Quantum phase transitions in condensed matter physics, with connections to string theory

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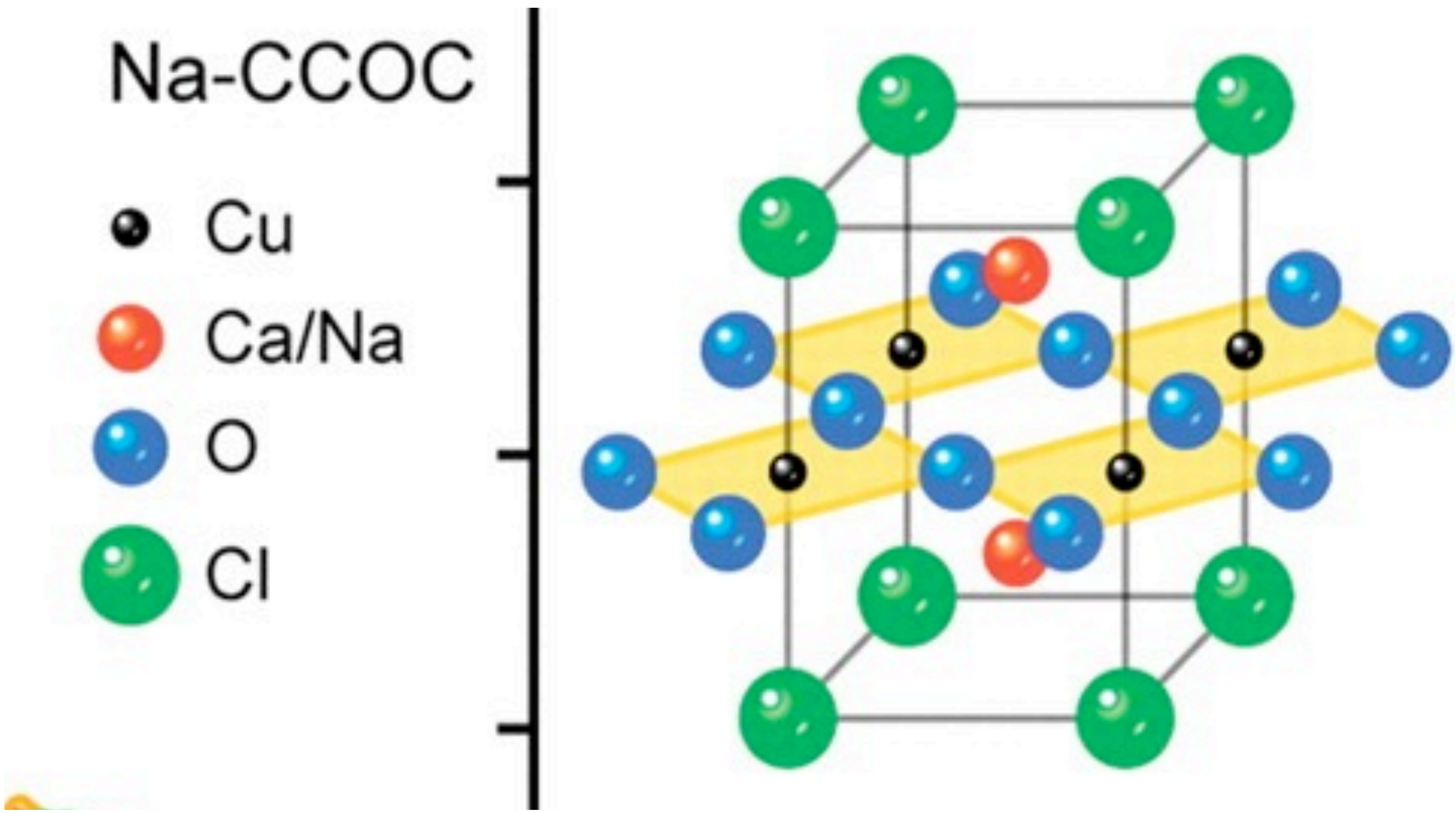
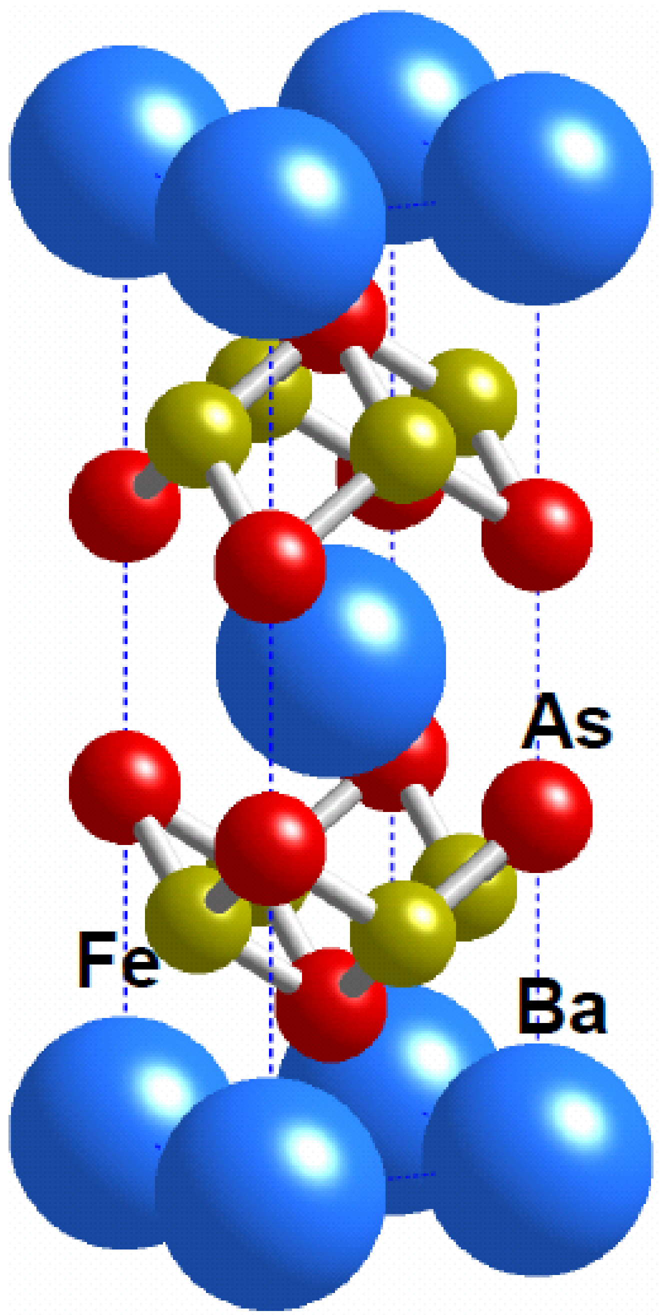
High temperature superconductors



