

Quantum criticality in condensed matter

Inaugural Symposium of the
Wolfgang Pauli Center
Hamburg, April 17, 2013

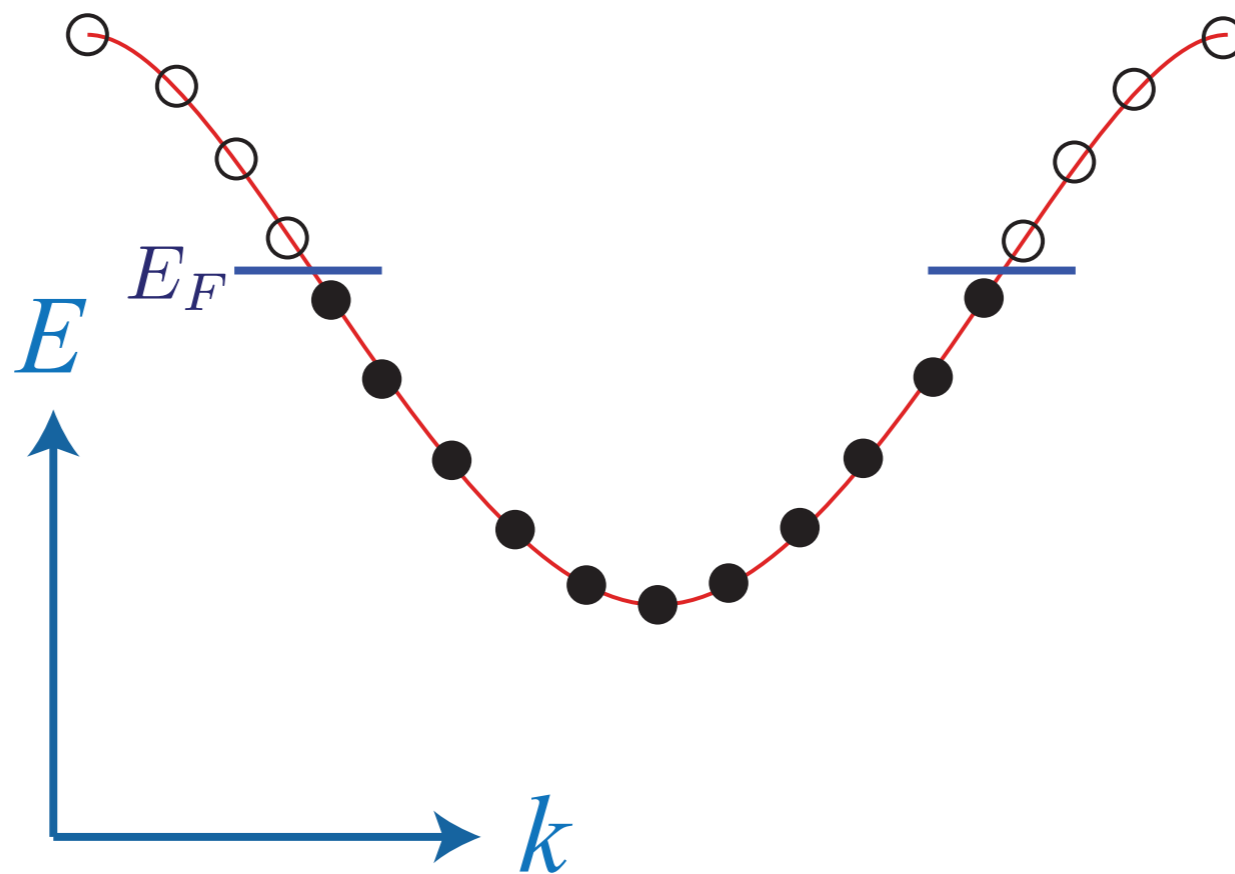
Subir Sachdev

SCIENTIFIC AMERICAN 308, 44 (JANUARY 2013)



Sommerfeld-Pauli-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states

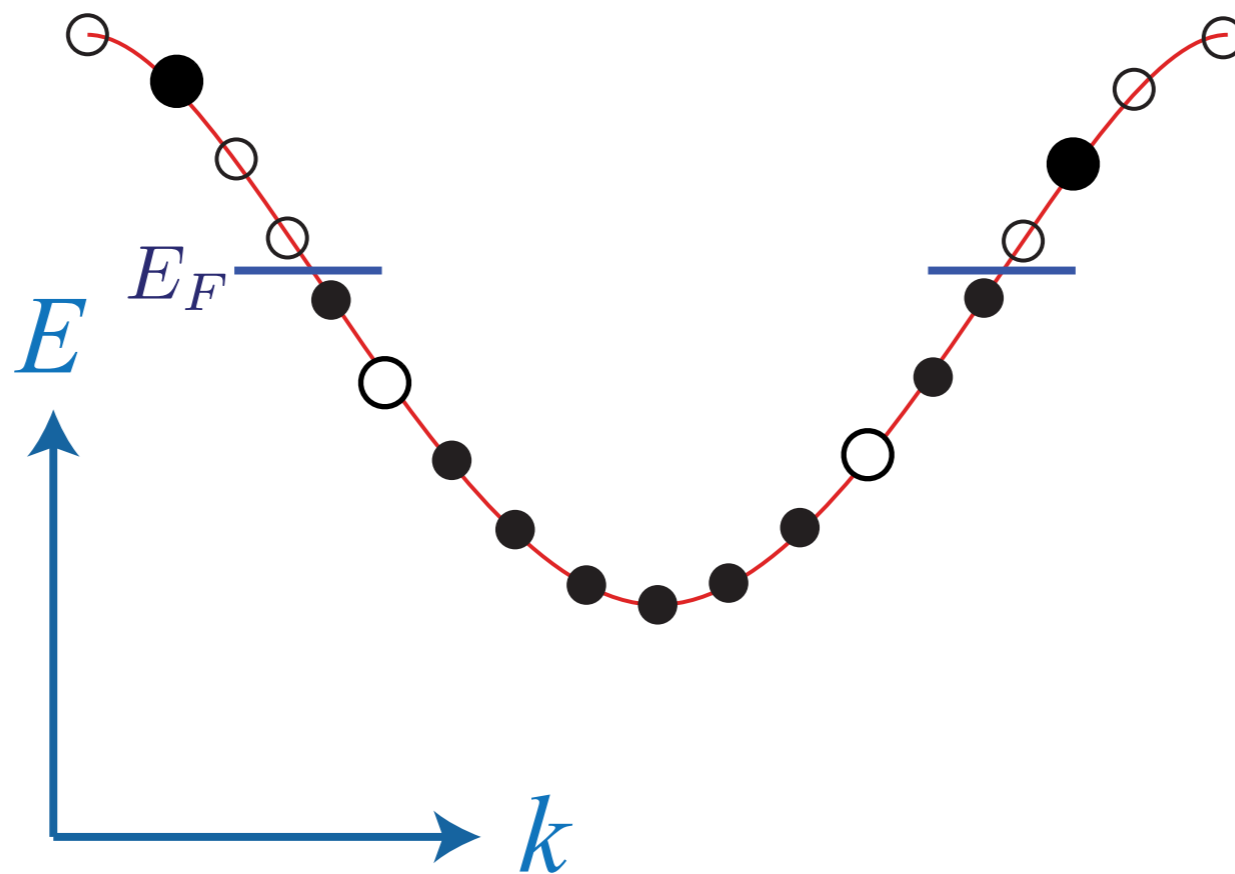
Metals



Boltzmann-Landau theory of dynamics of metals:

Long-lived **quasiparticles** (and **quasiholes**) have weak interactions which can be described by a Boltzmann equation

Metals



Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

many-particle

quantum entanglement,

and no quasiparticles

Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:

Conformal field theories and gauge-gravity duality

2. Metals with antiferromagnetism, and high temperature superconductivity

The pnictides and the cuprates

Outline

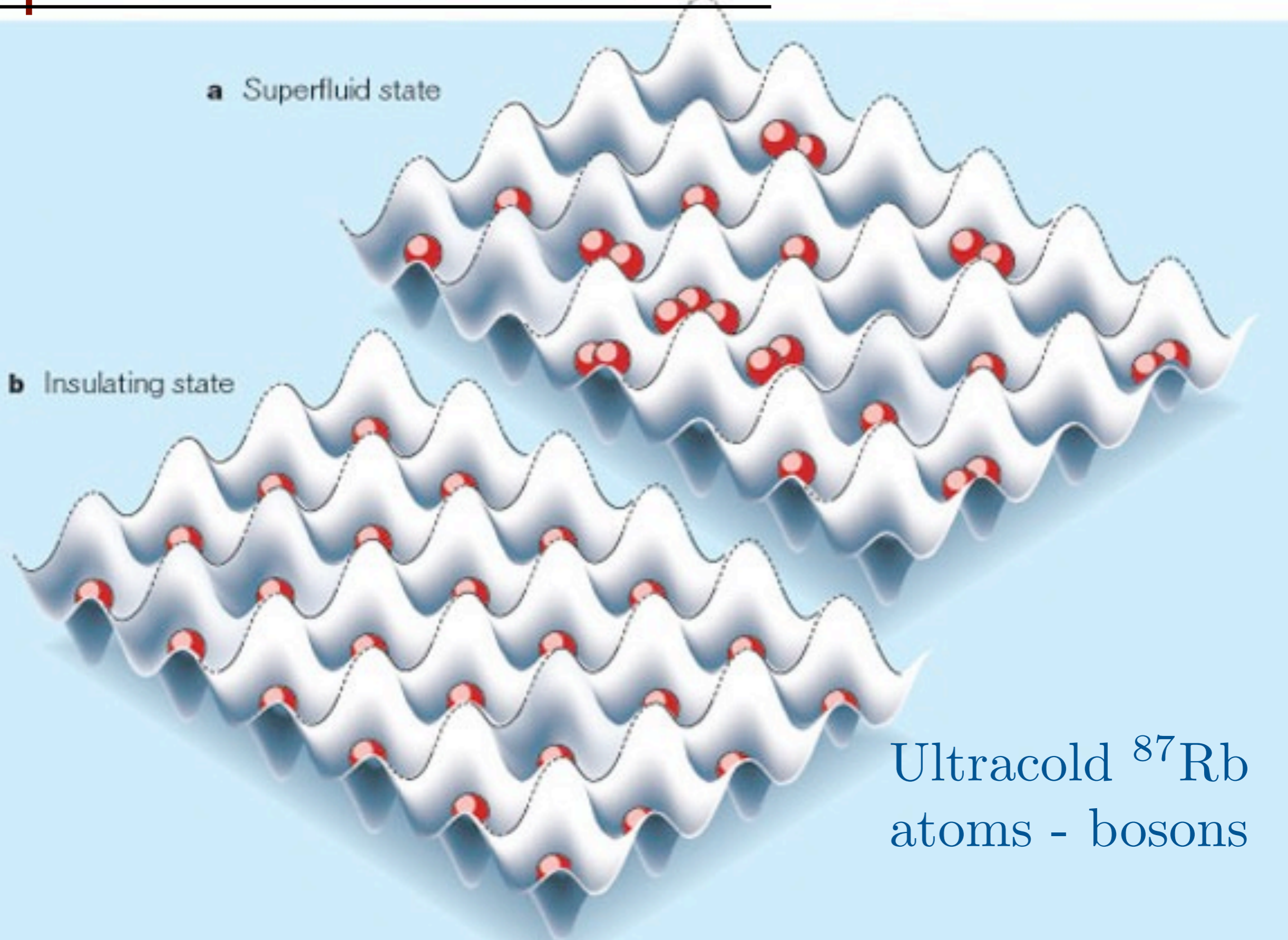
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2. Metals with antiferromagnetism, and high temperature superconductivity

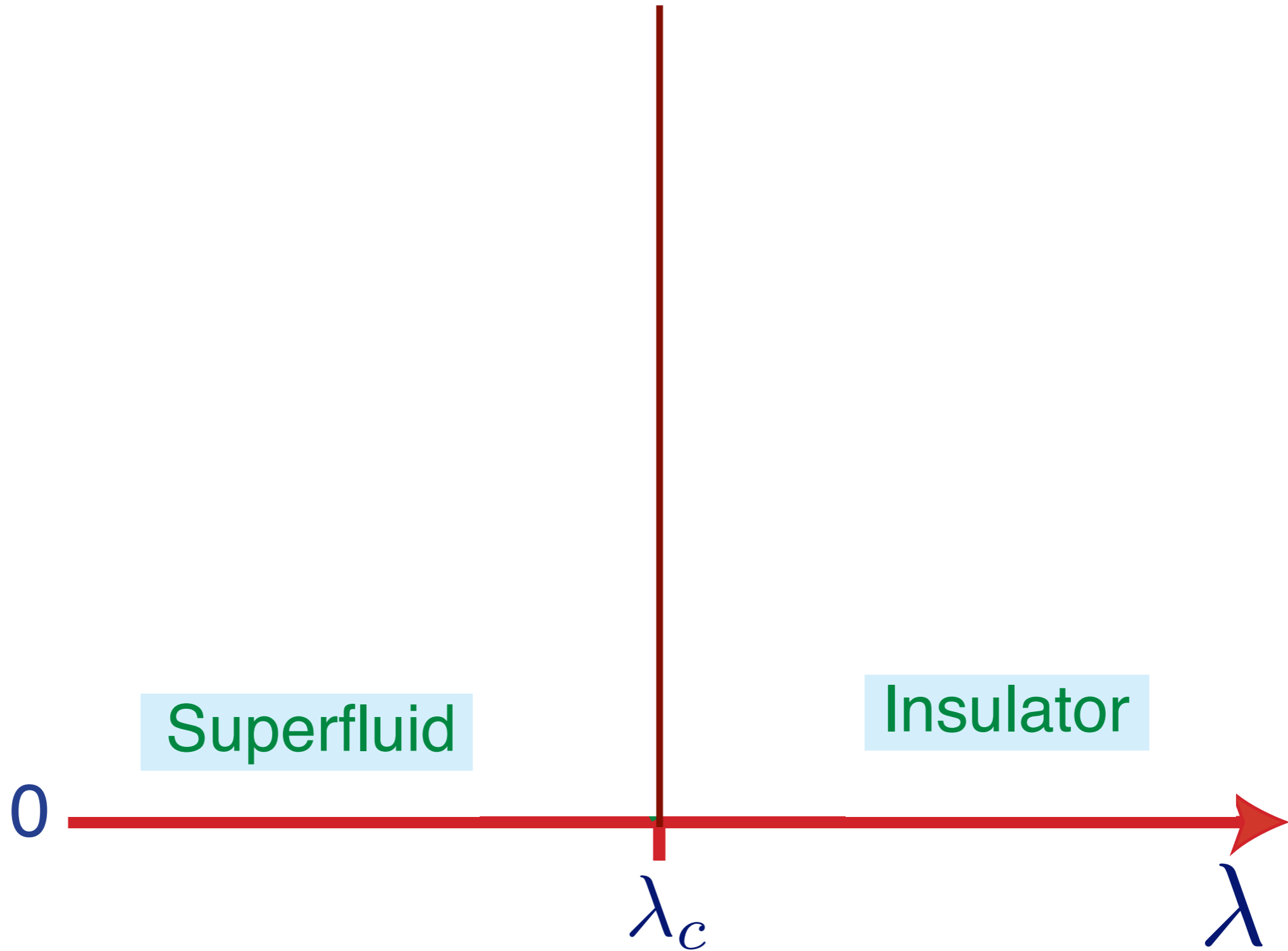
The pnictides and the cuprates

Superfluid-insulator transition

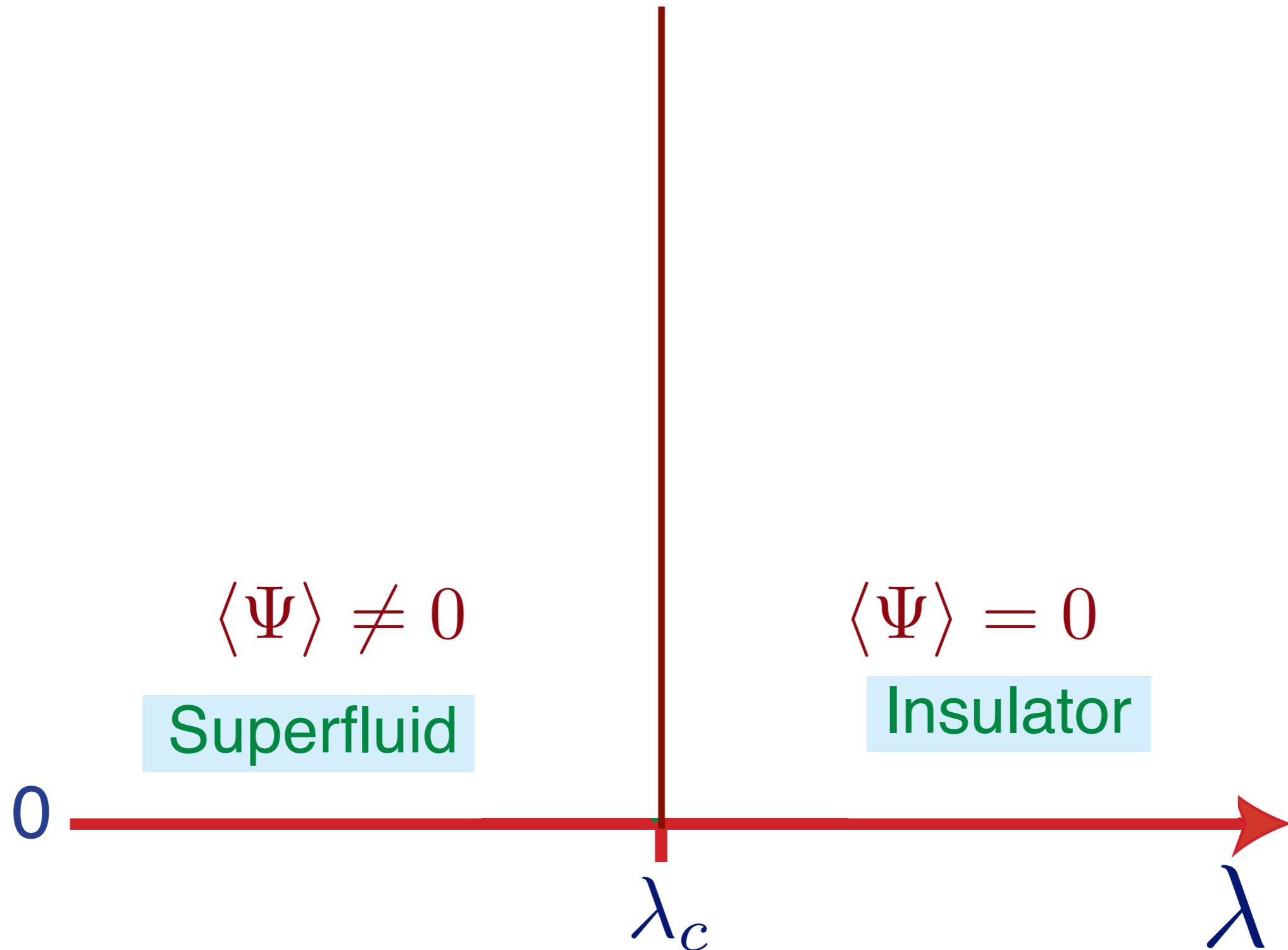


Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

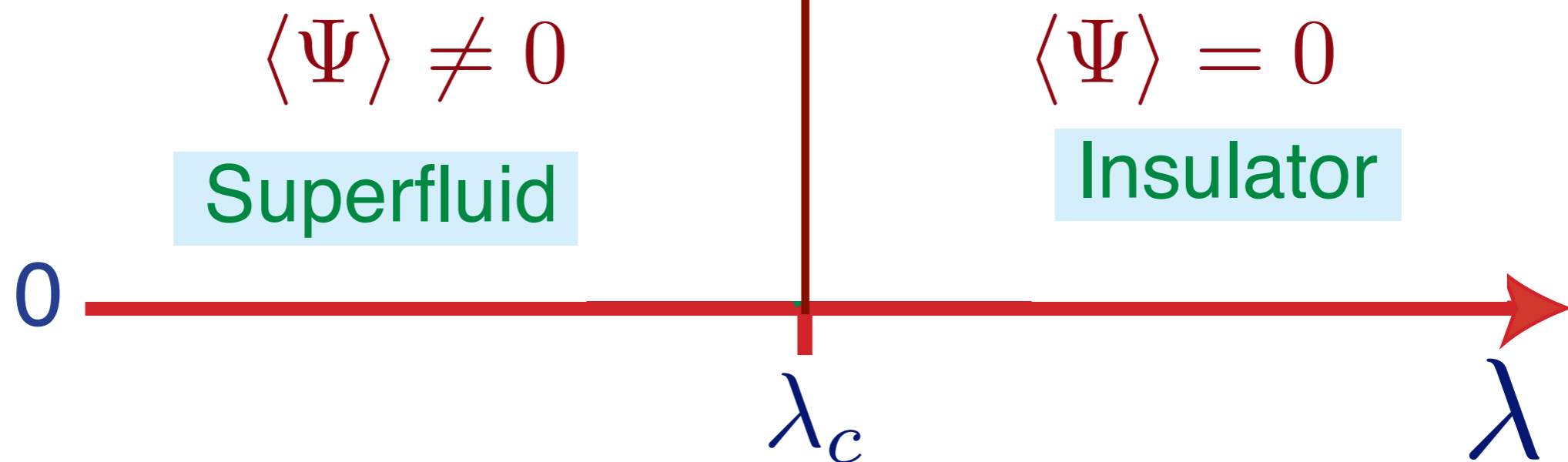


$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

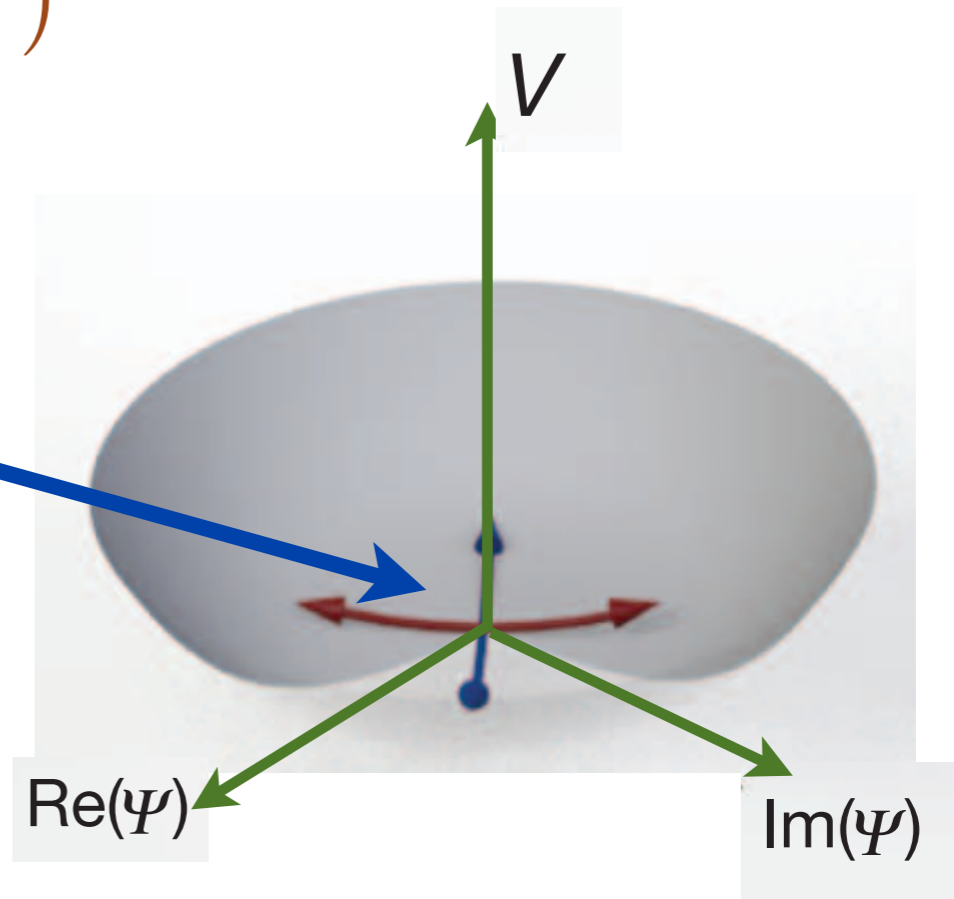
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$



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Particles and holes correspond to the 2 normal modes in the oscillation of Ψ about $\Psi = 0$.



$$\langle \Psi \rangle \neq 0$$

Superfluid

$$\langle \Psi \rangle = 0$$

Insulator

0

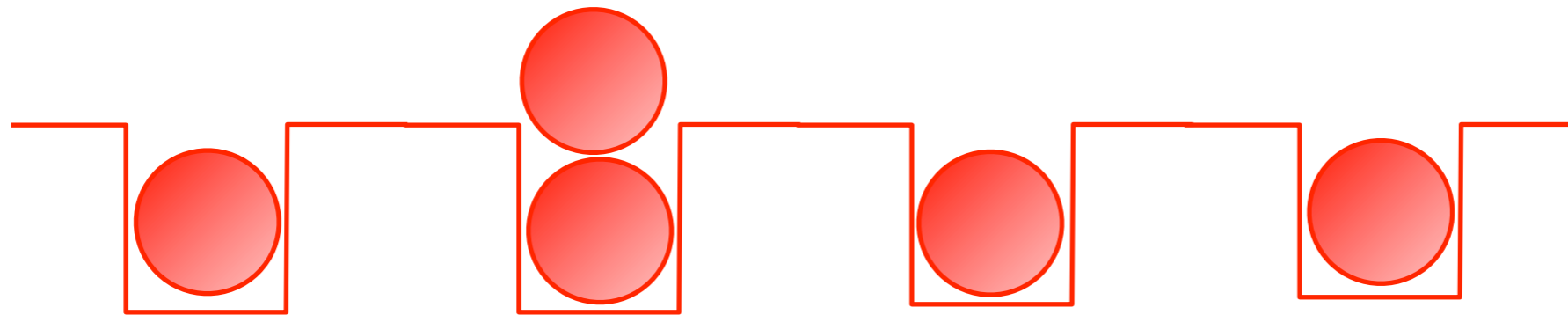
λ_c

λ



Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



Particles $\sim \Psi^\dagger$

Excitations of the insulator:

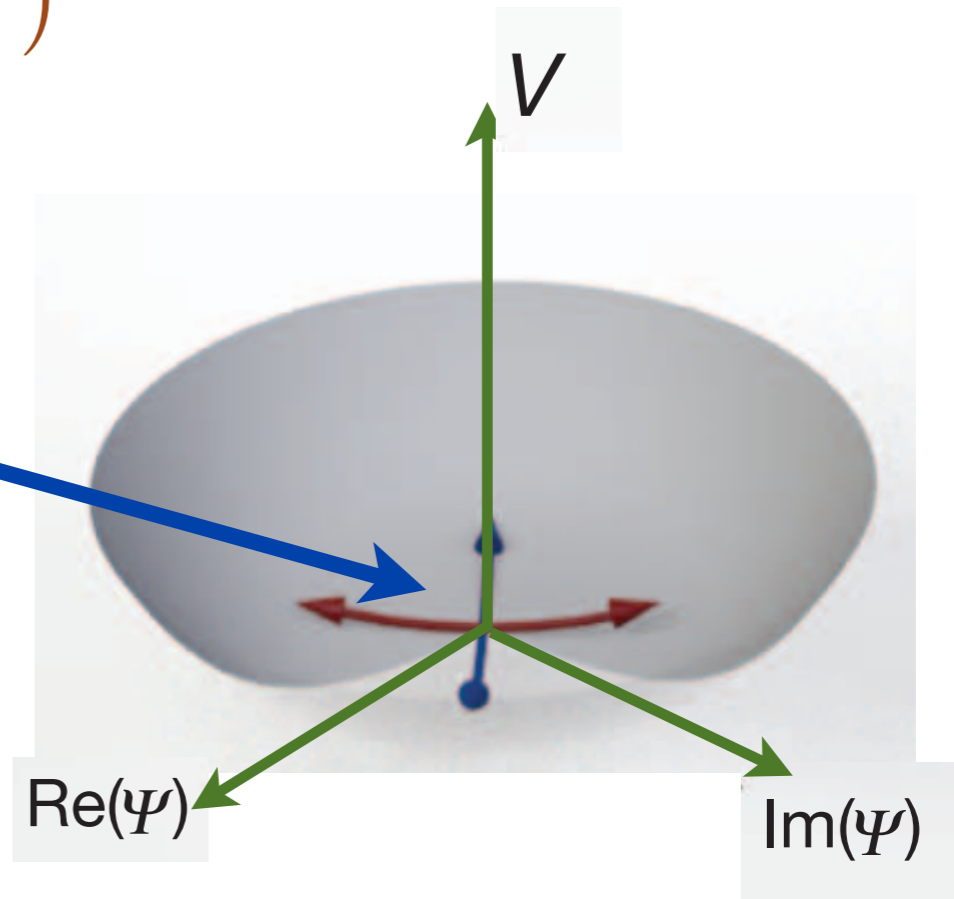


Holes $\sim \Psi$

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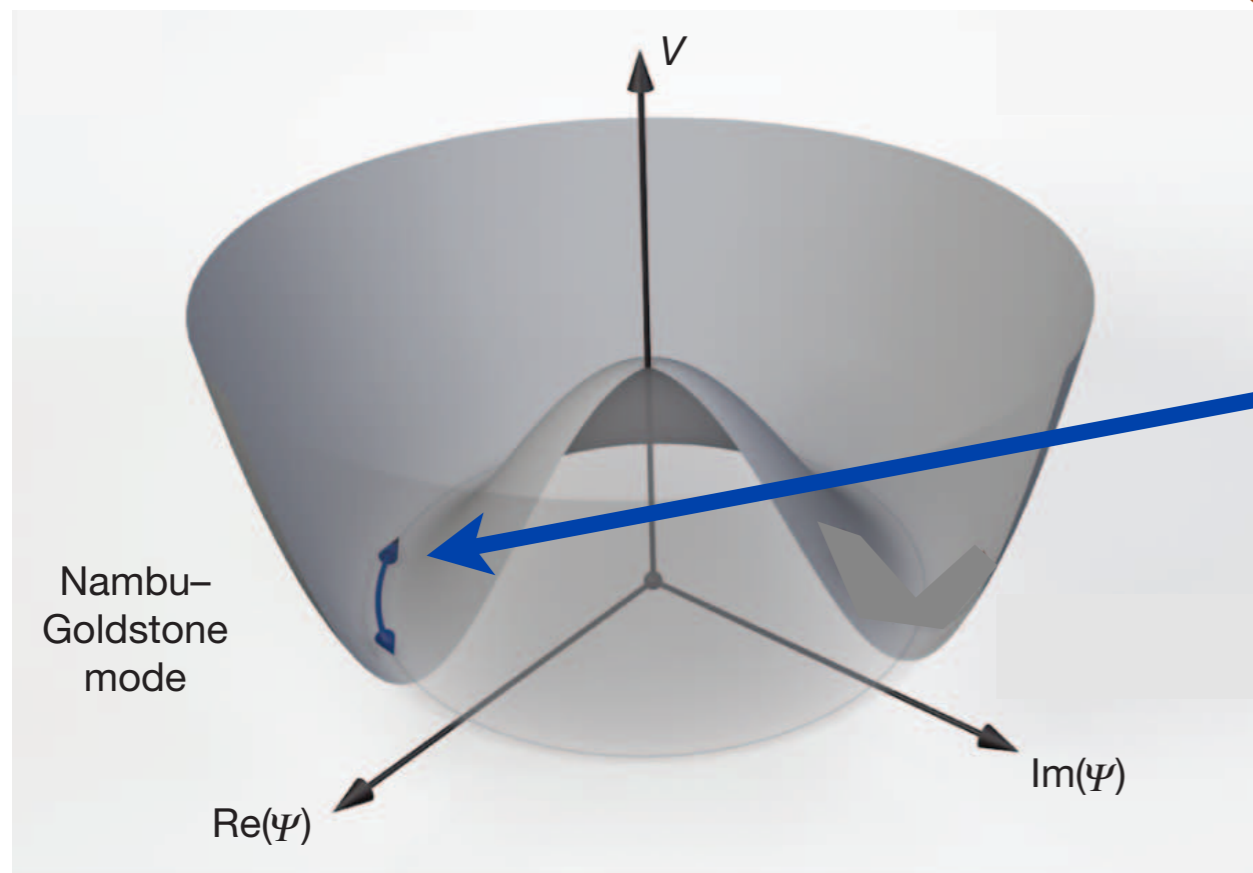
0

λ_c

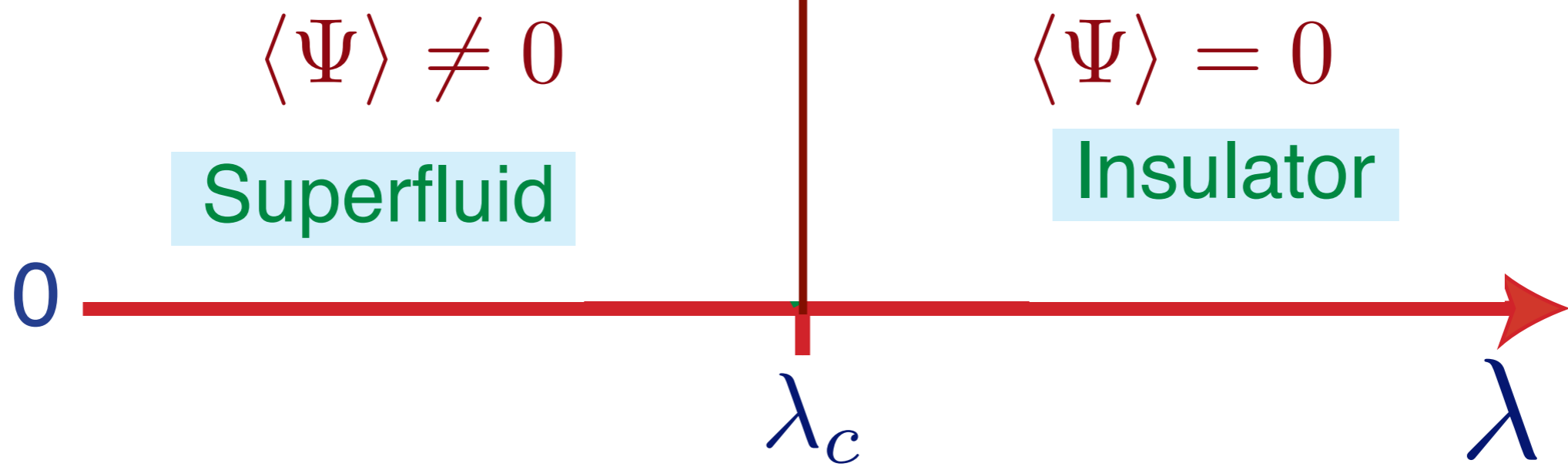
λ

$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$



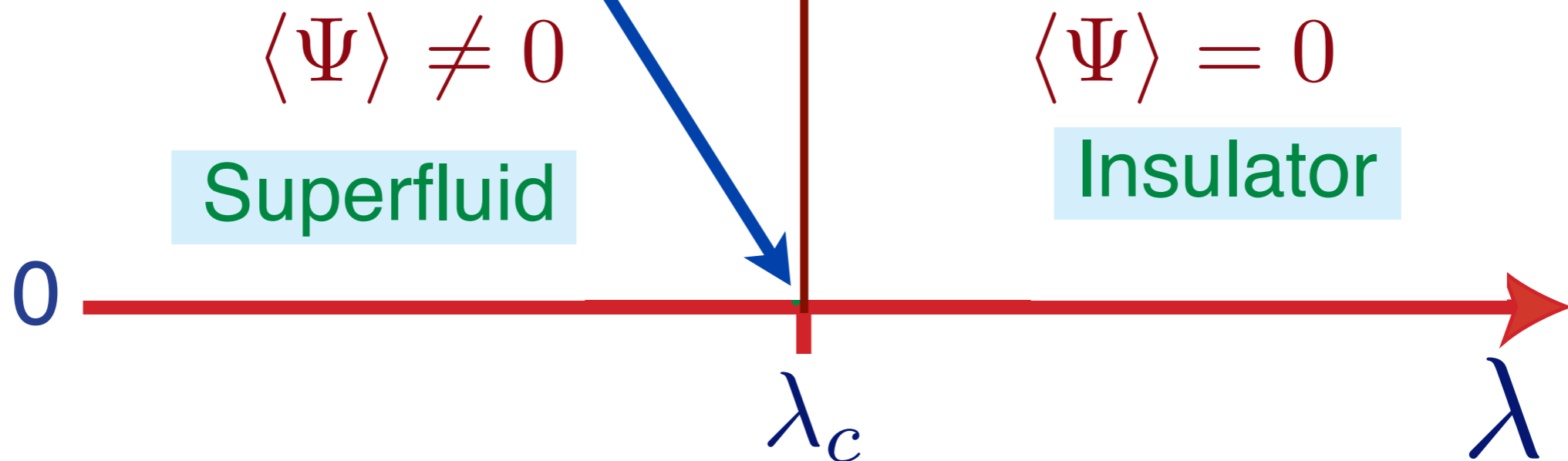
Nambu-Goldstone mode is the oscillation in the phase Ψ at a constant non-zero $|\Psi|$.



