

CFTs in 0+1 dimensions: implications for charged black holes and strange metals

Bootstrapping Nature:
Non-perturbative Approaches to Critical Phenomena
The Galileo Galilei Institute For Theoretical Physics
Firenze, October 20, 2022

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Talk online: sachdev.physics.harvard.edu



1. **SYK model: a solvable and generic 0+1 dimensional CFT**
2. **Charged black holes**
3. **Strange metals**
4. **From Yukawa-SYK models to a universal theory of strange metals**

SYK model:
A solvable and generic
0+1 dimensional CFT

Operators in a generic CFT_1 are similar to heavy ('black hole') operators in CFT_D with $D \geq 2$

The Sachdev-Ye-Kitaev (SYK) model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta=1}^N U_{\alpha\beta;\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\gamma} c_{\delta} - \mu \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}$$

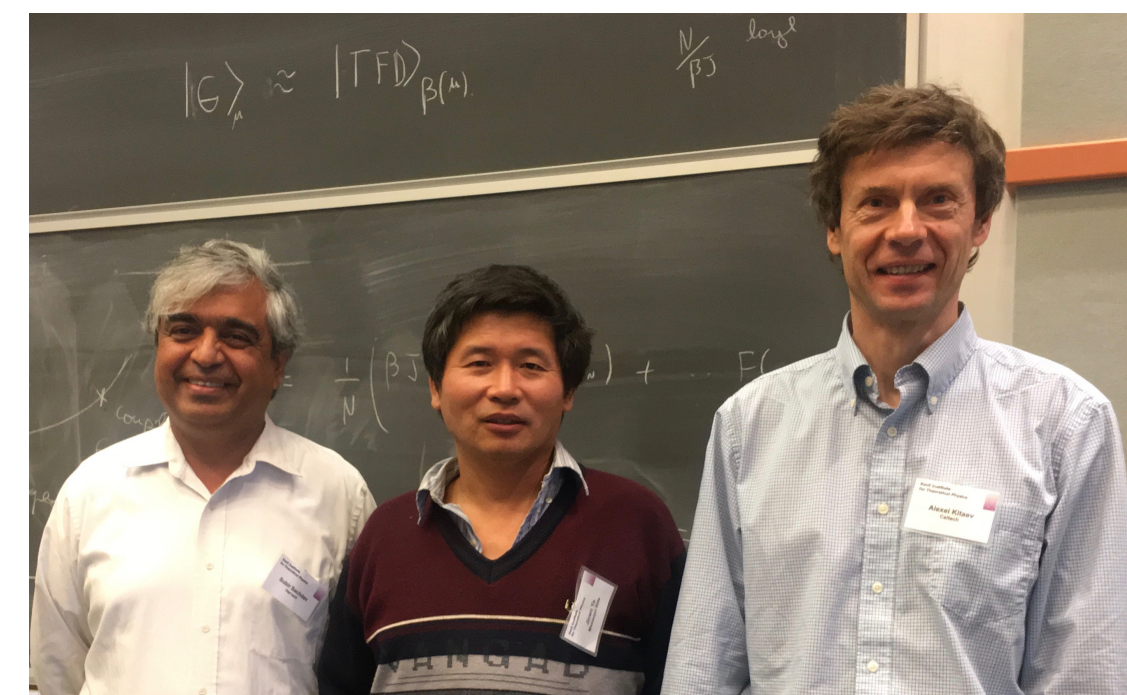
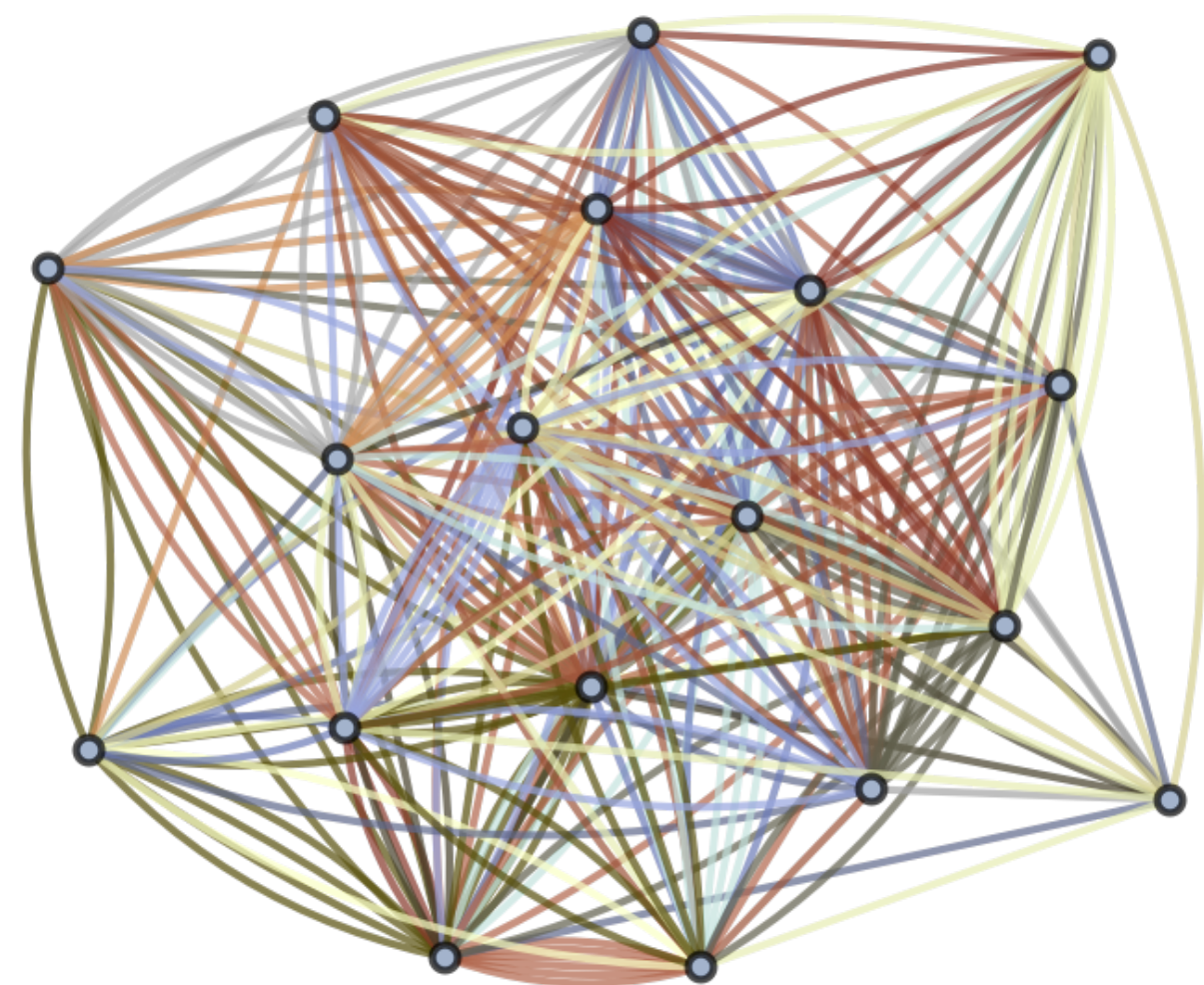
$$c_{\alpha} c_{\beta} + c_{\beta} c_{\alpha} = 0 \quad , \quad c_{\alpha} c_{\beta}^{\dagger} + c_{\beta}^{\dagger} c_{\alpha} = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_{\alpha} c_{\alpha}^{\dagger} c_{\alpha}; \quad [\mathcal{H}, \mathcal{Q}] = 0; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
 $N \rightarrow \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

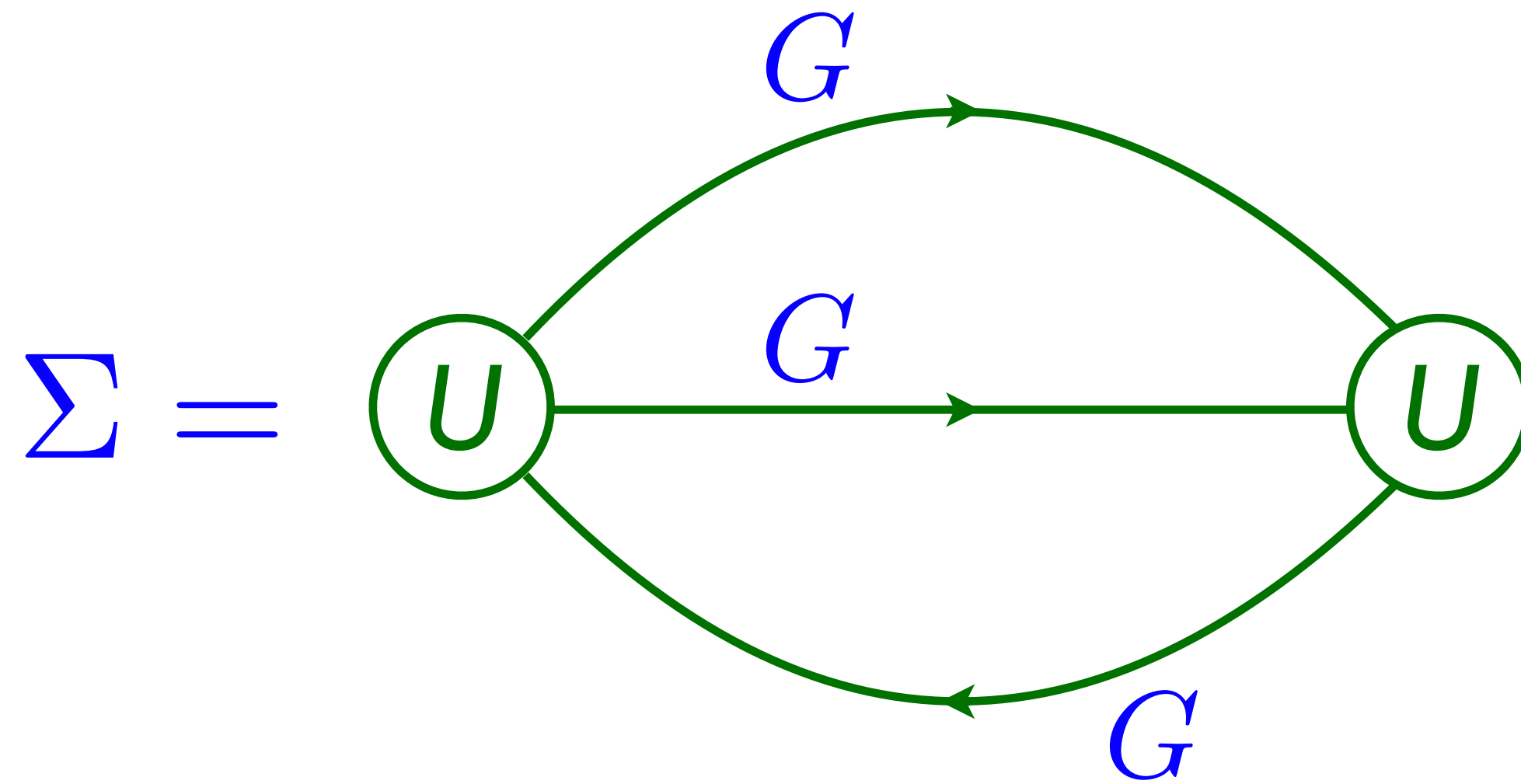


The Sachdev-Ye-Kitaev (SYK) model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = Q.$$

($i\omega + \mu \rightarrow 0$ Bootstrap equations!)



S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)



The complex SYK model

$$G_*(\tau) = -C \frac{e^{-2\pi\mathcal{E}T\tau}}{\sqrt{1 + e^{-4\pi\mathcal{E}}}} \left(\frac{T}{\sin(\pi T\tau)} \right)^{1/2}.$$

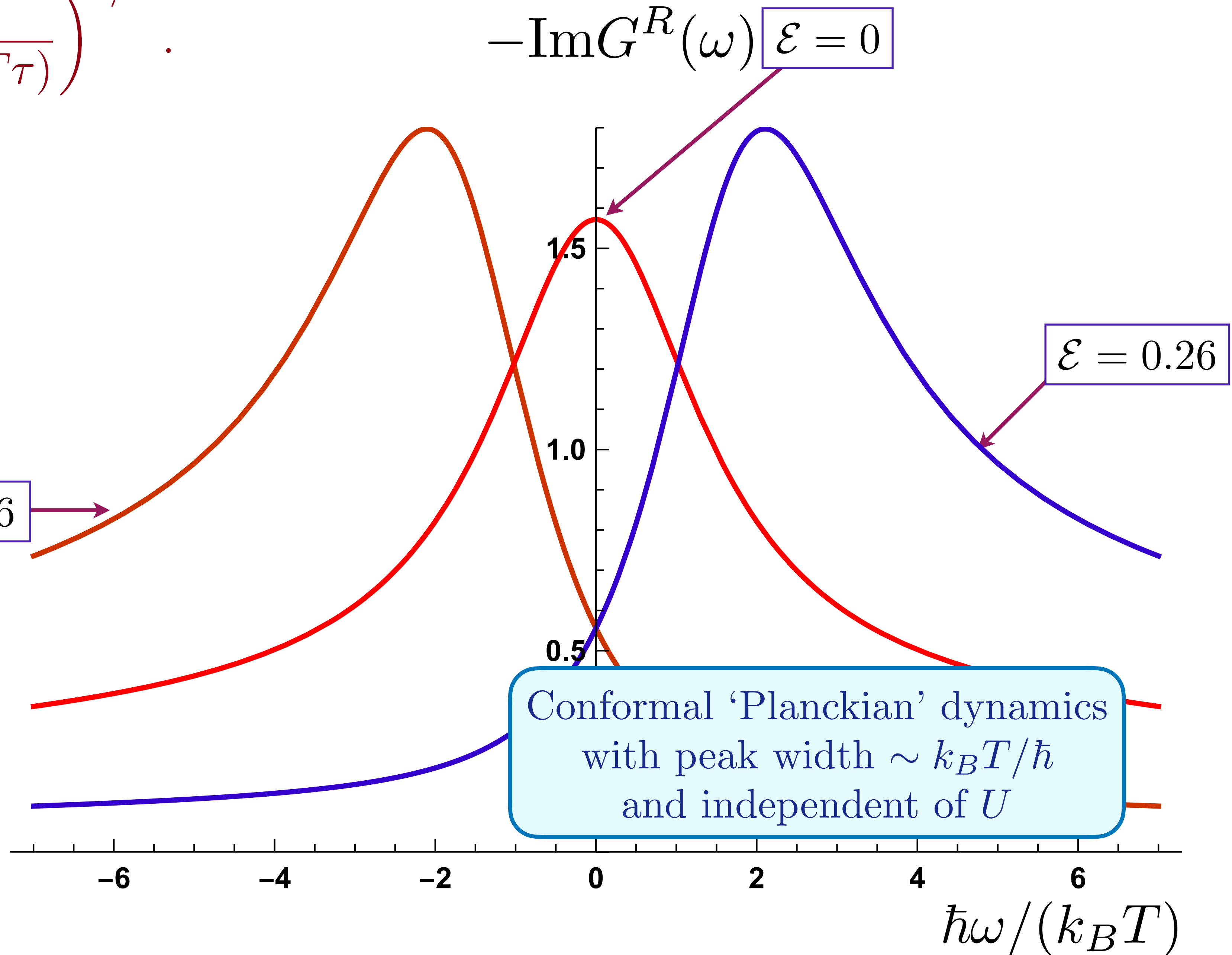
$$G_*^R(\omega) = \frac{-iC e^{-i\theta} \Gamma\left(\frac{1}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}{(2\pi T)^{1/2} \Gamma\left(\frac{3}{4} - \frac{i\omega}{2\pi T} + i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4 + \theta)}{\sin(\pi/4 - \theta)}$$

$$C = \left(\frac{\pi}{U^2 \cos(2\theta)} \right)^{1/4}$$

\mathcal{E} is a known function of Q
(Luttinger relation)

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)



The Sachdev-Ye-Kitaev (SYK) model

$$\begin{aligned} \mathcal{Z}(Q, T) &= \text{Tr}_Q \exp\left(-\frac{\mathcal{H}}{T}\right) = \exp(-F/T); \text{ Entropy } S = -\frac{\partial F}{\partial T}. \\ &\equiv \int_{E_0^-}^{\infty} D(E) e^{-E/T}; \quad D(E) = \sum_i \delta(E - E_i); \quad \mathcal{H} |\Psi_i\rangle = E_i |\Psi_i\rangle \end{aligned}$$

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A CFT in d spatial dimensions has an entropy density $S \sim T^d$. The SYK model is a 0+1 dimensional CFT, and we obtain a T -independent entropy:

$$\lim_{T \rightarrow 0} \lim_{N \rightarrow \infty} \frac{S}{N} = s_0$$

A. Georges, O. Parcollet, and S. Sachdev,
PRB **63**, 134406 (2001)

The constant s_0 is a known universal number. At $\mathcal{Q} = 1/2$, we have

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4} = 0.464847699170805107492692486833 \dots$$

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Conformal Perturbation theory

$$S = S_{\text{CFT}} + \sum_h \lambda_h \int_0^\beta d\tau O_h(\tau)$$

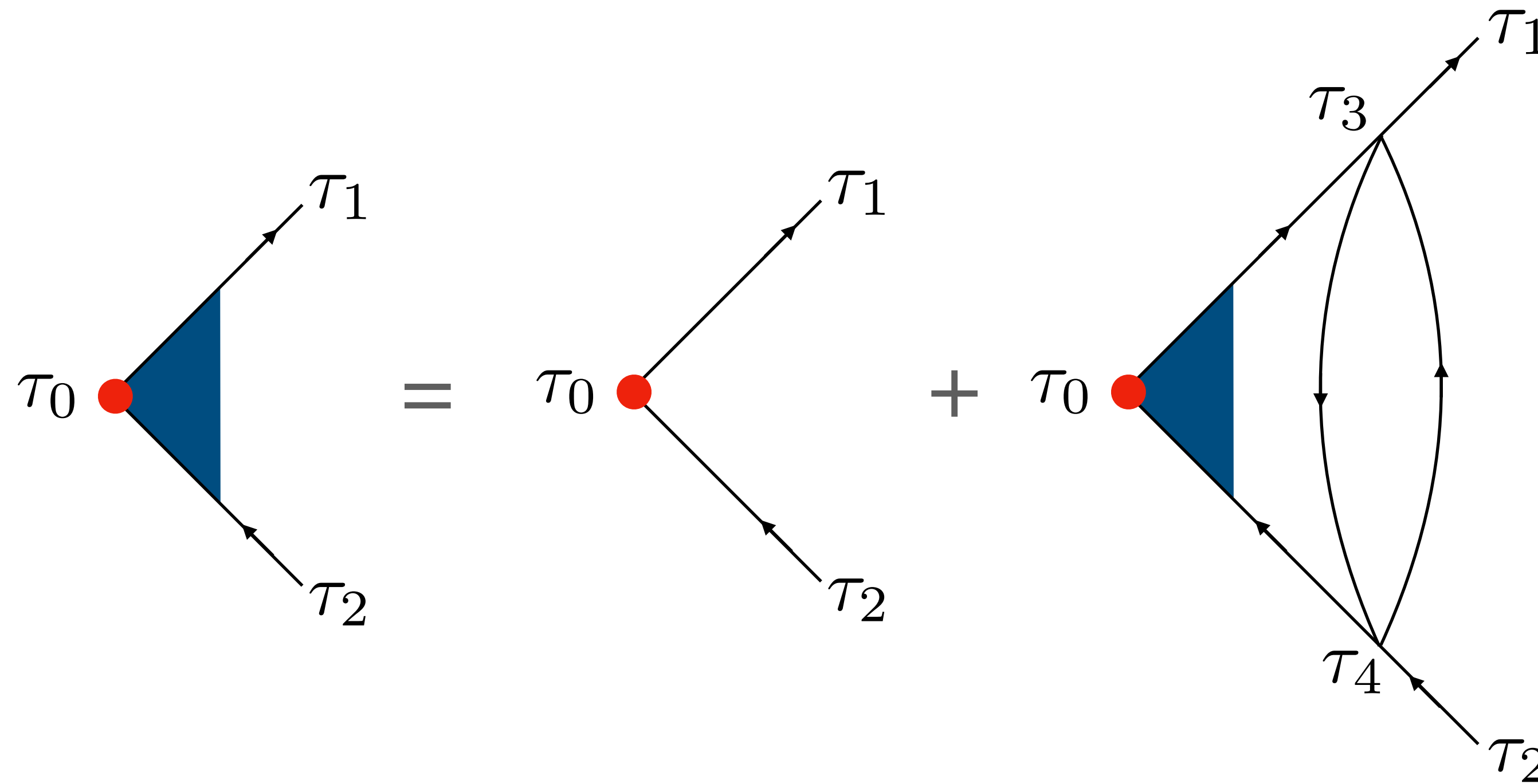
where $G_{\text{CFT}} = G_* \sim \text{sgn}(\tau)/\sqrt{|\tau|}$ and $\langle O_h(\tau)O_h(0) \rangle \sim 1/|\tau|^{2h}$

$$G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{|\tau|}} \left(1 + \sum_h \frac{g_h}{|\tau|^{h-1}} + \dots \right) , \quad S(T) = N \left[s_0 + \sum_h s_h T^{h-1} + \dots \right]$$

