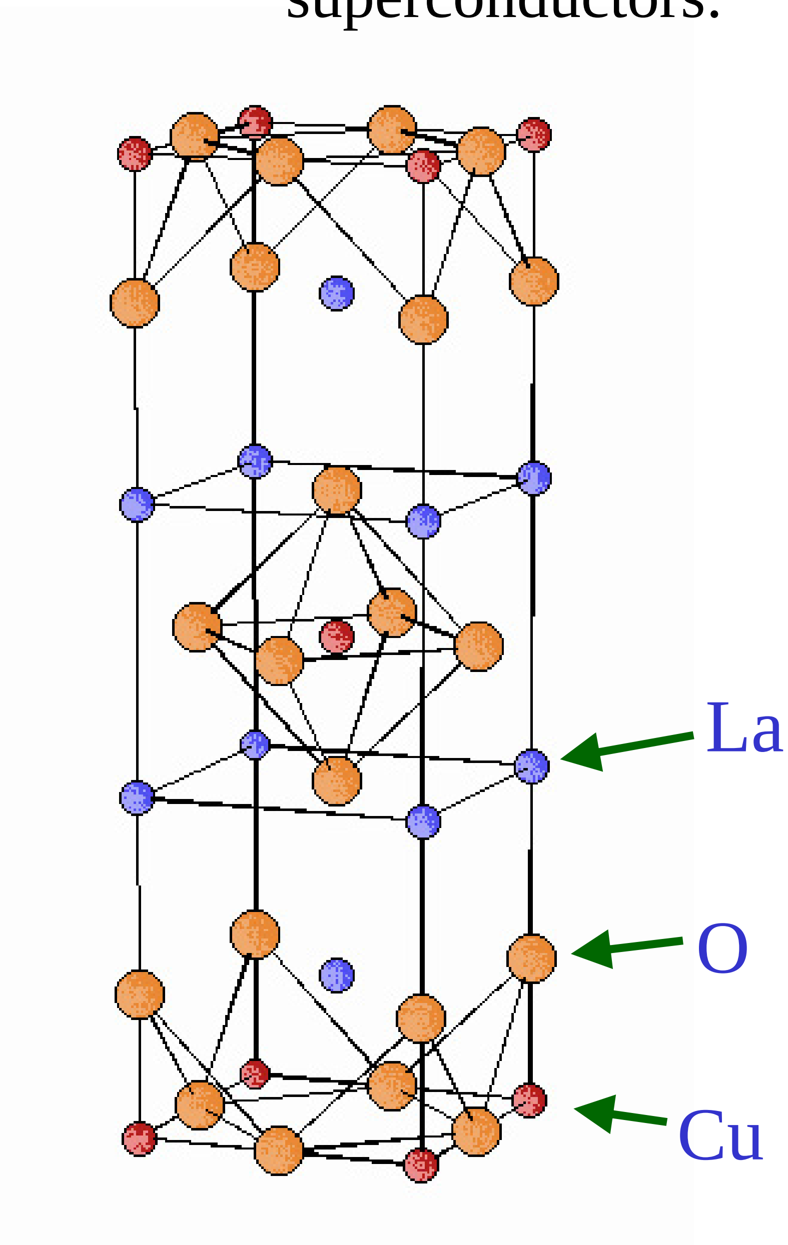
Quantum phase transitions: from Mott insulators to the cuprate superconductors

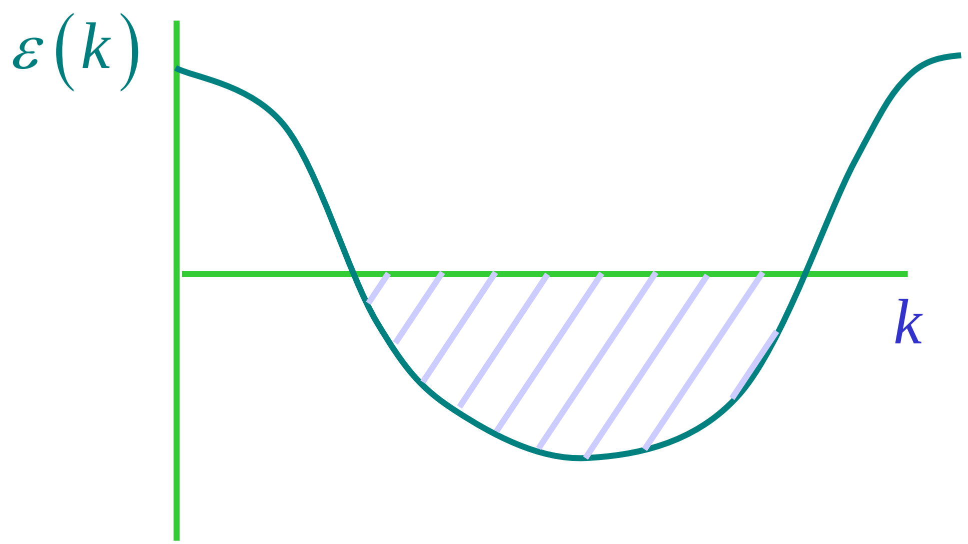
Colloquium article in *Reviews of Modern Physics* **75**, 913 (2003)

Leon Balents (UCSB)
Eugene Demler (Harvard)
Matthew Fisher (UCSB)
Kwon Park (Maryland)
Anatoli Polkovnikov (Harvard)
T. Senthil (MIT)
Ashvin Vishwanath (MIT)
Matthias Vojta (Karlsruhe)
Ying Zhang (Maryland)

Parent compound of the high temperature superconductors: La_2CuO_4



Band theory

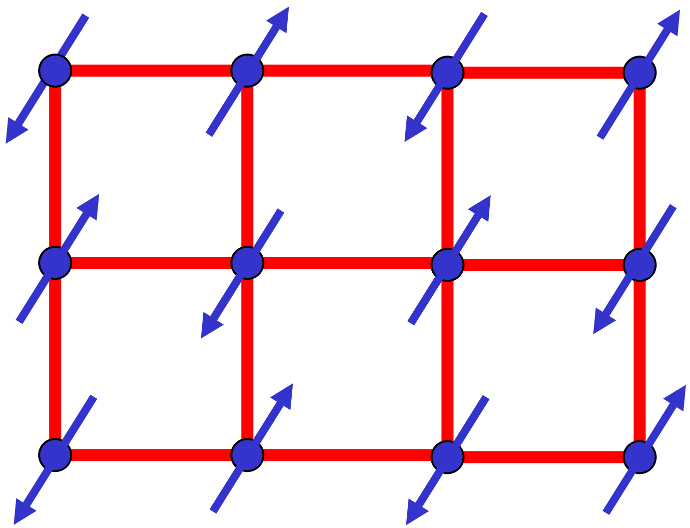


Half-filled band of Cu 3d orbitals – ground state is predicted by band theory to be a metal.

However, La_2CuO_4 is a very good insulator

Parent compound of the high temperature superconductors: La_2CuO_4

A Mott insulator



$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \Rightarrow$ spin operator with angular momentum $S=1/2$

Ground state has long-range spin density wave (Néel) order at wavevector $\boldsymbol{K}=(\pi,\pi)$

spin density wave order parameter:

$$\vec{\varphi} = \eta_i \frac{\vec{S}_i}{S} \quad ; \quad \eta_i = \pm 1 \text{ on two sublattices}$$

$$\langle \vec{\varphi} \rangle \neq 0$$

Parent compound of the high temperature superconductors: La_2CuO_4

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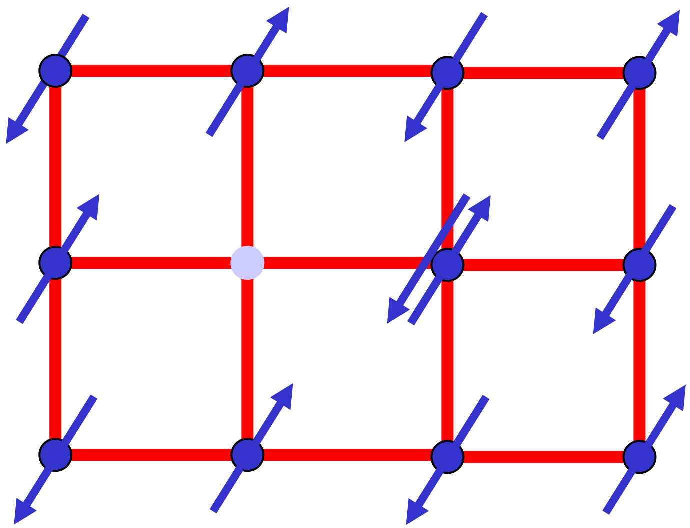

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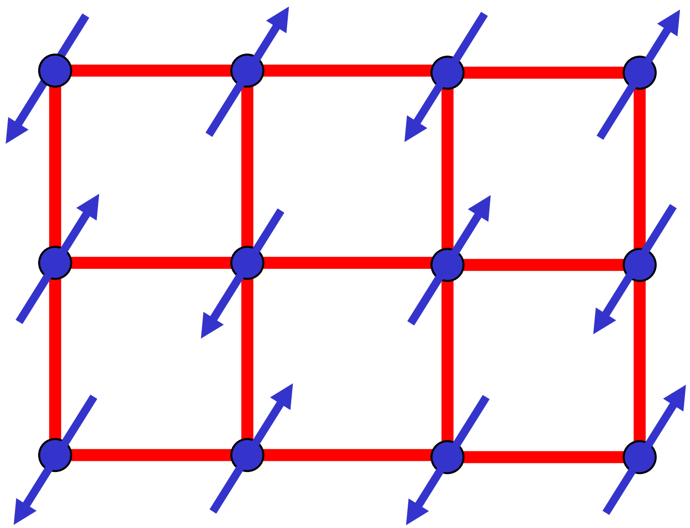

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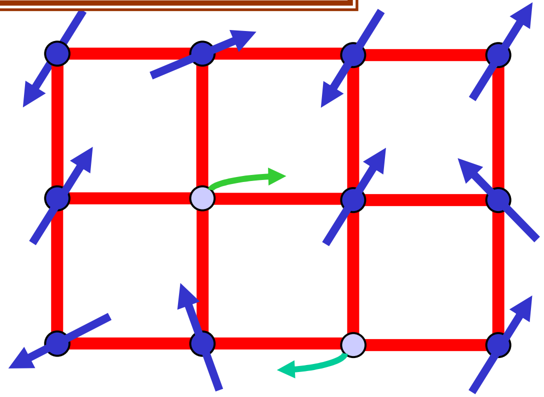
Superconductivity in a doped Mott insulator



Introduce mobile carriers of density δ
by substitutional doping of out-of-plane
ions *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$

Doped state is a paramagnet with $\langle \vec{\varphi} \rangle = 0$
and also a high temperature superconductor with
the BCS pairing order parameter $\langle \Psi_{\text{BCS}} \rangle \neq 0.$

$\Rightarrow$ With increasing δ, there must be one or more
quantum phase transitions involving

(i) onset of a non-zero $\langle \Psi_{\text{BCS}} \rangle$

(ii) restoration of spin rotation invariance by a transition
from $\langle \vec{\varphi} \rangle \neq 0$ to $\langle \vec{\varphi} \rangle = 0$

First study magnetic transition in Mott insulators…………

Outline

A. Magnetic quantum phase transitions in “dimerized” Mott insulators

Landau-Ginzburg-Wilson (LGW) theory

B. Mott insulators with spin S=1/2 per unit cell

Berry phases, bond order, and the breakdown of the LGW paradigm

C. Cuprate Superconductors

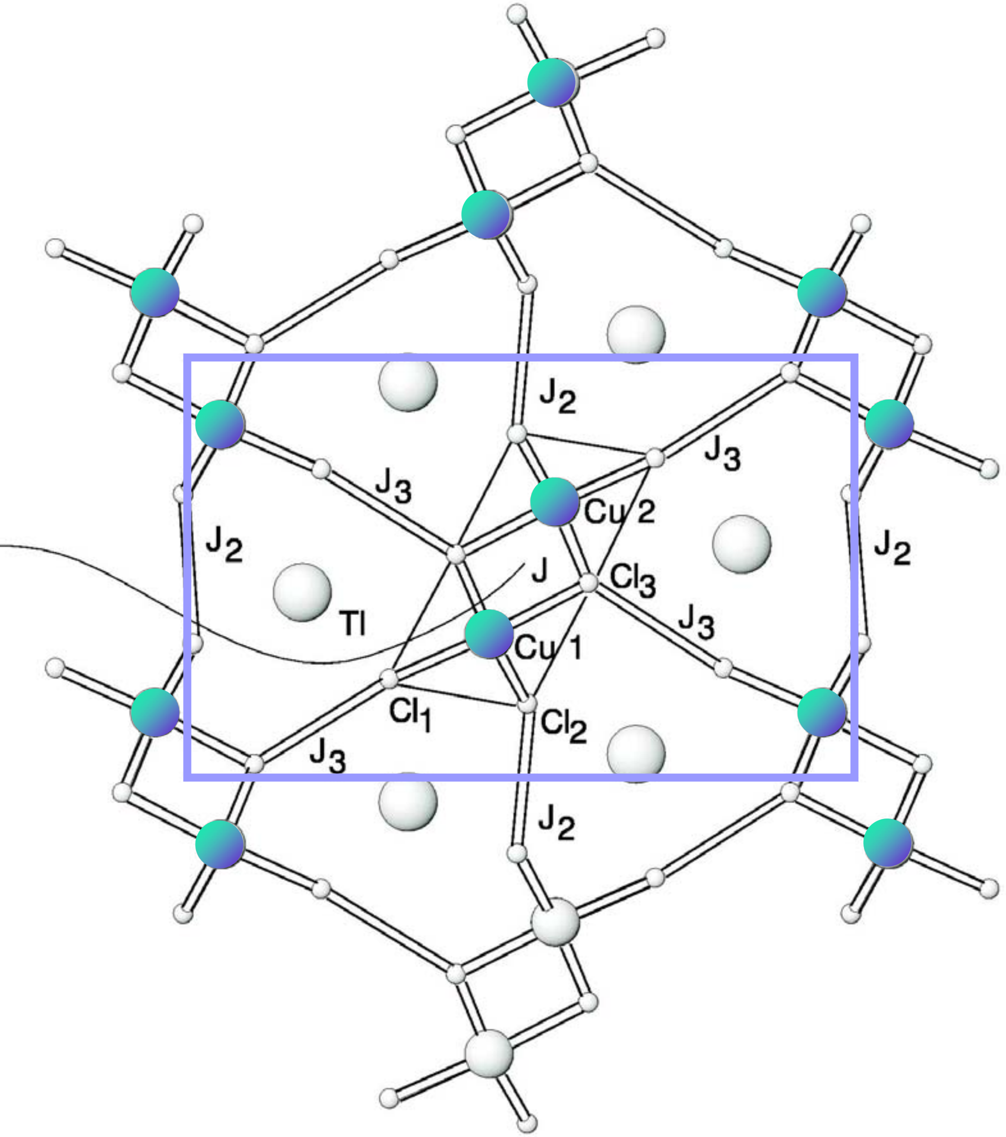
Competing orders and recent experiments

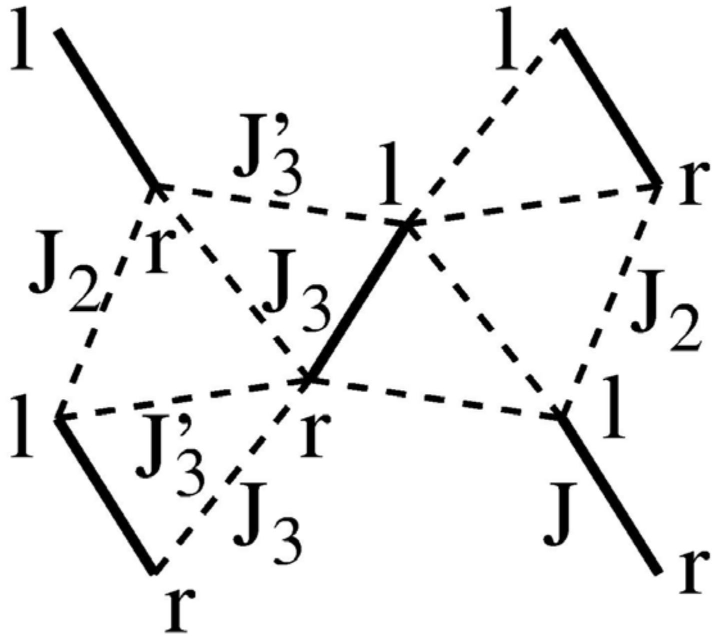
Magnetic quantum phase tranitions in "dimerized" Mott insulators:

Landau-Ginzburg-Wilson (LGW) theory:

Second-order phase transitions described by fluctuations of an ***order parameter*** *associated with a* ***broken symmetry***

$TlCuCl_3$



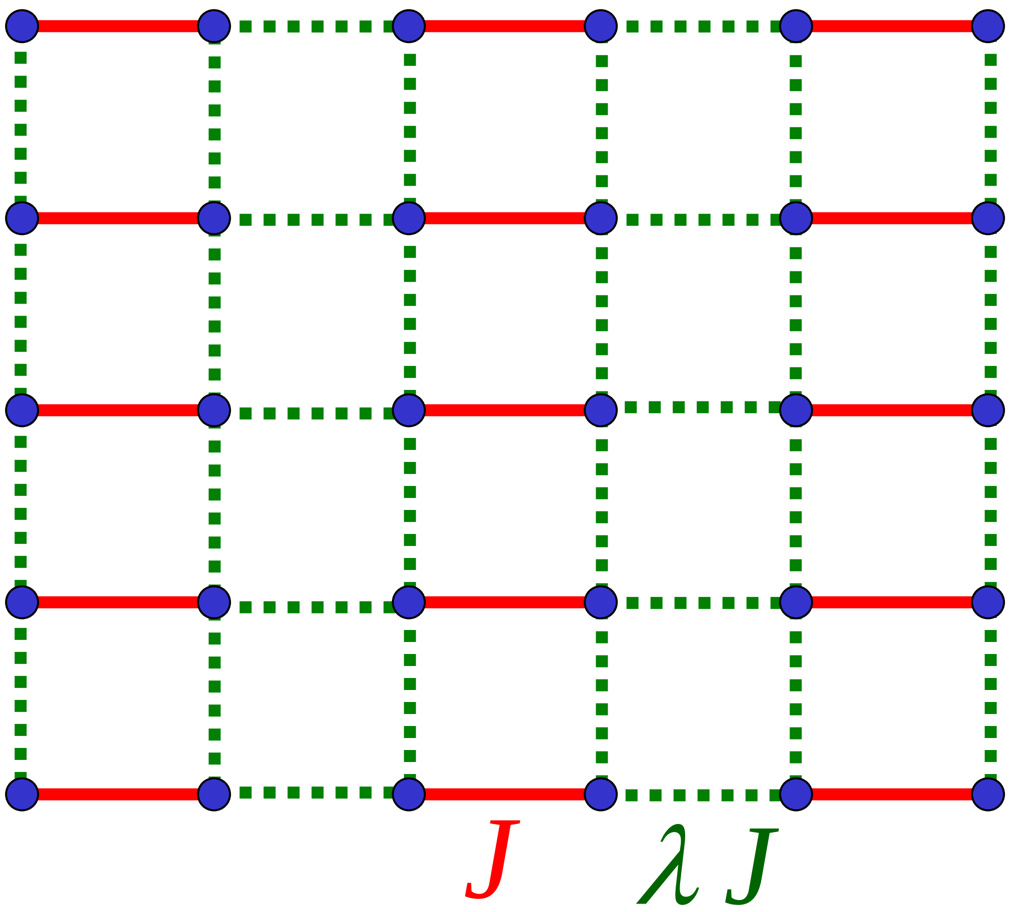


M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.

Coupled Dimer Antiferromagnet

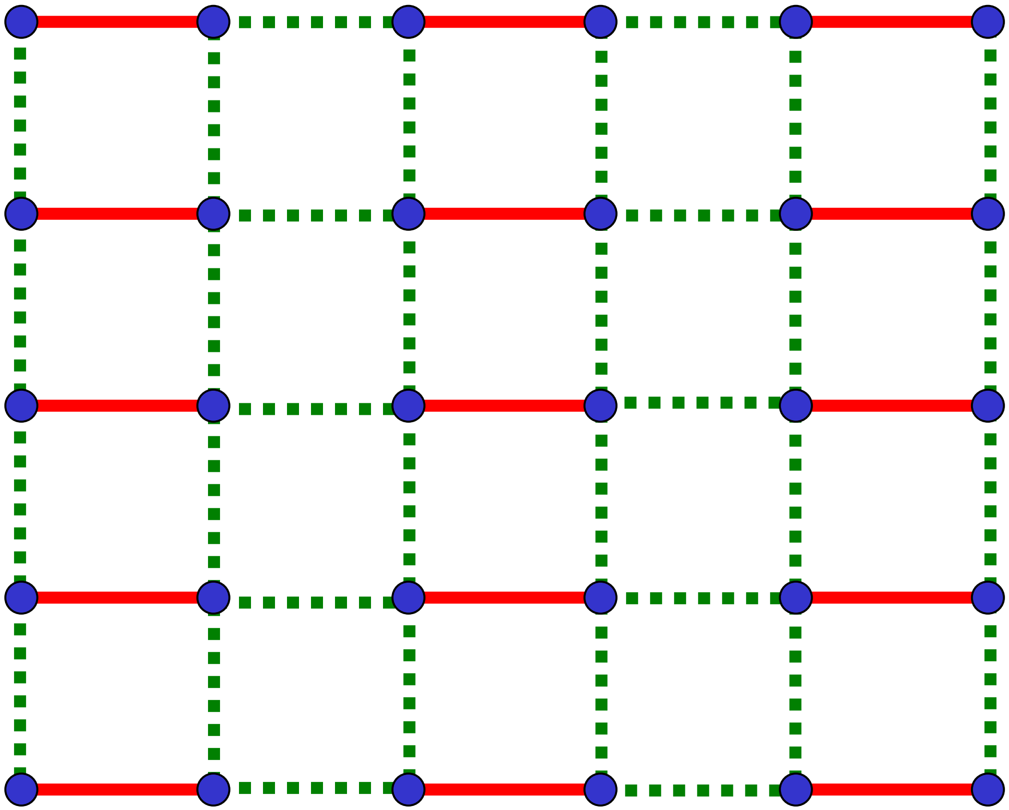
M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

S=1/2 spins on coupled dimers



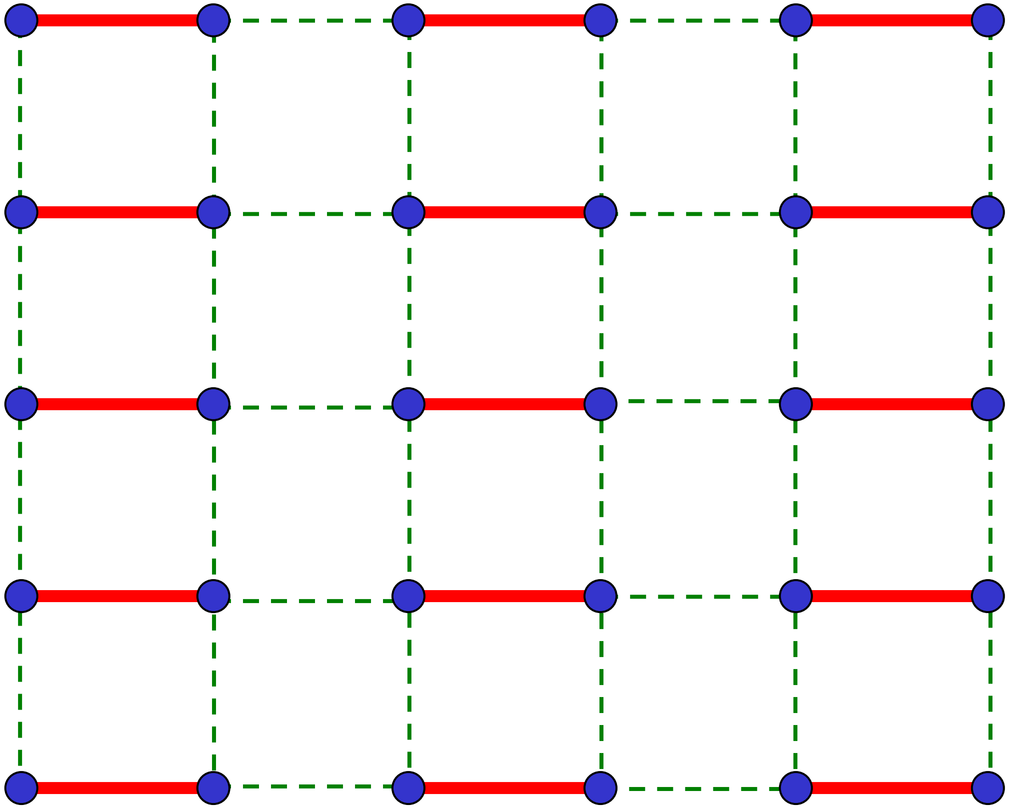
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$



λ close to 0

Weakly coupled dimers



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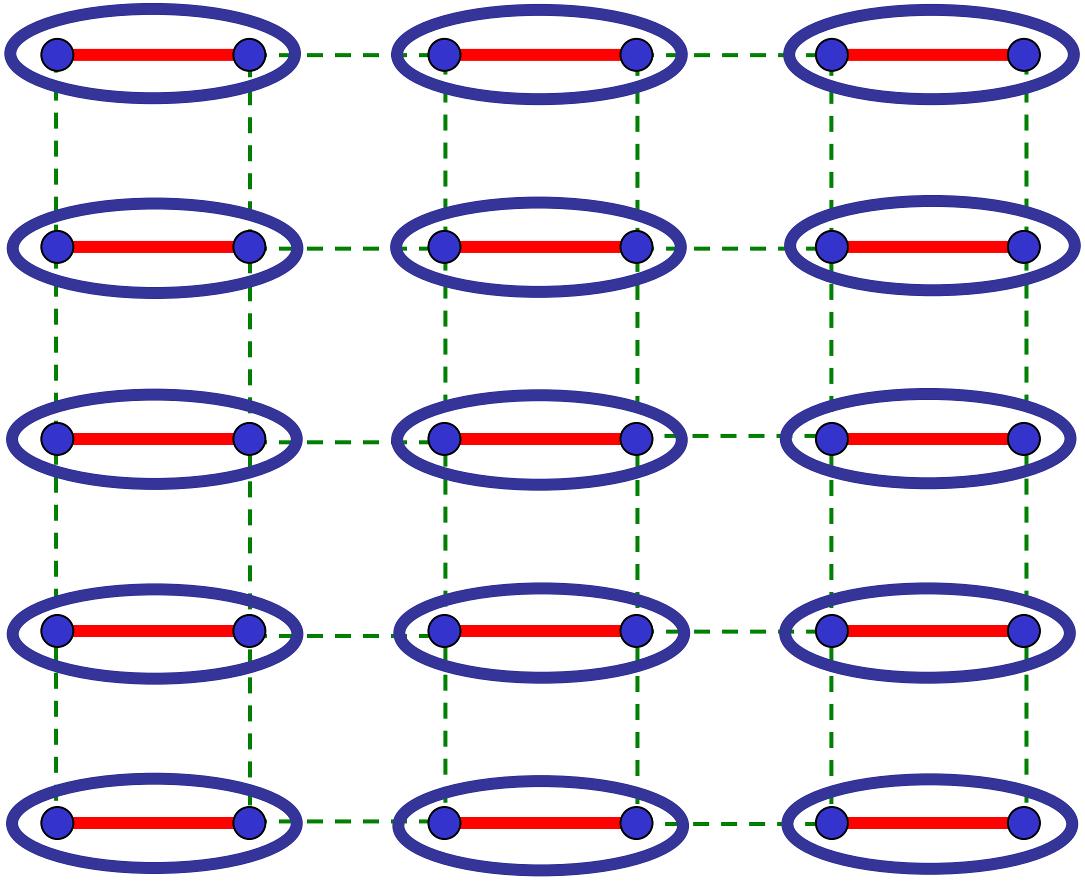
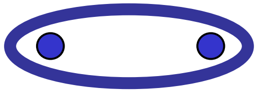




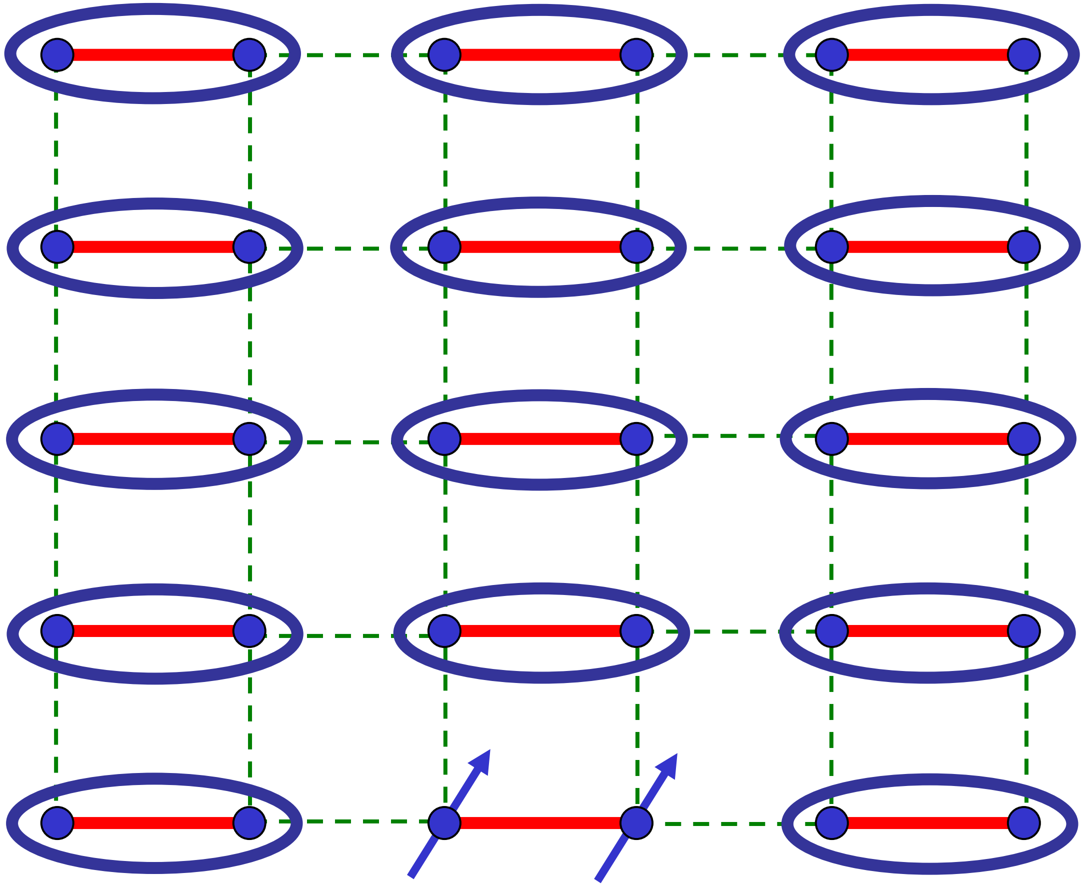
$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

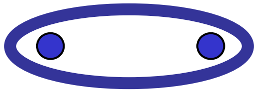
Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0, \; \left\langle \vec{\varphi} \right\rangle = 0$$

