SYK models of extremal black holes and strange metals

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Talk online: sachdev.physics.harvard.edu

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What are quasiparticles ?

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

What are quasiparticles ?

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where E_F is the Fermi energy.

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  where E_F is the Fermi energy.

- This time is much longer than the 'Planckian time' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

$$\tau_{\rm eq} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

1. Random matrix quasiparticle model

q=2, complex SYK

2. Matter without quasiparticles

q=4, complex SYK

3. Connections to black holes with AdS_2 horizons

4. Connections to strange metals

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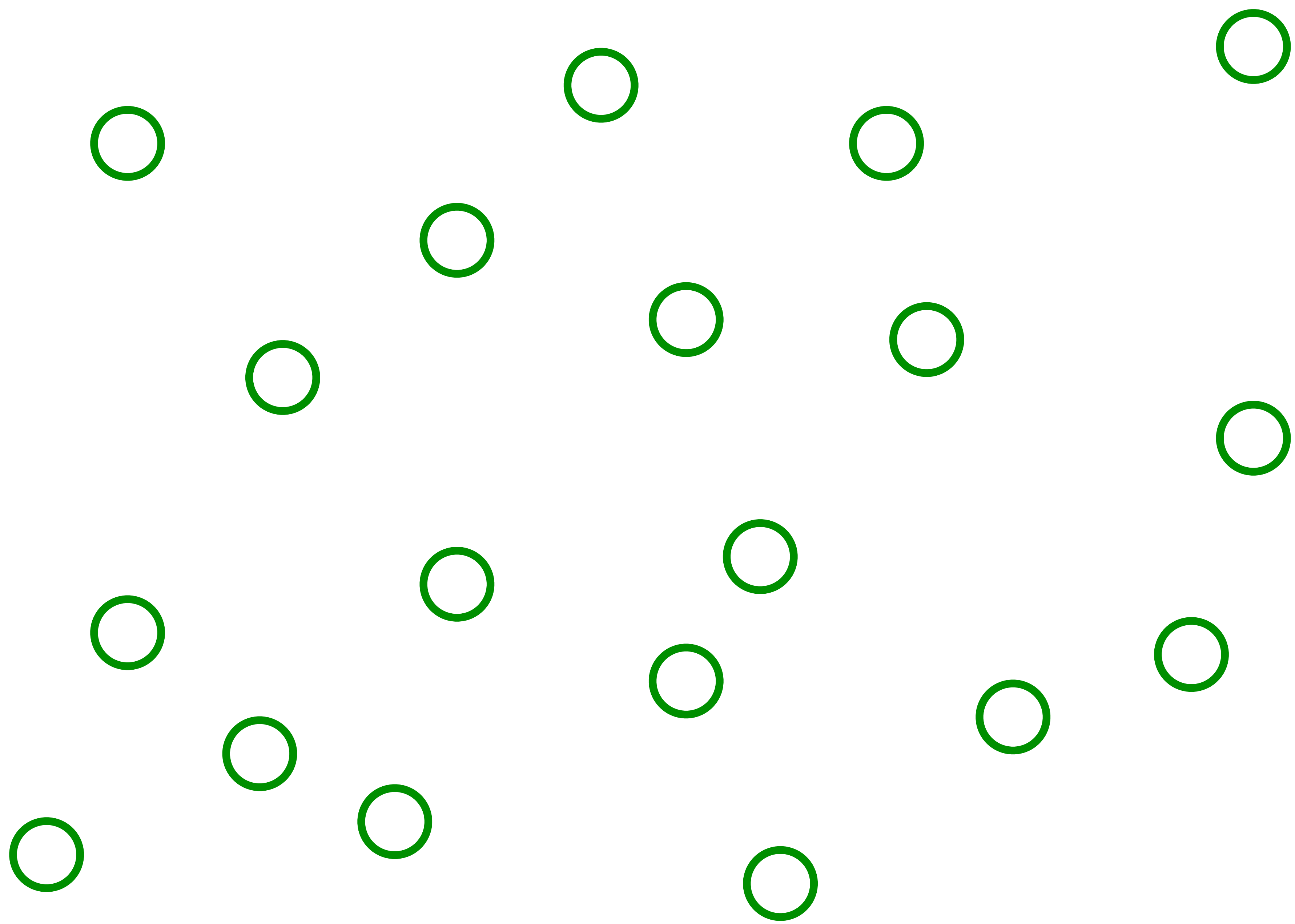
A simple model of a metal with quasiparticles



Pick a set of random positions

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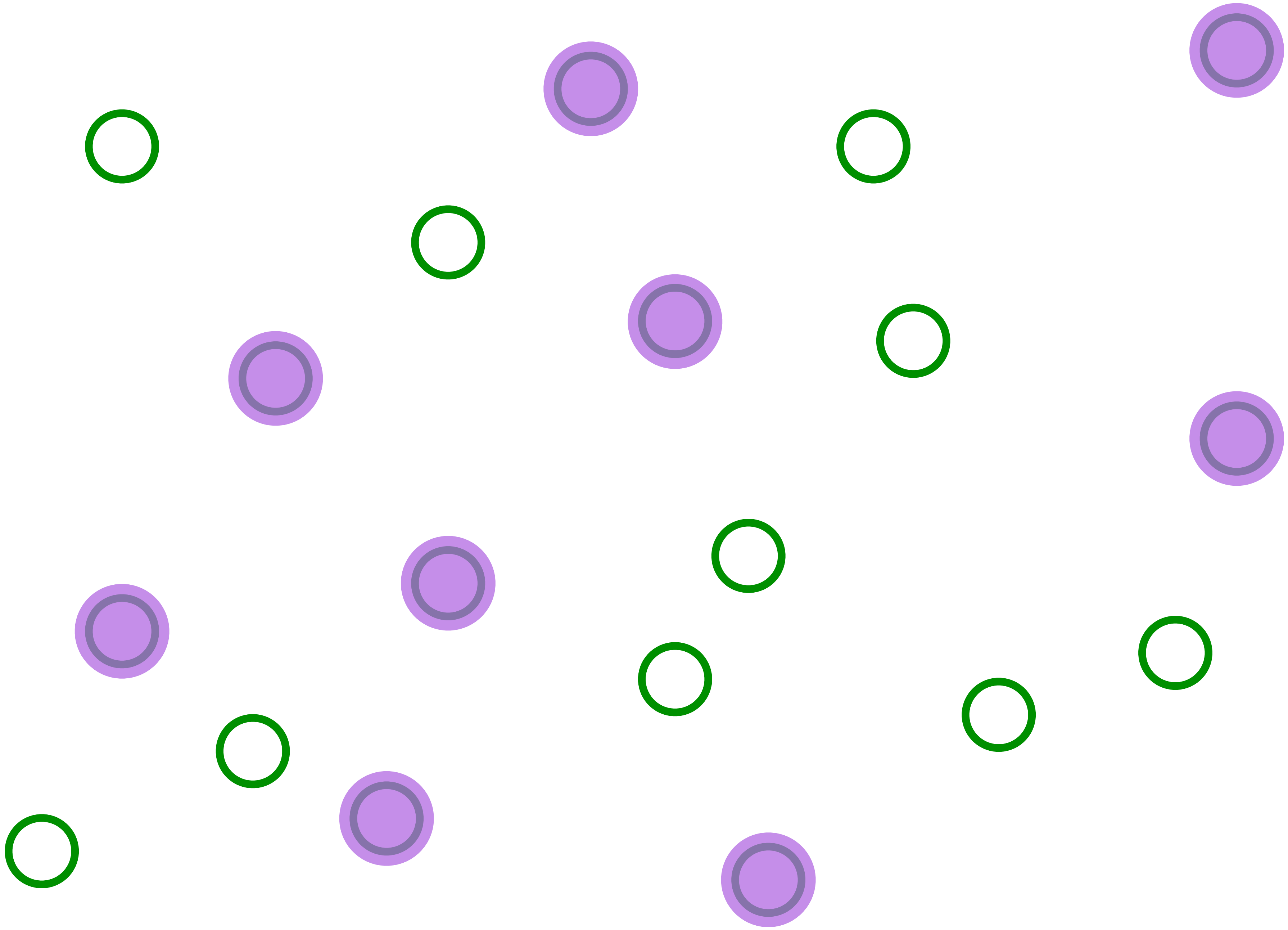

Place electrons randomly on some sites

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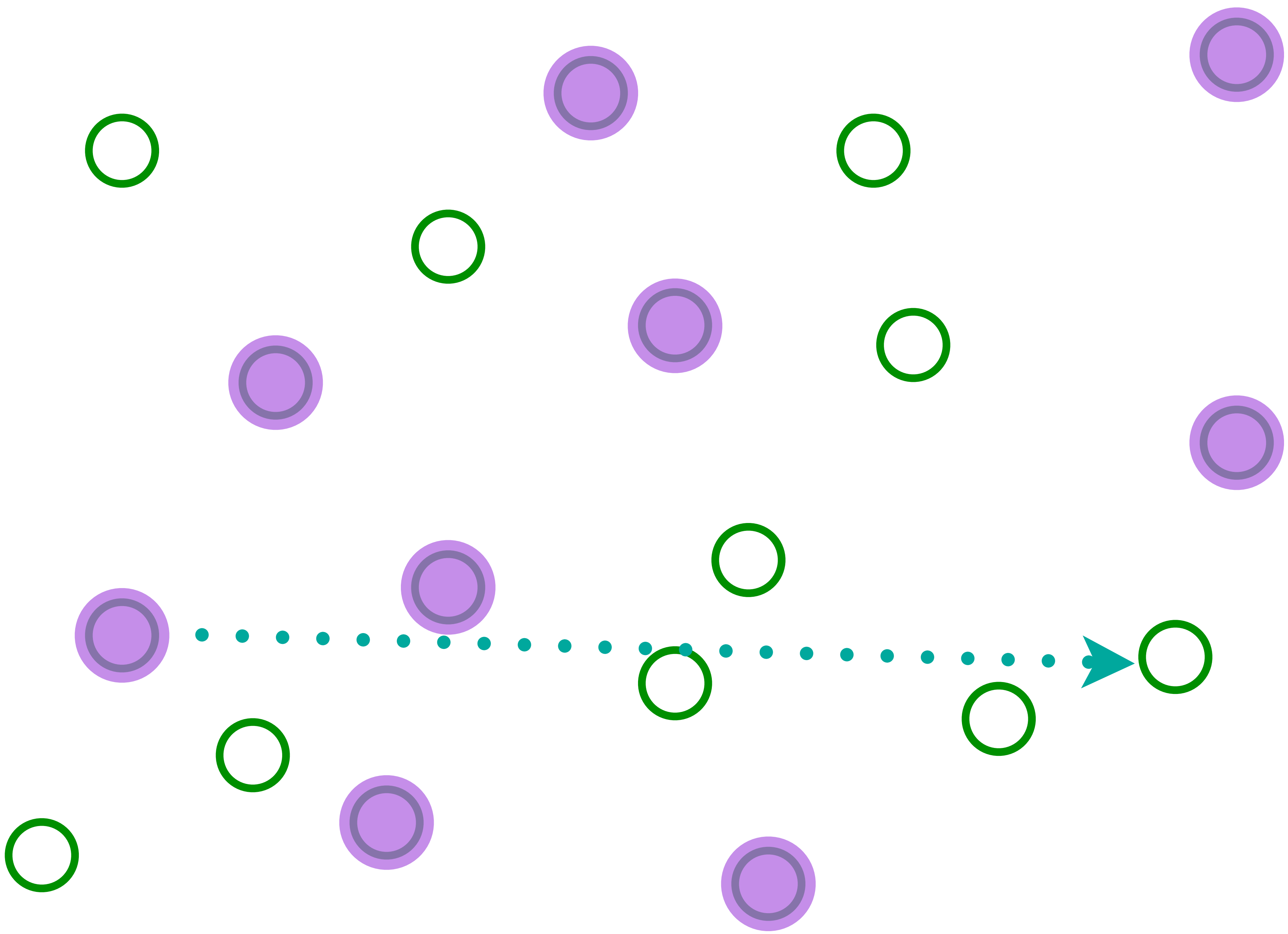

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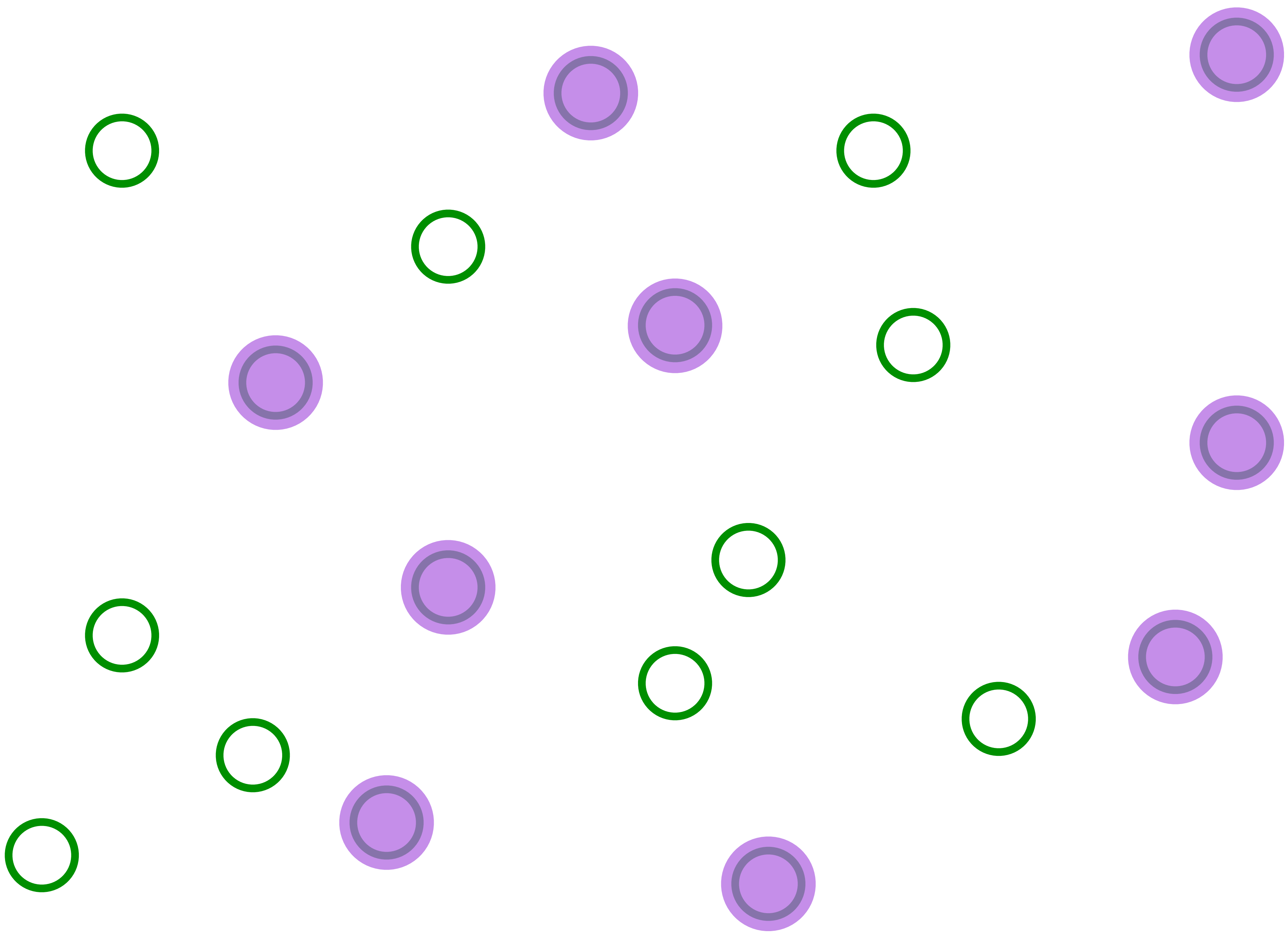

Electrons move one-by-one randomly

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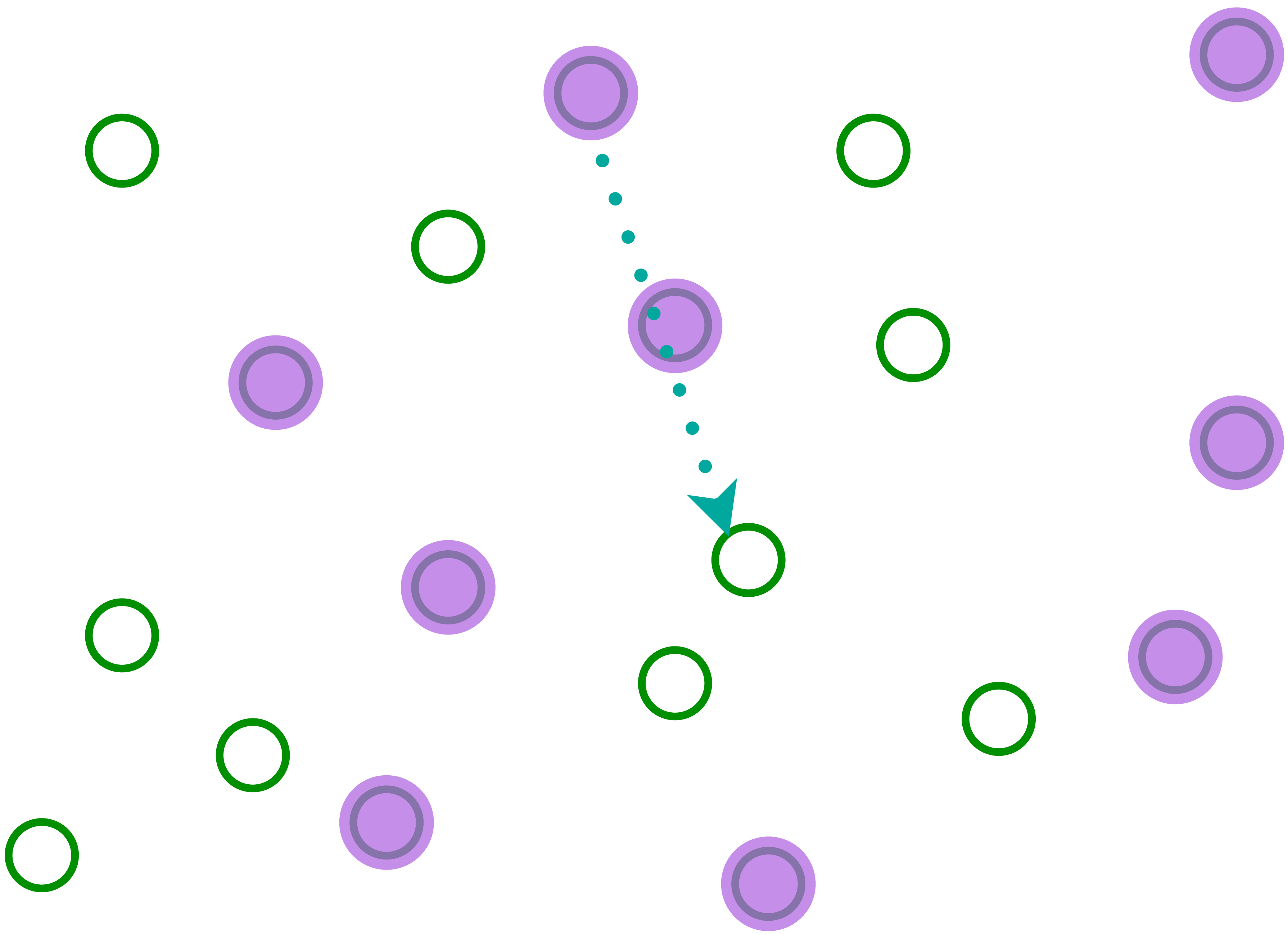

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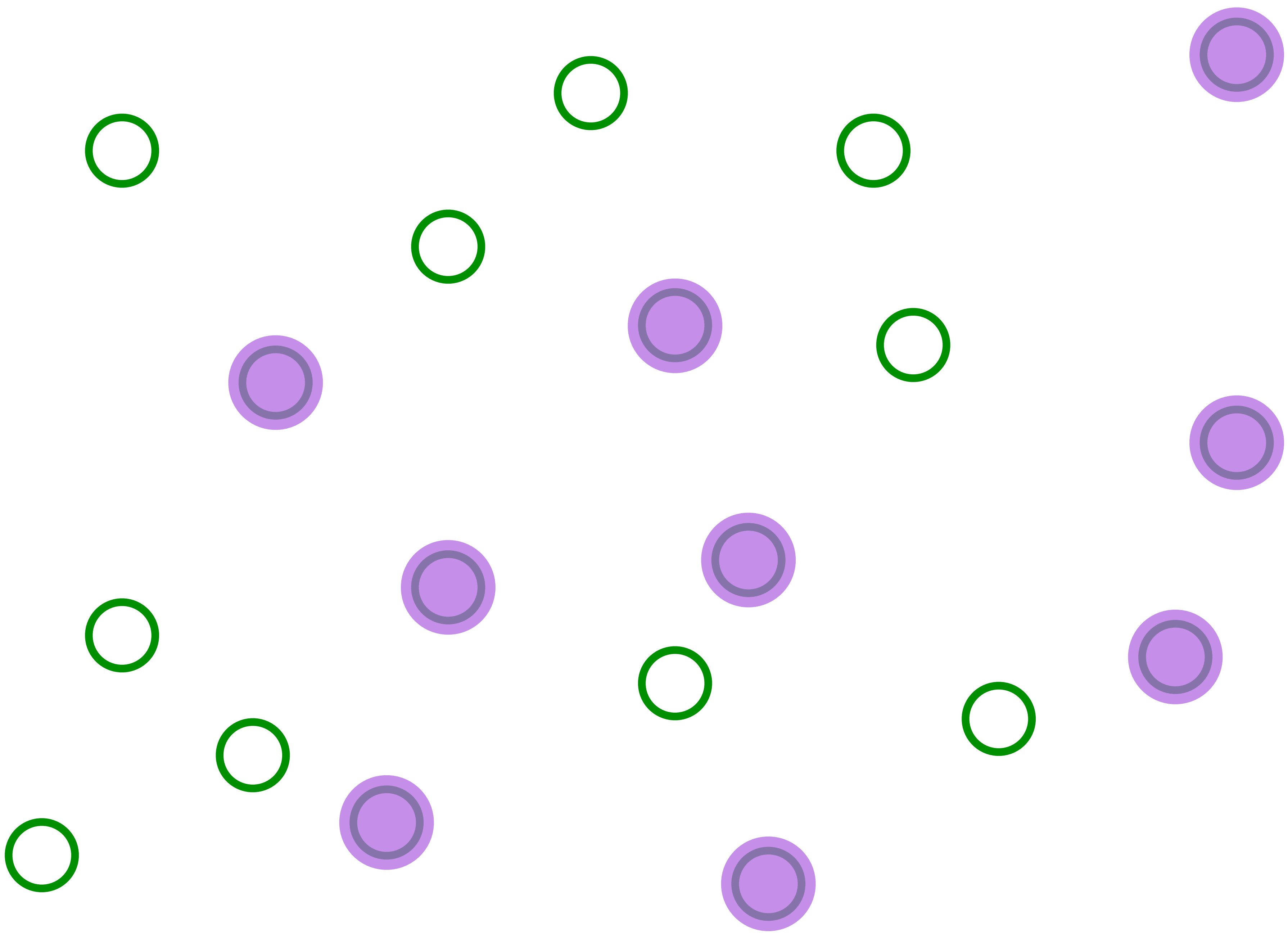

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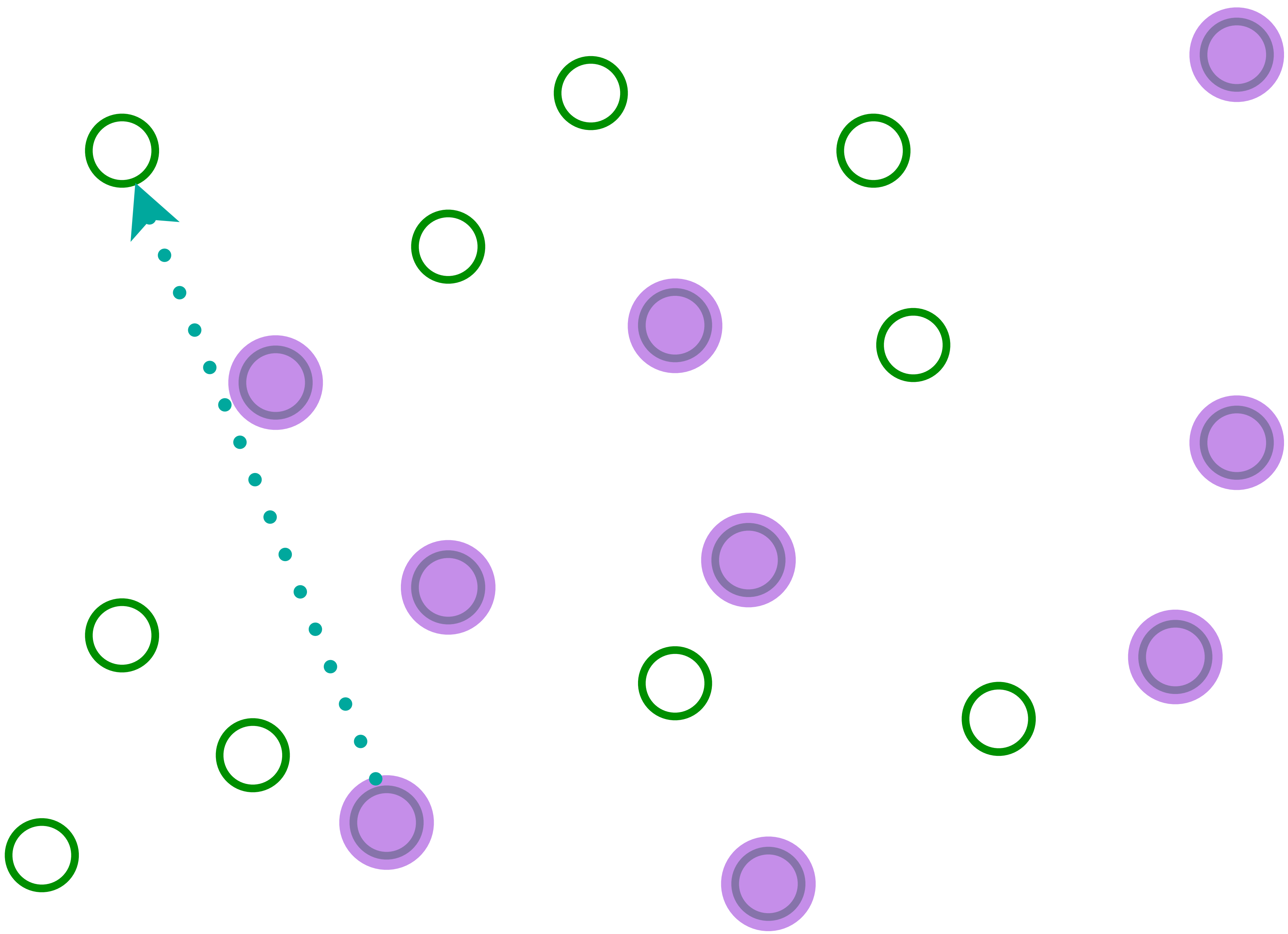

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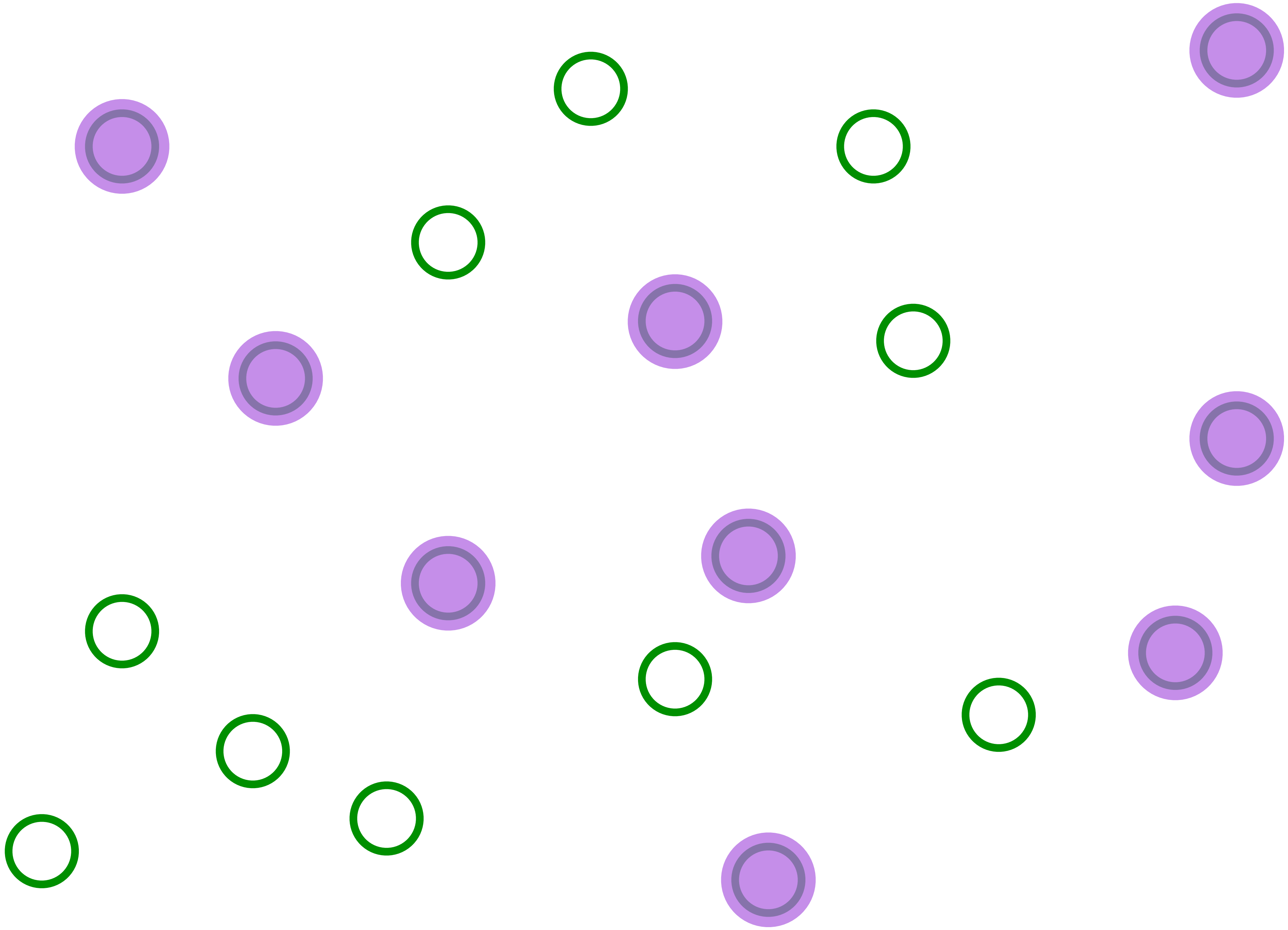

