Fermi-volume-changing quantum phase transitions and the cuprate phase diagram

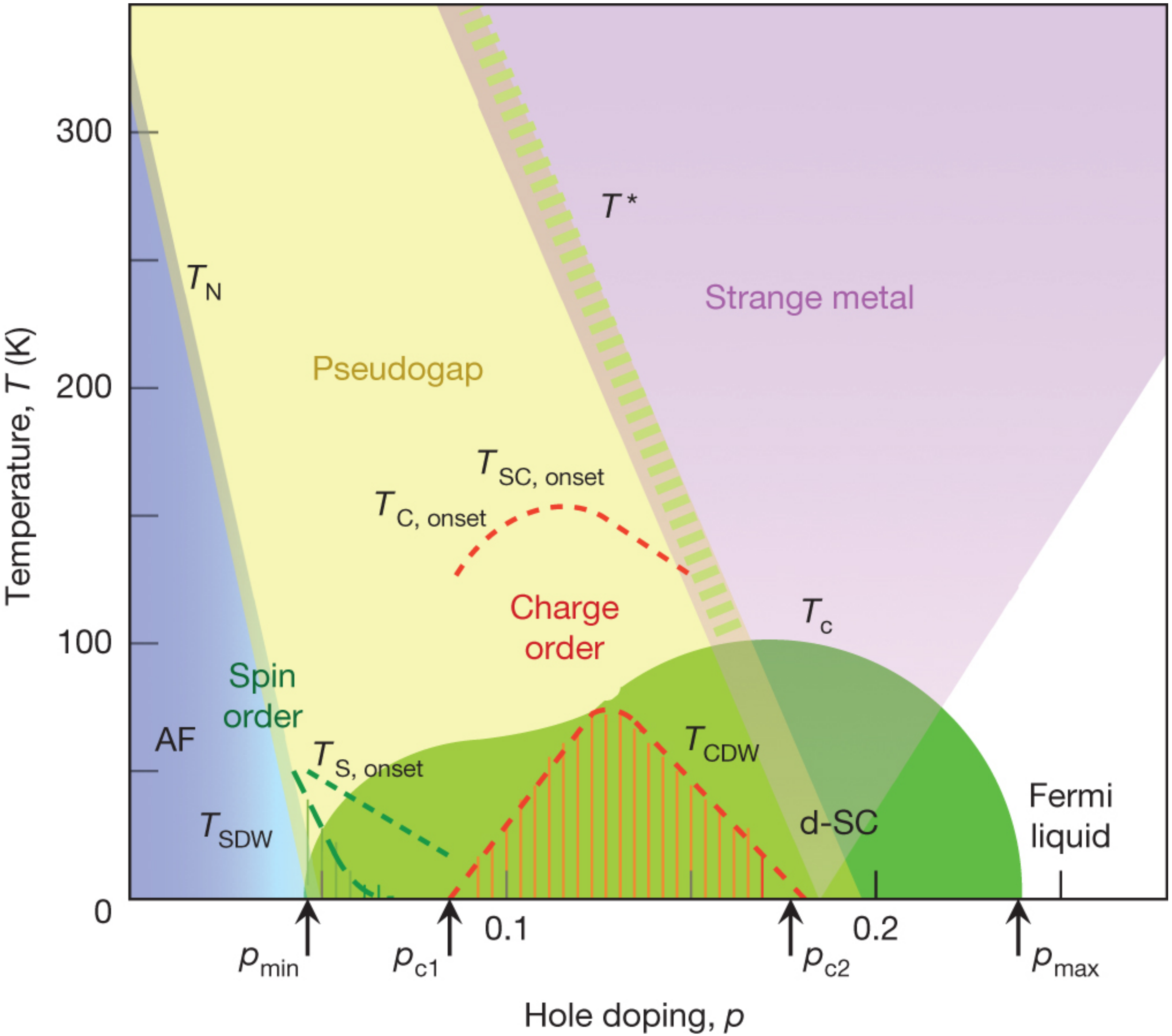
Florida State University
Tallahassee
January 31, 2025
Subir Sachdev

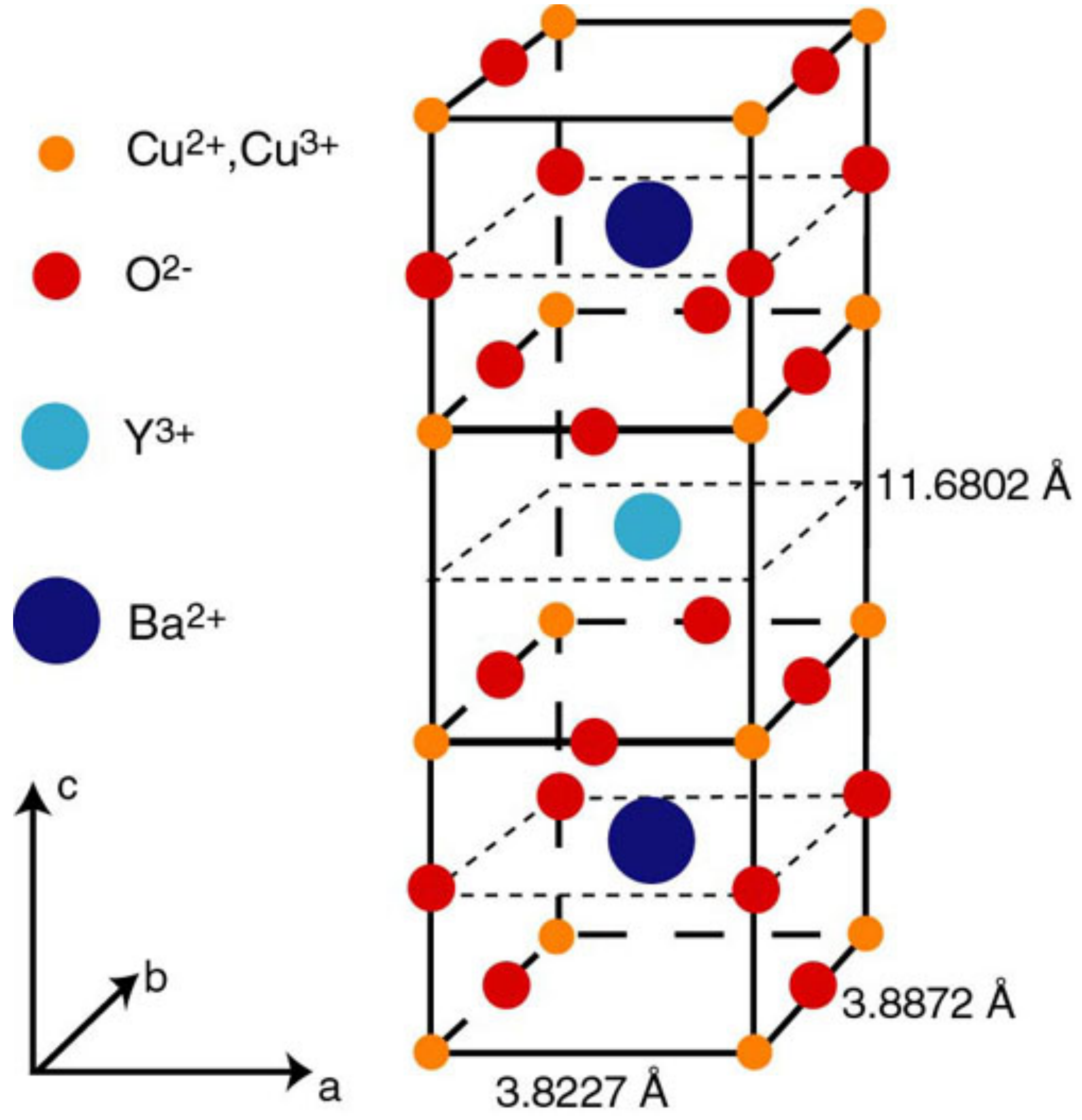
arXiv:2501.16417

Talk online: sachdev.physics.harvard.edu

PHYSICS
VE RI TAS
HARVARD

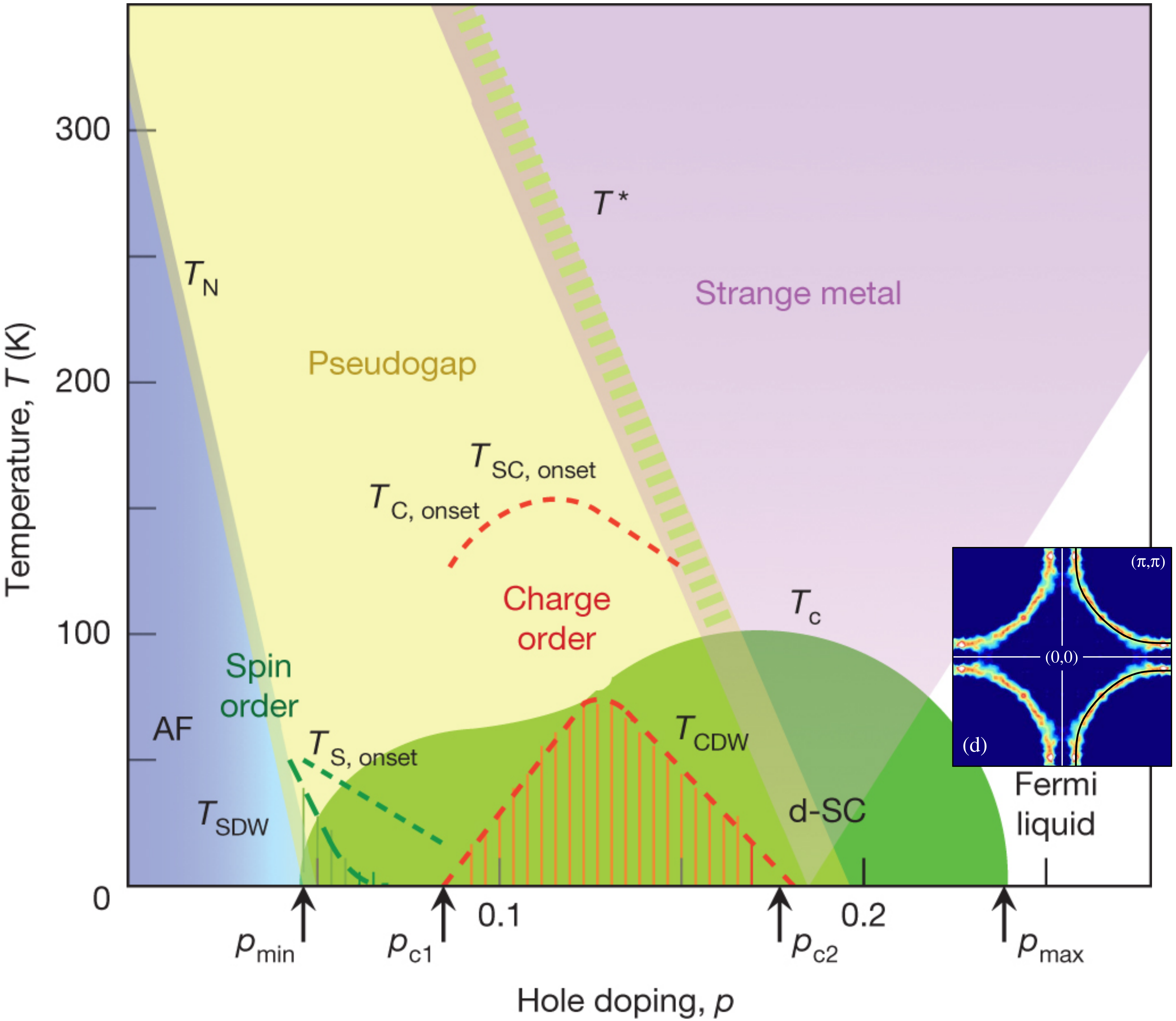
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





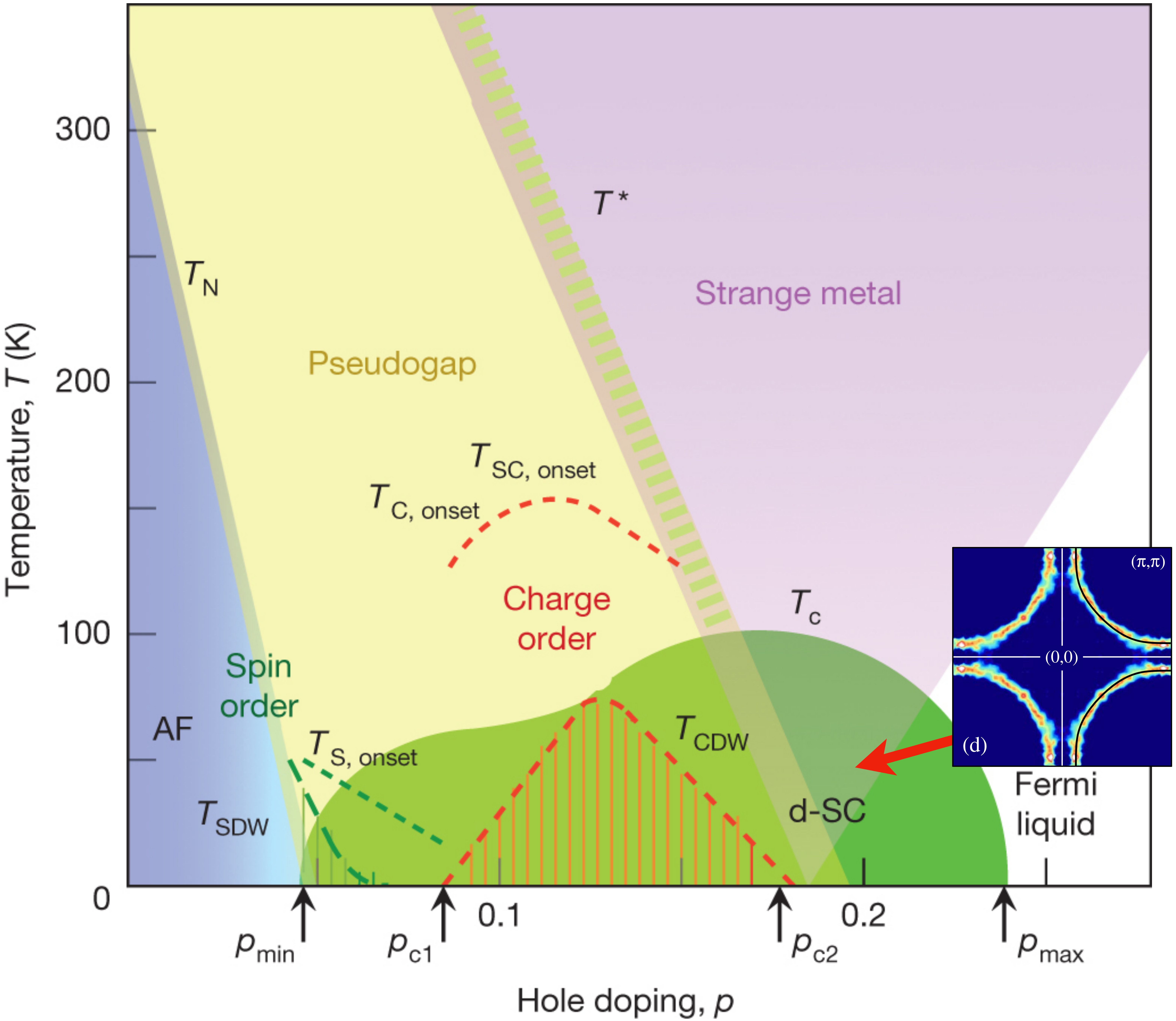
$YBa_2Cu_3O_{6+x}$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



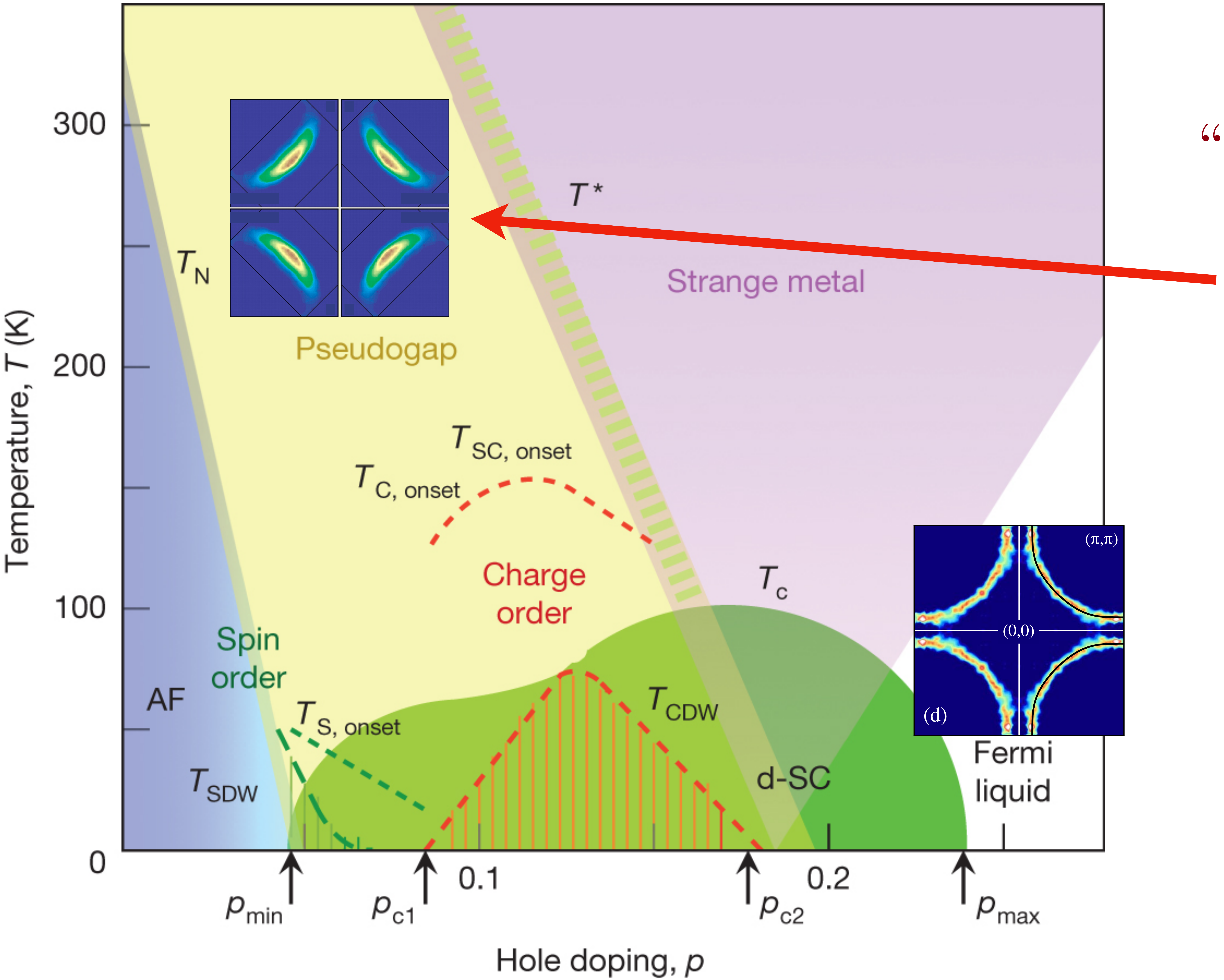
Fermi surface
as expected
in a model
of free electrons

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



BCS theory of
d-wave superconductivity
induced by
antiferromagnetic
spin fluctuations

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

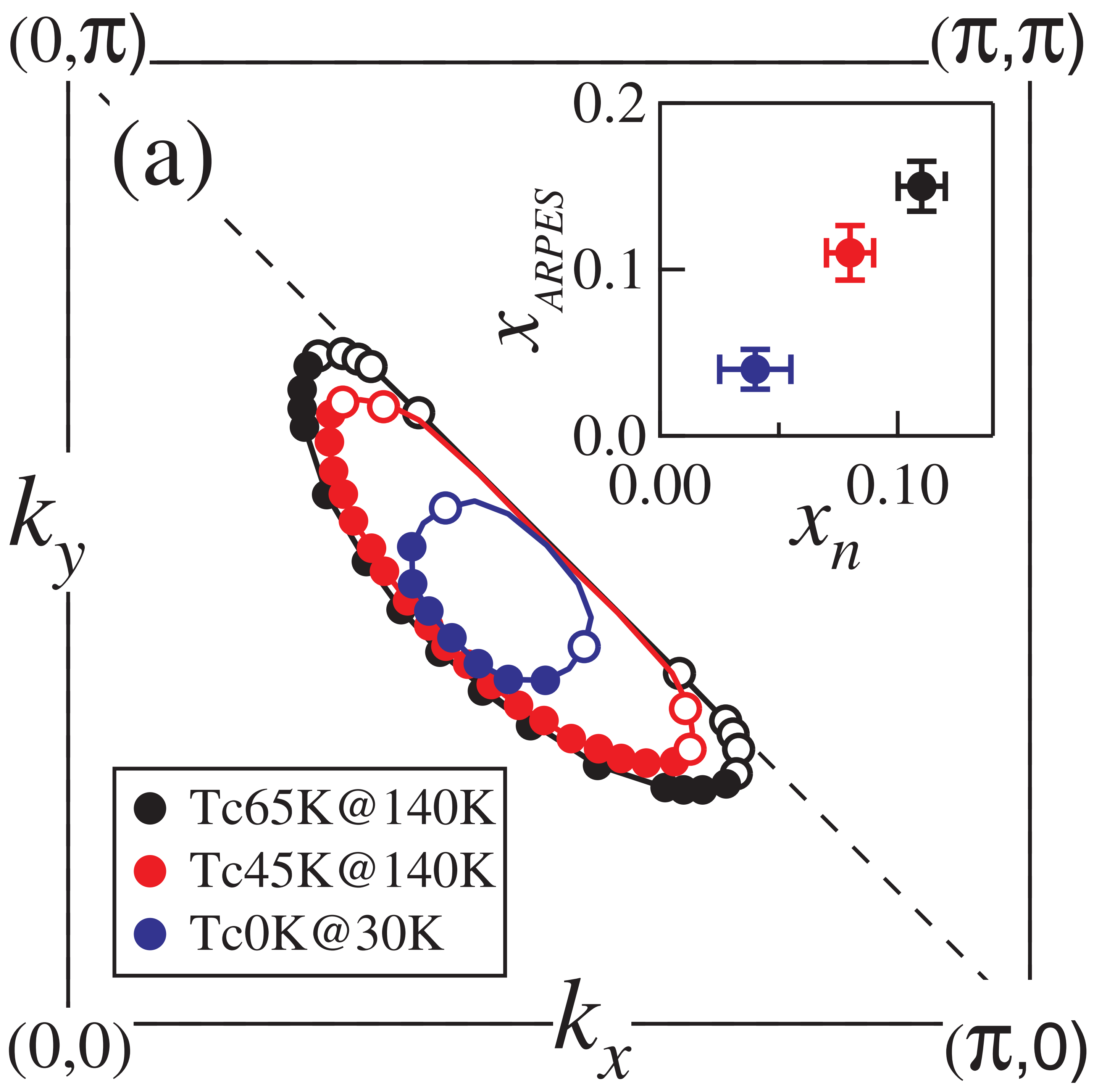


"Pseudogap metal"
Fermi surface
modified by
electron-electron
interactions

Requires
fractionalized
spin liquid
background

Photoemission expts in cuprates in pseudogap metal

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, *PRB* **73**, 174501 (2006); T. D. Stanescu and G. Kotliar, *PRB* **74**, 125110 (2006). C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, *PRL* **97**, 136401 (2006). S. Sakai, Y. Motome, M. Imada, *PRL* **102**, 056404 (2009).



Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

Non-Luttinger volume Fermi surface

The missing electrons are in a spin liquid!

Oshikawa's topological Luttinger argument implies that non-Luttinger Fermi surfaces must be accompanied by a background of fractionalized spinon excitations of a spin liquid

T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)
R. K. Kaul, A. Kolezhuk, M. Levin, S. S., T. Senthil, PRB **75** , 235122 (2007)
Y. Qi, S. S., PRB **81**, 115129 (2010)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Anisotropic damping and wave vector dependent susceptibility of the spin fluctuations in $La_{2-x}Sr_xCuO_4$ studied by resonant inelastic x-ray scattering

H. C. Robarts, M. Barthélemy, K. Kummer, M. García-Fernández, J. Li, A. Nag, A. C. Walters, K. J. Zhou, and S. M. Hayden

PHYSICAL REVIEW B **100**, 214510 (2019)

- Difficult to have intense paramagnons from a small Fermi surface.
- Spin waves only present at low energies in the presence of antiferromagnetic order
- Most natural interpretation is a spinon continuum, similar to that observed on the triangular lattice in $KYbSe_2$

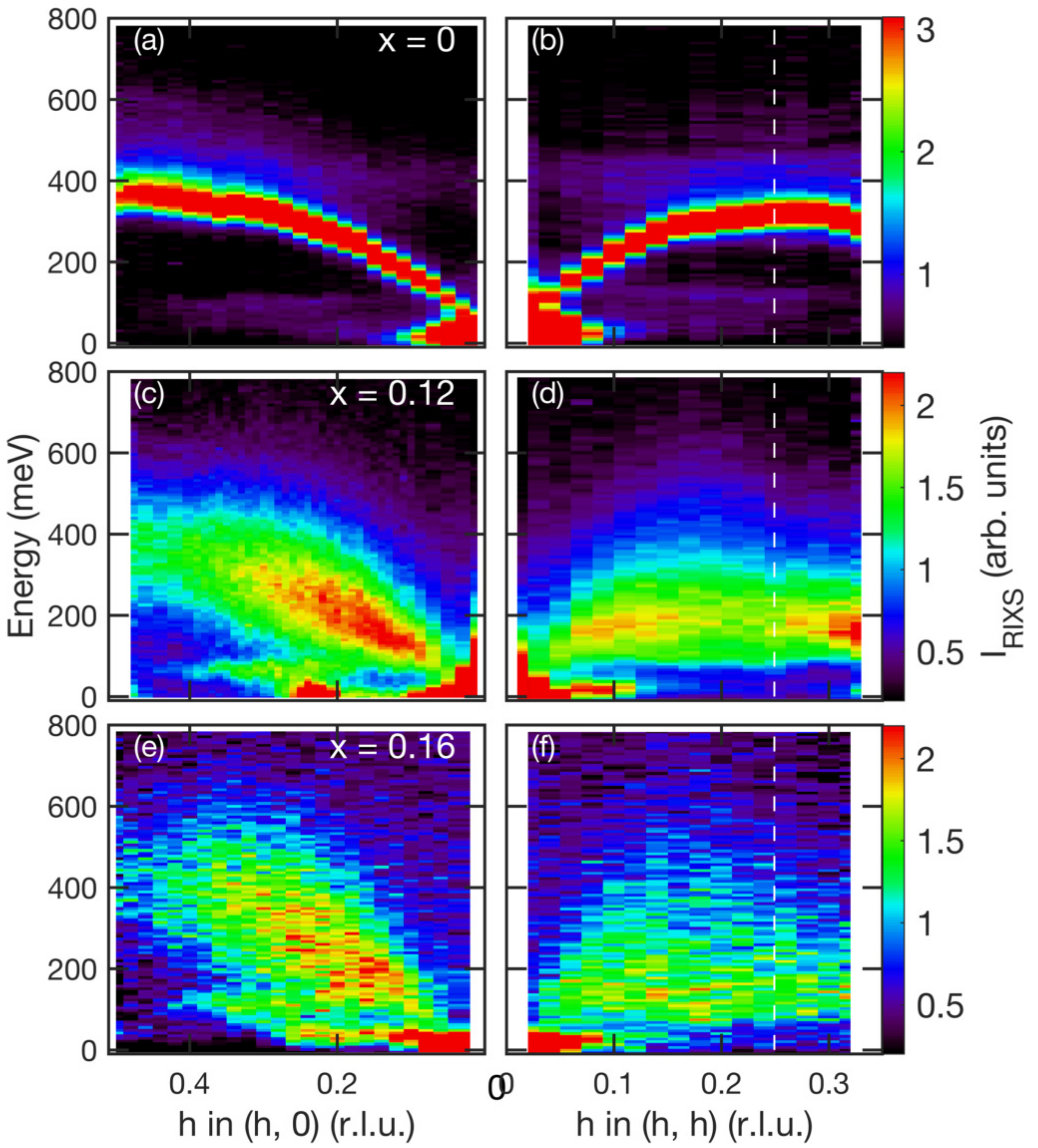
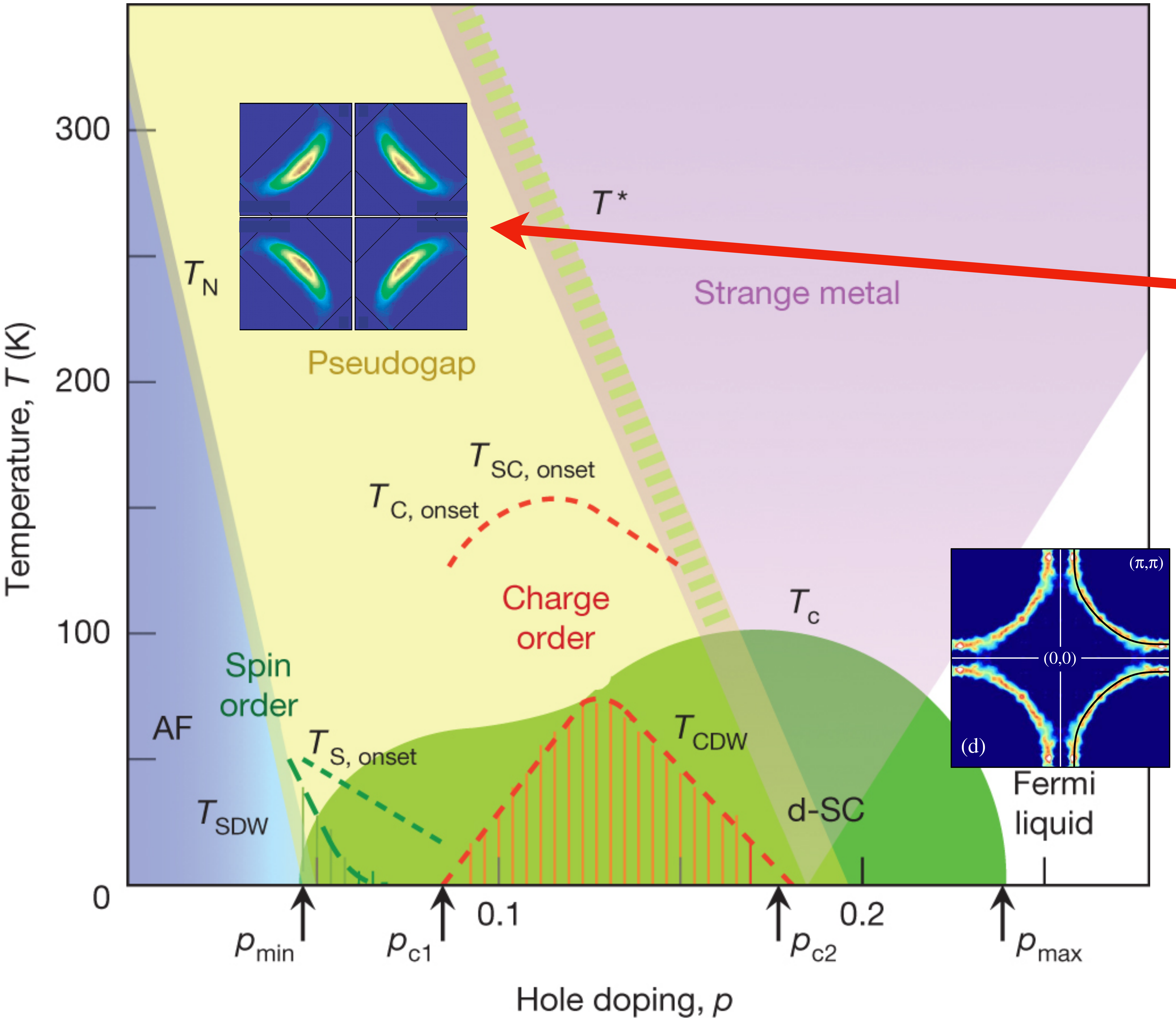


FIG. 2. I_{RIXS} intensity maps as a function of **Q** in LSCO $x = 0$ ($T \approx 20$ K), 0.12, and 0.16 ($T \approx 30$ K).

Pietro Bonetti, Maine Christos and S.S. (BCS), PNAS **121**, e2418633121 (2024)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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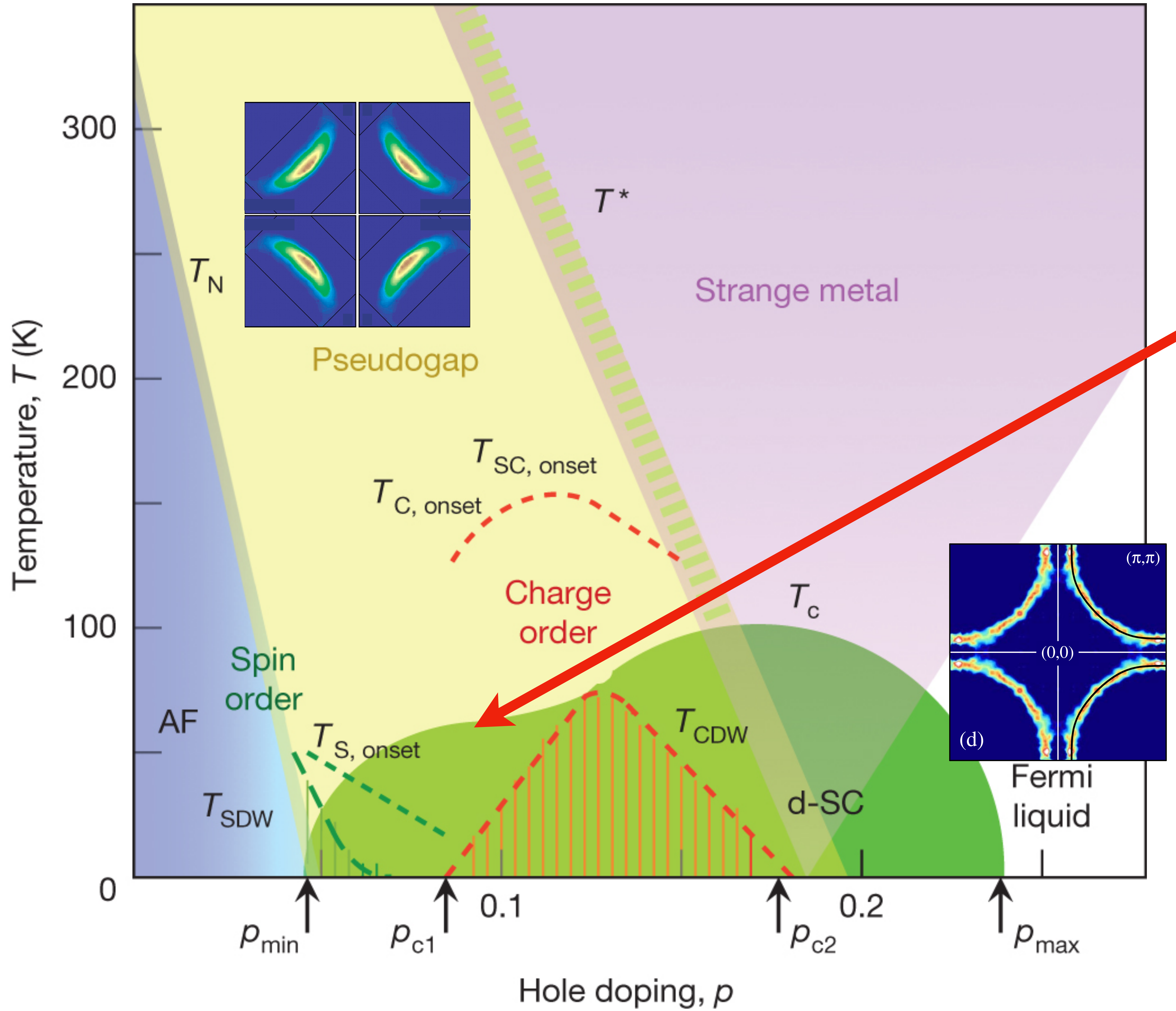
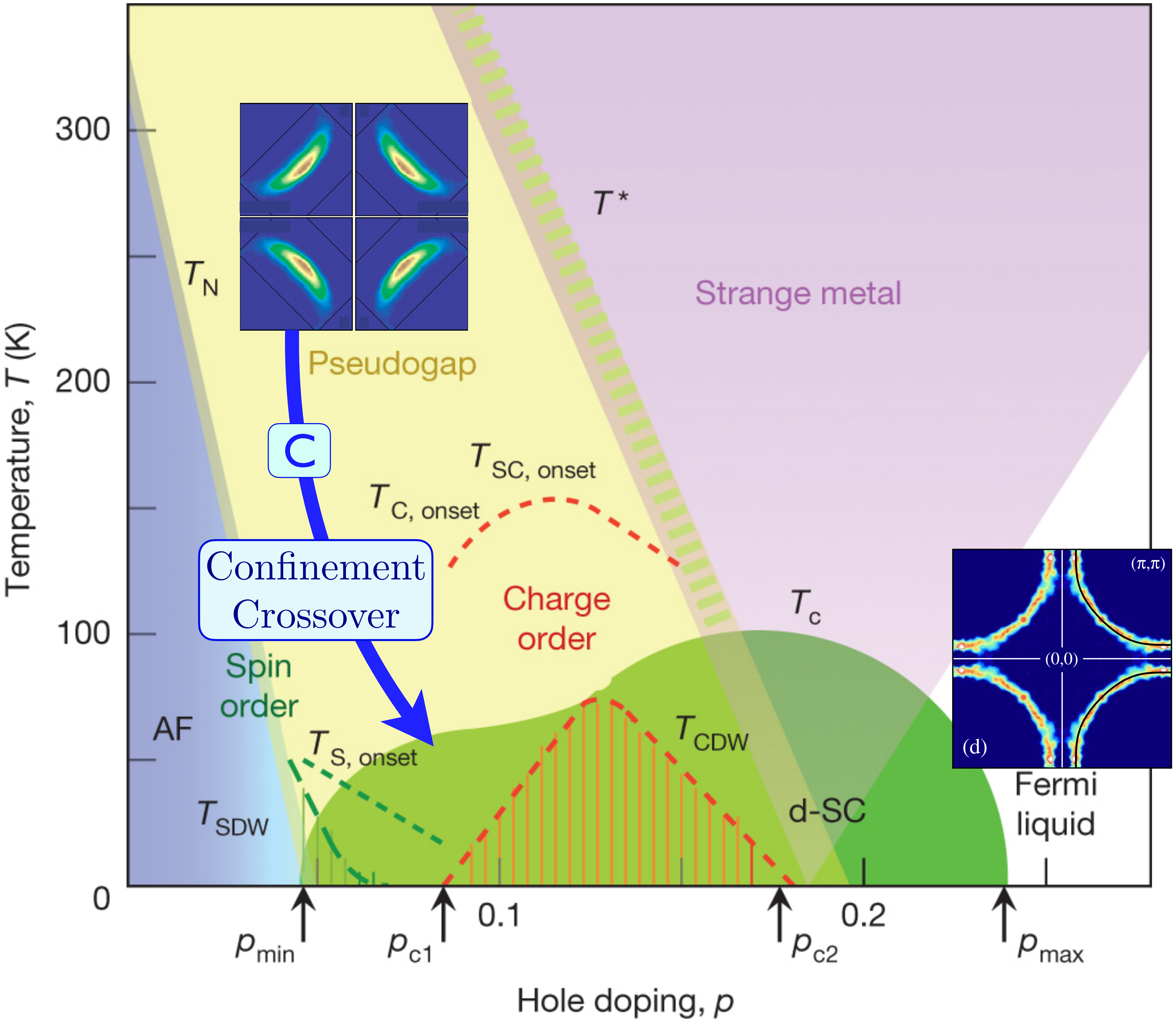
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



"Conventional" phases
with
symmetry breaking
and
without fractionalization

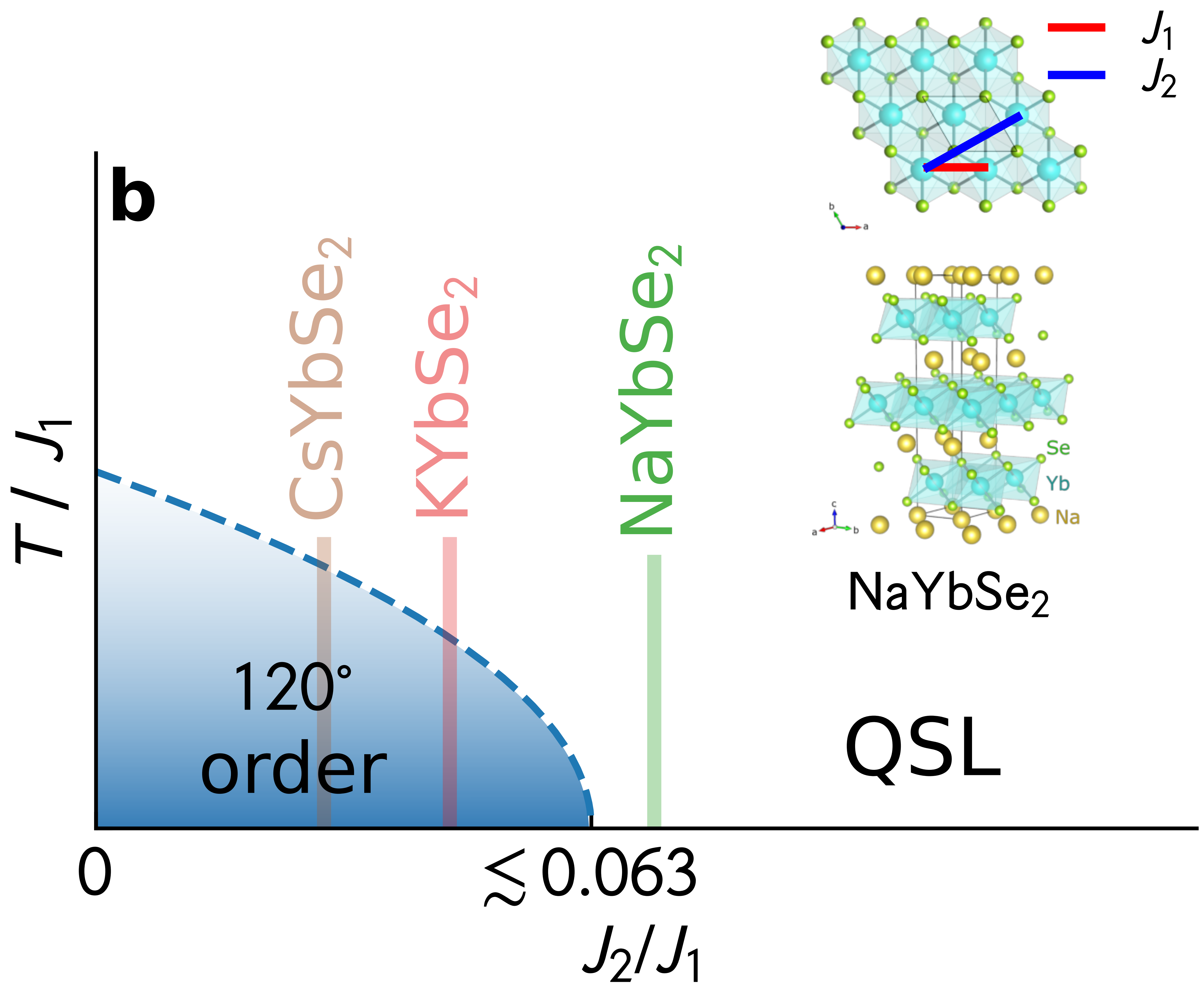
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