λ close to 0

Weakly coupled dimers





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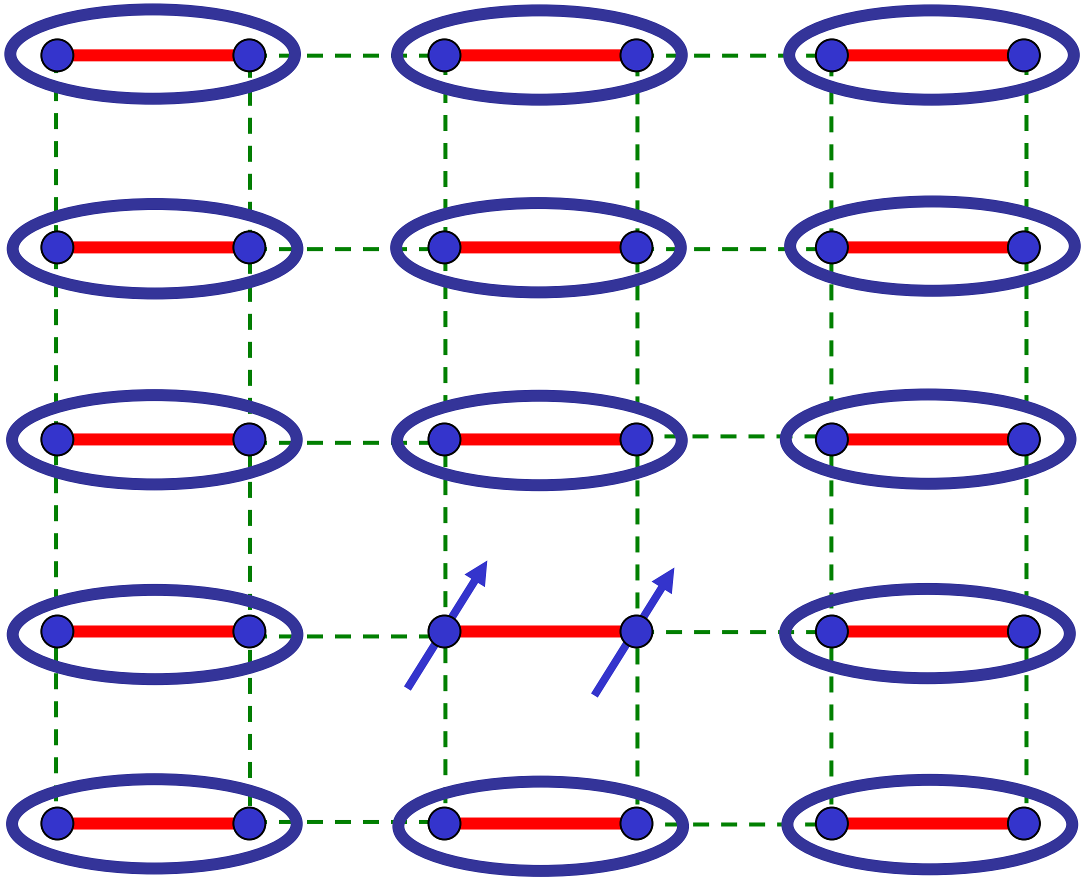
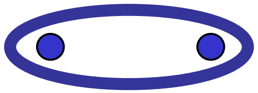
Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers





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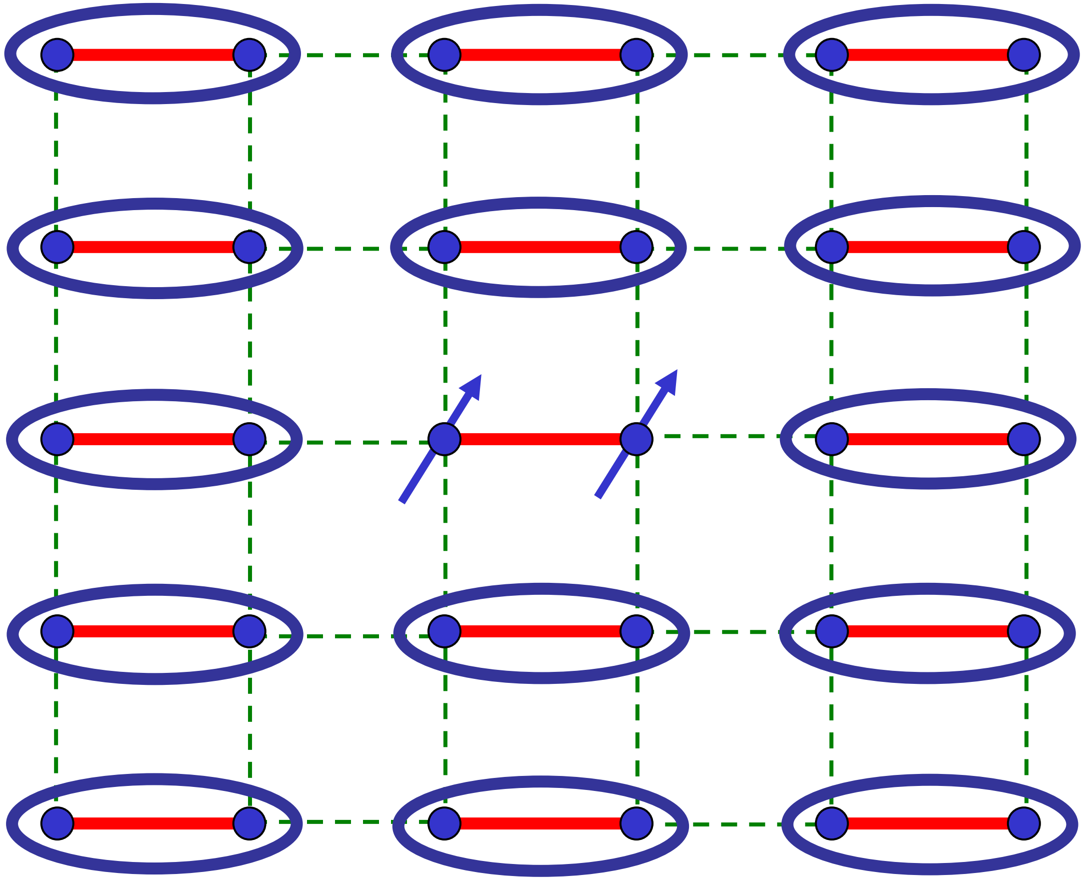

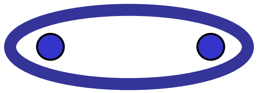 $= \frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big)$

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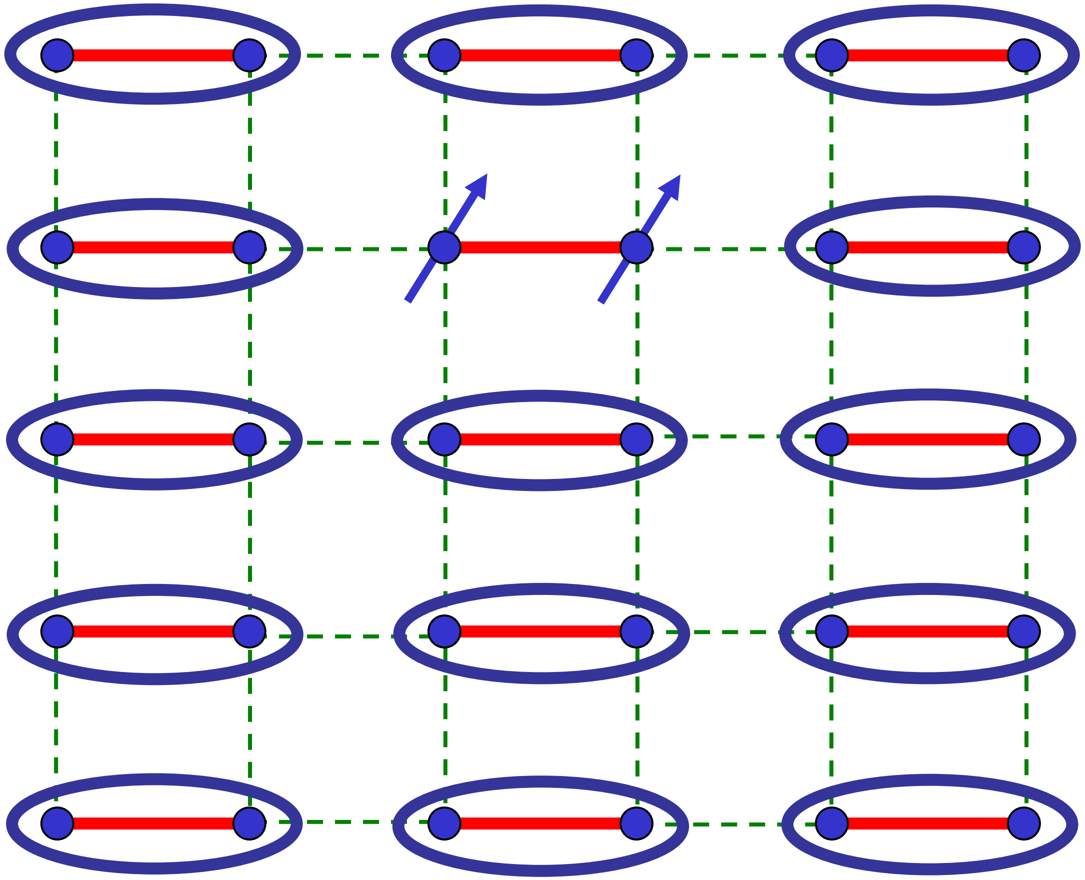
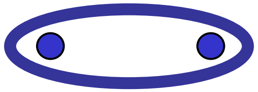
Weakly coupled dimers





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

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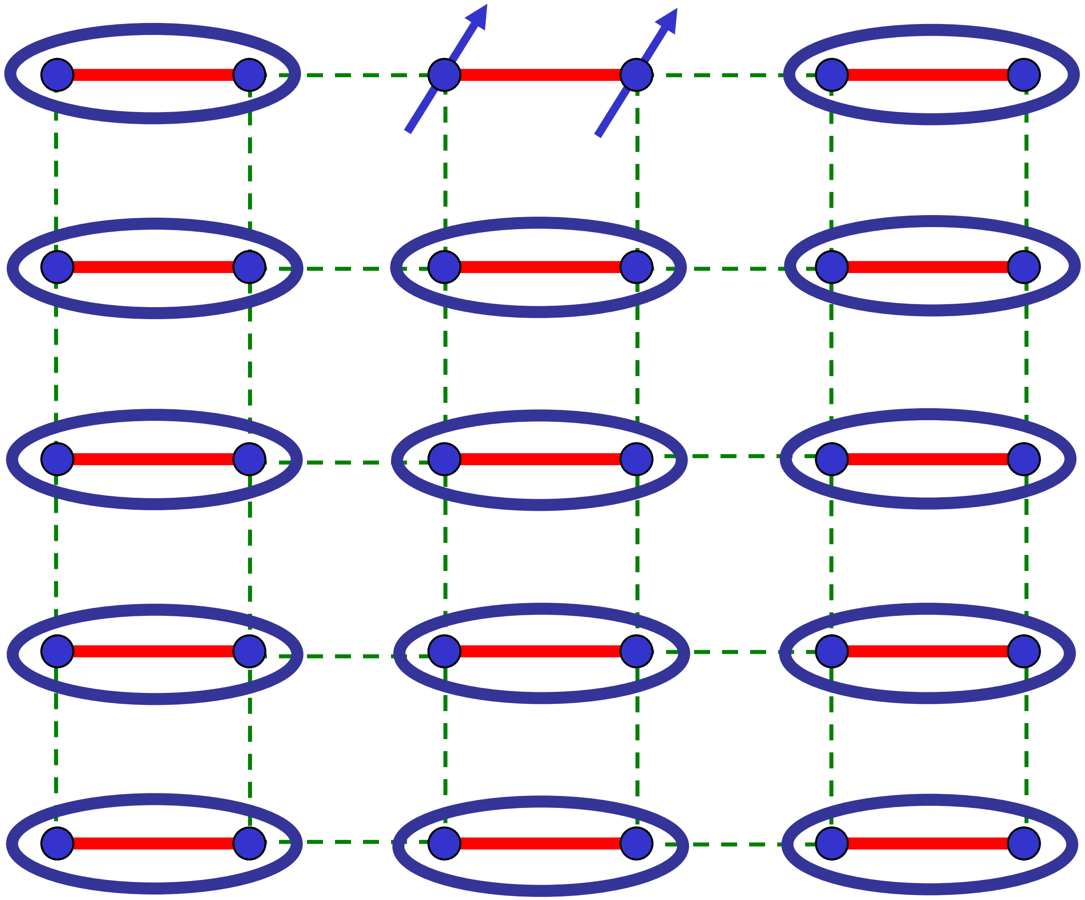
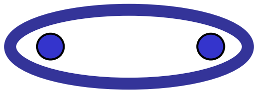
Weakly coupled dimers





$$= \frac{1}{\sqrt{2}} \Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \Big)$$

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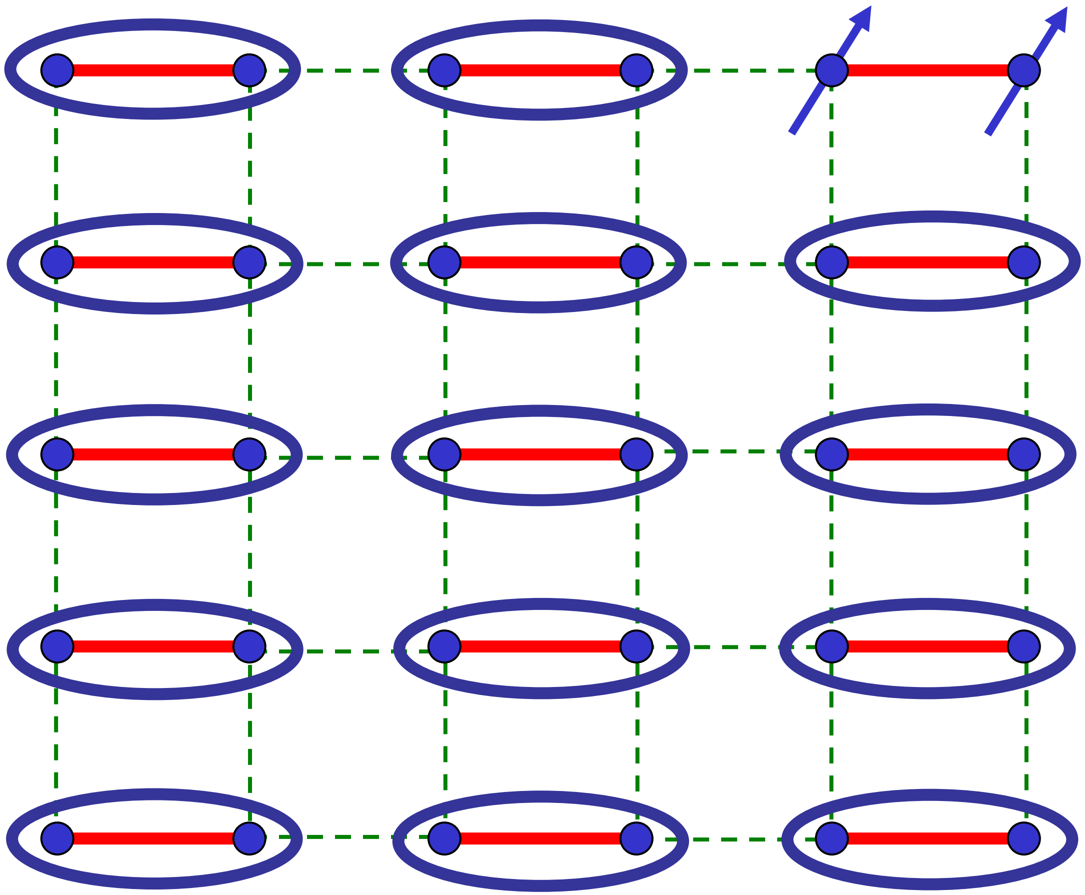
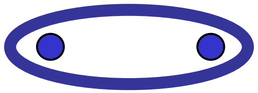




$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Excitation: S=1 ***triplon***

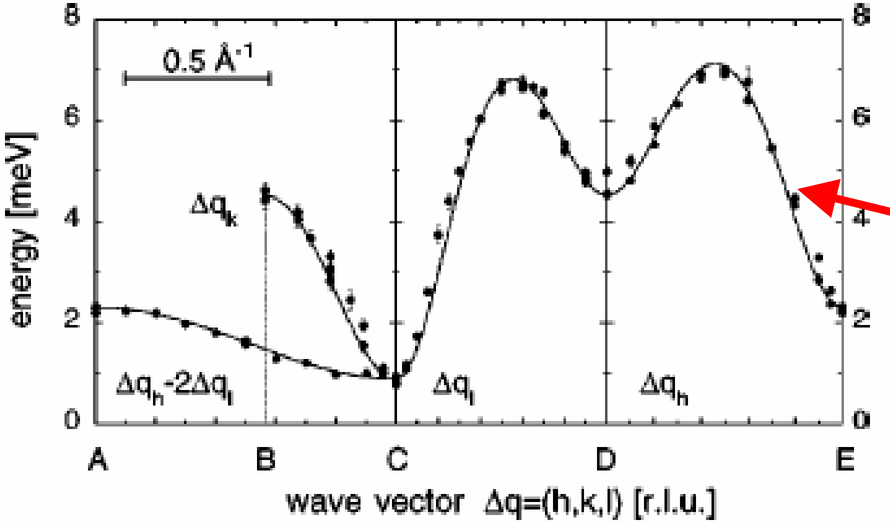
(***exciton***, spin collective mode)

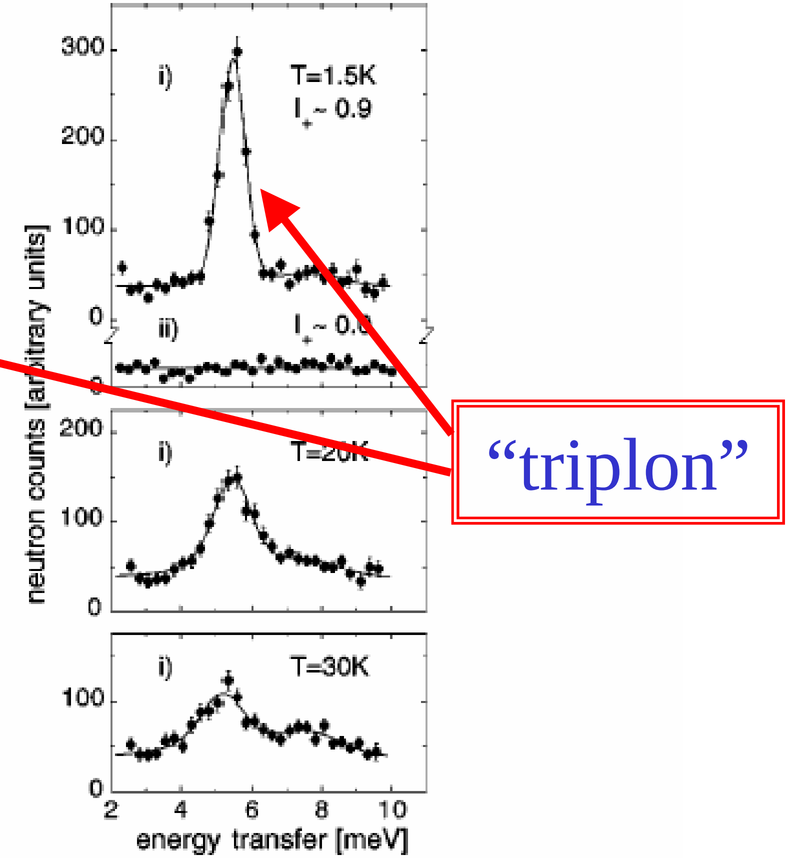
Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

$TlCuCl_3$





N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

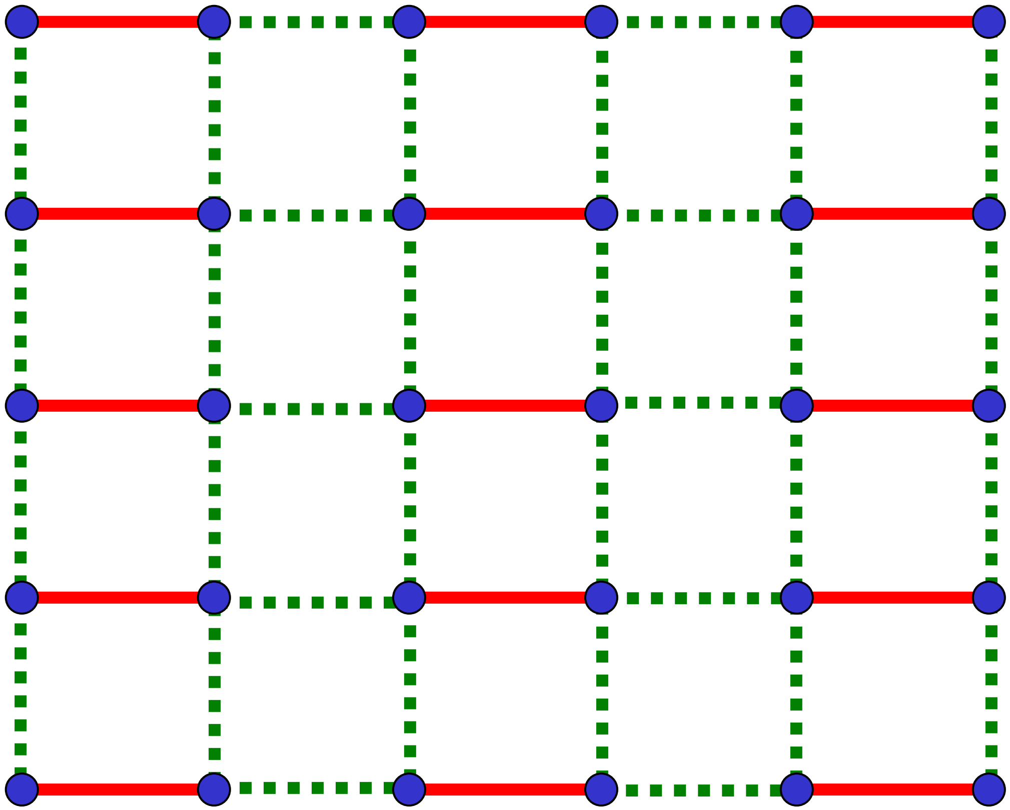
FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

For quasi-one-dimensional systems, the triplon linewidth takes the exact universal value $=1.20 k_B T e^{-\Delta/k_B T}$ at low T

K. Damle and S. Sachdev, *Phys. Rev.* B **57**, 8307 (1998)

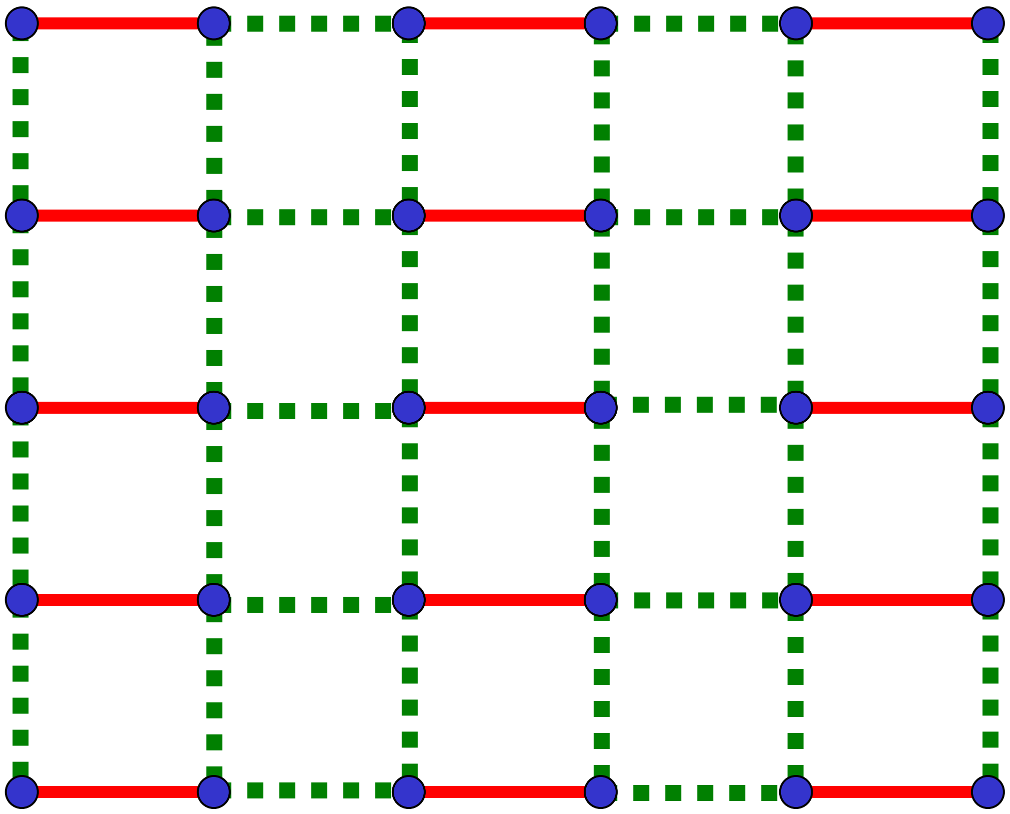
This result is in good agreement with observations in $CsNiCl_3$ (M. Kenzelmann, R. A. Cowley, W. J. L. Buyers, R. Coldea, M. Enderle, and D. F. McMorrow *Phys. Rev.* B **66**, 174412 (2002)) and Y_2NiBaO_5 (G. Xu, C. Broholm, G. Aeppli, J. F. DiTusa, T.Ito, K. Oka, and H. Takagi, preprint).

Coupled Dimer Antiferromagnet



λ close to 1

Weakly dimerized square lattice



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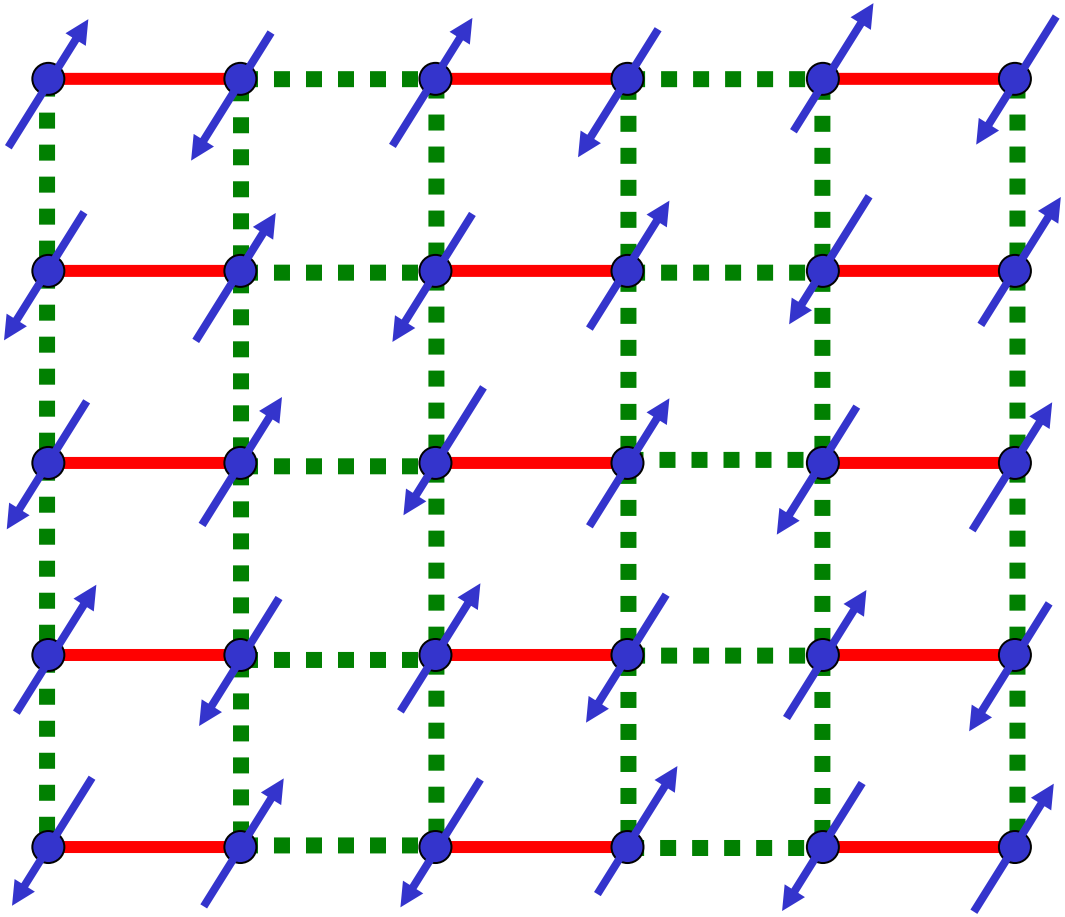


Excitations:
2 spin waves (***magnons***)

$$\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$$

Ground state has long-range spin density wave (Néel) order at wavevector ***K***= (π,π)

$$\langle \vec{\varphi} \rangle \neq 0$$

spin density wave order parameter: $\vec{\varphi} = \eta_i \frac{\vec{S}_i}{S}$; $\eta_i = \pm 1$ on two sublattices

$TlCuCl_3$

Neutron Diffraction Study of the Pressure-Induced Magnetic Ordering in the Spin Gap System $TlCuCl_3$

Akira OOSAWA*, Masashi FUJISAWA[1], Toyotaka OSAKABE, Kazuhisa KAKURAI and Hidekazu TANAKA[2]

Advanced Science Research Center, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195
[1]*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*
[2]*Research Center for Low Temperature Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*

(Received February 3, 2003)

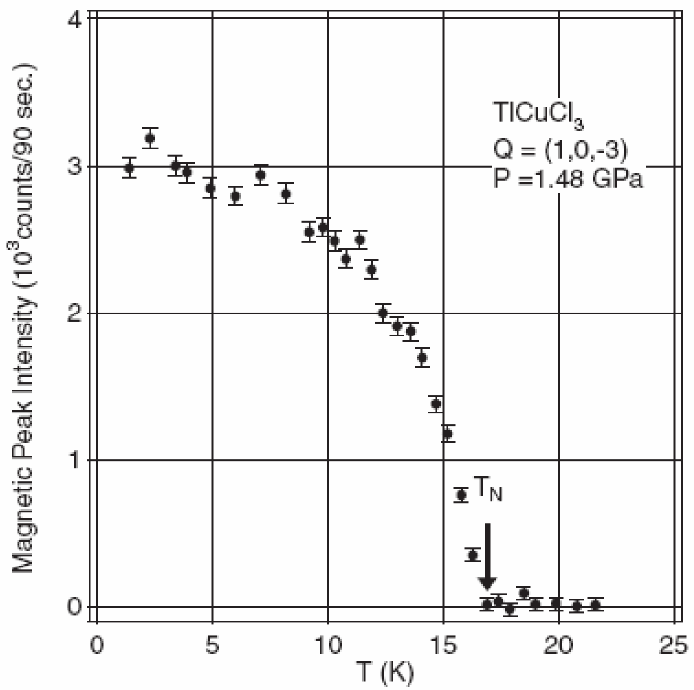


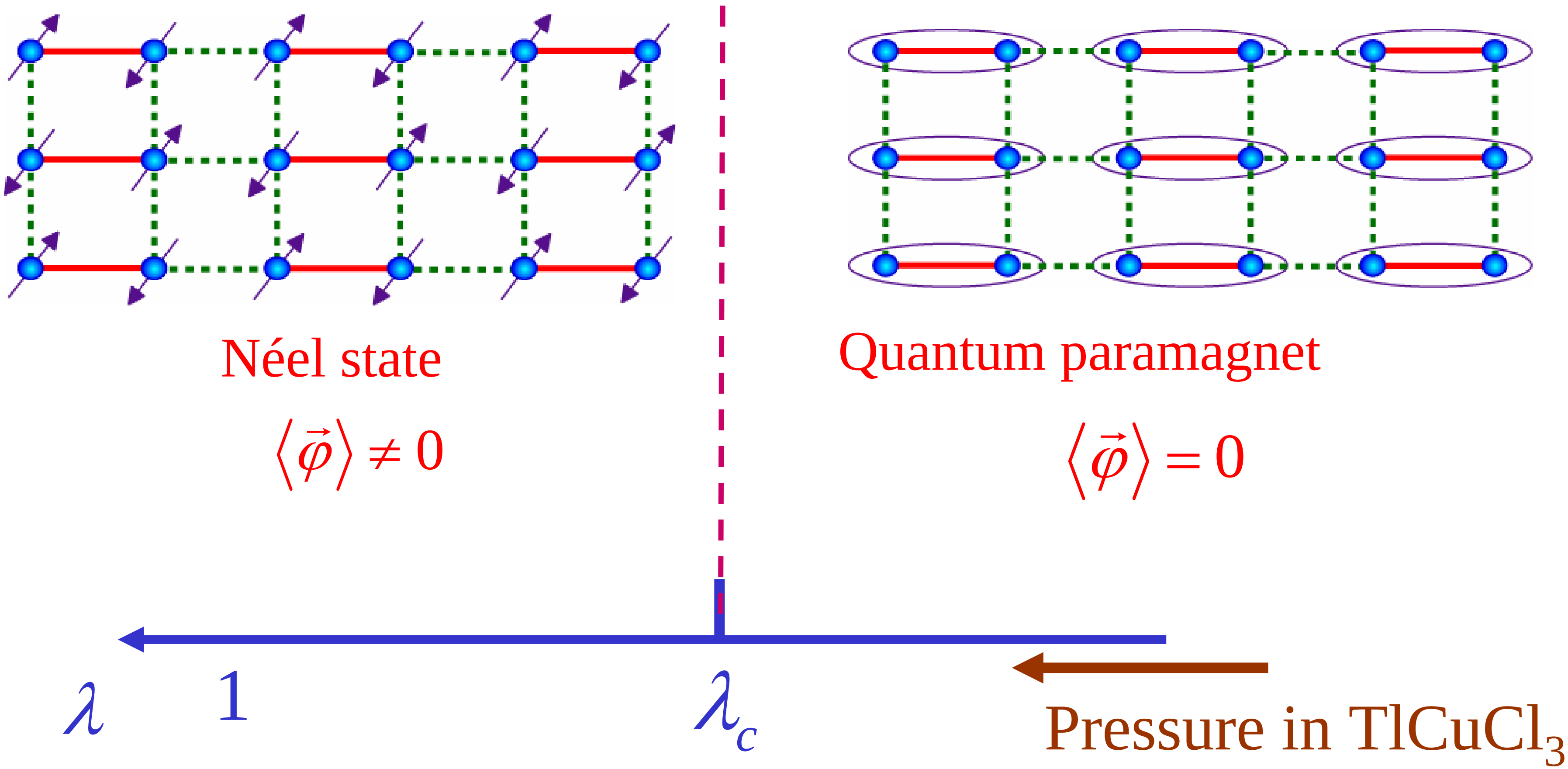
Fig. 3. Temperature dependence of the magnetic Bragg peak intensity for $Q = (1, 0, -3)$ reflection measured at $P = 1.48$ GPa in $TlCuCl_3$.

