A model of the cuprates: from the pseudogap metal, to d-wave superconductivity, and charge order

Superconductivity Gordon Research Conference
Interactions, Topology and Applications
Les Diablerets, May 3, 2023
Subir Sachdev

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang,
Mathias Scheurer, and S. S., arXiv:2302.07885
Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi,
and S.S., arXiv:2211.10452

Talk online: sachdev.physics.harvard.edu

PHYSICS
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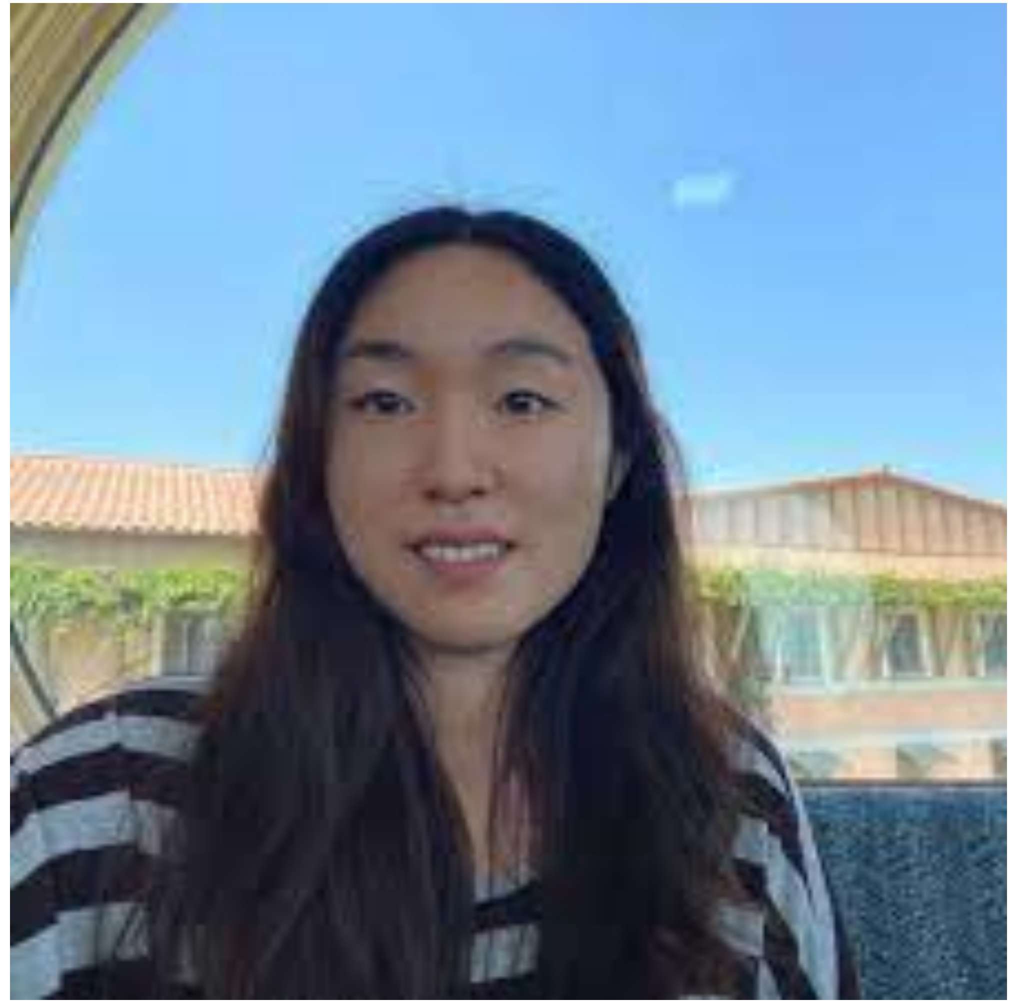
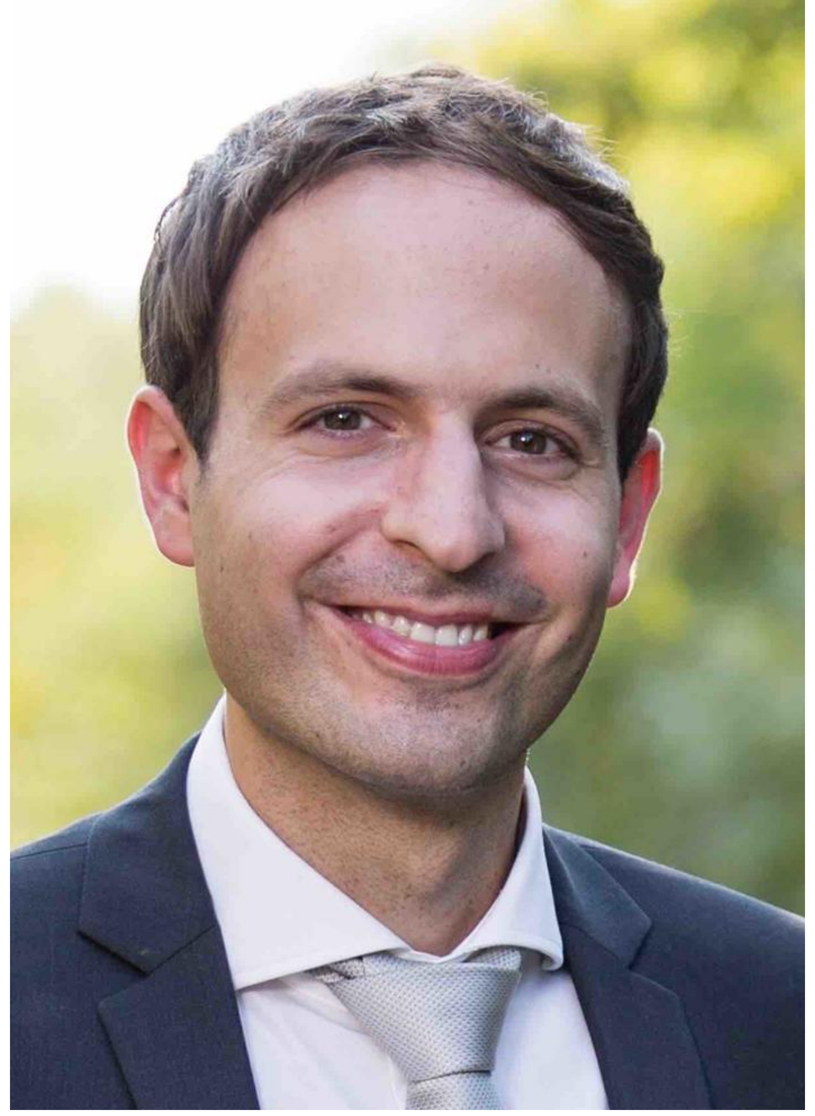
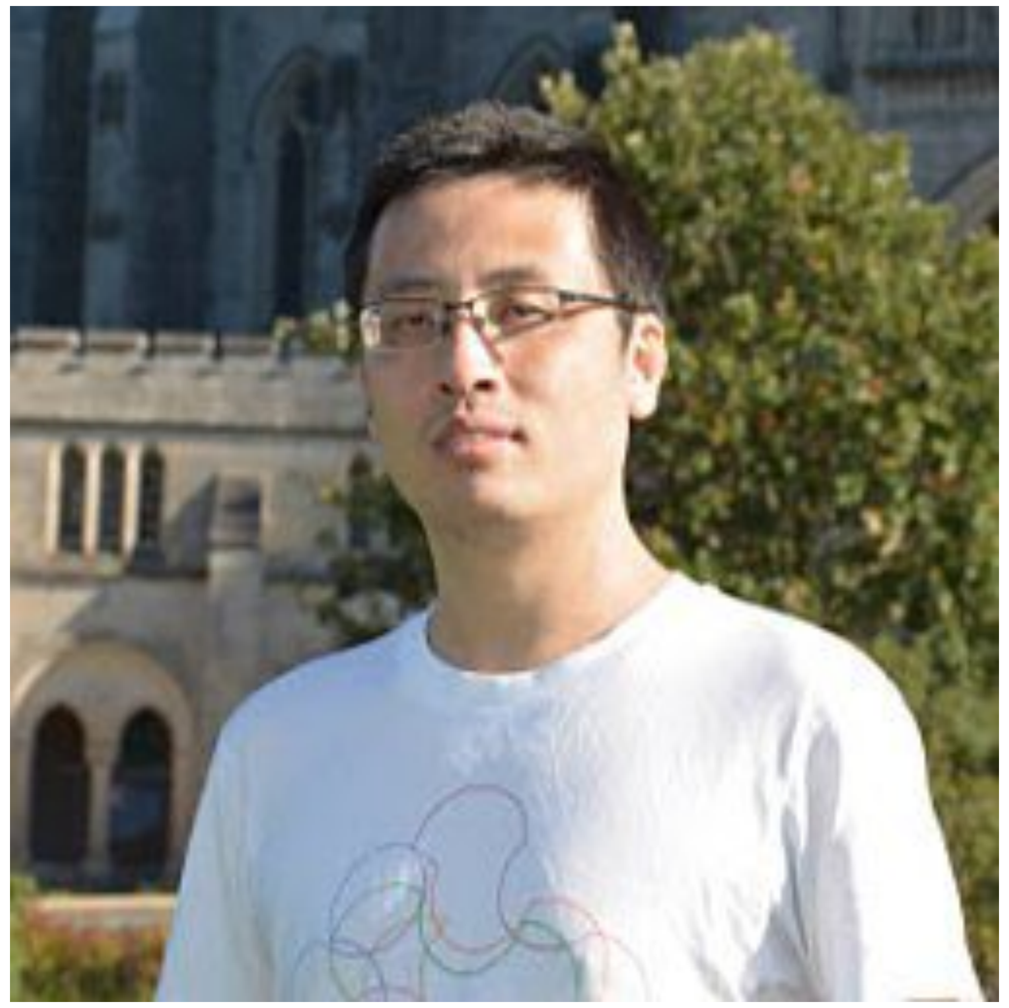
HARVARD


Maine Christos


Zhu-Xi Luo


Henry
Shackleton


Mathias Scheurer
Innsbruck→Stuttgart

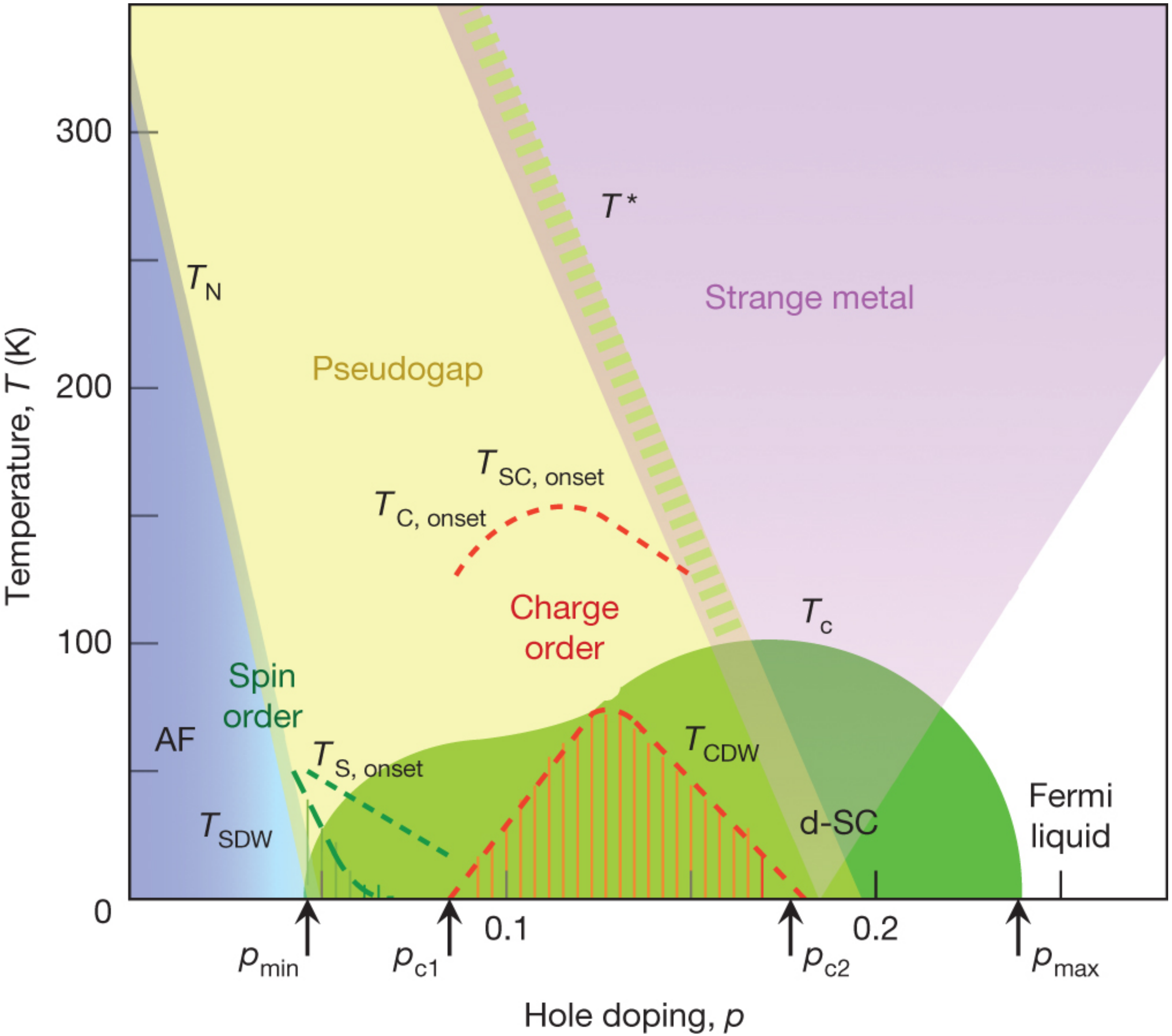

Ya-Hui Zhang
Johns Hopkins


Alexander
Nikolaenko


Darshan Joshi
TIFR Hyderabad

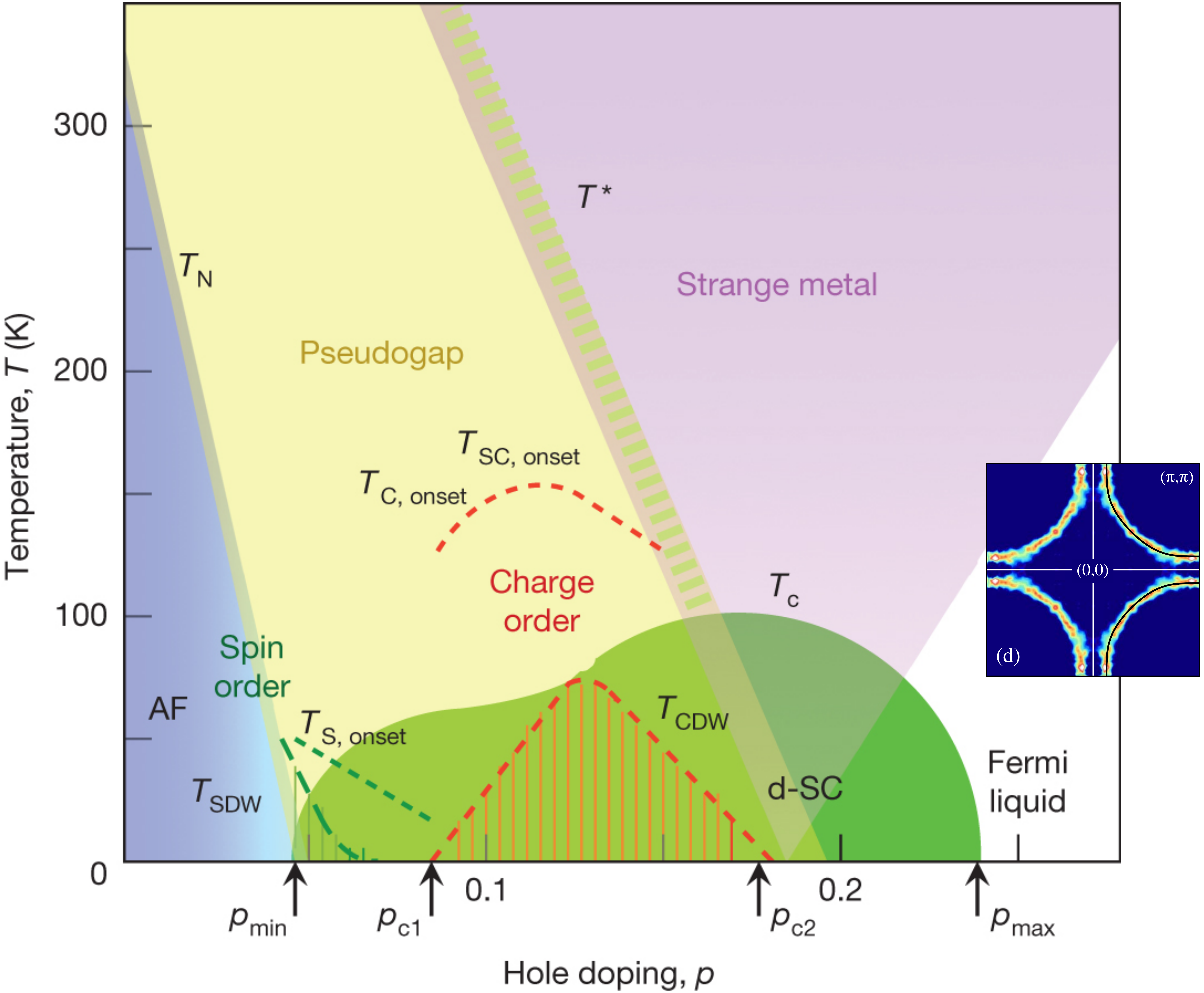

Jonas von Milczewski

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



Fermi liquid in the overdoped metal

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



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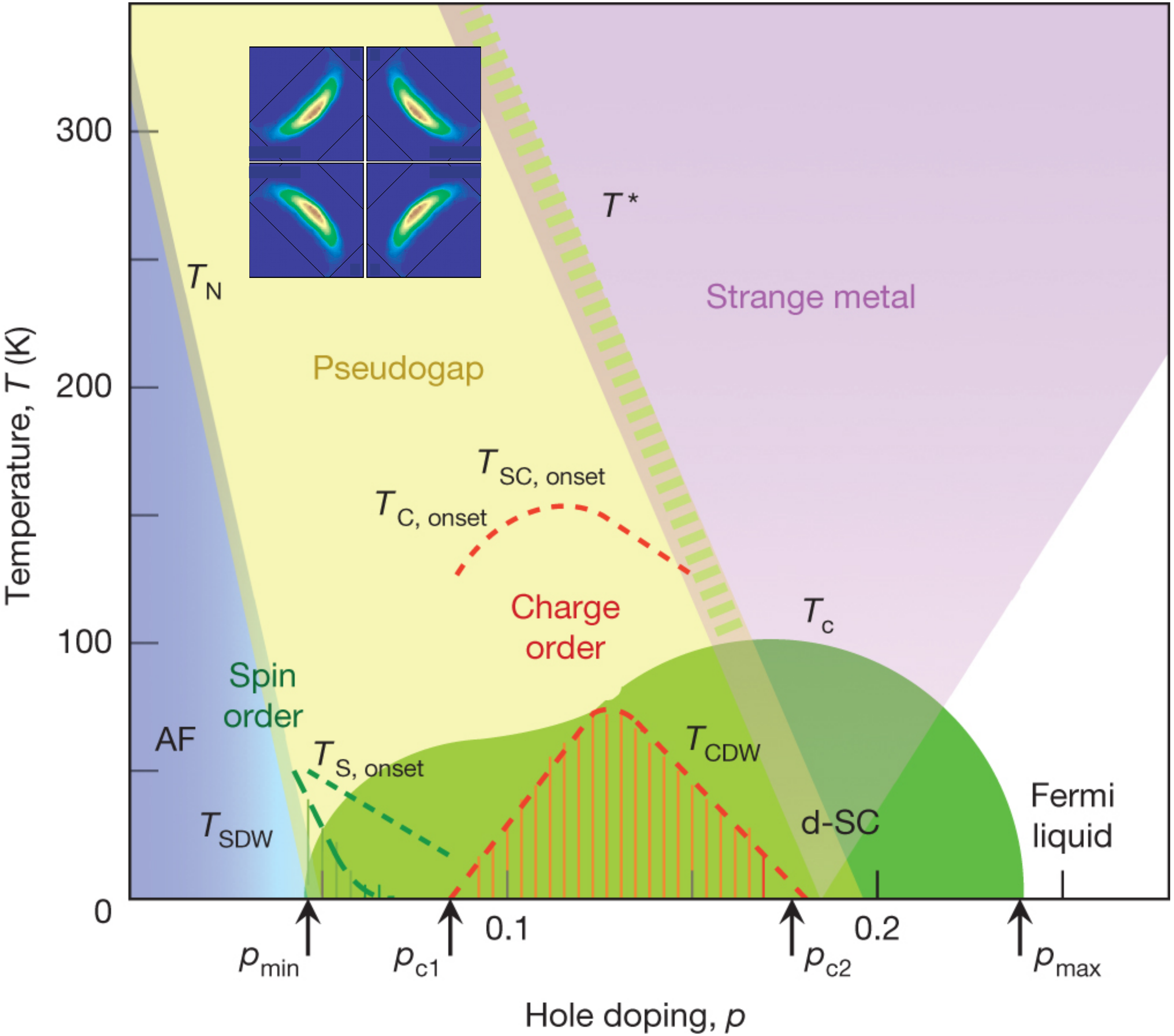

Theory for
"pseudogap metal"
with "Fermi arcs"?

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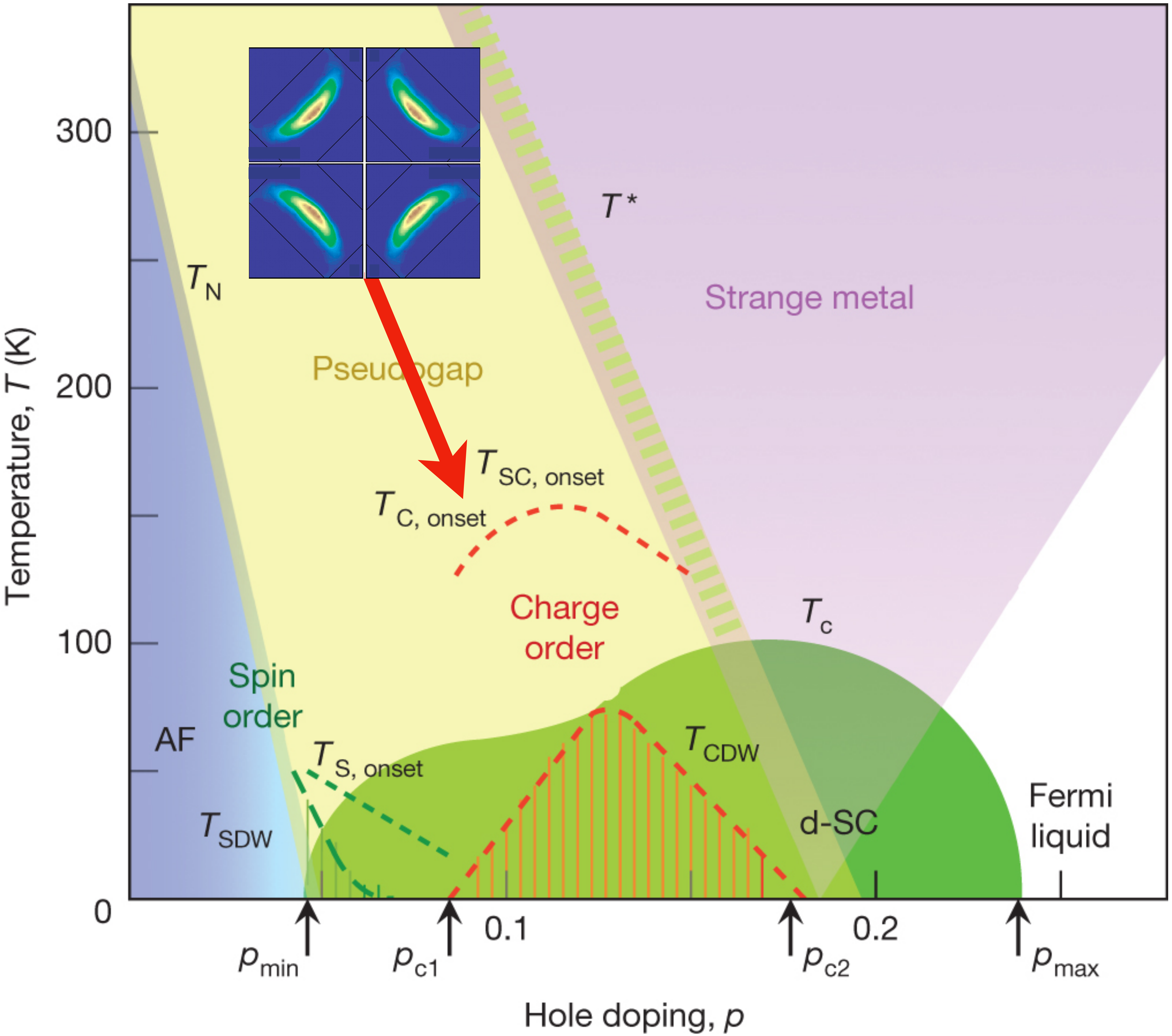

Needed: a theory for the onset of charge order and d-wave superconductivity from the pseudogap metal.

Why are T_c and T_{CDW} about the same?

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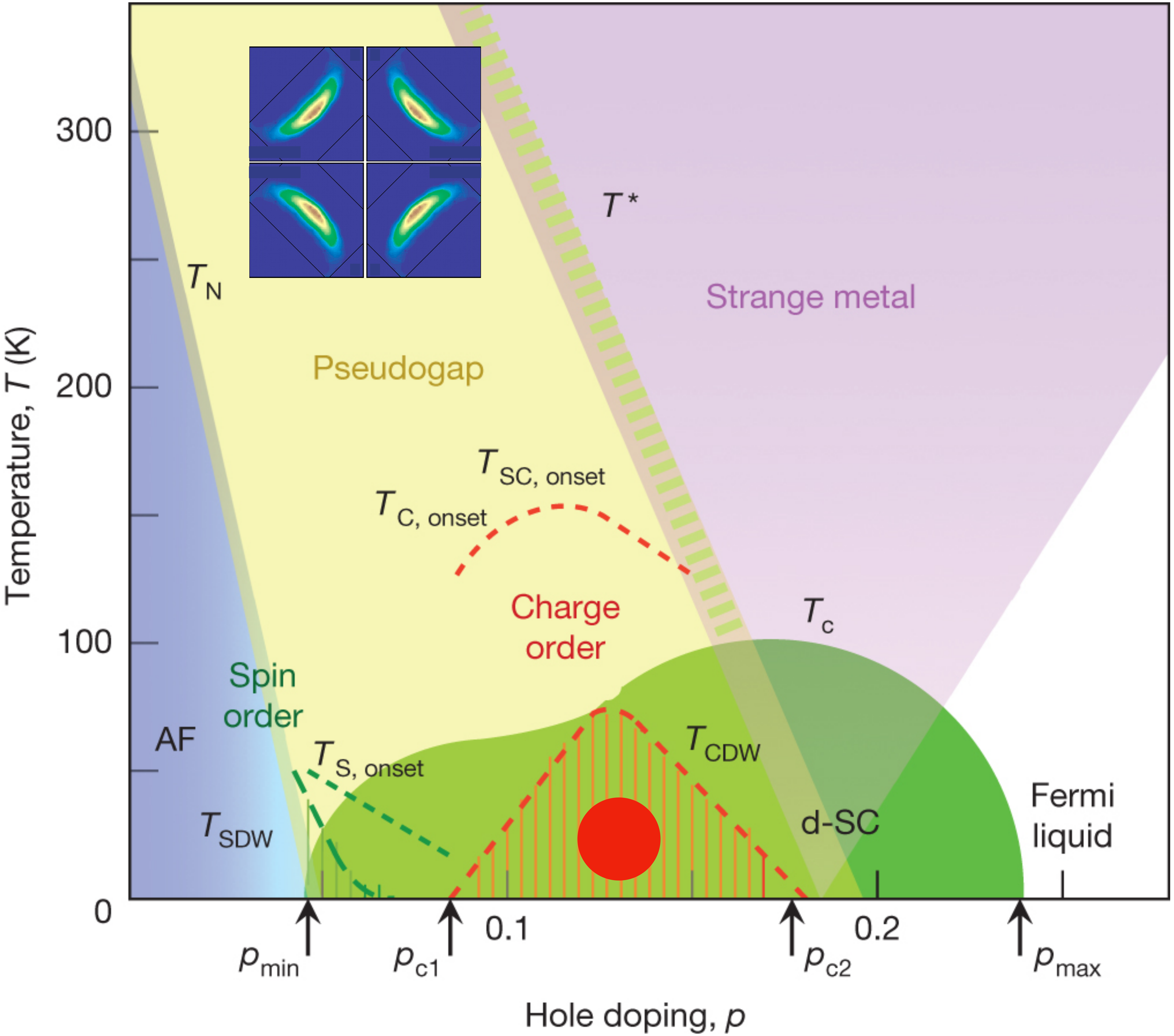

Quantum oscillations in the CDW phase at low T show only a single electron pocket.

This cannot be obtained in the theory of CDWs in a Fermi liquid.

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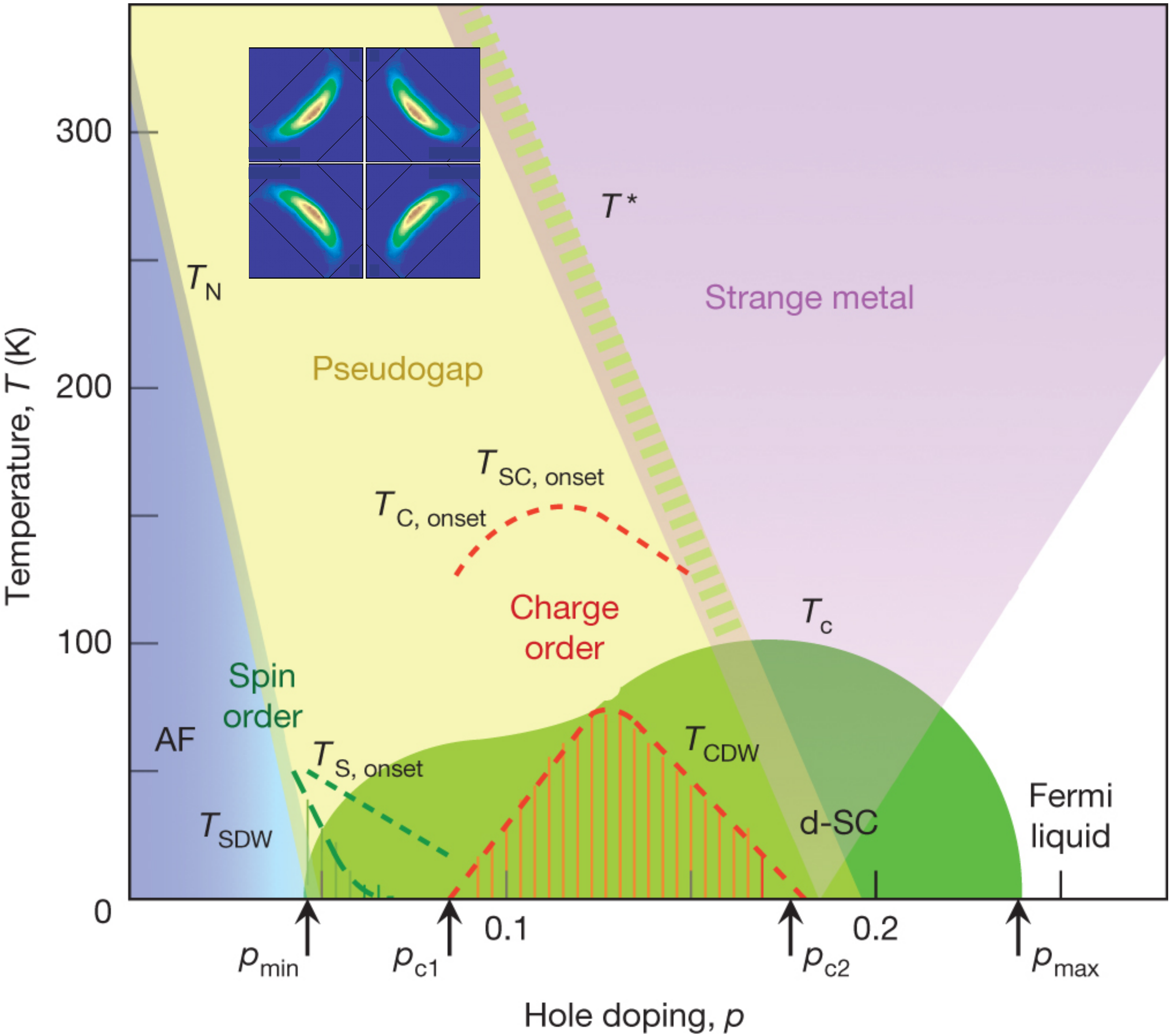

Theory for
"pseudogap metal"
with "Fermi arcs"?

Use the pseudogap metal
in place of the Fermi liquid
as the 'parent' to
conventional
d-wave superconductor,
charge density wave,
spin density wave,
pair density wave
...