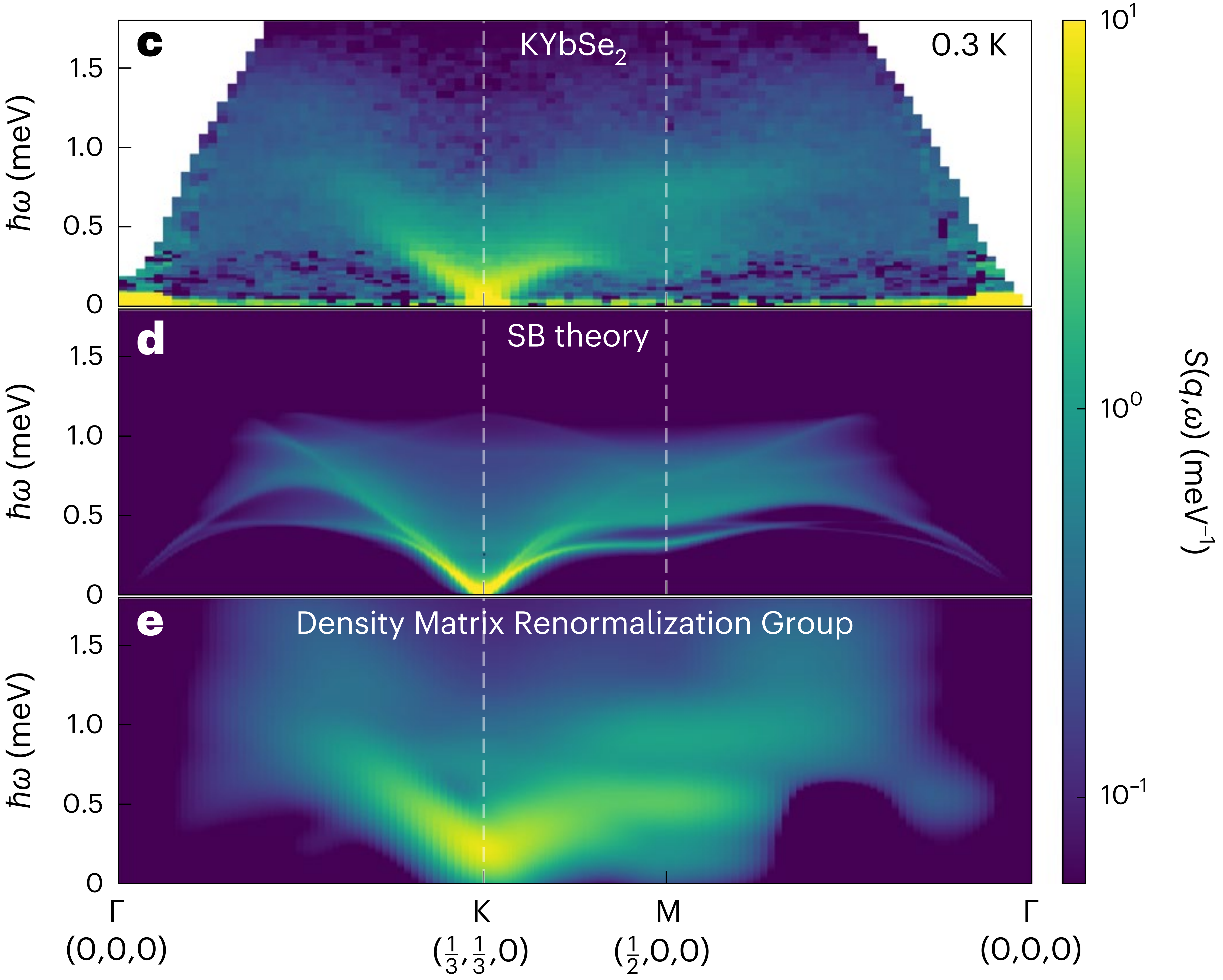


Proximate spin liquid and fractionalization in the triangular antiferromagnet $KYbSe_2$

A. O. Scheie[1], E. A. Ghioldi[2,3], J. Xing[4], J. A. M. Paddison[4], N. E. Sherman[5,6], M. Dupont[5,6], L. D. Sanjeewa[7,8], Sangyun Lee[9], A. J. Woods[9], D. Abernathy[1], D. M. Pajerowski[1], T. J. Williams[1], Shang-Shun Zhang[10], L. O. Manuel[3], A. E. Trumper[3], C. D. Pemmaraju[11], A. S. Sefat[4], D. S. Parker[4], T. P. Devereaux[11,12], R. Movshovich[9], J. E. Moore[5,6], C. D. Batista[2,13] & D. A. Tennant[2,13]

Nature Physics **20**, 74 (2024)





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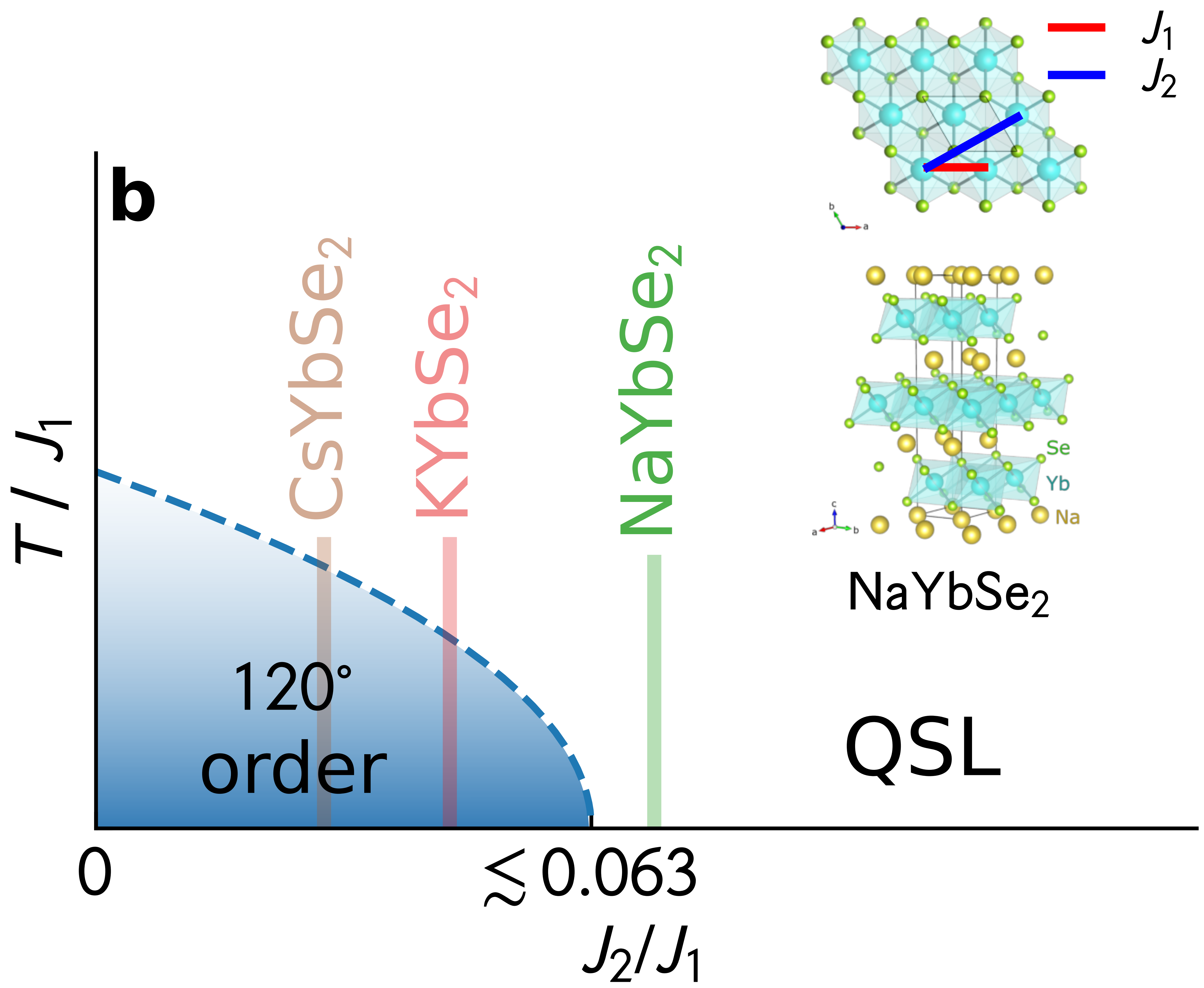
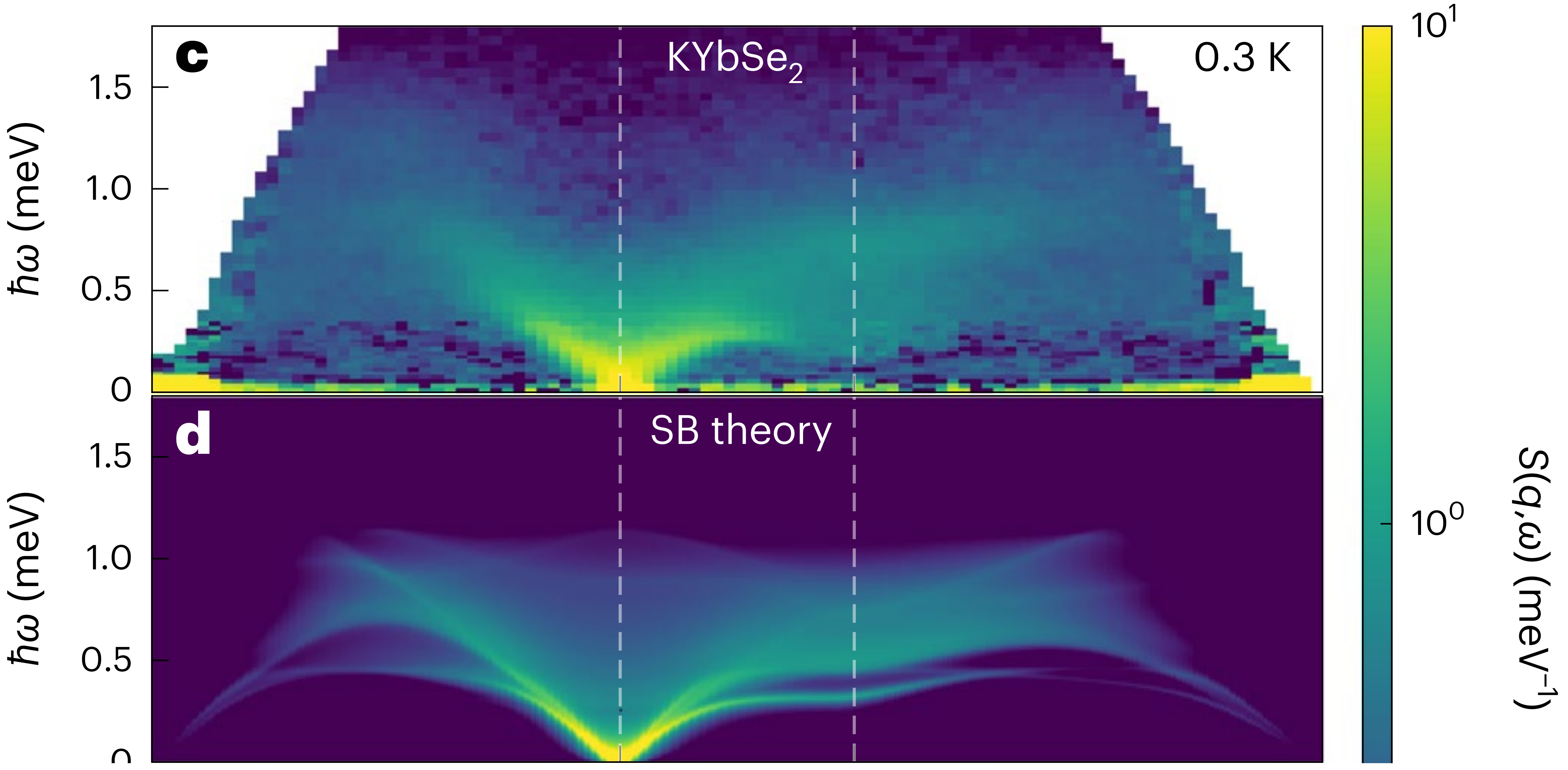
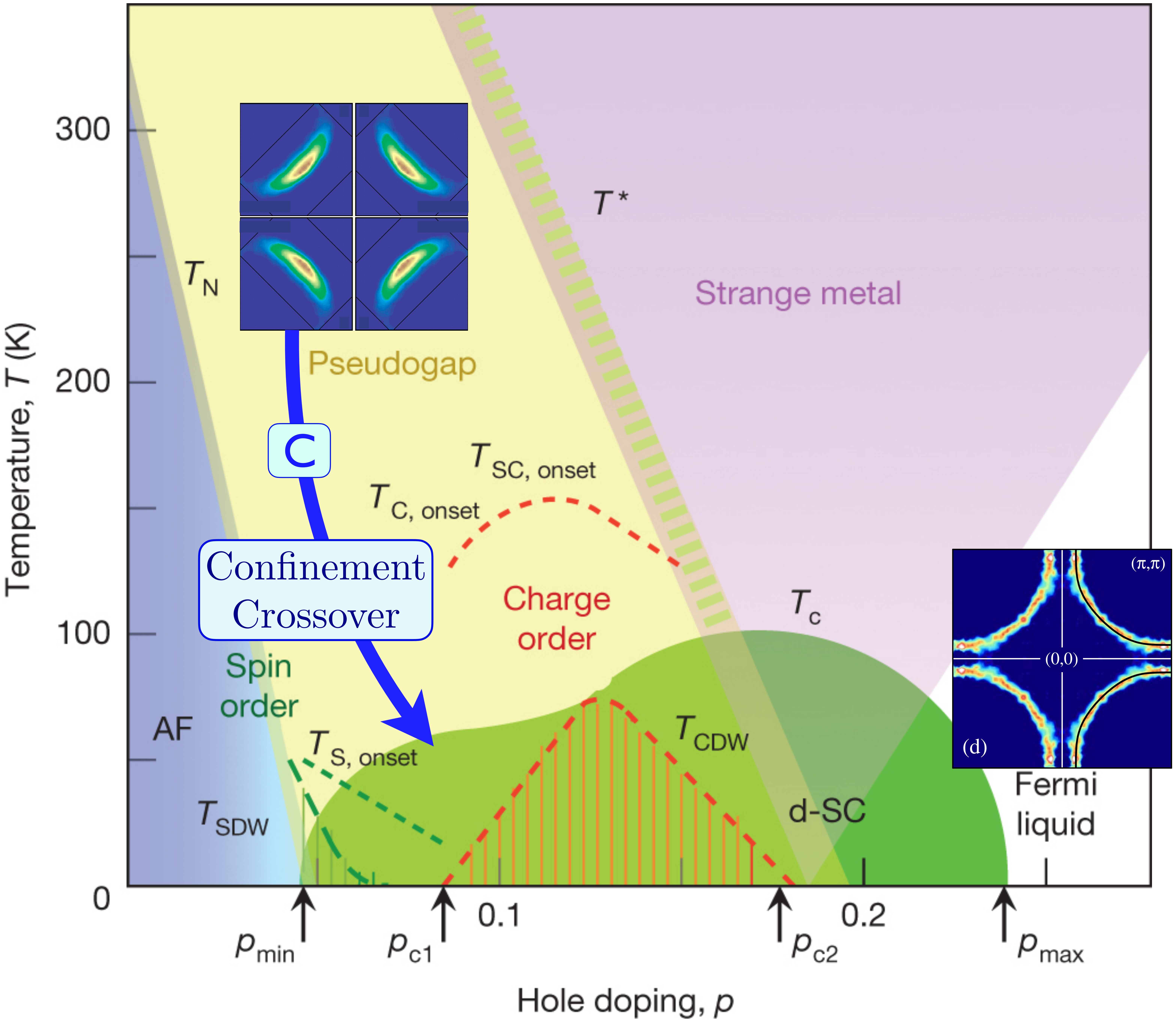


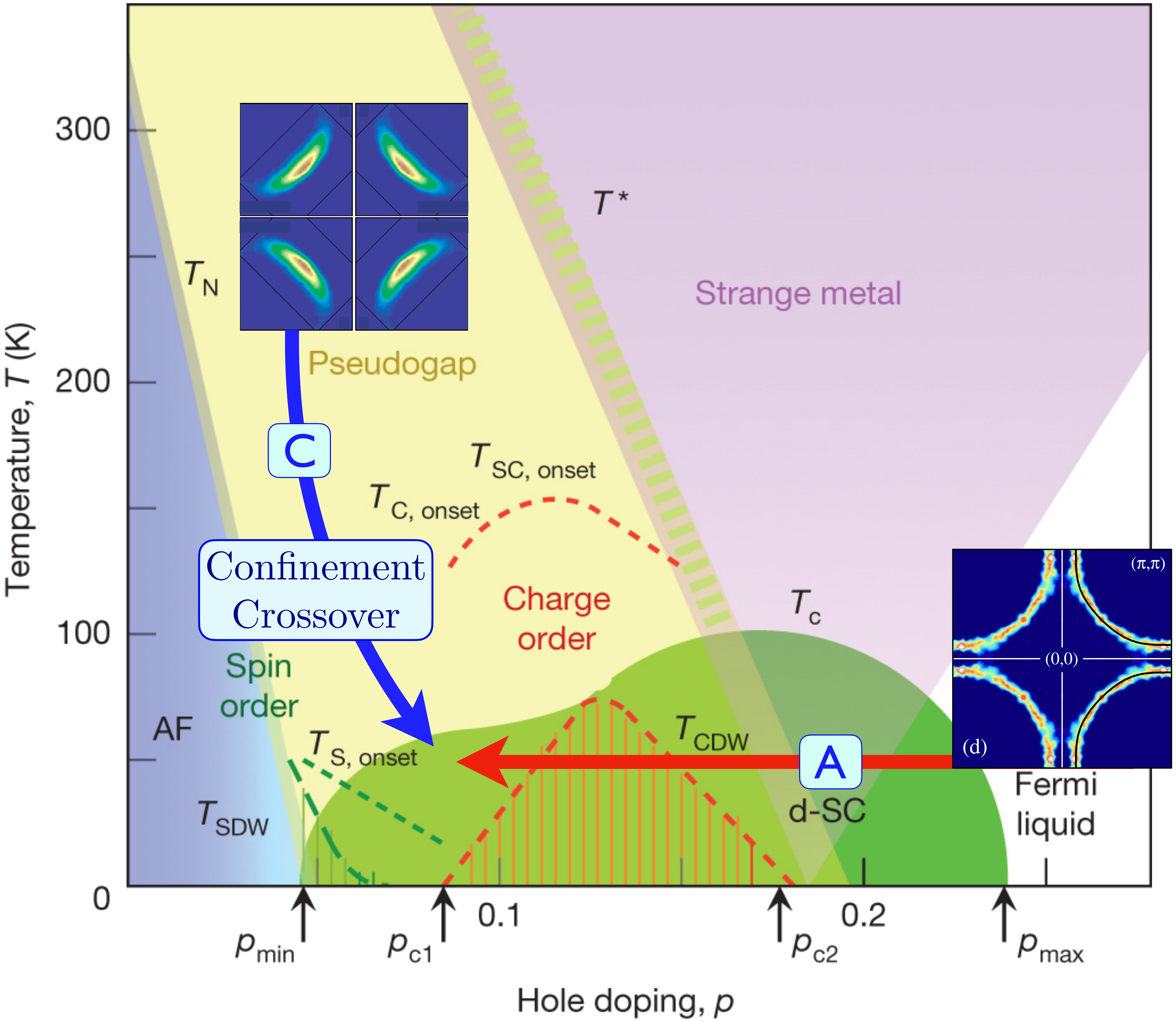


Neutron scattering observations "identify a diffuse continuum with a sharp lower bound within the measured spectra . . . The key features of the data are reproduced by Schwinger boson theory" of fractionalized spinons.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



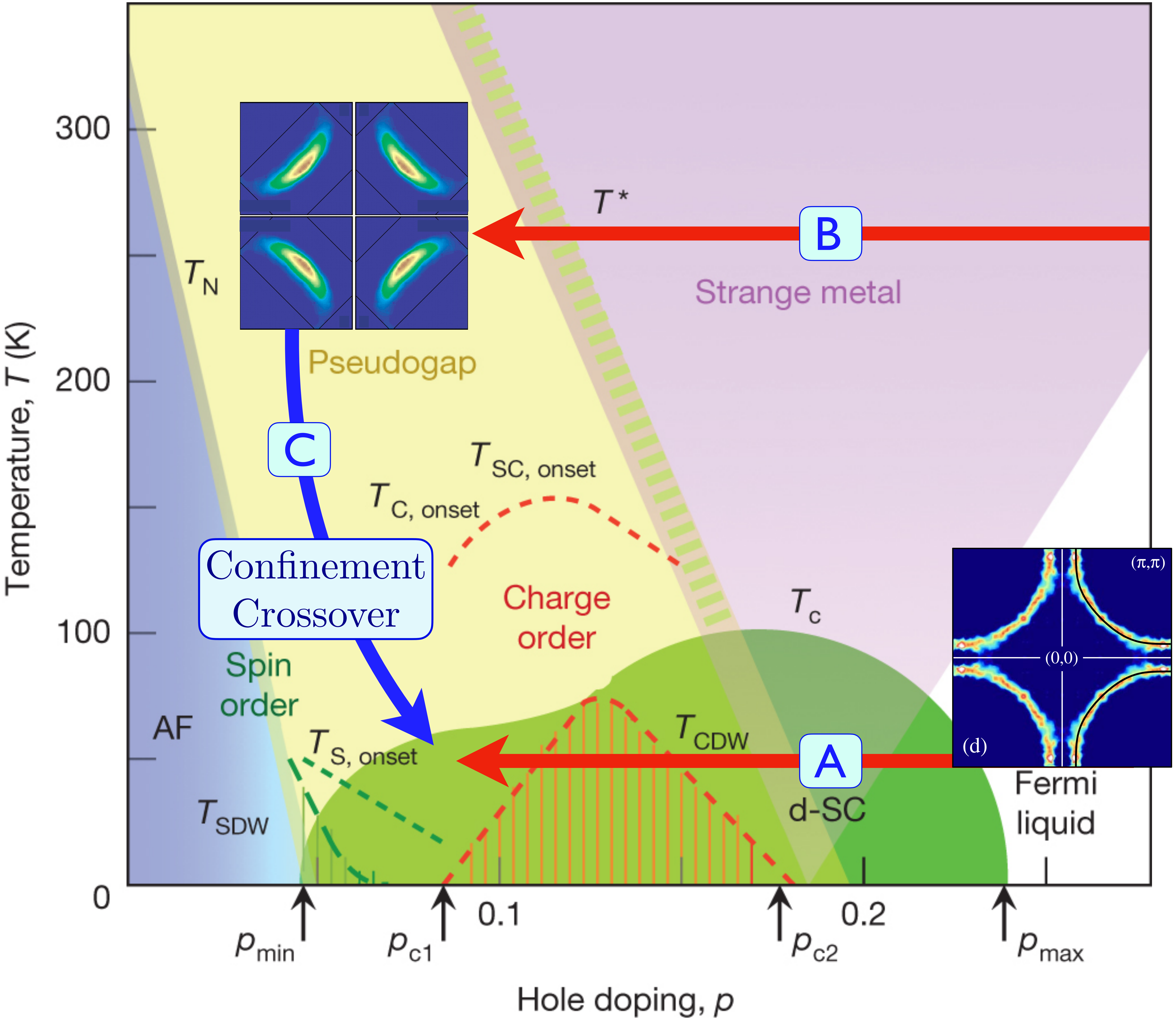
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
<u>with</u> symmetry breaking
and with spatial disorder.

FL-SDW QPT

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
without symmetry breaking
and with spatial disorder.

FL-FL* QPT

Requires fractionalization

Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

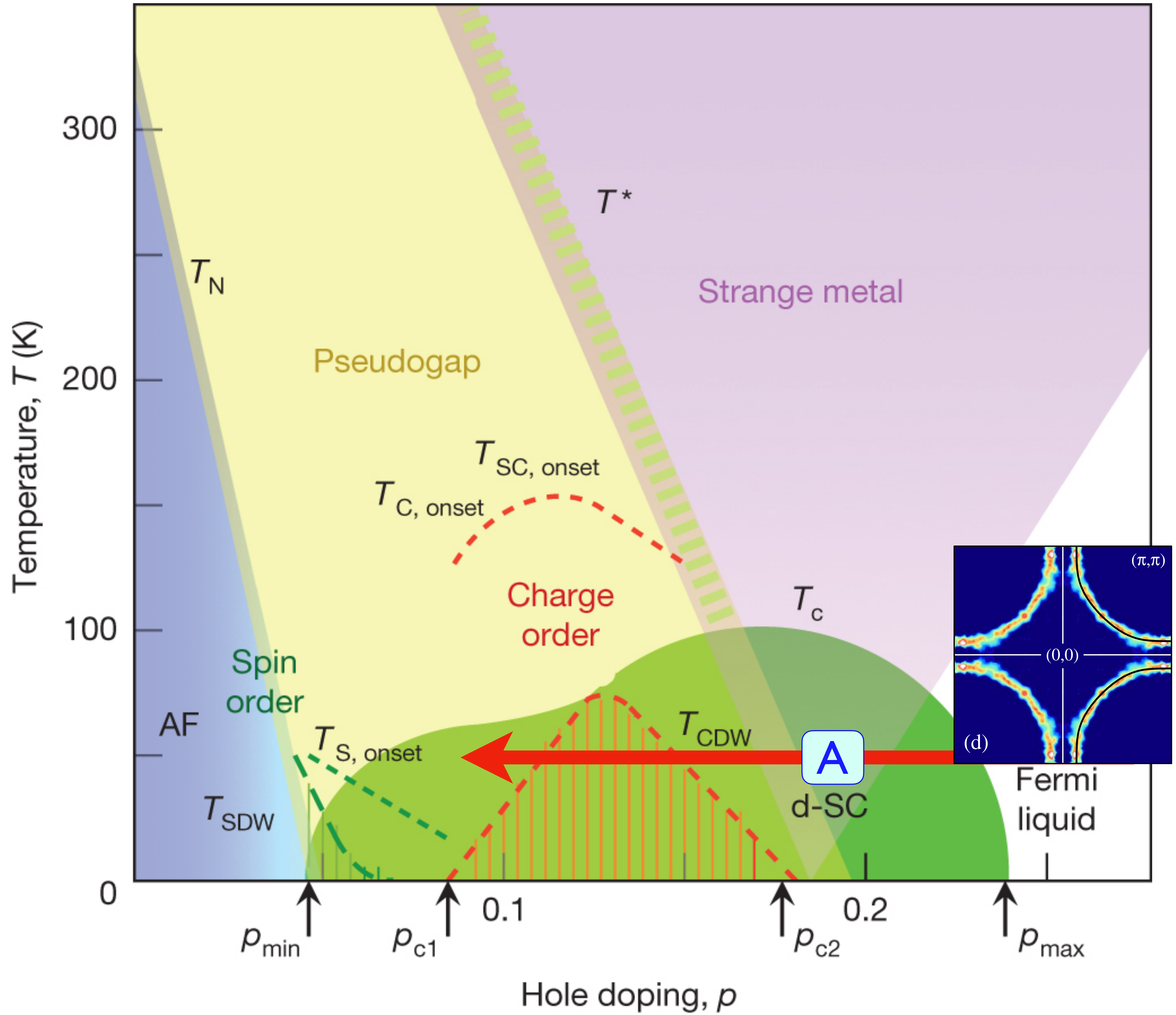
FL-SDW QPT

A. FL-SDW QPT

B. FL-FL* QPT

C. Confinement crossover

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



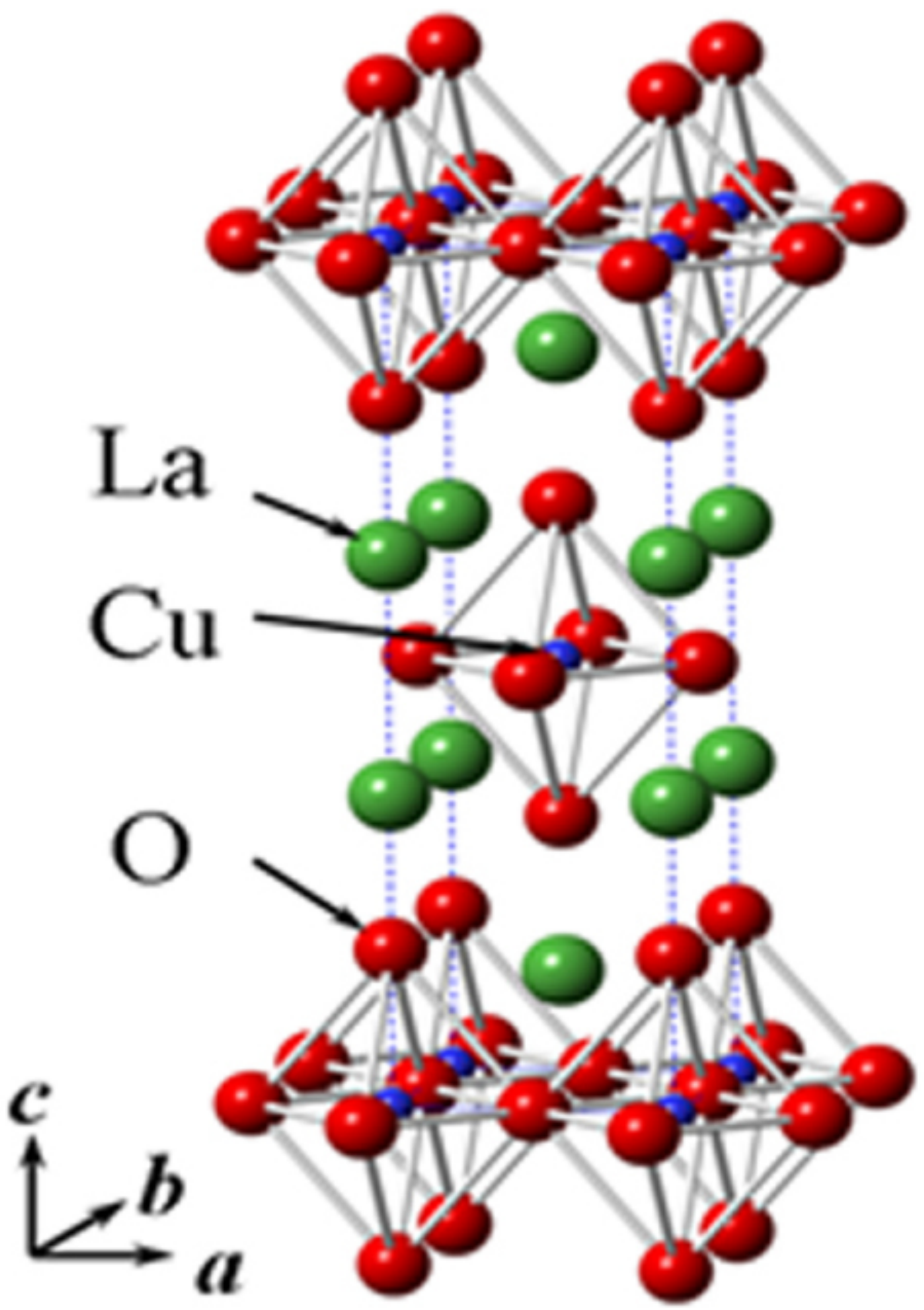
Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

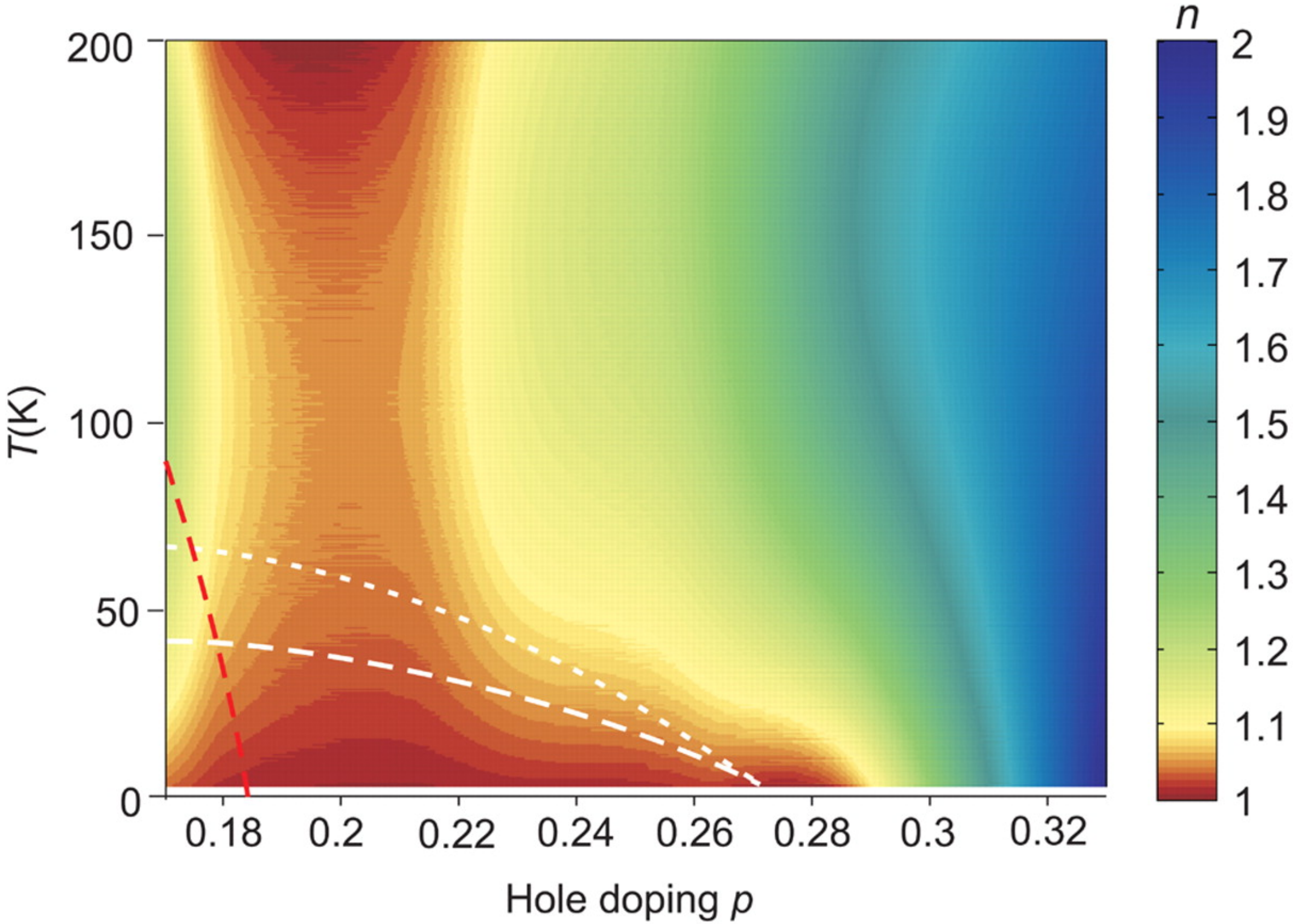
FL-SDW QPT

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009





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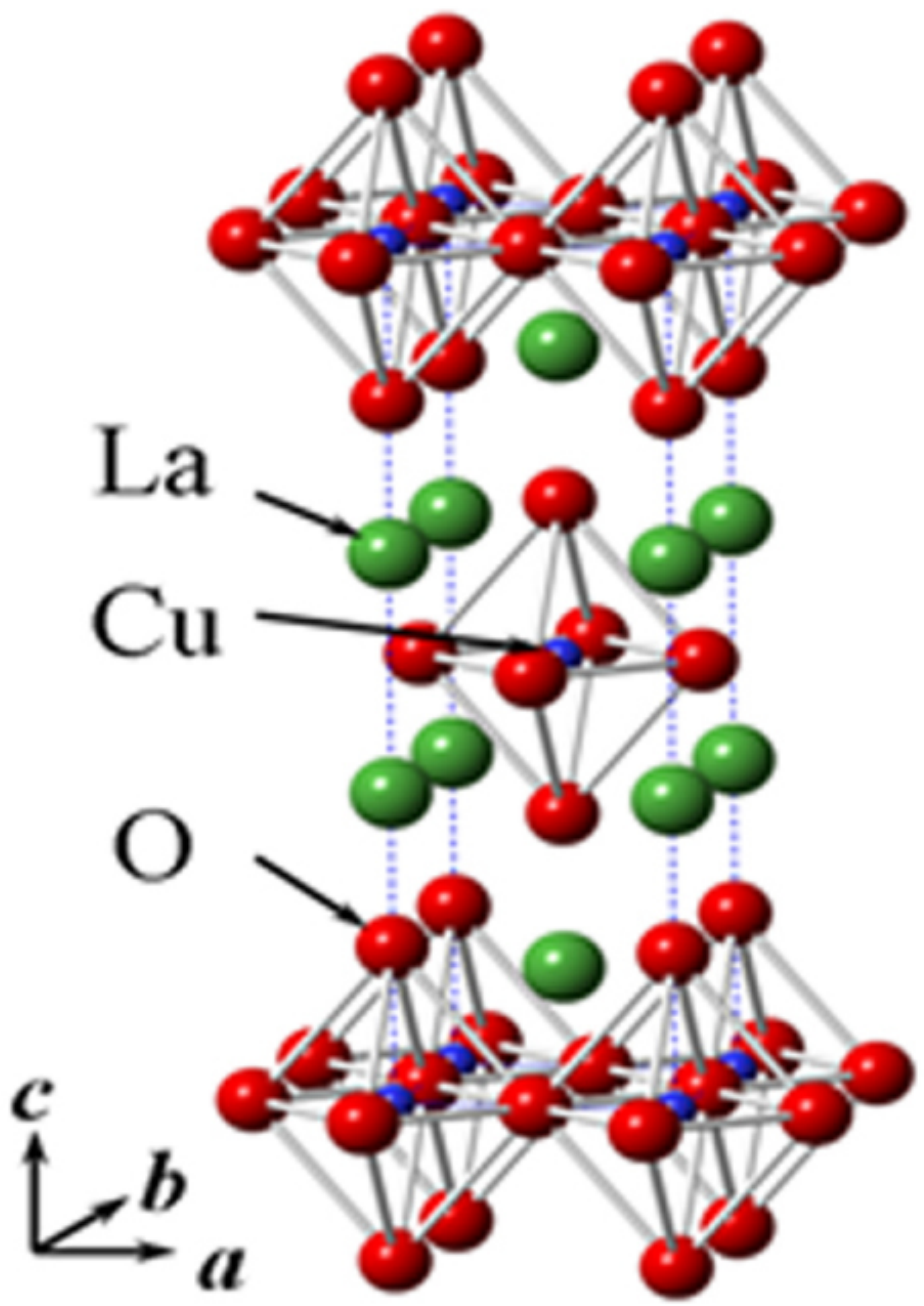
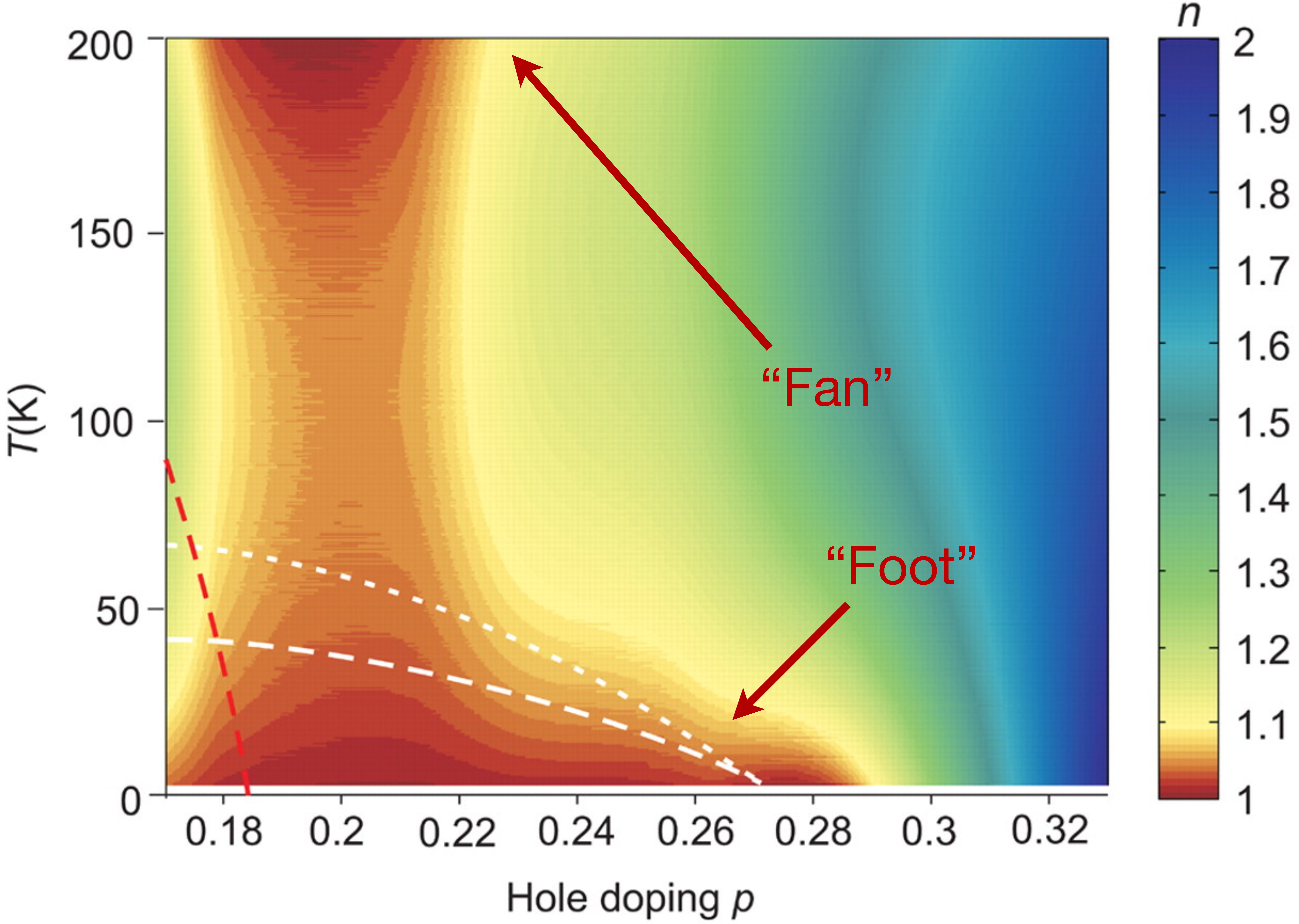
SCIENCE VOL 323 603 2009

FL-SDW QPT with Harris disorder provides a theory of the "foot"

