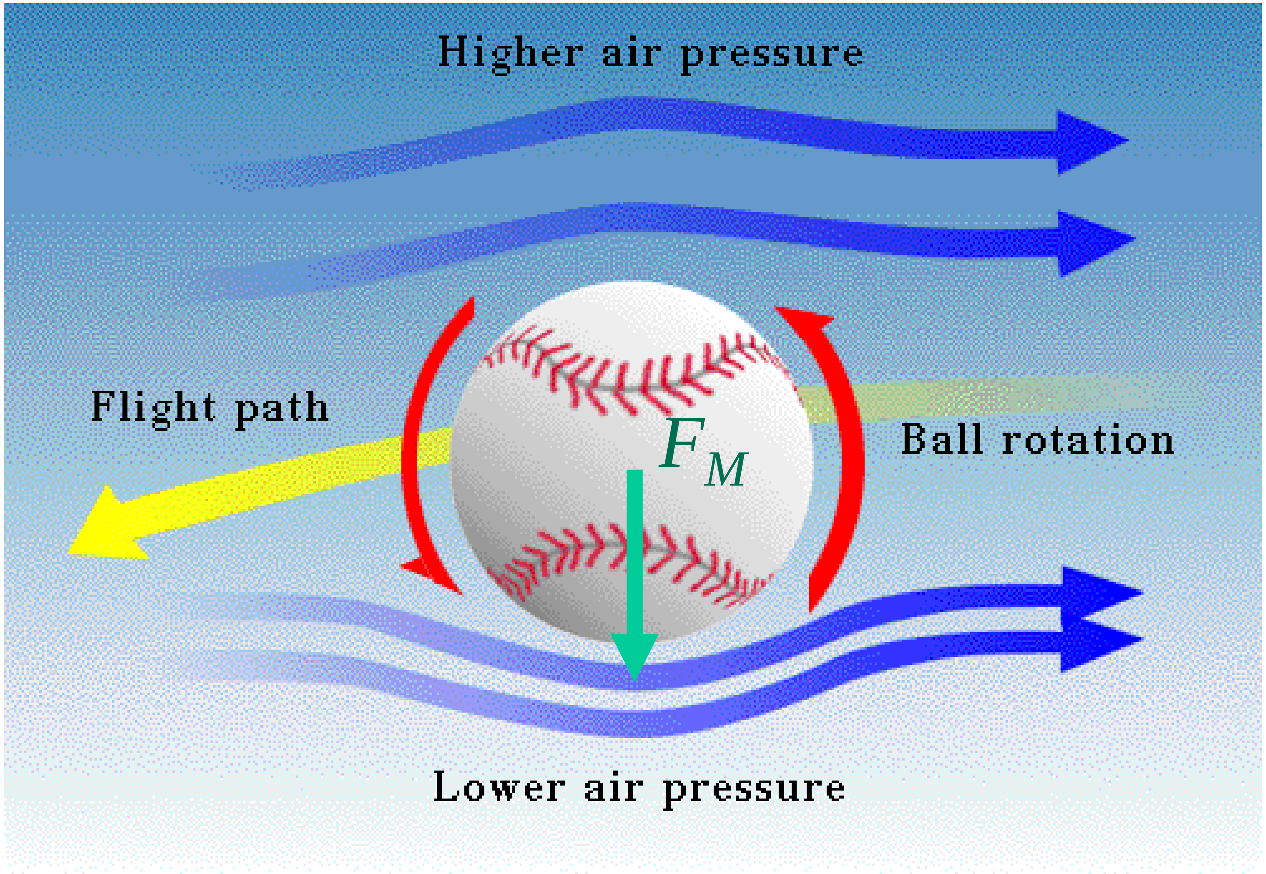


<u>Central question</u>:
In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these "particles" ?

In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of air}) \bullet (\text{velocity of ball}) \bullet (\text{circulation})$$

For a vortex in a superfluid, this is

$$\begin{aligned}\mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s\cdot d\mathbf{r}\right)\\ &= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\end{aligned}$$

where ρ = number density of bosons

$\mathbf{v}_s$ = local velocity of superfluid

$\mathbf{r}_v$ = position of vortex

For a vortex in a superfluid, this is

$$\mathbf{F}_M = (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s \cdot d\mathbf{r}\right)$$

$$= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right) \times \hat{\mathbf{z}}$$

$$= n\left(\mathbf{E} + \frac{d\mathbf{r}_v}{dt} \times \mathbf{B}\right)$$

where $\mathbf{E} = \rho\mathbf{v}_s \times \hat{\mathbf{z}}$ and $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

Dual picture:

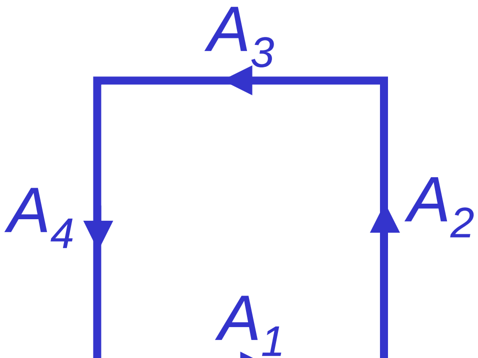
The vortex is a quantum particle with dual "electric" charge *n*, moving in a dual "magnetic" field of strength = *h*×(number density of Bose particles)

- The vortices are quantum particles moving in a periodic potential with the symmetry of the square lattice, and in the presence of a dual "magnetic" field of strength $= h\rho$, where ρ is the number density of bosons per unit cell.

- The vortex motion can be described by the effective Hofstadter Hamiltonian:

$$\mathcal{H}_v = -t \sum_{\langle ij \rangle} \left(e^{iA_{ij}} \varphi_i^* \varphi_j + \text{c.c.} \right)$$

where φ_i is an operator which annihilates a vortex particle at site i of a square lattice.



$$A_1 + A_2 + A_3 + A_4 = 2\pi f$$

where f is the boson filling fraction.

Bosons at filling fraction $f = 1$

- At f=1, the "magnetic" flux per unit cell is 2π, and the vortex does not pick up any phase from the boson density.

- The effective dual "magnetic" field acting on the vortex is zero, and the corresponding component of the Magnus force vanishes.

Bosons at rational filling fraction f=p/q

Quantum mechanics of the vortex "particle" in a periodic potential with *f* flux quanta per unit cell

Space group symmetries of Hofstadter Hamiltonian:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

Magnetic space group:

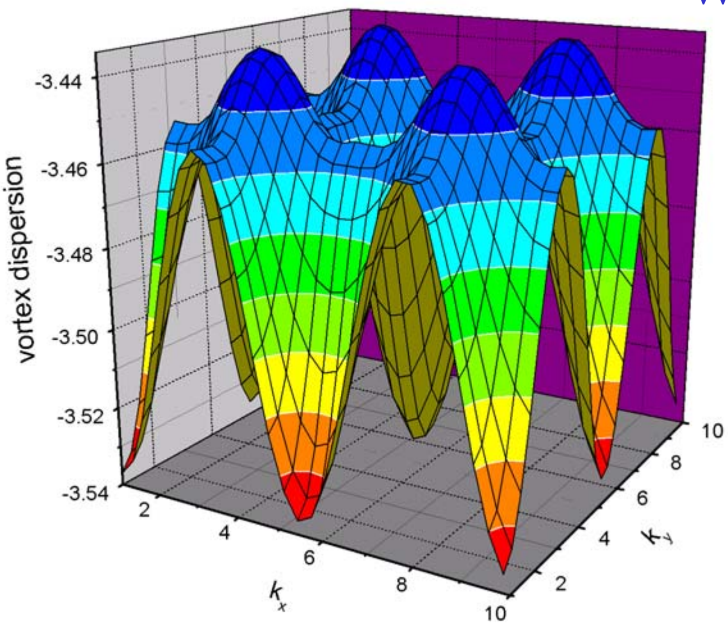
$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

The low energy vortex states must form a representation of this algebra

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

Hofstadter spectrum of the quantum vortex "particle" with field operator φ



At filling $f=p/q$, there are q species of vortices, φ_ℓ (with $\ell=1\ldots q$), associated with q degenerate minima in the vortex spectrum. These vortices realize the smallest, q-dimensional, representation of the magnetic algebra.

$$T_x : \varphi_\ell \rightarrow \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \rightarrow e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \rightarrow \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

