Where is the quantum critical point in the cuprate superconductors ?

PHYSICS

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Destruction of Neel order in the cuprates by electron doping,
R. K. Kaul, M. Metlitksi, S. Sachdev, and C. Xu,
Physical Review B **78**, 045110 (2008).

Paired electron pockets in the underdoped cuprates,
V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Competition between spin density wave order and superconductivity in the underdoped cuprates,
Eun Gook Moon and S. Sachdev,
arXiv:0905.2608

PHYSICS
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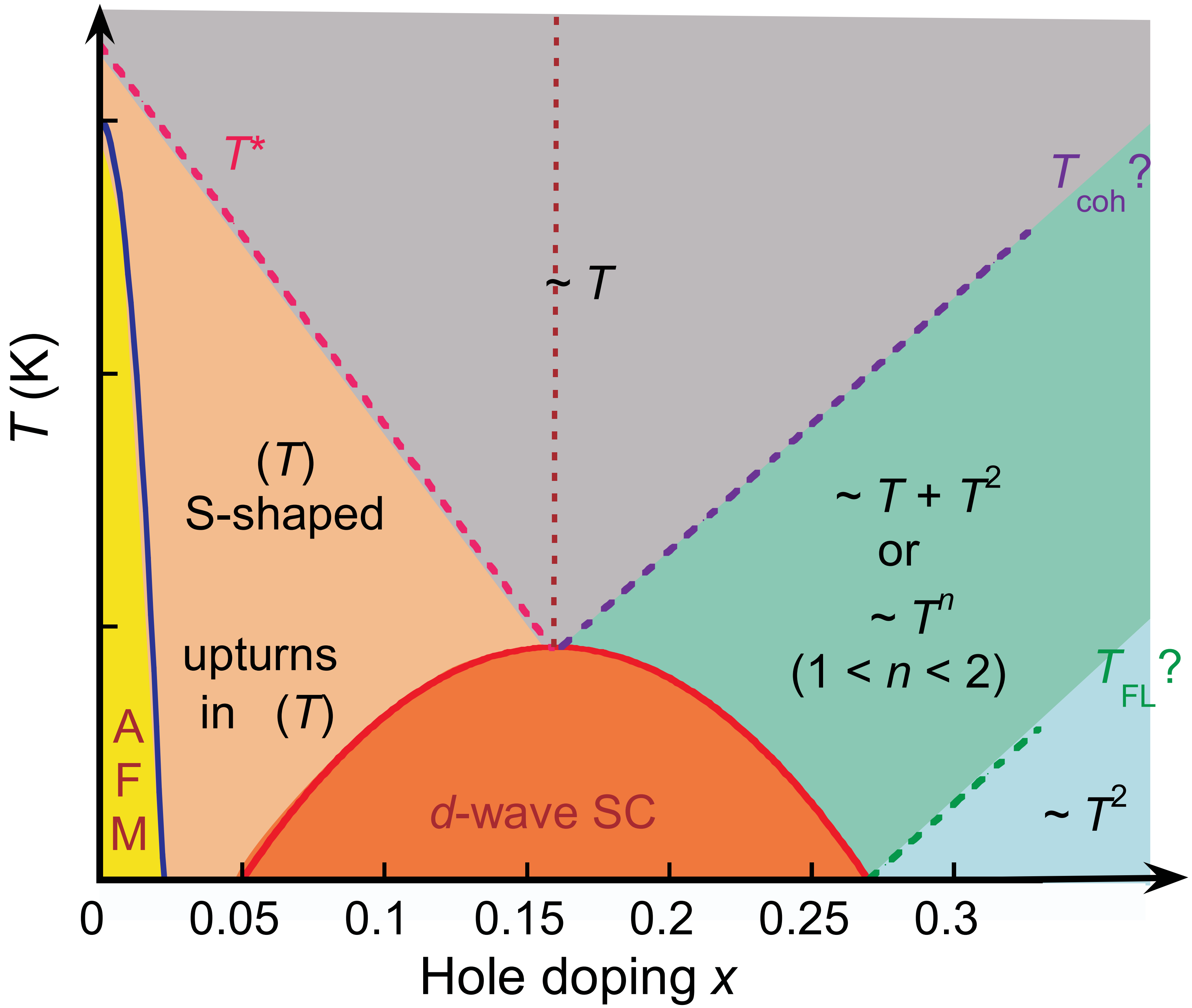
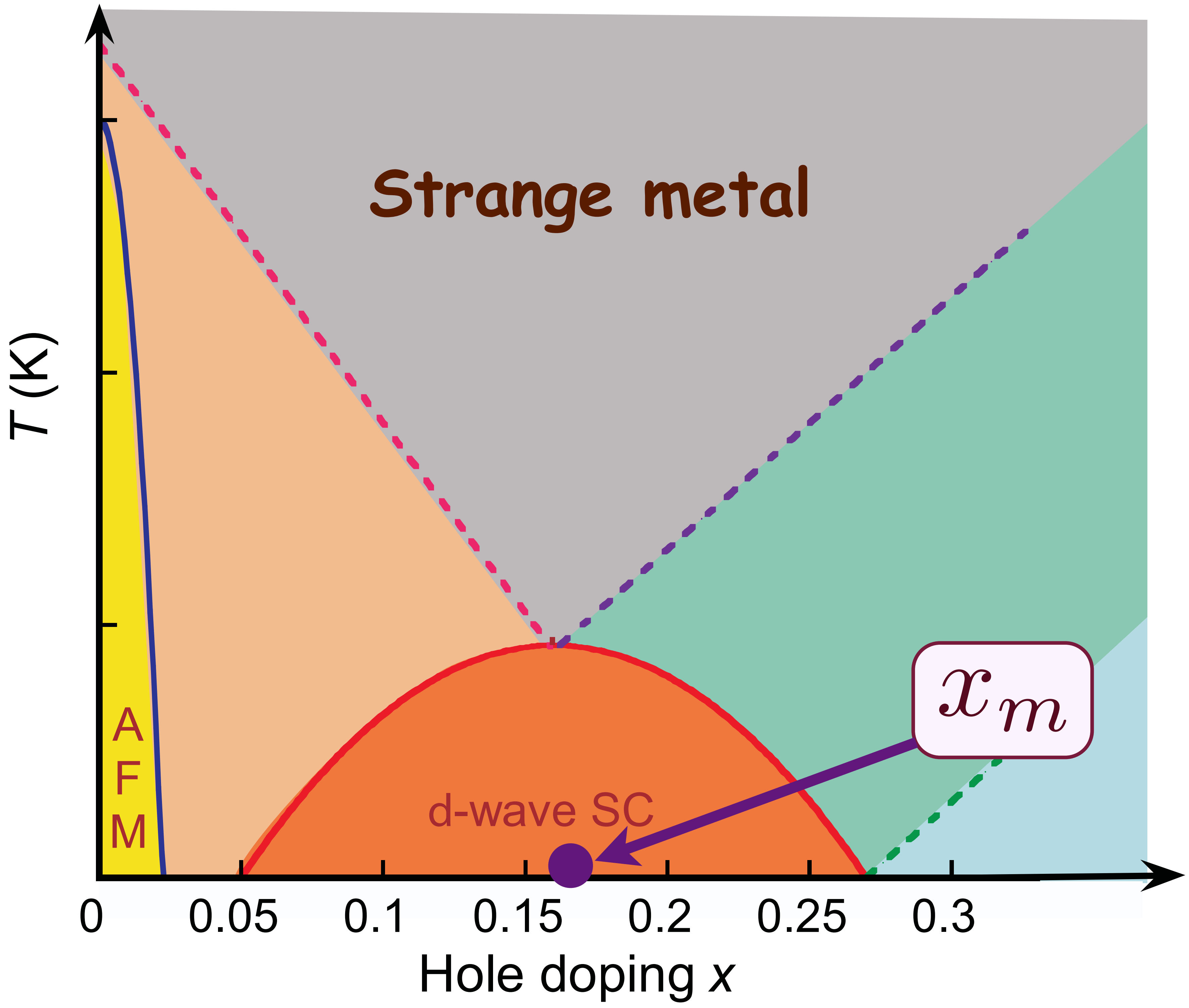
Crossovers in transport properties of hole-doped cuprates



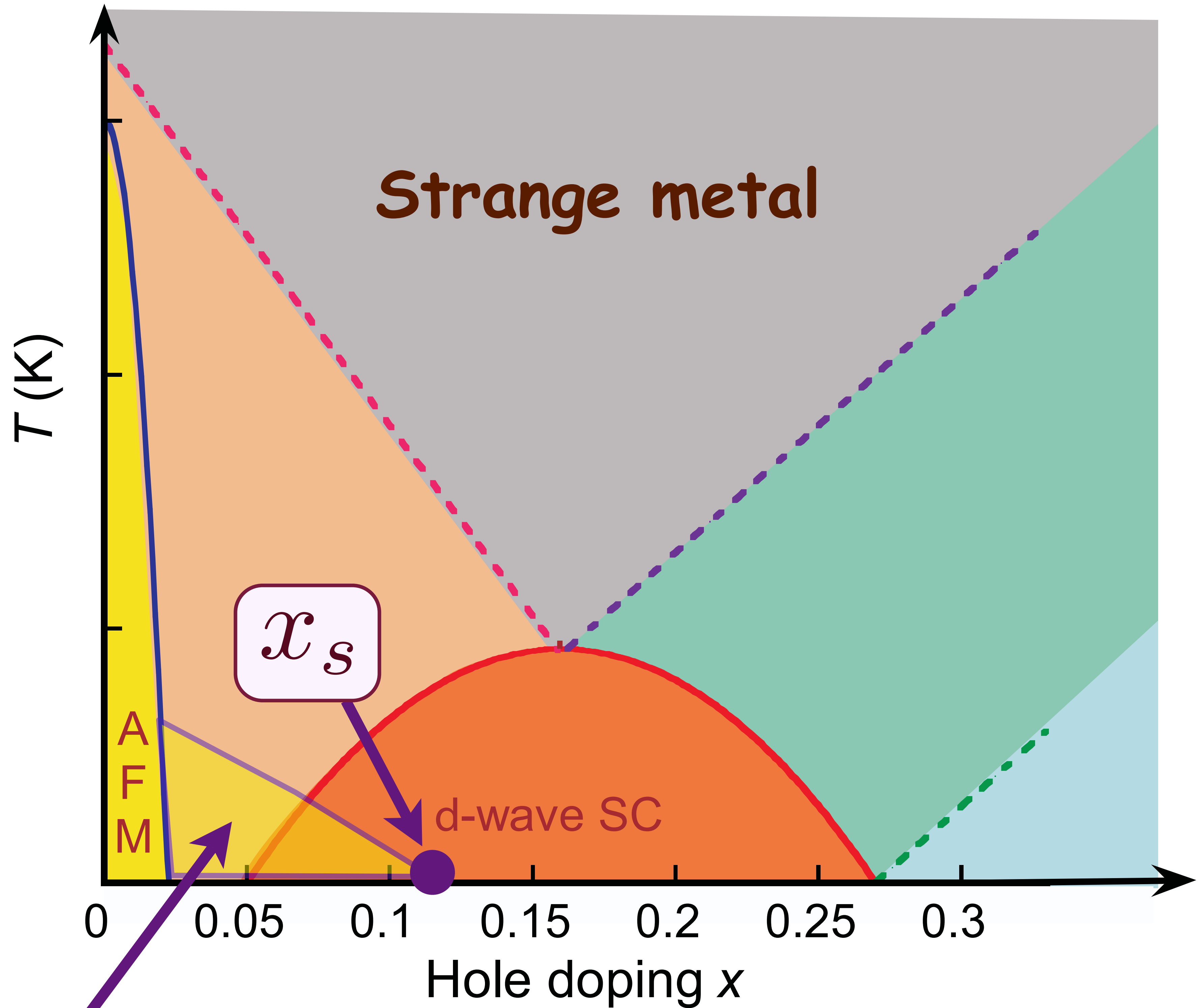
N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



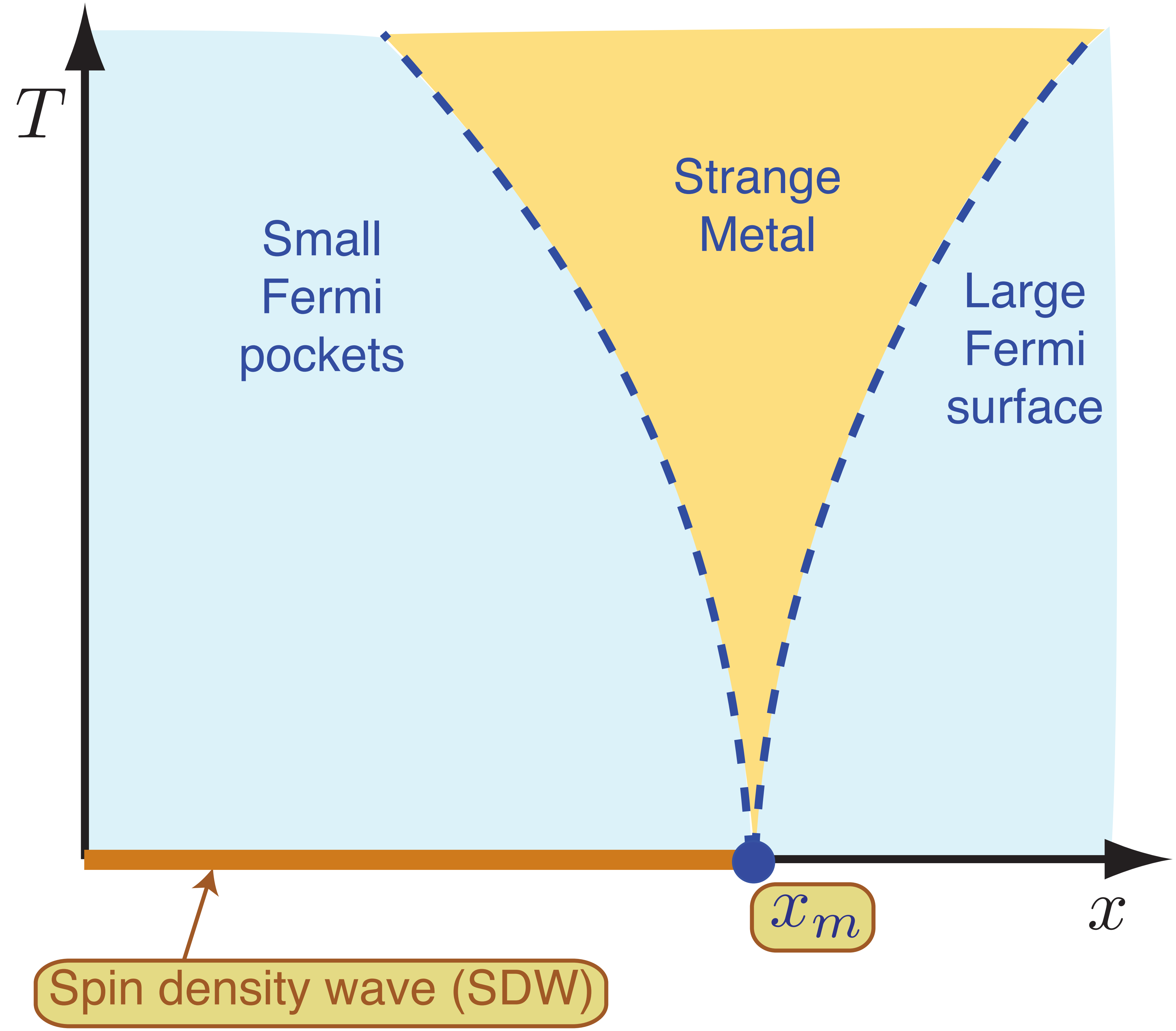
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



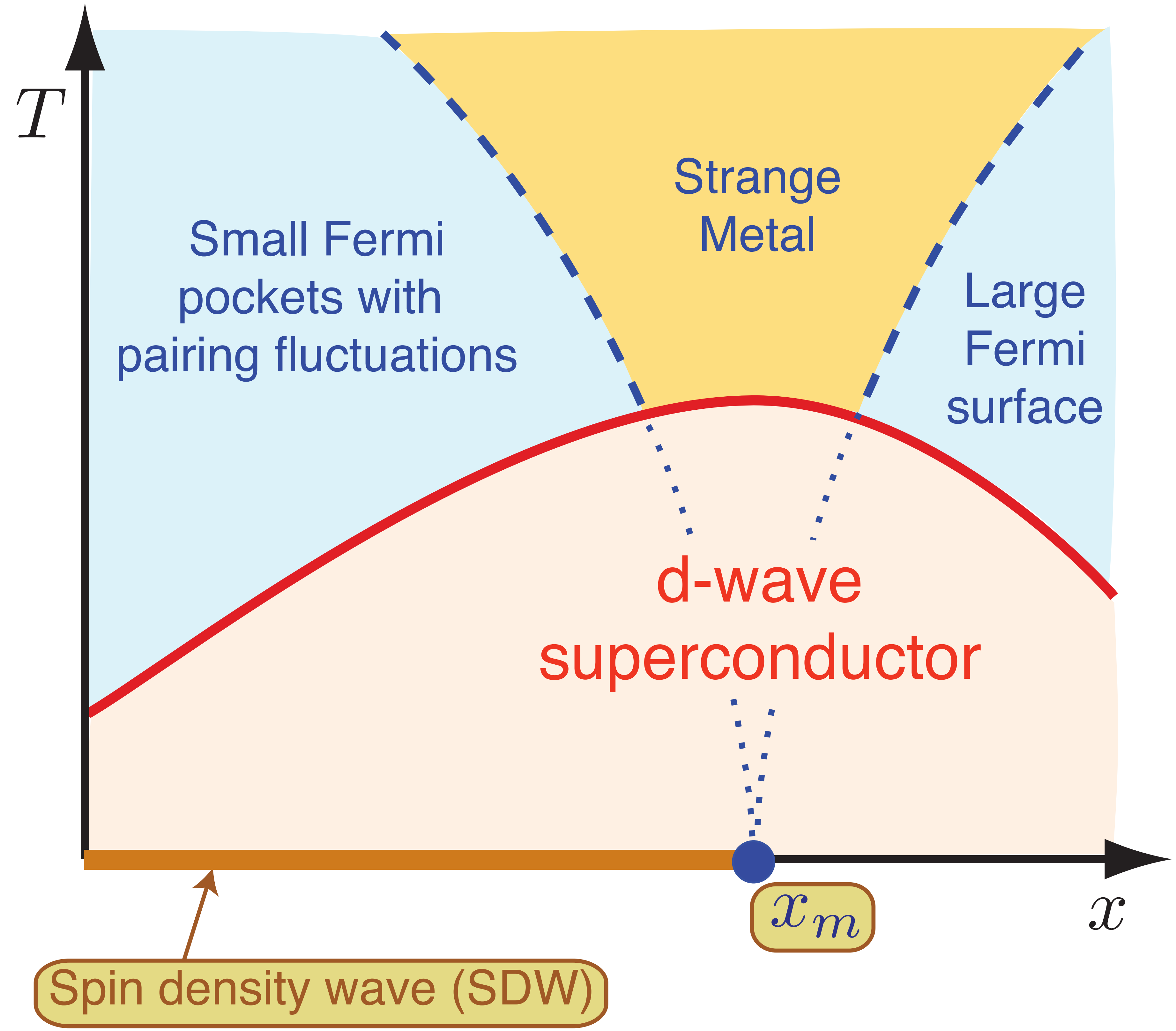
Spin and charge density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Theory of quantum criticality in the cuprates



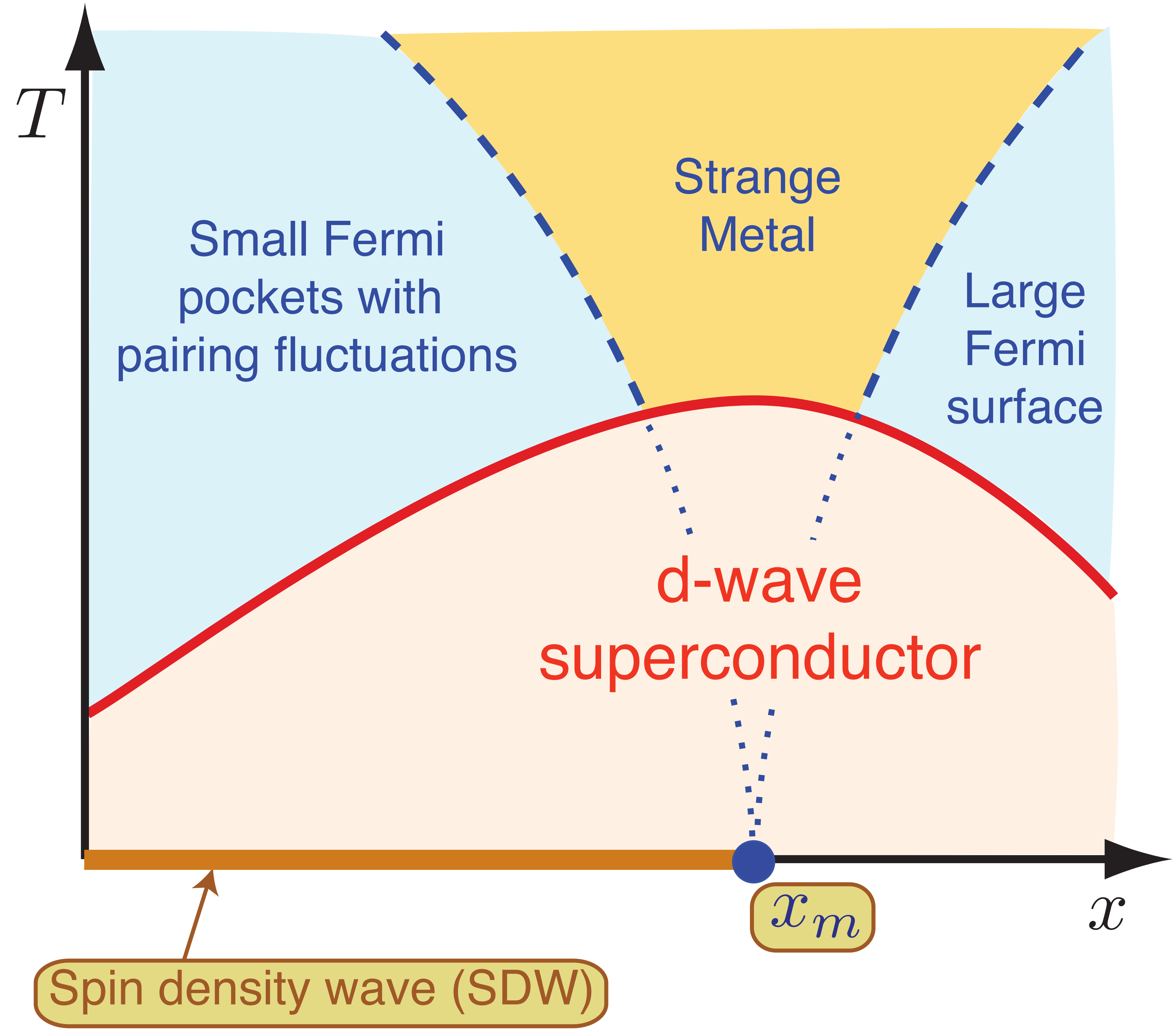
Underlying SDW ordering quantum critical point in metal at $x = x_m$

