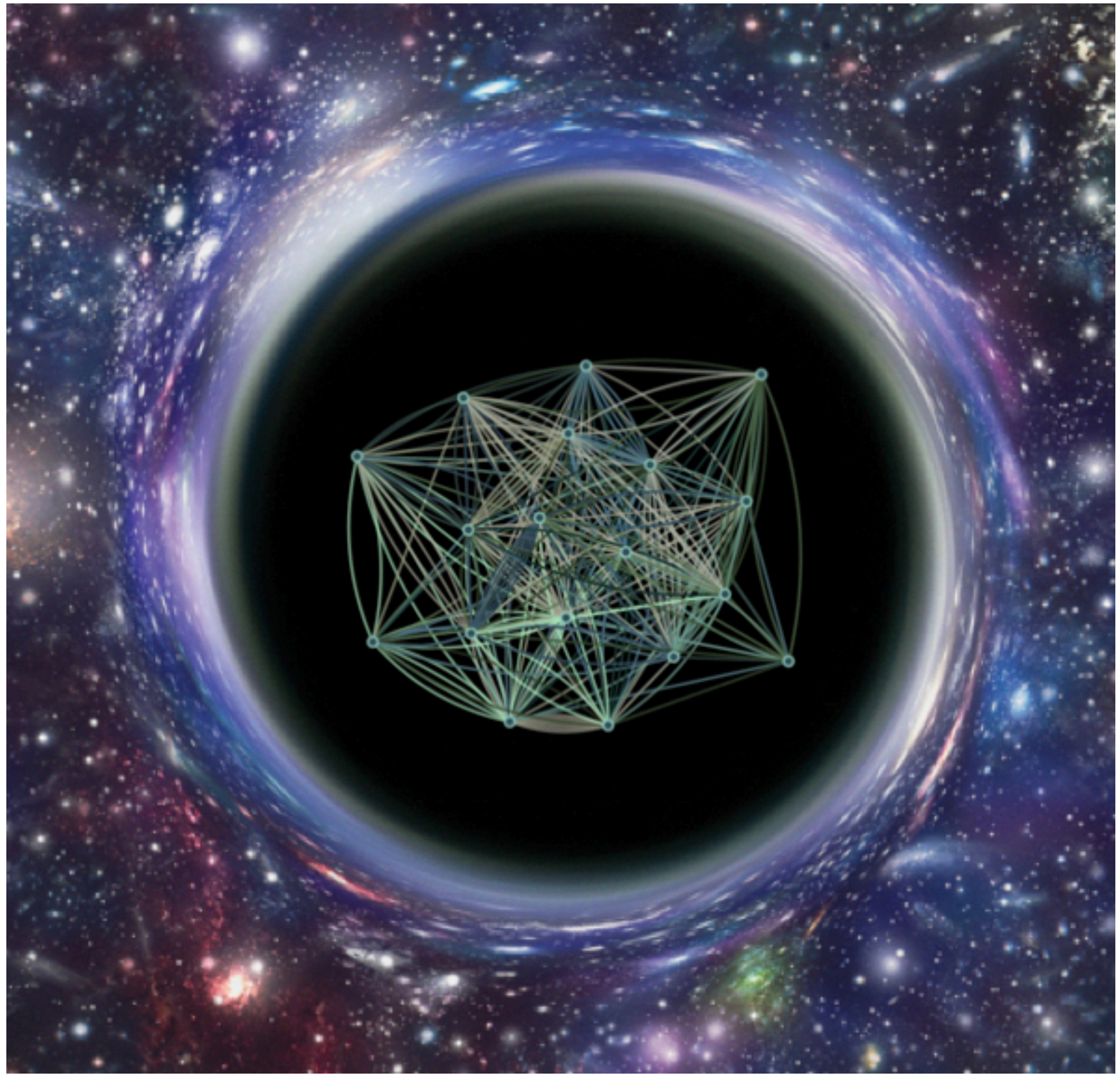
Quantum Gravity
in the Lab,
Google X,
November 16, 2019

Subir Sachdev

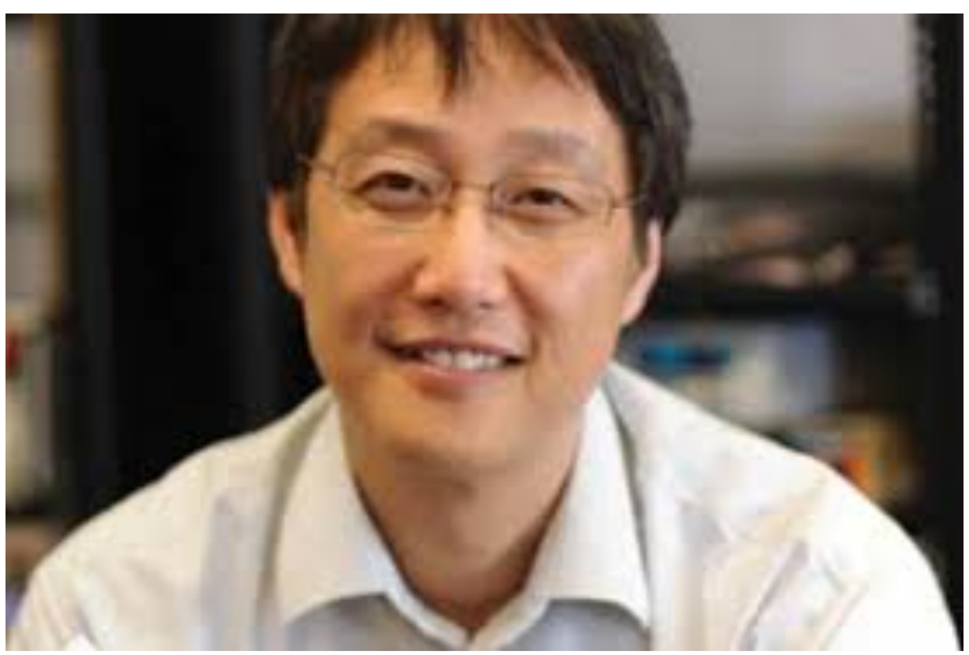


Talk online:
sachdev.physics.harvard.edu

# Quantum transport across SYK islands: probing Bekenstein-Hawking entropy in quantum matter experiments

Alex Kruchkov

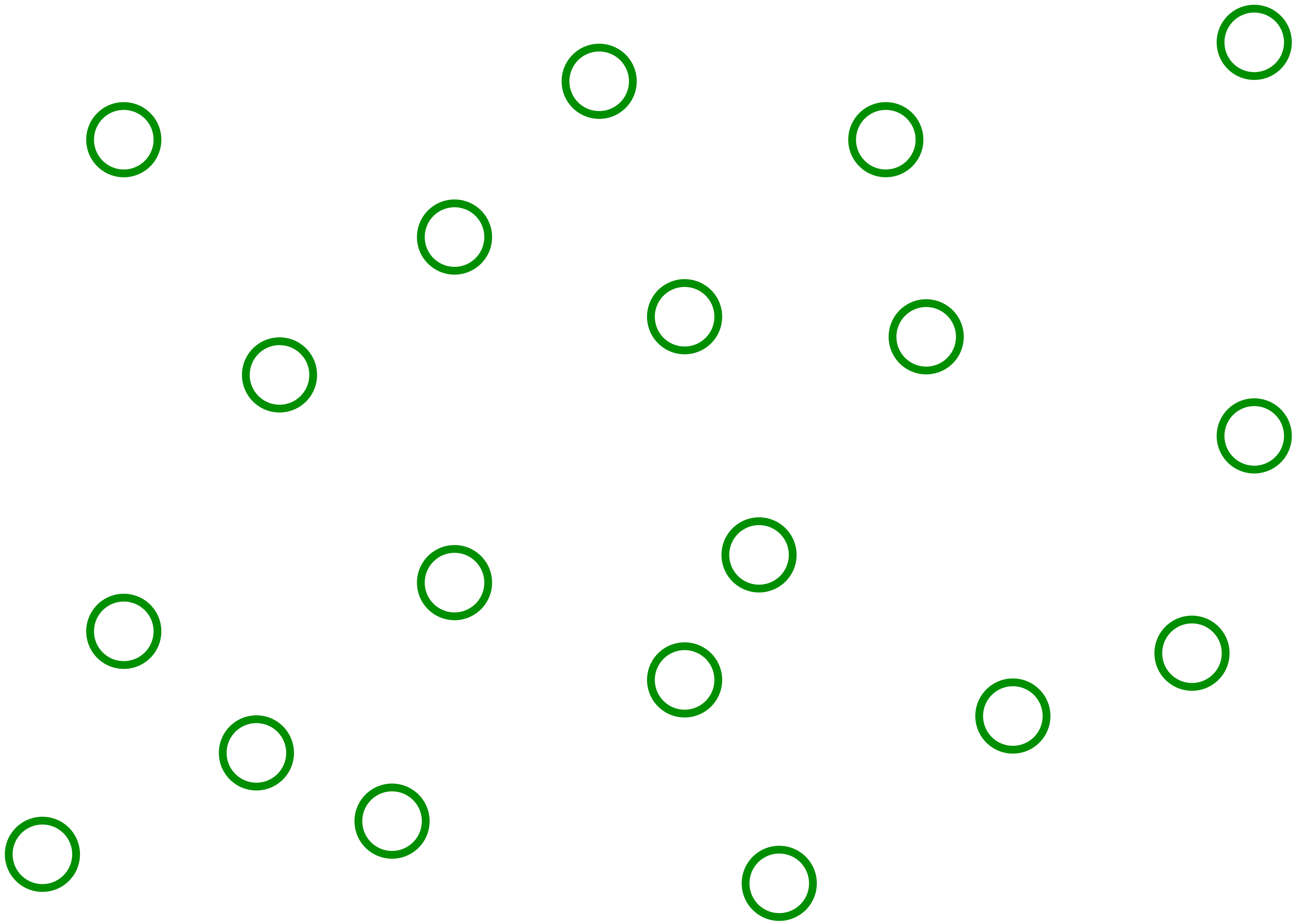
Aavishkar Patel



Philip Kim

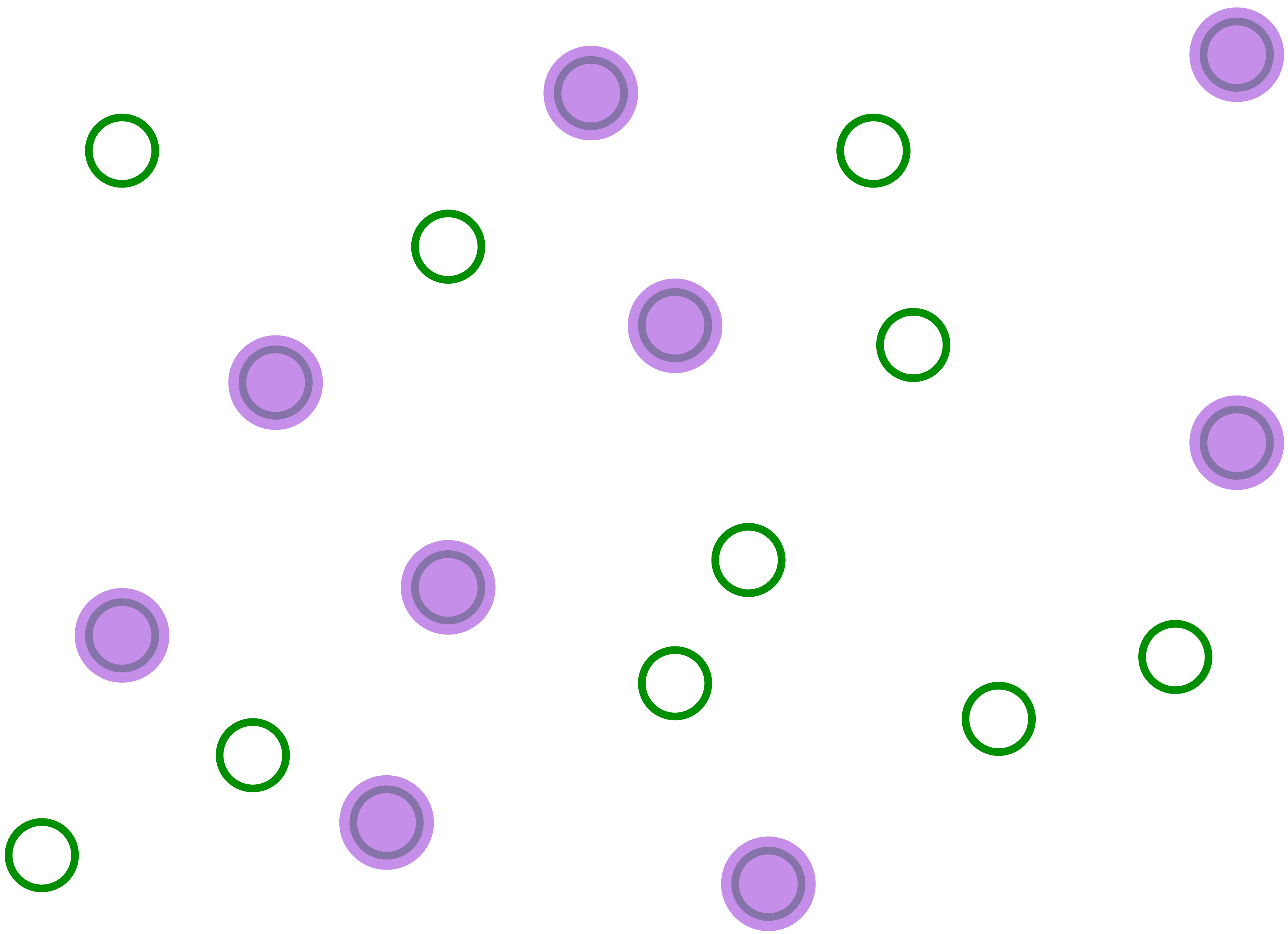
Amir Yacoby

# A simple model of a metal with quasiparticles



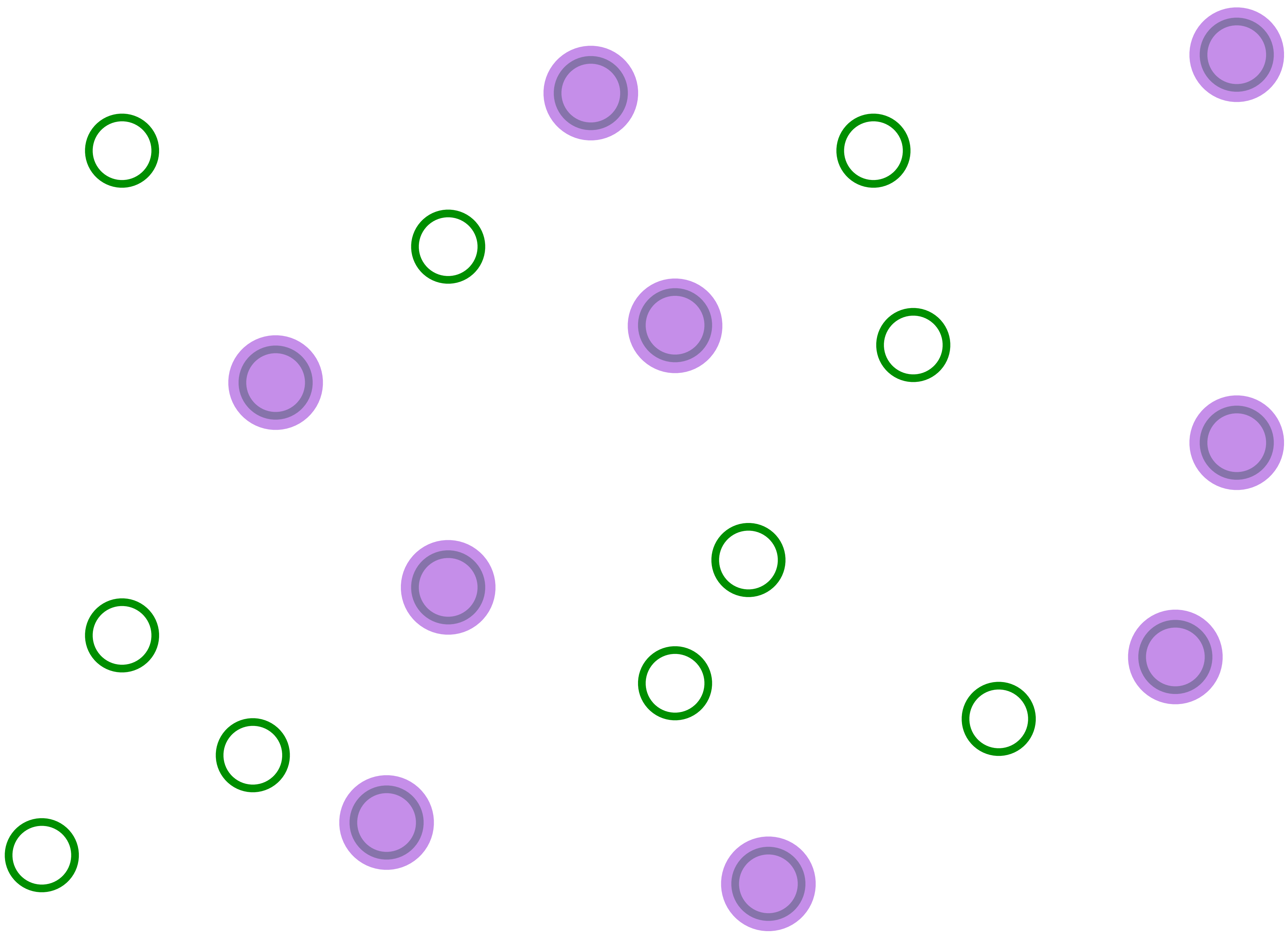
Pick a set of random positions

# A simple model of a metal with quasiparticles



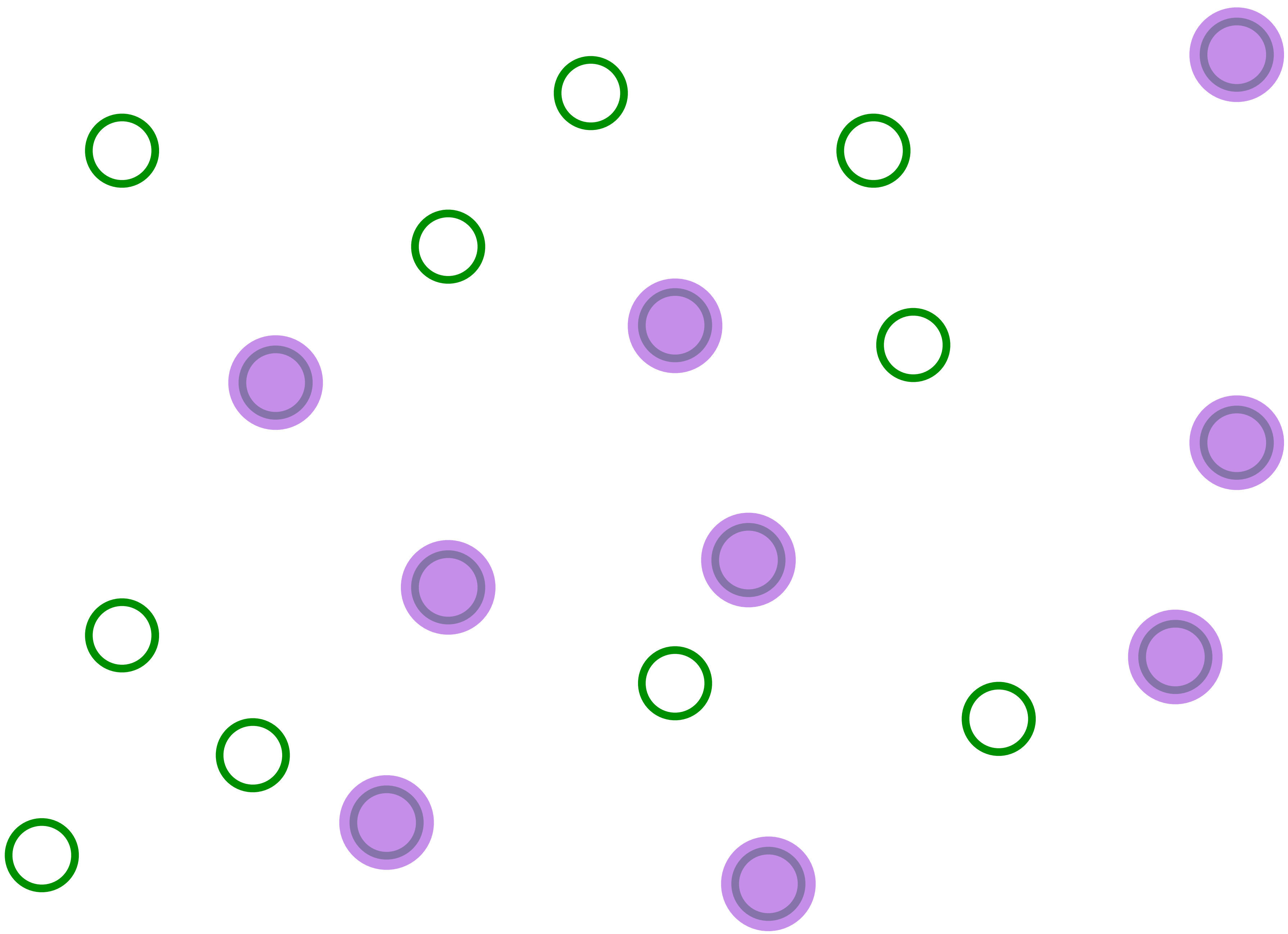
Place electrons randomly on some sites

# A simple model of a metal with quasiparticles



Electrons move one-by-one randomly

# A simple model of a metal with quasiparticles



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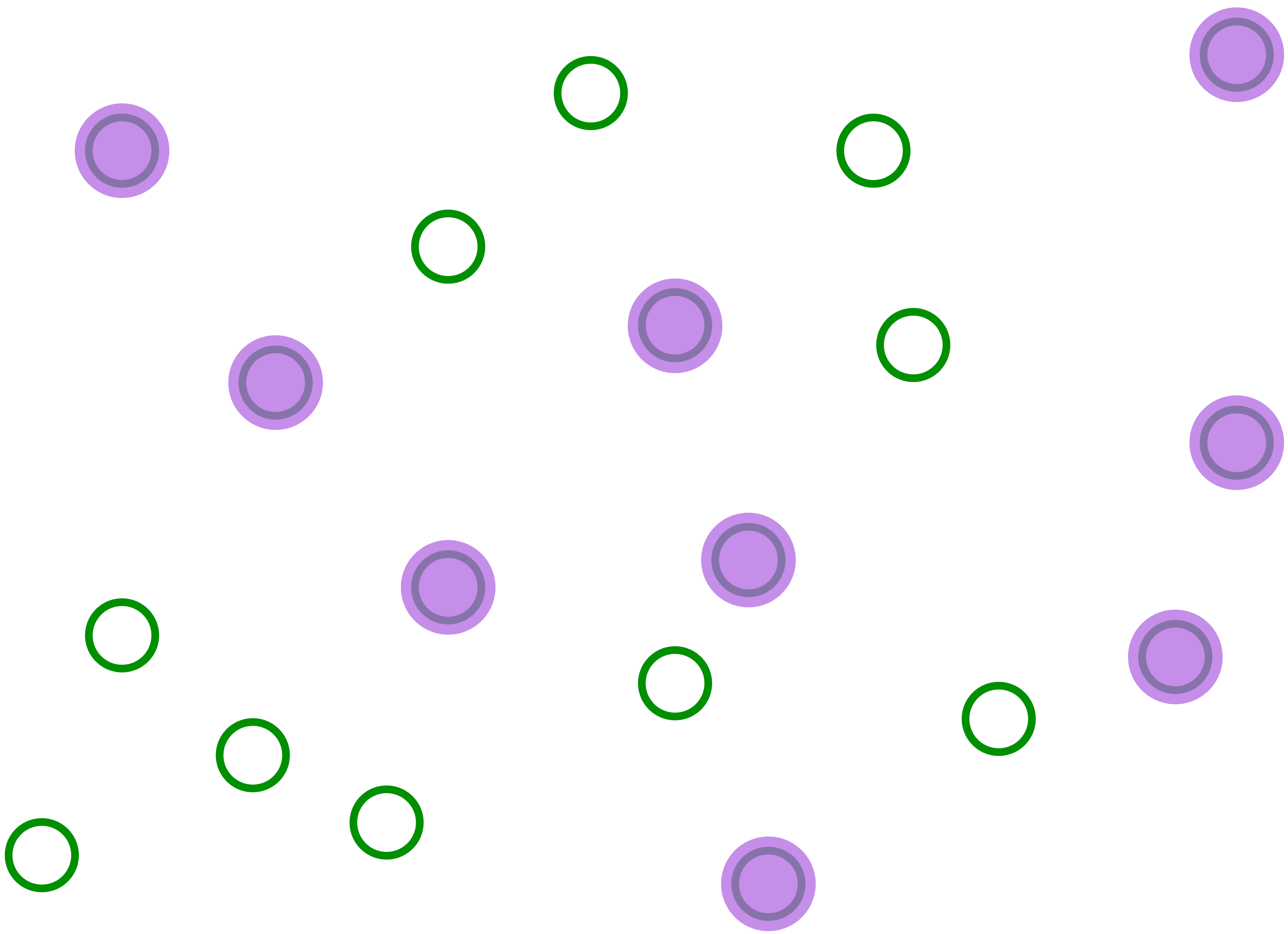

