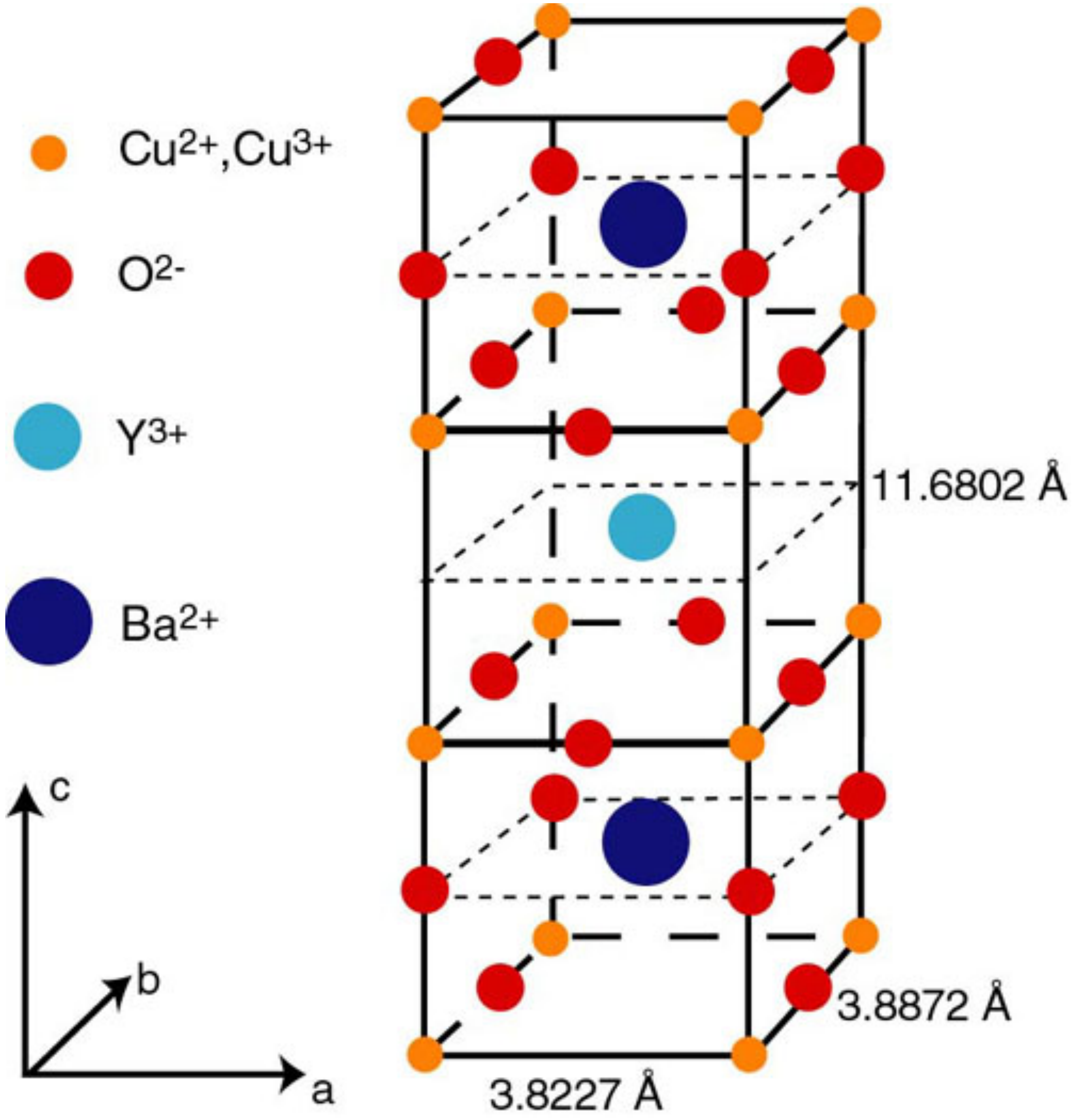
Entanglement in quantum matter: spin liquids and Sachdev-Ye-Kitaev models

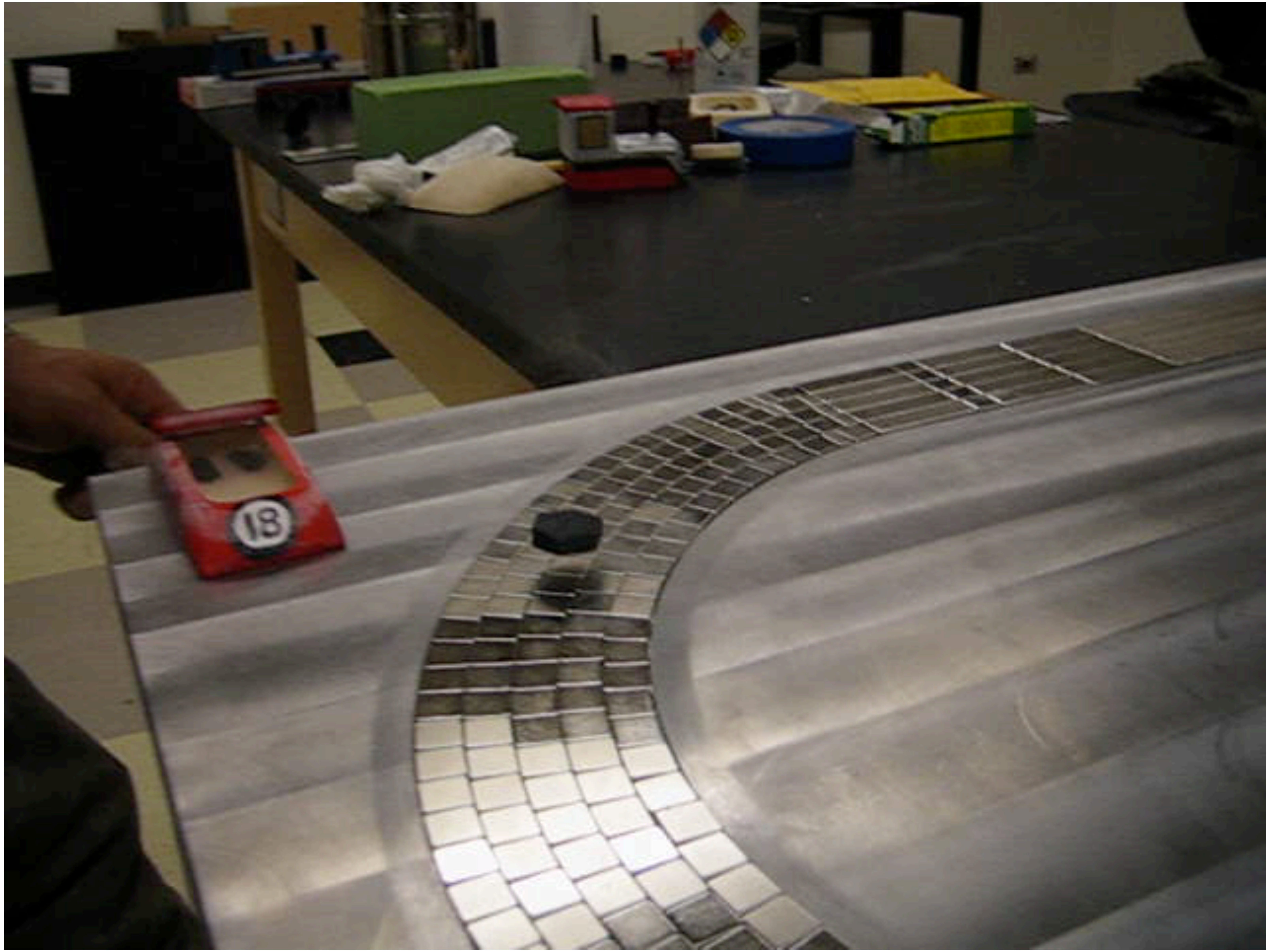
Florida State University, Tallahassee
January 30, 2025
Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

Cuprate high temperature superconductors



$$YBa_2Cu_3O_{6+x}$$



Nd-Fe-B magnets, YBaCuO superconductor

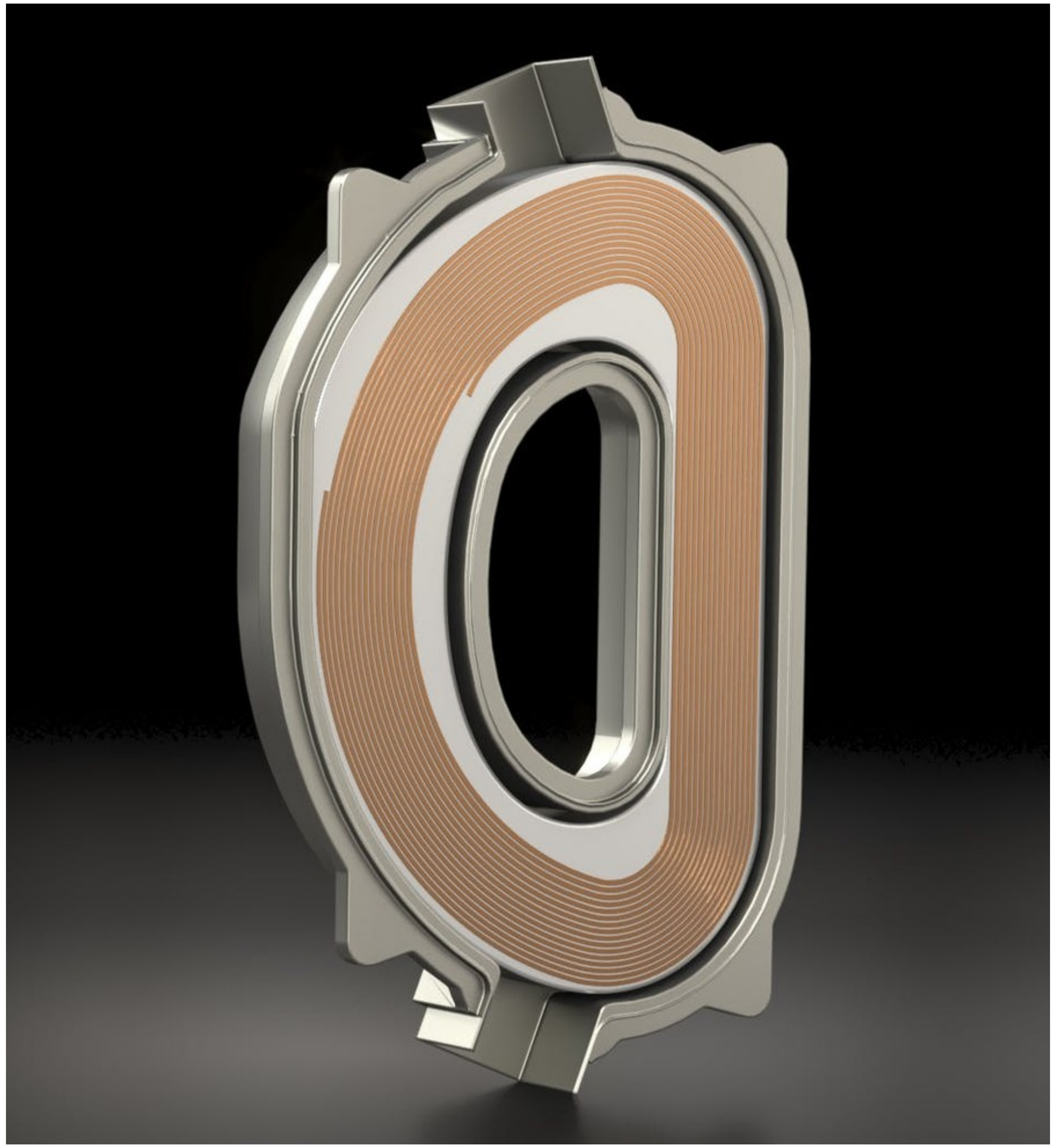
Julian Hetel and Nandini Trivedi, Ohio State University