J. Phys. Soc. Jpn **72**, 1026 (2003)

T=0

$\lambda_c = 0.52337(3)$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)



The method of bond operators (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B **41**, 9323 (1990)) provides a quantitative description of spin excitations in $TlCuCl_3$ across the quantum phase transition (M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, *Phys. Rev. Lett.* **89**, 077203 (2002))

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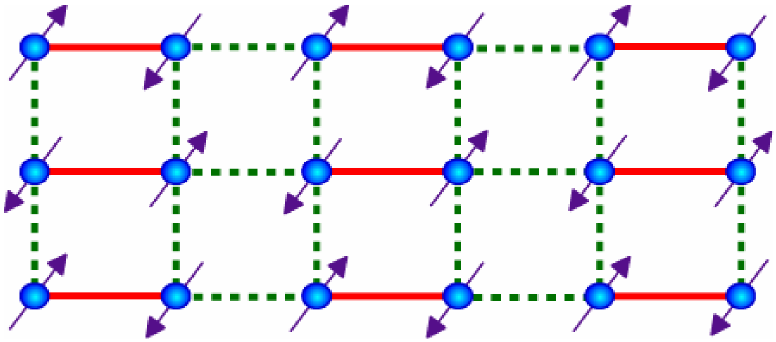
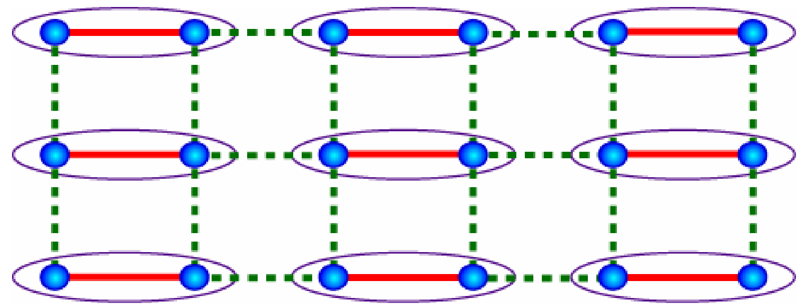




Néel state

$\langle \vec{\varphi} \rangle \neq 0$

Quantum paramagnet

$\langle \vec{\varphi} \rangle = 0$

Magnetic order as in La_2CuO_4

Electrons in charge-localized Cooper pairs

λ 1 λ_c

δ in cuprates ?

The method of bond operators (S. Sachdev and R.N. Bhatt, *Phys. Rev*. B **41**, 9323 (1990)) provides a quantitative description of spin excitations in $TlCuCl_3$ across the quantum phase transition (M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, *Phys. Rev. Lett*. **89**, 077203 (2002))

LGW theory for quantum criticality

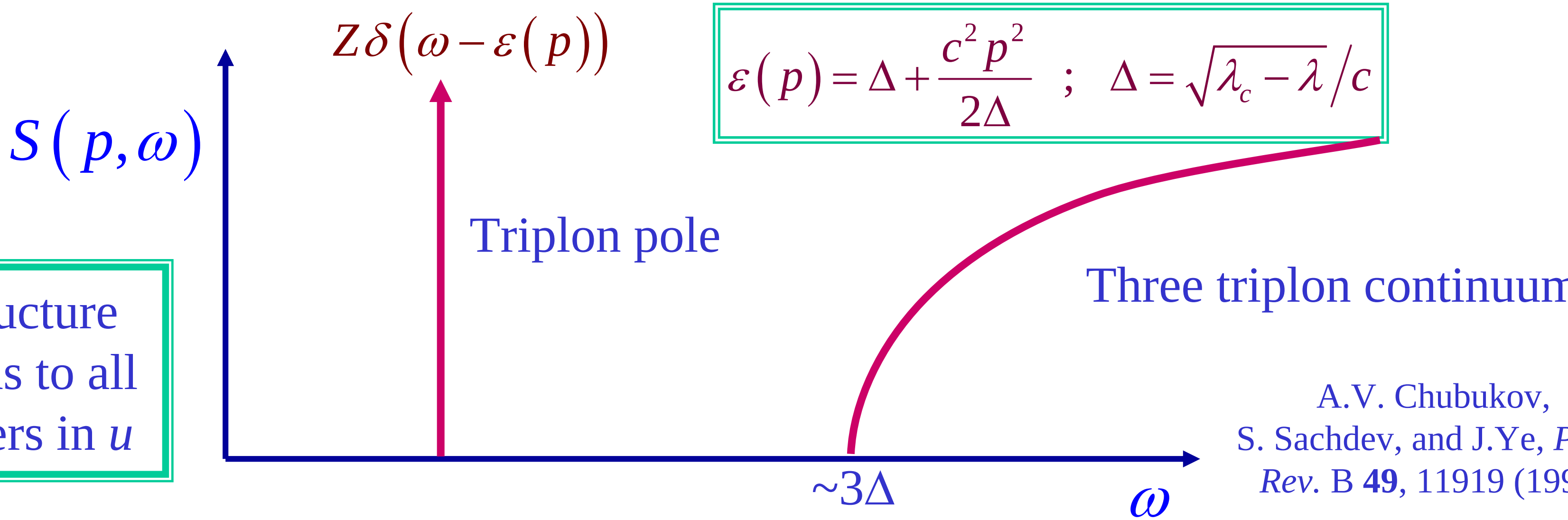
Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_\varphi = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + \frac{1}{c^2}\left(\partial_\tau \vec{\varphi}\right)^2 + \left(\lambda_c - \lambda\right)\vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

For $\lambda < \lambda_c$ oscillations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ lead to the following structure in the dynamic structure factor $S(p,\omega)$

$$\varepsilon(p) = \Delta + \frac{c^2 p^2}{2\Delta} \quad ; \quad \Delta = \sqrt{\lambda_c - \lambda}/c$$



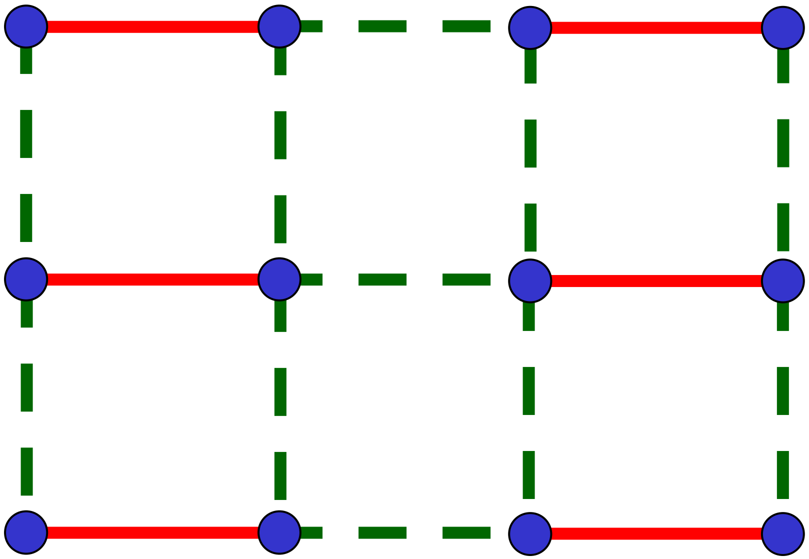
Structure holds to all orders in *u*

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

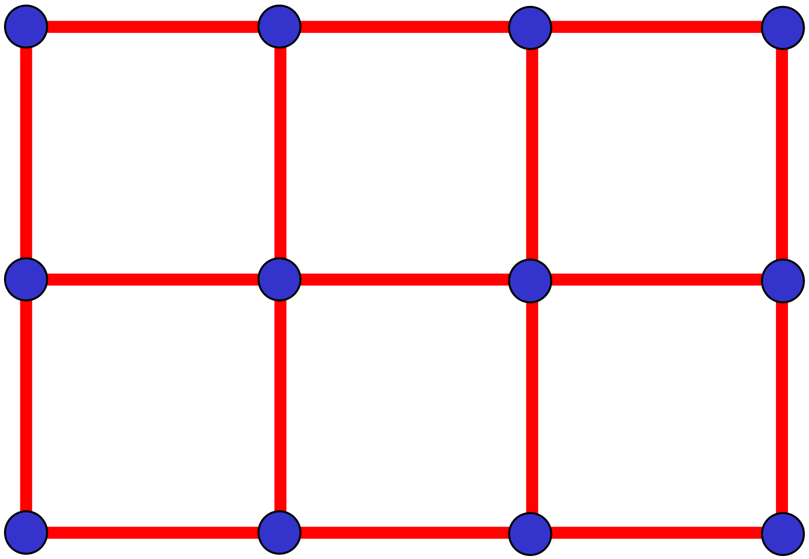
Mott insulators with spin S=1/2 per unit cell:

Berry phases, bond order, and the breakdown of the LGW paradigm

Mott insulator with two S=1/2 spins per unit cell



Mott insulator with one S=1/2 spin per unit cell



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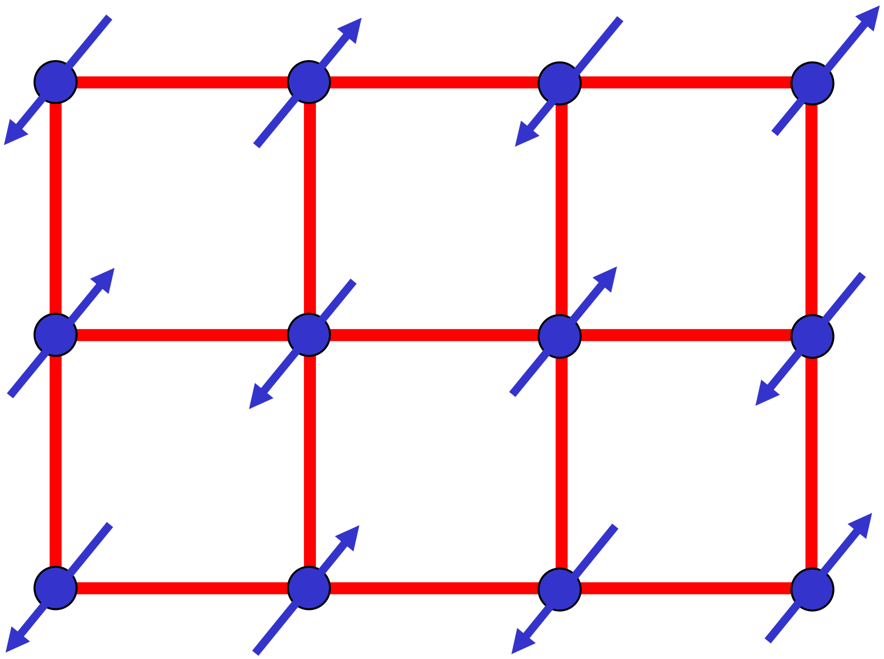
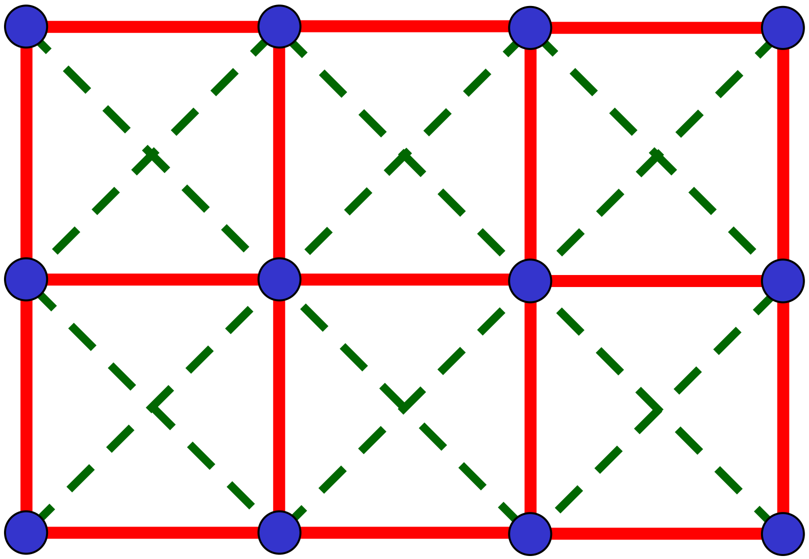


Ground state has Neel order with $\vec{\varphi} \neq 0$

Mott insulator with one S=1/2 spin per unit cell

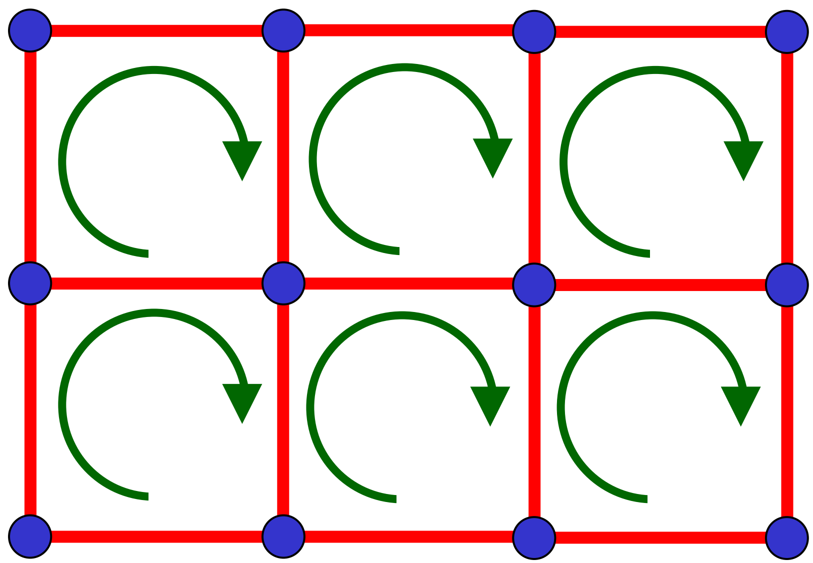


Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange. The strength of this perturbation is measured by a coupling g.

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Mott insulator with one S=1/2 spin per unit cell



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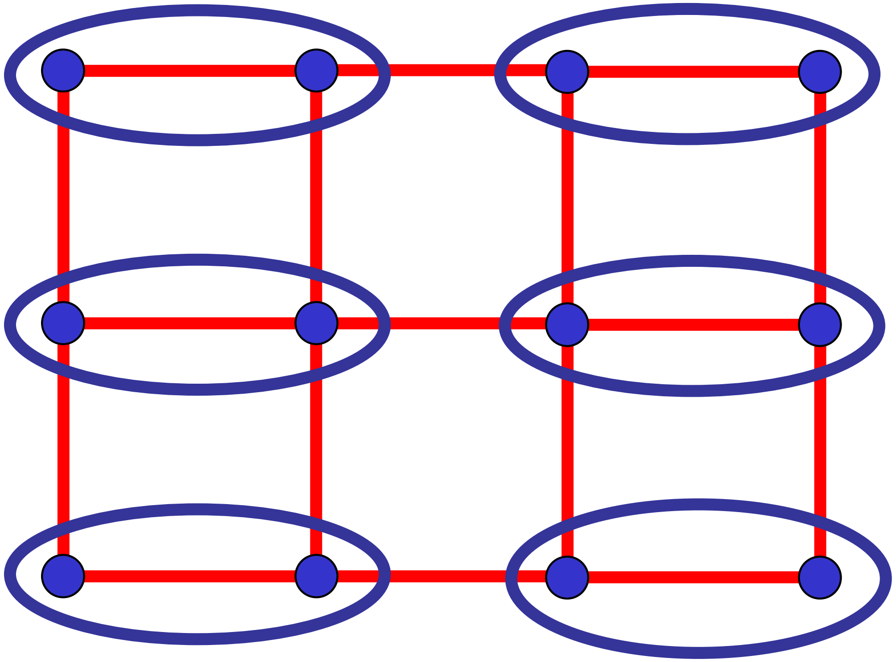
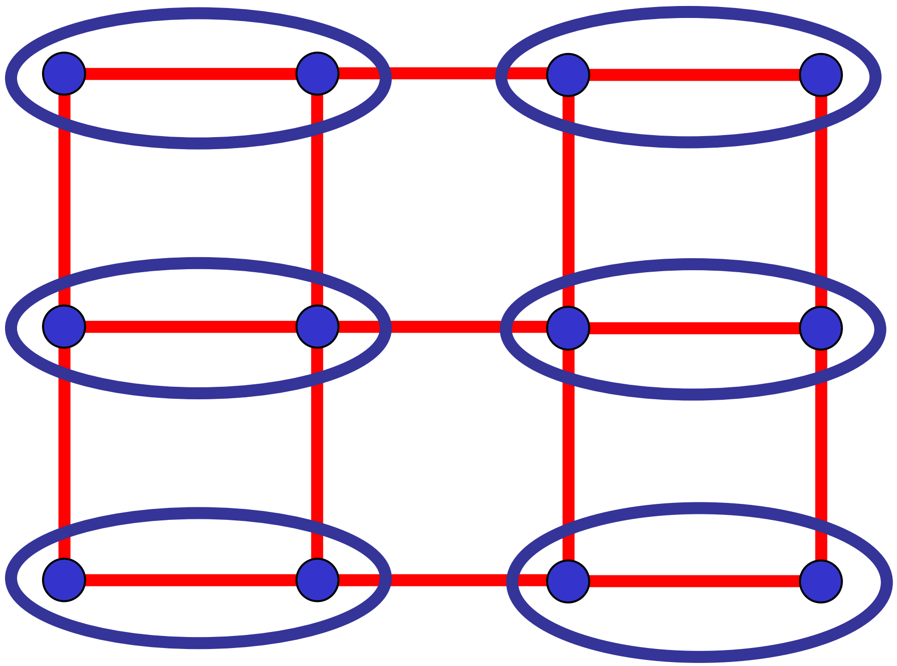
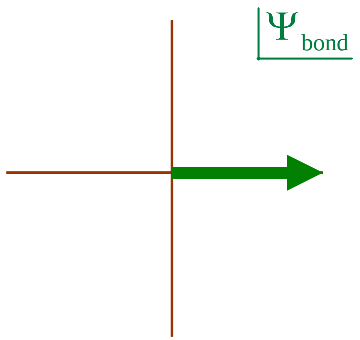
Mott insulator with one S=1/2 spin per unit cell



Possible large g paramagnetic ground state (Class A) with $\langle \vec{\varphi} \rangle = 0$

Mott insulator with one S=1/2 spin per unit cell





Possible large g paramagnetic ground state (Class A) with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

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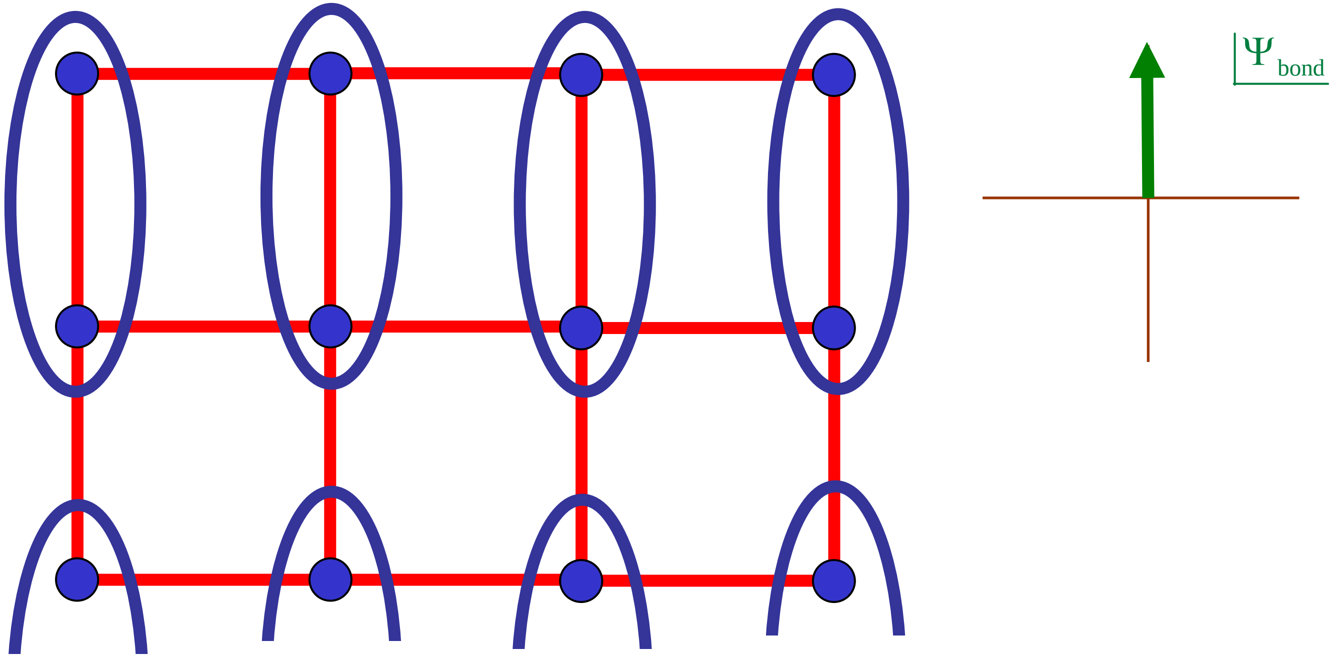

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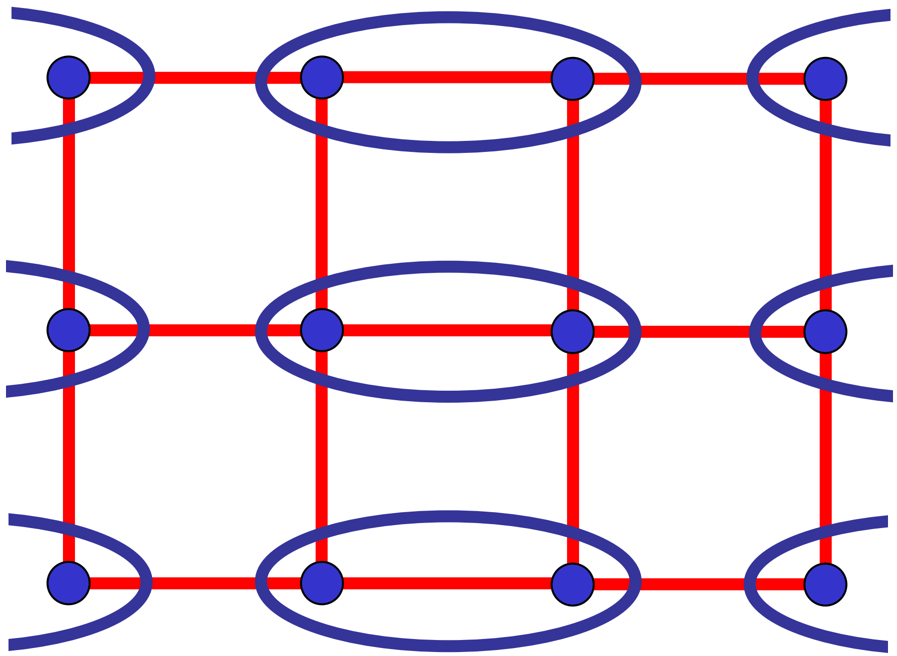

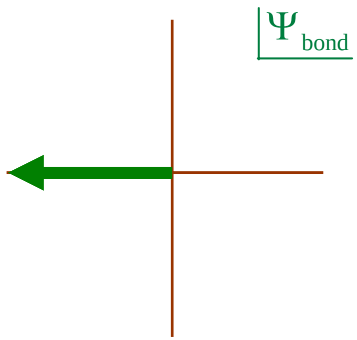

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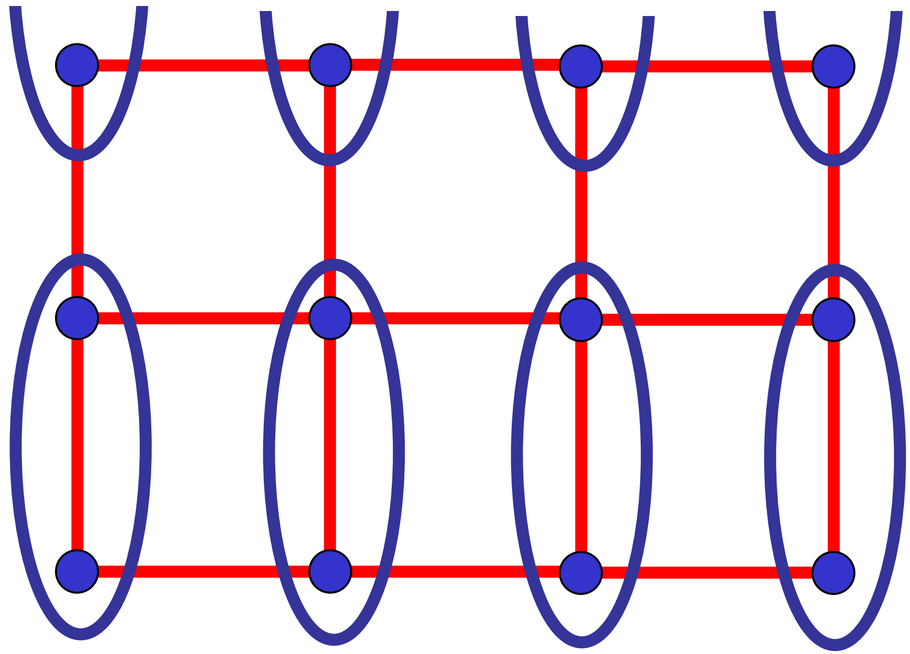

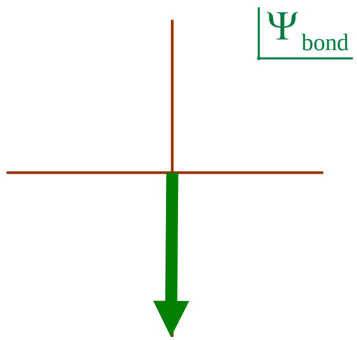

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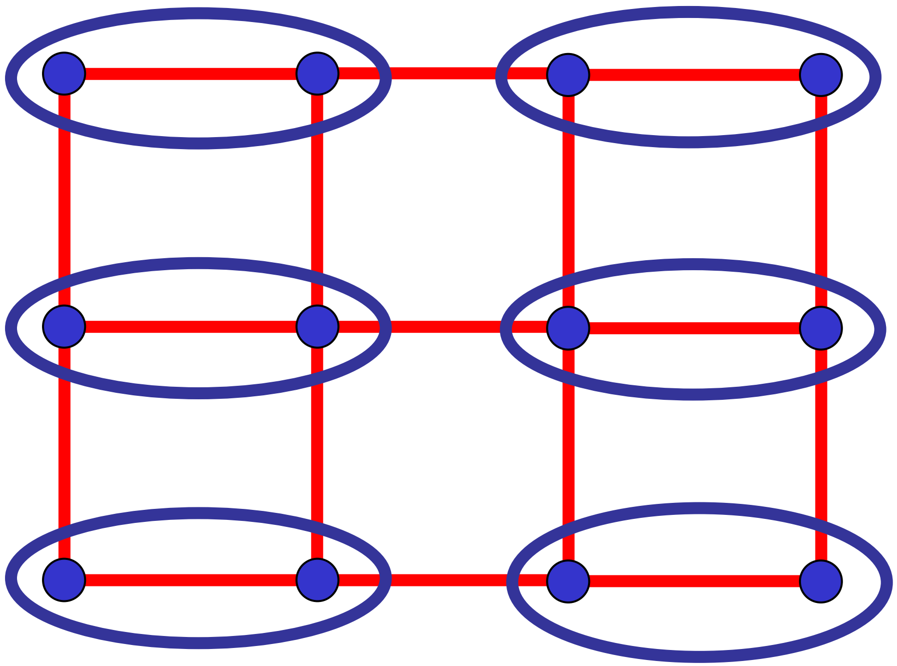