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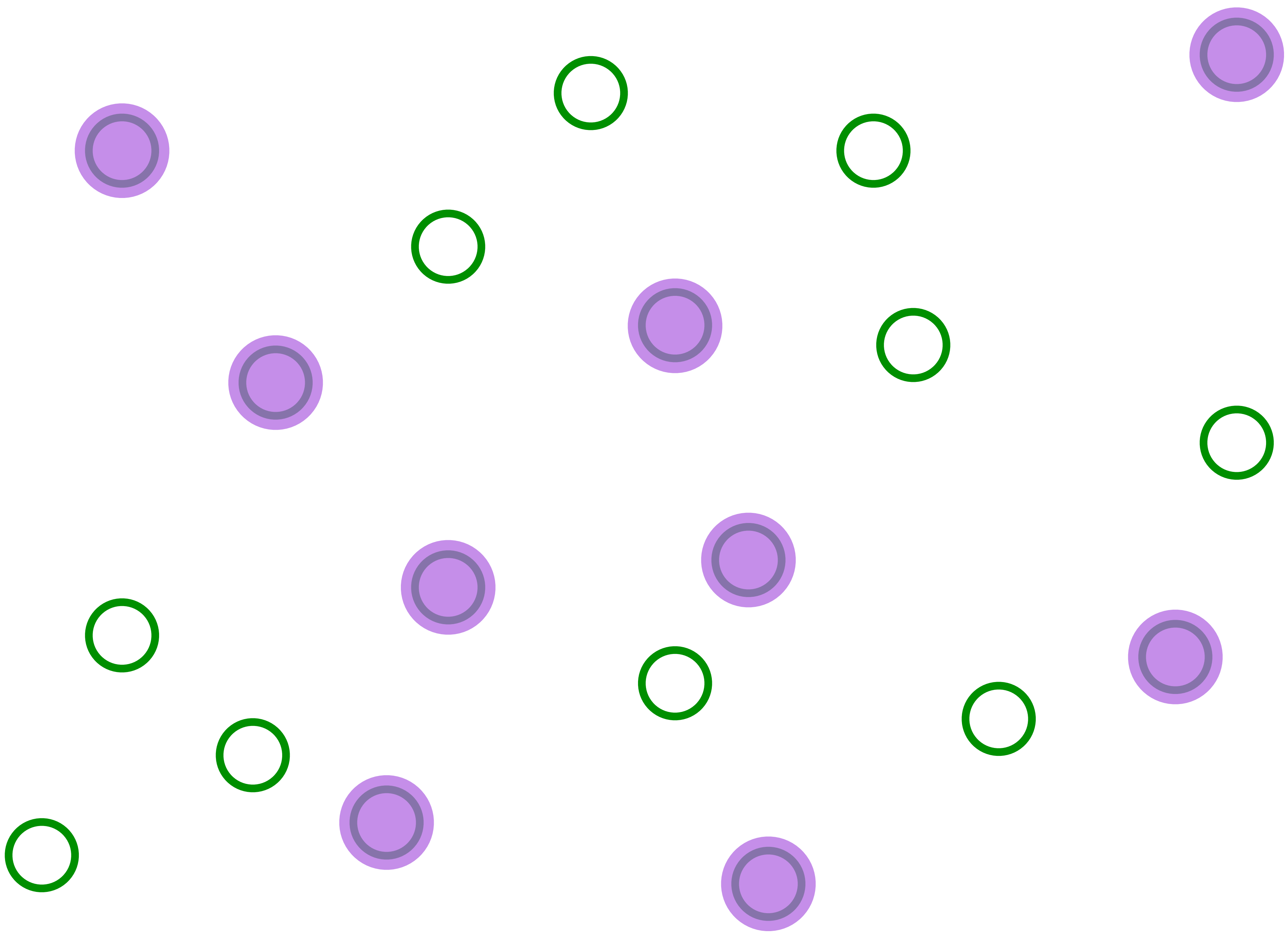

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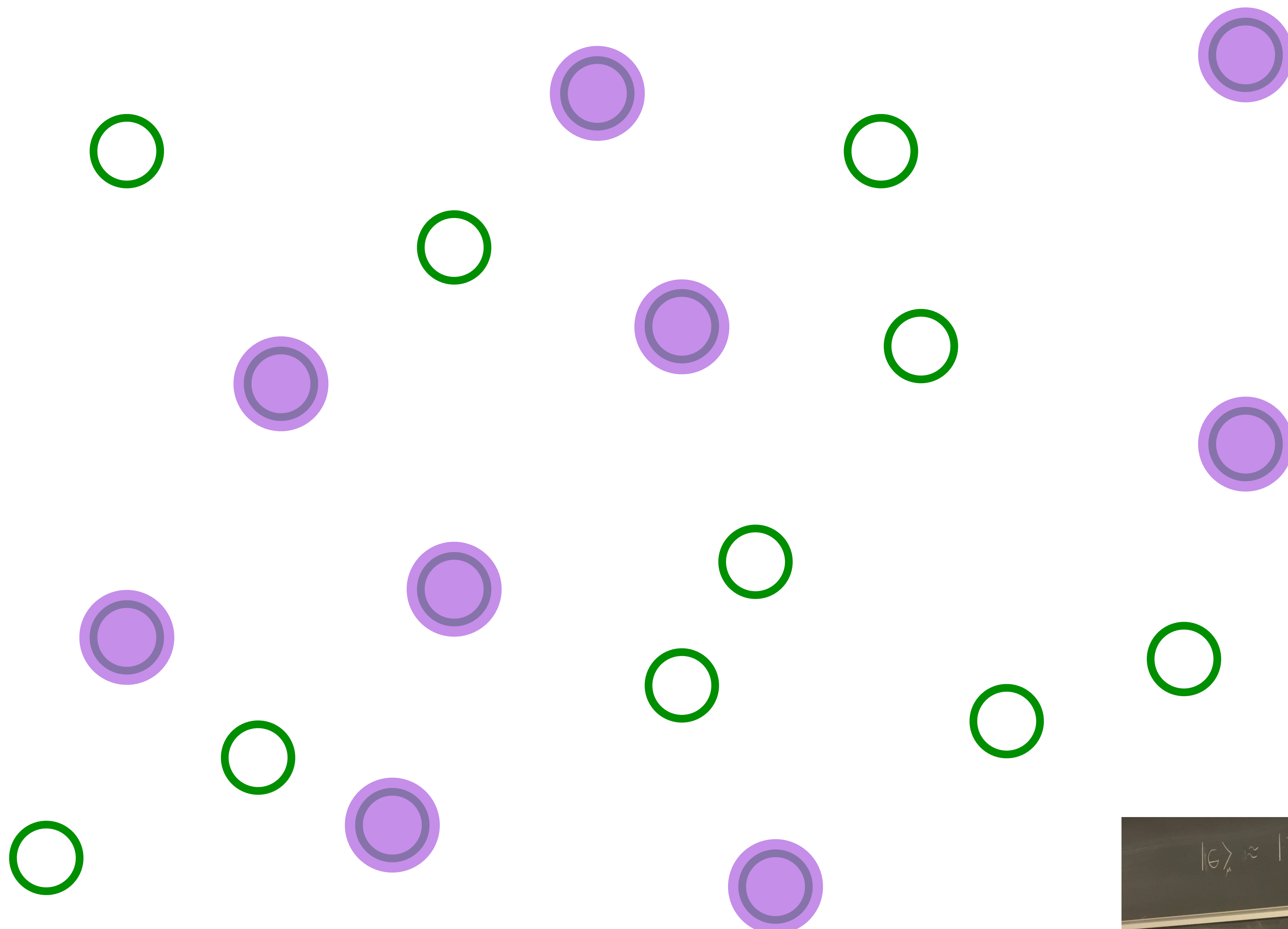
# The complex SYK model



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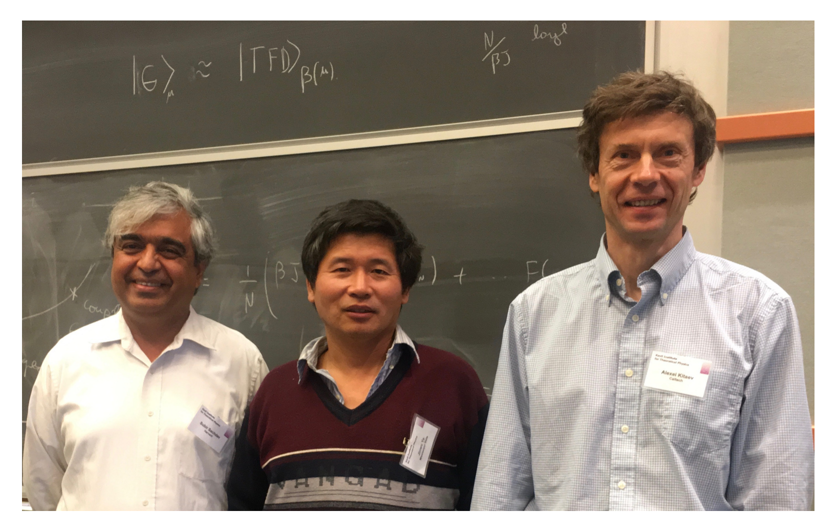

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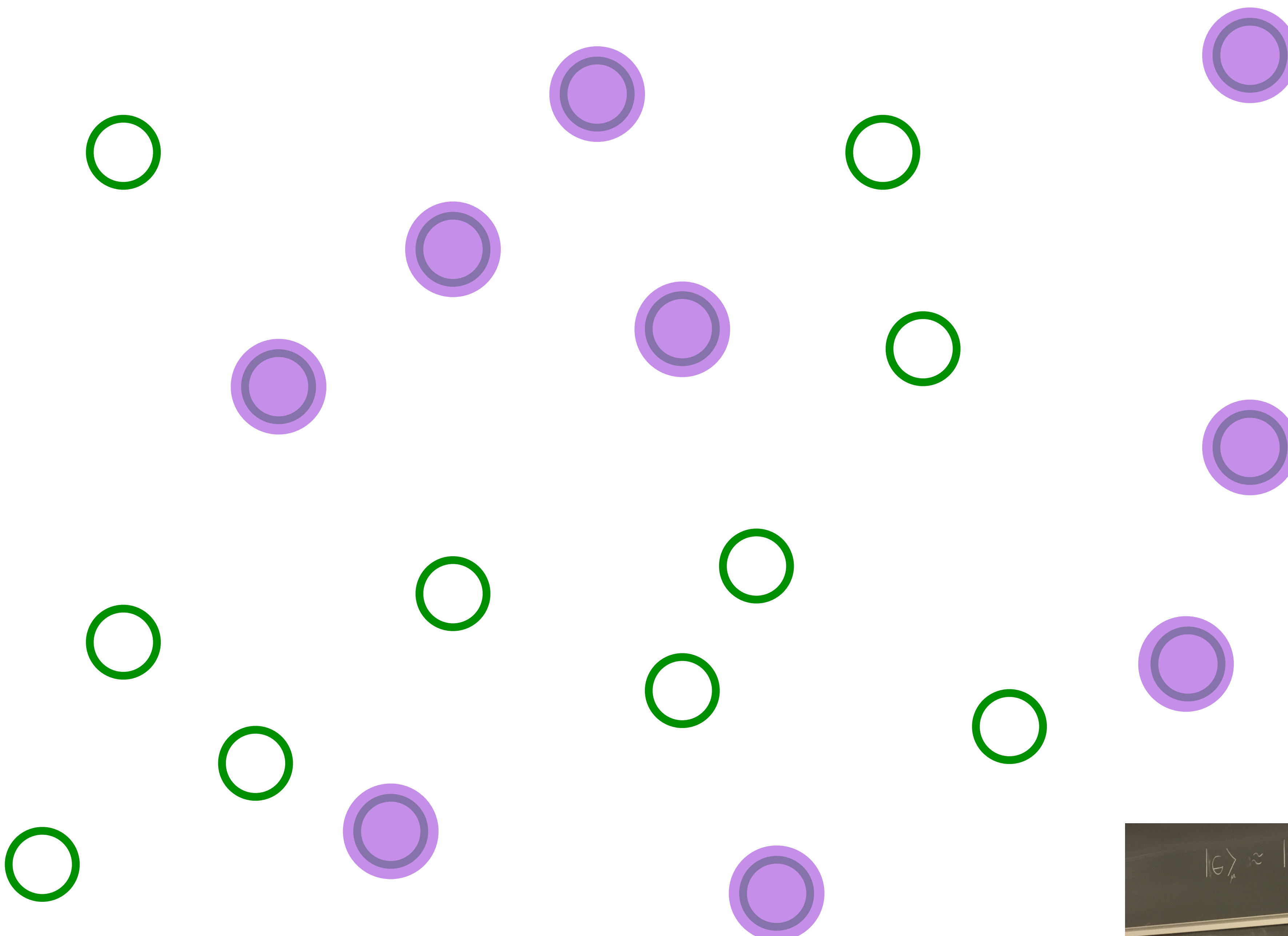
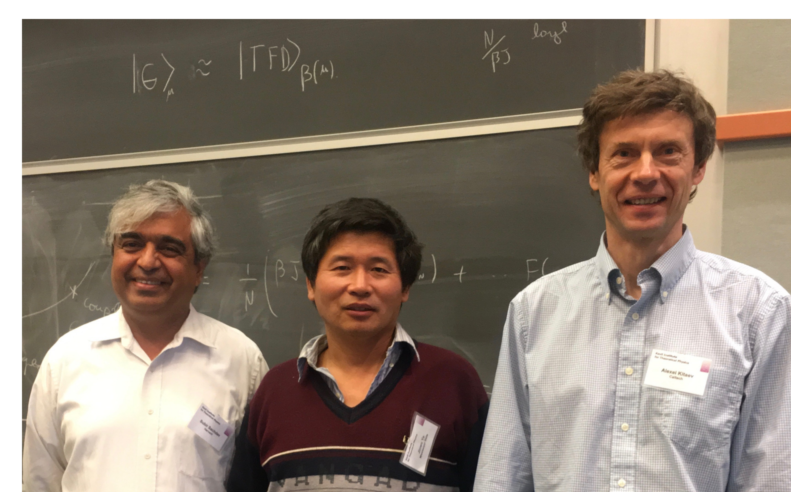


Entangle electrons pairwise randomly



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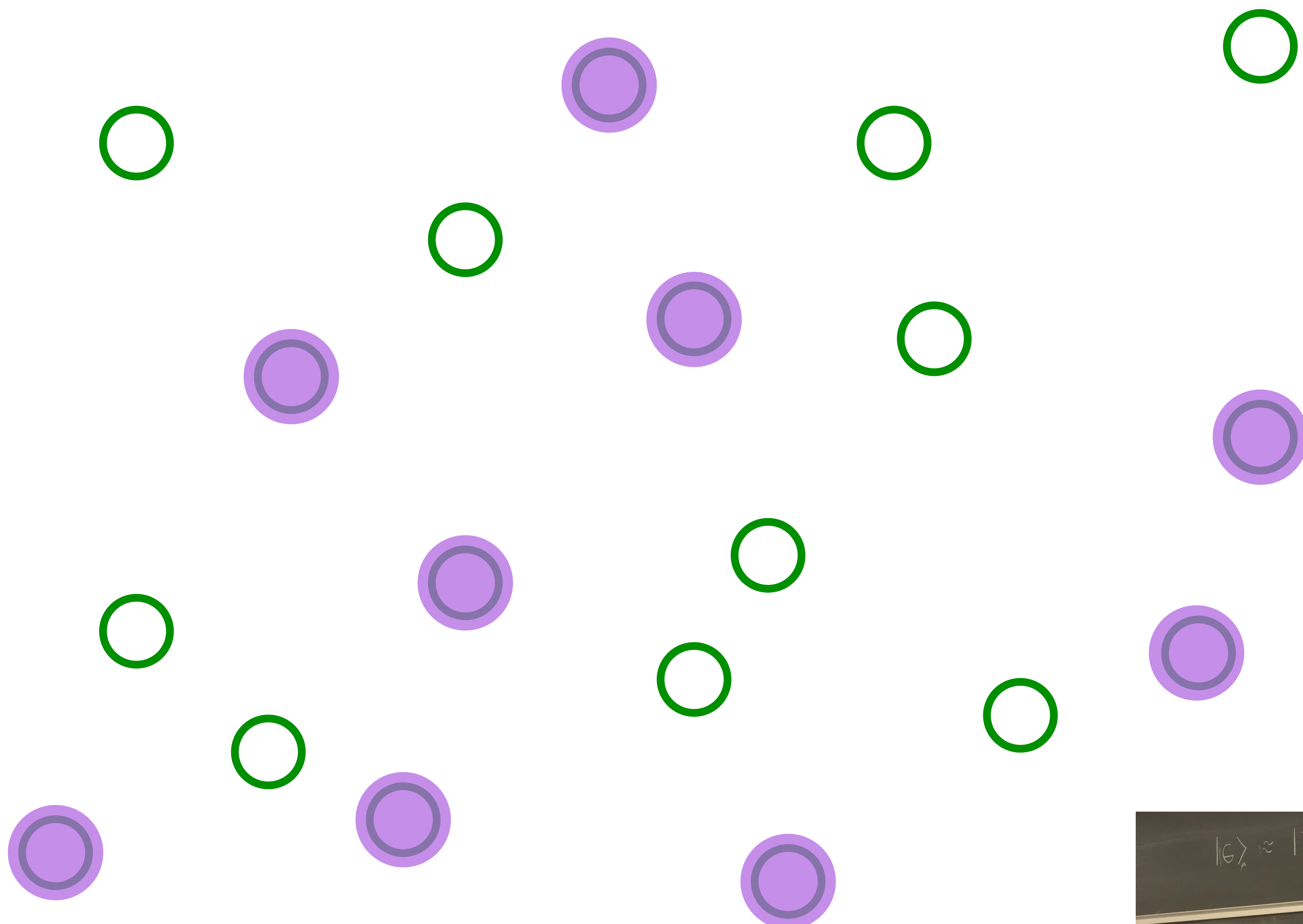

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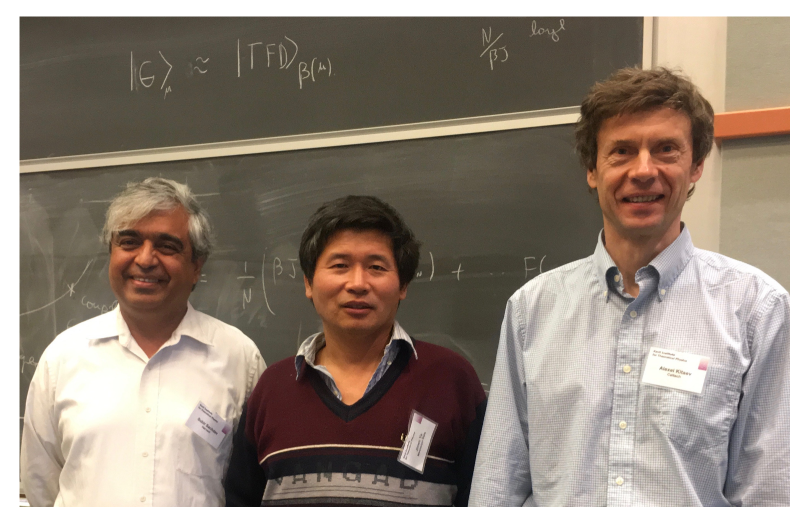