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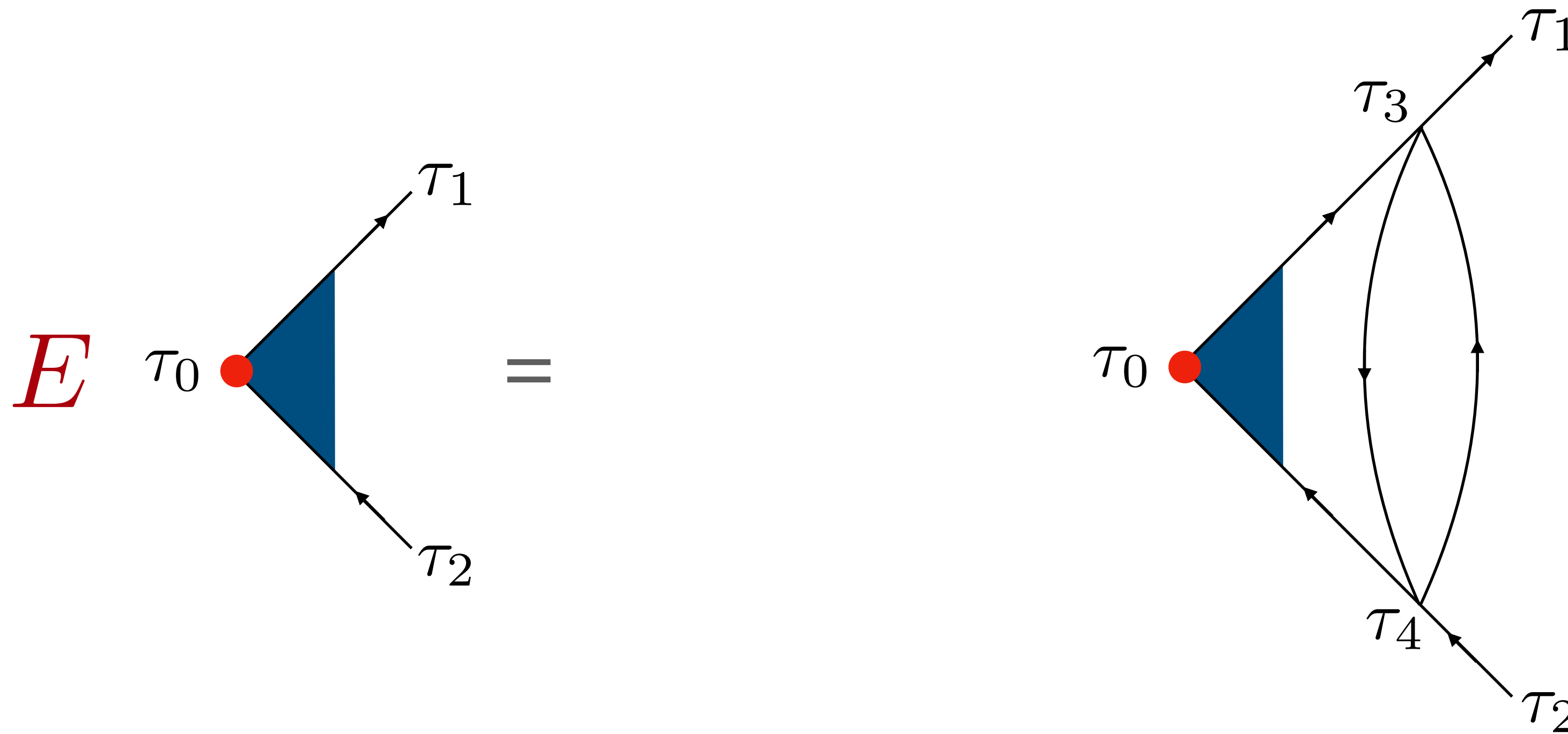
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(More
bootstrap
equations!)

Solution of eigenvalue equation with $E = 1$ yields a tower of O_h .

Conformal Perturbation theory

We define the three point function

$$v_h(\tau_1, \tau_2, \tau_0) = \langle c(\tau_1)c^\dagger(\tau_2)O_h(\tau_0) \rangle .$$

In the long time scaling limit, we can drop the bare first time on the right hand side, and obtain the eigenvalue equation

$$Ev(\tau_1, \tau_2, \tau_0) = \int d\tau_3 d\tau_4 K(\tau_1, \tau_2; \tau_3, \tau_4)v_h(\tau_3, \tau_4, \tau_0) ,$$

where the kernel K is

$$K(\tau_1, \tau_2; \tau_3, \tau_4) = -3U^2 G_*(\tau_{13})G_*(\tau_{24})G_*(\tau_{34})^2 ,$$

with $\tau_{ij} \equiv \tau_i - \tau_j$, and we are interested in the eigenvalue $E = 1$.

Conformal Perturbation theory

It is sufficient to solve the eigenvalue equation as $\tau_0 \rightarrow \infty$. Then, we can use the operator product expansion to write

$$c(\tau_1)c^\dagger(\tau_2) \sim \text{sgn}(\tau_{12}) \left[\frac{1}{|\tau_{12}|^{1/2}} + \sum_h \frac{c_h}{|\tau_{12}|^{1/2-h}} O_h(\tau_1) + \dots \right]$$

Inserting this into the definition of v , we conclude that $v \sim \text{sgn}(\tau_{12})/|\tau_{12}|^{1/2-h}$ as $\tau_0 \rightarrow \infty$. Then the eigenvalue equation simplifies to

$$E = -\frac{3 \tan(\pi h/2 - \pi/4)}{2h - 1} = 1.$$

There are an infinite number of solutions, and the lowest values are $h = 2, 3.77354\dots, 5.567946\dots, 7.63197\dots, \dots$. Consequently, the low T behavior of the entropy is

$$S(T) = N [s_0 + \gamma T + \gamma_2 T^{2.77354\dots} + \dots] .$$

The Sachdev-Ye-Kitaev (SYK) model

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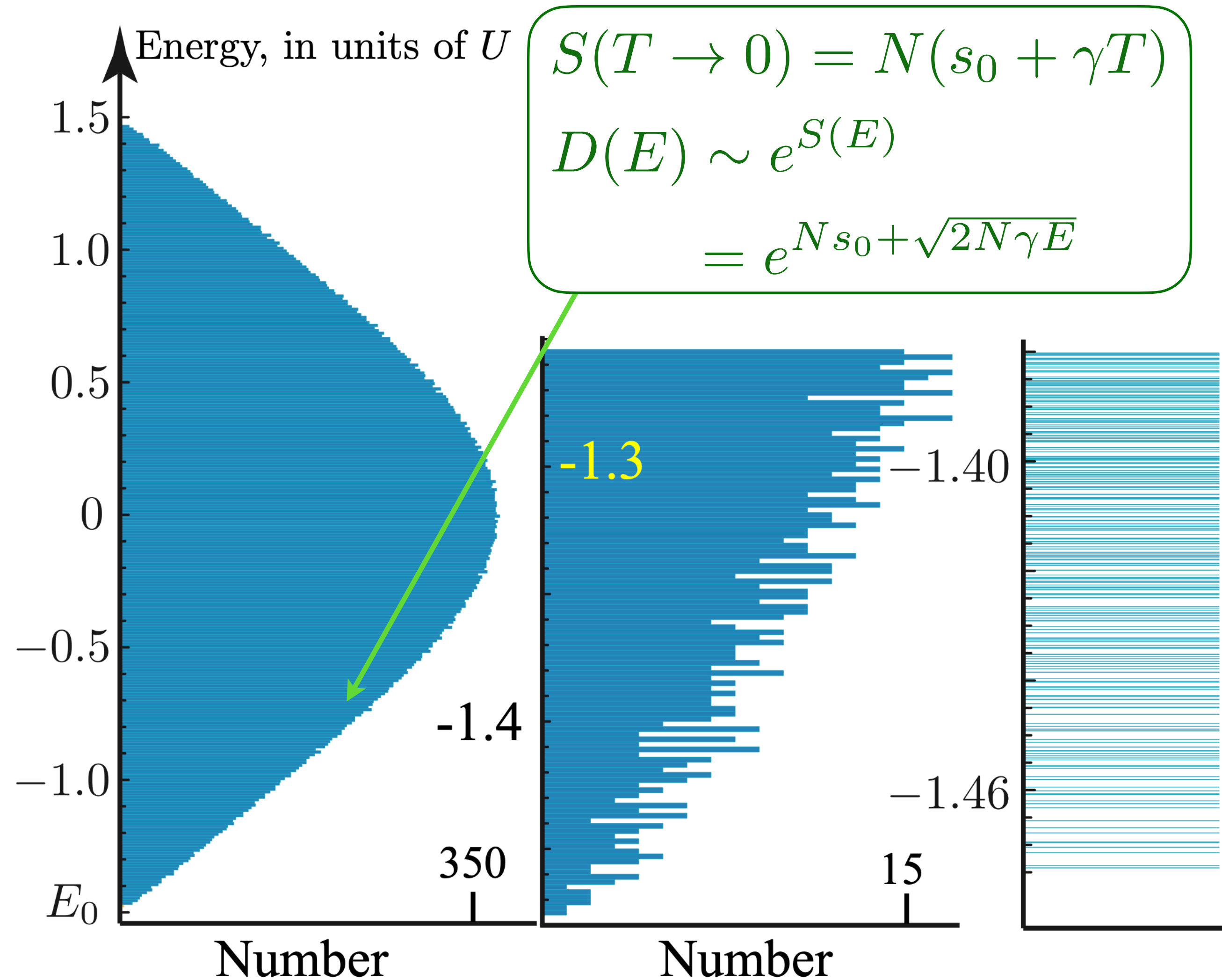
Conformal perturbations of saddle-point theory:

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$\gamma = \# / U$ is non-universal.

Many-body density of states

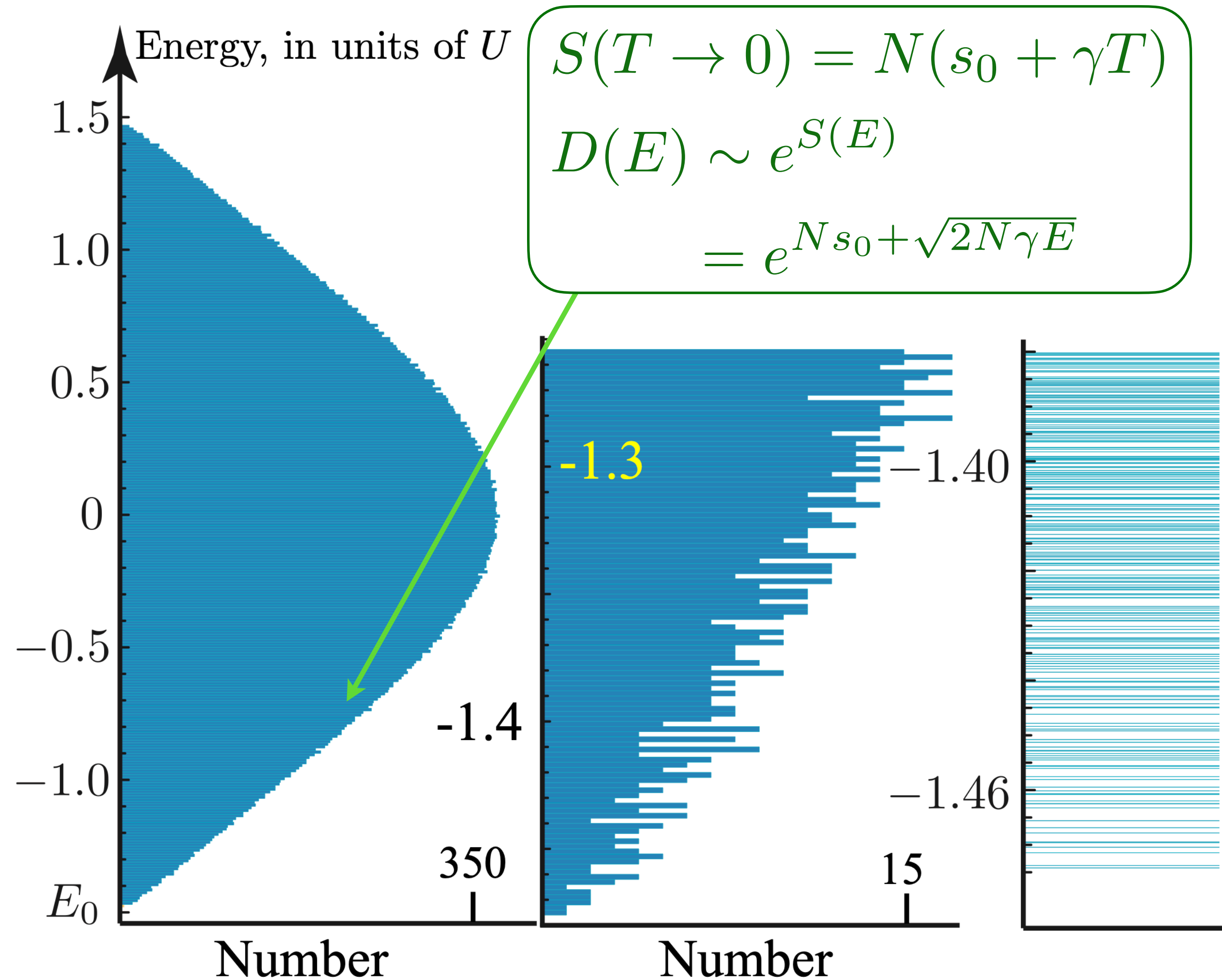
$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

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Energy level spacing $\sim e^{-N s_0}$!

Complex SYK model

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We will have a particular interest in the $h = 2$ operator. This operator is irrelevant for $U/N \ll T \ll U$, but dangerously irrelevant for $Ue^{-Ns_0} \ll T \ll U/N$.

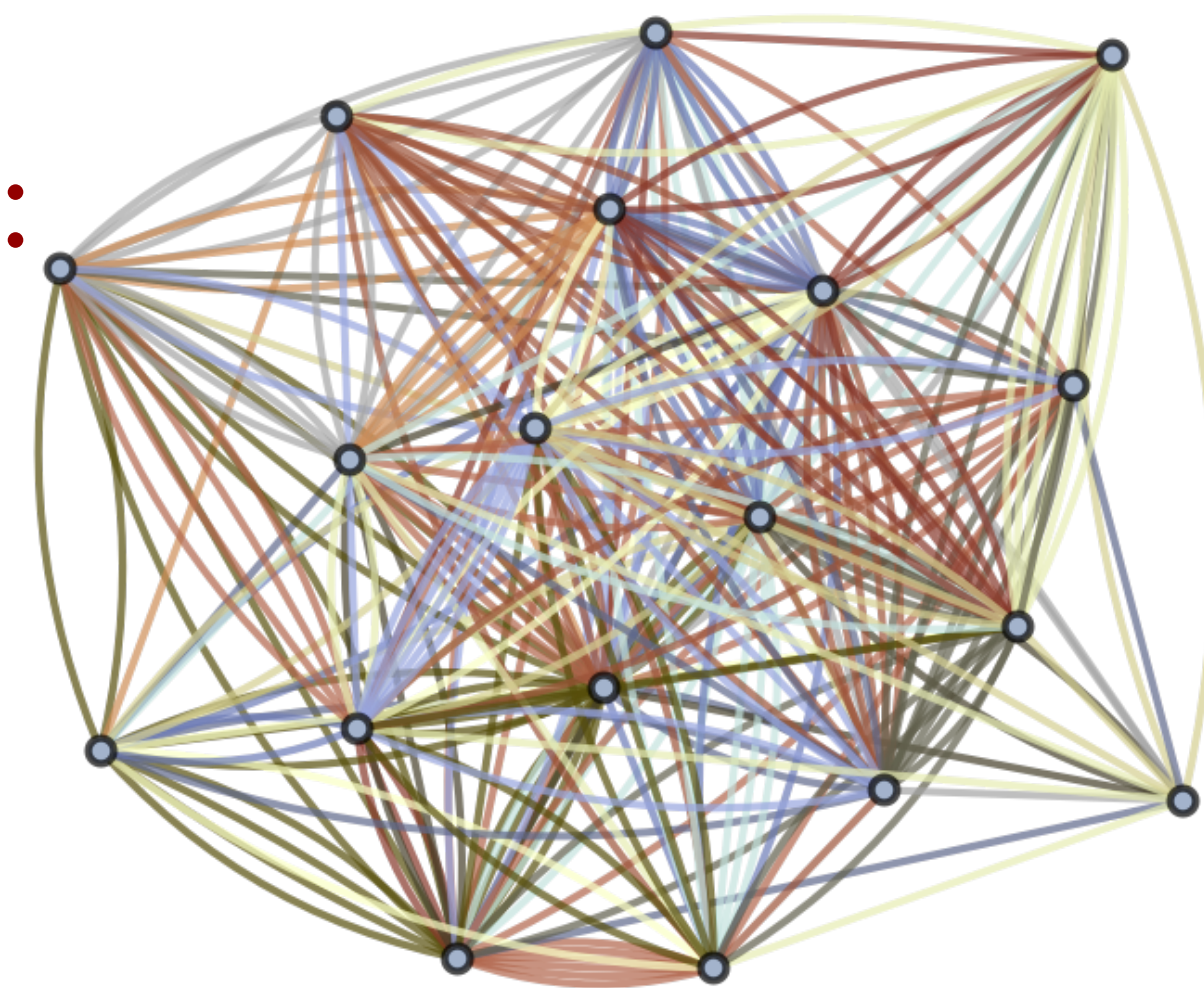
The Sachdev-Ye-Kitaev (SYK) model

The disorder-averaged partition is exactly this G - Σ theory:

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NI)$$

$$I = \ln \det [\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)]$$

$$+ \int d\tau_1 d\tau_2 [\Sigma(\tau_1, \tau_2)G(\tau_2, \tau_1) + (U^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)]$$



Saddle-point equations for $G(\tau_1 - \tau_2)$ and $\Sigma(\tau_1 - \tau_2)$:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau)G(-\tau)$$

$$G(\tau = 0^-) = Q.$$

G - Σ
path
integral

At frequencies $\ll U$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} e^{-i\phi(\sigma_1)+i\phi(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} e^{-i\phi(\sigma_1)+i\phi(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $\phi(\sigma)$ are arbitrary functions.

G - Σ
path
integral

Reparametrization and phase zero modes

We find the conformal saddle point, G_* , Σ_* , and only focus on the “Nambu-Goldstone” modes associated with breaking reparameterization and $U(1)$ gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_*(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-E_0/T + N s_0 - N I_{\text{eff}}[f, \phi]},$$

where $E_0 \propto N$ is the ground state energy.

$$\begin{aligned}
\mathcal{Z} &= \text{Tr} \exp \left(-\frac{\mathcal{H}}{k_B T} \right) \\
&\approx \exp \left(N \frac{s_0}{k_B} \right) \int \frac{\mathcal{D}f(\tau) \mathcal{D}\phi(\tau)}{||\text{SL}(2, \mathbb{R})||} \exp \left(-\frac{1}{\hbar} I_{\text{eff}} [f(\tau), \phi(\tau)] \right) \\
I_{\text{eff}} [f, \phi] &= \frac{NK}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T) \partial_\tau f)^2 - \frac{N\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T f(\tau)), \tau \},
\end{aligned}$$

where $f(\tau)$ is a monotonic map from $[0, 1/T]$ to $[0, 1/T]$. The conformal group in d spatial dimensions is $\text{SO}(d+2, 1)$, and $\text{PSL}(2, \mathbb{R}) \cong \text{SO}(2, 1)$, and the Schwarzian

$$\{g, \tau\} \equiv \frac{g'''}{g'} - \frac{3}{2} \left(\frac{g''}{g'} \right)^2,$$

vanishes for $g(\tau) = (a\tau + b)/(c\tau + d)$ a $\text{SL}(2, \mathbb{R})$ transformation ($ad - bc = 1$). The couplings K , γ , and \mathcal{E} can be related to thermodynamic derivatives.