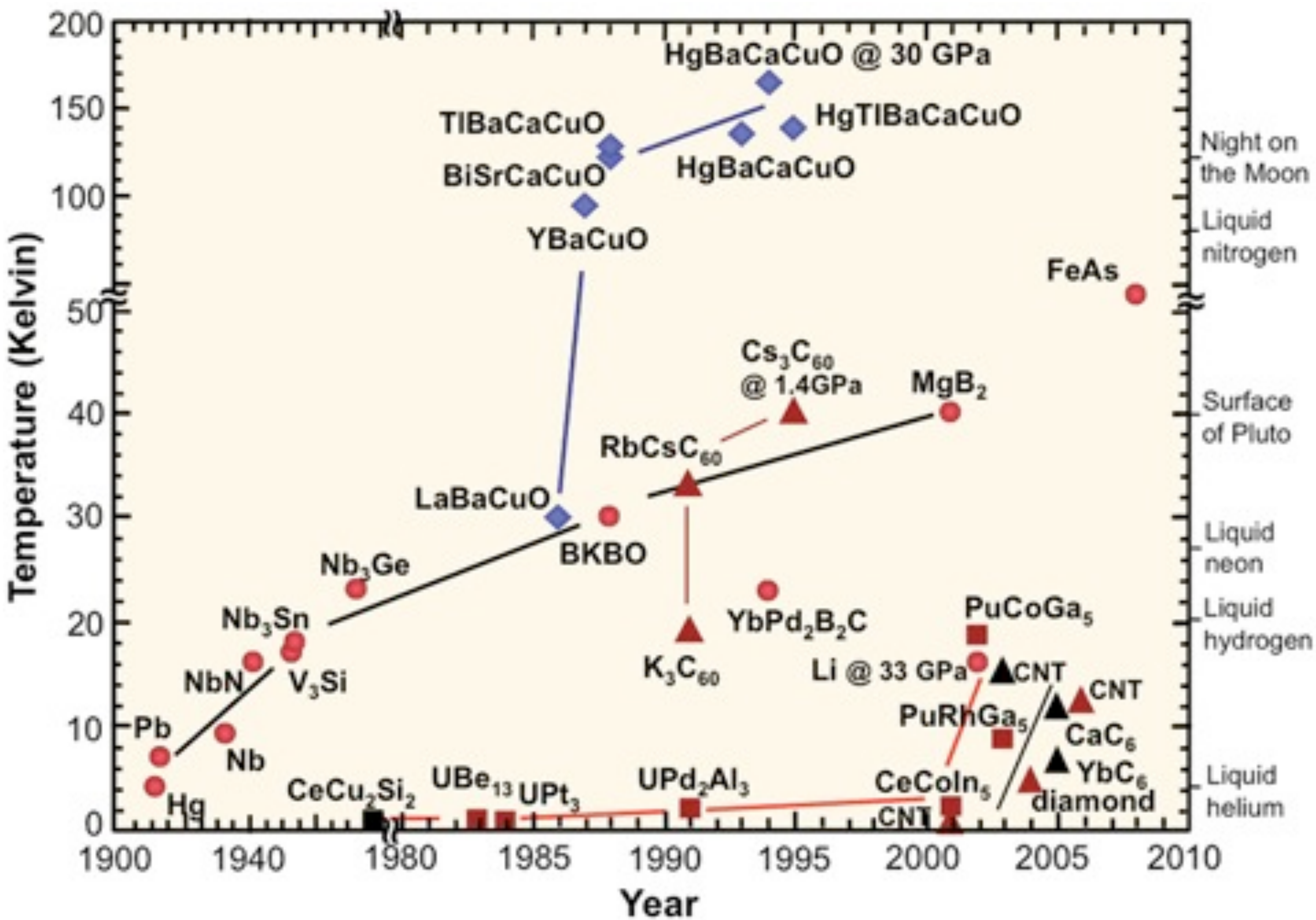
Cuprates

High temperature superconductors

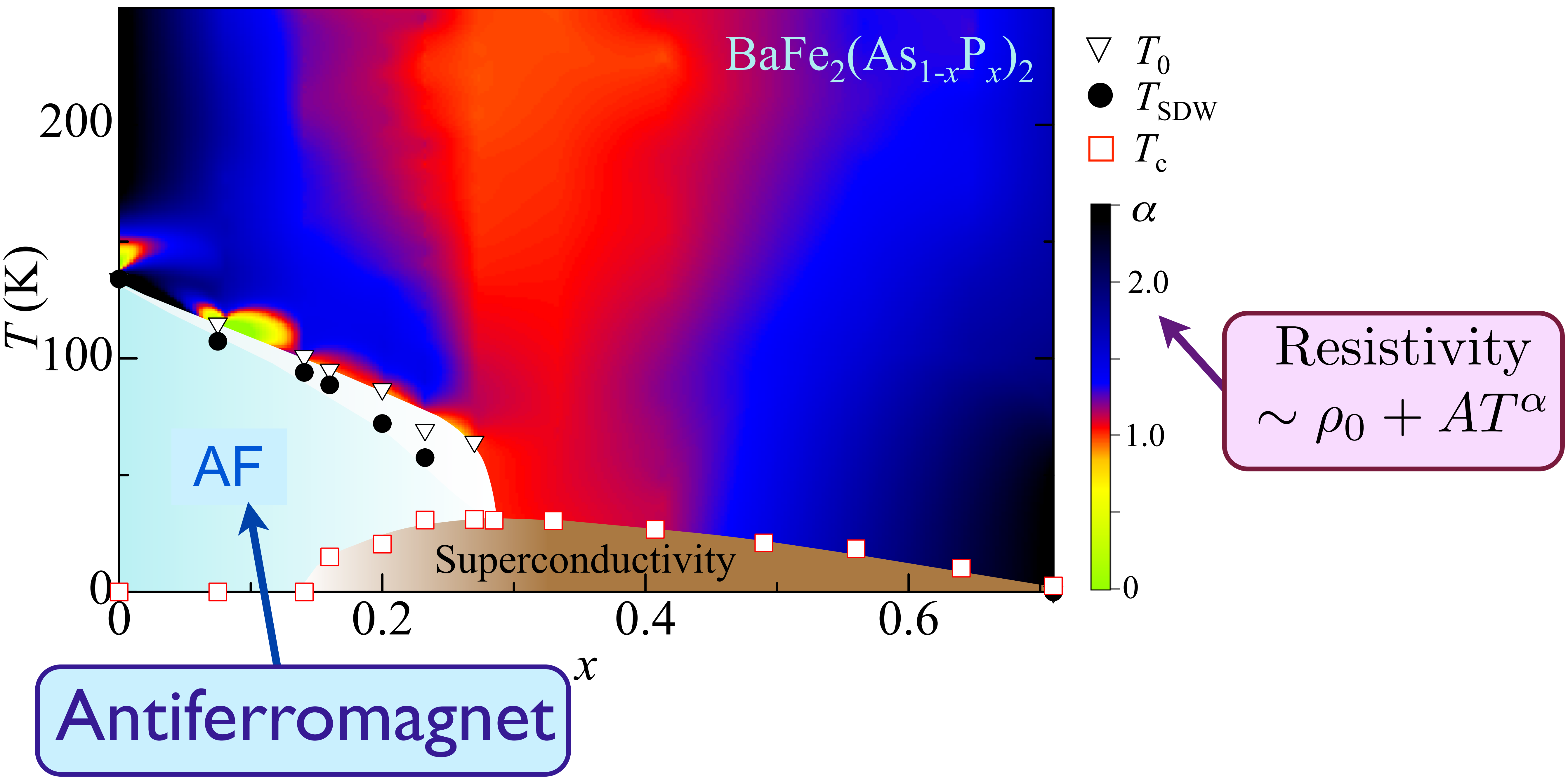


Pnictides

High temperature superconductors



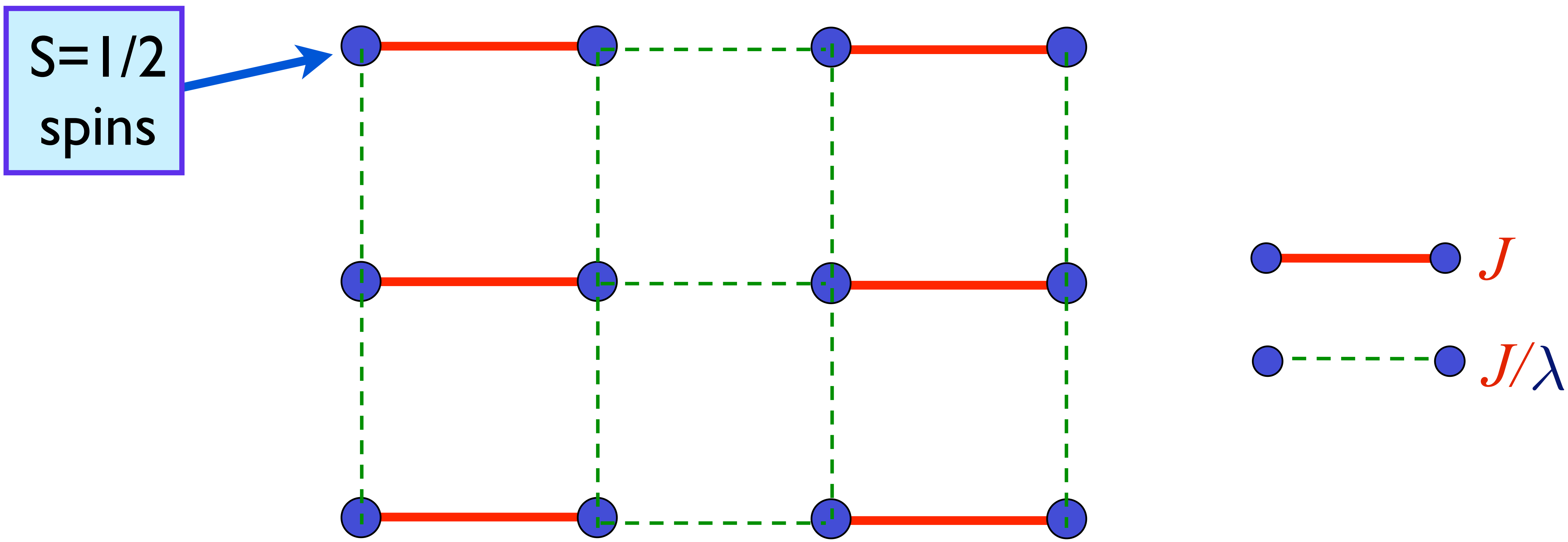
Temperature-doping phase diagram of the iron pnictides



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

Square lattice antiferromagnet

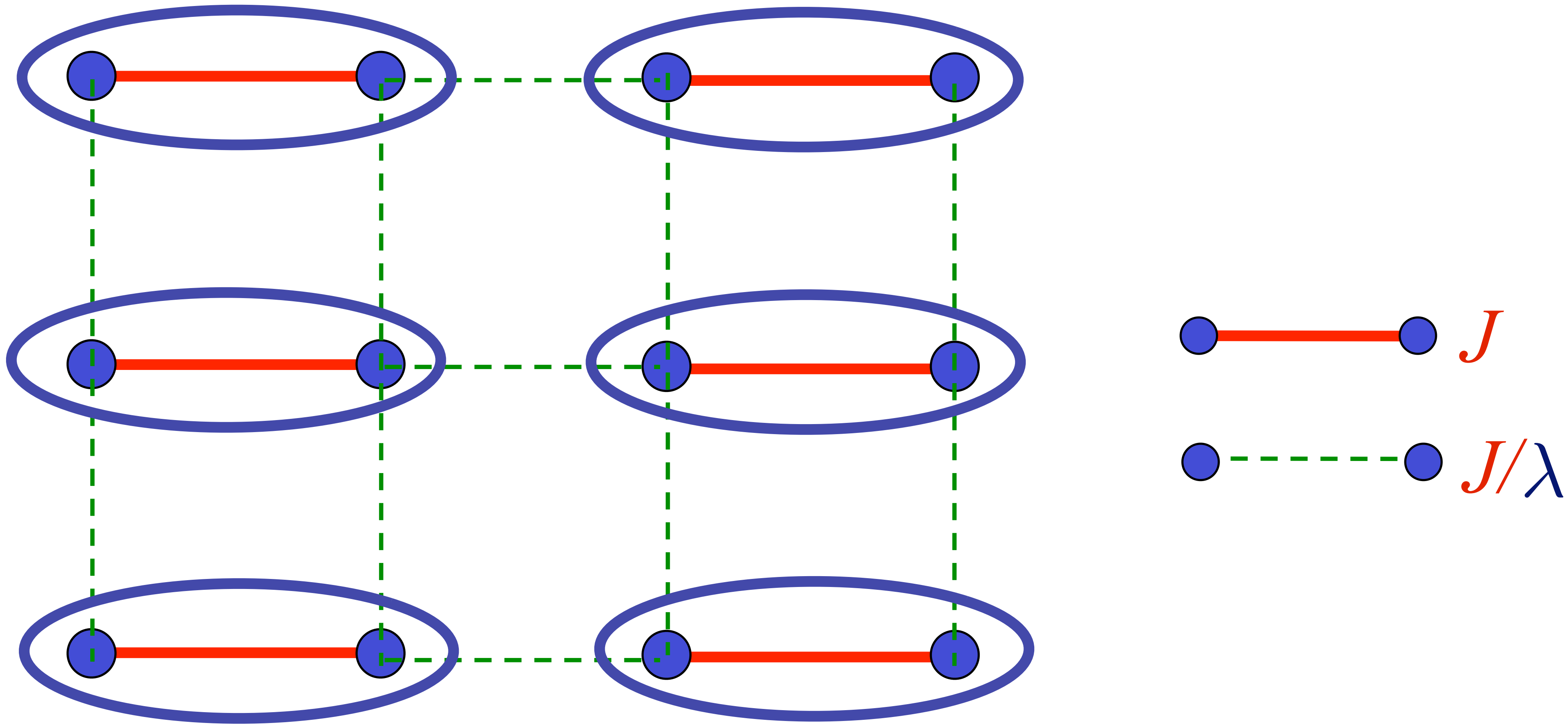
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

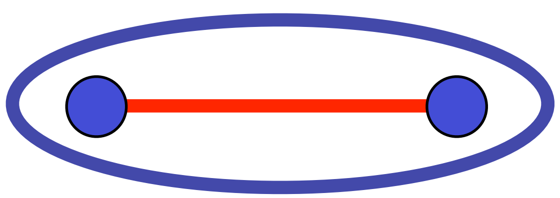


Examine ground state as a function of λ

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

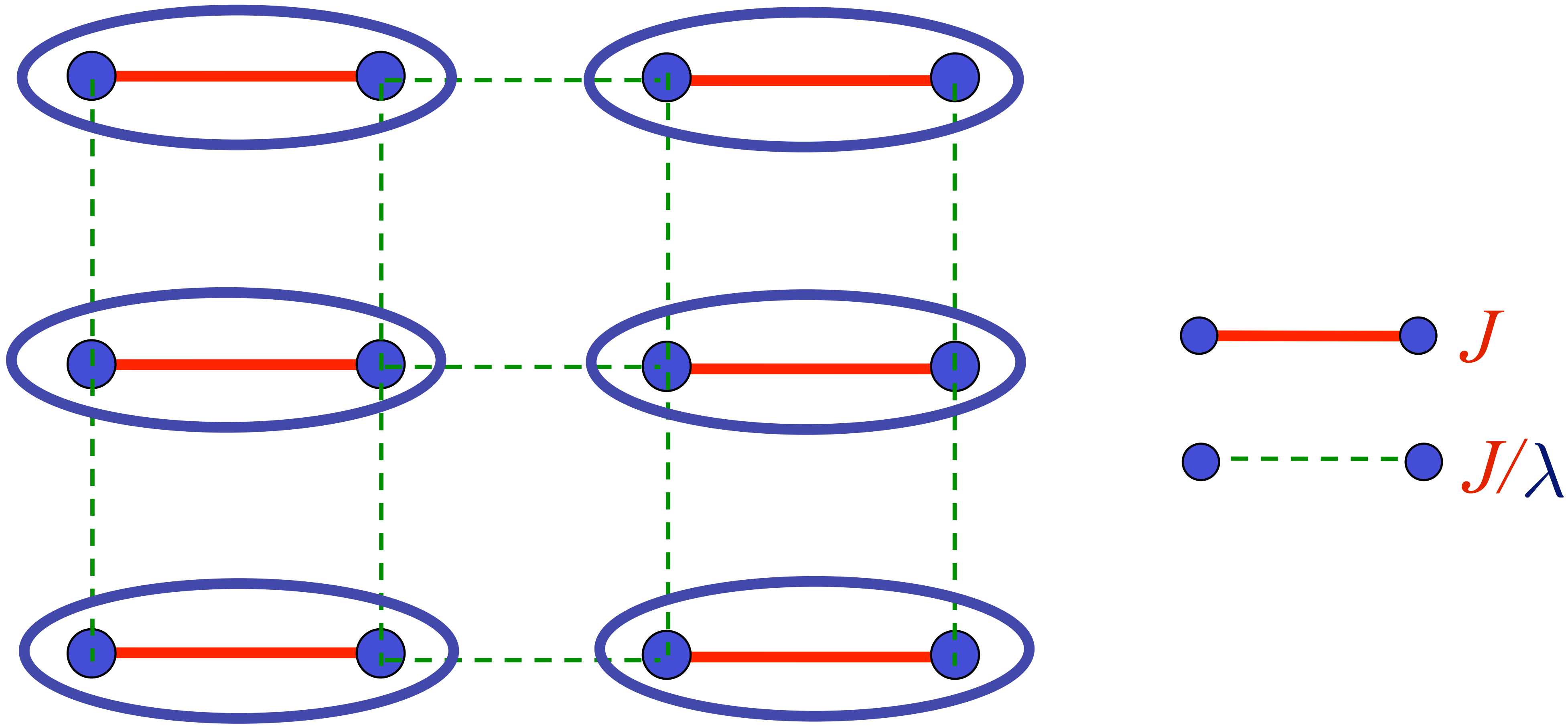


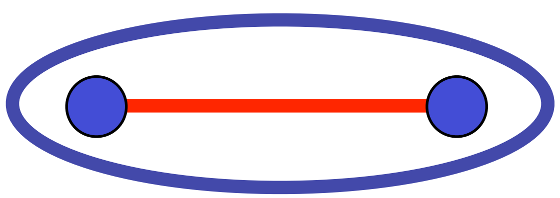
 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



