

Competing orders: beyond Landau-Ginzburg-Wilson theory

Colloquium article in *Reviews of Modern Physics* **75**, 913 (2003)

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Talk online: Google Sachdev

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Possible origins of the pseudogap in the cuprate superconductors:

- “Phase fluctuations”, “preformed pairs”

Complex order parameter: Ψ_{sc}

$\Psi_{sc} \rightarrow \Psi_{sc} e^{i\theta}$ symmetry encodes number conservation

- “Charge/valence-bond/pair-density/stripe” order

Order parameters: $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{r}}$

(density ρ represents any observable invariant under spin rotations, time-reversal, and spatial inversion)

$\rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\theta}$ encodes space group symmetry

- “Spin liquid”

Order parameters are not independent

Ginzburg-Landau-Wilson approach to competing order parameters:

$$F = F_{sc} [\Psi_{sc}] + F_{\text{charge}} [\rho_Q] + F_{\text{int}}$$

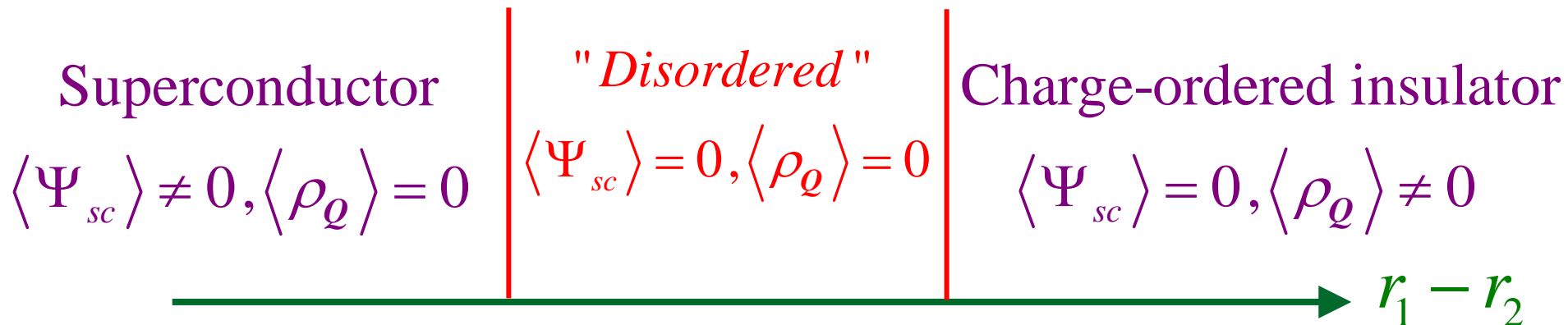
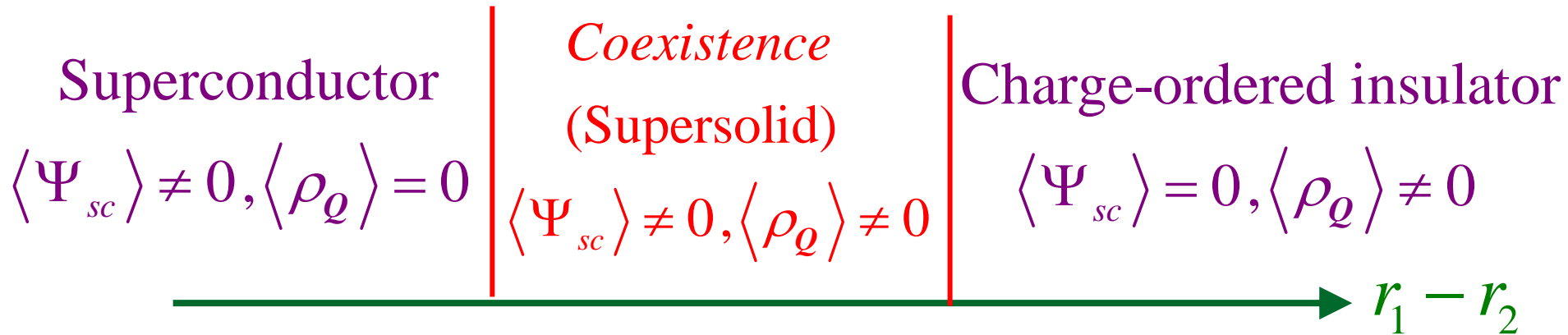
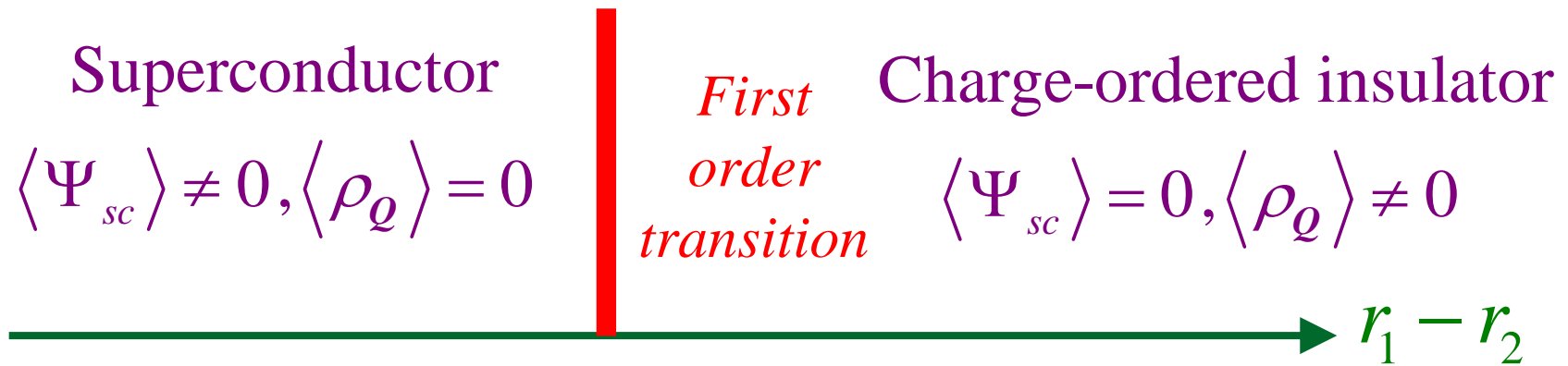
$$F_{sc} [\Psi_{sc}] = r_1 |\Psi_{sc}|^2 + u_1 |\Psi_{sc}|^4 + \dots$$

$$F_{\text{charge}} [\rho_Q] = r_2 |\rho_Q|^2 + u_2 |\rho_Q|^4 + \dots$$

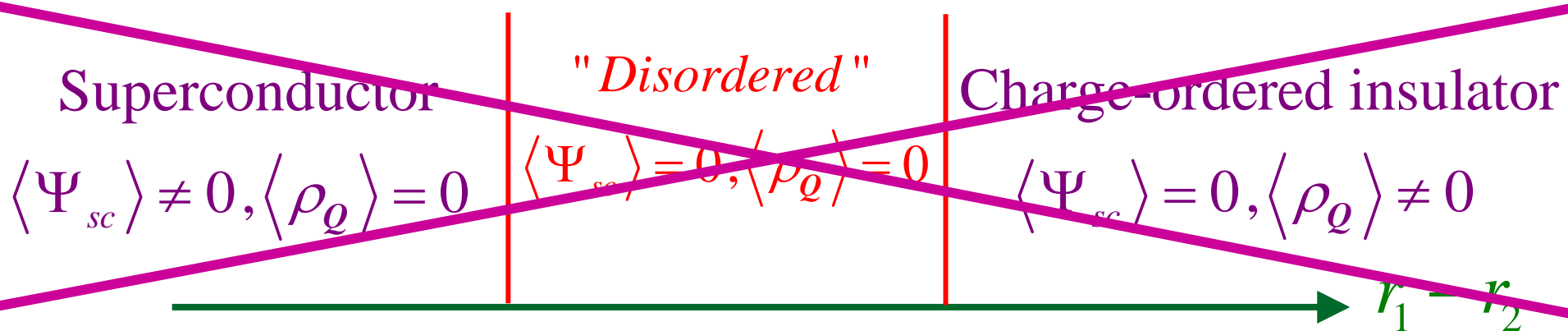
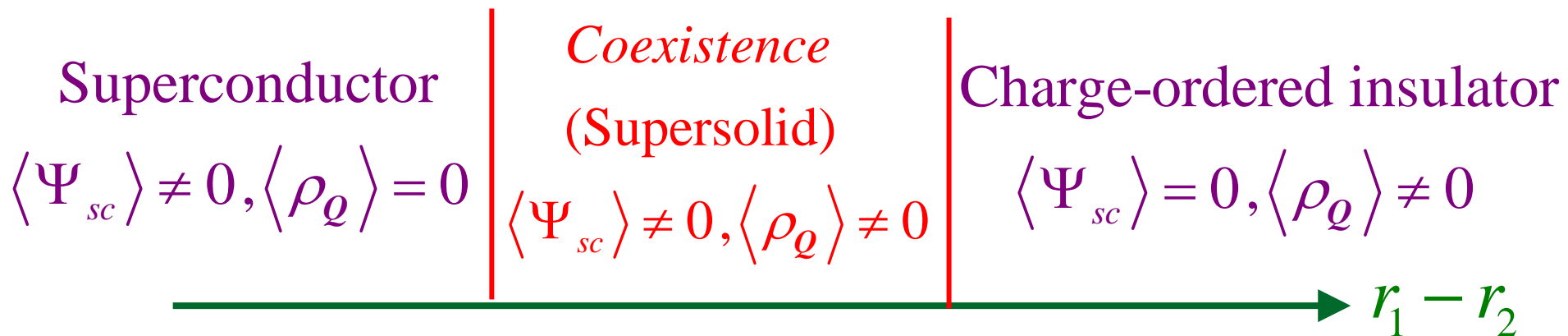
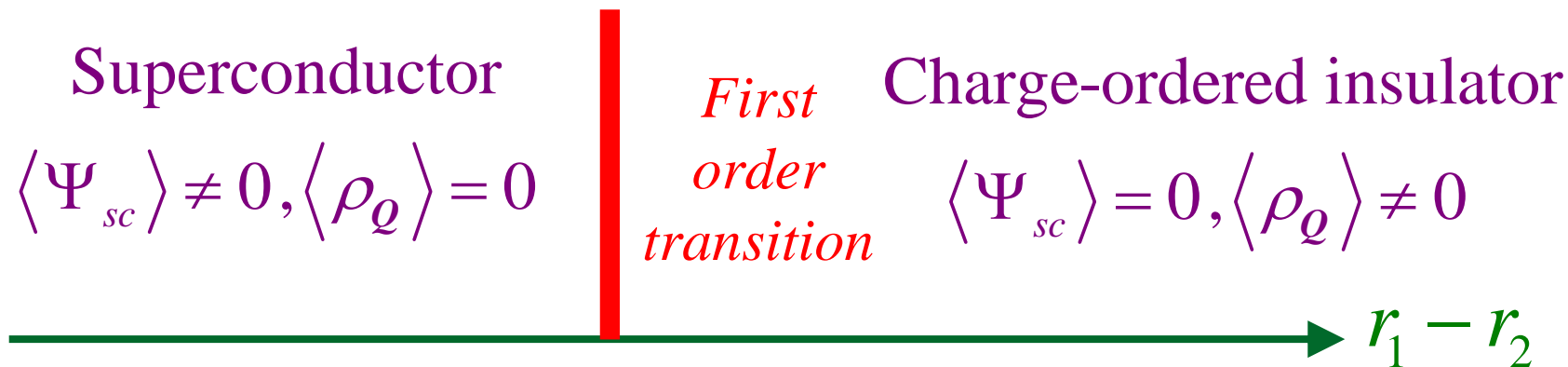
$$F_{\text{int}} = v |\Psi_{sc}|^2 |\rho_Q|^2 + \dots$$

Distinct symmetries of order parameters permit couplings only between their energy densities

Predictions of LGW theory



Predictions of LGW theory



Non-superconducting quantum phase must have some other “order”:

- Charge order in an insulator
- Fermi surface in a metal
- “Topological order” in a spin liquid
-

This requirement is not captured by LGW theory.

Outline

- A. Superfluid-insulator transitions of bosons on the square lattice at fractional filling
Dual vortex theory and the magnetic space group.