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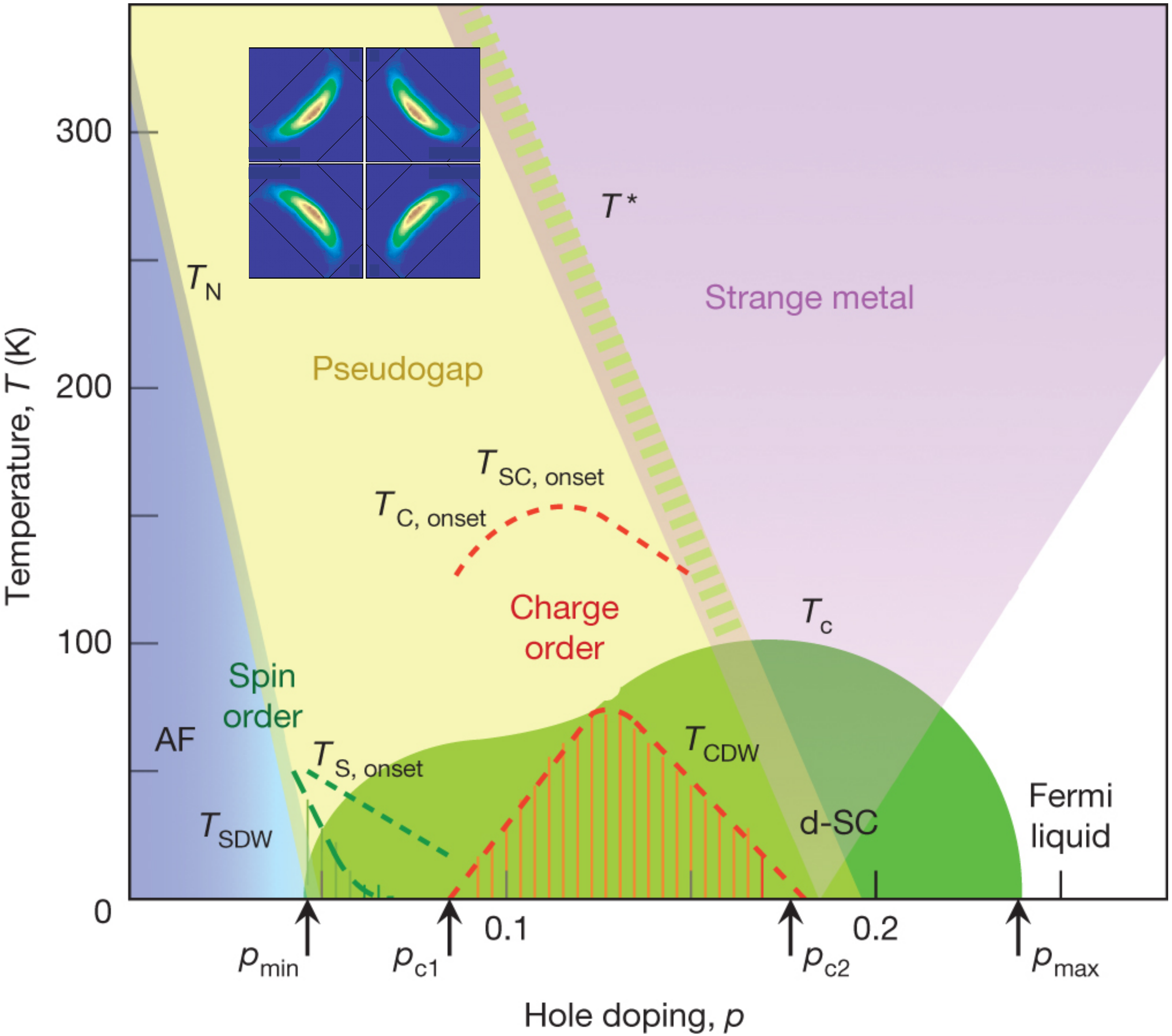
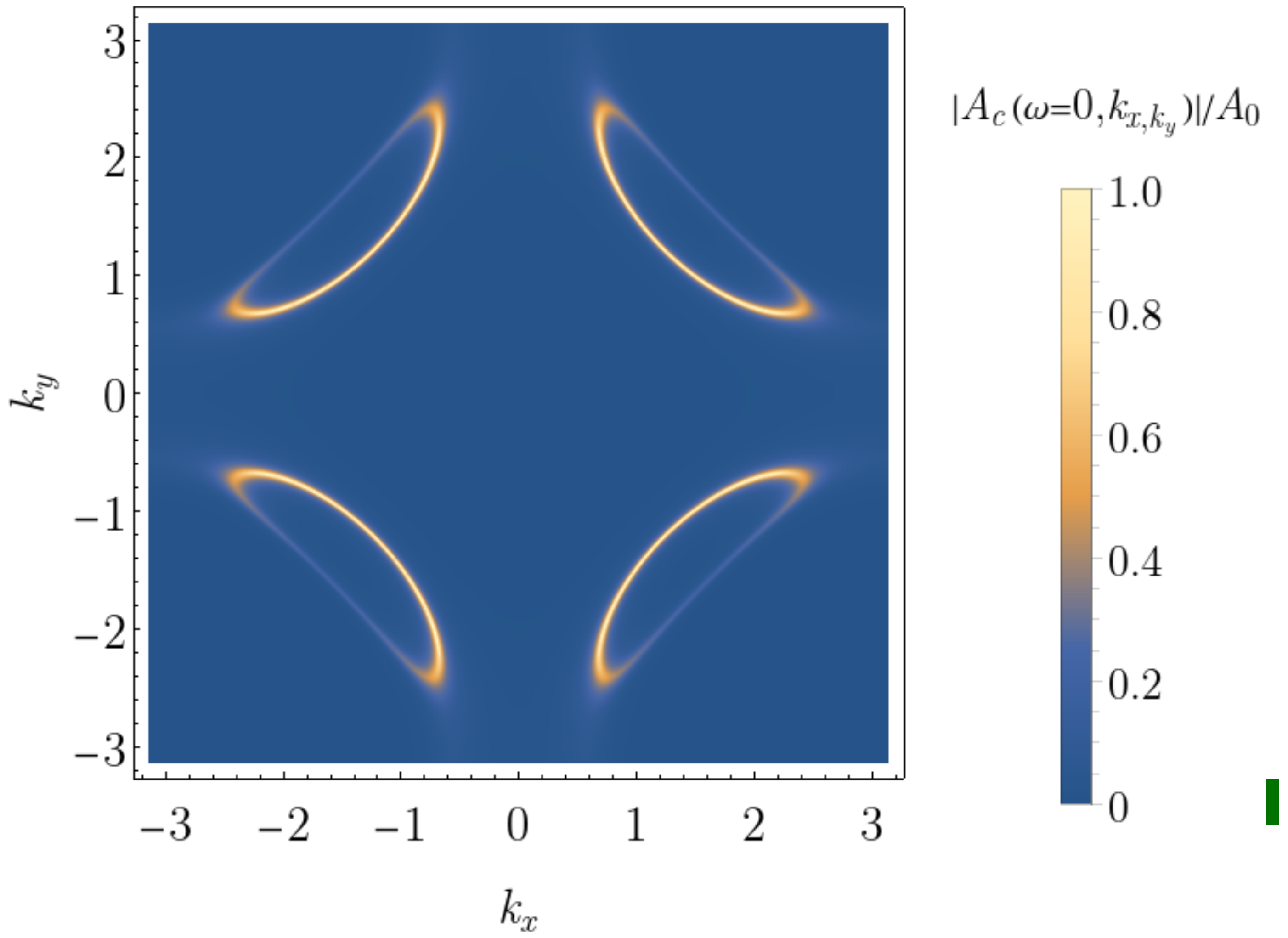




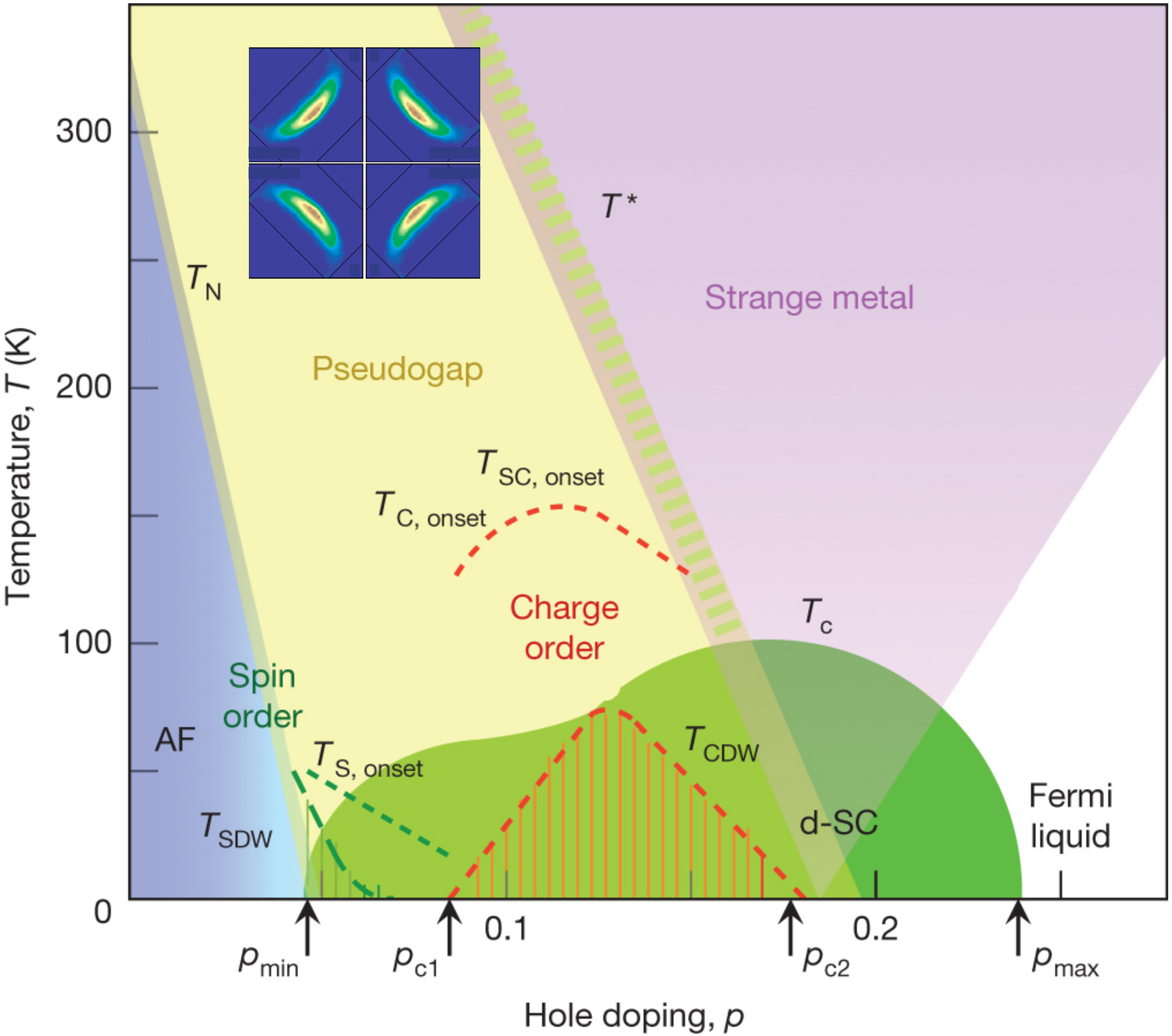
Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. Sachdev, PRB **105**, 075146 (2022)

Hole pocket Fermi surfaces
of size p with
charge e, spin-1/2 quasiparticles

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar, PRB **74**, 125110 (2006).
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, PRL **97**, 136401 (2006).
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009).
J. Skolimowski and M. Fabrizio, PRB **106**, 045109 (2022).

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



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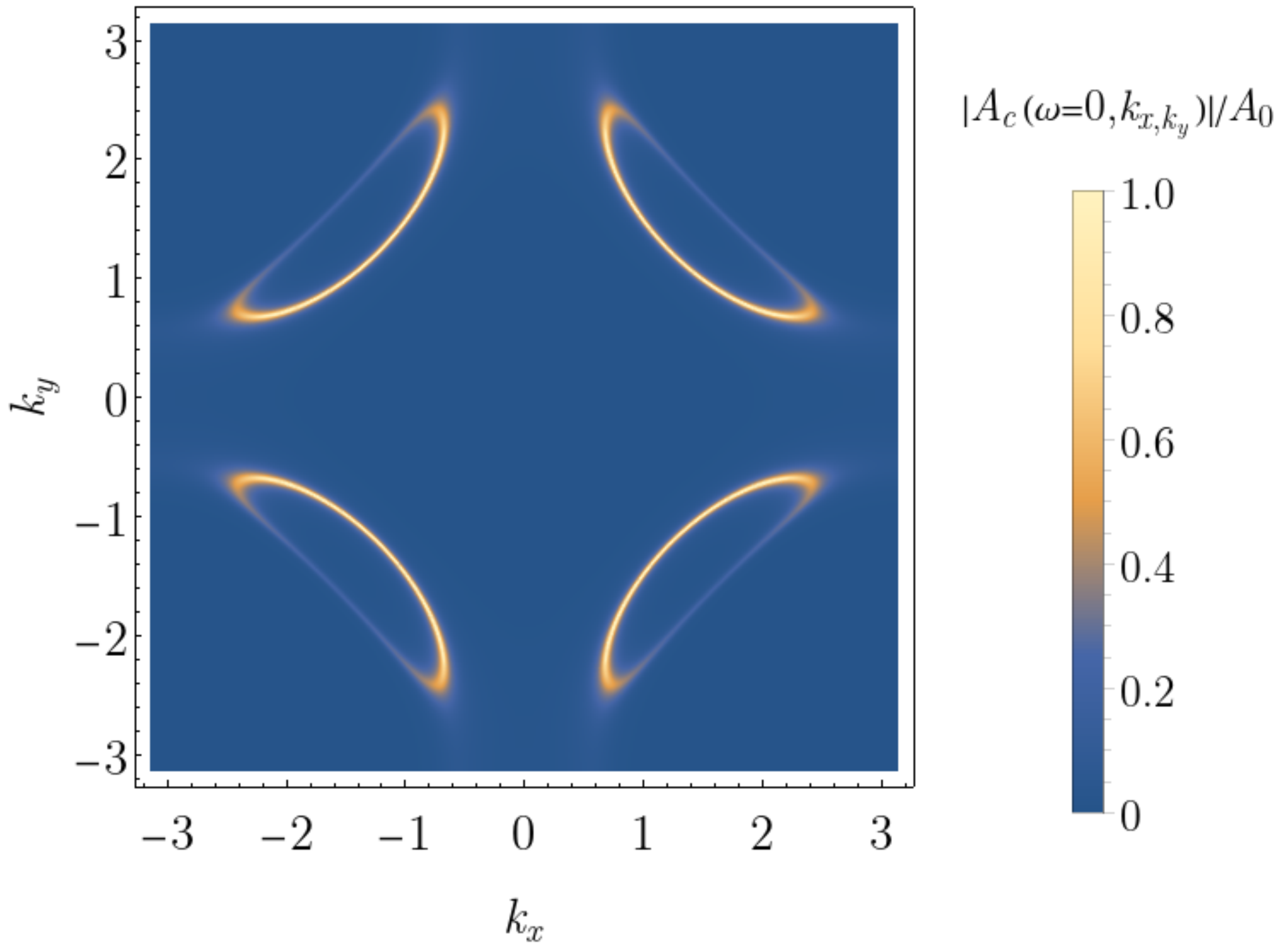


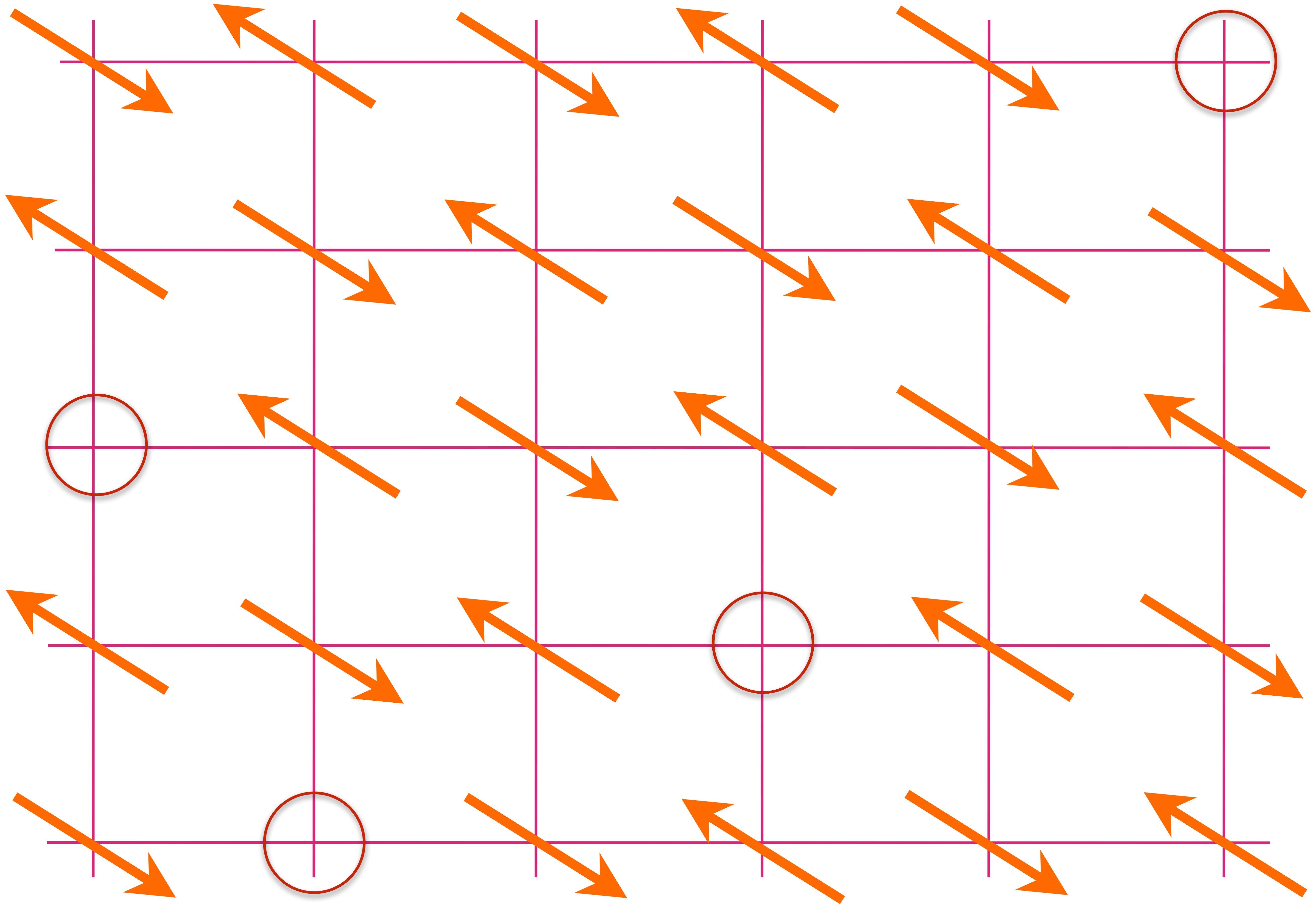
Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

$+$

'spectator' square lattice spin liquid *at half-filling.*

FL*: Spin liquid is *required* because the Fermi surface does not enclose the Luttinger volume $(1 + p)$.

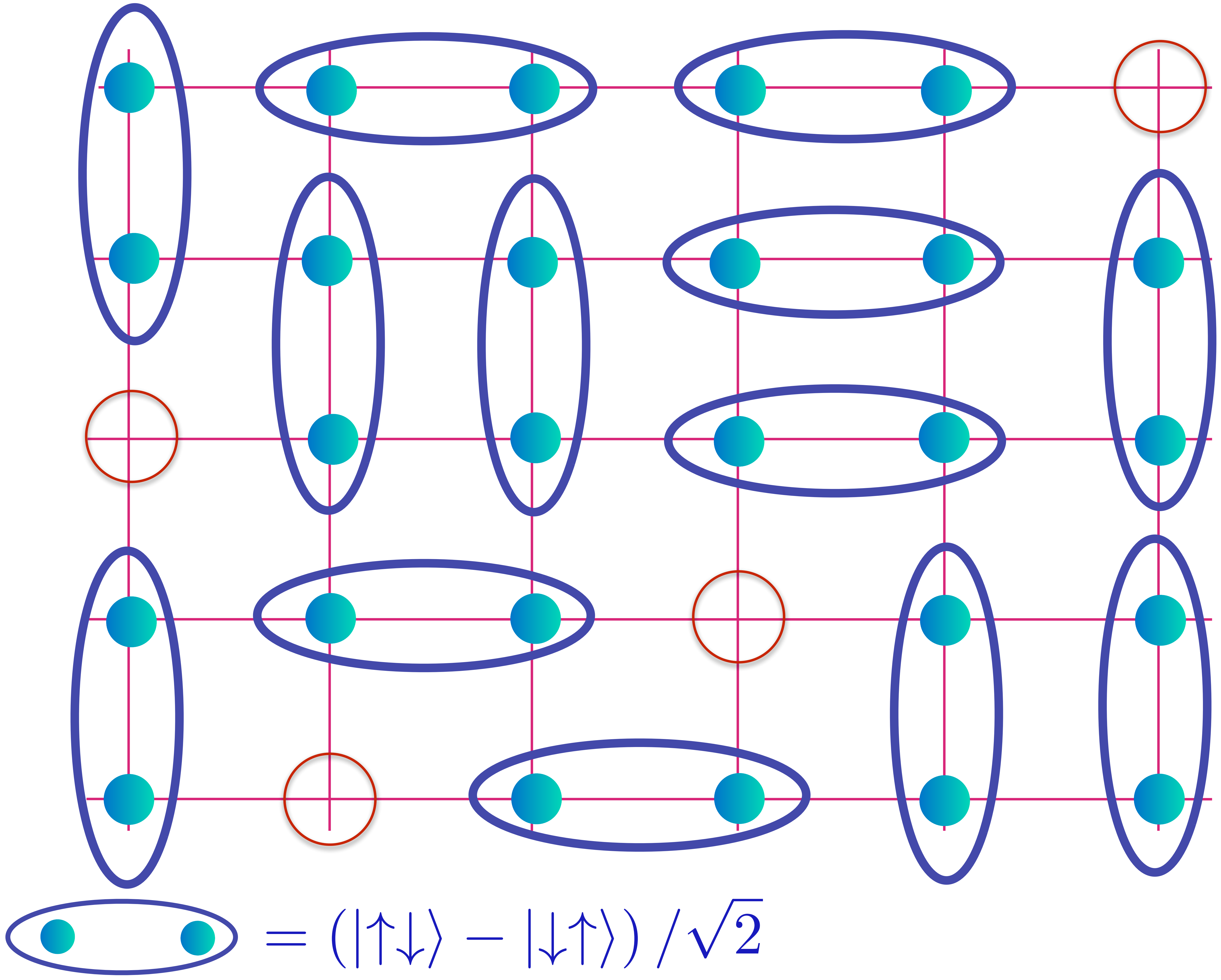
Earlier approach to FL* in a ***one-band*** model



Anti-ferromagnet with p holes per square

Holon metal

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)
D. Rokhsar and S.A. Kivelson, PRL **61**, 2376 (1988)



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Spin liquid with density *p* of spinless, charge +e "holons".

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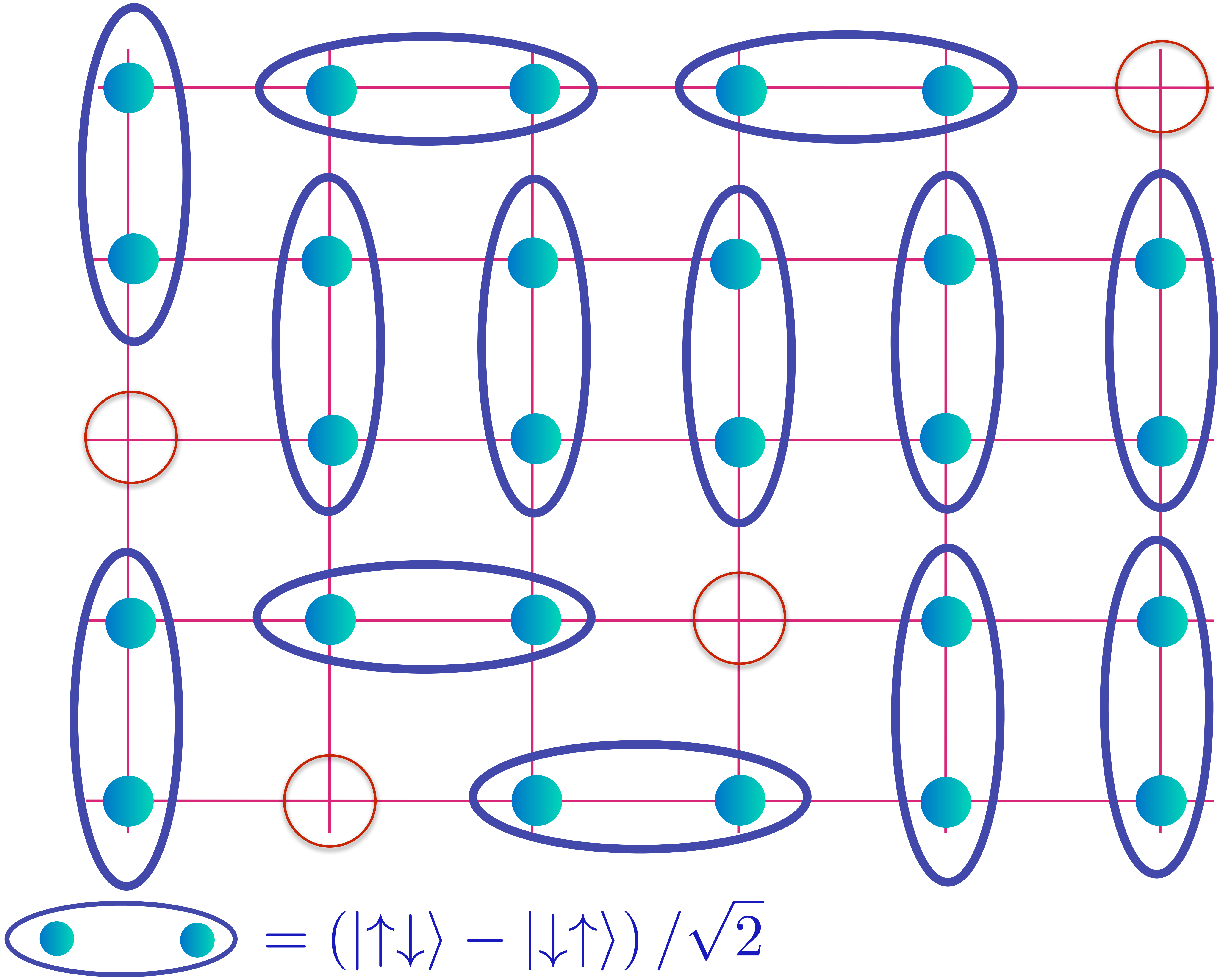

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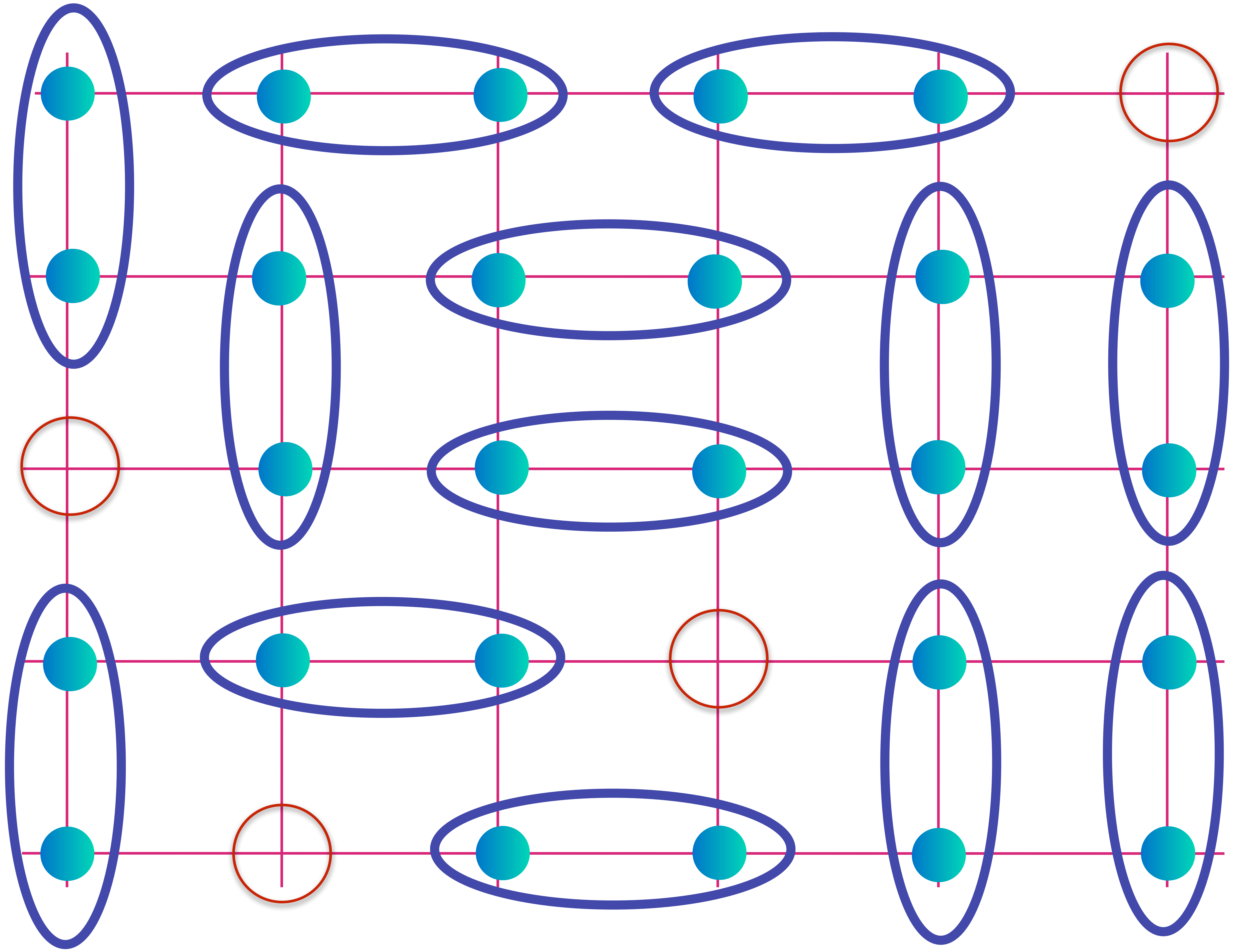

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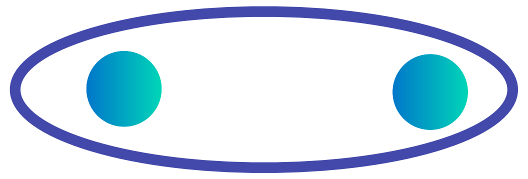

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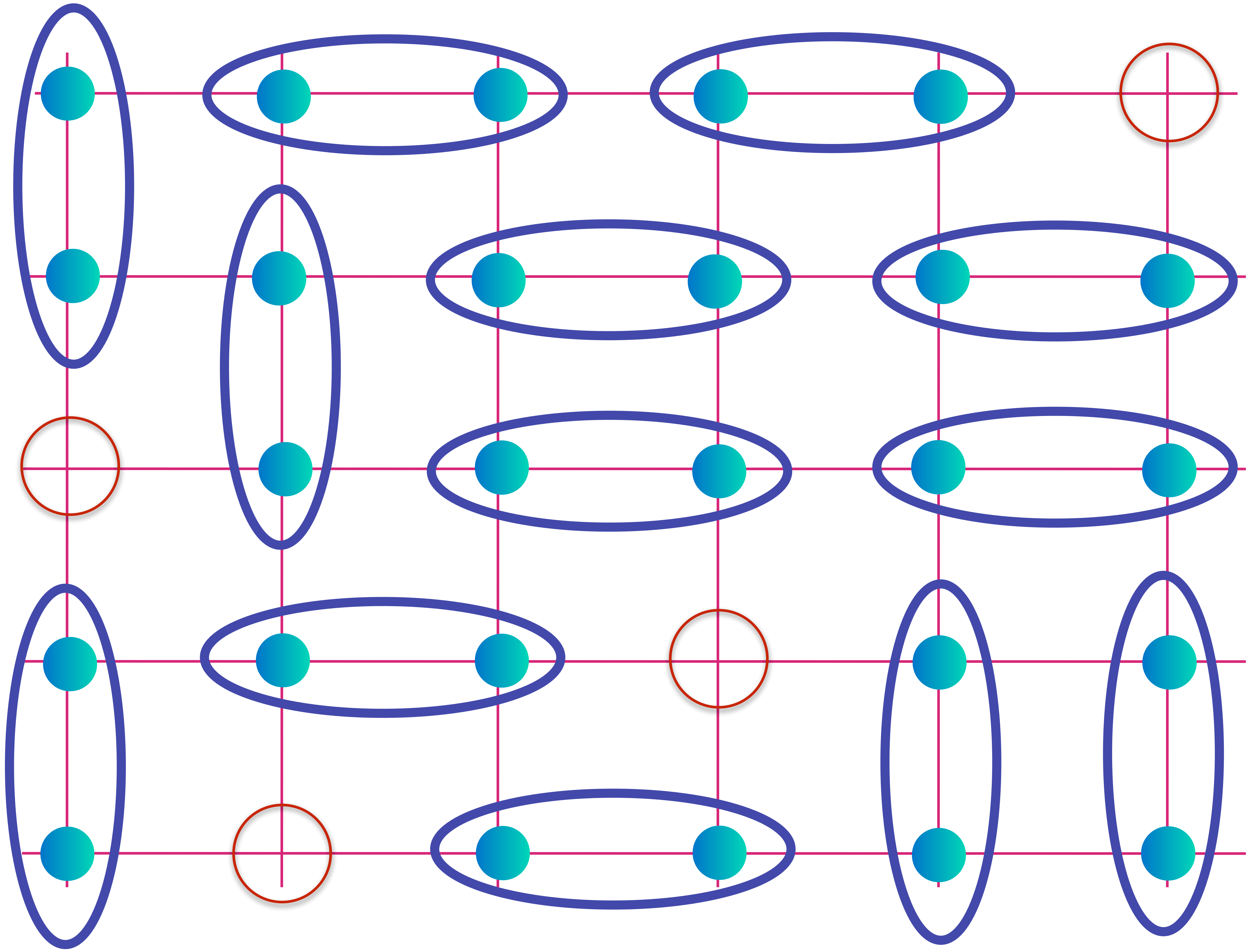

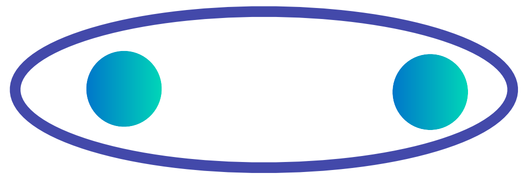

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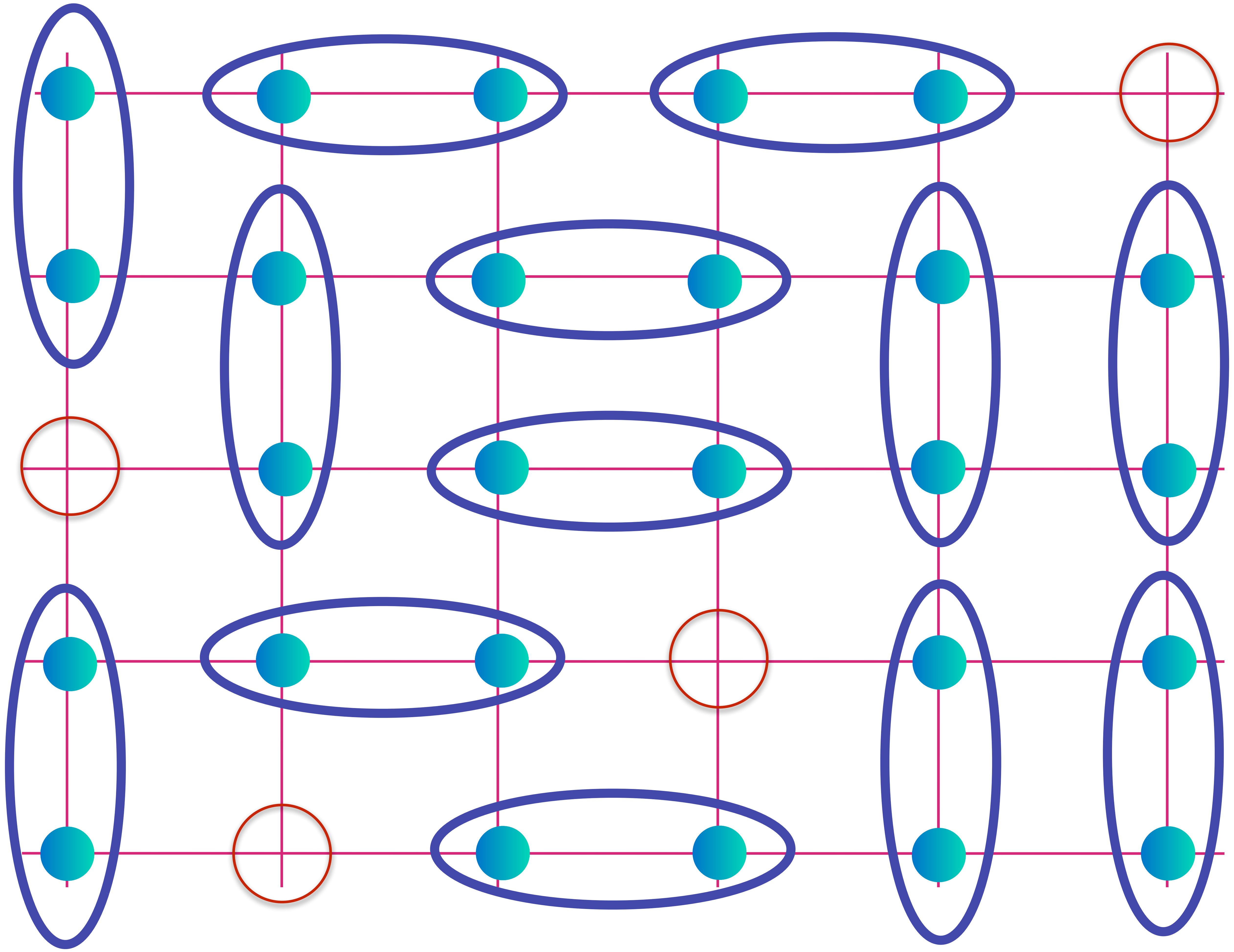

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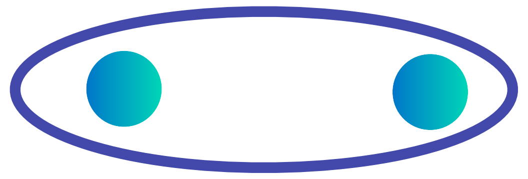

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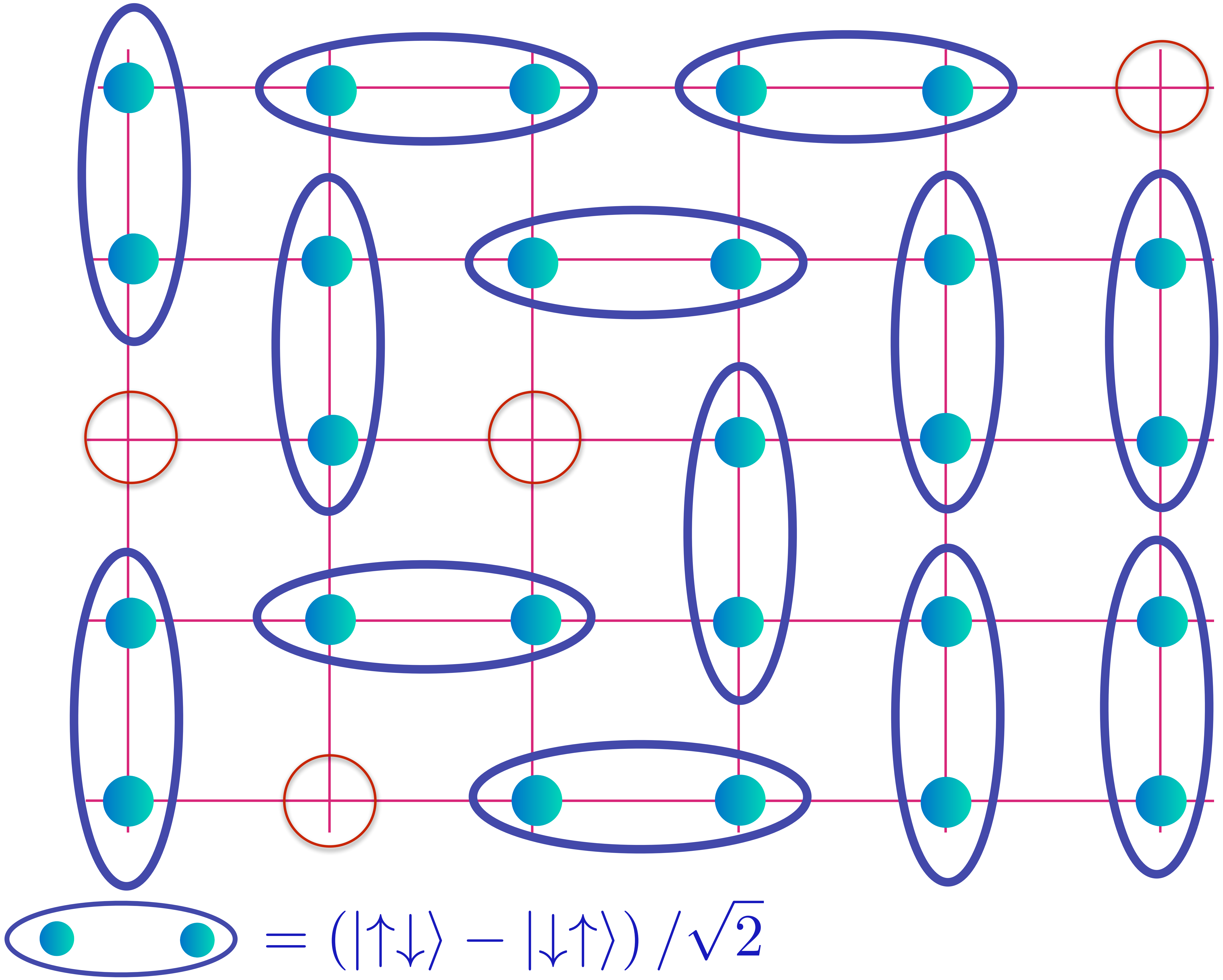