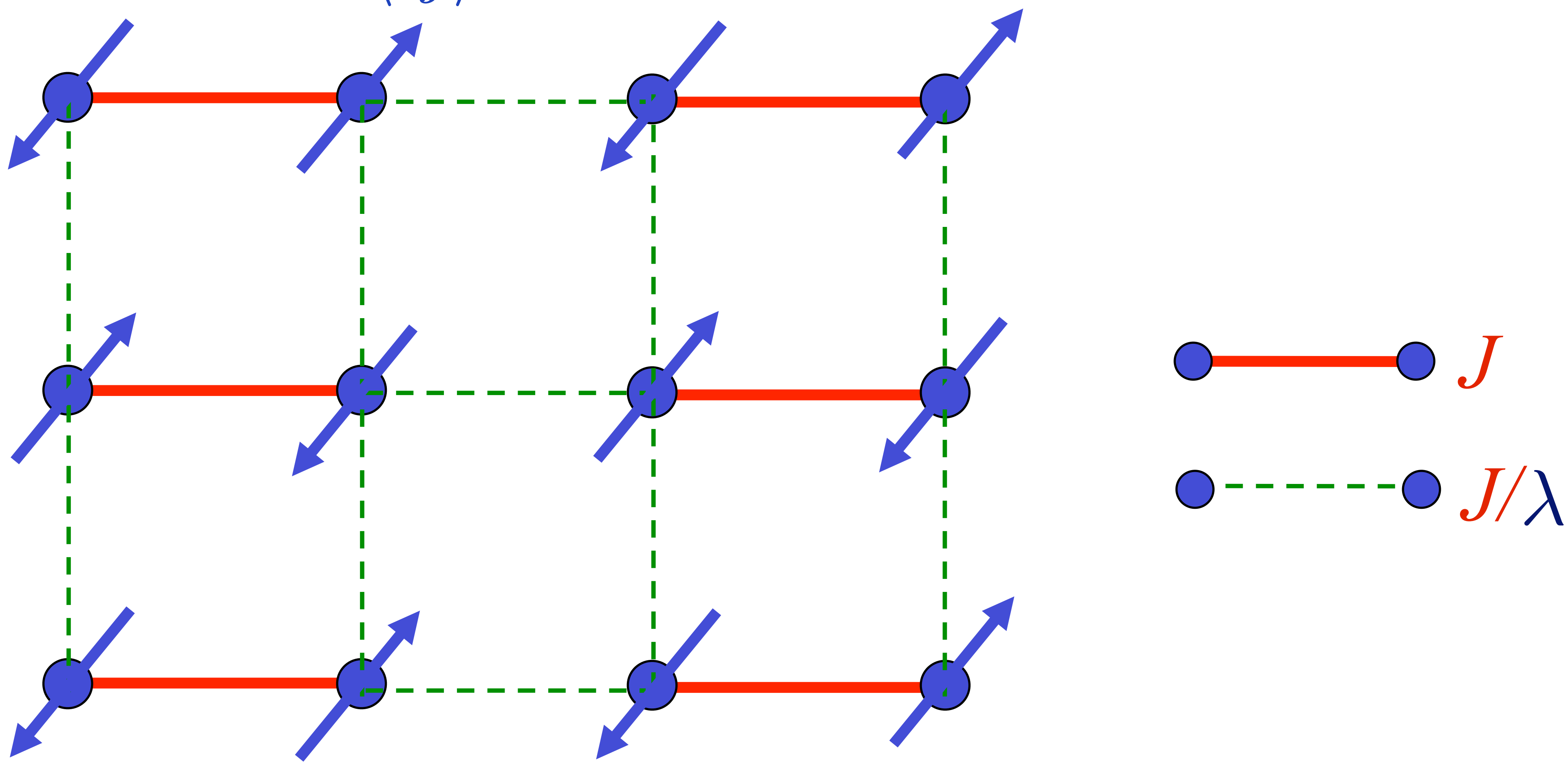


$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Square lattice antiferromagnet

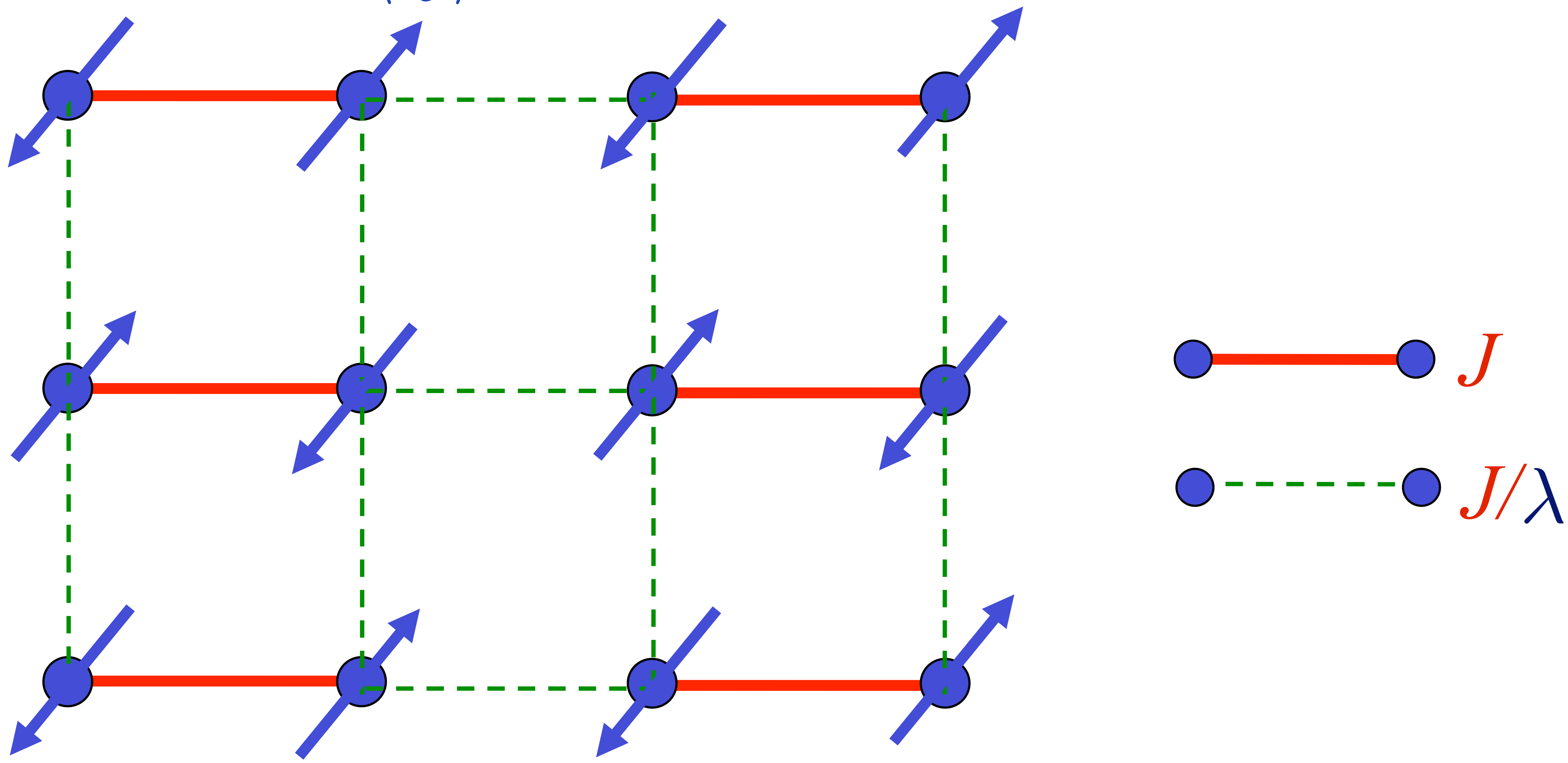
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

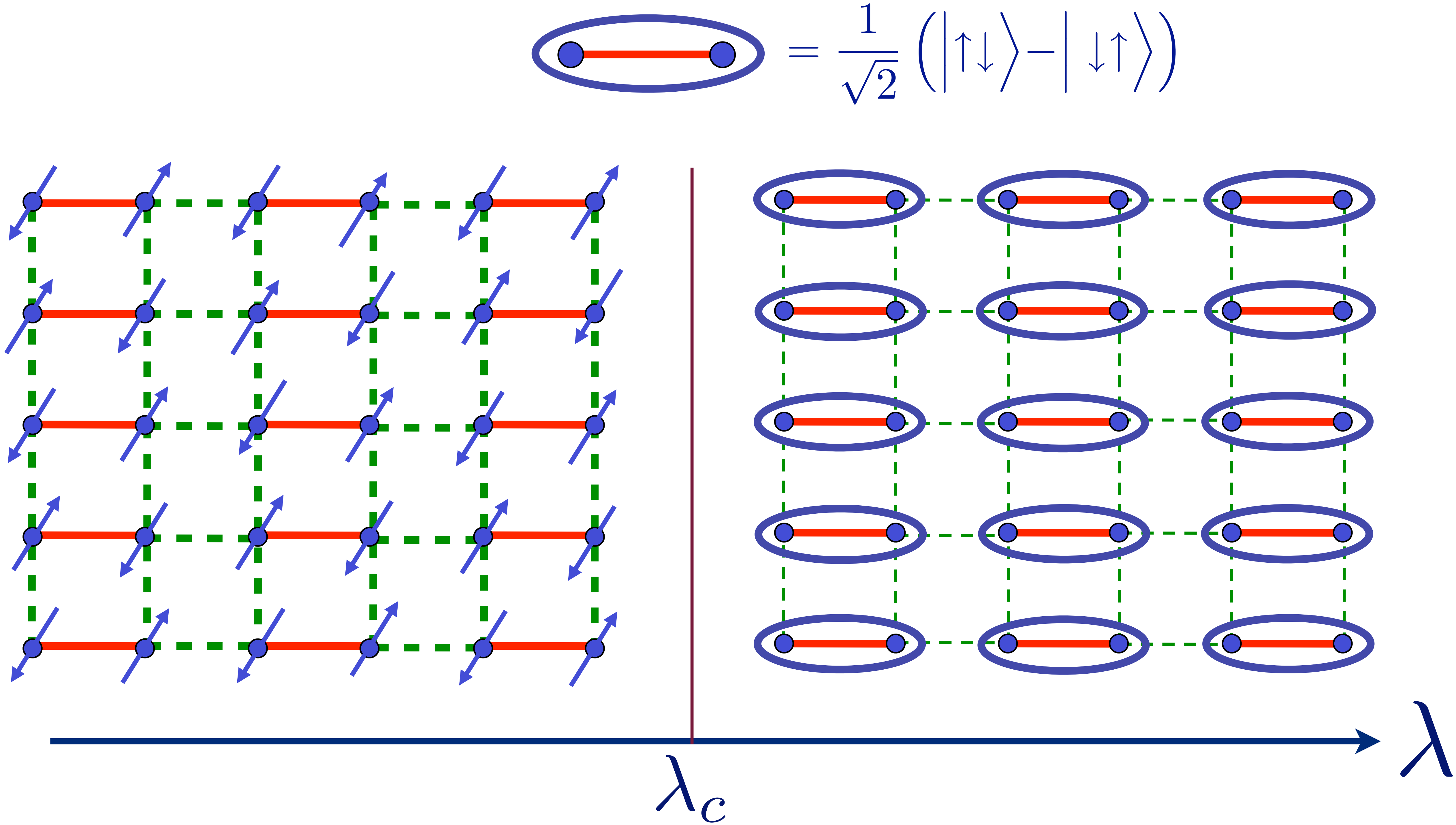
Square lattice antiferromagnet

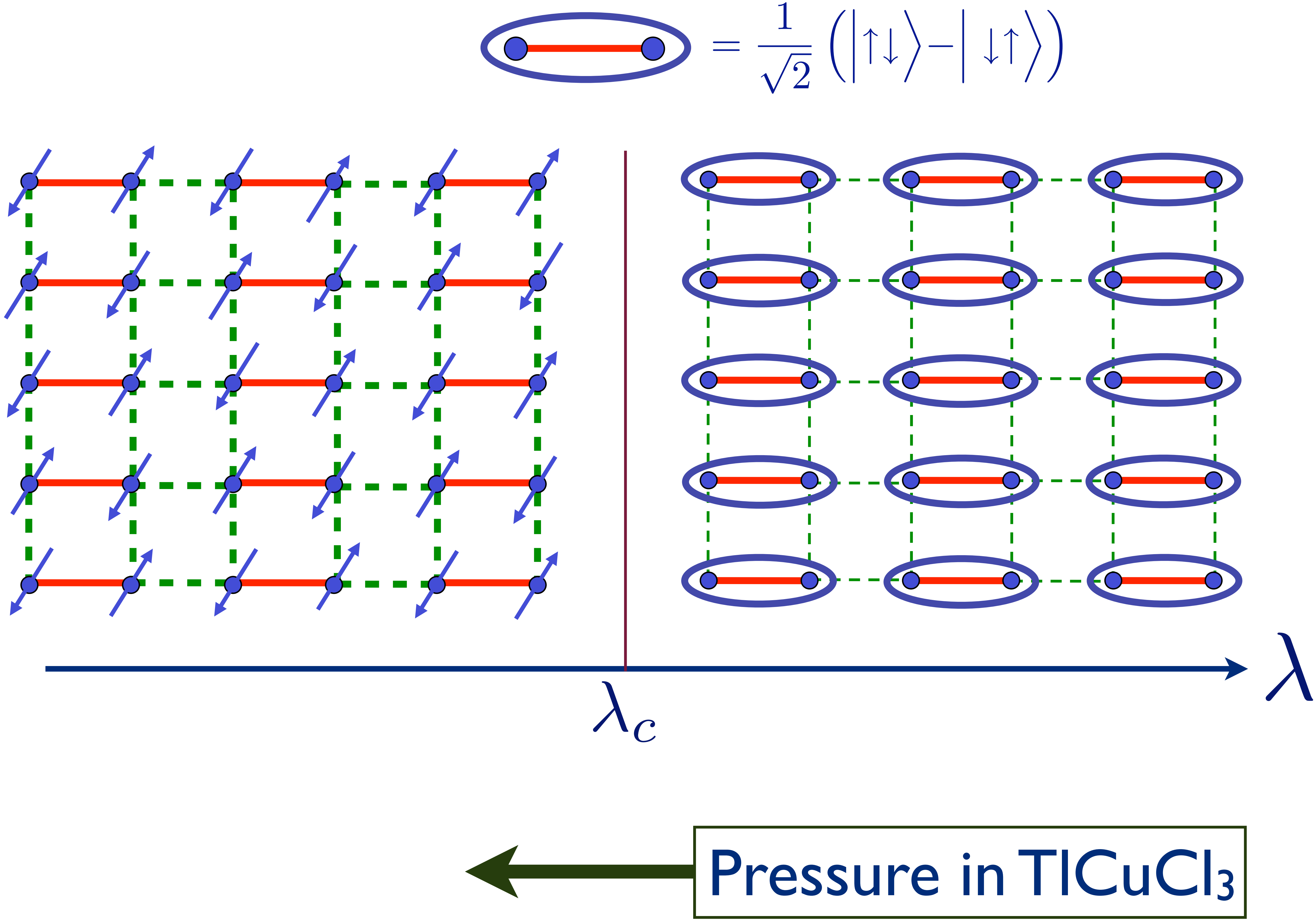
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

