Bekenstein-Hawking entropy and strange metals

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PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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Quantum matter without quasiparticles

1. A solvable model of an ordinary metal
2. A solvable model of a strange metal
3. Holography and charged black holes
4. The (slightly less) strange metal in graphene

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Infinite-range model of a Fermi liquid

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

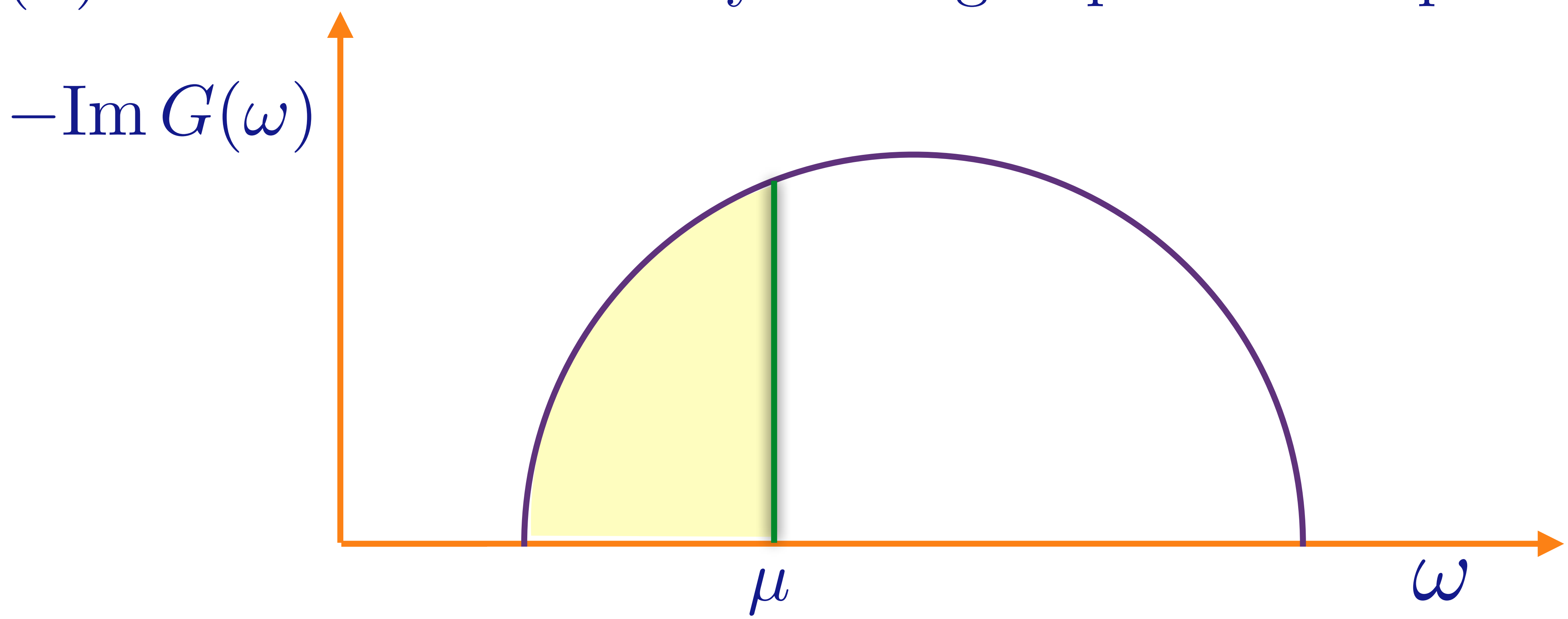
Fermions occupying the eigenstates of a *N* x *N* random matrix

Infinite-range model of a Fermi liquid

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

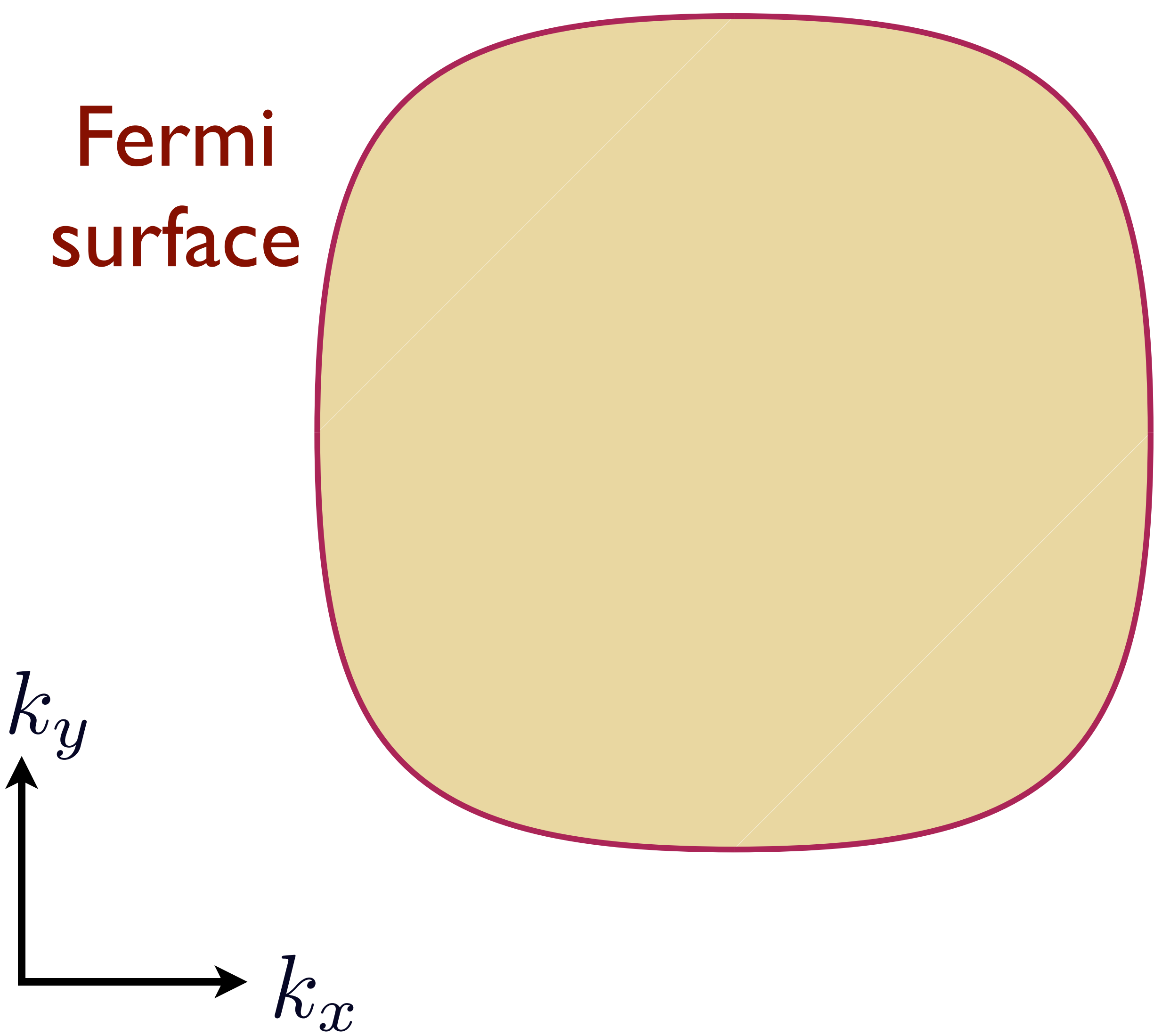
$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.



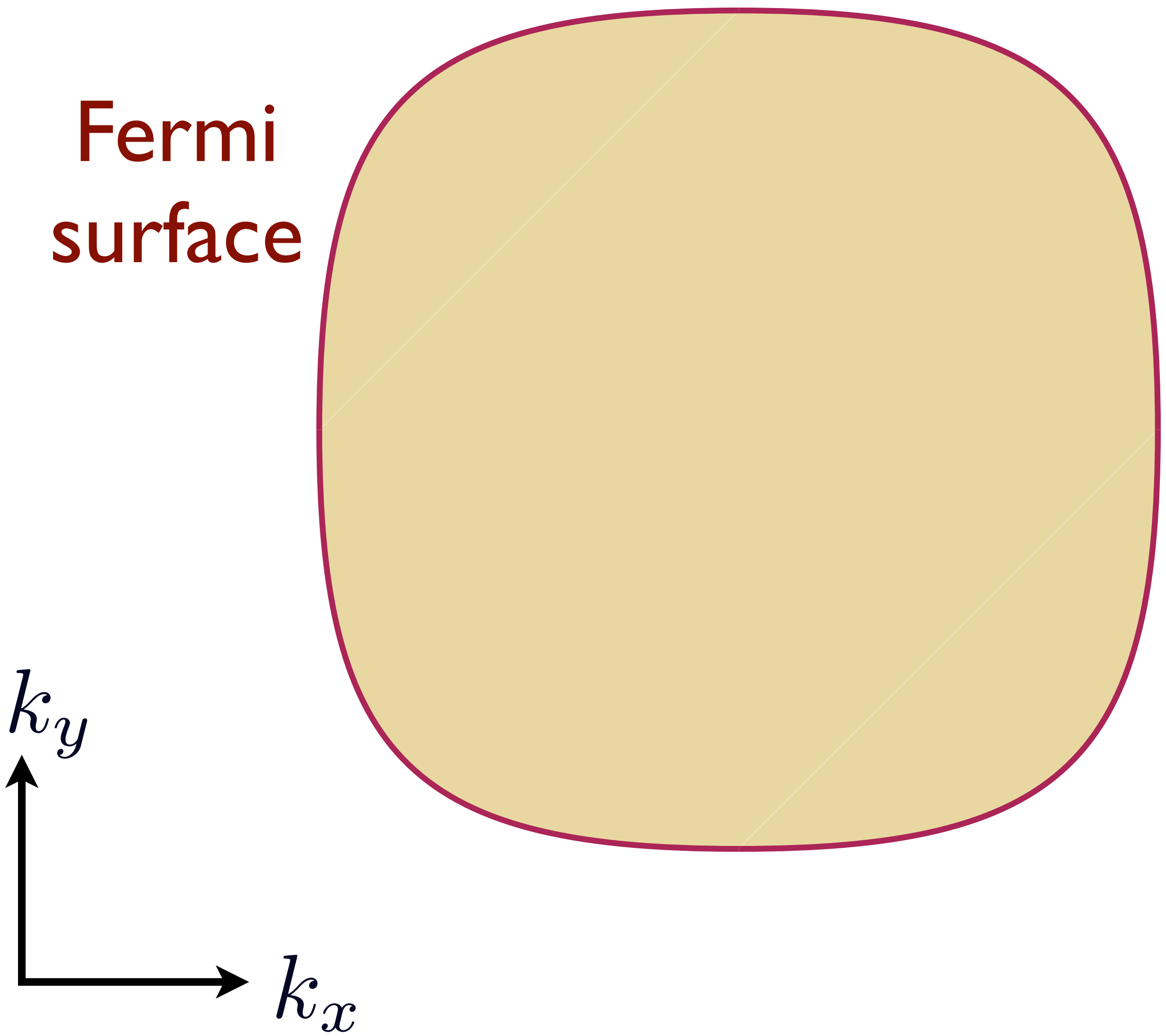
Fermions occupying eigenstates with a "semi-circular" density of states

Ordinary quantum matter: the Fermi liquid (FL)



- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface $= \mathcal{Q}$. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

Ordinary quantum matter: the Fermi liquid (FL)

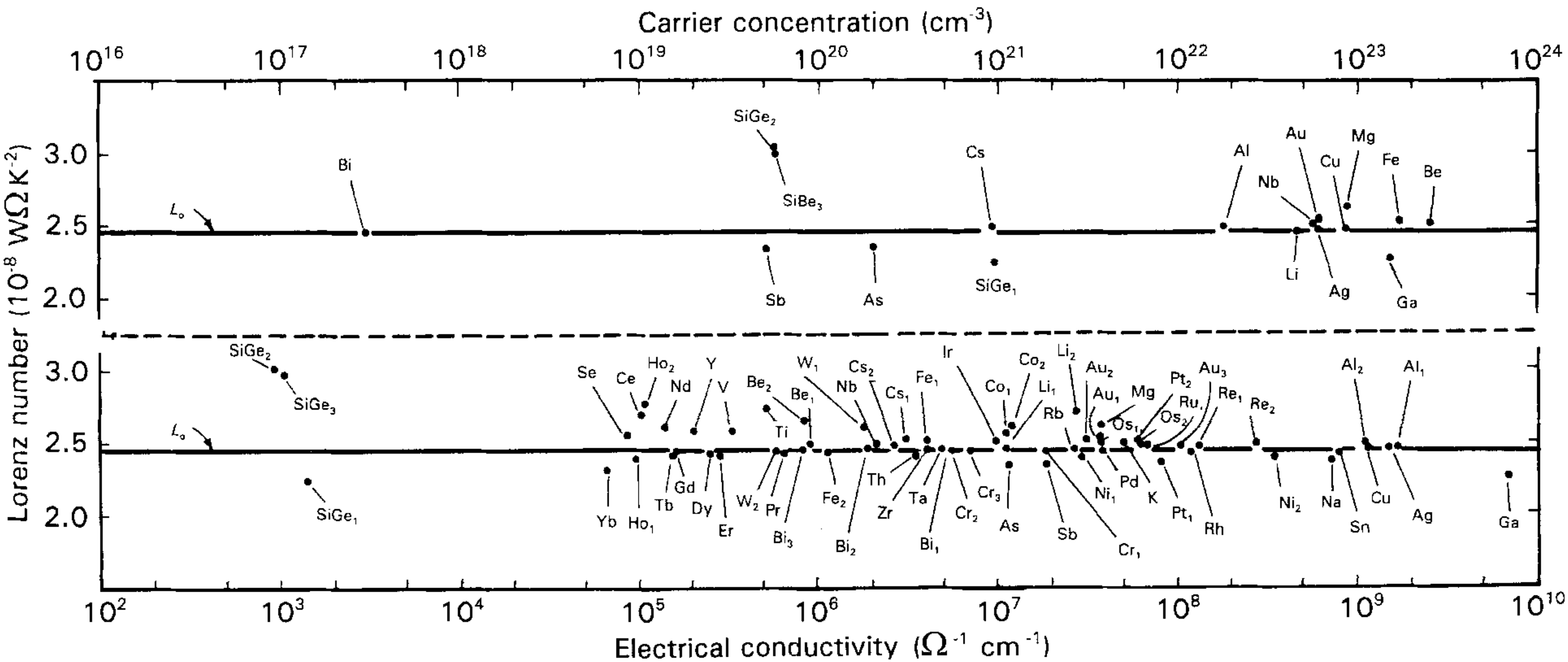


- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface $= \mathcal{Q}$. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

- $$\frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}} = \frac{\pi^2 k_B^2}{3e^2}$$

- Wiedemann-Franz law in a Fermi liquid:

$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8}\ \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$



G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

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Infinite-range model of a strange metal

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c^\dagger_{i\alpha} c_{i\beta} c^\dagger_{j\beta} c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c^\dagger_{j\beta} + c^\dagger_{j\beta} c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c^\dagger_{i\alpha} c_{i\alpha} = \mathcal{Q}$$

J_{ij} are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = J^2$
$N \to \infty$ at $M = 2$ yields spin-glass ground state.
$N \to \infty$ and then $M \to \infty$ yields critical strange metal

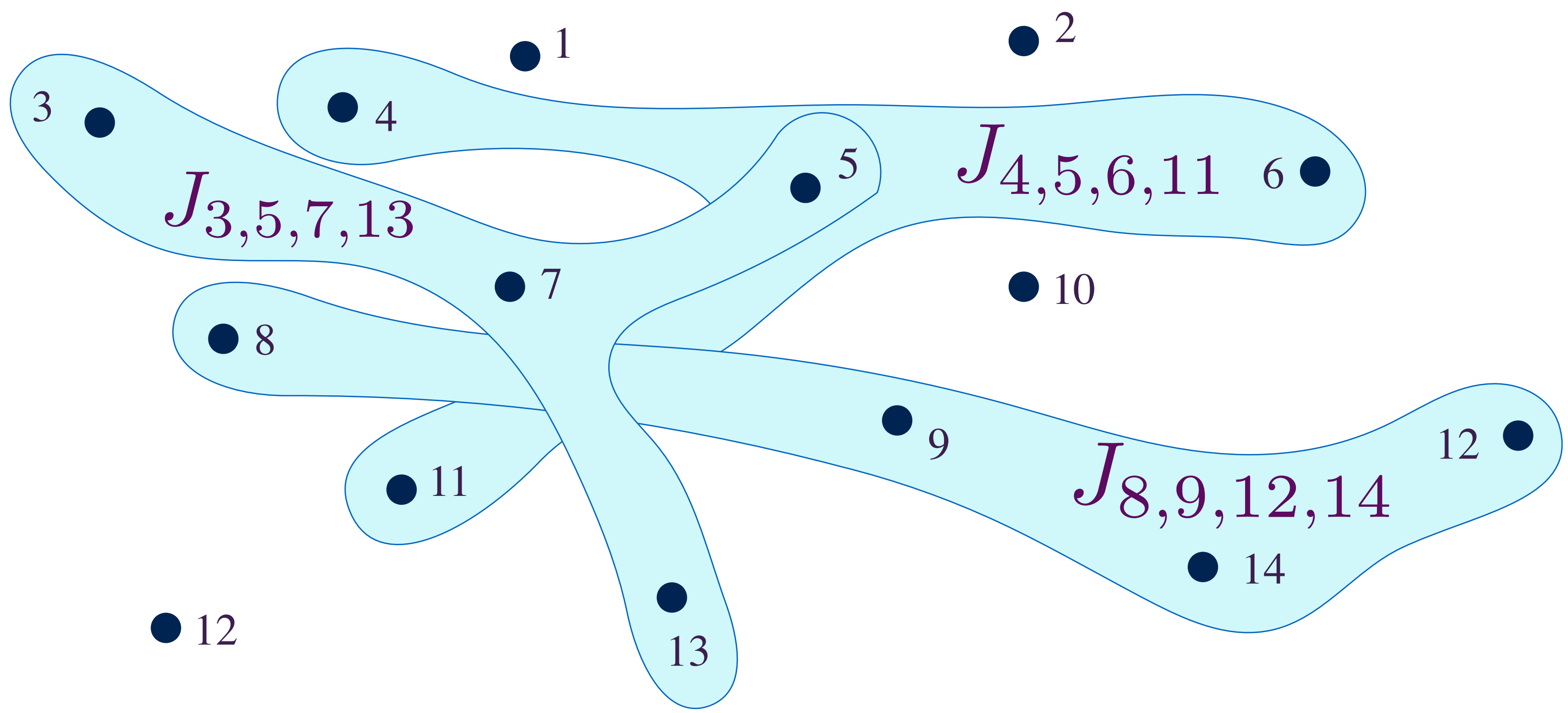
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range model of a strange metal

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields same critical strange metal; simpler to study numerically

A. Kitaev, unpublished; S. Sachdev, arXiv:1506.05111

Infinite-range strange metals

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. Let us also define $\widetilde{\Sigma}(z) = \Sigma(z) - \mu$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

Infinite-range strange metals

At frequencies $\ll J$, the equations for G and Σ can be written as

$$\int d\tau_2 G(\tau_1, \tau_2) \widetilde{\Sigma}(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)$$

$$\widetilde{\Sigma}(\tau_1, \tau_2) = -J^2 \left[G(\tau_1, \tau_2)\right]^2 G(\tau_2, \tau_1)$$

These equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\widetilde{\Sigma}(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \widetilde{\Sigma}(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, arXiv:1506.05111

These equations and invariances have similarities to those of the large N limit of quantum spins at the spatial boundary of a CFT2 (multi-channel Kondo problems)

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta
PRB **58**, 3794 (1998)

Infinite-range strange metals

Local fermion density of states

$$\rho(\omega) = -\mathrm{Im}\, G(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

$\mathcal{E}$ encodes the particle-hole asymmetry

While $\mathcal{E}$ determines the *low* energy spectrum, it is determined by the *total* fermion density $\mathcal{Q}$:

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right).$$

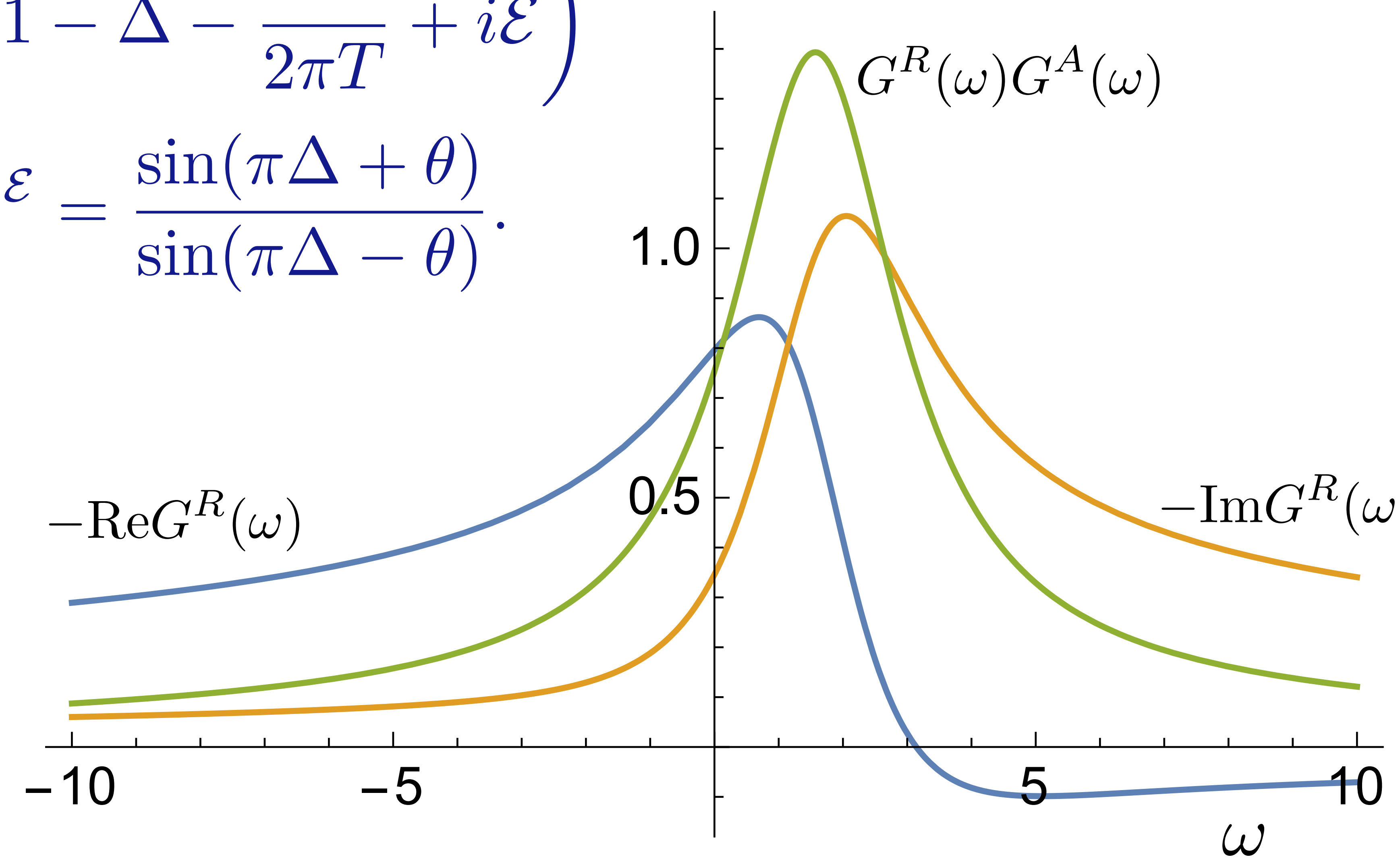
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

Infinite-range strange metals

At non-zero temperature, T, the Green's function also fully determined by $\mathcal{E}$.

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $\quad e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.



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Infinite-range strange metals

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$, and can be viewed as each site acquiring the universal boundary entropy of the multichannel Kondo problem.

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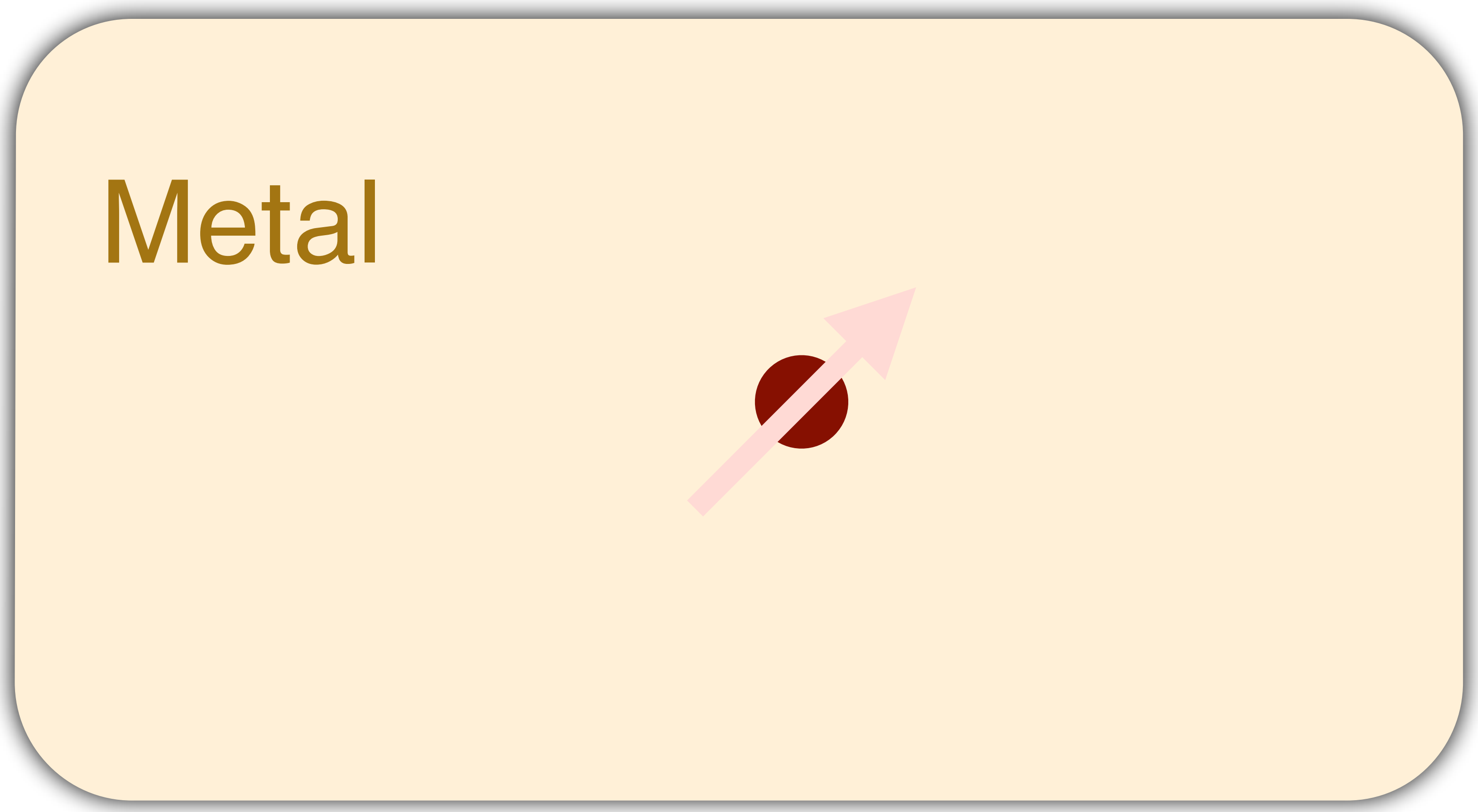
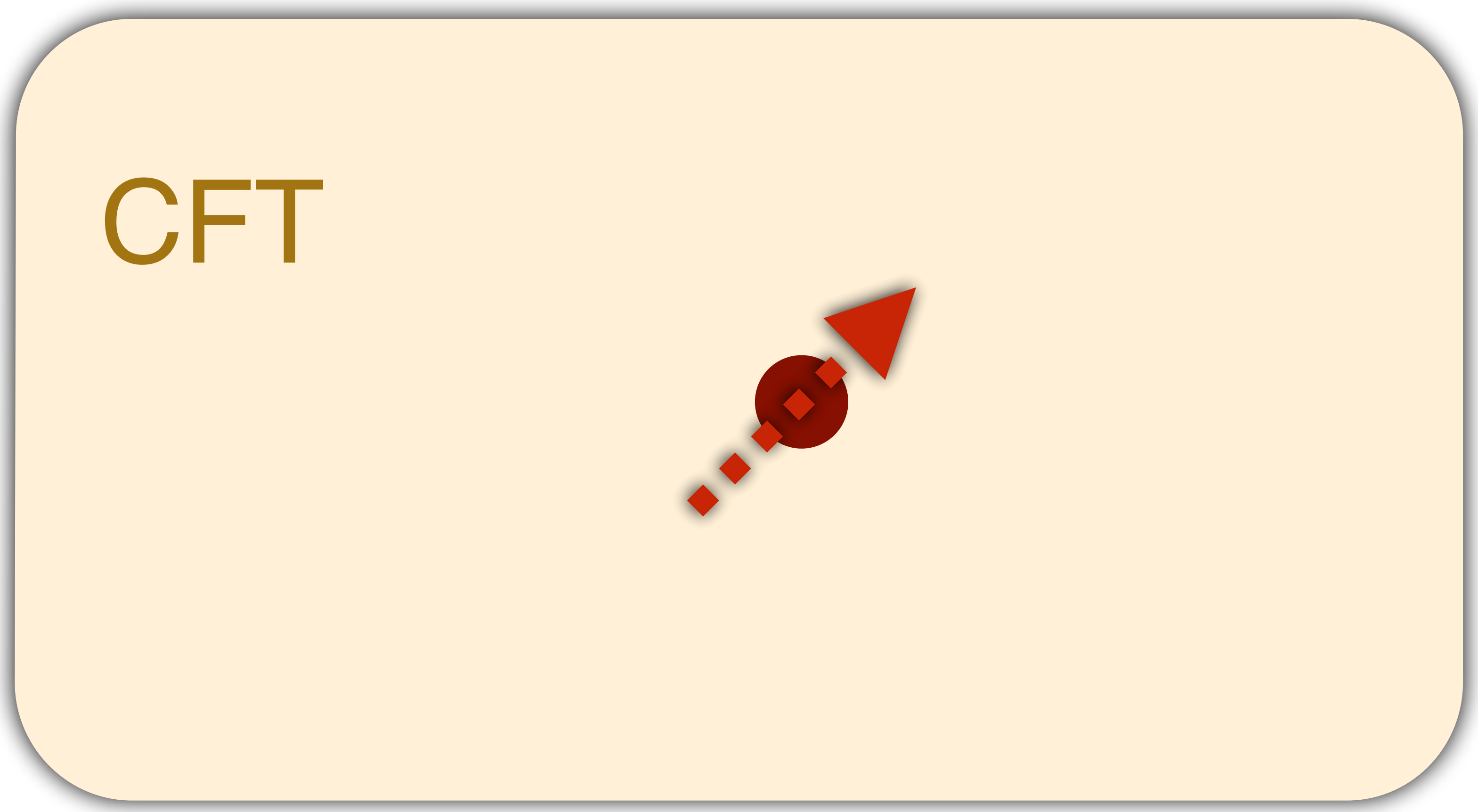
Free spin has entropy $\ln(2S+1)$

Kondo-screened
spin has
no entropy



N. Andrei and C. Destri, PRL **52**, 364 (1984).
A. M. Tsvelick, J. Phys. C **18**, 159 (1985).
I. Affleck and A. W. W. Ludwig, PRL **67**, 161 (1991).
S. Sachdev, C. Buragohain, and M. Vojta, Science **286**, 2479 (1999).

"Critically-screened" spin has "irrational" entropy



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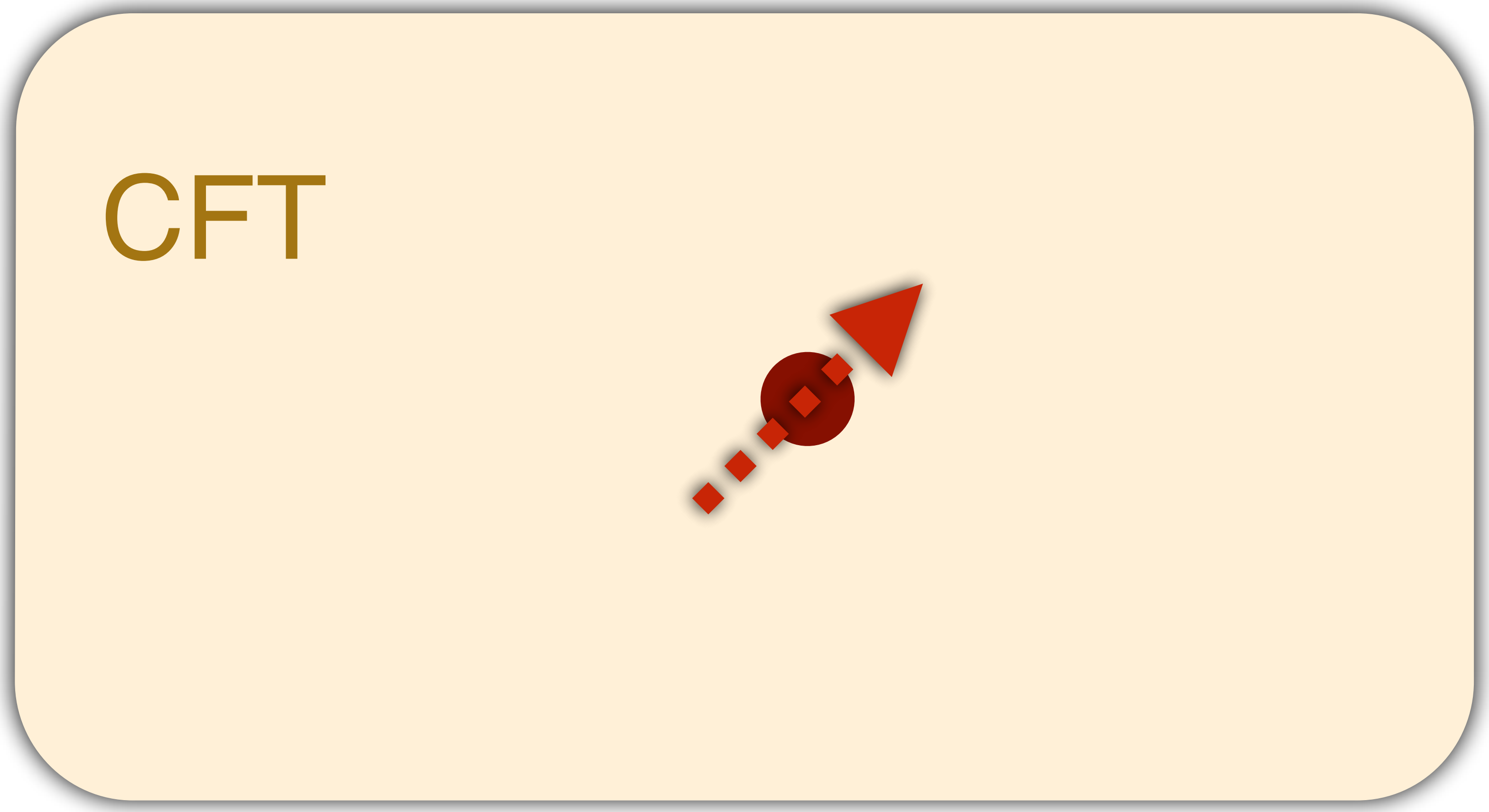

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S. Sachdev, C. Buragohain, and M. Vojta, Science **286**, 2479 (1999).

This entropy obeys

$$\left(\frac{\partial \mathcal{S}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

Note that $\mathcal{S}$ and $\mathcal{E}$ involve low-lying states, while $\mathcal{Q}$ depends upon *all* states, and details of the UV structure

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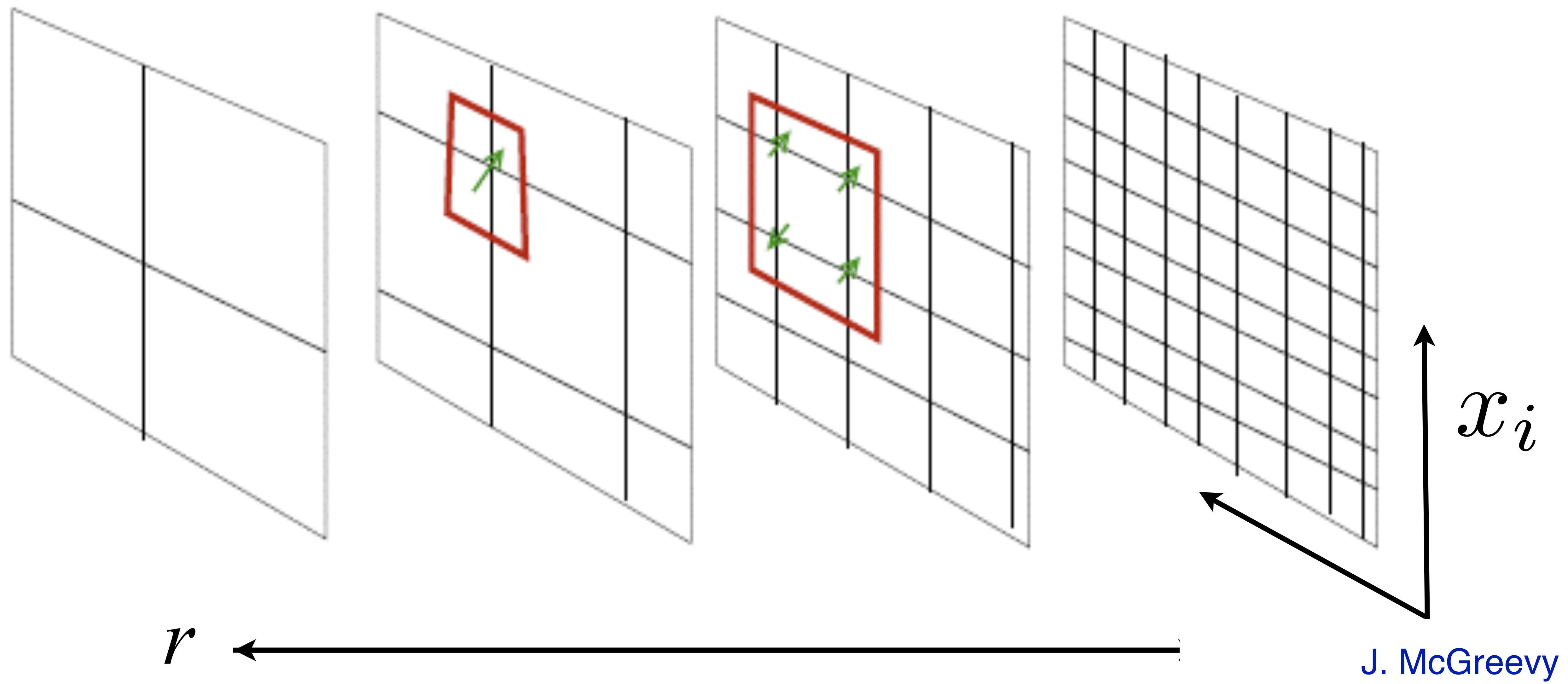
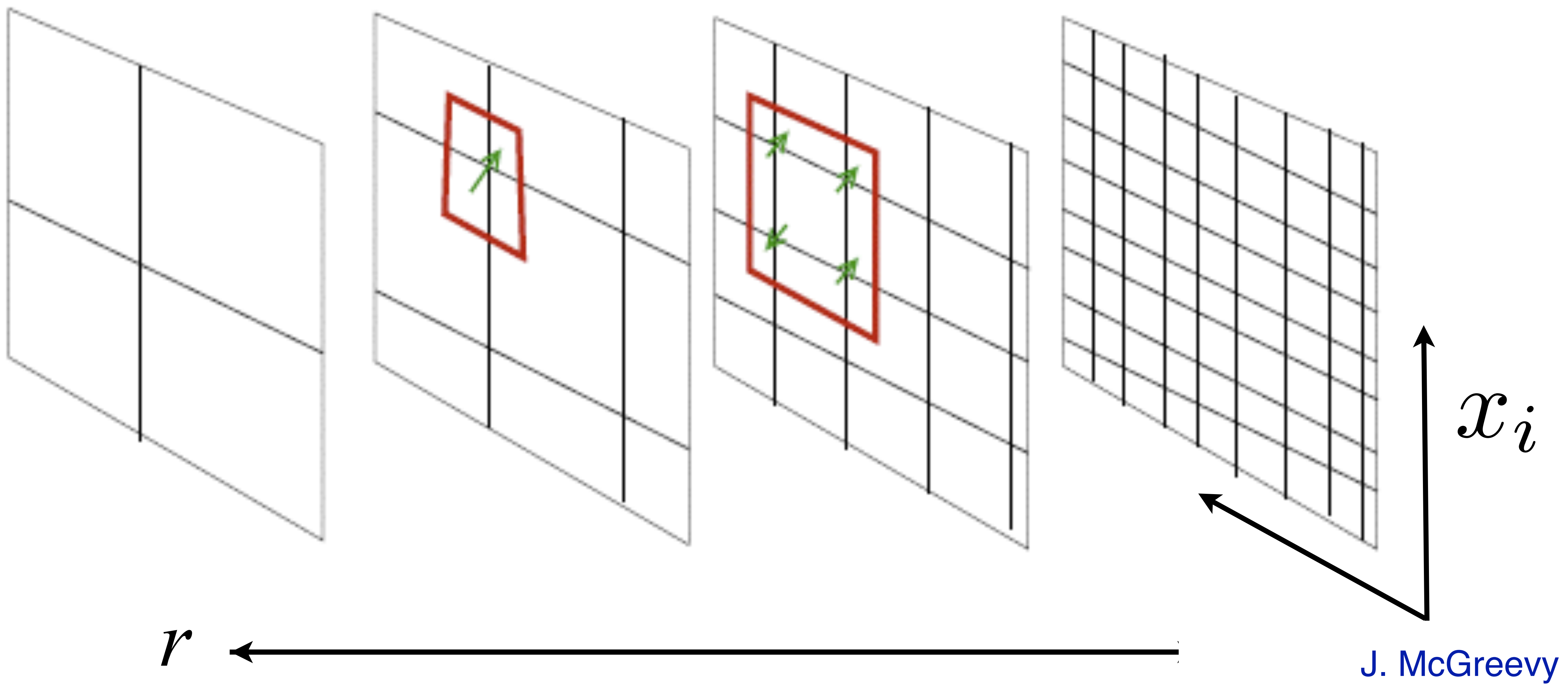
Holography



