Quantum spin liquids: from Rydberg atoms to the high temperature superconductors

University of Geneva

May 20, 2022

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Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR ADVANCED STUDY

PHYSICS HARVARD

1. Spin liquids and Z_2 gauge theory

2. Rydberg atoms as a Z_2 gauge theory

   Probing topological spin liquids

3. Paramagnon fractionalization theory of the pseudogap metal of the Hubbard model

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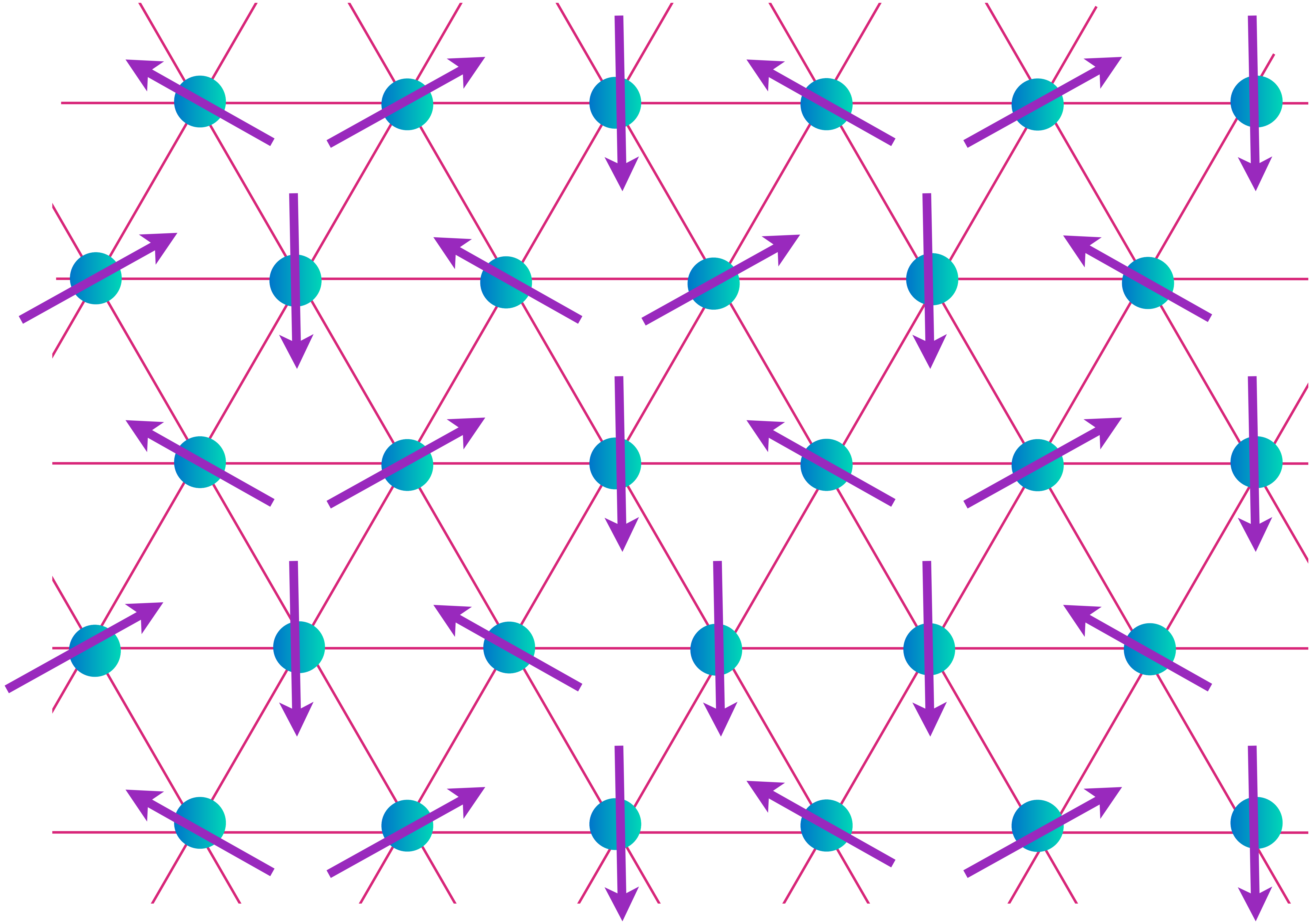
3. Paramagnon fractionalization theory of the pseudogap metal of the Hubbard model

Triangular lattice antiferromagnet

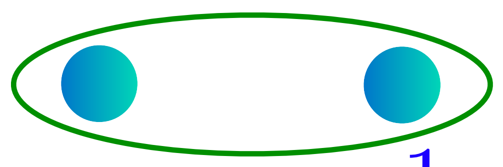
$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

Spin liquid: resonating valence bonds

Bosons at half-filling,
or a spin model with *S*=1/2 per unit cell



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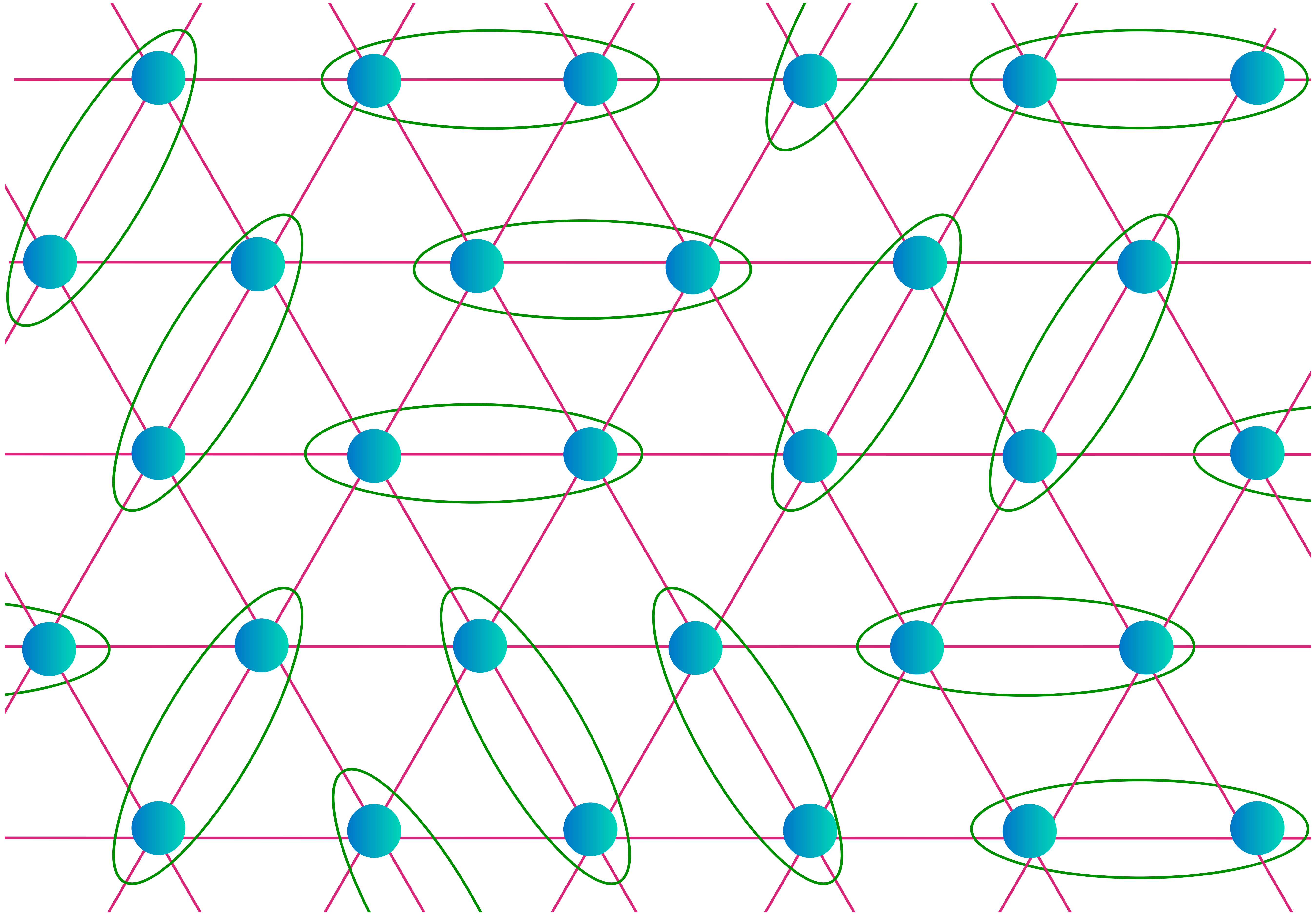

$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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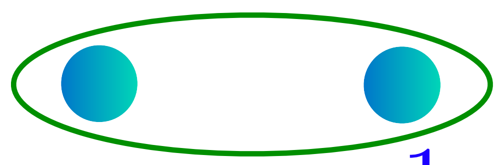

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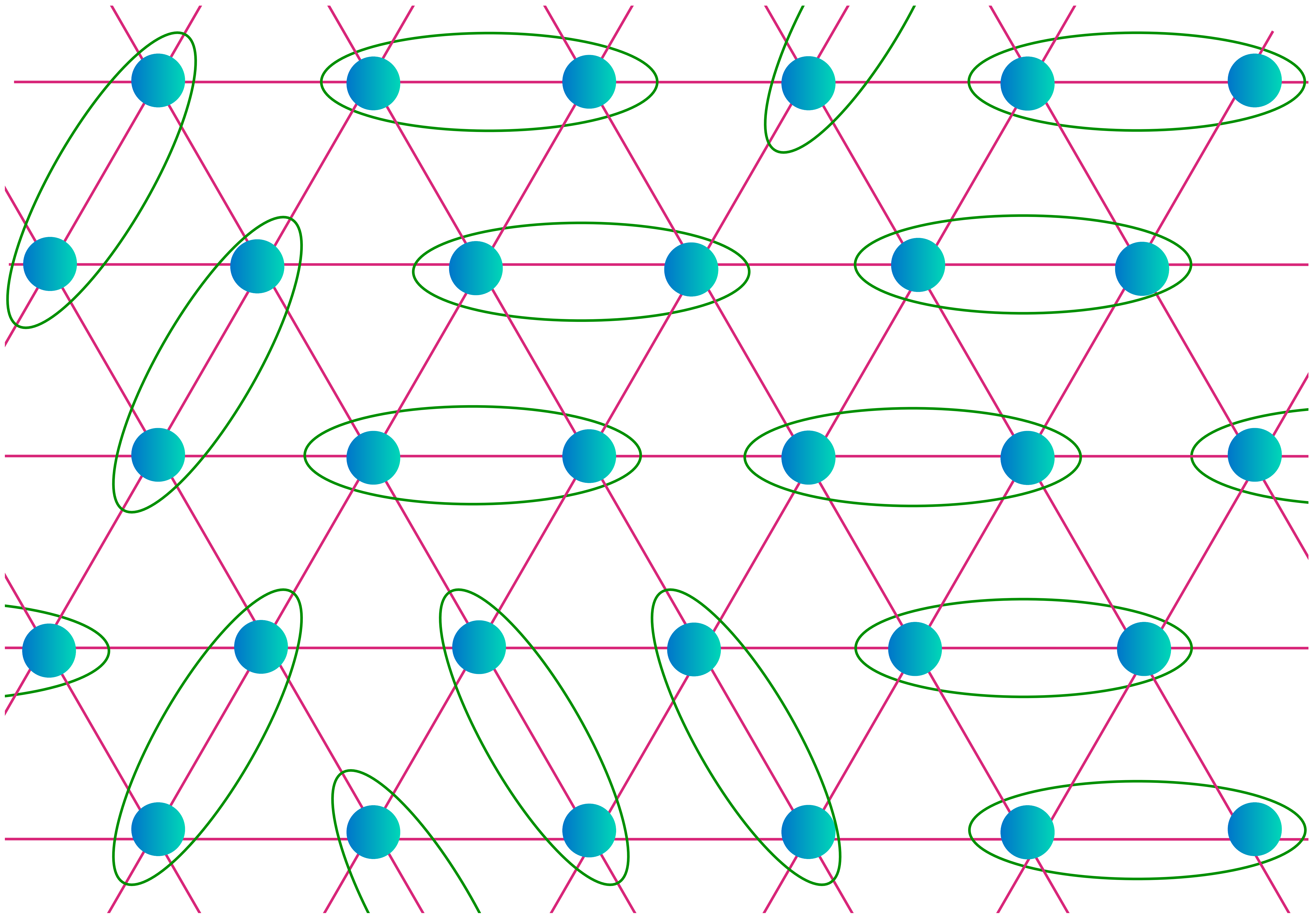

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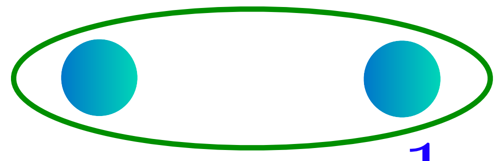

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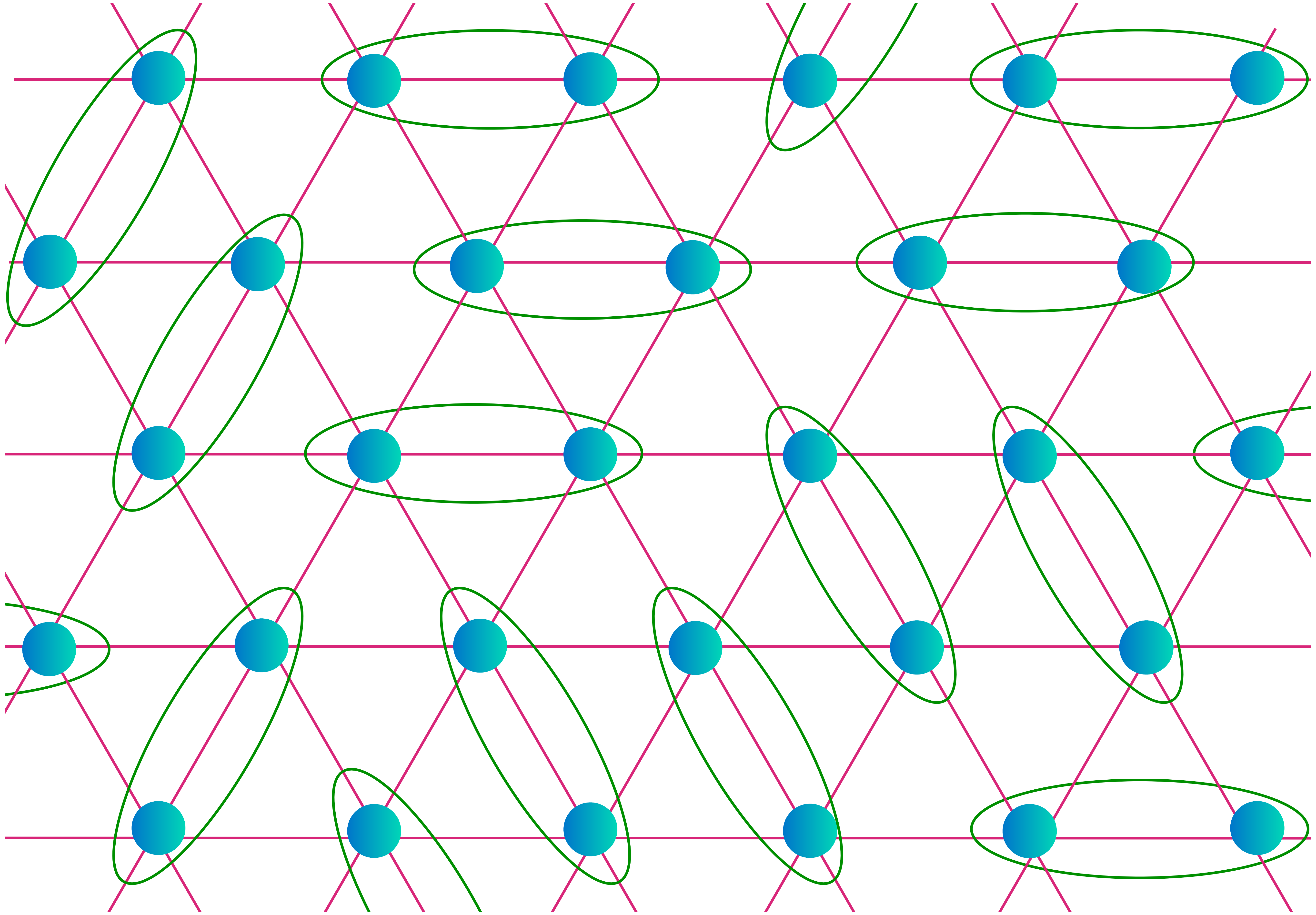

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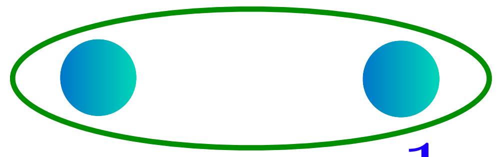

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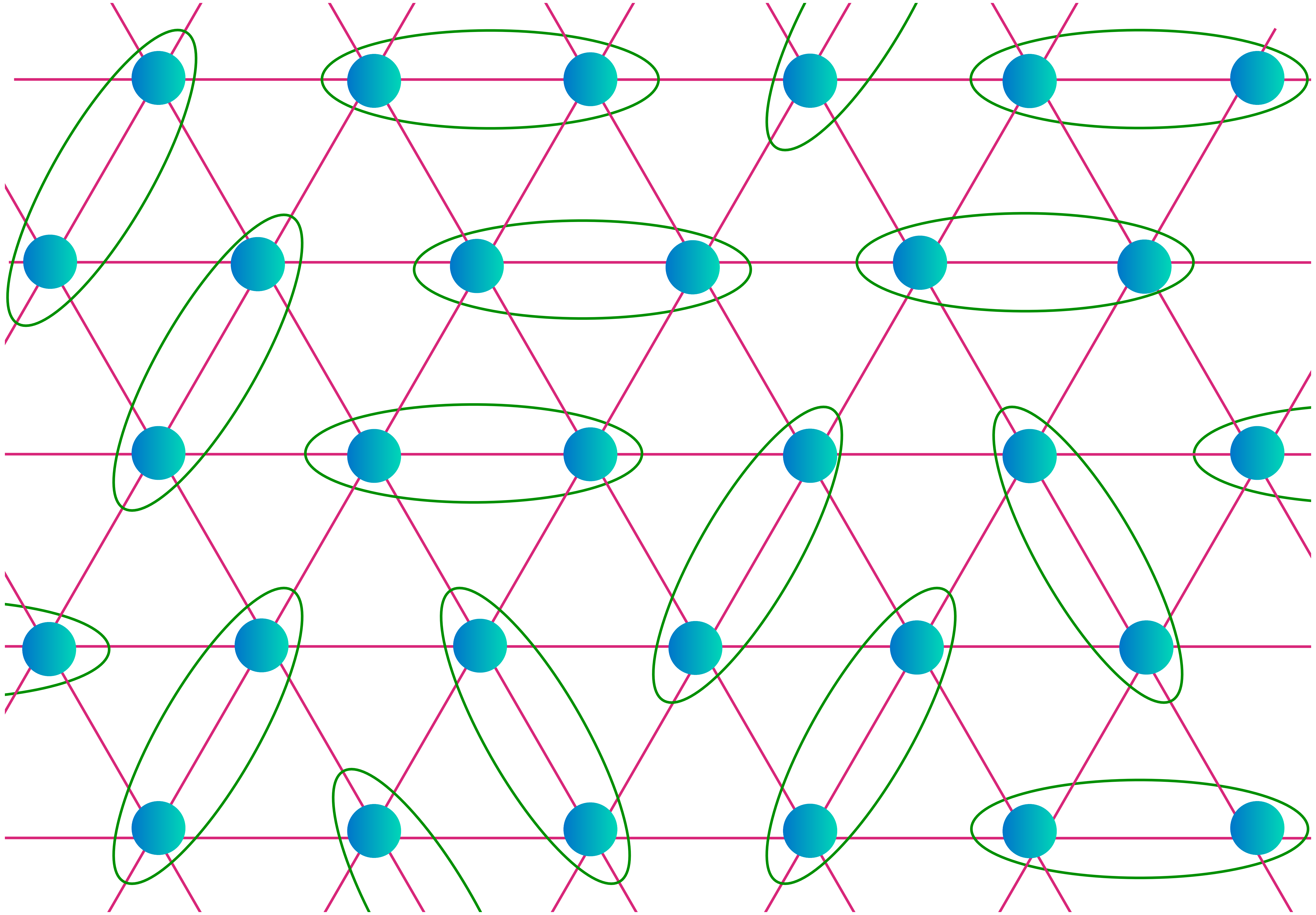

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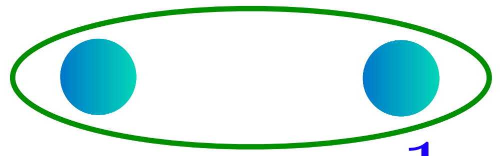

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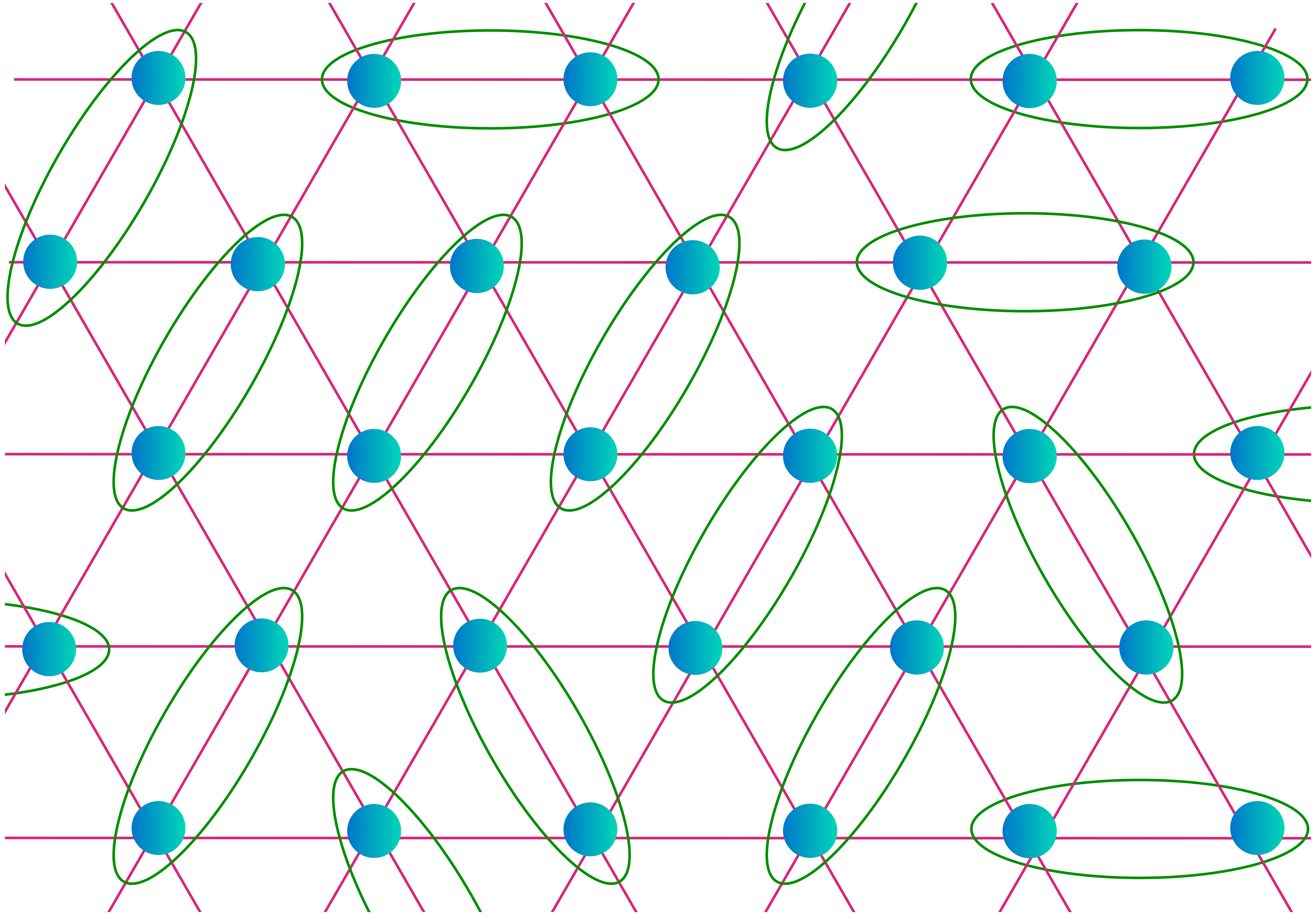

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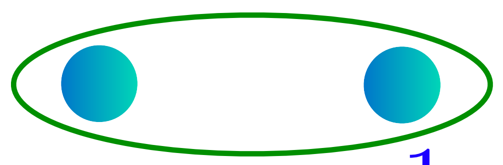

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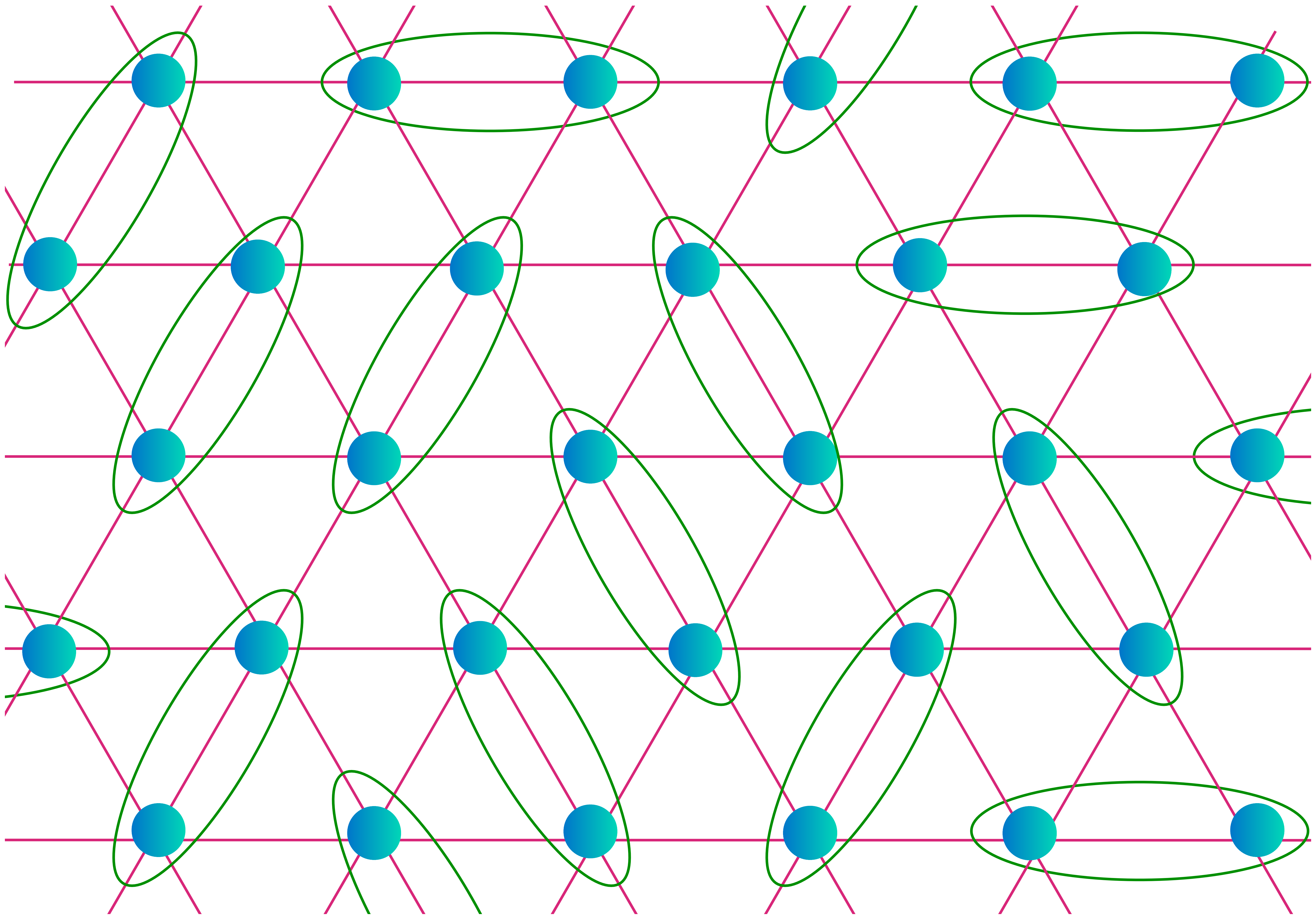

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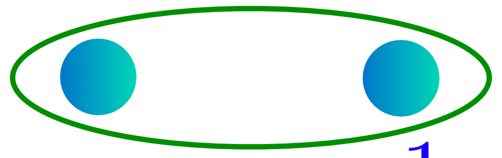
RVB: Z_2 spin liquid

Excitations with boson number 1/2



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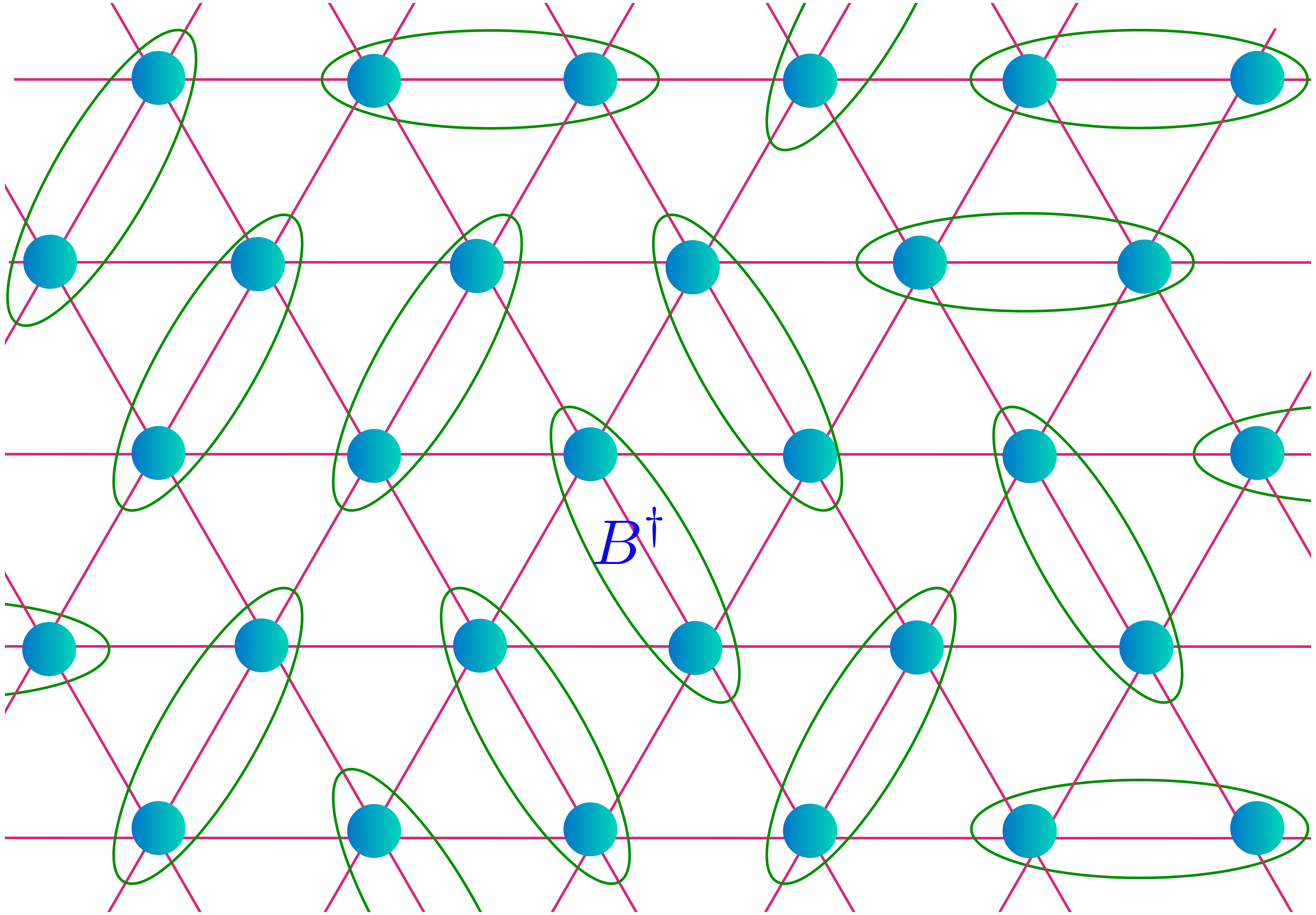
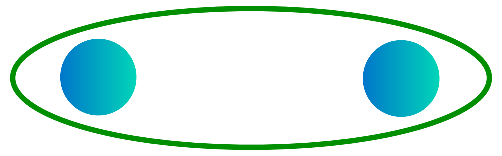
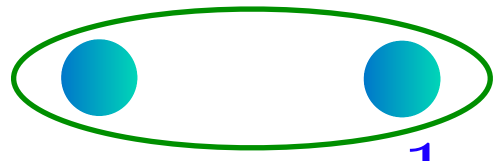


$B_2^\dagger$ 

$$= \frac{1}{\sqrt{2}} B_1^\dagger B_2^\dagger |0\rangle = \frac{1}{\sqrt{2}} |\uparrow\uparrow\rangle$$

RVB: Z_2 spin liquid

Excitations with boson number 1/2
a "spinon"



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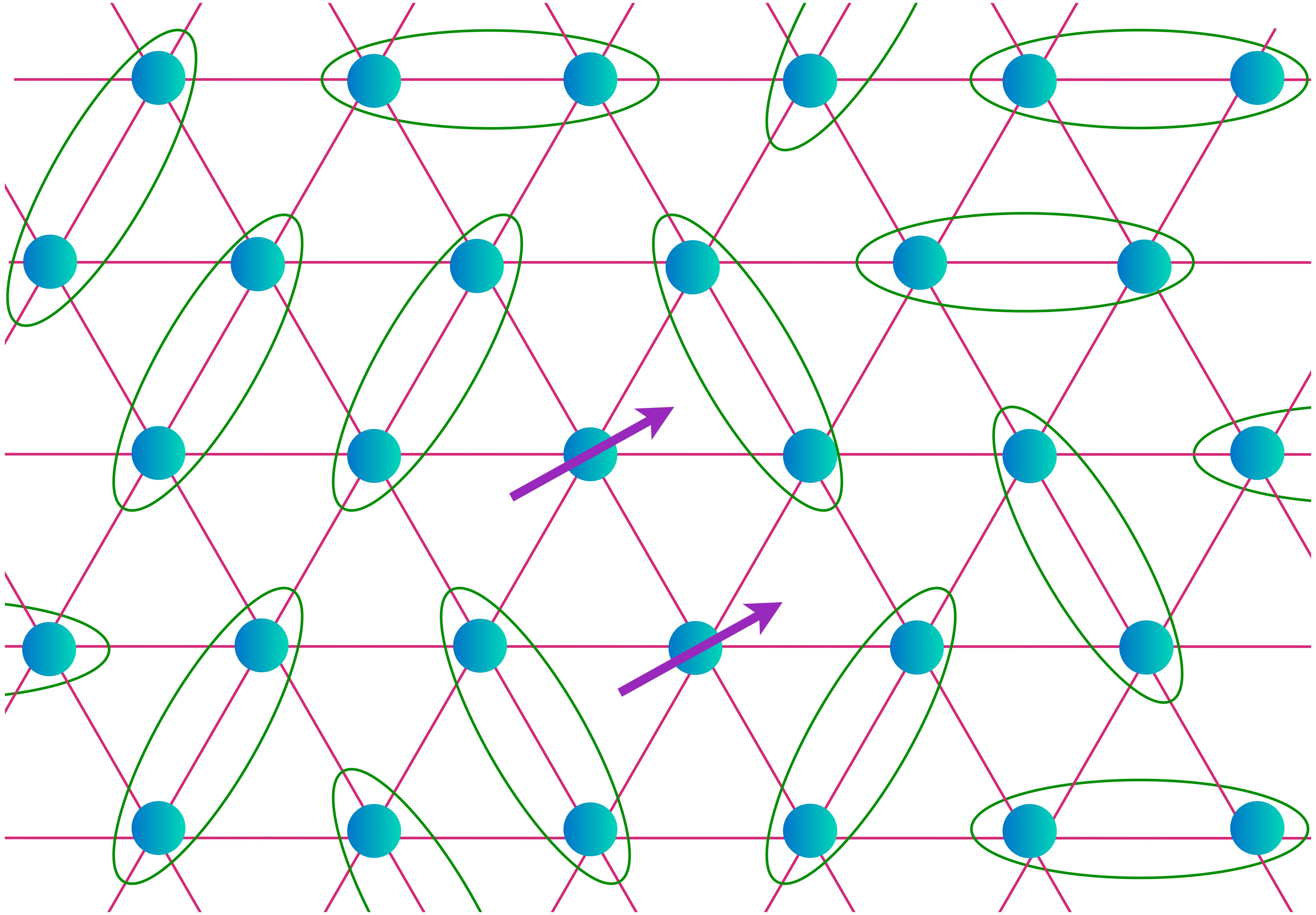

- The boson creation operator $B^\dagger$ creates a *pair* of spinons.
- A single spinon carries boson number $B^\dagger B = 1/2$: fractionalization!