Theory of quantum criticality in the cuprates



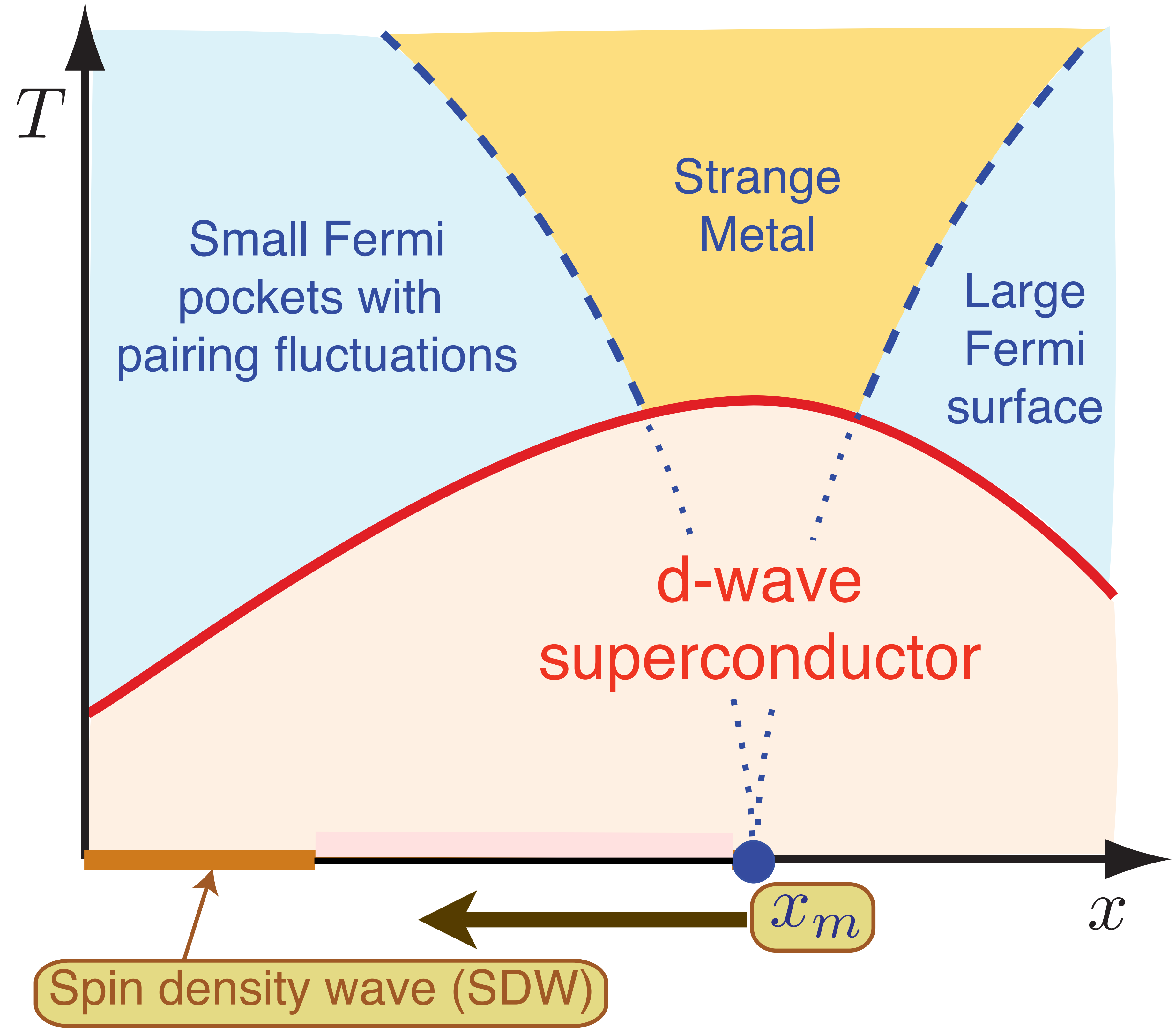
Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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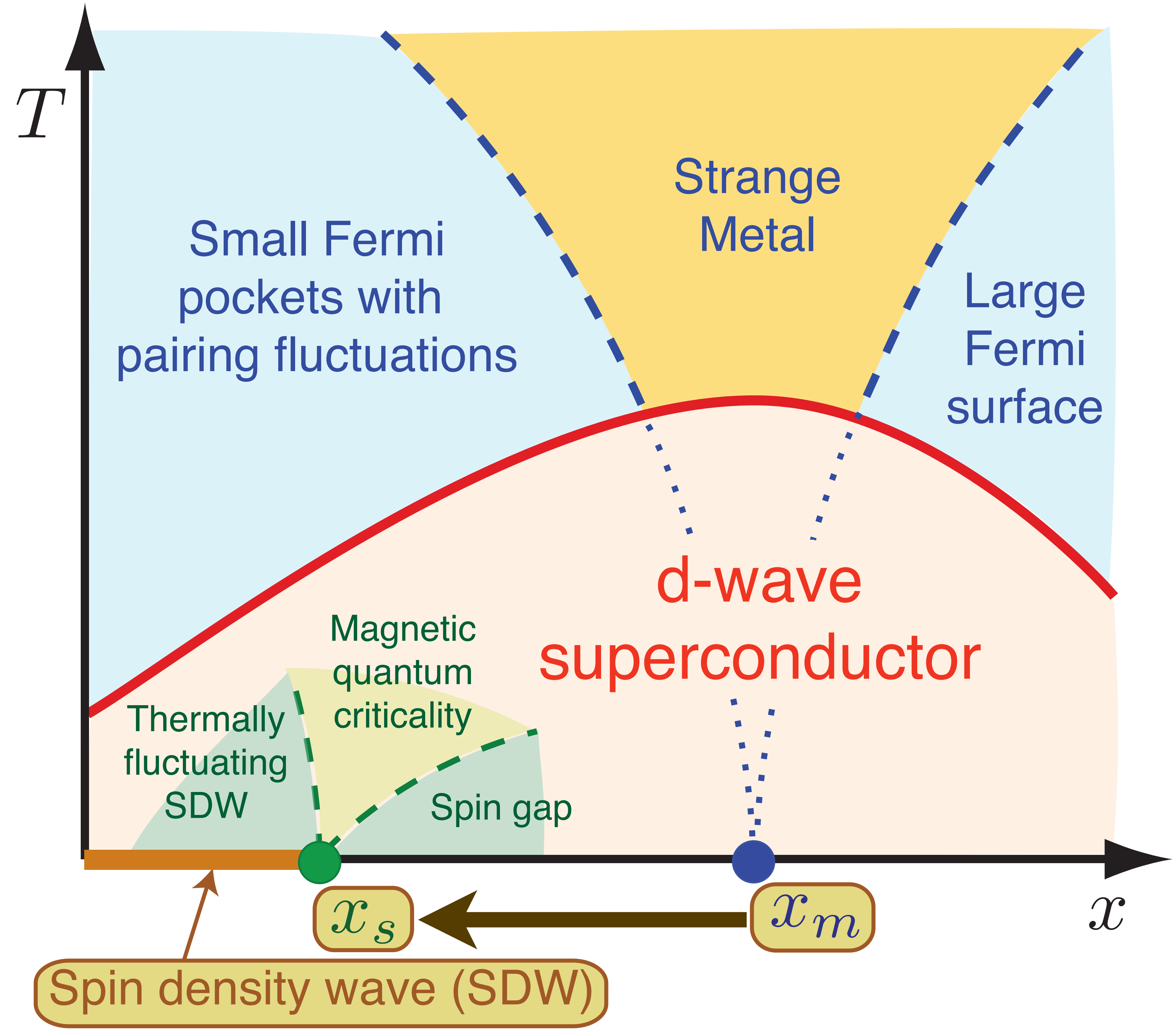

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Outline

Review:

1. Phenomenological quantum theory of competition between superconductivity and SDW order

2. Fermi surfaces in hole-doped cuprates

3. Superconductivity by SDW fluctuation exchange

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3. Superconductivity by SDW fluctuation exchange

New Results:

4. Superconductivity of fluctuating Fermi pockets in the underdoped cuprates

5. Quantum theory of competition between superconductivity and SDW order

Outline

Review:

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3. Superconductivity by SDW fluctuation exchange

1. Competition between SDW order and superconductivity

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2 r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{r}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 + \kappa\, \vec{\varphi}^2 \left|\psi\right|^2 \right] + \int d^2 r \left[\left|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi\right|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
see also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004)

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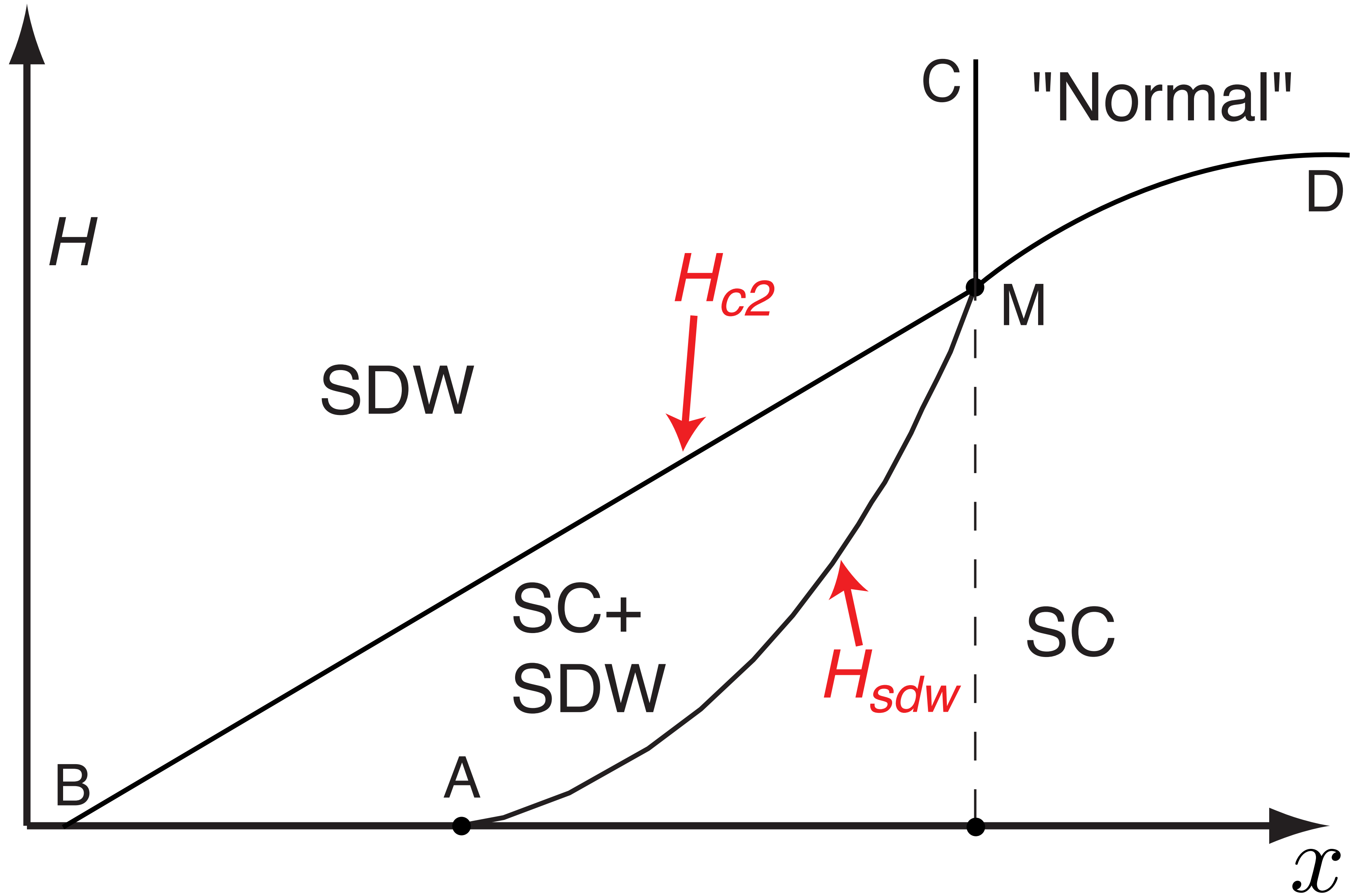

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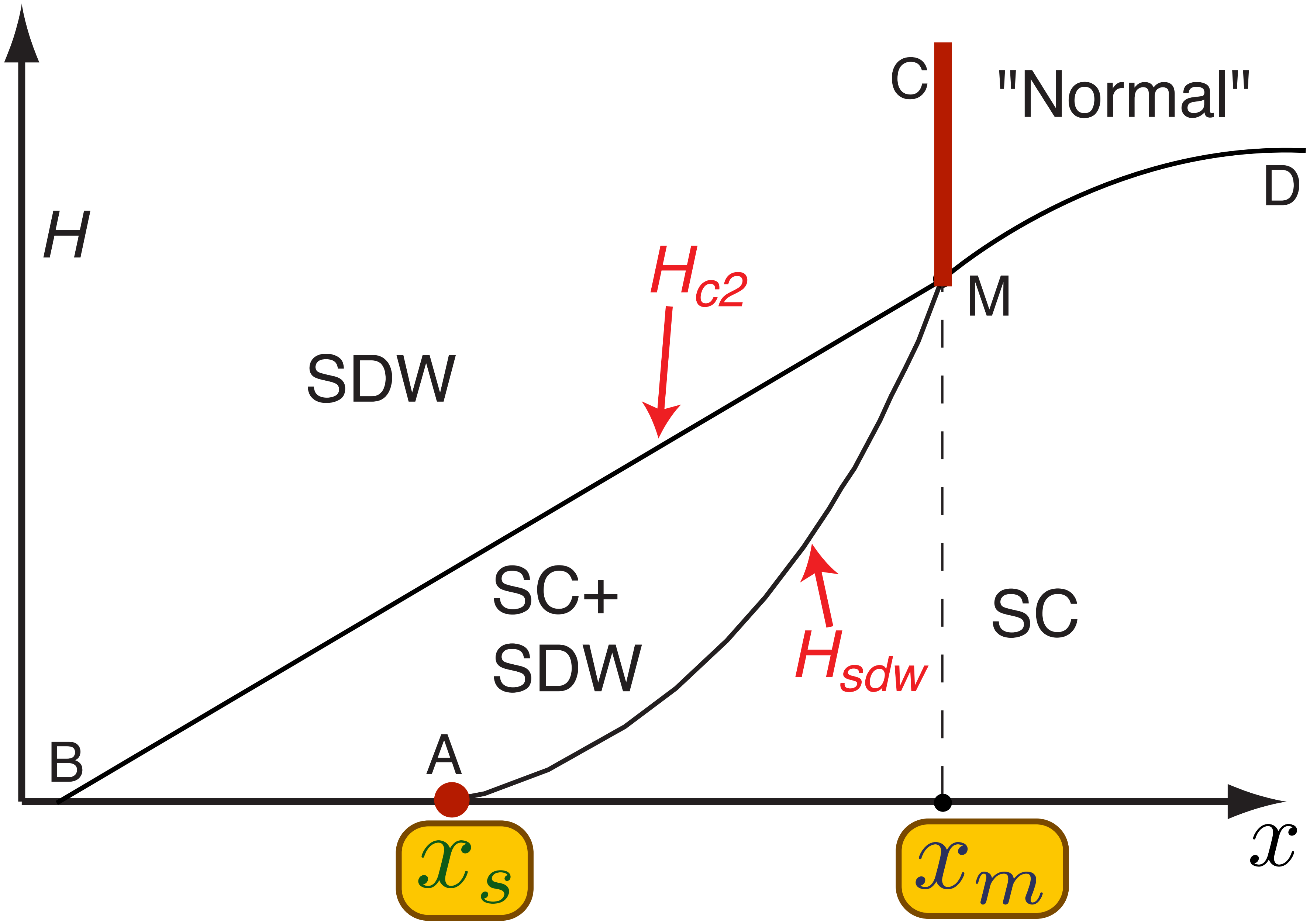

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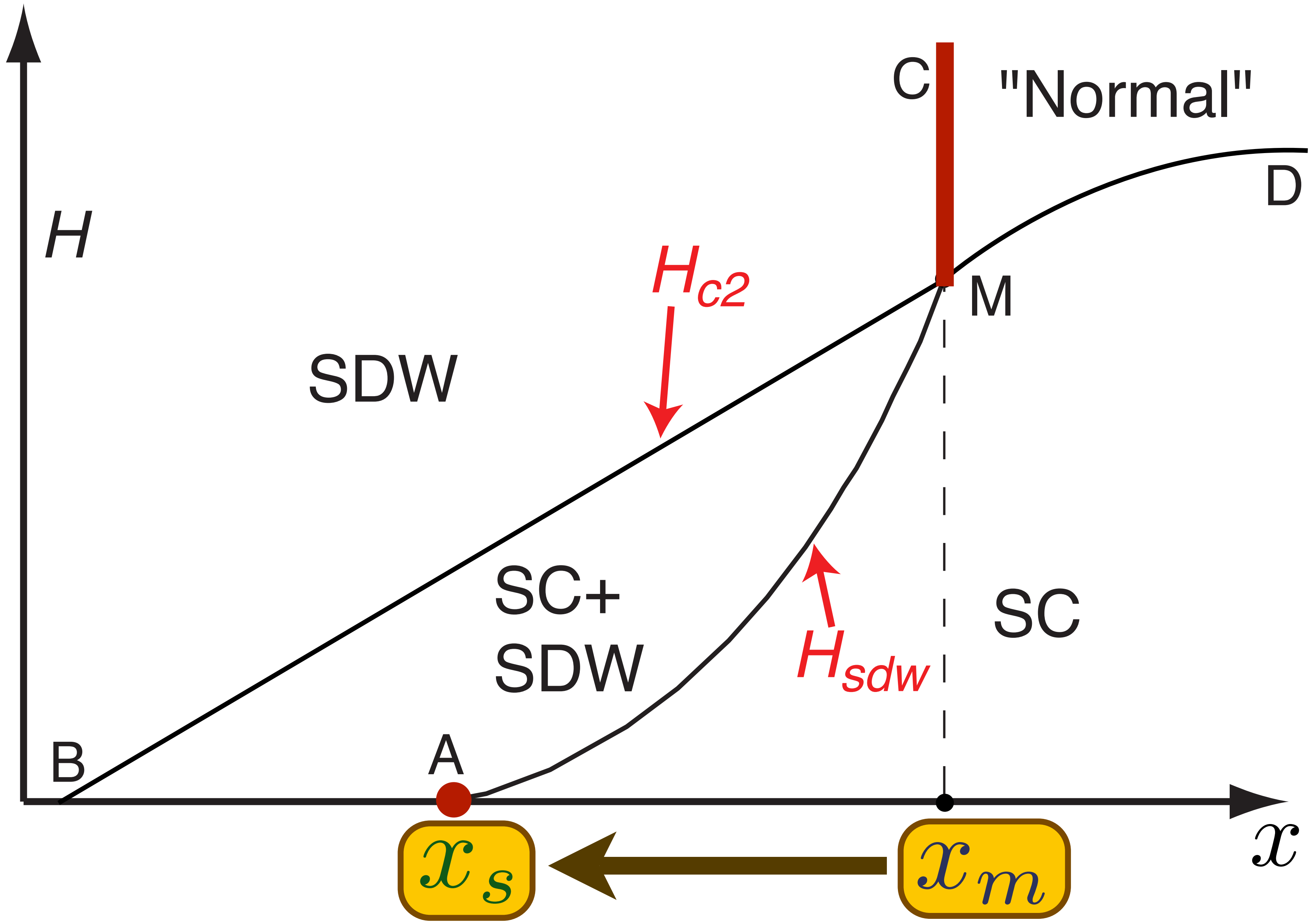

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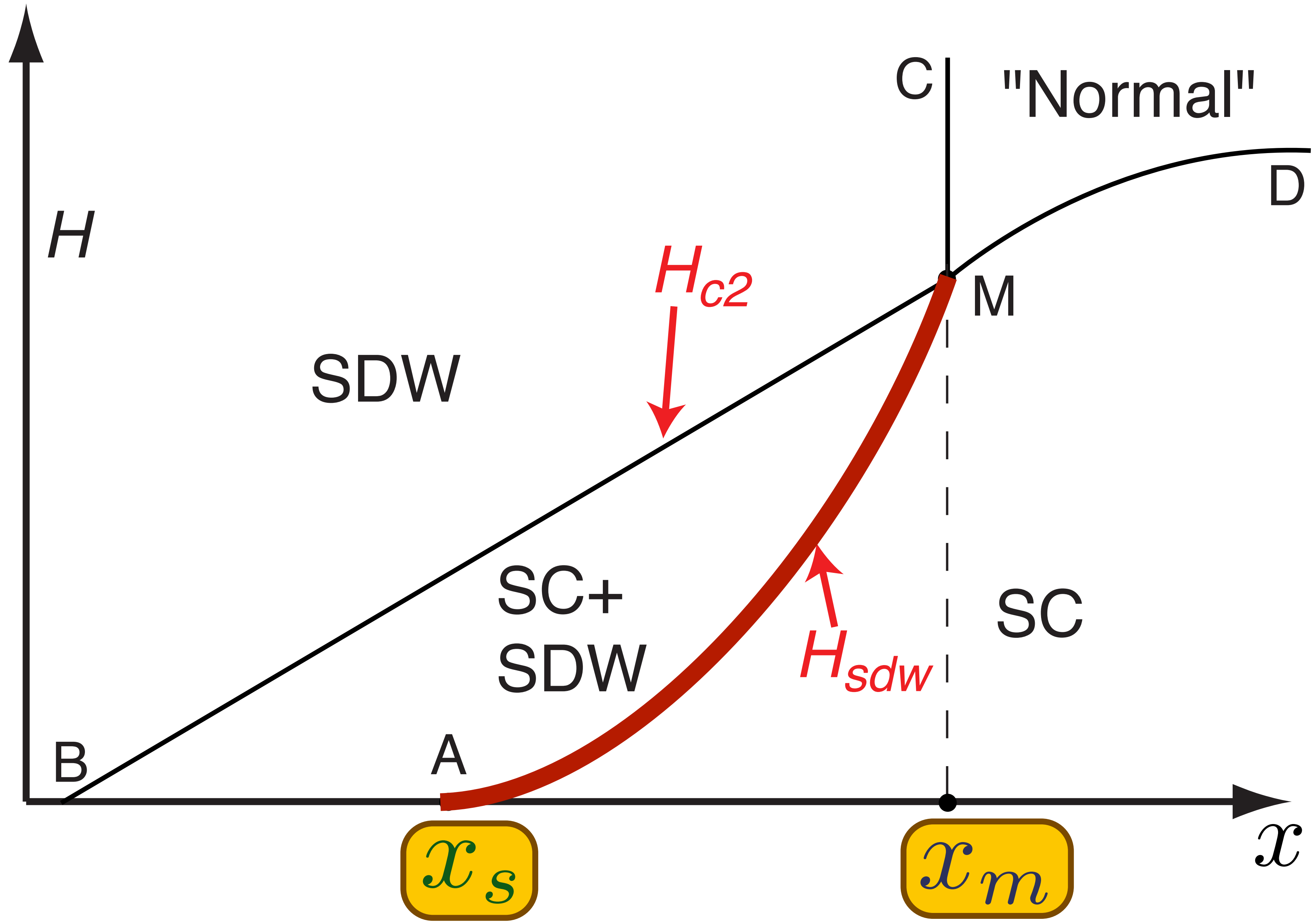
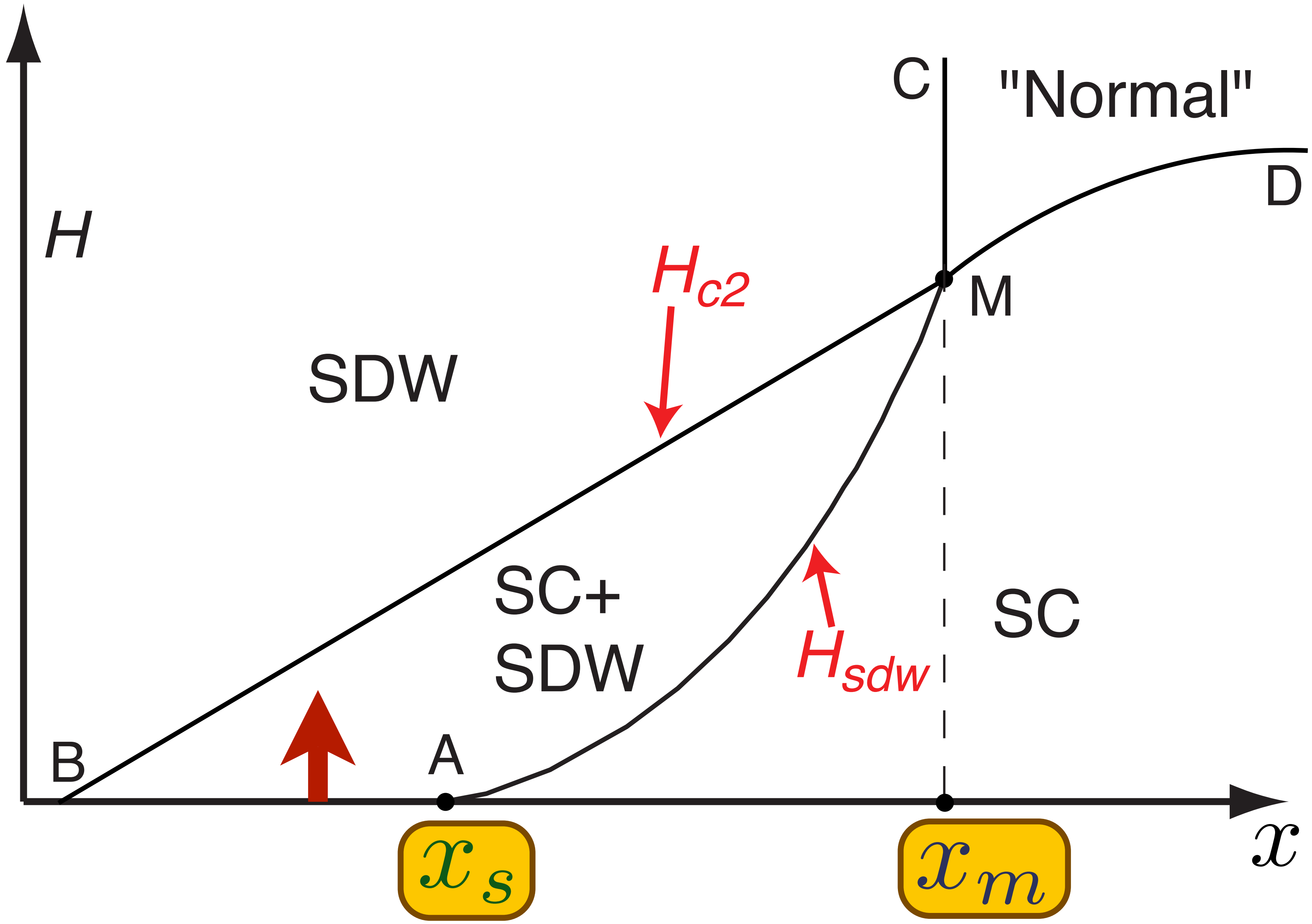


- For doping with $x_s < x < x_m$, SDW order appears at a quantum phase transition at $H = H_{sdw} > 0$.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



Neutron scattering on $La_{1.9}Sr_{0.1}CuO_4$
B. Lake *et al.*, *Nature* **415**, 299 (2002)