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

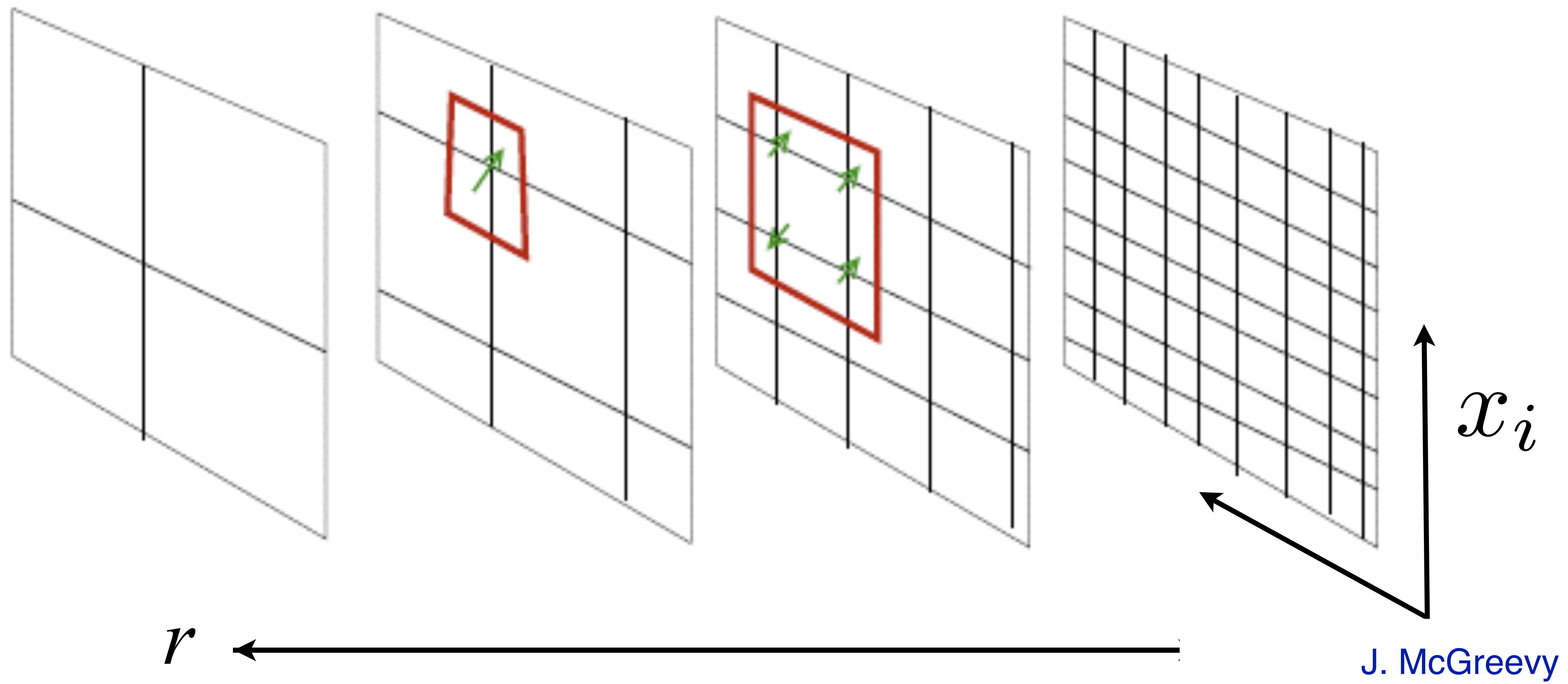
Holography



For a relativistic CFT in d spatial dimensions, the proper length, ds, in the holographic space is fixed by demanding the scale transformation $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

Holography



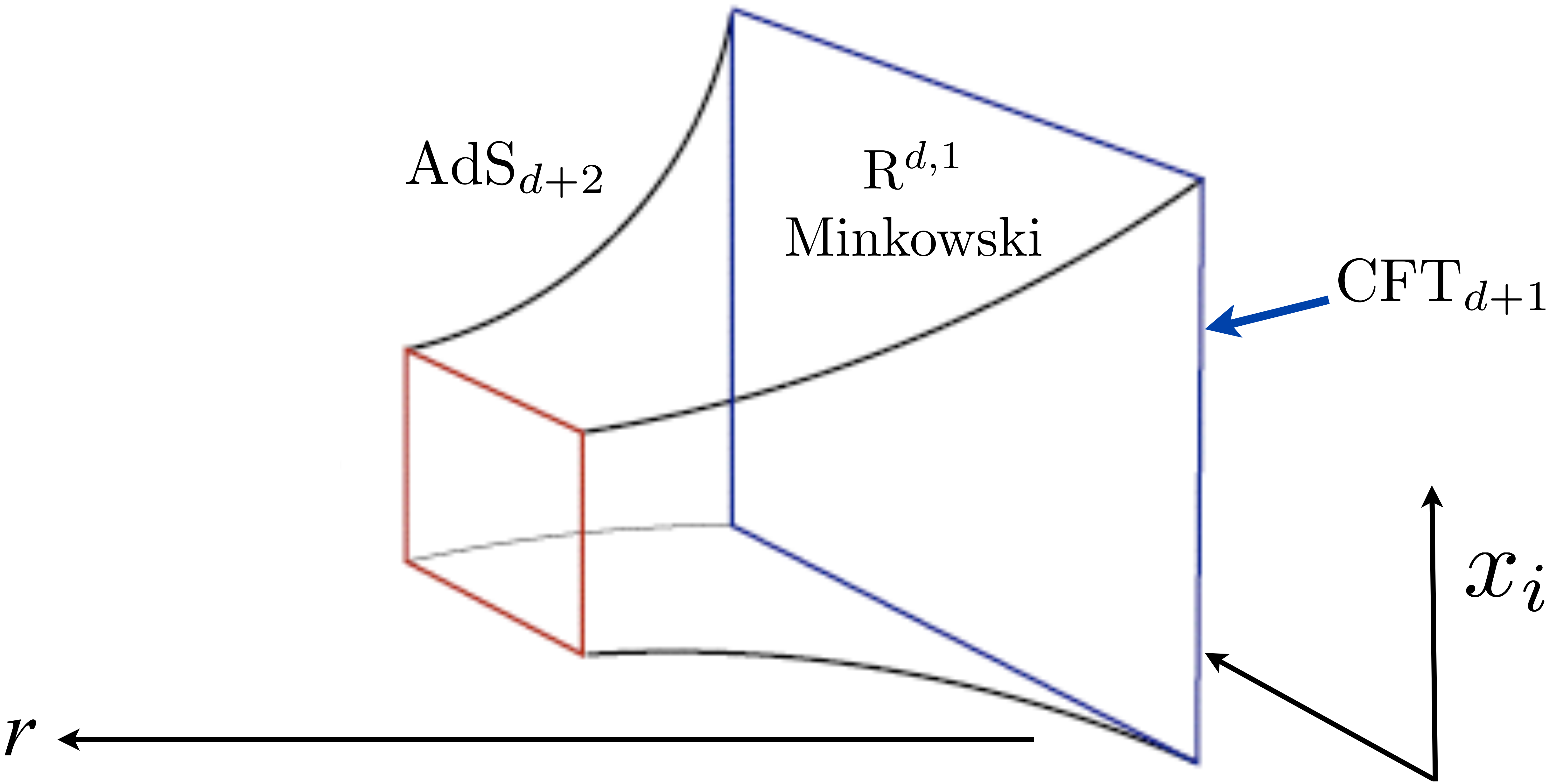
This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

This is the metric of anti-de Sitter space AdS_{d+2}.

AdS/CFT correspondence at zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + d\vec{x}^2\right]$$

AdS/CFT correspondence at non-zero temperature

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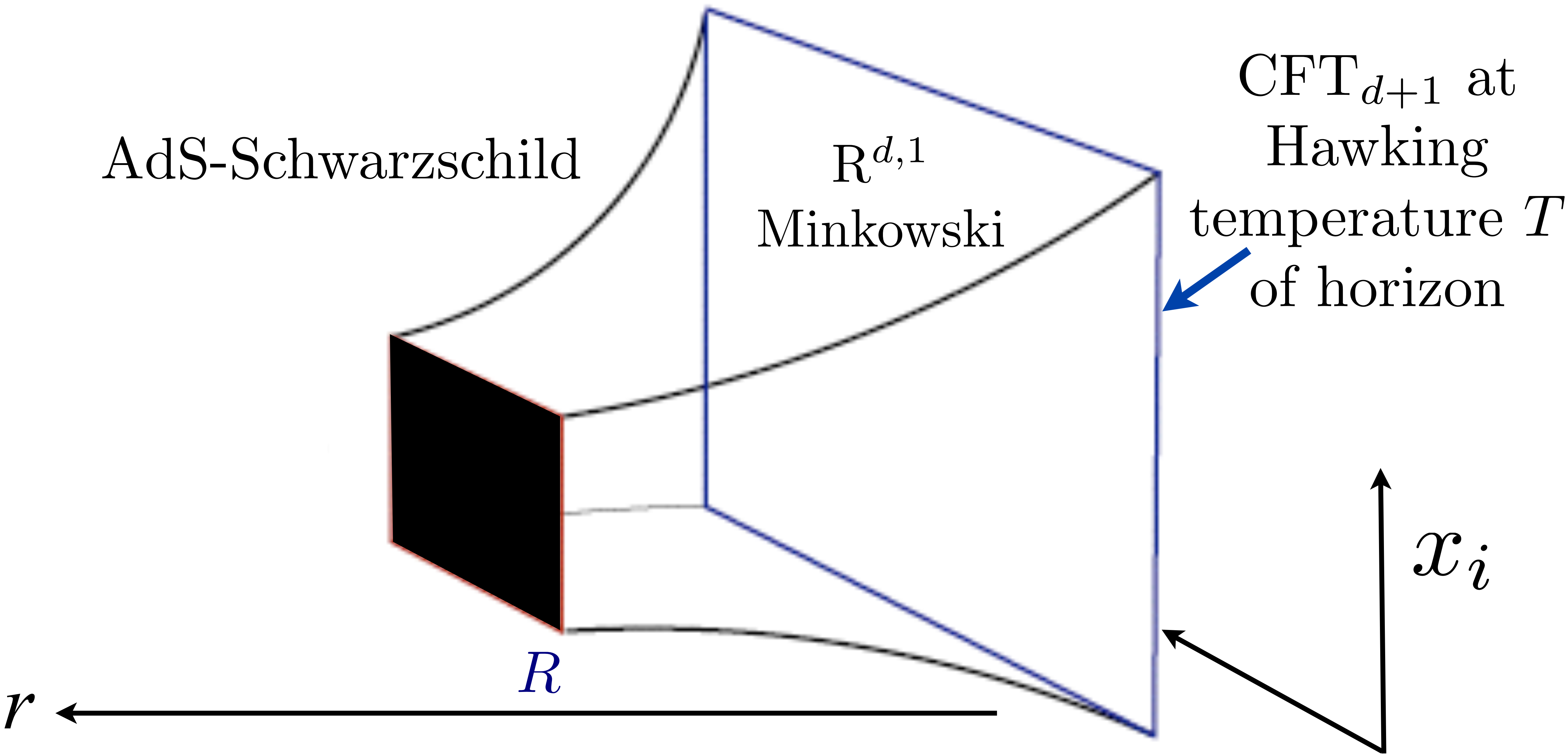

$$ds^2 = \left(\frac{L}{r}\right)^2\left[\frac{dr^2}{f(r)} - f(r)dt^2 + d\vec{x}^2\right]$$

with $f(r) = 1 - (r/R)^{d+1}$ and $T = (d+1)/(4\pi R)$.

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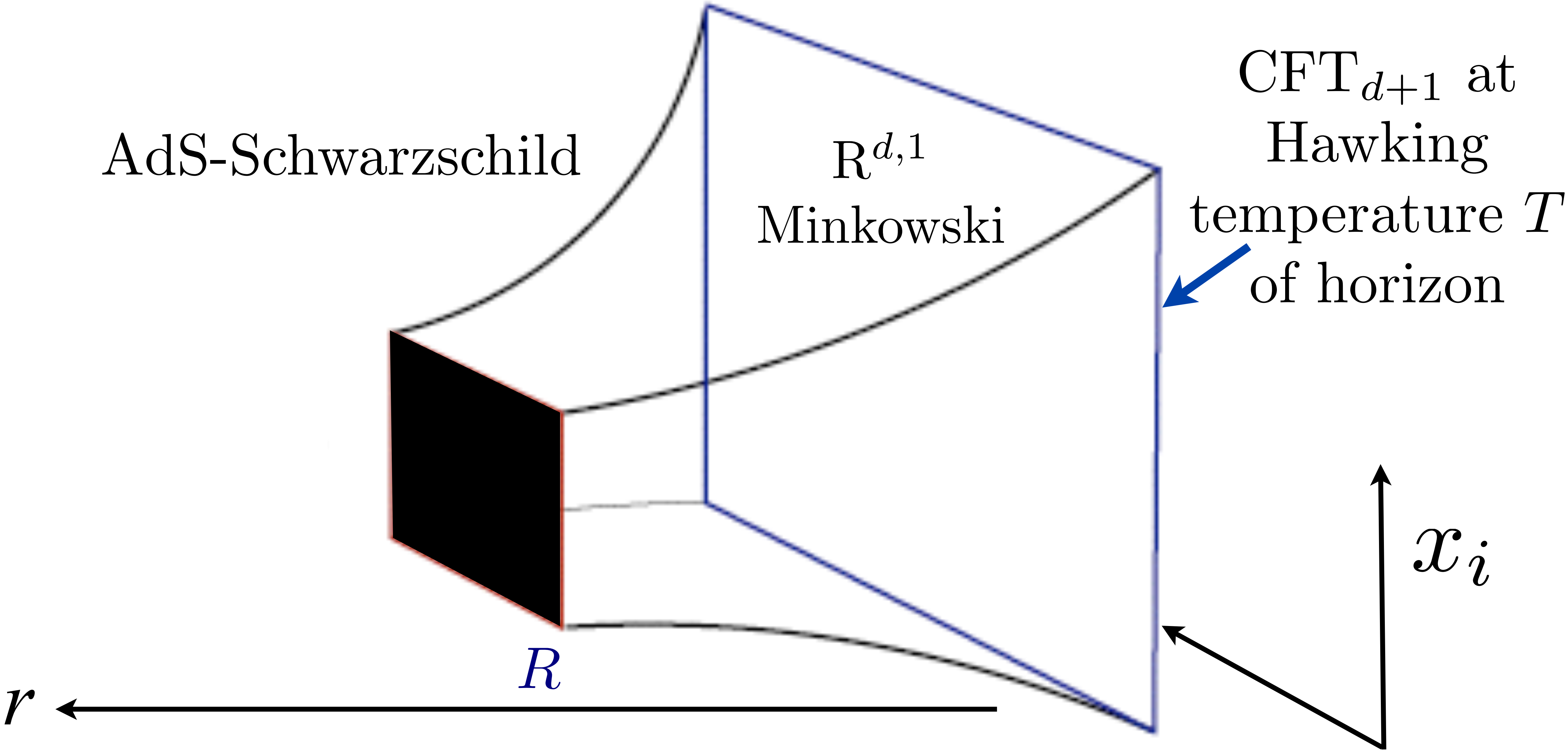

