Presence of Quantum Diffusion in Two Dimensions: Universal Resistance at the Superconductor-Insulator Transition

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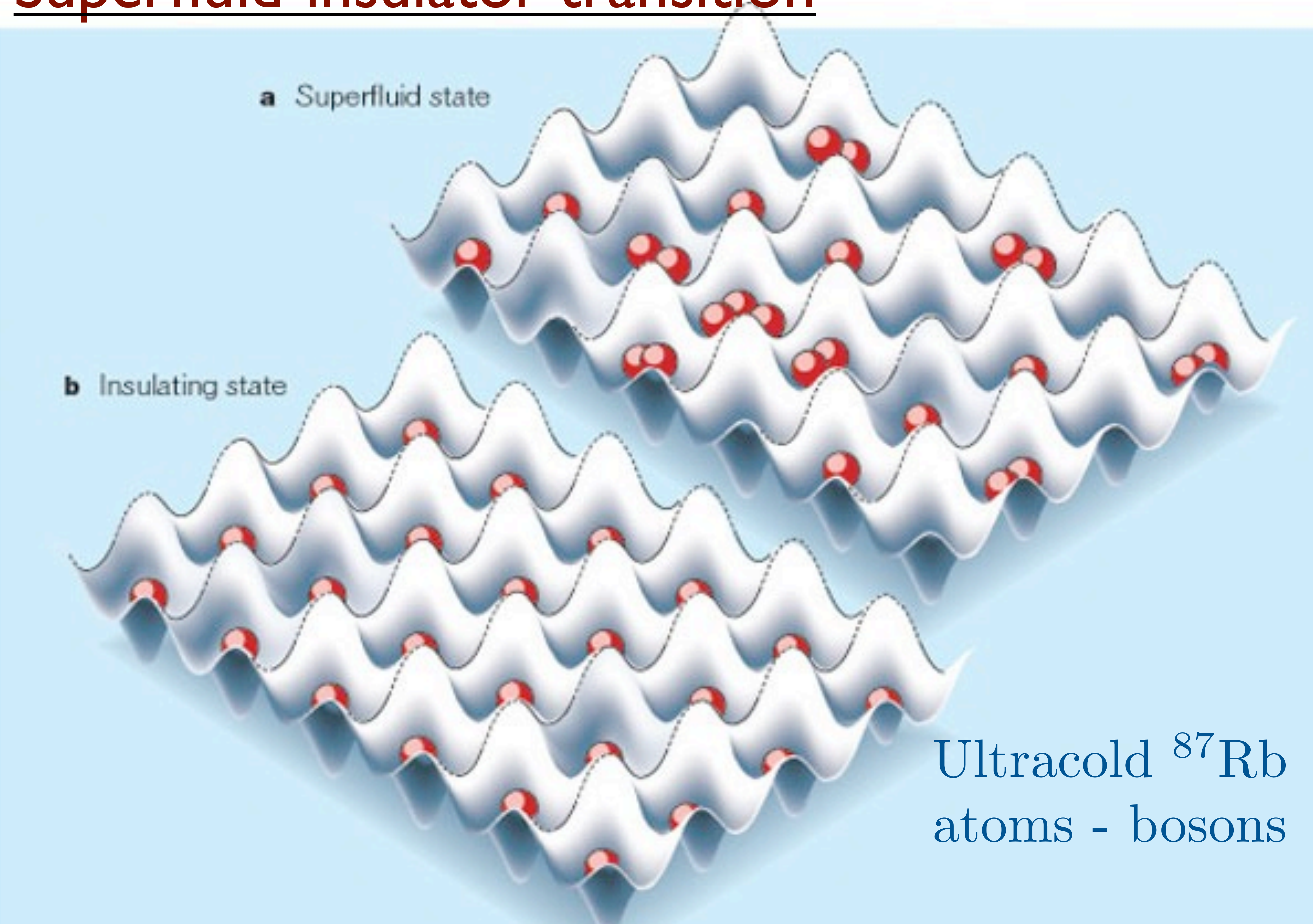
(Received 17 November 1989)

We argue that whenever the transition between the insulating and superconducting phases of a disordered two-dimensional Fermi system at zero temperature ($T=0$) is continuous, the system behaves like a normal metal right at the transition; i.e., the resistance has a finite, nonzero value at $T=0$. This value is *universal*—independent of all microscopic details. These features, consistent with recent measurements on disordered films, are hypothesized to apply to other 2D transitions at $T=0$, such as Anderson localization with spin-orbit coupling, and the quantum Hall effect.

Phys. Rev. Lett. 1990

Superfluid-insulator transition

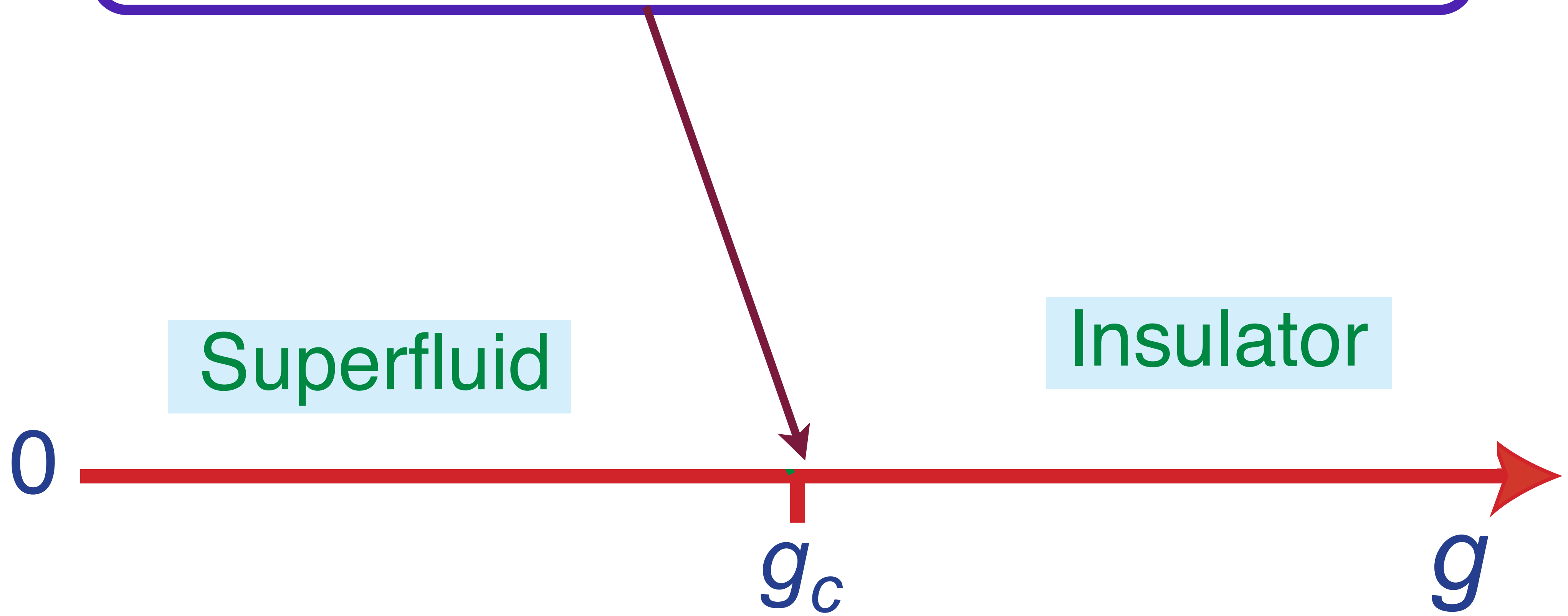


Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

$$\sigma = \frac{4e^2}{h}\Sigma$$

Σ, a universal number.



Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
thermal equilibration time, τ_R

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

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⚠ The worst possible state for quantum computation ⚠

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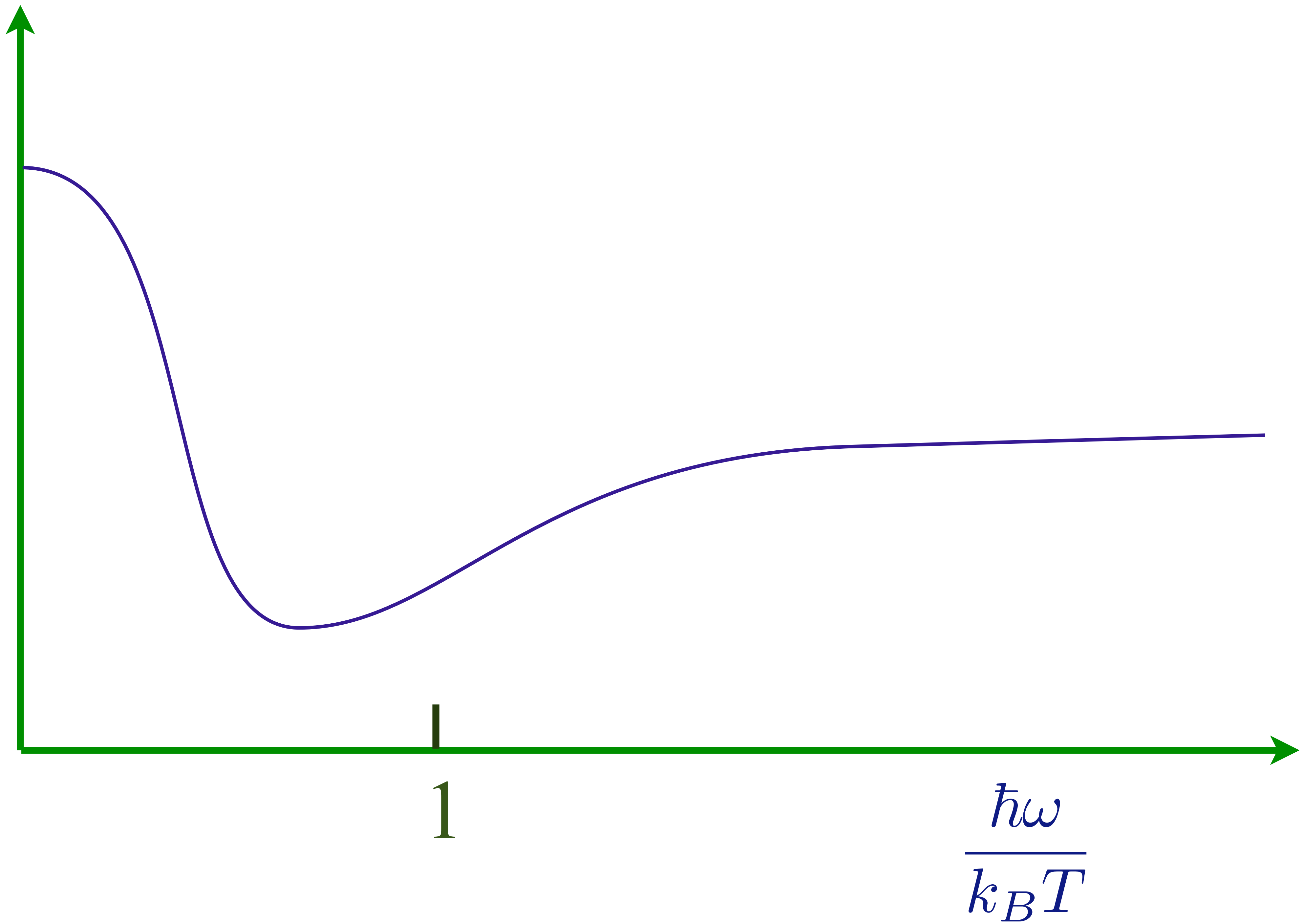

K. Damle and S. Sachdev, 1997

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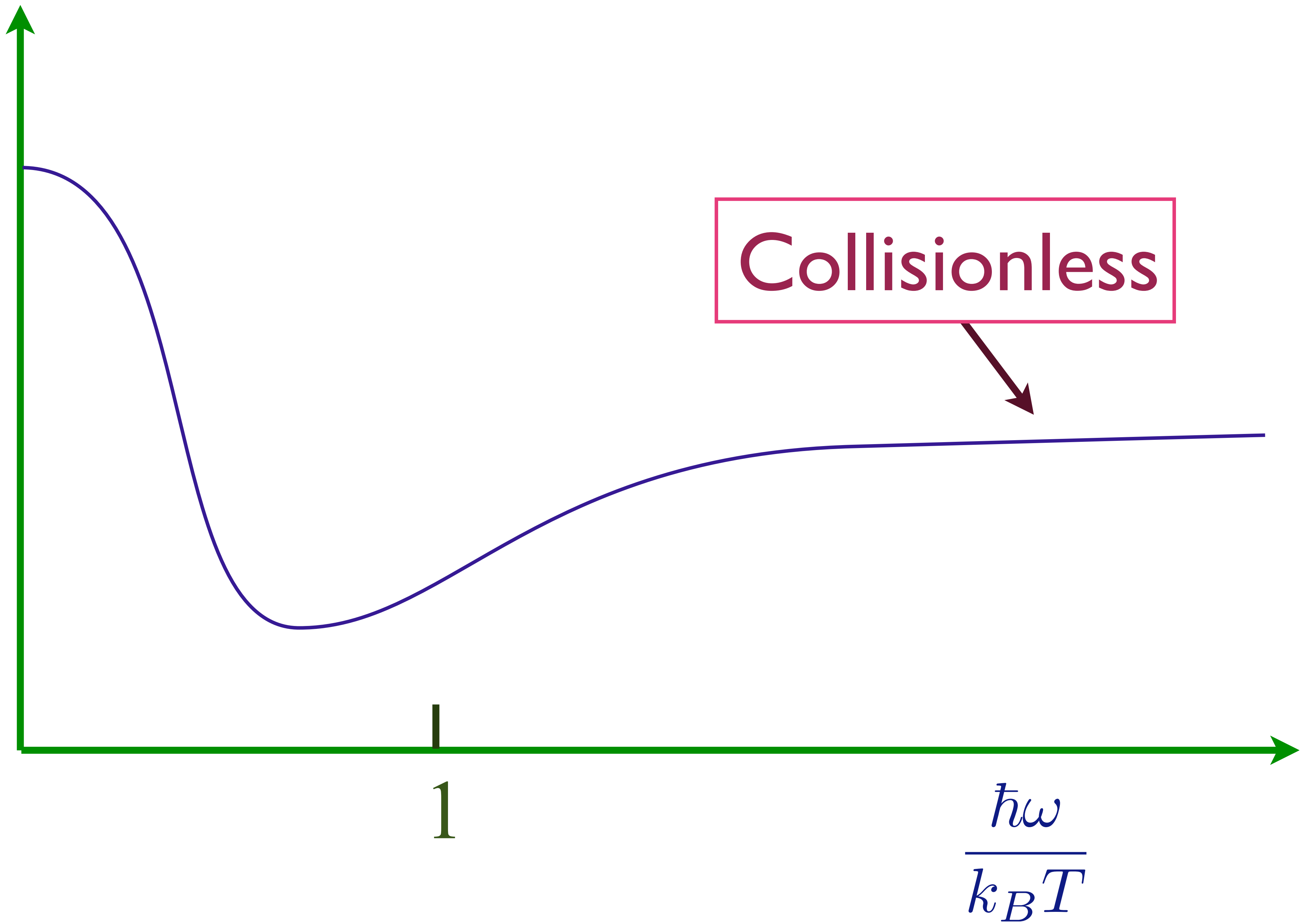

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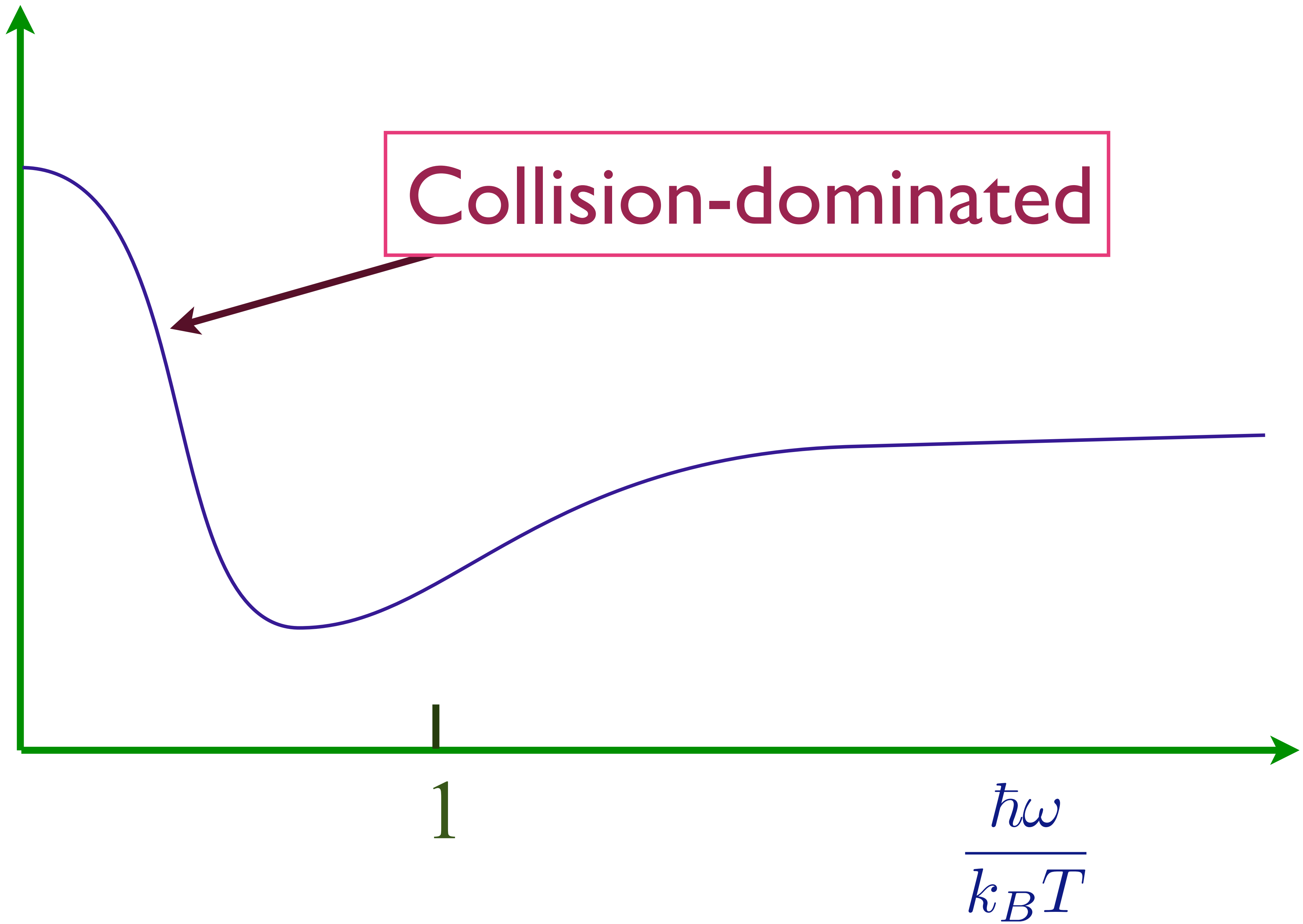
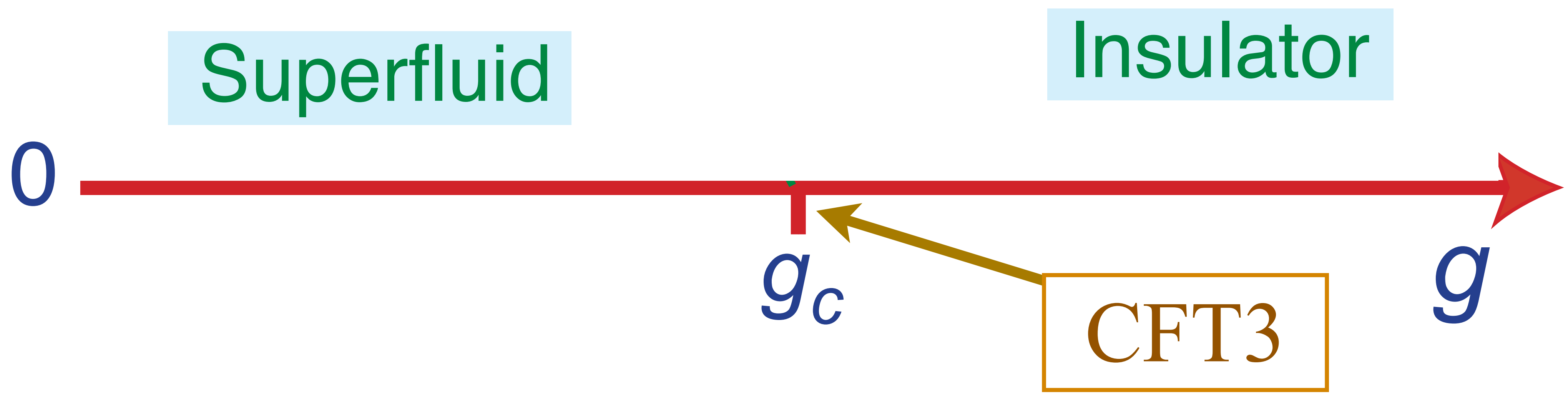


K. Damle and S. Sachdev, 1997

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\langle \psi \rangle \neq 0 \qquad\qquad \langle \psi \rangle = 0$$



Using the boson quasiparticle excitations of the insulator $\sim \psi$

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<u>is dual to</u>

Using the vortex quasiparticle excitations of the superfluid $\sim \varphi$

$$\mathcal{S}_{\rm dual} \;=\; \int d^3x \Big[|(\partial_\mu - i A_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + \frac{\tilde{u}}{2}|\varphi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$

- Now add Dirac fermions, generalize the gauge group to $\mathrm{SU}(N)$, and allow maximal supersymmetry in 2+1 dimensions.