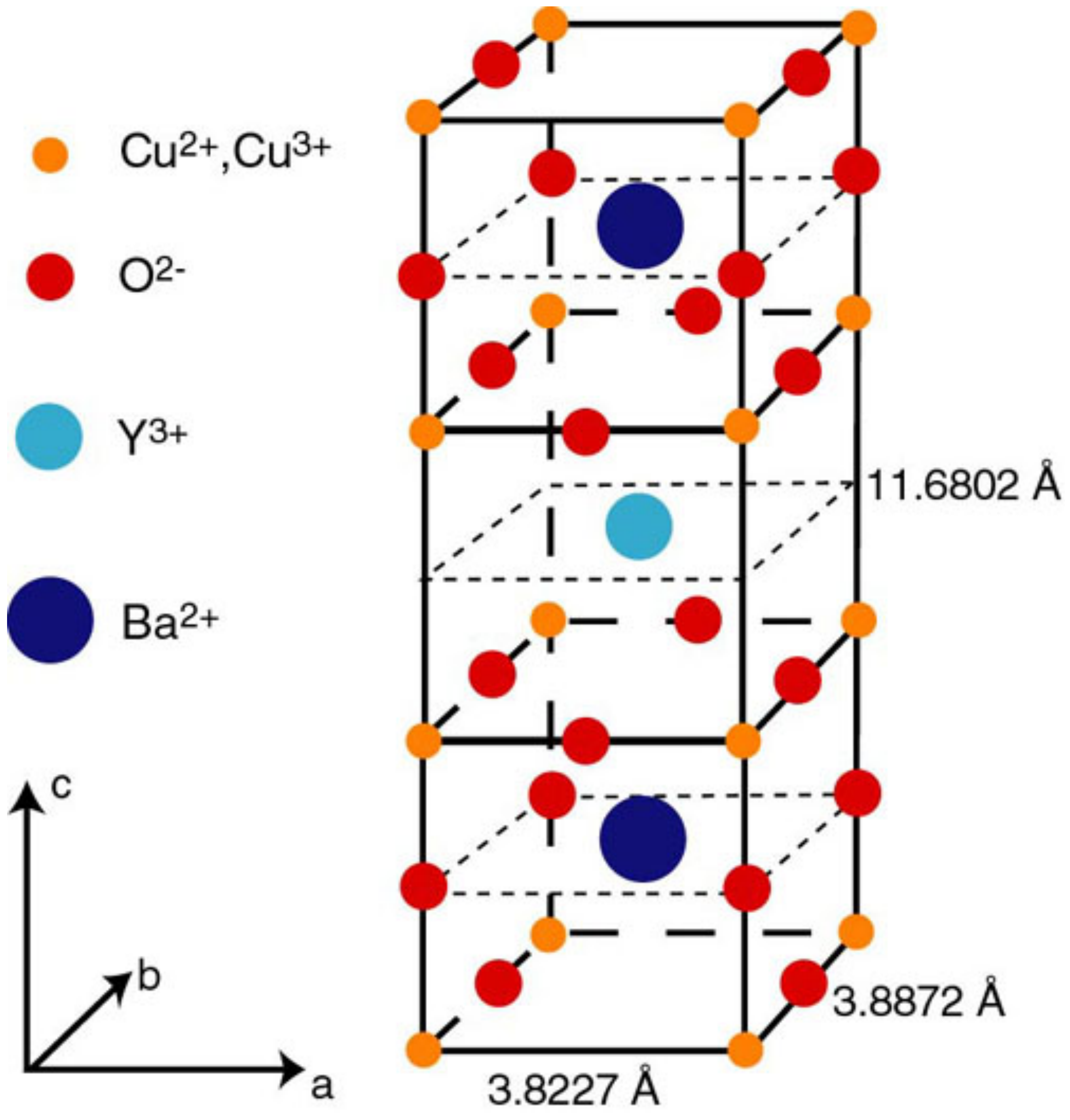
HTS Magnets: Enabling Technology

The surest path to limitless, clean, fusion energy

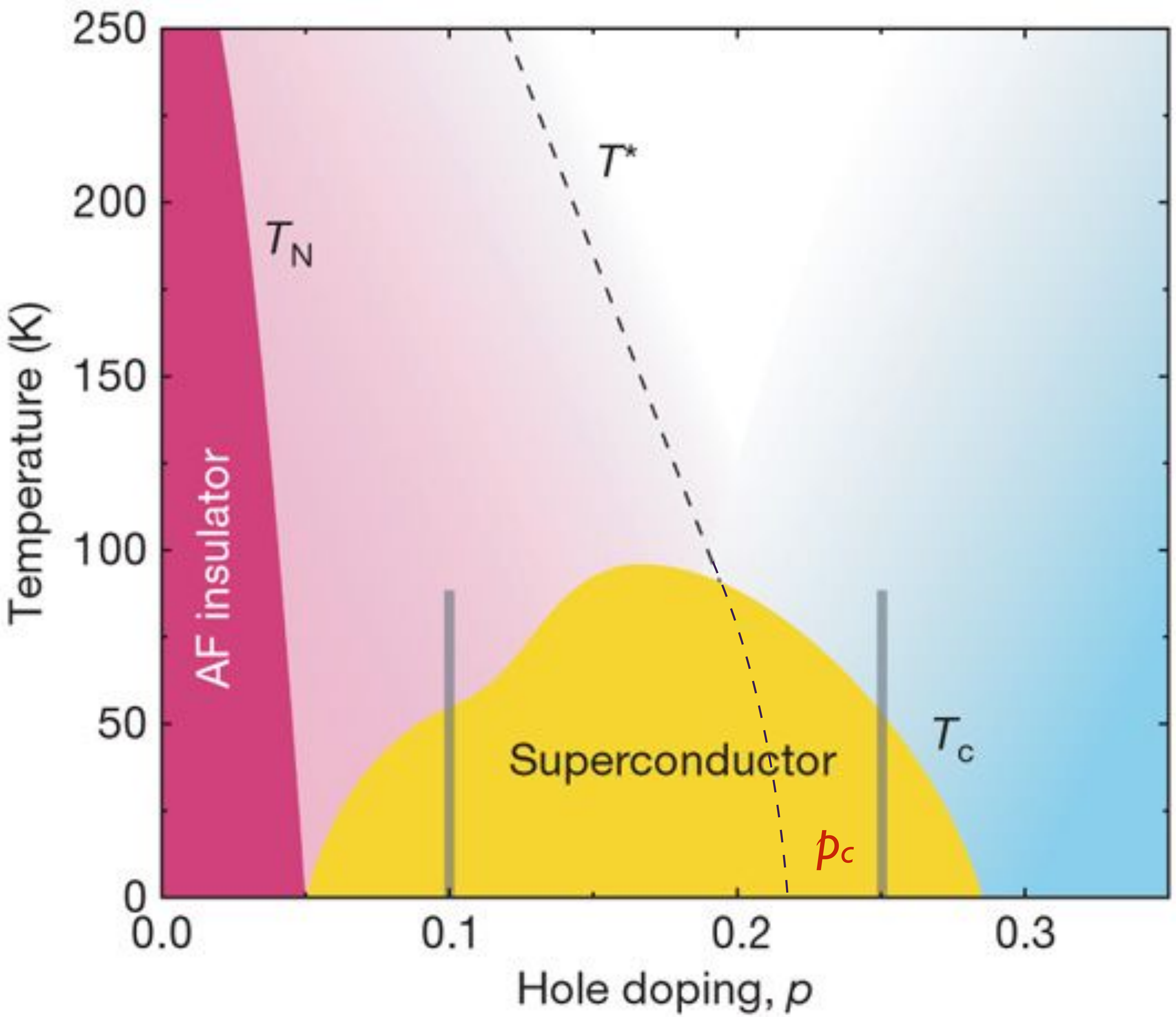
YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