Boson-vortex duality

The q φ_ℓ vortices characterize *both* superconducting and density wave orders

$$\underline{\text{Superconductor/insulator}} : \left\langle \varphi_\ell \right\rangle = 0 \Big/ \left\langle \varphi_\ell \right\rangle \neq 0$$

Boson-vortex duality

The q φ_ℓ vortices characterize *both* superconducting and density wave orders

Density wave order:

Status of space group symmetry determined by density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}$$

$$T_x : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{x}} \quad ; \quad T_y : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{y}}$$

$$R : \rho(\mathbf{Q}) \to \rho(R\mathbf{Q})$$

Field theory with projective symmetry

Degrees of freedom:

q complex φ_ℓ vortex fields

1 non-compact U(1) gauge field A_μ

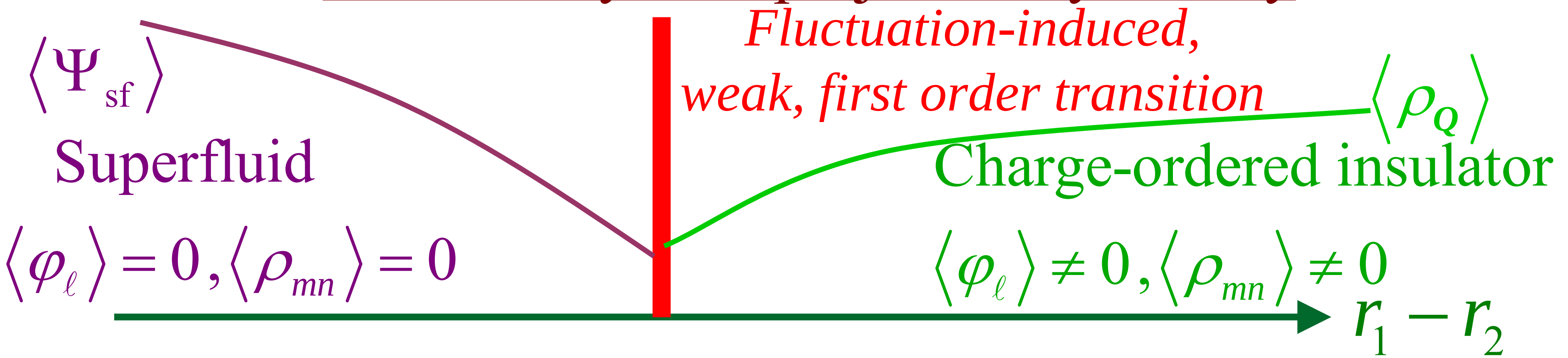
$$\mathcal{S} = \int d^2x d\tau \Big[\sum_\ell \left\{ \left|(\partial_\mu - iA_\mu)\varphi_\ell\right|^2 + s|\varphi_\ell|^2 \right\} + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 + \sum_{\ell mn} \gamma_{mn}\varphi_\ell^* \varphi_{\ell+m}^* \varphi_{\ell+n}\varphi_{\ell+m-n} \Big]$$

The projective symmetries constrain the couplings γ_{mn} to obey

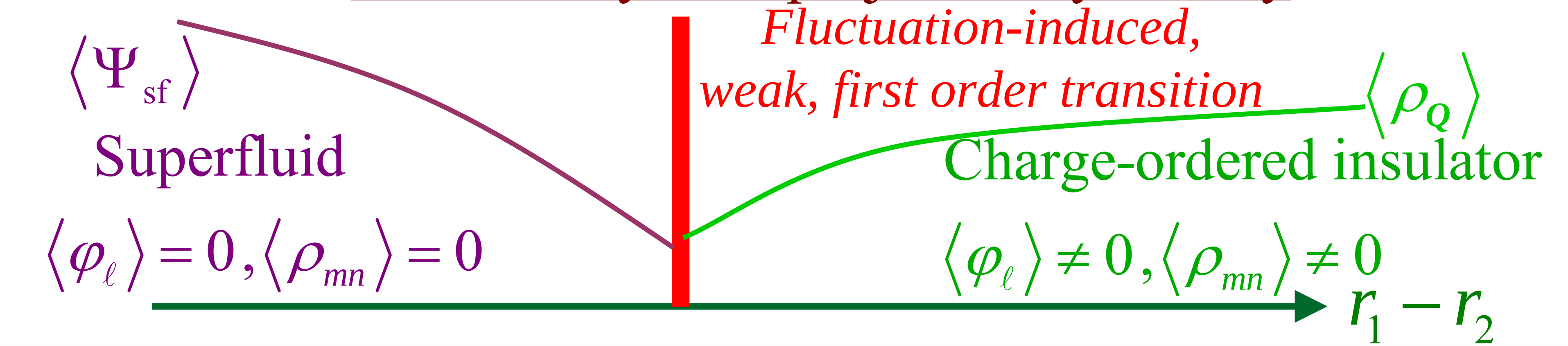
$$\gamma_{mn} = \gamma_{-m,-n} \;\; ; \;\; \gamma_{mn} = \gamma_{m,m-n} \;\; ; \;\; \gamma_{mn} = \gamma_{m-2n,-n}$$

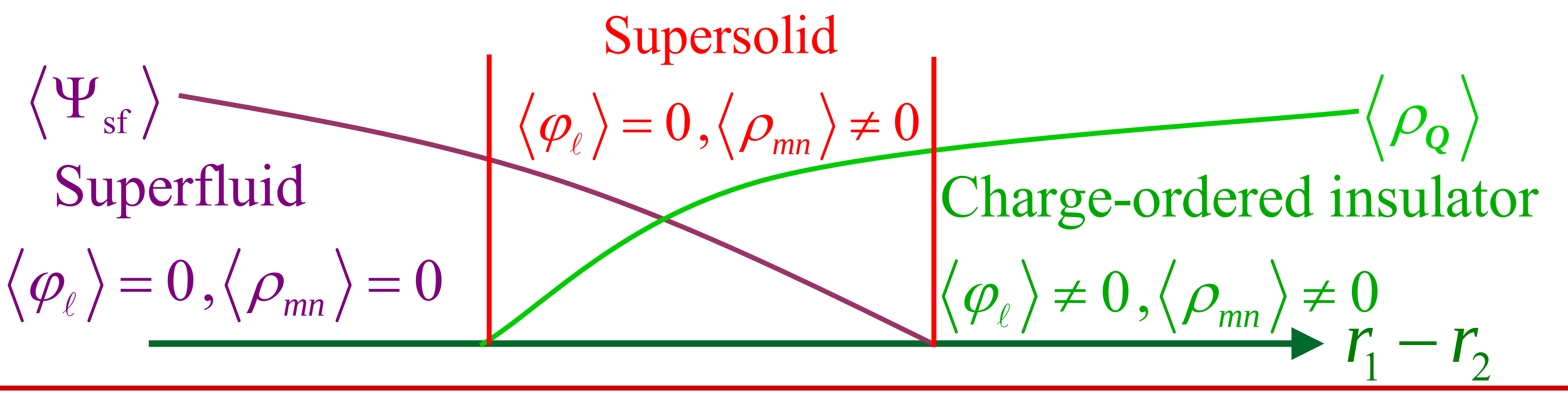
$$\gamma_{\bar{m}\bar{n}} = \frac{1}{q}\sum_{mn}\gamma_{mn} e^{-2\pi i f[n(\bar{m}-\bar{n})+\bar{n}(m-n)]}$$