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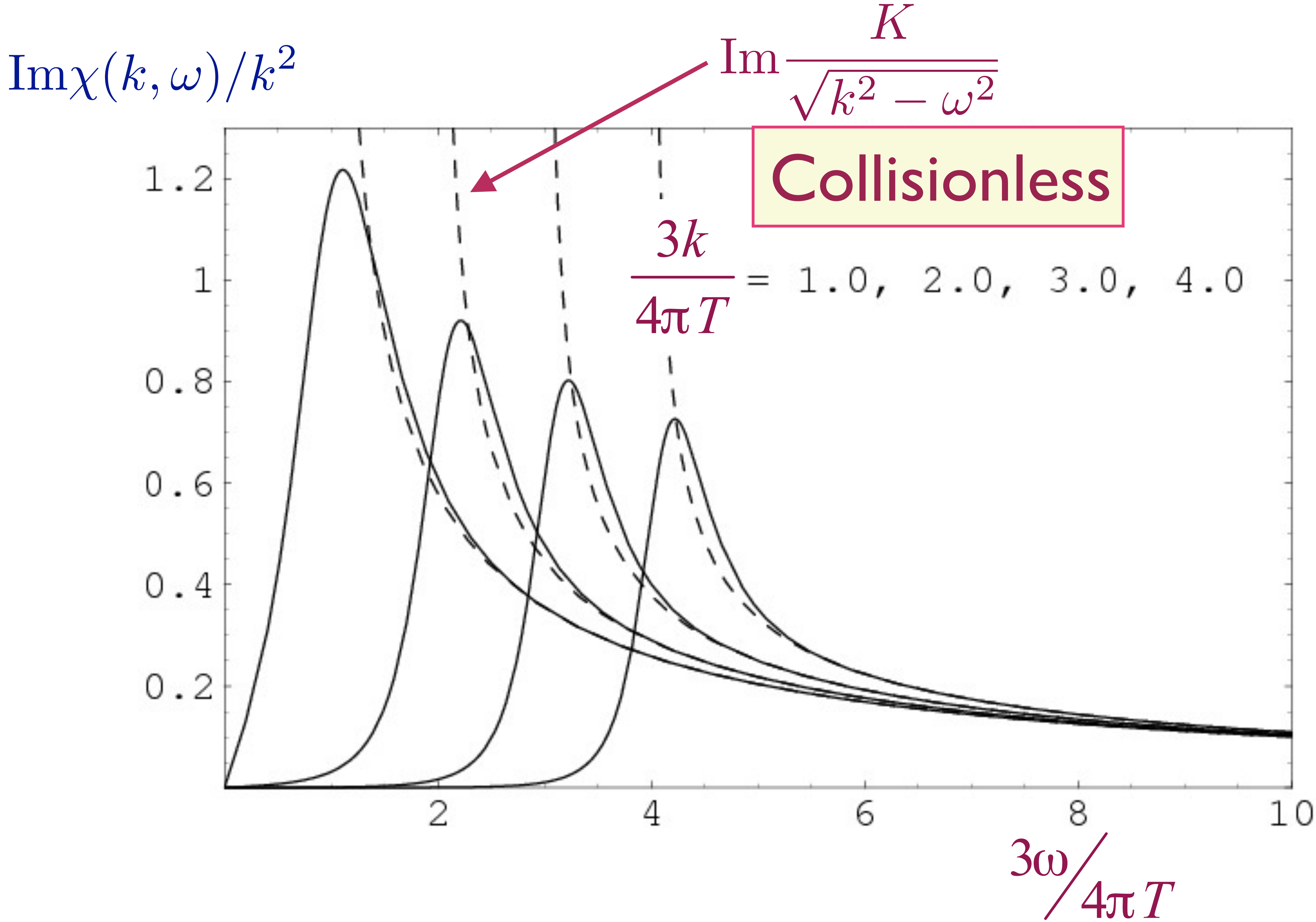
- Critical conductivity $\Sigma = \sqrt{2}N^{3/2}/3$ ("self-dual" value).

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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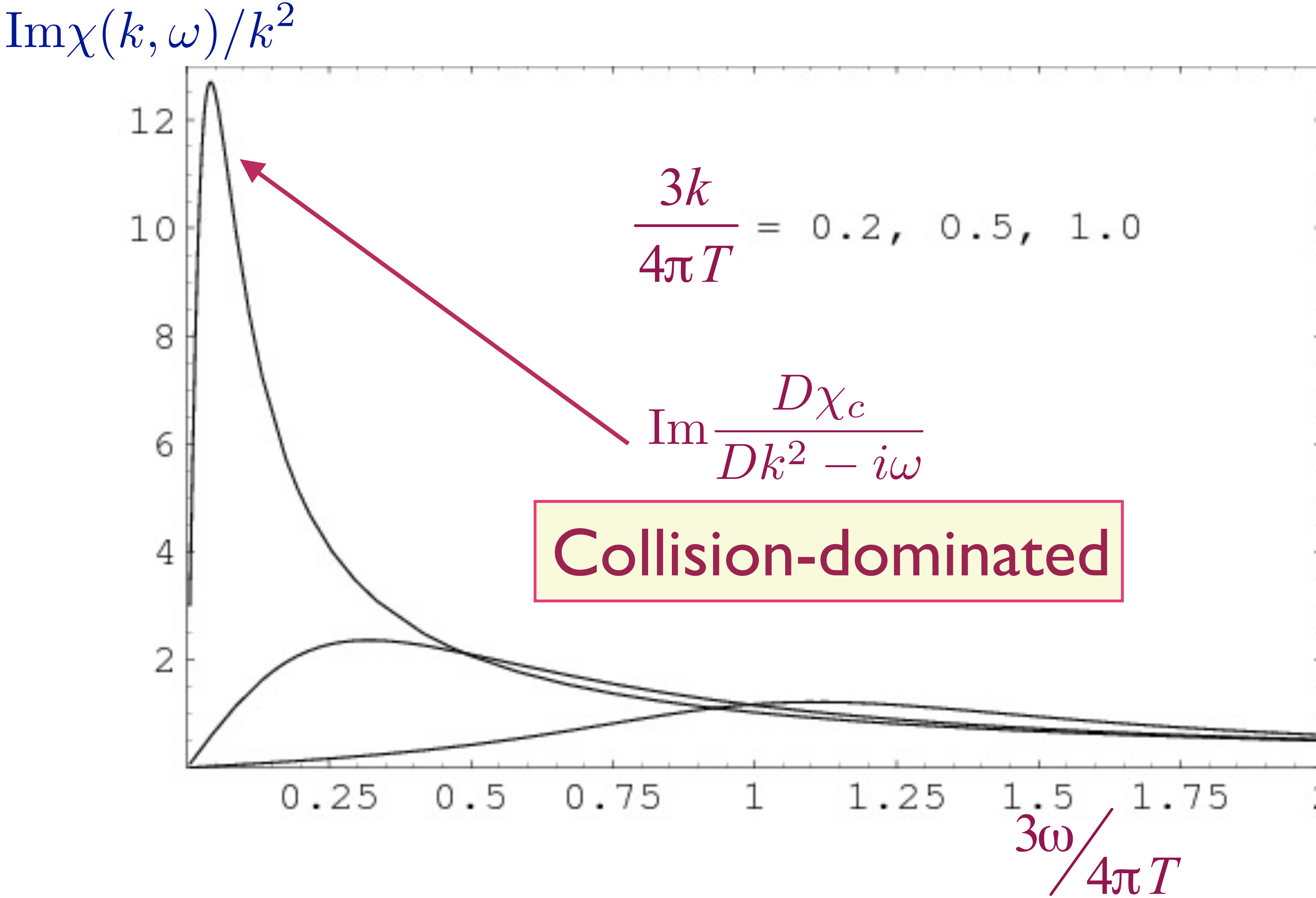

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Field theories in D spacetime dimensions are characterized by couplings g which obey the renormalization group equation

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Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $D+1$ dimensions.

At the RG fixed point, $\beta(g) = 0$, the D dimensional field theory is invariant under the scale transformation

$$x^\mu \to x^\mu / b \quad , \quad u \to b\, u$$

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This is an invariance of the *metric* of the theory in $D+1$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2/u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space AdS$_{D+1}$, and L is the AdS radius.

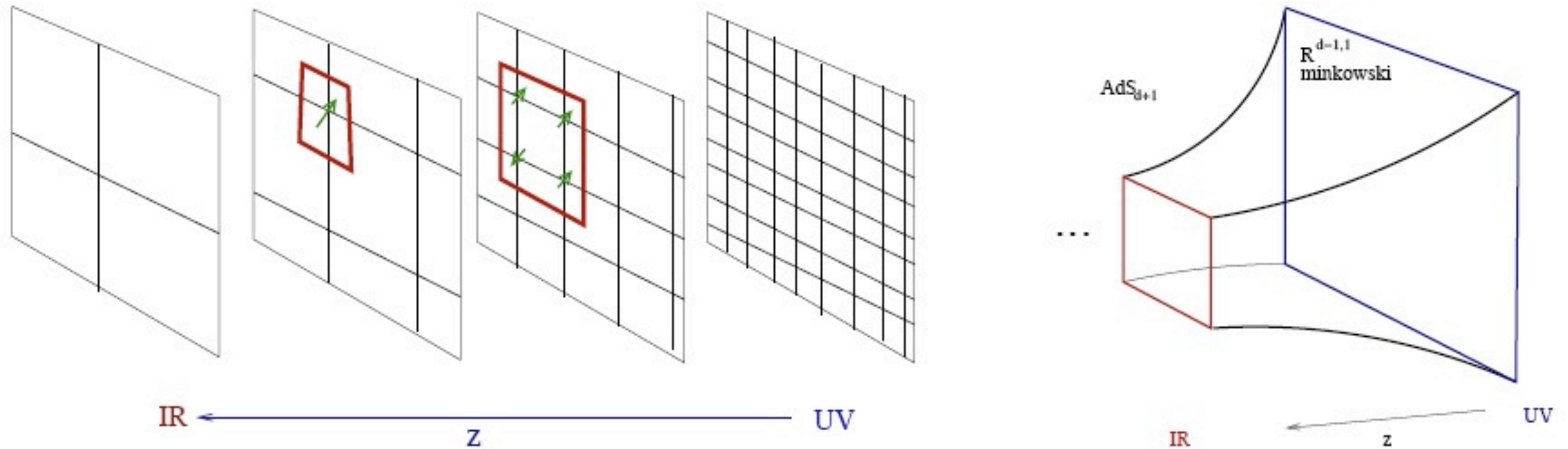

Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

J. McGreevy, arXiv0909.0518

Bonus: AdS_{D+1} is a solution of Einstein's equations with a negative cosmological constant, and is a symmetric space; the full group of symmetries of the metric is $\mathrm{SO}(D+1,1)$ (in Euclidean signature)

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$\mathrm{SO}(D+1,1)$ is the group of conformal transformations in D dimensions, and relativistic field theories at the RG fixed point are conformally invariant.

Higher Spin Gauge Theory and Holography: The Three-Point Functions

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Abstract

In this paper we calculate the tree level three-point functions of Vasiliev's higher spin gauge theory in AdS_4 and find agreement with the correlators of the free field theory of N massless scalars in three dimensions in the $O(N)$ singlet sector. This provides substantial evidence that Vasiliev theory is dual to the free field theory, thus verifying a conjecture of Klebanov and Polyakov. We also find agreement with the critical $O(N)$ vector model, when the bulk scalar field is subject to the alternative boundary condition such that its dual operator has classical dimension 2.

arXiv:0912.3462v2

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The quantum phase transitions of metals in two dimensions

Talk online: sachdev.physics.harvard.edu

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The quantum phase transitions of metals in two dimensions

Field theories with singularities along lines in momentum space: also being studied by the AdS/CFT correspondence

Talk online: sachdev.physics.harvard.edu

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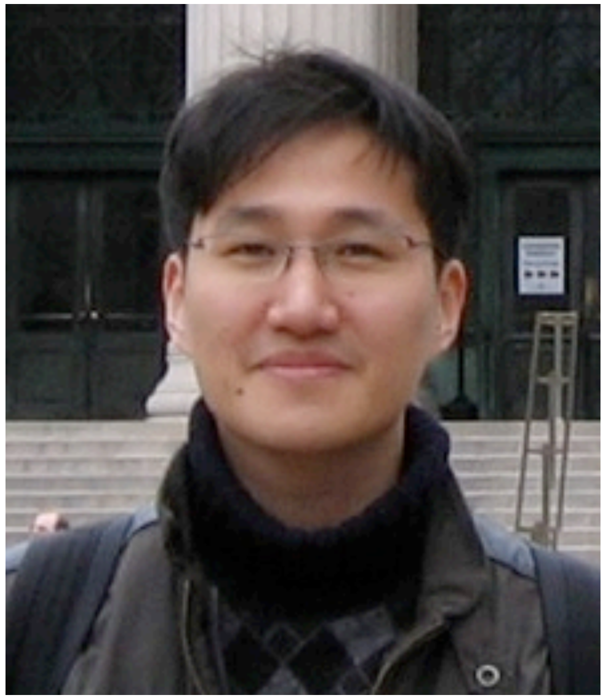

Max Metlitski, Harvard

arXiv:1001.1153, and to appear

Eun Gook Moon, Harvard

Phys. Rev. B 80, 035117 (2009), and to appear

PHYSICS
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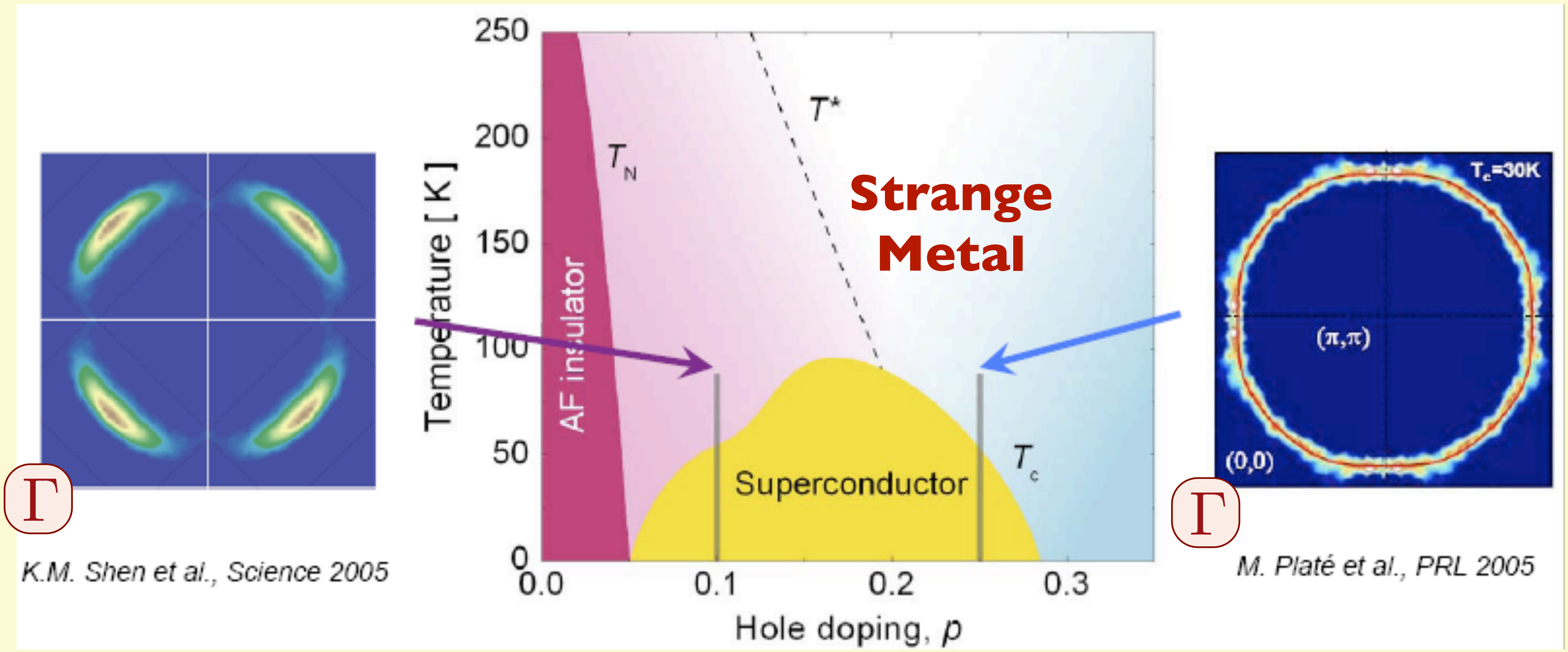
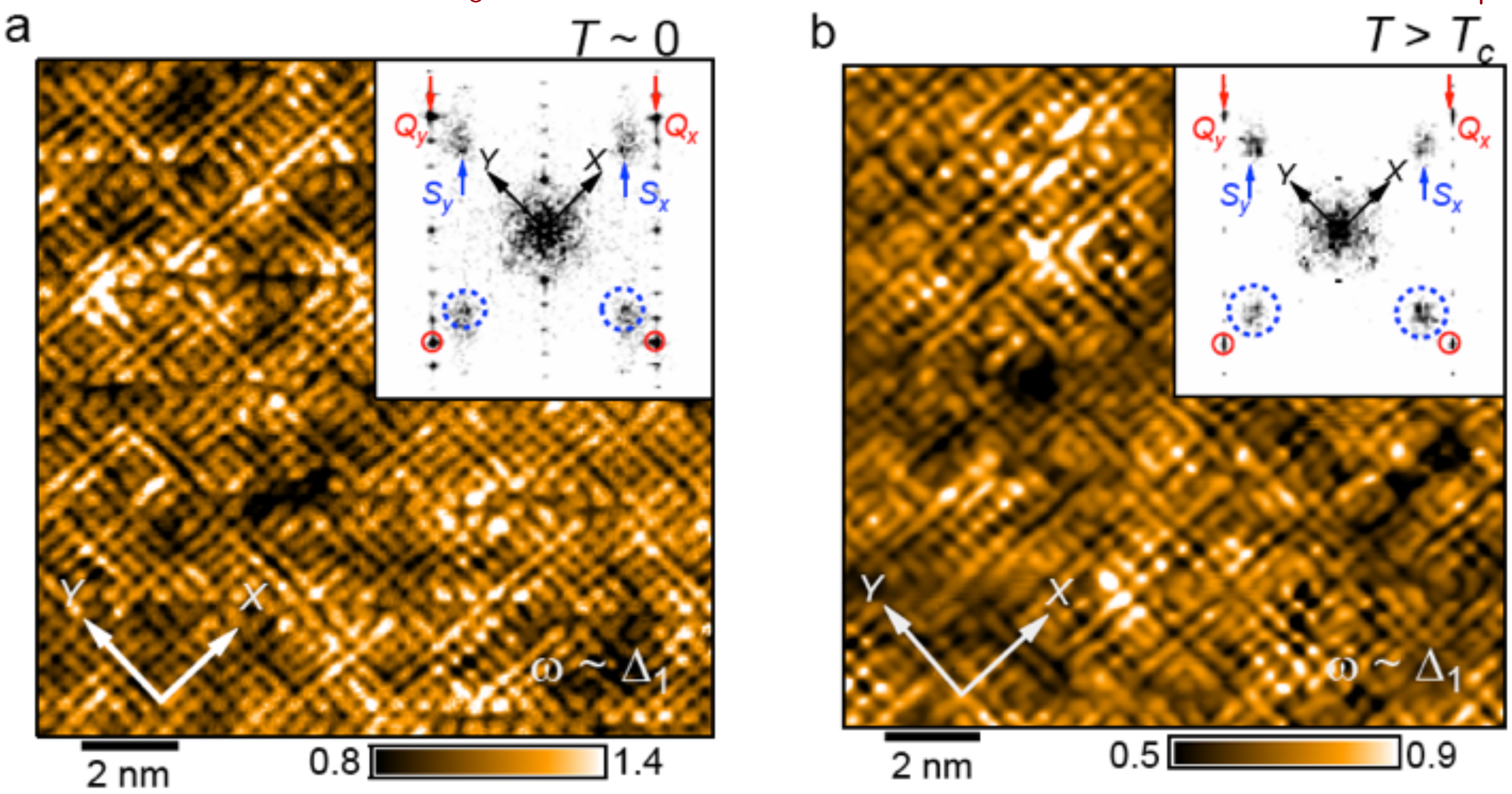
Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Smaller hole Fermi-pockets

Large hole Fermi surface

STM measurements of $Z(r)$, the energy asymmetry in density of states in $Bi_2Sr_2CaCu_2O_{8+\delta}$.