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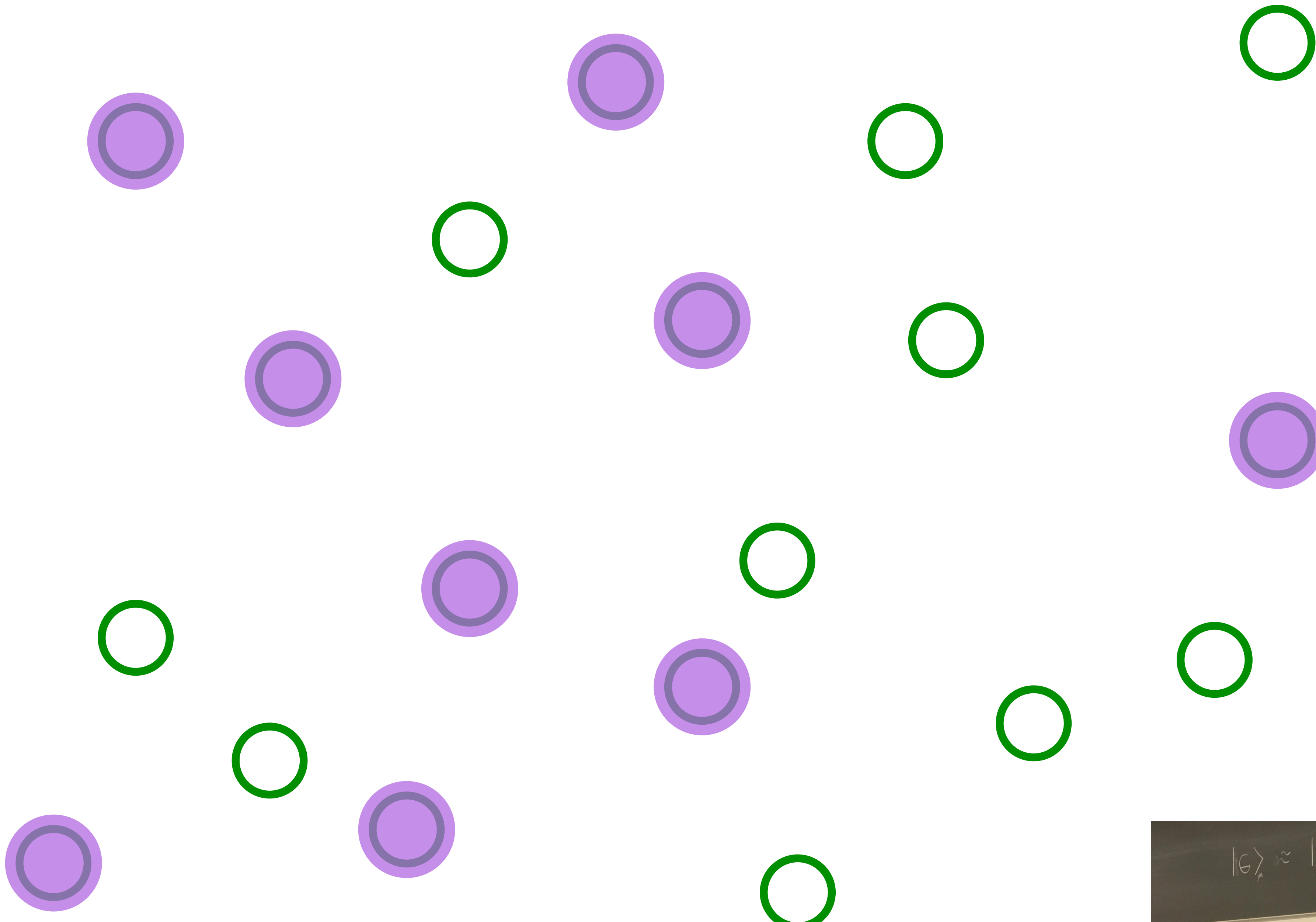

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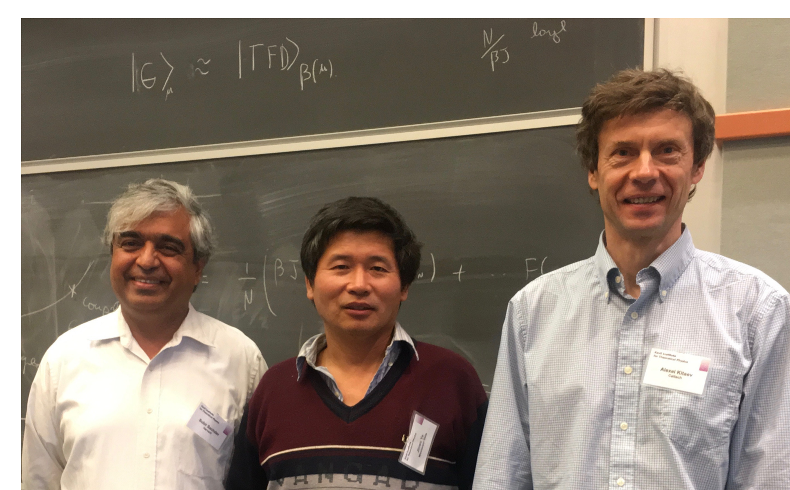

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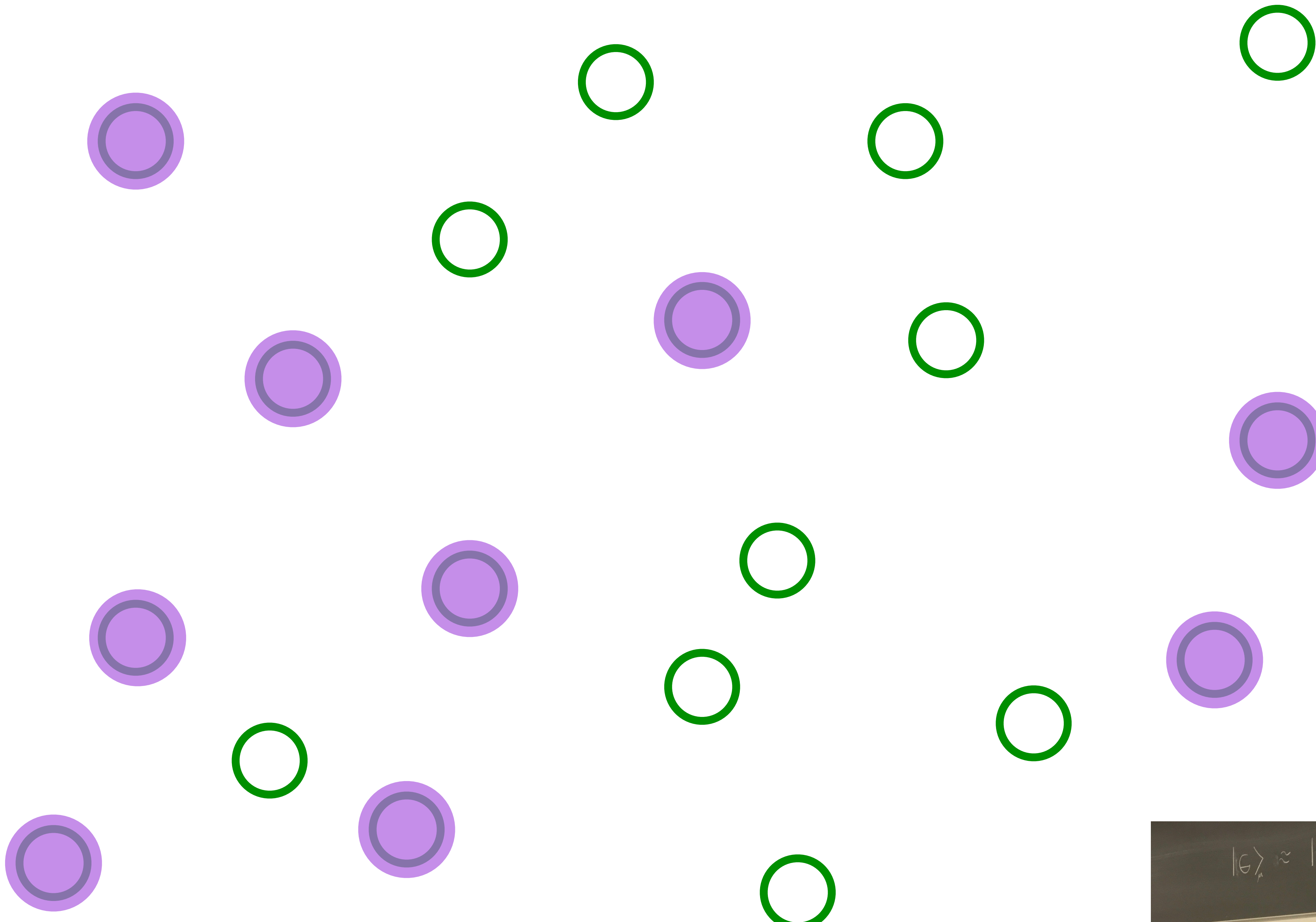

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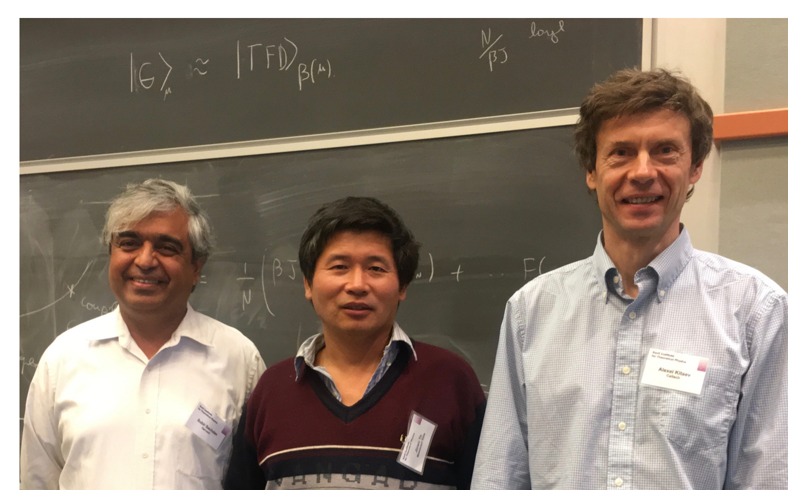

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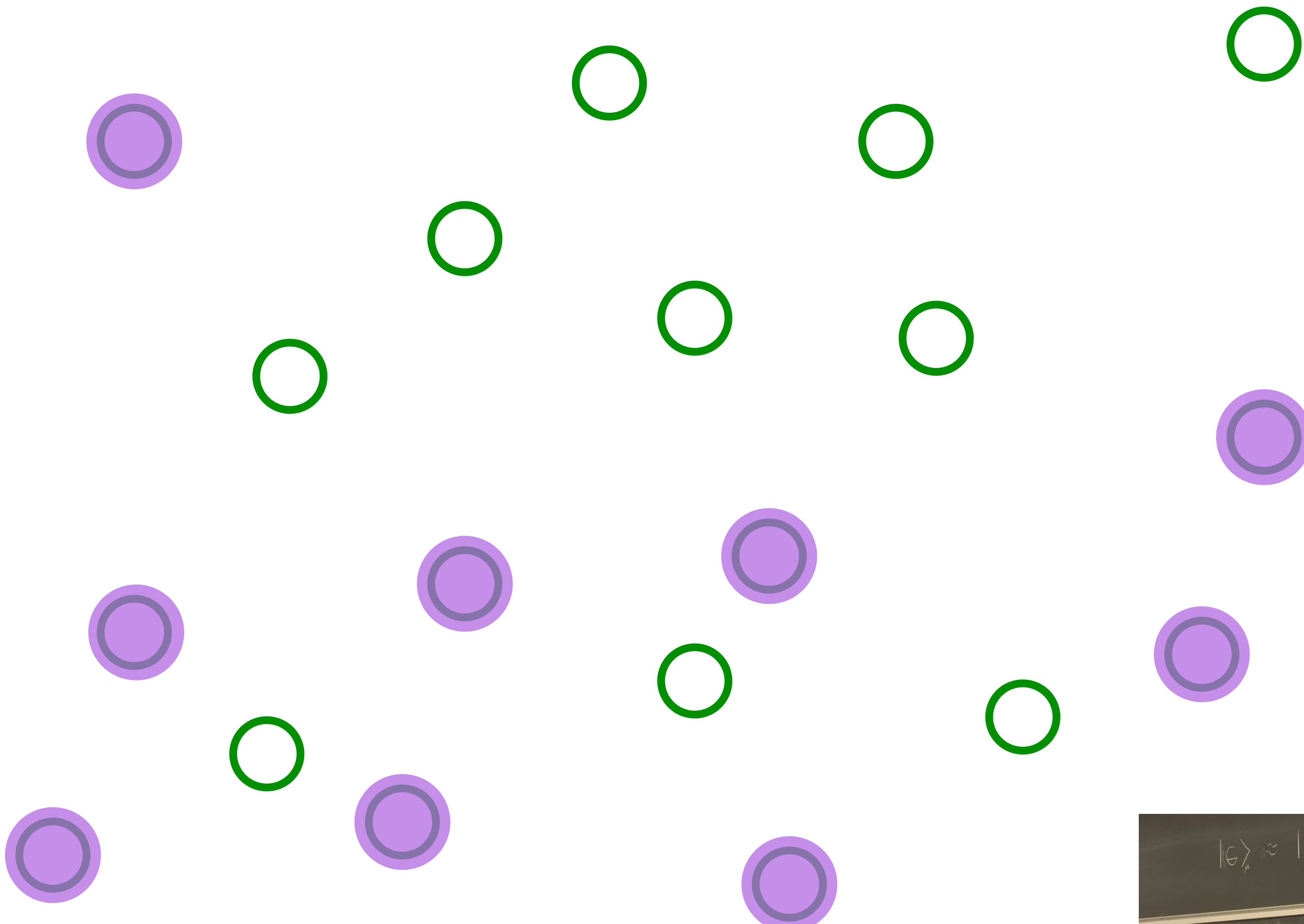

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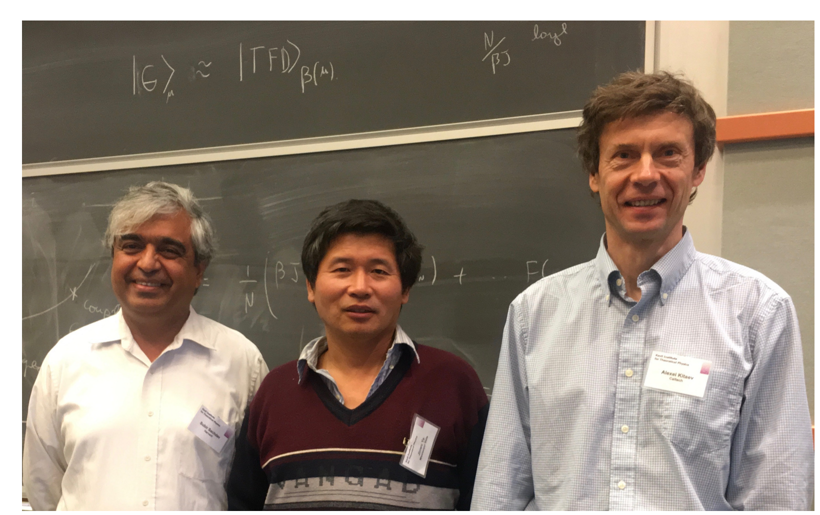

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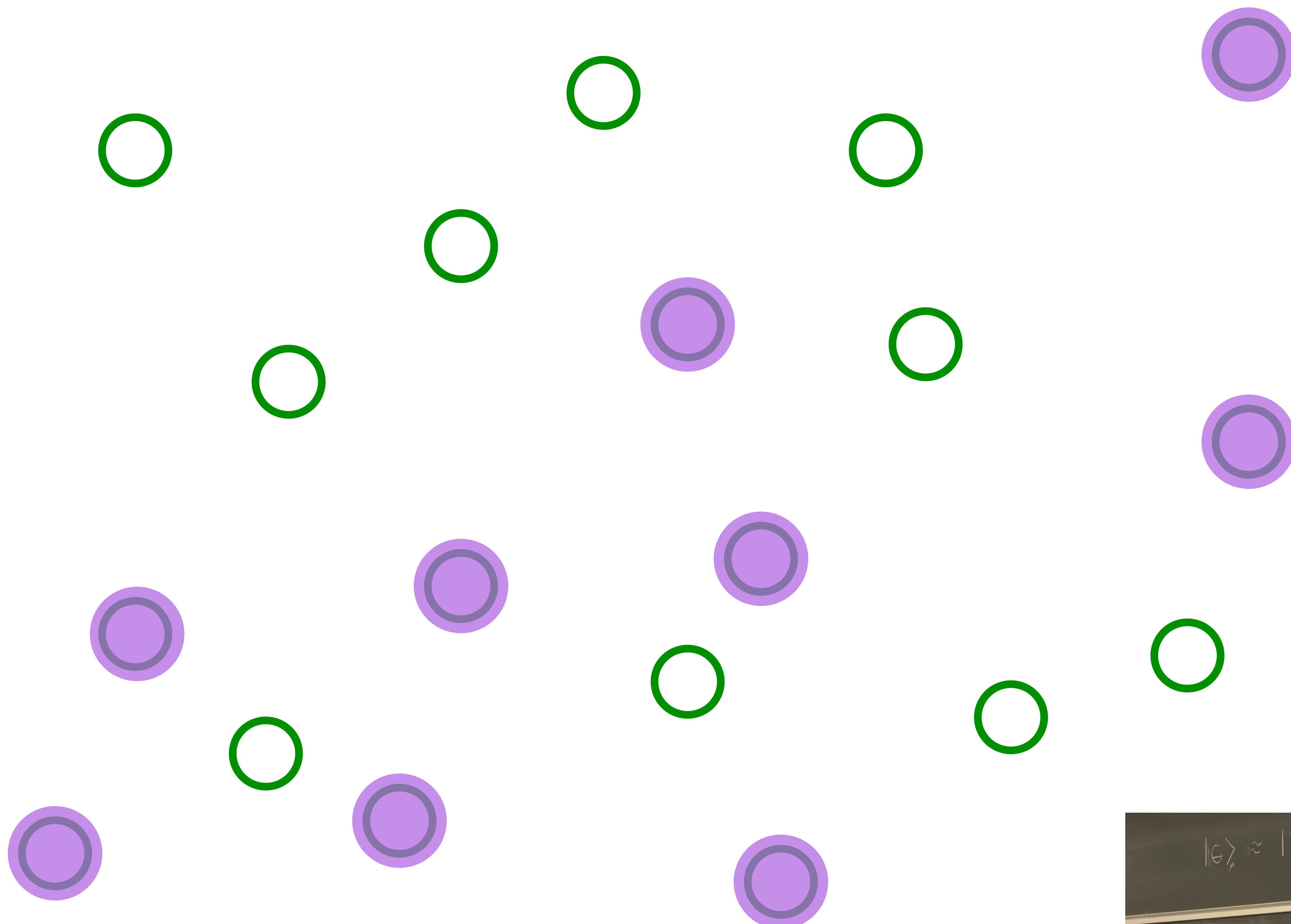

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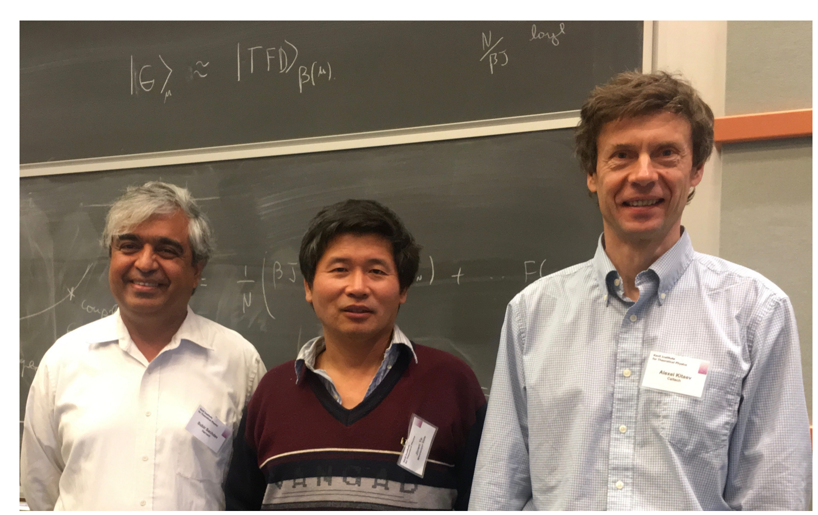

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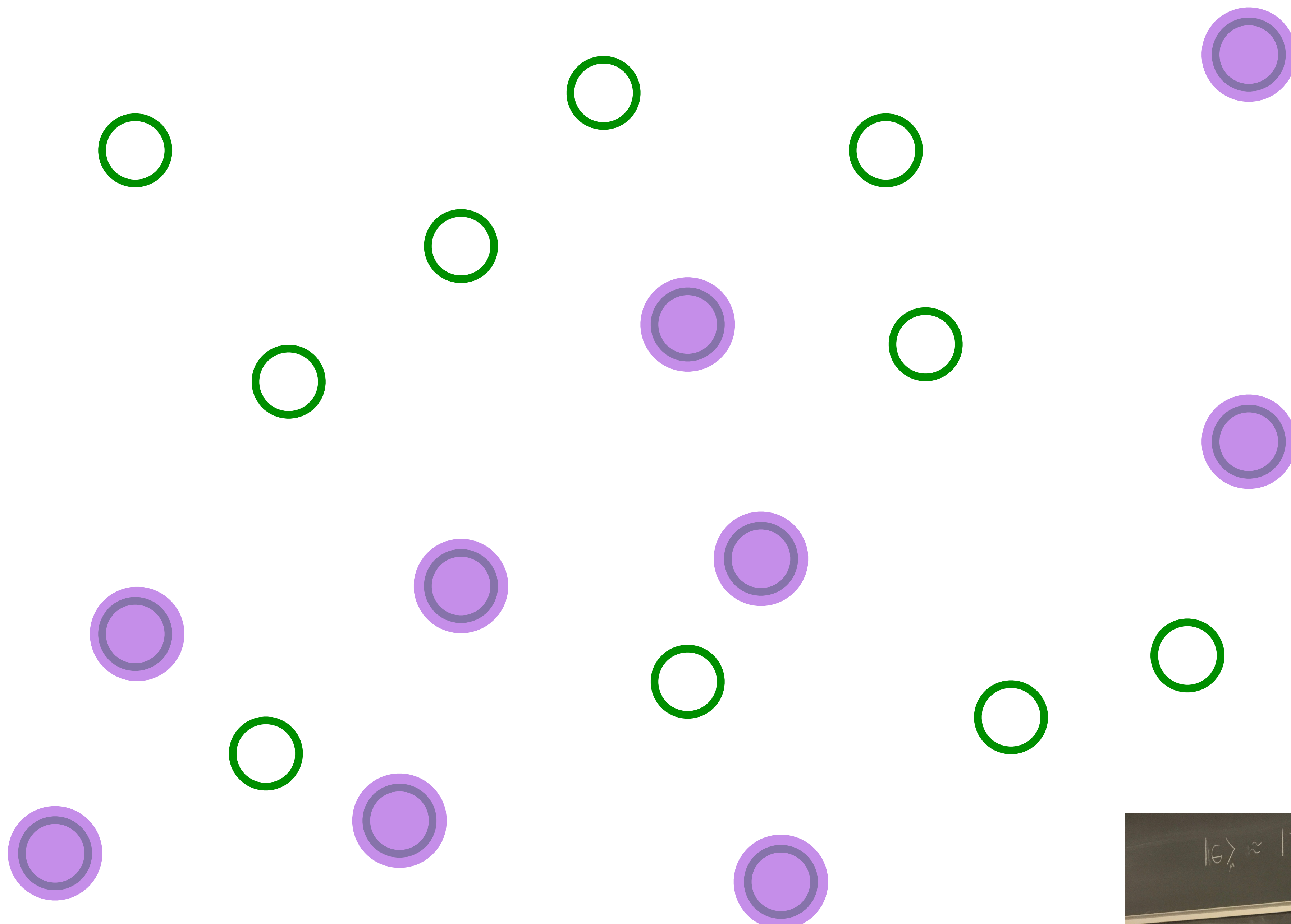
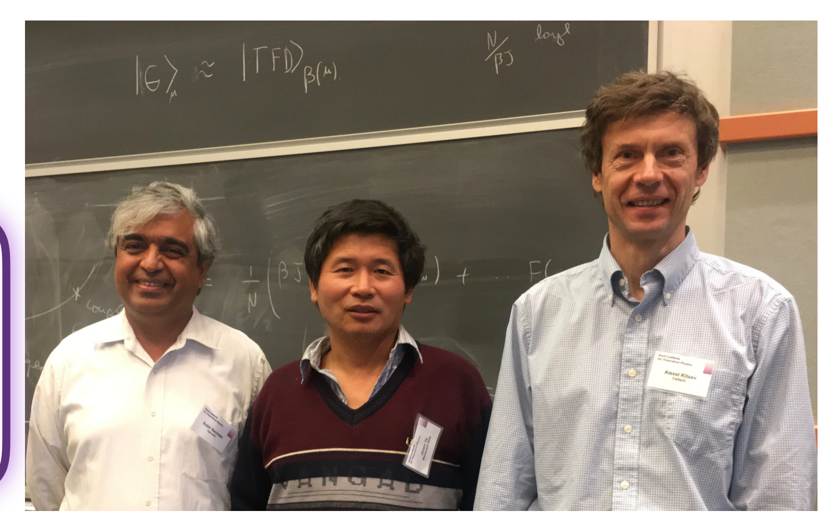


This describes both a strange metal and a black hole!