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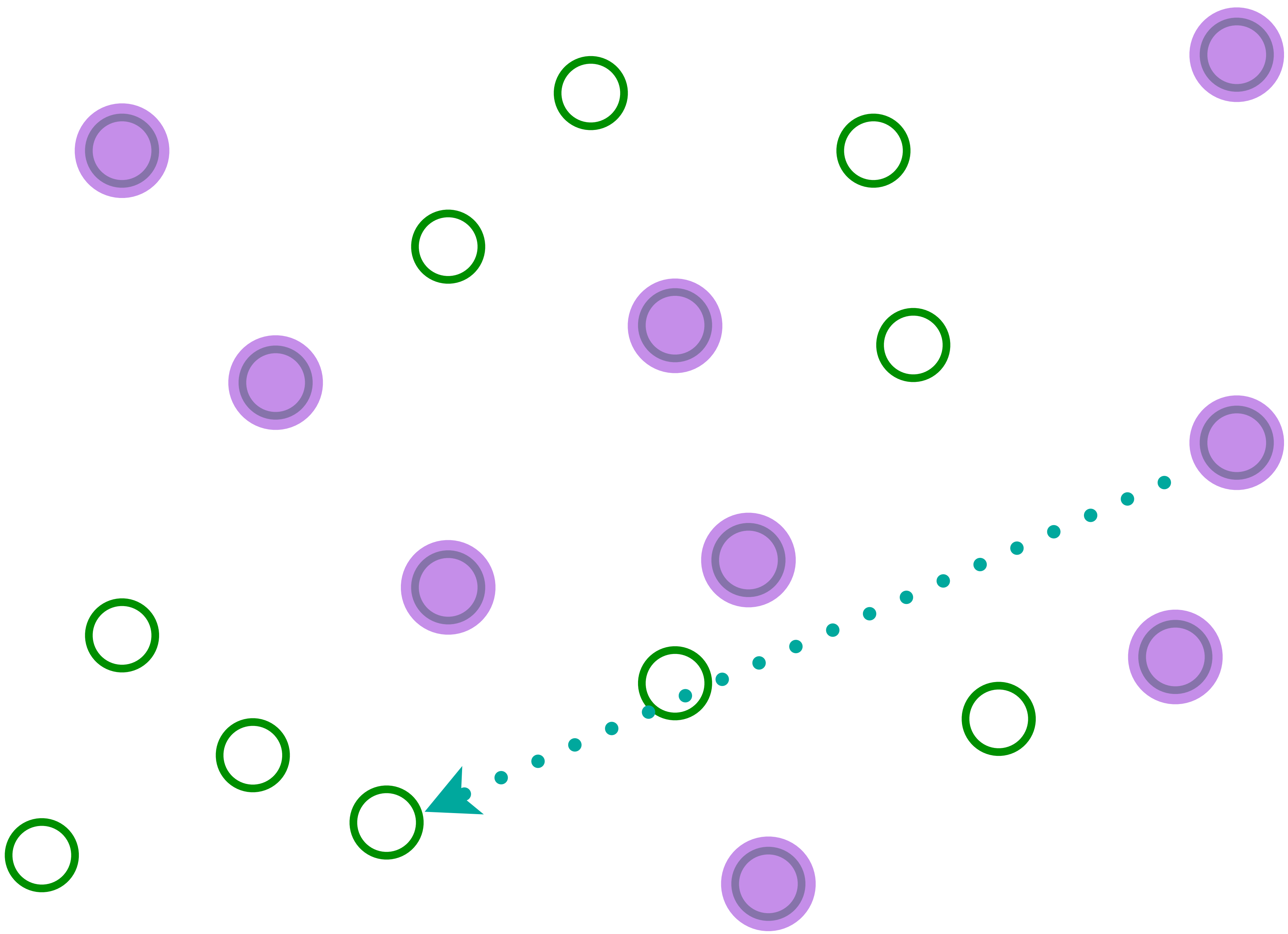

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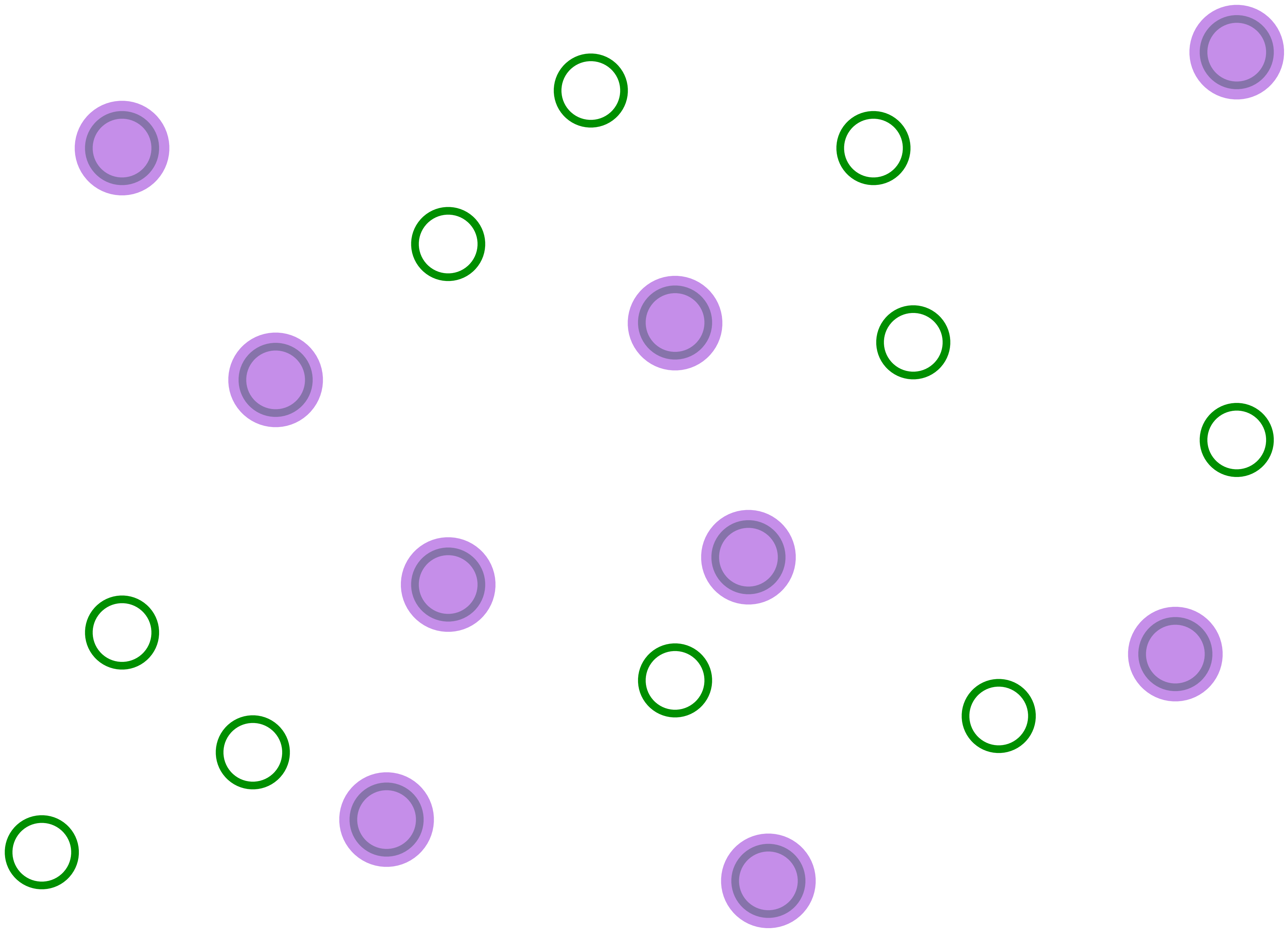


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a *N* x *N* random matrix

A simple model of a metal with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

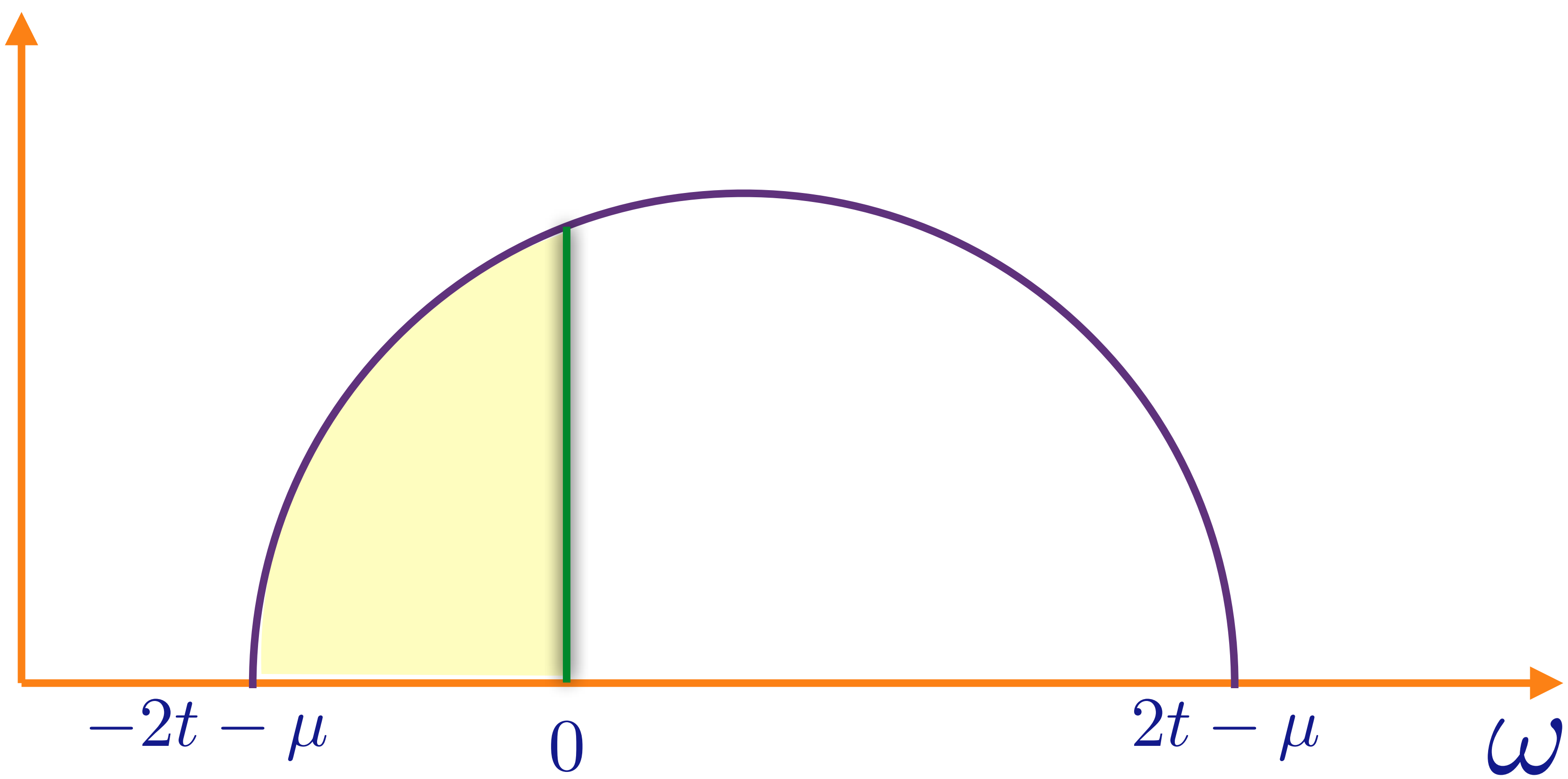
$$G(\tau) \equiv -T_\tau \left\langle c_i(\tau) c_i^\dagger(0) \right\rangle$$

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

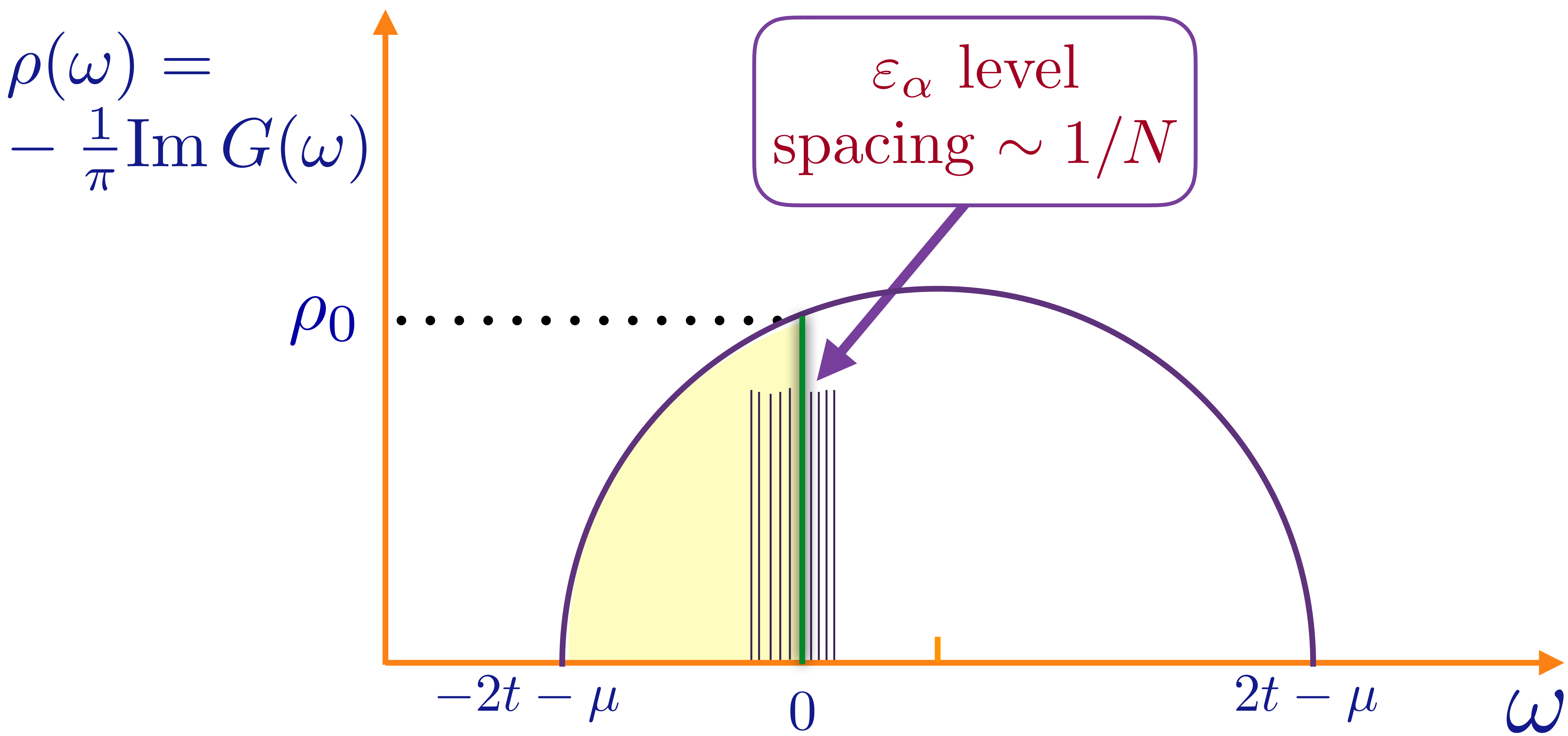
$G(\omega)$ can be determined by solving a quadratic equation.

$\rho(\omega) = -\frac{1}{\pi} \text{Im}\, G(\omega)$



A simple model of a metal with quasiparticles

Let ε_α be the N eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The single-particle density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$, and $\rho_0 \equiv \rho(\omega = 0)$.



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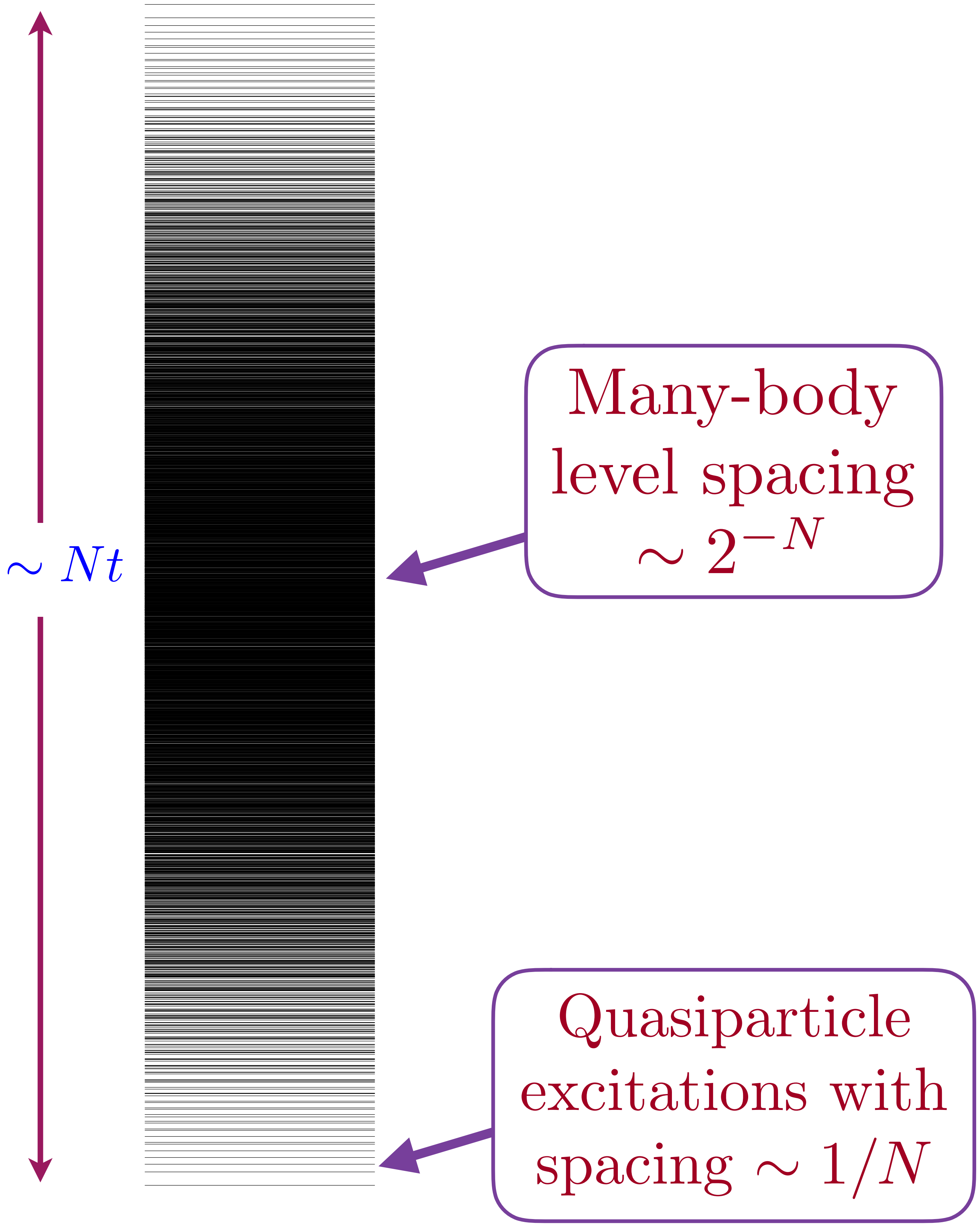

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

A simple model of a metal with quasiparticles

The grand potential $\Omega(T)$ at low T is (from the Sommerfeld expansion)

$$\Omega(T) - E_0 = N\left(-\frac{\pi^2}{6}\rho_0 T^2 + \mathcal{O}(T^4)\right) + \ldots$$

where $\rho_0 \equiv \rho(0)$ is the *single* particle density of states at the Fermi level.
From $\Omega(T)$ we can obtain the *many*-particle density of states $D(E)$

$$D(E) \sim \exp\left(\pi\sqrt{\frac{2N\rho_0(E-E_0)}{3}}\right),$$

$$E > E_0\ ,\ 1/N \ll \rho_0(E-E_0) \ll N$$

and $D(E) = 0$ for $E < E_0$. This is related to the asymptotic growth of the partitions of an integer,

$$p(n) \sim \exp(\pi\sqrt{2n/3})\,.$$

Near the lower bound, there are large sample-to-sample fluctuations due to variations in the lowest quasiparticle energies.

A simple model of a metal with quasiparticles

Now add weak interactions

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$. We compute the lifetime of a quasiparticle, τ_α, in an exact eigenstate $\psi_\alpha(i)$ of the free particle Hamitonian with energy ε_α. By Fermi's Golden rule, for ε_α at the Fermi energy

$$\begin{aligned} \frac{1}{\tau_\alpha} &= \pi U^2 \rho_0^2 \int d\varepsilon_\beta d\varepsilon_\gamma d\varepsilon_\delta f(\varepsilon_\beta)(1 - f(\varepsilon_\gamma))(1 - f(\varepsilon_\delta))\delta(\varepsilon_\alpha + \varepsilon_\beta - \varepsilon_\gamma - \varepsilon_\delta) \\ &= \frac{\pi^3 U^2 \rho_0^2}{4} T^2 \end{aligned}$$

where ρ_0 is the density of states at the Fermi energy, and $f(\epsilon) = 1/(e^{\epsilon/T} + 1)$ is the Fermi function.

Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as $\sim T^{-2}$ at the Fermi level.

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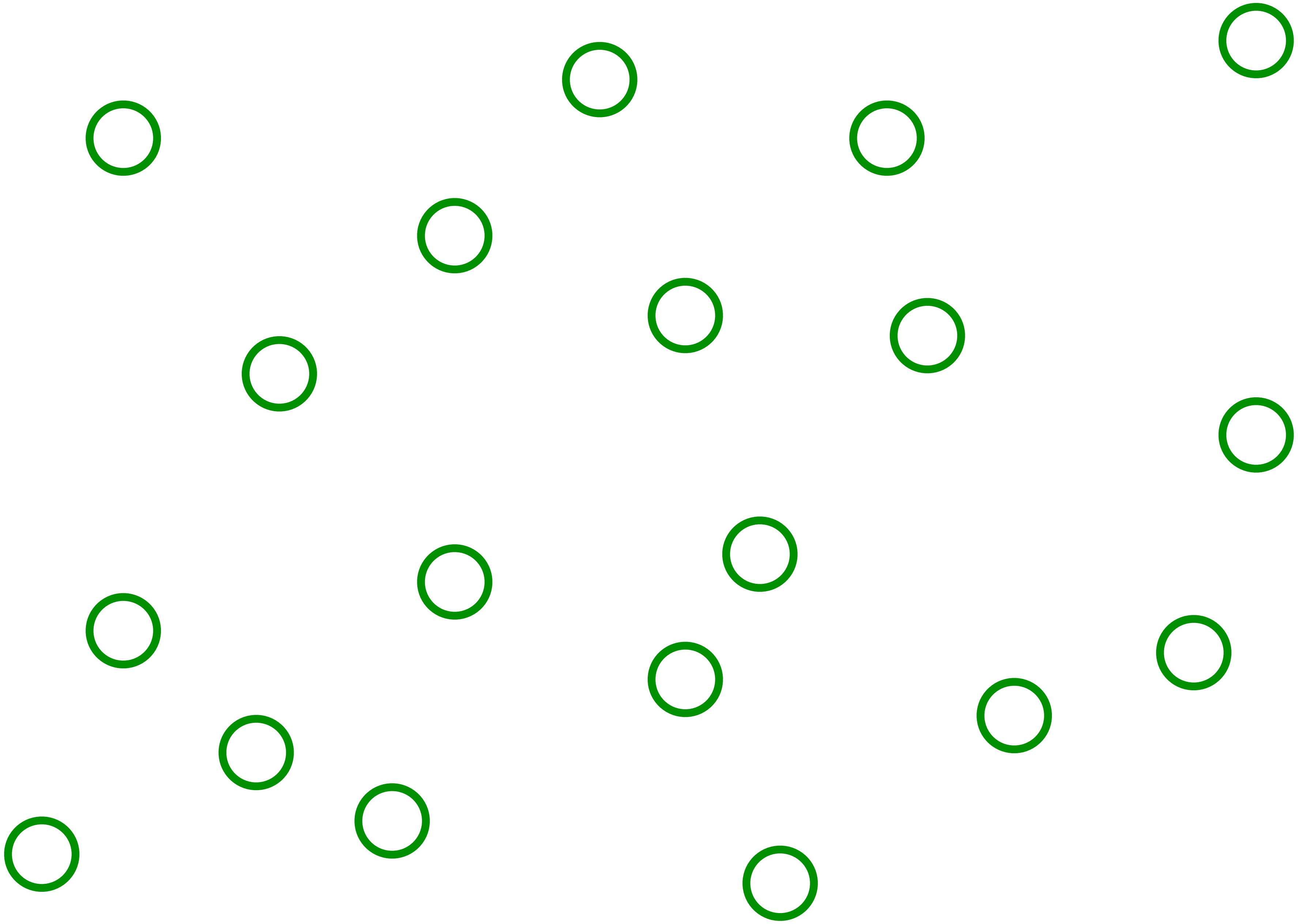
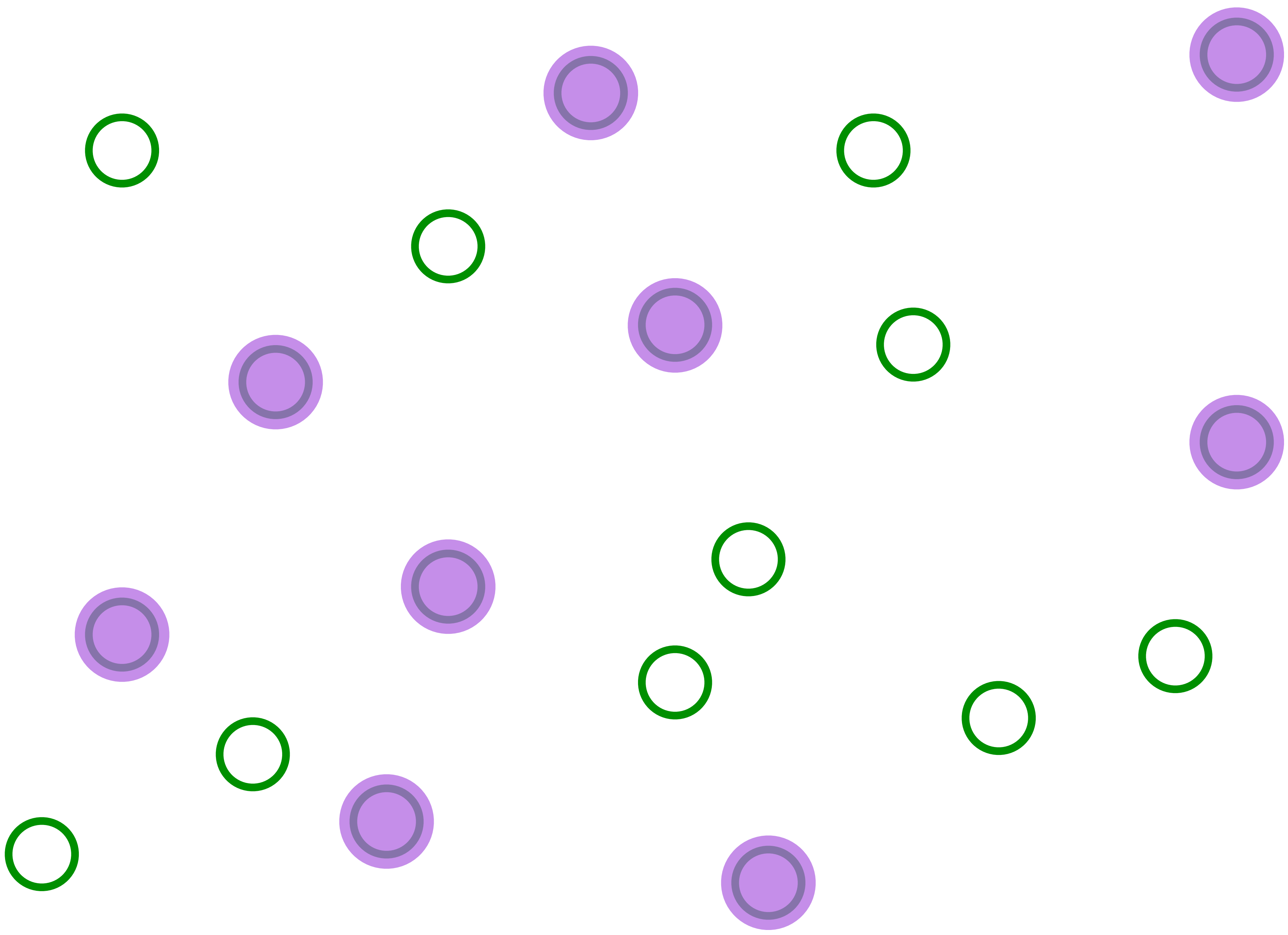
The Sachdev-Ye-Kitaev (SYK) model