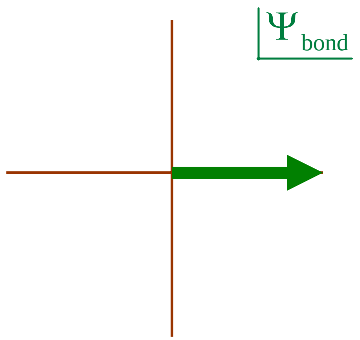

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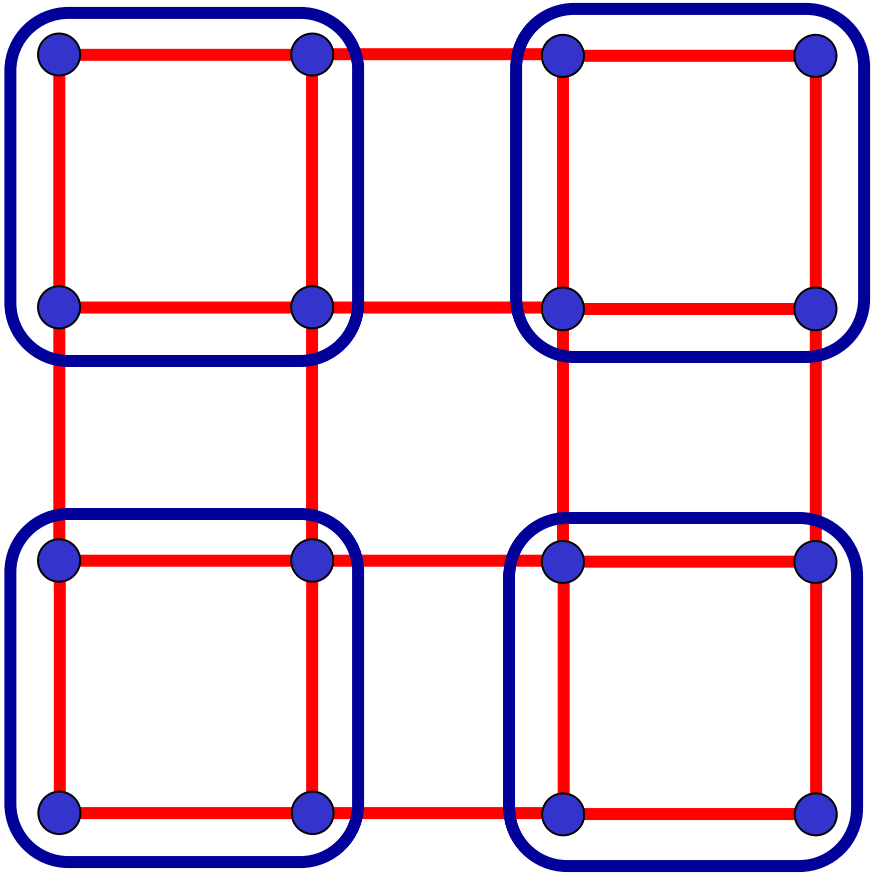

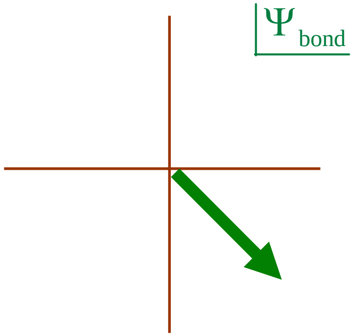

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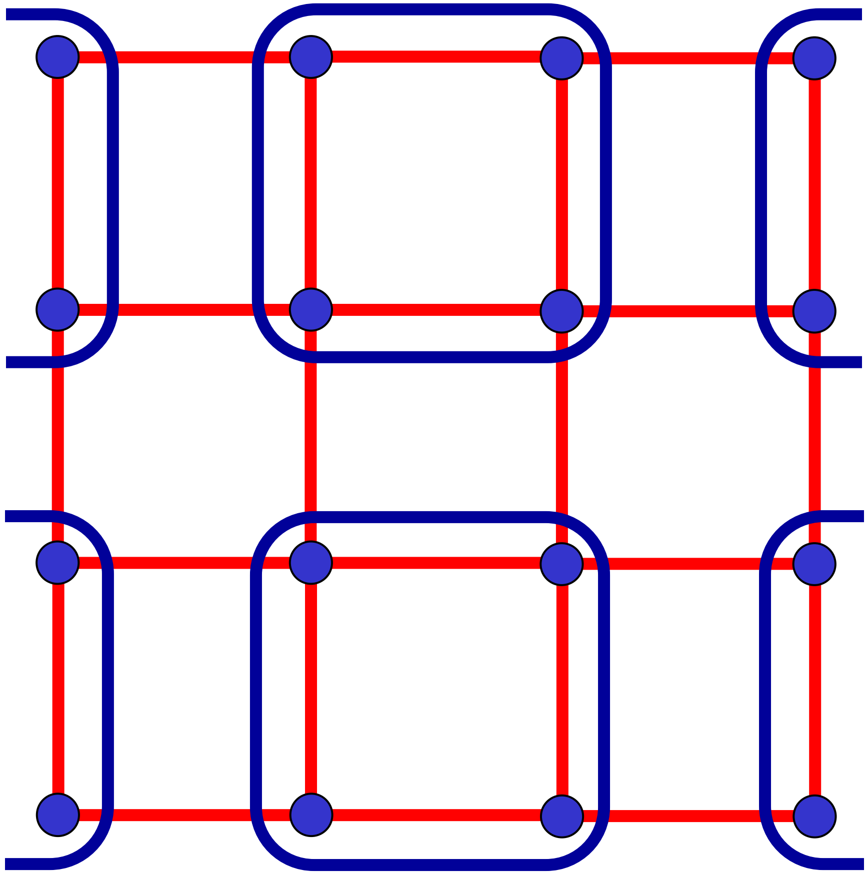

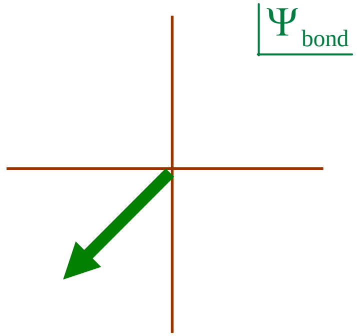

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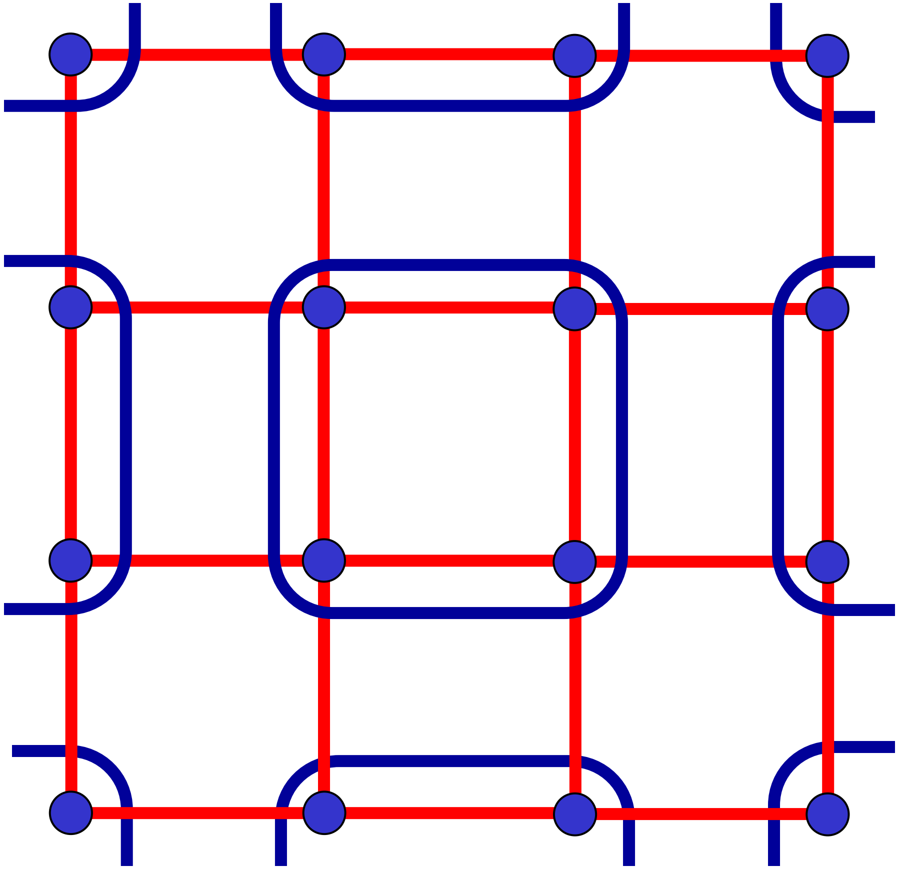

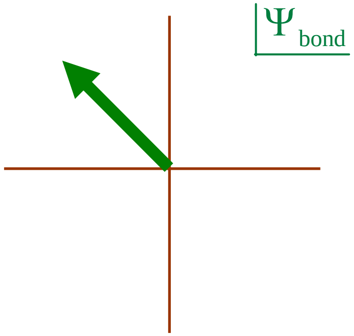

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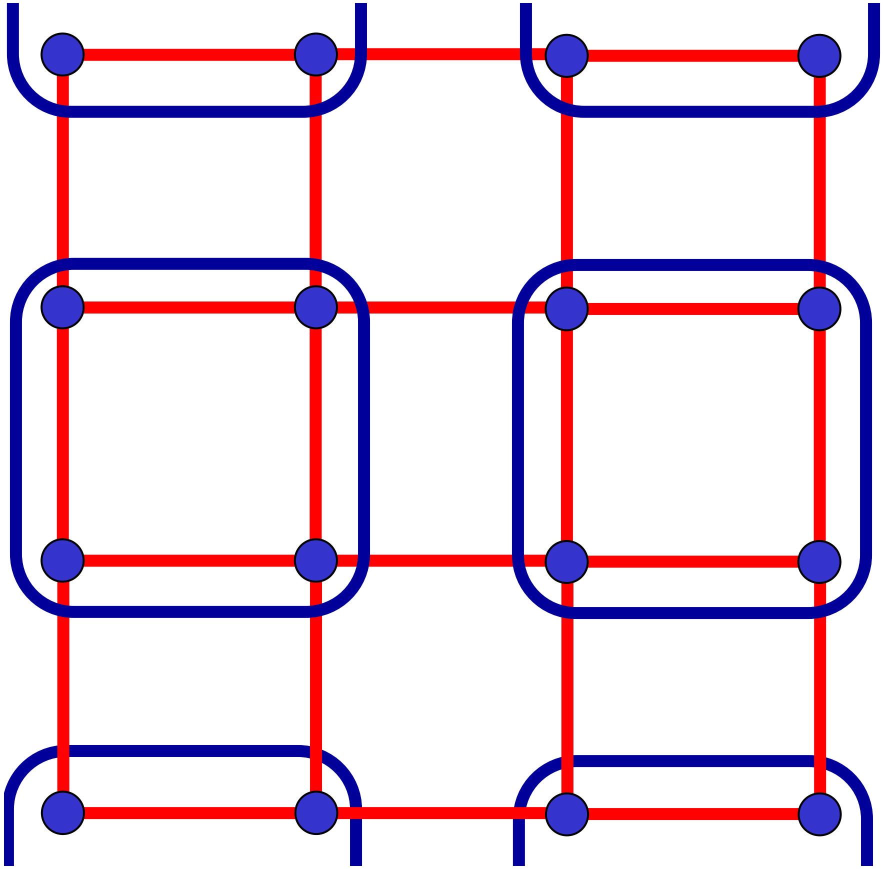

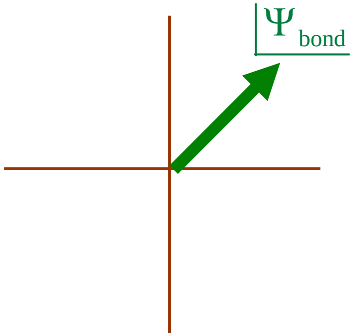

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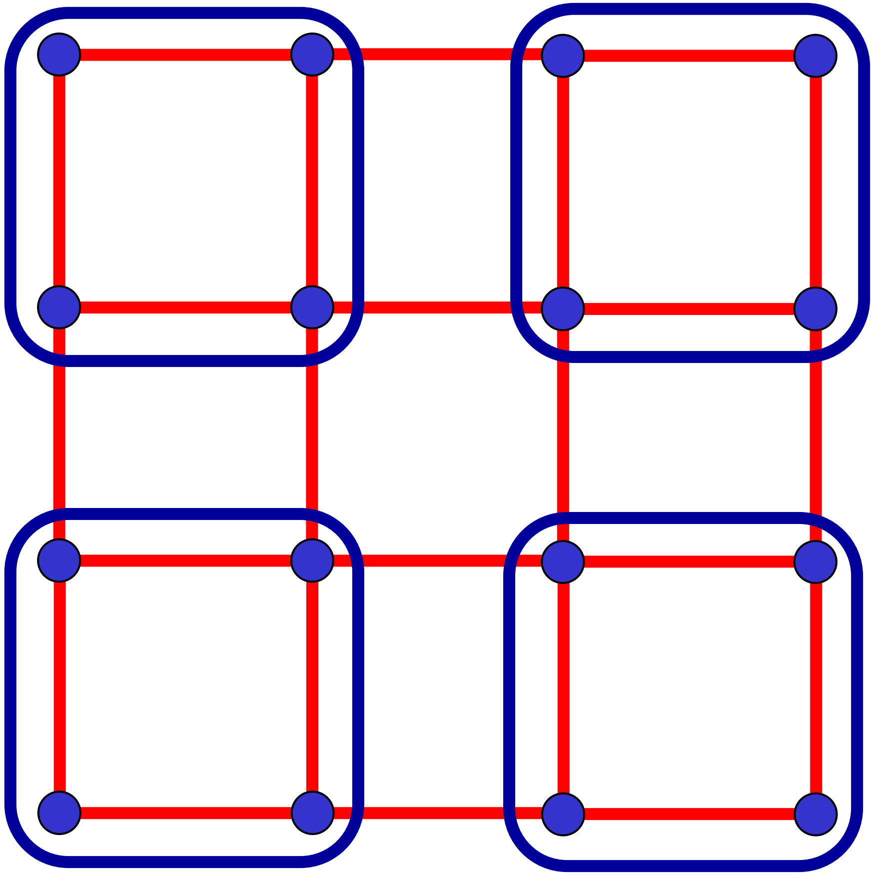

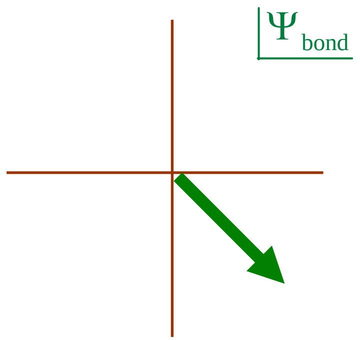

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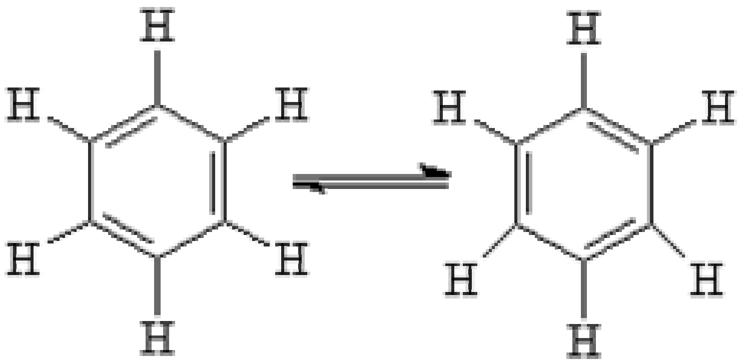
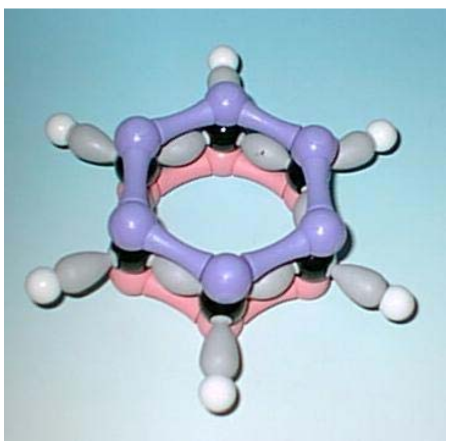
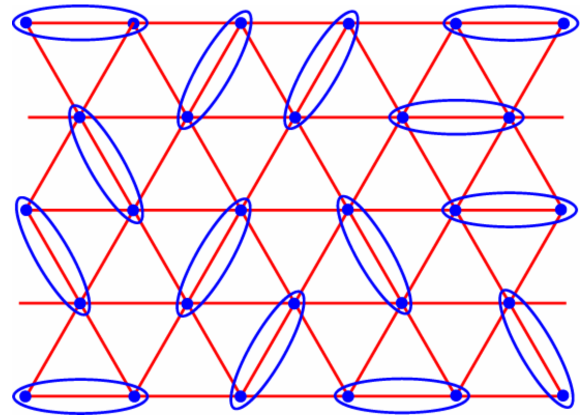
Resonating valence bonds





Resonance in benzene leads to a symmetric configuration of valence bonds
(*F. Kekulé, L. Pauling*)



Different valence bond pairings resonate with each other, leading to a resonating valence bond *liquid*, (Class B paramagnet) with $\langle \Psi_{\text{bond}} \rangle = 0$

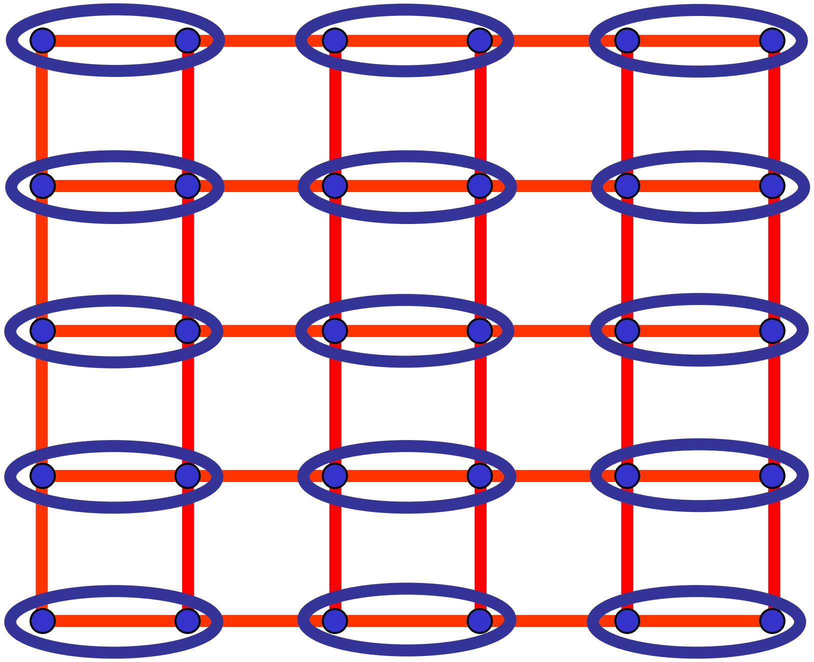
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974); P.W. Anderson 1987

Such states are associated with non-collinear spin correlations, Z_2 gauge theory, and topological order.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991); X. G. Wen, *Phys. Rev.* B **44**, 2664 (1991).

Excitations of the paramagnet with non-zero spin

$\left\langle \Psi_{\text{bond}} \right\rangle \neq 0$; Class A



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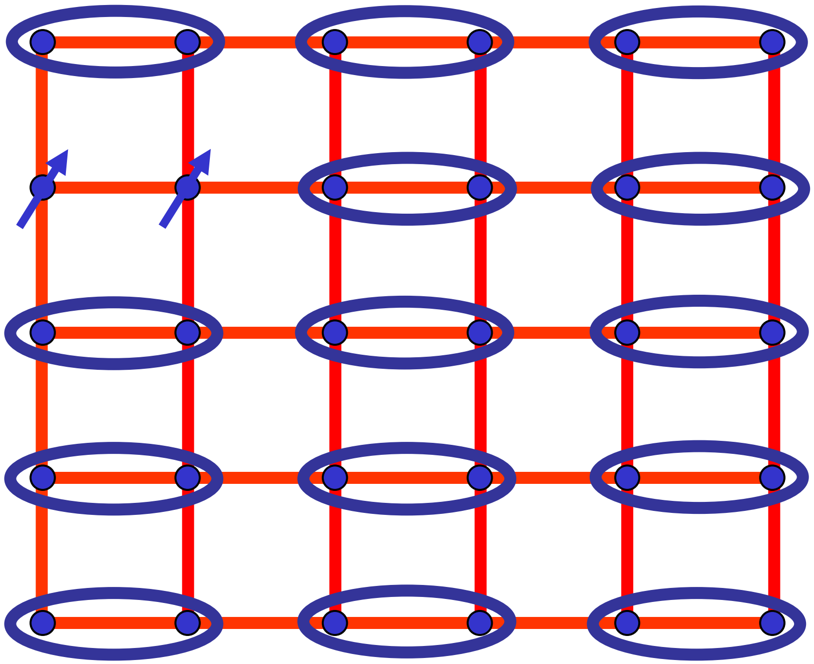

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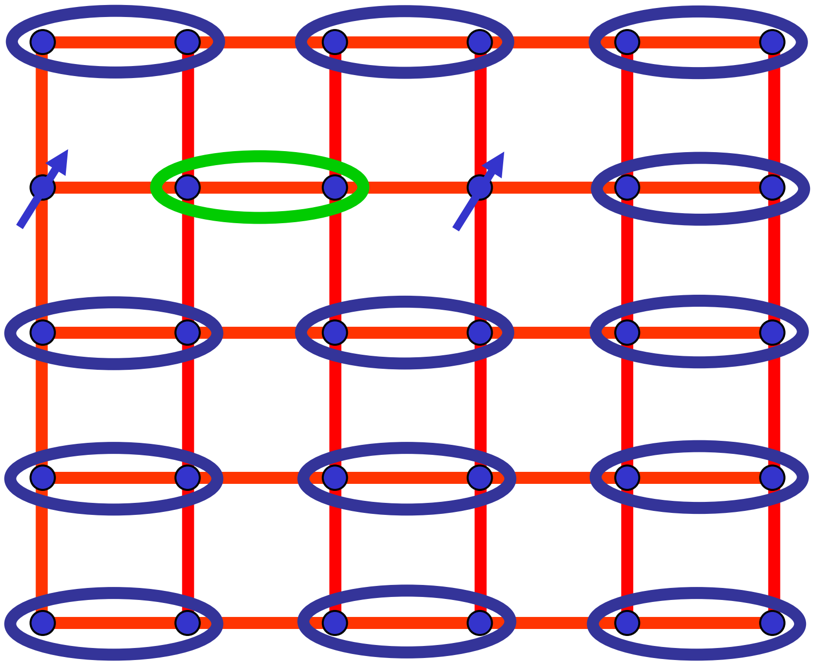

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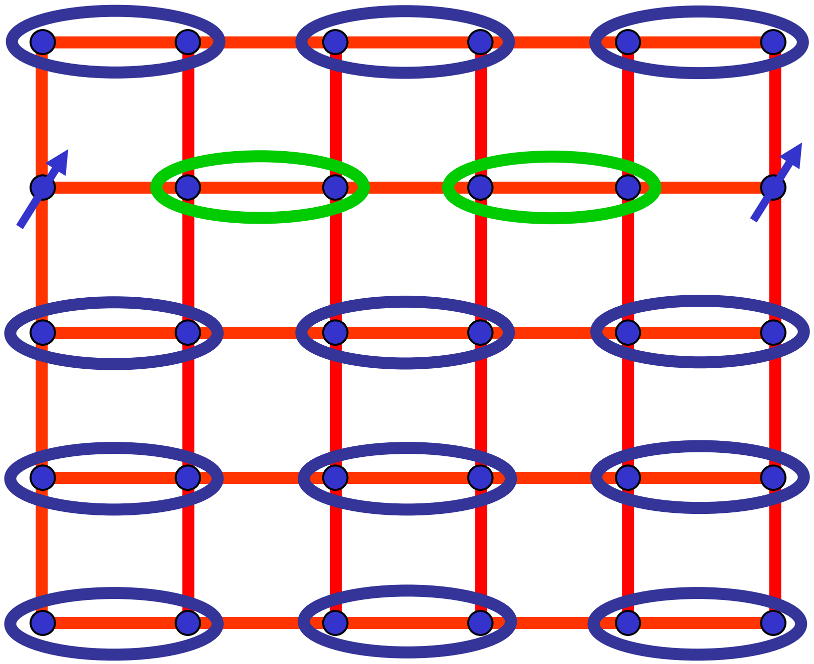

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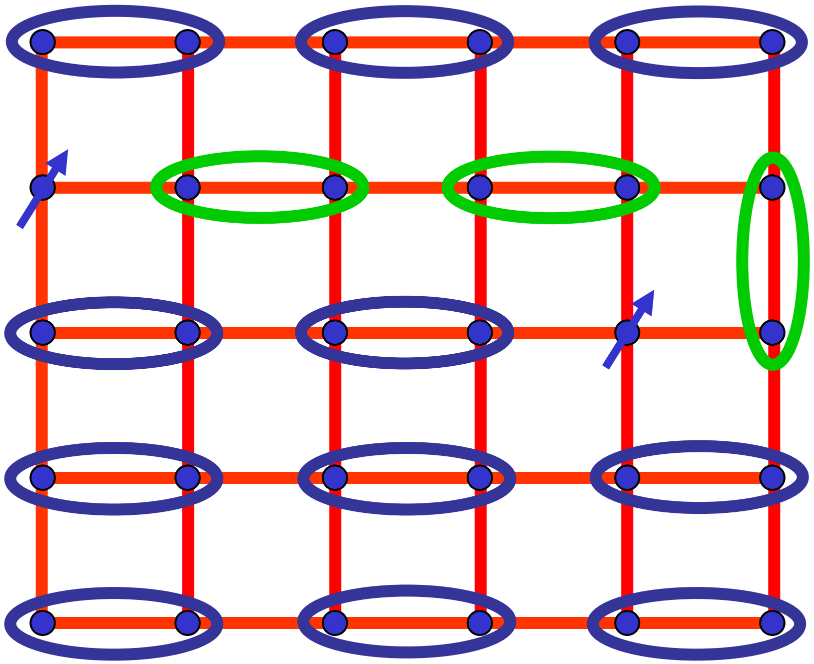

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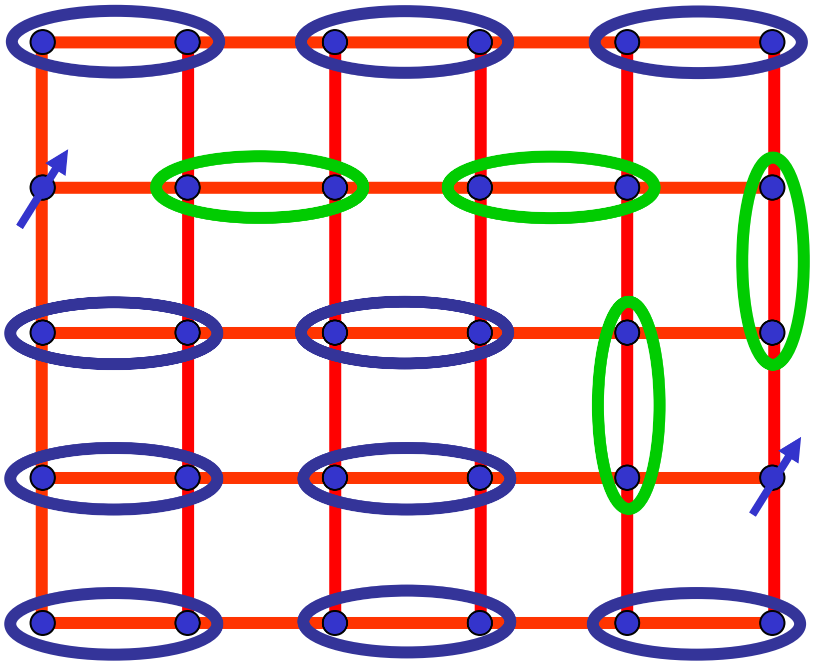

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$$\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

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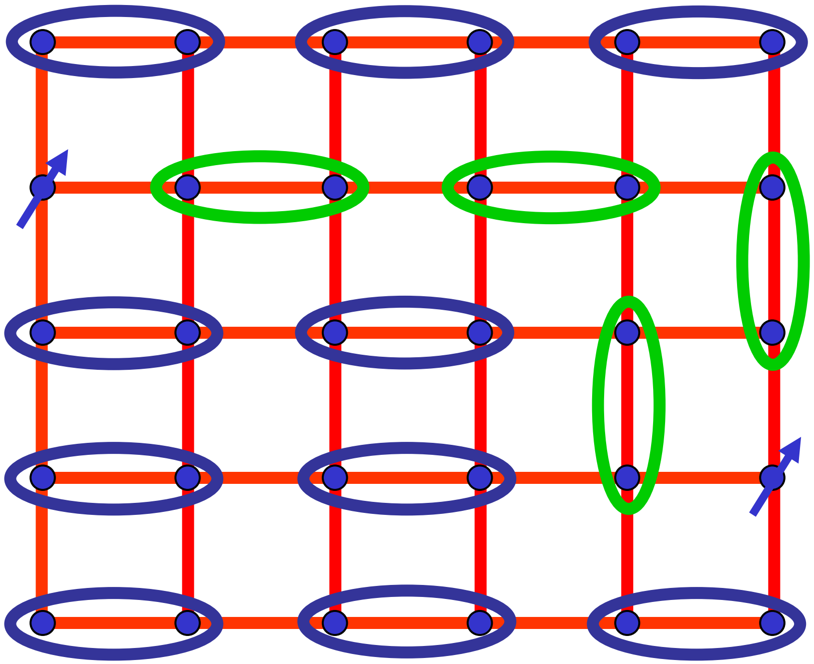

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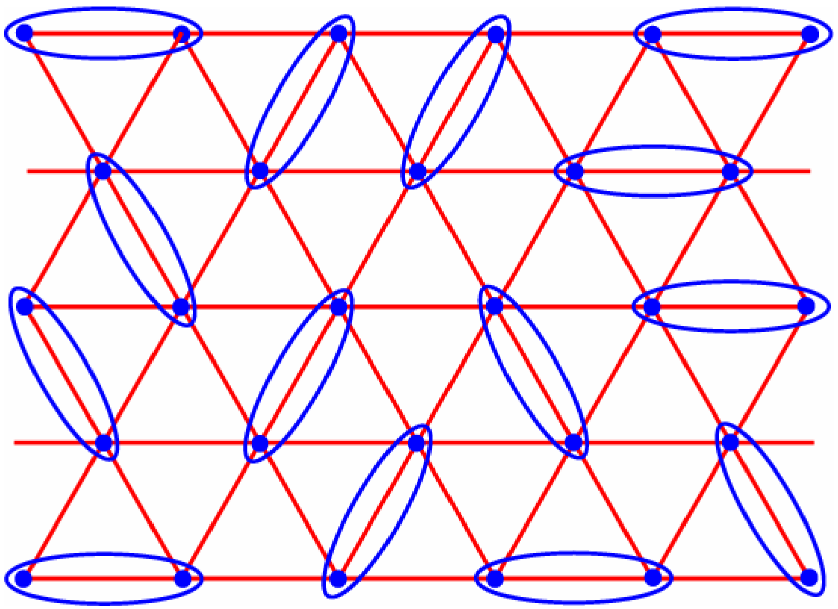