Entropy density of CFT_{d+1}, $\mathcal{S} \sim T^d$

Bekenstein-Hawking entropy density, $\mathcal{S}_{\mathrm{BH}} \sim T^d$

AdS/CFT correspondence at non-zero temperature

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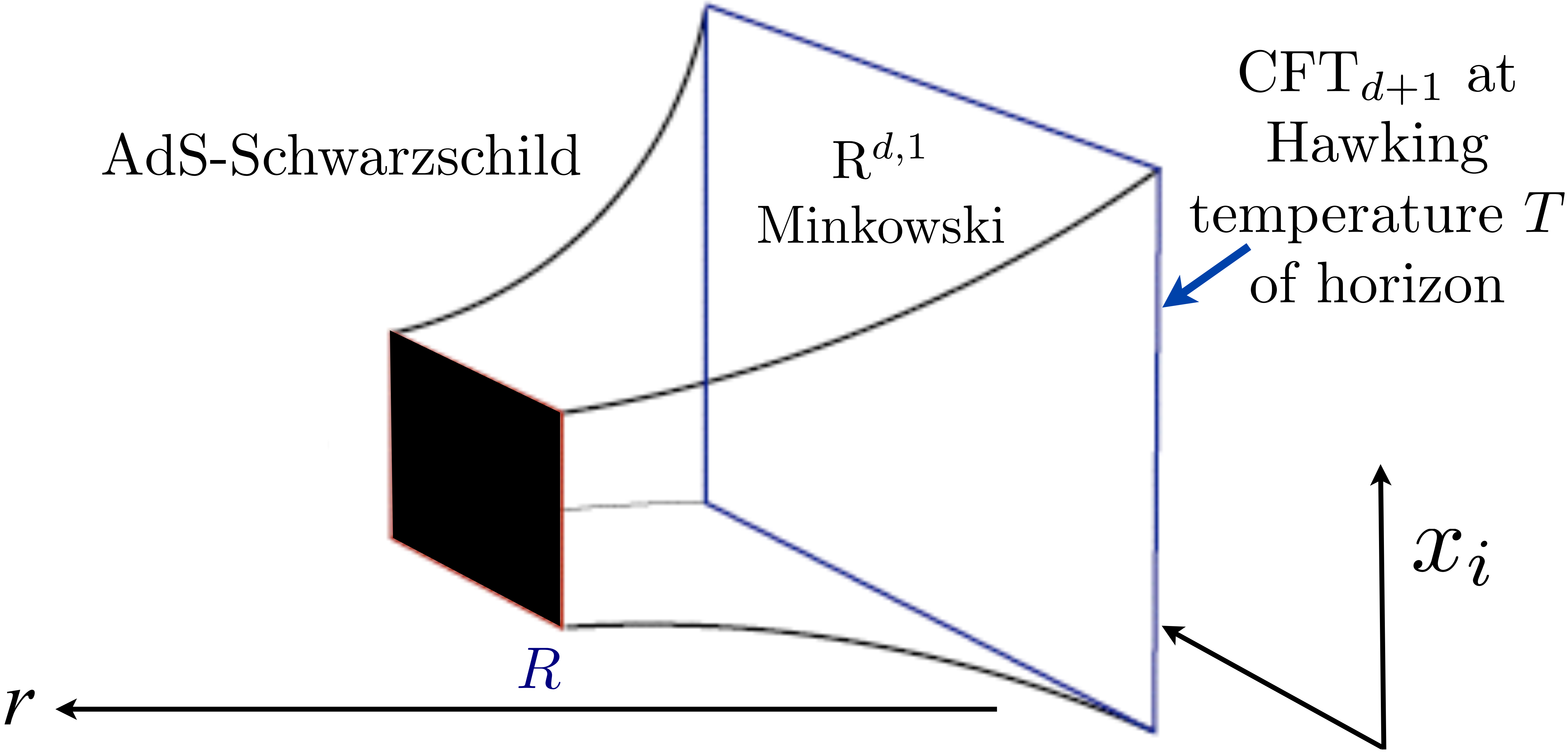
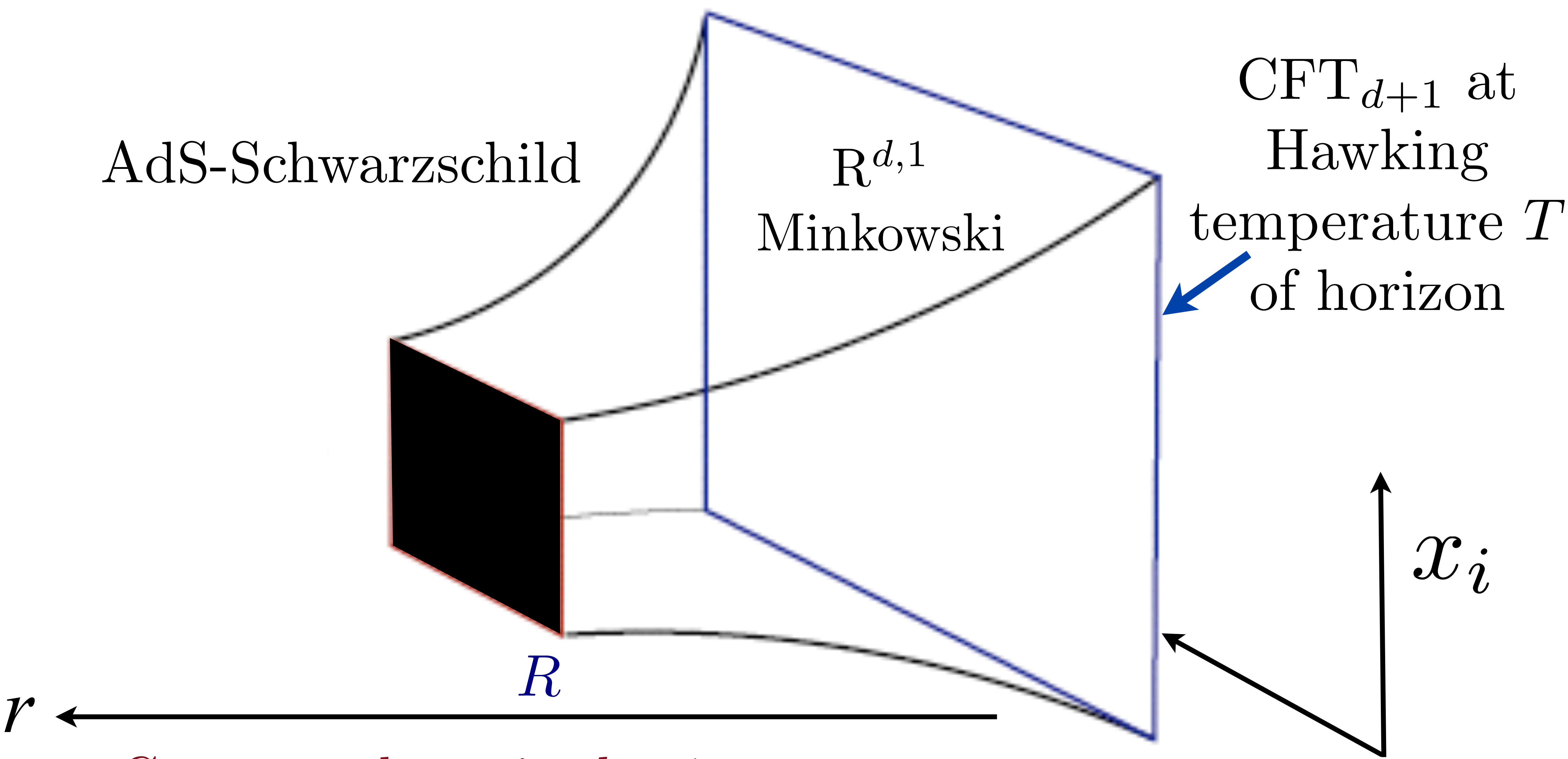


For SU(N) SYM in $d = 3$, $\mathcal{S}_{\mathrm{BH}} = (\pi^2/2)N^2T^3$. But there is (still) no confirmation of this from a field-theory computation on SYM.

Gubser, Klebanov, Peet 96

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} \right) \right]$



Correspondence in $d = 1$:

$$\mathcal{S} = \mathcal{S}_{\mathrm{BH}} = \frac{\pi}{3} c T,$$

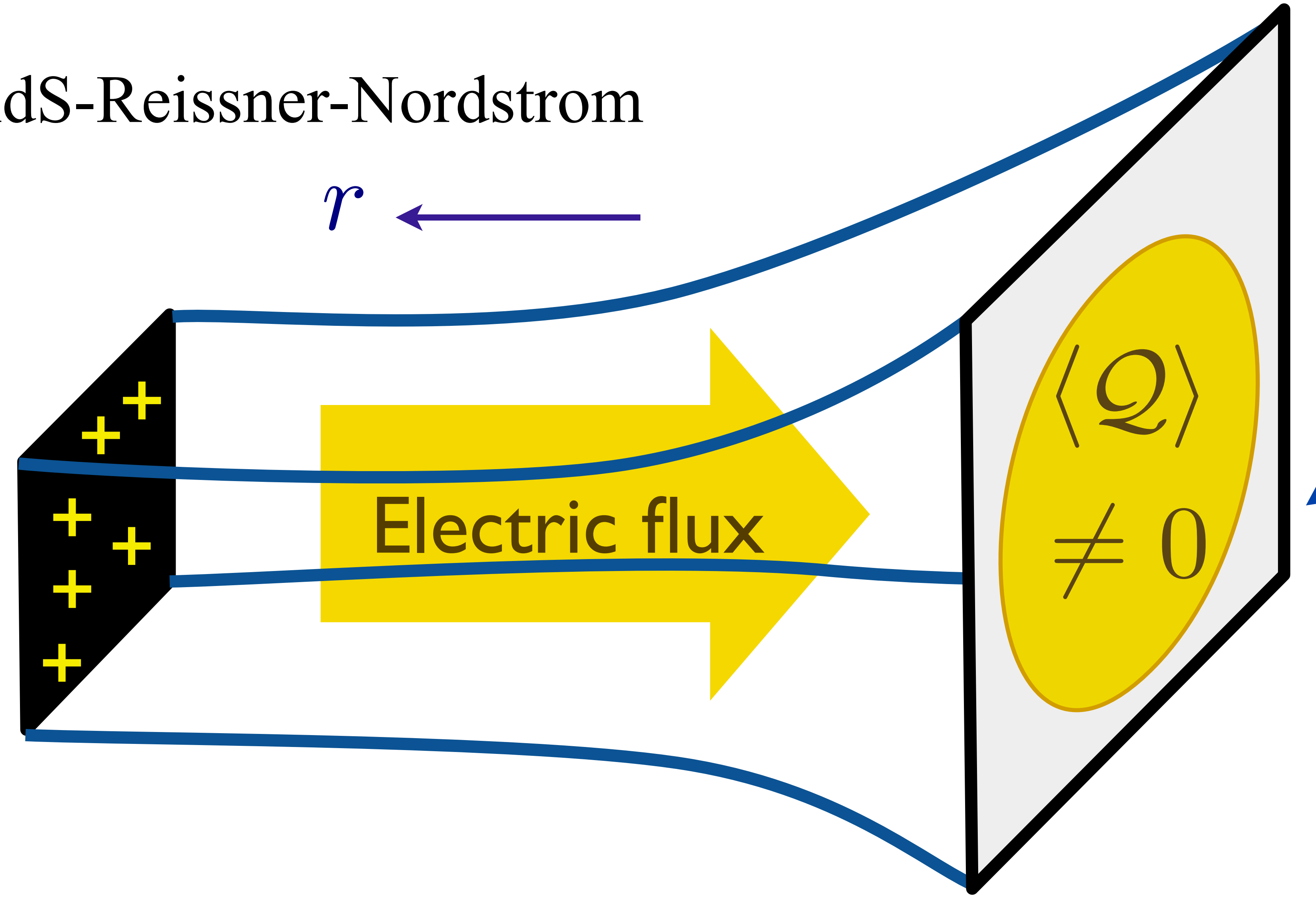
where $c = 12\pi L/\kappa^2$ is the central charge of the CFT_2.

Strominger 97

Charged black branes

Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2}F^2\right)\right]$

AdS-Reissner-Nordstrom



Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

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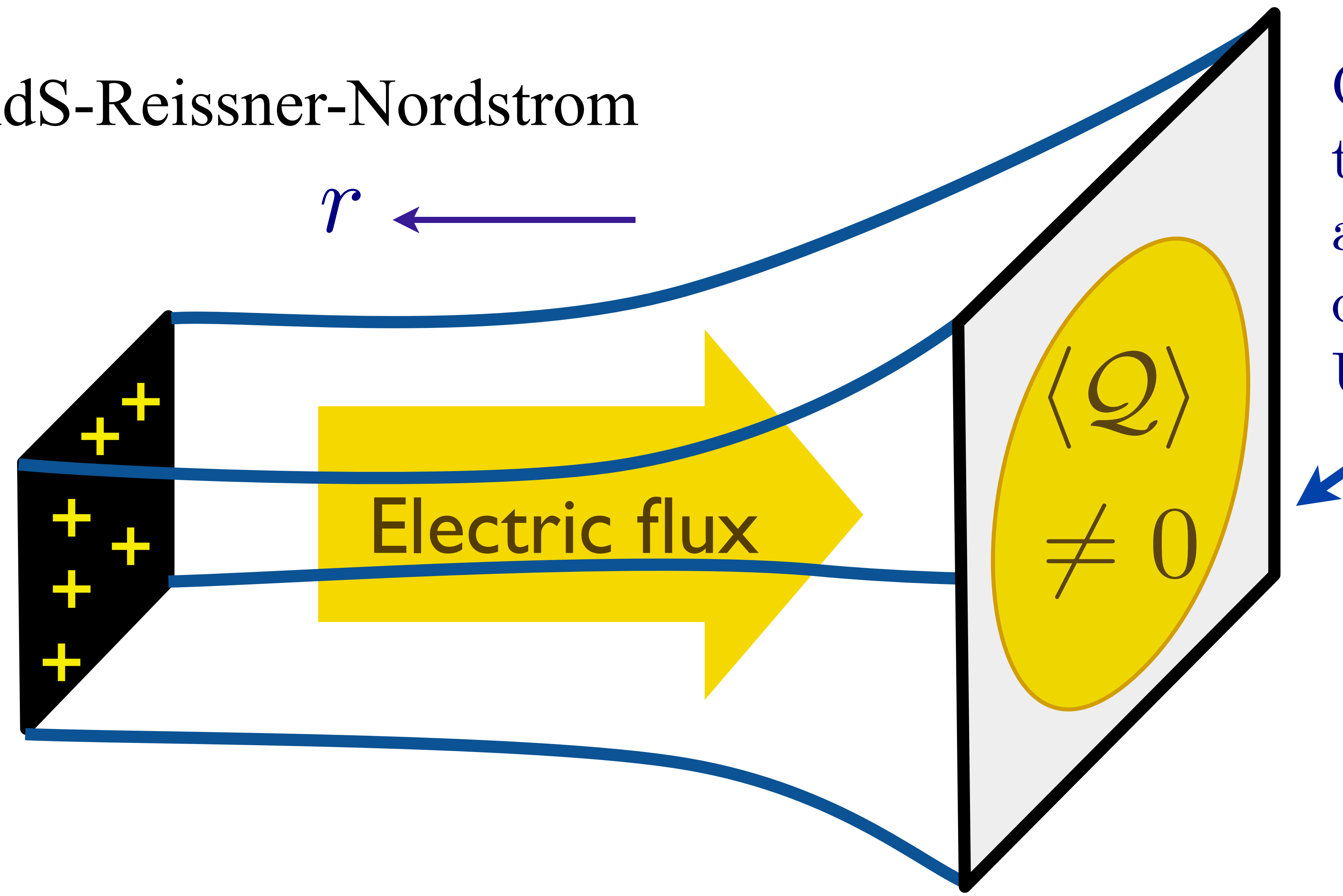
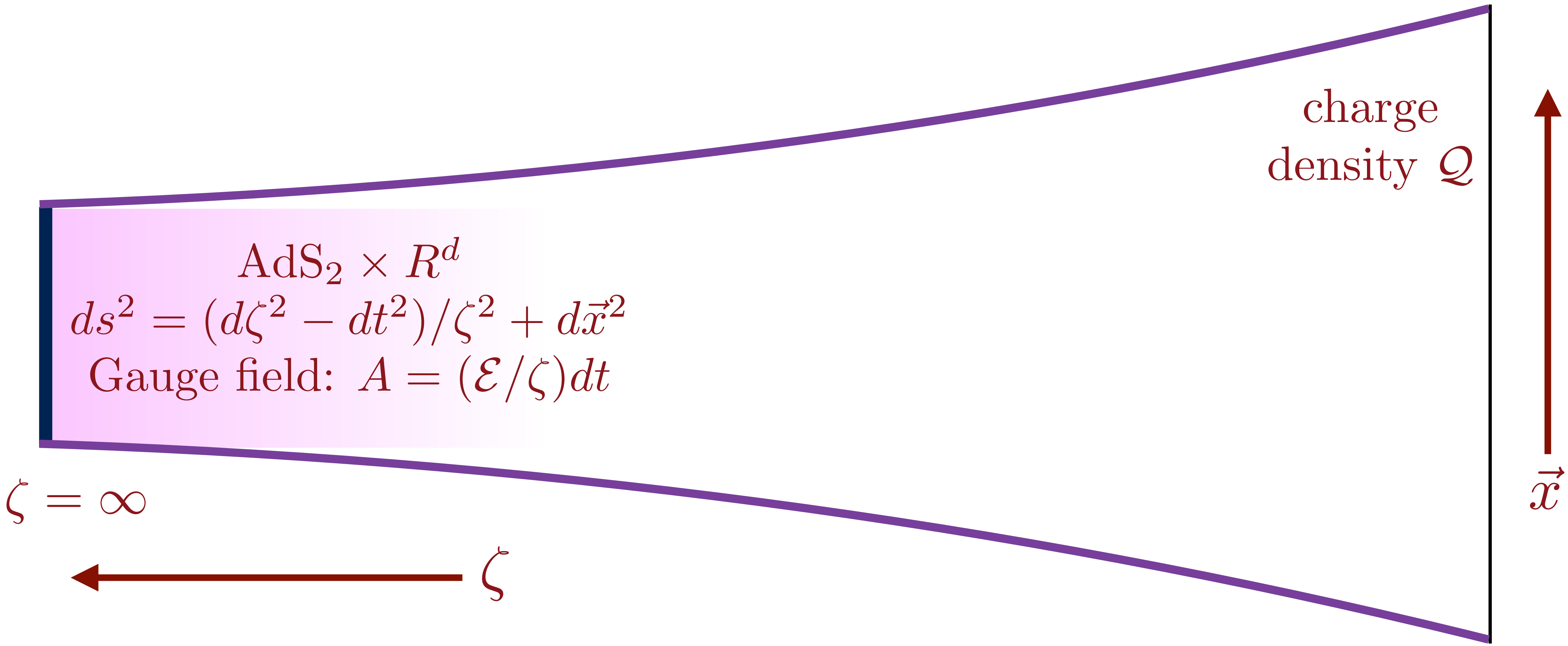


Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

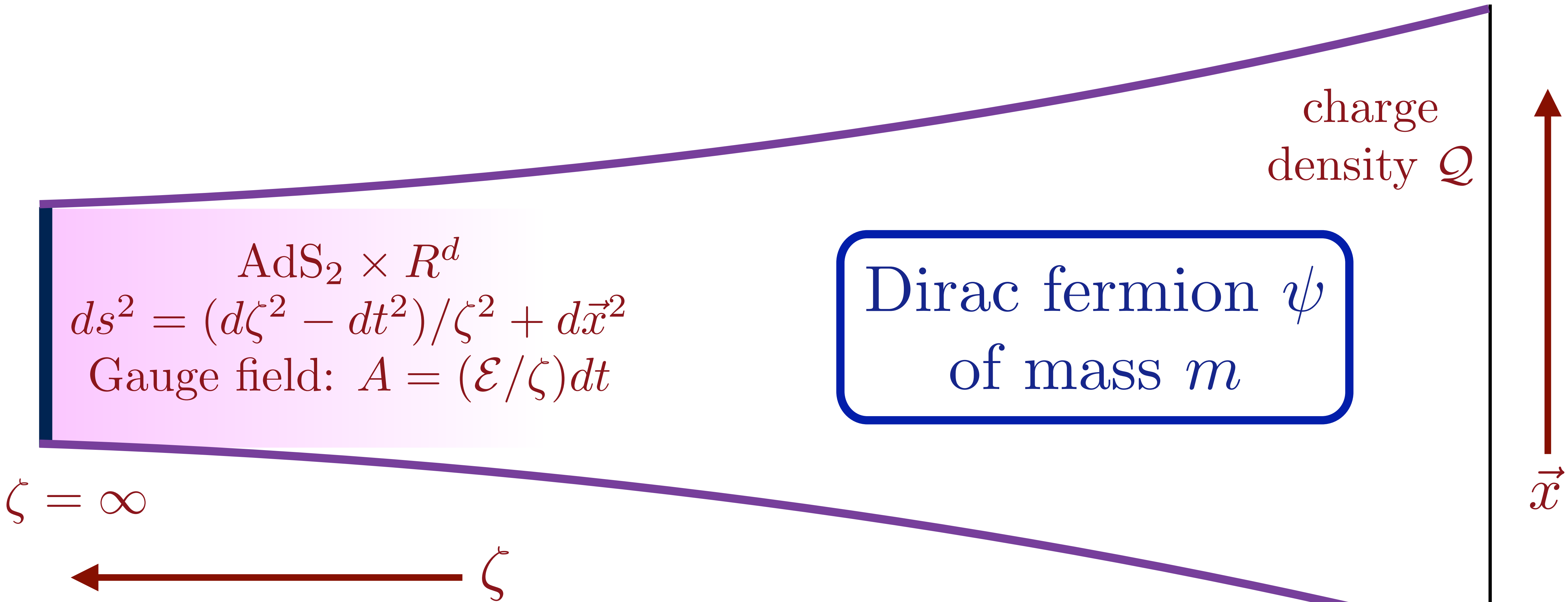
A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

General Relativity of charged black branes



- Near-horizon metric is AdS_2, with near-horizon electric field $\mathcal{E}$.