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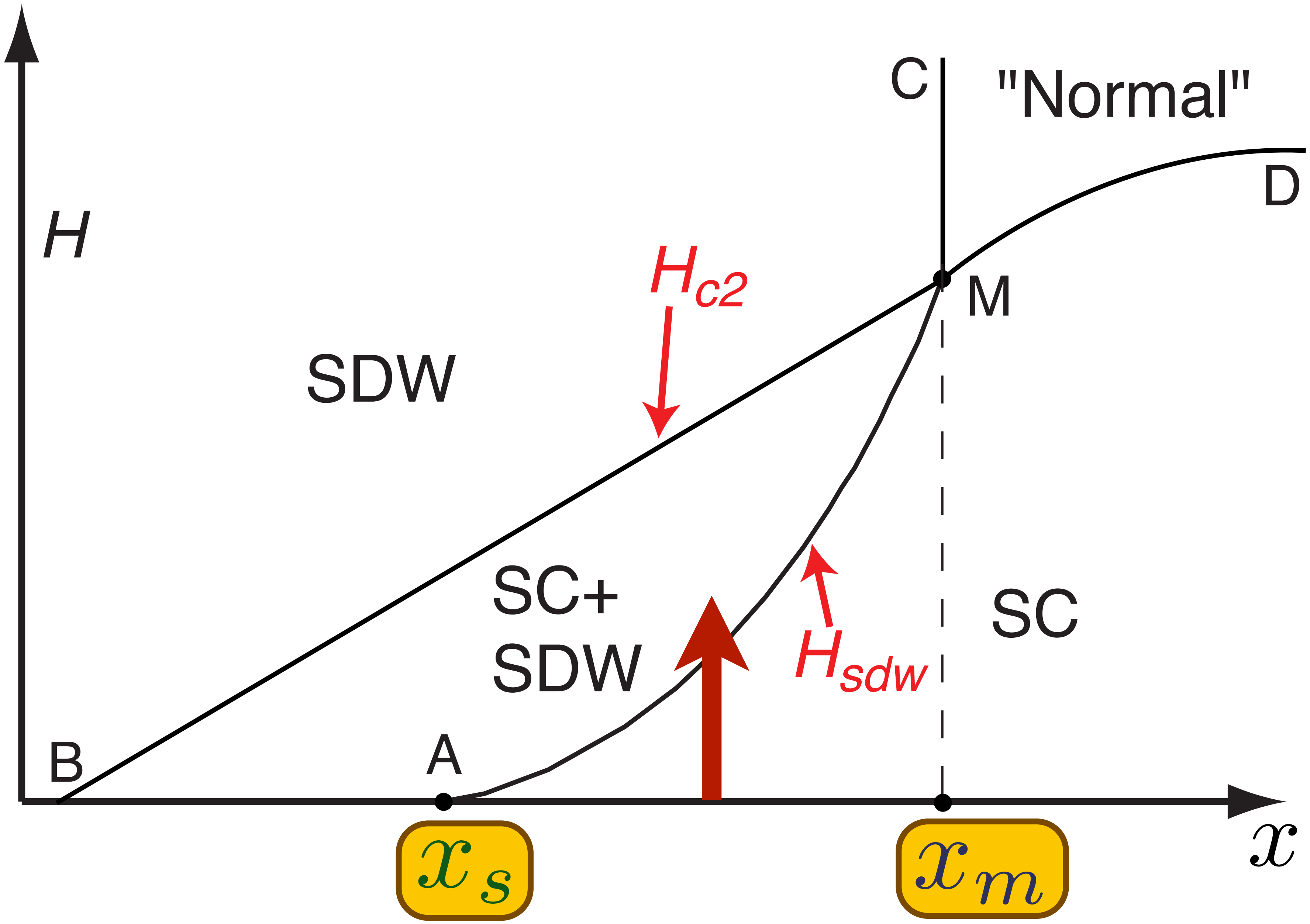
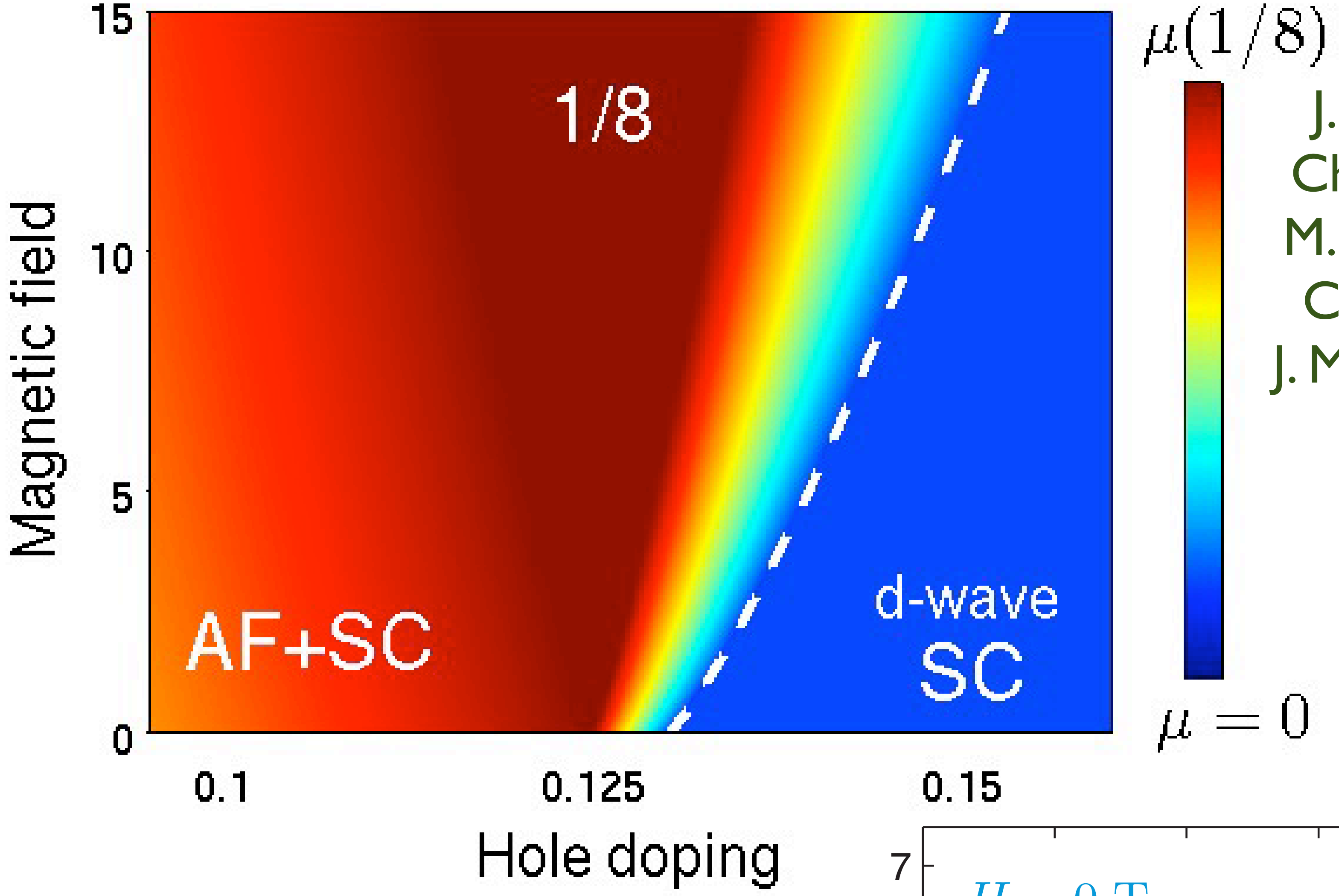
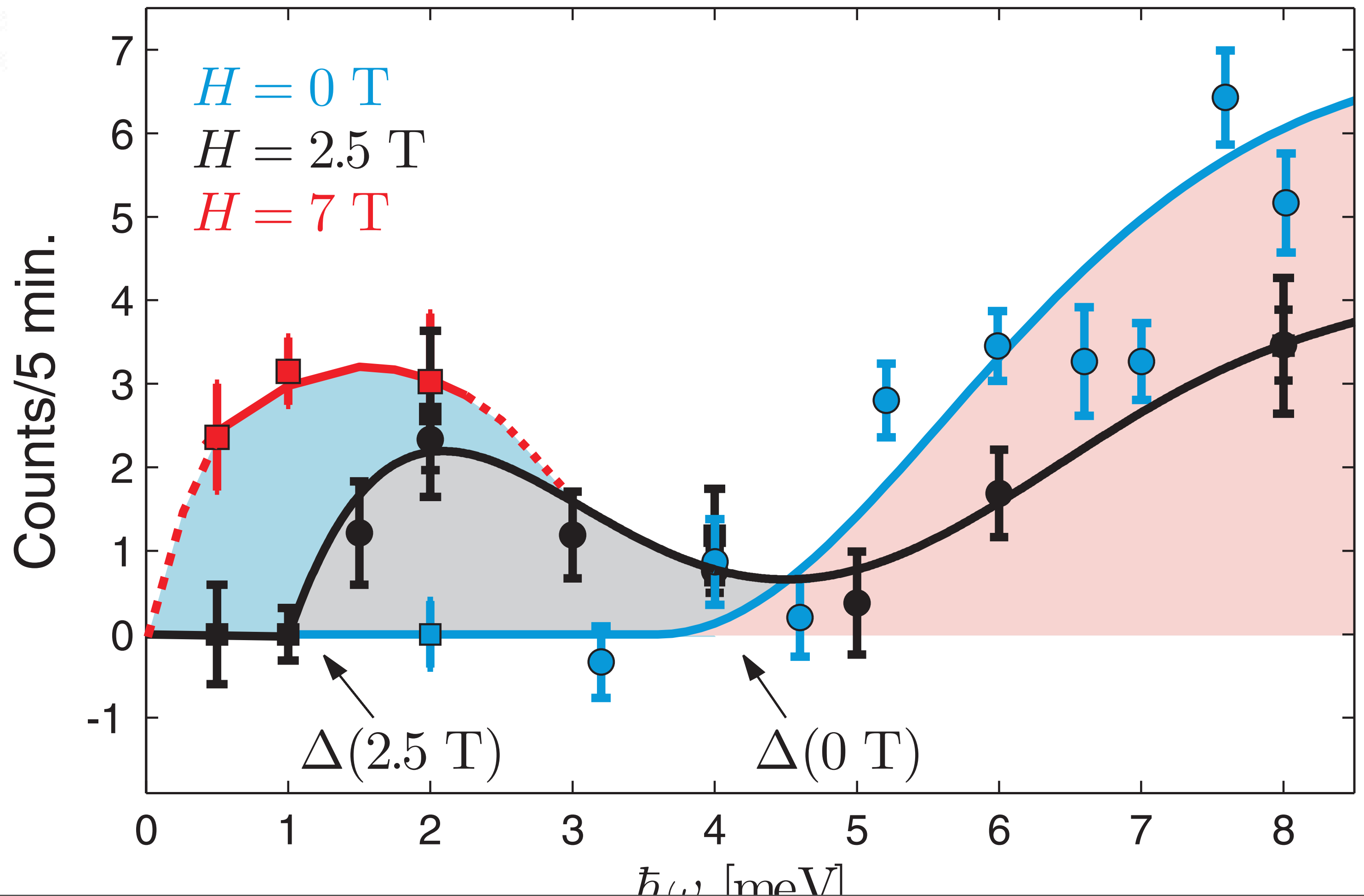


Neutron scattering on $La_{1.855}Sr_{0.145}CuO_4$
J. Chang *et al.*, *Phys. Rev. Lett.* 102, 177006 (2009).

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

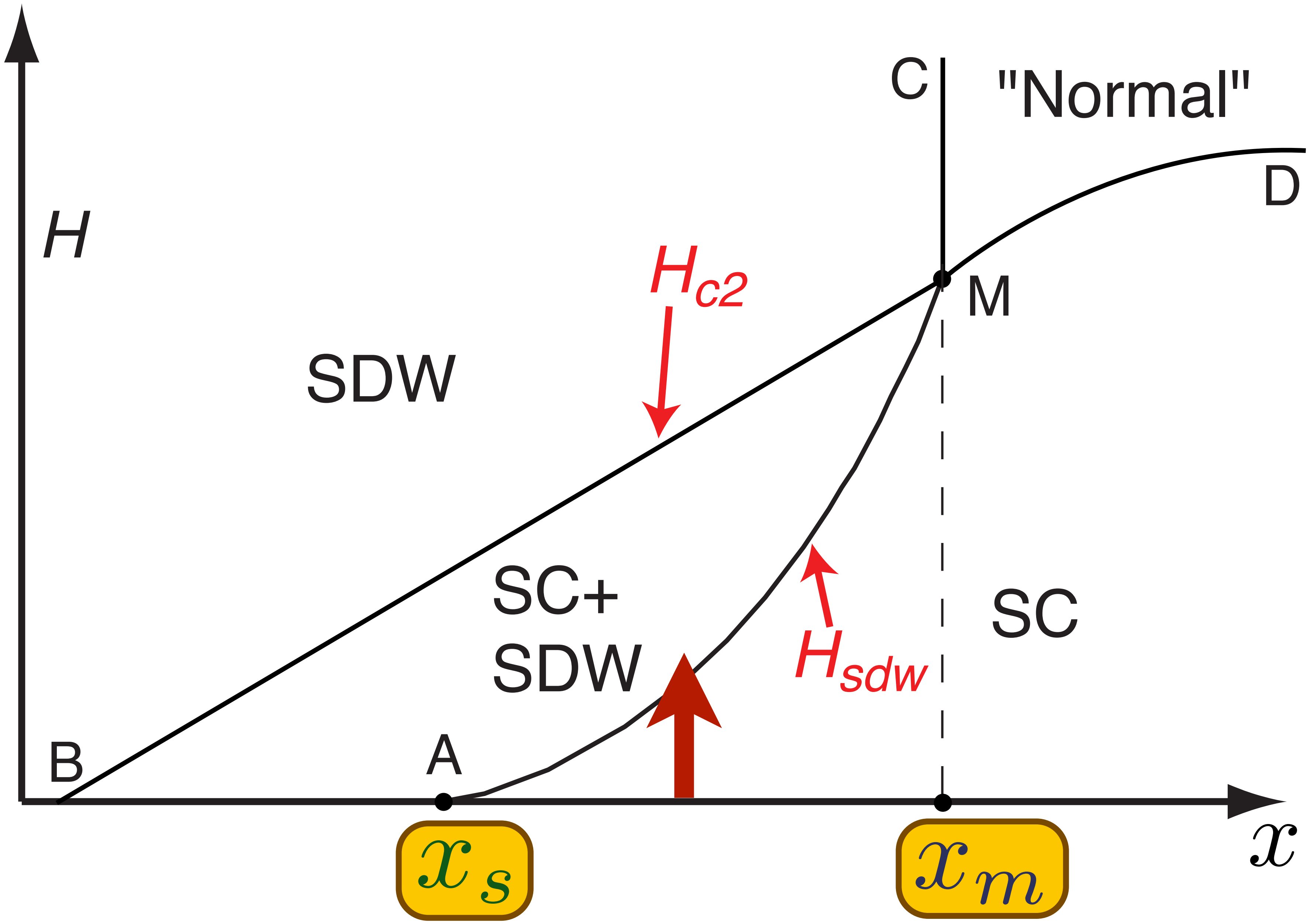


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).



J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



Neutron scattering on $YBa_2Cu_3O_{6.45}$
D. Haug *et al.*, arXiv:0902.3335

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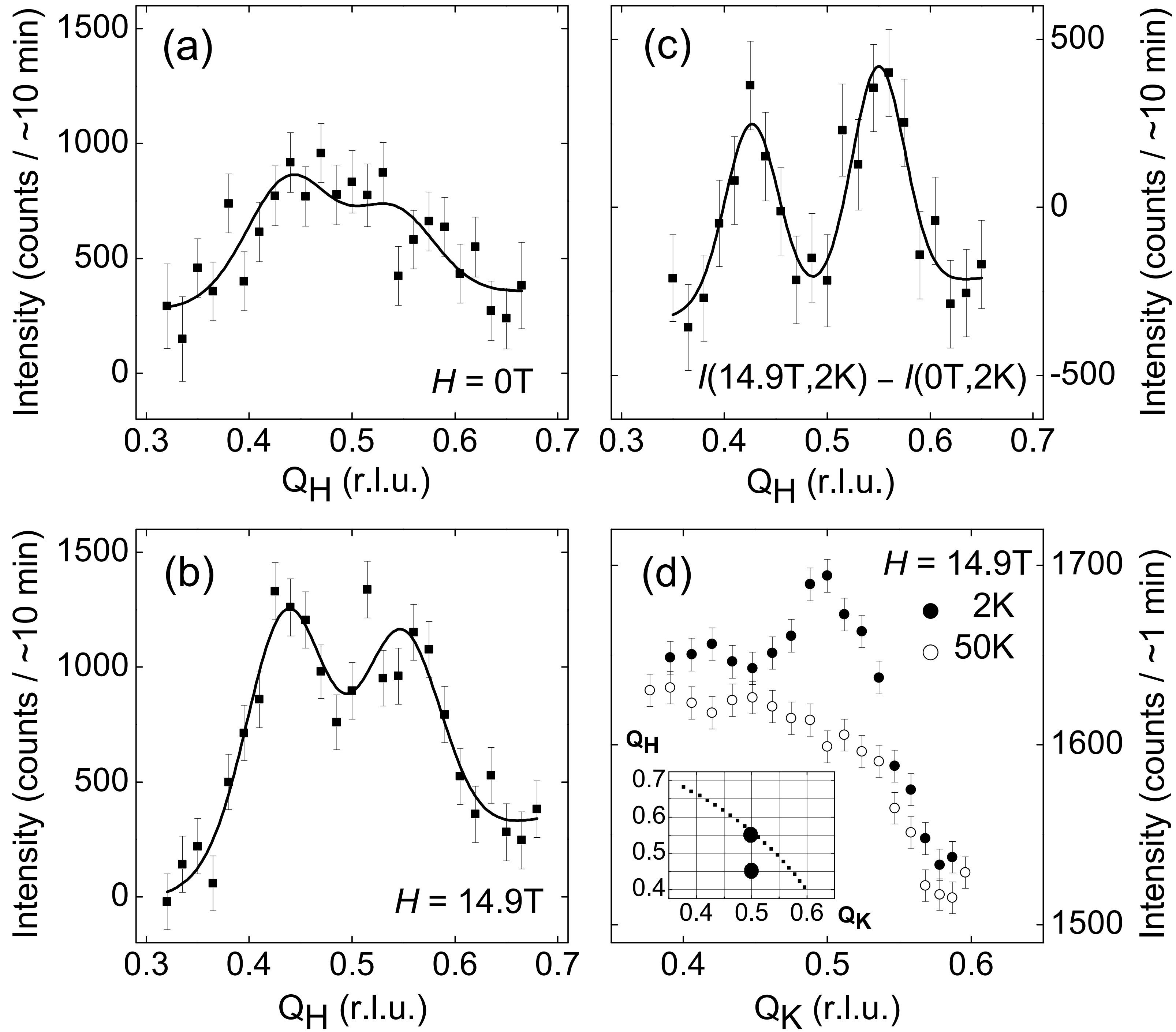
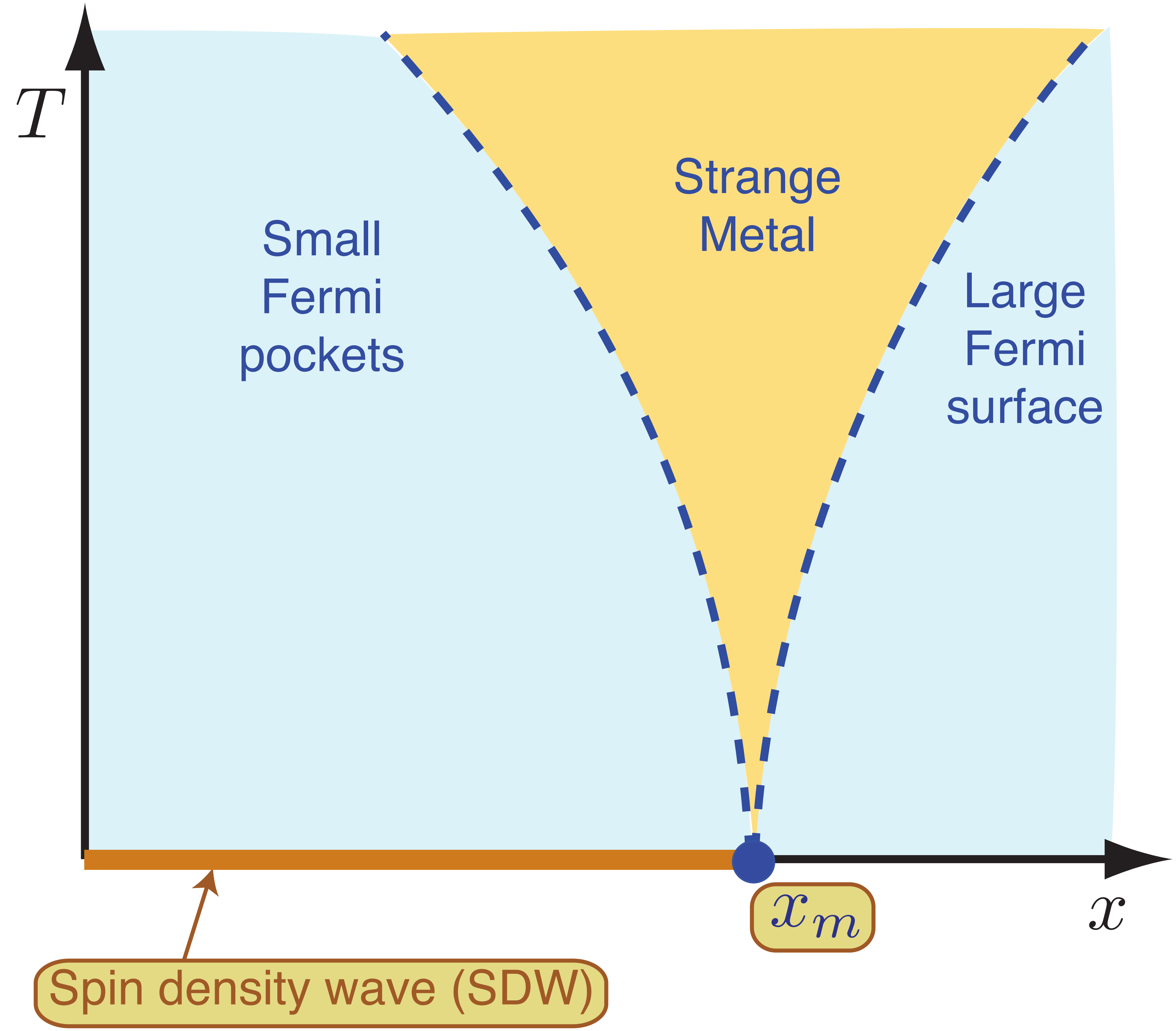


D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *arXiv:0902.3335*.

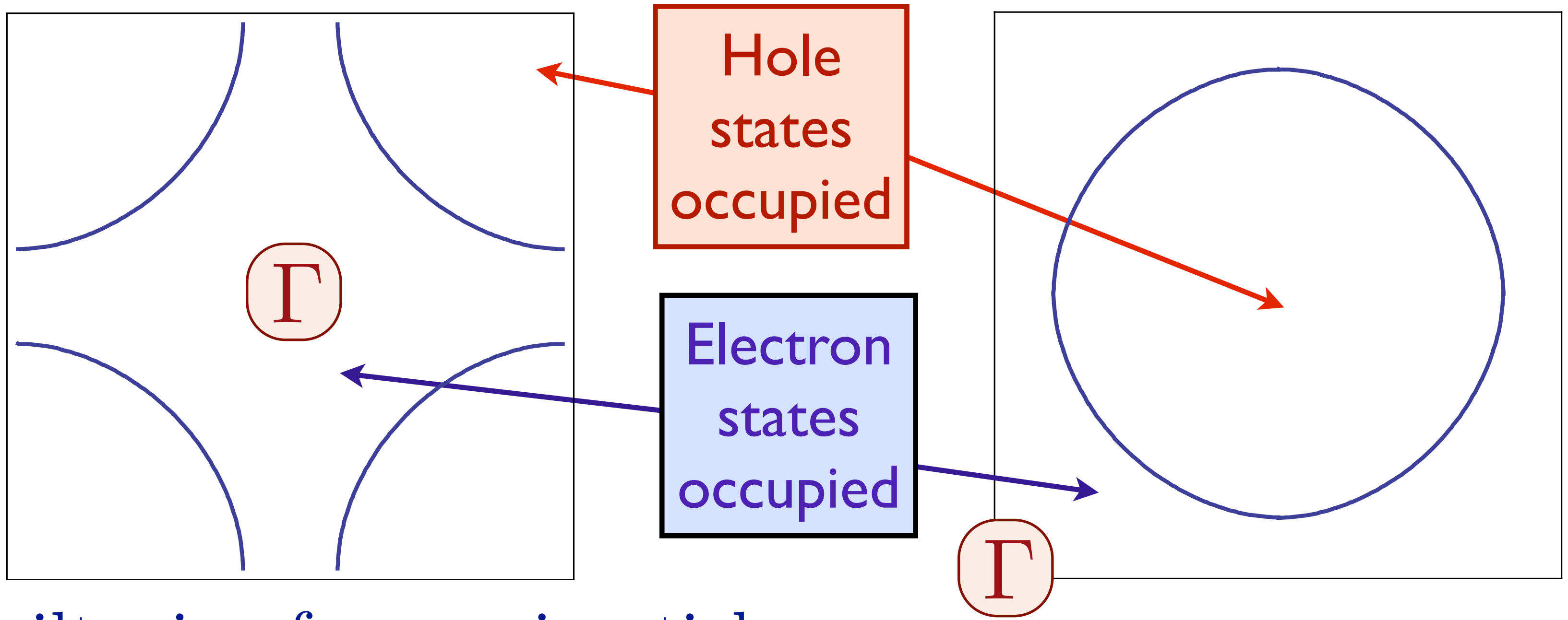
2. Fermi surfaces in hole-doped cuprates

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in metal at $x = x_m$

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-p) & \text{for hole-doping } p \\ 2\pi^2(1+x) & \text{for electron-doping } x \end{cases}$$

The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

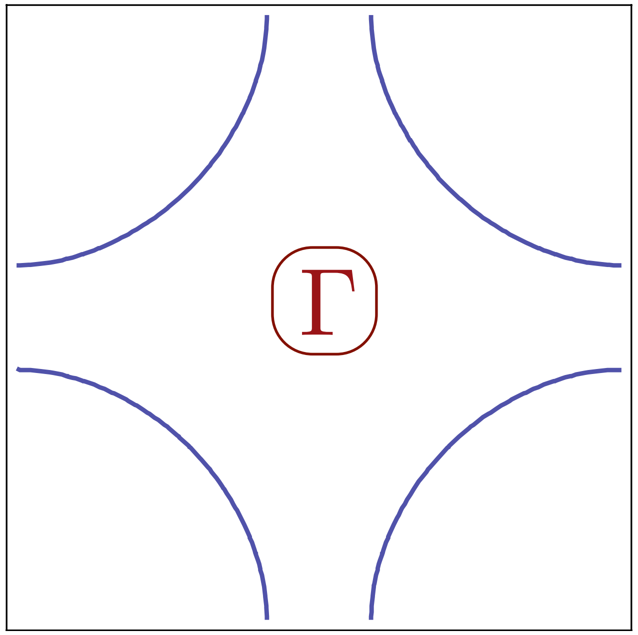
$$H_{\rm sdw} = -\vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^\dagger_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\rm sdw}$ for $\vec{\varphi} = (0, 0, \varphi)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

This leads to the Fermi surfaces shown in the following slides for electron and hole doping.