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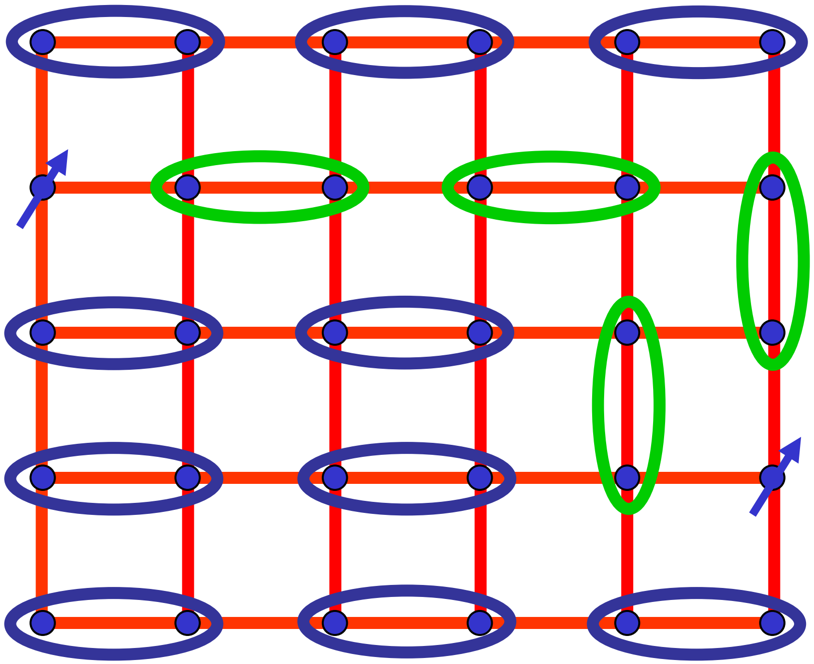

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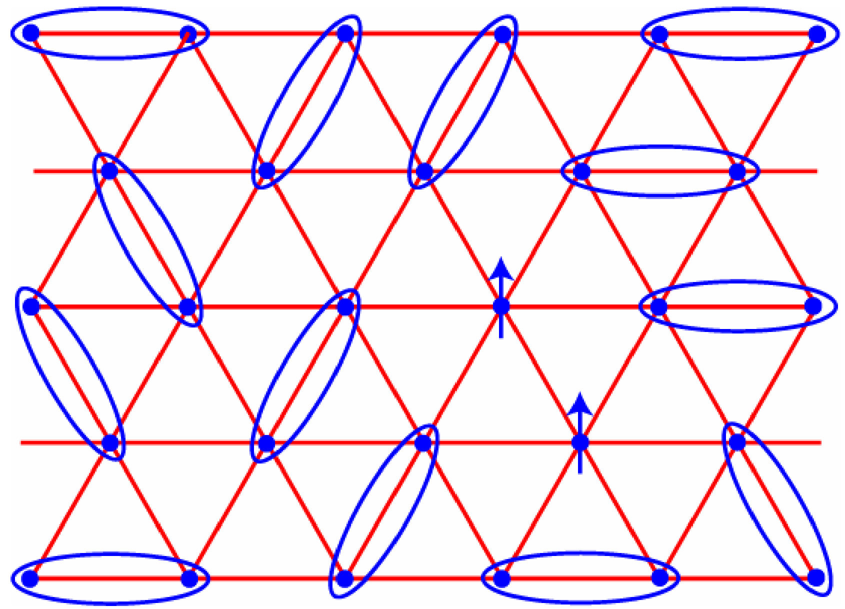

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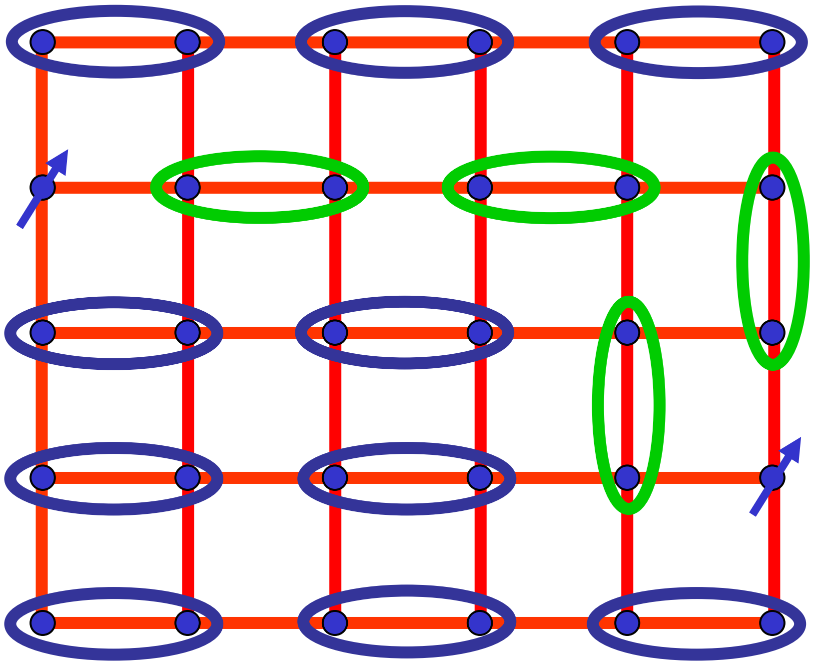

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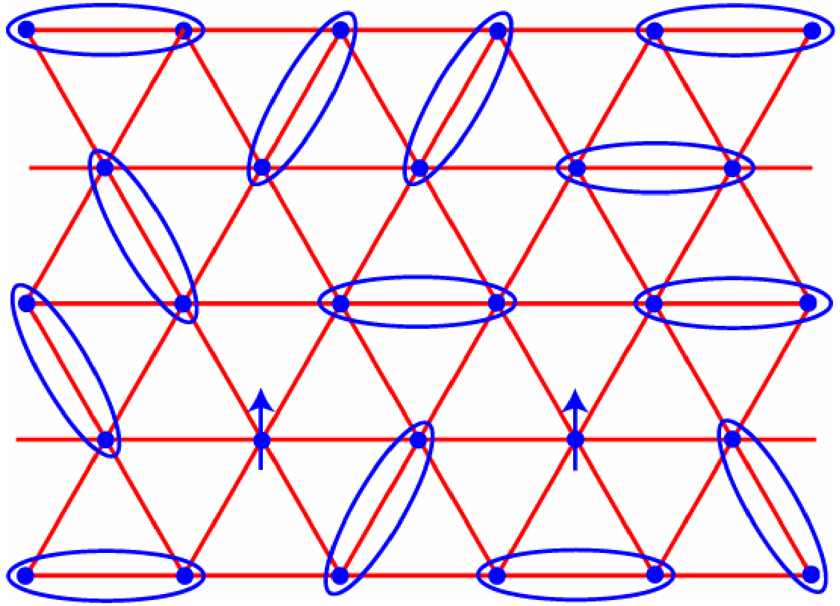

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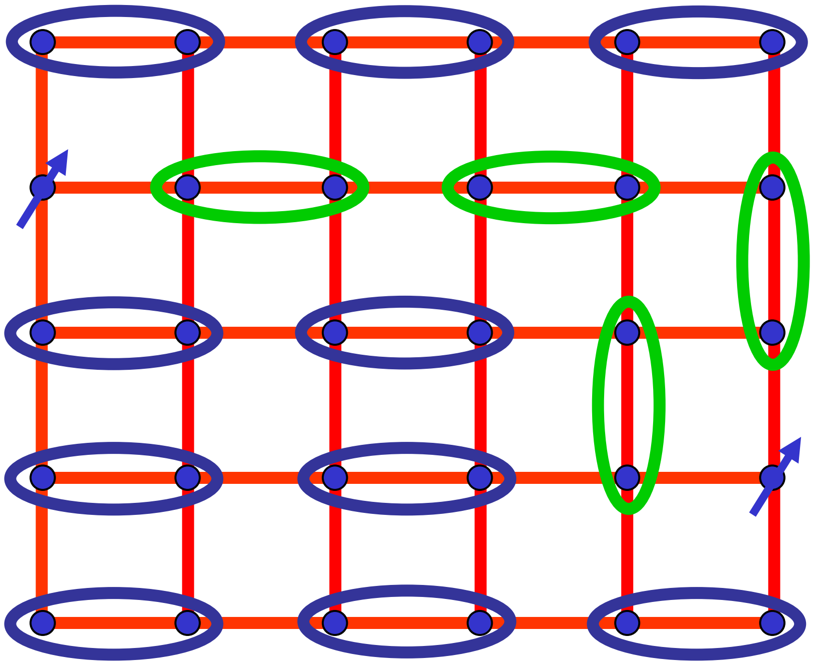

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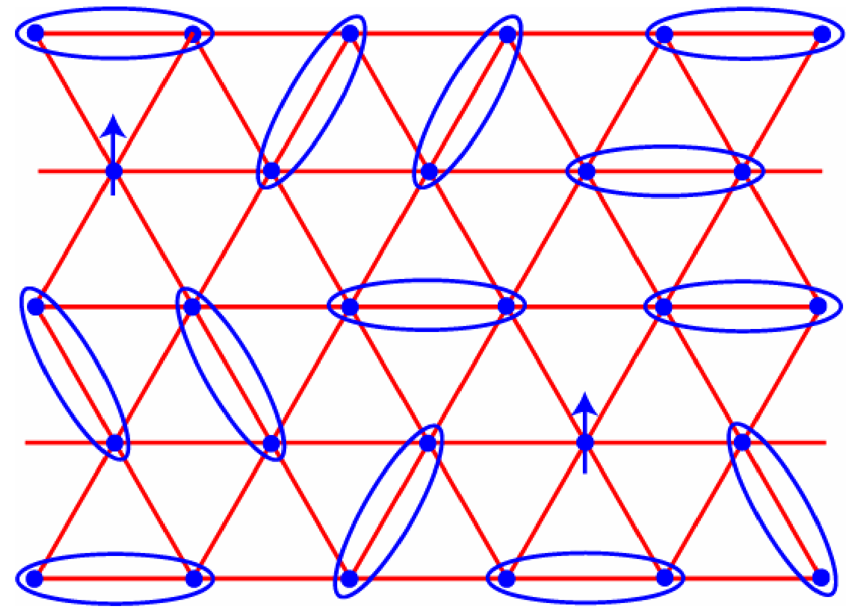
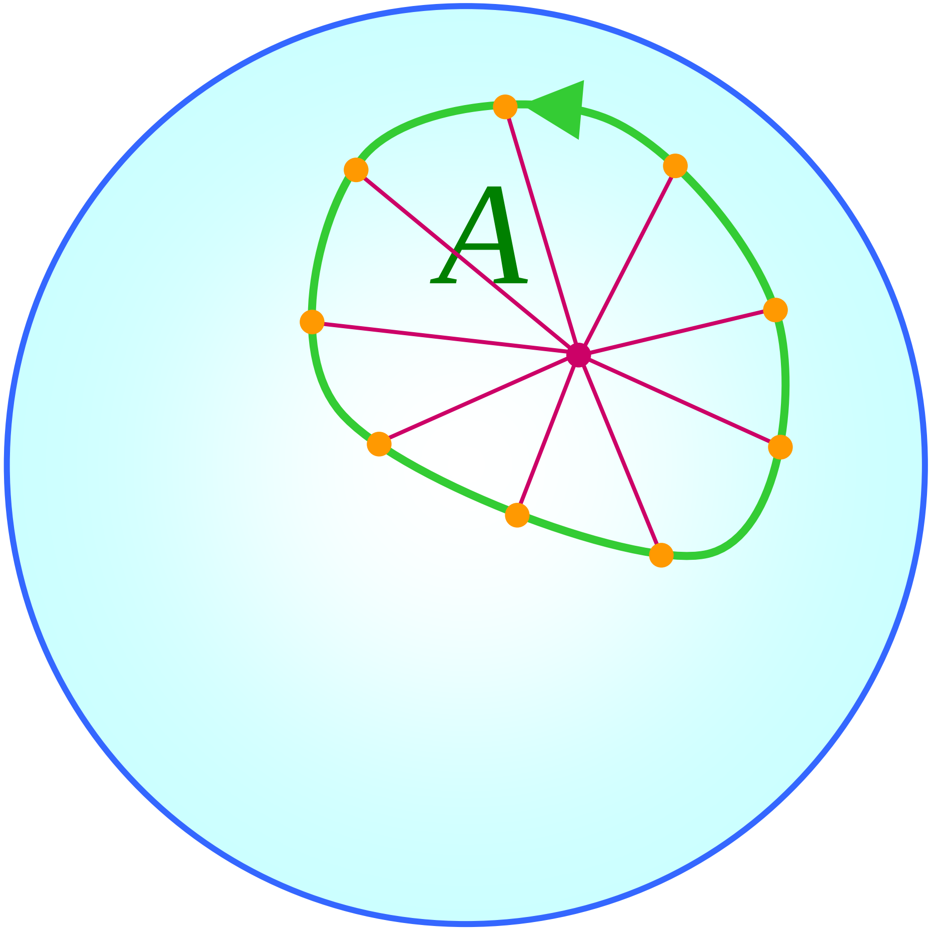


S=1/2 ***spinons*** can propagate independently across the lattice

Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



$$e^{iSA}$$

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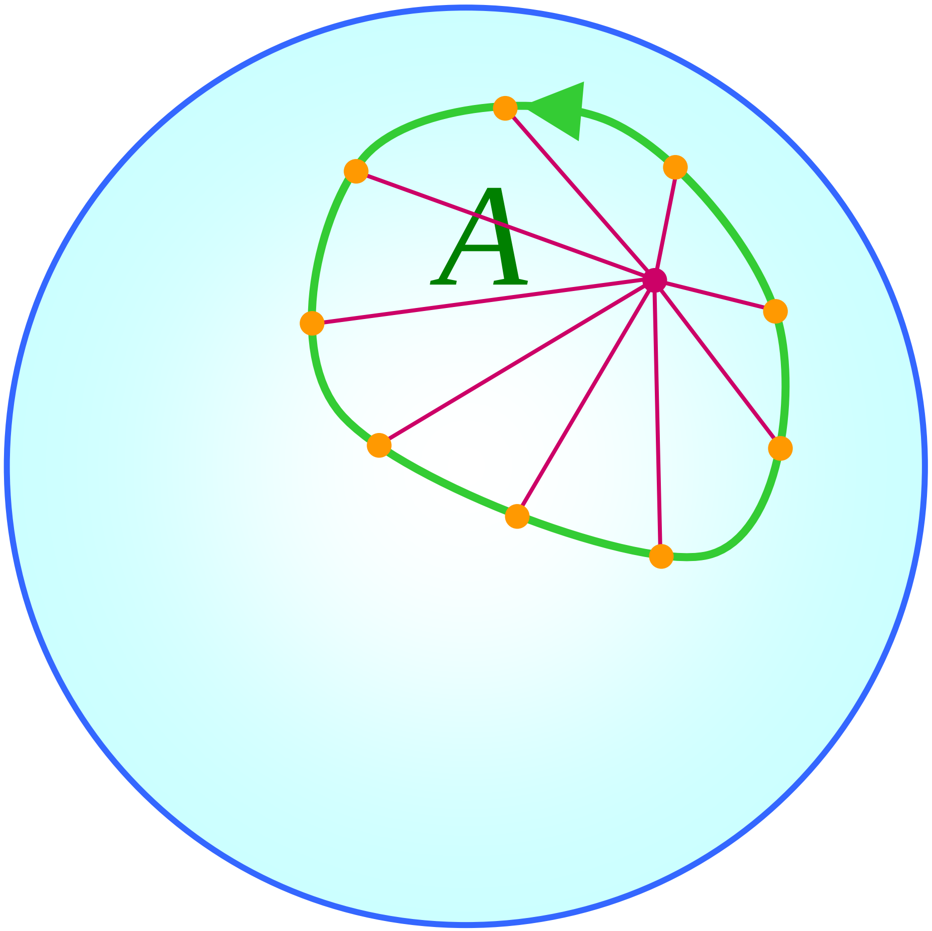

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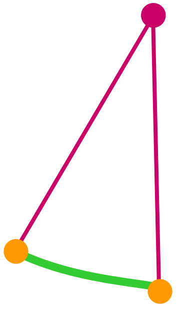
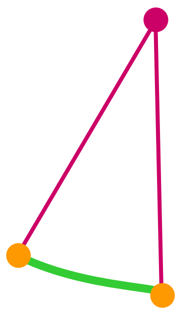


Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*



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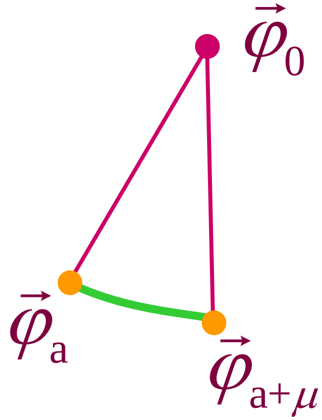

$(\mu = x, y, \tau)$

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$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

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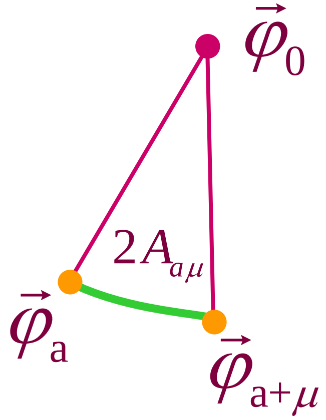

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

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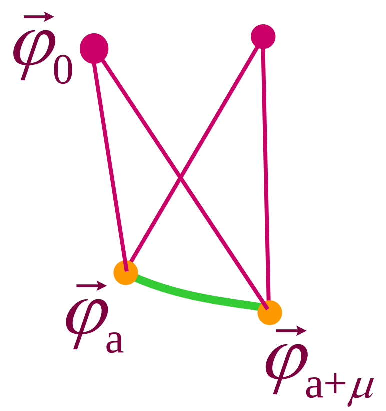

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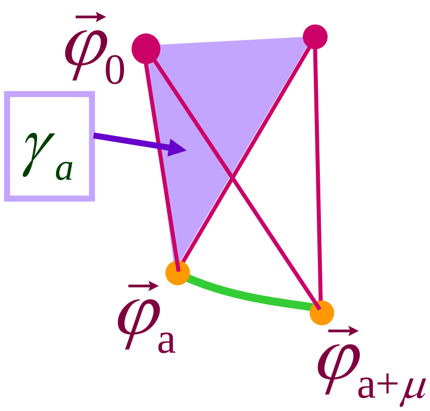

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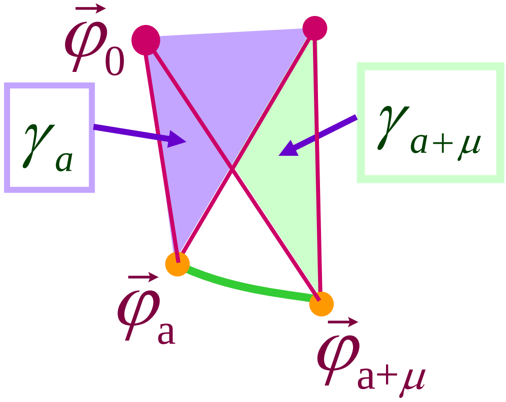

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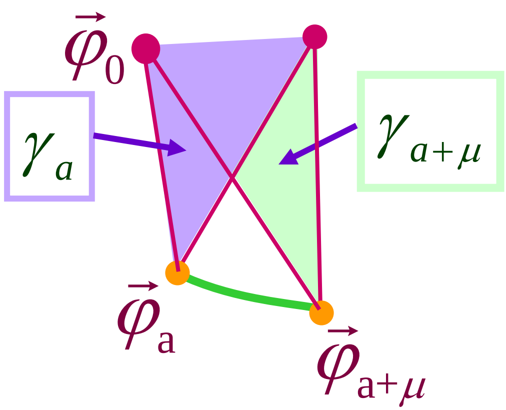
$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_a$, $\vec{\varphi}_{a+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$

$$2A_{a\mu} \rightarrow 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

Change in choice of $\vec{\varphi}_0$ is like a "gauge transformation"



S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

Recall $\vec{\varphi}_a = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_a = (0,0,1)$ in classical Neel state;

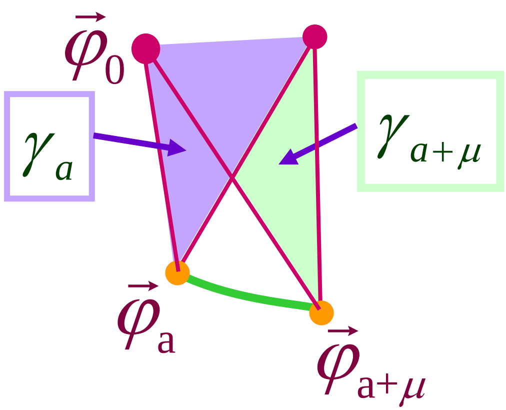
$\eta_a \rightarrow \pm 1$ on two square sublattices ;

$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_a$, $\vec{\varphi}_{a+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$

$$2A_{a\mu} \rightarrow 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

Change in choice of $\vec{\varphi}_0$ is like a "gauge transformation"

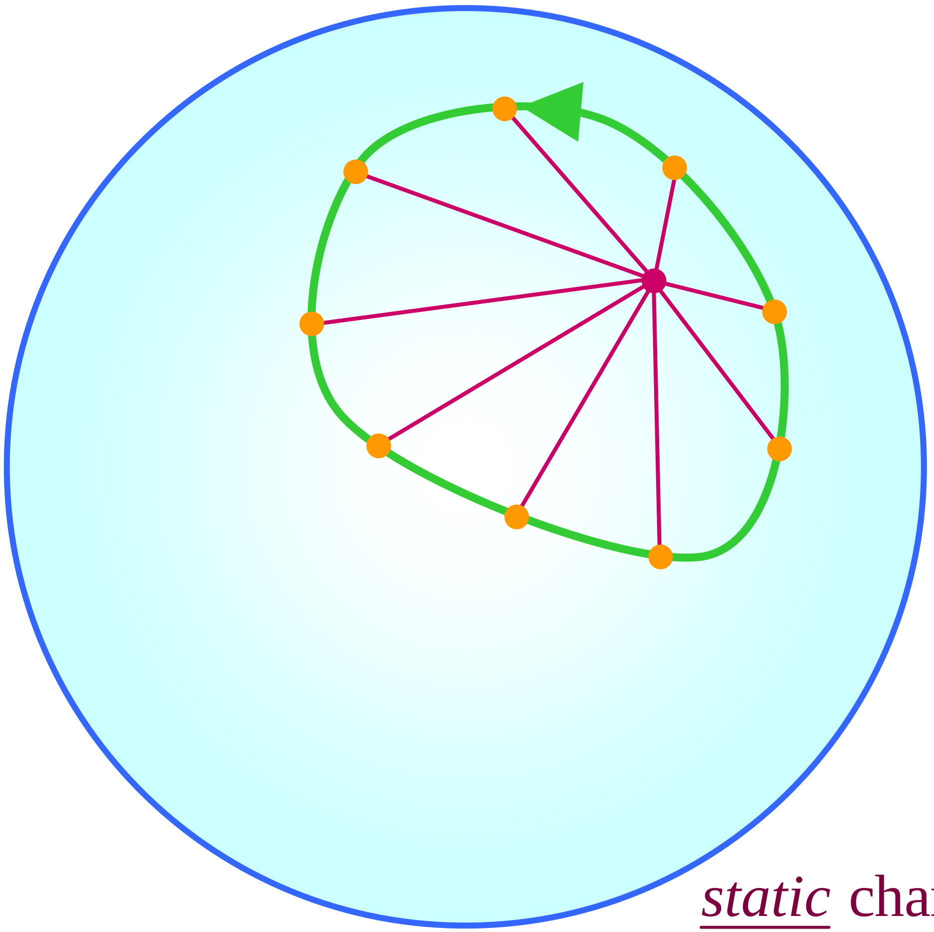


The area of the triangle is uncertain modulo 4π, and the action has to be invariant under $A_{a\mu} \rightarrow A_{a\mu} + 2\pi$

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



$$\exp\left(i \sum_a \eta_a A_{a\tau} \right)$$

Sum of Berry phases of all spins on the square lattice.

$$= \exp\left(i \sum_{a,\mu} J_{a\mu} A_{a\mu} \right)$$

with "current" $J_{a\mu}$ of

static charges ± 1 on sublattices

Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$

Modulus of weights in partition function: those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Berry phases lead to large cancellations between different time histories $\rightarrow$ need an effective action for $A_{a\mu}$ at large g

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) + i \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 3 spacetime dimensions with static charges ± 1 on two sublattices.

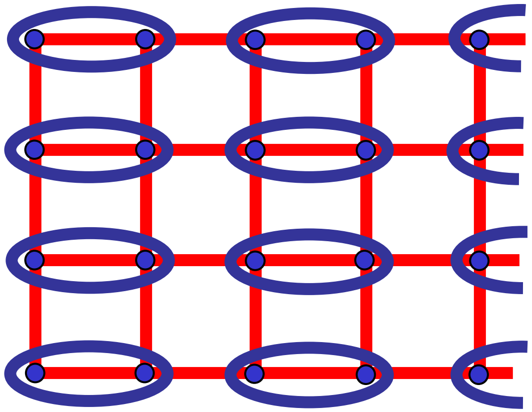
Analysis by a duality mapping shows that this theory is *always* in a phase with $\langle \Psi_{\text{bond}} \rangle \neq 0$ (Class A paramagnet). The gauge theory is in a *confining* phase (spinons are confined and only S=1 triplons propagate).

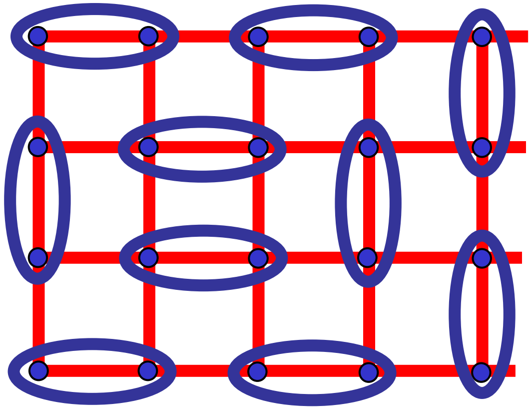
Proliferation of monopoles in the presence of Berry phases.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett*. B **4**, 1043 (1990).

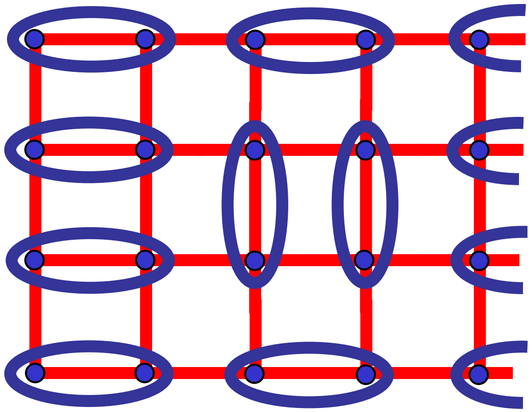
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