# The complex SYK model

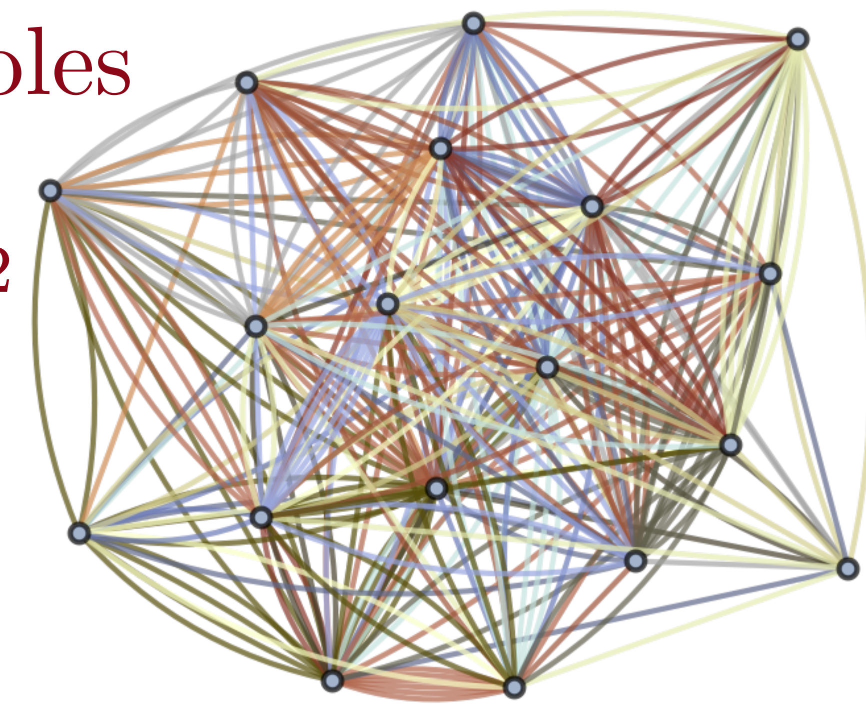
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta} \, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

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Flat band

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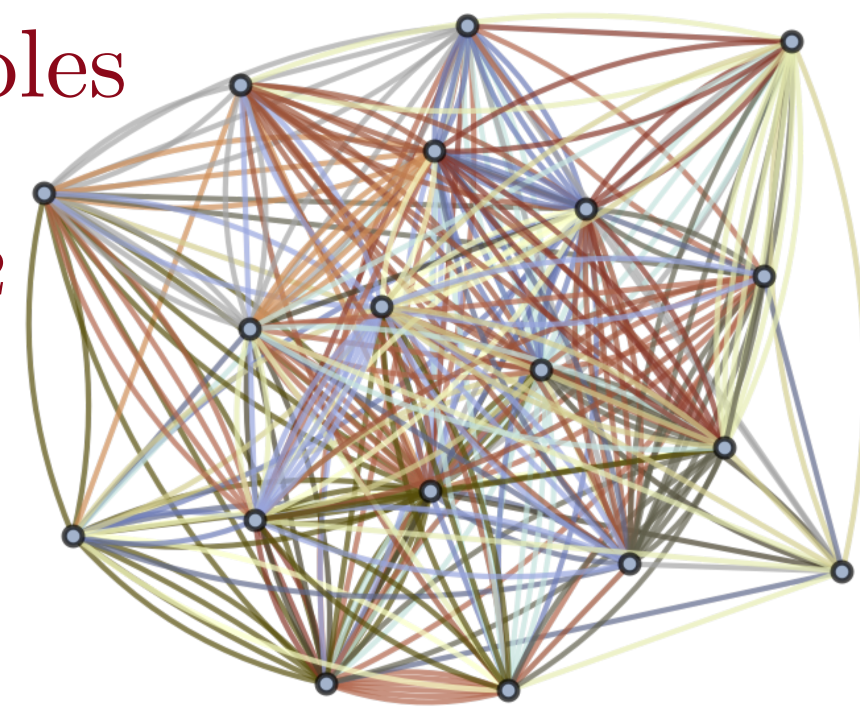

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Random interactions

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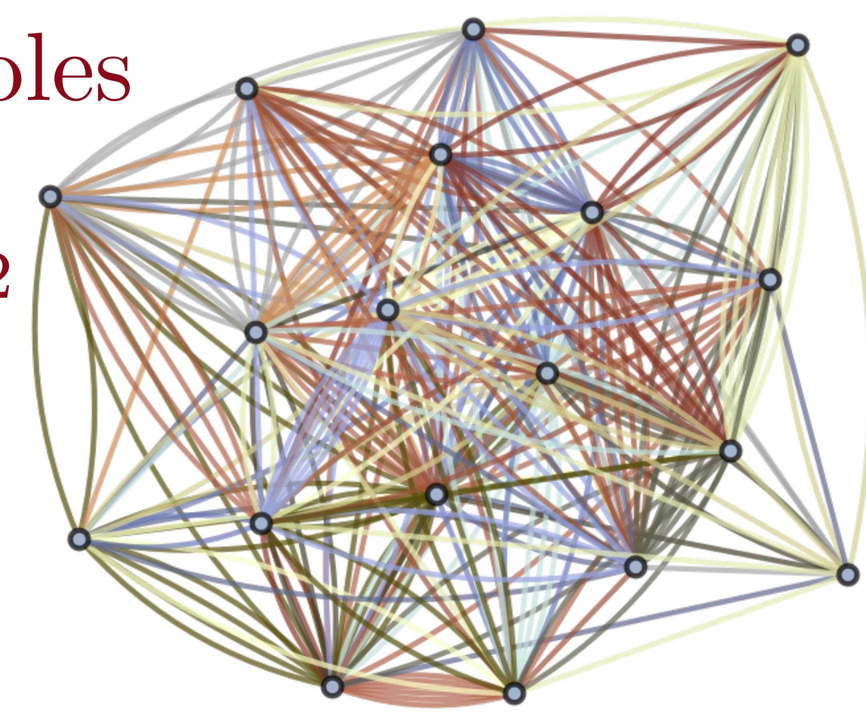

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Random interactions

Flat band

Density

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

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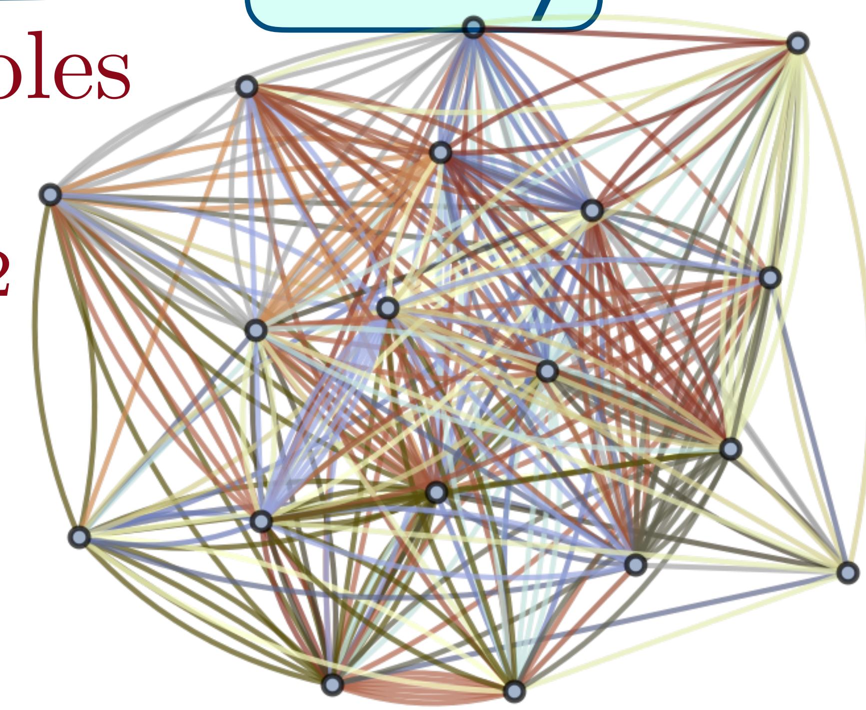

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$$G(\tau) \sim \begin{cases} -\tau^{-2\Delta} & \tau > 0 \\ e^{-2\pi\mathcal{E}}(-\tau)^{-2\Delta} & \tau < 0 \end{cases} \quad , \quad T = 0$$

where $\Delta = 1/4$, and the particle-hole asymmetry $\mathcal{E}$ is known exactly as a function of $\mathcal{Q}$ via a Luttinger relation.

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)
A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)
Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, arXiv: 1910.14099

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**Note:** In an ordinary metal (Fermi liquid), there is no particle-hole asymmetry at long times:

$$G(\tau) \sim \begin{cases} -\tau^{-1} & \tau > 0 \\ (-\tau)^{-1} & \tau < 0 \end{cases} \quad , \quad T = 0$$

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- There is a non-zero extensive entropy as $T \to 0$

$$\lim_{T\to 0}\lim_{N\to\infty}\frac{S}{N} = S_0(\mathcal{Q}) \neq 0$$

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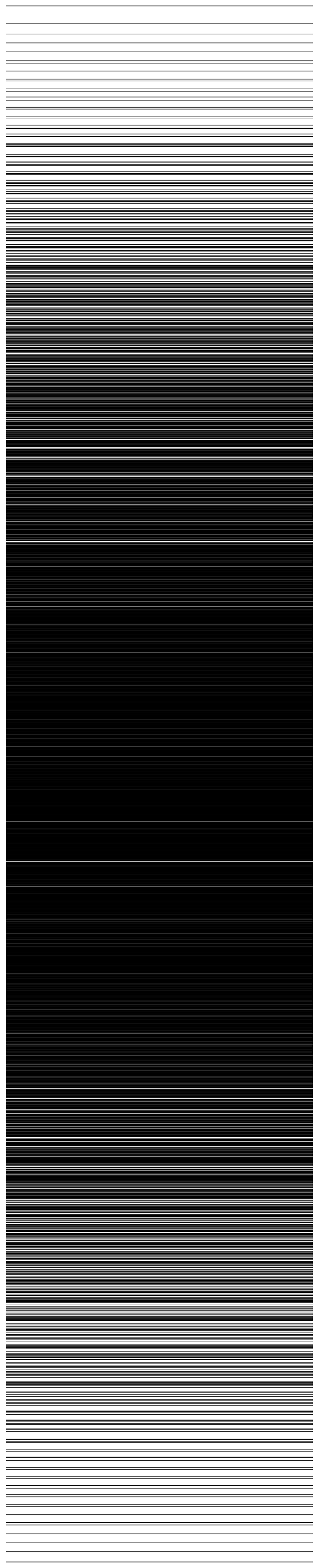
# A simple model of a metal with quasiparticles



Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$

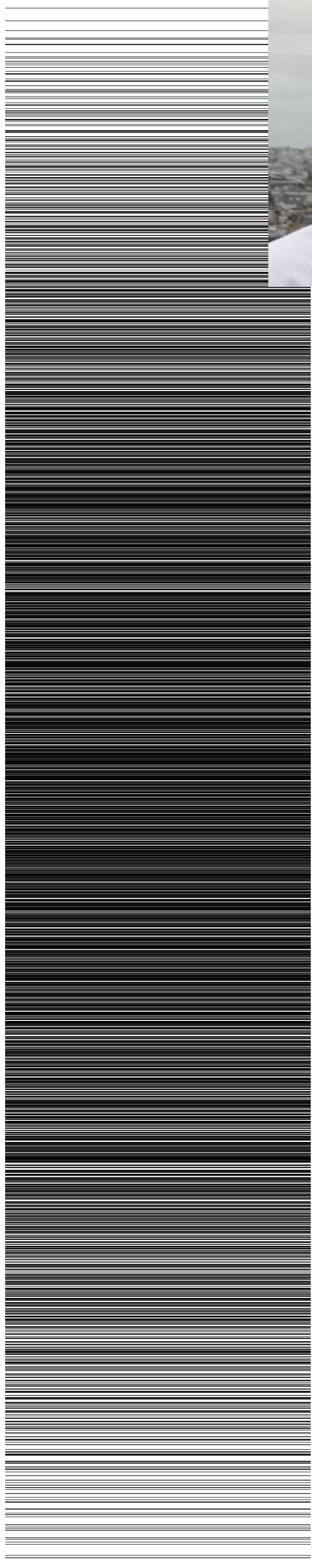
There are $2^N$ many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha ,$$

where $n_\alpha = 0, 1$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $\varepsilon_\alpha$ have a level spacing $\sim 1/N$.

# The SYK model

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are $2^N$ many body levels with energy $E$, which do not admit a quasiparticle decomposition. Shown are all values of $E$ for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N S_0$ with

$$\begin{aligned} S_0 &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\ &< \ln 2 \end{aligned}$$

where $G$ is Catalan's constant, for the half-filled case $Q = 1/2$.

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

# The complex SYK model

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$$\boxed{\frac{dS_0}{d\mathcal{Q}} = 2\pi\mathcal{E}}$$


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- All properties described so far apply to charged black holes with $\mathrm{AdS}_2$ horizons, with $\mathcal{E}$ a dimensionless measure of the electric field on the horizon. The relation $(*)$ is obtained from the Einstein-Maxwell equations (A. Sen, 2005).

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010); PRX **5,** 041025 (2015)

## SYK "derivation" of $dS_0/d\mathcal{Q} = 2\pi\mathcal{E}$

- At $T > 0$, conformal invariance implies the electron Green's function

$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{1/2} \quad , \quad 0 < \tau < 1/T$$

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- Use the Maxwell relation

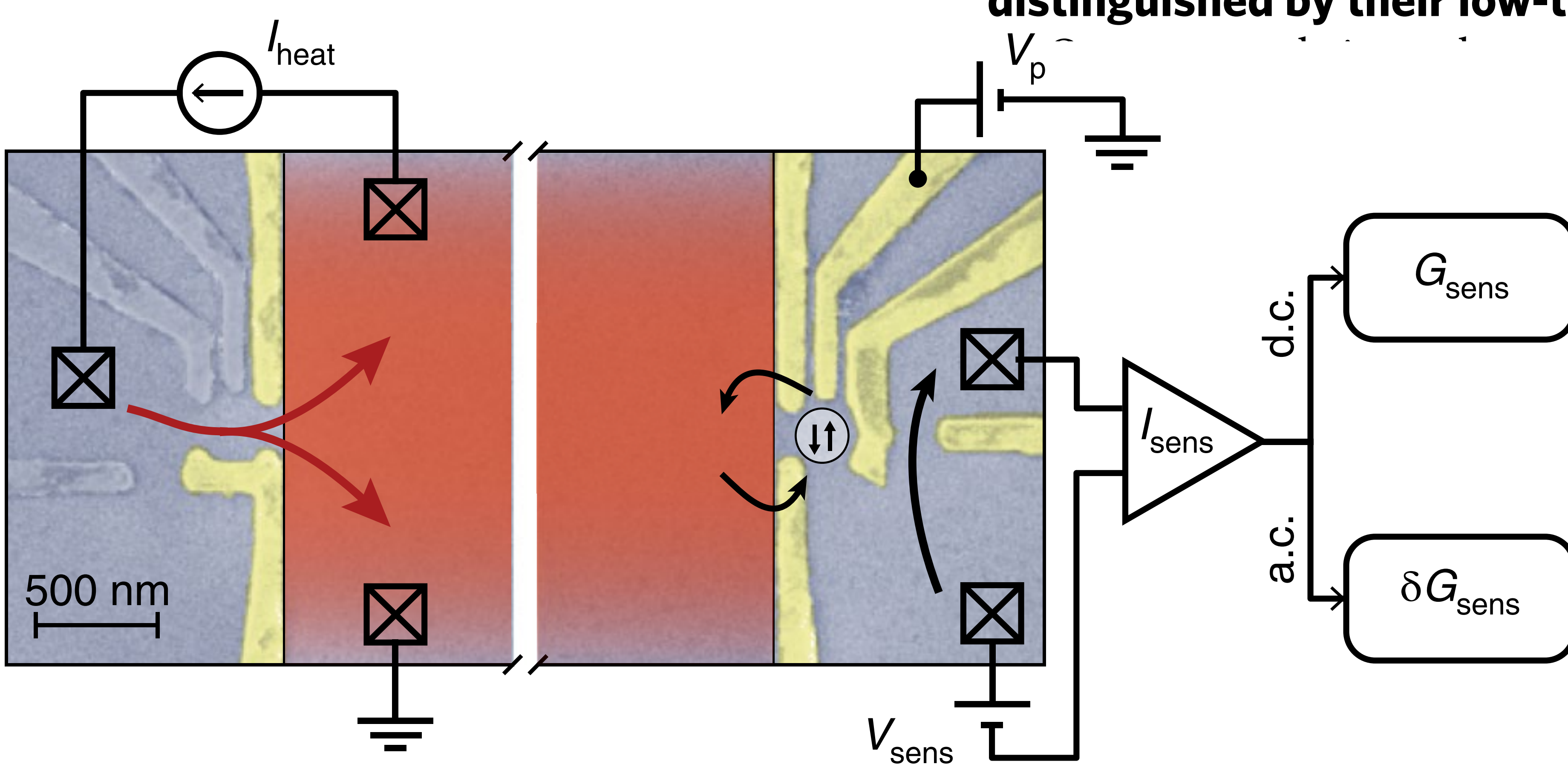
$$\boxed{\frac{1}{N}\left(\frac{\partial S}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}}}$$

# Direct entropy measurement in a mesoscopic quantum system

Nikolaus Hartman, Christian Olsen, Silvia Lüscher, Mohammad Samani, Saeed Fallahi, Geoffrey C. Gardner, Michael Manfra, and Joshua Folk

Nature Physics 14, 1083 (2018)

**The entropy of an electronic system offers important insights into the nature of its quantum mechanical ground state. This is particularly valuable in cases where the state is difficult to identify by conventional experimental probes, such as conductance. Traditionally, entropy measurements are based on bulk properties, such as heat capacity, that are easily observed in macroscopic samples but are unmeasurably small in systems that consist of only a few particles[1,2]. Here, we develop a mesoscopic circuit to directly measure the entropy of just a few electrons, and demonstrate its efficacy using the well-understood spin statistics of the first, second and third electron ground states in a GaAs quantum dot[3-8]. The precision of this technique, quantifying the entropy of a single spin-1/2 to within 5% of the expected value of $k_B \ln 2$, shows its potential for probing more exotic systems. For example, entangled states or those with non-Abelian statistics could be clearly distinguished by their low-temperature entropy[9-13].**




$$\frac{1}{N}\left(\frac{\partial S}{\partial Q}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_Q$$