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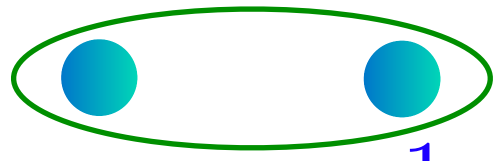

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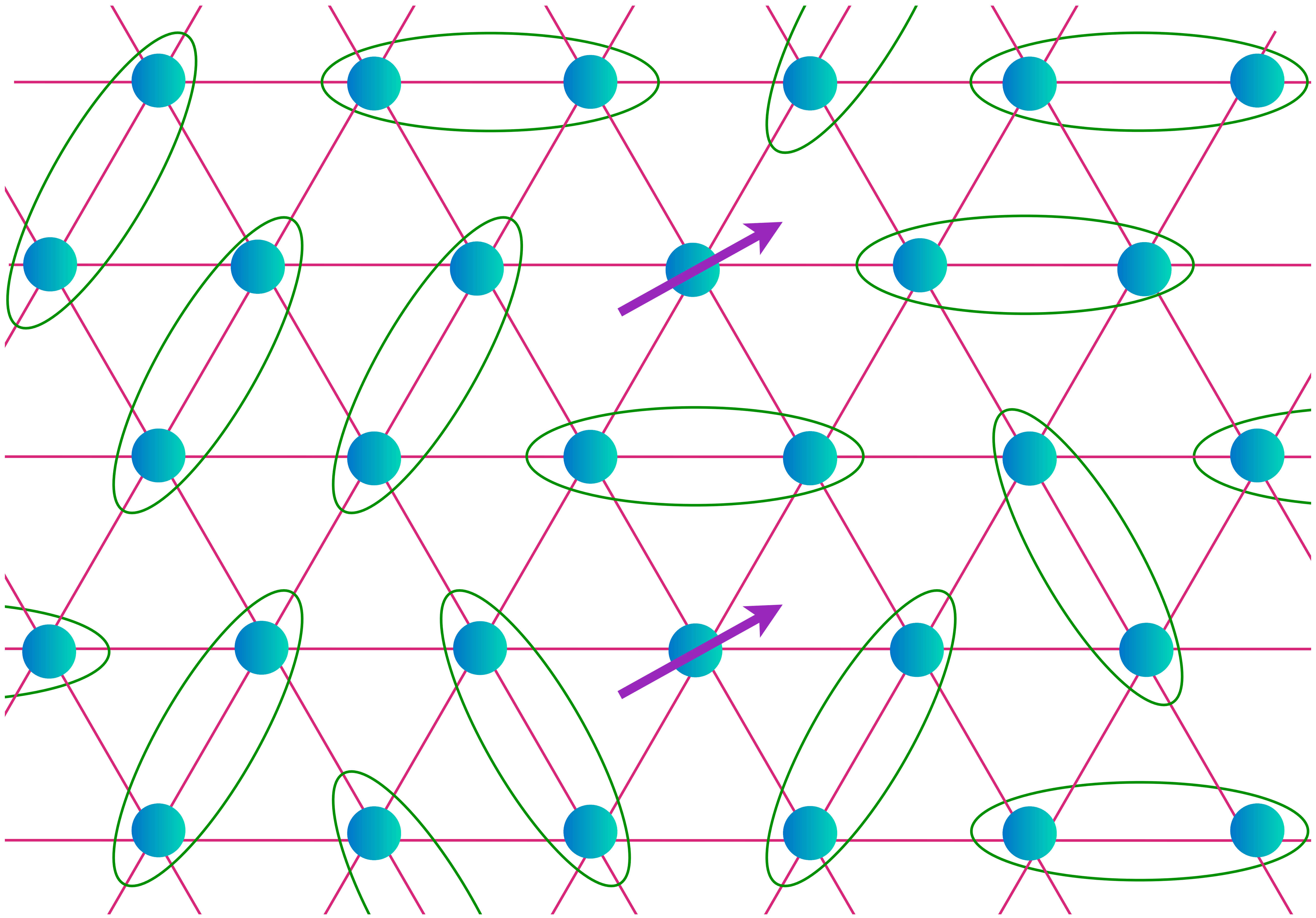

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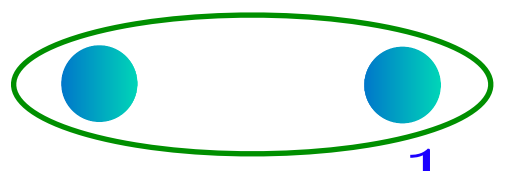

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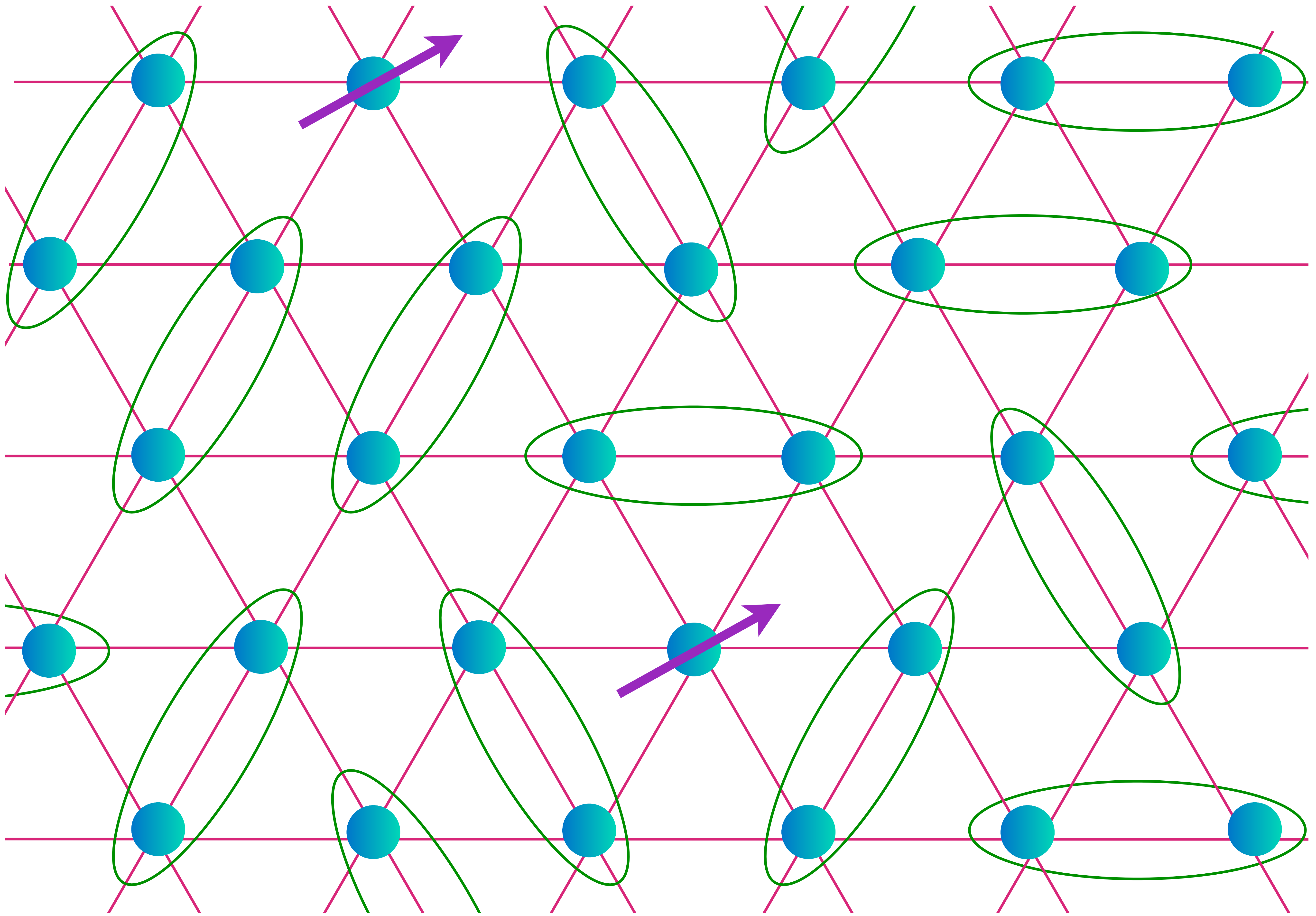

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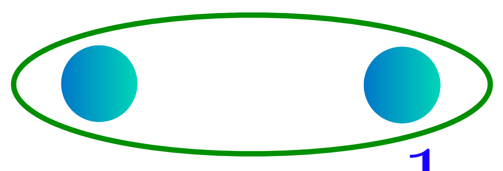

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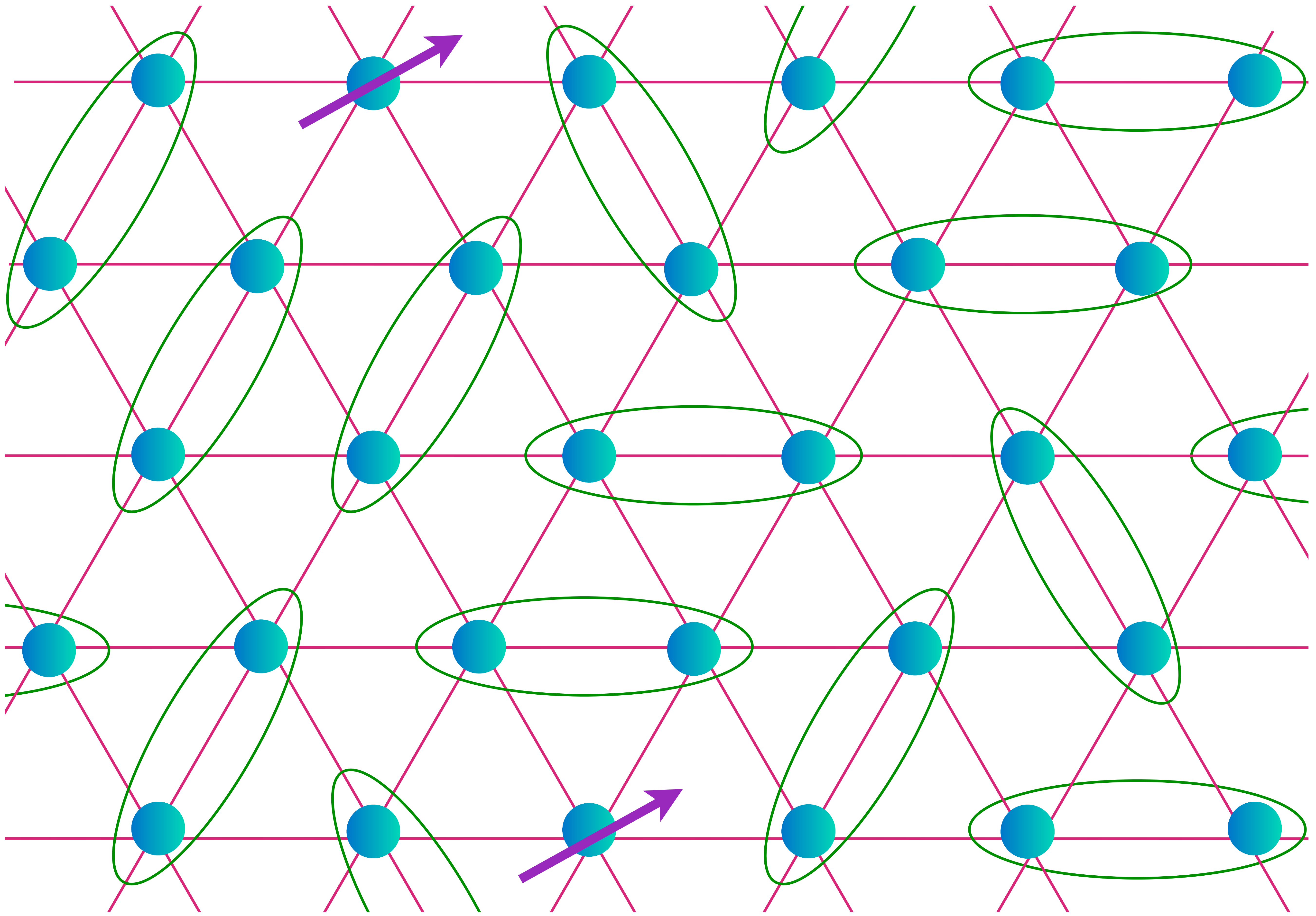

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Excitations with boson number 0
a vison (*m* particle)

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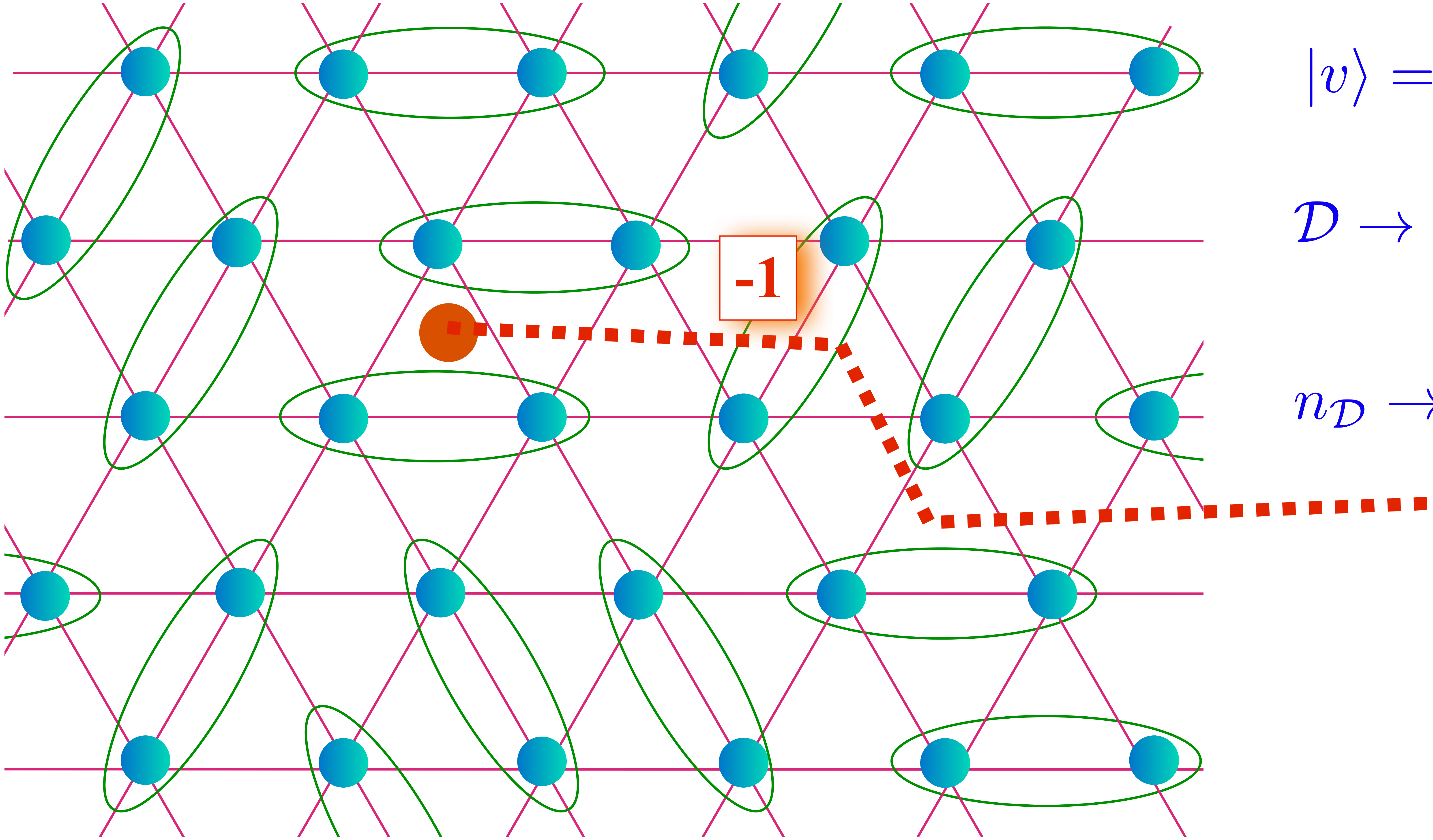

$$|v\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} (-1)^{n_{\mathcal{D}}} |\mathcal{D}\rangle$$

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N. Read and B. Chakraborty, Phys. Rev. B 40, 7133 (1989)

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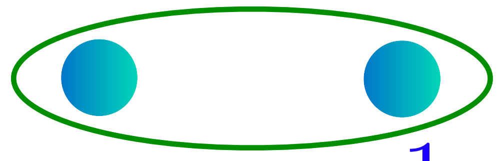

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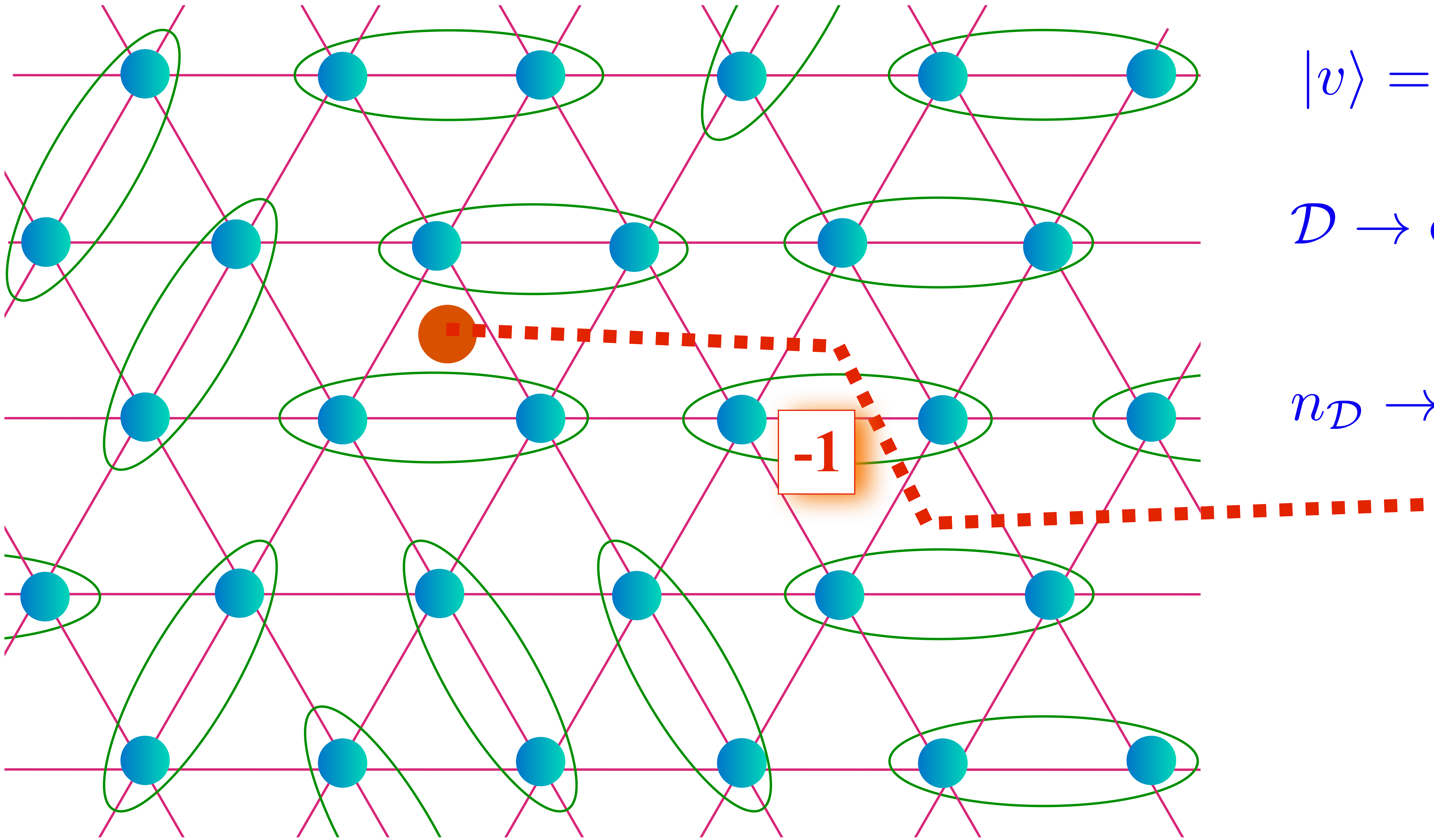

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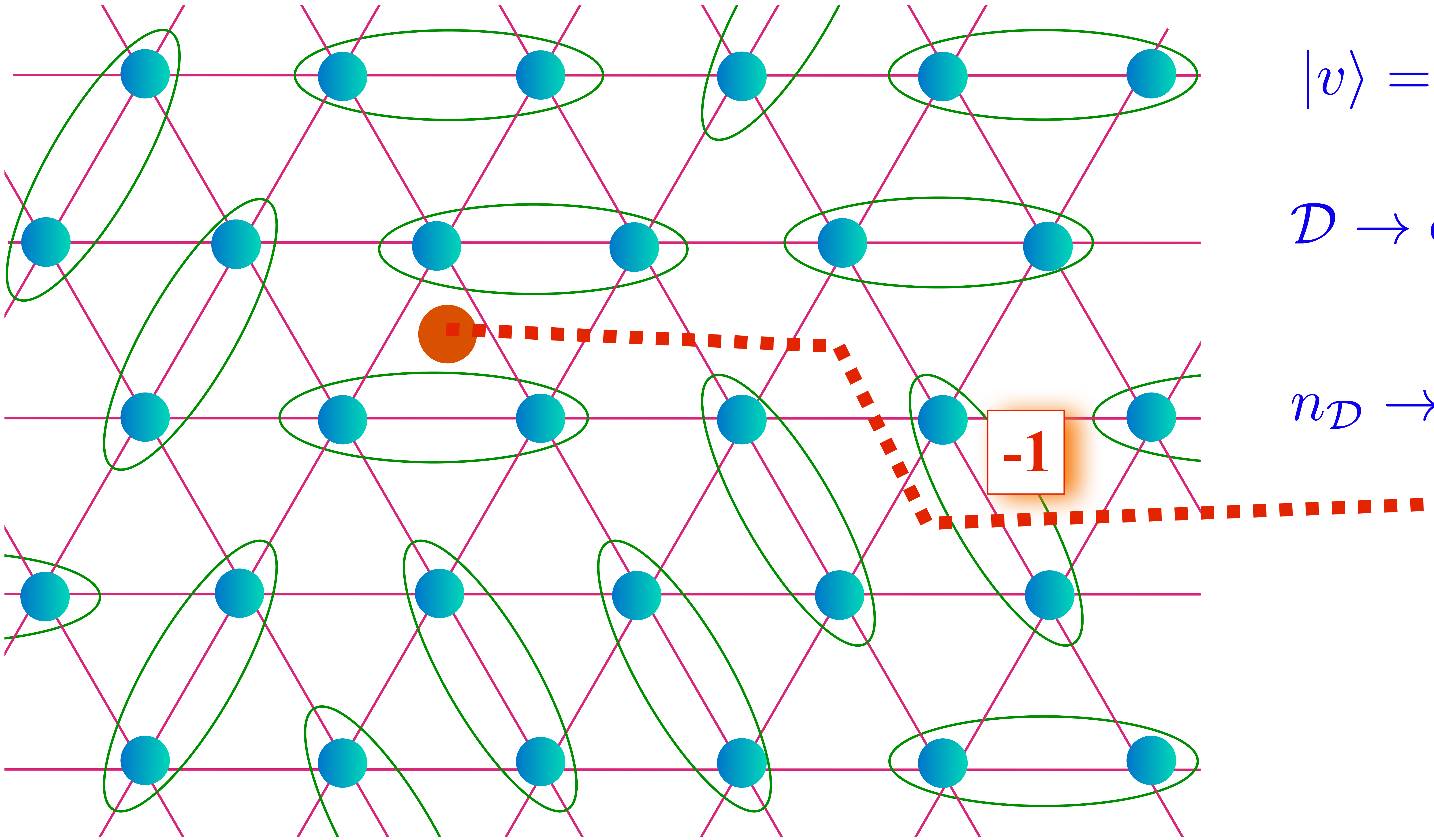

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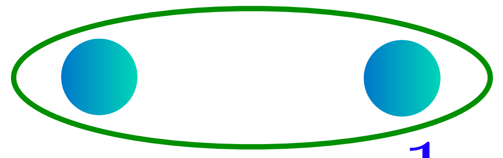

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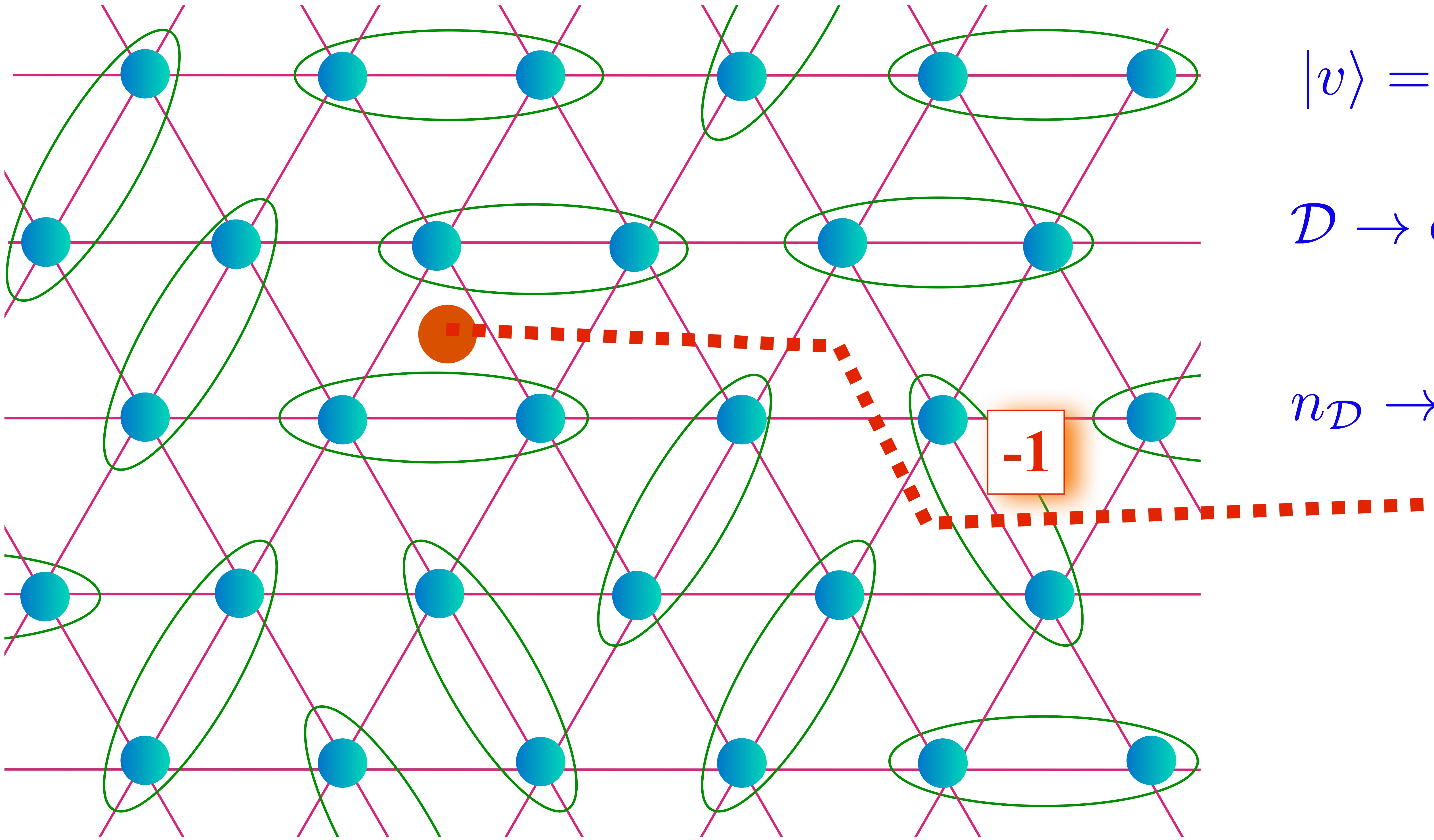

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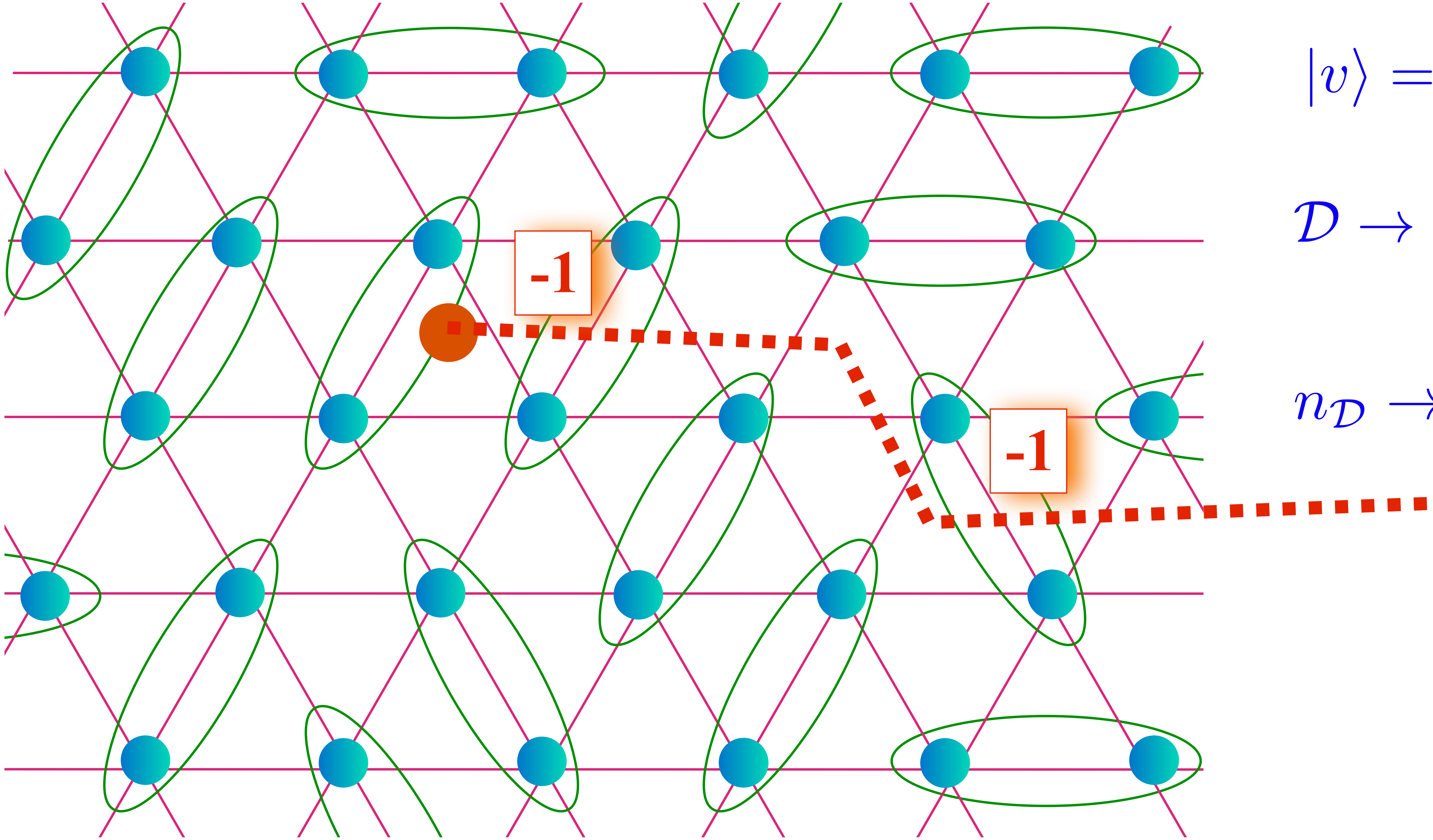

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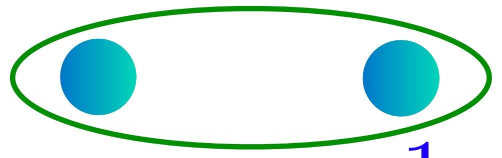

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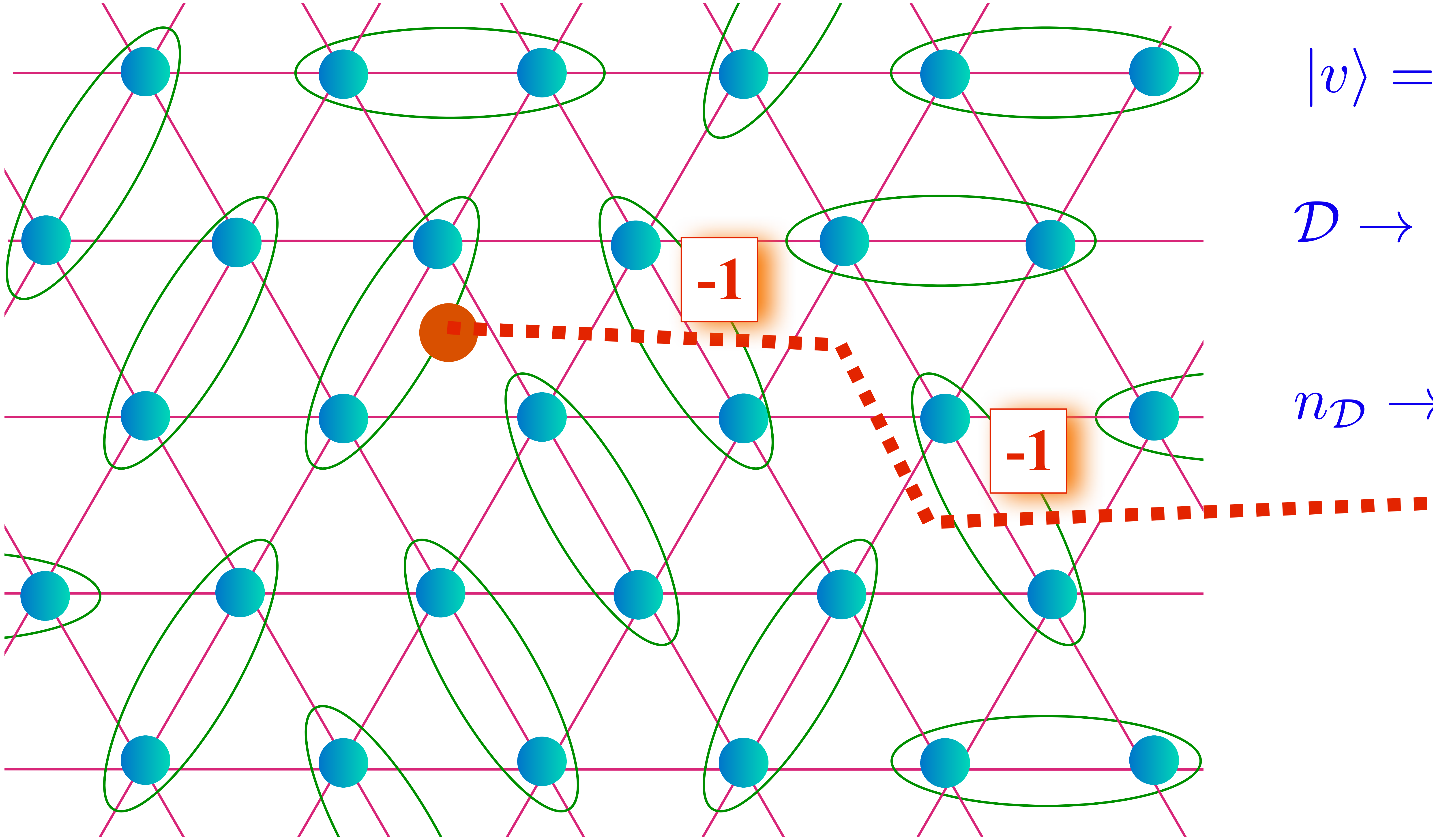

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RVB: Z_2 spin liquid

Read and Sachdev (1990); Wen (1991)

The simplest stable spin liquid (which need not break time-reversal) is the deconfined phase of a $\mathbb{Z}_2$ gauge theory. There are 'spinon' excitations which carry unit $\mathbb{Z}_2$ electric charges, and 'vison' excitations which carry π $\mathbb{Z}_2$ magnetic flux.

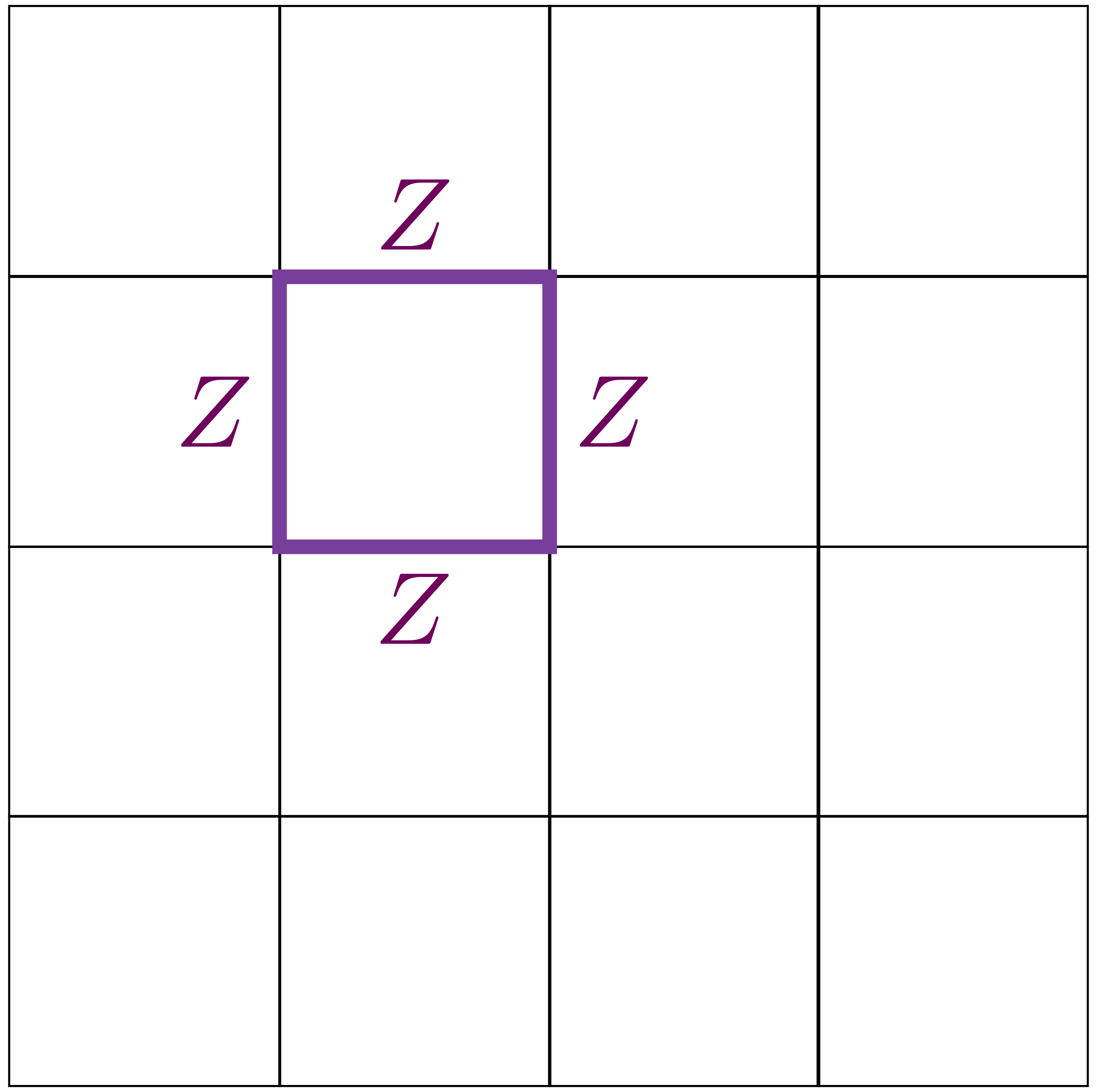
Anyon	e (spinon)	ϵ (spinon)	m (vison)
Boson number	$1/2$	$1/2$	0
Self-statistics	boson	fermion	boson

Any pair of e, ϵ, m are mutual semions.

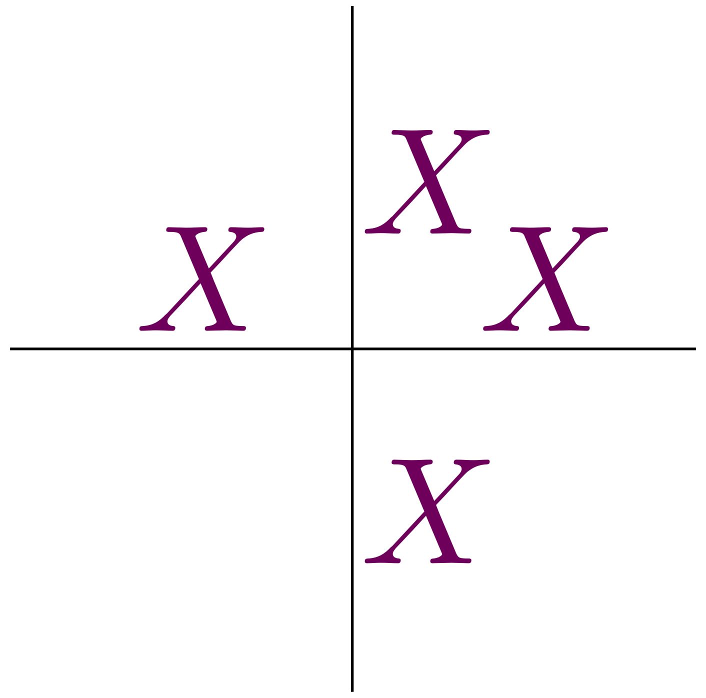
These anyons are 'topological': they cannot be created individually by any local operator, and their existence implies a four-fold ground state degeneracy on a large torus.

Pure Z_2 gauge theory: only describes vison sector

$$\mathcal{H}_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} Z_\ell - g \sum_\ell X_\ell$$



$G_i =$



$$[\mathcal{H}_{\mathbb{Z}_2}, G_i] = 0$$

$G_i = (-1)^{2S}$ for spin S antiferromagnets

For $S = 1/2$ we have an 'odd' spin liquid.

R. Jalabert and S. Sachdev, Physical Review B **44**, 686 (1991)

S. Sachdev and M. Vojta, Journal of the Physical Society of Japan **69,** Suppl. B, 1 (2000); cond-mat/9910231

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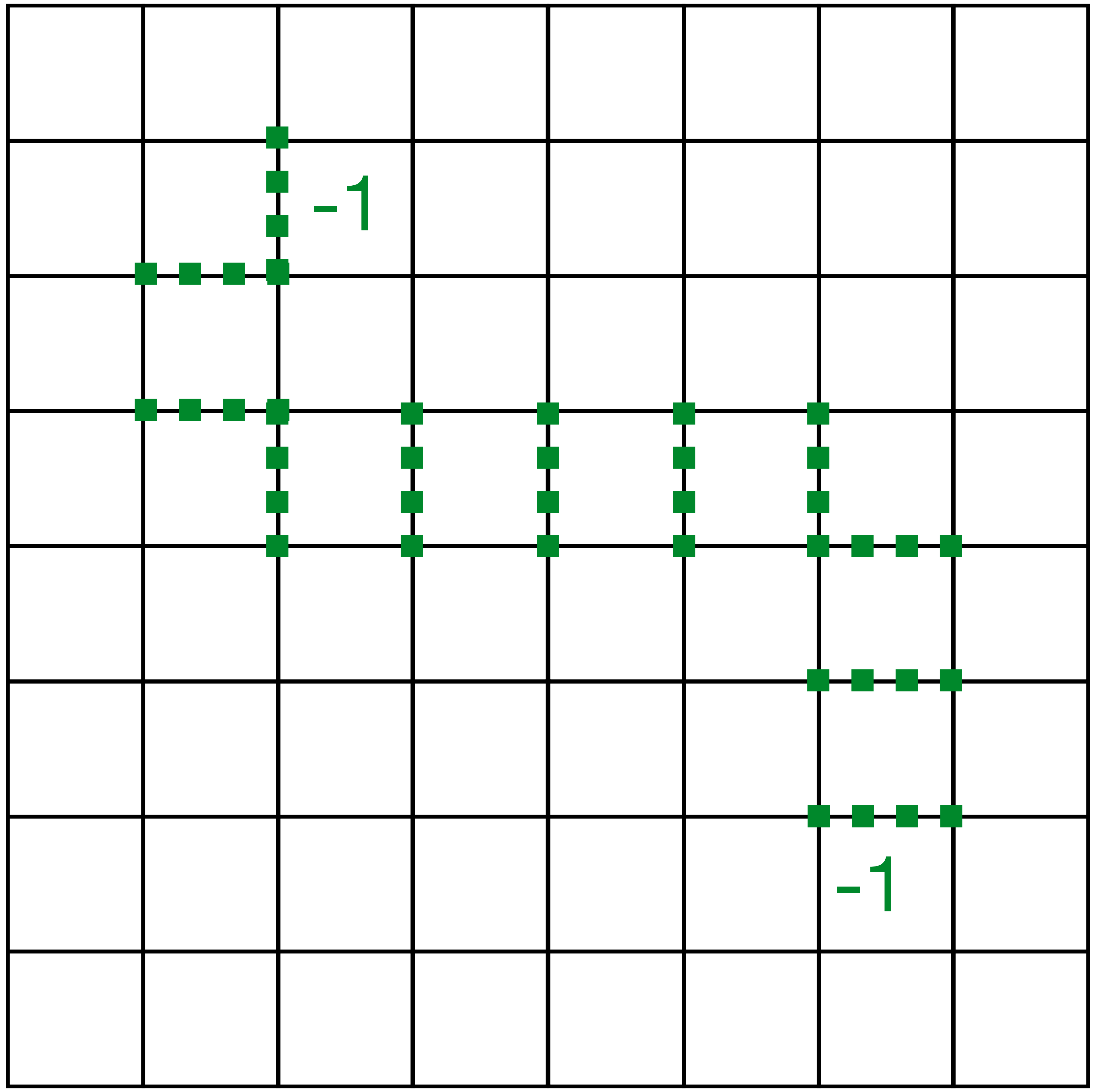

Two visons (indicated by the -1's in the plaquettes) connected by an invisible string. The dashed lines indicate the links, ℓ, on which $Z_\ell = -1$. The plaquettes with an even number of dashed lines on their edges carry no $\mathbb{Z}_2$ fluxes, and so are 'invisible'.

Read and Sachdev (1990); Wen (1991)

Z_2 gauge theory with matter: describes visons and spinons

$$\mathcal{H}_{\mathbb{Z}_2} = -K \sum_{\square} \prod_{\ell \in \square} Z_\ell - g \sum_{\ell} X_\ell$$

$$- J \sum_{\ell \in (i,j)} \tau_i^z Z_\ell \tau_j^z - h \sum_i \tau_\ell^x$$

$$G_i = \tau_i^x \prod_{\ell \in i} X_\ell \quad , \quad [\mathcal{H}_{\mathbb{Z}_2}, G_i] = 0$$

Now we choose $G_i = 1$ and $\text{sgn}(h) = (-1)^{2S}$.

The τ_i^x operator creates a $\mathbb{Z}_2$ electric charge – a 'spinon' which has mutual semionic statistics with a vison.