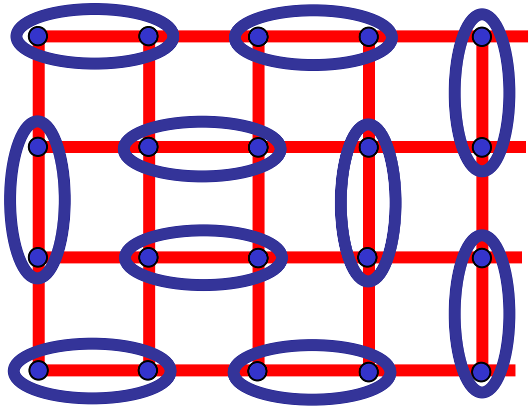
Ordering by quantum fluctuations



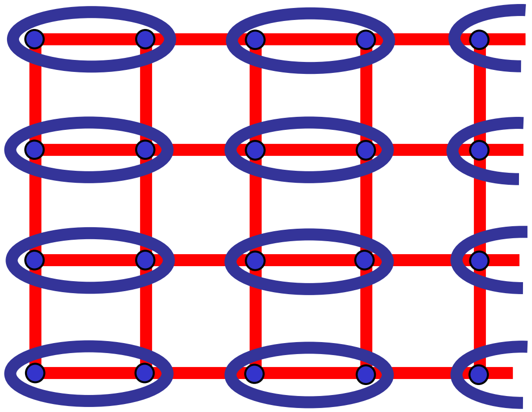


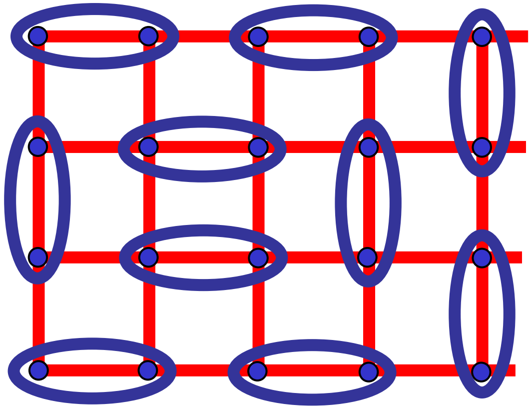
Ordering by quantum fluctuations



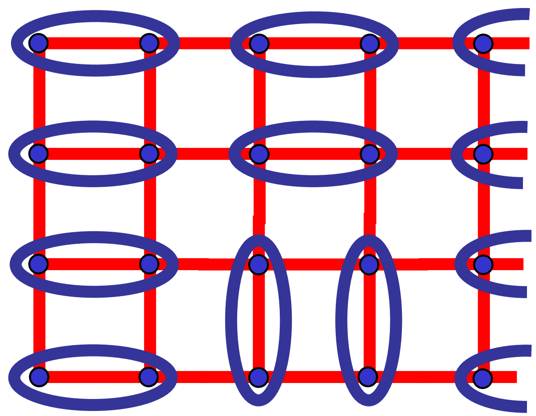


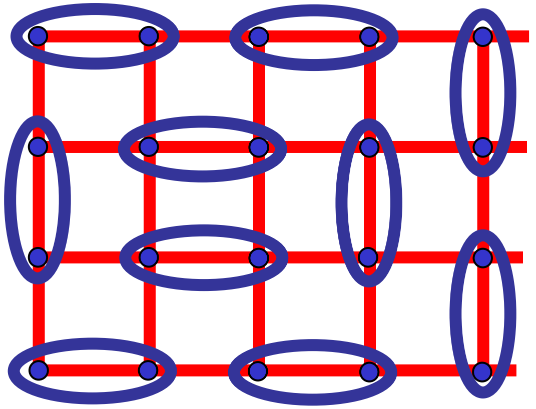
Ordering by quantum fluctuations



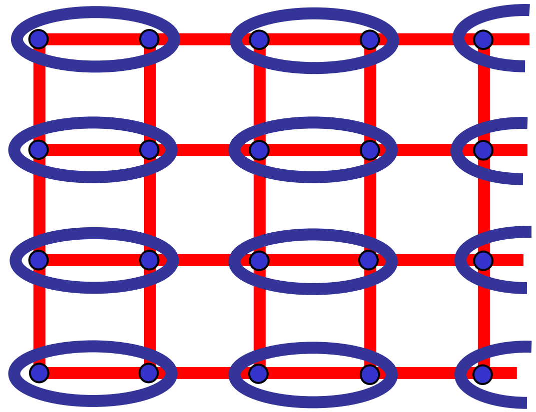


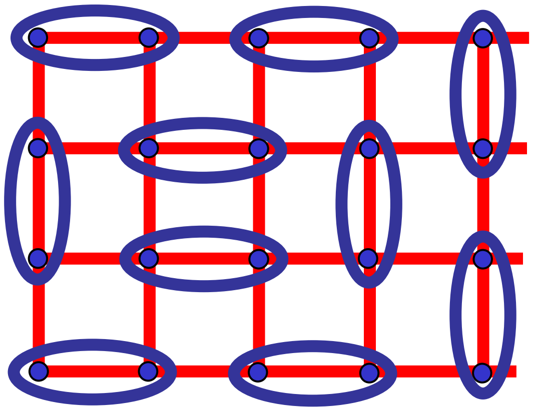
Ordering by quantum fluctuations



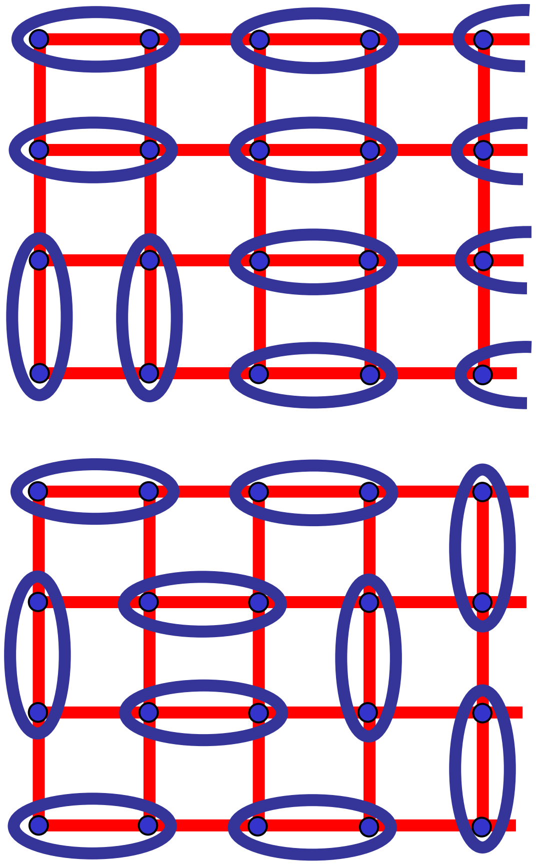


Ordering by quantum fluctuations

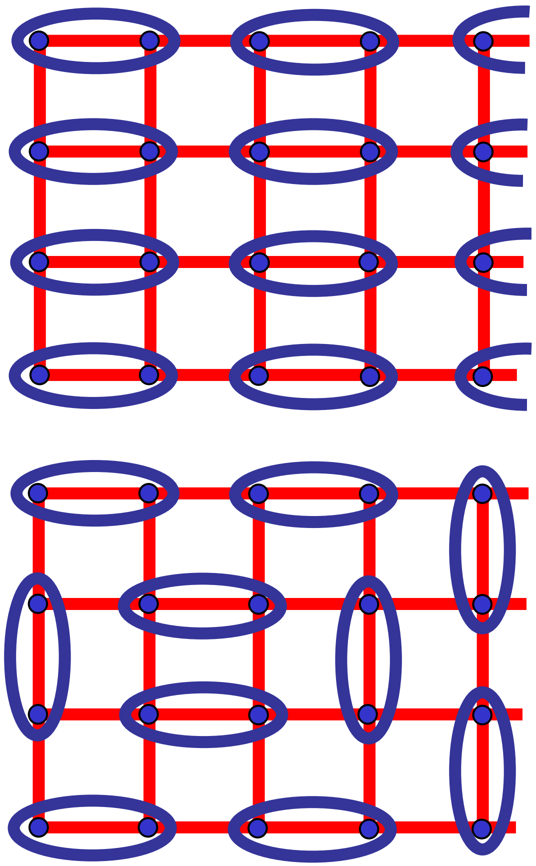




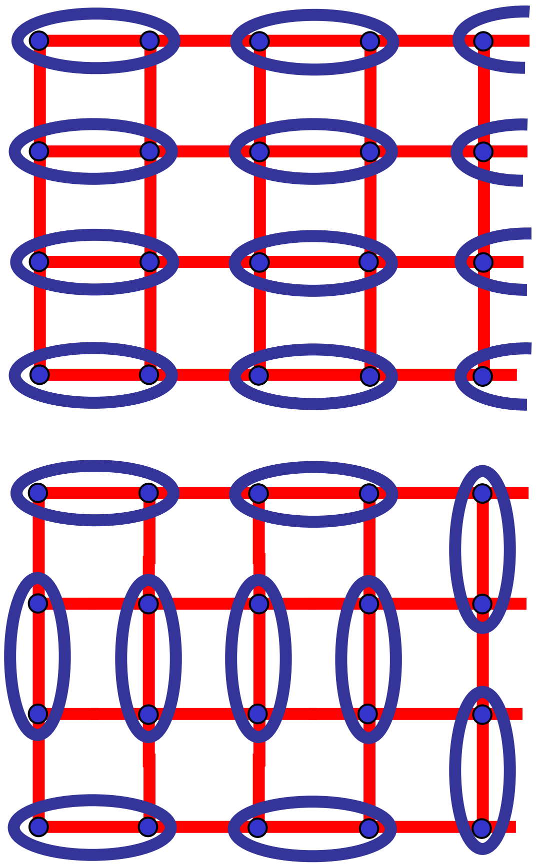
Ordering by quantum fluctuations



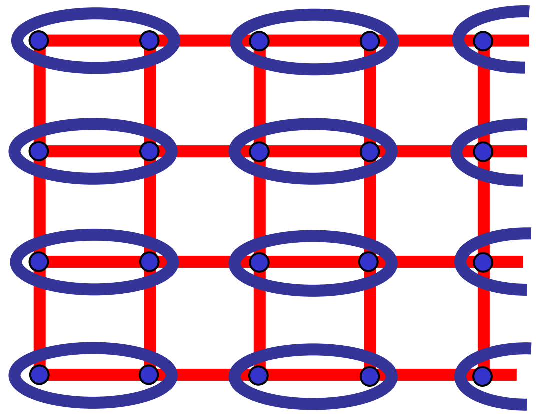
Ordering by quantum fluctuations

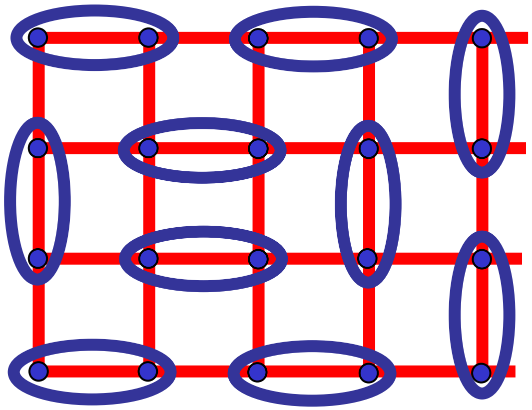


Ordering by quantum fluctuations

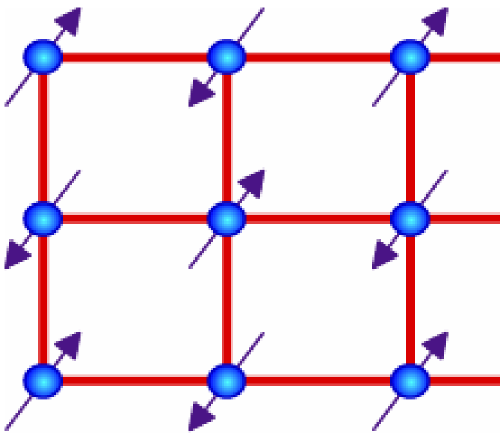


Ordering by quantum fluctuations





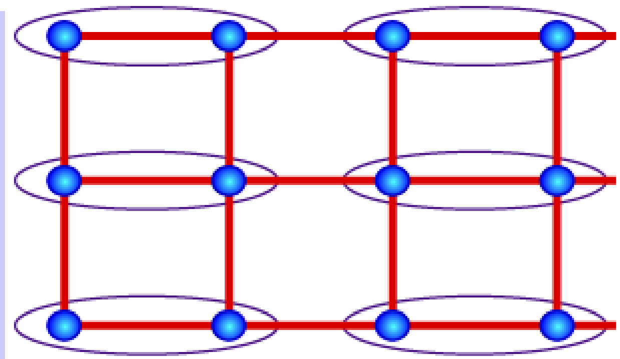
$$Z=\prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2-1\right)\exp\left(\frac{1}{g}\sum_{a,\mu}\vec{\varphi}_a\cdot\vec{\varphi}_{a+\mu}+i\sum_a \eta_a A_{a\tau}\right)$$



Neel order

$\langle\vec{\varphi}\rangle \neq 0$

?



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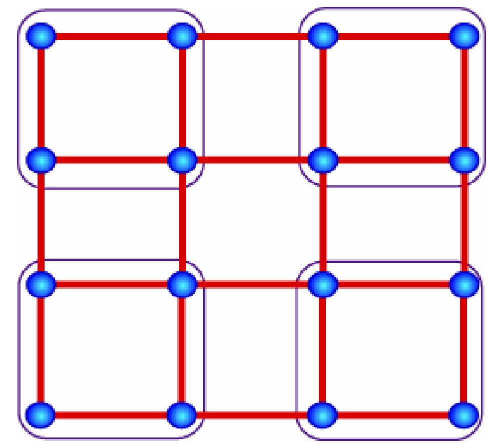


Bond order

$\langle\Psi_{\text{bond}}\rangle \neq 0$

Not present in LGW theory of $\vec{\varphi}$ order

0

g

Naïve approach: add bond order parameter to LGW theory "by hand"

Neel order

$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{bond}} \rangle = 0$

First order transition

Bond order

$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{bond}} \rangle \neq 0$

g

Neel order

$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{bond}} \rangle = 0$

Coexistence

$\langle \vec{\varphi} \rangle \neq 0,$

$\langle \Psi_{\text{bond}} \rangle \neq 0$

Bond order

$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{bond}} \rangle \neq 0$

g

Neel order

$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{bond}} \rangle = 0$

"disordered"

$\langle \vec{\varphi} \rangle = 0,$

$\langle \Psi_{\text{bond}} \rangle = 0$

Bond order

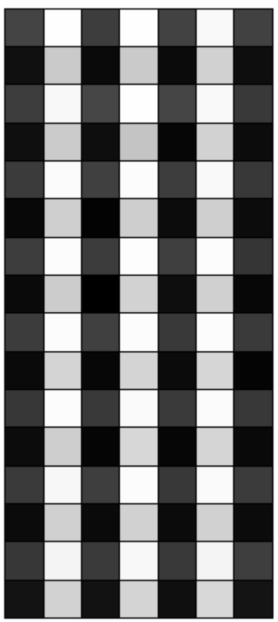
$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{bond}} \rangle \neq 0$

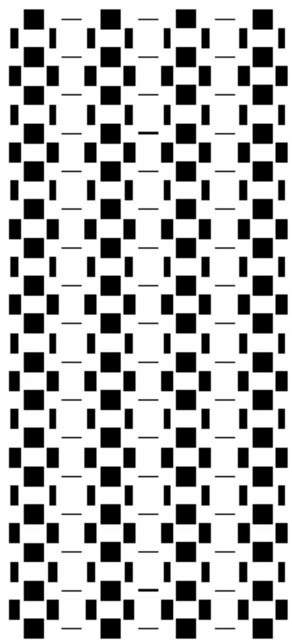
g

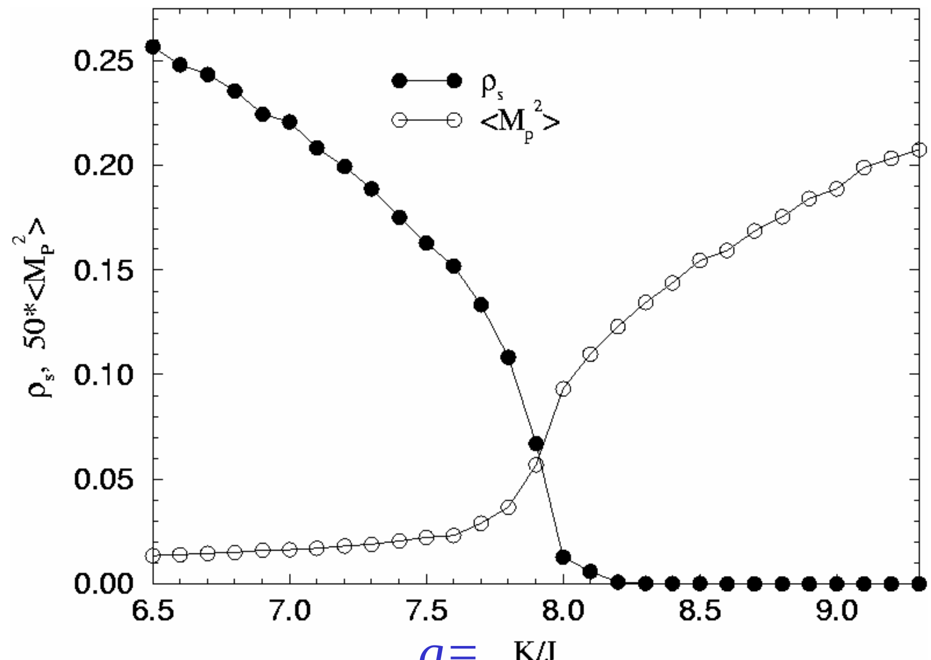
Bond order in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett*. **89**, 247201 (2002)

First *large scale* (> 8000 spins) numerical study of the destruction of Neel order in a *S=1/2* antiferromagnet with full square lattice symmetry



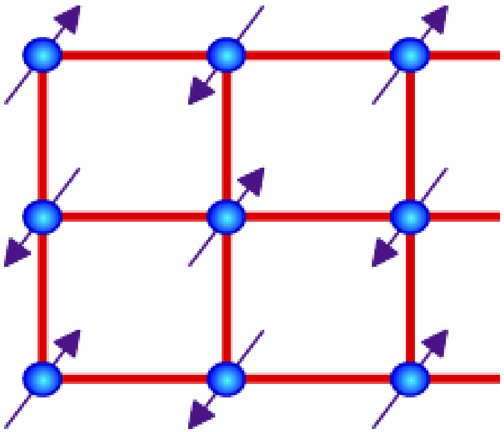




$g=$

$$H = 2J\sum_{\langle ij\rangle}\left(S_i^x S_j^x + S_i^y S_j^y\right) - K\sum_{\langle ijkl\rangle\subset\square}\left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+\right)$$

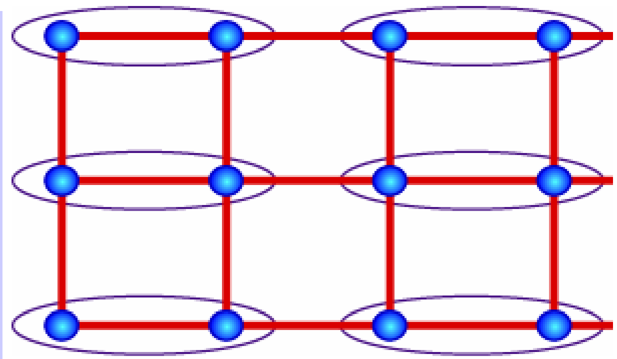
$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$



Neel order

$\langle \vec{\varphi} \rangle \neq 0$

?



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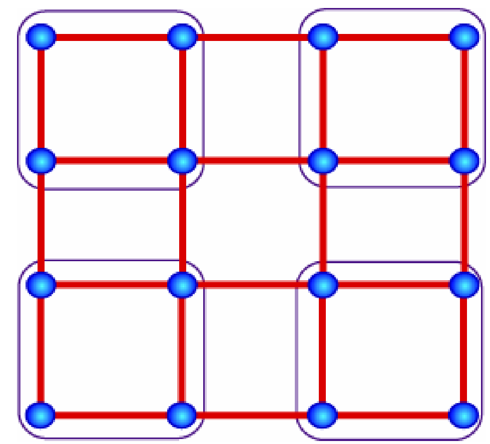


Bond order

$\langle \Psi_{\text{bond}} \rangle \neq 0$

Not present in LGW theory of $\vec{\varphi}$ order

0 g

Theory of a second-order quantum phase transition between Neel and bond-ordered phases

At the quantum critical point:

- $A_\mu \rightarrow A_\mu + 2\pi$ periodicity can be ignored

(Monopoles interfere destructively and are dangerously irrelevant).

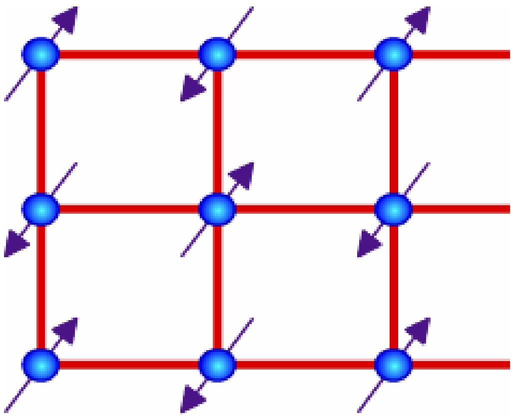
- $S = 1/2$ *spinons* z_α, with $\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$, are globally propagating degrees of freedom.

Second-order critical point described by emergent fractionalized degrees of freedom (A_μ and z_α); Order parameters ($\vec{\varphi}$ and Ψ_{bond}) are "composites" and of secondary importance

S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990); G. Murthy and S. Sachdev, *Nuclear Physics* B **344**, 557 (1990); C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001); S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002); O. Motrunich and A. Vishwanath, cond-mat/0311222.

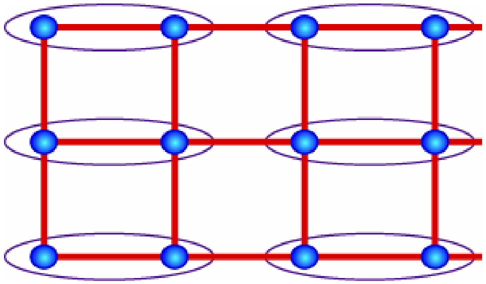
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Phase diagram of S=1/2 square lattice antiferromagnet



Neel order

$\langle \vec{\varphi} \rangle \sim \langle z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$



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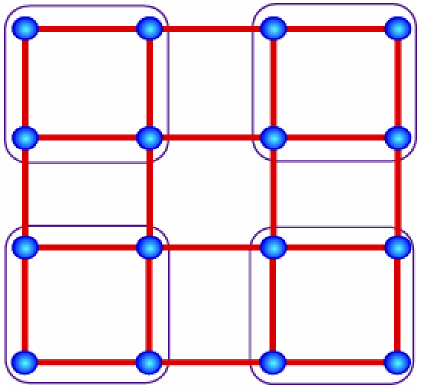


Bond order $\langle \Psi_{\text{bond}} \rangle \neq 0$

(associated with condensation of monopoles in A_μ),

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

g

Second-order critical point described by

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + r\left|z_\alpha\right|^2 + \frac{u}{2}\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

at its critical point $r = r_c$, where A_μ is *non-compact*

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).
S. Sachdev cond-mat/0401041.

Mott insulators with spin S=1/2 per unit cell:

Berry phases, bond order, and the breakdown of the LGW paradigm

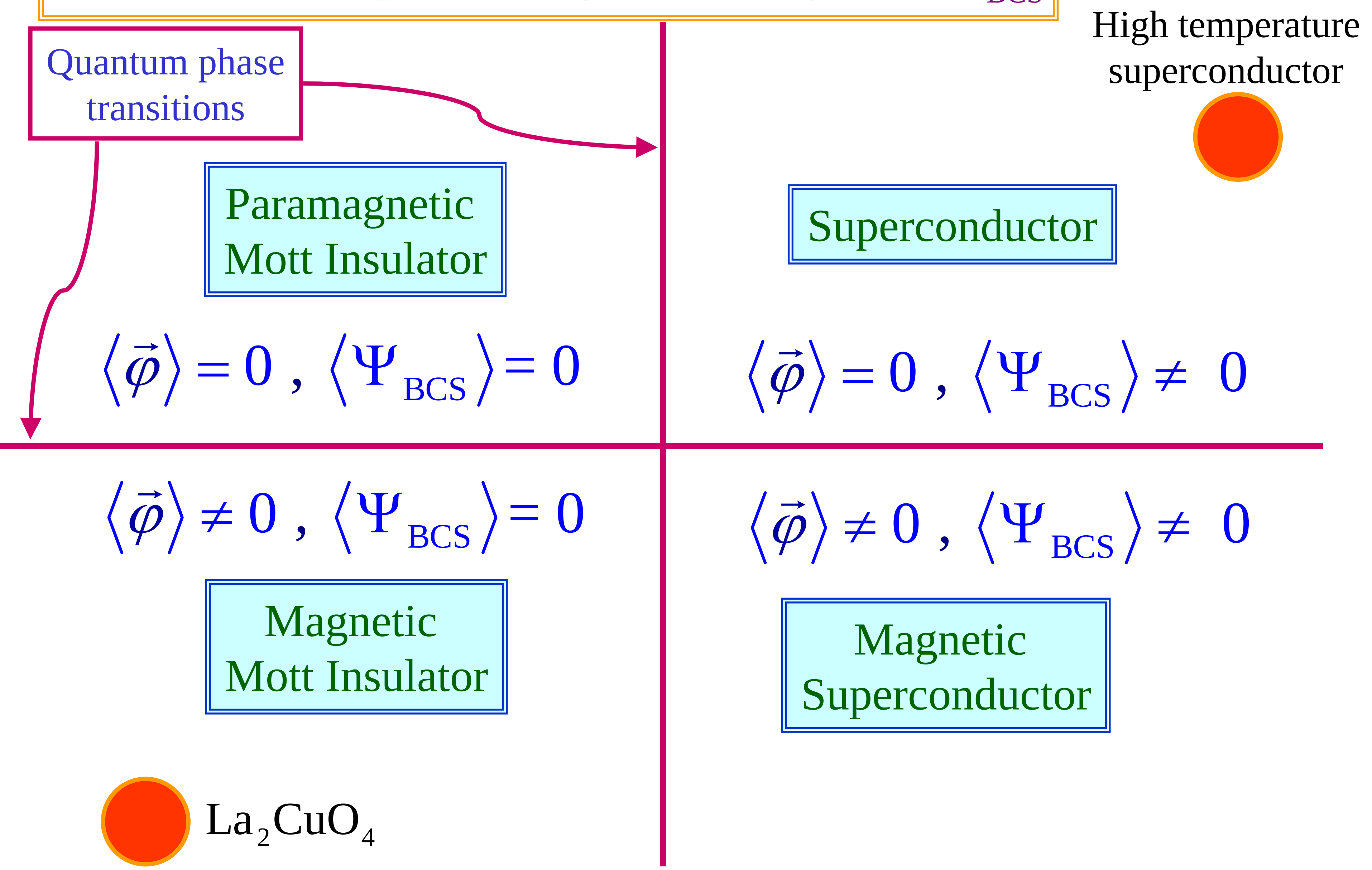
Order parameters/broken symmetry

\+

Emergent gauge excitations, fractionalization.

Cuprate superconductors:
Competing orders and recent experiments

Minimal LGW phase diagram with $\vec{\varphi}$ and Ψ_{BCS}



Minimal LGW phase diagram with $\vec{\varphi}$ and Ψ_{BCS}

High temperature superconductor

Quantum phase transitions

Paramagnetic Mott Insulator

$\langle \vec{\varphi} \rangle = 0 \,,\ \langle \Psi_{\text{BCS}} \rangle = 0$

Superconductor

$\langle \vec{\varphi} \rangle = 0 \,,\ \langle \Psi_{\text{BCS}} \rangle \neq 0$

$\langle \vec{\varphi} \rangle \neq 0 \,,\ \langle \Psi_{\text{BCS}} \rangle = 0$

Magnetic Mott Insulator

$\langle \vec{\varphi} \rangle \neq 0 \,,\ \langle \Psi_{\text{BCS}} \rangle \neq 0$

Magnetic Superconductor

La_2CuO_4

Spin density wave order $K \neq (\pi, \pi)$

Spirals……*Shraiman, Siggia*

Stripes……..*Zaanen, Kivelson*…..

Quantum phase transitions

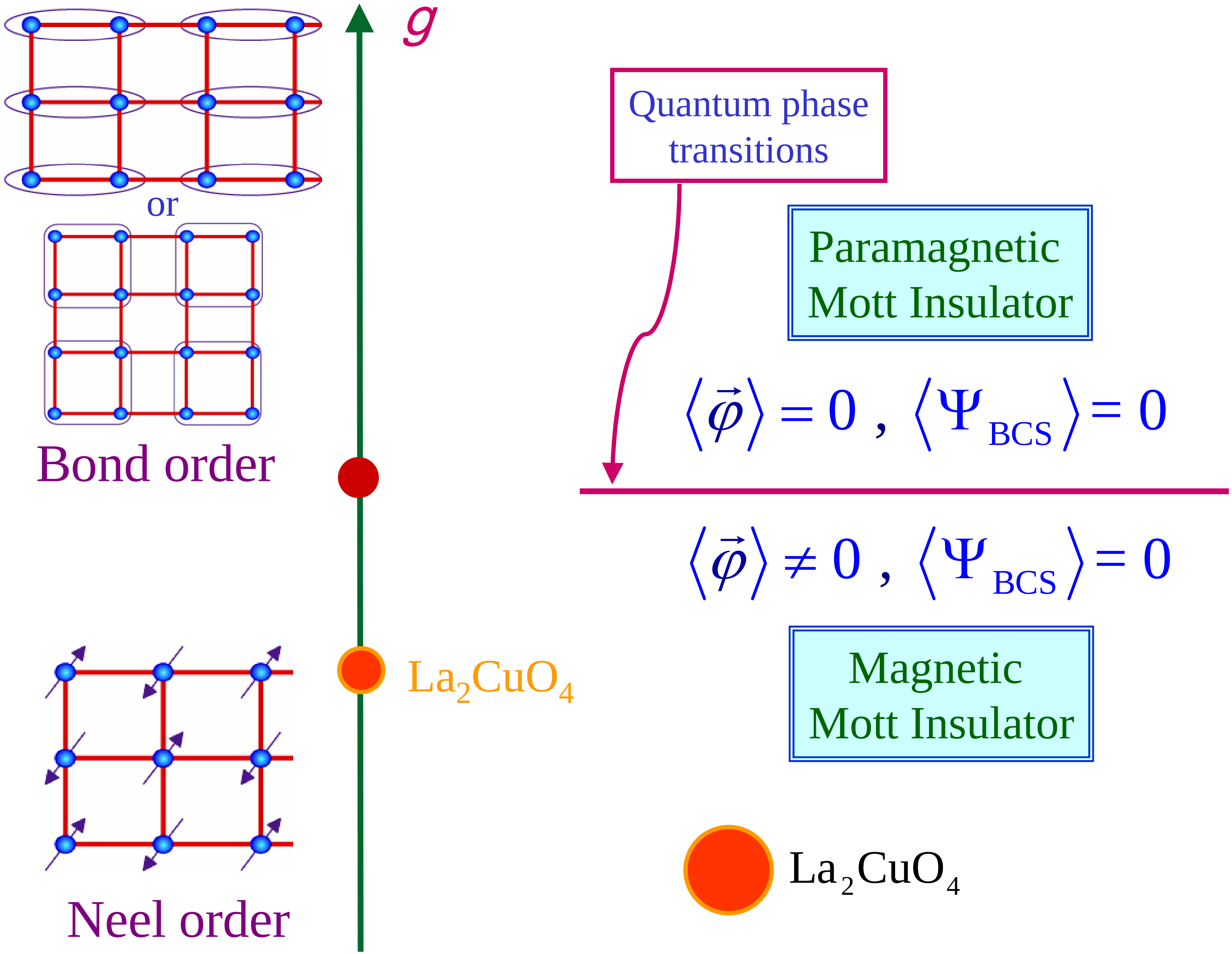
Paramagnetic Mott Insulator

$$\langle \vec{\varphi} \rangle = 0 \,,\; \langle \Psi_{BCS} \rangle = 0$$

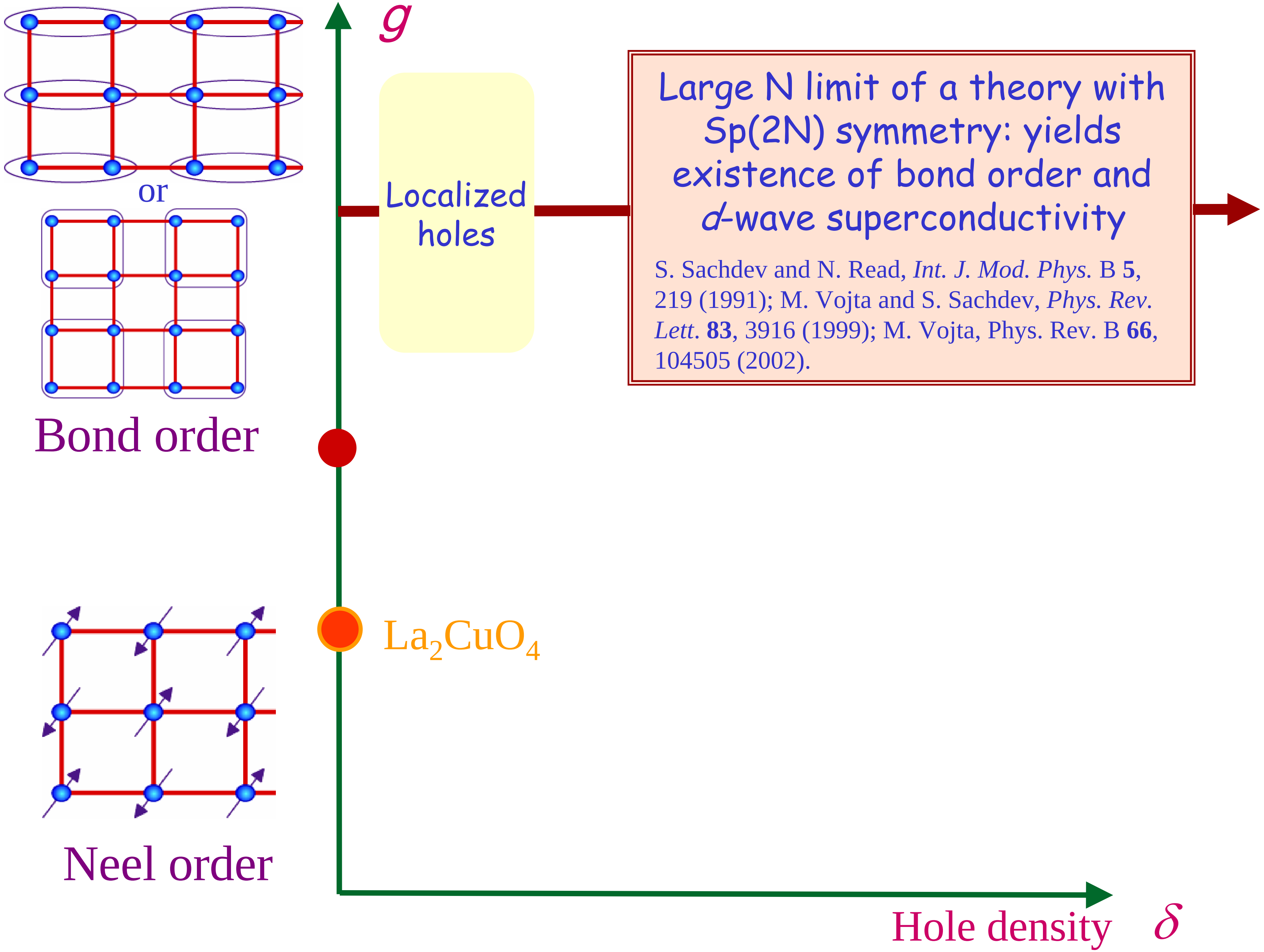
$$\langle \vec{\varphi} \rangle \neq 0 \,,\; \langle \Psi_{BCS} \rangle = 0$$