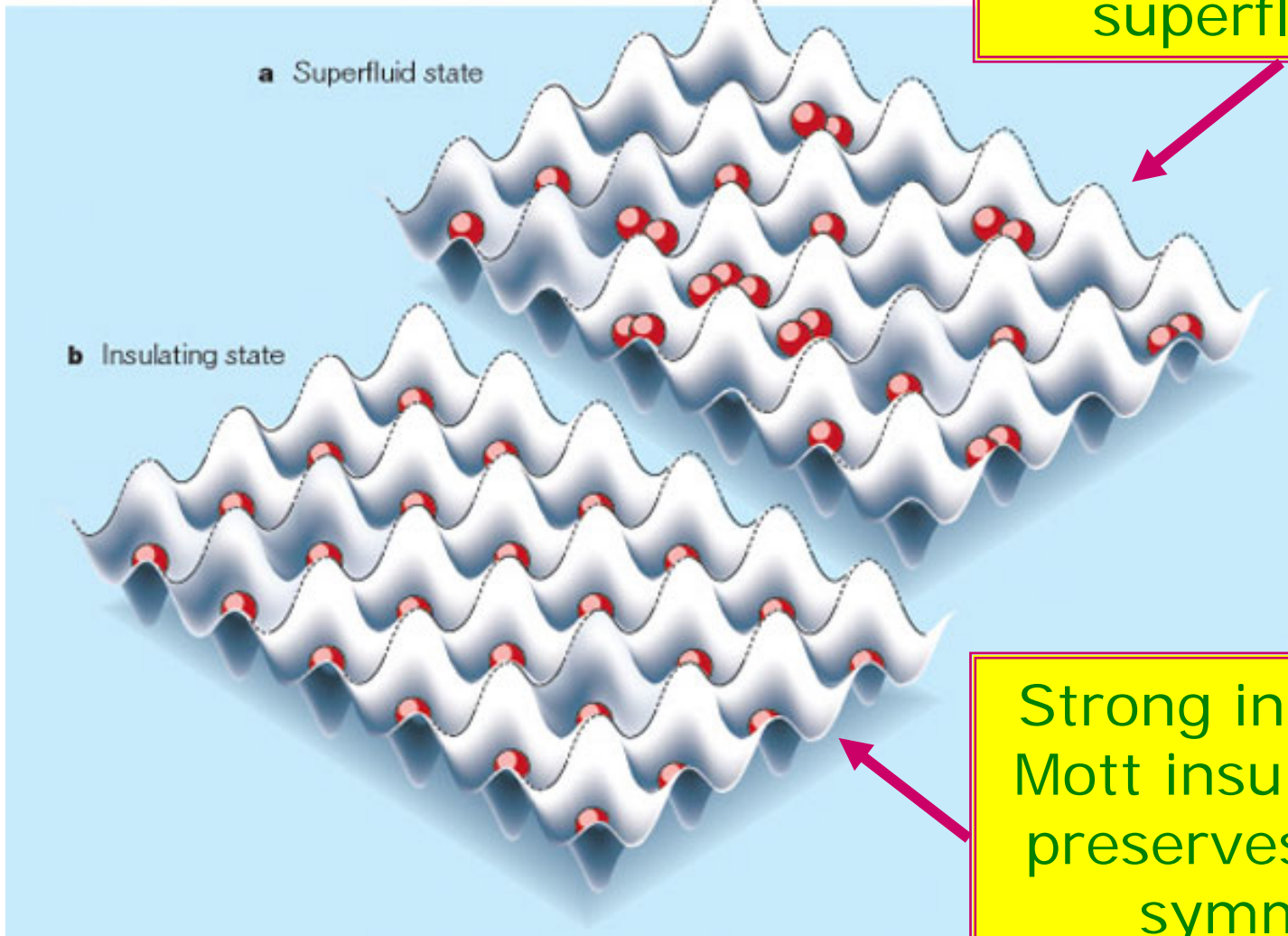
- B. Application to a short-range pairing model for the cuprate superconductors
Charge order and d-wave superconductivity in an effective theory for the spin $S=0$ sector.

- C. Implications for STM

A. Superfluid-insulator transitions of bosons
on the square lattice at fractional filling

*Dual vortex theory and
the magnetic space group.*

Bosons at density $f = 1$



Weak interactions:
superfluidity

Strong interactions:
Mott insulator which
preserves all lattice
symmetries

LGW theory: continuous quantum transitions between these states

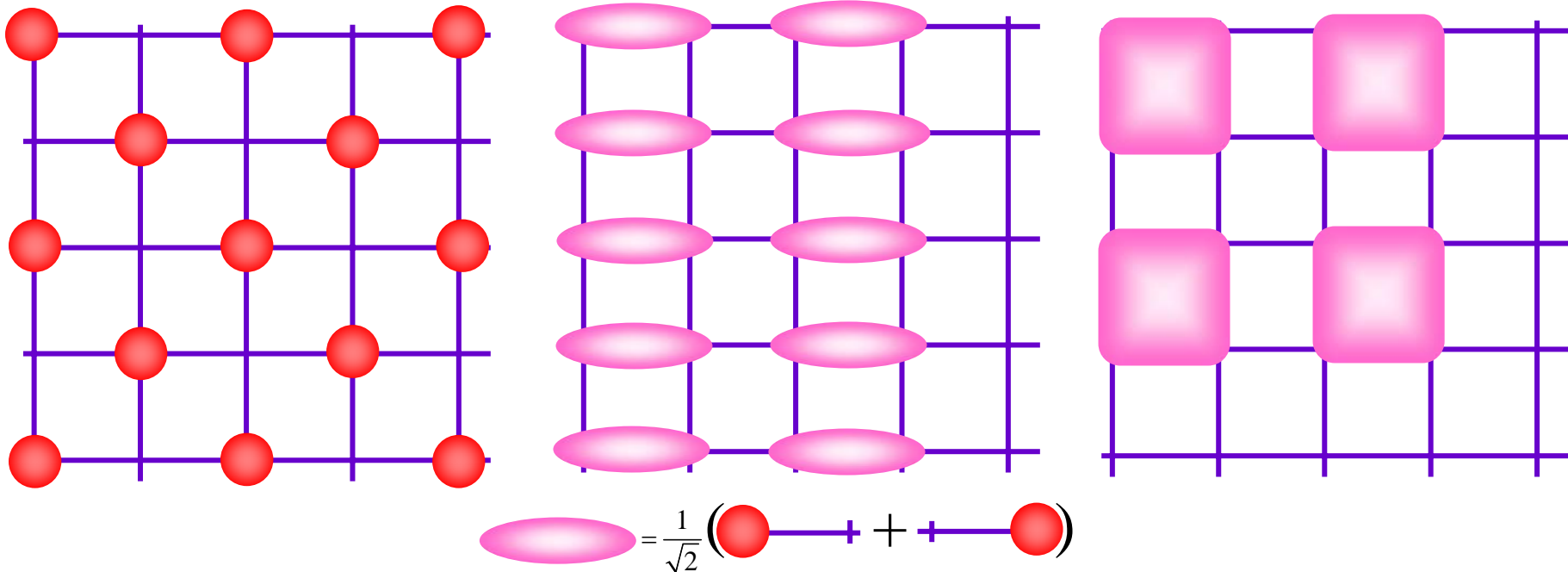
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Bosons at density $f = 1/2$

Weak interactions: superfluidity

$$\langle \Psi_{sc} \rangle \neq 0$$

Strong interactions: Candidate insulating states

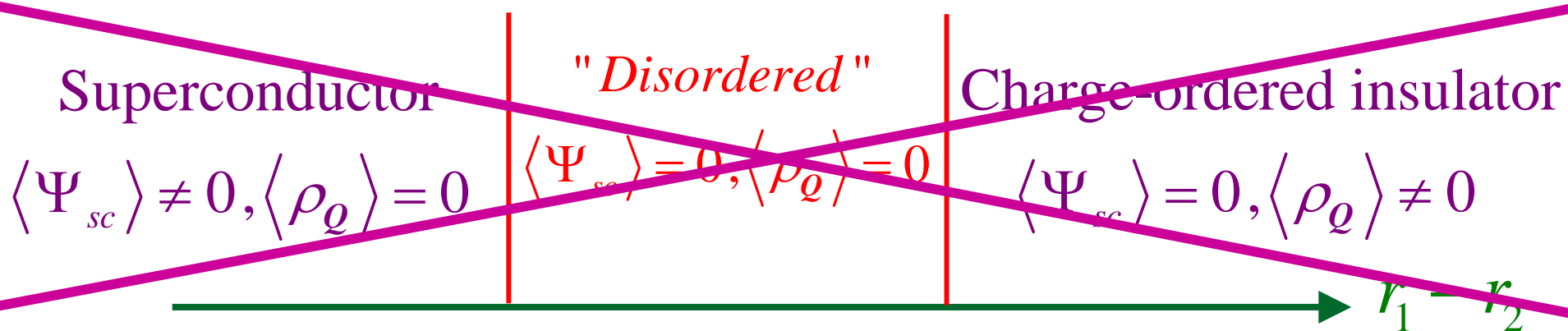
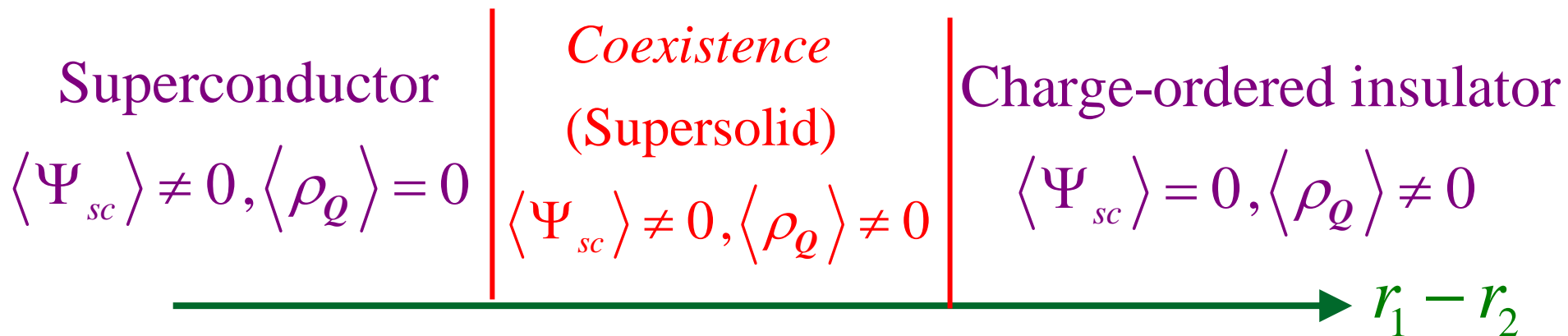
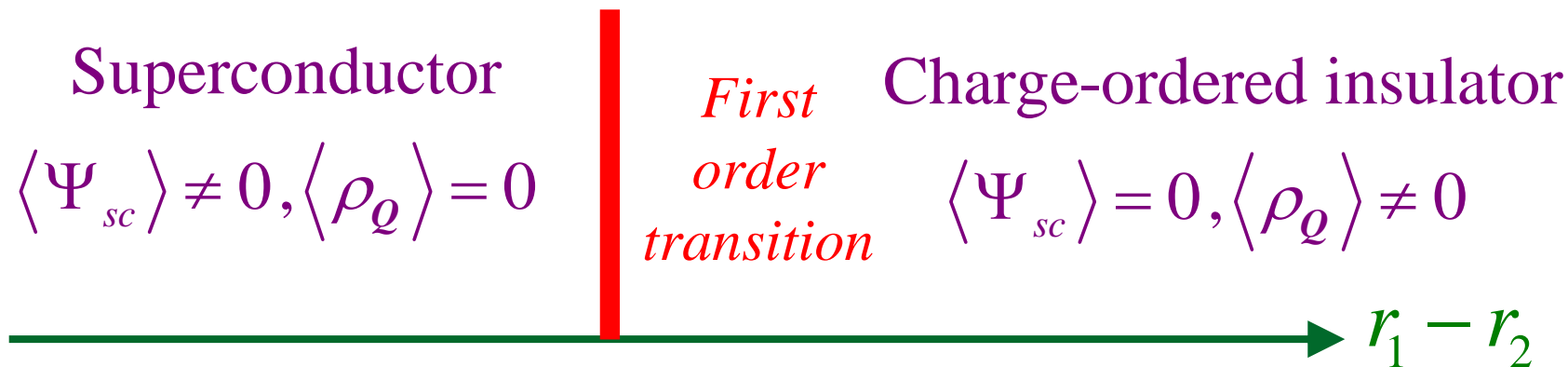


All insulating phases have density-wave order $\rho(\mathbf{r}) = \sum_{\mathbf{q}} \rho_{\mathbf{q}} e^{i\mathbf{q} \cdot \mathbf{r}}$ with $\langle \rho_{\mathbf{q}} \rangle \neq 0$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev. B* **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

