

Unveiling the order of the high temperature superconductors

Harvard University
March 31, 2014

Subir Sachdev

Talk online: sachdev.physics.harvard.edu



Theorists at Harvard



Max Metlitski



Rolando
La Placa



Andrea Allais



Johannes
Bauer



Debanjan
Chowdhury



Jay Deep
Sau

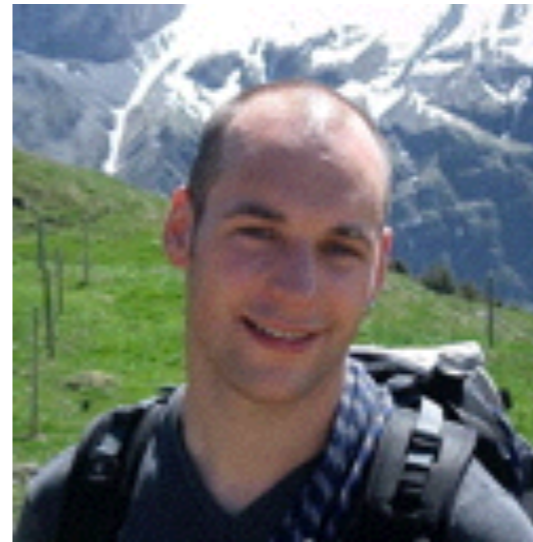
Experimentalists at Harvard



Yang He



Yi Yin



Martin Zech



Anjan
Soumyanarayanan



Ilija Zelkovic



Michael Yee



Jennifer Hoffman

Experimentalists at Cornell



Kazuhiro Fujita
Cornell/ BNL



Mohammad Hamidian
Cornell / BNL

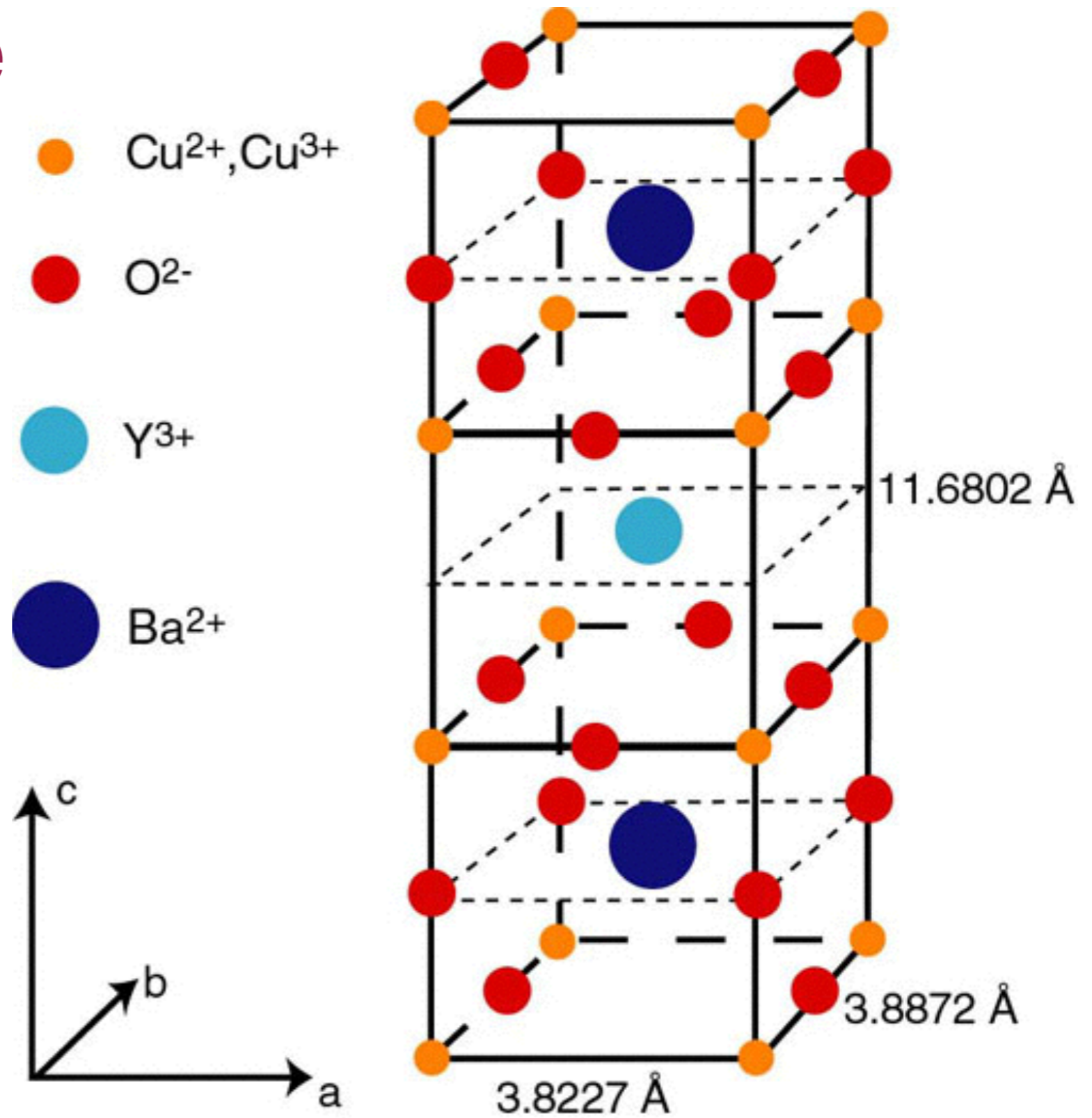


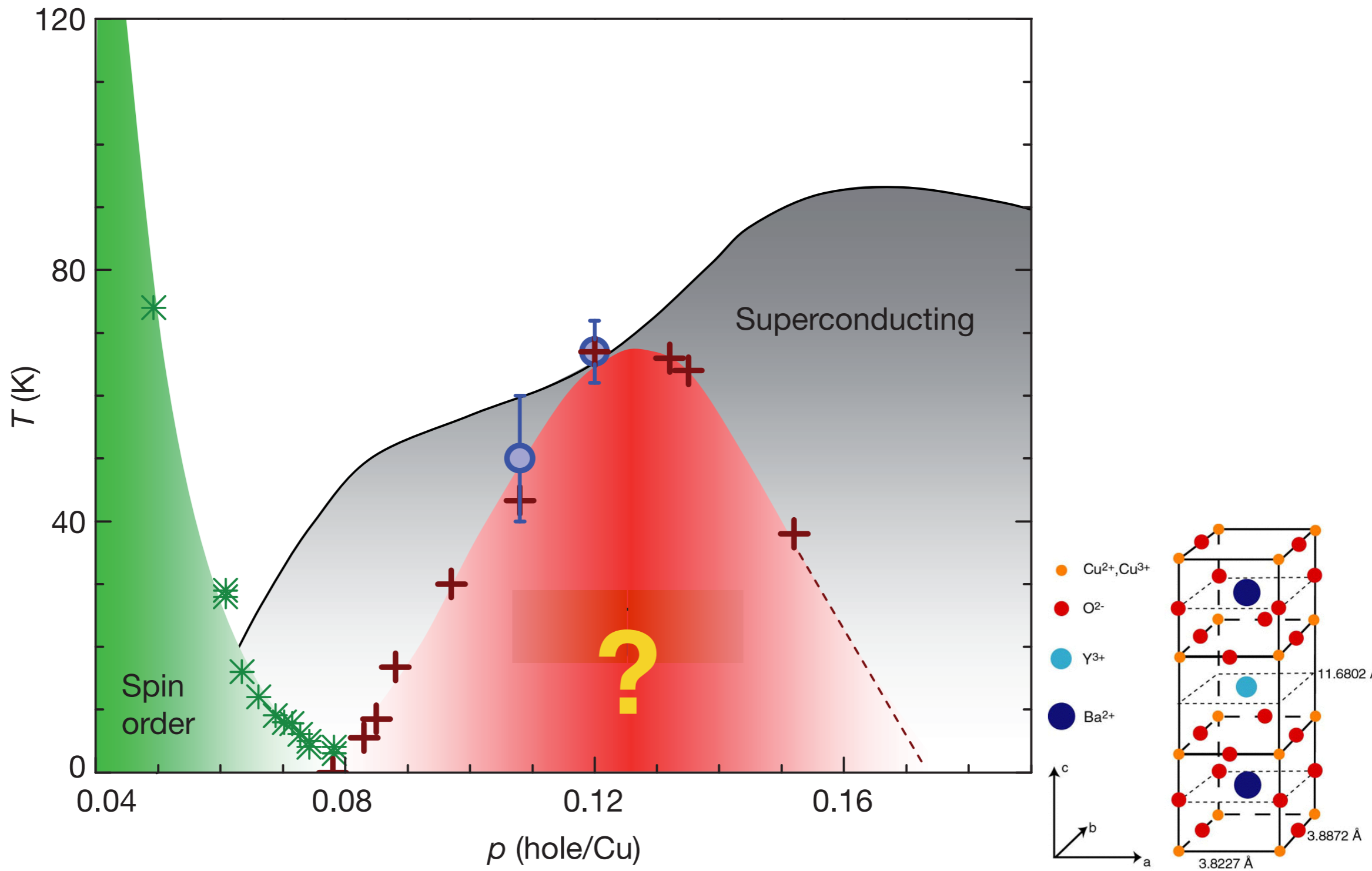
Stephen Edkins
Cornell / St Andrews



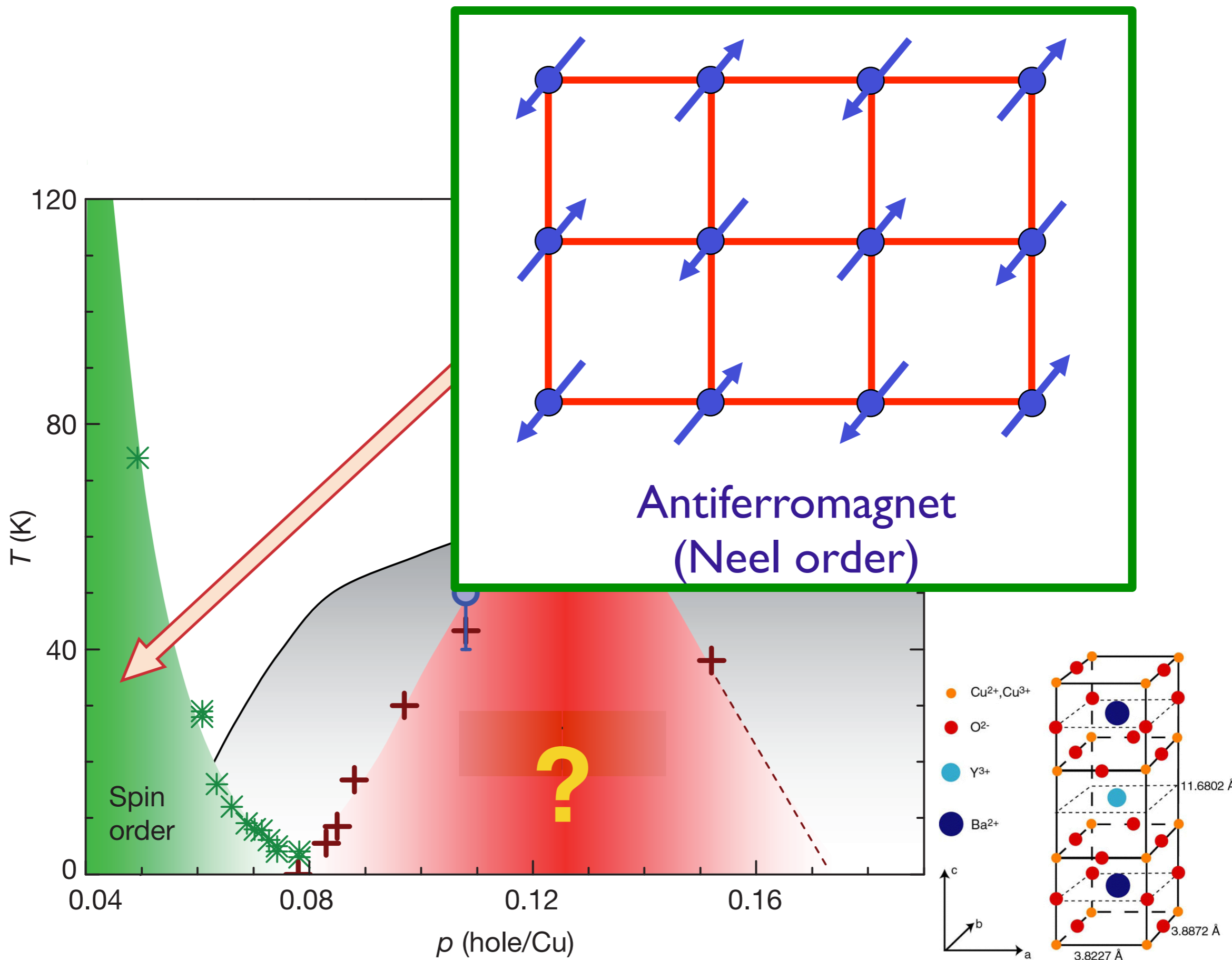
J. C. Seamus Davis

High temperature superconductors

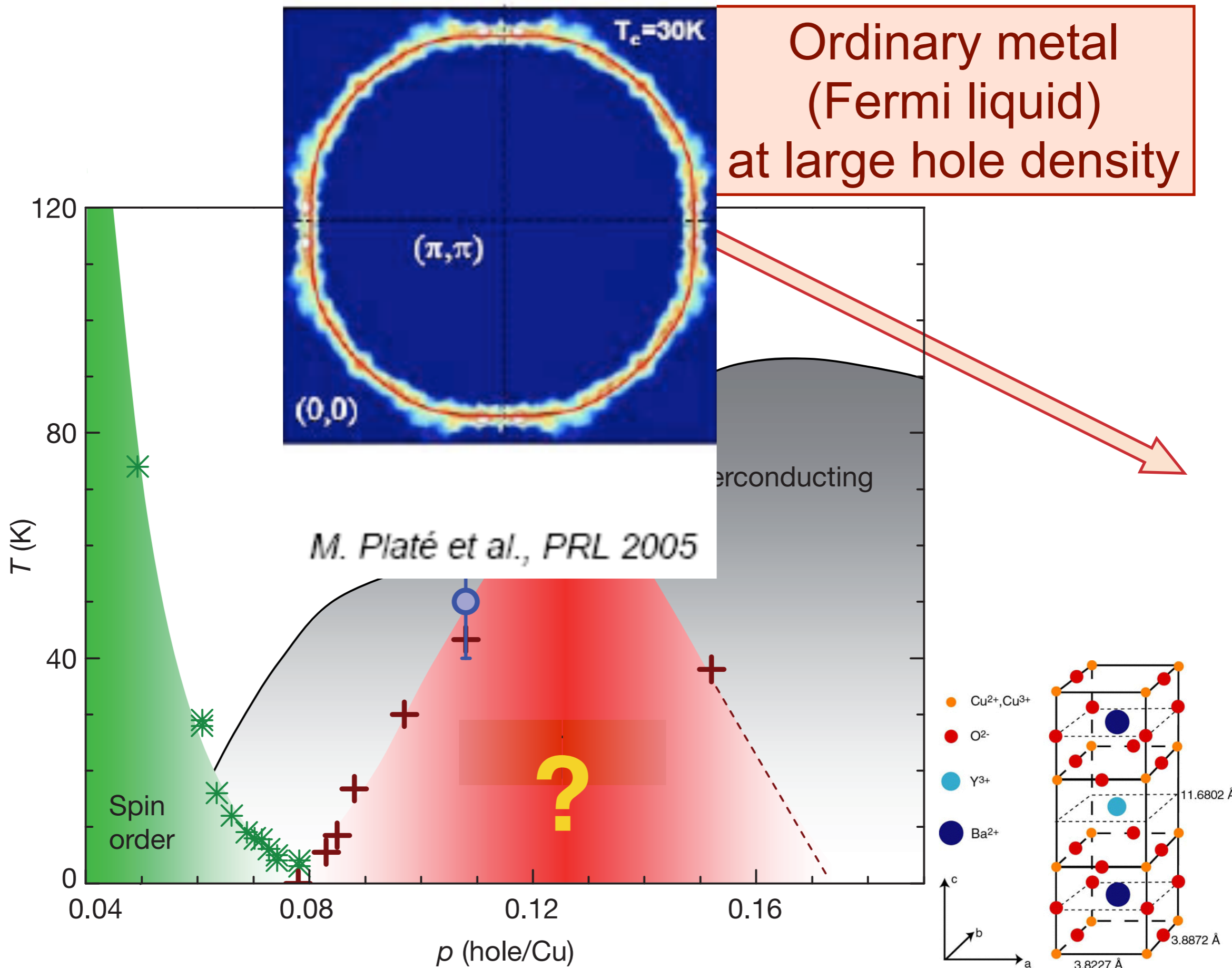




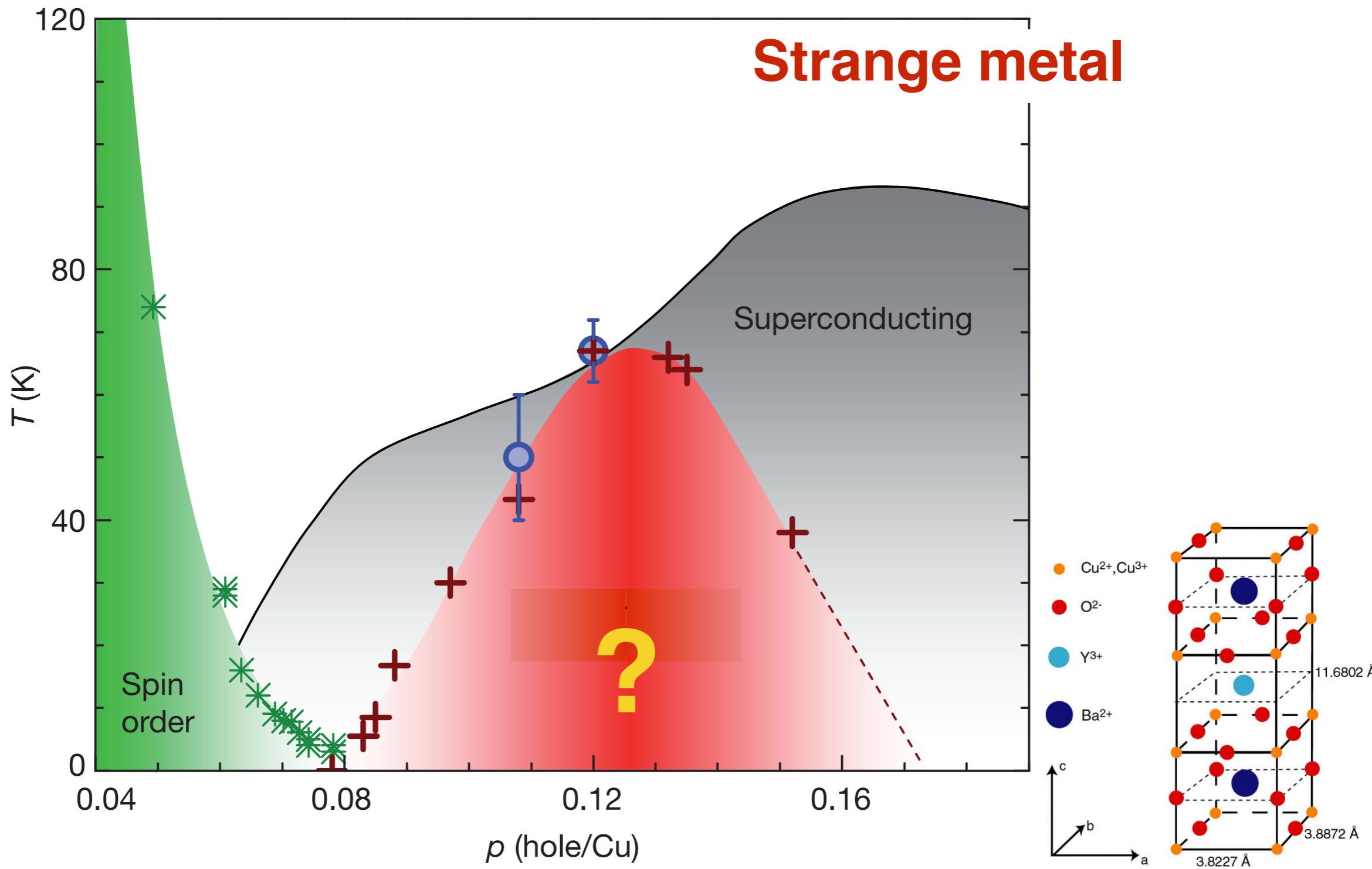
T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



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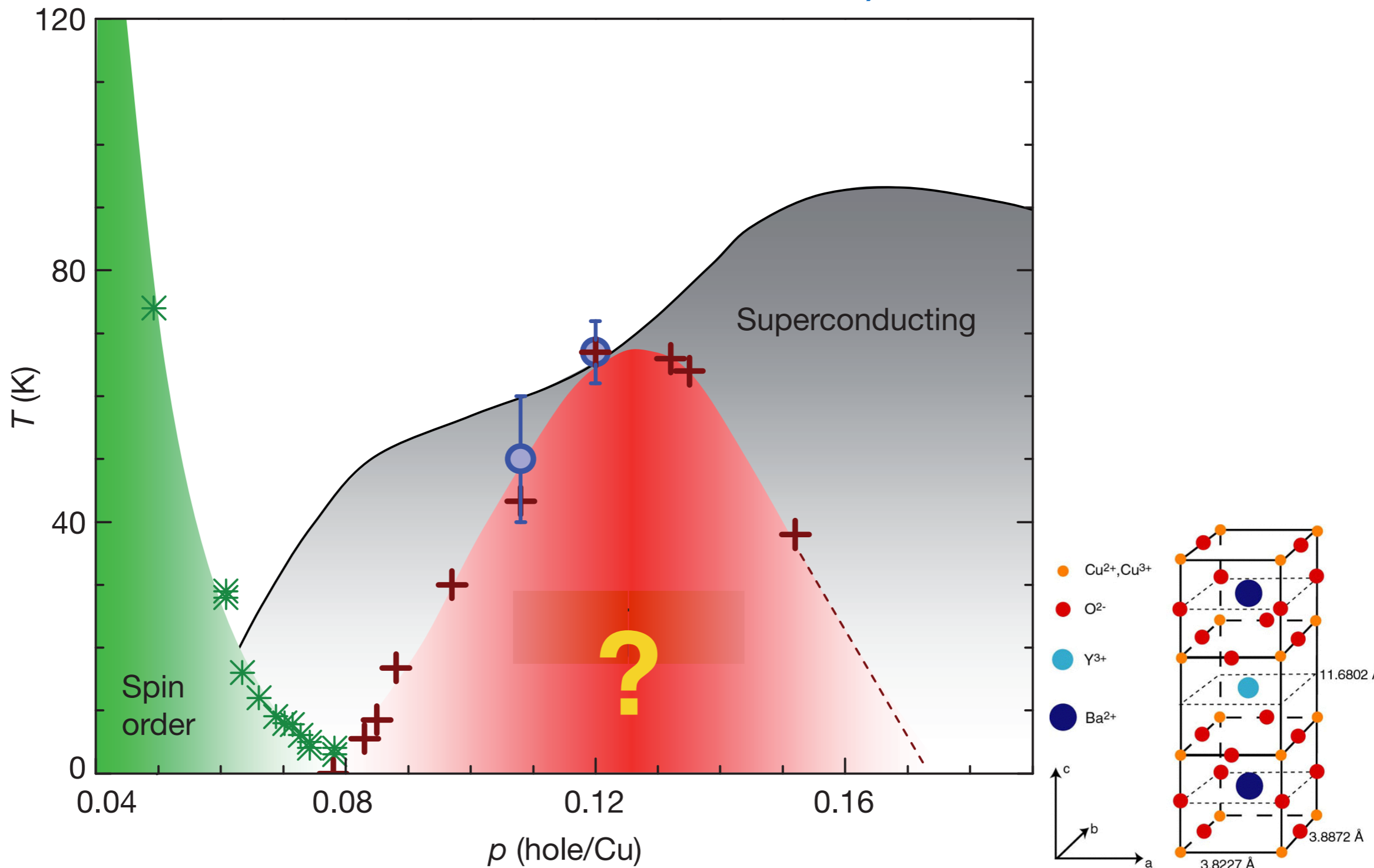


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(In the La-based superconductors, the region with magnetic order overlaps with the intermediate region - I will not discuss these materials)



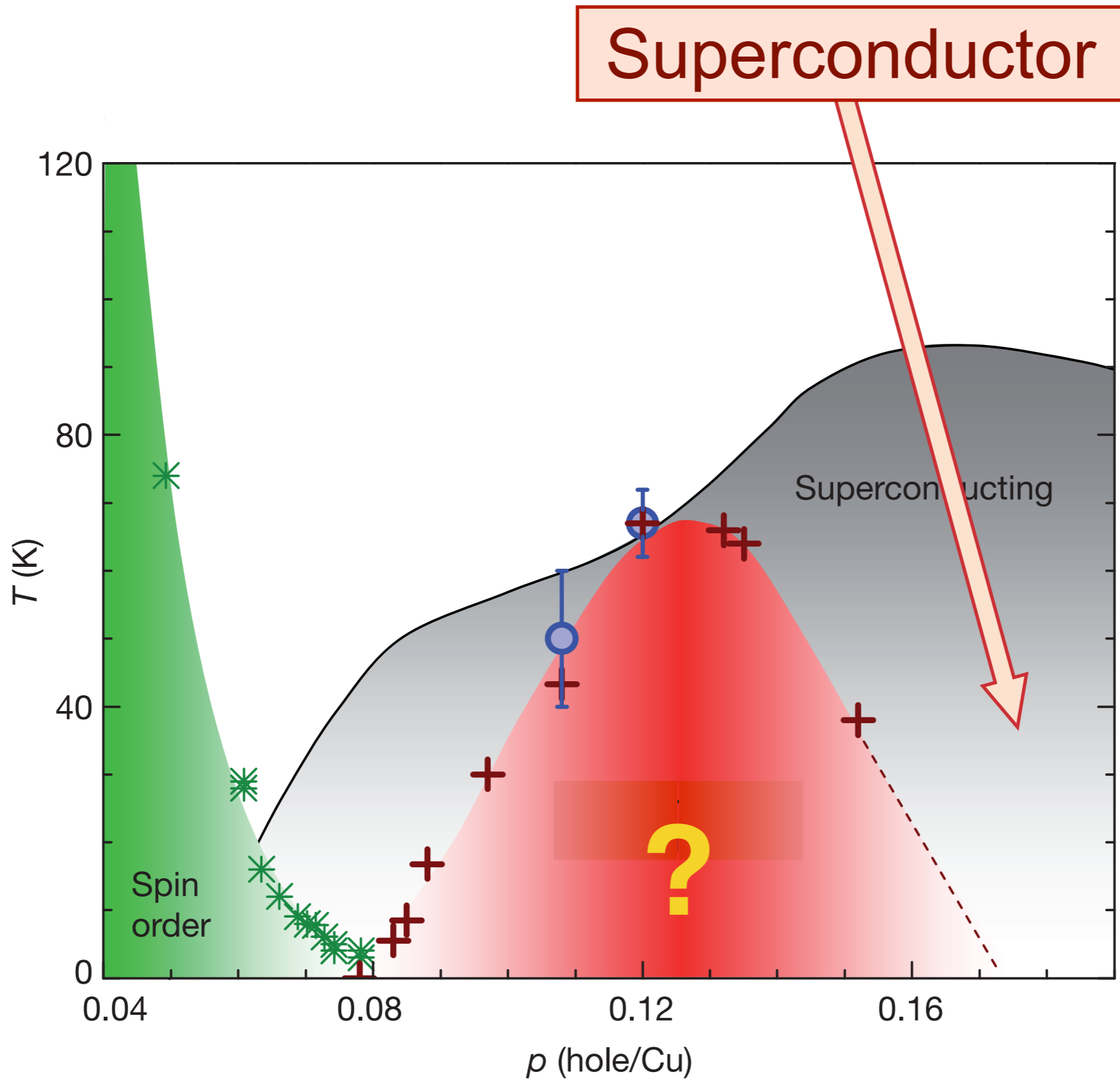
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Outline

1. Antiferromagnetism in metals and *d*-wave superconductivity
2. Low hole density state:
d-wave bond order
3. Evolution of Fermi surface

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Superconductivity: Bose condensation of Cooper pairs of electrons

$$\varepsilon^{\alpha\beta} \left\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\beta}^{\dagger}(\mathbf{r}_2) \right\rangle = \left[P(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{SC} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right)$$

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow} = -\varepsilon^{\downarrow\uparrow} = 1; \quad \varepsilon^{\uparrow\uparrow} = \varepsilon^{\downarrow\downarrow} = 0$$

Superconductivity: Bose condensation of Cooper pairs of electrons

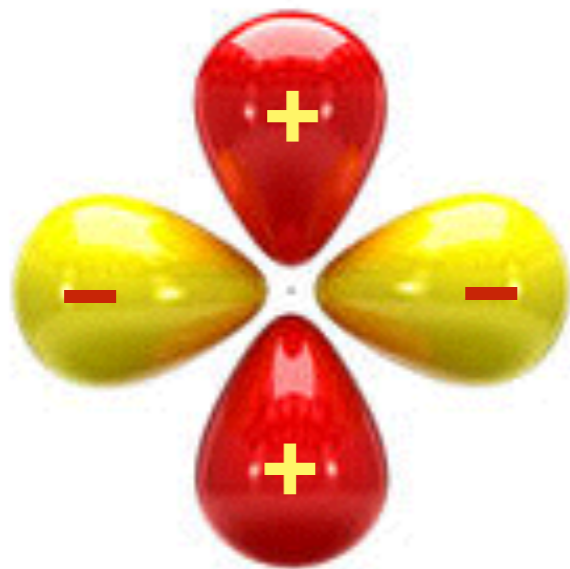
$$\varepsilon^{\alpha\beta} \left\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\beta}^{\dagger}(\mathbf{r}_2) \right\rangle = \left[P(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{SC} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right)$$

Nearly constant condensate wavefunction
(superconducting order parameter)

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Superconductivity: Bose condensation of Cooper pairs of electrons

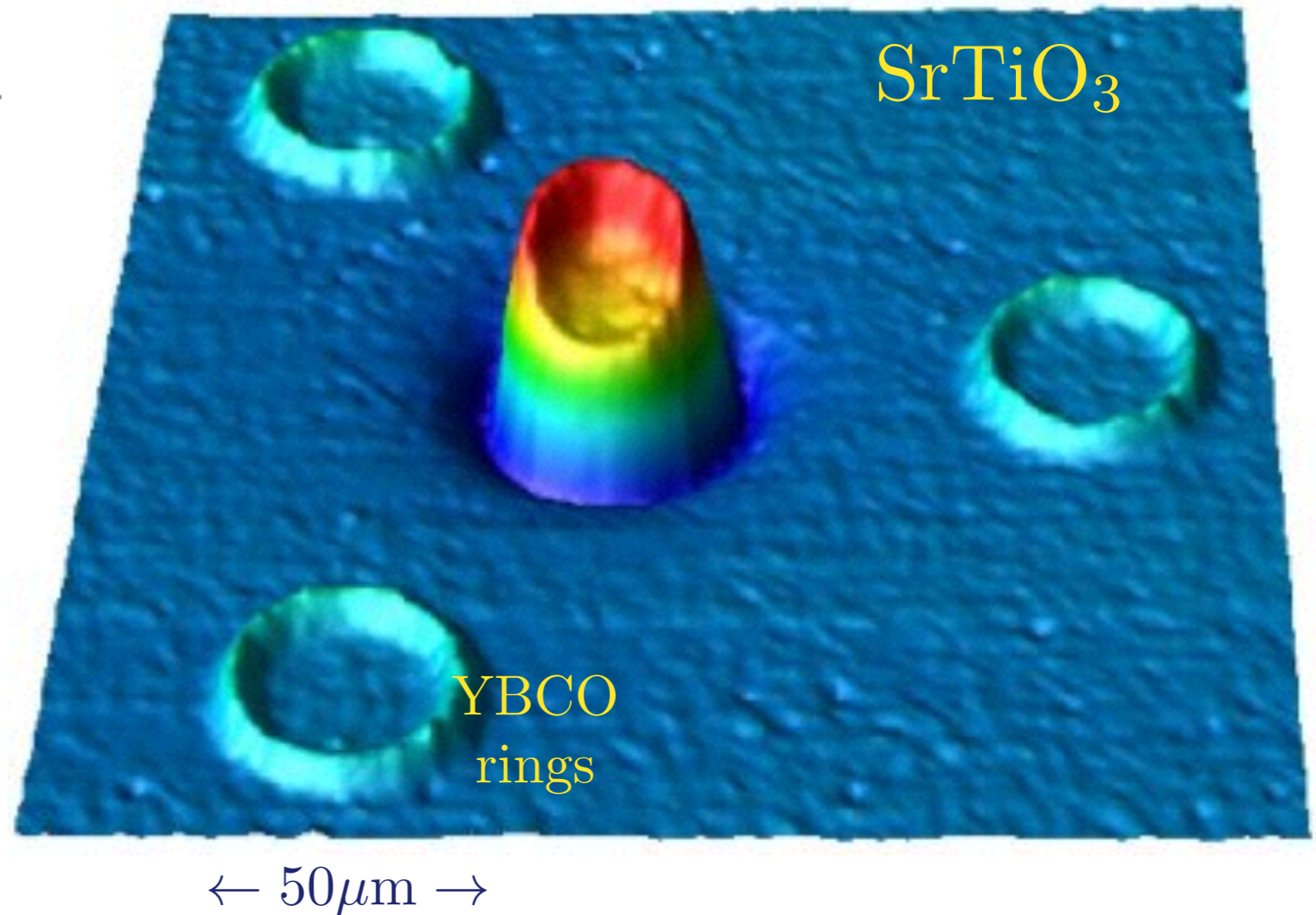
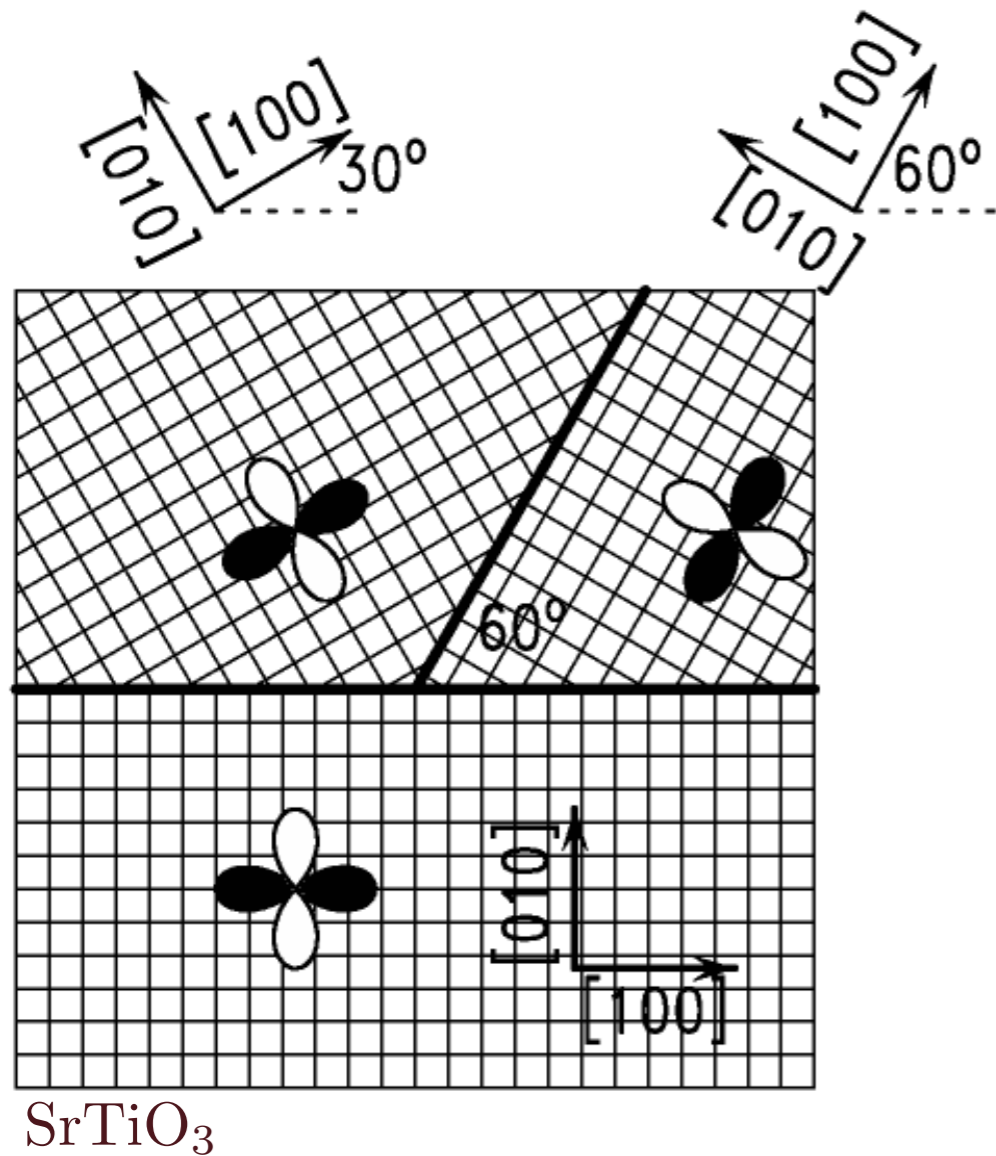
$$\varepsilon^{\alpha\beta} \left\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\beta}^{\dagger}(\mathbf{r}_2) \right\rangle = \left[P(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{SC} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right)$$



Internal Cooper-pair wavefunction.
Has *d*-wave form in cuprates

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow} = -\varepsilon^{\downarrow\uparrow} = 1; \quad \varepsilon^{\uparrow\uparrow} = \varepsilon^{\downarrow\downarrow} = 0$$

Phase-sensitive measurement of the *d*-wave symmetry of Cooper pairs



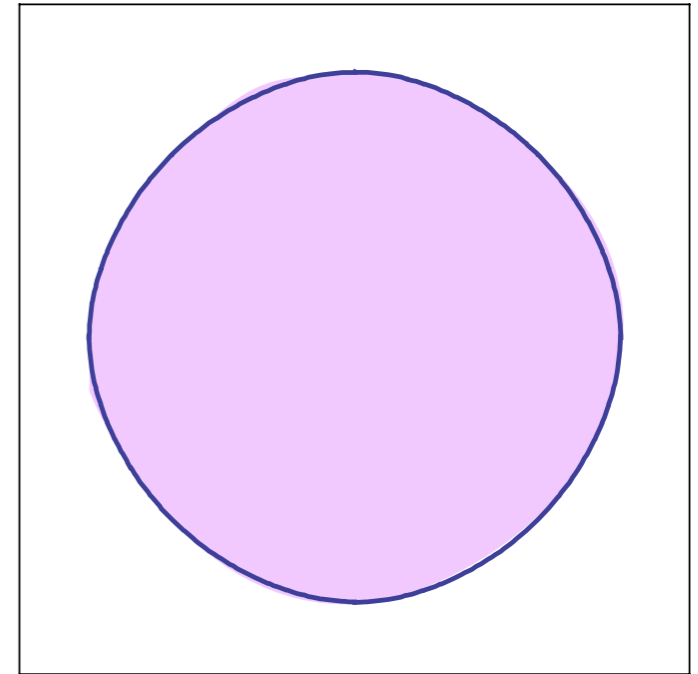
Pairing Symmetry and Flux Quantization in a Tricrystal Superconducting Ring of YBa₂Cu₃O_{7- δ}

C. C. Tsuei, J. R. Kirtley, C. C. Chi,* Lock See Yu-Jahnes, A. Gupta, T. Shaw, J. Z. Sun, and M. B. Ketchen
IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598

Phys. Rev. Lett. **73**, 593 (1994)

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface

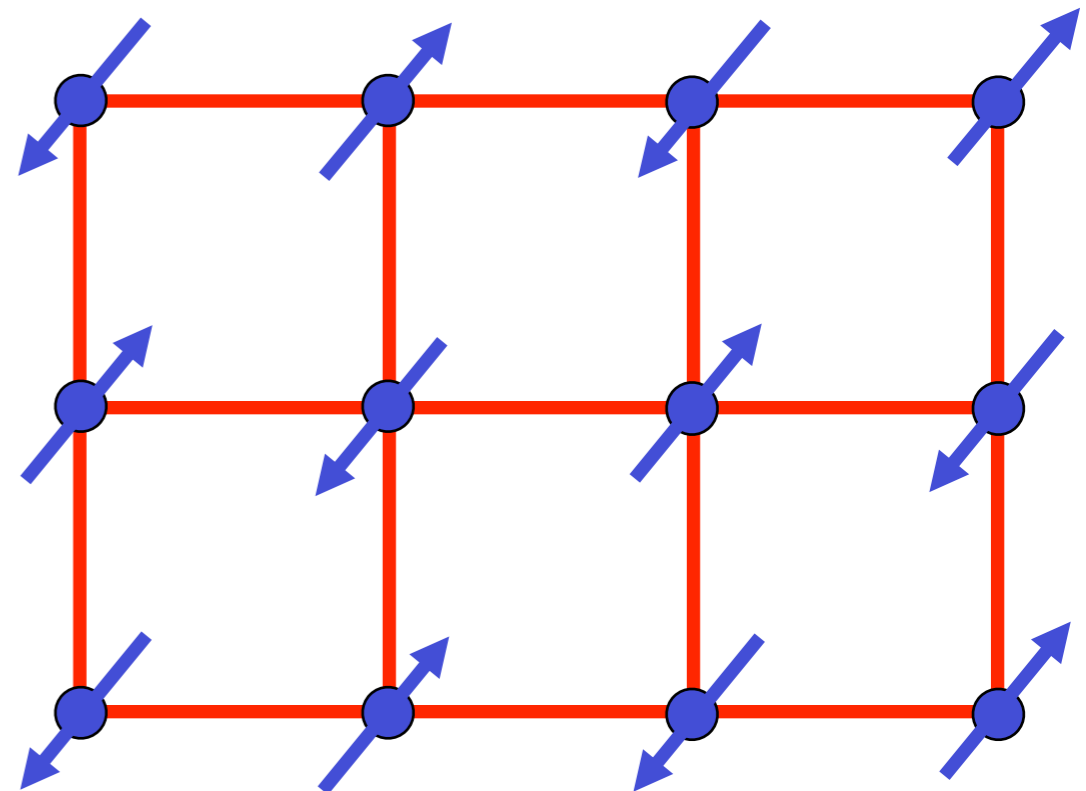


+

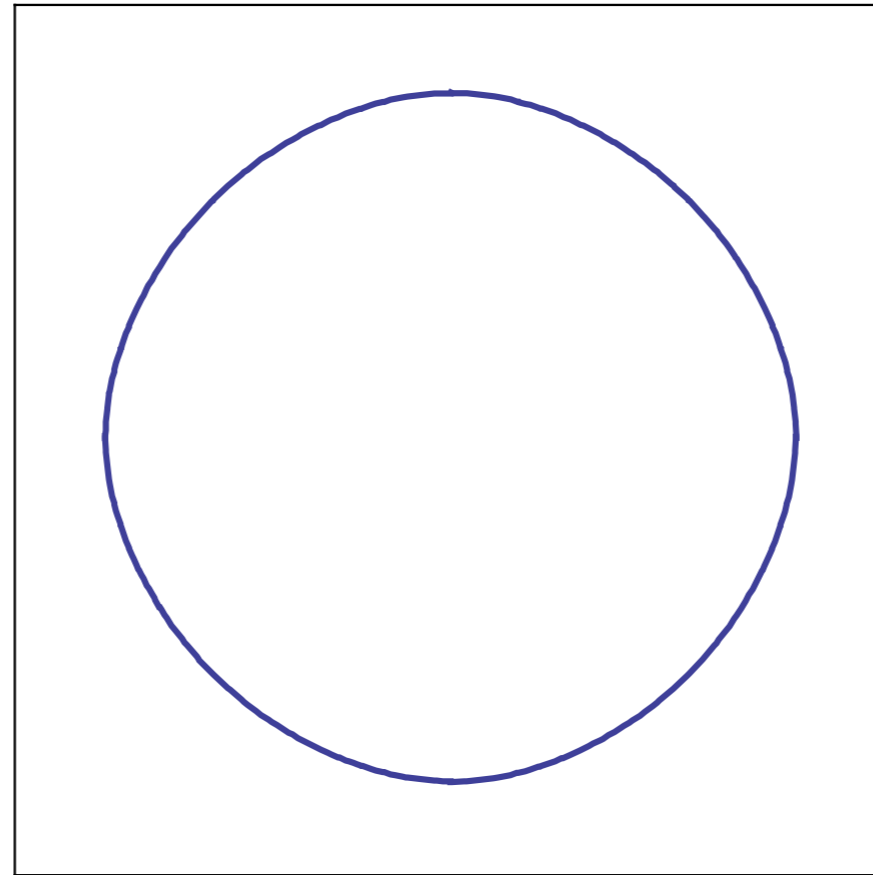
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where $\mathbf{K} = (\pi, \pi)$ is the ordering
wavevector.

