

Strange metals, black holes, and graphene

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Talk online: sachdev.physics.harvard.edu



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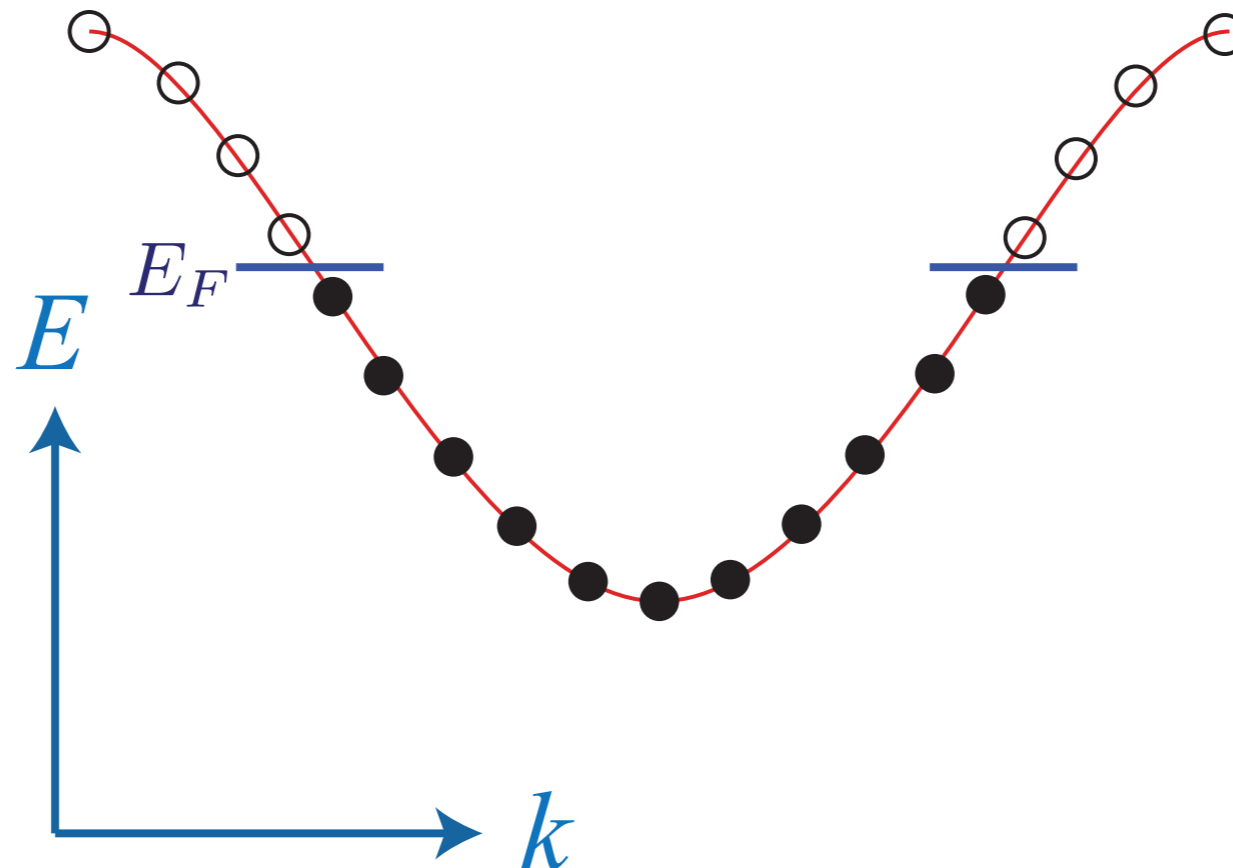


HARVARD

Foundations of quantum many body theory:

I. Ground states connected adiabatically to independent electron states

Metals

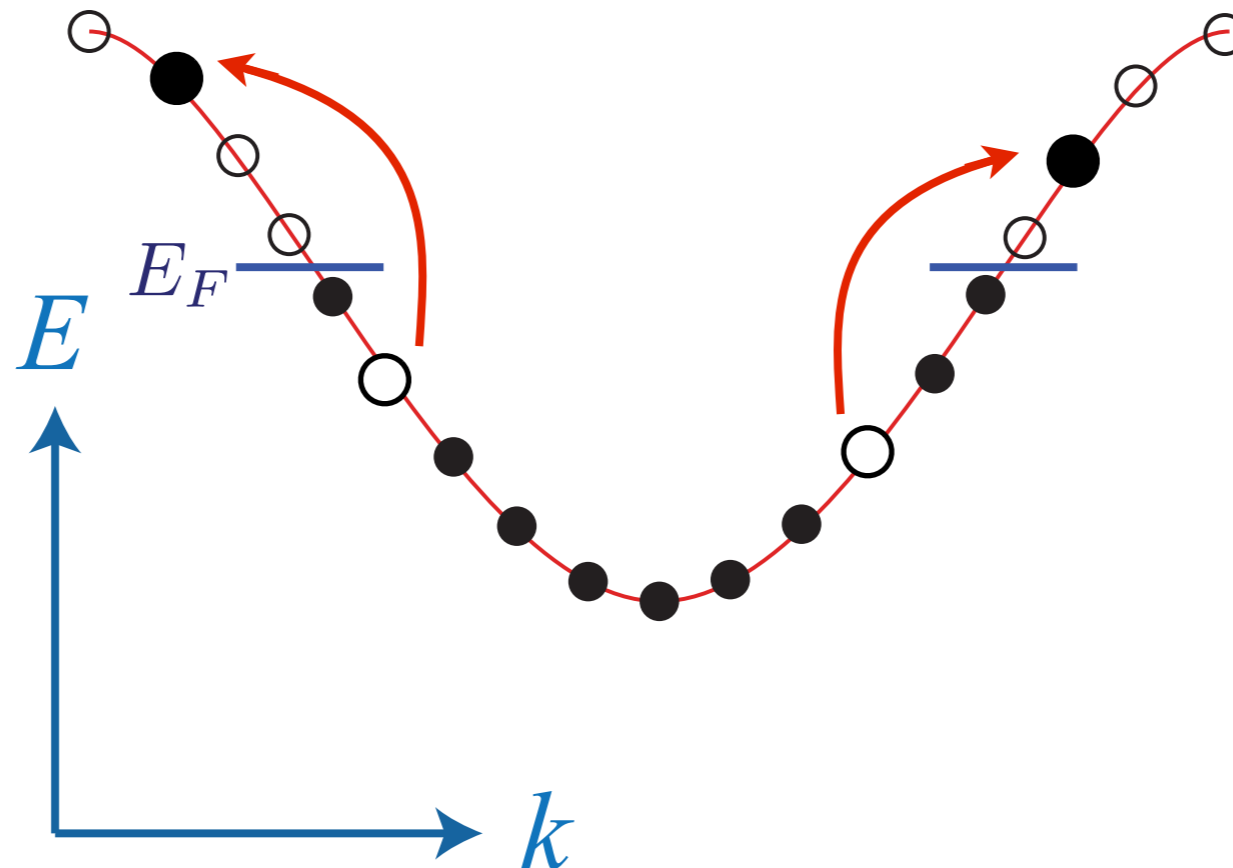


Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

2. Boltzmann-Landau theory of quasiparticles

Metals



Entangled phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement
2. Boltzmann-Landau theory of quasiparticles

Topological quantum matter:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Entangled phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement
2. No quasiparticles

Strange metals with no quasiparticles:

Such metals are found, most prominently, in certain high temperature superconductors.

Entangled phases of quantum matter:

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Strange metals with no quasiparticles:

Such metals are found, most prominently, in certain high temperature superconductors.

But how can we be sure that no quasiparticles exist in a given system? Perhaps there are some entangled quasiparticles inaccessible to current experiments.....

Local thermal equilibration or phase coherence time, τ_φ :

- There is an *lower bound* on τ_φ in all many-body quantum systems of order $\hbar/(k_B T)$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

and the lower bound is realized by systems *without* quasiparticles.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T ;
e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

A bound on quantum chaos:

- The time over which a many-body quantum system becomes “chaotic” is given by $\tau_L = 1/\lambda_L$, where λ_L is the “Lyapunov exponent” determining memory of initial conditions. This LYAPUNOV TIME obeys the rigorous lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

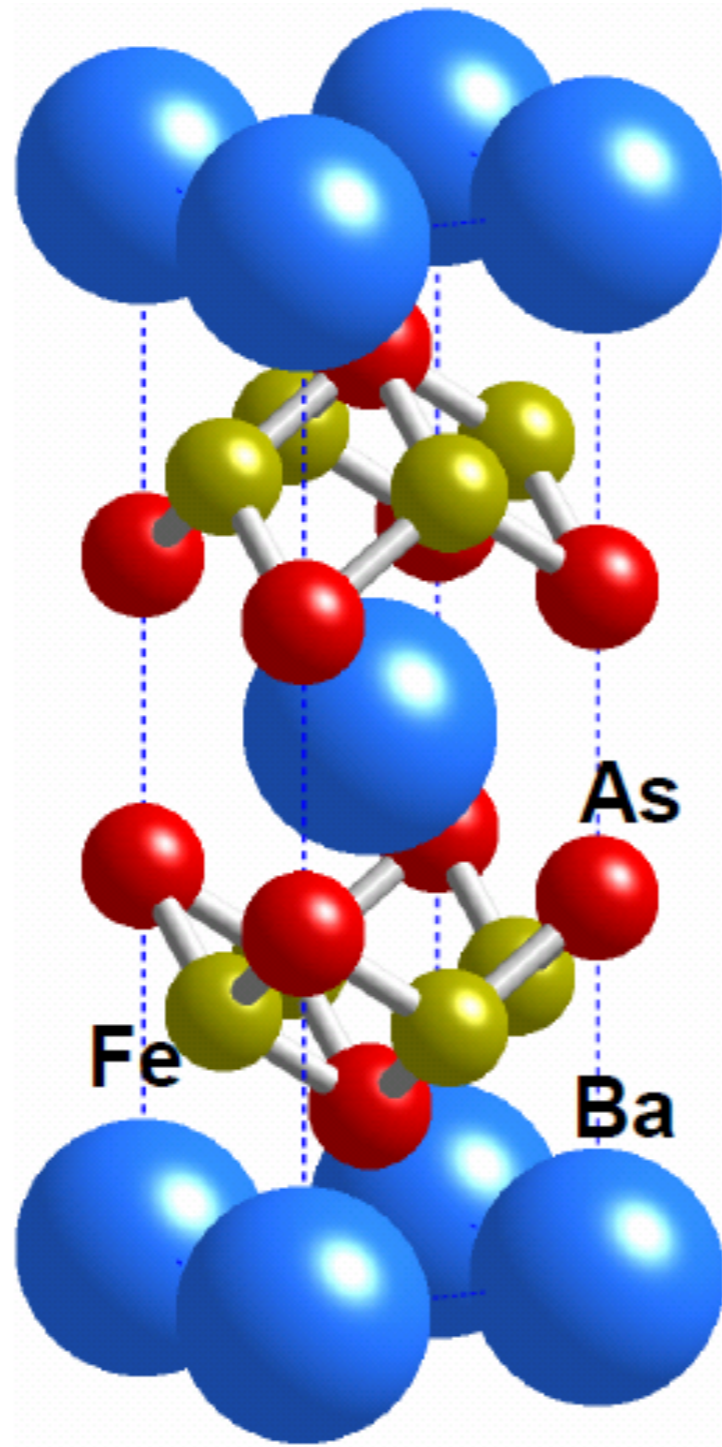
J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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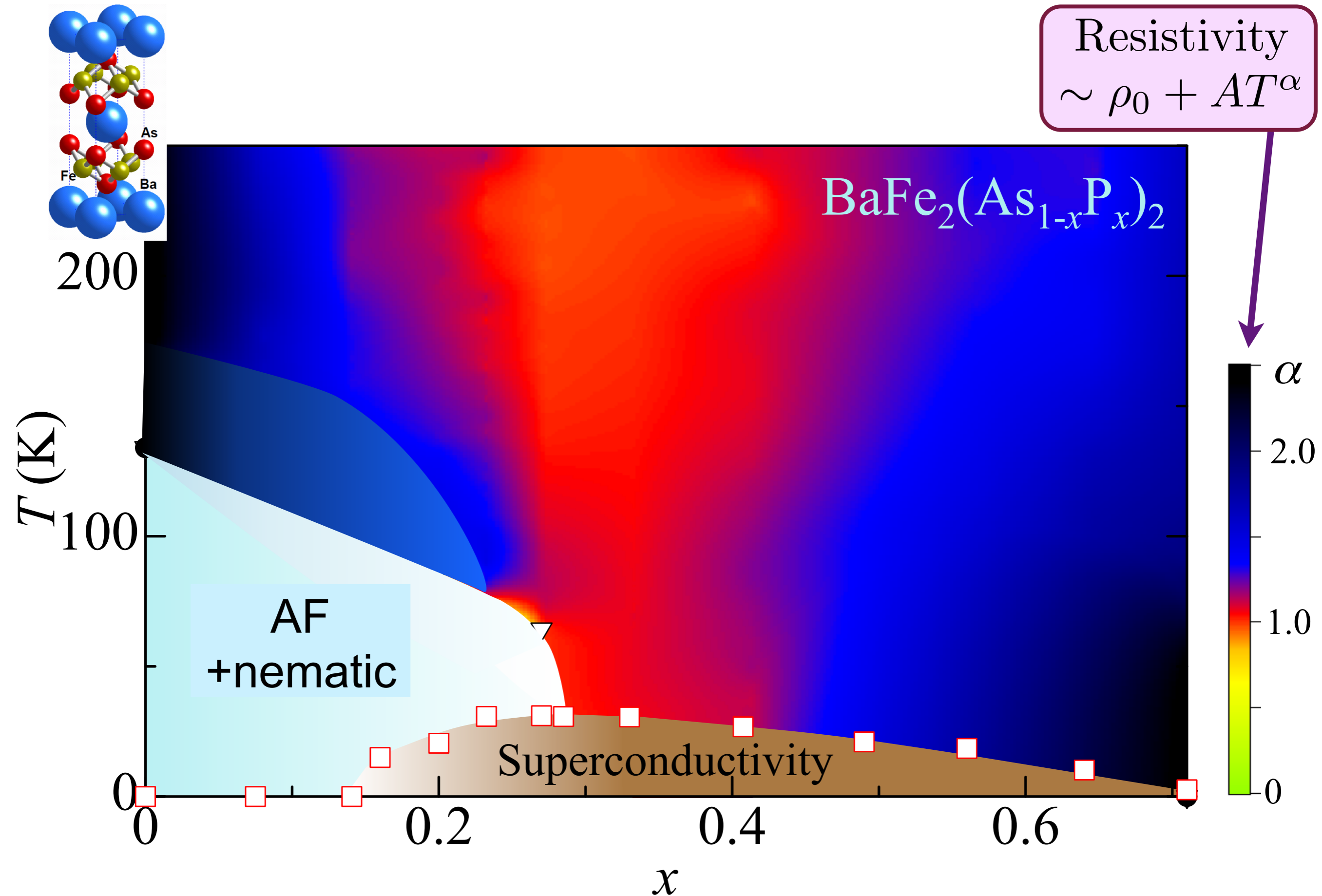
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Quantum matter without quasiparticles
 \approx fastest possible many-body quantum chaos

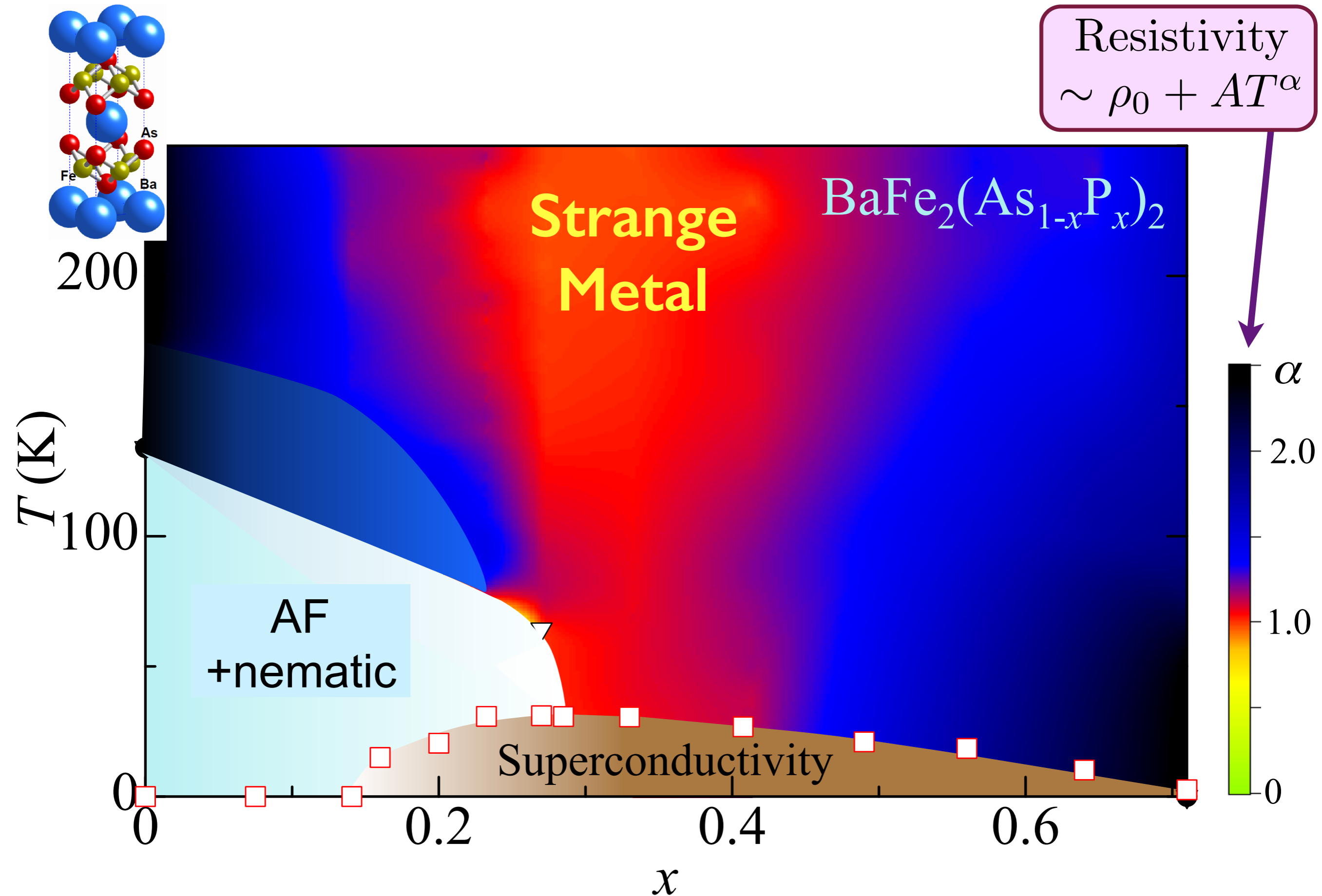


A high
temperature
superconductor



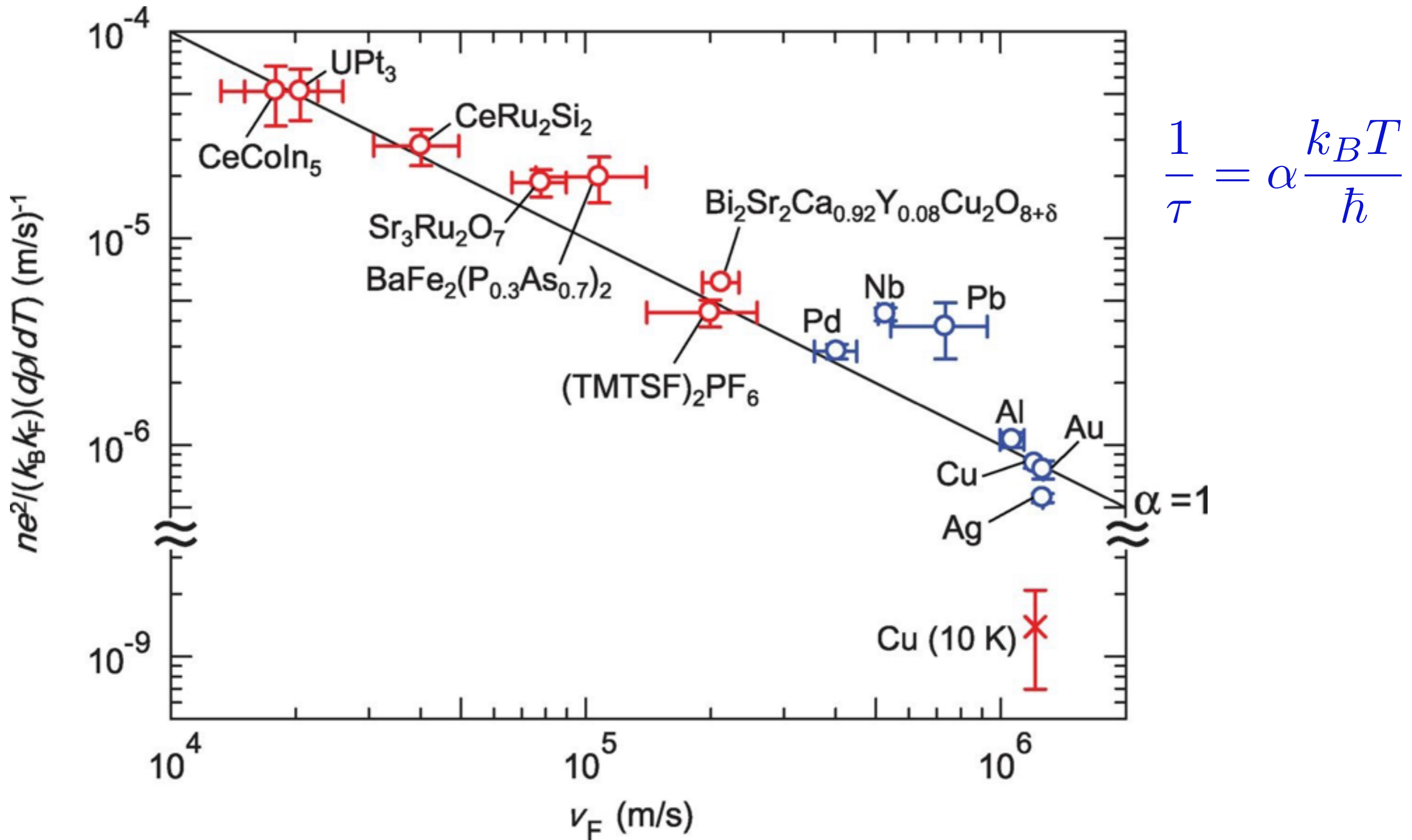


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,
 H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,
Physical Review B **81**, 184519 (2010)

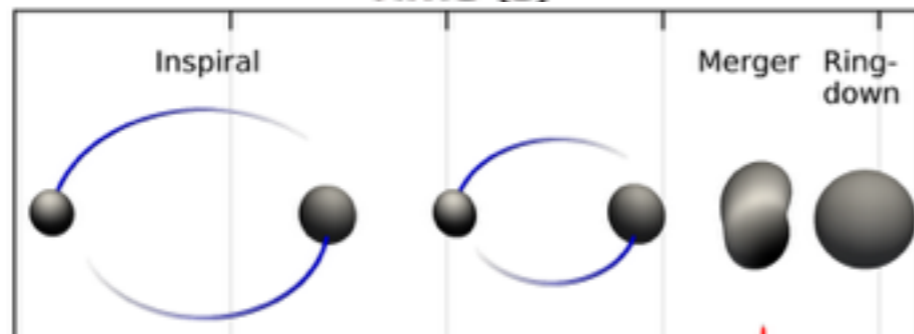
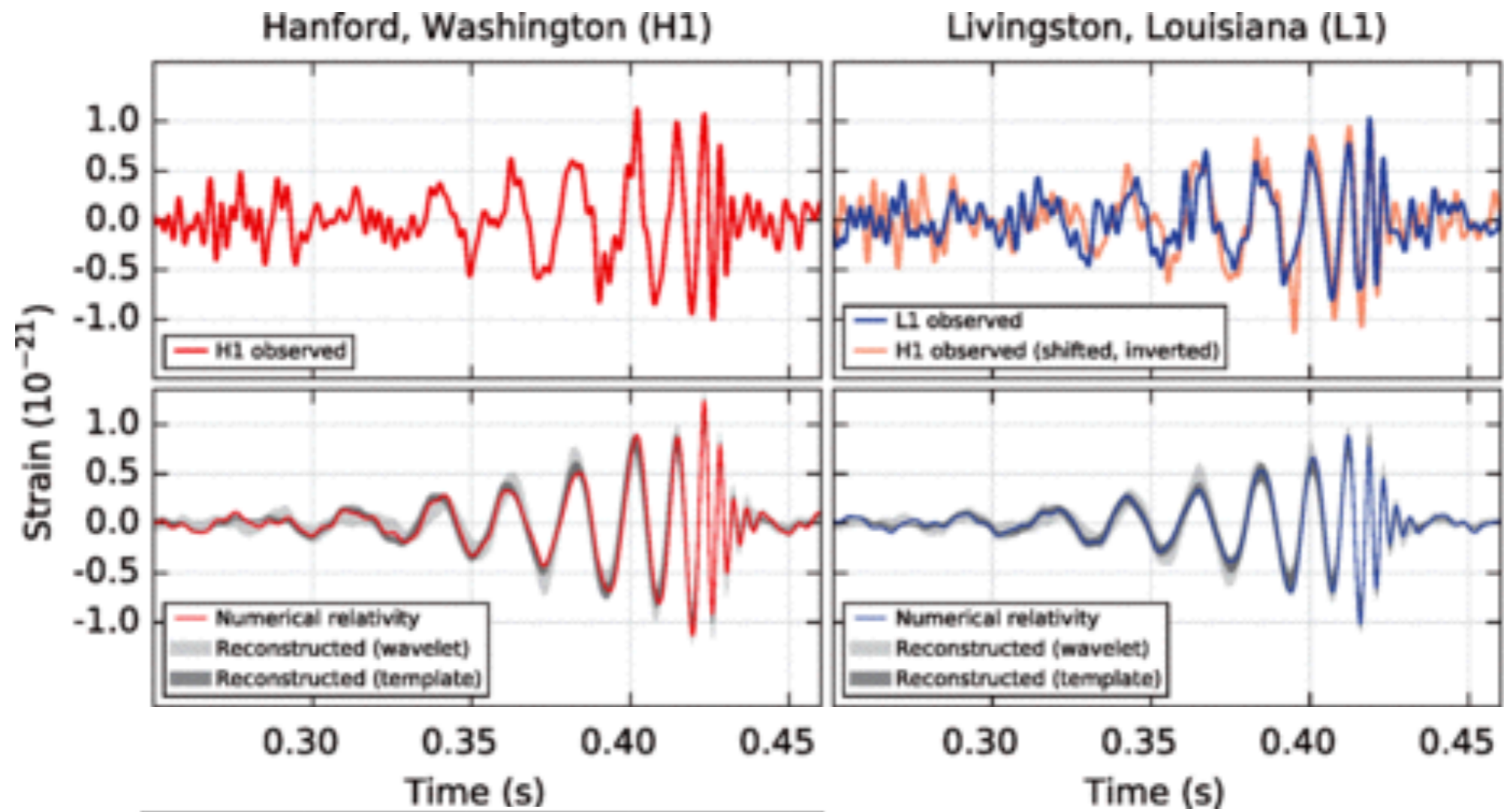