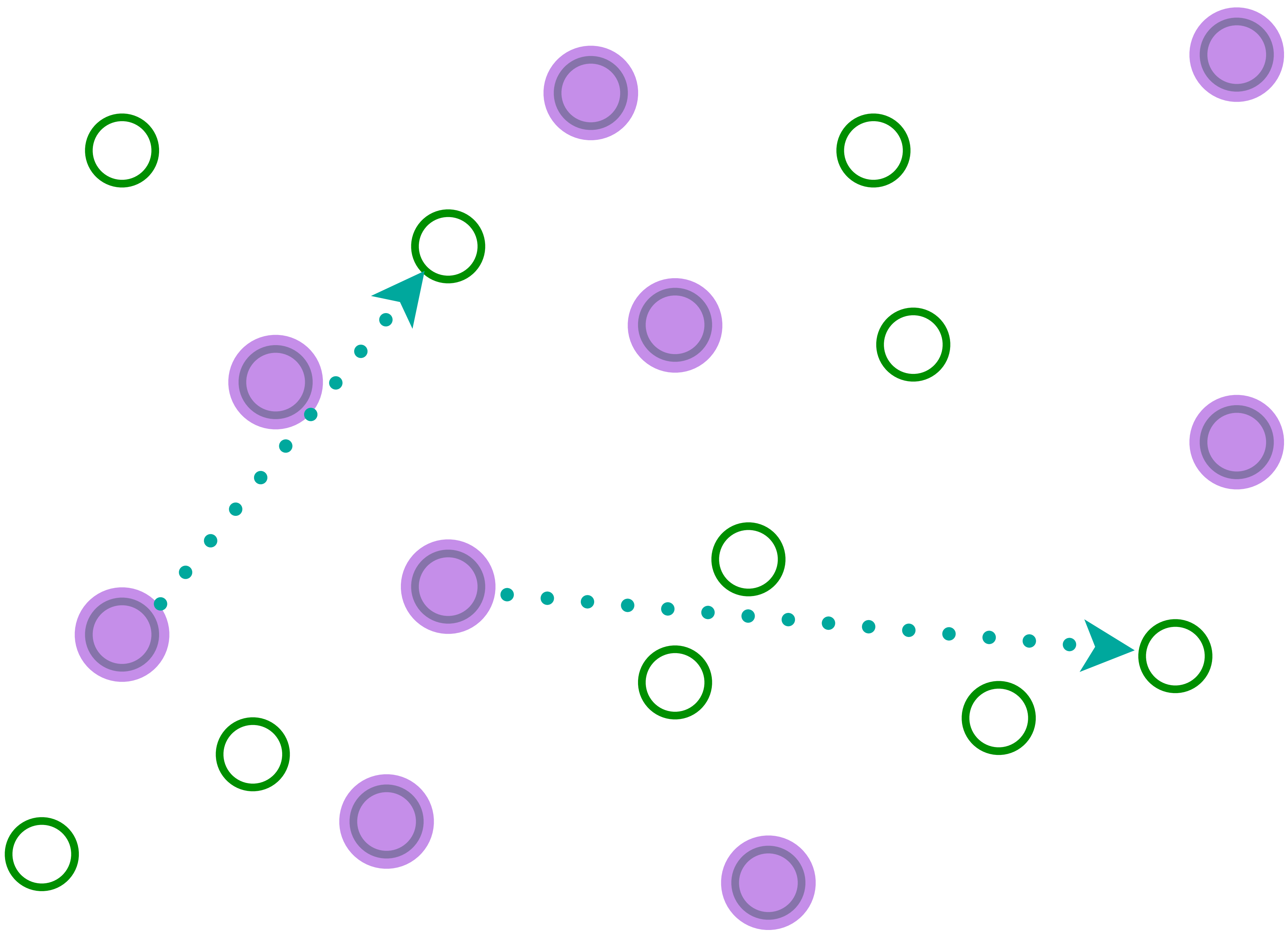
Pick a set of random positions

The SYK model



Place electrons randomly on some sites

The SYK model



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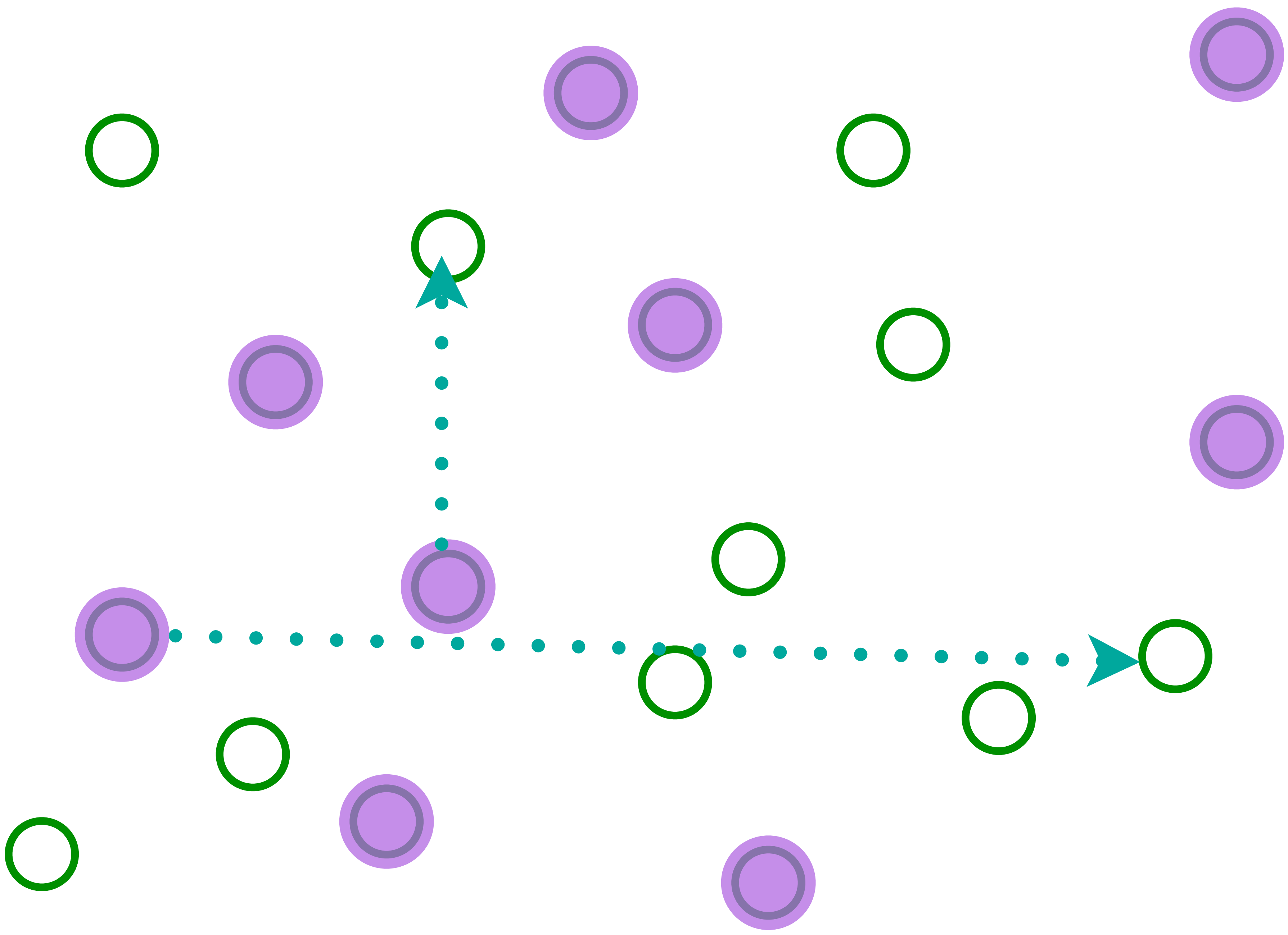

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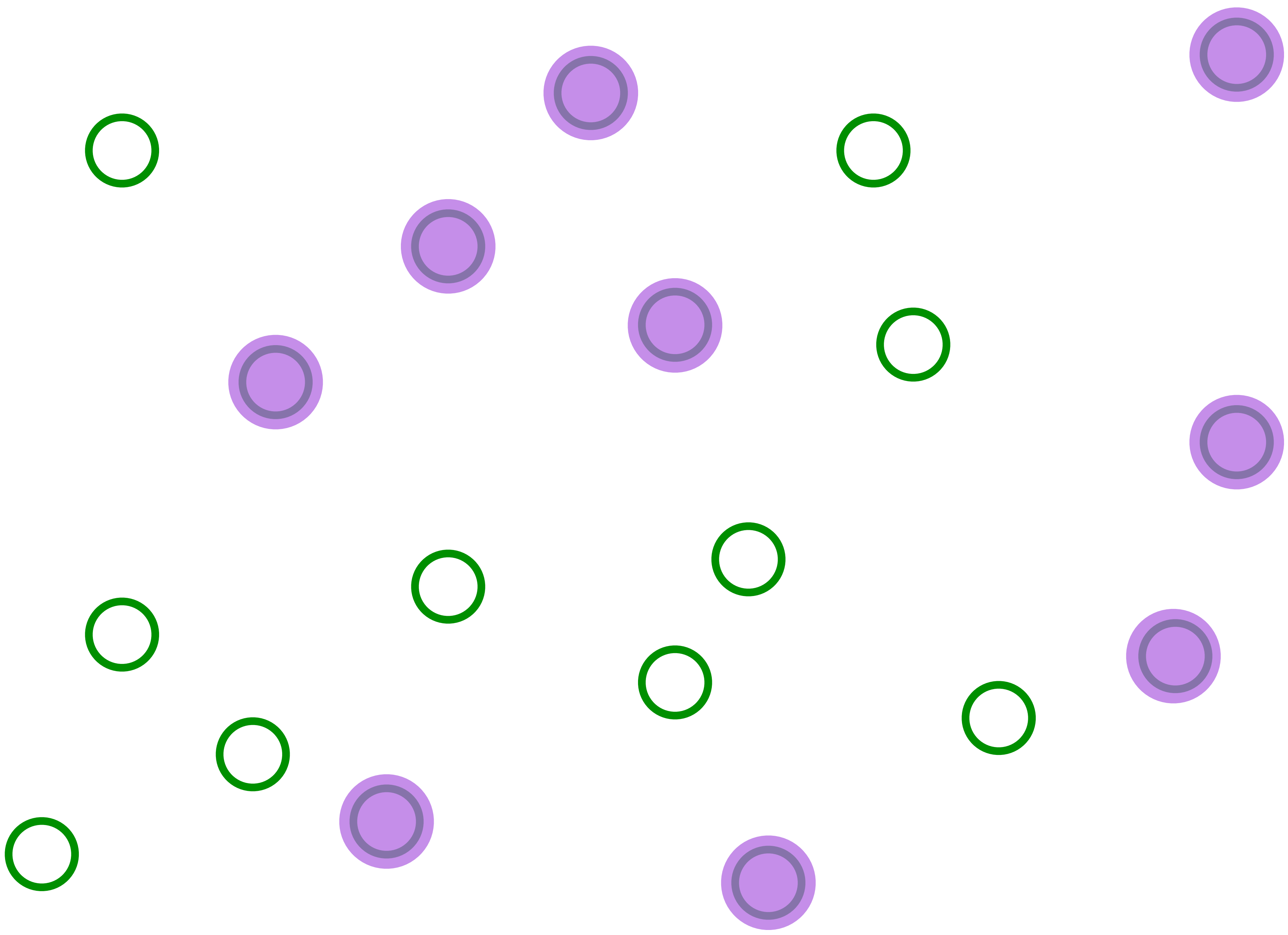

Entangle electrons pairwise randomly

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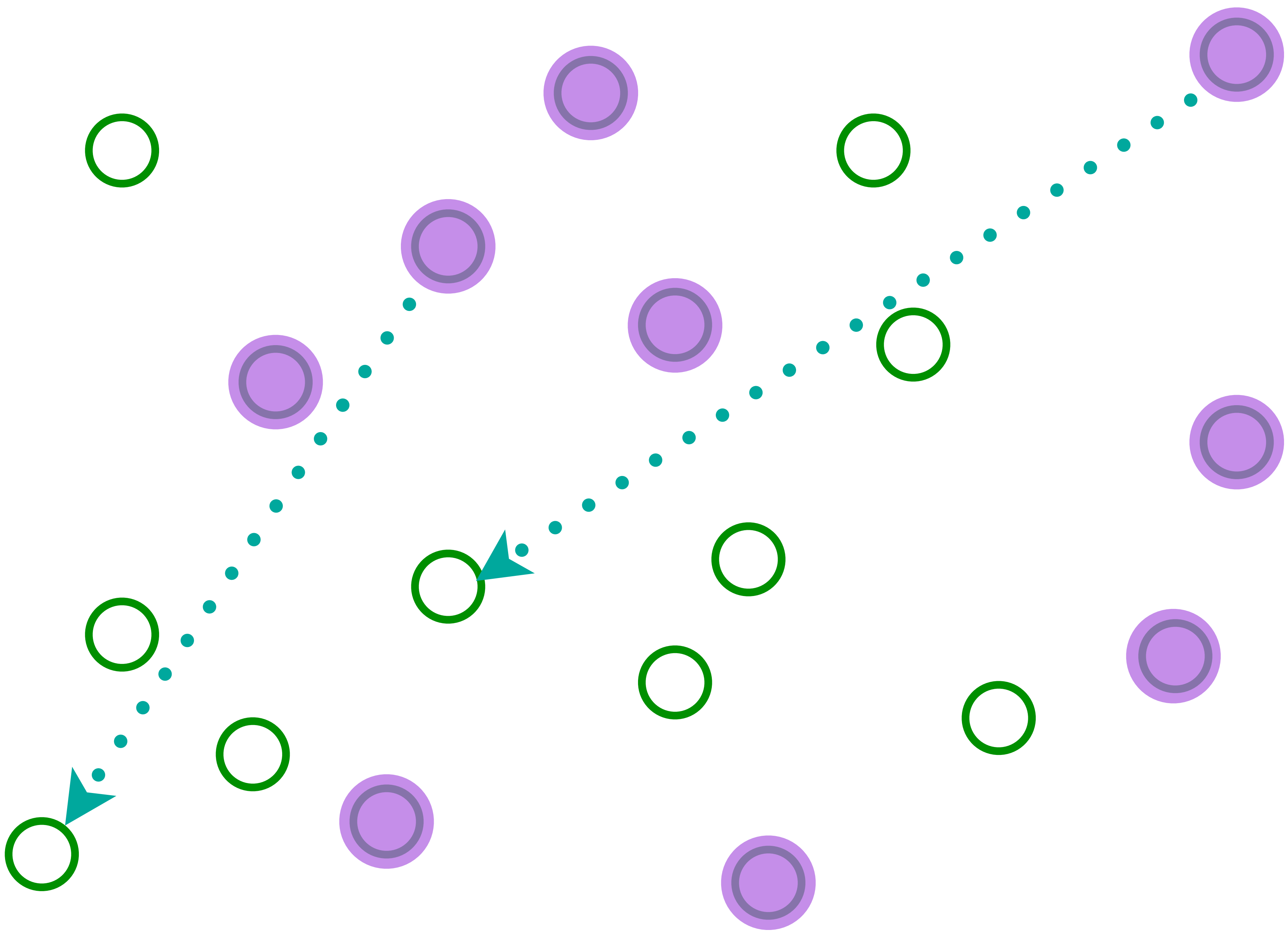

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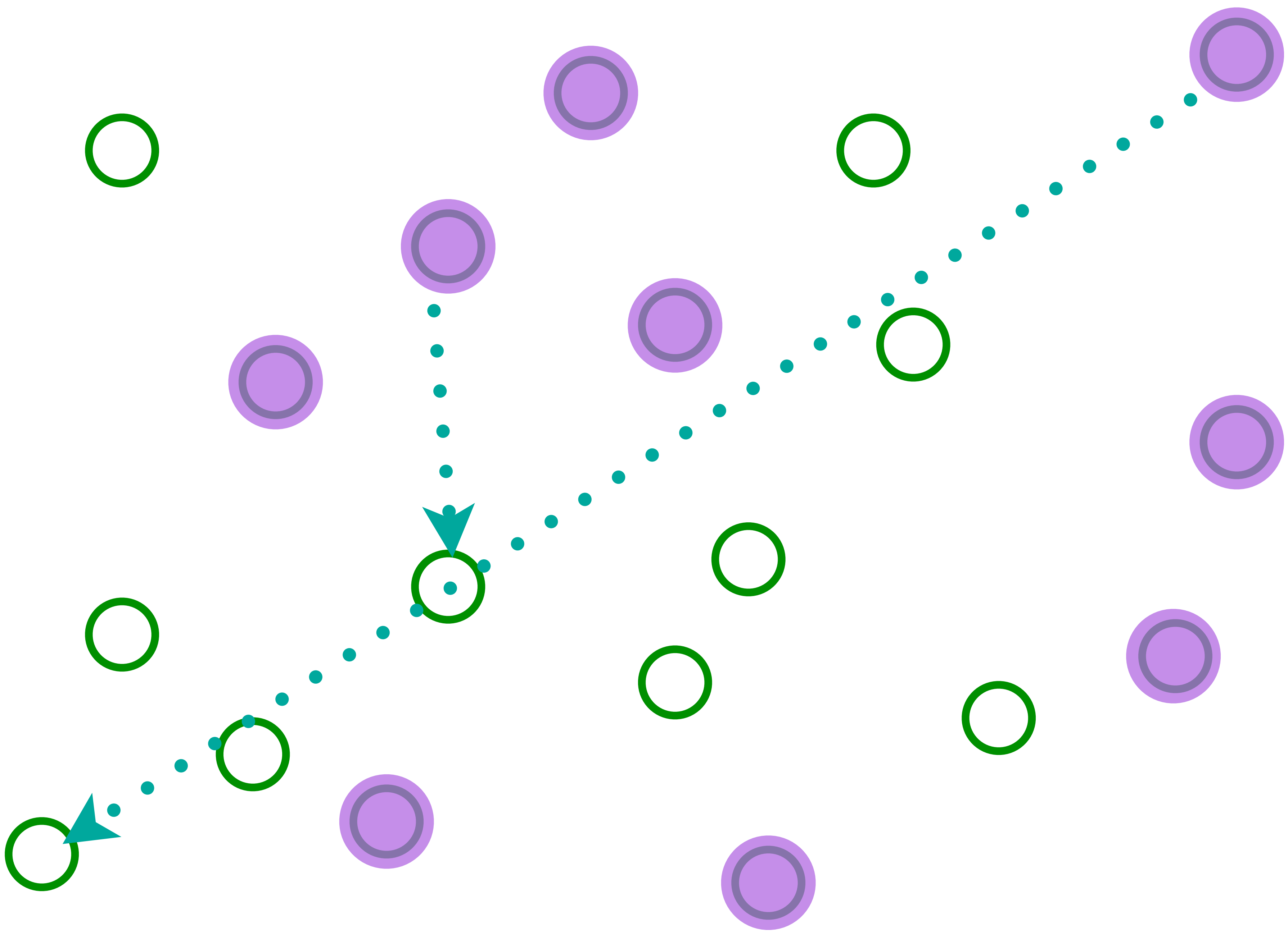

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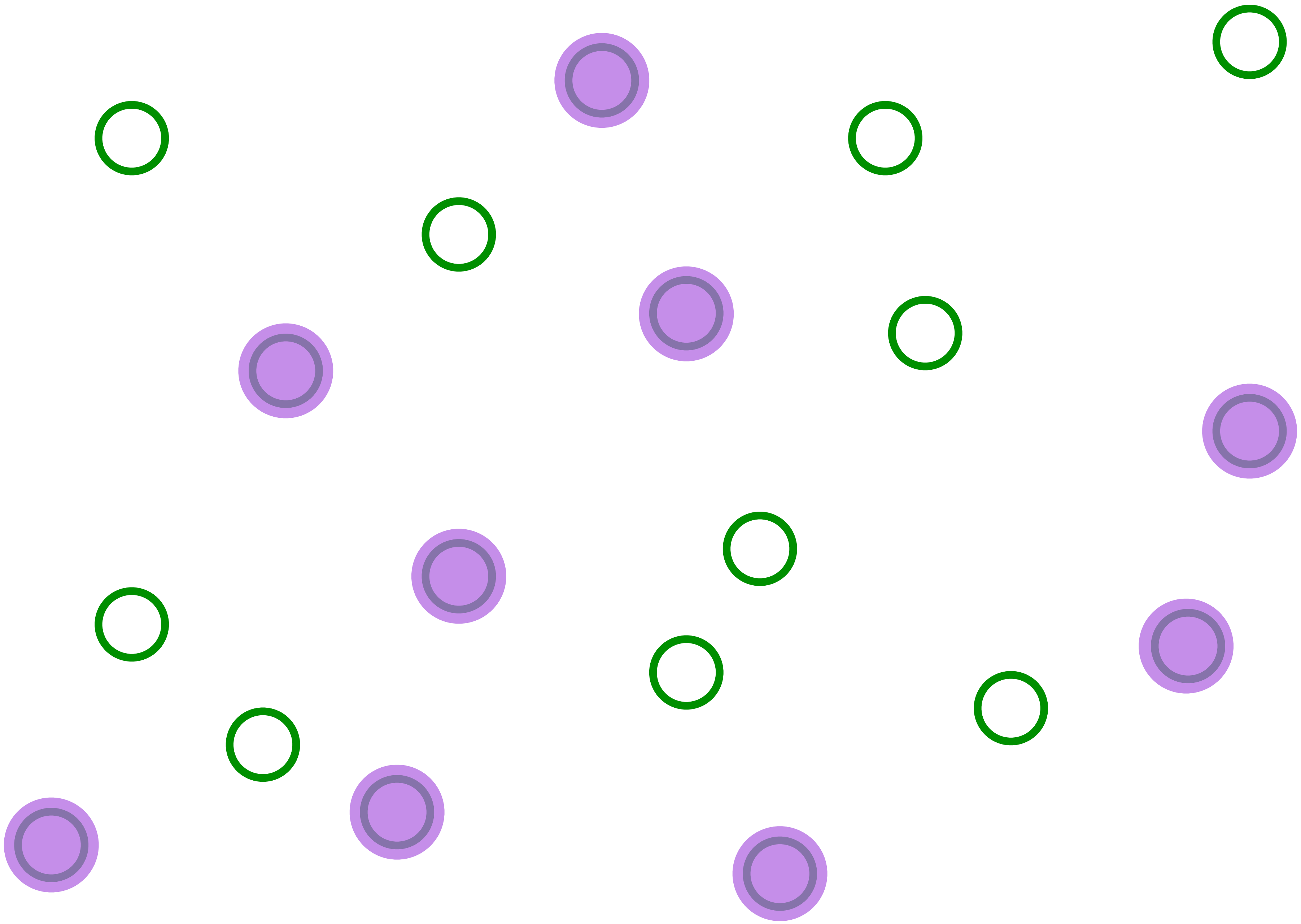

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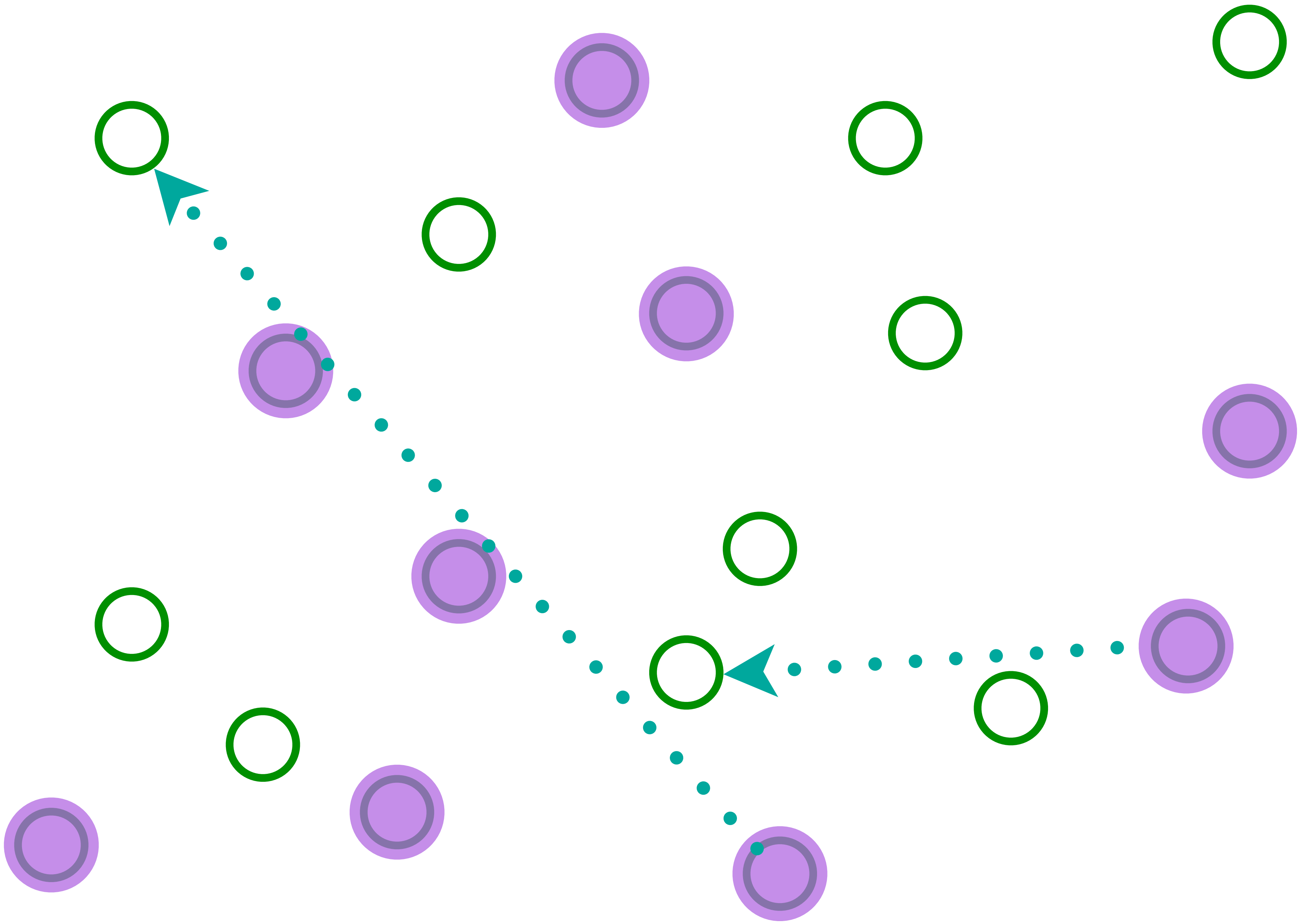

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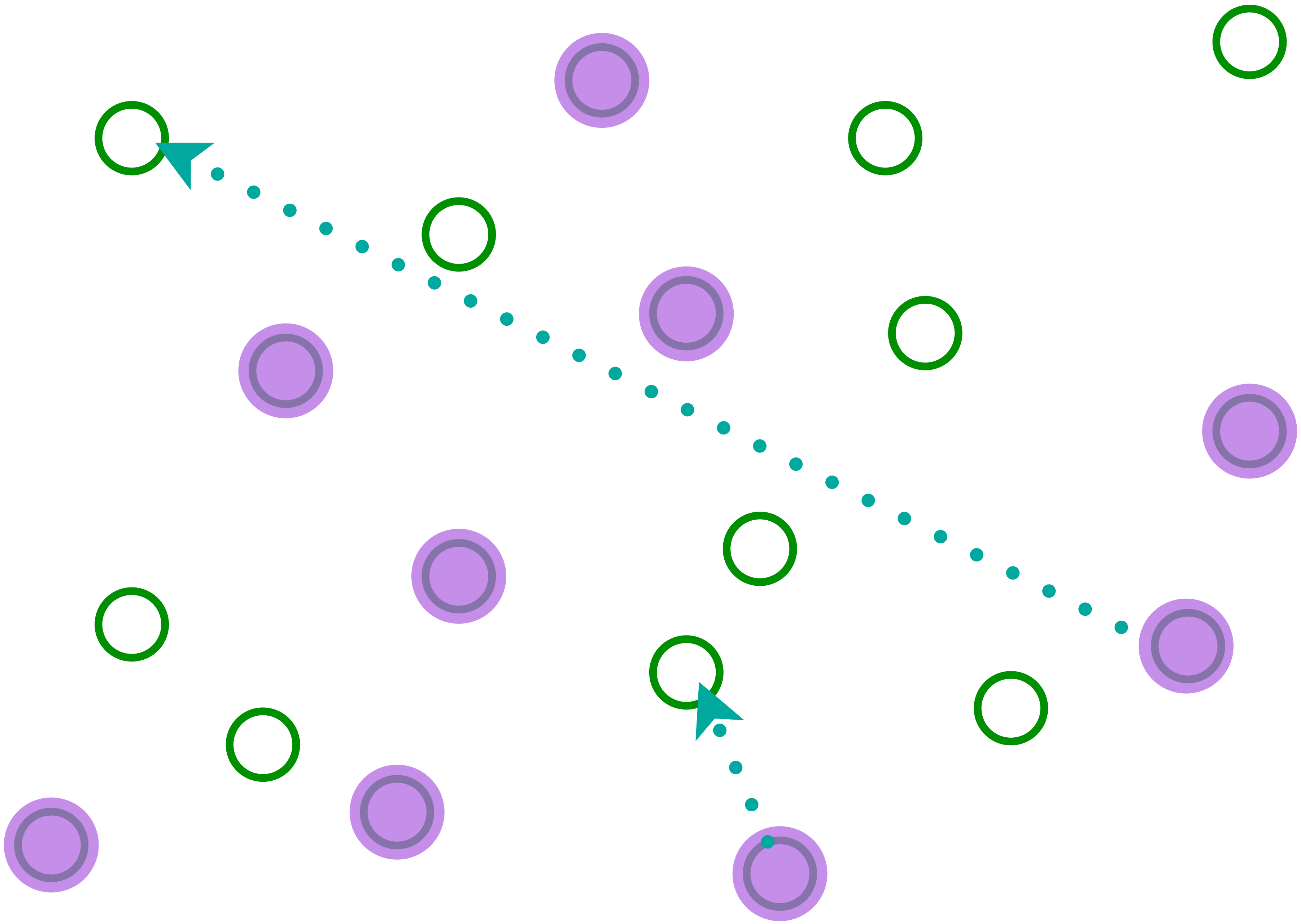