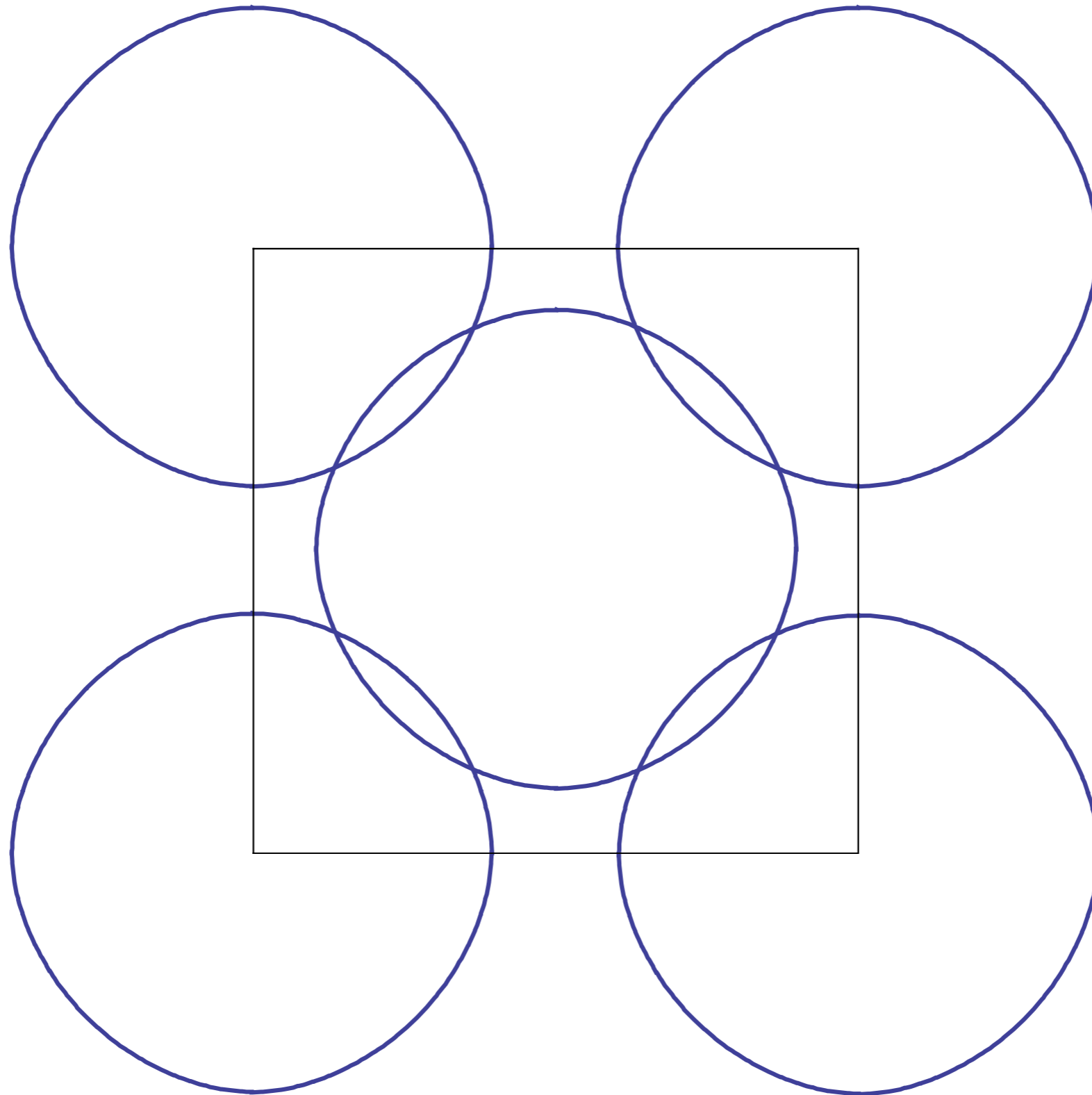


Fermi surface+antiferromagnetism



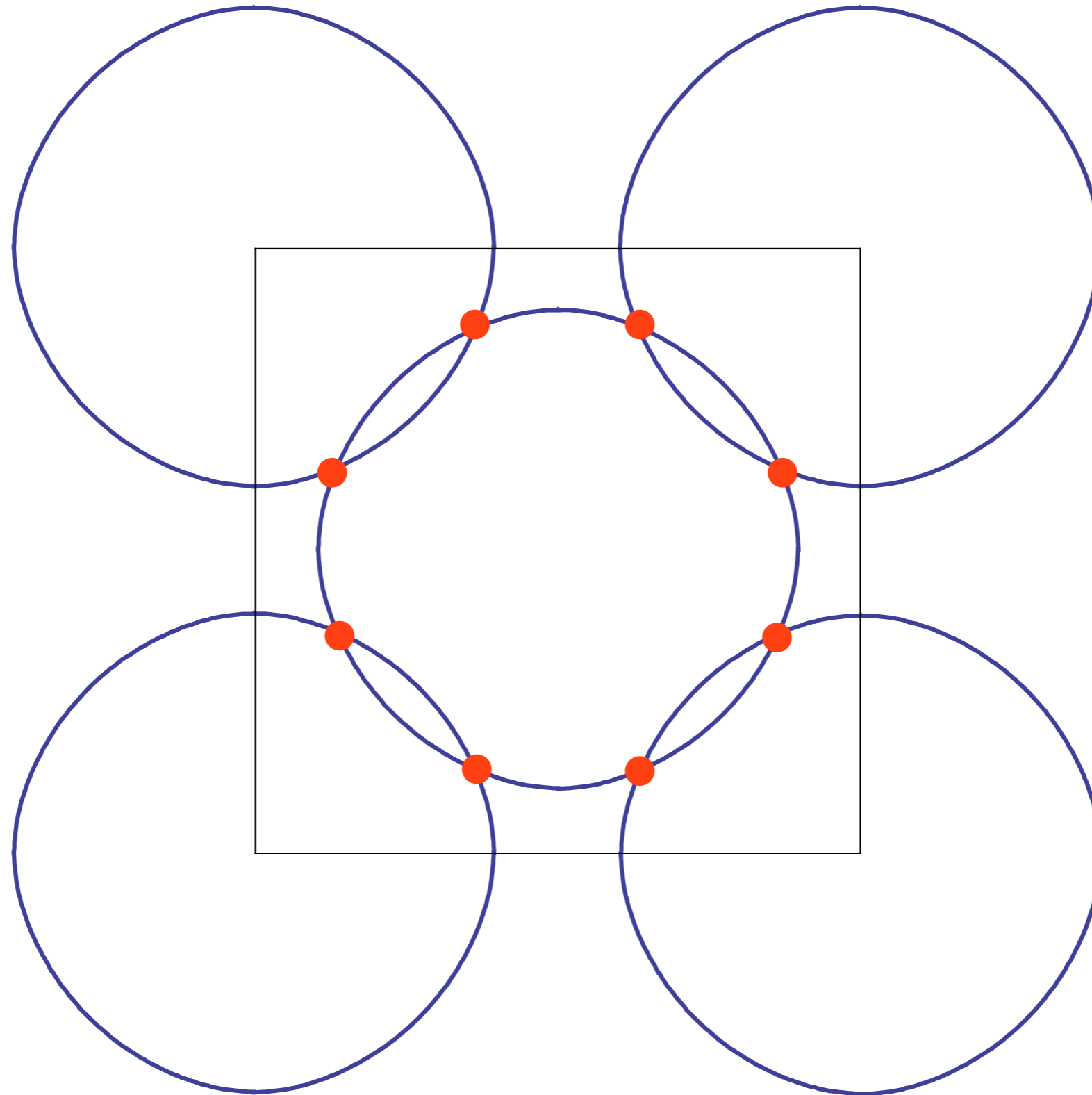
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



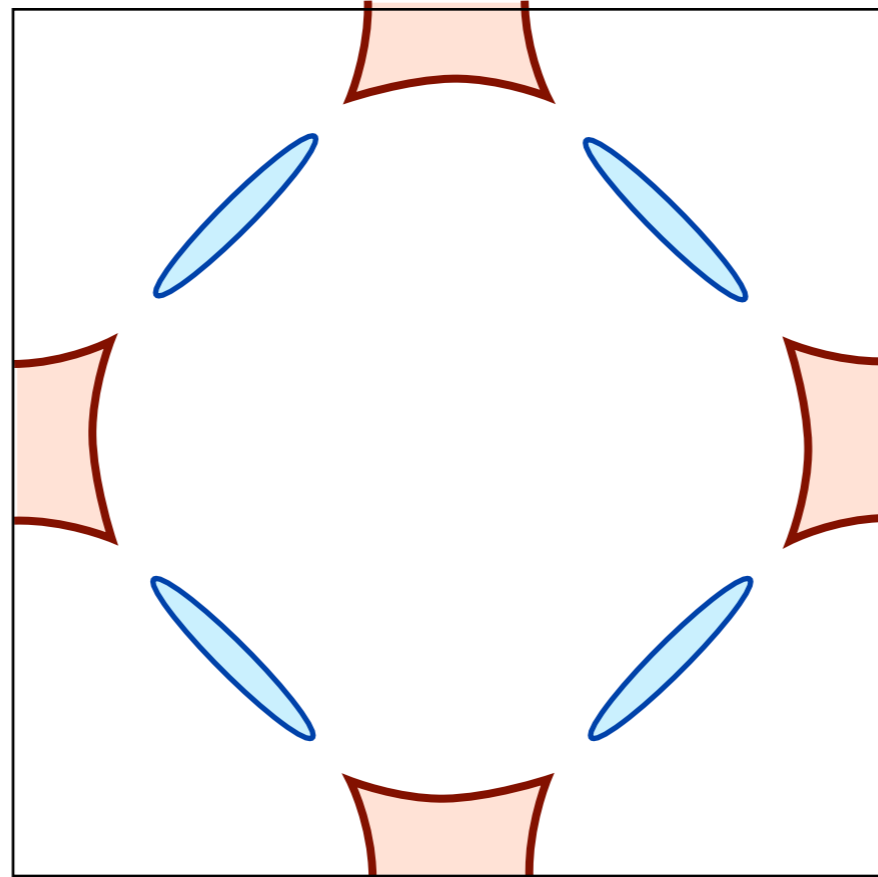
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



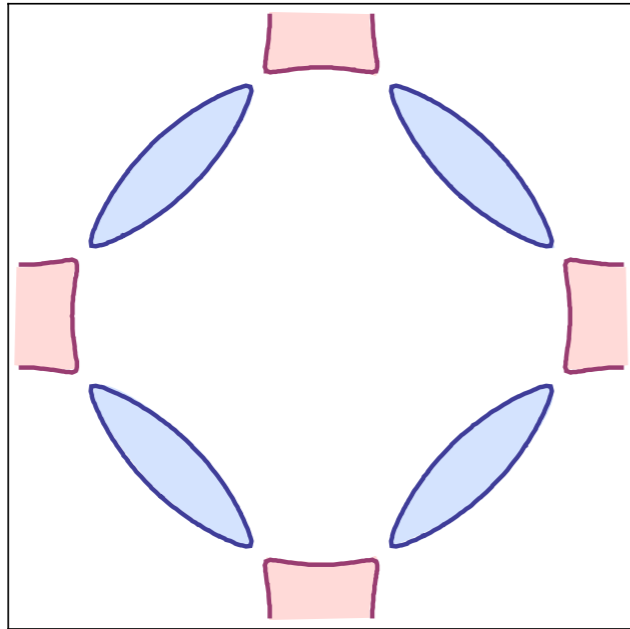
“Hot” spots

Fermi surface+antiferromagnetism



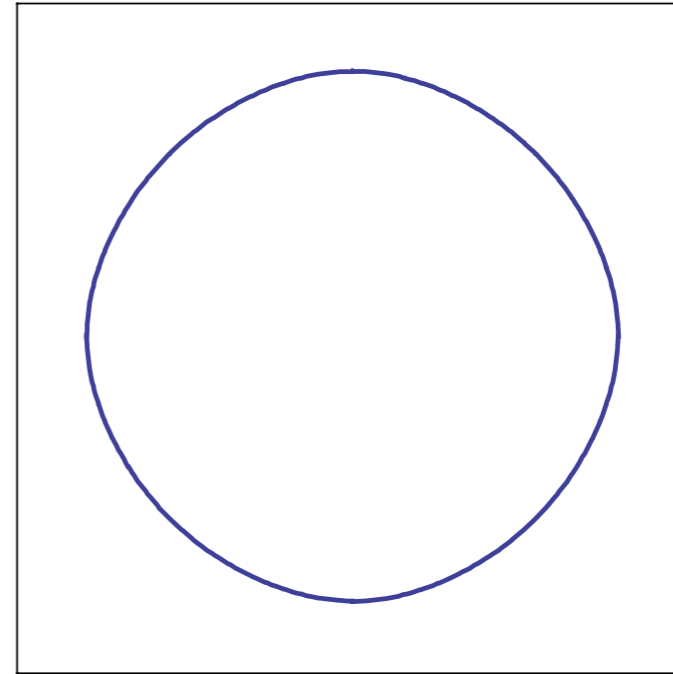
Electron and hole pockets in
antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

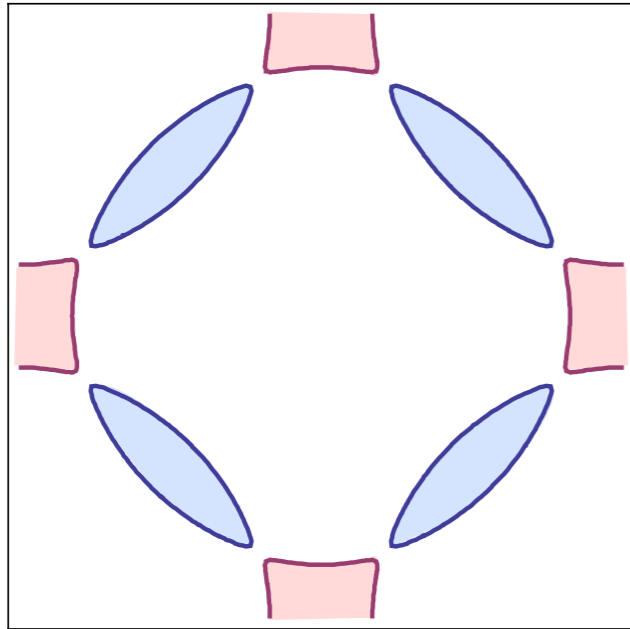


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

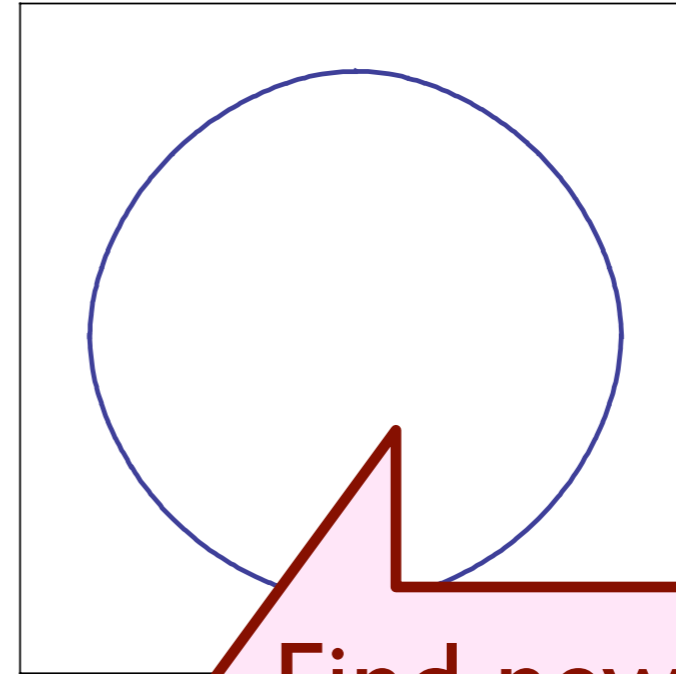
r

Quantum phase transition with onset of antiferromagnetism in a metal



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
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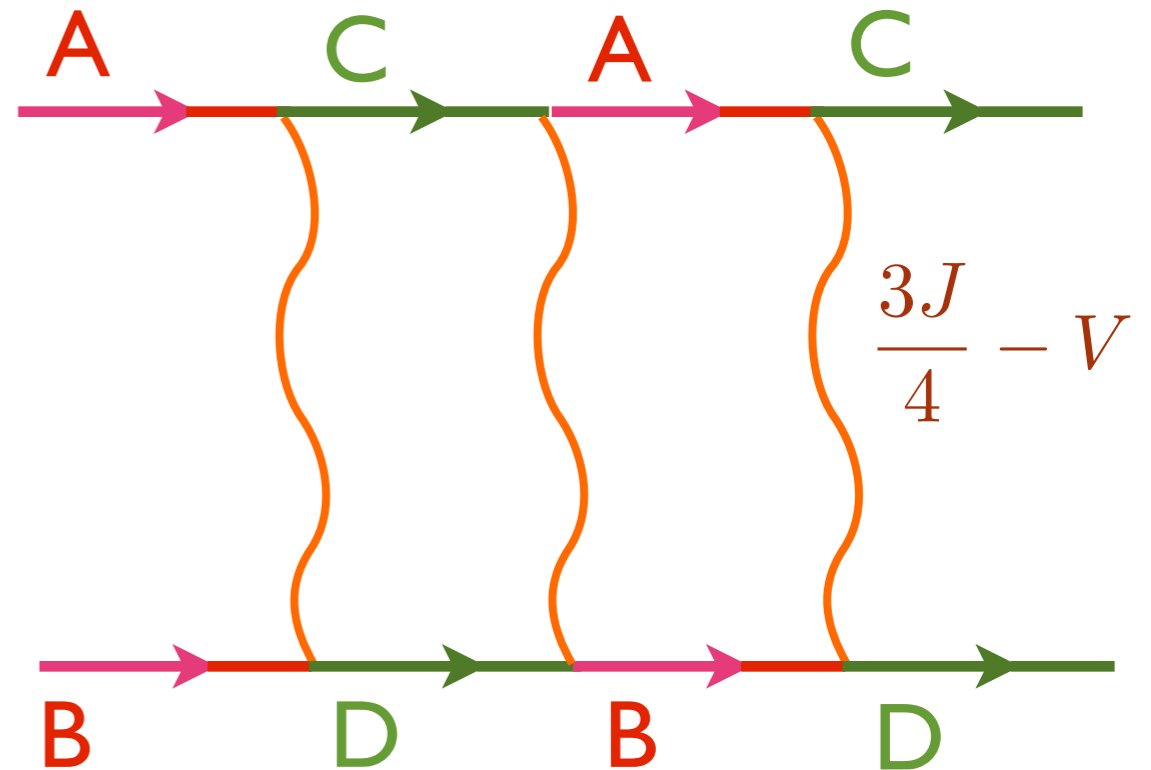
Metal with "large"
Fermi surface

Find new instabilities
upon approaching
critical point

r

Pairing “glue” from antiferromagnetic fluctuations

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$

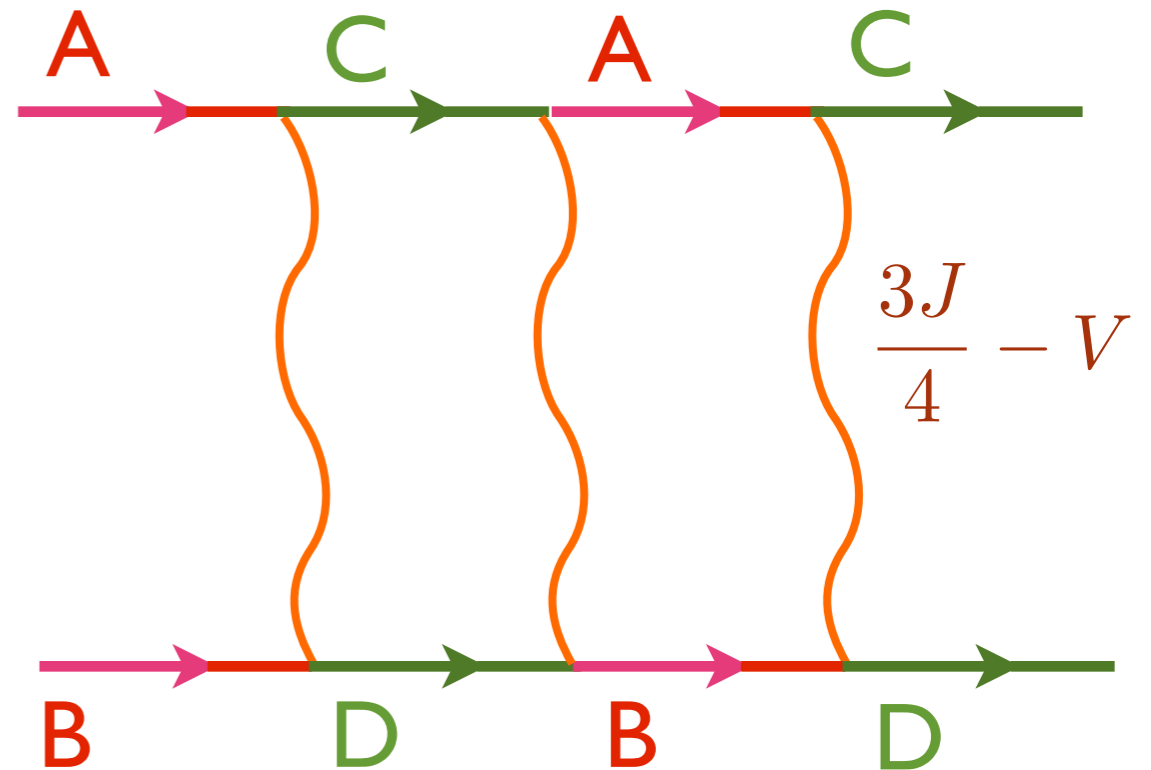


V.J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)
 D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)
 K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

Pairing “glue” from antiferromagnetic fluctuations

Electron hopping

$$\begin{aligned}
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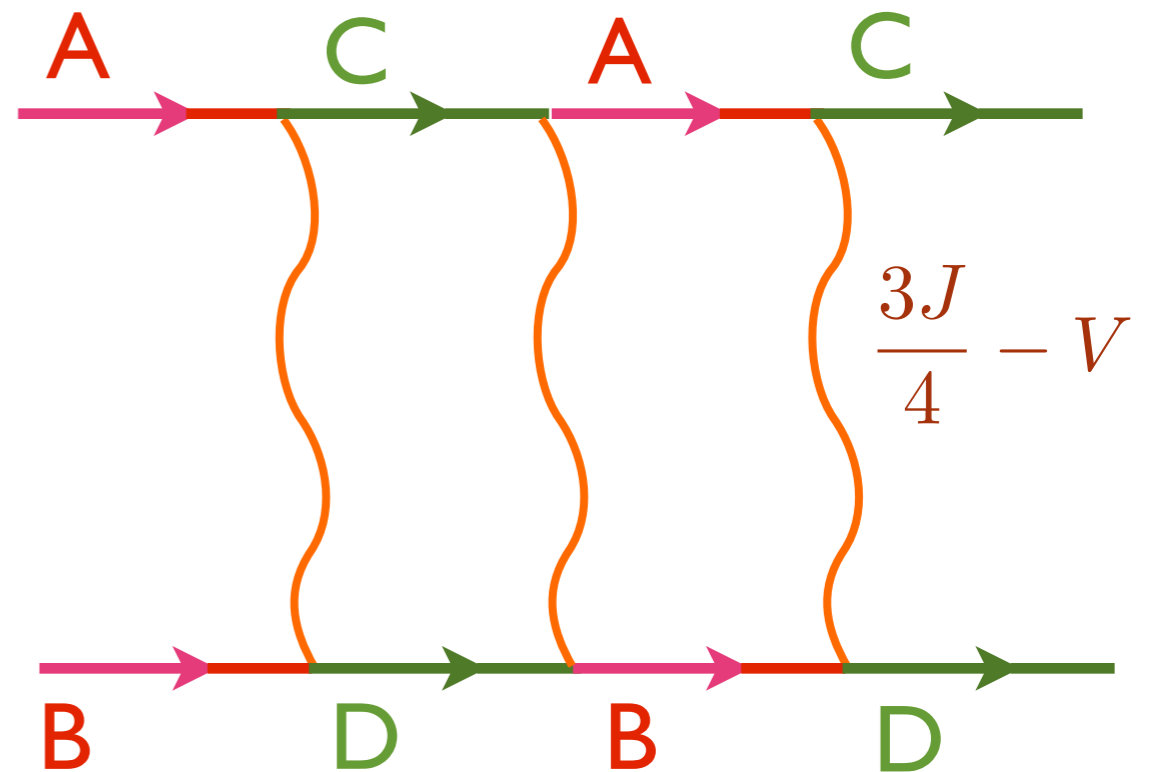


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Antiferromagnetic
exchange
interaction

Theory



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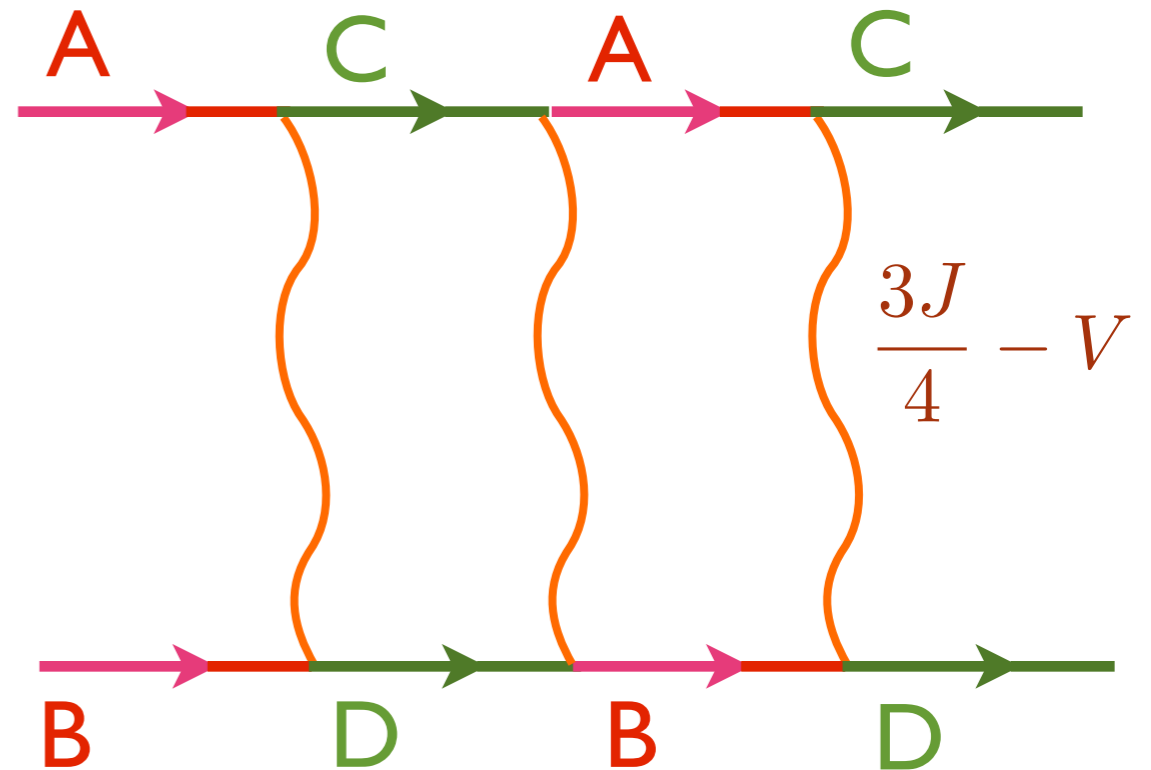
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 \end{aligned}$$

Coulomb repulsion

Theory

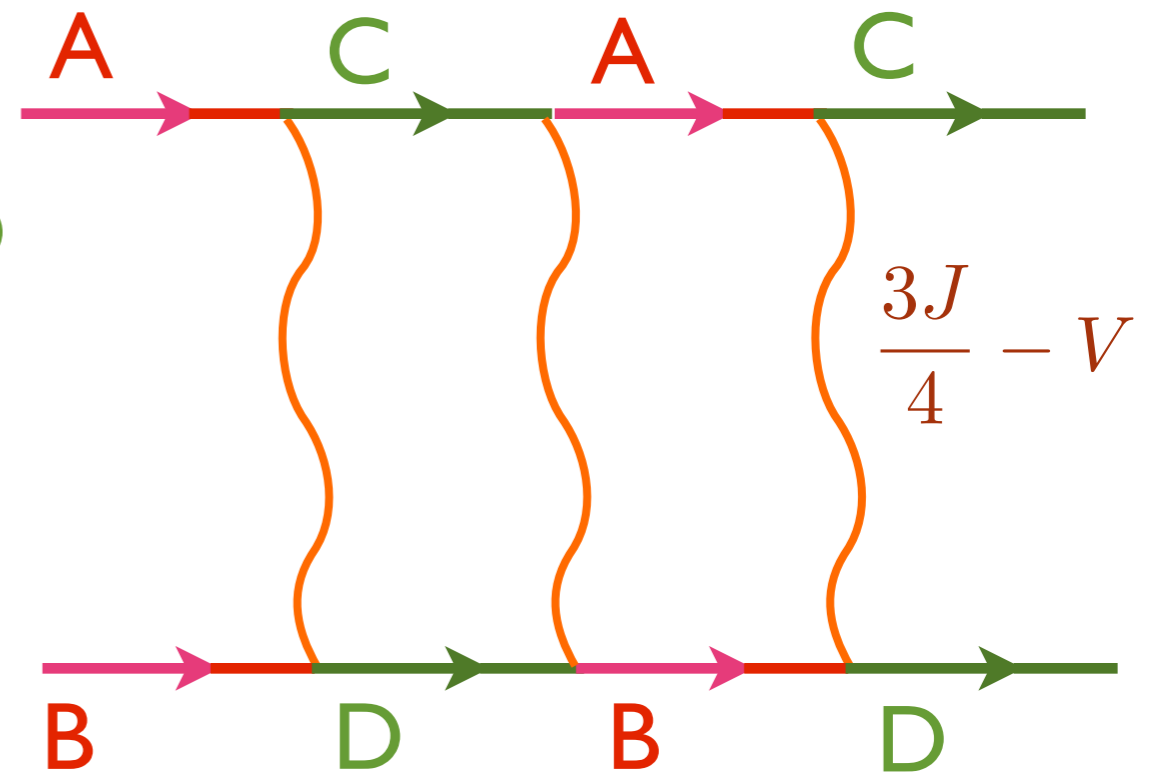
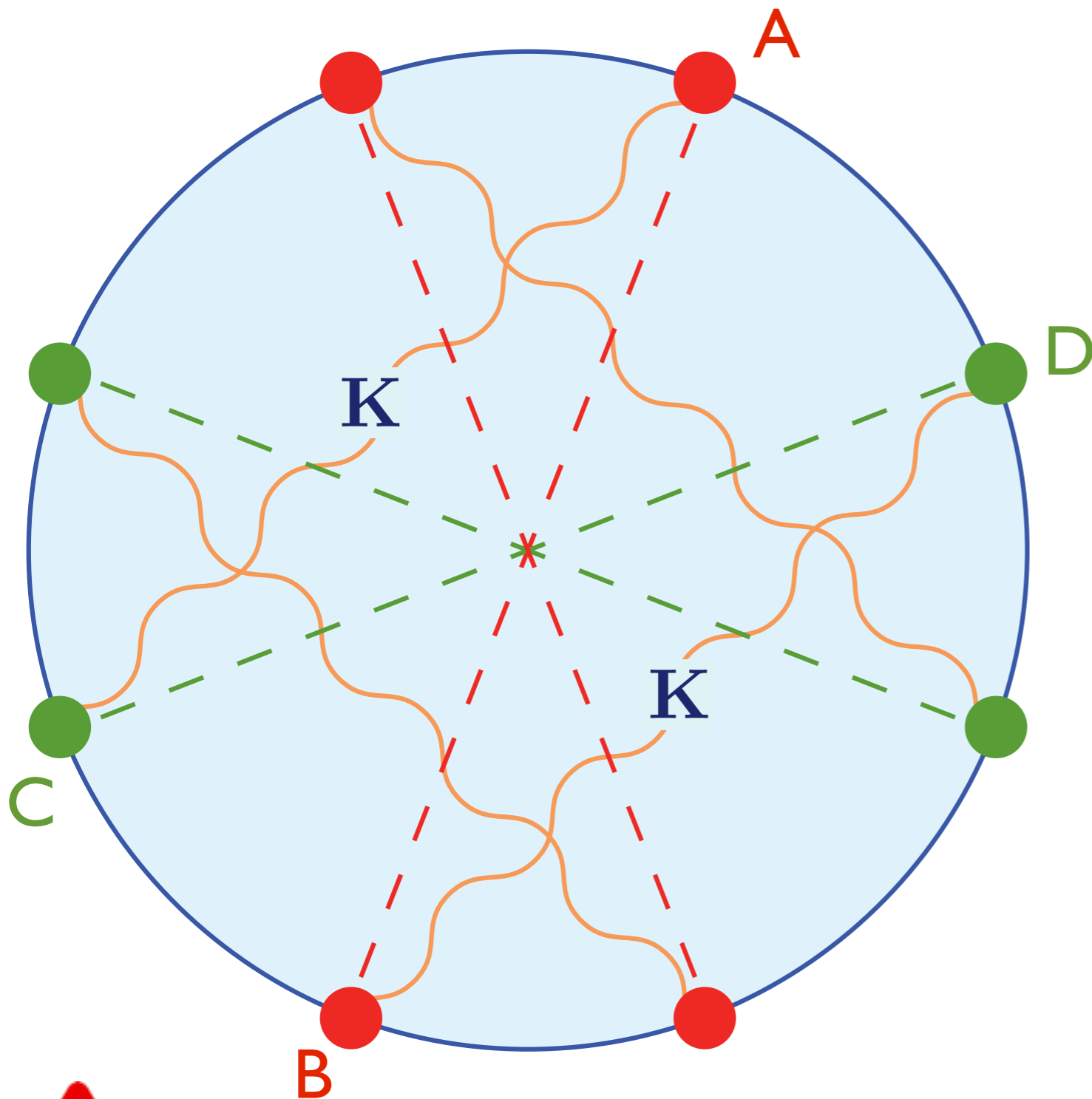


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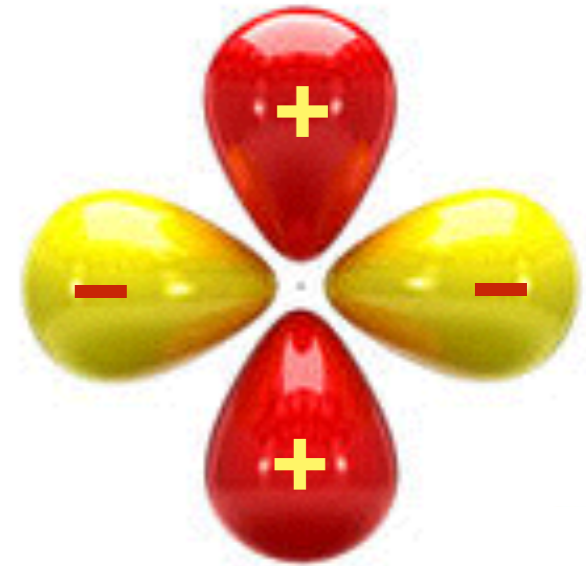
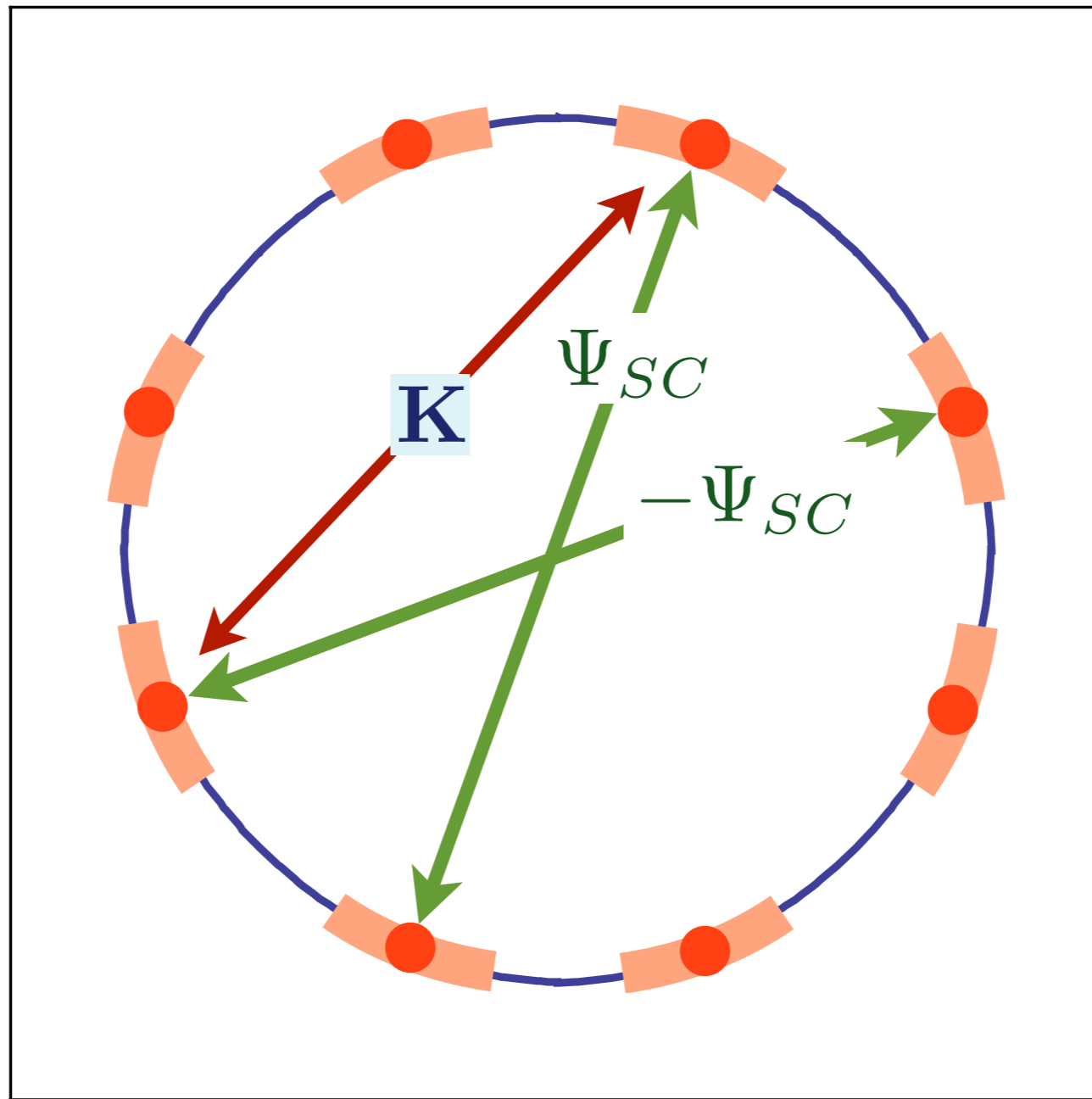
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Pairing “glue” from antiferromagnetic fluctuations



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$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} (\cos k_x - \cos k_y) \Psi_{SC}$$