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Quantum phases of Rydberg atoms on a kagome lattice,
Rhine Samajdar, Wen Wei Ho, Hannes Pichler, M. D. Lukin, and S. S.,
Proceedings of the National Academy of Sciences **118**, e2015785118 (2021); arXiv:2011.12295

Emergent Z_2 gauge theories and topological excitations in Rydberg atom arrays,
Rhine Samajdar, Darshan G. Joshi, Yanting Teng, and S. S., arXiv:2204.00632

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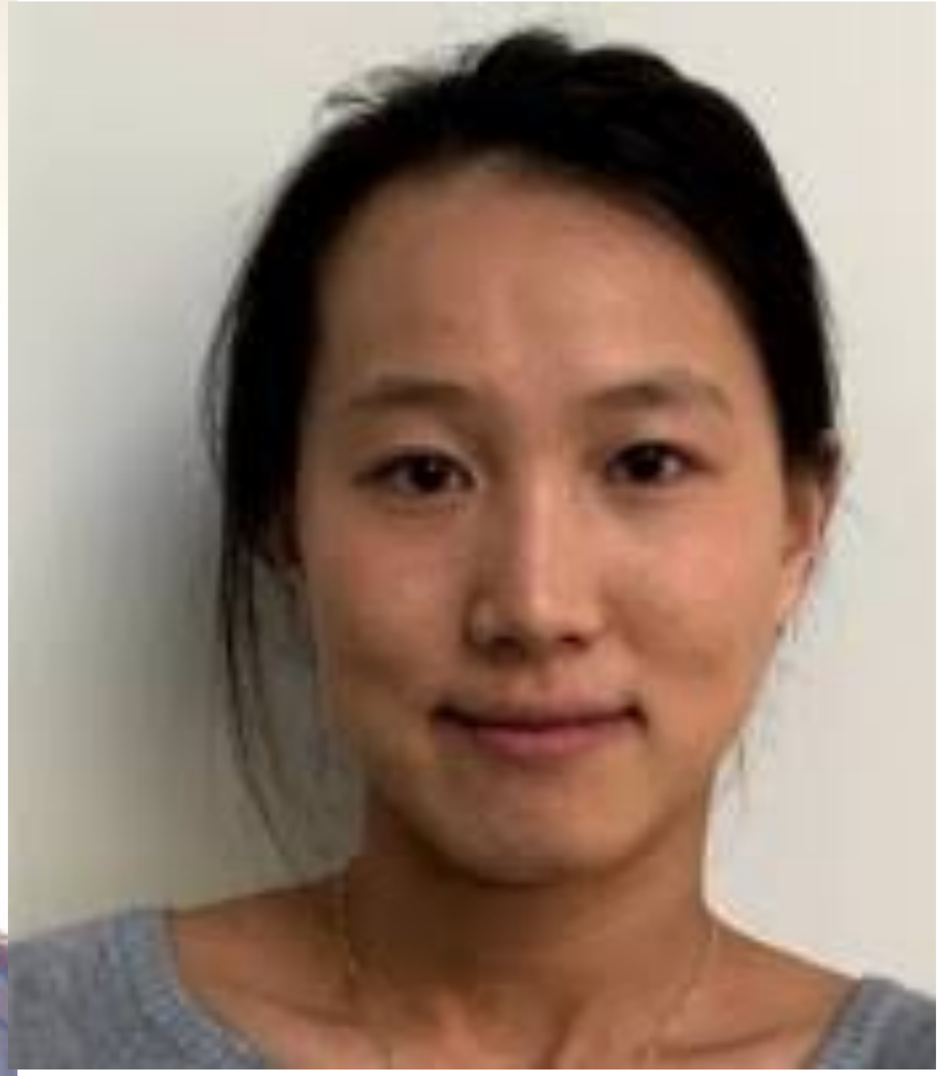
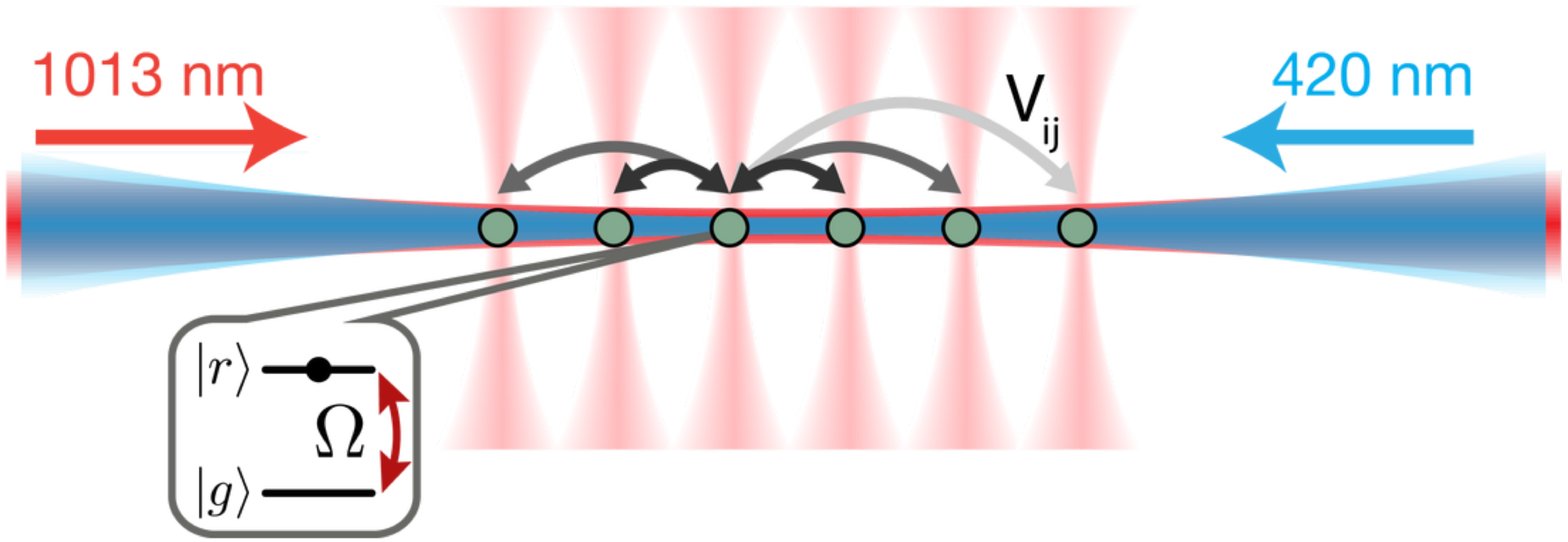
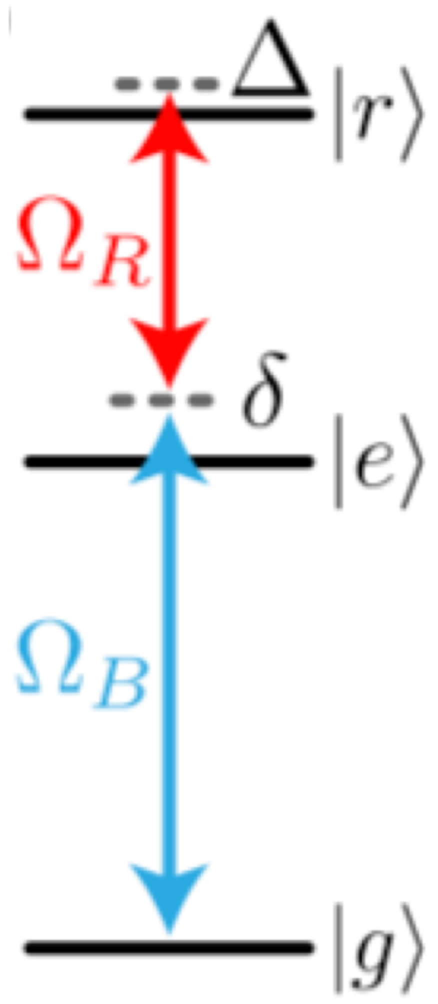


Yanting Teng

QPTs in a Rydberg quantum simulator





$$|g\rangle \equiv |0\rangle$$

$$|r\rangle \equiv b^\dagger |0\rangle$$

$$\mathcal{H} = \sum_\ell \left[\frac{\Omega}{2} \left(b_\ell + b_\ell^\dagger \right) - \Delta n_\ell \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_\ell n_{\ell'}$$

$$V_{|\ell - \ell'|} \sim \frac{1}{|\ell - \ell'|^6}$$

$$n_\ell \equiv b_\ell^\dagger b_\ell$$

$n_\ell = 0, 1$ ‘hard core’ bosons

FSS model

S. Sachdev, K. Sengupta, and S.M. Girvin, PRB **66**, 075128 (2002)
P. Fendley, K. Sengupta, S. Sachdev, PRB **69**, 075106 (2004)

FSS model on the kagome lattice

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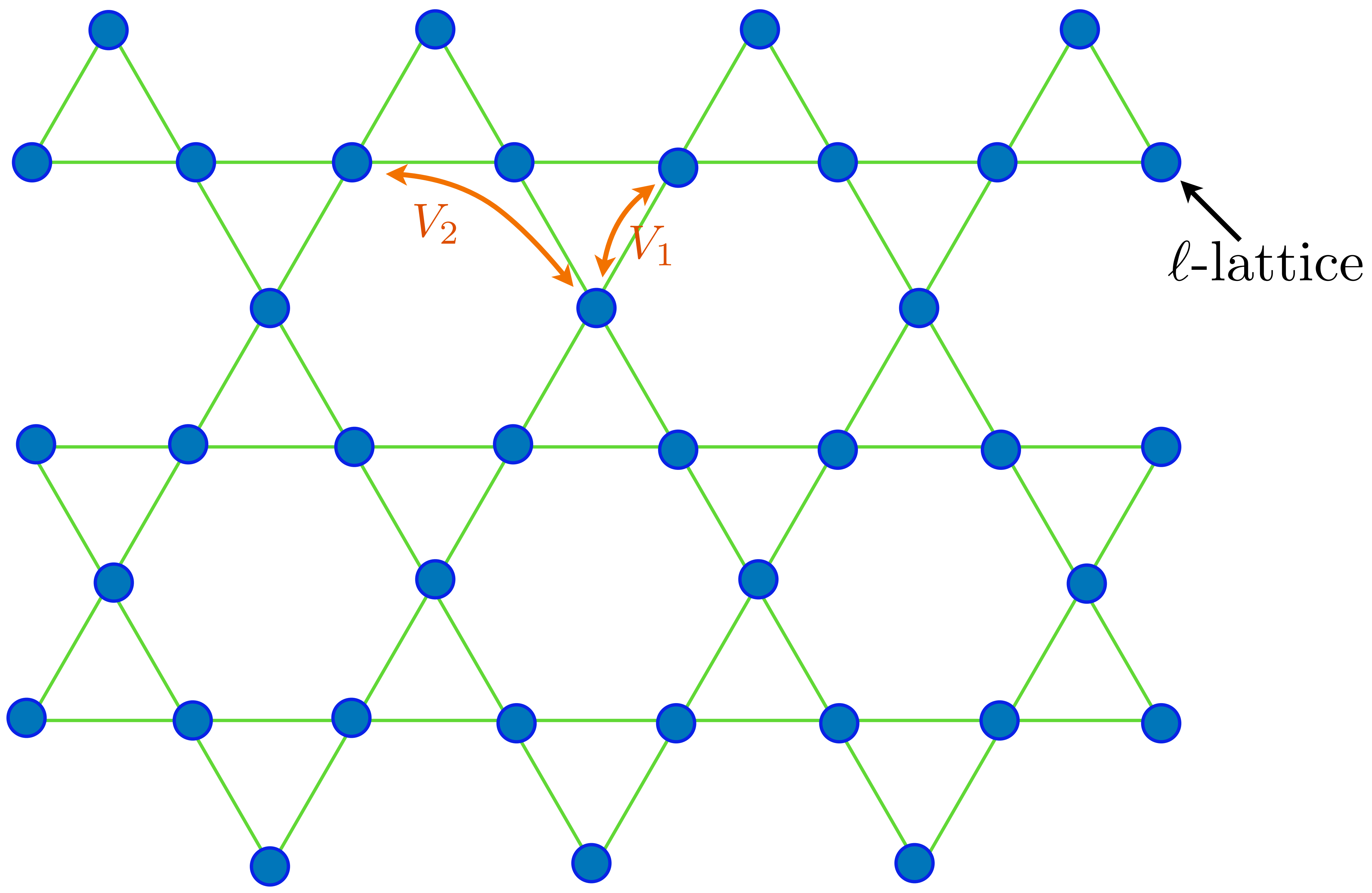
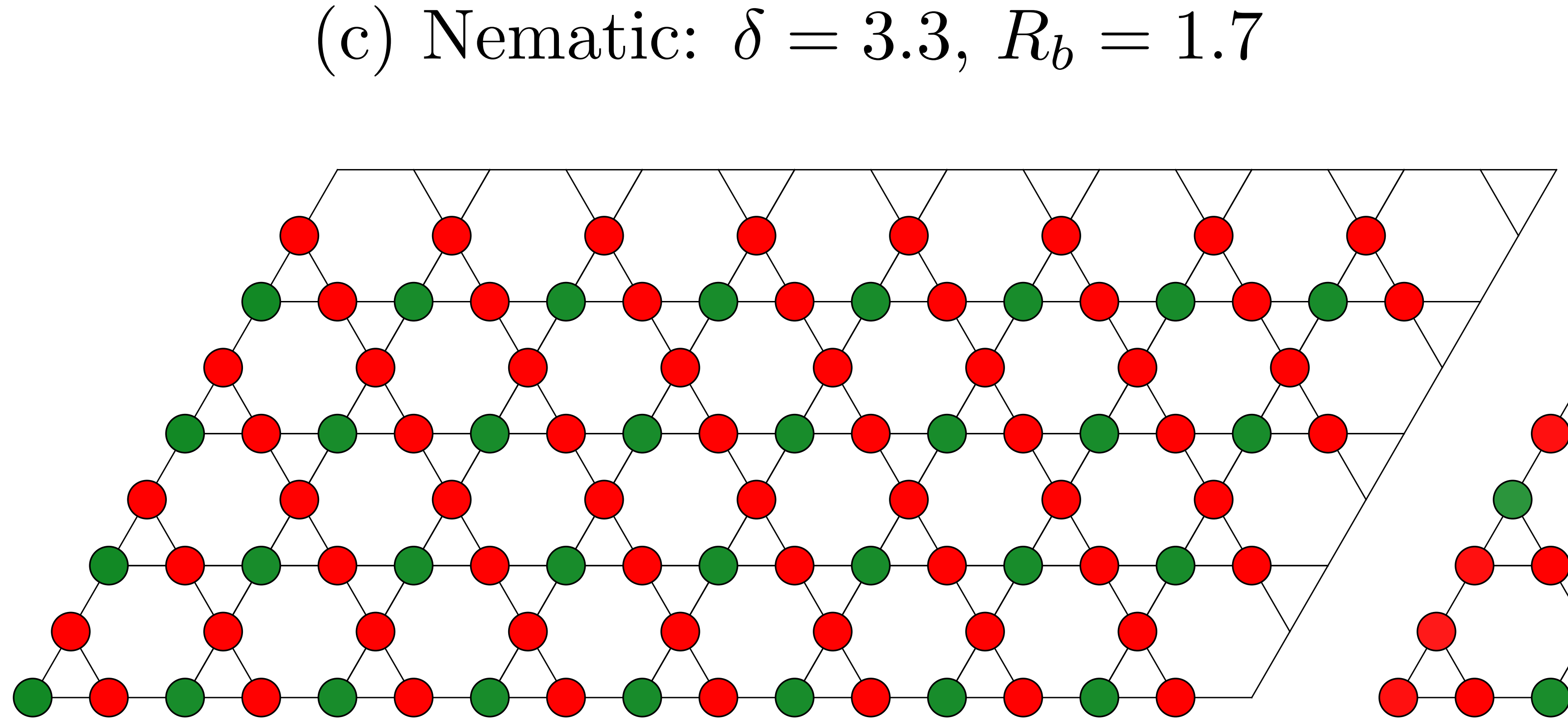
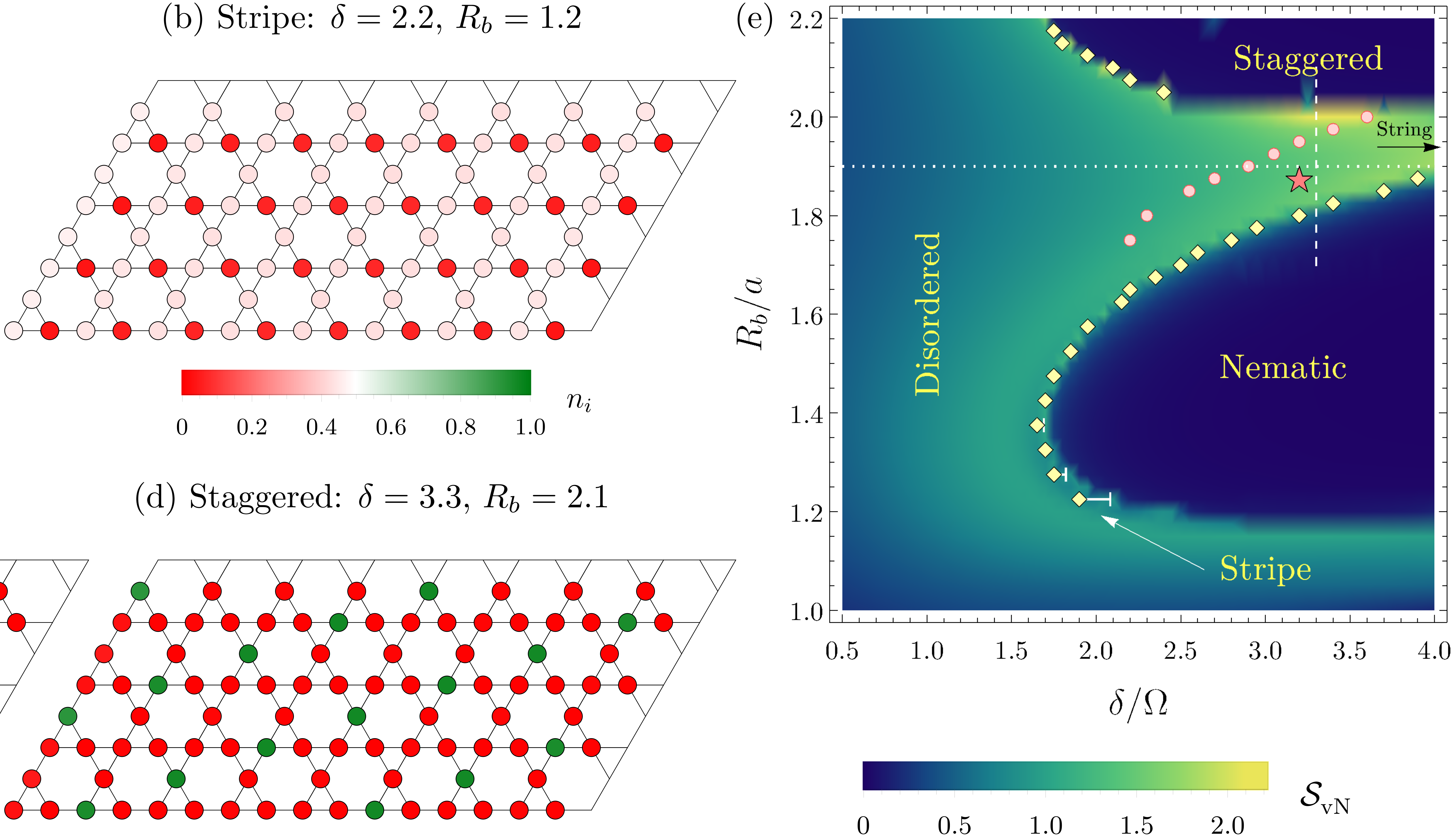


Rydberg atoms on site-kagome lattice: theory





R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **118**, e2015785118 (2021)

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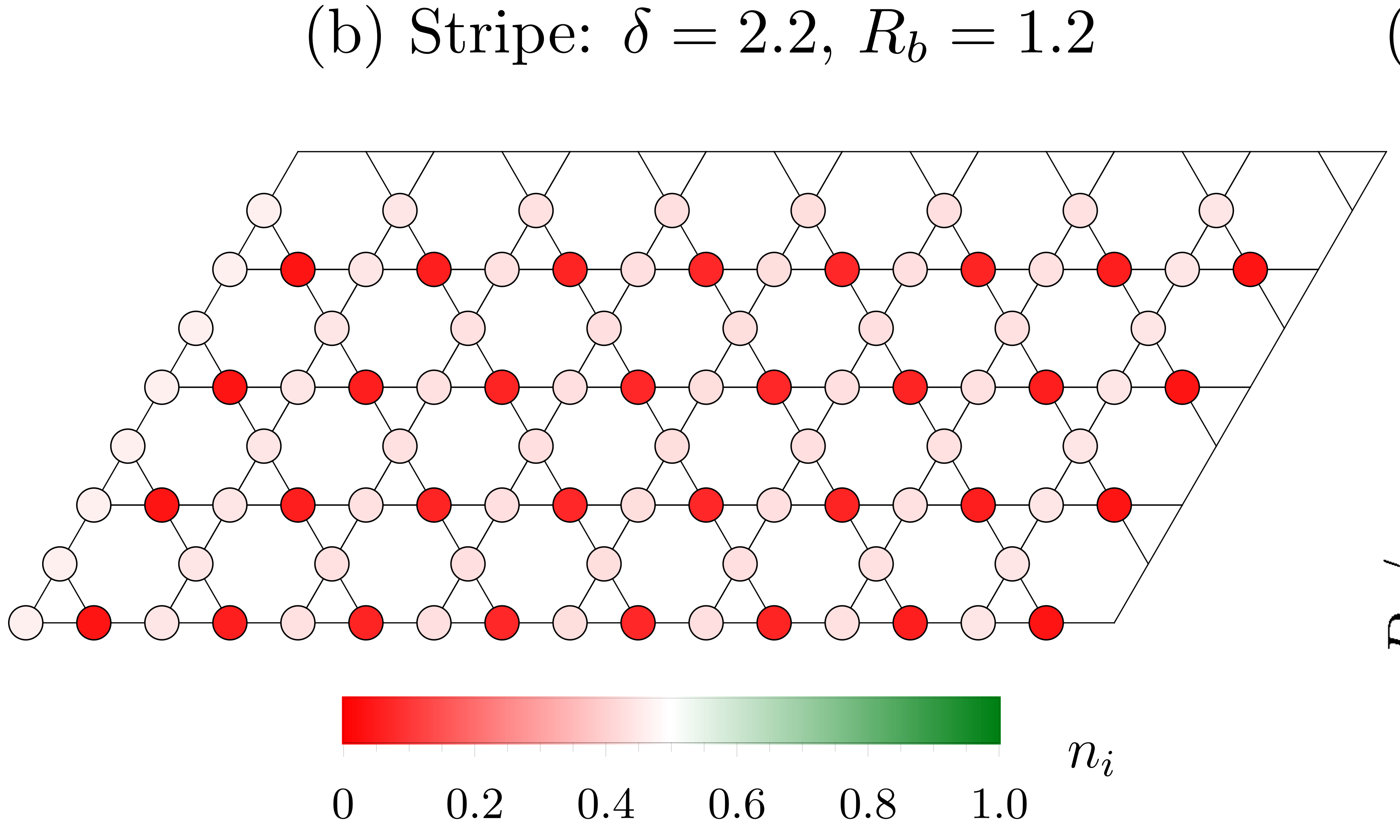

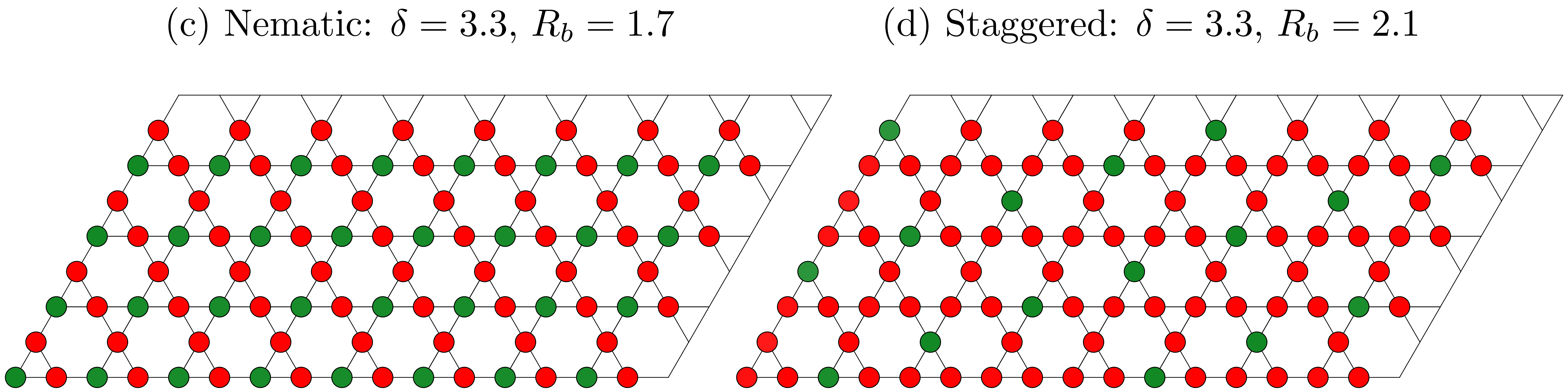

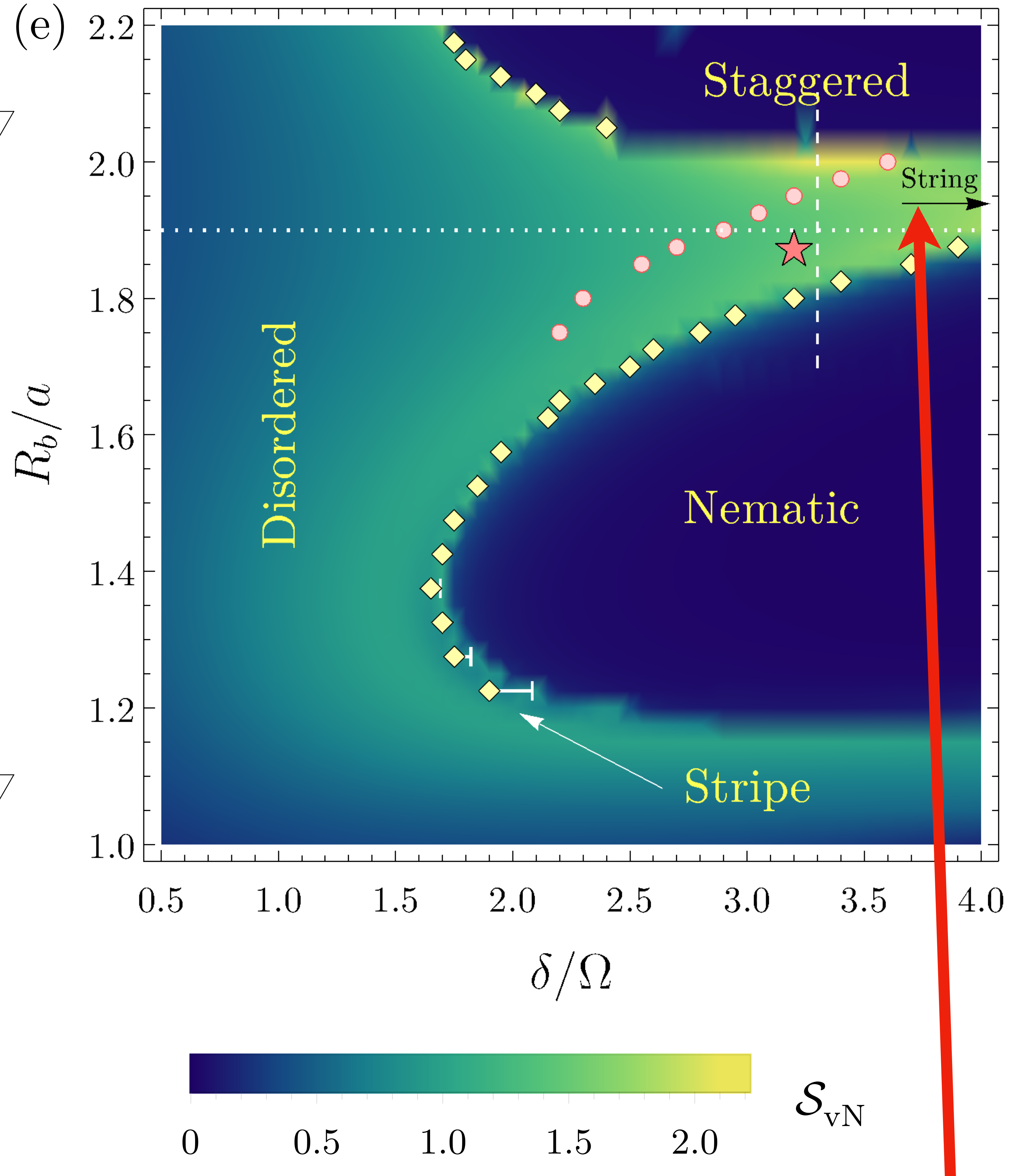

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **118**, e2015785118 (2021)

Topological spin liquid described by emergent $\mathbb{Z}_2$ gauge theory ?

From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell} \left[\frac{\Omega}{2} \left(b_\ell + b_\ell^\dagger \right) - \Delta n_\ell \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_\ell n_{\ell'}$$

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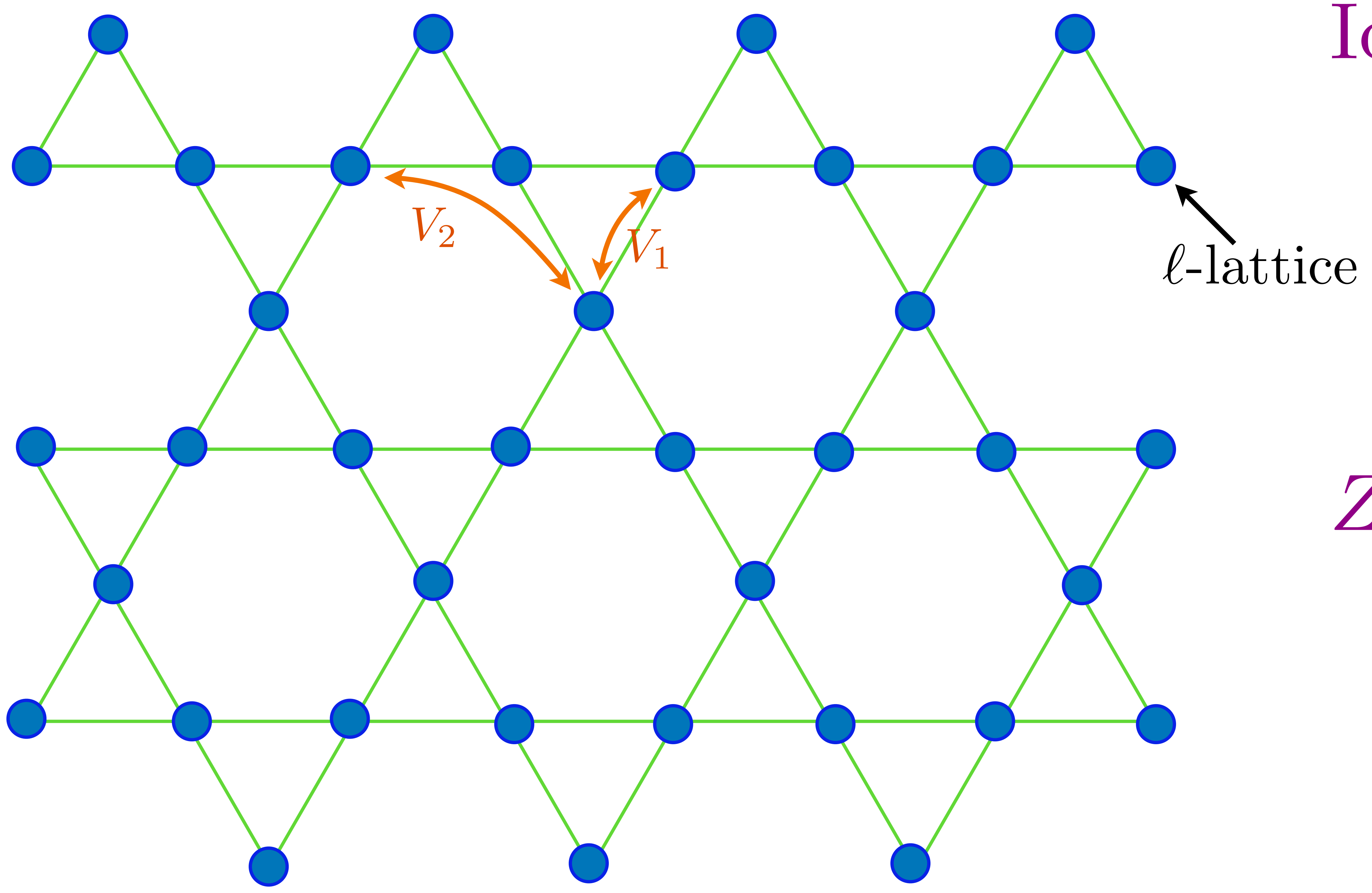

Identify hard core bosons with a qubit X, Y, Z

$$b_\ell + b_\ell^\dagger \Leftrightarrow Z_\ell$$
$$n_\ell \Leftrightarrow (1 - X_\ell)/2$$

Z will become the $\mathbb{Z}_2$ gauge field

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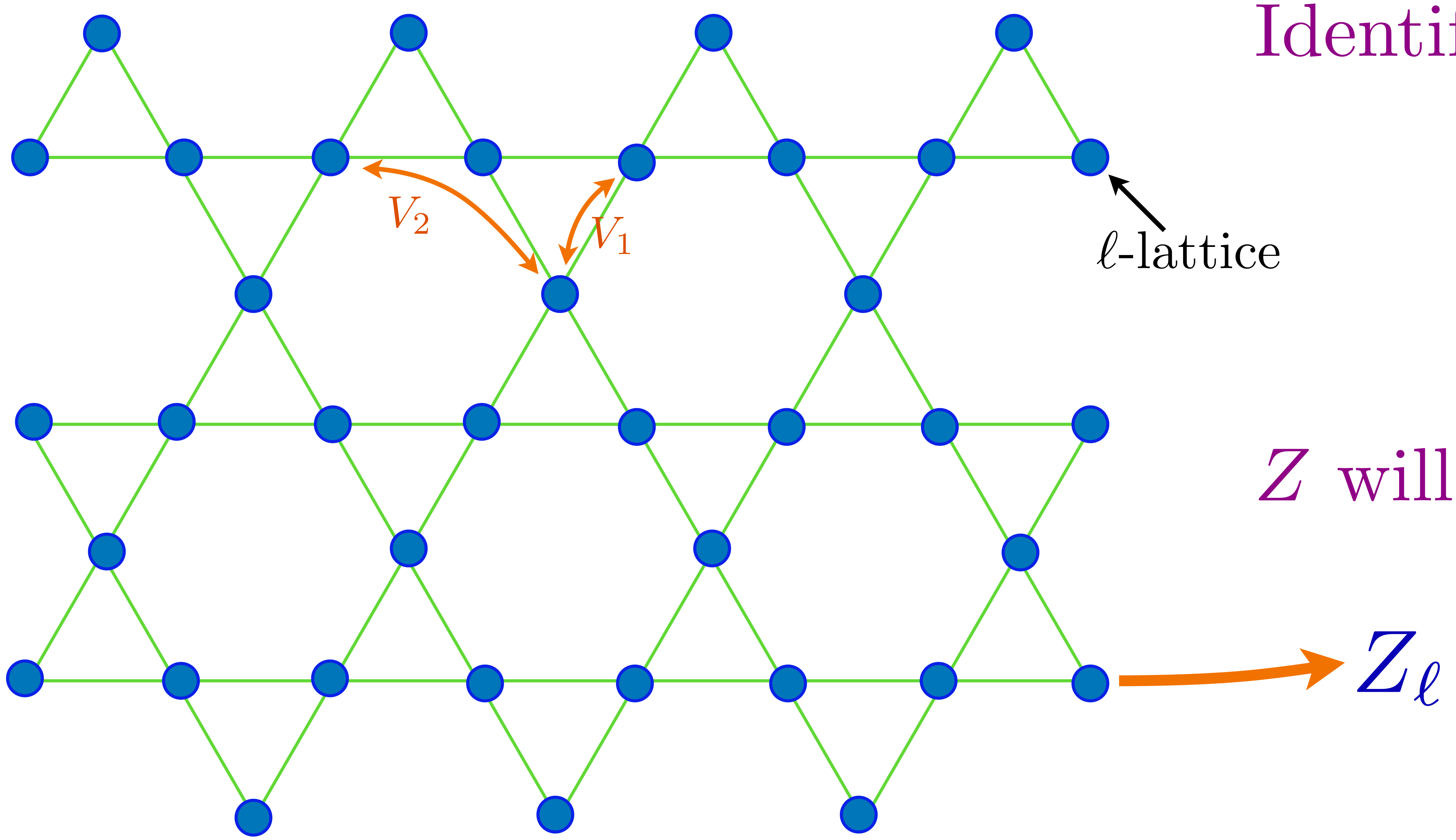
$$\mathcal{H} = \sum_{\ell} \left[\frac{\Omega}{2} Z_\ell + \frac{\Delta}{2} X_\ell \right] + \sum_{\ell < \ell'} \frac{V_{|\ell - \ell'|}}{4} (1 - X_\ell)(1 - X_{\ell'})$$



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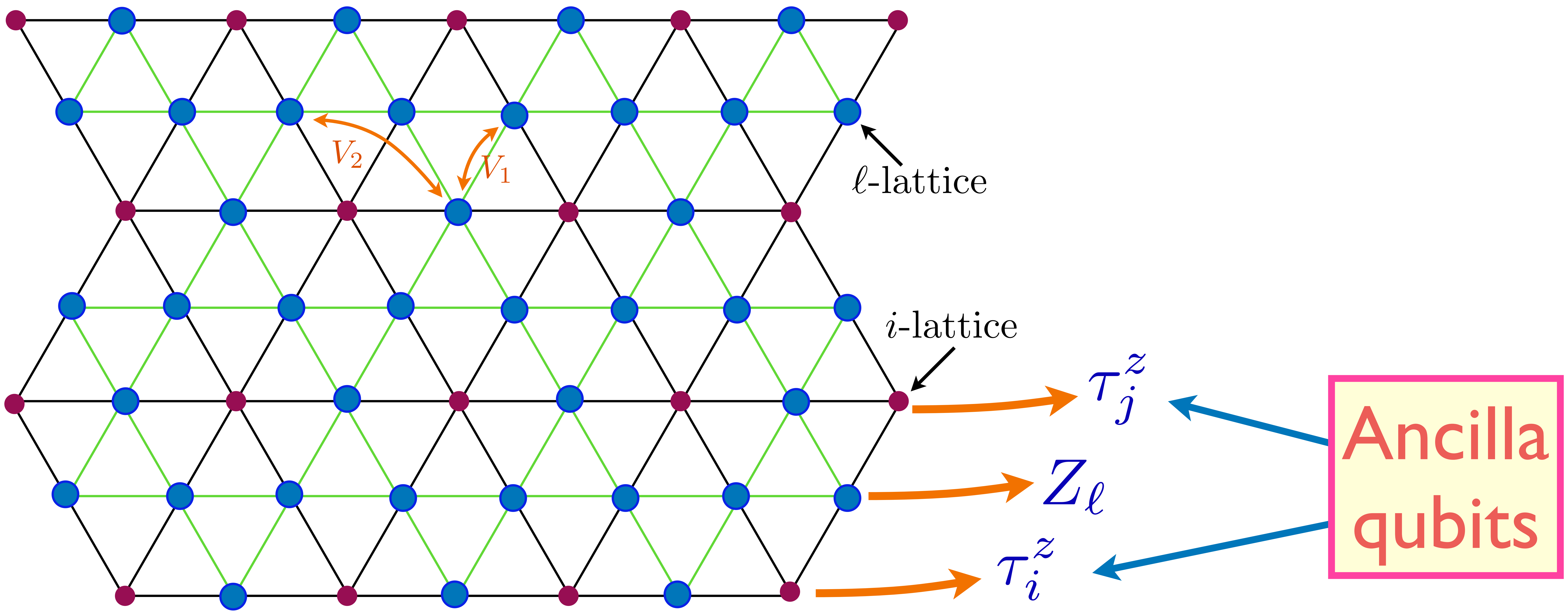
$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[\frac{\Omega}{2} \tau_i^z \, Z_\ell \, \tau_j^z + \frac{\Delta}{2} X_\ell \right] + \sum_{\ell < \ell'} \frac{V_{|\ell - \ell'|}}{4} (1 - X_\ell)(1 - X_{\ell'})$$

Introduce $\mathbb{Z}_2$ matter fields on 'i sites'. Gauge invariance: $\tau_i^z \to \varrho_i \tau_i^z$, $Z_{ij} \to \varrho_i Z_{ij} \varrho_j$, $\tau_i^x \to \tau_i^x$, $X_\ell \to X_\ell$, $\varrho_i = \pm 1$. Gauss law constraint: $G_i = \tau_i^x \prod_{\ell \in i} X_\ell = 1$.