Field theory with projective symmetry

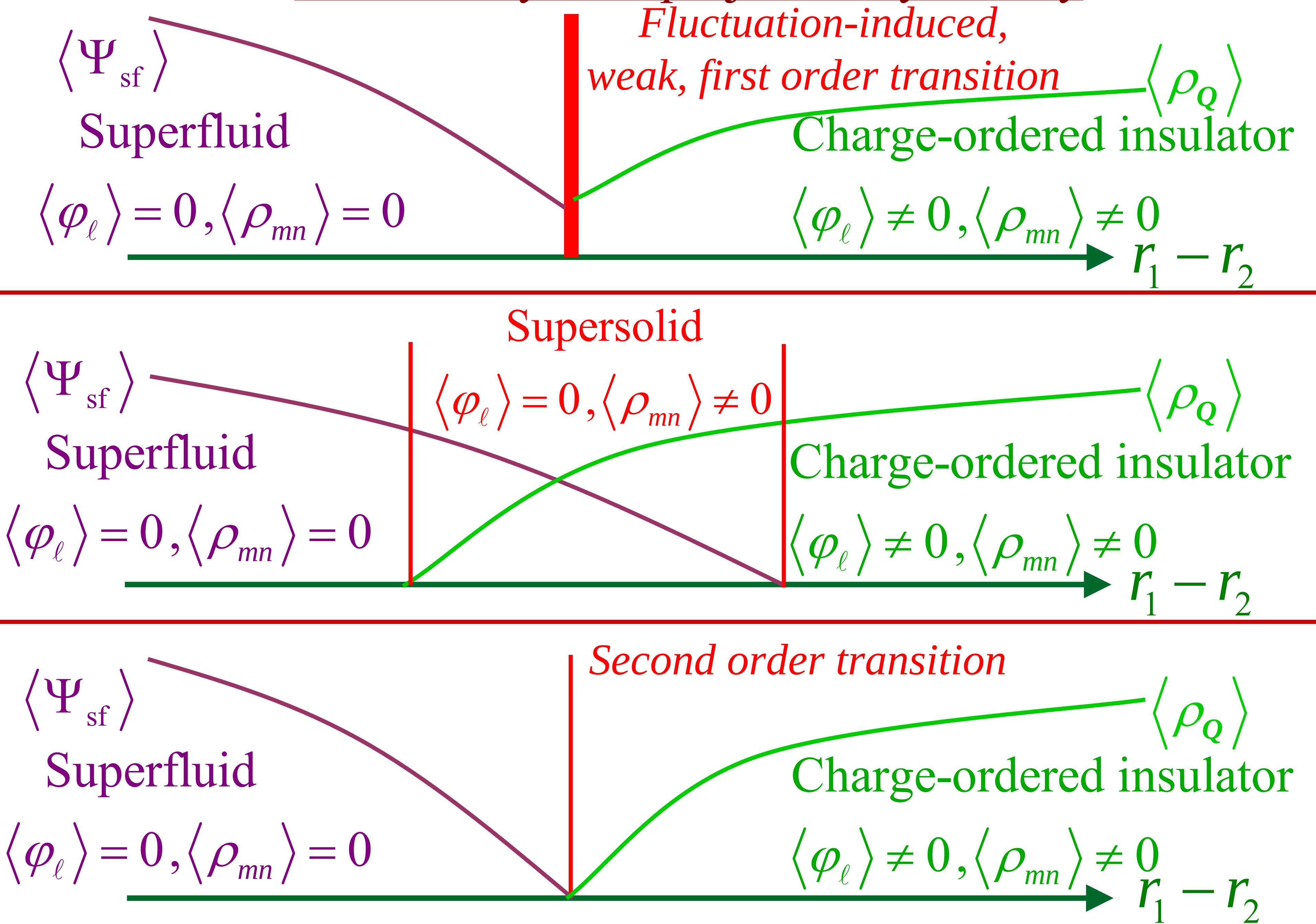


Field theory with projective symmetry





Field theory with projective symmetry

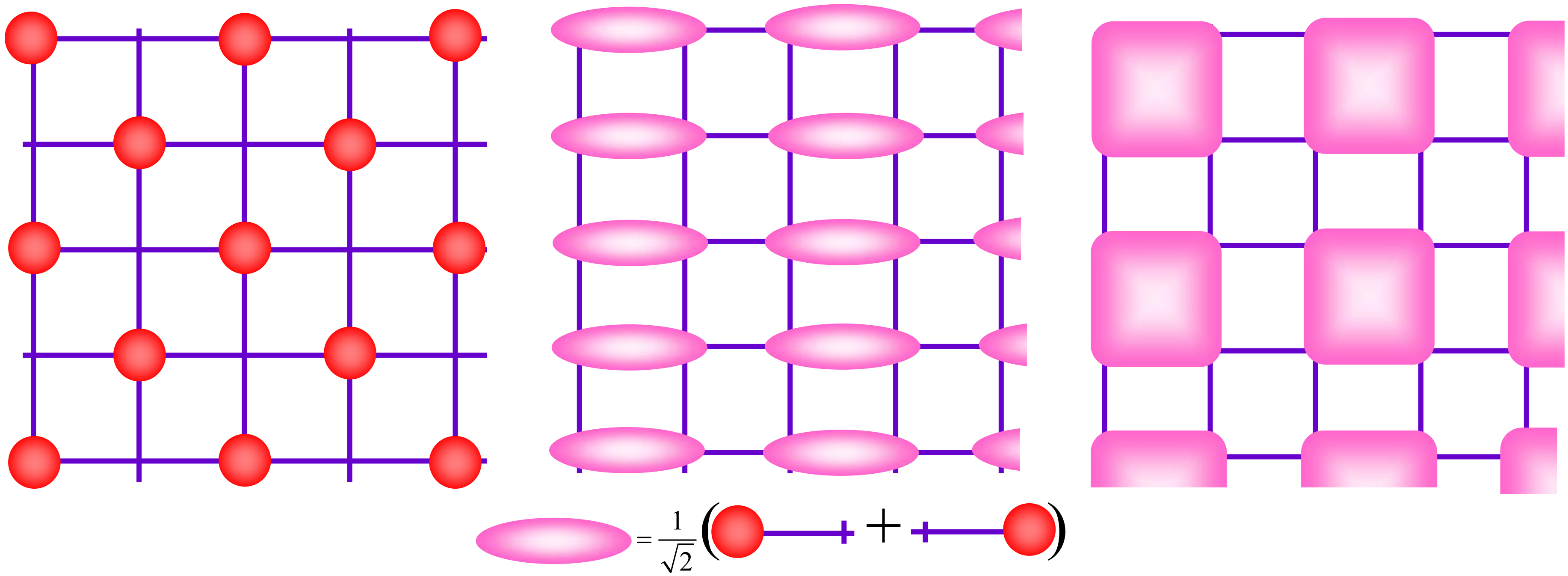


Vortices in a superfluid near a Mott insulator at filling $f=p/q$

- The excitations of the superfluid are described by the quantum mechanics of q flavors of low energy vortices moving in zero dual "magnetic" field.

- The orientation of the vortex in flavor space implies a particular configuration of density-wave order in its vicinity.