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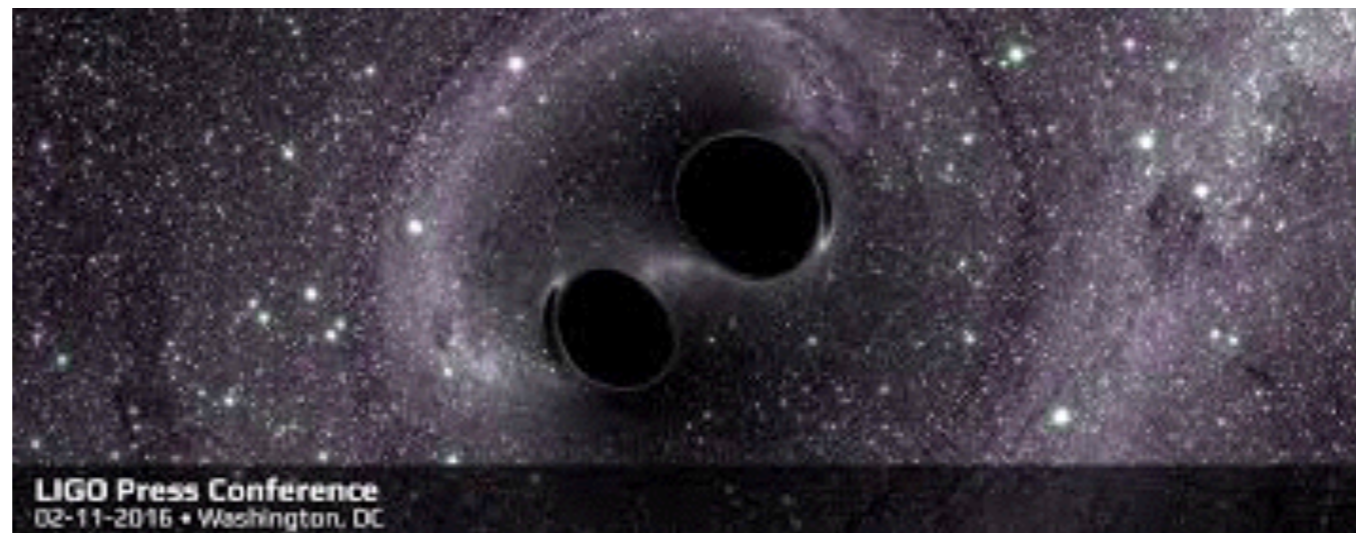
Strange metals

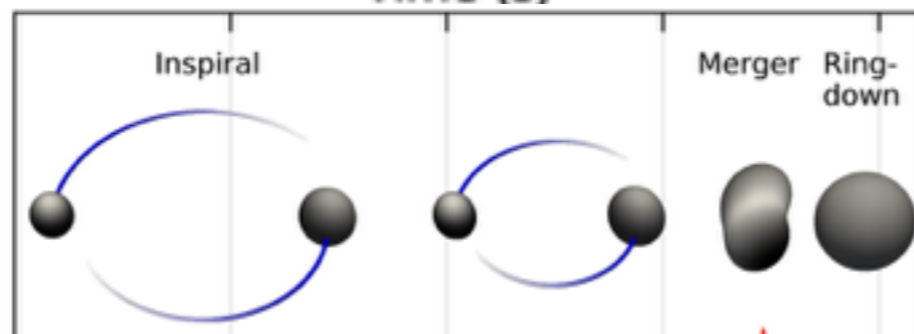
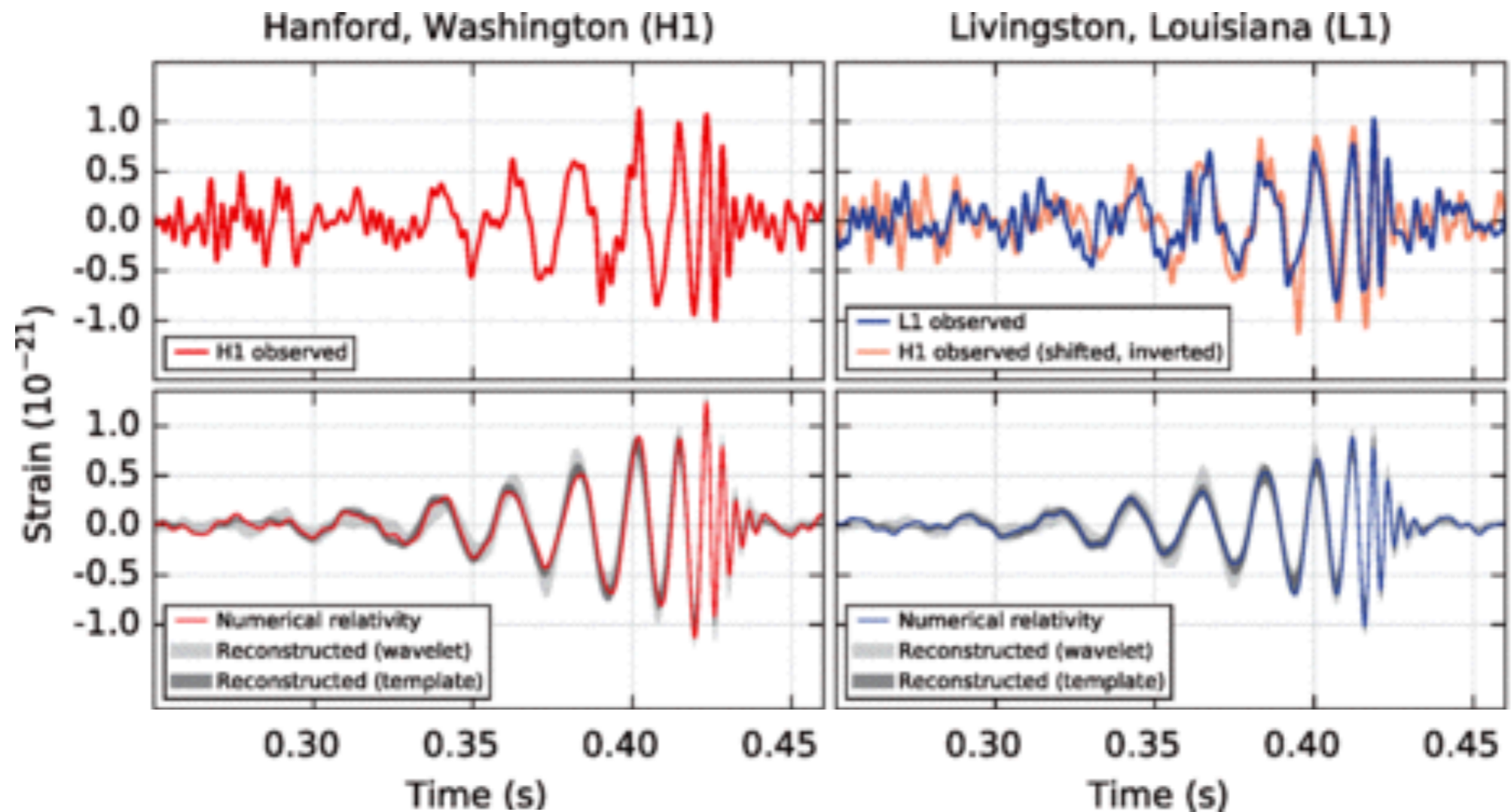


J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie, *Science*. **339**, 804 (2013)



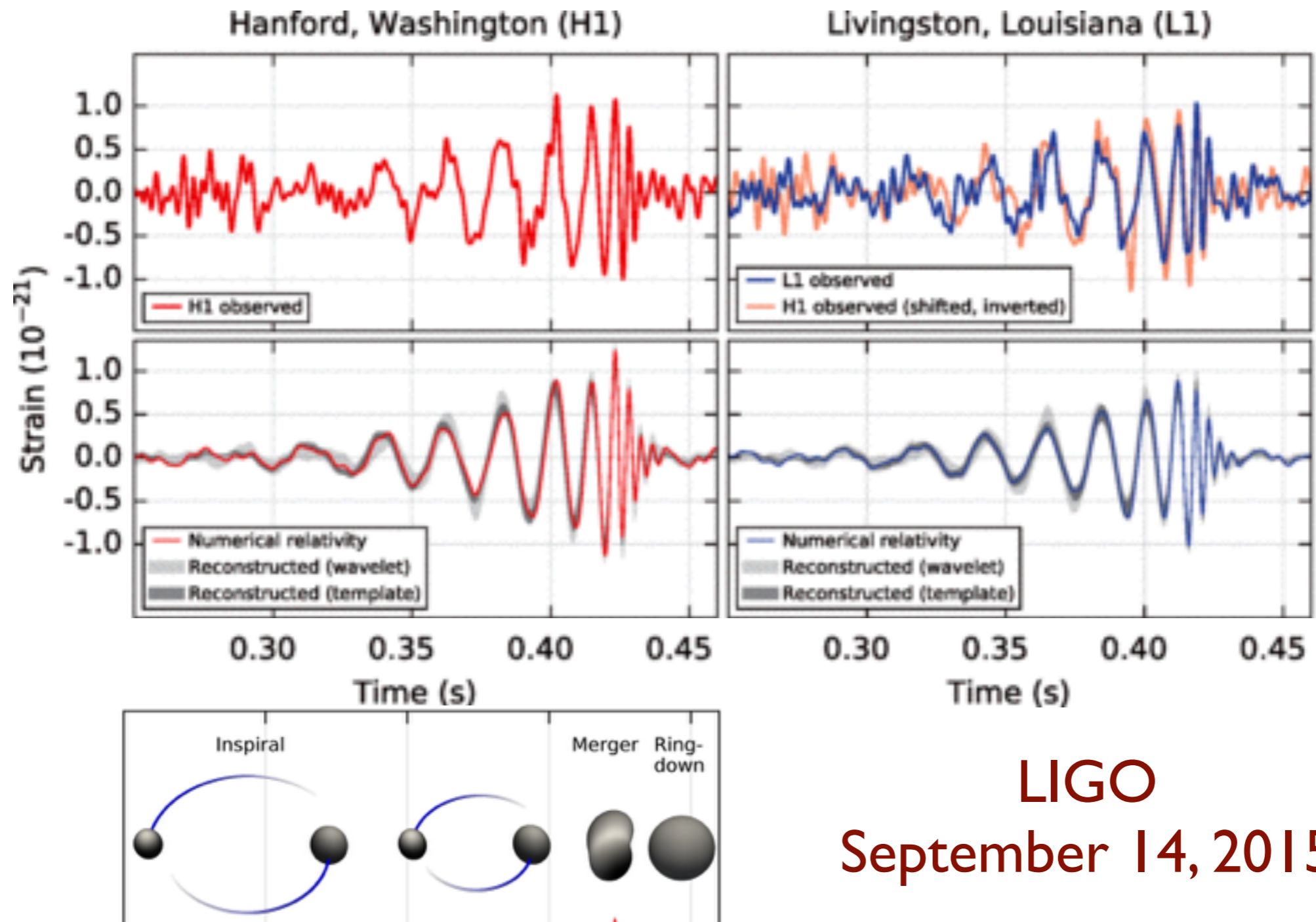
LIGO
September 14, 2015





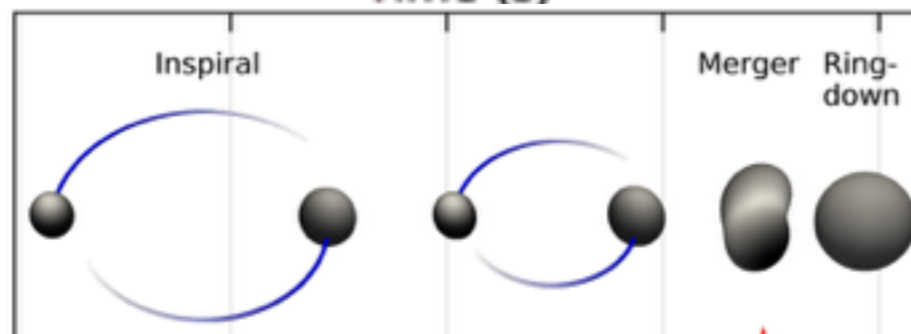
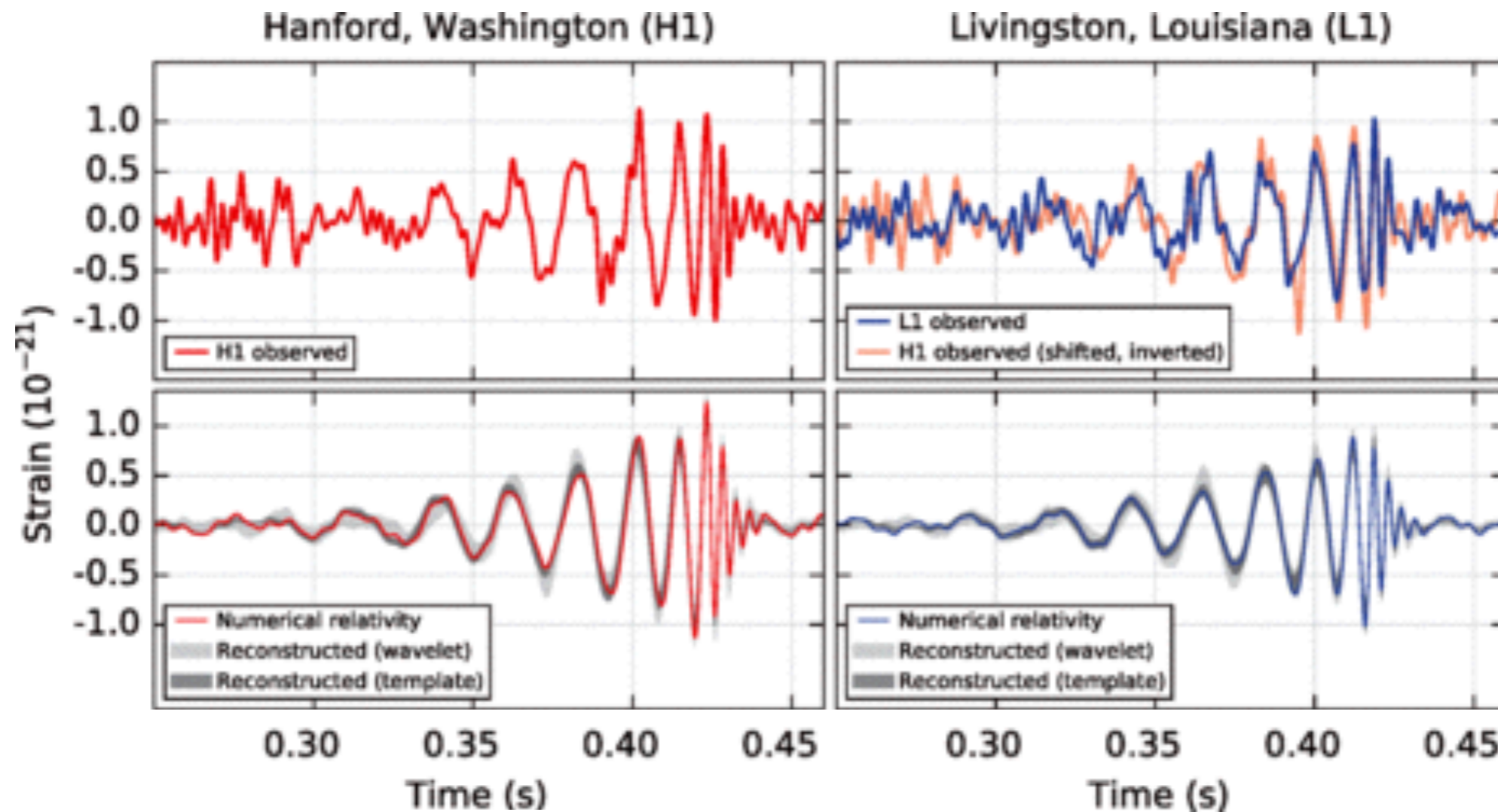
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- Black holes have a “ring-down” time, τ_r , in which they radiate energy, and stabilize to a ‘featureless’ spherical object. This time can be computed in Einstein’s general relativity theory.



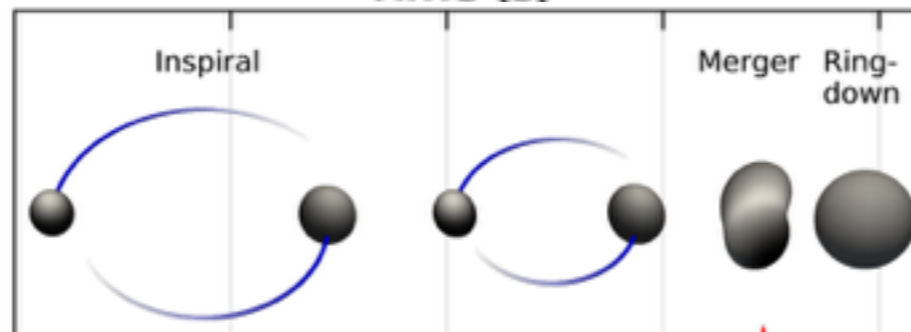
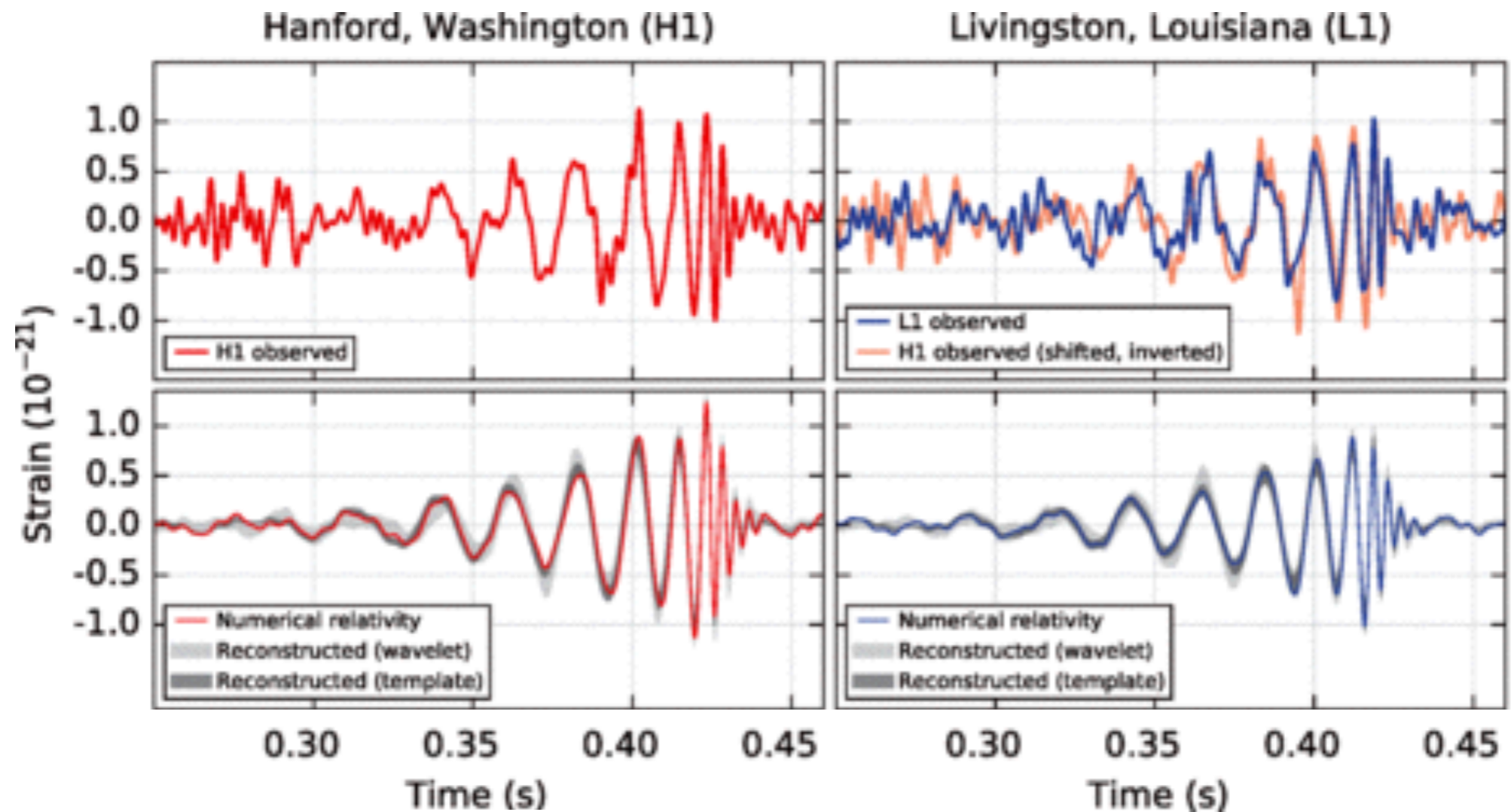
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- ‘Featureless’ black holes have a Bekenstein-Hawking entropy, and a Hawking temperature, T_H .



LIGO
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- Expressed in terms of the Hawking temperature, the ring-down time is $\tau_r \sim \hbar / (k_B T_H)$!



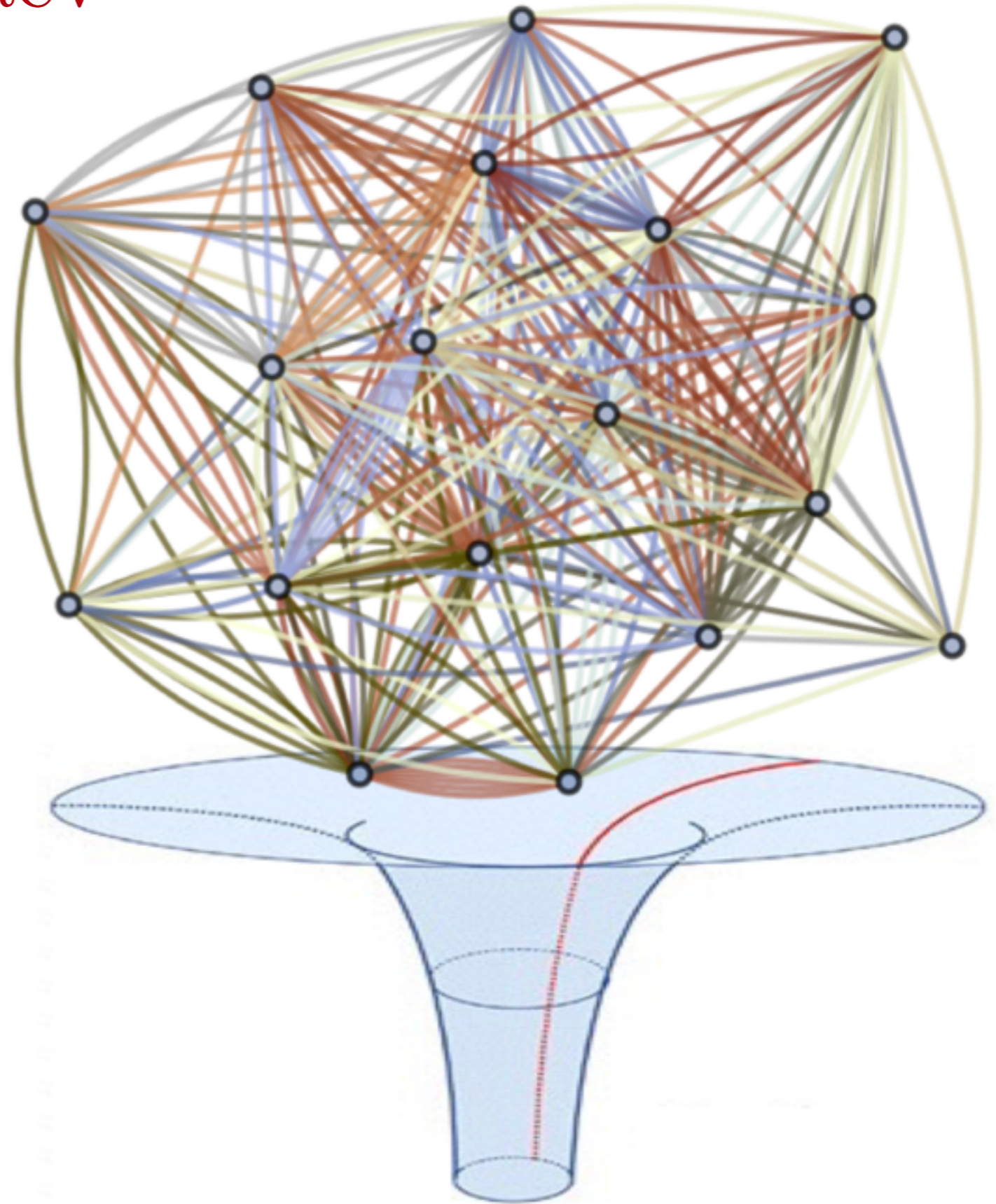
LIGO
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- Expressed in terms of the Hawking temperature, the ring-down time is $\tau_r \sim \hbar / (k_B T_H)$!
- For this black hole $T_H \approx 1$ nK, $\tau_r = 7.7$ milliseconds. (Radius of black hole = 183 km; Mass of black hole = 62 solar masses.)

- Is there a connection between strange metals and black holes?
- Why do they have the same equilibration time $\sim \hbar/(k_B T)$?