R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, S. Sachdev, PNAS **118**, e2015785118 (2021)

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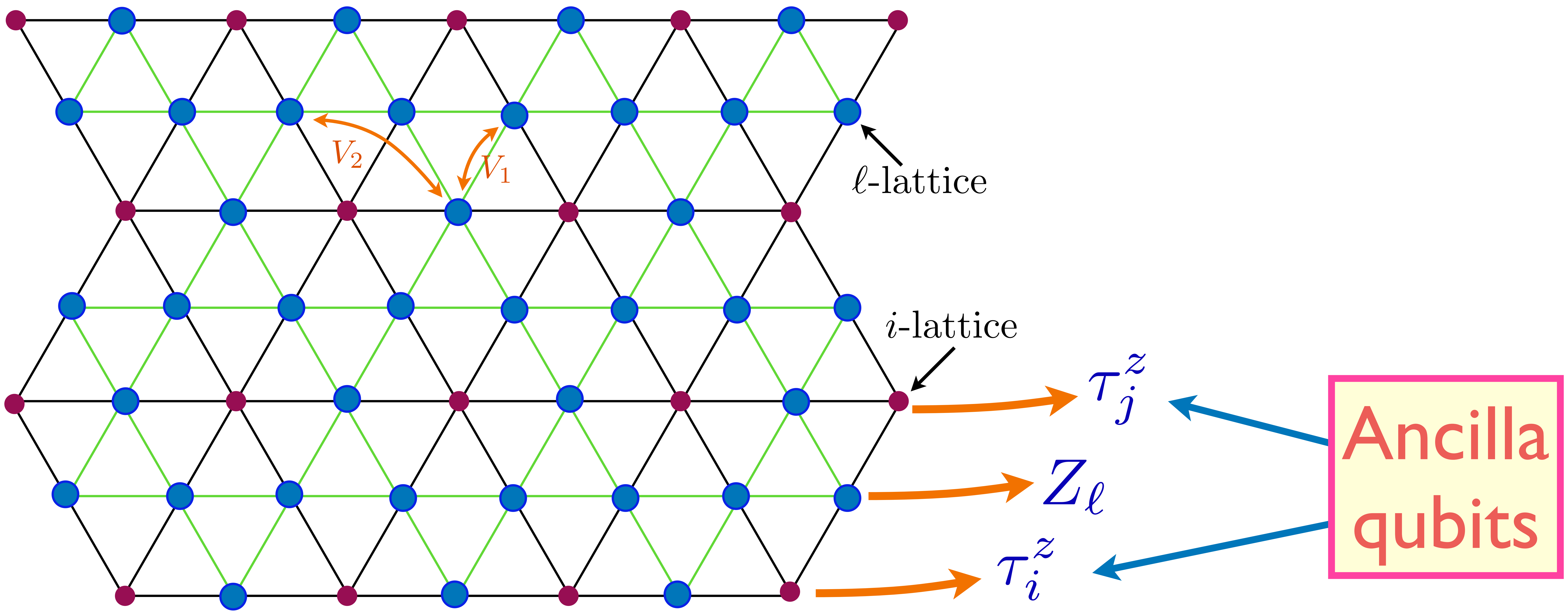
The K_{loop} terms are generated in a large V expansion: ‘resonance’ between Rydberg states can stabilize a phase with deconfined $\mathbb{Z}_2$ gauge charges *i.e.* a $\mathbb{Z}_2$ spin liquid



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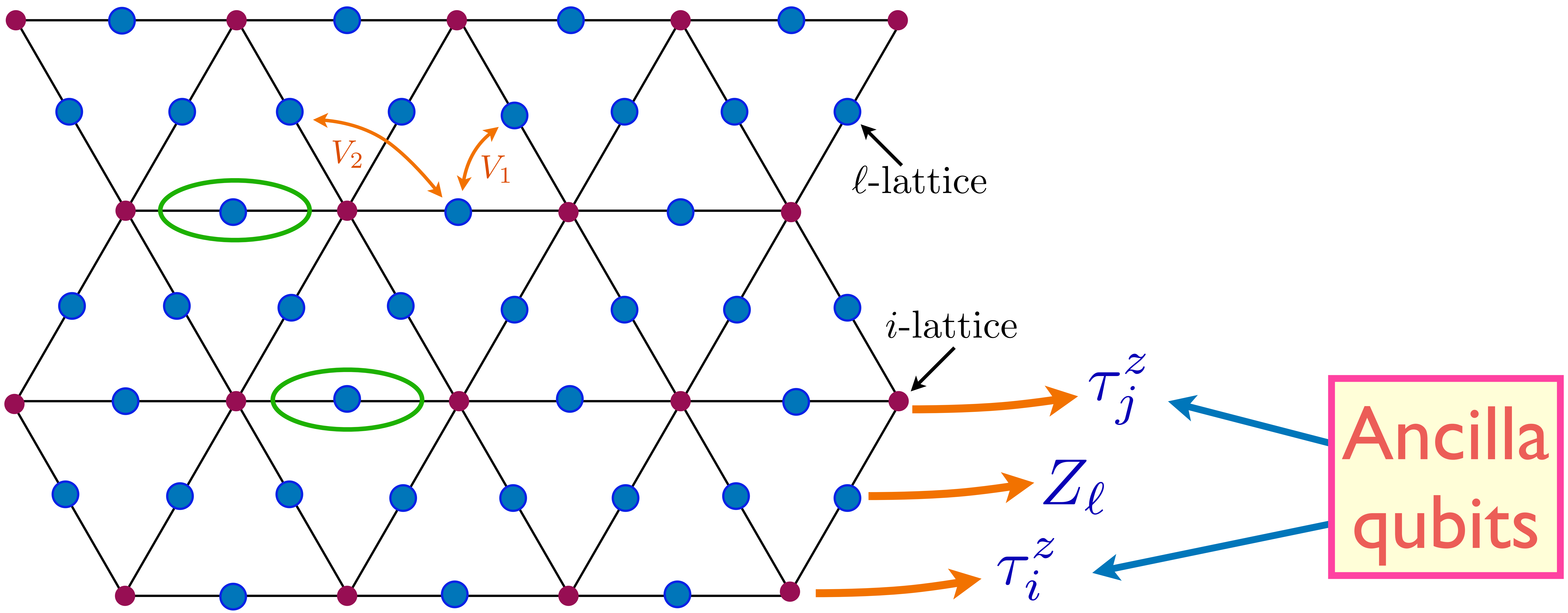

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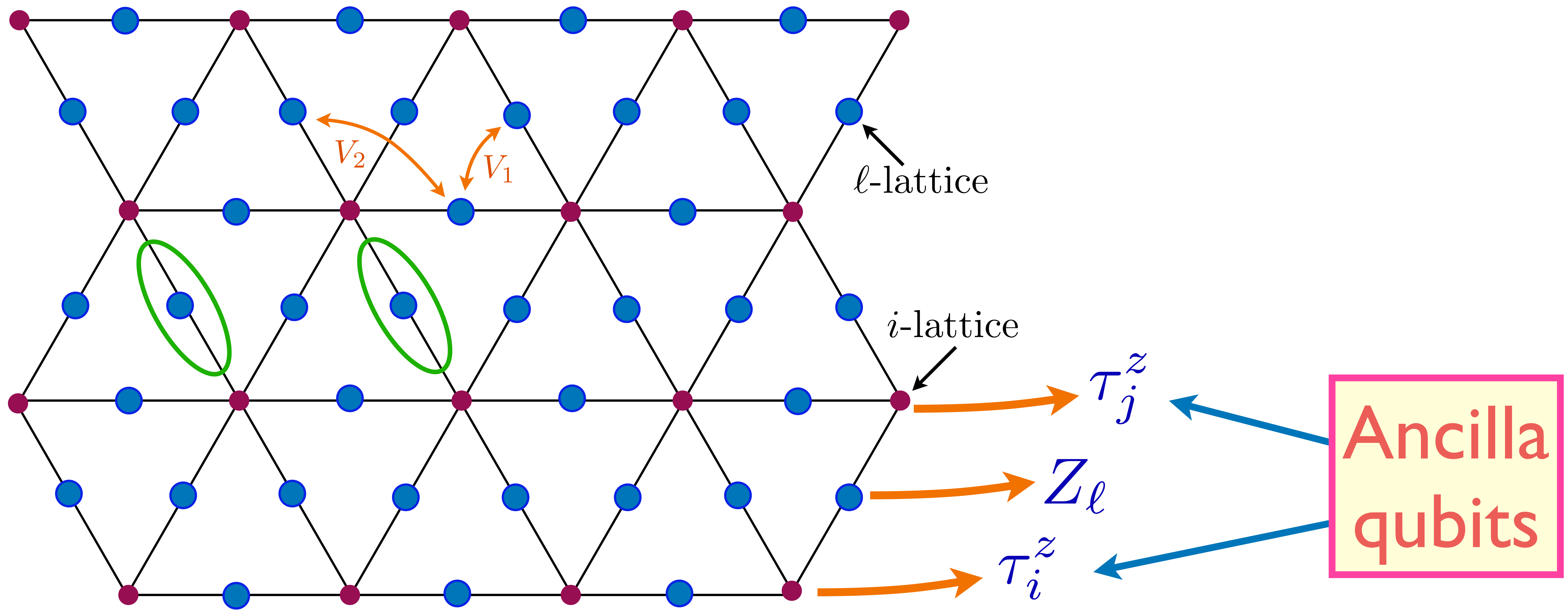

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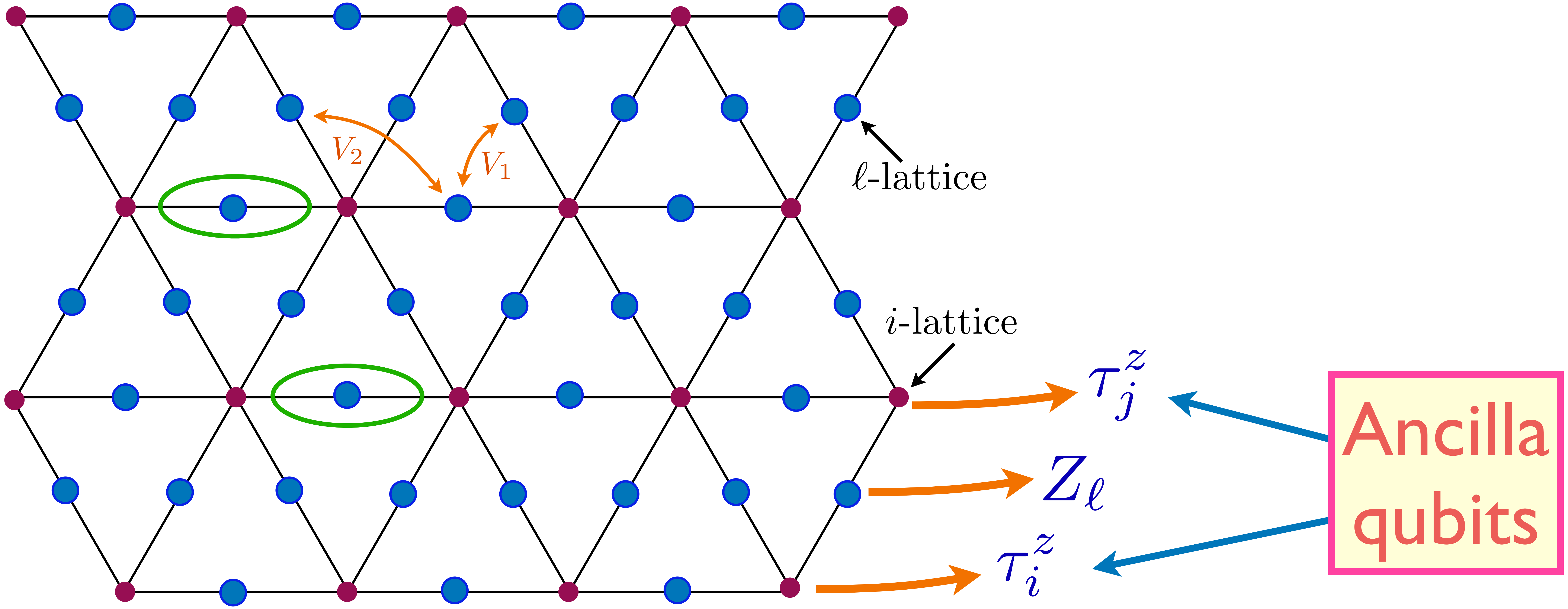

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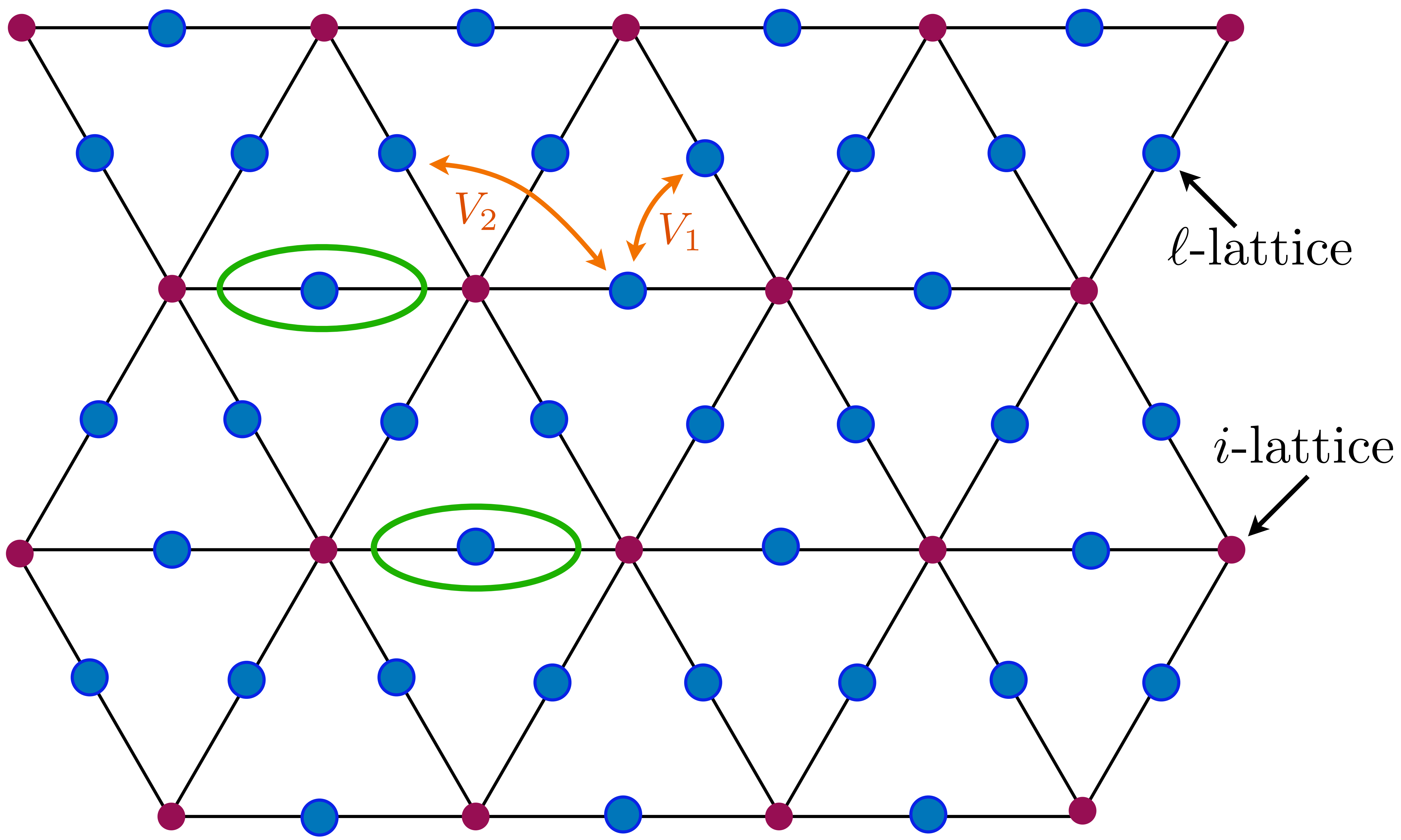

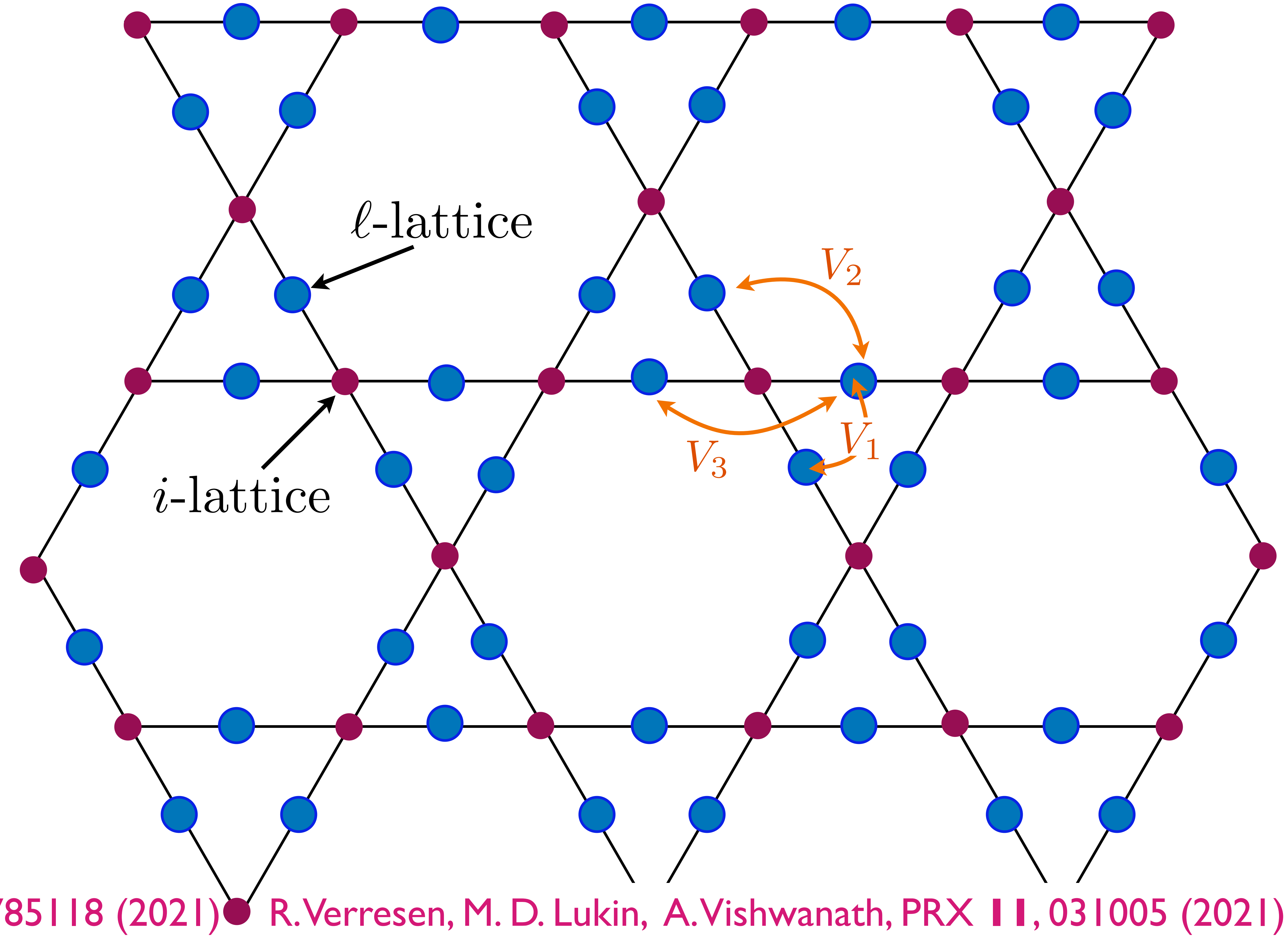

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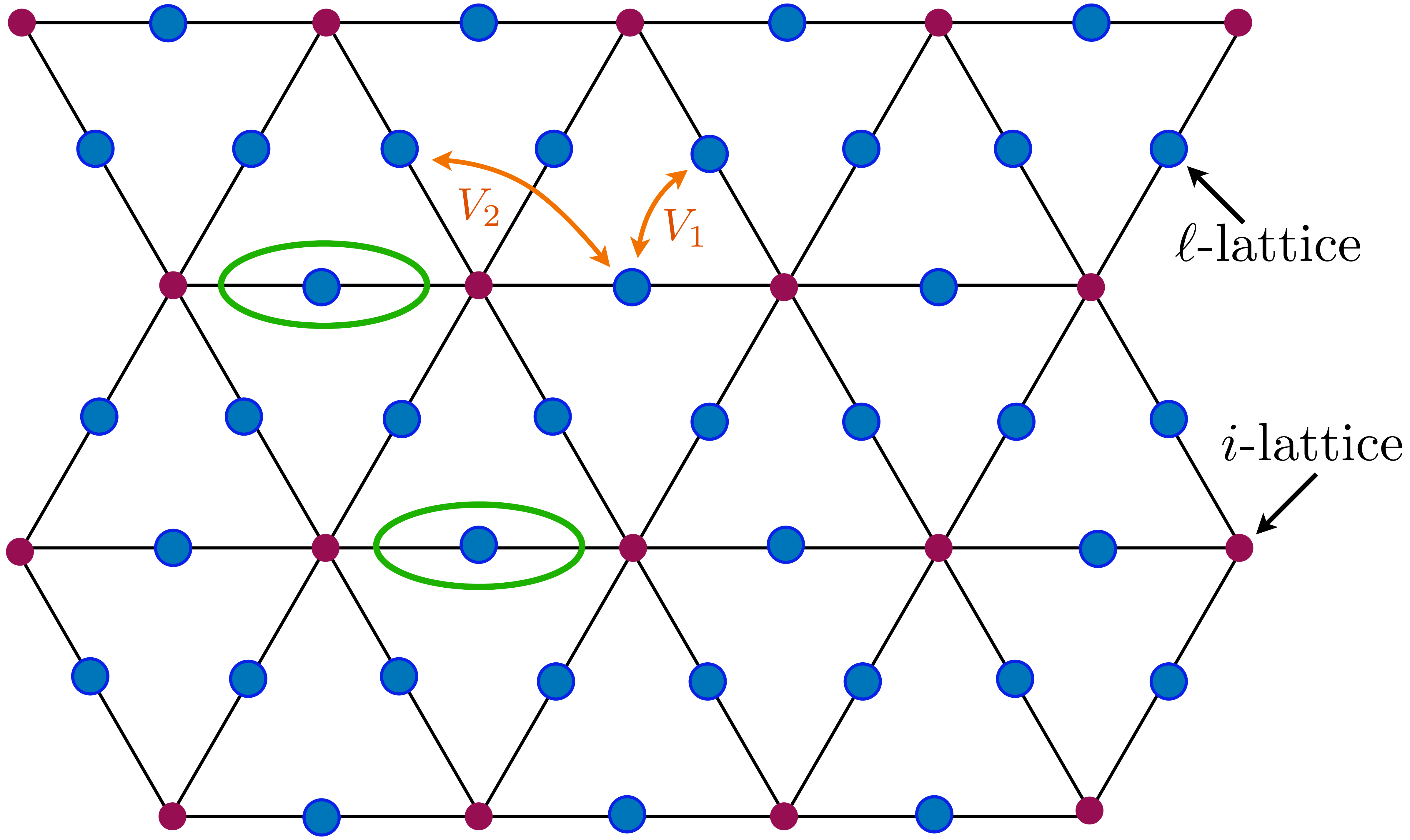

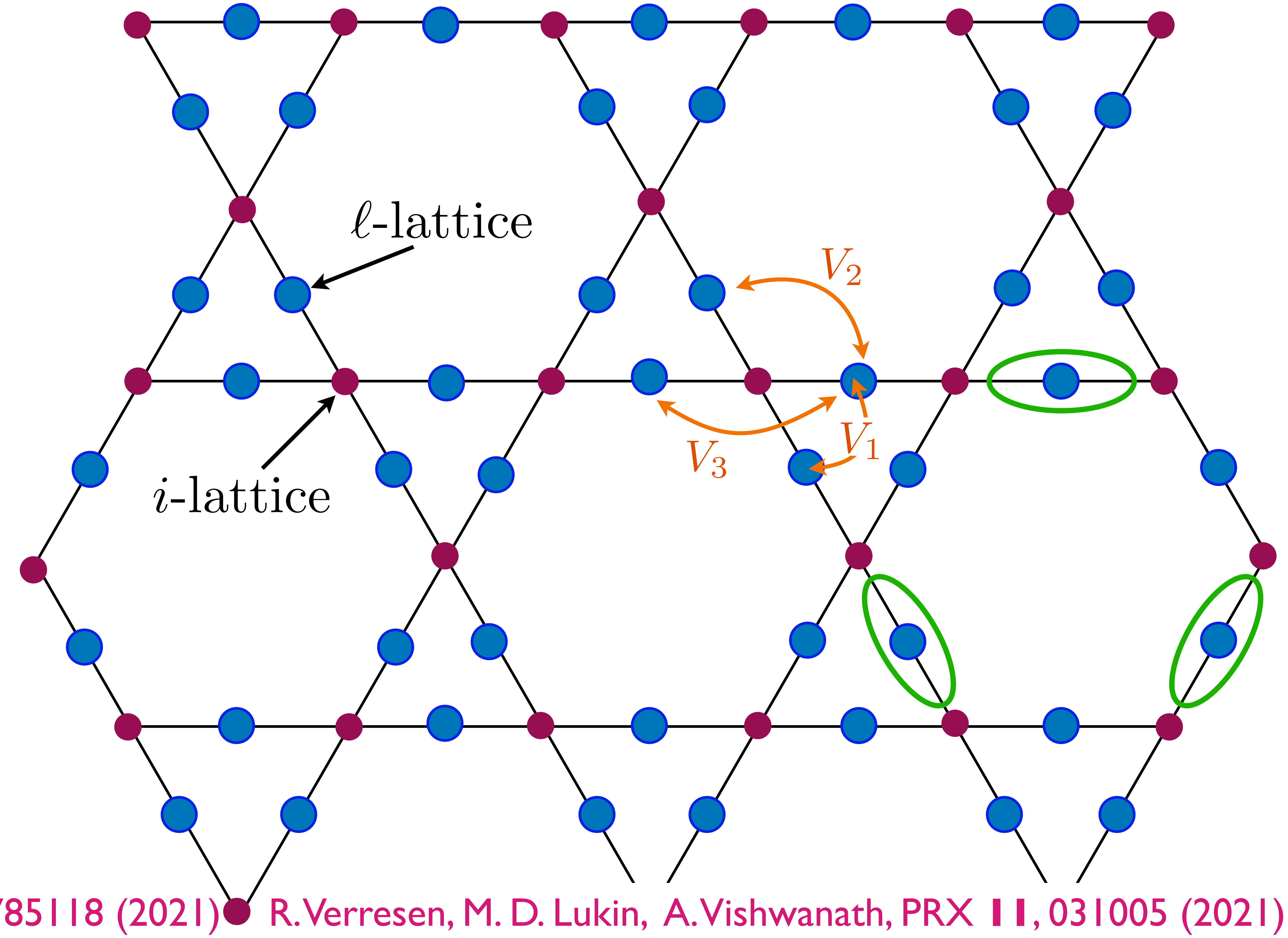

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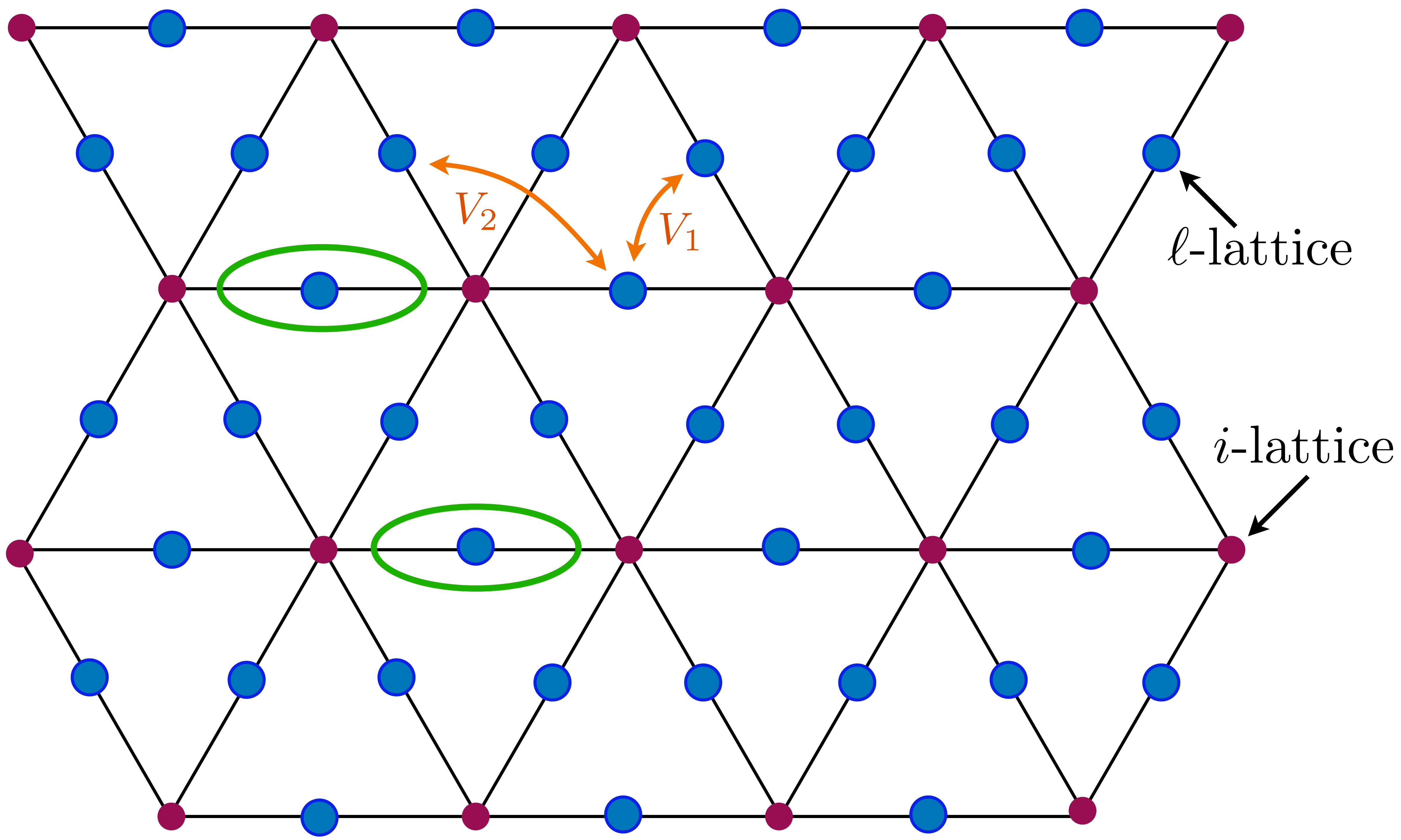

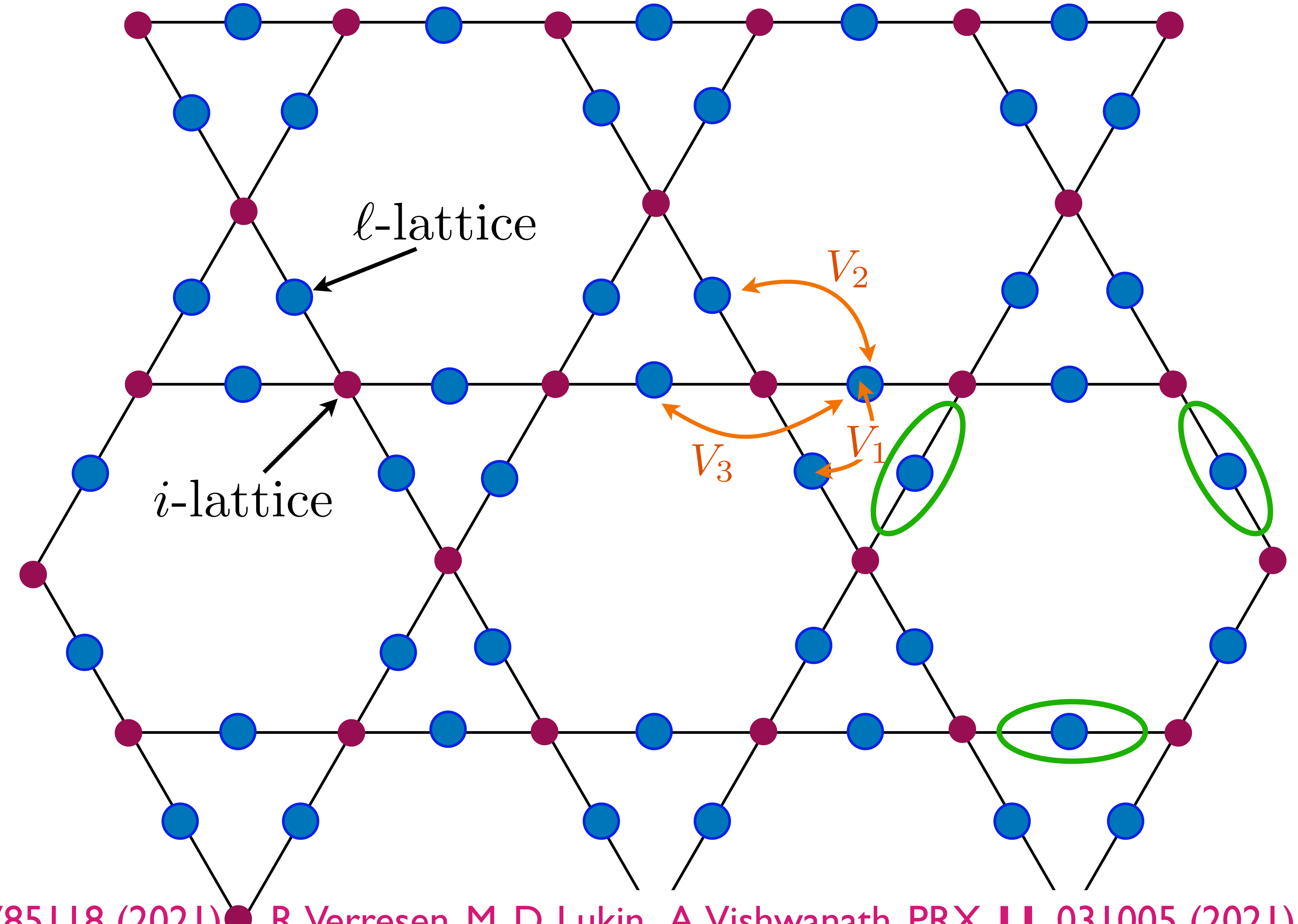

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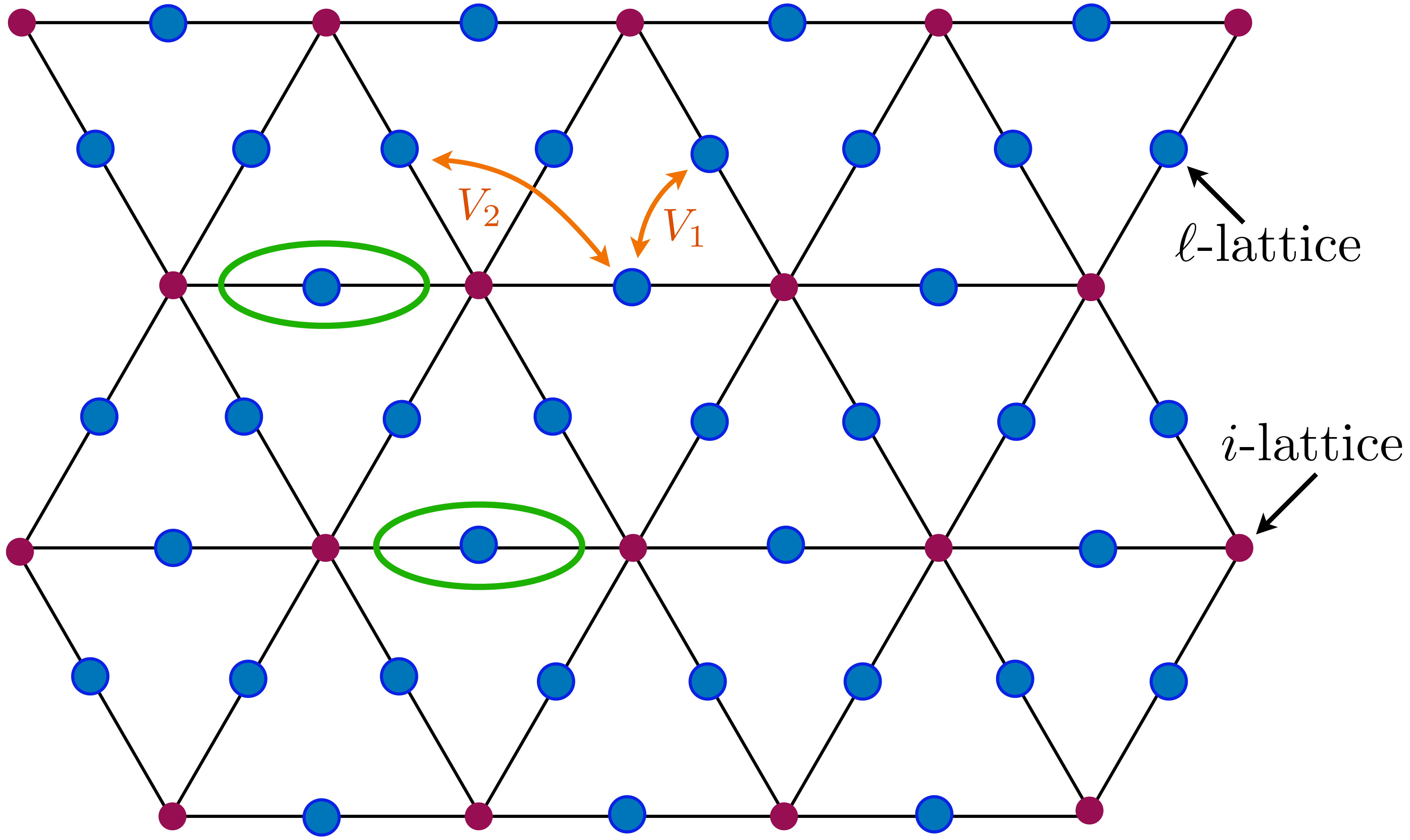

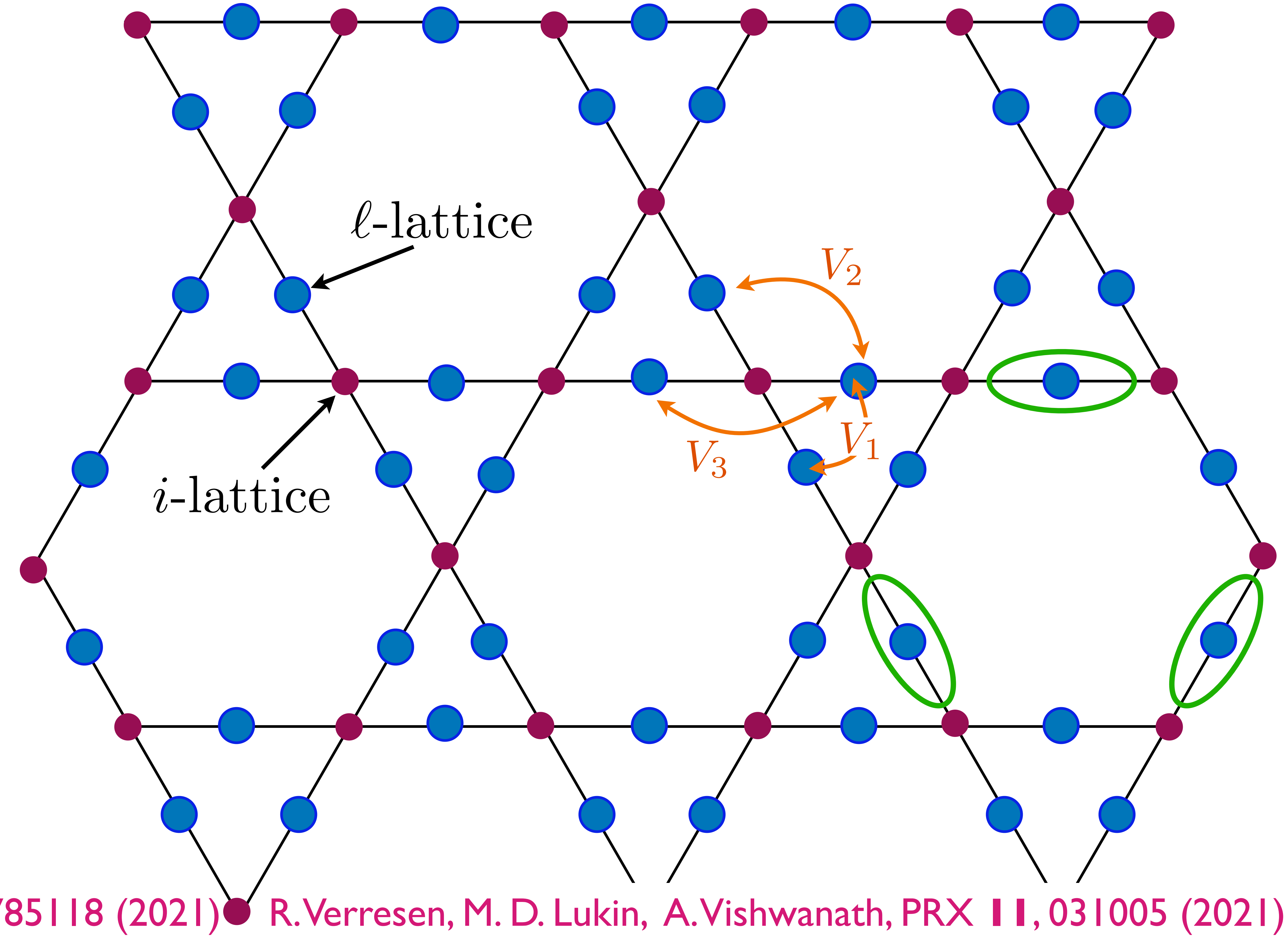

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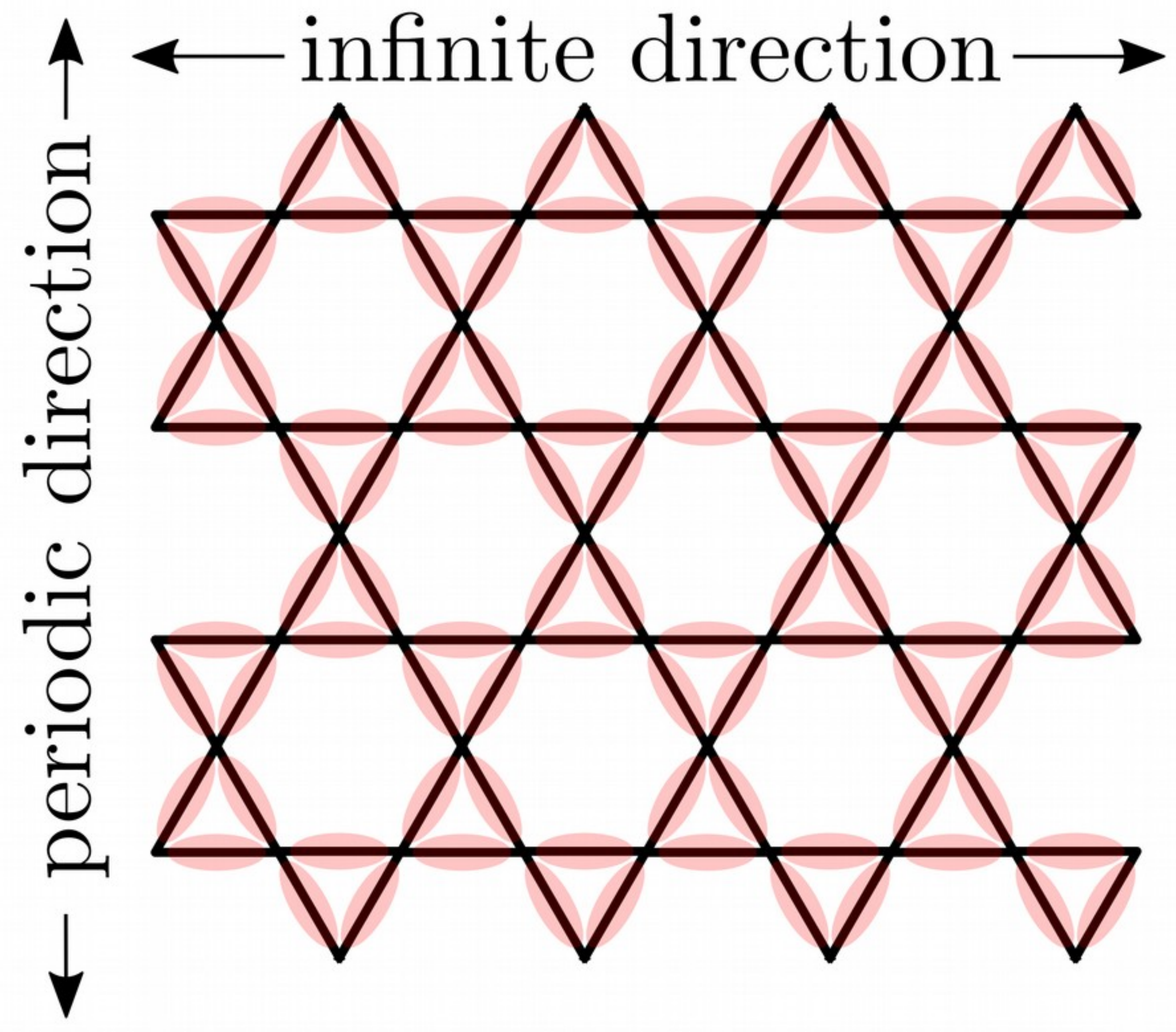
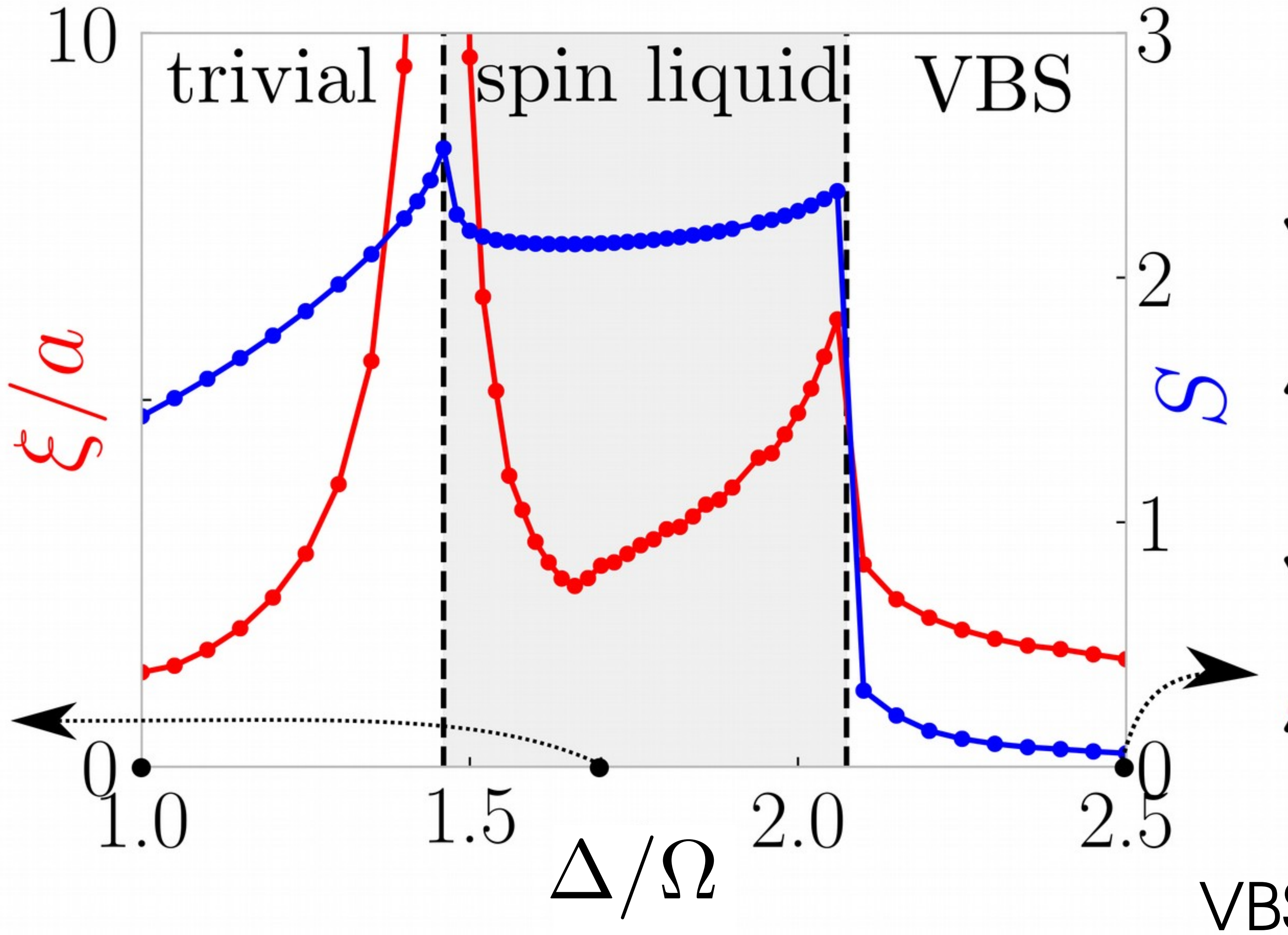
Rydberg atoms on link-kagome lattice: theory

$$\mathcal{H} = \sum_j \left[\frac{\Omega}{2} \left(b_j + b_j^\dagger \right) - \Delta n_j \right] + \sum_{i<j} V_{|i-j|} n_i n_j \,, \quad n_j \equiv b_j^\dagger b_j = 0, 1.$$