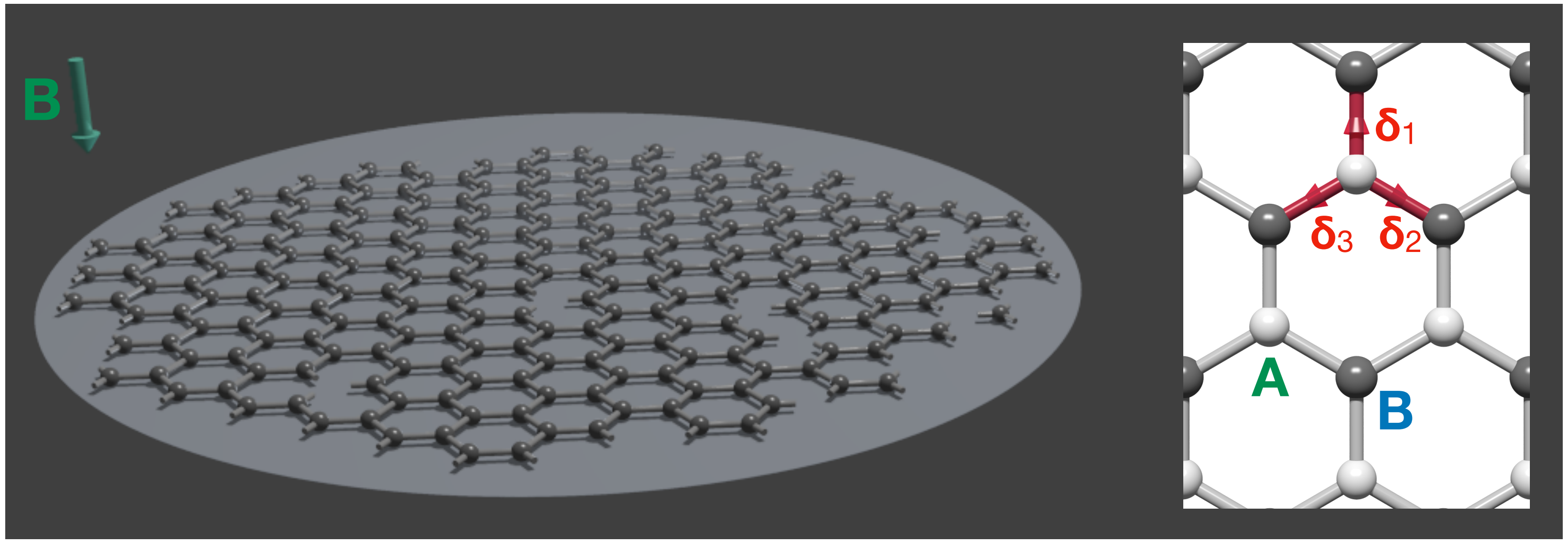
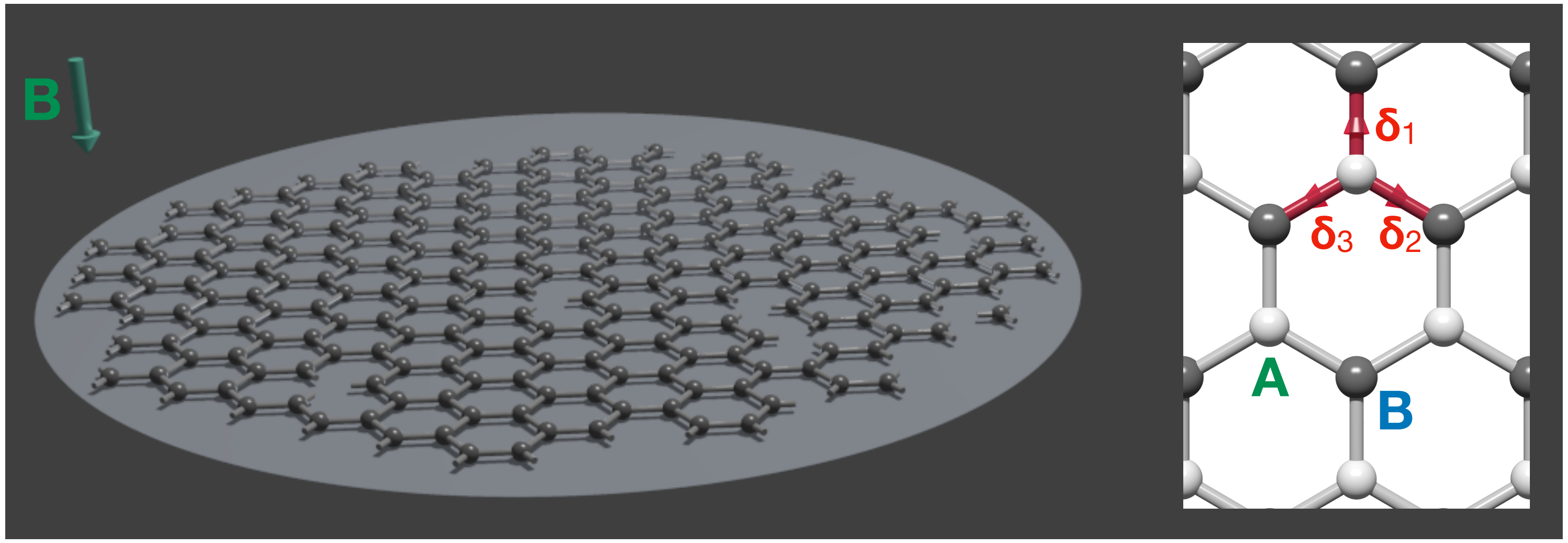



FIG. 1. Schematic depiction of the proposed device. Irregular shaped graphene flake in applied magnetic field $B$ forms the (0+1) dimensional many-body system equivalent to a black hole in (1+1) anti-de Sitter space. Inset: lattice structure of graphene with A and B sublattices marked and nearest neighbor vectors denoted by $\boldsymbol{\delta}_j$.

A. Chen, R. Ilan, F. de Juan, D. I. Pikulin, and M. Franz, *Phys. Rev. Lett.* **121**, 036403 (2018)

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- Measure the chemical potential of a SYK graphene flake. A linear-in-$T$ dependence of $\mu$ as $T \to 0$ is direct evidence for an extensive entropy at zero temperature.

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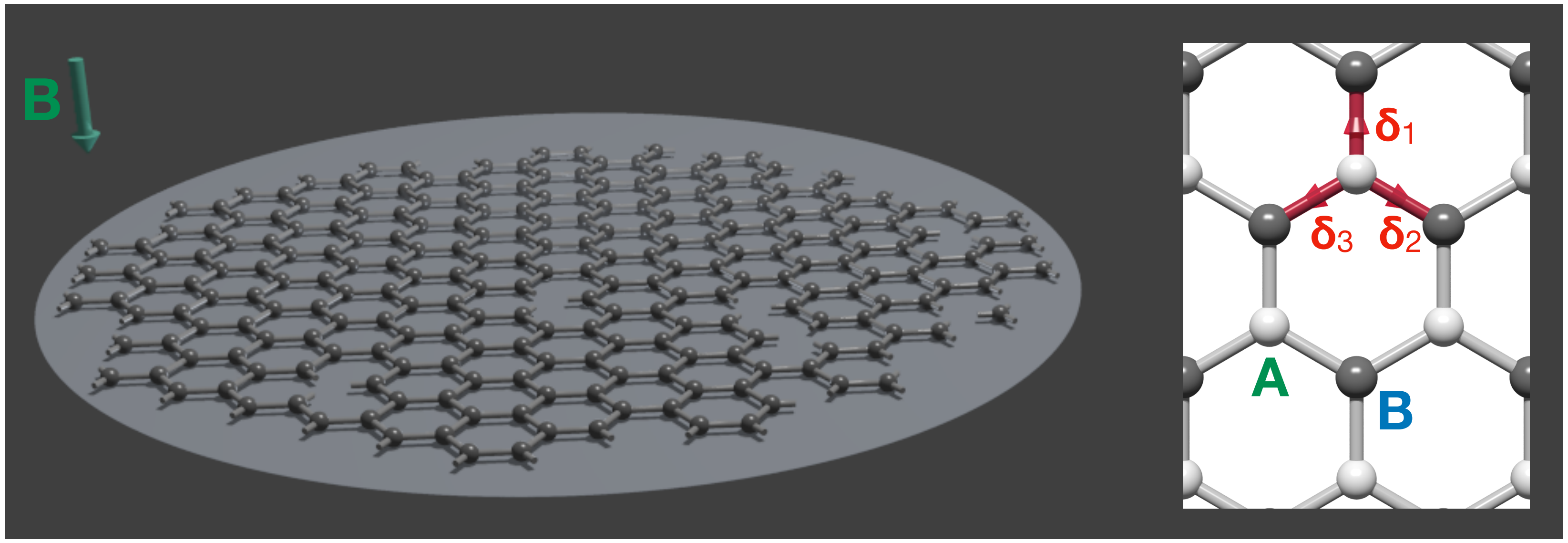


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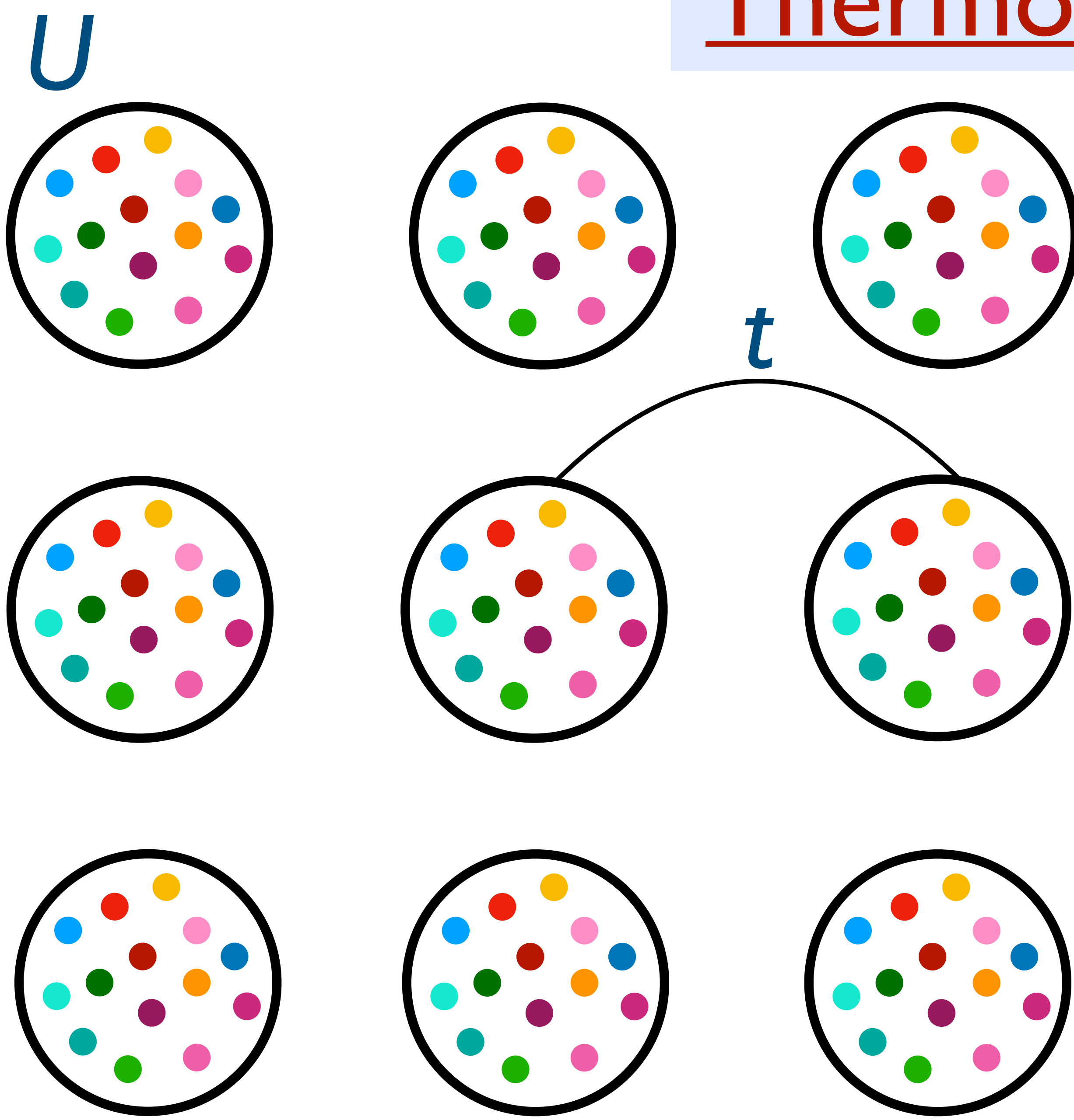
# Thermopower

Apply a temperature difference $\Delta T$, and measure the voltage difference $\Delta V$, while no current is flowing. The thermopower is

$$\Theta = \frac{\Delta V}{\Delta T} = \frac{k_B}{e} \times (\text{a number})$$

# Thermopower




Lattice of SYK islands

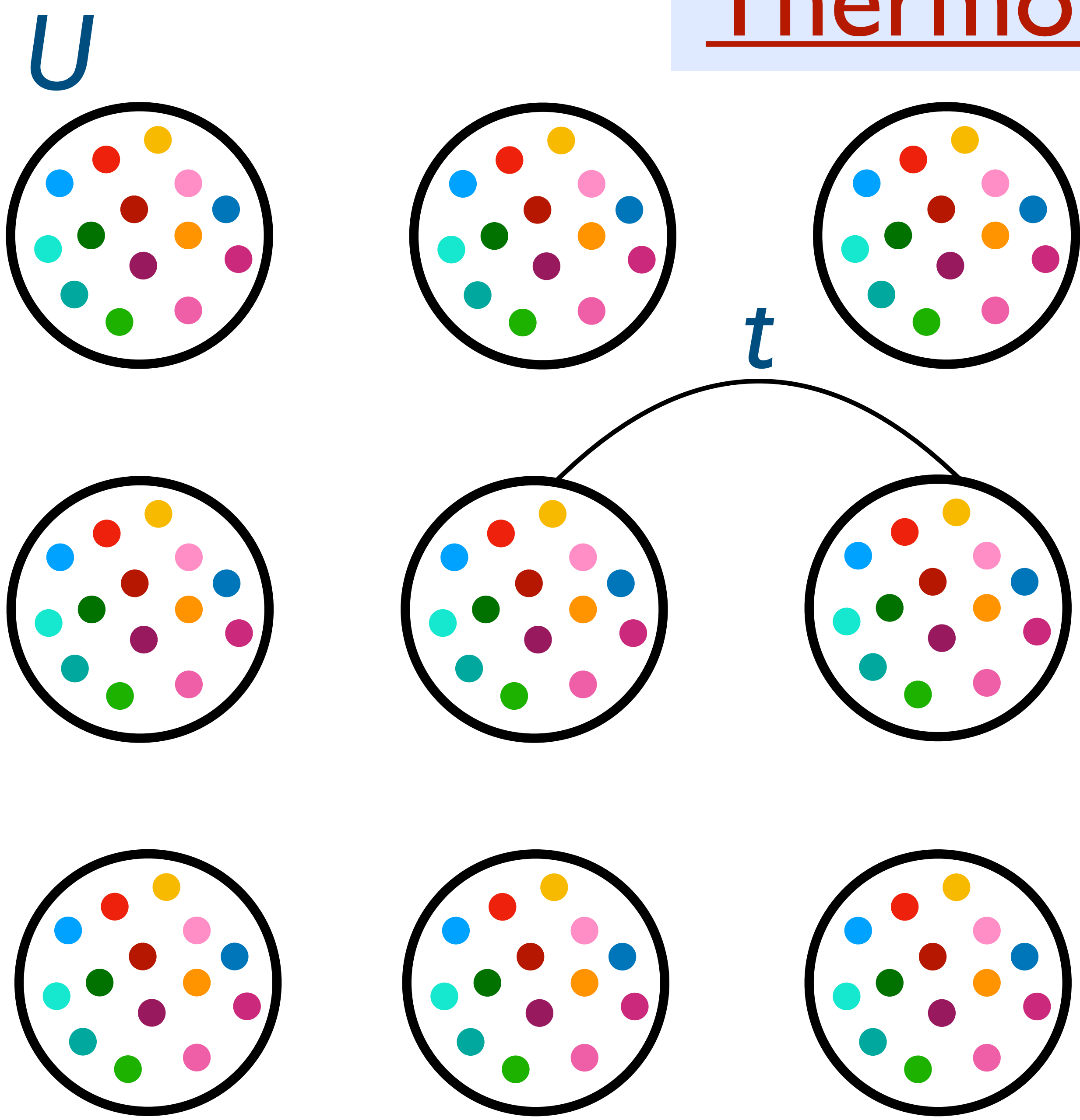
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017)

A. A. Patel, J. McGreevy, D. P. Arovas, S. Sachdev, PRX **8**, 021049 (2018)

$$f(\omega) = \frac{1}{e^{\hbar\omega/k_B T} + 1}$$

$$\Theta = \frac{\hbar}{eT} \frac{\int d\omega\, \omega \left(-\partial f/\partial\omega\right) \left[\mathrm{Im} G(\omega)\right]^2}{\int d\omega \left(-\partial f/\partial\omega\right) \left[\mathrm{Im} G(\omega)\right]^2} = \frac{k_B}{e} 2\pi\mathcal{E} = \frac{1}{e}\frac{dS_0}{dQ}$$

# Thermopower

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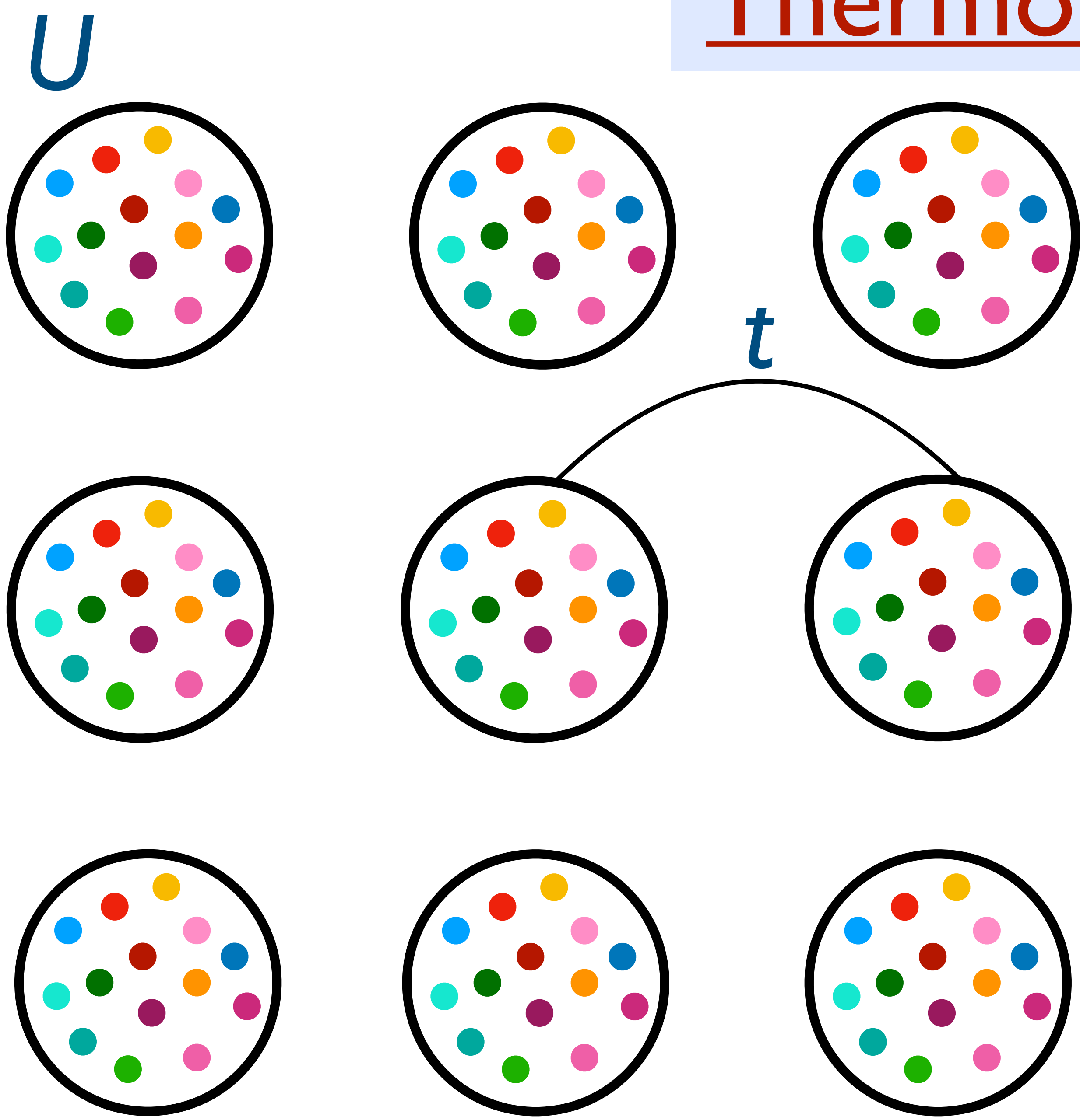
$\Theta = (1/e) dS_0/dQ$ is a general property of charged black holes

# Thermopower

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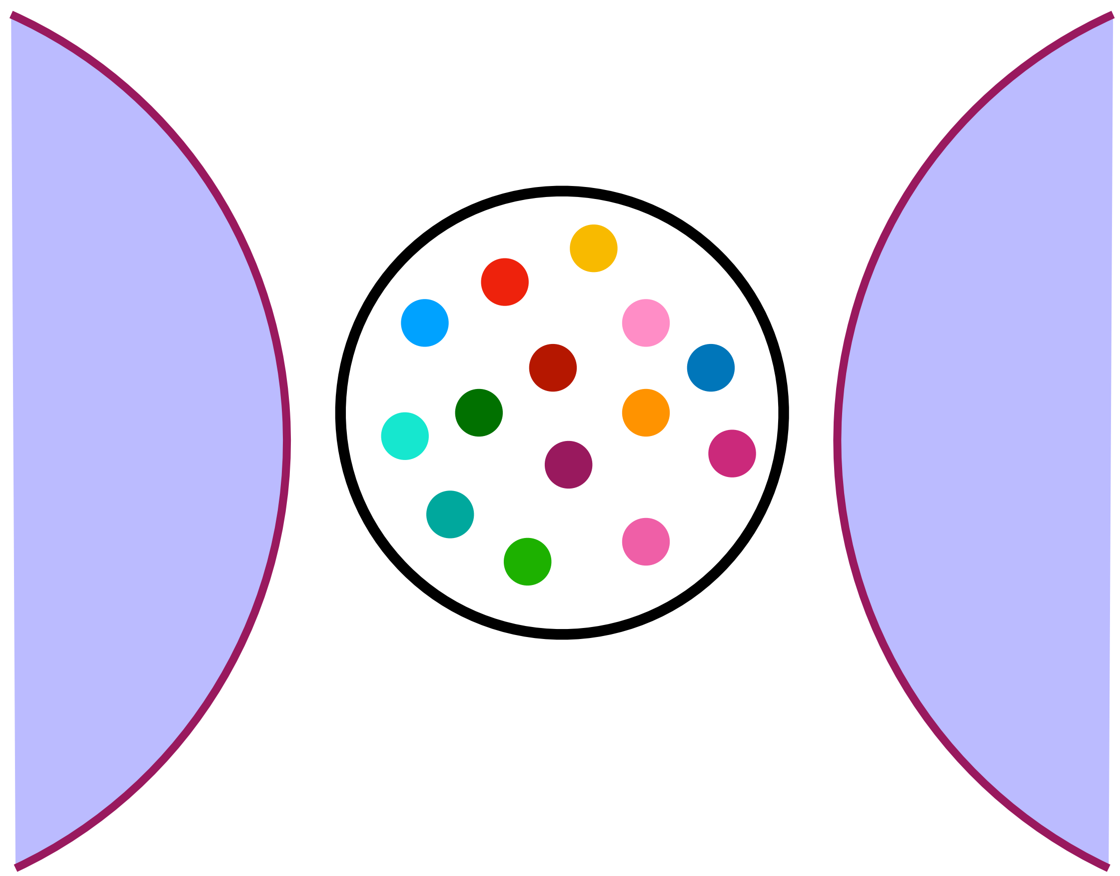
Thermopower is non-zero as $T \to 0$