The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Has a dual representation as a black hole

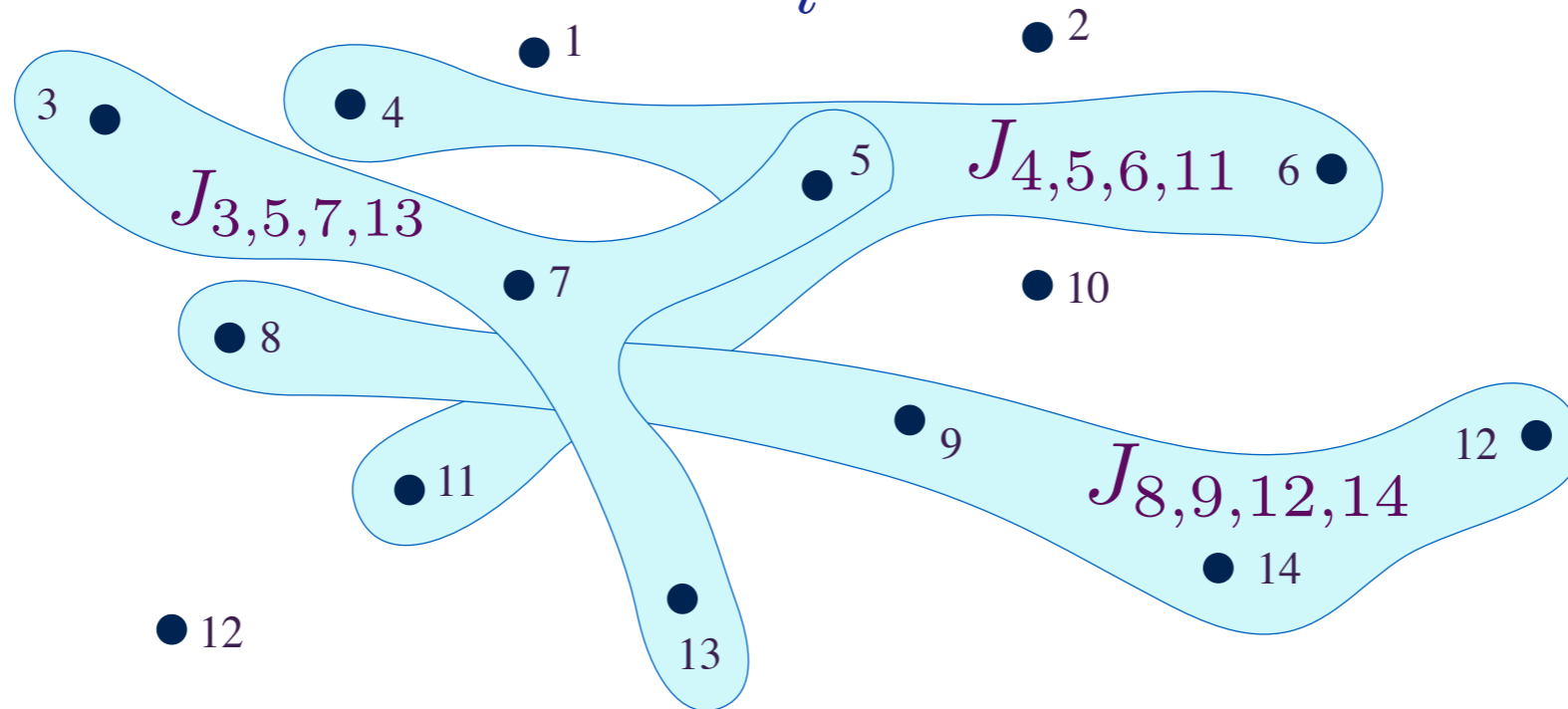


Infinite-range (SYK) model of a strange metal

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;kl}$ are independent random variables with $\overline{J_{ij;kl}} = 0$ and $\overline{|J_{ij;kl}|^2} = J^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

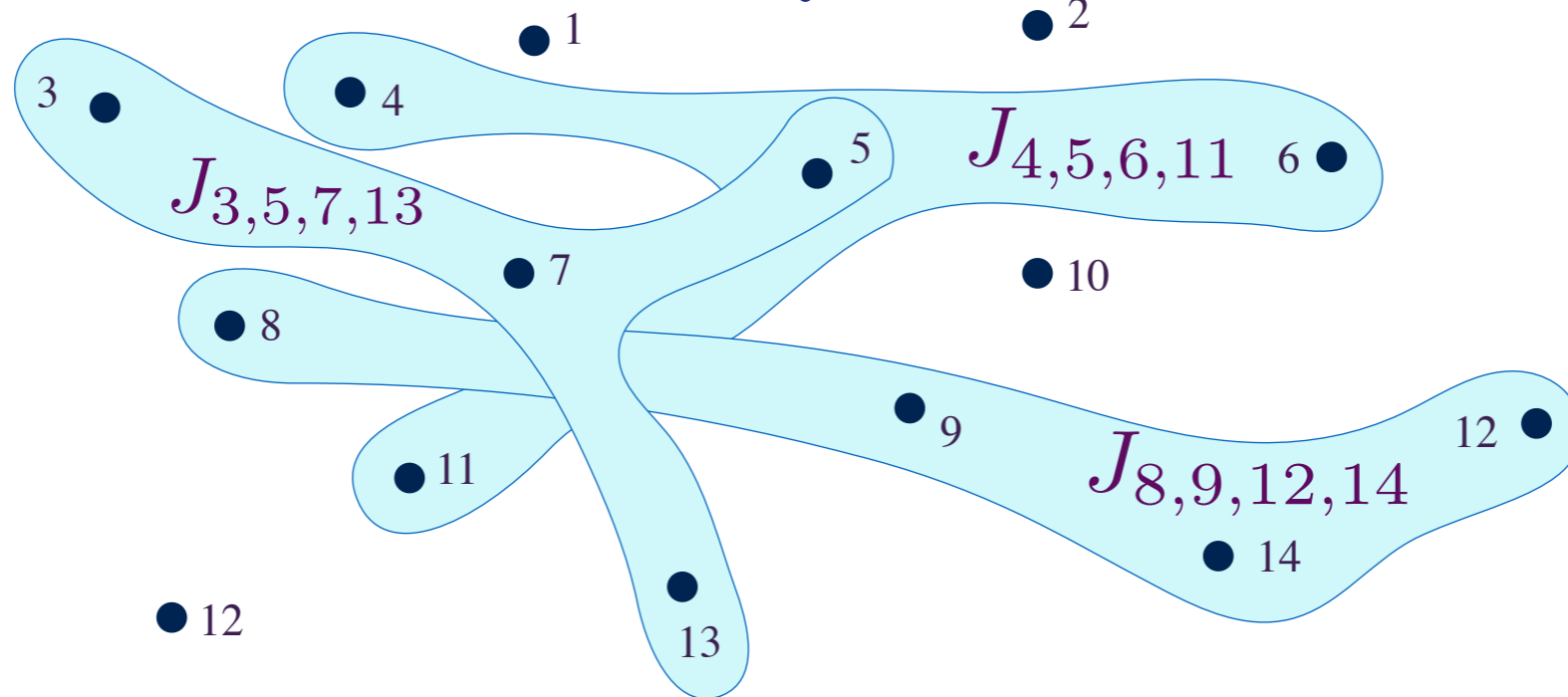
A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)

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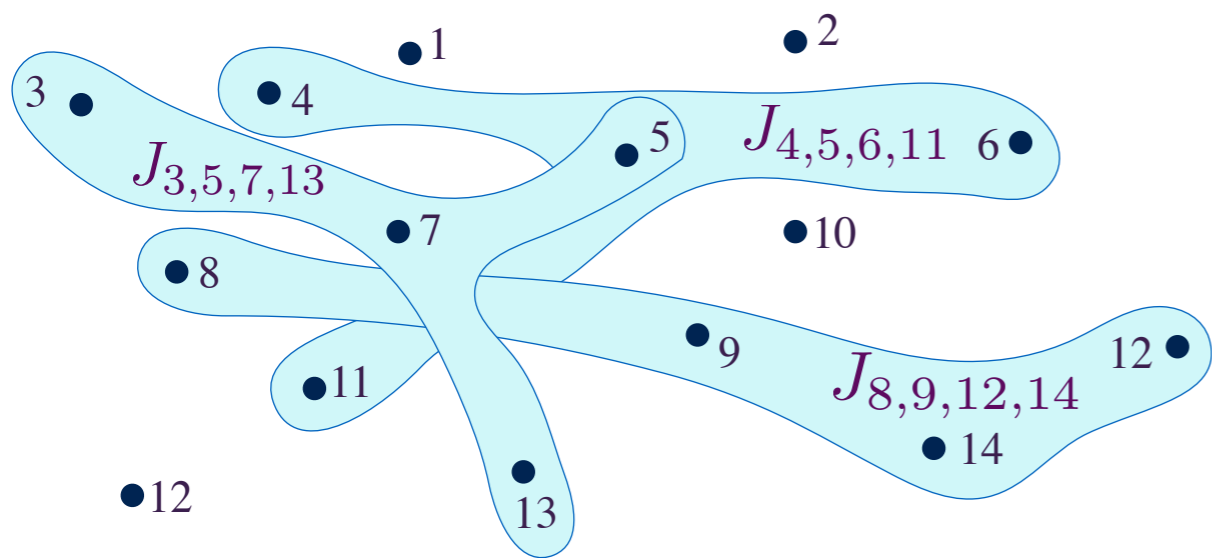


A fermion can move only by entangling with another fermion:
the Hamiltonian has “nothing but entanglement”.

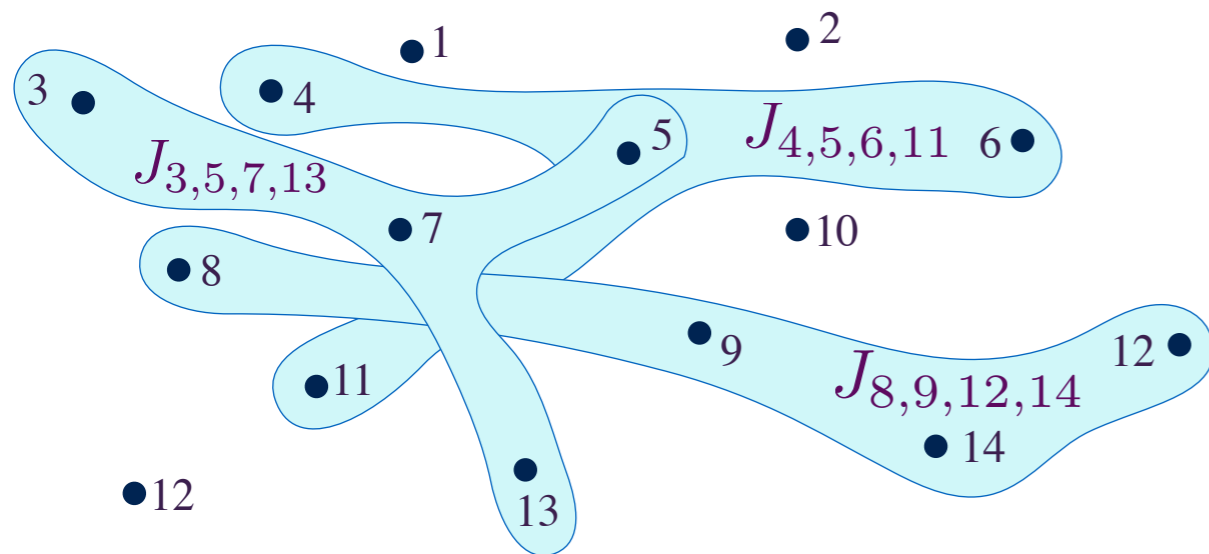
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Local fermion density of states

$$\rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

and “conformal” extension to $T > 0$.

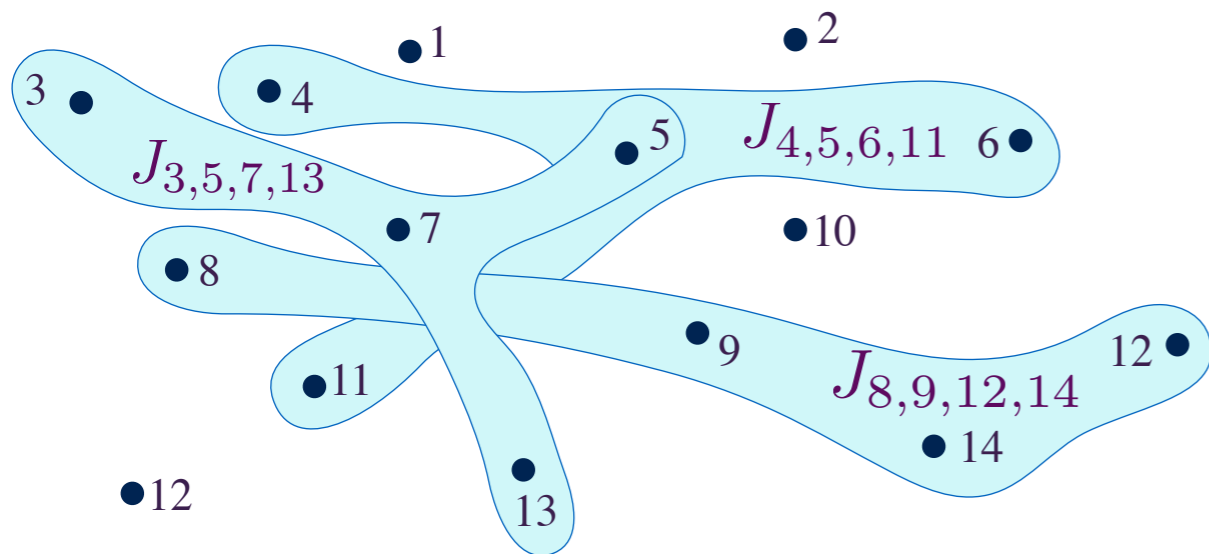
Known ‘equation of state’
determines \mathcal{E} as a function of Q

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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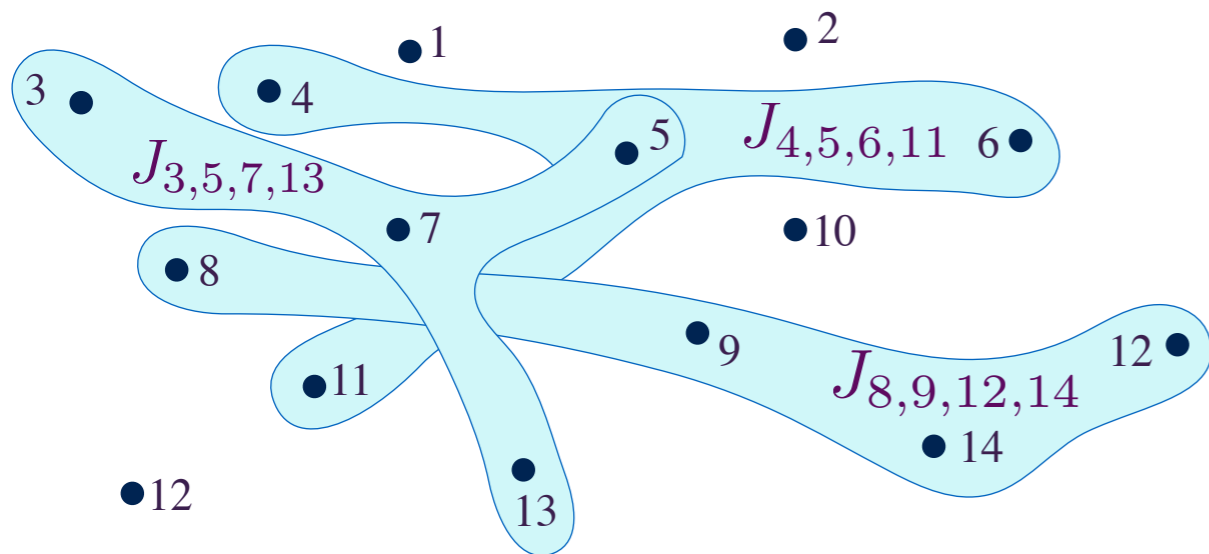
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Microscopic zero temperature
entropy density, \mathcal{S} , obeys

$$\frac{\partial \mathcal{S}}{\partial Q} = 2\pi\mathcal{E}$$

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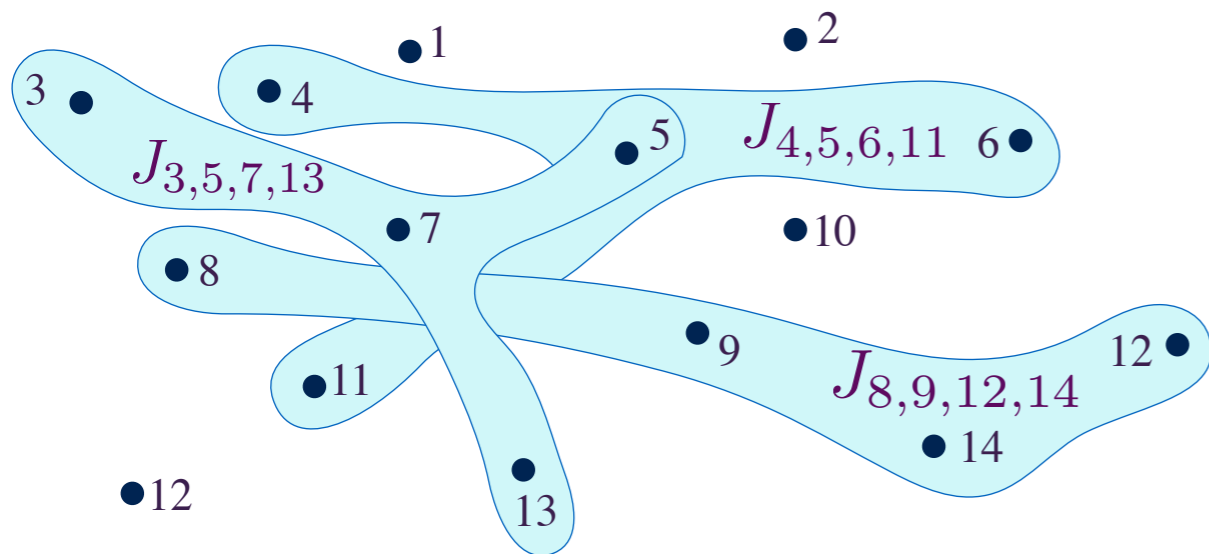
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The SYK strange metal is
holographically dual to the
gravity theory of the AdS_2
near-horizon geometry of
charged black holes

S. Sachdev,
Phys. Rev. Lett. **105**, 151602 (2010)

A. Georges, O. Parcollet, and S. Sachdev
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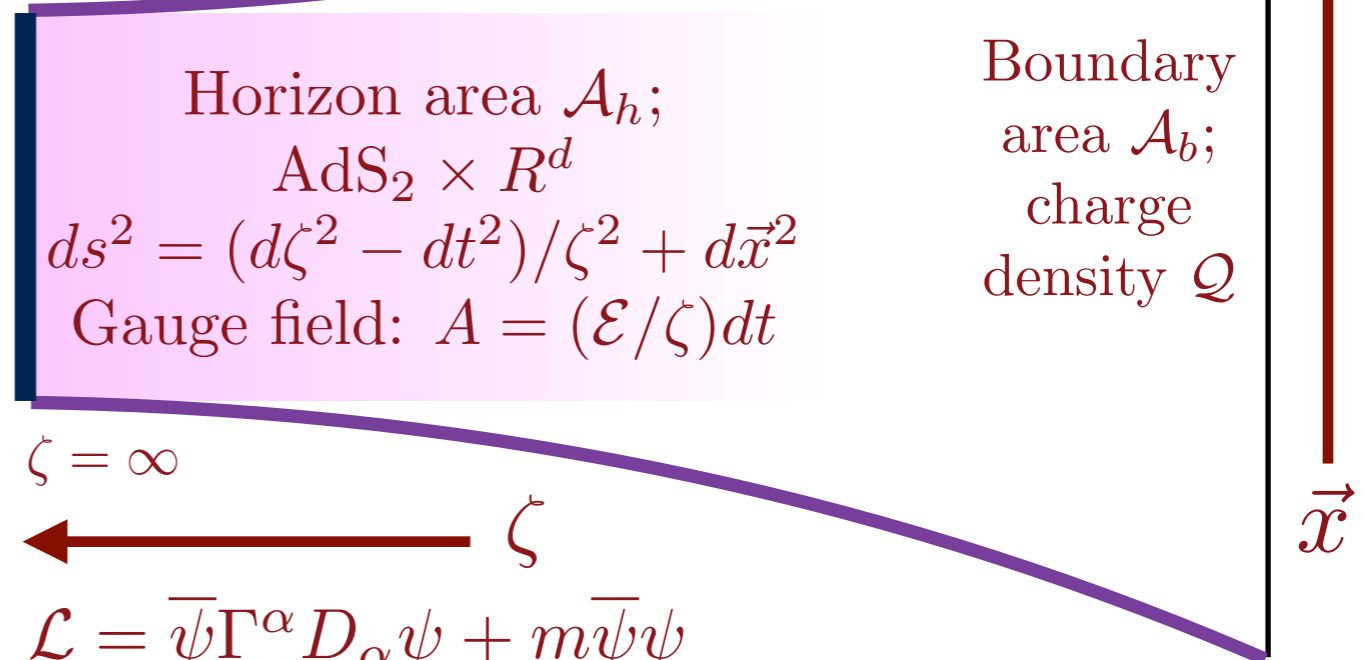
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Einstein-Maxwell theory
+ cosmological constant



Horizon area \mathcal{A}_h ;
 $\text{AdS}_2 \times R^d$
 $ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$
Gauge field: $A = (\mathcal{E}/\zeta)dt$

Boundary
area \mathcal{A}_b ;
charge
density Q

$\zeta = \infty$

ζ

$$\mathcal{L} = \bar{\psi} \Gamma^\alpha D_\alpha \psi + m \bar{\psi} \psi$$

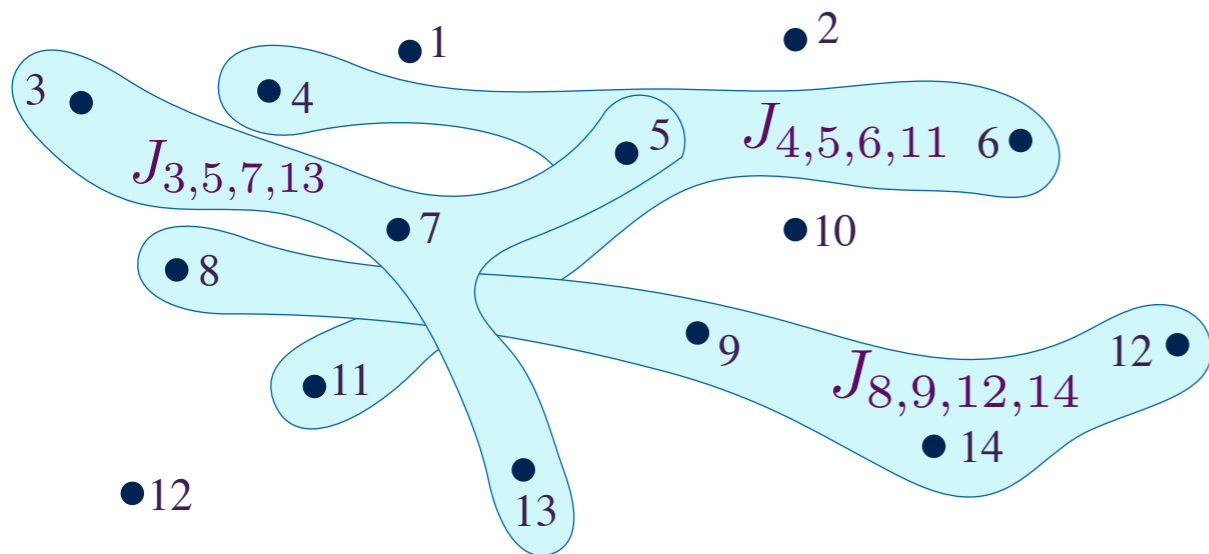
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and identical conformal extension to $T > 0$.