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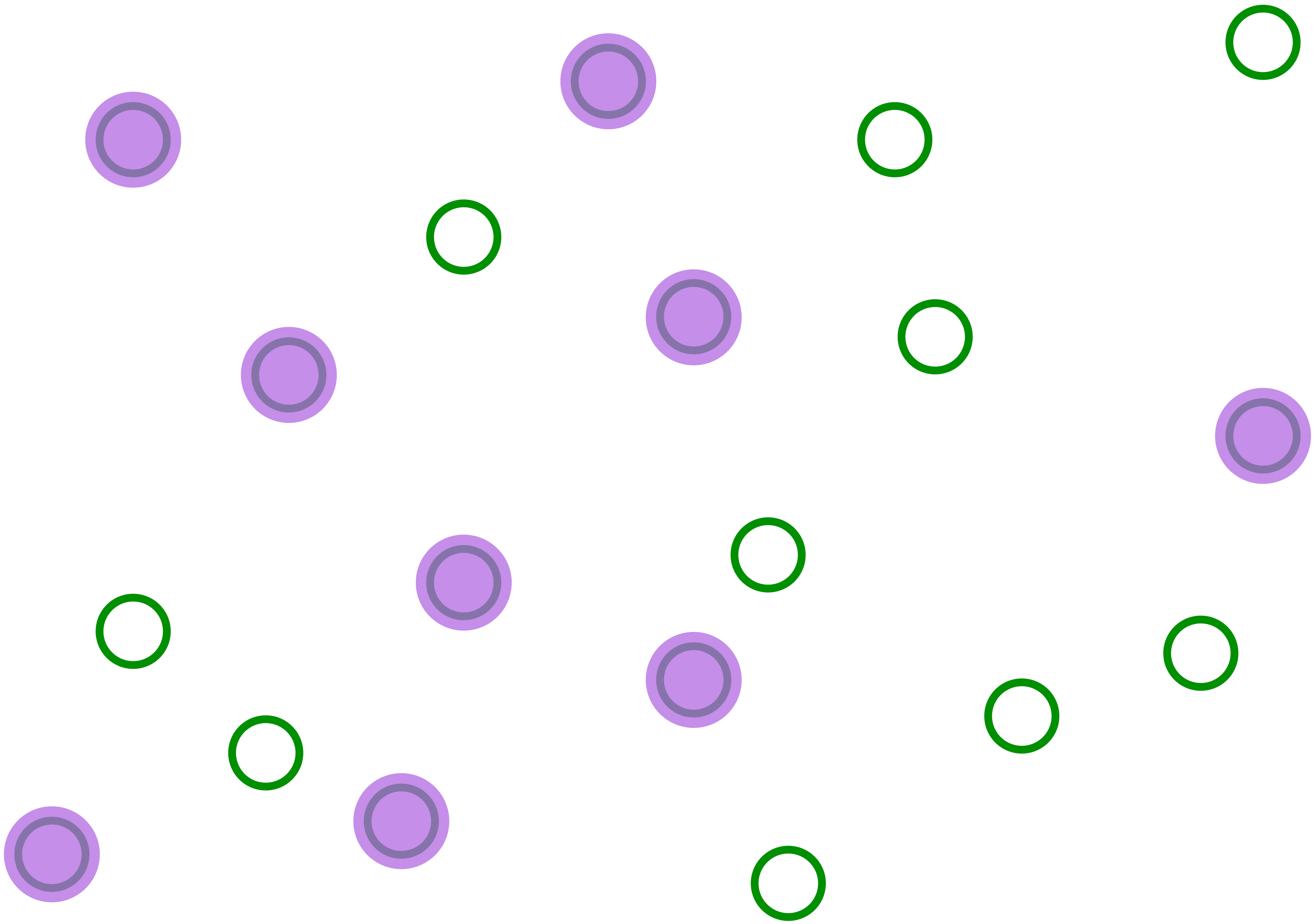

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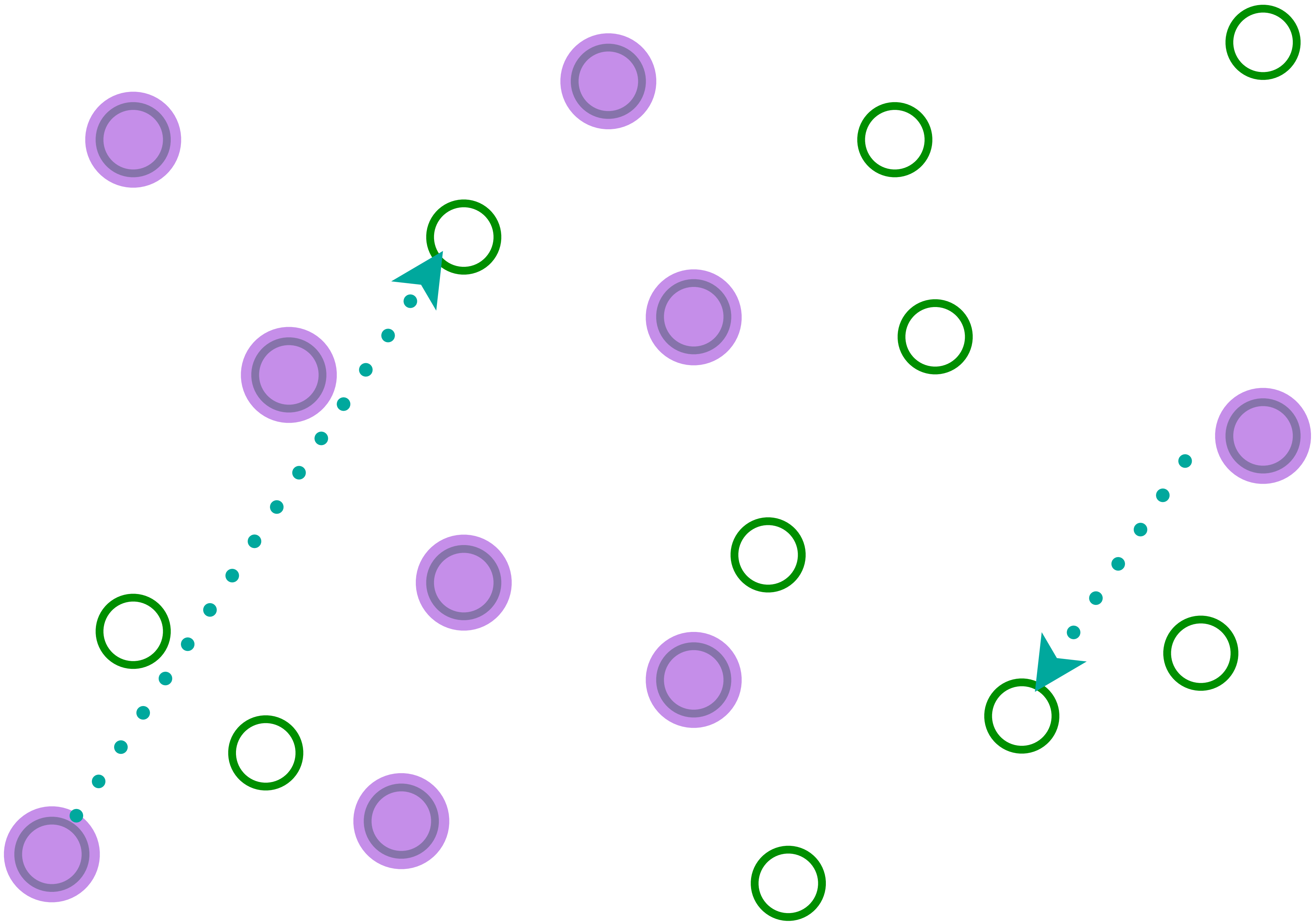

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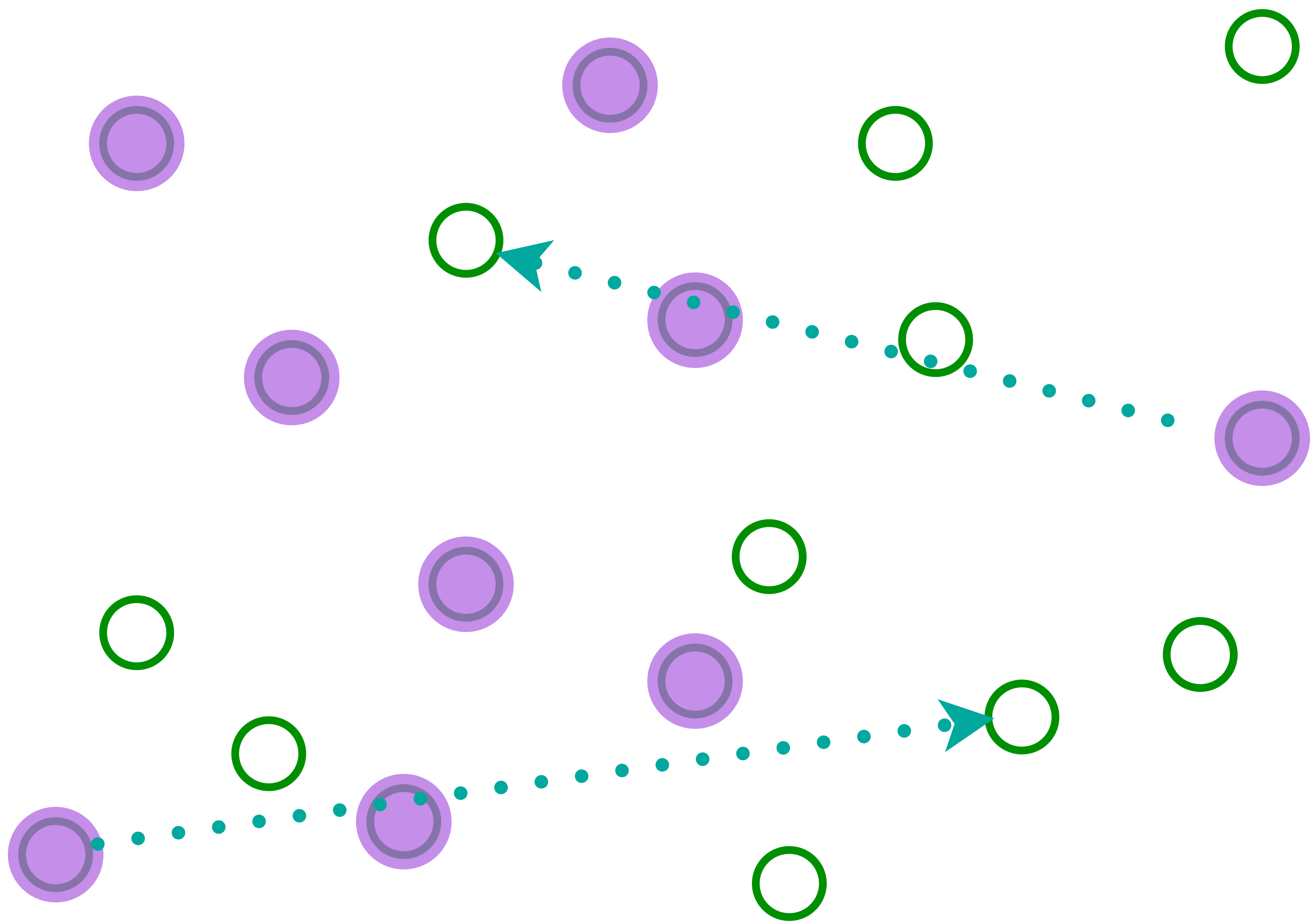

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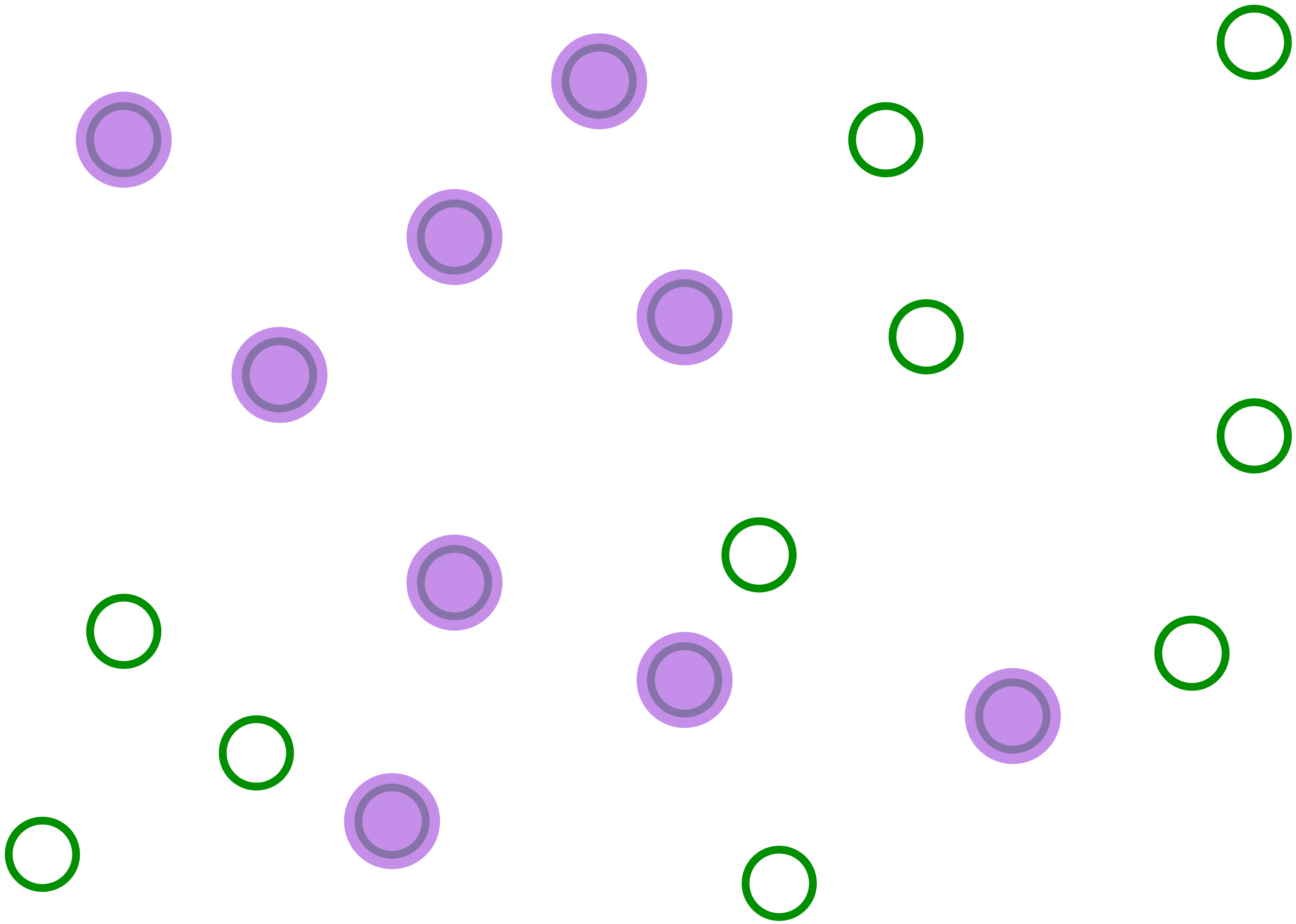

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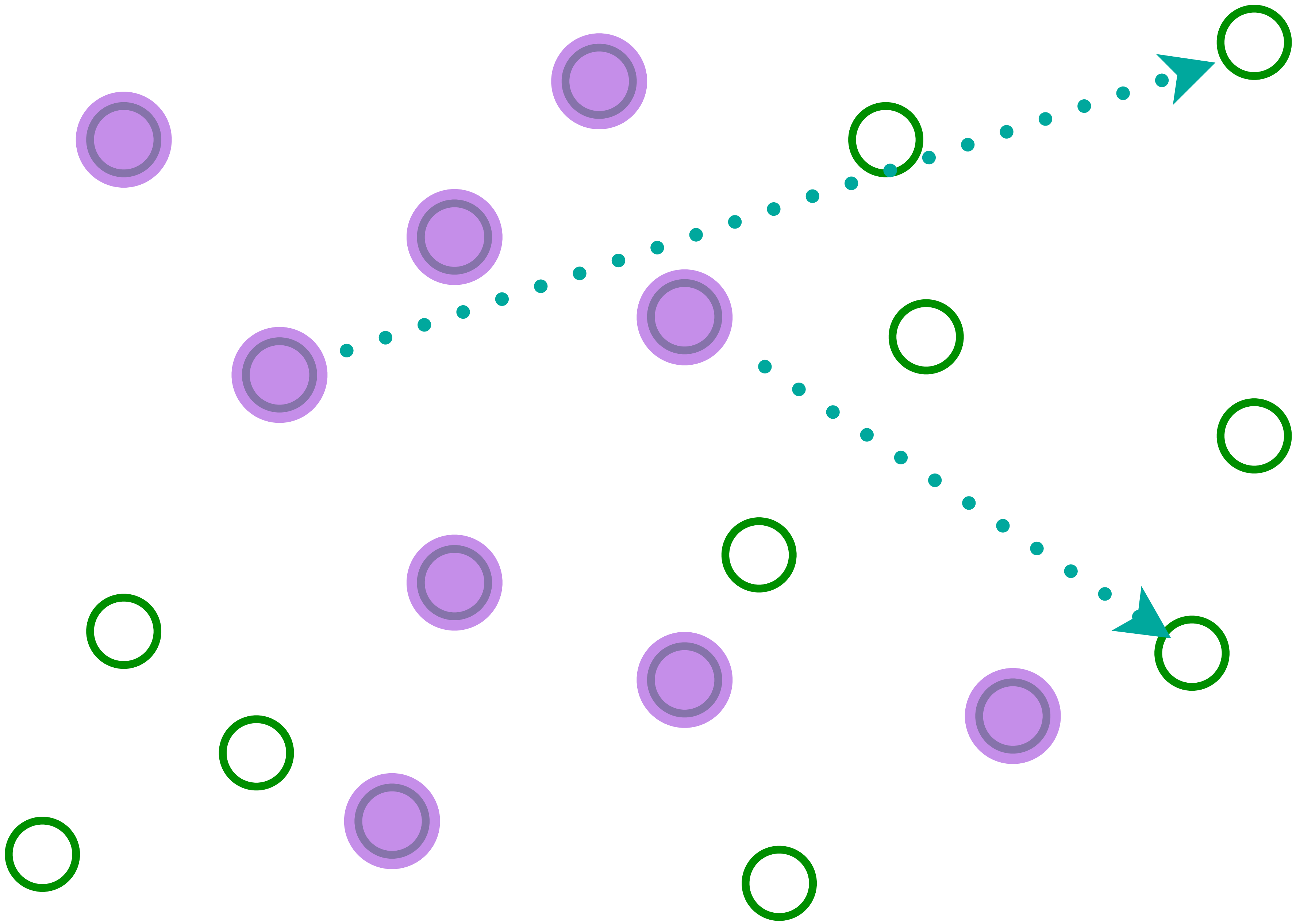

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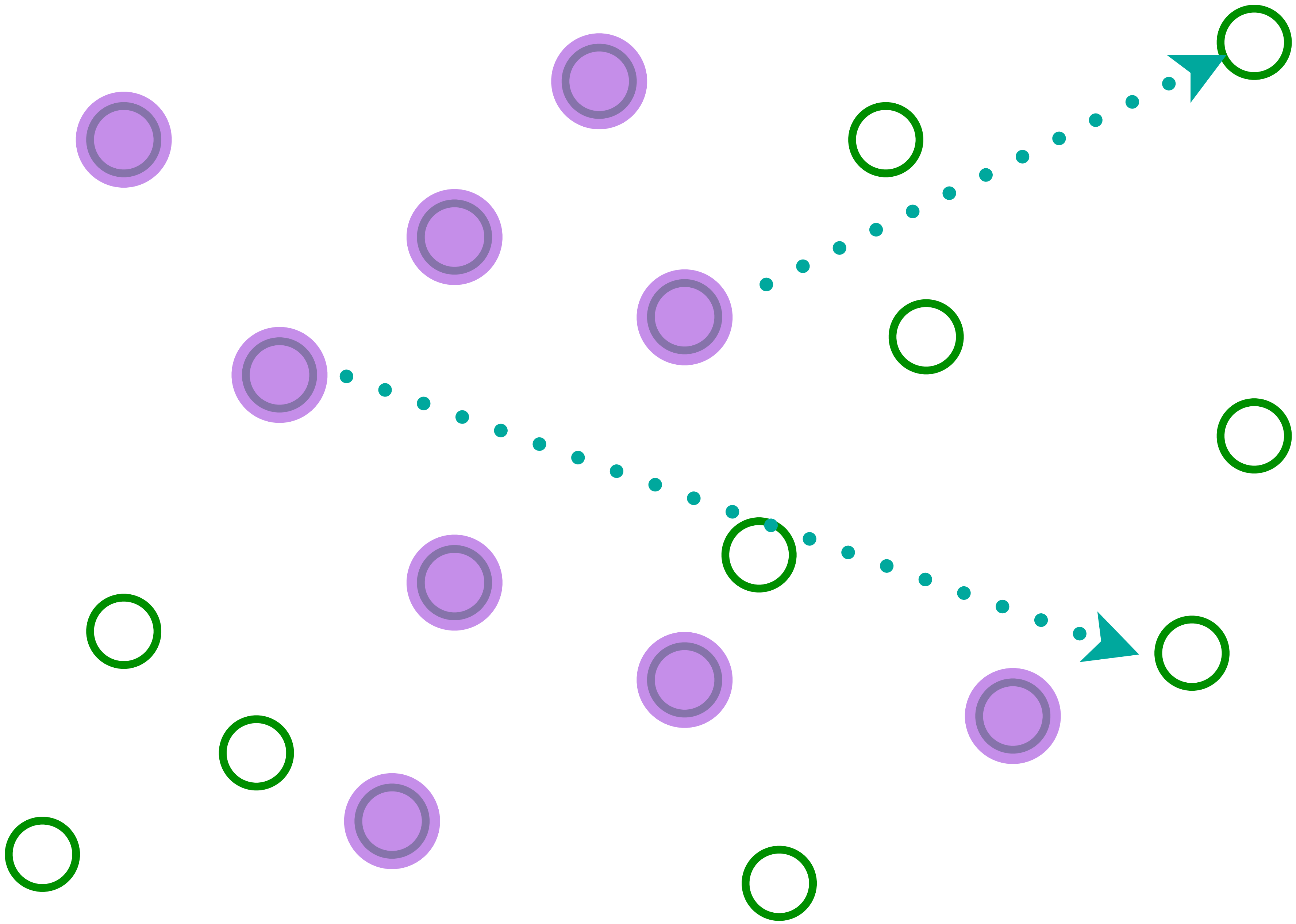

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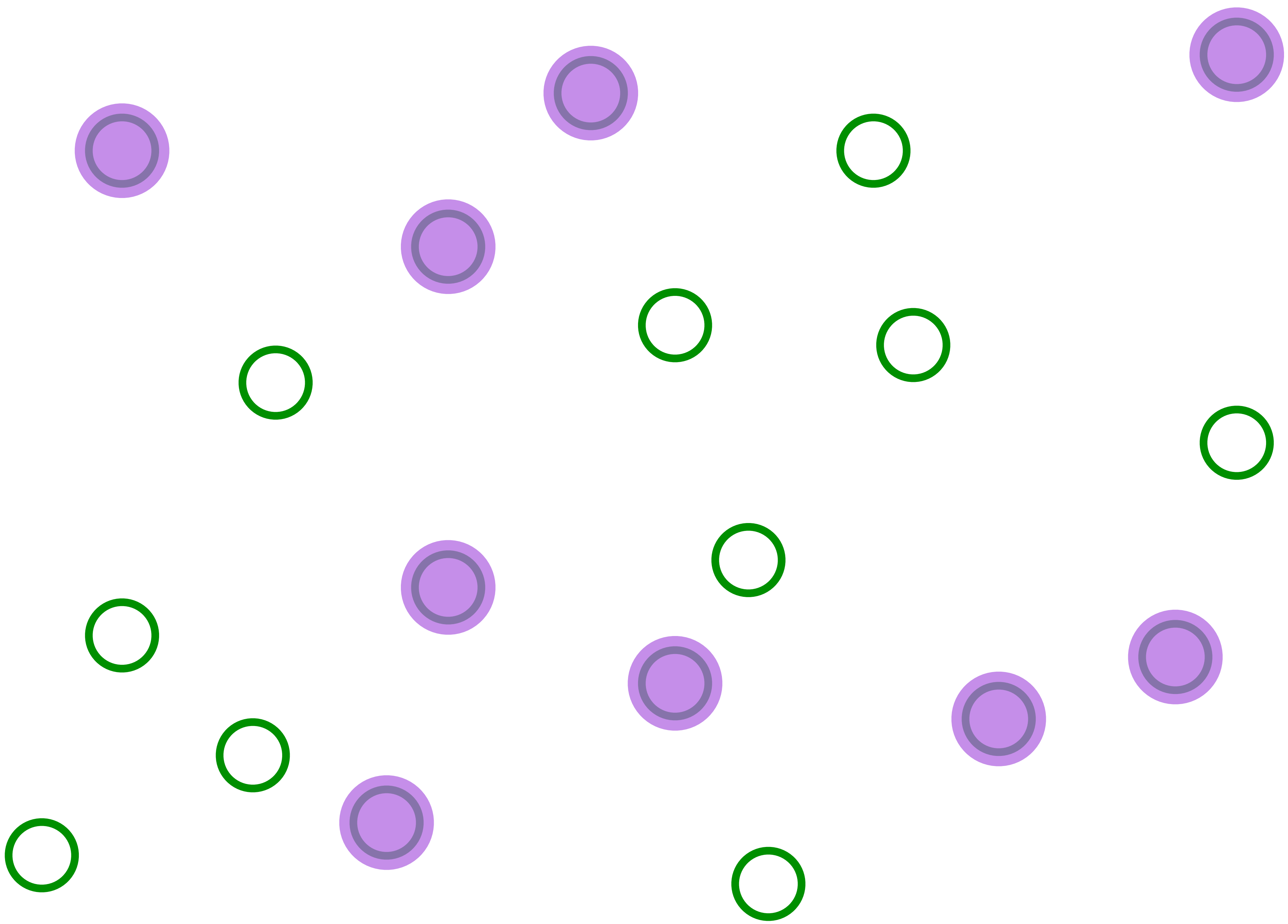

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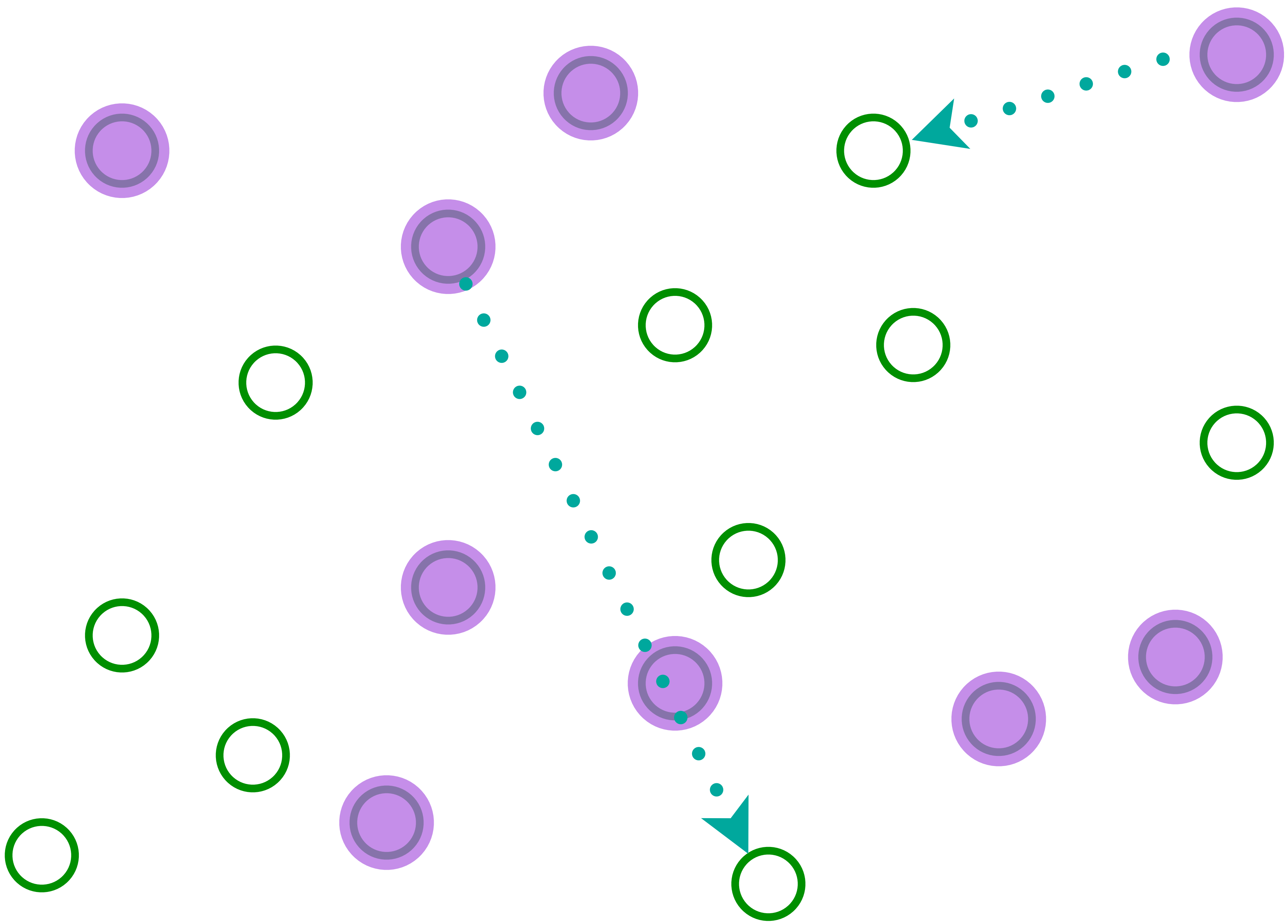