M. J. Lawler, K. Fujita, Jhinhwan Lee, A. R. Schmidt, Y. Kohsaka, Chung Koo Kim, H. Eisaki, S. Uchida, J. C. Davis, J. P. Sethna, and Eun-Ah Kim, preprint

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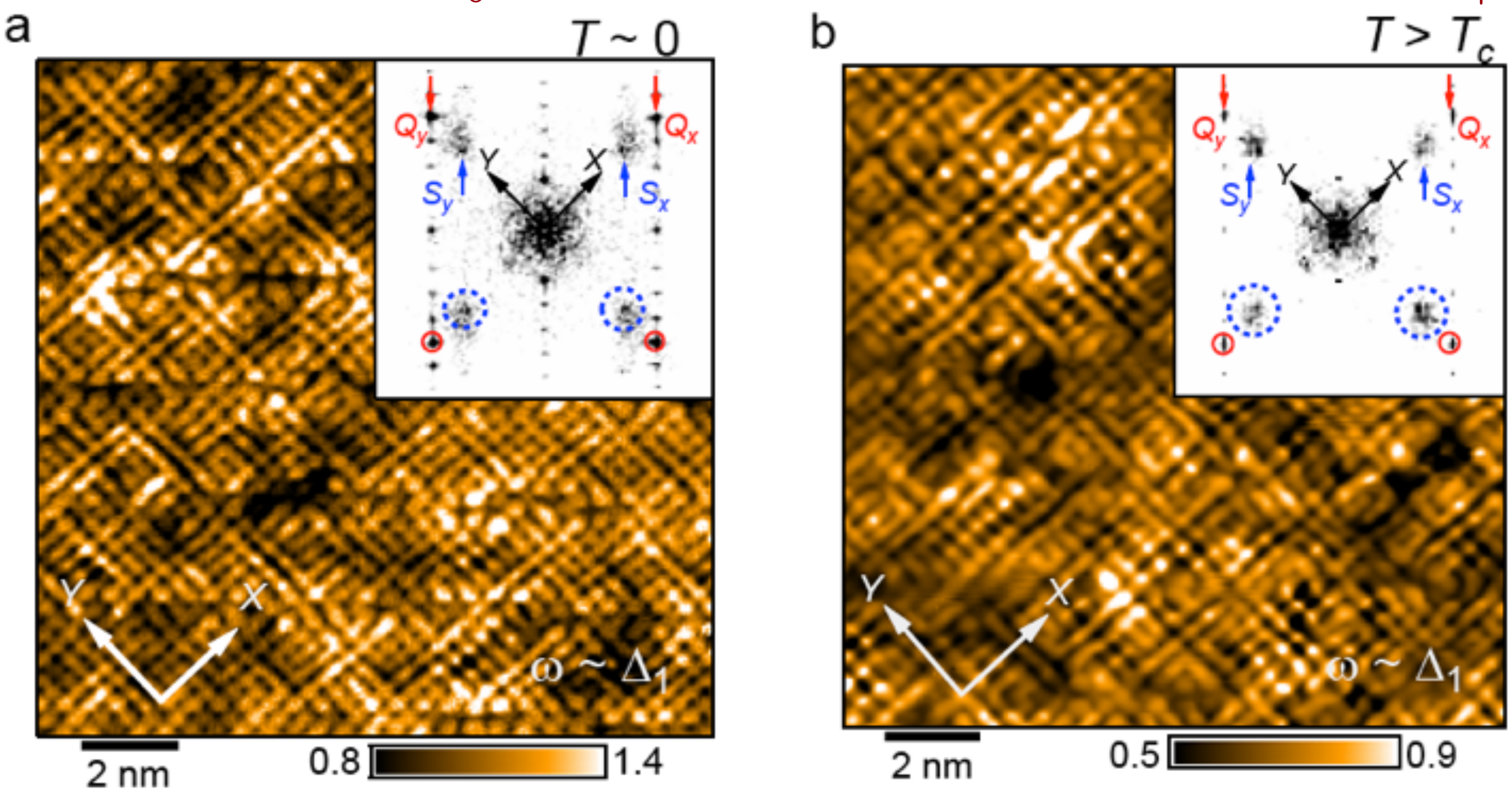

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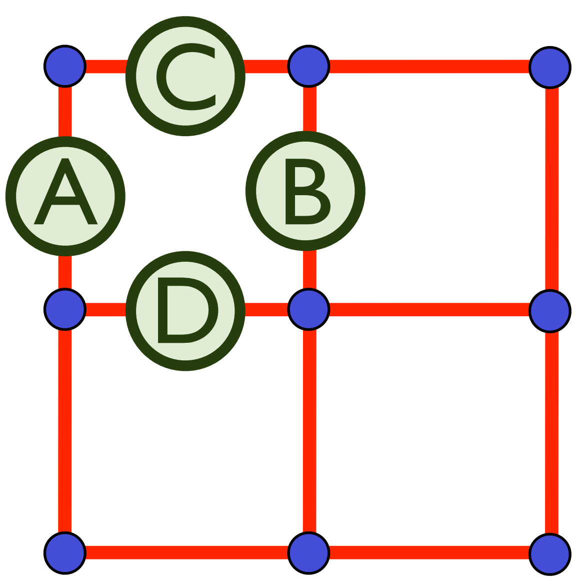

$$O_N = Z_A + Z_B - Z_C - Z_D$$

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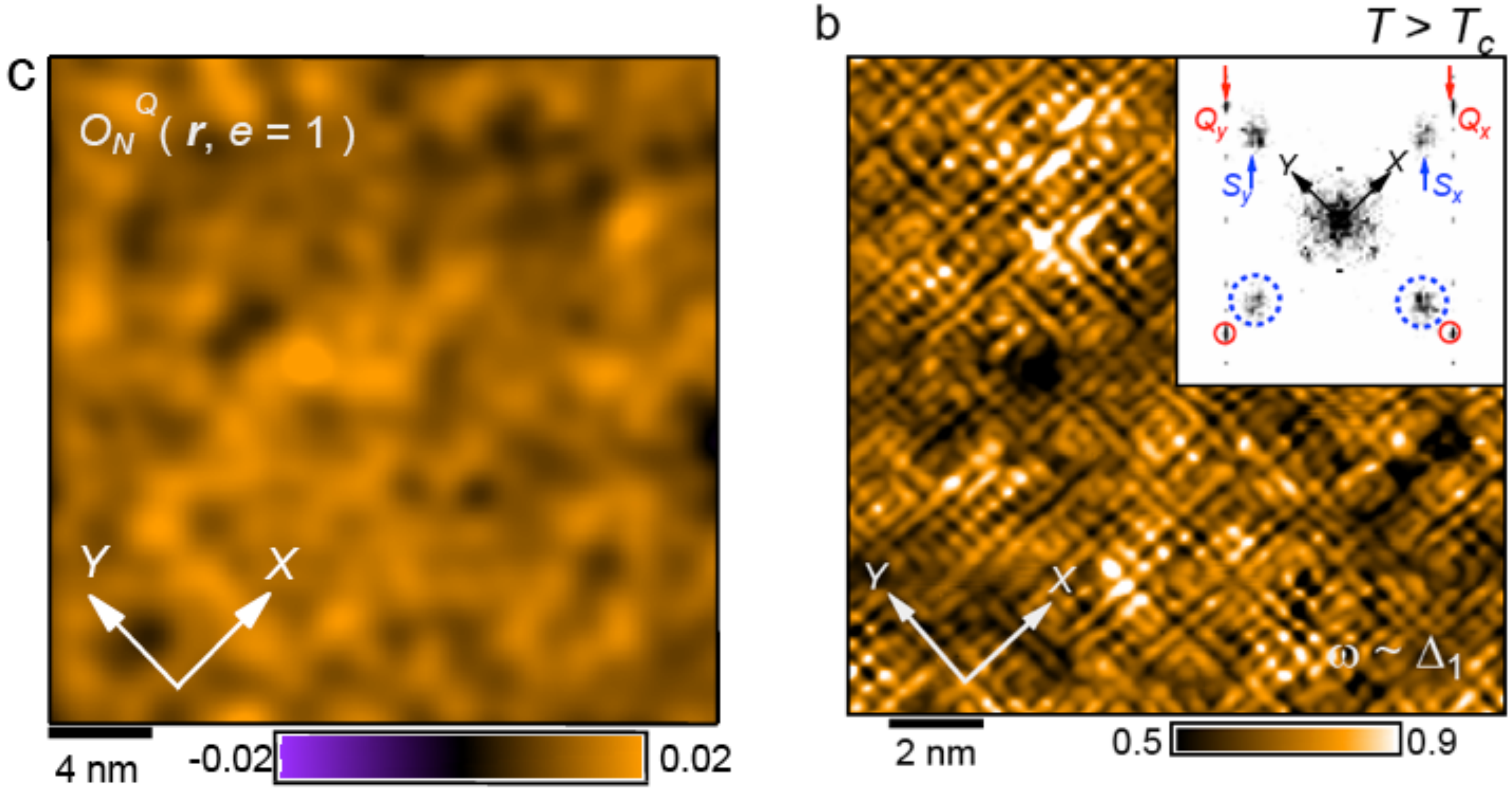

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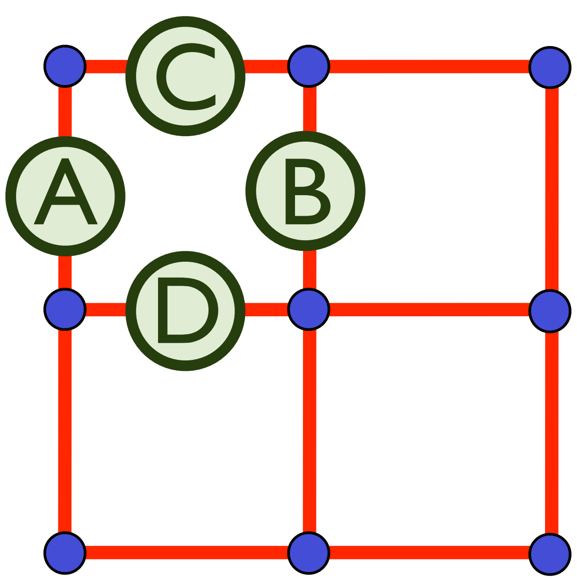
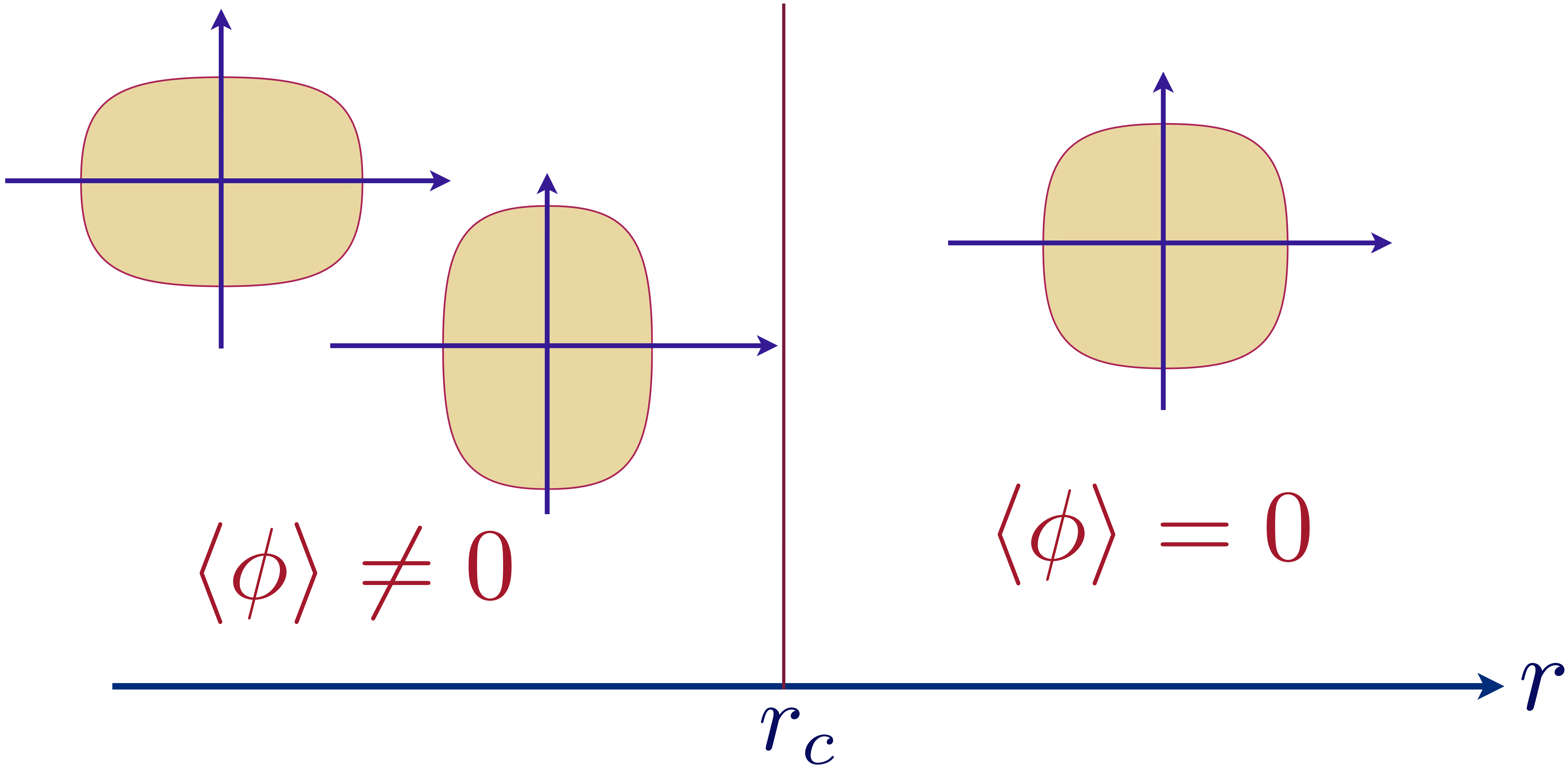


$$O_N = Z_A + Z_B - Z_C - Z_D$$

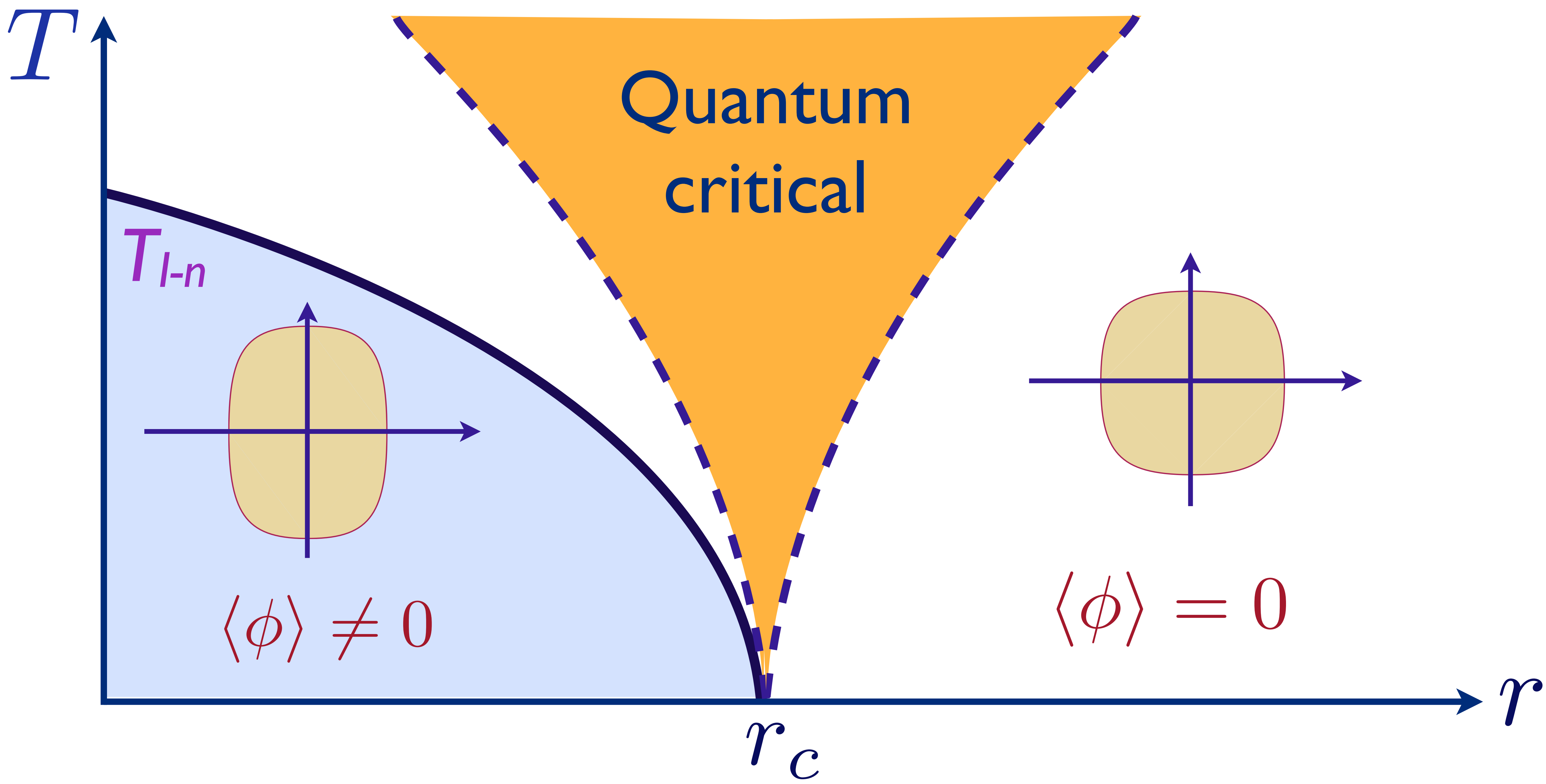
Strong anisotropy of electronic states between *x* and *y* directions: Electronic "Ising-nematic" order

Quantum criticality of Ising-nematic order



Phase diagram as a function of coupling r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