‘Equation of state’ relating \mathcal{E}
and Q depends upon the geometry
of spacetime far from the AdS_2

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell$$



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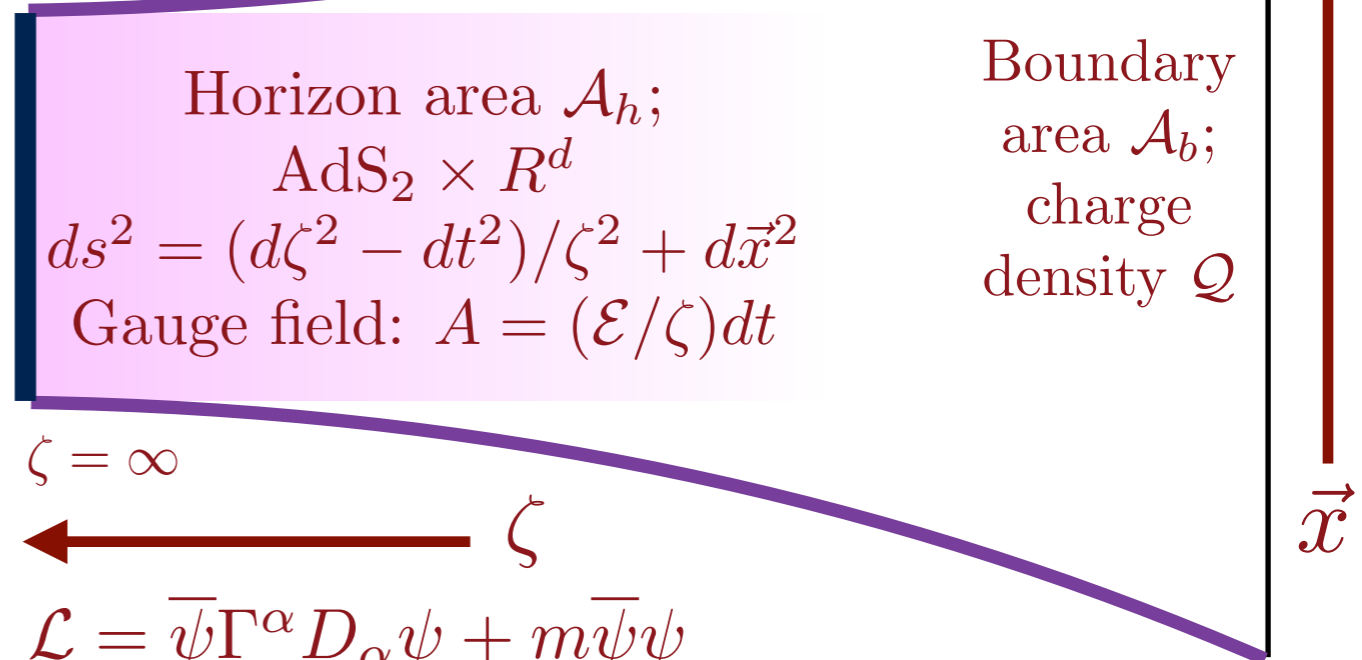
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$$\zeta$$

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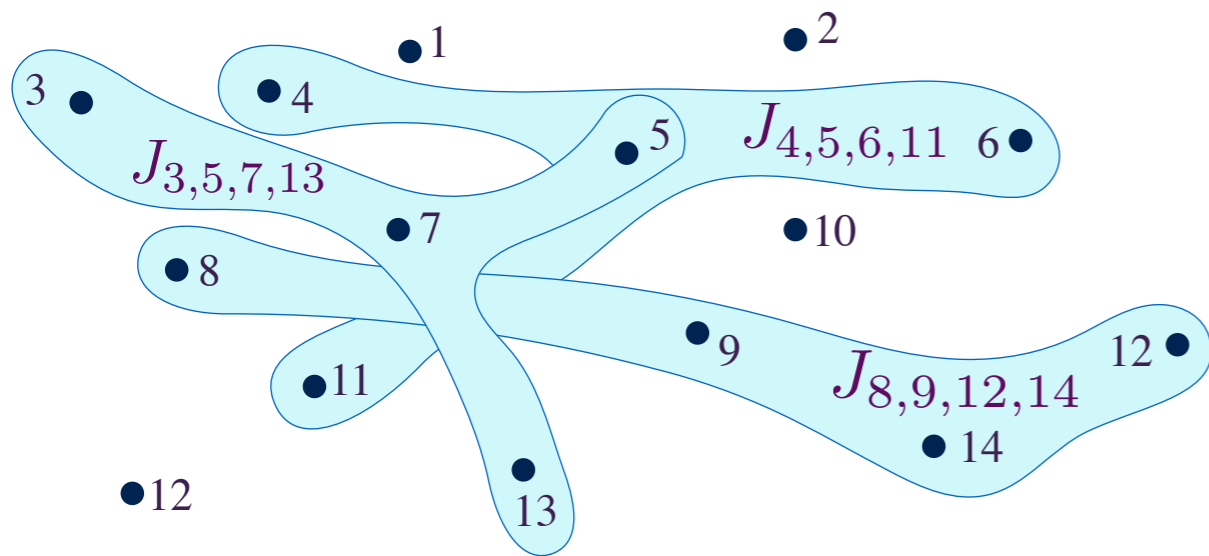
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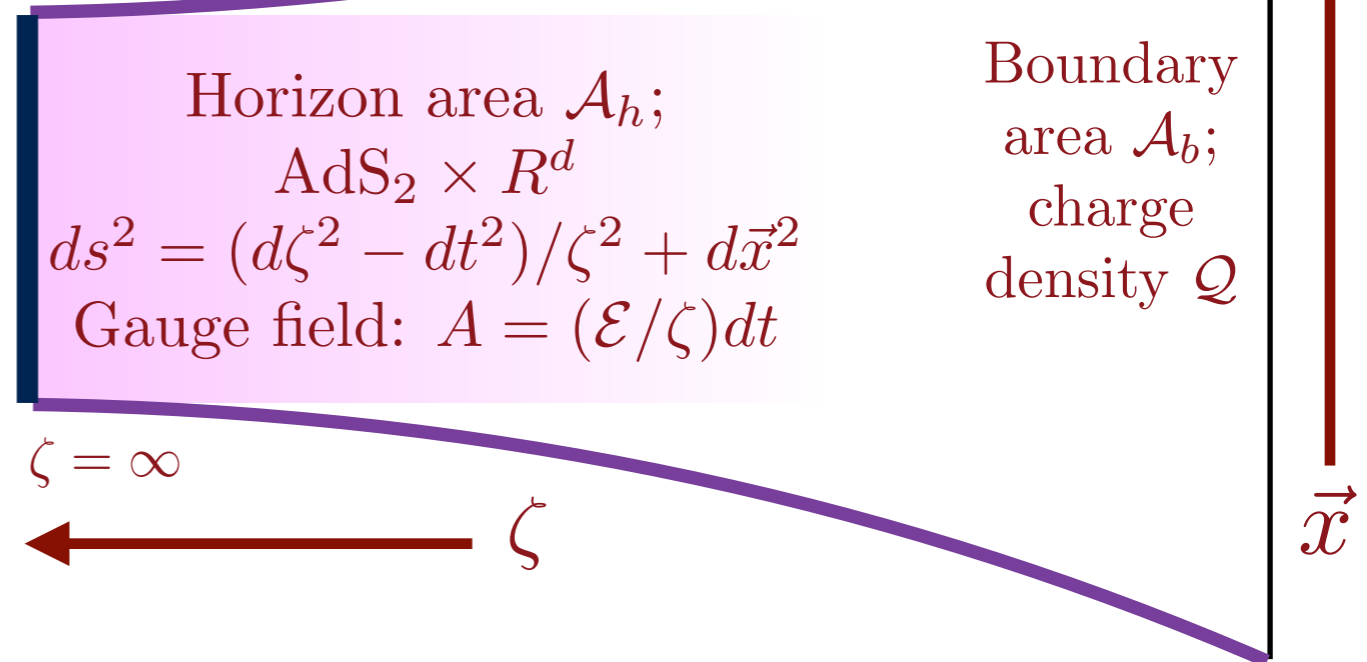
Black hole thermodynamics (classical general relativity) yields

$$\frac{\partial \mathcal{S}_{\text{BH}}}{\partial Q} = 2\pi\mathcal{E}$$

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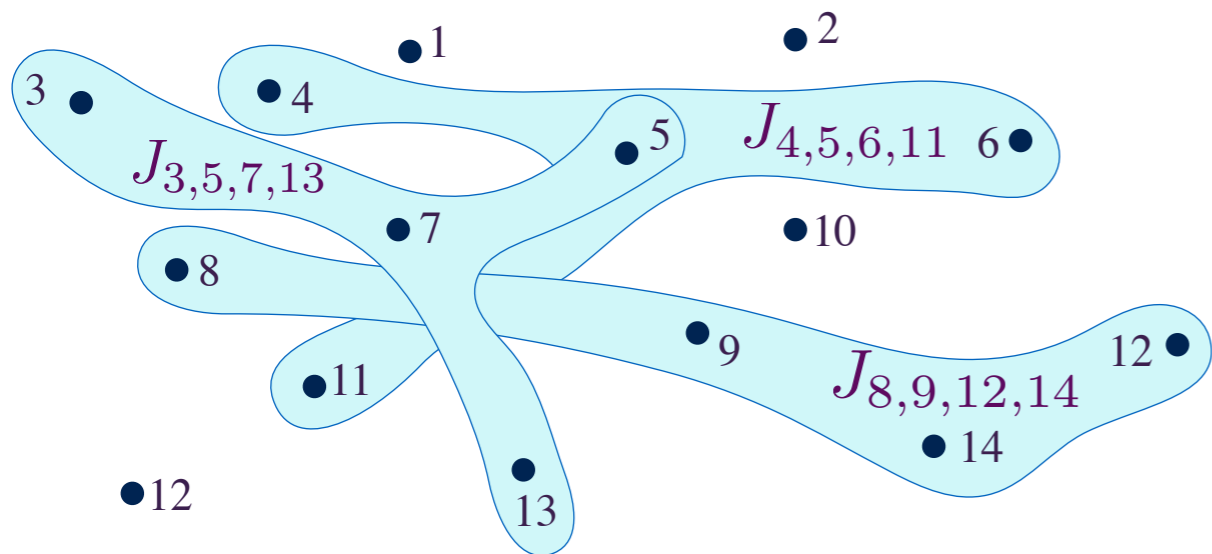
Einstein-Maxwell theory
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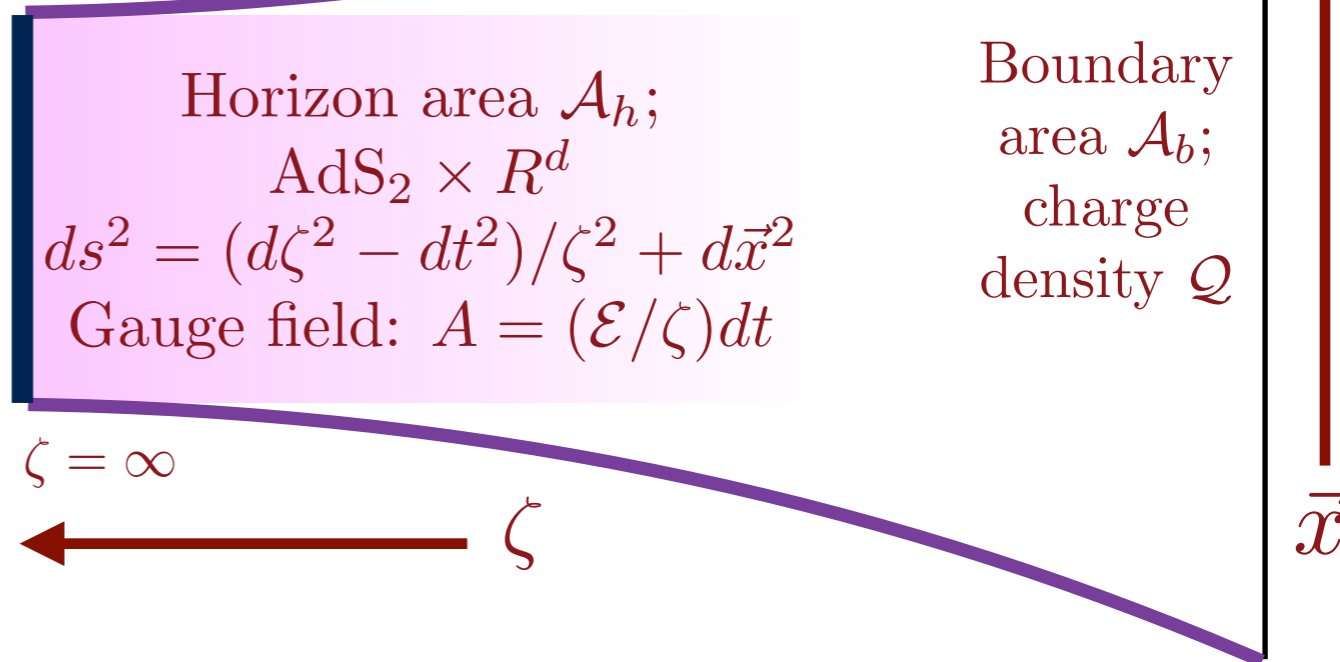
- The two models have the same symmetry at low energies: reparameterization invariance spontaneously broken to $SL(2, \mathbb{R})$. At low energies they are described by the same effective action (the “Schwarzian”) for a scalar field which is the remnant on the graviton on AdS_2 .

A. Kitaev, unpublished; A. Almheiri and J. Polchinski, JHEP 1511 (2015) 014; J. Polchinski and V. Rosenhaus, arXiv: 1601.06768;
J. Maldacena and D. Stanford, arXiv: 1604.07818; K. Jensen, arXiv: 1605.06098;
J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv: 1606.03438; A. Almheiri and B. Kang, arXiv: 1606.04108;
A. Jevicki, K. Suzuki, and J. Yoon, arXiv: 1603.06246; A. Jevicki and K. Suzuki, arXiv: 1608.7567

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N J_{ij;k\ell} c_i^\dagger c_j^\dagger c_k c_\ell$$



Einstein-Maxwell theory
+ cosmological constant



- The scrambling times of the SYK model and of black holes in Einstein gravity saturate the bound on quantum chaos

$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

S. Shenker and D. Stanford, arXiv:1306.0622; J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409; A. Kitaev, unpublished; J. Maldacena and D. Stanford, arXiv:1604.07818; K. Jensen, arXiv:1605.06098.

- Is there a connection between strange metals and black holes?
- Why do they have the same equilibration time $\sim \hbar/(k_B T)$?

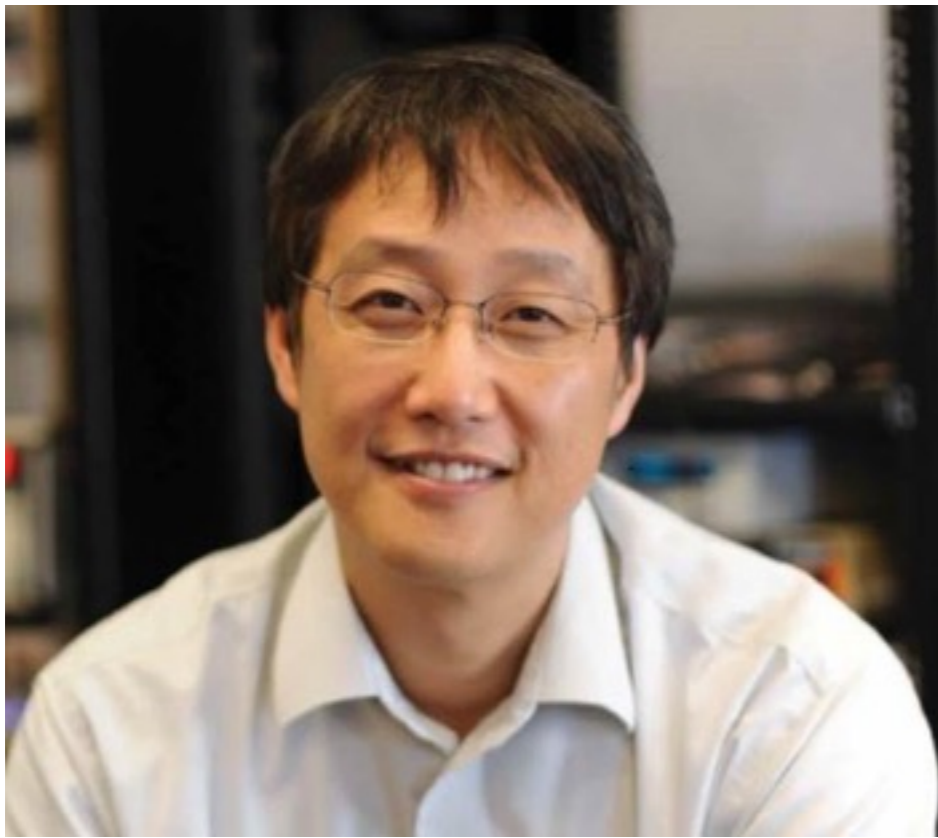
- Is there a connection between strange metals and black holes?
Yes, *e.g.* the SYK model.
- Why do they have the same equilibration time $\sim \hbar/(k_B T)$?
Strange metals don't have quasiparticles and thermalize rapidly;
Black holes are “fast scramblers”.

- Graphene

Strange metal transport

Theoretical predictions inspired by holography

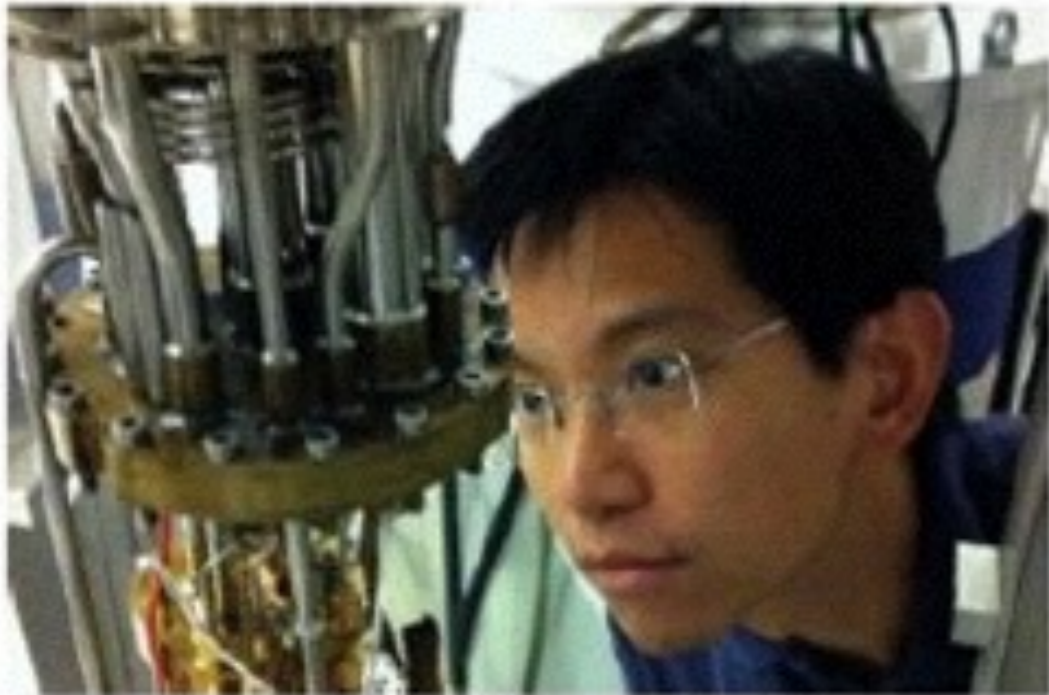
Comparison with experiments



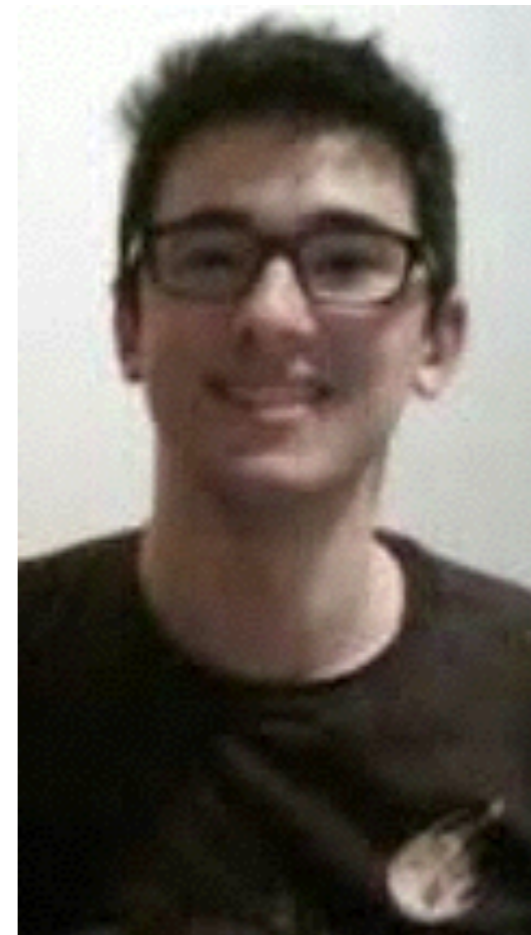
Philip Kim



Jesse Crossno

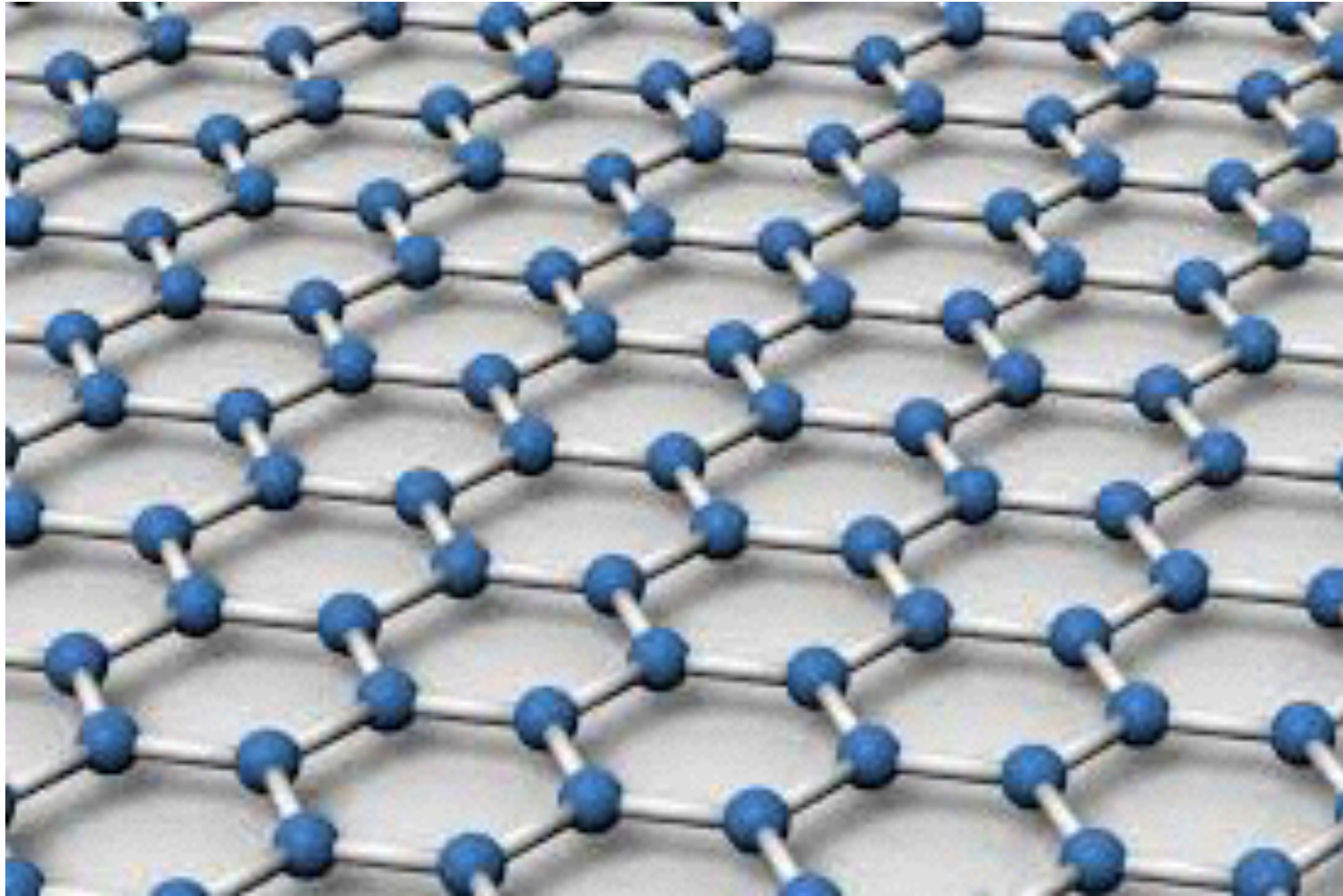


Kin Chung Fong

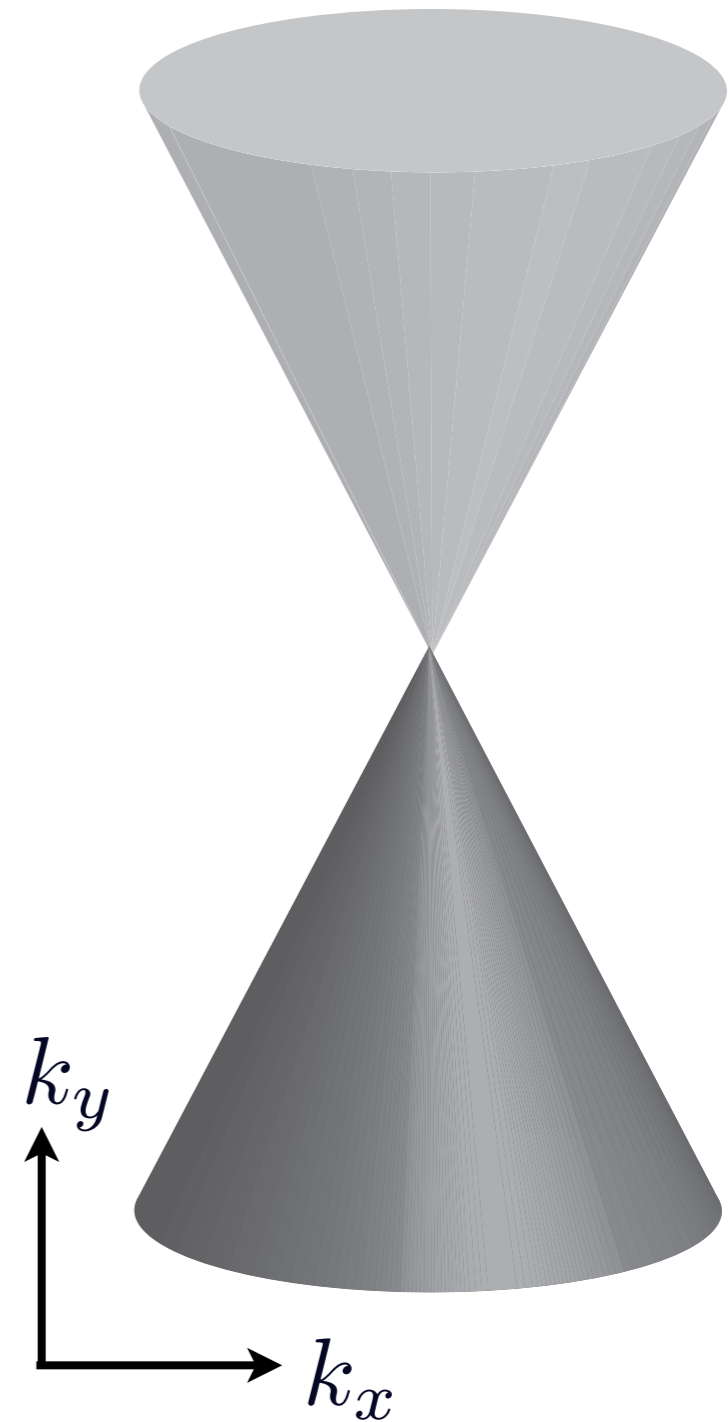


Andrew Lucas

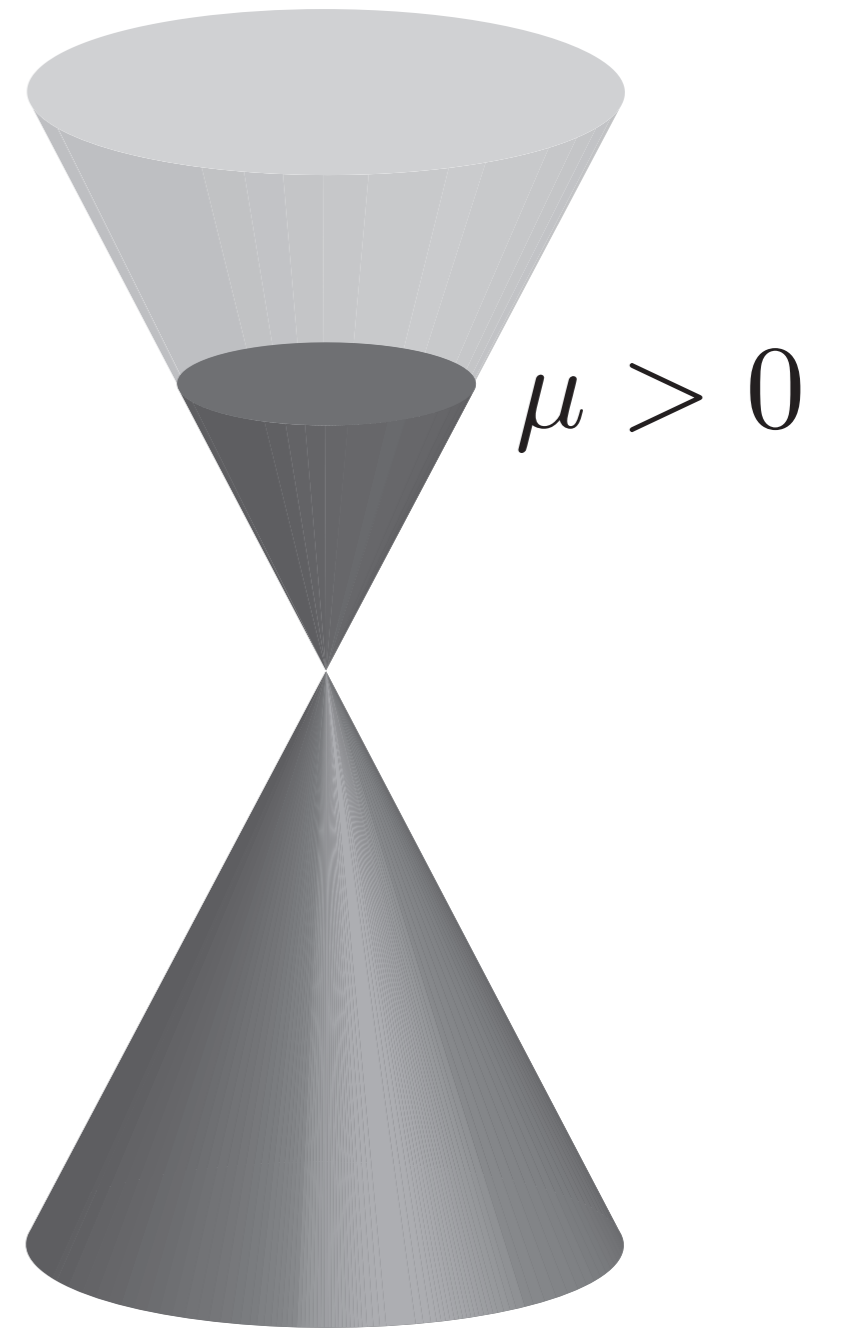
Graphene



Same “Hubbard” model as for ultracold atoms, but for electrons on the honeycomb lattice