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

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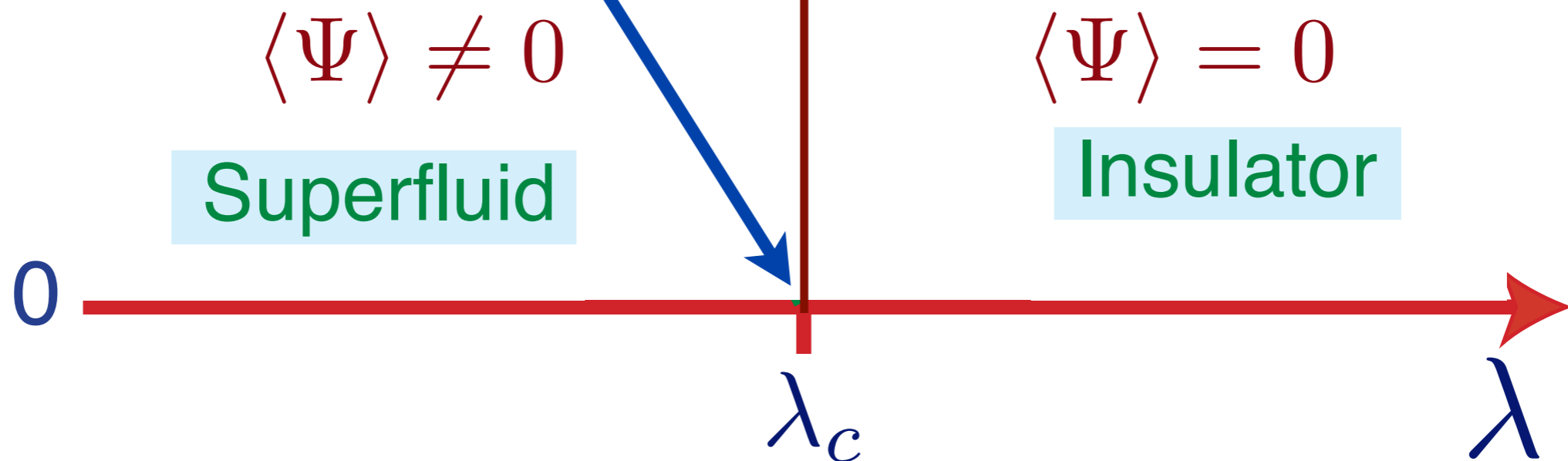
A conformal field theory
in 2+1 spacetime dimensions:
a CFT3



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

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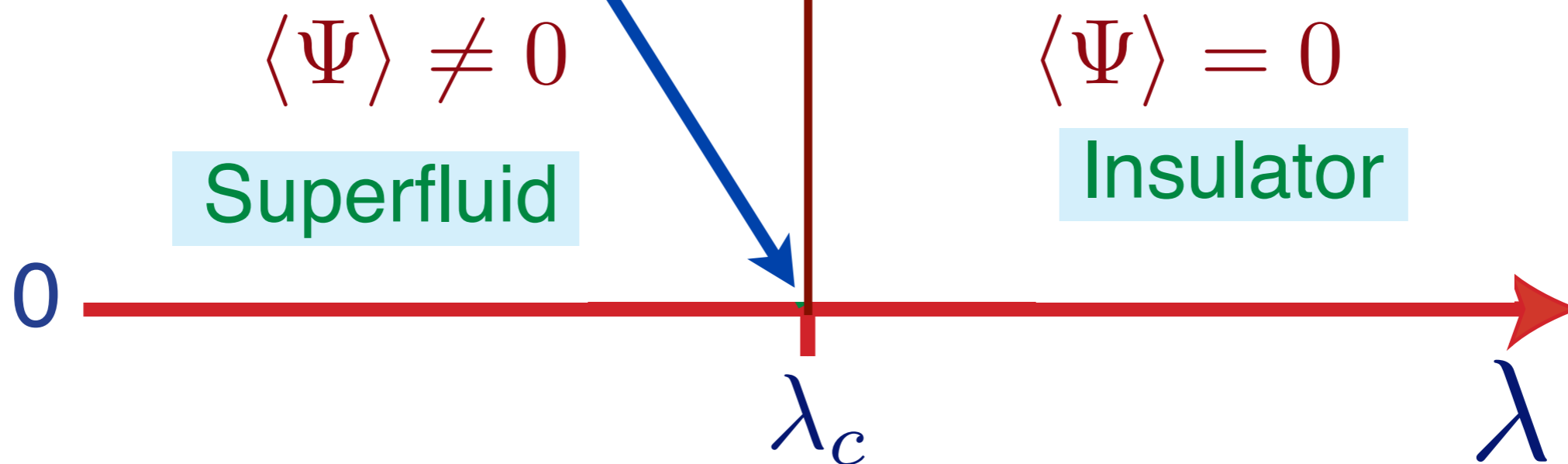
Quantum state with
complex, many-body,
“long-range” quantum entanglement



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

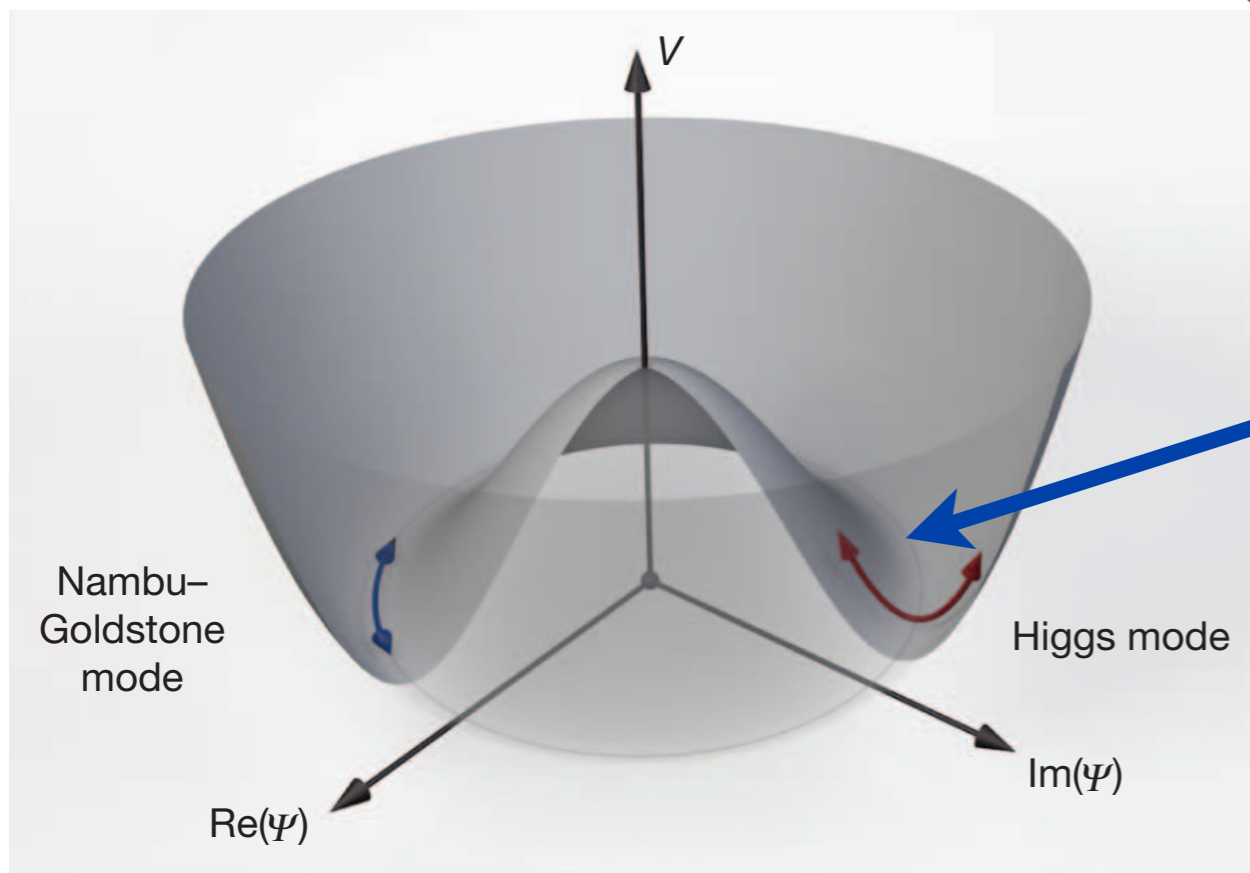
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$

No well-defined normal modes,
or particle-like excitations



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$



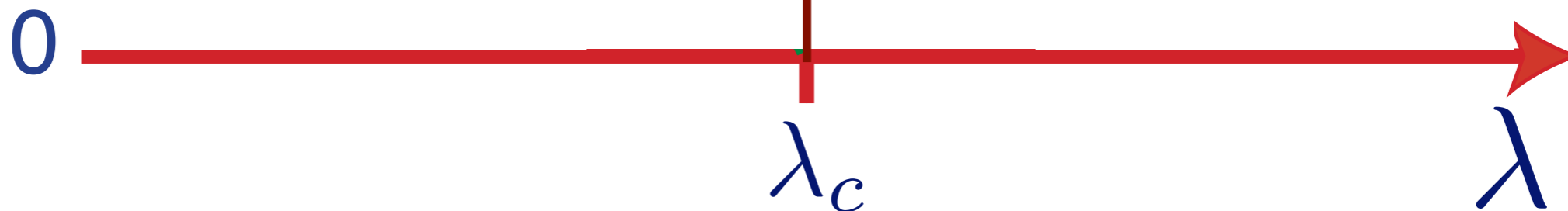
Higgs mode is the oscillation in the amplitude $|\Psi|$. This decays rapidly by emitting pairs of Nambu-Goldstone modes.

$$\langle \Psi \rangle \neq 0$$

Superfluid

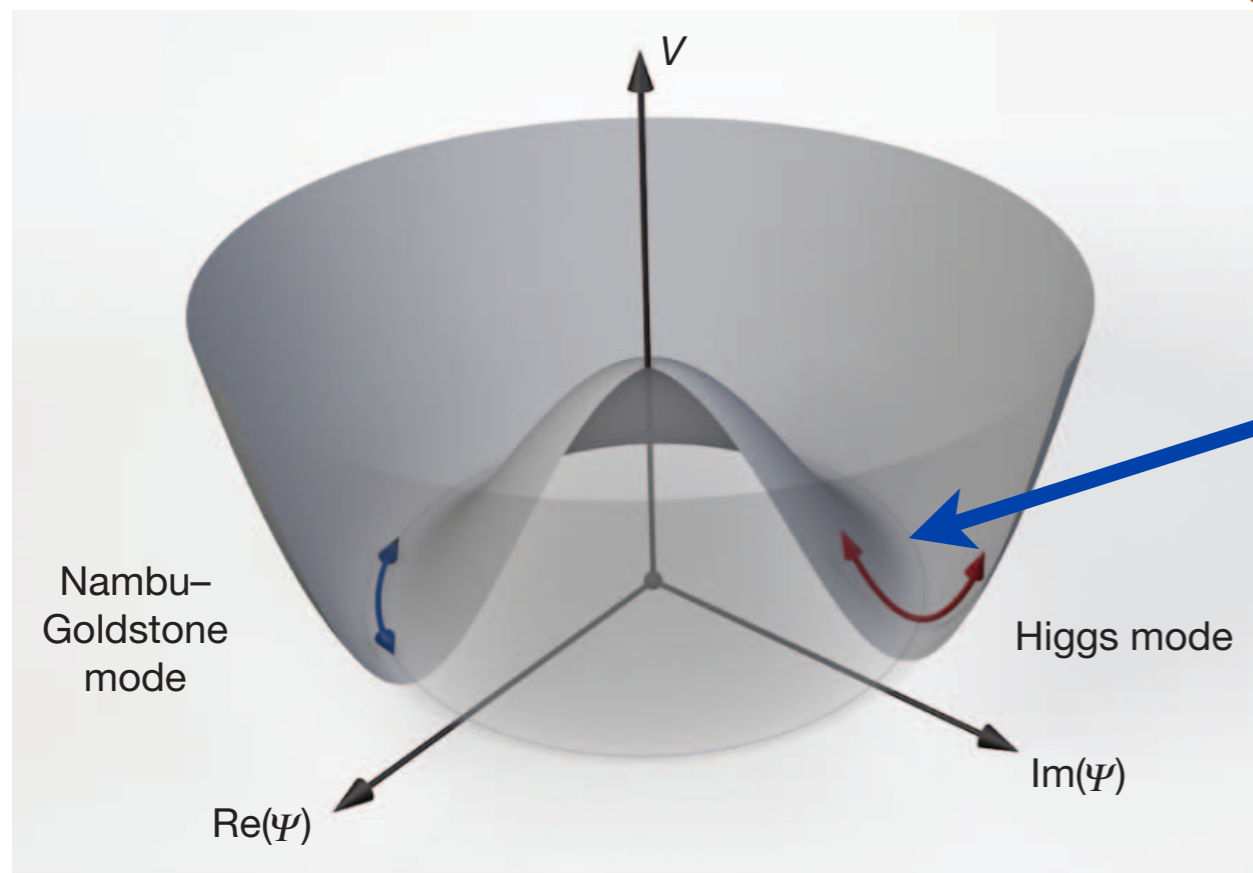
$$\langle \Psi \rangle = 0$$

Insulator



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

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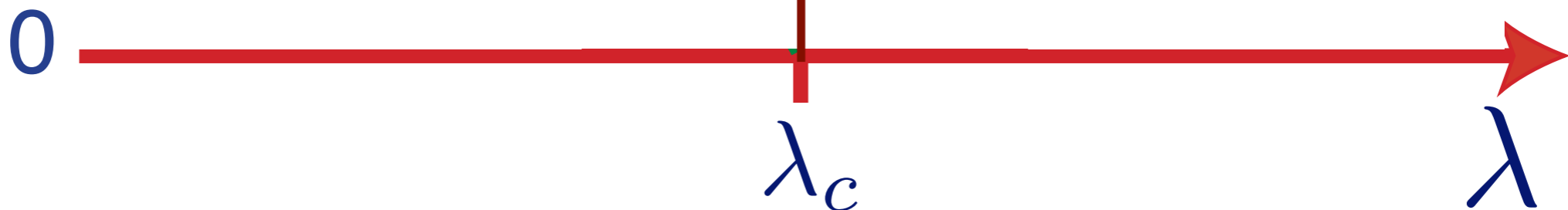
Despite rapid decay, there is a well-defined Higgs “quasi-normal mode”. This is associated with a pole in the lower-half of the complex frequency plane.

$$\langle \Psi \rangle \neq 0$$

Superfluid

$$\langle \Psi \rangle = 0$$

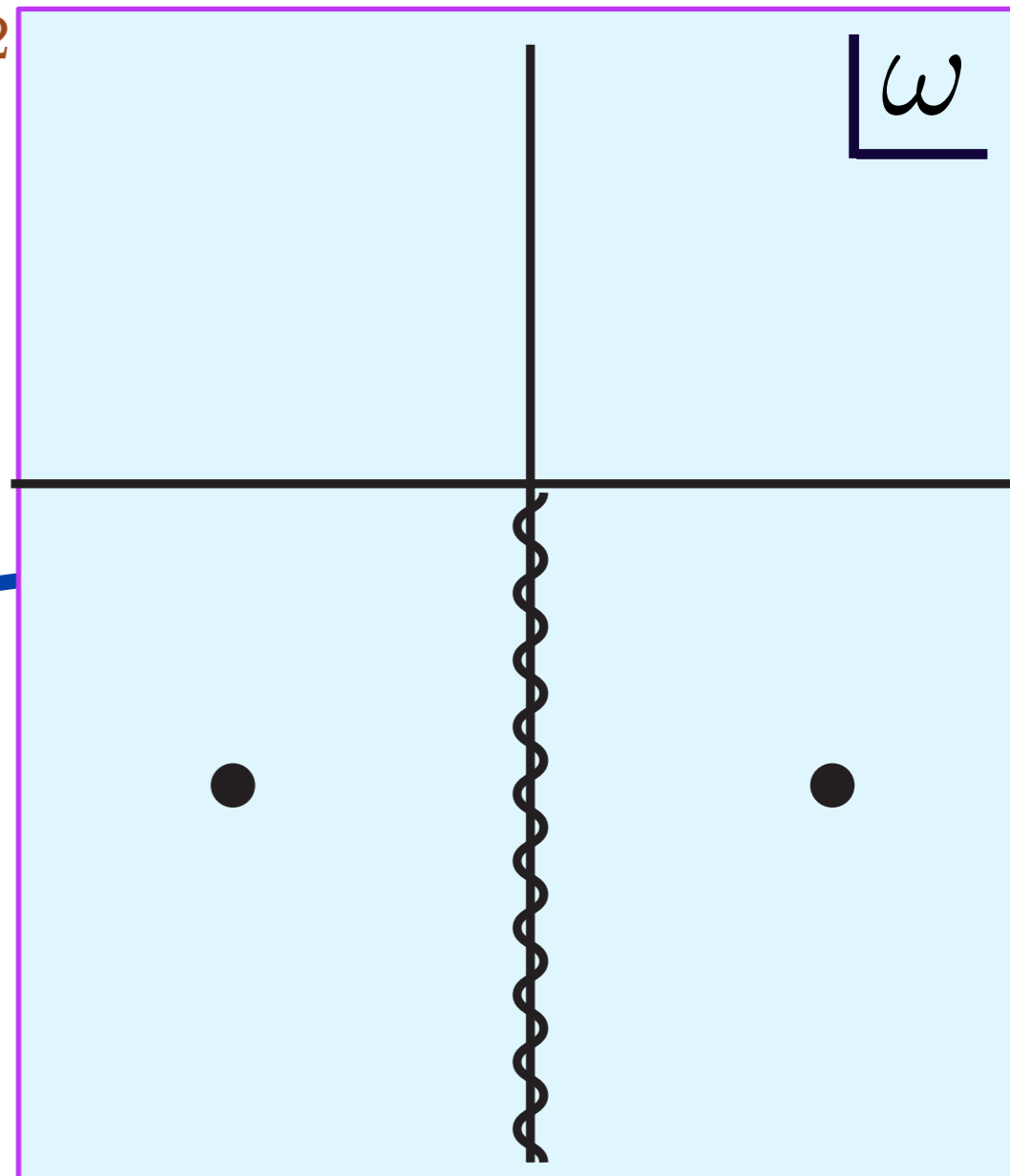
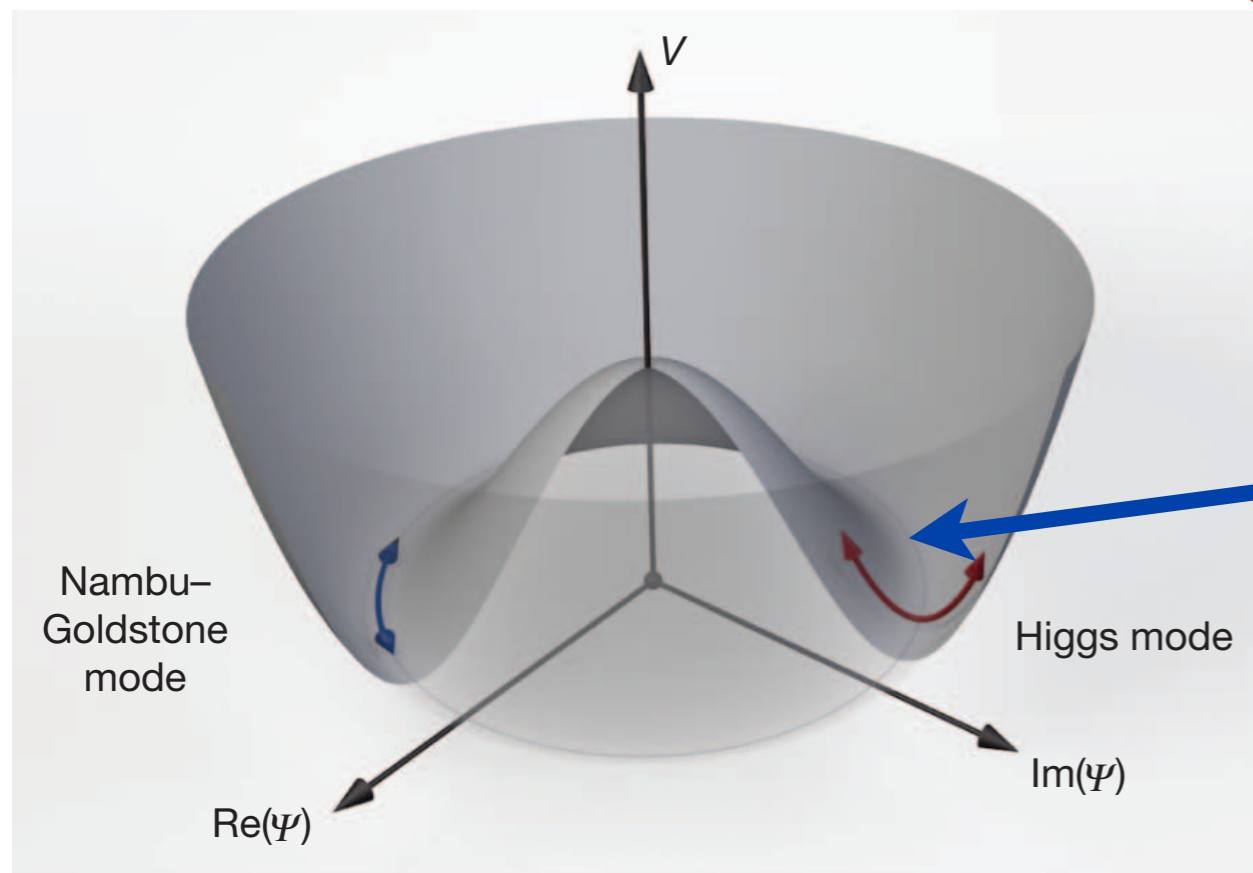
Insulator



D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

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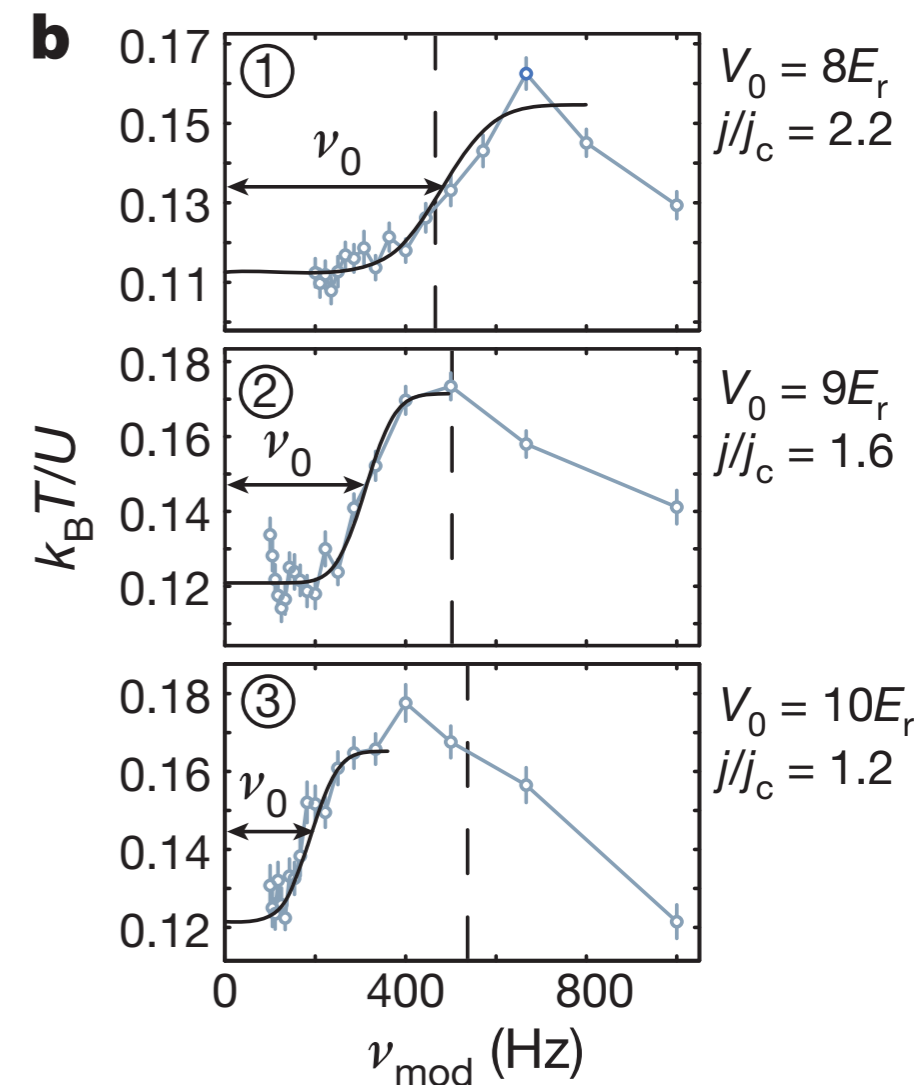
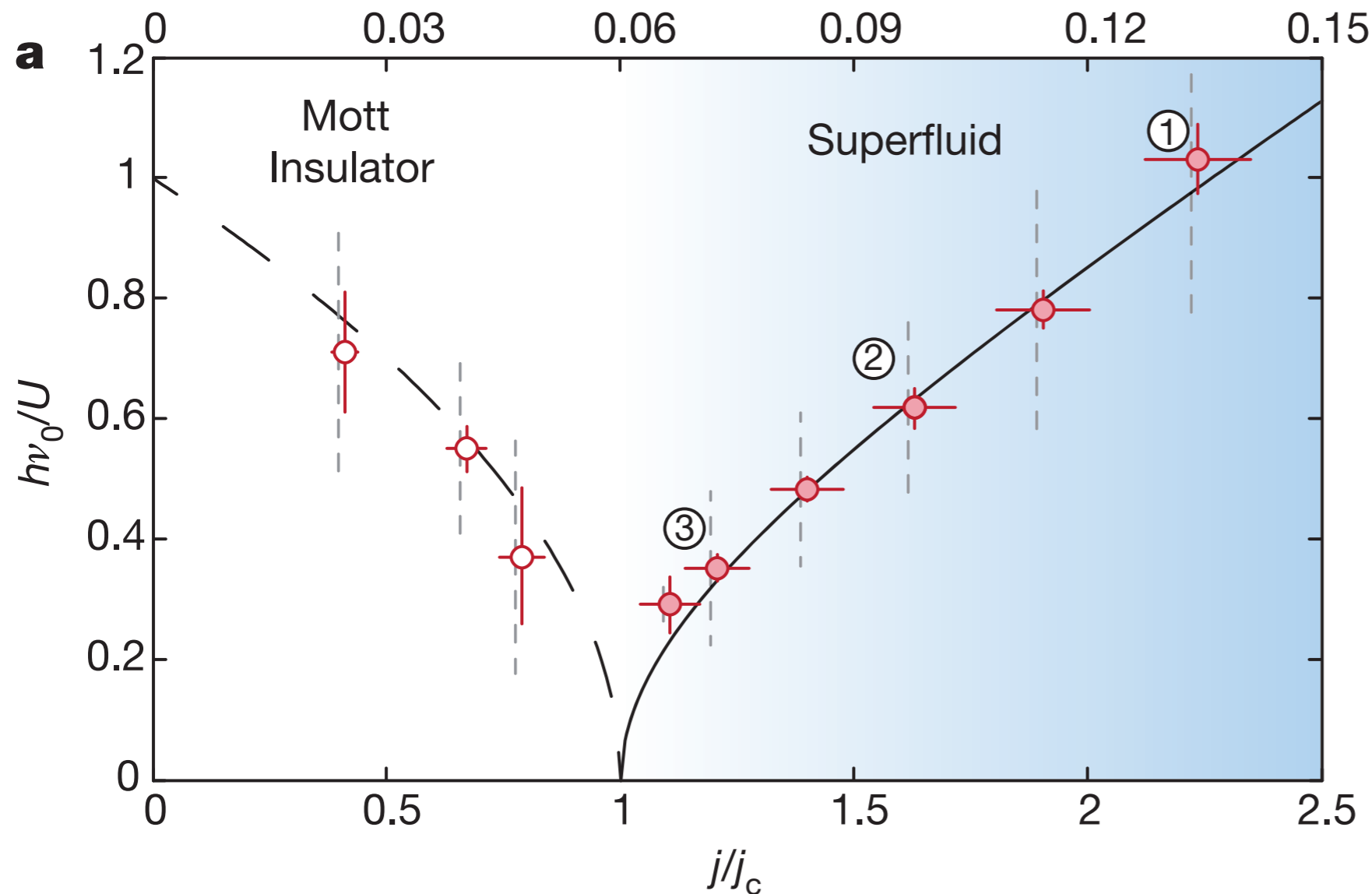
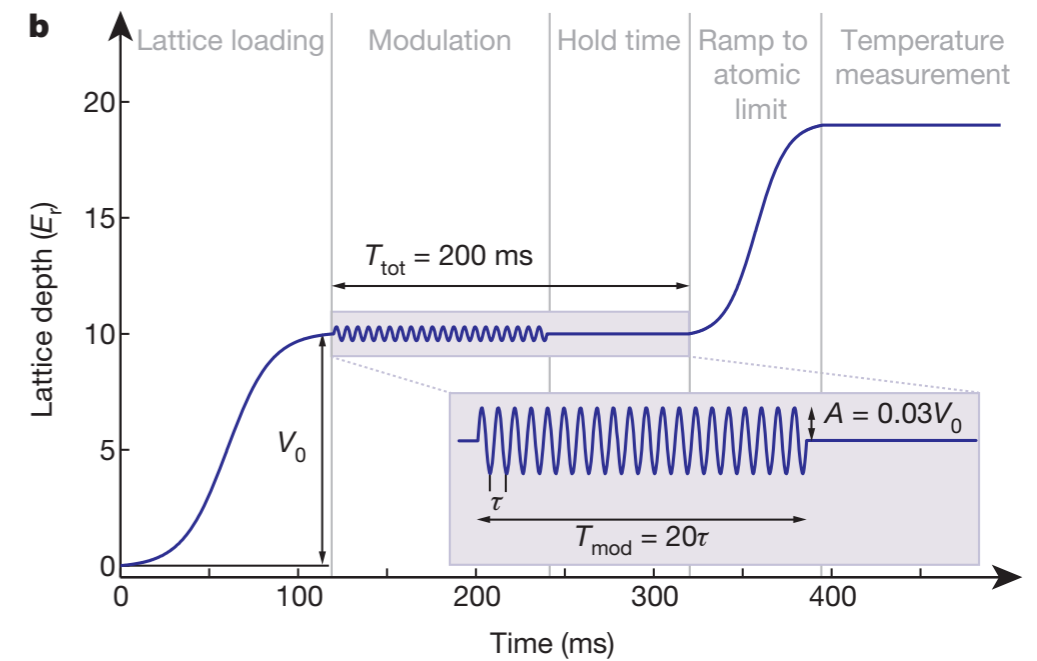
D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).
The Higgs quasi-normal mode is at the frequency

$$\frac{\omega_{\text{pole}}}{\Delta} = -i \frac{4}{\pi} + \frac{1}{N} \left(\frac{16 (4 + \sqrt{2} \log (3 - 2\sqrt{2}))}{\pi^2} + 2.46531203396 i \right) + \mathcal{O} \left(\frac{1}{N^2} \right)$$

where Δ is the particle gap at the complementary point in the “paramagnetic” state with the same value of $|\lambda - \lambda_c|$, and $N = 2$ is the number of vector components of Ψ . The universal answer is a consequence of the strong interactions in the CFT3

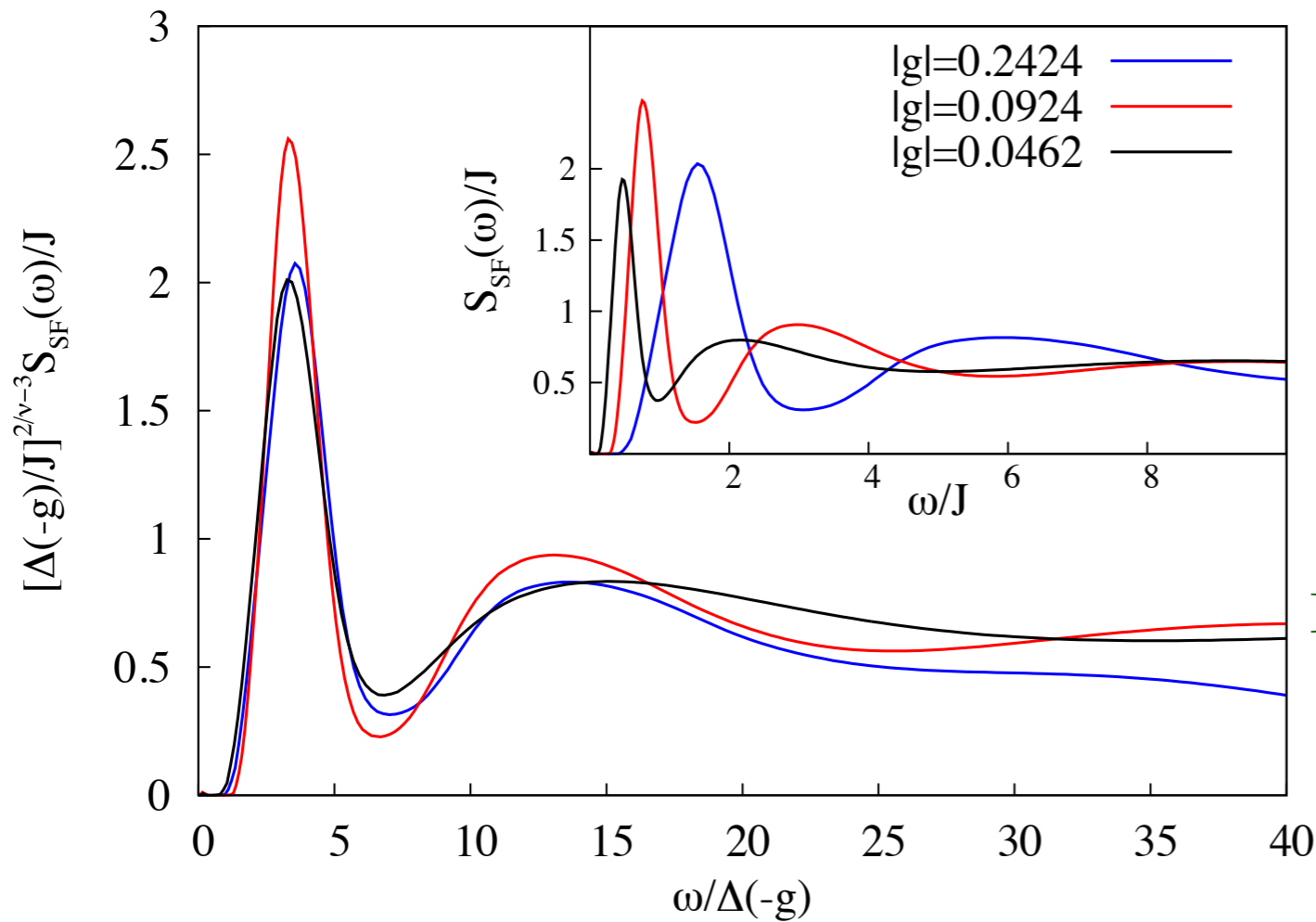
Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole



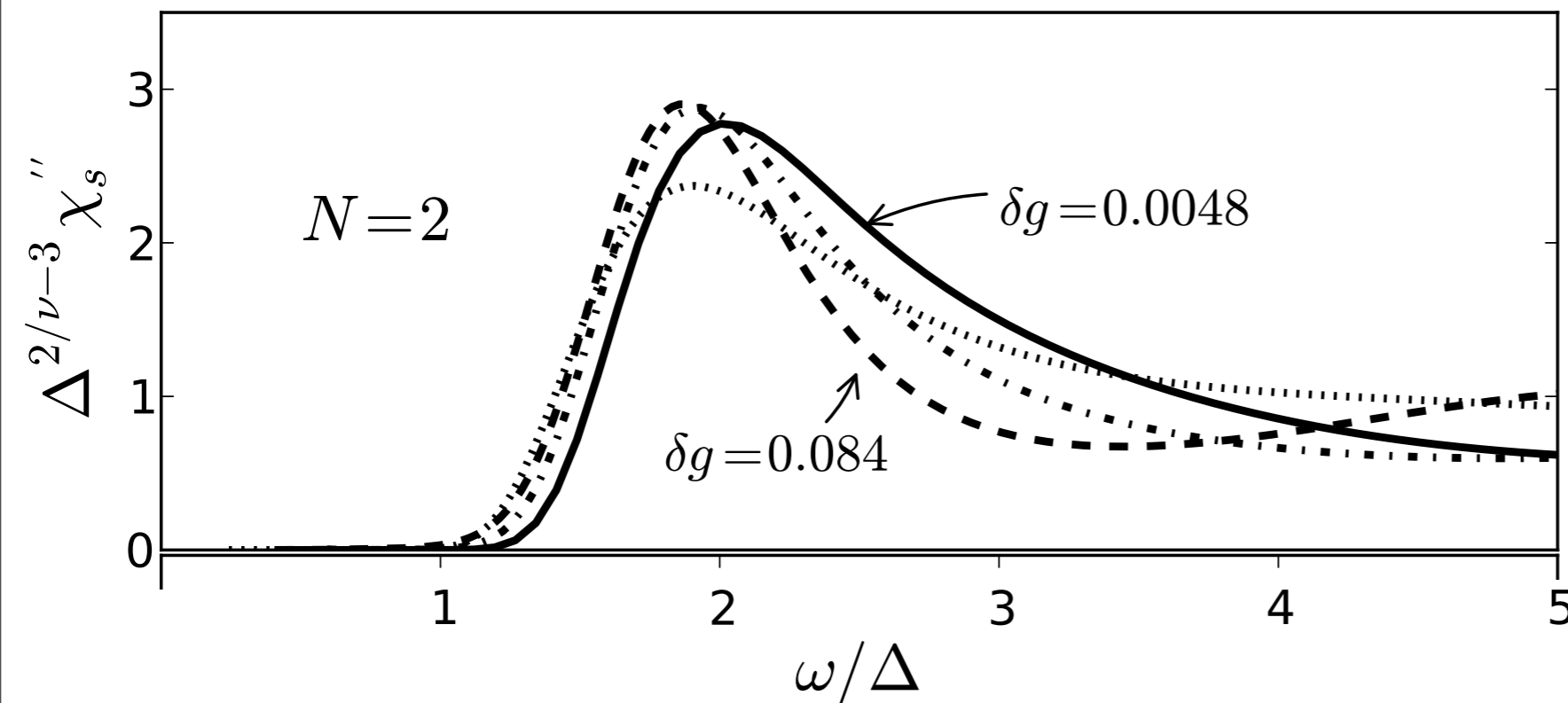
Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