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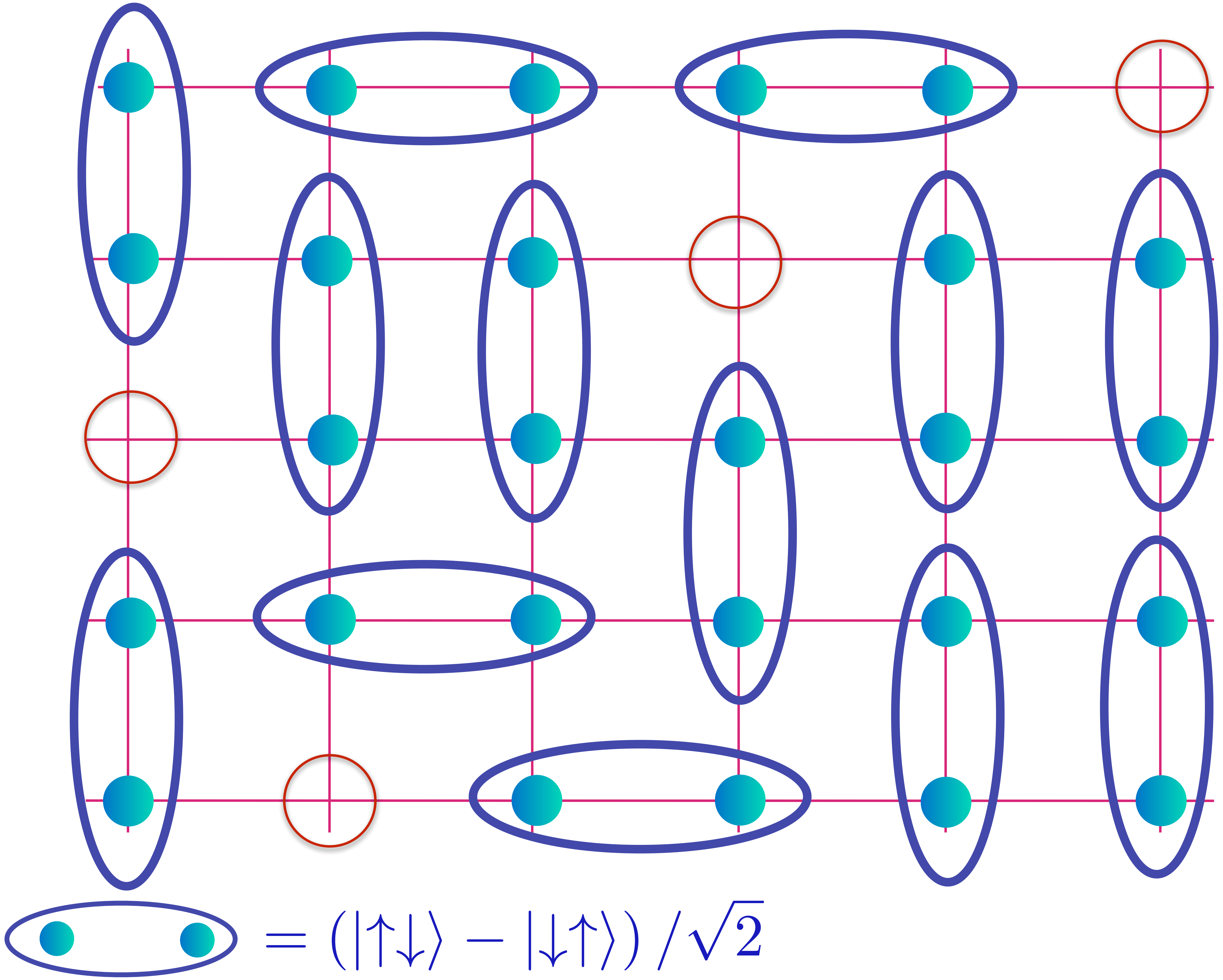

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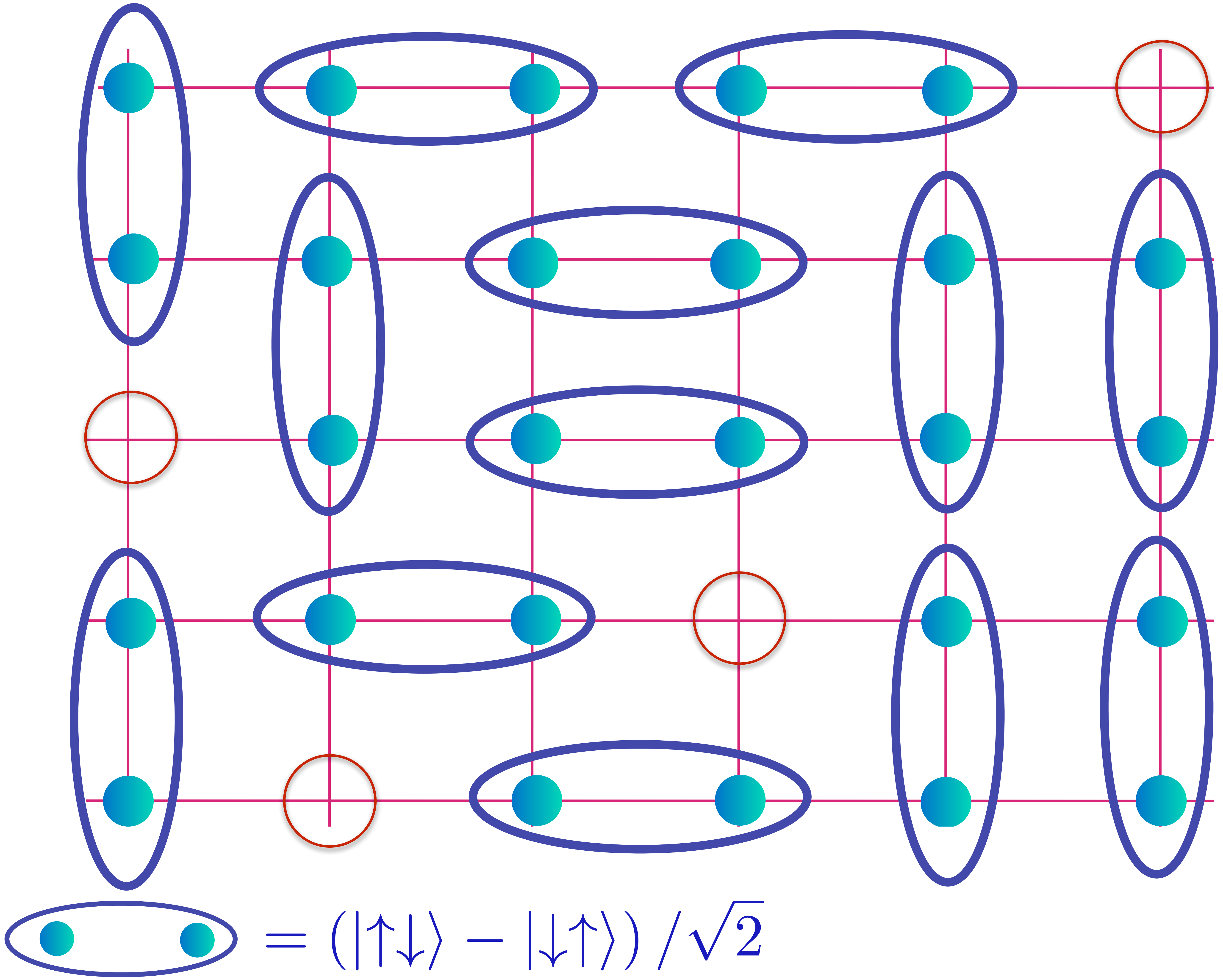

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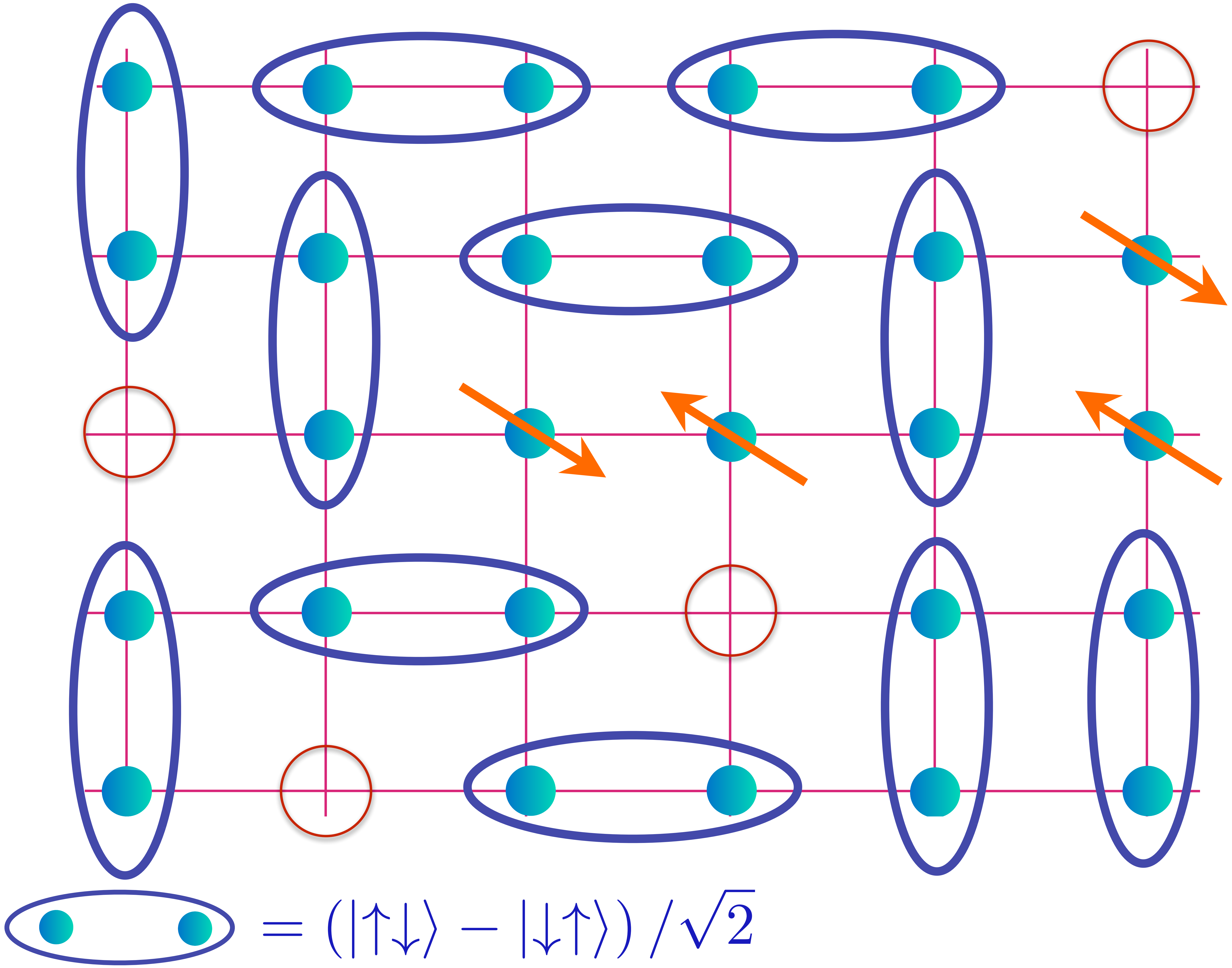

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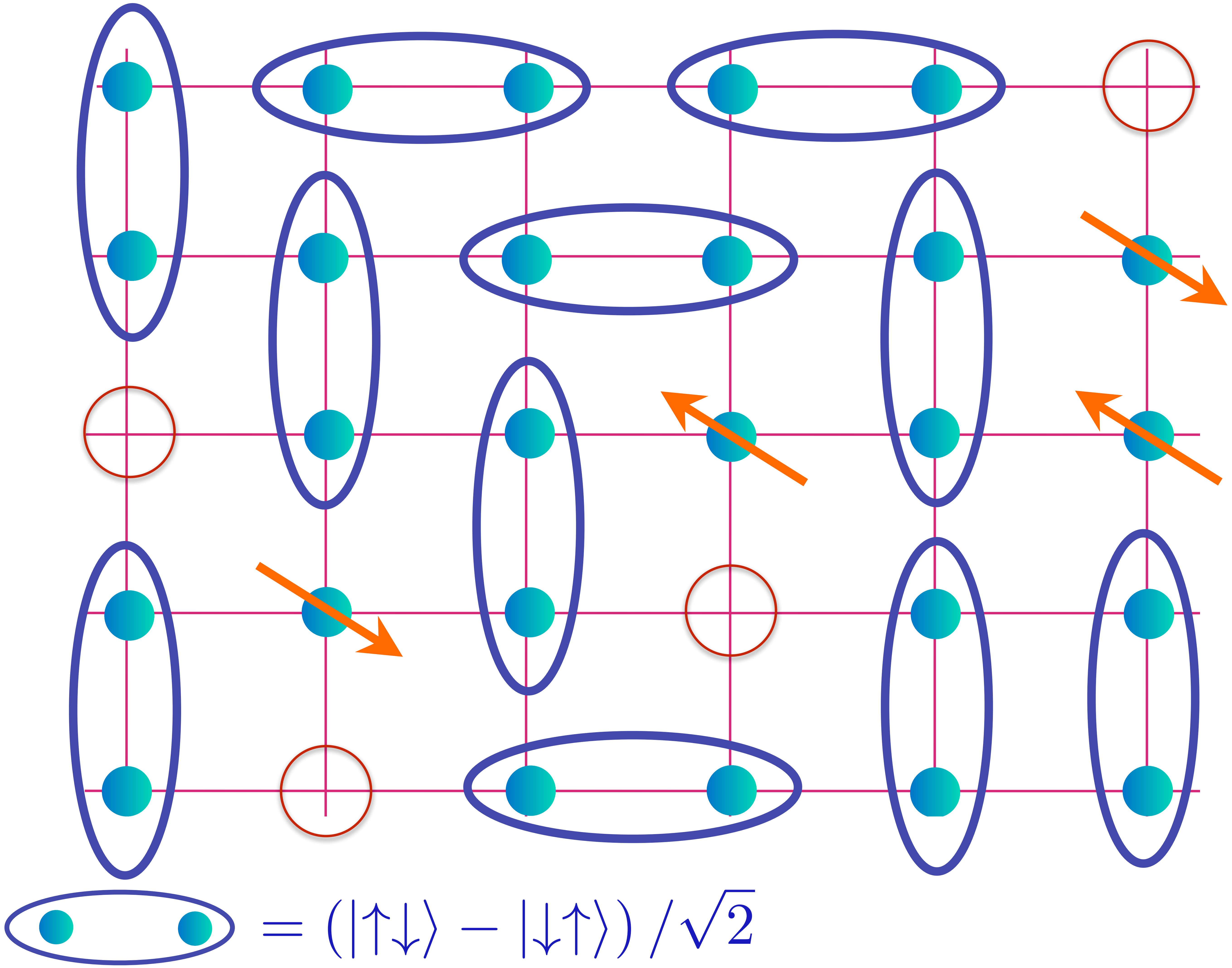

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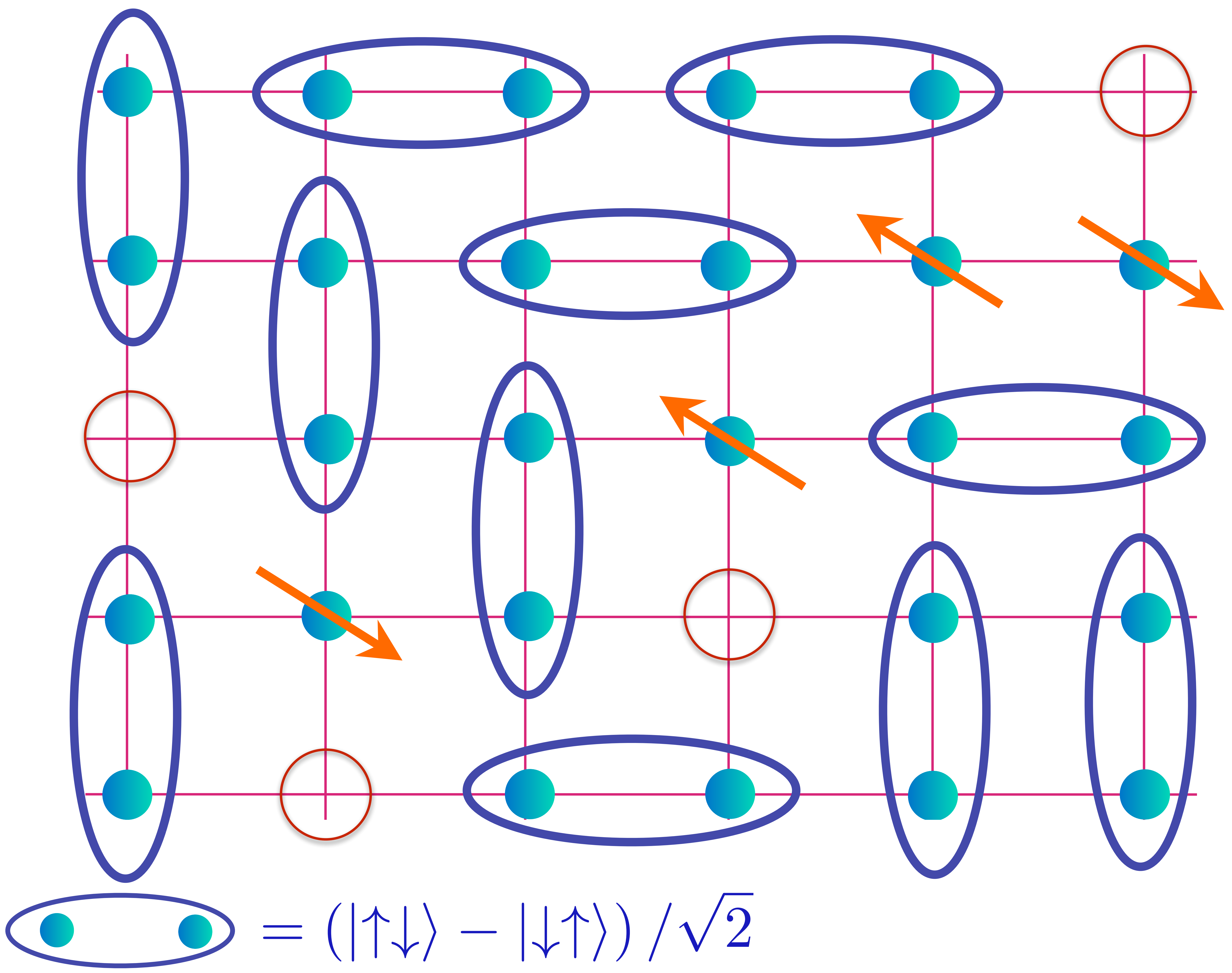

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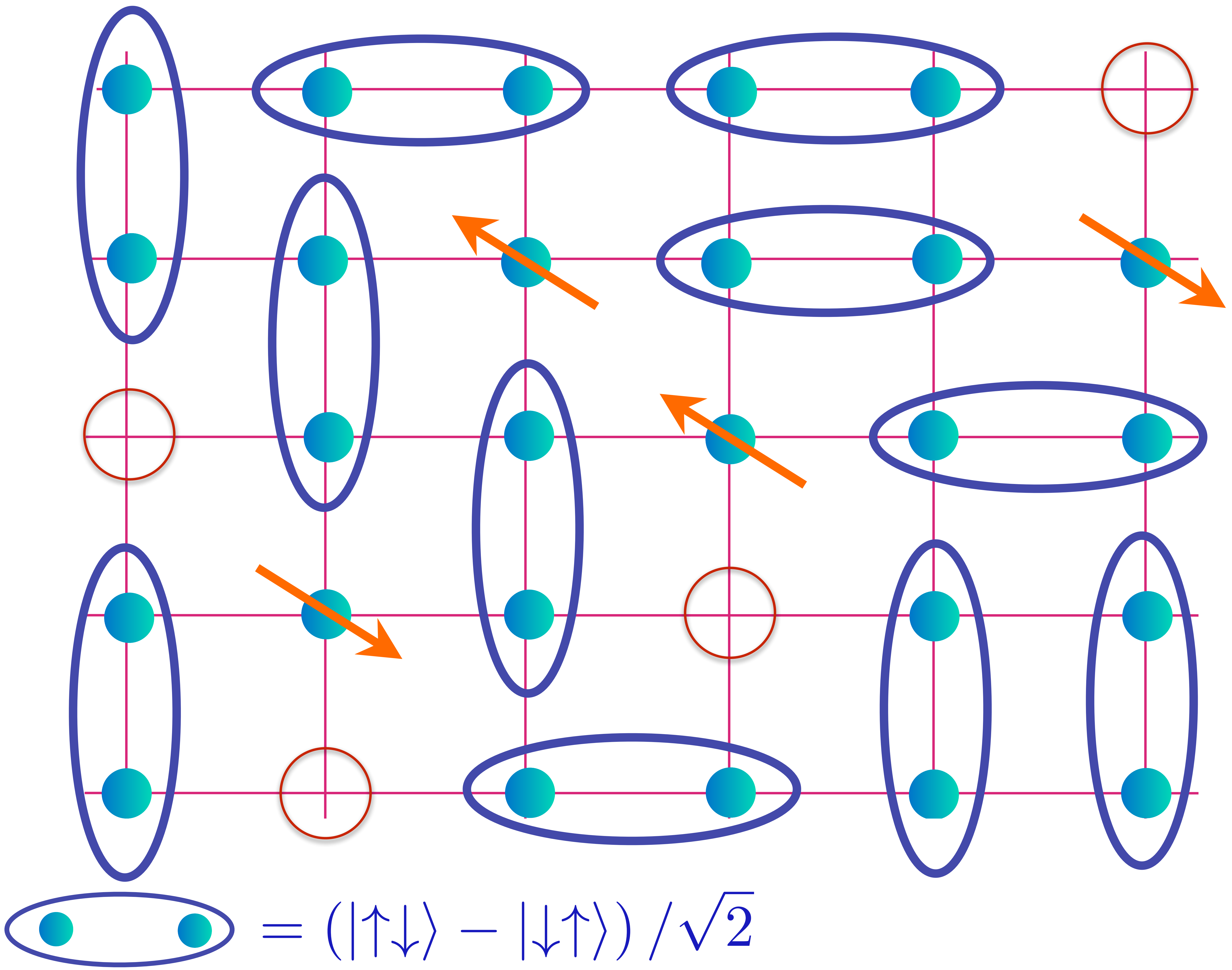

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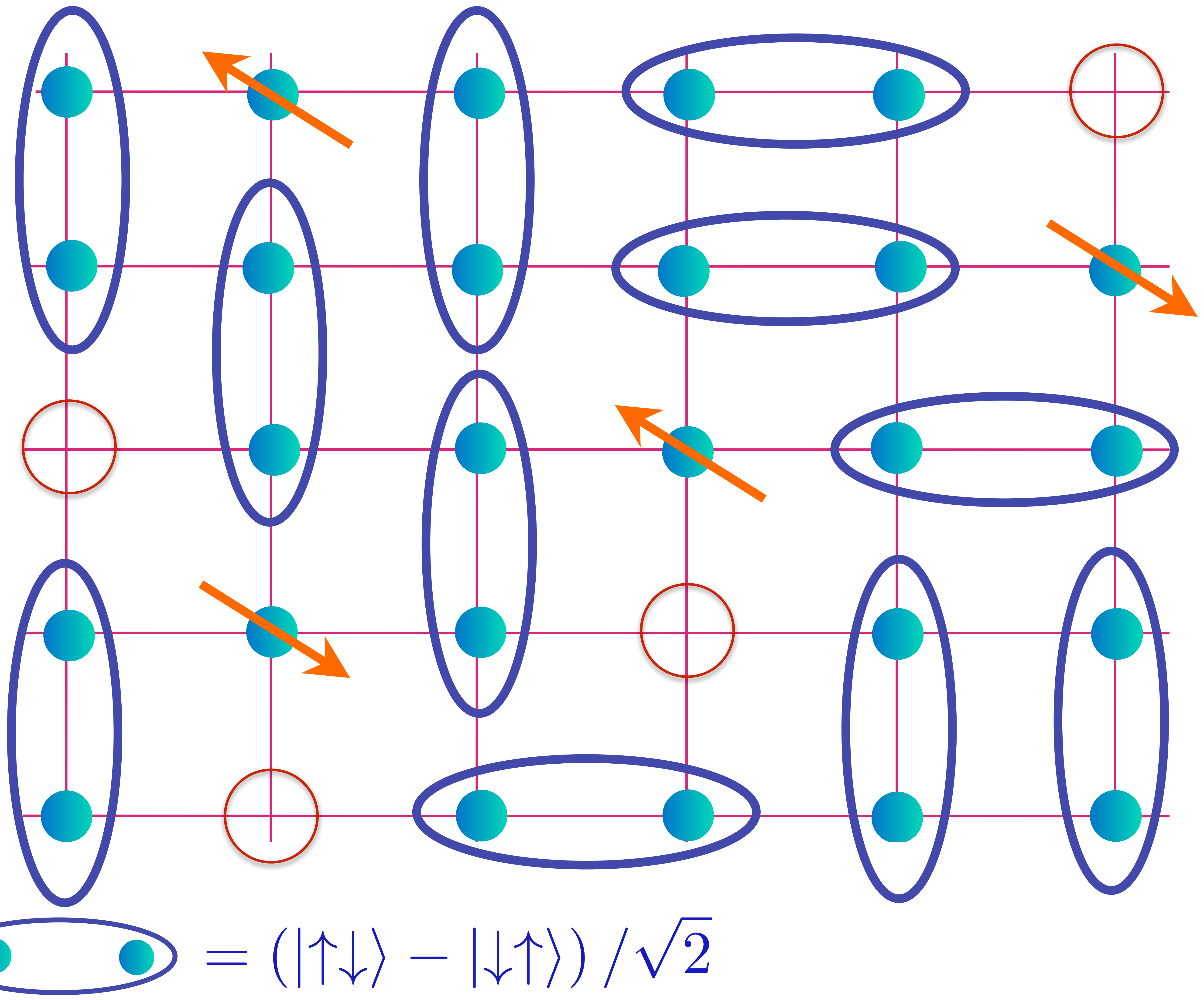

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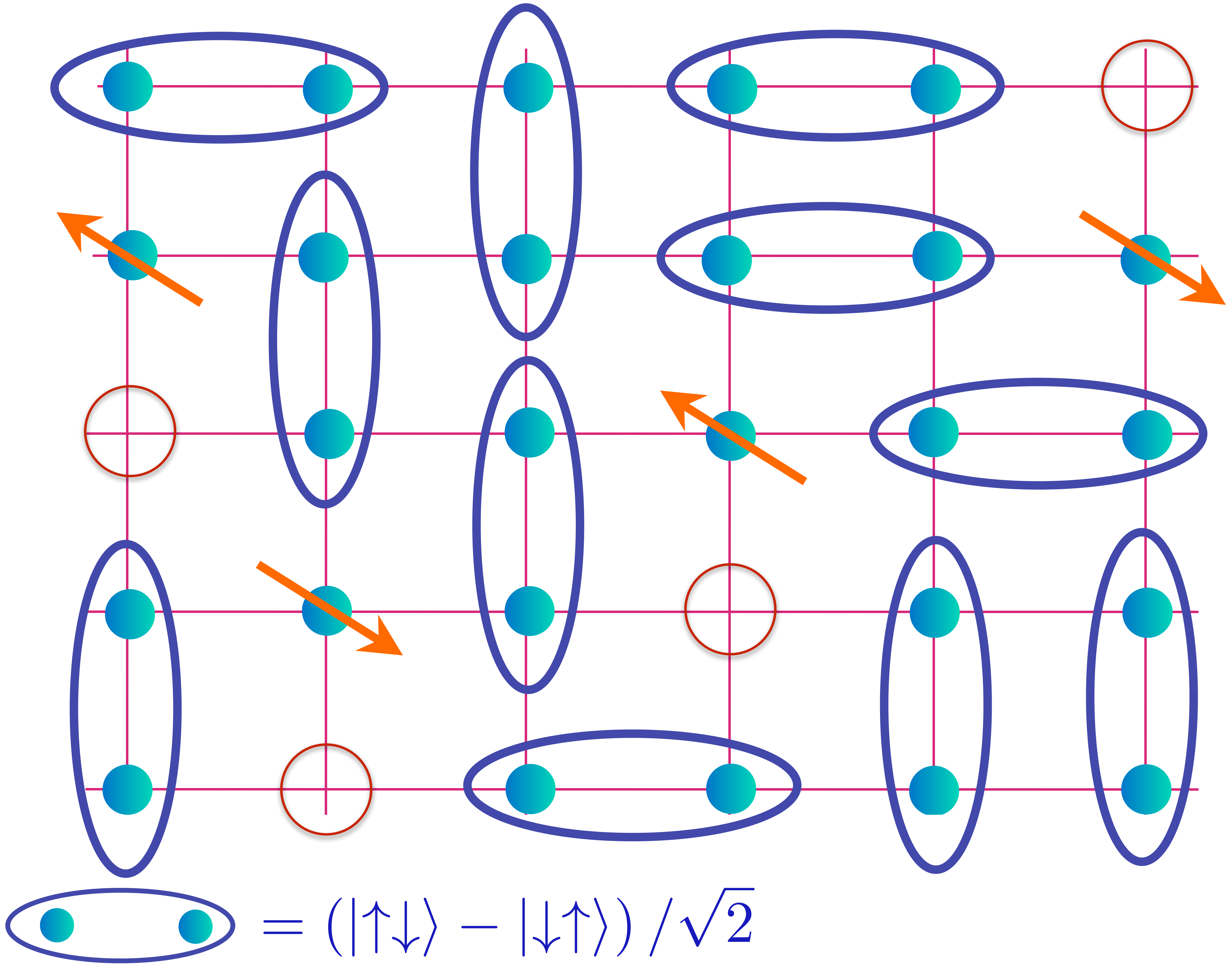

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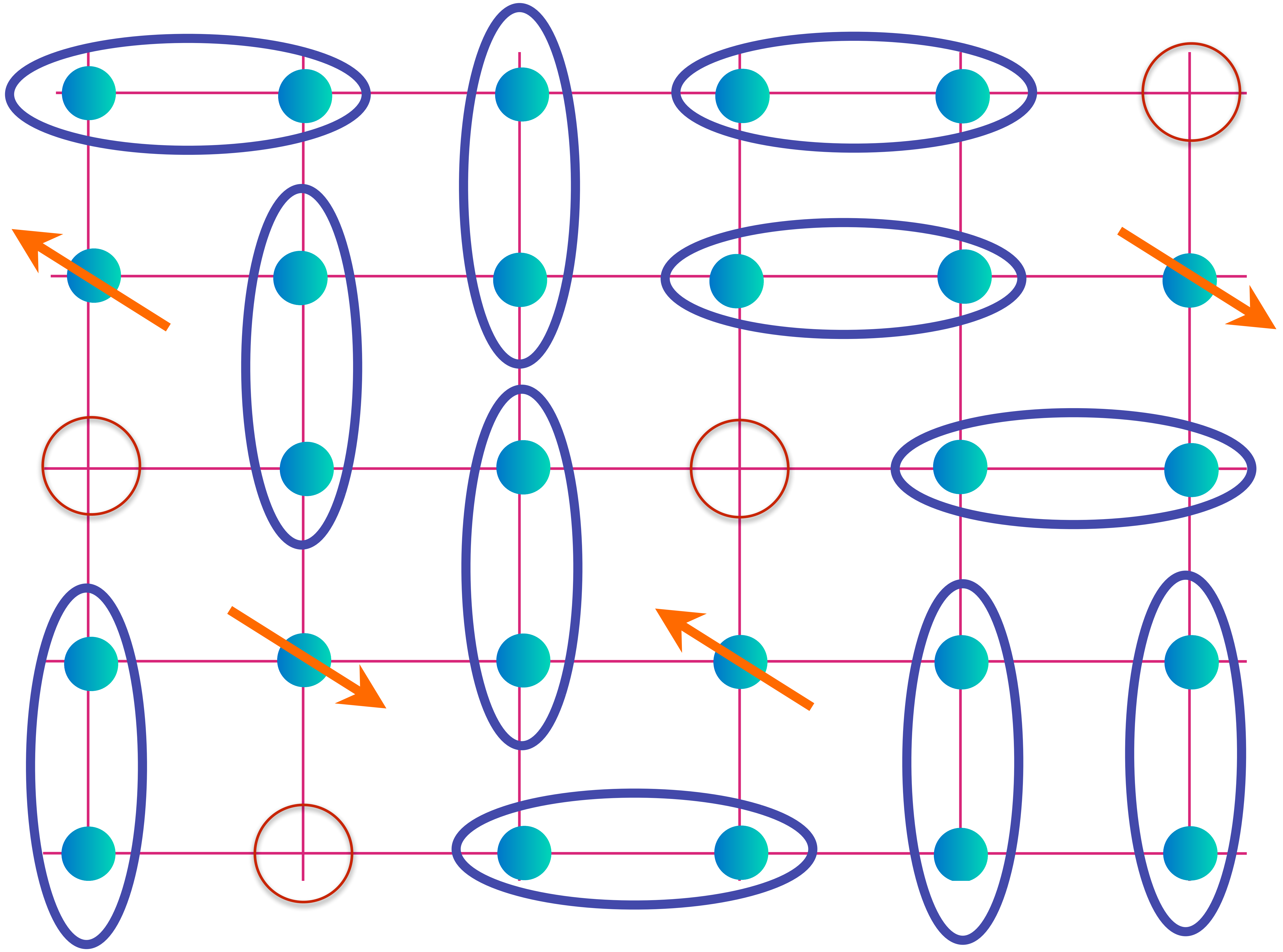

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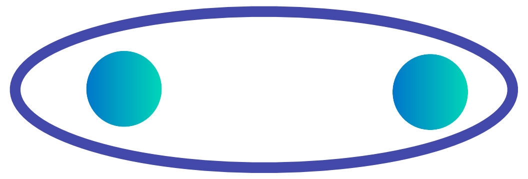

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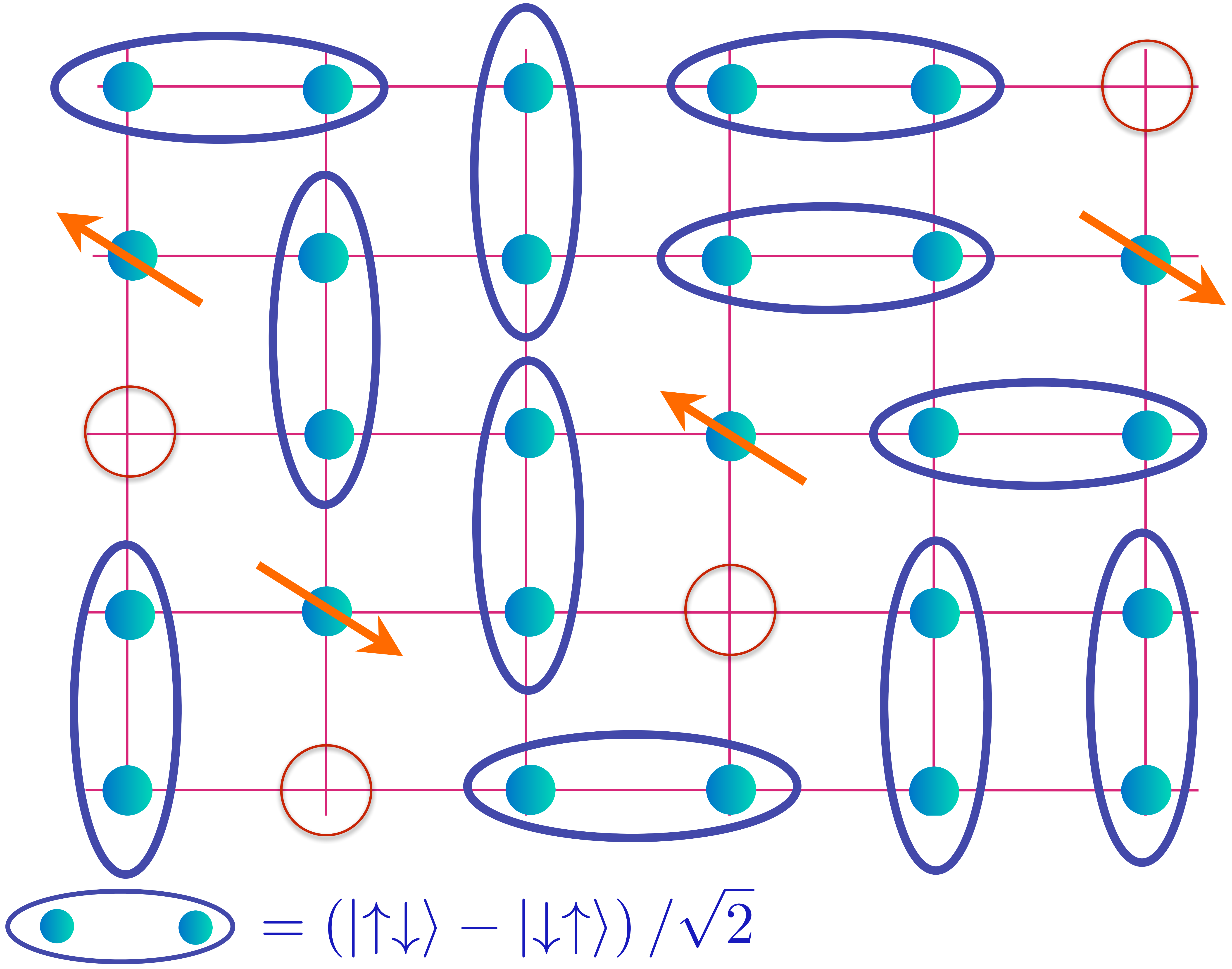