Mott insulators obtained by condensing vortices at $f = 1/2$



Charge density wave (CDW) order

Valence bond solid (VBS) order

Valence bond solid (VBS) order

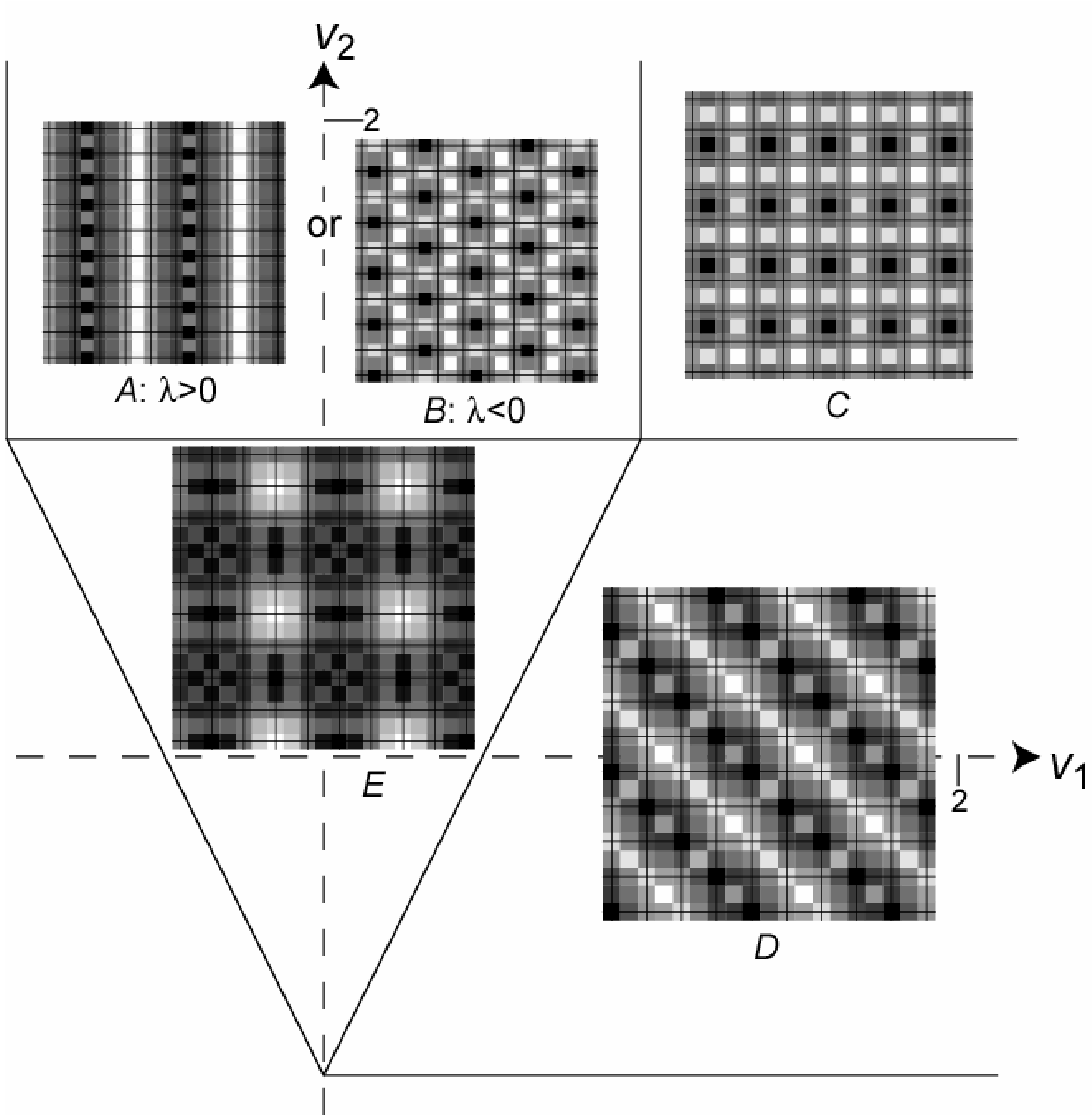
Can define a common CDW/VBS order using a generalized "density" $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$

All insulators have $\langle \Psi \rangle = 0$ and $\langle \rho_{\mathbf{Q}} \rangle \neq 0$ for certain $\mathbf{Q}$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Mott insulators obtained by condensing vortices at $f = 1/4, 3/4$



$a \times b$ unit cells;

q/a, q/b, ab/q,

all integers

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

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I. Statement of the problem

A. Antiferromagnets

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Berry phases imply that vortices carry "flavor"

III. The cuprate superconductors

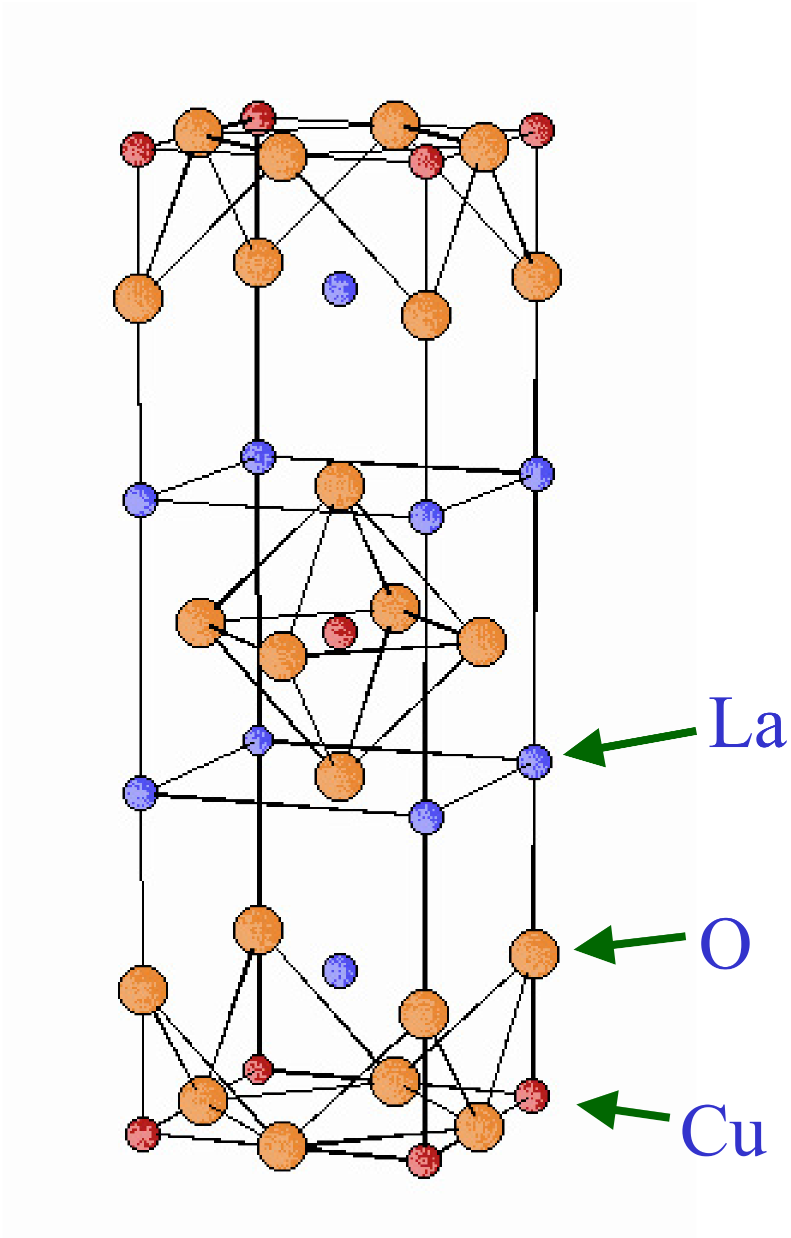
Detection of vortex flavors ?

IV. Defects in the antiferromagnet

Hedgehog Berry phases and VBS order

III. The cuprate superconductors

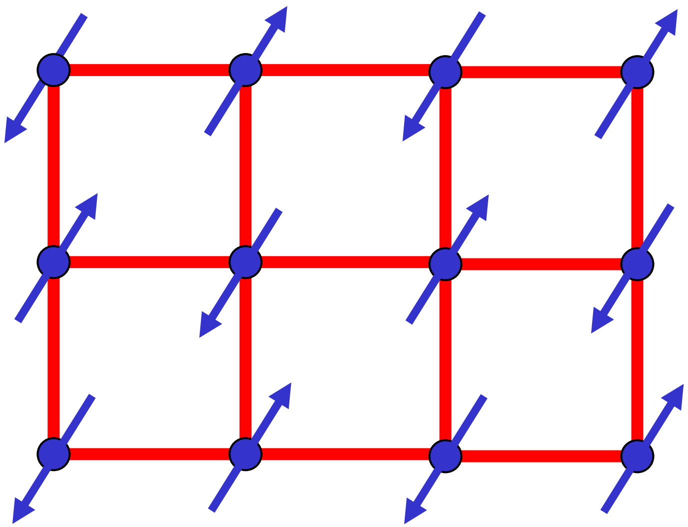
Detection of dual vortex wavefunction in STM experiments ?