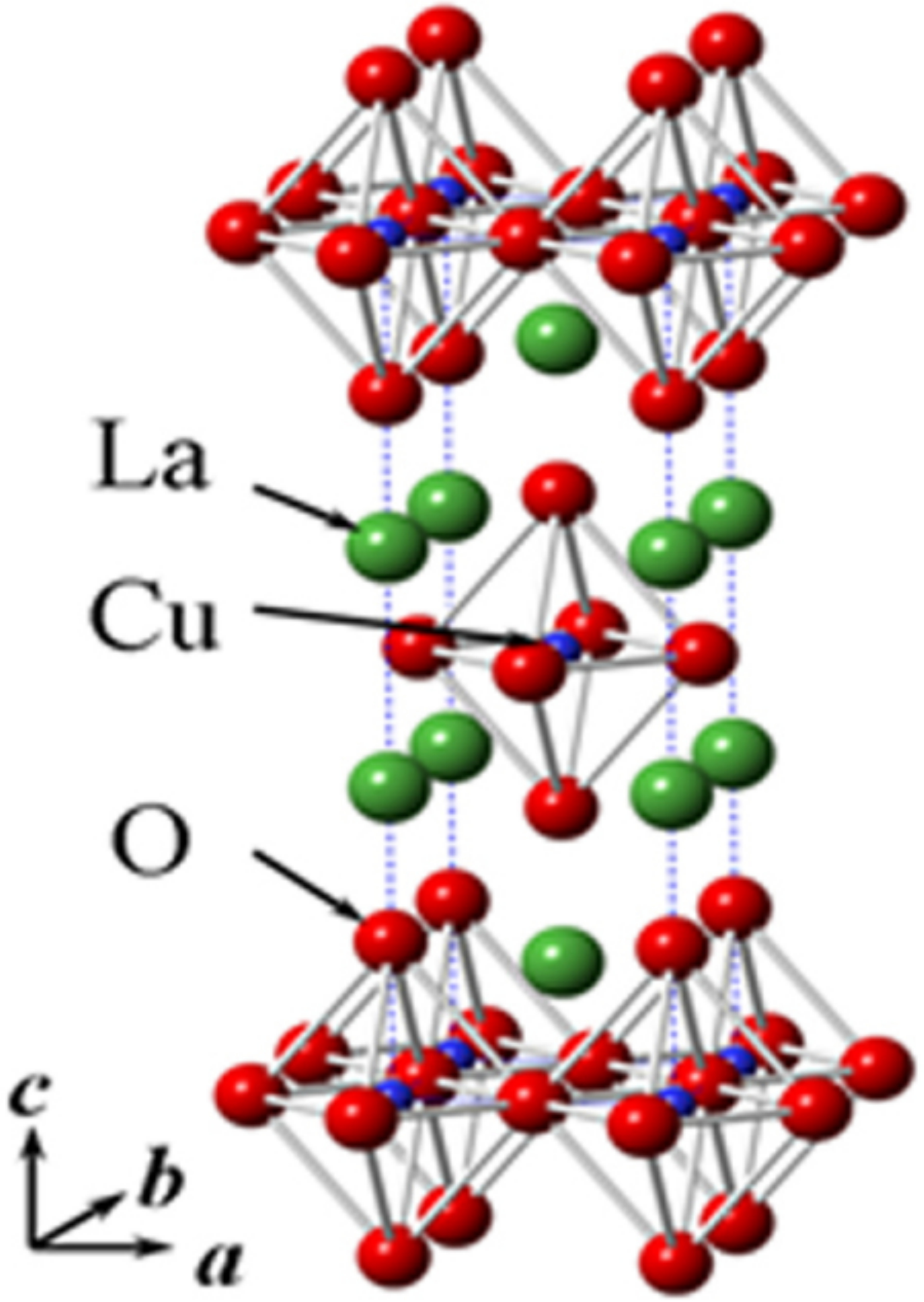


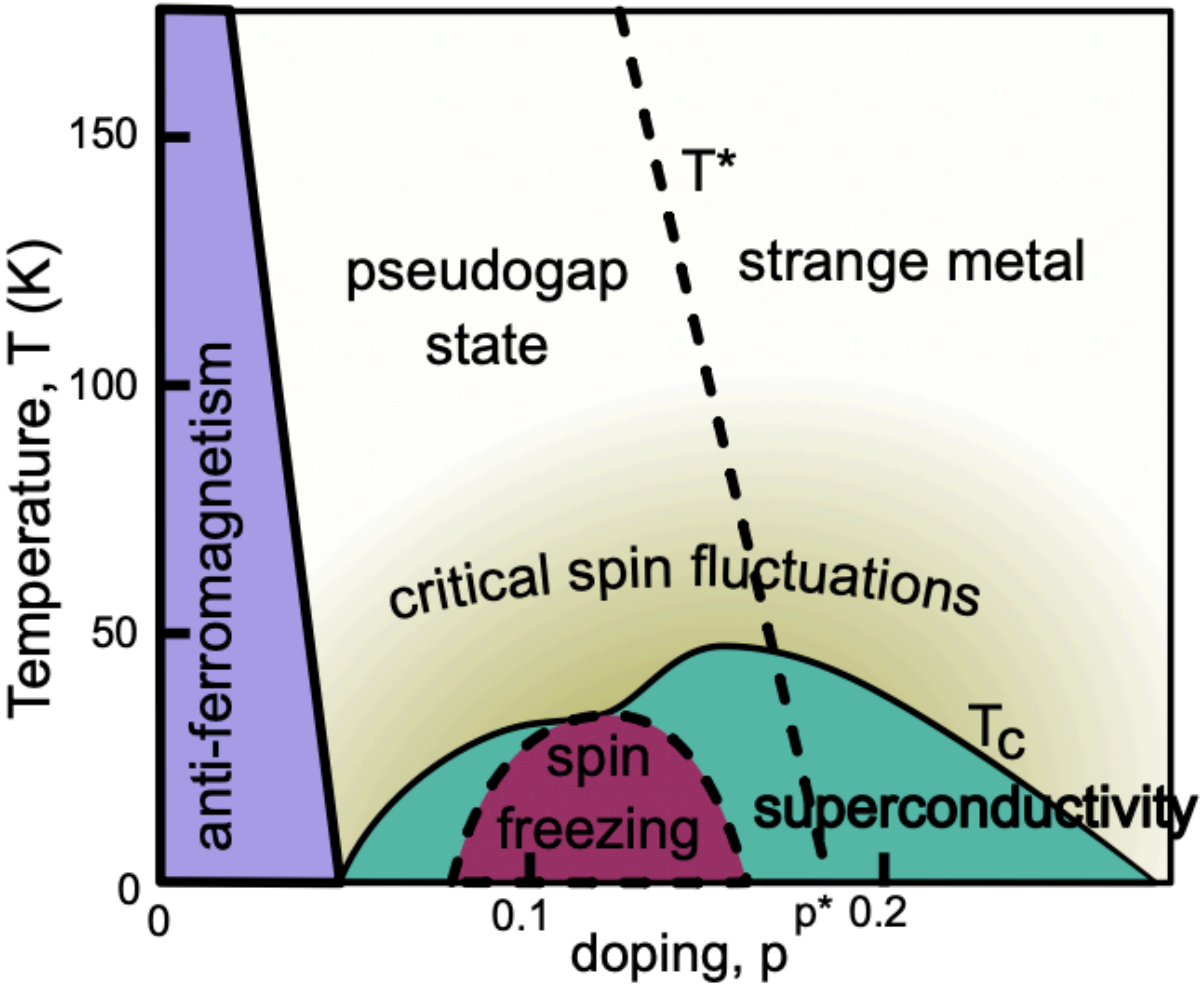




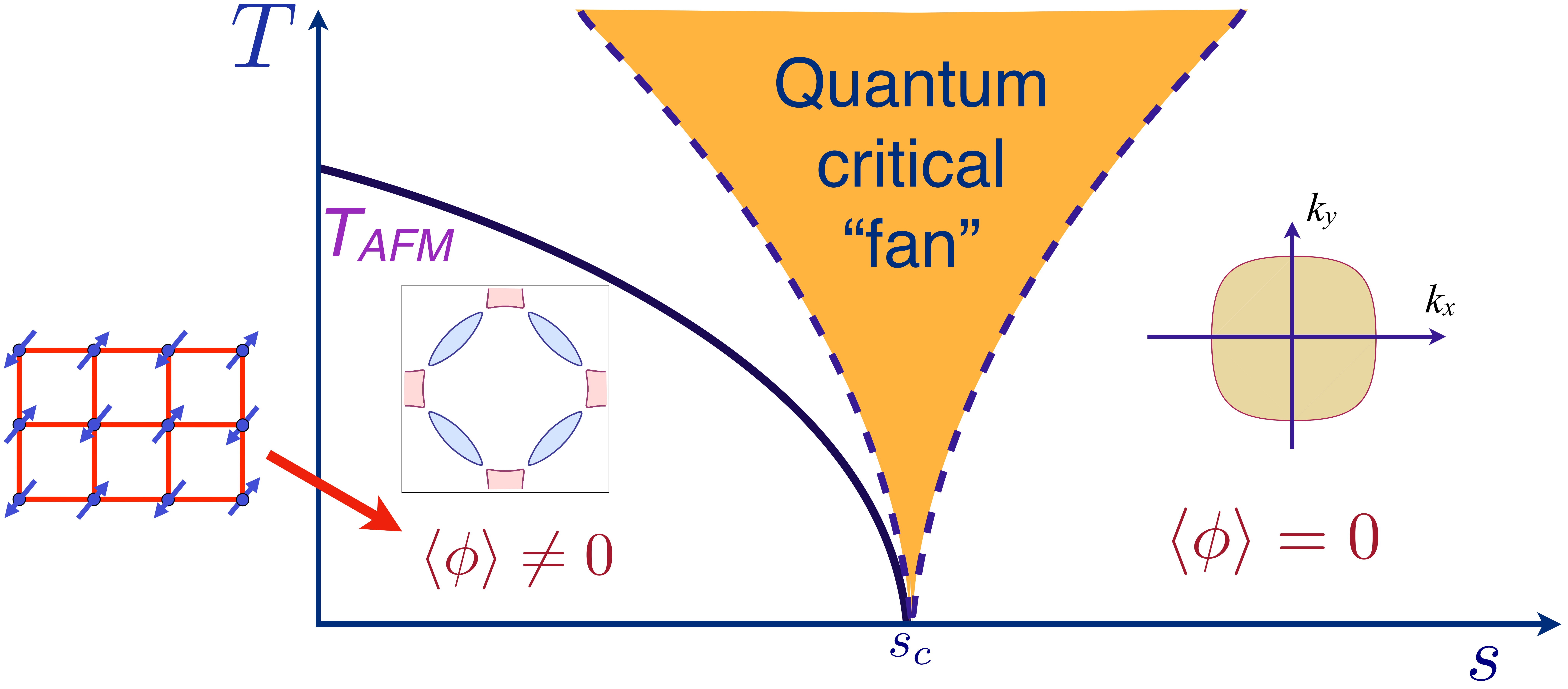
Neutron scattering in LSCO

Jacopo Radaelli, Aavishkar A. Patel, ….S. S., Stephen Hayden, to appear

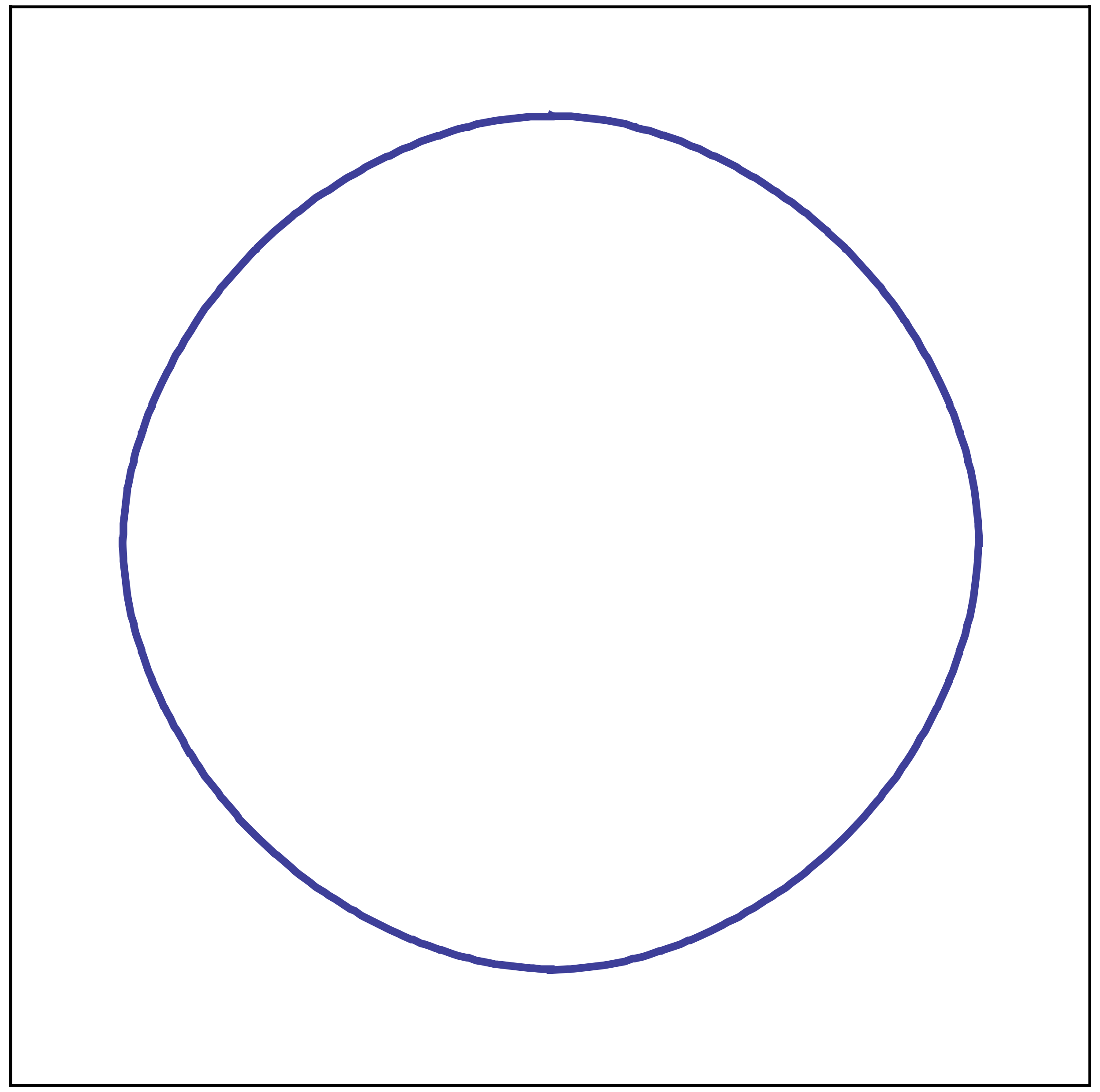




Fermi surface reconstruction from spin density wave (SDW) order

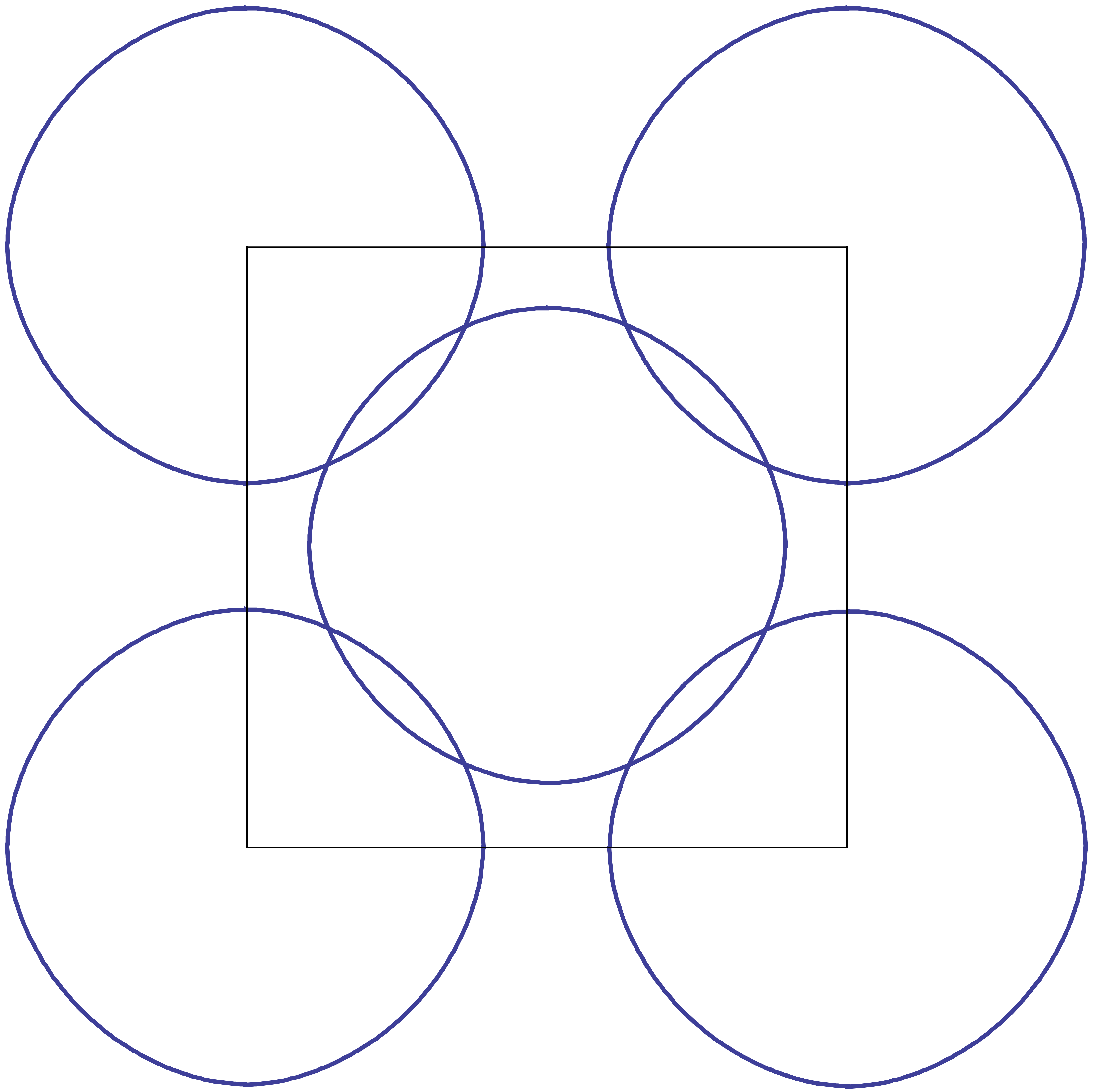


Fermi surface+antiferromagnetism



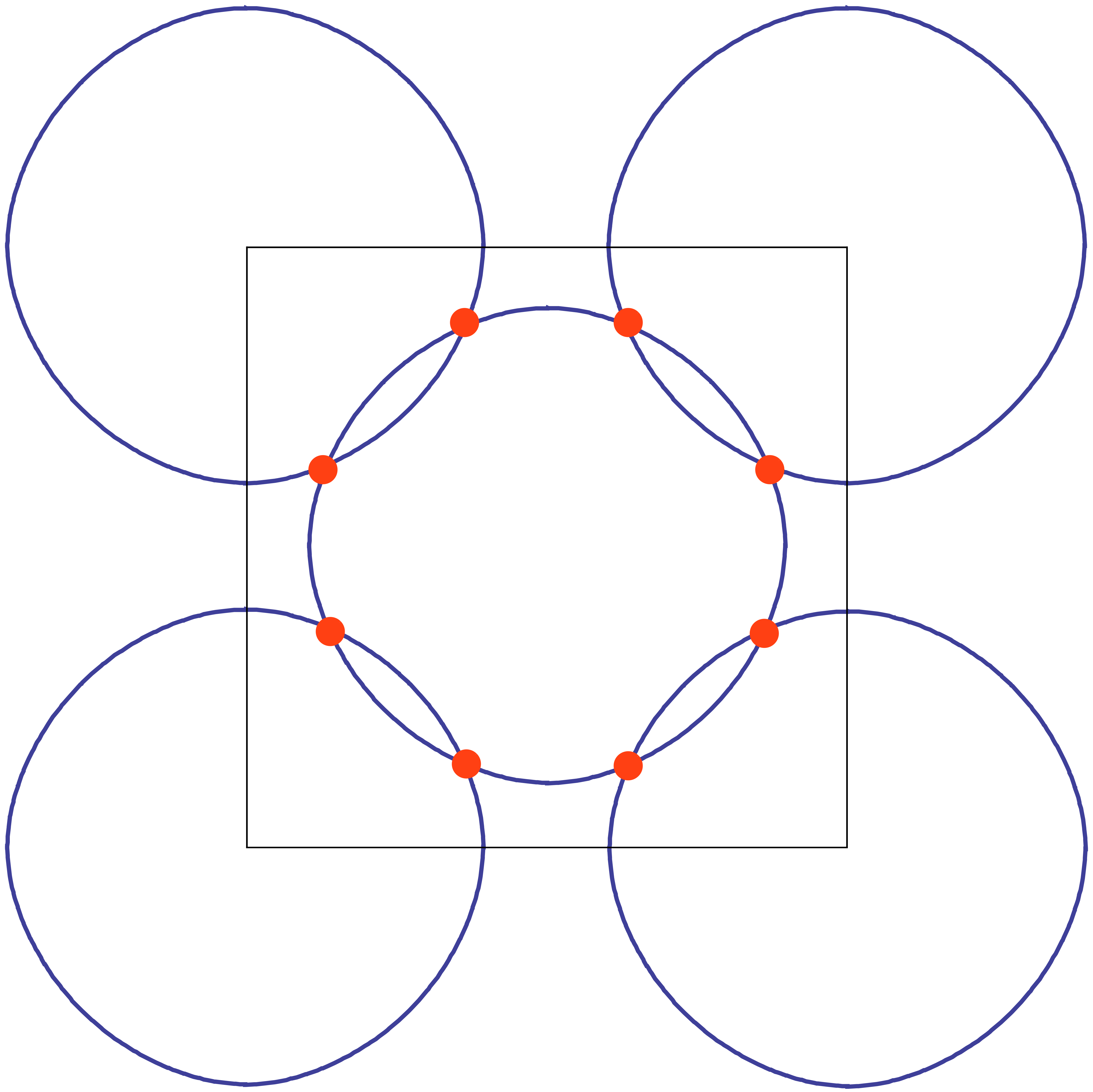
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



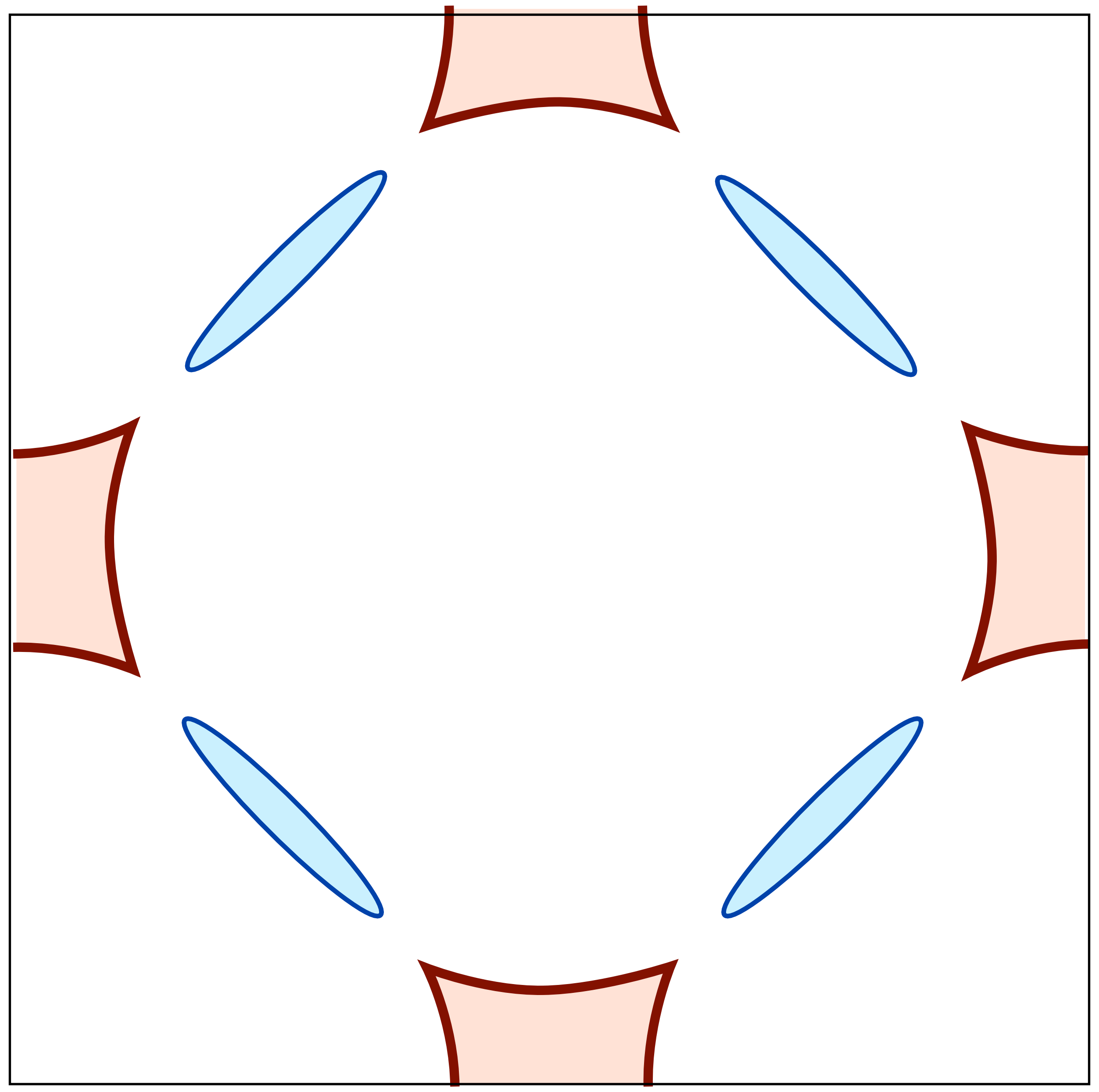
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



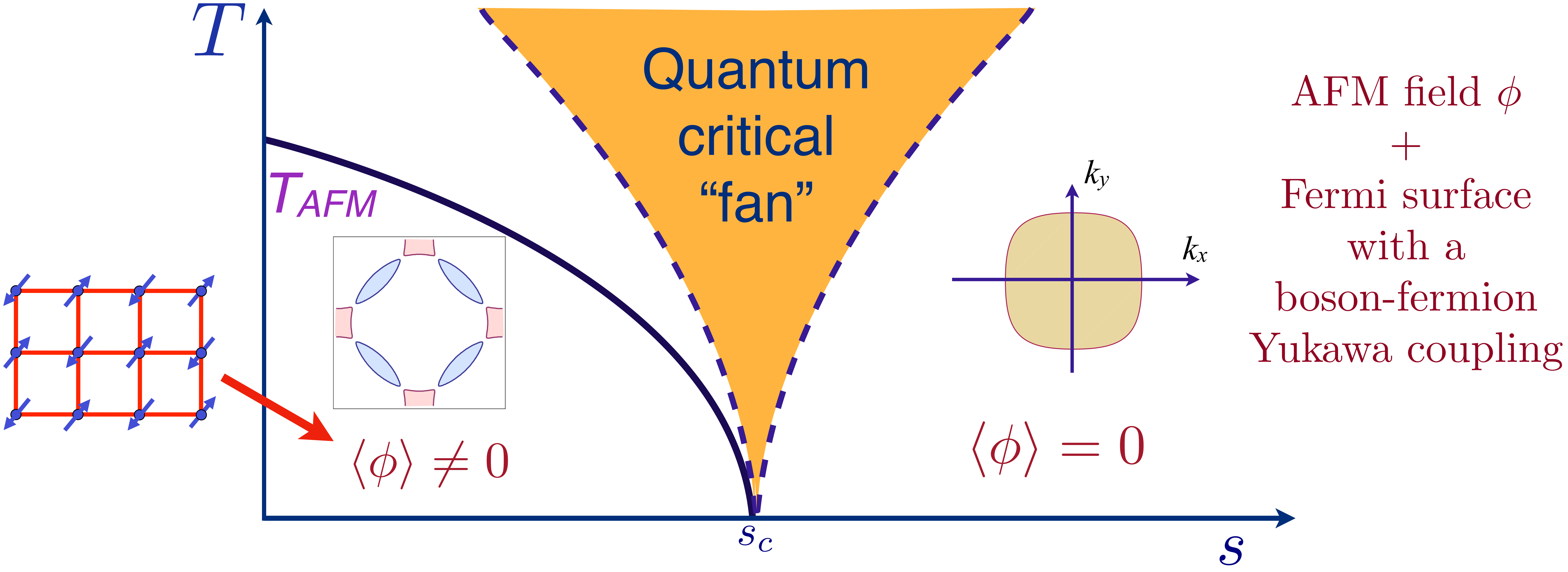
“Hot” spots

Fermi surface+antiferromagnetism



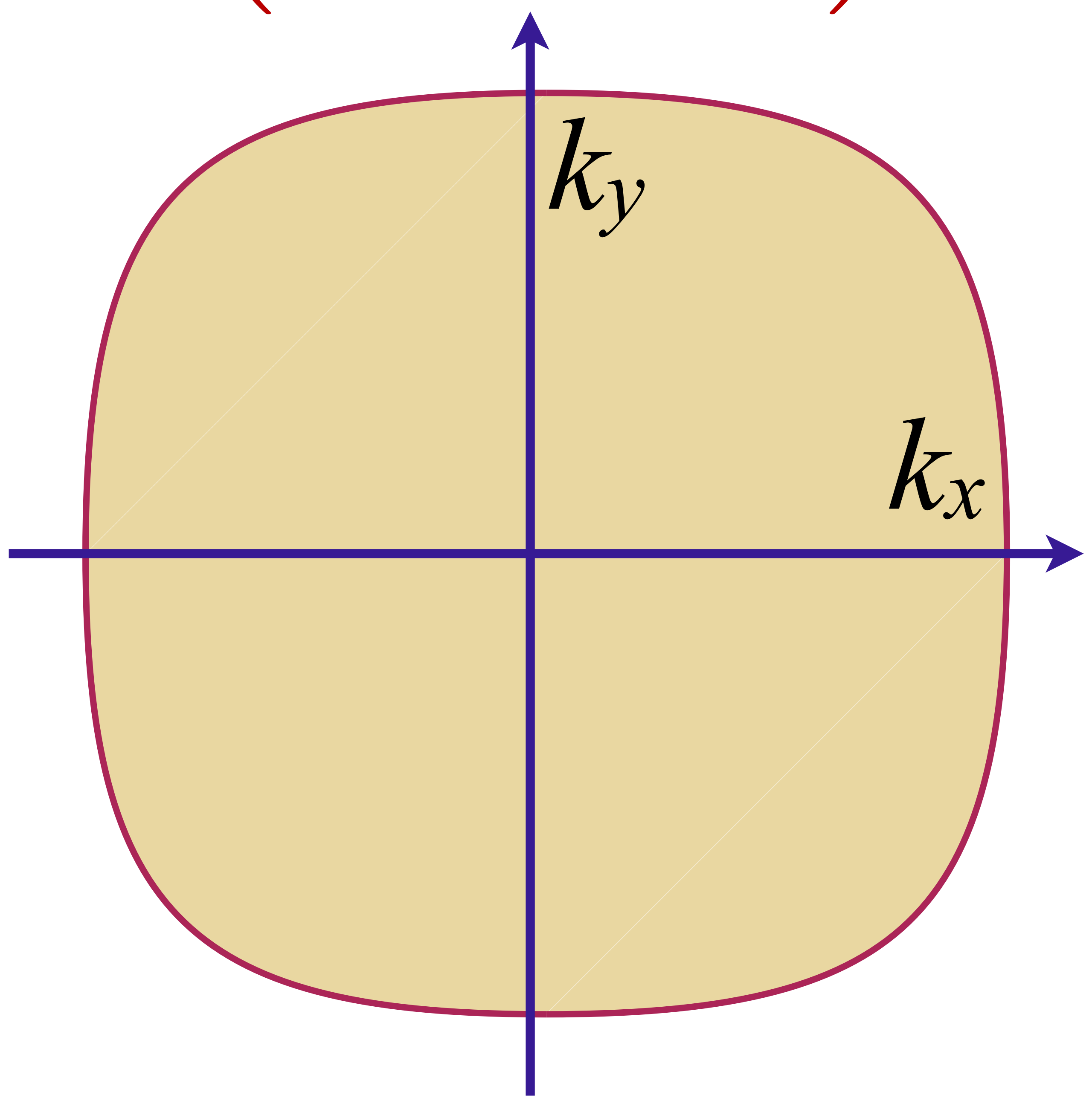
Electron and hole pockets in antiferromagnetic phase with $\langle\phi\rangle \neq 0$

Fermi surface reconstruction from spin density wave (SDW) order



Fermi surface + critical boson with no spatial disorder

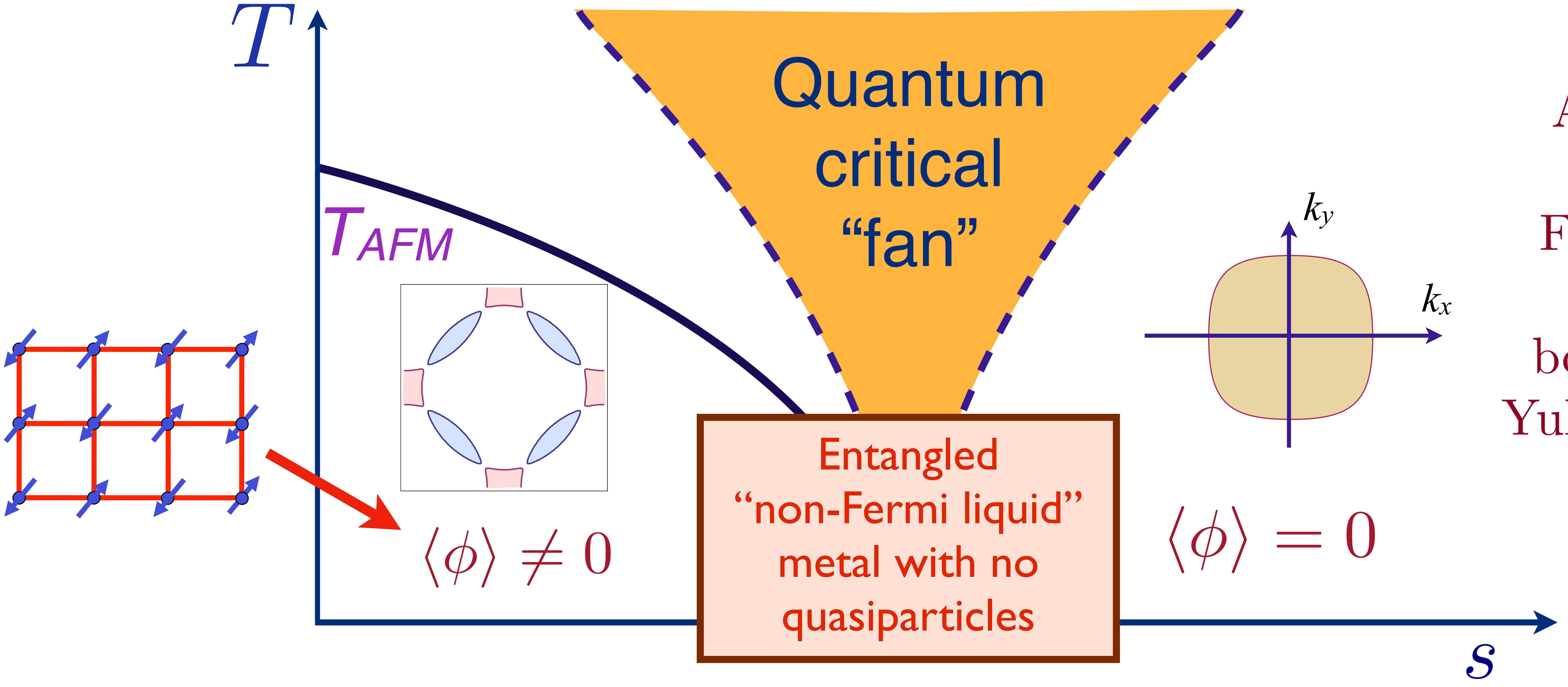
$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

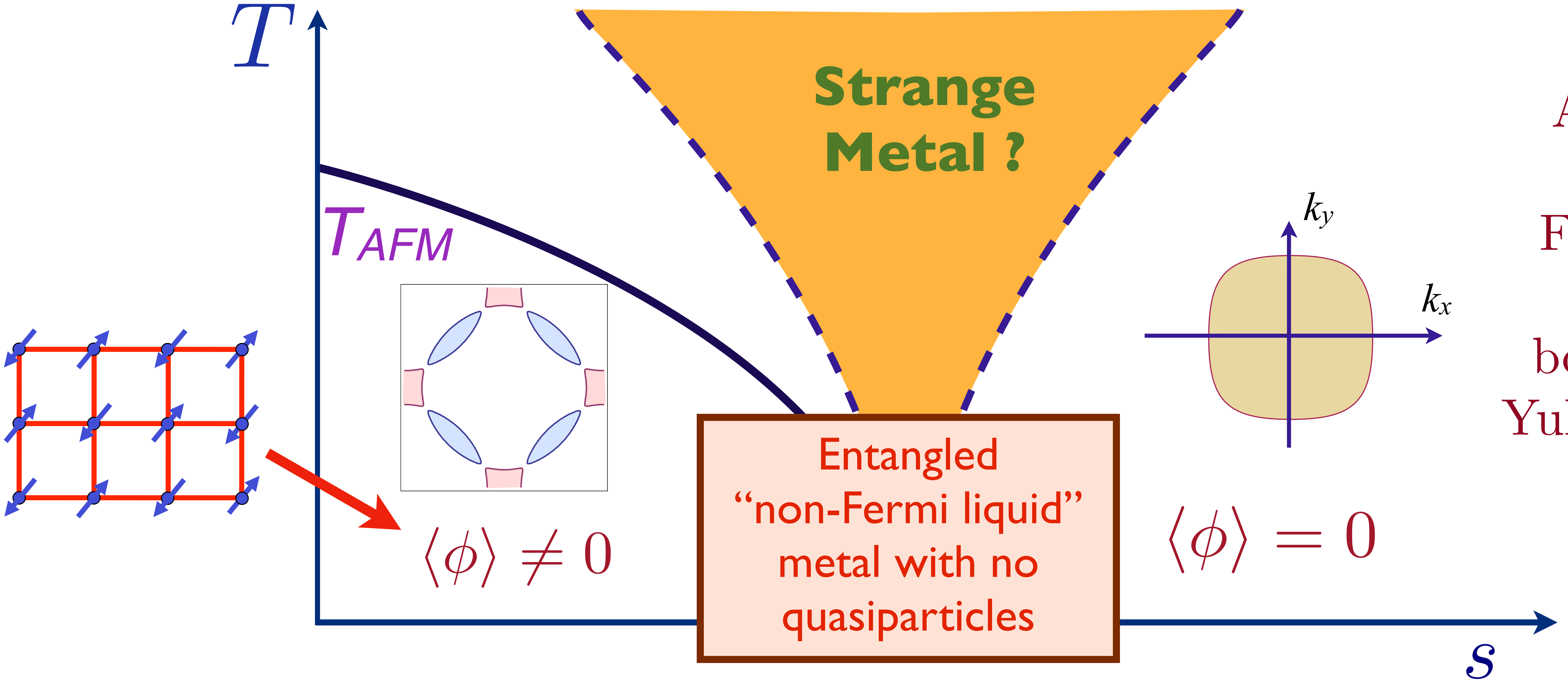
$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface reconstruction from spin density wave (SDW) order



AFM field ϕ
+
Fermi surface
with a
boson-fermion
Yukawa coupling

Fermi surface reconstruction from spin density wave (SDW) order



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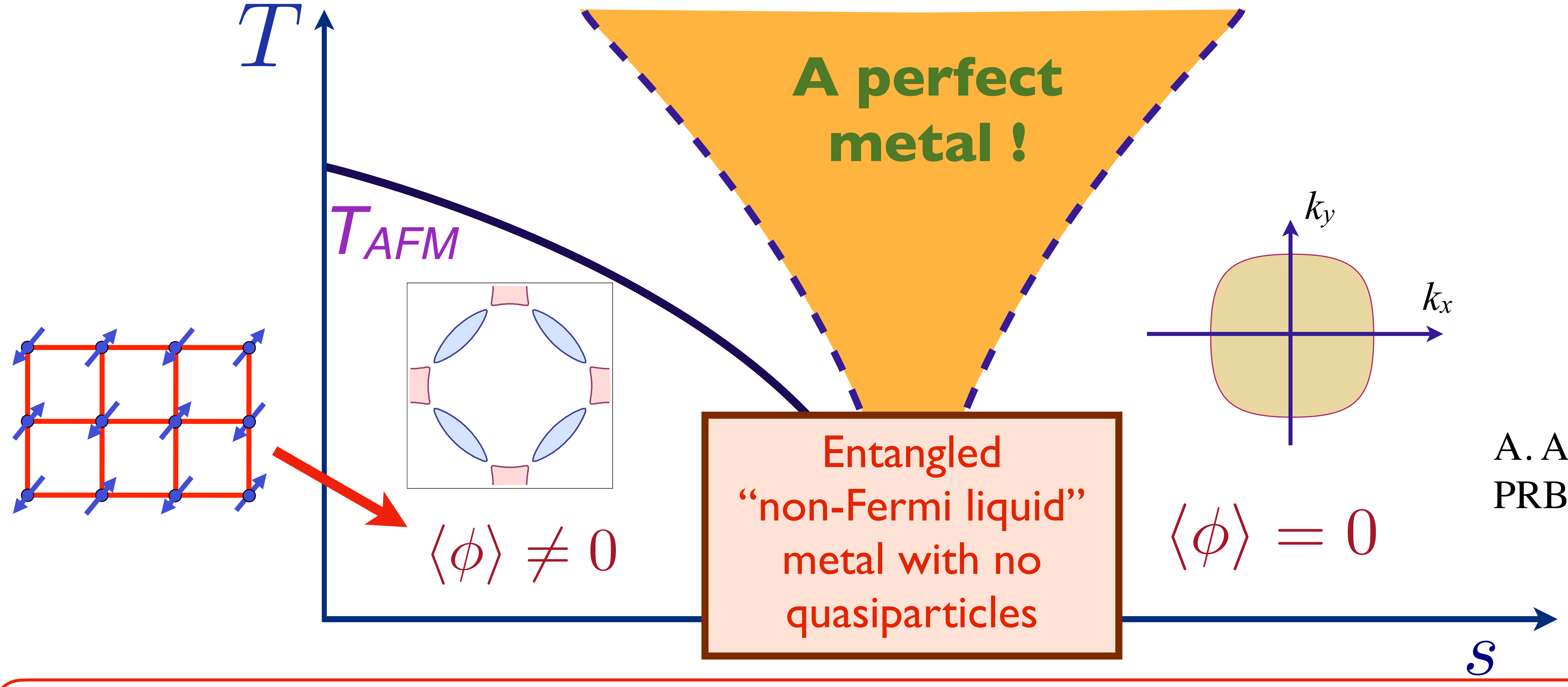
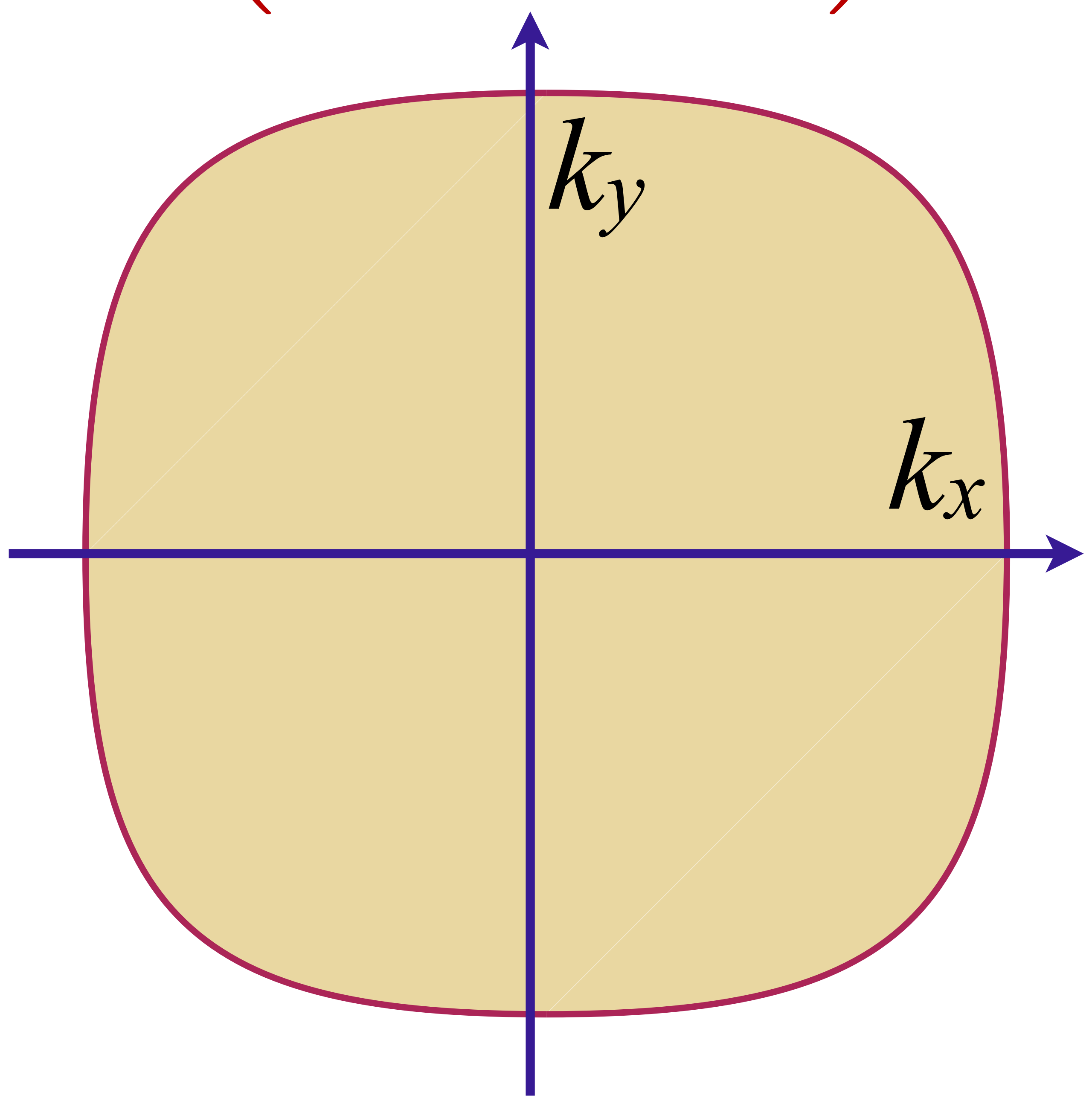


A. A. Patel and S. S., PRB **90**, 165146 (2014)

Extreme drag: the fermions c "drag" the bosons ϕ as they move, and so electrical current does not relax, even though strong c-ϕ scattering leads to absence of c quasiparticles.