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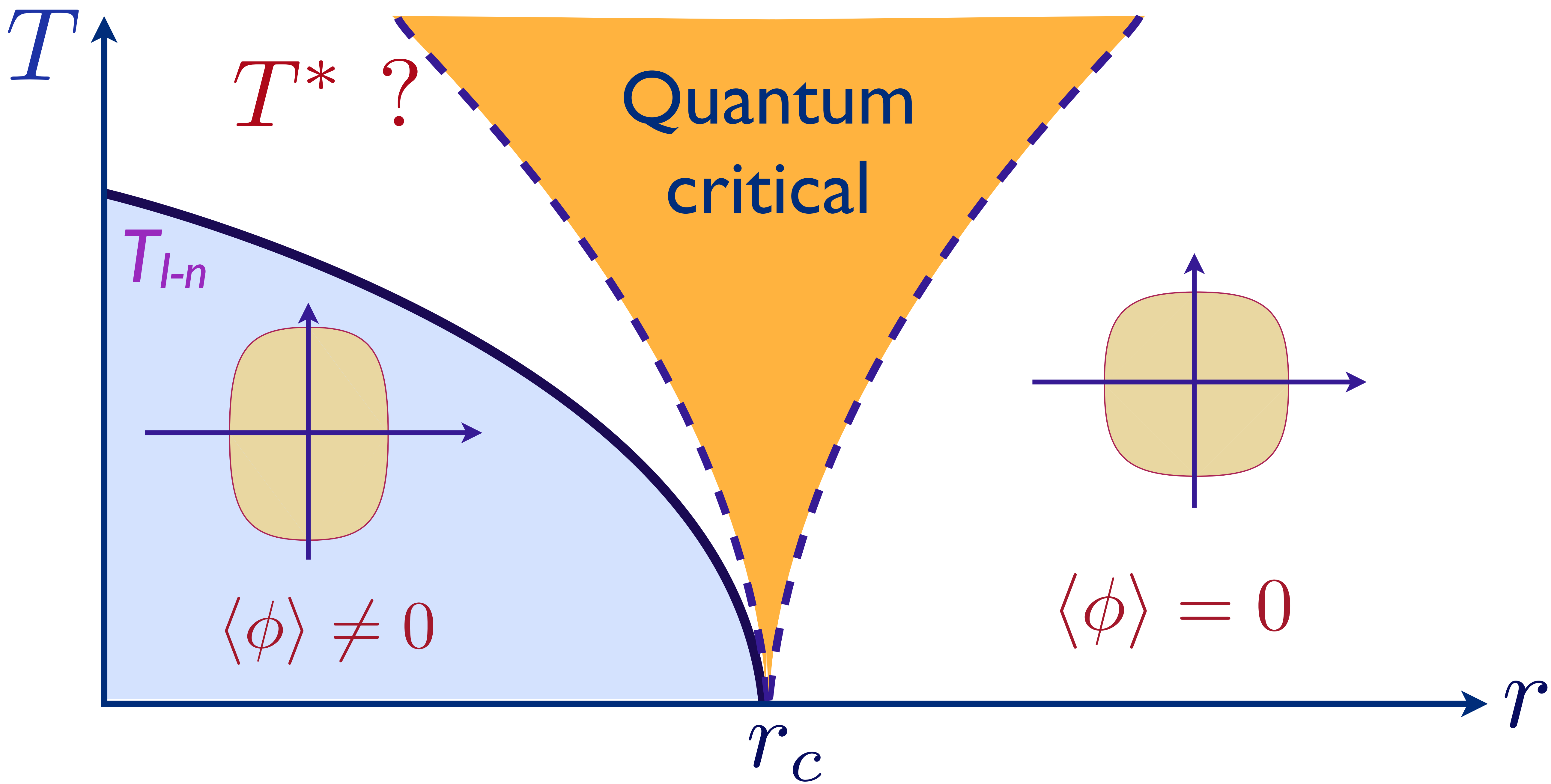

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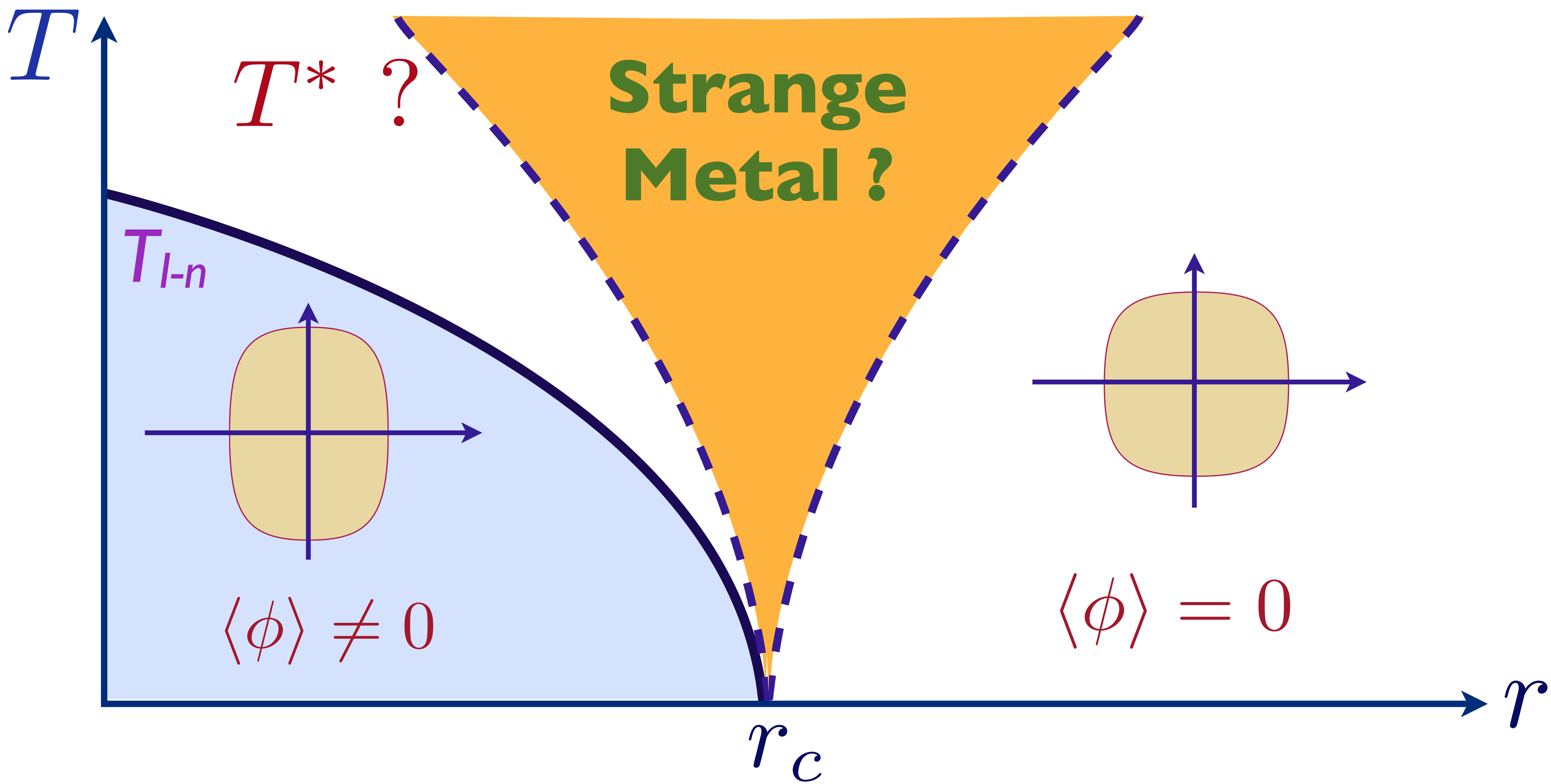
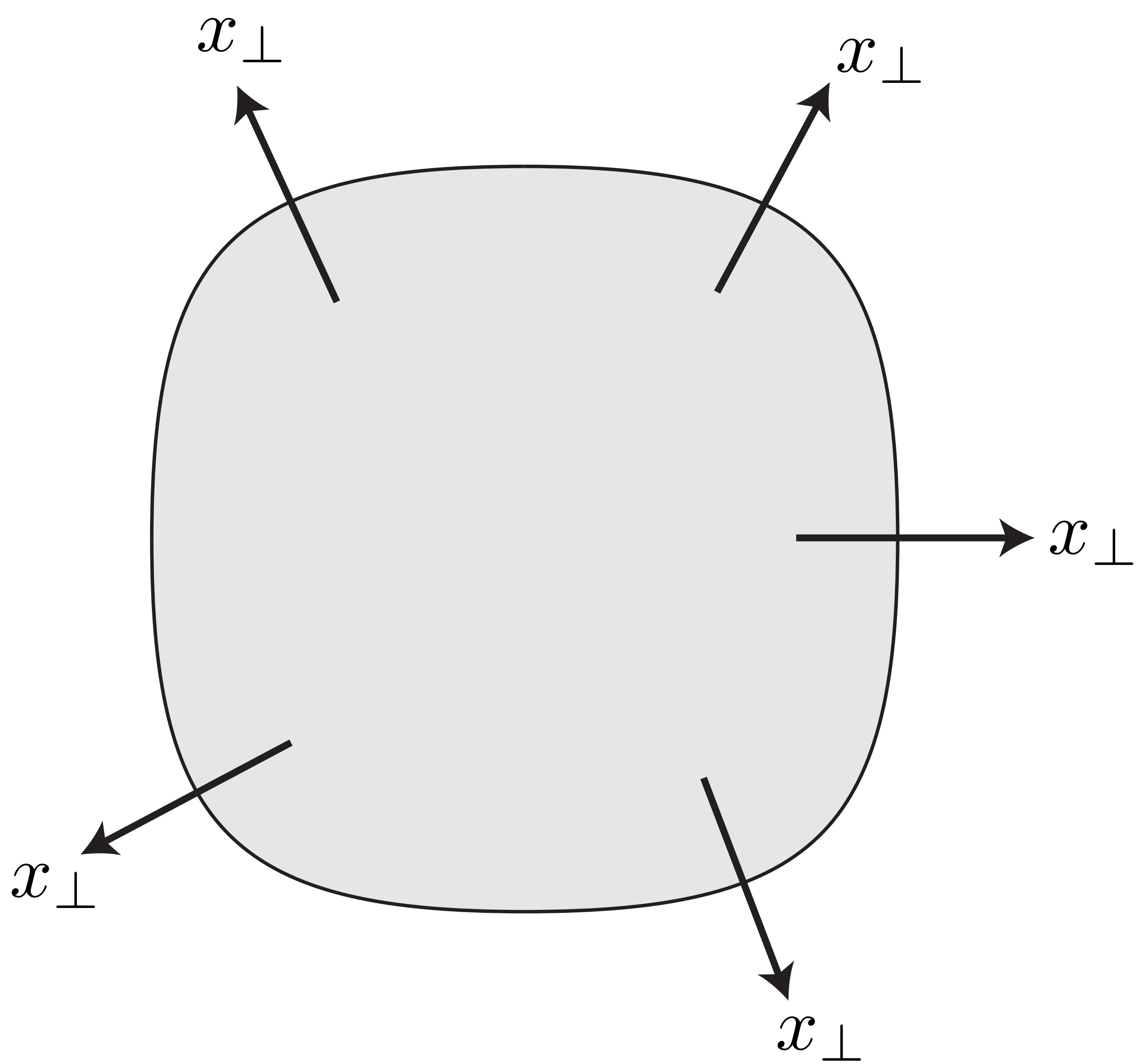


Phase diagram as a function of T and r

Fermi liquid theory



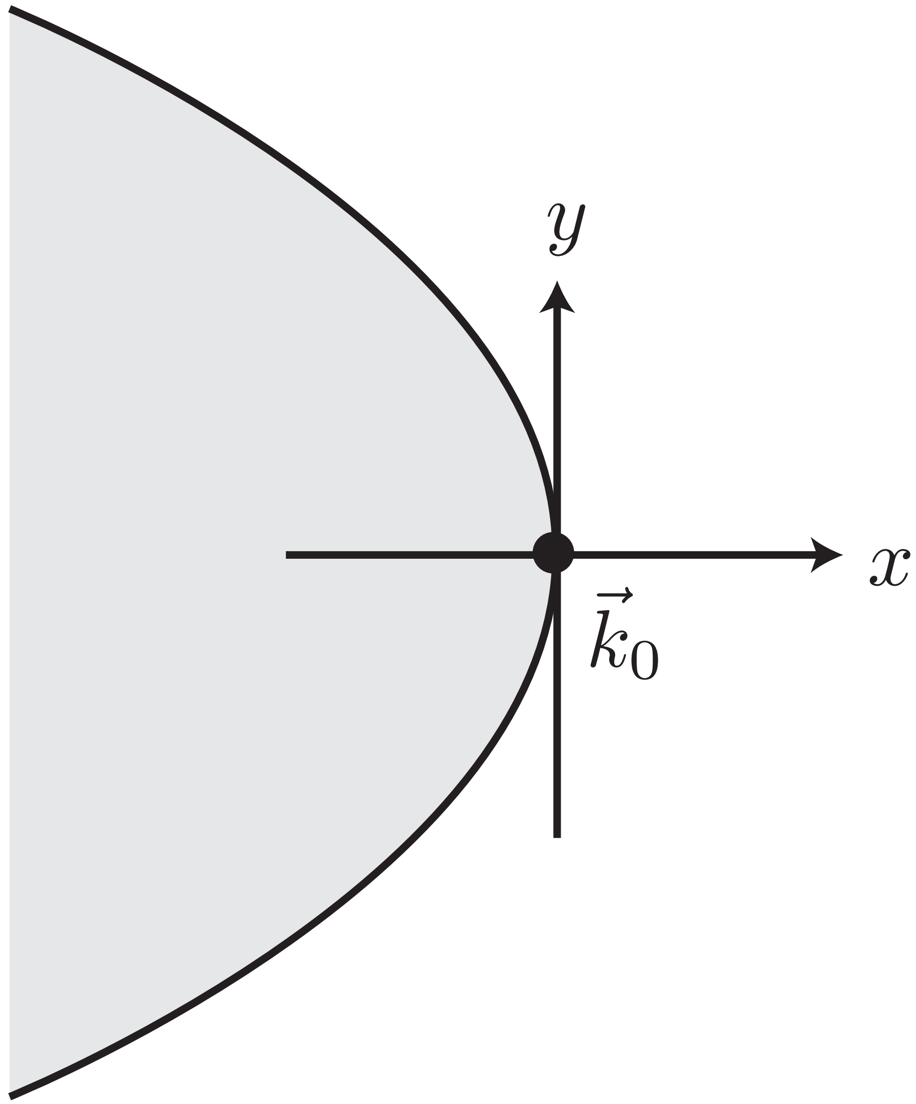
$$\mathcal{S}_{\rm FL} = \int d\Omega_{\hat{n}} \int dx_\perp \psi^\dagger_{\hat{n}a}(x_\perp) \left(\frac{\partial}{\partial \tau} - i v_F(\hat{n}) \frac{\partial}{\partial x_\perp} \right) \psi_{\hat{n}a}(x_\perp)$$

Infinite number of 1+1 dimensional chiral fermions

Fermi liquid theory

Focus on one extended patch, and scale towards a fixed $\vec{k}_0$ while keeping the Fermi velocity v_F and Fermi surface curvature, κ fixed.

Requires a 2+1 dimensional field theory at every $\vec{k}_0$.

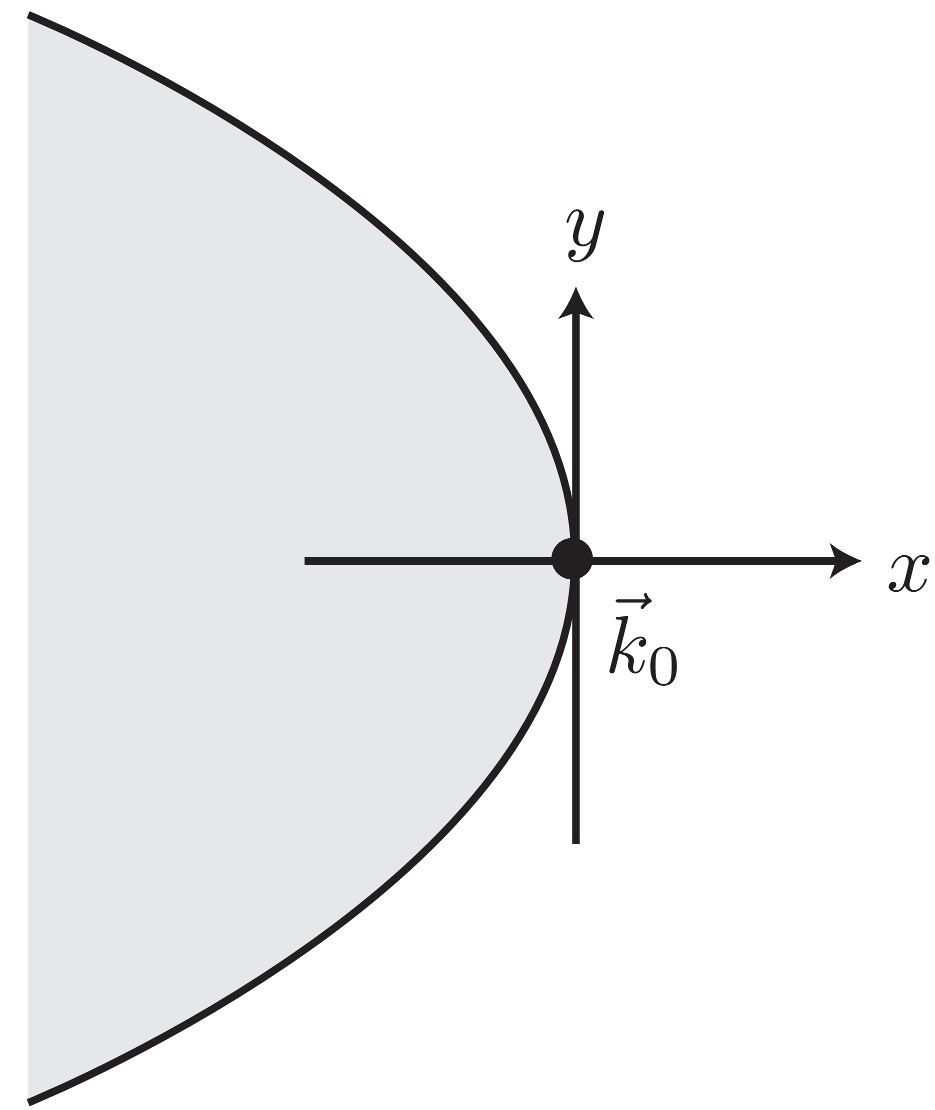


$$\mathcal{S}_0 = \int d\tau \int dx \int d^{d-1}y \; \psi_a^\dagger \left(\zeta \frac{\partial}{\partial \tau} - i v_F \frac{\partial}{\partial x} - \frac{\kappa}{2} \nabla_y^2 \right) \psi_a$$

$$\mathcal{S}_1 = u_0 \int d\tau \int dx \int d^{d-1}y \; \psi_a^\dagger \psi_b^\dagger \psi_b \psi_a .$$

Fermi liquid theory

$$
\begin{aligned}
x' &= x/s^2 \\
y' &= y/s \\
\tau' &= \tau/s^2 \\
\psi' &= \psi s^{(d+1)\ell/2} \\
u_0' &= u_0 s^{(1-d)\ell}
\end{aligned}
$$



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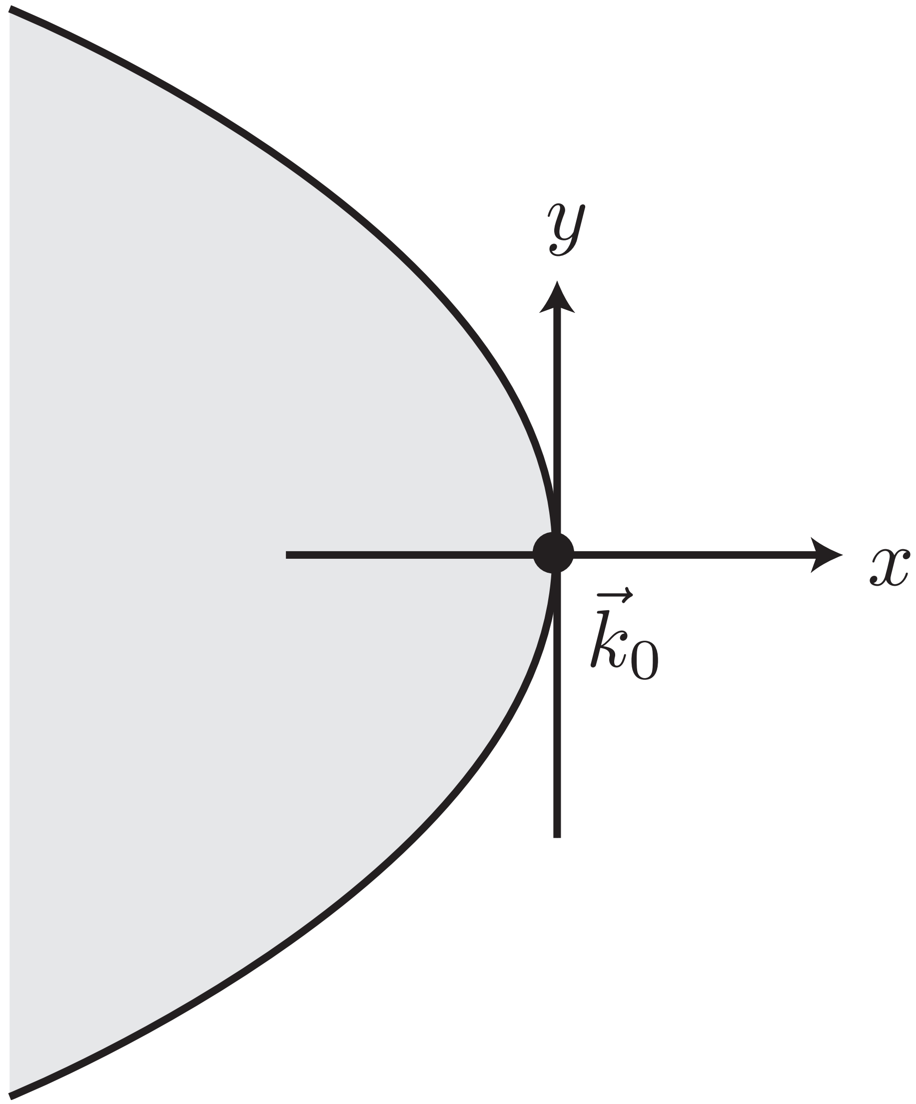