La_2CuO_4

La_2CuO_4

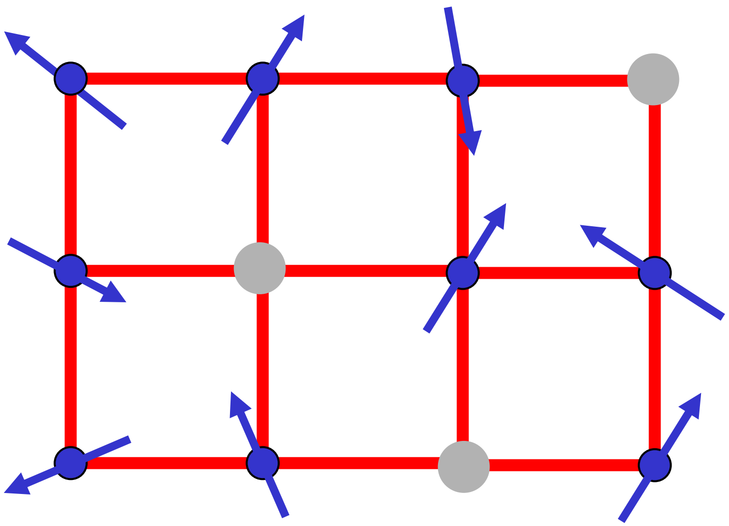
Mott insulator: square lattice antiferromagnet



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$La_{2-\delta}Sr_{\delta}CuO_4$

Superfluid: condensate of paired holes



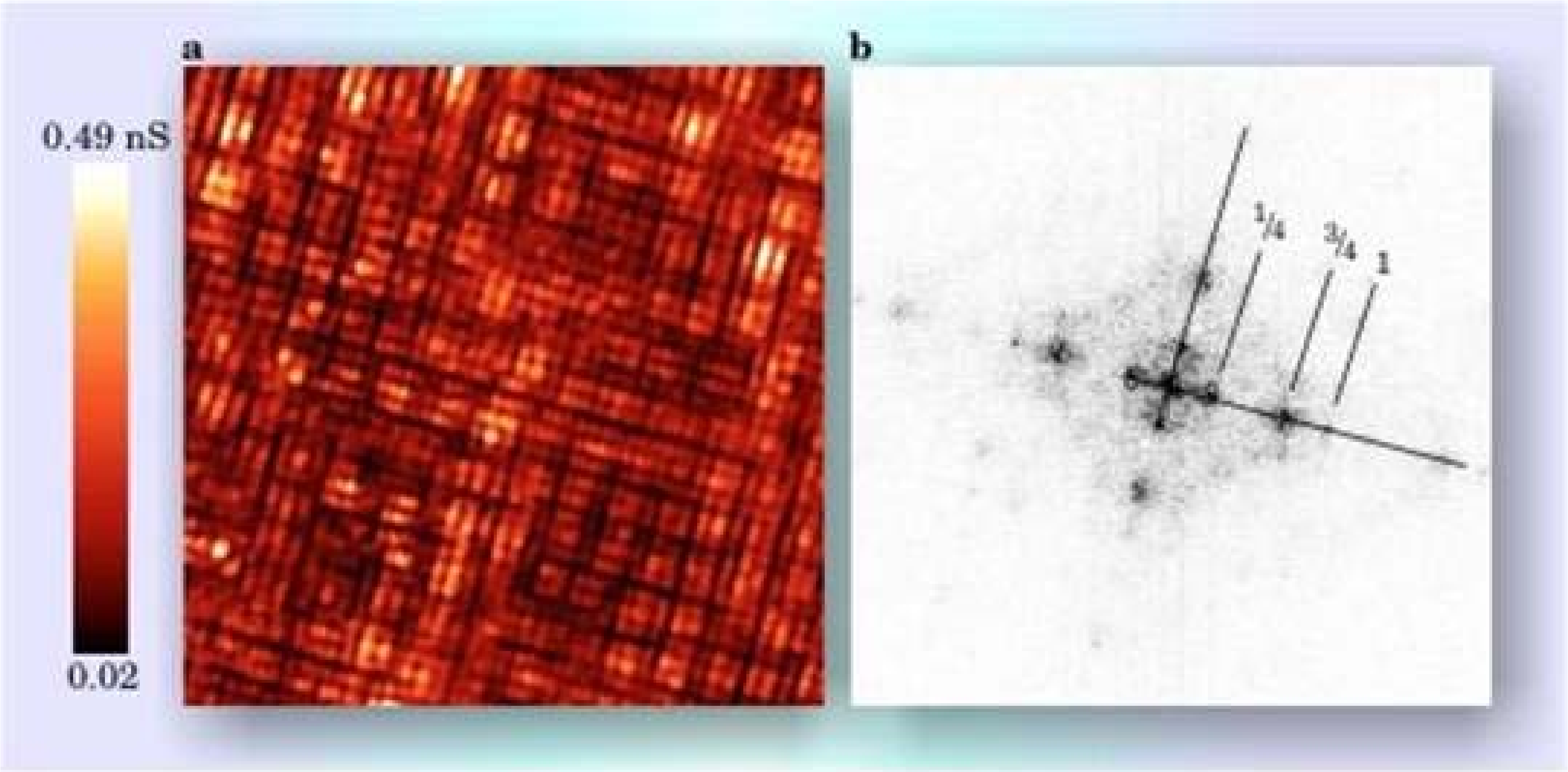
$$\langle \vec{S} \rangle = 0$$

<u>Many</u> experiments on the cuprate superconductors show:

- Tendency to produce modulations in spin singlet observables at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

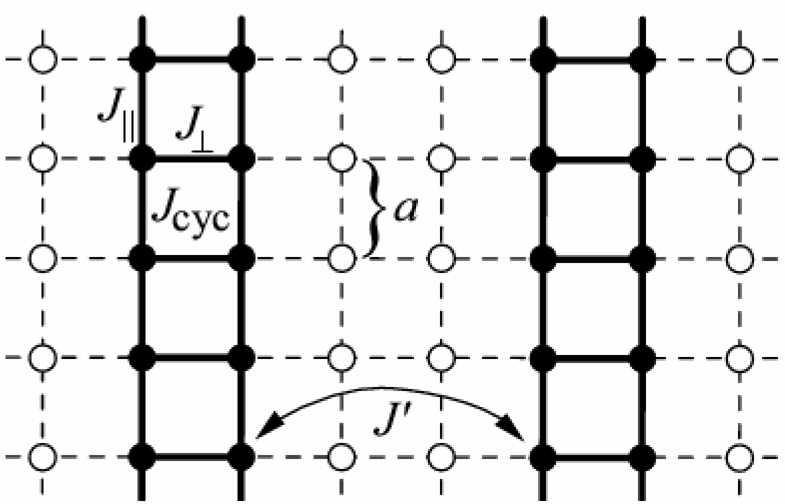
- Proximity to a Mott insulator at hole density $\delta=1/8$ with long-range charge modulations at wavevectors $(2\pi/a)(1/4,0)$ and $(2\pi/a)(0,1/4)$.