Spin density wave theory in hole-doped cuprates



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

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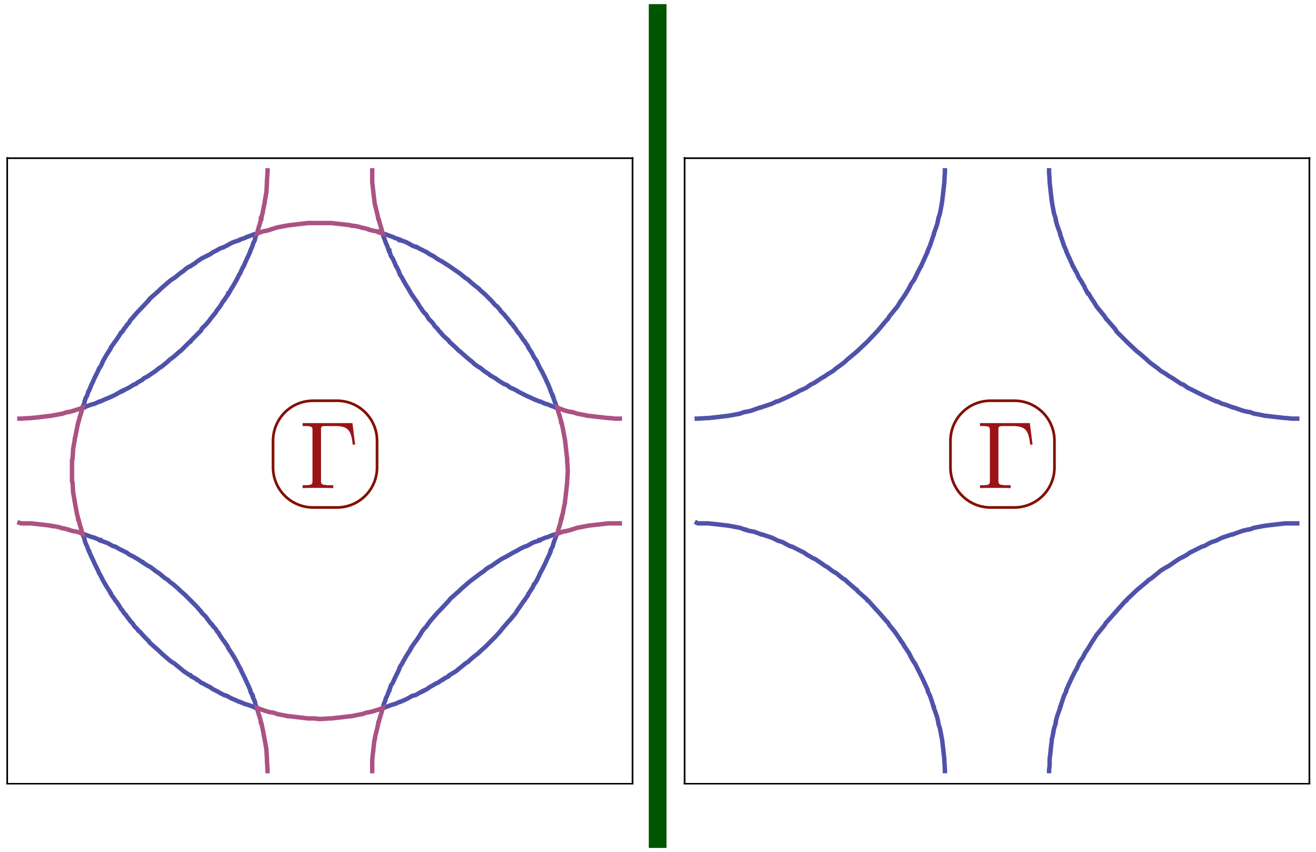

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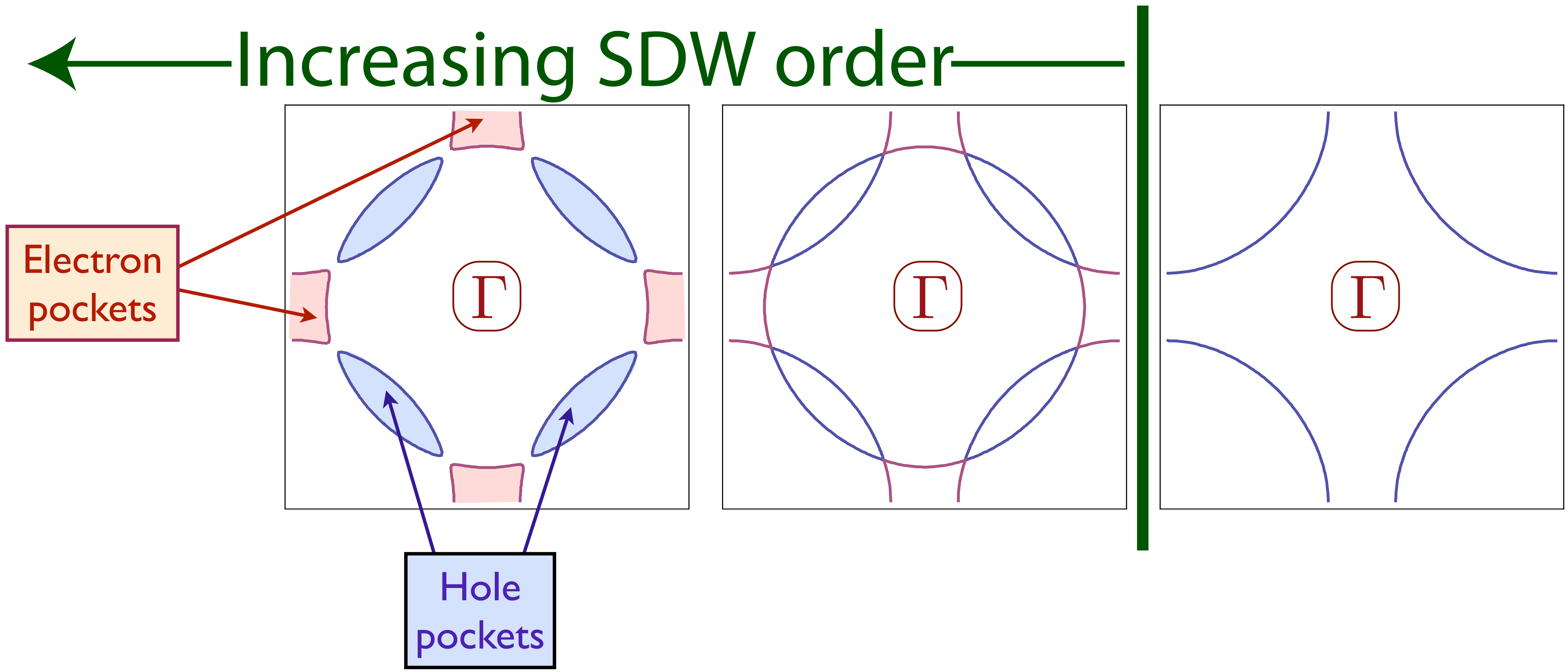

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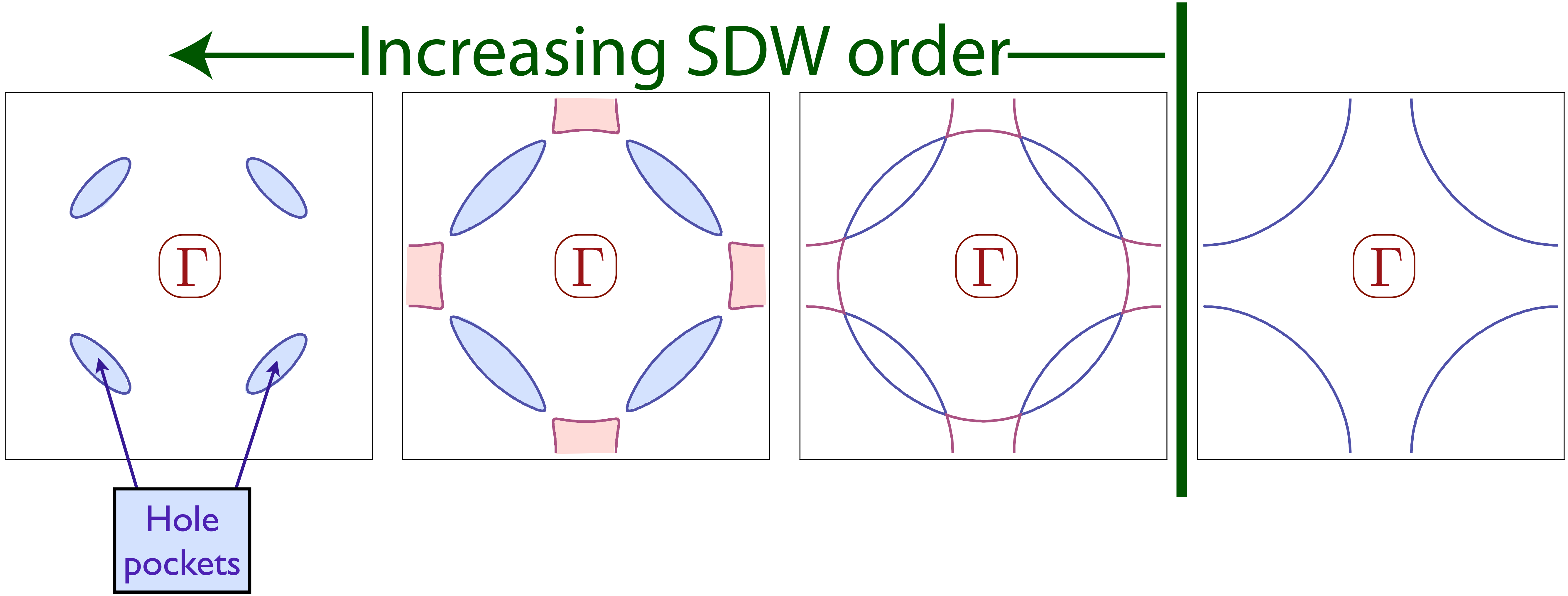

SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

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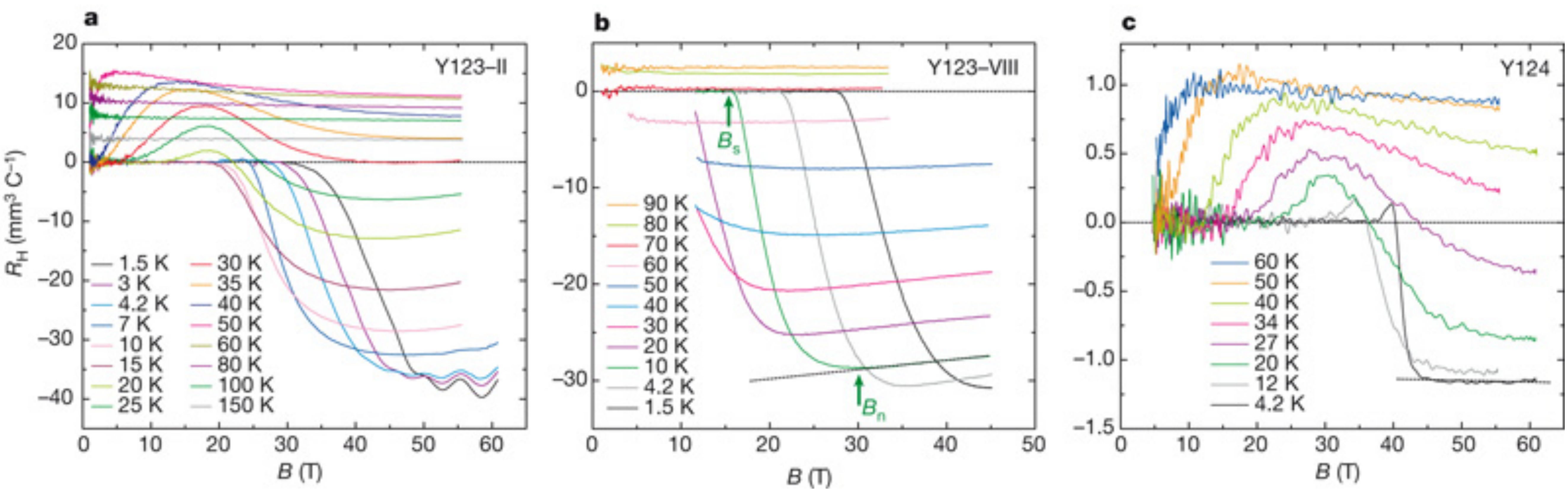
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

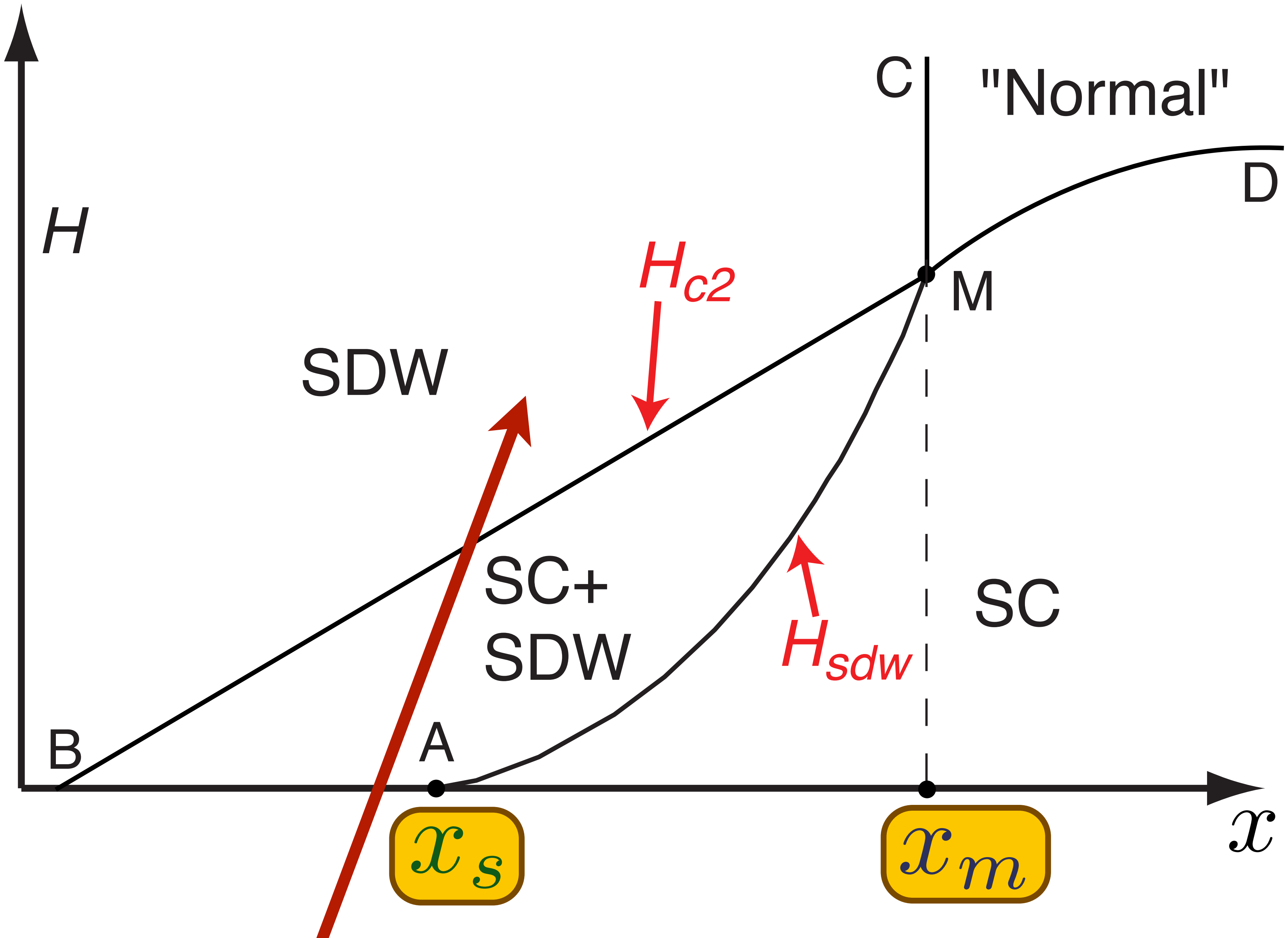
David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

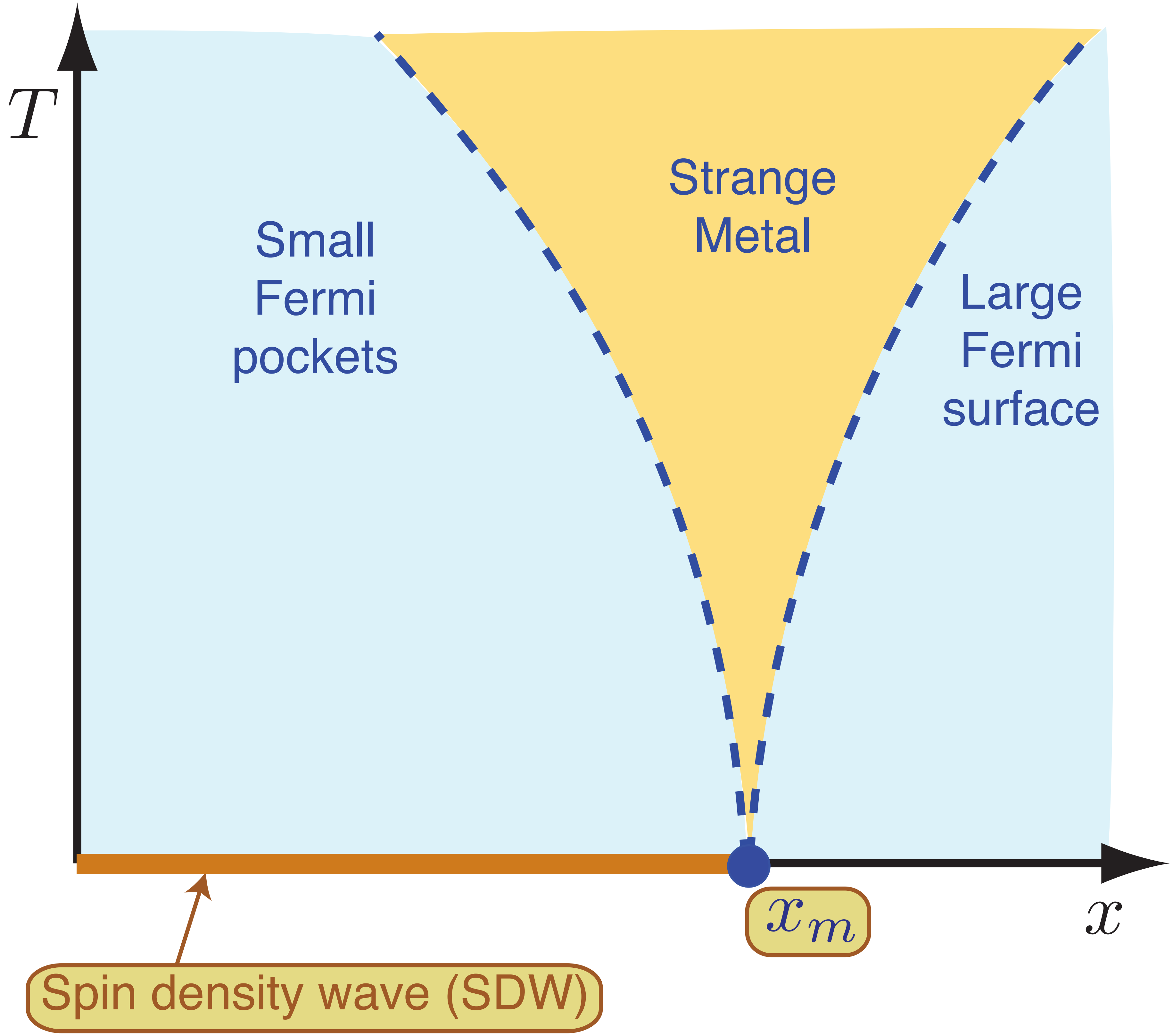


Quantum oscillations without Zeeman splitting

N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

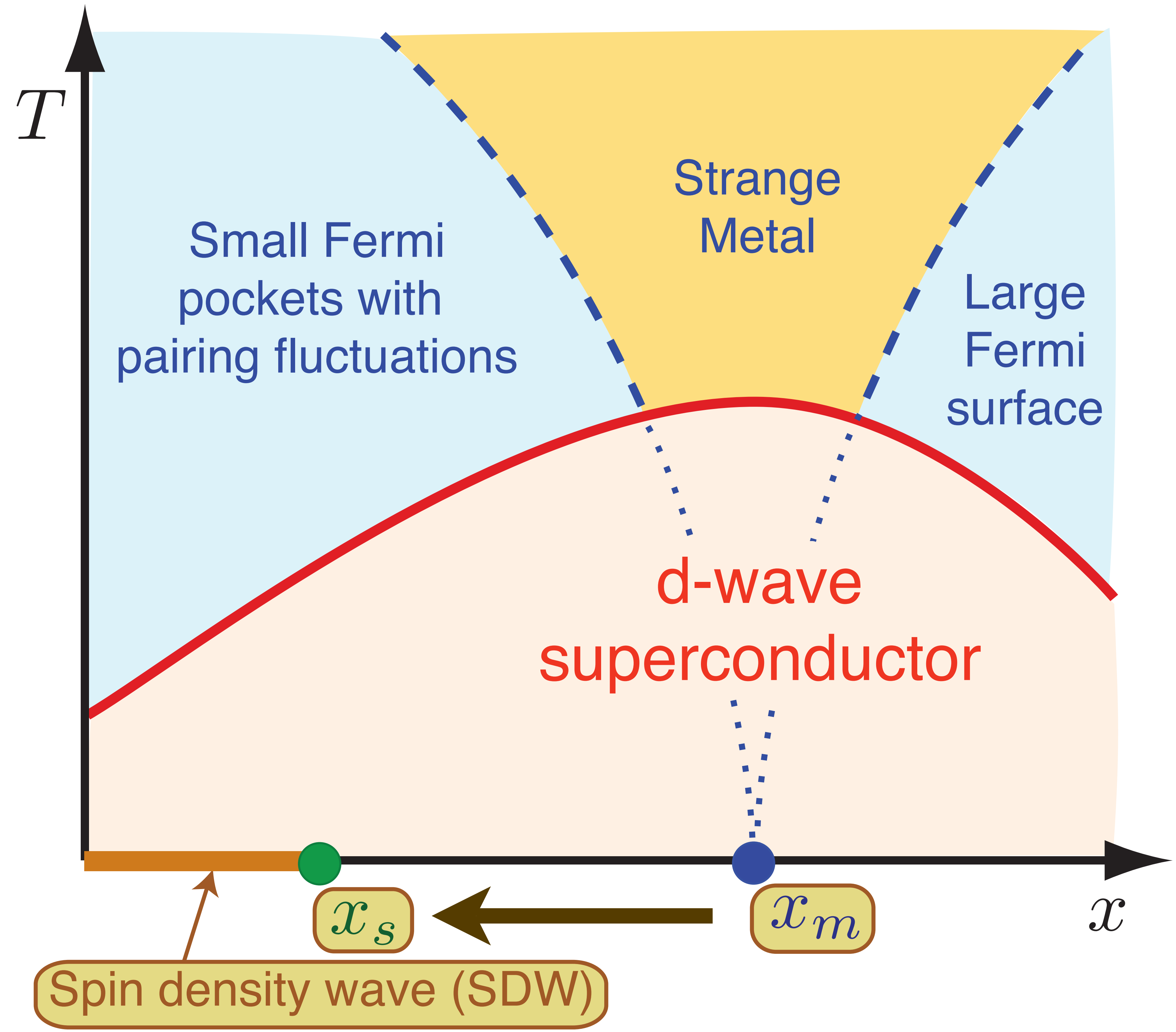
3. Superconductivity by SDW fluctuation exchange

Theory of quantum criticality in the cuprates



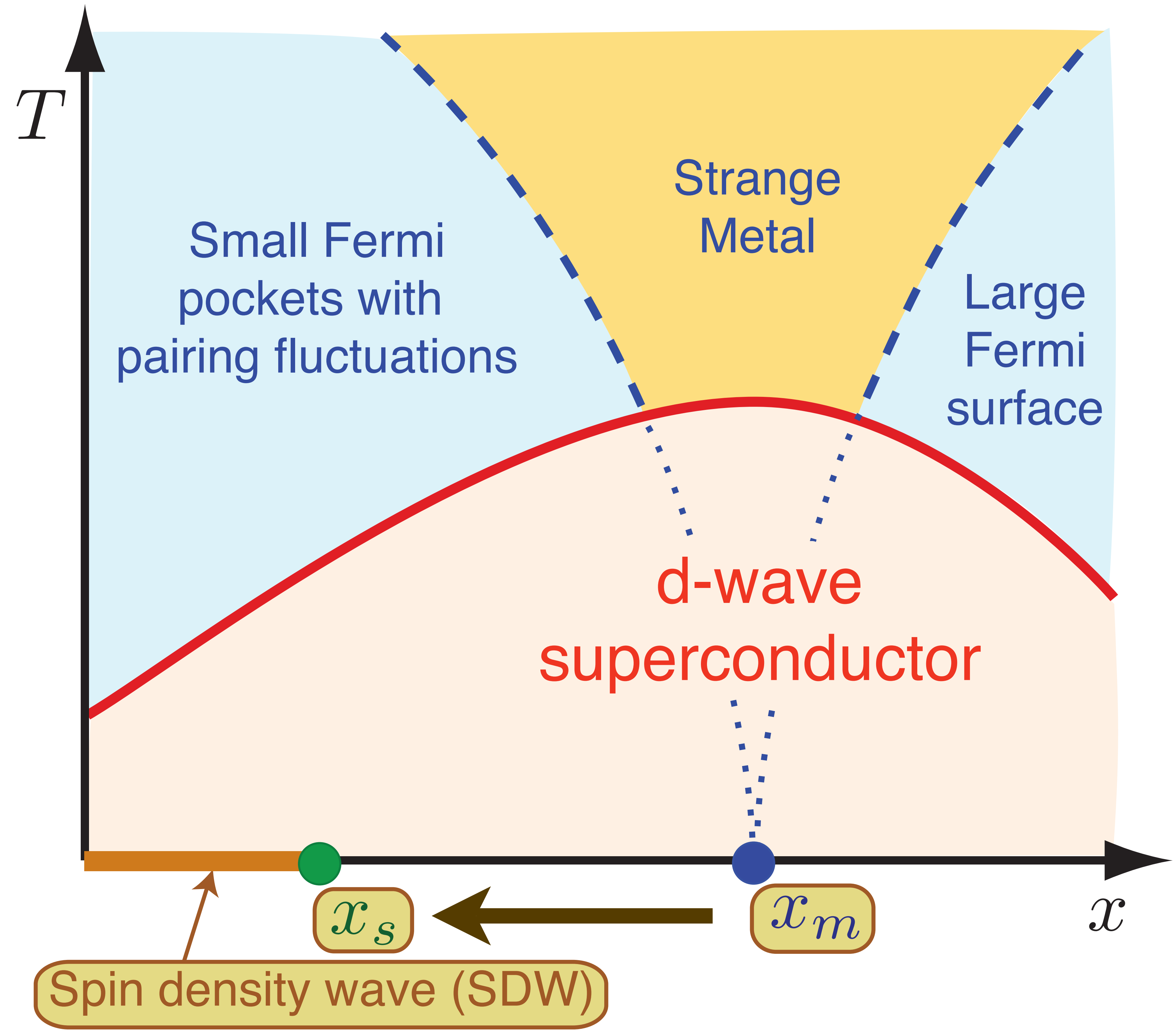
Underlying SDW ordering quantum critical point in metal at $x = x_m$