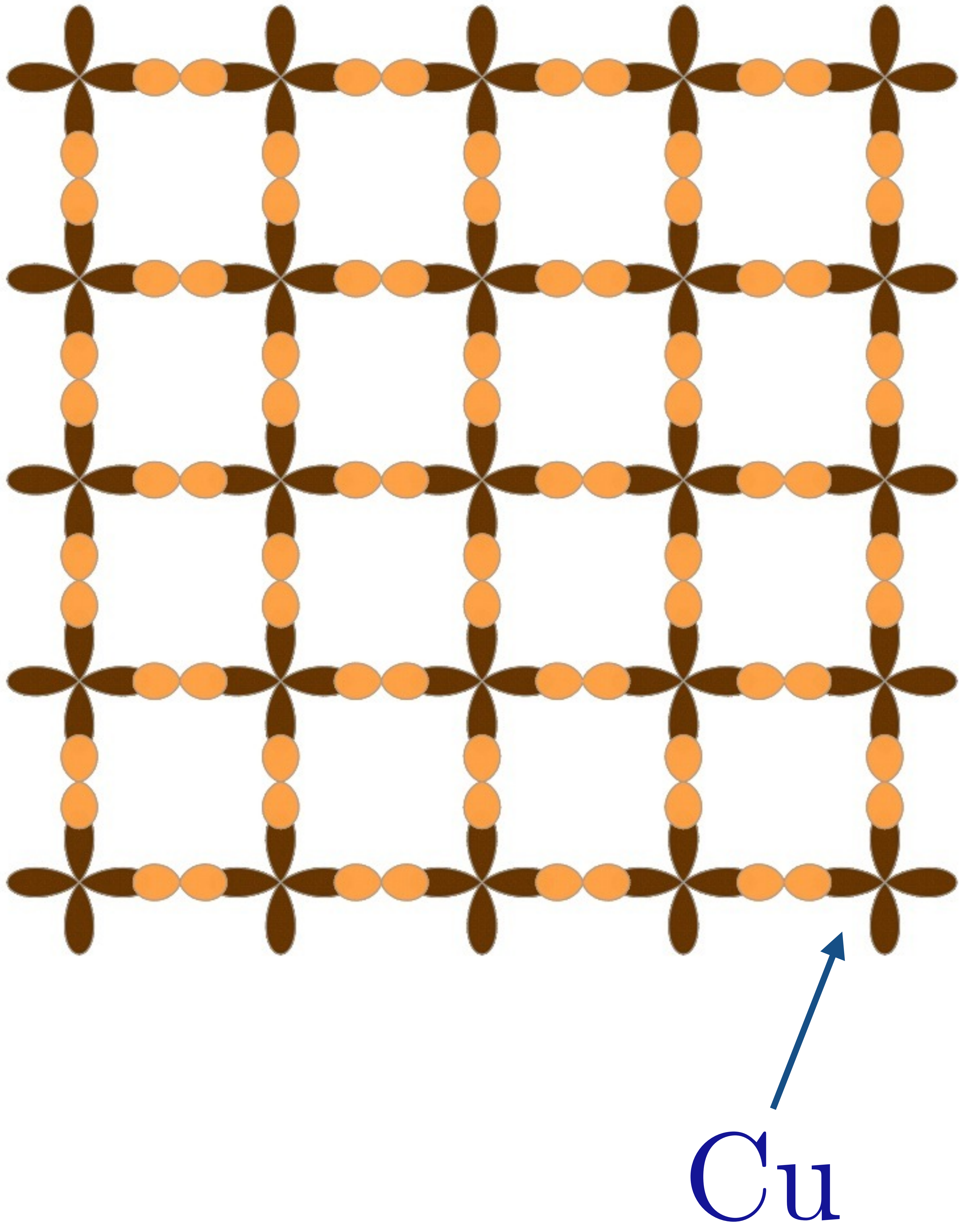
Commonwealth Fusion Systems

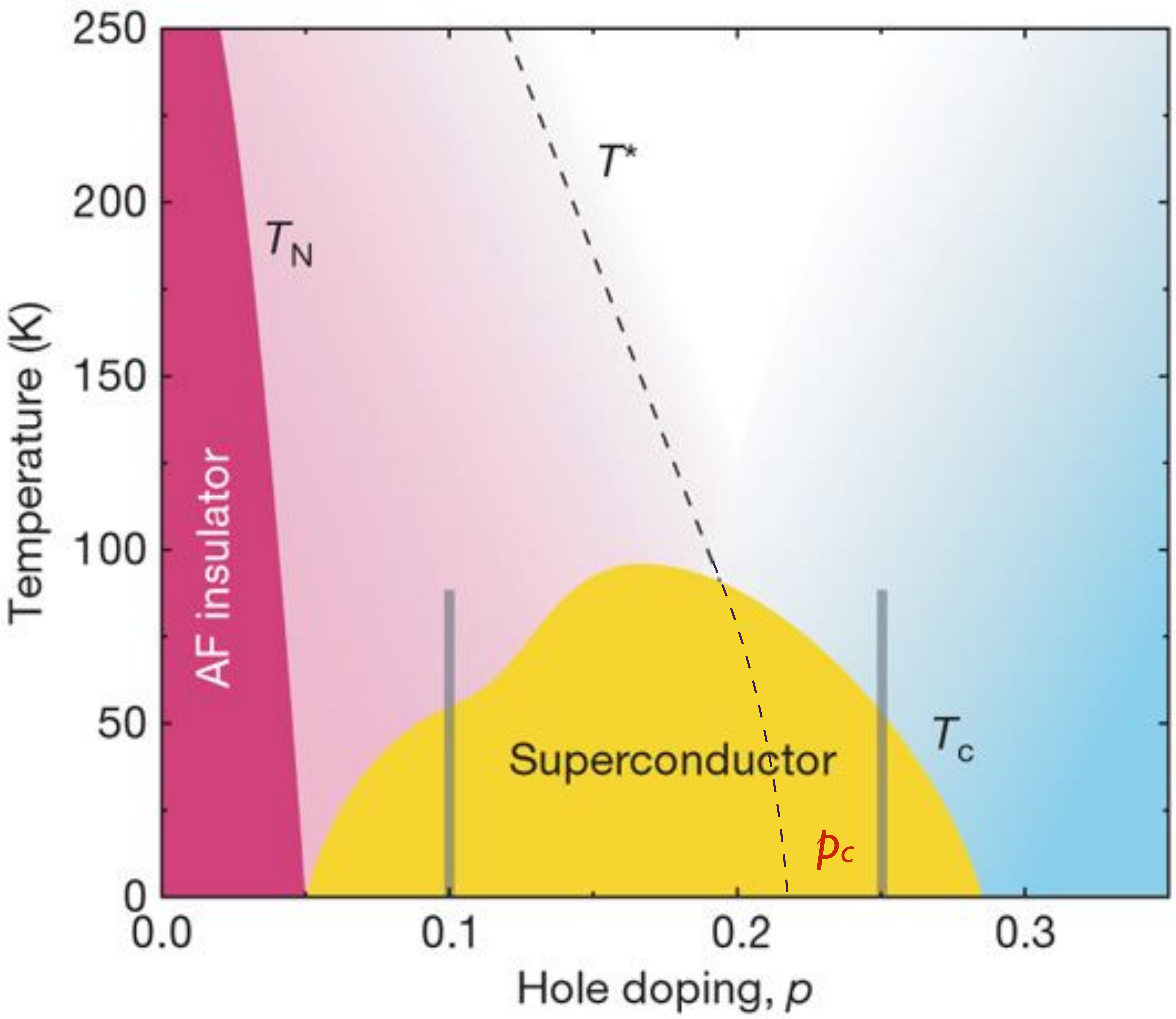



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c


Cu


250
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0.0
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AF insulator
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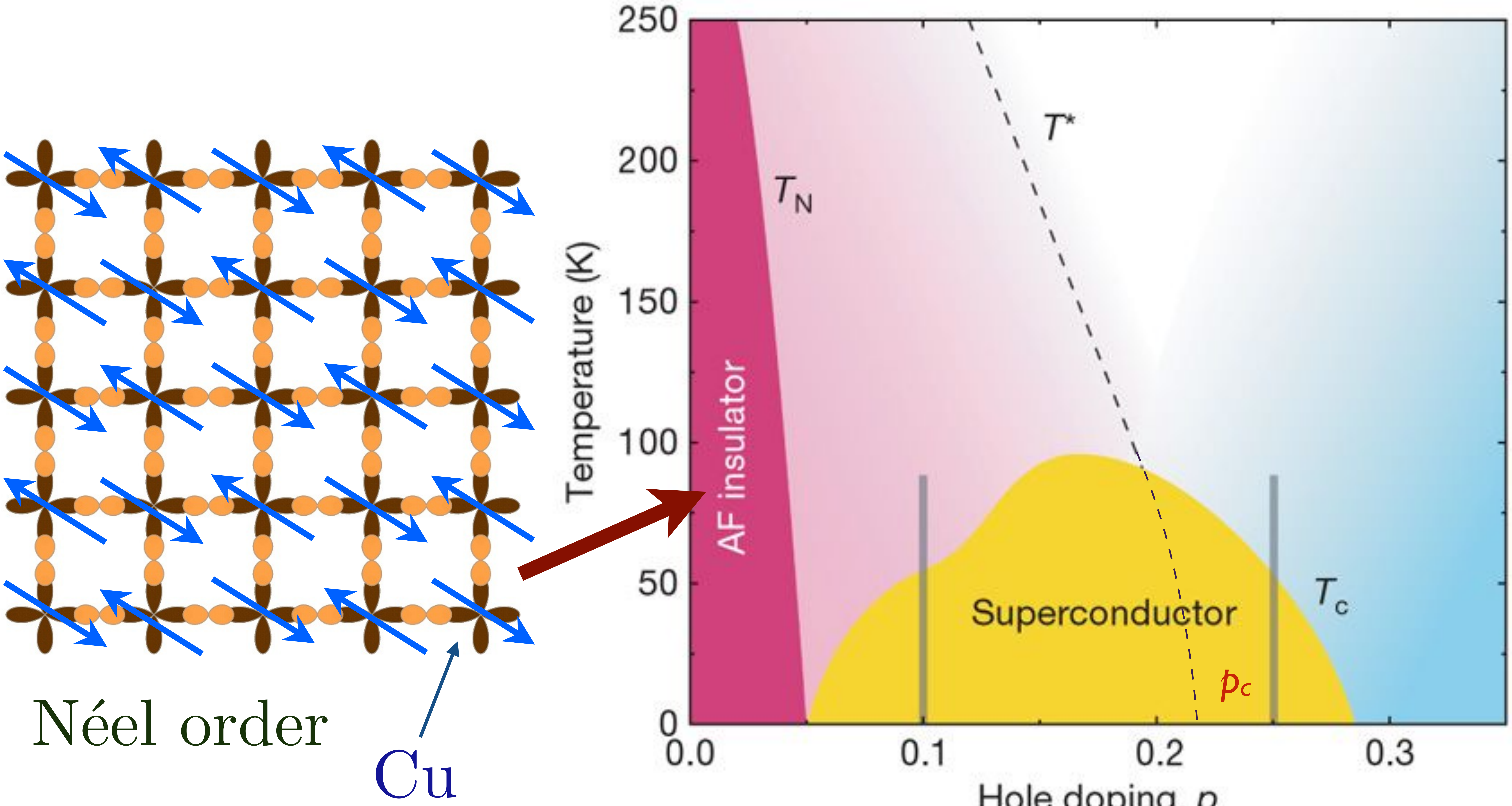
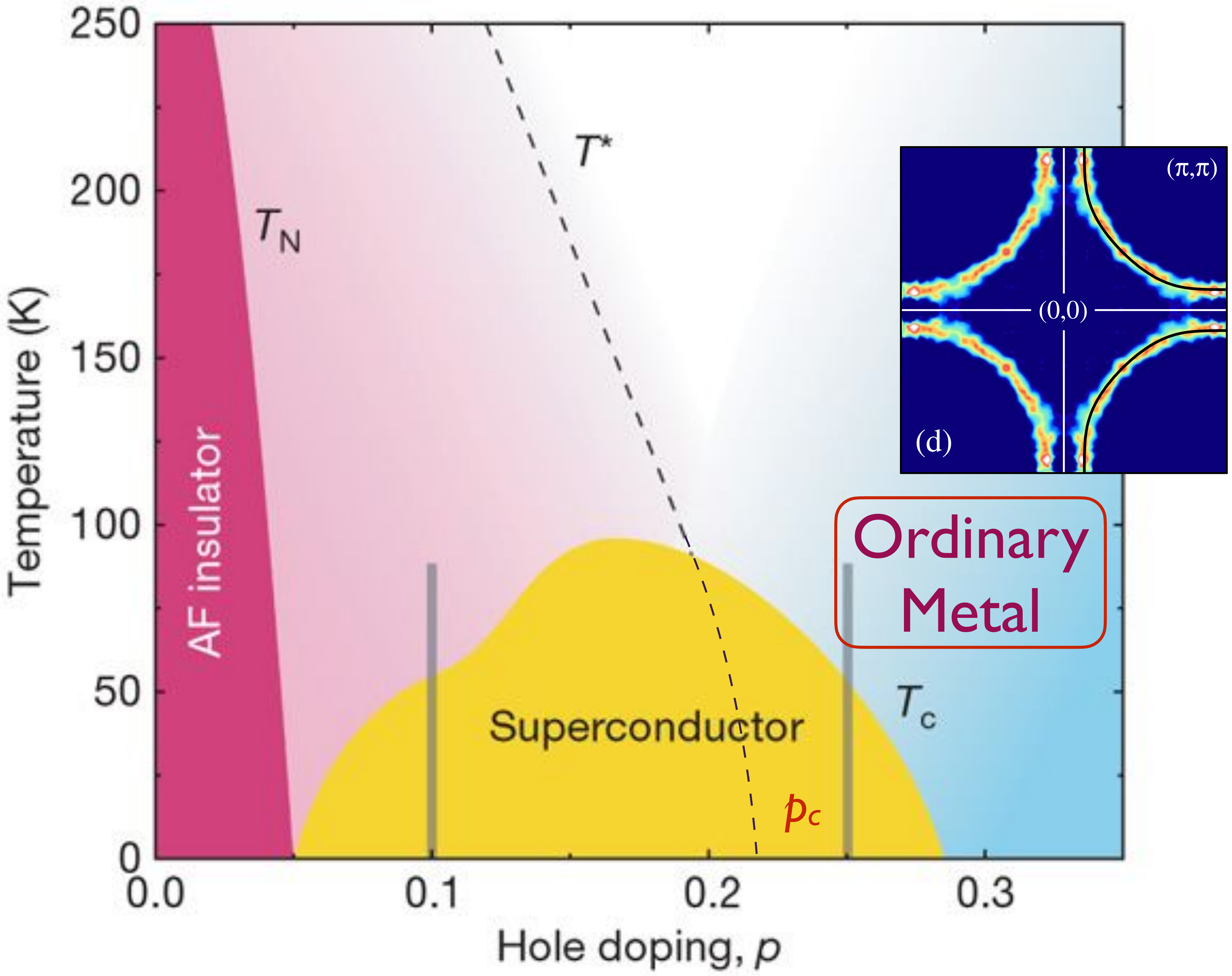

Néel order
Cu
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Superconductor
T_c
p_c

Ordinary metal:

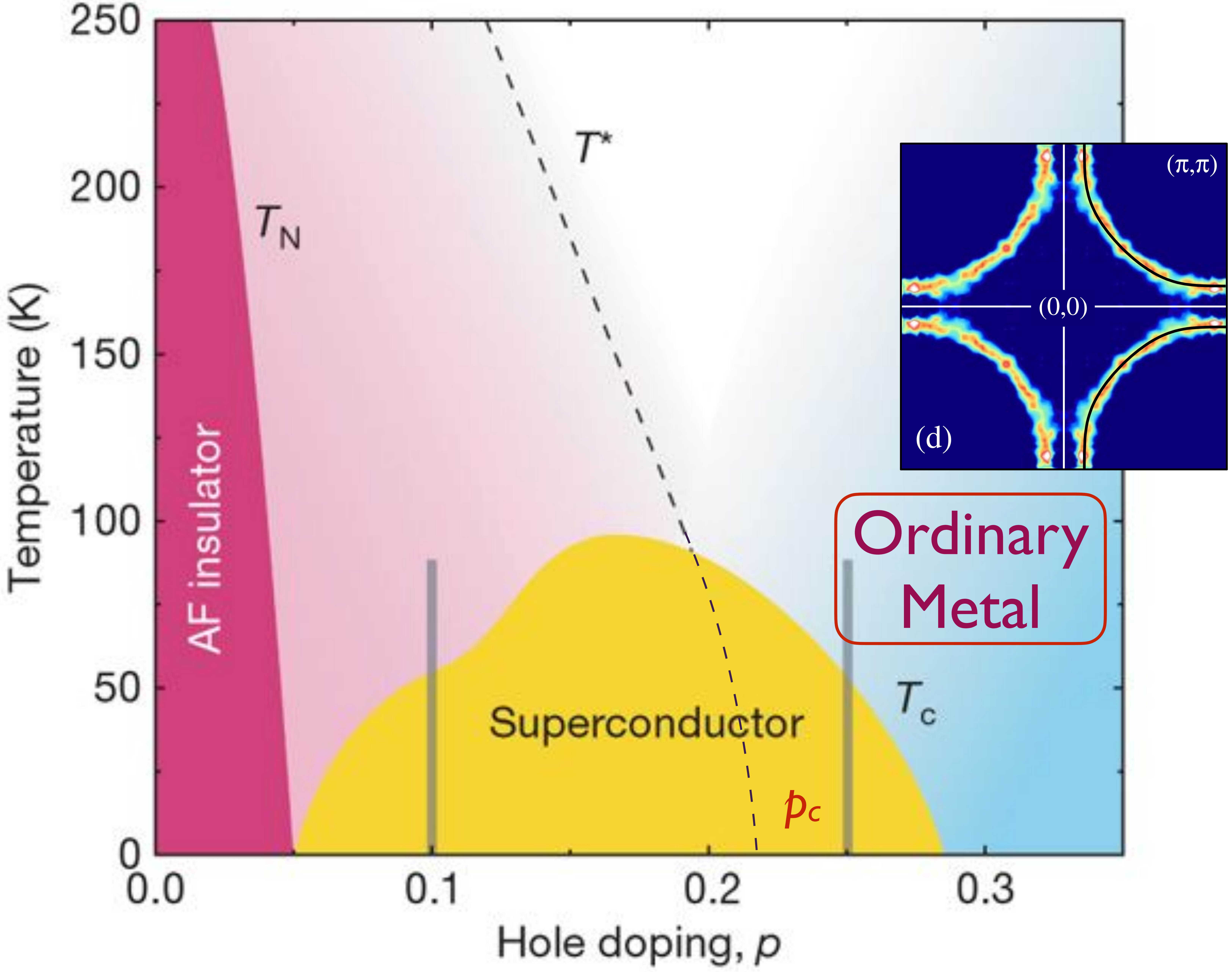
Nearly-free
gas of fermions, with a
Fermi surface between
empty and full states
(Sommerfeld, 1927).



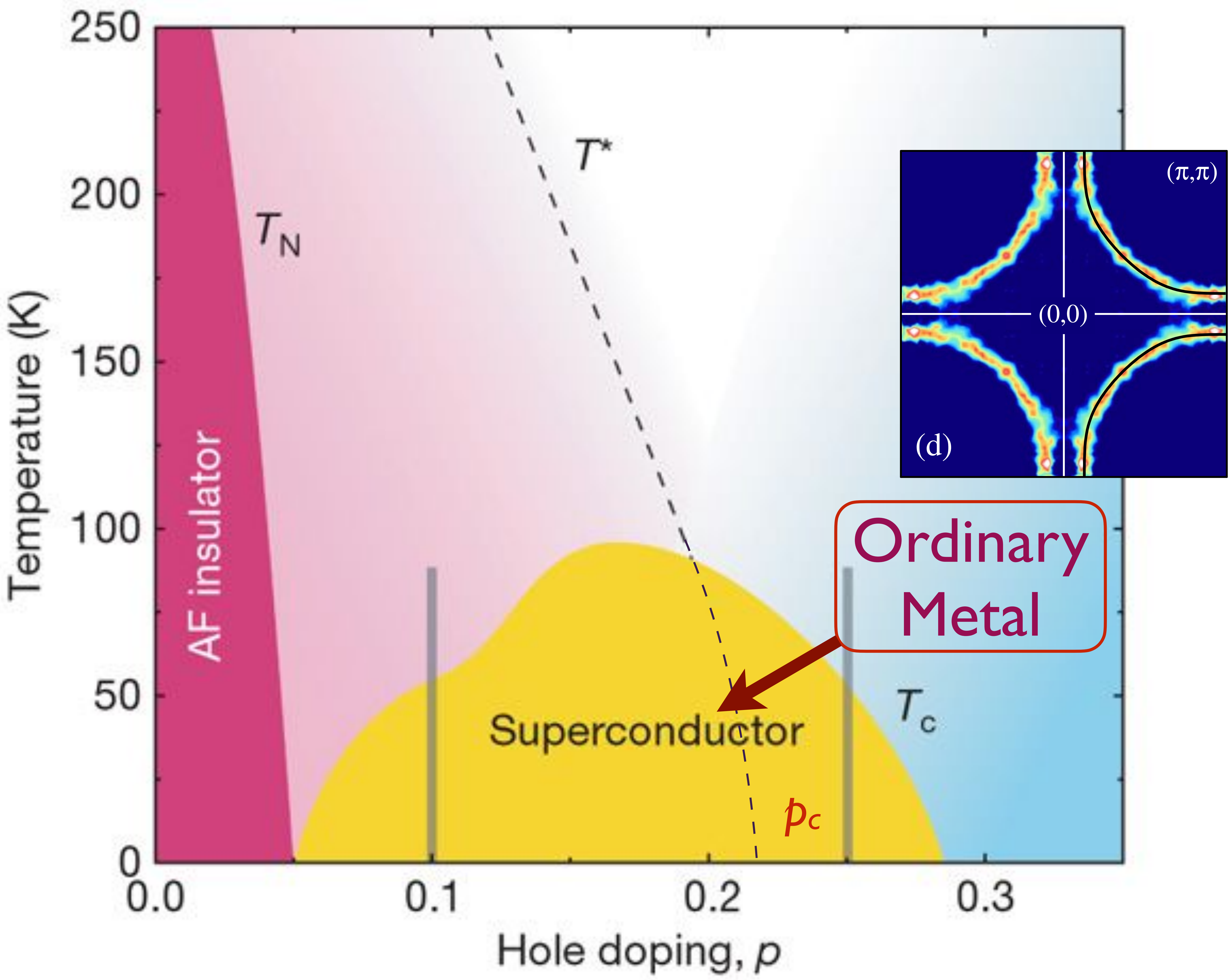
Ordinary metal:

Nearly-free
gas of fermions, with a
Fermi surface between
empty and full states
(Sommerfeld, 1927).

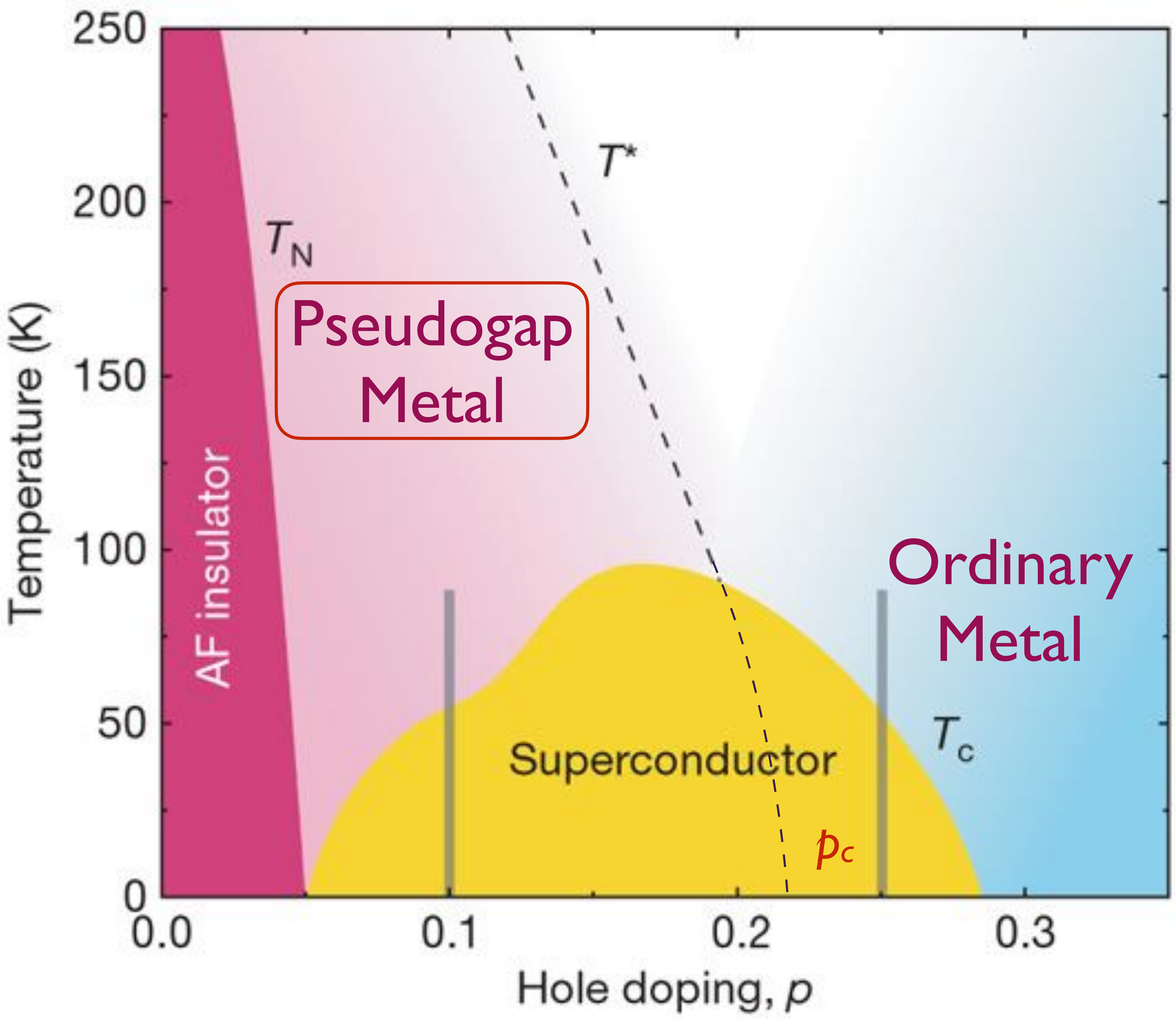
Area enclosed by the
Fermi surface is the
same as that for free
fermions
(Luttinger 1960).