The cuprate superconductor $Ca_{2-x}Na_xCuO_2Cl_2$

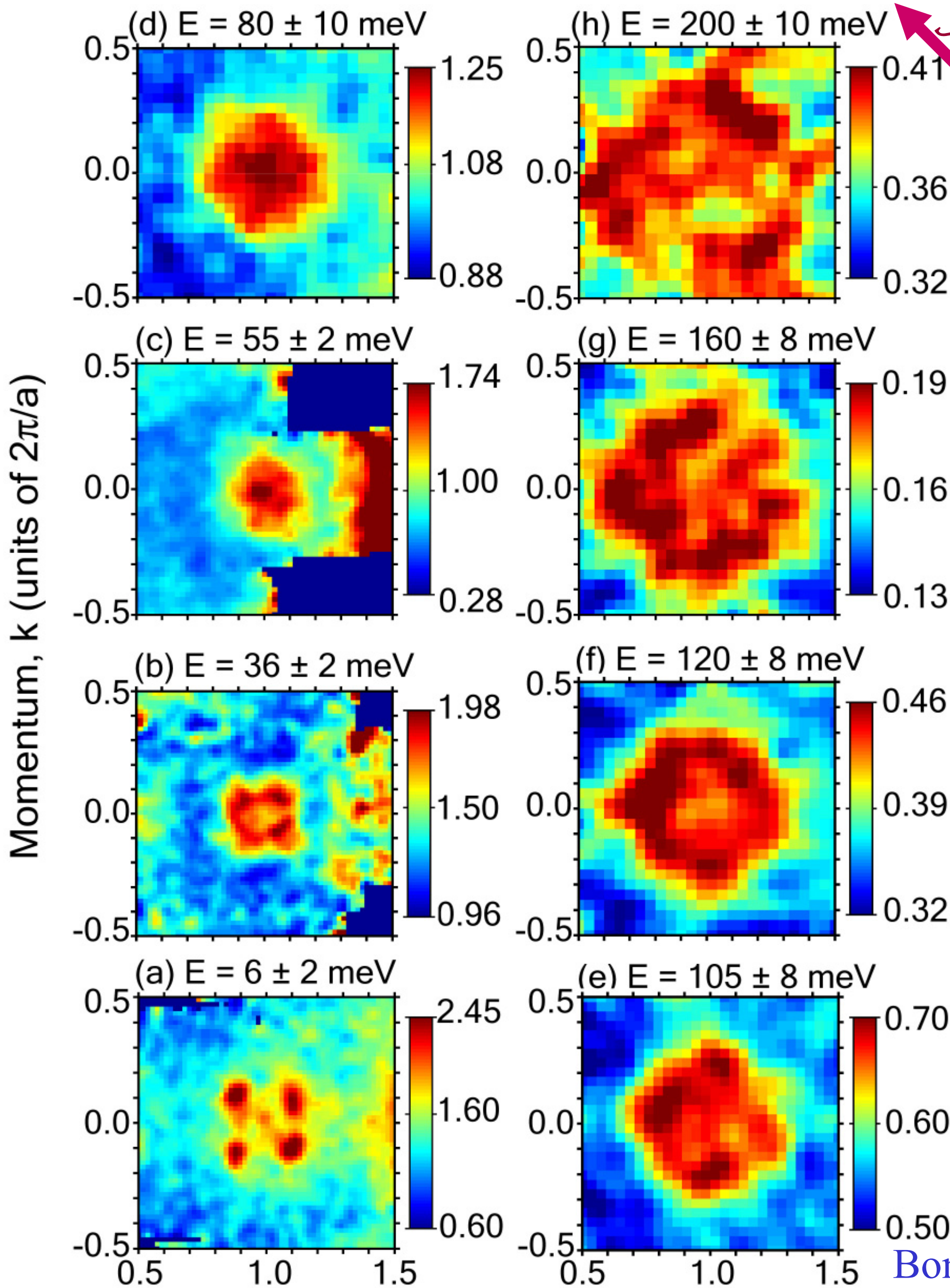


T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004).

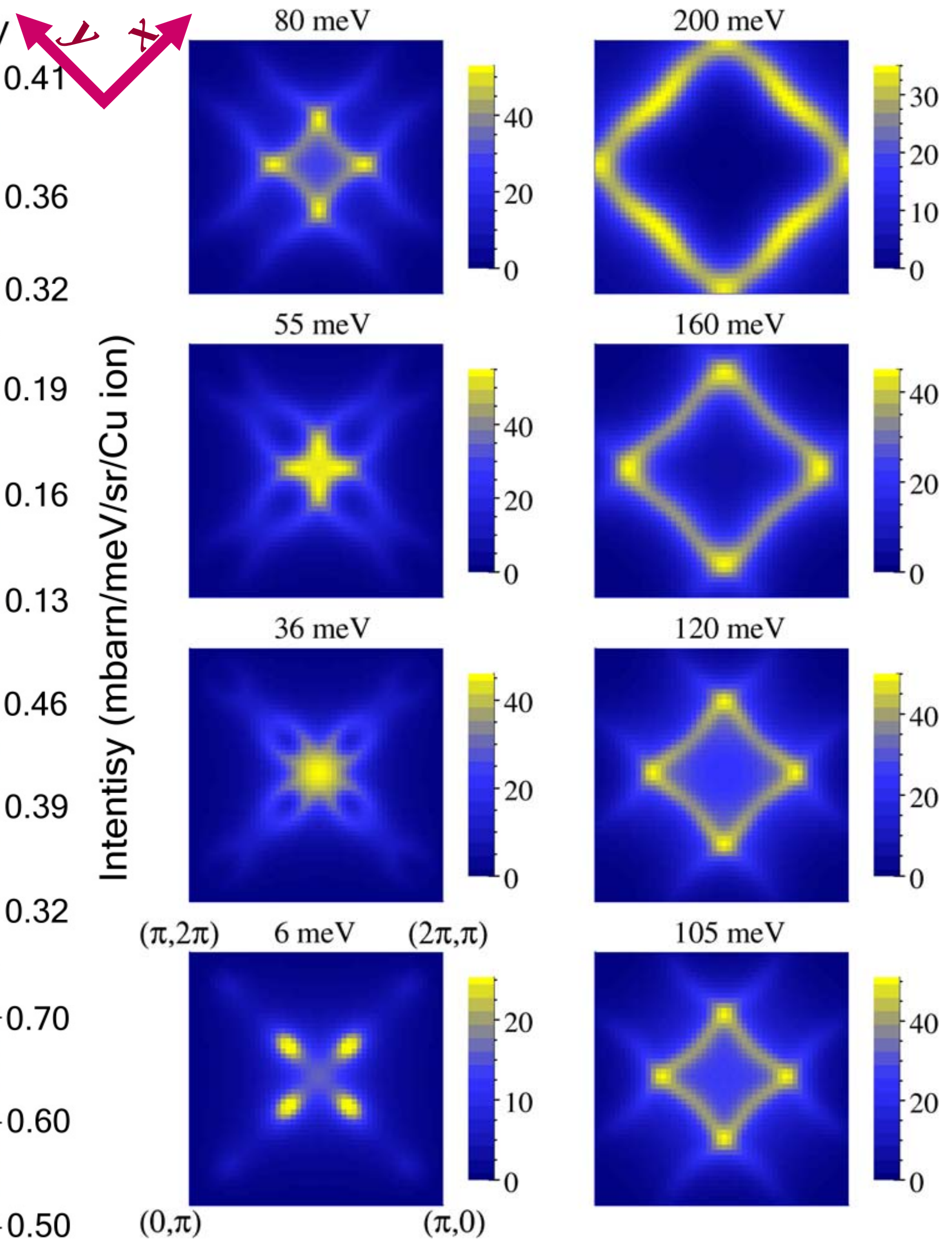
Possible structure of VBS order



This strucuture also explains spin-excitation spectra in neutron scattering experiments



Tranquada *et al.*, *Nature* **429**, 534 (2004)



Bond operator (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B 41, 9323 (1990)) theory of coupled-ladder model, M. Vojta and T. Ulbricht, *Phys. Rev. Lett.* **93**, 127002 (2004)

<u>Many</u> experiments on the cuprate superconductors show:

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Do vortices in the superfluid "know" about these "density" modulations ?

Consequences of our theory:

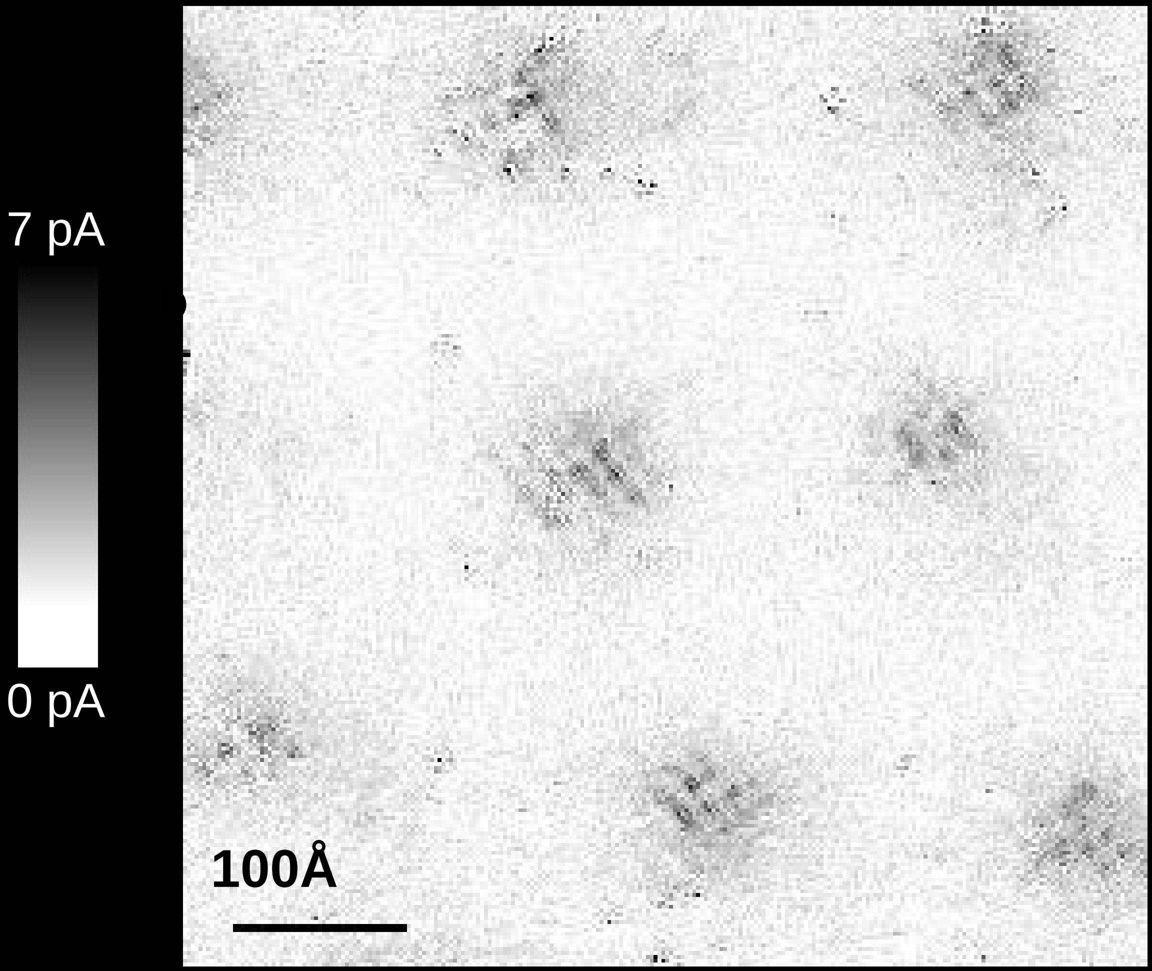
Information on VBS order is contained in the vortex flavor space