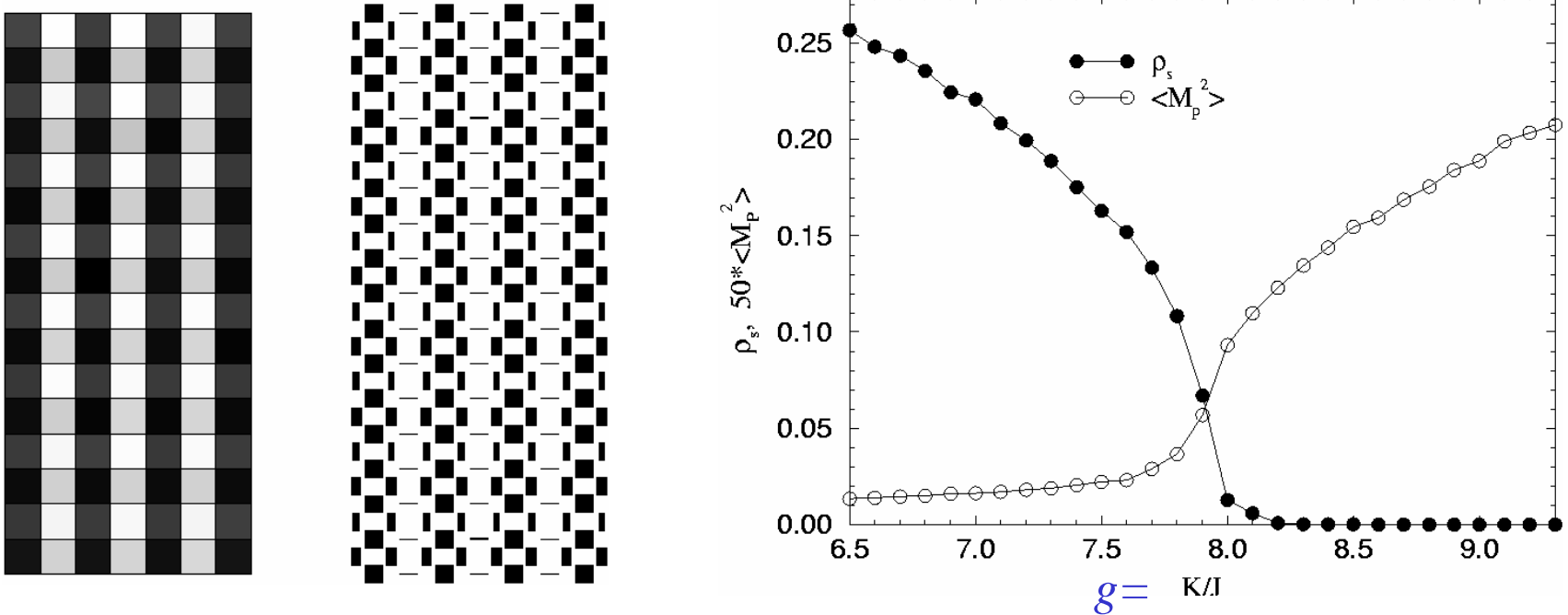
Predictions of LGW theory



Superfluid insulator transition of hard core bosons at $f=1/2$

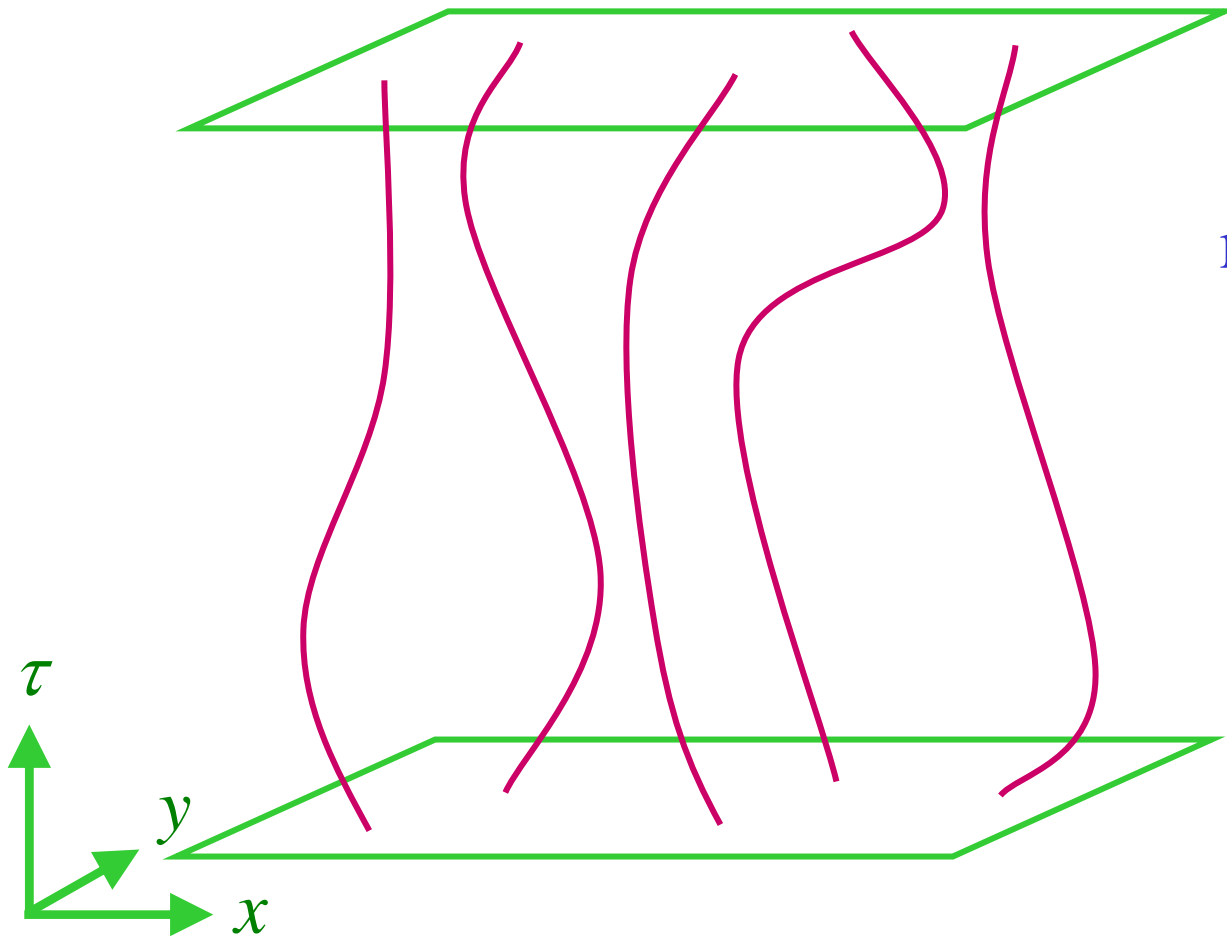
A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002)

Large scale (> 8000 sites) numerical study of the destruction of superfluid order at half filling with full square lattice symmetry



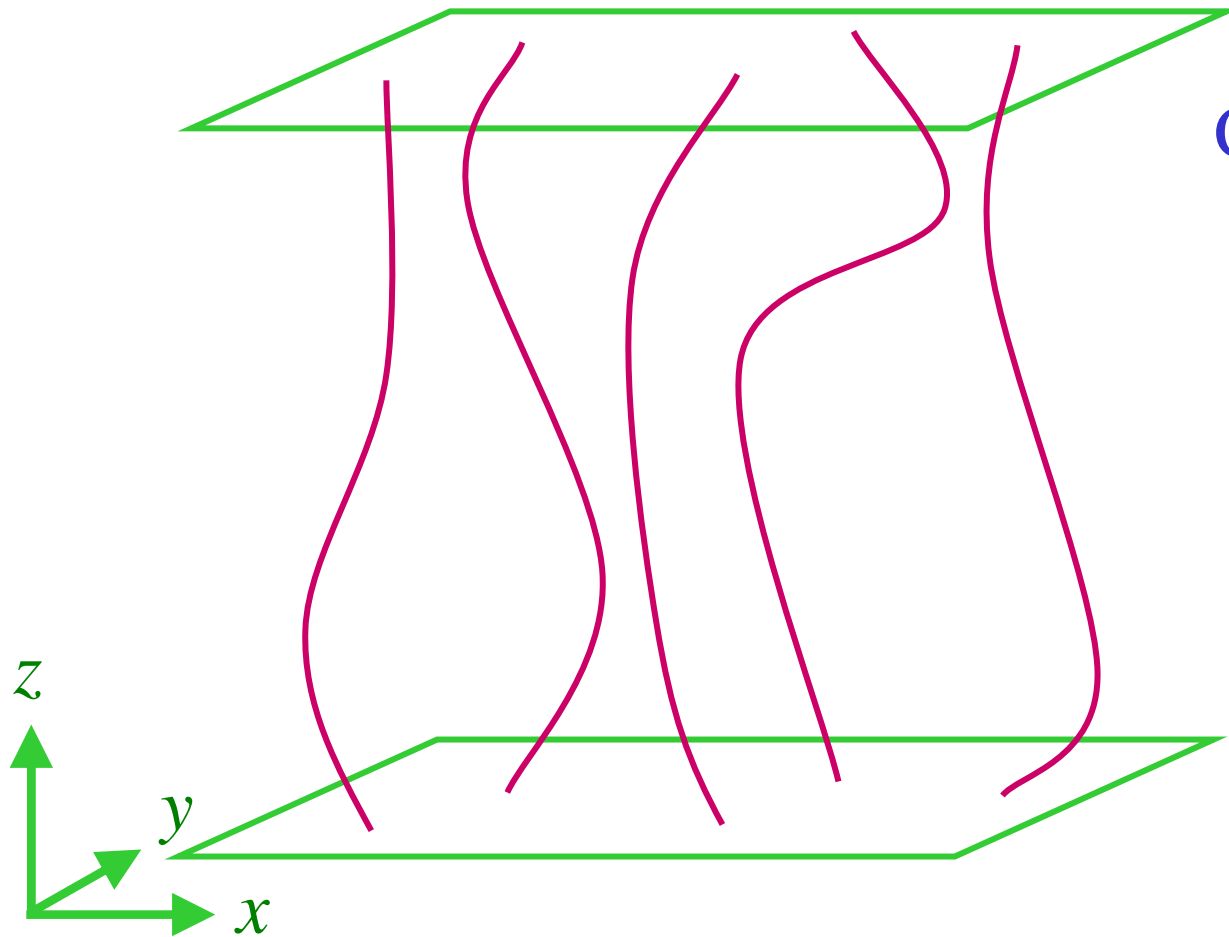
$$H = J \sum_{\langle ij \rangle} (S_i^+ S_j^- + S_i^- S_j^+) - K \sum_{\langle ijkl \rangle \subset \square} (S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+)$$

Boson-vortex duality



Quantum
mechanics of two-
dimensional
bosons: world
lines of bosons in
spacetime

Boson-vortex duality



Classical statistical mechanics of a “dual” three-dimensional superconductor: vortices in a “magnetic” field

Strength of “magnetic” field = density of bosons
= f flux quanta per plaquette

Boson-vortex duality

Statistical mechanics of dual superconductor is invariant under the square lattice space group:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

Magnetic space group:

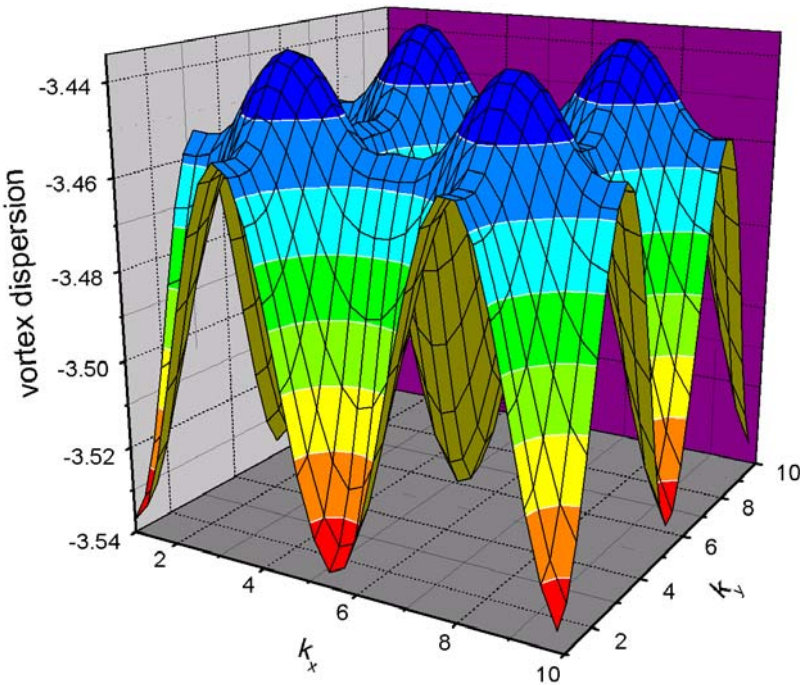
$$T_x T_y = e^{2\pi i f} T_y T_x \ ;$$

$$R^{-1} T_y R = T_x \ ; \ R^{-1} T_x R = T_y^{-1} \ ; \ R^4 = 1$$

Strength of “magnetic” field = density of bosons
= f flux quanta per plaquette

Boson-vortex duality

Hofstadter spectrum of dual “superconducting” order



At density $f = p / q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \dots q$), associated with q gauge-equivalent regions of the Brillouin zone

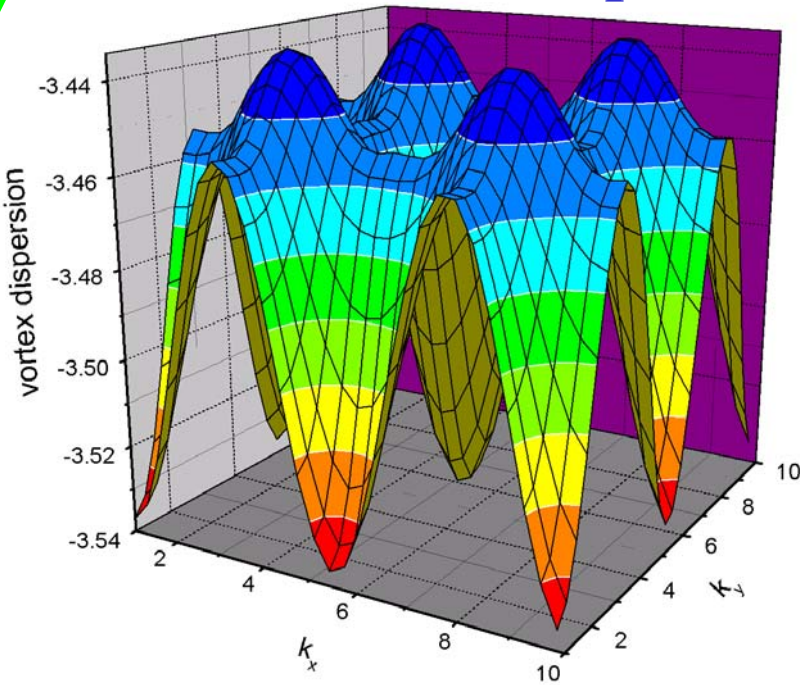
Magnetic space group:

$$T_x T_y = e^{2\pi i f} T_y T_x ;$$

$$R^{-1} T_y R = T_x ; \quad R^{-1} T_x R = T_y^{-1} ; \quad R^4 = 1$$

Boson-vortex duality

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At density $f = p / q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \dots q$), associated with q gauge-equivalent regions of the Brillouin zone

The q vortices form a *projective* representation of the space group

$$T_x : \varphi_\ell \rightarrow \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \rightarrow e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \rightarrow \frac{1}{\sqrt{q}} \sum_{m=1}^q \varphi_m e^{2\pi i \ell m f}$$