**d-wave superconductor:
sign-changing pairing amplitude**



V. J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)
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Outline

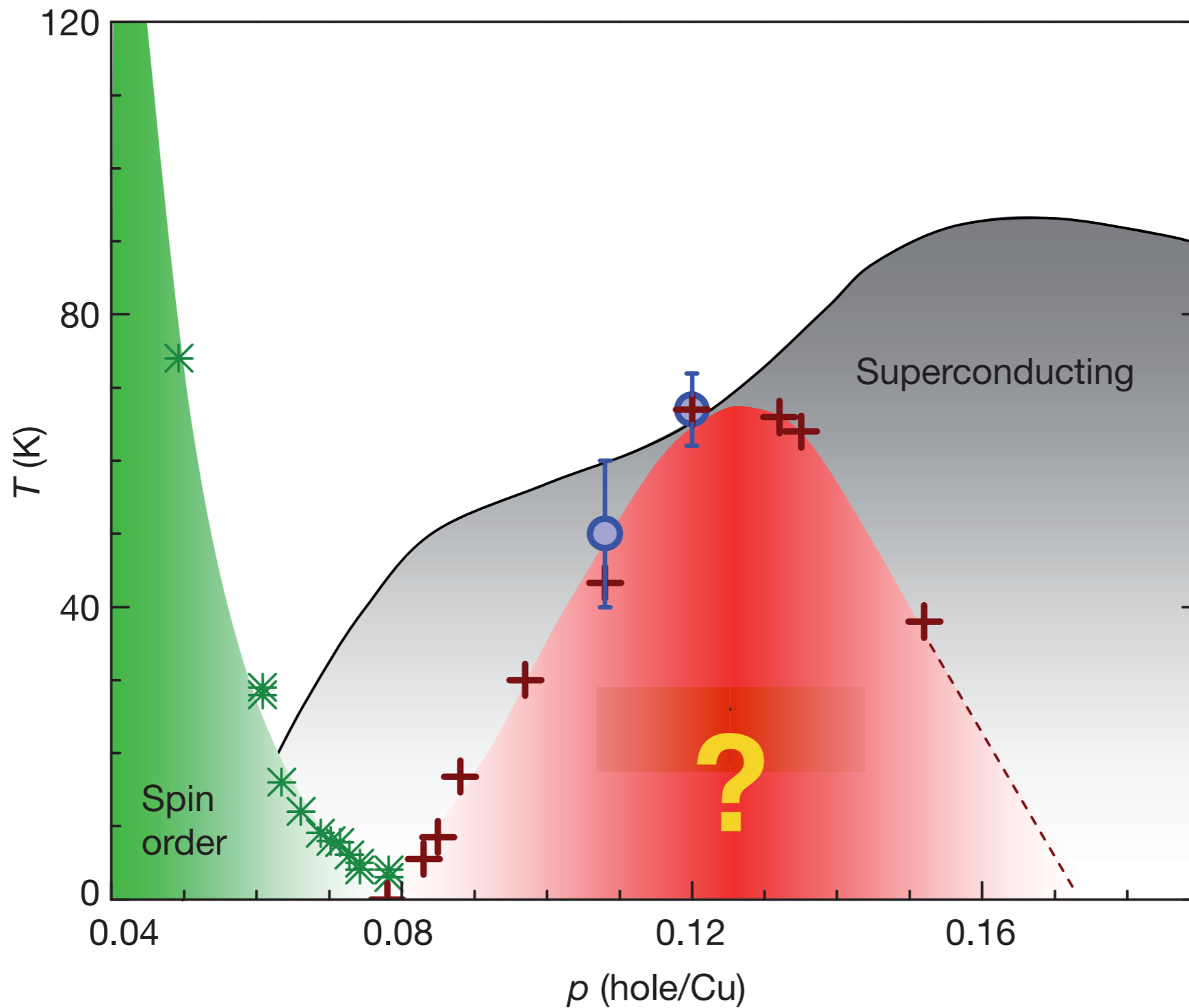
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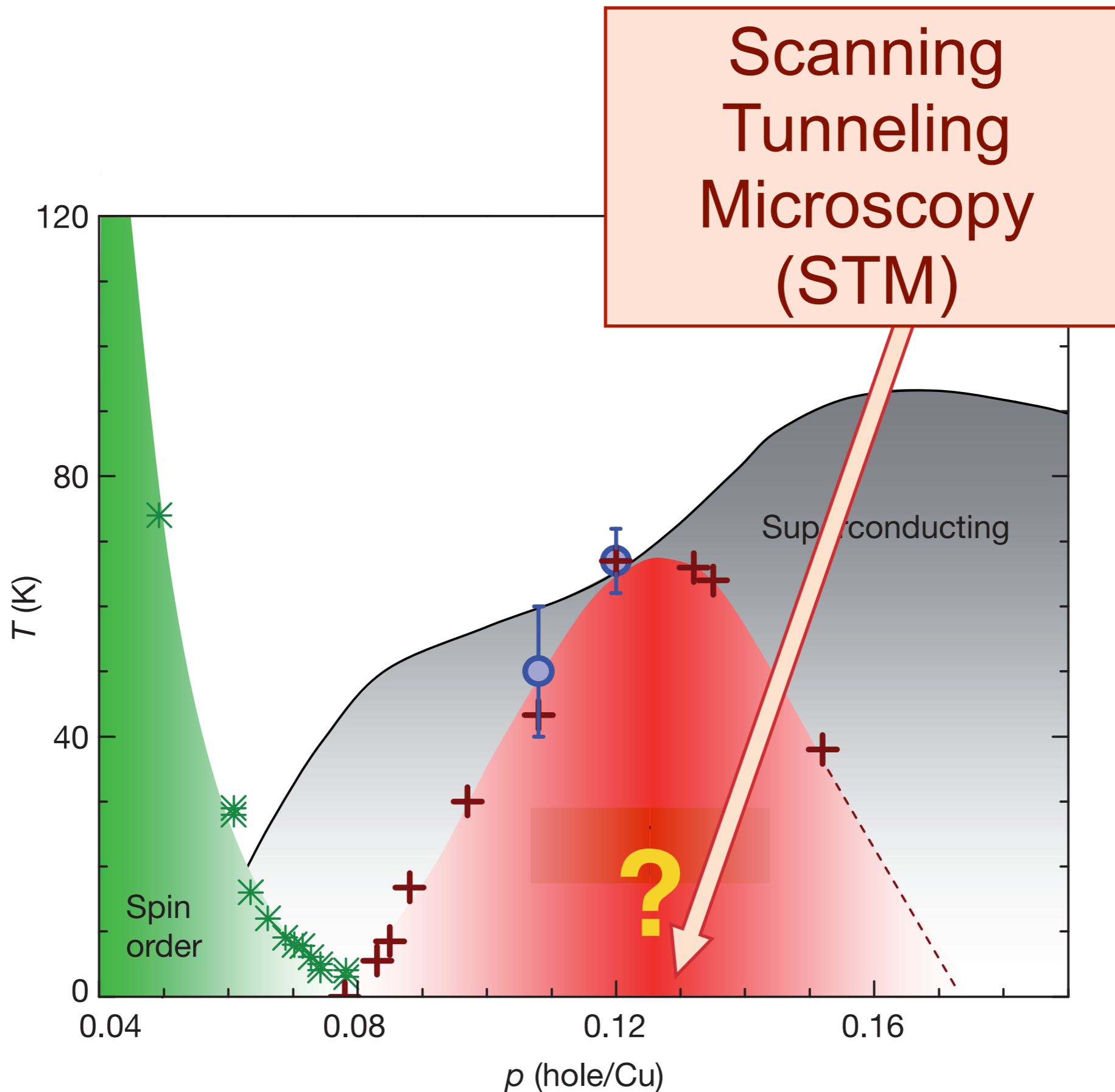
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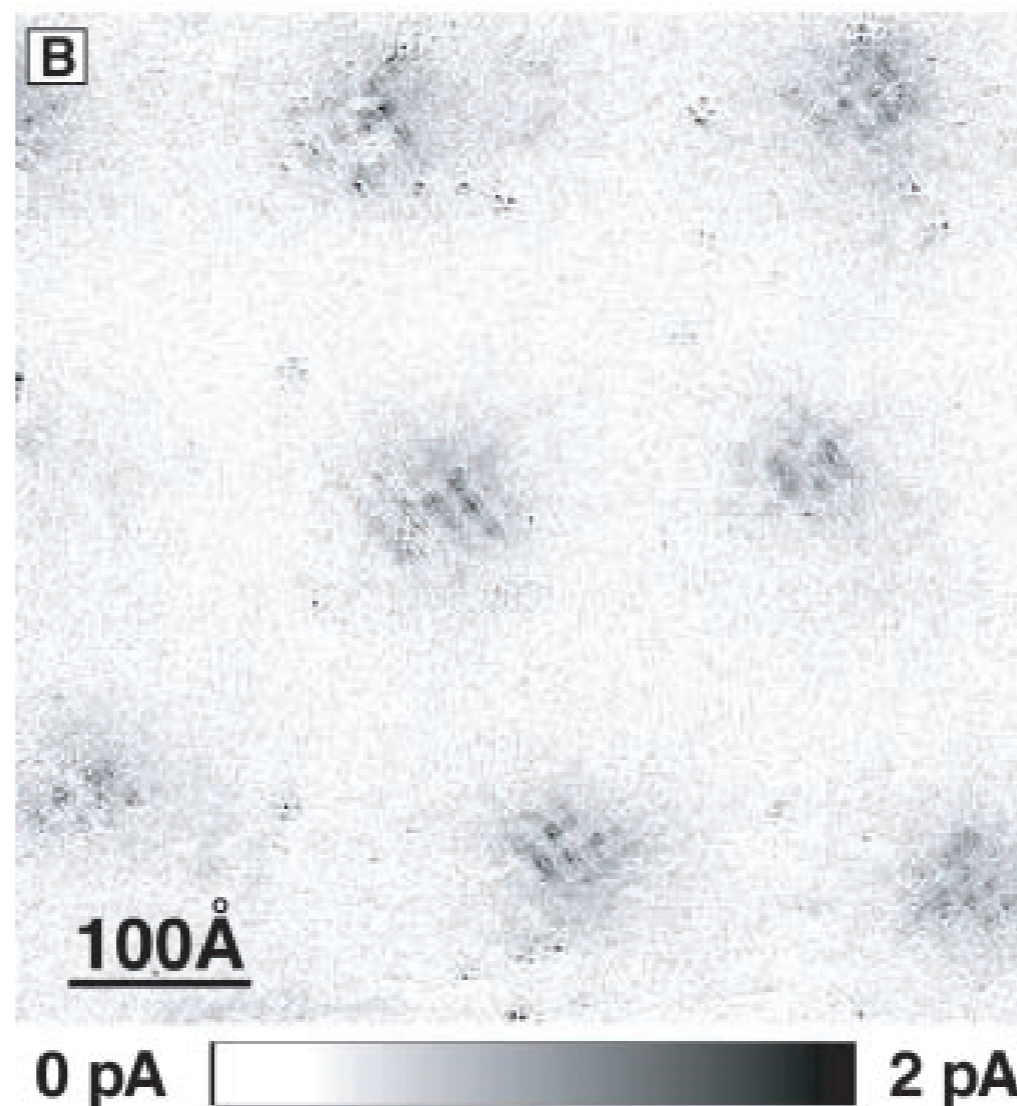
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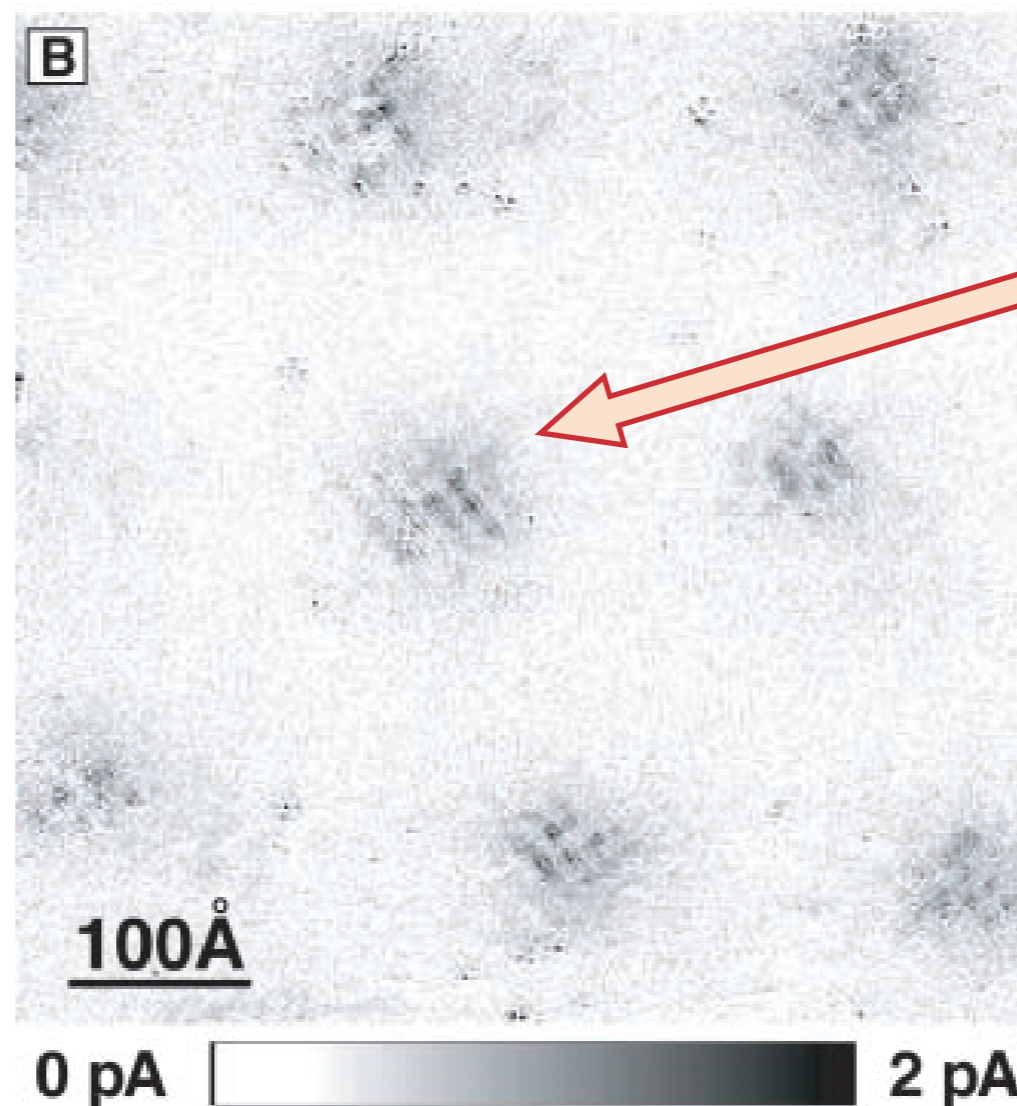
A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan,
H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

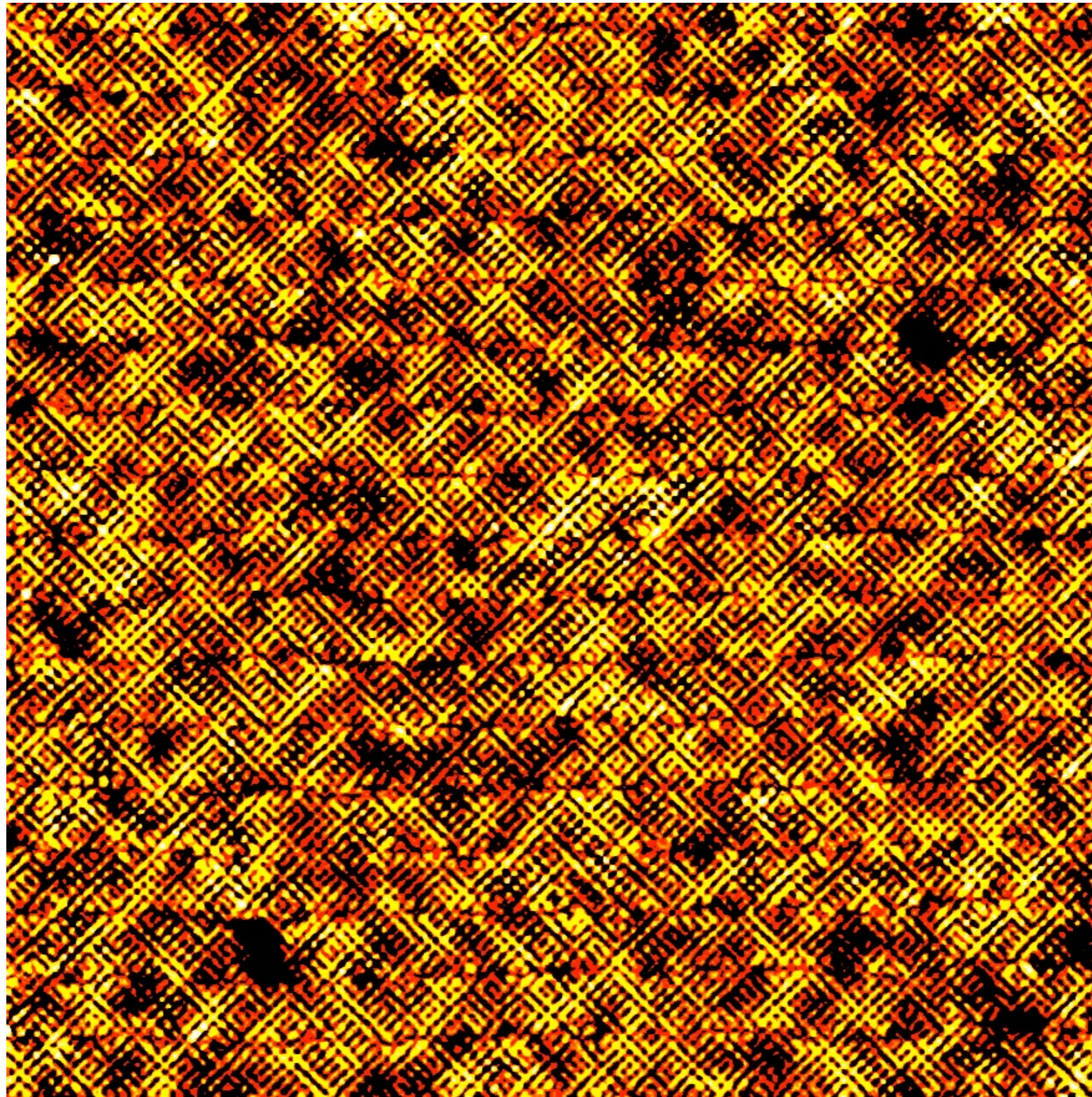


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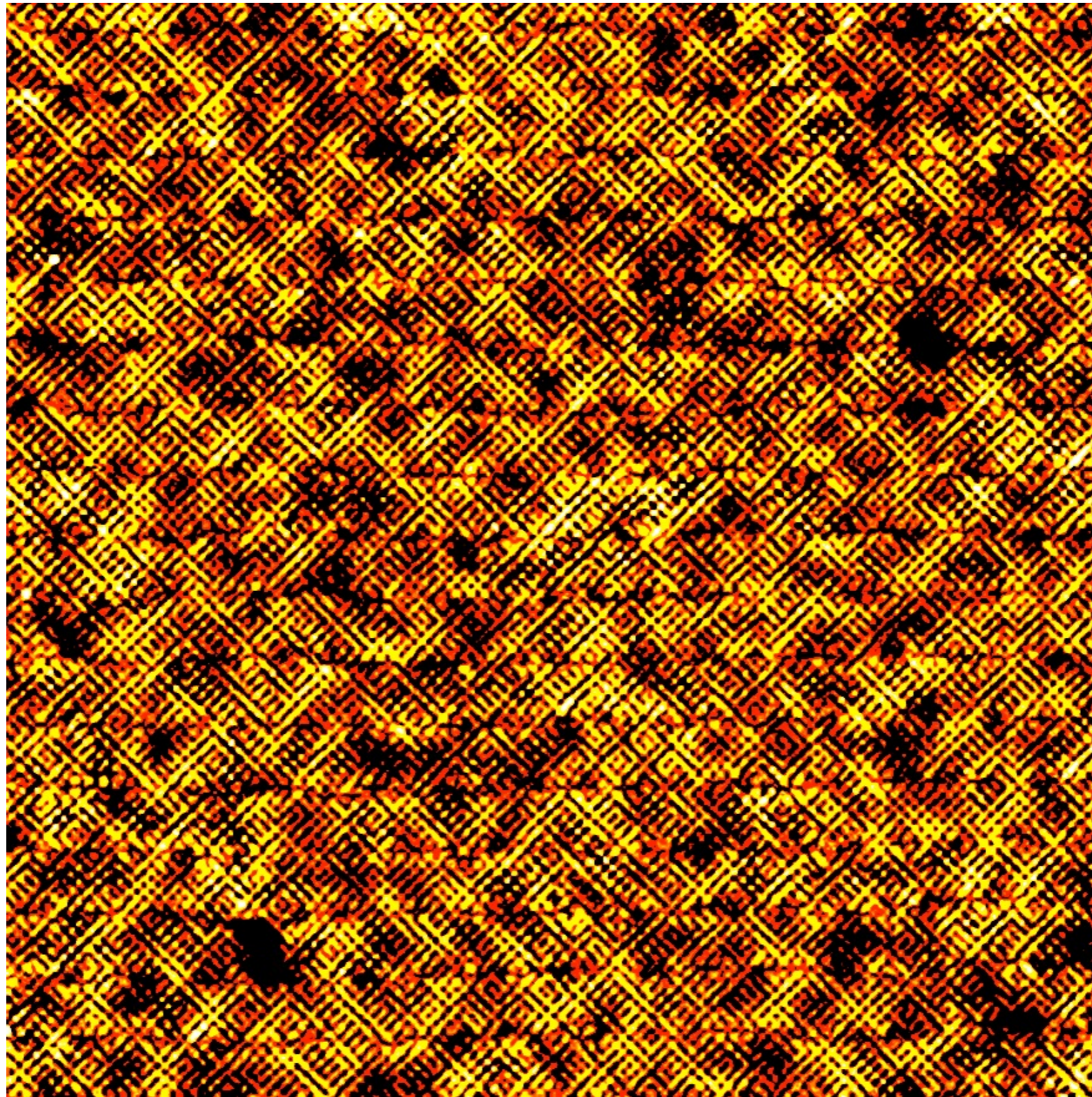
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A charge density
wave (CDW)
around vortex
cores ?



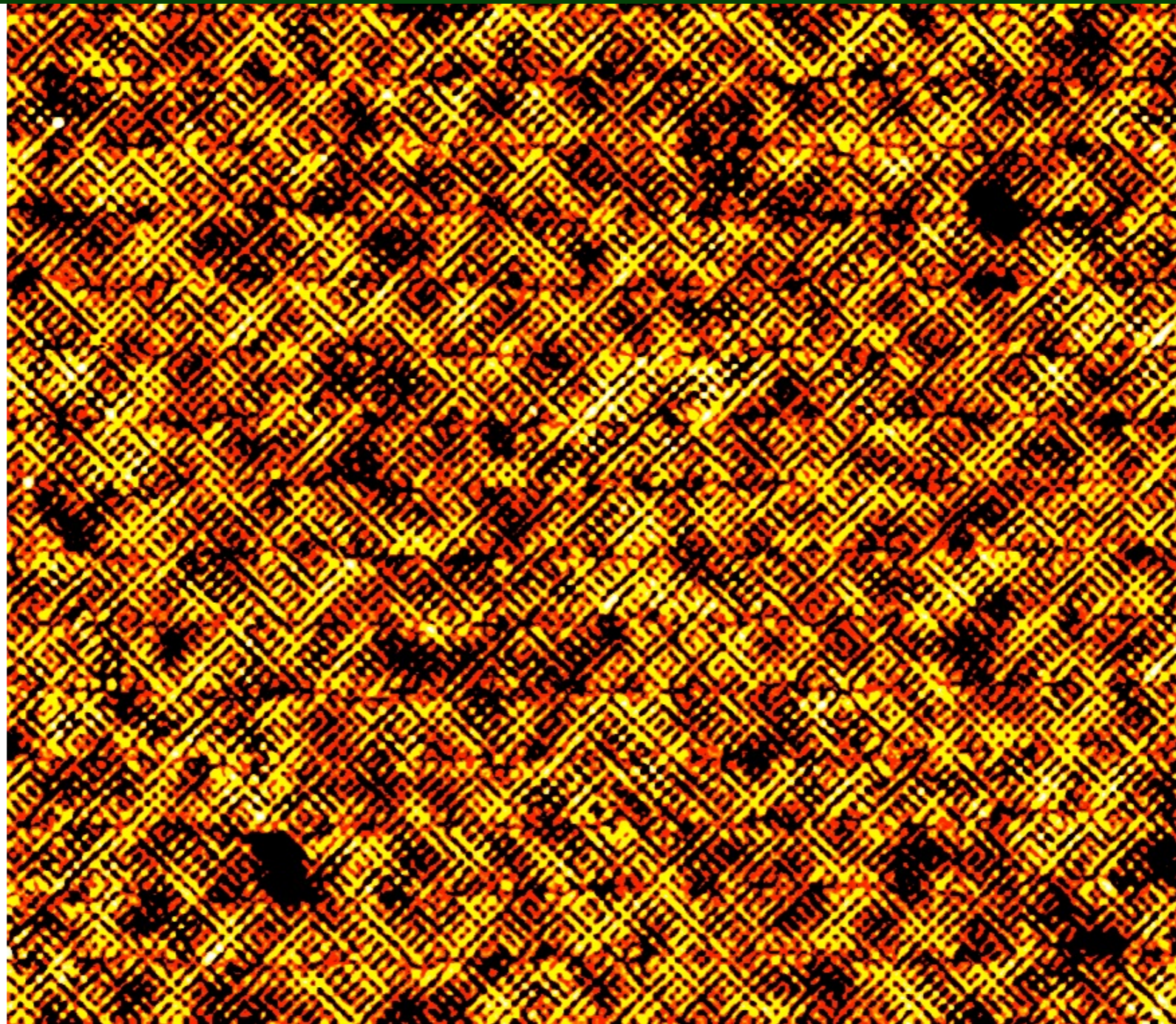
“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007).



(My “cosmic background” map)

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Fourier transform yields non-dispersing peaks at
 $Q \approx 2\pi(\pm 1/4, 0)$ and $\approx 2\pi(0, \pm 1/4)$



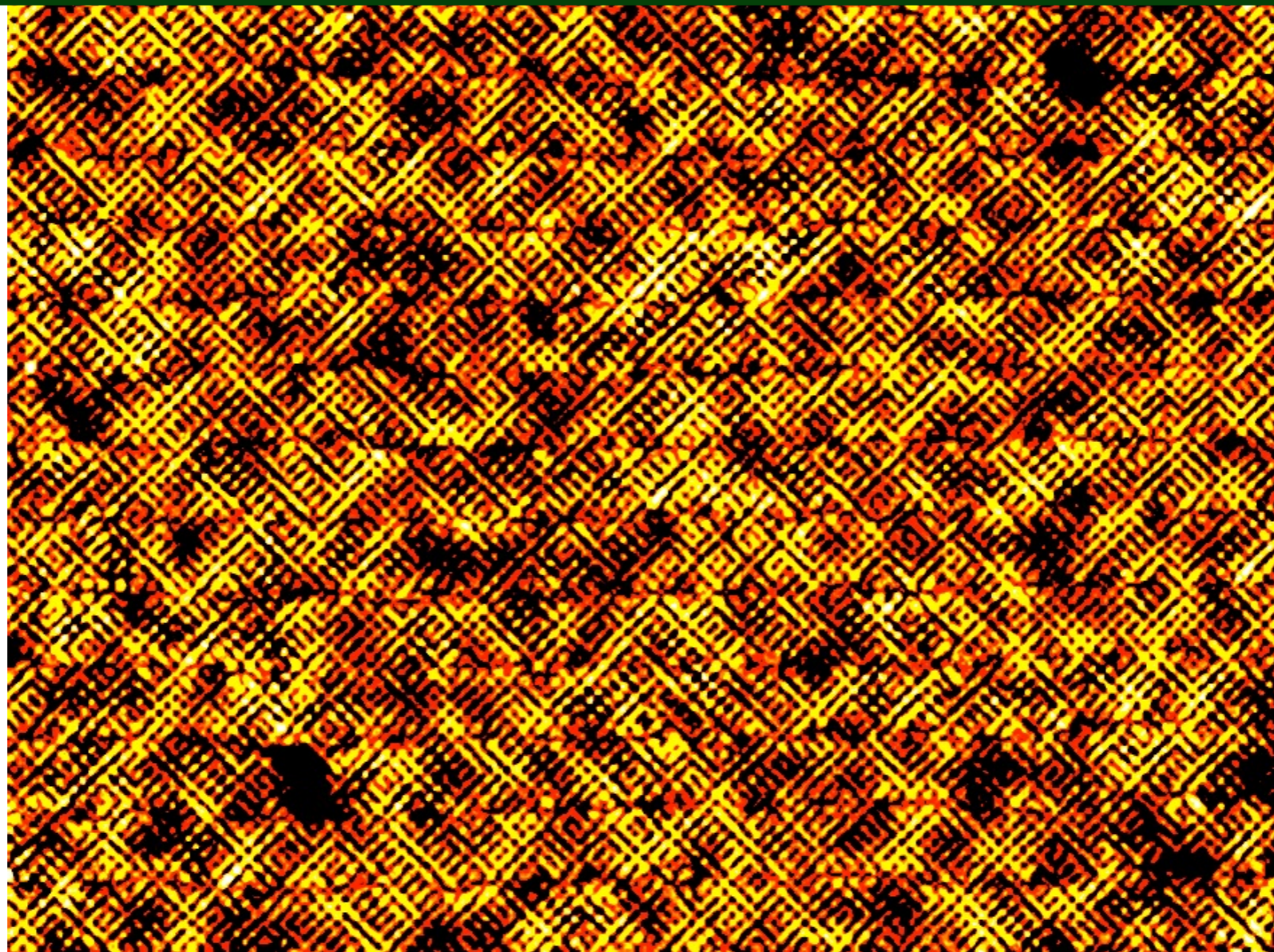
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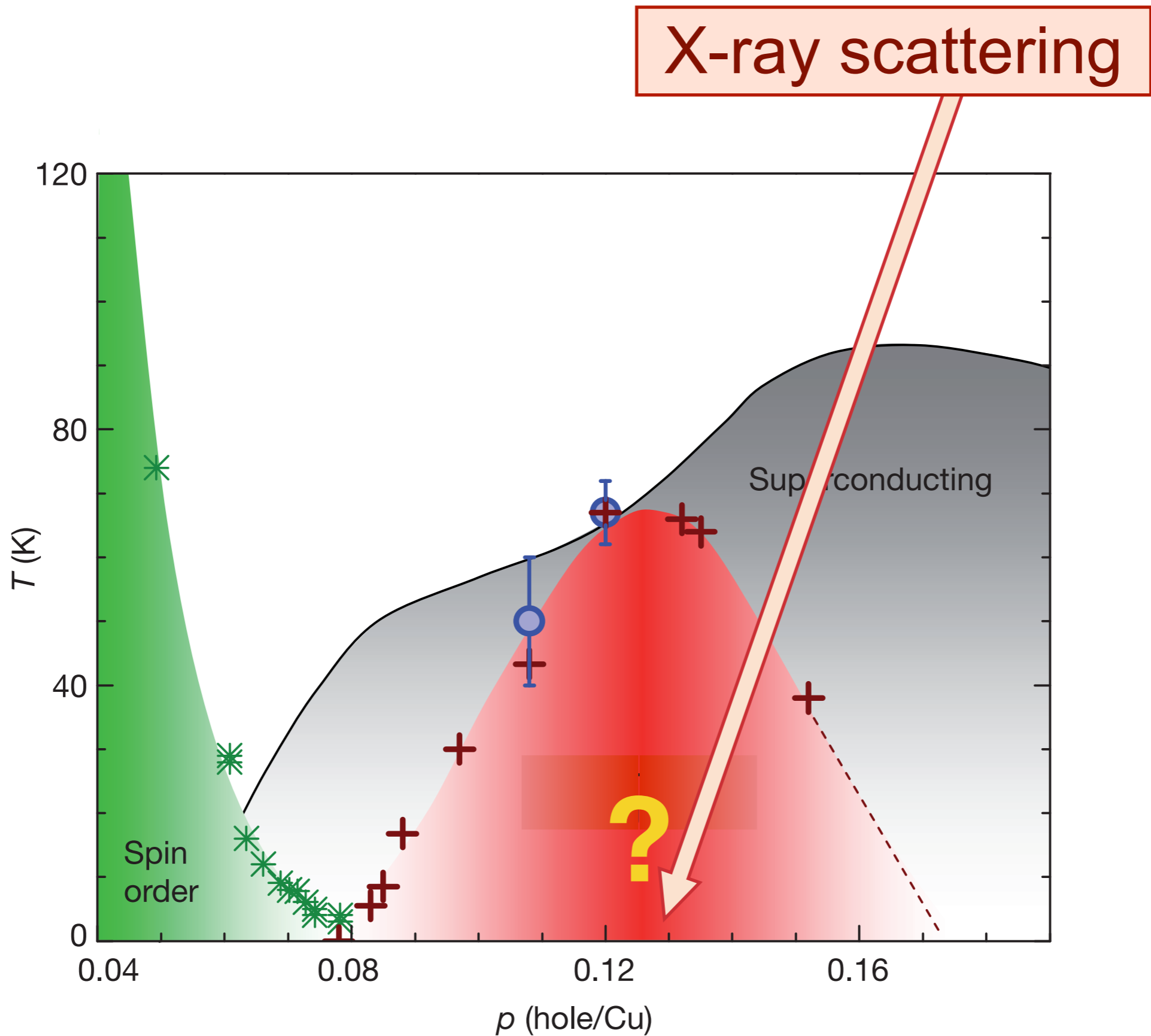
$$\mathbf{Q} \approx 2\pi(\pm 1/4, 0) \text{ and } \approx 2\pi(0, \pm 1/4)$$

But there seems to be much more structure in the map ...

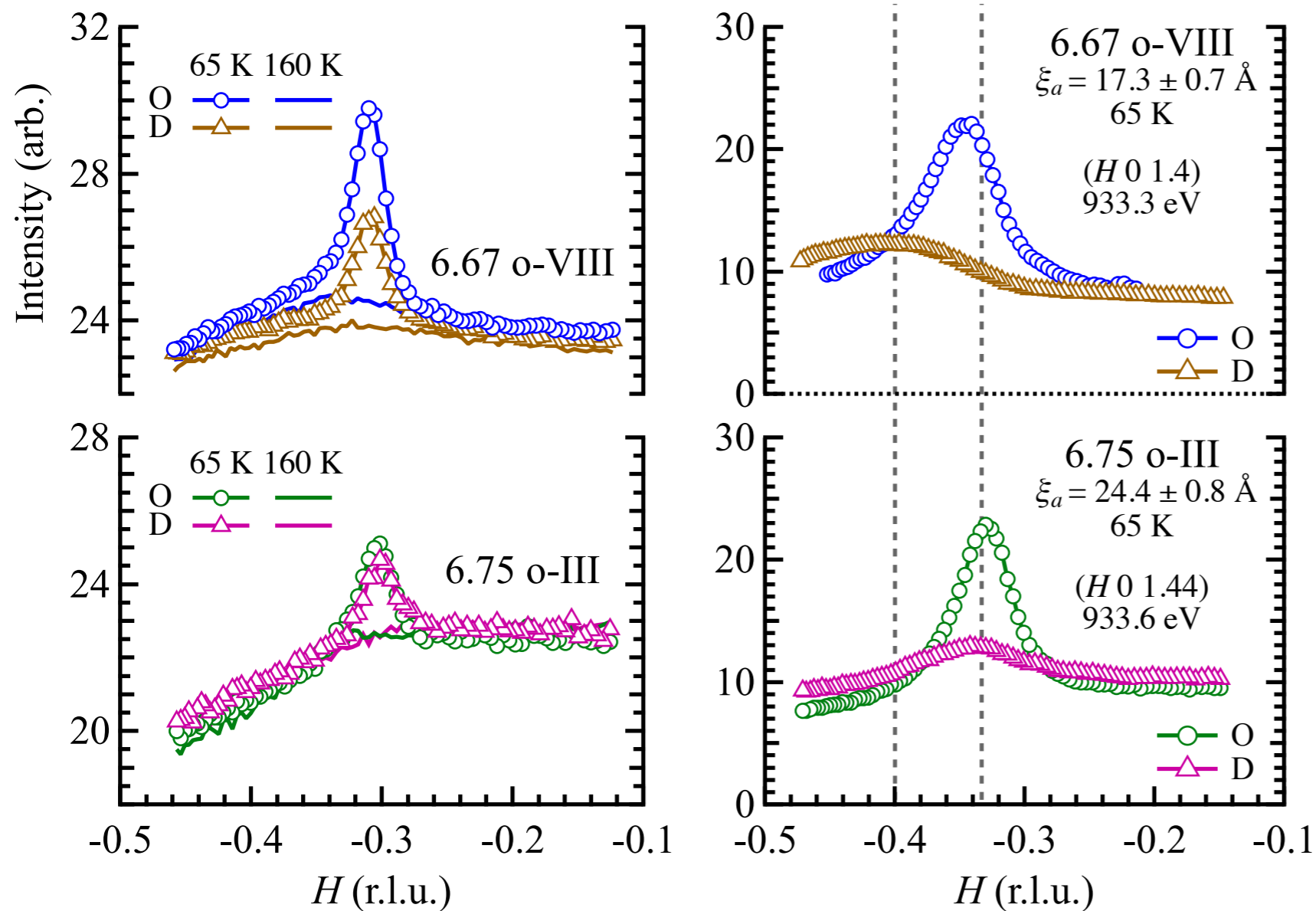


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A. J. Achkar, X. Mao, Christopher McMahon, R. Sutarto, F. He, Ruixing Liang, D. A. Bonn, W. N. Hardy, and D. G. Hawthorn, arXiv:1312.6630

Many groups have observed CDW peaks in X-ray scattering over the past 2 years.

These X-ray wavevector is found to agree with the wavevector obtained from STM. R. Comin, A. Frano, M. M. Yee, Y. Yoshida, H. Eisaki, E. Schierle, E. Weschke, R. Sutarto, F. He, A. Soumyanarayanan, Yang He, M. Le Tacon, I.S. Elfimov, J. E. Hoffman, G.A. Sawatzky, B. Keimer, A. Damascelli, Science **343**, 390 (2014).

Charge density wave (CDW) order

$$\langle c_{\alpha}^{\dagger}(\mathbf{r})c_{\alpha}(\mathbf{r}) \rangle = \Psi_{CDW}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$

Charge density wave (CDW) order

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CDW wavevector \mathbf{Q}

Charge density wave (CDW) order

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Nearly constant CDW order parameter

Generalized charge density wave (CDW) order

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\alpha}(\mathbf{r}_2) \rangle$$
$$= \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{CDW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}$$



Nearly constant CDW order parameter

Generalized charge density wave (CDW) order

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“Polarization” function $\mathcal{P}(\mathbf{r}) = \int \frac{d^2 k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$

Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y) \dots$$

Generalized charge density wave (CDW) order

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\alpha}(\mathbf{r}_2) \rangle = \left[\mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{CDW} \left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}$$

“Polarization” function $\mathcal{P}(\mathbf{r}) = \int \frac{d^2 k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$

Time-reversal symmetry requires $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$.

We expand

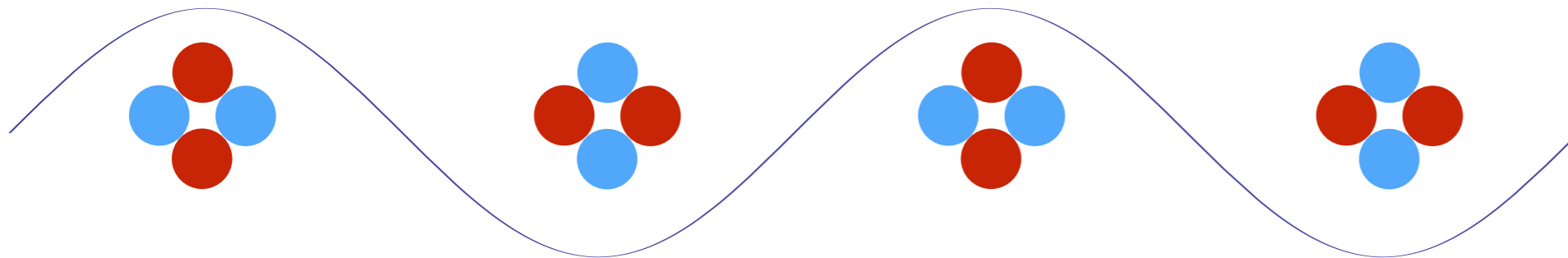
$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y) \dots$$



A CDW with $\mathcal{P}_d \neq 0$ is nearly degenerate with SC near the quantum-critical point for antiferromagnetism in a metal.

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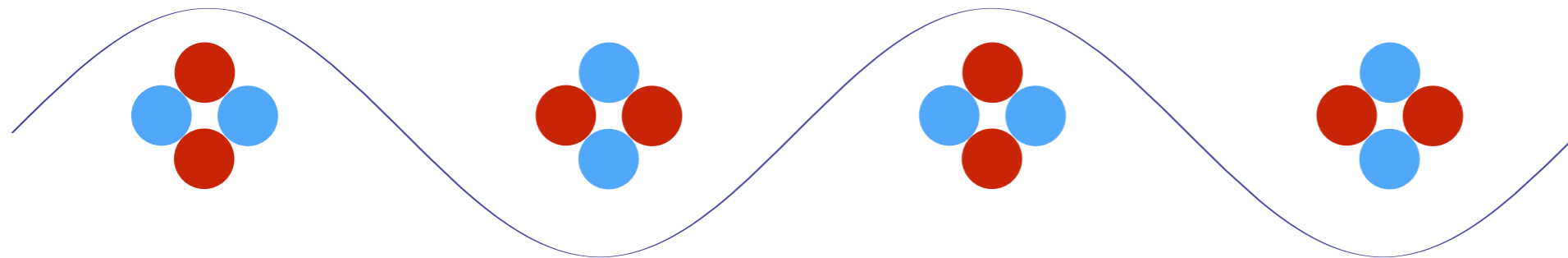


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M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010)

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(Adapted from a picture of a gravity wave)



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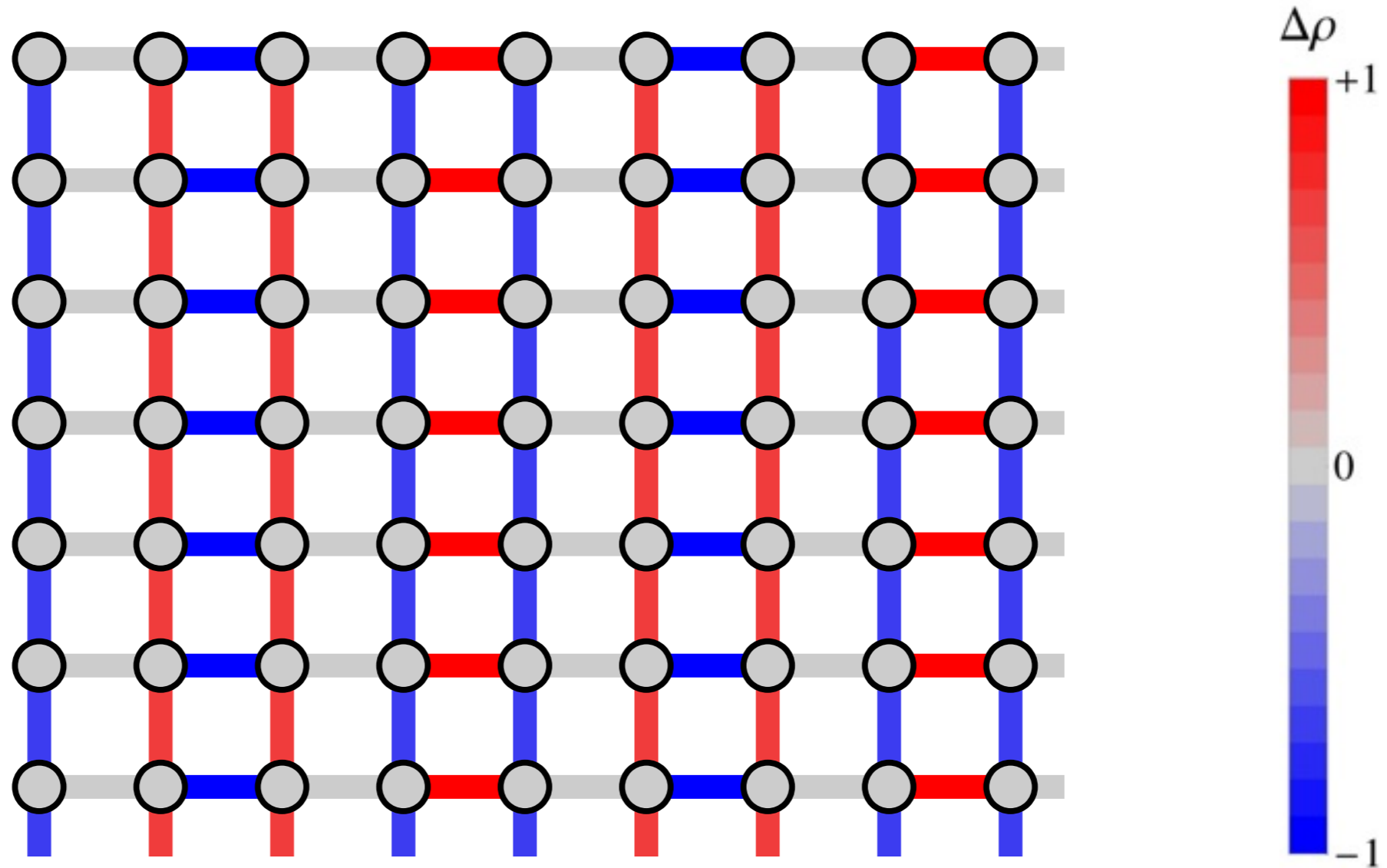
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010)

d -wave bond order.