Density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}$$

Each pinned vortex in the superfluid has a halo of density wave order over a length scale ≈ the zero-point quantum motion of the vortex. This scale diverges upon approaching the insulator

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



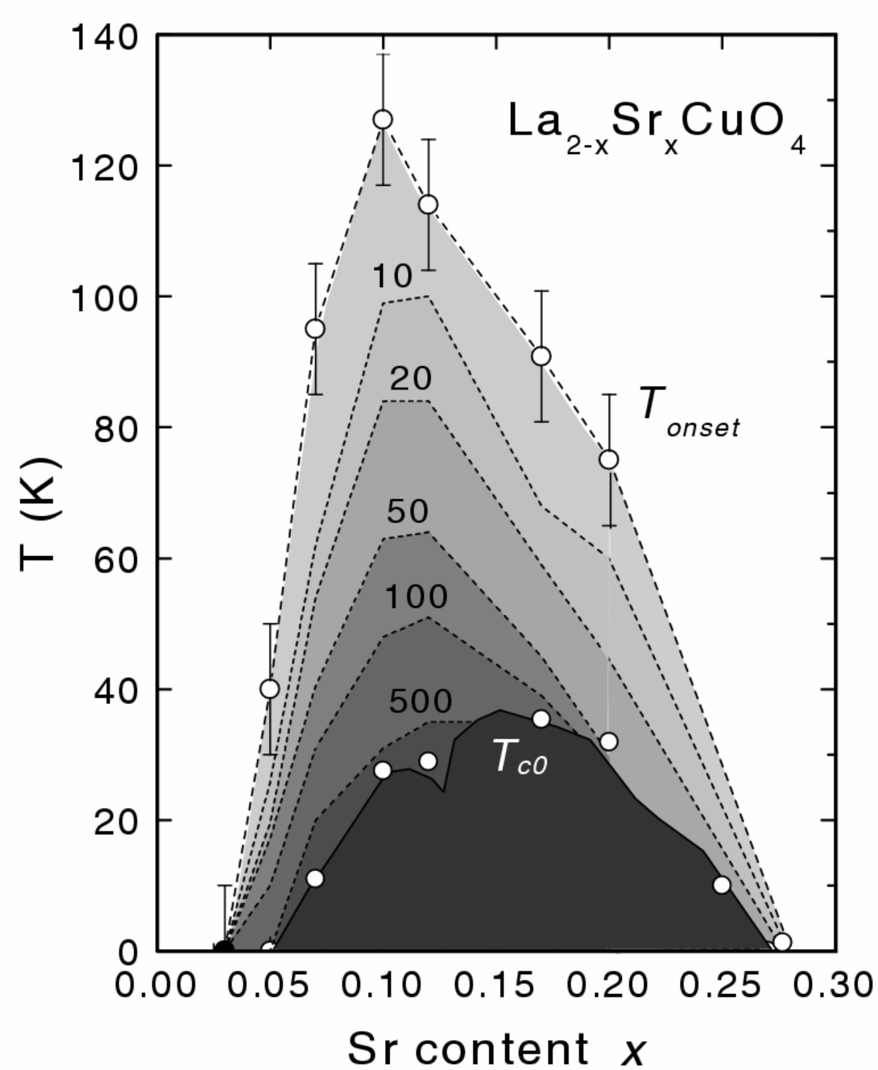
Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Prediction of VBS order near vortices: K. Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

Measurements of Nernst effect are well explained by a model of a liquid of vortices and anti-vortices

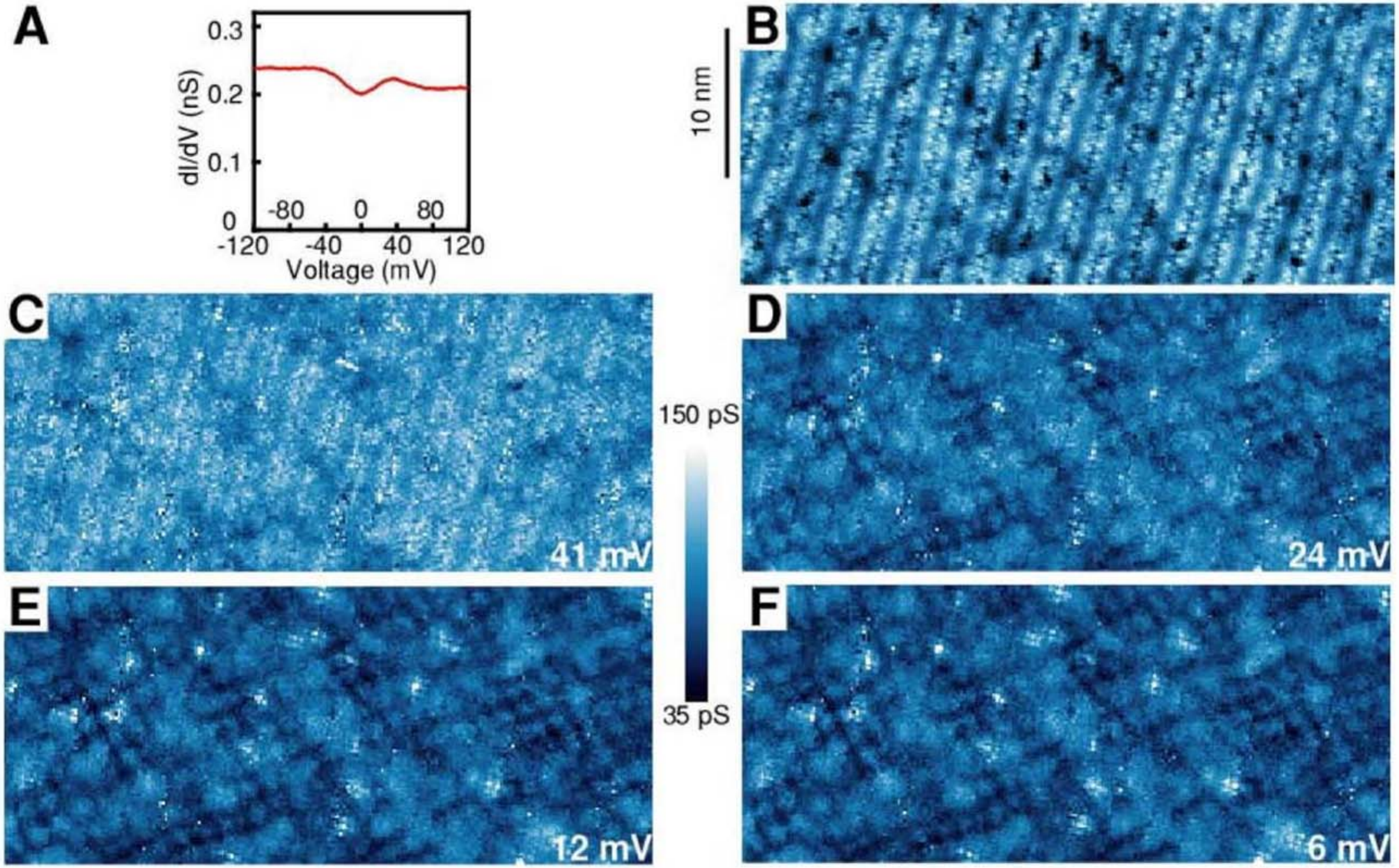


N. P. Ong, Y. Wang, S. Ono, Y. Ando, and S. Uchida, *Annalen der Physik* **13**, 9 (2004).

Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, *Science* **299**, 86 (2003).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