Boson-vortex duality

The φ_ℓ fields characterize *both* superconducting and charge order

Superconductor/insulator : $\langle \varphi_\ell \rangle = 0 / \langle \varphi_\ell \rangle \neq 0$

Charge order:

Status of space group symmetry determined by

density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m, n)$

$$\rho_{\mathbf{Q}_{mn}} = e^{i\pi mnf} \sum_{\ell=1}^q \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell m f}$$

$$T_x : \rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\mathbf{Q} \cdot \hat{x}} \quad ; \quad T_y : \rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\mathbf{Q} \cdot \hat{y}}$$

$$R : \rho(\mathbf{Q}) \rightarrow \rho(R\mathbf{Q})$$

Boson-vortex duality

The φ_ℓ fields characterize *both* superconducting and charge order

Competition between superconducting and charge orders:

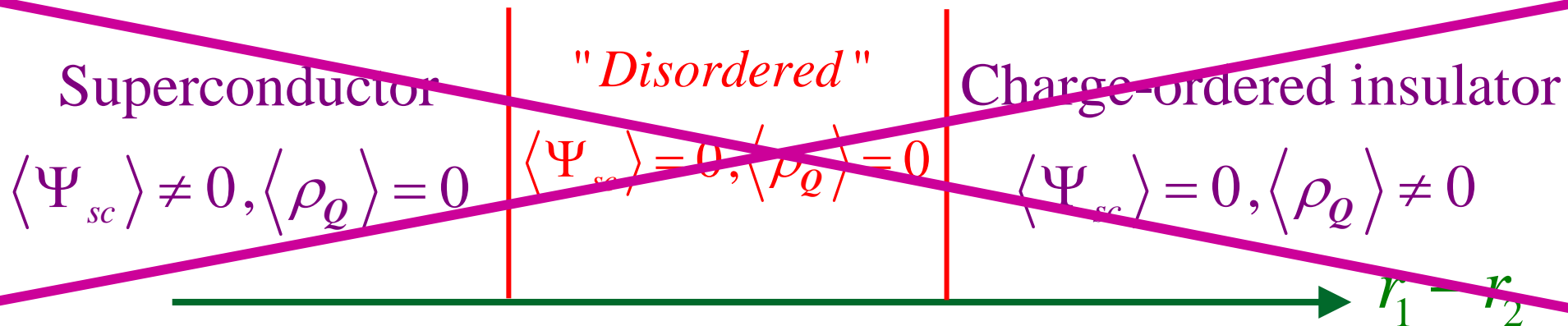
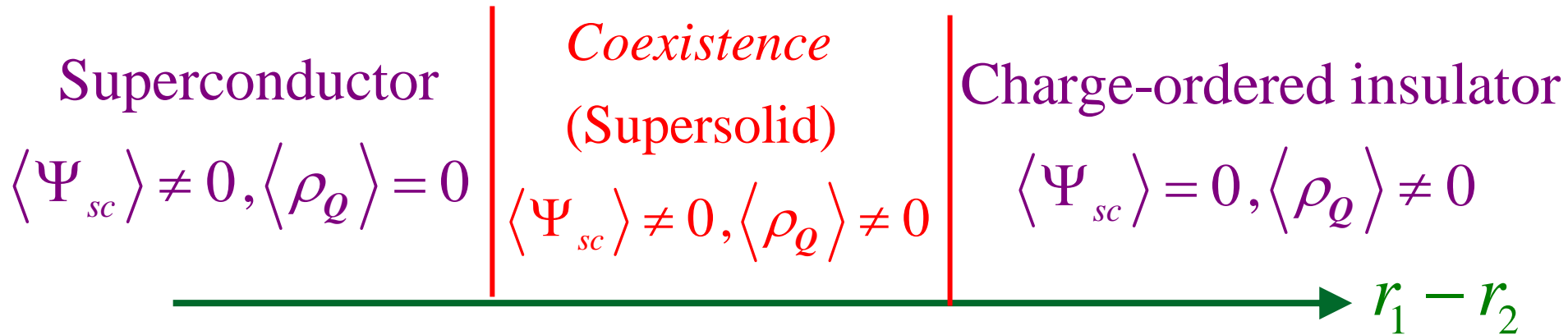
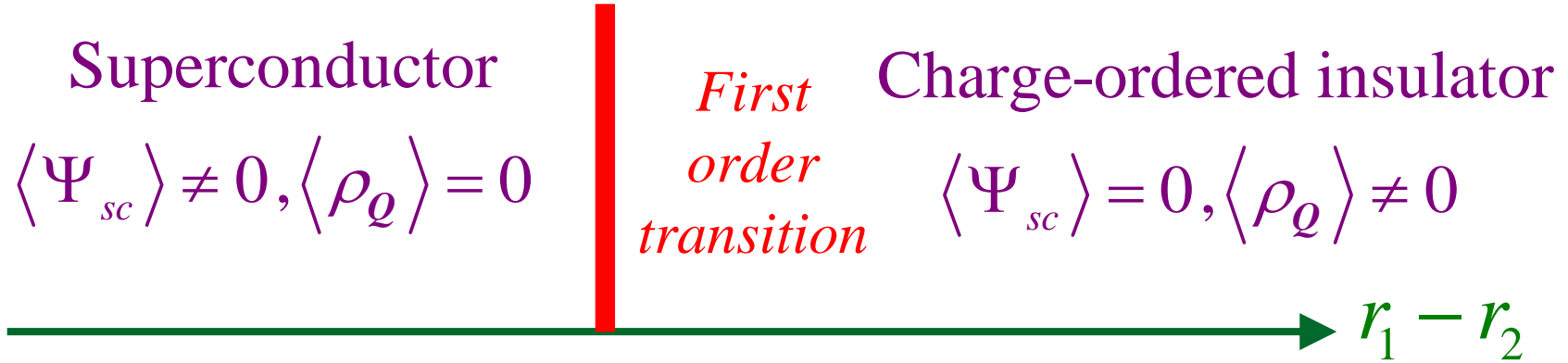
"*Extended* LGW" theory of the φ_ℓ fields with the action invariant under the projective transformations:

$$T_x : \varphi_\ell \rightarrow \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \rightarrow e^{2\pi i l f} \varphi_\ell$$

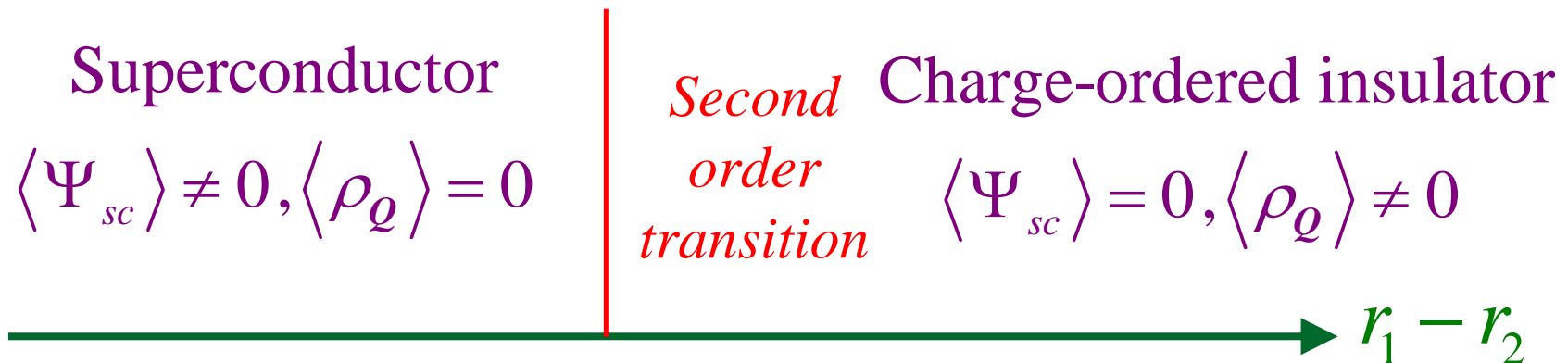
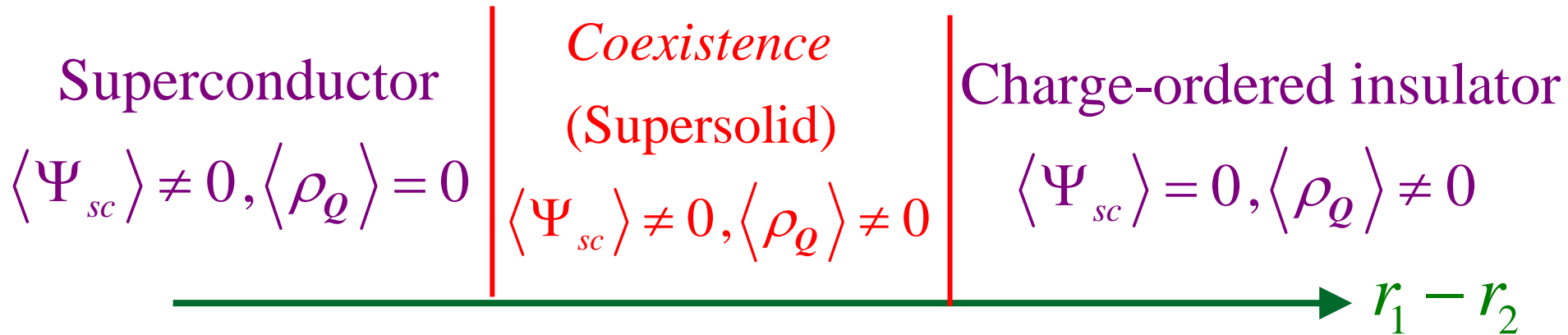
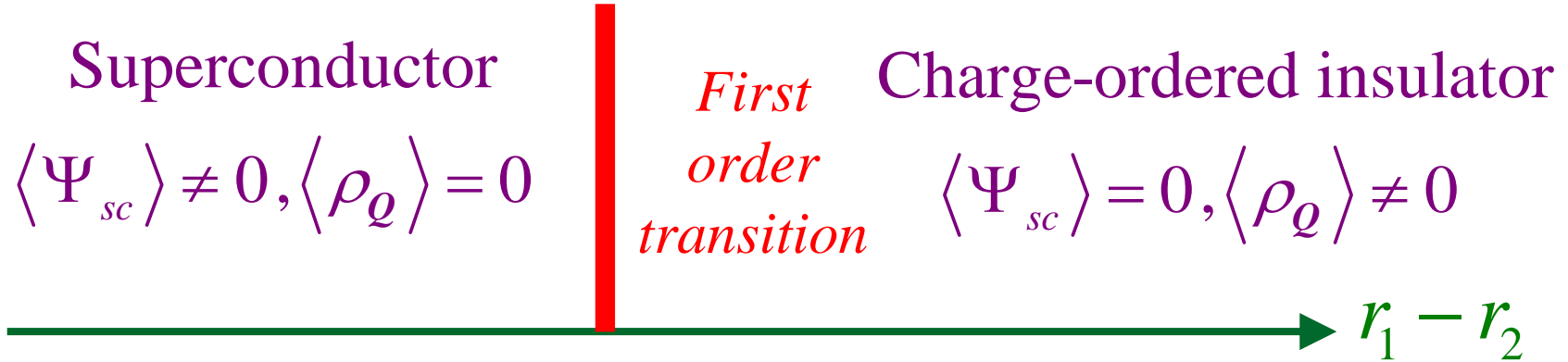
$$R : \varphi_\ell \rightarrow \frac{1}{\sqrt{q}} \sum_{m=1}^q \varphi_m e^{2\pi i l m f}$$

Immediate benefit: There is no intermediate
“disordered” phase with neither order
(or without “topological” order).