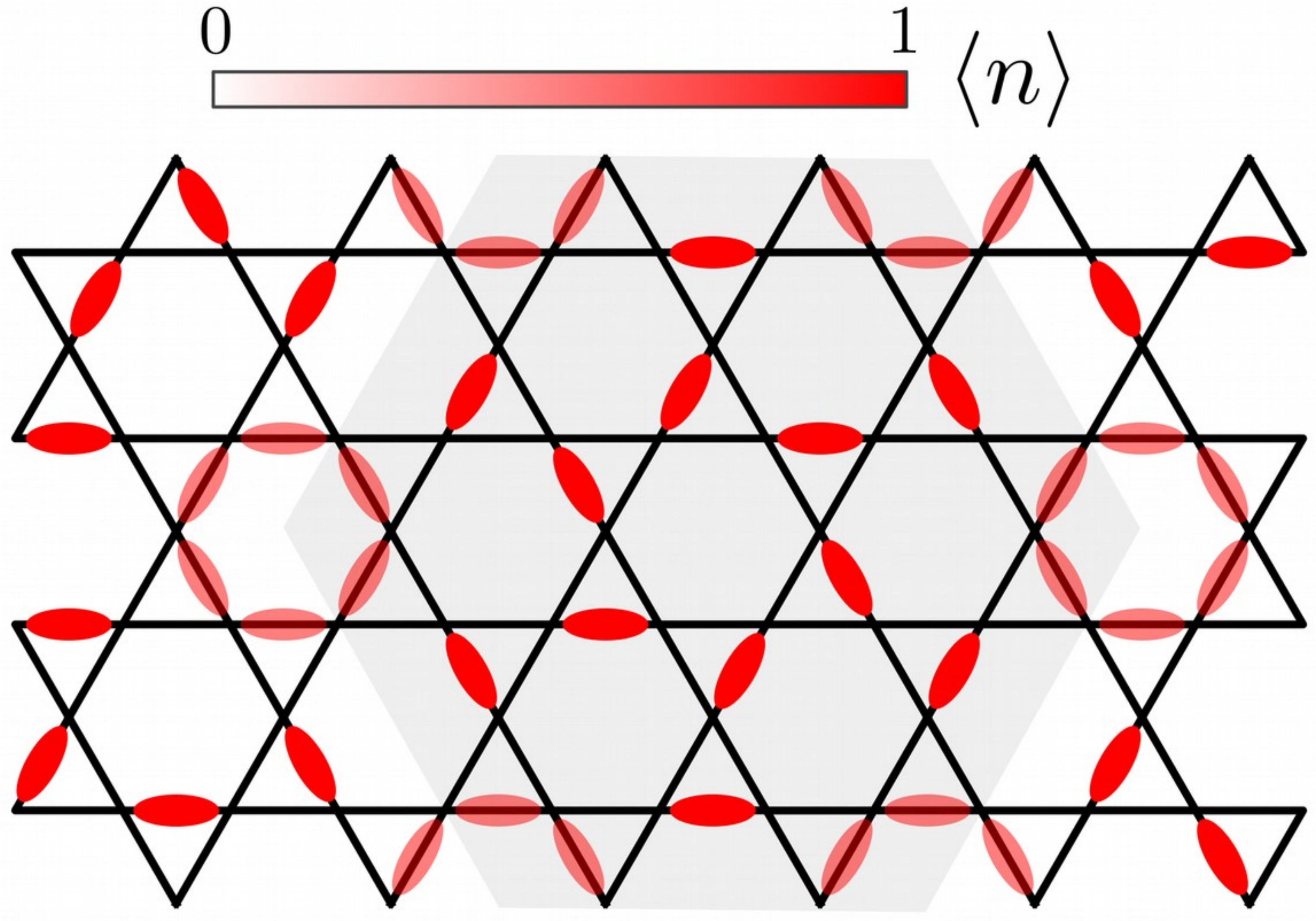
The sites j are on the links of the kagome lattice.
Examine the PXP model, $V_{\text{nearest neighbor}} = \infty$, other $V_k = 0$.



intermediate featureless phase

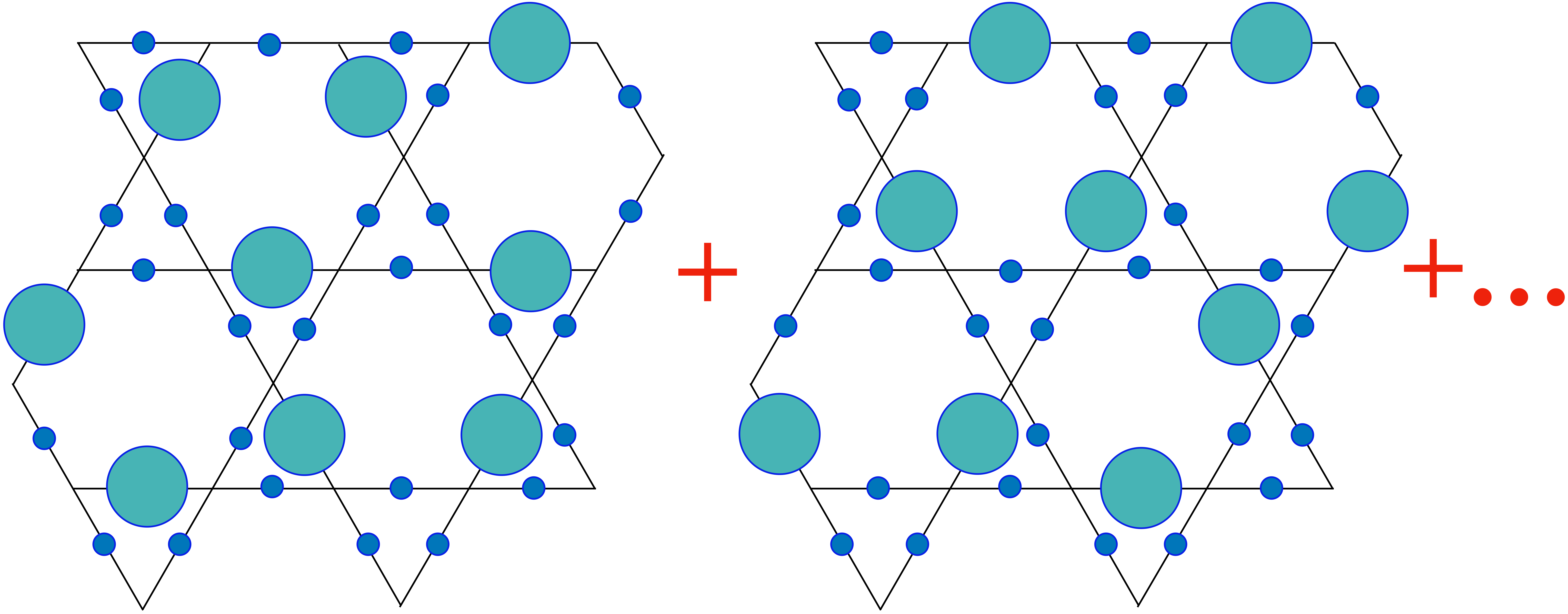


DMRG



VBS pattern studied before in context of Heisenberg model on kagome lattice (Nikolic and Senthil, Singh and Huse, ...)

R. Verresen, M. D. Lukin, A. Vishwanath, PRX **11**, 031005 (2021)



Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, Science **374**, 1242 (2021).

Rydberg atoms
on the
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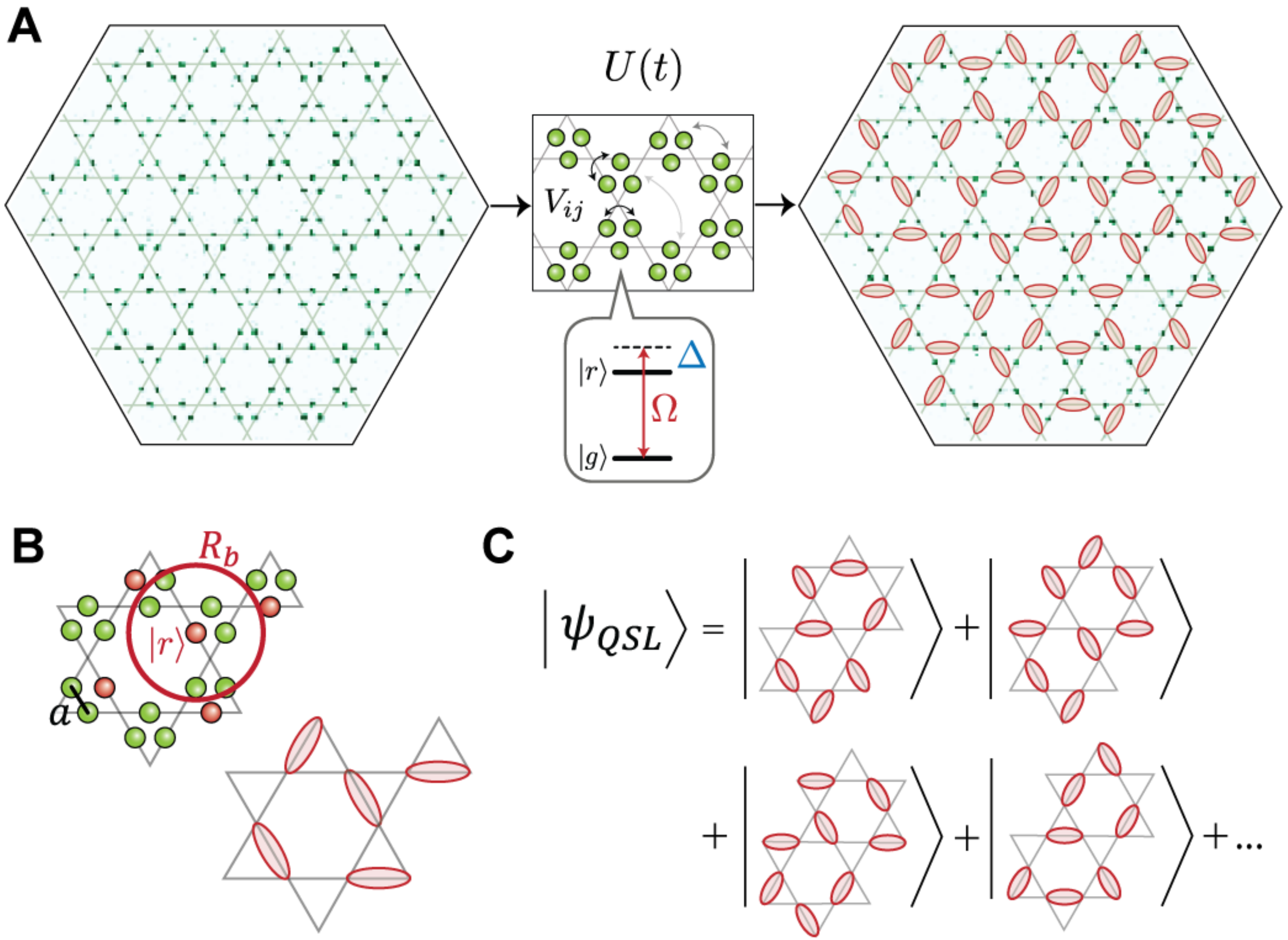
experiment



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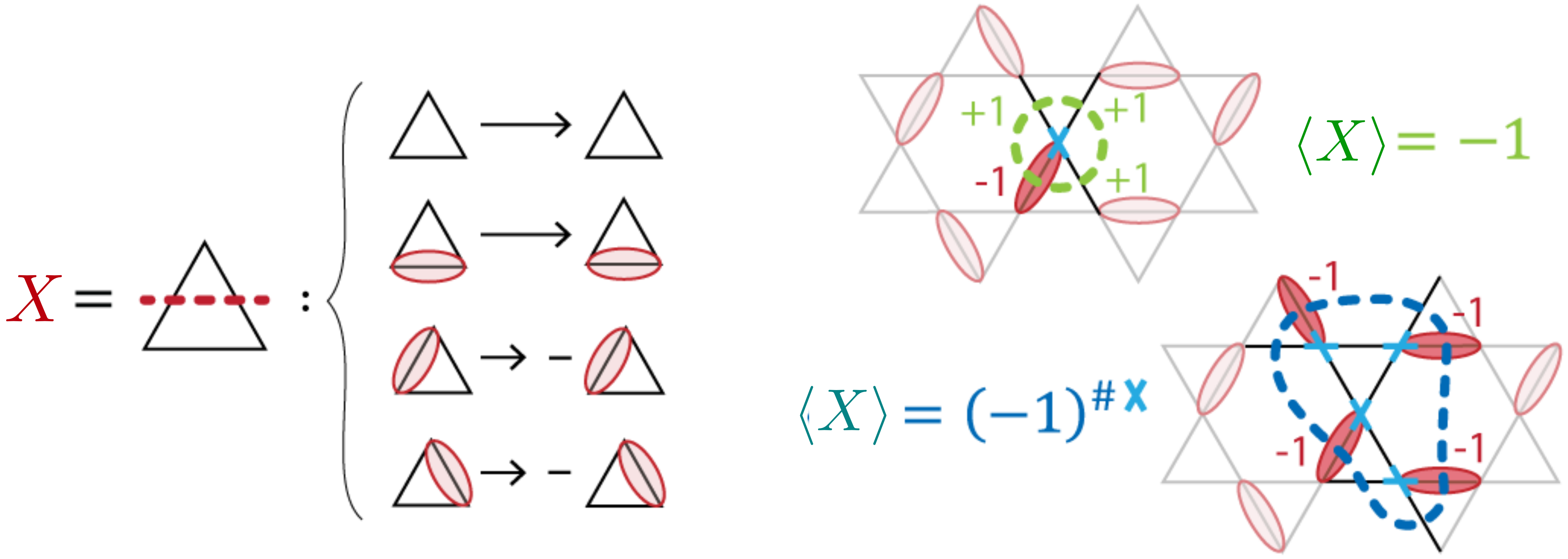
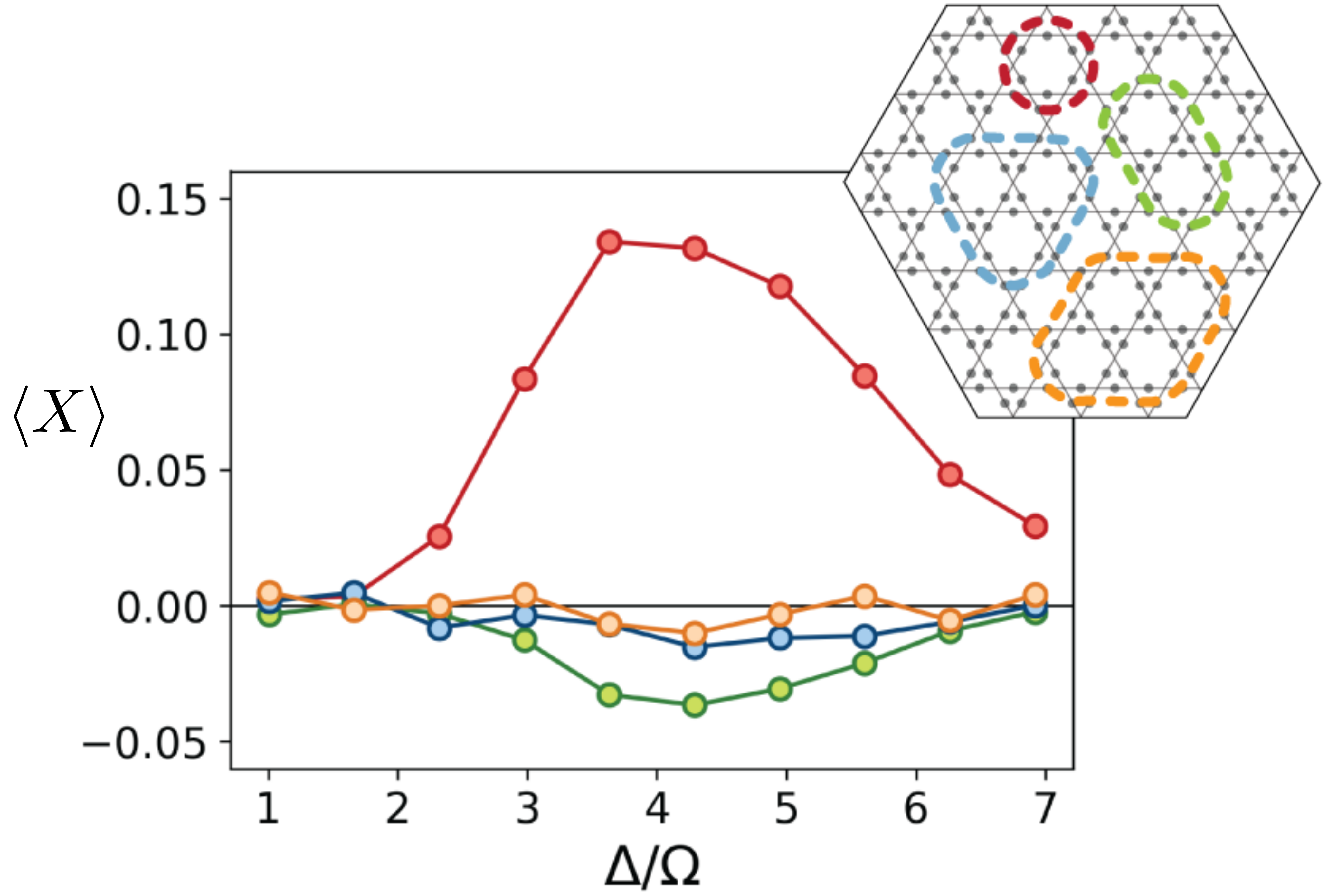


Measurement of
the topological
X operator
$= \prod_{\text{loop}} X_\ell$.
Detects close-packed dimers.



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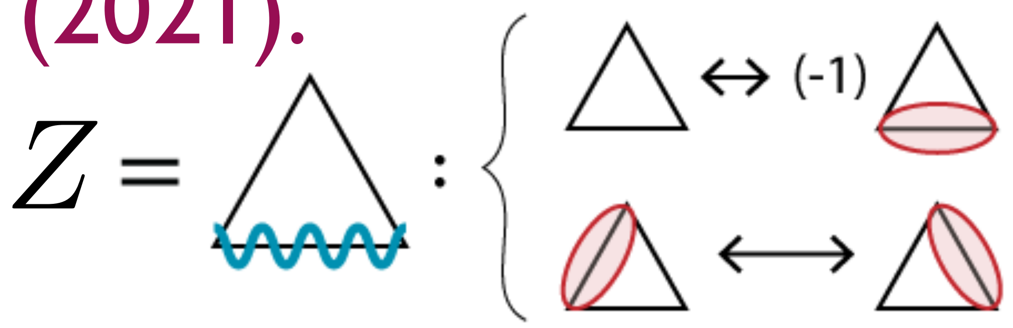

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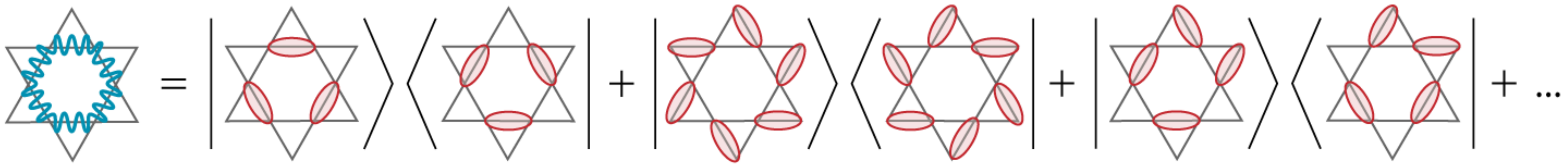
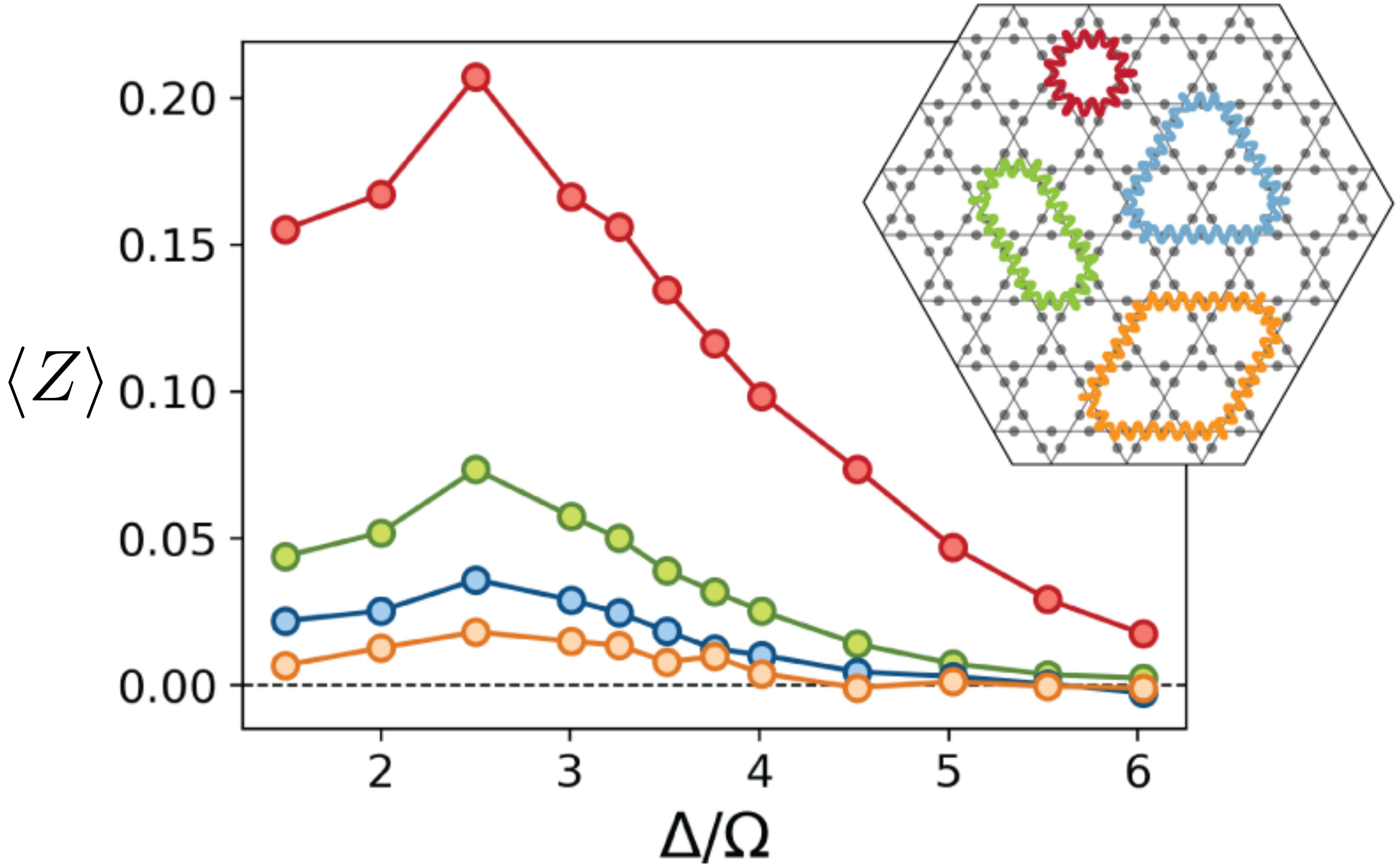


Measurement of
the topological
Z operator.
Detects resonance
between dimer loops.



1. Spin liquids and Z_2 gauge theory

2. Rydberg atoms as a Z_2 gauge theory

   Probing topological spin liquids

3. Paramagnon fractionalization theory of the pseudogap metal of the Hubbard model

Yahui Zhang

Alexander
Nikolaenko

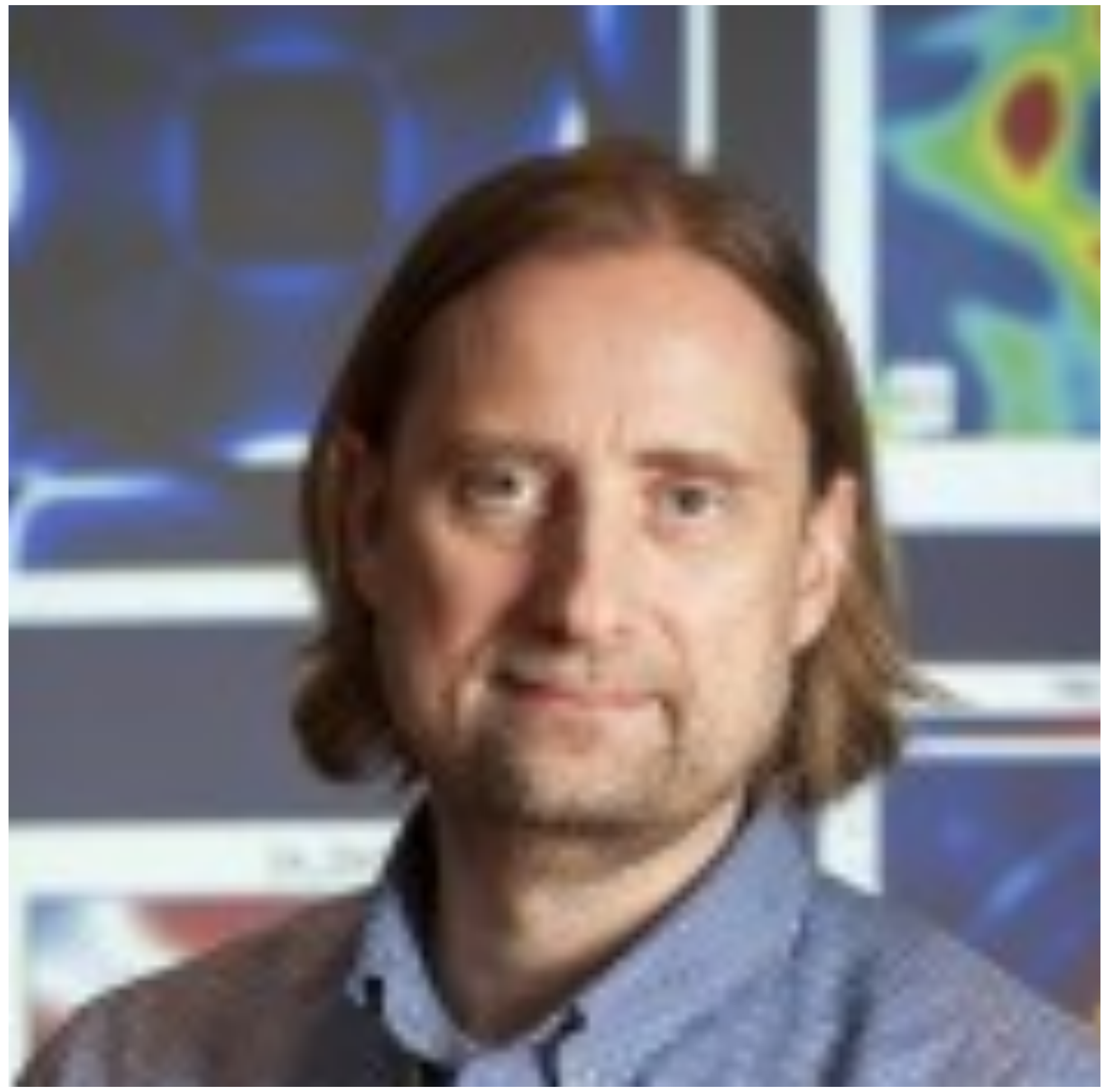
Maria
Tikhanovskaya

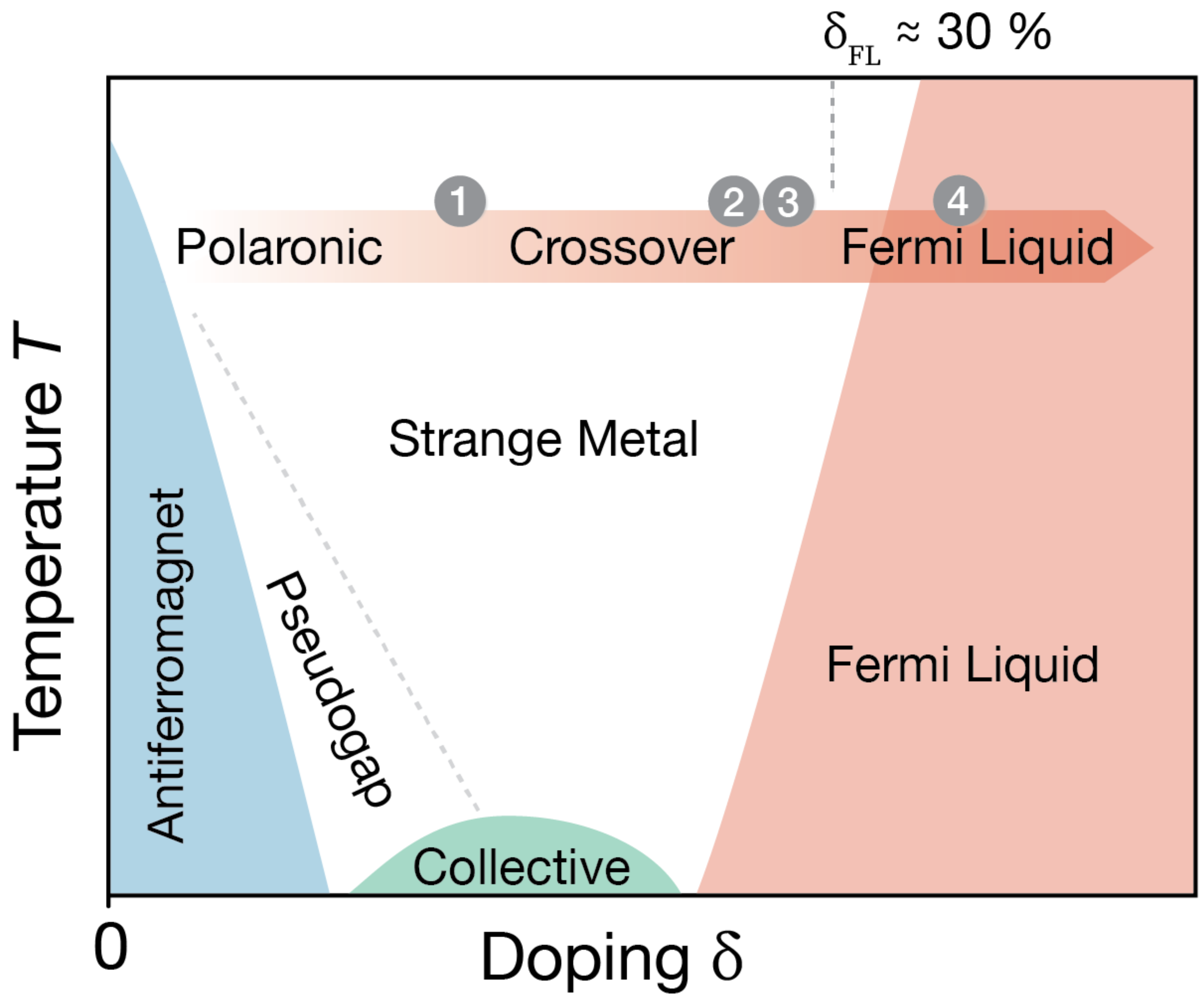
Eric Mascot

arXiv: 2001.09159
arXiv: 2103.05009

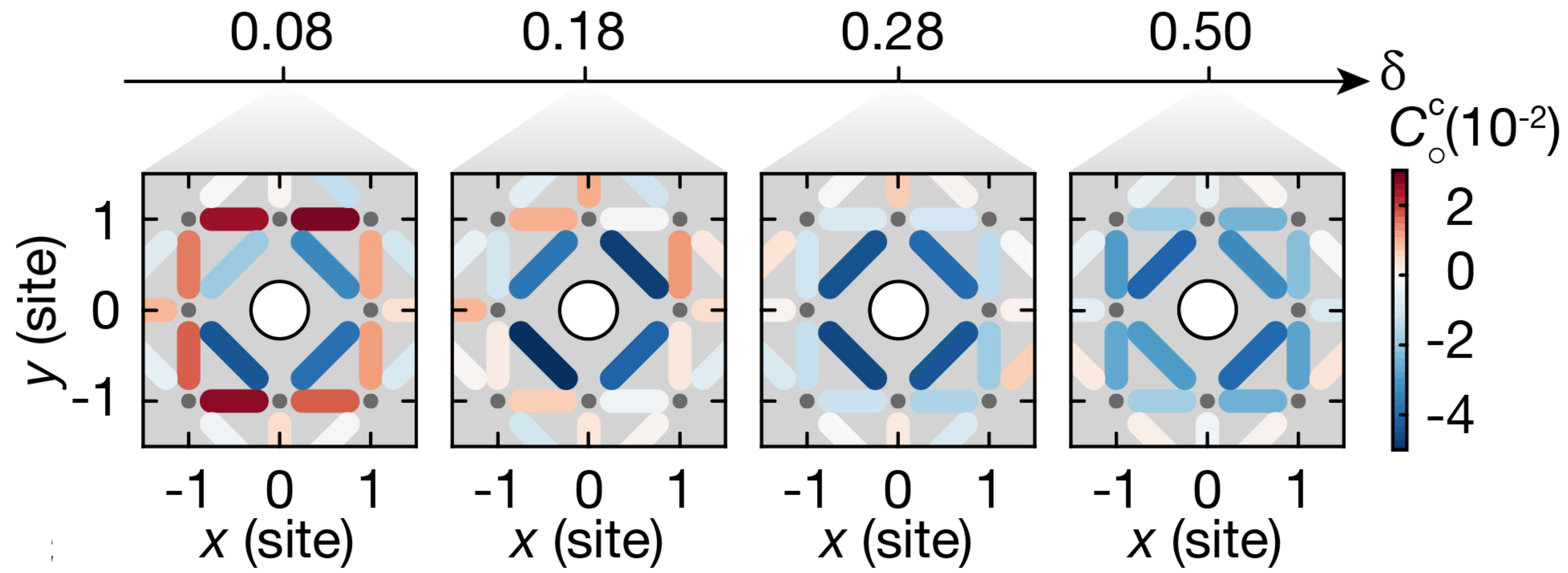
arXiv: 2006.01140
arXiv: 2111.13703

Dirk Morr





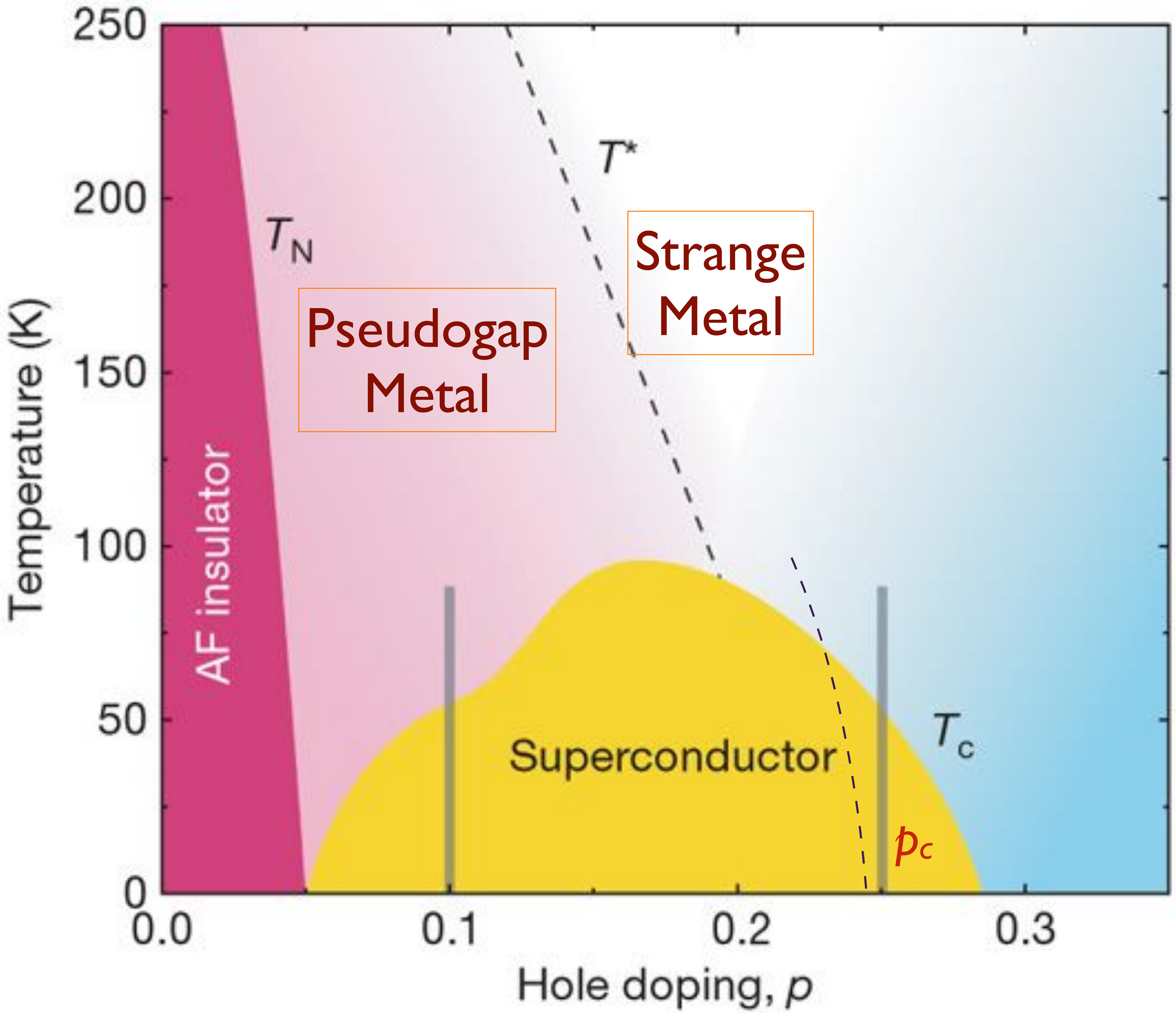
Ultracold fermionic atoms in optical lattices



Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

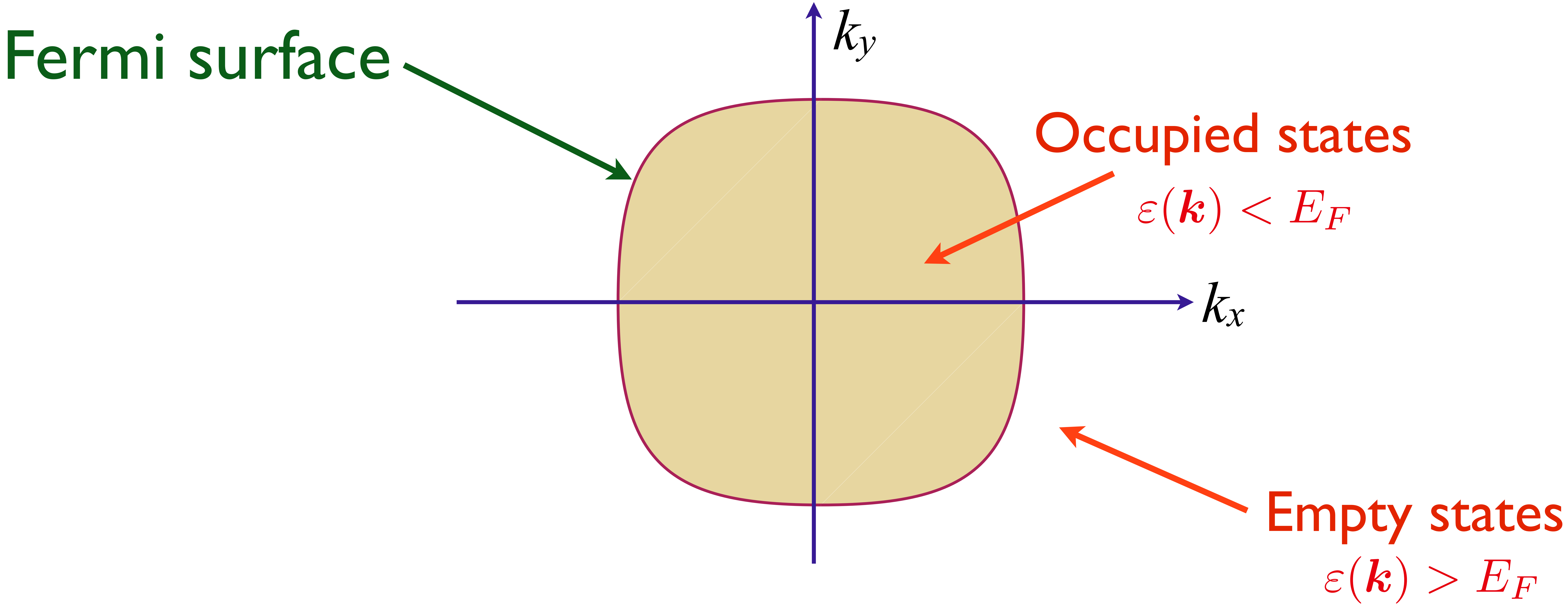
Science **374** (2021) 82



250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
Pseudogap Metal
T*
Strange Metal
Superconductor
T_c
p_c