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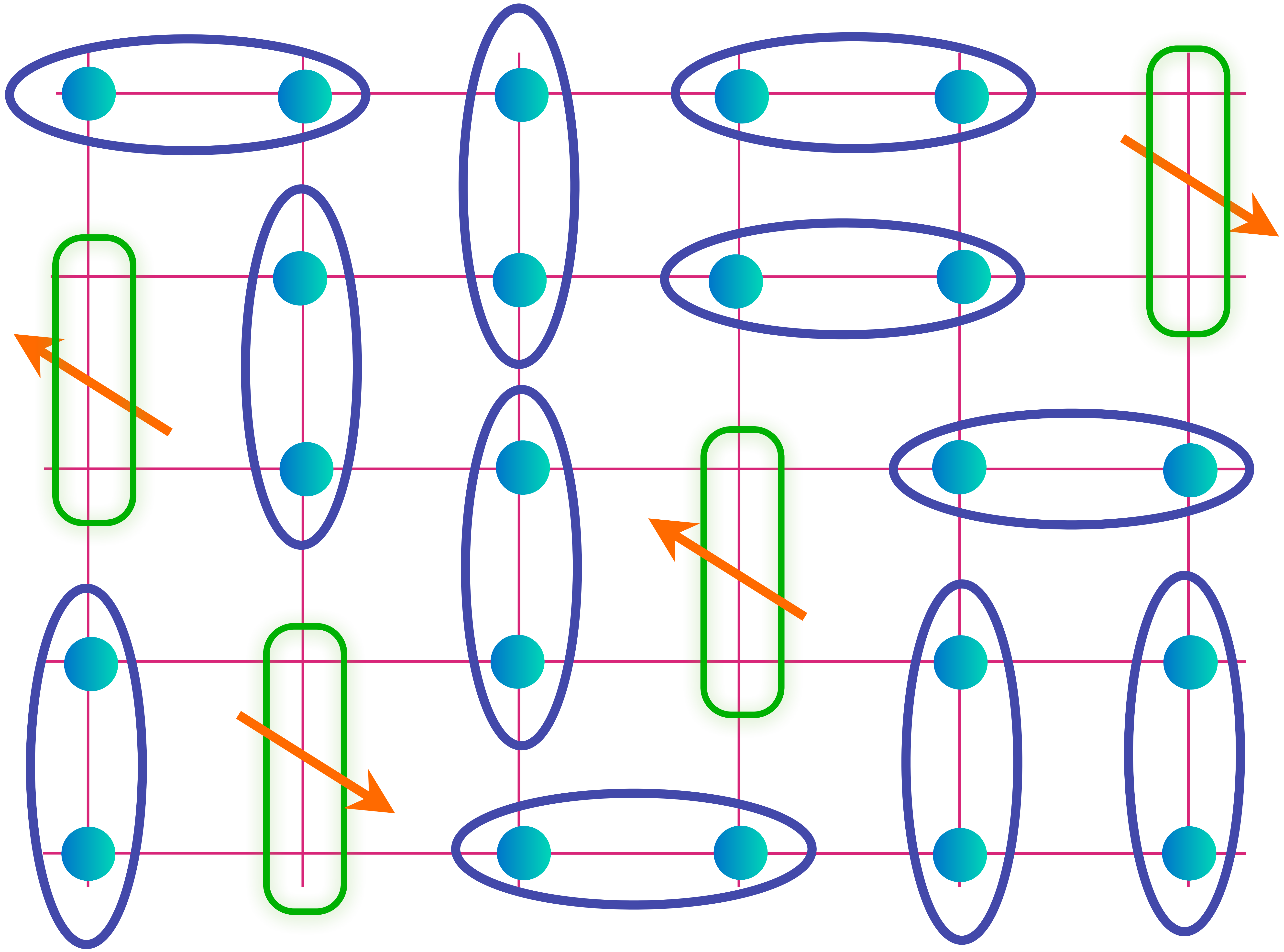
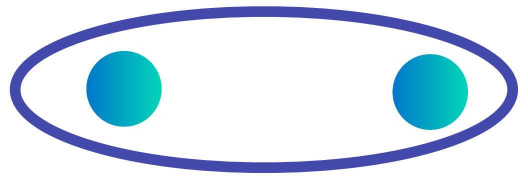
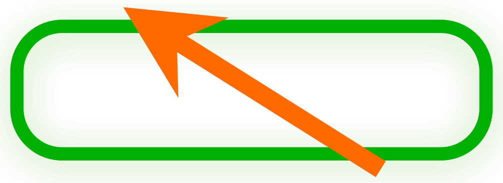
Earlier approach to FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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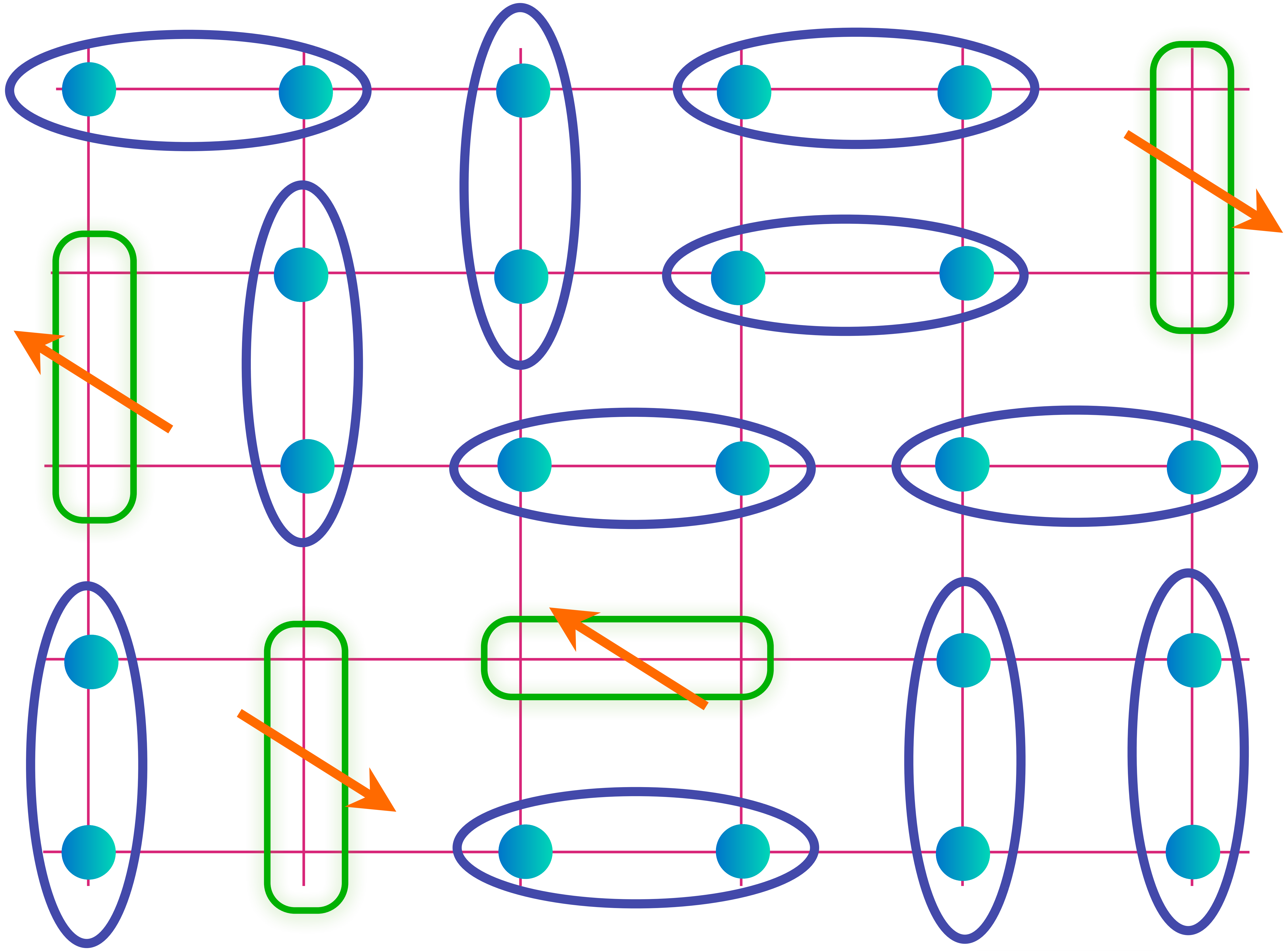
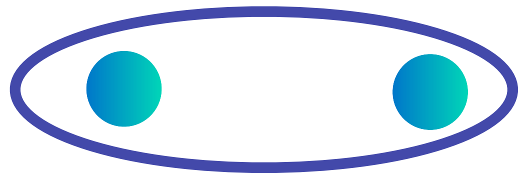


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields



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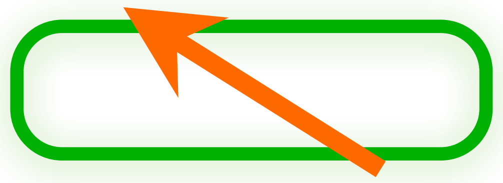

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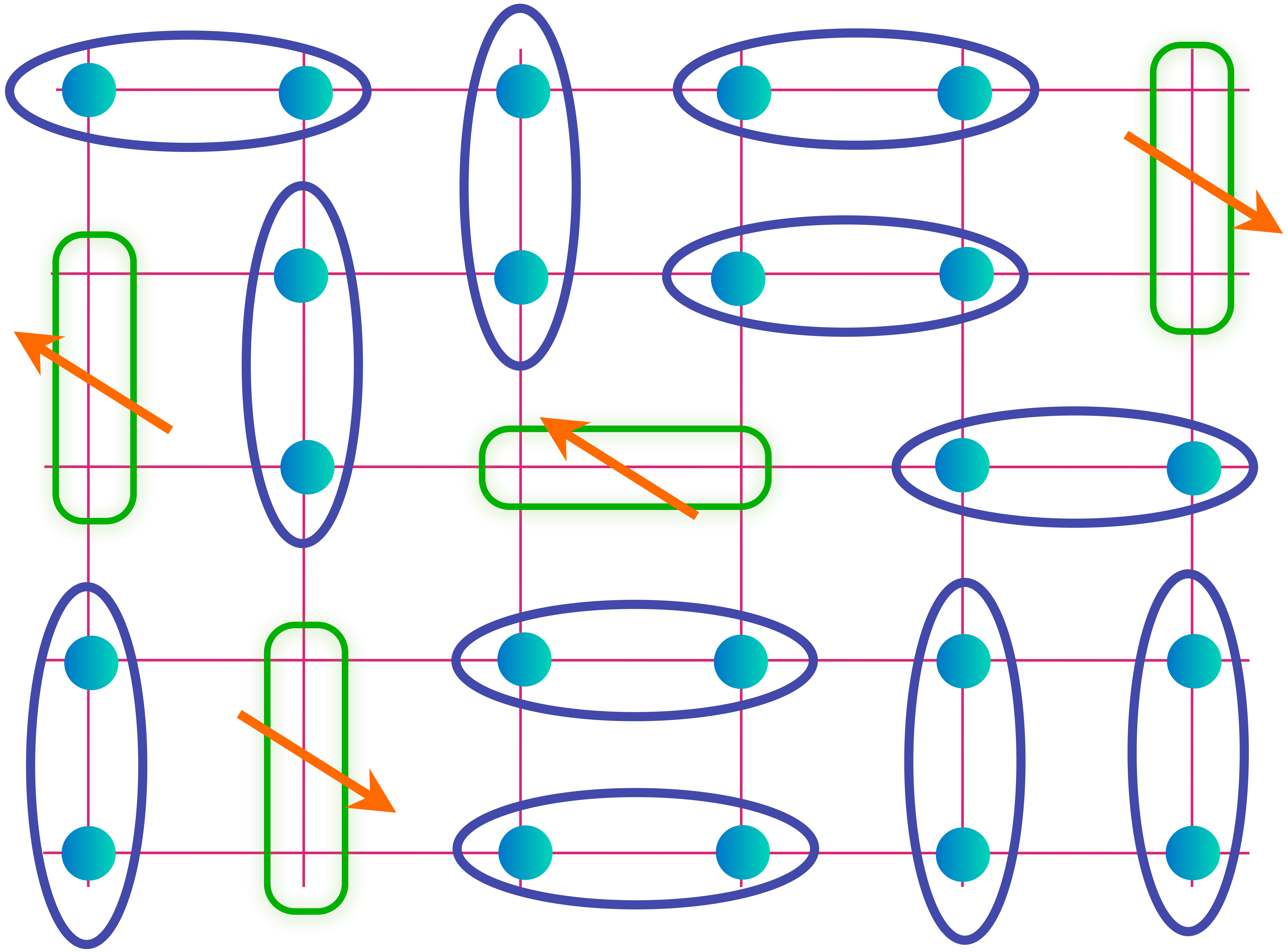

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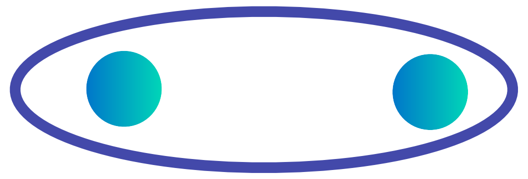
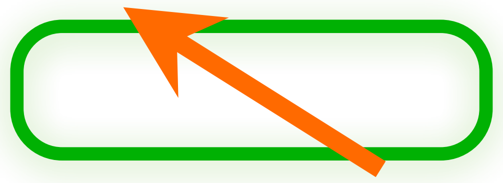
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Earlier approach to FL* in a ***one-band*** model

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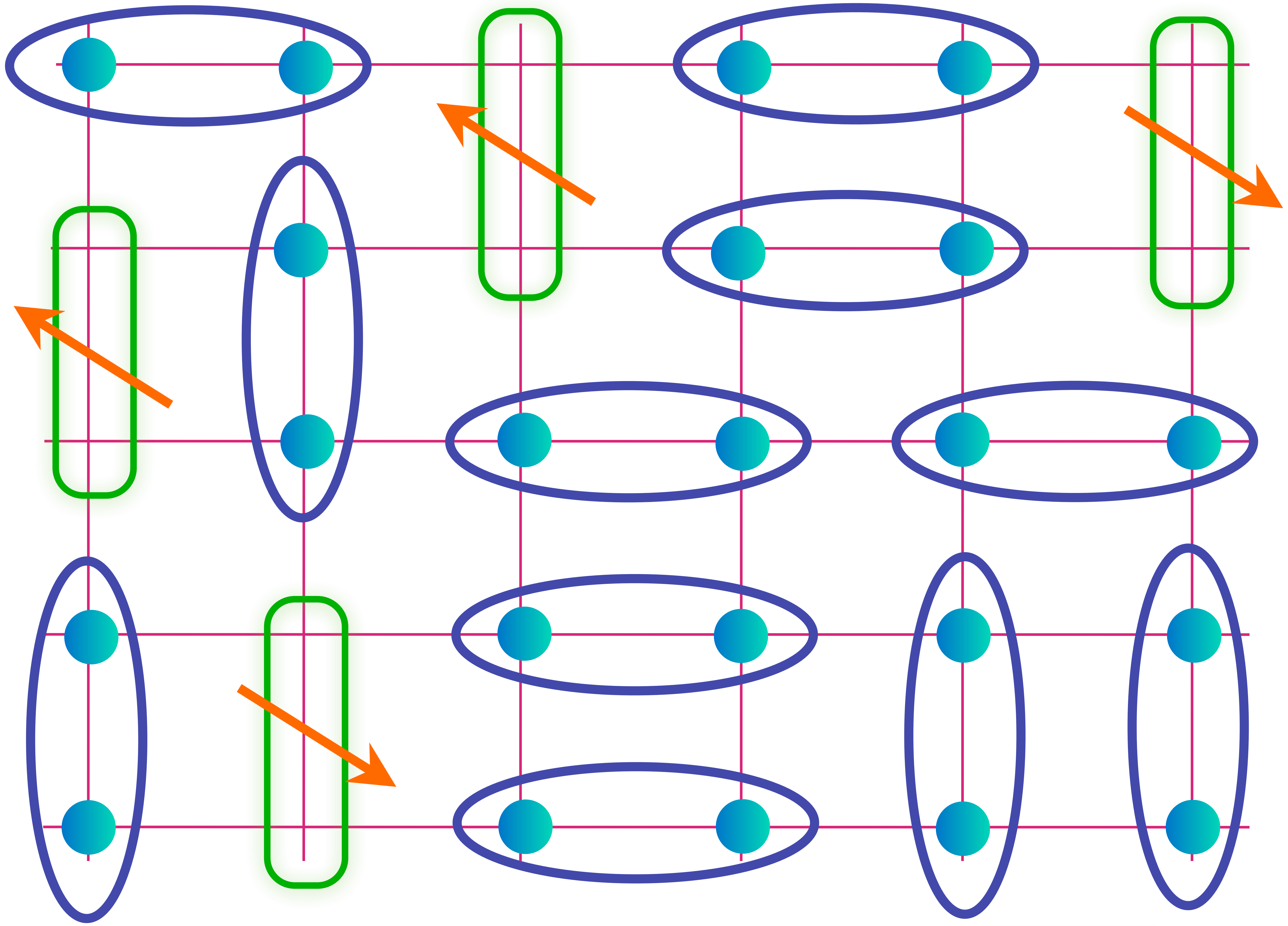
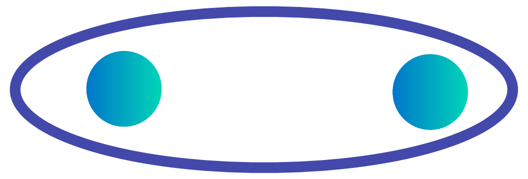
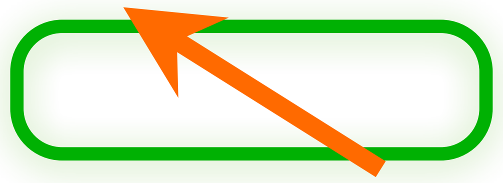
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with
electron-like
quasiparticles
on a Fermi
surface of size
p, and
emergent
gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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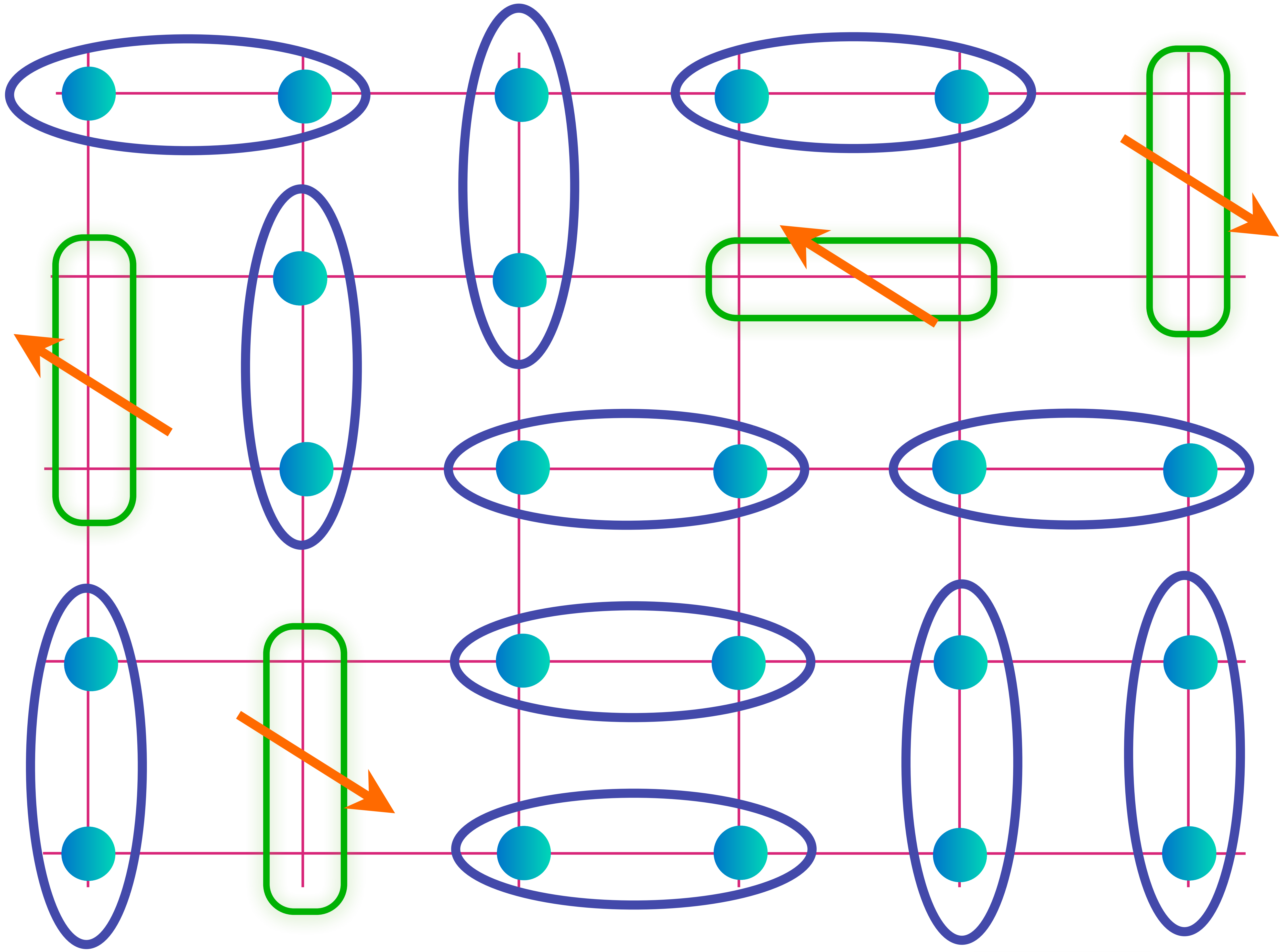
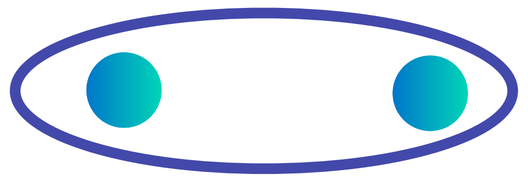
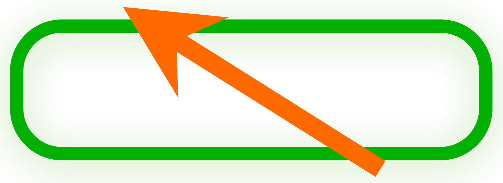


Metal with electron-like quasiparticles on a Fermi surface of size *p*, and emergent gauge fields



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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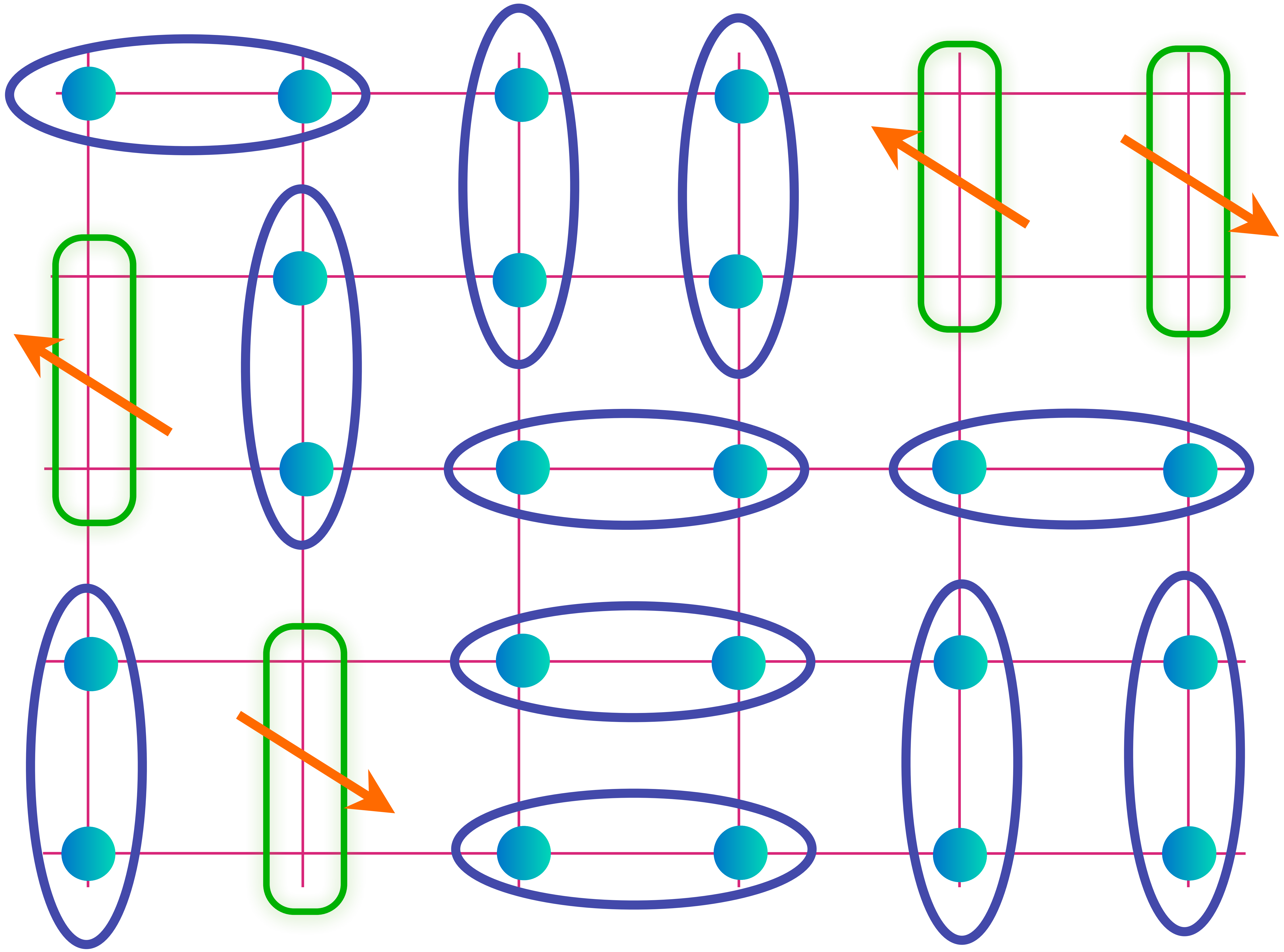
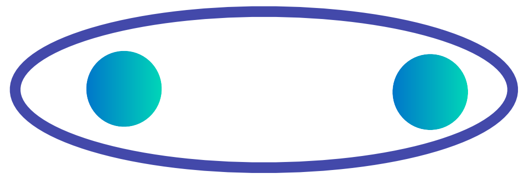


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields



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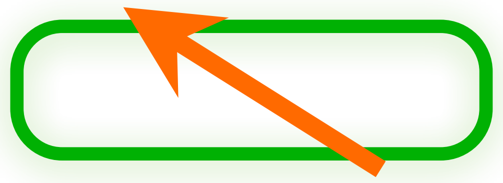

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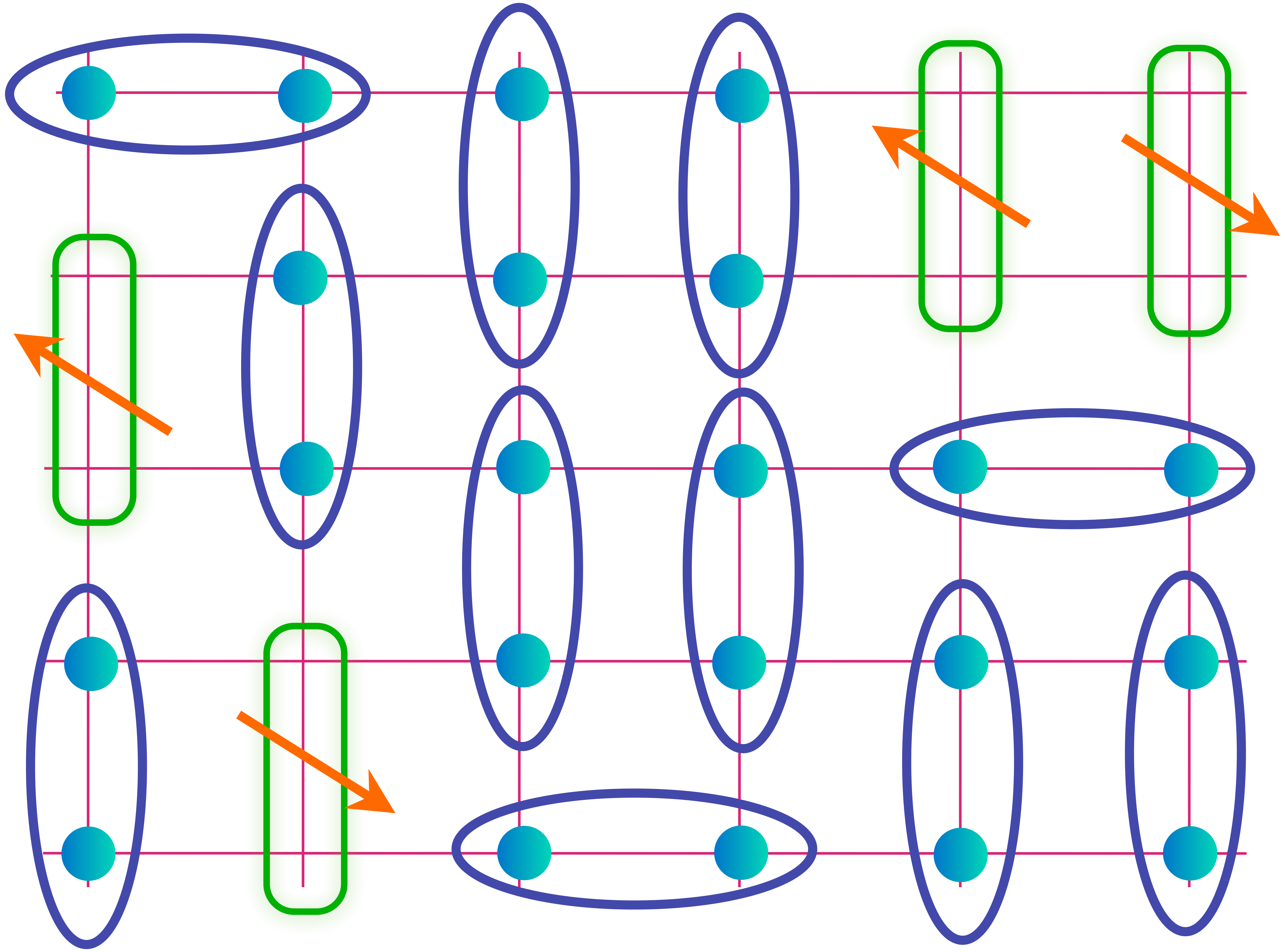
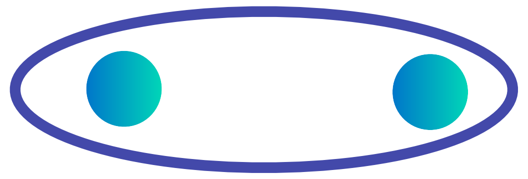


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields



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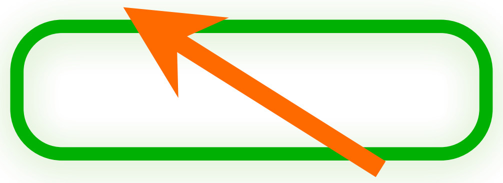

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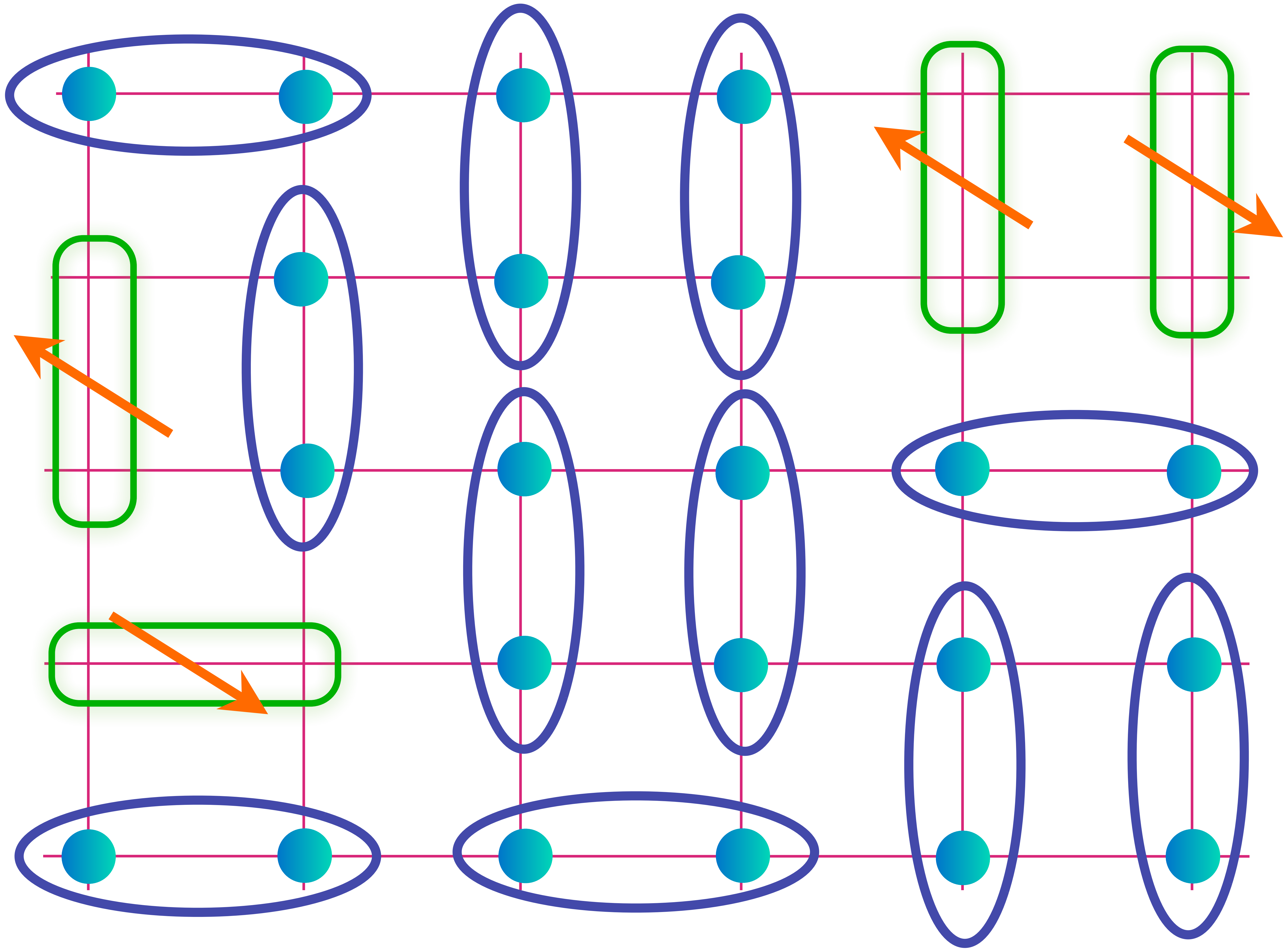

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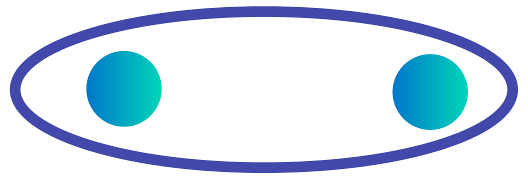