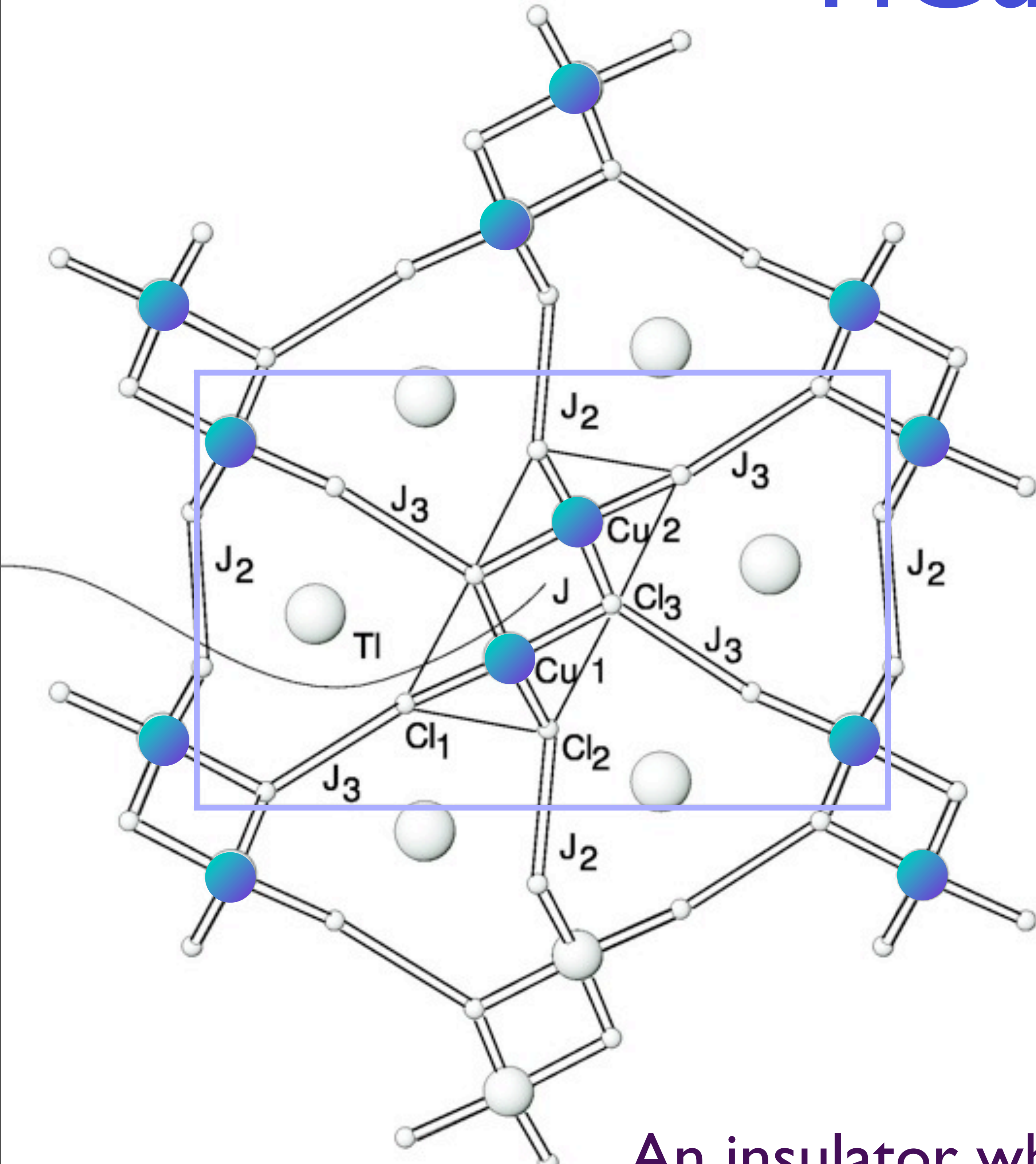
No EPR pairs

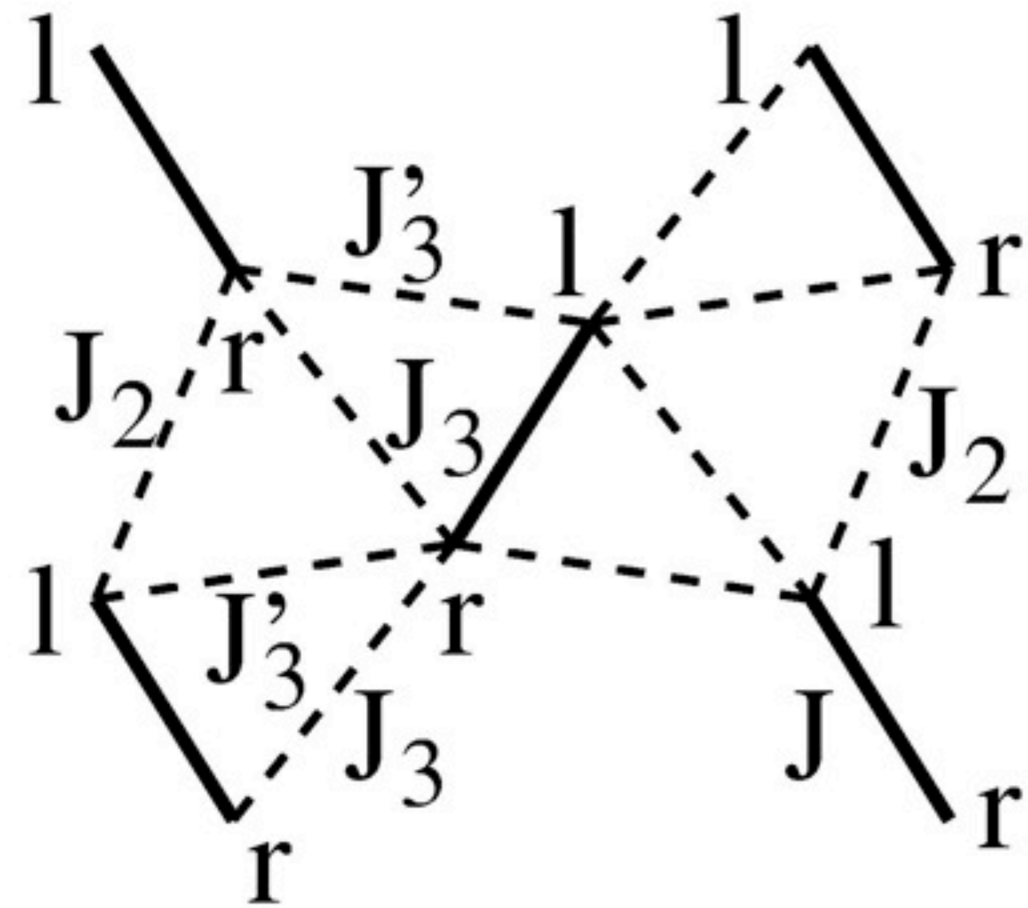

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ



A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

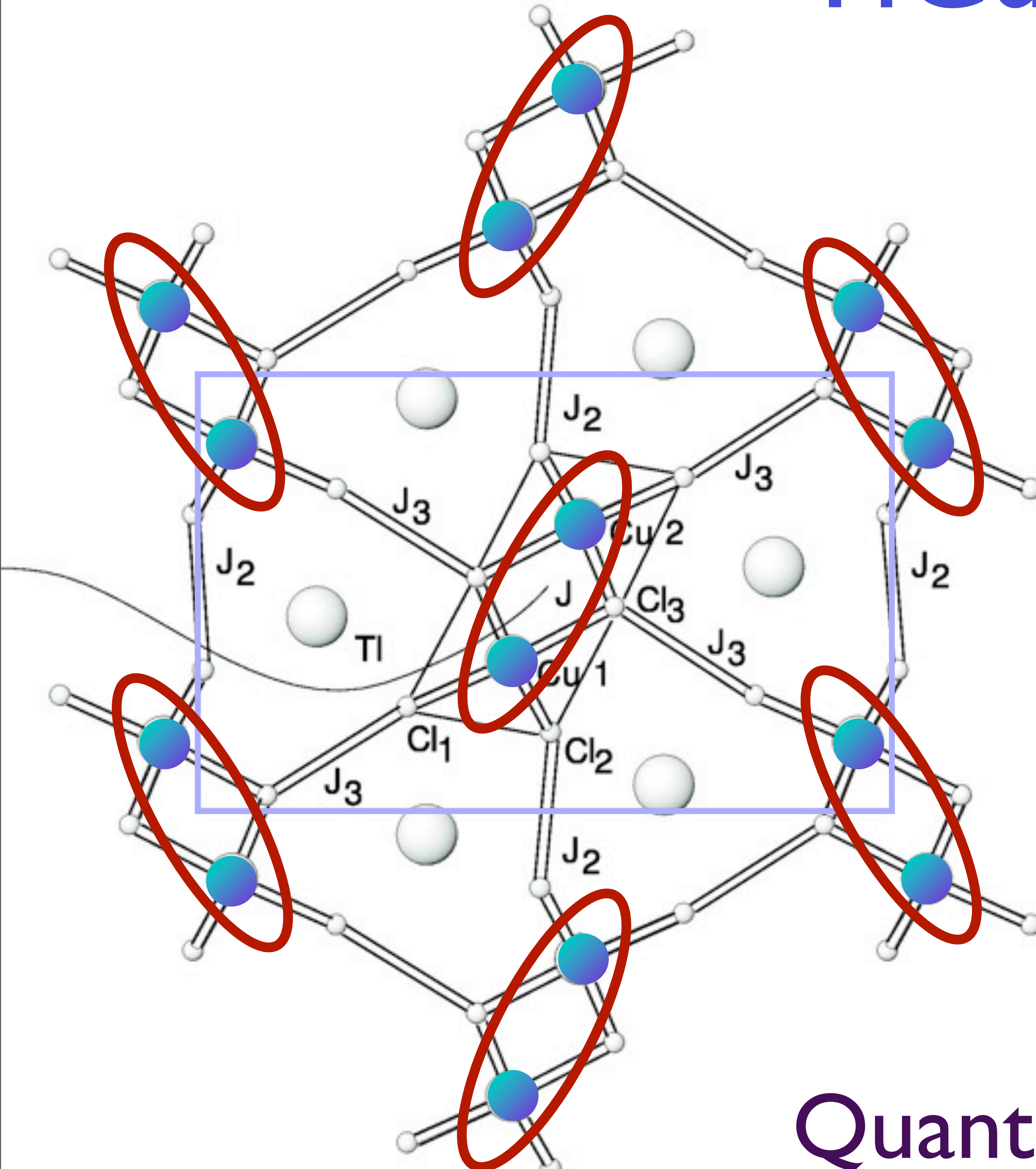
$TlCuCl_3$

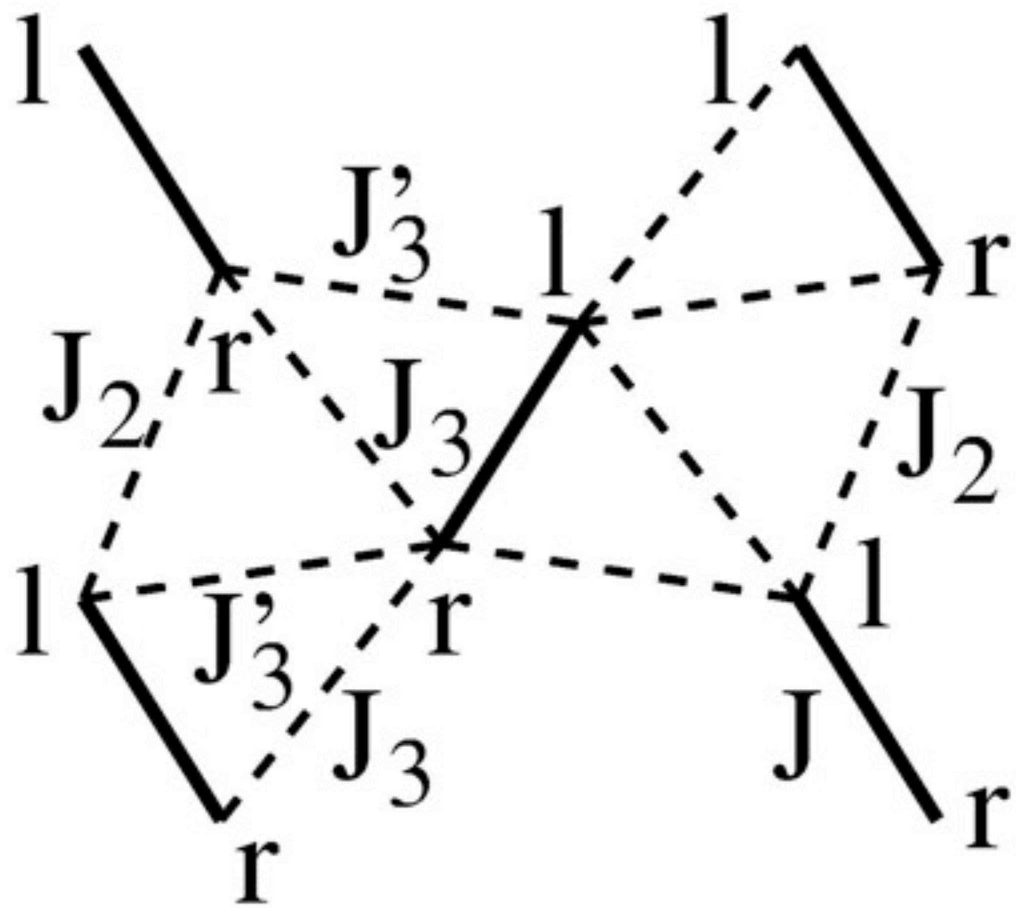




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

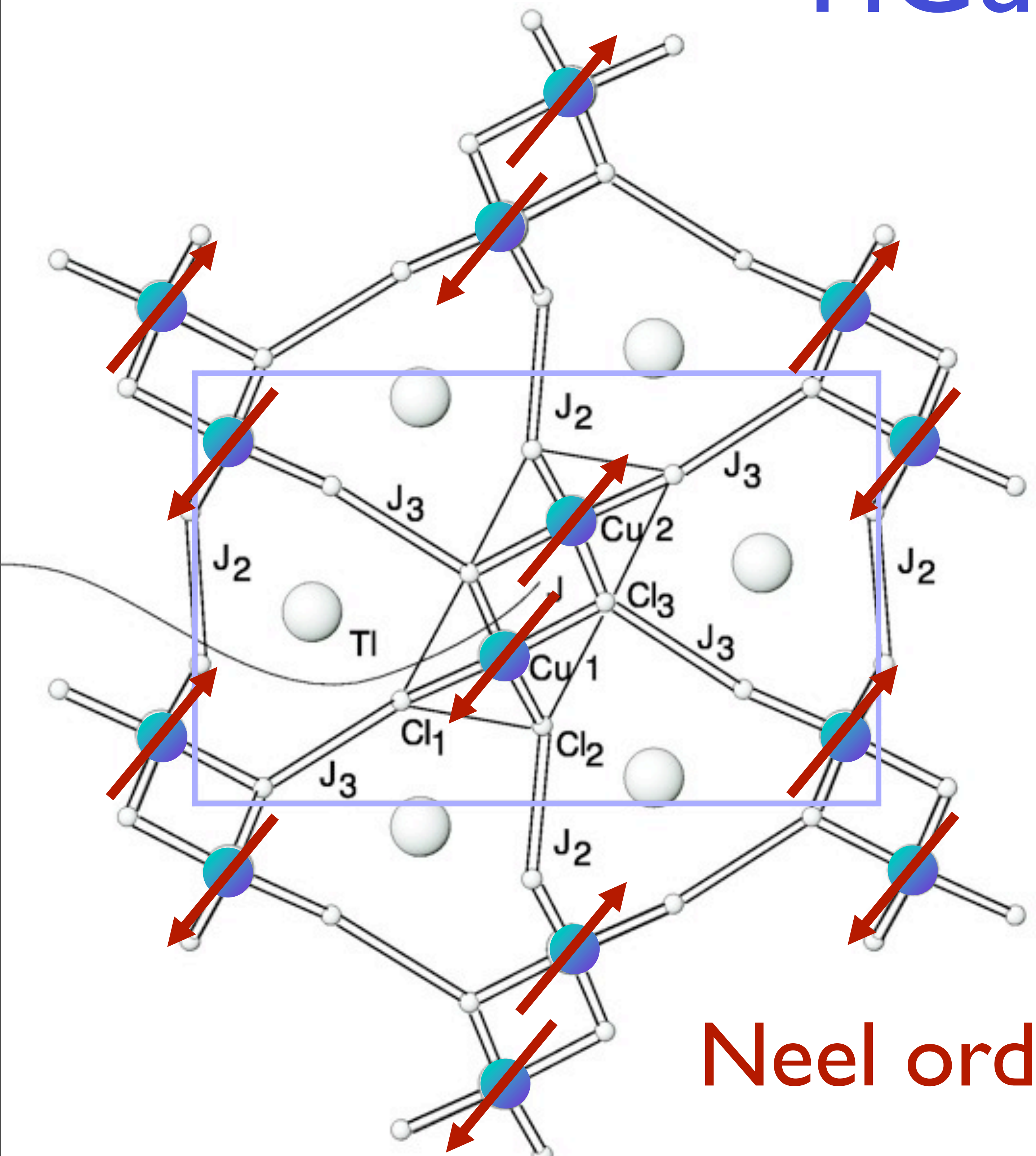
$TlCuCl_3$

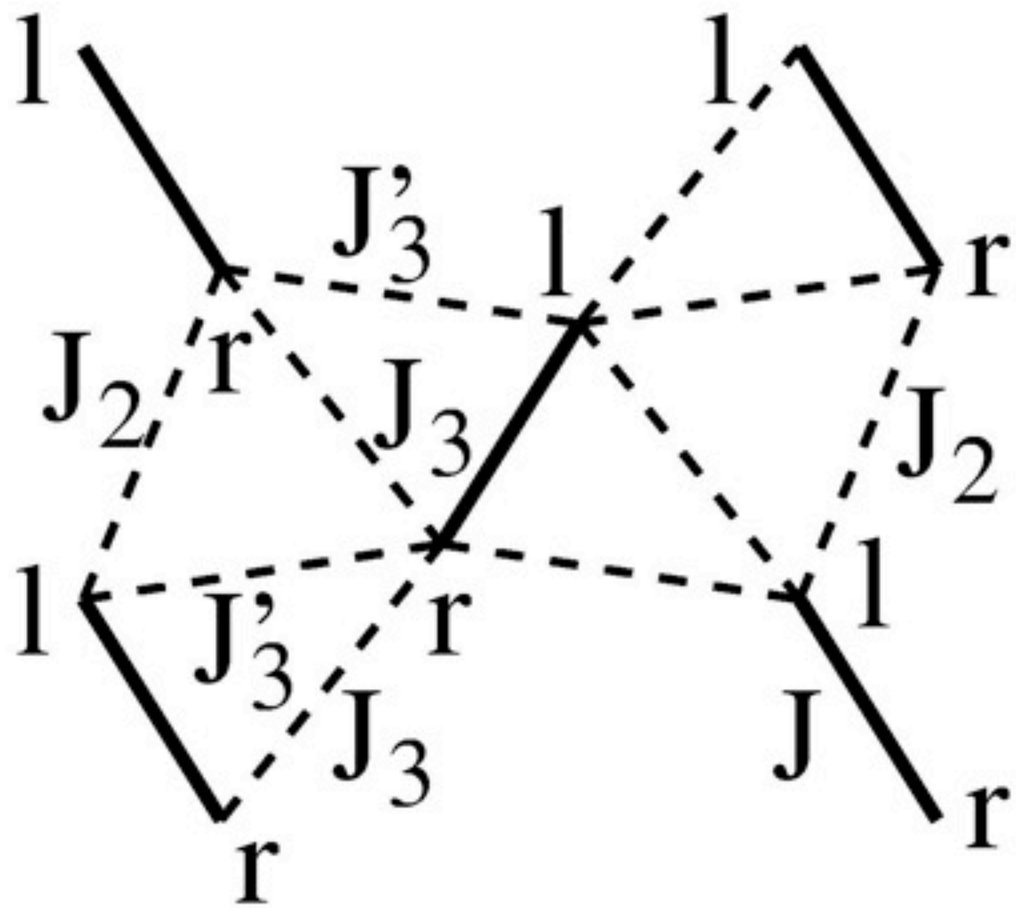