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Conformal perturbations of saddle-point theory:

$$S = N(s_0 + \gamma T) \quad \Rightarrow \quad D(E) \sim \exp\left(N s_0 + \sqrt{2N\gamma E}\right)$$

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Exact path integral over Schwarzian quantum gravity theory:

$$S = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{U}{T}\right) - \frac{\ln N}{2} \quad \Rightarrow \quad D(E) \sim N^{-1} \exp(N s_0) \sinh(\sqrt{2N\gamma E})$$

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J. S. Cotler et al.,
JHEP 05 (2017) 118

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Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, JHEP 02 (2020) 157

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

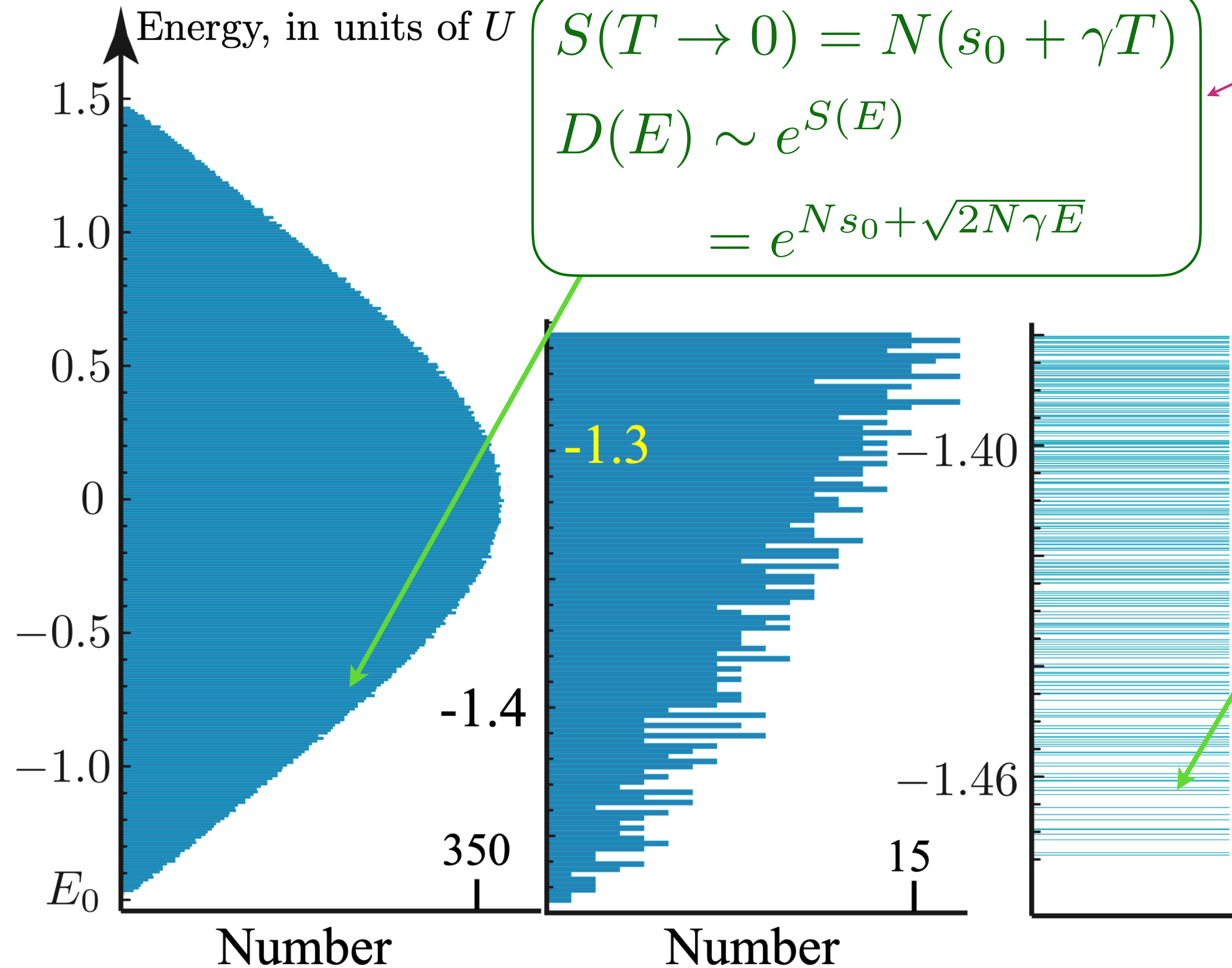
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Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$S(T \rightarrow 0) = N(s_0 + \gamma T)$$

$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

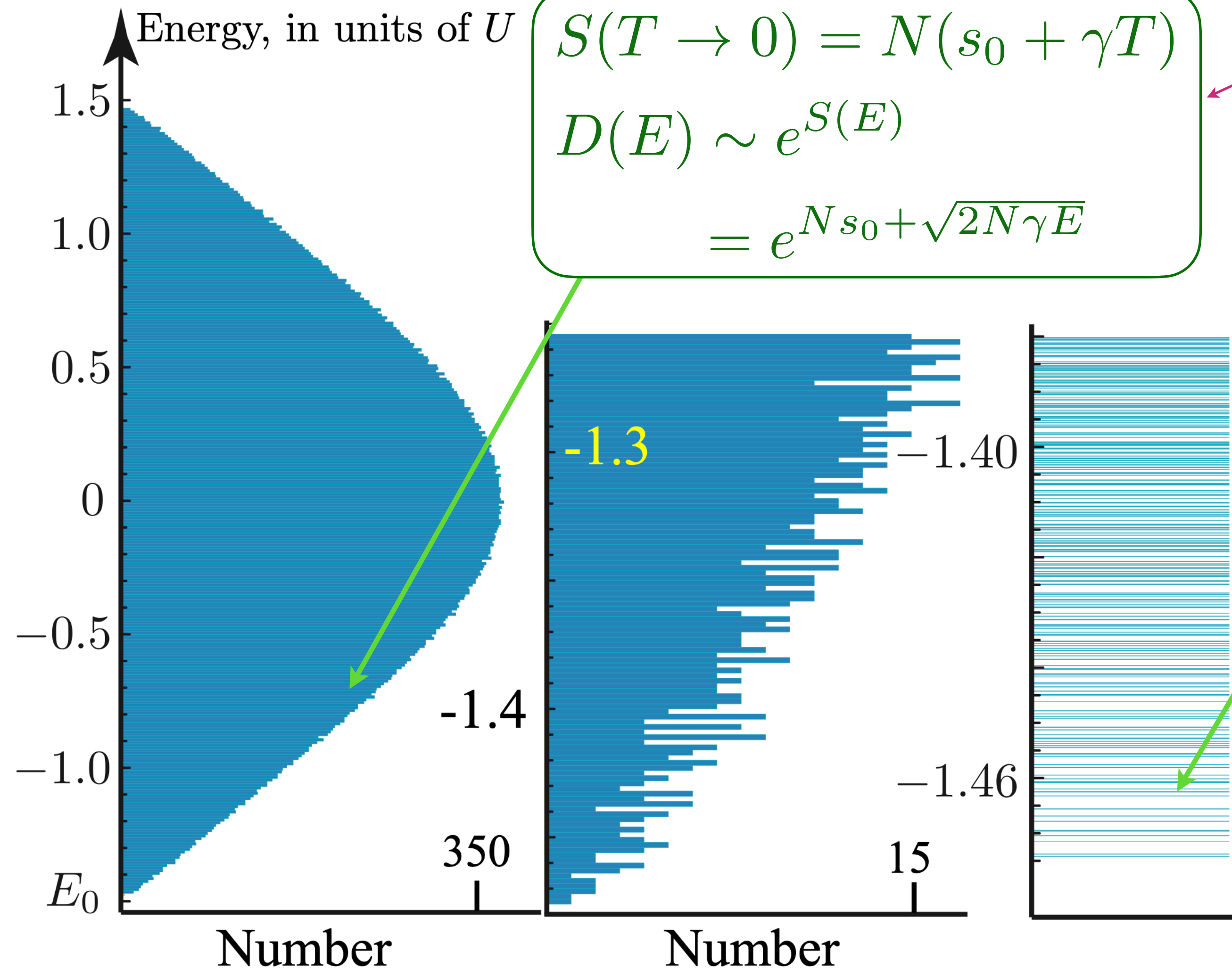
$$D(E) \sim N^{-1} e^{N s_0} \sinh(\sqrt{2N\gamma E})$$

$$D(E) \sim e^{N s_0} \sqrt{2\gamma E / N}$$

Complex SYK model

Many-body density of states

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$$S(T \rightarrow 0) = N(s_0 + \gamma T)$$

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$$= e^{N s_0 + \sqrt{2N\gamma E}}$$

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$$D(E) \sim e^{N s_0} \sqrt{2\gamma E / N}$$

No exponentially large degeneracy, but exponentially small level spacing!
 No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

Complex SYK model

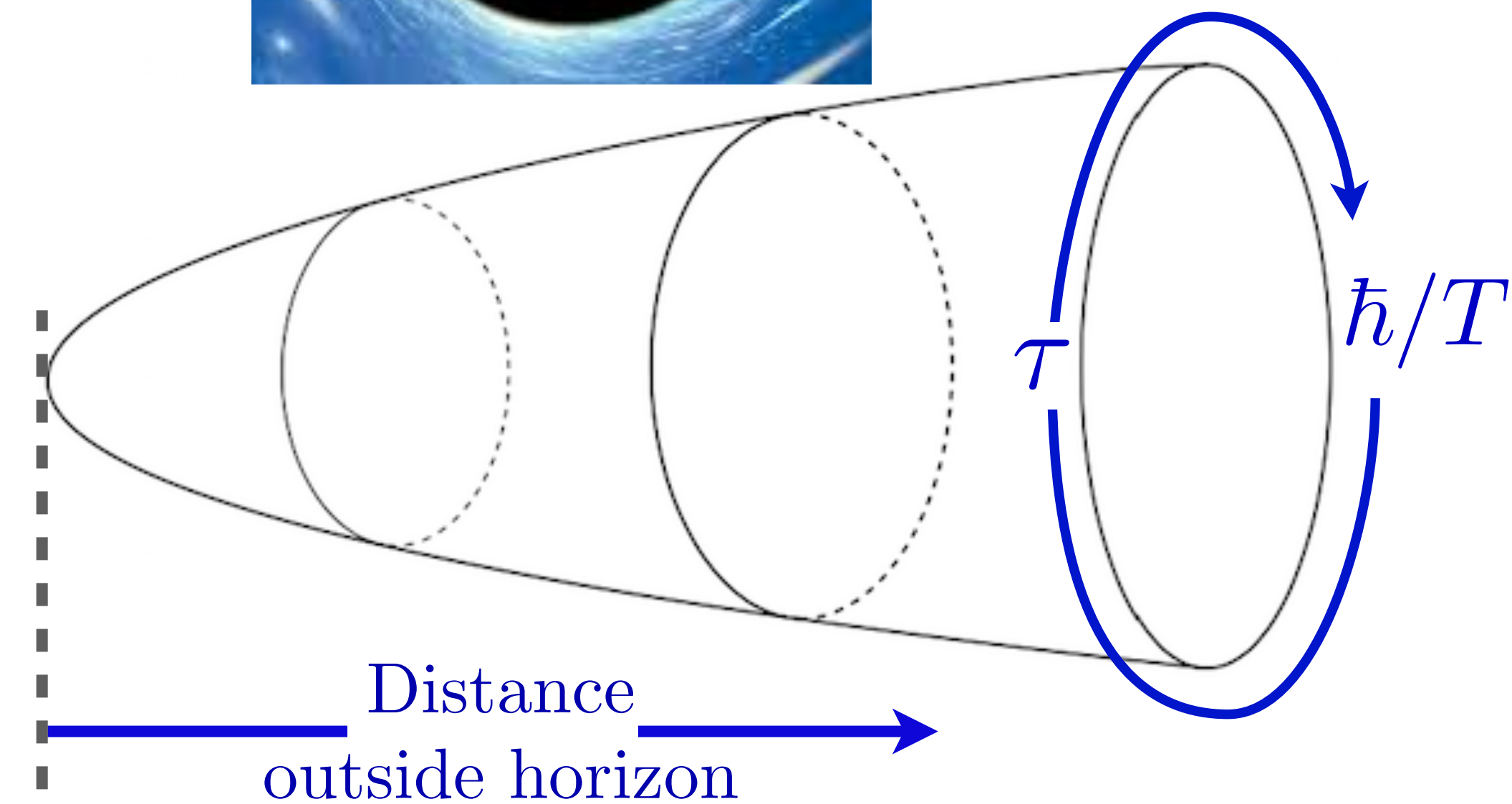
Charged black holes

S.S., arXiv:2205.02285

Thermodynamics of quantum black holes with charge Q :



$$\mathcal{Z}(Q, T) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right)$$



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$$= \exp(S_{BH}) \times \left(\dots????\dots \right)$$

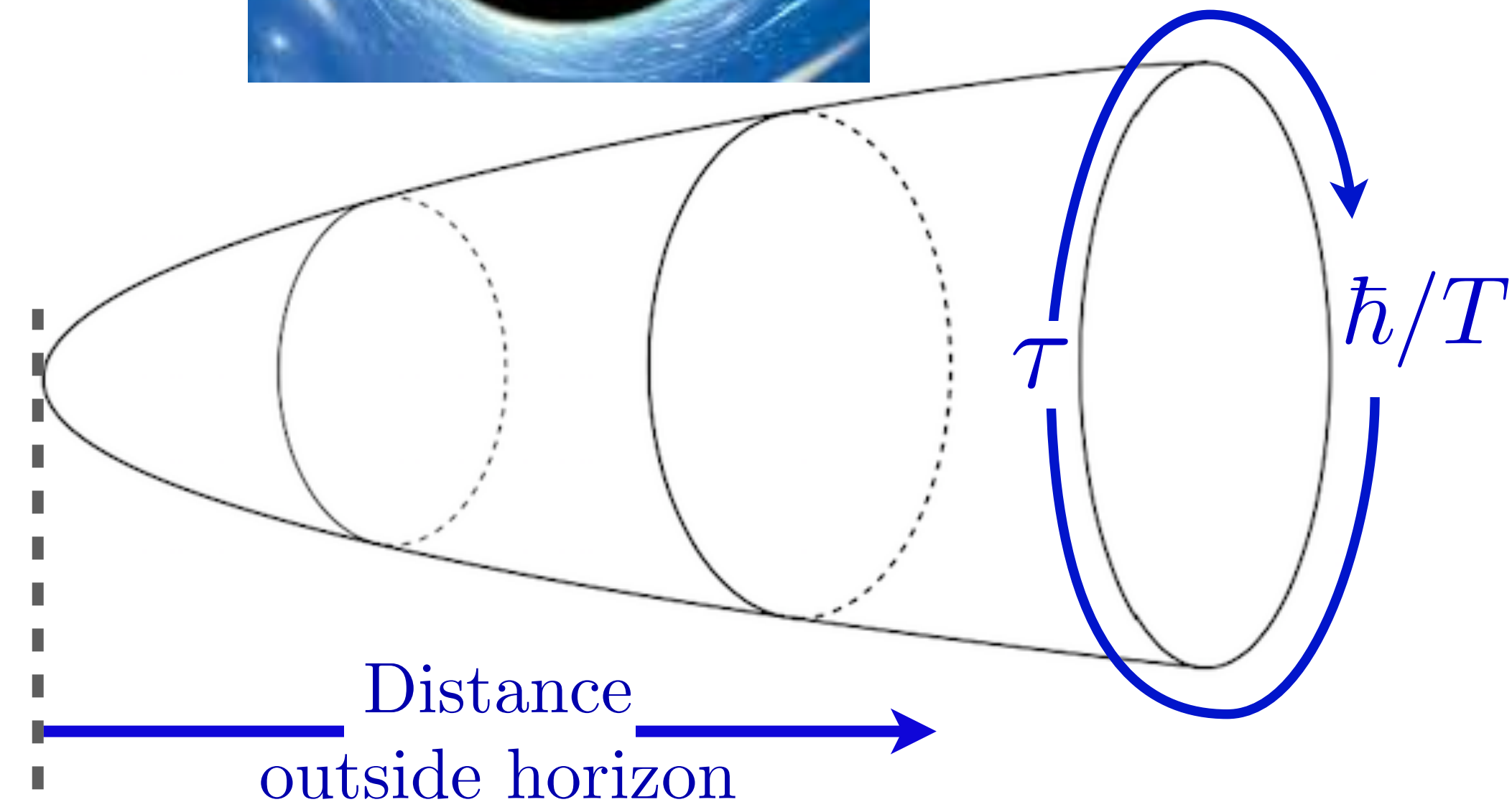
Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)



$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c} \right)$$

$A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



Thermodynamics of quantum black holes with charge Q :



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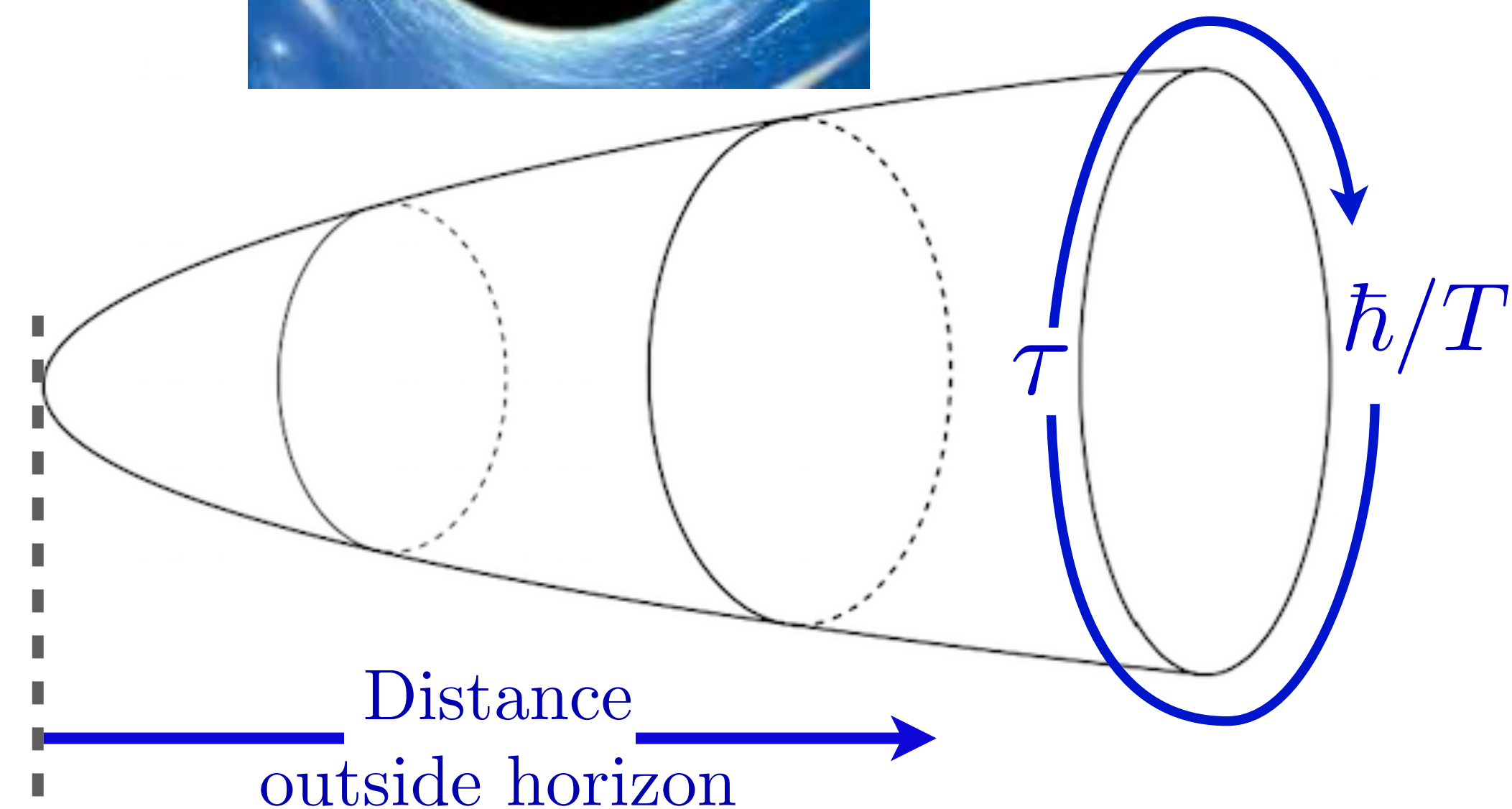


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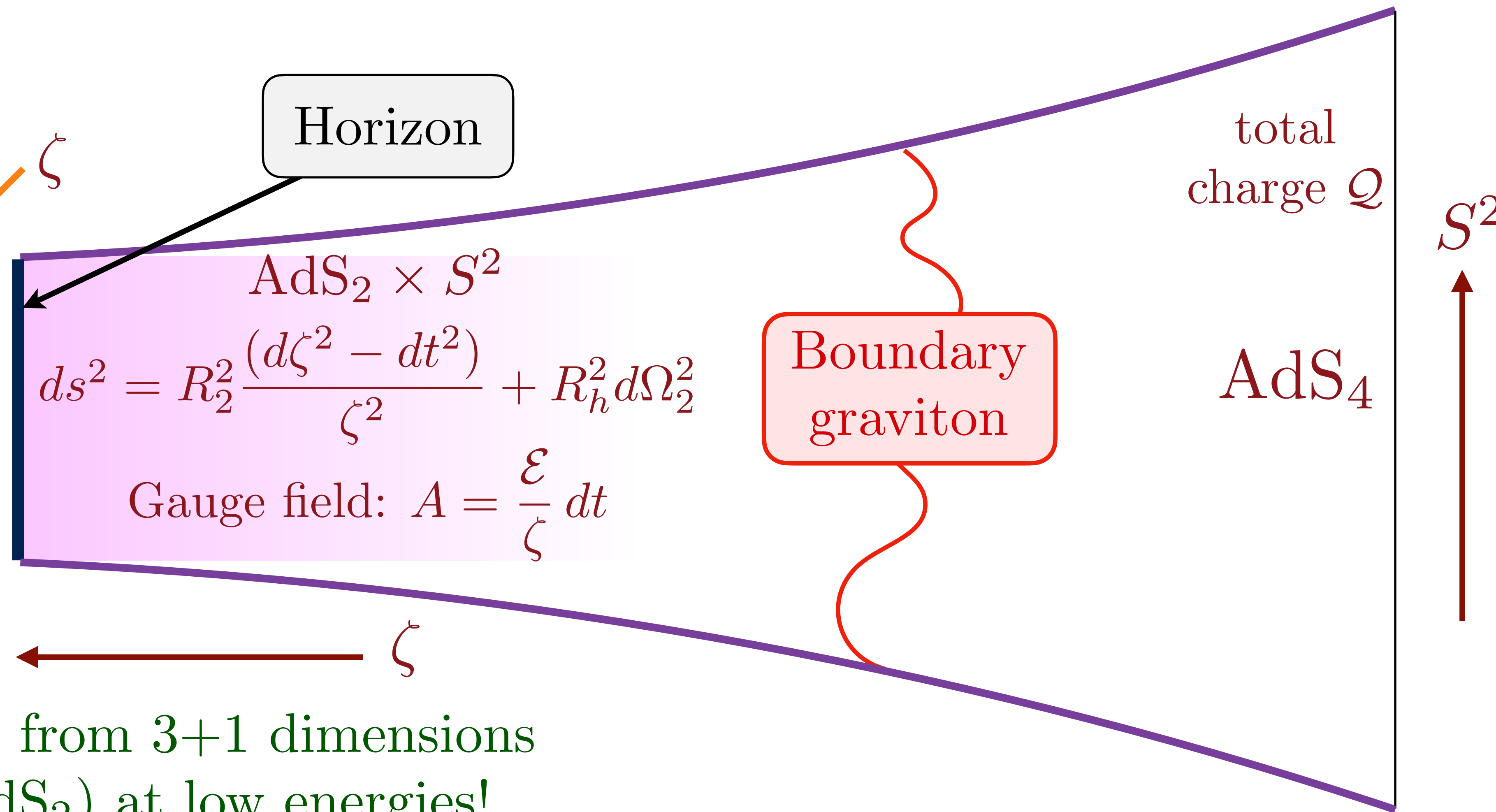
$A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

Note the similarity to the large N entropy of the SYK model!
(along with other similarities)

Sachdev PRL 2010



Reissner-Nordstrom black hole of Einstein-Maxwell theory



Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

The isometry group of AdS_2 is the 0+1 dimensional conformal group $SL(2, \mathbb{R})$.