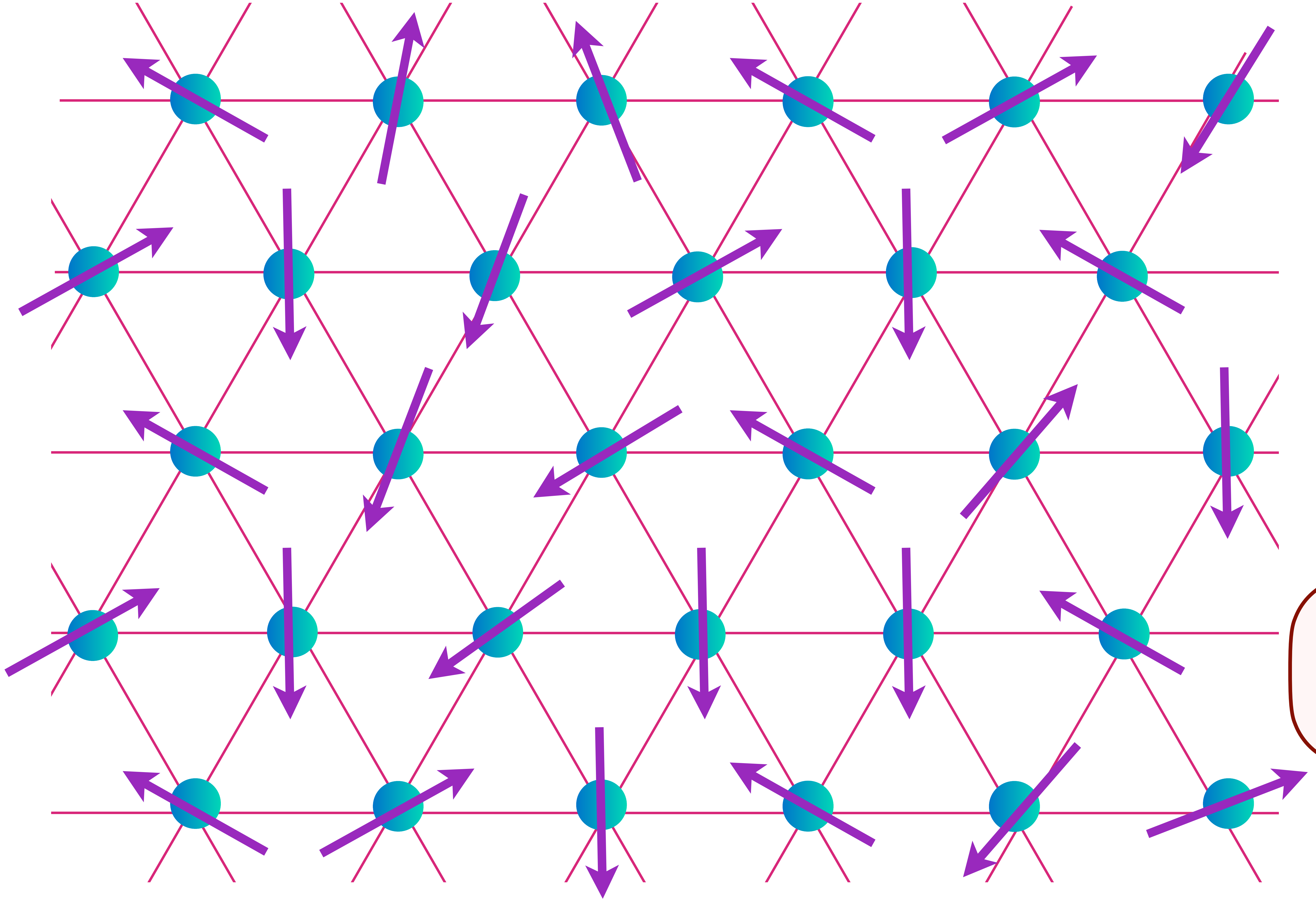
Luttinger relation

Electrons move with momentum **k** through the lattice with dispersion $\varepsilon(\boldsymbol{k})$

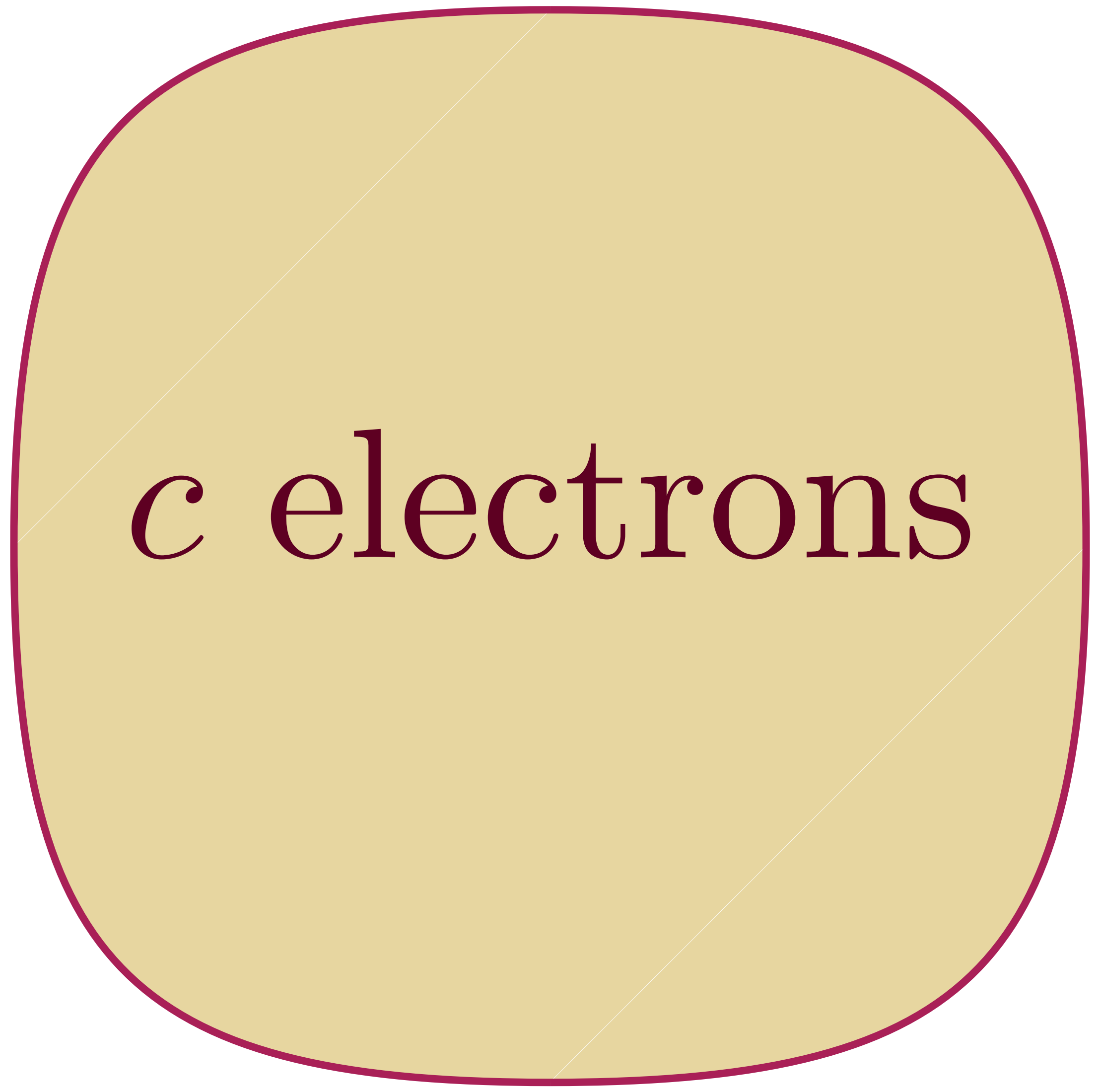


$$2 \times \frac{\text{Volume inside Fermi surface}}{(2\pi)^d} = \text{density of electrons (mod 2)}$$

Kondo lattice



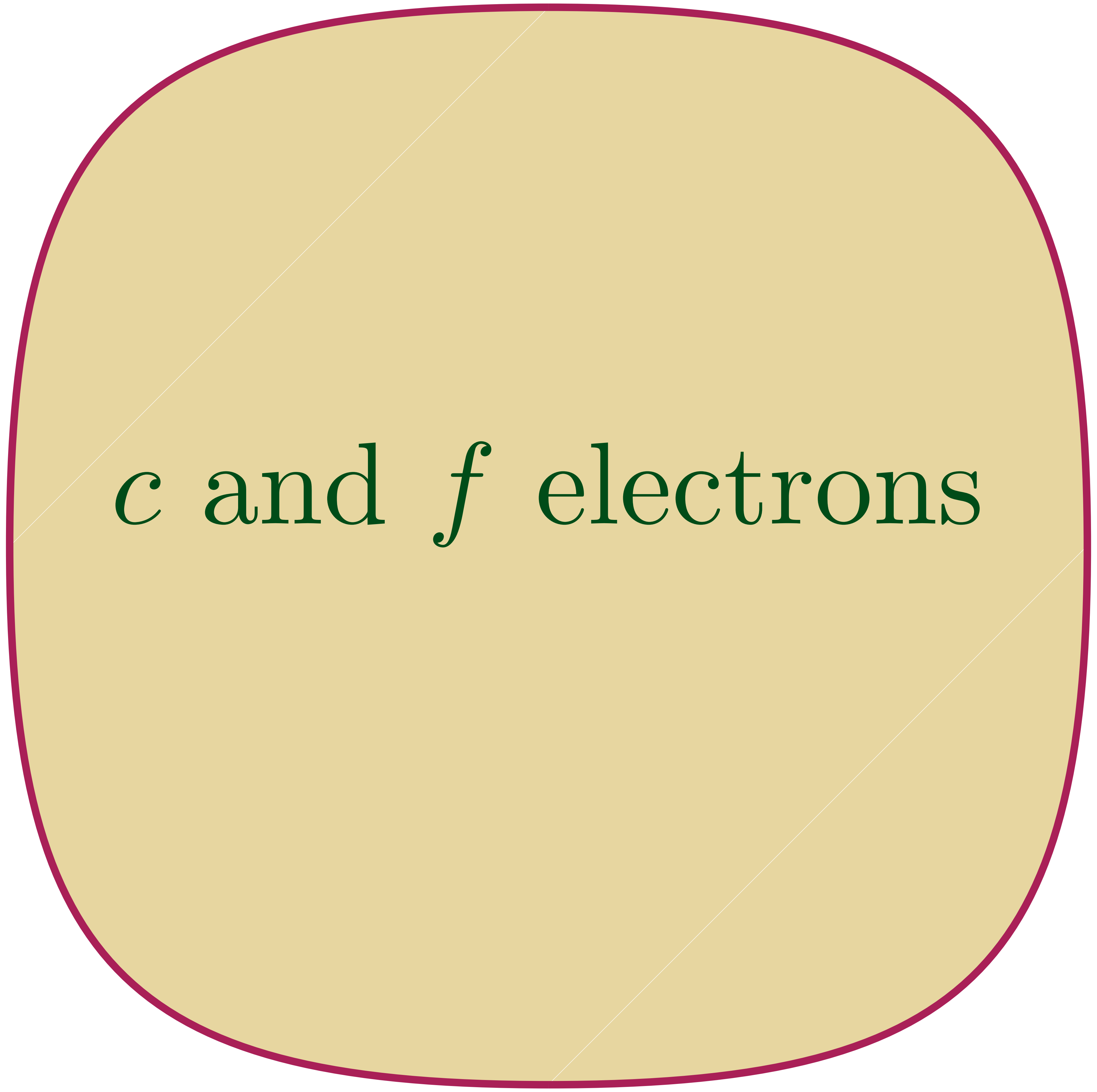
Kondo exchange J_K



Density of the electrons per unit cell $= 1 + p$

f electrons

Kondo lattice: HFL phase



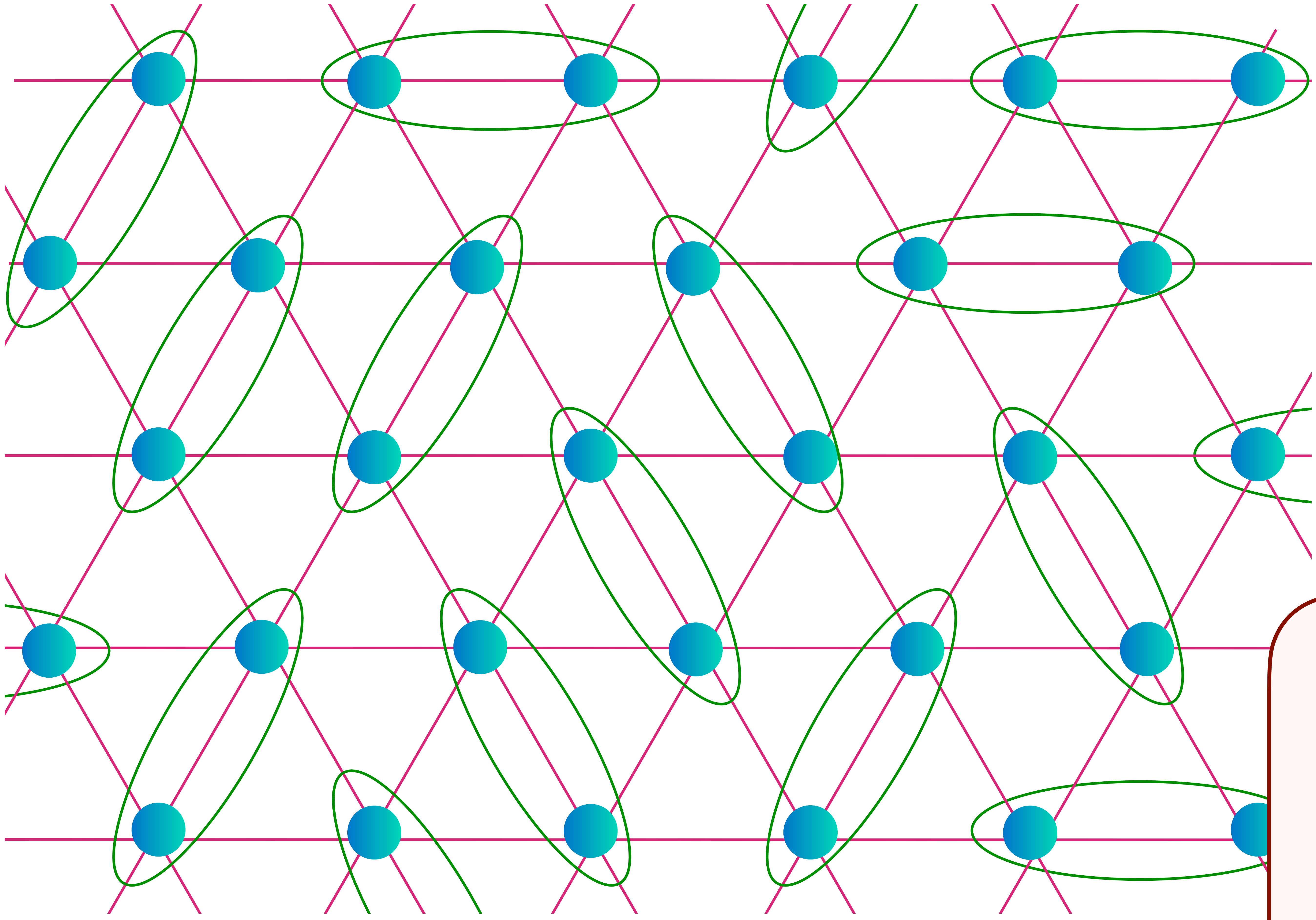
Kondo
exchange
J_K

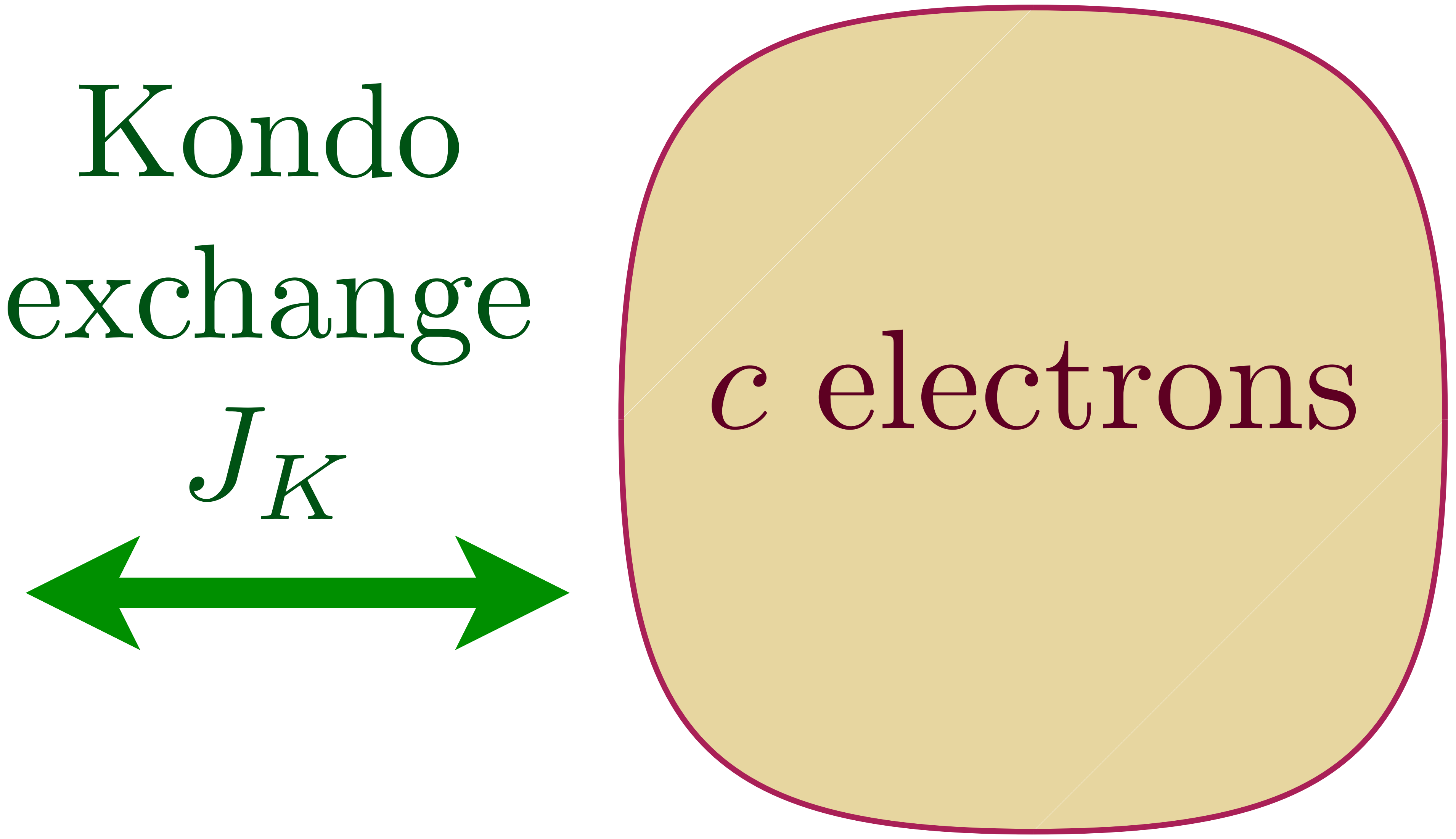
Density of the electrons
per unit cell $= 1 + p$,
Fermi surface size $= 1 + p$.
Luttinger volume "large" Fermi surface.

$$|\text{HFL}\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } (c, f)\rangle$$

Kondo lattice: FL* phase

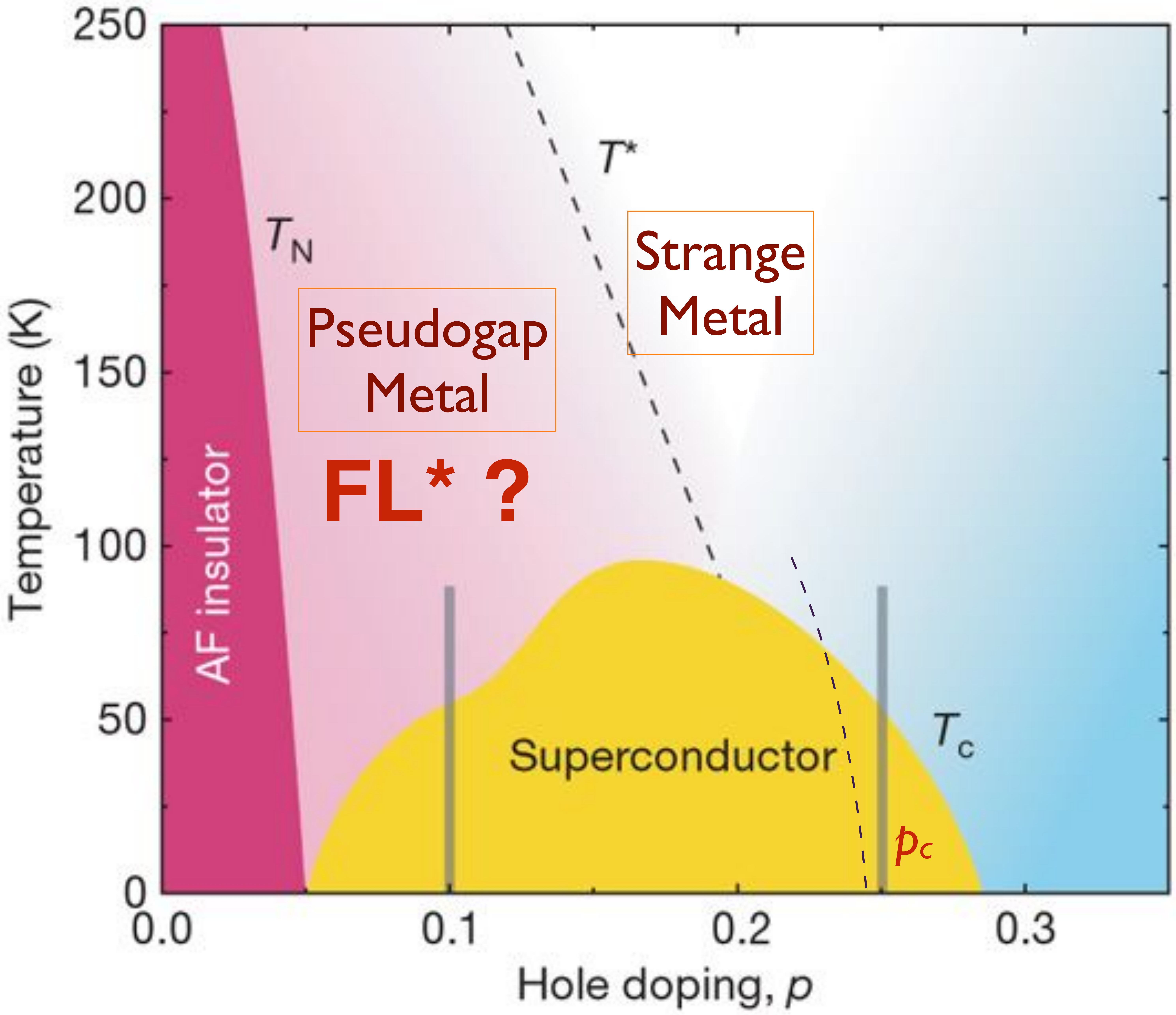
T. Senthil, M. Vojta, and S. Sachdev, PRL **90**, 216403 (2003)





f electrons

Density of the electrons
per unit cell $= 1 + p$,
Fermi surface size $= p$.
Remaining electrons form
an 'odd' spin liquid.
Non-Luttinger "small" Fermi surface is
stable to all orders in J_K.



Can a FL* state in a *single-band* Hubbard model describe the pseudogap metal over an intermediate temperature range, along with a crossover/transition to confinement at lower temperatures?

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\sigma}^{\dagger} c_{i\sigma}$$

We use the operator equation (valid on each site i):

$$U\left(n_{\uparrow} - \frac{1}{2}\right)\left(n_{\downarrow} - \frac{1}{2}\right) = -\frac{2U}{3}\boldsymbol{S}^2 + \frac{U}{4}$$

Then we decouple the interaction via

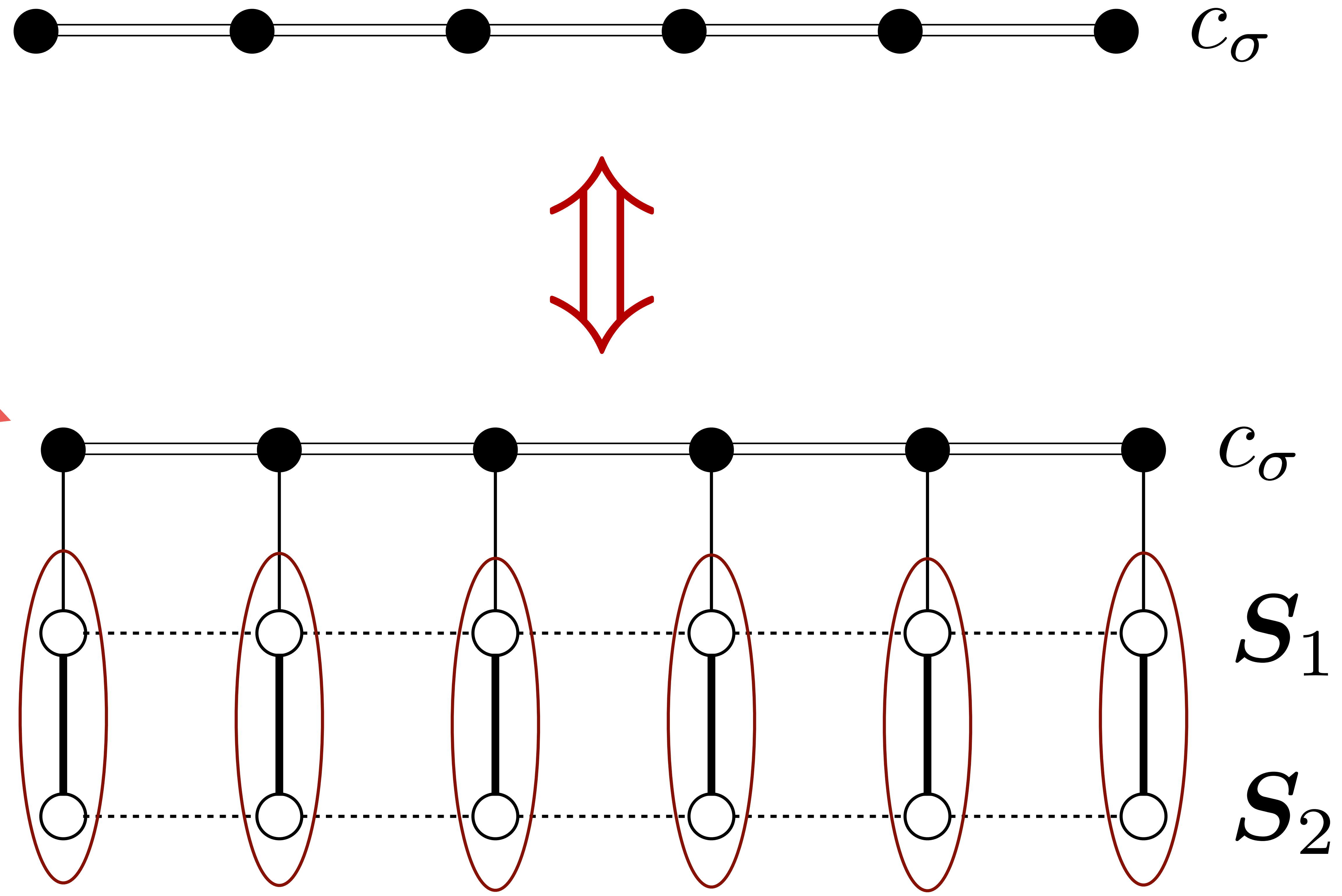
$$\exp\left(\frac{2U}{3}\sum_i \int d\tau \boldsymbol{S}_i^2\right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U}\boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^{\dagger} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'}\right]\right)$$

This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

Paramagnon theory of the Hubbard model

Free electrons of density 1-p

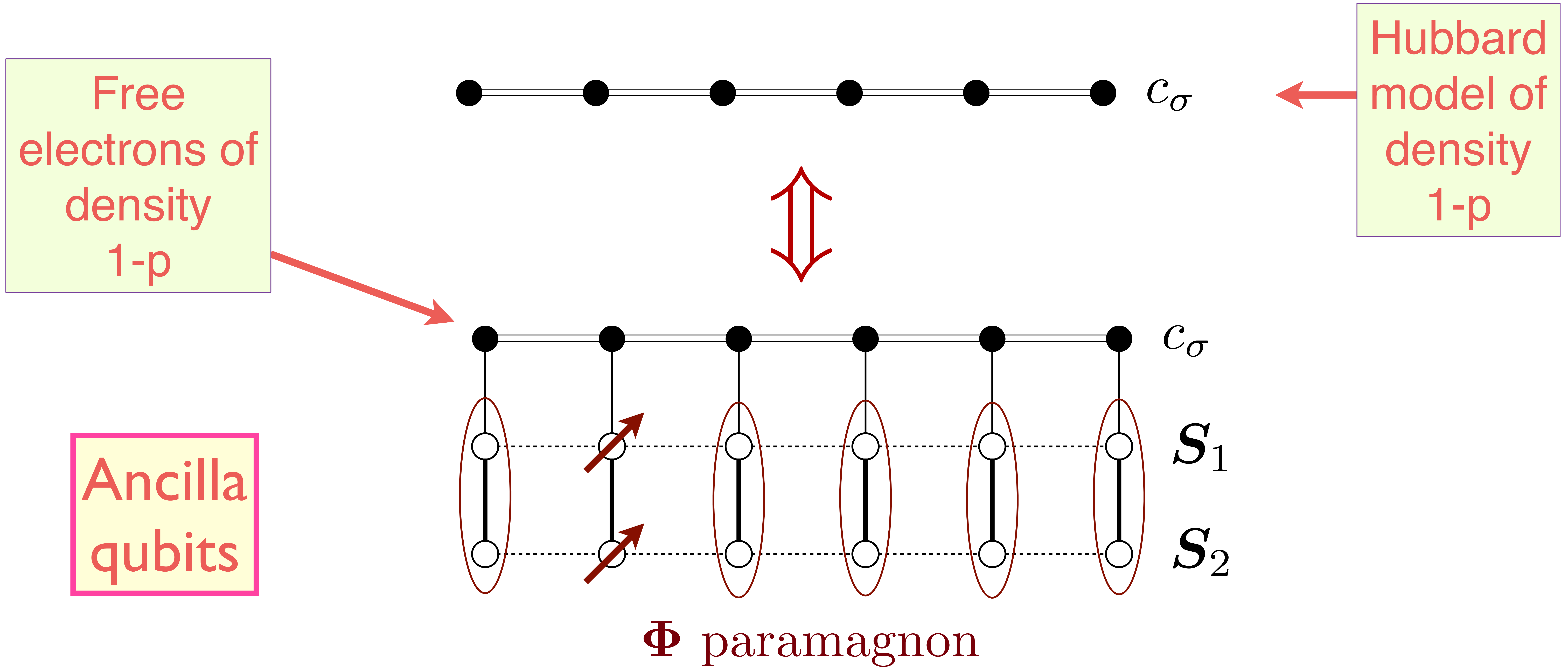
Hubbard model of density 1-p



Ancilla qubits

$$\mathcal{H}_{\text{paramagnon}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

Paramagnon theory of the Hubbard model



$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} - \lambda \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

Paramagnon theory of the Hubbard model

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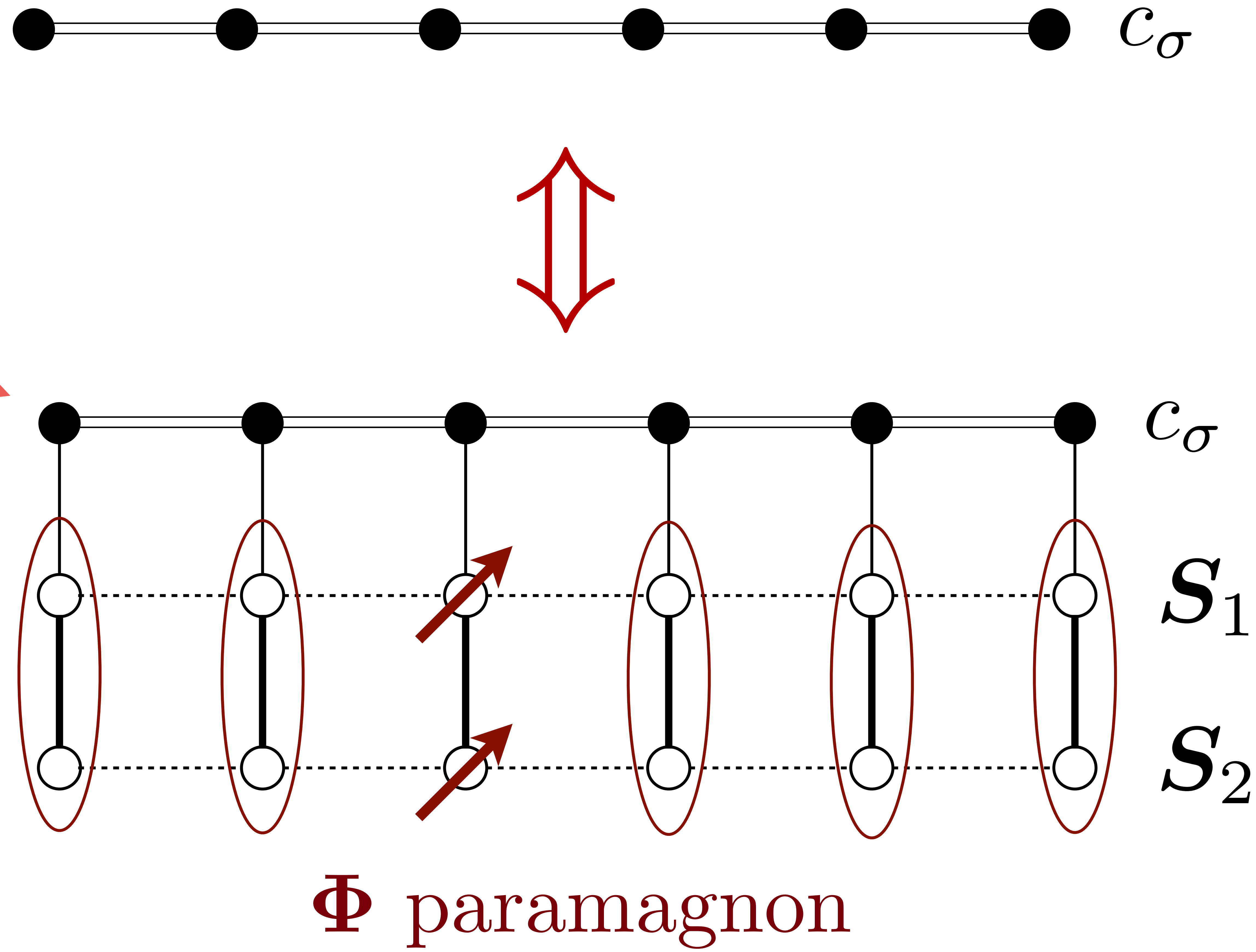

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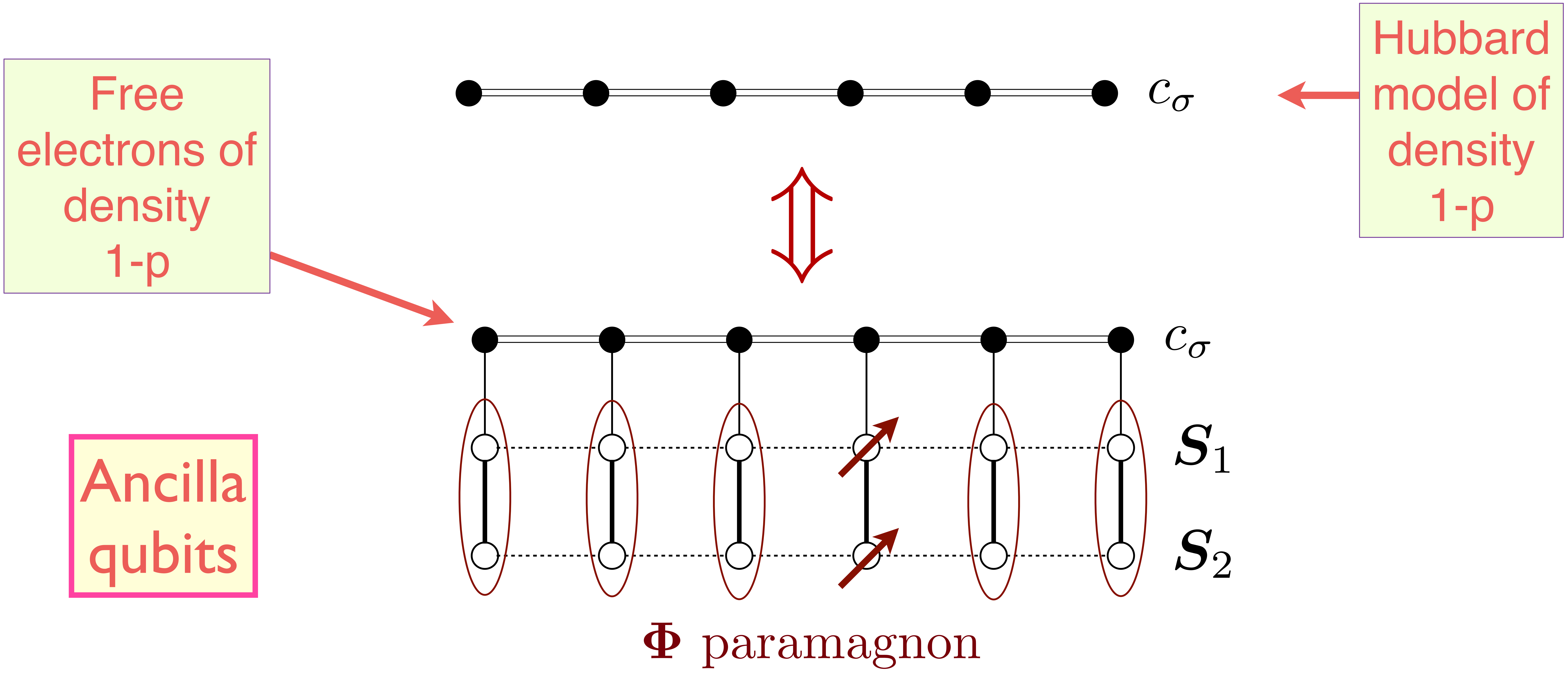

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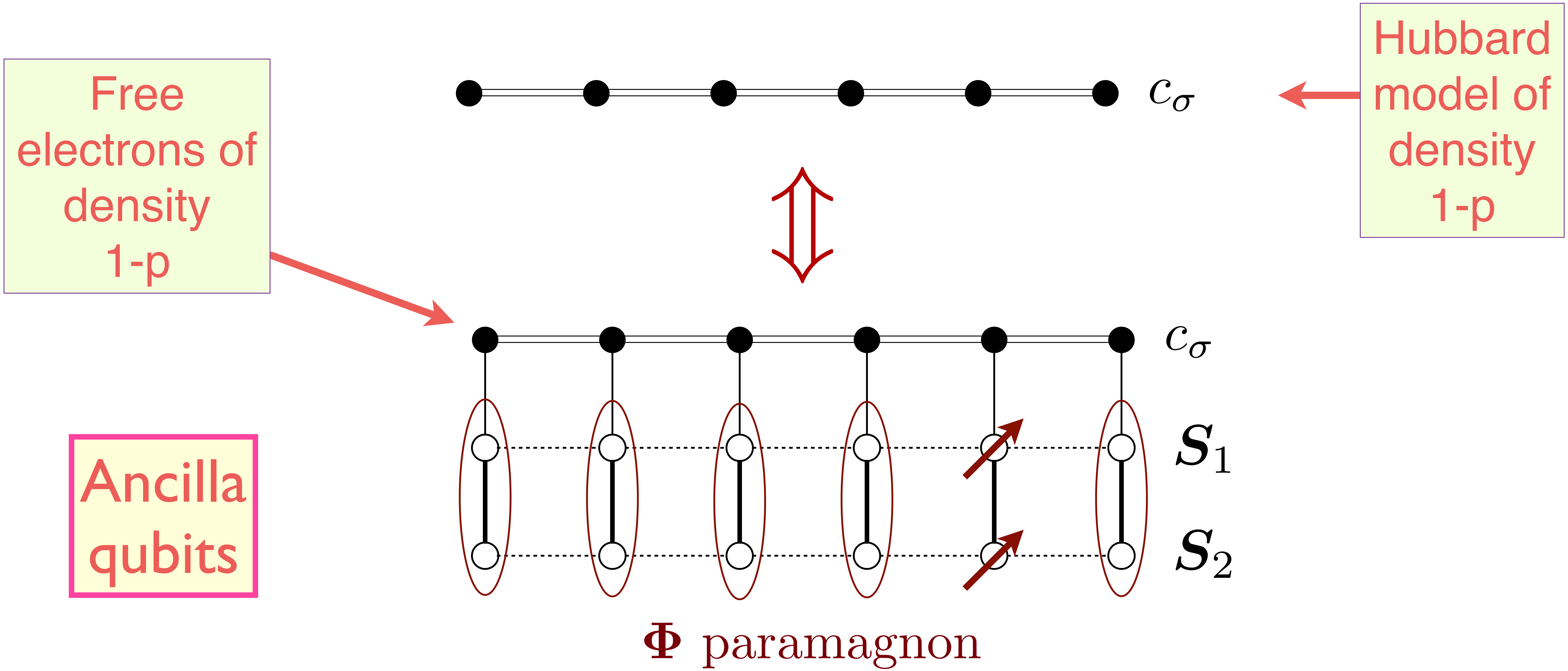
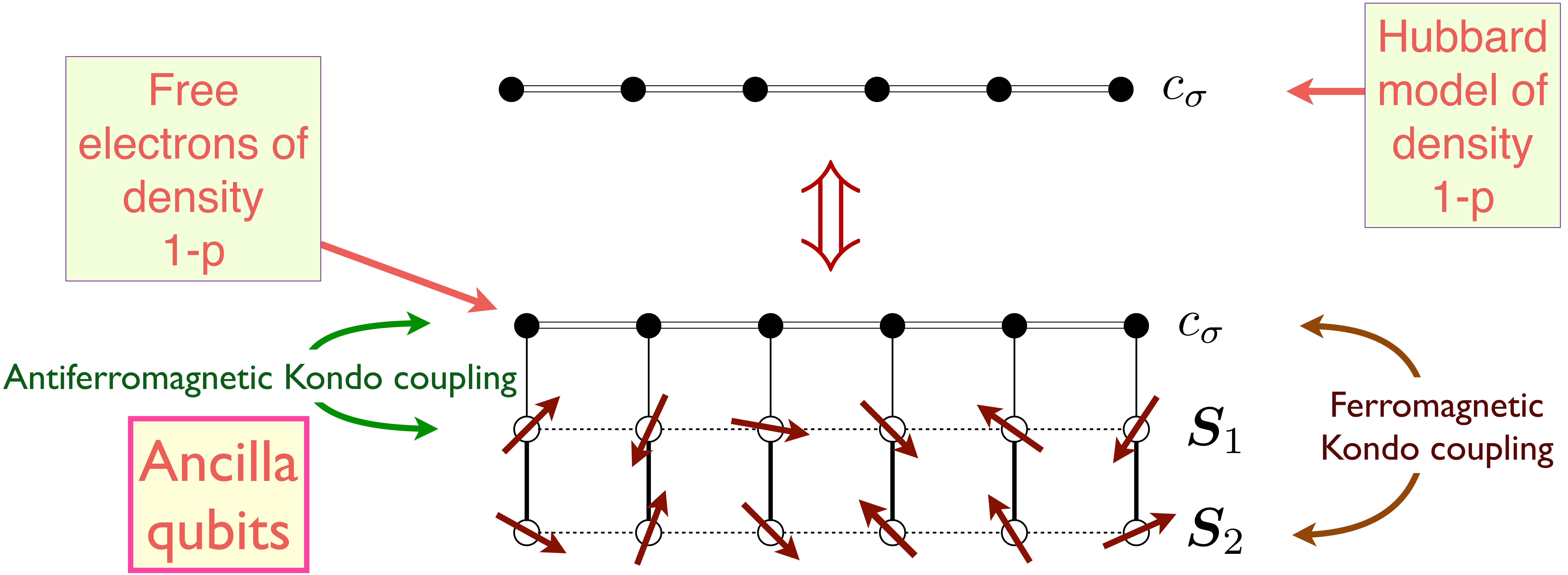