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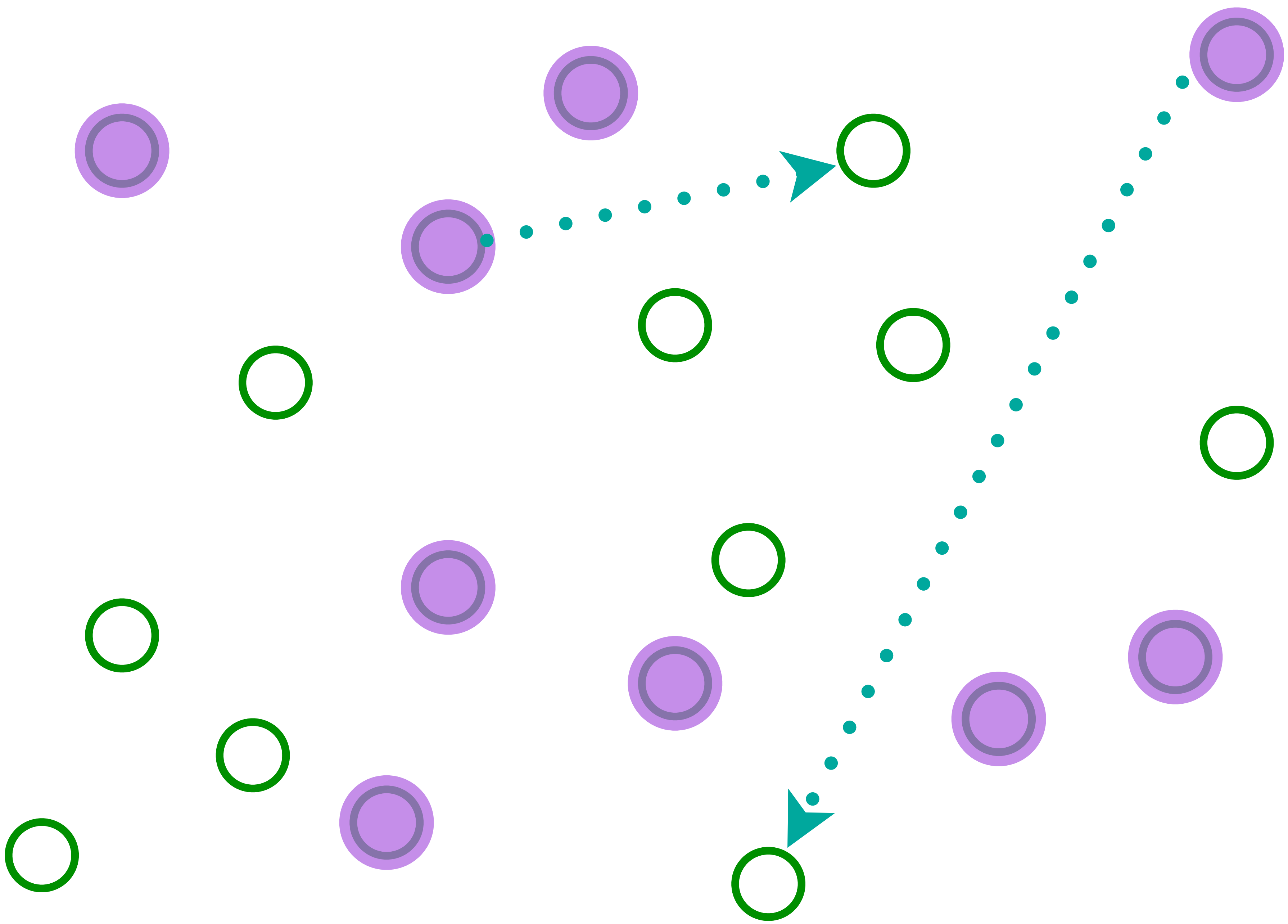

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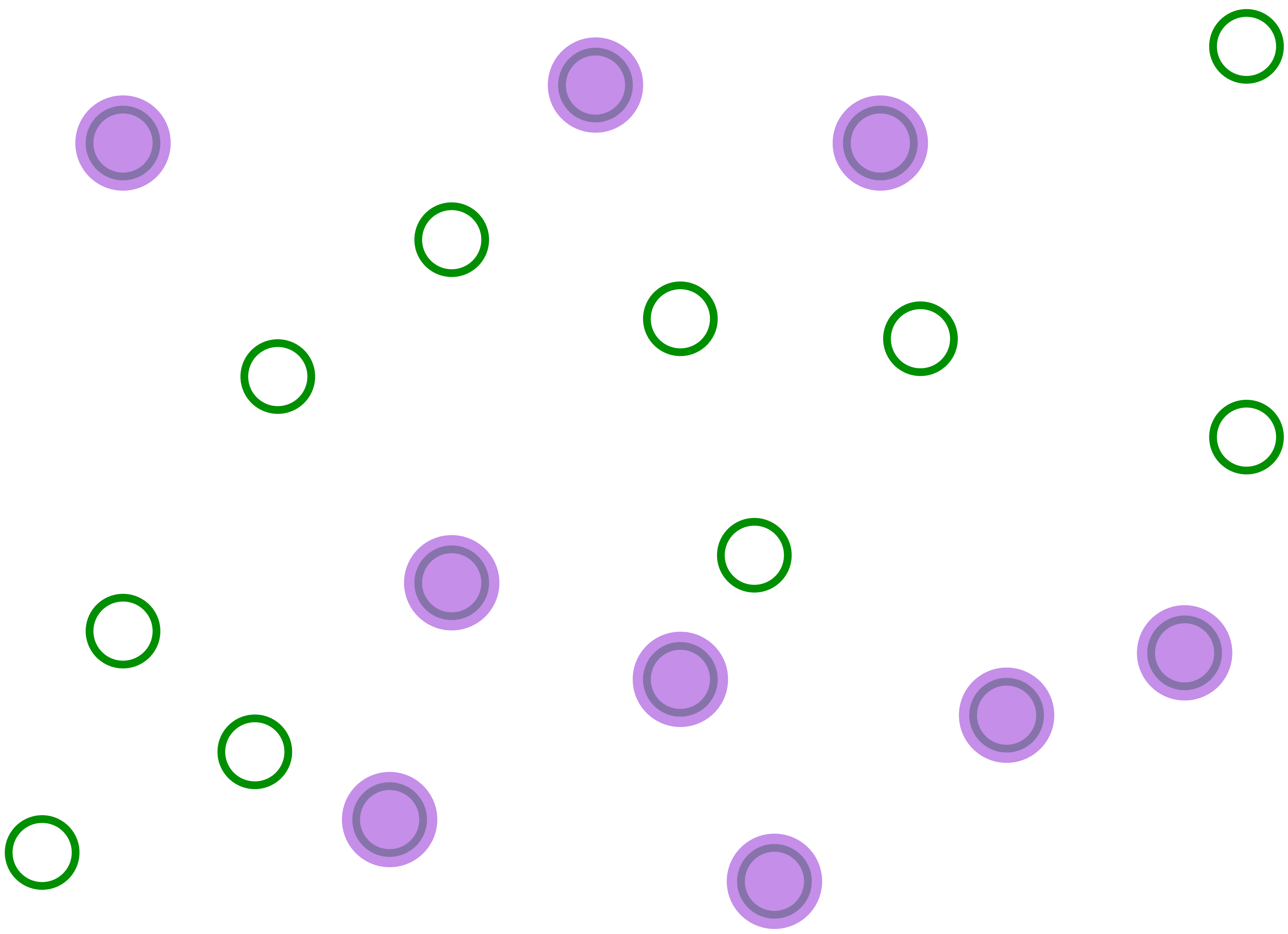

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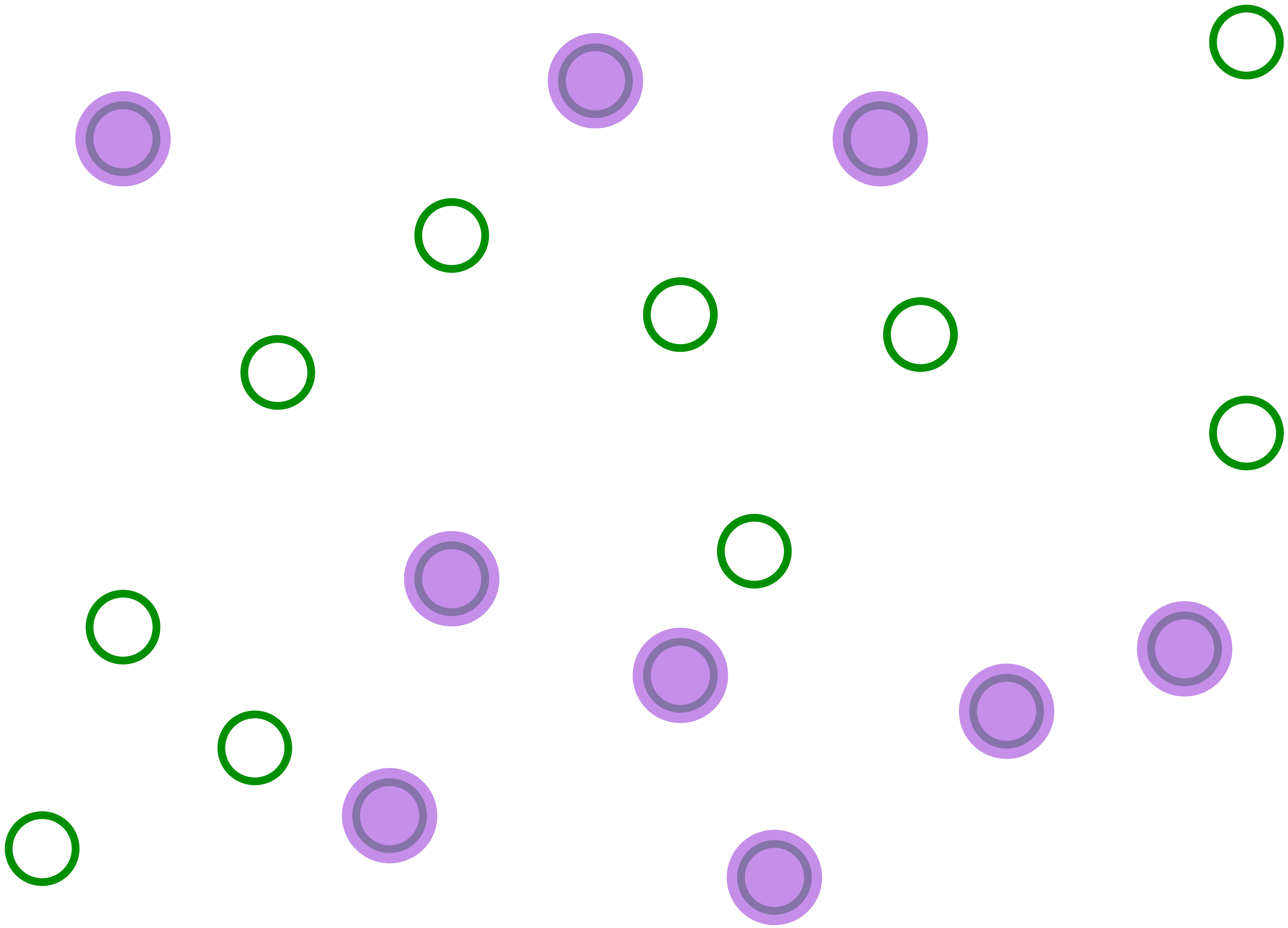

This describes both a strange metal and a black hole!

The SYK model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

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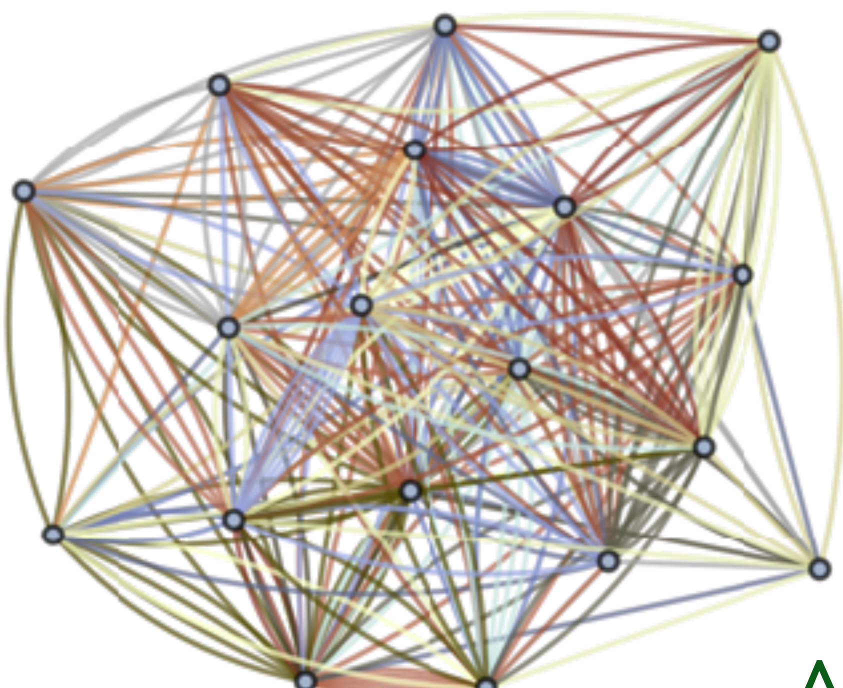
$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

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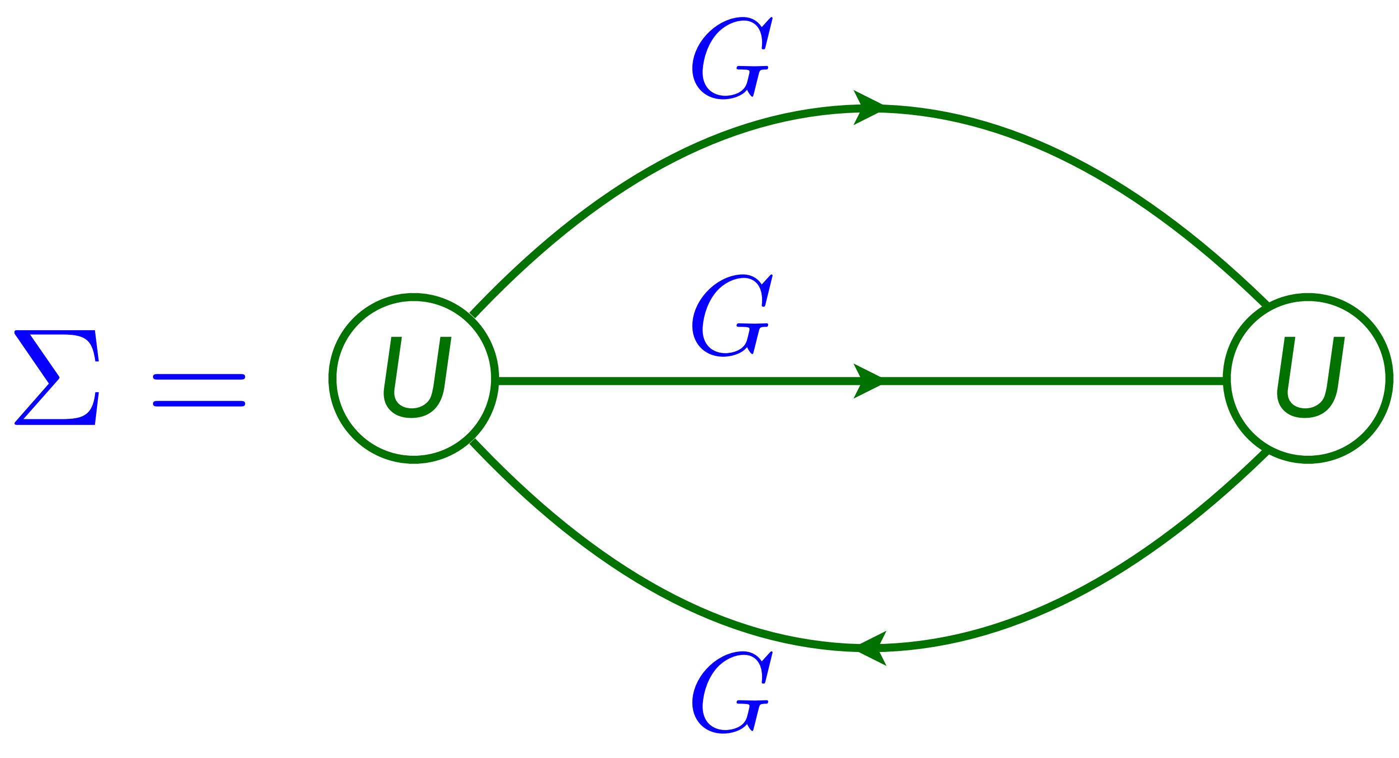
$$G(\tau = 0^-) = \mathcal{Q}.$$



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Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{e^{i(\pi/4+\theta)}}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{Ae^{-i(\pi/4+\theta)}}{\sqrt{z}}$$

where $A = (\pi/U^2 \cos(2\theta))^{1/4}$. The value of θ is universally related to $\mathcal{Q}$ by a Luttinger-Ward functional analysis similar to that used to establish the Luttinger theorem of Fermi liquid theory:

$$\mathcal{Q} = \frac{1}{2} - \frac{\theta}{\pi} - \frac{\sin(2\theta)}{4}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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$$G(\tau = 0^-) = \mathcal{Q}.$$

At $T > 0$, we obtain a solution with a conformal structure

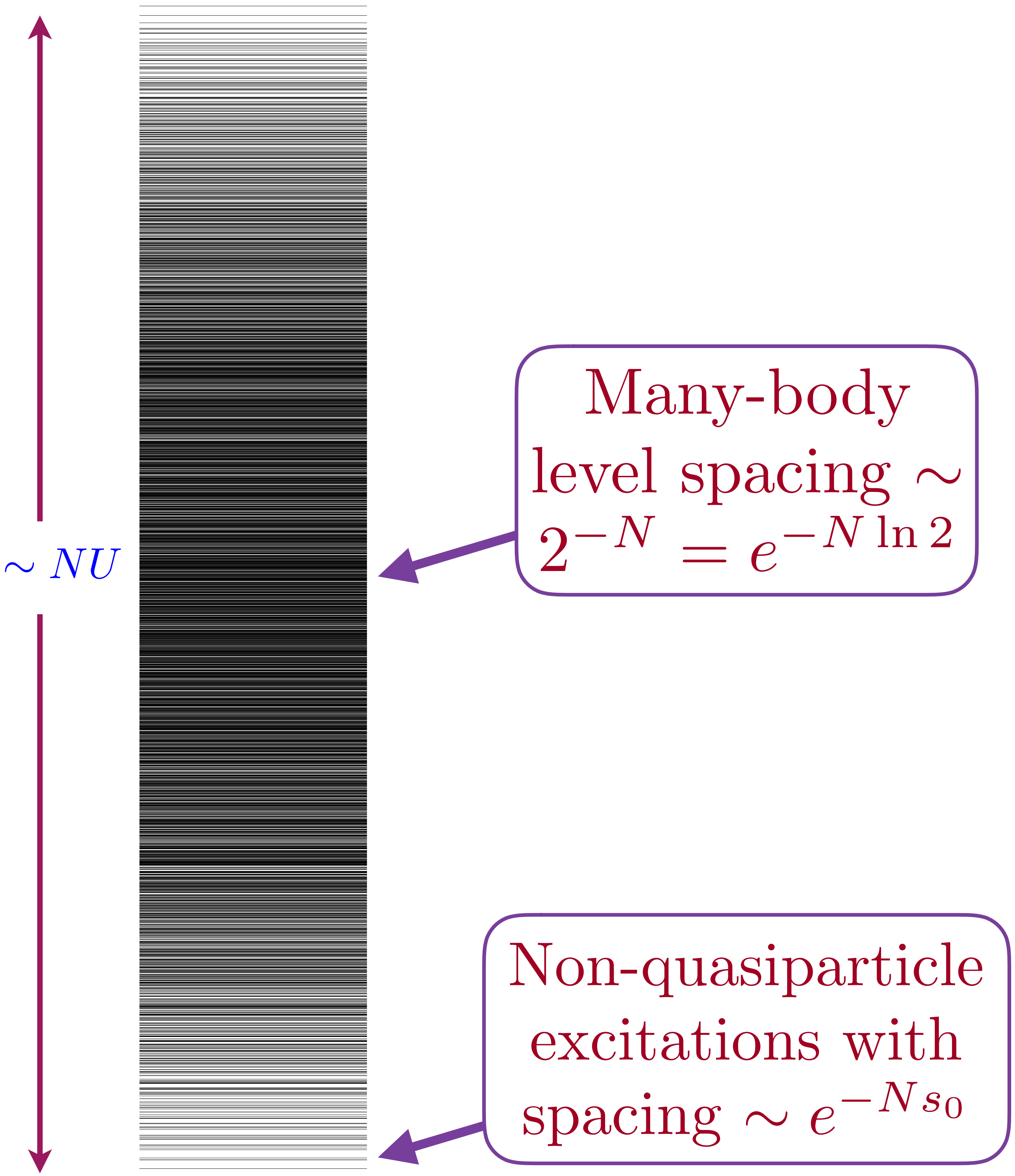
$$G(\tau) = -A \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1 + e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T \tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T \,,$$

where the 'particle-hole asymmetry' is determined by $\mathcal{E}$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}.$$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The SYK model



There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}\,.$$

At $\mathcal{Q} = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model

The low T expansion of the grand potential is

$$\Omega(T) - E_0 = N\left[-s_0 T - \frac{1}{2}(\gamma + 4\pi^2\mathcal{E}^2 K)T^2 + \mathcal{O}(T^3)\right] + 2T\ln\left(\frac{U}{T}\right)\ldots$$

From the grand potential, we can obtain the low energy, the *many*-particle density of states $D(E)$

$$D(E) = \sum_{p=-\infty}^{\infty} e^{2\pi p\mathcal{E}}\, d\left(E - \frac{p^2}{2NK}\right)$$

where $N\mathcal{Q} + p$ is the integer fermion number, $d(E) = 0$ for $E < E_0$, and

$$d(E) \sim \exp\left(N s_0\right) \sinh\left(\sqrt{2N\gamma(E - E_0)}\right) \,,$$

$$E > E_0 \,,\; e^{-cN} \ll \gamma(E - E_0) \ll N \,.$$

There are exponentially more low energy states than for the quasiparticle case, and $D(E)$ self-averages down to energies exponentially small in N.

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, arXiv:1611.04650;
S. Sachdev, PRX **5**, 041025 (2015); R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849 ;
A.M. Garcia-Garcia and J.J.M. Verbaarschot, arXiv:1701.06593; D. Bagrets, A. Altland, and A. Kamenev, arXiv:1702.08902;
D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S.J. Suh, arXiv:1711.08467; Yingfei Gu and S. Sachdev, unpublished.

A simple model of a metal with quasiparticles

The grand potential $\Omega(T)$ at low T is (from the Sommerfeld expansion)

$$\Omega(T) - E_0 = N\left(-\frac{\pi^2}{6}\rho_0 T^2 + \mathcal{O}(T^4)\right) + \ldots$$

where $\rho_0 \equiv \rho(0)$ is the *single* particle density of states at the Fermi level.
From $\Omega(T)$ we can obtain the *many*-particle density of states $D(E)$

$$D(E) \sim \exp\left(\pi\sqrt{\frac{2N\rho_0(E-E_0)}{3}}\right) ,$$

$$E > E_0\ ,\ 1/N \ll \rho_0(E-E_0) \ll N$$

and $D(E) = 0$ for $E < E_0$. This is related to the asymptotic growth of the partitions of an integer,

$$p(n) \sim \exp(\pi\sqrt{2n/3})\,.$$

Near the lower bound, there are large sample-to-sample fluctuations due to variations in the lowest quasiparticle energies.

The SYK model

The low T expansion of the grand potential is

$$\Omega(T) - E_0 = N\left[-s_0 T - \frac{1}{2}(\gamma + 4\pi^2\mathcal{E}^2 K)T^2 + \mathcal{O}(T^3)\right] + 2T\ln\left(\frac{U}{T}\right)\ldots$$

From the grand potential, we can obtain the low energy, the *many*-particle density of states $D(E)$

$$D(E) = \sum_{p=-\infty}^{\infty} e^{2\pi p\mathcal{E}}\, d\left(E - \frac{p^2}{2NK}\right)$$

where $N\mathcal{Q} + p$ is the integer fermion number, $d(E) = 0$ for $E < E_0$, and

$$d(E) \sim \exp\left(N s_0\right) \sinh\left(\sqrt{2N\gamma(E - E_0)}\right) \,,$$

$$E > E_0 \;,\; e^{-cN} \ll \gamma(E - E_0) \ll N \,.$$

There are exponentially more low energy states than for the quasiparticle case, and $D(E)$ self-averages down to energies exponentially small in N.

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, arXiv:1611.04650; S. Sachdev, PRX **5**, 041025 (2015); R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849 ; A.M. Garcia-Garcia and J.J.M. Verbaarschot, arXiv:1701.06593; D. Bagrets, A. Altland, and A. Kamenev, arXiv:1702.08902; D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S.J. Suh, arXiv:1711.08467; Yingfei Gu and S. Sachdev, unpublished.

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The structure of $d(E)$ suggests that the low energy states have split into $\sim e^{N s_0}$ super-selection sectors

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, arXiv:1611.04650;
S. Sachdev, PRX **5**, 041025 (2015); R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849 ;
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These results rely on a low energy theory with emergent time reparameterization (conformal) and U(1) gauge invariance, which are spontaneously broken down to SL(2,R) and global U(1) symmetries.

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, arXiv:1611.04650; S. Sachdev, PRX **5**, 041025 (2015); R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849 ; A.M. Garcia-Garcia and J.J.M. Verbaarschot, arXiv:1701.06593; D. Bagrets, A. Altland, and A. Kamenev, arXiv:1702.08902; D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S.J. Suh, arXiv:1711.08467; Yingfei Gu and S. Sachdev, unpublished.

The SYK model

No quasiparticles

- Rapid local thermal equilibration (of fermion correlators) in a 'Planckian' time

$$\tau_{\rm eq} \sim \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

Established by solution of Schwinger-Keldysh equations for a quench.

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Established by solution of Schwinger-Keldysh equations for a quench.