Fermi surface + critical boson with no spatial disorder

$$c_{\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$

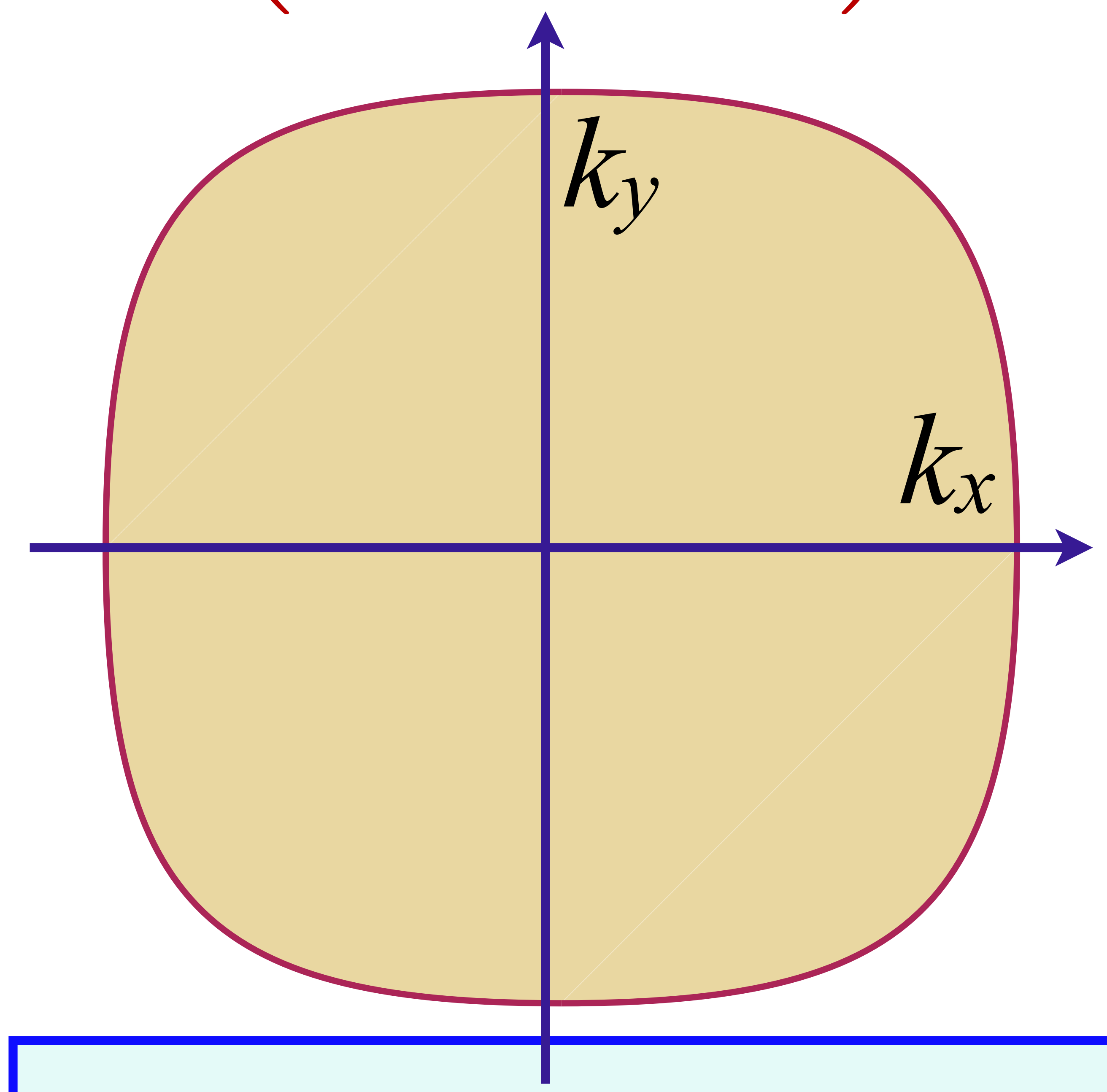


$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c_{\sigma}^{\dagger}(\boldsymbol{r})\tau_{\sigma\sigma'}^{a}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface + critical boson with potential disorder

$$c_{\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



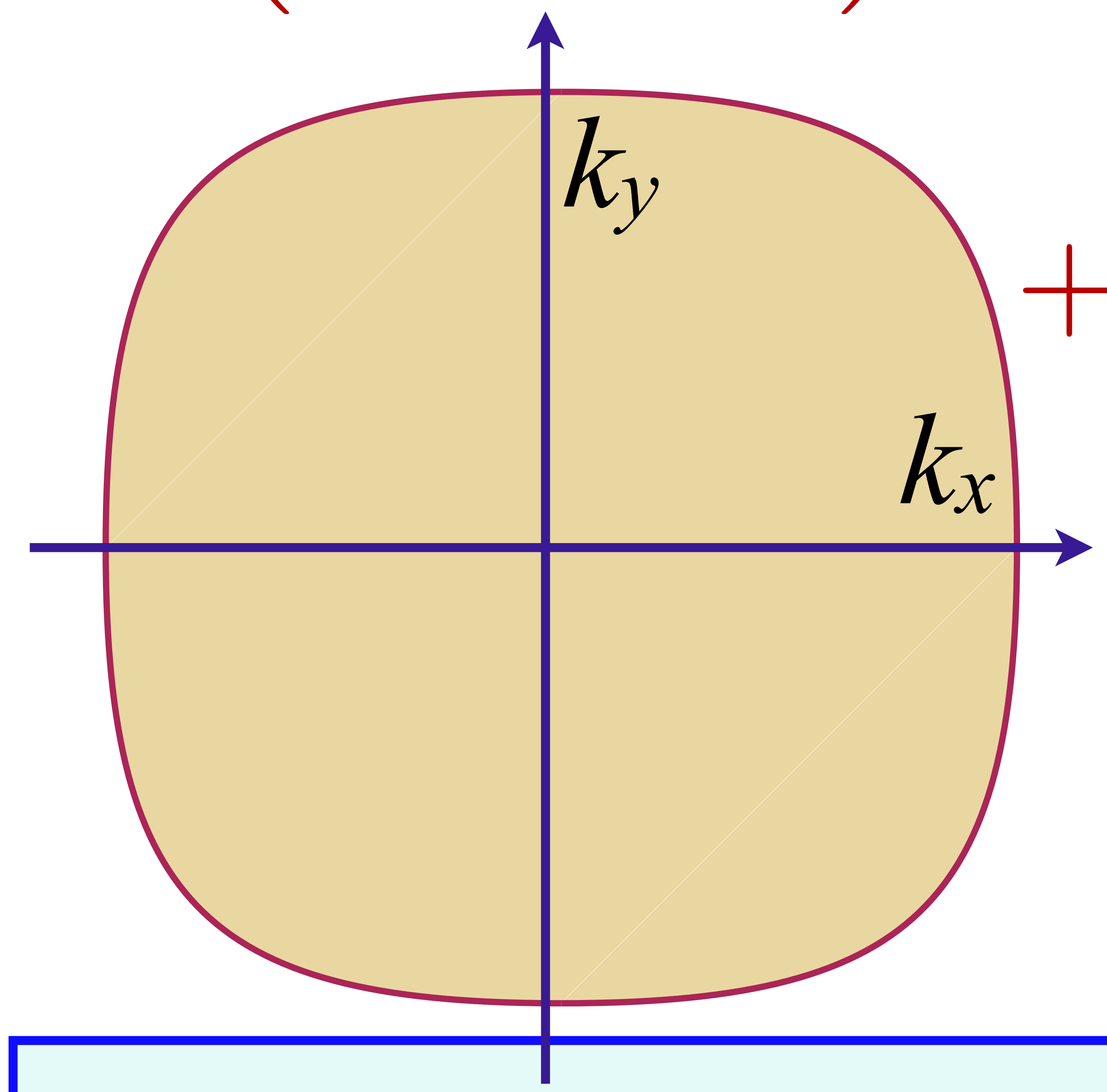
$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c_\sigma^\dagger(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c_\sigma^\dagger(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Fermi surface + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})][\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\left[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})\right]^2+u\left[\phi(\boldsymbol{r})\right]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})}=0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')}=\delta s^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

RG analysis (Harris criterion) shows that $\delta s(\boldsymbol{r})$ is most relevant disorder.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \Big(\frac{s + \delta s_j}{2} \phi_{ja}^2 + \frac{u}{4M} \left(\phi_{ja}^2\right)^2 \Big) \Big]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j \left(\gamma |\Omega| + \Omega^2 / c^2 \right) |\phi_{ja}(i\Omega)|^2 \,,$$

where $a = 1 \ldots M$ is a flavor index for an order parameter with $\mathrm{O}(M)$ symmetry.

J. A. Hoyos, Chetan Kotabage, Thomas Vojta
Phys. Rev. Lett. **99**, 230601 (2007)
T. Vojta, J.A. Hoyos, Priyanka Mohan, Rajesh Narayanan,
J. Phys.: Condens. Matter **23**, 094206 (2011)

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where $a = 1 \ldots M$ is a flavor index for an order parameter with $\mathrm{O}(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\overline{\mathcal{S}}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\widetilde{\delta s_j}}{2} \phi_{ja}^2 \Big]$$

Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev,
Phys. Rev. Lett. **101**, 035701 (2008).

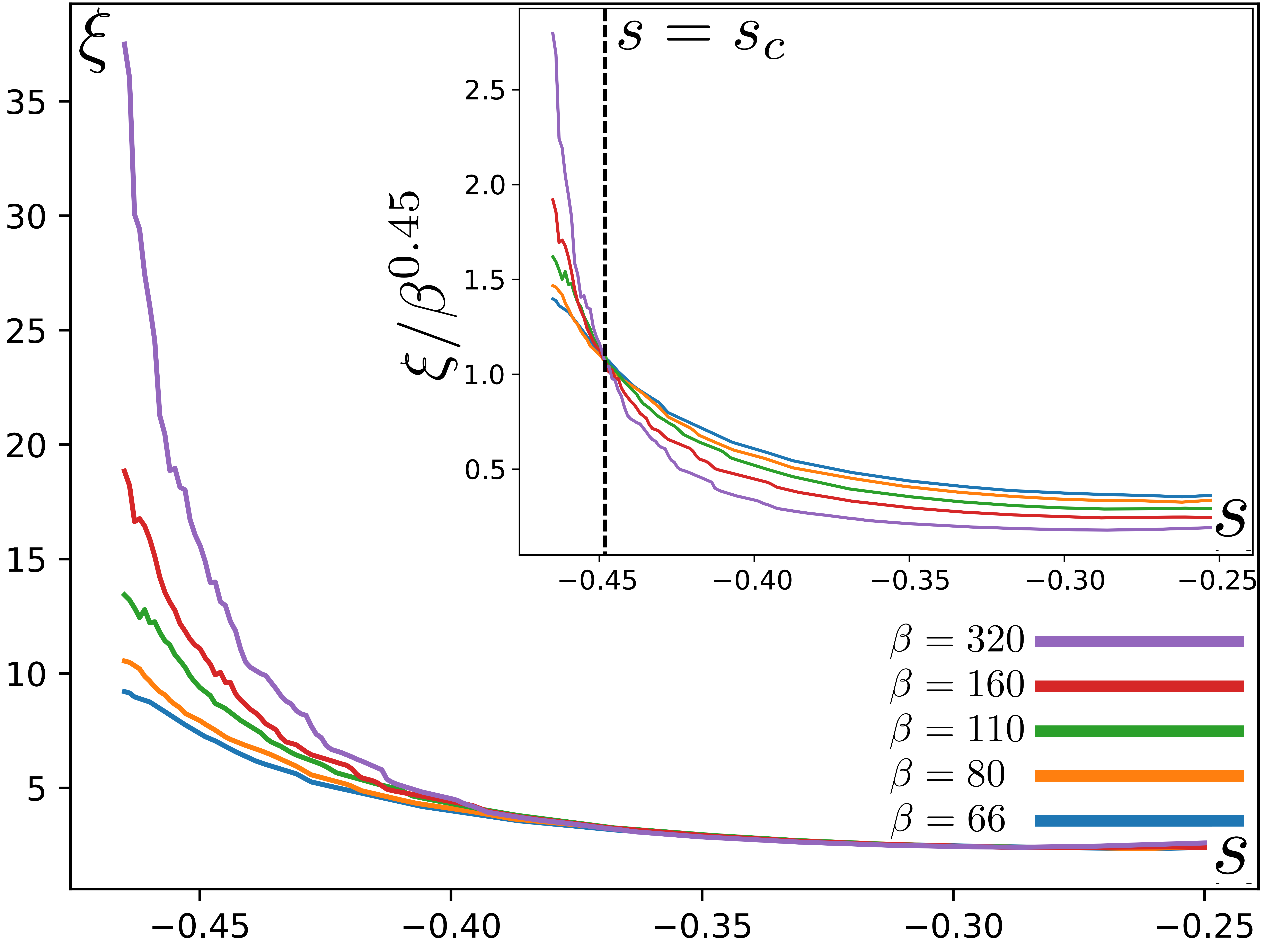
$$\widetilde{\delta s_j} = s + \delta s_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\overline{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + \delta s_j + uT \sum_\Omega \sum_\alpha \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma |\Omega| + \Omega^2 / c^2 + e_\alpha}$$

where e_α and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\overline{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \ldots L^2$ for a $L \times L$ sample.

Aavishkar A. Patel, Peter Lunts, S.S., PNAS **121**, e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

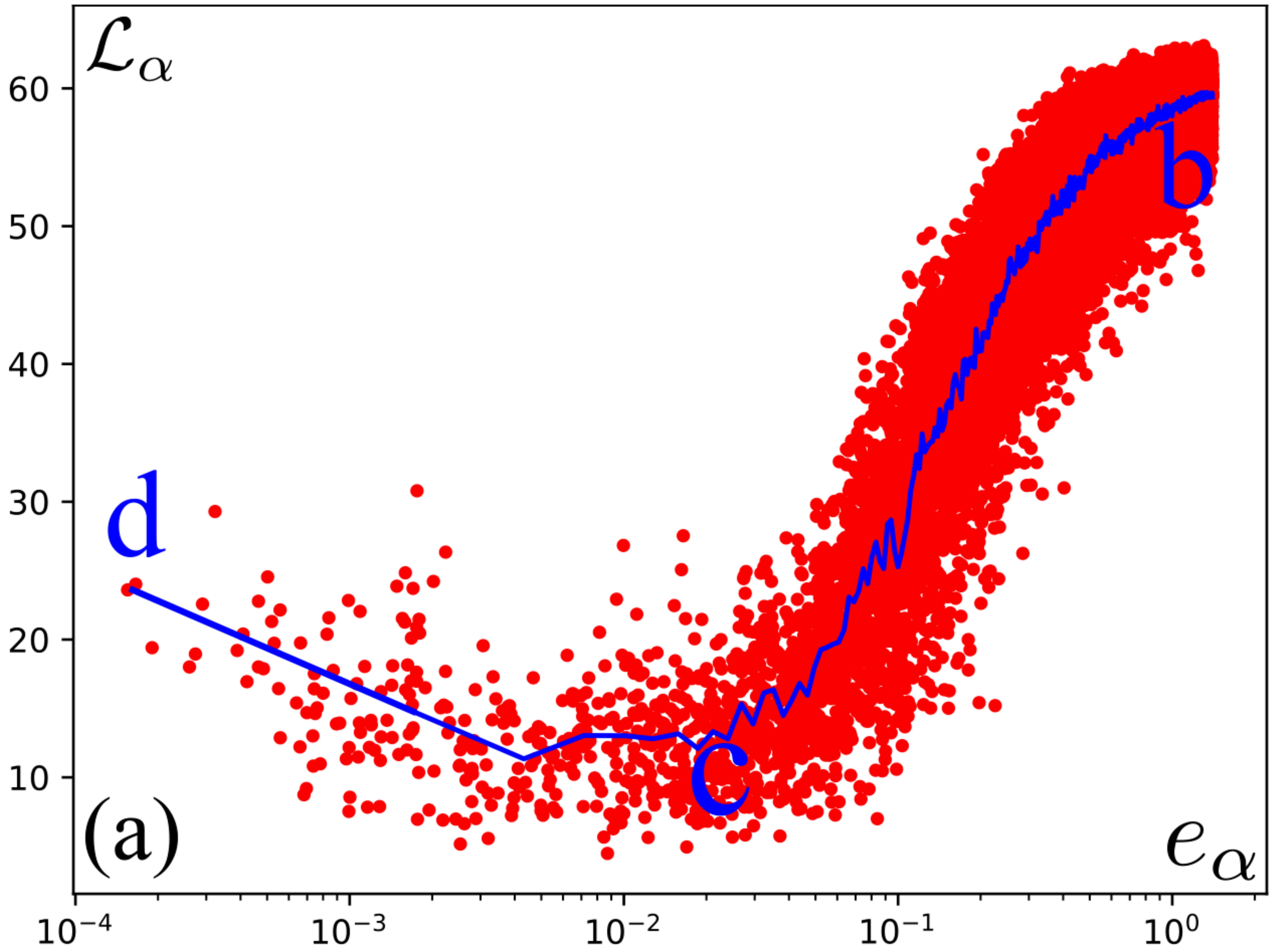
ϕ correlation length ξ



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$



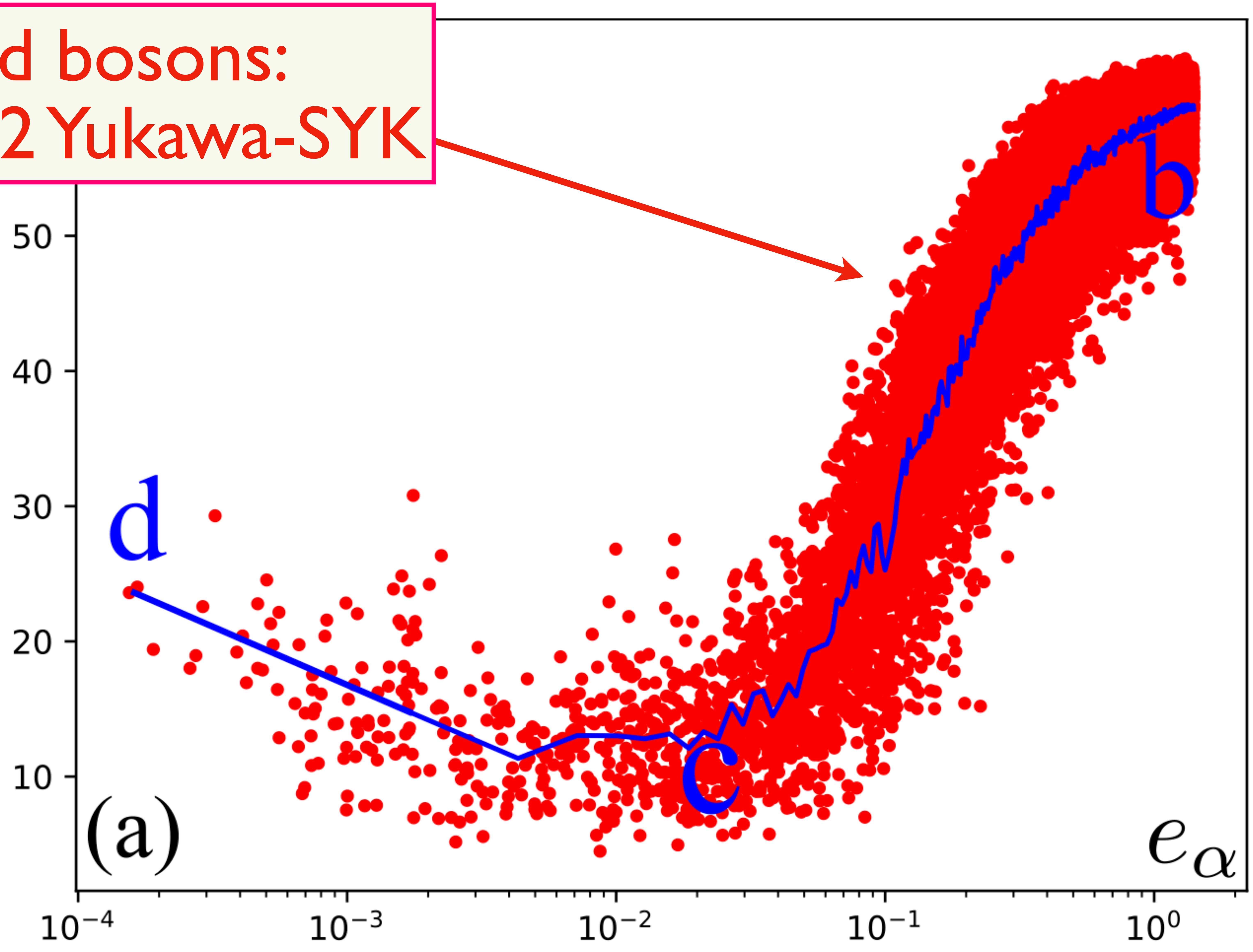
QMC:
Aavishkar A. Patel,
Peter Lunts, and
Michael Albergo,
arXiv:2410.05365

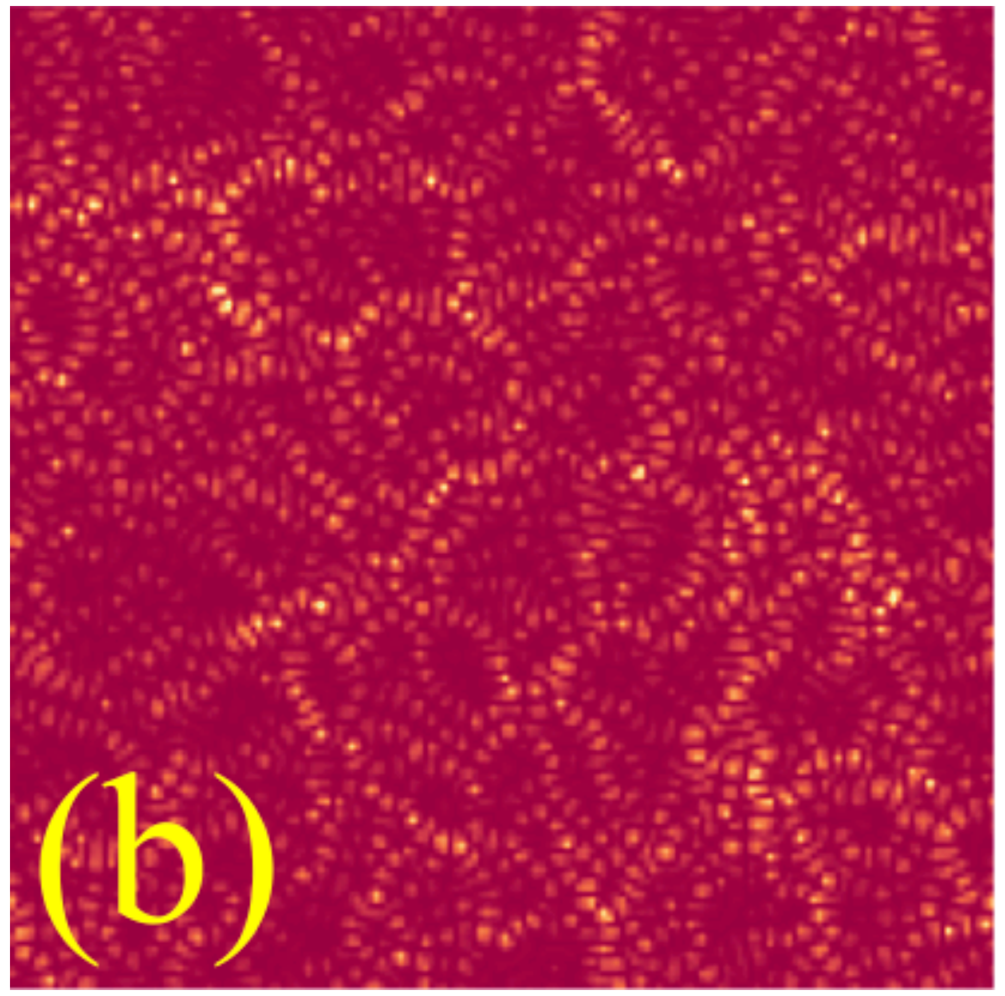
Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$

Extended bosons:
physics of *d*=2 Yukawa-SYK





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Aavishkar A. Patel,
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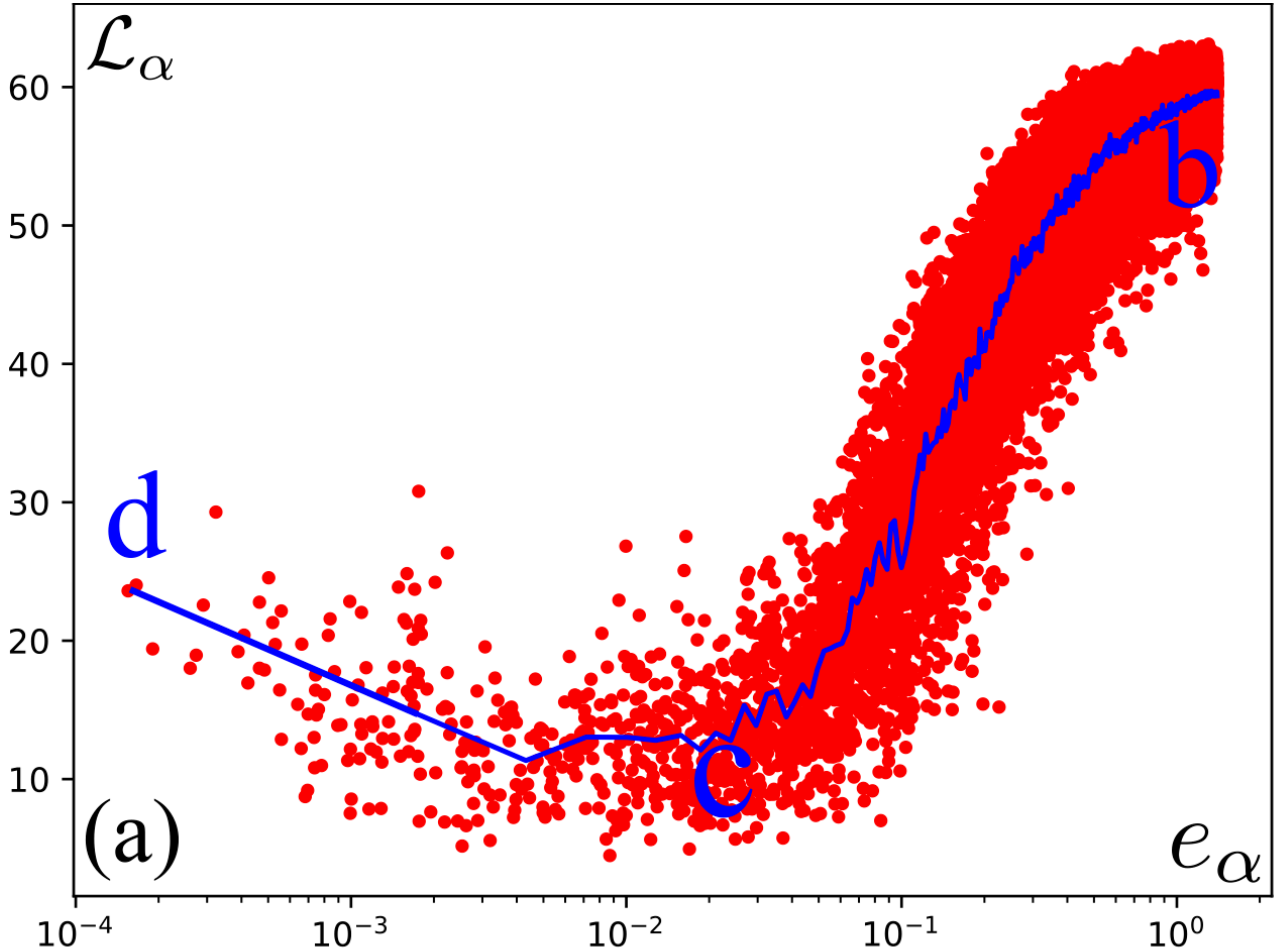
arXiv:2410.05365

Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

QMC:
Aavishkar A. Patel,
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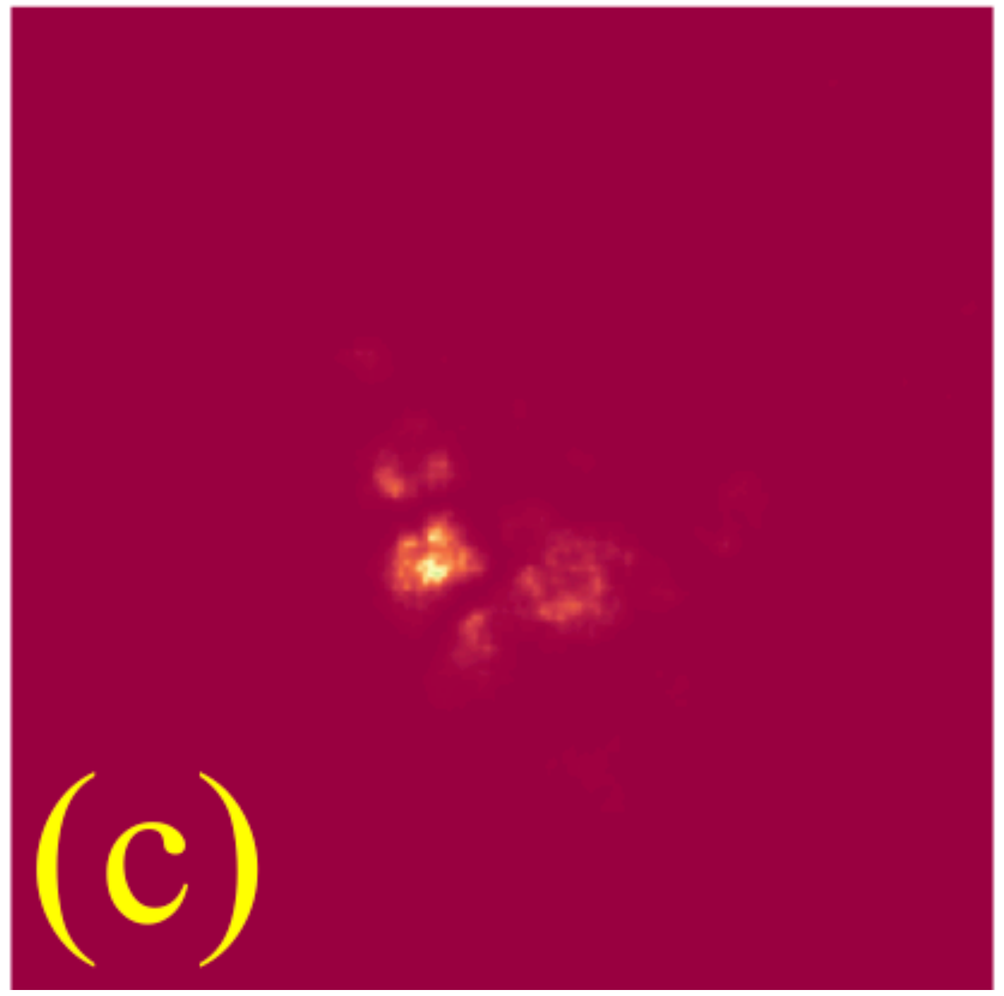
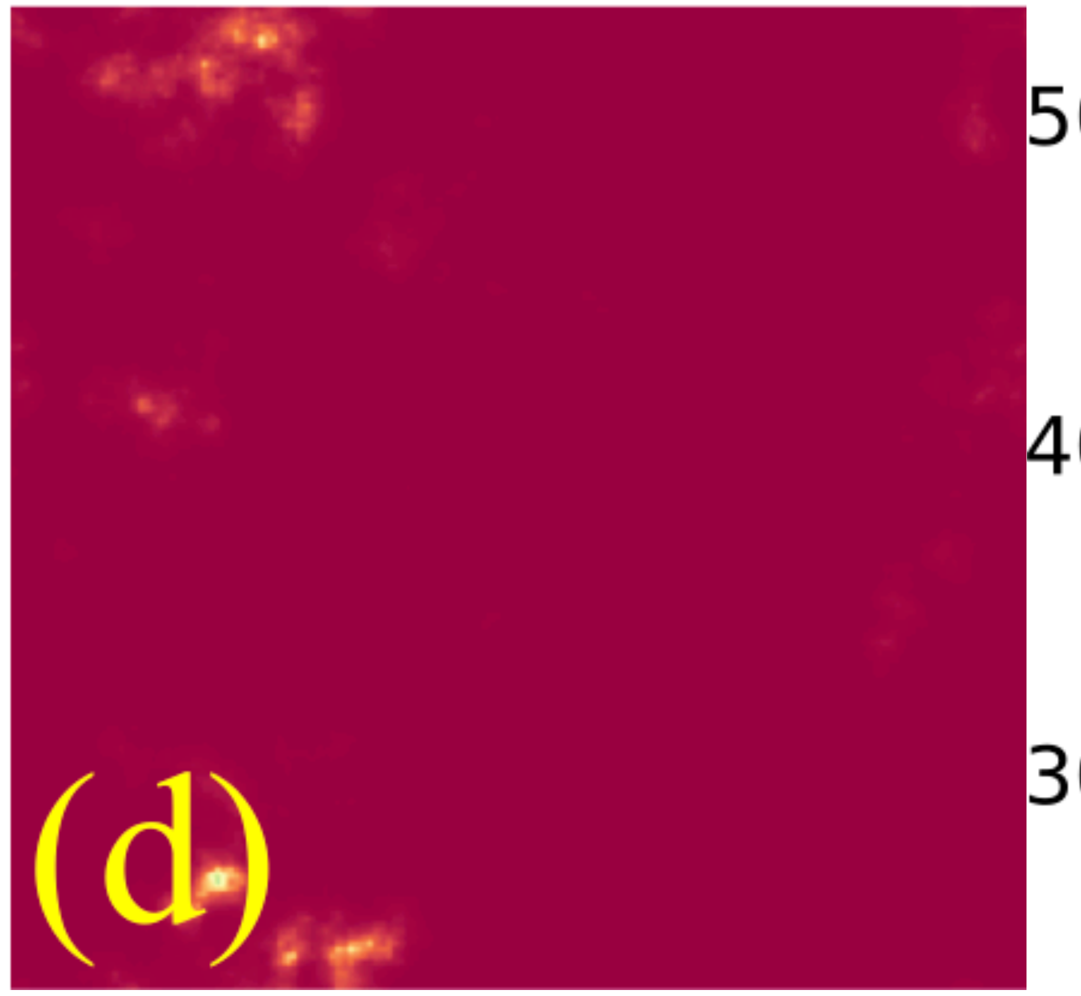
arXiv:2410.05365

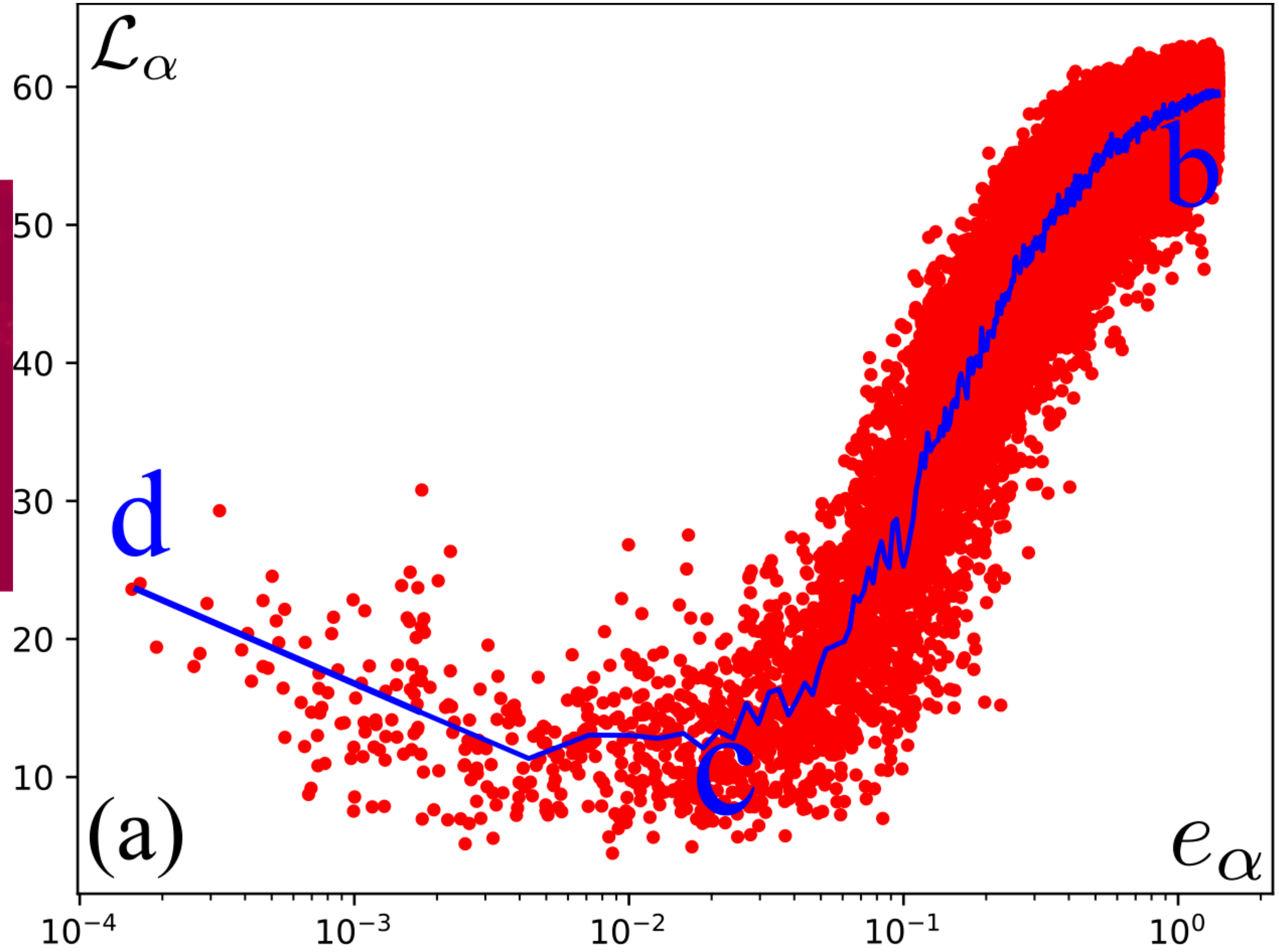


Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$



Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)