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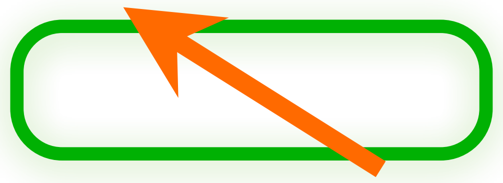

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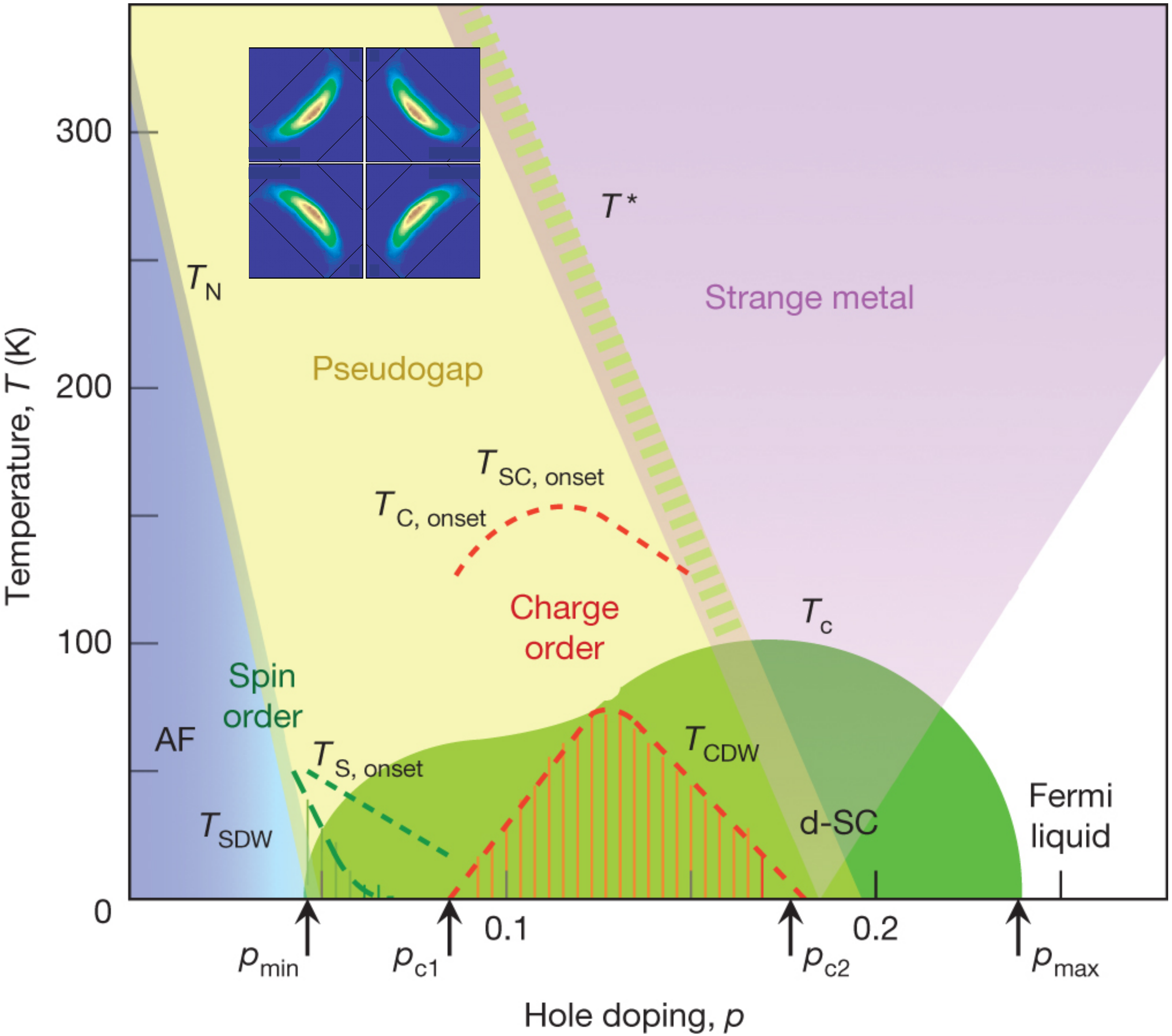
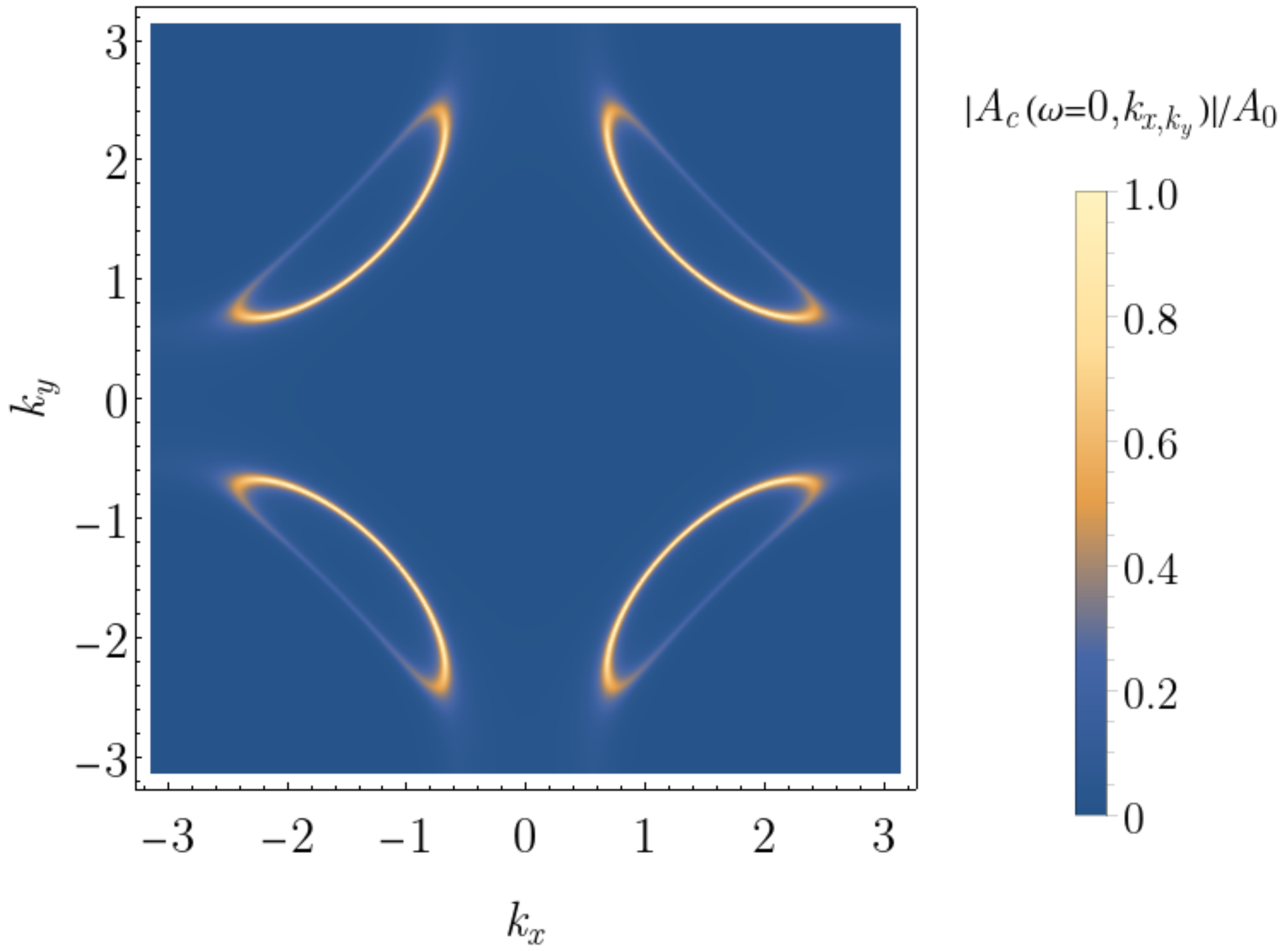
B Keimer *et al.* *Nature* **518**, 179-186 (2015)





Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

E. Mascot, A. Nikolaenko, M.Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. Sachdev, PRB **105**, 075146 (2022)

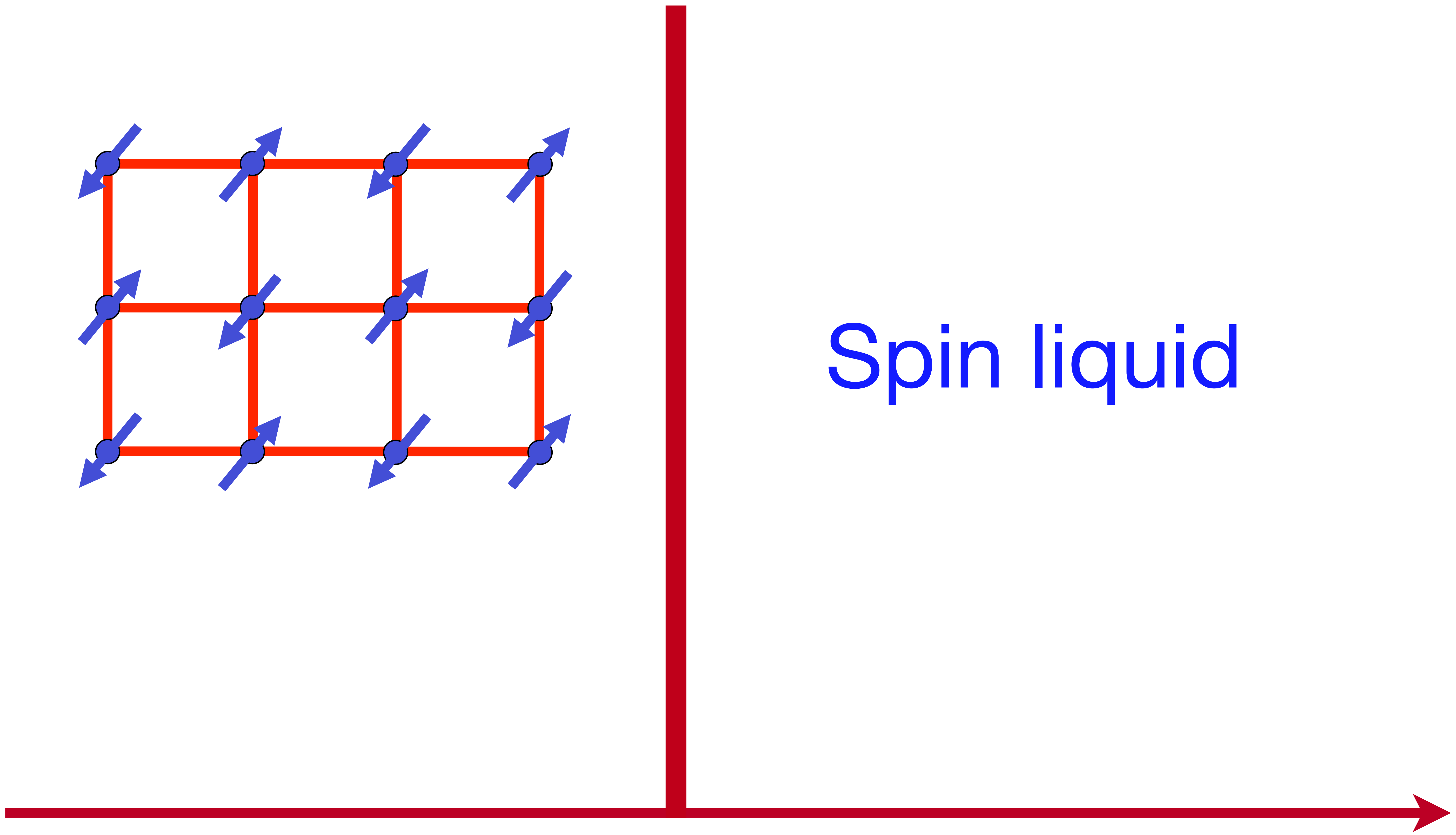
Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

$+$

'spectator' square lattice spin liquid *at half-filling.*

But which spin liquid?

Insulating S=1/2 antiferromagnet



Spin liquid

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

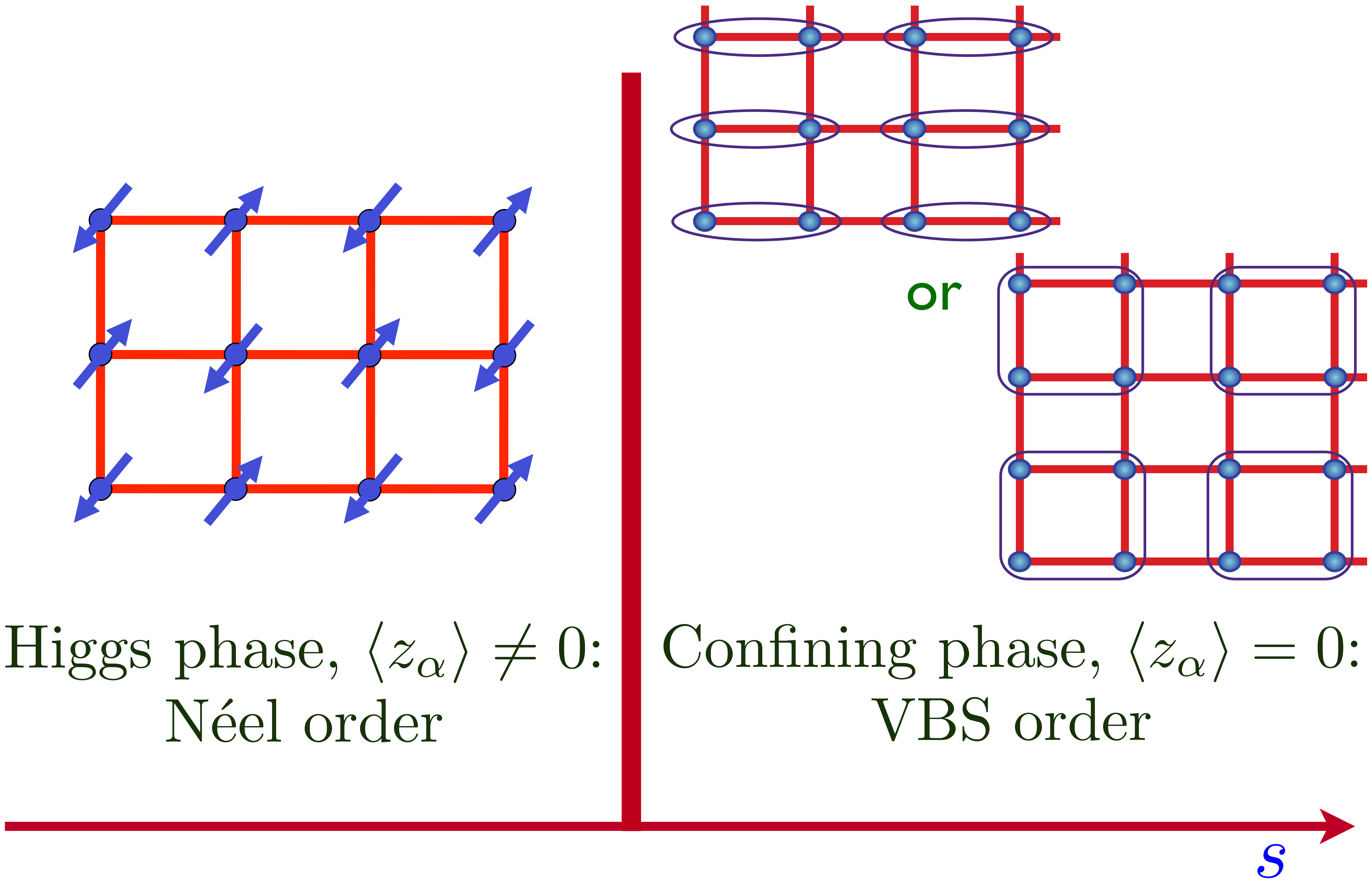
Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \qquad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

D.P. Arovas and A. Auerbach, PRB **38**, 316 (1988)

Insulating S=1/2 antiferromagnet



Higgs phase, $\langle z_\alpha \rangle \neq 0$:
Néel order

Confining phase, $\langle z_\alpha \rangle = 0$:
VBS order

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

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Mean-field spin liquid
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Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. S., PRL **62**, 1694 (1989)

Insulating S=1/2 antiferromagnet

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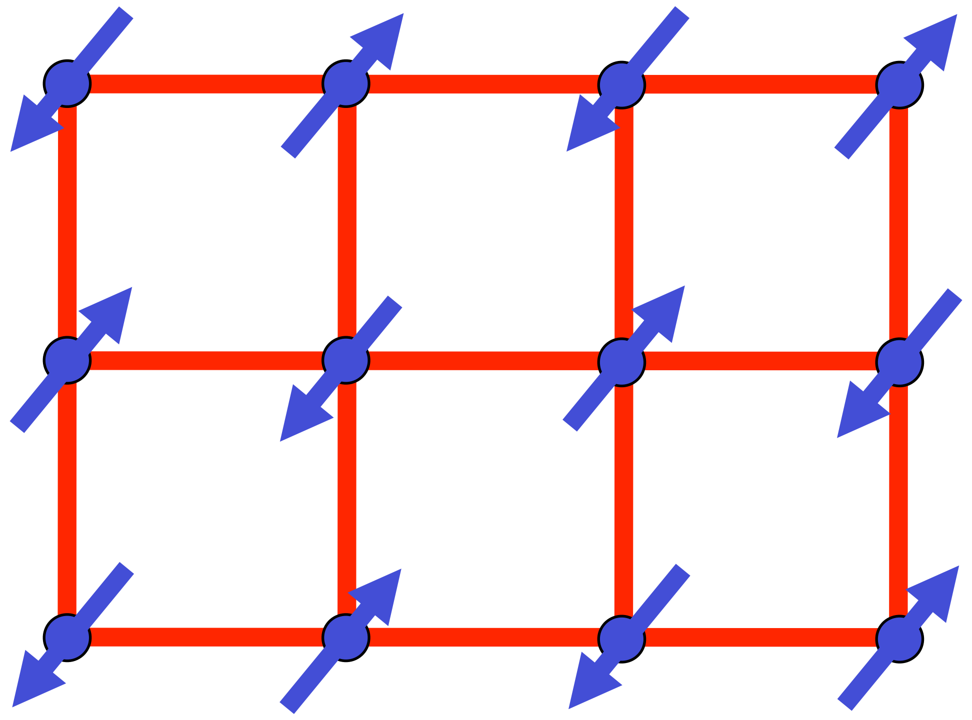

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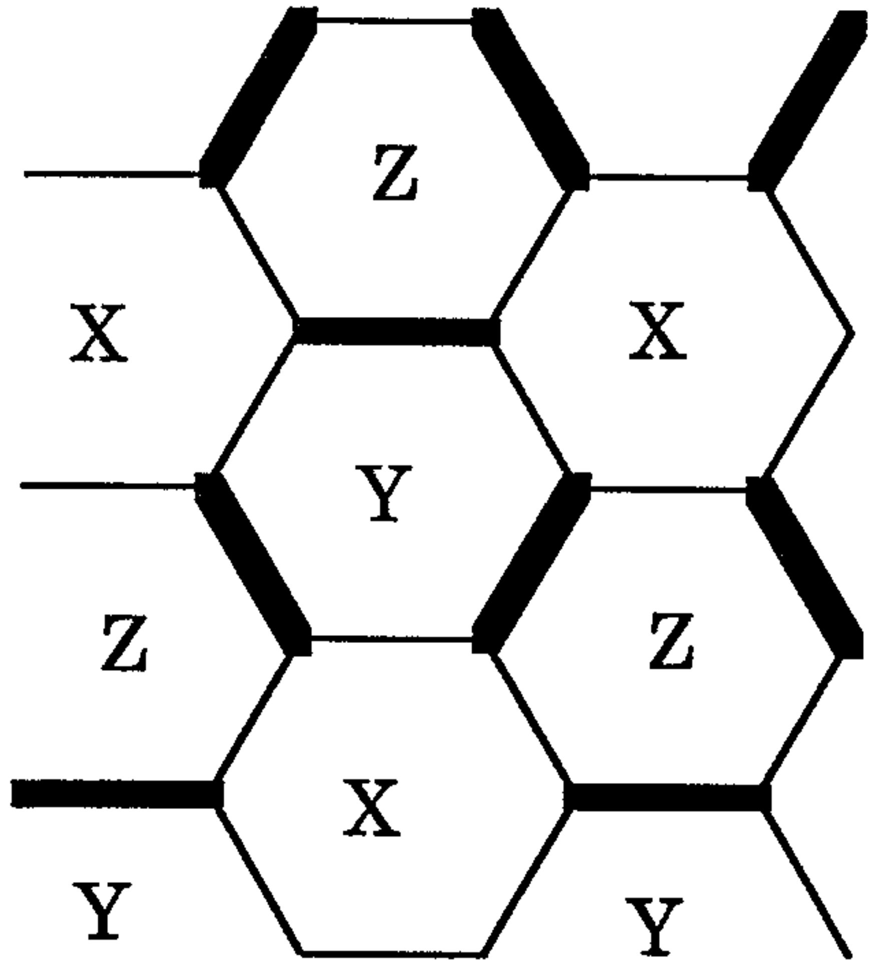


FIG. 8. The dual sublattices X, Y, Z of the honeycomb lattice. The thickness of the links signifies the structure of the spin-Peierls order for $n_c \neq 0$ mod 3; links with equal thickness have equal values of $\langle \hat{S}(i)\hat{S}(i+\hat{\eta})\rangle$.

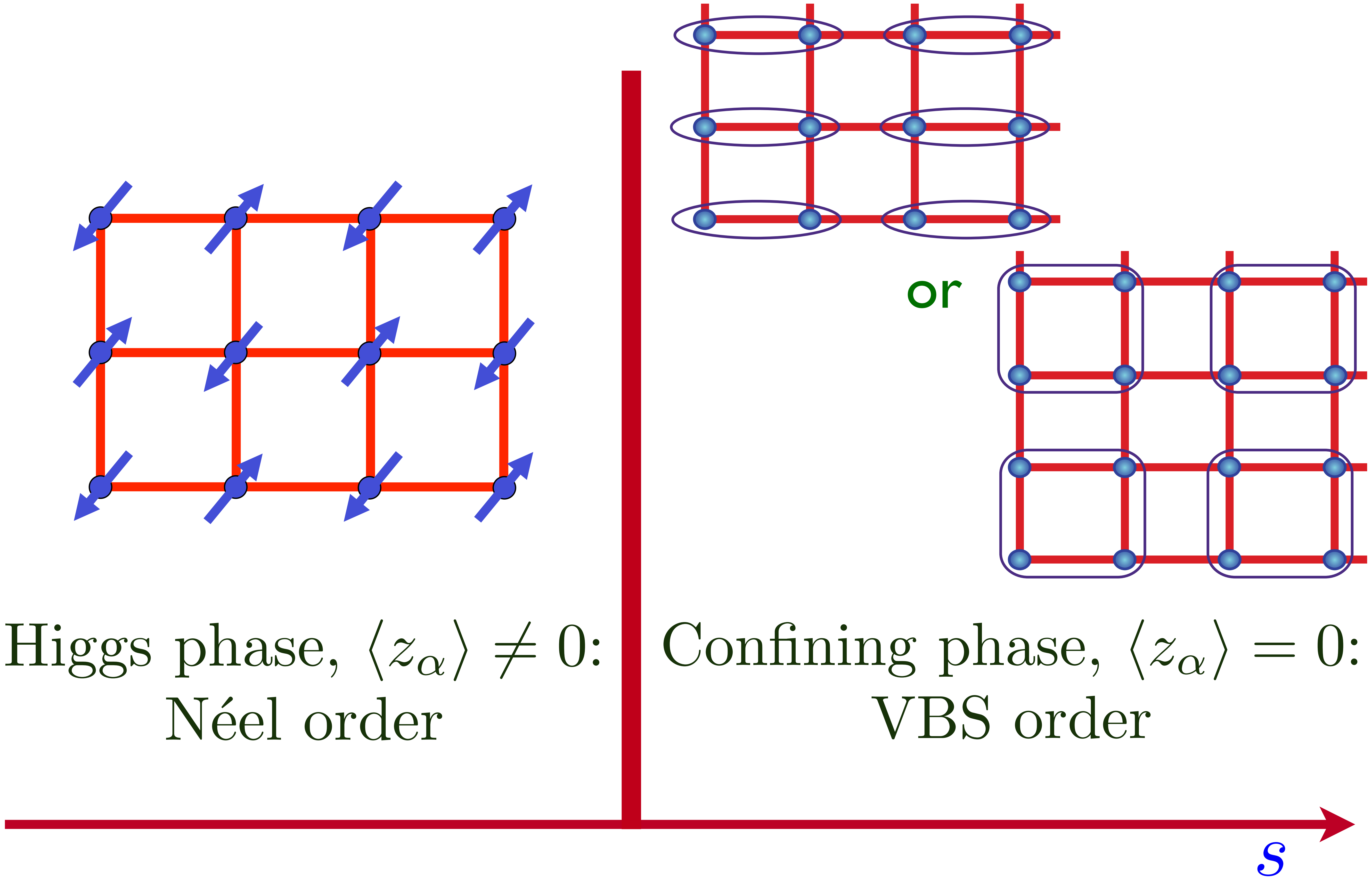
Kekule/T-IVC order on
the honeycomb lattice

s

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. S., PRL **62**, 1694 (1989)

Insulating S=1/2 antiferromagnet



Higgs phase, $\langle z_\alpha \rangle \neq 0$:
Néel order

Confining phase, $\langle z_\alpha \rangle = 0$:
VBS order

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

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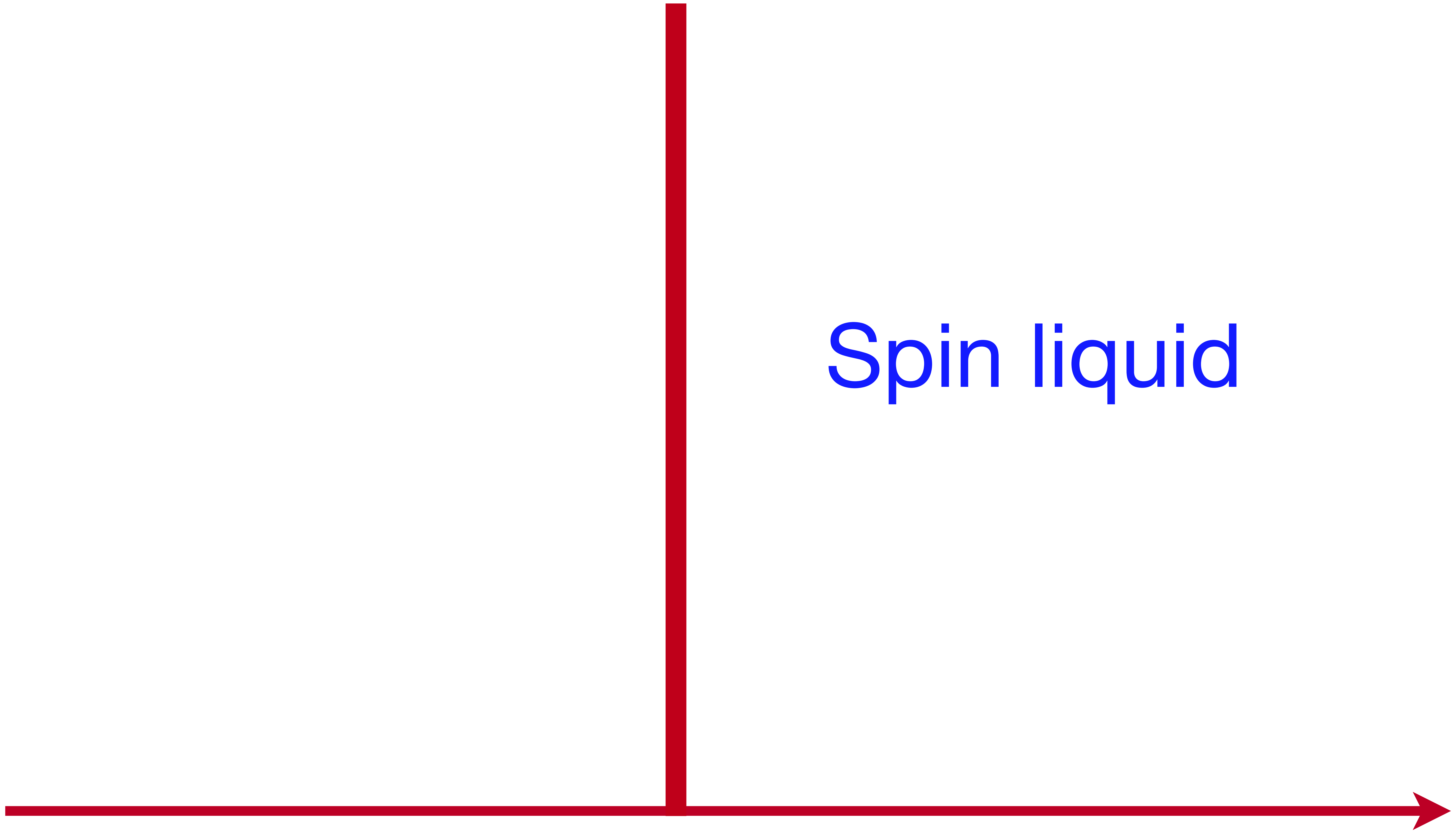

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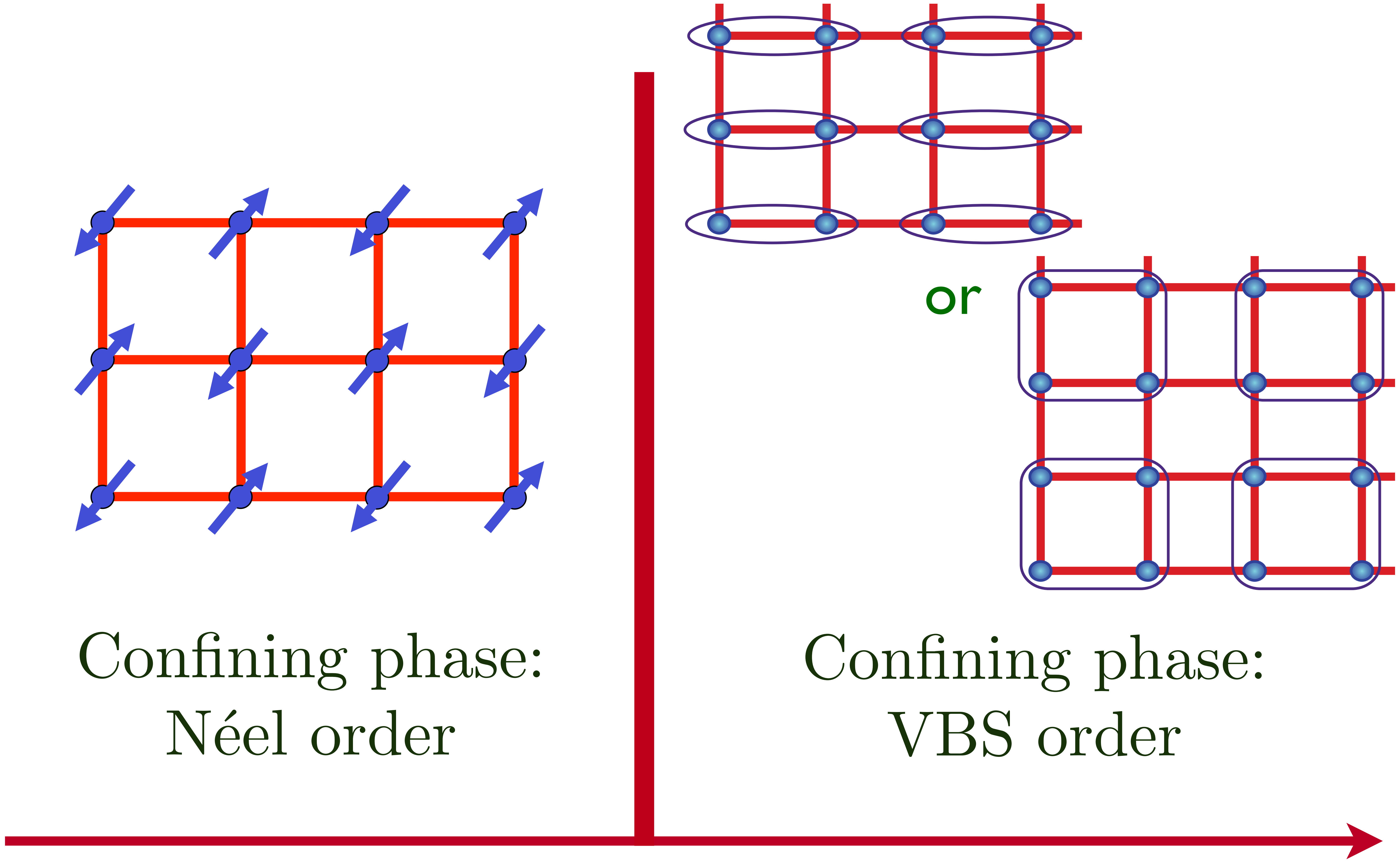
π-flux mean-field theory
with gapless spinons at 2 Dirac points.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

Insulating S=1/2 antiferromagnet



or

Confining phase:
Néel order

Confining phase:
VBS order

$$\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + \ldots$$

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

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with gapless spinons at 2 Dirac points.
Low energy theory of $N_f = 2$
Dirac fermions Ψ_s coupled to
an emergent $\mathrm{SU}(2)_N$ gauge field.
Confining order parameters
are Néel and VBS states,
with a global $\mathrm{SO}(5)_f$ symmetry!