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$$

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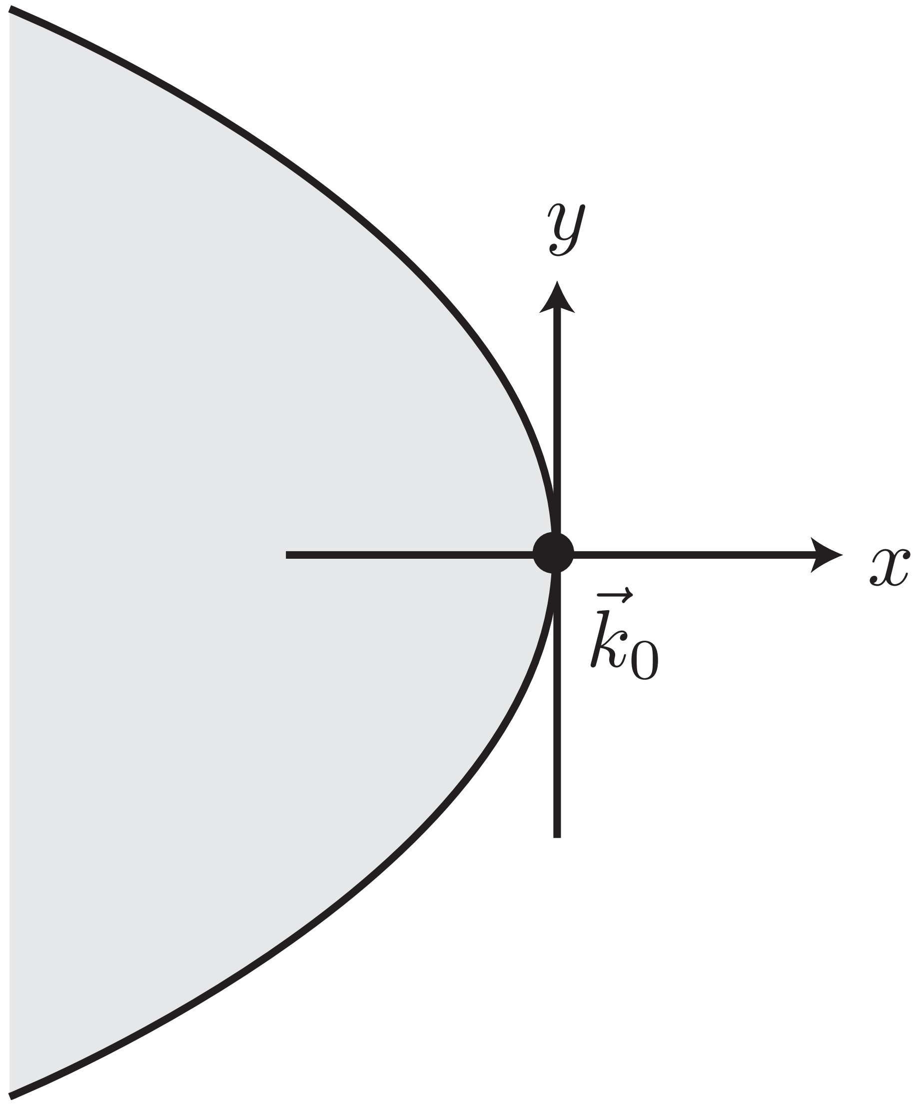
$$

Fermi liquid theory

$$\mathrm{Im}\Sigma(k,\omega) = \begin{cases} \omega^2 \dfrac{u_0^2}{4\pi v_F^2 \kappa} \Lambda^{d-2} & , \; d > 2 \\ \\ \omega^2 \dfrac{u_0^2}{4\pi v_F^2 \kappa} \ln(|\omega|) & , \; d = 2 \end{cases}$$



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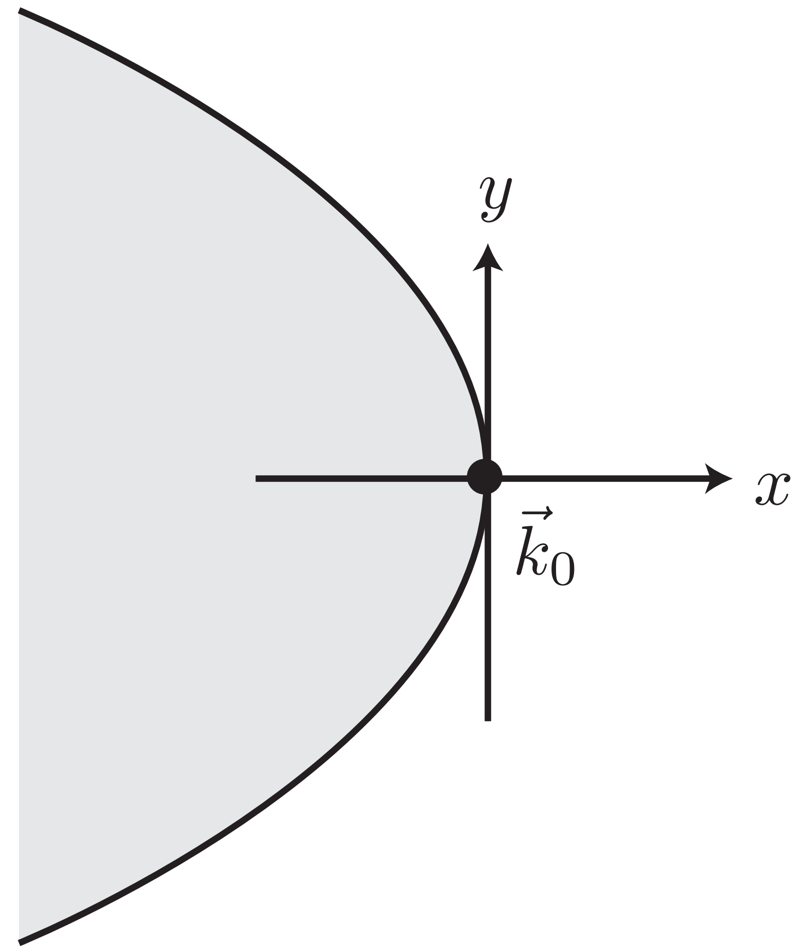
Fermi liquid theory



The theory has an exact "Galilean" symmetry:

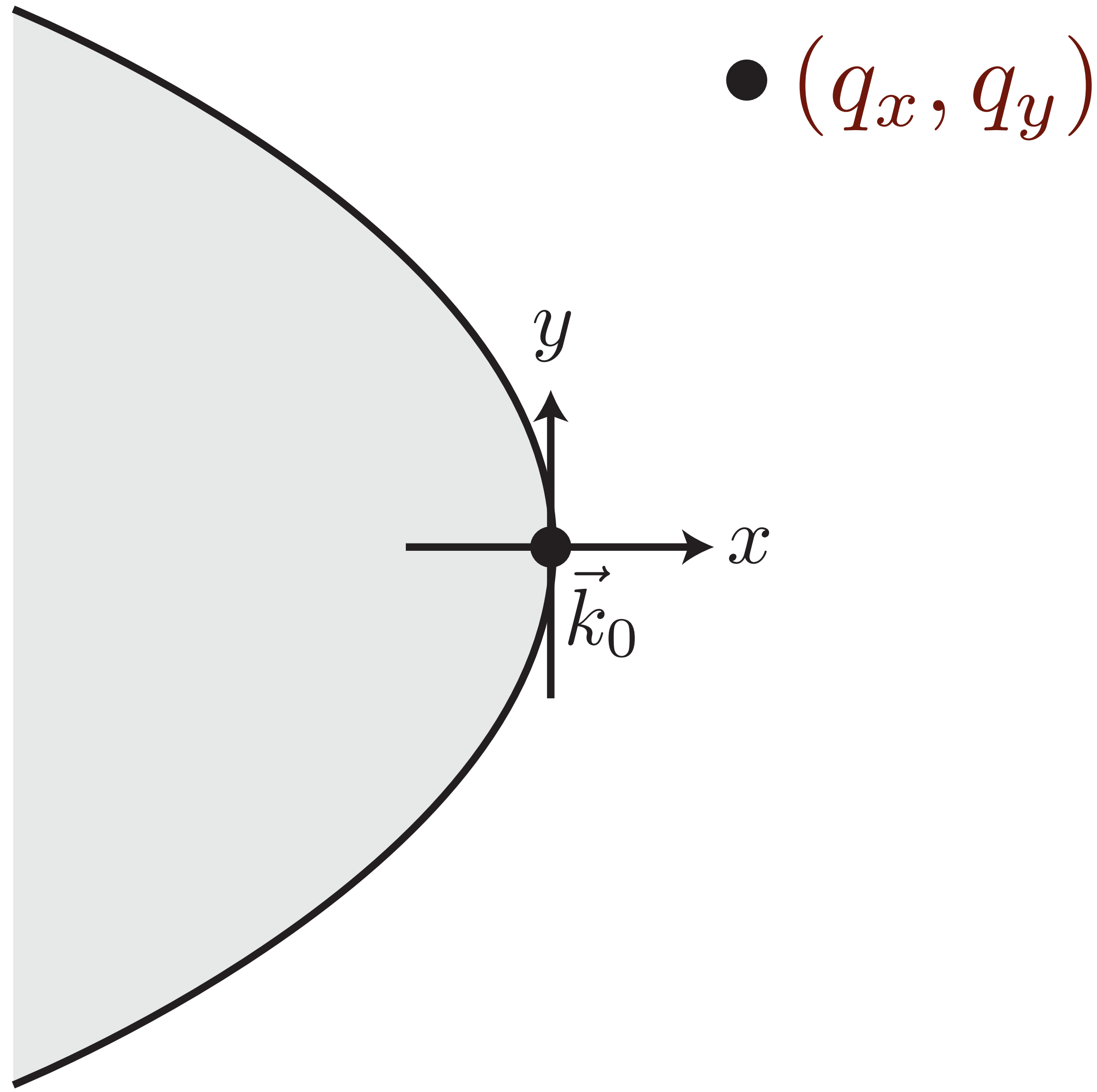
$$\psi(x,\vec{y}) \to \exp\left(-i\frac{v_F}{\kappa}\left(\vec{\theta}\cdot\vec{y}+\frac{\theta^2}{2}x\right)\right)\psi(x,\vec{y}+\vec{\theta}\,x)$$

This implies for the fermion Green's function

$$G(q_x,\vec{q}_y,\omega) = G(\varepsilon_q,\omega)$$

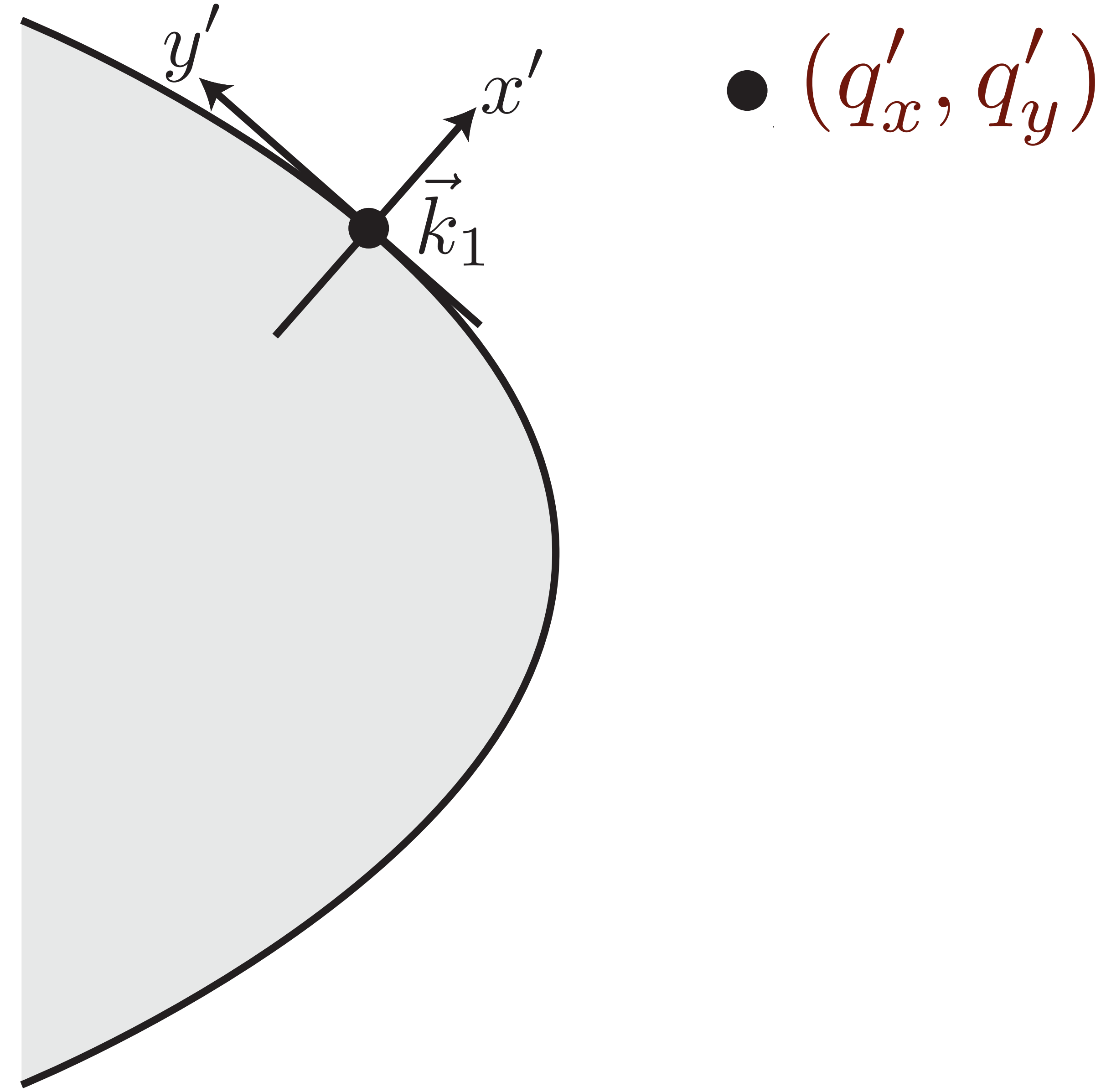
where $\varepsilon_k = v_F q_x + \kappa q_y^2/2$.