Thermodynamics of quantum black holes with charge Q :



$$\mathcal{Z}(Q, T) = \int \mathcal{D}g_{\mu\nu} \mathcal{D}A_{\mu} \exp \left(-\frac{1}{\hbar} I_{\text{Einstein gravity+Maxwell EM}}^{(3+1)}[g_{\mu\nu}, A_{\mu}] \right)$$

Saddle-point:

$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c} \right)$$

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$$S_{BH}(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar} \left(1 + \frac{2(\pi A_0)^{1/2} T}{\hbar c} \right)$$

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$$S(T \rightarrow 0, Q) = \frac{A(T)c^3}{4G\hbar} - \frac{3}{4} \ln \left(\frac{\hbar c^5}{GT^2} \right)$$

The $\ln T$ term is the SYK/boundary-graviton correction to Bekenstein-Hawking.

There is also a
 $-\frac{559}{180} \ln \left(\frac{A_0 c^3}{\hbar G} \right)$
 term from other massless
 modes; Sen (2011)
 Iliesiu, Murthy, Turiaci (2022)

Black hole questions and answers

Can we find a quantum simulation of the inside of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy computed outside the black hole?

Black hole questions and answers

Can we find a quantum simulation of the inside of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy computed outside the black hole?

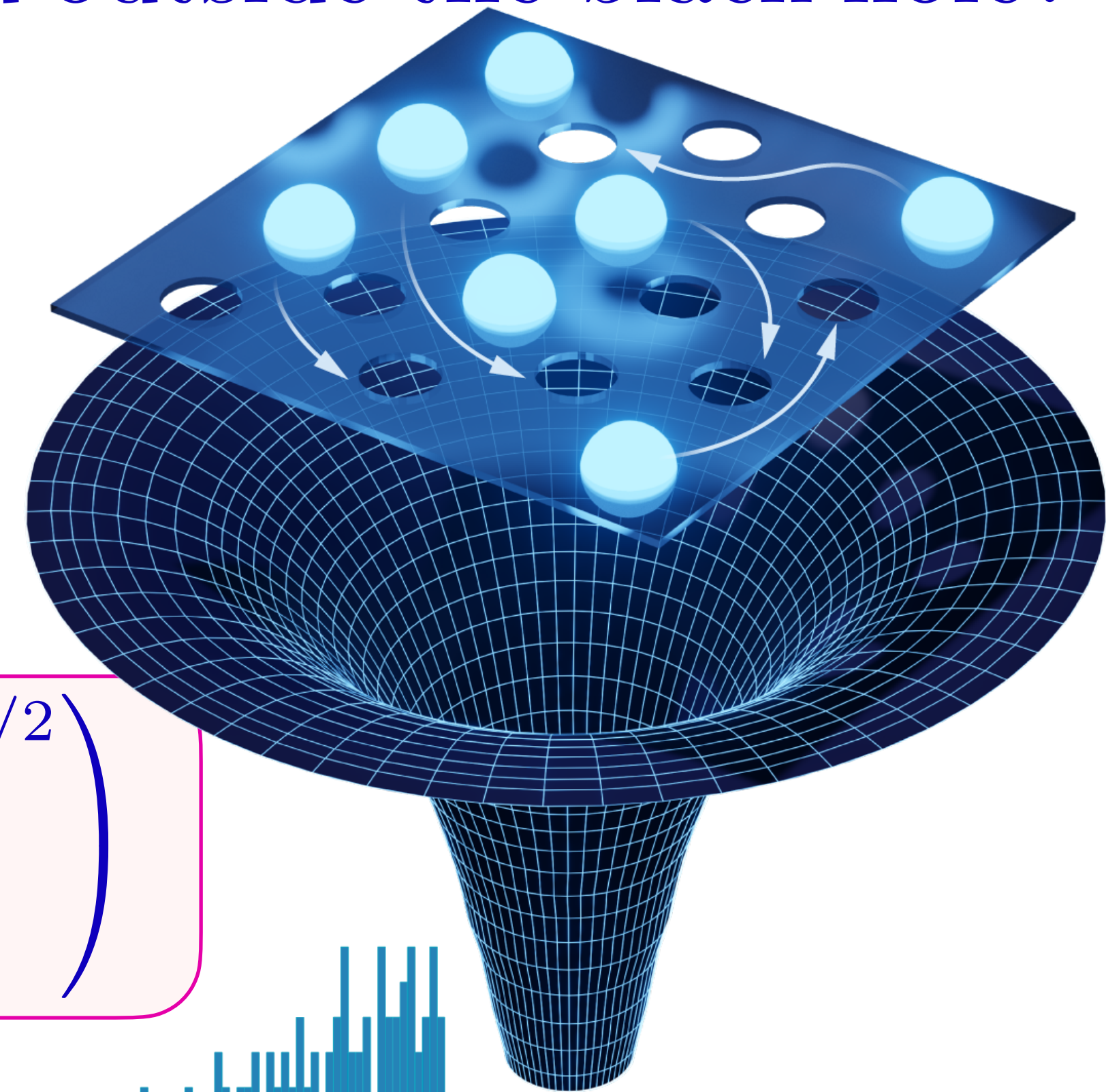
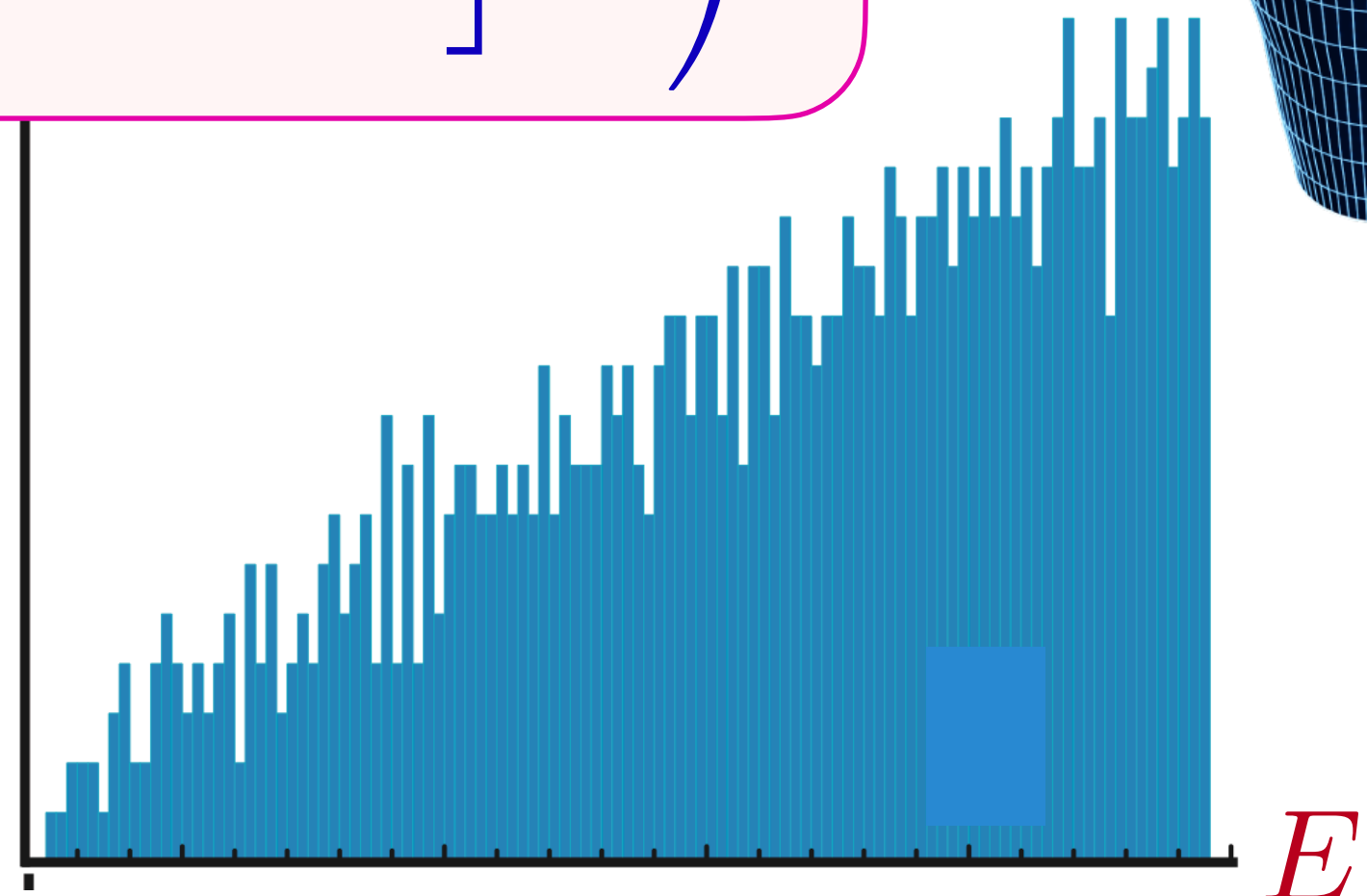
Yes, for charged black holes:

- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G} \right)^{-347/90} \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.

$D(E)$



Black hole questions and answers

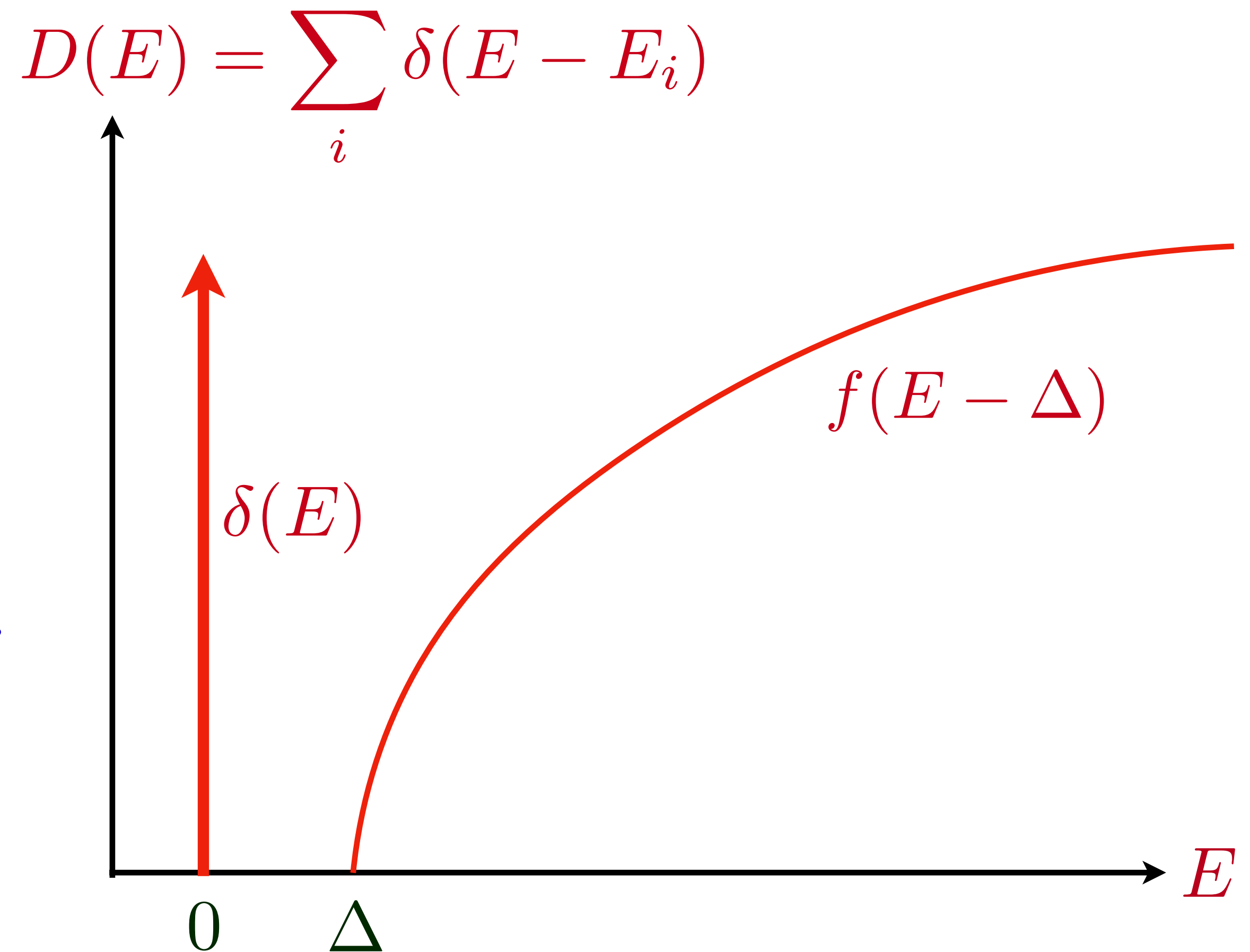
Can we find a quantum simulation of the inside of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy computed outside the black hole?

Yes, for charged black holes:

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \delta(E) + \theta(E - \Delta) f(E - \Delta) + \dots$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.