Observation of Higgs quasi-normal mode in quantum Monte Carlo



Scaling of spectral response functions predicted in D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, and Nikolay Prokof'ev, arXiv:1301.3139

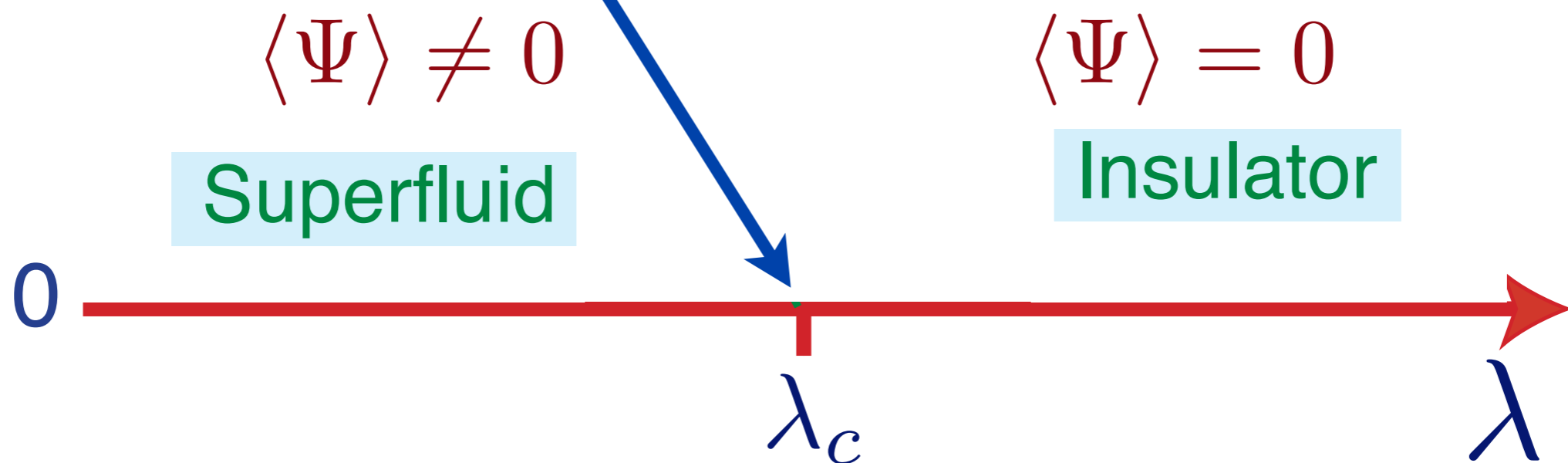


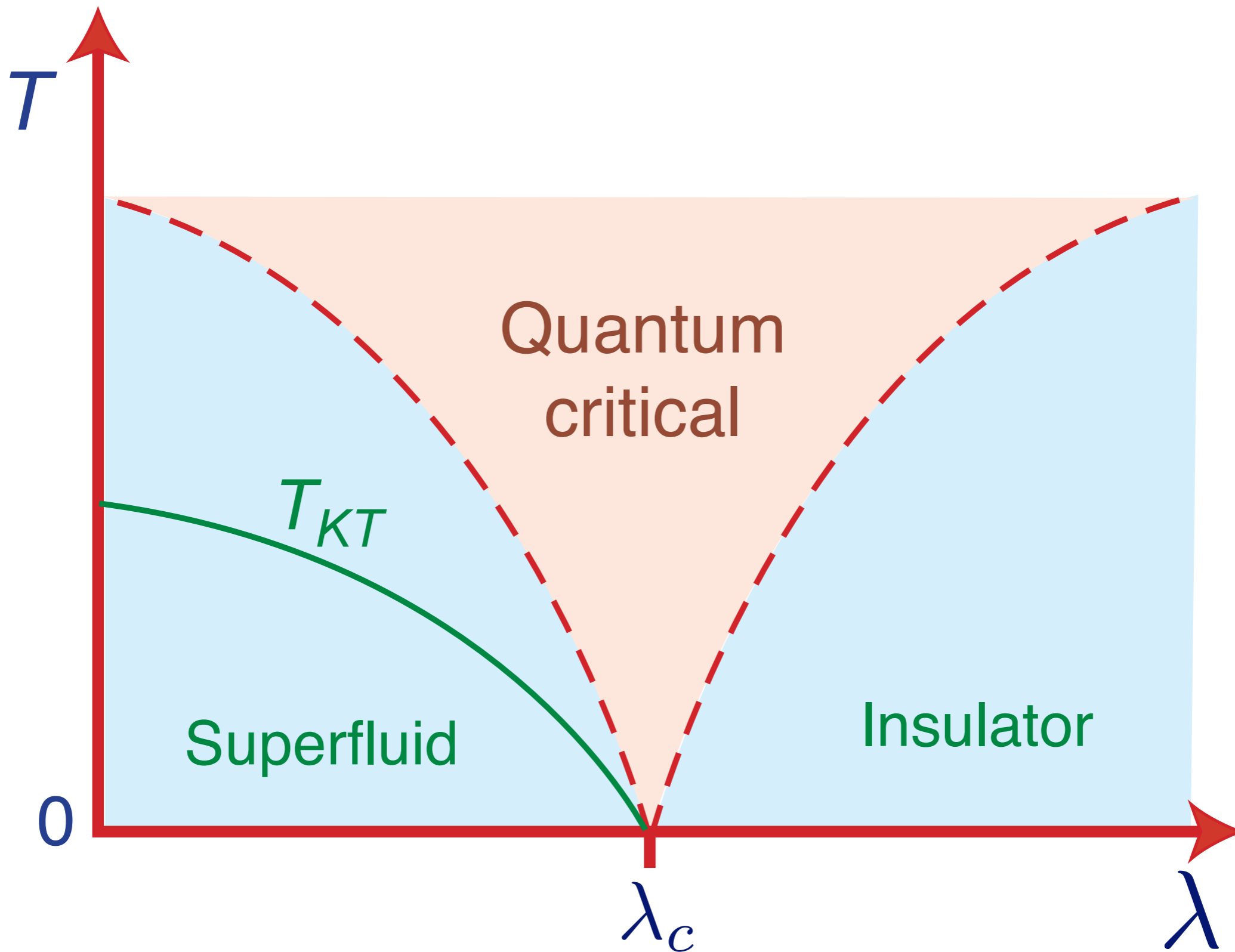
Snir Gazit, Daniel Podolsky, and Assa Auerbach, arXiv:1212.3759

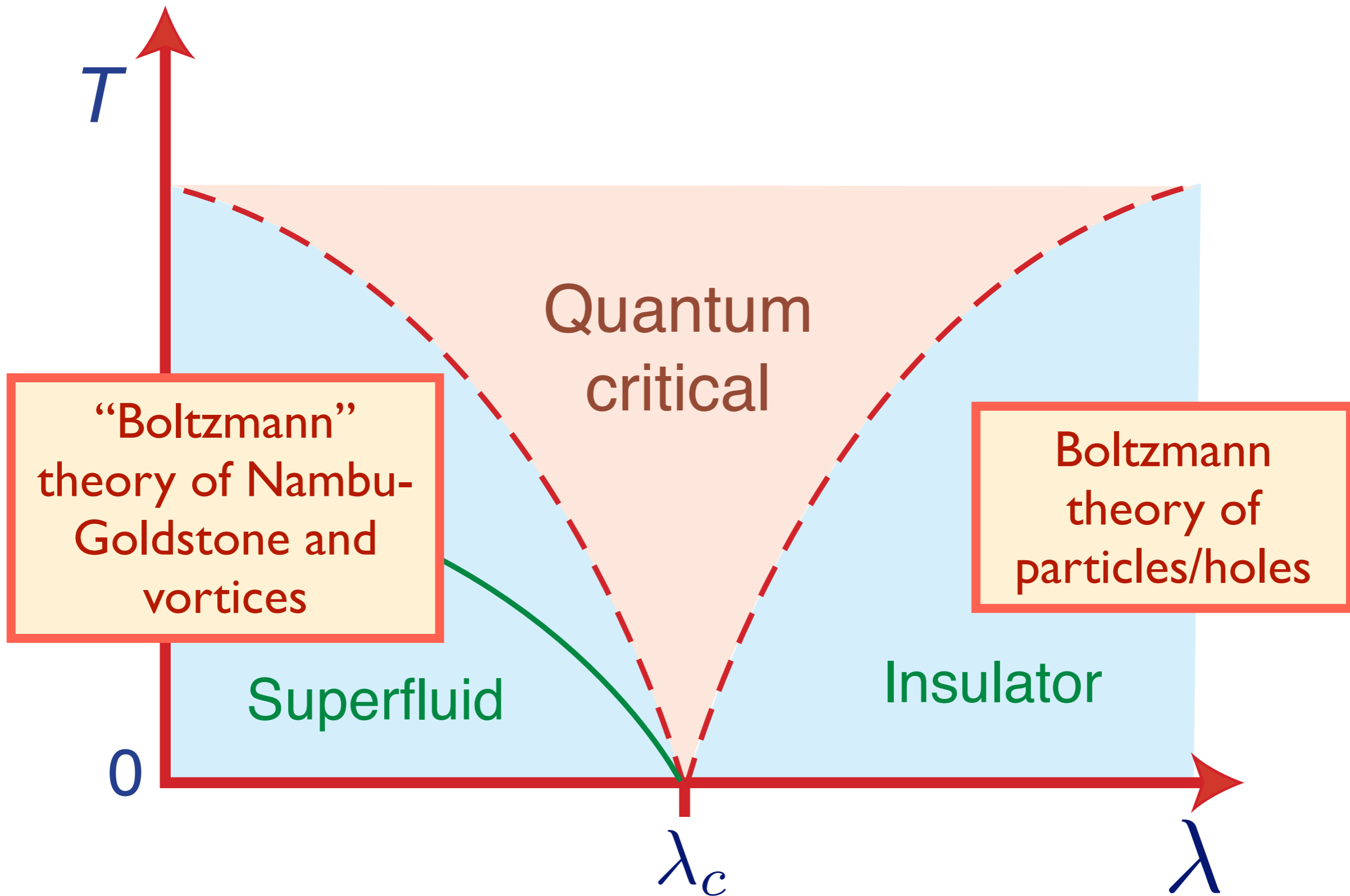
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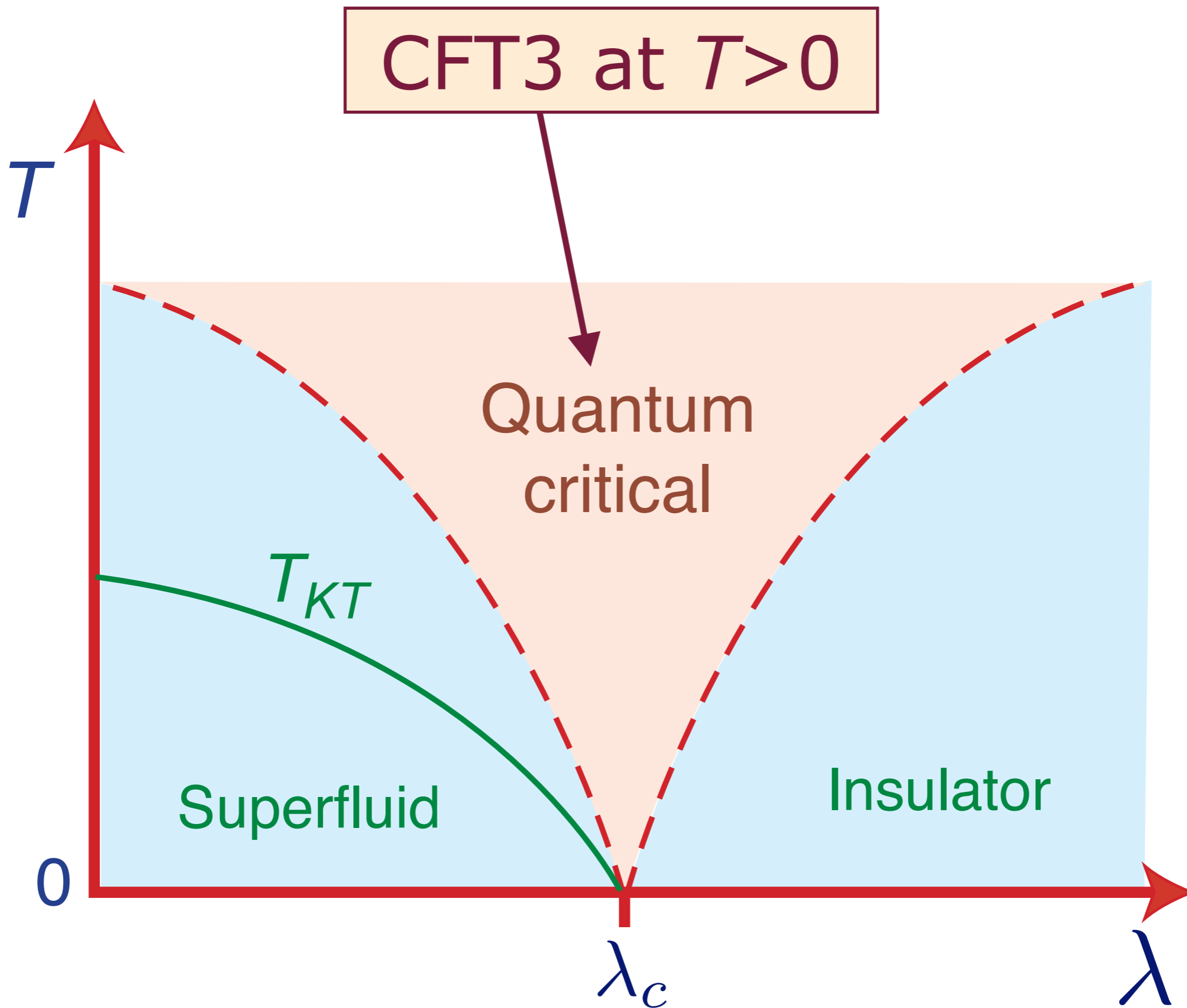
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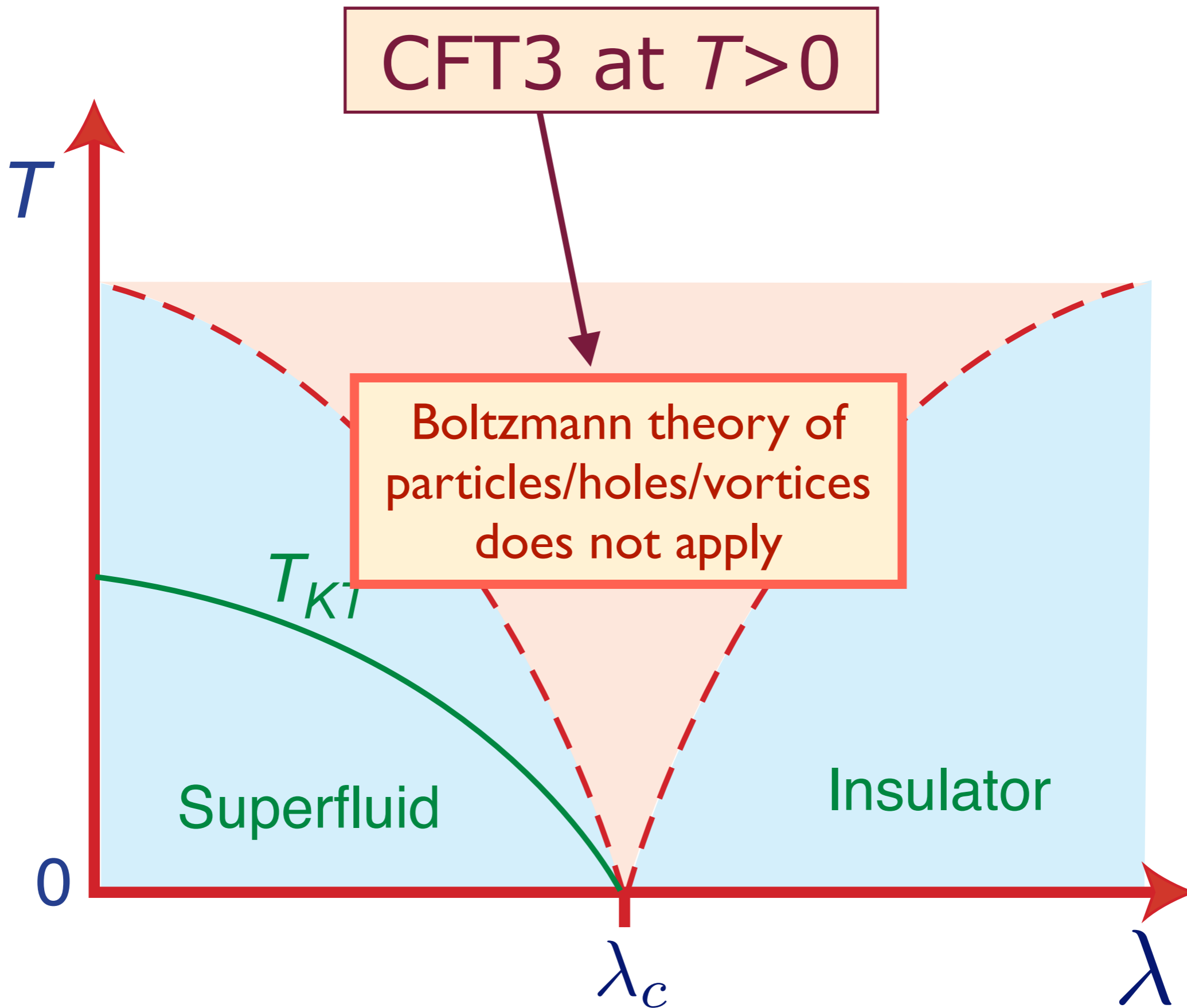
A conformal field theory
in 2+1 spacetime dimensions:
a CFT3











CFT3 at $T > 0$

Boltzmann theory of particles/holes/vortices does not apply

Superfluid

Insulator

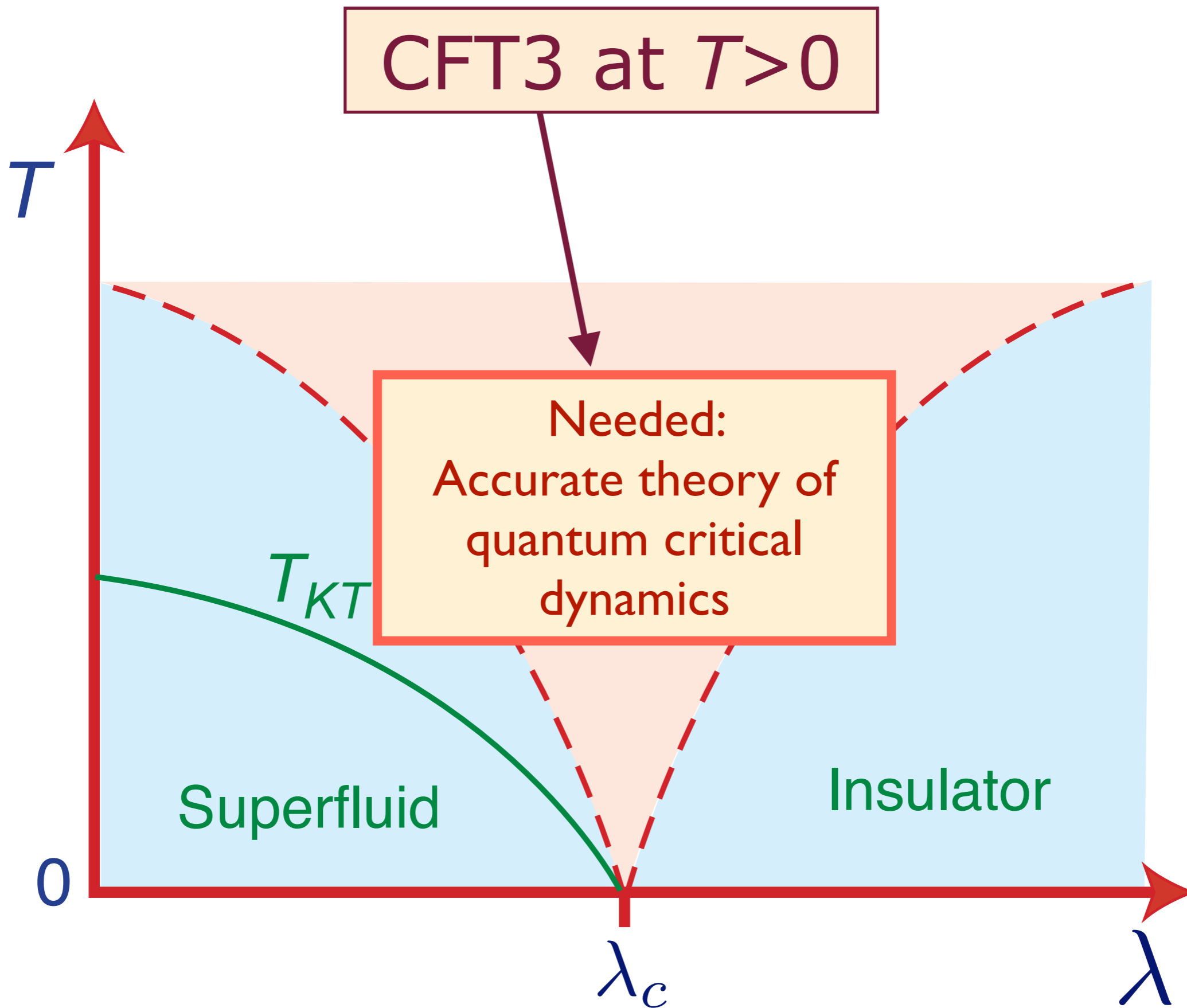
T_{KT}

λ_c

λ

T

0



Quantum critical dynamics

Quantum “*nearly perfect fluid*”
with shortest possible *local* equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C} \frac{\hbar}{k_B T}$$

where \mathcal{C} is a *universal* constant.

Response functions are characterized by poles in LHP
with $\omega \sim k_B T / \hbar$.

These poles (quasi-normal modes) appear naturally in
the holographic theory.

(Analogous of Higgs quasi-normal mode.)

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999).

Quantum critical dynamics

Transport co-efficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Conductivity

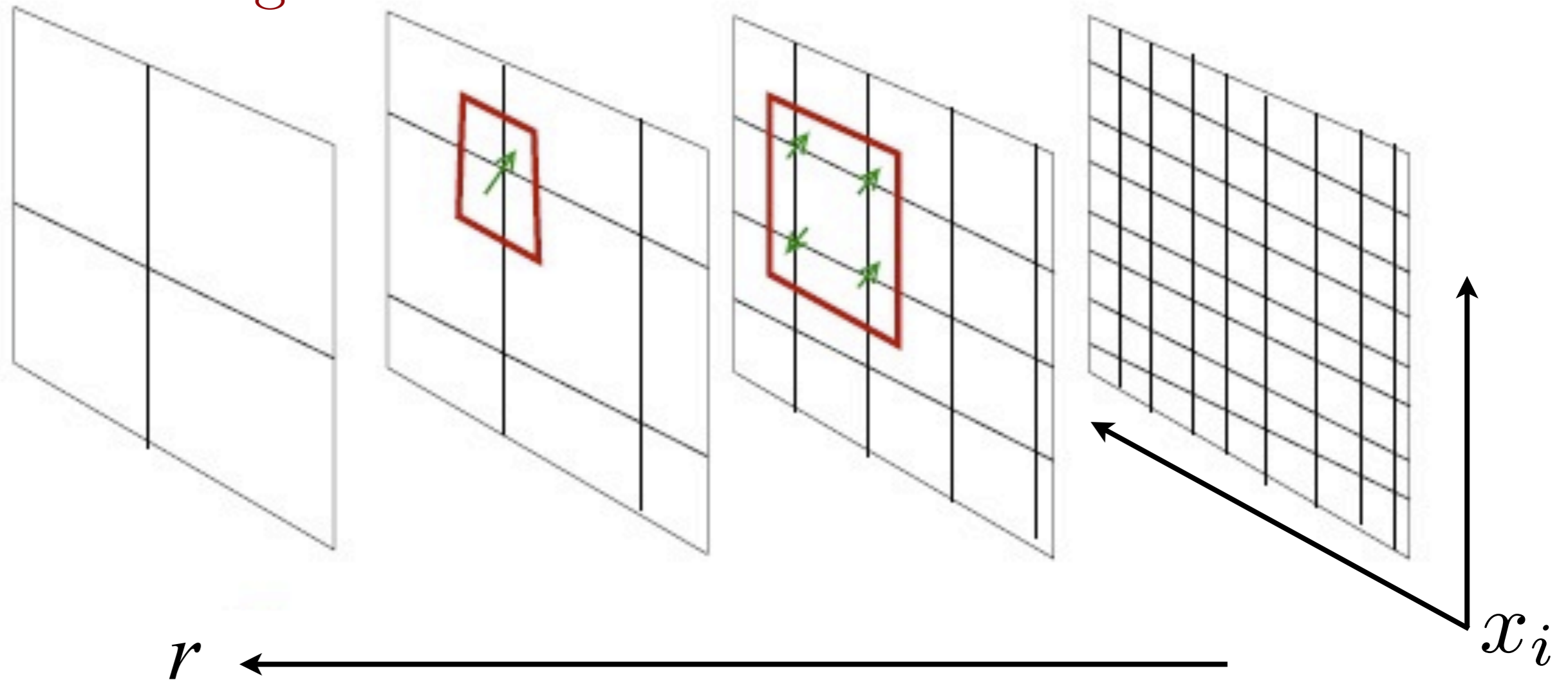
$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)]$$

(Q is the “charge” of one boson)

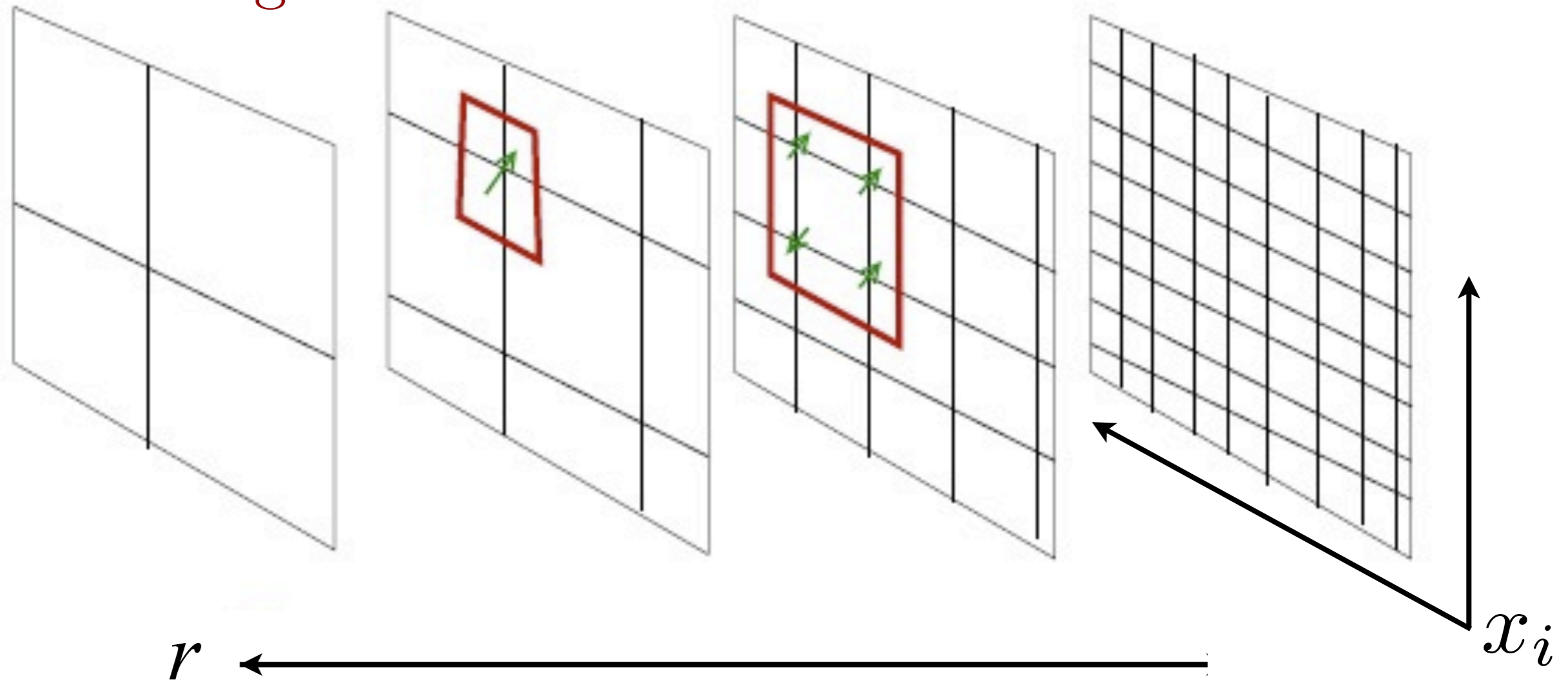
M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett.* **64**, 587 (1990)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Renormalization group: \Rightarrow Follow coupling constants of quantum many body theory as a function of length scale r



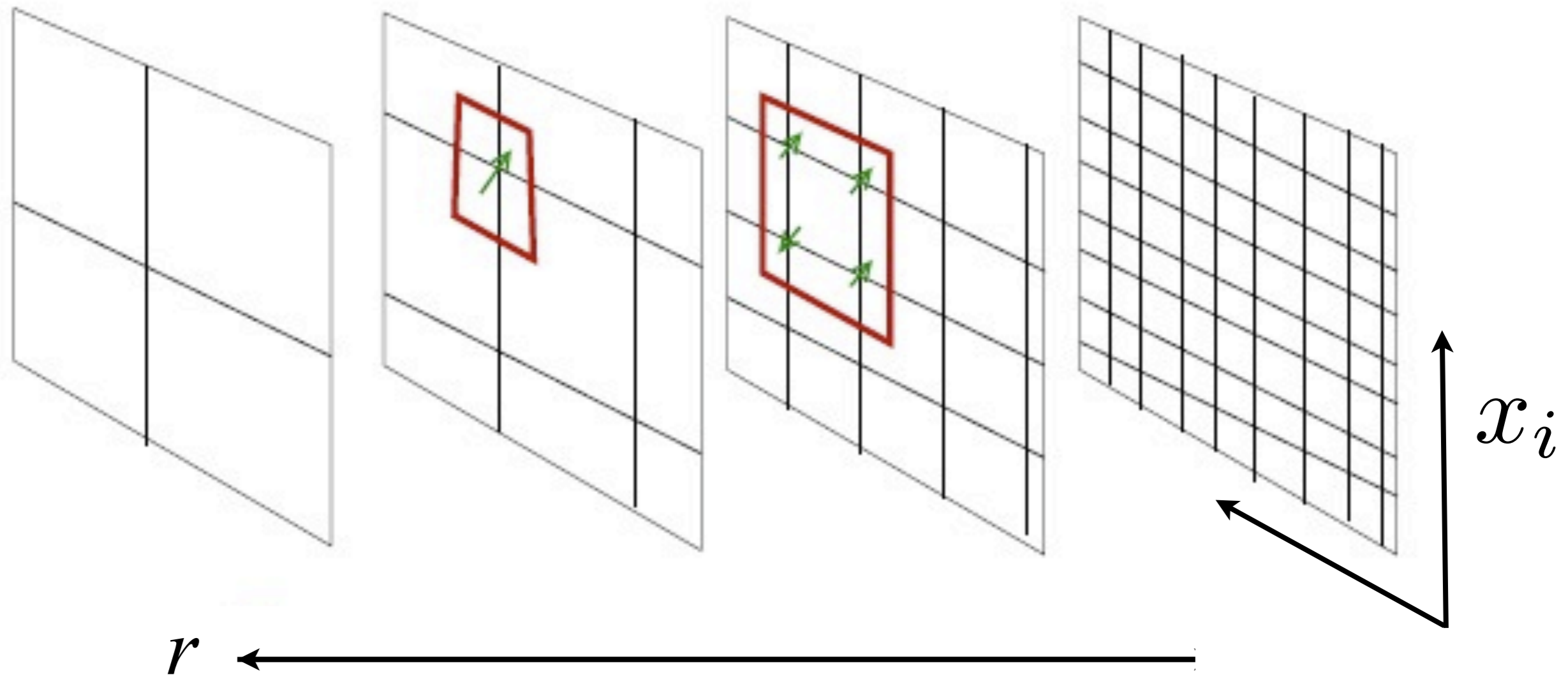
Renormalization group: \Rightarrow Follow coupling constants of quantum many body theory as a function of length scale r



Key idea: \Rightarrow Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

J. McGreevy, arXiv0909.0518

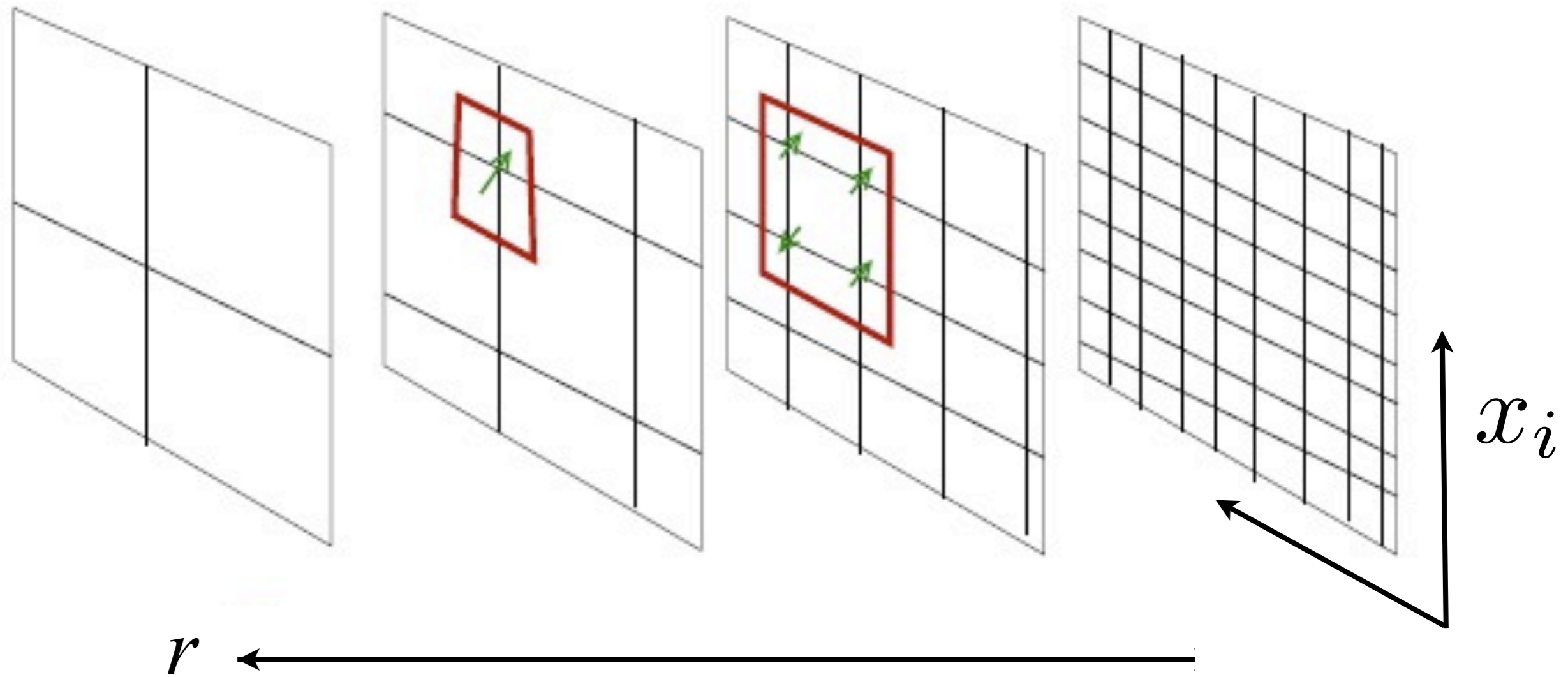
Holography



For a relativistic CFT in d spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \dots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

Holography

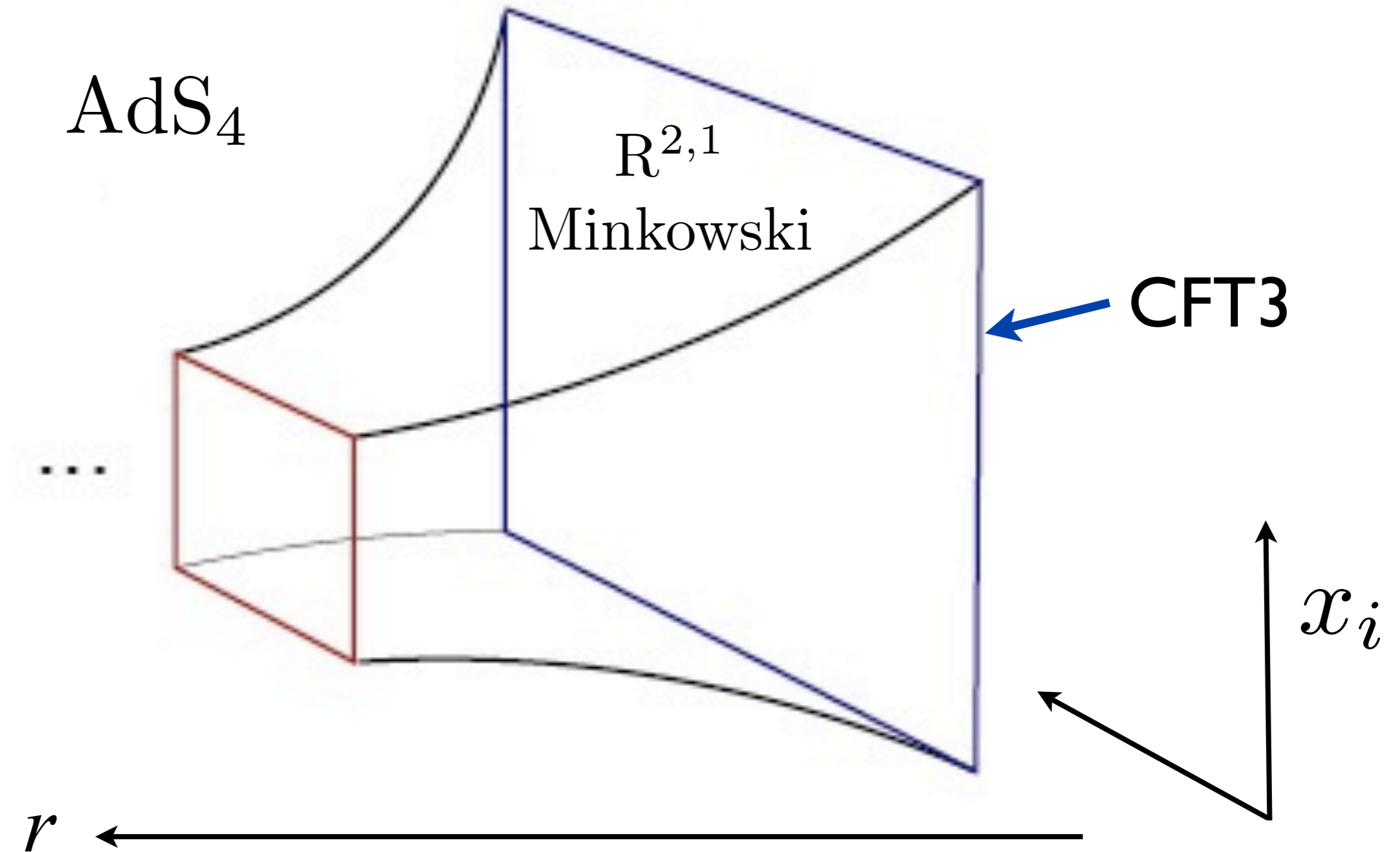


This gives the unique metric

$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

This is the metric of anti-de Sitter space AdS_{d+2} .