Theory of quantum criticality in the cuprates



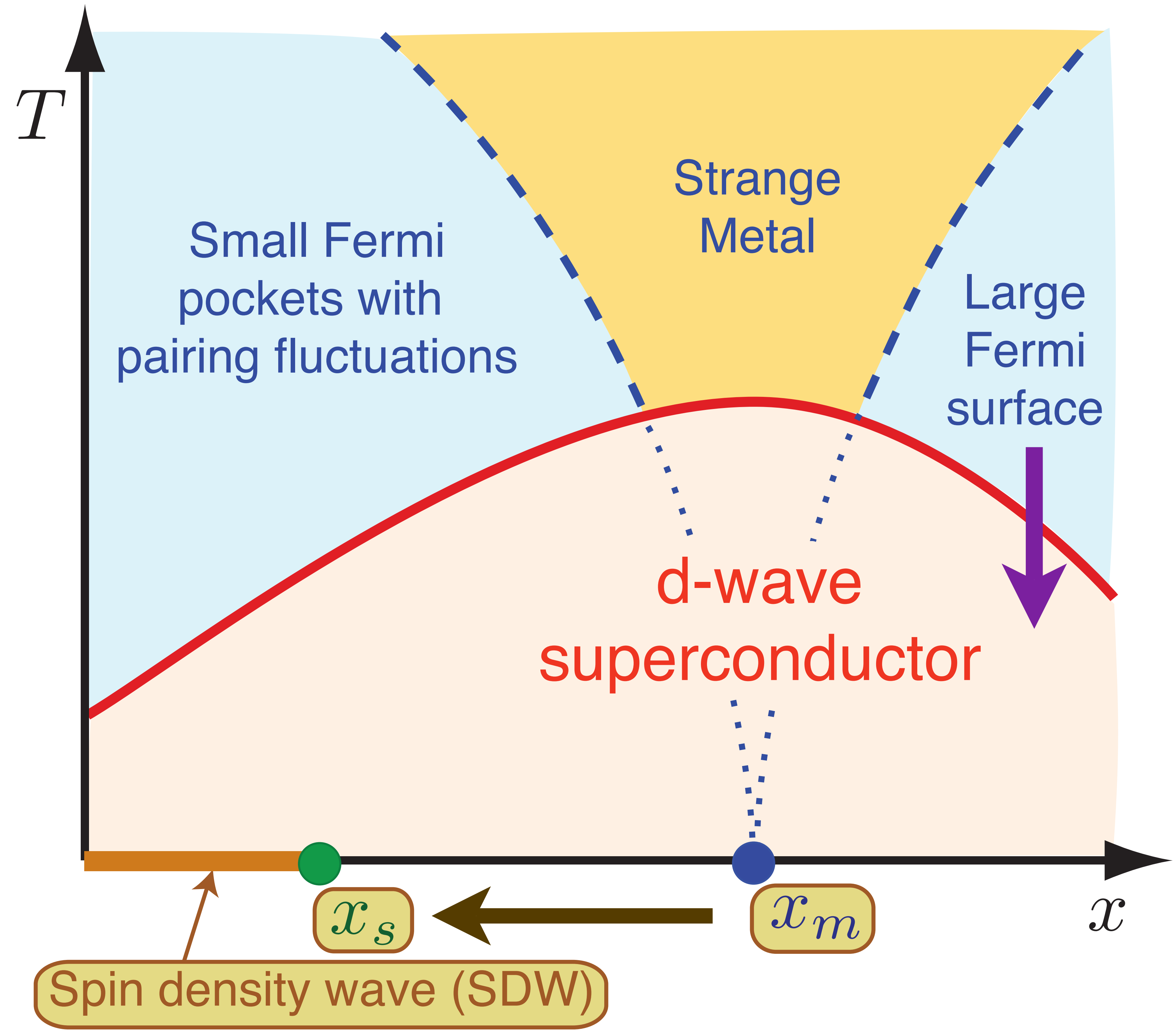
Onset of d-wave superconductivity hides the critical point at $x = x_m$ and moves it to $x = x_s < x_m$

Theory of quantum criticality in the cuprates



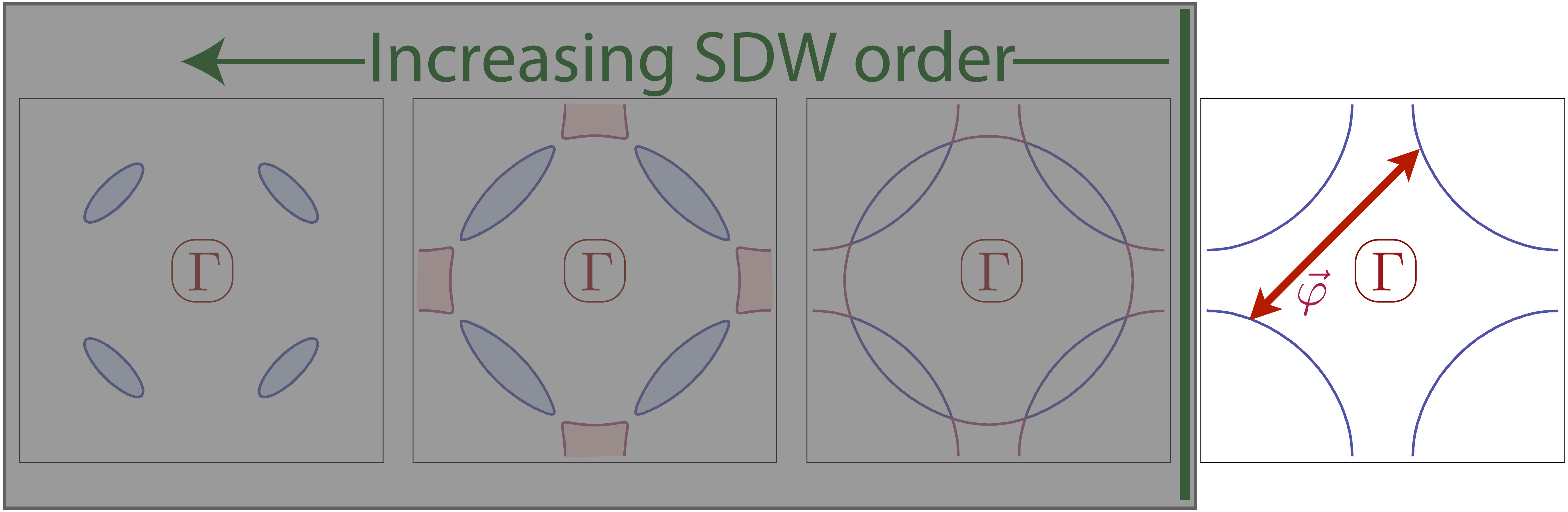
Theory of the onset of d-wave superconductivity from a large Fermi surface

Theory of quantum criticality in the cuprates



Theory of the onset of d-wave superconductivity from a large Fermi surface

Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

David Pines, Douglas Scalapino

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = -\sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

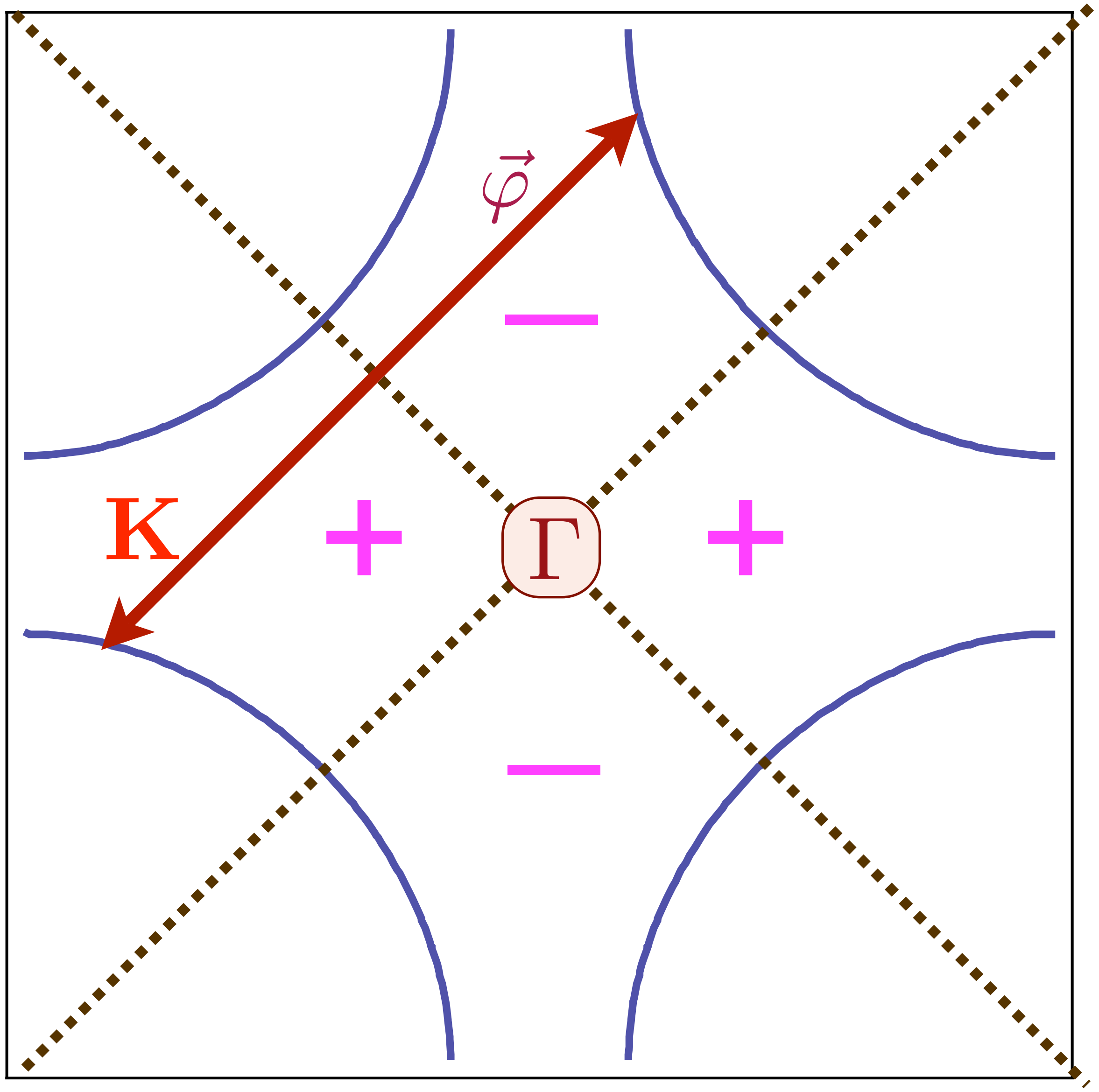
$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^{\dagger}_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q} - \mathbf{K})^2},$$

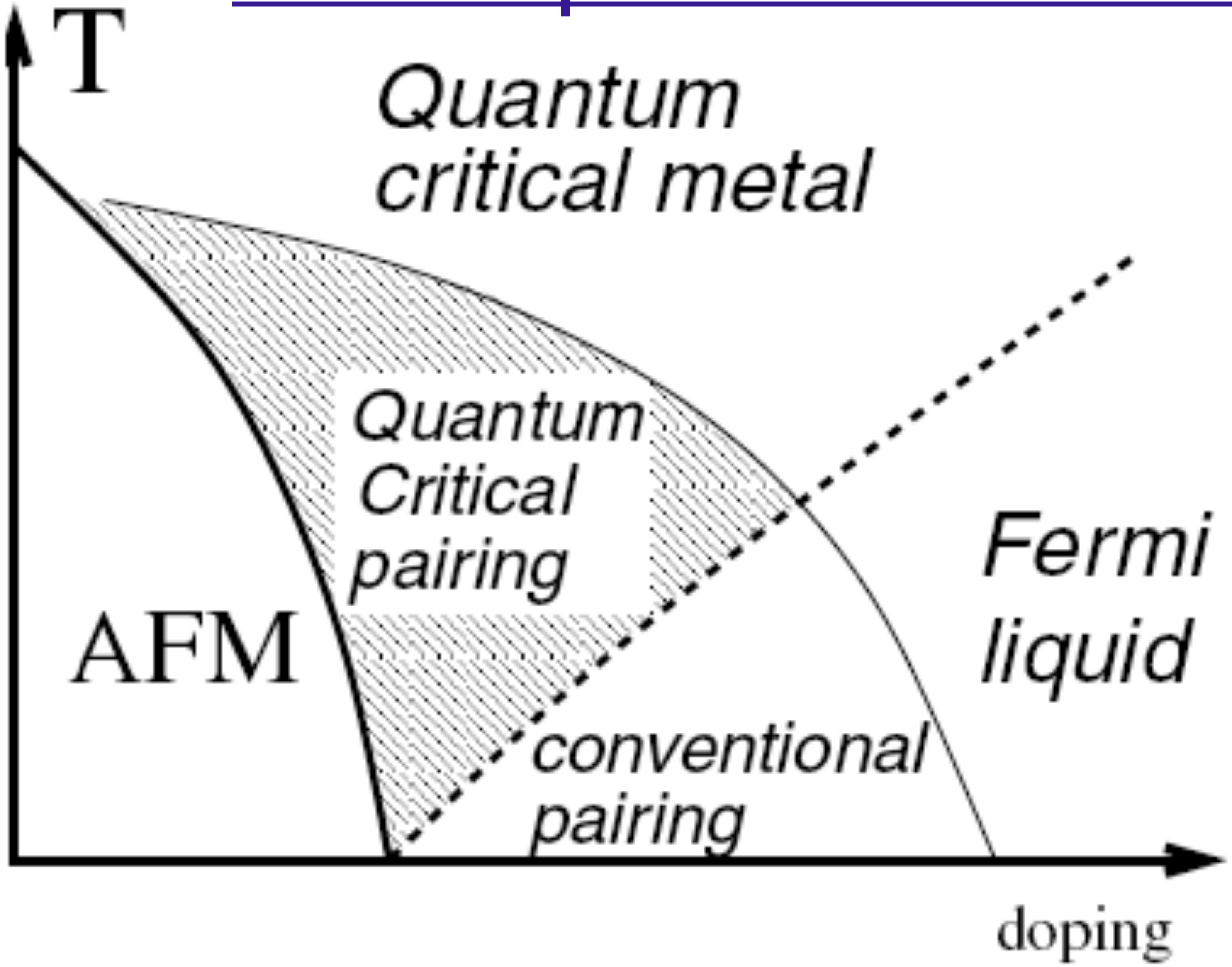
with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

d-wave pairing of the large Fermi surface



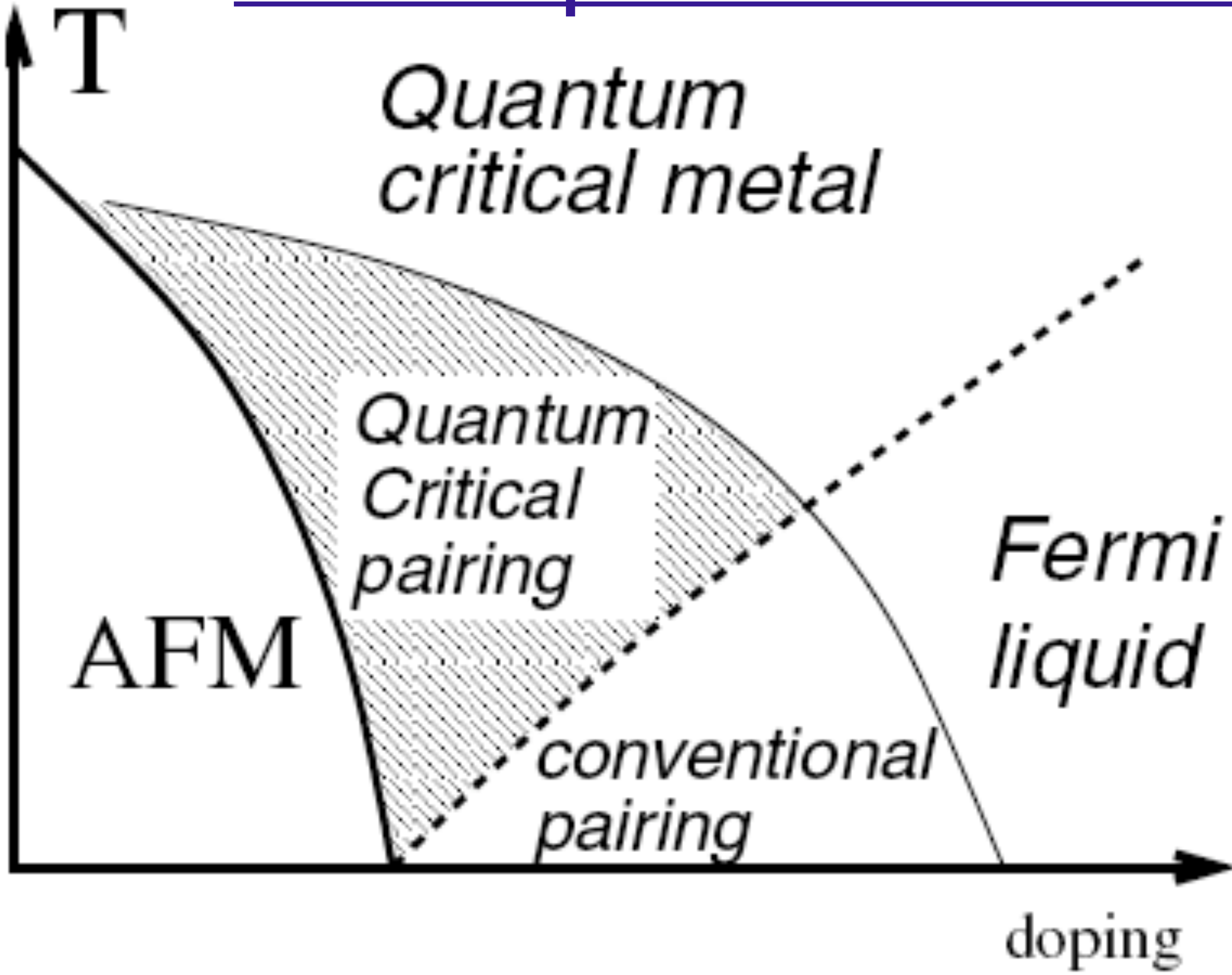
$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



- T_c *increases* upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.
- No simple mechanism for nodal-anti-nodal dichotomy.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Outline

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New Results:

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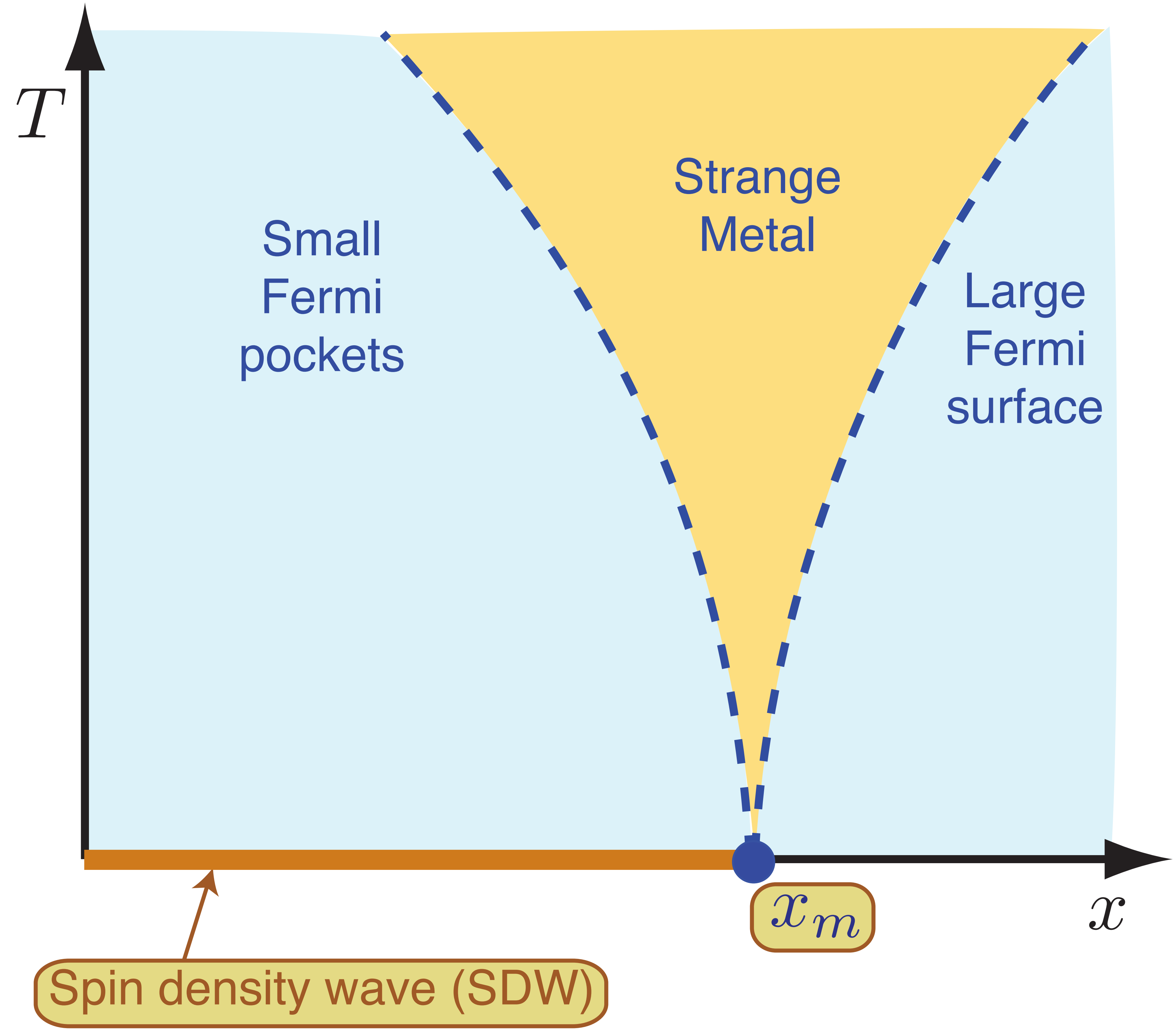
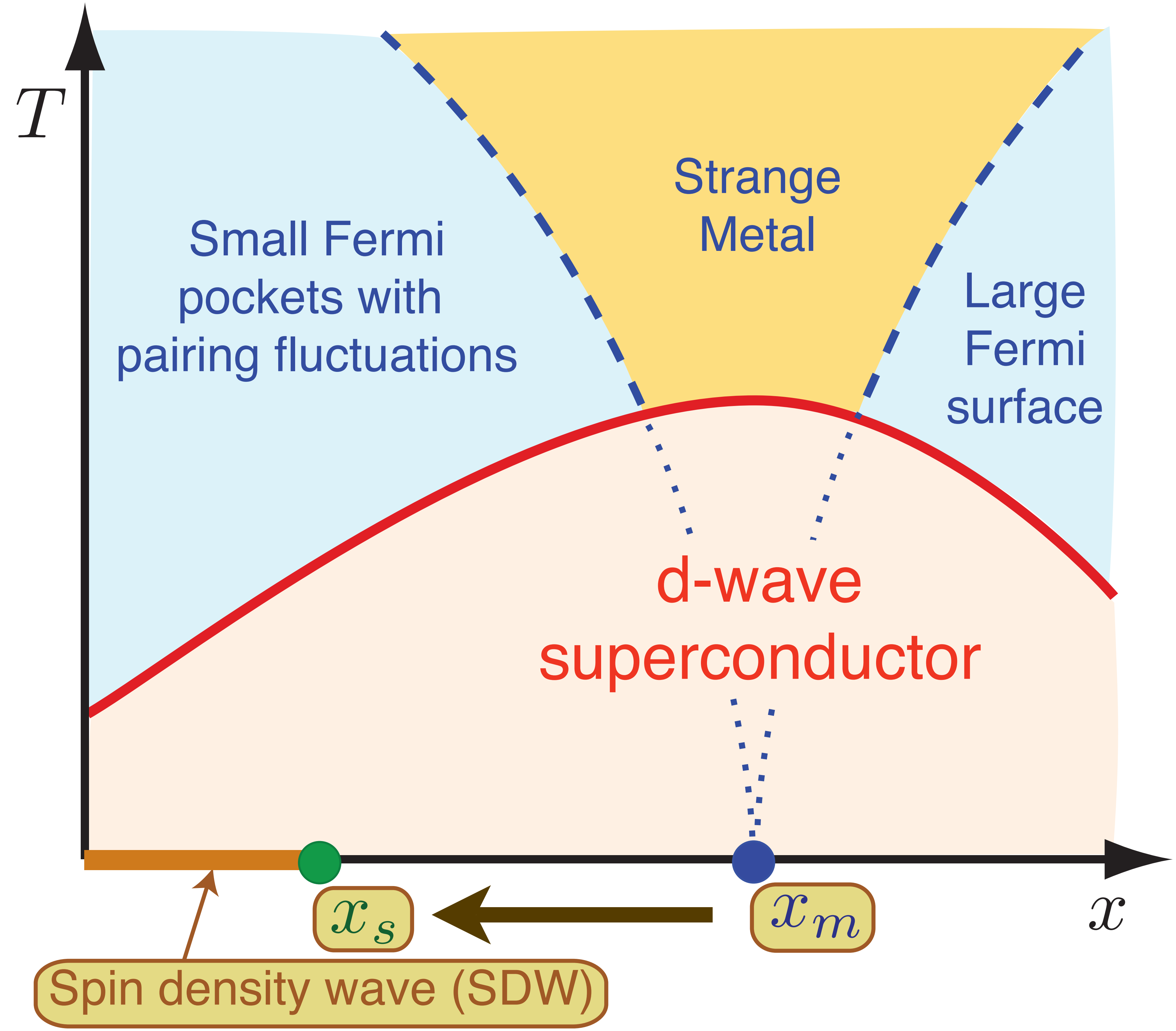
4. Superconductivity of fluctuating Fermi pockets in the underdoped cuprates