# Thermopower



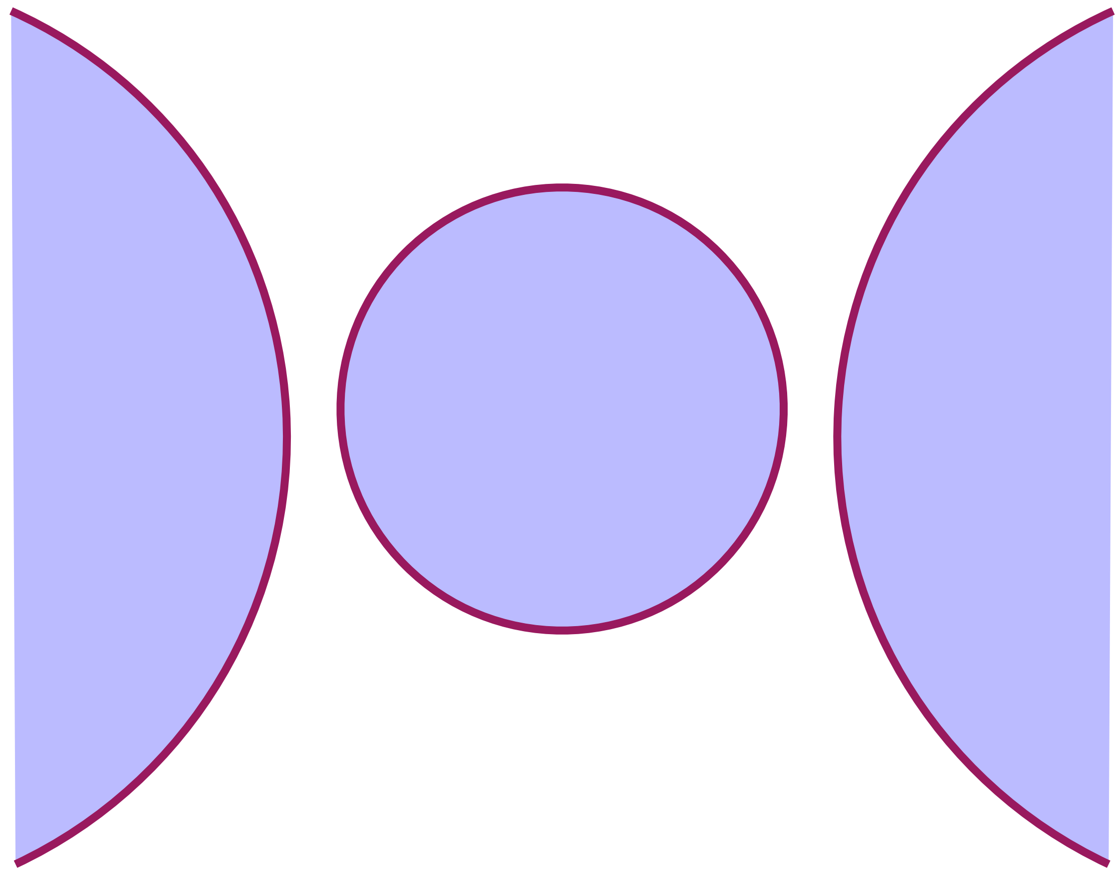
One SYK island very weakly coupled to normal metal leads

$$f(\omega) = \frac{1}{e^{\hbar\omega/k_B T} + 1}$$

$$\Theta = \frac{\hbar}{eT} \frac{\int d\omega\, \omega\, (-\partial f/\partial\omega)\, [\mathrm{Im} G(\omega)]}{\int d\omega\, (-\partial f/\partial\omega)\, [\mathrm{Im} G(\omega)]} = \frac{k_B}{e} \frac{4\pi\mathcal{E}}{3} = \frac{2}{3e} \frac{dS_0}{dQ}$$

Thermopower is non-zero as $T \to 0$

# Thermopower

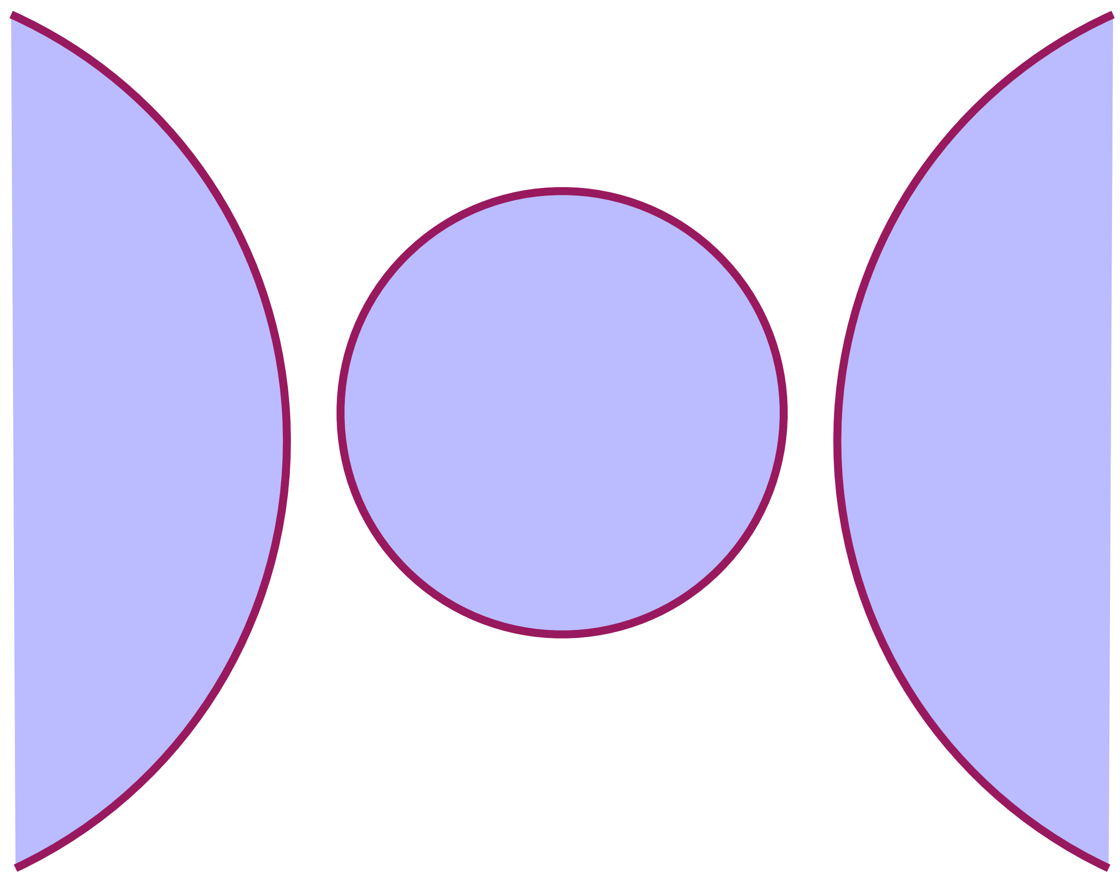


One normal metal very weakly coupled to normal metal leads

$$f(\omega) = \frac{1}{e^{\hbar\omega/k_B T} + 1}$$

$$\Theta = \frac{\hbar}{eT} \frac{\int d\omega\, \omega\, (-\partial f/\partial\omega)\, [\mathrm{Im} G(\omega)]}{\int d\omega\, (-\partial f/\partial\omega)\, [\mathrm{Im} G(\omega)]} = \frac{k_B}{e} \frac{\pi^2 \rho'(E_F) k_B T}{3\rho(E_F)} = \frac{1}{e} \frac{dS}{dQ}$$

# Thermopower



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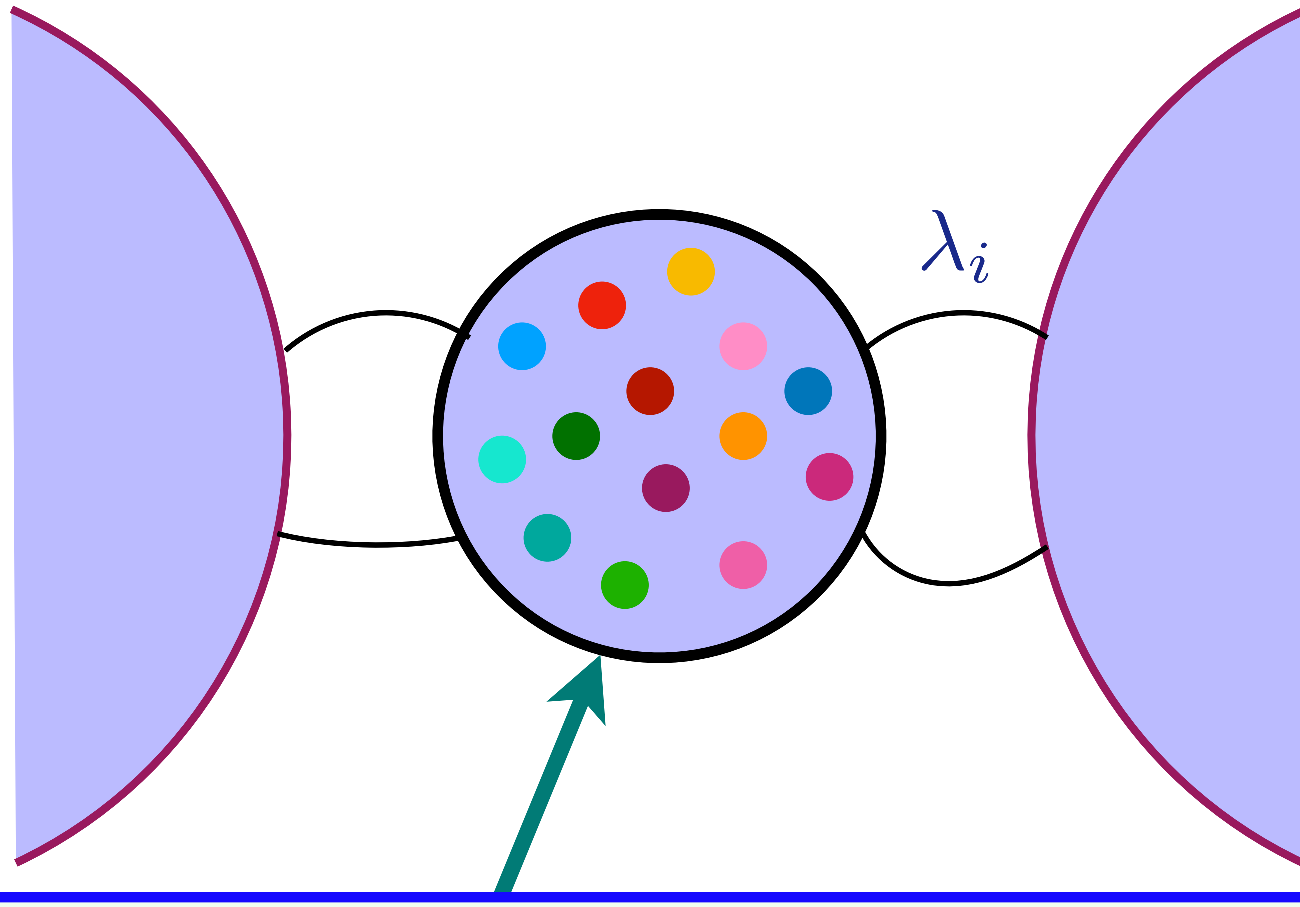
Thermopower vanishes linearly as $T \to 0$

# Thermopower

## A more realistic model

N. Gnezdilov, J. Hutasoit,
C. Beenakker
PRB **98**, 081413 (2018)



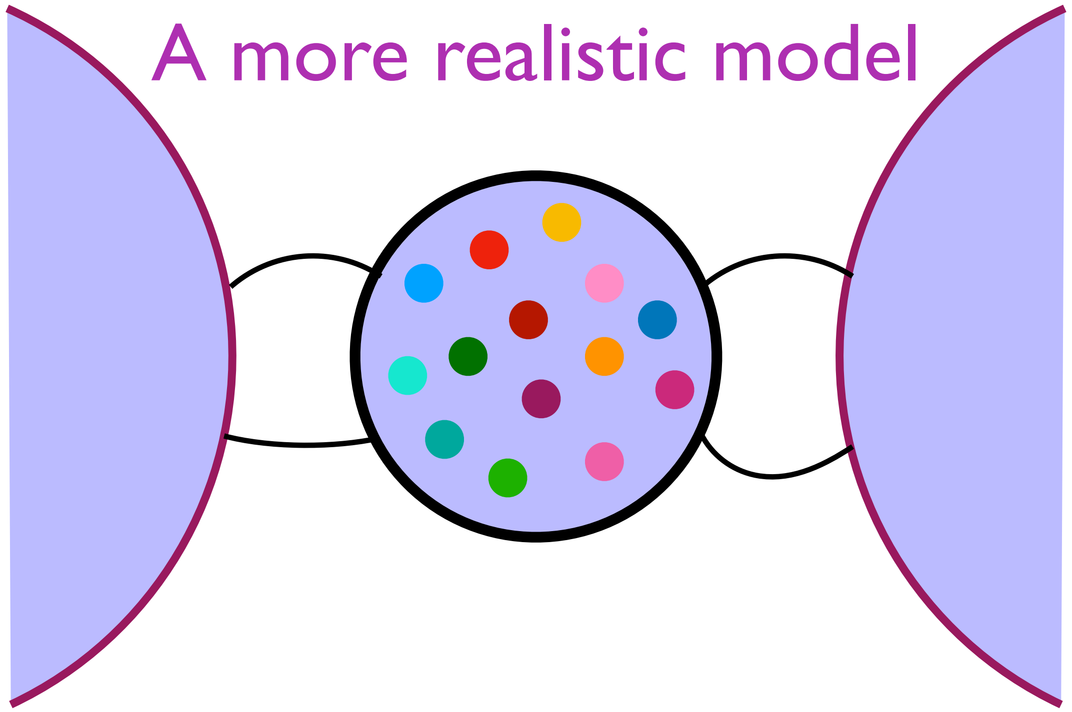

$$\Gamma = \pi \rho_{\rm lead} \sum_i |\lambda_i|^2$$

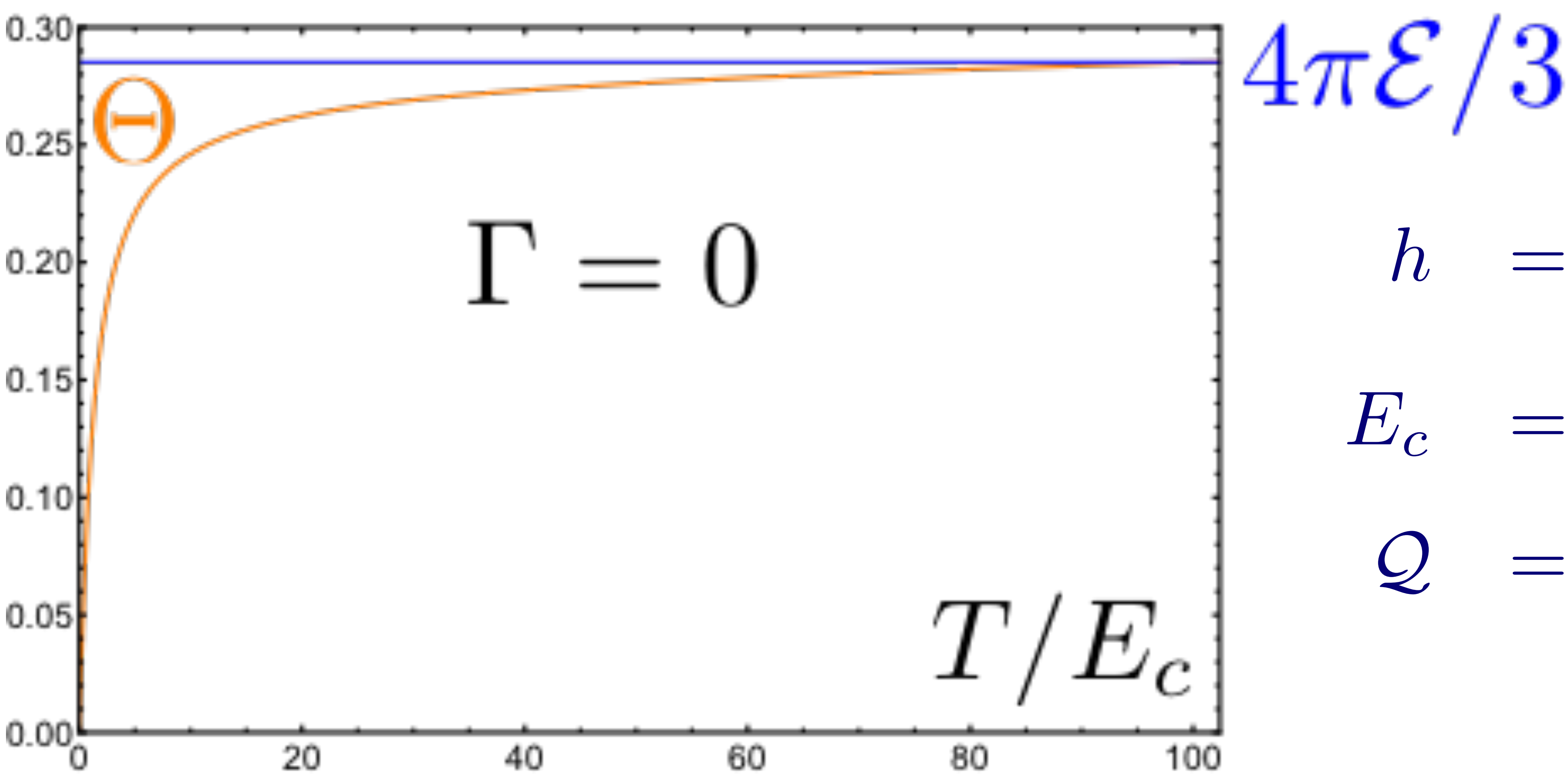
$$\overline{|h_{\alpha\beta}|^2} = h^2$$

$$H = \frac{1}{\sqrt{N}} \sum_{\alpha\beta=1}^{N} h_{\alpha\beta} c_\alpha^\dagger c_\beta + \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

$$\Theta = \frac{\hbar}{eT} \frac{\mathcal{L}_1}{\mathcal{L}_0} \quad , \quad \mathcal{L}_n = \int d\omega\, \omega^n \left(-\partial f/\partial\omega\right) \frac{{\rm Im} G(\omega)}{|1 + i\Gamma G(\omega)|^2}$$