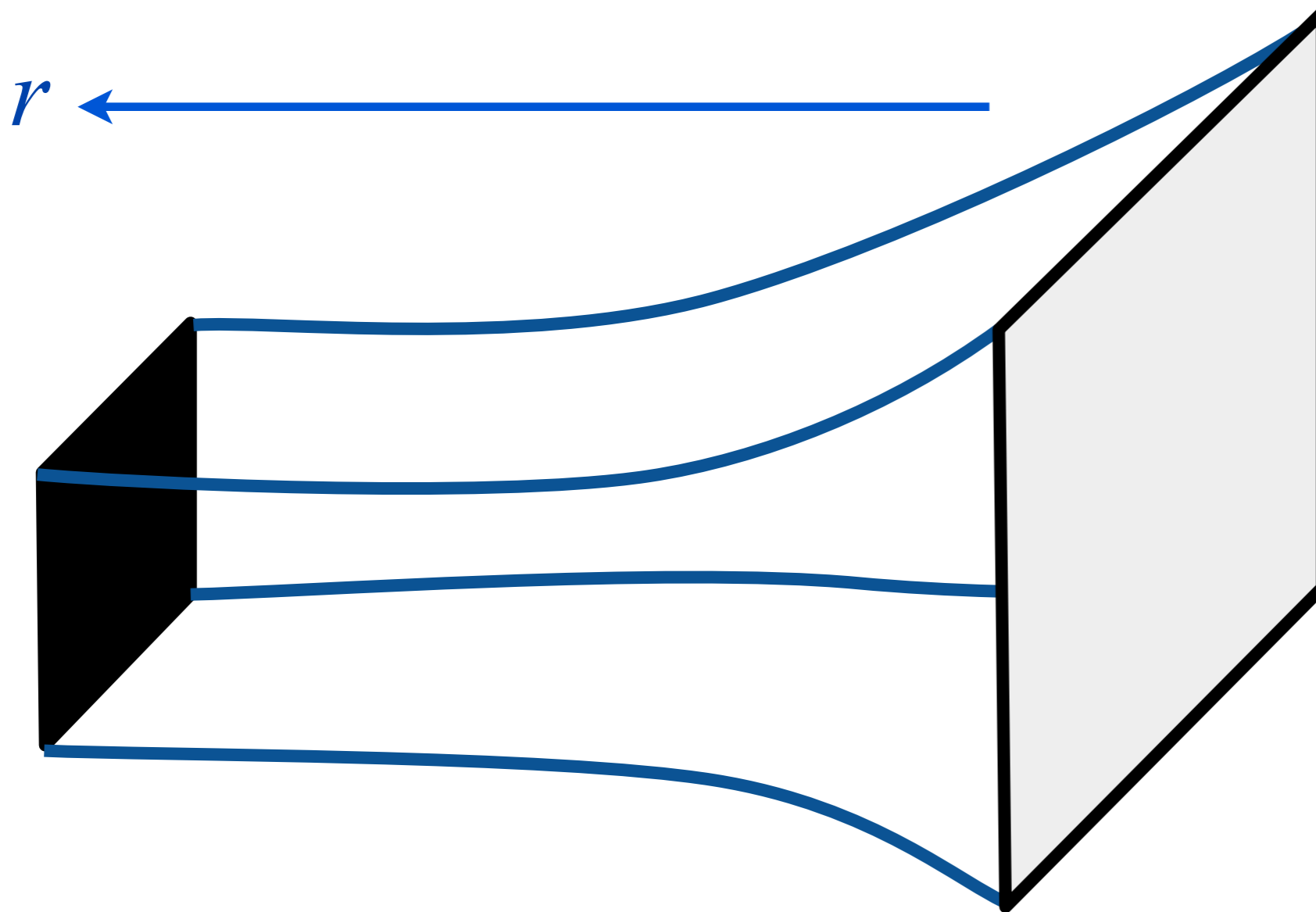
AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

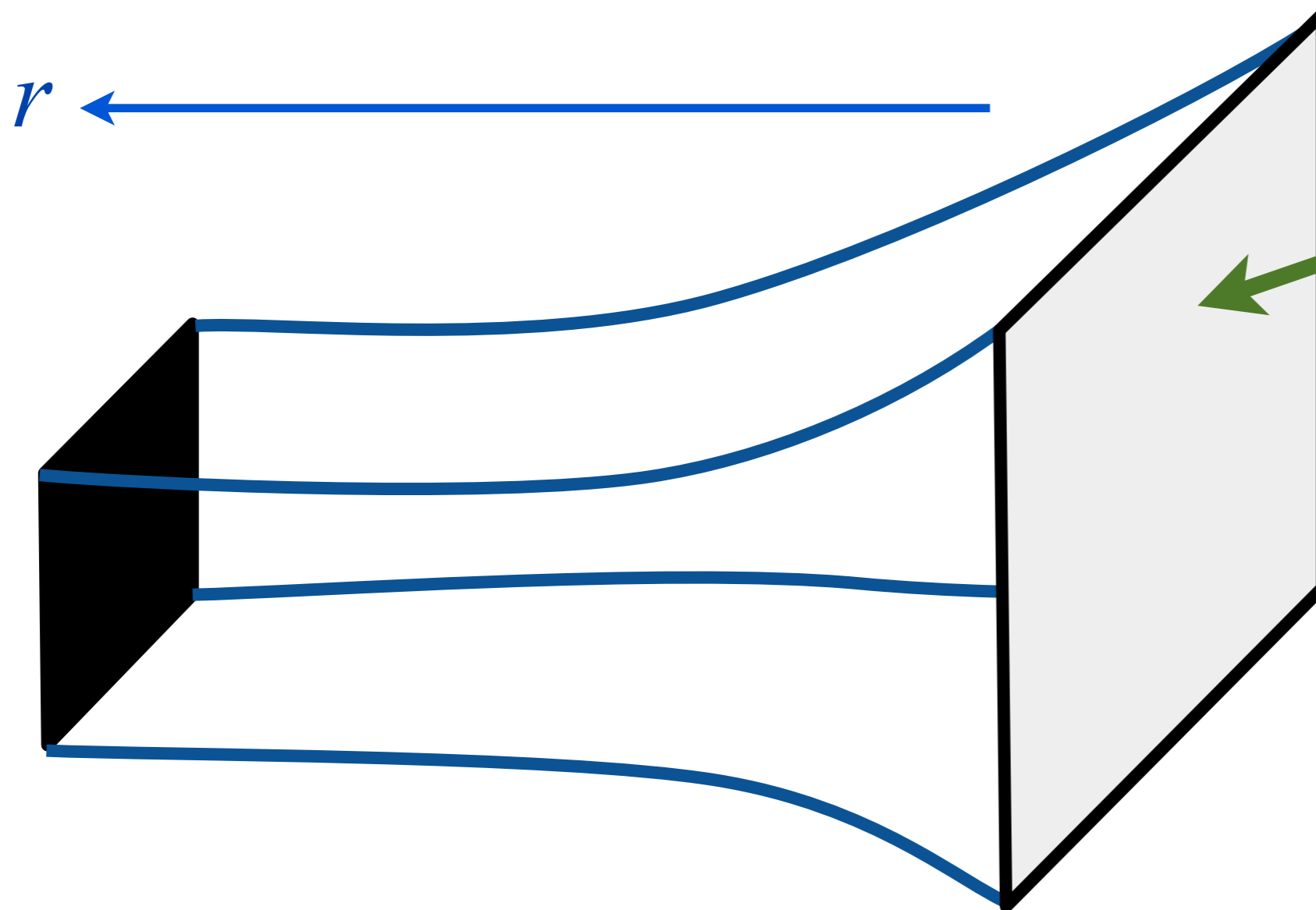
Gauge-gravity duality at non-zero temperatures



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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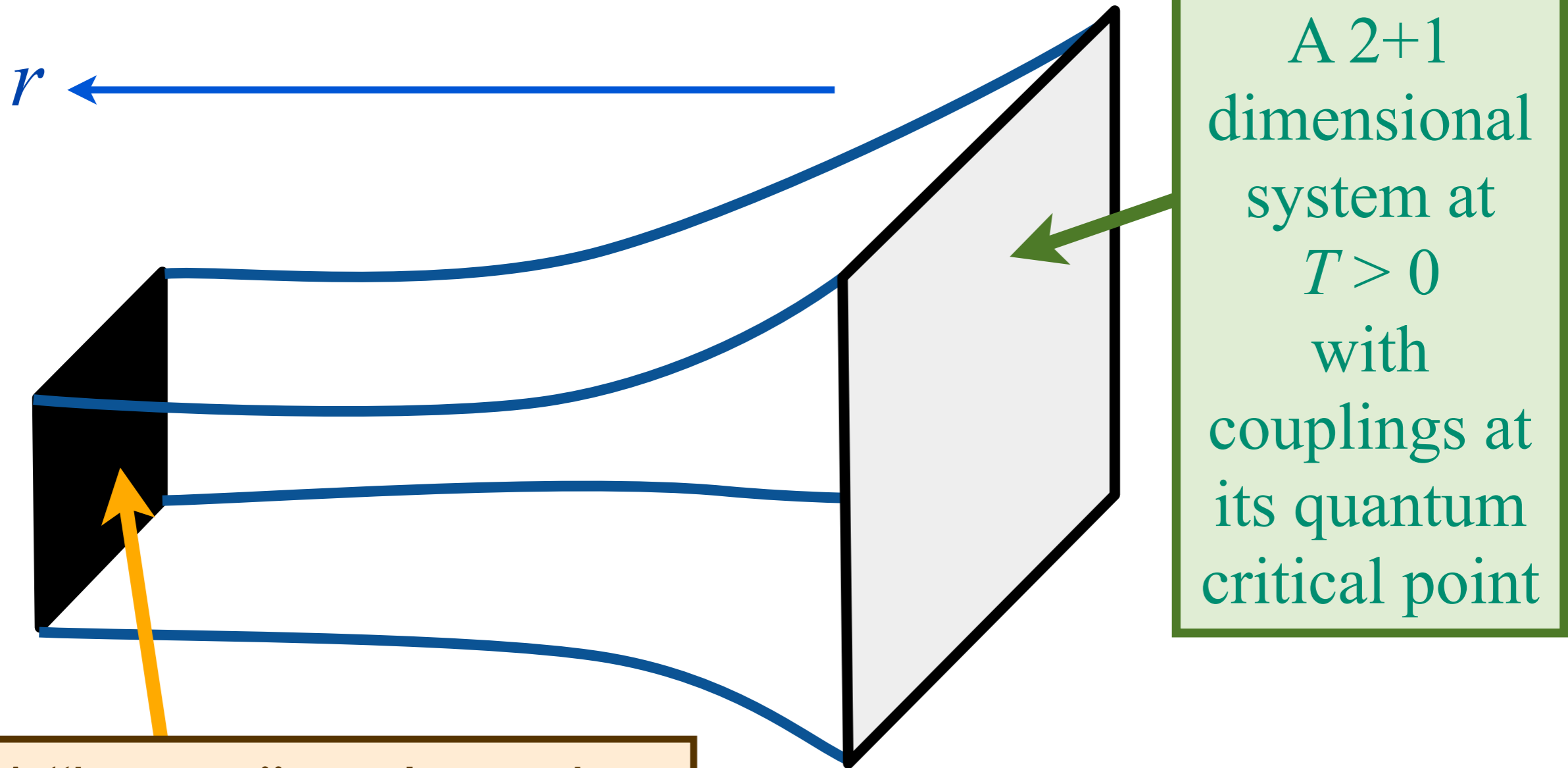
Gauge-gravity duality at non-zero temperatures



A 2+1
dimensional
system at
 $T > 0$
with
couplings at
its quantum
critical point

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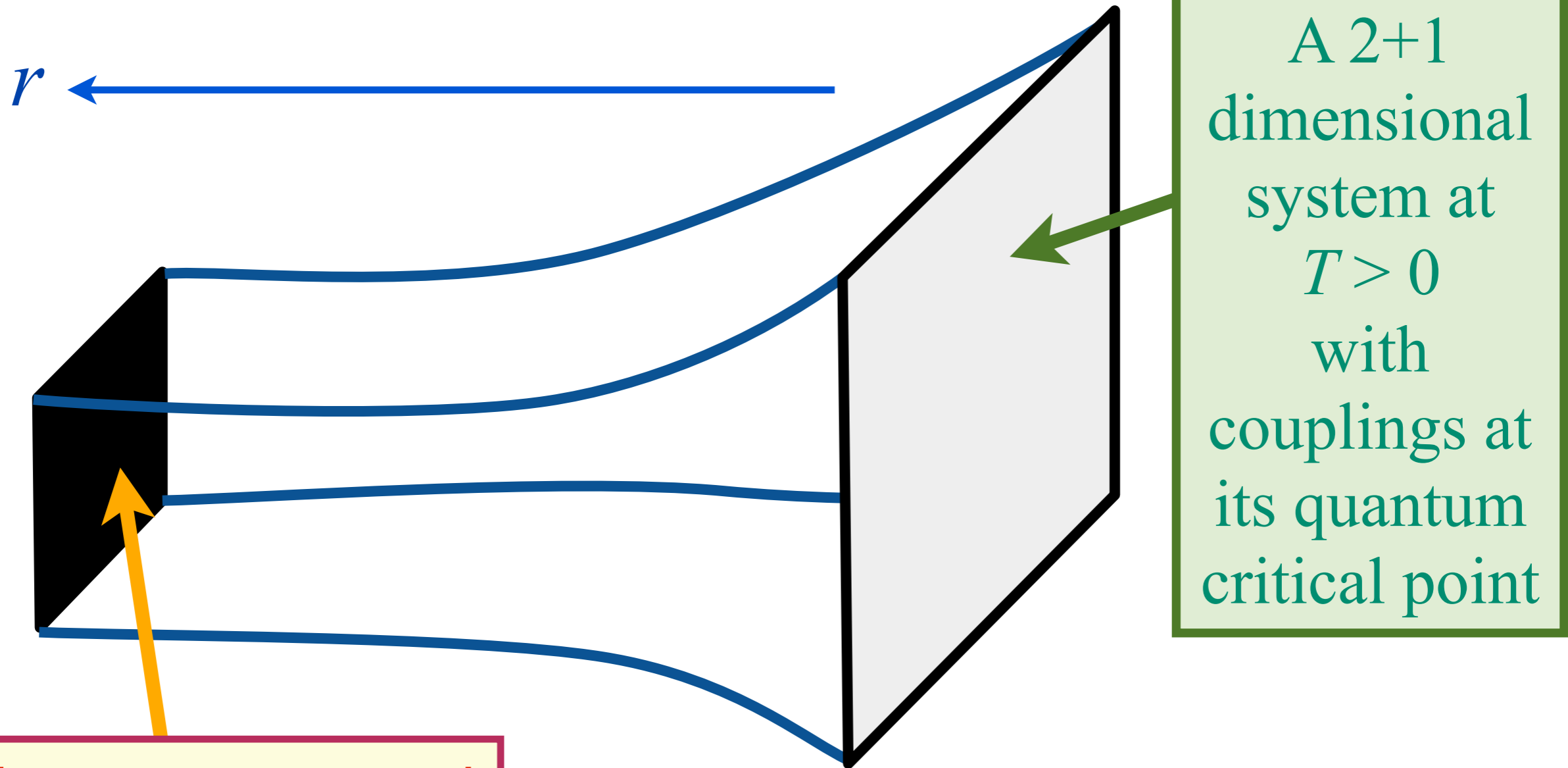
Gauge-gravity duality at non-zero temperatures



A “horizon”, similar to the surface of a black hole !

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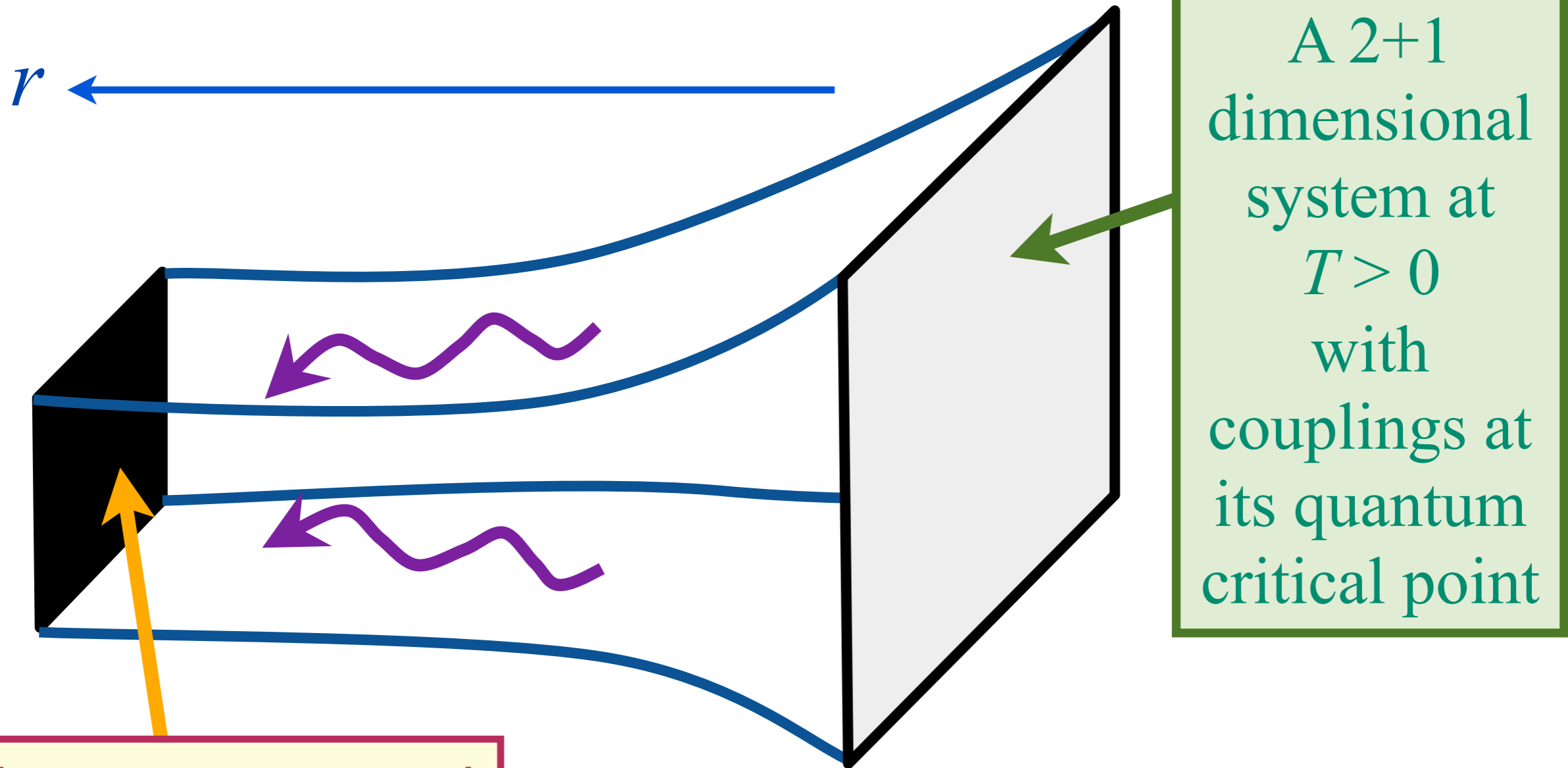
Gauge-gravity duality at non-zero temperatures



A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Gauge-gravity duality at non-zero temperatures

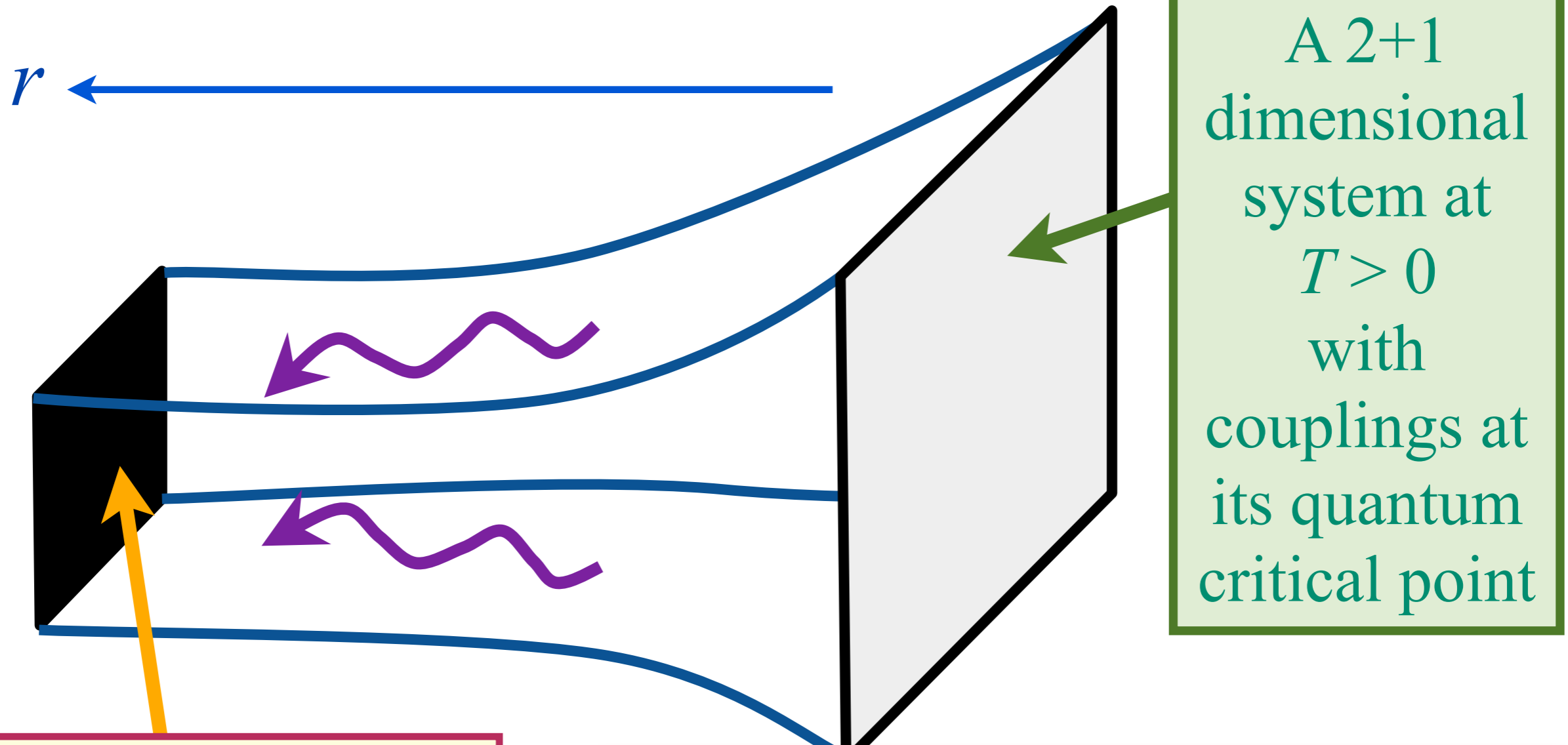


A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point

Quasi-normal modes of quantum criticality = waves falling into black hole

Gauge-gravity duality at non-zero temperatures



AdS₄ theory of quantum criticality

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

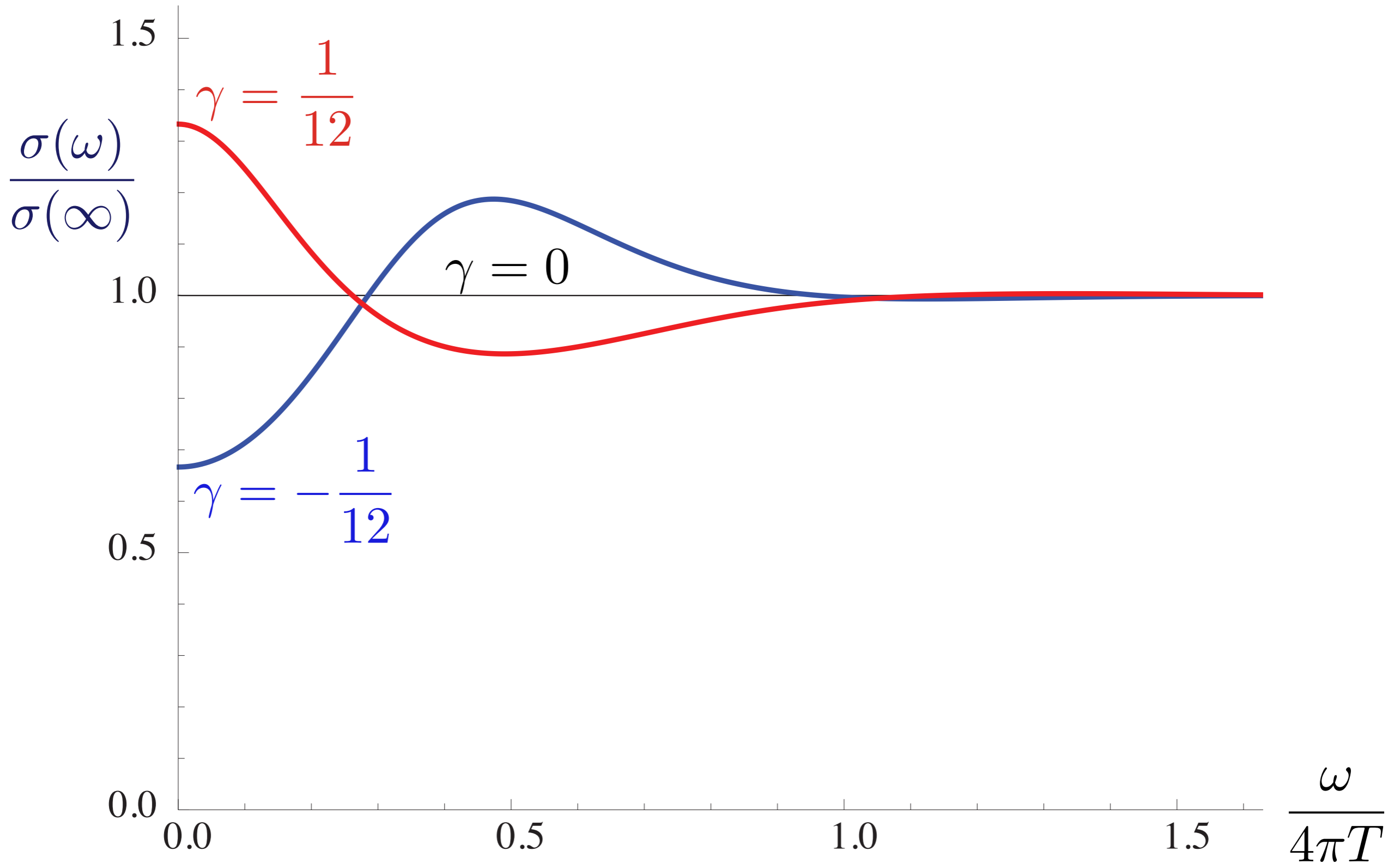
$$\mathcal{S}_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[\frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] + \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right],$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current J_μ and the stress energy tensor $T_{\mu\nu}$, and a 3-point T, J, J correlator.

R. C. Myers, S. Sachdev, and A. Singh, *Phys. Rev. D* **83**, 066017 (2011)

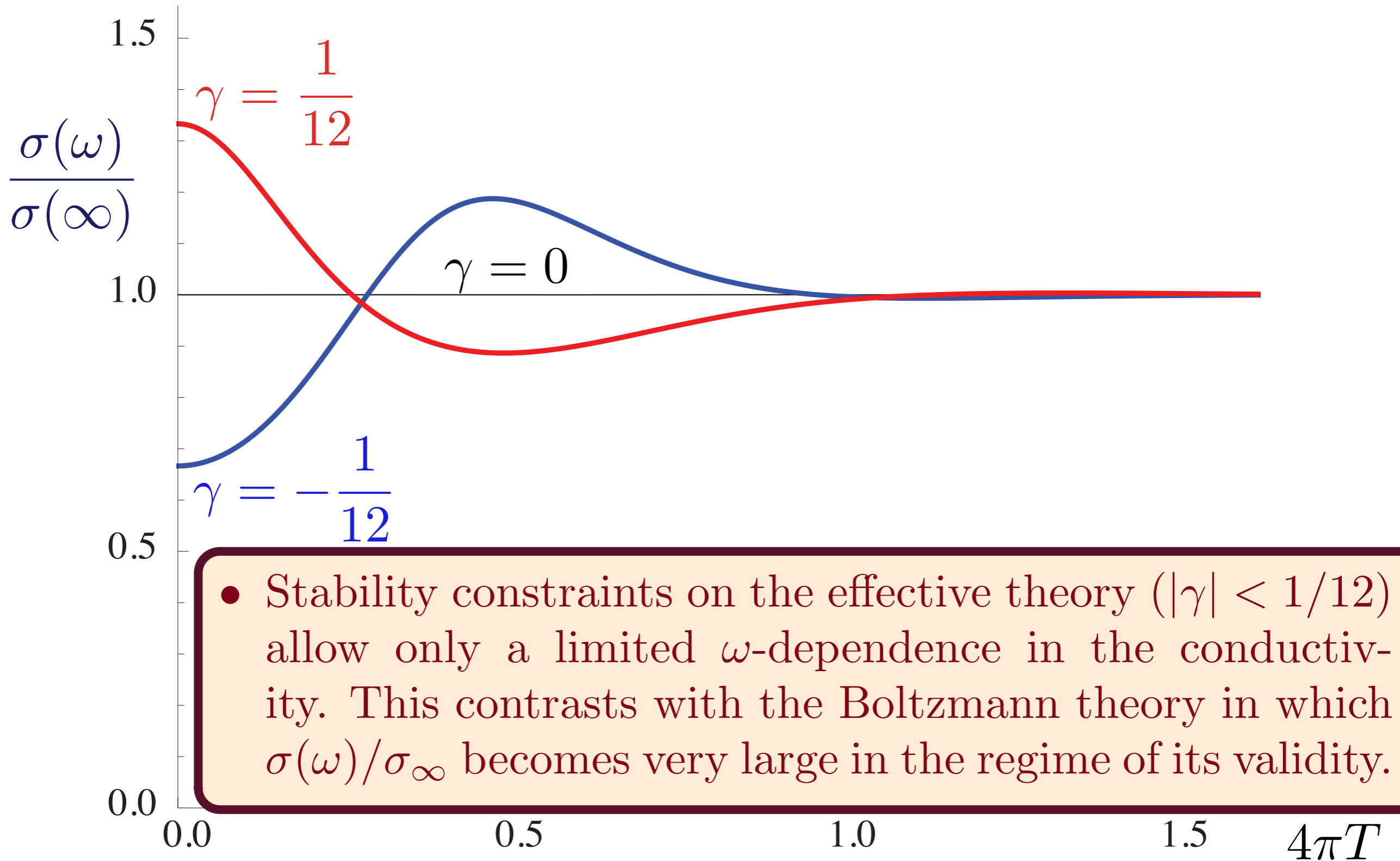
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Phys. Rev. B* **87**, 085138 (2013)

AdS₄ theory of quantum criticality



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS₄ theory of quantum criticality



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AdS₄ theory of quantum criticality

PRL **95**, 180603 (2005)

PHYSICAL REVIEW LETTERS

week ending
28 OCTOBER 2005

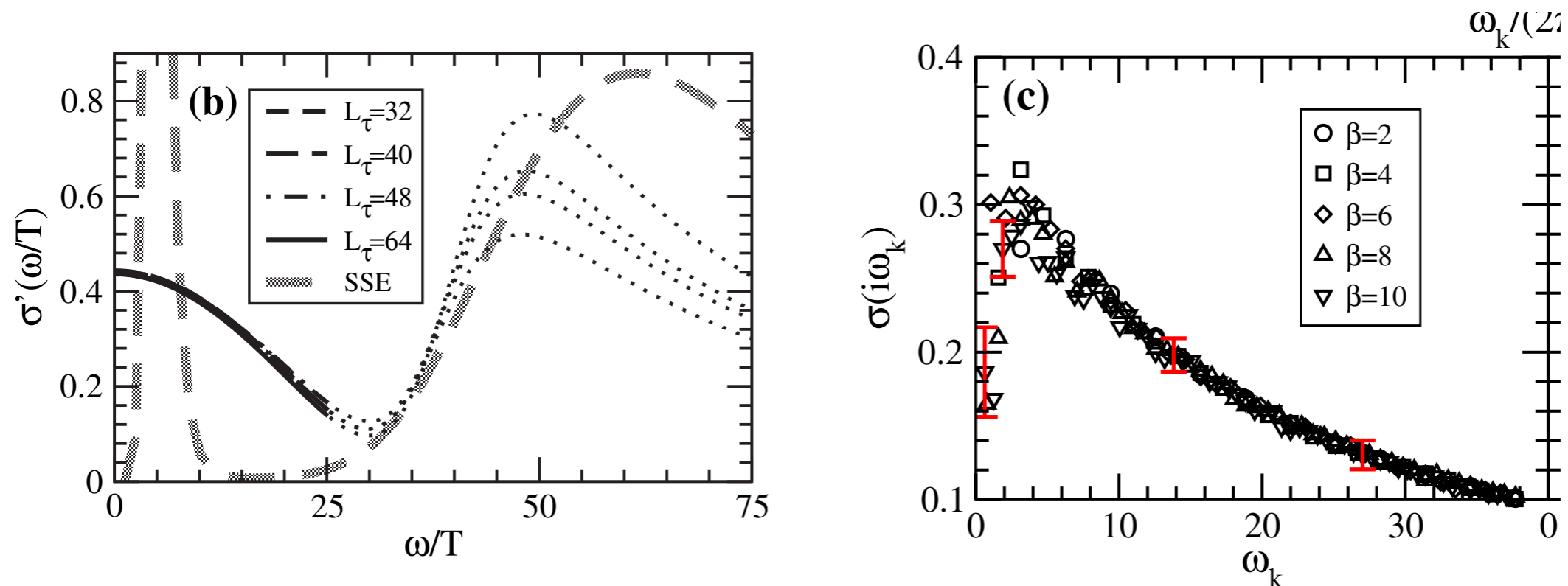
Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

Jurij Šmakov and Erik Sørensen

Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada

(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature T . We find clear evidence for *deviations* from ω_k scaling of the conductivity towards ω_k/T scaling at low Matsubara frequencies ω_k . By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with ω/T at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5)Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.



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QMC yields $\sigma(0)/\sigma_\infty \approx 1.36$

Holography yields $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.