M. Heydemann, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2020

L.V. Iliesiu, S. Murthy, G. J. Turiaci, 2022

Black hole questions and answers

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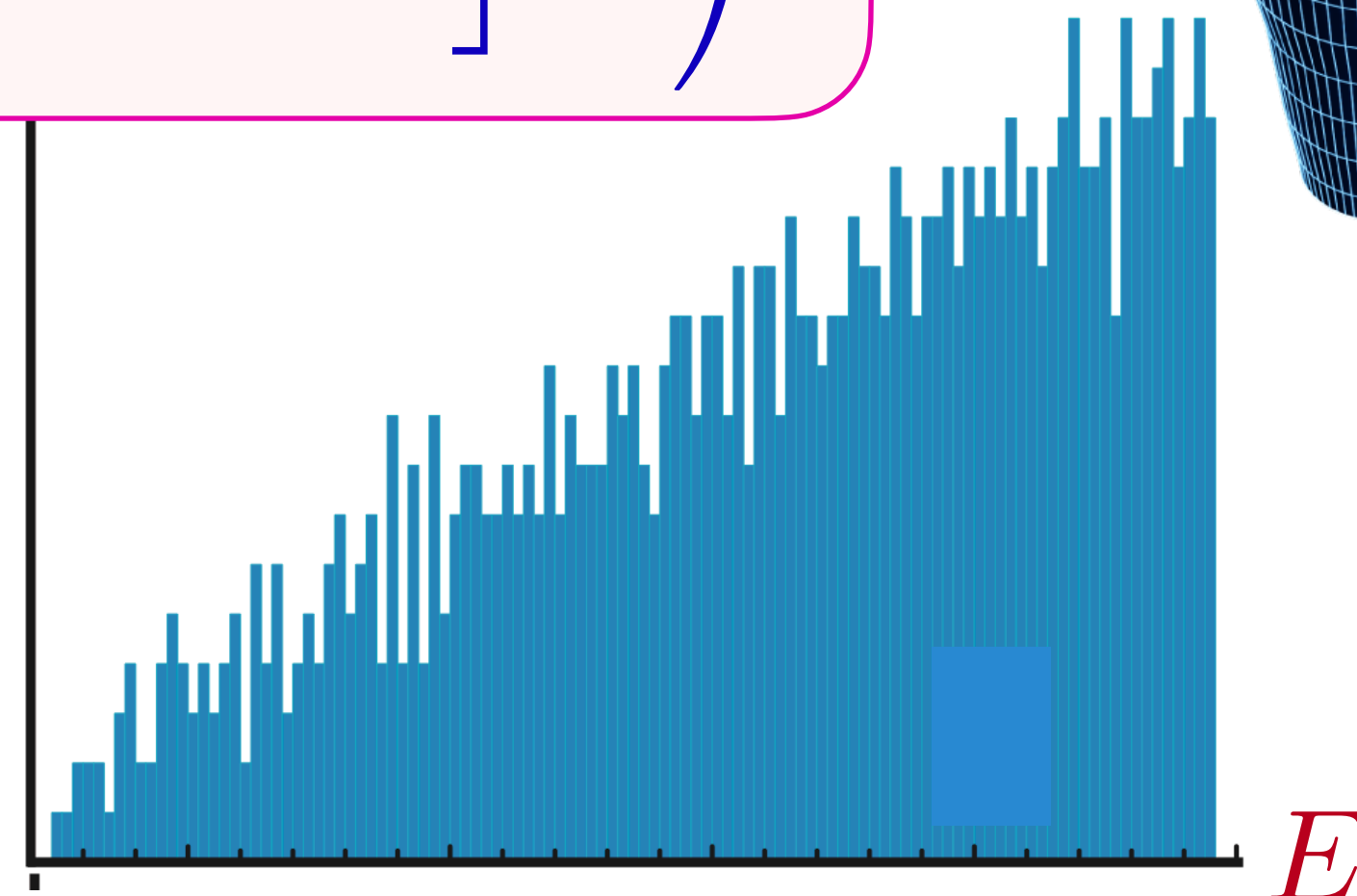
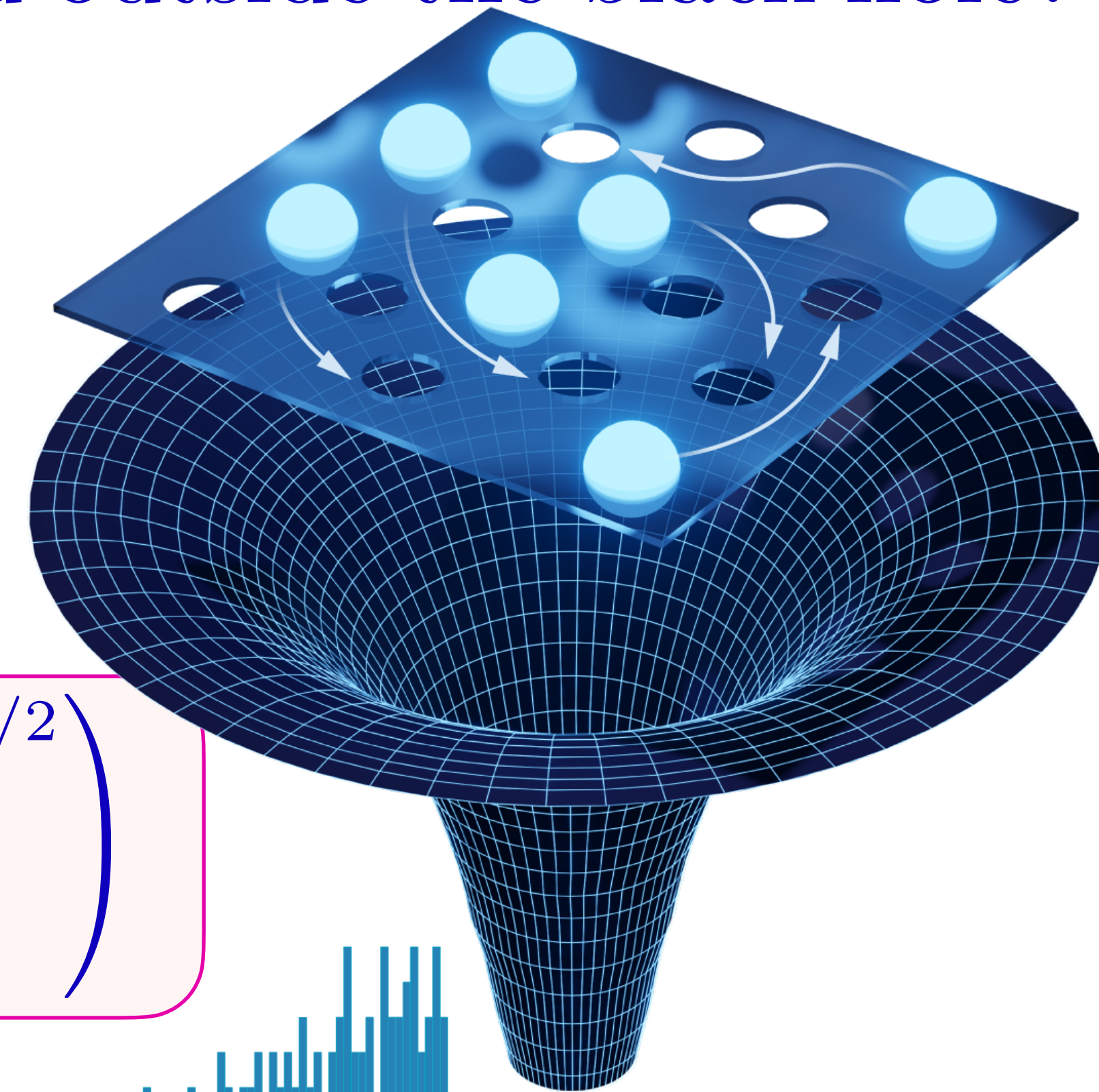
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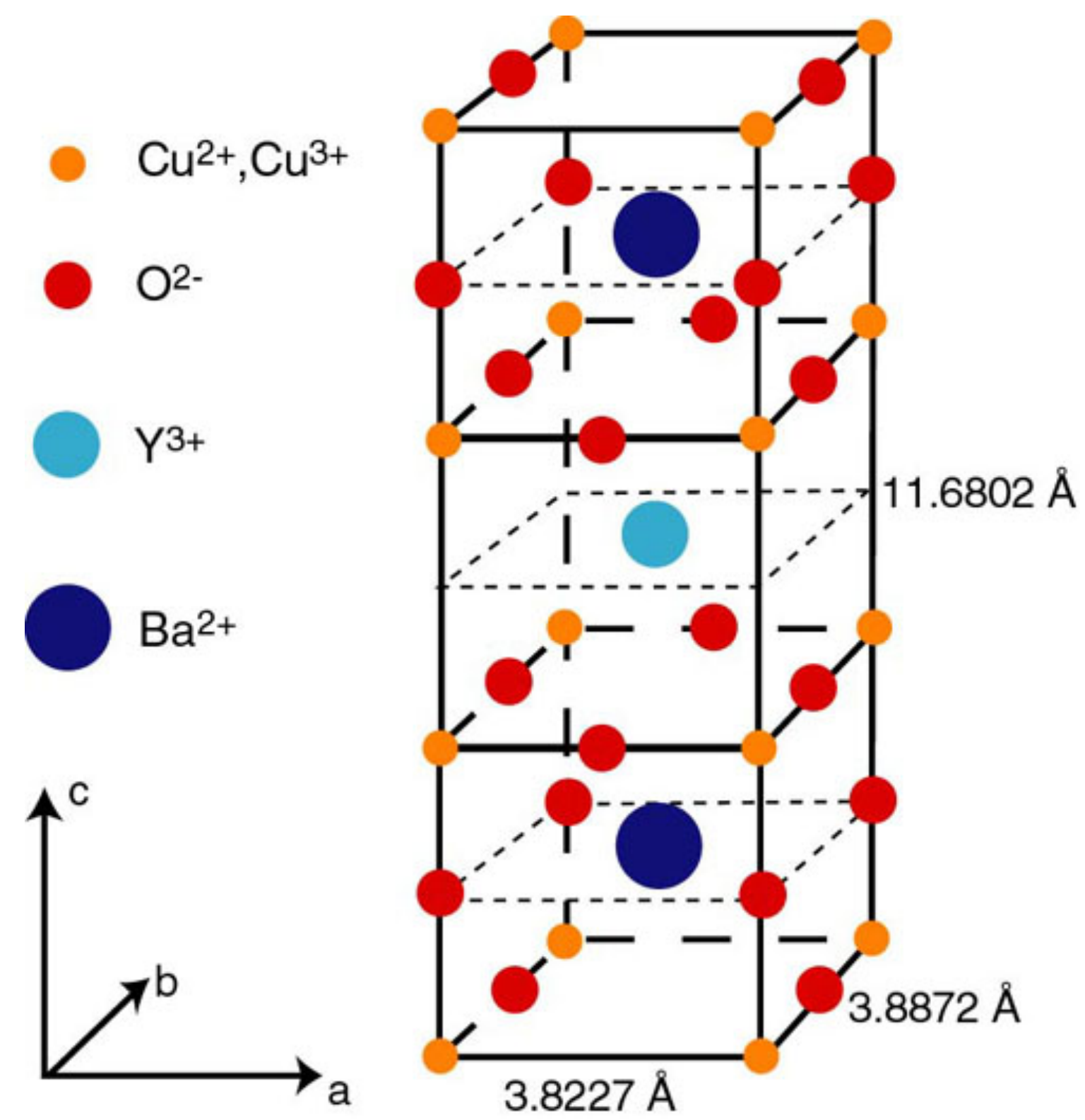
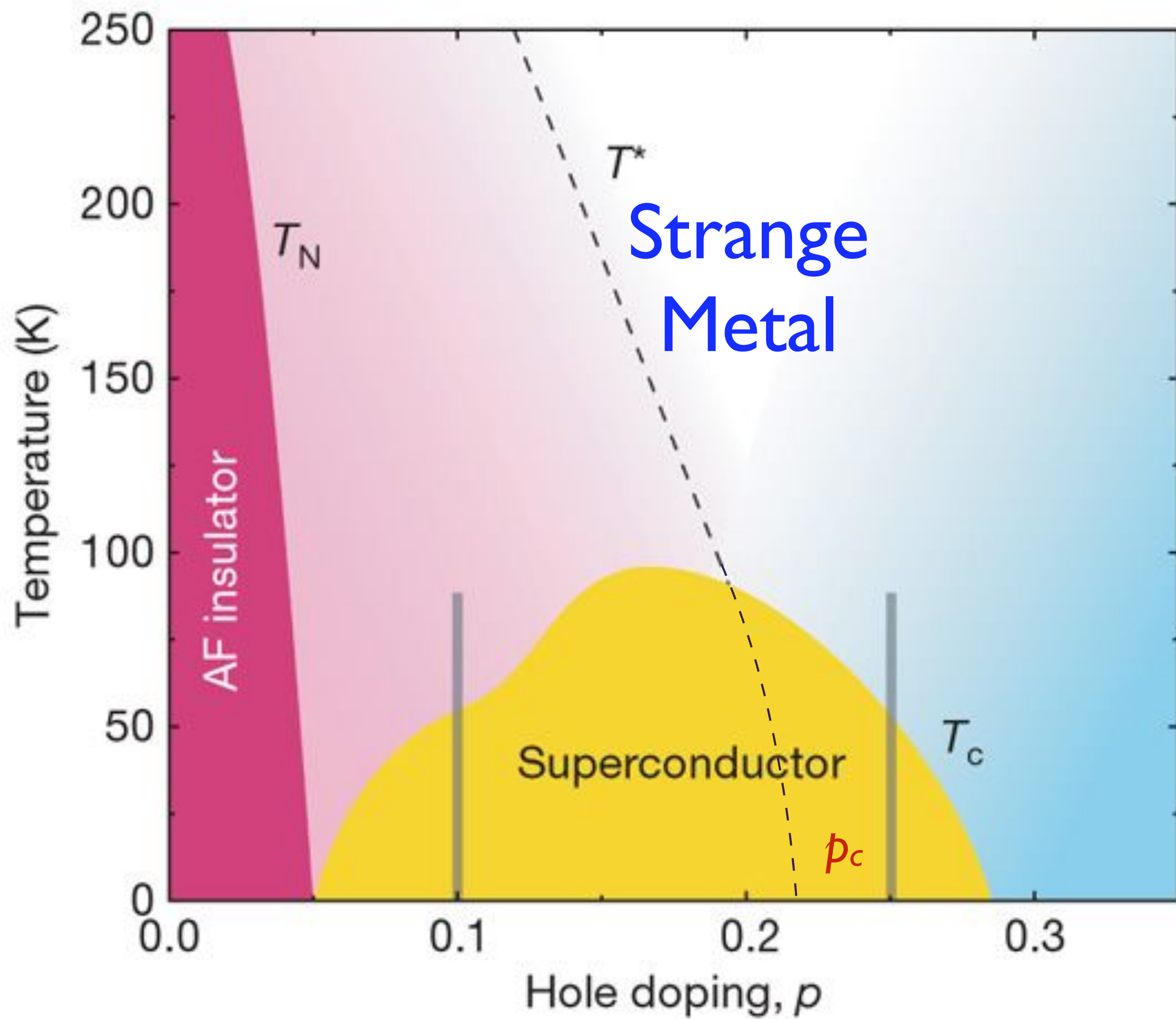
$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G} \right)^{-347/90} \exp \left(\frac{A_0 c^3}{4\hbar G} \right) \sinh \left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

- ‘Wormhole’ contributions to this quantum simulation have led to an understanding of the Page curve of entanglement entropy of evaporating black holes.

Saad, Shenker, Stanford (2019)



Strange metals



Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \dots$ as $T \rightarrow 0$
and $\rho(T) < h/e^2$ (in $d = 2$).
Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \rightarrow 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\frac{1}{\tau_{\text{trans}}(\omega)} - i\omega \frac{m_{\text{trans}}^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_{\sigma} \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon.....A. Georges, arXiv:2205.04030

4. Photoemission: nearly “marginal Fermi liquid” electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_{\Sigma} \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2$$

T.J. Reber....D. Dessau,
Nature Communications **10**, 5737 (2019)

From
Yukawa-SYK models
to a universal theory of
strange metals



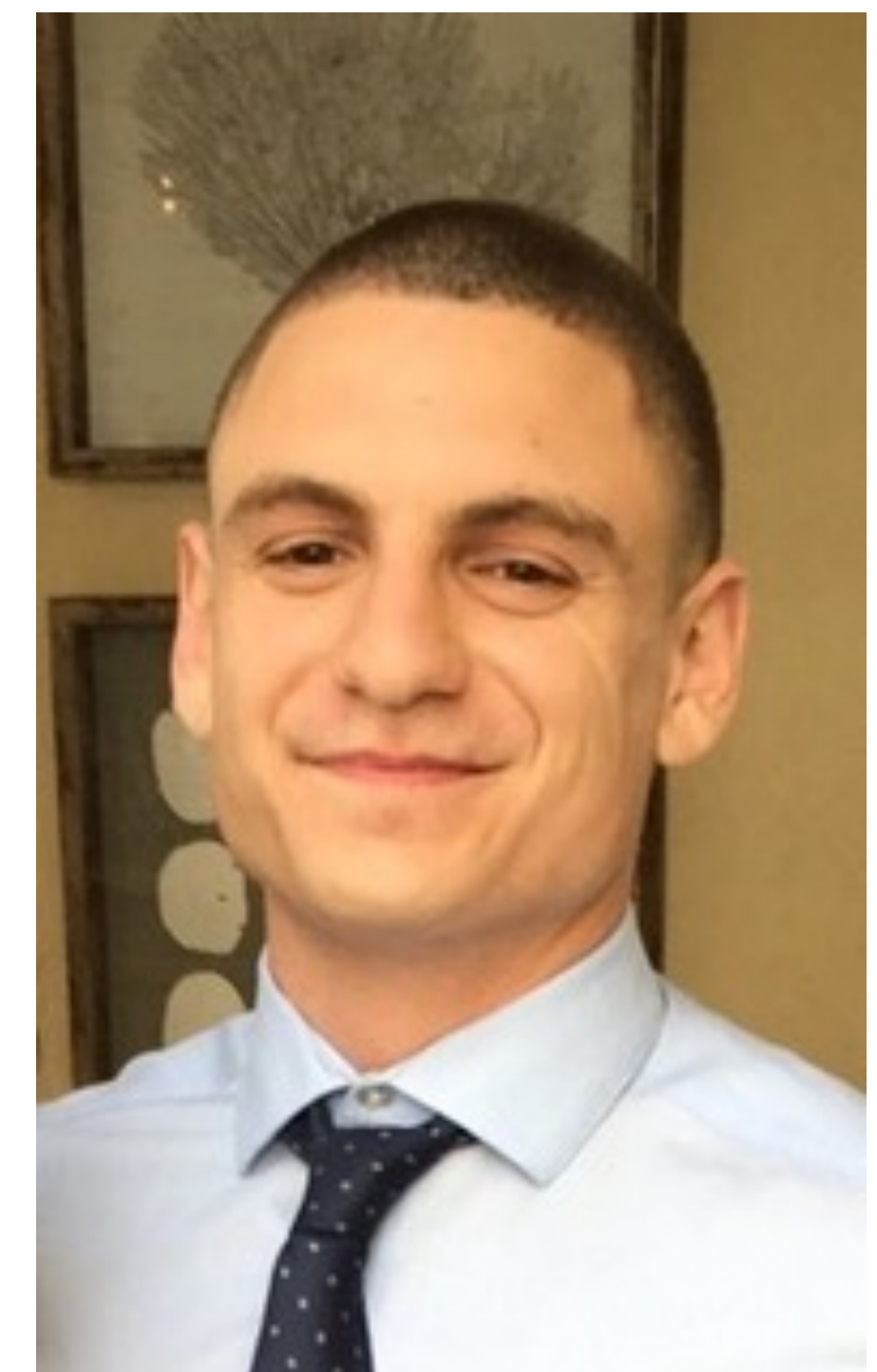
Aavishkar Patel

Flatiron Institute, NYC



Haoyu Guo

Harvard



Ilya Esterlis

Harvard → Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell,$$

$g_{ij\ell}$ independent random numbers with zero mean. Large N limit leads to Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)

A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)

E. Marcus and S. Vandoren, JHEP 01, 166 (2018)

Yuxuan Wang, PRL **124**, 017002 (2020)

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)

E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763

Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)

W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)

I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

The disorder-averaged partition function is a G - Σ - D - Π theory:

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}G \mathcal{D}\Sigma \mathcal{D}D \mathcal{D}\Pi \exp(-N S_{\text{all}}) \\ S_{\text{all}} &= -\ln \det(\partial_\tau - \mu + \Sigma) + \frac{1}{2} \ln \det(-\partial_\tau^2 + \omega_0^2 - \Pi) \\ &+ \int d\tau \int d\tau' \left[-\Sigma(\tau'; \tau) G(\tau, \tau') + \frac{1}{2} \Pi(\tau' \tau) D(\tau, \tau') \right. \\ &\quad \left. + \frac{g^2}{2} G(\tau, \tau') G(\tau', \tau) D(\tau, \tau') \right]. \end{aligned}$$

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi + \sum_\ell \frac{1}{2} (\pi_\ell^2 + \omega_0^2 \phi_\ell^2) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$
$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

$$G(i\omega) \sim -i \operatorname{sgn}(\omega) |\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

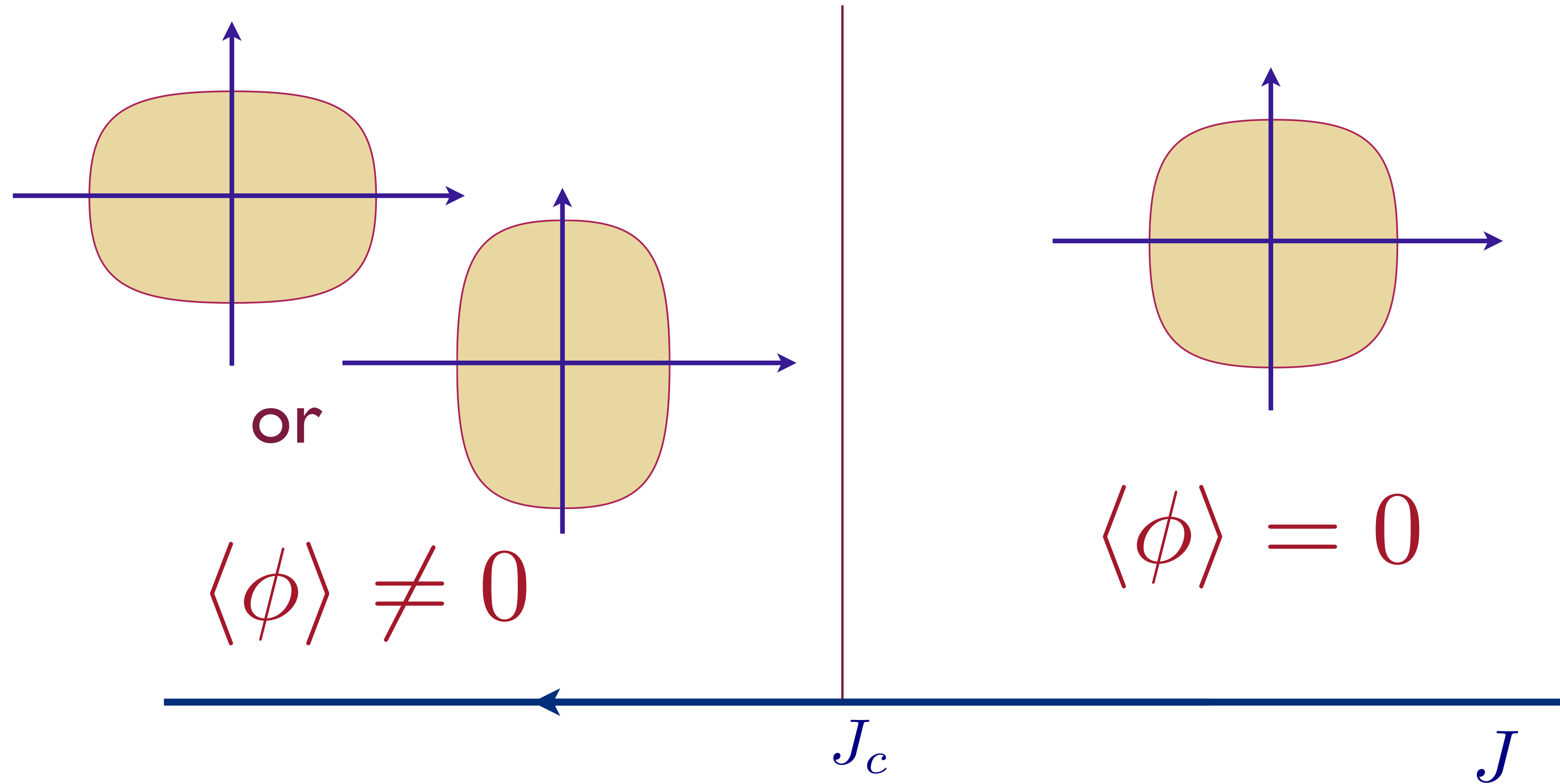
A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037 \dots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)

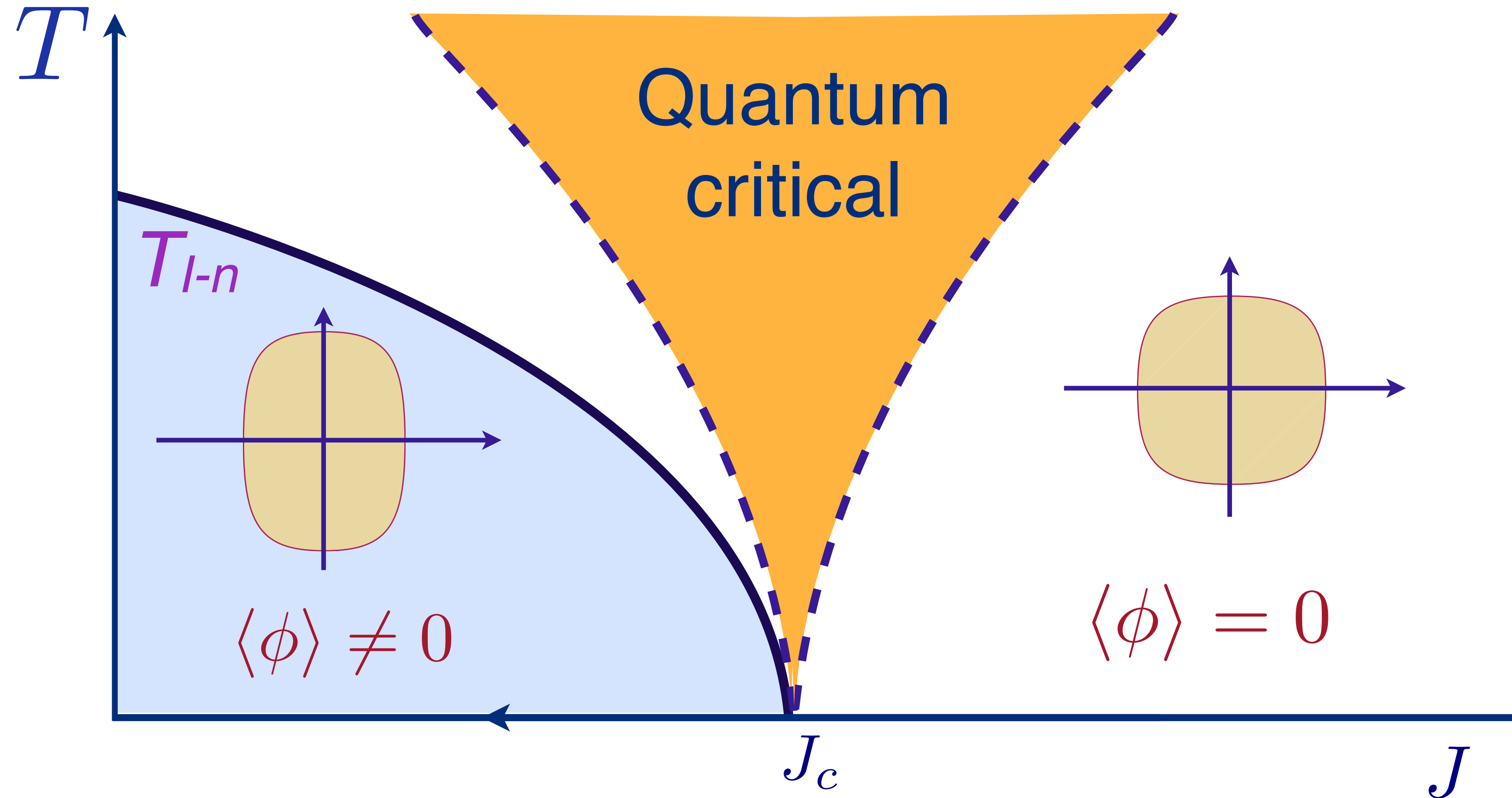
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Quantum criticality of Ising-nematic ordering in a metal