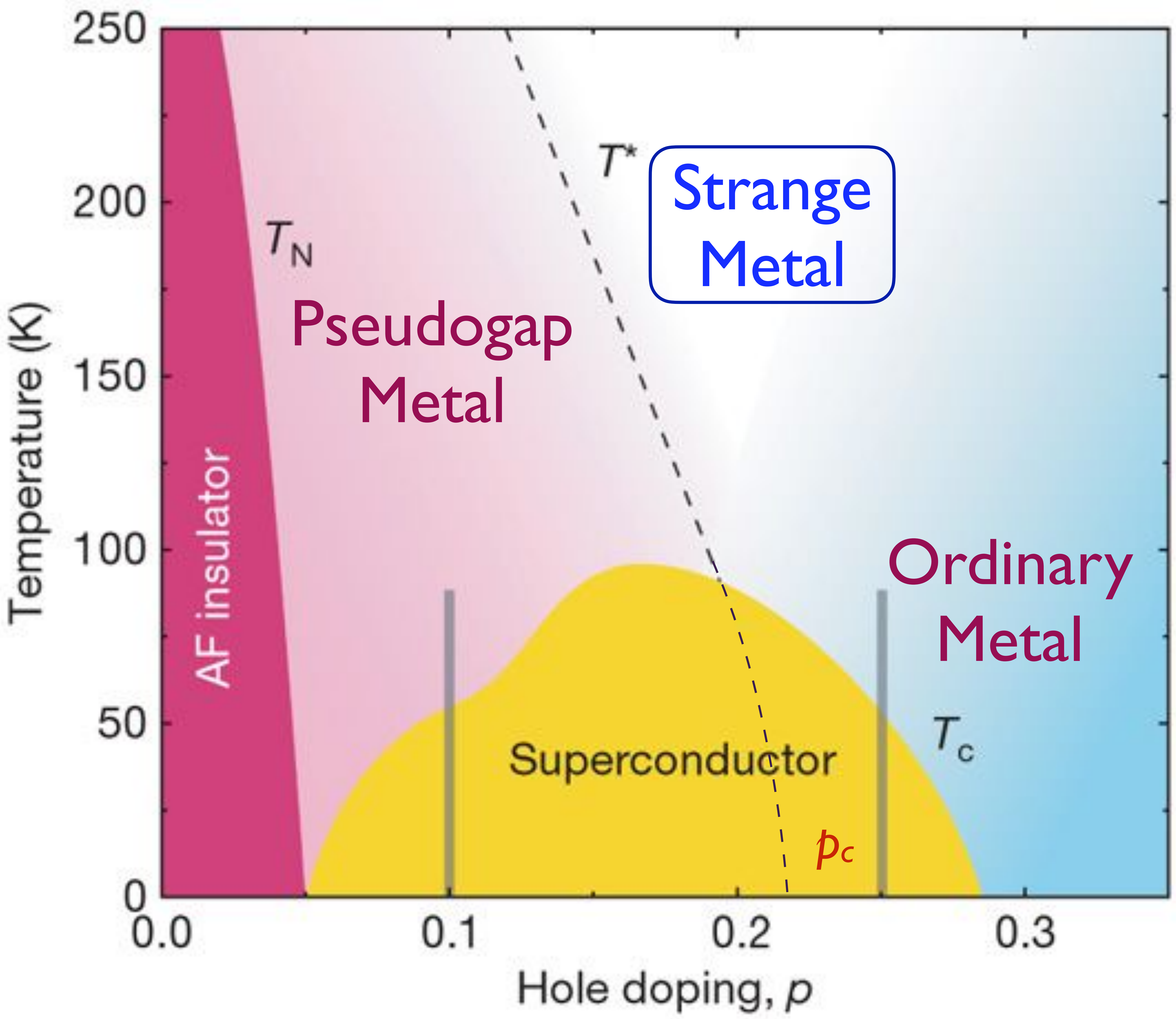
<u>Superconductor:</u>
Bose-Einstein condensation of fermion pairs (Bardeen, Cooper, Schrieffer 1957)



<u>Pseudogap metal:</u>
many-particle entanglement similar to that in a spin liquid



Strange metal: many-particle entanglement similar to that in Sachdev-Ye-Kitaev models

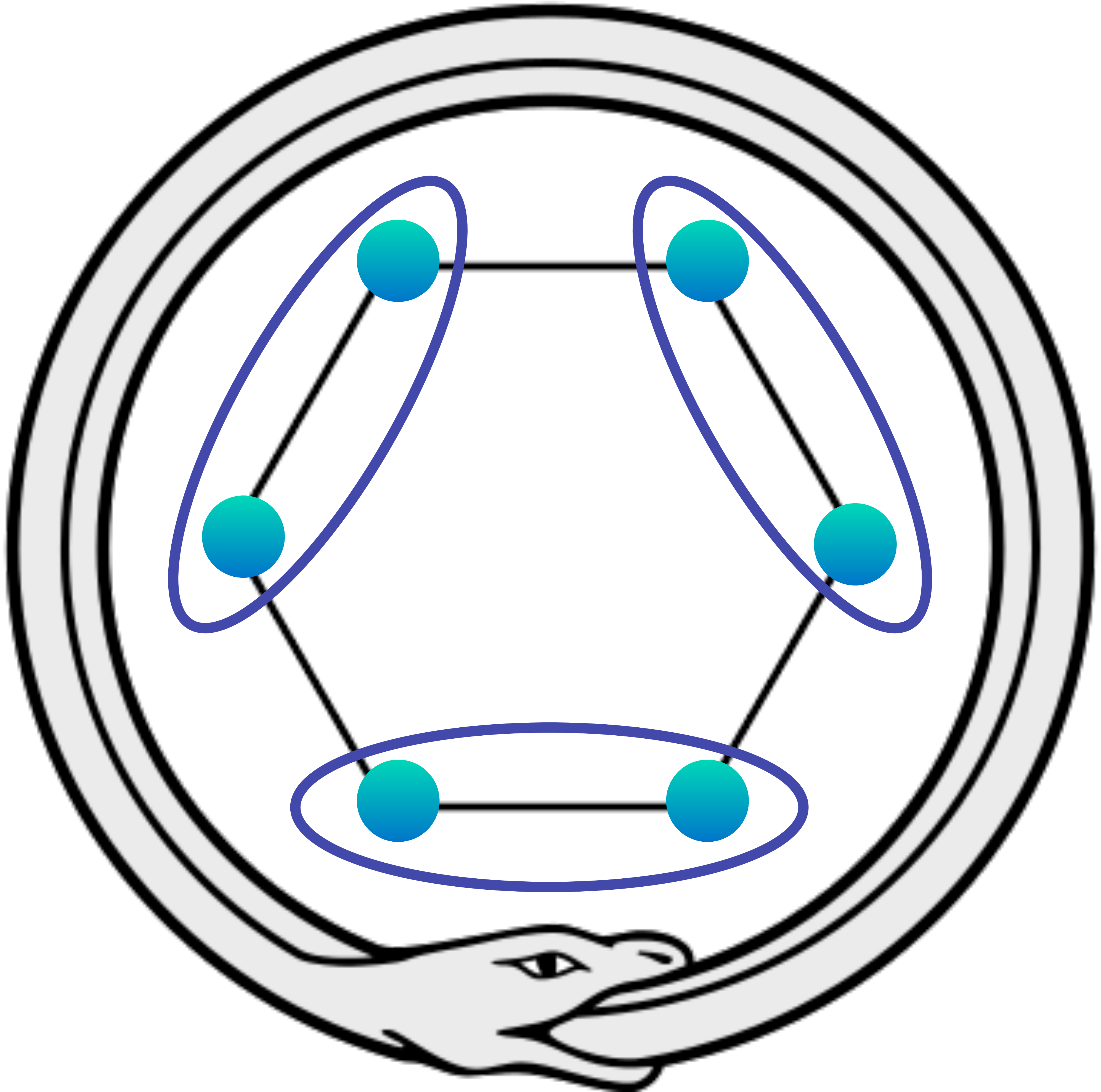


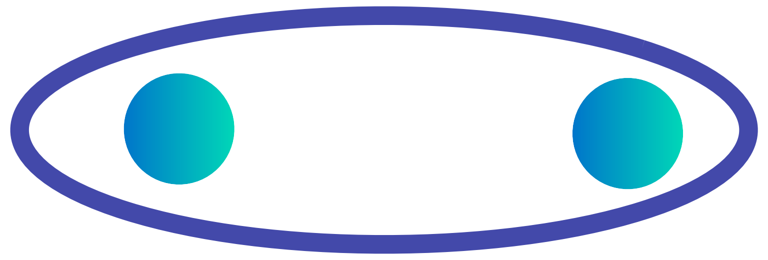
Entanglement of stationary electrons.

The simplest spin liquid: Z_2

Kekule's spooky dream (1865)

Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail*





$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Benzene

*Wikipedia

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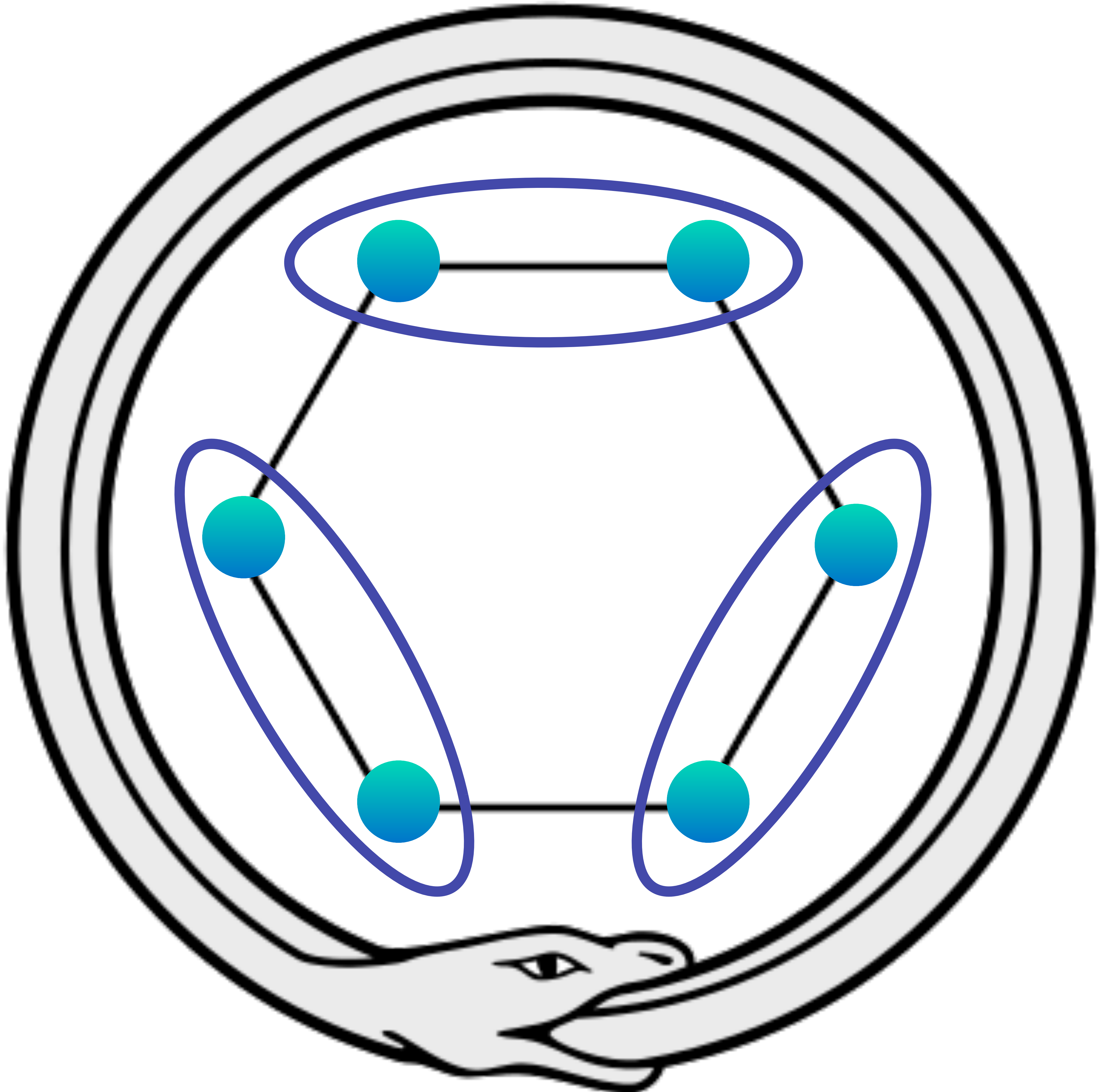
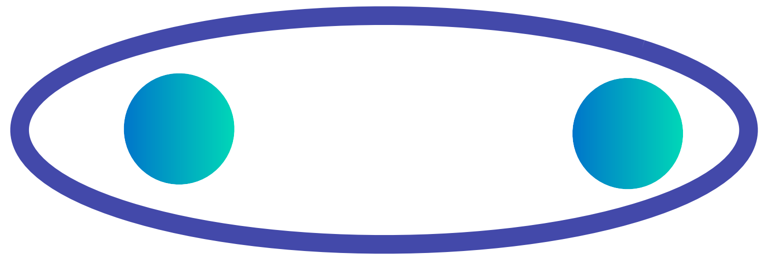