Theory of quantum criticality in the cuprates



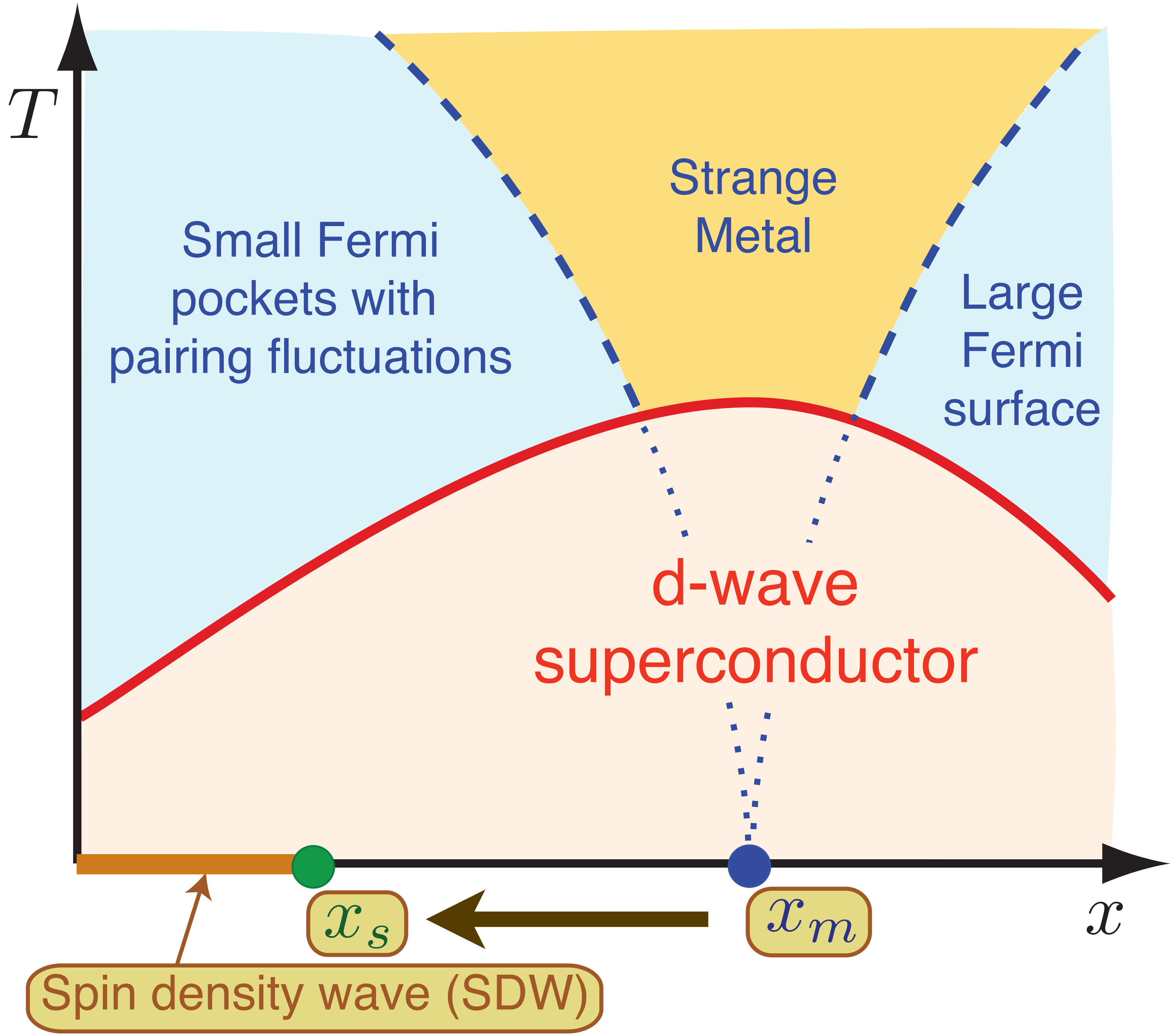
Underlying SDW ordering quantum critical point in metal at $x = x_m$

Theory of quantum criticality in the cuprates



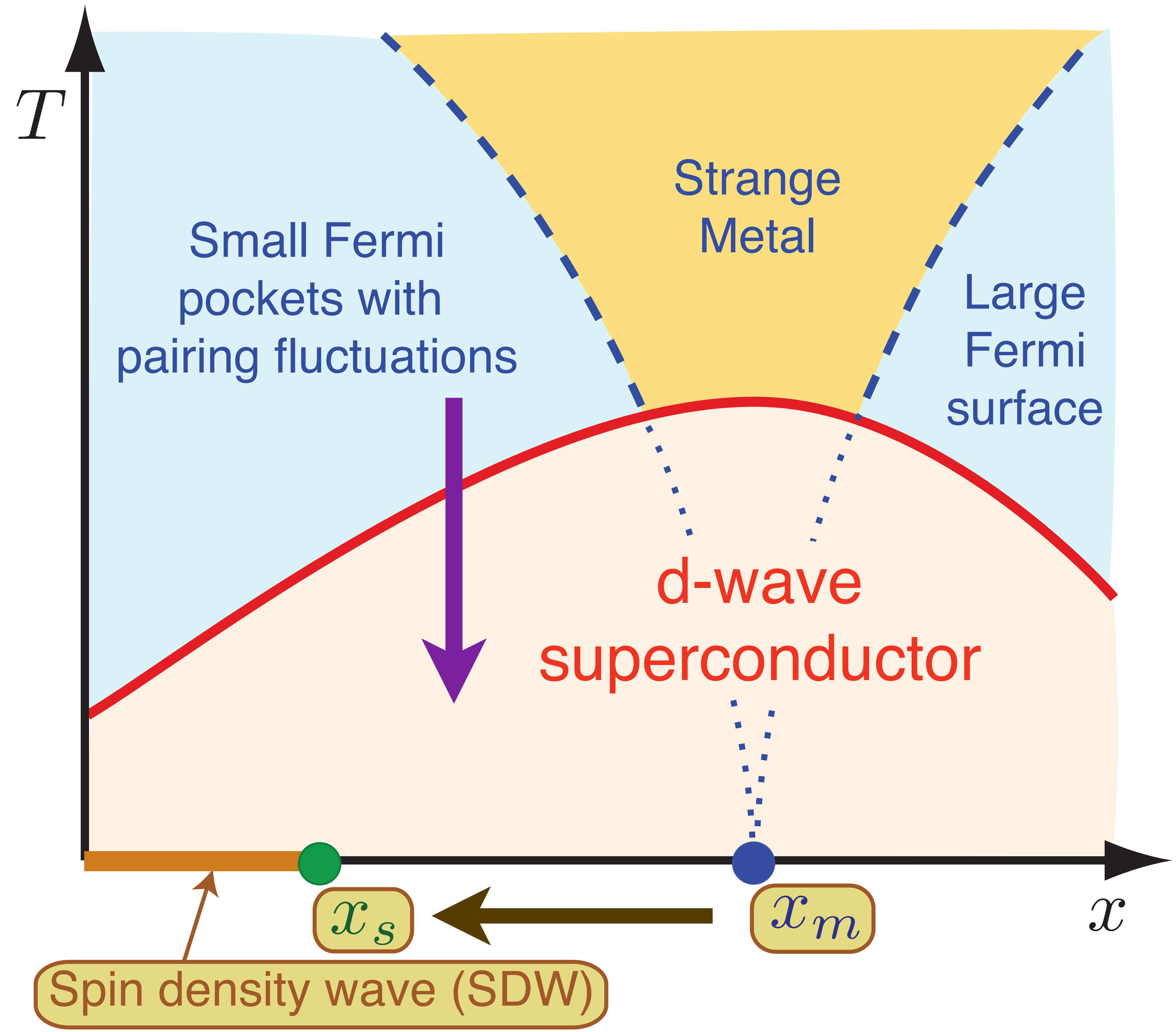
Onset of d-wave superconductivity hides the critical point at $x = x_m$ and moves it to $x = x_s < x_m$

Theory of quantum criticality in the cuprates



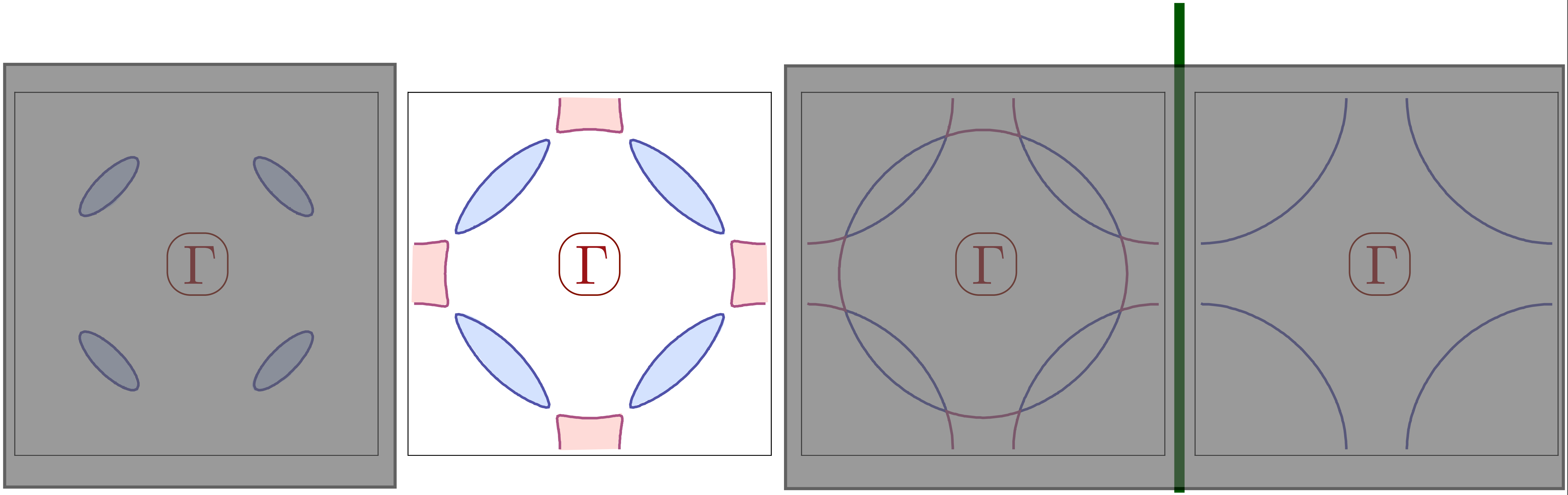
Theory of the onset of d-wave superconductivity from small Fermi pockets

Theory of quantum criticality in the cuprates



Theory of the onset of d-wave superconductivity from small Fermi pockets

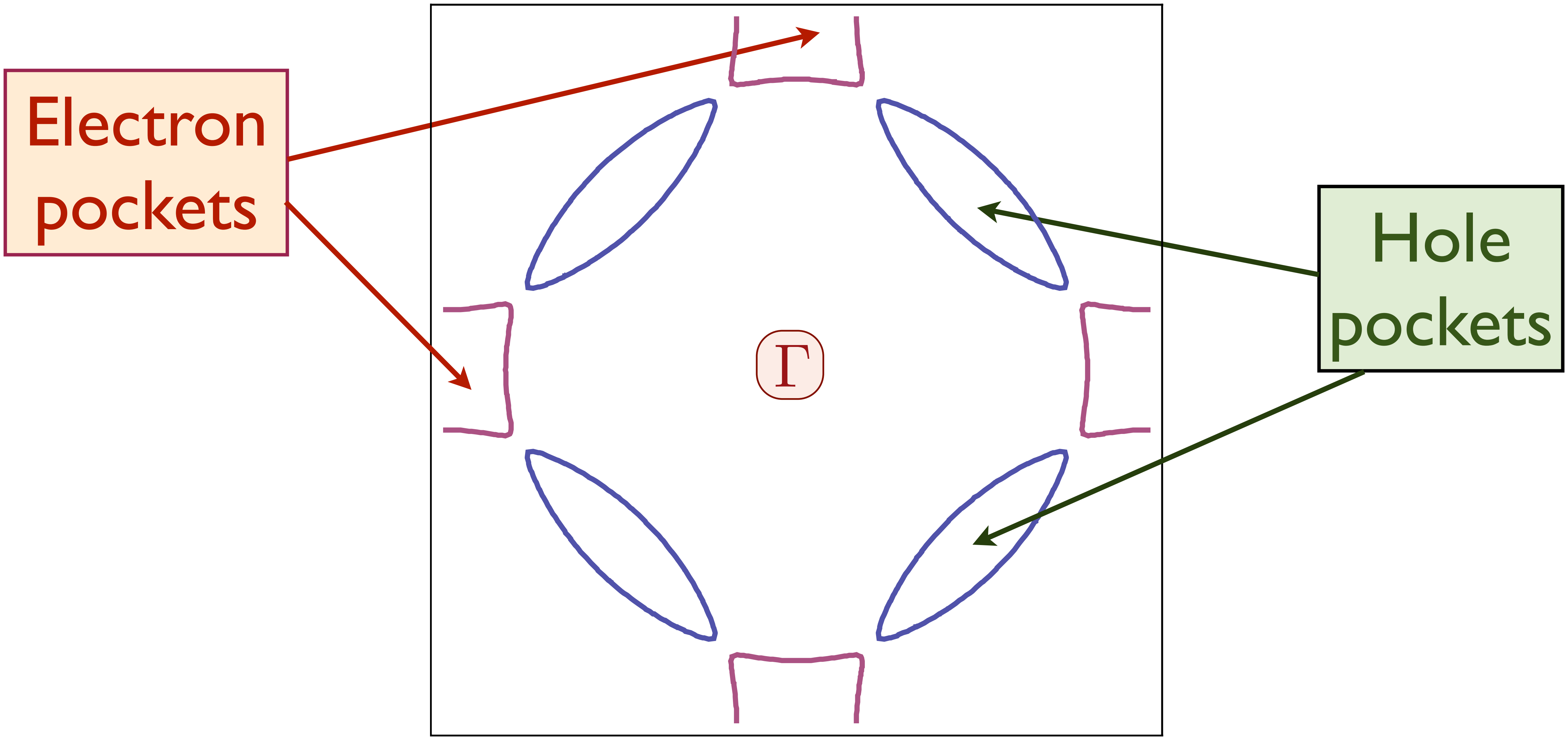
Fermi pockets in hole-doped cuprates



Begin with SDW ordered state, and focus on fluctuations in the *orientation* of $\vec{\varphi}$, by using a unit-length bosonic spinor z_α

$$\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

Charge carriers in the lightly-doped cuprates with Neel order



Spin density wave theory for electrons near $(0,\pi)$ and $(\pi,0)$

Let us write $c_{(0,\pi)\alpha} = c_{1\alpha}$, $c_{(\pi,0)\alpha} = c_{2\alpha}$ and $\varepsilon_{(0,\pi)} = \varepsilon_{(\pi,0)} = \varepsilon_0$. Then the Hamiltonian for $\vec{\varphi} = (0,0,\varphi)$ with $\varphi > 0$ is

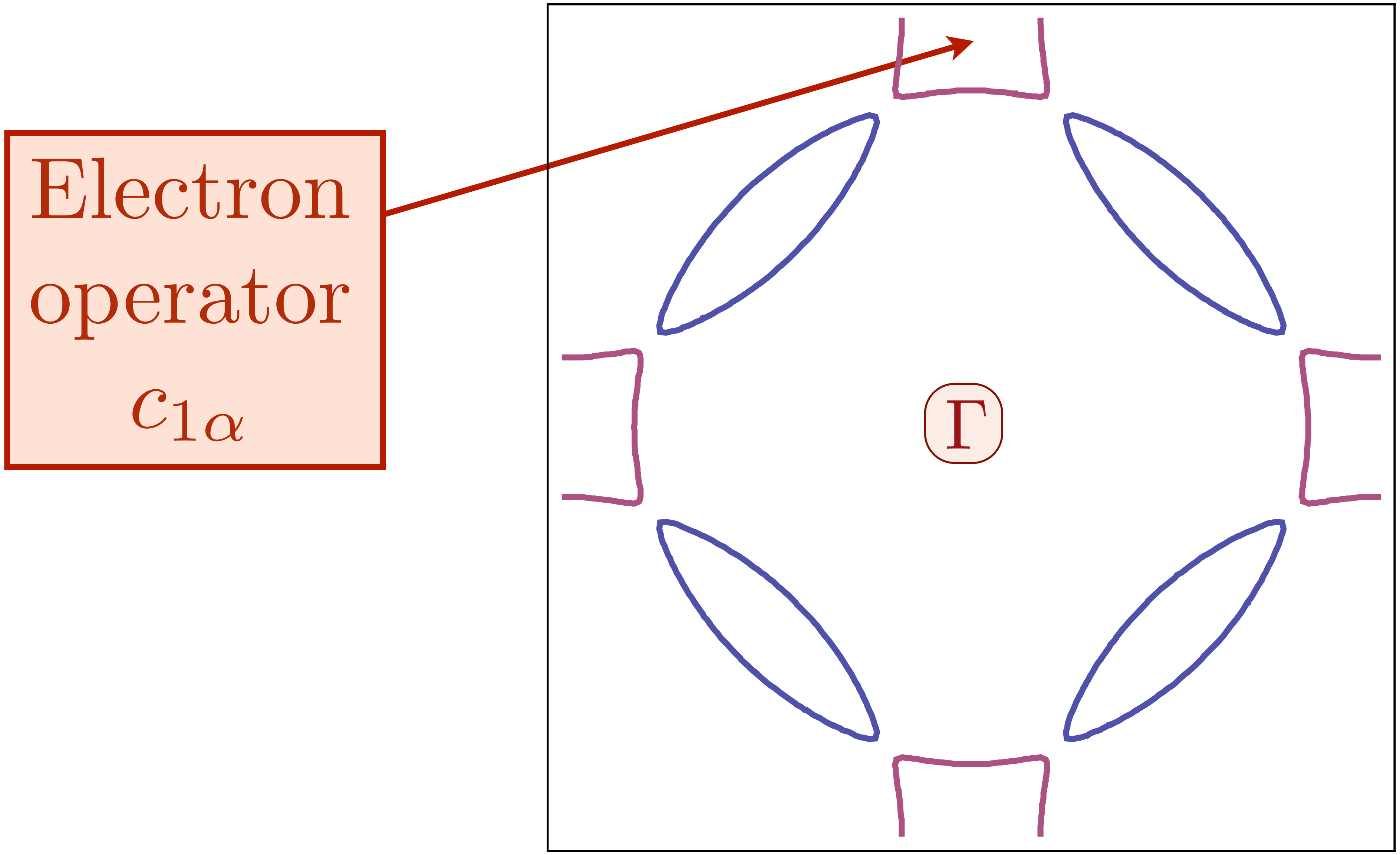
$$\begin{aligned} H_0 + H_{\rm sdw} &= \varepsilon_0 \left(c_{1\alpha}^\dagger c_{1\alpha} + c_{2\alpha}^\dagger c_{2\alpha} \right) \\ &\quad - \varphi \left(c_{1\uparrow}^\dagger c_{2\uparrow} - c_{1\downarrow}^\dagger c_{2\downarrow} + c_{2\uparrow}^\dagger c_{1\uparrow} - c_{2\downarrow}^\dagger c_{1\downarrow} \right) \end{aligned}$$

We diagonalize this by writing

$$H_0 + H_{\rm sdw} = (\varepsilon_0 - \varphi) \left(g_+^\dagger g_+ + g_-^\dagger g_- \right) + (\varepsilon_0 + \varphi) \left(h_+^\dagger h_+ + h_-^\dagger h_- \right)$$

where

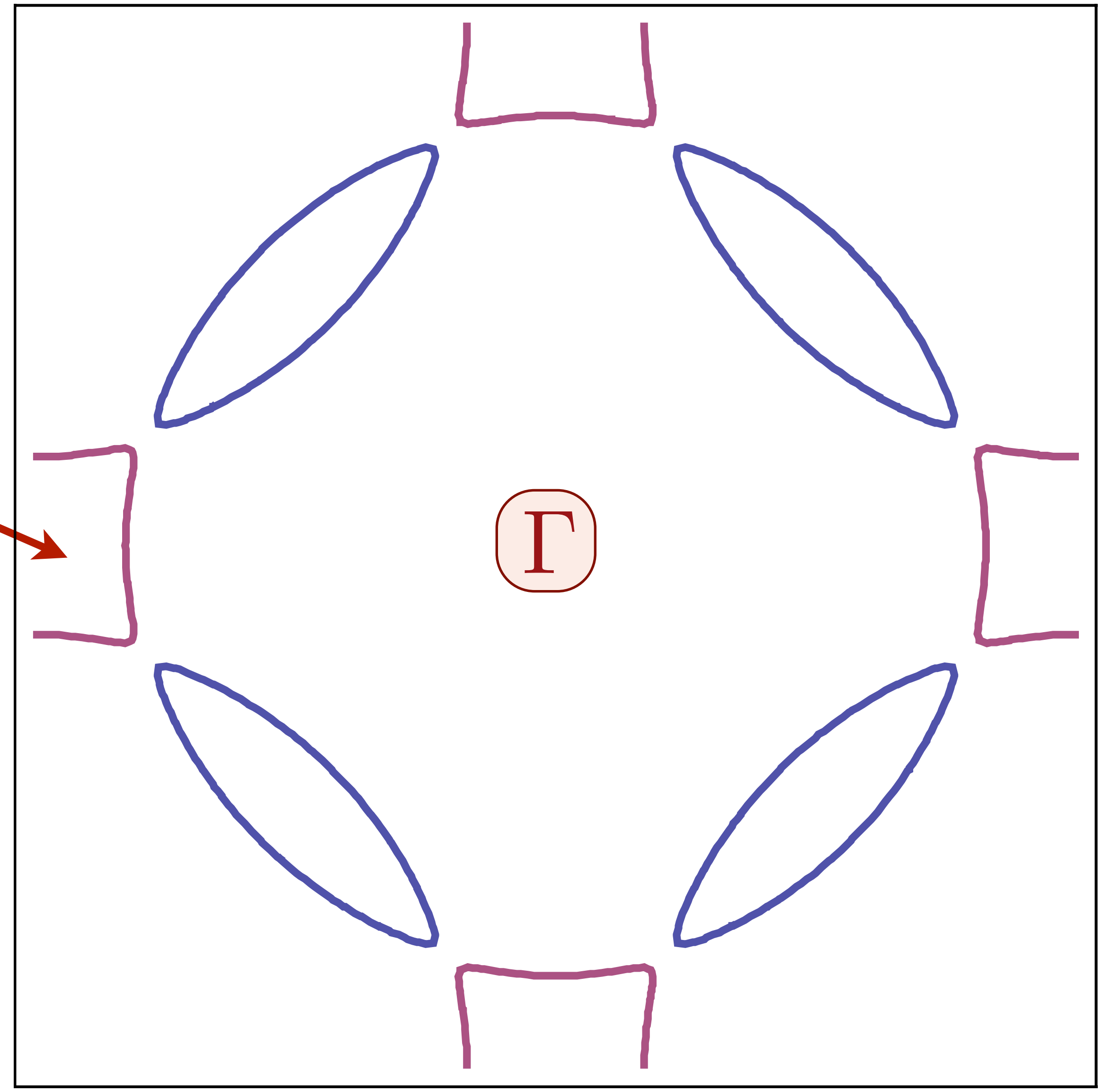
$$\begin{aligned} c_{1\uparrow} &= (g_+ + h_+)/\sqrt{2} \\ c_{2\uparrow} &= (g_+ - h_+)/\sqrt{2} \\ c_{1\downarrow} &= (g_- + h_-)/\sqrt{2} \\ c_{2\downarrow} &= (-g_- + h_-)/\sqrt{2} \end{aligned}$$



For a uniform SDW order with $\vec{\varphi} = (0, 0, \varphi)$, write

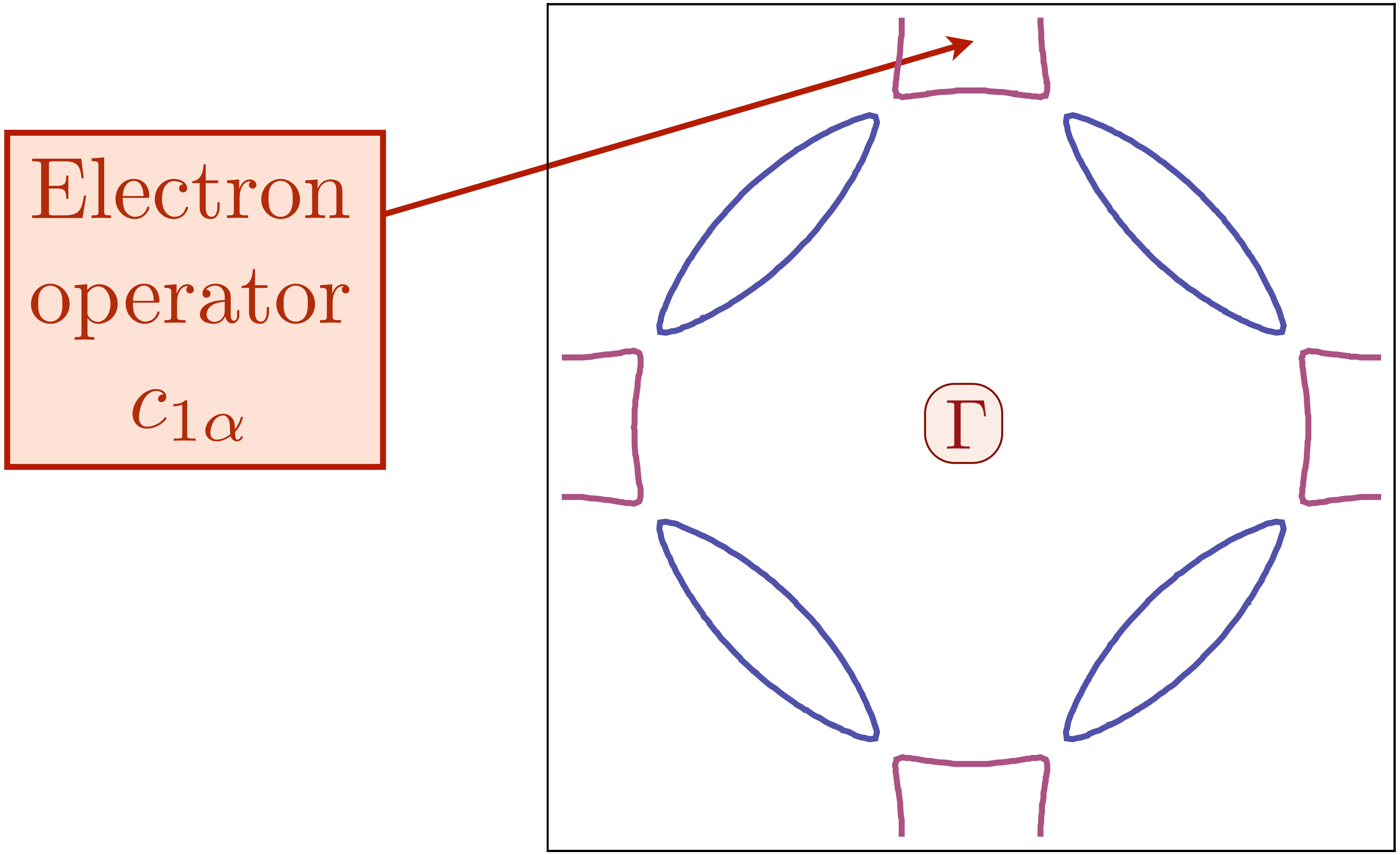
$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \quad \begin{pmatrix} g_+ \\ g_- \end{pmatrix}$$

Electron operator $c_{2\alpha}$



SDW theory also specifies electrons at second pocket for $\vec{\varphi} = (0, 0, \varphi)$