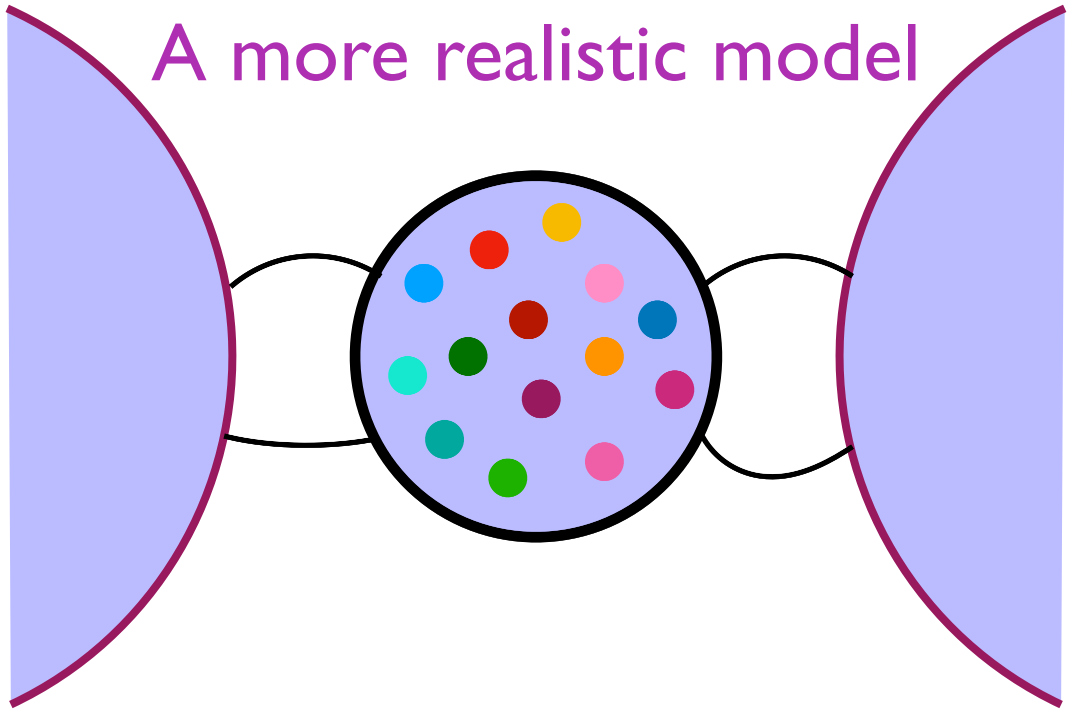
# Thermopower



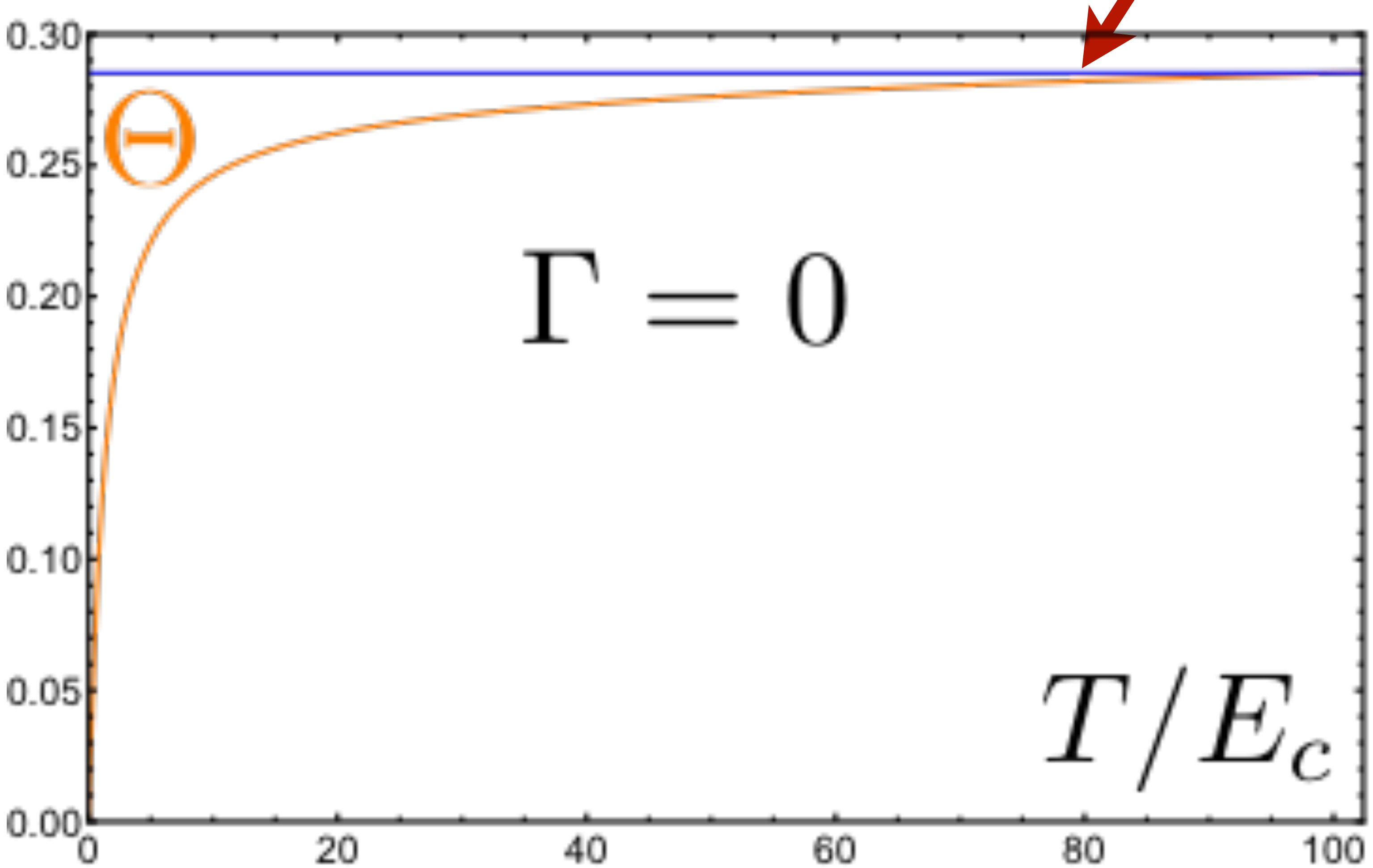




$$h = U/10^2$$
$$E_c = \frac{h^2}{U} = U/10^4$$
$$\mathcal{Q} = 1/3$$

# Thermopower




Regime of $T$-independent $\Theta$ for $E_c < T < U$ is evidence for SYK/black hole entropy




$4\pi\mathcal{E}/3$

$$
\begin{aligned}
h &= U/10^2 \\
E_c &= \frac{h^2}{U} = U/10^4 \\
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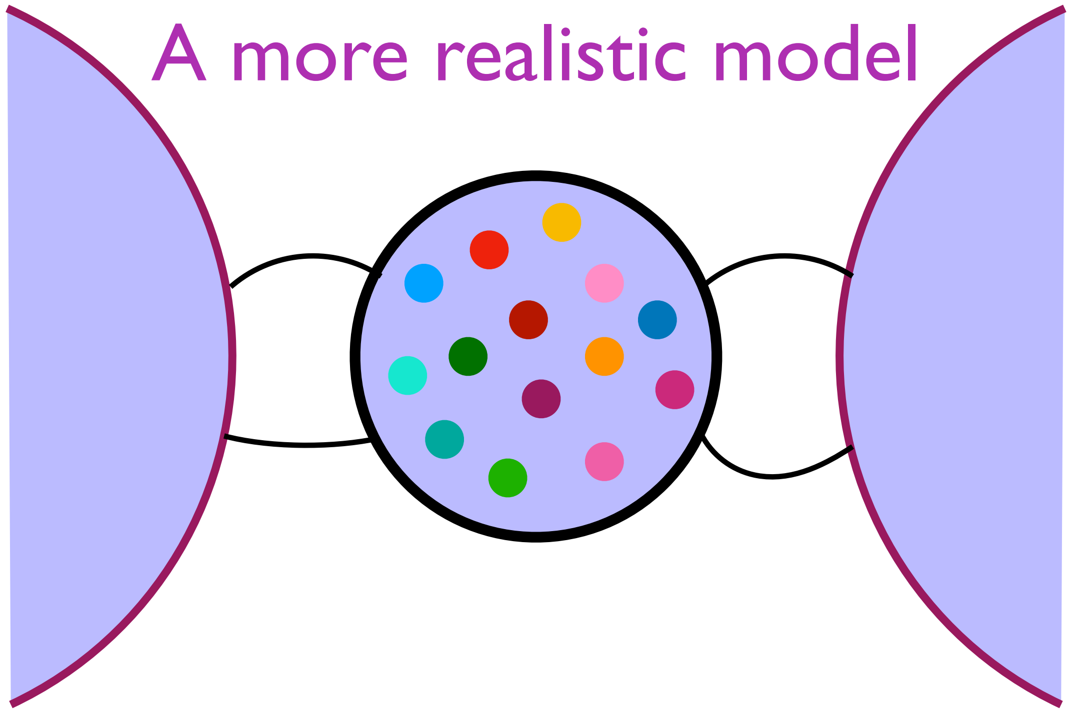
\end{aligned}
$$

# Thermopower

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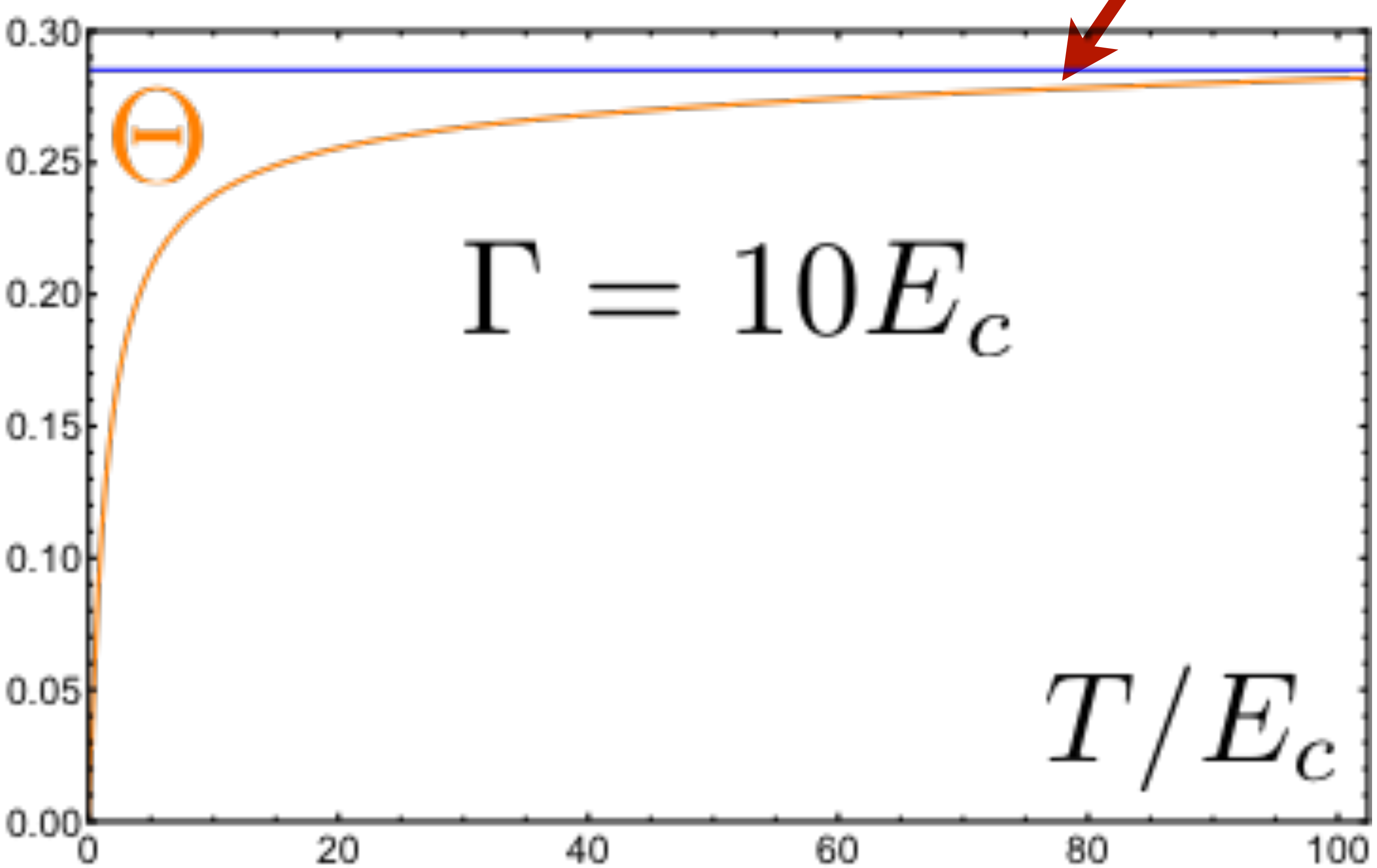


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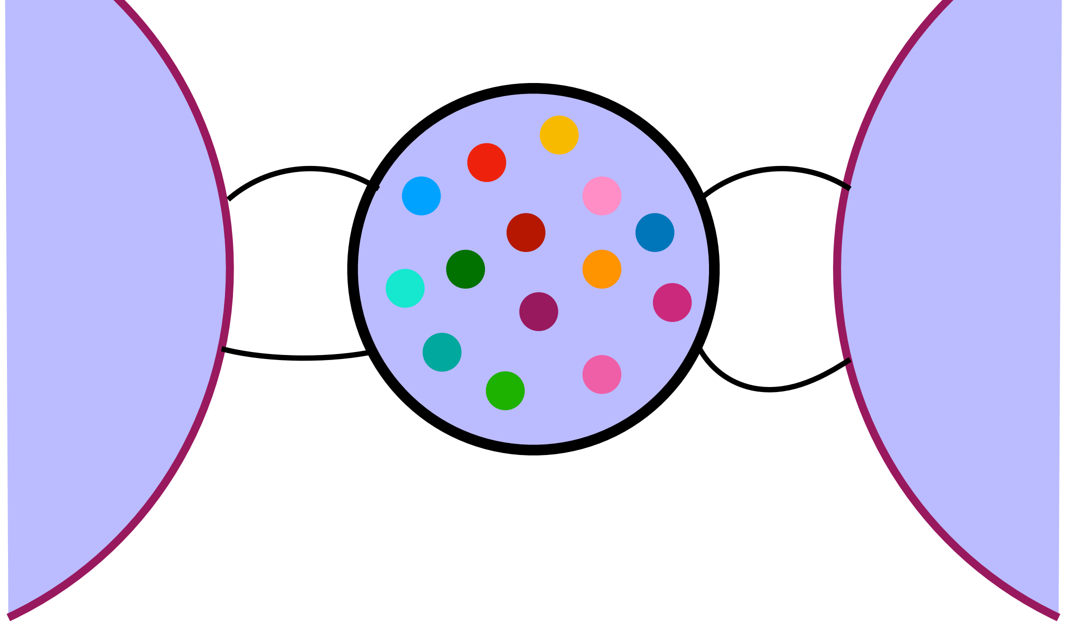
# Thermopower

A more realistic model



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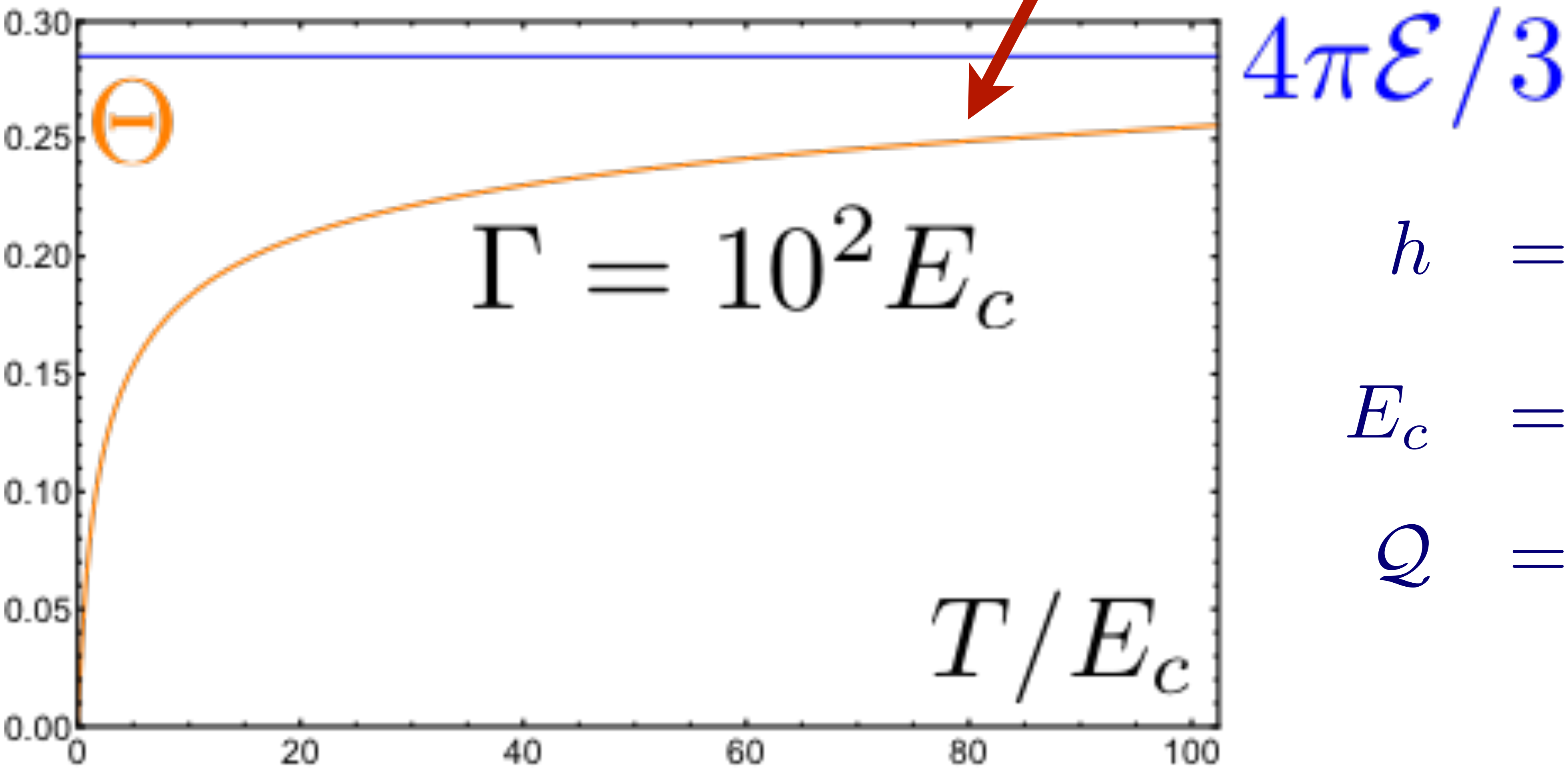


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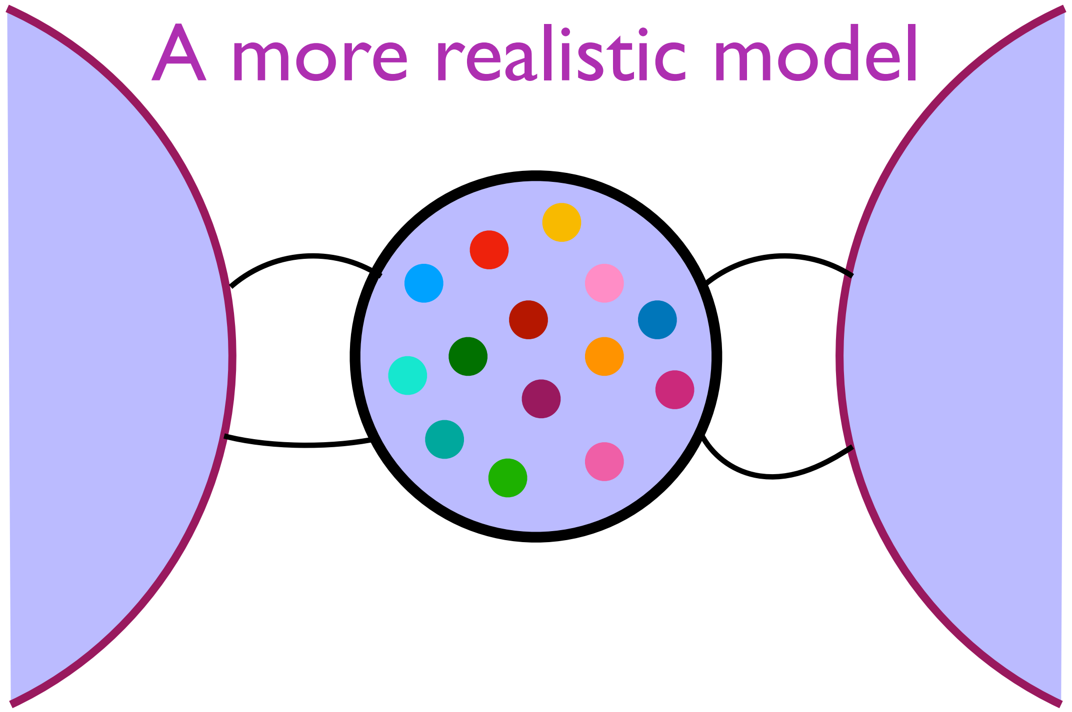


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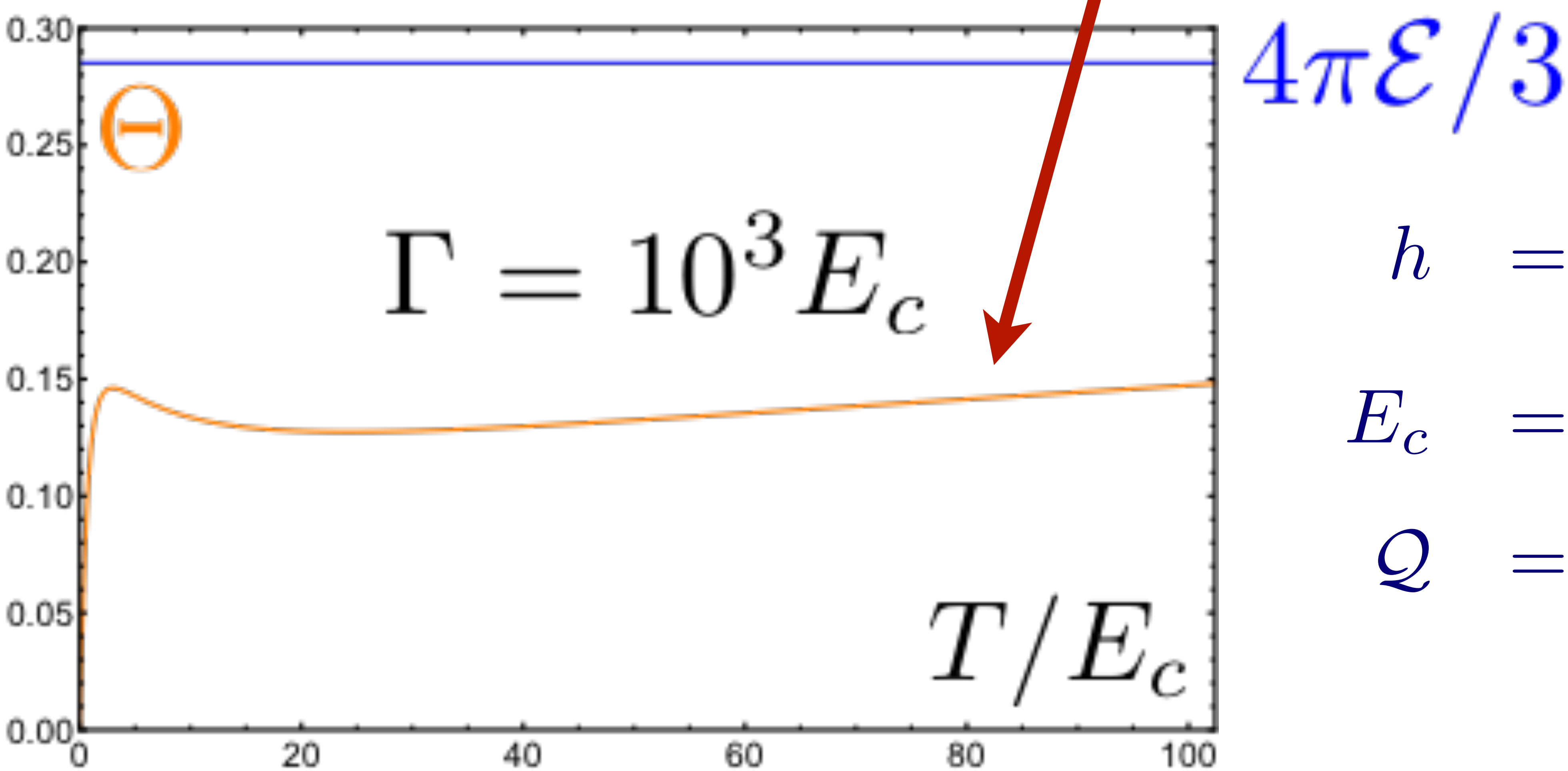