Pomeranchuk instability as a function of coupling J

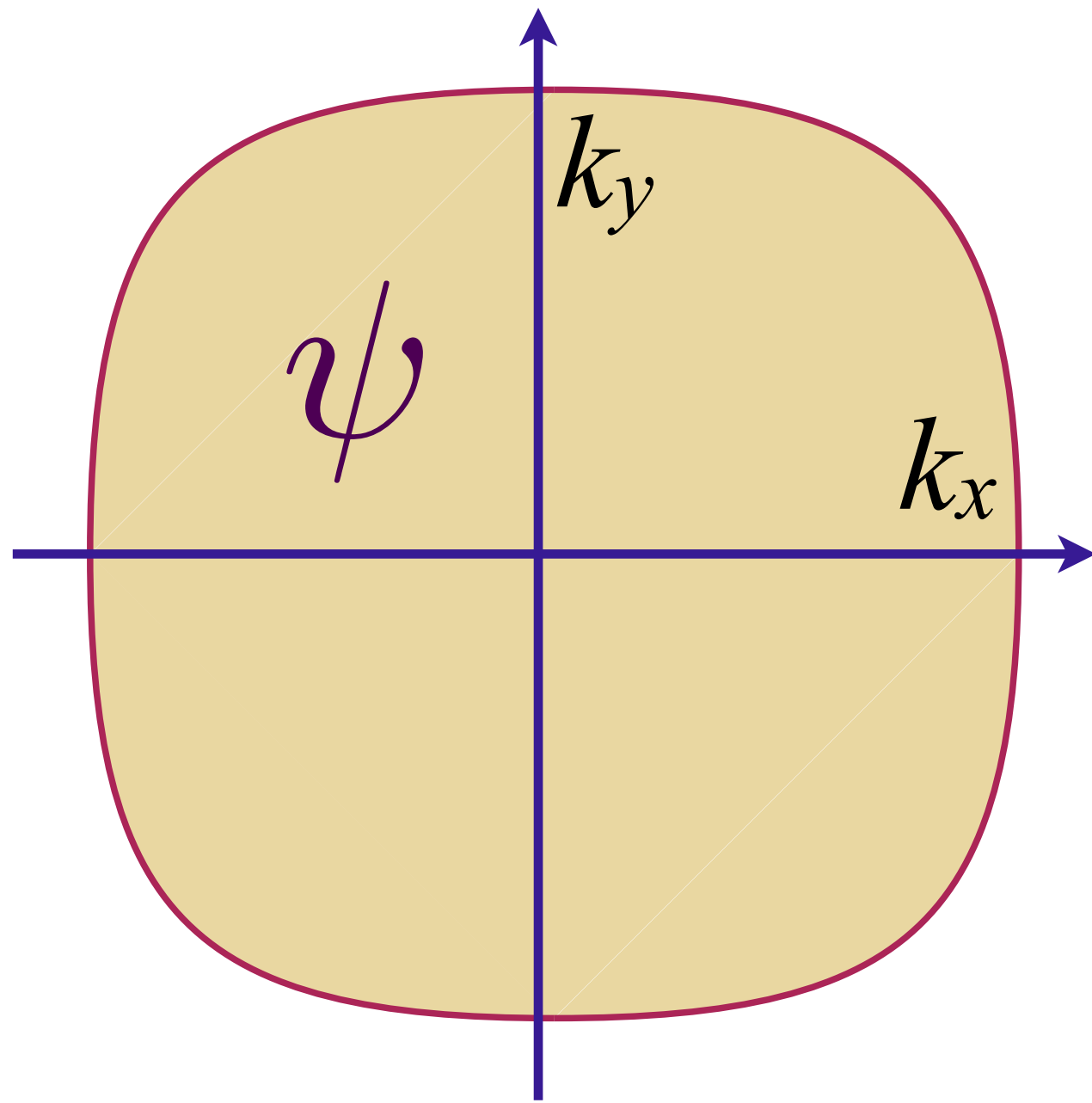
Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and J

Fermi surface

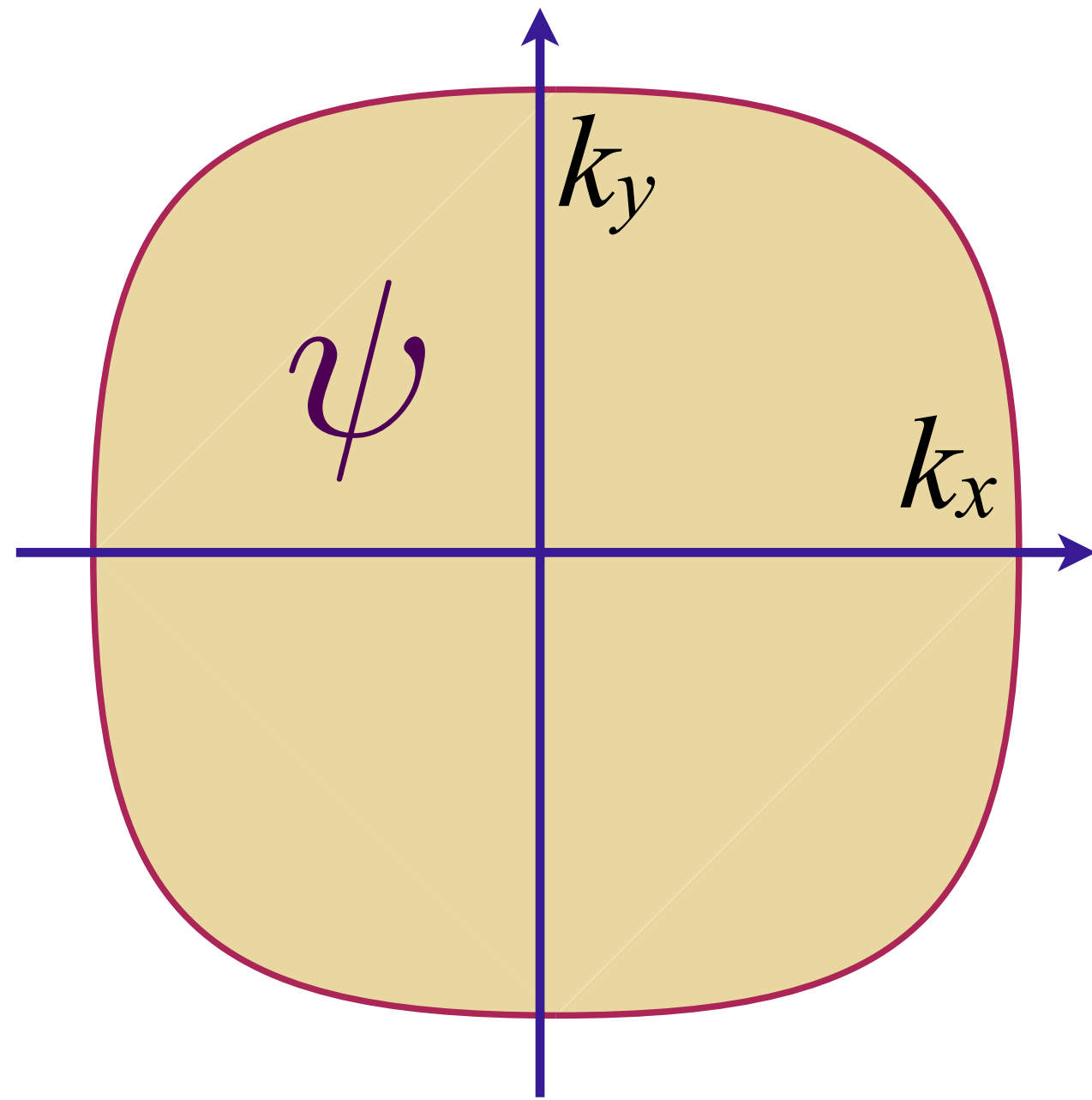
$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



$$-J \psi^\dagger(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \psi(\mathbf{r})$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



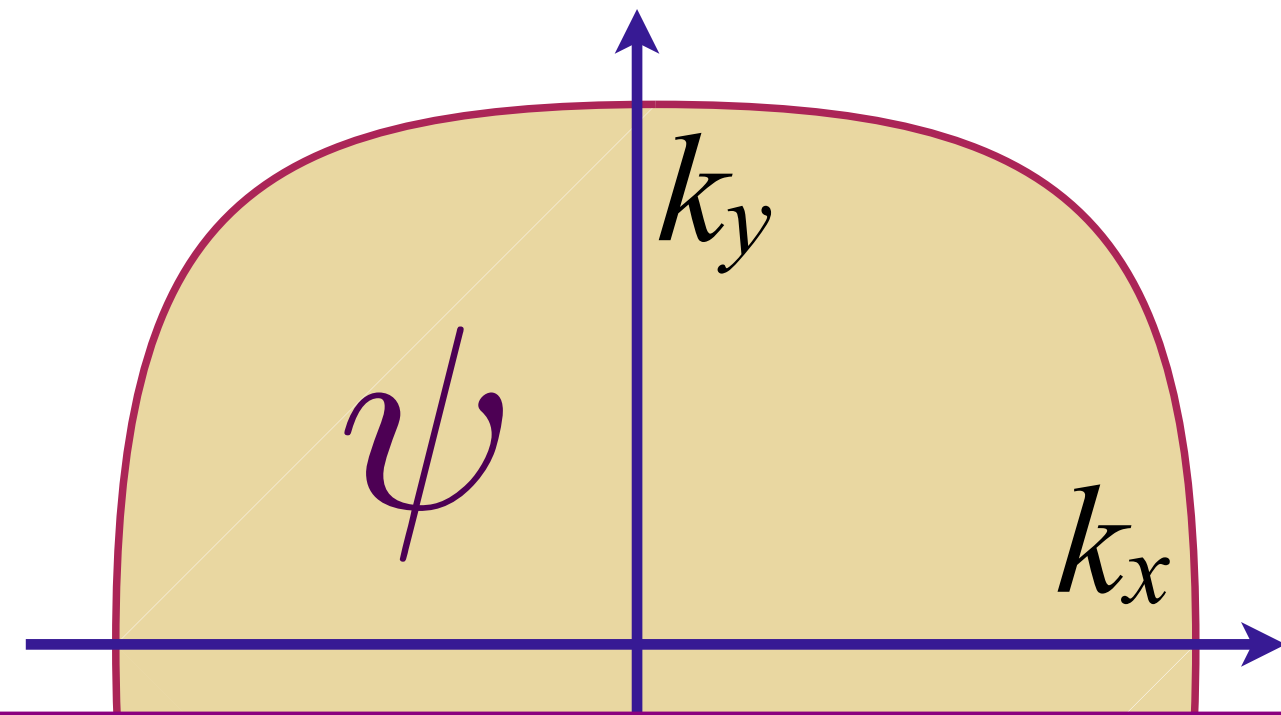
a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

Fermi surface coupled to a critical boson

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$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

Solve in a large N limit with Yukawa coupling

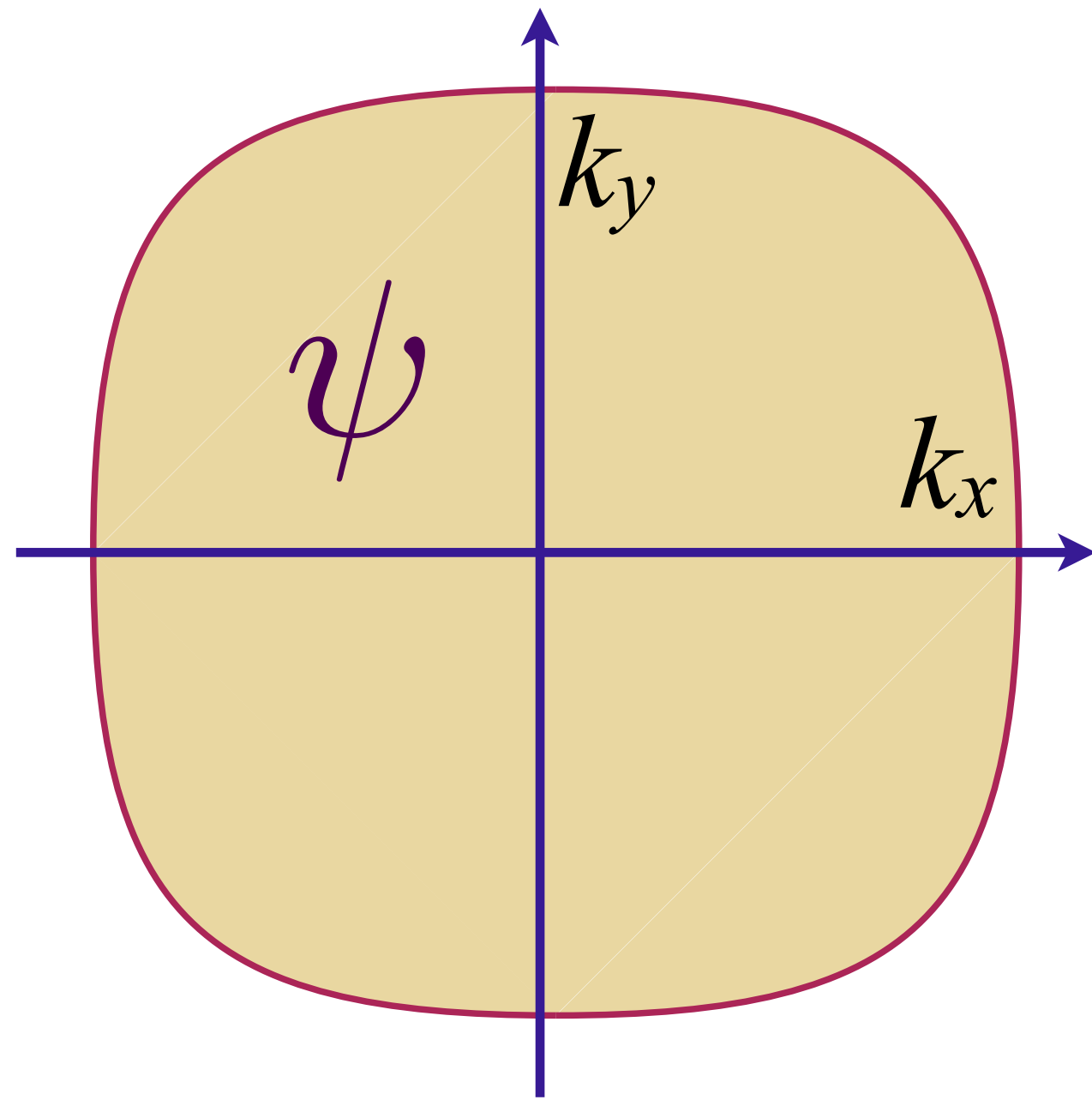
$$\frac{g_{ijl}}{N} \int d^2r d\tau \psi_i^\dagger(\mathbf{r}, \tau) \psi_j(\mathbf{r}, \tau) \phi_l(\mathbf{r}, \tau) \quad , \quad \overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

to obtain Eliashberg solution for electron (G) and boson (D) Green's functions at small ω :

$$\Sigma(\hat{\mathbf{k}}, i\omega) \sim -i \text{sgn}(\omega) |\omega|^{2/3}, \quad G(\mathbf{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) - \Sigma(\hat{\mathbf{k}}, i\omega)}, \quad D(\mathbf{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

$$\text{Re} [\sigma(\omega)] = D\delta(\omega) + \dots$$

a critical boson ϕ
e.g. Ising-nematic order

$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)

D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)

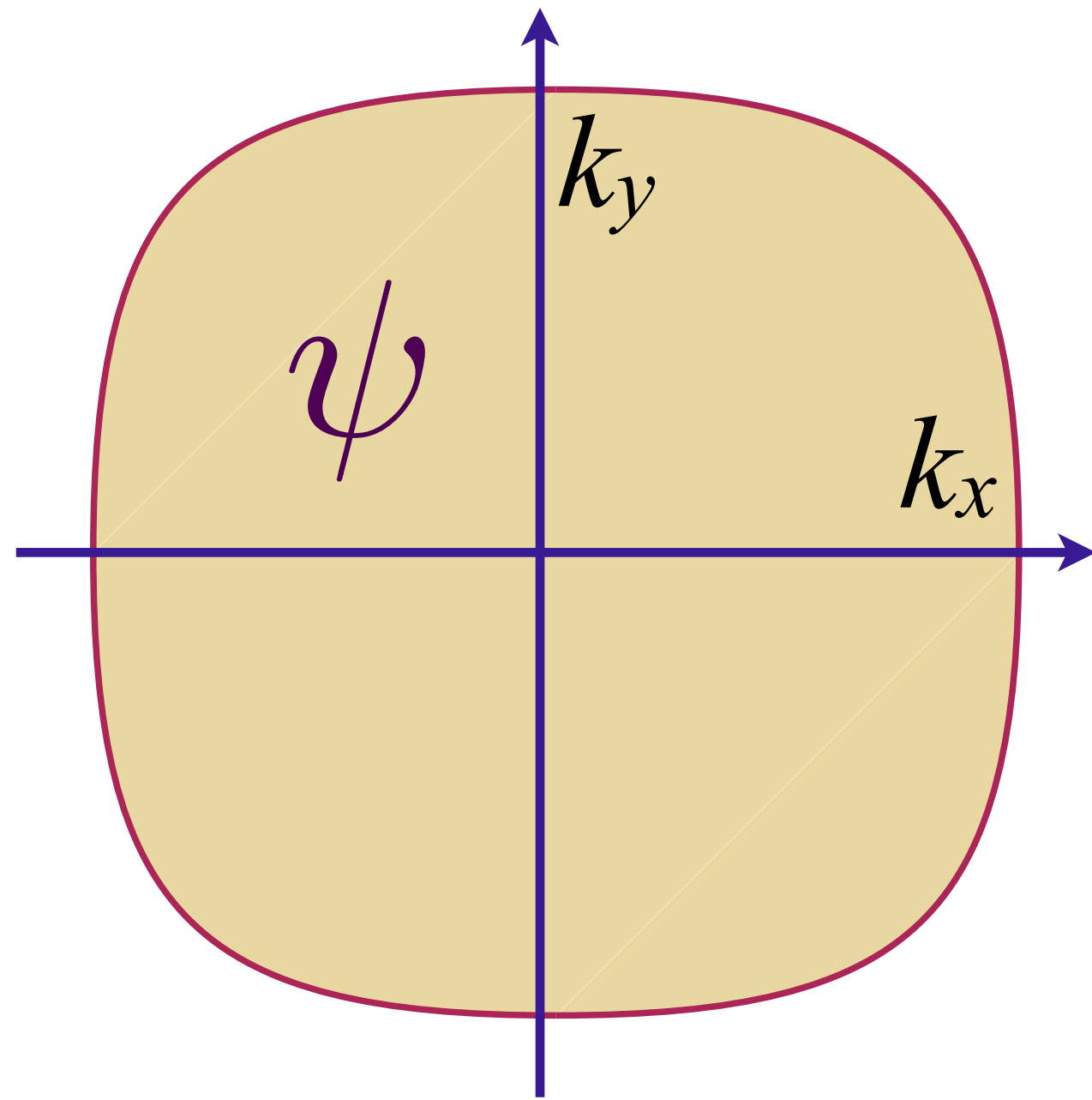
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)

A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r})$$

Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \dots \quad (\omega^{-2/3} \text{ term has vanishing co-efficient})$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)

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Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)