 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

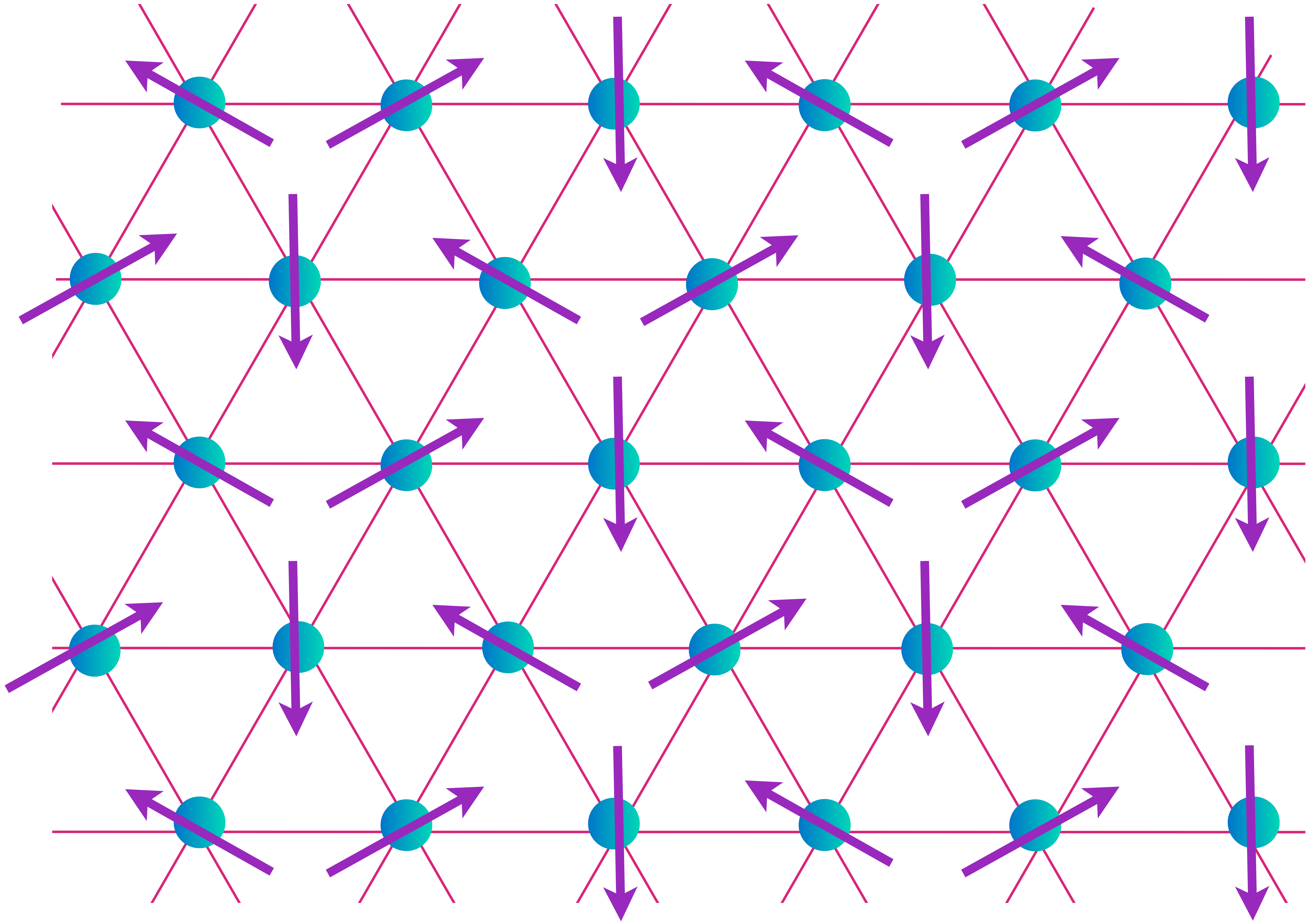
Benzene

*Wikipedia

Triangular lattice antiferromagnet

Spin model with S=1/2 per unit cell

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



$$[S_\alpha, S_\beta] = i\epsilon_{\alpha\beta\gamma} S_y$$

$$S_\alpha^2 = S(S+1);$$

$$S = 1/2$$

$$S_z \left|\uparrow\right\rangle = (1/2) \left|\uparrow\right\rangle$$

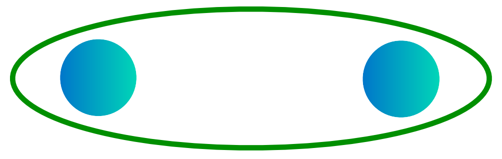
$$S_z \left|\downarrow\right\rangle = -(1/2) \left|\downarrow\right\rangle$$

on each site i

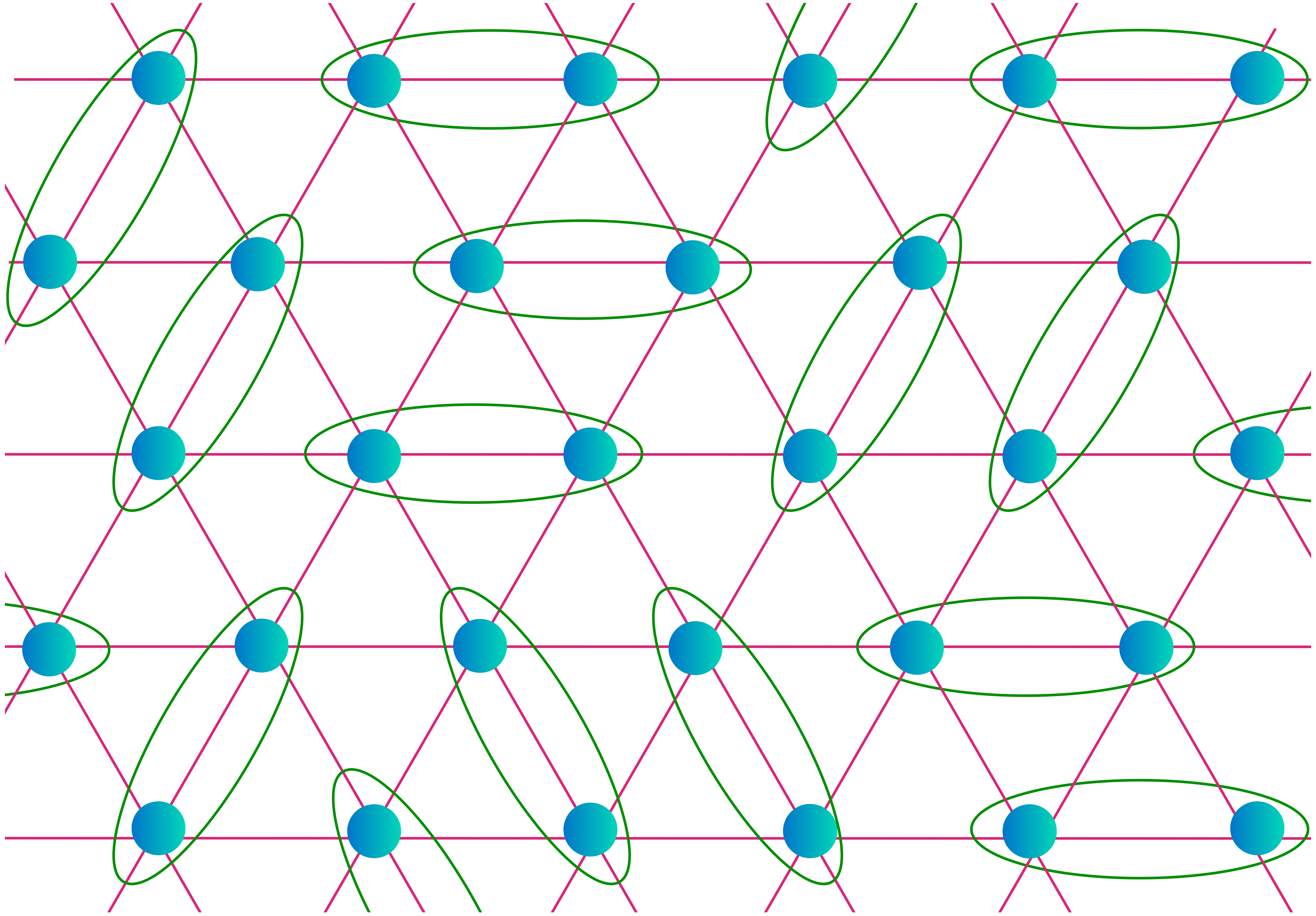
Nearest-neighbor model has non-collinear Neel order

Spin liquid: resonating valence bonds

Spin model with S=1/2 per unit cell



$$= \frac{1}{\sqrt{2}} \left(| \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \right)$$



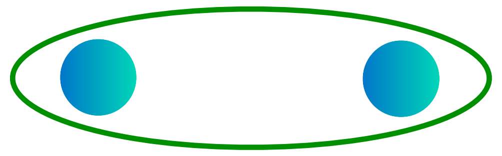
$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