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

$$\mathcal{P}(\mathbf{k}) = \cos(k_x) - \cos(k_y) \quad \text{and} \quad \mathbf{Q} = (\pi/2, 0)$$



This d -wave bond order was first discussed in
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

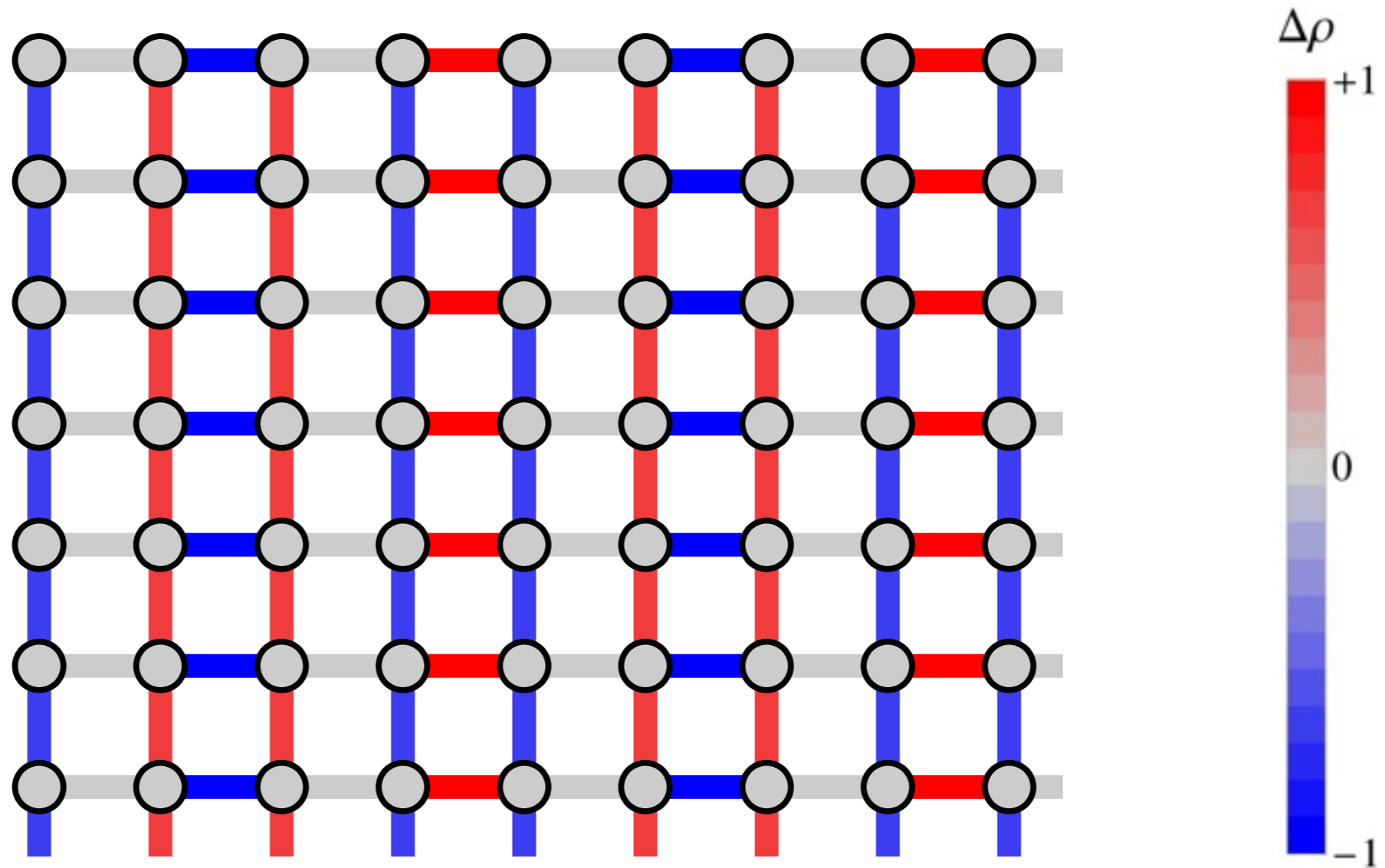
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$$\mathcal{P}(\mathbf{k}) = \cos(k_x) - \cos(k_y) \quad \text{and} \quad \mathbf{Q} = (\pi/2, 0)$$

Density wave on horizontal bonds has a phase-shift of π relative to the wave on vertical bonds



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Pseudospin symmetry of the exchange interaction

$$H_J = J \vec{S}_1 \cdot \vec{S}_2$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

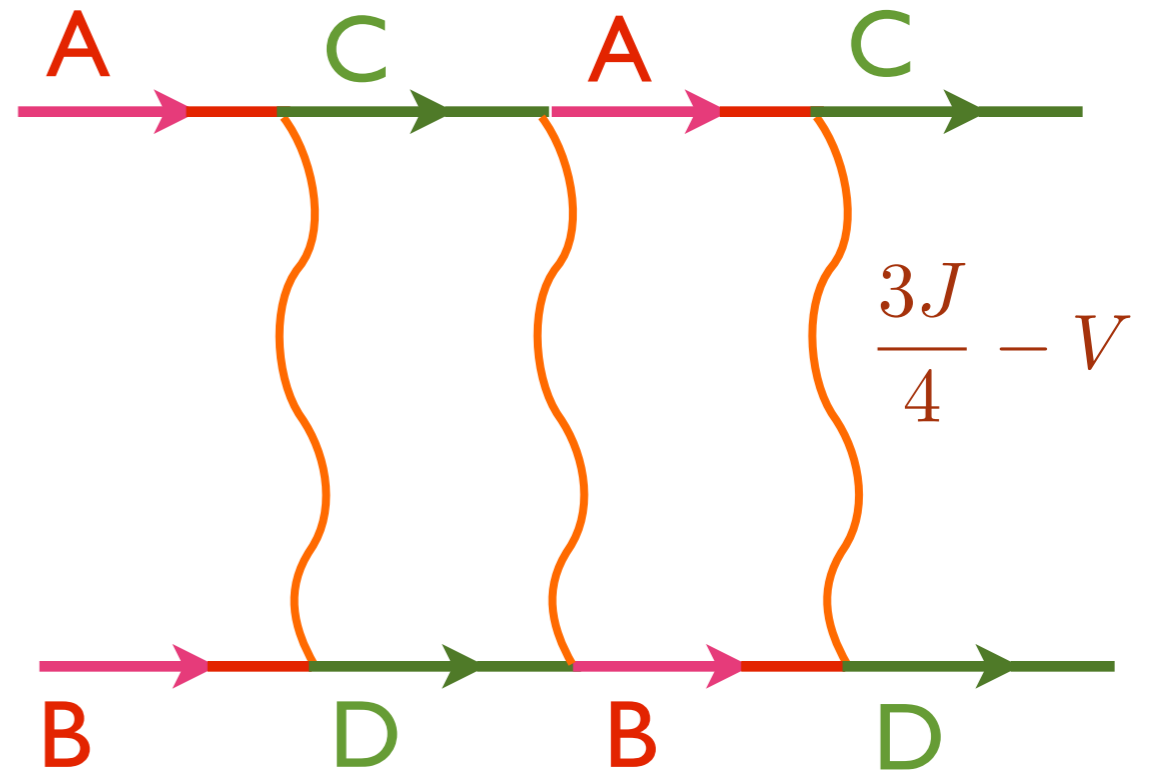
Then we can write

$$H_J = \frac{1}{8} J \left(\Psi_{1\alpha a}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{1\beta a} \right) \cdot \left(\Psi_{2\gamma b}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{2\delta b} \right)$$

where a, b are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site $\Psi_{i\alpha a} \rightarrow U_{i,ab} \Psi_{i\alpha b}$

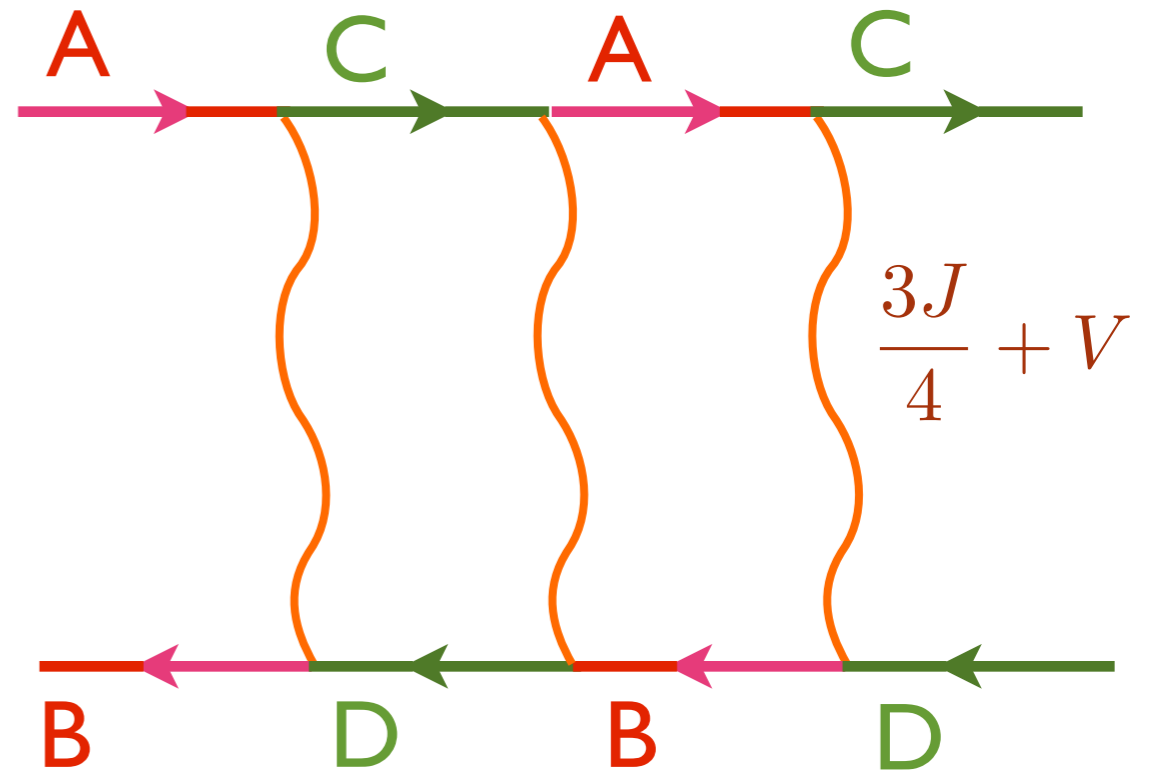
Pairing “glue” from antiferromagnetic fluctuations

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$



Same “glue” leads to *d*-wave particle-hole pairing !

$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$

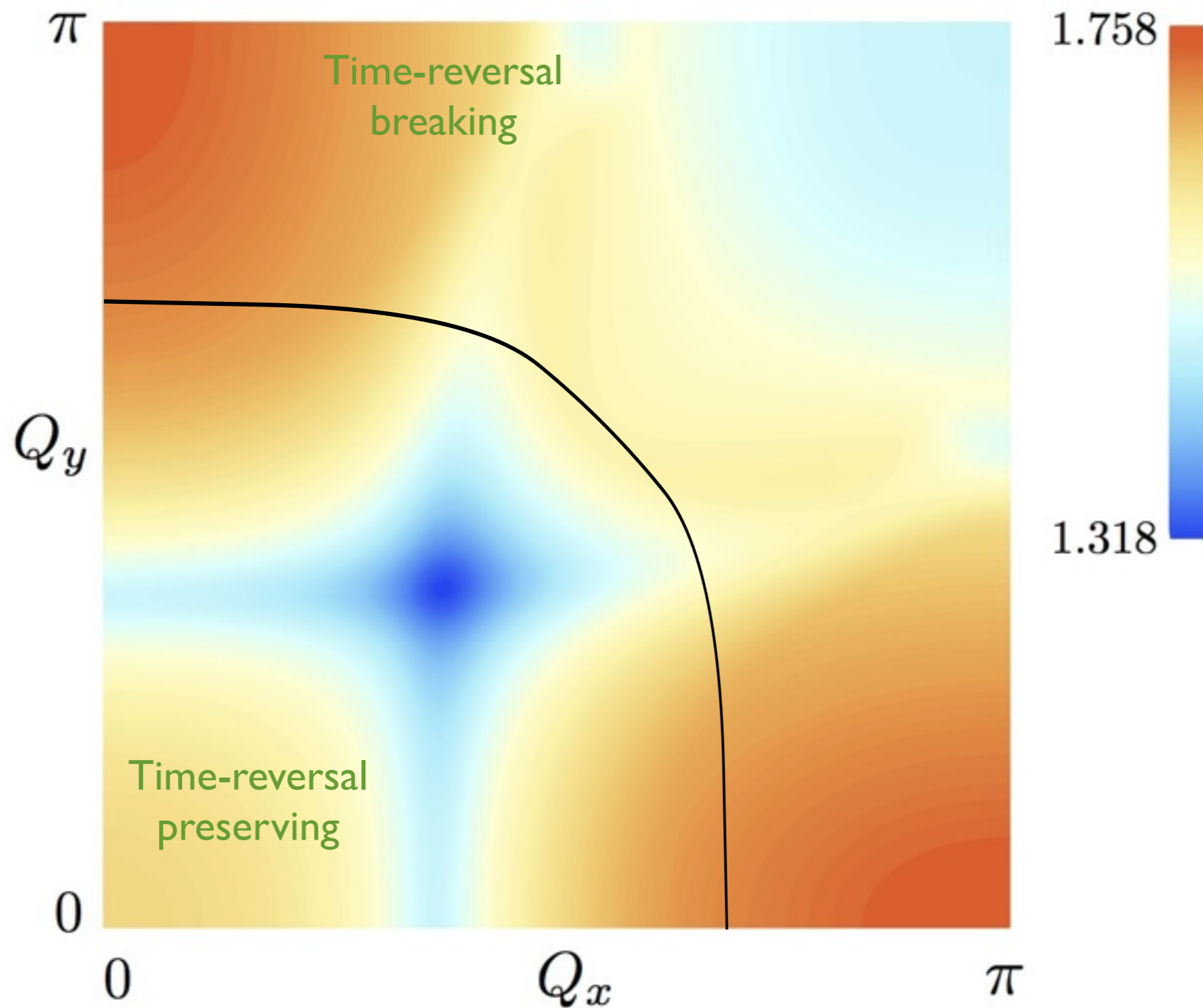


M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)

J. C. Davis and Dung-Hai Lee, *Proc. Natl. Acad. Sci.* **110**, 17623 (2013)

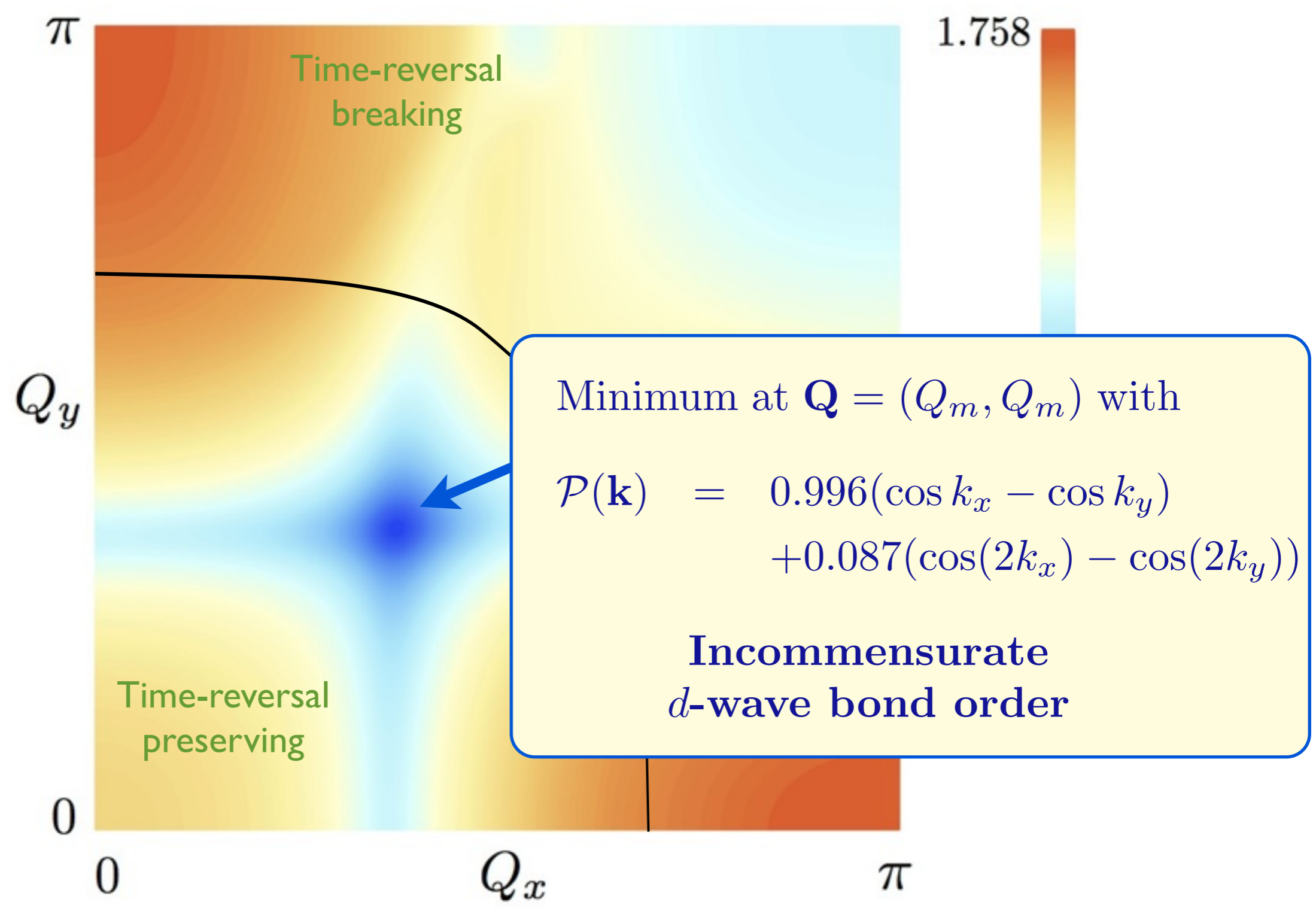
J. D. Sau and S. Sachdev, *Phys. Rev. B* **89**, 075129 (2014)



Eigenvalues, $\lambda(\mathbf{Q})$, of the spin-singlet, particle-hole propagator.

The corresponding eigenvector is $\mathcal{P}(\mathbf{k})$ and this leads to the order

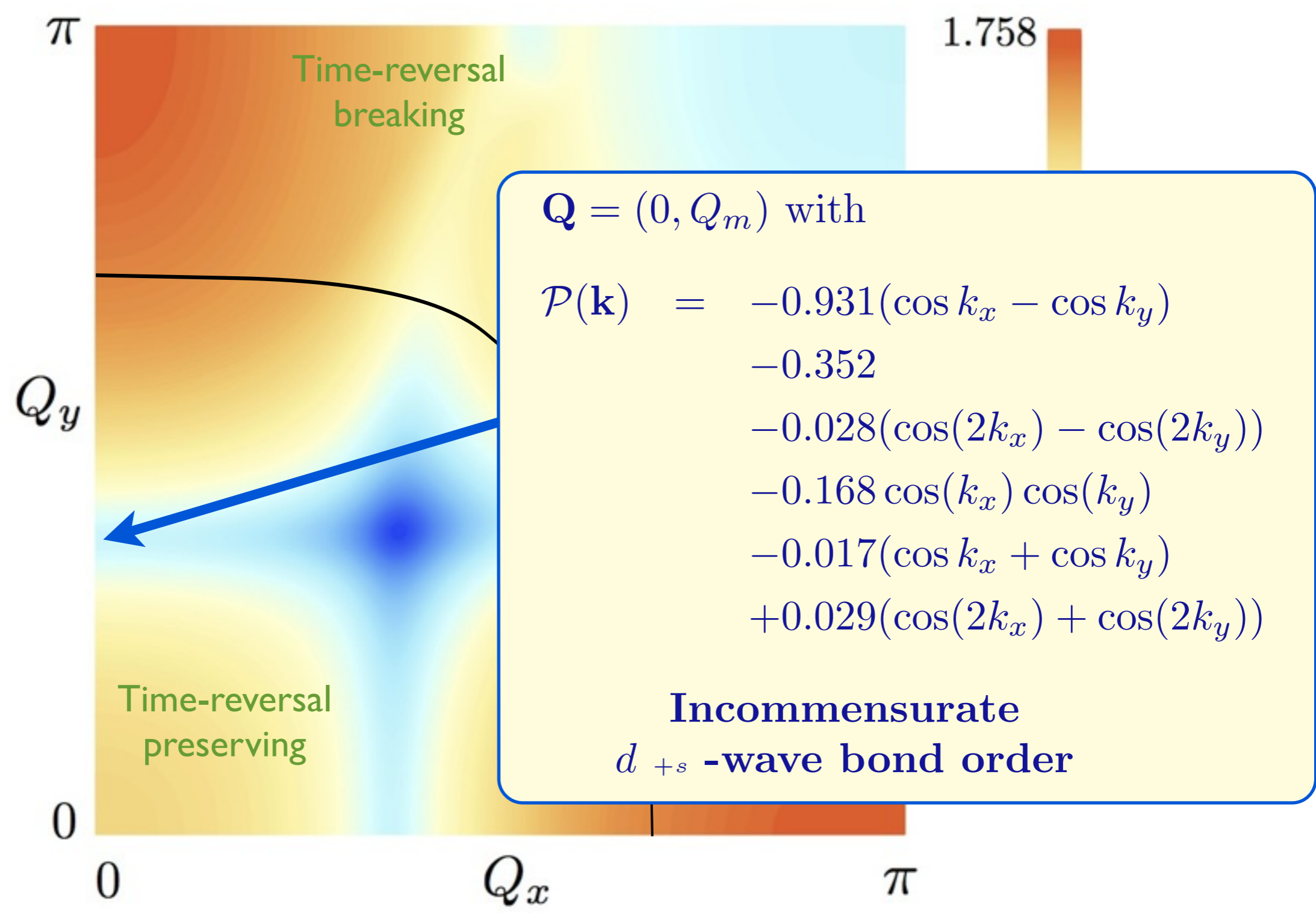
$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j) / 2}$$



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$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j) / 2}$$



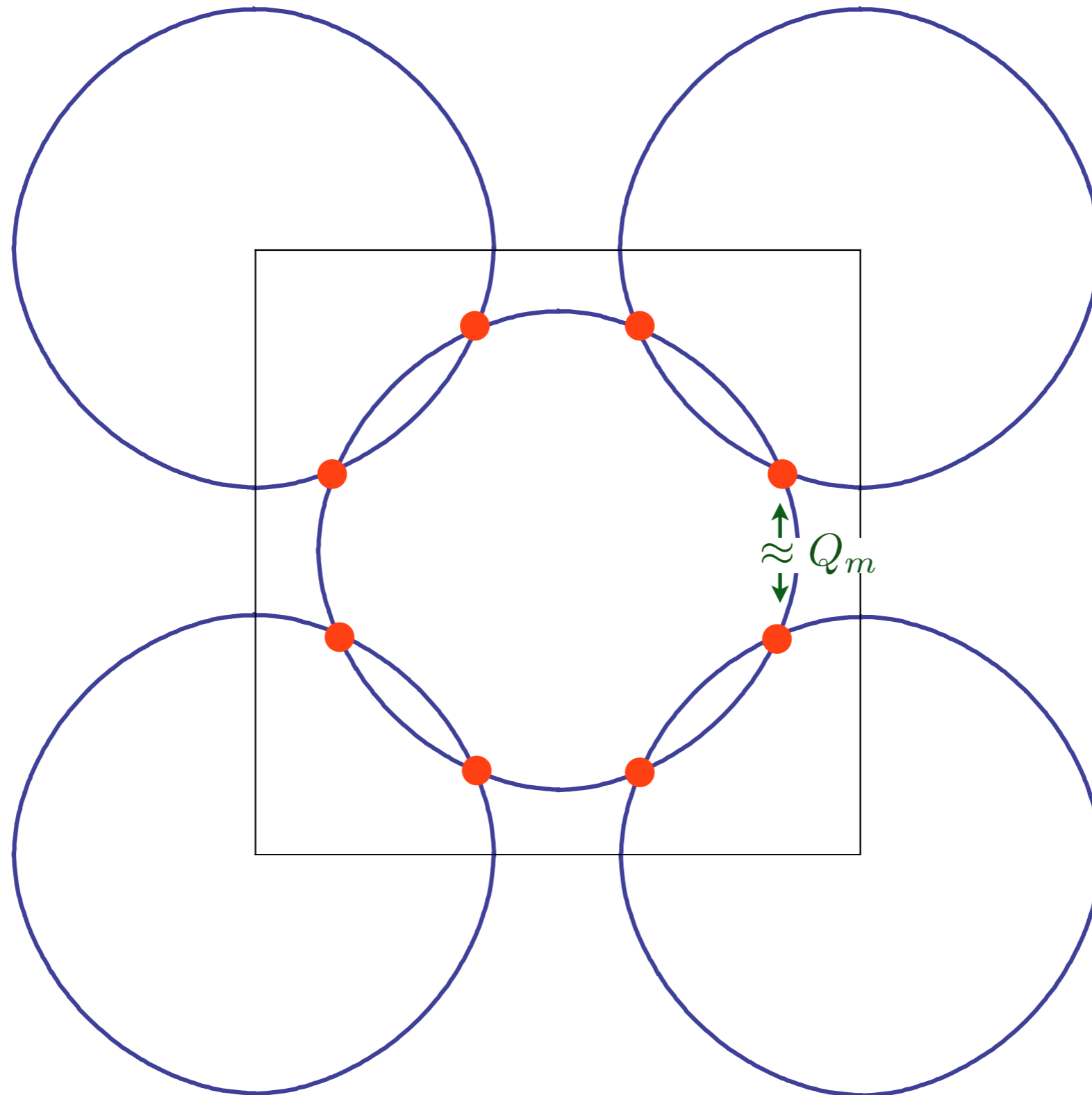


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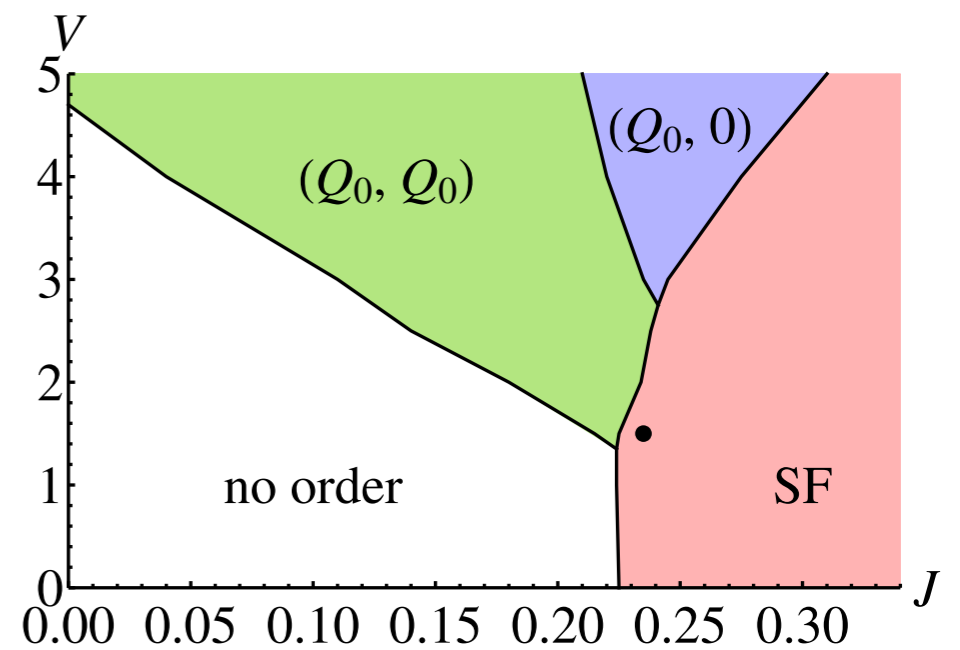
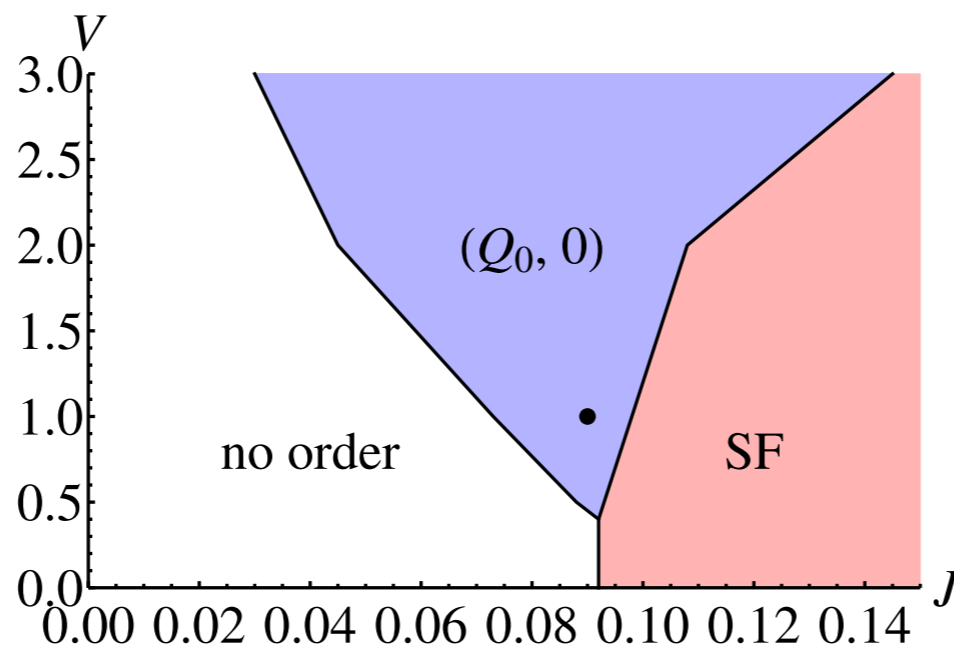
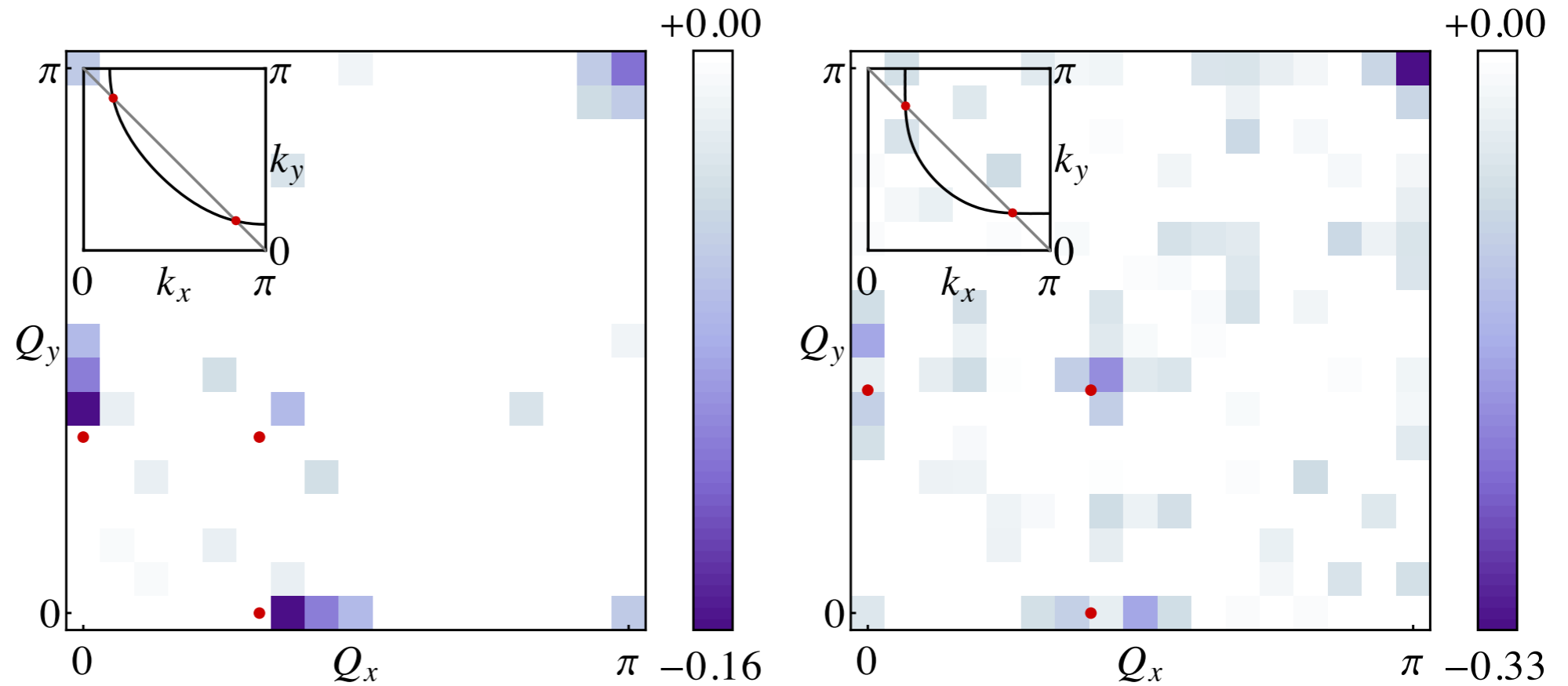


Fermi surface+antiferromagnetism



Q_m is approximately the separation between hotspots.

Results of a variational Monte Carlo computation on a wavefunction with double-occupancy projected out.
 A. Allais, J. Bauer, and S. Sachdev, arXiv:1402.4807



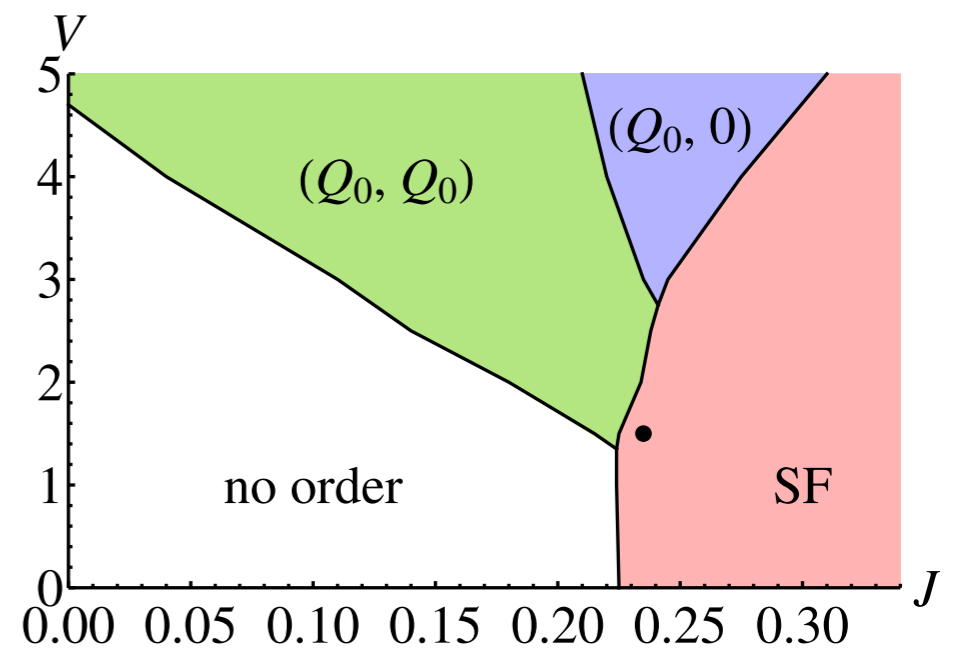
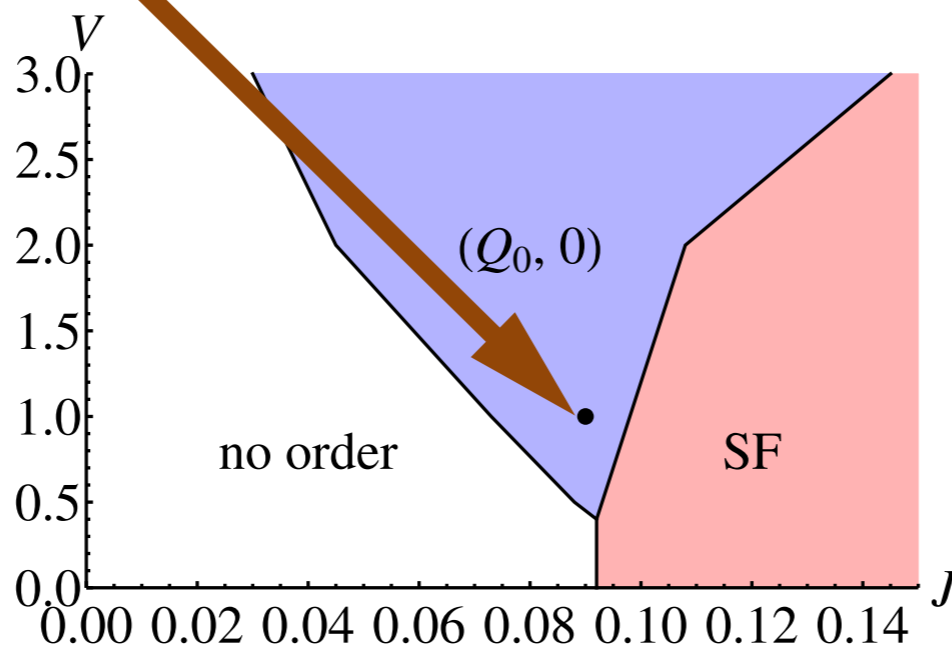
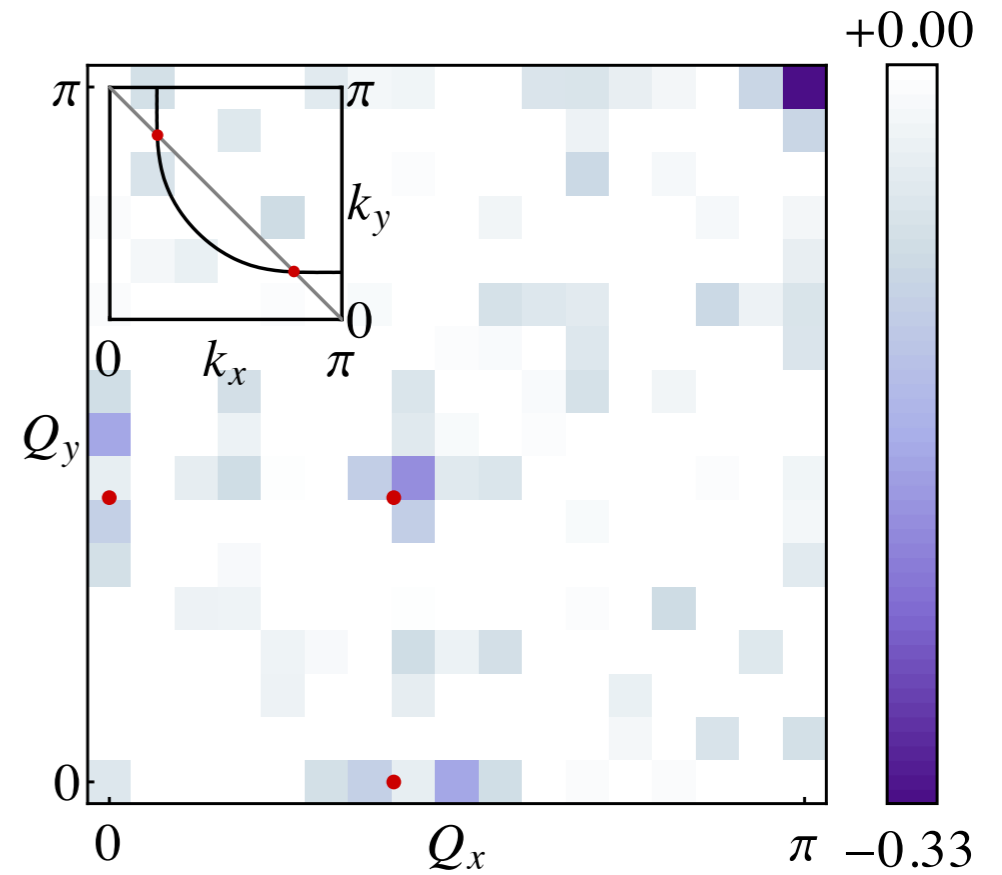
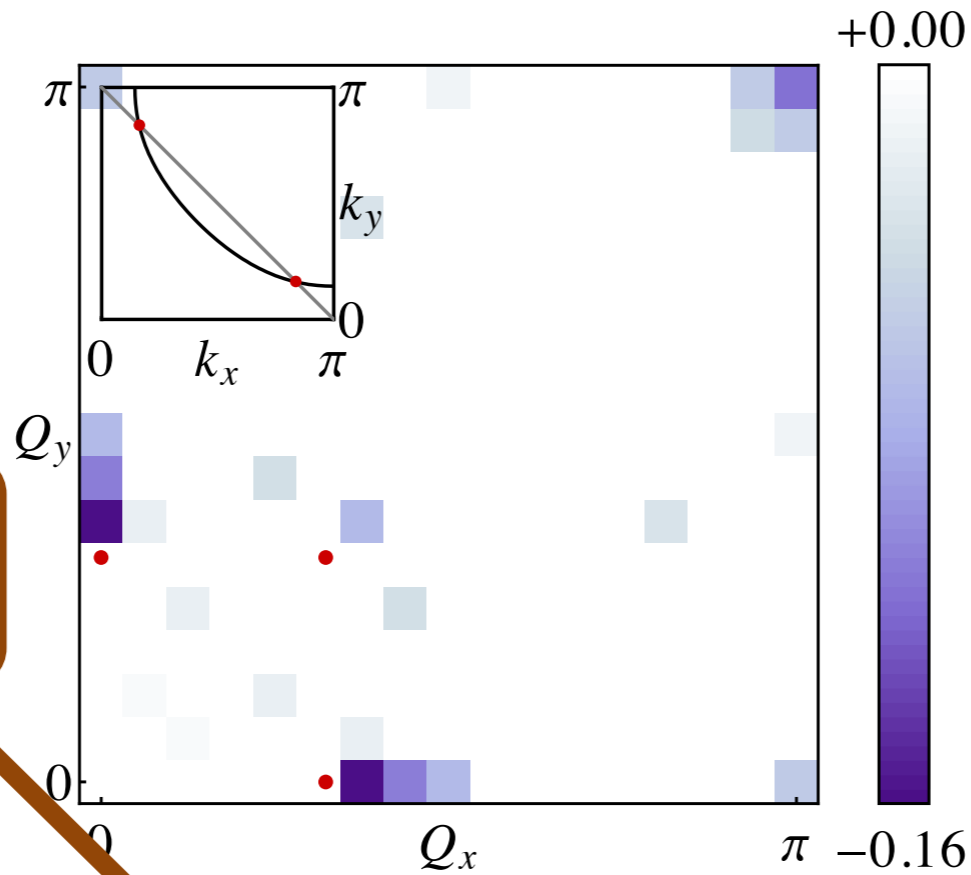
Andrea Allais

Results of a variational Monte Carlo computation on a wavefunction with double-occupancy projected out.
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Q-plot above at this value of J, V



Andrea Allais



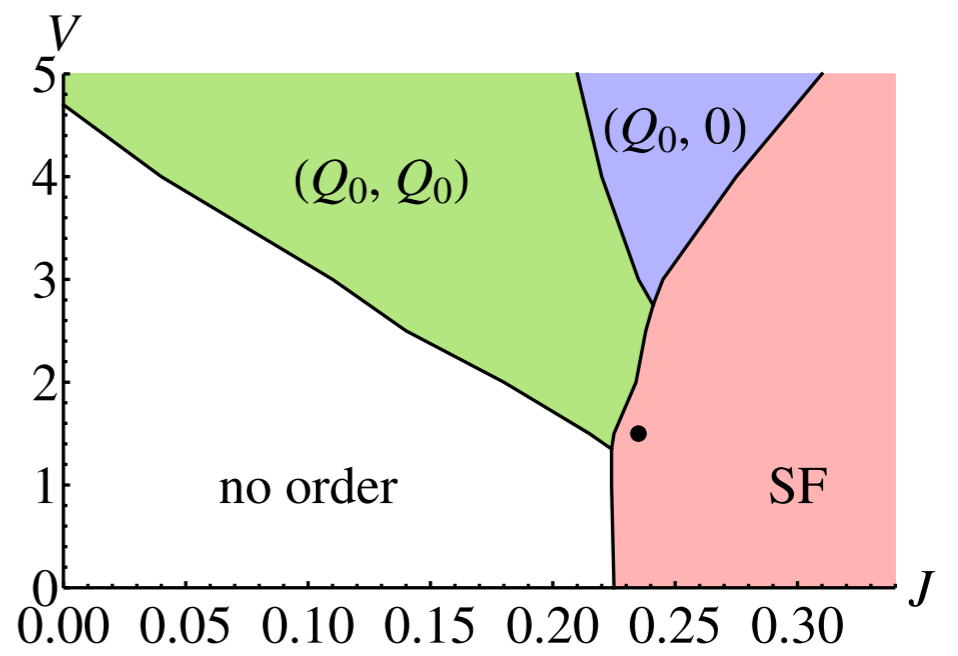
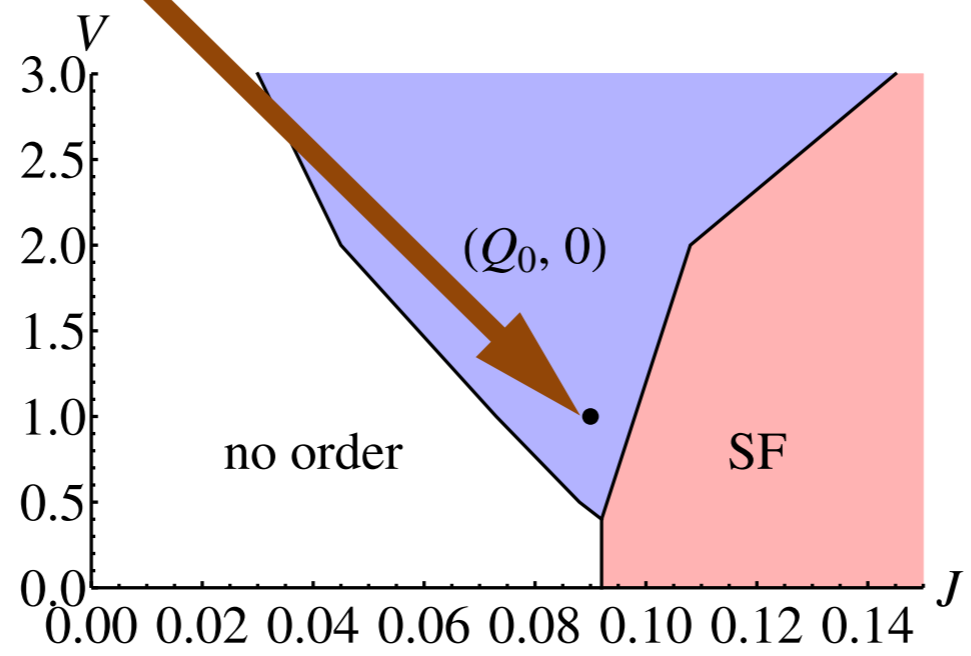
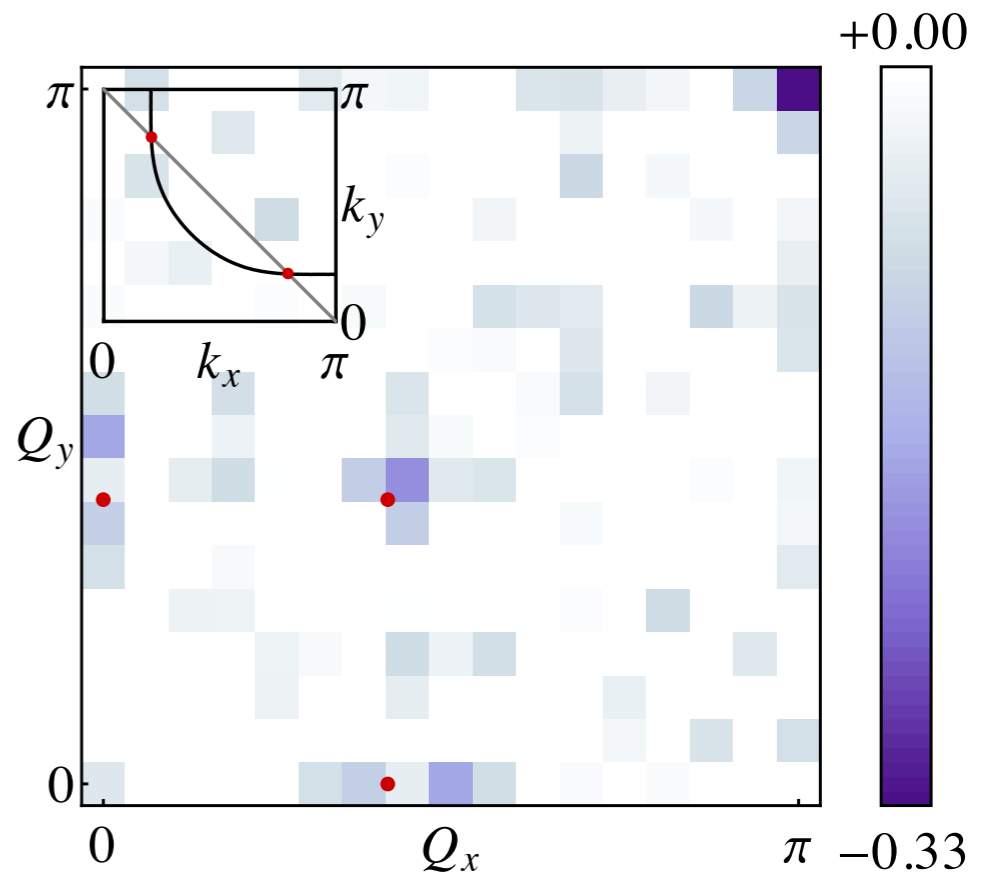
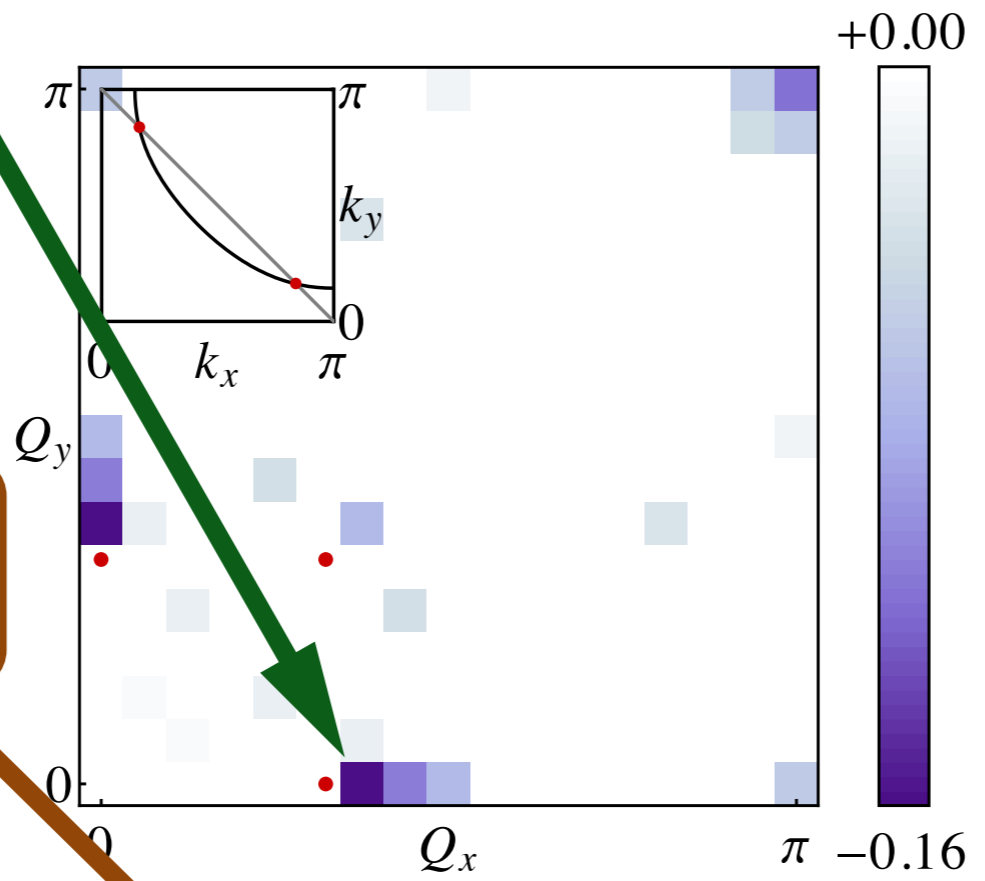
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Q of the lowest energy state. Predominantly *d* wave