Fermi liquid theory



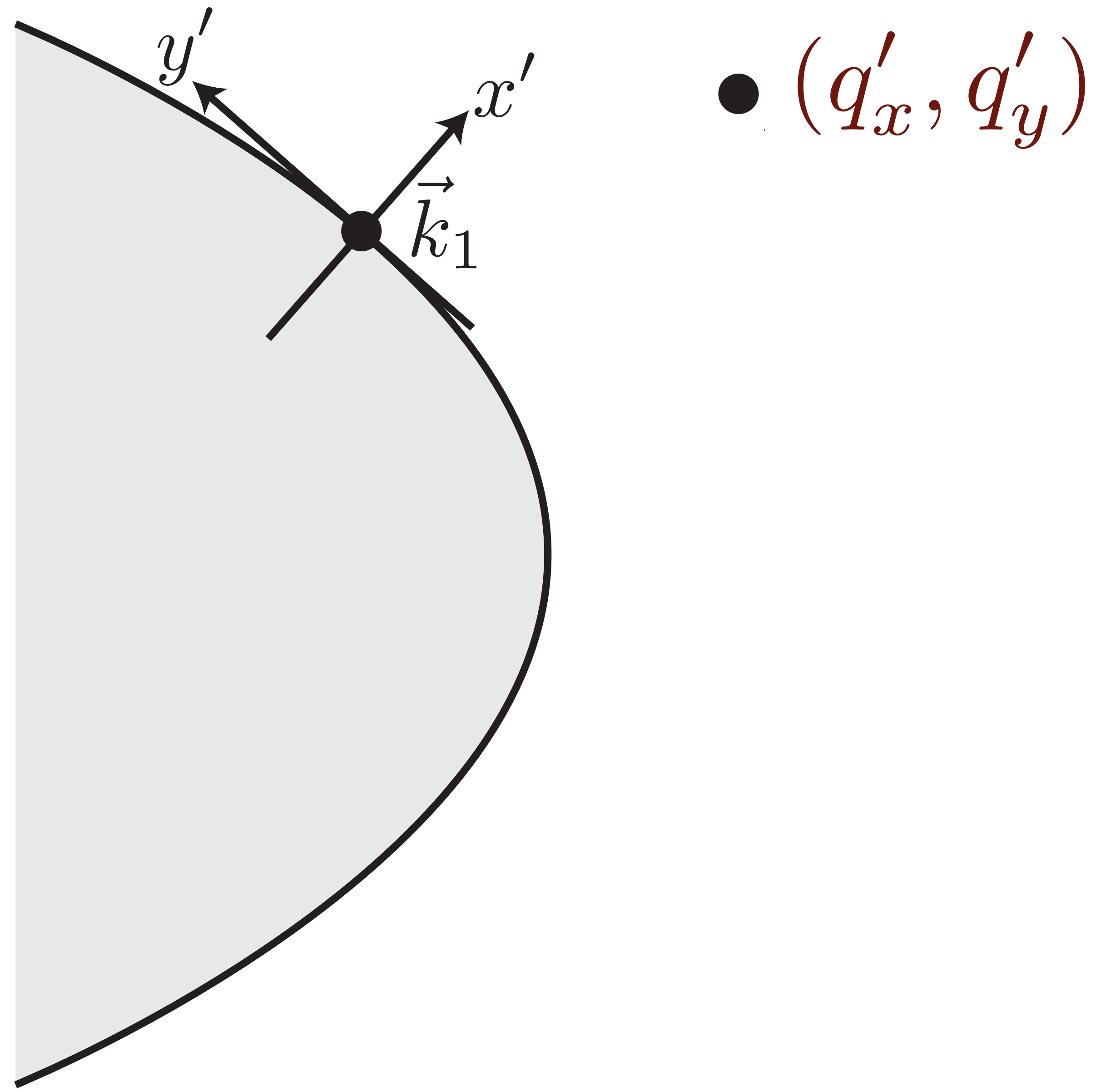
Field theory defined at $\vec{k}_0$

Fermi liquid theory



$$\text{Field theory defined at } \vec{k}_1$$

Fermi liquid theory



$\varepsilon_{q'} = \varepsilon_q$ ensures compatibility

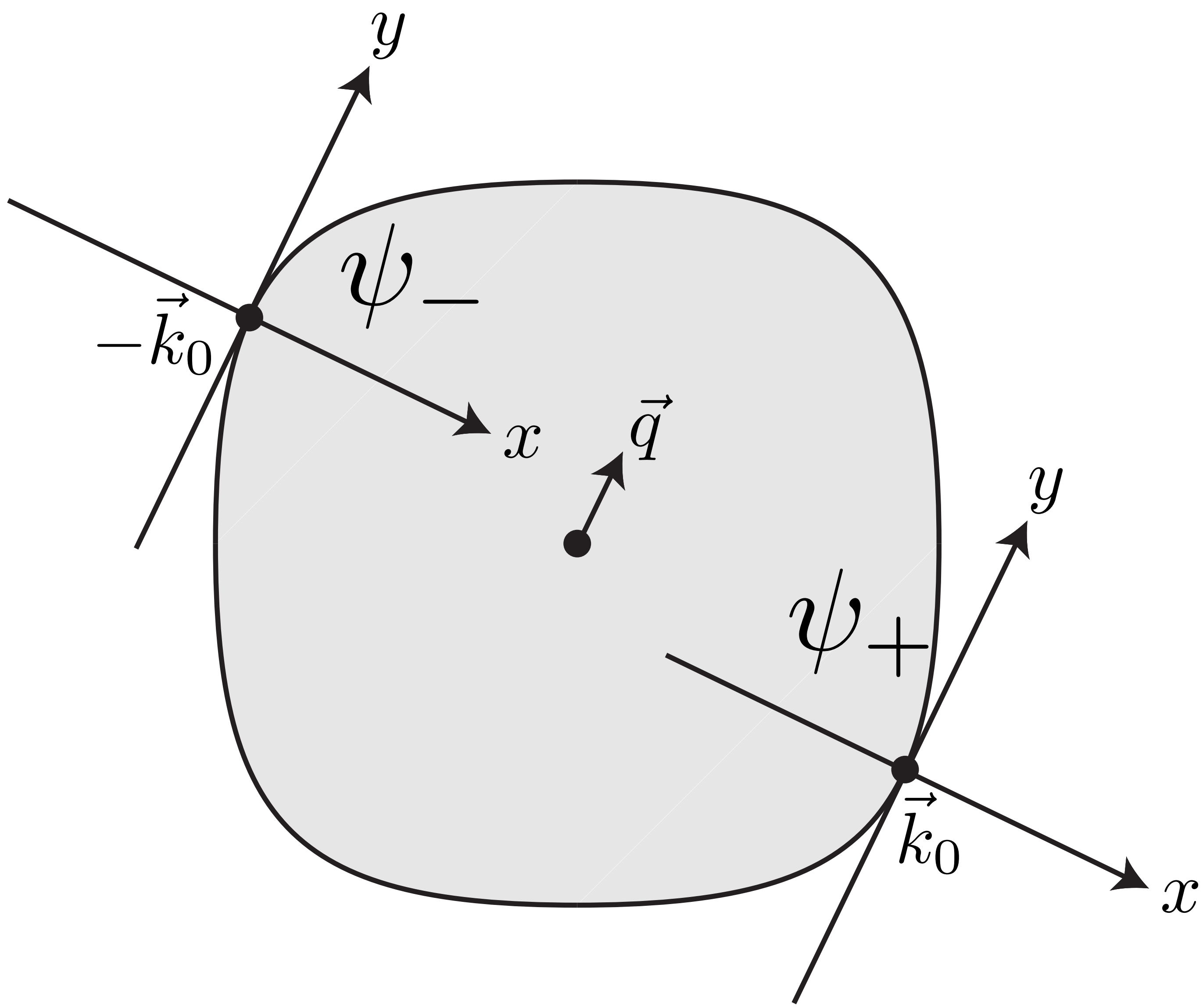
of redundant 2+1 dimensional field theories.

Quantum criticality of Ising-nematic transition

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (r - r_c)\phi^2 + u\phi^4 \right]$$

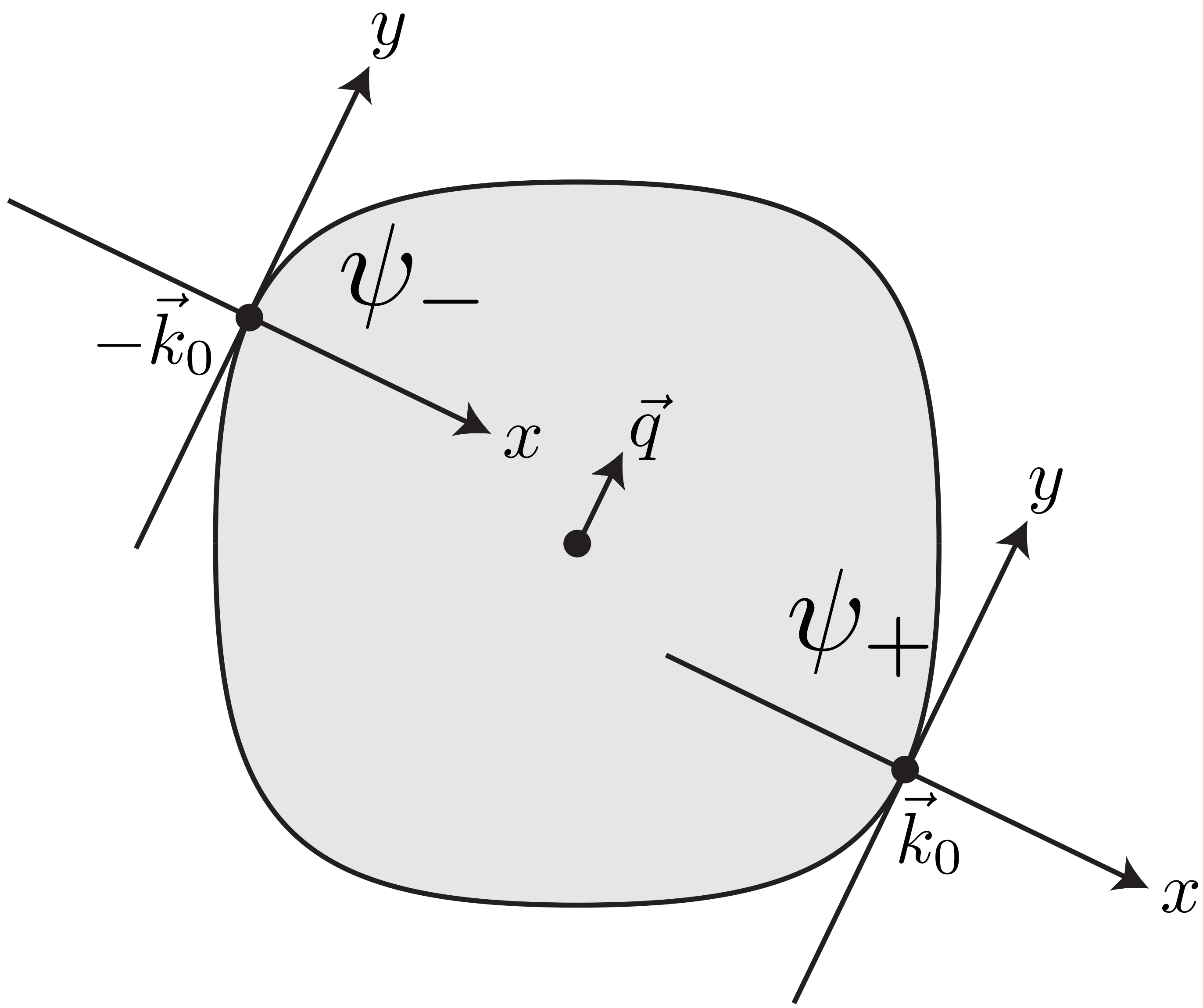
$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$



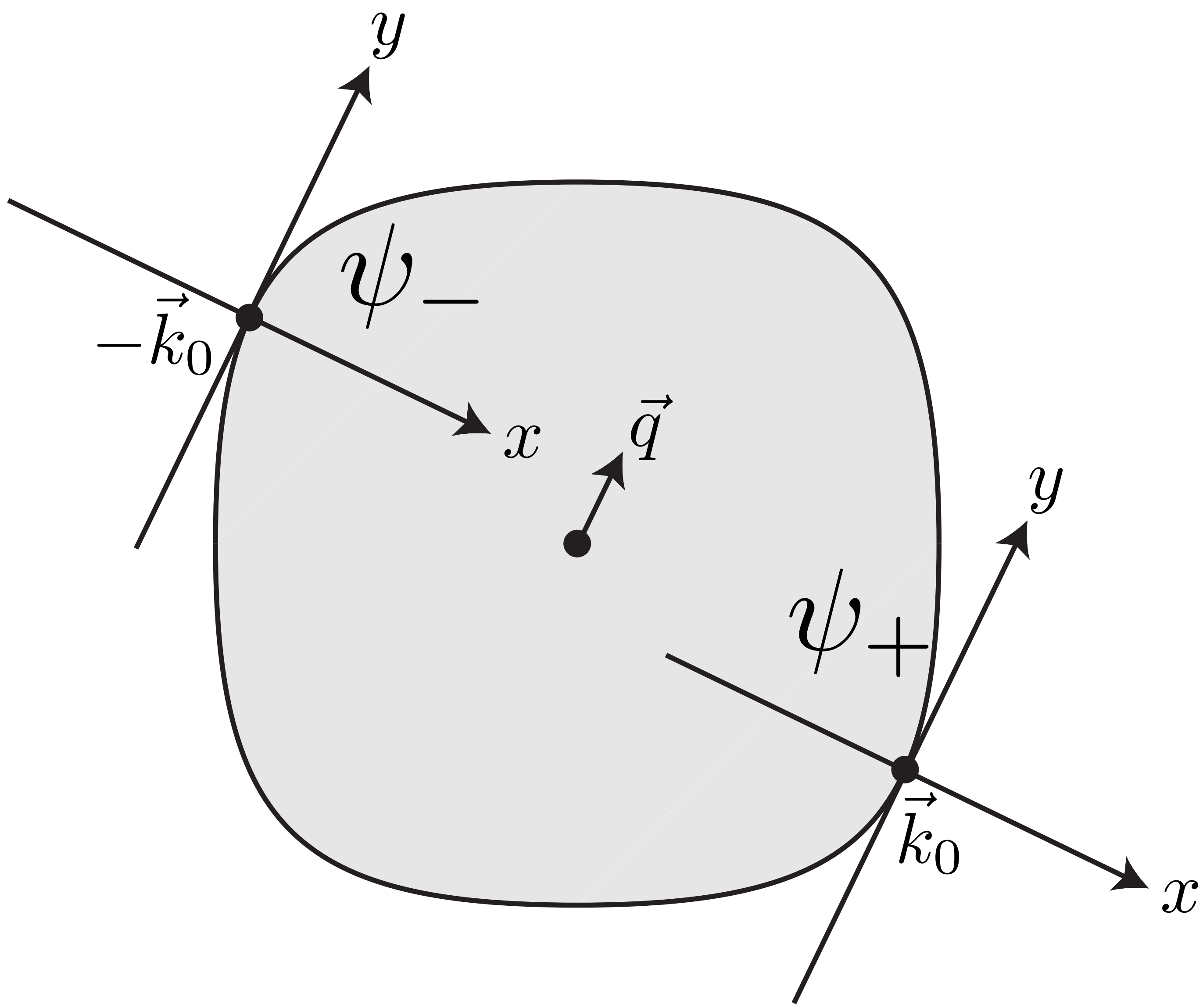
A ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Expand fermion kinetic energy at wavevectors about $\vec{k}_0$



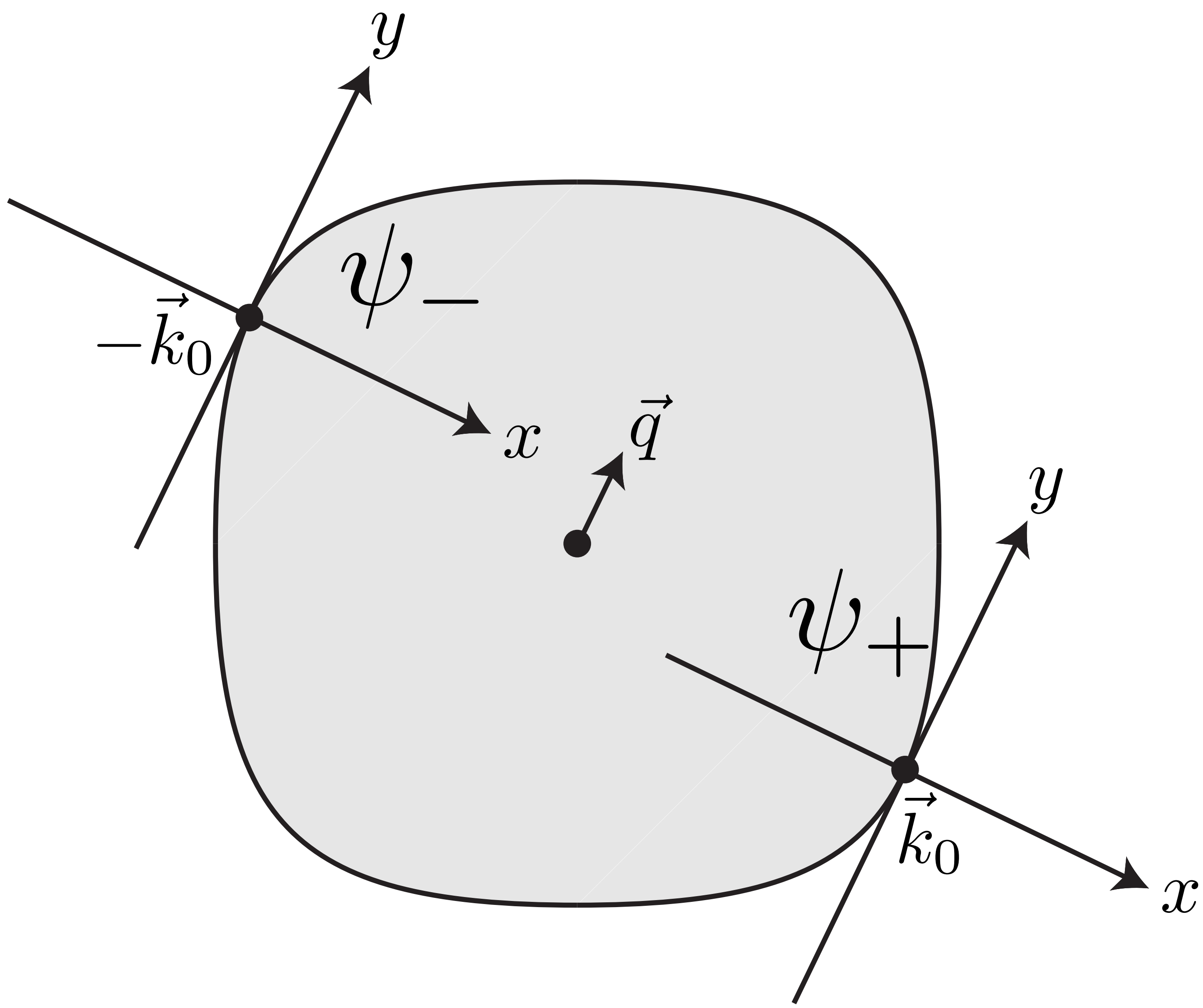
$$
\begin{aligned}
\mathcal{L} \;=\;& \psi_+^\dagger \left(\zeta\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\zeta\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- \\
& - \lambda\phi\left(\psi_+^\dagger\underline{\psi_+ + \psi_-^\dagger}\psi_-\right) + \frac{1}{2g}\left(\partial_y\phi\right)^2 + \frac{r}{2}\phi^2
\end{aligned}
$$

Theory of Ising-nematic transition



$$\mathcal{L} = \psi_+^\dagger \left(\zeta\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\zeta\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \lambda\phi\left(\psi_+^\dagger\psi_+ - \psi_-^\dagger\psi_-\right) + \frac{1}{2g}\left(\partial_y\phi\right)^2 + \frac{r}{2}\phi^2$$

Theory of a Fermi surface minimally coupled to an Abelian or non-Abelian gauge field with $\phi \sim A_x$



$$\mathcal{L} = \psi_+^\dagger \left(\zeta\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\zeta\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$
$$- \lambda\phi\left(\psi_+^\dagger\underline{\psi_+ + \psi_-^\dagger}\psi_-\right) + \frac{1}{2g}\left(\partial_y\phi\right)^2 + \frac{r}{2}\phi^2$$

Theory of Ising-nematic transition

$$
\begin{aligned}
\mathcal{L} &= \psi_+^\dagger \left(\zeta \partial_\tau - i \partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\zeta \partial_\tau + i \partial_x - \partial_y^2\right) \psi_- \\
&\quad - \lambda \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g} \left(\partial_y \phi\right)^2 + \frac{r}{2} \phi^2
\end{aligned}
$$

After tuning the single parameter $r \sim \lambda - \lambda_c$, and sending $\zeta \to 0$, $\mathcal{L}$ describes a critical theory with no coupling constants. There is a separate copy of this critical theory for each direction $\hat{q}$. This theory has 2 independent exponents z and η, and the correlation length and susceptibility exponents are given by

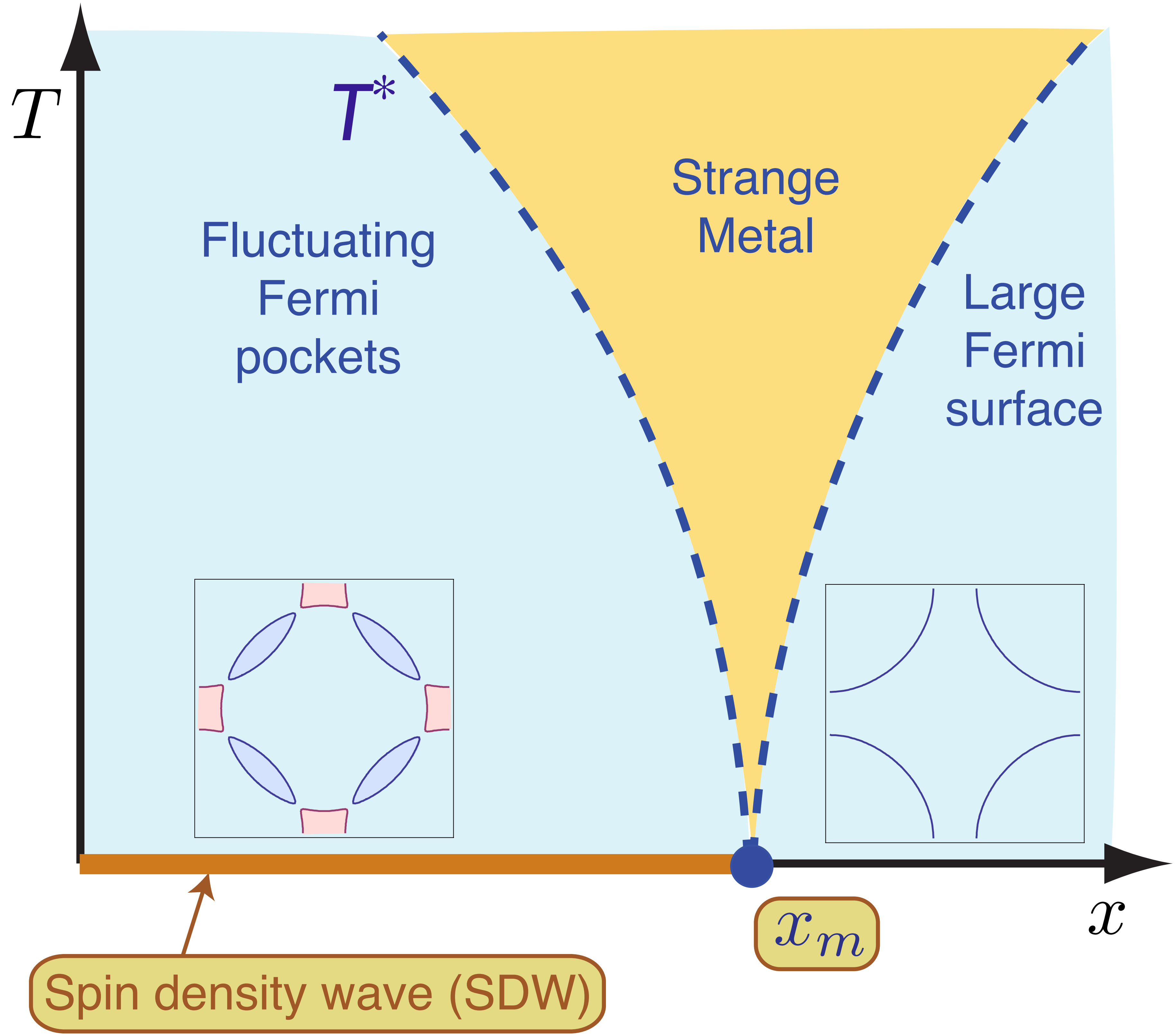
$$
\nu = \frac{1}{z-1} \quad ; \quad \gamma = 1
$$