Maximum possible holographic value $\sigma(0)/\sigma_\infty = 1.33$

W. Witzack-Krempa and S. Sachdev, arXiv:1302.0847

Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

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Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations

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Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute OPE co-efficients of operators of the CFT

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- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
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Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:

Conformal field theories and gauge-gravity duality

2. Metals with antiferromagnetism, and high temperature superconductivity

The pnictides and the cuprates

Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:

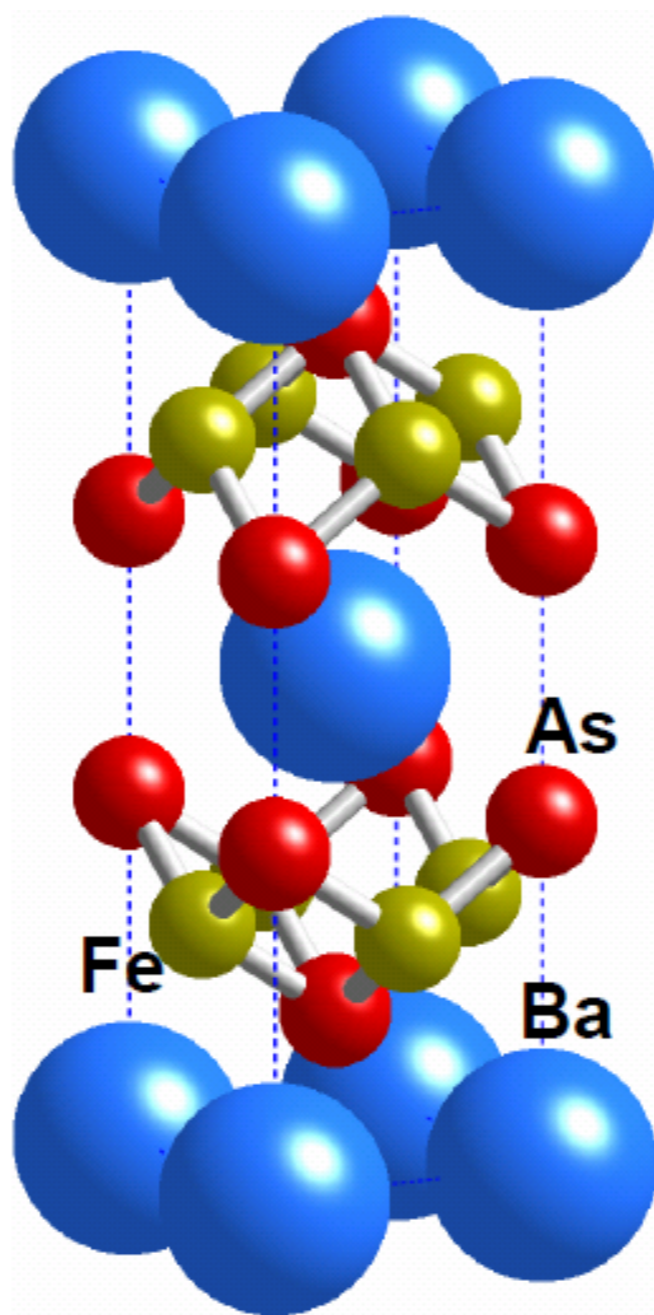
Conformal field theories and gauge-gravity duality

2. Metals with antiferromagnetism, and high temperature superconductivity

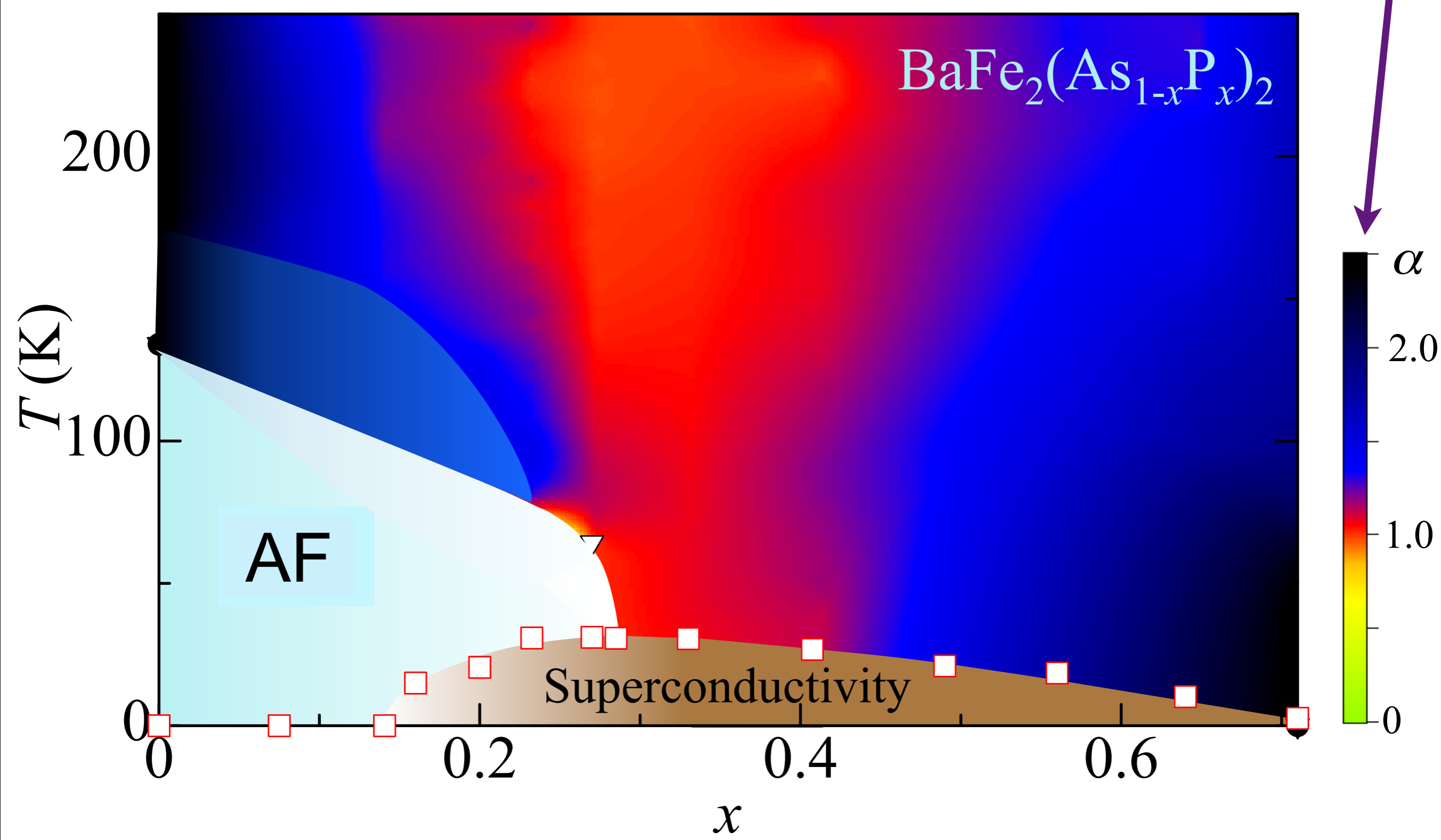
The pnictides and the cuprates

Iron pnictides:

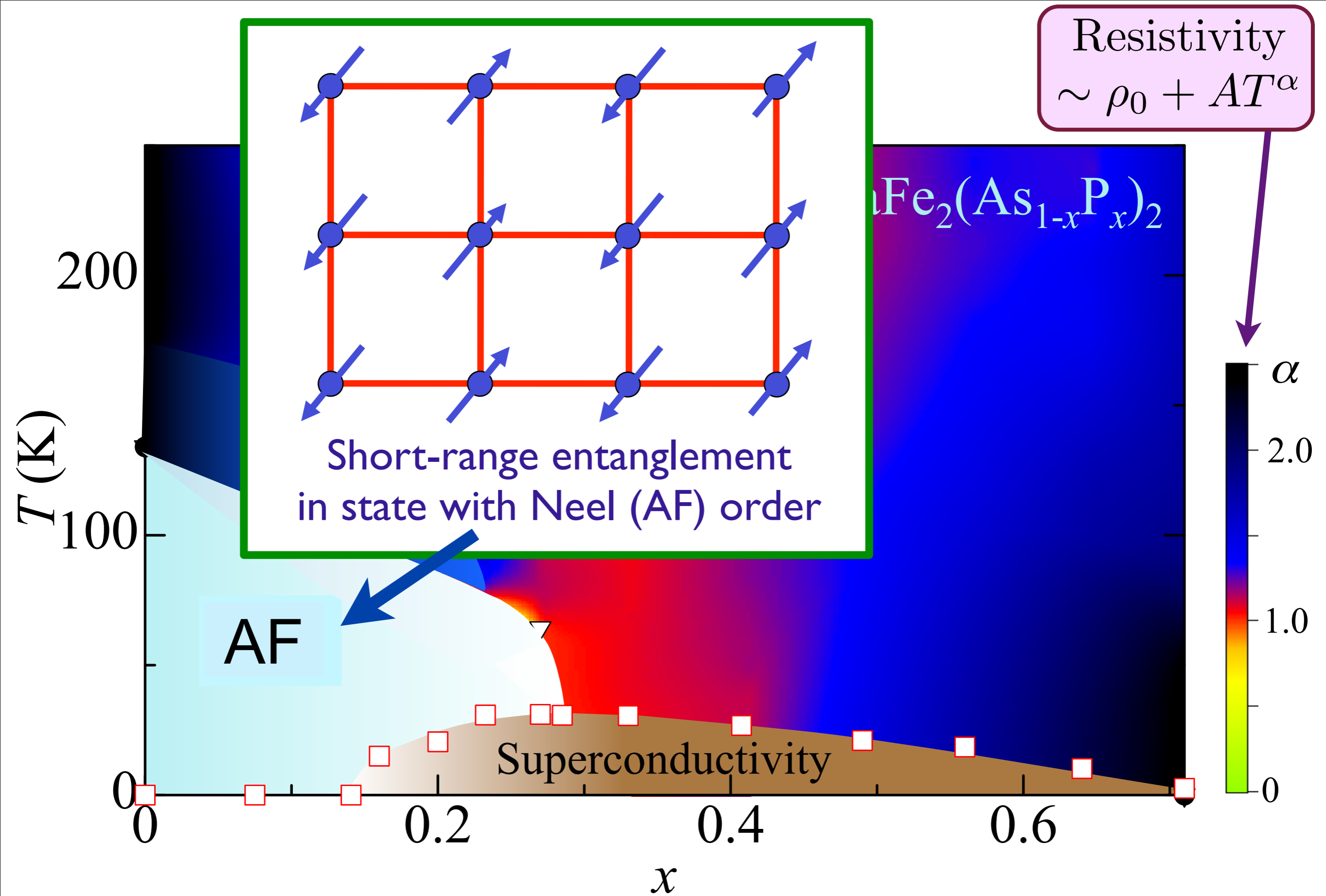
a new class of high temperature superconductors



Resistivity
 $\sim \rho_0 + AT^\alpha$

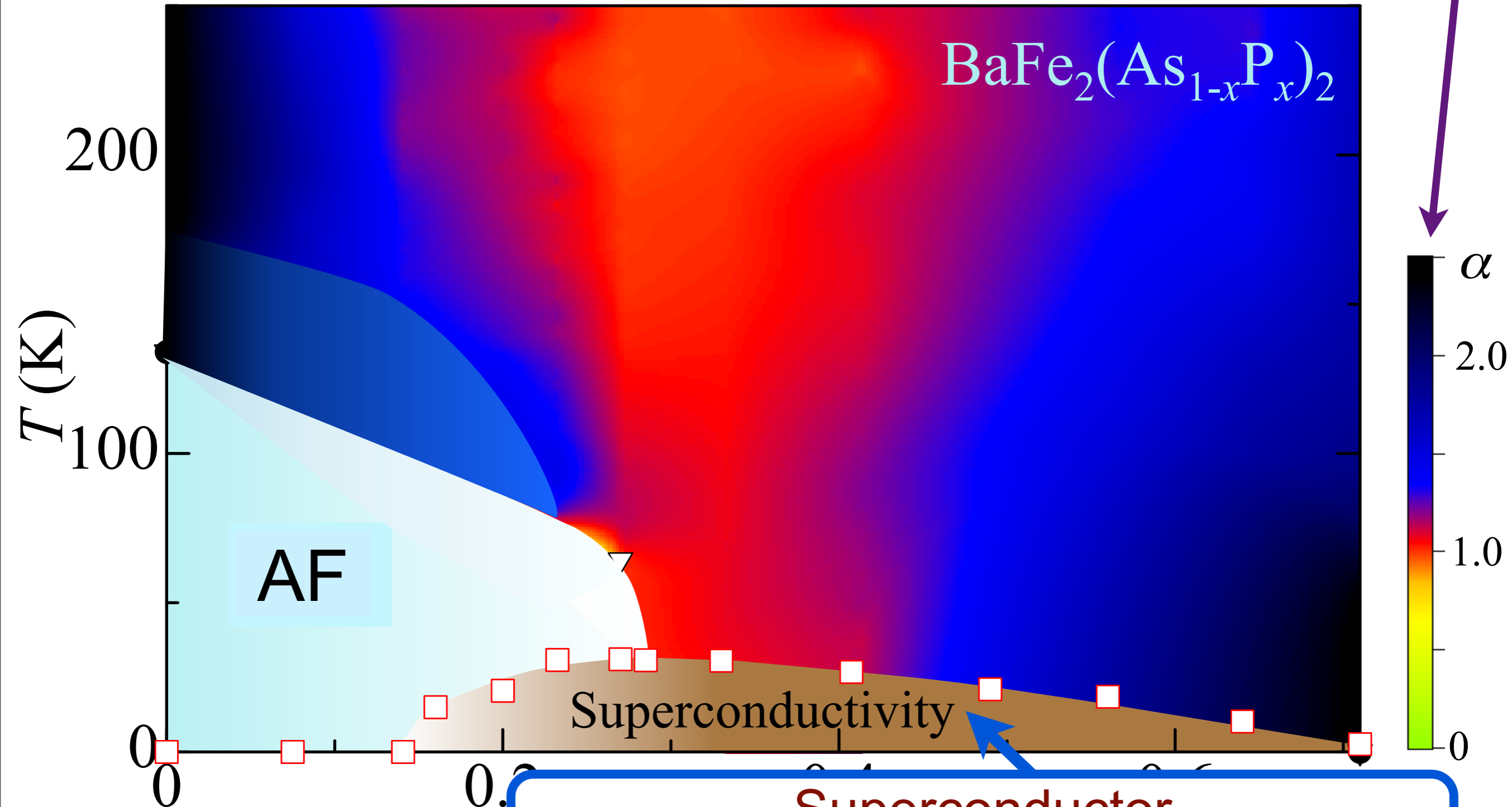


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
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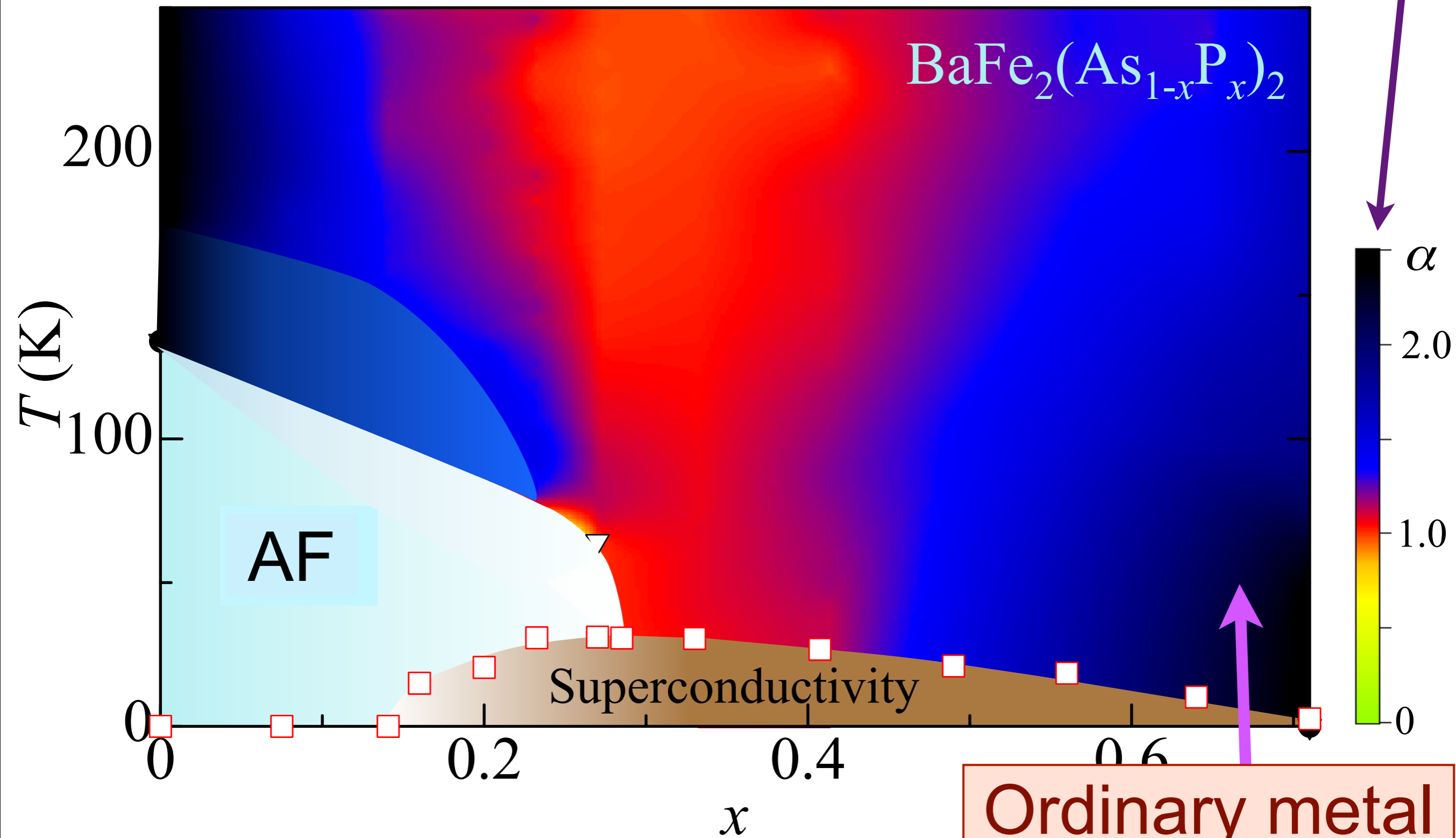
Resistivity
 $\sim \rho_0 + AT^\alpha$



Superconductor
Bose condensate of pairs of electrons
Short-range entanglement

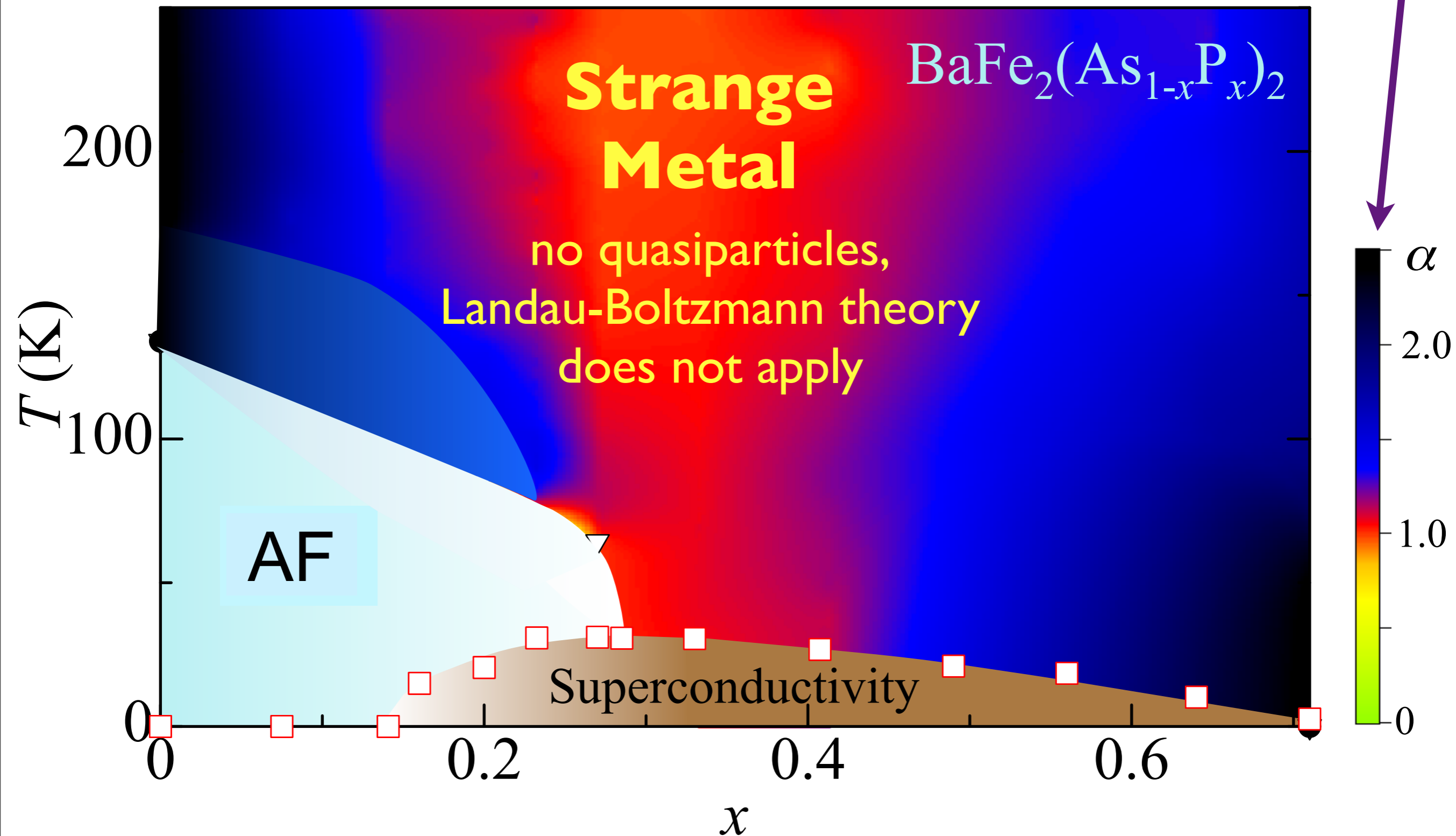
S. Kasahara, T. Shiba
H. Ike

Resistivity
 $\sim \rho_0 + AT^\alpha$



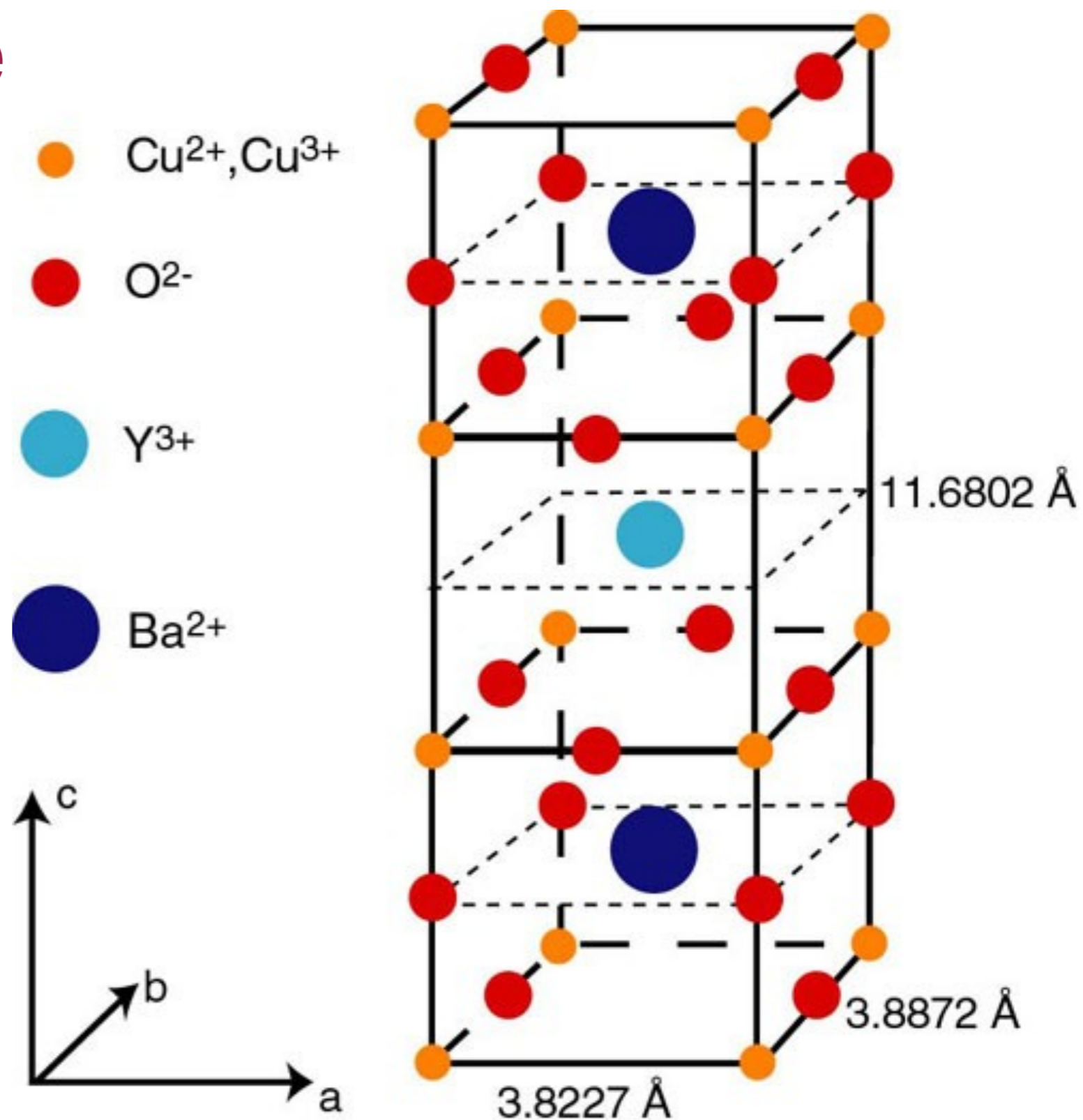
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. O.
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Ma
Physical Review B **81**, 184519 (2010)

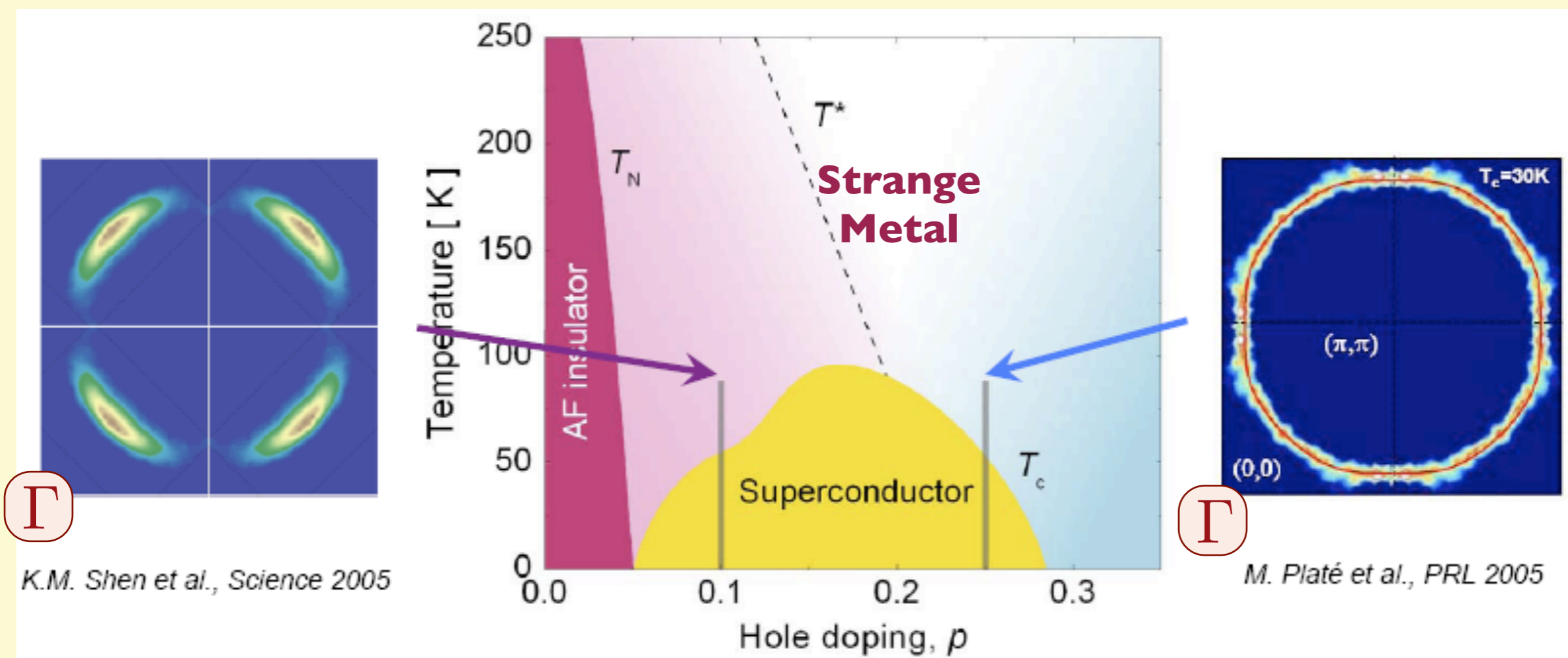
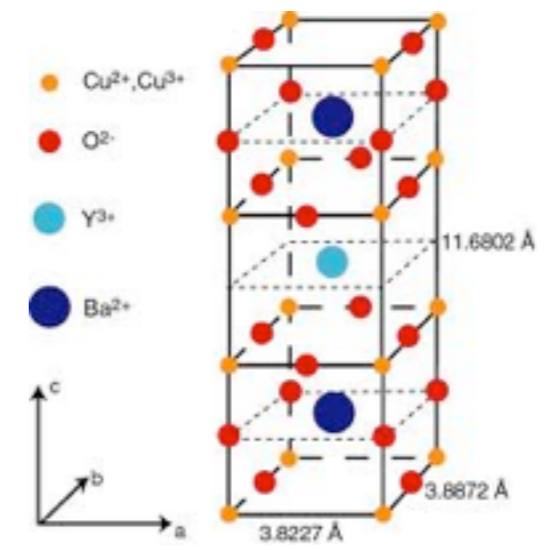
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S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)

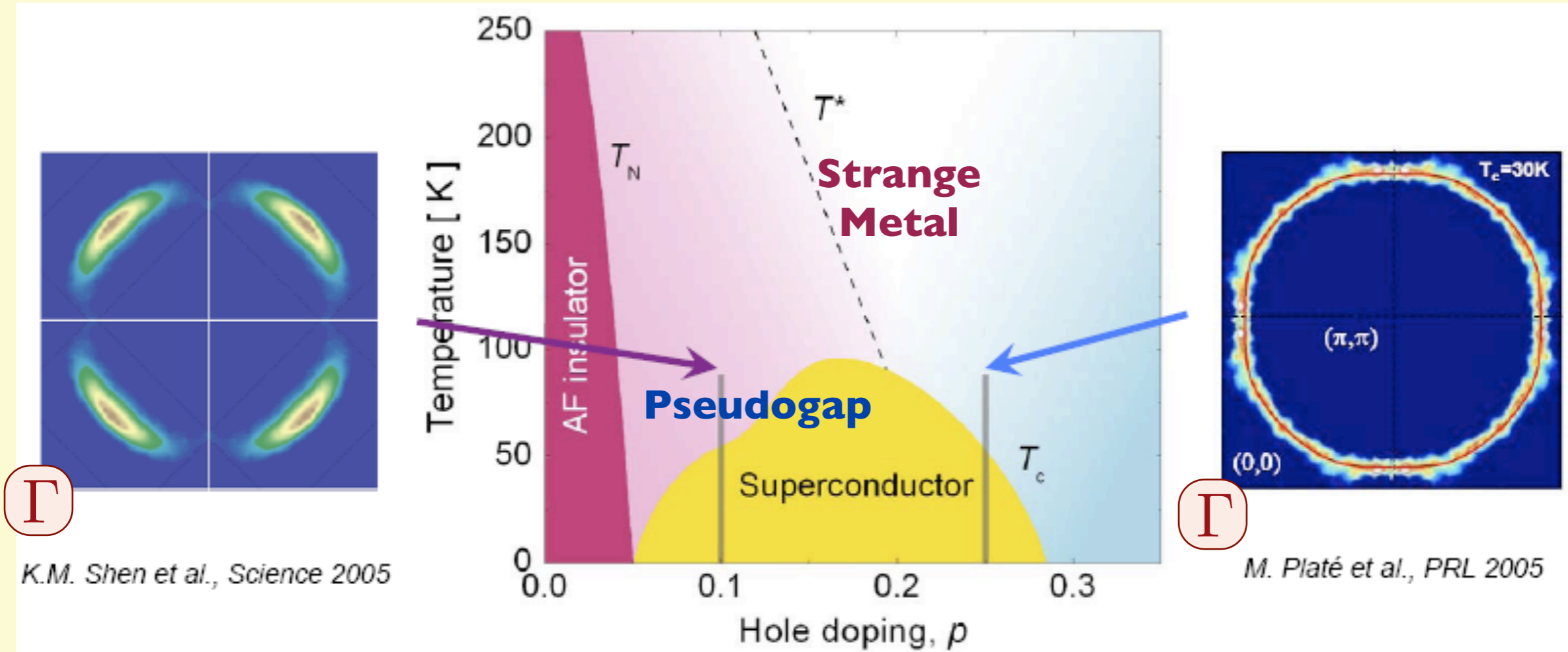
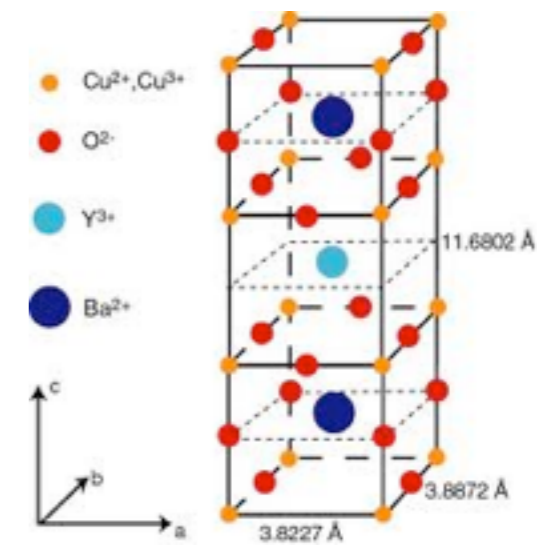
High temperature superconductors