Q-plot above at this value of J, V



Andrea Allais



See also:

Feedback of SC fluctuations on the wavevector of the charge order

D. Chowdhury and S. Sachdev, to appear



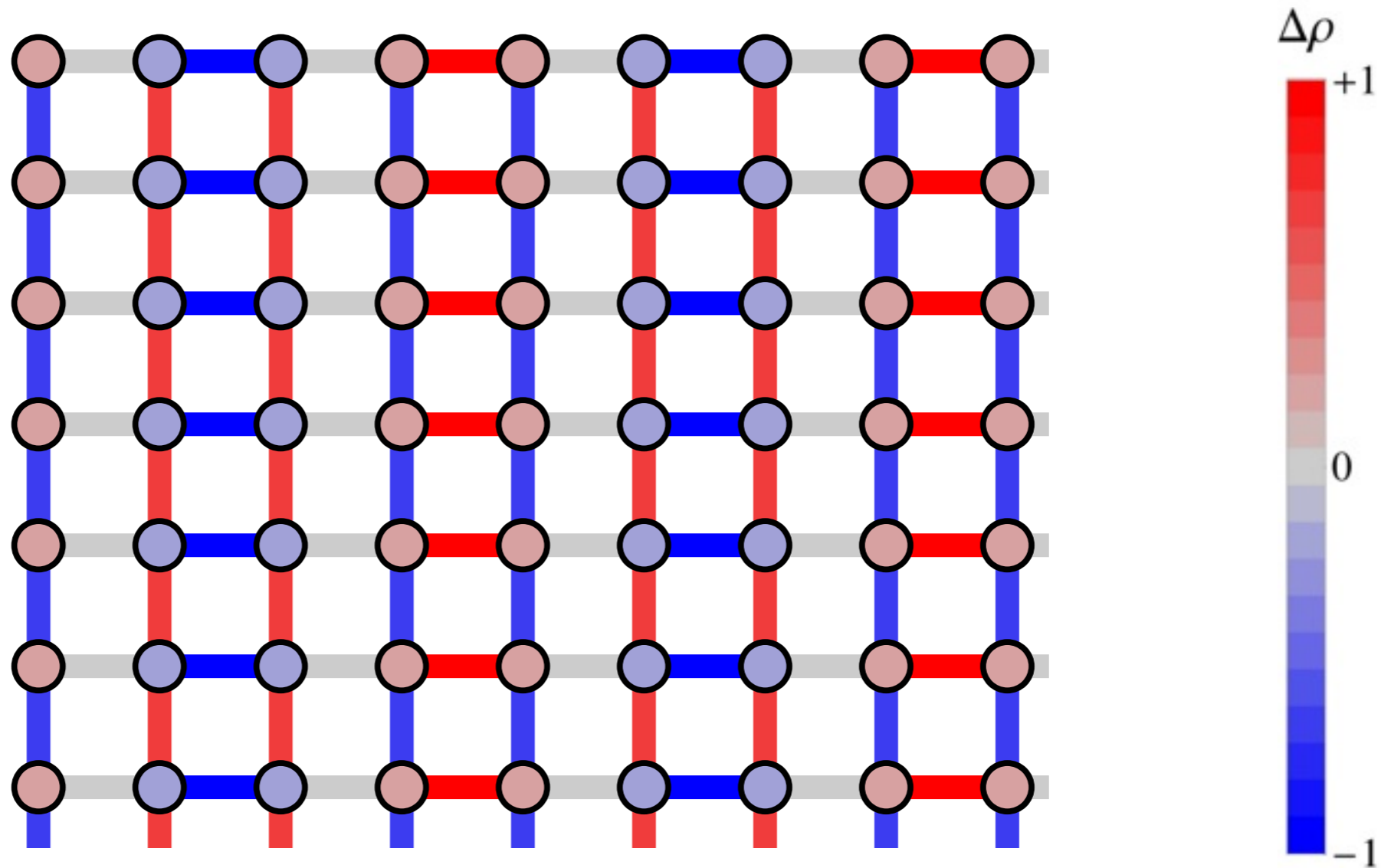
d -wave bond order.

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and i, j nearest neighbors.

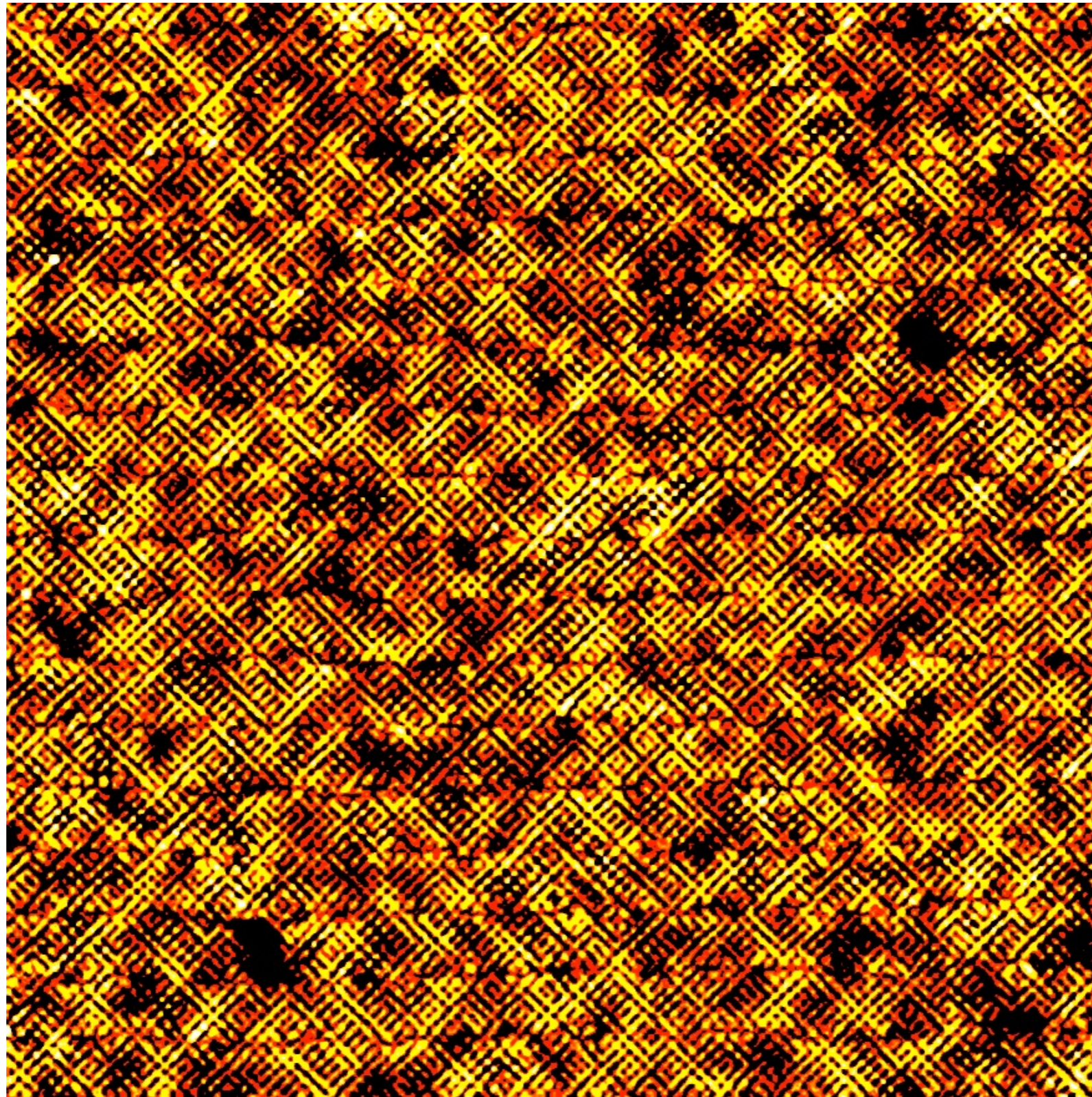
$$P_{ij} = \left[\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$$

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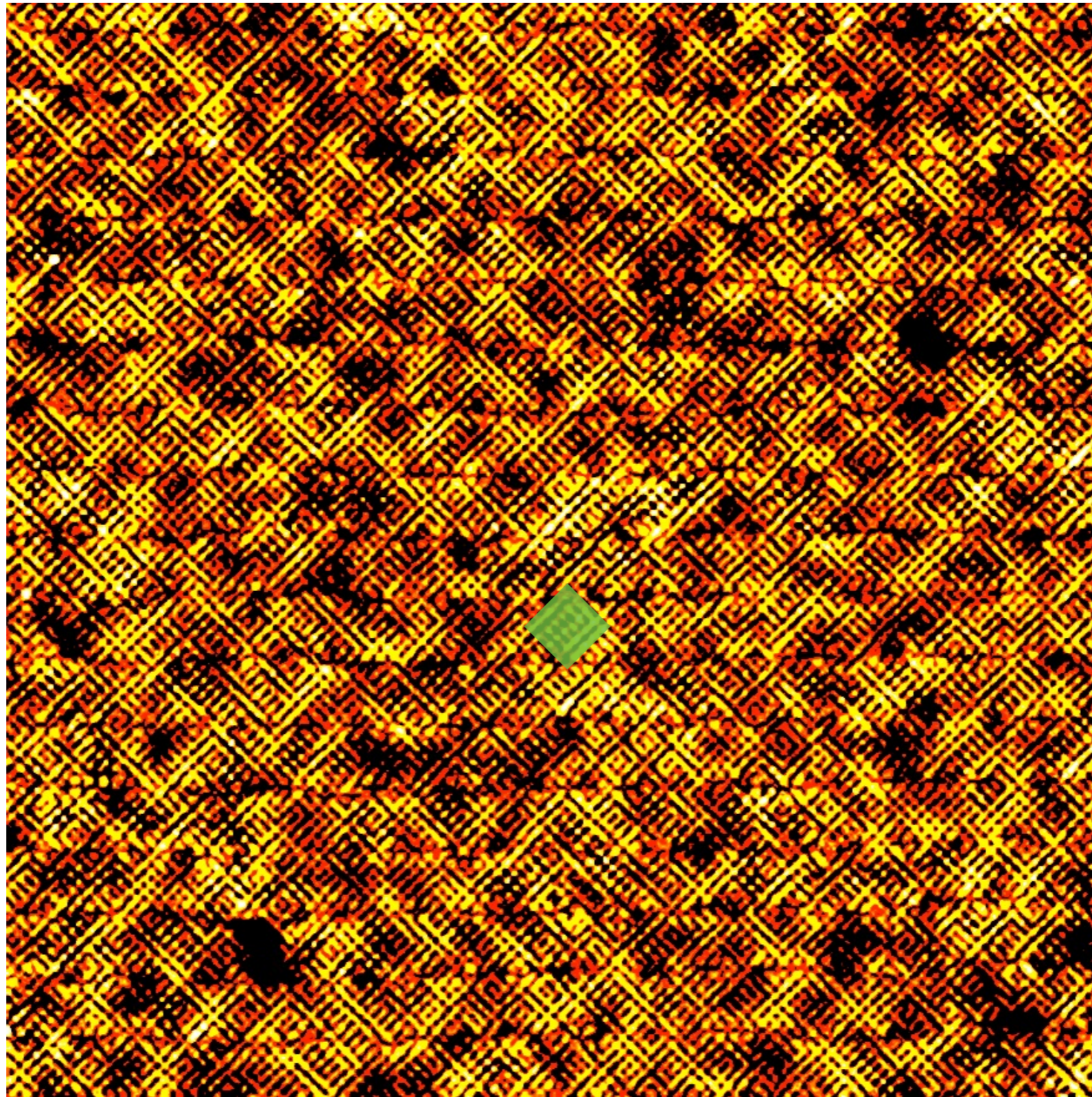


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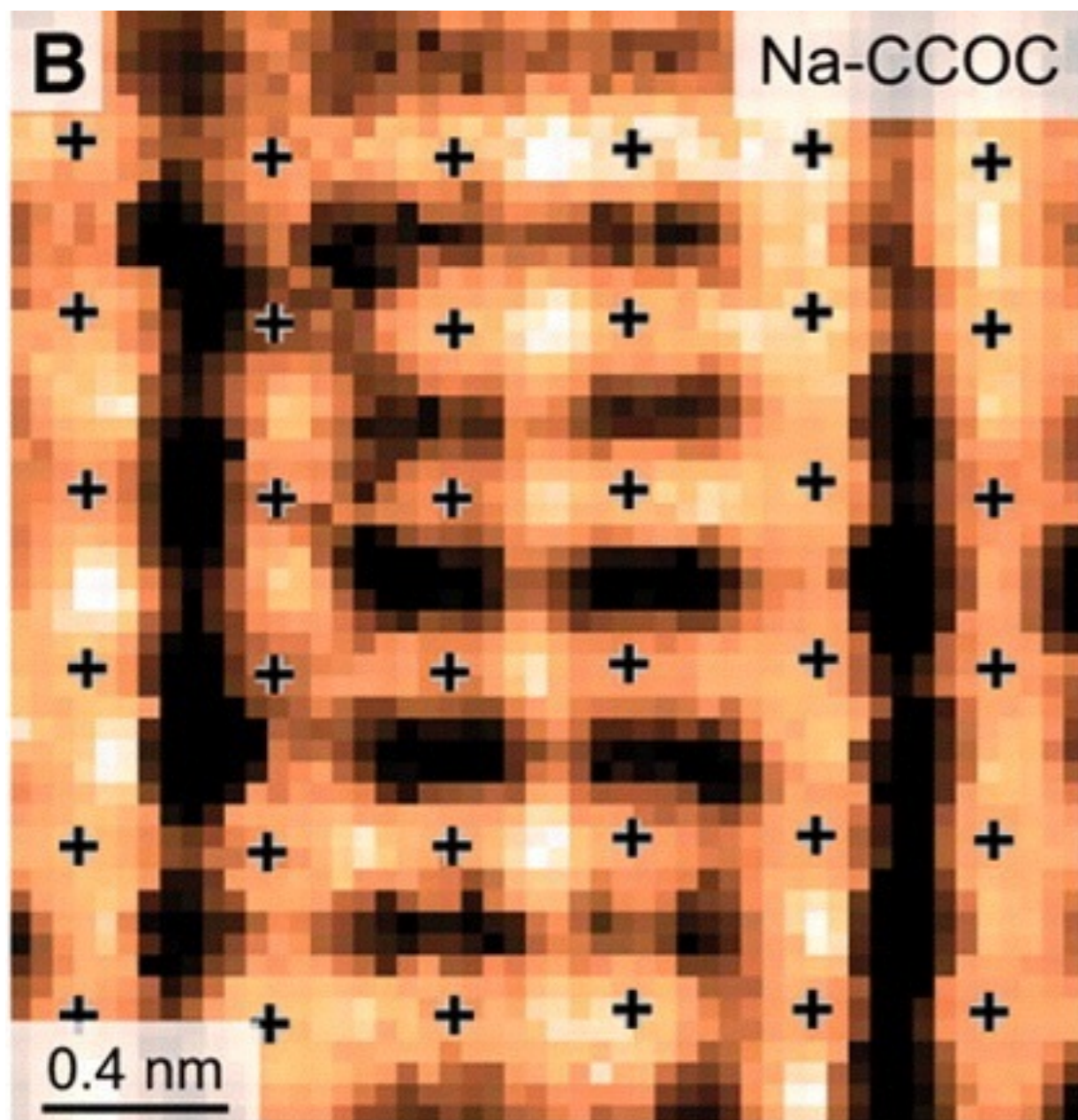
(My “cosmic
background”
map)

“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007).

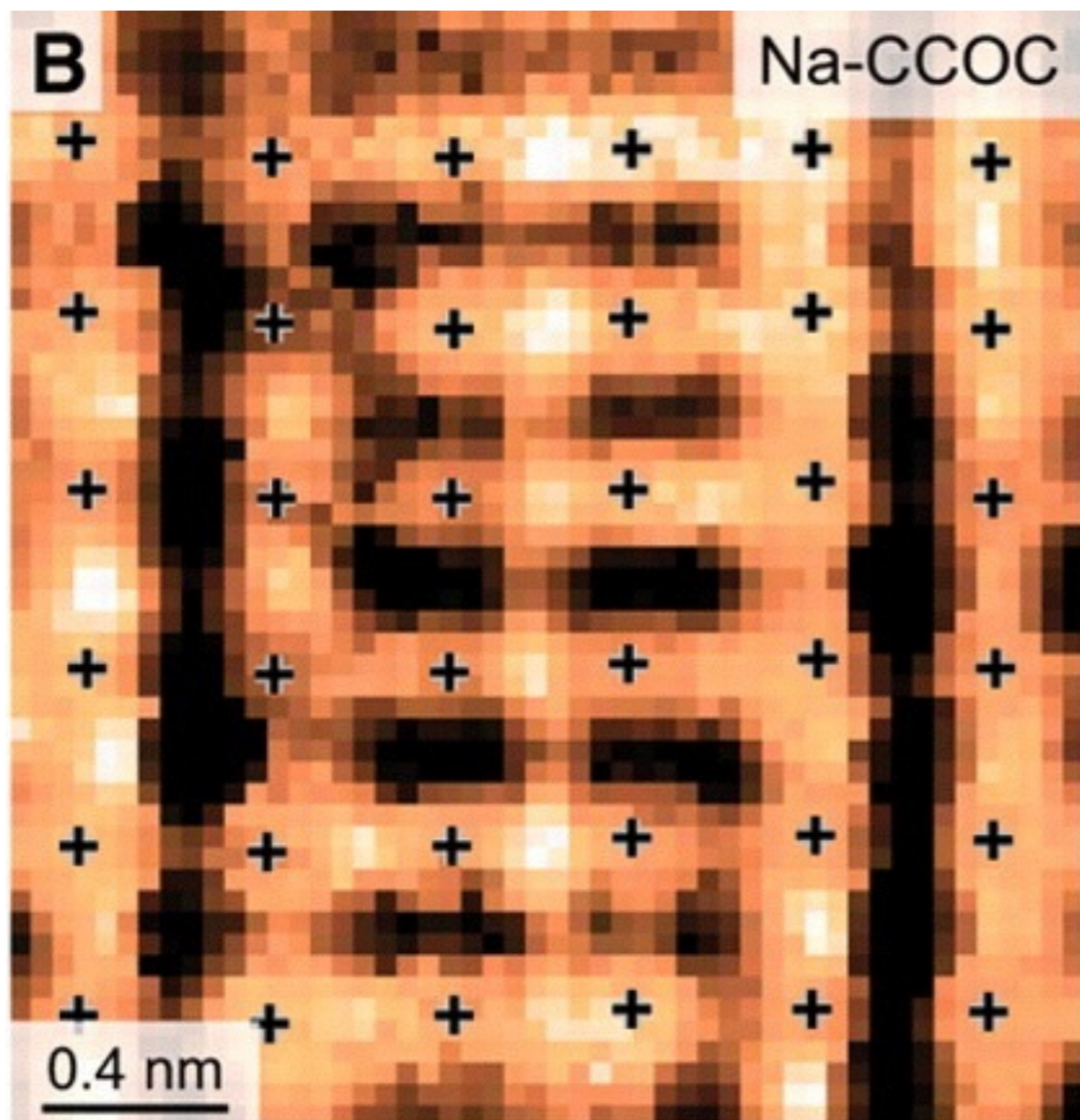


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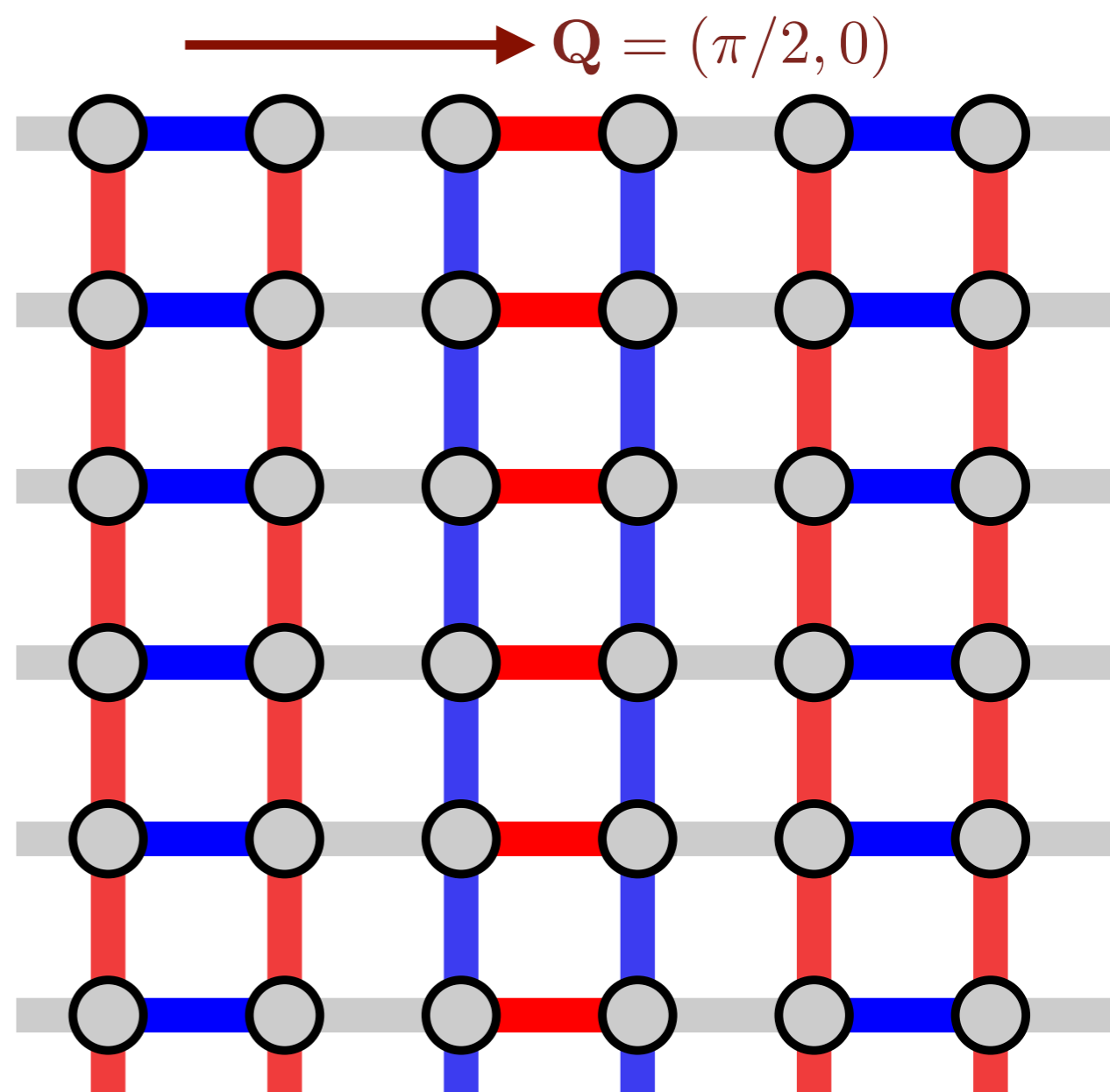
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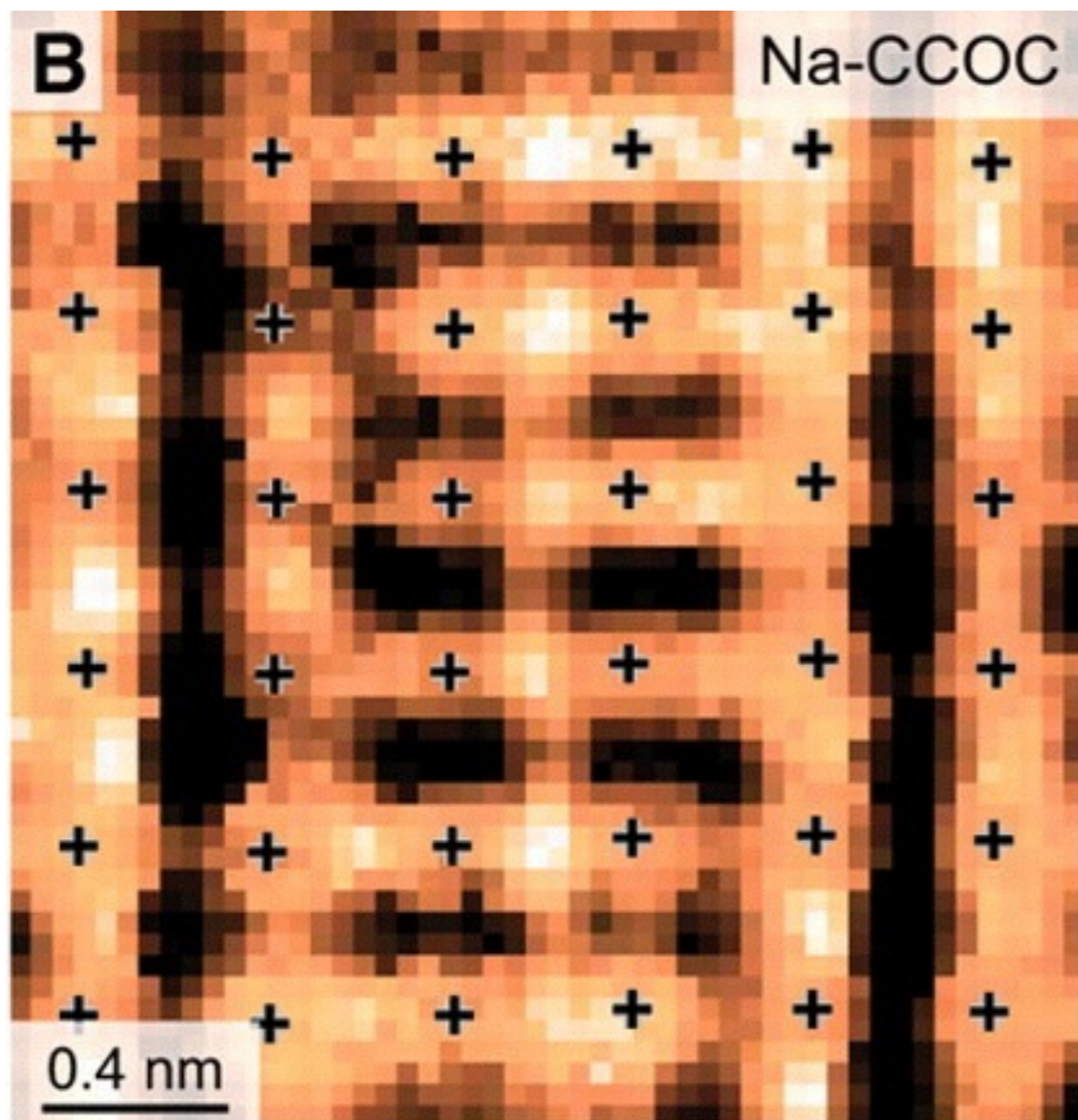
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



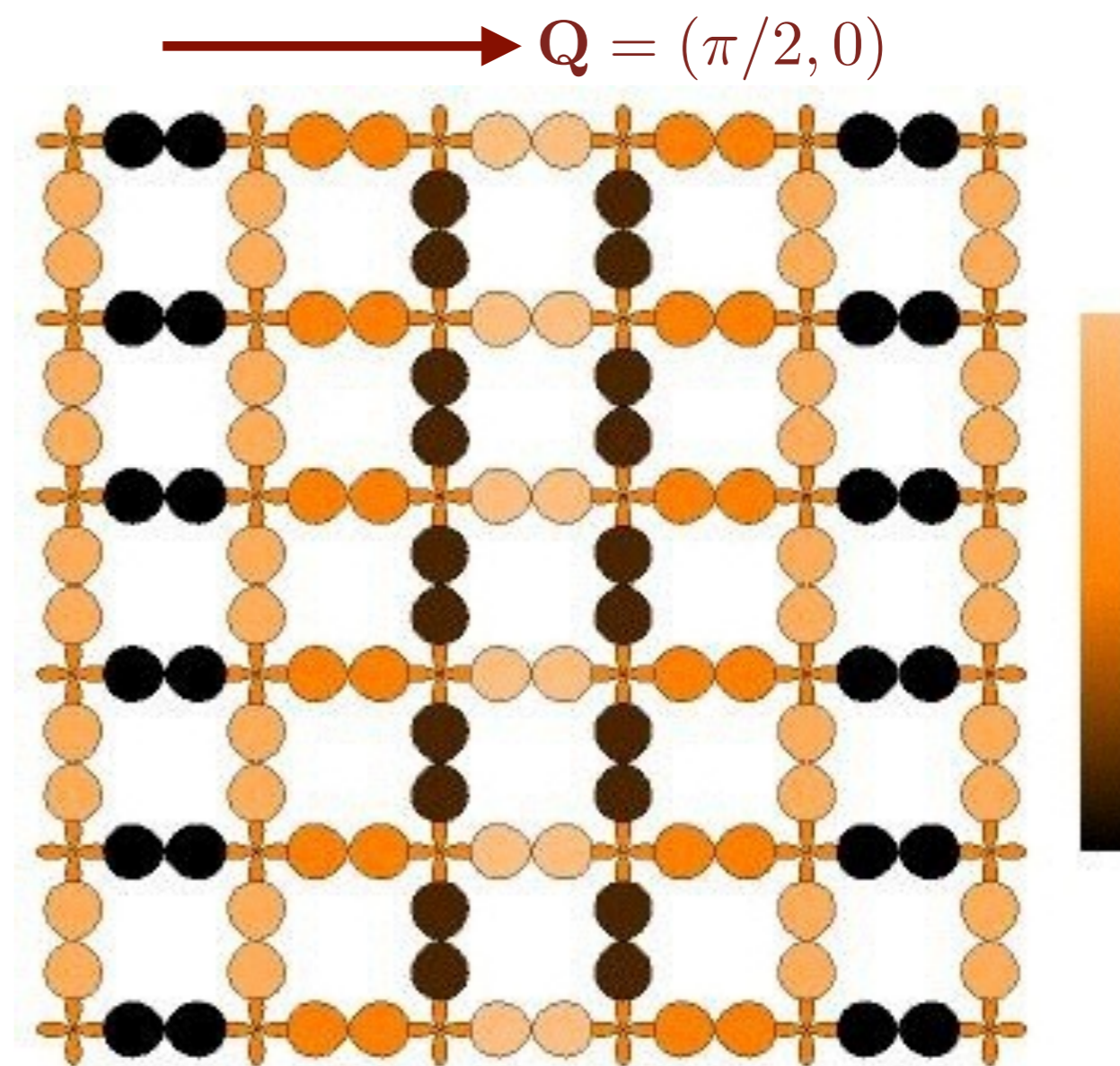
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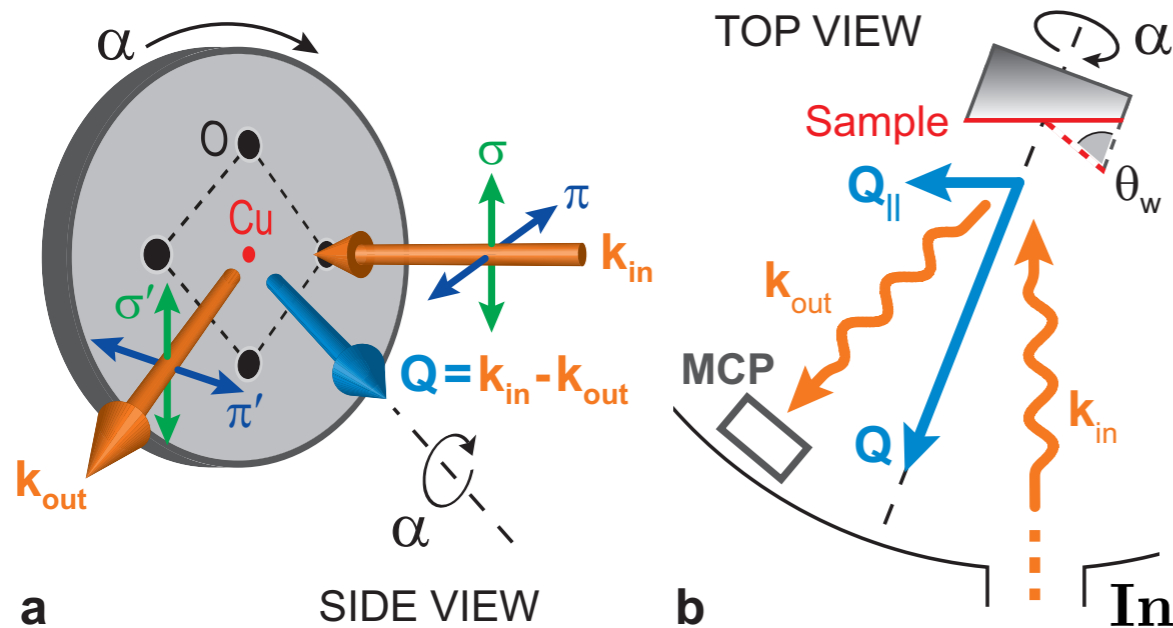


d-wave bond order

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S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

The symmetry of charge order in the cuprates

R. Comin, R. Sutarto, F. He, E. da Silva Neto, L. Chauviere, A. Frano, R. Liang, W.N. Hardy, D.A. Bonn, Y. Yoshida, H. Eisaki, J. E. Hoffman, B. Keimer, G.A. Sawatzky, and A. Damascelli, arXiv:1402.5415

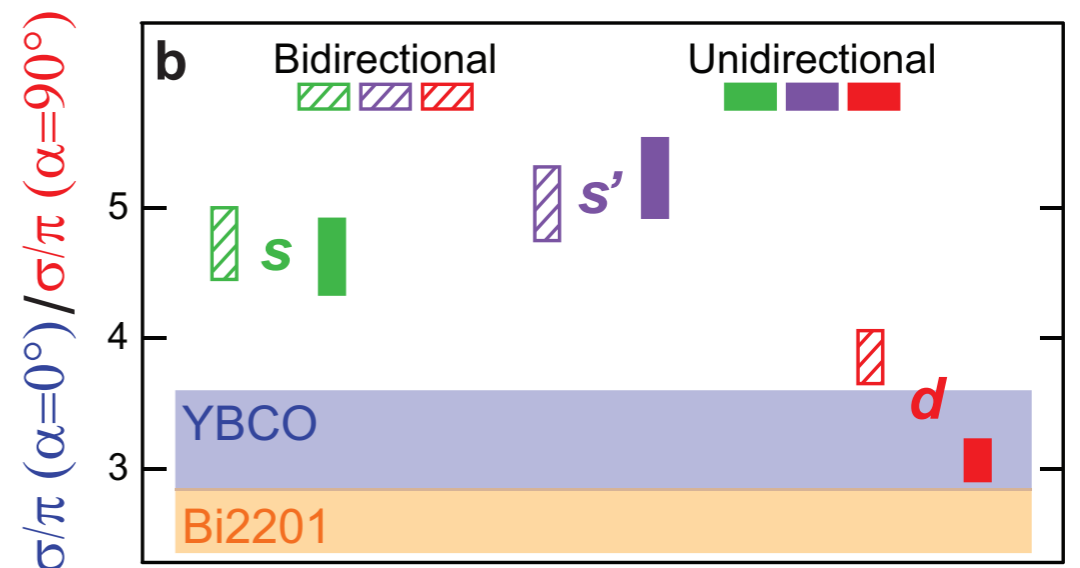
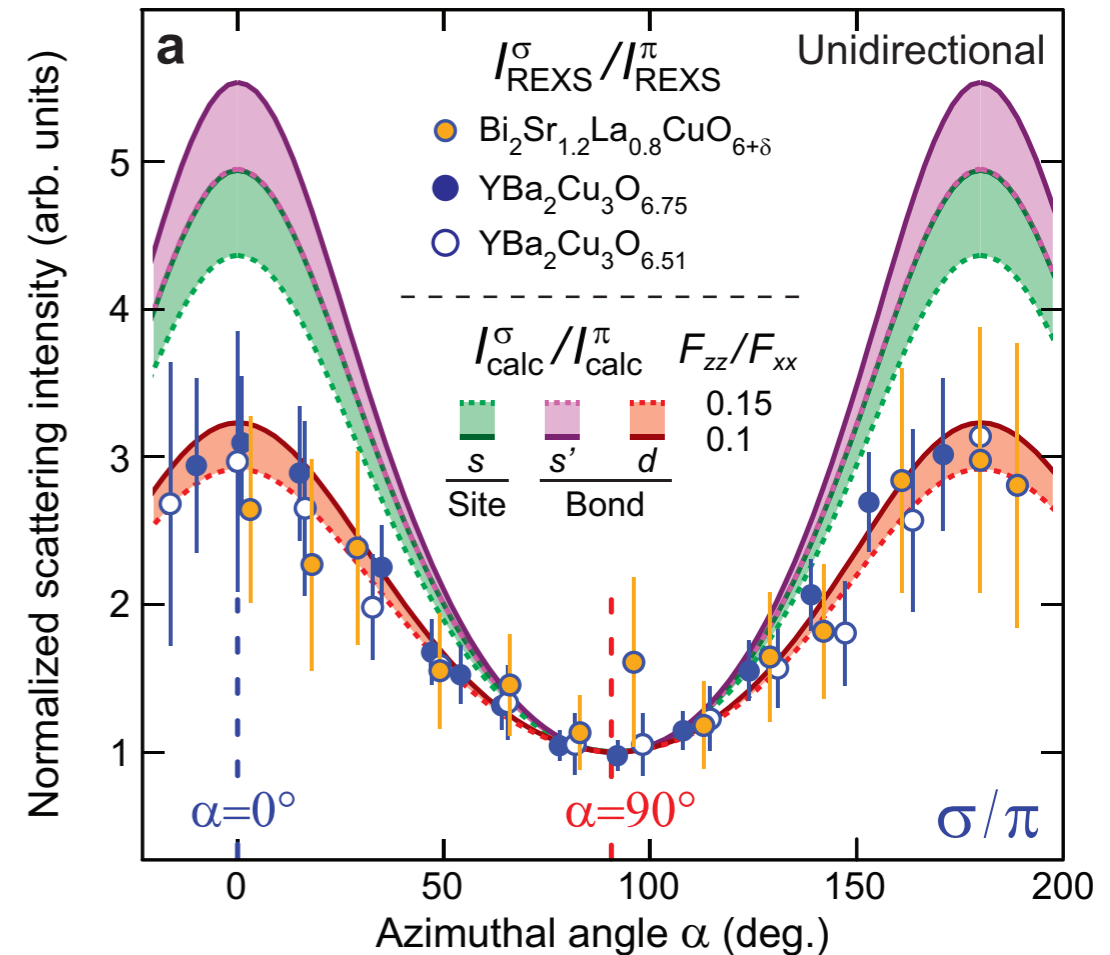
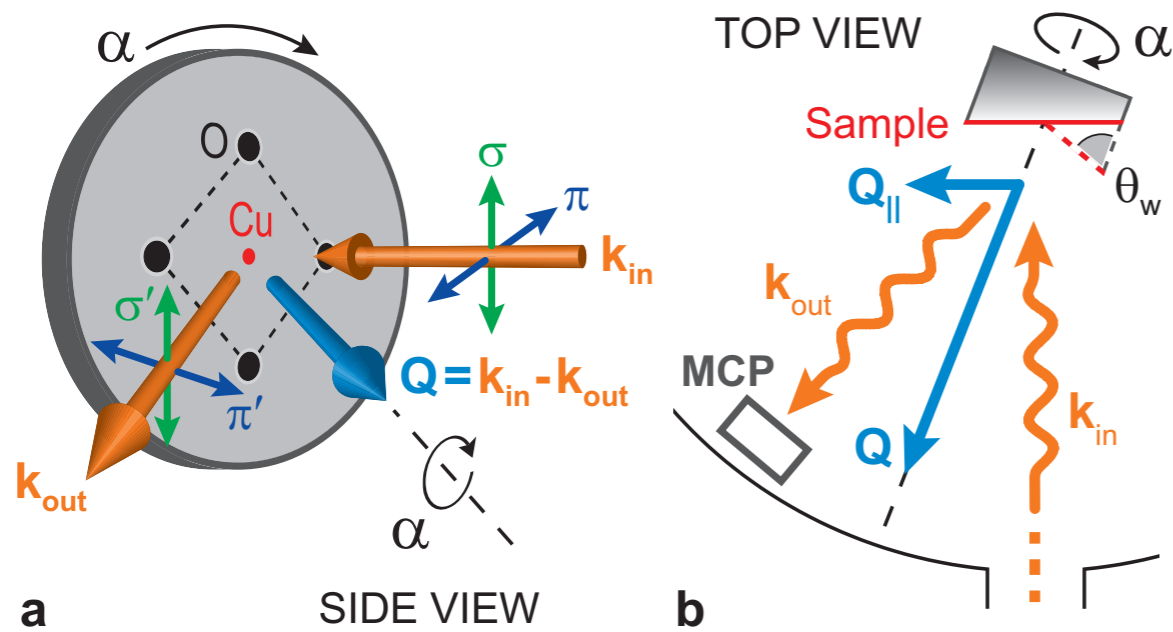


Polarization and angular analysis of X-ray scattering at $\mathbf{Q} = (Q_0, 0)$ in Bi2201 and YBCO

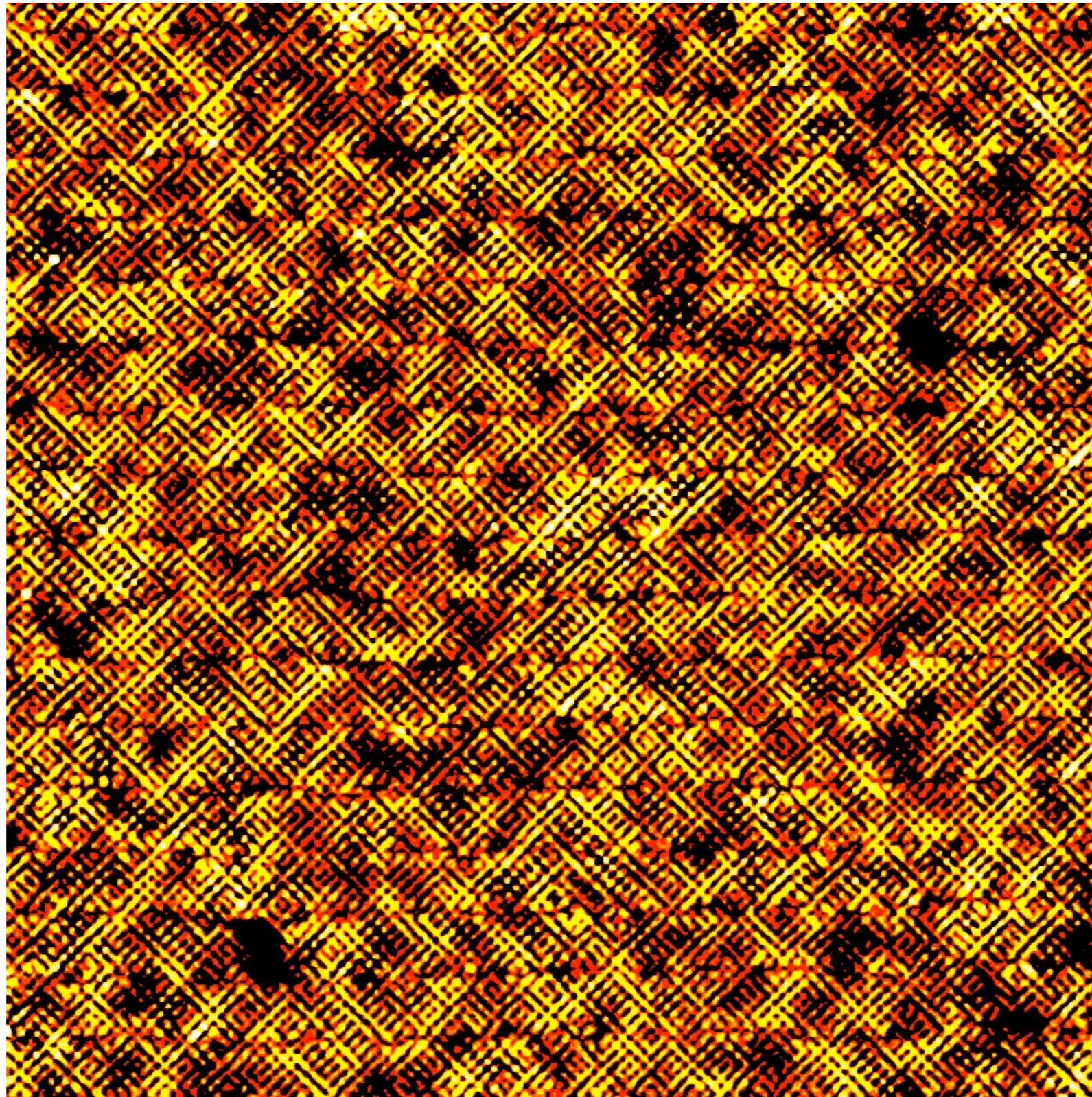
In addition, by adopting a special experimental geometry, we also resolve the *intra-unit-cell* symmetry of the charge ordered state, which is revealed to be a ***d-wave bond-order***. These results represent a fundamental advancement in our microscopic description of charge order in cuprates, and provide crucial insights for the understanding of its origin and interplay with superconductivity and magnetism.