Quantum paramagnet at ambient pressure

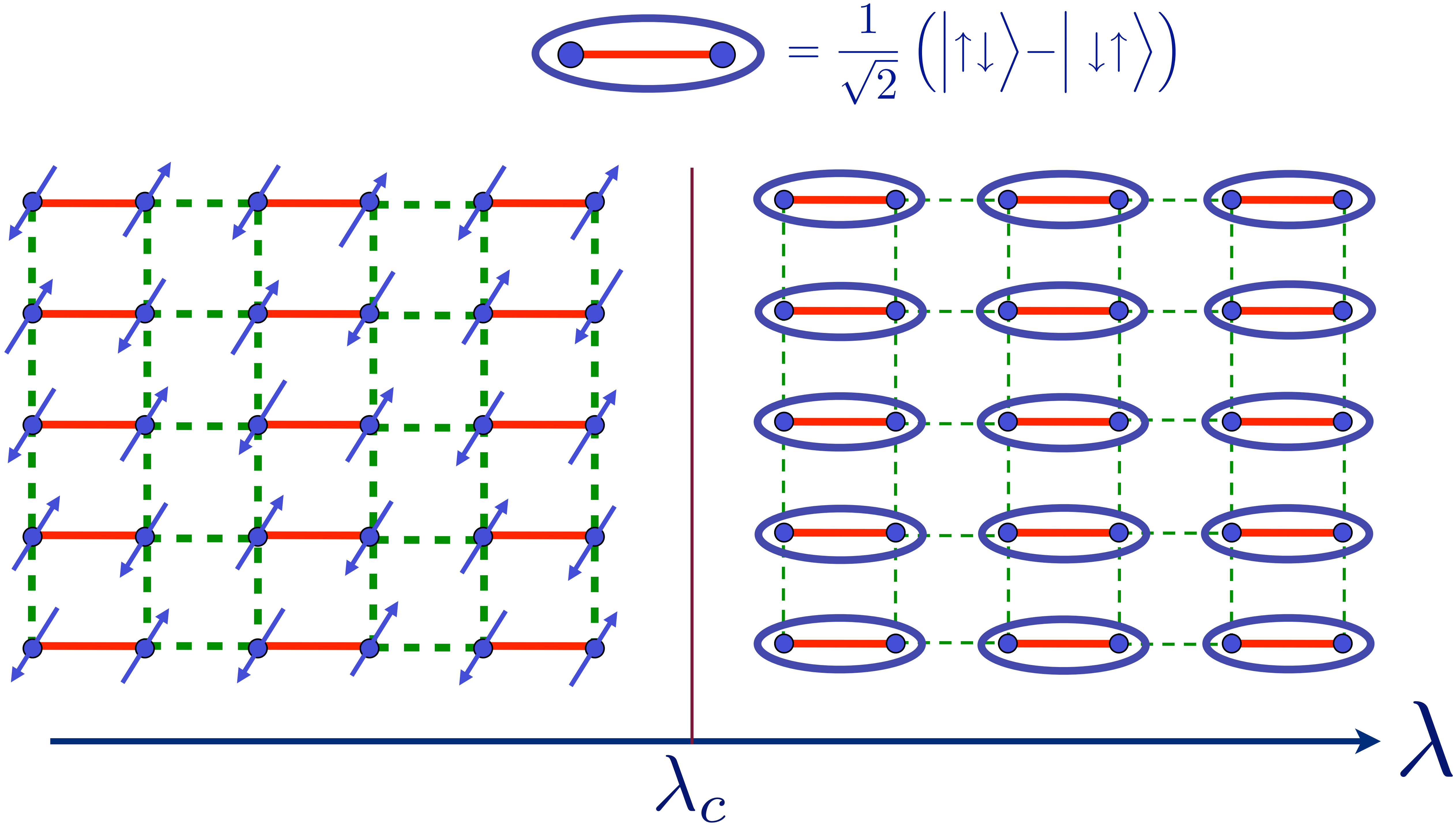
$TlCuCl_3$

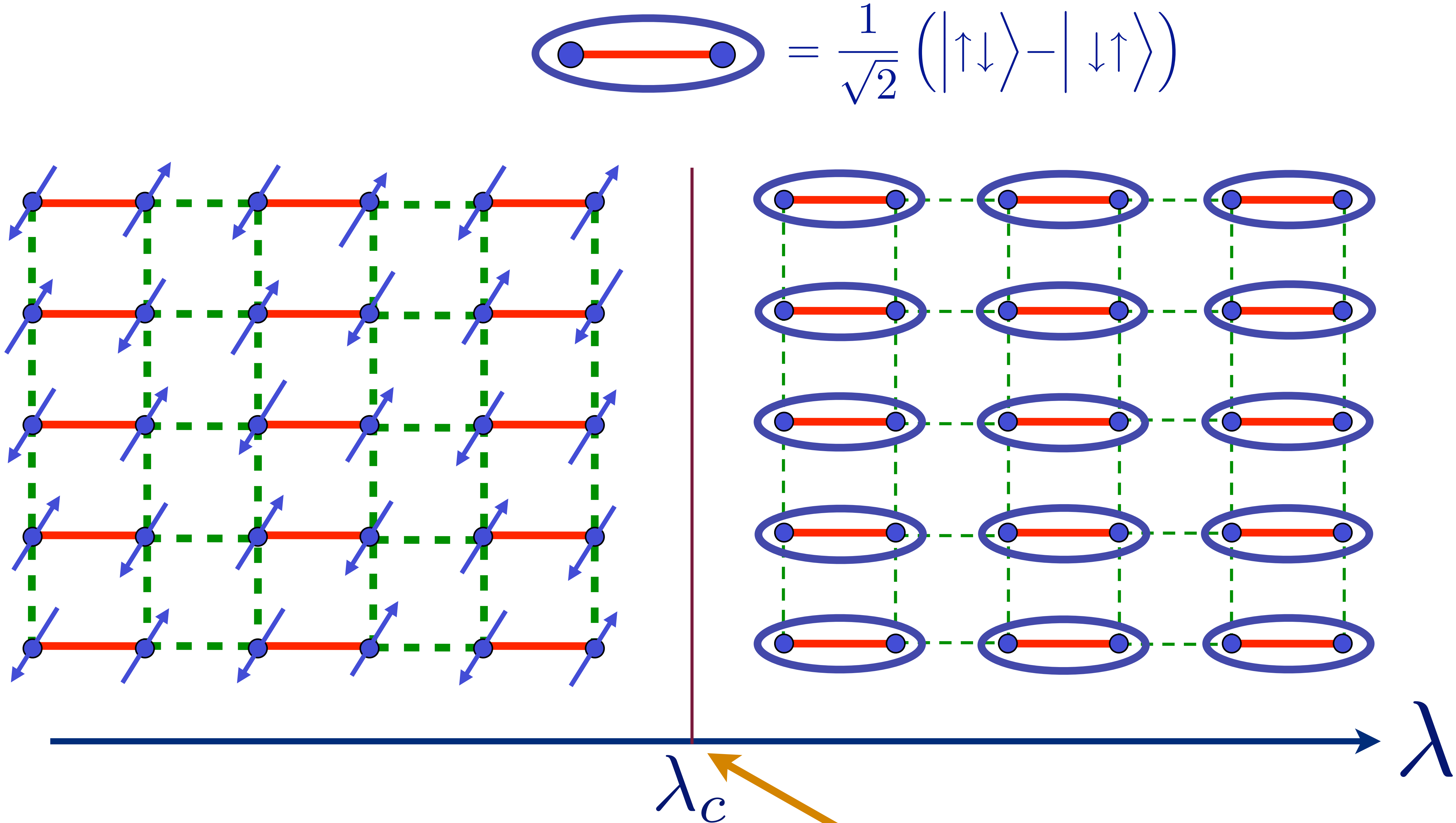




Neel order under pressure

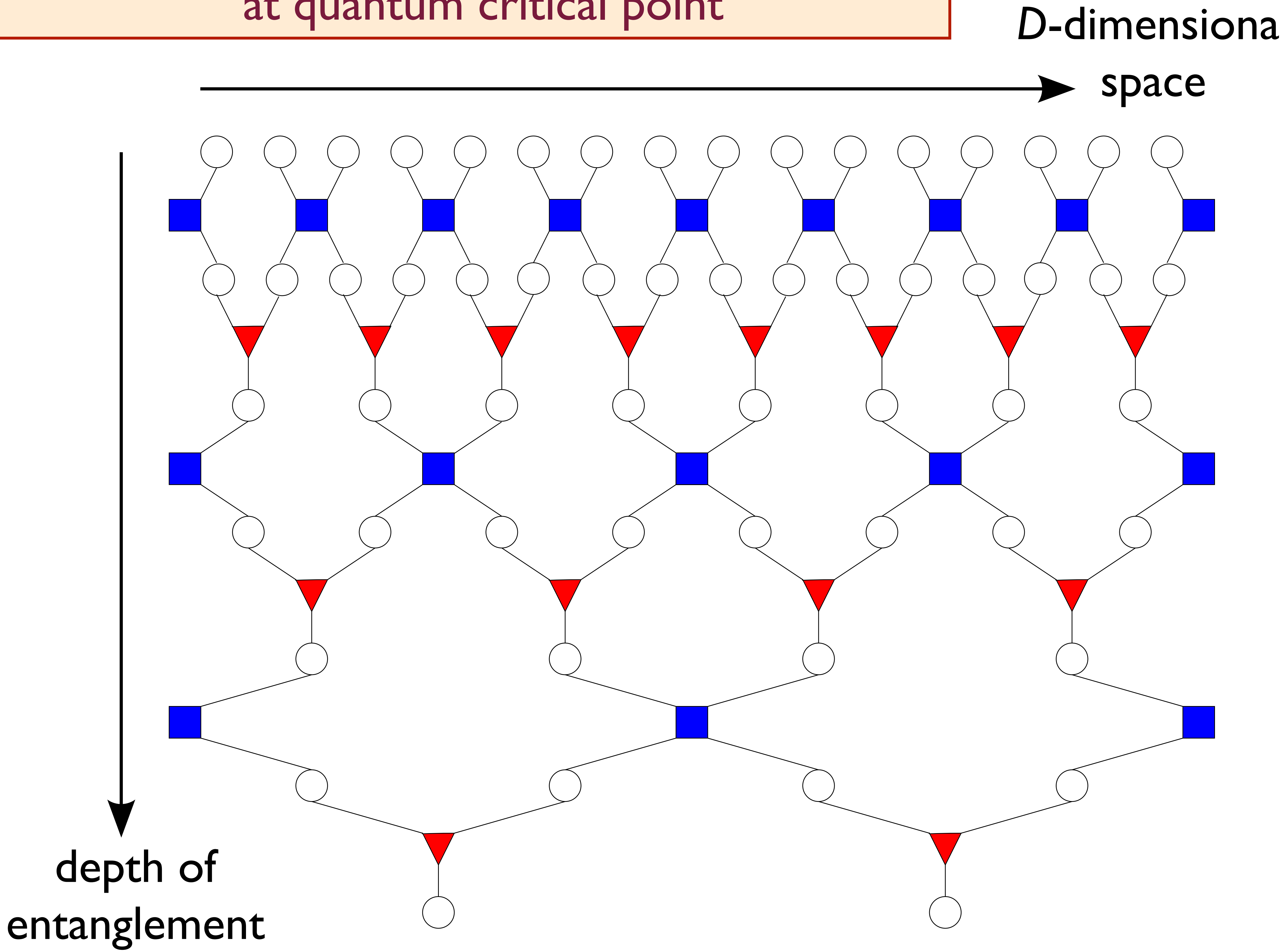
A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).


$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ



Quantum critical point with non-local entanglement in spin wavefunction