STM measurements observe "density" modulations with a period of $\approx$ 4 lattice spacings

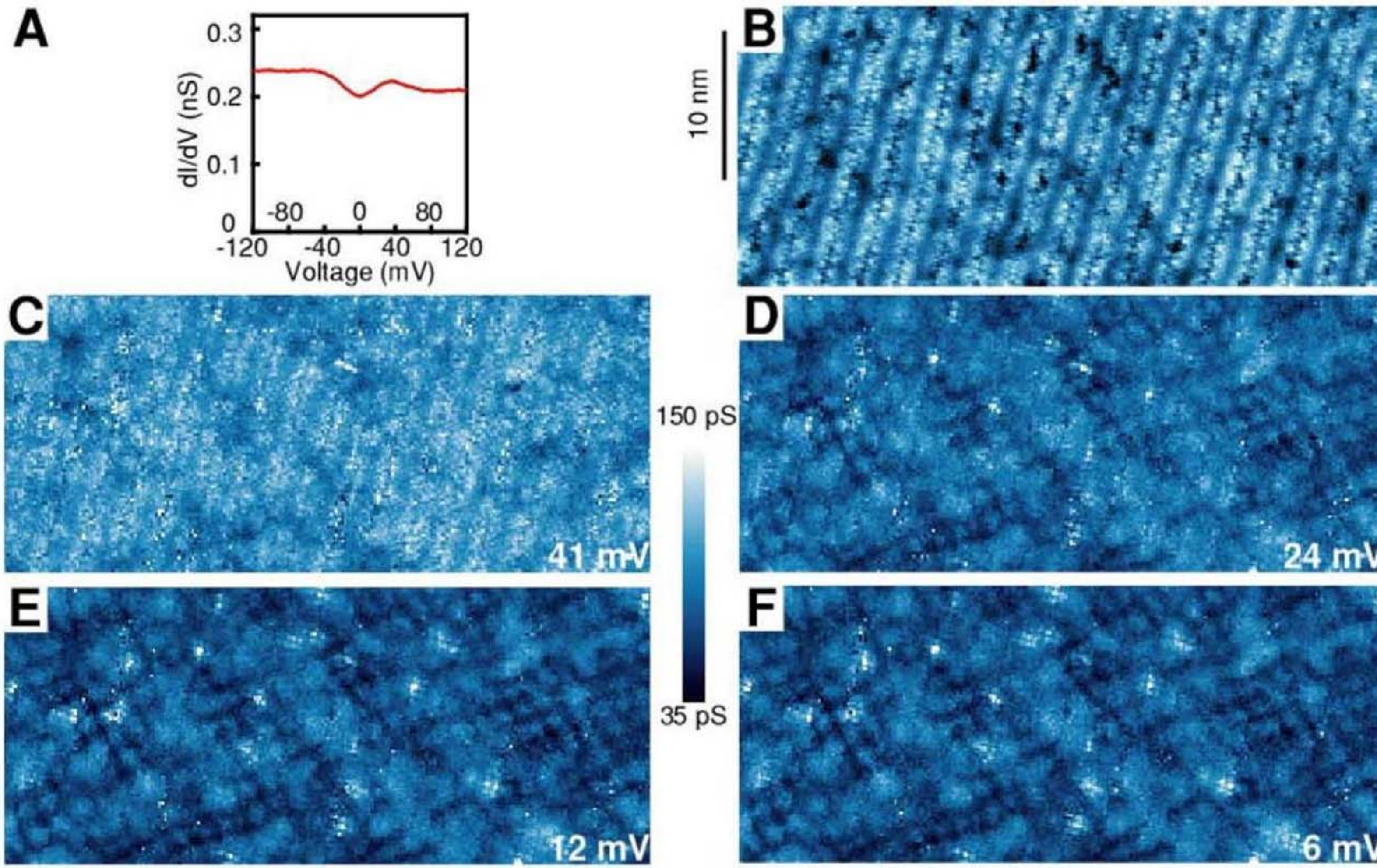


LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ at 100 K.

M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

Our theory: modulations arise from pinned vortex-anti-vortex pairs – these thermally excited vortices are also responsible for the Nernst effect



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Superfluids near Mott insulators

The Mott insulator has average Cooper pair density, $f = p/q$ per site, while the density of the superfluid is close (but need not be identical) to this value

- Dual description using vortices with flux *h/(2e)* which come in multiple (usually q) "flavors"
- The lattice space group acts in a projective representation on the vortex flavor space.
- These flavor quantum numbers provide a distinction between superfluids: they constitute a "quantum order"
- Any pinned vortex must chose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.

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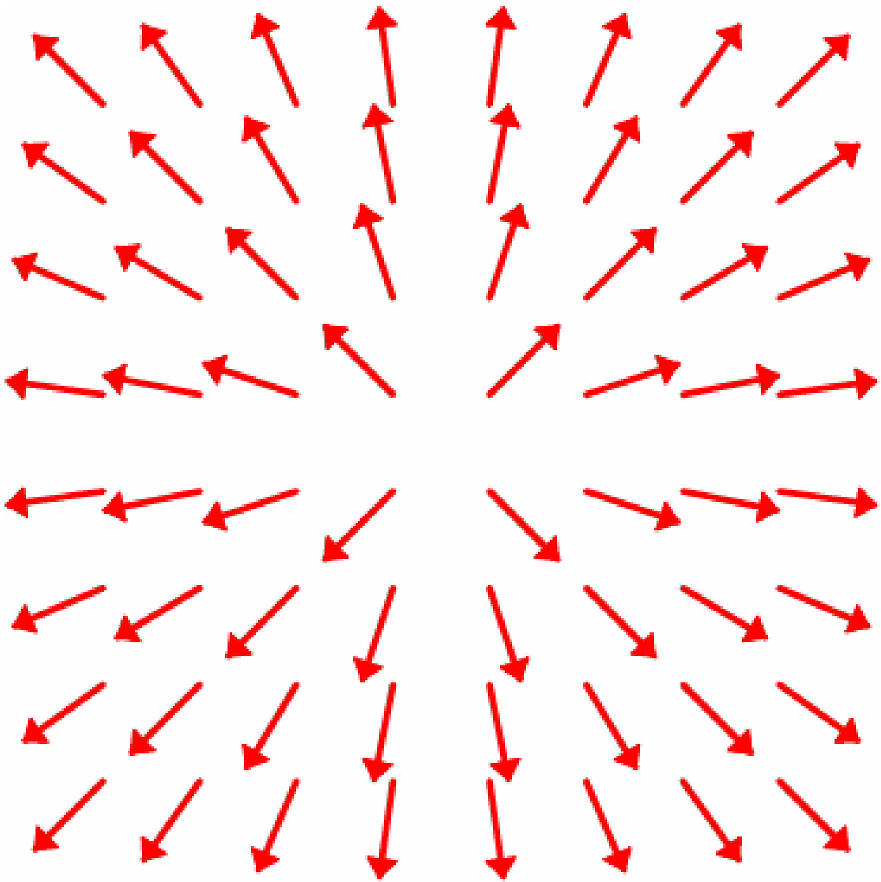
Hedgehog Berry phases and VBS order

Defects in the Neel state



$$\vec{\varphi} \propto \hat{r}$$

Monopoles are points in spacetime, representing tunnelling events

- The Berry phases of the spins attach a non-trivial phase factor to each hedgehog tunnelling event

F. D. M. Haldane, *Phys. Rev. Lett.* **61**, 1029 (1988)

- Consequently, the hedgehog creation operator, $v^\dagger$, transforms non-trivially under square lattice space group operations (for spin $S = 1/2$):

$$T_x \; : \; v^\dagger \to -iv \quad ; \quad T_y \; : \; v^\dagger \to iv \quad ; \quad R \; : \; v^\dagger \to iv^\dagger$$

- These transformation properties allow the remarkable identification

$$v \sim e^{-i\pi/4} \Psi_{\rm vbs}$$

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

So the defects in the Neel order are linked to the VBS order parameter.

- Condensation of hedgehogs induces long-range VBS order, and eliminates a purely "quantum disordered" state with vector spin excitations.