The symmetry of charge order in the cuprates

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Δ_{CDW}	Probability levels P (%)	
	Bidirectional	Unidirectional
Δ_s	30.3	38.8
$\Delta_{s'} (\cos k_x + \cos k_y)$	12.0	6.0
$\Delta_d (\cos k_x - \cos k_y)$	81.8	87.6



(My “cosmic
background”
map)

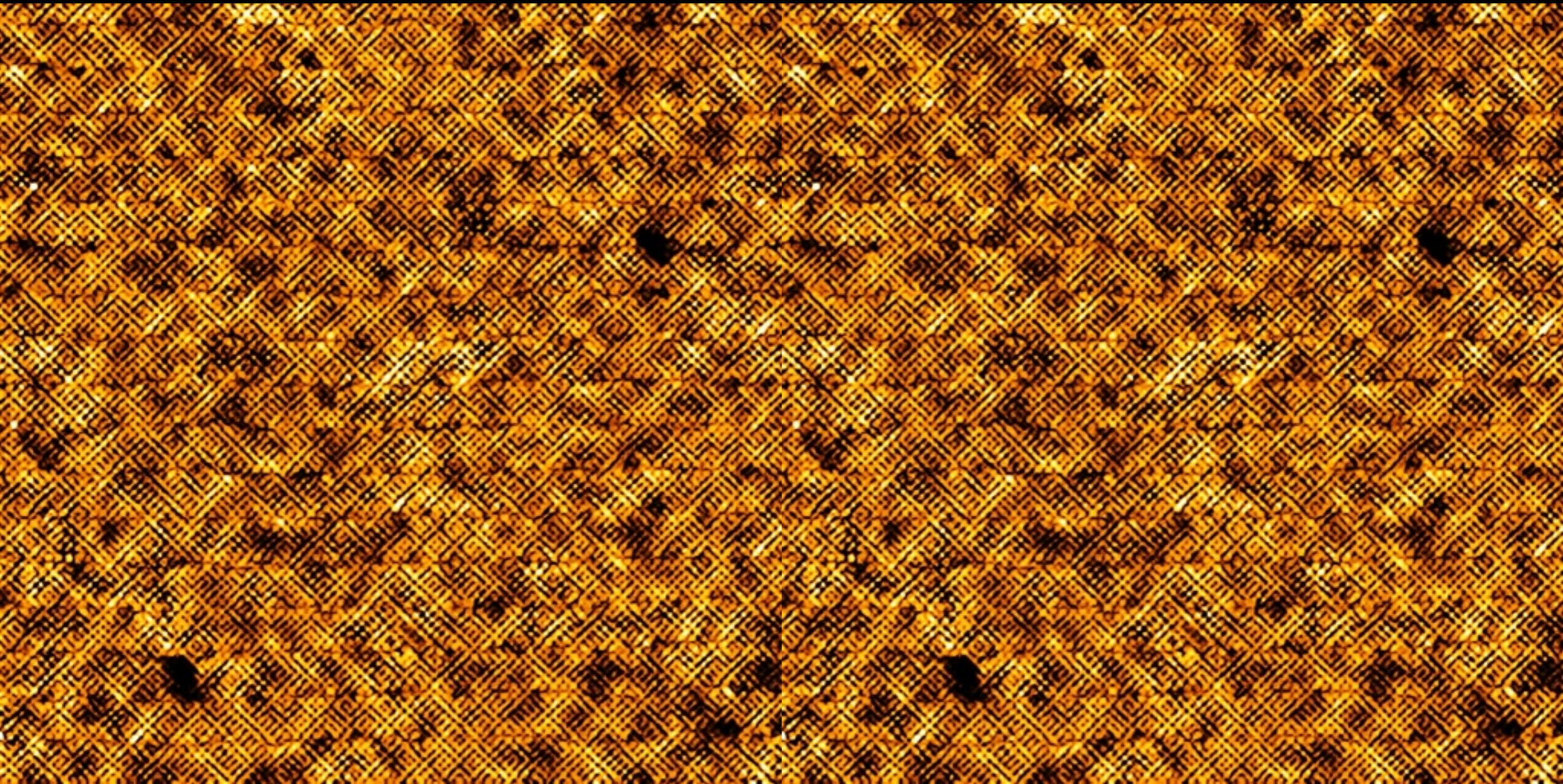
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UD45K
BSCCO

$R(r, 150\text{mV})$

$R(r, 150\text{mV})$

$R(r, 150\text{mV})$



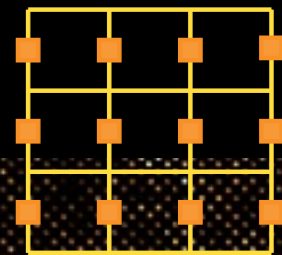
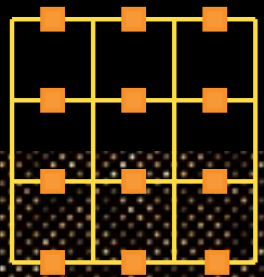
Note that these are identical images.

UD45K

$R(r=0, 150\text{mV})$

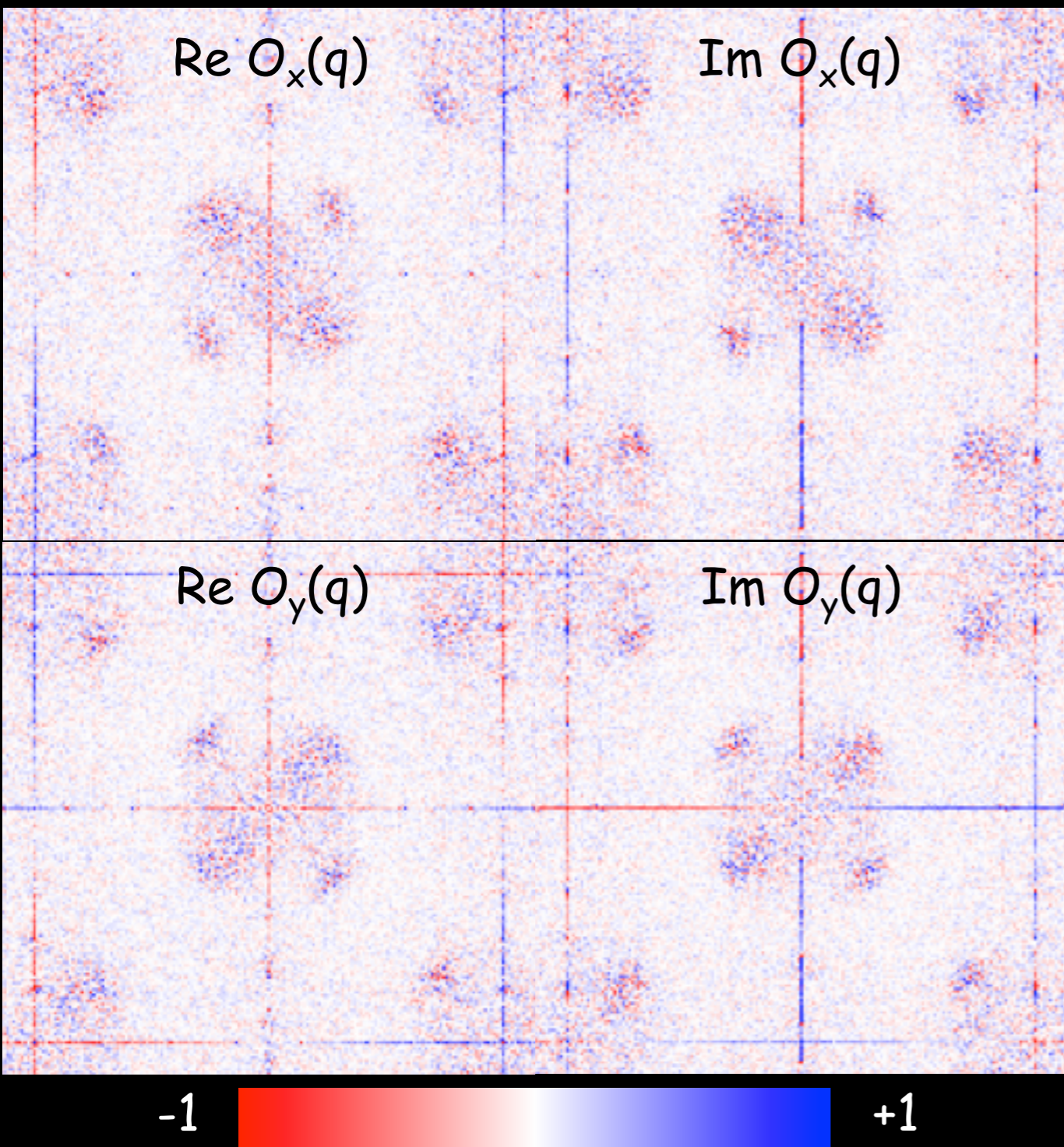
$R(r=O_x, 150\text{mV})$

$R(r=O_y, 150\text{mV})$



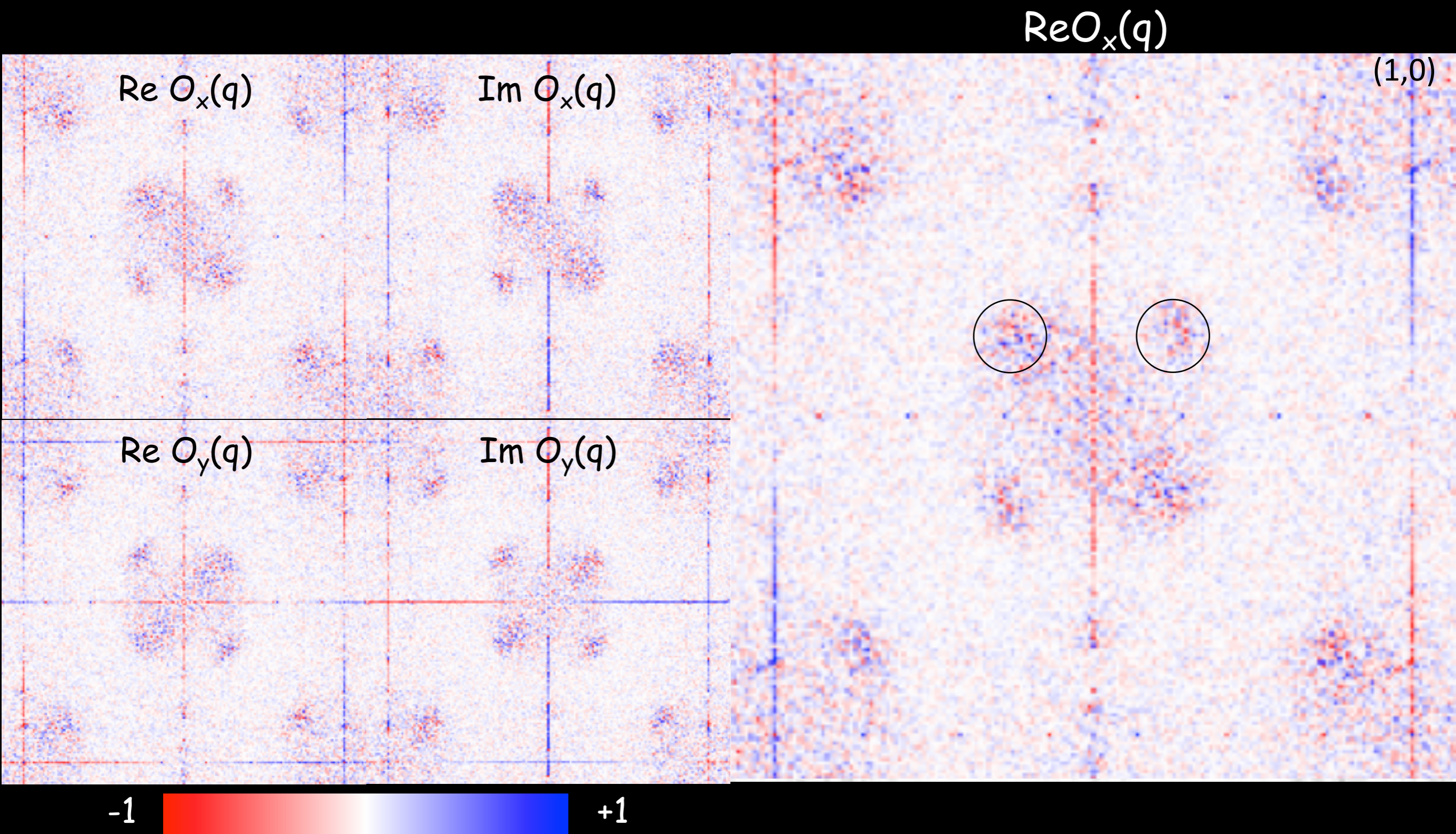
UD45K

Q-space electronic structure

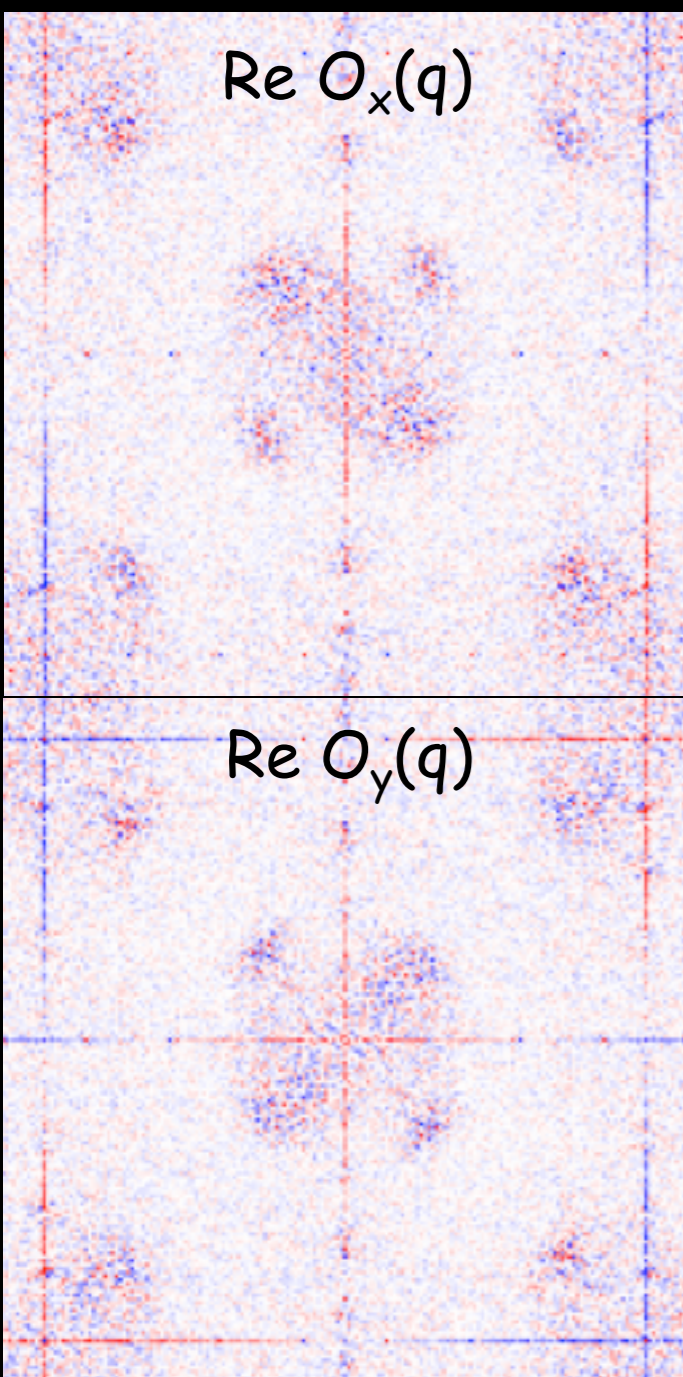


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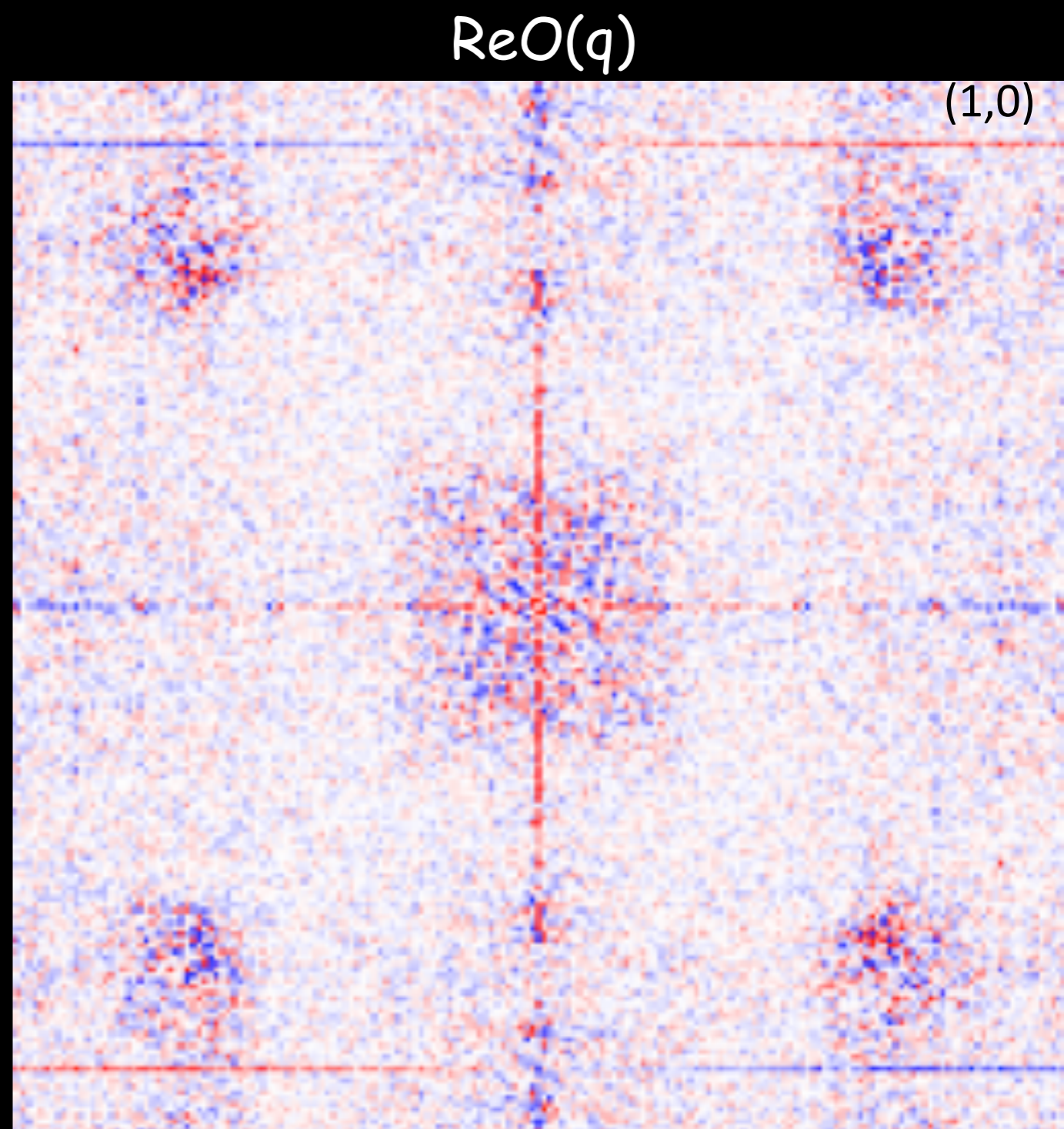
Broad (0,Q) and (Q,0) DW Features



UD45K

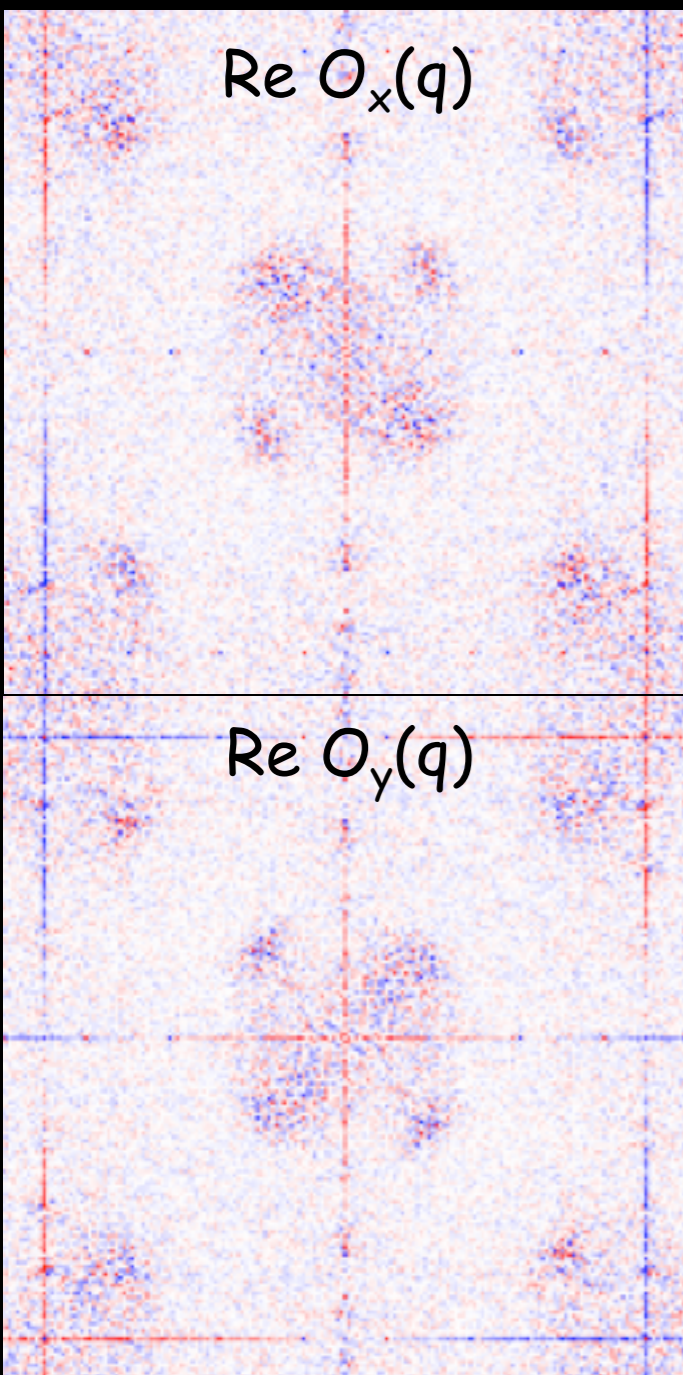


$\text{Re } O_x + \text{Re } O_y$

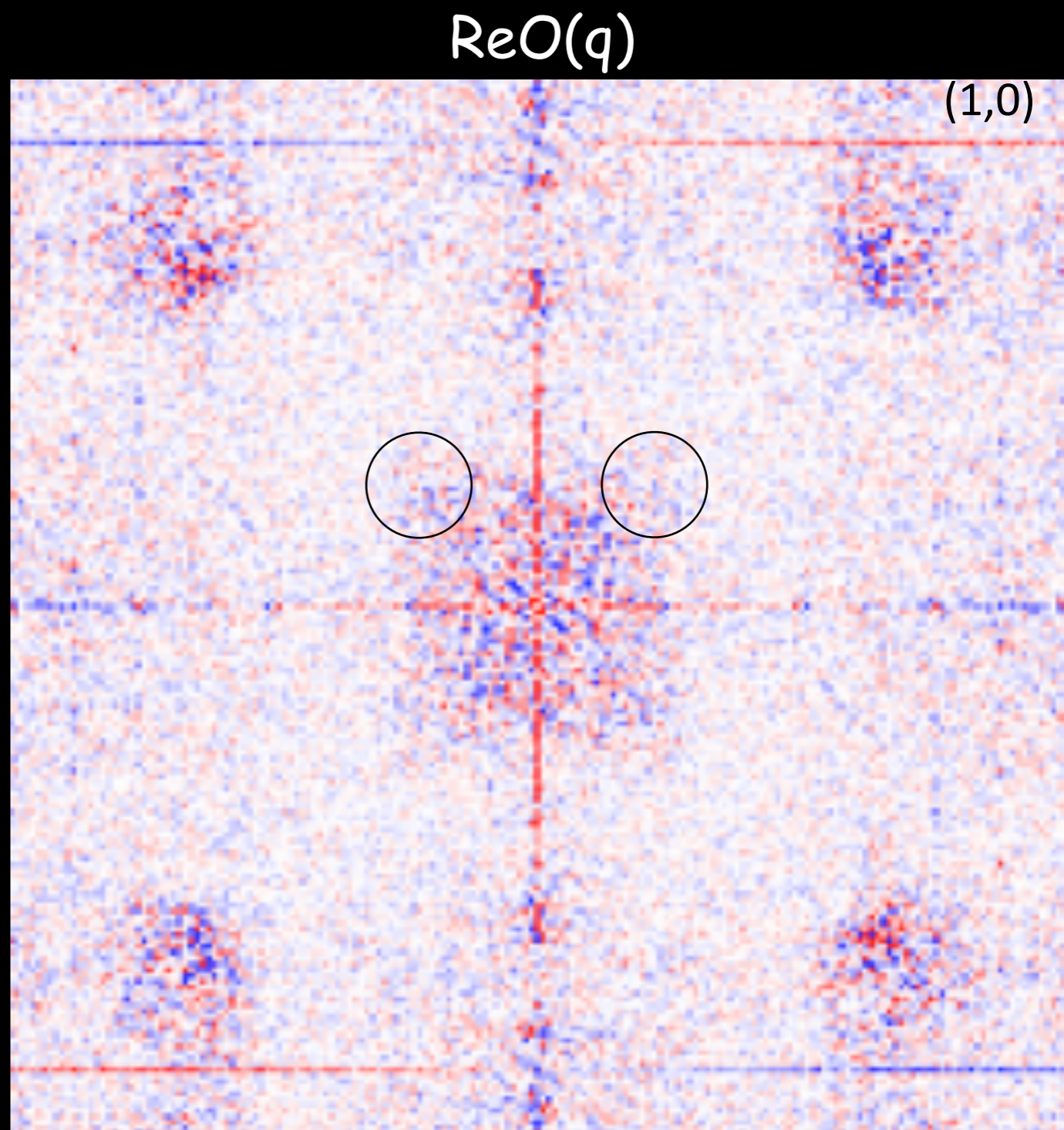


UD45K

$\text{Re}Q_x(0,Q)$ Vs $\text{Re}O_y(0,Q)$



$\text{Re} O_x + \text{Re} O_y$



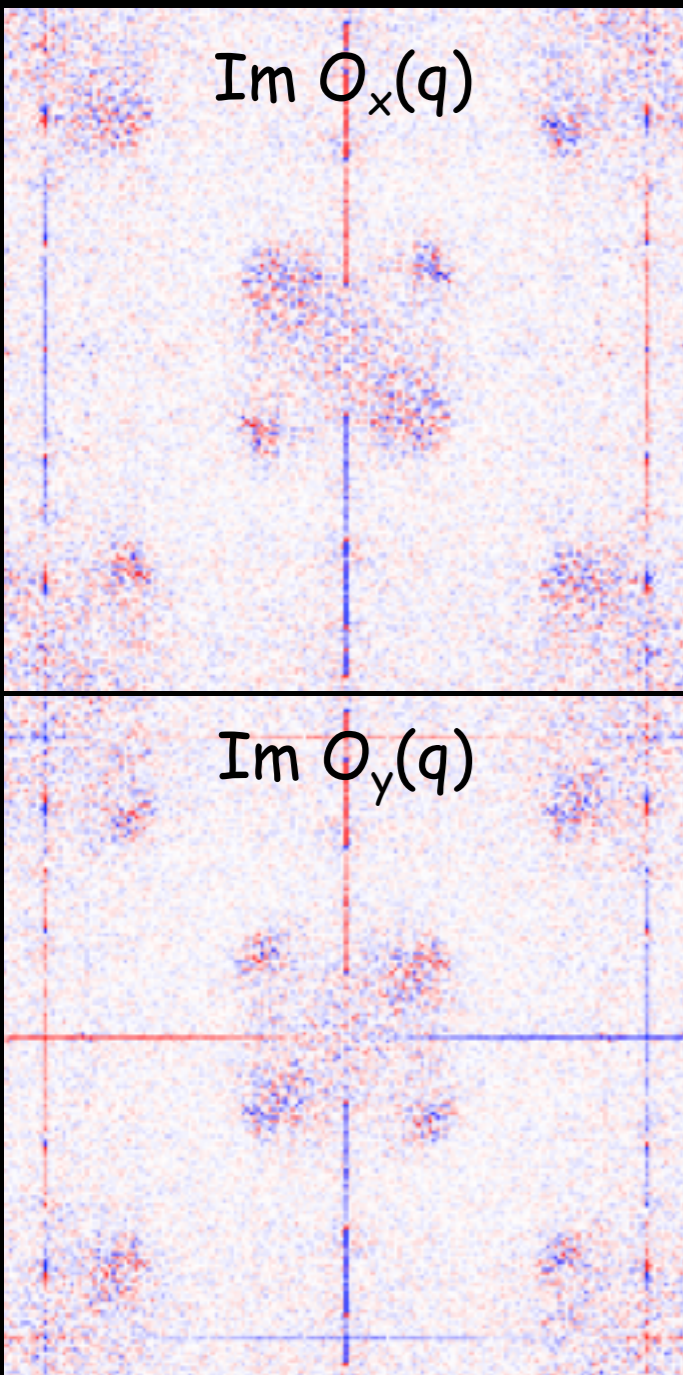
$$\text{Re}O_x(Q,0) + \text{Re}O_y(Q,0) \approx 0$$

$$\text{Re}O_x(0,Q) + \text{Re}O_y(0,Q) \approx 0$$

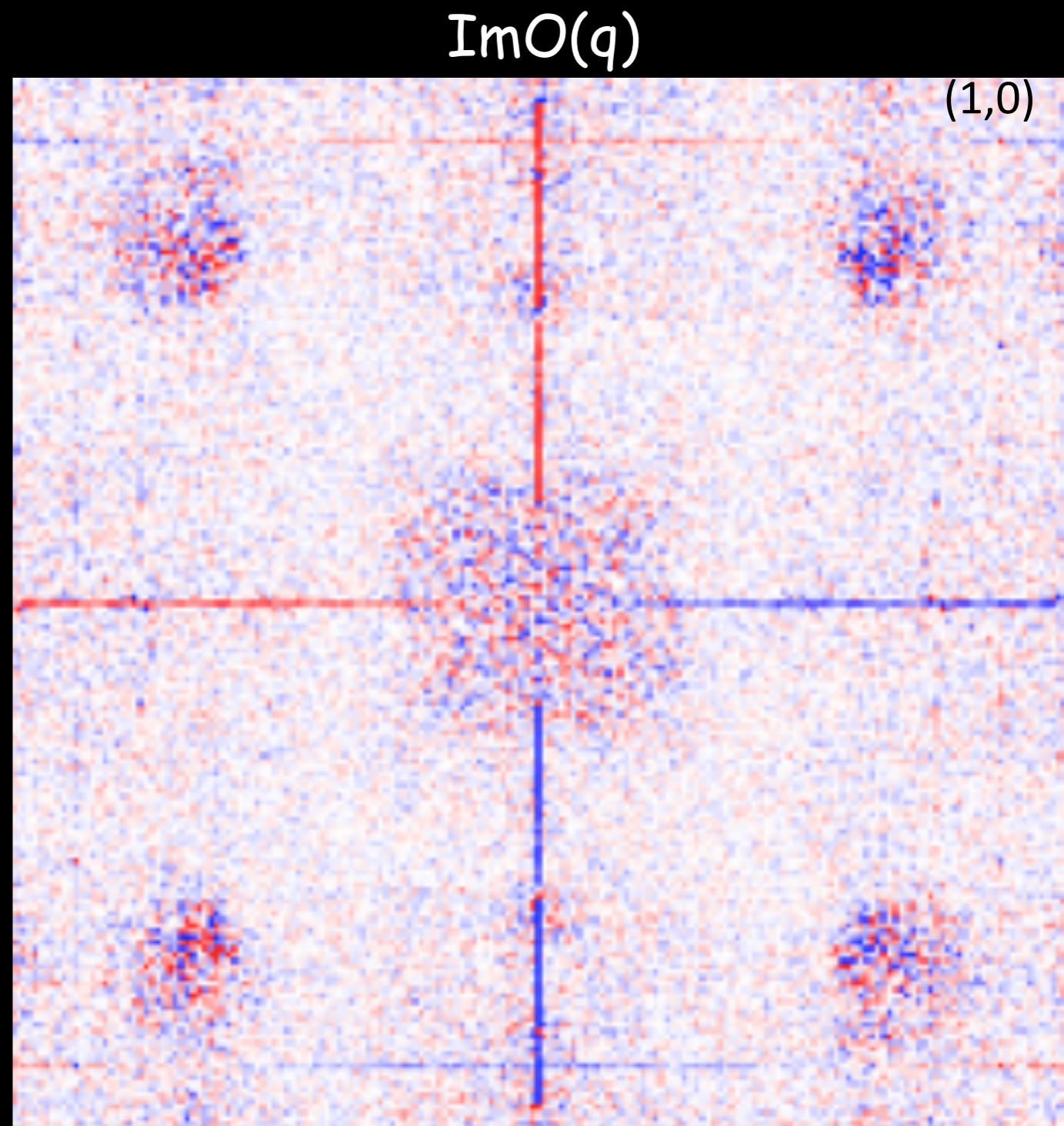


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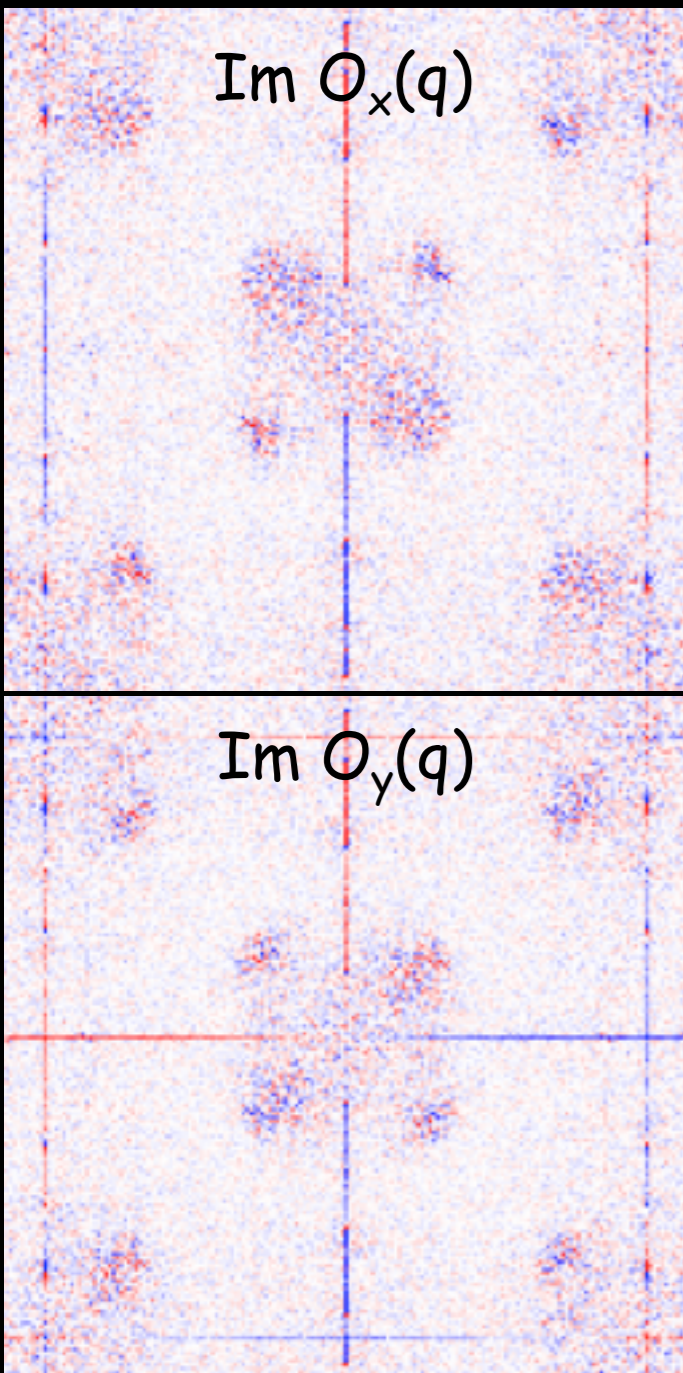
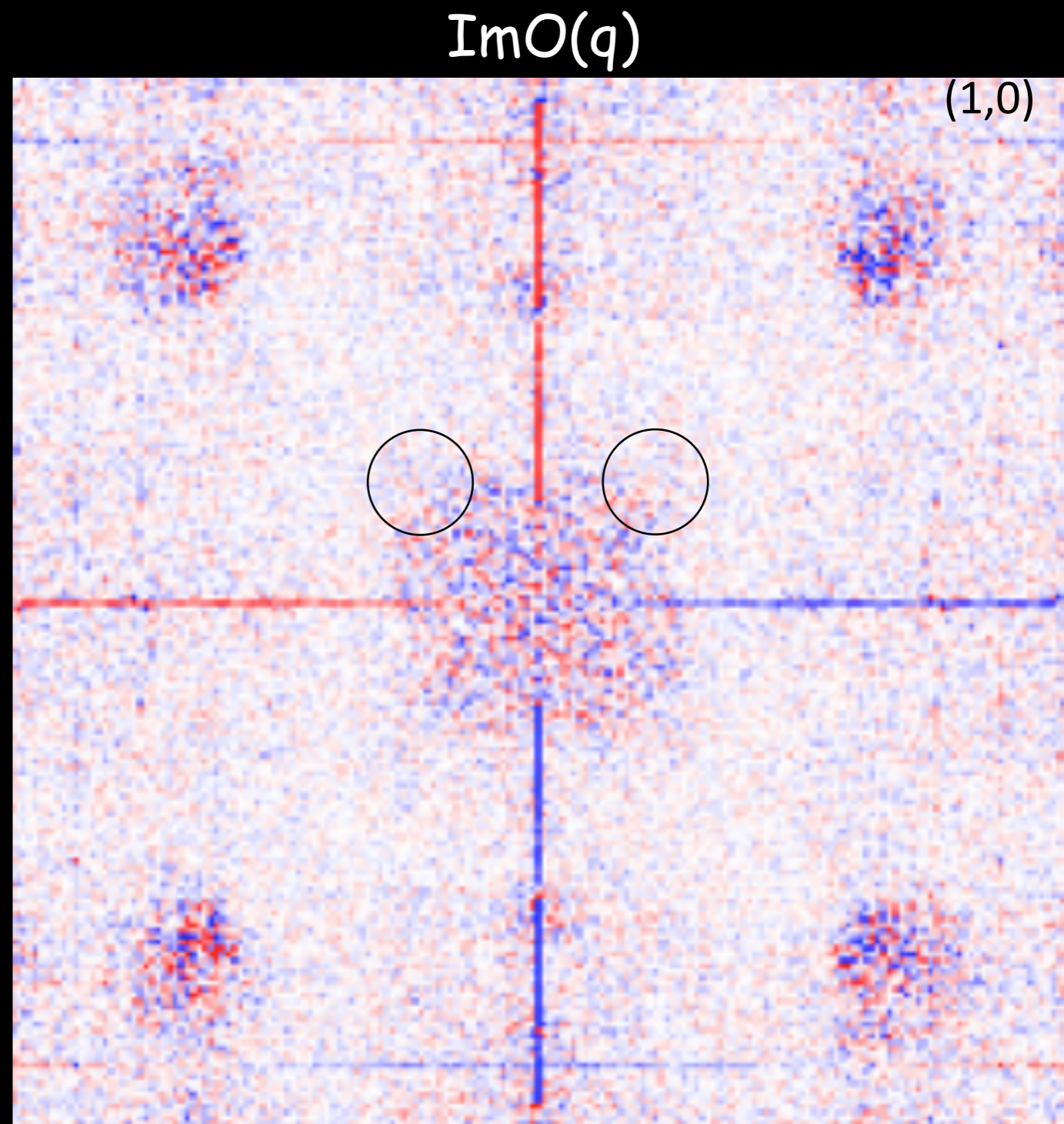
$\text{Im}Q_x(0,Q)$ Vs $\text{Im}O_y(0,Q)$