Analysis of "extended LGW" theory of projective representation

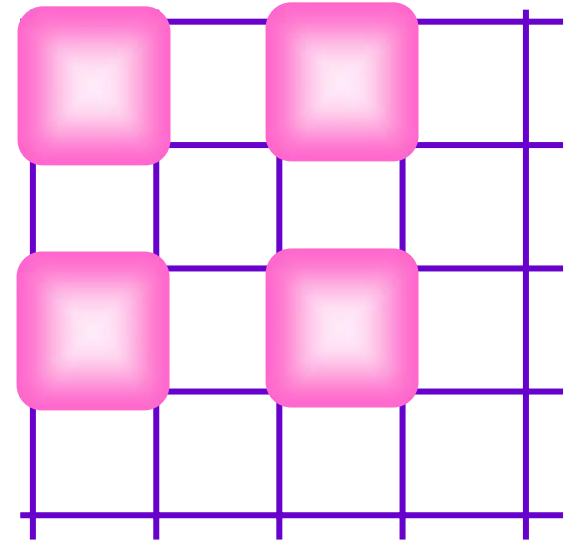
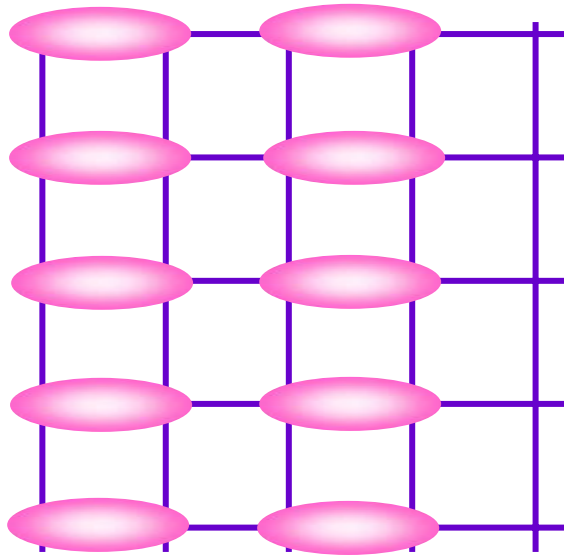
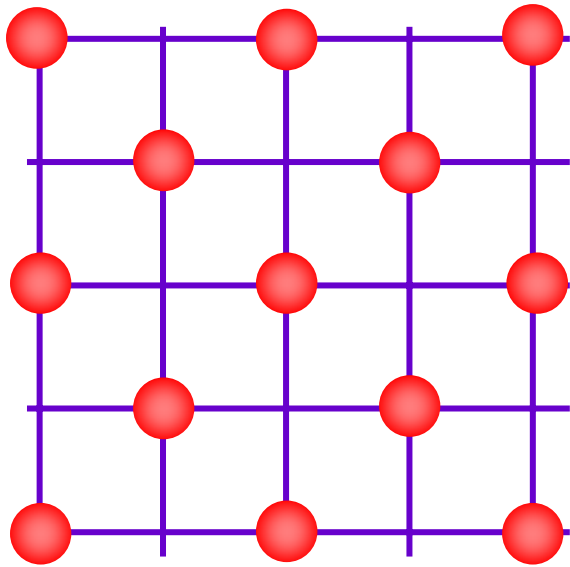


Analysis of “extended LGW” theory of projective representation



Analysis of “extended LGW” theory of projective representation

Spatial structure of insulators for $q=2$ ($f=1/2$)

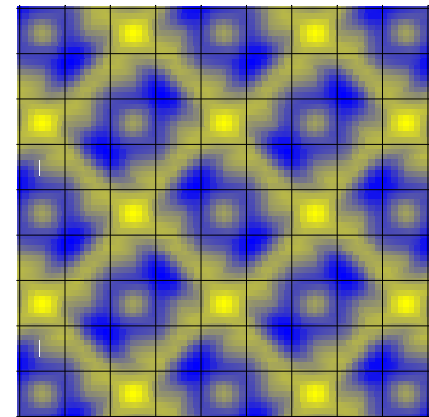
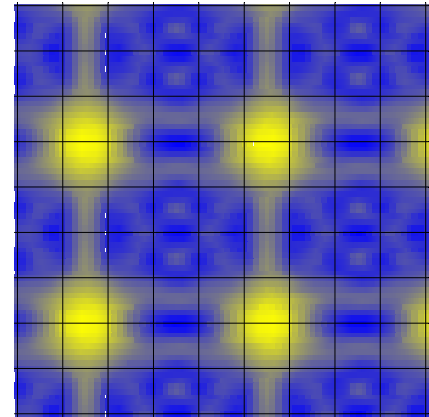
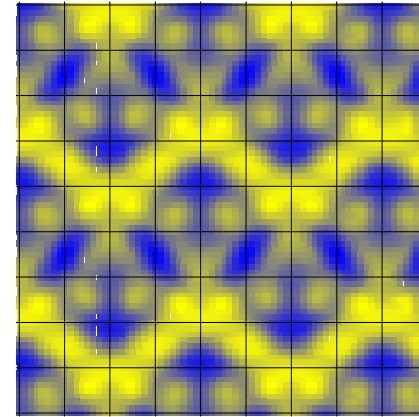
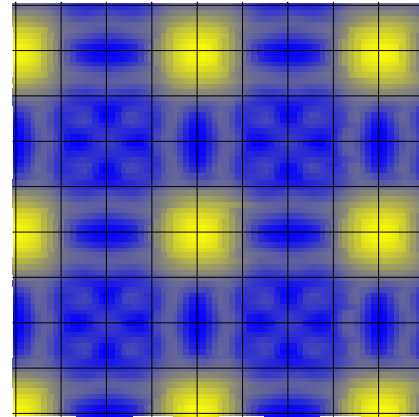
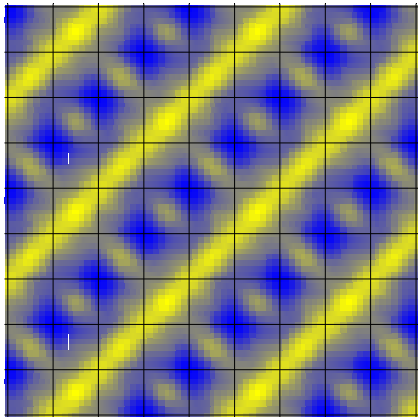
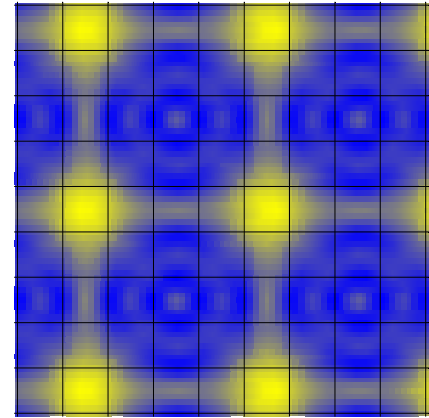
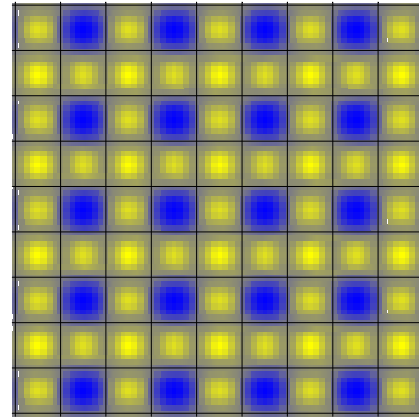
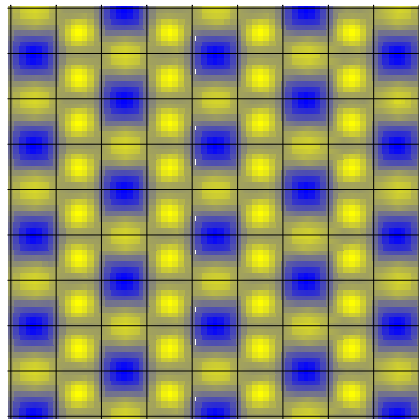
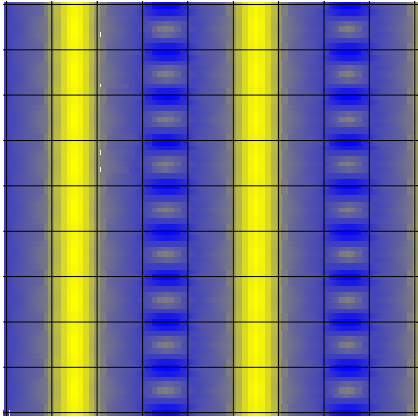


$$\text{pink oval} = \frac{1}{\sqrt{2}} (\text{red circle} + + + \text{red circle})$$

$a \times b$ unit cells; $\frac{q}{a}$, $\frac{q}{b}$, $\frac{ab}{q}$, all integers

Analysis of “extended LGW” theory of projective representation

Spatial structure of insulators for $q=4$ ($f=1/4$ or $3/4$)



$a \times b$ unit cells; $\frac{q}{a}$, $\frac{q}{b}$, $\frac{ab}{q}$, all integers

B. Application to a short-range pairing model for the cuprate superconductors

*Charge order and d-wave superconductivity in
an effective theory for the spin $S=0$ sector.*

A convenient derivation of the effective theory of short-range pairs is provided by the doped quantum dimer model

$$\begin{aligned}
 H_{dqd} = & J \sum_{\square} (| \begin{array}{cc} \bullet & \bullet \\ | & | \\ \bullet & \bullet \end{array} \rangle \langle \begin{array}{cc} \bullet & \bullet \\ \text{---} & \text{---} \\ \bullet & \bullet \end{array} | + | \begin{array}{cc} \bullet & \bullet \\ \text{---} & \text{---} \\ \bullet & \bullet \end{array} \rangle \langle \begin{array}{cc} | & | \\ \bullet & \bullet \end{array} |) \\
 - t \sum_{\triangle} (| \begin{array}{cc} \circ & \bullet \\ | & | \\ \bullet & \bullet \end{array} \rangle \langle \begin{array}{cc} \bullet & \bullet \\ \text{---} & \text{---} \\ \circ & \bullet \end{array} | + | \begin{array}{cc} \bullet & \bullet \\ \text{---} & \text{---} \\ \bullet & \circ \end{array} \rangle \langle \begin{array}{cc} \circ & \bullet \\ | & | \\ \bullet & \bullet \end{array} |) - \dots
 \end{aligned}$$

Density of holes = δ

E. Fradkin and S. A. Kivelson, *Mod. Phys. Lett. B* **4**, 225 (1990).

Duality mapping of doped dimer model shows:

(a) Superfluid, insulator, and supersolid ground states of a theory which obeys the magnetic algebra

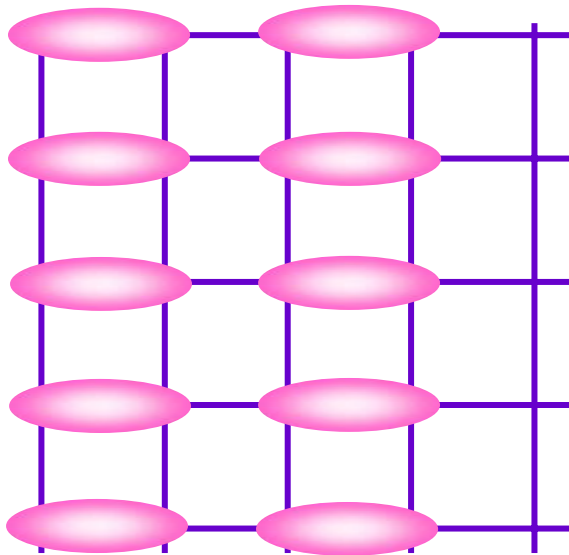
$$T_x T_y = e^{2\pi i f} T_y T_x$$

$$\text{with } f = \frac{1-\delta}{2}$$

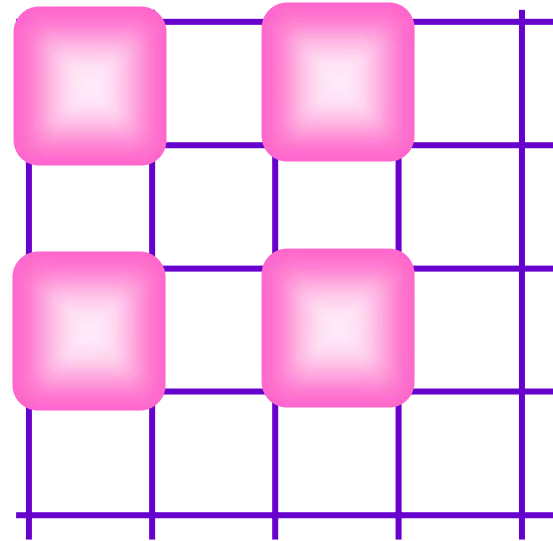
Duality mapping of doped dimer model shows:

(b) At $\delta = 0$, the ground state is a Mott insulator with valence-bond-solid (VBS) order. This associated with $f=1/2$ and the algebra

$$T_x T_y = -T_y T_x$$



or



Duality mapping of doped dimer model shows:

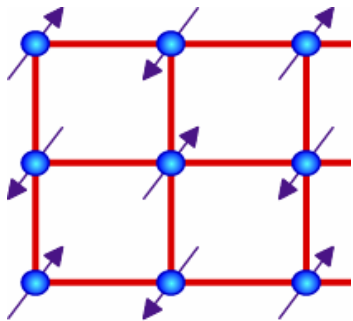
- (c) At larger δ , the ground state is a d -wave superfluid. The structure of the “extended LGW” theory of the competition between superfluid and solid order is identical to that of bosons on the square lattice with density f . These bosons can therefore be viewed as d -wave Cooper pairs of electrons. The phase diagrams of part (A) can therefore be applied here.

$$T_x T_y = e^{2\pi i f} T_y T_x$$

$$\text{with } f = \frac{1-\delta}{2}$$

Global phase diagram

g = parameter controlling strength of quantum fluctuations in a semiclassical theory of the destruction of Neel order