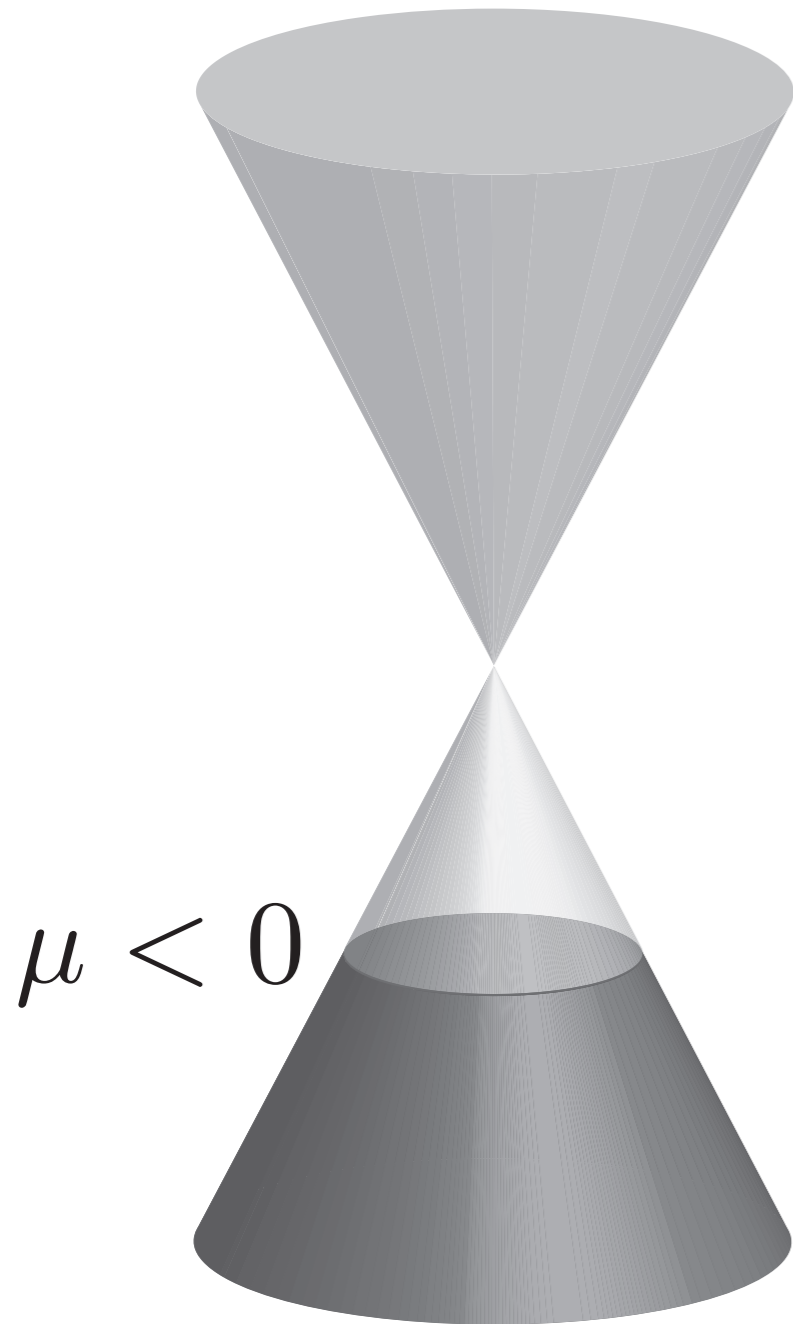


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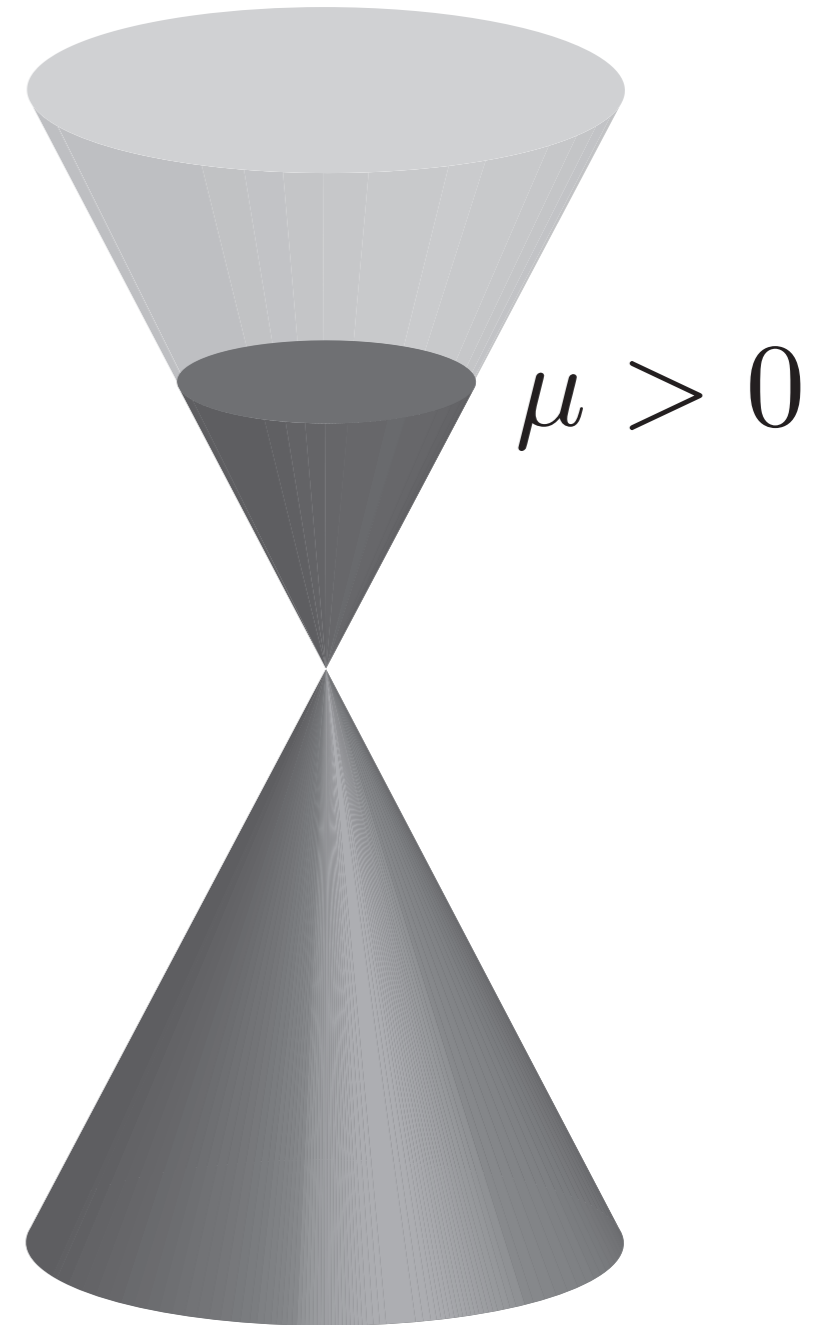


**Electron
Fermi surface**

Graphene

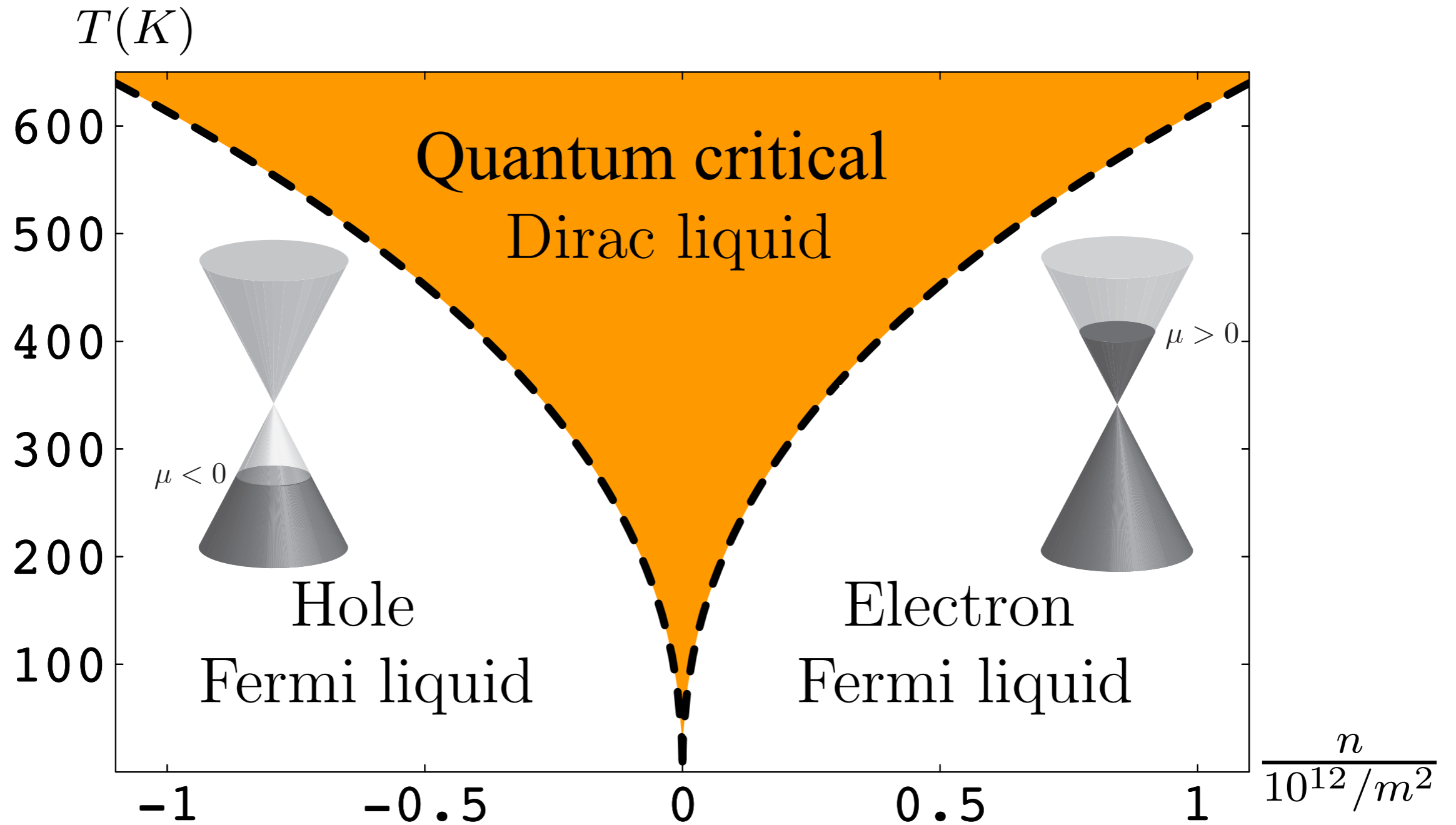


**Hole
Fermi surface**



**Electron
Fermi surface**

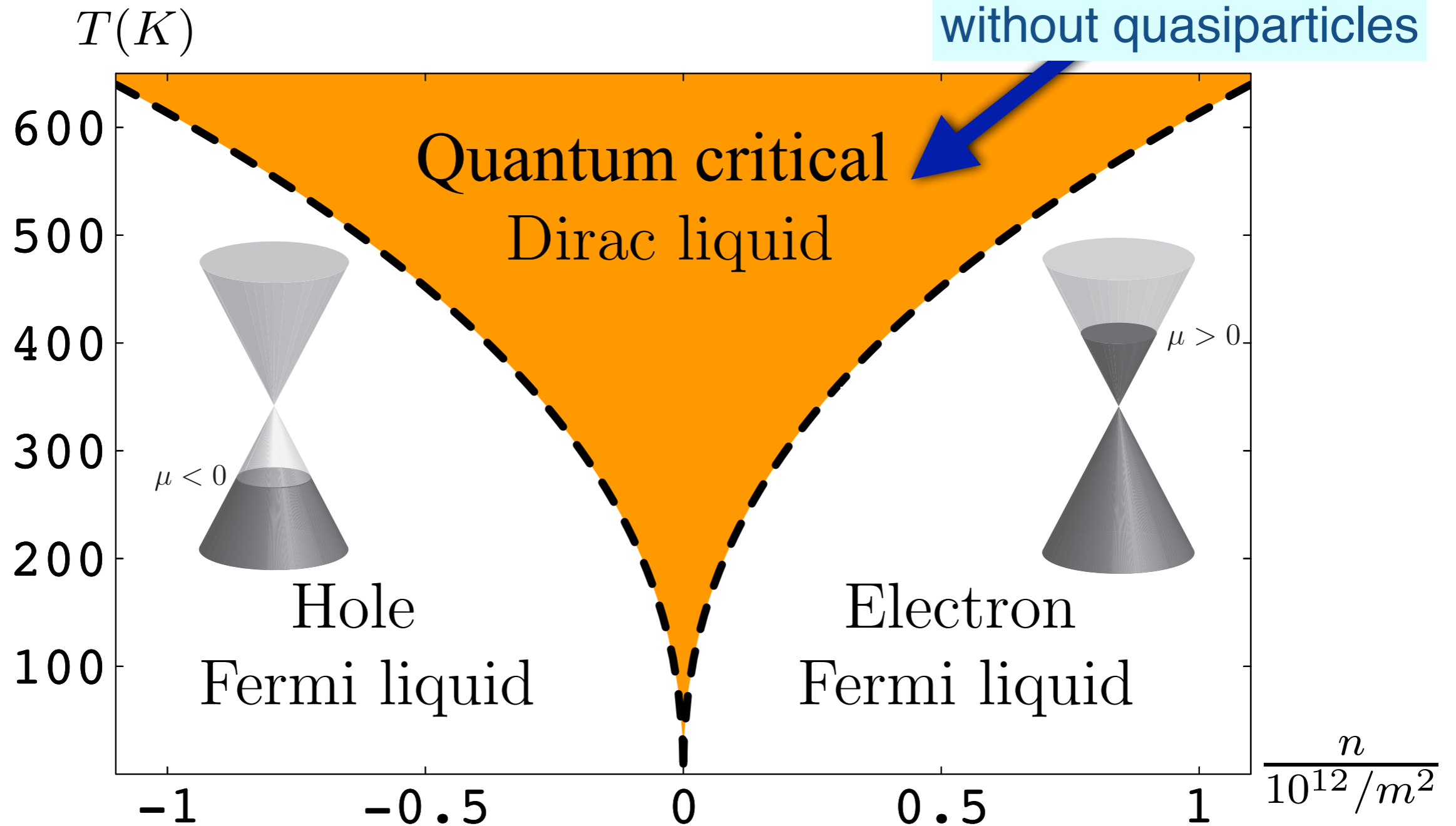
Graphene



D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

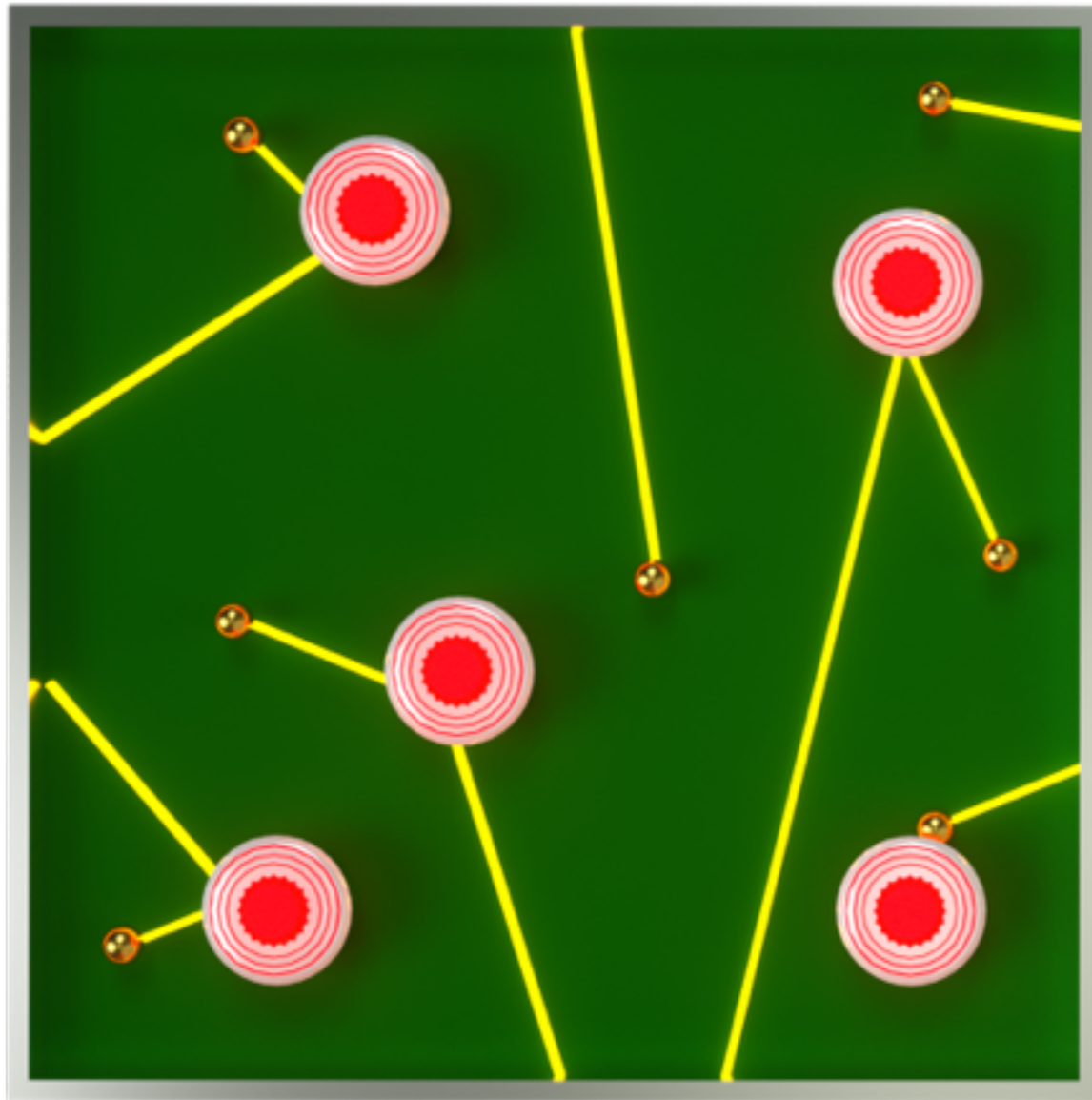
Graphene

Predicted
“strange metal”
without quasiparticles

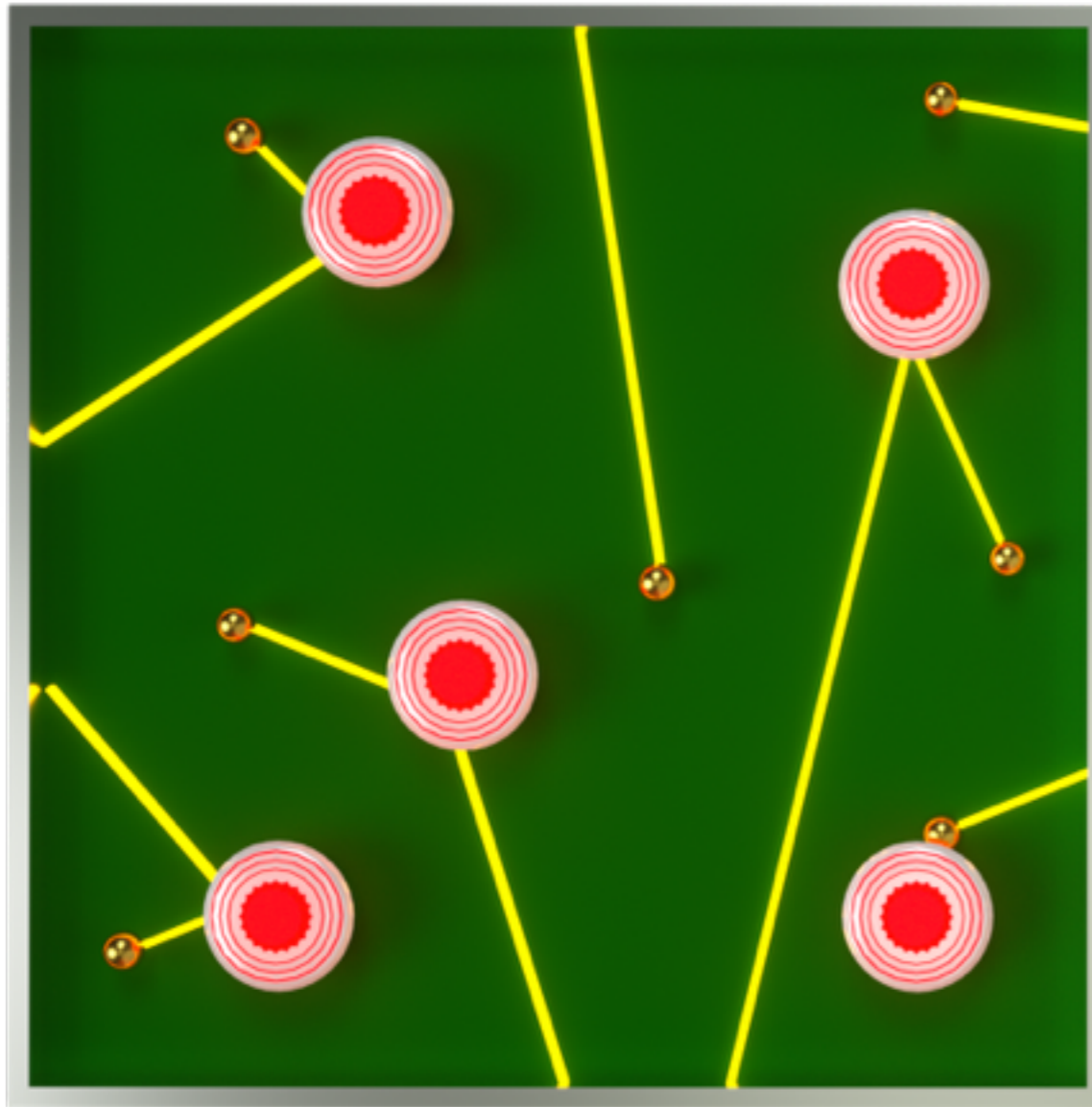


M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)