A. Tanaka and X. Hu, PRL **95**, 036402 (2005); T. Senthil and M.P.A. Fisher PRB **74**, 064405 (2006); Ying Ran and X.-G. Wen, cond-mat/0609620

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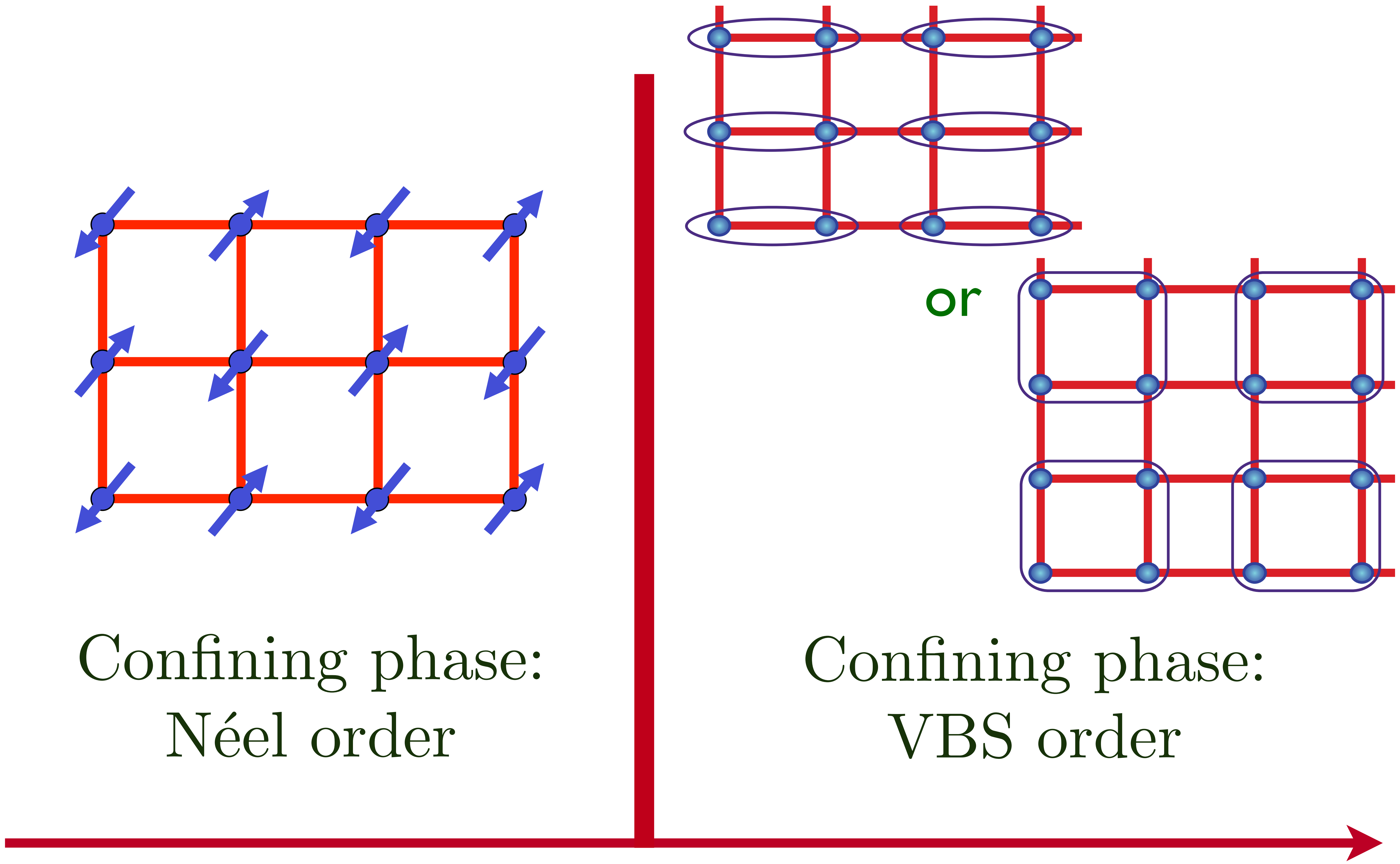

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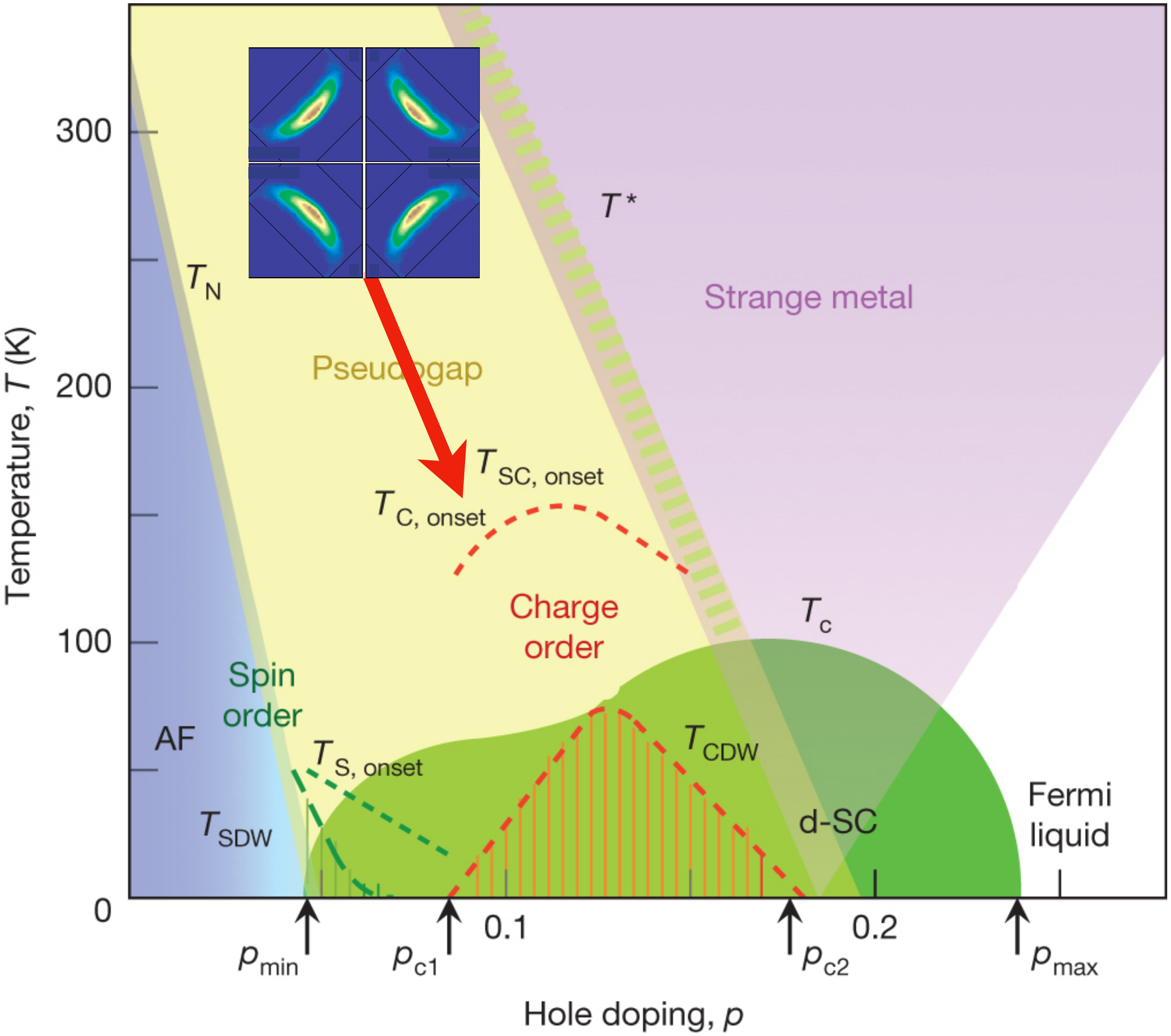
Dual to $\mathbb{CP}^1$ U(1) gauge theory.

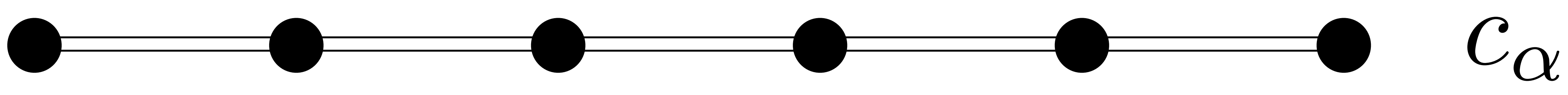
A. Tanaka and X. Hu, PRL **95**, 036402 (2005); T. Senthil and M.P.A. Fisher PRB **74**, 064405 (2006); Ying Ran and X.-G. Wen, cond-mat/0609620
C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, Physical Review X **7**, 031051 (2017)

B Keimer *et al.* *Nature* **518**, 179-186 (2015)



A theory for the confinement
of fractionalized excitations
in the π-flux spin liquid
(which is dual to the
$\mathbb{CP}^1$ spin liquid)
from
electrically charged excitations.

Ancilla theory of the Hubbard model



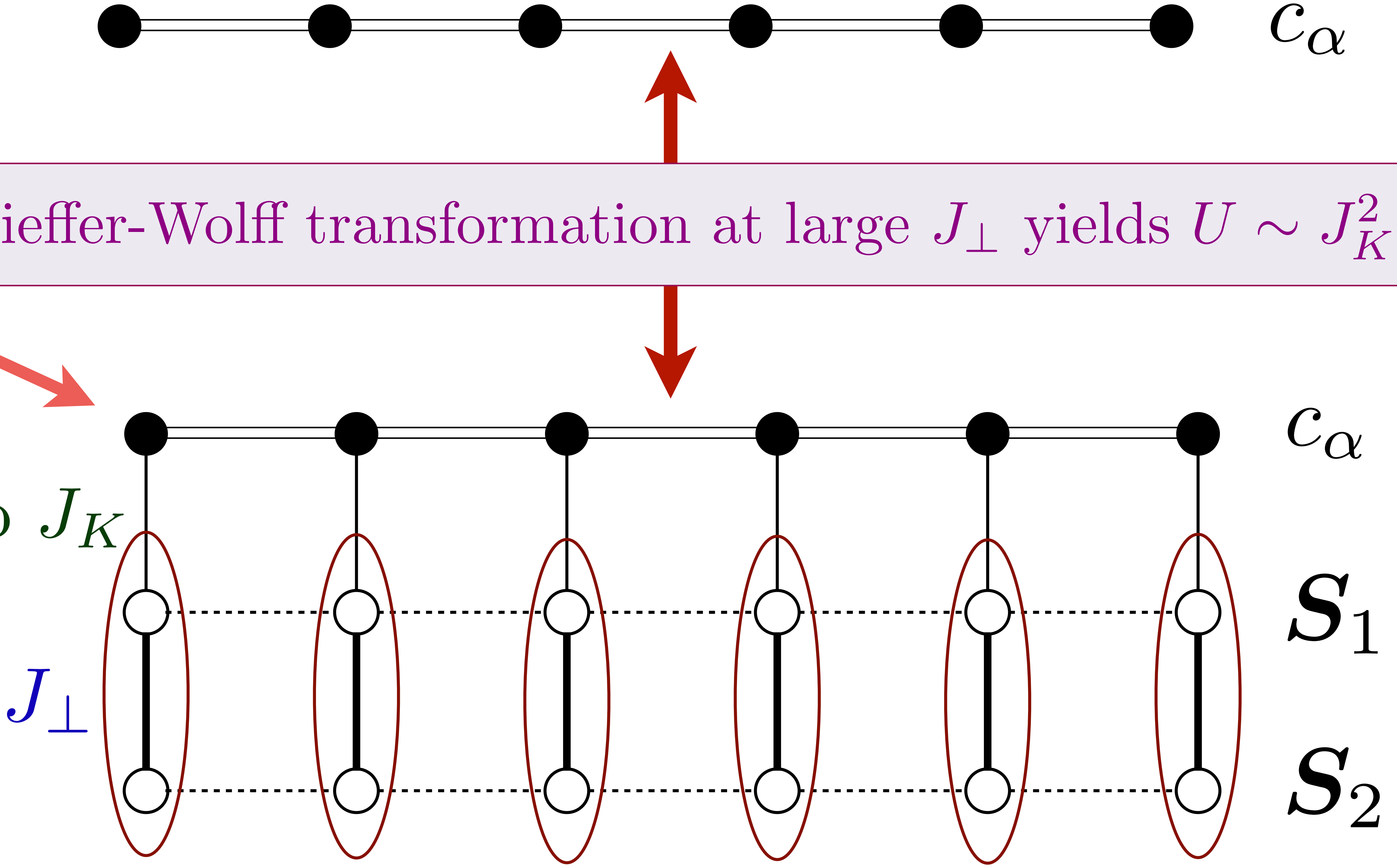
Hubbard model of hole density 1+p

Ancilla theory of the Hubbard model

Hubbard model of hole density 1+p

Free holes of density 1+p

Schrieffer-Wolff transformation at large $J_\perp$ yields $U \sim J_K^2/J_\perp$



Antiferromagnetic Kondo J_K

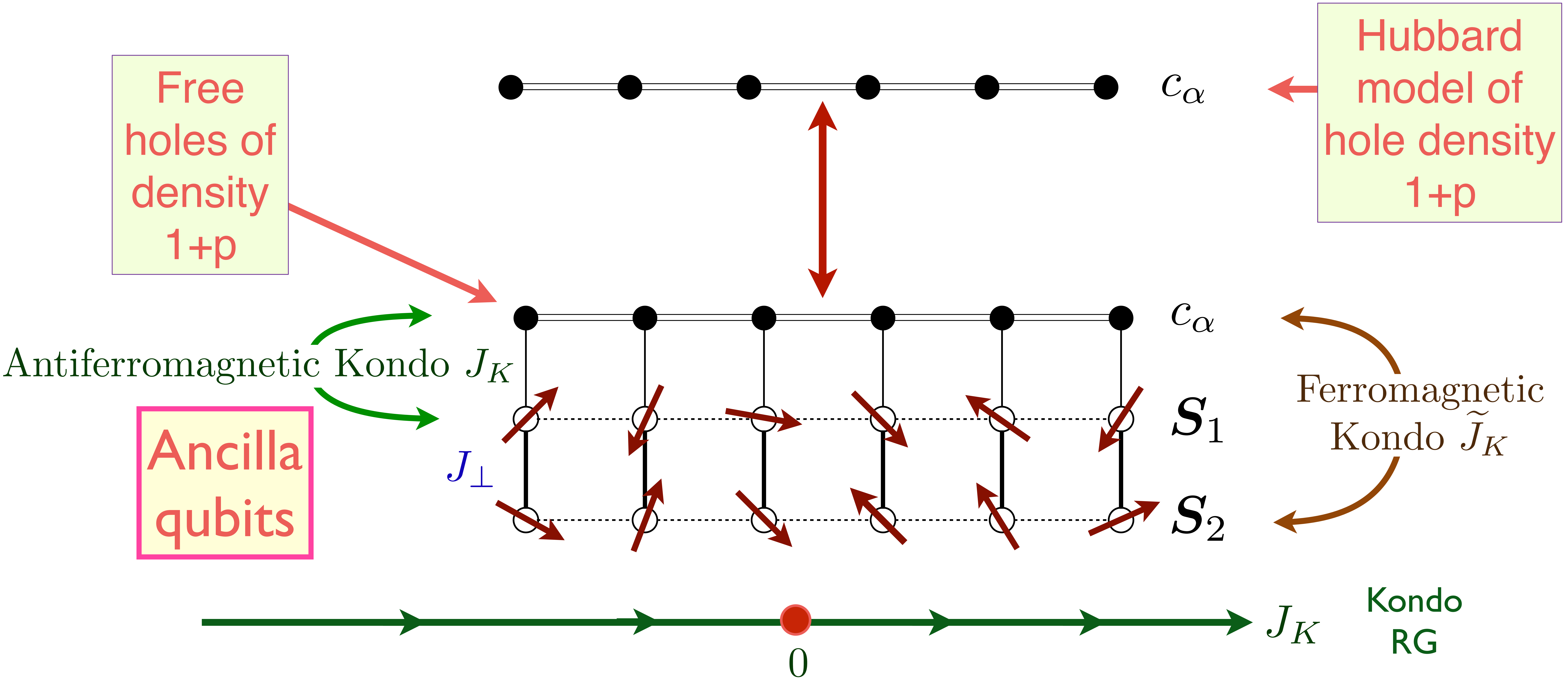
Ancilla qubits

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model



Free holes of density 1+p

Hubbard model of hole density 1+p

Antiferromagnetic Kondo J_K

Ancilla qubits

Ferromagnetic Kondo $\widetilde{J}_K$

Kondo RG

$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^{\dagger} c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^{\dagger} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

Large
Fermi surface.
Size: $1+p$

Trivial
insulator

FL

doping p

Ancilla theory of the Hubbard model

Small Fermi surface.
Size $1+p+1$
$= p \pmod 2$

Spin liquid

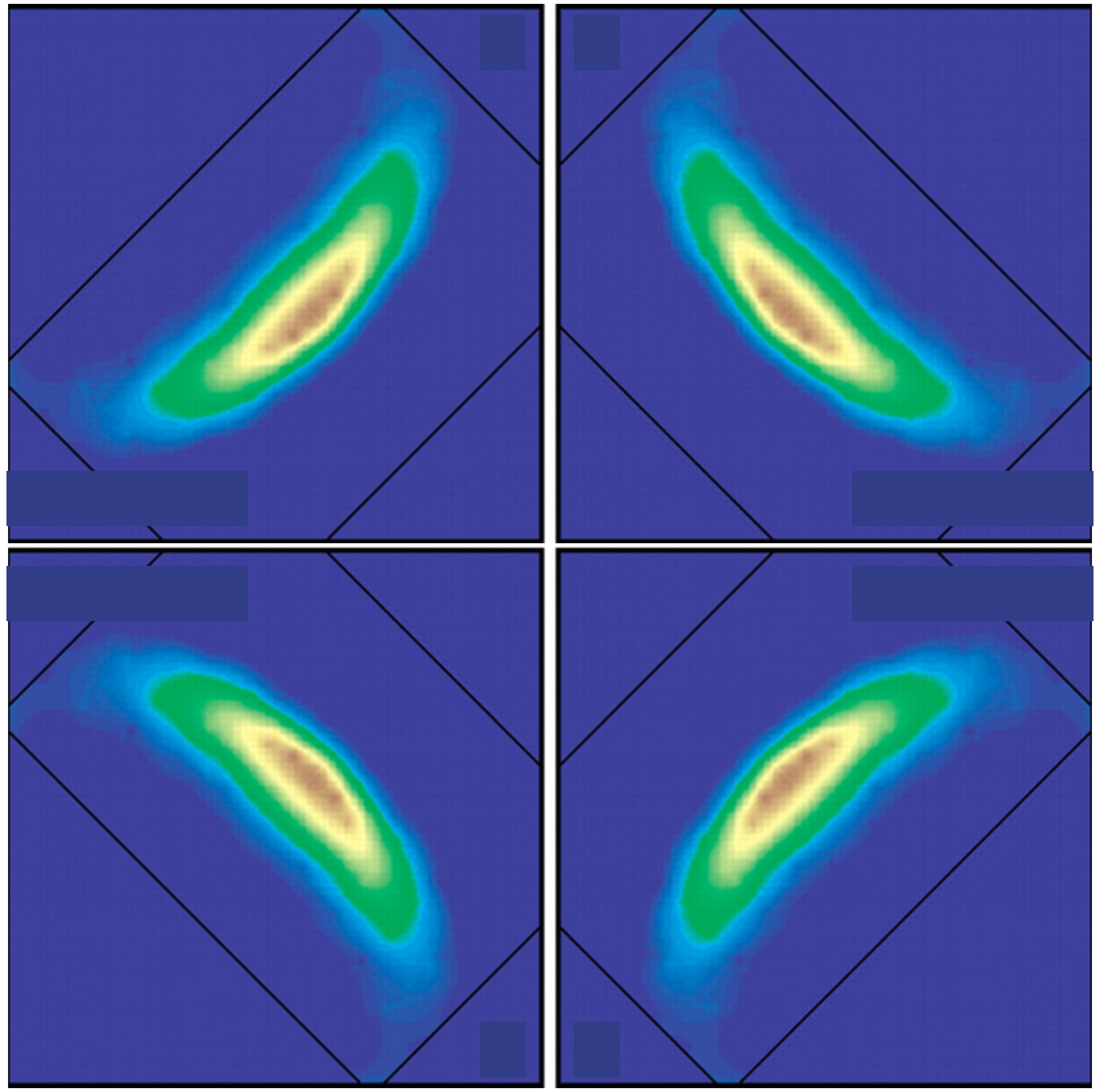
Large
Fermi surface.
Size: $1+p$

Trivial
insulator

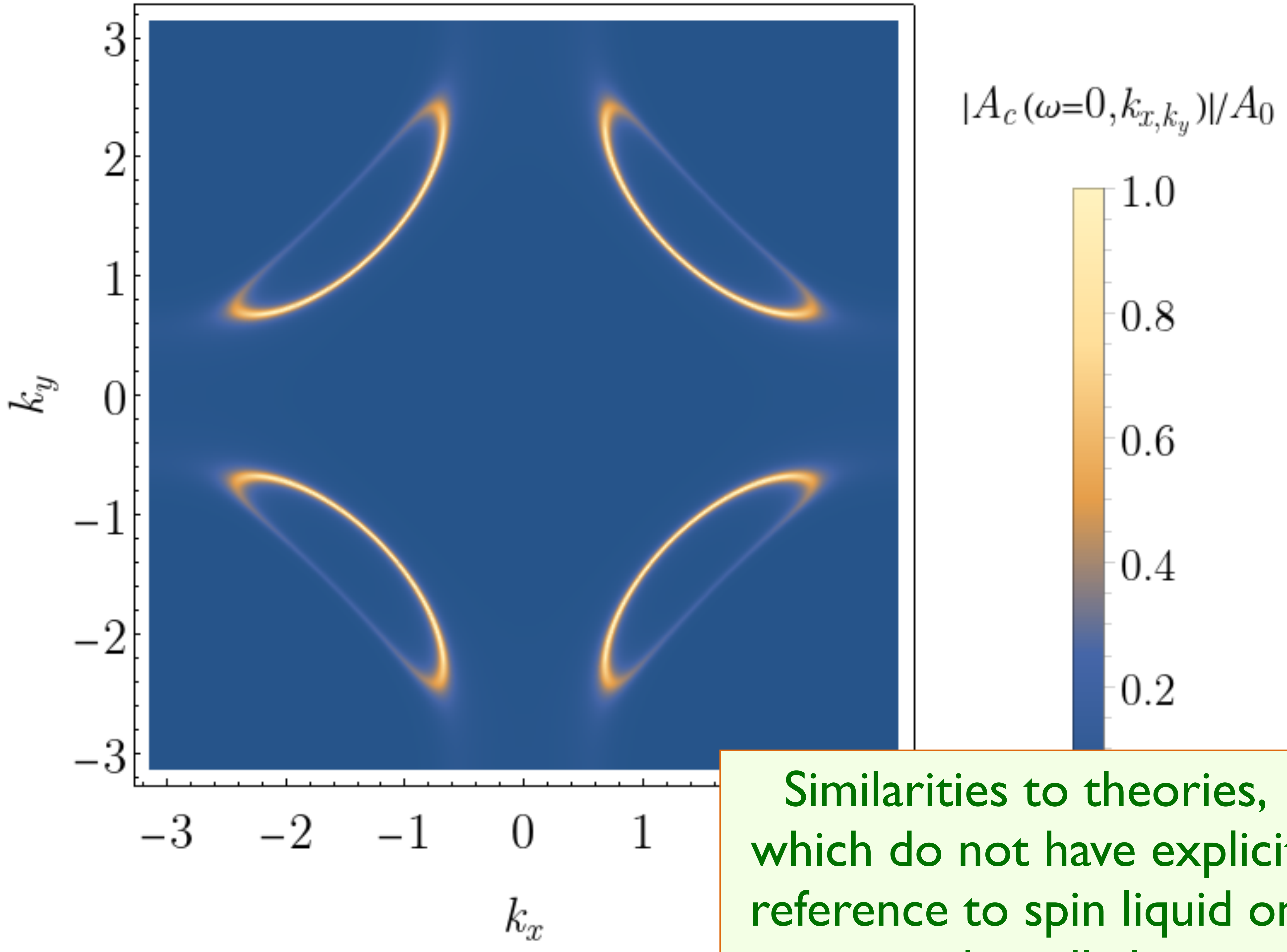
doping p

Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid
$\oplus$
Spin Liquid

Photoemission at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$



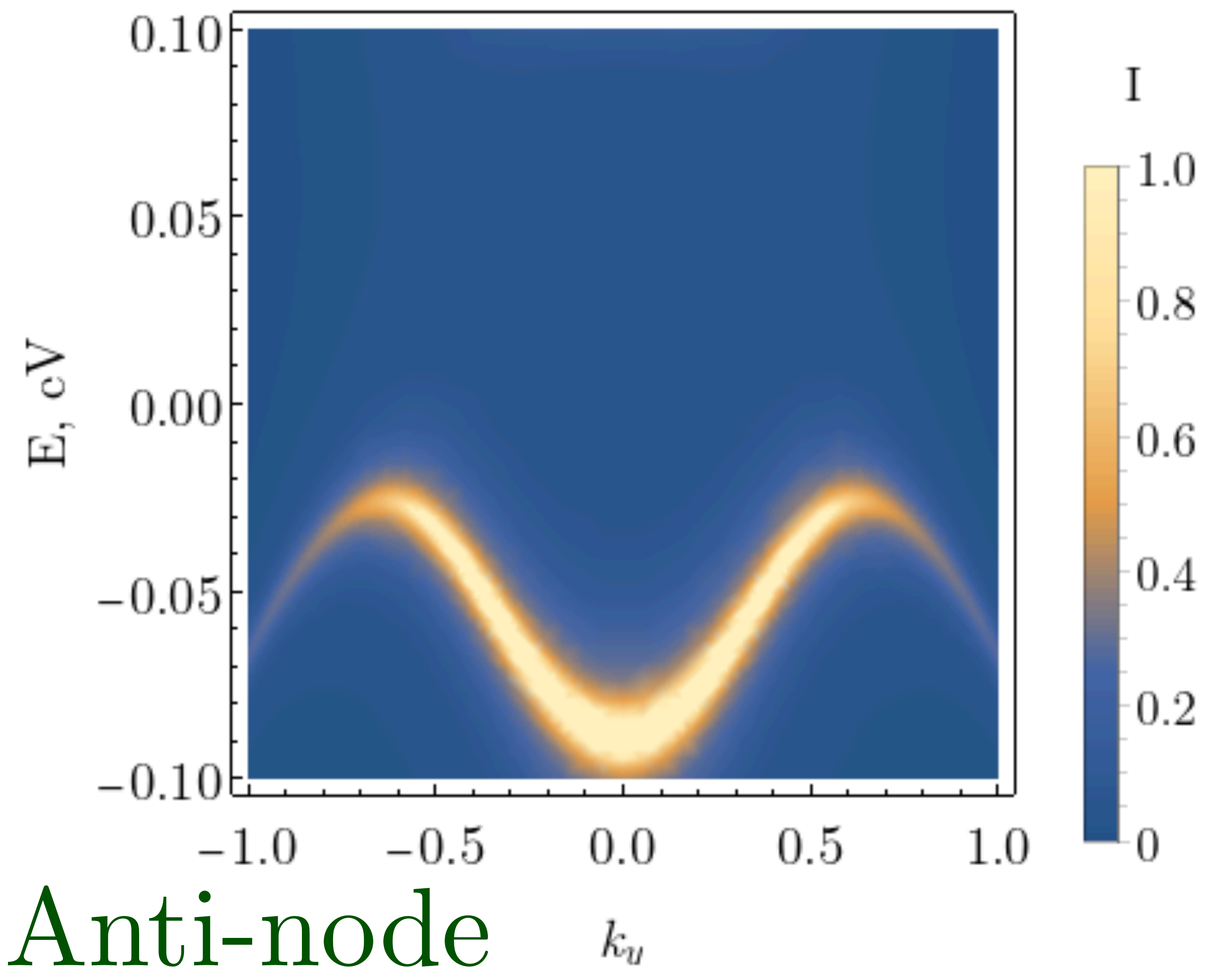
“Fermi arcs”

Similarities to theories, which do not have explicit reference to spin liquid on second ancilla layer

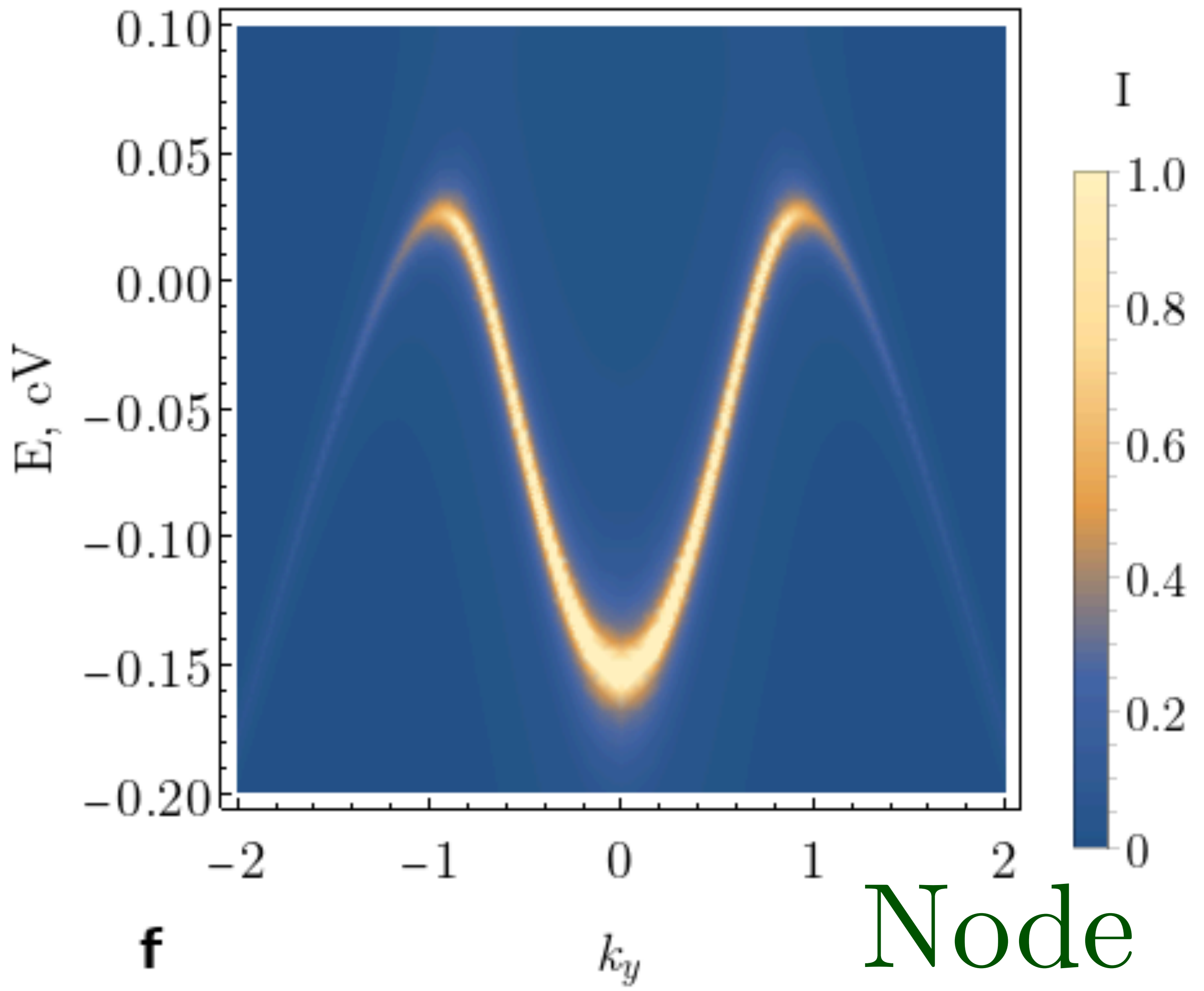
Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006)
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009)

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

FL* in a ***one-band*** model

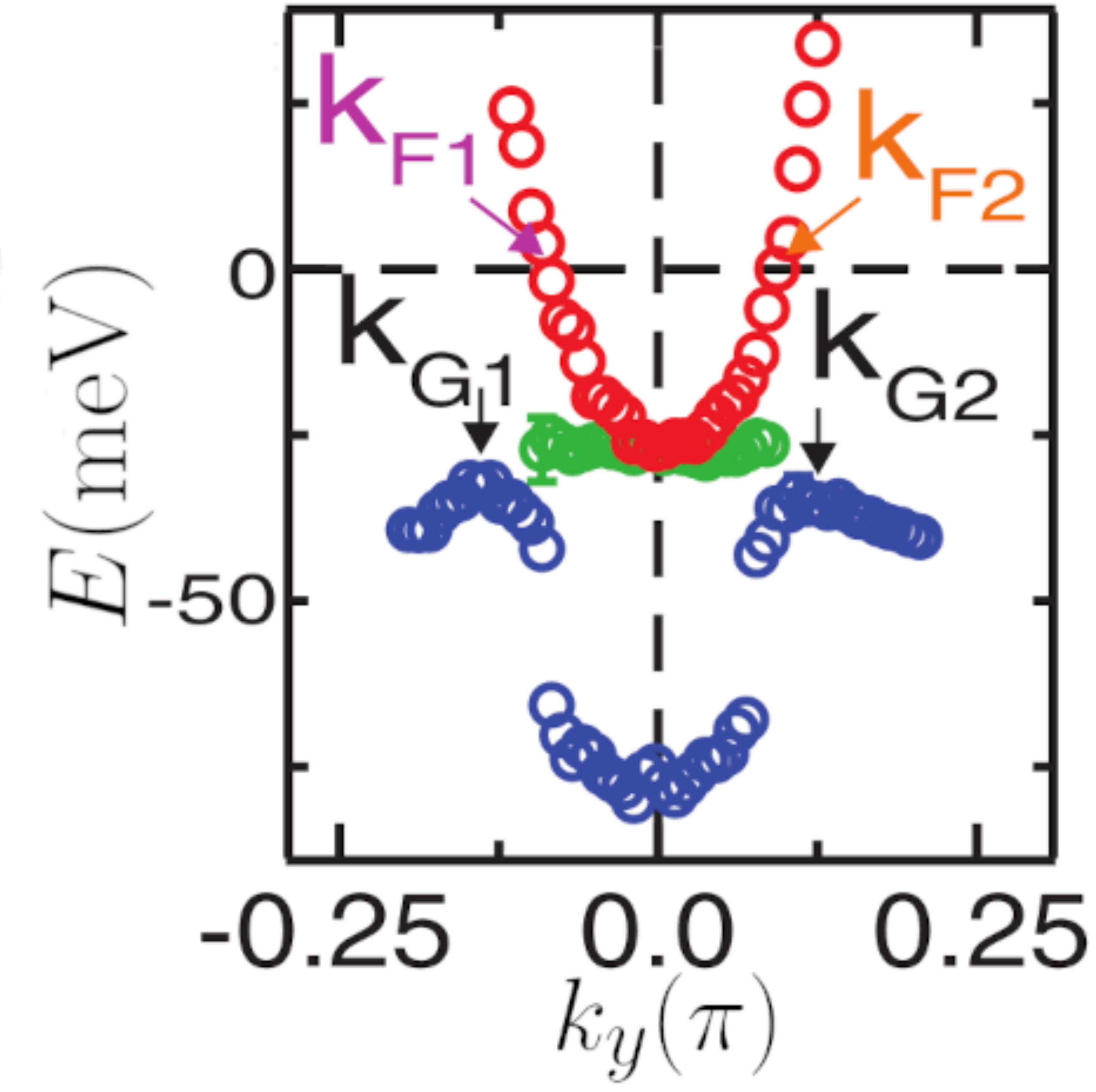


Anti-node

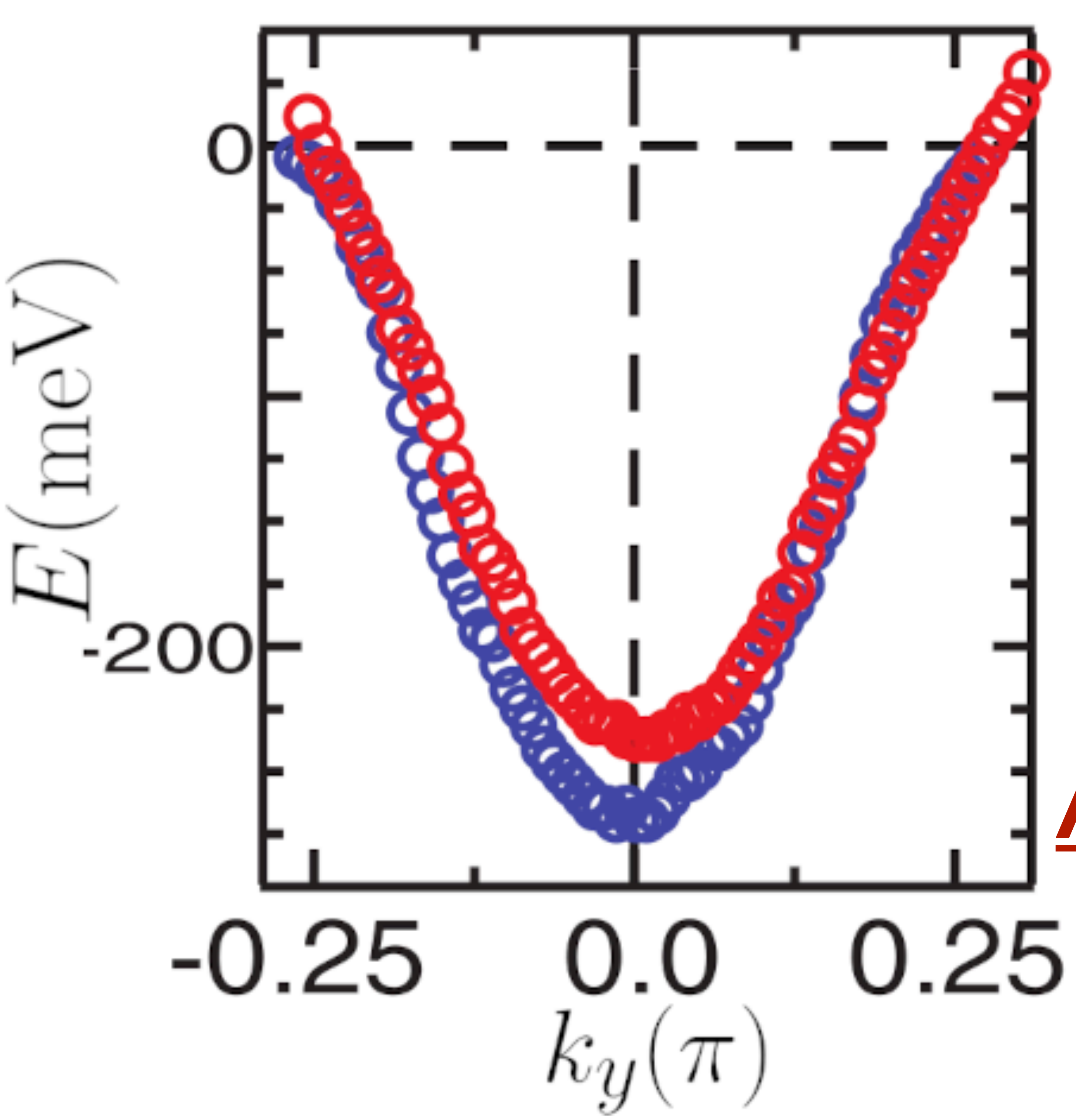


Node

Second ancilla layer is needed to describe MDC and EDC



f



R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

ARPES on Bi2201

Similarities to theories, which do not have explicit reference to spin liquid on second ancilla layer

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang,
PRB **73**, 174501 (2006)
S. Sakai, Y. Motome, M. Imada,
PRL **102**, 056404 (2009)