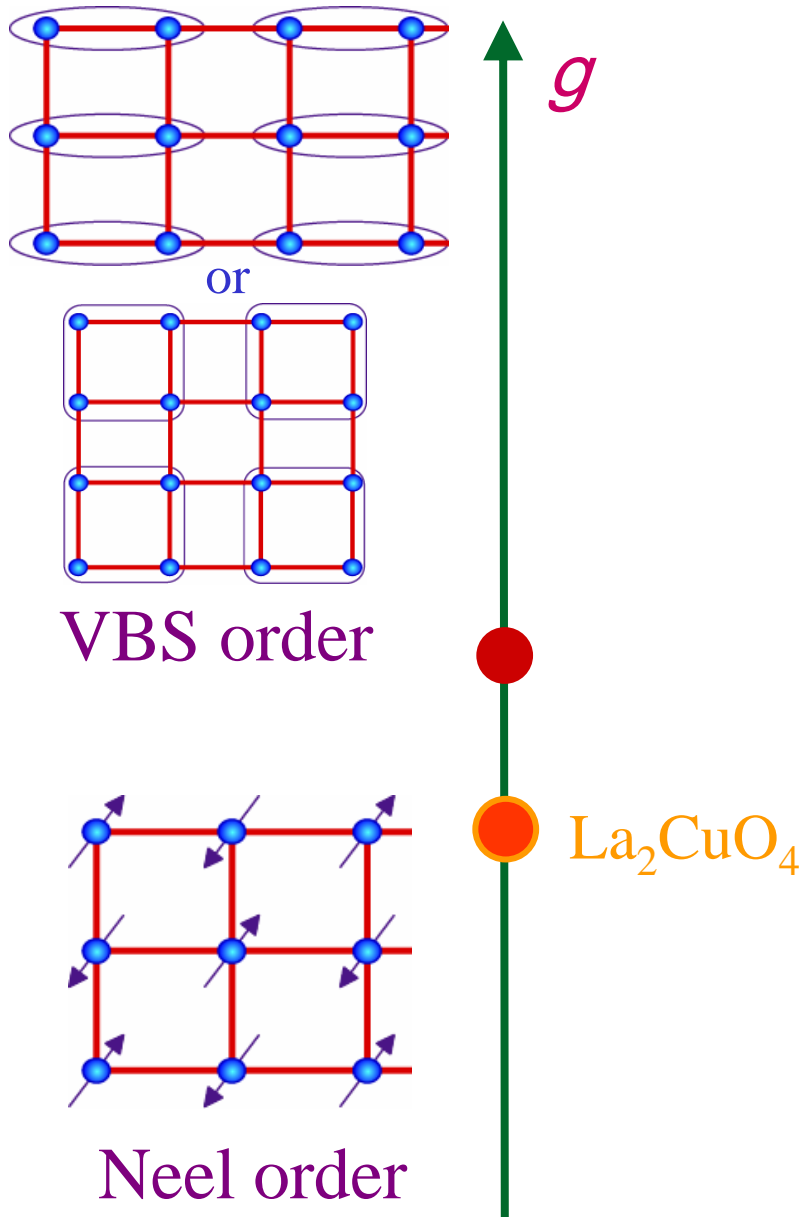


Neel order



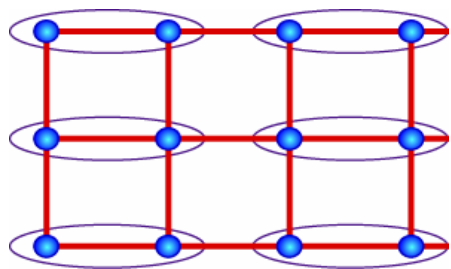
La_2CuO_4

Global phase diagram

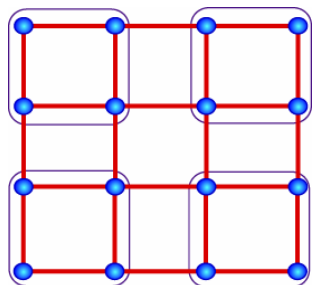


N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

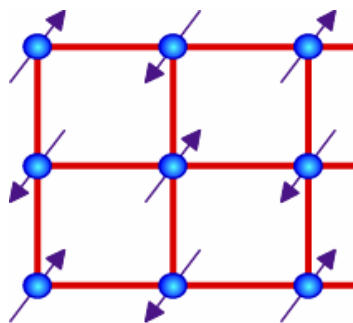
Global phase diagram



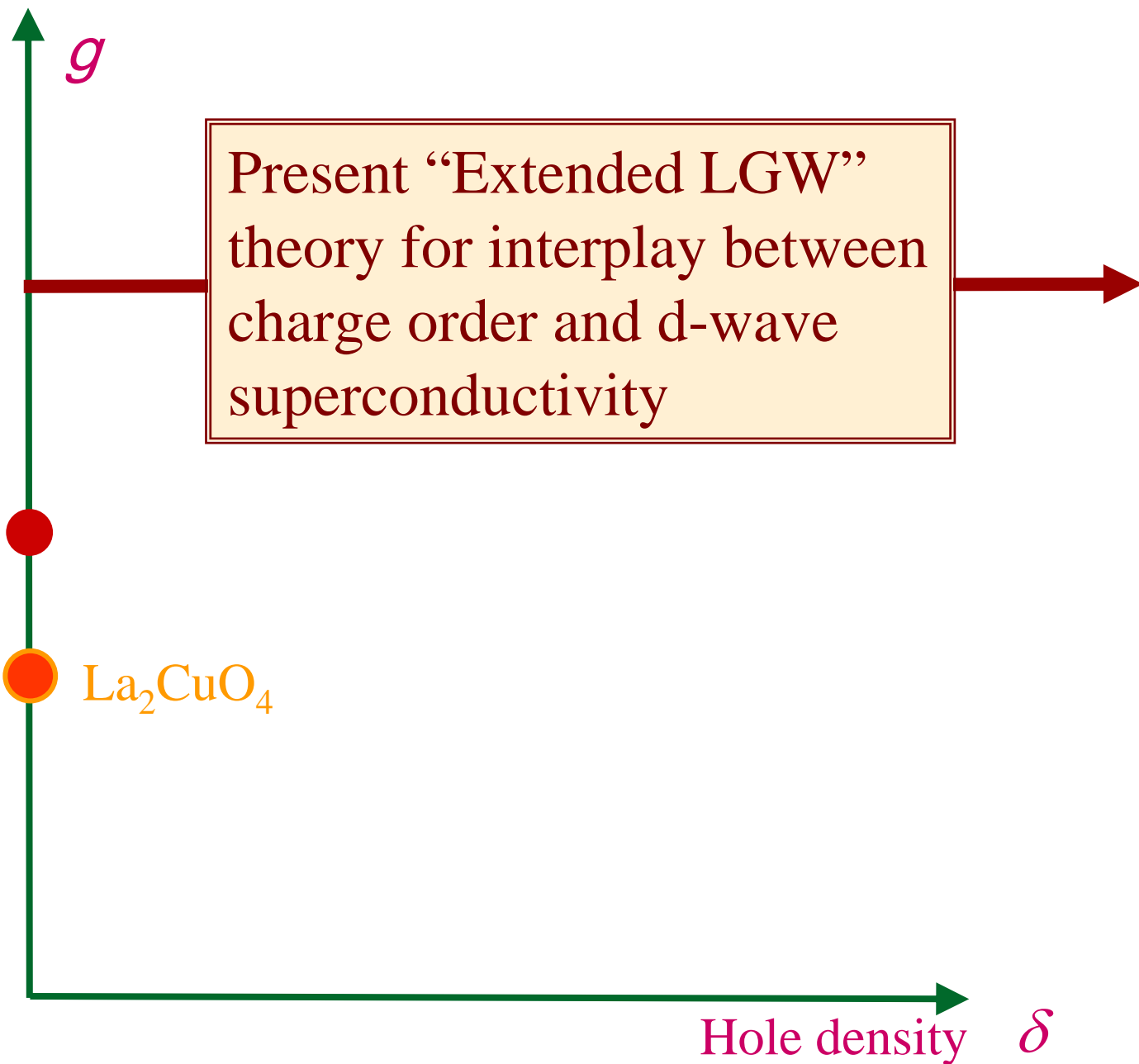
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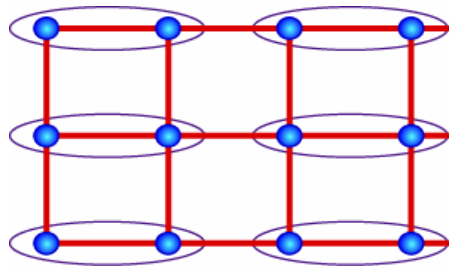
VBS order



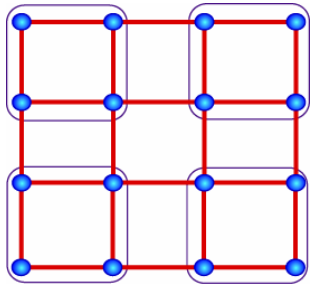
Neel order



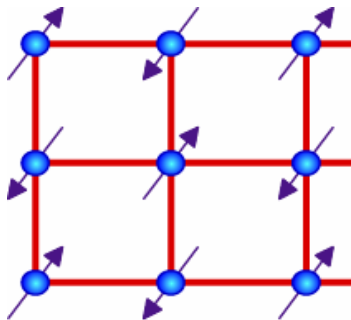
Global phase diagram



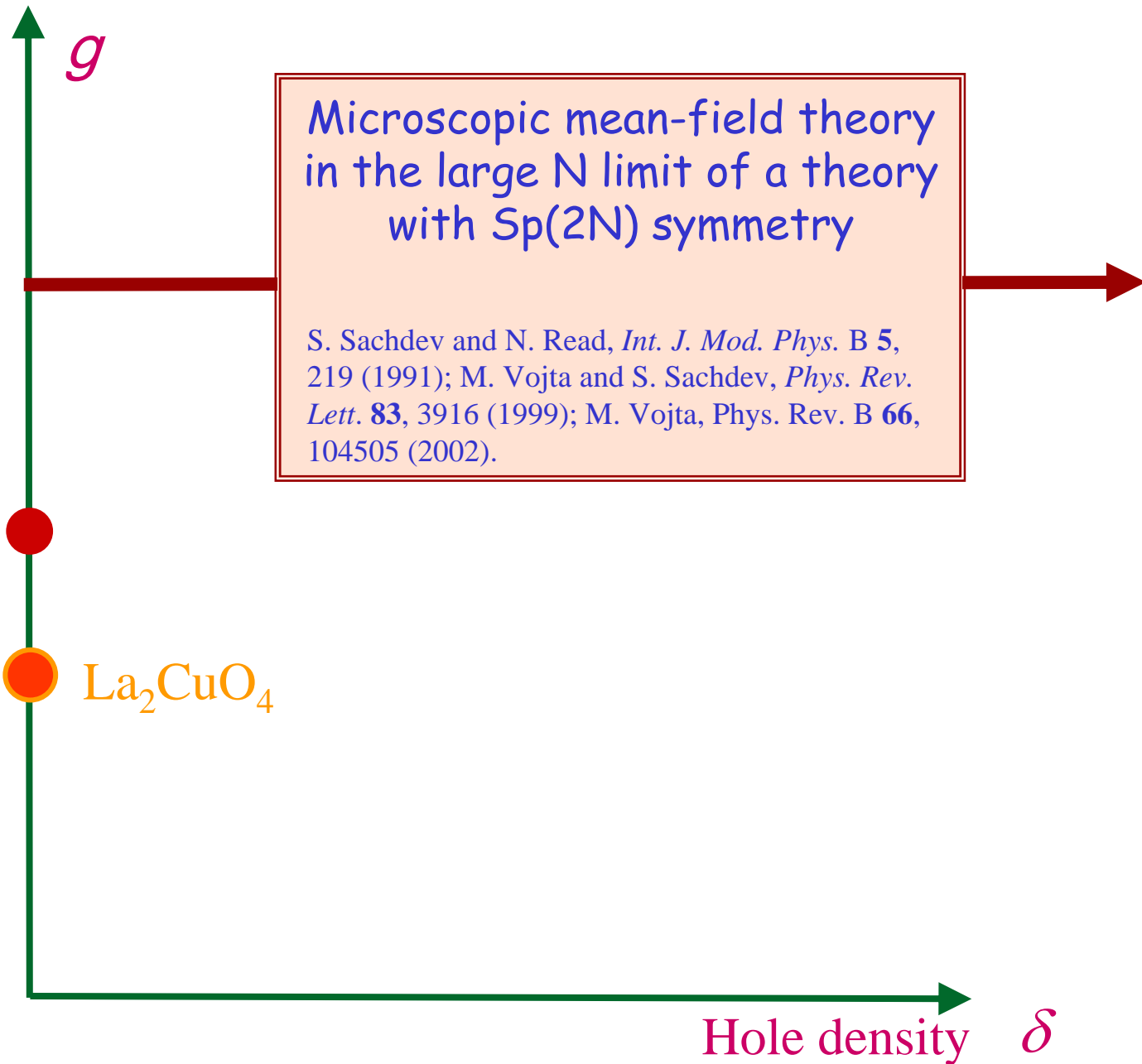
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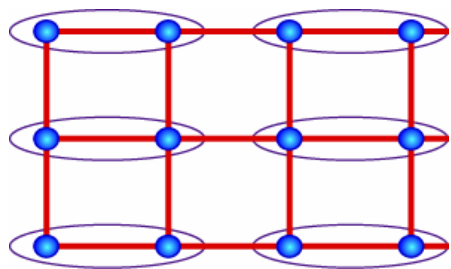
VBS order