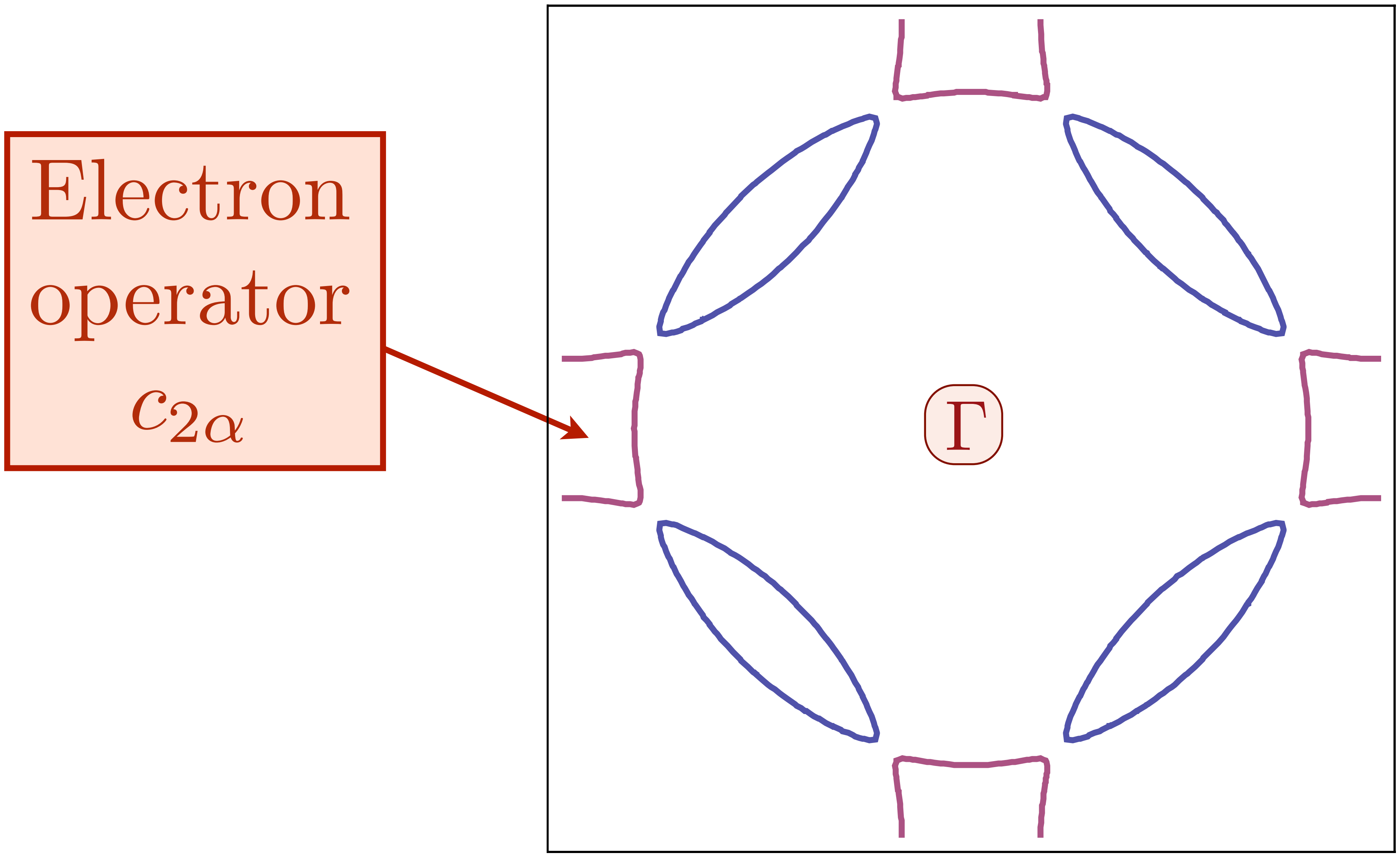
$$\begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \begin{pmatrix} g_+ \\ -g_- \end{pmatrix} ,$$



For a spacetime dependent SDW order, $\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$,

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix} \quad ; \quad \mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.$$

So $g_\pm$ are the "up/down" electron operators
in a rotating reference frame defined by the local SDW order



For a spacetime dependent SDW order, $\vec{\varphi} = z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta$,

$$\begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ -g_- \end{pmatrix} \quad ; \quad \mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z^*_\downarrow \\ z_\downarrow & z^*_\uparrow \end{pmatrix}.$$

Same SU(2) matrix also rotates electrons in second pocket.

Fluctuating pocket theory for electrons near $(0, \pi)$ and $(\pi, 0)$

Summarizing, in the low energy theory, the $c_{1,2\alpha}$ are expressed in terms of the $g_\pm$ fermions and the z_α by

$$\begin{aligned} c_{1\uparrow} &= z_\uparrow g_+ - z_\downarrow^* g_- \\ c_{2\uparrow} &= z_\uparrow g_+ + z_\downarrow^* g_- \\ c_{1\downarrow} &= z_\downarrow g_+ + z_\uparrow^* g_- \\ c_{2\downarrow} &= z_\downarrow g_+ - z_\uparrow^* g_- \end{aligned}$$

Note that this invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\phi} z_\alpha \quad ; \quad g_+ \rightarrow e^{-i\phi} g_+ \quad ; \quad g_- \rightarrow e^{i\phi} g_-,$$

which must be obeyed by the effective action for z_α and $g_\pm$.

Fluctuating pocket theory for electrons near $(0, \pi)$ and $(\pi, 0)$

We will show that in the resulting theory, the $g_\pm$ are unstable to a simple s-wave pairing with

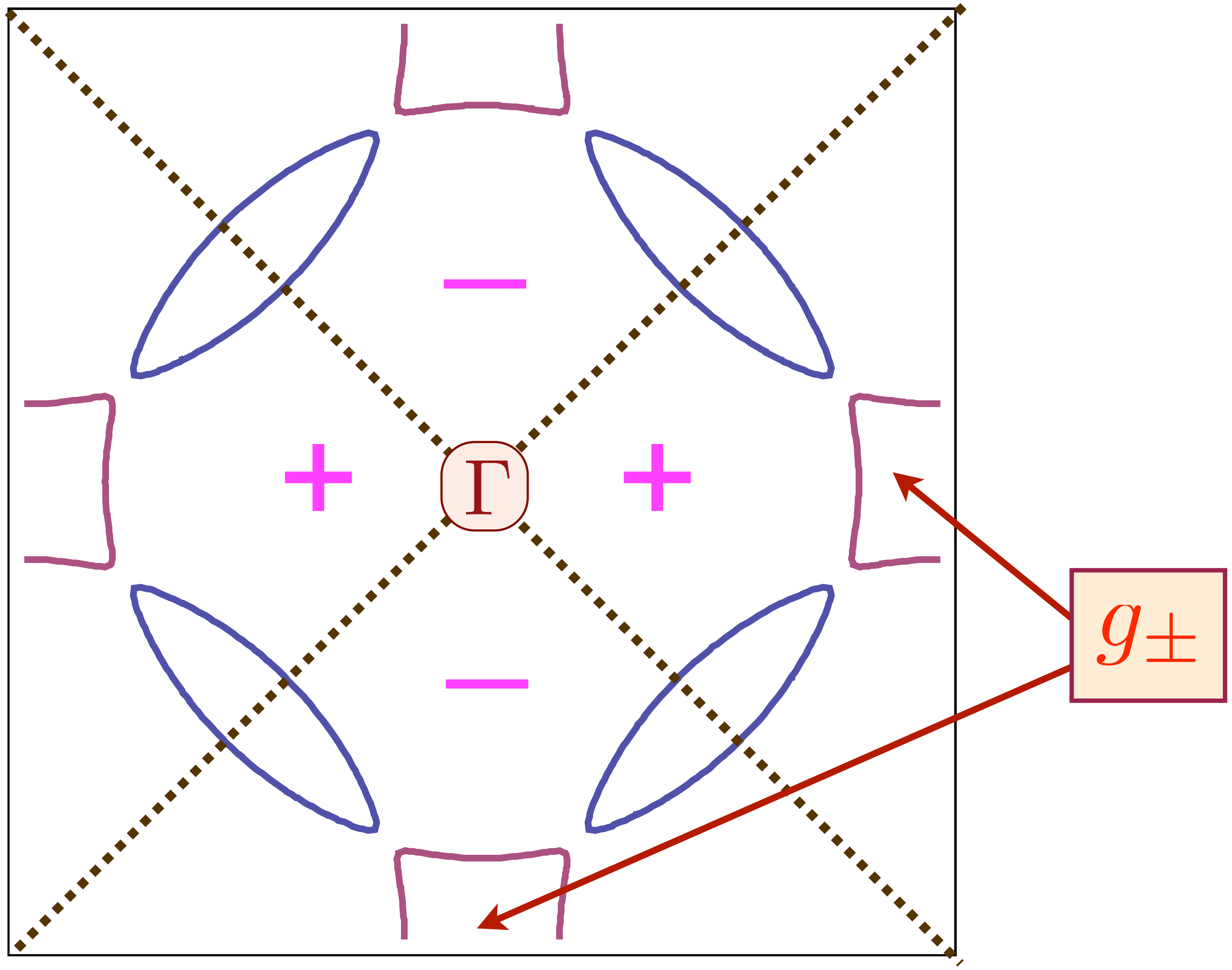
$$\langle g_+ g_- \rangle = \Delta$$

For the physical electron operators, this pairing implies

$$\begin{aligned} \langle c_{1\uparrow} c_{1\downarrow} \rangle &= \Delta \left\langle |z_\alpha|^2 \right\rangle \\ \langle c_{2\uparrow} c_{2\downarrow} \rangle &= -\Delta \left\langle |z_\alpha|^2 \right\rangle \end{aligned}$$

i.e. d-wave pairing !

Strong pairing of the $g_\pm$ electron pockets



$$\langle g_+ g_- \rangle = \Delta$$

Low energy theory for spinless, charge $-e$ fermions $g_\pm$, and spinful, charge 0 bosons z_α:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_z + \mathcal{L}_g \\ \mathcal{L}_z &= \frac{1}{t}\Big[|(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2|\nabla - i\mathbf{A})z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1)\Big] \\ &+ \text{Berry phases of monopoles in } A_\mu.\end{aligned}$$

CP^1 field theory for z_α and an emergent U(1) gauge field A_μ. Coupling t tunes the strength of SDW orientation fluctuations.

Low energy theory for spinless, charge $-e$ fermions $g_\pm$, and spinful, charge 0 bosons z_α:

$$
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&+ \text{ Berry phases of monopoles in } A_\mu.
\end{aligned}
$$

CP^1 field theory for z_α and an emergent U(1) gauge field A_μ. Coupling t tunes the strength of SDW orientation fluctuations.

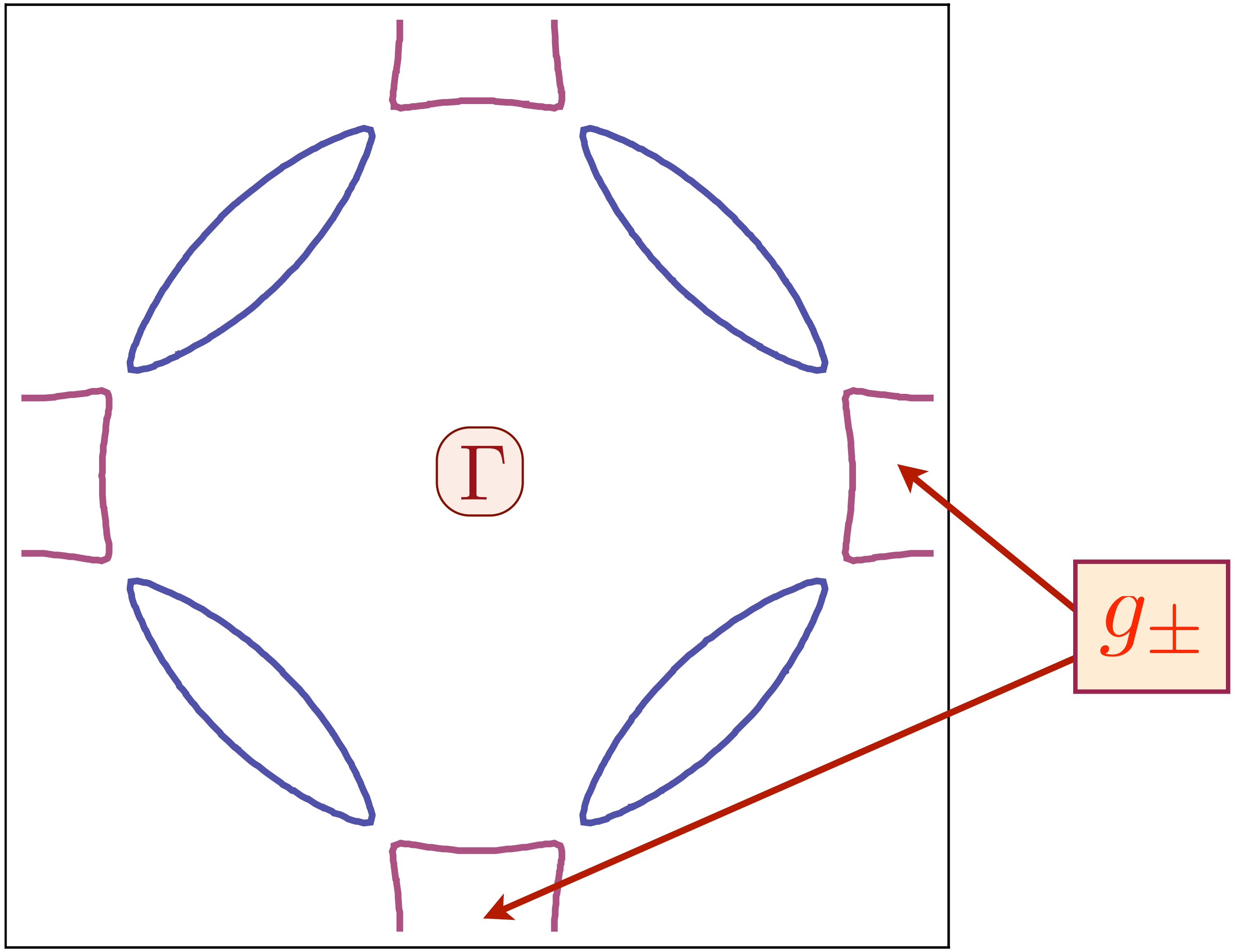
$$
\begin{aligned}
\mathcal{L}_g &= g_+^\dagger\left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu\right]g_+ \\
&\quad + g_-^\dagger\left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu\right]g_-
\end{aligned}
$$

Two Fermi surfaces coupled to the emergent U(1) gauge field A_μ with opposite charges

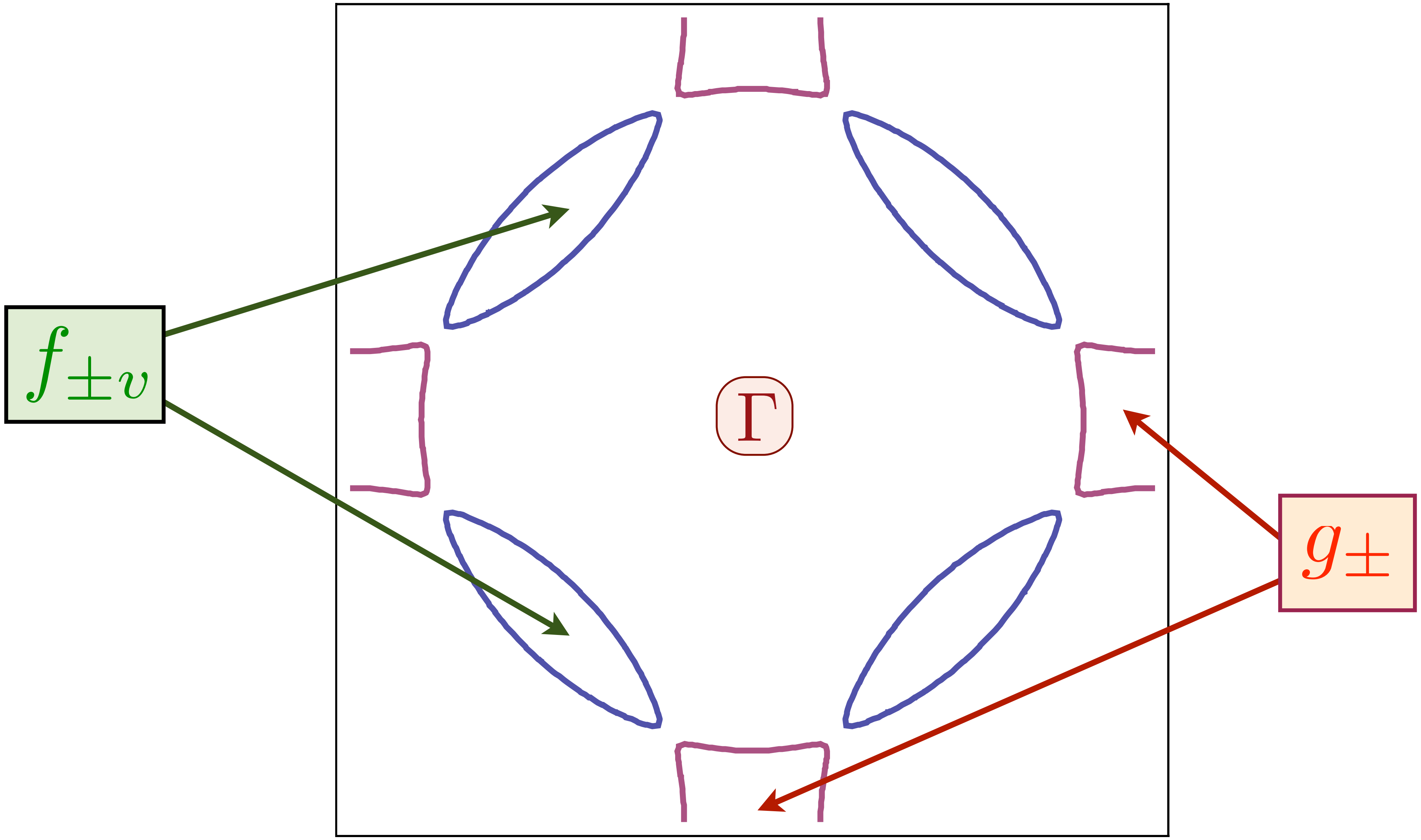
Strong pairing of the $g_\pm$ electron pockets

- Gauge forces lead to a s-wave paired state with a T_c of order the Fermi energy of the pockets. Inelastic scattering from low energy gauge modes lead to very singular $g_\pm$ self energy, but is *not* pair-breaking.

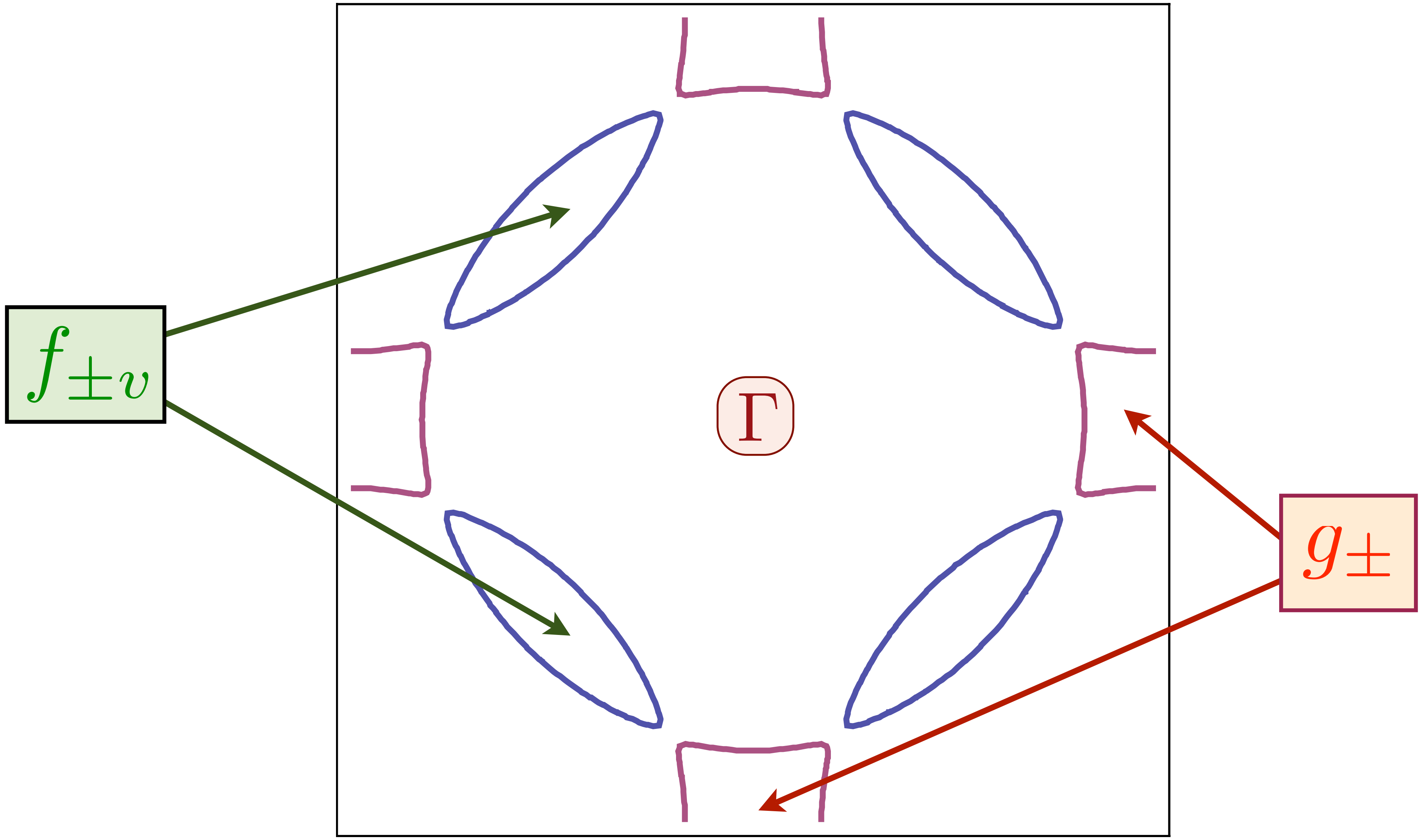
$$\langle g_+ g_- \rangle = \Delta$$



Γ
$g_\pm$



$f_{\pm v}$
Γ
$g_{\pm}$



Low energy theory for spinless, charge $+e$ fermions $f_{\pm v}$:

$$\mathcal{L}_f = \sum_{v=1,2} \left\{ f_{+v}^{\dagger} \left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu \right] f_{+v} + f_{-v}^{\dagger} \left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu \right] f_{-v} \right\}$$

Weak pairing of the $f_\pm$ hole pockets

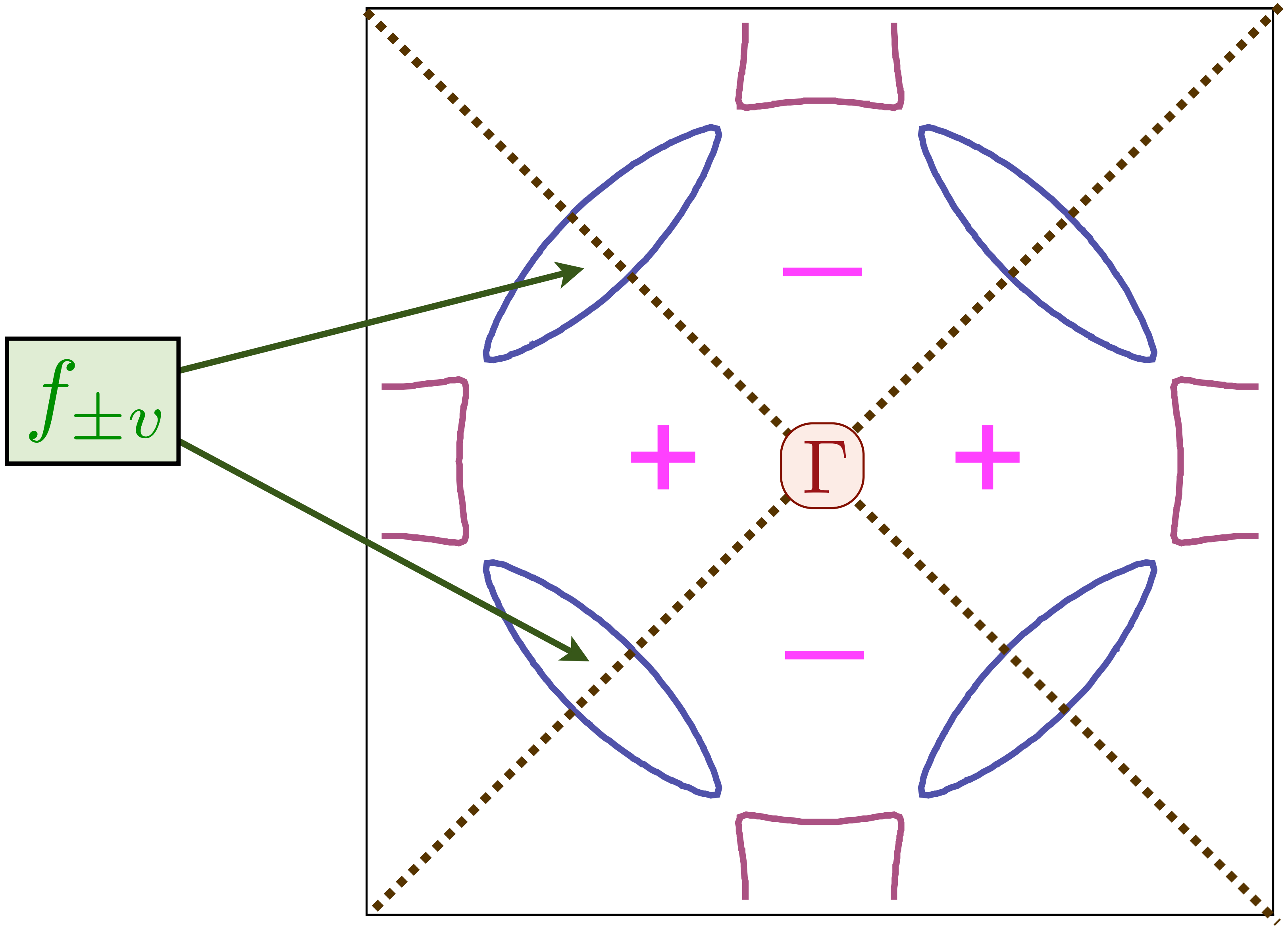
$$\mathcal{L}_{\text{Josephson}} = iJ\left[g_+ g_-\right]\left[f_{+1}\overleftrightarrow{\partial}_x f_{-1} - f_{+1}\overleftrightarrow{\partial}_y f_{-1} \right.$$
$$\left. + f_{+2}\overleftrightarrow{\partial}_x f_{-2} + f_{+2}\overleftrightarrow{\partial}_y f_{-2}\right] + \text{H.c.}$$

V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, Phys. Rev. B **55**, 3173 (1997).