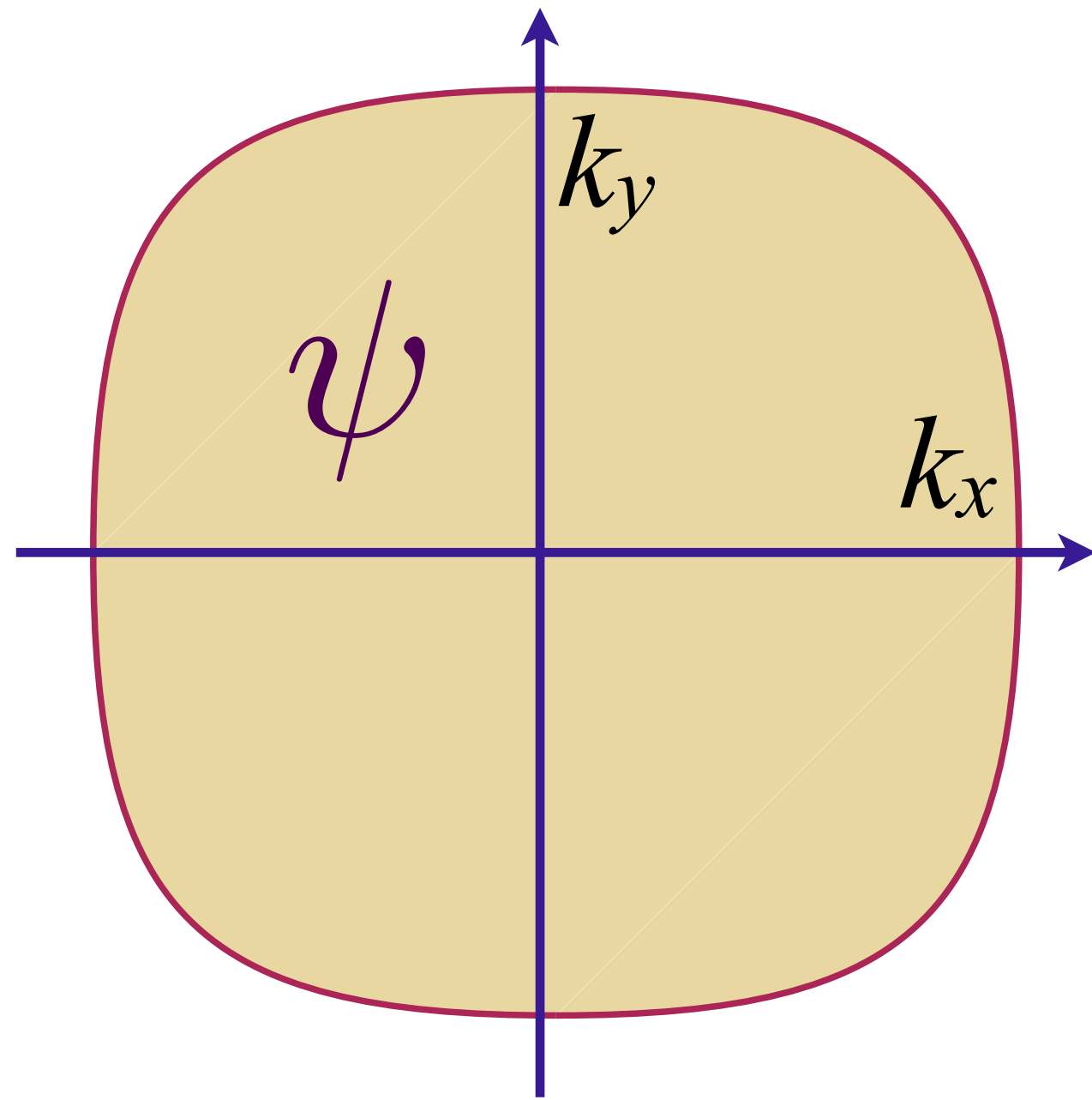
Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

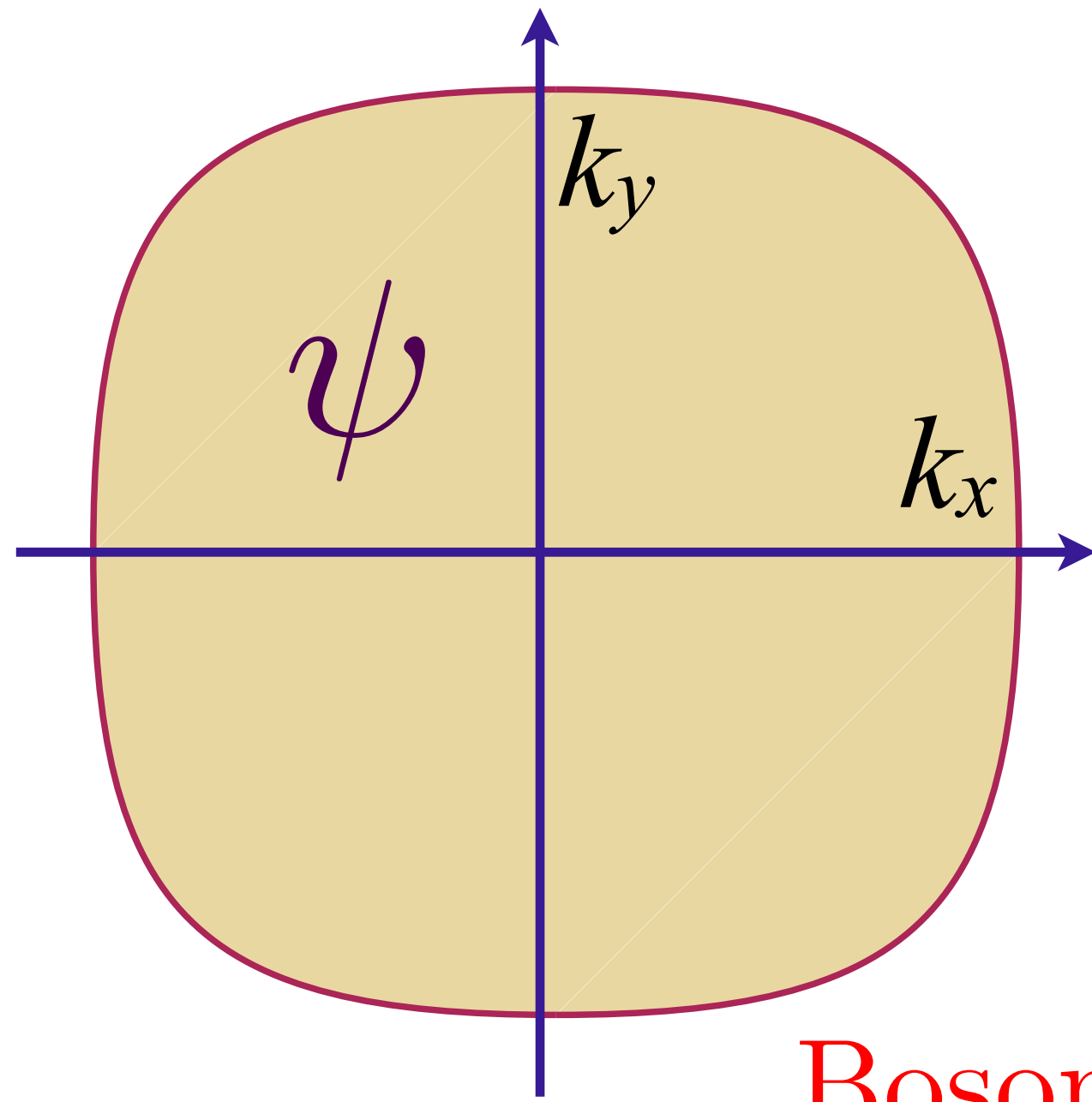
$$\begin{aligned} & \frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) \\ & + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r}) \end{aligned}$$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')}$ = $v^2\delta(\mathbf{r} - \mathbf{r}')$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

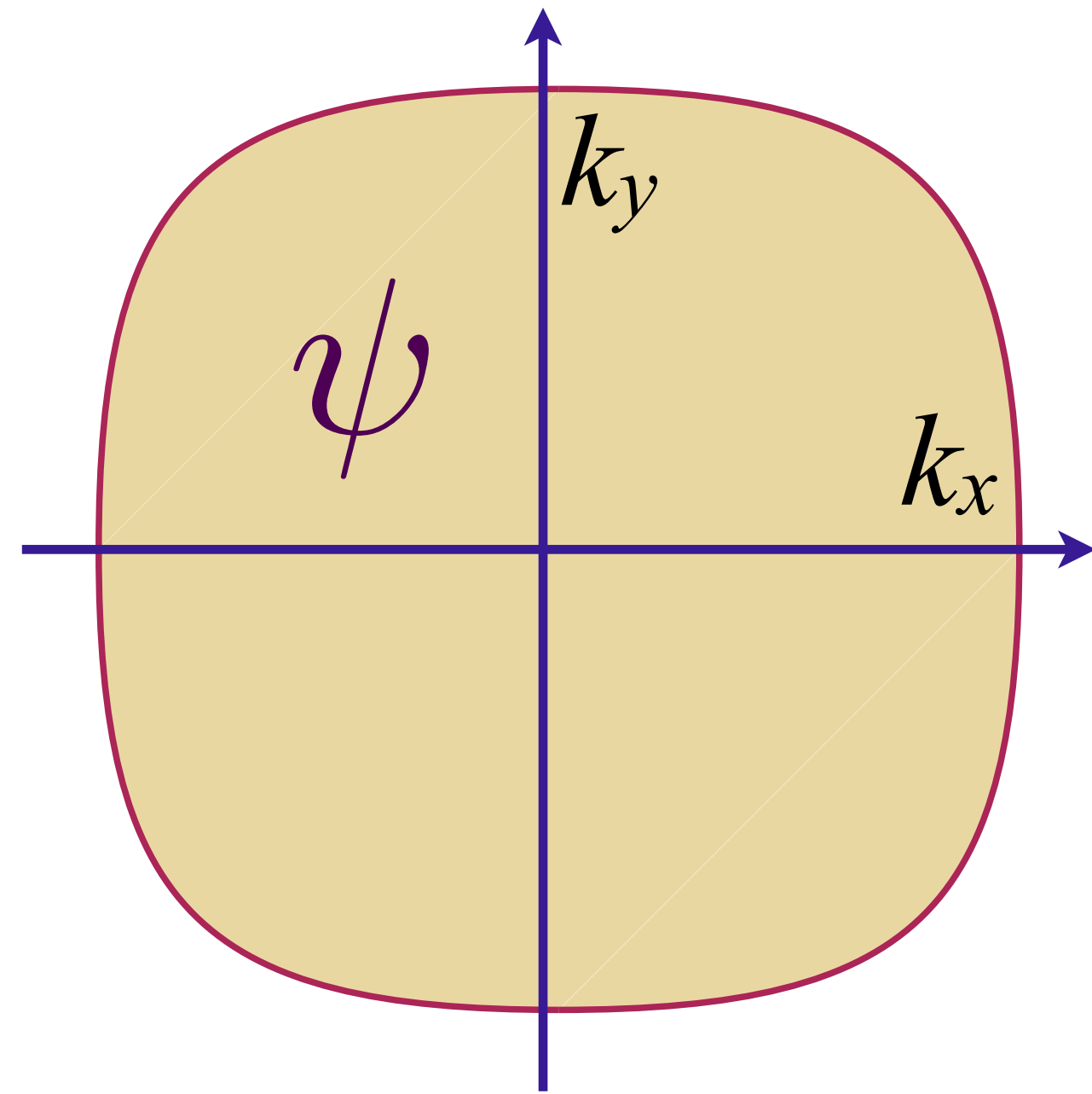
Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\frac{g^2}{v^2}\omega \ln(1/|\omega|)$; $\frac{1}{\tau(\omega)} \sim \frac{g^2}{v^2}|\omega|$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\frac{[\phi(\mathbf{r})]^2}{J} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

But resistivity and optical conductivity are like a Fermi liquid,
with residual resistivity $\sim v^2$.

Haoyu Guo, Aavishkar Patel, Ilya Esterlis, S.S. PRB **106**, 115151 (2022)

T. C. Wu, Y. Liao, and M. S. Foster, PRB **106**, 155108 (2022)

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

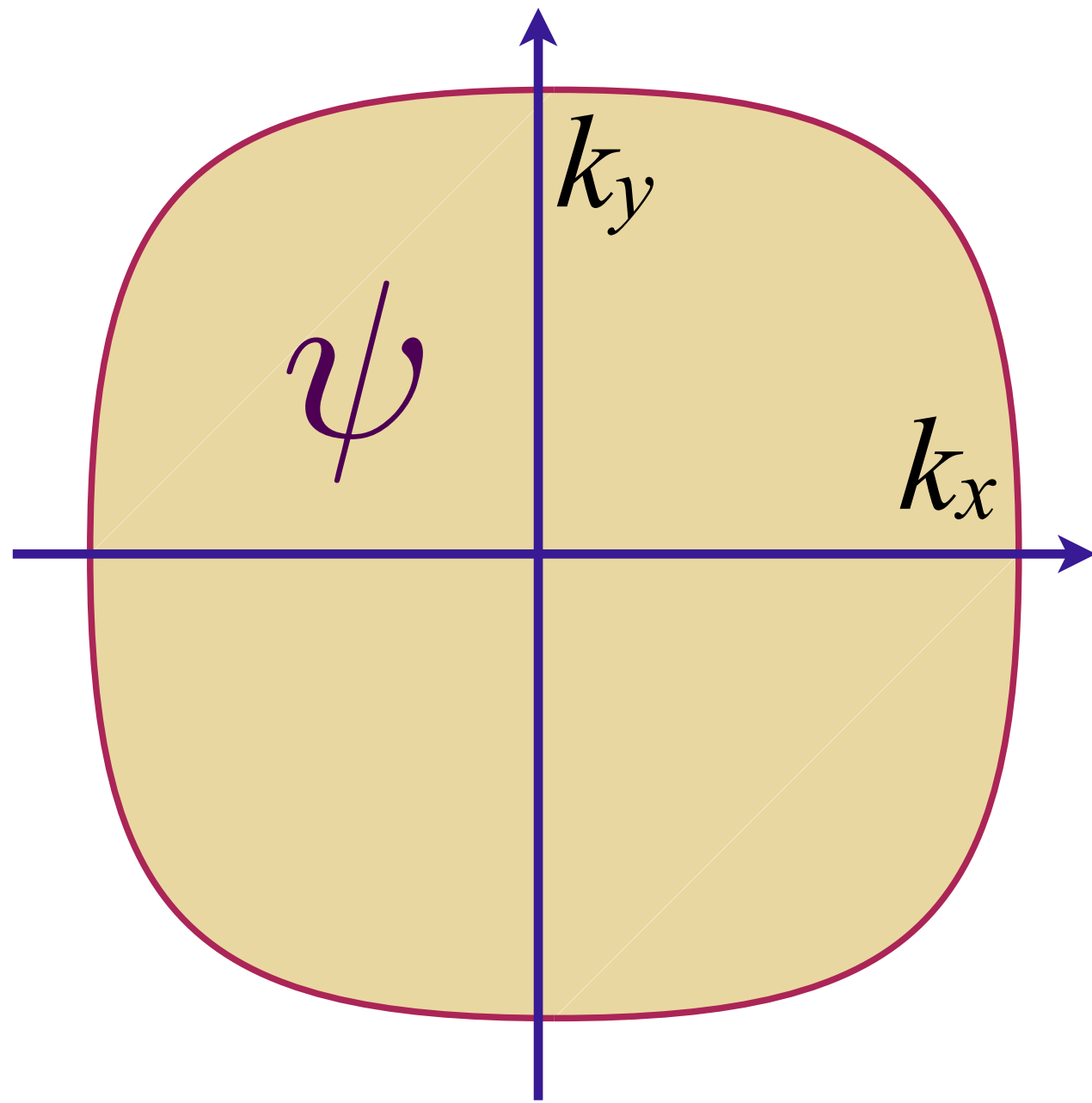
Potential disorder ν

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\mathbf{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\mathbf{k}) \right) \psi_{\mathbf{k}}$$

a critical boson ϕ
e.g. Ising-nematic order

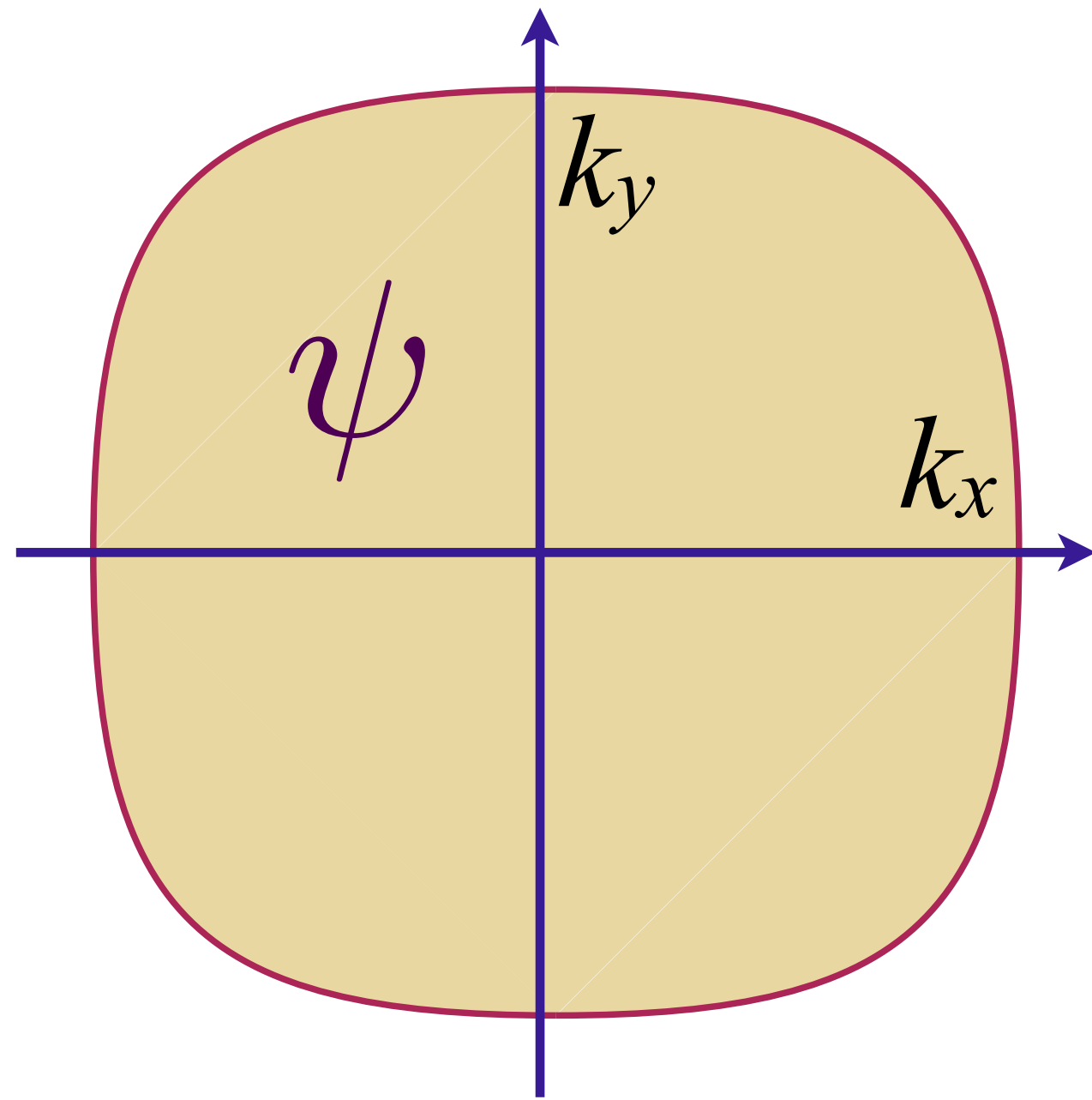


$$\frac{[\phi(\mathbf{r})]^2}{J + J'(\mathbf{r})} + \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\phi(\mathbf{r}) + v(\mathbf{r})\psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

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$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

ϕ^2 “mass” disorder $J'(\mathbf{r})$ is strongly relevant;
 rescale ϕ to move disorder to the Yukawa coupling;

Spatially random Yukawa coupling $g'(\mathbf{r})$ with $\overline{g'(\mathbf{r})} = 0$, $\overline{g'(\mathbf{r})g'(\mathbf{r}')} = g'^2 \delta(\mathbf{r} - \mathbf{r}')$

Spatially random potential $v(\mathbf{r})$ with $\overline{v(\mathbf{r})} = 0$, $\overline{v(\mathbf{r})v(\mathbf{r}')} = v^2 \delta(\mathbf{r} - \mathbf{r}')$

Fermi surface coupled to a critical boson with disorder

All results are obtained from the large N saddle-point and response functions of this G - Σ - D - Π theory:

$$\mathcal{Z} = \int \mathcal{D}G \mathcal{D}\Sigma \mathcal{D}D \mathcal{D}\Pi \exp(-N S_{\text{all}})$$

$$\begin{aligned} S_{\text{all}} = & -\ln \det(\partial_\tau + \varepsilon(\mathbf{k}) - \mu + \Sigma) + \frac{1}{2} \ln \det(-\partial_\tau^2 + \mathbf{q}^2 + m_b^2 - \Pi) \\ & + \int d\tau d^2r \int d\tau' d^2r' \left[-\Sigma(\tau', \mathbf{r}'; \tau, \mathbf{r}) G(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{1}{2} \Pi(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \right. \\ & + \frac{g^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') + \frac{v^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) \delta(\mathbf{r} - \mathbf{r}') \\ & \left. + \frac{g'^2}{2} G(\tau, \mathbf{r}; \tau', \mathbf{r}') G(\tau', \mathbf{r}'; \tau, \mathbf{r}) D(\tau, \mathbf{r}; \tau', \mathbf{r}') \delta(\mathbf{r} - \mathbf{r}') \right]. \end{aligned}$$

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Saddle-point equations

$$\Sigma(\tau, \mathbf{r}) = g^2 D(\tau, \mathbf{r}) G(\tau, \mathbf{r}) + v^2 G(\tau, \mathbf{r}) \delta^2(\mathbf{r}) + g'^2 G(\tau, \mathbf{r}) D(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

$$\Pi(\tau, \mathbf{r}) = -g^2 G(-\tau, -\mathbf{r}) G(\tau, \mathbf{r}) - g'^2 G(-\tau, \mathbf{r}) G(\tau, \mathbf{r}) \delta^2(\mathbf{r}),$$