Tensor network representation of entanglement at quantum critical point



Characteristics of quantum critical point

- Long-range entanglement

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- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").

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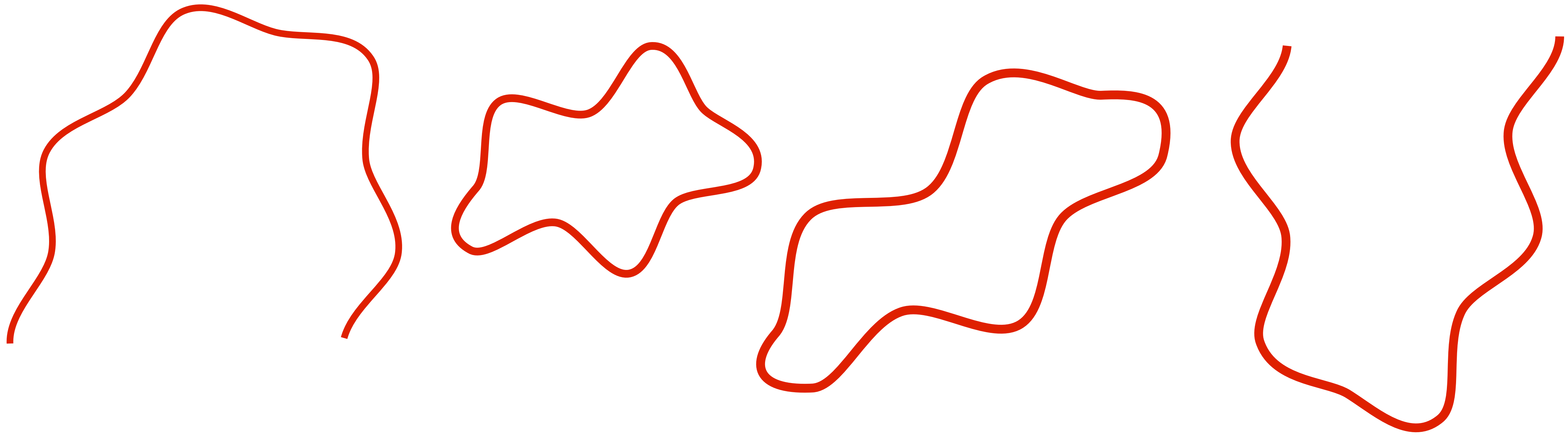
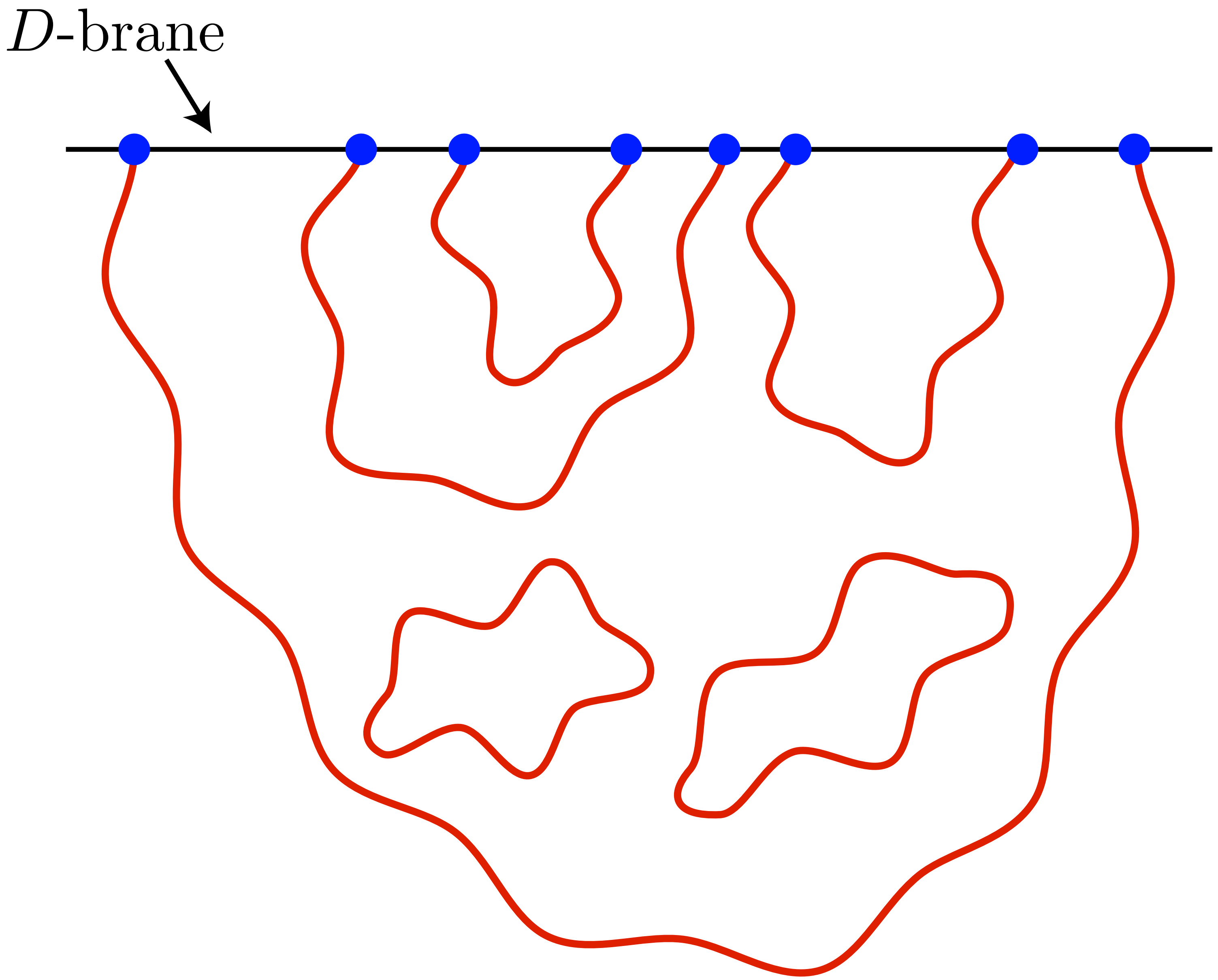
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of “light”).