$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - \lambda \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

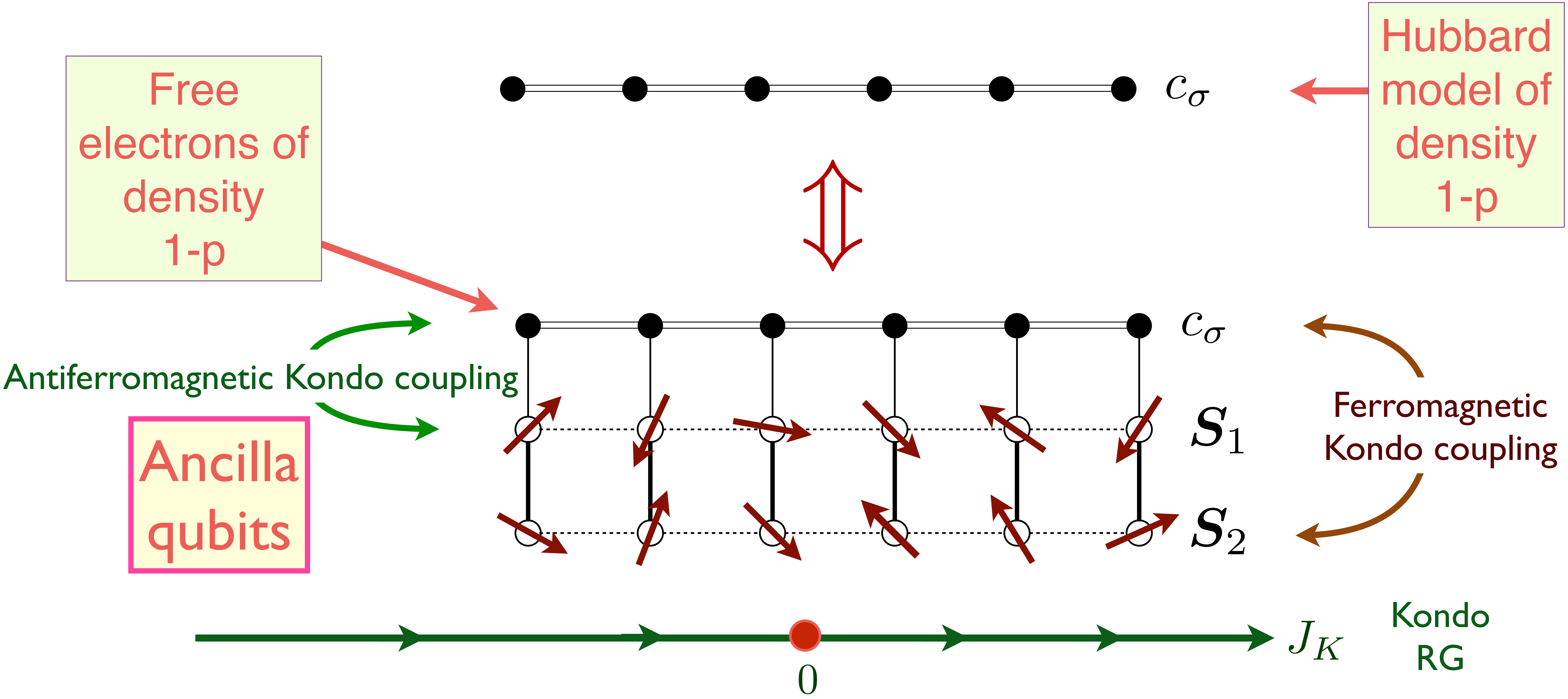
Paramagnon fractionalization theory of the Hubbard model



$$\boldsymbol{\Phi}_i = \frac{1}{\sqrt{3}} \left(\boldsymbol{S}_{2i} - \boldsymbol{S}_{1i} \right)$$

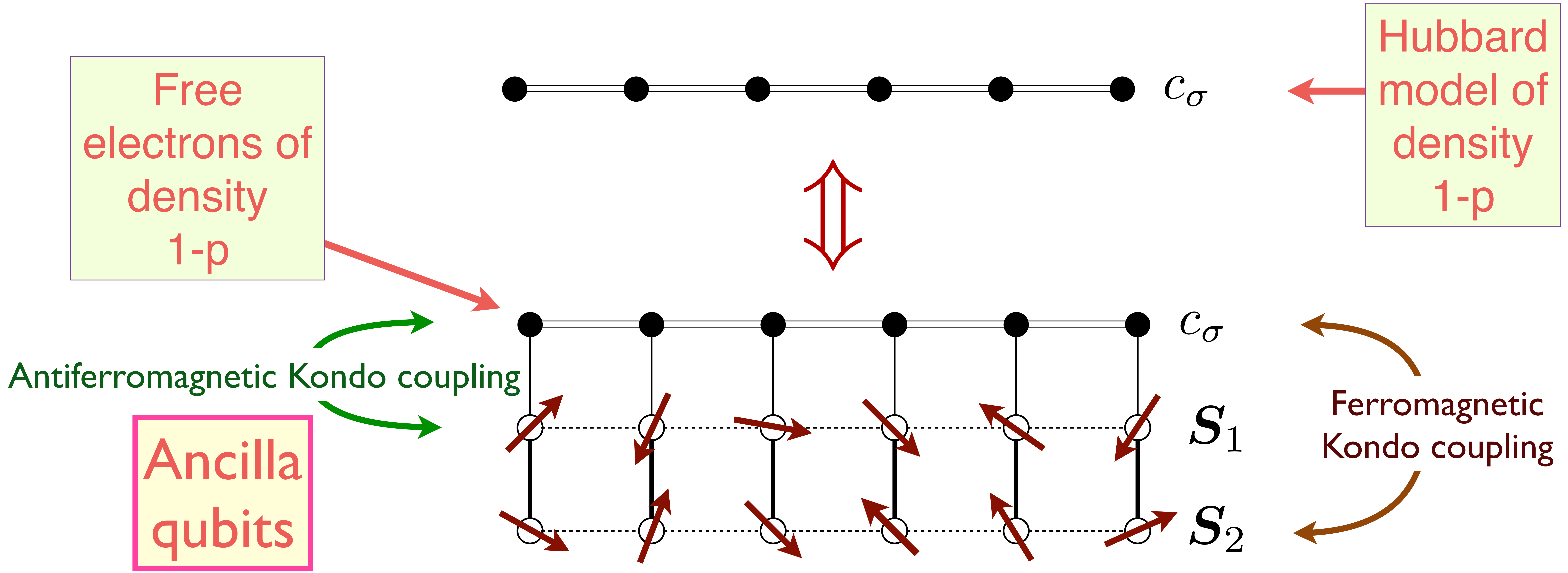
$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} - \lambda \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{\Phi}_i + \ldots$$

Paramagnon fractionalization theory of the Hubbard model



$$\mathcal{H}_{\rm paramagnon} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + J_K \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + -\widetilde{J}_K \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{2i} + \ldots$$

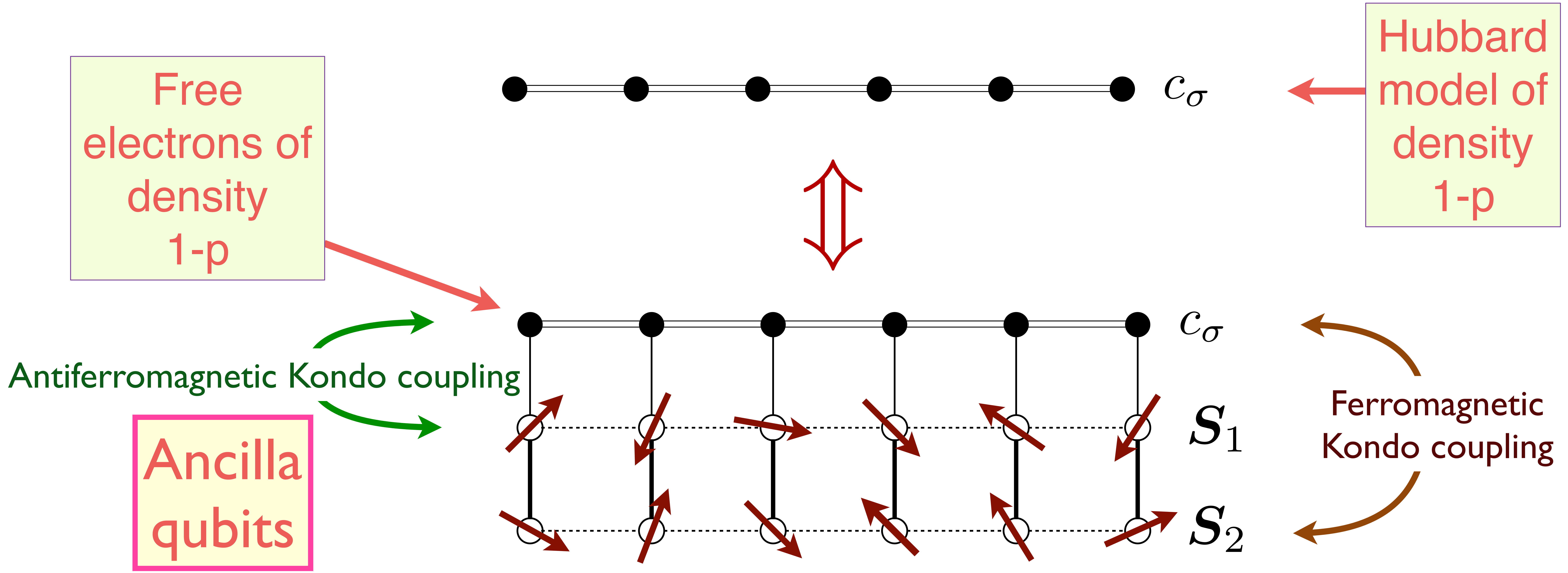
Paramagnon fractionalization theory of the Hubbard model



A FL* state is realized when the antiferromagnetic Kondo coupling dominates, and the c_σ and $\boldsymbol{S}_1$ form a "large" Fermi surface of hole density

$$(1+p)+1 = 2+p = p \text{ mod } 2!$$

Paramagnon fractionalization theory of the Hubbard model



A FL* state is realized when the antiferromagnetic Kondo coupling dominates, and the c_σ and $\boldsymbol{S}_1$ form a "large" Fermi surface of hole density

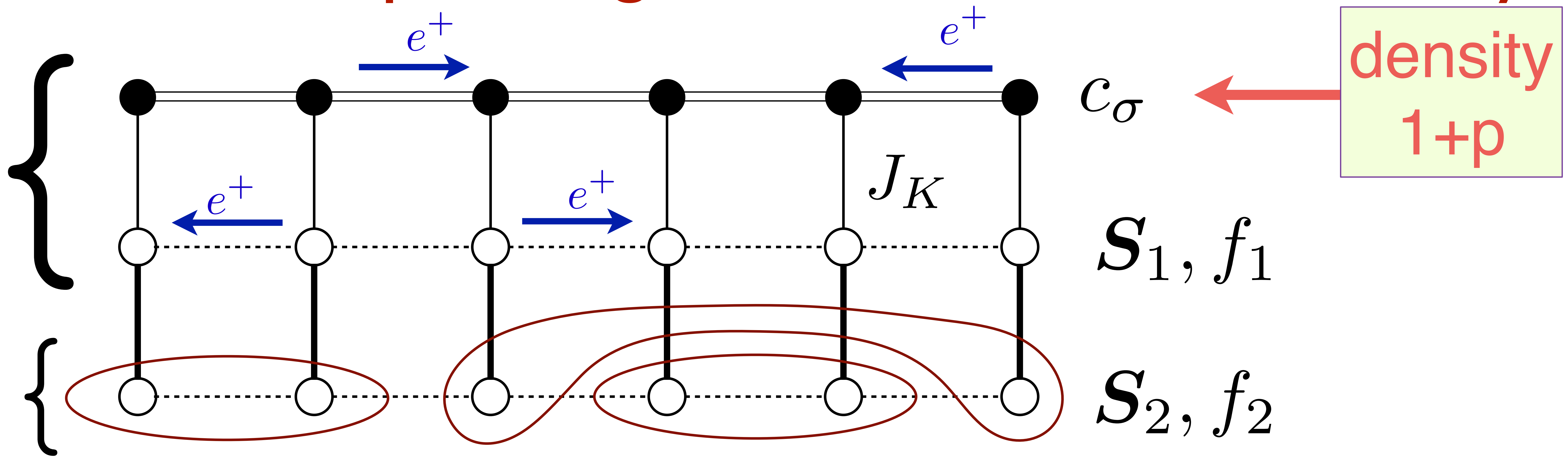
$$(1+p)+1 = 2+p = p \bmod 2!$$

The $\boldsymbol{S}_2$ must form an 'odd' spin liquid which does not break translational symmetry, to obtain a metal with a non-Luttinger volume Fermi surface.

Trial wavefunctions in the paramagnon fractionalization theory

Small Fermi surface.
Size $2+p \cong p$

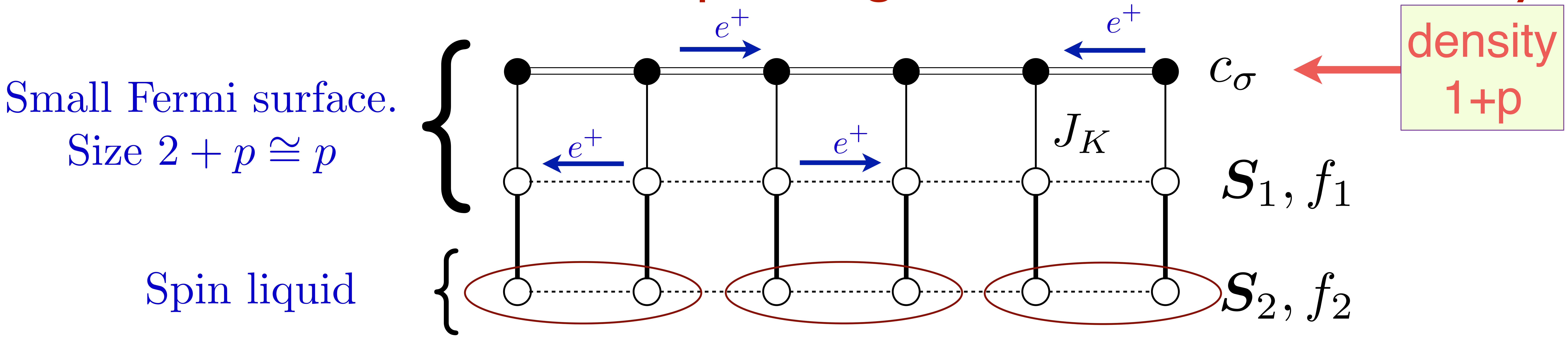
Spin liquid



Small Fermi surface of size p

$$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } f_1, f_2] \bowtie |\text{Slater determinant of } (c, f_1)\rangle \otimes |\text{Slater determinant of } f_2\rangle$$

Trial wavefunctions in the paramagnon fractionalization theory



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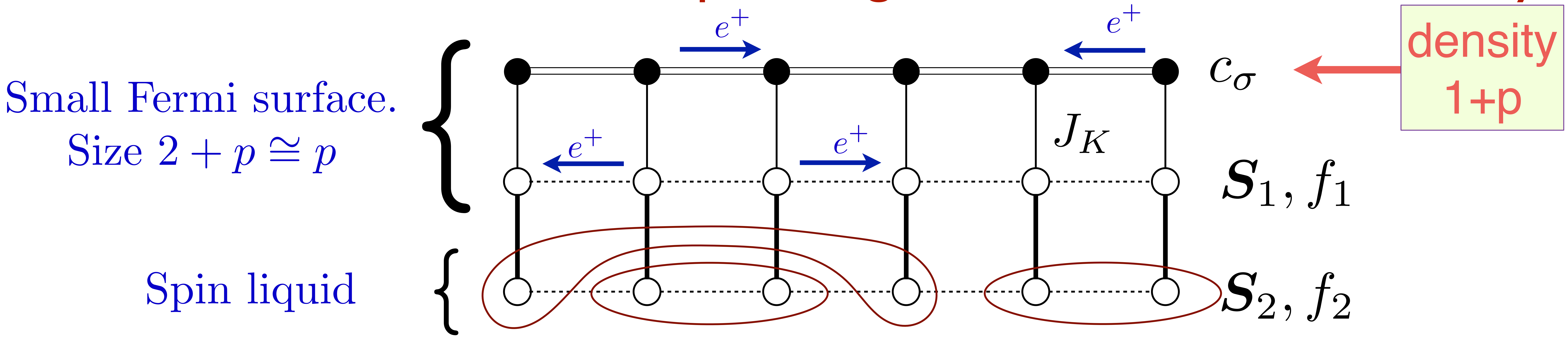
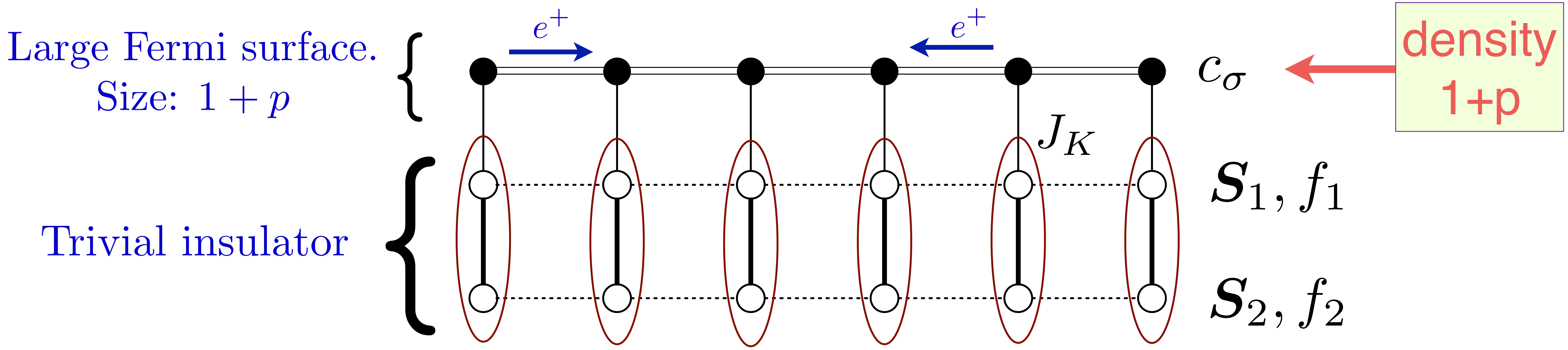
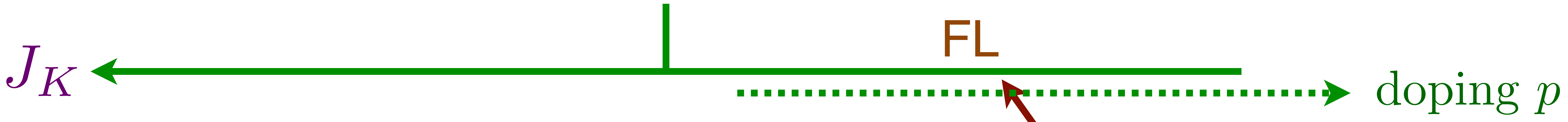


Small Fermi surface of size p

$$
\begin{aligned}
|\mathrm{FL}*\rangle = {} & [\text{Projection onto rung singlets of } f_1, f_2] \\
& \bowtie |\text{Slater determinant of } (c, f_1)\rangle \\
& \otimes |\text{Slater determinant of } f_2\rangle
\end{aligned}
$$

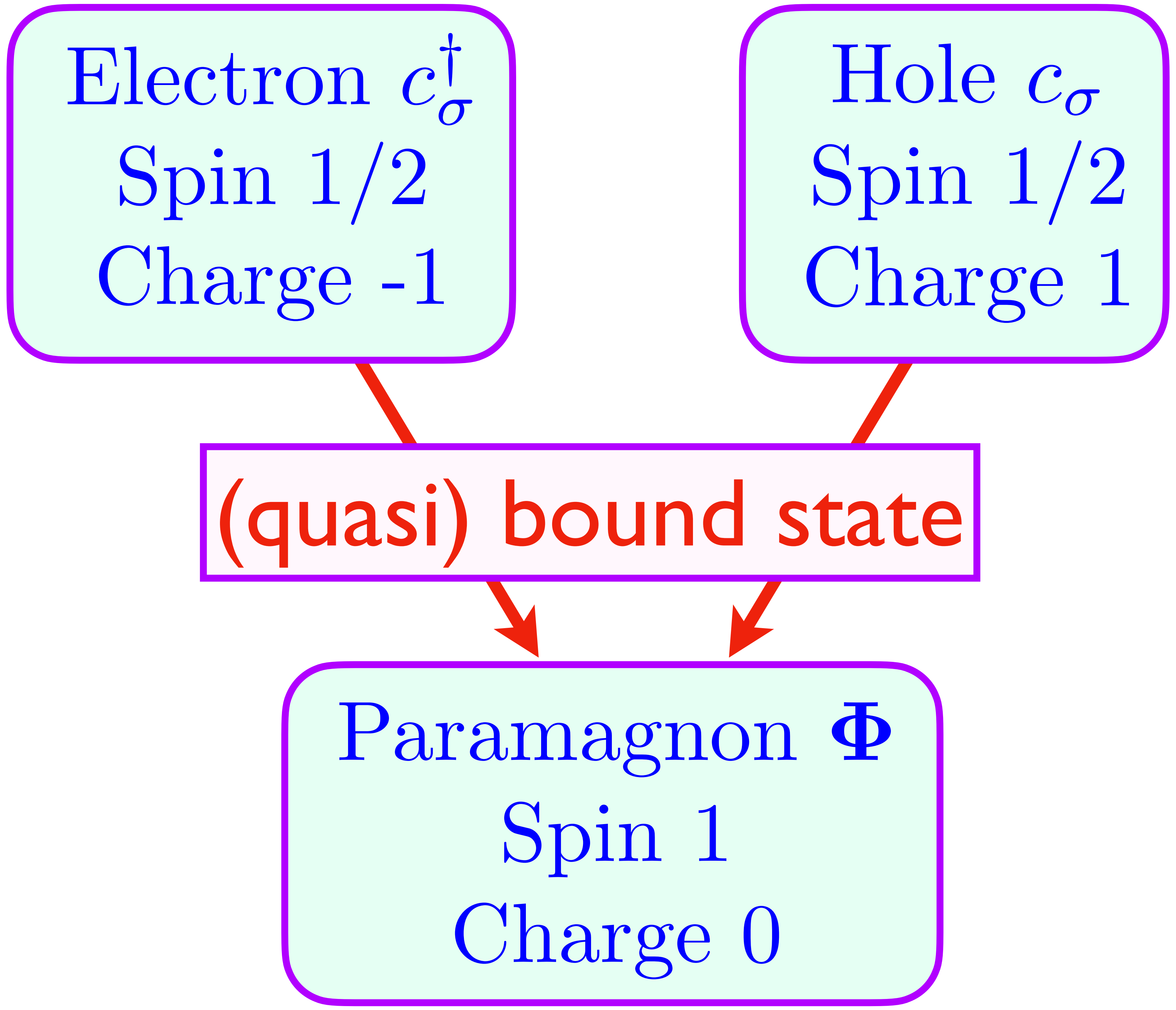
Trial wavefunctions in the paramagnon fractionalization theory



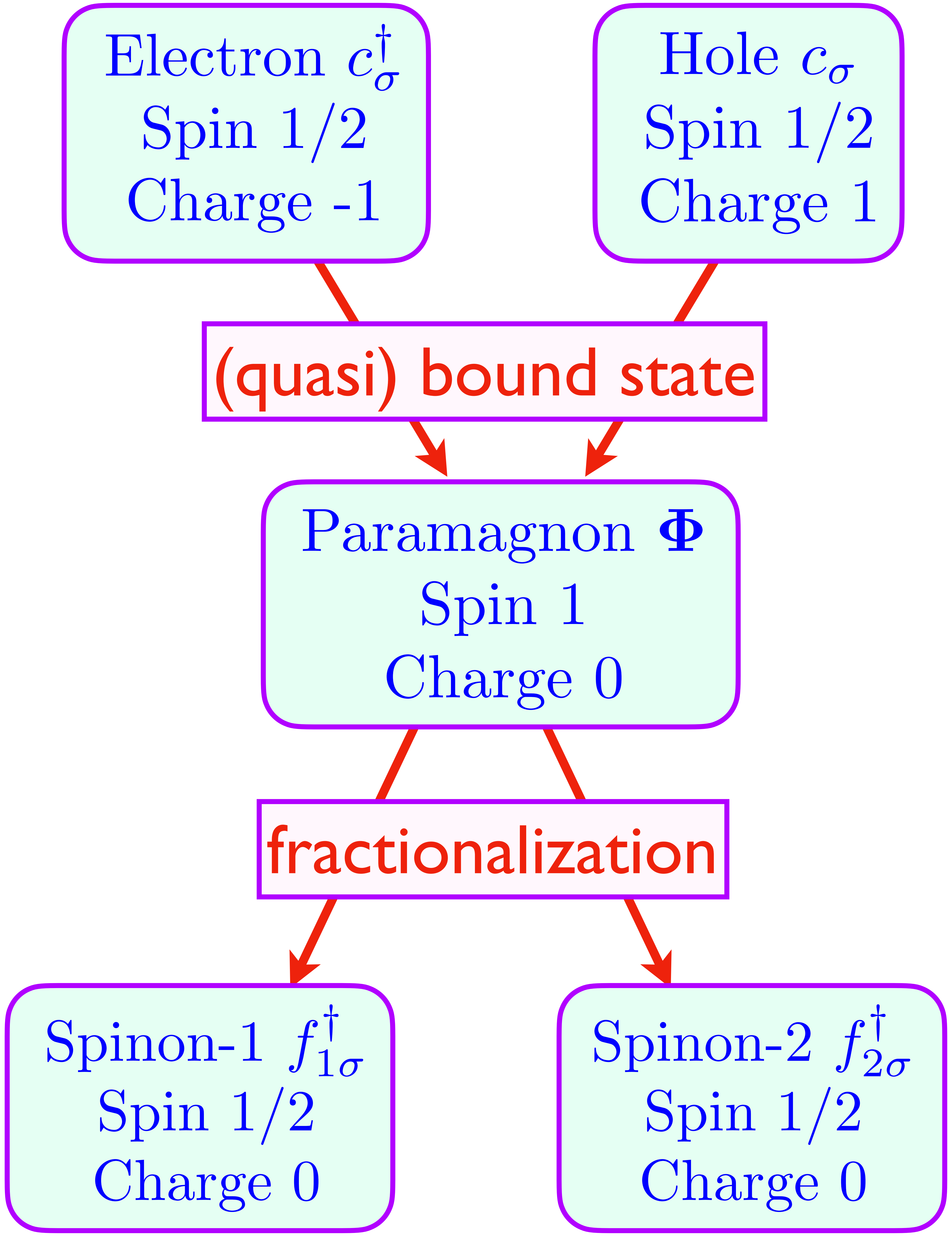


Large Fermi surface of size $1+p$

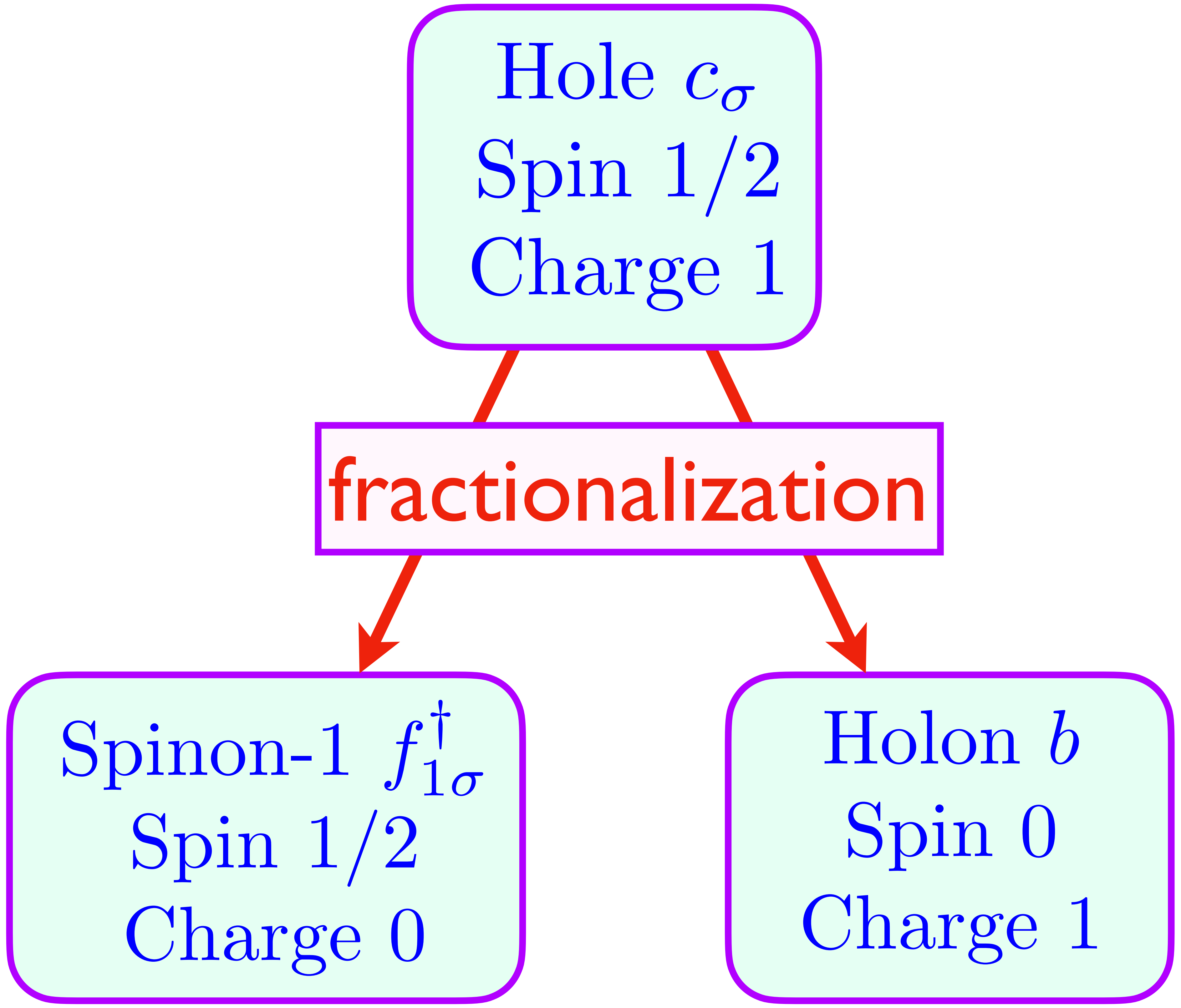
$$|\mathrm{FL}\rangle = |\text{Rung singlets of } f_1, f_2\rangle \otimes |\text{Slater determinant of } c\rangle$$