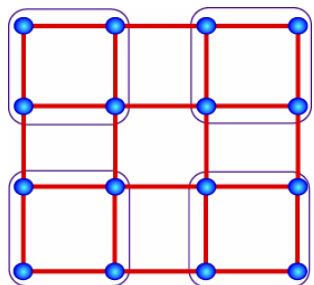
Neel order



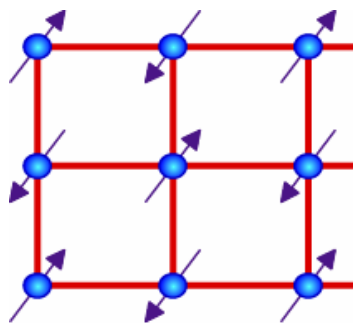
Global phase diagram



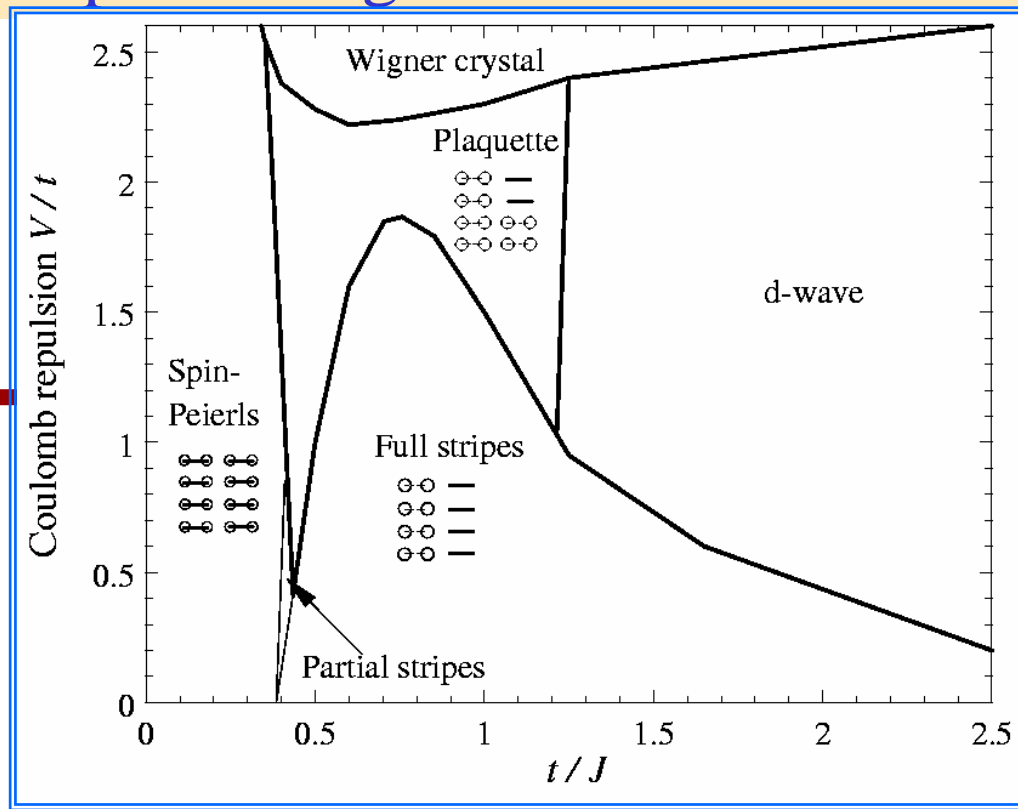
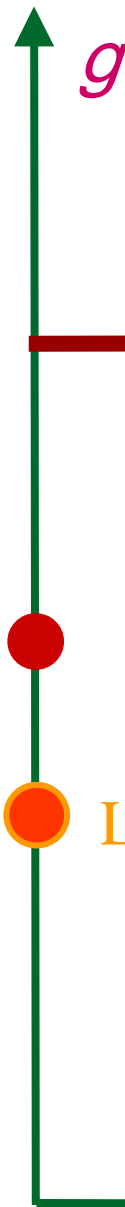
or



VBS order



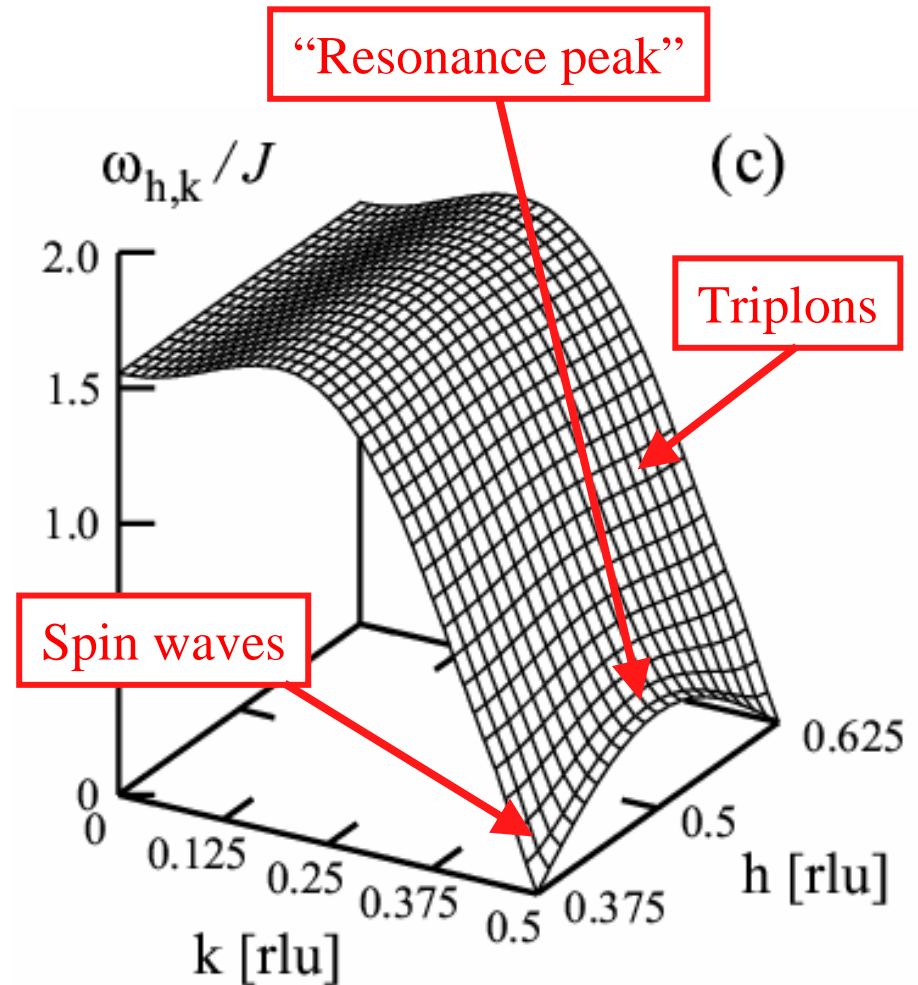
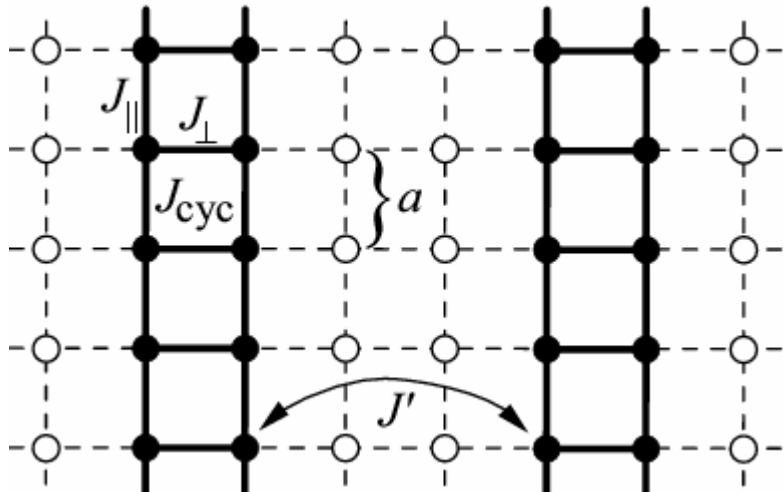
Neel order



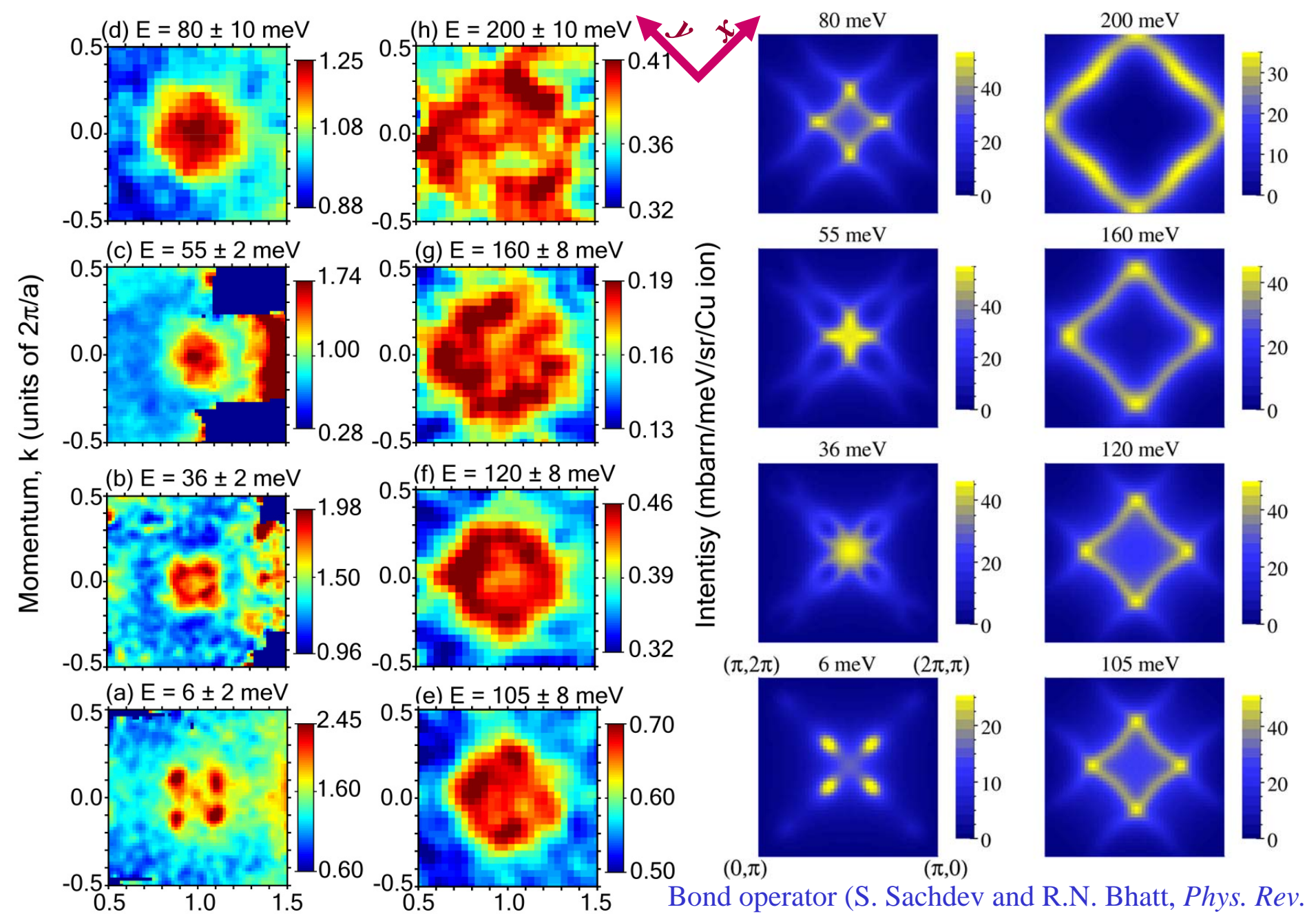
La_2CuO_4

Hole density δ

Spin excitations in a coupled ladder model

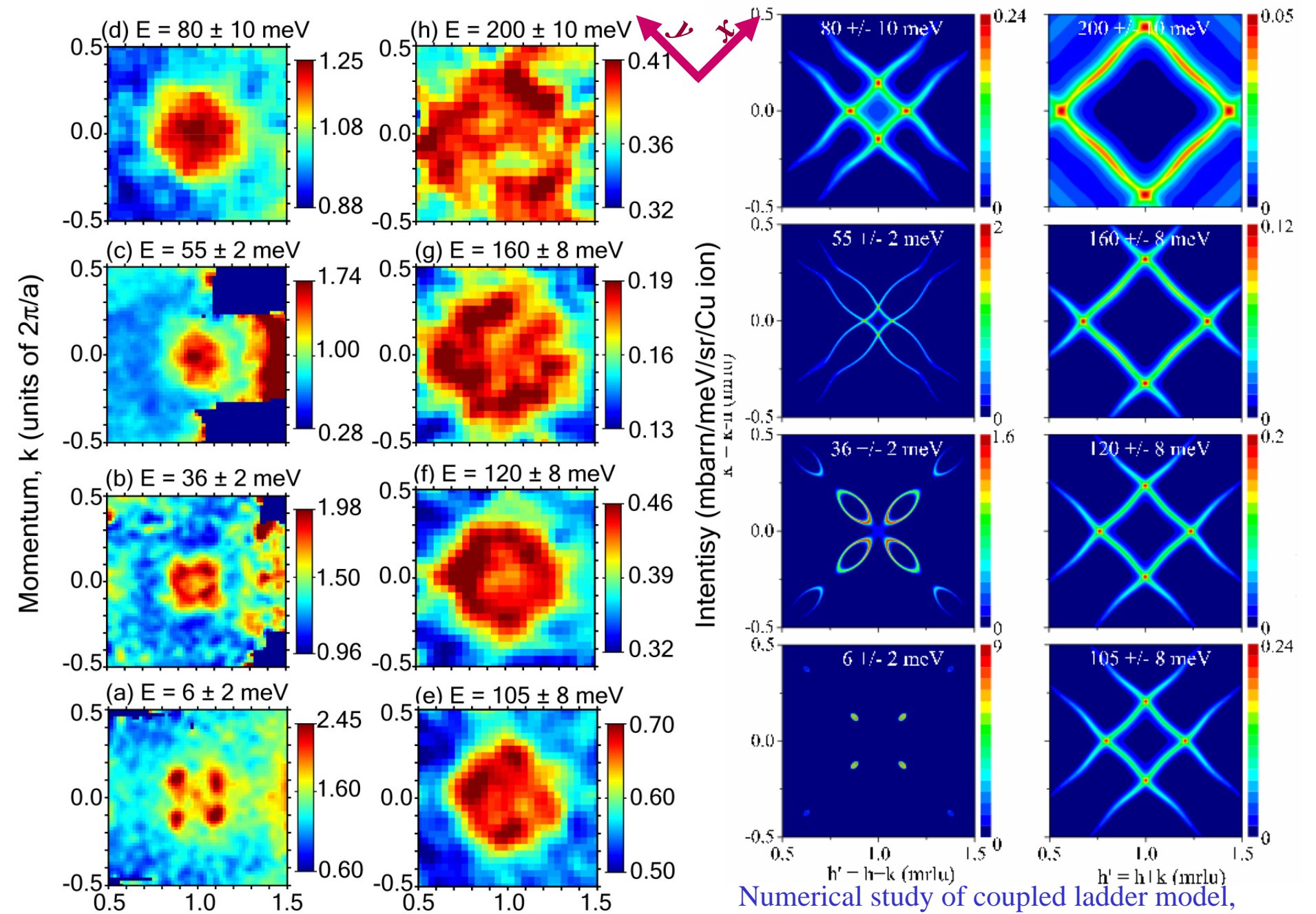


- M. Vojta and T. Ulbricht, cond-mat/0402377
- G.S. Uhrig, K.P. Schmidt, and M. Grüninger, cond-mat/0402659
- M. Vojta and S. Sachdev, unpublished.