Magnetic Mott Insulator

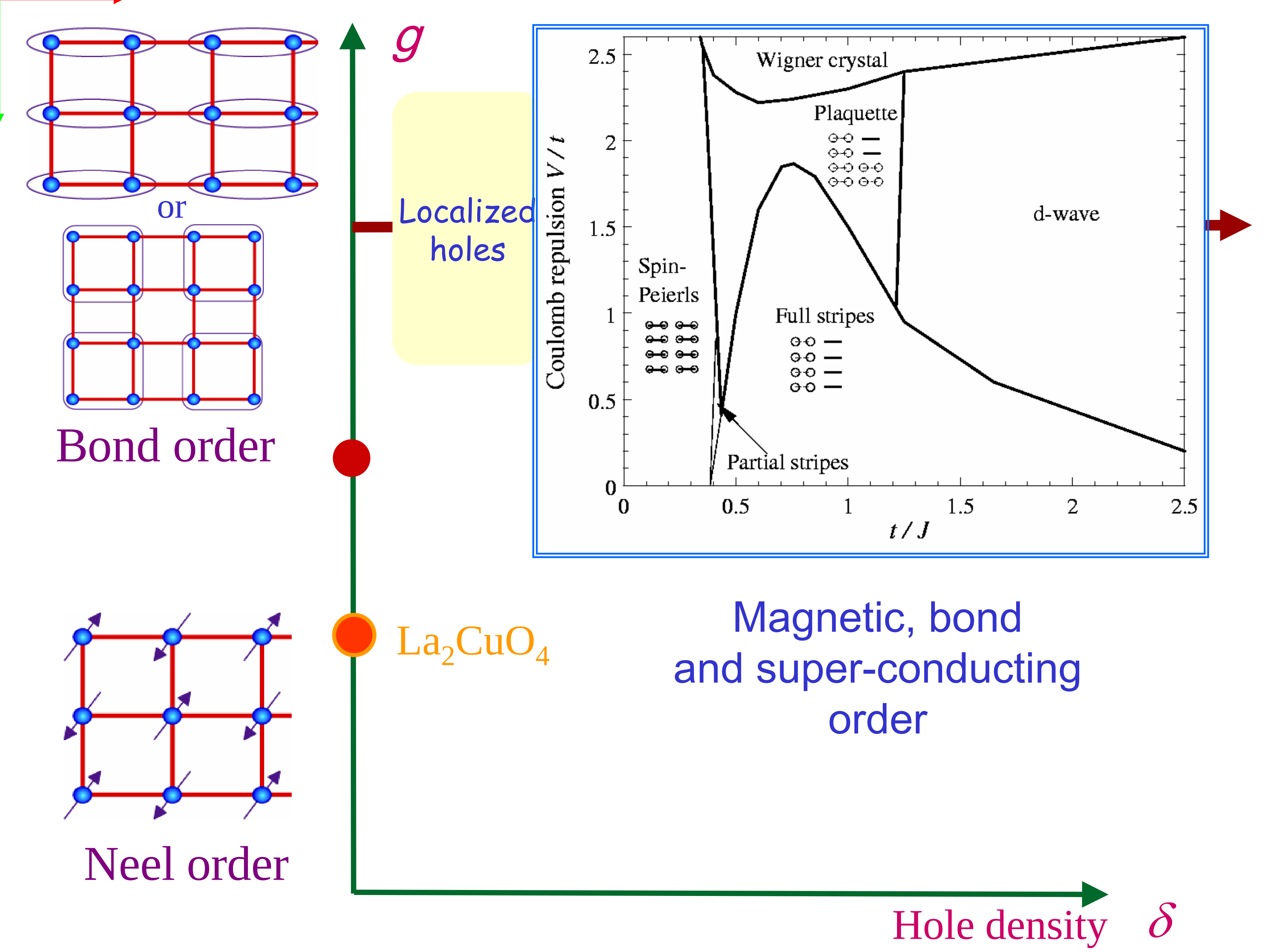
La_2CuO_4



g
or
Bond order
Neel order
La_2CuO_4
Quantum phase transitions
Paramagnetic Mott Insulator
$\langle \vec{\varphi} \rangle = 0 \, , \, \langle \Psi_{BCS} \rangle = 0$
$\langle \vec{\varphi} \rangle \neq 0 \, , \, \langle \Psi_{BCS} \rangle = 0$
Magnetic Mott Insulator
La_2CuO_4



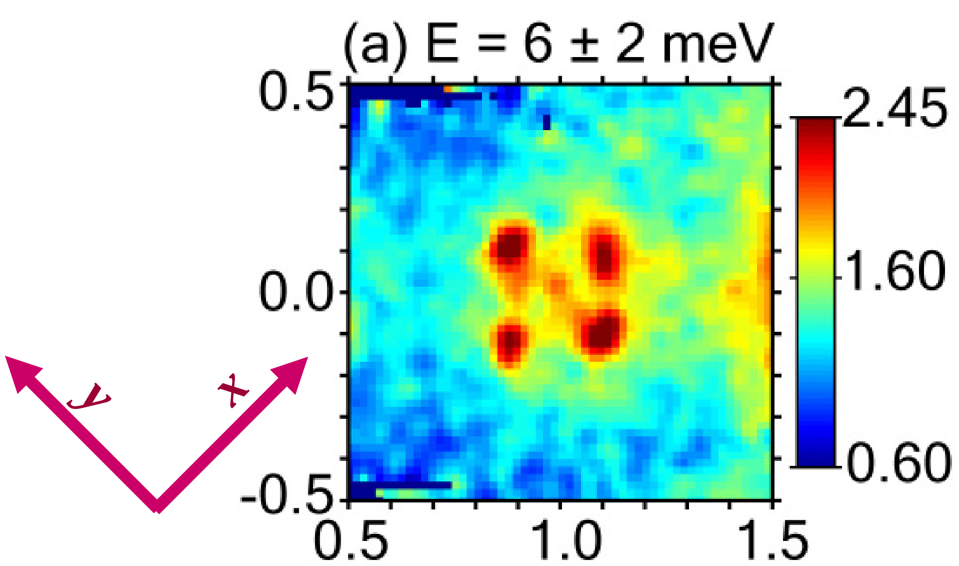
g
or
Bond order
Neel order
Localized holes
Large N limit of a theory with Sp(2N) symmetry: yields existence of bond order and d-wave superconductivity
S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991); M. Vojta and S. Sachdev, Phys. Rev. Lett. 83, 3916 (1999); M. Vojta, Phys. Rev. B 66, 104505 (2002).
La$_2$CuO$_4$
Hole density
δ



g
Bond order
or
Localized holes
Wigner crystal
Plaquette
d-wave
Spin-Peierls
Full stripes
Partial stripes
Coulomb repulsion V / t
t / J
0
0.5
1
1.5
2
2.5
La2CuO4
Neel order
Magnetic, bond and super-conducting order
Hole density δ

Neutron scattering measurements of $La_{15/8}Ba_{1/8}CuO_4$ (Zurich oxide)

J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, cond-mat/0401621



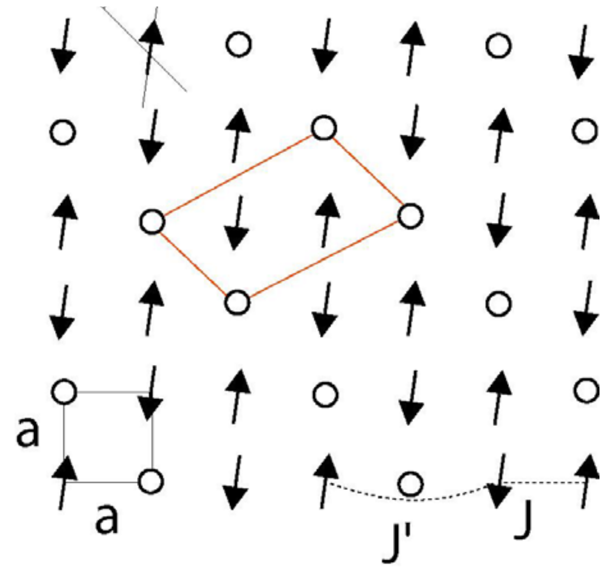
Scattering off spin density wave order with

$$\left\langle \vec{S}_i \right\rangle = \vec{N} \cos\left(\boldsymbol{Q} \bullet \boldsymbol{r}_i + \alpha \right)$$

$$\boldsymbol{Q} = 2\pi \left(\frac{1}{2}, \frac{1}{2} \pm \frac{1}{8} \right) \text{ and } 2\pi \left(\frac{1}{2} \pm \frac{1}{8}, \frac{1}{2} \right)$$

At higher energies, semiclassical theory predicts that peaks lead to spin-wave ("light") cones.

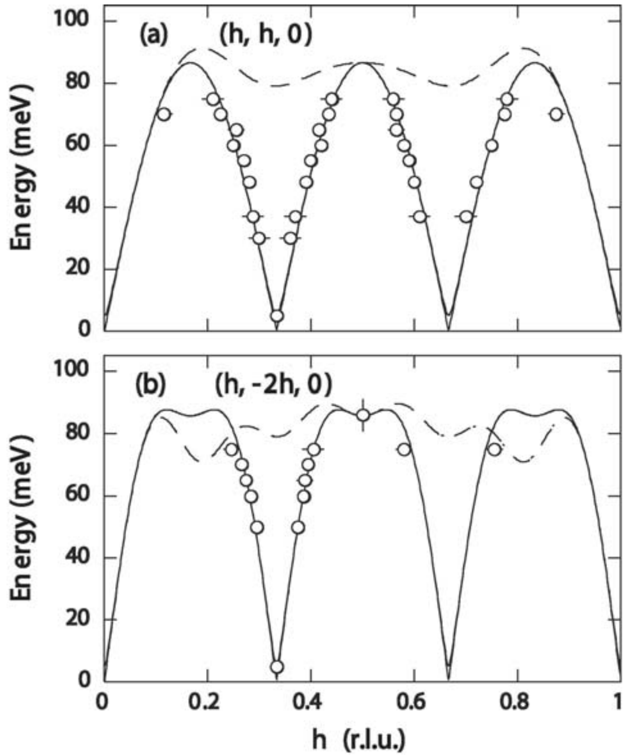
$La_{5/3}Sr_{1/3}NiO_4$



$$\text{Ni has } S = 1; \;\; \boldsymbol{Q} = 2\pi\left(\frac{1}{2} \pm \frac{1}{6}, \frac{1}{2} \pm \frac{1}{6}\right)$$

A. T. Boothroyd, D. Prabhakaran, P. G. Freeman, S.J.S. Lister, M. Enderle, A. Hiess, and J. Kulda, *Phys. Rev. B* **67**, 100407 (2003).

$La_{5/3}Sr_{1/3}NiO_4$

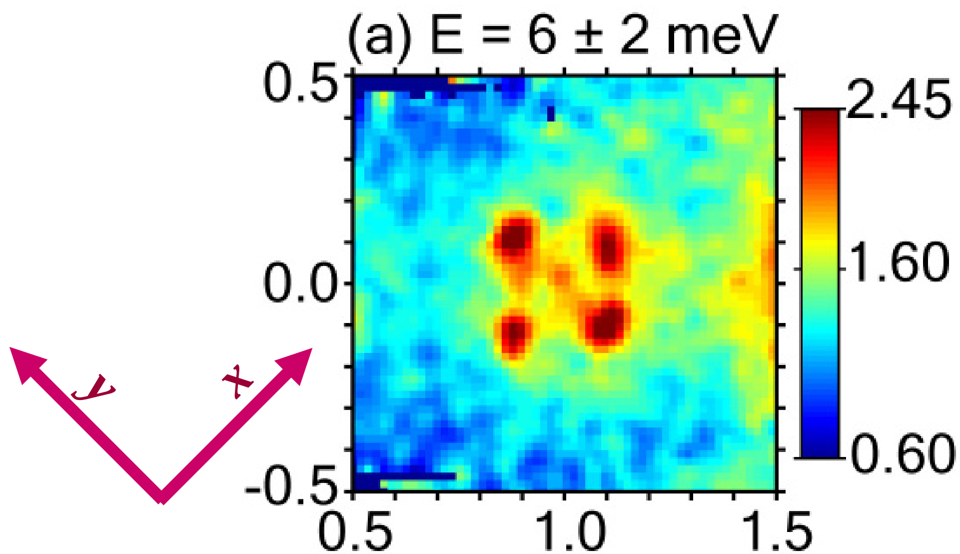


Spin waves: J=15 meV, J'=7.5meV

A. T. Boothroyd, D. Prabhakaran, P. G. Freeman, S.J.S. Lister, M. Enderle, A. Hiess, and J. Kulda, *Phys. Rev. B* **67**, 100407 (2003).

Neutron scattering measurements of $La_{15/8}Ba_{1/8}CuO_4$ (Zurich oxide)

J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, cond-mat/0401621



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$$\left\langle \vec{S}_i \right\rangle = \vec{N} \cos\left(\boldsymbol{Q} \bullet \boldsymbol{r}_i + \alpha\right)$$

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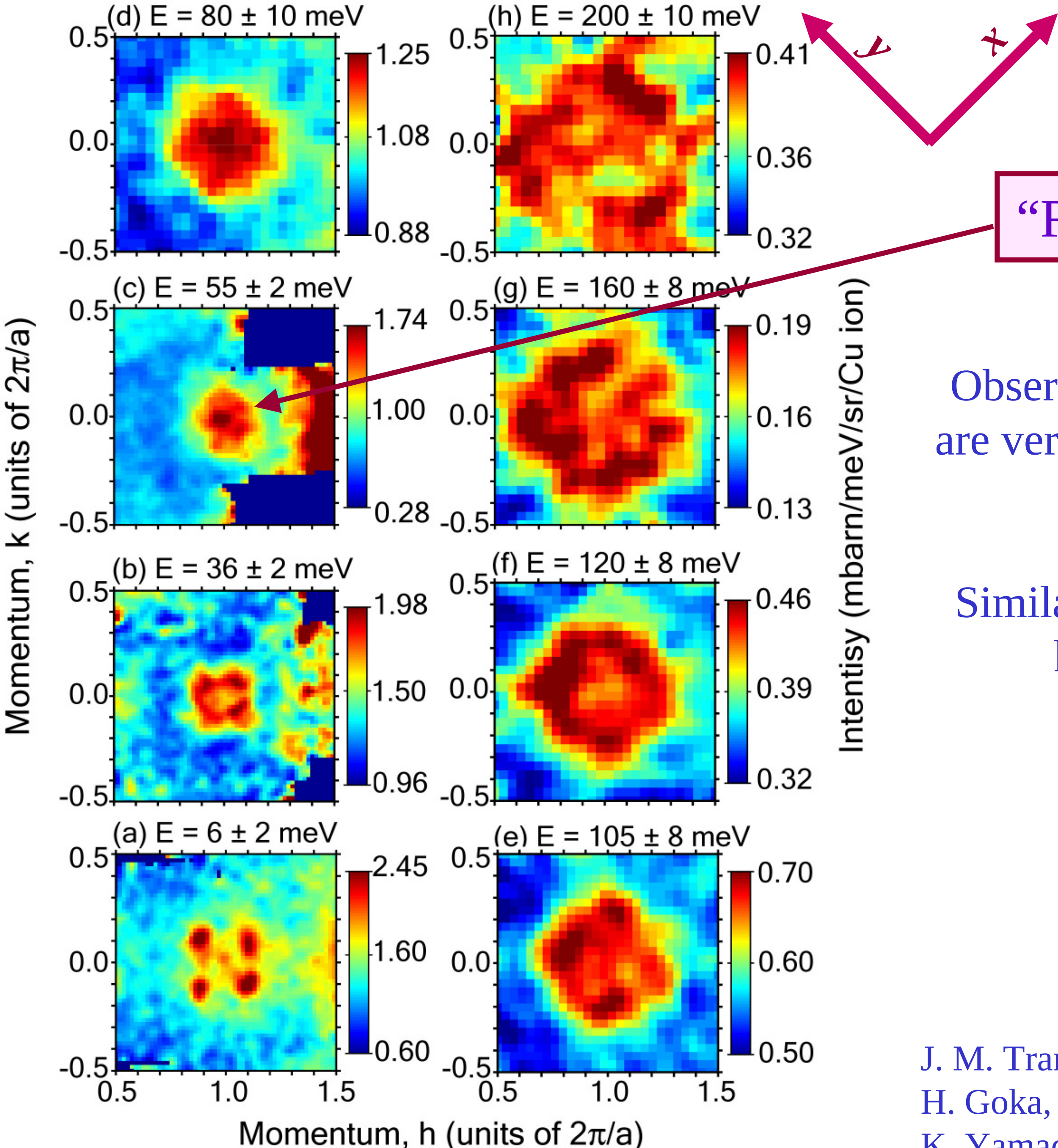
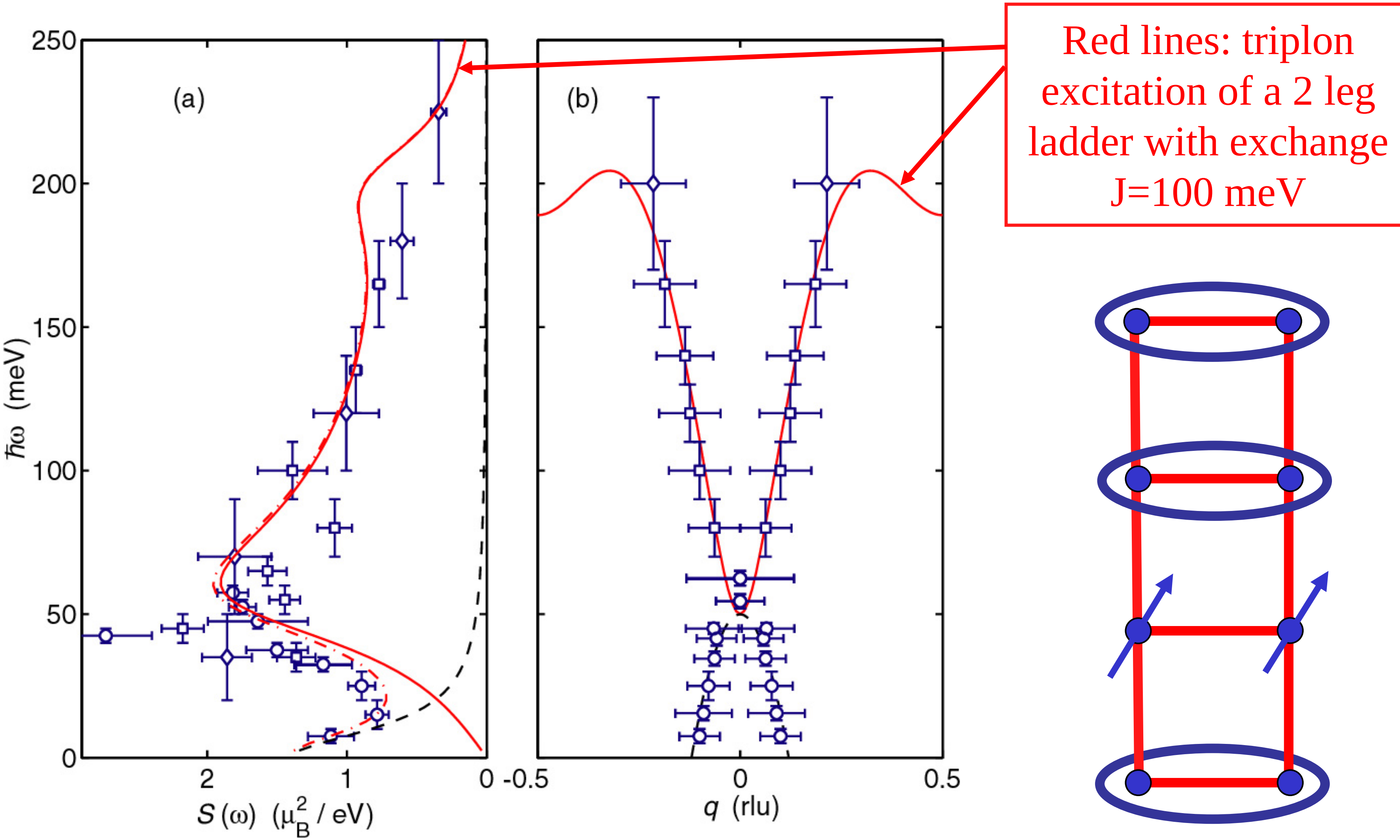
At higher energies, semiclassical theory predicts that peaks lead to spin-wave ("light") cones.



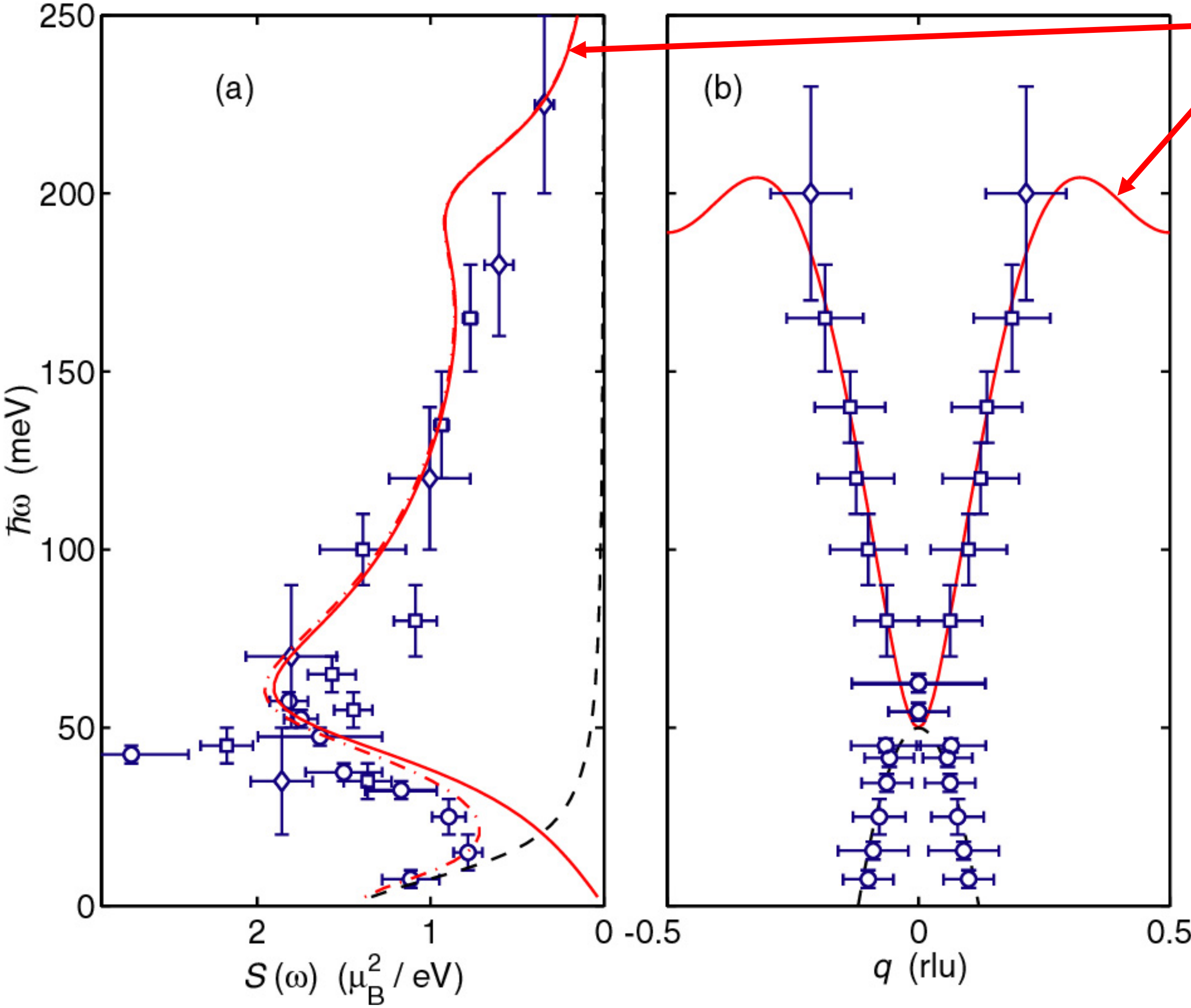
Observations in $La_{15/8}Ba_{1/8}CuO_4$ are very different and do not obey spin-wave model.

Similar spectra are seen in most hole-doped cuprates.

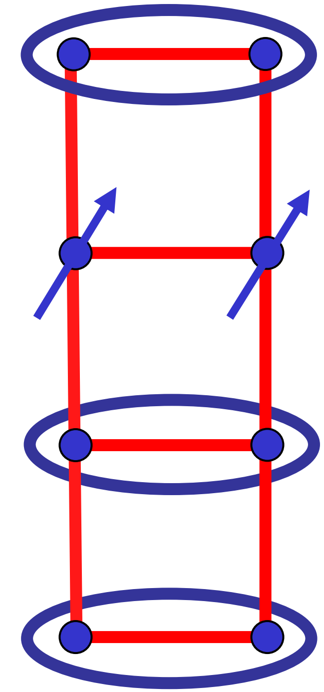
J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, cond-mat/0401621



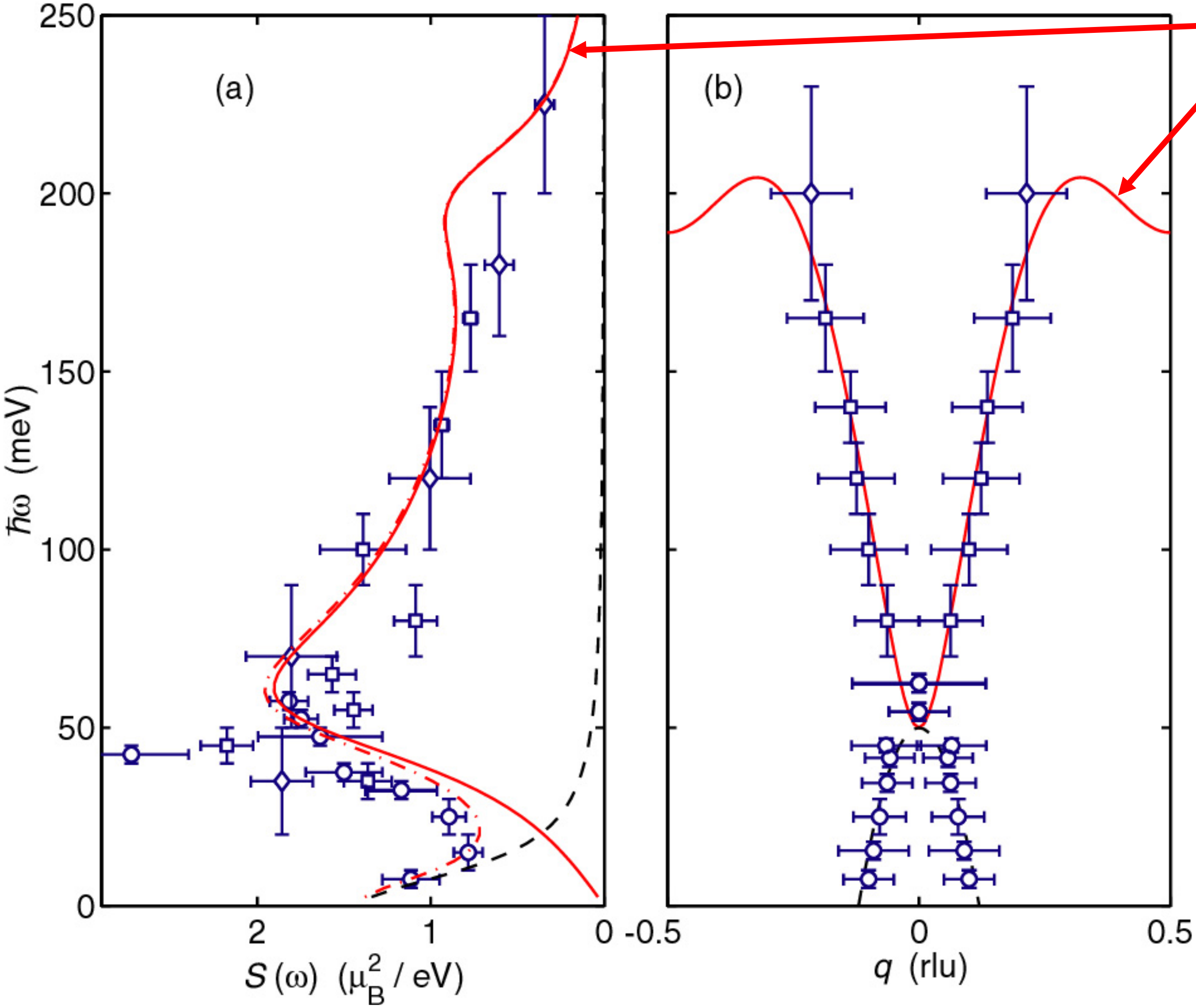
J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, cond-mat/0401621



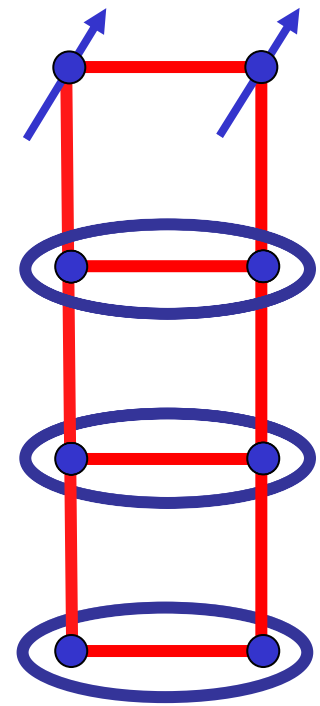
Red lines: triplon excitation of a 2 leg ladder with exchange J=100 meV



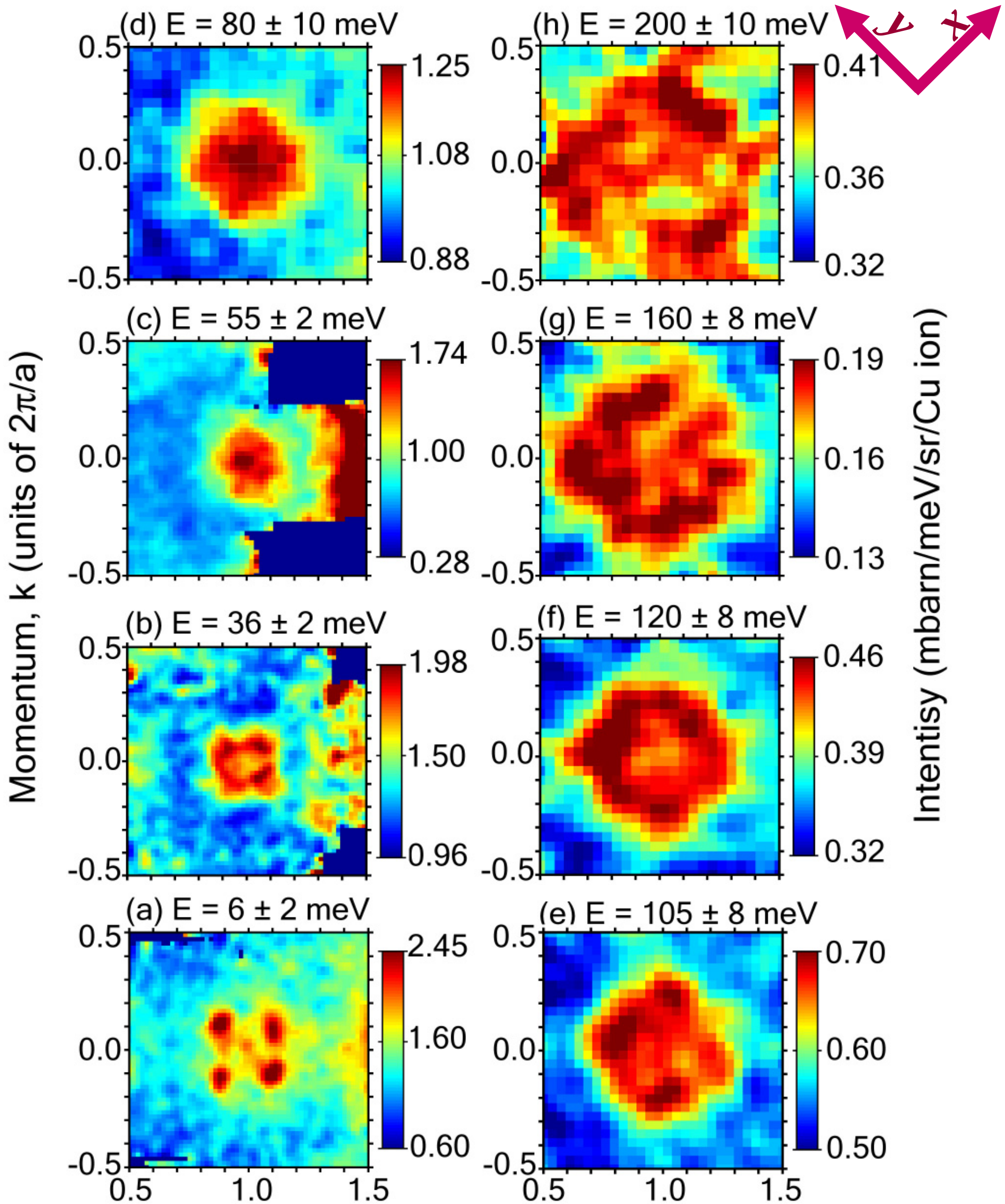
J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, cond-mat/0401621