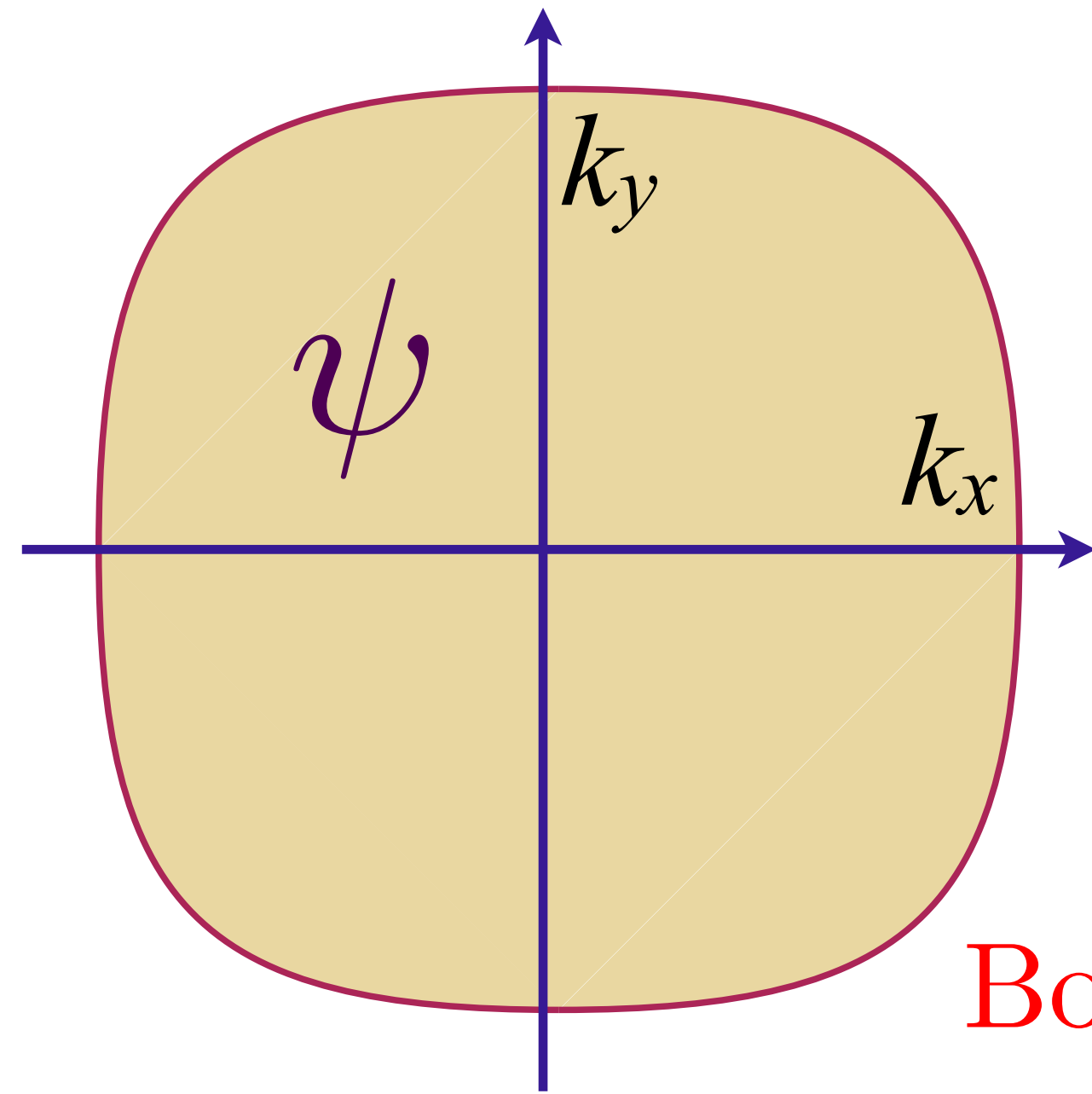
$$G(i\omega, \mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega, \mathbf{k})},$$

$$D(i\Omega, \mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega, \mathbf{q})}.$$

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e.g. Ising-nematic order



$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

Fermion self energy:

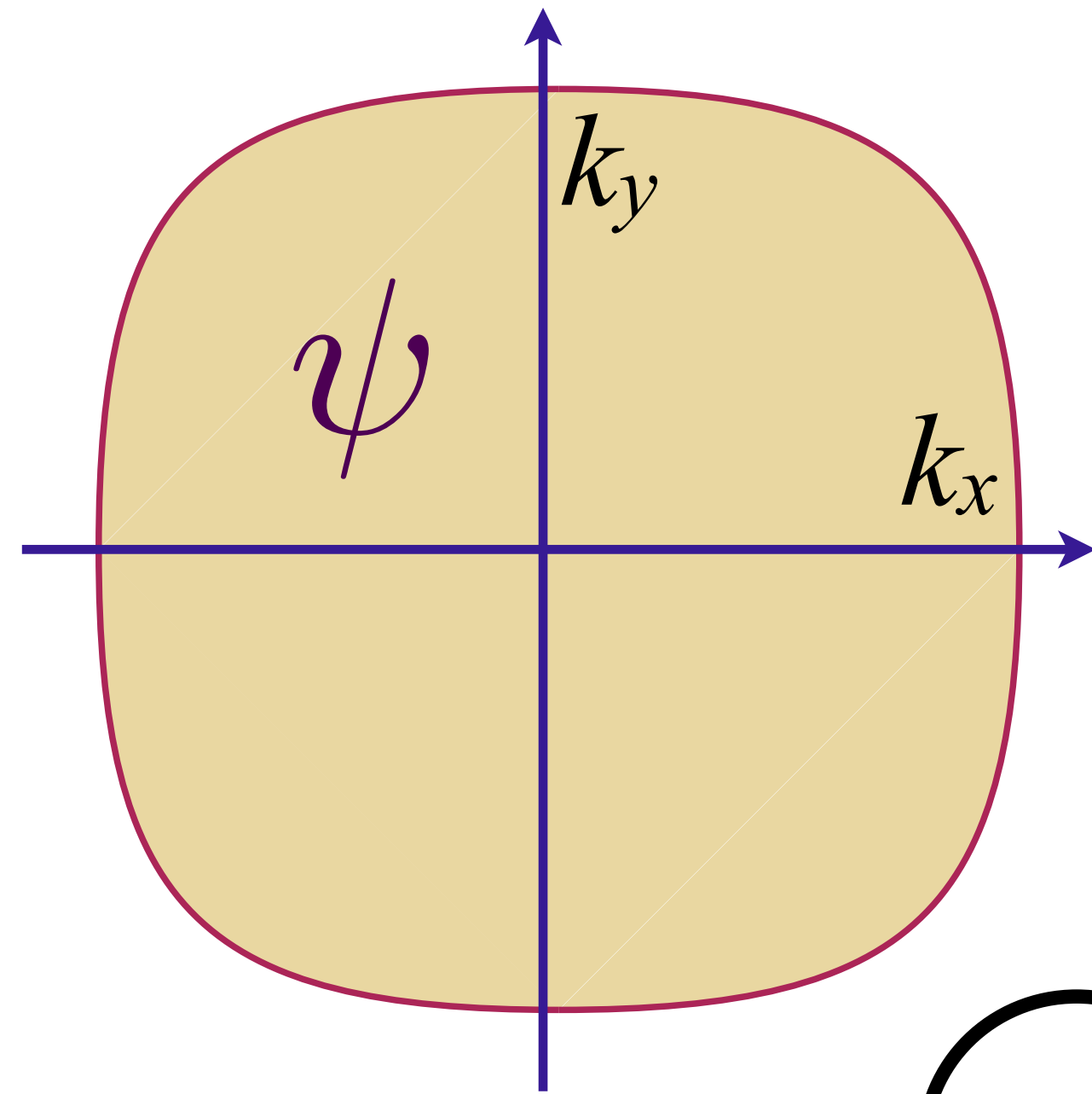
$$\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i \left(\frac{g^2}{v^2} + g'^2 \right) \omega \ln(1/|\omega|); \quad \frac{1}{\tau(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega|$$

Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat

Fermi surface coupled to a critical boson with disorder

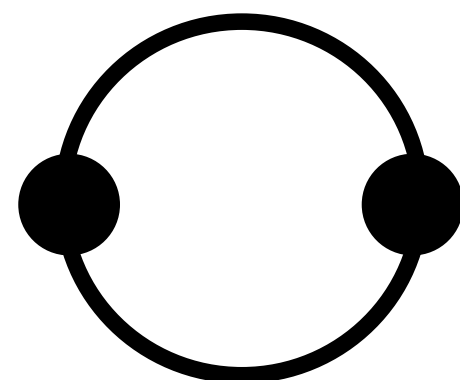
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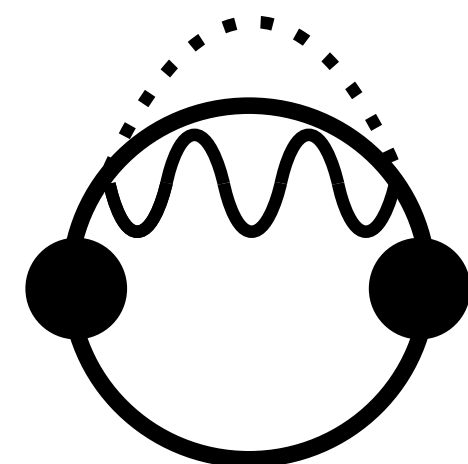


$$[\phi(\mathbf{r})]^2 + [g + g'(\mathbf{r})] \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \phi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \psi(\mathbf{r})$$

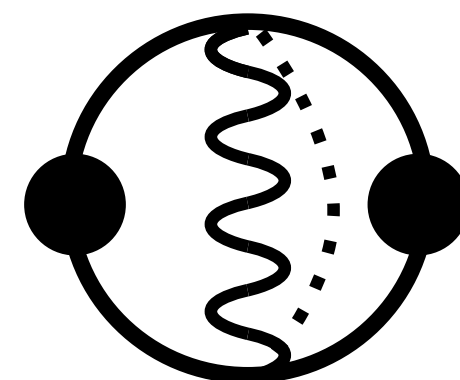
Conductivity:



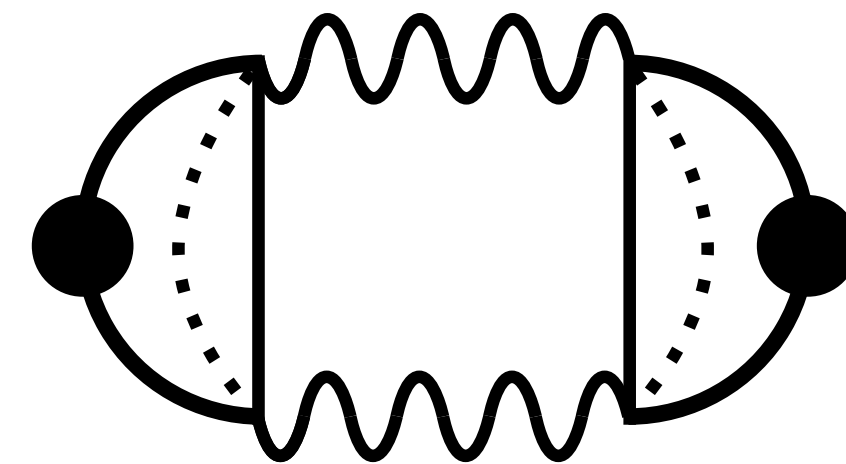
(a)
 σ_v



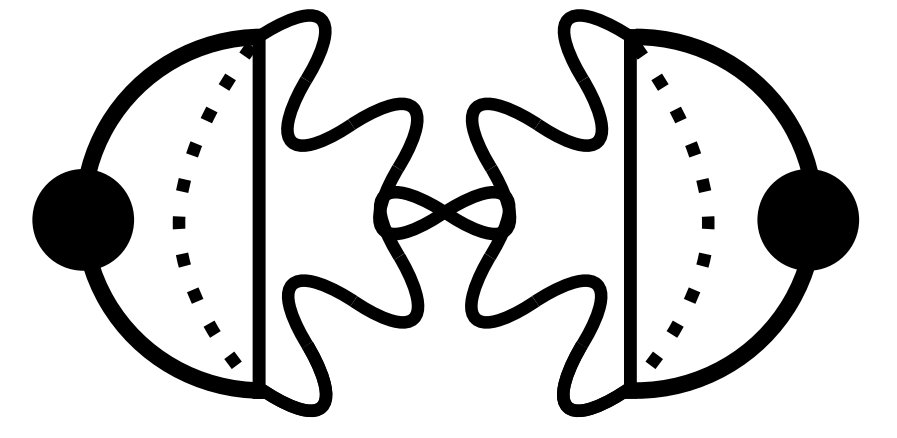
(b)
 $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$



(c)
 $\sigma_{V,g}$



(d)



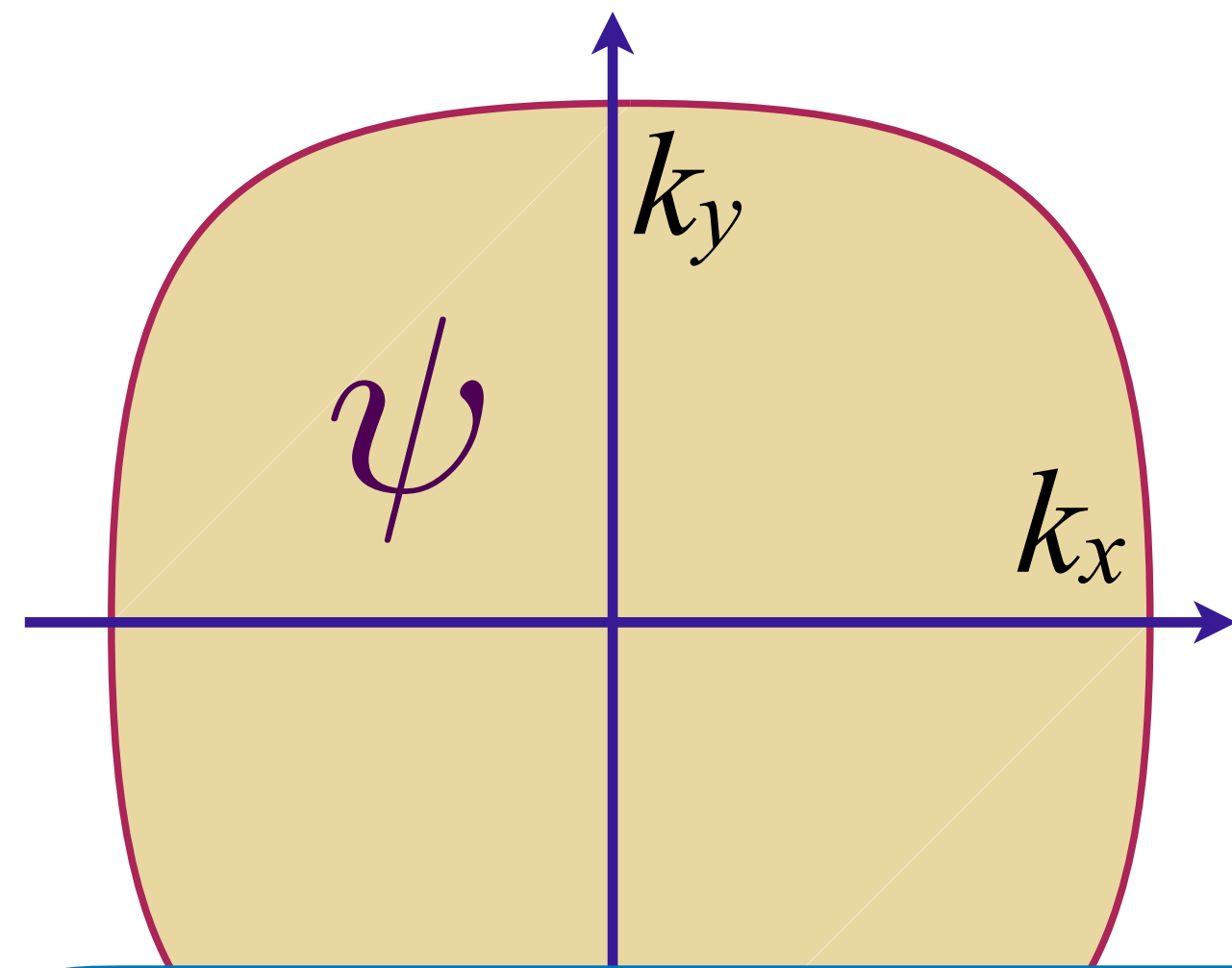
(e)

+ all ladders and bubbles.....

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Conductivity: $\sigma(\omega) \sim [1/\tau_{\text{trans}}(\omega) - i\omega m_{\text{trans}}^*(\omega)/m]^{-1}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m_{\text{trans}}^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2 ; Linear-in- T resistivity determined by g'^2 ;
 Transport insensitive to g ; Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

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Interaction disorder g'

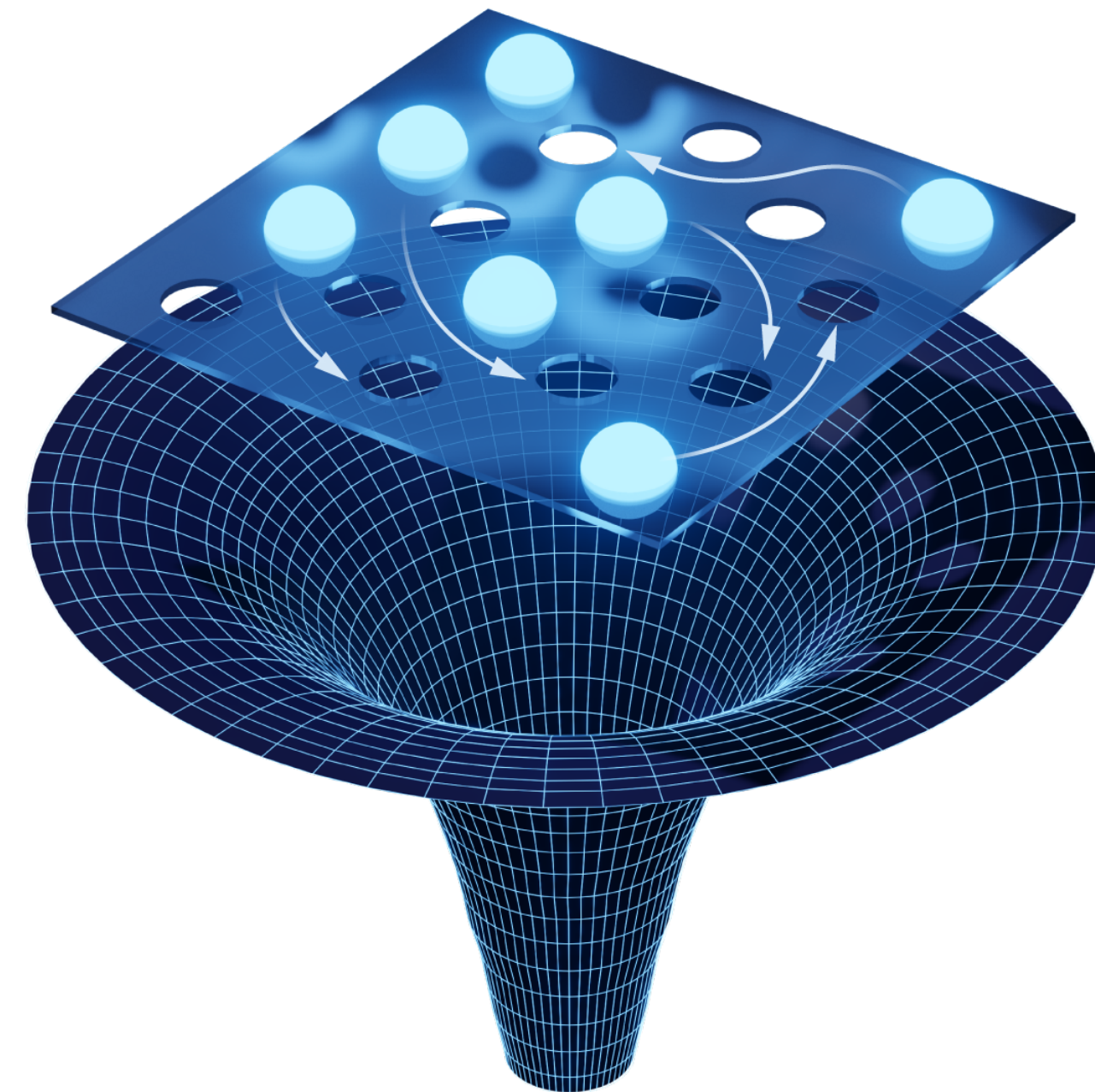
A marginal Fermi liquid AND a strange metal

Summary

- SYK: a solvable toy model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

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- SYK: a solvable toy model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.
- Toy SYK model captures the correct universal low energy quantum theory of charged black holes, and provides a Hamiltonian realization of black hole microstates.
- Linear- T resistivity, $T \ln(1/T)$ specific heat, $\sim 1/\omega$ optical conductivity, and marginal Fermi liquid electron spectrum *all* arise from a SYK-like model with spatially random interactions in a two-dimensional quantum-critical metal.