Smaller hole Fermi-pockets

Large hole Fermi surface

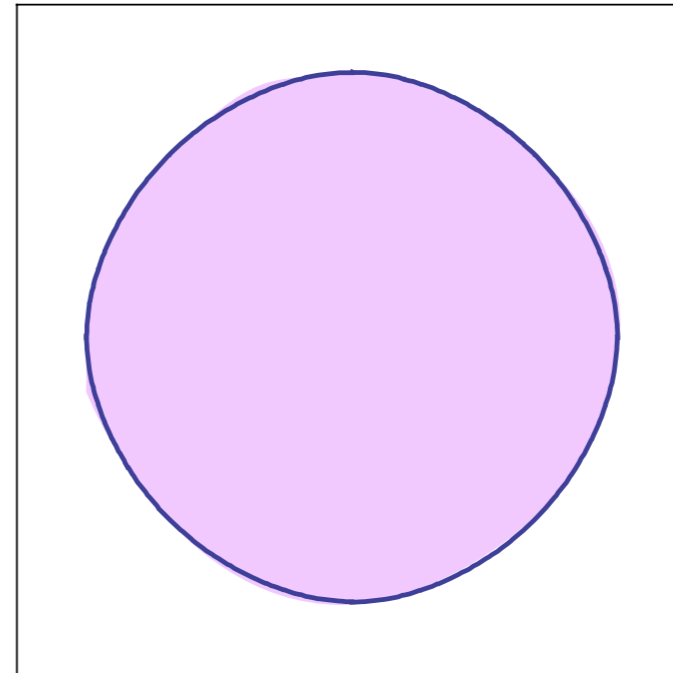


Smaller hole Fermi-pockets

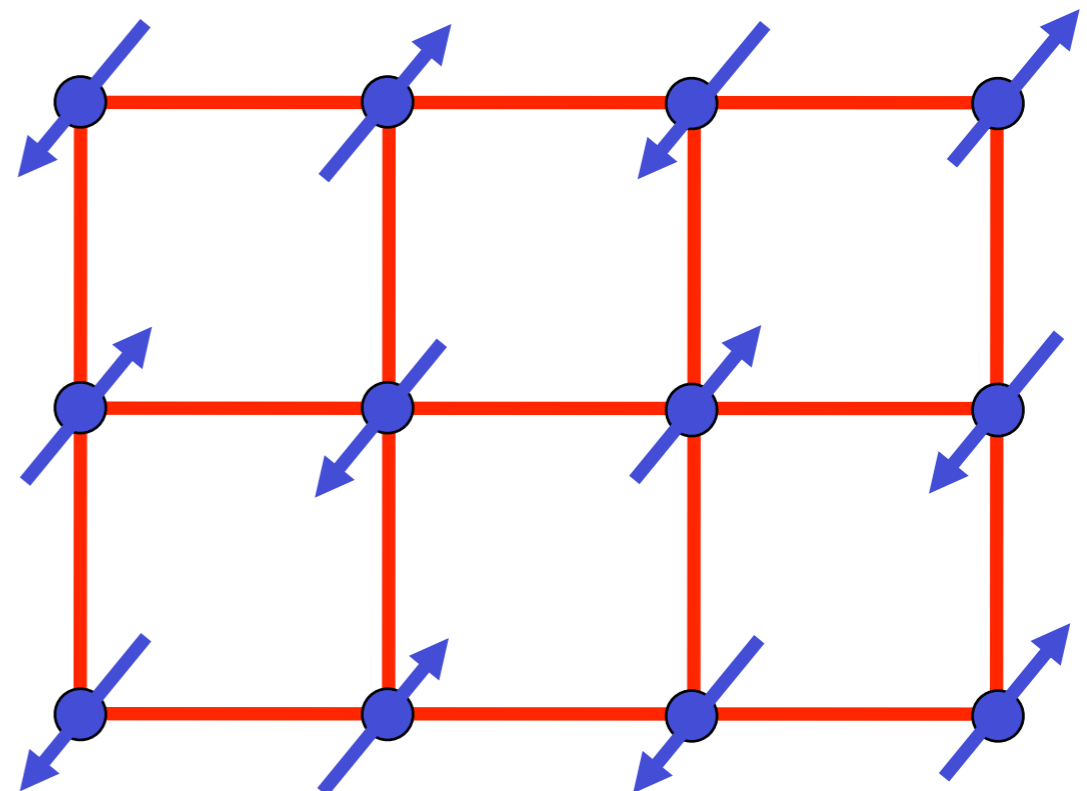
Large hole Fermi surface

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface



+

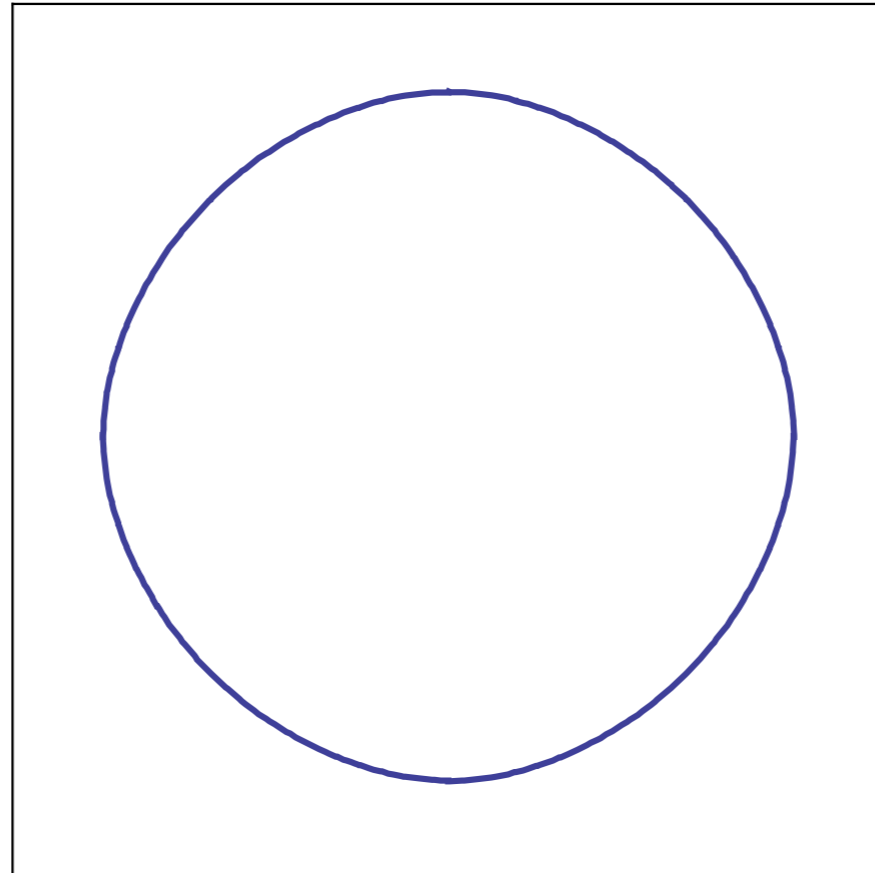


The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

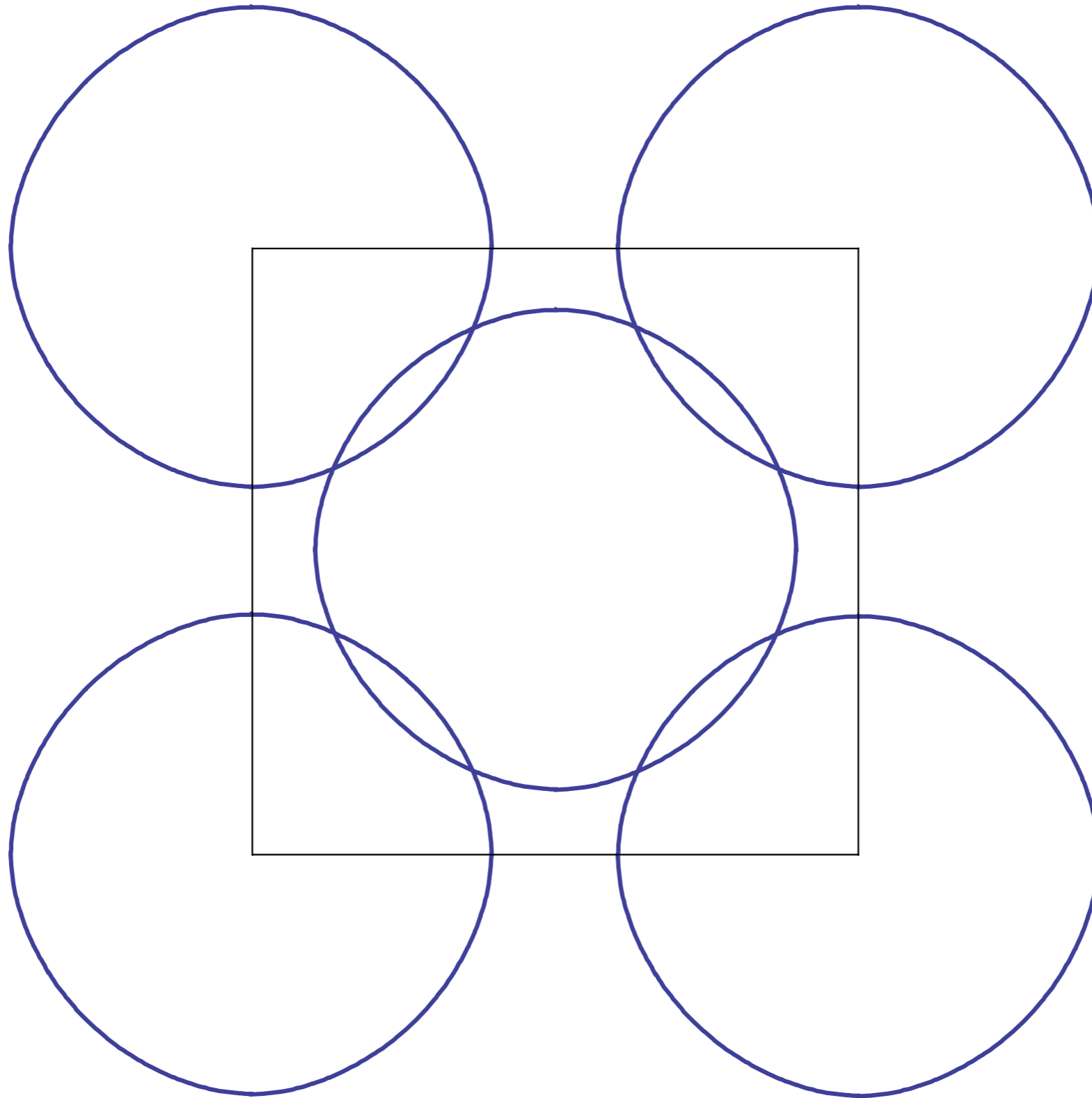
where \mathbf{K} is the ordering wavevector.

Fermi surface+antiferromagnetism



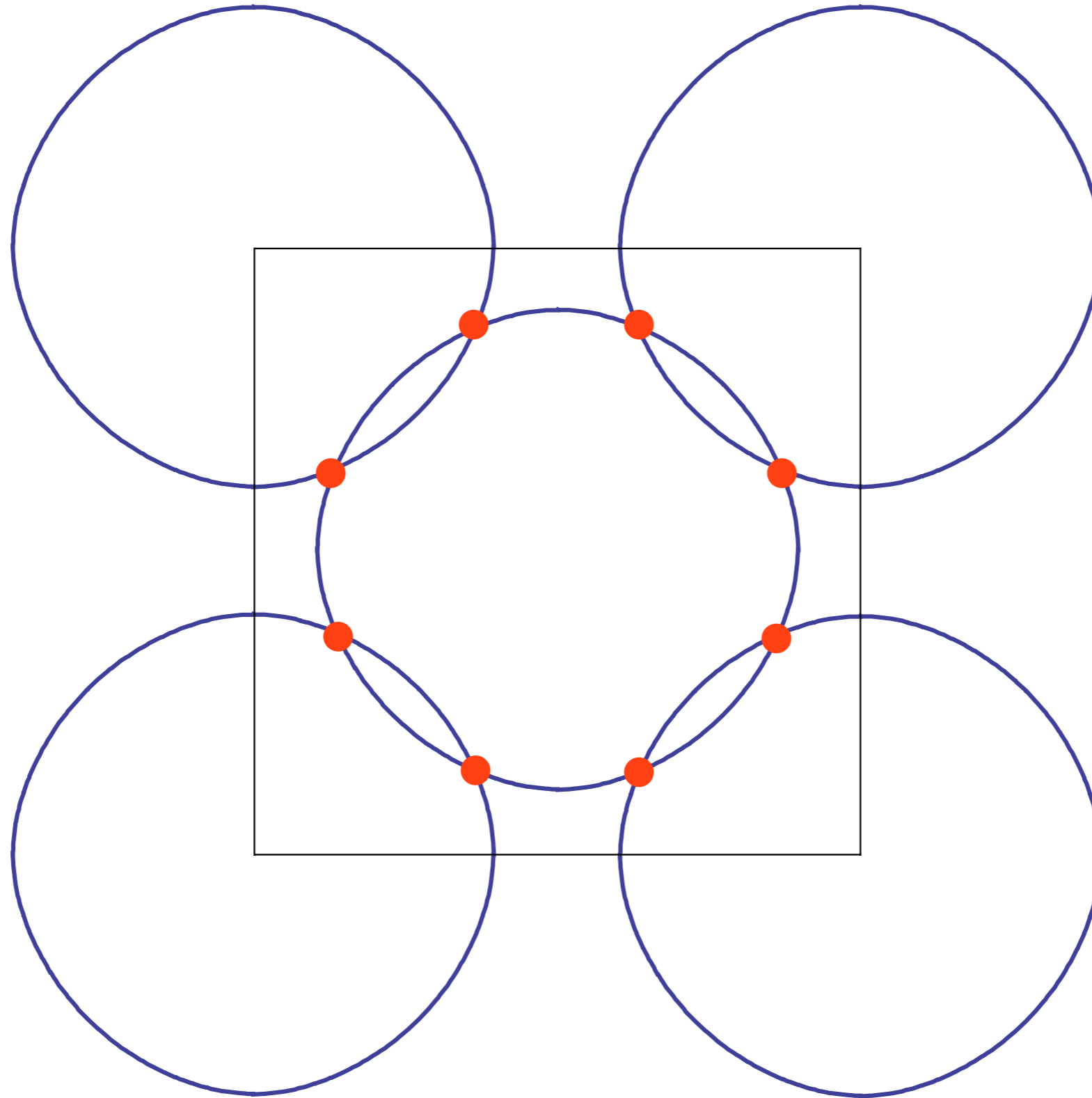
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



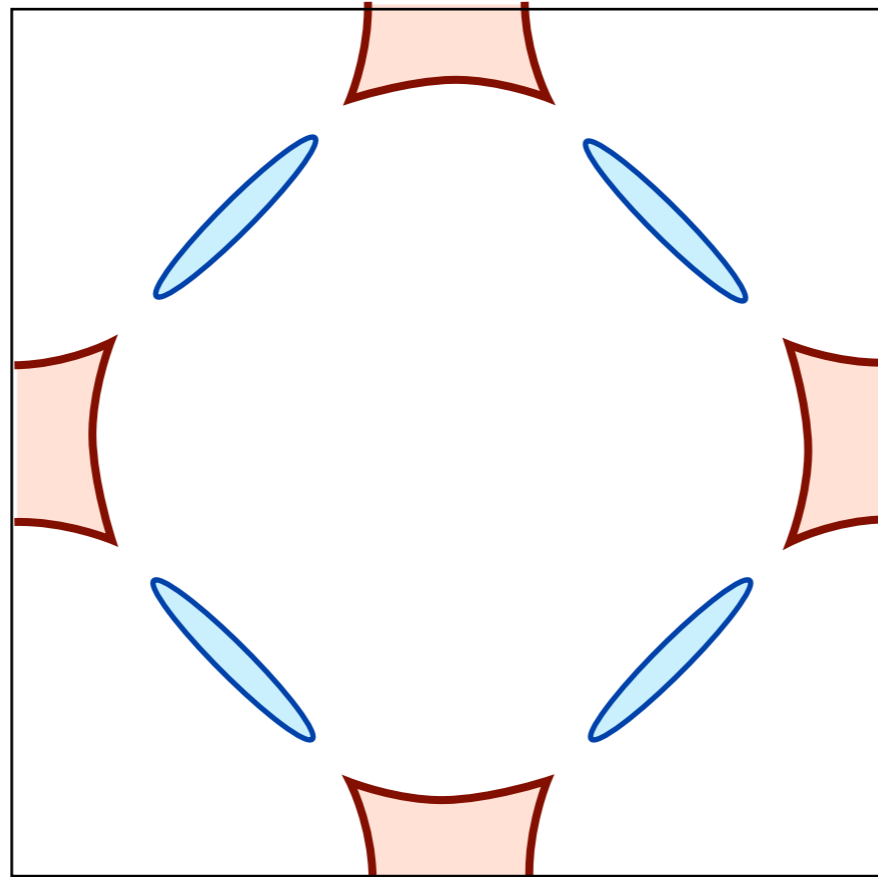
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



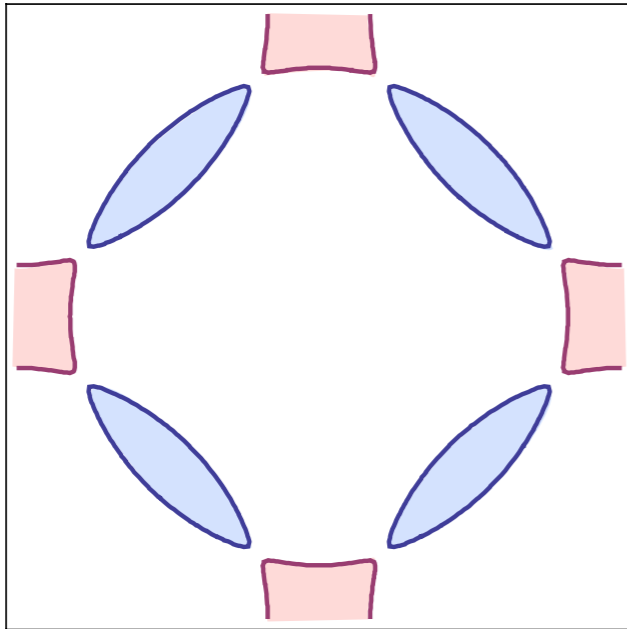
“Hot” spots

Fermi surface+antiferromagnetism



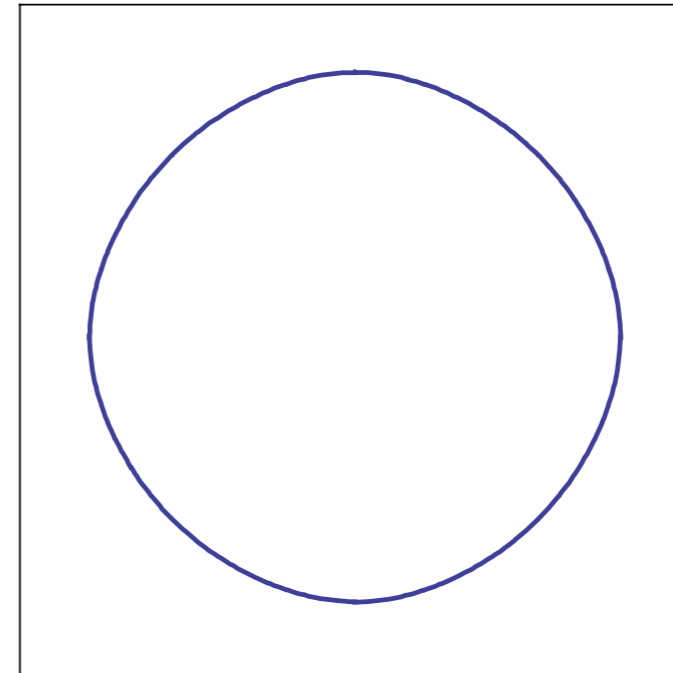
Electron and hole pockets in
antiferromagnetic phase
with antiferromagnetic order parameter $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

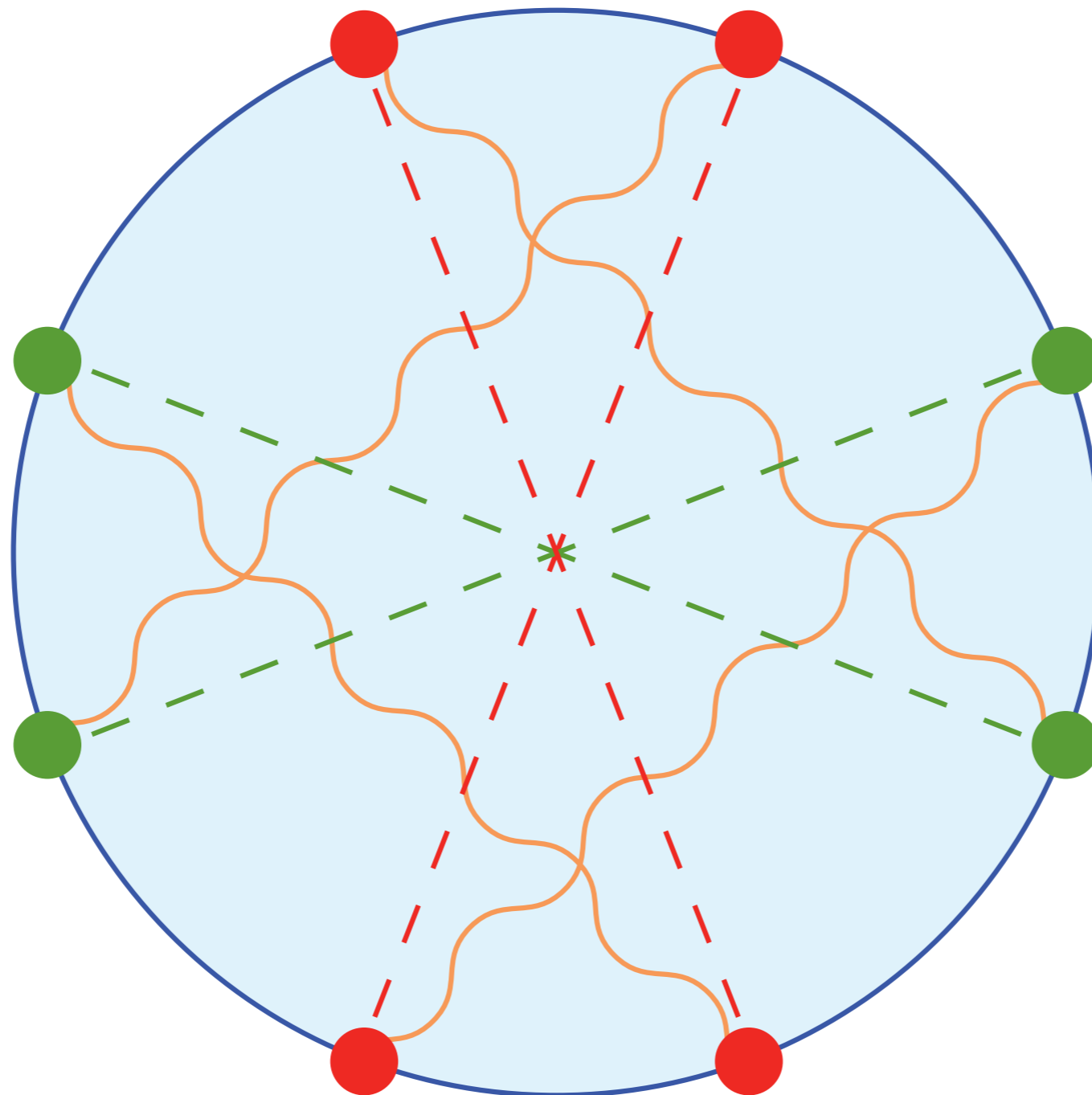


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

r

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



Unconventional pairing at and near hot spots

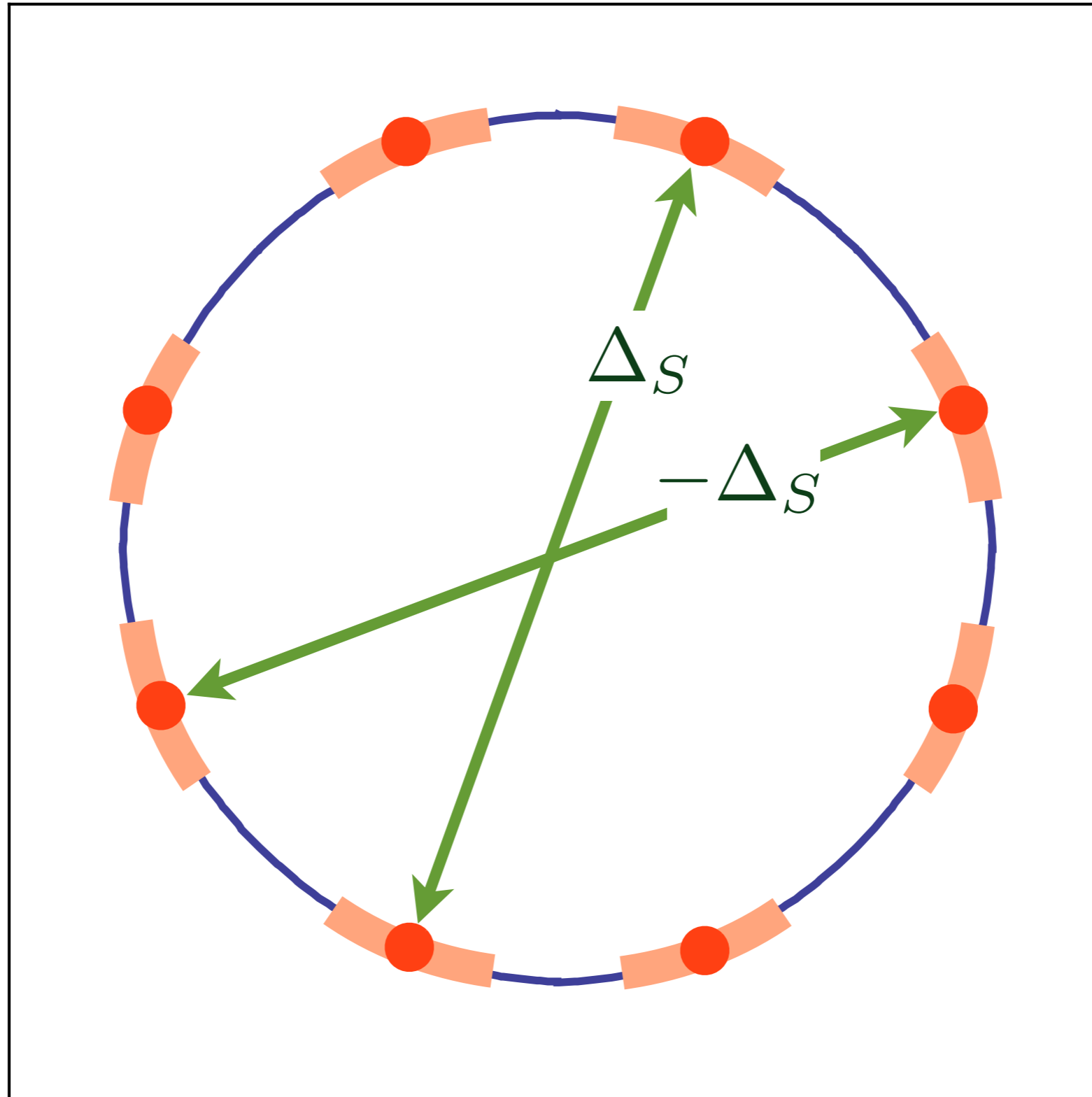
$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)

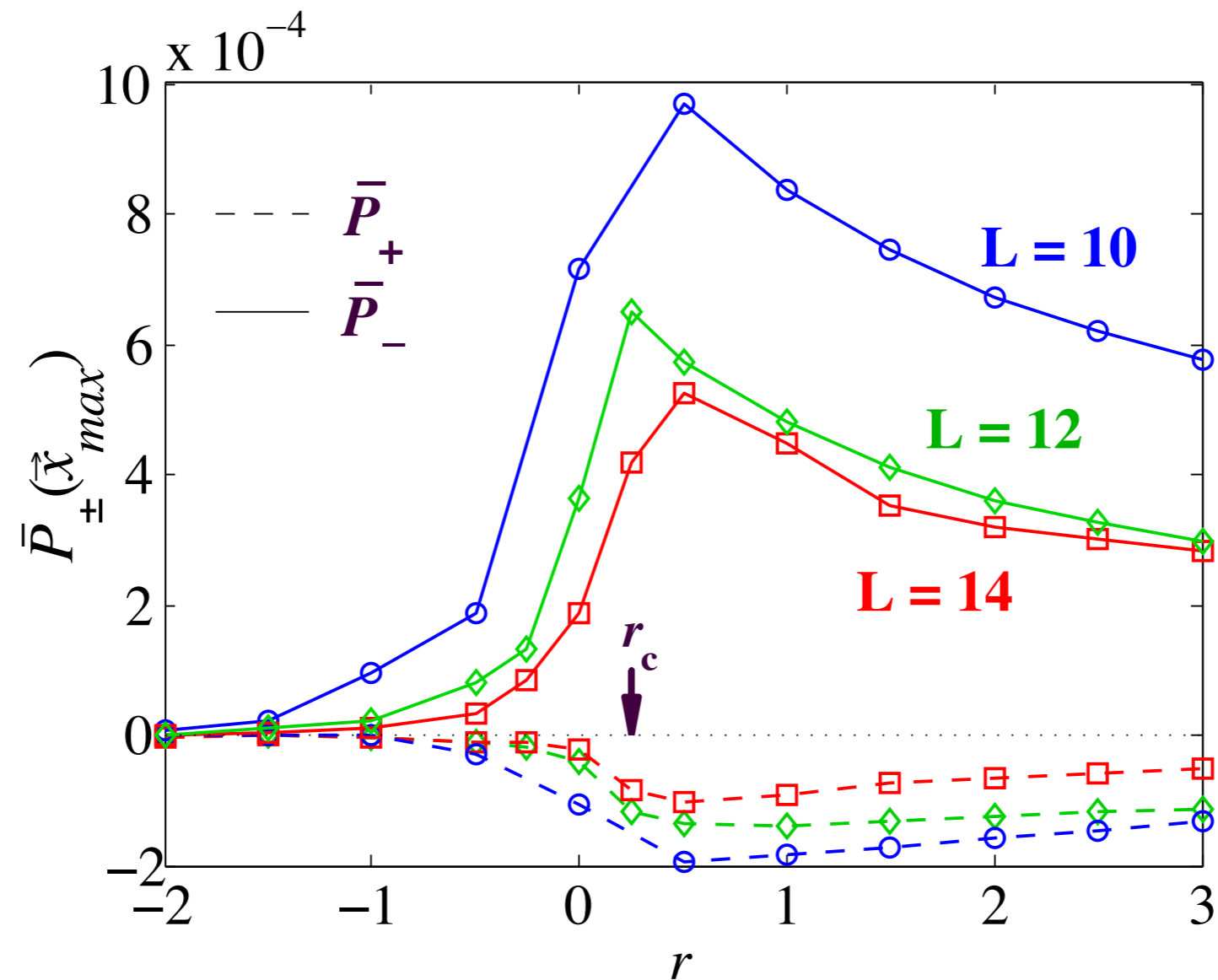
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

S. Raghu, S. A. Kivelson, and D. J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



Unconventional pairing at and near hot spots

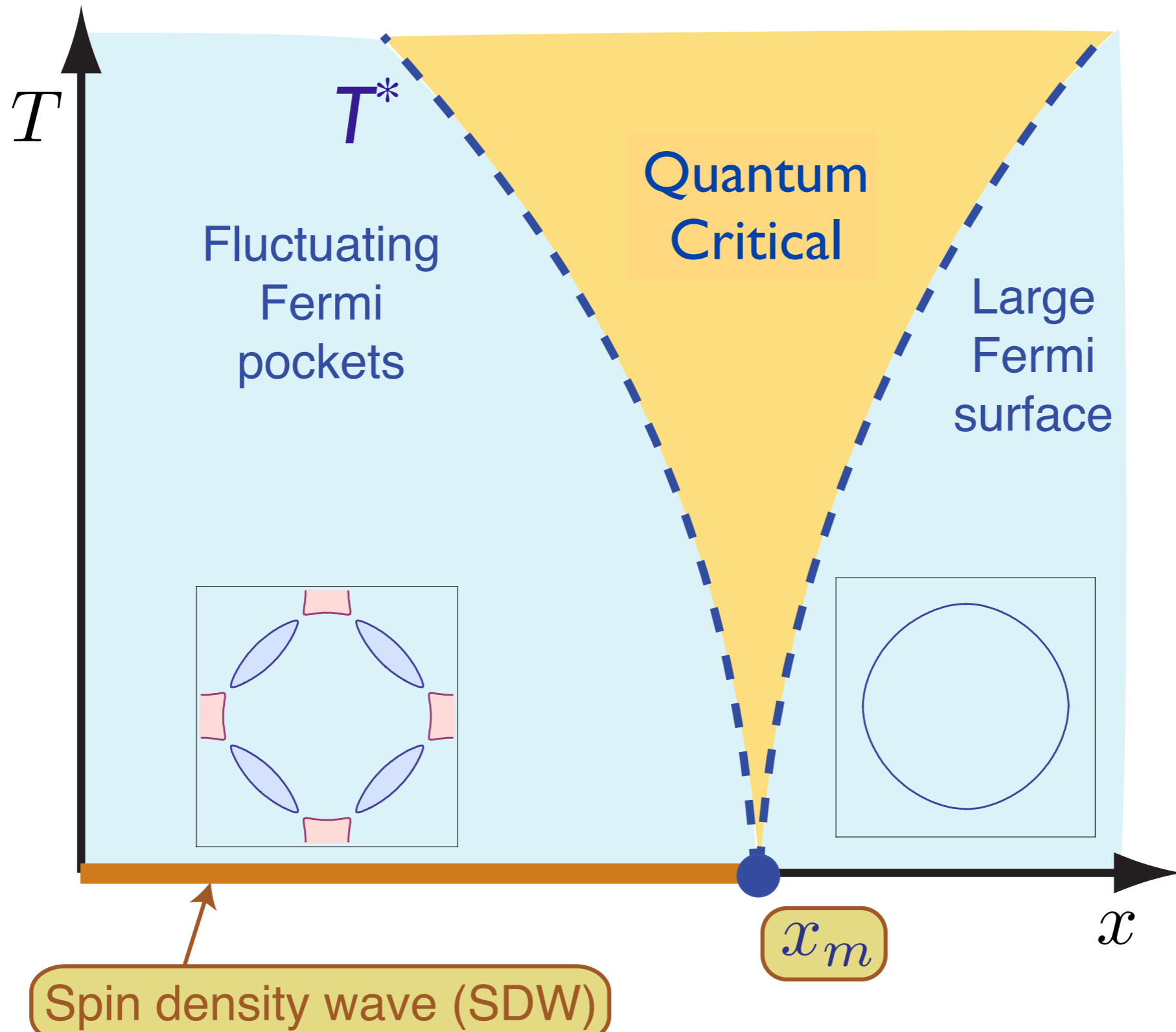
Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals



s/d pairing amplitudes P_{+}/P_{-}
as a function of the tuning parameter r

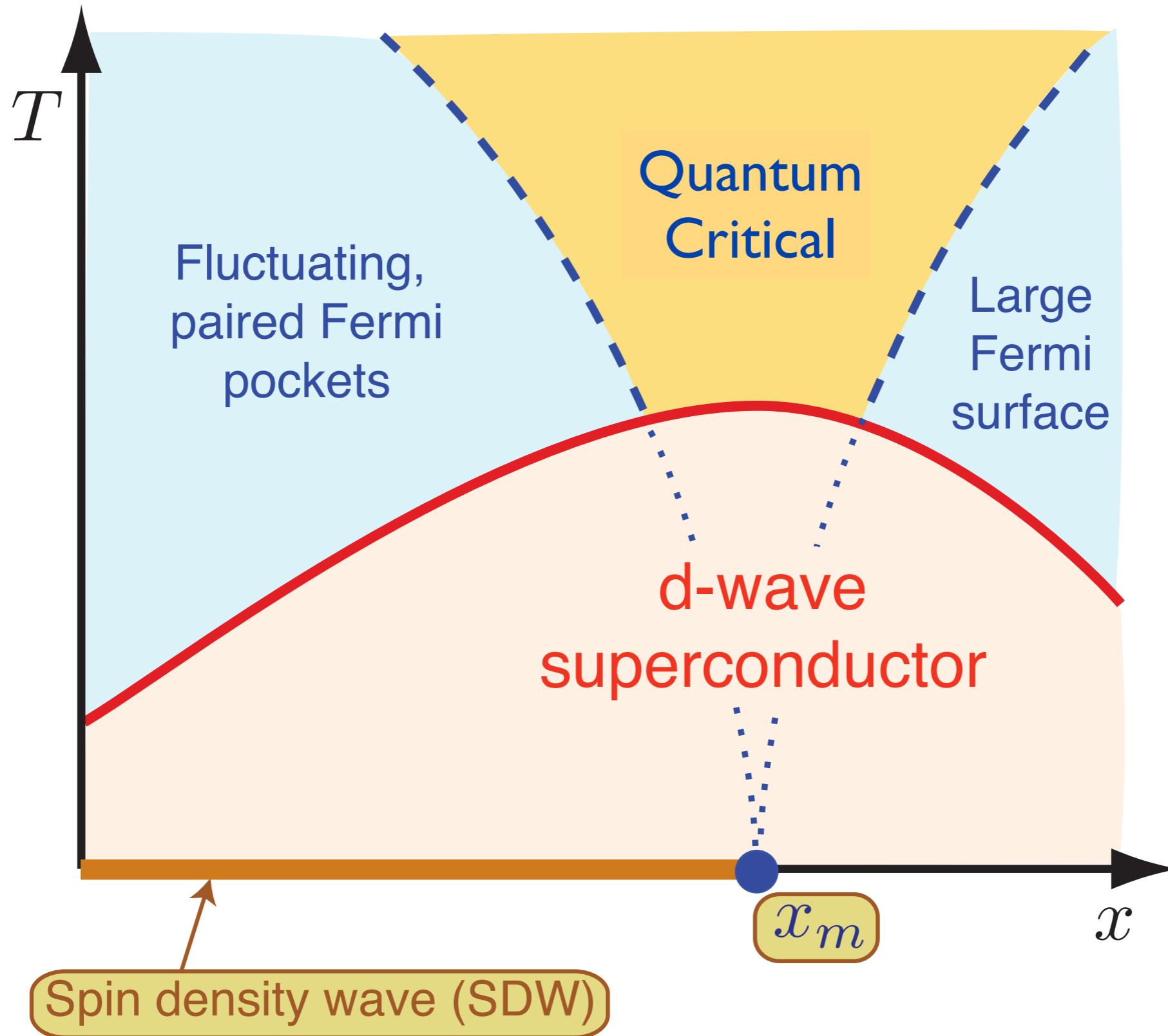
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Fermi surface+antiferromagnetism



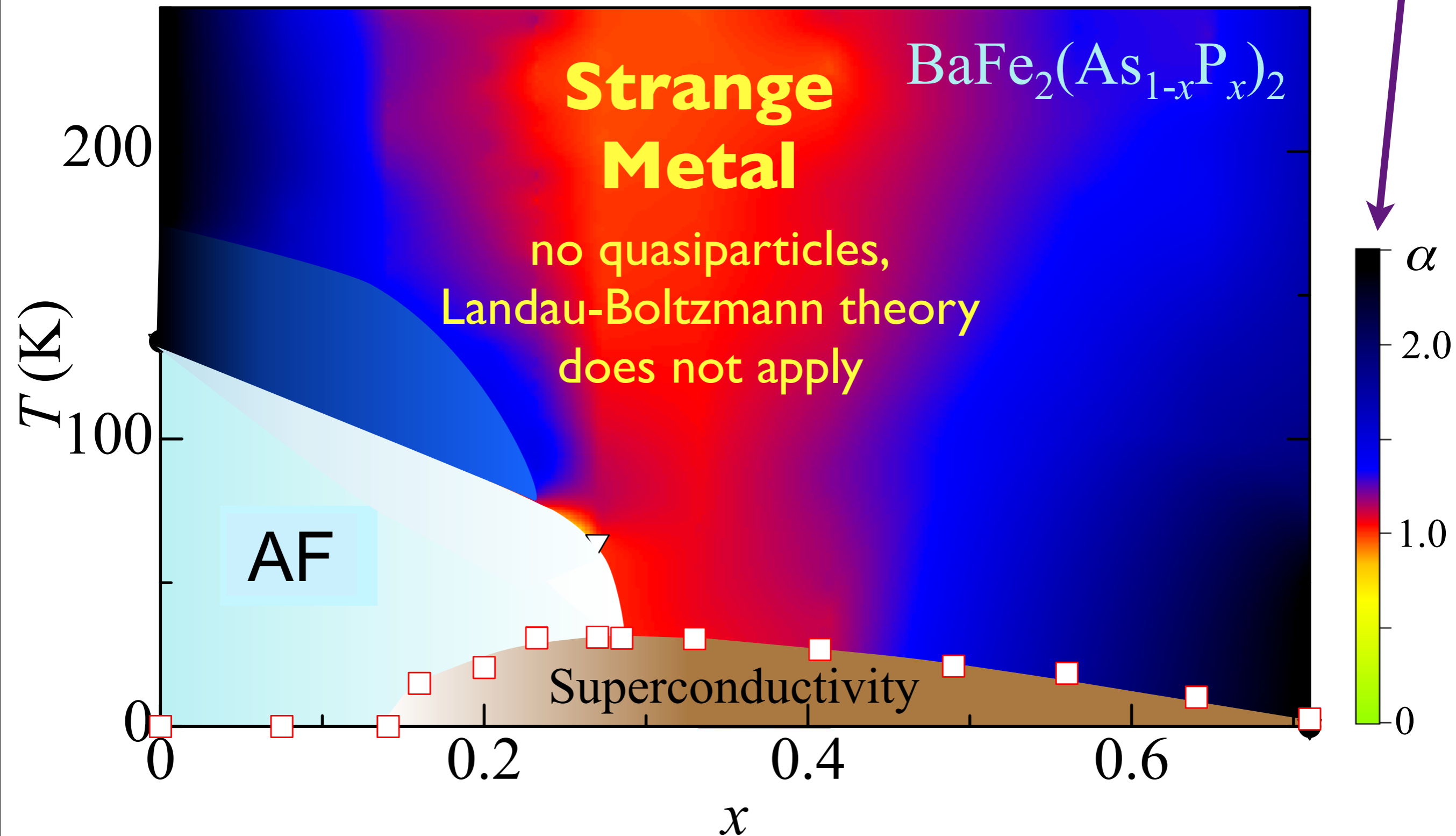
Underlying SDW ordering quantum critical point
in metal at $x = x_m$