Quantum fields on charged black branes



Boundary Green's function of ψ at $T = 0$

$$\mathrm{Im}G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-(1-2\Delta)} & , \ \omega < 0. \end{cases}$$

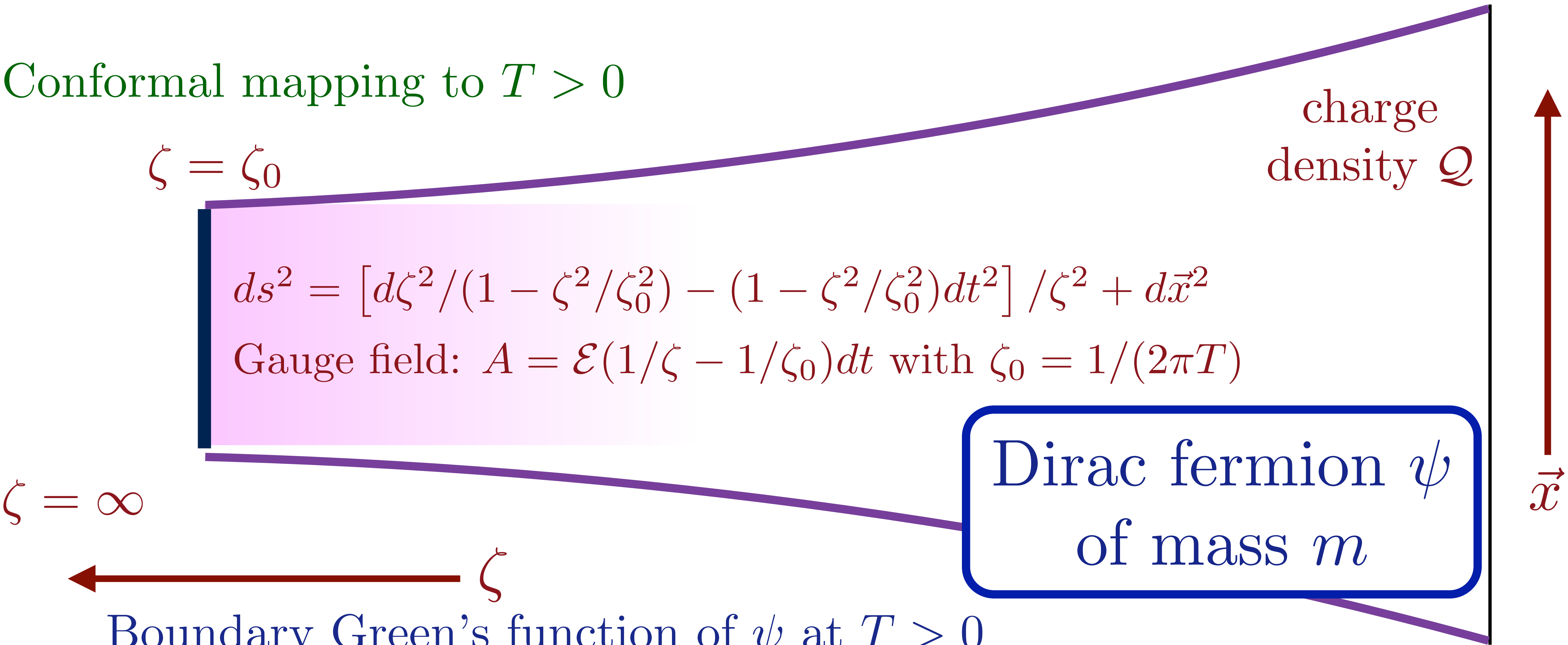
where the fermion scaling dimension Δ is a function of m

$\mathcal{E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)

Quantum fields on charged black branes

Conformal mapping to $T > 0$



Boundary Green's function of ψ at $T > 0$

is fully determined by $\mathcal{E}$

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

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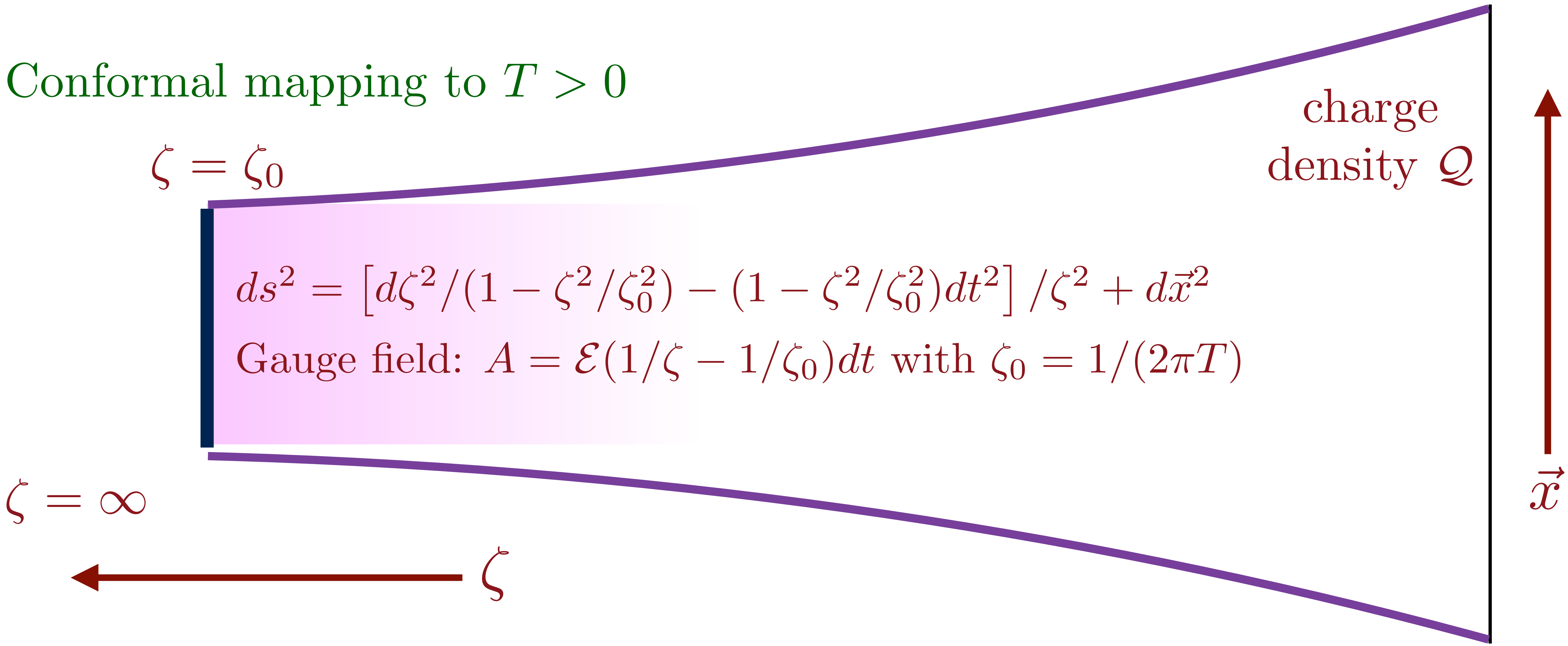
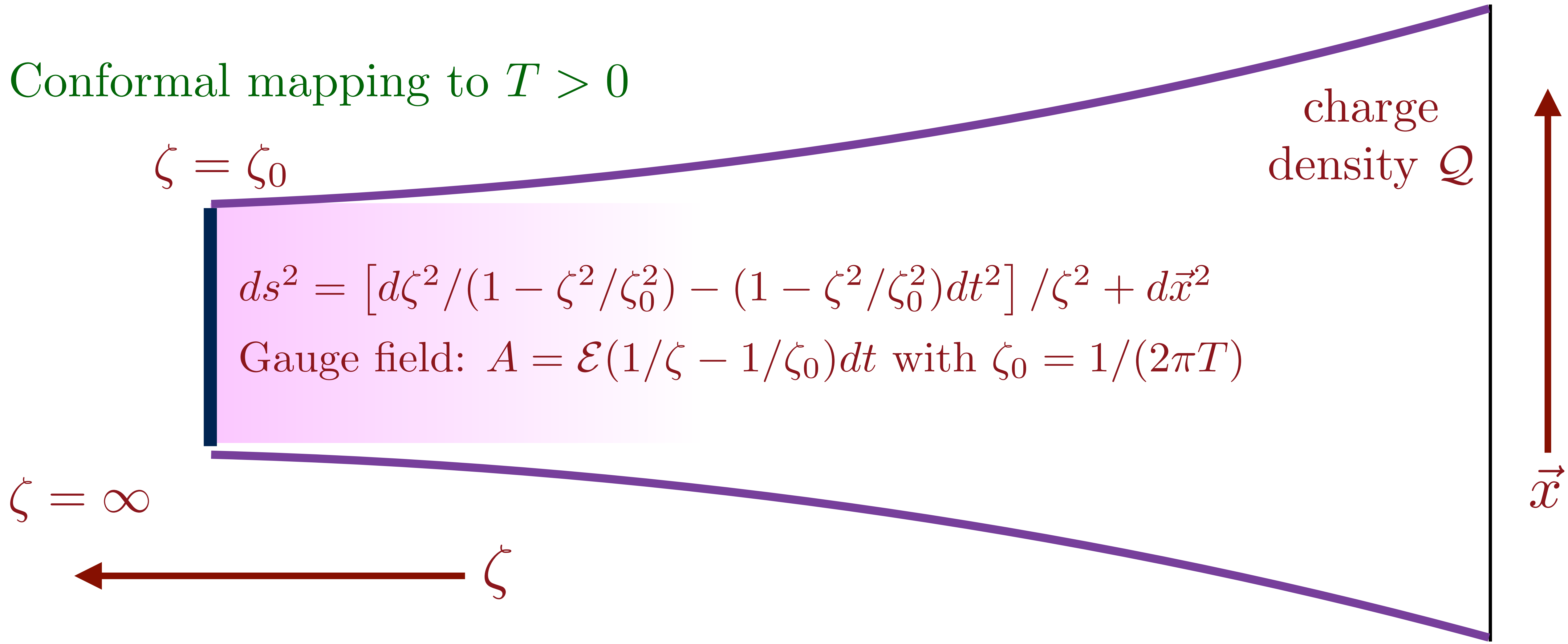


- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{BH}$.

General Relativity of charged black branes



- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $\mathcal{S}_{BH}$.
- Using Gauss's Law, it can be shown that $\mu(T) = -2\pi\mathcal{E}T+$ constant as $T \to 0$.
- Using a thermodynamic Maxwell relation (also obeyed by gravity),

A. Sen
hep-th/0506177
S. Sachdev
1506.05111

$$\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

General Relativity of charged black branes

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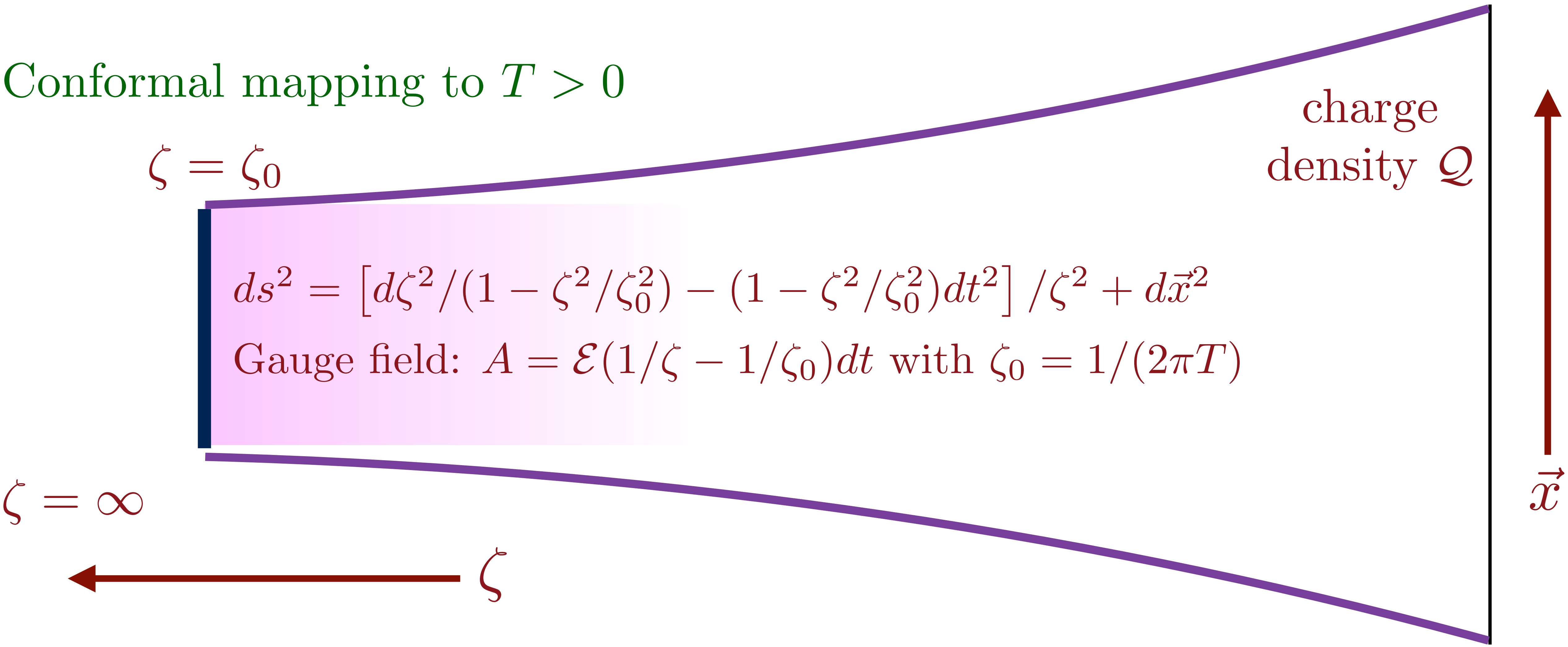

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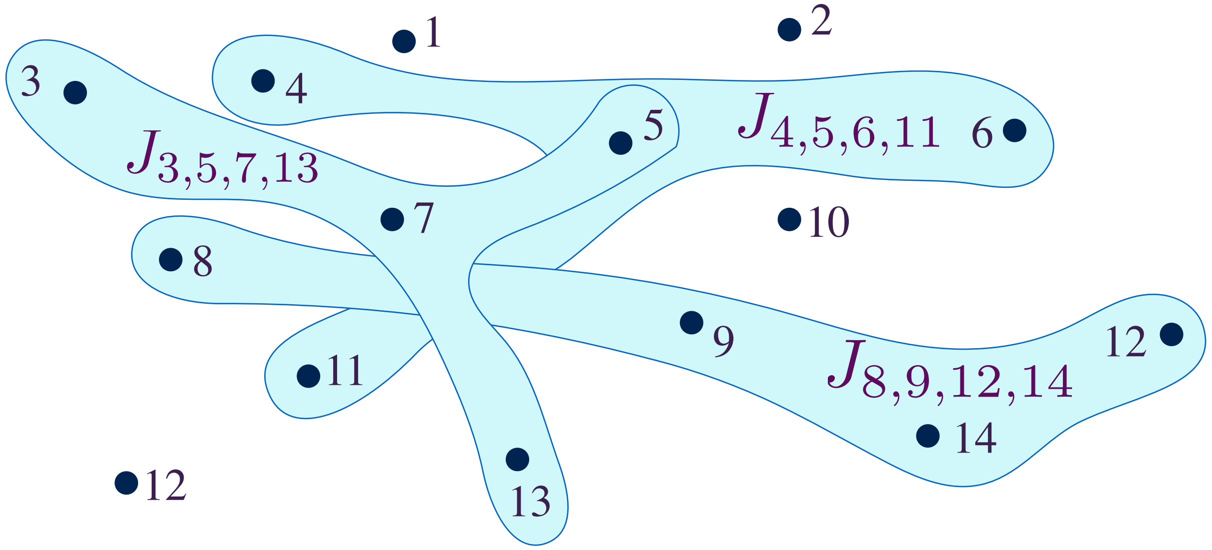
- Using a thermodynamic Maxwell relation (also obeyed by gravity)

$$\left(\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

Also obeyed by Wald entropy in higher-derivative gravity.

A. Sen
hep-th/0506177
S. Sachdev
1506.05111

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$



$$\mathcal{Q} = \frac{1}{N} \sum_i \left\langle c_i^\dagger c_i \right\rangle .$$

Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}}\, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

Known ‘equation of state’ determines $\mathcal{E}$ as a function of $\mathcal{Q}$

Microscopic zero temperature entropy density, $\mathcal{S}$, obeys

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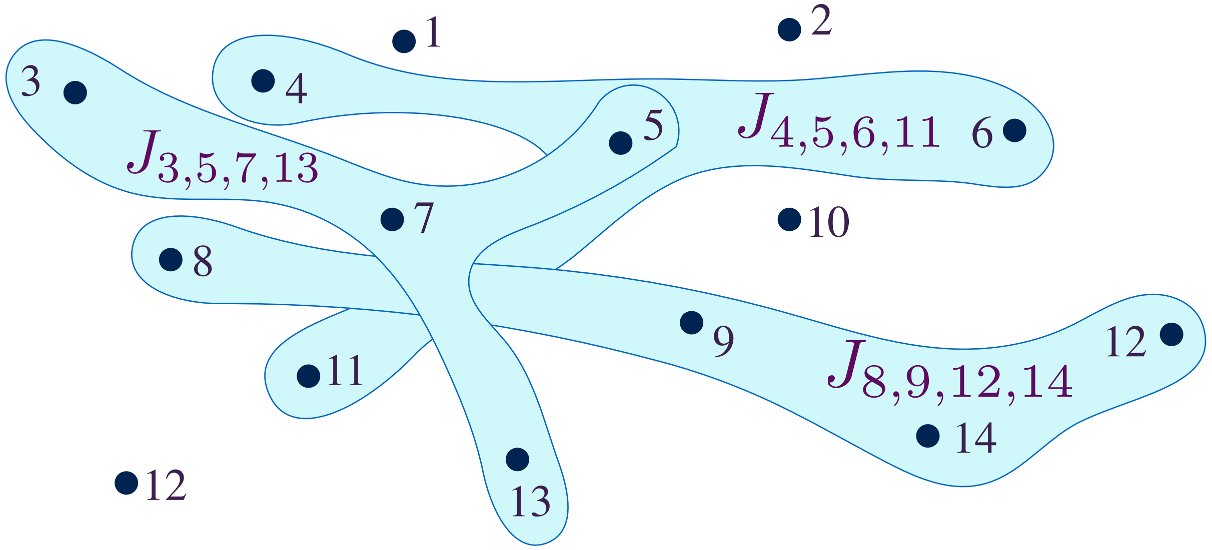

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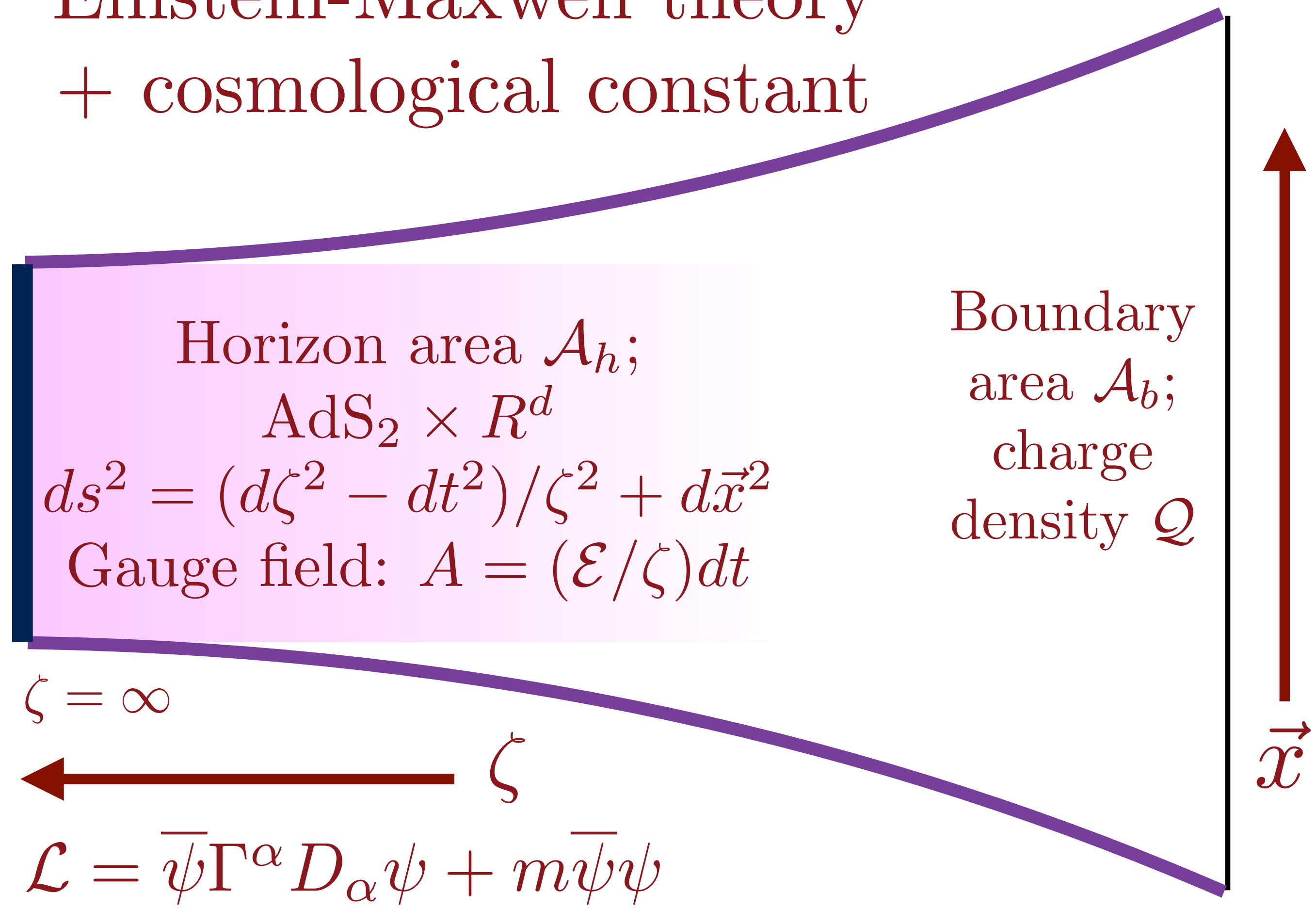
Einstein-Maxwell theory
+ cosmological constant



$$\mathcal{L} = \overline{\psi} \Gamma^\alpha D_\alpha \psi + m \overline{\psi} \psi$$