- Presence of quasiparticles should slow down thermalization, so *all* quantum systems obey

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

Absence of quasiparticles $\Leftrightarrow$ Fastest possible thermalization

1. Random matrix quasiparticle model

q=2, complex SYK

2. Matter without quasiparticles

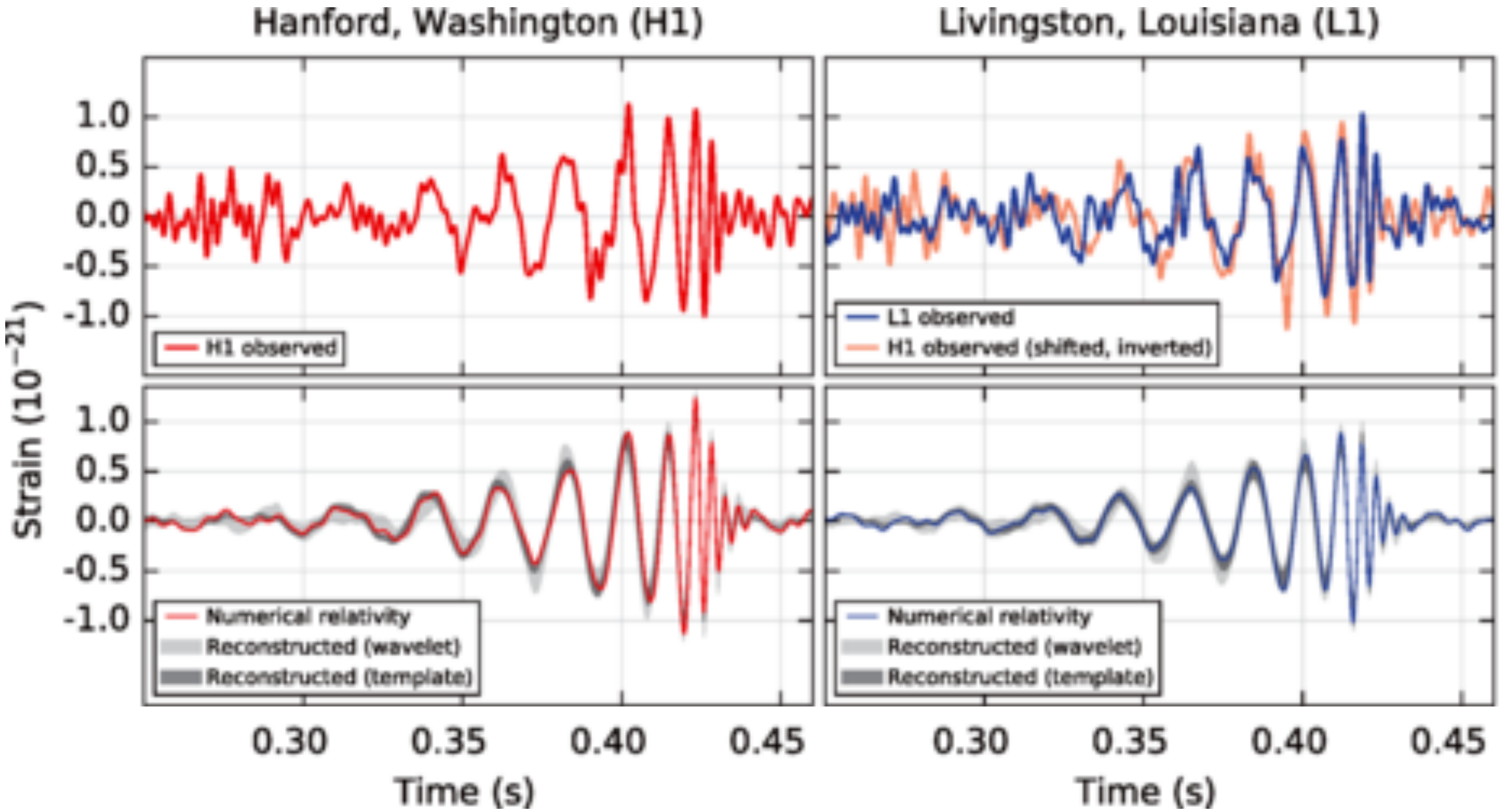
q=4, complex SYK

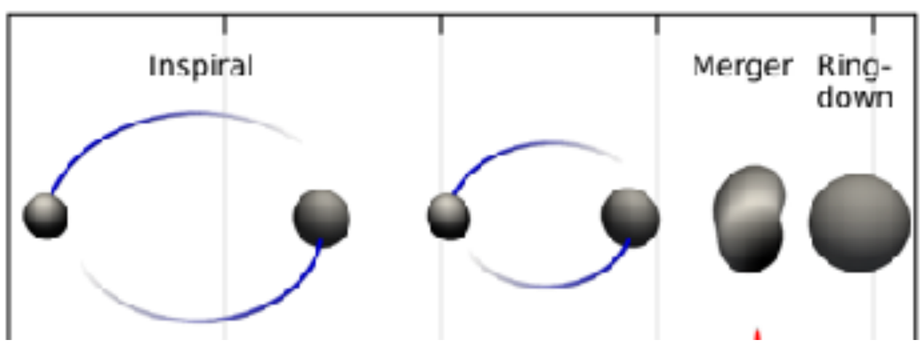
3. Connections to black holes with AdS_2 horizons

4. Connections to strange metals

Black holes

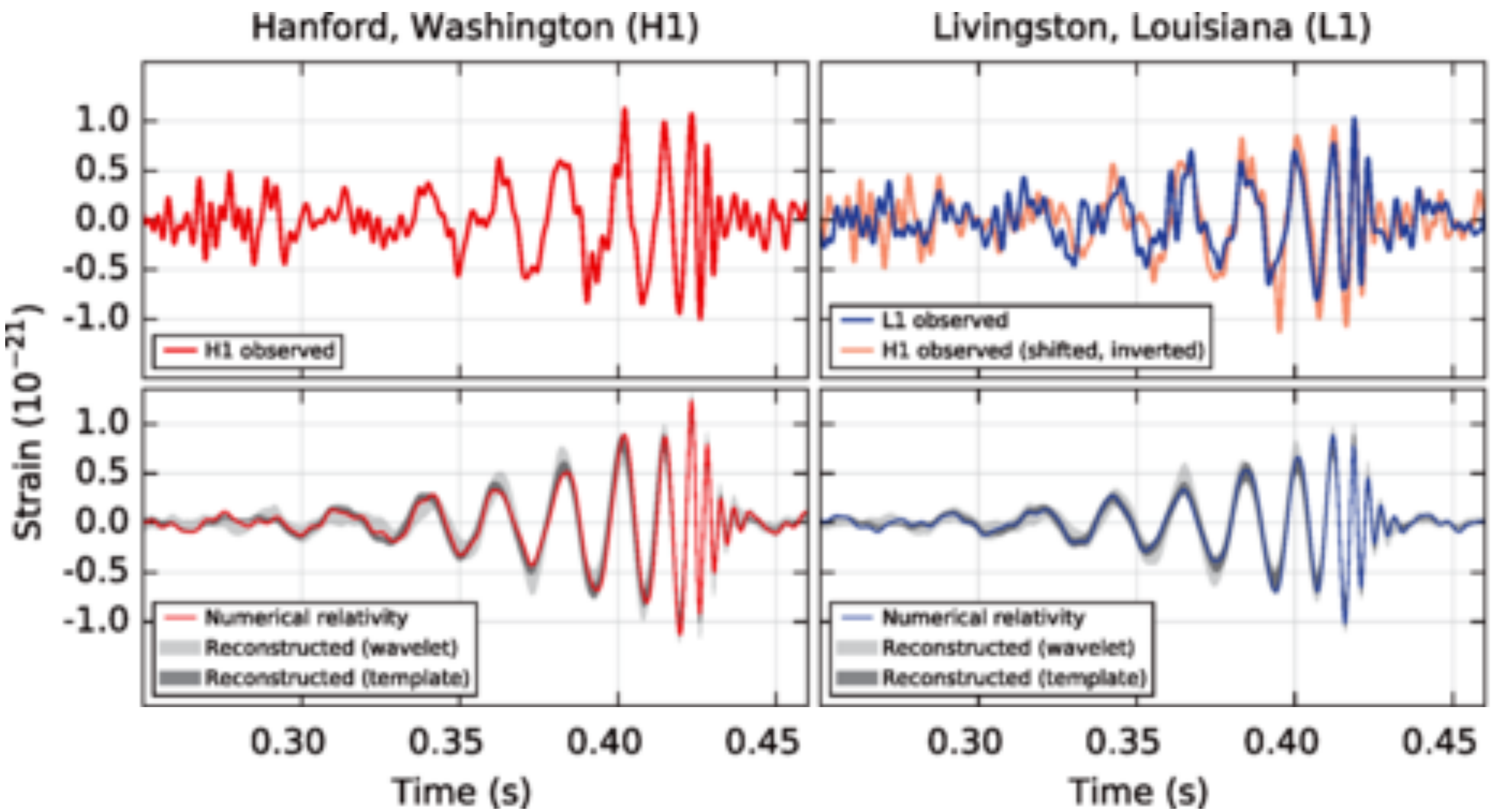
- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area.

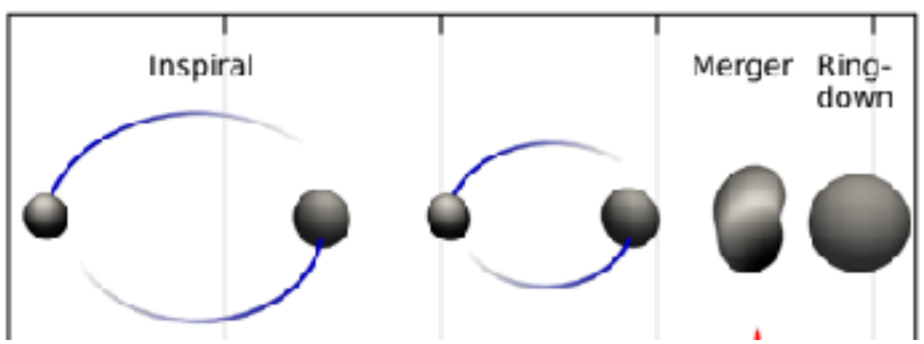




LIGO
September 14, 2015

- The ring-down is predicted by General Relativity to happen in a time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds.





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- The ring-down is predicted by General Relativity to happen in a time $\dfrac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously this happens to equal $\dfrac{\hbar}{k_B T_H}$: so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!

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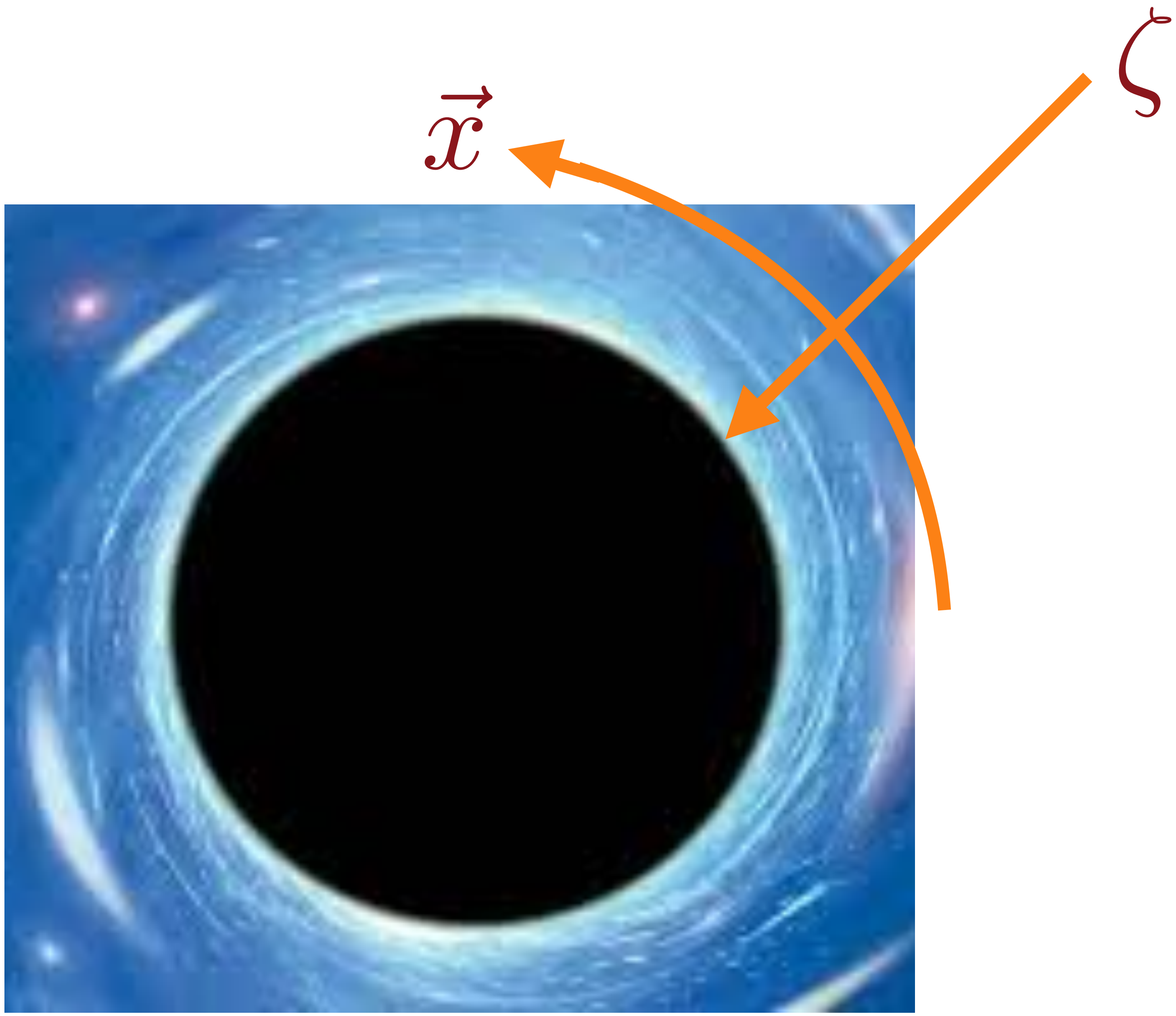
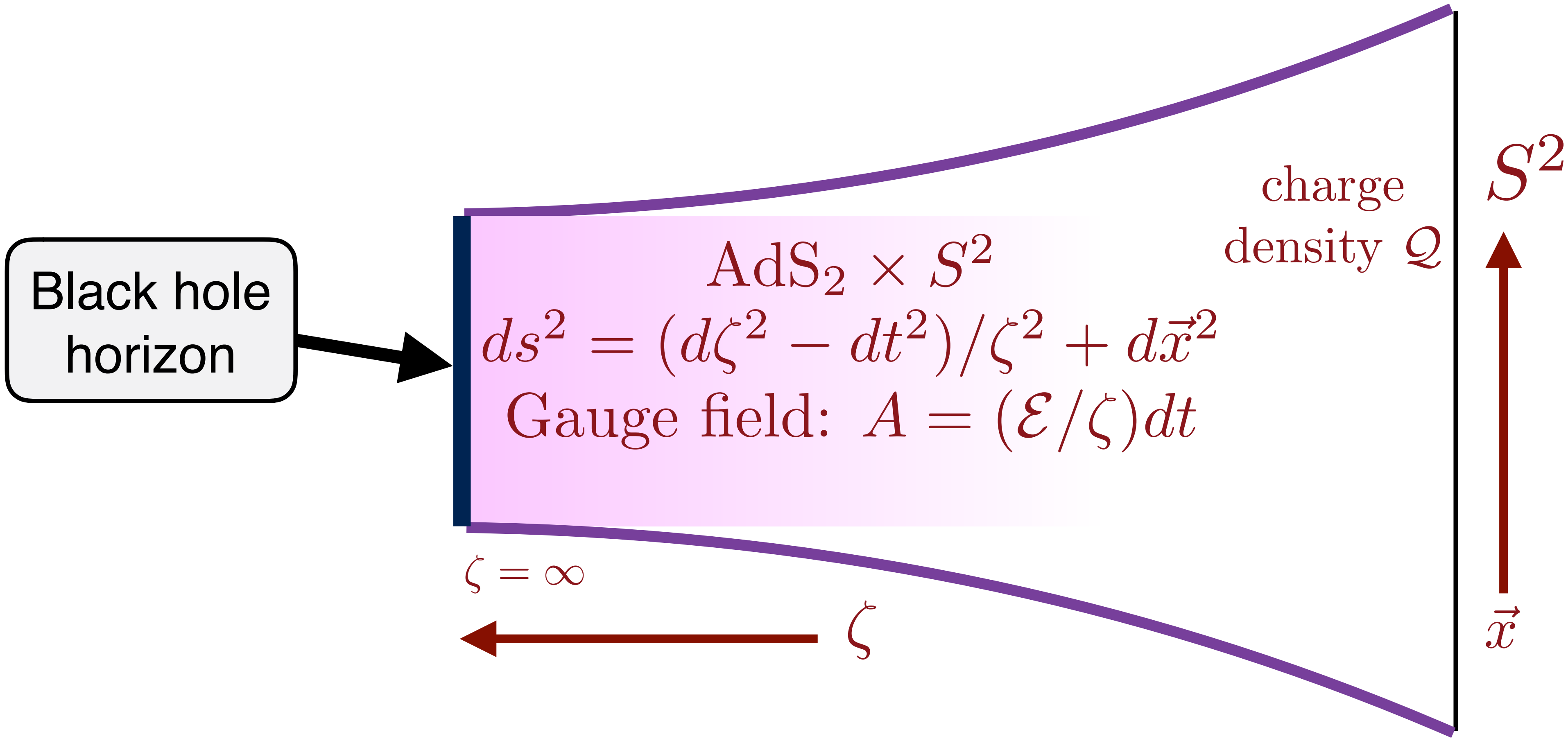
<u>Holography</u>:
Quantum black holes "look like"
quantum many-particle systems
without quasiparticle excitations,
residing "on" the surface of the black hole



Consider a charged black hole with the smallest possible mass: the extremal limit. Zoom in to the near-horizon region at low energies. In this limit, the quantum theory lives in one space (ζ) and one time dimension

SYK models and black holes



The near-horizon region of an extremal charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system

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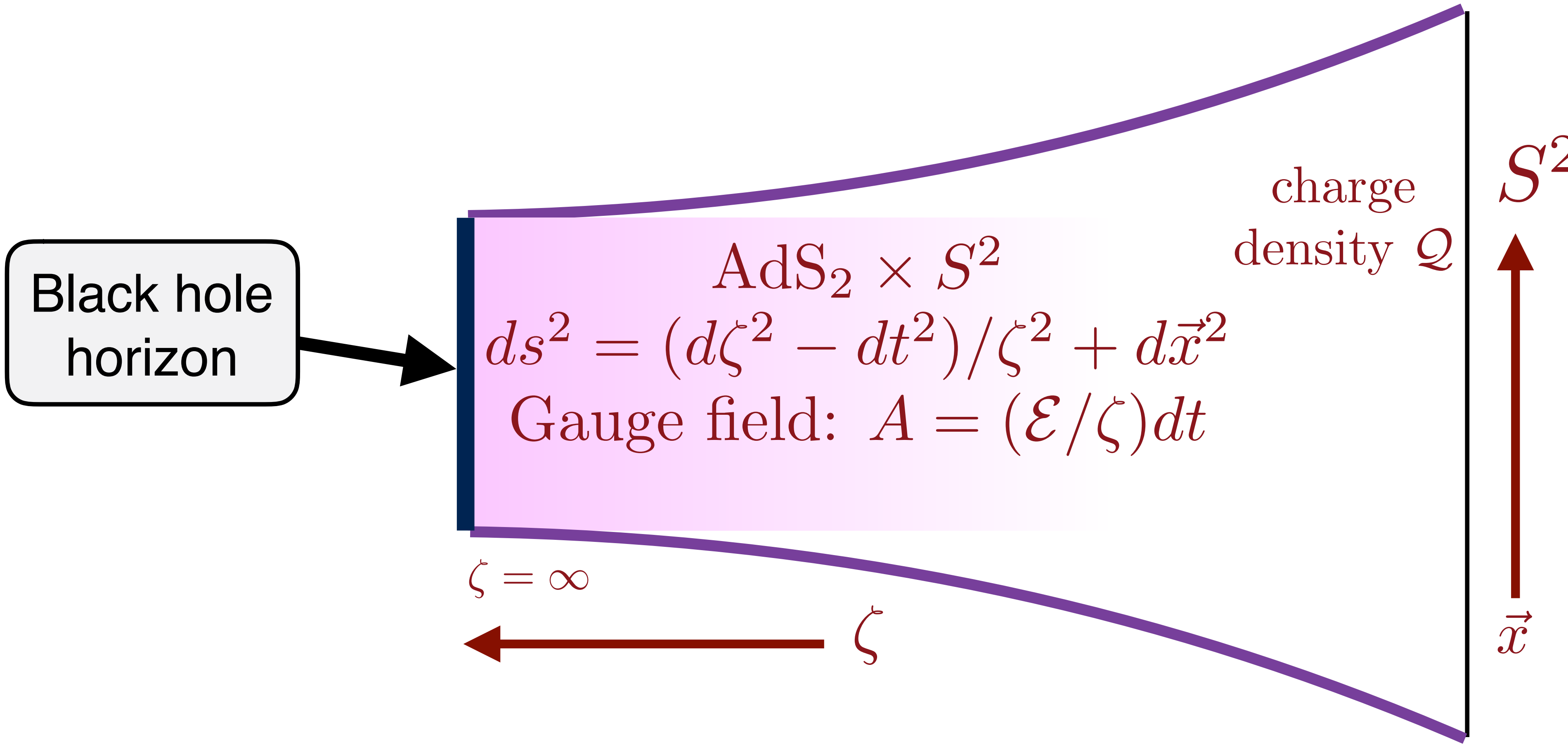

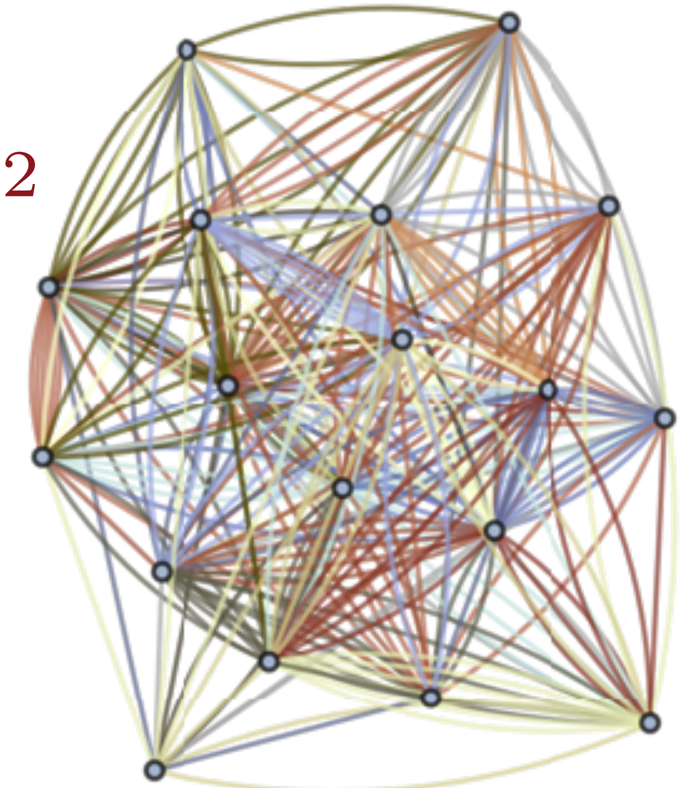

The near-horizon region of an extremal charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system

This turns out to be the SYK model

Phys. Rev. Lett. **105**, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $\mathrm{AdS}_2 \times \mathrm{R}^2$ physics of Reissner-Nordström black holes.

SYK models and black holes

- Reparameterization invariance is a defining property of Einstein's theory of gravity

- In imaginary time, AdS_2 is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

$ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2$ is invariant under

$$\tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \text{ with } ad - bc = 1.$$

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Their identical symmetries lead to the same low energy "Schwarzian" theory for the SYK model and extremal charged black holes !

A. Kitaev, 2015
J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

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Starting *both* from the theory of SYK model, or from a semi-classical quantization of the Einstein-Maxwell theory of extremal charged black holes, we obtain the same low T expansion of the grand potential at large N:

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S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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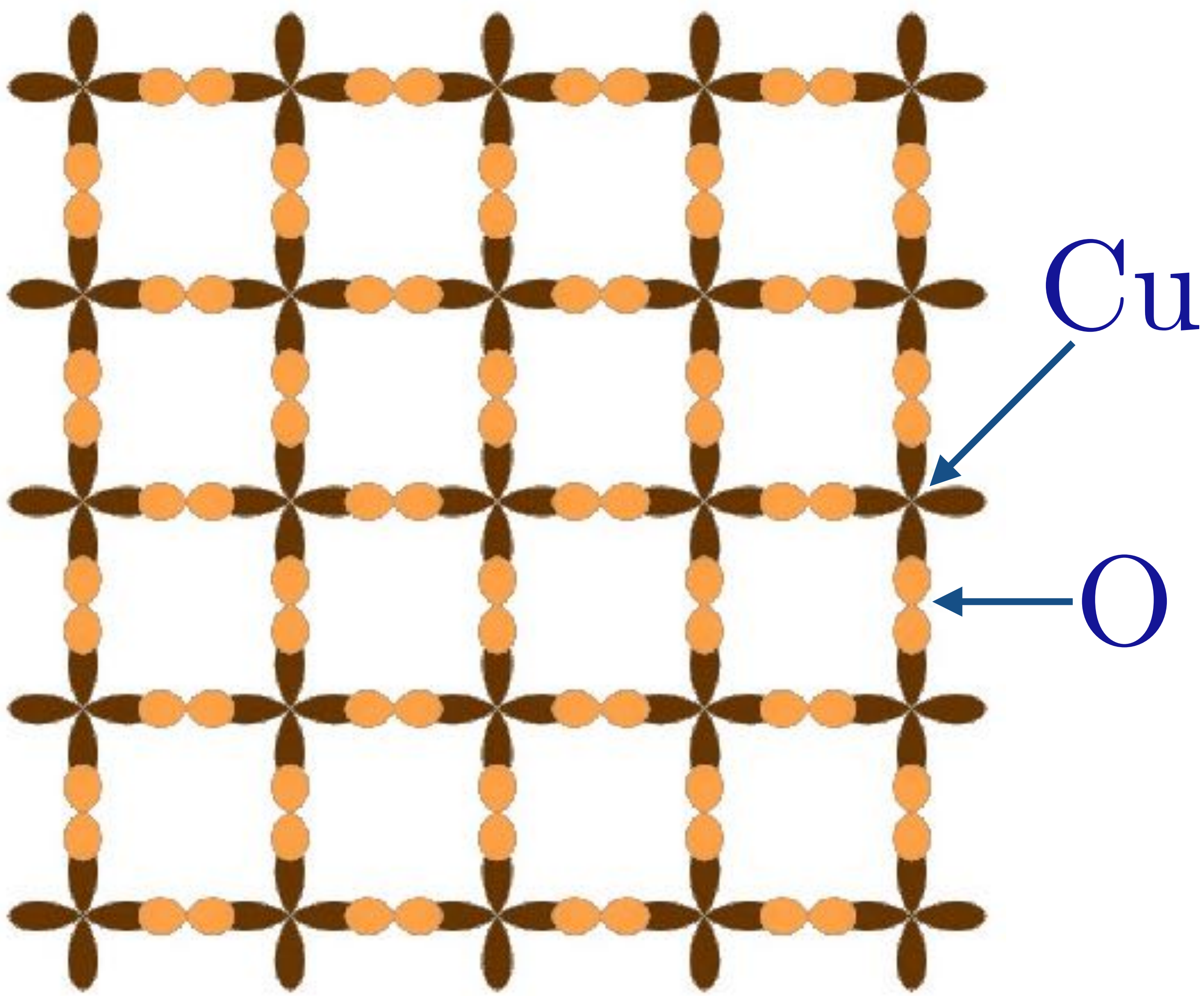
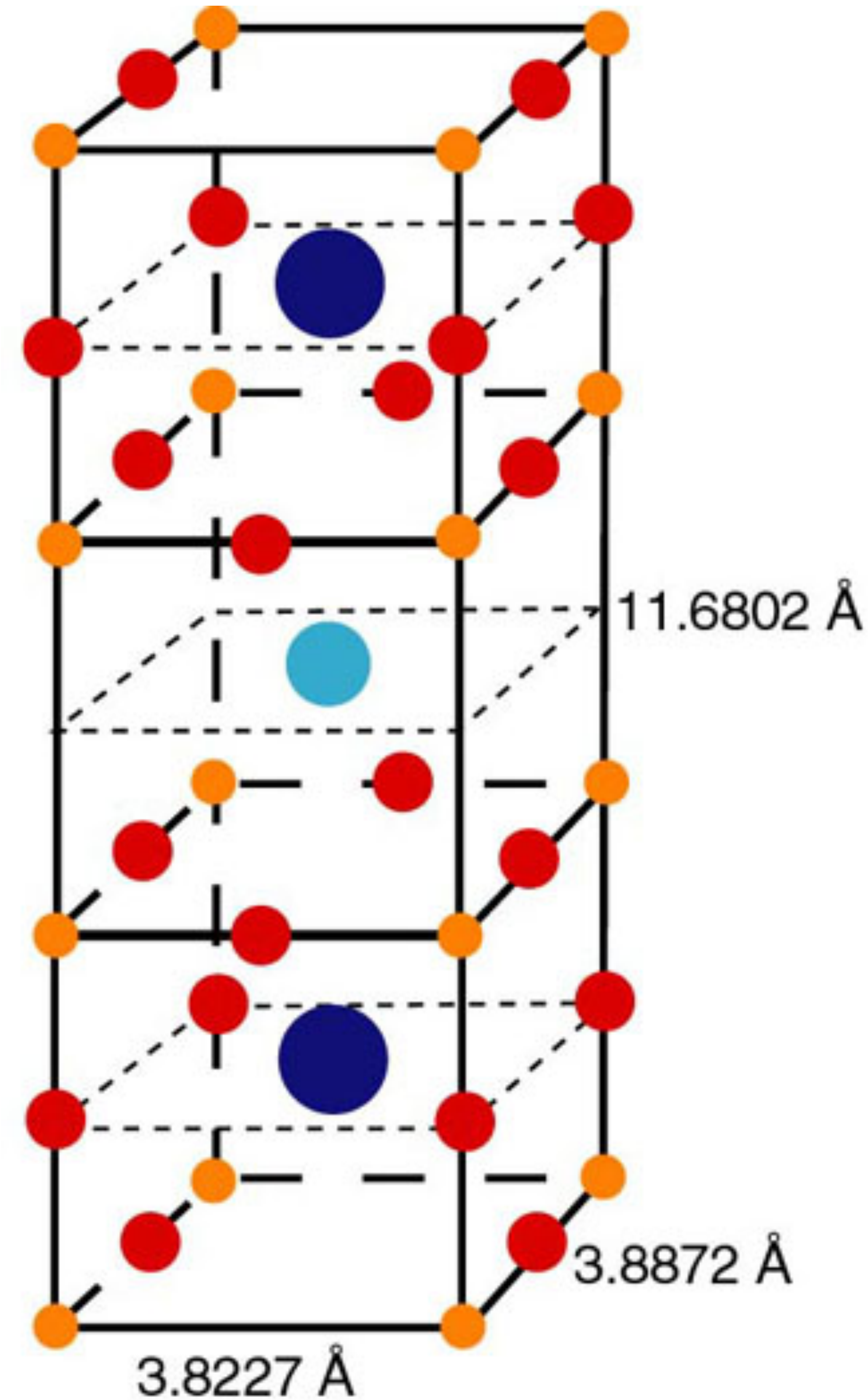
High temperature superconductors