QMC:
Aavishkar A. Patel,
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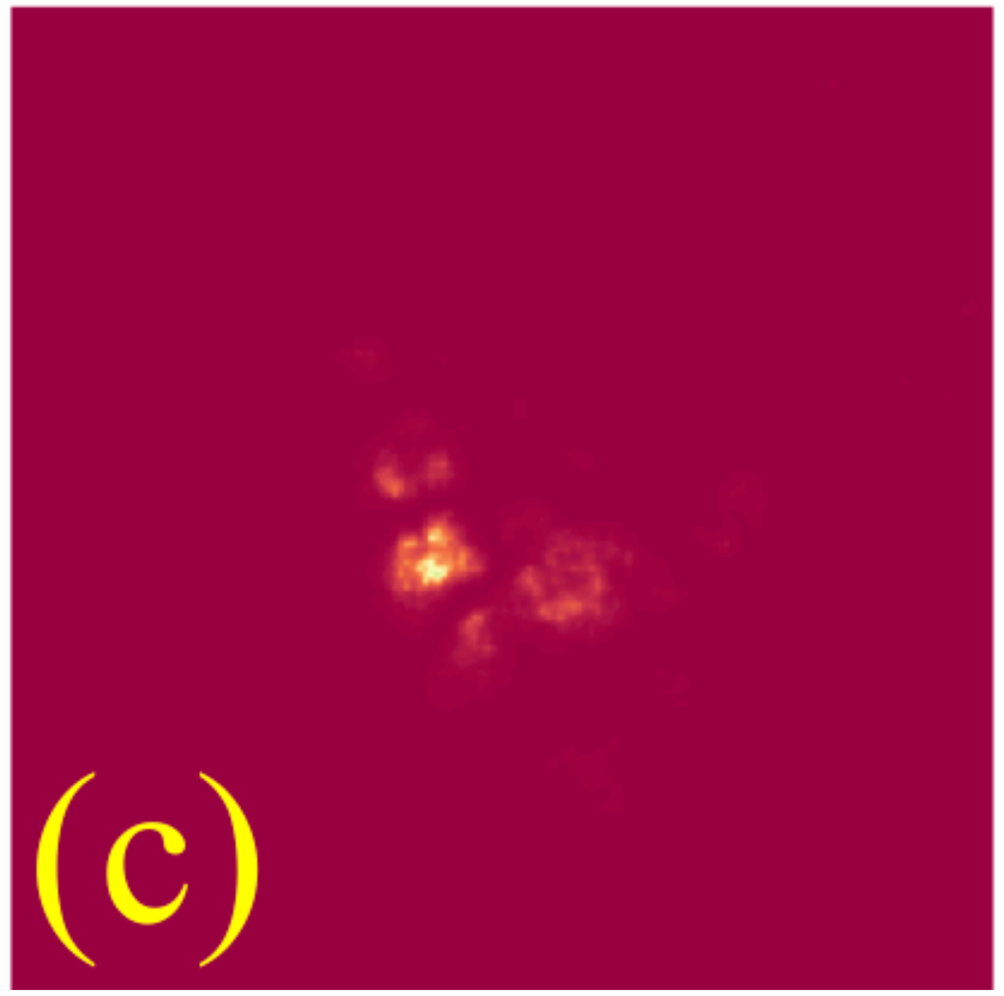
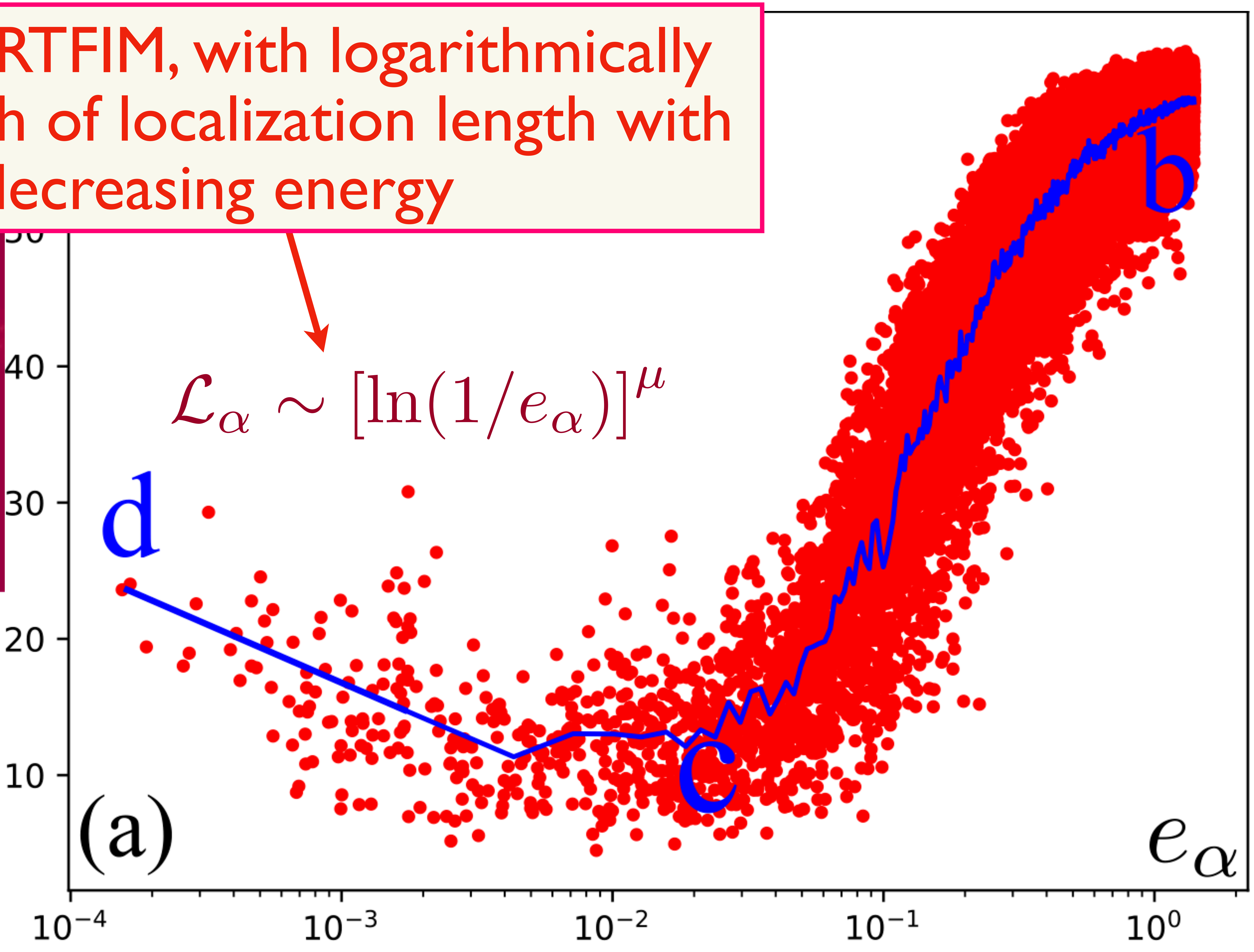
arXiv:2410.05365

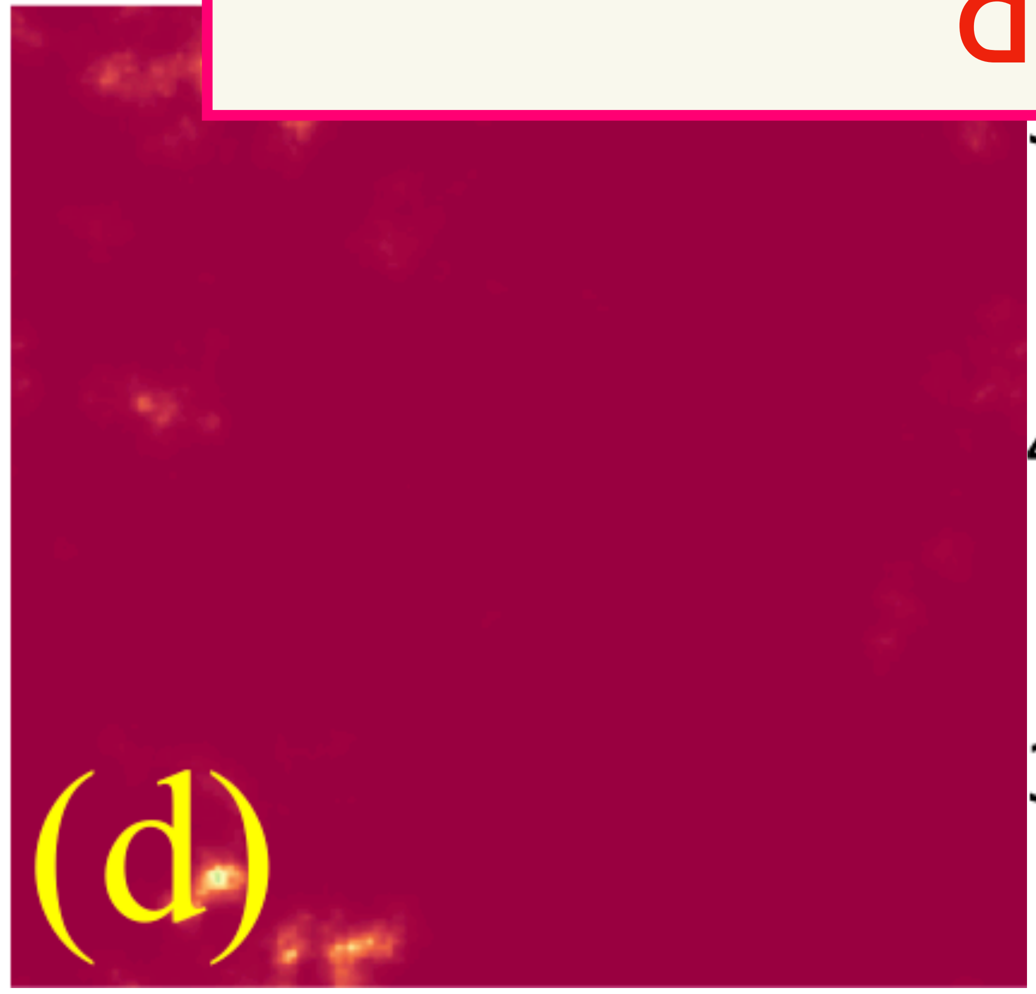


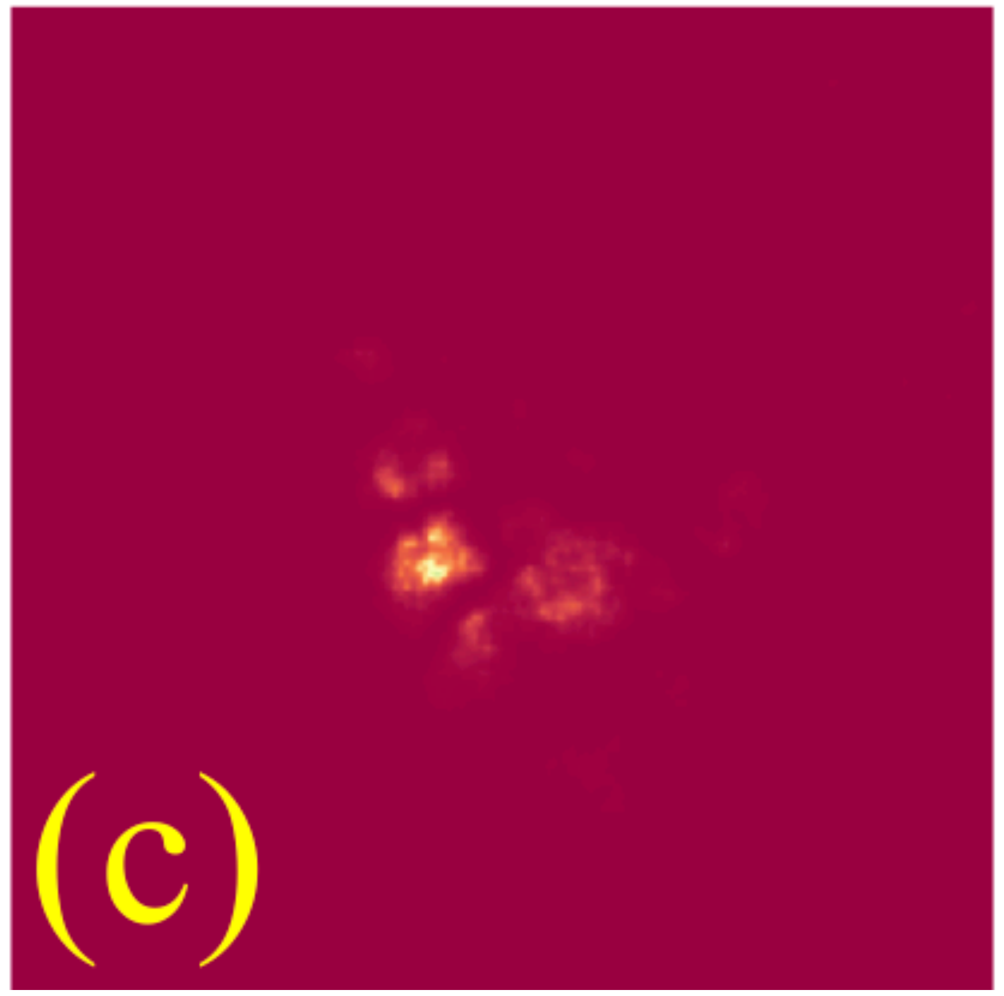
Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$

Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy







Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

J. A. Hoyos, C. Kotabage, T. Vojta PRL **99**, 230601 (2007)

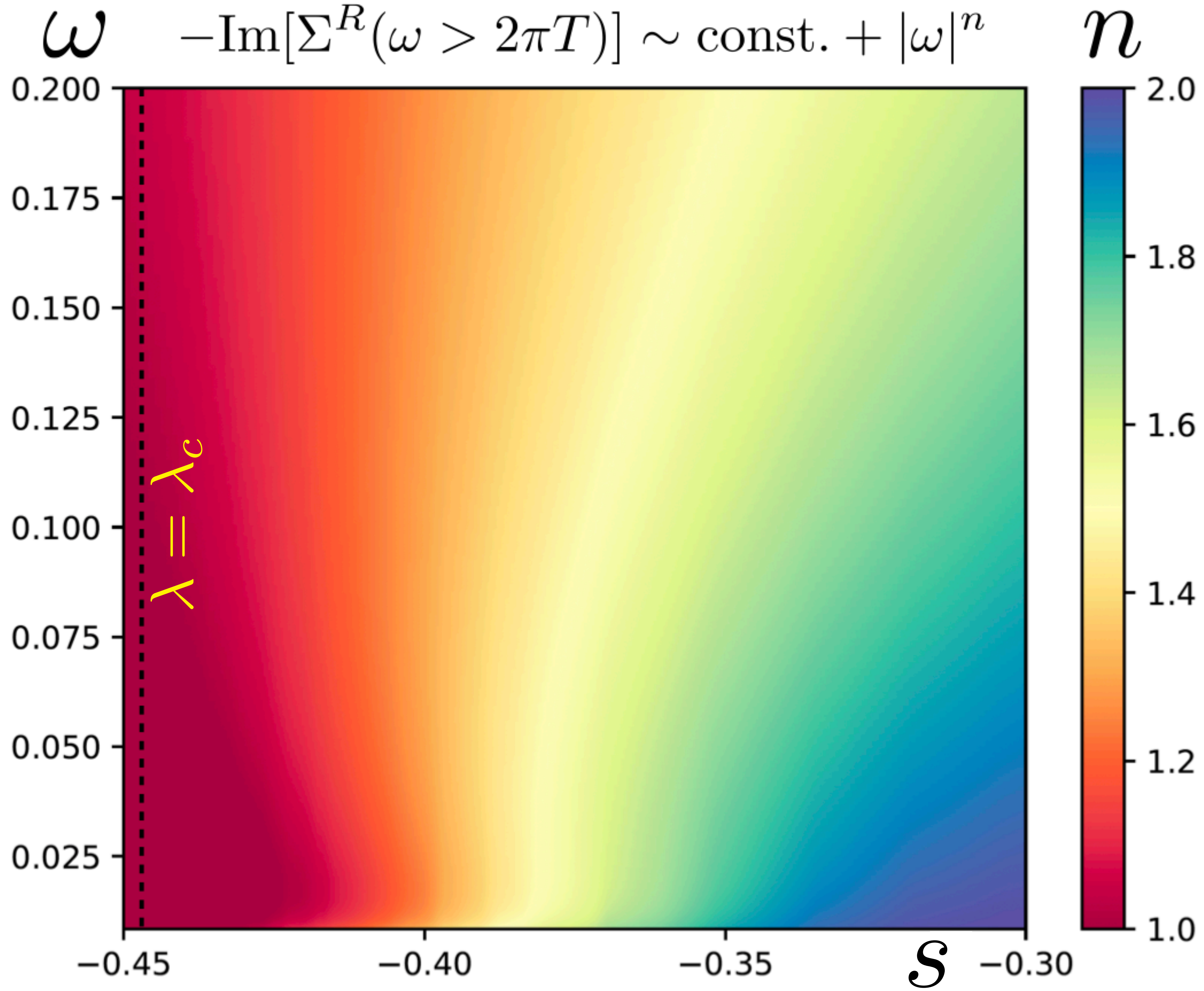
QMC: Aavishkar A. Patel, Peter Lunts, and Michael Albergo, arXiv:2410.05365

Bosonic eigenmodes in random mass Hertz theory

Transport scattering rate

$$\Sigma(i\omega) = -i\pi g'^2 \mathcal{N}_0 \frac{T}{L^2} \sum_{\alpha,\Omega} \frac{\text{sgn}(\omega+\Omega)}{\gamma|\Omega| + \Omega^2/c^2 + e_\alpha}\,.$$

$$-\text{Im}[\Sigma^R(\omega > 2\pi T)] \sim \text{const.} + |\omega|^n$$



$L = 160, \beta = 800$, 10 disorder samples

Extended region in λ with $n \approx 1$ - a strange metal *phase*

Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

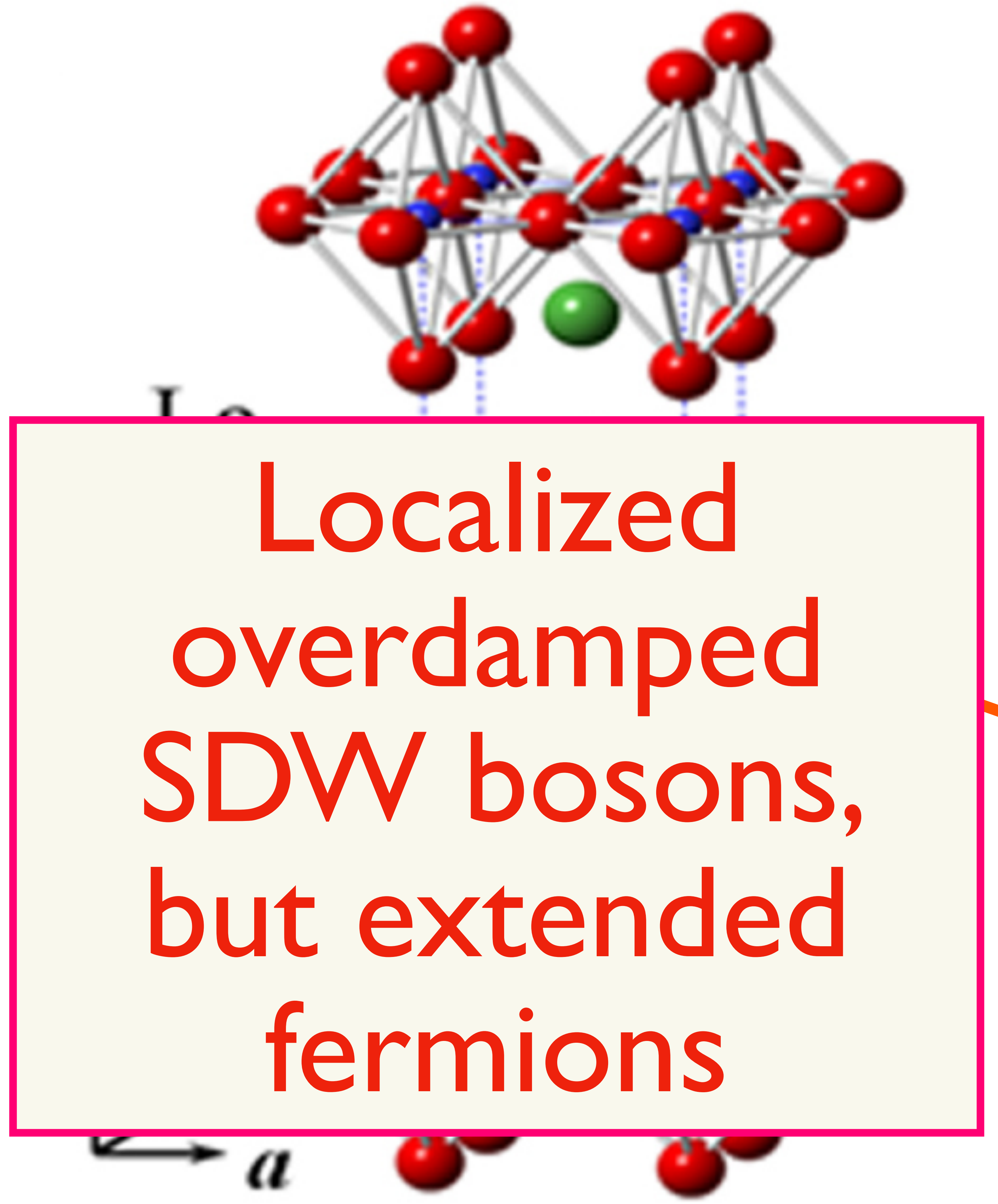
See also: QMC results of Aavishkar A. Patel, Peter Lunts, Michael S. Albergo, arXiv:2410.05365

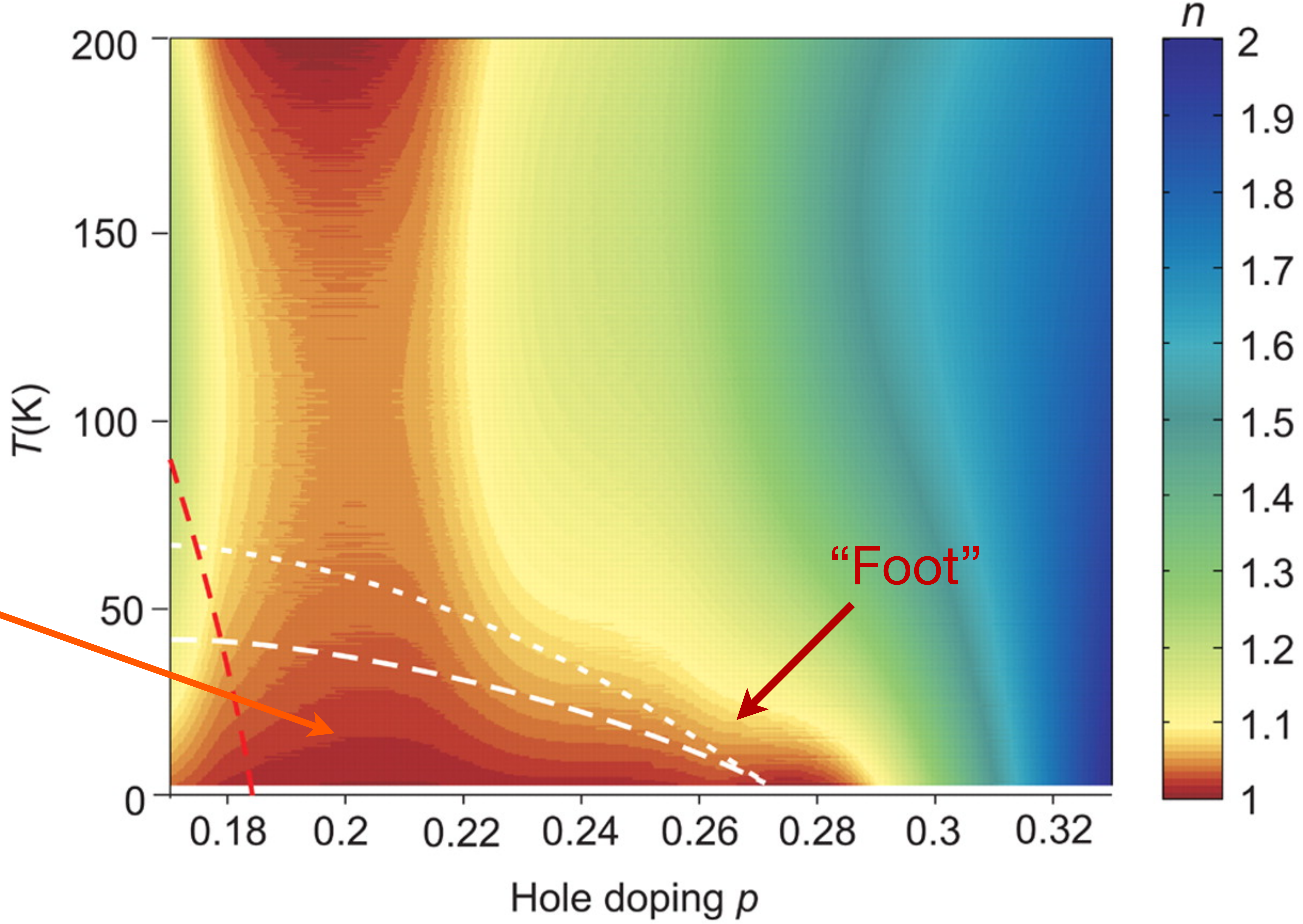
Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

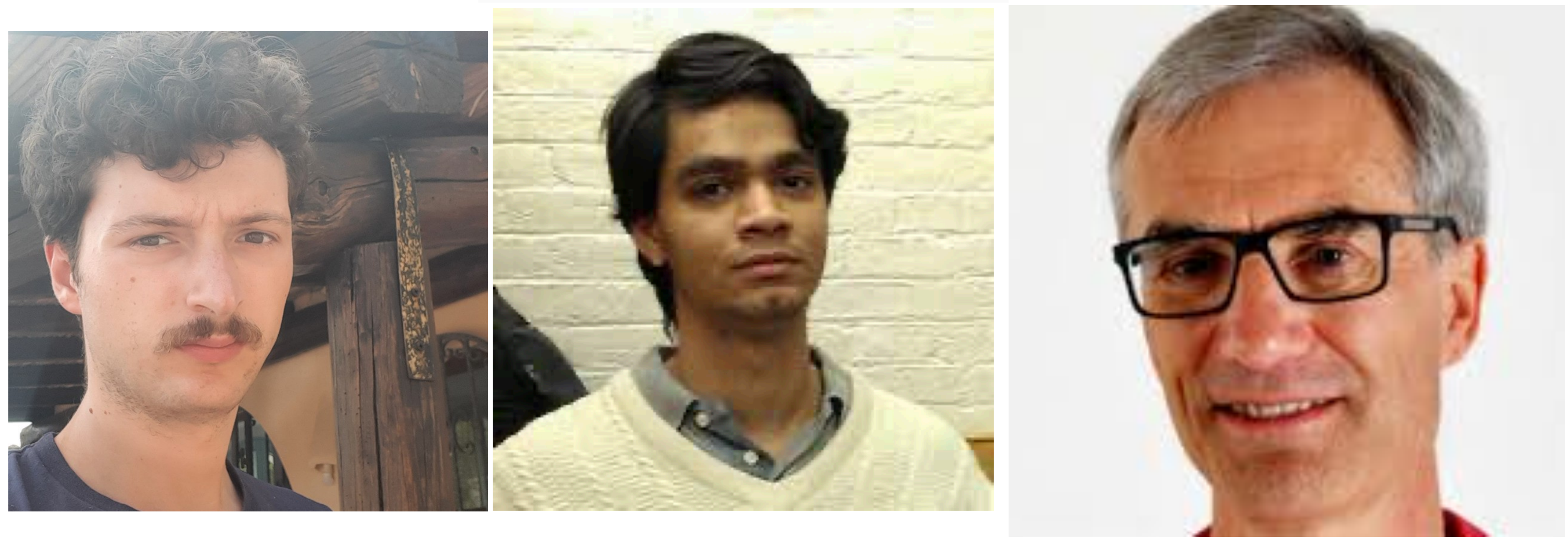
R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009

FL-SDW QPT with Harris disorder provides a theory of the "foot"

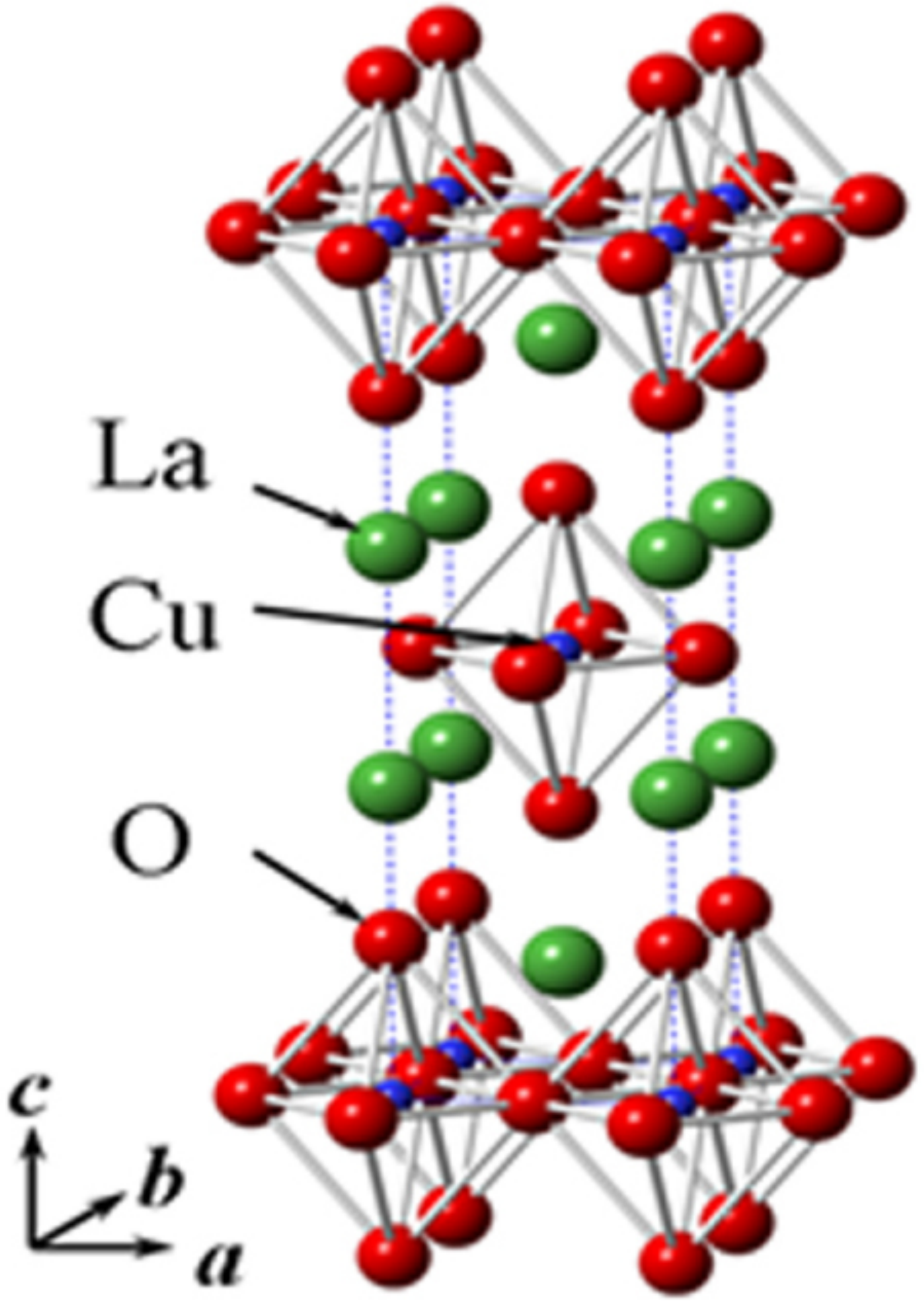


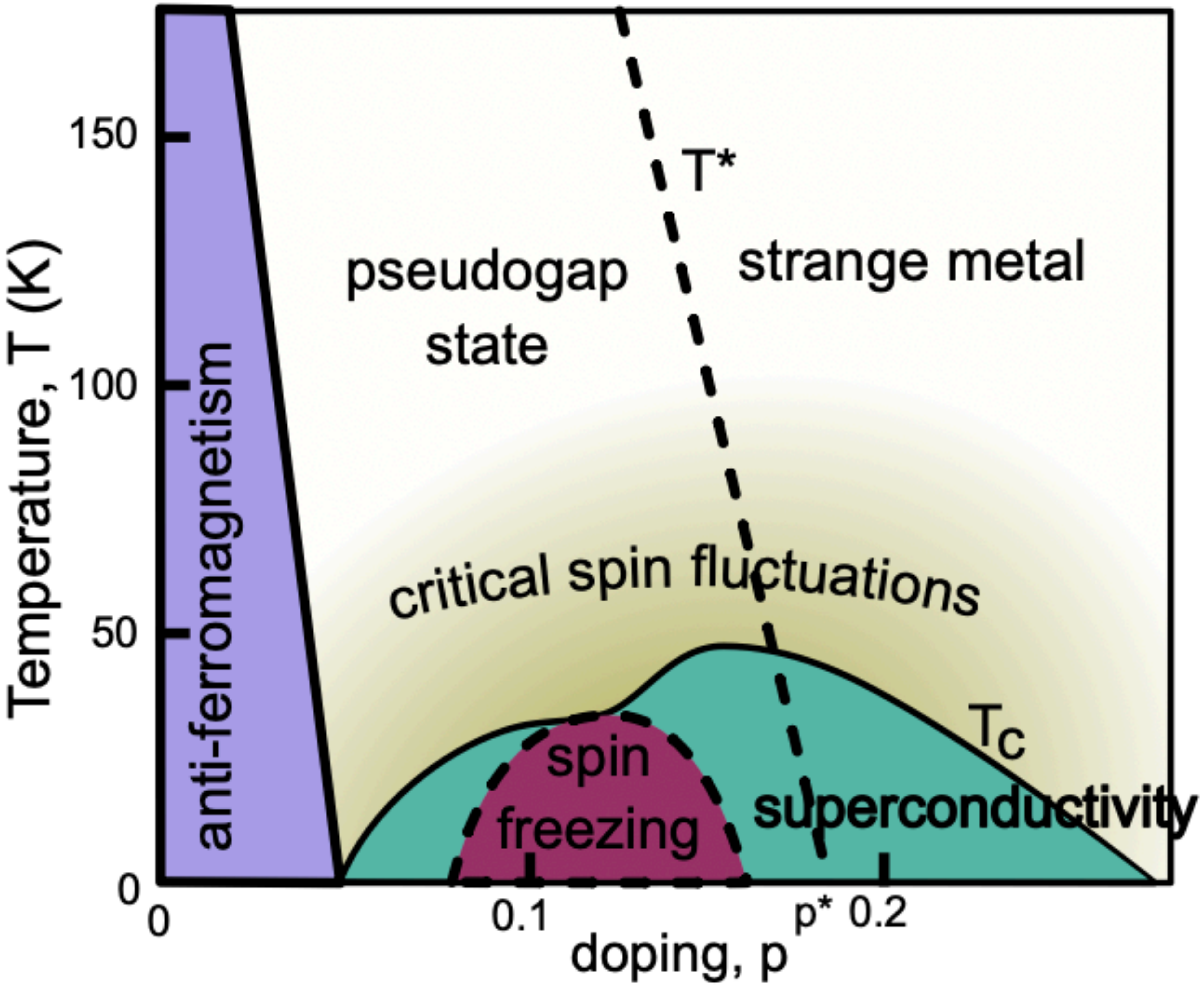




Neutron scattering in LSCO

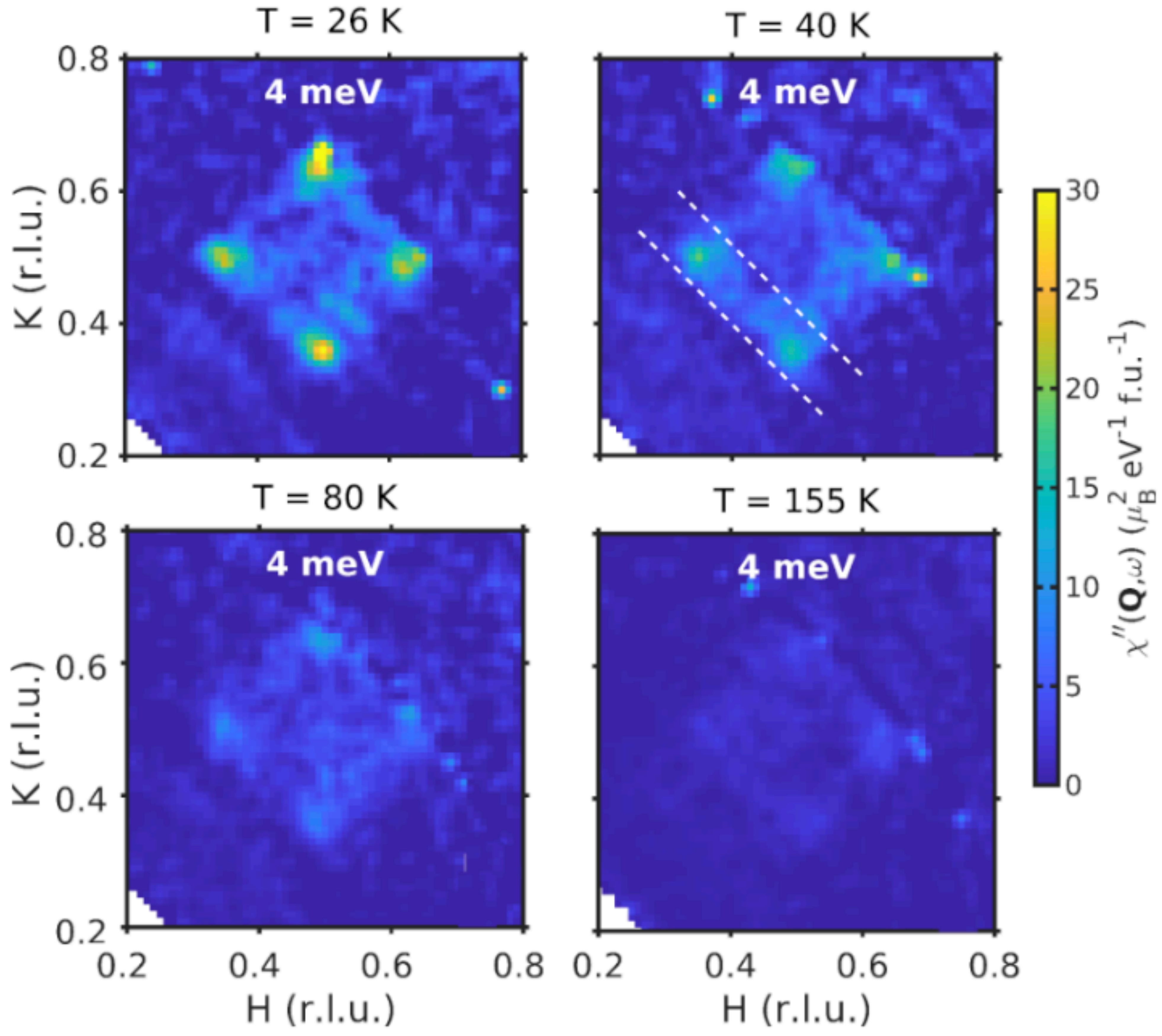
Jacopo Radaelli, Aavishkar A. Patel, ….S. S., Stephen Hayden, to appear

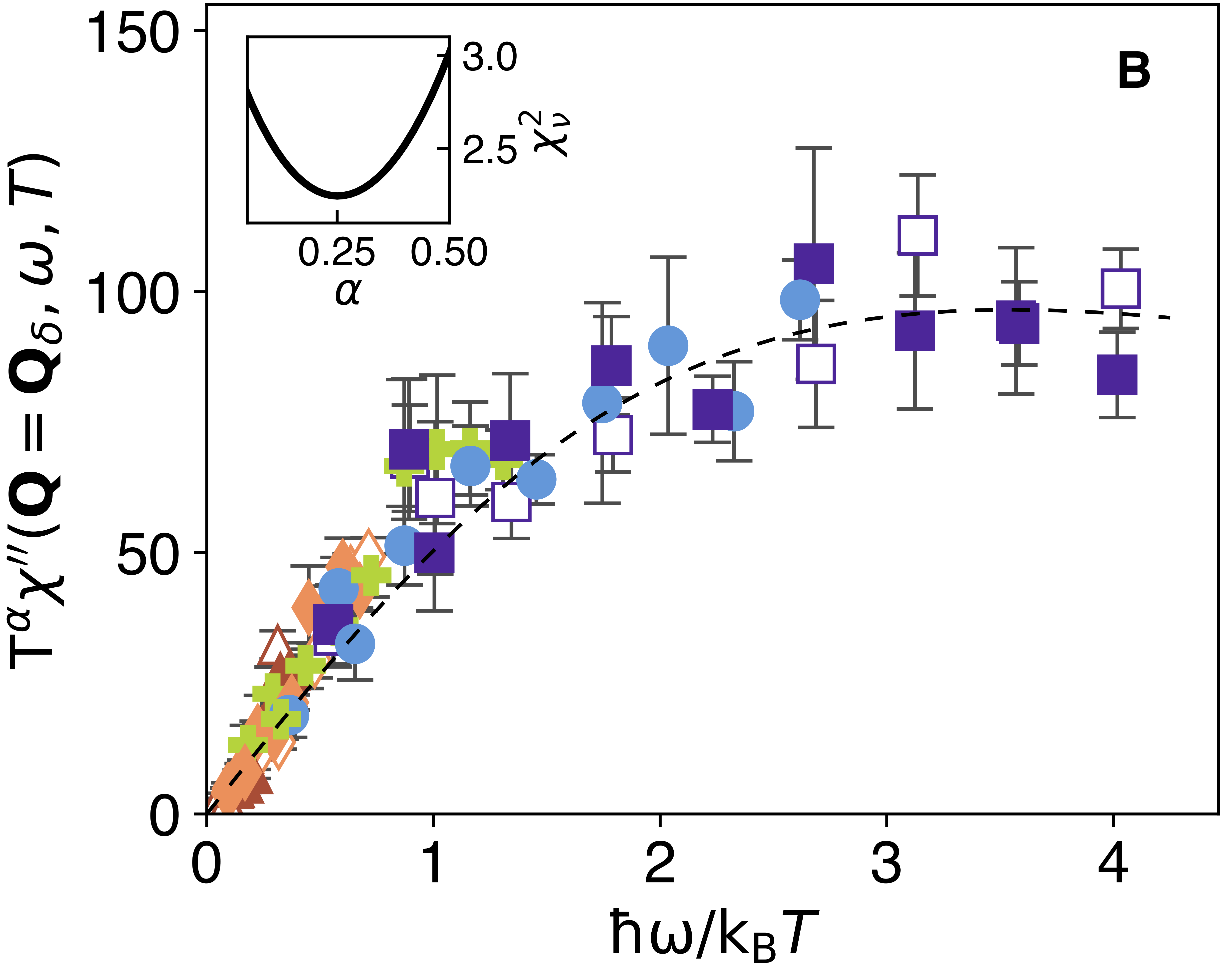




Neutron scattering in LSCO

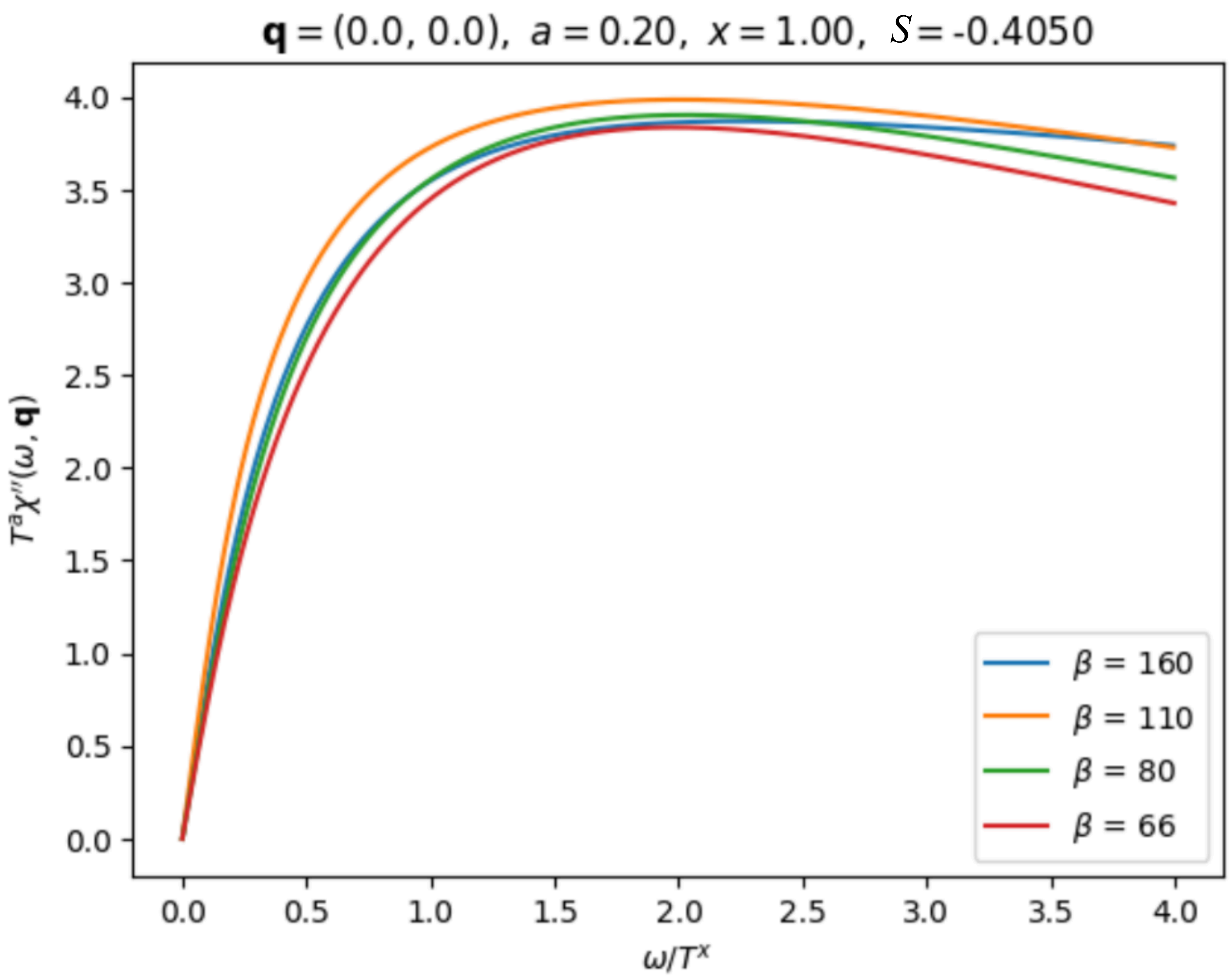
Jacopo Radaelli, Aavishkar A. Patel, ….S. S., Stephen Hayden, to appear