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'Equation of state' relating $\mathcal{E}$ and $\mathcal{Q}$ depends upon the geometry of spacetime far from the AdS_2

Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\rm BH}}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$

A. Sen, arXiv:hep-th/0506177; S. Sachdev, arXiv:1506.0511

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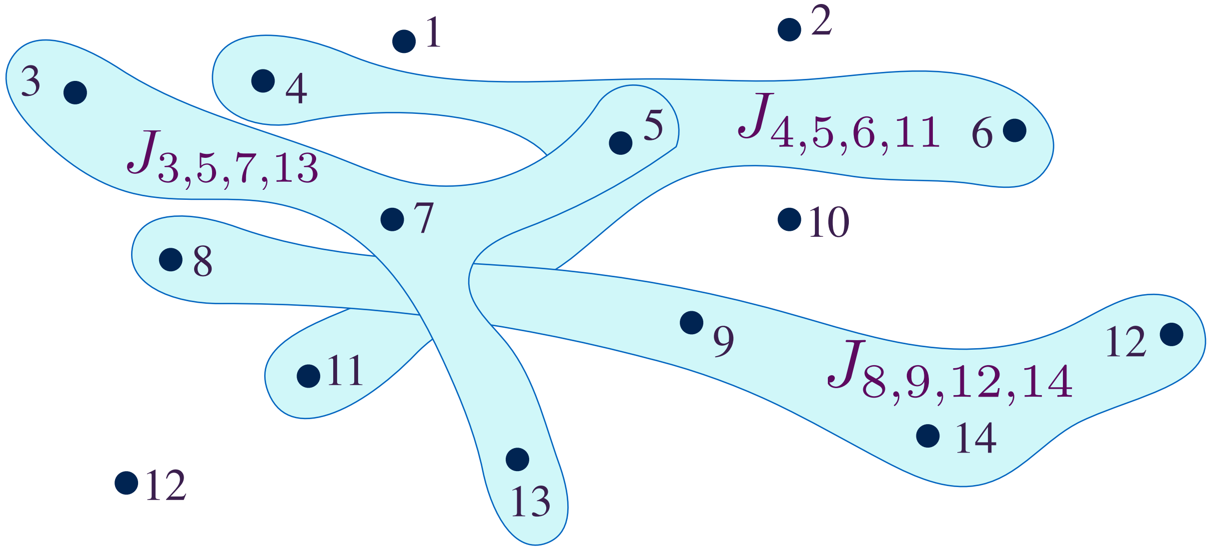

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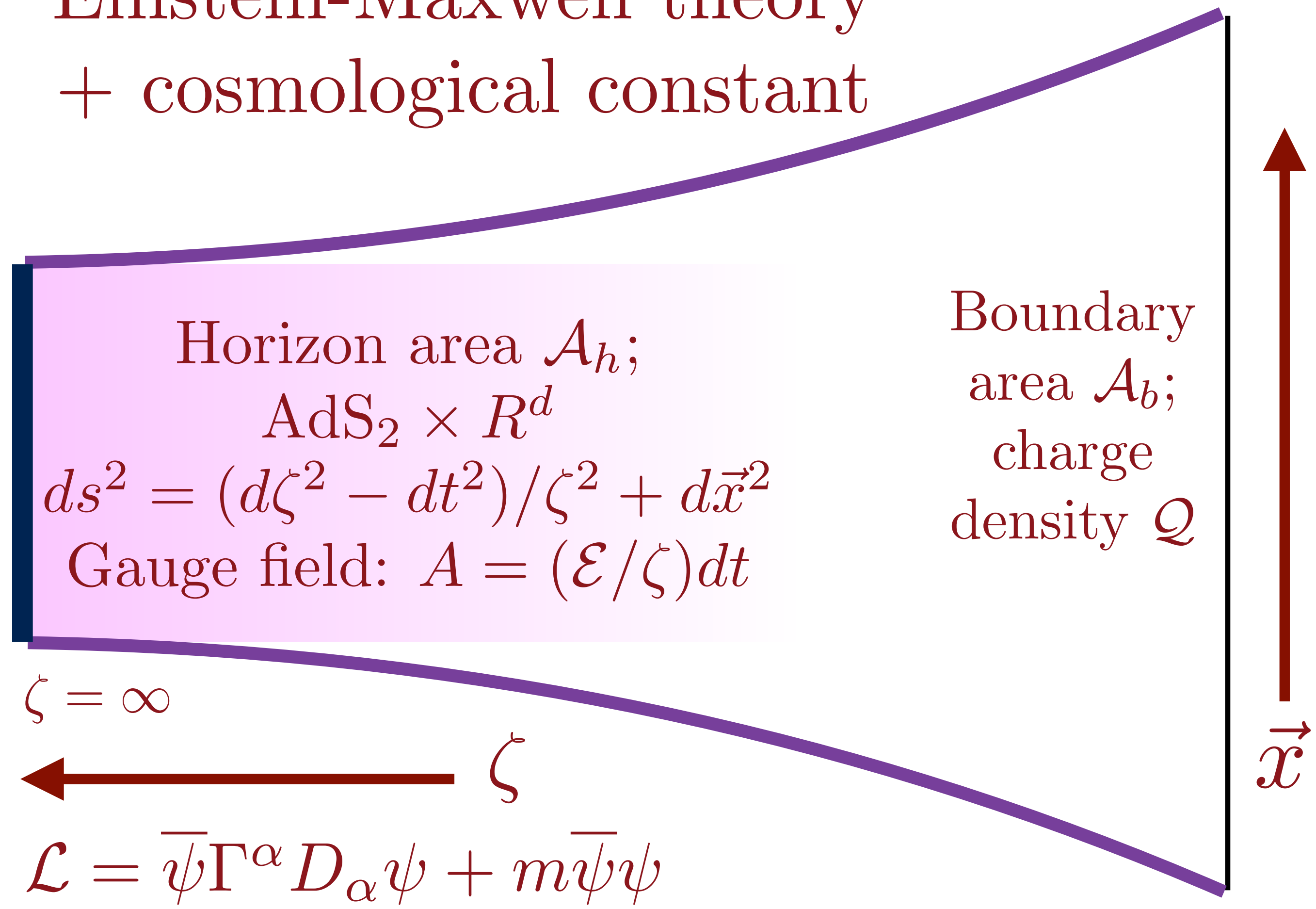

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Evidence for AdS_2 gravity dual of H

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S. Sachdev, arXiv:1506.05111

Quantum matter without quasiparticles

1. A solvable model of an ordinary metal
2. A solvable model of a strange metal
3. Holography and charged black holes
4. The (slightly less) strange metal in graphene

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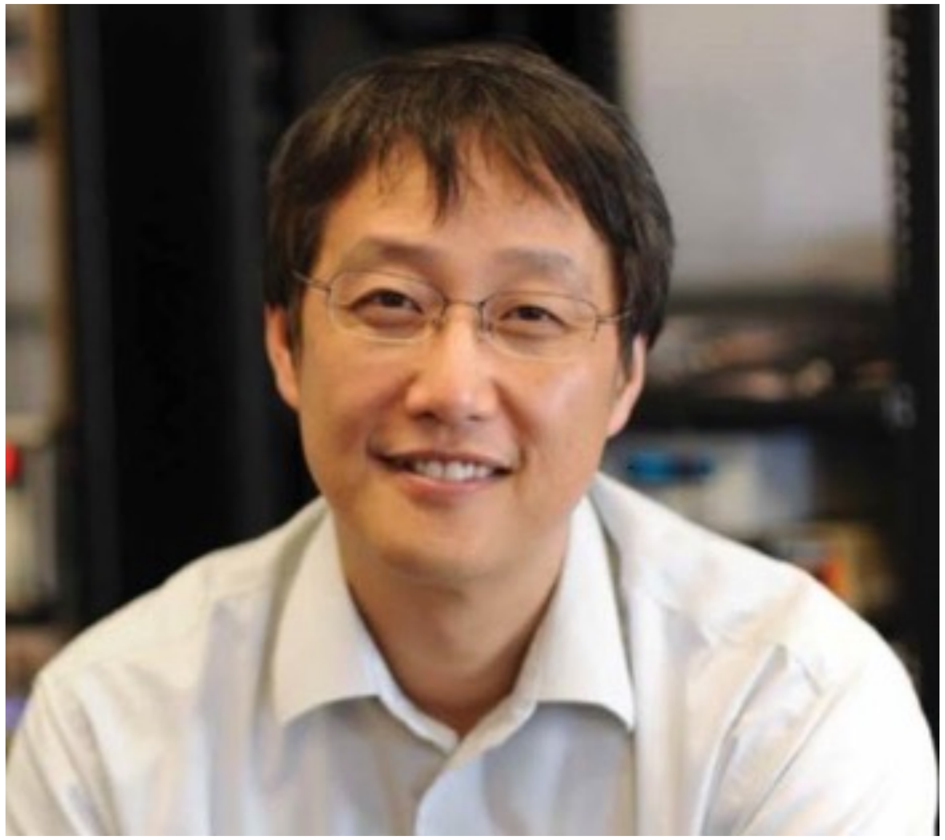
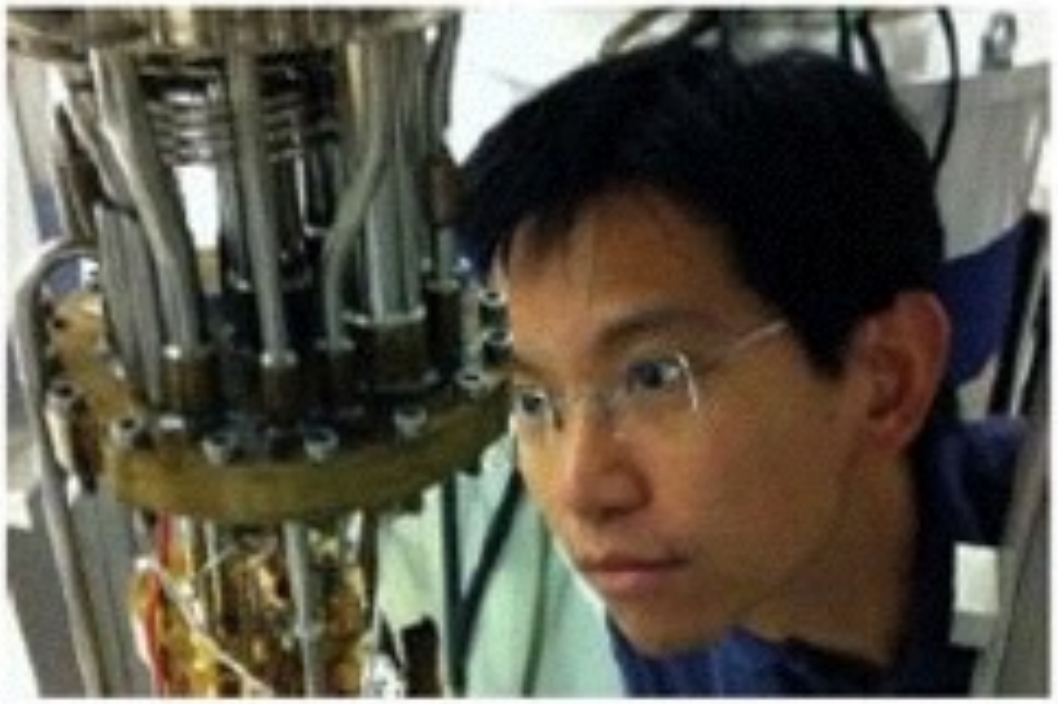
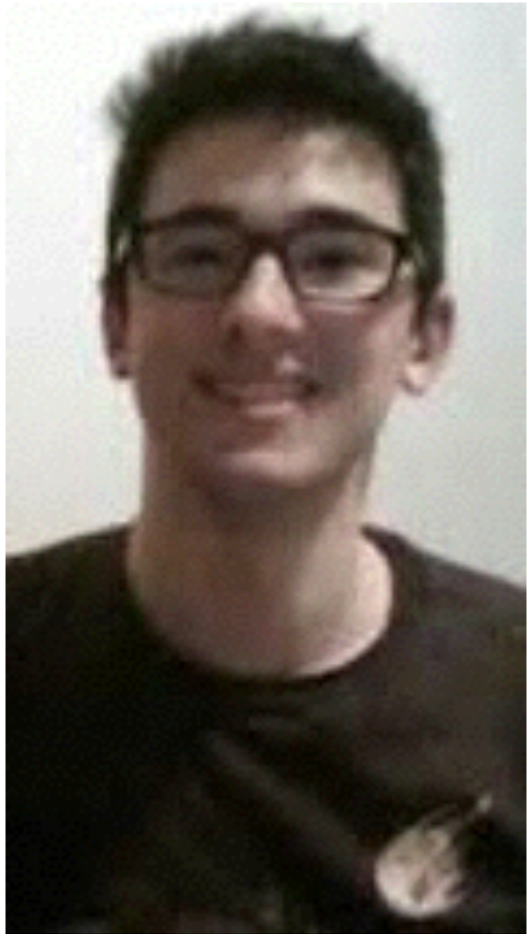