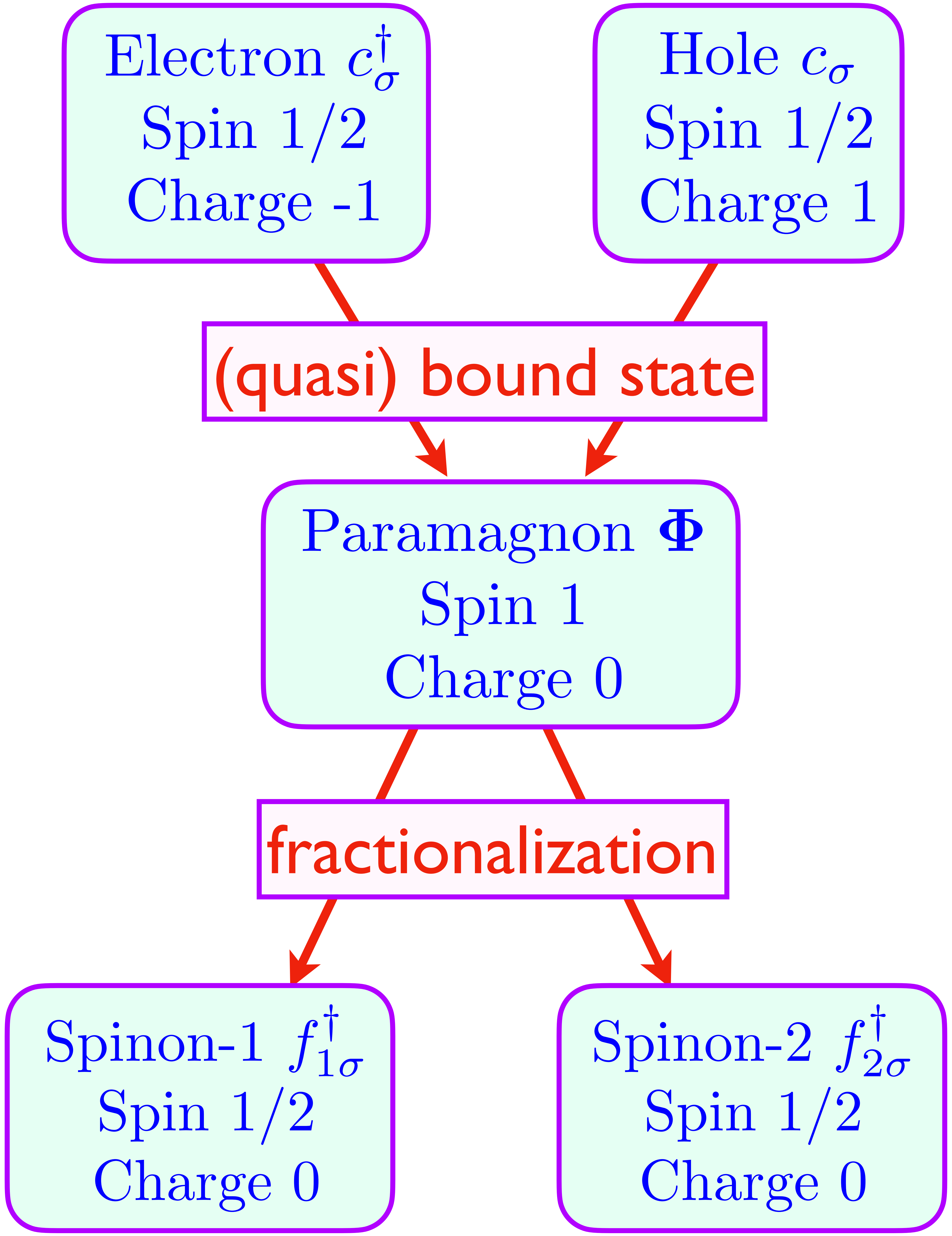
Paramagnon fractionalization



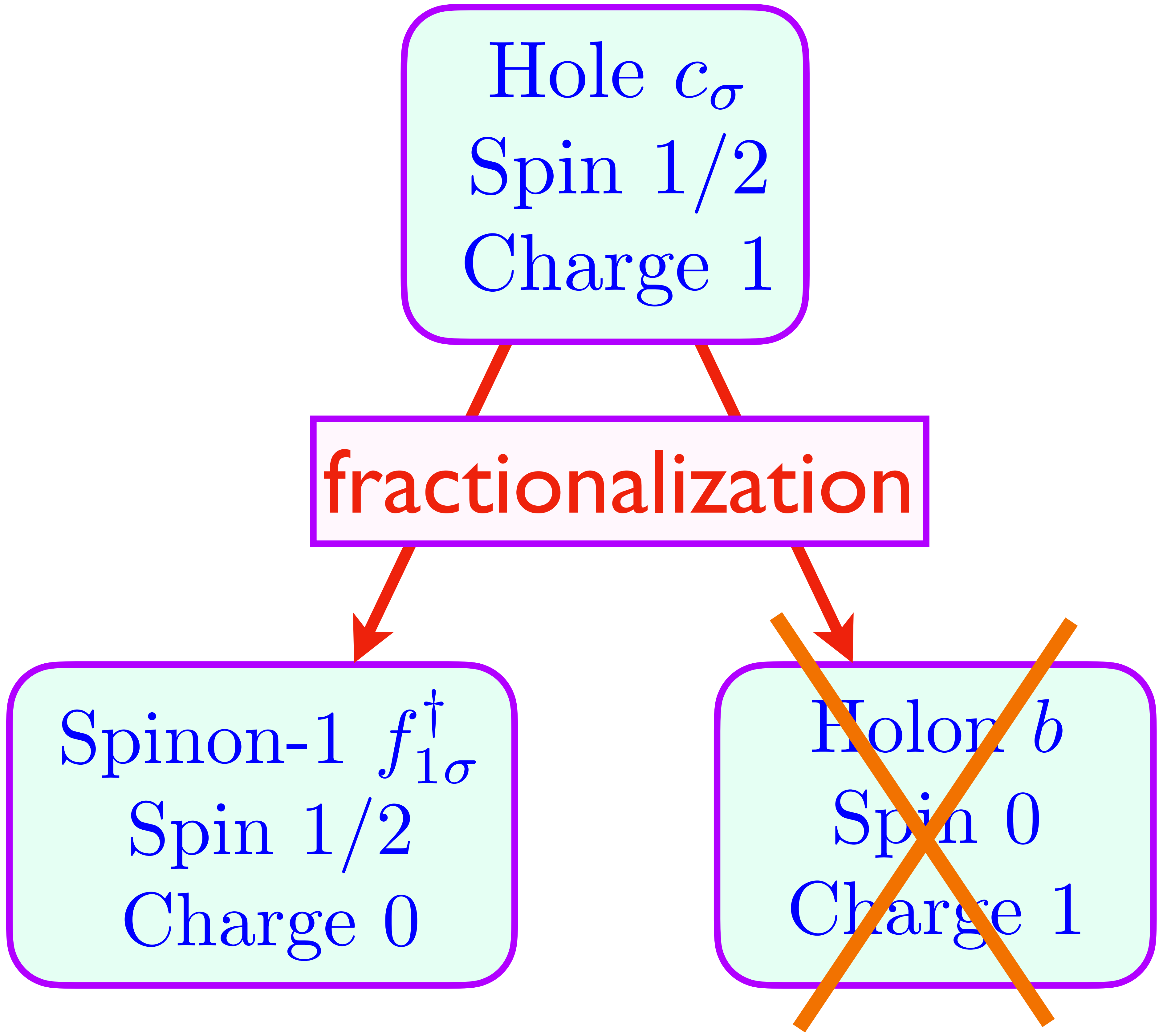
Paramagnon fractionalization



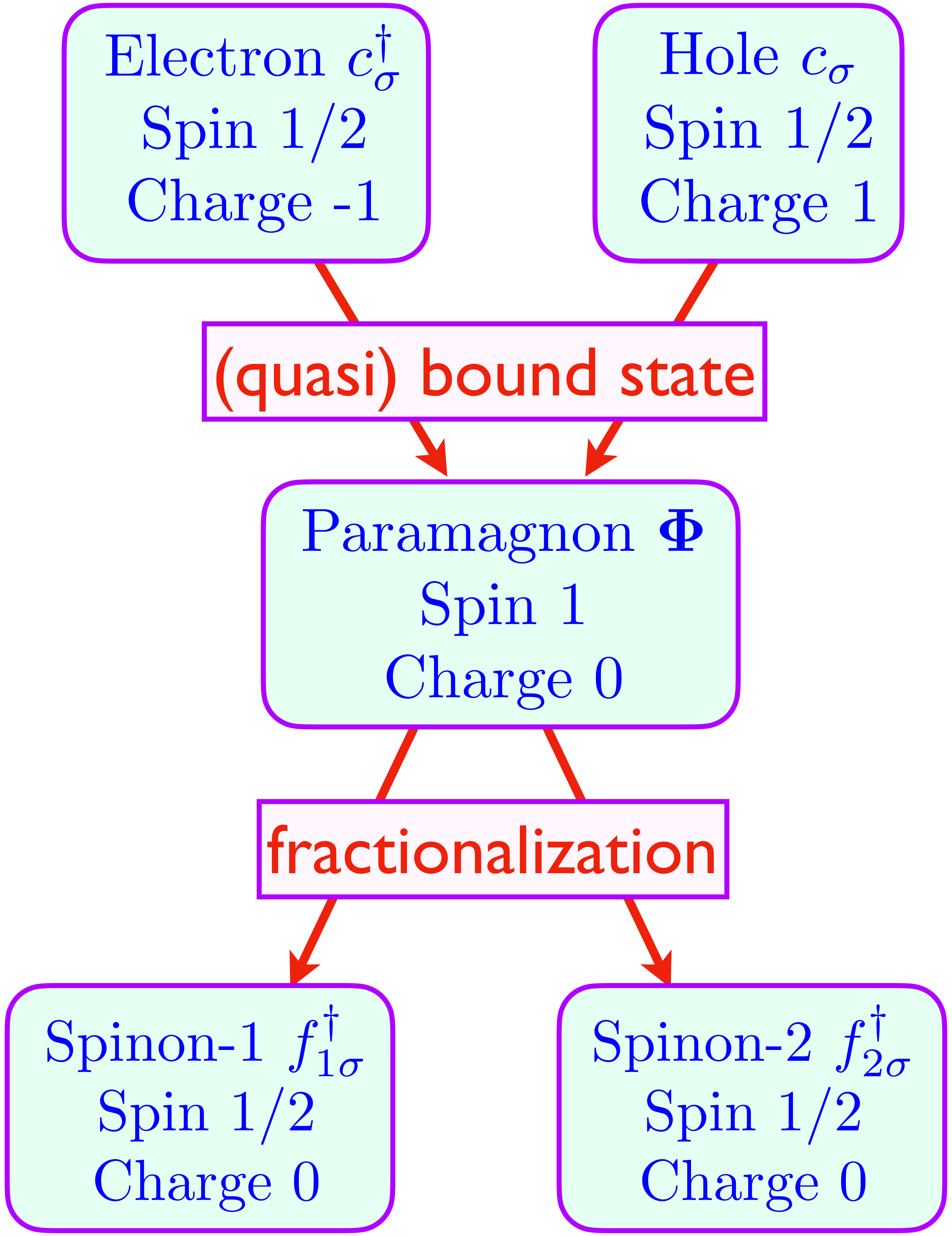
Electron fractionalization



Paramagnon fractionalization



Electron fractionalization

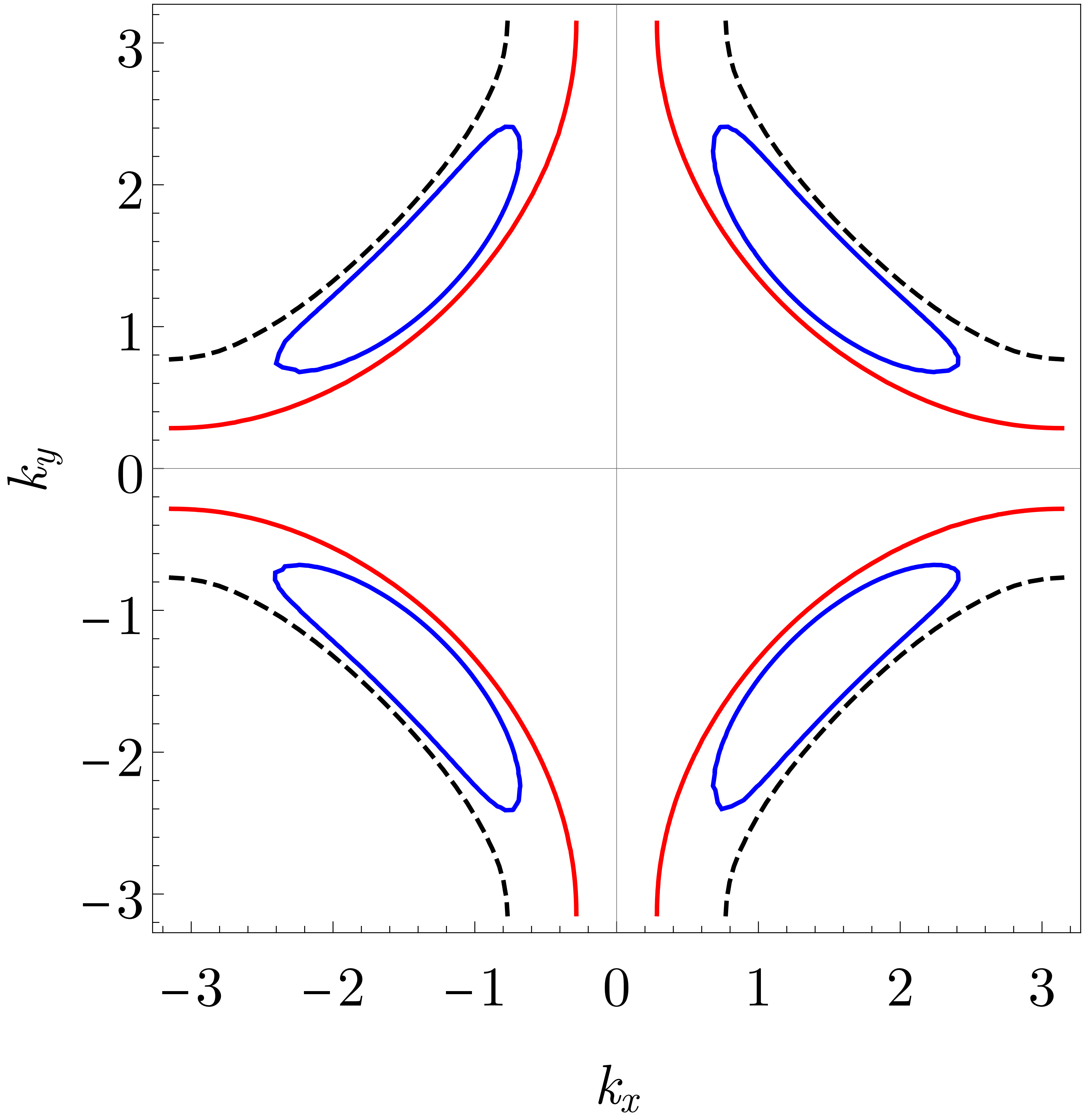


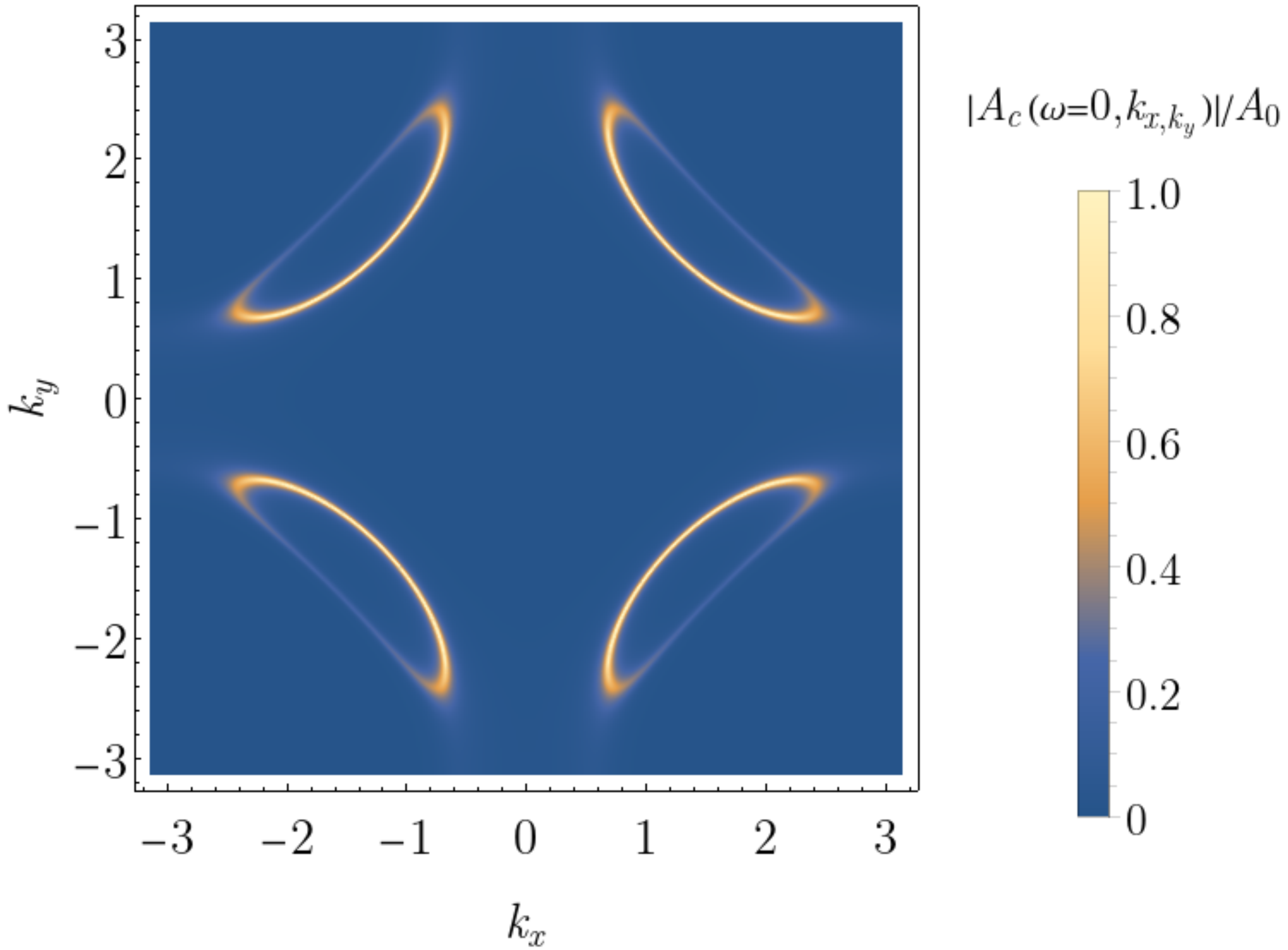
Paramagnon fractionalization

Don't fractionalize the electron; fractionalize the paramagnon!

FL* in a ***one-band*** model

"Fermi arc" spectral functions

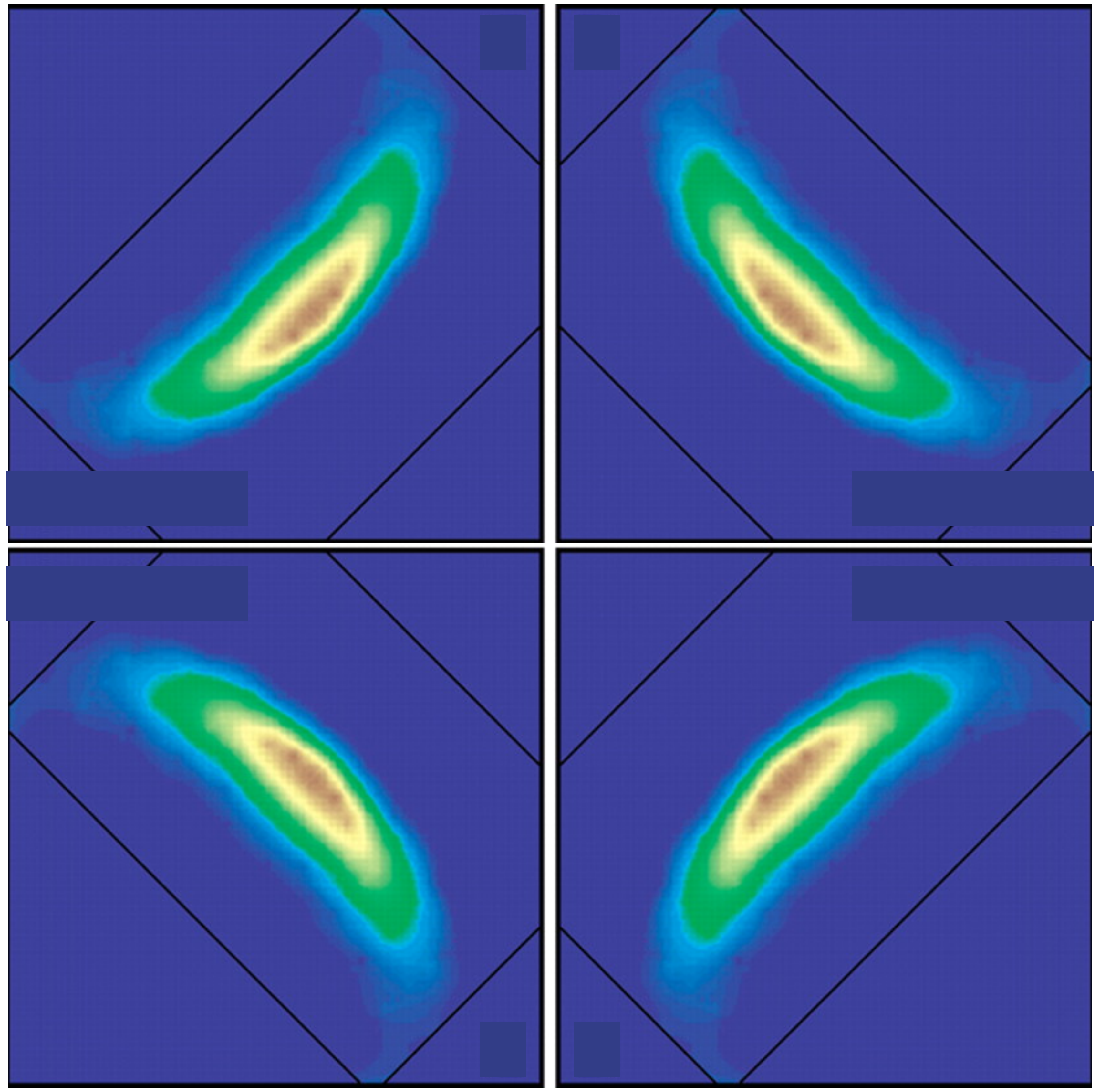




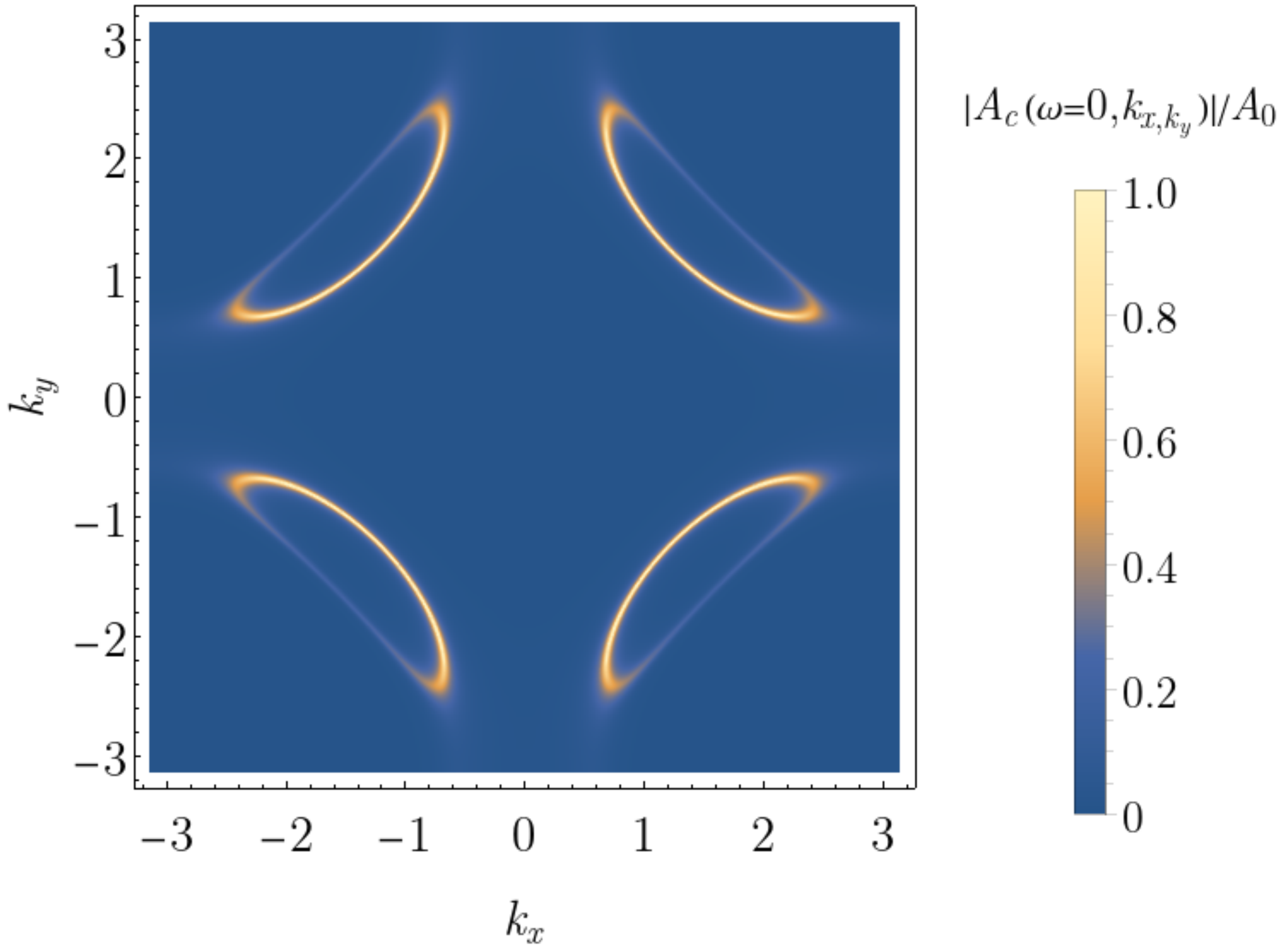
FL* Hamiltonian: $[(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2] \times \mathrm{U}(1)_{\mathrm{em}}$ is broken to $\mathrm{U}(1)_{\mathrm{diag}}$ by Higgs condensate Φ:

$$H = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} + \sum_i \Phi \left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_\sigma \right)$$

Photoemission at small p



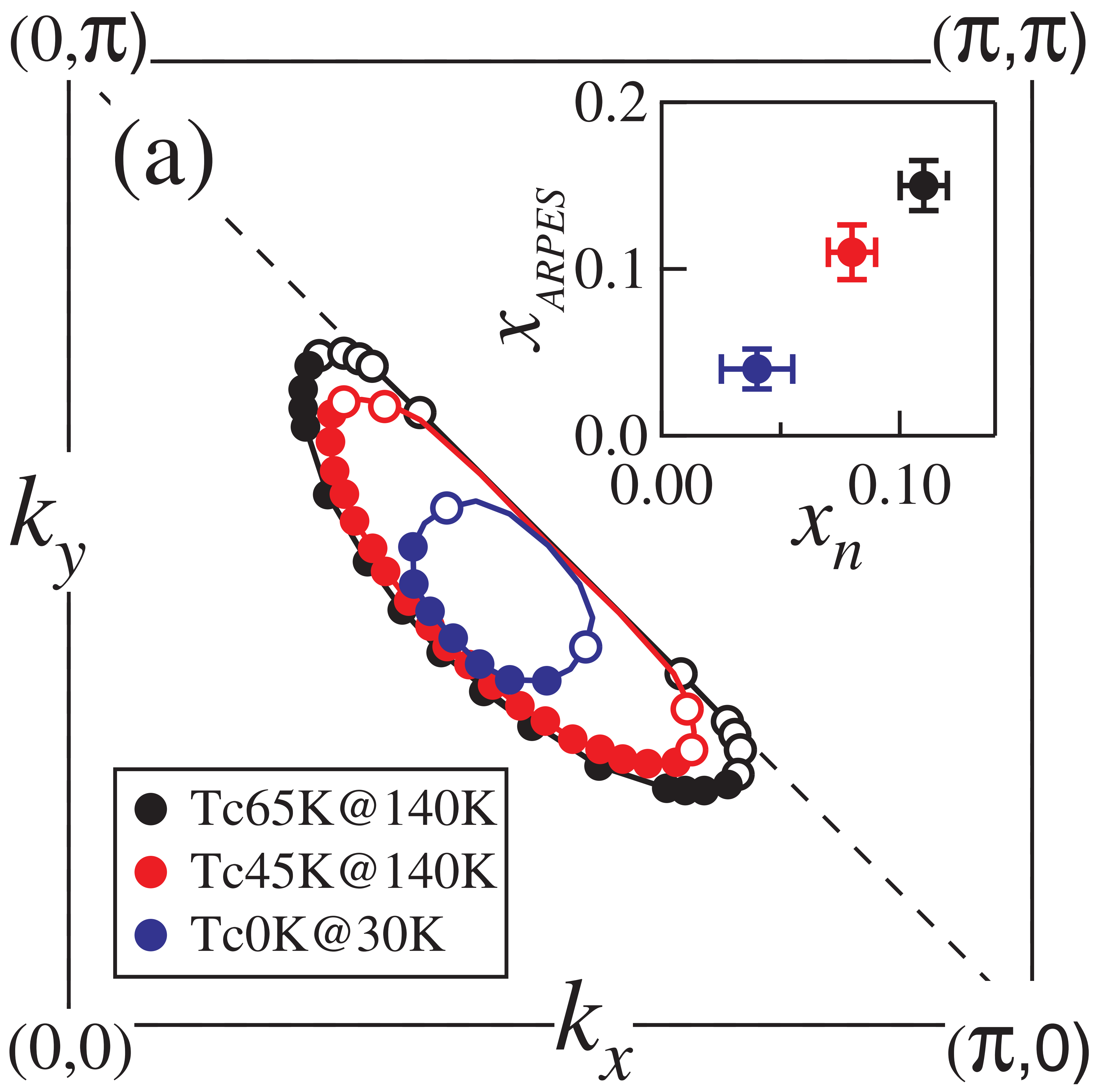
$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$



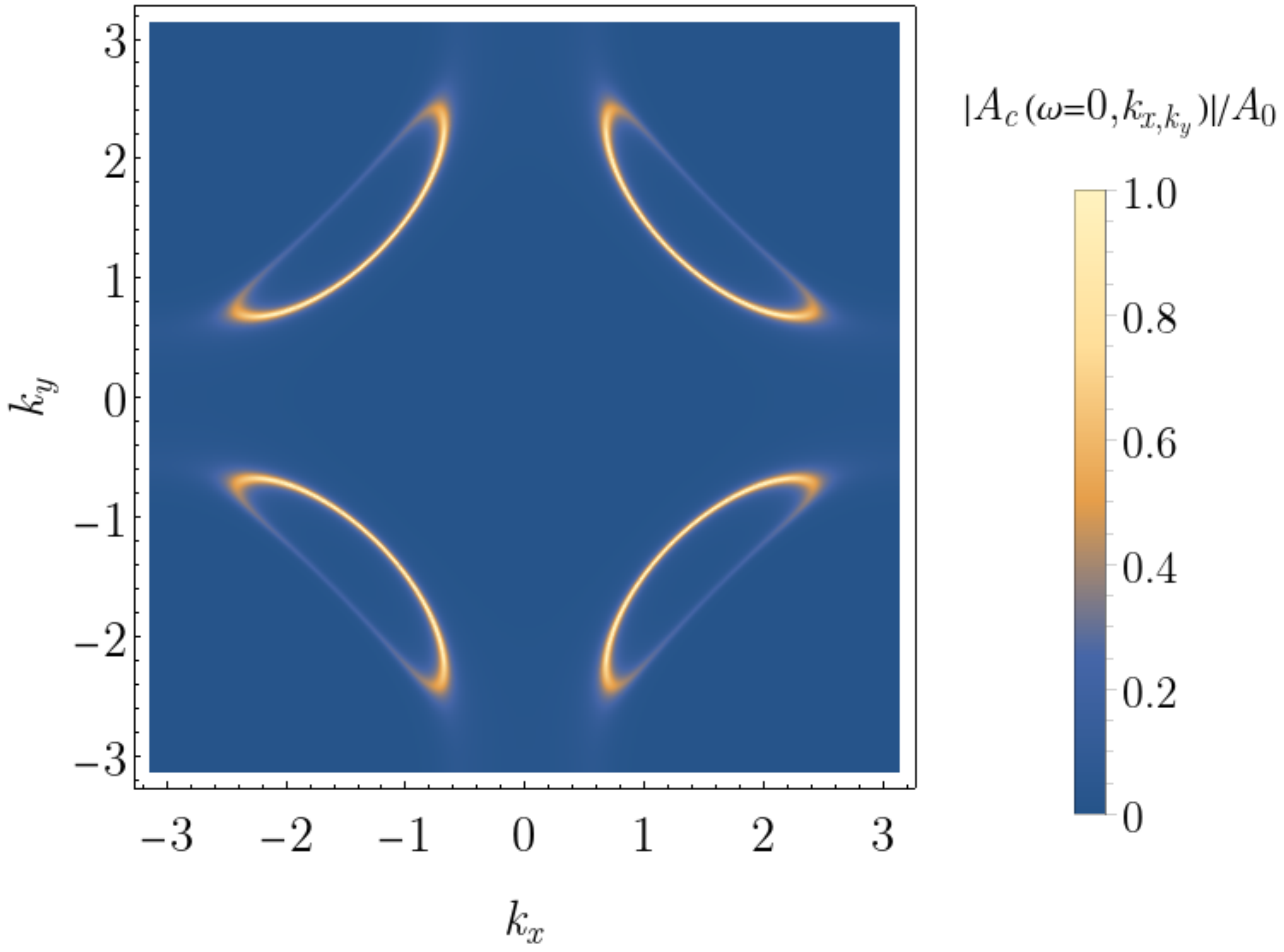
"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

Photoemission at small p



“*Fermi pockets*”

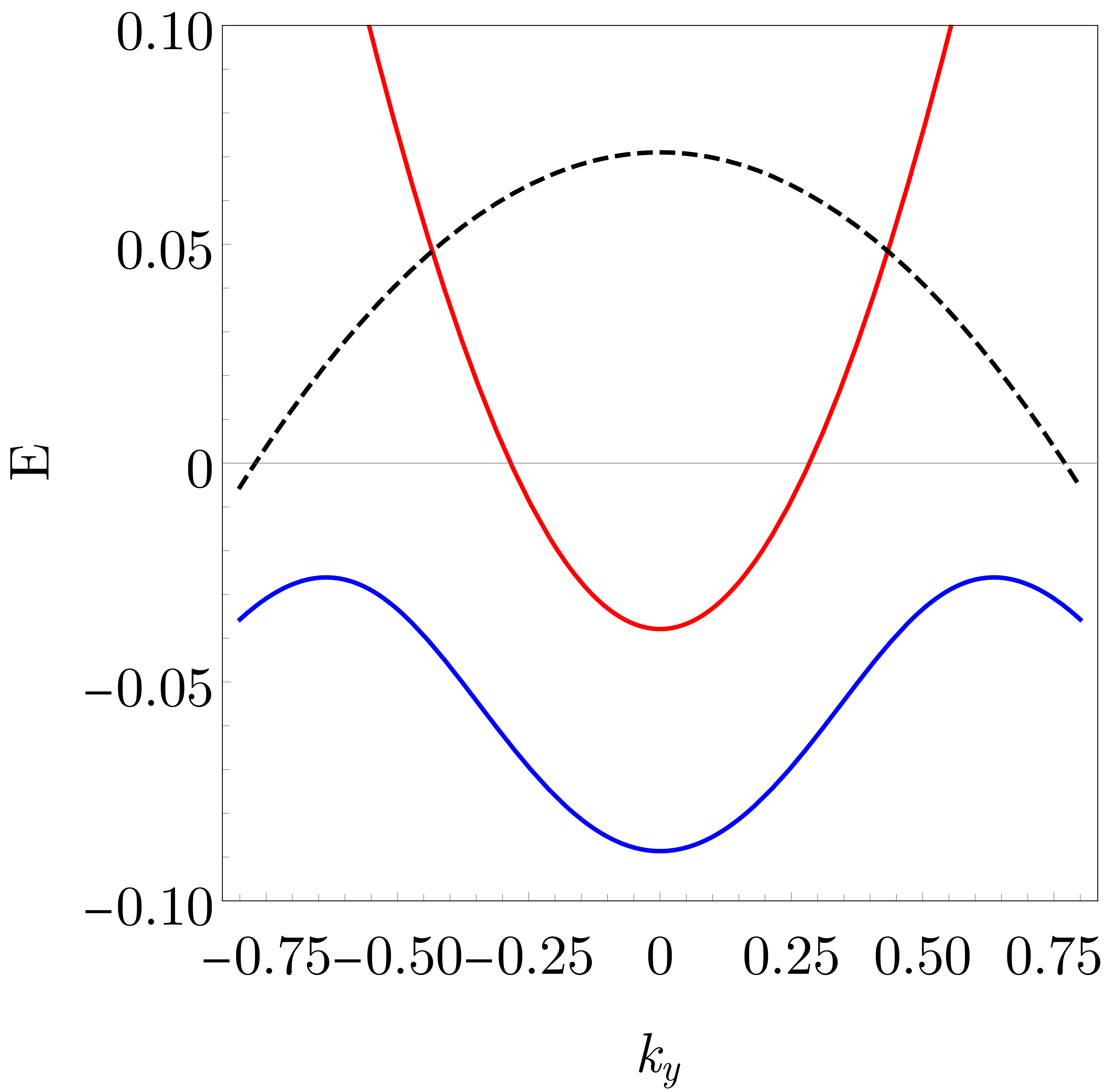


Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

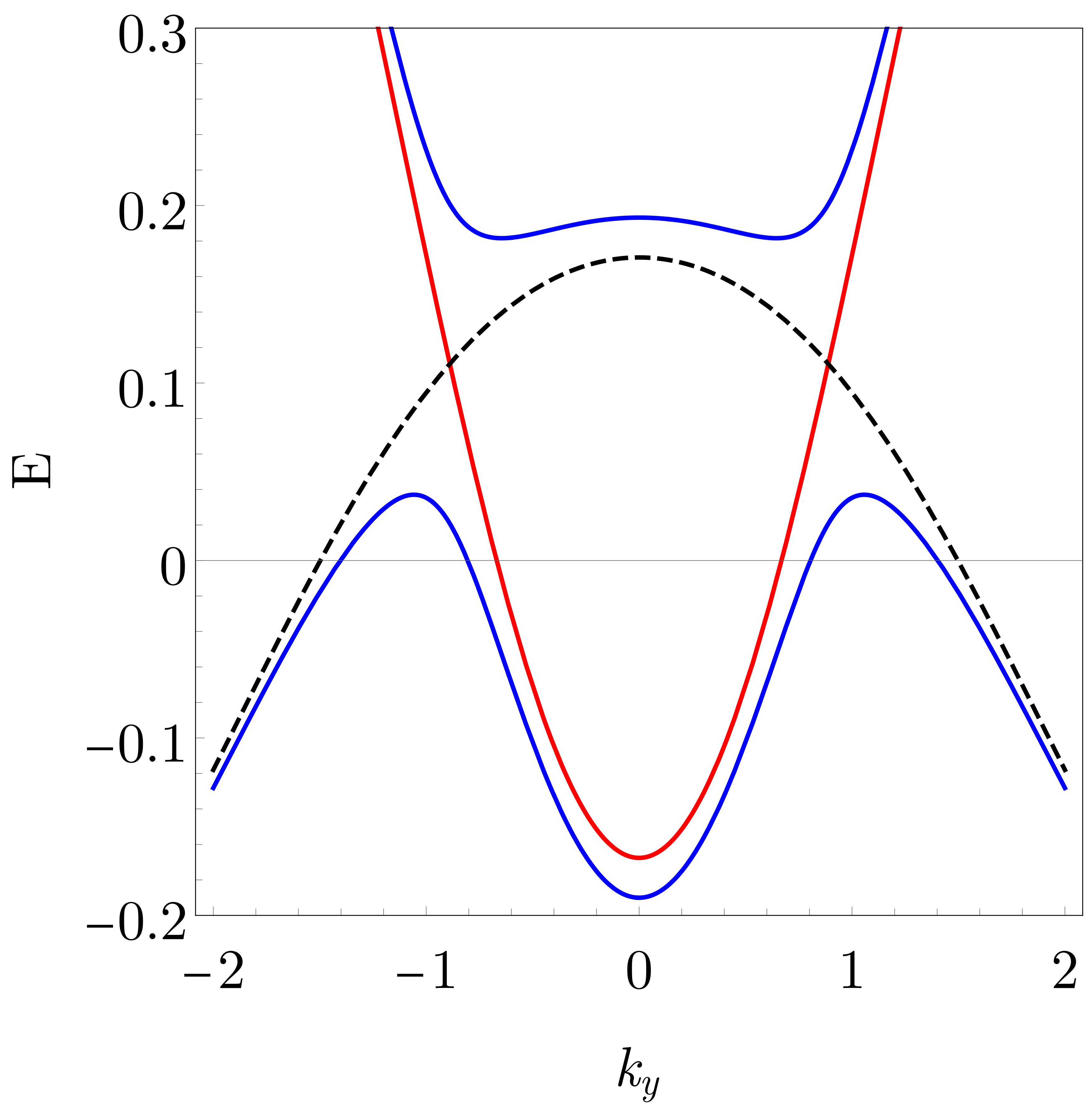
FL* in a ***one-band*** model

Electronic dispersion

Anti-node: $k_x = \pi$



Node: $k_x = 2$



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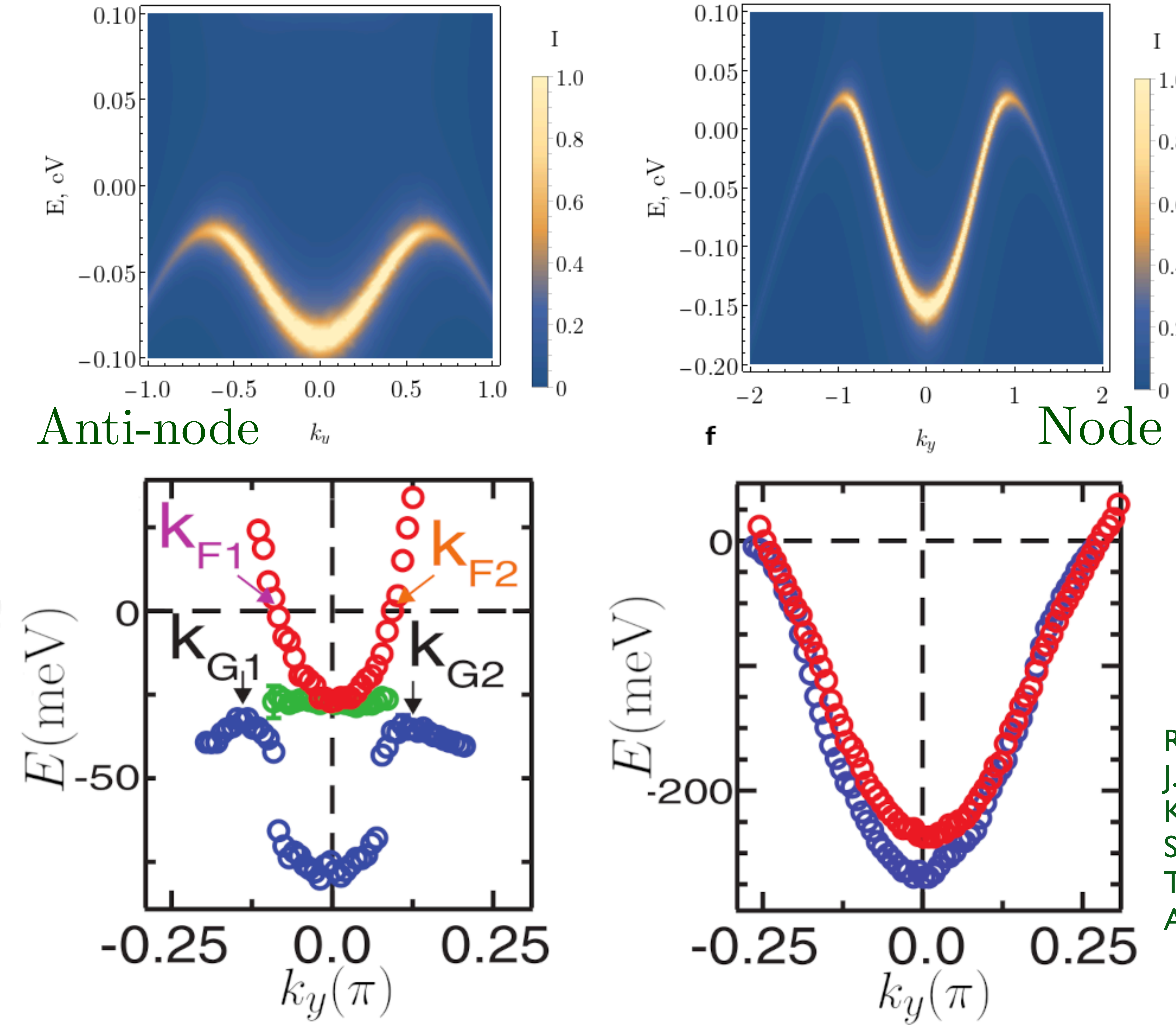
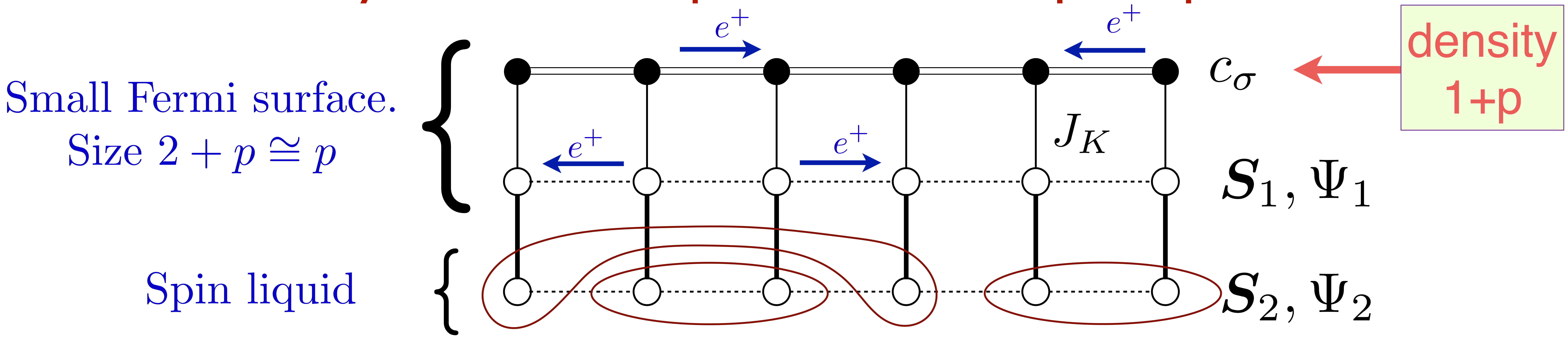


FL* in a ***one-band*** model

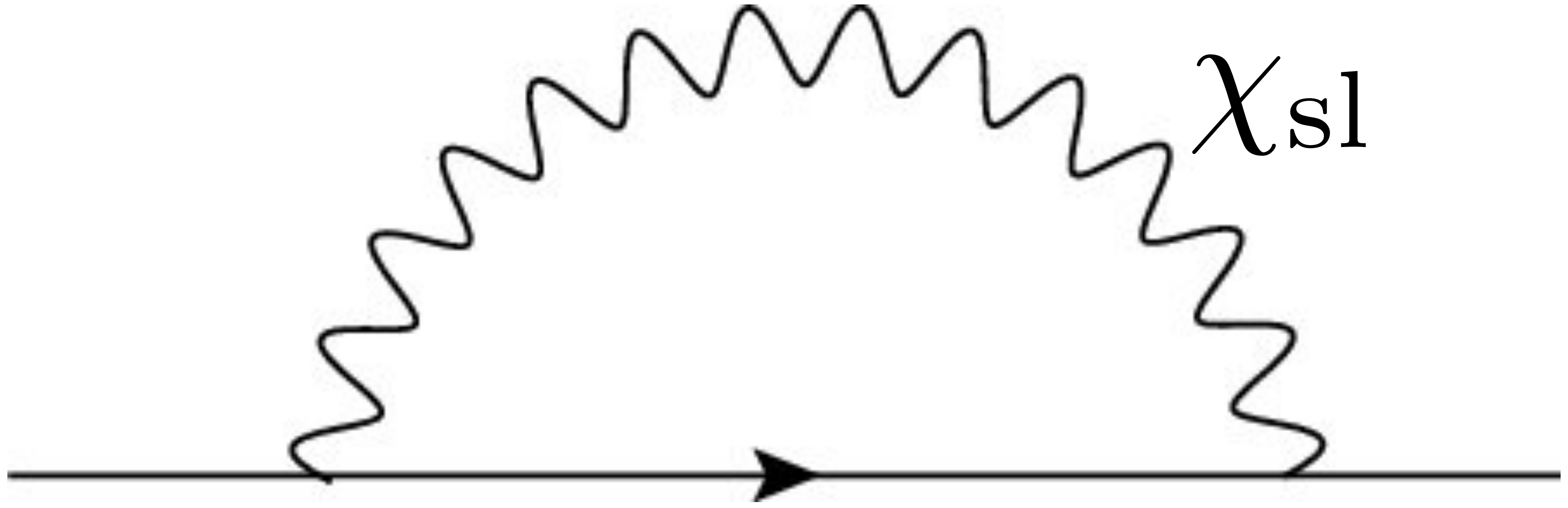
ARPES on Bi2201

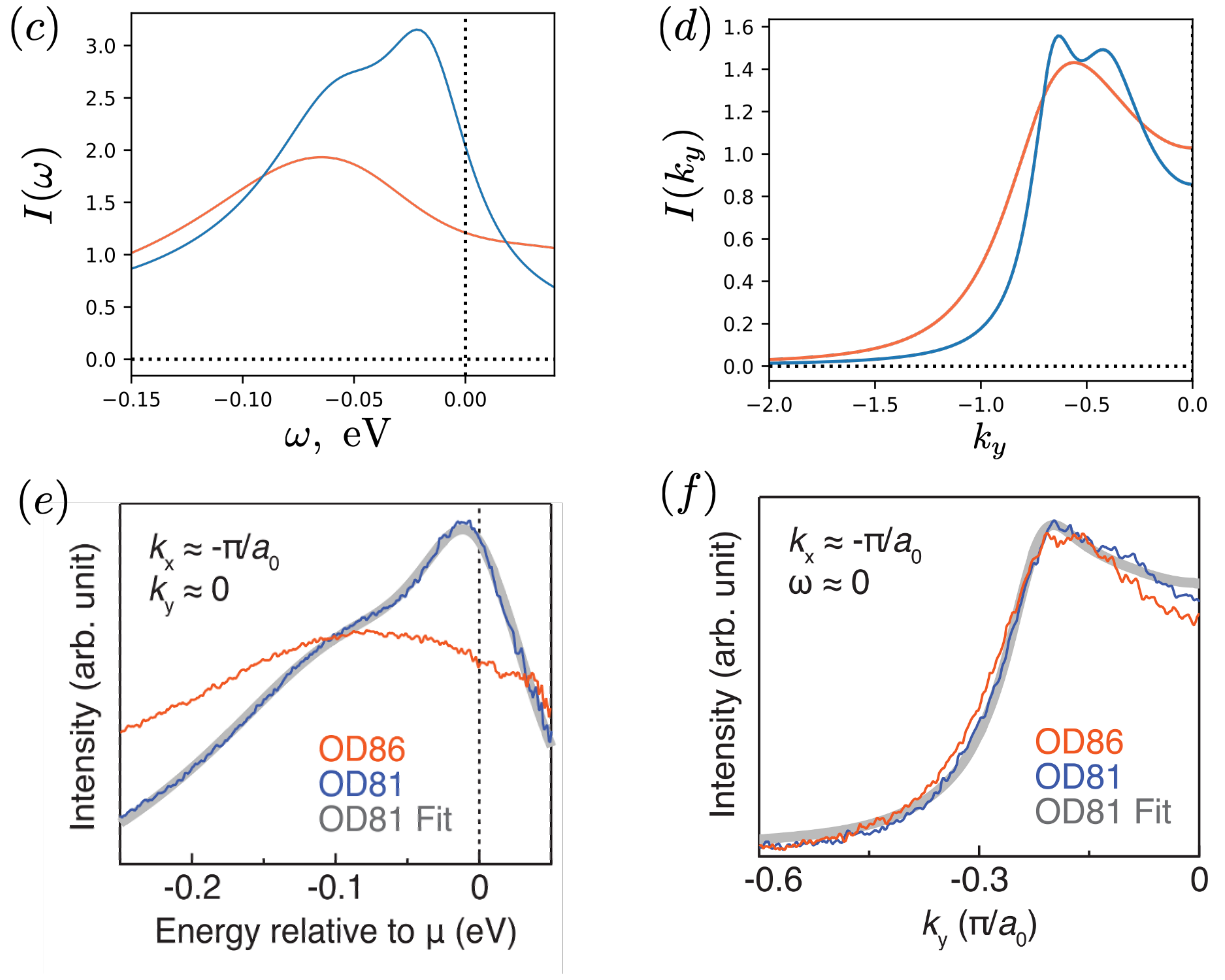
R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

Dynamic consequences of the spin liquid



The only singular gauge fluctuations are those in the spin liquid of the Ψ_2. We can compute their influence on the electronic spectrum perturbatively in the exchange couplings in terms of the dynamic spin susceptibility $\chi_{\rm sl}$.





Antinodal EDC and MDC

(c,d) Theory with SYK spin liquid in Ψ_2 layer. Similar EDC obtained by gapless $\mathbb{Z}_2$ spin liquid

(e,f) Experiments on Bi2212 by S.-D. Chen, M. Hashimoto, Y. He, D. Song, K.-J. Xu, J.-F. He, T. P. Devereaux, H. Eisaki, D.-H. Lu, J. Zaanen, and Z.-X. Shen, Science **366**, 1099 (2019).

Summary

- Probing $\mathbb{Z}_2$ spin liquid with Rydberg atoms:
  Two-state Rydberg atoms on the kagome and ruby lattices can be written *exactly* as a $\mathbb{Z}_2$ gauge theory. Evidence for intermediate scale deconfinement of a $\mathbb{Z}_2$ gauge theory on the ruby lattice.

Summary

- Probing $\mathbb{Z}_2$ spin liquid with Rydberg atoms:
  Two-state Rydberg atoms on the kagome and ruby lattices can be written *exactly* as a $\mathbb{Z}_2$ gauge theory. Evidence for intermediate scale deconfinement of a $\mathbb{Z}_2$ gauge theory on the ruby lattice.

- Paramagnon fractionalization theory of FL* for the pseudogap metal of the cuprate high temperature superconductors:
  Don't fractionalize the mobile electron, but fractionalize the paramagnon into 'ancilla qubits'.
  Predicts electronic spectra in good agreement with observations in *both* nodal and anti-nodal regions.
  $(\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{SU}(2)_S)/\mathbb{Z}_2$ theory for transition from FL* to FL.