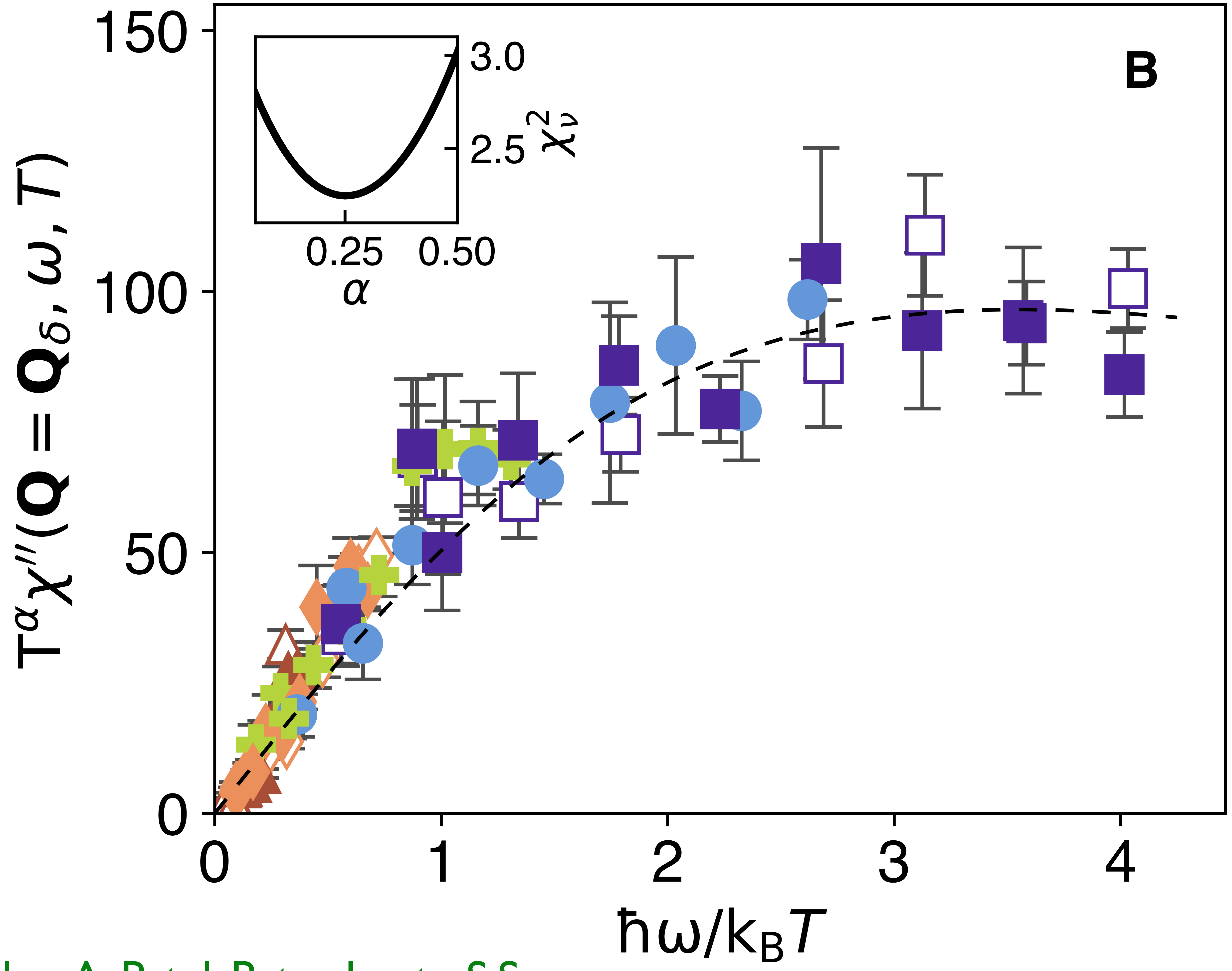




Neutron scattering in LSCO

Jacopo Radaelli, Aavishkar A. Patel, ….S. S., Stephen Hayden, to appear





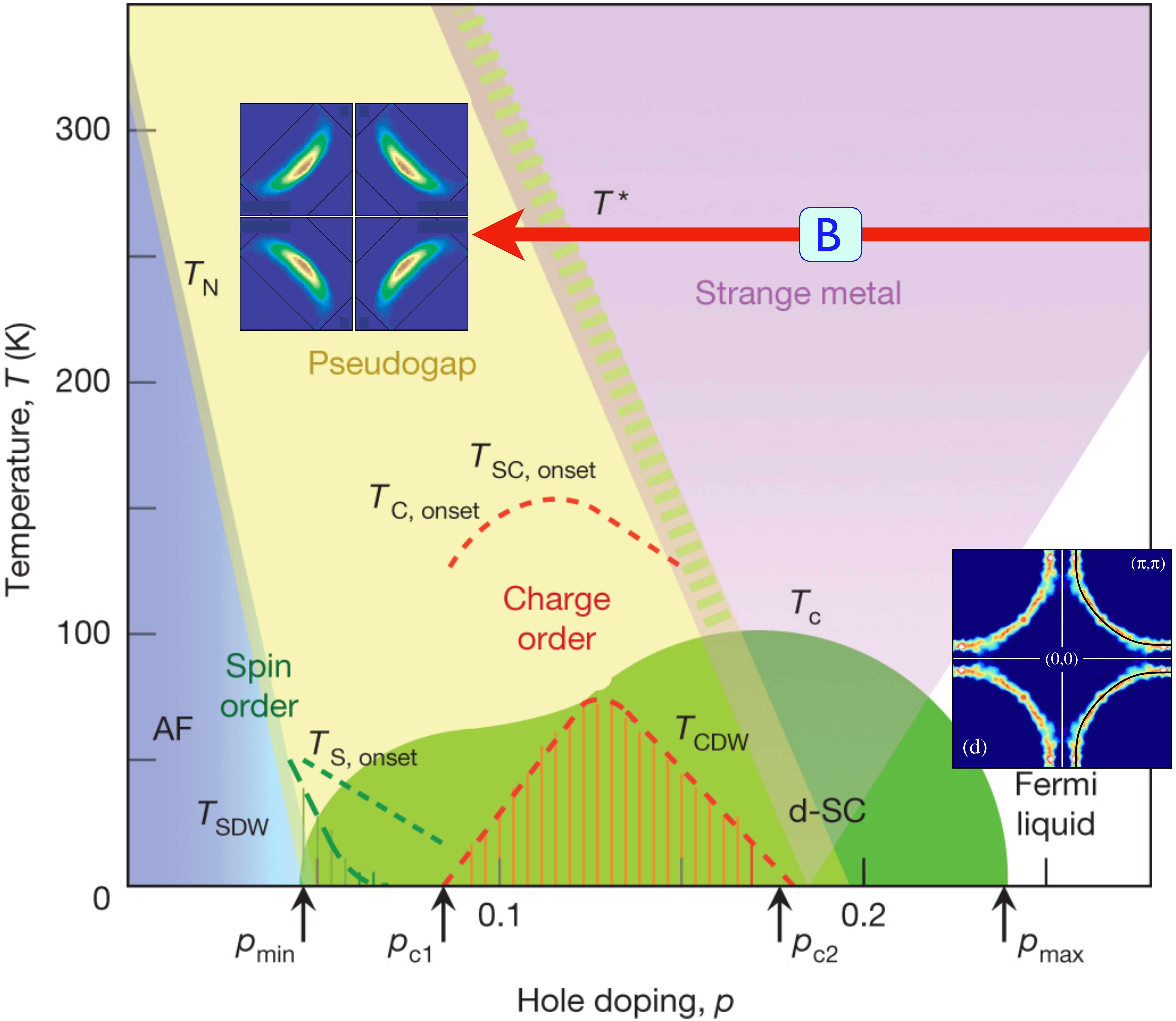
Aavishkar A. Patel, Peter Lunts, S.S.,
PNAS **121**, e2402052121 (2024)

A. FL-SDW QPT

B. FL-FL* QPT

C. Confinement crossover

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



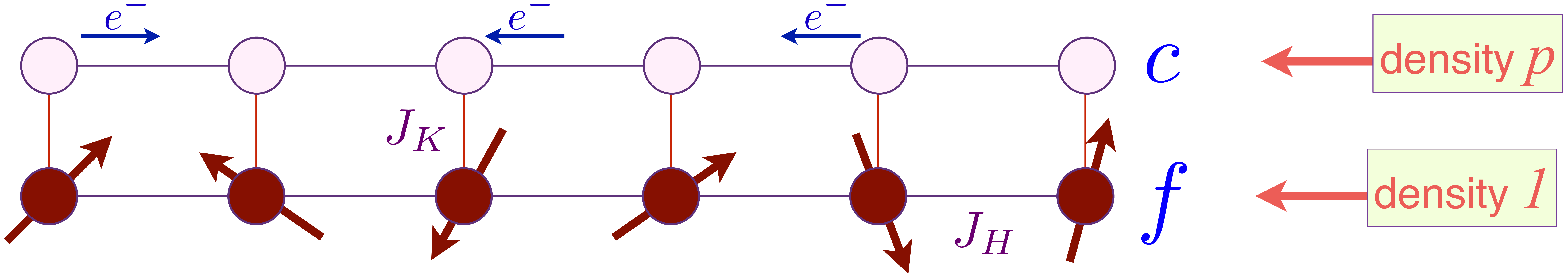
Fermi-volume-changing QPT
without symmetry breaking
and with spatial disorder.

FL-FL* QPT

Requires fractionalization

Fermi-volume-changing QPT in the Kondo lattice

$$\mathcal{H}_{KL} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_i J_K\, c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_i + \sum_{\langle ij \rangle} J_H\, \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



Assume J_H is chosen so that at $J_K = 0$
the $\boldsymbol{S}_i$ spins have a fractionalized spin liquid ground state.

Represent $\boldsymbol{S}_i$ by fermionic spinons: $\boldsymbol{S}_i = \frac{1}{2} f^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} f_{i\sigma'}$

$$\sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} = 1 \text{ for all } i.$$

Fermi-volume-changing QPT in the Kondo lattice

T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fermi-volume-changing QPT in a one-band model

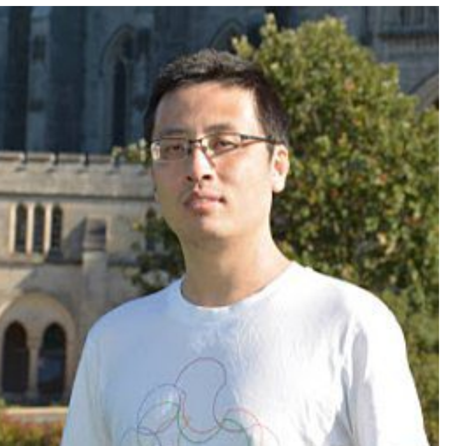
Ya-Hui Zhang and S. S., *PRR* **2**, 023172 (2020)

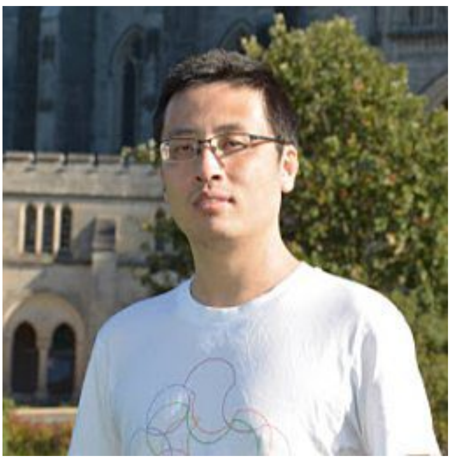
Pseudogap metal = Kondo Lattice Heavy Fermi Liquid $\oplus$ Spin Liquid

Fractionalized excitations of layer S_1 confined by condensation of Higgs boson $\Phi \sim f_{1\sigma}^\dagger c_\sigma$.

Frationalized excitations of layer S_2 in the π-flux spin liquid with massless Dirac spinons and SU(2) gauge field, dual to $\mathbb{CP}^1$ U(1) gauge theory with critical bosonic spinons.

Ya-Hui Zhang





Kondo lattice

FL* $\langle \Phi \rangle = 0$ | $\langle \Phi \rangle \neq 0$ FL

0 — Small Fermi surface of size p | Large Fermi surface of size $1+p$ — J_K

One-band model

FL* $\langle \Phi \rangle \neq 0$ | $\langle \Phi \rangle = 0$ FL

J_K — Small Fermi surface of size p | Large Fermi surface of size $1+p$ — 0

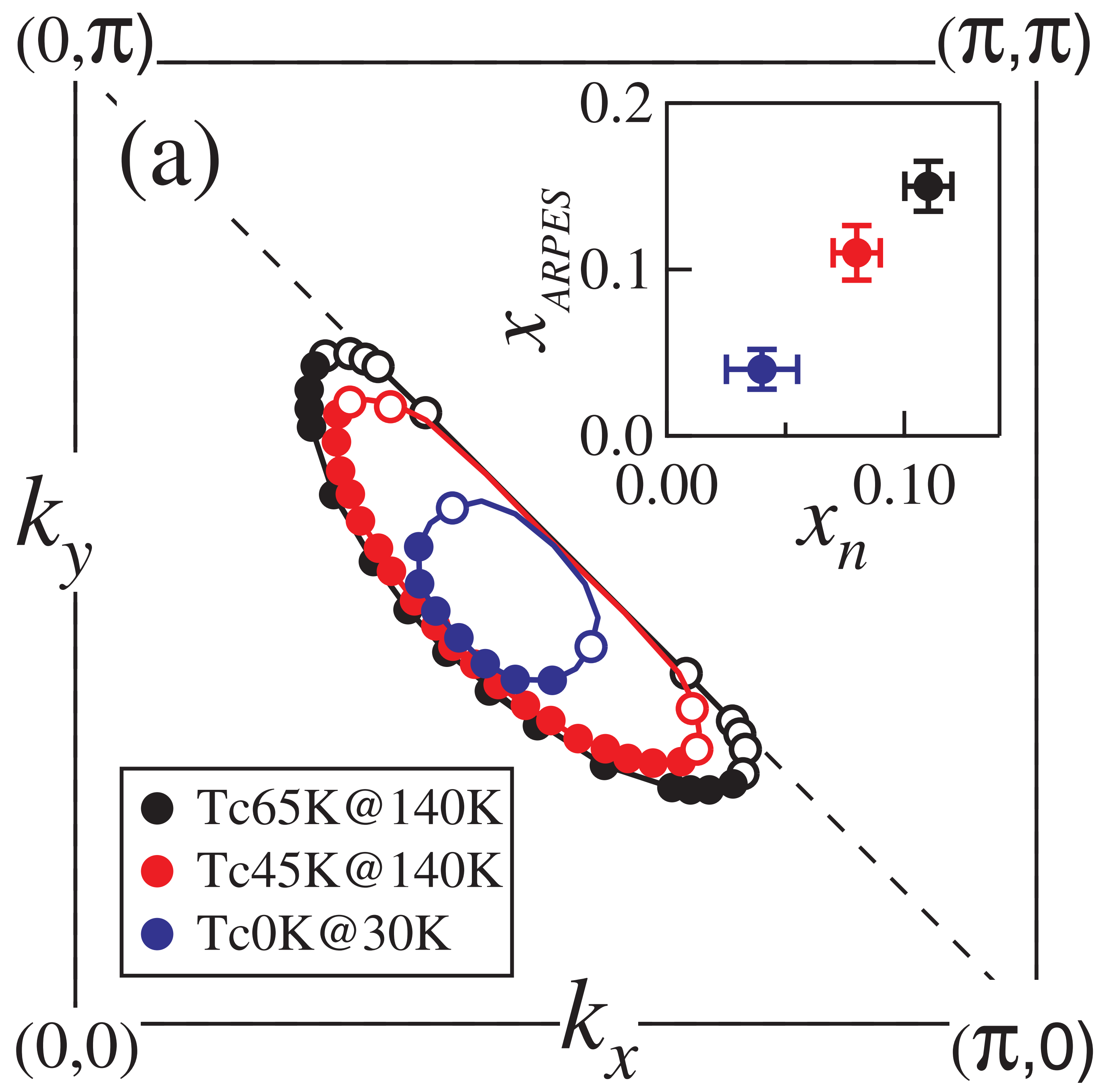
One-band model has an **'inverted' Kondo lattice transition** in a theory using a bilayer of ancilla qubits

Ya-Hui Zhang and S. Sachdev, PRR **2**, 023172 (2020)

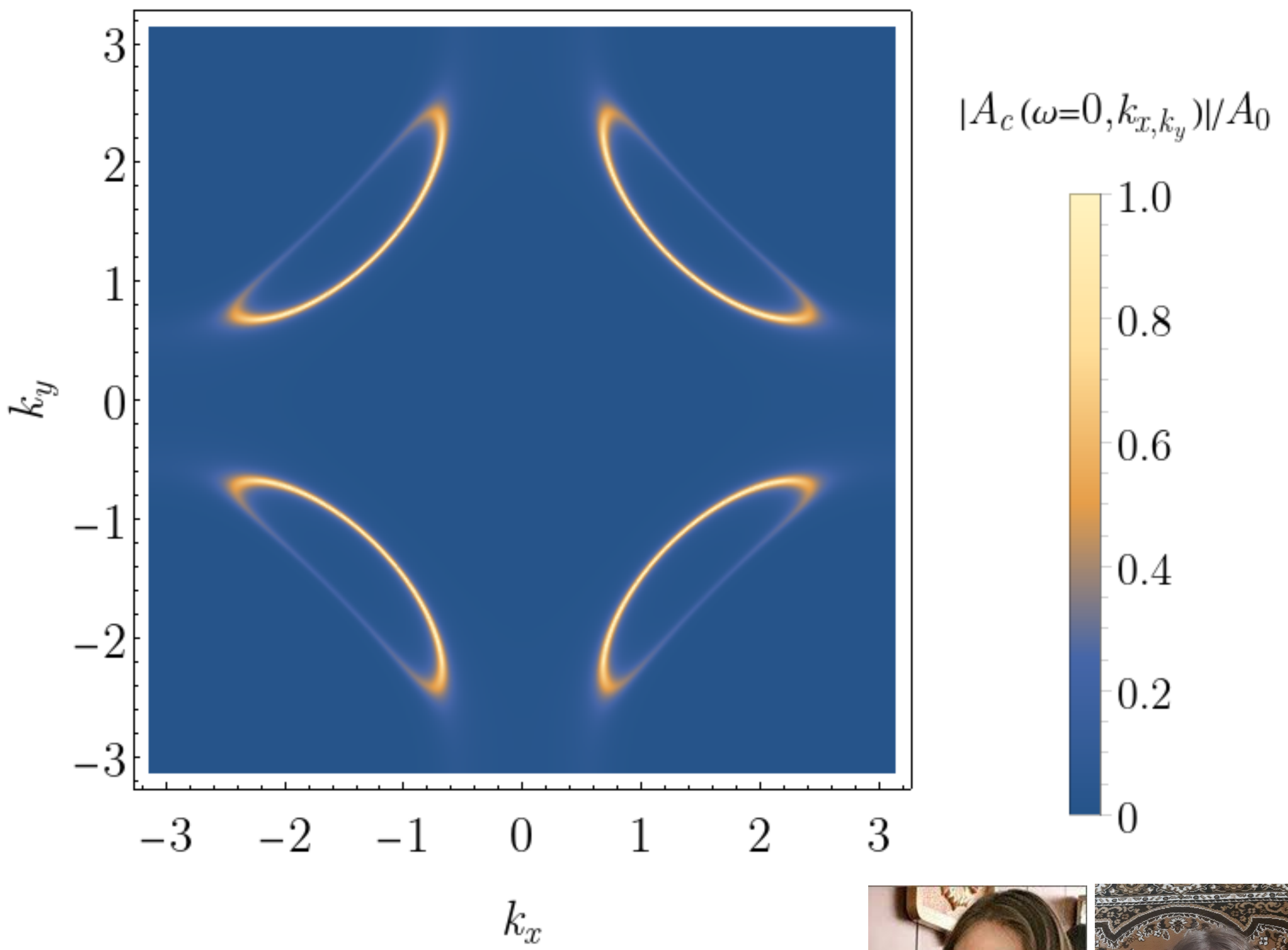
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Ancilla theory of photoemission

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, *PRB* **73**, 174501 (2006); T. D. Stanescu and G. Kotliar, *PRB* **74**, 125110 (2006). C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, *PRL* **97**, 136401 (2006). S. Sakai, Y. Motome, M. Imada, *PRL* **102**, 056404 (2009).



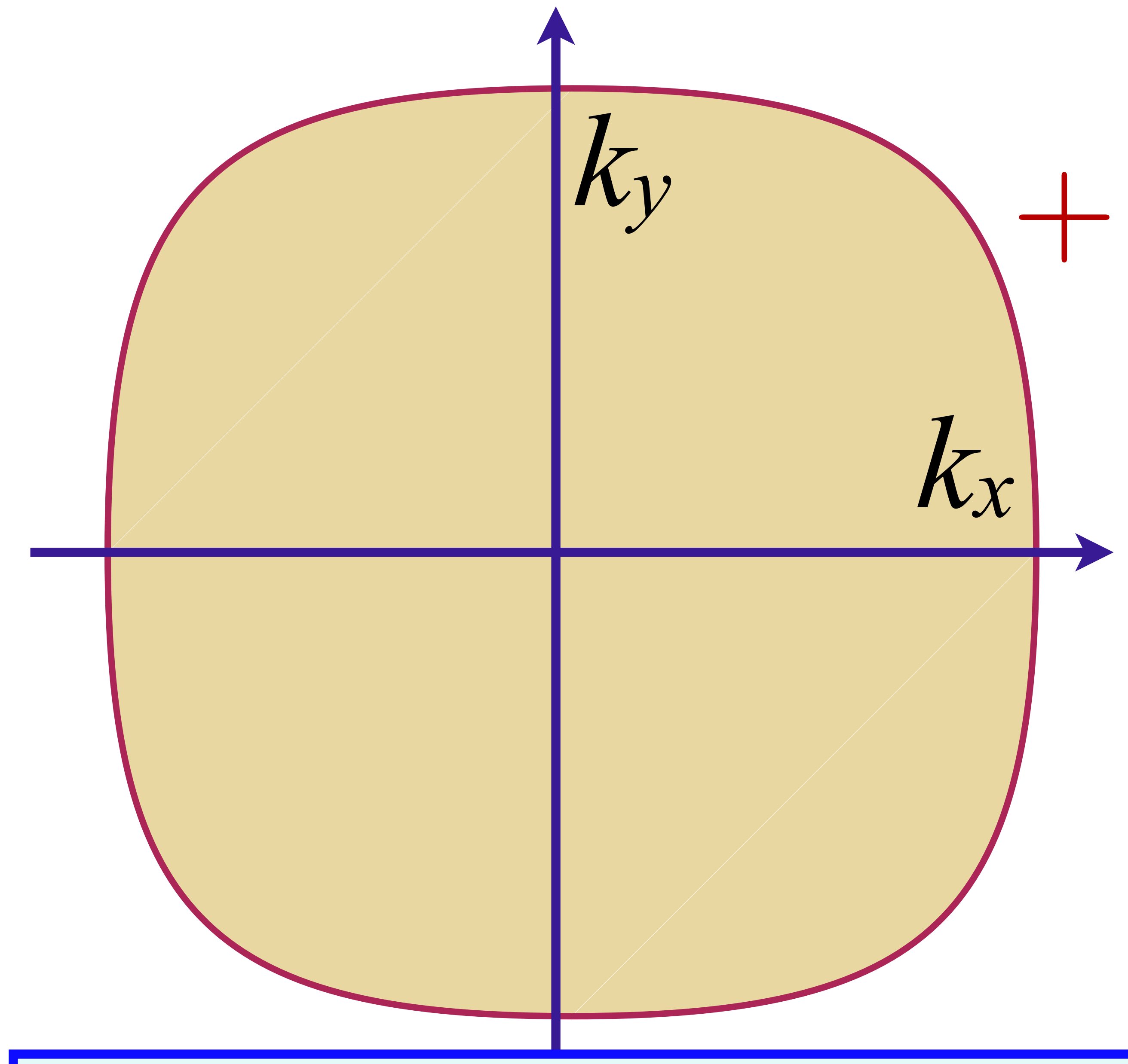
Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).



$$H_{\rm mf} = -\sum_{i,j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \sum_{i,j} t_{1,ij} f_{1i\sigma}^\dagger f_{1j\sigma} - \sum_i \Phi \left(c_{i\sigma}^\dagger f_{1i\sigma} + f_{1i\sigma}^\dagger c_{i\sigma} \right)$$

Kondo lattice + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})]\,[\Phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})f_\sigma(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

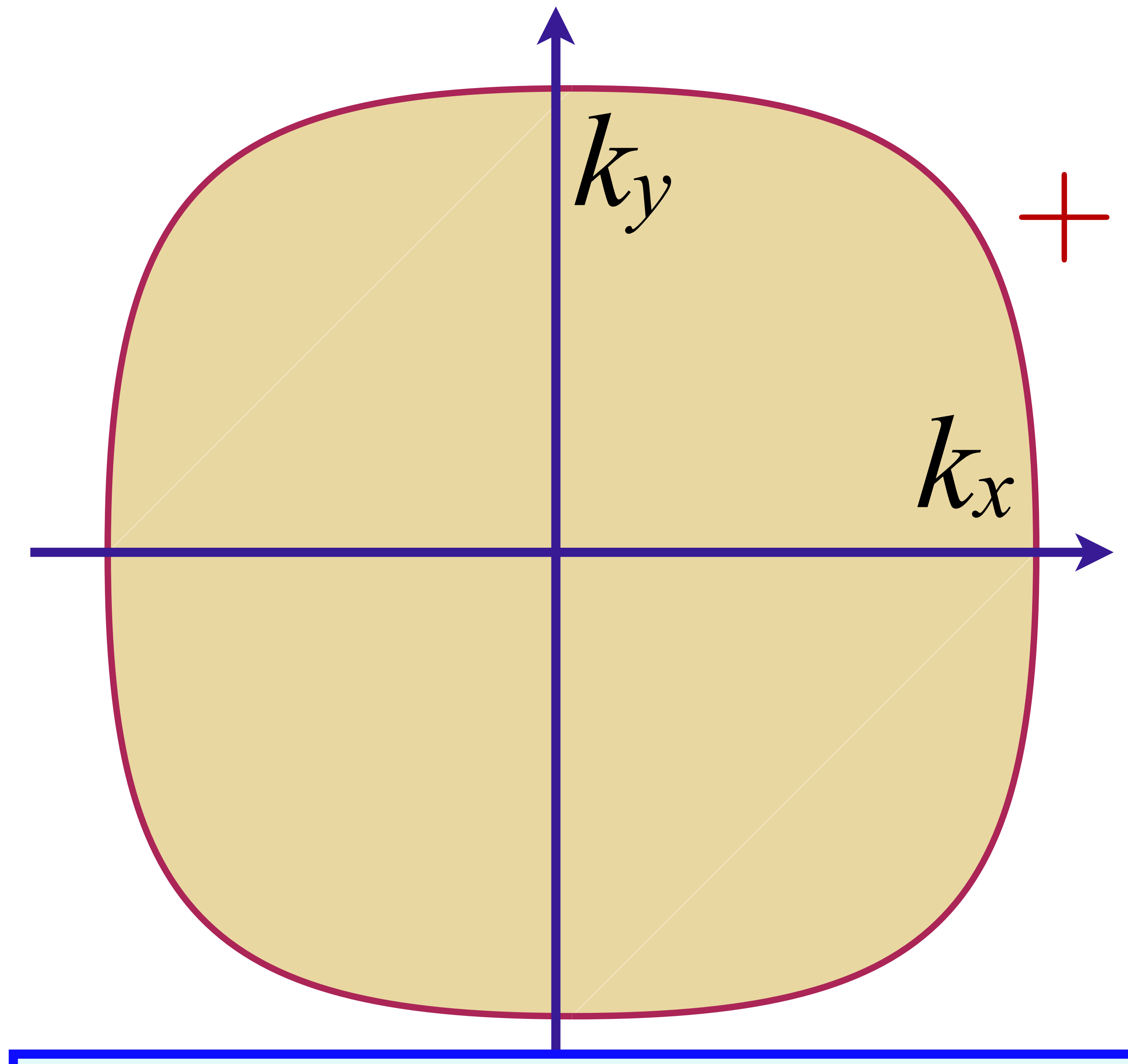
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})}=0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')}=\delta s^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

$v(\boldsymbol{r})$ leads to elastic scattering of c_σ and ‘Altshuler-Aronov’ corrections; localization of c_σ only at long length scales, not relevant for experiments

Kondo lattice + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+[s+\delta s(\boldsymbol{r})]\,[\Phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})f_\sigma(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

Rescale Φ

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random mass $\delta s(\boldsymbol{r})$ with $\overline{\delta s(\boldsymbol{r})}=0$, $\overline{\delta s(\boldsymbol{r})\delta s(\boldsymbol{r}')}=\delta s^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

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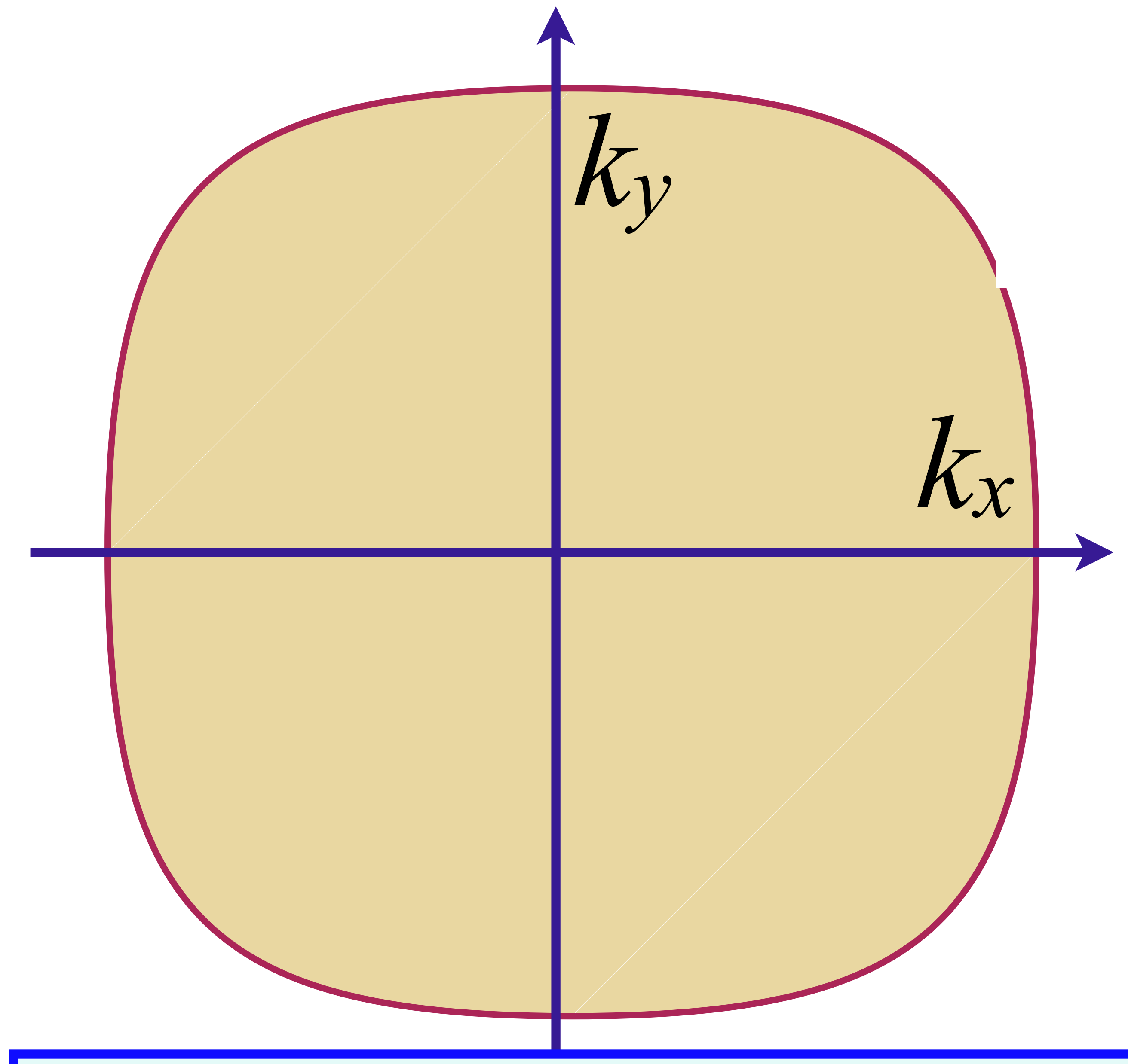
Kondo lattice + critical boson with potential and interaction disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+s\,[\Phi(\boldsymbol{r})]^2+[g+g'(\boldsymbol{r})]\,c^\dagger_\sigma(\boldsymbol{r})f_\sigma(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

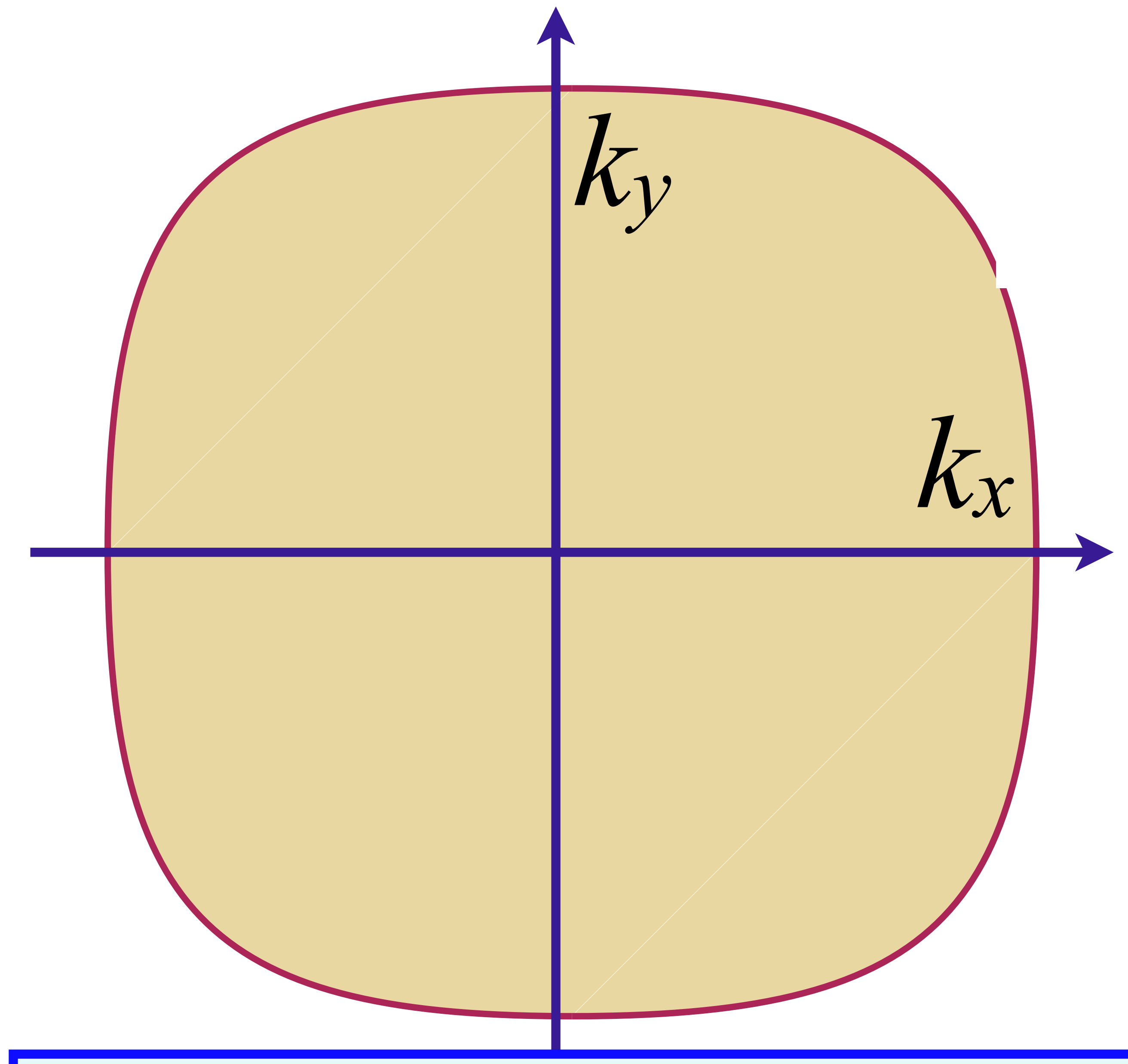
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})}=0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')}=g'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

$g'(\boldsymbol{r})$ creates inhomogeneity in the position of QCP (Harris disorder): the two-dimensional Yukawa-Sachdev-Ye-Kitaev model.

Kondo lattice + critical boson with potential and interaction disorder

$$c_{\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}+f_{\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon_1(\boldsymbol{k})\right)f_{\boldsymbol{k}\sigma}$$



$$+s\,[\Phi(\boldsymbol{r})]^2+[g+g'(\boldsymbol{r})]\,c_{\sigma}^{\dagger}(\boldsymbol{r})f_{\sigma}(\boldsymbol{r})\,\Phi(\boldsymbol{r})+\text{H.c.}$$

$$+K\,[\nabla_{\boldsymbol{r}}\Phi(\boldsymbol{r})]^2+u\,[\Phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c_{\sigma}^{\dagger}(\boldsymbol{r})c_{\sigma}(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

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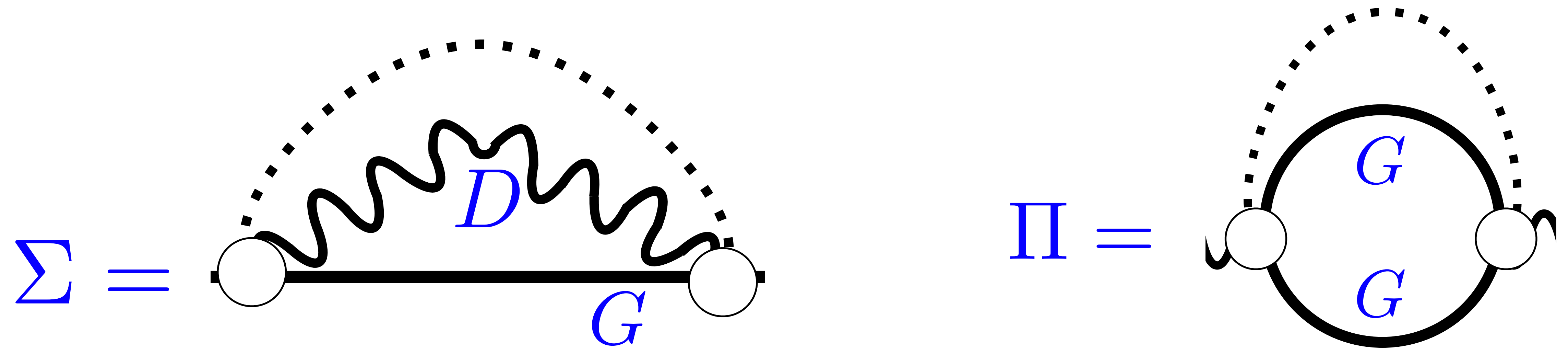
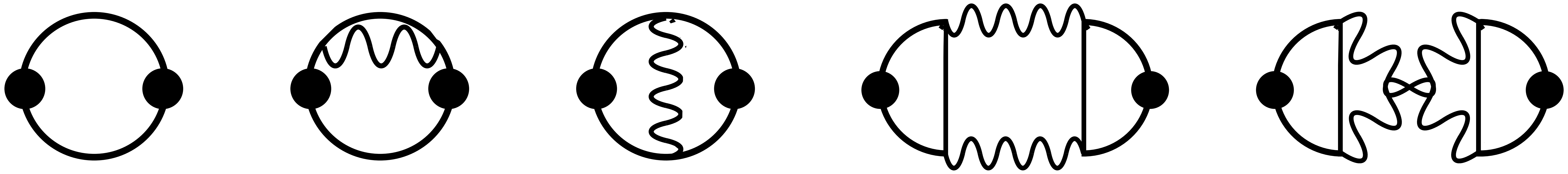
Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})}=0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')}=g'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Analyze 2d-YSYK model in a self-averaging manner as in the SYK model.
Should be applicable as long as eigenmodes of $\Phi(\boldsymbol{r})$ are extended.