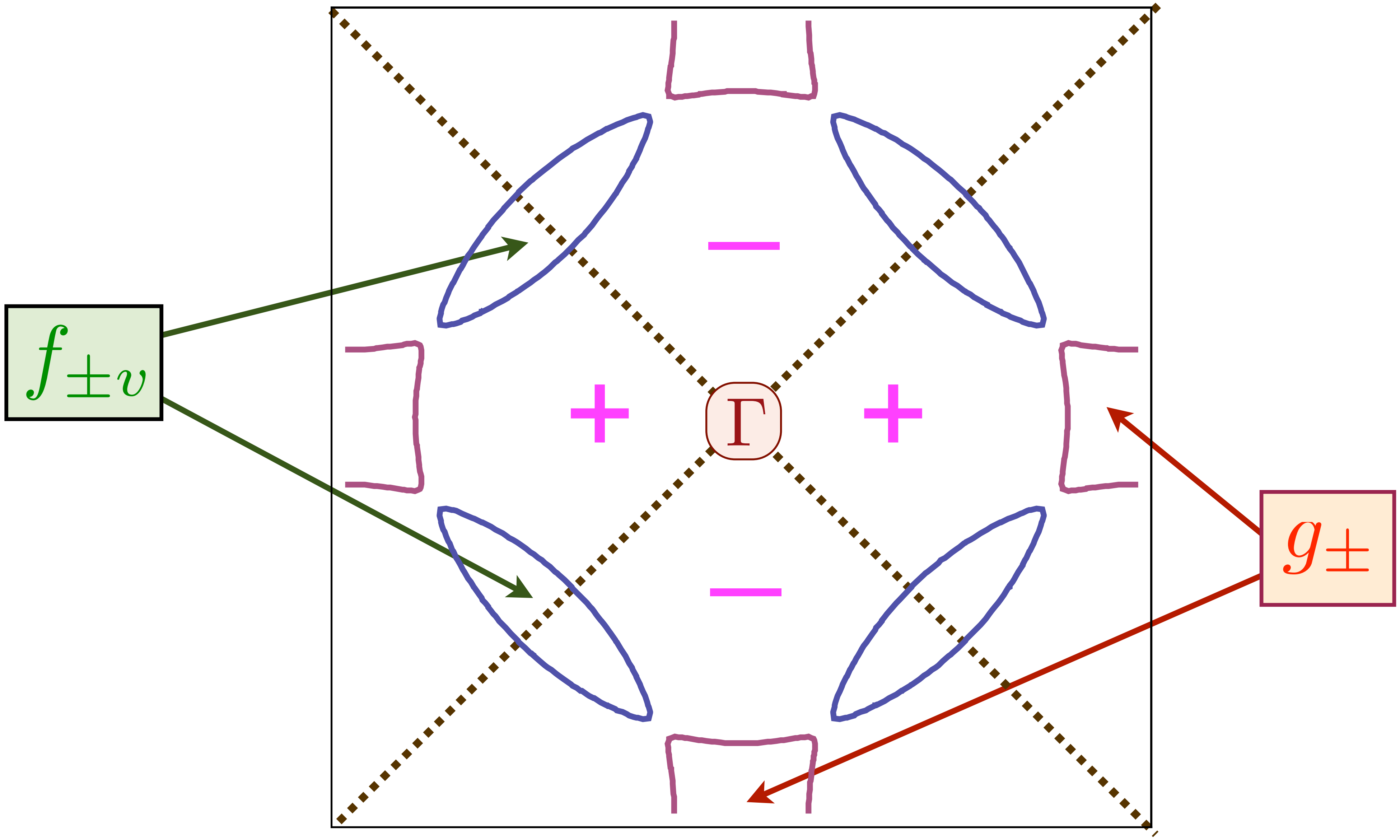
Proximity Josephson coupling J to $g_\pm$ fermions leads to p-wave pairing of the $f_{\pm v}$ fermions. The A_μ gauge forces are pair-breaking, and so the pairing is weak.

$$\begin{aligned}
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k})\rangle &\sim (k_x - k_y) J \langle g_+ g_-\rangle; \\
\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &\sim (k_x + k_y) J \langle g_+ g_-\rangle; \\
\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &= 0,
\end{aligned}$$

Weak pairing of the $f_\pm$ hole pockets



$$\begin{aligned}
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k})\rangle &\sim (k_x - k_y) J \langle g_+ g_- \rangle; \\
\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &\sim (k_x + k_y) J \langle g_+ g_- \rangle; \\
\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &= 0,
\end{aligned}$$



d-wave pairing of the electrons is associated with

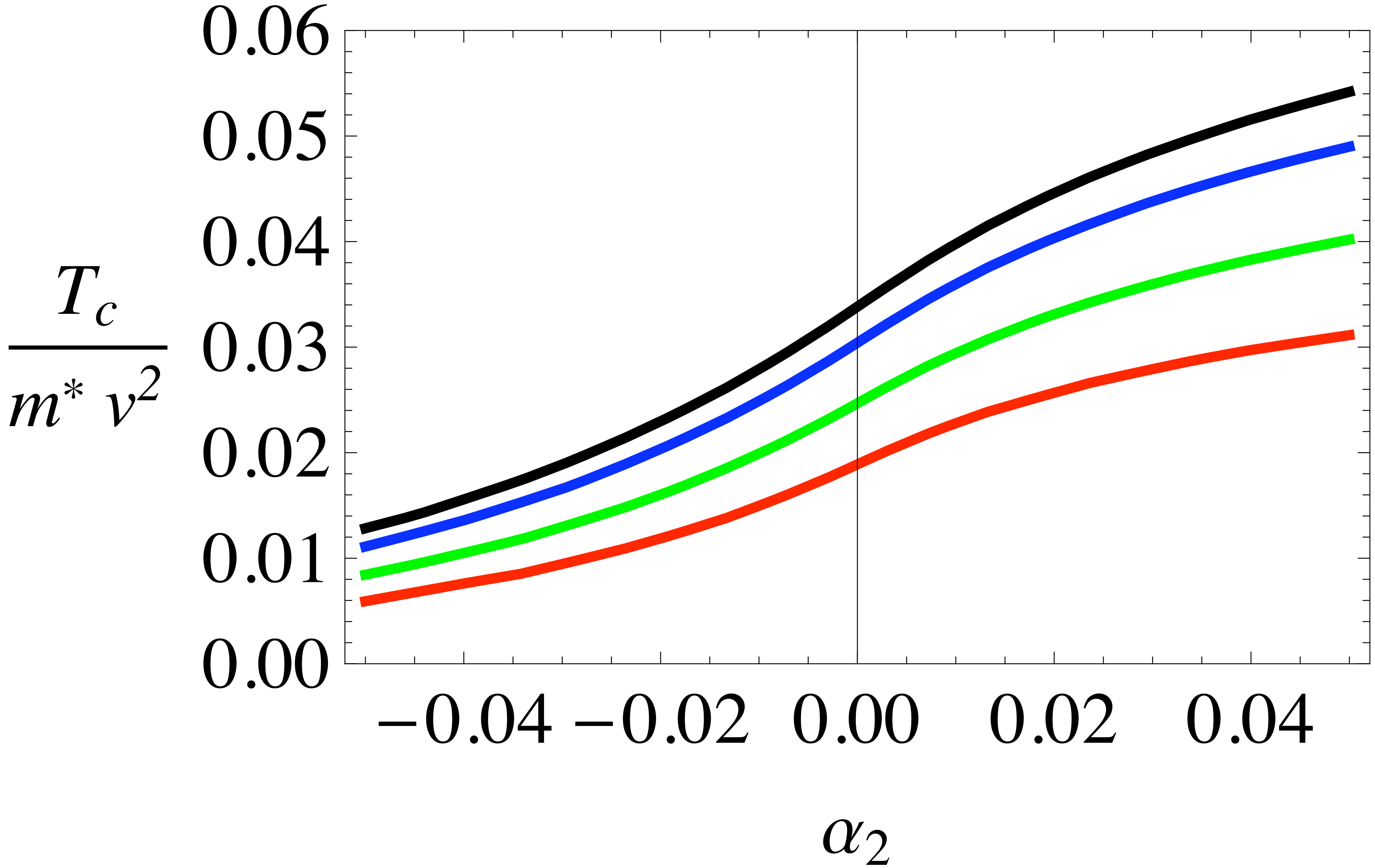
- Strong s-wave pairing of $g_{\pm}$
- Weak p-wave pairing of $f_{\pm v}$.

5. Quantum theory of competition between superconductivity and SDW order

Minimal, universal theory for competition between superconductivity and SDW order

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_z + \mathcal{L}_g \\
\mathcal{L}_z &= \frac{1}{t}\left[|(\partial_\tau - iA_\tau) z_\alpha|^2 + v^2 |\nabla - i\mathbf{A}) z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1) \right] \\
\mathcal{L}_g &= g_+^\dagger \left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu \right] g_+ \\
&\quad + g_-^\dagger \left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu \right] g_-
\end{aligned}$$

- T_c *decreases* upon approaching the SDW transition. SDW and SC orders compete.



Two dimensionless parameters in Eliashberg theory:

$$\alpha_1 \equiv \frac{\hbar k_F}{m^* v} \quad ; \quad \alpha_2 \equiv \left(\frac{1}{t_c} - \frac{1}{t}\right) \frac{1}{m^*}.$$

The red, green, blue, and black lines correspond to $\alpha_1^2/2 = E_F/(m^* v^2) = 0.16, 0.21, 0.26, 0.29$. The energy scale $m^* v^2 \approx 110$ meV.

Back-action of the onset of superconductivity on SDW order

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_z + \mathcal{L}_g \\
\mathcal{L}_z &= \frac{1}{t}\Big[|(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2|\nabla - i\mathbf{A})z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1)\Big] \\
\mathcal{L}_g &= g_+^\dagger\left[(\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - i\mathbf{A})^2 - \mu\right]g_+ \\
&\quad + g_-^\dagger\left[(\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + i\mathbf{A})^2 - \mu\right]g_- \\
&\quad - \Delta g_+^\dagger g_-^\dagger - \Delta g_- g_+
\end{aligned}
$$

Determine the critical coupling, t_c, for the onset of SDW order as a function of the superconducting pairing gap, Δ.

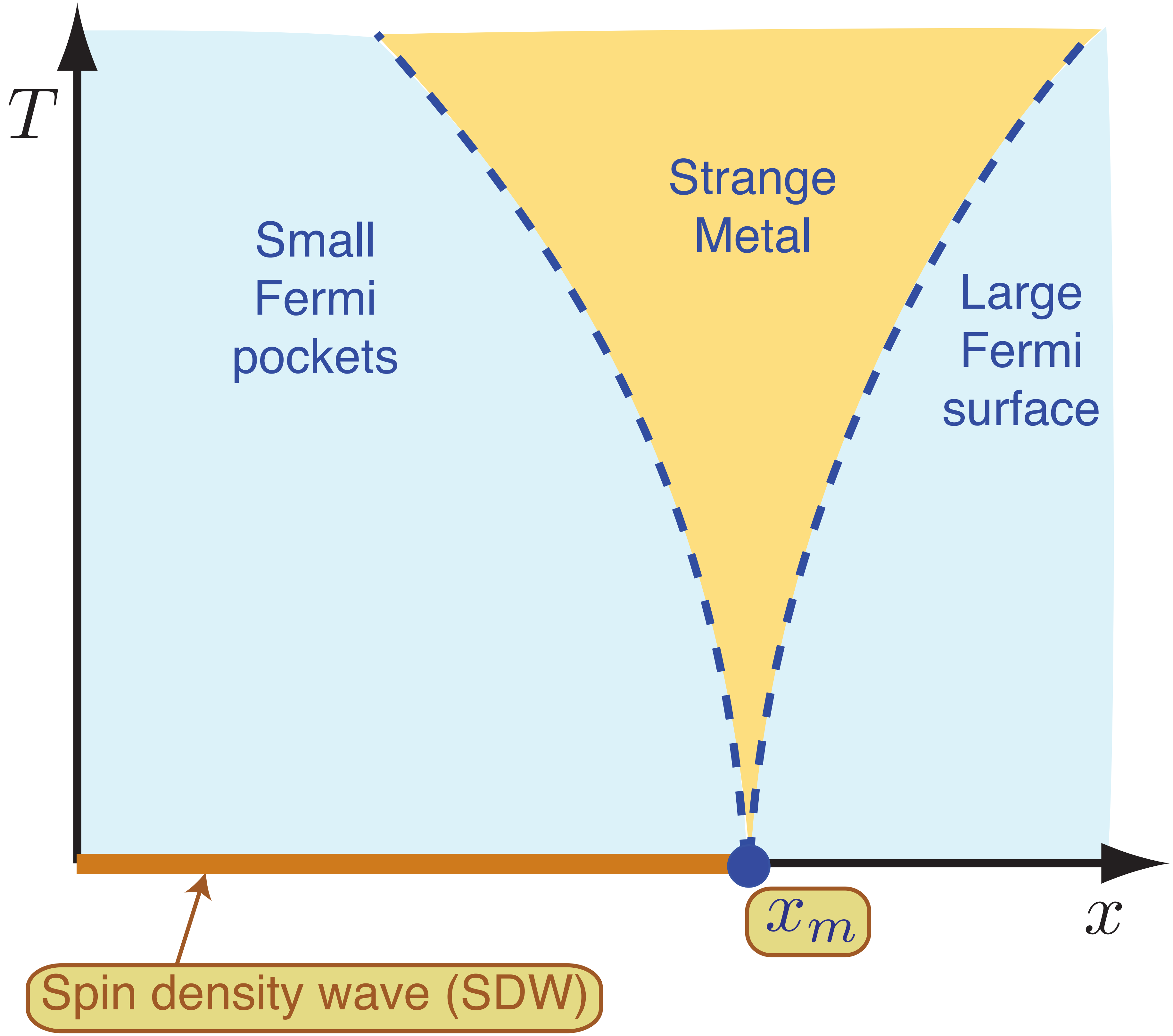
Primary coupling between SDW order and superconductivity is via the gauge fluctuations:

$$
\begin{aligned}
\frac{1}{t_c(\Delta)} &= \frac{1}{t_c^0} + \frac{1}{N} F(\Delta); \\
F(\Delta) &= \int \frac{d^2 q d\omega}{8\pi^3} \frac{q^2}{8(\omega^2 + v^2 q^2)^{1/2}} \left[\frac{1}{(\omega^2 + v^2 q^2) D_1(q,\omega)} \right. \\
&\qquad \left. + \frac{1}{q^2 D_2(q,\omega) + \omega^2 D_1(q,\omega)} \right]
\end{aligned}
$$

where D_1 and D_2 are the longitudinal and transverse gauge propagators.

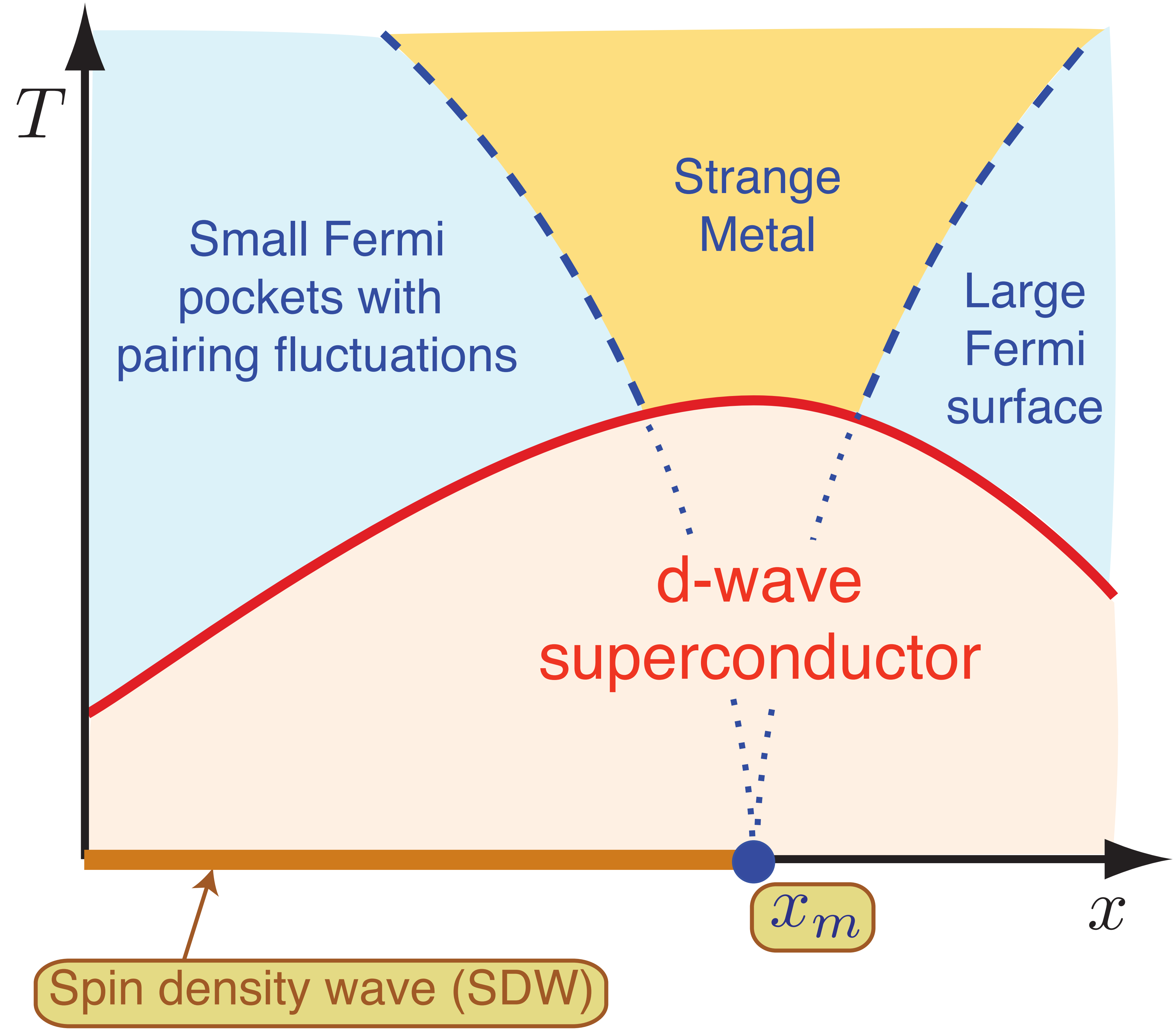
The opening of the fermion gap, Δ, decreases screening of gauge fluctuations, and so gauge fluctuations are *enhanced* in the superconductor. This leads to a suppression of SDW order as Δ increases, and so t_c is a monotonically decreasing function of Δ.

Theory of quantum criticality in the cuprates



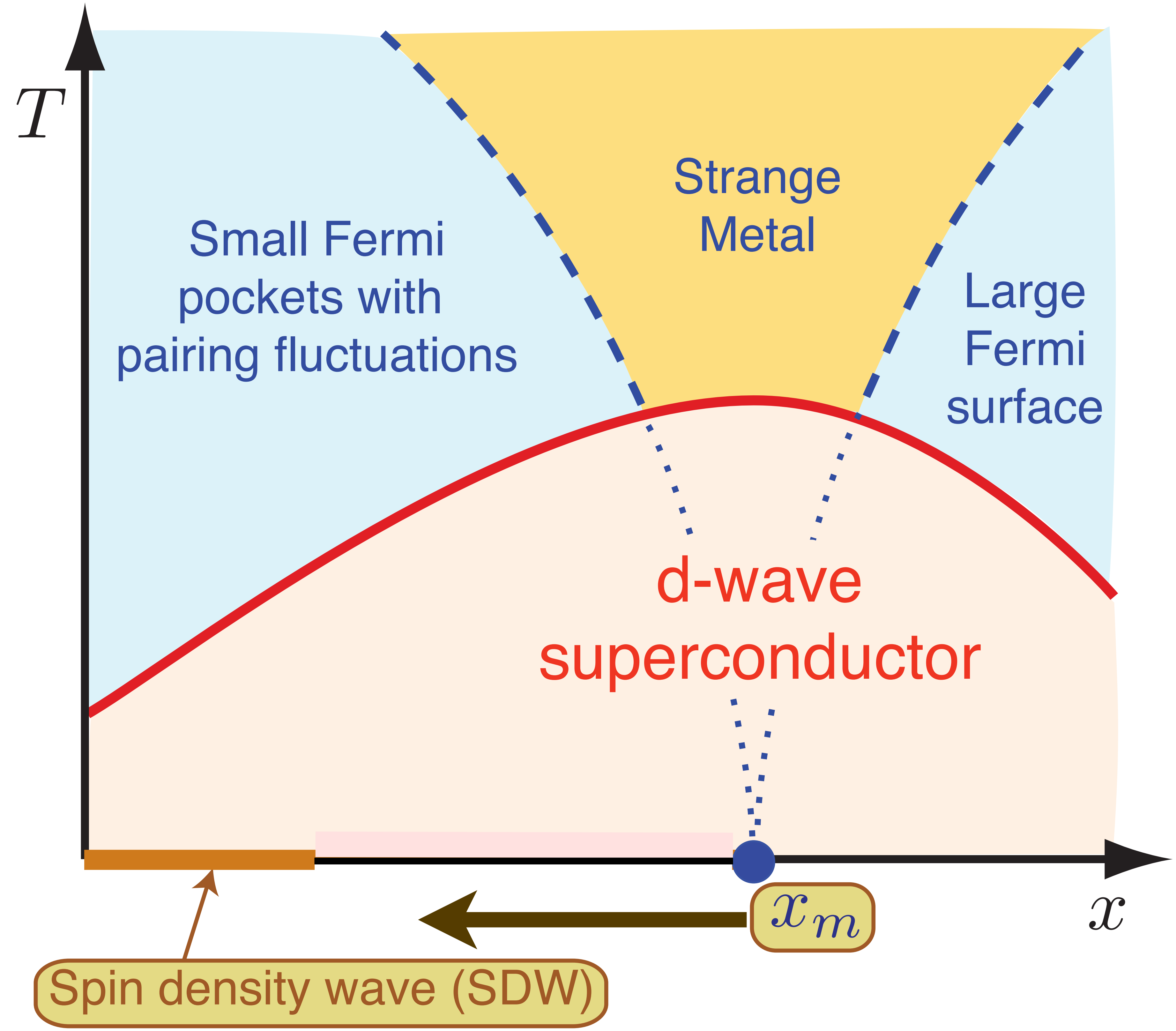
Underlying SDW ordering quantum critical point in metal at $x = x_m$

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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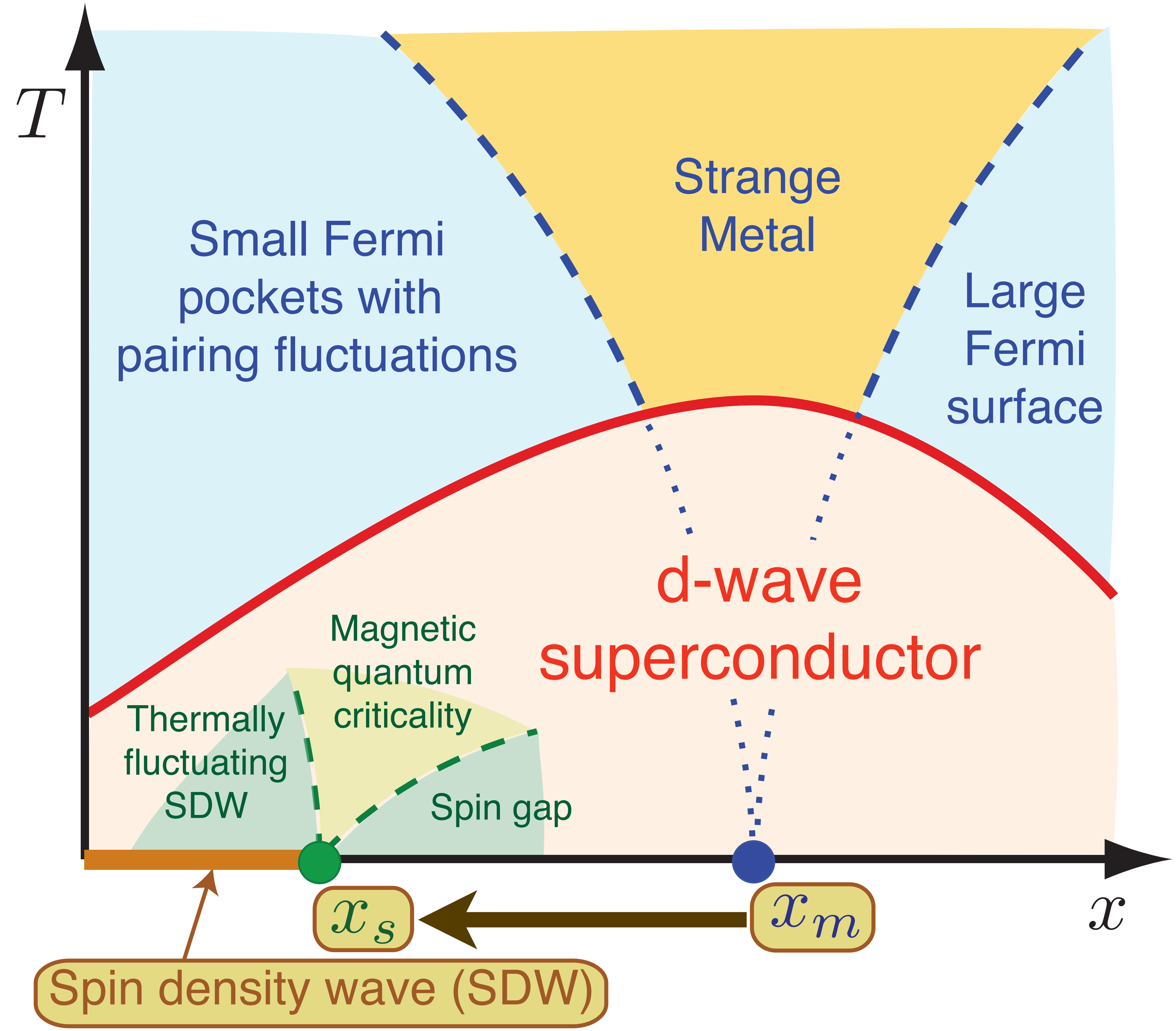
Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Conclusions

- ★ Gauge theory for pairing in the underdoped cuprates, describing "angular" fluctuations of spin-density-wave order
- ★ Natural route to *d*-wave pairing with strong pairing at the antinodes and weak pairing at the nodes
- ★ Explains characteristic "competing order" features of field-doping phase diagram: SDW order is more stable in the metal than in the superconductor.