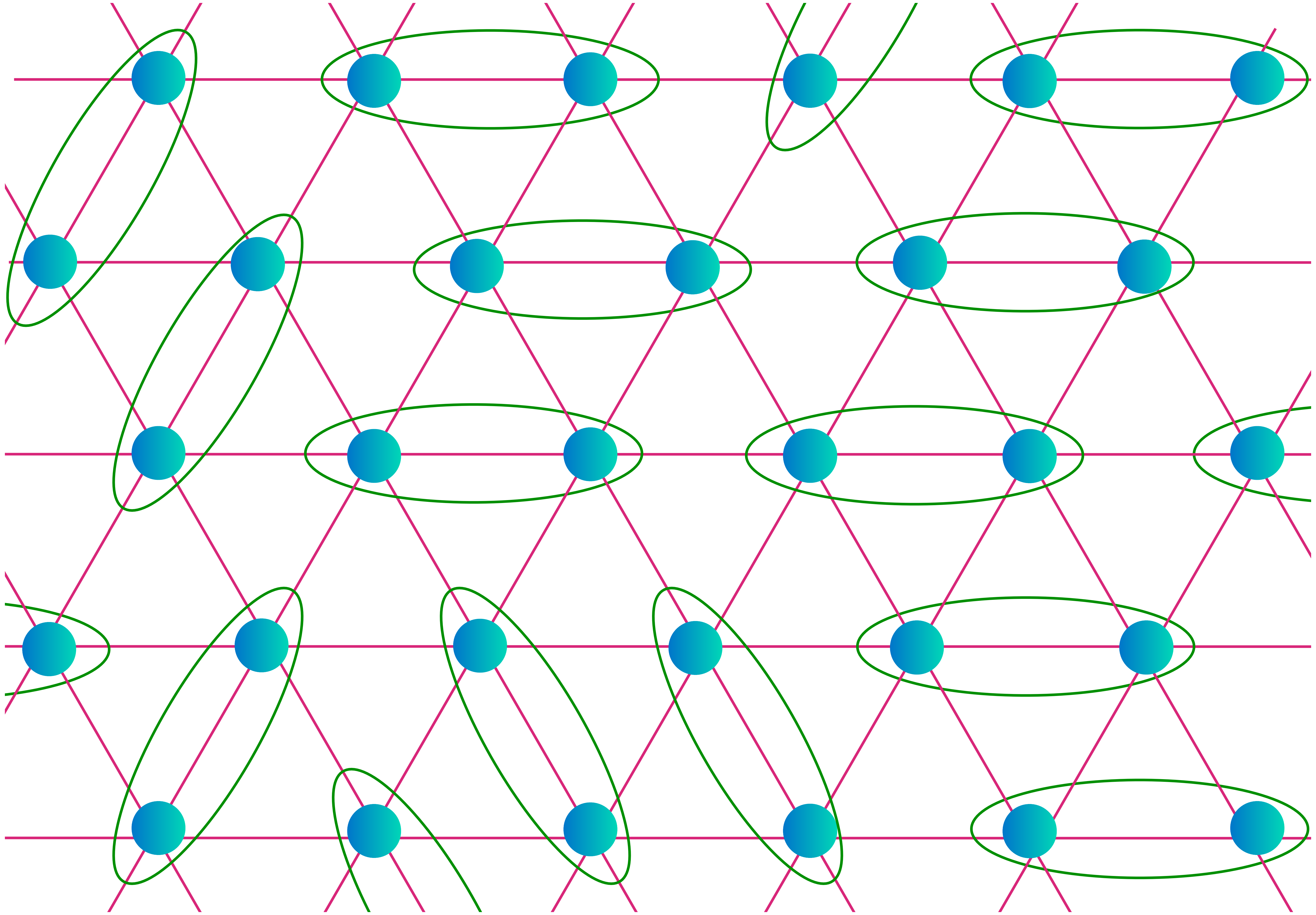
$\mathcal{D} \rightarrow$ dimer covering of lattice

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

Spin liquid: resonating valence bonds

Spin model with S=1/2 per unit cell

 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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Spin liquid: resonating valence bonds

Spin model with $S=1/2$ per unit cell

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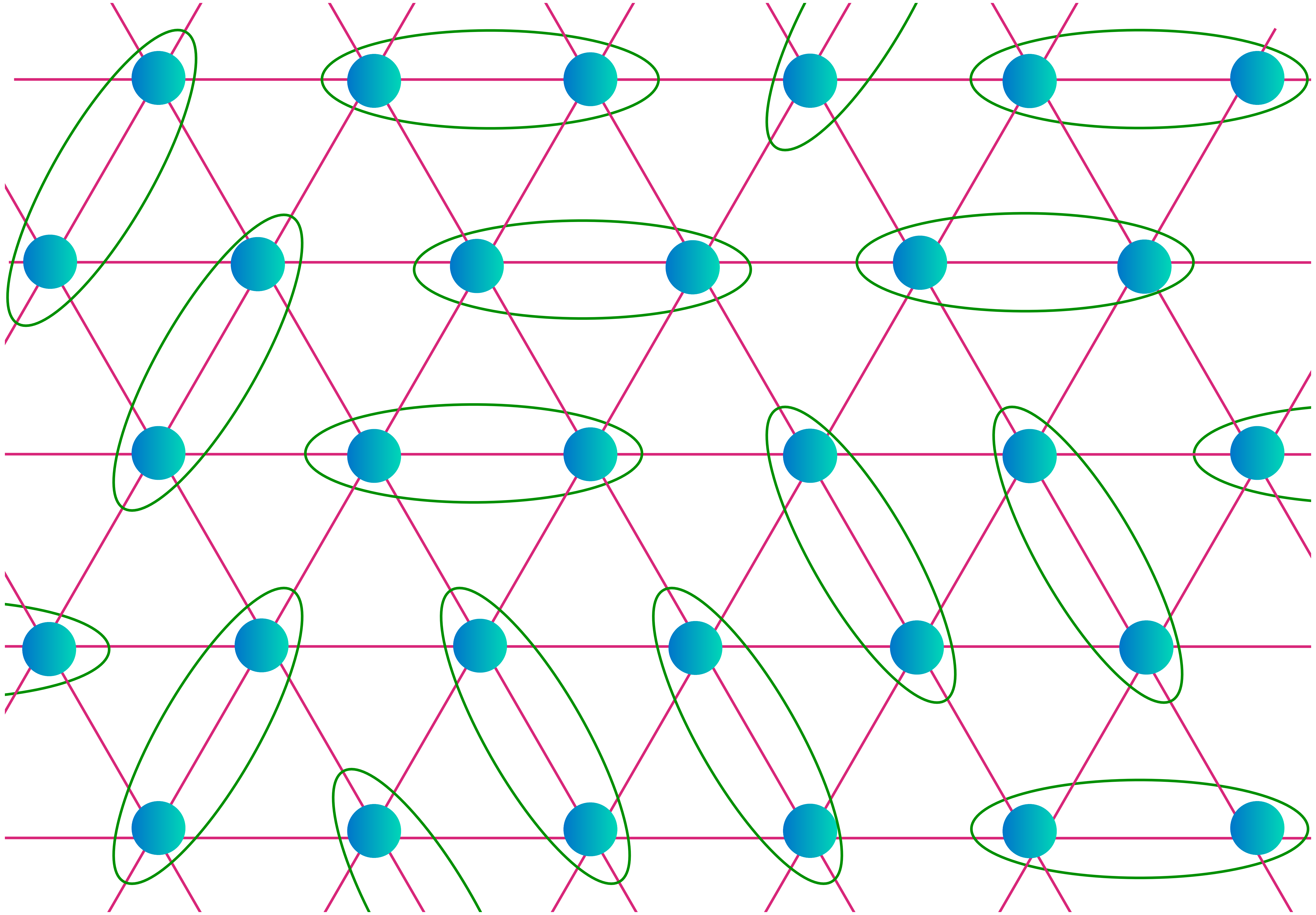

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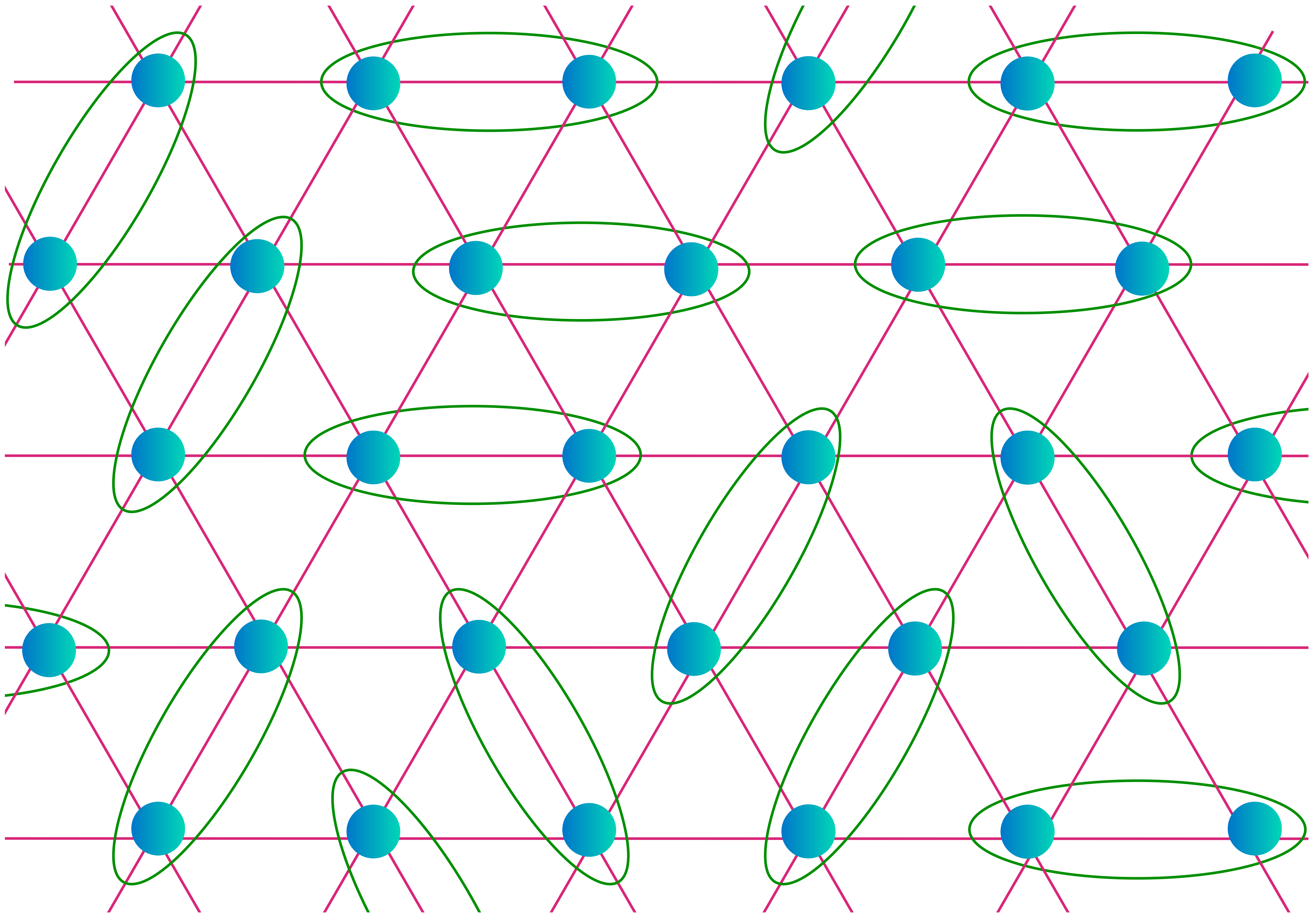

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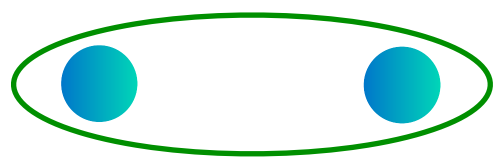