Philip Kim

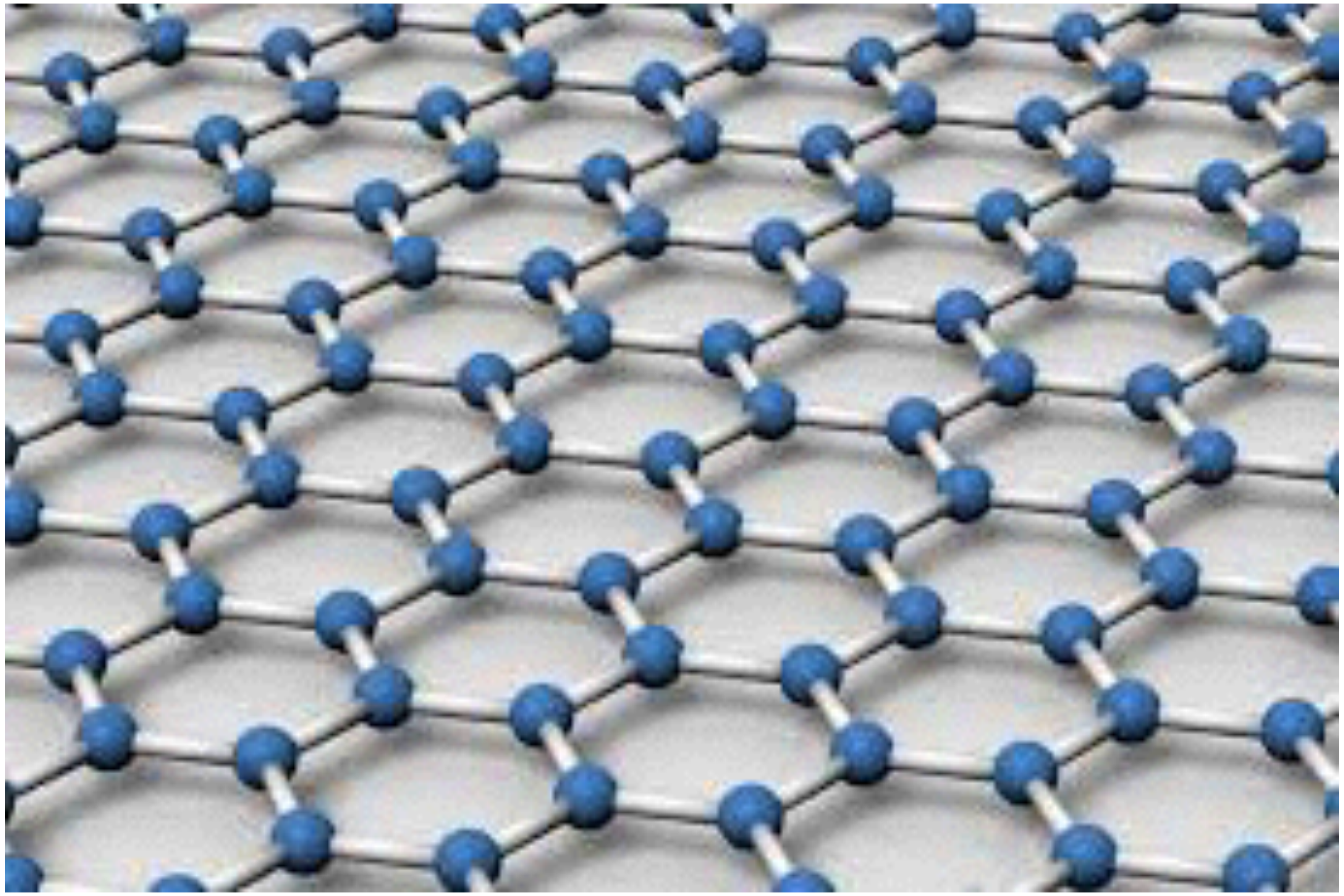
Jesse Crossno

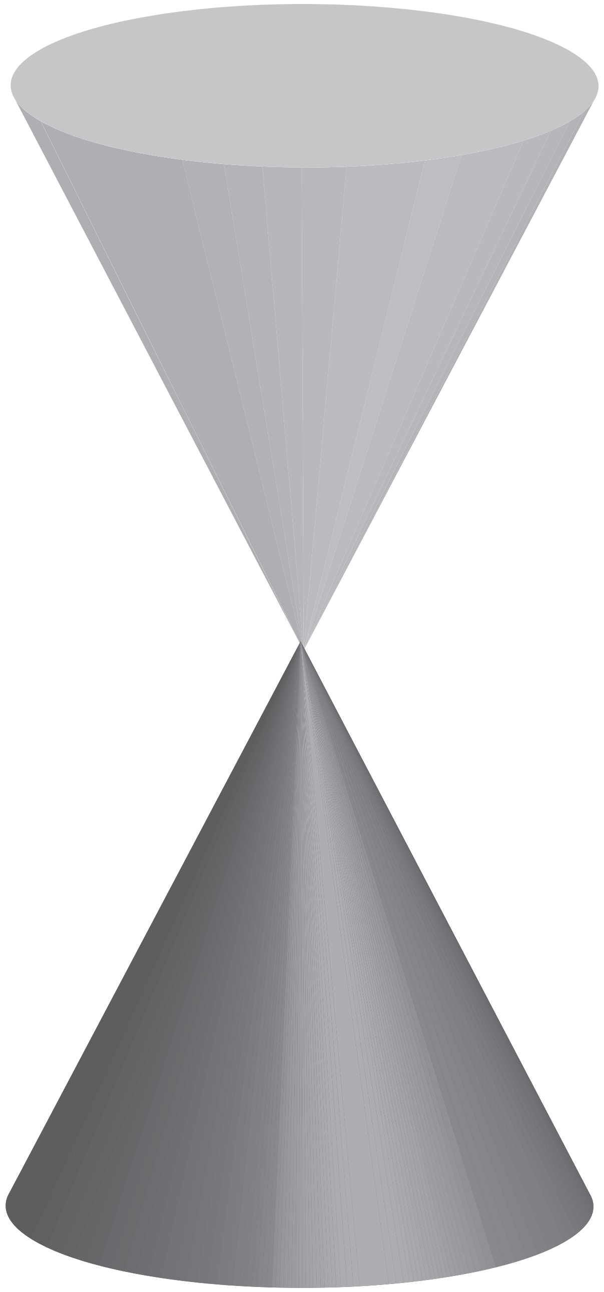


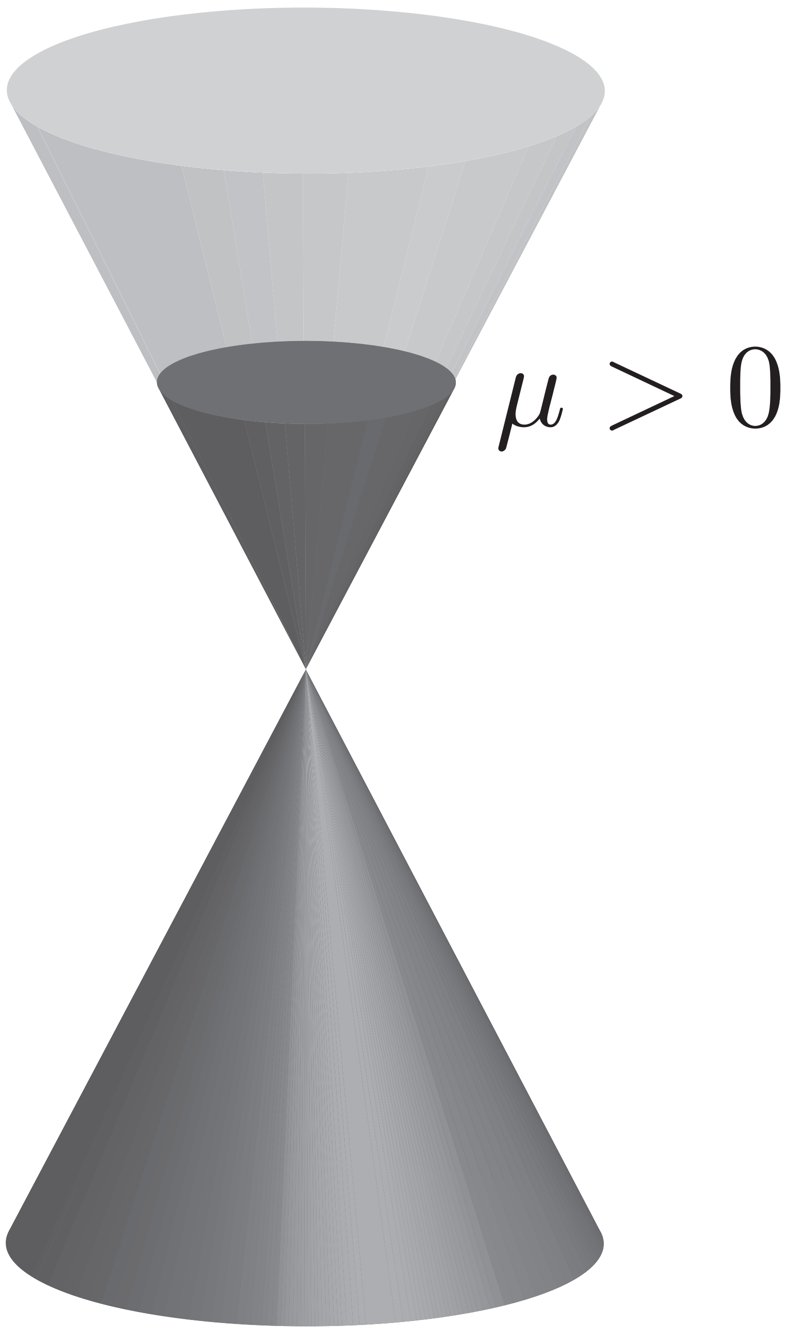
Kin Chung Fong



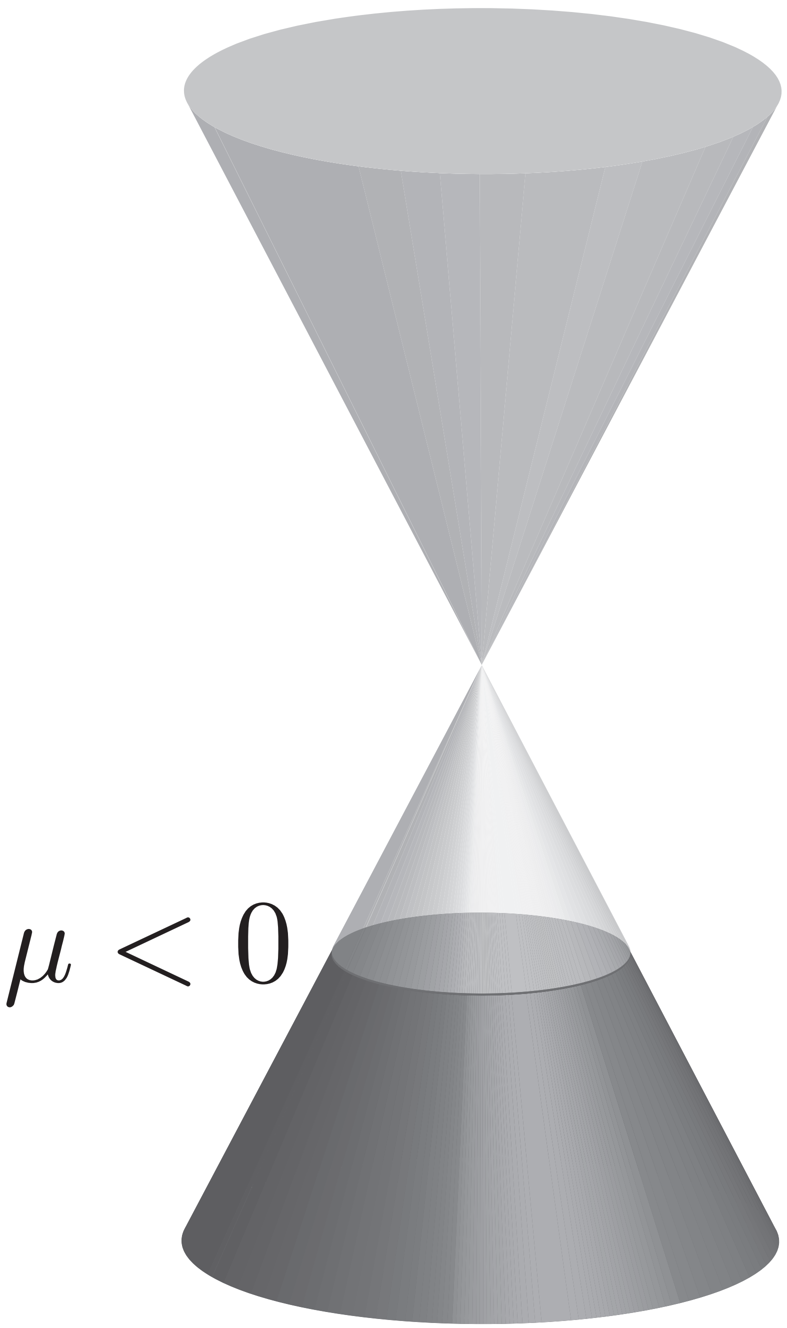
Andrew Lucas



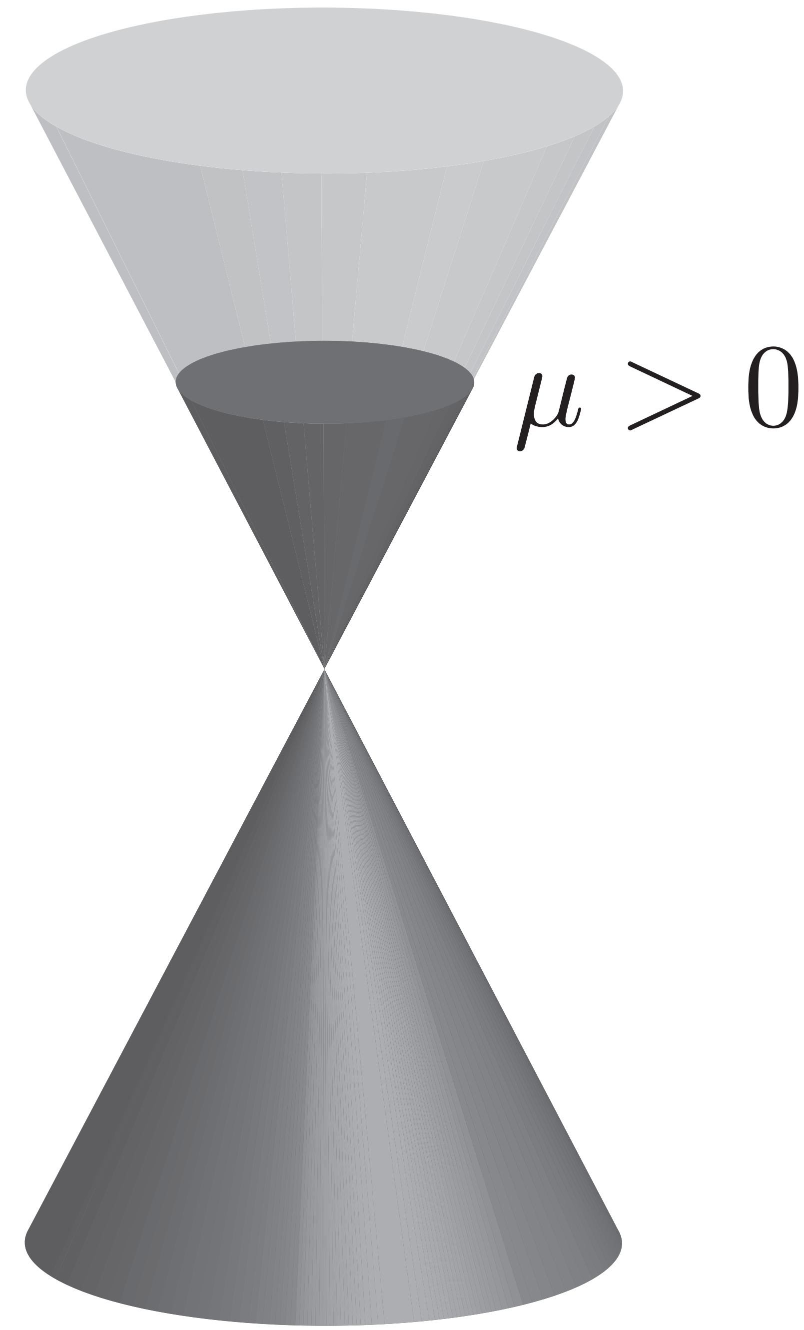




Electron
Fermi surface



Hole
Fermi surface



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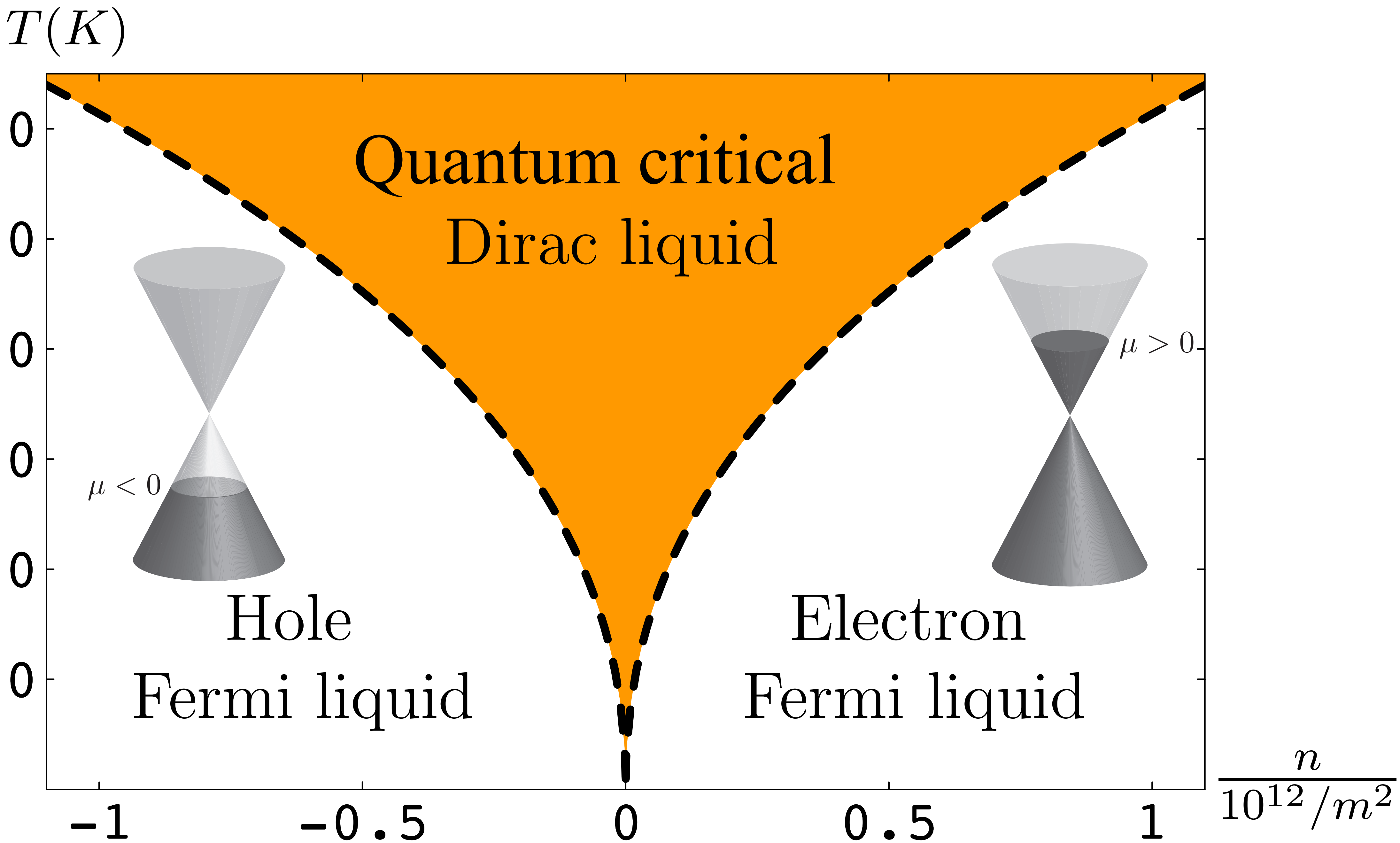

$T(K)$
600
500
400
300
200
100
Quantum critical
Dirac liquid
$\mu < 0$
$\mu > 0$
Hole
Fermi liquid
Electron
Fermi liquid
-1
-0.5
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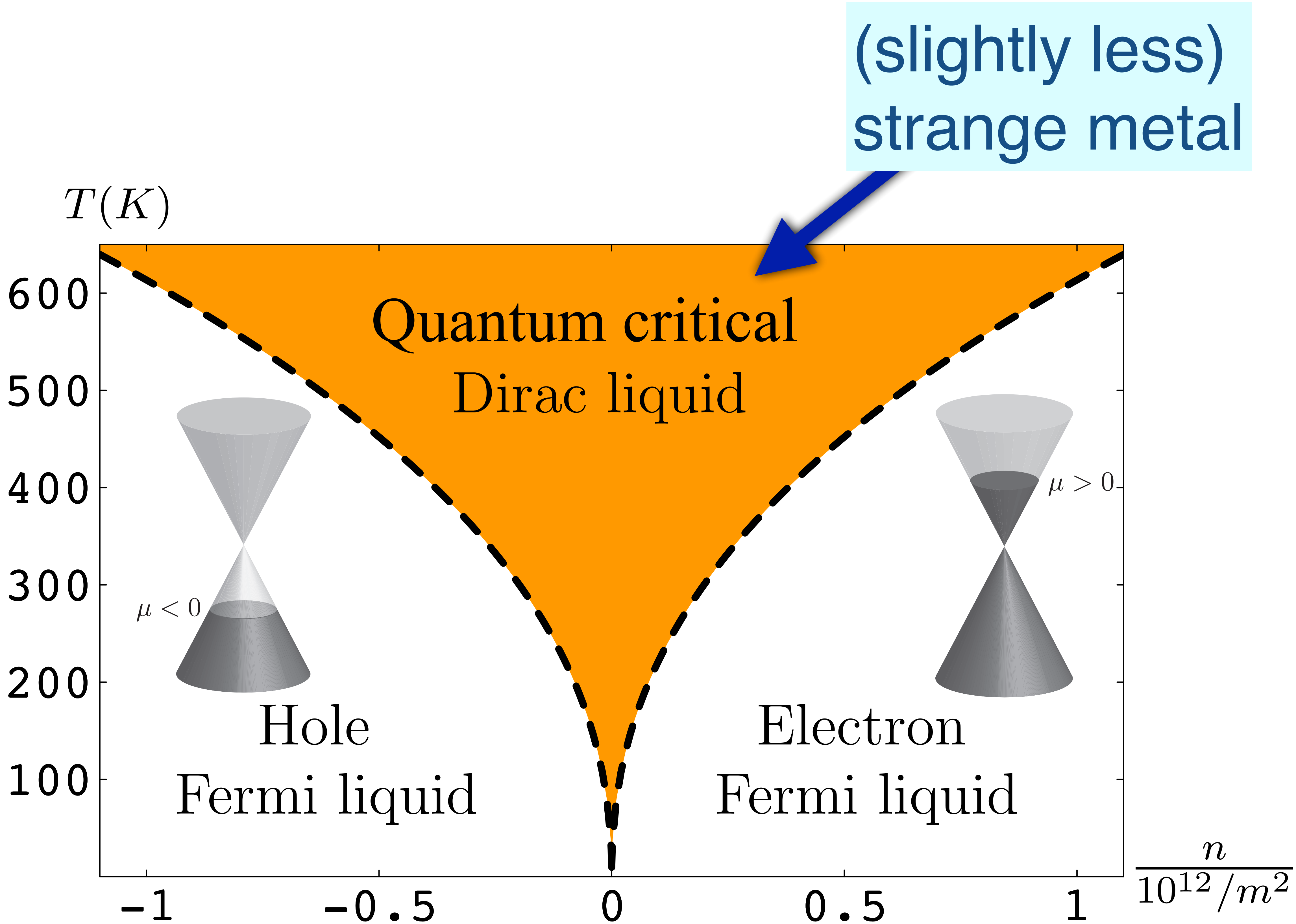
$\frac{n}{10^{12}/m^2}$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Transport in Strange Metals

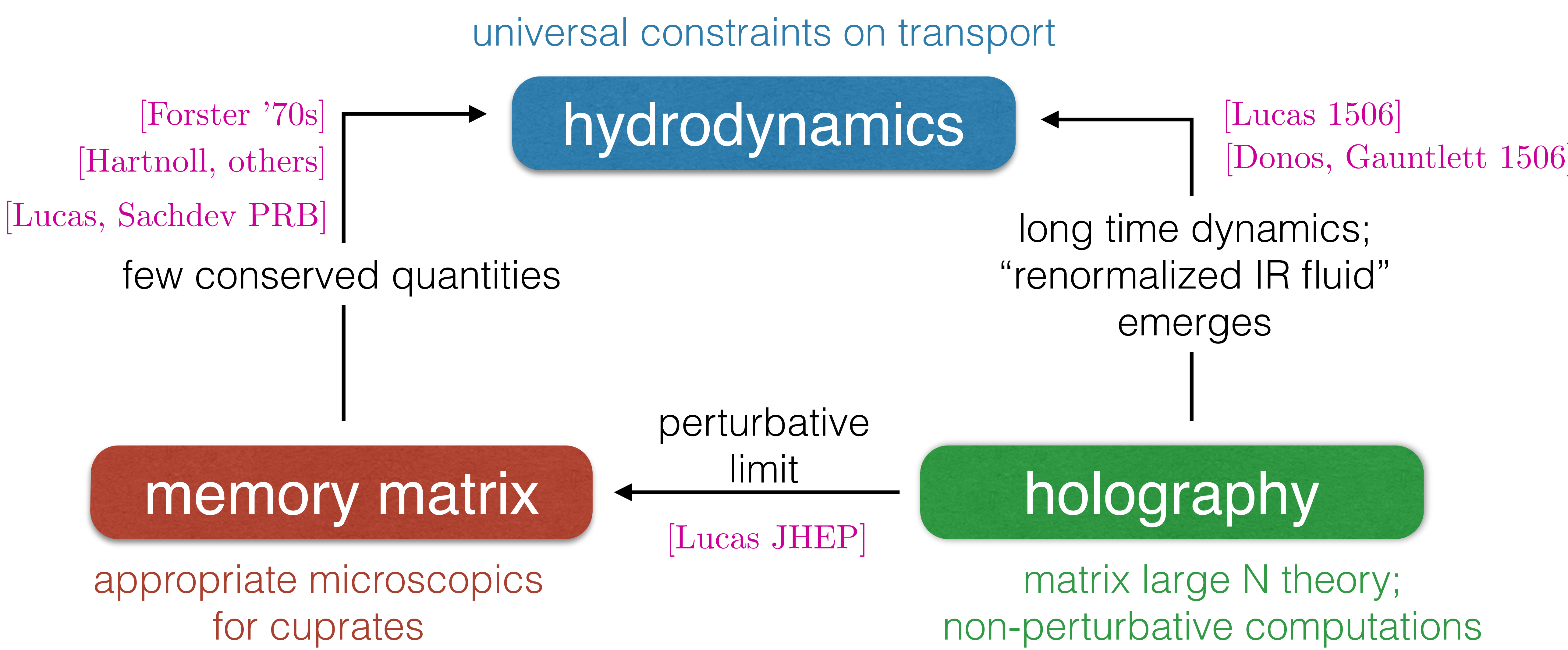

figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Transport in Strange Metals