Bond operator (S. Sachdev and R.N. Bhatt, *Phys. Rev. B* 41, 9323 (1990)) theory of coupled-ladder model,
 M. Vojta and T. Ulbricht, cond-mat/0402377

J. M. Tranquada *et al.*, *Nature* **429**, 534 (2004)



Numerical study of coupled ladder model,
 G.S. Uhrig, K.P. Schmidt, and M. Grüninger,
 cond-mat/0402659

C. Implications for STM

Pinning of charge order in a superconductor

The φ_ℓ fields are fluctuating, and we can use the action

$$S = \int d^2 r d\tau \left(L_0 [\varphi_\ell] + L_{\text{pin}} [\varphi_\ell] \right)$$

$$L_0 [\varphi_\ell] = \sum_\ell \left(\left| \partial_\mu \varphi_\ell \right|^2 + s \left| \varphi_\ell \right|^2 \right) + \dots$$

$$L_{\text{pin}} [\varphi_\ell] = V_{\text{pin}}(r) \sum_{\mathcal{Q}} \rho_{\mathcal{Q}} e^{i\mathcal{Q} \cdot r} \quad \text{with} \quad \rho_{\mathcal{Q}_{mn}} = e^{i\pi mnf} \sum_{\ell=1}^q \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell m f}$$

Pinning leads to modulations at

$$\text{waveve} \mathcal{Q}_{mn} = \frac{2\pi p}{q} (m, n) = 2\pi f (m, n)$$

Charge order in a magnetic field

The φ_ℓ fields are fluctuating, and we can use the action

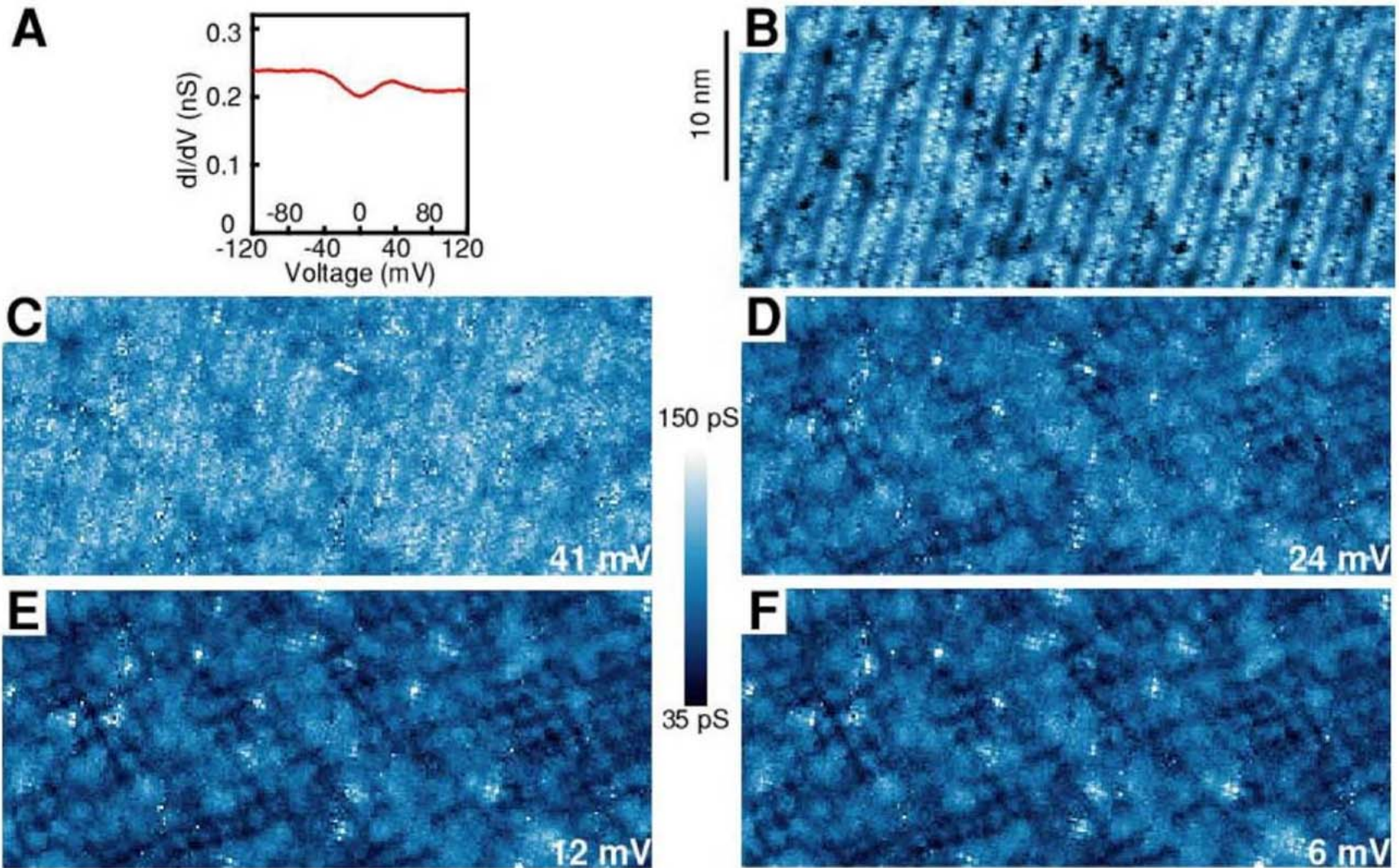
$$S = \int d^2 r d\tau \left(L_0 [\varphi_\ell] + L_{\text{pin}} [\varphi_\ell] \right)$$

$$L_0 [\varphi_\ell] = \sum_\ell \left(\left| \partial_\mu \varphi_\ell \right|^2 + s \left| \varphi_\ell \right|^2 \right) + \dots$$

$$L_{\text{pin}} [\varphi_\ell] = V_{\text{pin}}(r) \sum_{\mathcal{Q}} \rho_{\mathcal{Q}} e^{i\mathcal{Q} \cdot r} \quad \text{with} \quad \rho_{\mathcal{Q}_{mn}} = e^{i\pi mnf} \sum_{\ell=1}^q \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell m f}$$

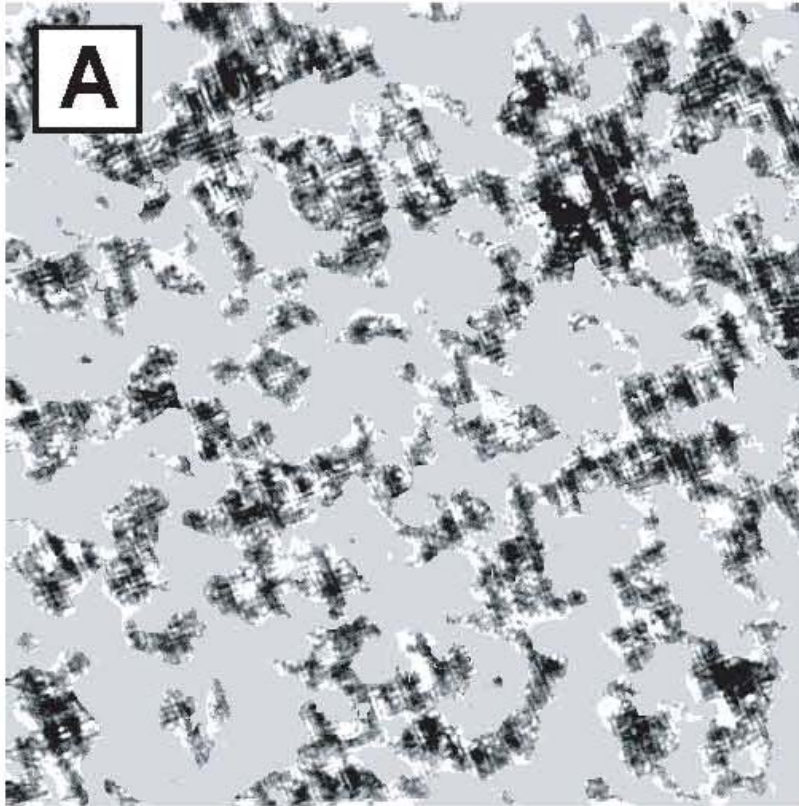
Recompute modulation in *same* theory but
in sector with φ_ℓ "charge" = number of vortices

The projective transformations of vortices imply that
each pinned vortex has a halo charge order modulations:
this can be computed with no additional parameters !



LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ at 100 K.

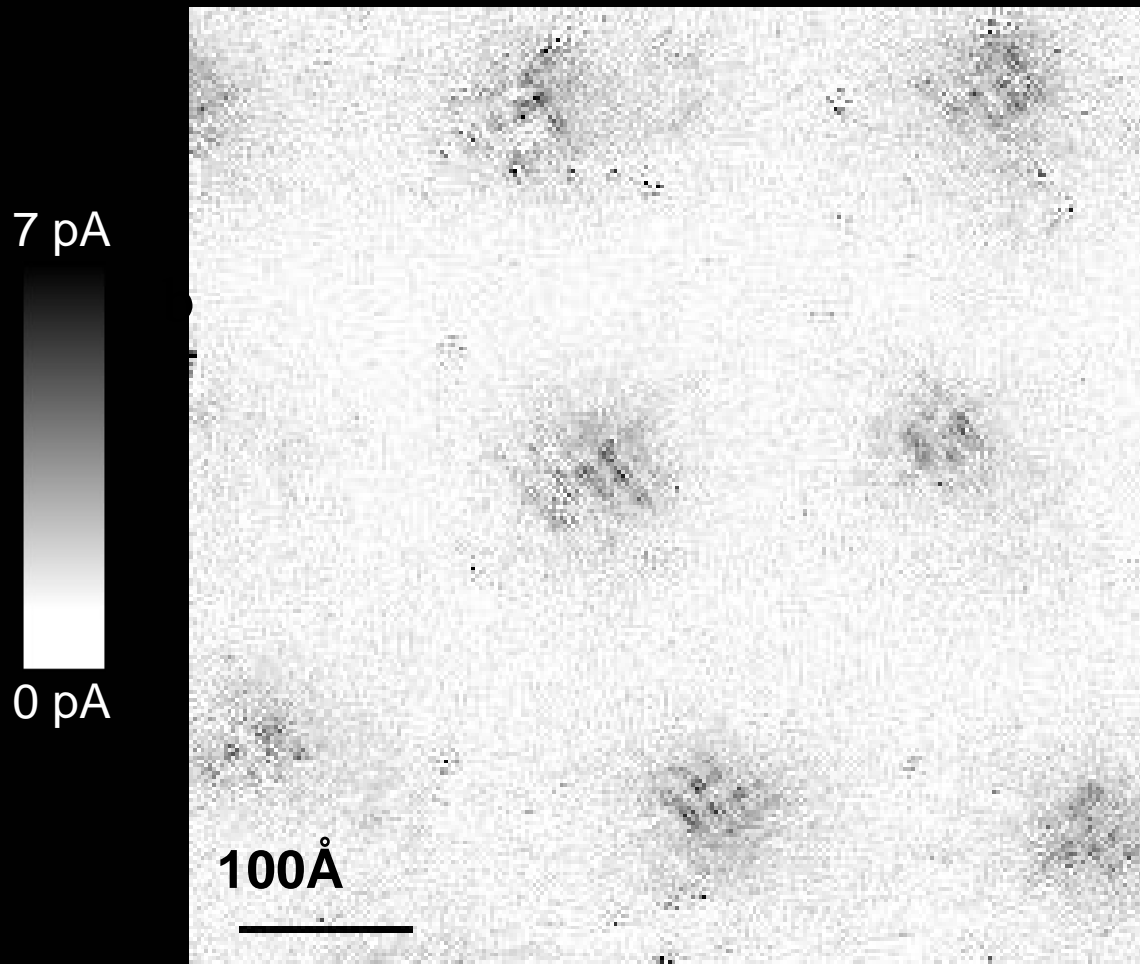
M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).



Energy integrated
LDOS (between 65
and 150 meV) of
strongly underdoped
 $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ at
low temperatures,
showing only regions
without
superconducting
“coherence peaks”

K. McElroy, D.-H. Lee, J. E. Hoffman, K. M Lang, E. W. Hudson, H. Eisaki,
S. Uchida, J. Lee, J.C. Davis, cond-mat/0404005.

Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV



**J. Hoffman E. W. Hudson, K. M. Lang,
V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida,
and J. C. Davis, *Science* 295, 466 (2002).**

Conclusions

- I. Description of the competition between superconductivity and charge order in term of defects (vortices). Theory naturally excludes “disordered” phase with no order.
- II. Vortices carry the quantum numbers of *both* superconductivity *and* the square lattice space group (in a projective representation).
- III. Pinned vortices are intricately linked to pinned charge order.
- IV. Future theory: $S=1/2$ and $S=1$ excitations.