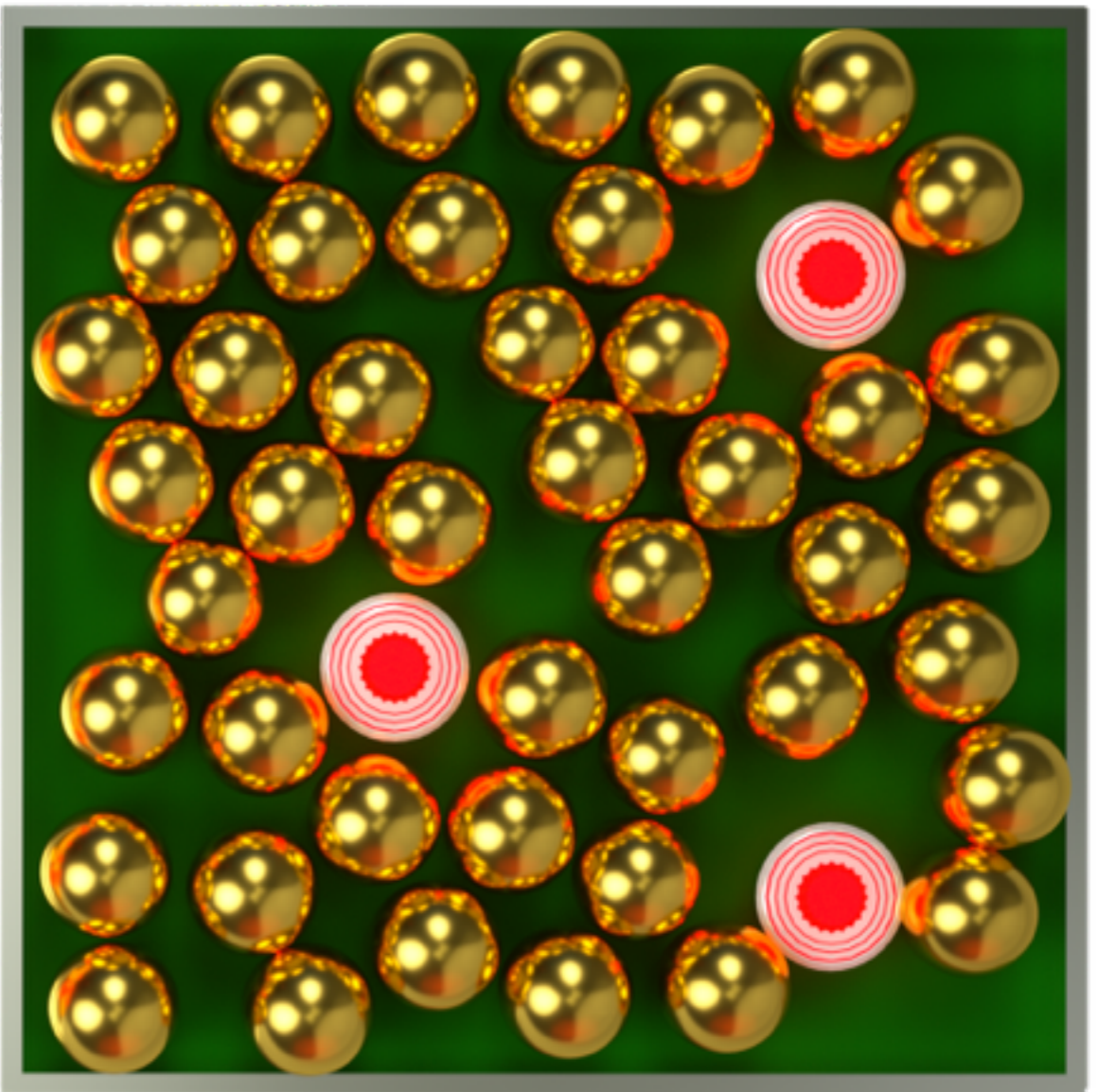
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)



Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events

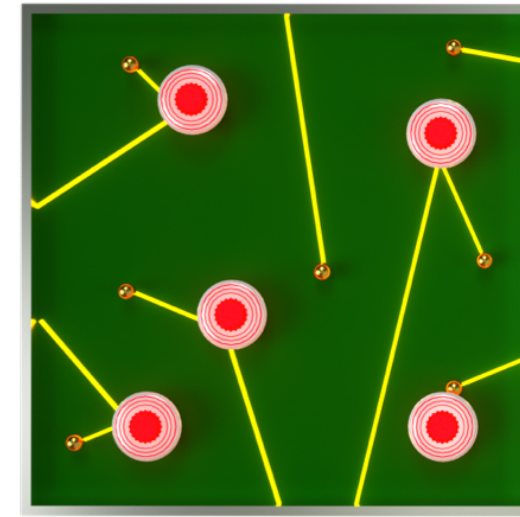


Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events



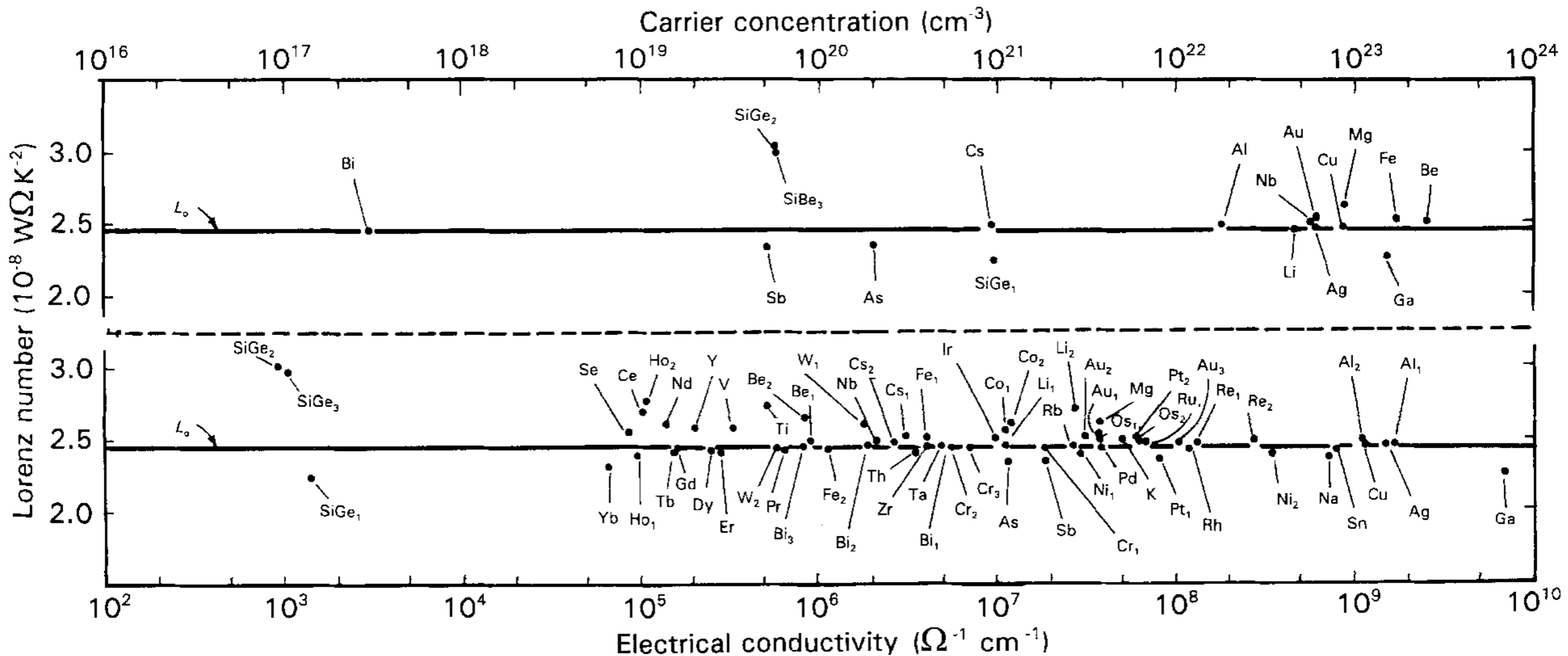
Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron “liquid” then “flows” around impurities

Thermal and electrical conductivity with quasiparticles

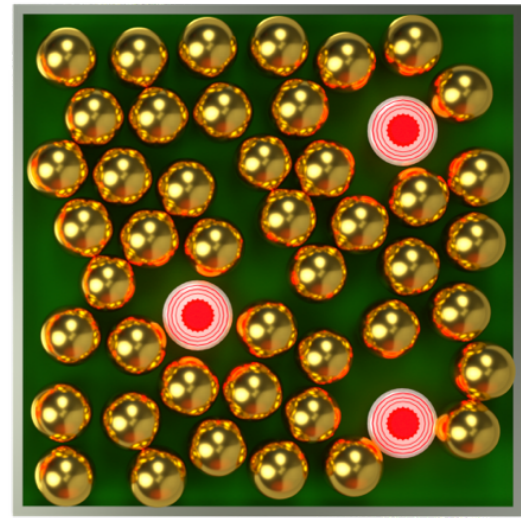


- Wiedemann-Franz law in a Fermi liquid:

$$L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \frac{\text{W} \cdot \Omega}{\text{K}^2}.$$



Transport in Strange Metals



For a strange metal
with a “relativistic” Hamiltonian,
hydrodynamic, holographic,
and memory function methods yield

$$\text{Lorentz ratio } L = \kappa / (T\sigma) \\ = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 Q^2 \tau_{\text{imp}} / (\mathcal{H} \sigma_Q)\right)^2}$$

$Q \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density

$\sigma_Q \rightarrow$ quantum critical conductivity

$\tau_{\text{imp}} \rightarrow$ momentum relaxation time from impurities.

Note that for a clean system ($\tau_{\text{imp}} \rightarrow \infty$ first),

the Lorentz ratio diverges $L \sim 1/Q^4$,

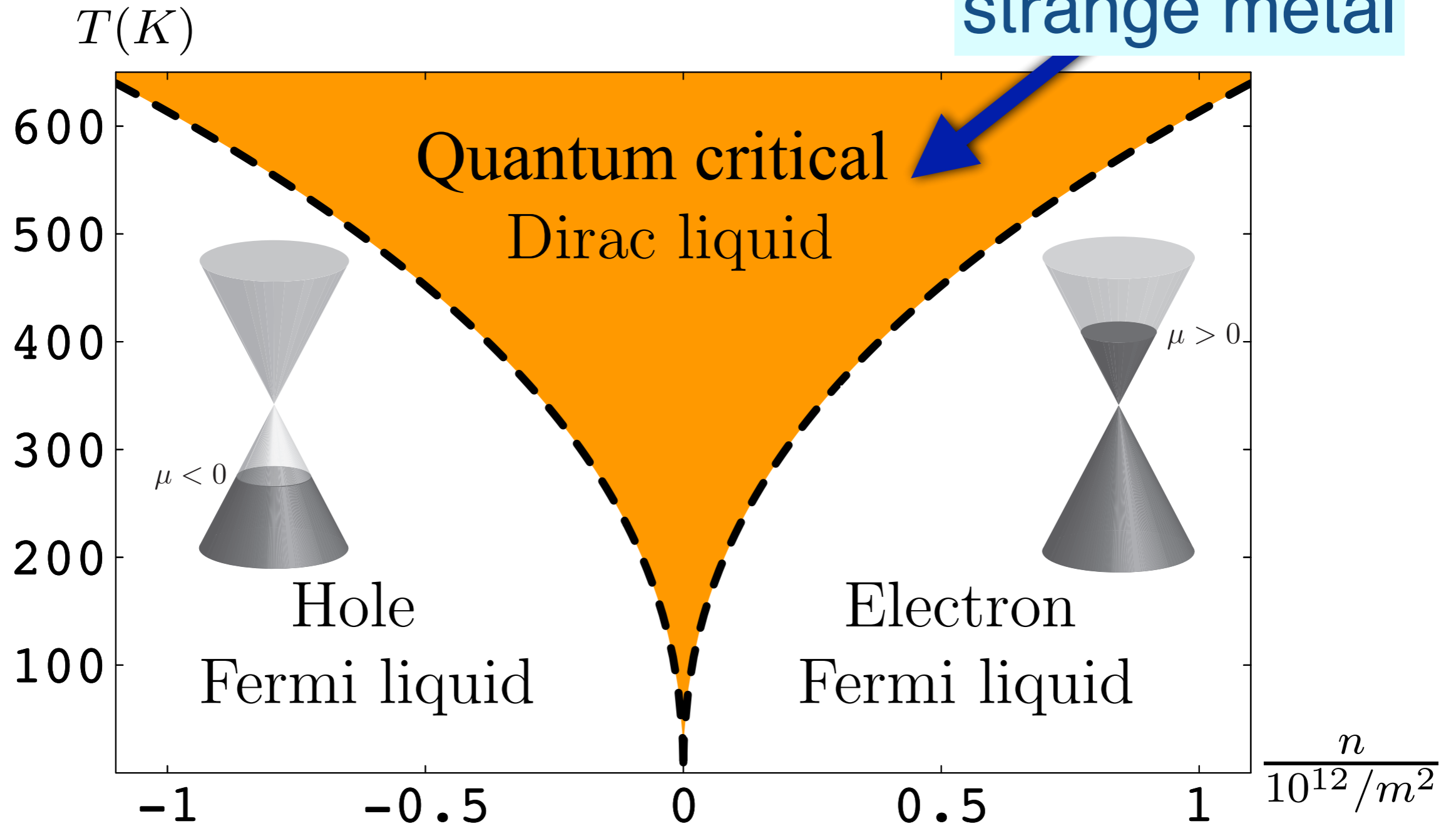
as we approach “zero” electron density at the Dirac point.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene

Predicted
strange metal

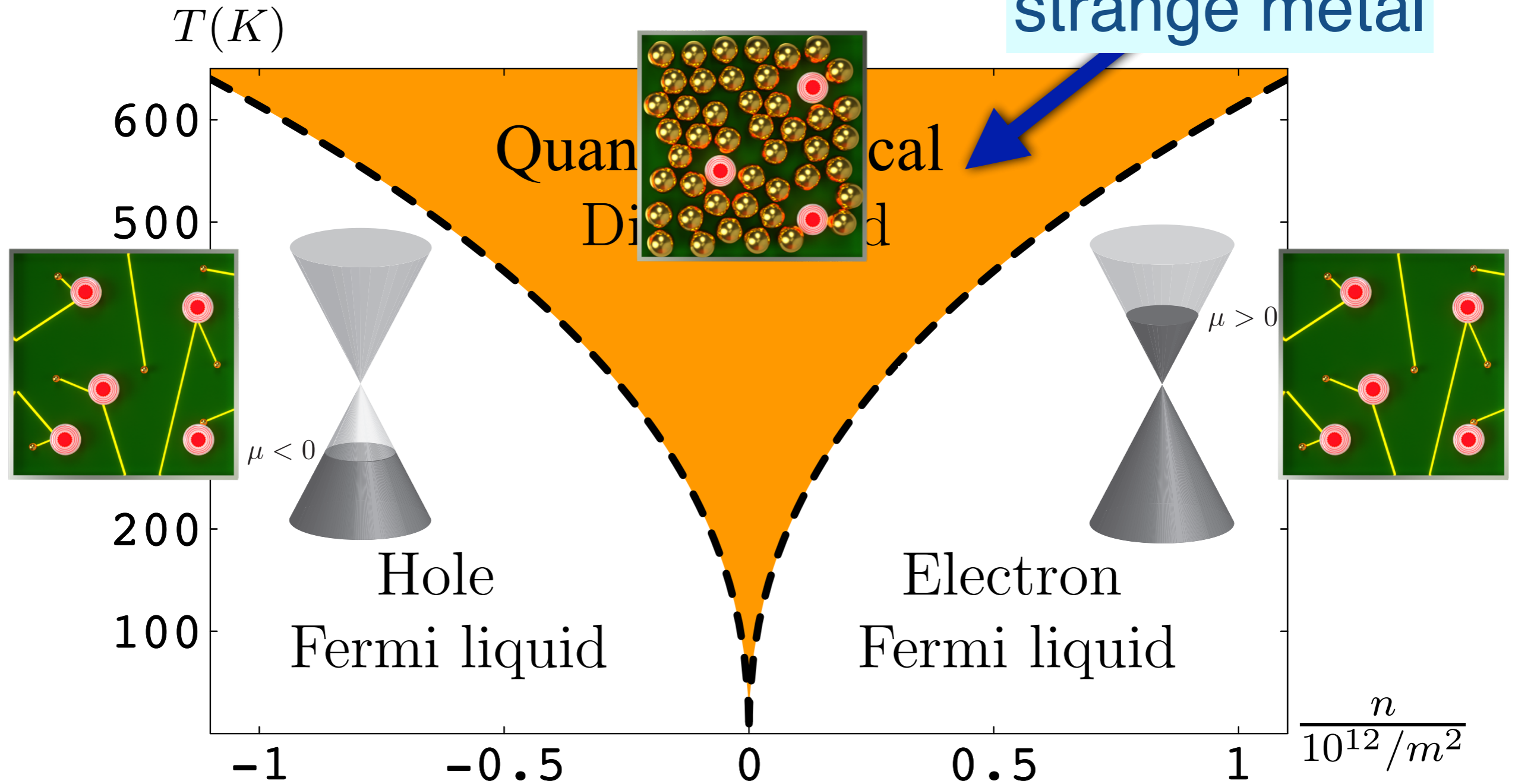


M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)

M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Graphene

Predicted
strange metal

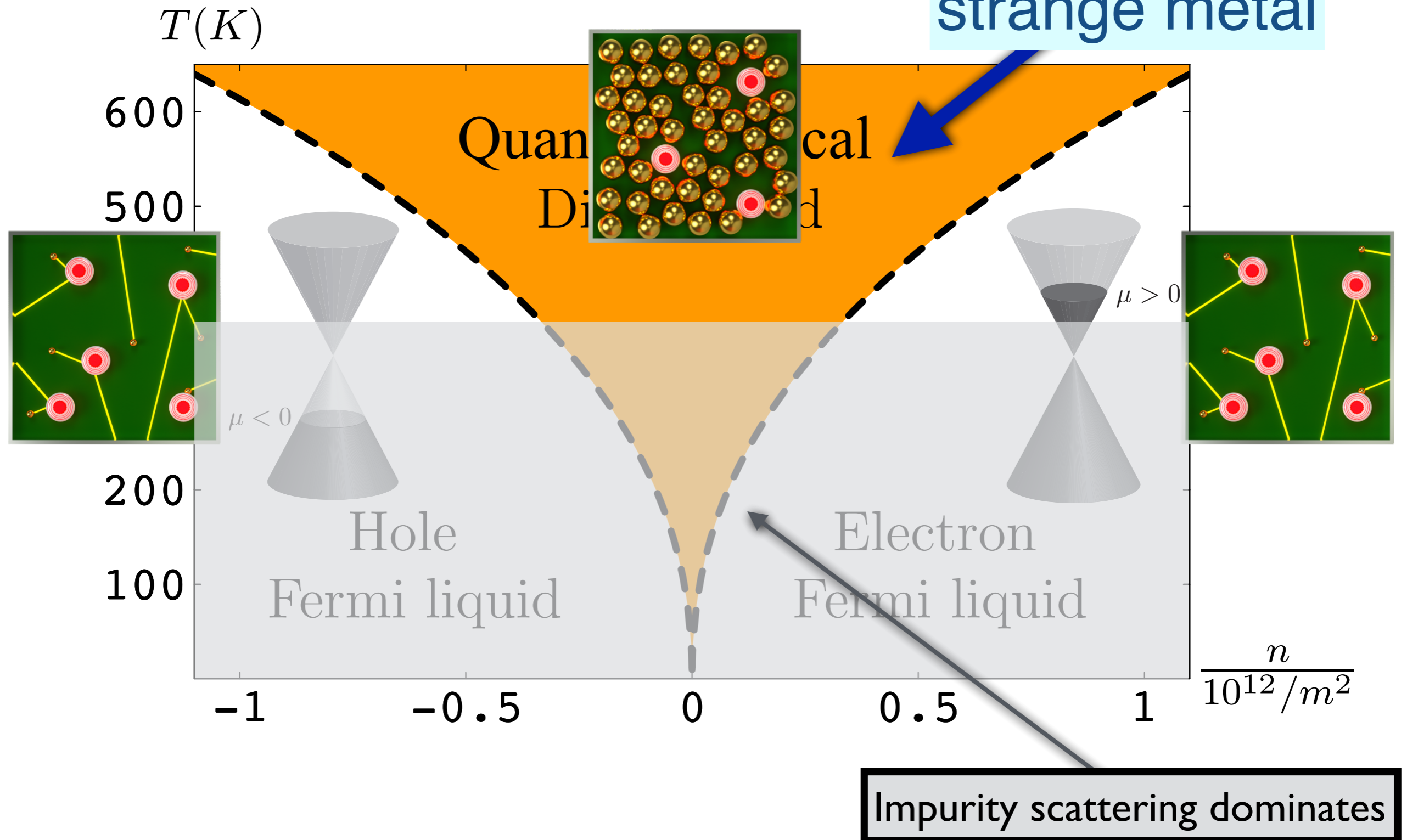


M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)

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Graphene

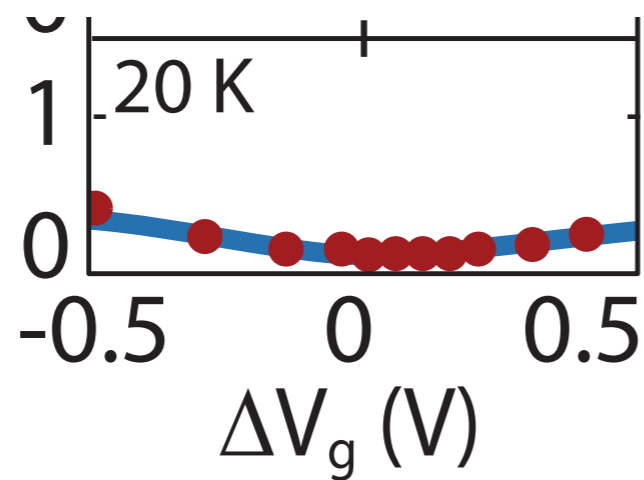
Predicted
strange metal



M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)

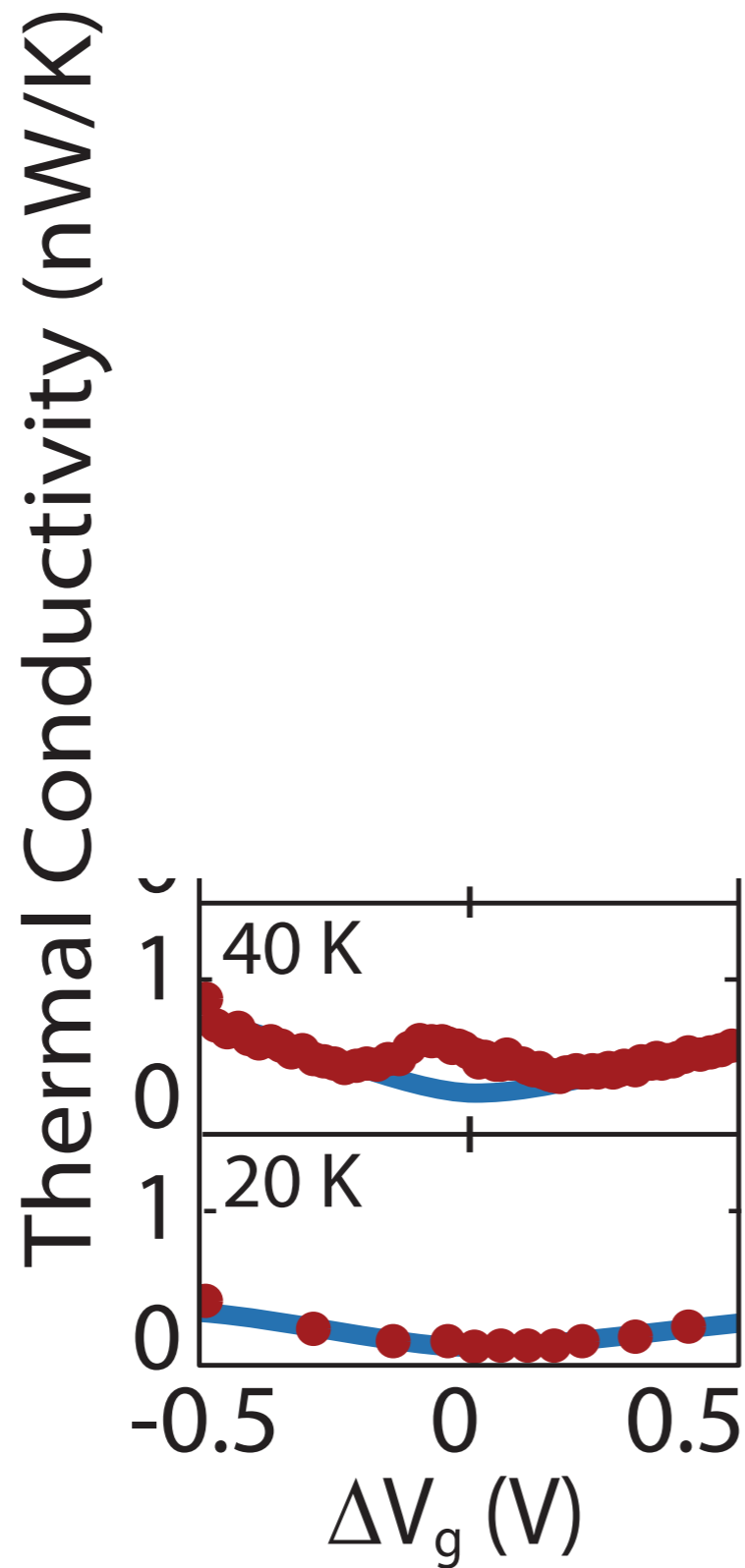
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

Thermal Conductivity (nW/K)