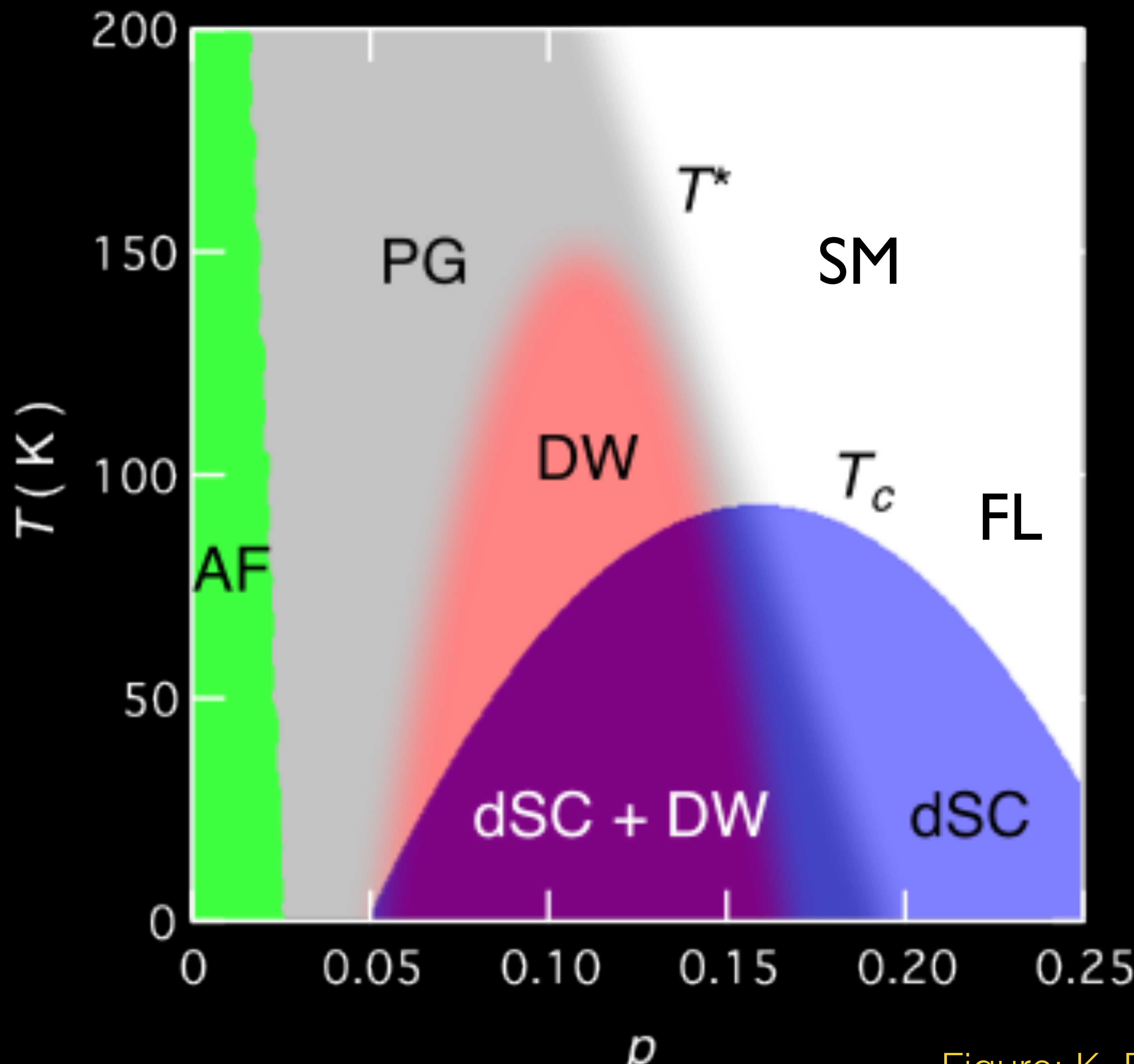


CuO_2 plane

Described by a Hubbard model on the Cu sites



$YBa_2Cu_3O_{6+x}$



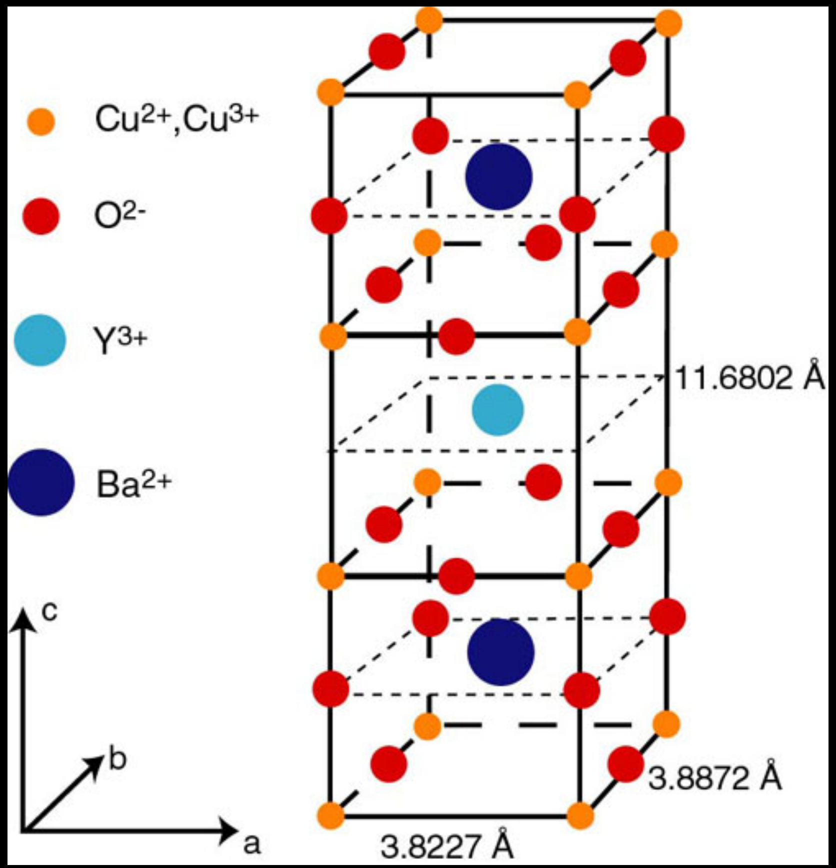


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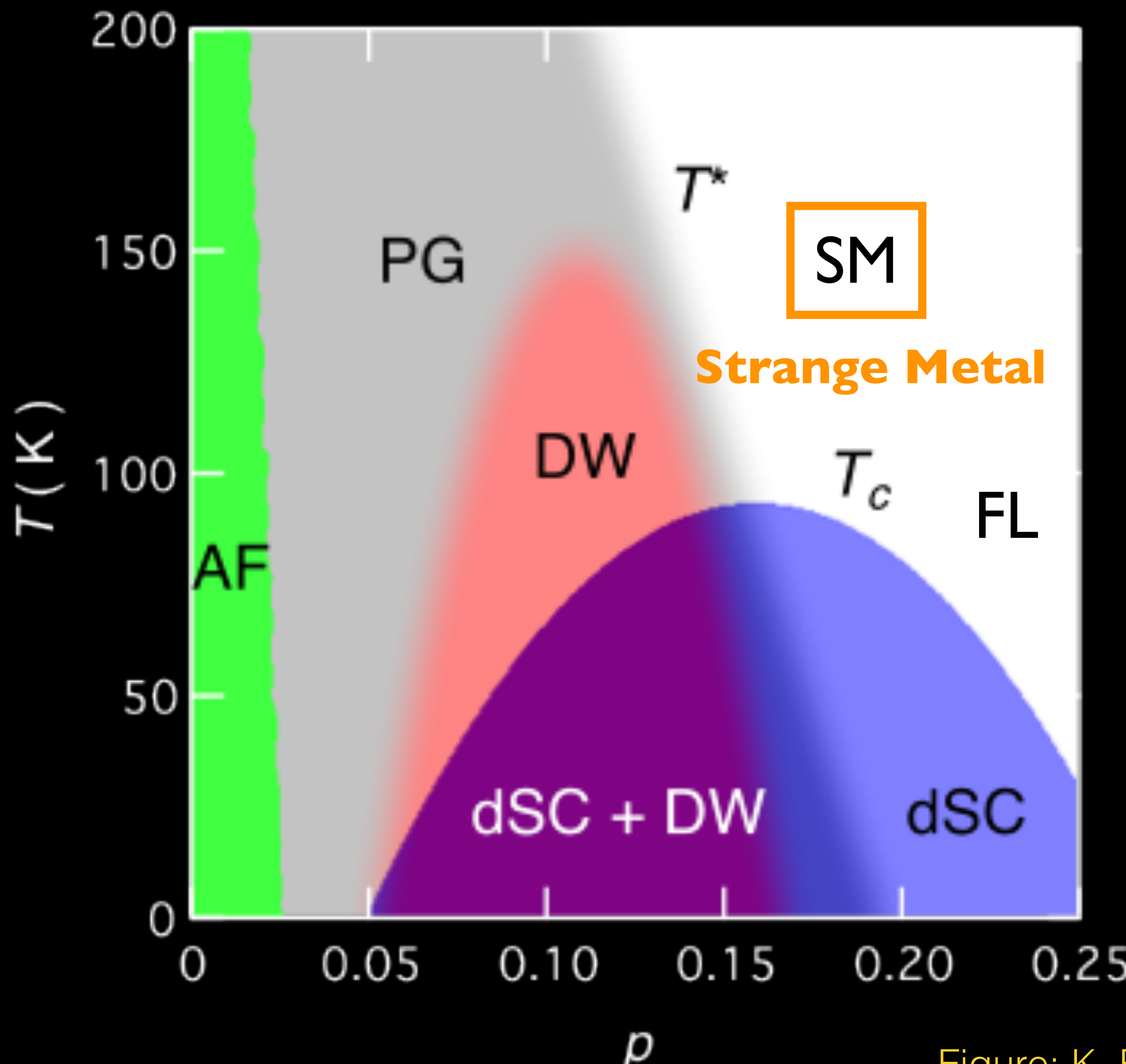
Figure: K. Fujita and J. C. Seamus Davis



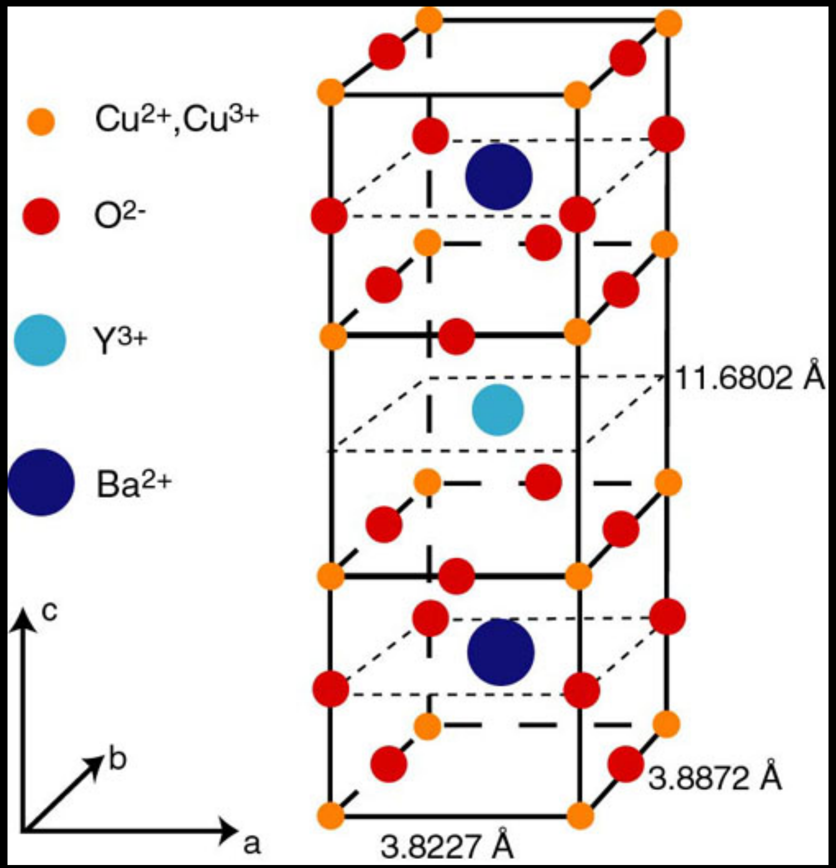


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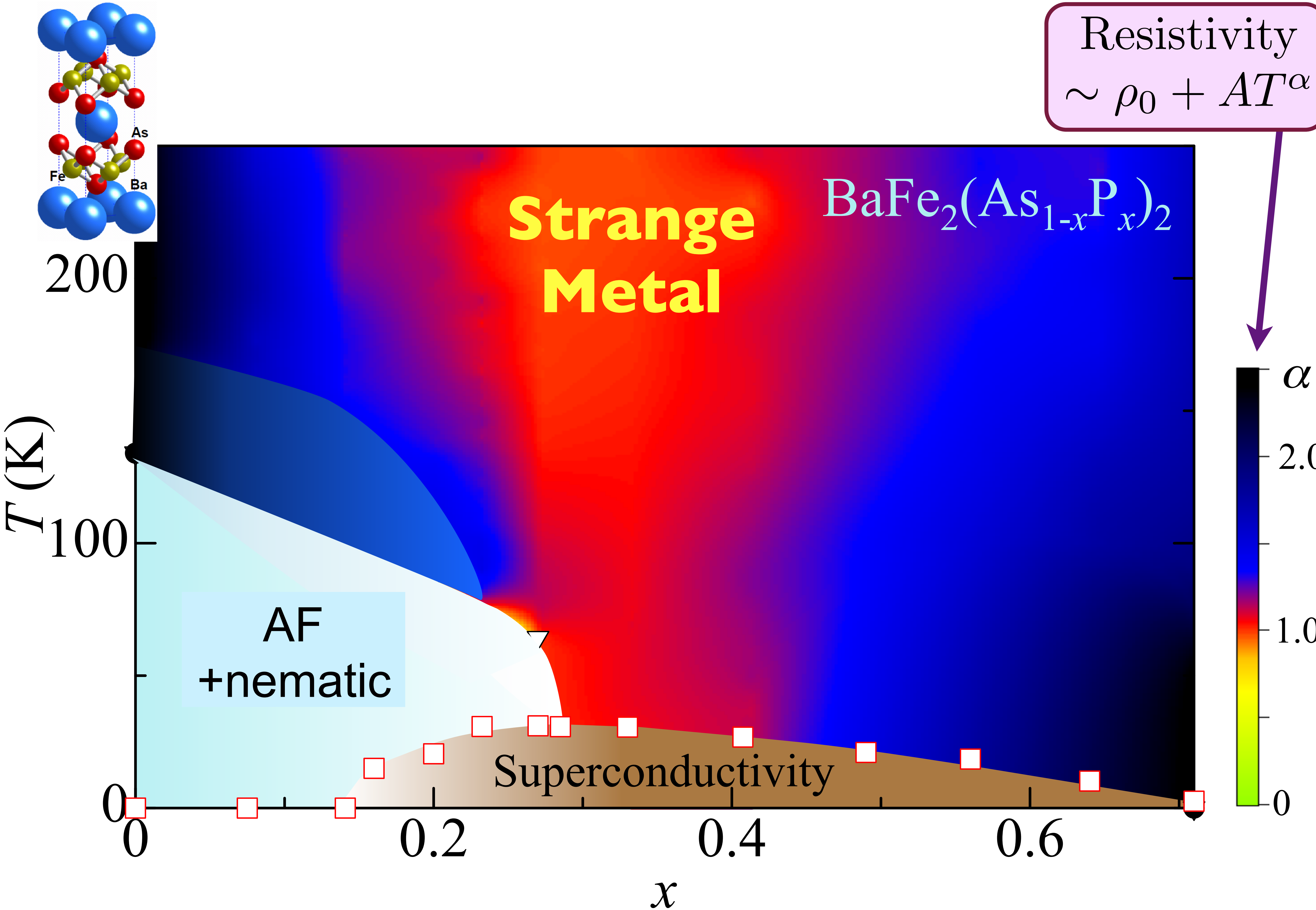
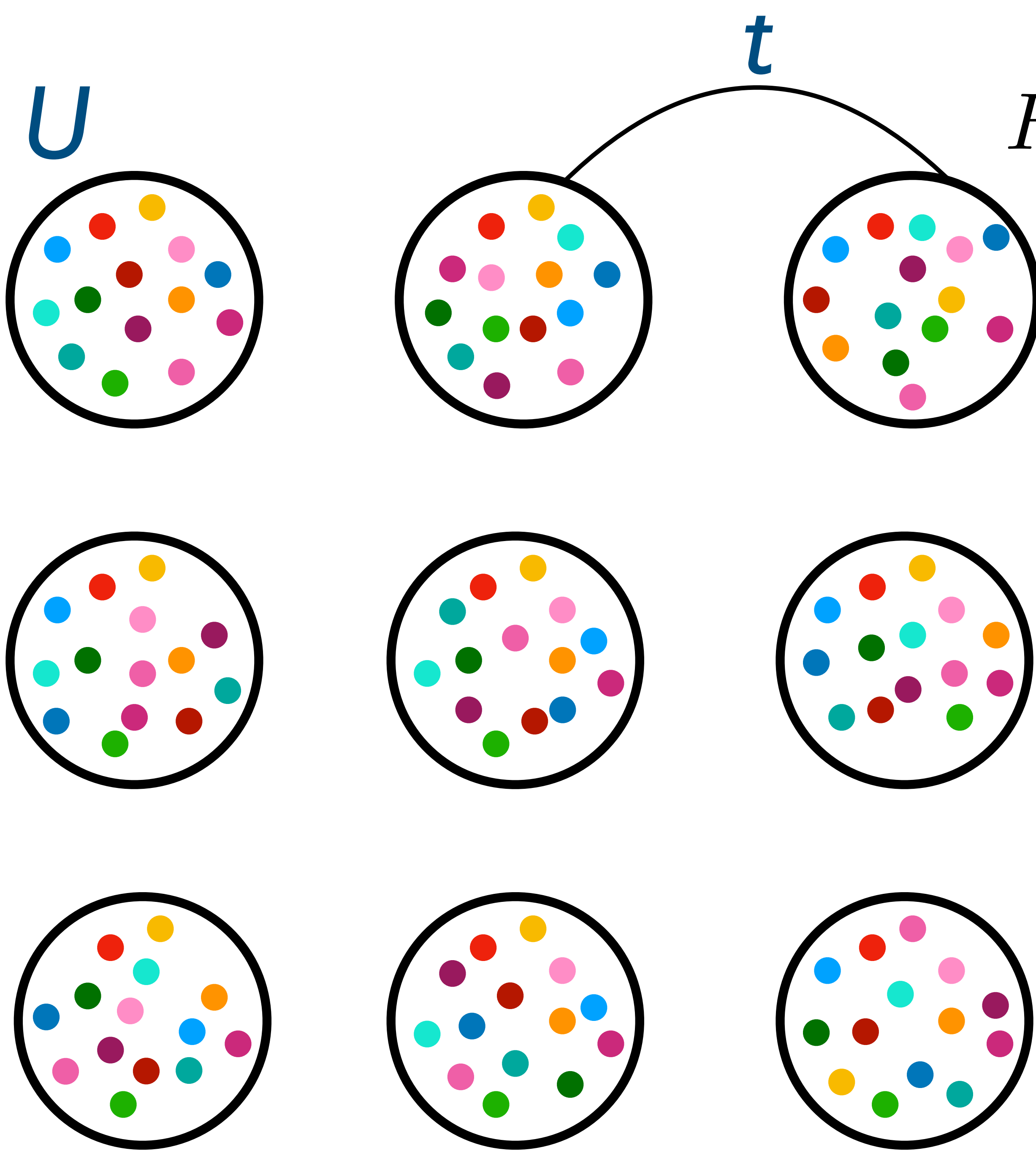
Figure: K. Fujita and J. C. Seamus Davis



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Coupled SYK Islands

SYK quantum islands of electrons with random hopping between them.



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx}$$

$$+ \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

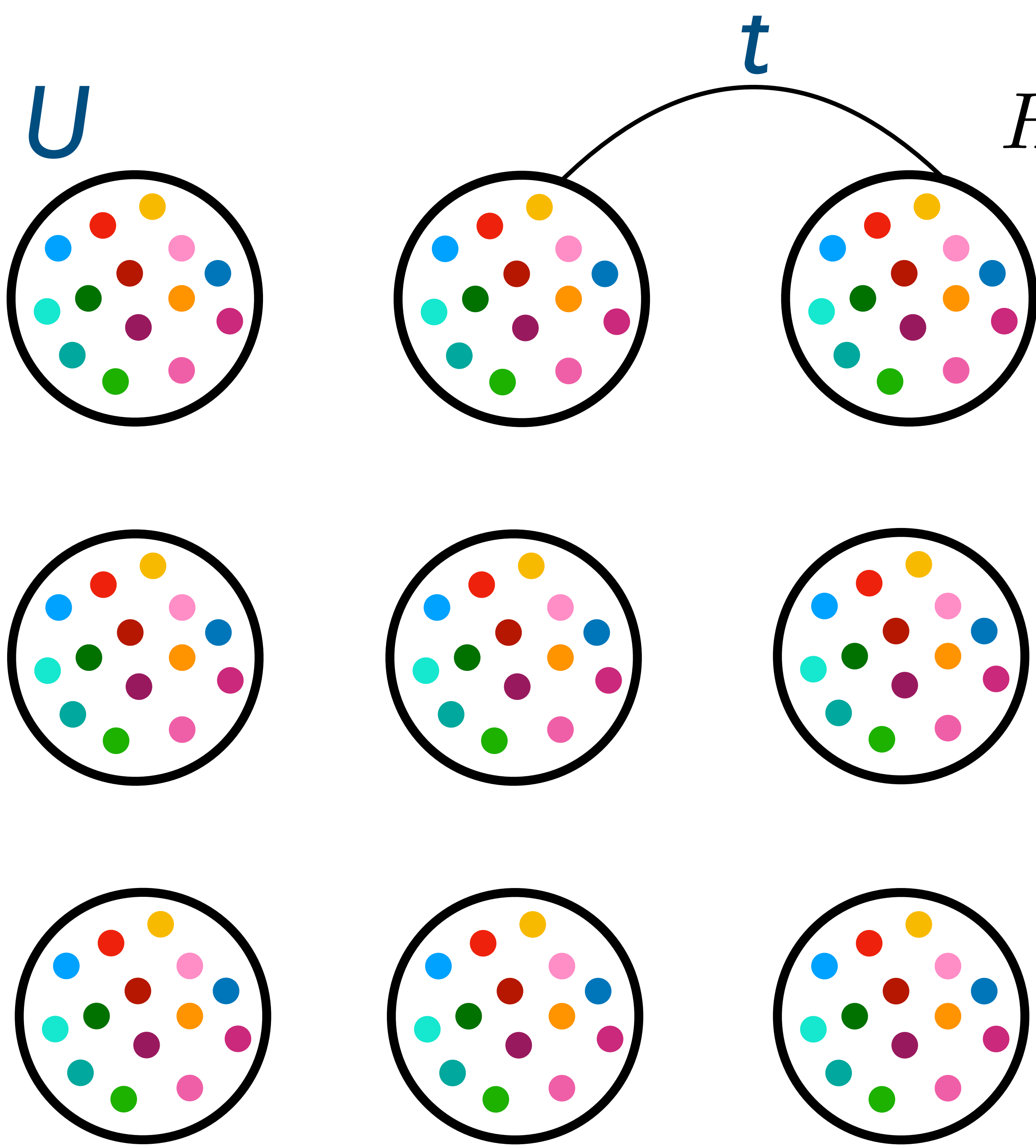
$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands

Can also use non-random *t*, and the same *U* on all "islands".



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$$+ \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij} \qquad c^\dagger_{i,x} c_{j,x'}$$

Pengfei Zhang, PRB **96**, 205138 (2017)

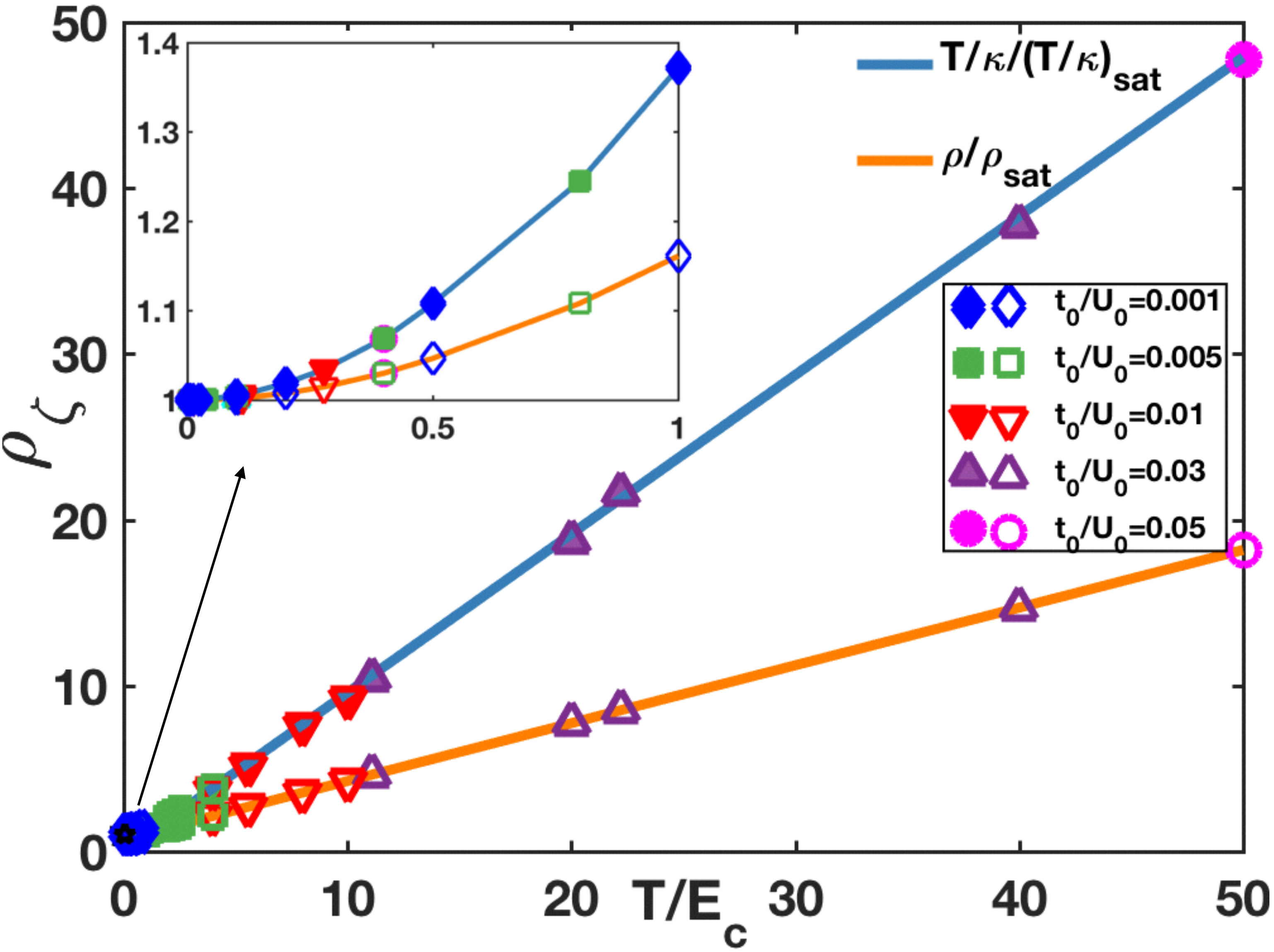
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands

Low 'coherence' scale

$$E_c \sim \frac{t_0^2}{U}$$



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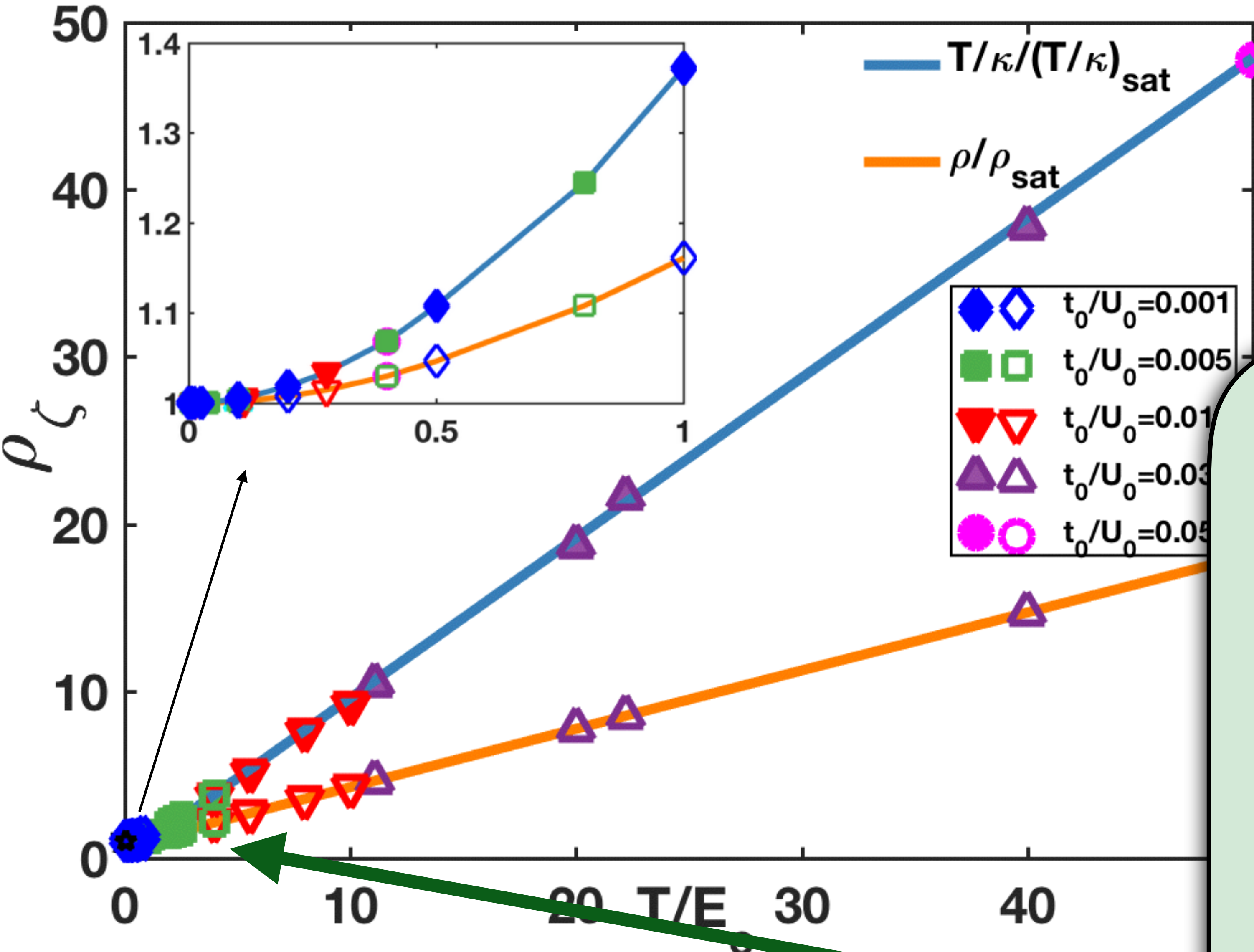