- The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a **CFT3**

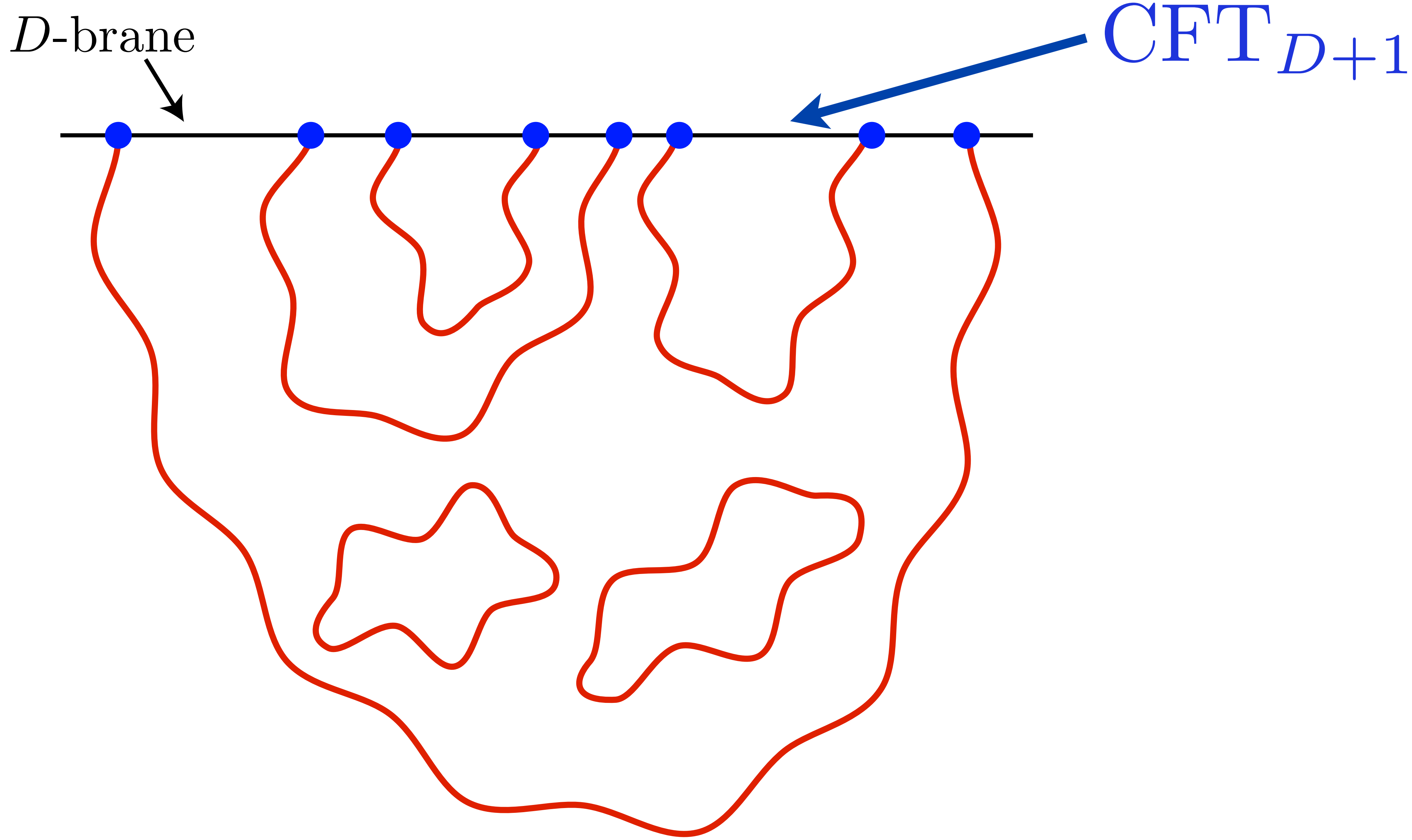
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.



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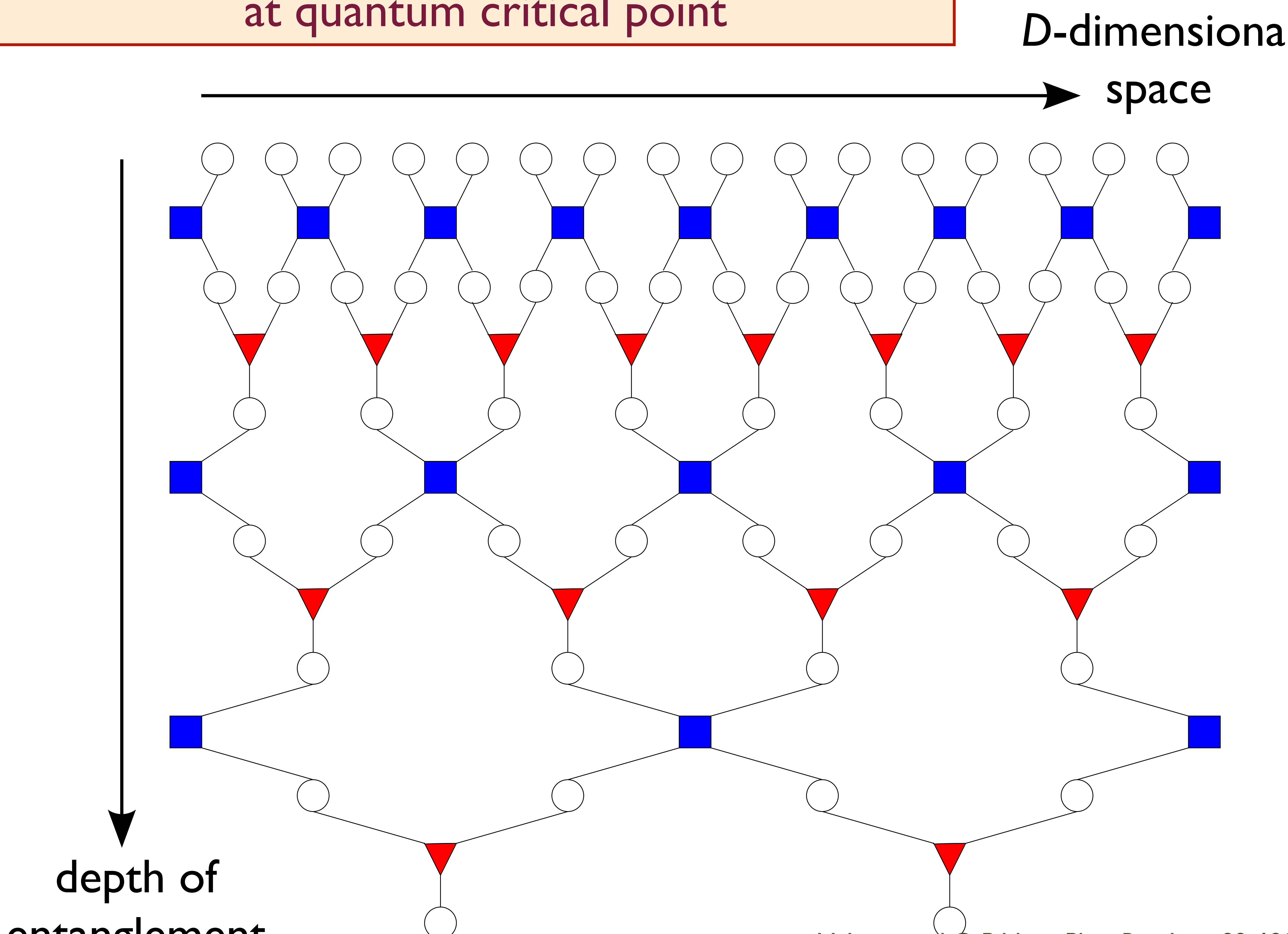
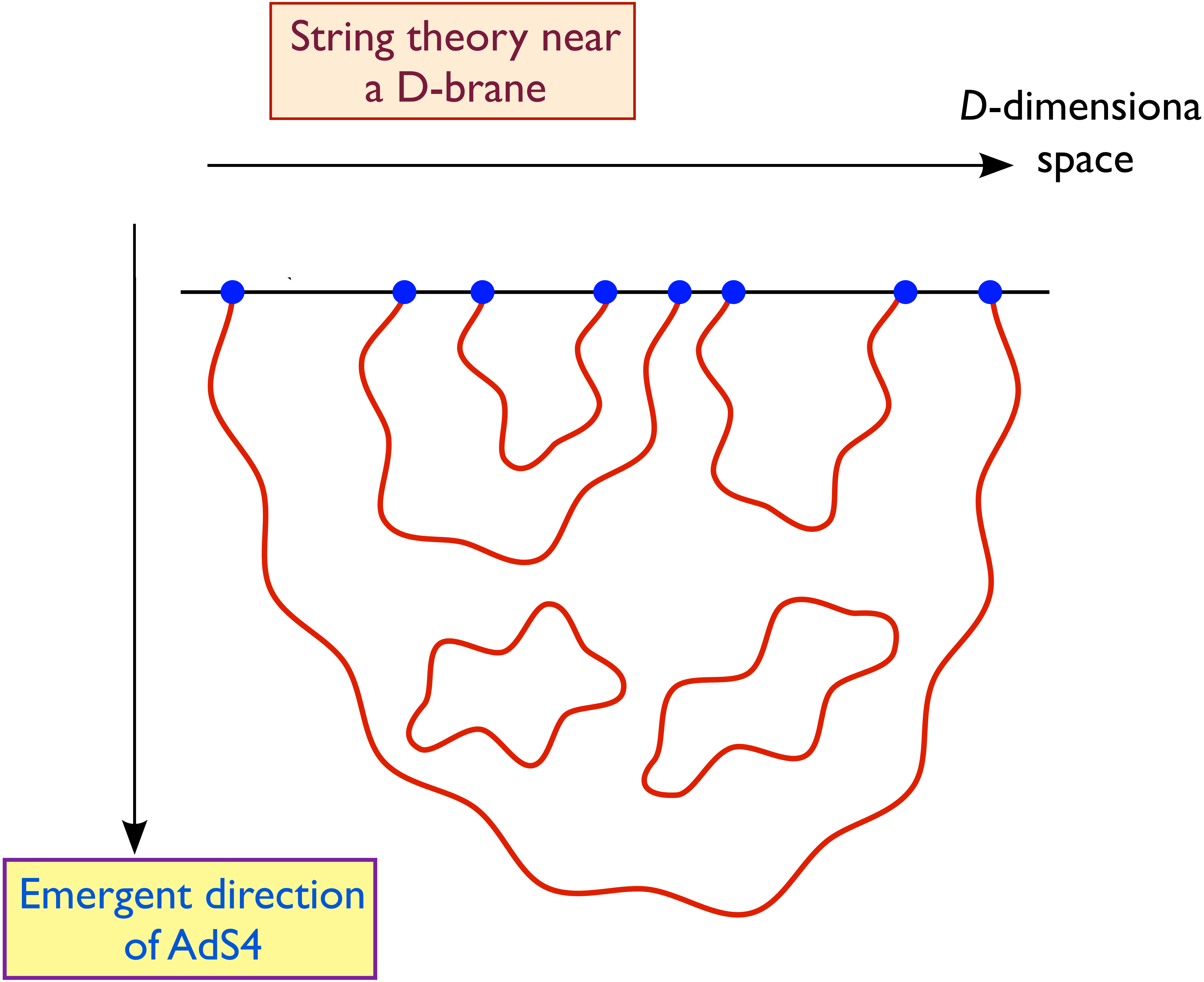
- In $D = 2$, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

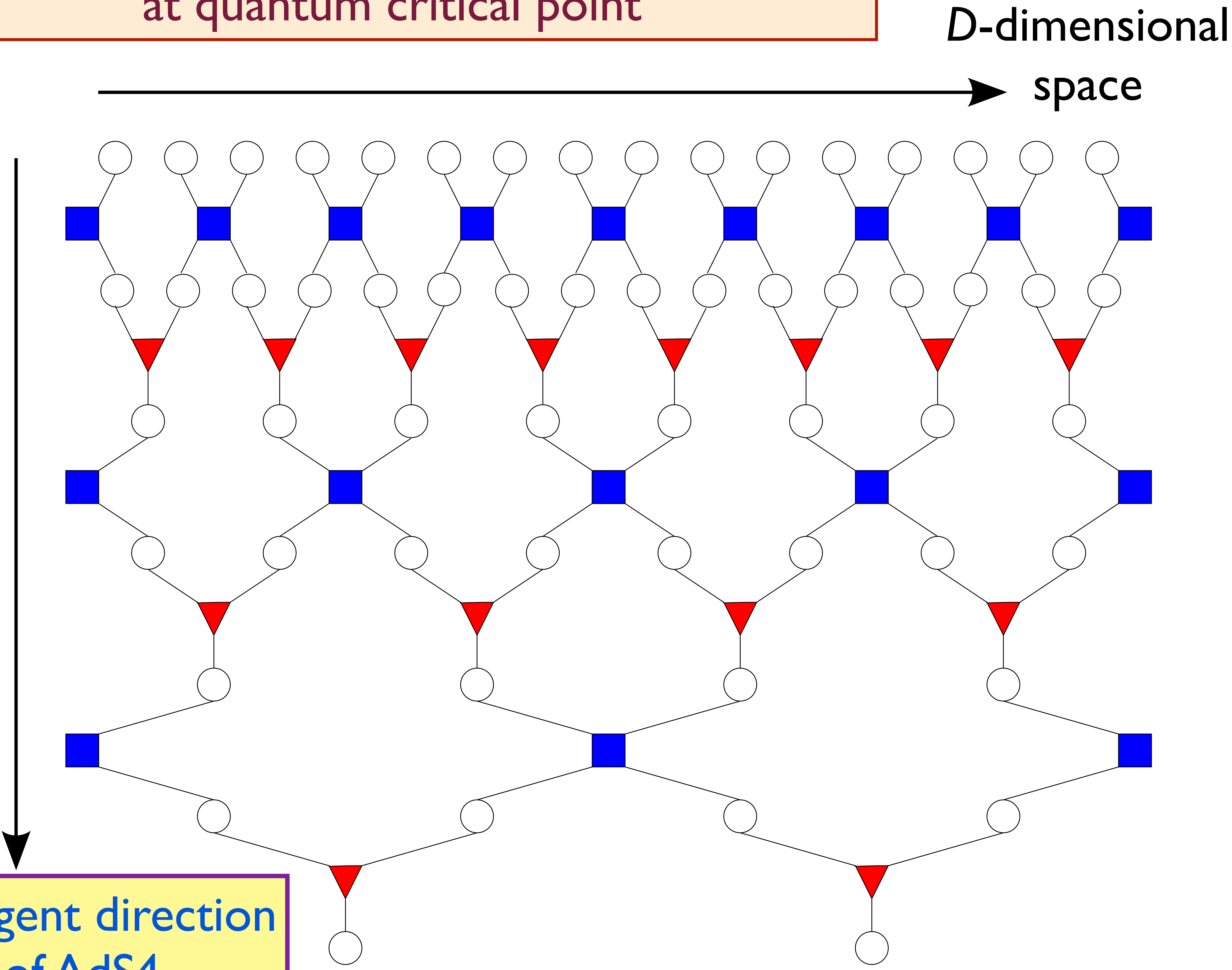
Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)