Ancilla theory of the Hubbard model

Small Fermi surface.
Size $1 + p + 1$
$= p \pmod 2$

Spin liquid

e^+

J_K

$J_\perp$

c_α

$\boldsymbol{S}_1, f_1$

$\boldsymbol{S}_2, f$

Large
Fermi surface.
Size: $1 + p$

Trivial
insulator

FL*

FL

J_K

doping p

Pseudogap metal: $\langle c^\dagger_\alpha f_{1\alpha} \rangle \neq 0$

f_α form π-flux spin liquid with $\mathrm{SU}(2)_N$ gauge field

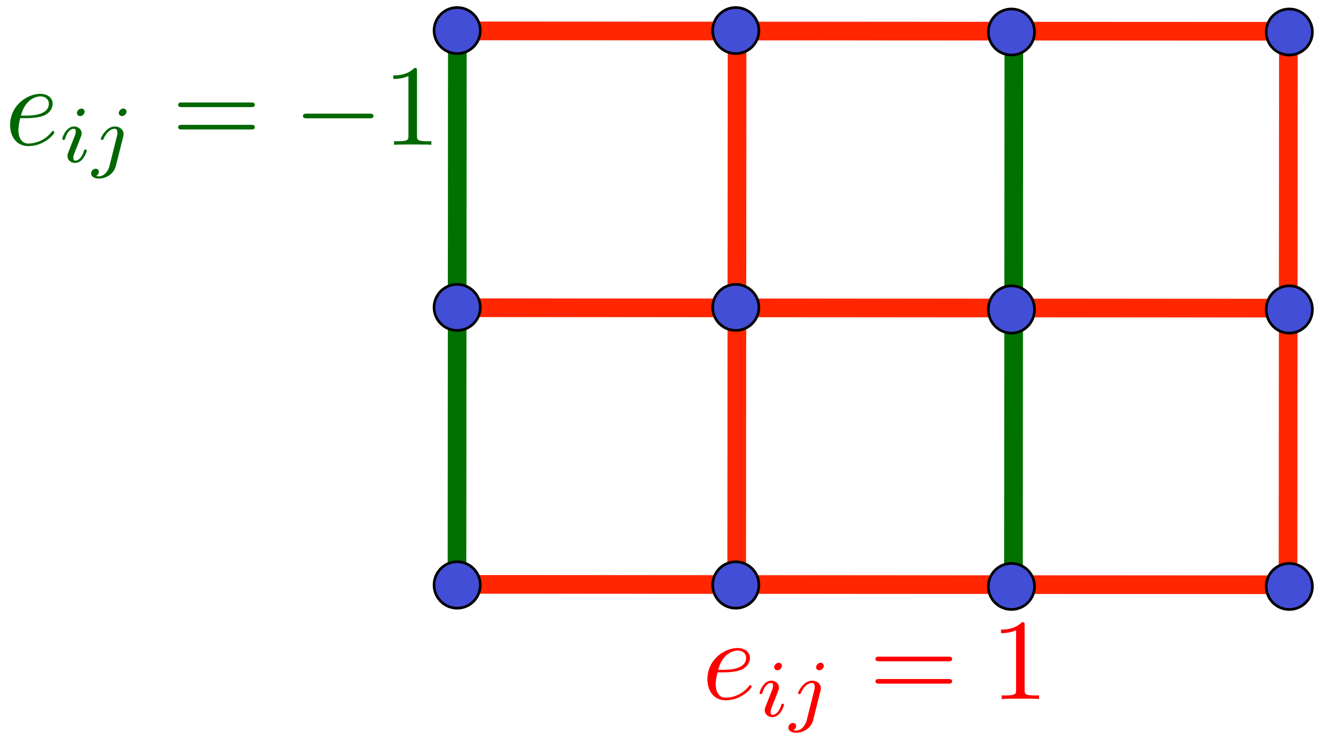
Charge e, $\mathrm{SU}(2)_N$ fundamental, Higgs boson $B \sim \begin{pmatrix} f^\dagger_{1\alpha} f_\alpha \\ \varepsilon_{\alpha\beta} f^\dagger_{1\alpha} f^\dagger_\beta \end{pmatrix}$

Boson with same quantum numbers in X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996)

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right)$$

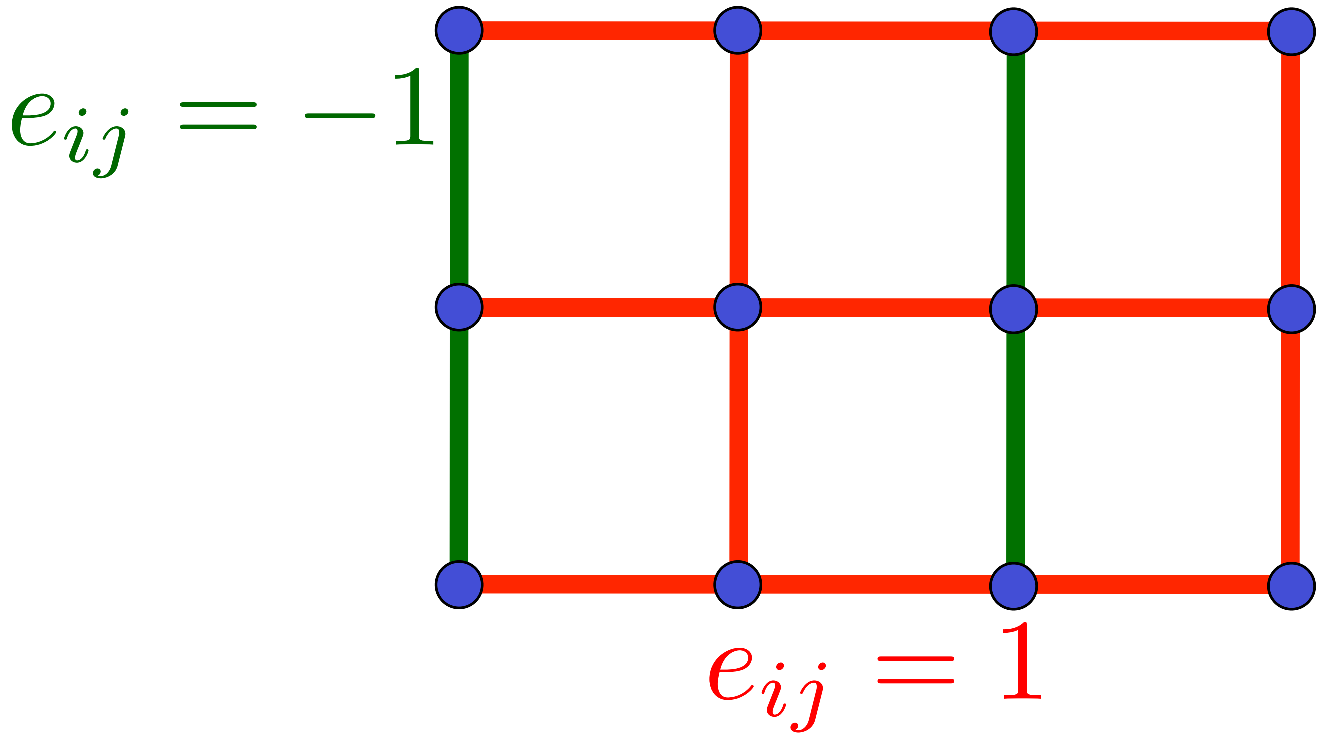


Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ\sum_{\langle ij\rangle} e_{ij}\left(f^\dagger_{i\alpha}f_{j\alpha} - f^\dagger_{j\alpha}f_{i\alpha}\right) = \boxed{iJ\sum_{\langle ij\rangle} e_{ij}\left(\Psi^\dagger_i U_{ij}\Psi_j - \Psi^\dagger_j U_{ji}\Psi_i\right)} ; \quad \Psi_i = \left(\begin{array}{c} f_{i\uparrow} \\ f^\dagger_{i\downarrow}\end{array}\right)$$

H_f is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; U_{ij} is the SU(2)$_N$ gauge field.



Confinement of SU(2)$_N$ gauge theory by charge fluctuations

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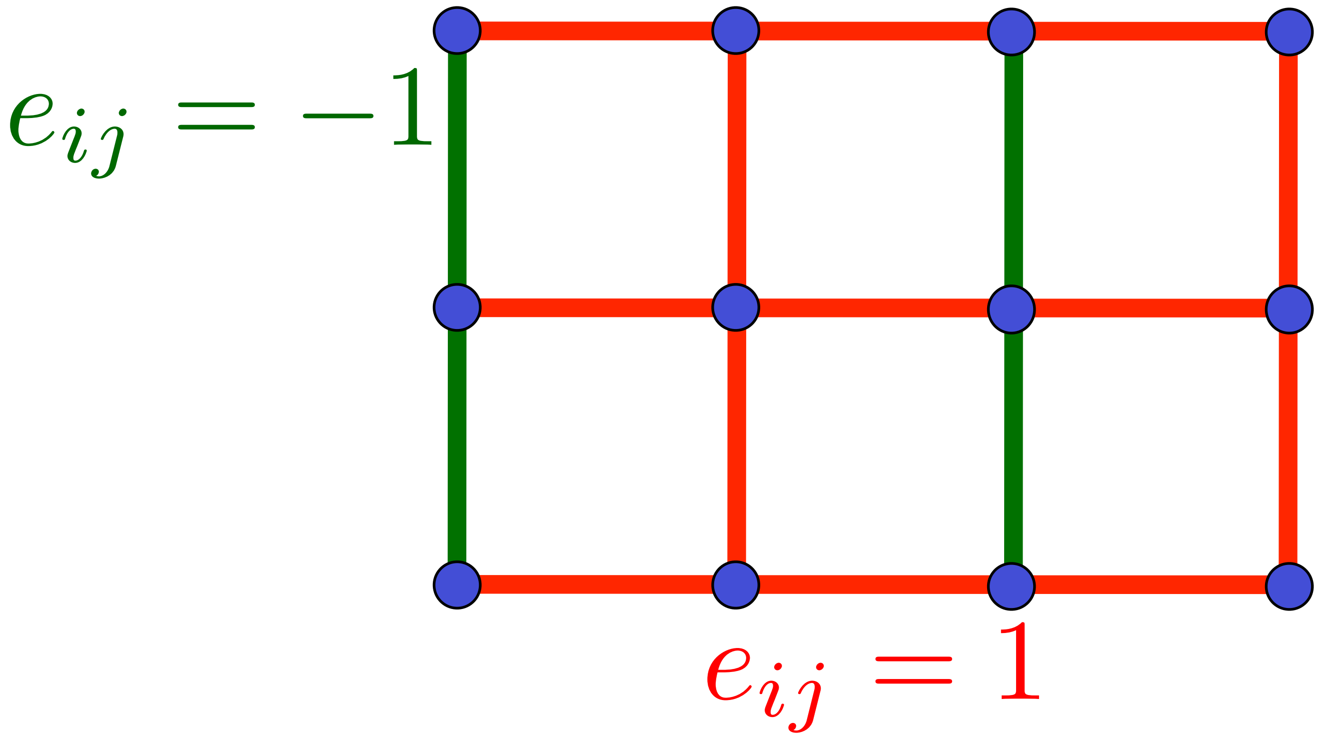


- The nearest-neighbor effective Hamiltonian for charge e, SU(2)$_N$ fundamental boson B_i is constrained by the fact that the composite of B_i and Ψ_i is an electron:

$$\boxed{H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \ldots}$$

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g \sum_{\langle \boldsymbol{ij} \rangle} \left|\Delta_{\boldsymbol{ij}}\right|^2$$

$$+ J_1 \sum_{\langle \boldsymbol{ij} \rangle} Q_{\boldsymbol{ij}}^2 + K_1 \sum_{\langle \boldsymbol{ij} \rangle} J_{\boldsymbol{ij}}^2 .$$

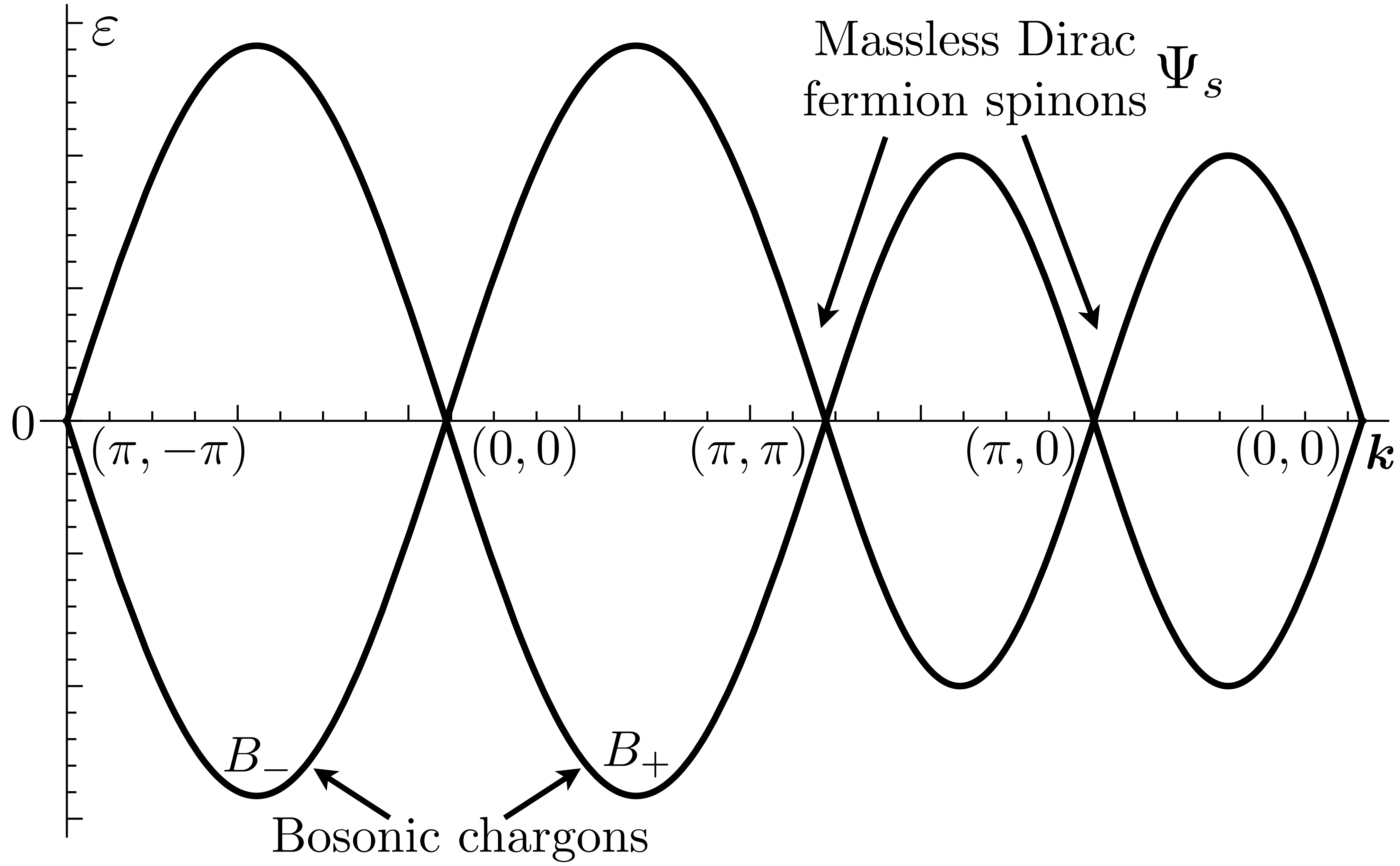
site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

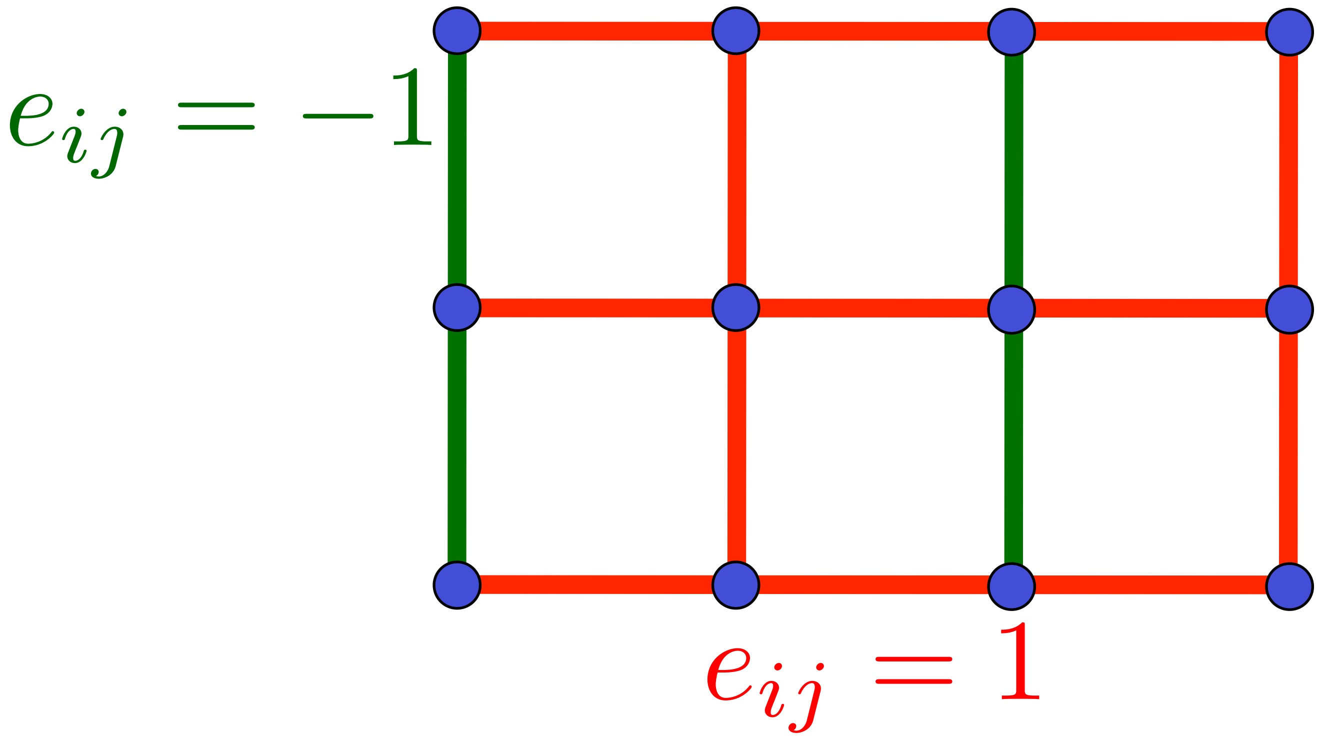
bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

bond current: $i \left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

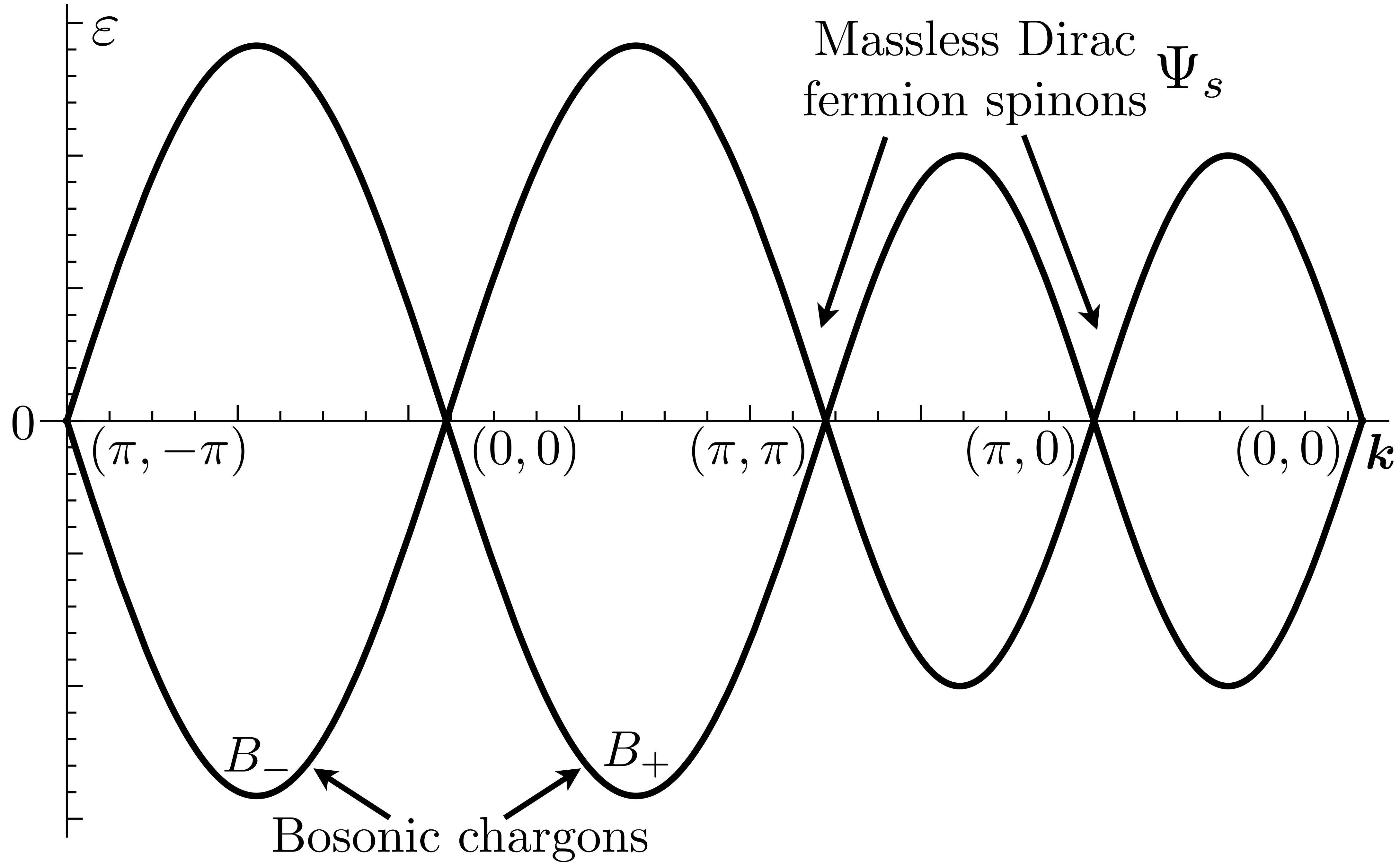
Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$.

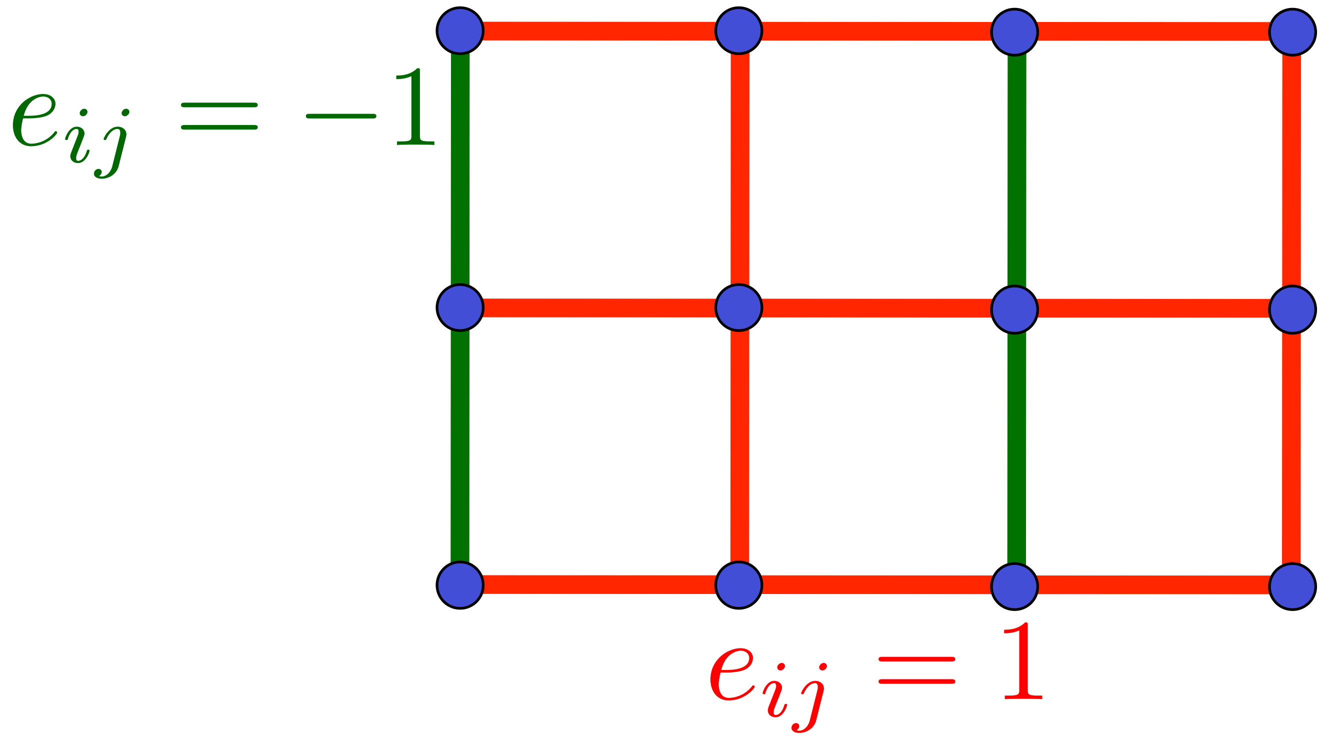
Confinement of SU(2)$_N$ gauge theory by charge fluctuations





Confinement of $SU(2)_N$ gauge theory by charge fluctuations

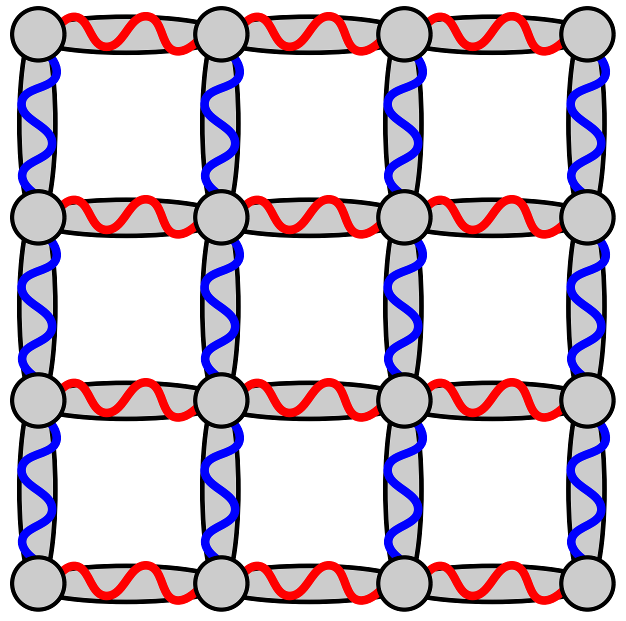




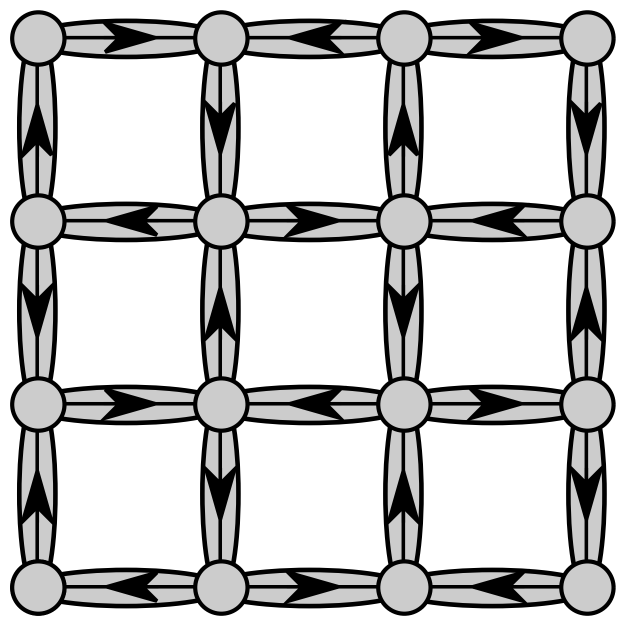
$SU(2)_N$ gauge-invariant and $SU(2)$ spin invariant order parameters of Higgs phases:

$$\begin{aligned}
x\text{-CDW}: \quad & \rho_{(\pi,0)} = B^*_{a+}B_{a+} - B^*_{a-}B_{a-} \\
y\text{-CDW}: \quad & \rho_{(0,\pi)} = B^*_{a+}B_{a-} + B^*_{a-}B_{a+} \\
d\text{-density wave}: \quad & D = i\left(B^*_{a+}B_{a-} - B^*_{a-}B_{a+}\right) \\
d\text{-wave superconductor}: \quad & \Delta = \varepsilon_{ab}B_{a+}B_{b-}
\end{aligned}$$

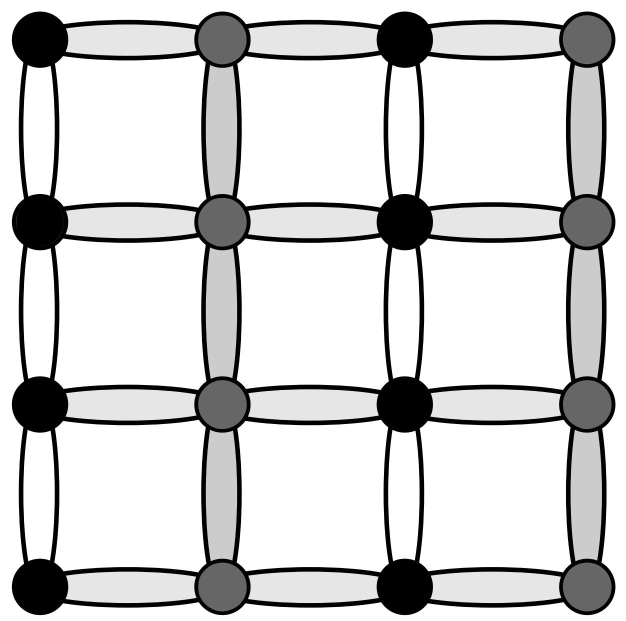
d-wave SC



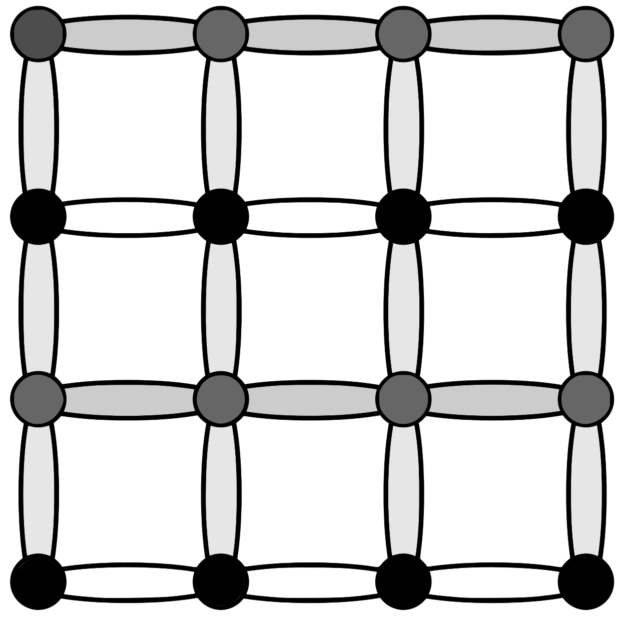
d-density



(π,0) stripe

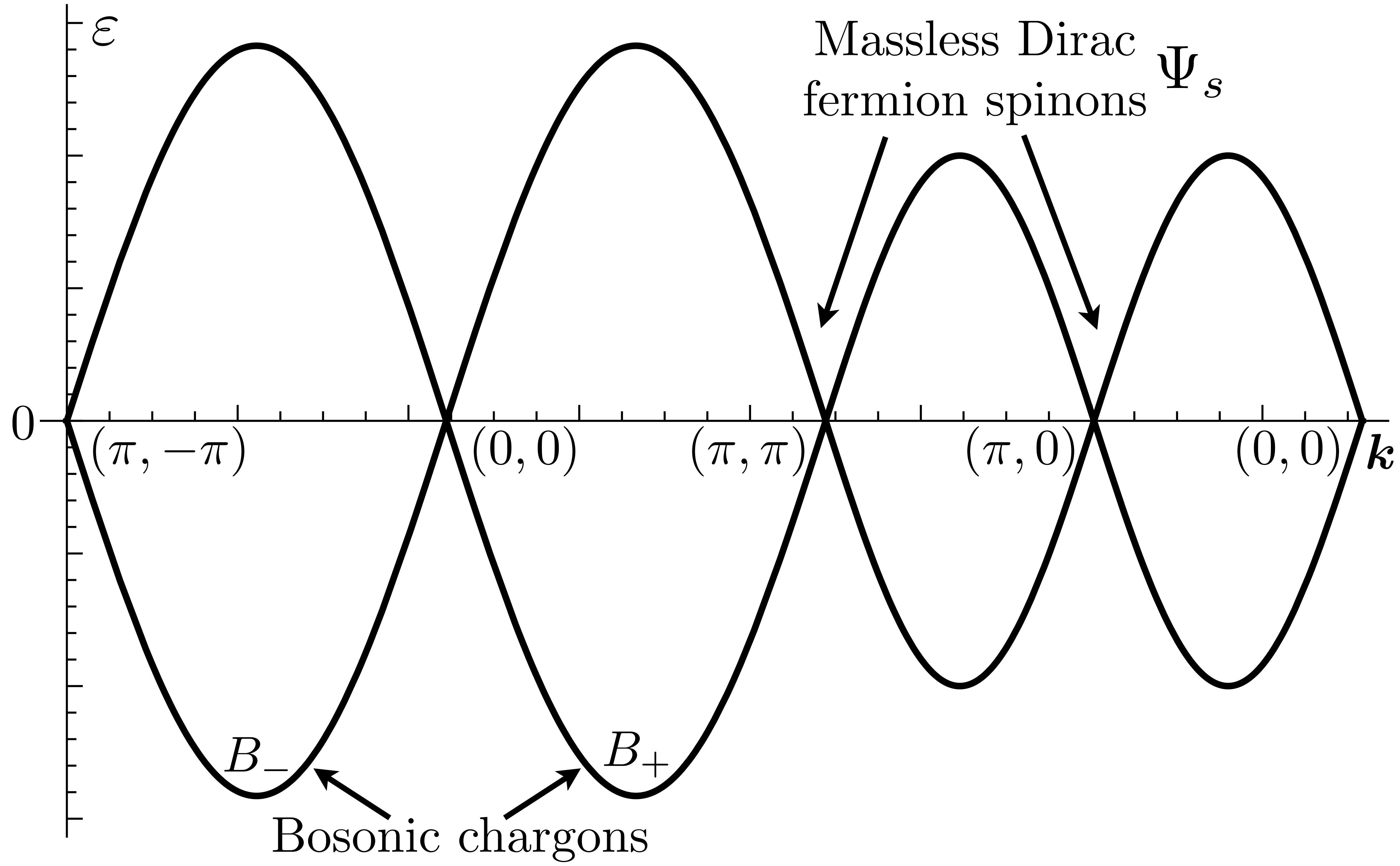


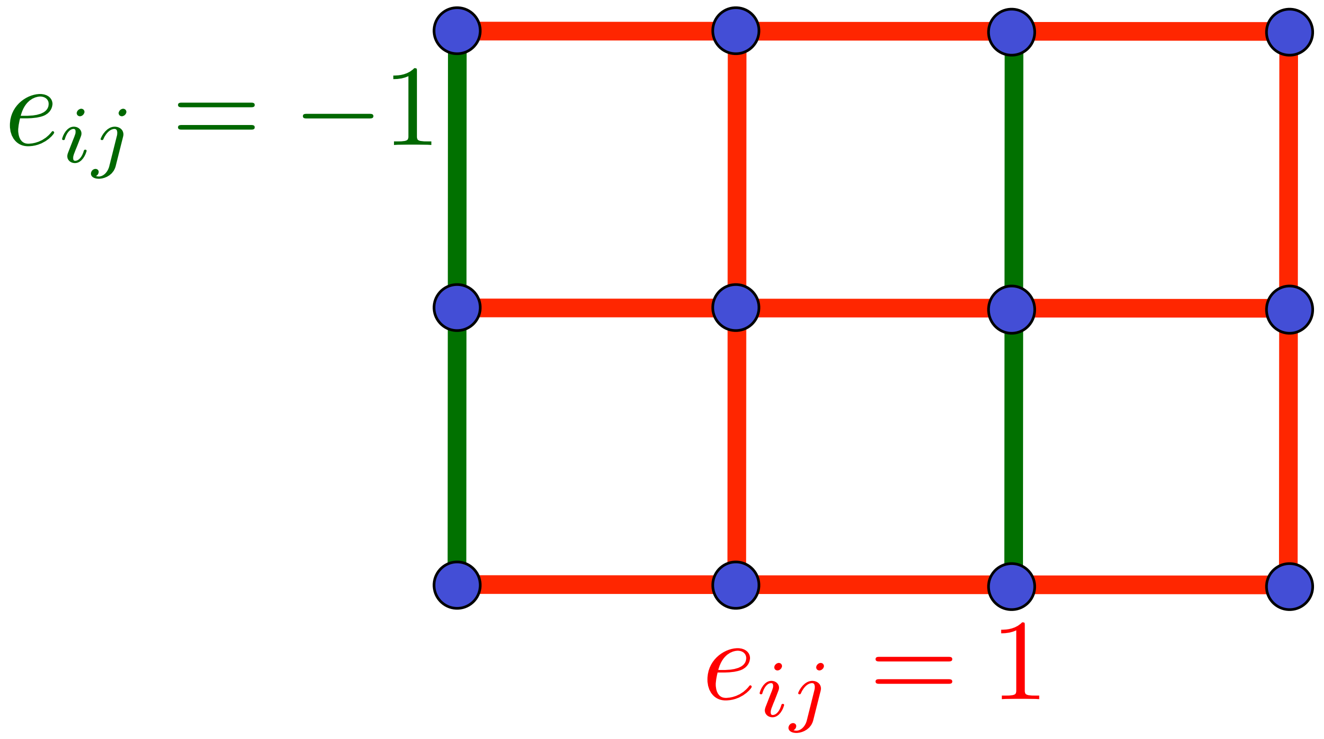
(0,π) stripe



The $\mathcal{O}(B^2_{a\pm})$ terms in the energy have a $SO(5)_b$ rotation symmetry between these orders.