Fermi surface+antiferromagnetism



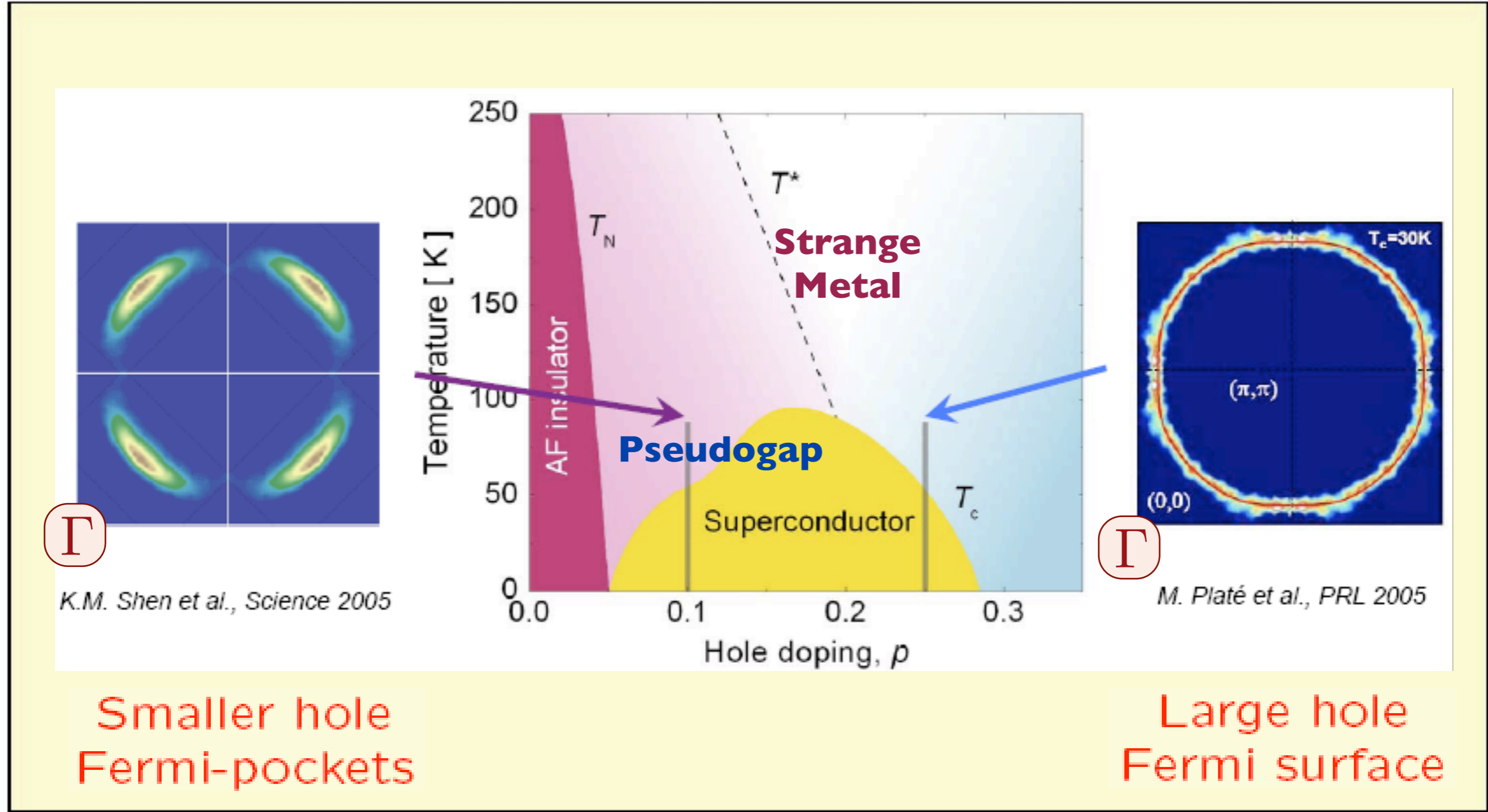
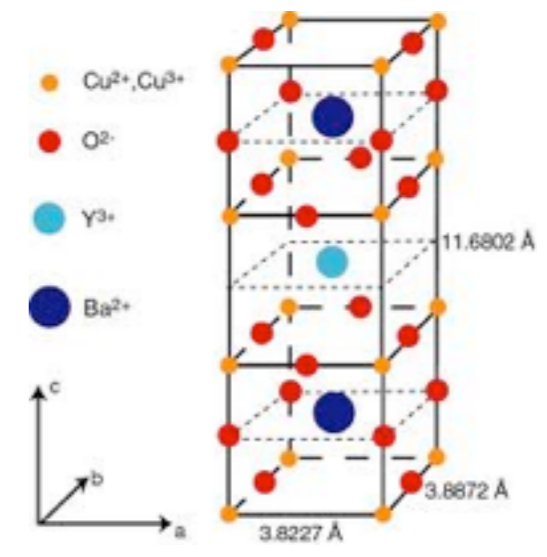
QCP for the onset of SDW order is actually within a superconductor

Resistivity
 $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)

What about the pseudogap ?



- There is an approximate pseudospin symmetry in metals with antiferromagnetic spin correlations.
- The pseudospin partner of d -wave superconductivity is an incommensurate d -wave bond order
- These orders form a pseudospin doublet, which is responsible for the “pseudogap” phase.

M. A. Metlitski and S. Sachdev, Phys. Rev. B **85**, 075127 (2010)

T. Holder and W. Metzner, Phys. Rev. B **85**, 165130 (2012)

C. Husemann and W. Metzner, Phys. Rev. B **86**, 085113 (2012)

K. B. Efetov, H. Meier, and C. Pépin, arXiv:1210.3276.

S. Sachdev and R. La Placa, arXiv:1303.2114

Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i < j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

with $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$ is the antiferromagnetic exchange interaction.
Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

$$H_J = \frac{1}{8} \sum_{i < j} J_{ij} \left(\Psi_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{j\delta} \right)$$

which is invariant under independent SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha} \rightarrow U_i \Psi_{i\alpha}$$

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of H_J .

- I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
- E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
- P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

Pseudospin symmetry of the exchange interaction

$$H_{tJ} = - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

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- I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
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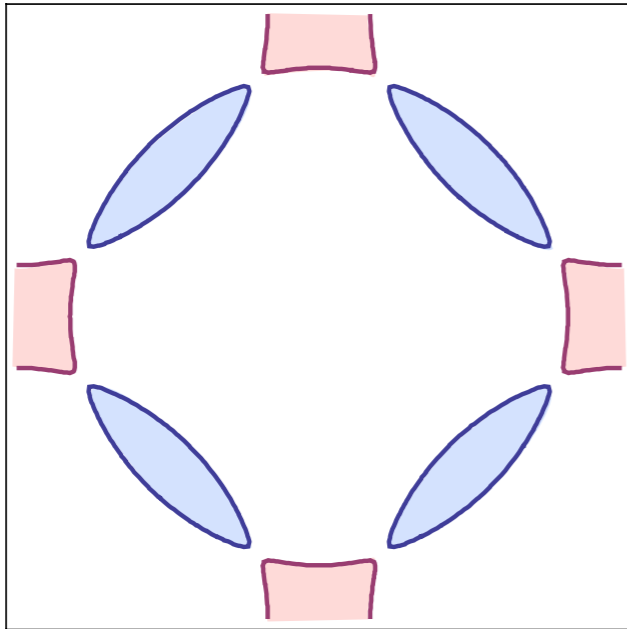
$$H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left(\Psi_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left(\Psi_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{j\delta} \right)$$

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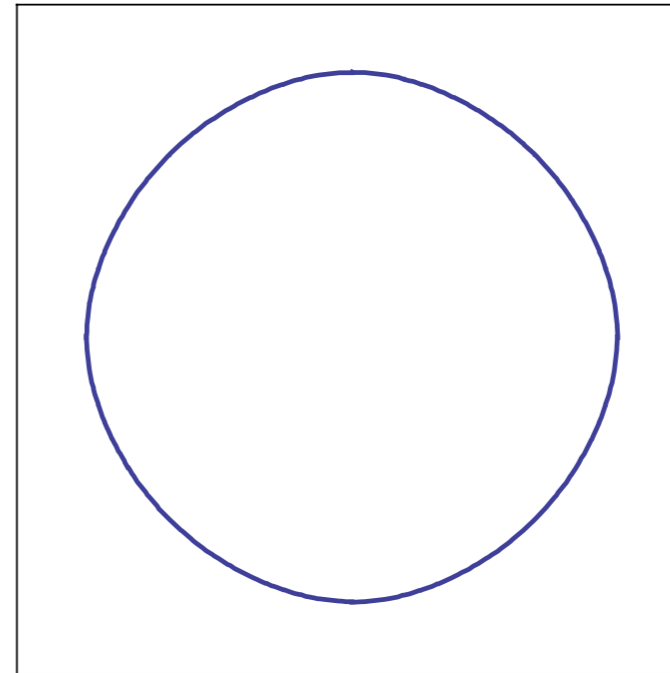
We will start with the Néel state, and find important consequences of the pseudospin symmetry in metals with antiferromagnetic correlations.

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

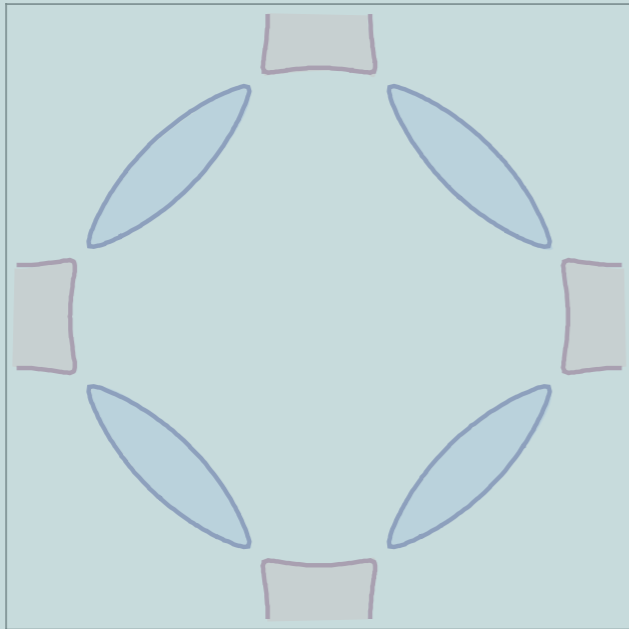


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

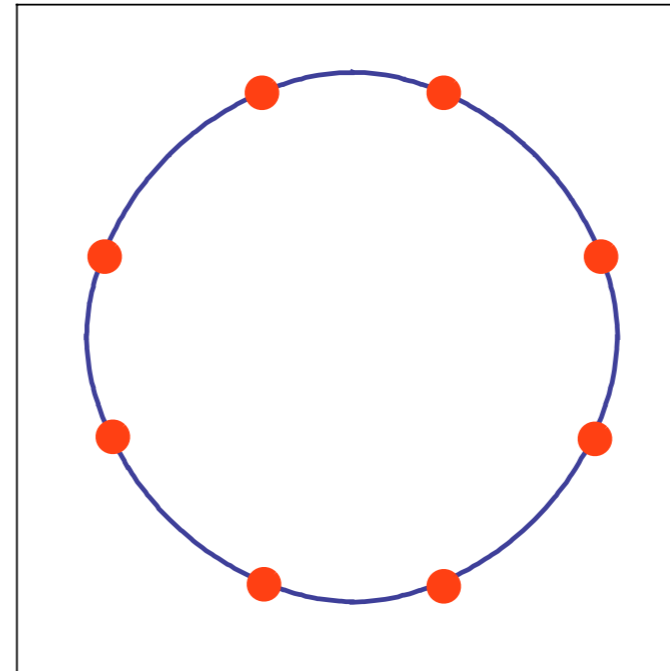
r

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets



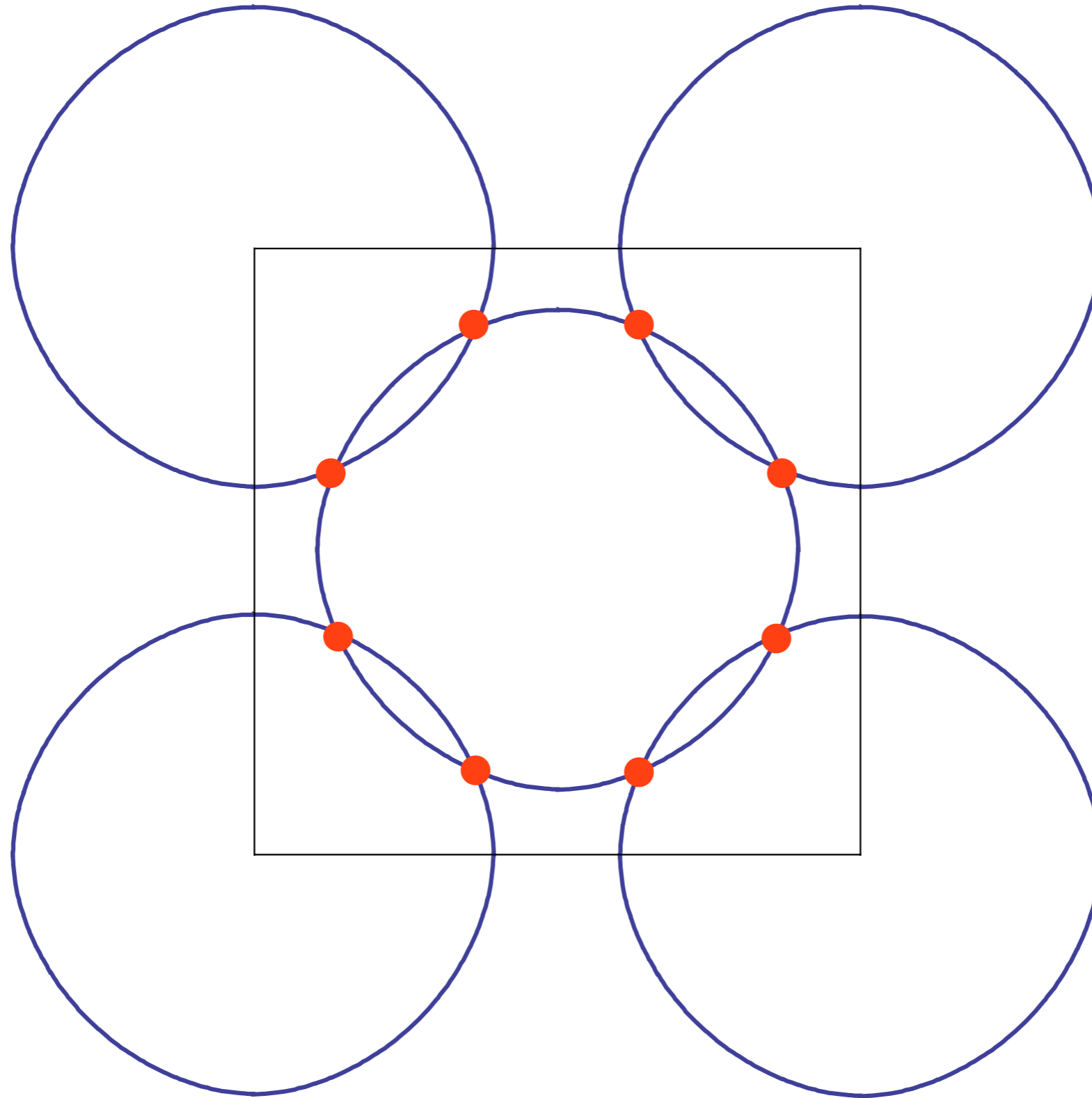
$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

Focus on
this
region

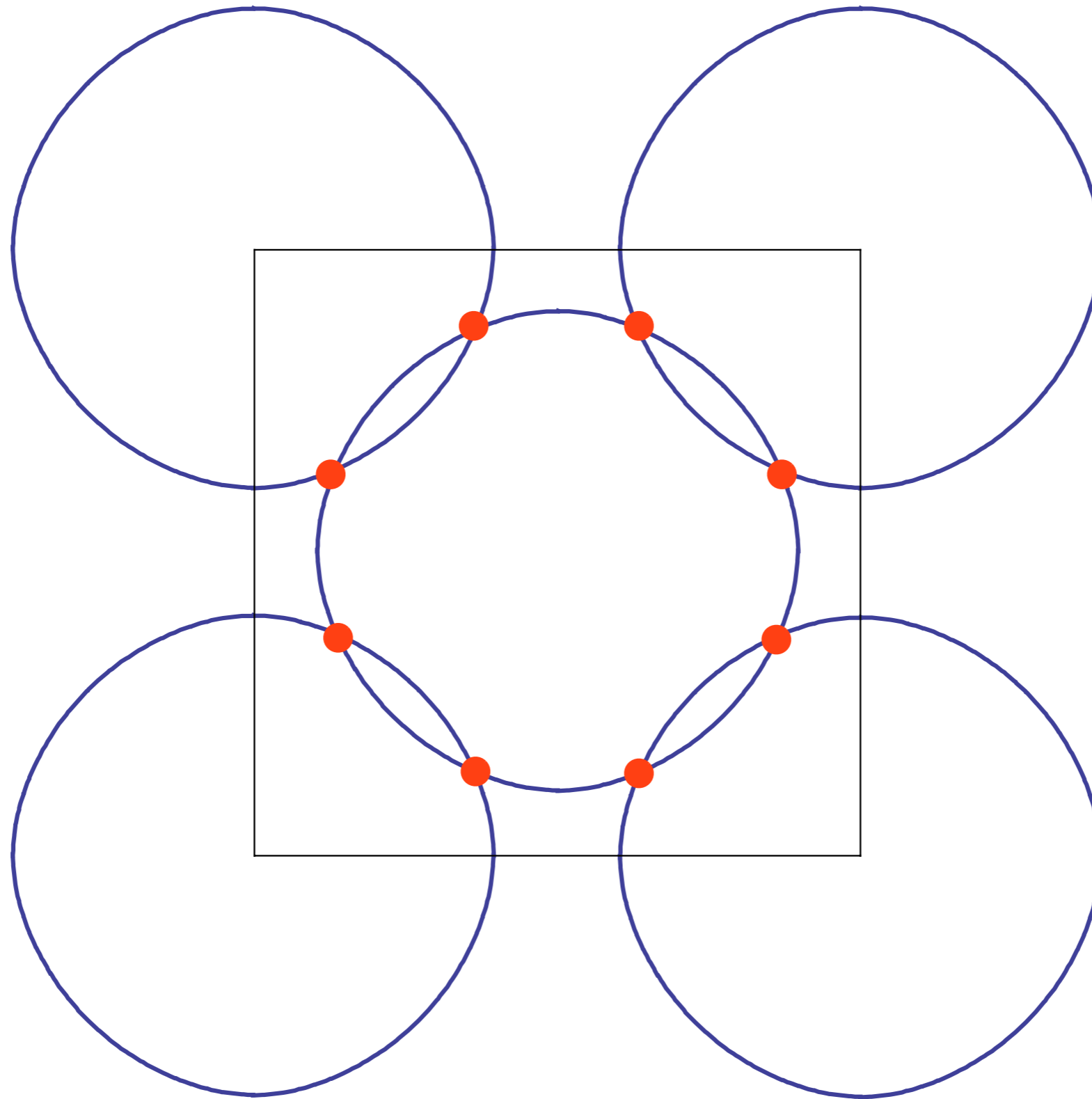
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Fermi surface+antiferromagnetism



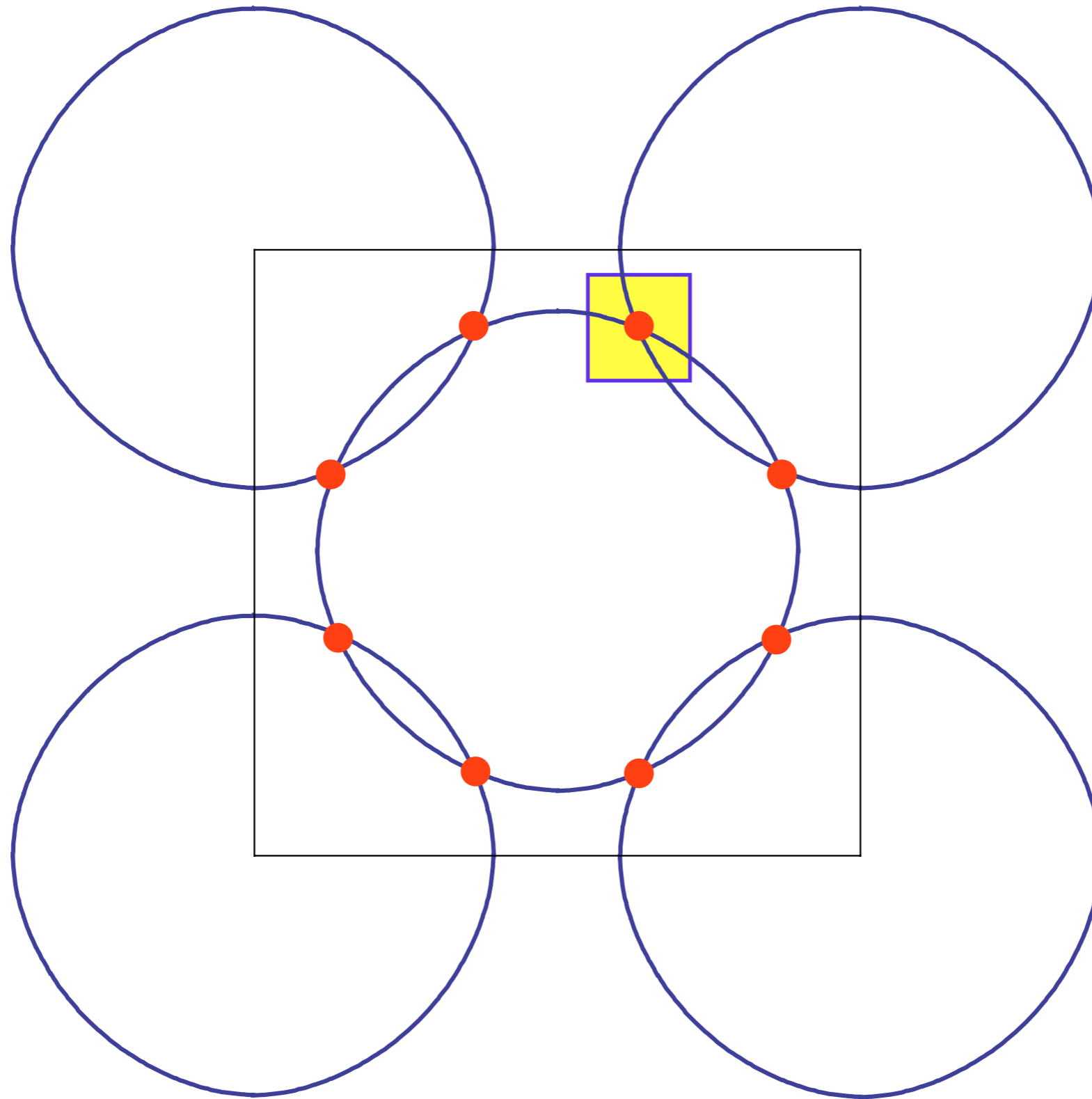
“Hot” spots

Fermi surface+antiferromagnetism



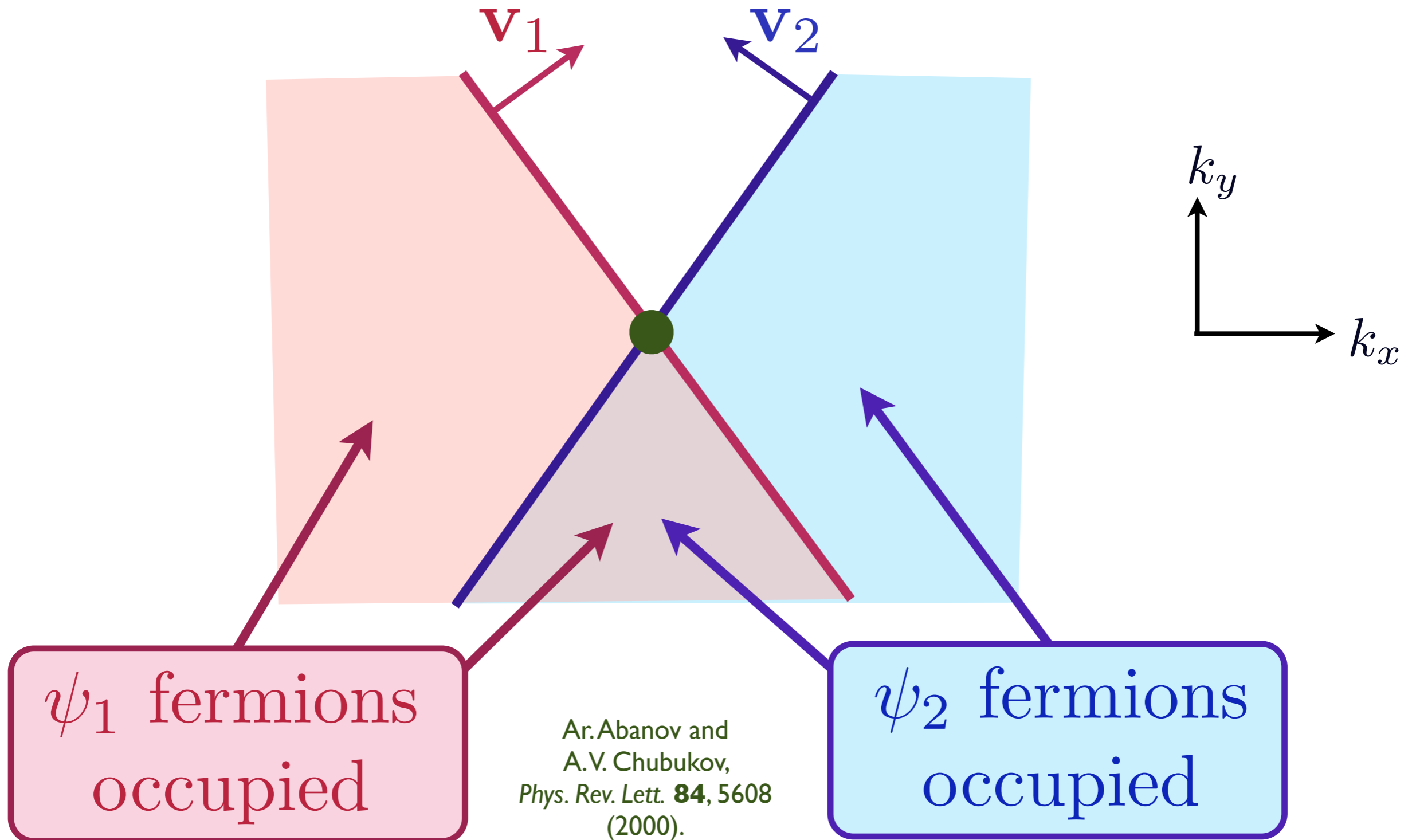
Low energy theory for critical point near hot spots

Fermi surface+antiferromagnetism

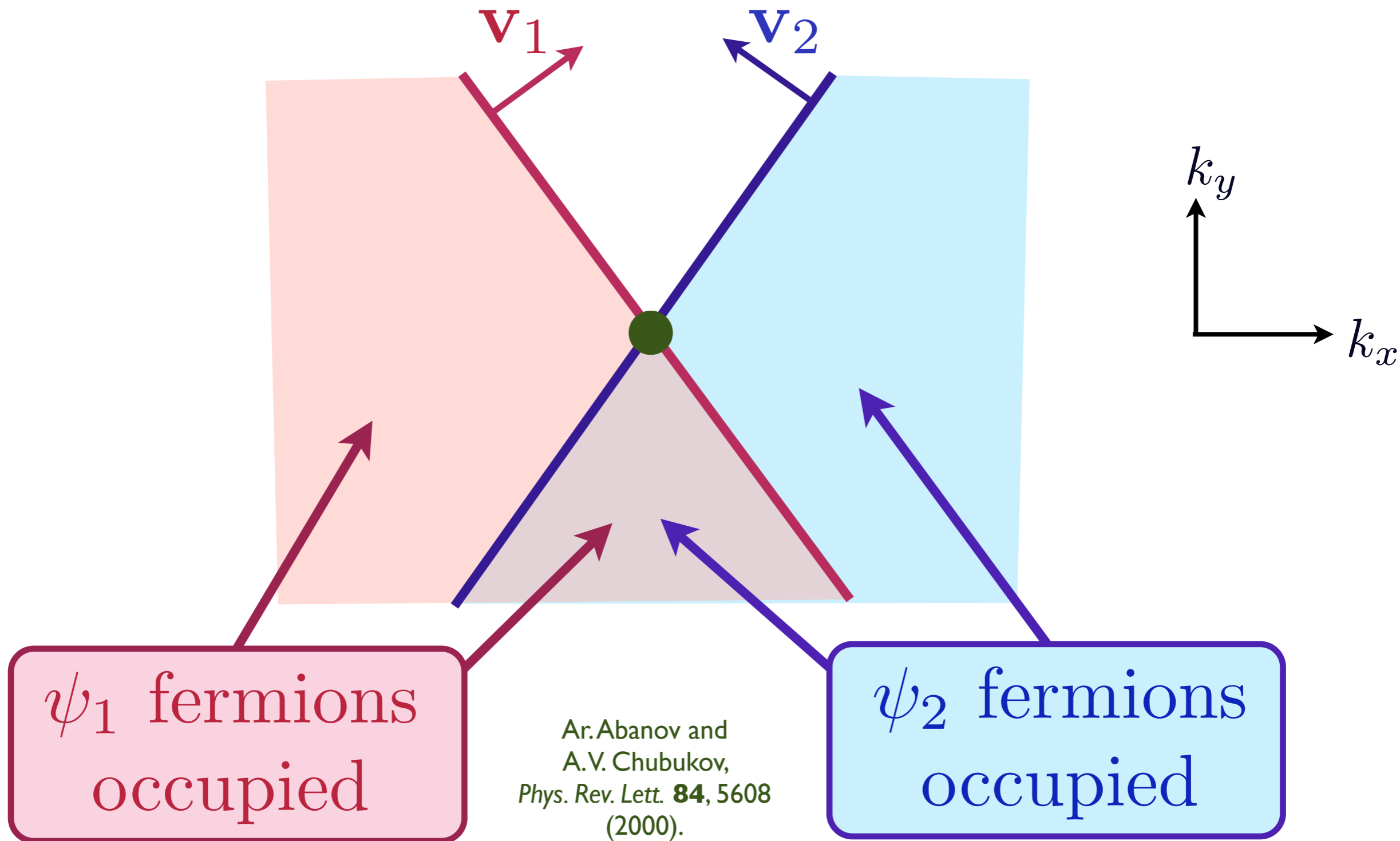


Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



$$\mathcal{S} = \int d^2r d\tau \left[\psi_{1\alpha}^\dagger (\partial_\tau - i\mathbf{v}_1 \cdot \nabla_r) \psi_{1\alpha} + \psi_{2\alpha}^\dagger (\partial_\tau - i\mathbf{v}_2 \cdot \nabla_r) \psi_{2\alpha} \right. \\
 \left. + \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \right]$$



$$\mathcal{S} = \int d^2r d\tau \left[\psi_{1\alpha}^\dagger (\partial_\tau - i\mathbf{v}_1 \cdot \nabla_r) \psi_{1\alpha} + \psi_{2\alpha}^\dagger (\partial_\tau - i\mathbf{v}_2 \cdot \nabla_r) \psi_{2\alpha} + \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \right]$$



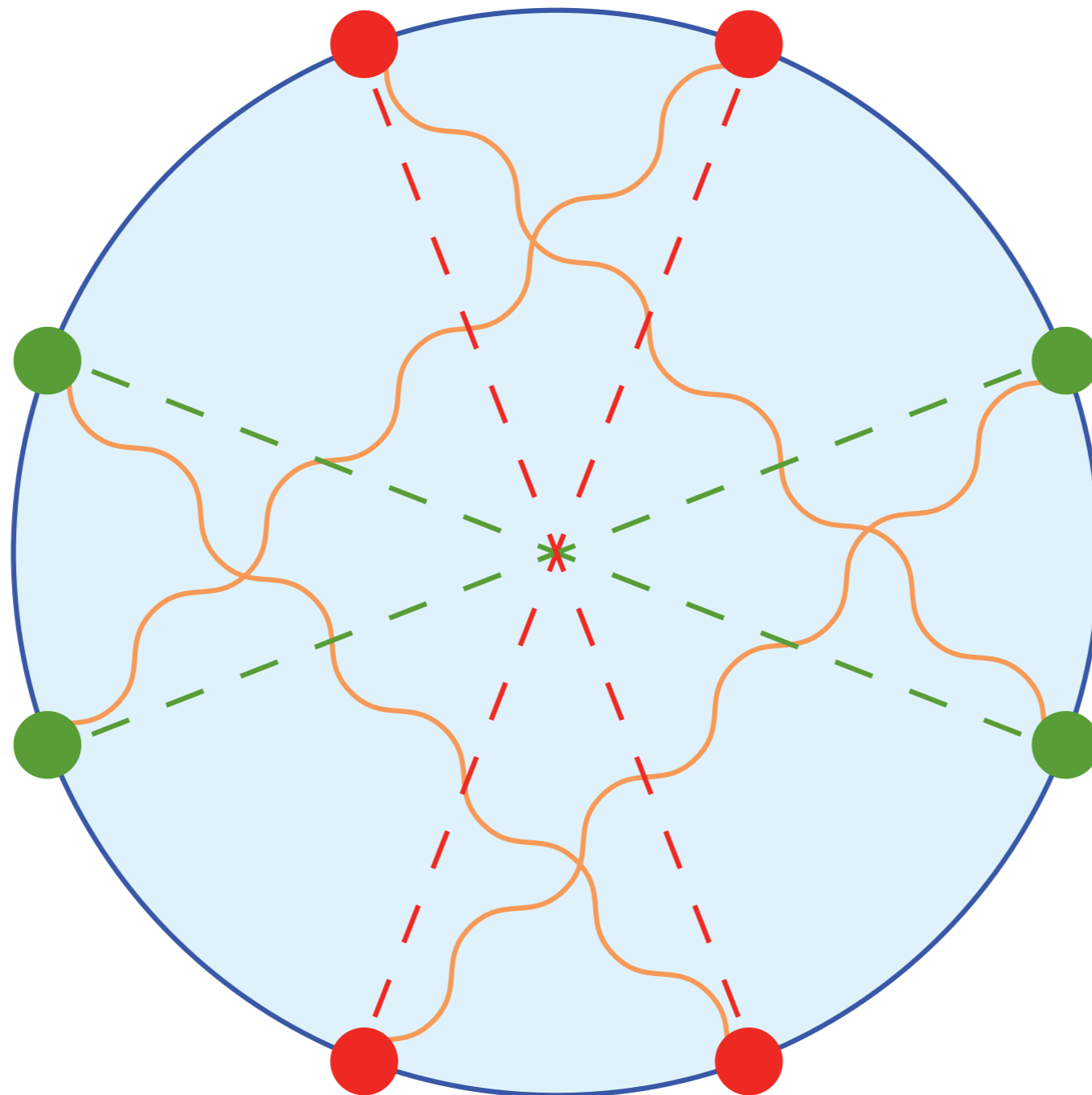
This low-energy theory is invariant under independent SU(2) pseudospin rotations on each pair of hot-spots: there is a global SU(2) × SU(2) × SU(2) × SU(2) pseudospin symmetry.