The fermion and order parameter Green's functions obey the scaling forms

$$
G(\vec{q}, \omega) = \xi^{2-\eta} \Phi_\psi \left((q_x + q_y^2) \xi^2, \omega \xi^z \right) \; ; \; D(\vec{q}, \omega) = \xi^{z-1} \Phi_\phi \left(q_y \xi, \omega \xi^z \right)
$$

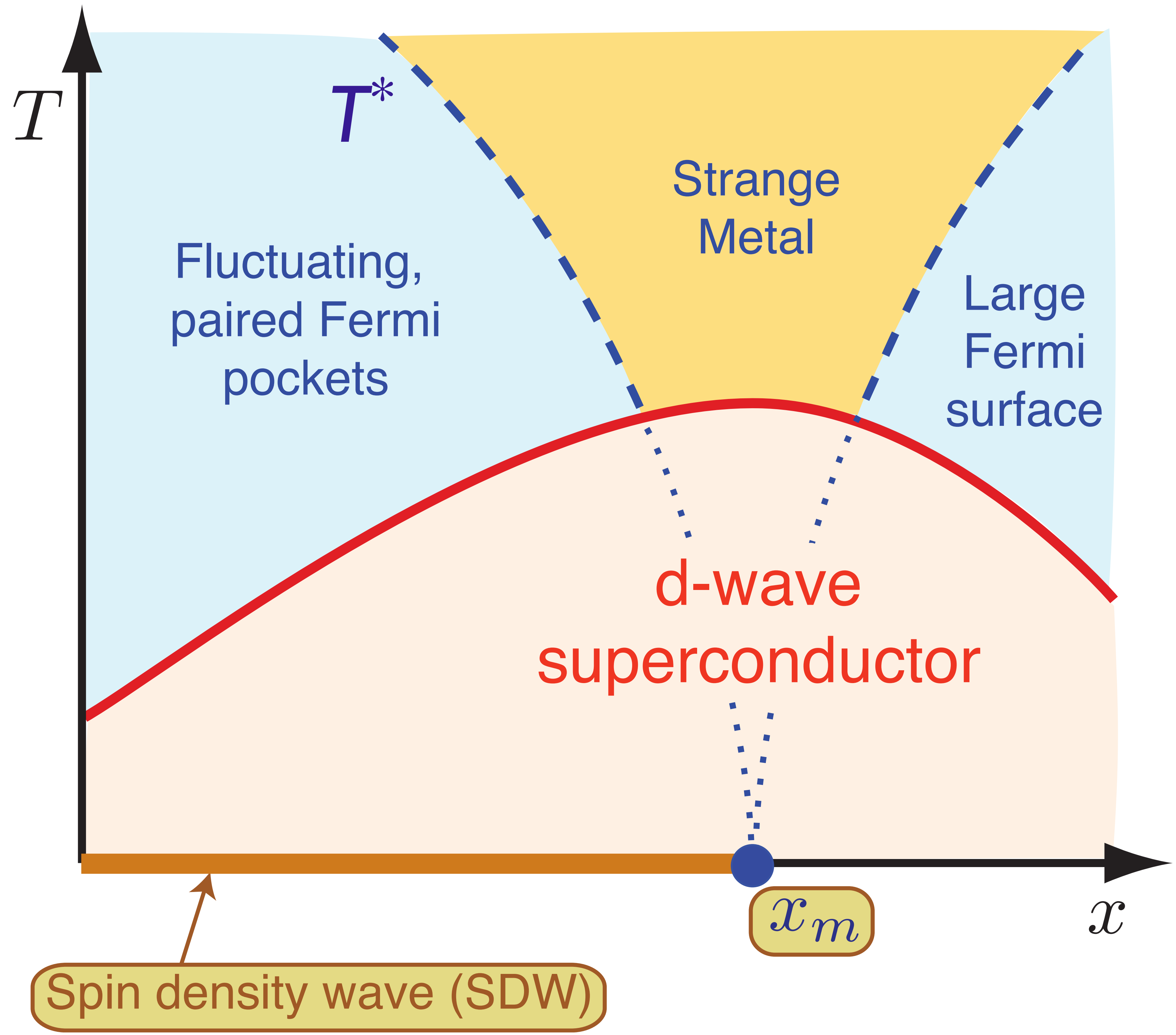
We have computed the exponents to three loops, and find $z = 3$ and $\eta = 0.06824$ at this order.

Theory of quantum criticality in the cuprates



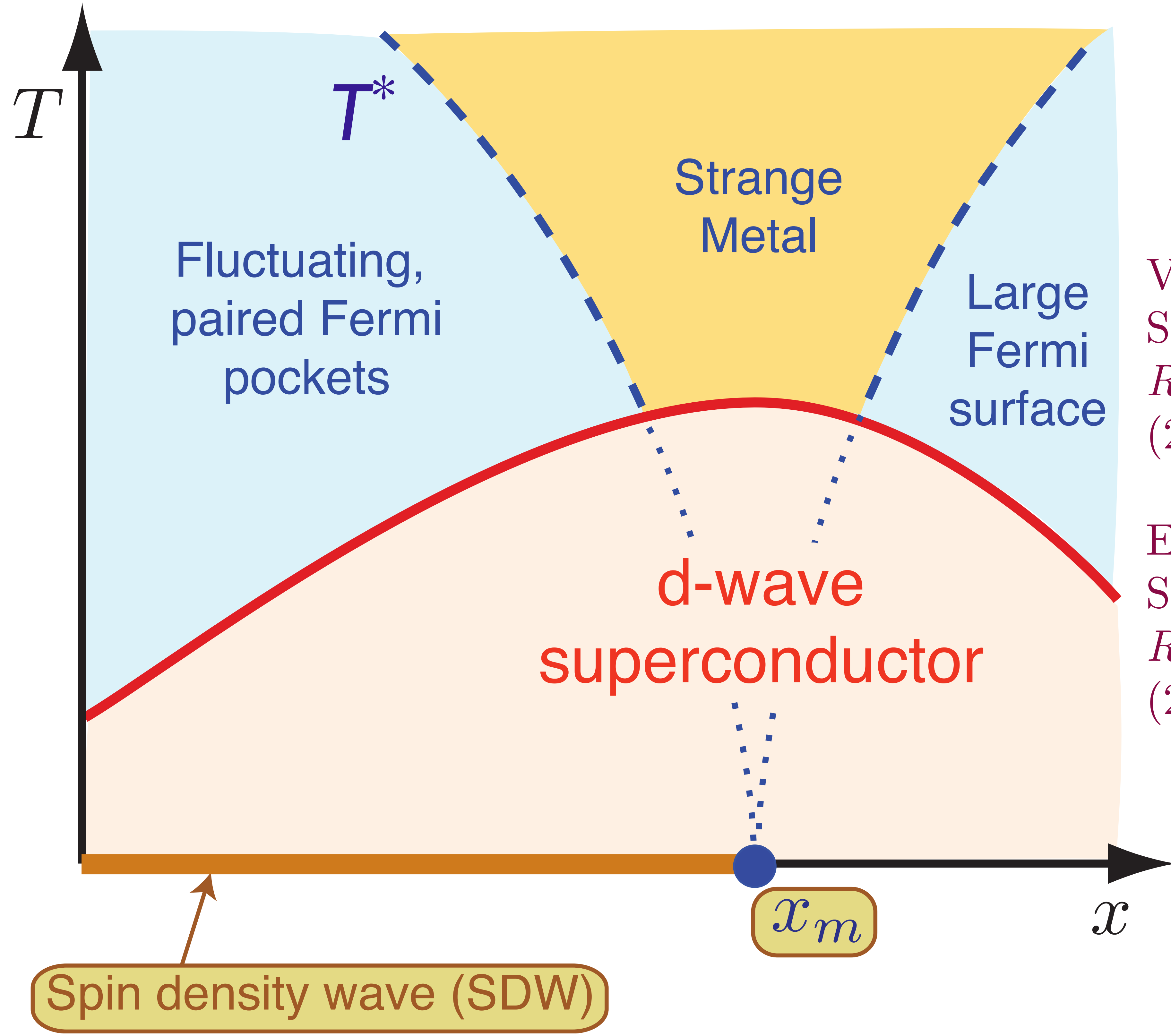
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$; has associated Ising-nematic order

Theory of quantum criticality in the cuprates



Onset of d-wave superconductivity hides the SDW critical point $x = x_m$

Theory of quantum criticality in the cuprates

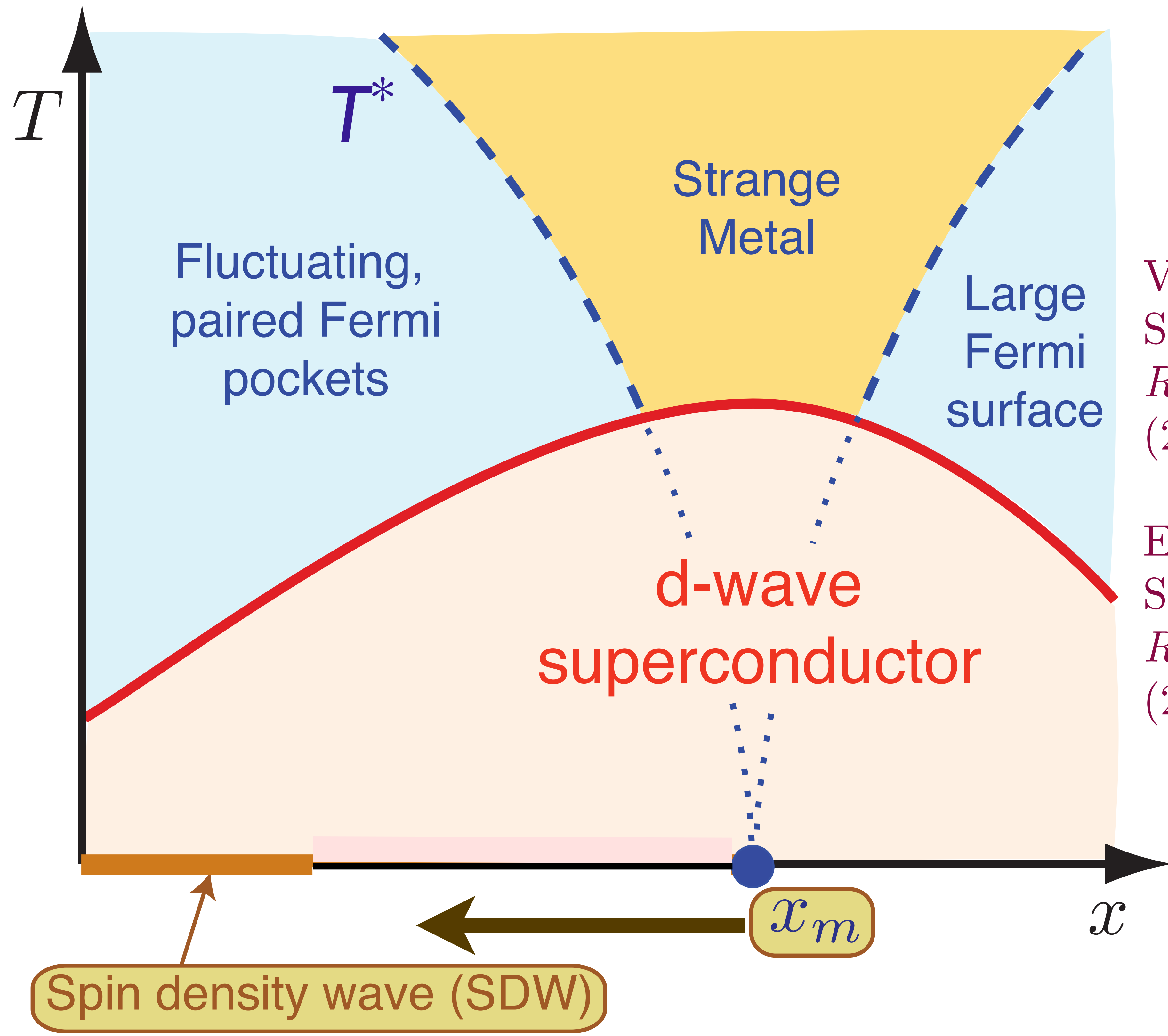


V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual SDW quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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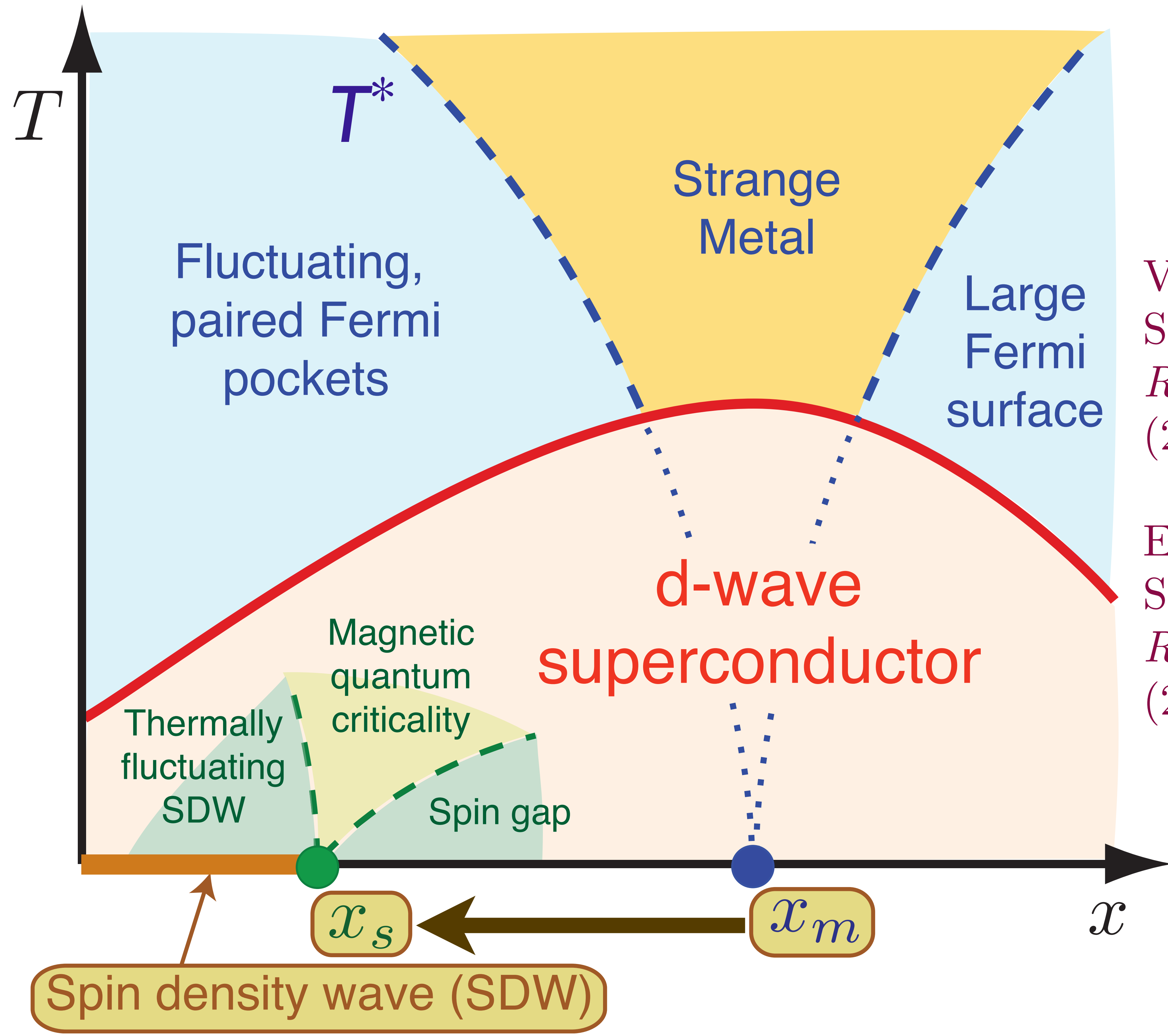

V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

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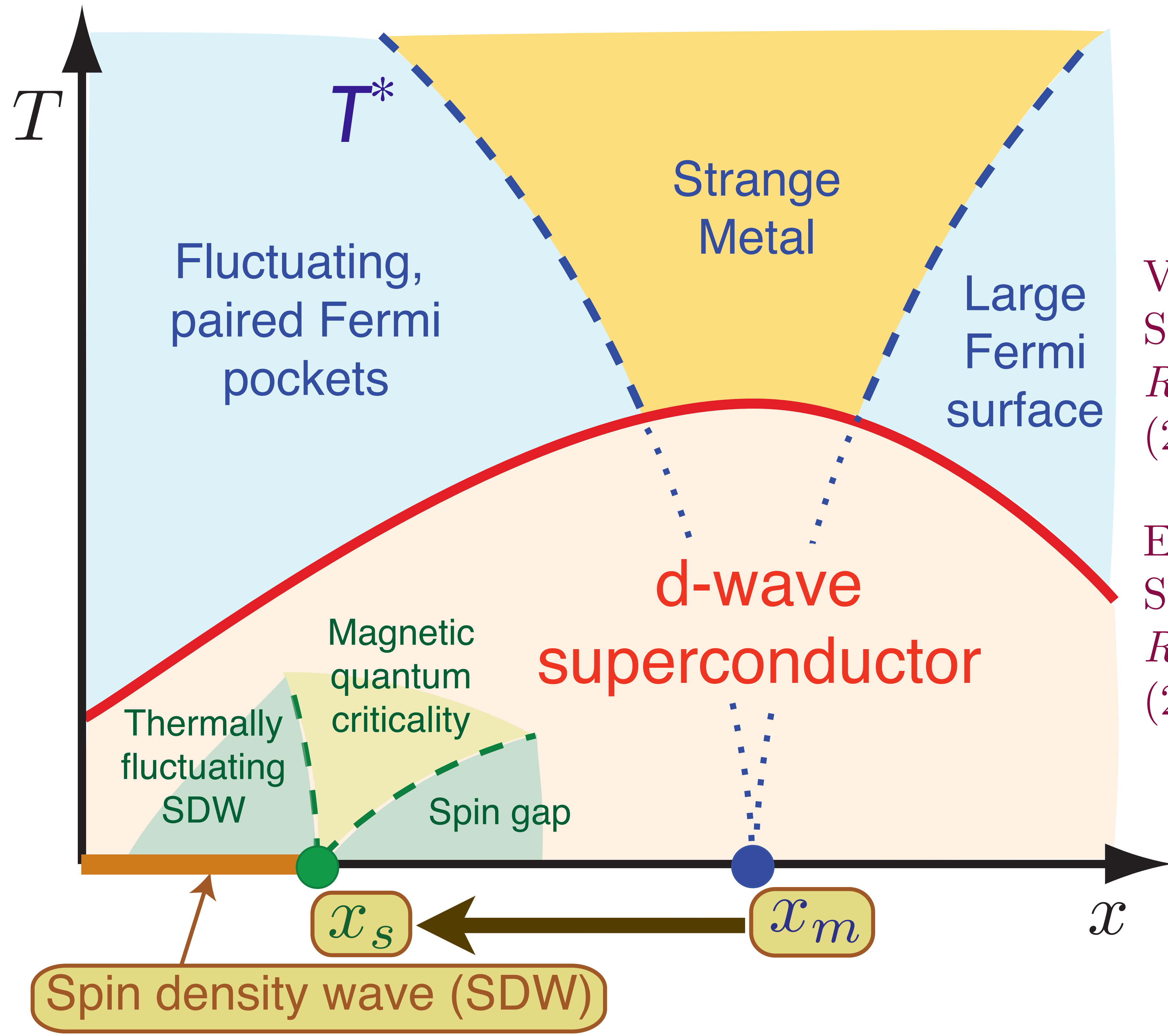


V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Physics of competition: *d*-wave SC and SDW "eat up" same pieces of the large Fermi surface.