2d-YSYK model: Fermi surface + critical boson with interaction disorder





Residual resistivity is determined by v^2

Linear-in-T resistivity determined by g'^2

Transport insensitive to g

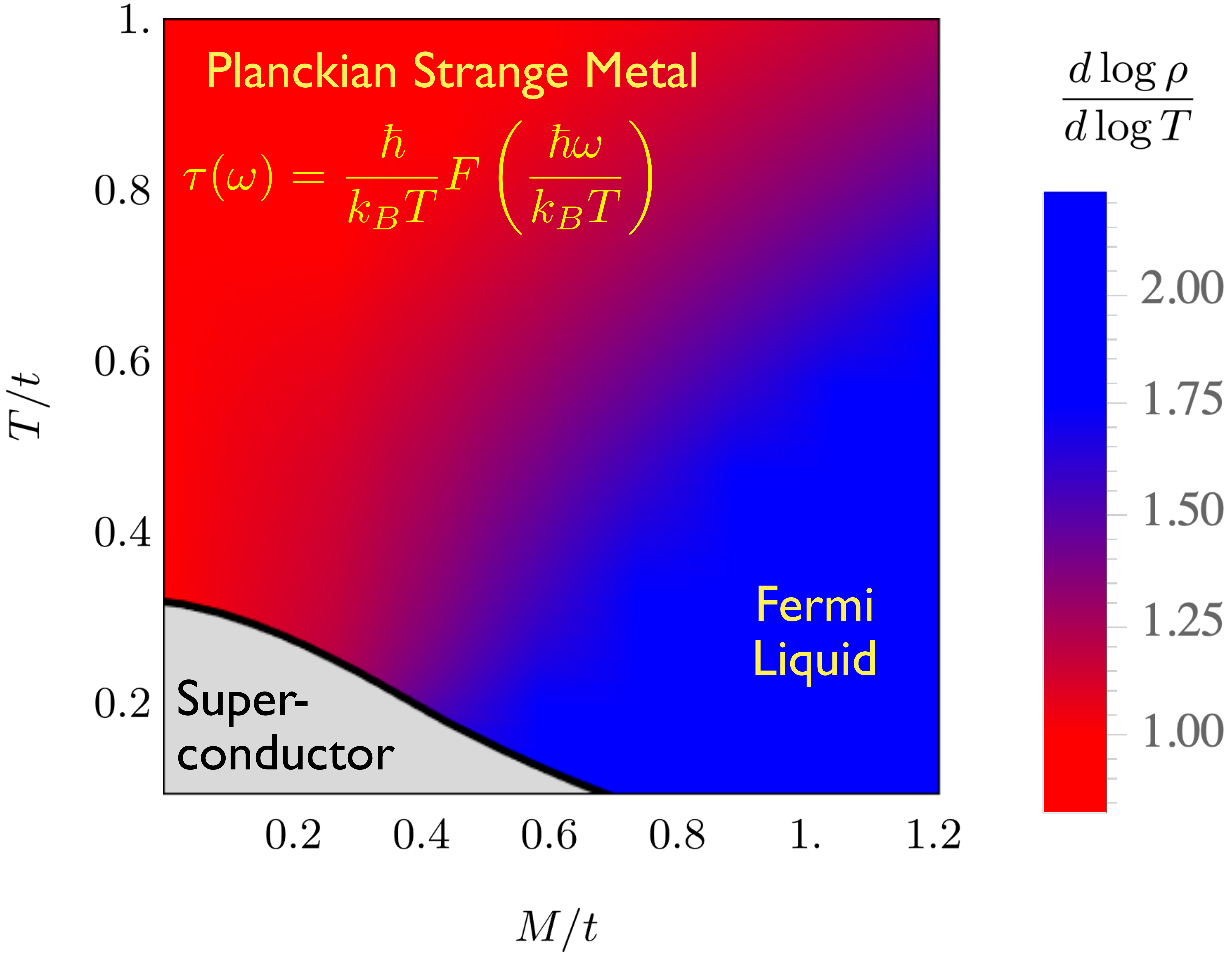
Marginal Fermi liquid self energy $\Sigma \sim \omega \ln \omega$

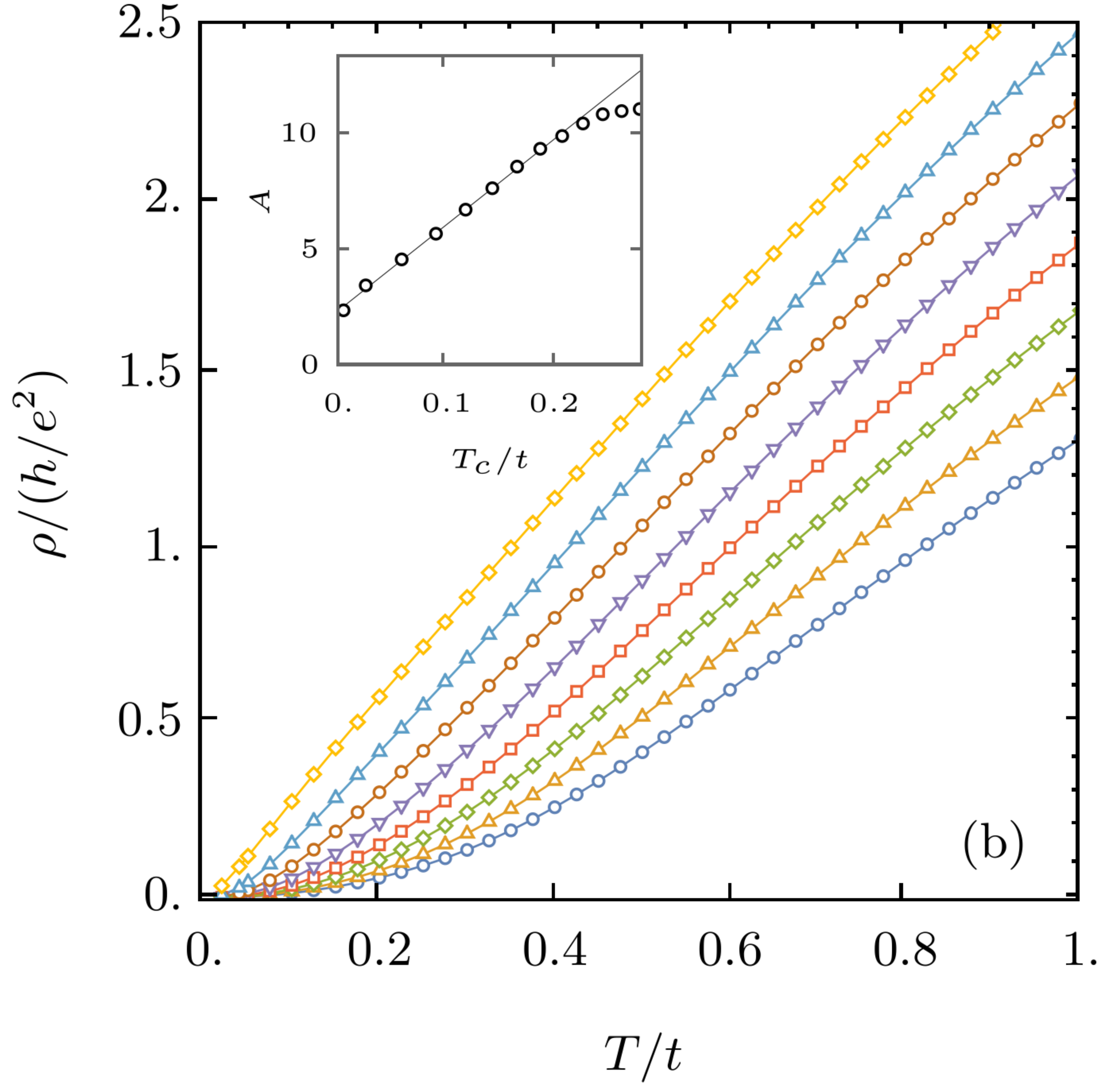
$T \ln(1/T)$ specific heat

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)



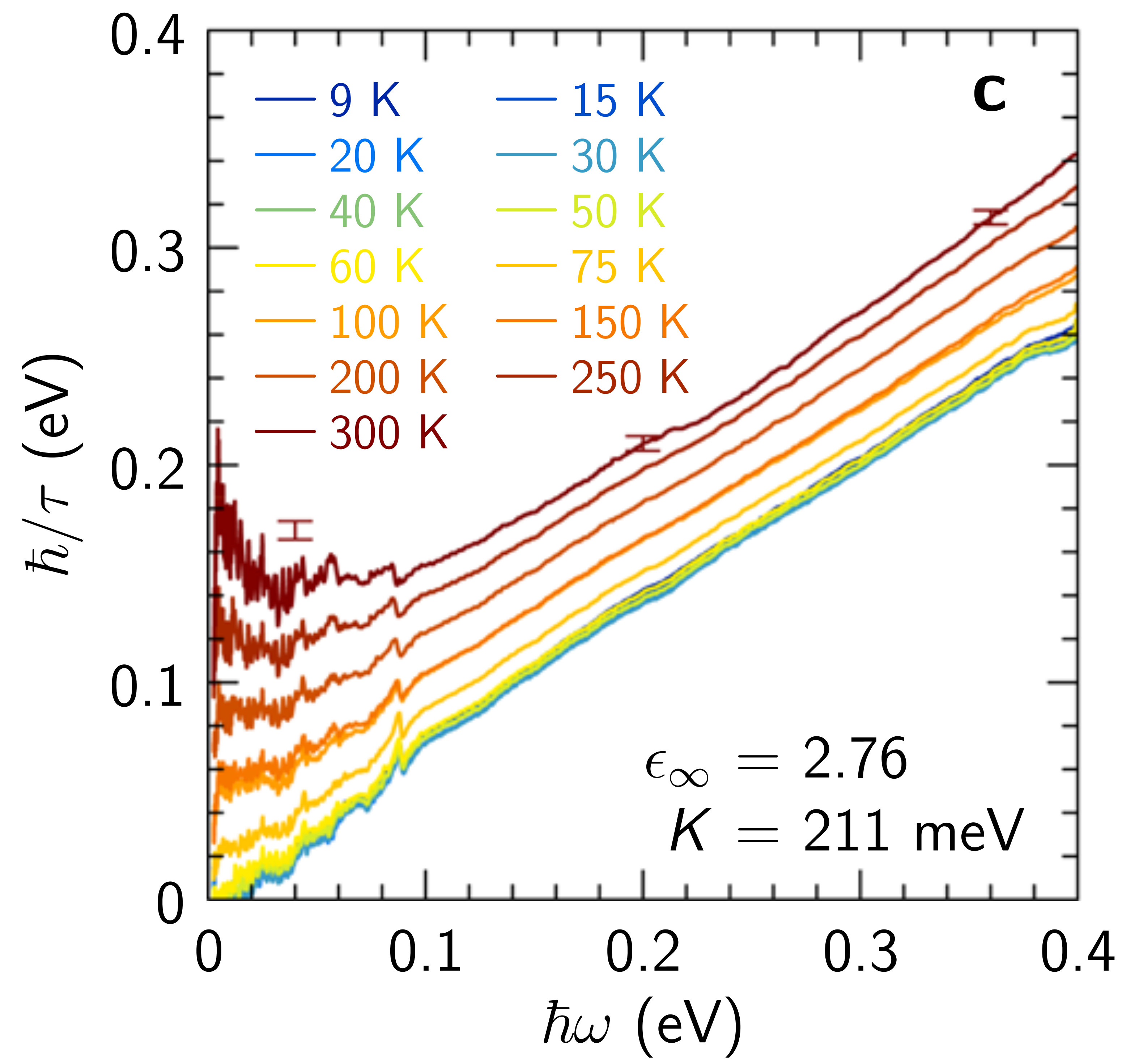


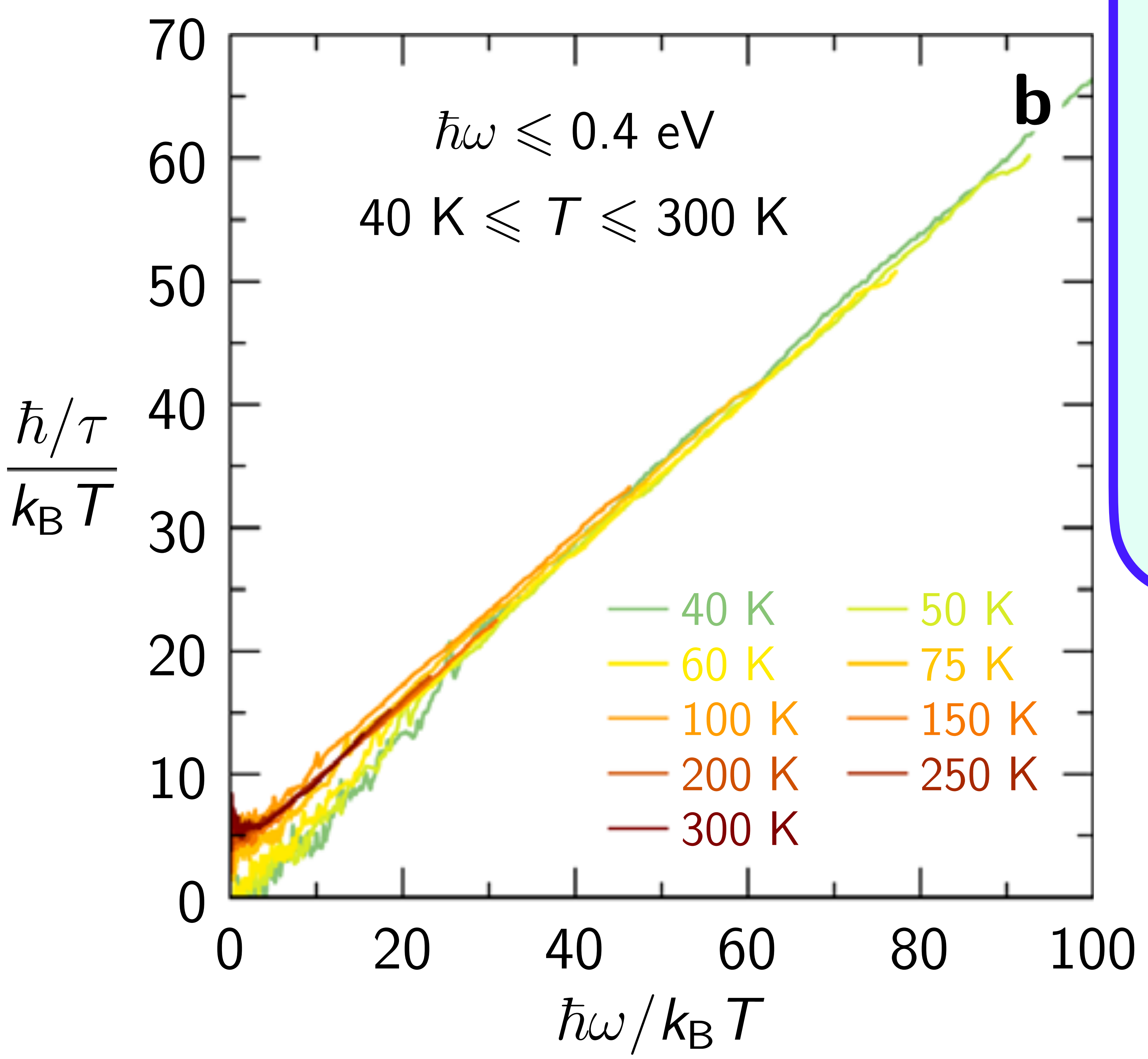
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy

$S(T \to 0) \sim T\ln(1/T)$.

$La_{2-x}Sr_xCuO_4$

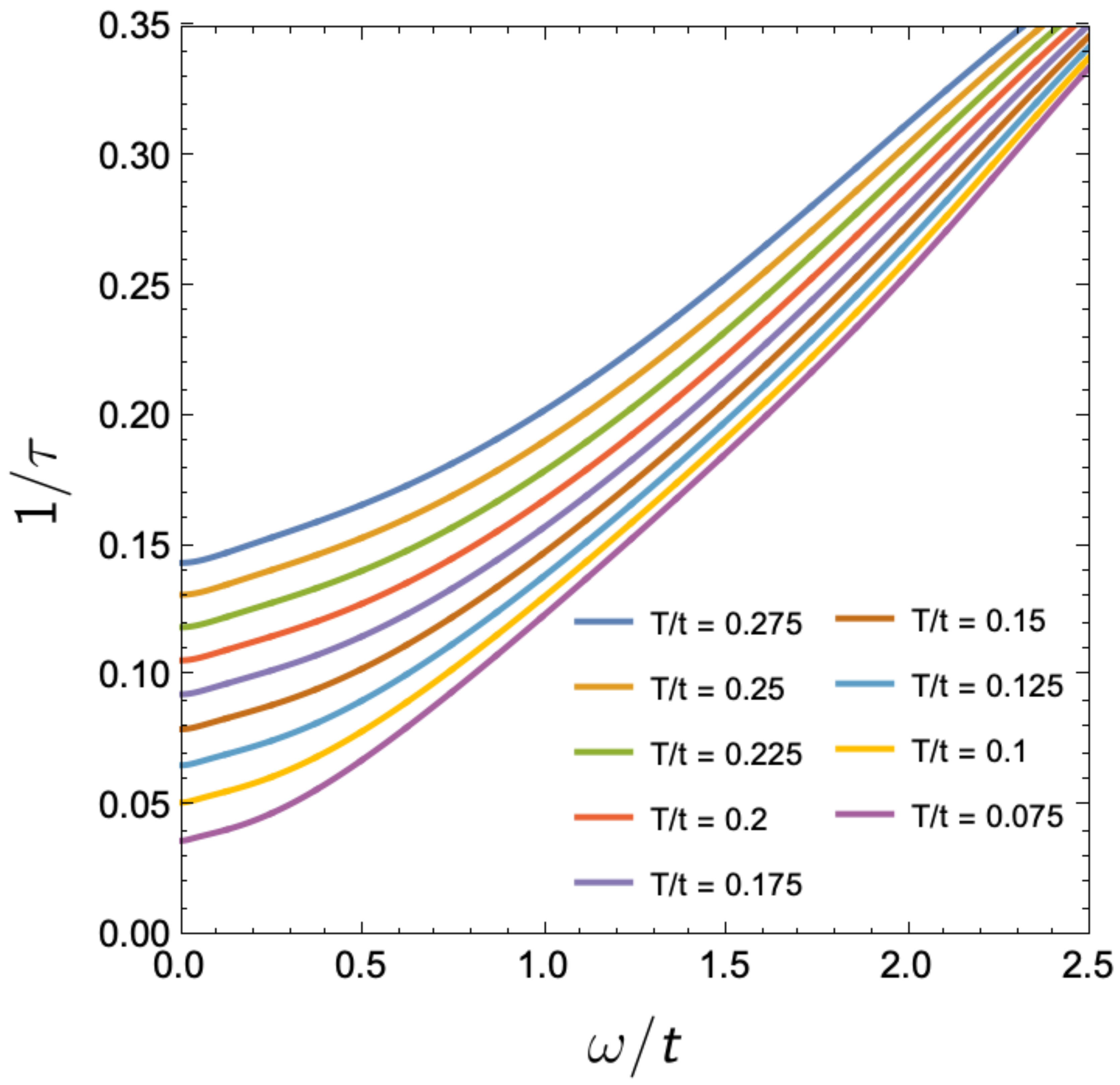
$p = 0.24$

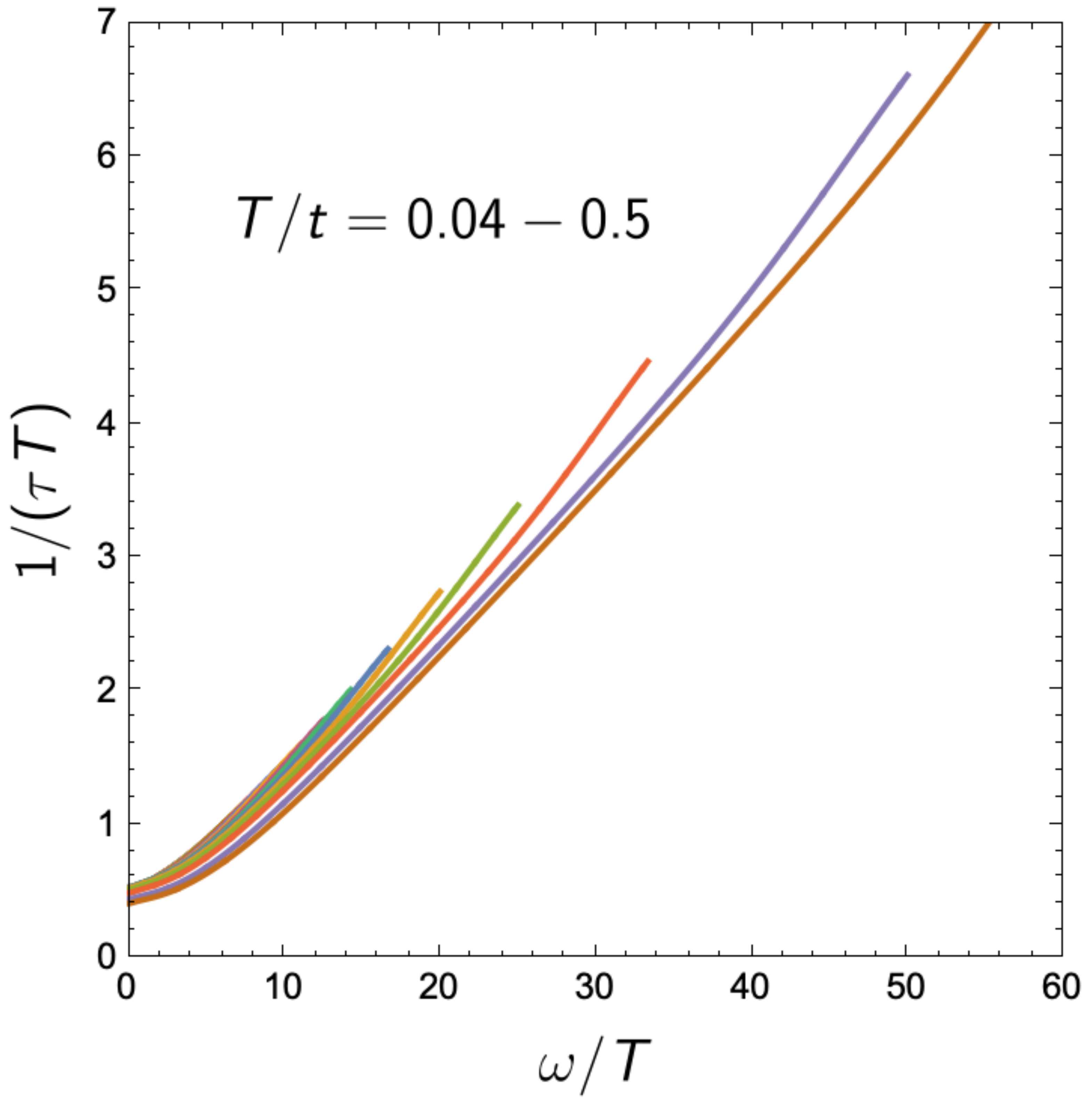
$T_c = 19$ K

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \dfrac{m^*(\omega)}{m} + i \dfrac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

and entropy $S(T \to 0) \sim T \ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

Seebeck Coefficient in a Cuprate Superconductor: Particle-Hole Asymmetry in the Strange Metal Phase and Fermi Surface Transformation in the Pseudogap Phase

A. Gourgout,[1,*] G. Grissonnanche,[1,2,3,*,†] F. Laliberté,[1] A. Ataei,[1] L. Chen,[1] S. Verret,[1] J.-S. Zhou,[4] J. Mravlje,[5] A. Georges,[6,7,8,9] N. Doiron-Leyraud,[1] and Louis Taillefer[1,10,‡]

PHYSICAL REVIEW X **12**, 011037 (2022)

Skewed non-Fermi liquids and the Seebeck effect

Antoine Georges[1,2,3,4] and Jernej Mravlje[5]

PRR **3**, 043132 (2021)

The particle-hole asymmetry in the Φ propagator (for the FL*-FL transition) $\sim 1/(-i\omega + q^2 + \gamma|\omega| + \alpha)$ leads to a **skewed marginal Fermi liquid**.

P. Lunts, A.A. Patel, and S.S., arXiv:2412.15330

The ϕ propagator (for the SDW-FL transition) does not have the $-i\omega$ term, and so is *not* skewed.

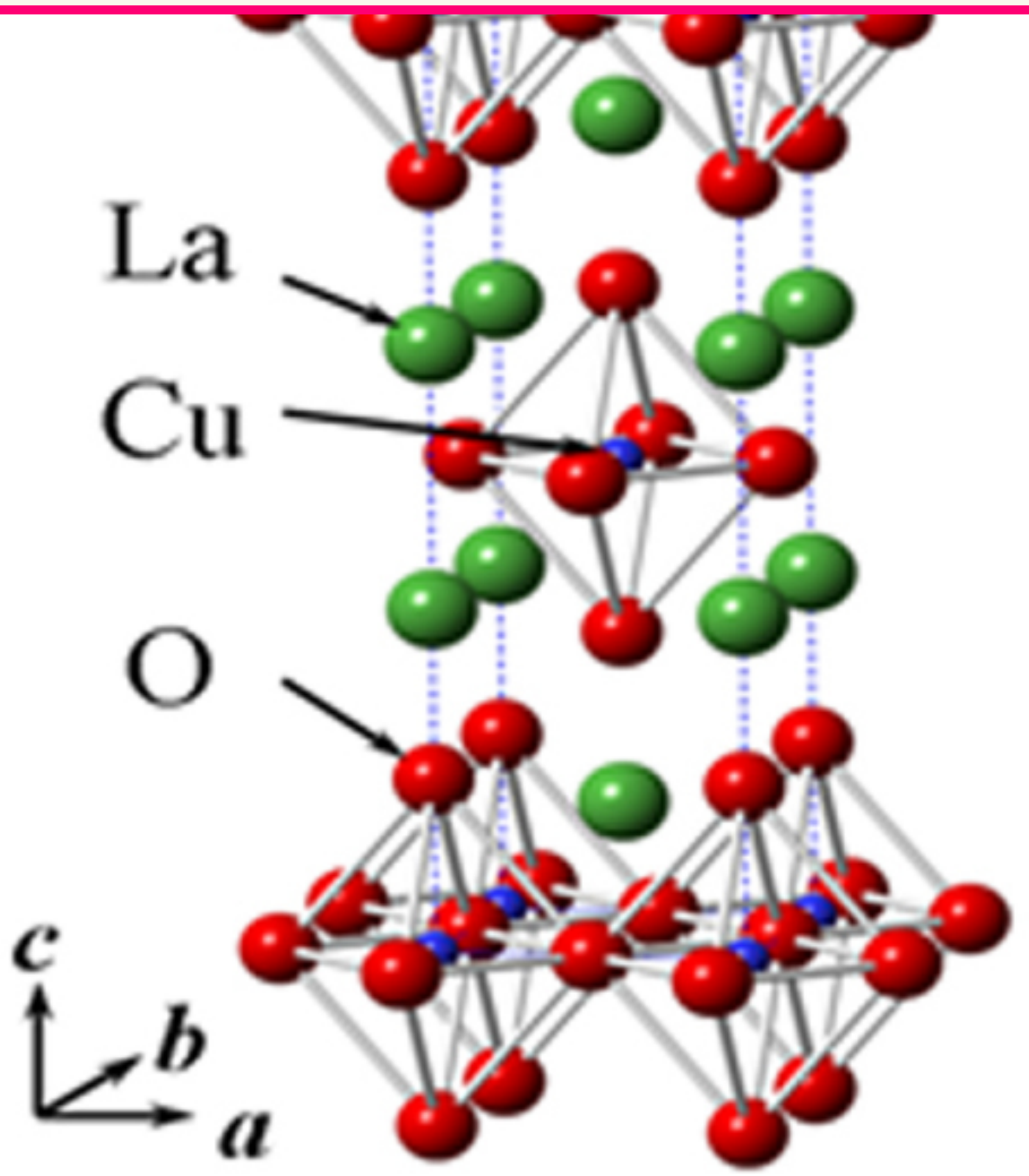
Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

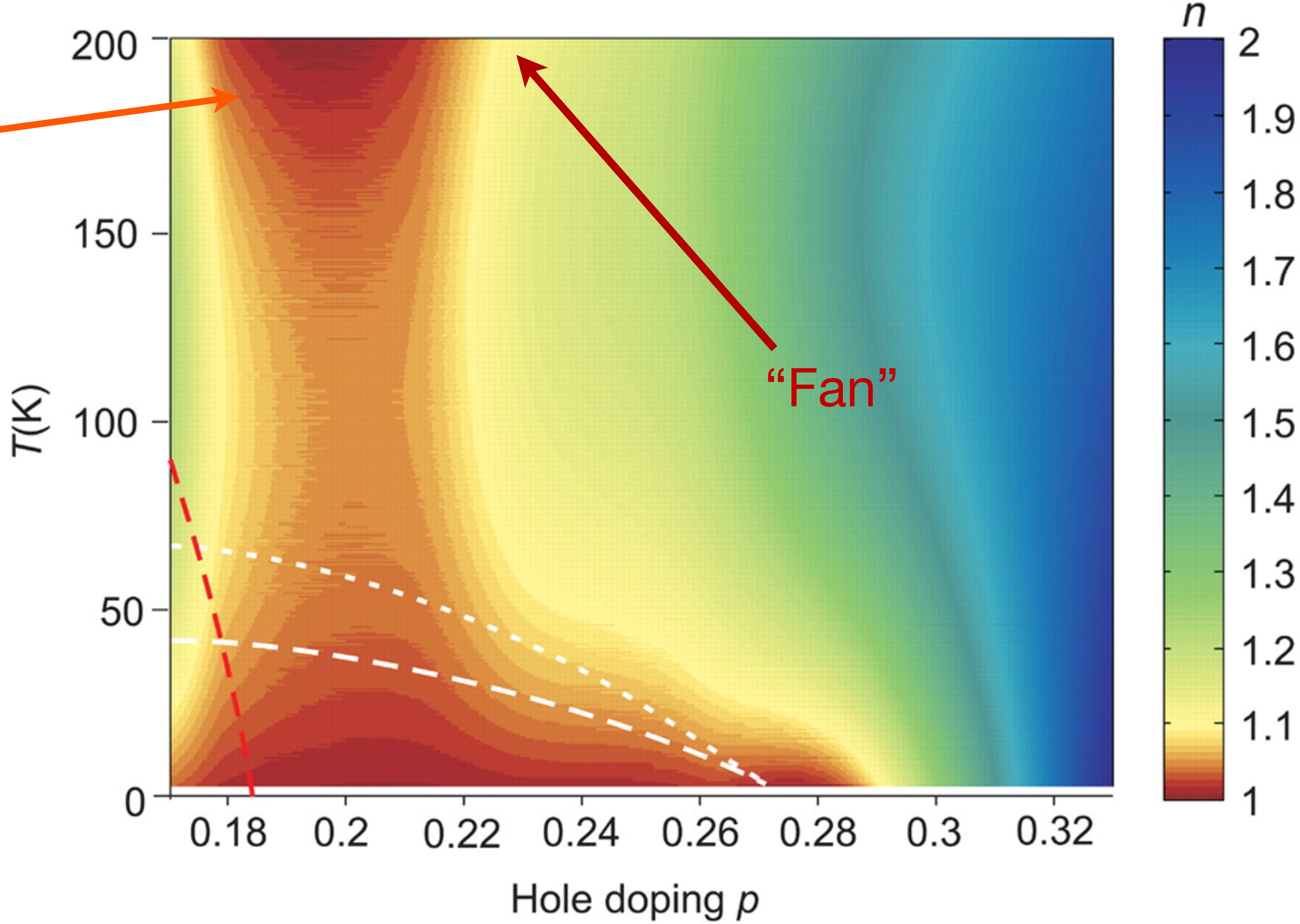
R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4*] H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009

2dYSYK theory of FL-FL* QPT provides a theory of the "fan"

Extended fermions and Higgs bosons





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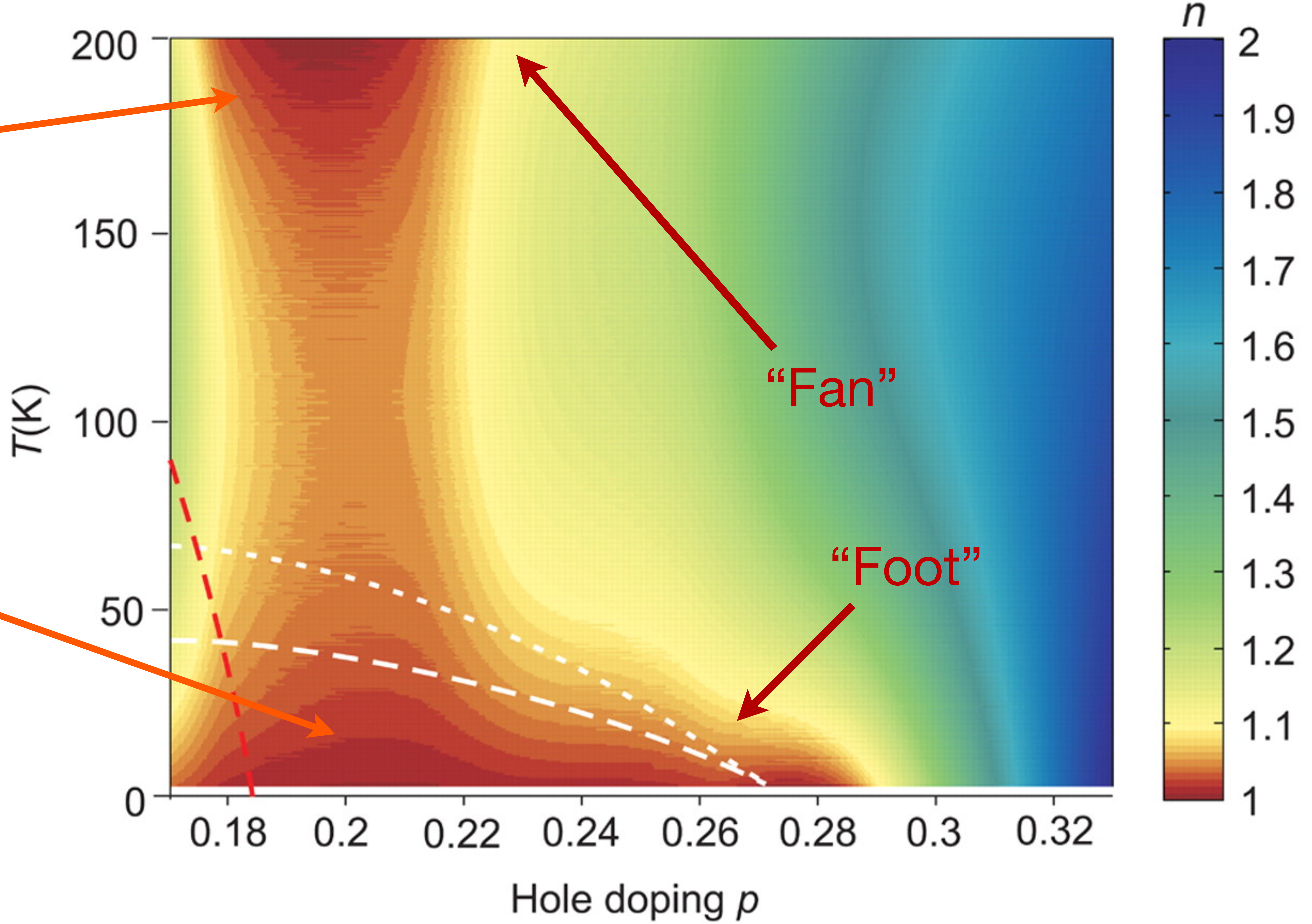
SCIENCE VOL 323 603 2009

FL-SDW QPT with Harris disorder provides a theory of the "foot"

Extended fermions and Higgs bosons

Localized overdamped SDW bosons, but extended fermions

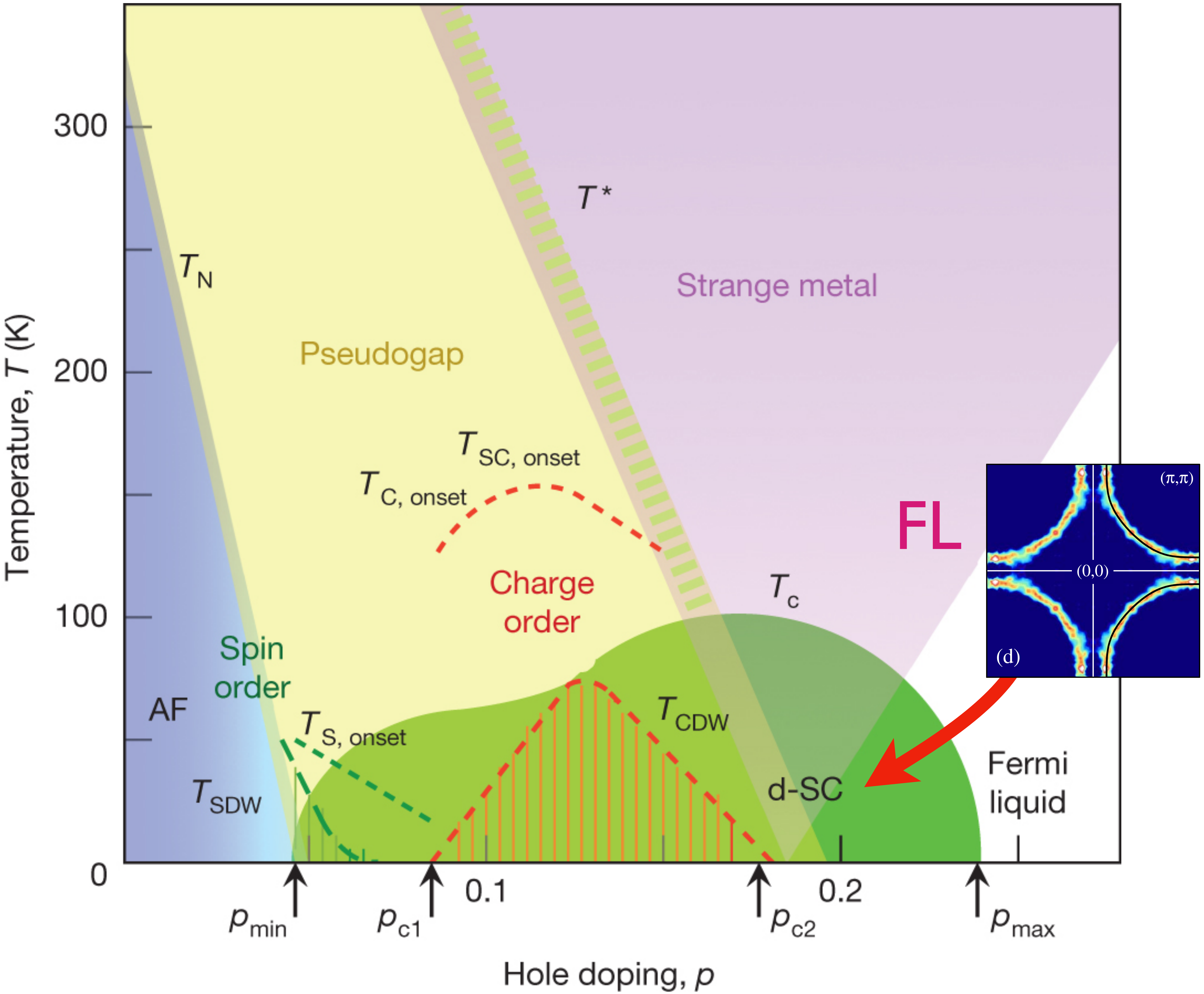


A. FL-SDW QPT

B. FL-FL* QPT

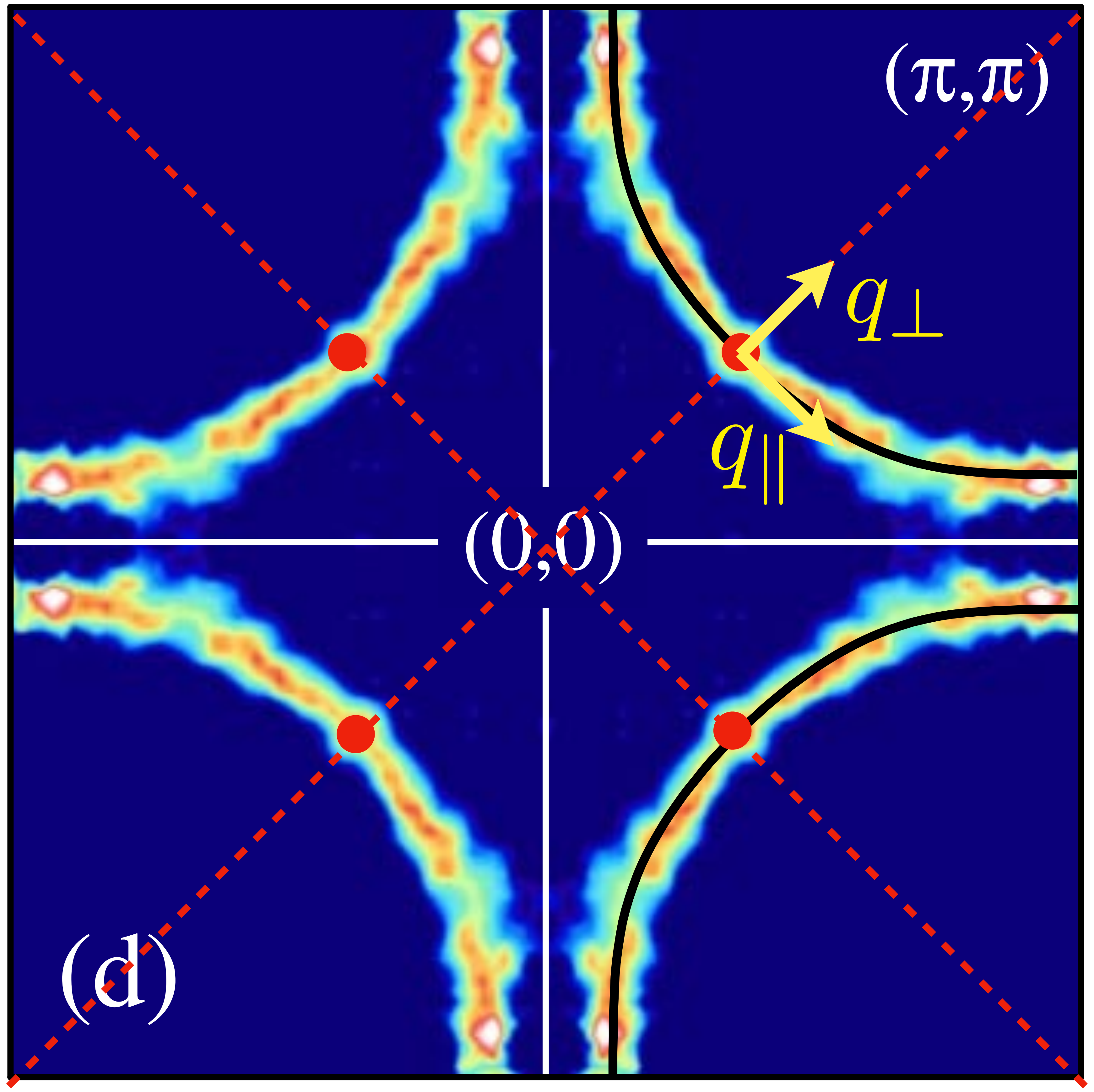
C. Confinement crossover

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



BCS-type theory of *d*-wave superconductivity (and charge order) induced by antiferromagnetic spin fluctuations.