String theory near a D-brane
D-dimensional space
Emergent direction of AdS4

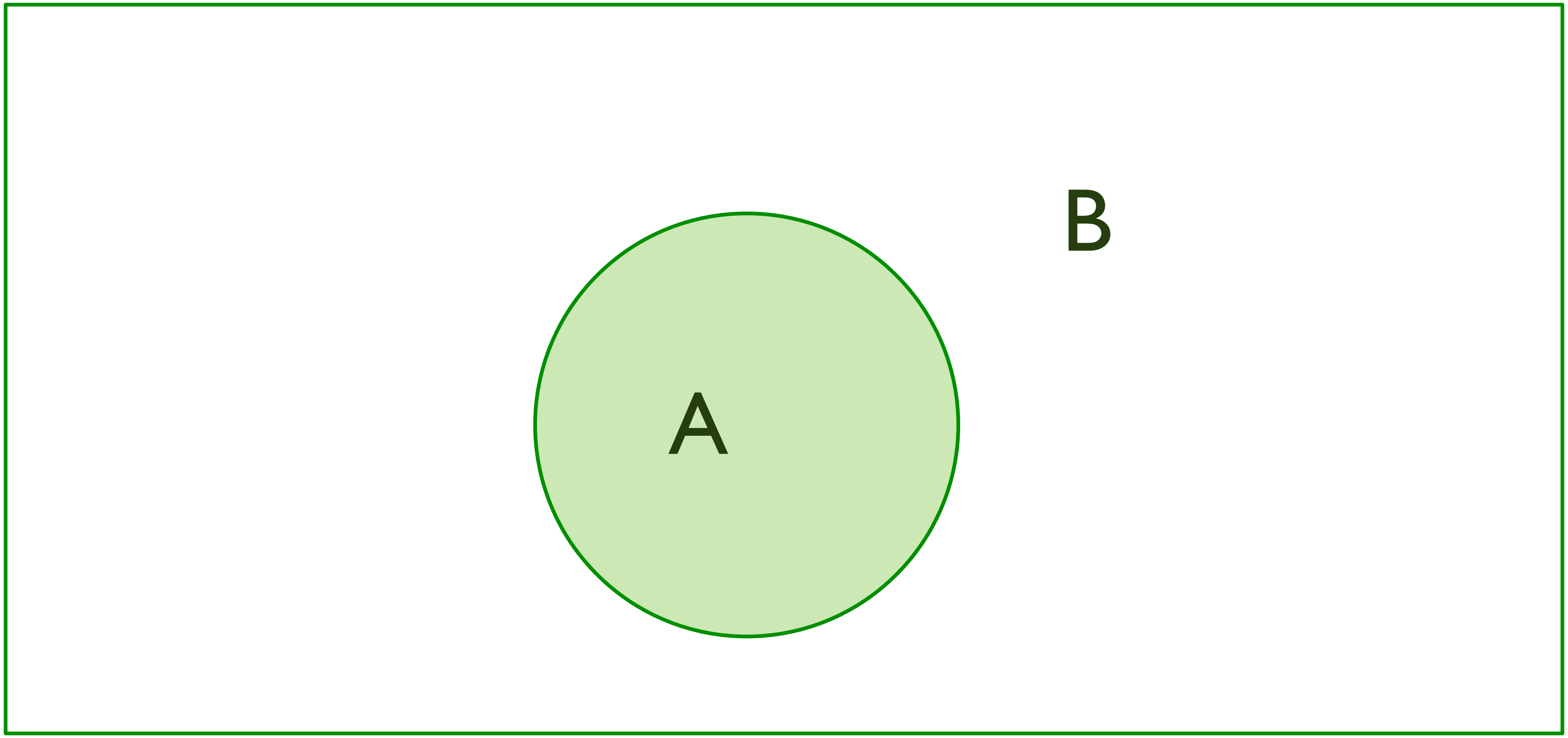
Tensor network representation of entanglement at quantum critical point



Emergent direction of AdS4

Brian Swingle, arXiv:0905.1317

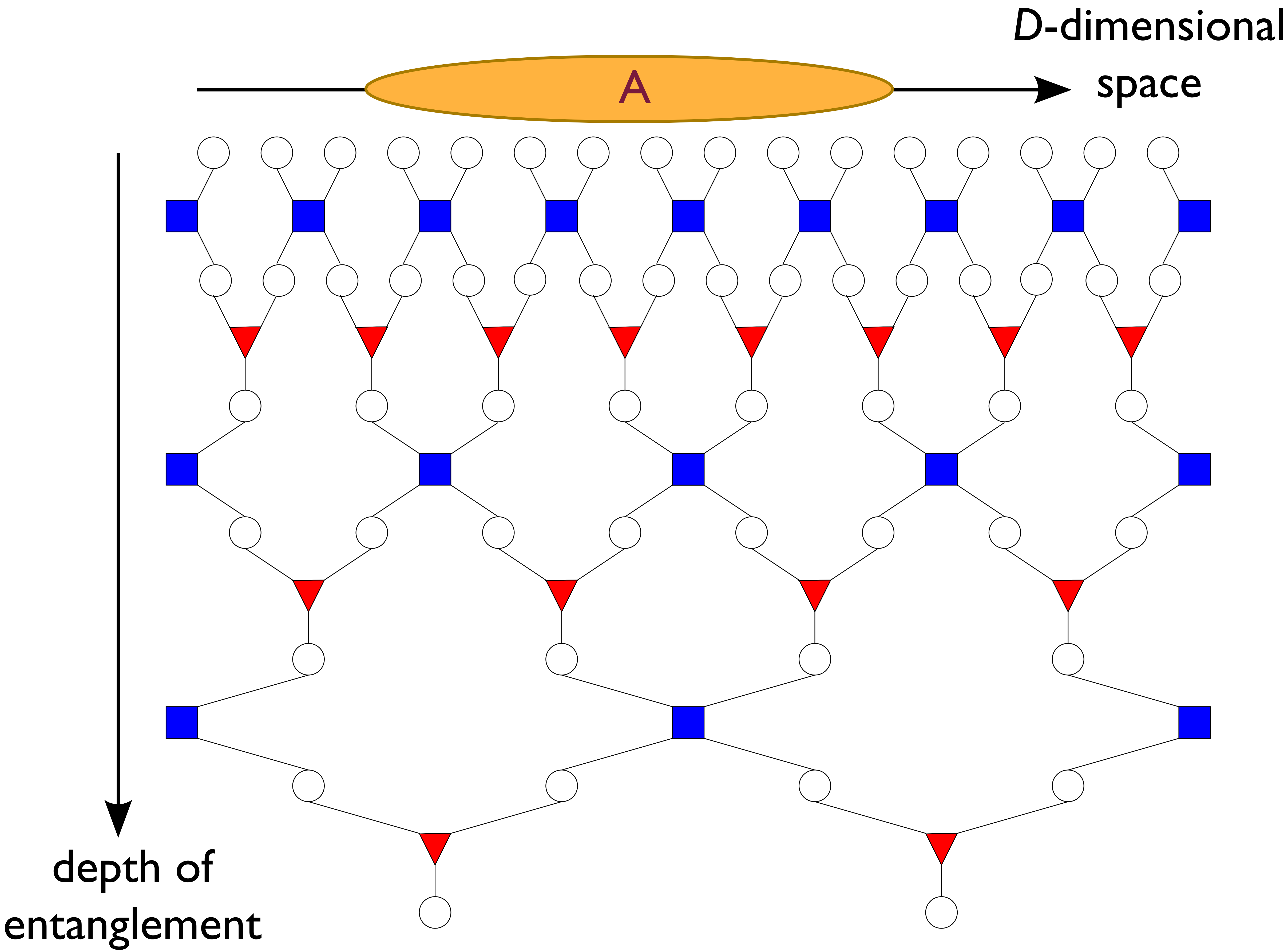
Entanglement entropy



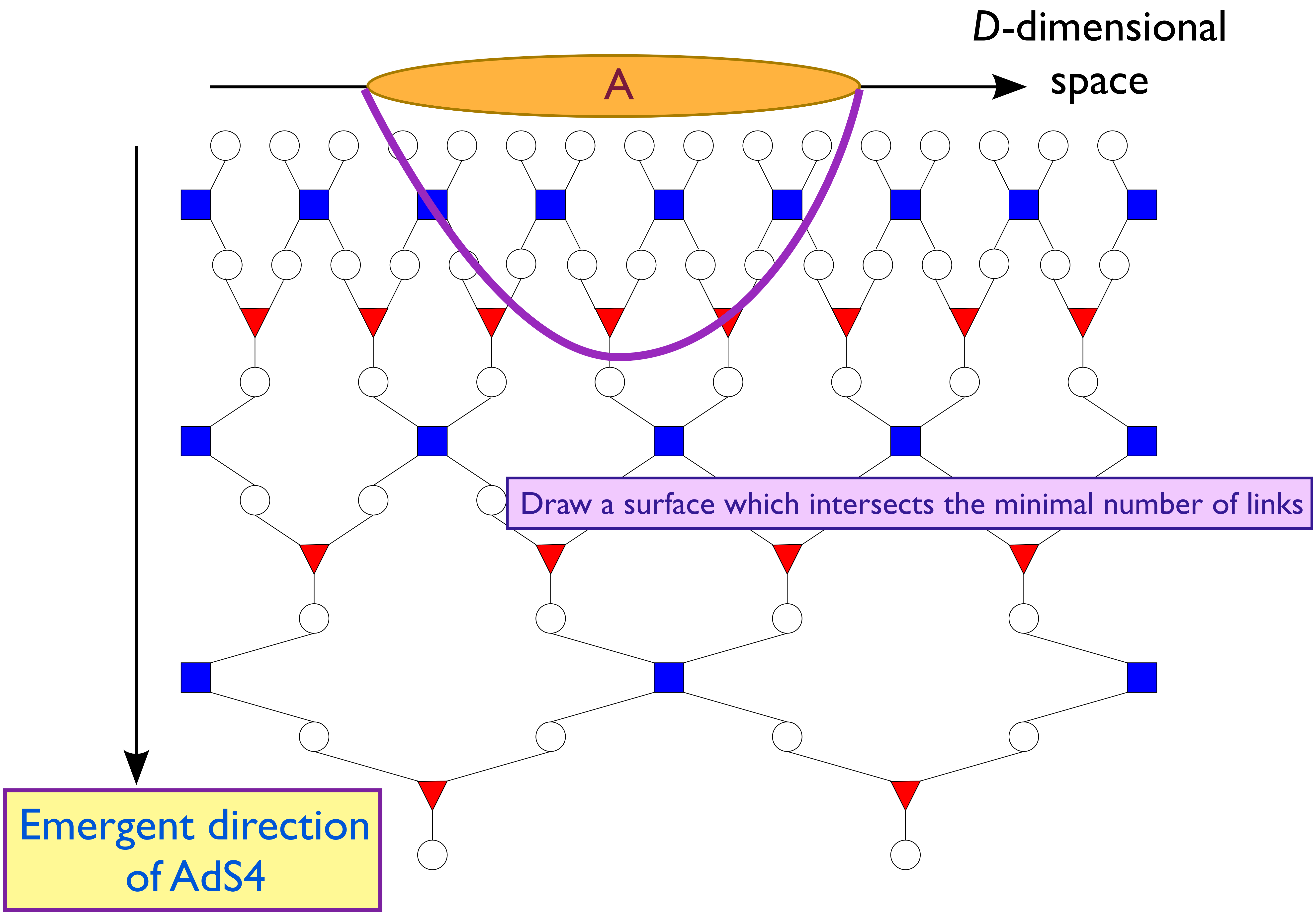
$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entangled entropy



Entanglement entropy



Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

This can be seen both the string and tensor-network pictures

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317

Rudro Rana Biswas

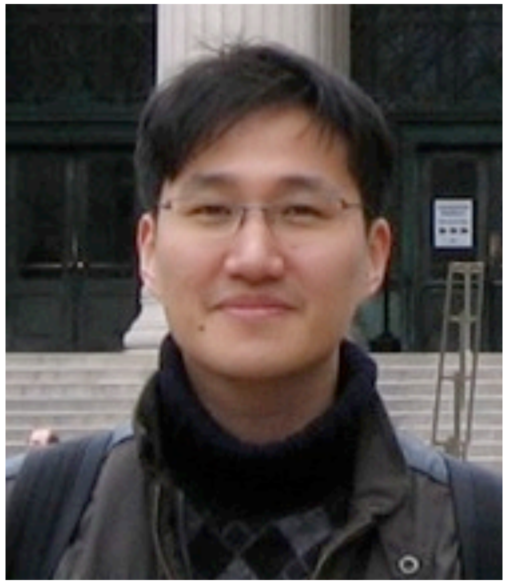
- Interactions and impurities in graphene
- Topological insulators
- Spin liquids with Majorana Fermi surfaces

Yejin Huh

- Nematic ordering *d*-wave superconductors
- Phases of the kagome antiferromagnet

Max Metlitski

- Entanglement at quantum critical points
- Field theory of AF onset in metals
- Field theory of non-Fermi liquids



Eun Gook Moon

- Gases of ultracold fermionic atoms
- Competing order parameters
  at quantum critical points
- Non-Fermi liquids and superconductivity

Susanne Pielawa

- Dynamics of ultracold atoms
- Novel phases of atoms in tilted lattices