For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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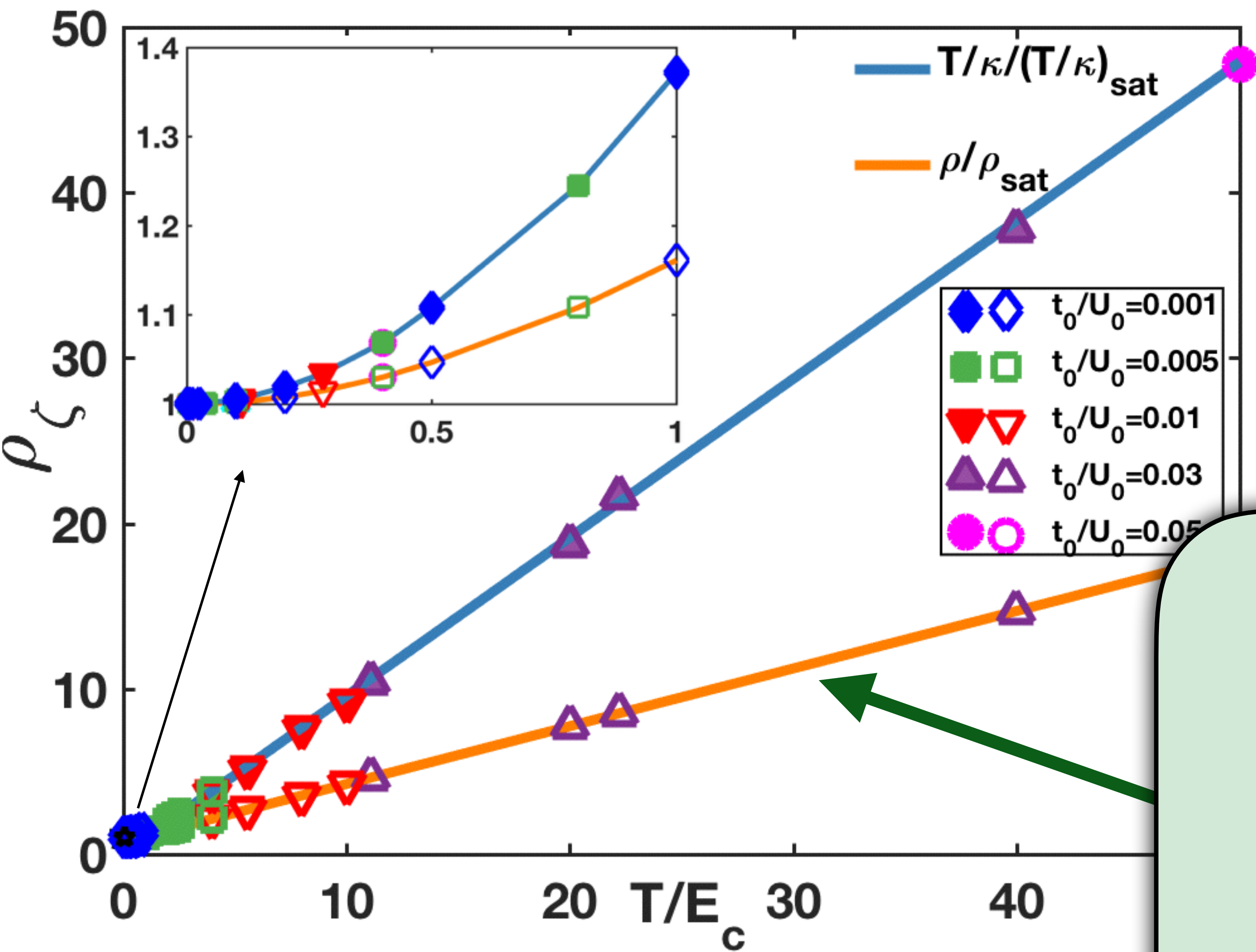


For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right) \quad , \quad s = s_0$$

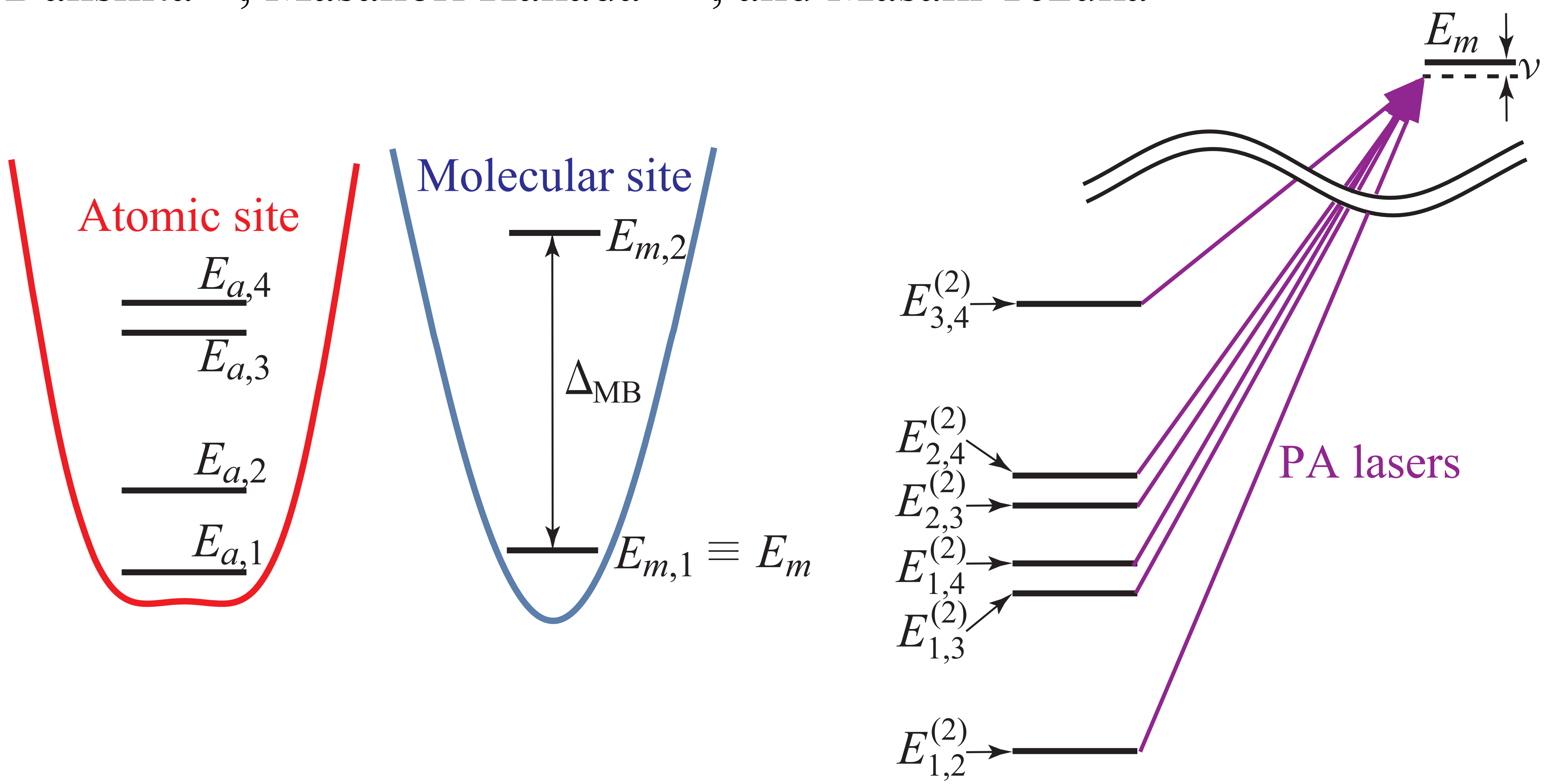
Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Other experimental realizations

Creating and probing the Sachdev–Ye–Kitaev model with ultracold gases: Towards experimental studies of quantum gravity

Ippei Danshita[1,*], Masanori Hanada[1,2,3], and Masaki Tezuka[4]



Prog. Theor. Exp. Phys. **2017**, 083I01

Other experimental realizations

PRL **119,** 040501 (2017) PHYSICAL REVIEW LETTERS

Digital Quantum Simulation of Minimal AdS/CFT

L. García-Álvarez,[1] I. L. Egusquiza,[2] L. Lamata,[1] A. del Campo,[3] J. Sonner,[4] and E. Solano[1,5]

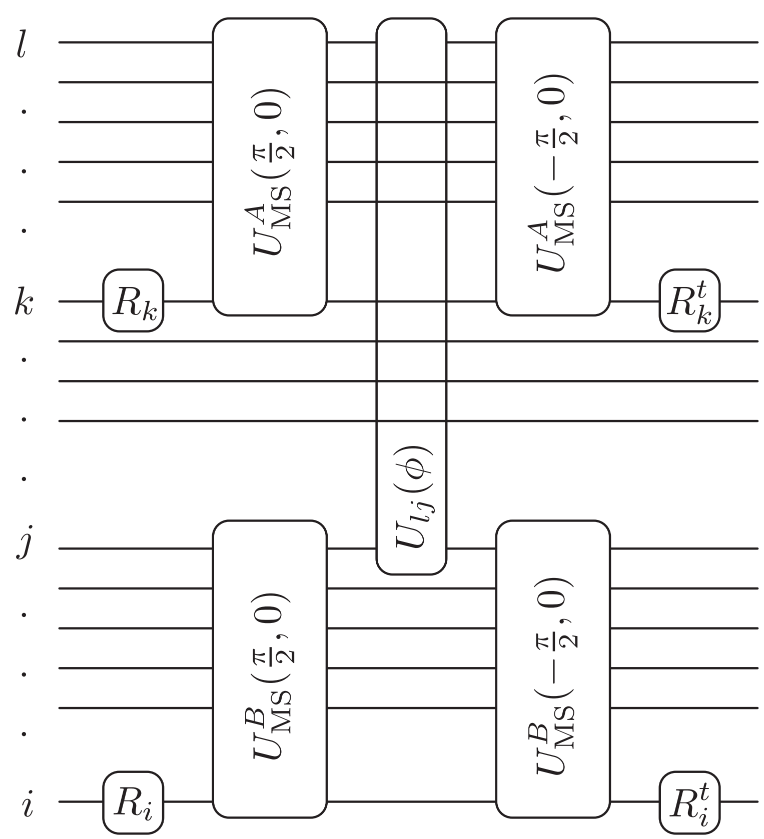


FIG. 1. Engineering many-body interactions in trapped-ion qubits. Operation sequence of single-qubit and multiqubit gates, inside a Trotter step, acting on trapped-ion qubits to generate a generic interaction term (13). The single-qubit rotations R_i and R_k act on qubits i and k, respectively, and the phase ϕ of the two-qubit entangling gate, $U_{ij}(\phi)$, must be chosen adequately in order to produce the desired combination of $\alpha_i \alpha_j \alpha_k \alpha_l$ in the interaction.

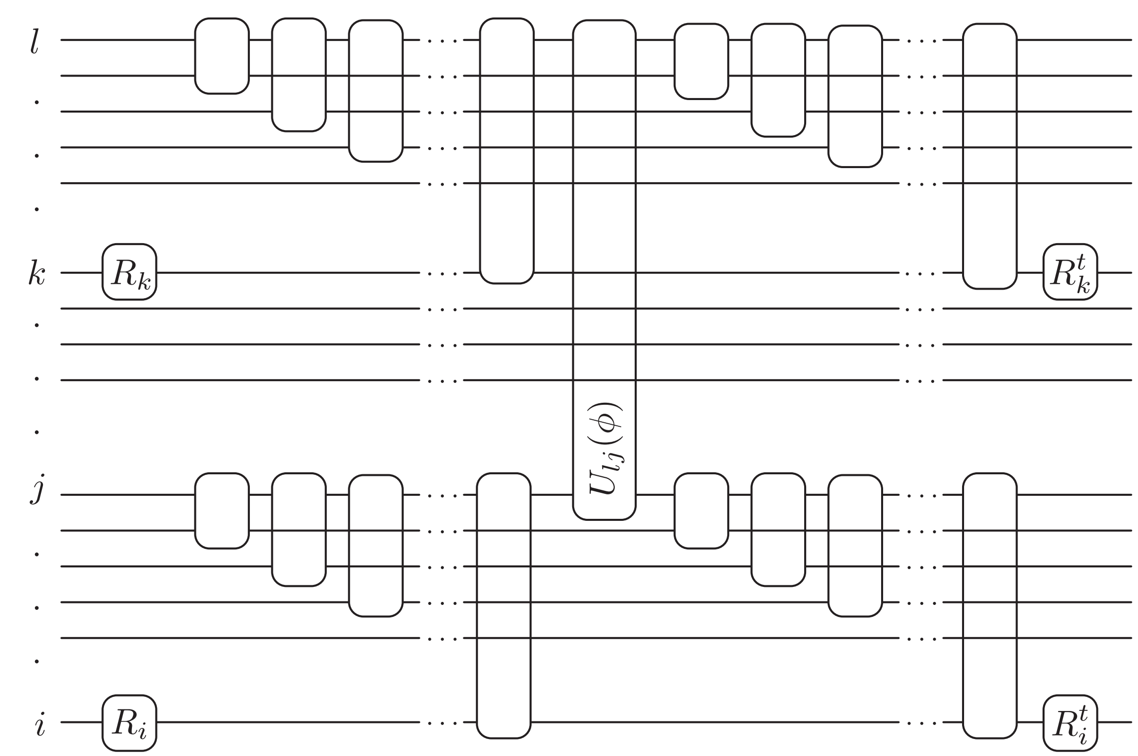
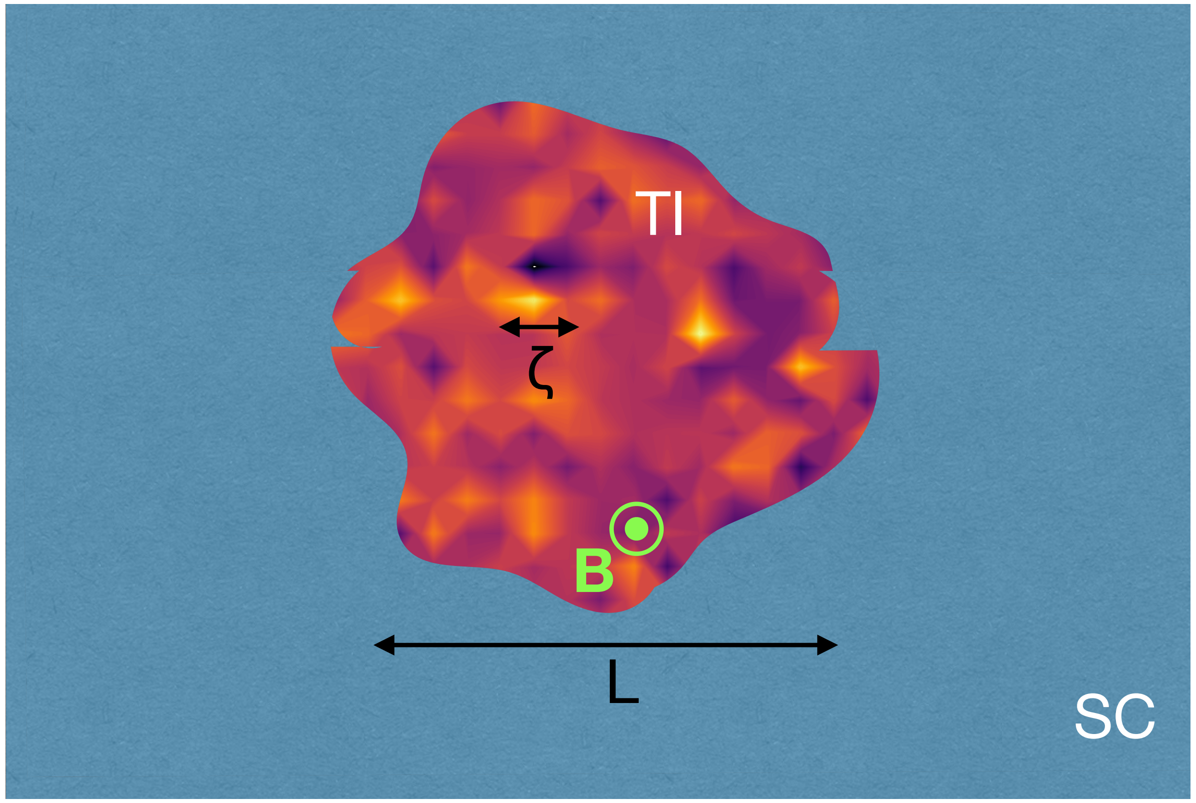


FIG. 2. Engineering many-body interactions in superconducting circuits. We consider sets of two-qubit gates and their inverses, which involve qubits l and j with the rest of the qubits included in the σ^z strings of the interaction. Thus, a set of n two-qubit gates takes on the role of the Mølmer-Sø rensen gate in the trapped-ion protocol. Note that two-qubit gates between distant qubits may be performed by a set of SWAP gates and an entangling gate between nearest-neighbor qubits.

Other experimental realizations



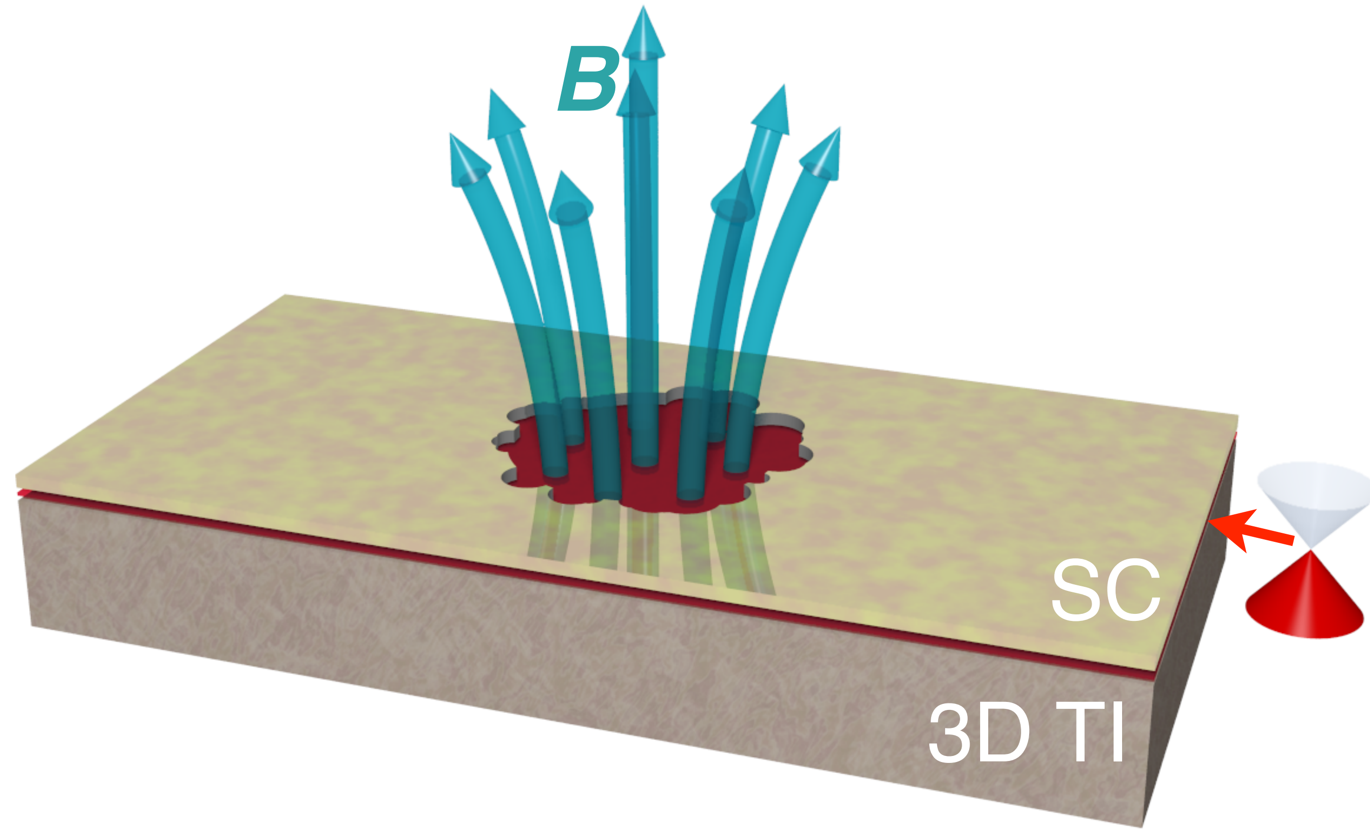
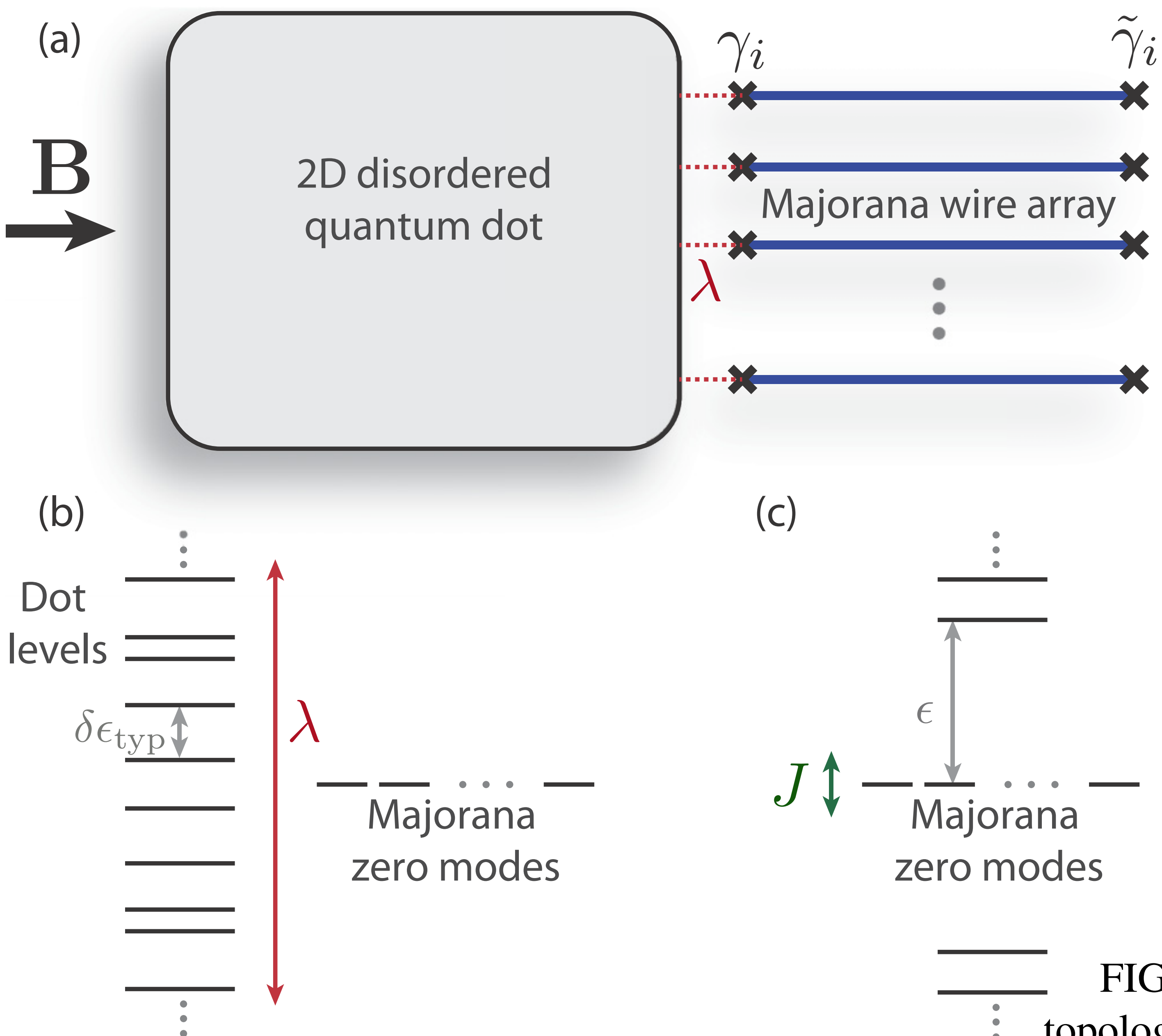


FIG. 2. Schematic depiction of the device proposed in Ref. [31]: a 3D topological insulator (TI) covered by a thin layer of an ordinary superconductor (SC). A hole with an irregular shape is fabricated in the SC layer and threaded with magnetic flux Φ. The colored region represents the hole of linear size L. The colorscale inside the hole illustrates the typical charge density distribution associated with a pair of MZMs with the characteristic disorder length scale ζ.

D. I. Pikulin and M. Franz, Phys. Rev. X **7**, 031006 (2017)

Other experimental realizations



Approximating the Sachdev-Ye-Kitaev model with Majorana wires

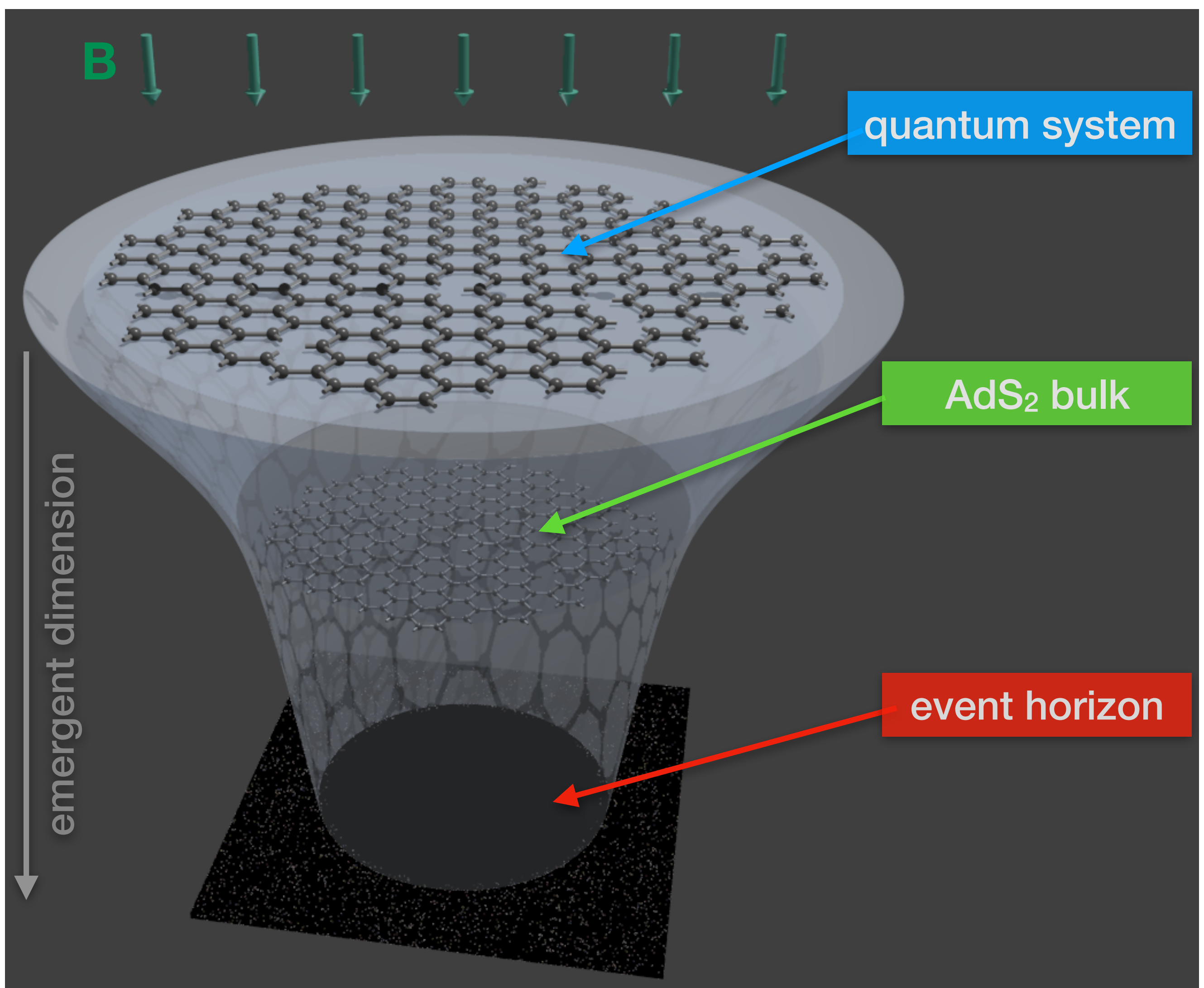
Aaron Chew, Andrew Essin, and Jason Alicea, Phys. Rev. B **96**, 121119 (2017)

FIG. 1. (a) Device that approximates the SYK model using topological wires interfaced with a 2D quantum dot. The dot mediates disorder and four-fermion interactions among Majorana modes $\gamma_{1,\ldots,N}$ inherited from the wires, while Majorana bilinears are suppressed by an approximate time-reversal symmetry. (b) Energy levels pre hybridization. The dot-Majorana hybridization energy λ is large compared to $N\delta\epsilon_{\rm typ}$, where N is the number of Majorana modes and $\delta\epsilon_{\rm typ}$ is the typical dot level spacing; this maximizes leakage into the dot. (c) Energy levels post hybridization. The N absorbed Majorana modes enhance the energy ϵ to the next excited dot state via level repulsion; four-Majorana interactions occur on a scale $J < \epsilon$.

Other experimental realizations

Quantum holography in a graphene flake with an irregular boundary

Electrons in clean macroscopic samples of graphene exhibit an astonishing variety of quantum phases when strong perpendicular magnetic field is applied. These include integer and fractional quantum Hall states as well as symmetry broken phases and quantum Hall ferromagnetism. Here we show that mesoscopic graphene flakes in the regime of strong disorder and magnetic field can exhibit another remarkable quantum phase described by holographic duality to an extremal black hole in two dimensional anti-de Sitter space. This phase of matter can be characterized as a maximally chaotic non-Fermi liquid since it is described by a complex fermion version of the Sachdev-Ye-Kitaev model known to possess these remarkable properties.



A. Chen, R. Ilan, F. De Juan, D.I. Pikulin, and M. Franz, arXiv:1802.00802

Quantum matter without quasiparticles

- Rapid local thermal equilibration (of fermion correlators) in a 'Planckian' time

$$\tau_{\rm eq} \sim \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

- Presence of quasiparticles should slow down thermalization, so *all* quantum systems obey

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

Absence of quasiparticles $\Leftrightarrow$ Fastest possible thermalization

Quantum matter without quasiparticles

- Planckian dynamics is realized in the 'solvable' SYK models

- Black holes thermalize in a time $\sim \hbar/(k_B T_H)$, where T_H is the Hawking temperature.

- A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, describes the quantum dynamics of

  - the SYK models
  - black holes with near-extremal AdS_2 horizons