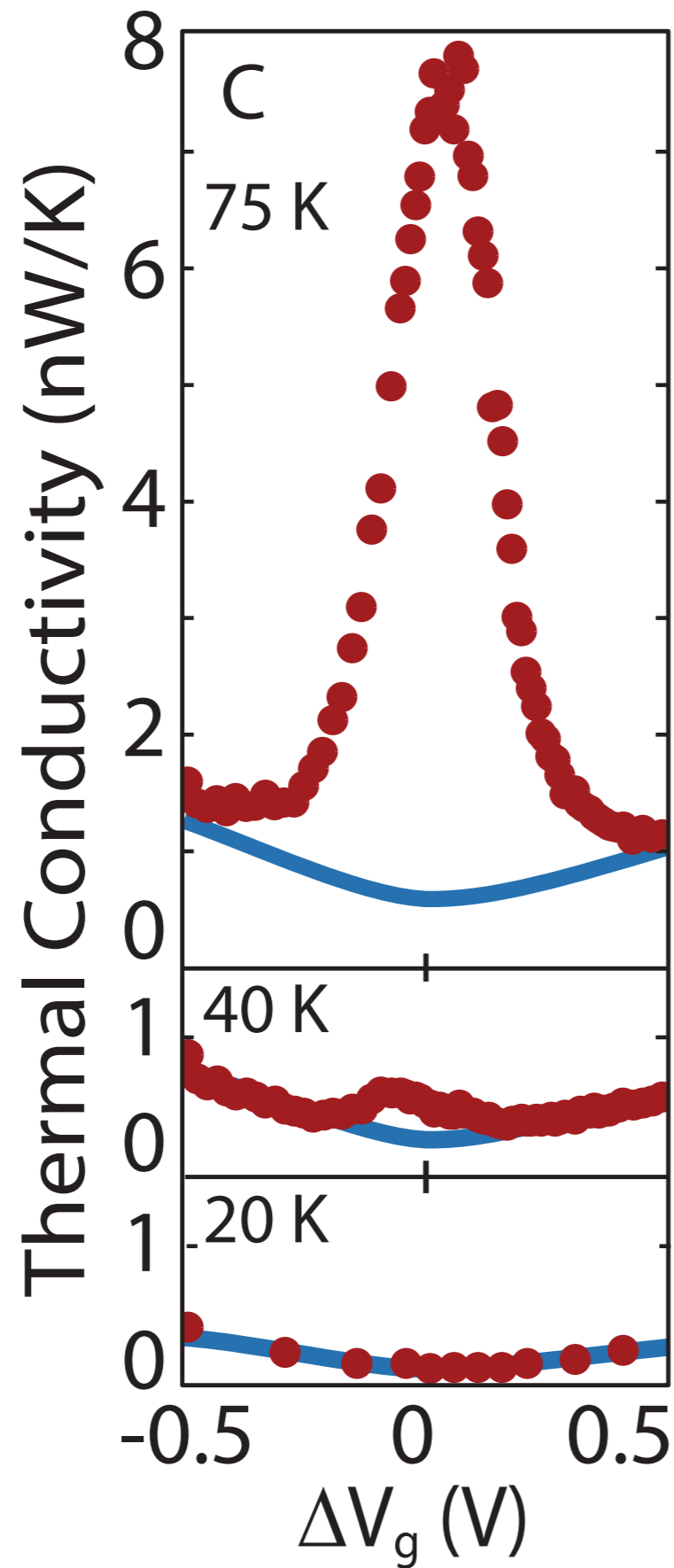
Red dots: data

Blue line: value for $L = L_0$



Red dots: data

Blue line: value for $L = L_0$

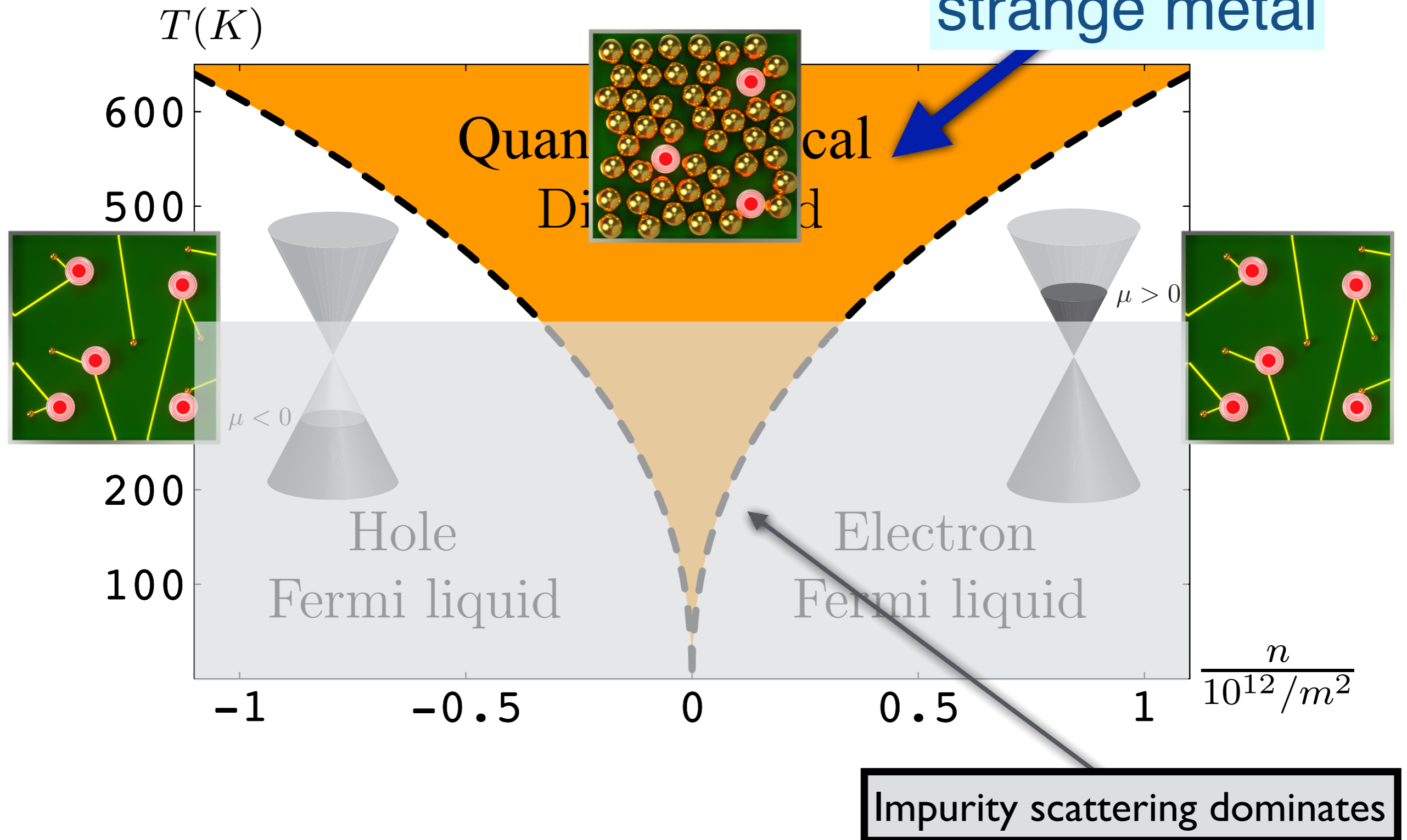


Red dots: data

Blue line: value for $L = L_0$

Graphene

Predicted
strange metal

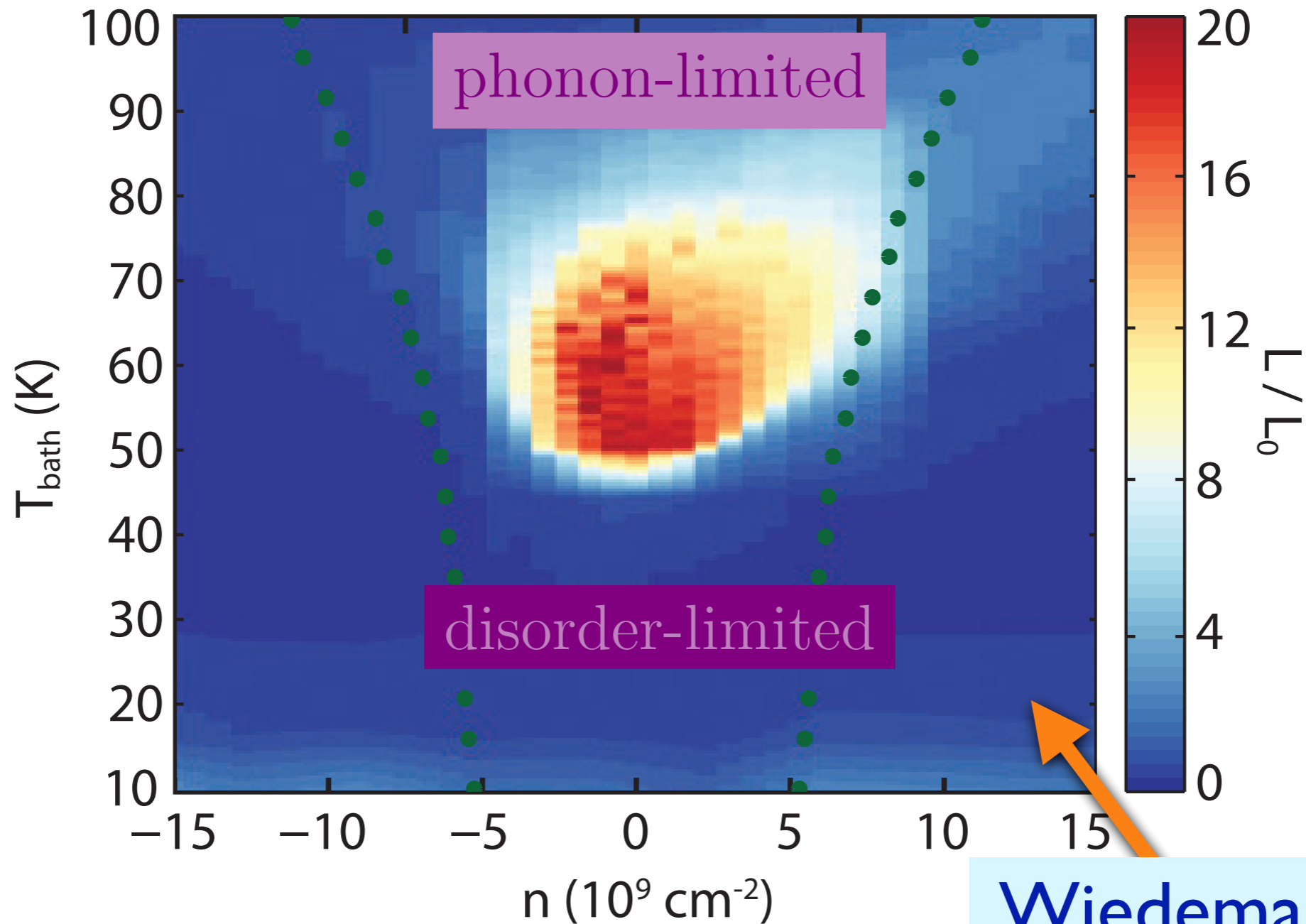
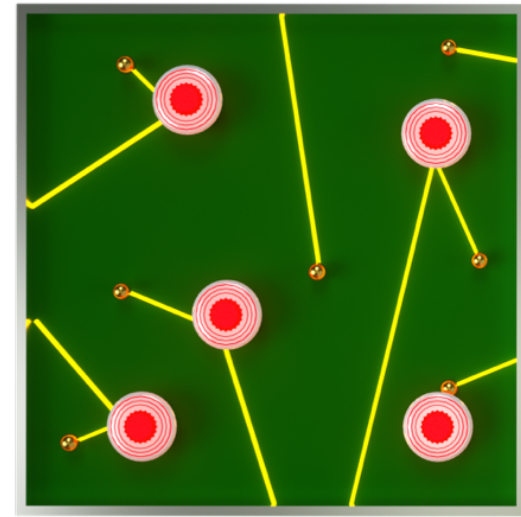


M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)

M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

J. Crossno et al., Science **351**, 1058 (2016)

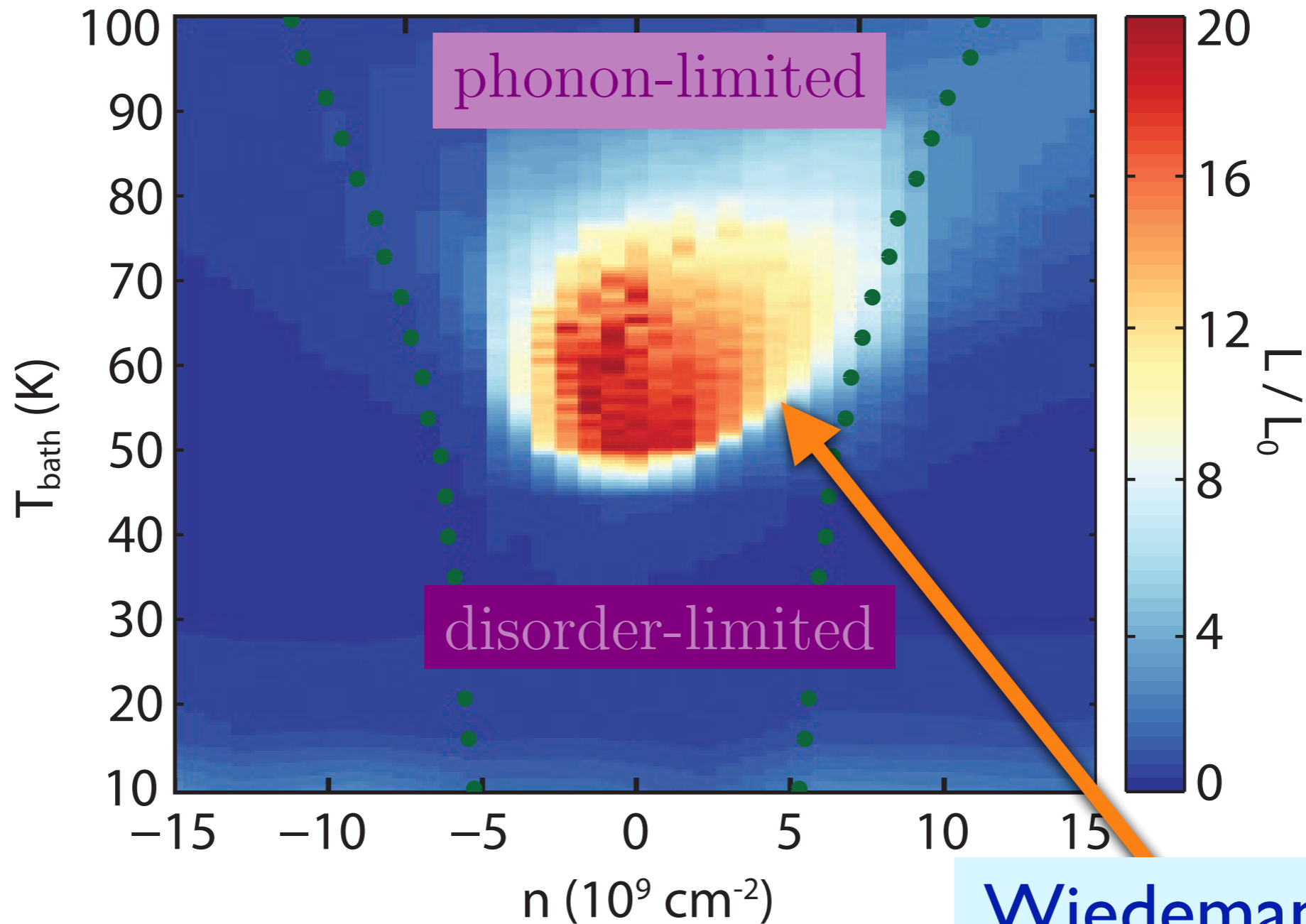
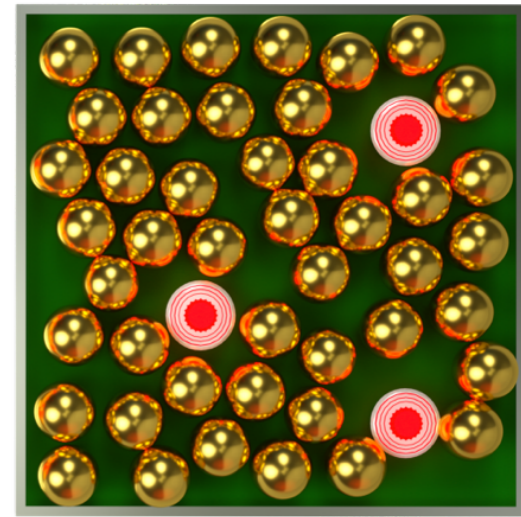
Strange metal in graphene



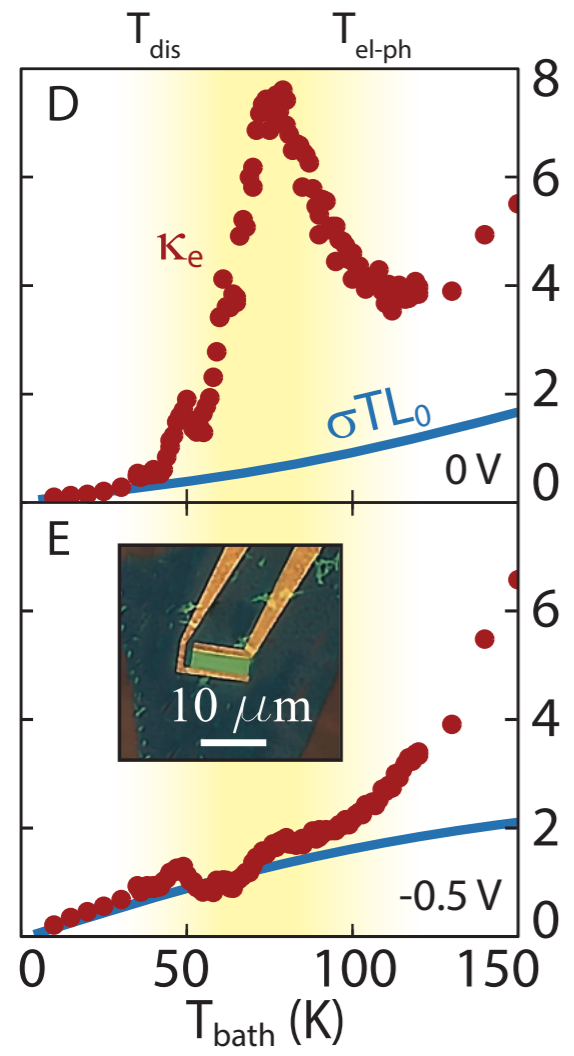
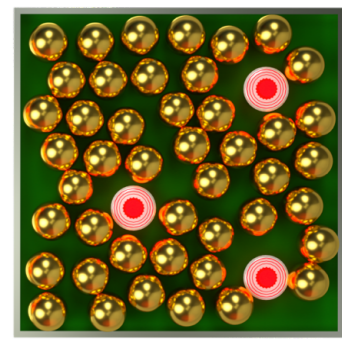
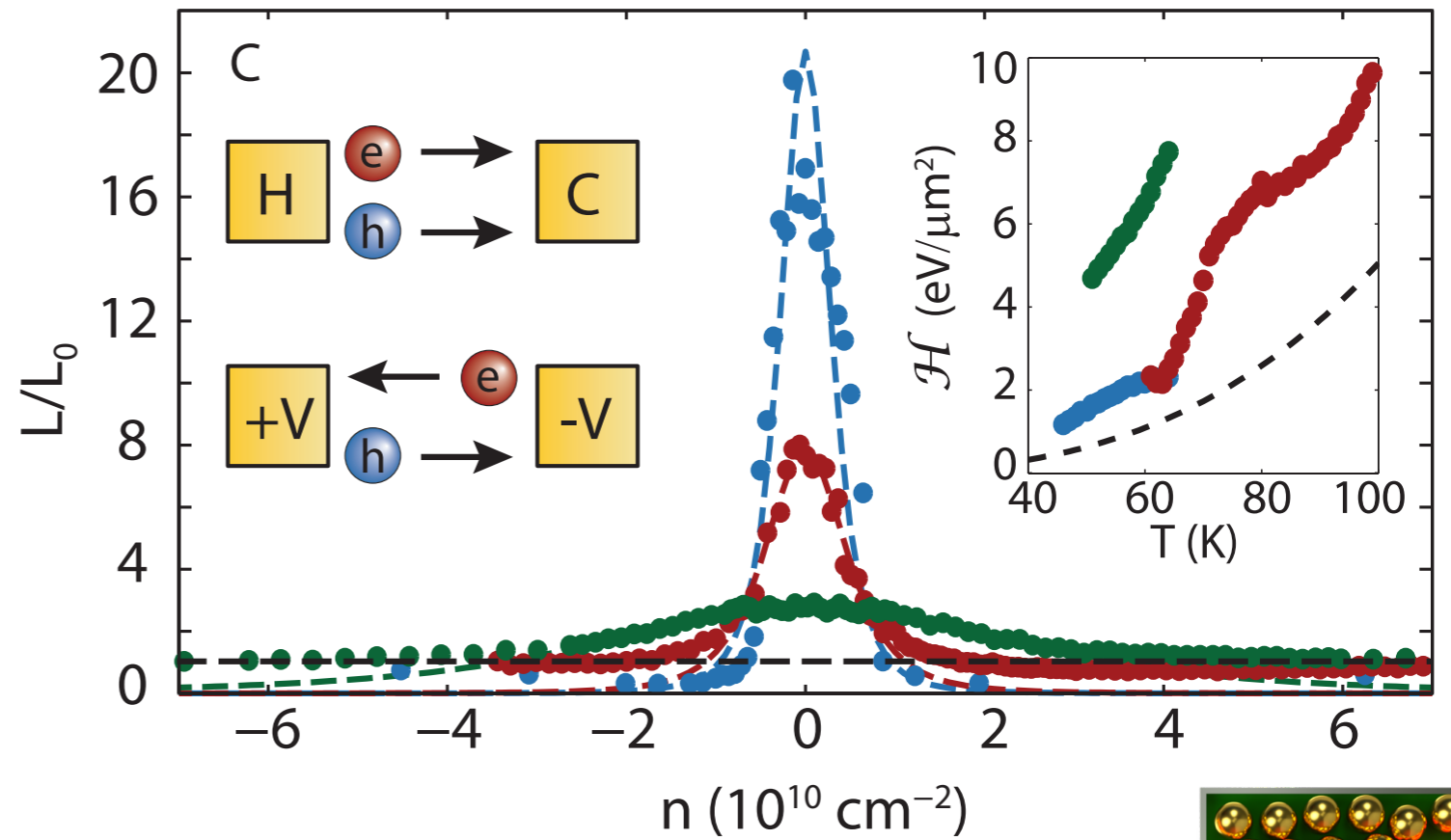
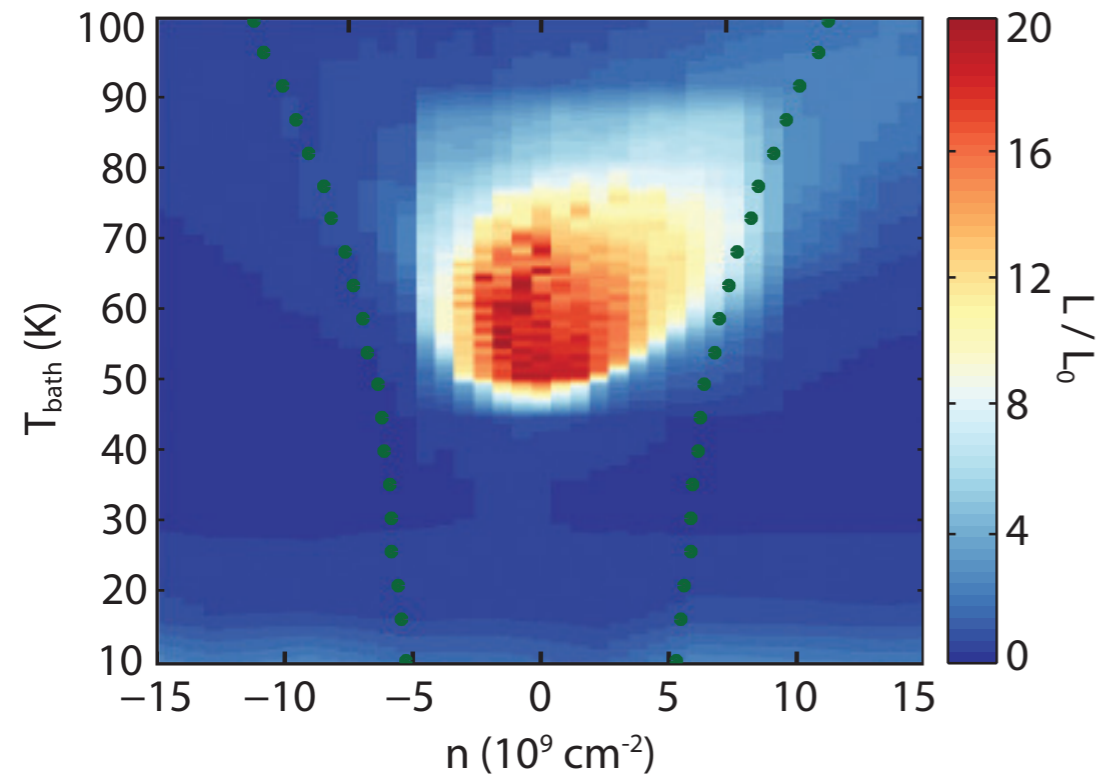
Wiedemann-Franz
obeyed

J. Crossno et al., Science **351**, 1058 (2016)

Strange metal in graphene



**Wiedemann-Franz
violated !**



Lorentz ratio $L = \kappa / (T\sigma)$

$$= \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{(1 + e^2 v_F^2 Q^2 \tau_{\text{imp}} / (\mathcal{H} \sigma_Q))^2}$$