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by

$$L = \frac{\pi^2 k_B^2}{3e^2}$$

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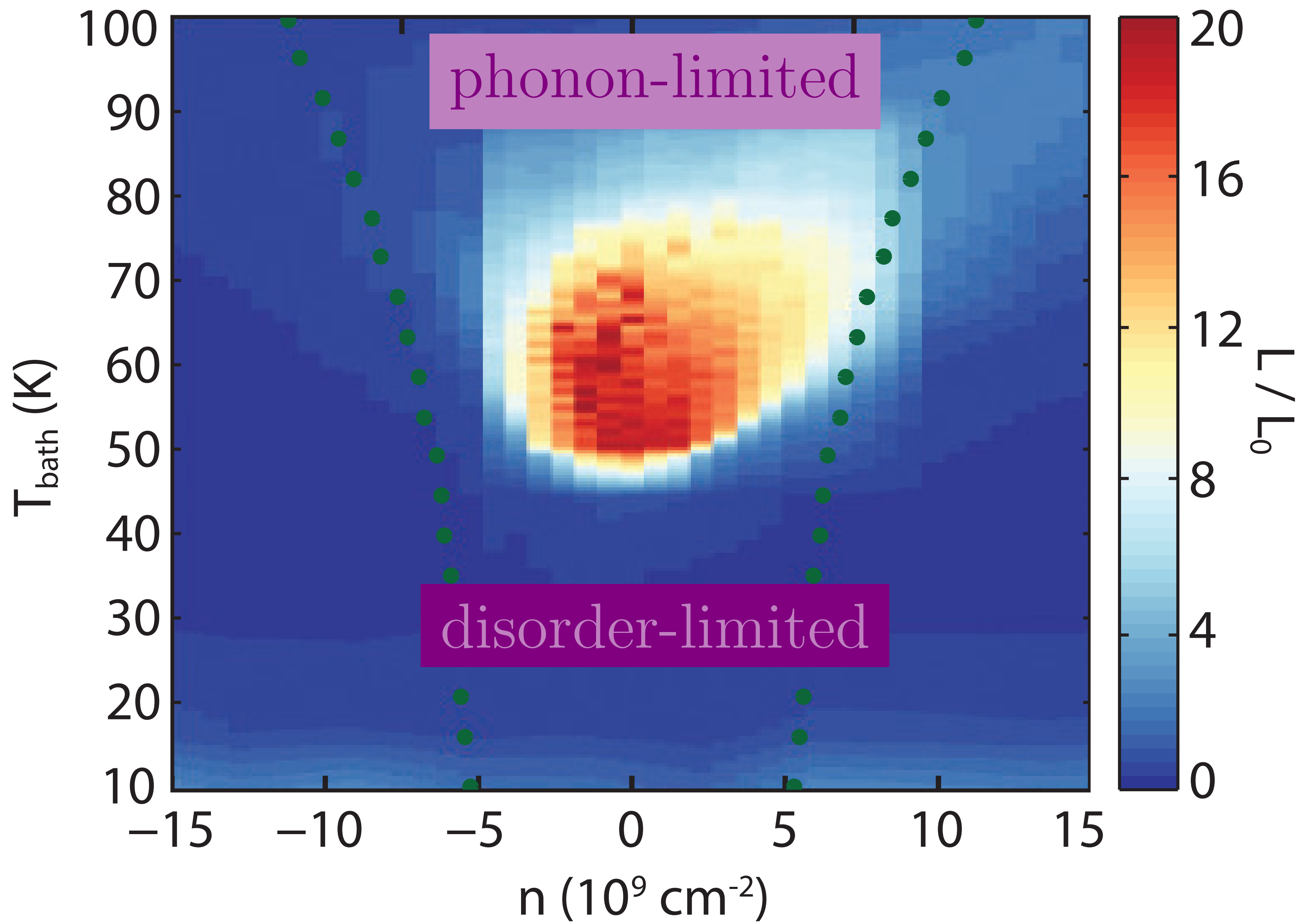
In contrast, for a strange metal with a "relativistic" Hamiltonian, L diverges as the charge density $\mathcal{Q} \to 0$, and then the impurity momentum relaxation time $\tau_{\rm imp} \to \infty$:

$$L = \frac{\mathcal{H}\tau_{\rm imp}}{T^2\sigma_Q} \frac{1}{\left(1 + \mathcal{Q}^2\tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2},$$

where $\mathcal{H}$ is the enthalpy density. This divergence happens because at $\mathcal{Q} = 0$, κ diverges as $\tau_{\rm imp} \to \infty$, while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Wiedemann-Franz Law Violations in Experiment



[Crossno *et al, submitted*]

Dirac Fluid in Graphene

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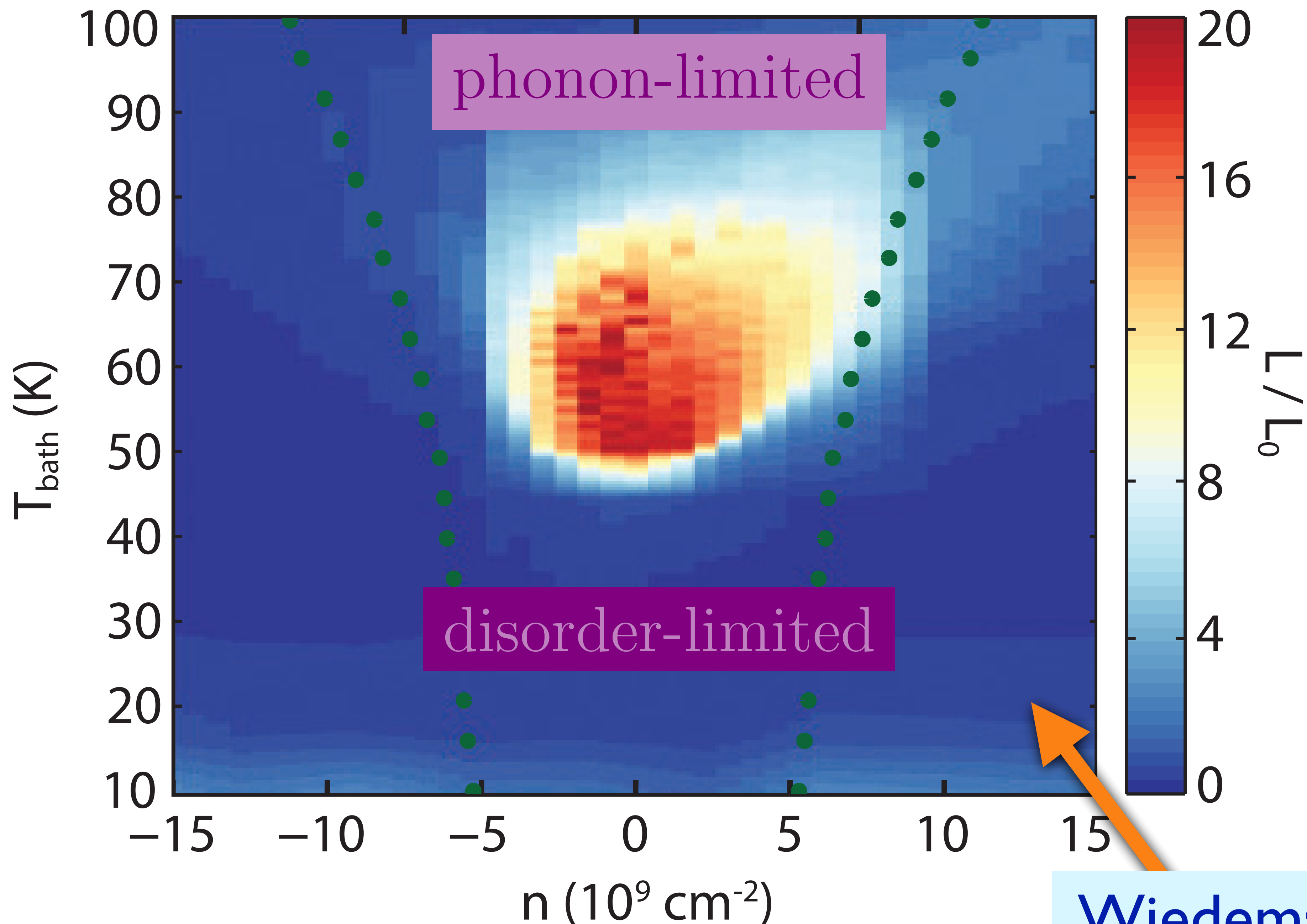

Wiedemann-Franz obeyed

[Crossno *et al, submitted*]

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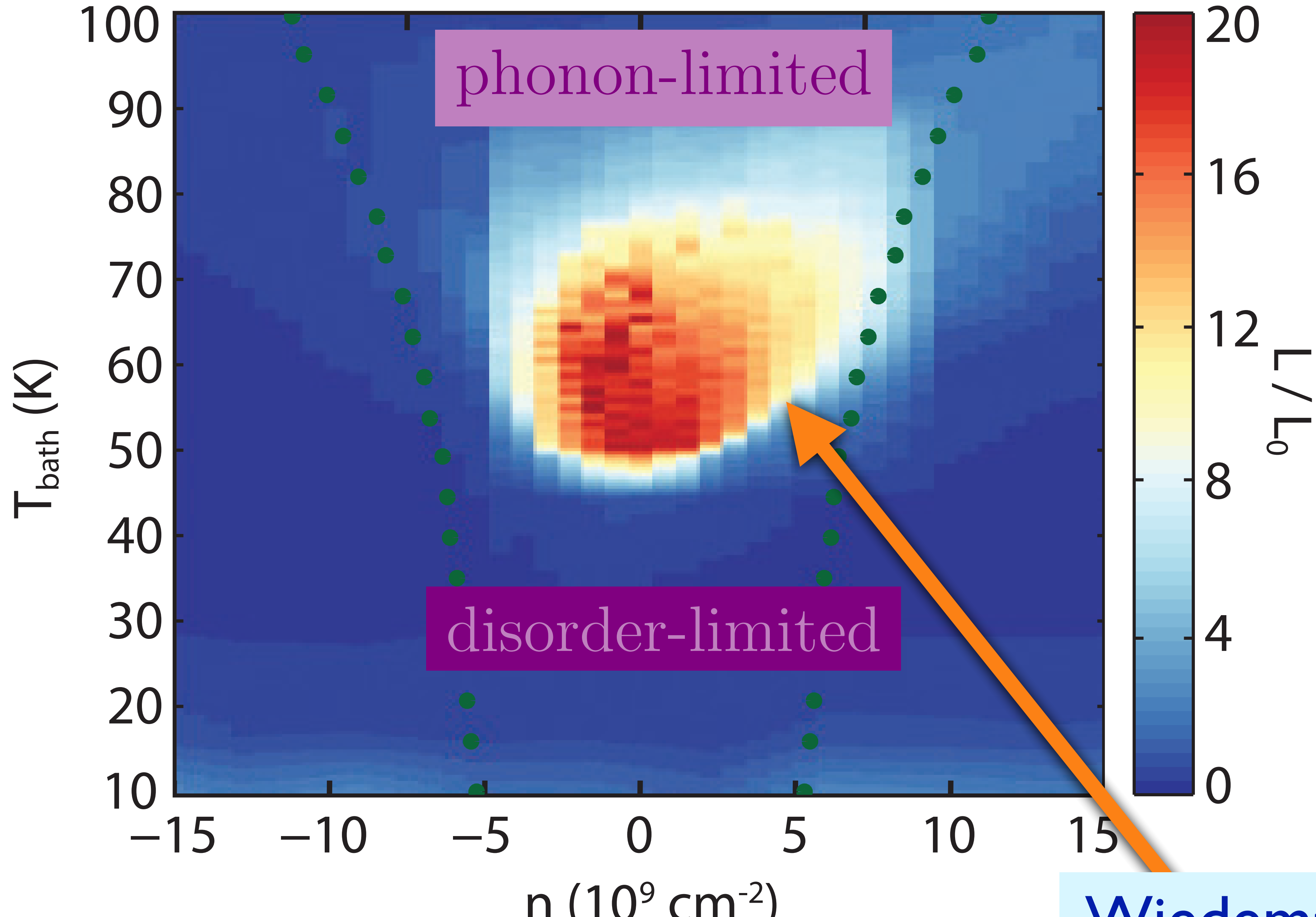


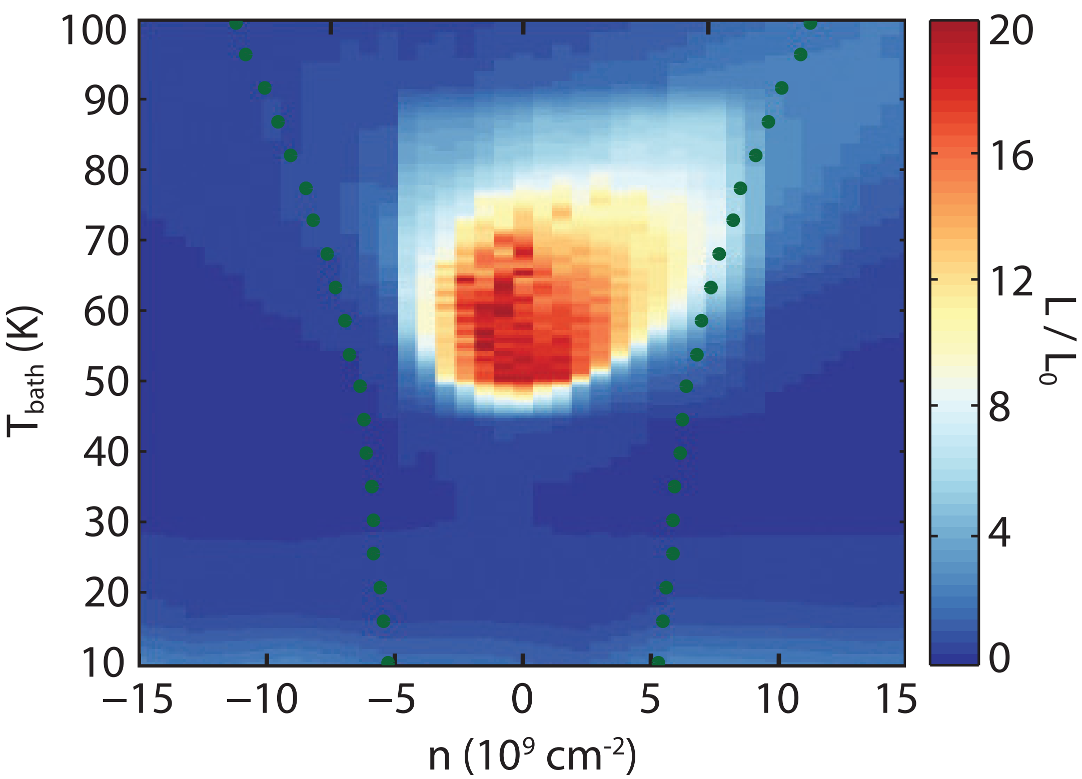
Wiedemann-Franz violated !

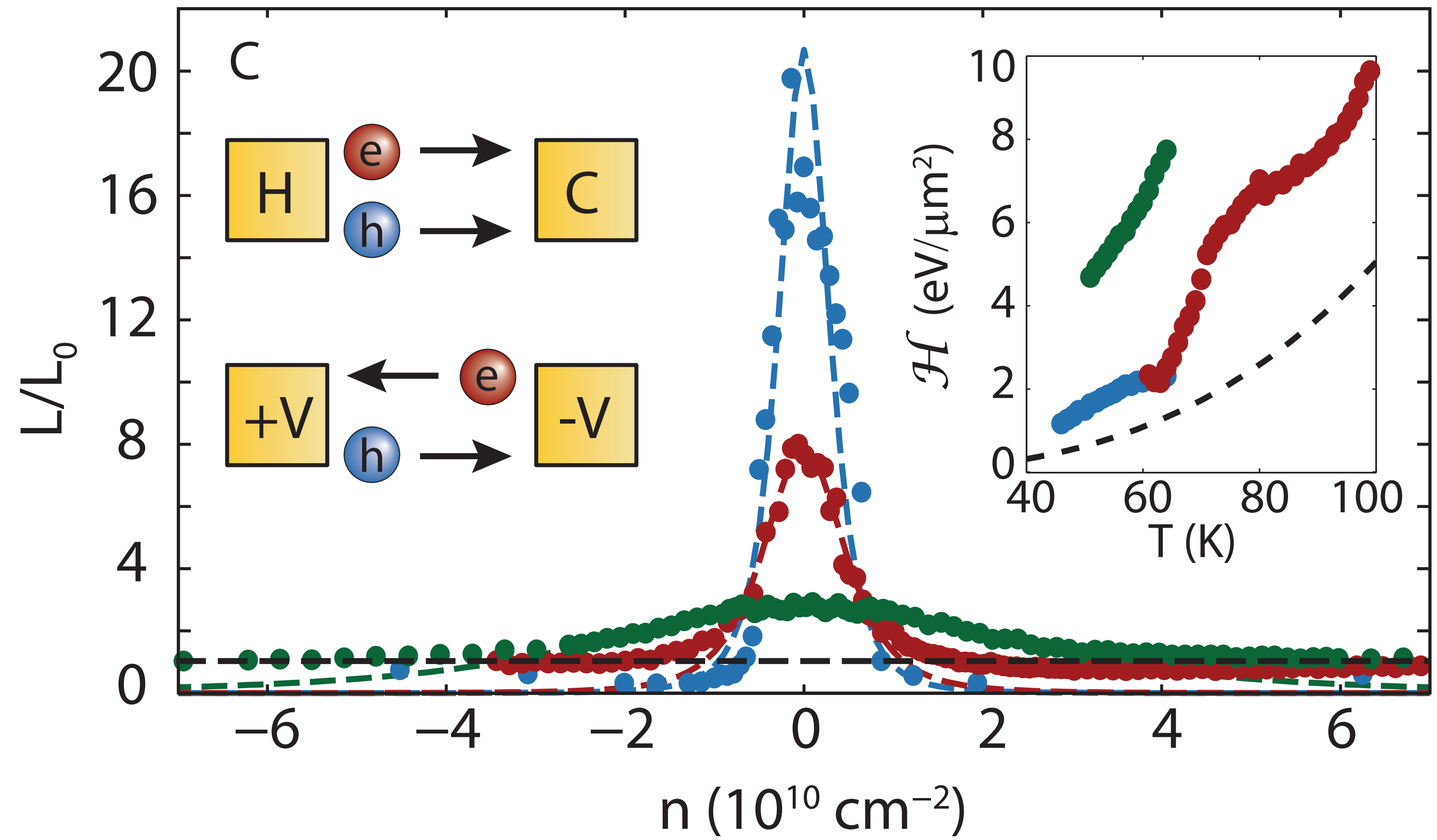
[Crossno *et al, submitted*]

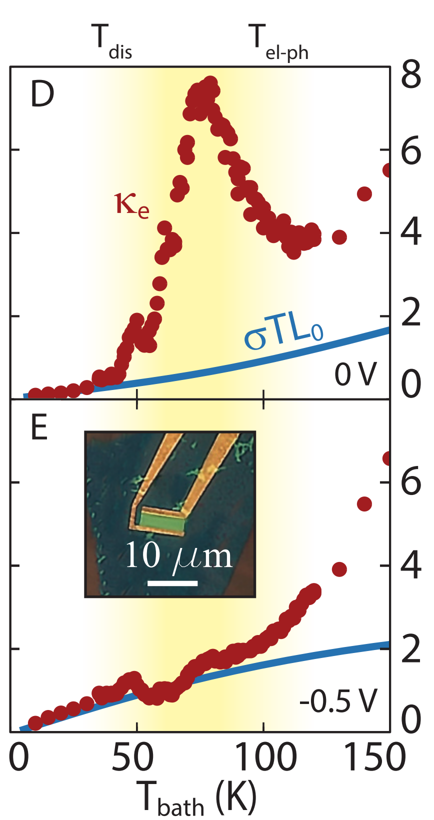
(submitted)

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2,*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] and Kin Chung Fong[5,†]







$$\text{Lorentz ratio } L = \kappa/(T\sigma) = \frac{\mathcal{H}\tau_{\text{imp}}}{T^2\sigma_Q}\frac{1}{\left(1+\mathcal{Q}^2\tau_{\text{imp}}/(\mathcal{H}\sigma_Q)\right)^2}$$

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