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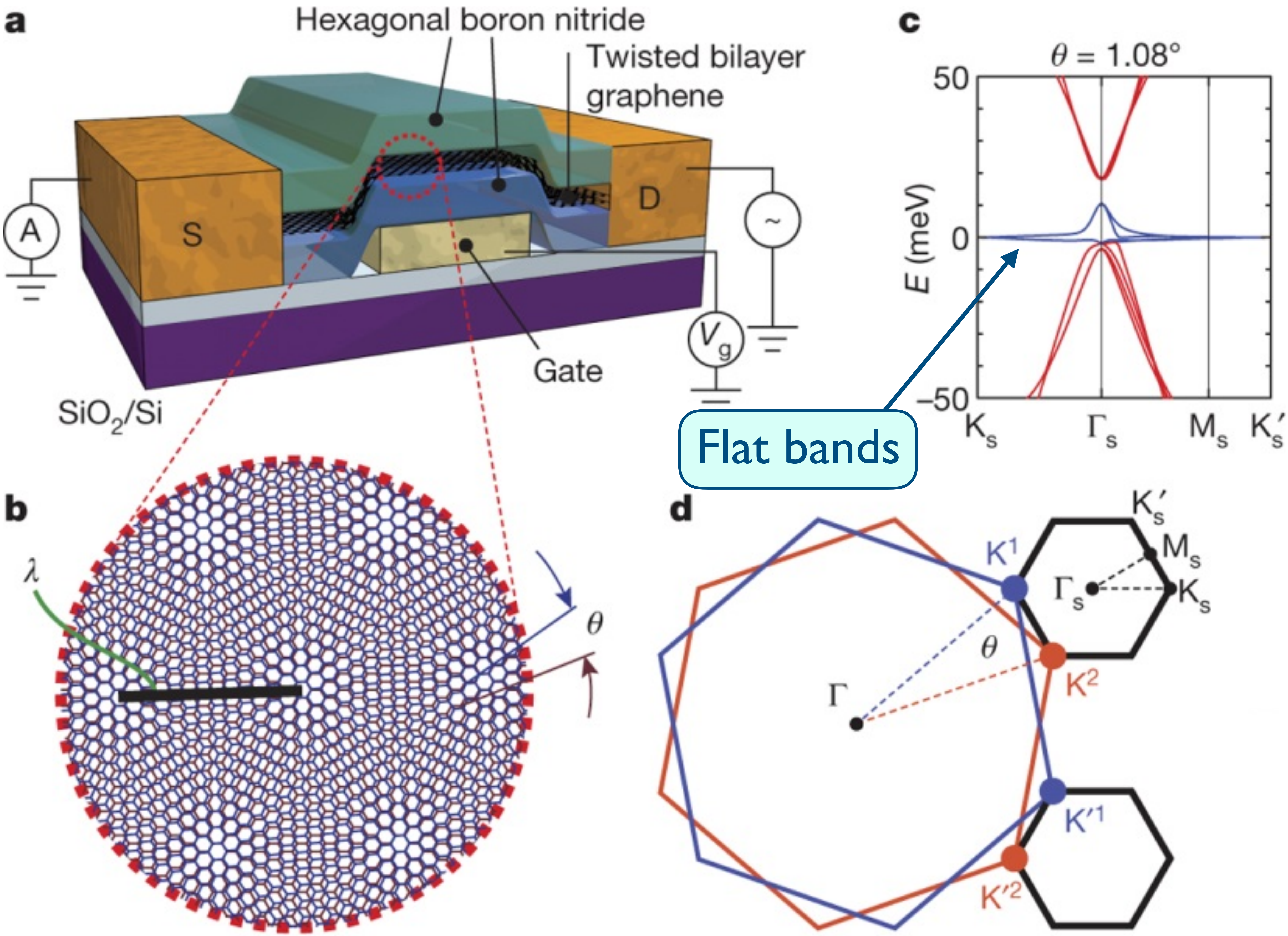
# Planckian metals and resonant SYK models

Aavishkar Patel

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

# Twisted bilayer graphene




Cao et al., Nature **556**, 80 (2018)

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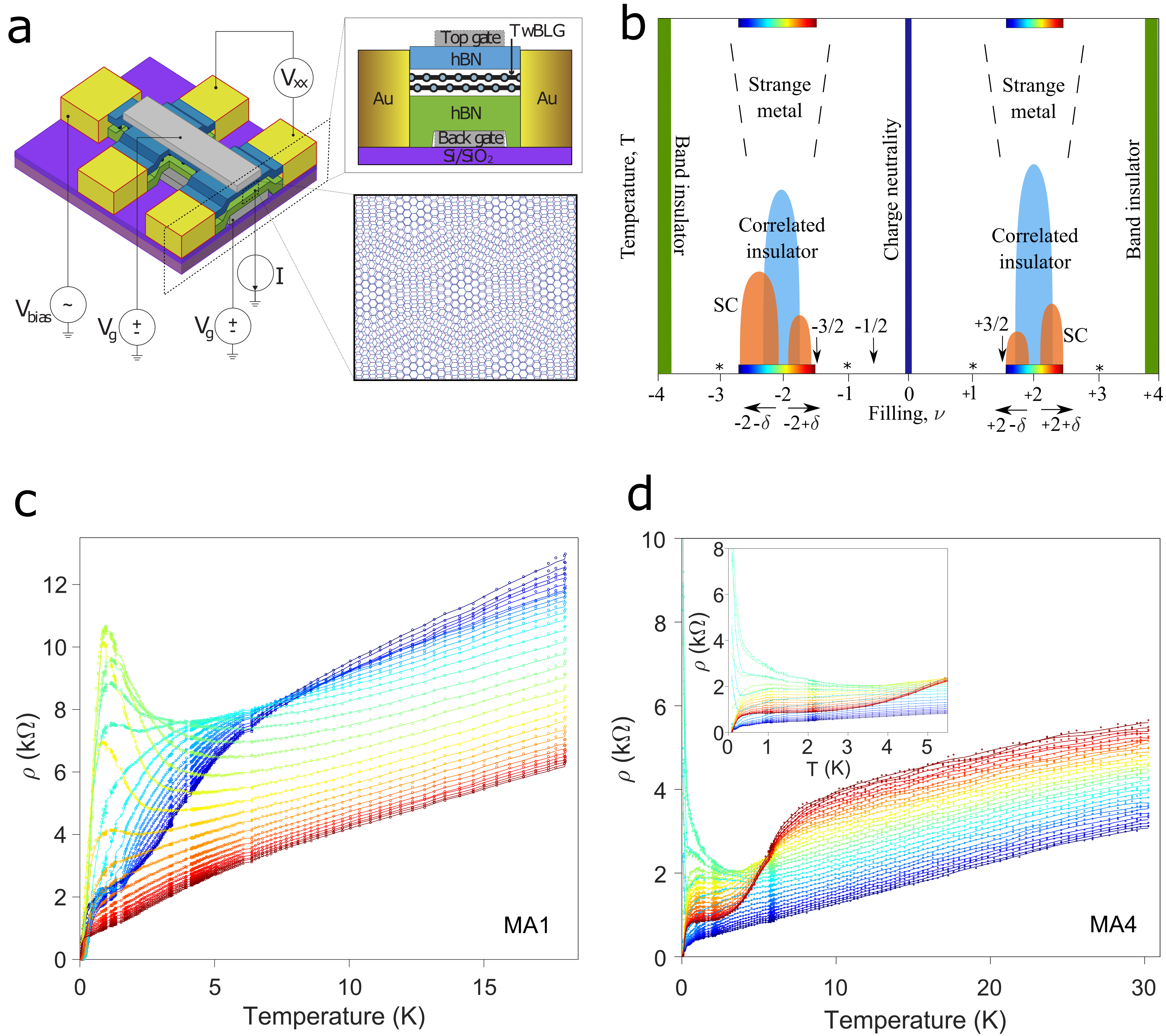



Cao *et al.*, arXiv:1901.03710

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, $\rho$, is

$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau}$$

with a universal scattering rate

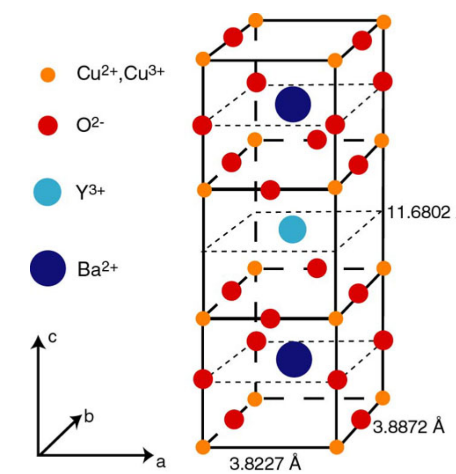
$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$


independent of the strength of interactions!

| Material | | $n$ ($10^{27}$ m$^{-3}$) | $m^*$ ($m_0$) | $A_1 / d$ ($\Omega$ / K) | $h / (2e^2 T_F)$ ($\Omega$ / K) | $\alpha$ |
|---|---|---|---|---|---|---|
| Bi2212 | $p = 0.23$ | 6.8 | 8.4 ± 1.6 | 8.0 ± 0.9 | 7.4 ± 1.4 | 1.1 ± 0.3 |
| Bi2201 | $p \sim 0.4$ | 3.5 | 7 ± 1.5 | 8 ± 2 | 8 ± 2 | 1.0 ± 0.4 |
| LSCO | $p = 0.26$ | 7.8 | 9.8 ± 1.7 | 8.2 ± 1.0 | 8.9 ± 1.8 | 0.9 ± 0.3 |
| Nd-LSCO | $p = 0.24$ | 7.9 | 12 ± 4 | 7.4 ± 0.8 | 10.6 ± 3.7 | 0.7 ± 0.4 |
| PCCO | $x = 0.17$ | 8.8 | 2.4 ± 0.1 | 1.7 ± 0.3 | 2.1 ± 0.1 | 0.8 ± 0.2 |
| LCCO | $x = 0.15$ | 9.0 | 3.0 ± 0.3 | 3.0 ± 0.45 | 2.6 ± 0.3 | 1.2 ± 0.3 |
| TMTSF | $P = 11$ kbar | 1.4 | 1.15 ± 0.2 | 2.8 ± 0.3 | 2.8 ± 0.4 | 1.0 ± 0.3 |

**Slope of *T*-linear resistivity vs Planckian limit in seven materials.**

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$




A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

# Flat band metal

For a dispersionless SYK model

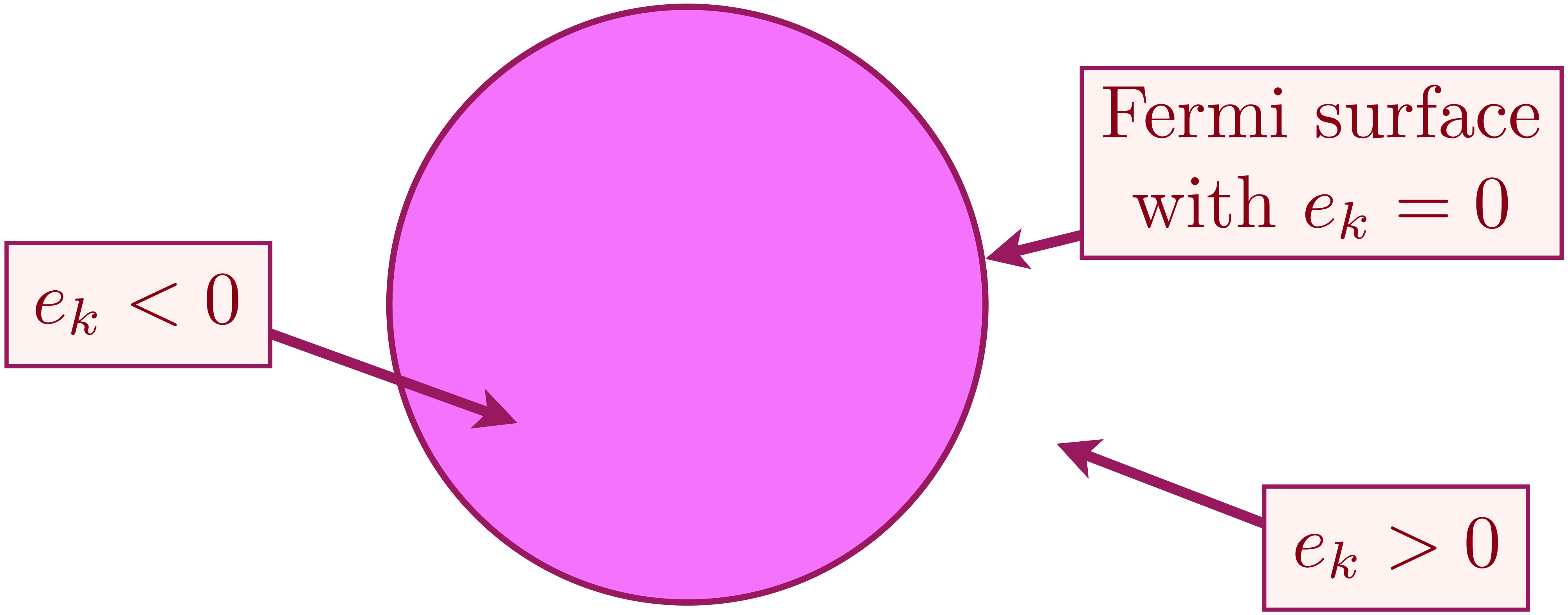
$$\langle c_\alpha(\tau) c_\alpha^\dagger(0) \rangle \sim e^{-(e/U) 2\pi \mathbb{C} T \tau} \times \left( \frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

Fermions have energy $e = -\mu$, and for $|e| \ll U$, we have the particle-hole asymmetry $\mathcal{E} = \mathbb{C} e/U$ with $\mathbb{C} = 0.41$.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

# Adding dispersion




- All electrons in the (flat band) SYK model have the same $e$
- In a more realistic metal, the electrons have a dispersion $e_k$ ($k$ is momentum), and $e_k = 0$ is the Fermi surface.

# Flat band metal

For a dispersionless SYK model

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# Planckian metal ansatz with dispersion


For a strongly-interacting metal with underlying quasiparticle dispersion $e_k$ ($k$ is the momentum)

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle \sim e^{-(e_k/U)2\pi \mathbb{C} T\tau} \times \left( \frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

At $e_k = 0$ we have a ‘remnant Fermi surface’ with a particle-hole symmetric spectral function.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

# Planckian metal ansatz with dispersion


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No free parameters—everything is determined by the (underlying) quasiparticle dispersion $e_k$, and the interaction strength $U$.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

## Resistivity of a Planckian metal as $T \to 0$

From the Kubo formula,

$$\sigma = \frac{e^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{de}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[ \text{Im}\, G^R_{\text{SYK}} \left( e, \frac{\omega}{T} \right) \right]^2 \text{sech}^2 \left( \frac{\omega}{2T} \right)$$

where the Fermi surface is defined by $e_k = 0$, $\boldsymbol{v}_F = \nabla_{\boldsymbol{k}} e_k$ on the Fermi surface, and

$$m^* = \frac{d\, V_{FS}}{\oint_{FS} |\boldsymbol{v}_F|} \,,$$

with $d$ the spatial dimensionality, and $V_{FS}$ is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual $m^*$.

Evaluating the integrals, we find

$$\rho = \frac{m^*}{ne^2} \, 2.71 \mathbb{C} \, \frac{k_B T}{\hbar} \quad , \quad \text{using } \mathcal{E} = \mathbb{C} e/U \,,$$

where $n = V_{FS}/(2\pi)^d$ is the density.

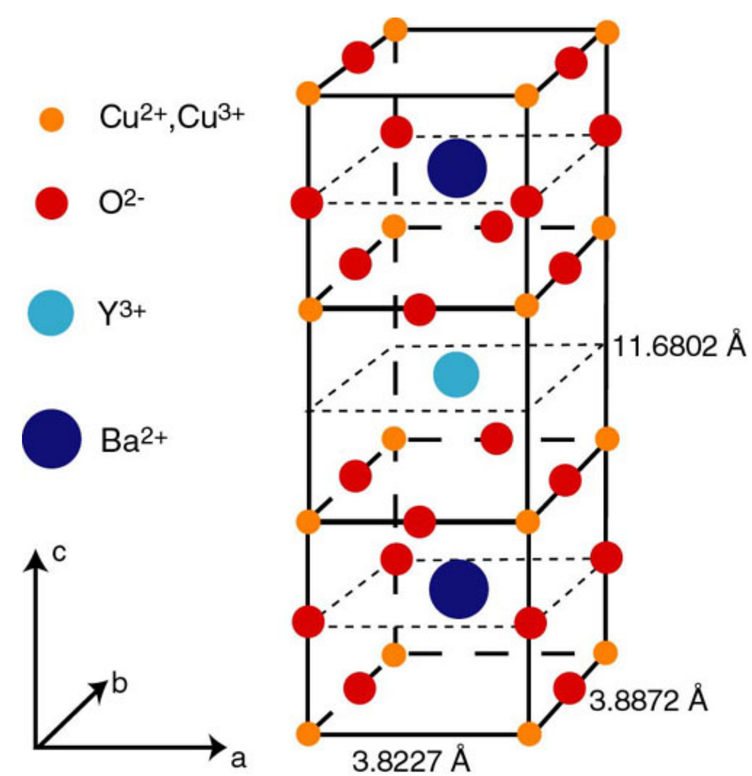
A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

# Resistivity of a Planckian metal as $T \to 0$

$$\rho = \frac{m^*}{ne^2} \, 2.71 \mathbb{C} \, \frac{k_B T}{\hbar}$$

Note that all explicit dependence on $U$ has cancelled out!

Choosing $\mathbb{C} = 0.41$ as in the SYK model, we have the prefactor $2.71\mathbb{C} = 1.11$.


Aavishkar Patel

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

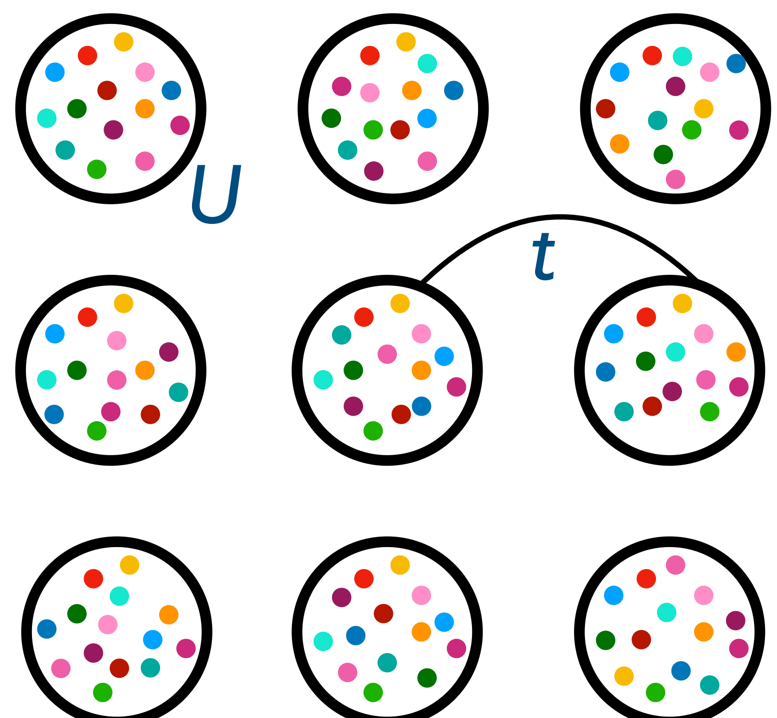
# A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$

$$+ \sum_{k\alpha} e_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

$e_k$ has a bandwidth $W$.




- Disordered Fermi liquid for $T < W^2/U$ with resistivity $\sim T^2$.
- Incoherent, bad metal for $W^2/U < T < U$ with linear-in-$T$ resistivity $\sim (h/e^2)(T/(W^2/U))$.

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2 \left[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8)\right]$$


Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999); Yingfei Gu, Xiao-Liang Qi, D. Stanford, JHEP (2017) 125

# Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$
$$+ \sum_{k\alpha} e_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

$e_k$ has a bandwidth $W$.

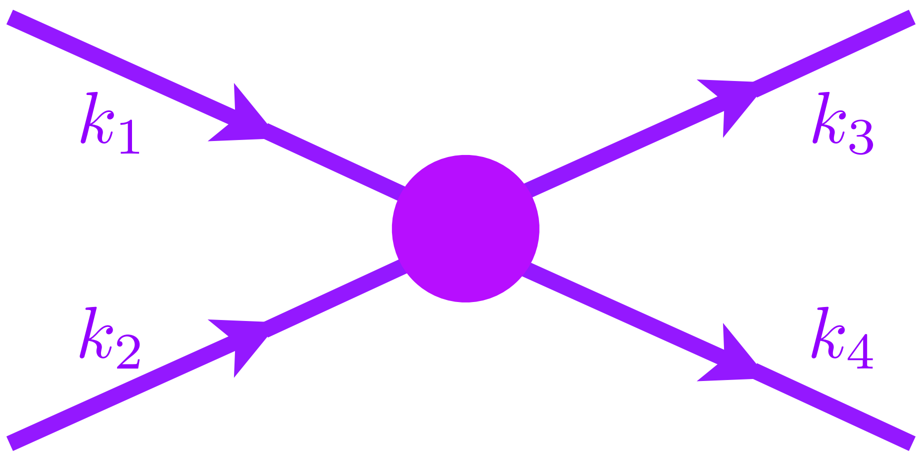
We examine a model with weaker $W \lesssim U$, but impose a **resonance condition**.

This leads to a solution which obeys the Planckian ansatz as $T \to 0$.

$$\overline{U(k_1, k_2, k_3, k_4)U^*(k_5, k_6, k_7, k_8)} =$$
$$U^2 \Big[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8)\Big]$$
$$\times \Big[\delta(e_{k_1} + e_{k_2} - e_{k_3} - e_{k_4}) + \delta(e_{k_5} + e_{k_6} - e_{k_7} - e_{k_8})\Big]$$

This implies off-site interactions with correlations which decay with a power-law in space.

# Resonant SYK model




Interactions with $e_{k_1} + e_{k_2} \neq e_{k_3} + e_{k_4}$ are non-resonant: we "integrate these out" in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion $e_k$, which we have already accounted for.

Keep only the **interactions resonant in the bare quasiparticle energy** with $e_{k_1} + e_{k_2} = e_{k_3} + e_{k_4}$ and account for them with a self-consistent SYK-like analysis.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

- Experimental routes to measuring zero temperature entropy $S_0$ of SYK (charged black holes):
  - Linear-in-$T$ behavior of chemical potential $\mu$ at low $T$.
  - Non-zero thermopower $\Theta \propto dS_0/d\mathcal{Q}$ at low $T$.
- Resonant SYK models described Planckian/strange metal transport in correlated materials at low $T$.