$\text{Im} O_x + \text{Im} O_y$



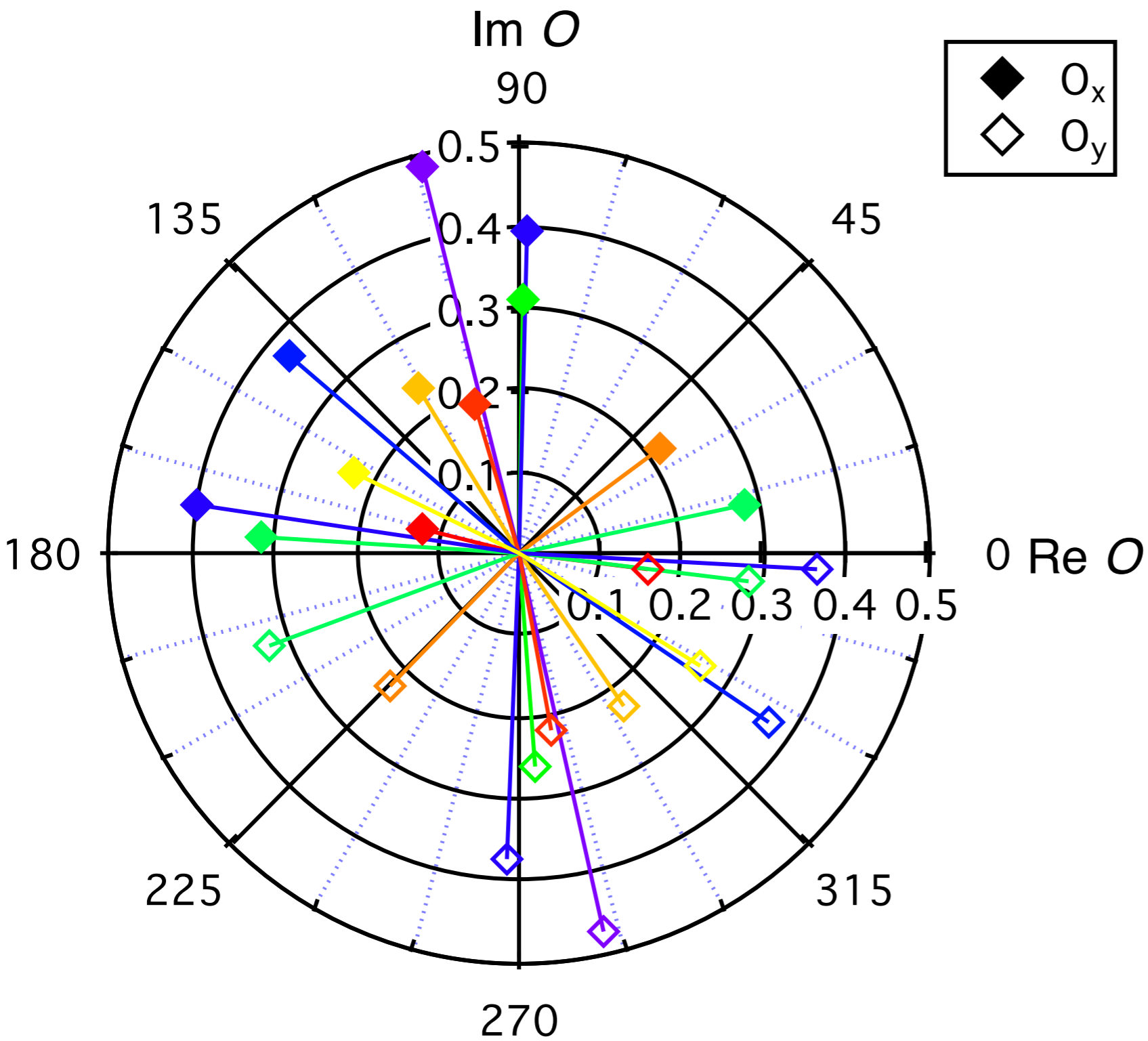
UD45K

 $\text{Im}Q_x(0,Q)$ Vs $\text{Im}O_y(0,Q)$  $\text{Im} O_x + \text{Im} O_y$ 

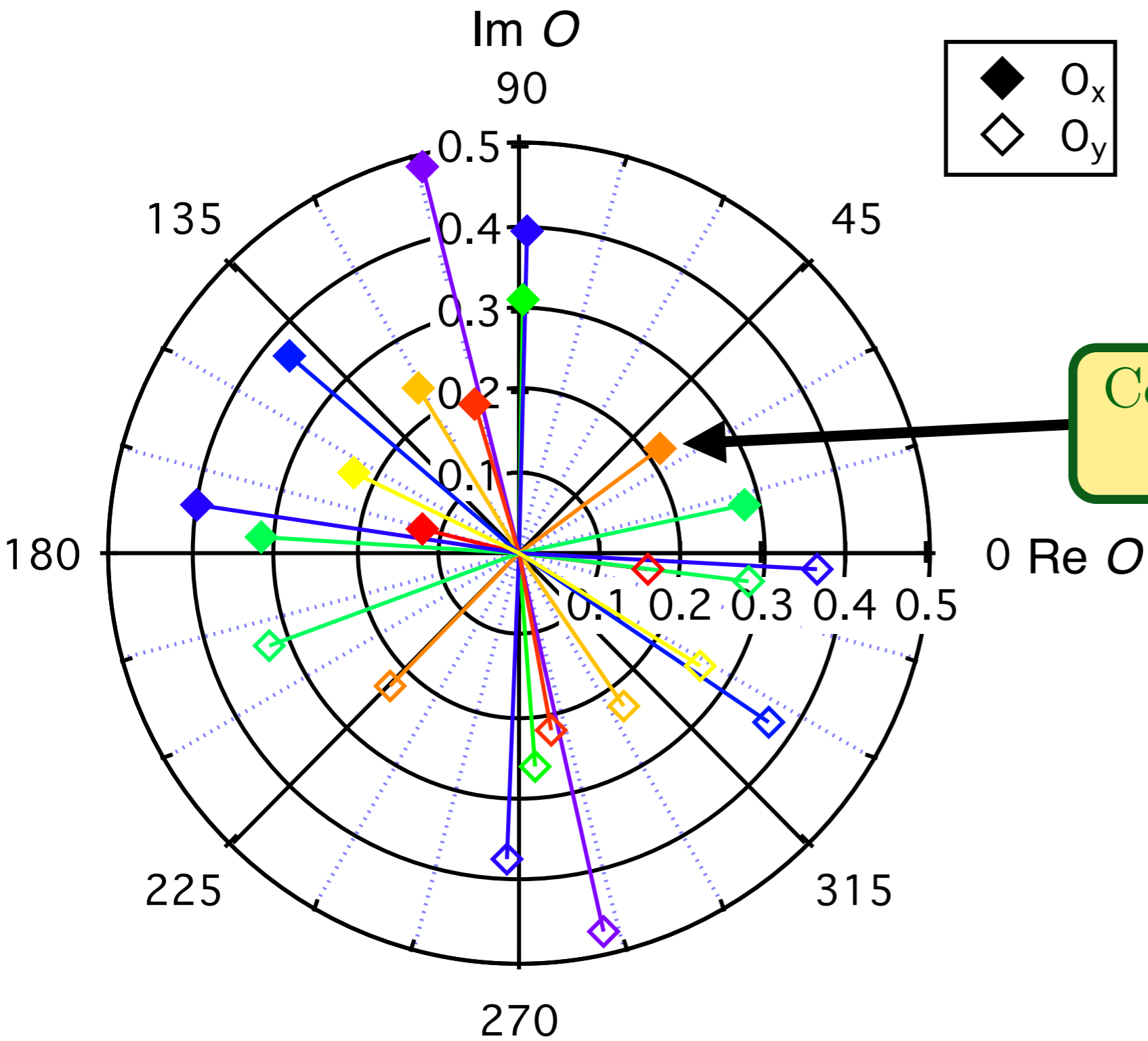
$$\begin{aligned} \text{Im}O_x(0,Q) + \text{Im}O_y(0,Q) &\approx 0 \\ \text{Im}O_x(Q,0) + \text{Im}O_y(Q,0) &\approx 0 \end{aligned}$$



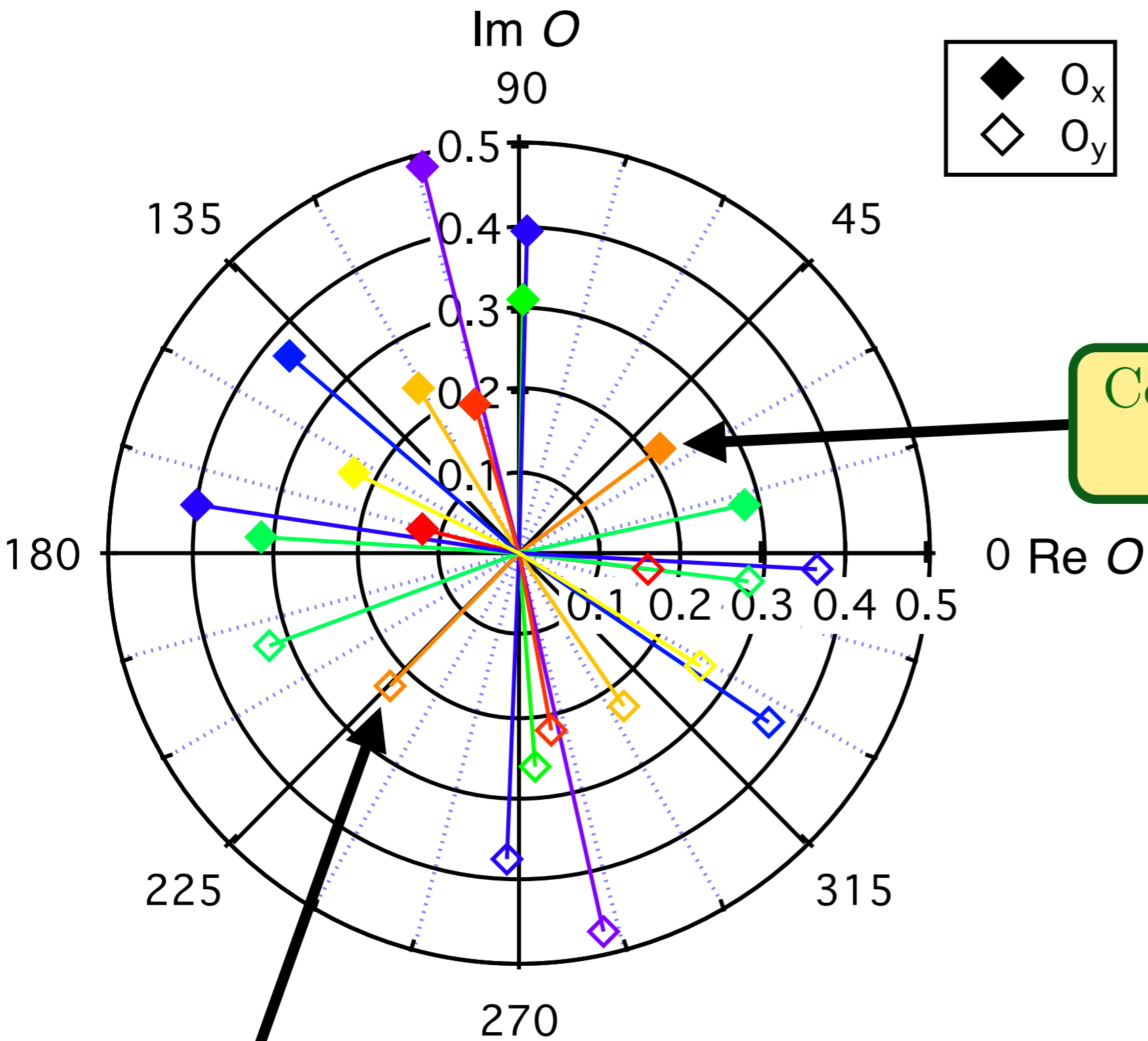
**Phase-sensitive
measurement of the
d symmetry of
density wave order**



Phase-sensitive measurement of the d symmetry of density wave order



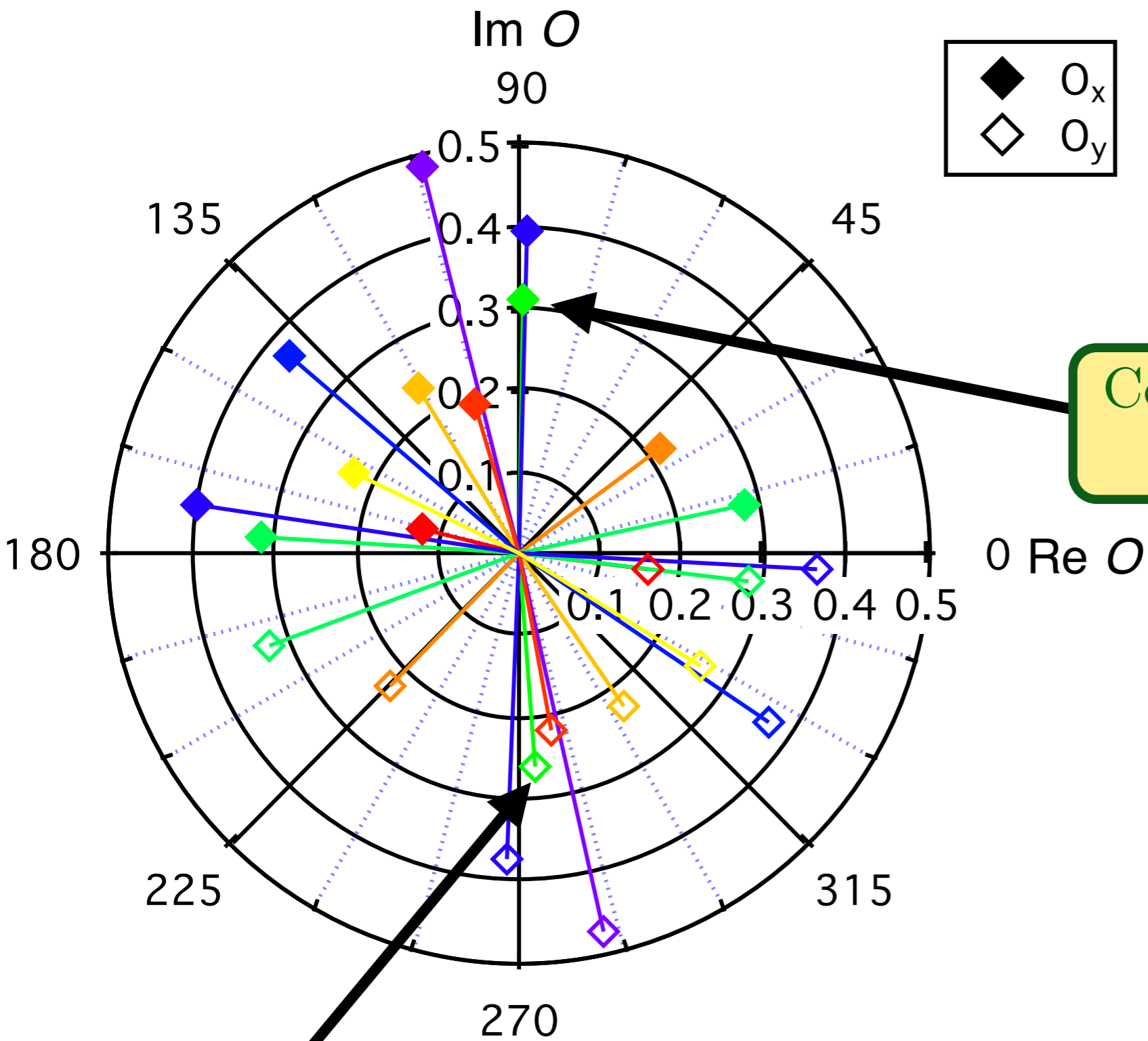
Phase-sensitive measurement of the d symmetry of density wave order



Complex value of O_x at a pixel

Complex value of O_y at same pixel

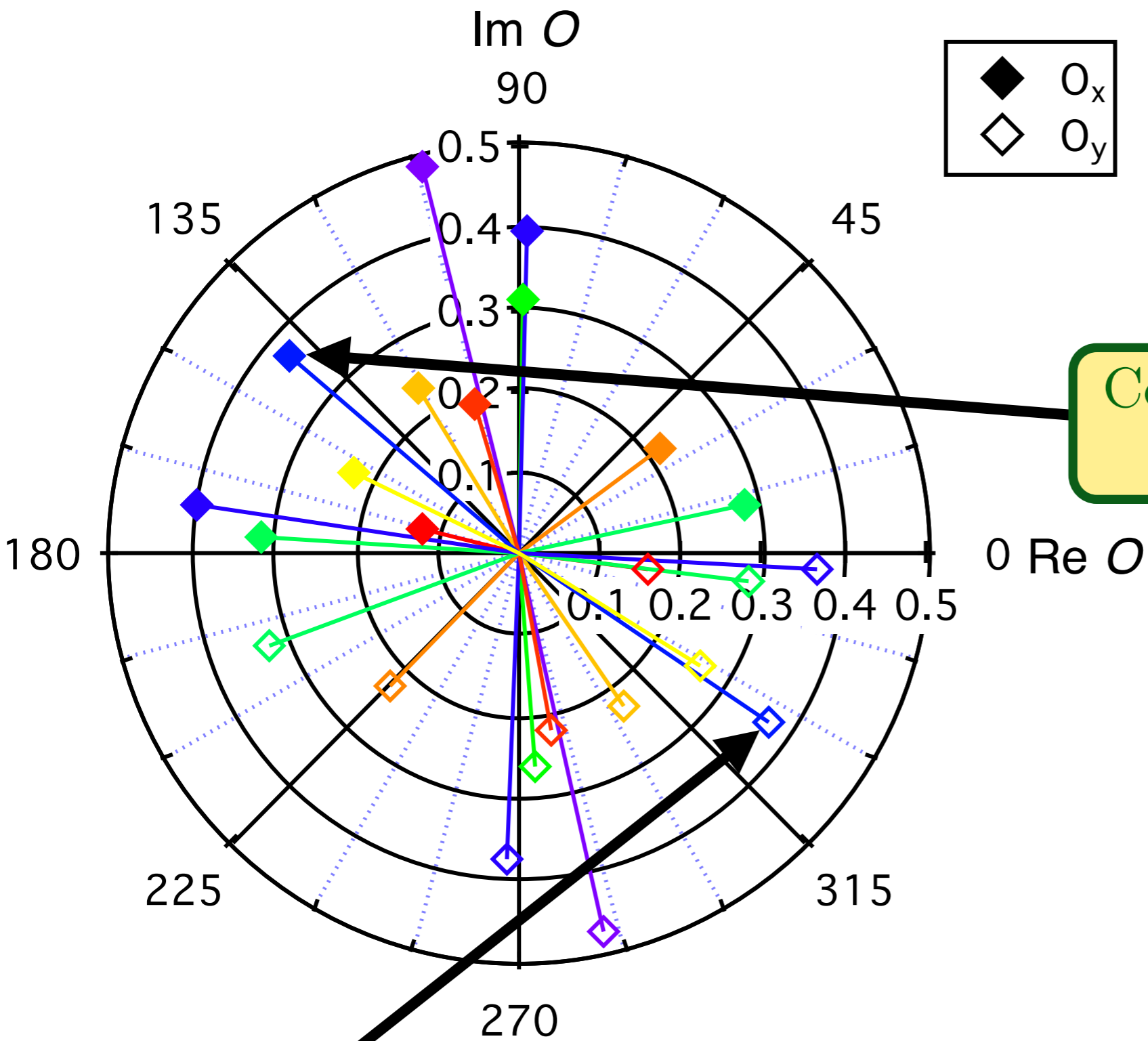
Phase-sensitive measurement of the d symmetry of density wave order



Complex value of O_x at a pixel

Complex value of O_y at same pixel

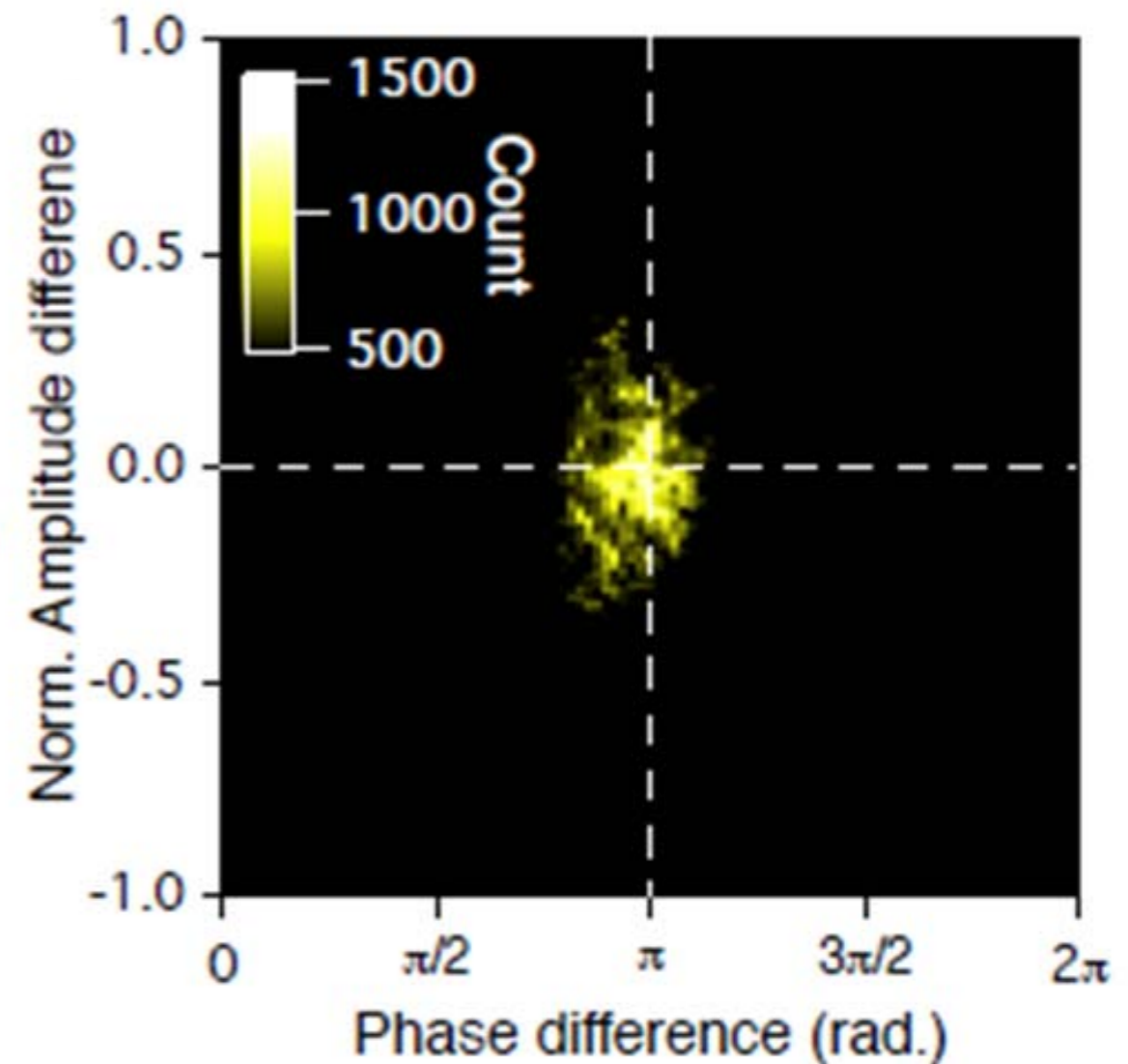
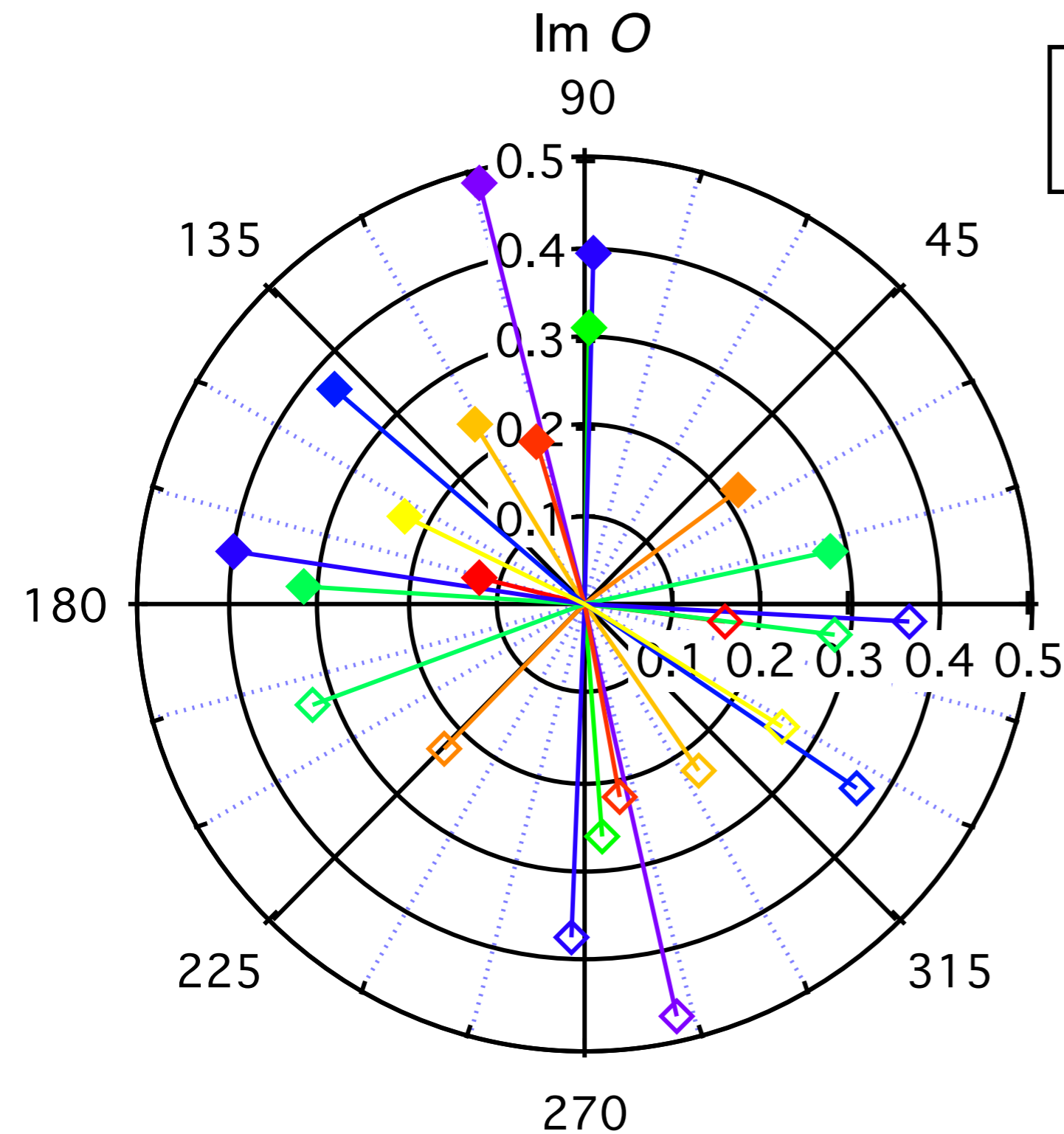
Phase-sensitive measurement of the d symmetry of density wave order

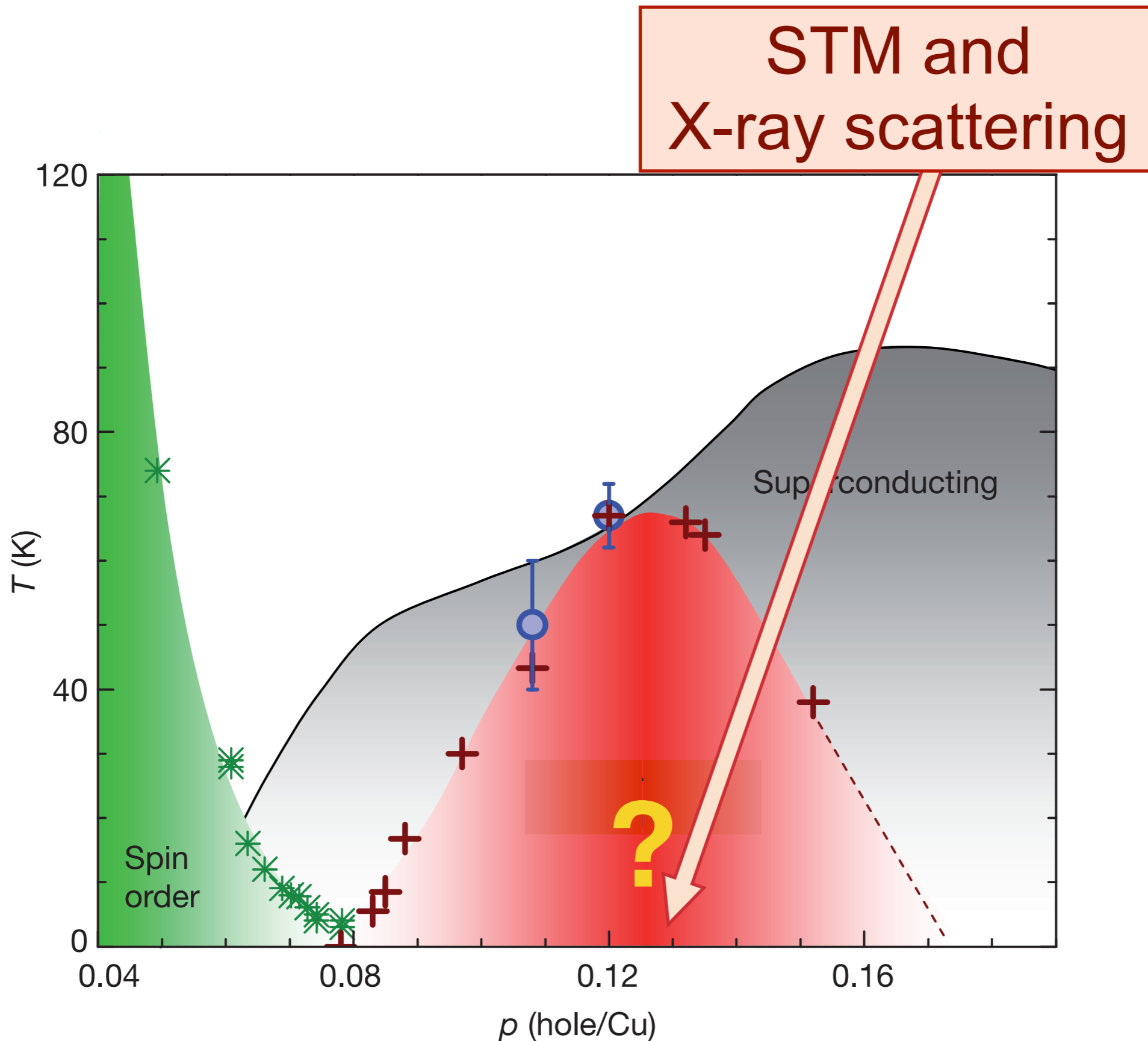


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Complex value of O_y at same pixel

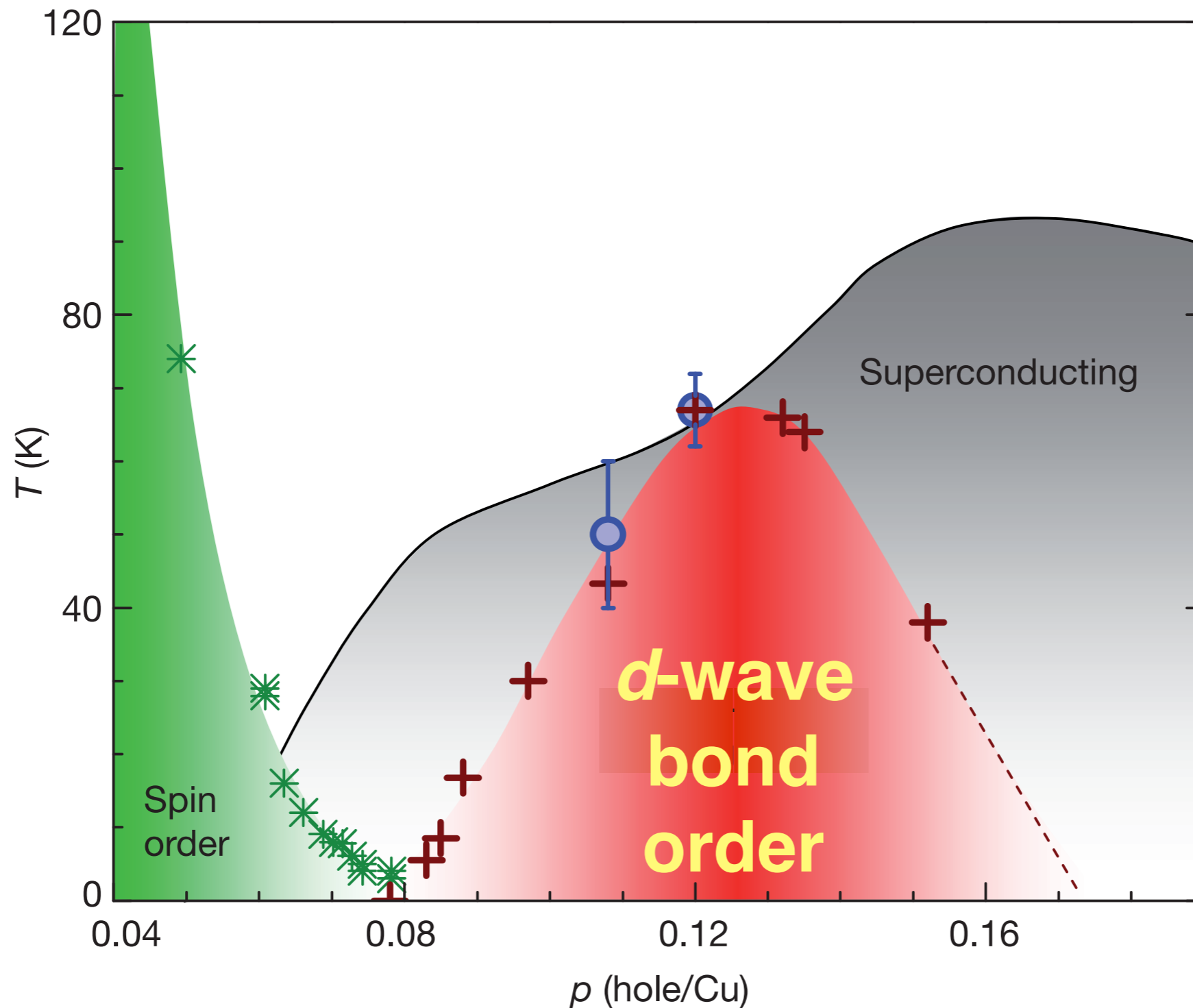
Phase-sensitive measurement of the d symmetry of density wave order





T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).

Phase-sensitive measurement of the d symmetry of density wave order

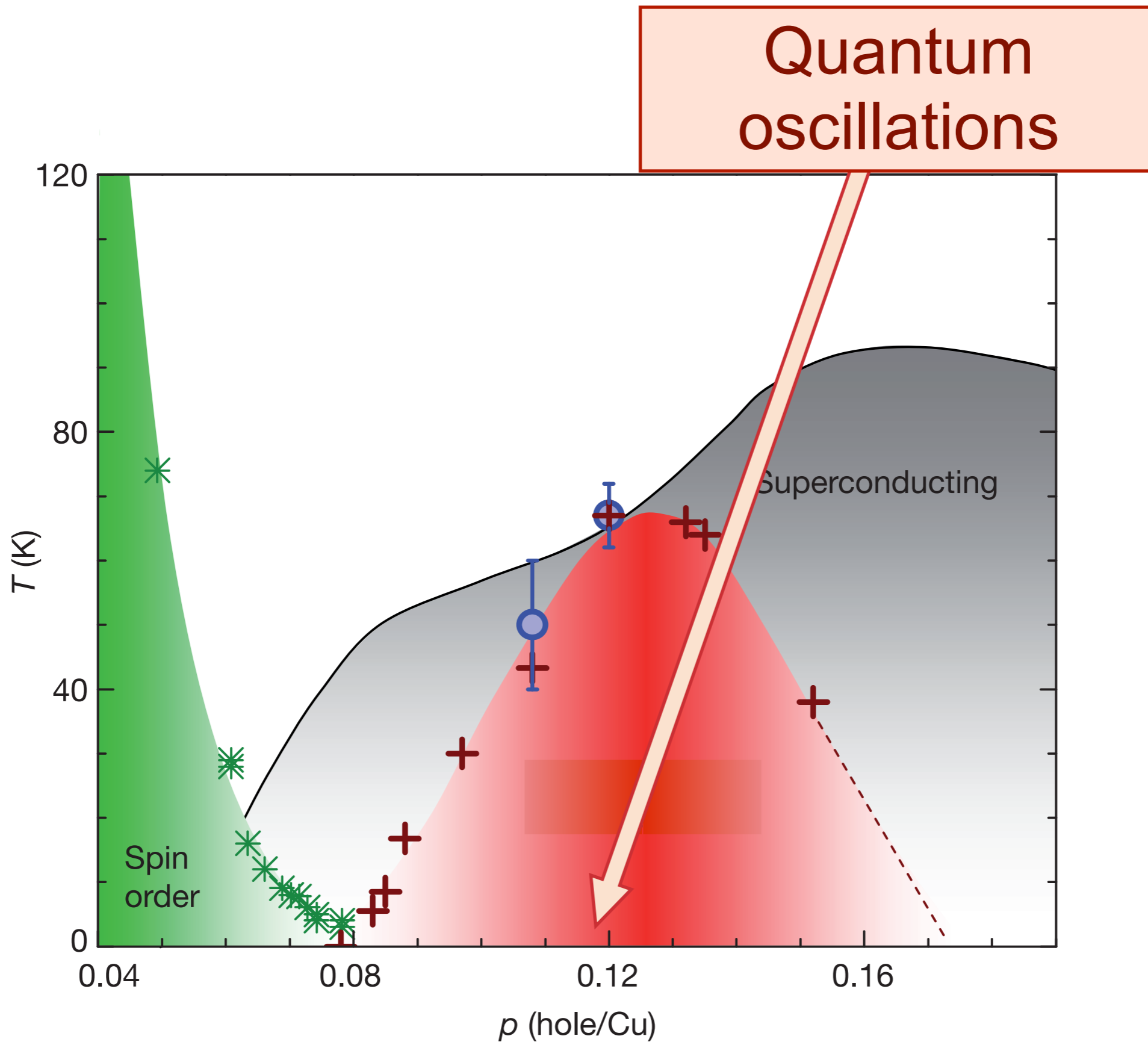


Outline

1. Antiferromagnetism in metals and *d*-wave superconductivity
2. Low hole density state:
d-wave bond order
3. Evolution of Fermi surface

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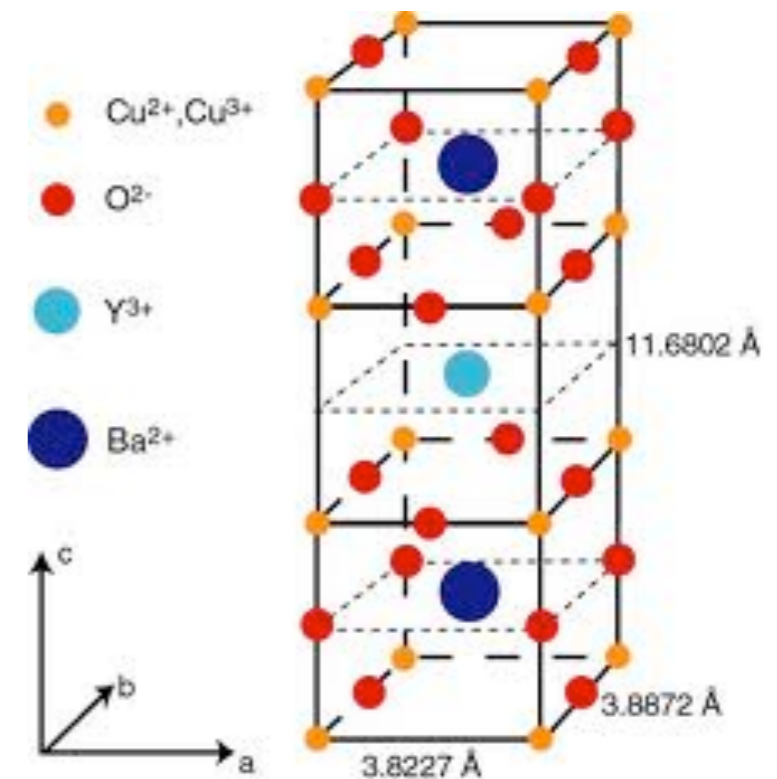
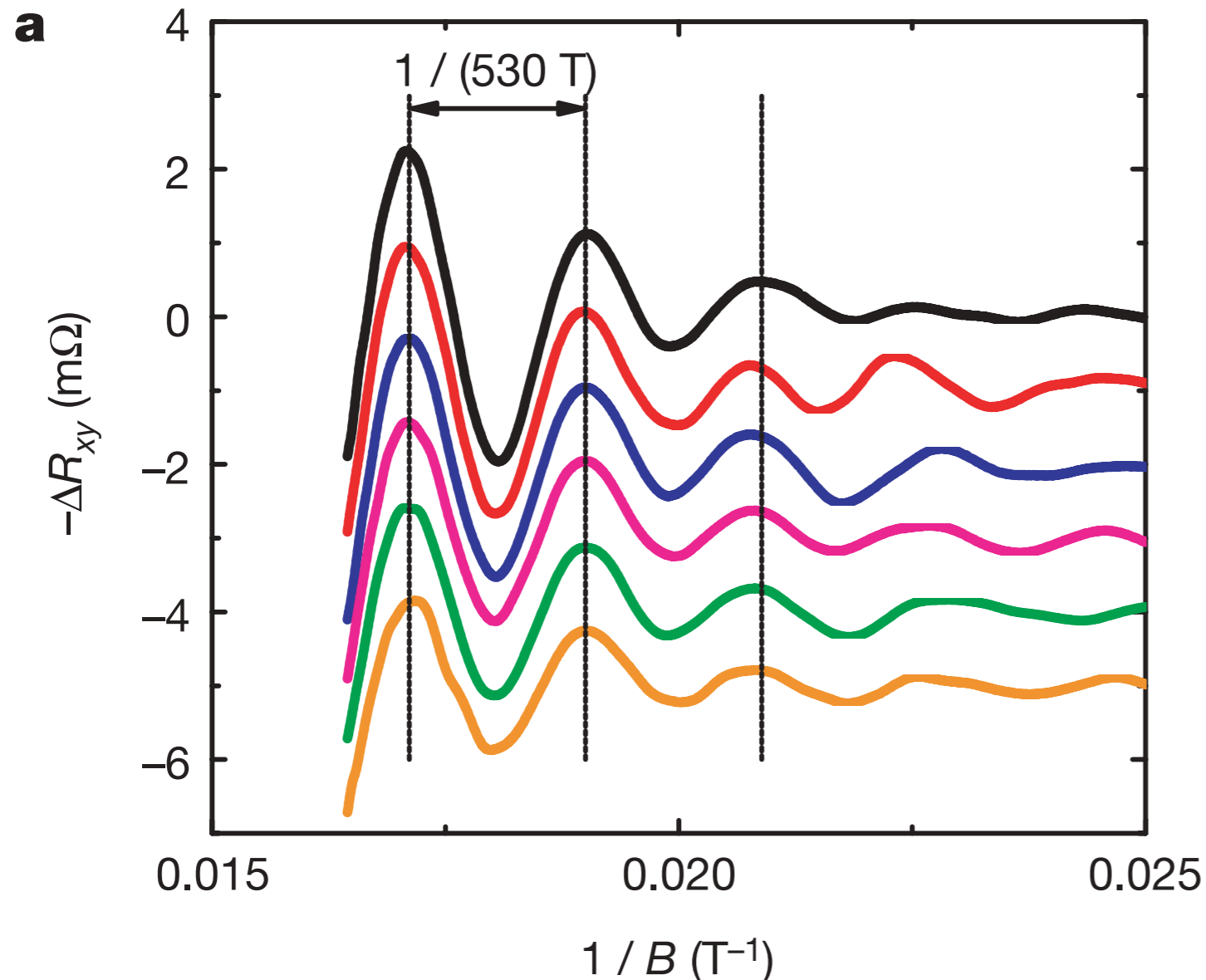