FL→dSC



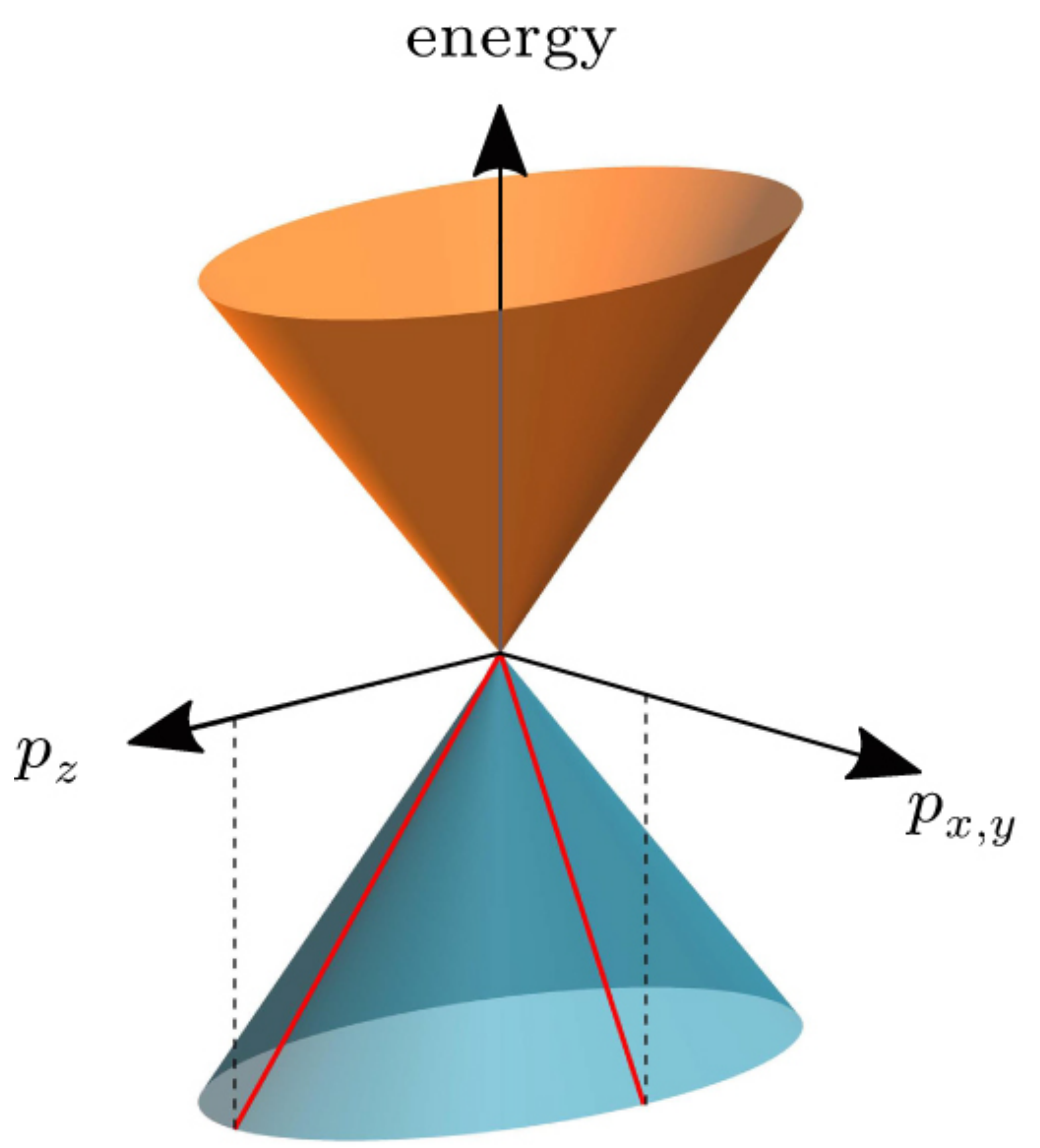
BCS/Bogoliubov quasiparticles in a d-wave superconductor

$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$
$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

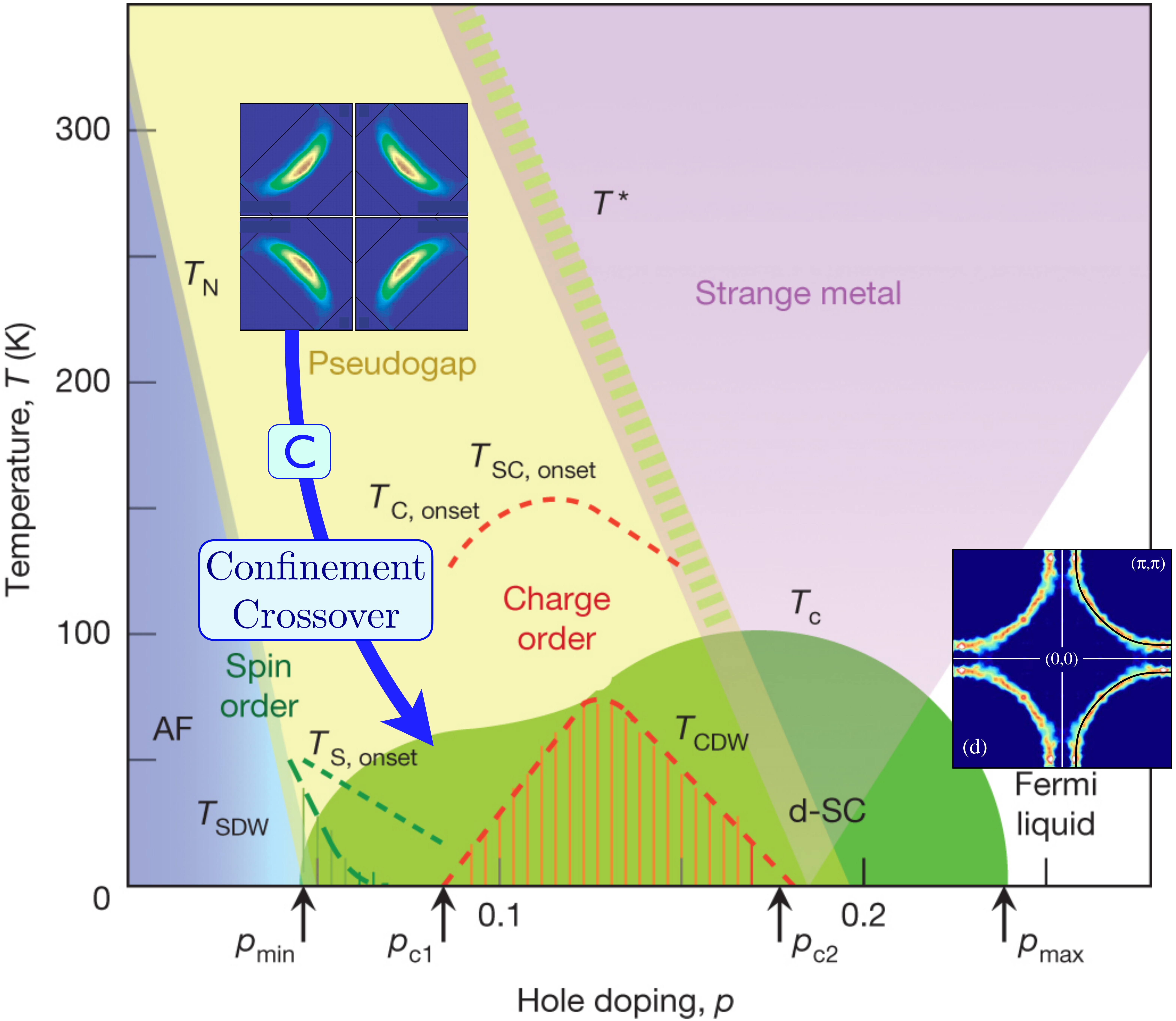
4 nodal points where

$$E_{\boldsymbol{k}_0+\boldsymbol{q}} = \left(v_F^2 q_\perp^2 + v_\Delta^2 q_{||}^2\right)^{1/2}$$

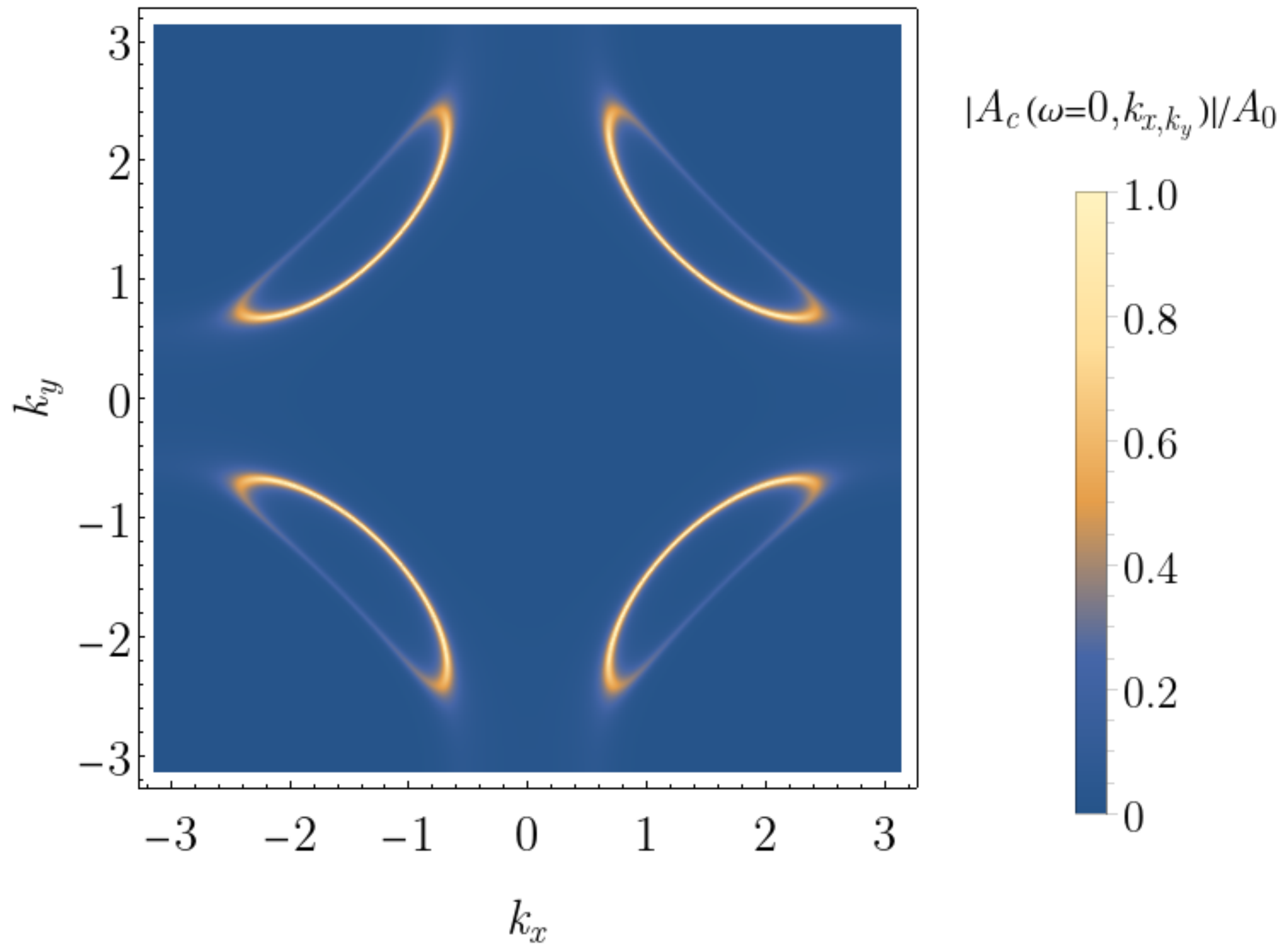
with $v_F \gg v_\Delta$.



Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

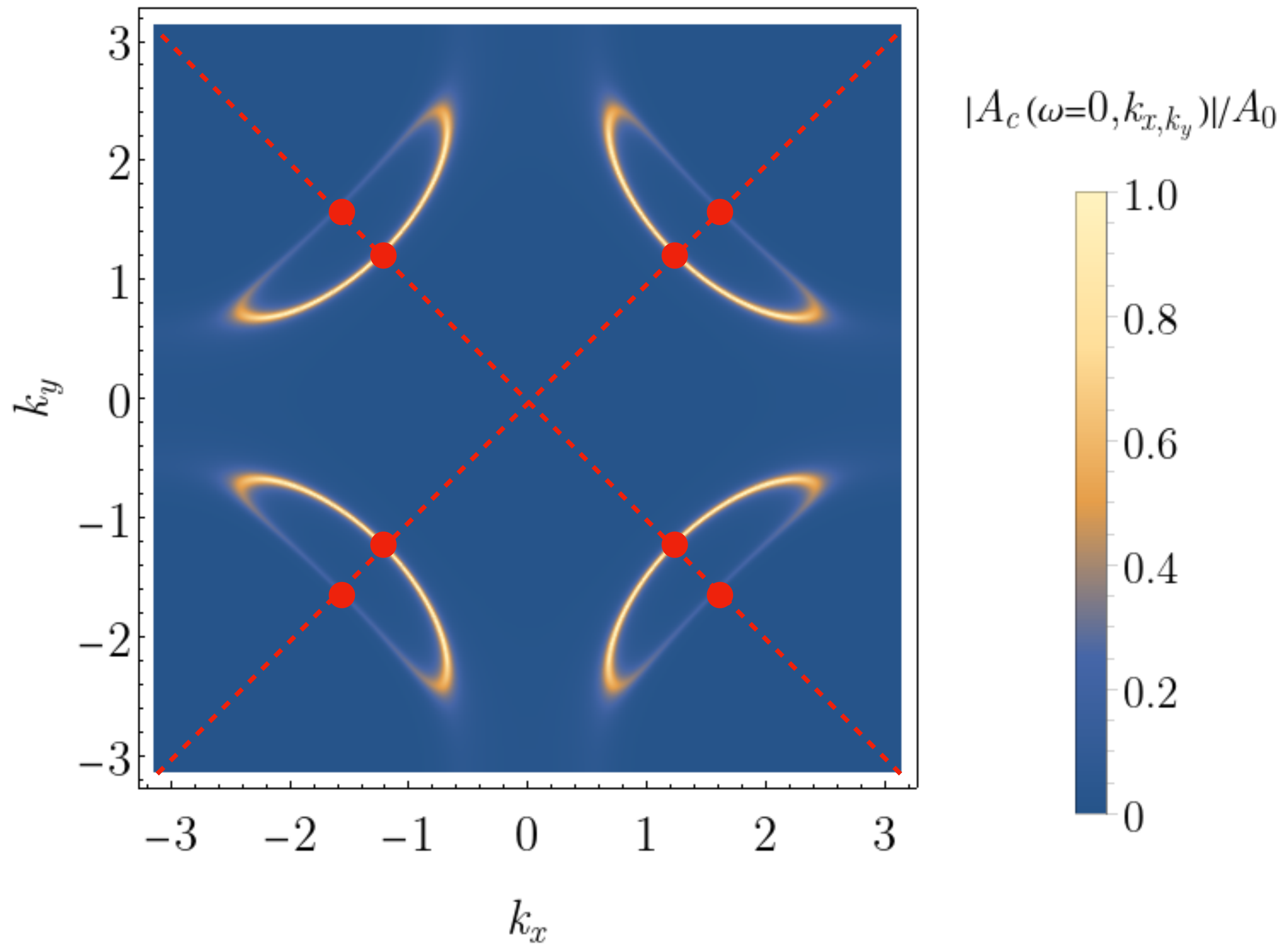


FL*



E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

FL*→dSC*

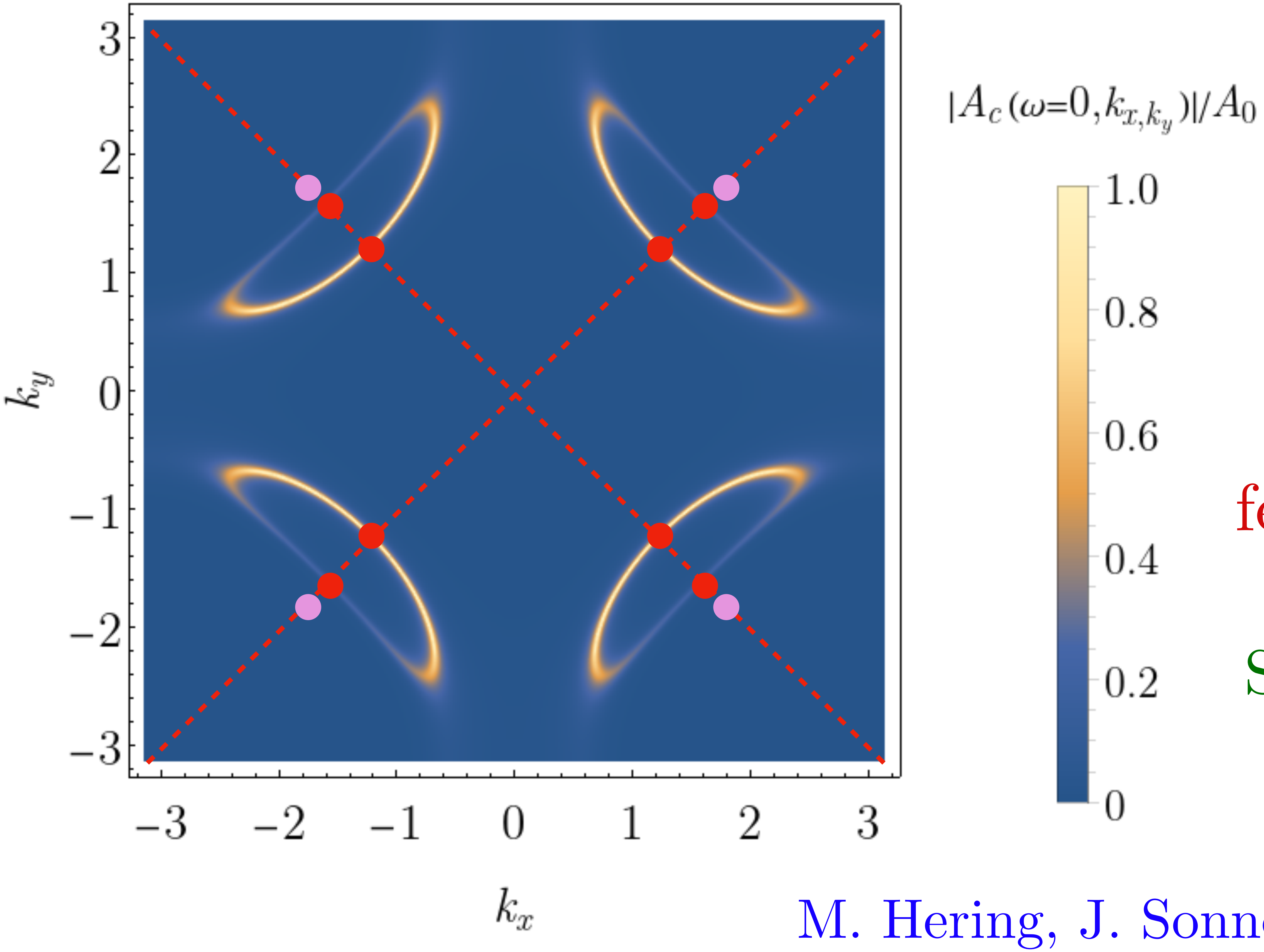


$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$

$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

Adding d-wave pairing
to the hole pockets
leads to 8 nodal points???

FL*→dSC*



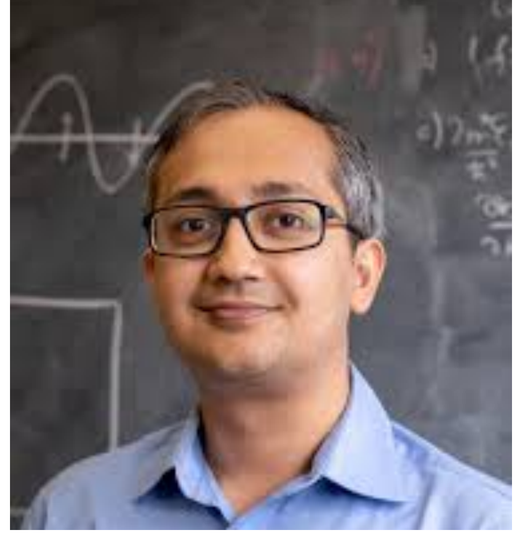
8 nodal points of
Bogoliubov quasiparticles
from the Fermi pockets
and
4 nodal points of
fermionic Dirac spinons from
the π-flux spin liquid

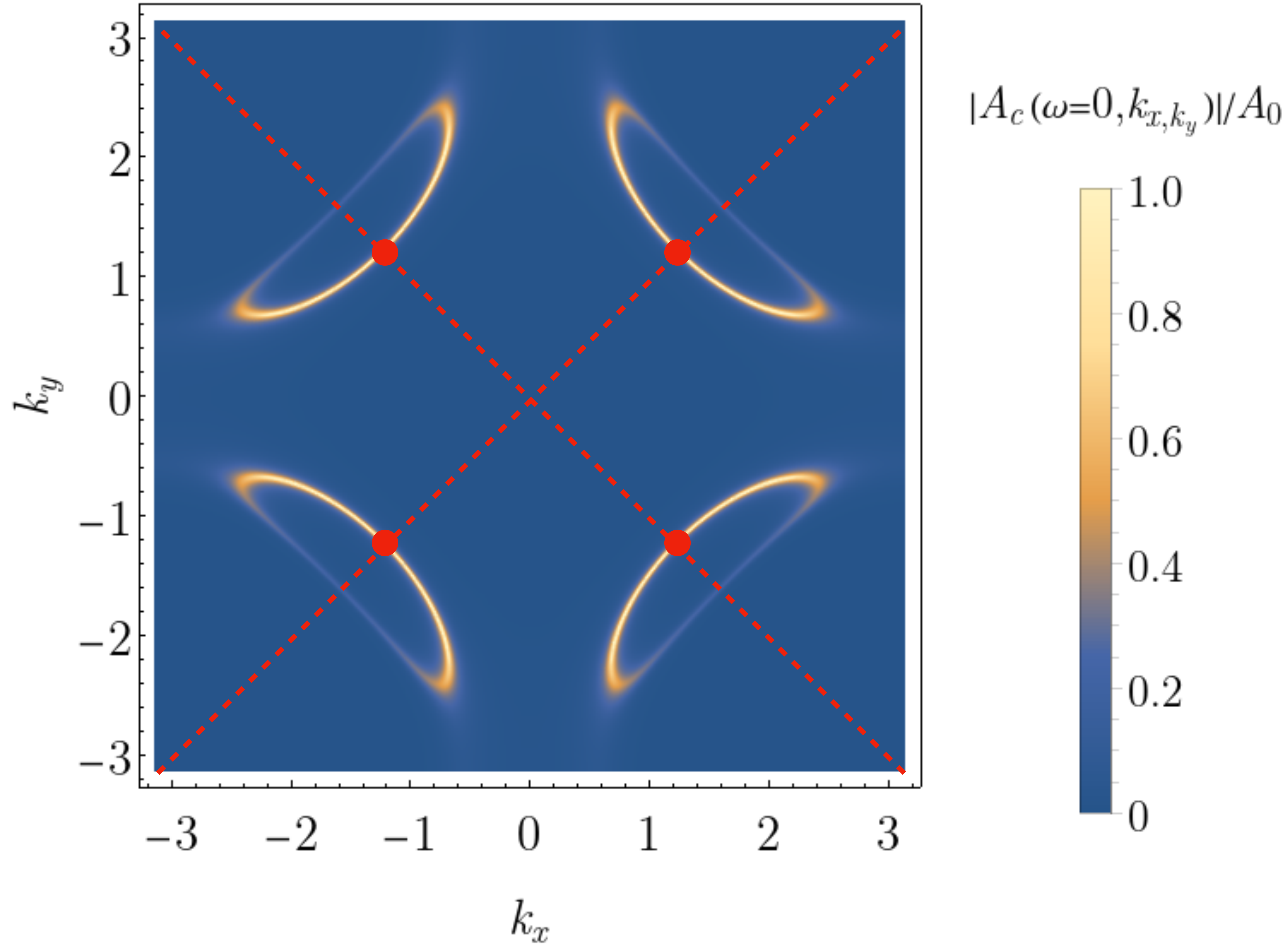
Such a spin liquid is required
to be present
in the background for FL*:

M. Hering, J. Sonnenschein, Y. Iqbal and J. Reuther,
PRB **99**, 100405 (2019)

FL*→dSC

Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev,
npj Quantum Materials **9**, 4 (2024)





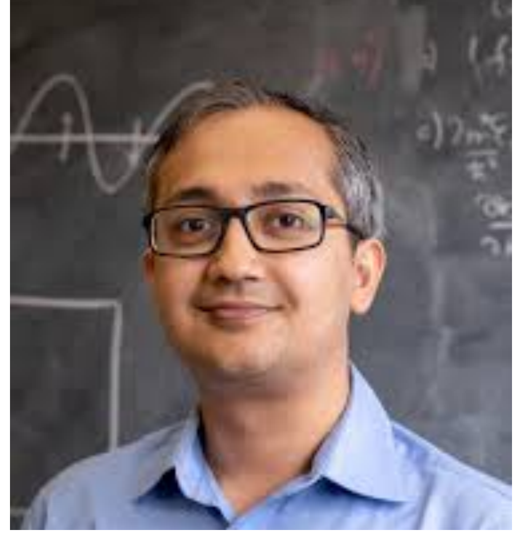
d-wave superconductor obtained by condensing charge-e, SU(2) fundamental boson B.

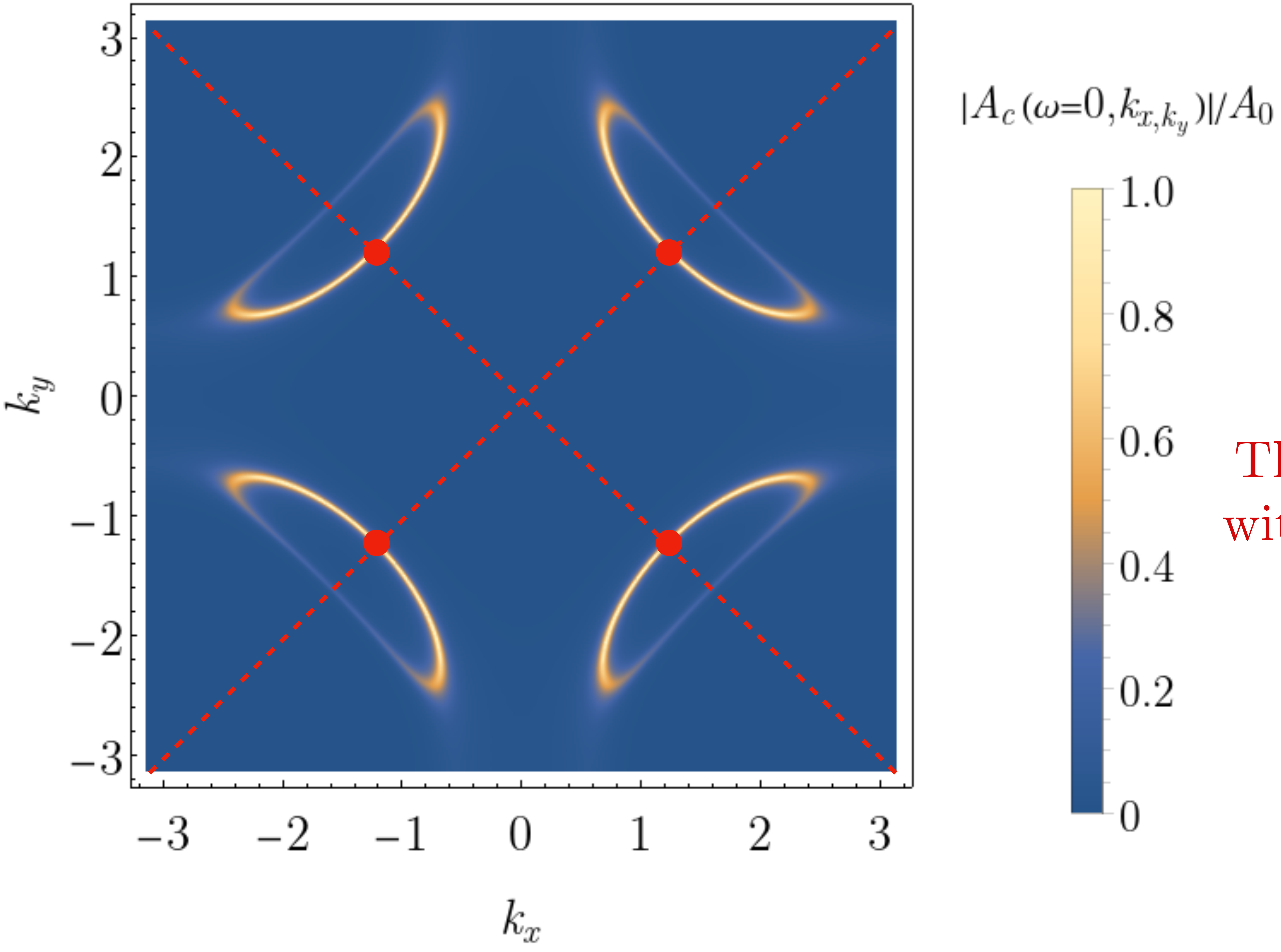
The B Higgs condensate allows spinons and Bogoliubov quasiparticles
to hybridize.

8 nodal points annihilate each other, leaving 4 nodal points with anisotropic velocities, just as in a BCS d-wave state.

FL*→dSC

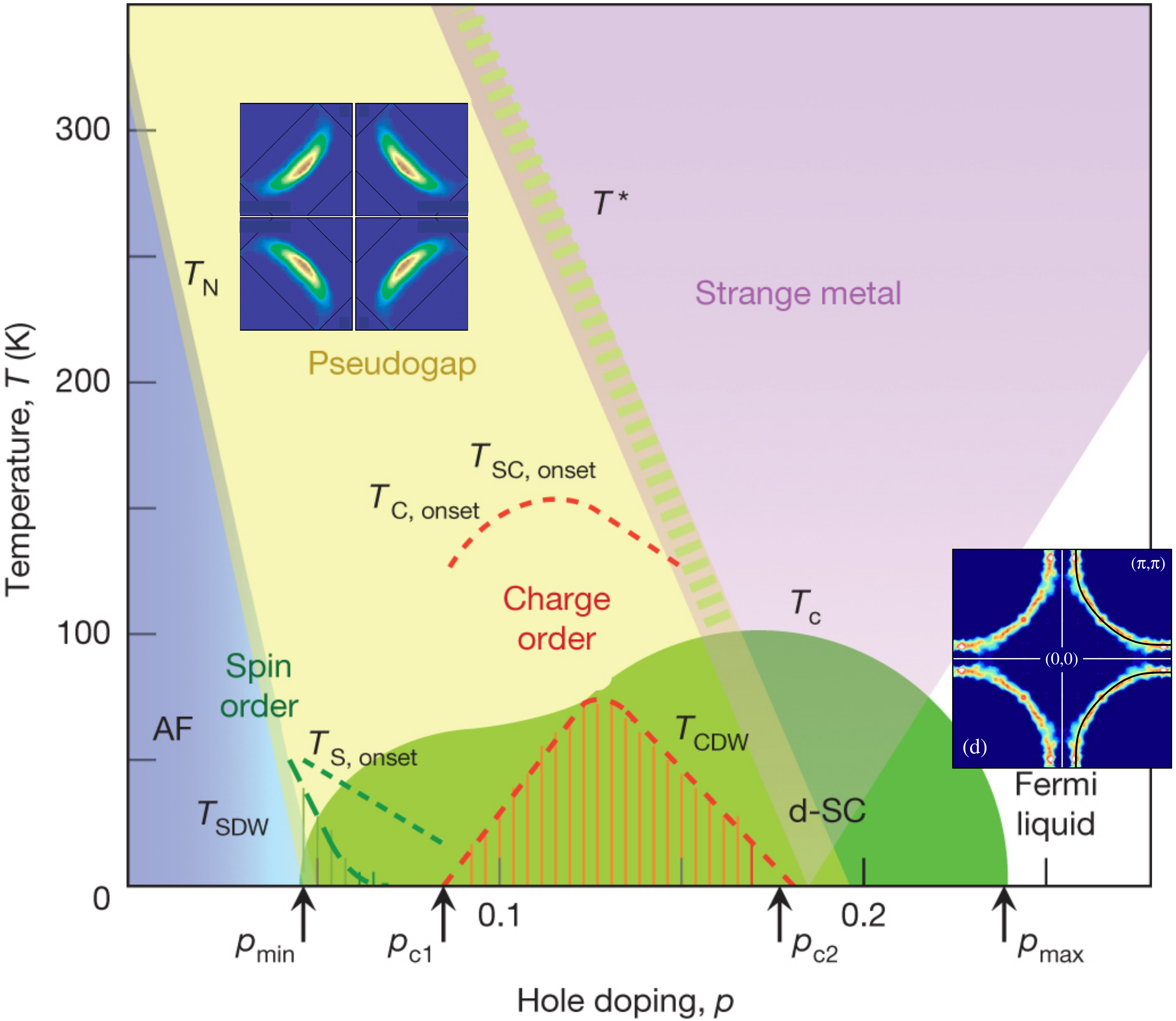
Shubhayu Chatterjee and S. Sachdev,
PRB **94**, 205117 (2016)
Maine Christos and S.Sachdev,
npj Quantum Materials **9**, 4 (2024)



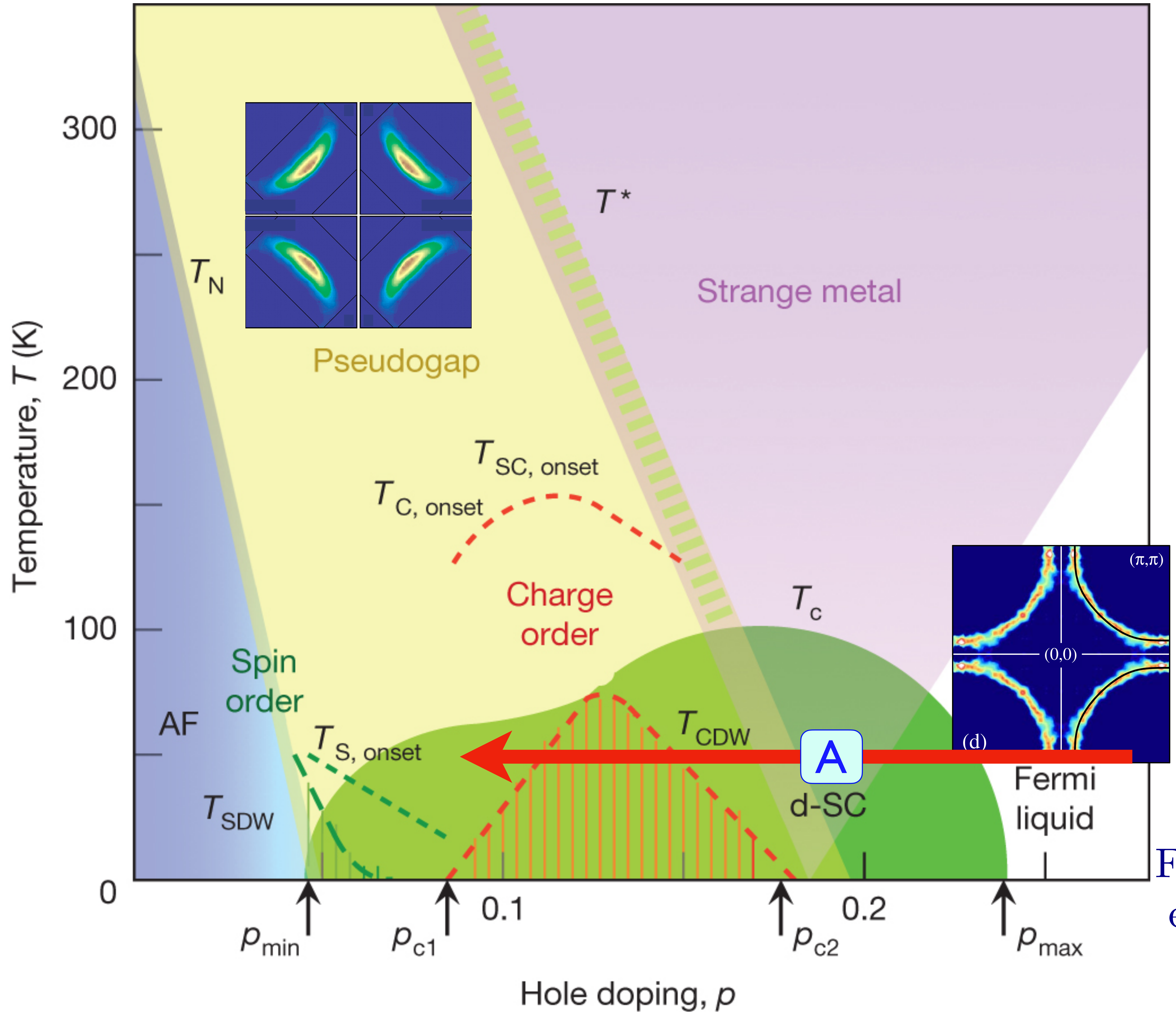


The spinons do *not* become the Bogoliubov quasiparticles, they *annihilate* the unwanted Bogoliubov quasiparticles. This leads to a d-wave superconductor with 4 nodal Bogoliubov quasiparticles, with $v_F \gg v_\Delta$, consistent with observations.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



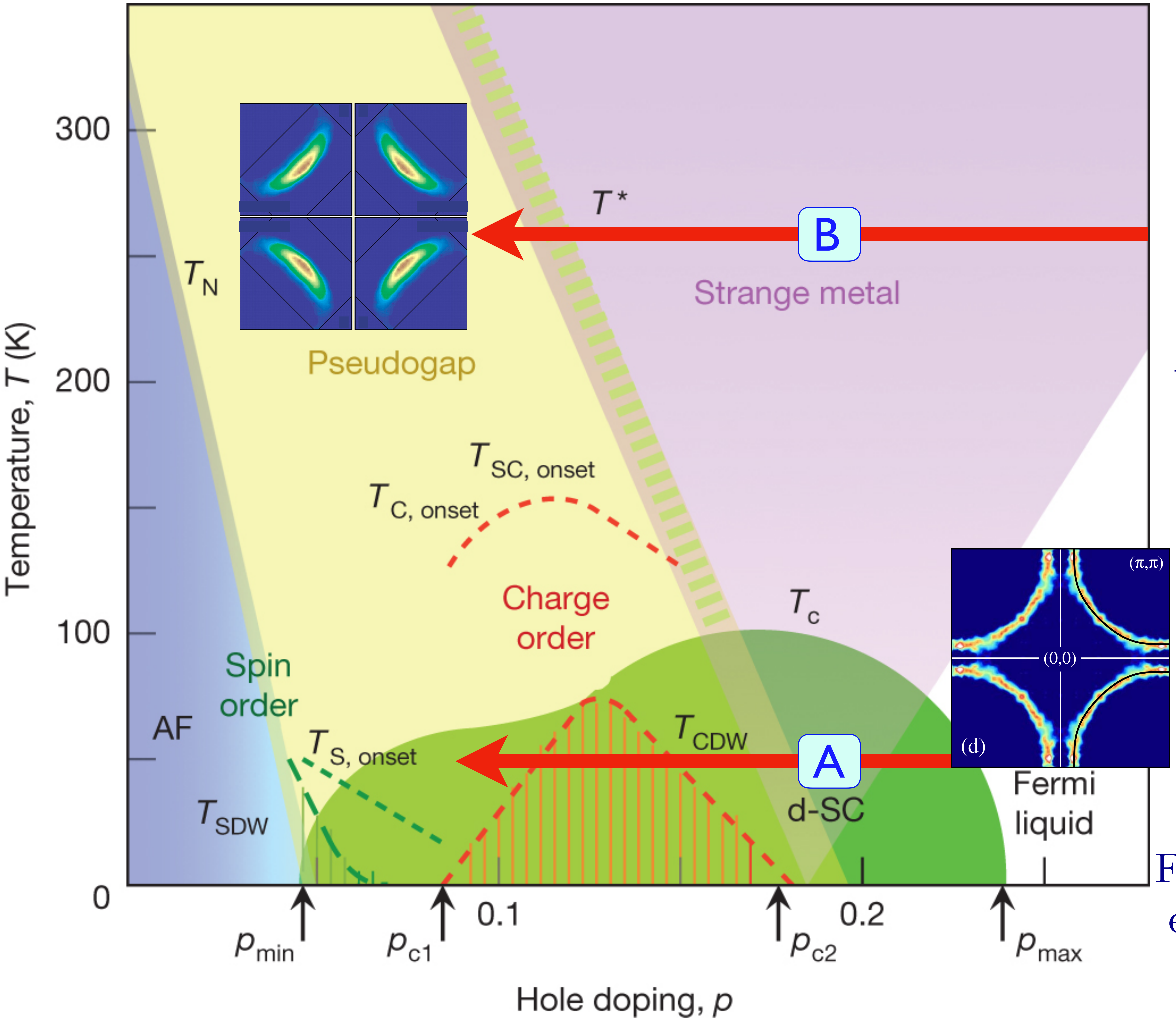
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Fermi-volume-changing QPT
with symmetry breaking
and with spatial disorder.

FL-SDW QPT with Harris disorder with extended fermions and localized bosons provides a theory of the "foot"

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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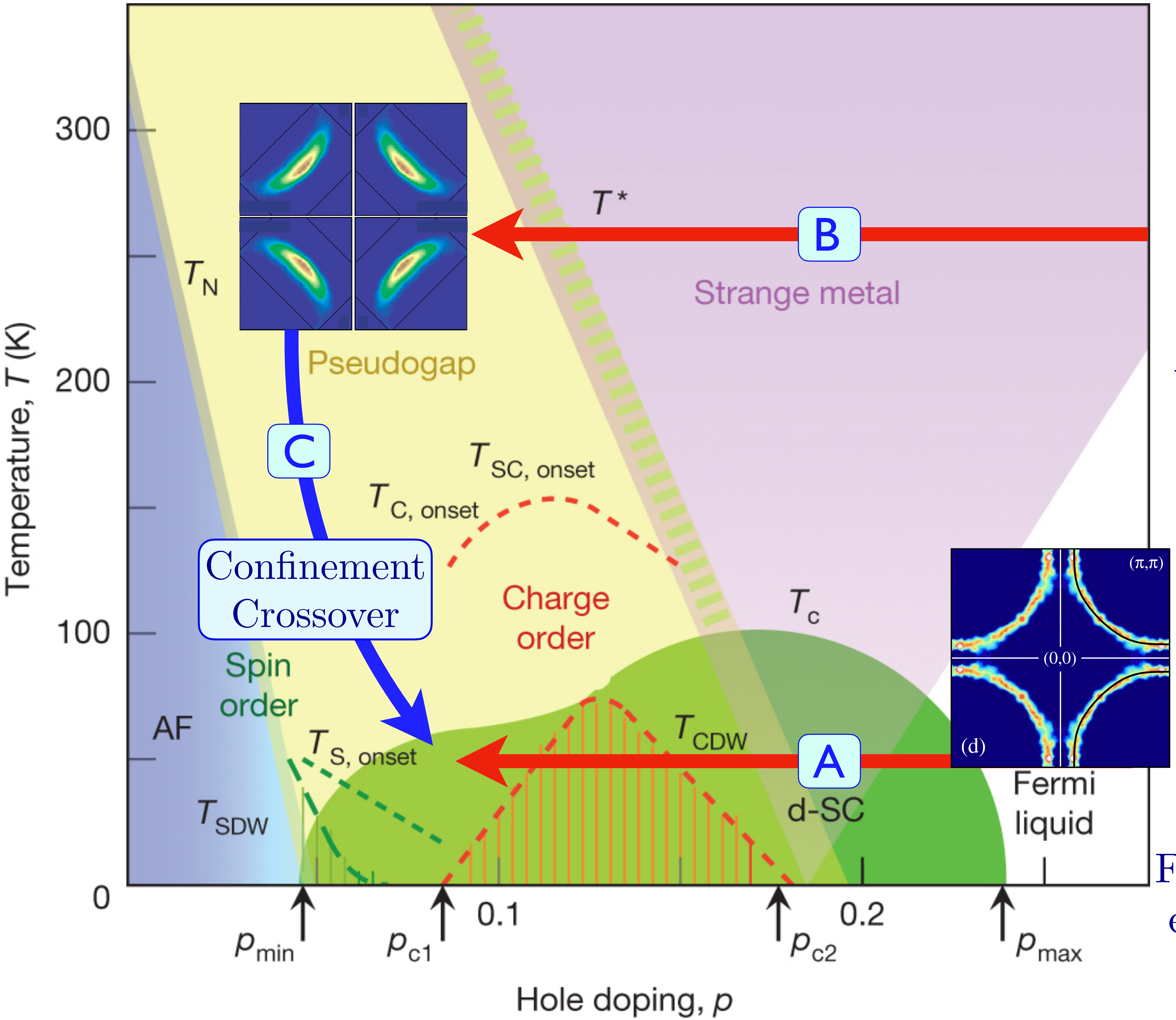

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