Shift of ordering critical point in a metal due to onset of superconductivity

SDW ordering:

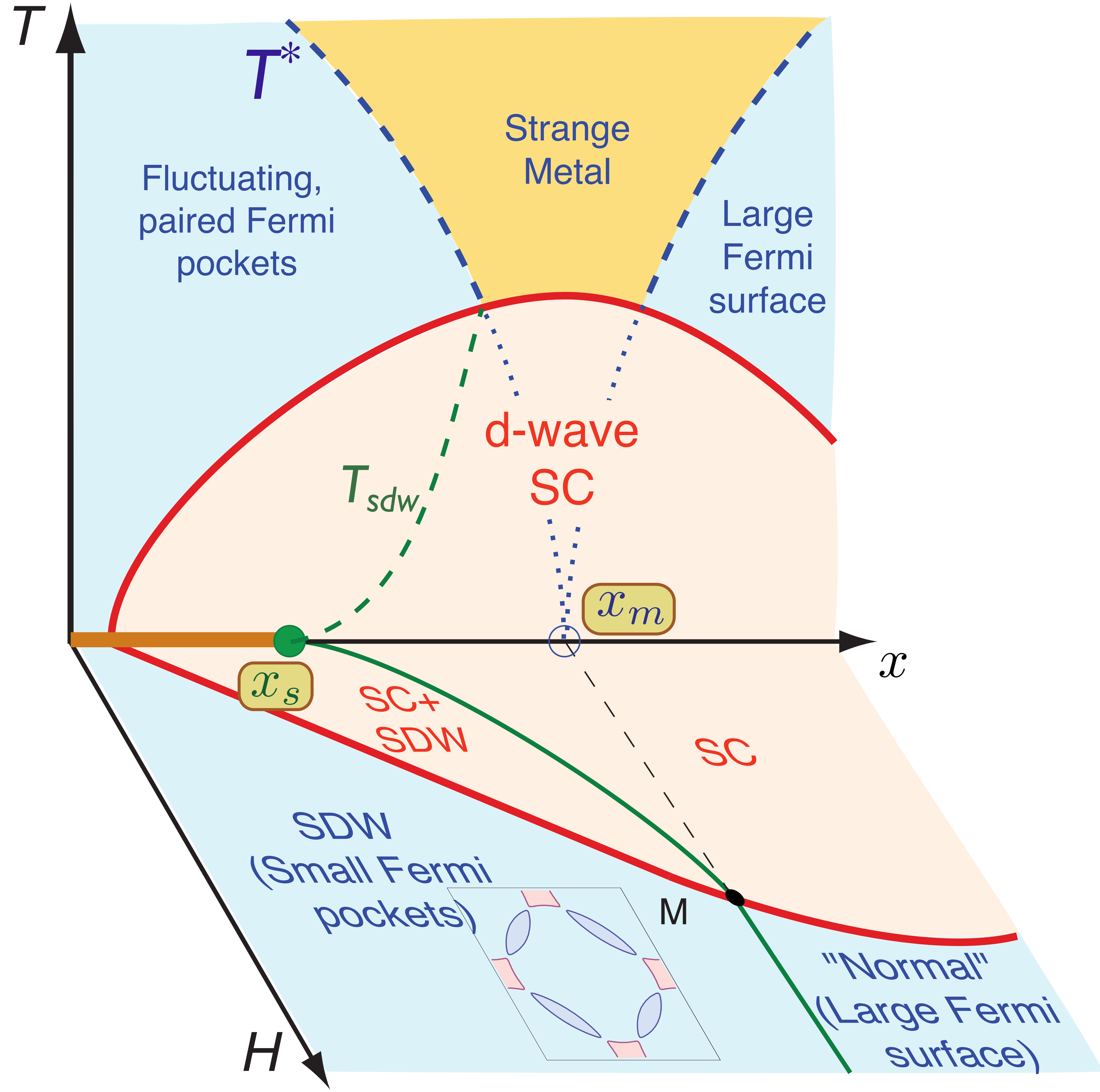
$$x_s = x_m - c|\Delta|$$

Ising-nematic ordering:

$$\tilde{x}_s = \tilde{x}_m - \tilde{c}|\Delta|^2$$

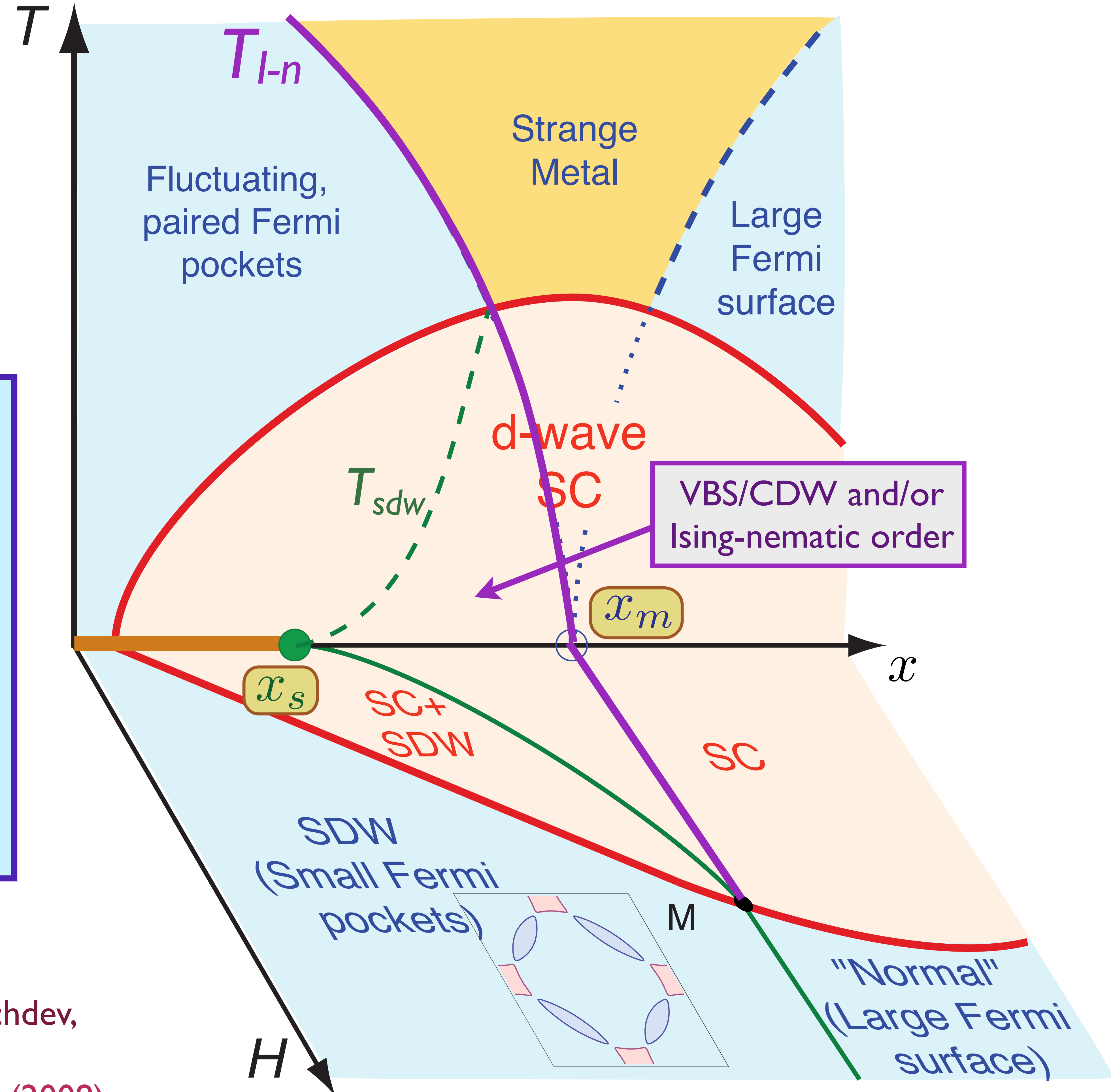
Δ is the fermion pairing gap.

E.G. Moon and S. Sachdev, to appear



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

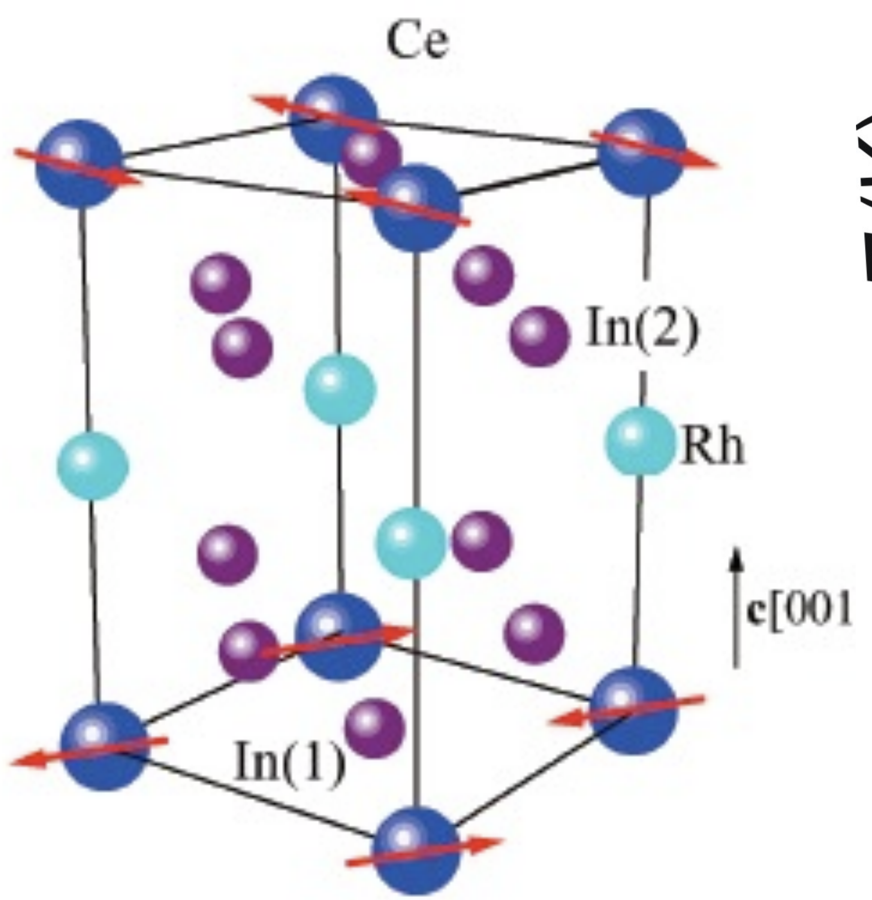
E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

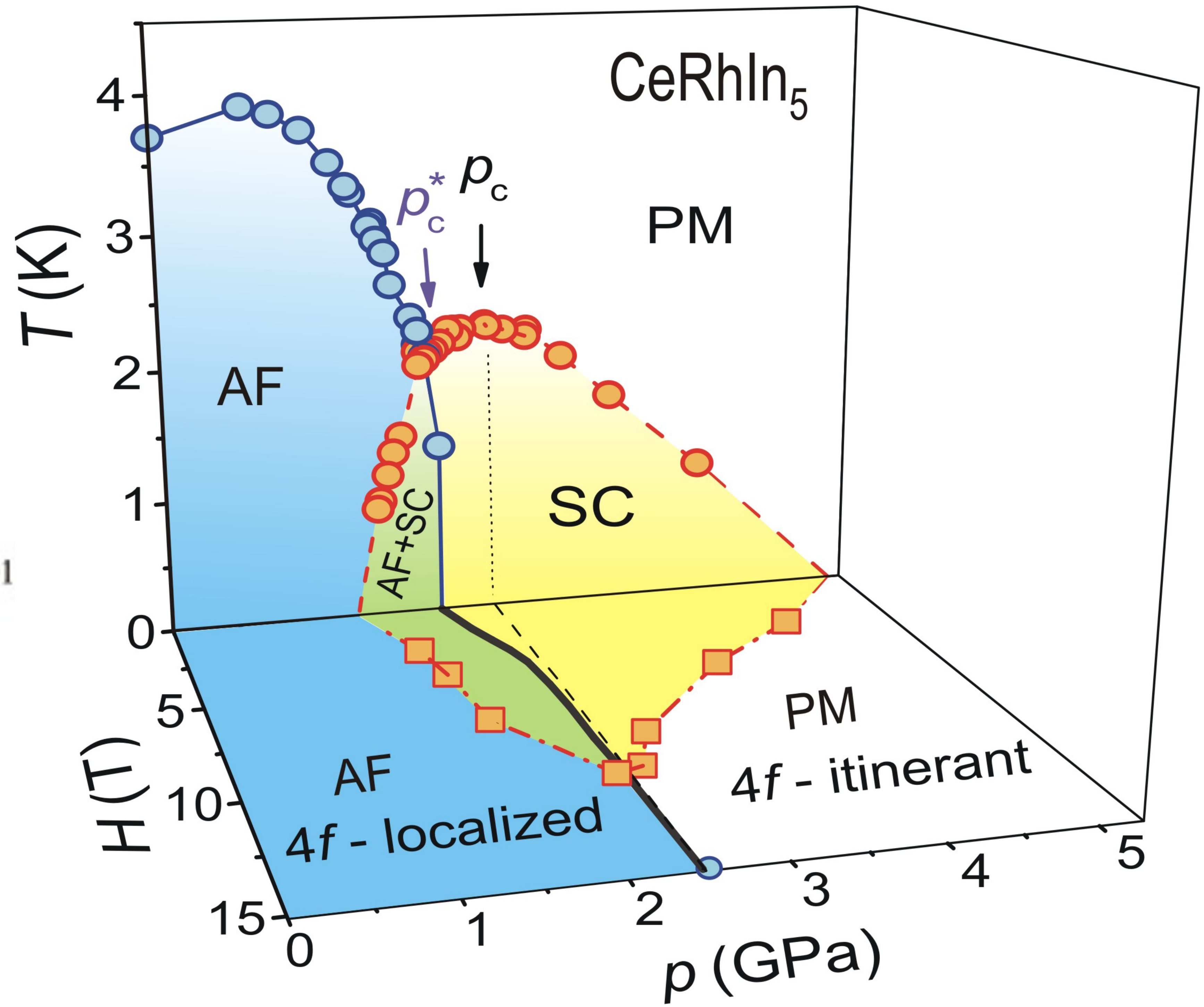


Onset of superconductivity disrupts SDW order, but VBS/CDW/ Ising-nematic ordering can survive

R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).

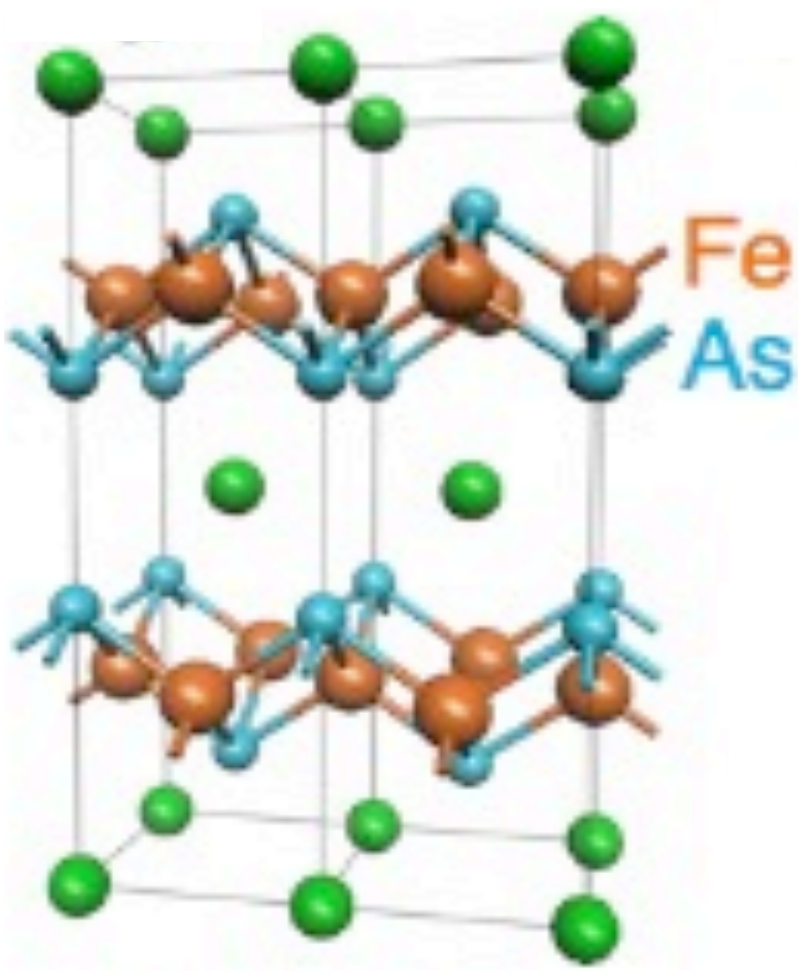
Similar phase diagram for $CeRhIn_5$



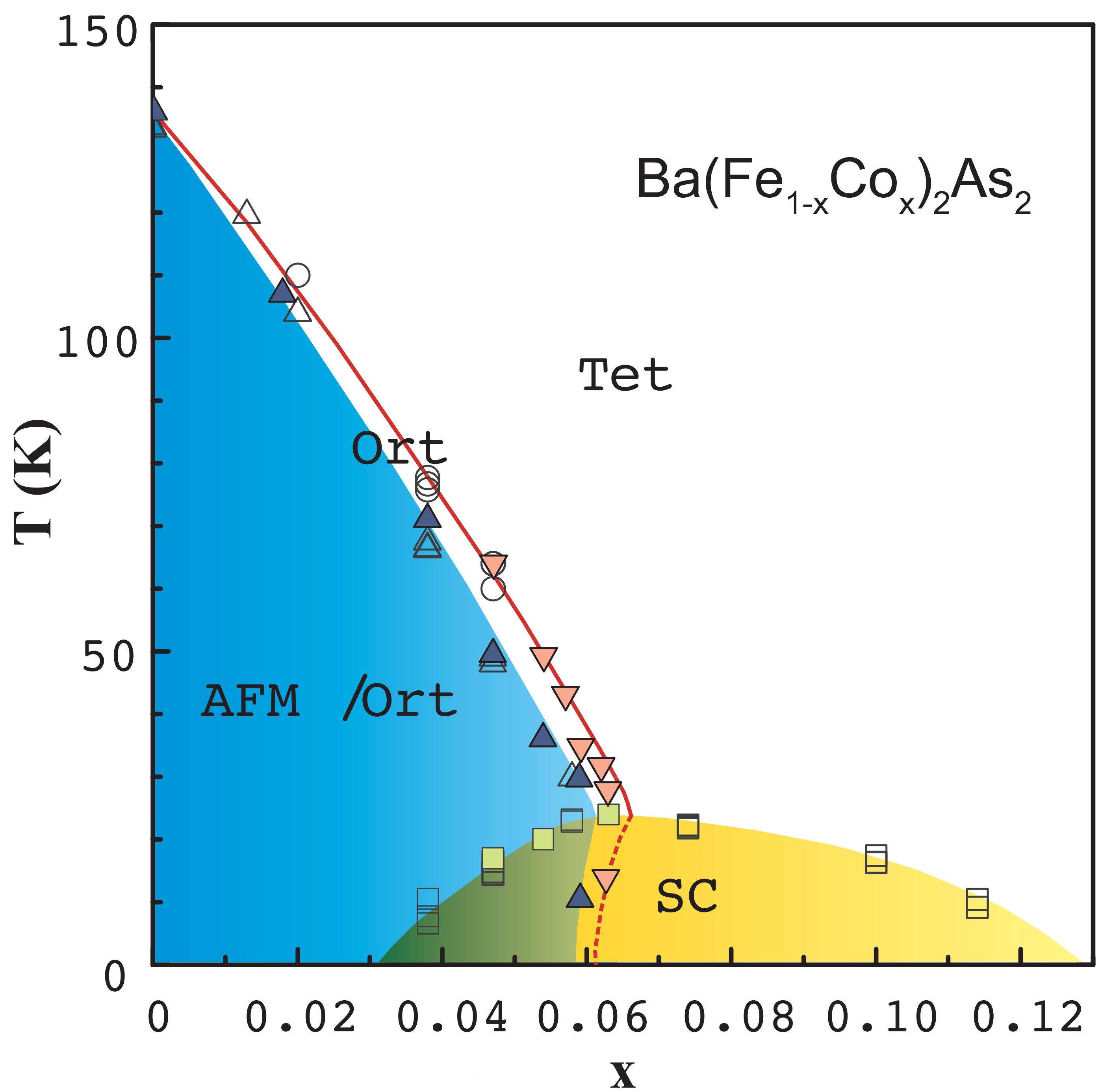


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Similar phase diagram for the pnictides



BaFe$_2$As$_2$
Unit cell



S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, arXiv:0911.3136.