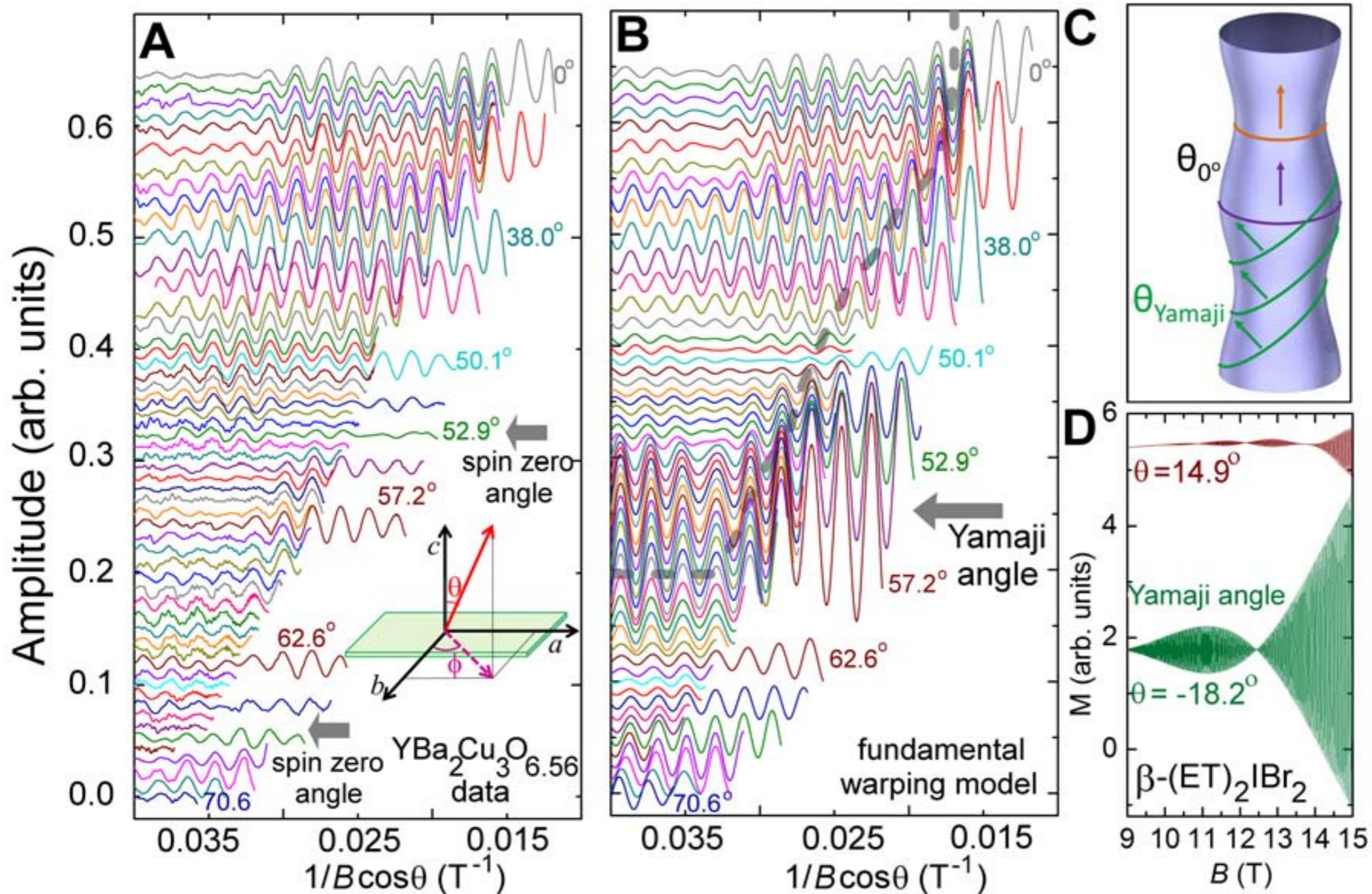
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Quantum oscillations and the Fermi surface in an underdoped high- T_c superconductor

Nicolas Doiron-Leyraud¹, Cyril Proust², David LeBoeuf¹, Julien Levallois², Jean-Baptiste Bonnemaïson¹, Ruixing Liang^{3,4}, D. A. Bonn^{3,4}, W. N. Hardy^{3,4} & Louis Taillefer^{1,4}

Nature 447, 565 (2007)

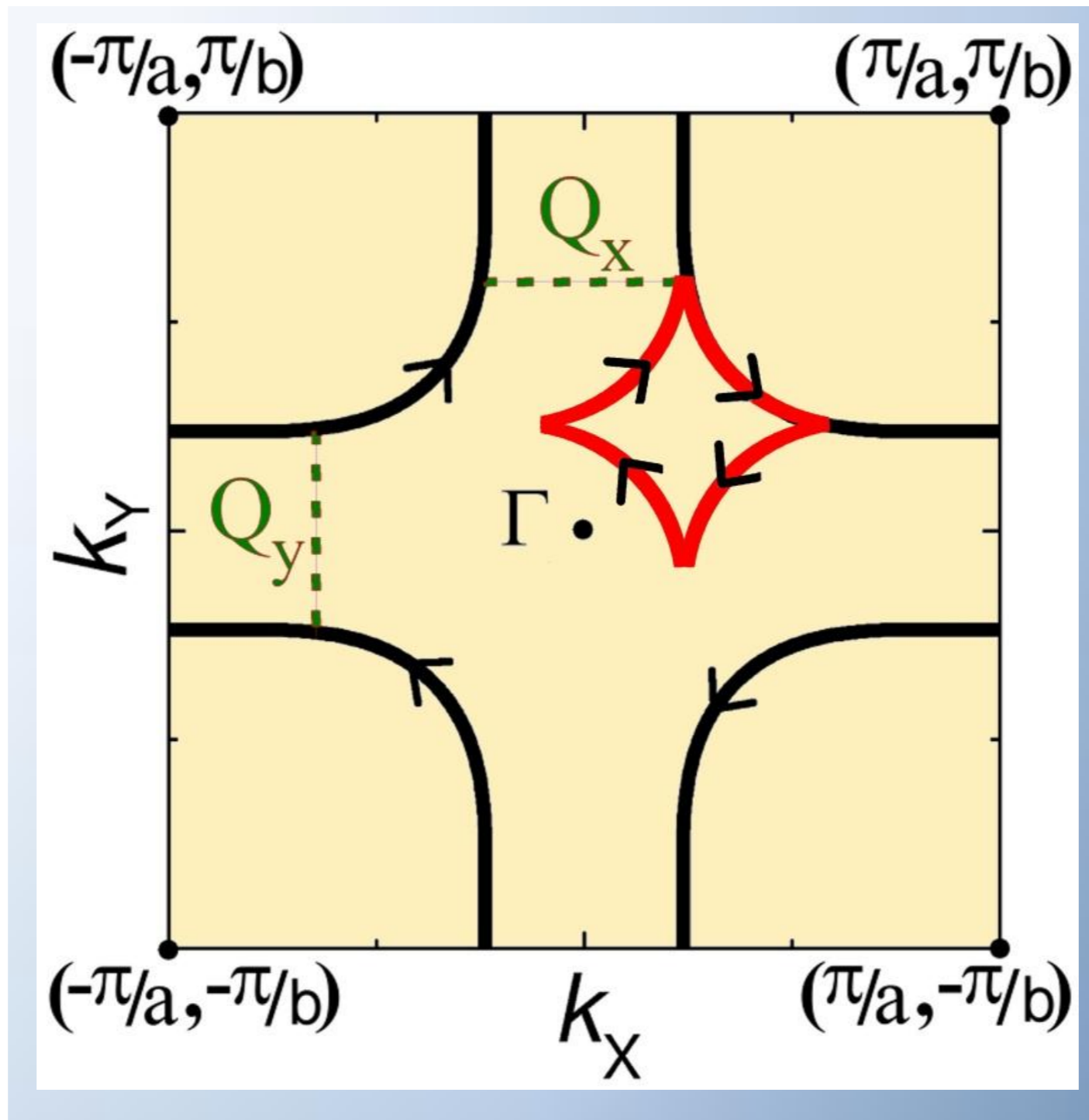




Twofold twisted Fermi surface from staggered order in an underdoped high T_c superconductor

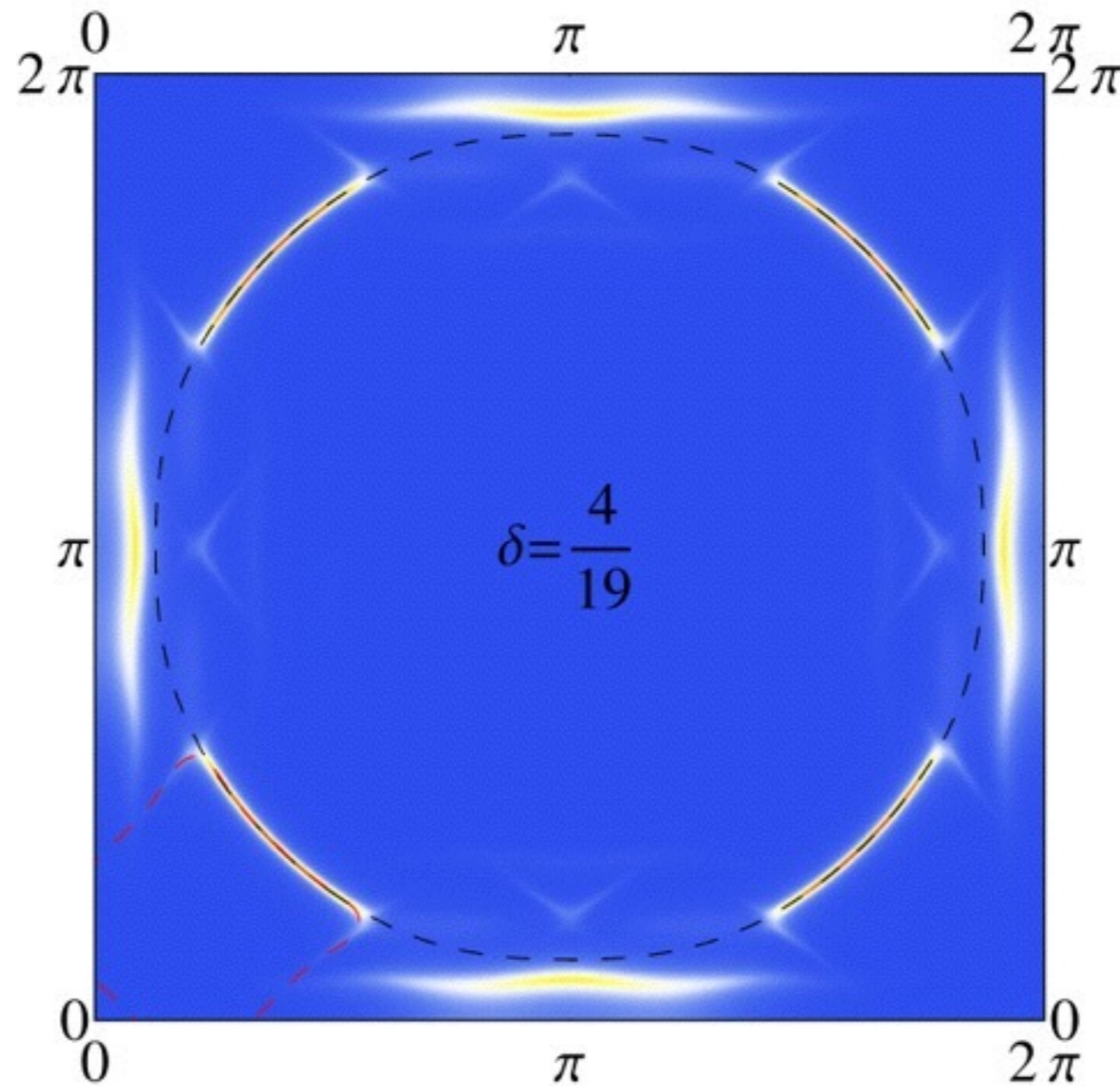
APS March meeting 2013 B2.00004

Suchitra E. Sebastian,^{1*} N. Harrison,² F. F. Balakirev,² M. M. Altarawneh,^{2,3} Ruixing Liang,^{4,5} D. A. Bonn,^{4,5} W. N. Hardy,^{4,5} G. G. Lonzarich,¹



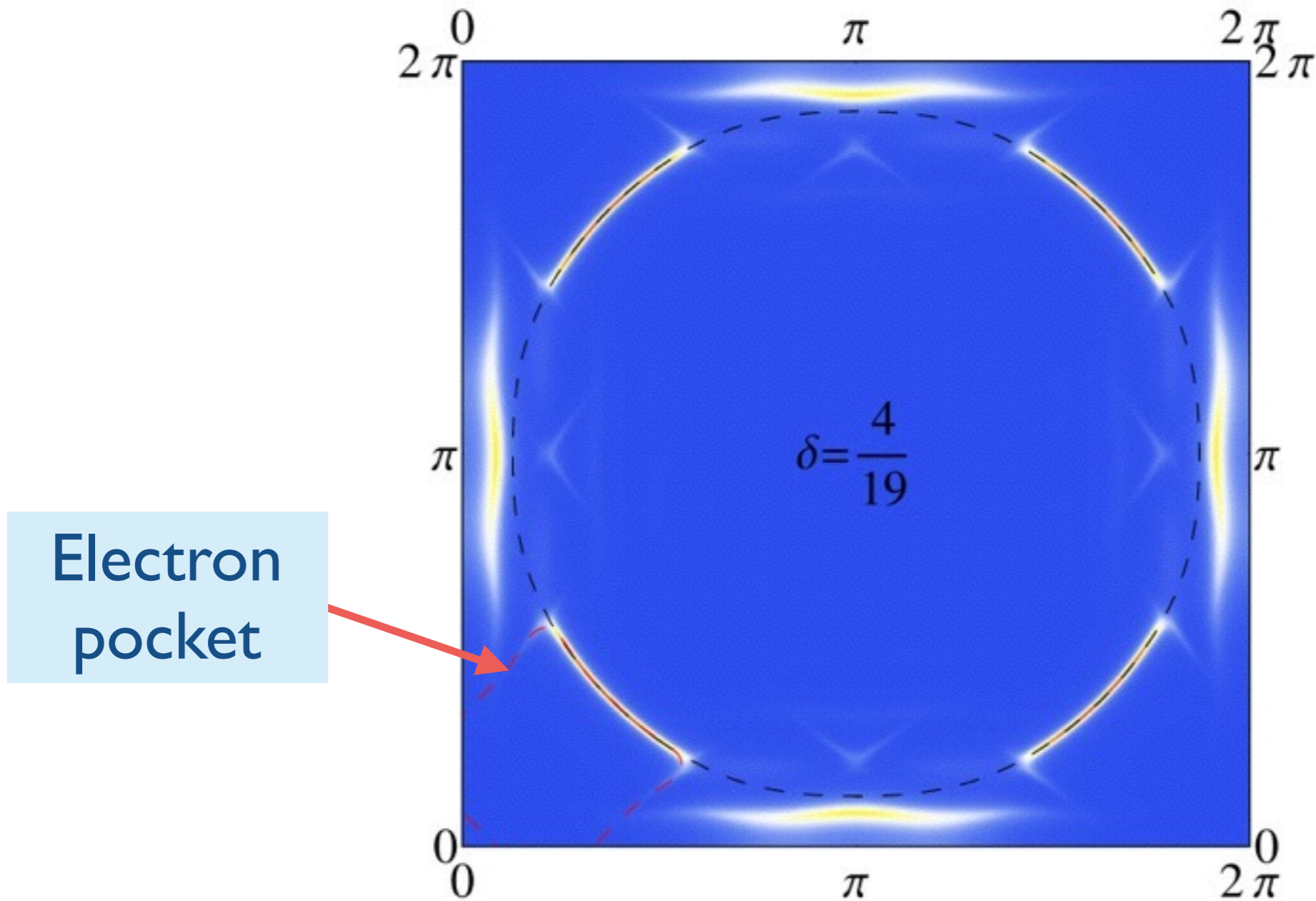
N. Harrison and S. E. Sebastian, Phys. Rev. Lett. **106**, 226402 (2011).

Electron spectral function in the presence of charge order
at $\mathbf{Q} = 2\pi(\delta, 0)$ and $\mathbf{Q} = 2\pi(0, \delta)$



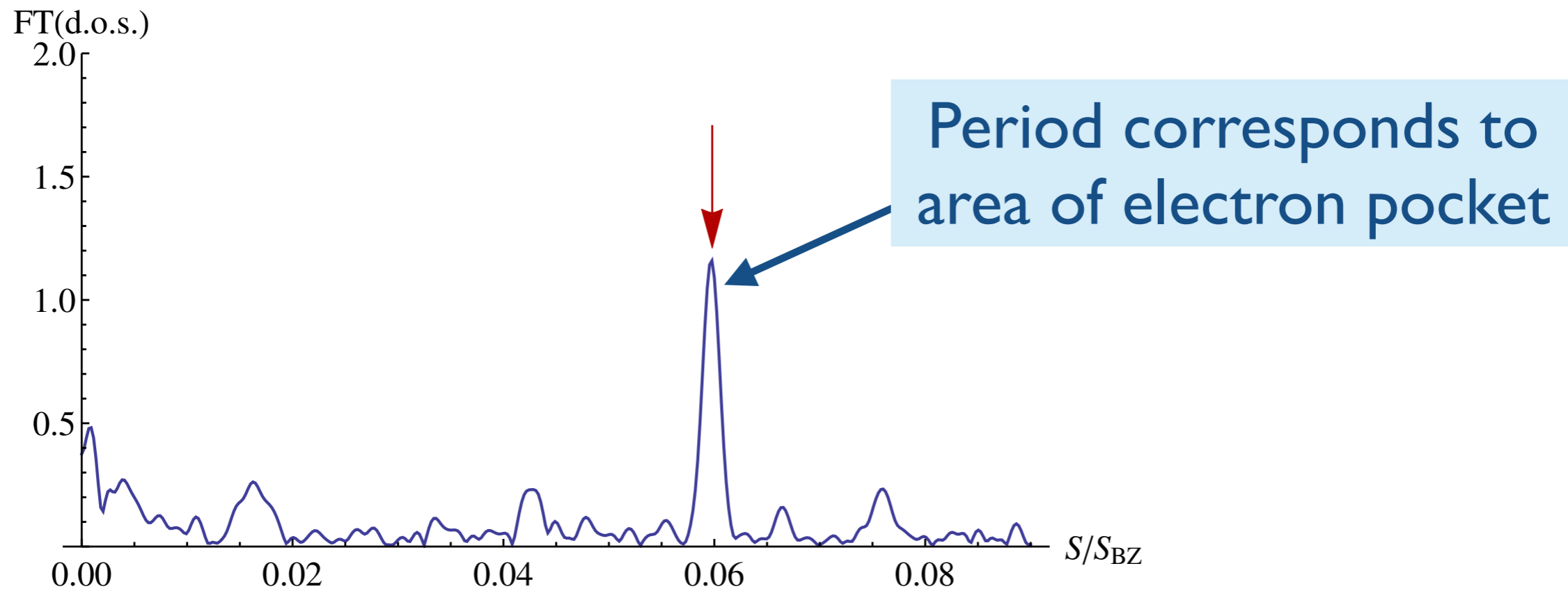
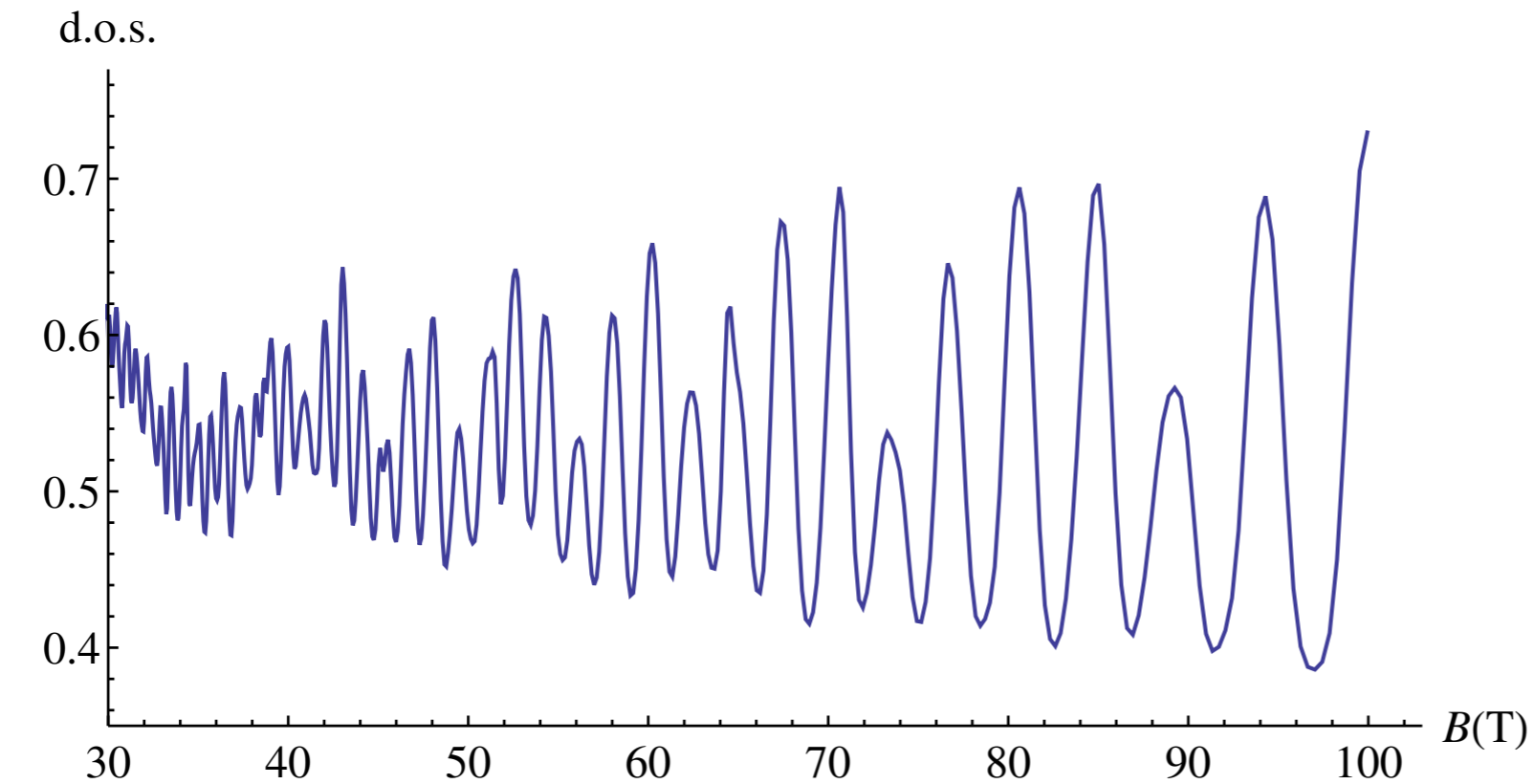
S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)
A. Allais, D. Chowdhury, and S. Sachdev, to appear

Electron spectral function in the presence of charge order
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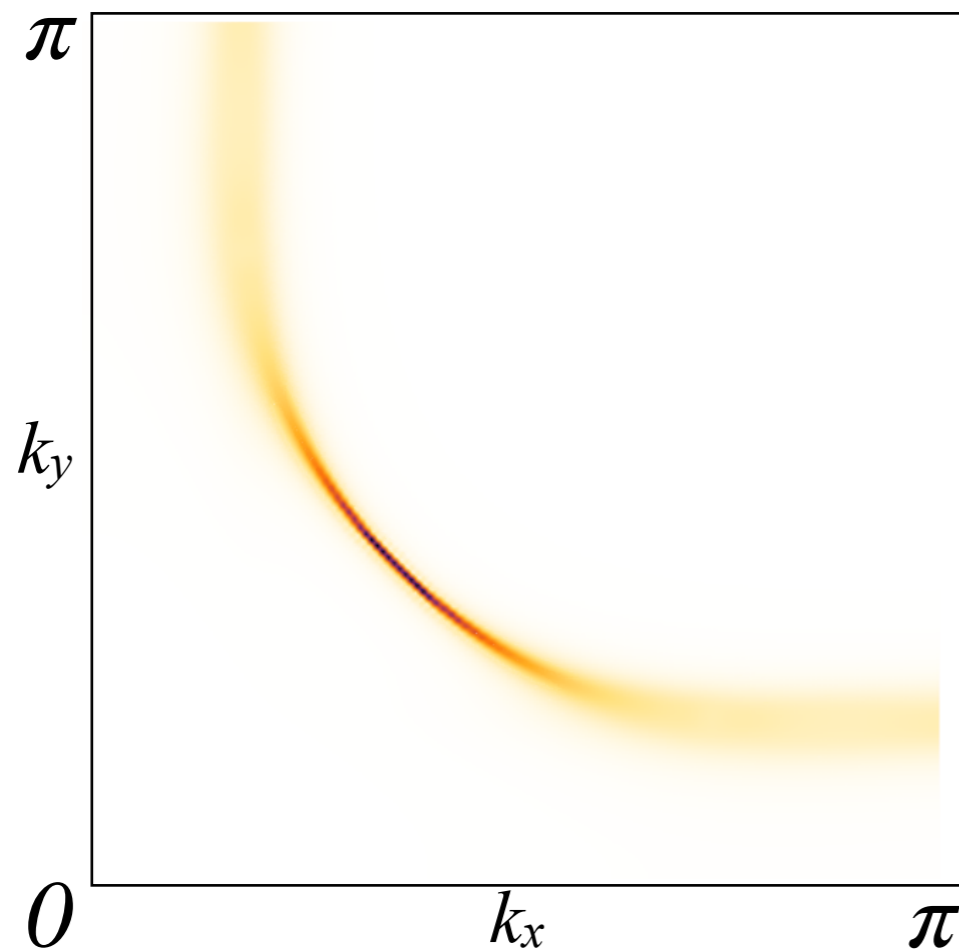
S. Sachdev and R. LaPlaca *Phys. Rev. Lett.* **111**, 027202 (2013)
A. Allais, D. Chowdhury, and S. Sachdev, to appear

Quantum oscillation in the presence of charge order at $\mathbf{Q} = 2\pi(\delta, 0)$ and $\mathbf{Q} = 2\pi(0, \delta)$

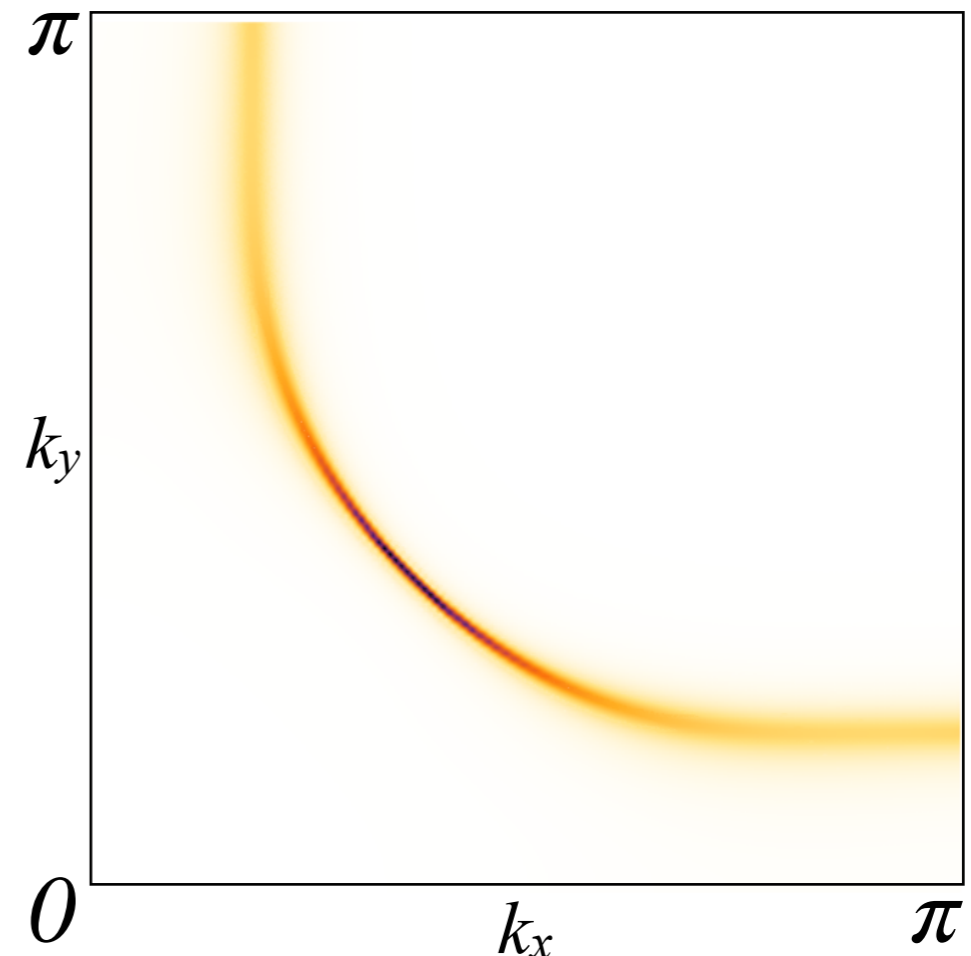


Electron spectral function with fluctuating charge order and superconductivity at temperatures above T_c .

SC+BO
T=0.05

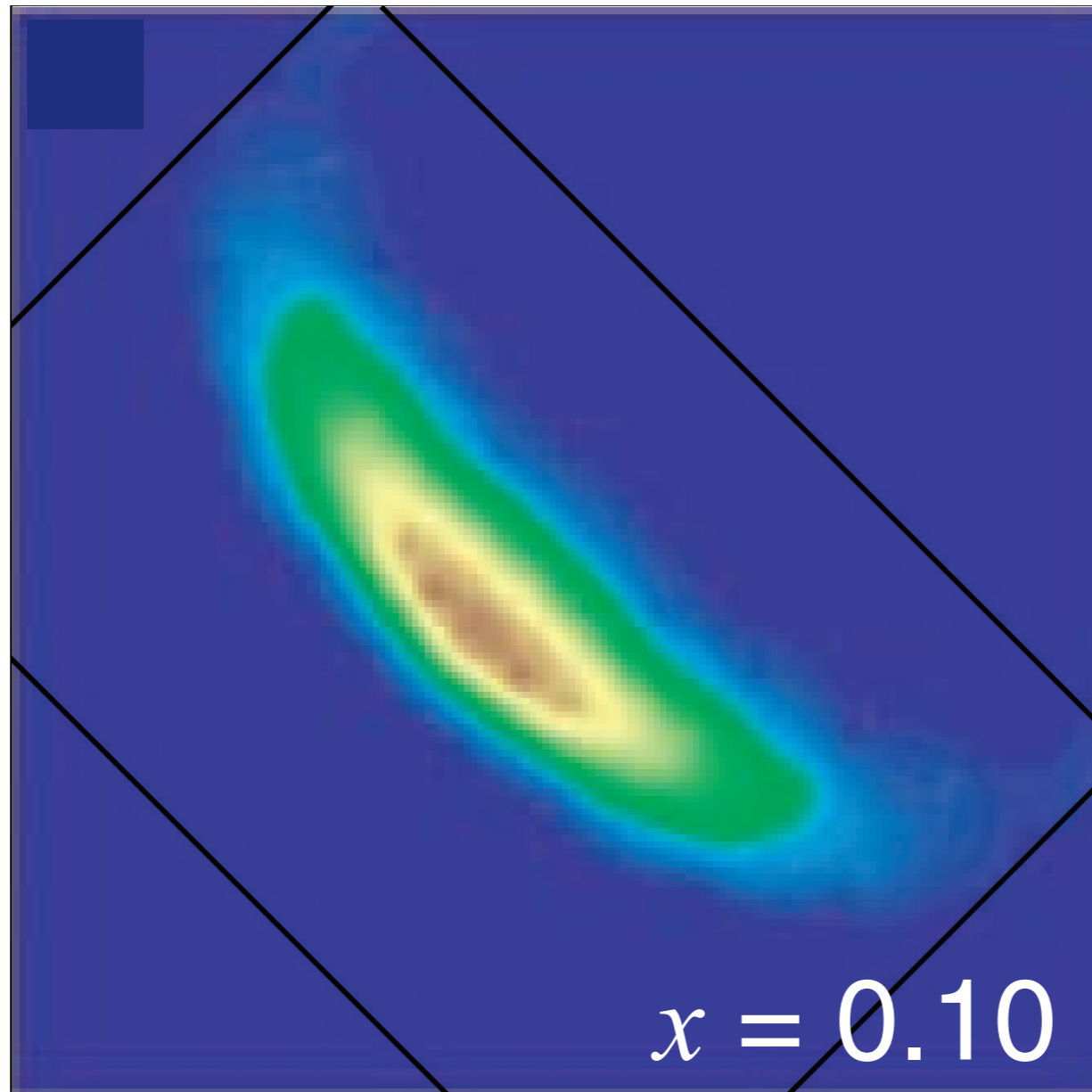


SC+BO
T=0.10



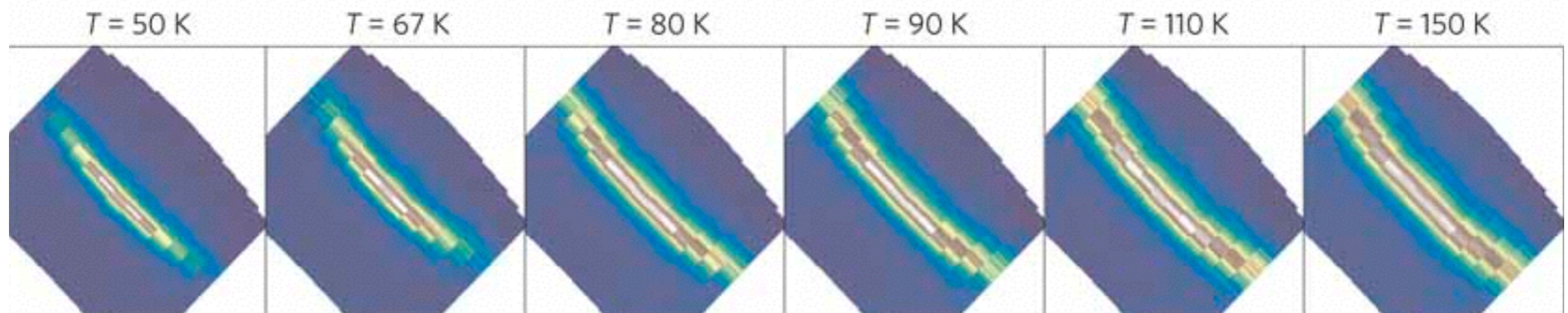
Spectral functions $A(k, \omega=0)$ in the presence of fluctuating SC and BO (with wave vectors $(Q,0)$, $(0,Q)$) at two different temperatures. Notice how spectral weight fills up in the antinodal regions as a function of increasing temperature.

A. Allais, D. Chowdhury, and S. Sachdev, to appear

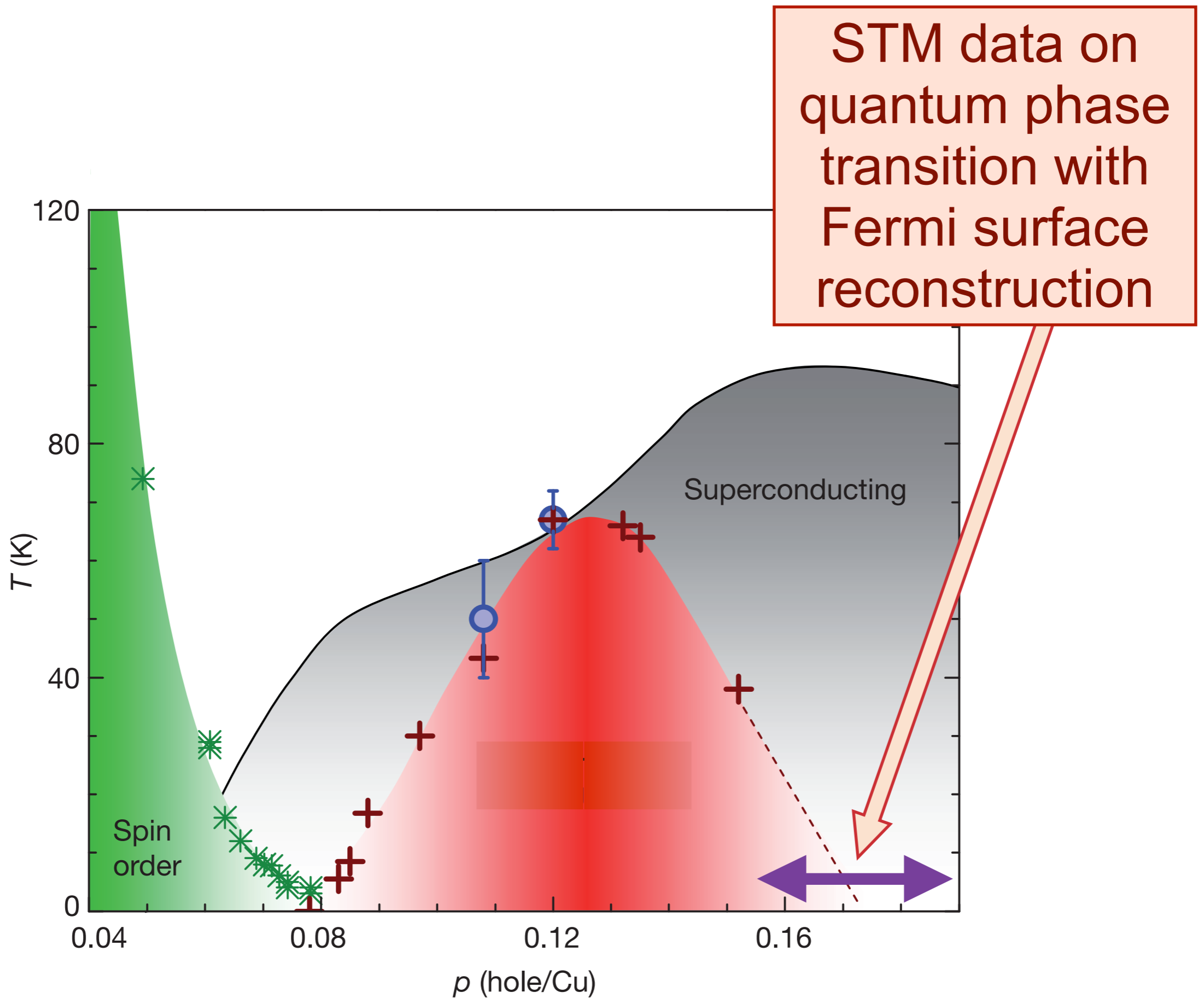


**Nodal Quasiparticles and
Antinodal Charge Ordering in
 $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$**

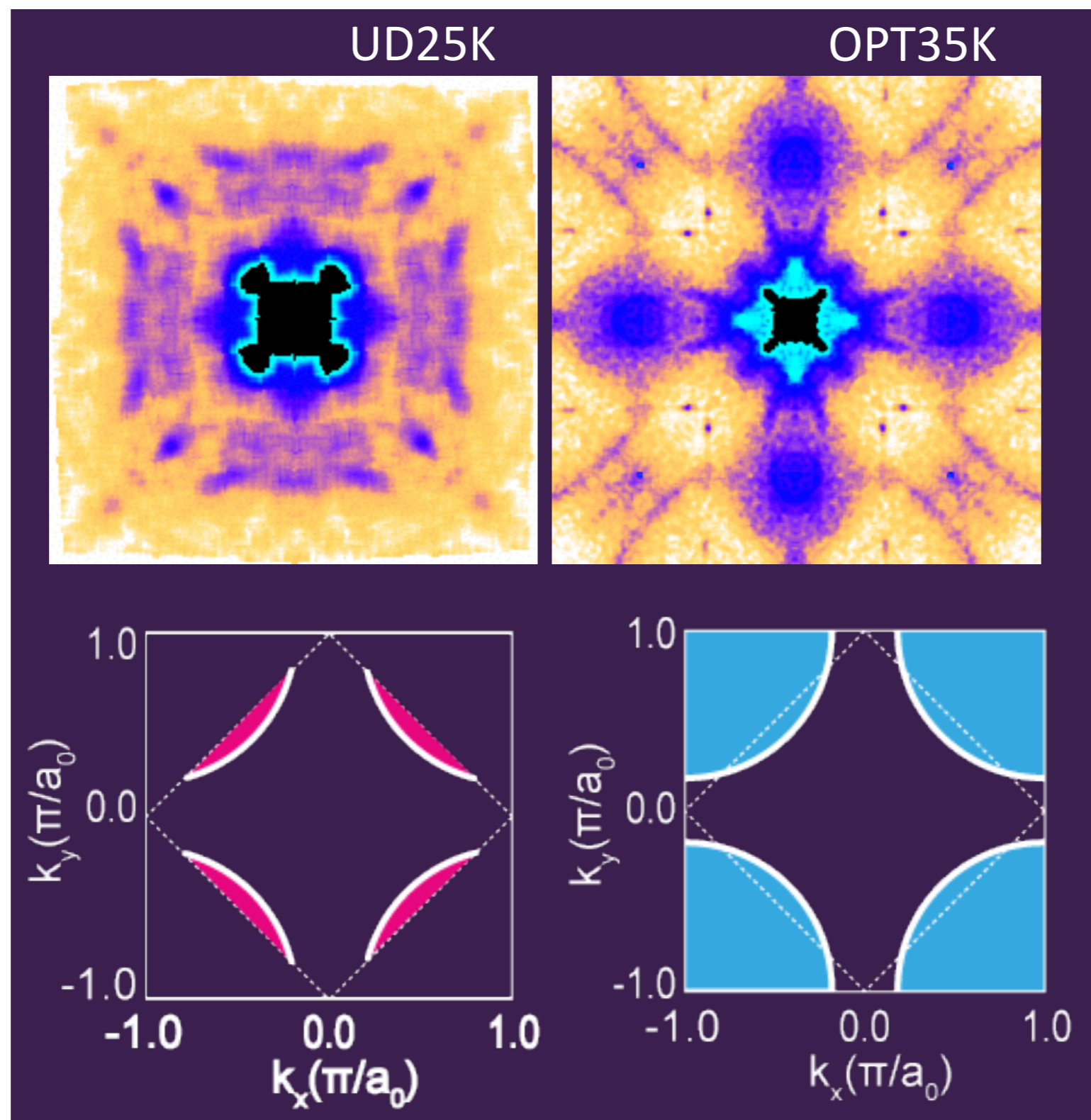
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle,
W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi,
Z.-X. Shen, *Science* **307**, 901 (2005).



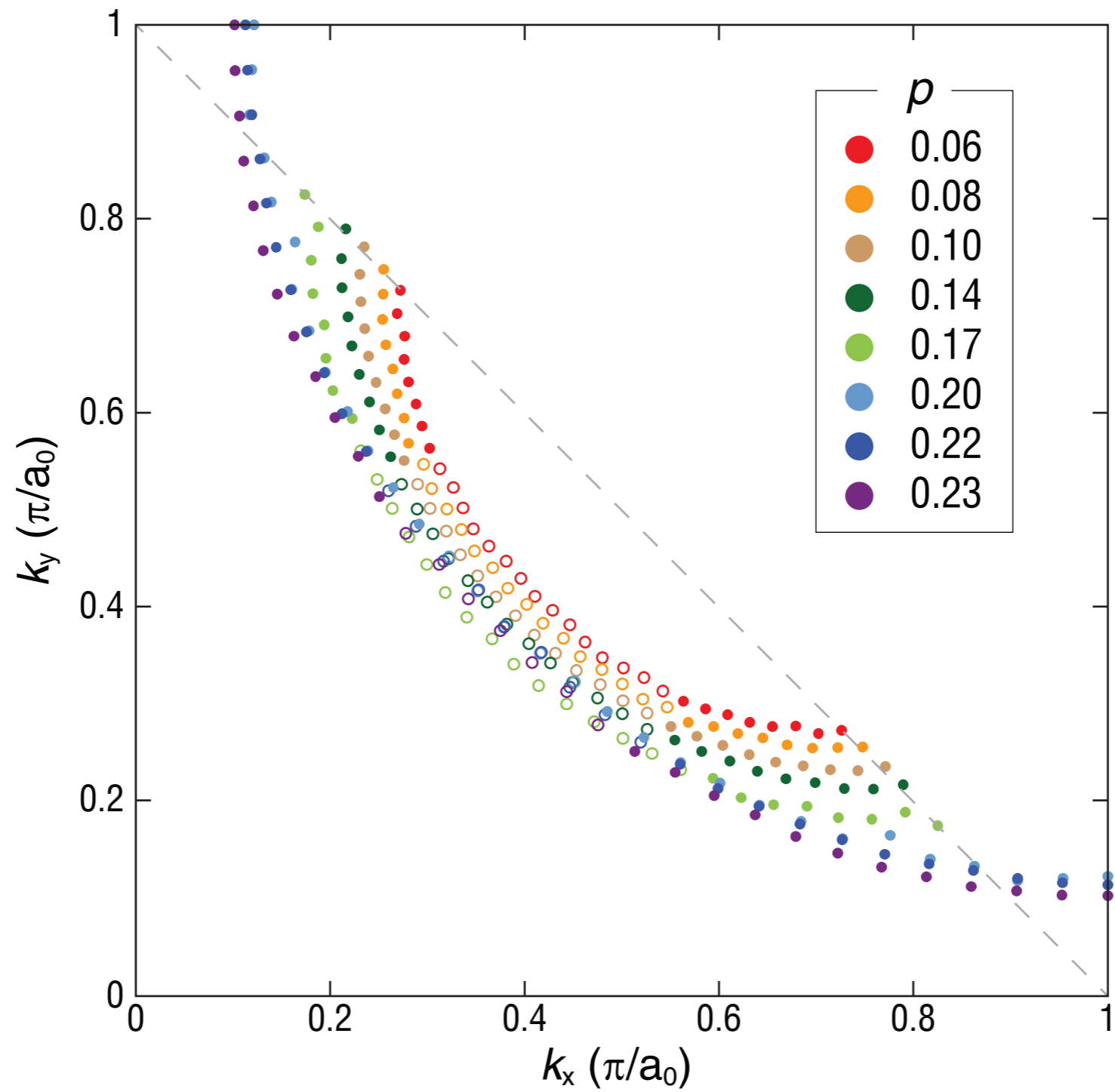
T. J. Reber, N. C. Plumb, Z. Sun, Y. Cao, Q. Wang, K. McElroy, H. Iwasawa, M. Arita, J. S. Wen, Z. J. Xu, G. Gu, Y. Yoshida, H. Eisaki, Y. Aiura, and D. S. Dessau, *Nature Physics* **8**, 606 (2012)



T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, J. E. Hoffman, arXiv:1305.2778, *Science*, to appear.



K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, and J. C. Davis, *Science*, to appear.

Conclusions

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