ψ_1 fermions occupied

ψ_2 fermions occupied

M.A. Metlitski and S. Sachdev,
 Phys. Rev. B **85**, 075127 (2010)

V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



Unconventional pairing at and near hot spots

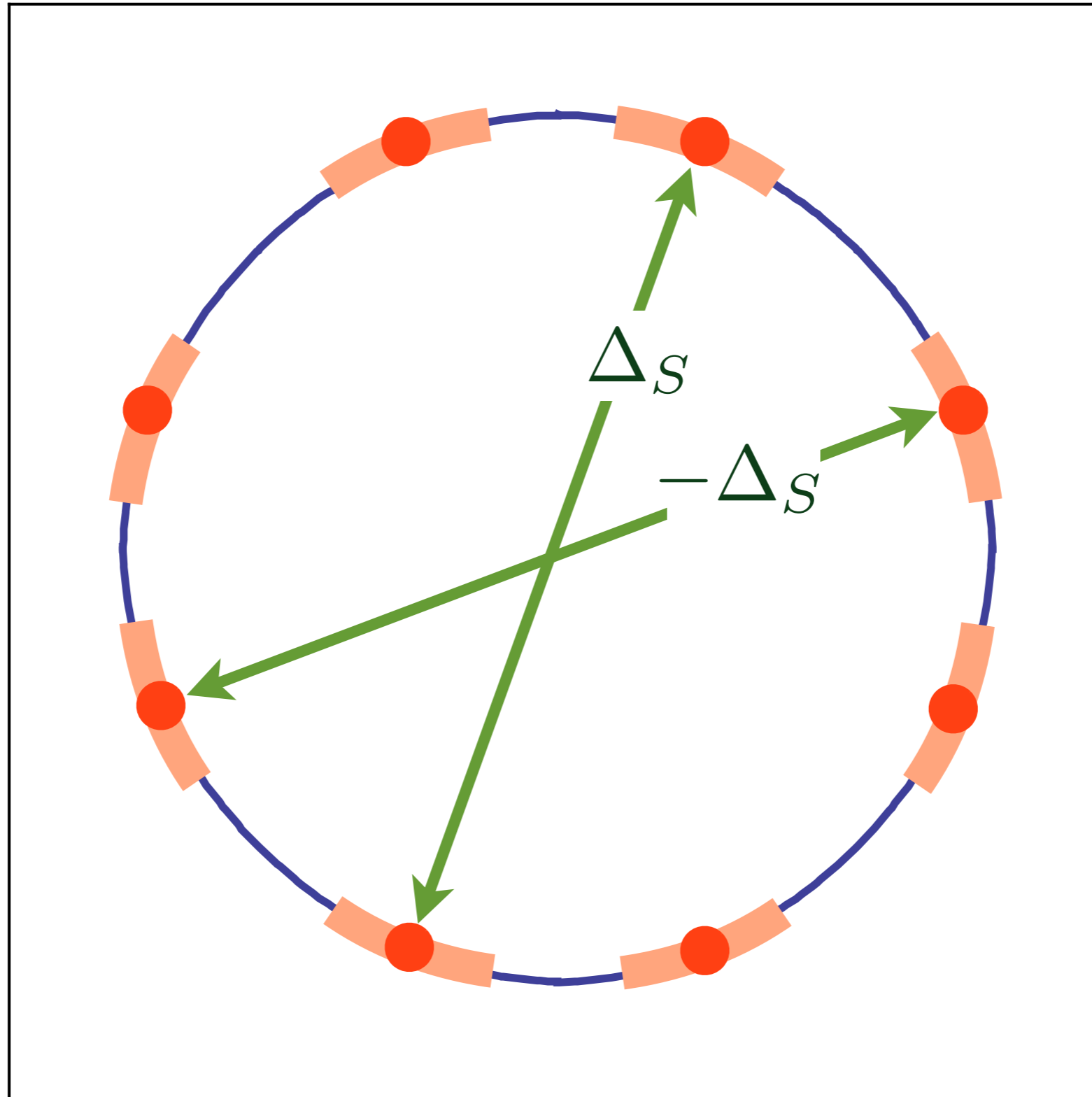
$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$

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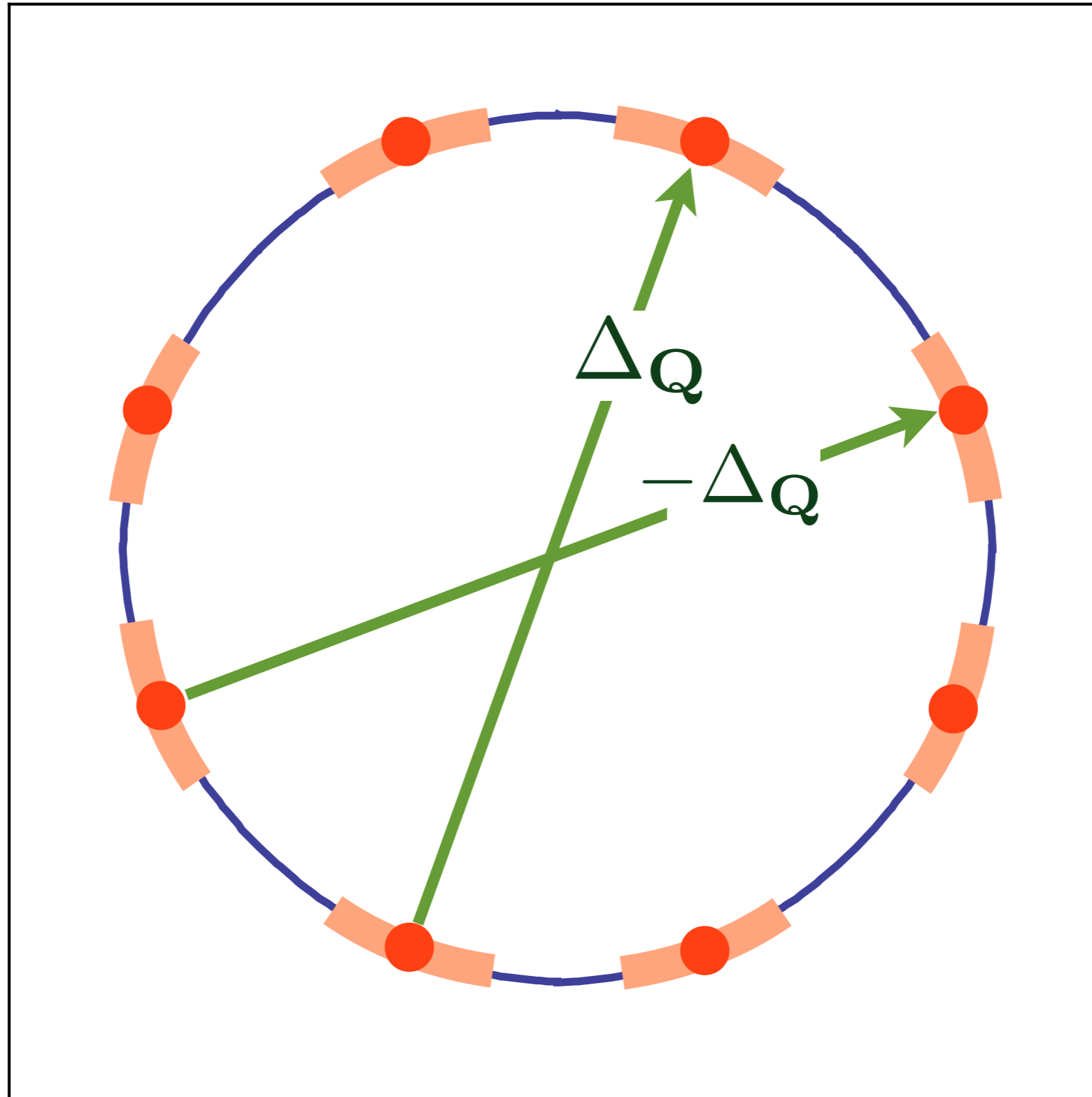
S. Raghu, S. A. Kivelson, and D. J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)



Unconventional pairing at and near hot spots

$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

After
pseudospin
rotation on
half the
hot-spots



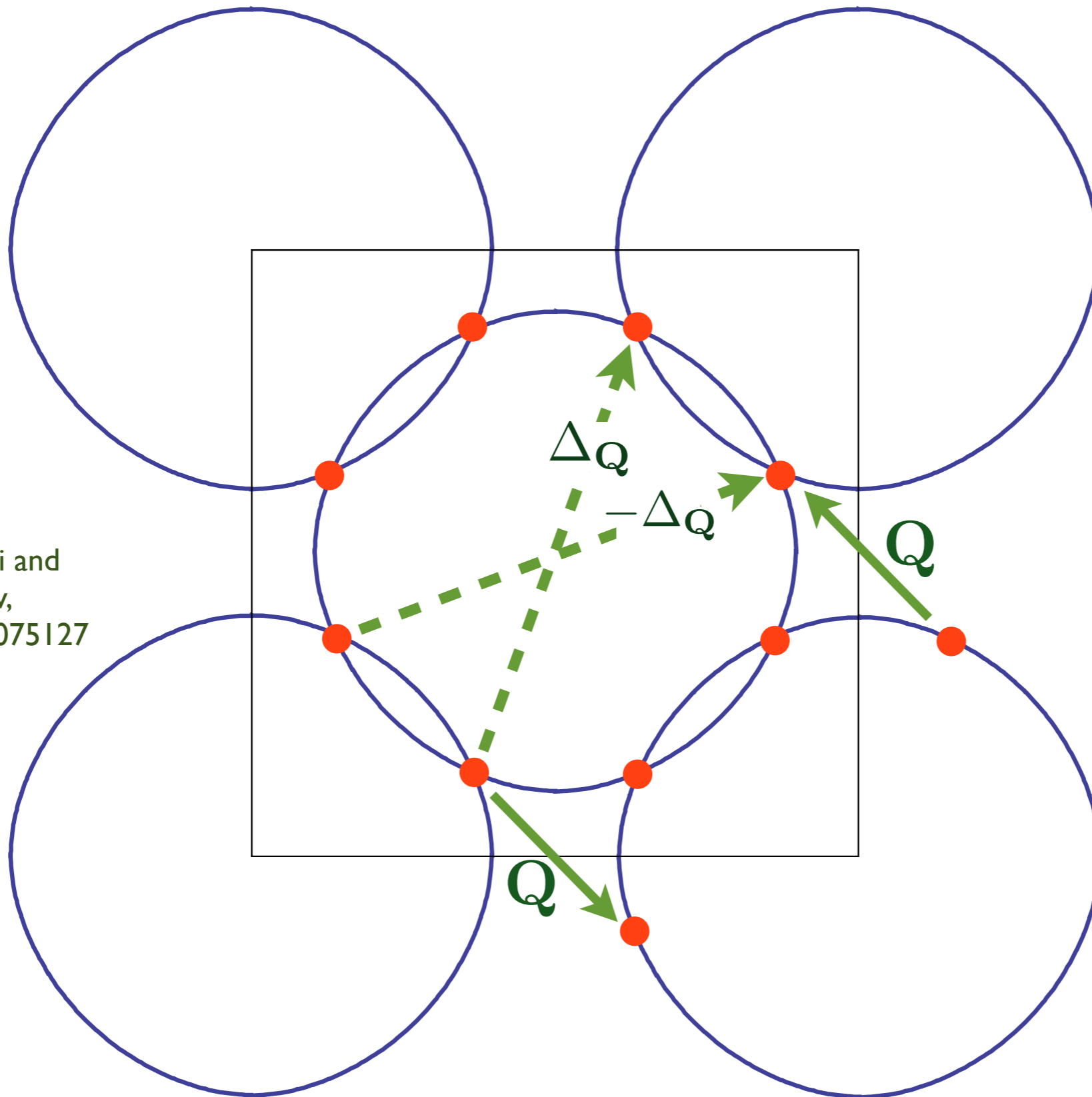
\mathbf{Q} is ' $2k_F$ '
wavevector

M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)

Unconventional particle-hole pairing at and near hot spots

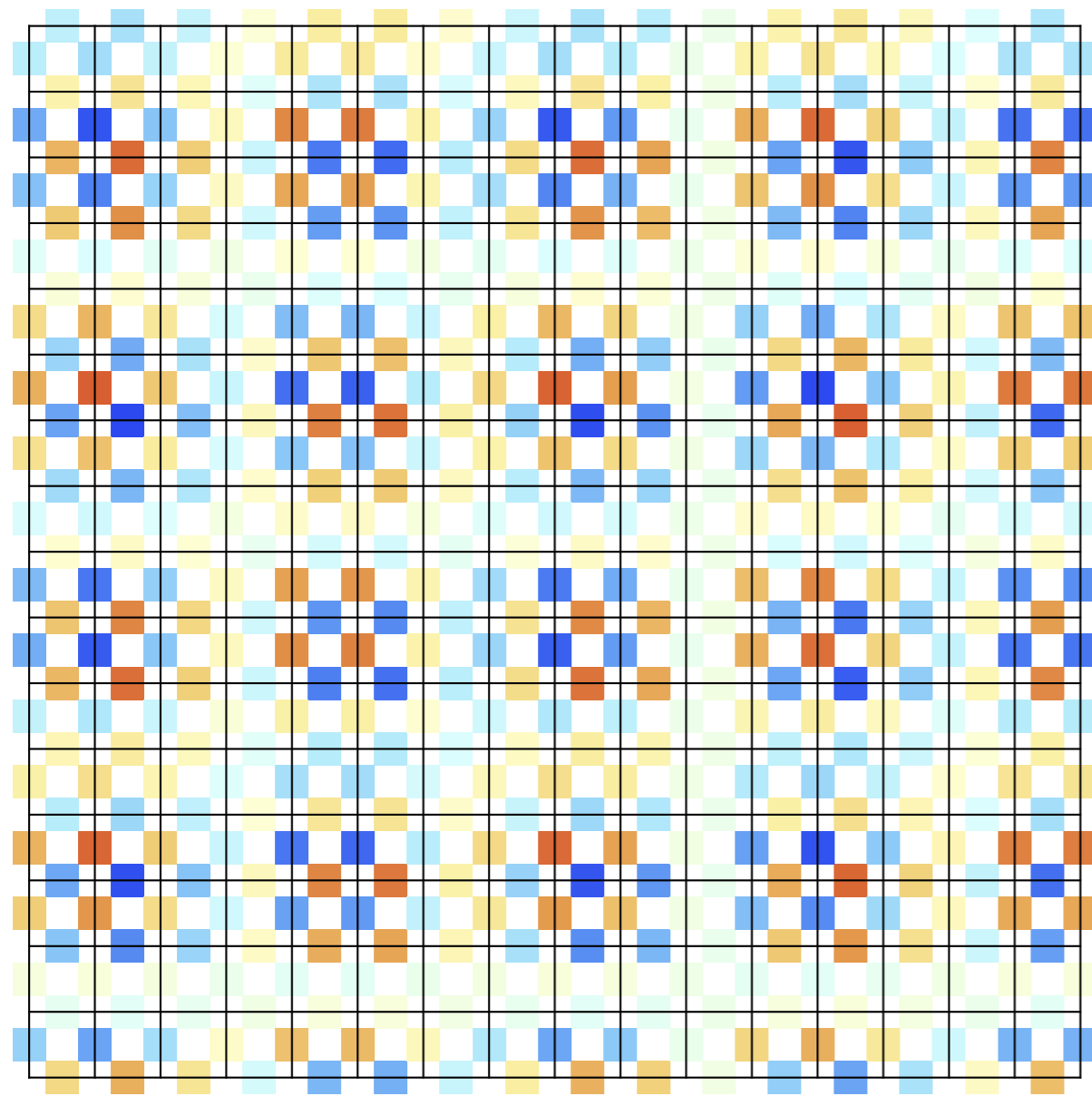
Incommensurate d -wave bond order

M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)



$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \Delta_Q (\cos k_x - \cos k_y)$$

Incommensurate d -wave bond order



“Bond density”
measures amplitude
for electrons to be
in spin-singlet
valence bond.

M.A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)

$$\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \rangle = \sum_{\mathbf{Q}} \sum_{\mathbf{k}} e^{i\mathbf{Q}\cdot(\mathbf{r}+\mathbf{s})/2} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{s})} \langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle$$

where \mathbf{Q} extends over $\mathbf{Q} = (\pm Q_0, \pm Q_0)$ with $Q_0 = 2\pi/(7.3)$ and

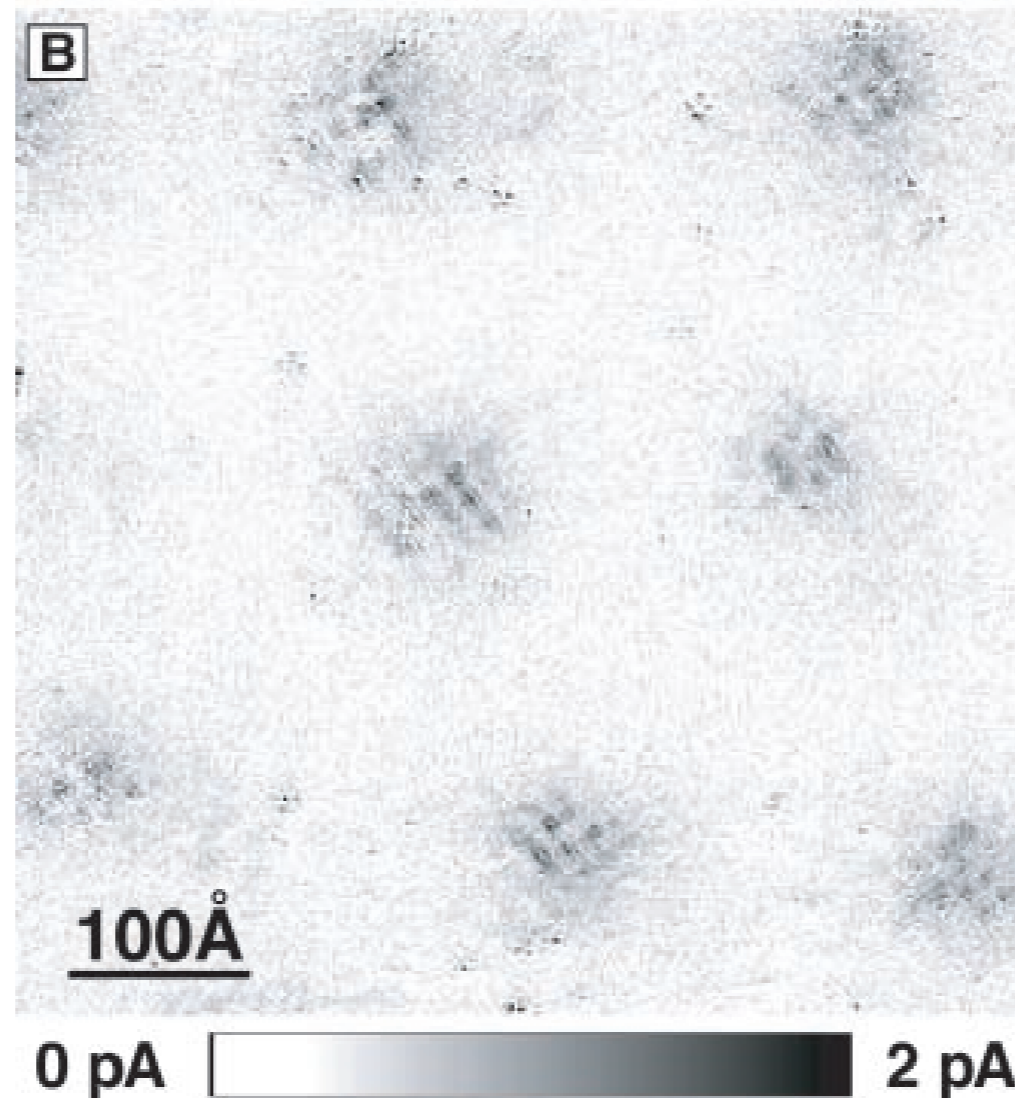
$$\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Note $\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \rangle$ is non-zero *only* when \mathbf{r}, \mathbf{s} are nearest neighbors.

A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

J. E. Hoffman,¹ E. W. Hudson,^{1,2*} K. M. Lang,¹ V. Madhavan,¹
H. Eisaki,^{3†} S. Uchida,³ J. C. Davis^{1,2‡}

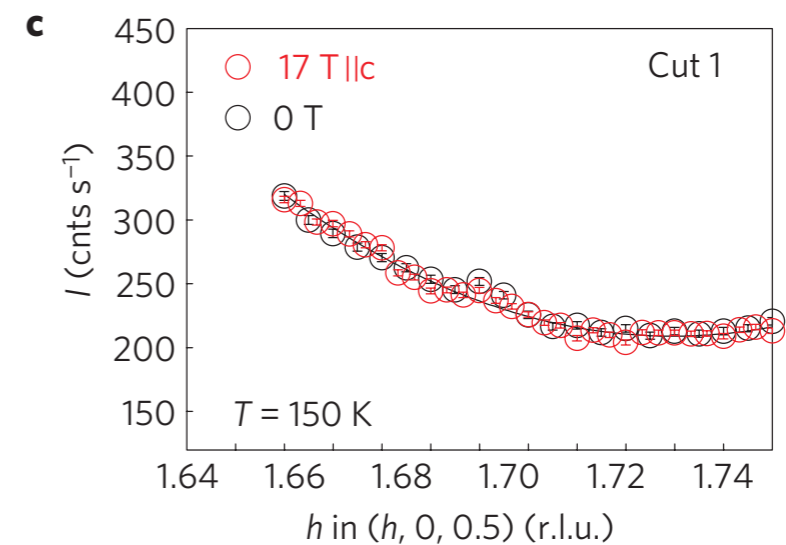
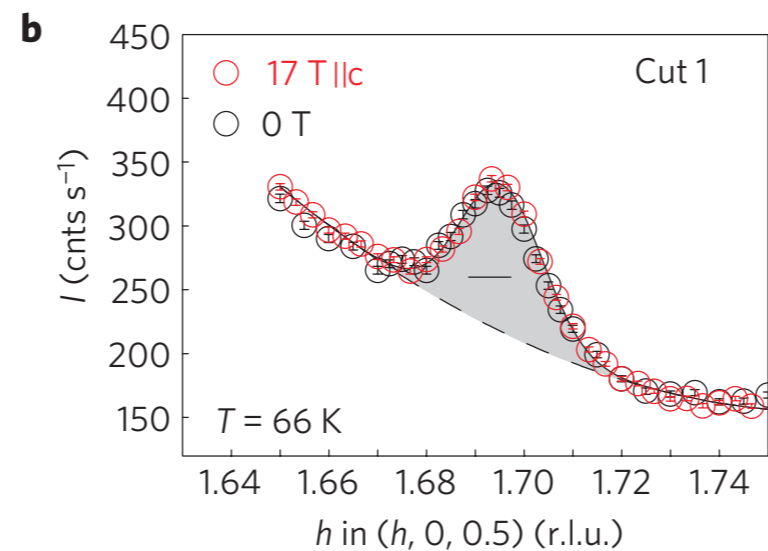
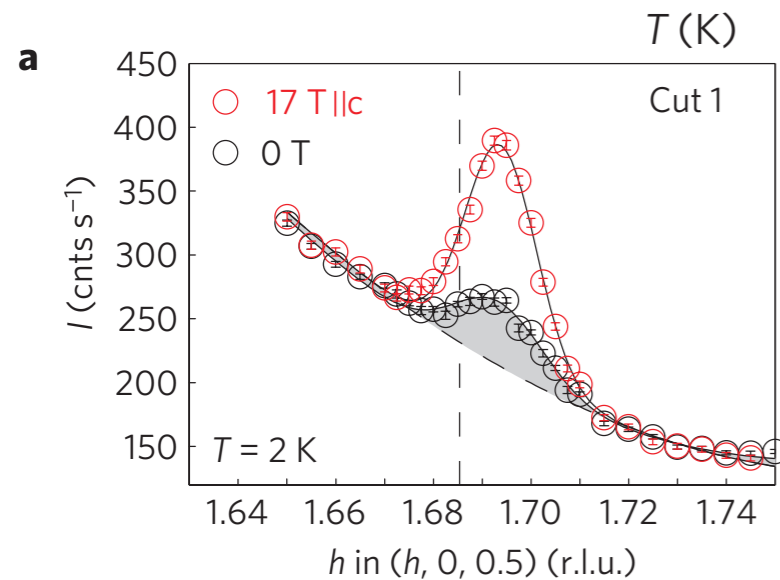
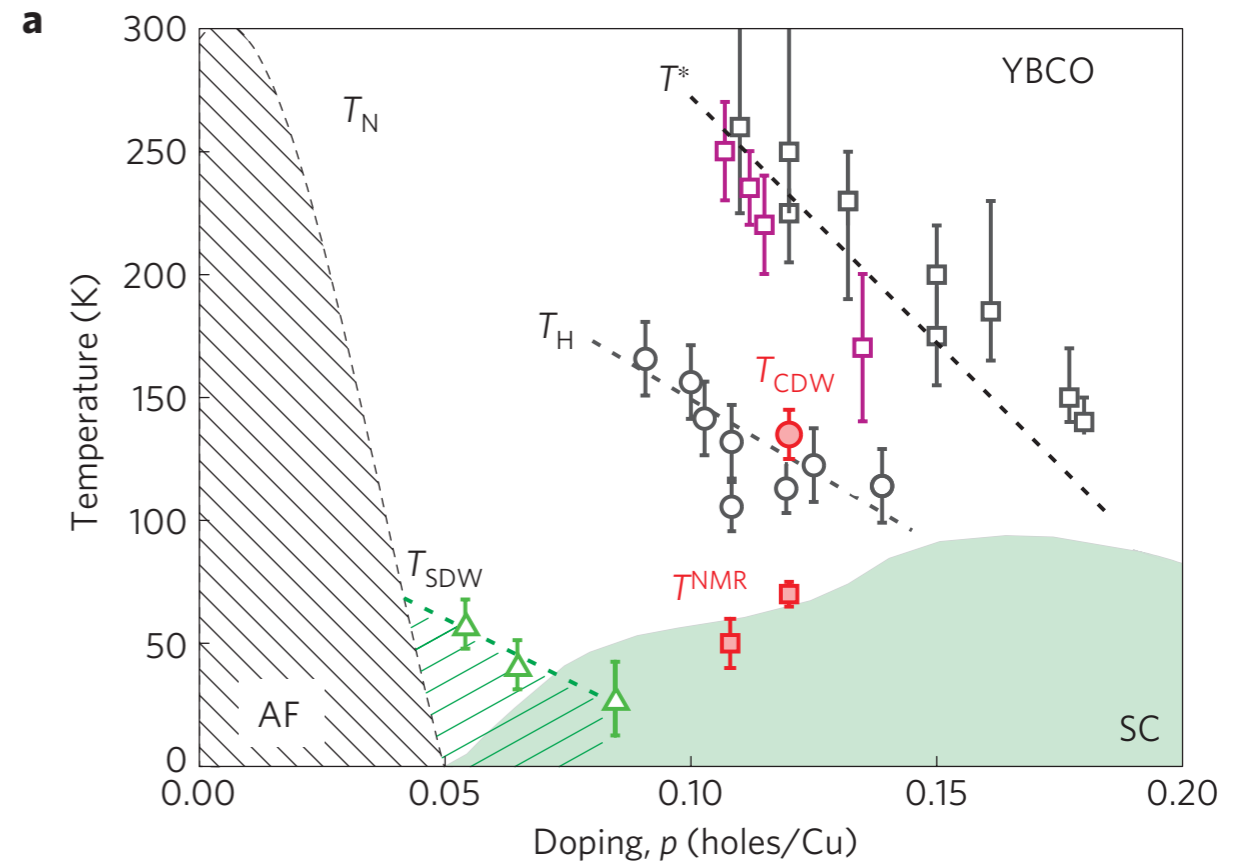
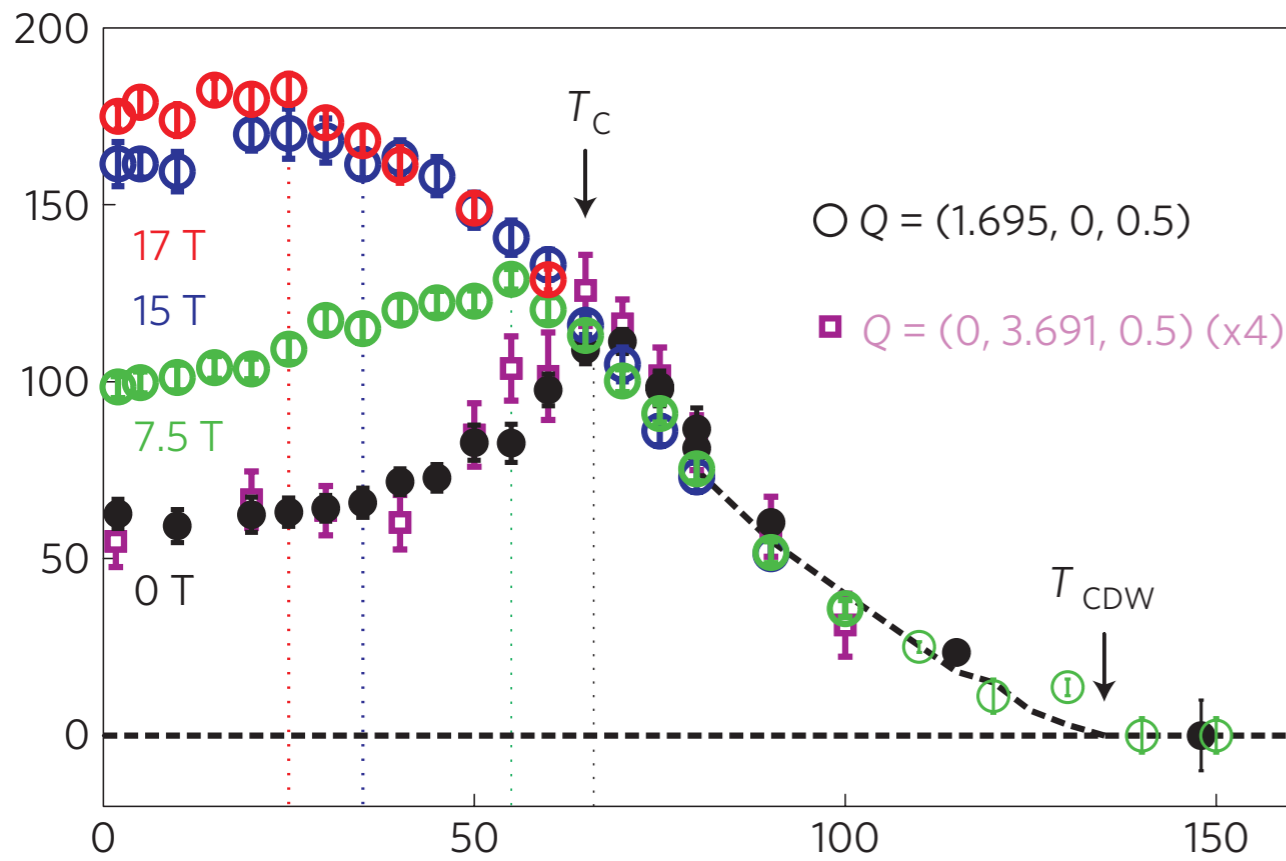
SCIENCE VOL 295 18 JANUARY 2002



Direct observation of competition between superconductivity and charge density wave order in $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$

NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

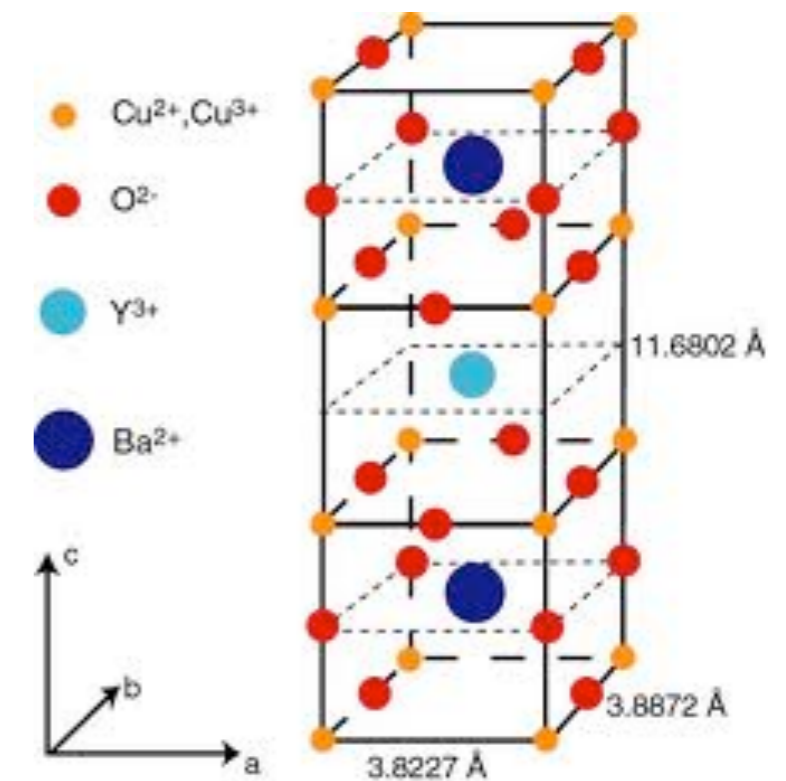
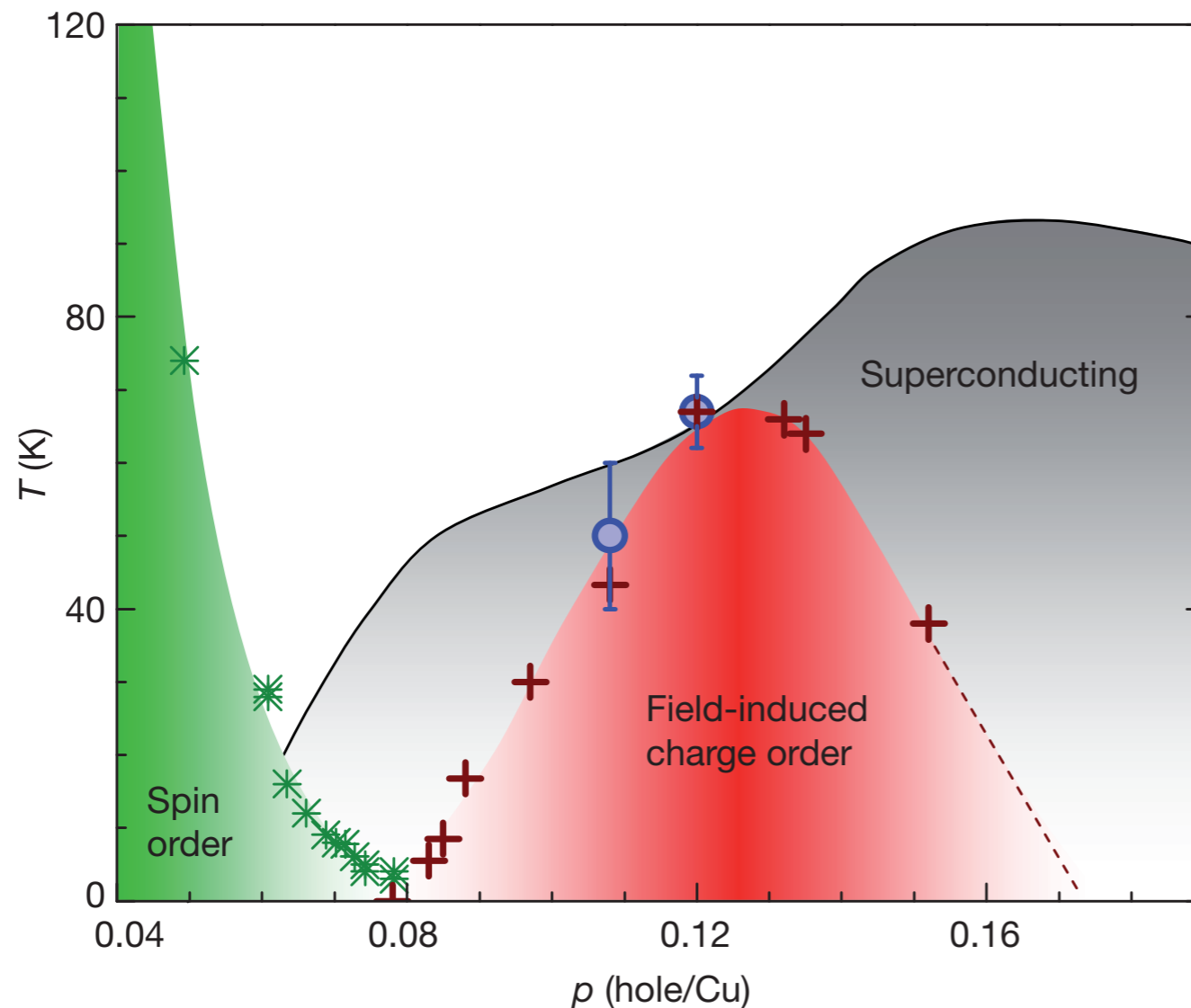
J. Chang^{1,2*}, E. Blackburn³, A. T. Holmes³, N. B. Christensen⁴, J. Larsen^{4,5}, J. Mesot^{1,2}, Ruixing Liang^{6,7}, D. A. Bonn^{6,7}, W. N. Hardy^{6,7}, A. Watenphul⁸, M. v. Zimmermann⁸, E. M. Forgan³ and S. M. Hayden⁹



Magnetic-field-induced charge-stripe order in the high-temperature superconductor $\text{YBa}_2\text{Cu}_3\text{O}_y$

Tao Wu¹, Hadrien Mayaffre¹, Steffen Krämer¹, Mladen Horvatić¹, Claude Berthier¹, W. N. Hardy^{2,3}, Ruixing Liang^{2,3}, D. A. Bonn^{2,3} & Marc-Henri Julien¹

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191



Summary

Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points using the methods of gauge-gravity duality.
- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.
- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Summary

Antiferromagnetism in metals and the high temperature superconductors

- Antiferromagnetic quantum criticality leads to d -wave superconductivity (supported by sign-problem-free Monte Carlo simulations)
- Metals with antiferromagnetic spin correlations have nearly degenerate instabilities: to d -wave superconductivity, and to a charge density wave with a d -wave form factor. This is a promising explanation of the pseudogap regime.