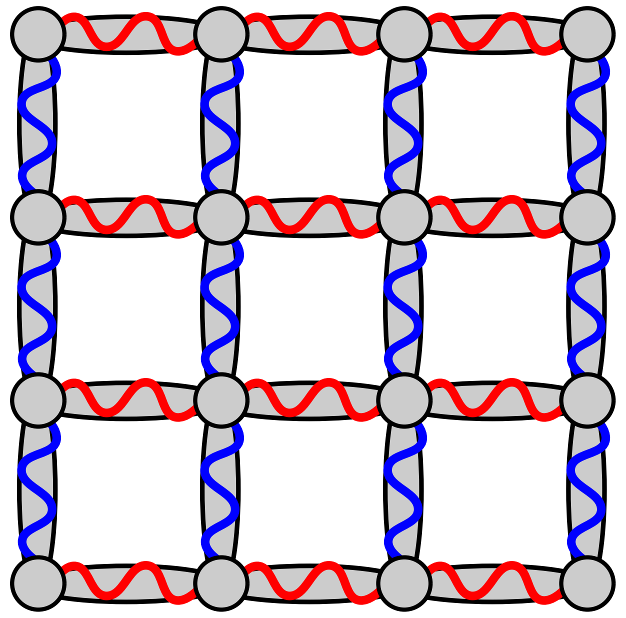
Confinement of $SU(2)_N$ gauge theory by charge fluctuations



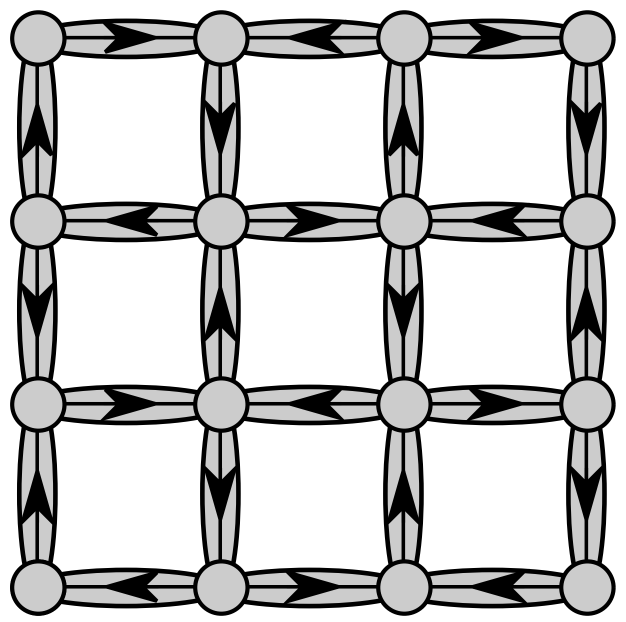


The B_{av}
$(a \to SU(2)_N$ gauge, $v \to$ valley)
are the "square roots" of
conventional
d-wave superconductor,
charge density wave,
pair density wave
…

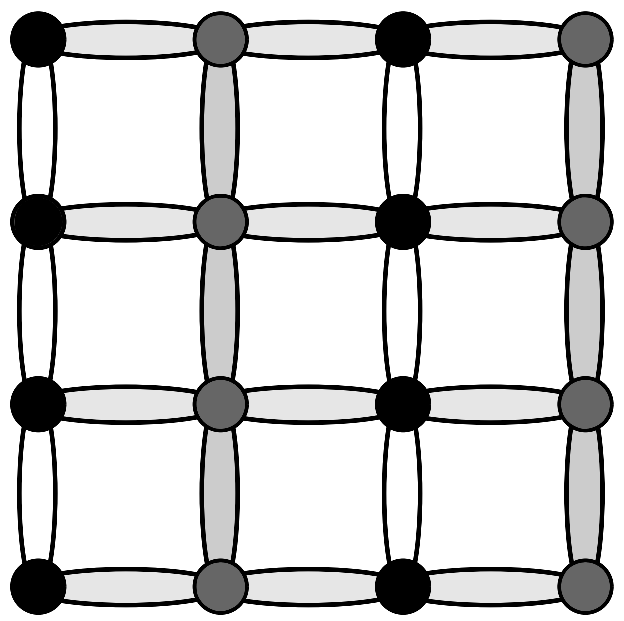
d-wave SC



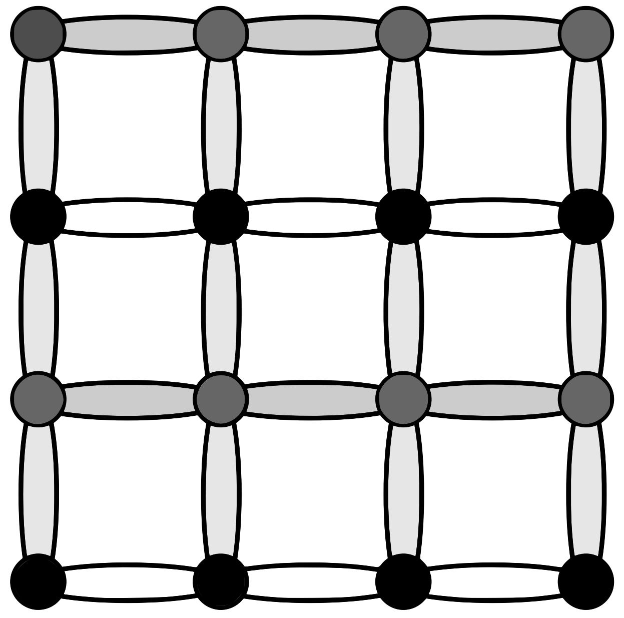
d-density



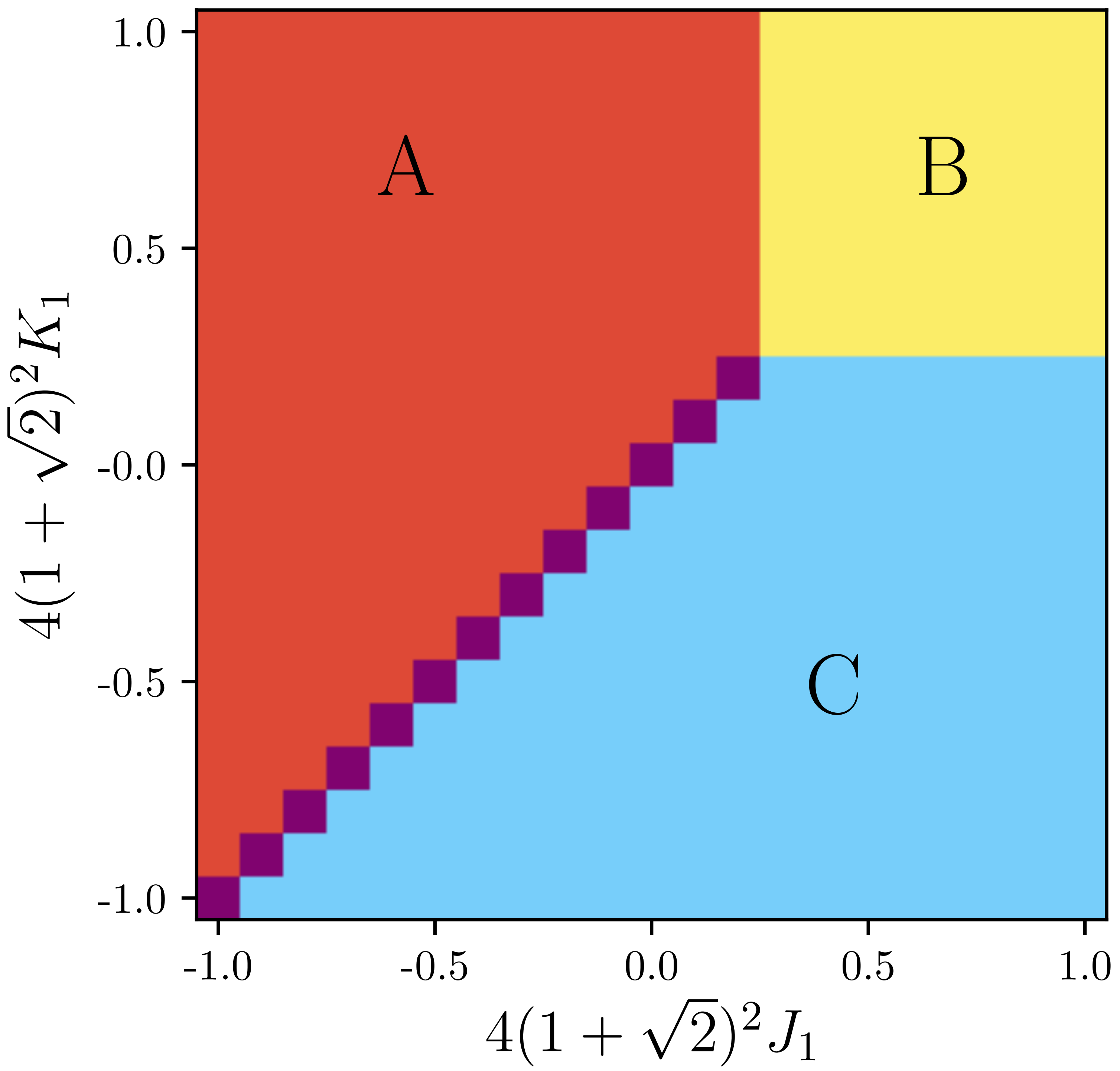
(π,0) stripe



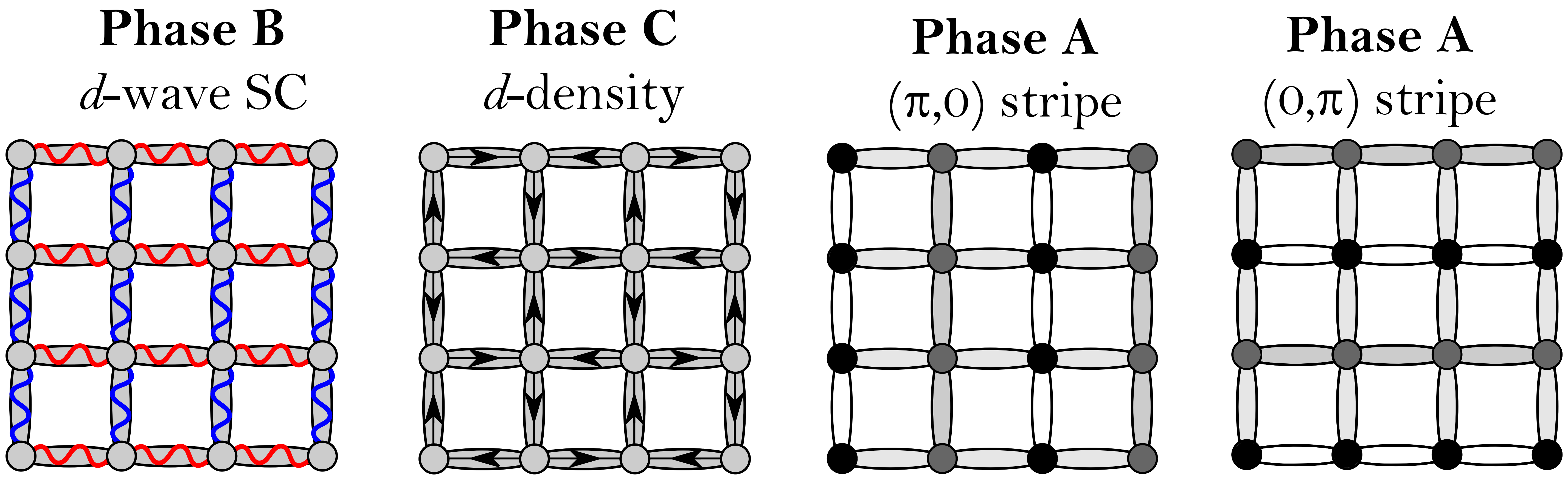
(0,π) stripe



Confinement of SU(2)$_N$ gauge theory by charge fluctuations



$$\langle B \rangle \neq 0$$



Global phase diagram of SU(2)$_N$ gauge theory

$\langle B \rangle \neq 0$

$\mathrm{SO}(5)_b$

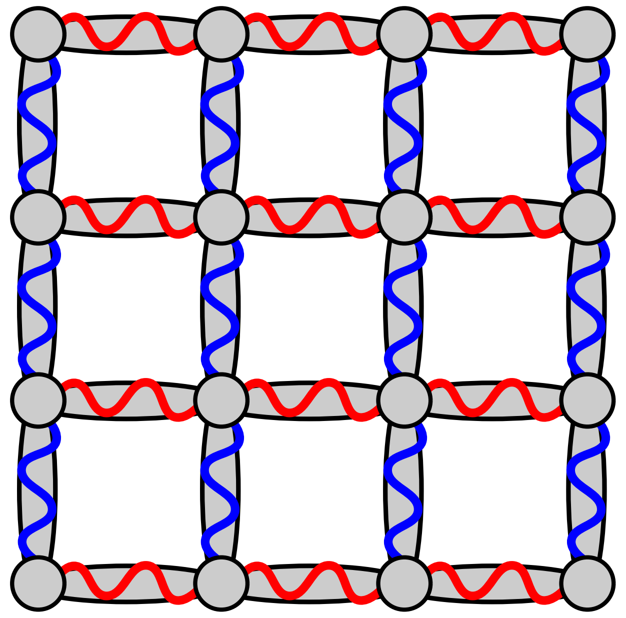
$\langle B \rangle = 0$

$\mathrm{SO}(5)_f$

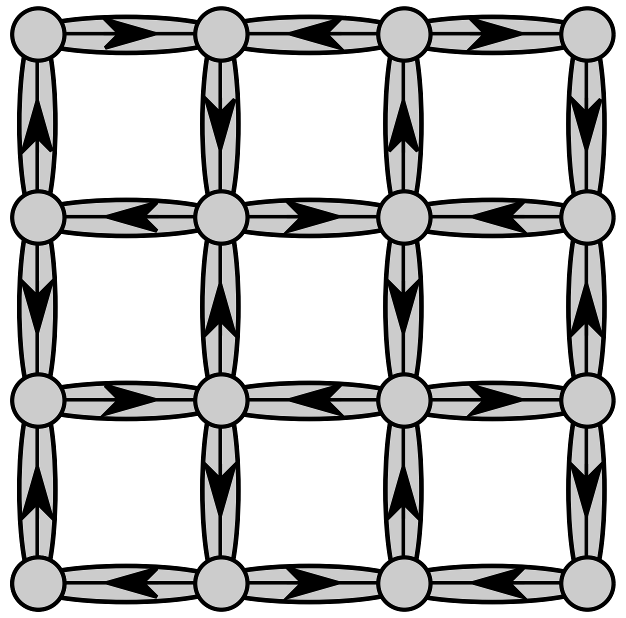
Confining phase:
Néel order

Confining phase:
VBS order

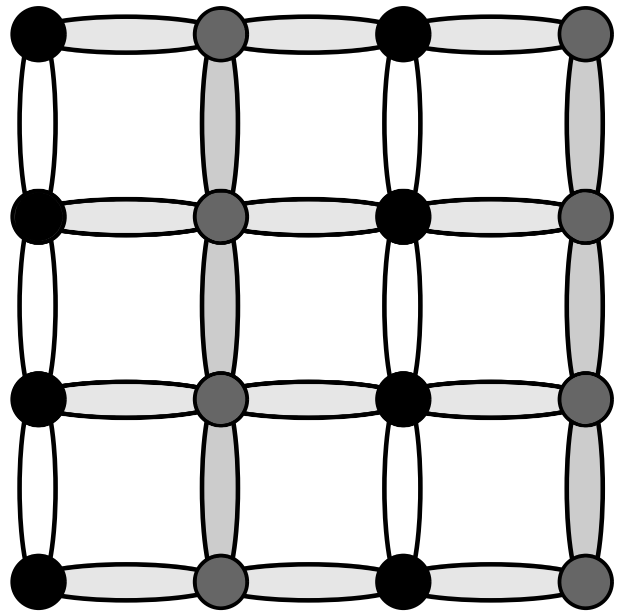
Phase B
d-wave SC



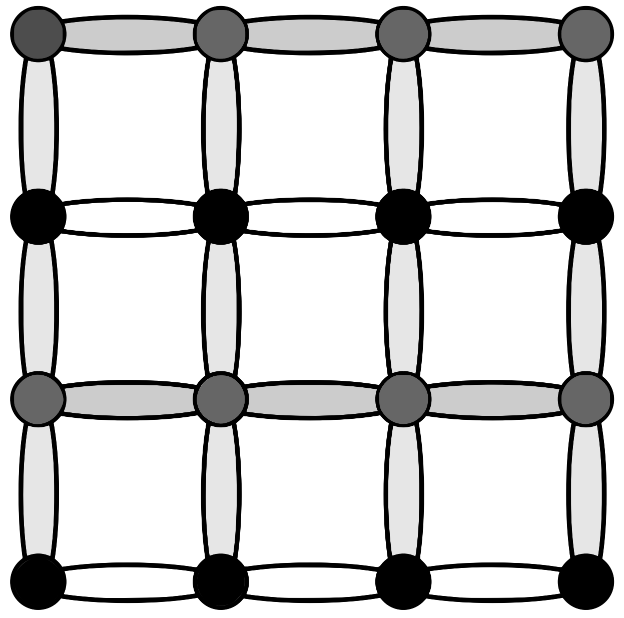
Phase C
d-density



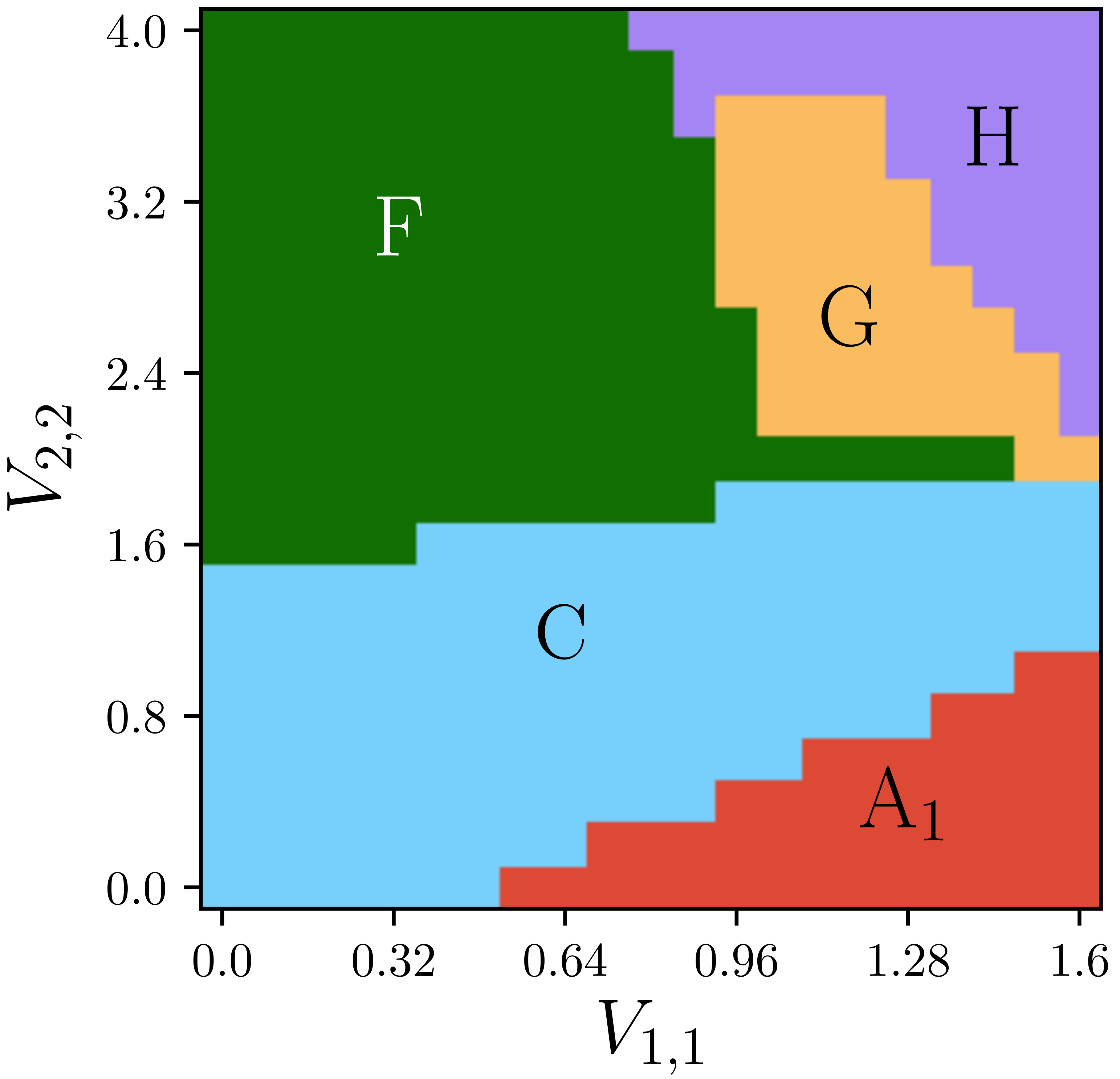
Phase A
(π,0) stripe



Phase A
(0,π) stripe



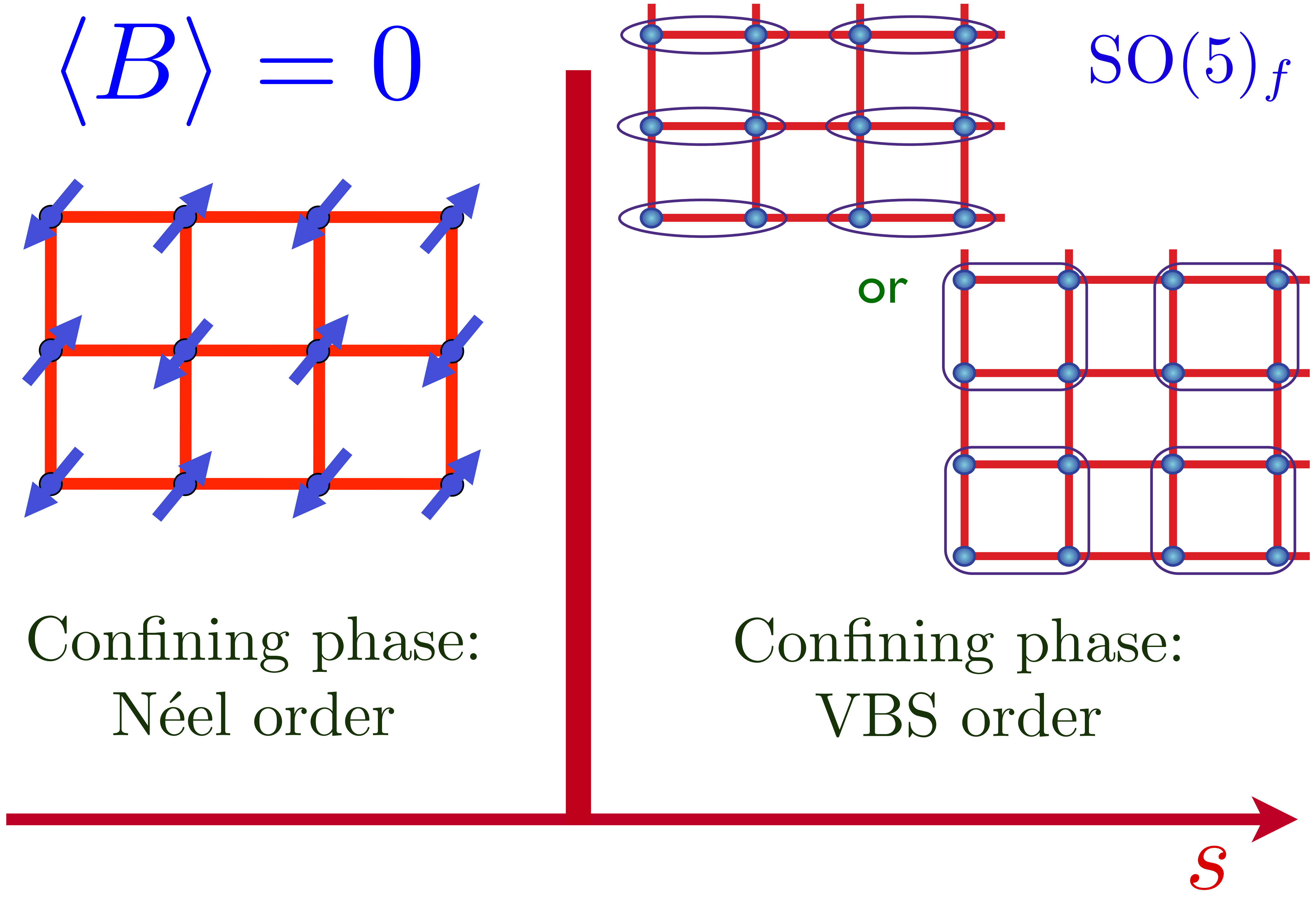
Global phase diagram of $SU(2)_N$ gauge theory



$\langle B \rangle \neq 0$

Including further-neighbor couplings in B

$\langle B \rangle = 0$



$SO(5)_f$

or

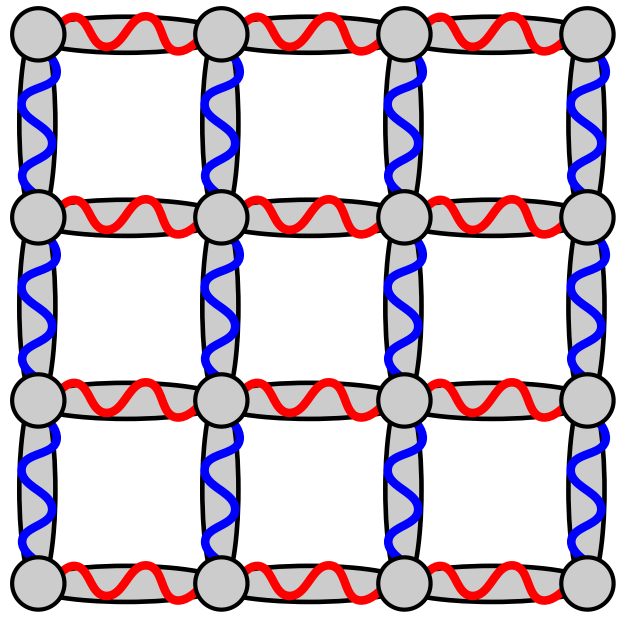
Confining phase:
Néel order

Confining phase:
VBS order

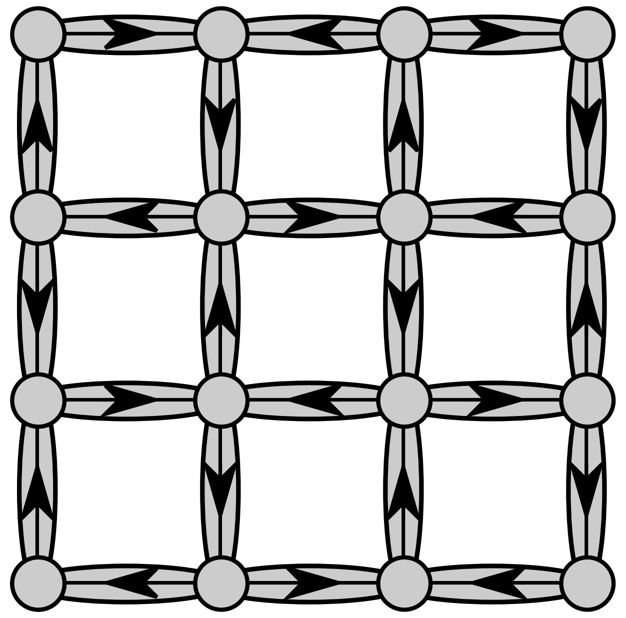
s

r

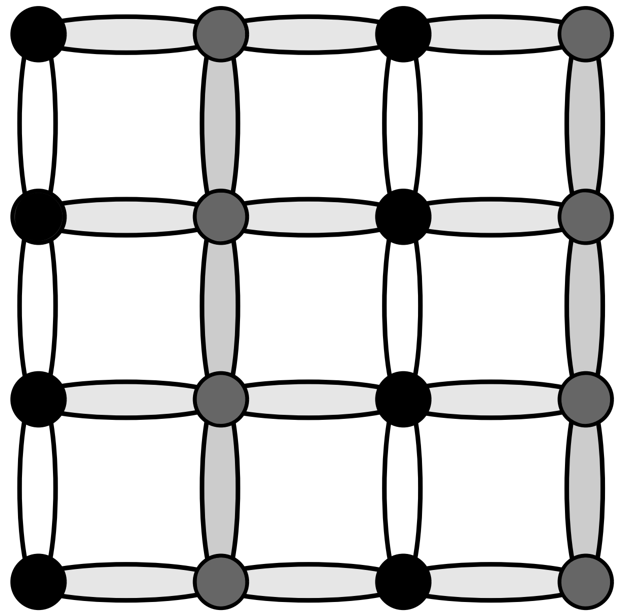
Phase B
d-wave SC



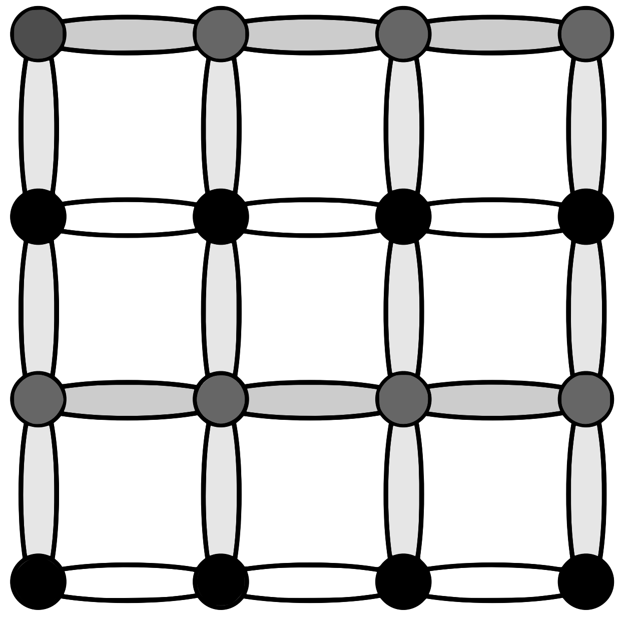
Phase C
d-density



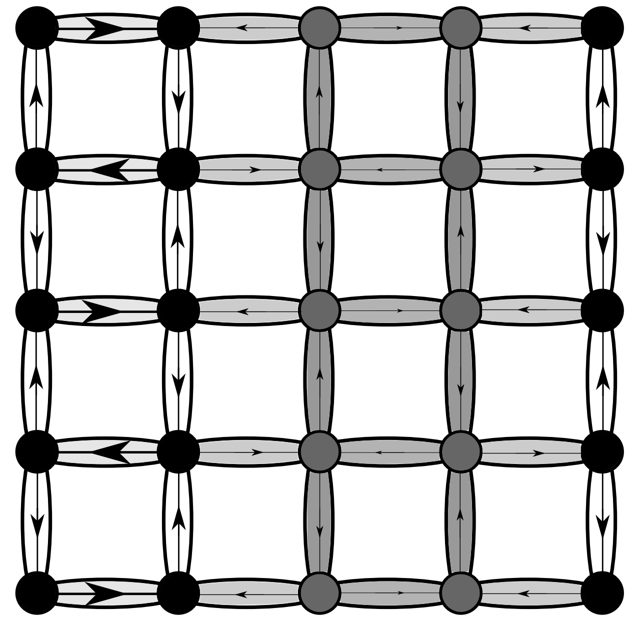
Phase A
(π,0) stripe



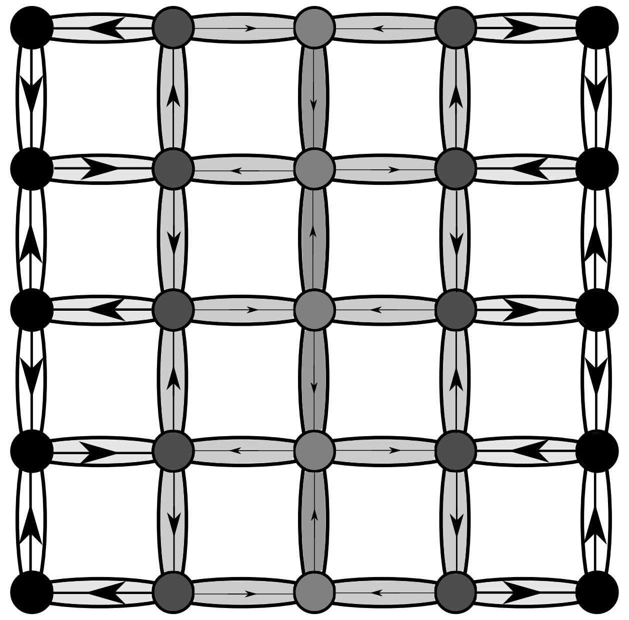
Phase A
(0,π) stripe

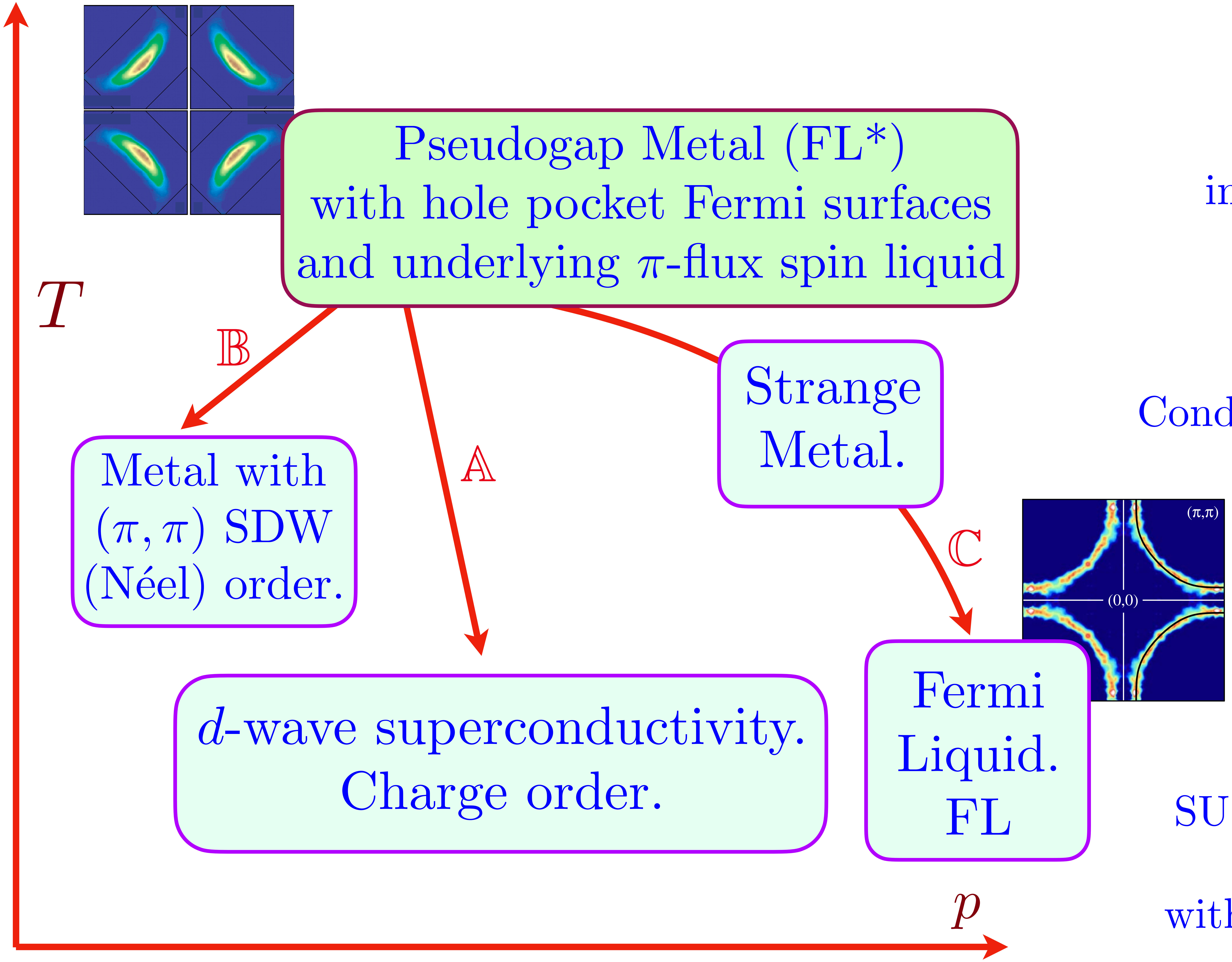


Phase F
Period 4 bond centered
stripe + d-density



Phase G
Period 4 site centered
stripe + d-density





Pseudogap Metal (FL*)
with hole pocket Fermi surfaces
and underlying π-flux spin liquid

Metal with
(π, π) SDW
(Néel) order.

Strange
Metal.

d-wave superconductivity.
Charge order.

Fermi
Liquid.
FL

Arrow $\mathbb{A}$

Condensation of B
in SU$(2)_N$ gauge theory.

Arrow $\mathbb{B}$

Condensation of z_α in dual $\mathbb{CP}^1$
U(1) gauge theory.

Arrow $\mathbb{C}$

SU$(2)_s \times$U$(1)_a$ gauge theory
in ancilla model
with Higgs field $H_{\alpha a} \sim c_\alpha^\dagger f_{1a}$