Red lines: triplon excitation of a 2 leg ladder with exchange J=100 meV



J. M. Tranquada, H. Woo, T. G. Perring, H. Goka, G. D. Gu, G. Xu, M. Fujita, and K. Yamada, cond-mat/0401621



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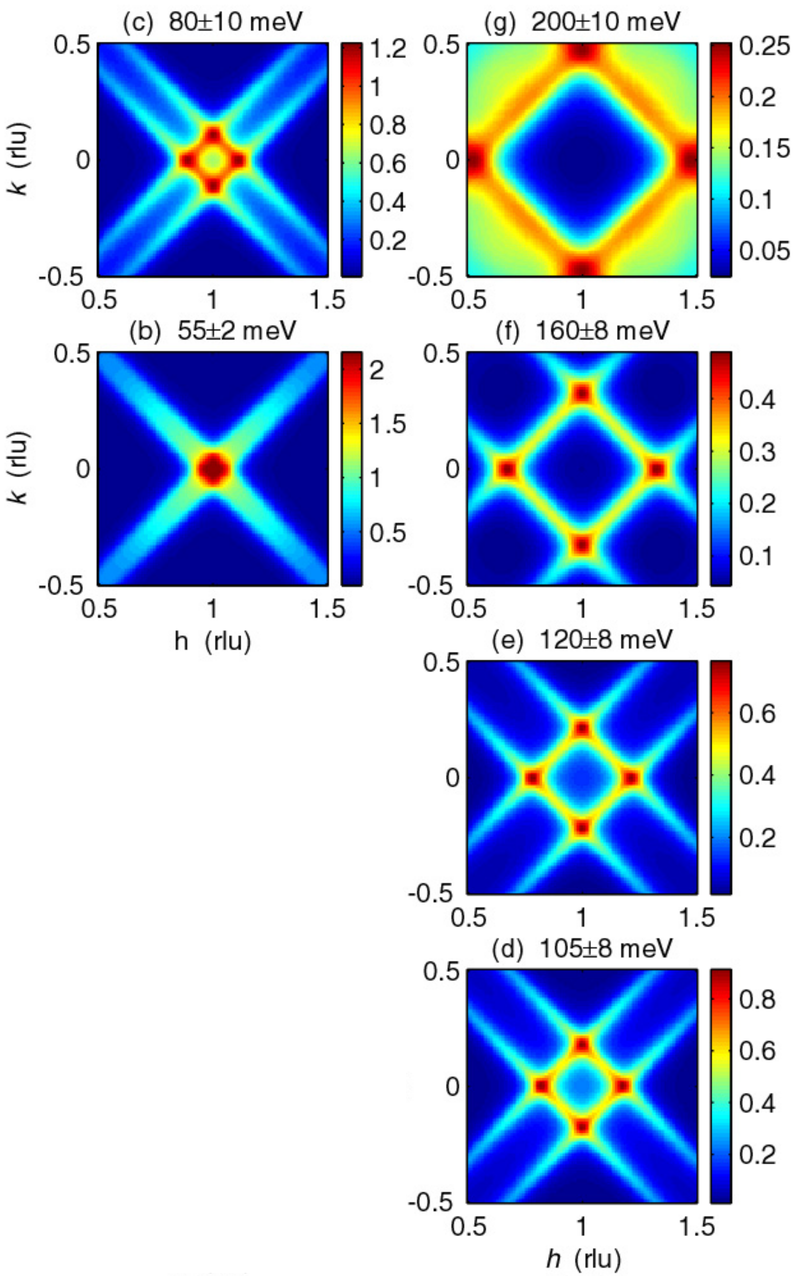


Spectrum of a two-leg ladder

Proposals of

- M. Vojta and T. Ulbricht, cond-mat/0402377
- G.S. Uhrig, K.P. Schmidt, and M. Grüninger, cond-mat/0402659
- M. Vojta and S. Sachdev, to appear.

Magnetic excitations display a crossover from *spin-waves* (at low energies) to *triplons* (at high energies), and main features can be described by proximity to a magnetic quantum phase transition in the presence of period 4, *static*, bond order.

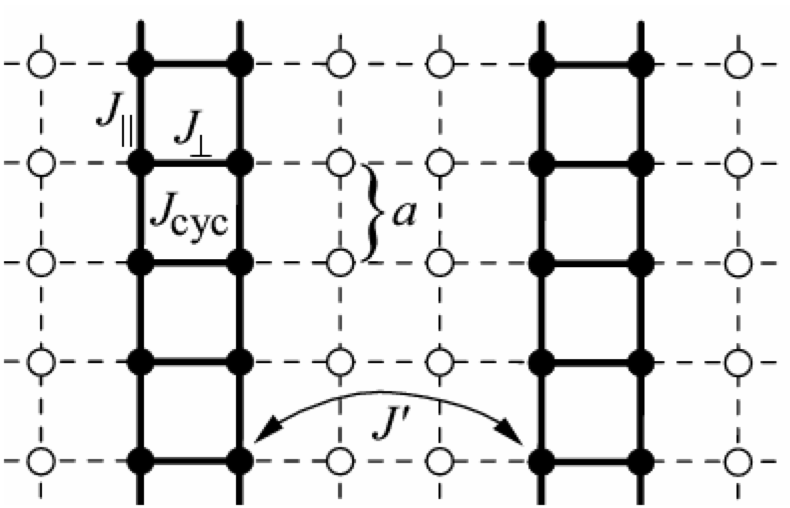
LGW action for magnetic order parameter $\vec{\varphi}_i = e^{i\boldsymbol{K}\cdot\boldsymbol{r}_i}\vec{S}_i$ $(\boldsymbol{K} = (\pi,\pi))$:

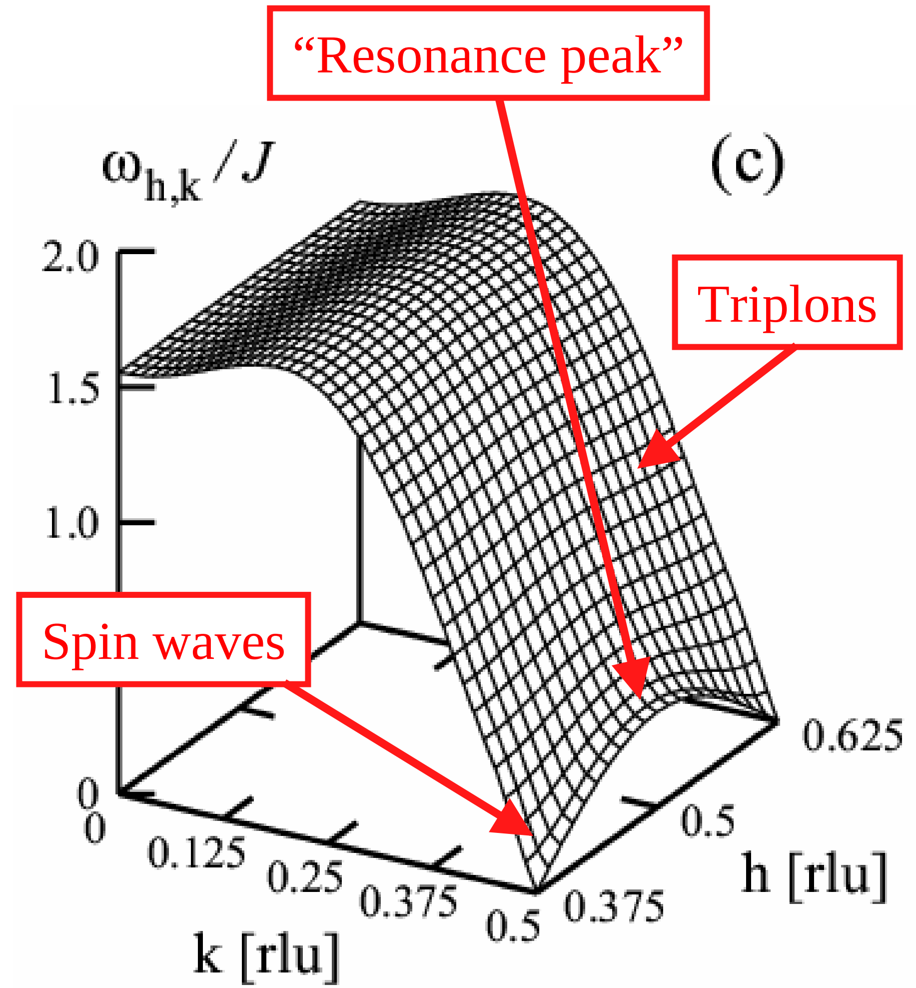
$$S_\varphi = \int d\tau \sum_i \left\{ \frac{1}{2}\left(c^2 \left(\partial_\tau \vec{\varphi}_i\right)^2 + \left(g - g_c\right)\vec{\varphi}_i^2 \right) + \frac{u}{4!}\left(\vec{\varphi}_i^2\right)^2 \right\}$$

$$+ \int d\tau \sum_{\langle ij \rangle} \left\{ \left(\vec{\varphi}_i - \vec{\varphi}_j\right)^2 - P(i,j)\,\vec{\varphi}_i \cdot \vec{\varphi}_j \right\}$$

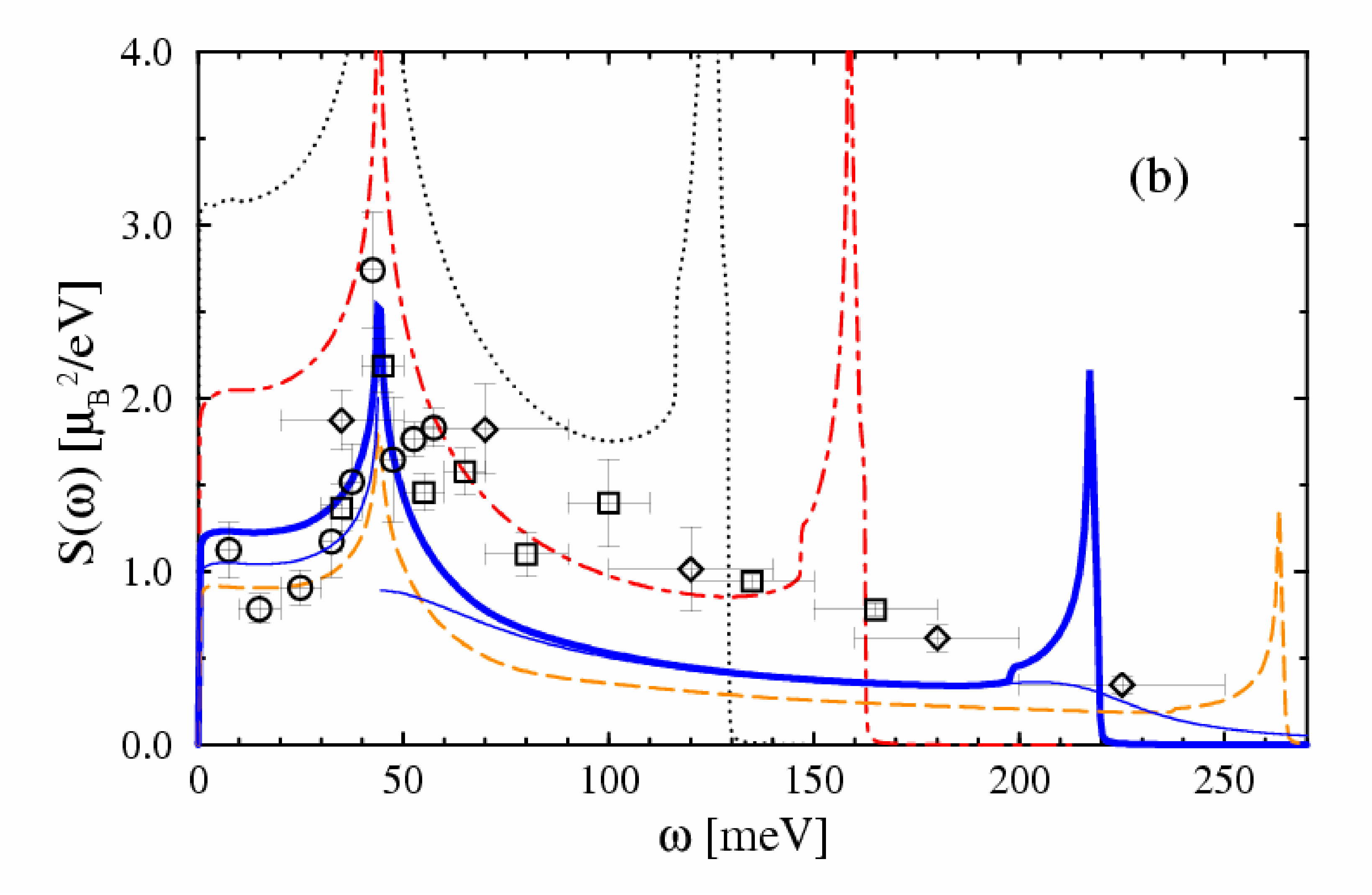
$$P(i,j) = \Psi_{\text{bond}} e^{i\boldsymbol{Q}\cdot\left(\boldsymbol{r}_i + \boldsymbol{r}_j\right)} + \text{c.c.}$$

Possible simple microscopic model of bond order

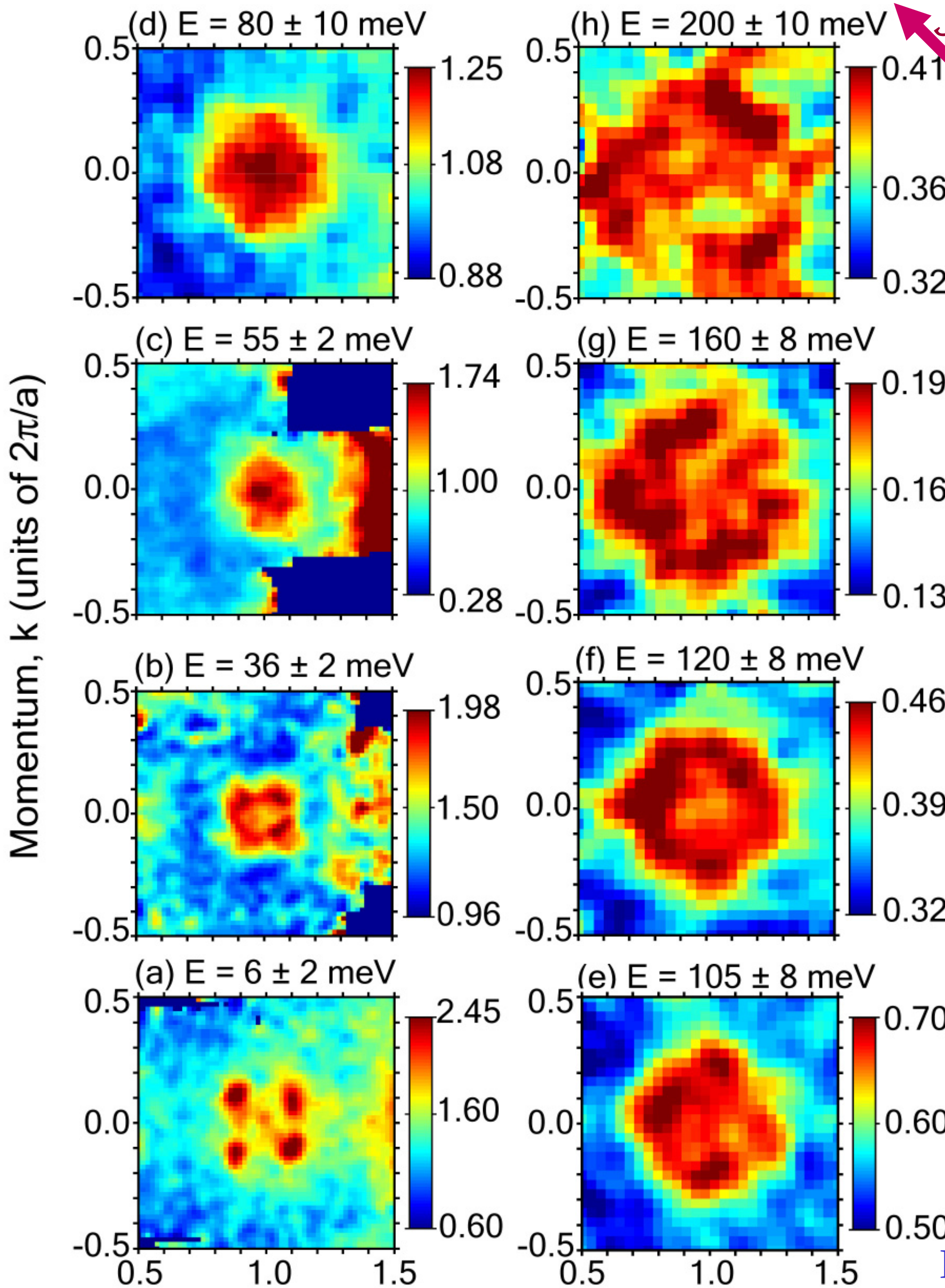




G.S. Uhrig, K.P. Schmidt, and
M. Grüninger, cond-mat/0402659



G.S. Uhrig, K.P. Schmidt, and M. Grüninger, cond-mat/0402659



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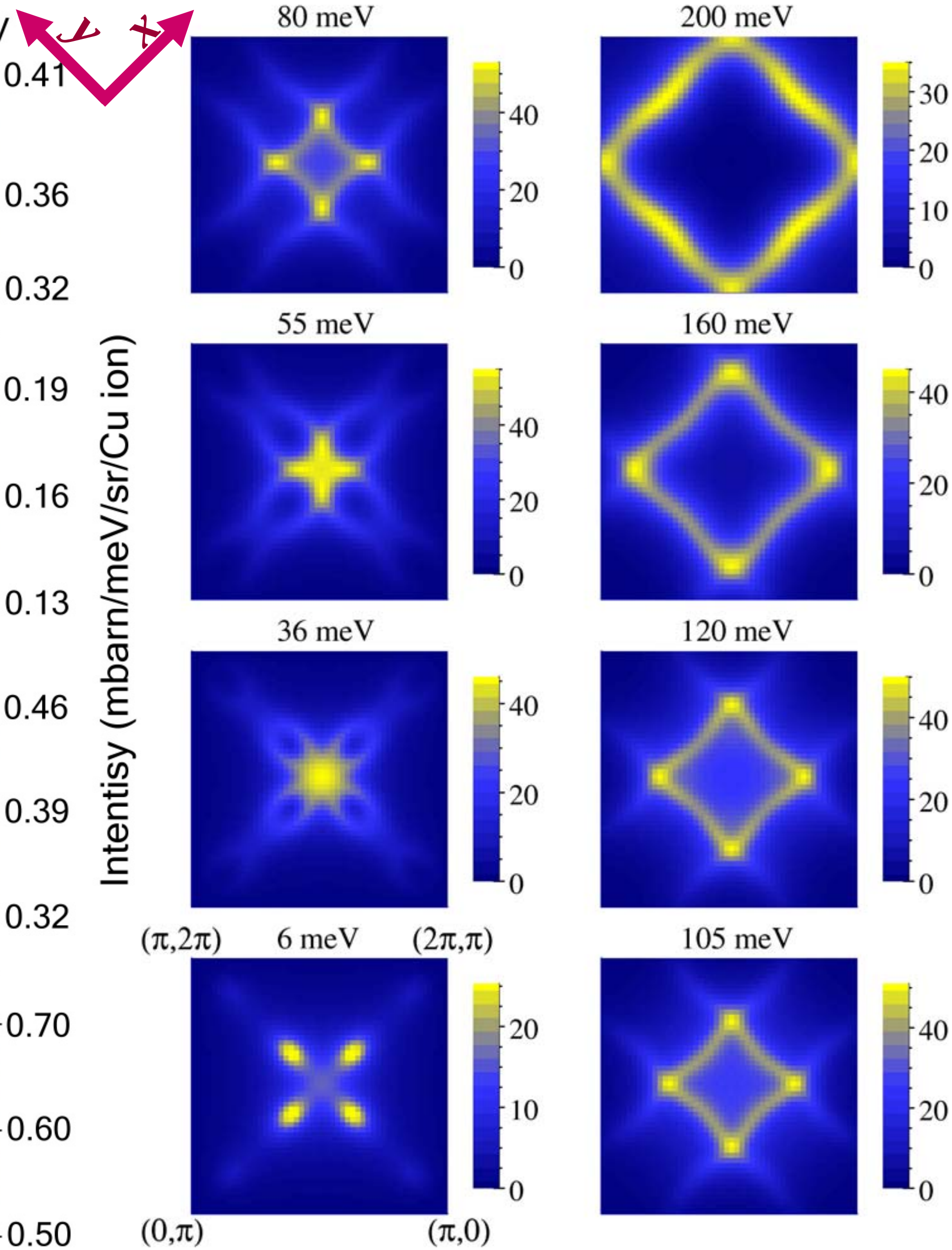
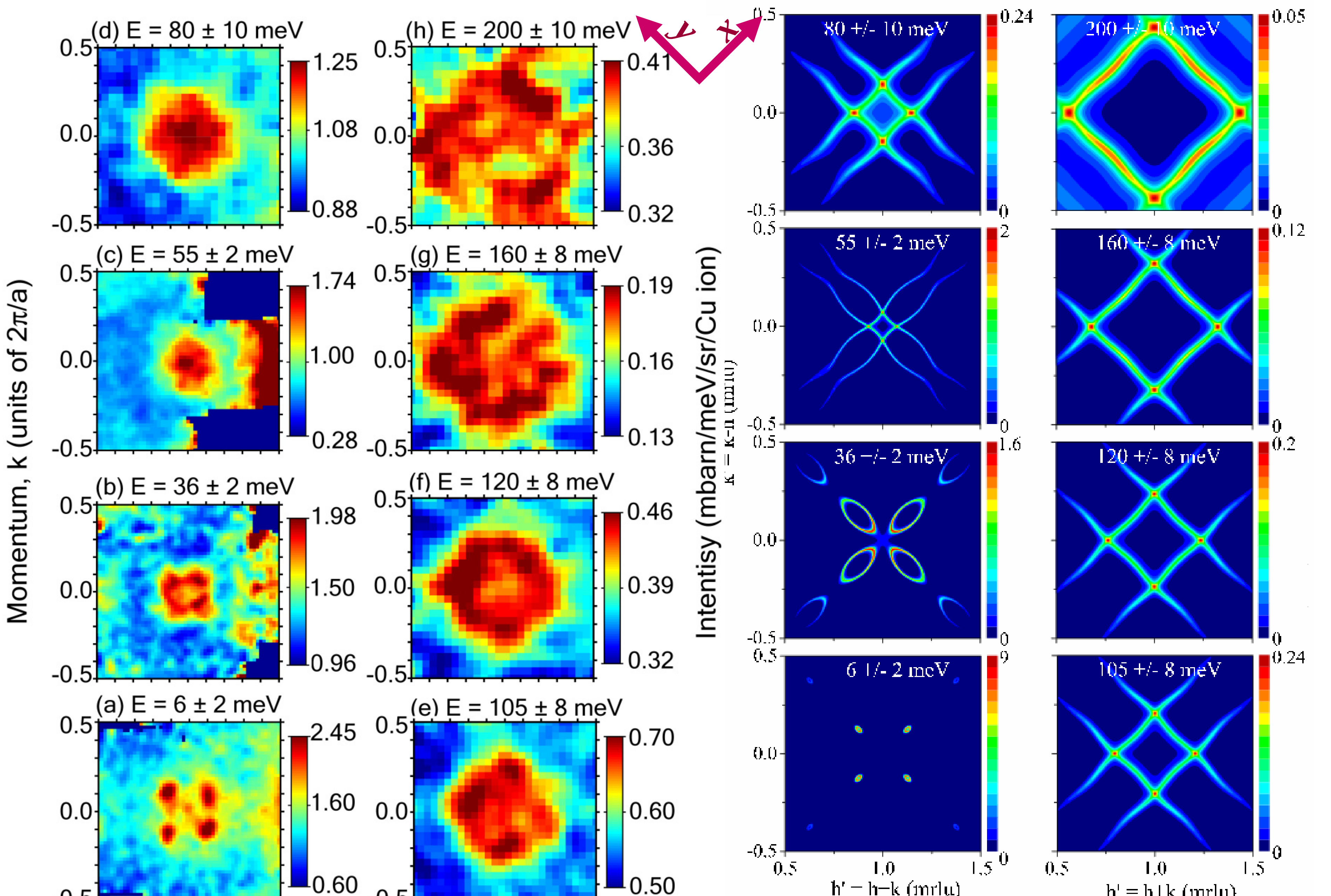
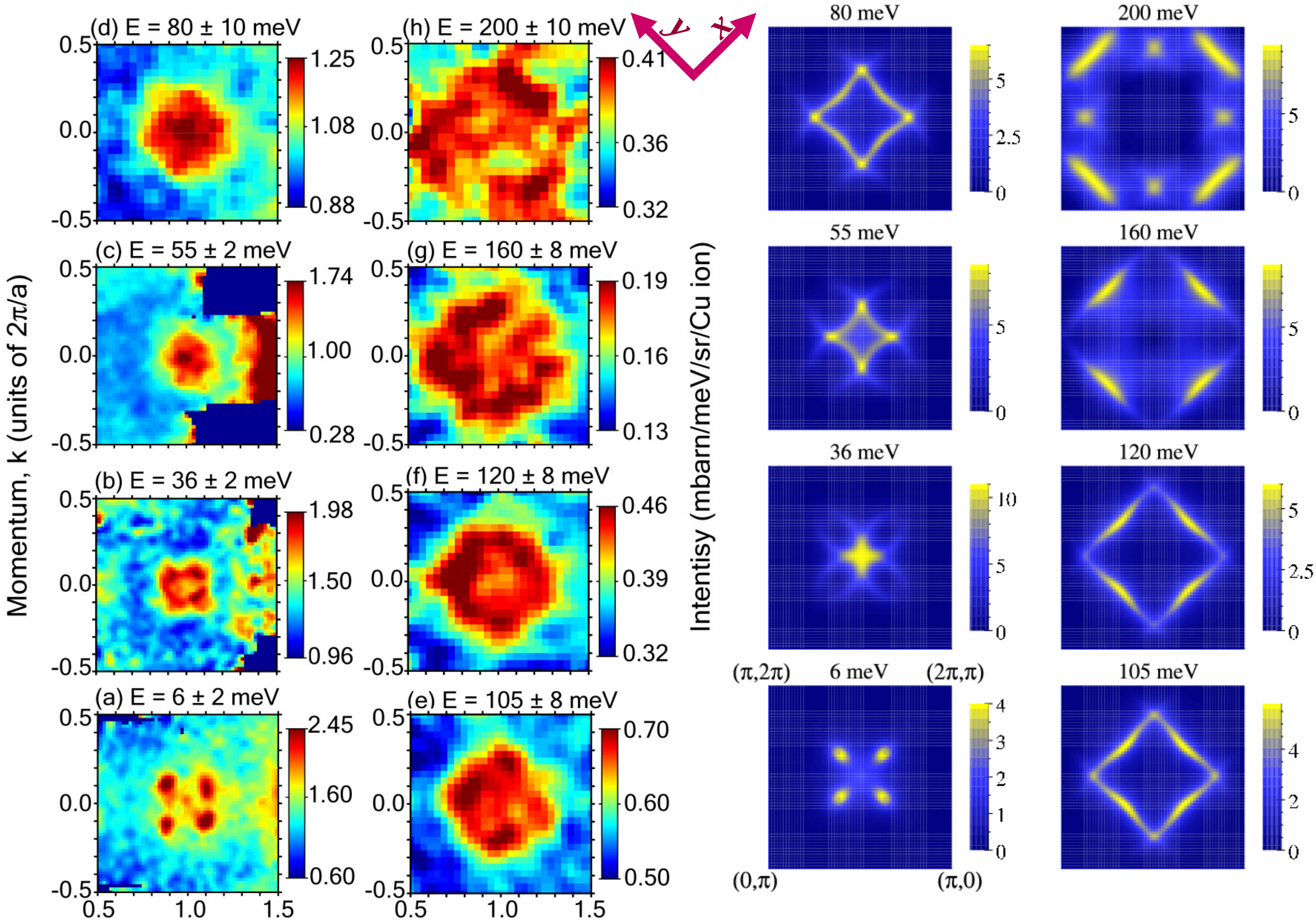


Bond operator (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B 41, 9323 (1990)) theory of coupled-ladder model, M. Vojta and T. Ulbricht, cond-mat/0402377



J. M. Tranquada *et al.*, cond-mat/0401621

Numerical study of coupled ladder model, G.S. Uhrig, K.P. Schmidt, and M. Grüninger, cond-mat/0402659



J. M. Tranquada *et al.*, cond-mat/0401621

LGW theory of magnetic criticality in the presence of *static* bond order, M. Vojta and S. Sachdev, to appear.

Conclusions

I. Theory of quantum phase transitions between magnetically ordered and paramagnetic states of Mott insulators:

A. *Dimerized Mott insulators*: Landau-Ginzburg-Wilson theory of fluctuating magnetic order parameter.

B. S=1/2 square lattice: Berry phases induce bond order, and LGW theory breaks down. Critical theory is expressed in terms of emergent fractionalized modes, and the *order parameters are secondary*.

Conclusions

II. Competing spin-density-wave/bond/superconducting orders in the hole-doped cuprates.

- Main features of spectrum of excitations in LBCO modeled by LGW theory of quantum critical fluctuations in the presence of static bond order across a wide energy range.

- Predicted magnetic field dependence of spin-density-wave order observed by neutron scattering in LSCO. E. Demler, S. Sachdev, and Y. Zhang, *Phys.Rev. Lett.* **87**, 067202 (2001); B. Lake *et al. Nature*, **415**, 299 (2002); B. Khaykhovich *et al.* Phys. Rev. B **66**, 014528 (2002).

- Predicted pinned bond order in vortex halo consistent with STM observations in BSCCO. K. Park and S. Sachdev *Phys. Rev.* B **64**, 184510 (2001); Y. Zhang, E. Demler and S. Sachdev, *Phys. Rev.* B **66,** 094501 (2002); J.E. Hoffman et al. *Science* **295**, 466 (2002).

- Energy dependence of LDOS modulations in BSCCO best modeled by modulations in bond variables. M. Vojta, *Phys. Rev.* B **66**, 104505 (2002); D. Podolsky, E. Demler, K. Damle, and B.I. Halperin, *Phys. Rev.* B **67**, 094514 (2003); C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, *Phys. Rev.* B **67**, 014533 (2003).

Conclusions

III. Breakdown of LGW theory of quantum phase transitions with magnetic/bond/superconducting orders in doped Mott insulators ?