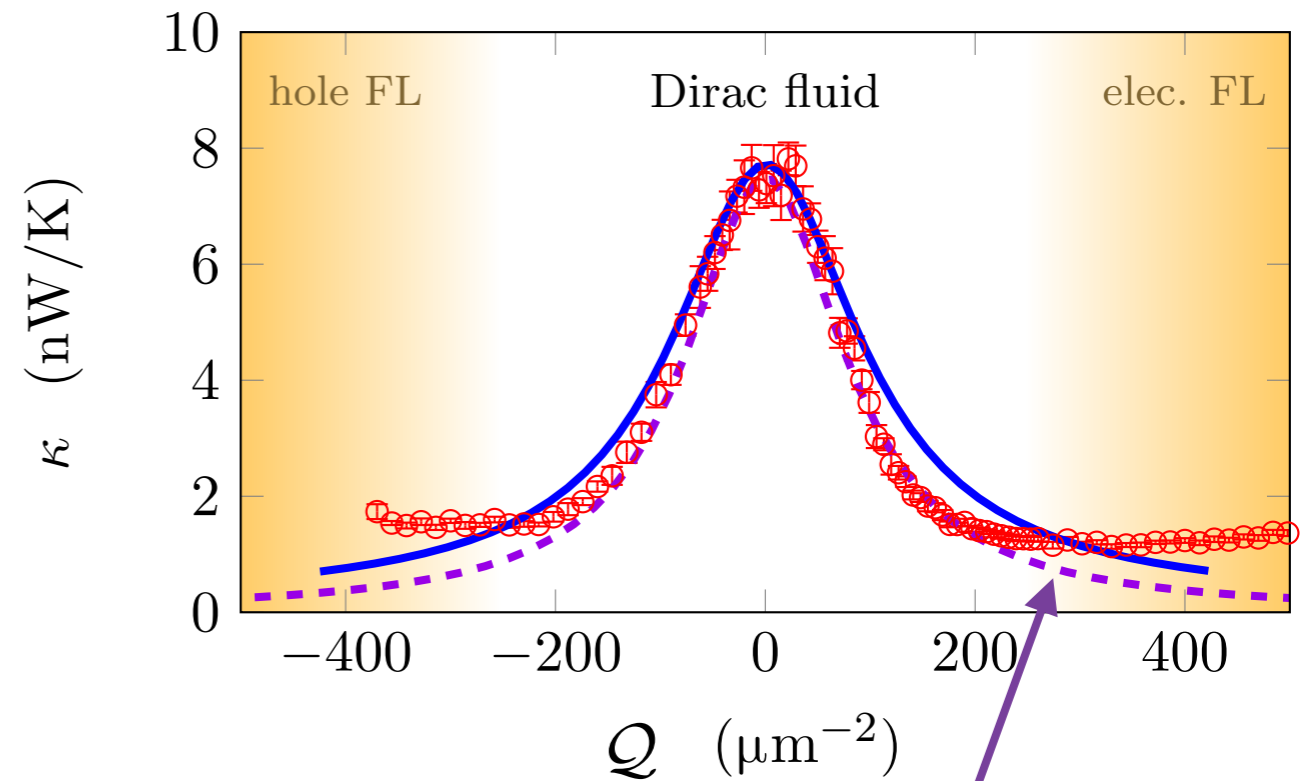
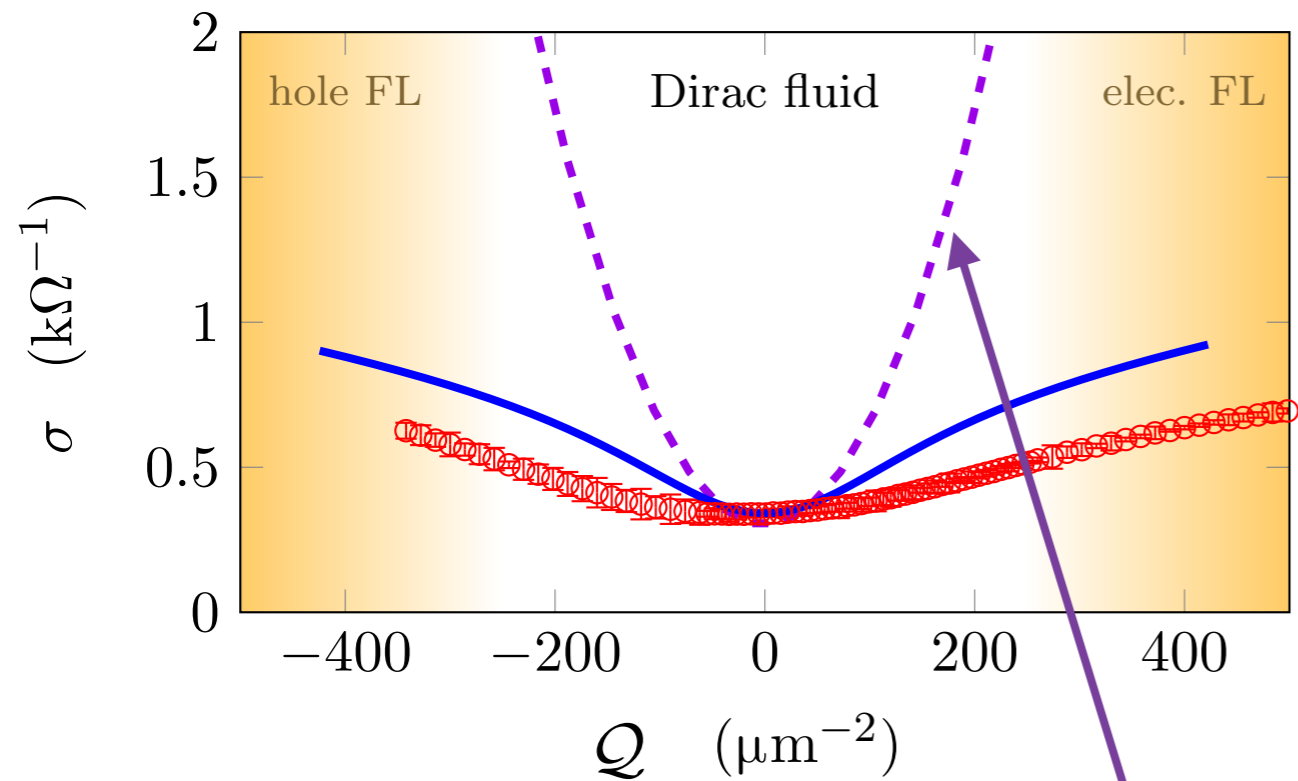
$Q \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density

$\sigma_Q \rightarrow$ quantum critical conductivity

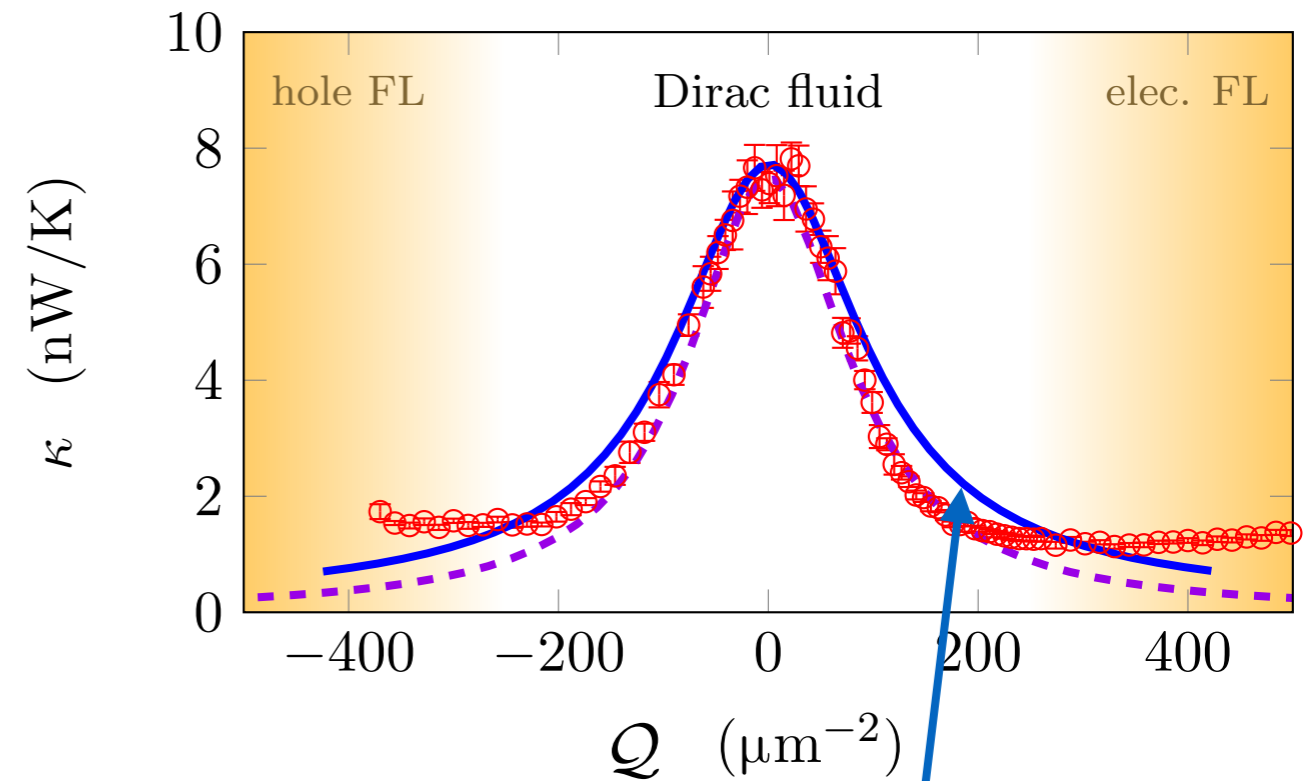
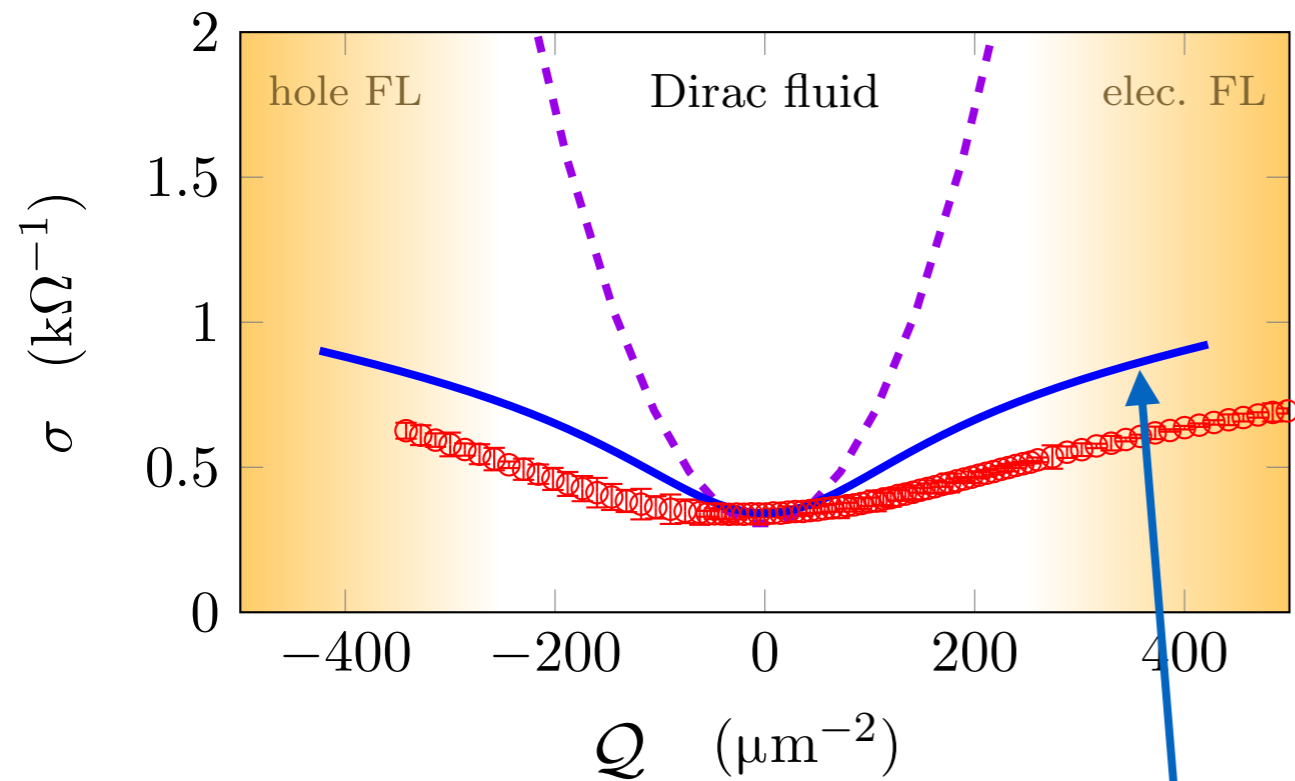
$\tau_{\text{imp}} \rightarrow$ momentum relaxation time from impurities

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

J. Crossno et al., Science **351**, 1058 (2016)



Comparison to theory with a single momentum relaxation time τ_{imp} . Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity

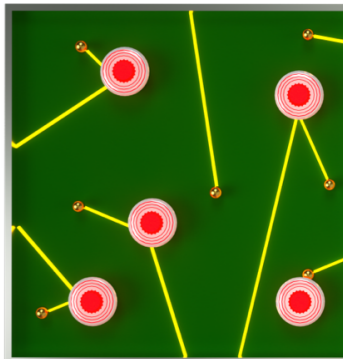
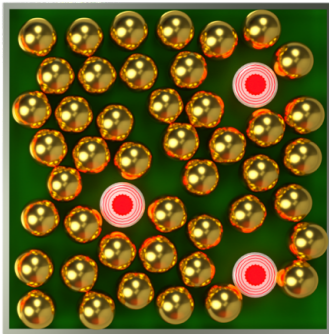
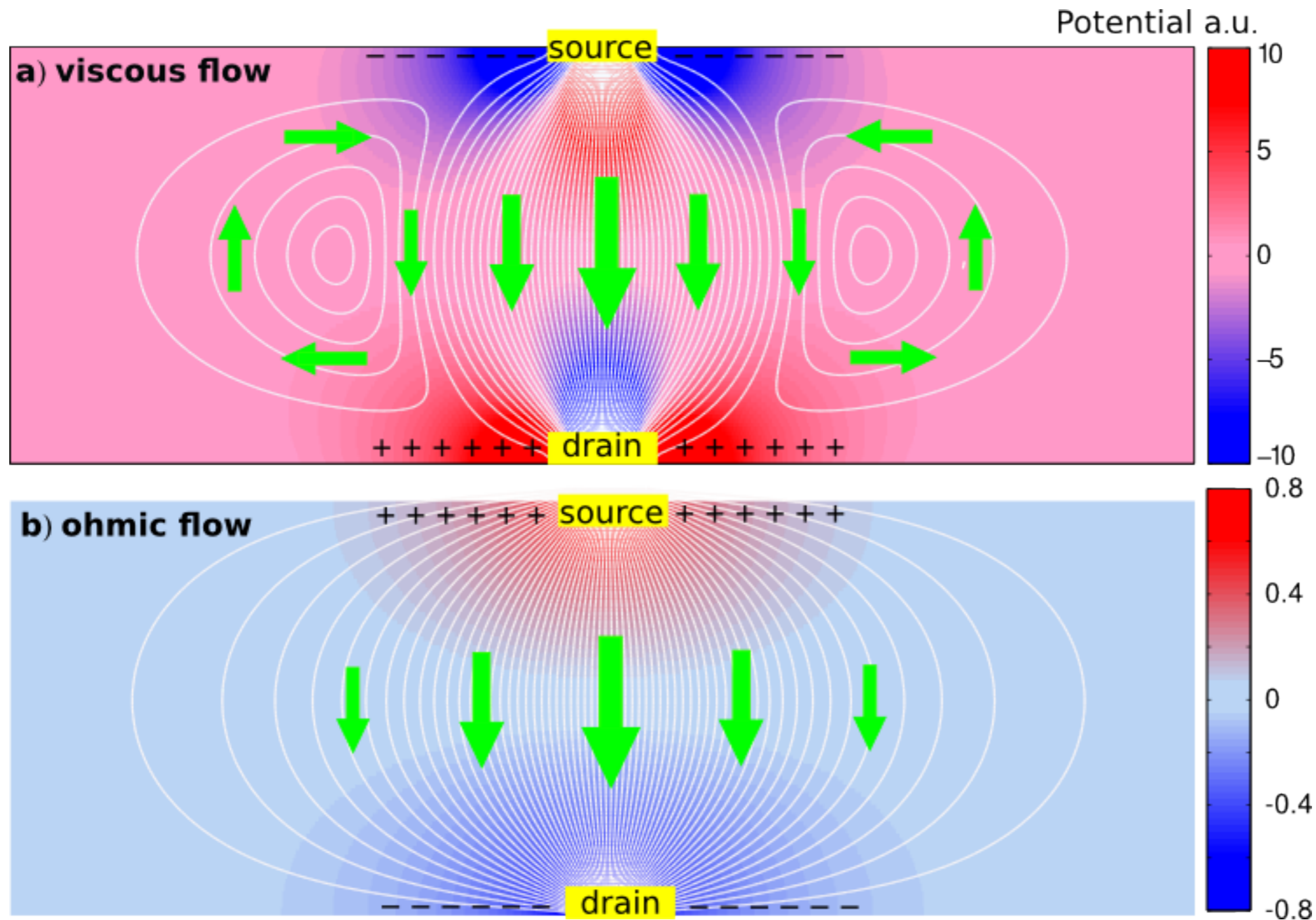


Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.

Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The T dependencies of other parameters also agree well with expectation.

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene



L. Levitov and G. Falkovich, arXiv:1508.00836, *Nature Physics online*

Strange metal in graphene

Science 351, 1055 (2016)

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin¹, I. Torre^{2,3}, R. Krishna Kumar^{1,4}, M. Ben Shalom^{1,5}, A. Tomadin⁶, A. Principi⁷, G. H. Auton⁵, E. Khestanova^{1,5}, K. S. Novoselov⁵, I. V. Grigorieva¹, L. A. Ponomarenko^{1,4}, A. K. Geim¹, M. Polini^{3,6}

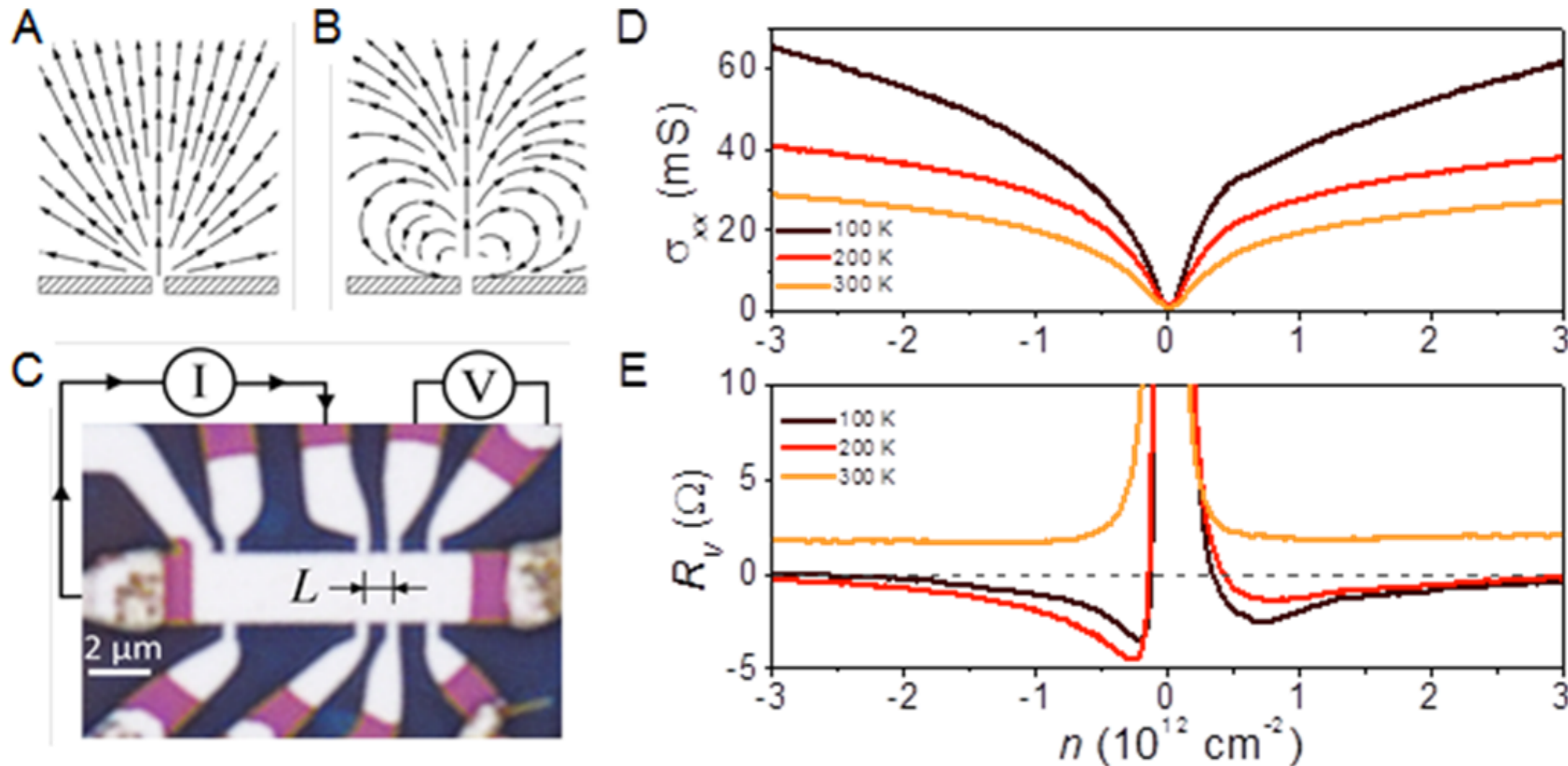
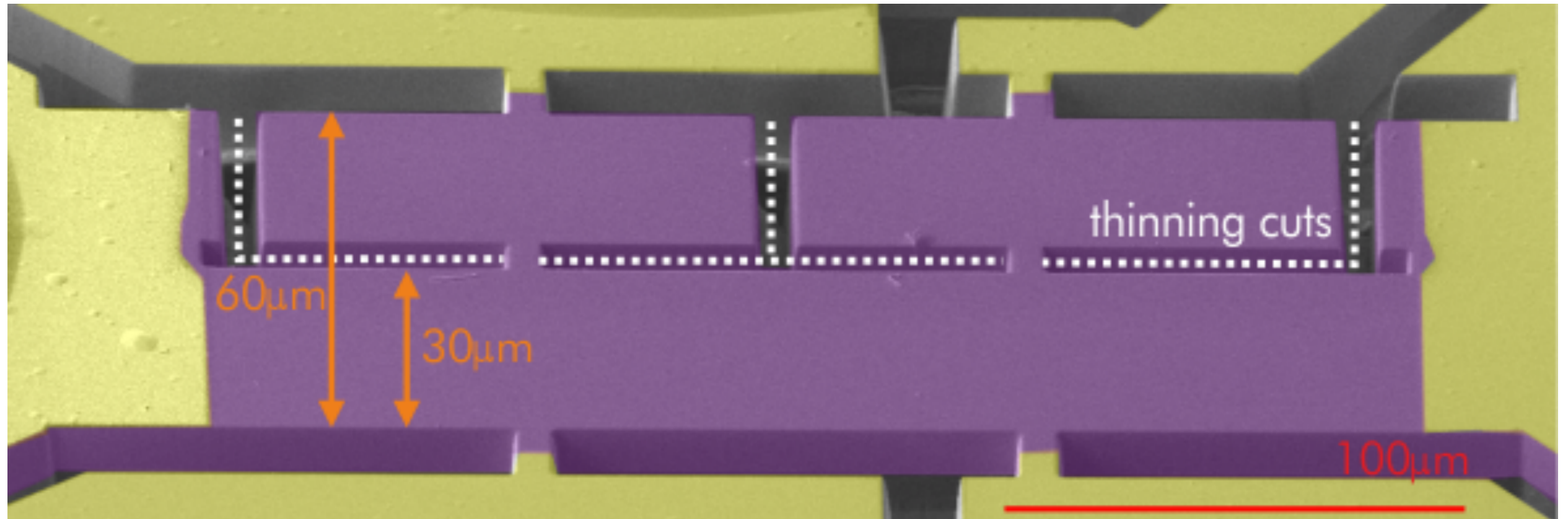


Figure 1. Viscous backflow in doped graphene. (a,b) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (a) and a viscous Fermi liquid (b). (c) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (d,e) Longitudinal conductivity σ_{xx} and R_V for this device as a function of n induced by applying gate voltage. $I = 0.3 \mu\text{A}$; $L = 1 \mu\text{m}$. For more detail, see Supplementary Information.

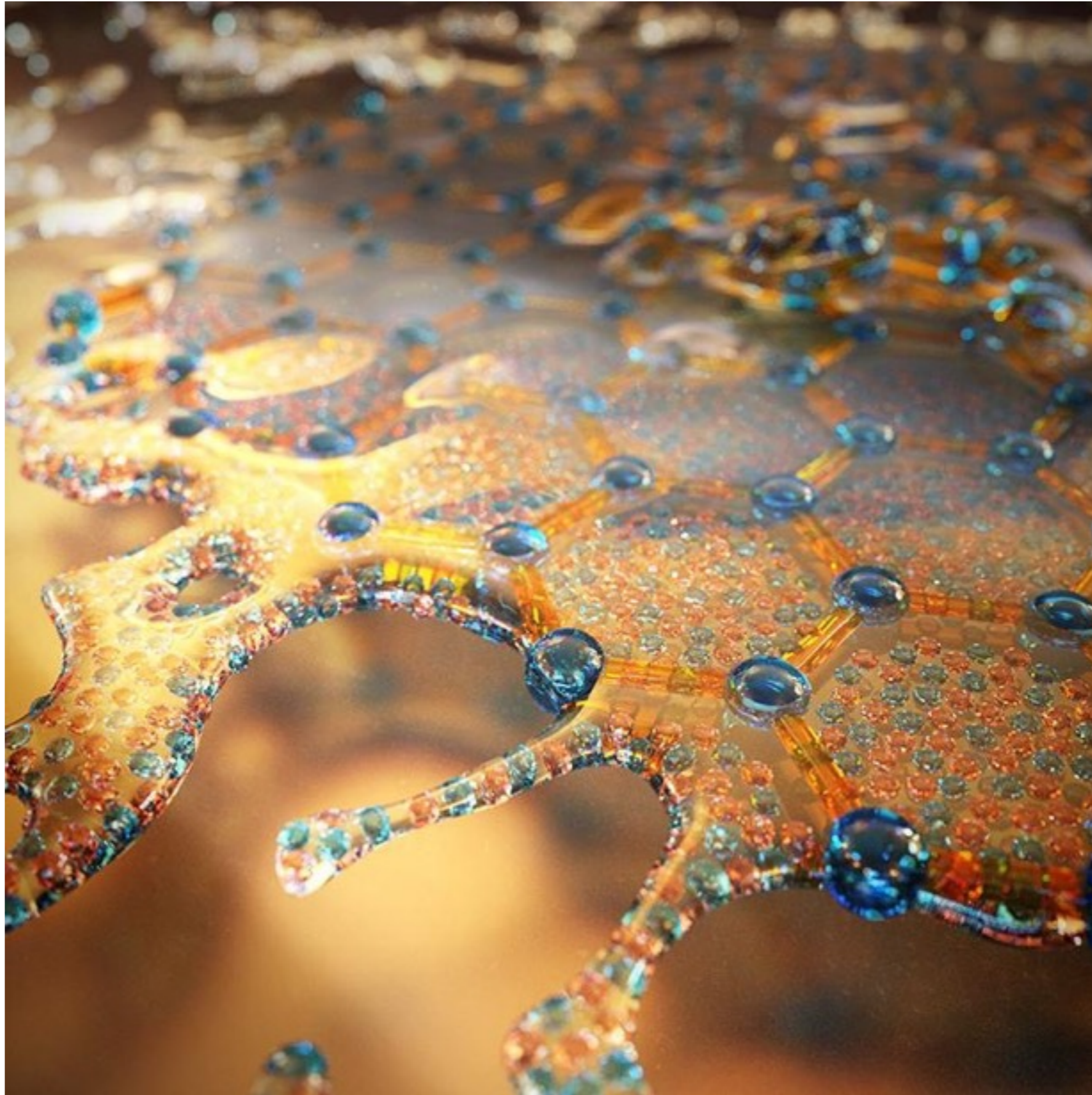
Signature of Navier-Stokes hydrodynamic flow in PdCoO₂



Experiment: Successively narrow the channel in factors of 2, measuring the resistance after every step.

P.J.W. Moll, P. Kushwaha, N. Nandi, B. Schmidt and A.P. Mackenzie, Science 351, 1061 (2016)

Graphene: “a metal that behaves like water”



Entangled quantum matter without quasiparticles

- Is there a connection between strange metals and black holes?
Yes, *e.g.* the SYK model.
- Why do they have the same equilibration time $\sim \hbar/(k_B T)$?
Strange metals don't have quasiparticles and thermalize rapidly;
Black holes are “fast scramblers”.
- Theoretical predictions for strange metal transport in graphene agree well with experiments