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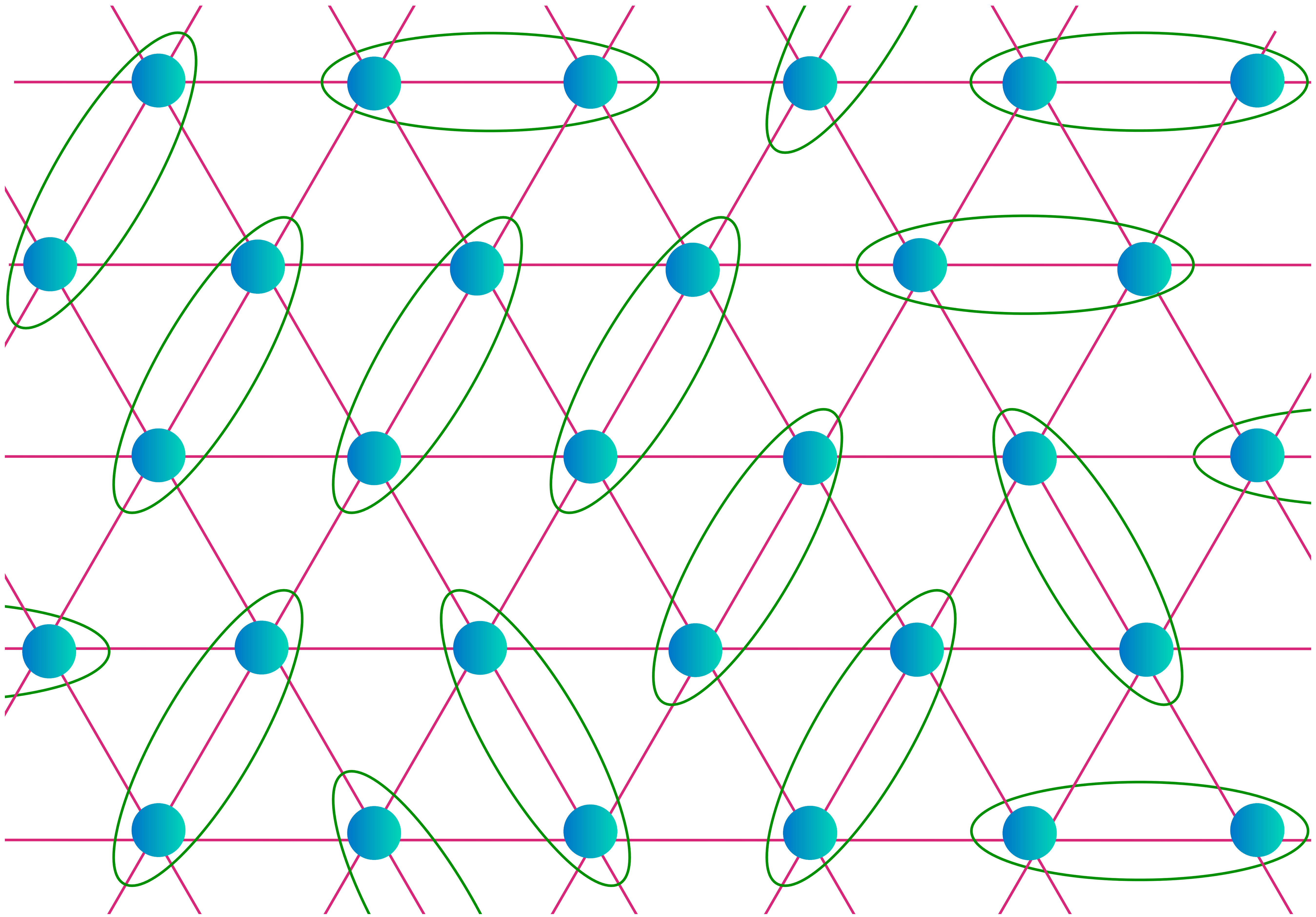

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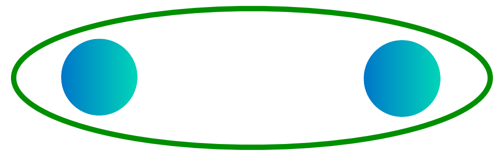
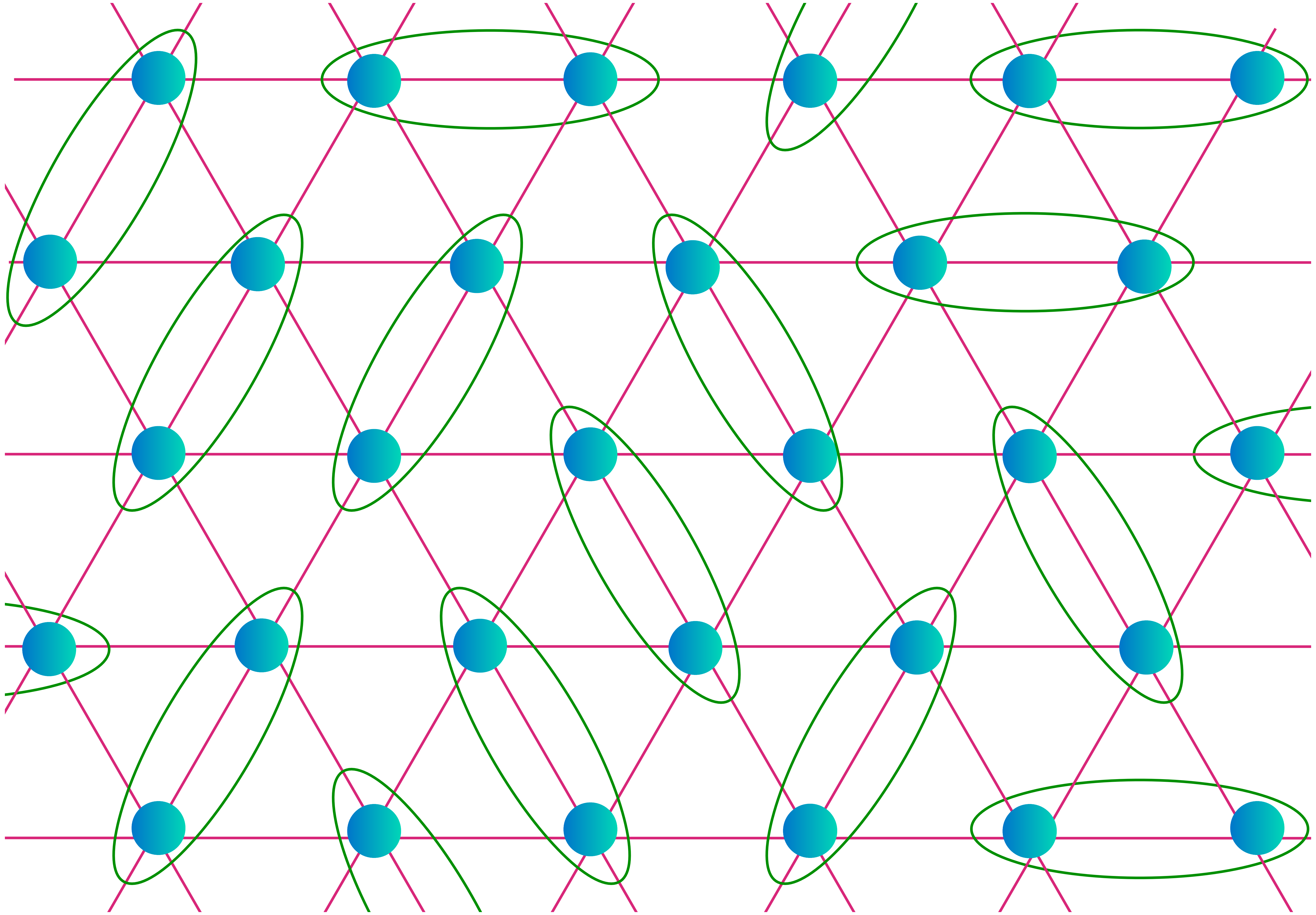
 $= \frac{1}{\sqrt{2}} \left(| \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle \right)$



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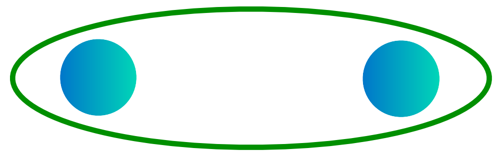

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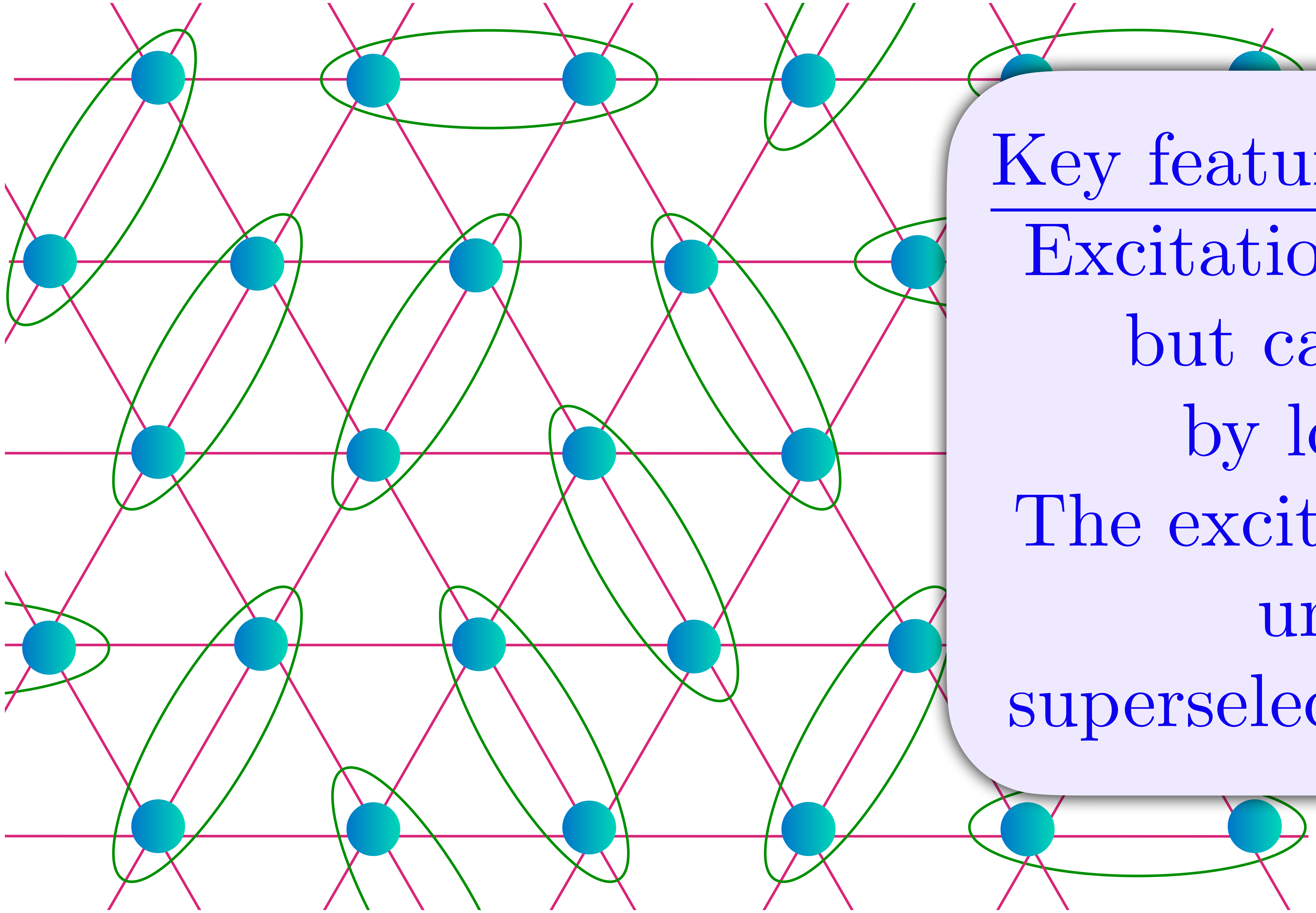
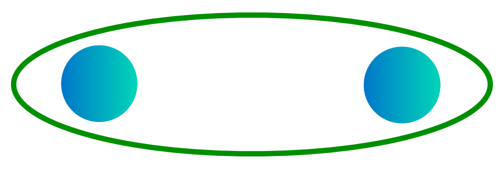


Key feature: fractionalization.
Excitations are particle-like, but cannot be created by local operators.
The excitations are classified under distinct superselection/anyon sectors.

RVB: Z_2 spin liquid

Fractionalized excitations: a "spinon" with spin S=1/2



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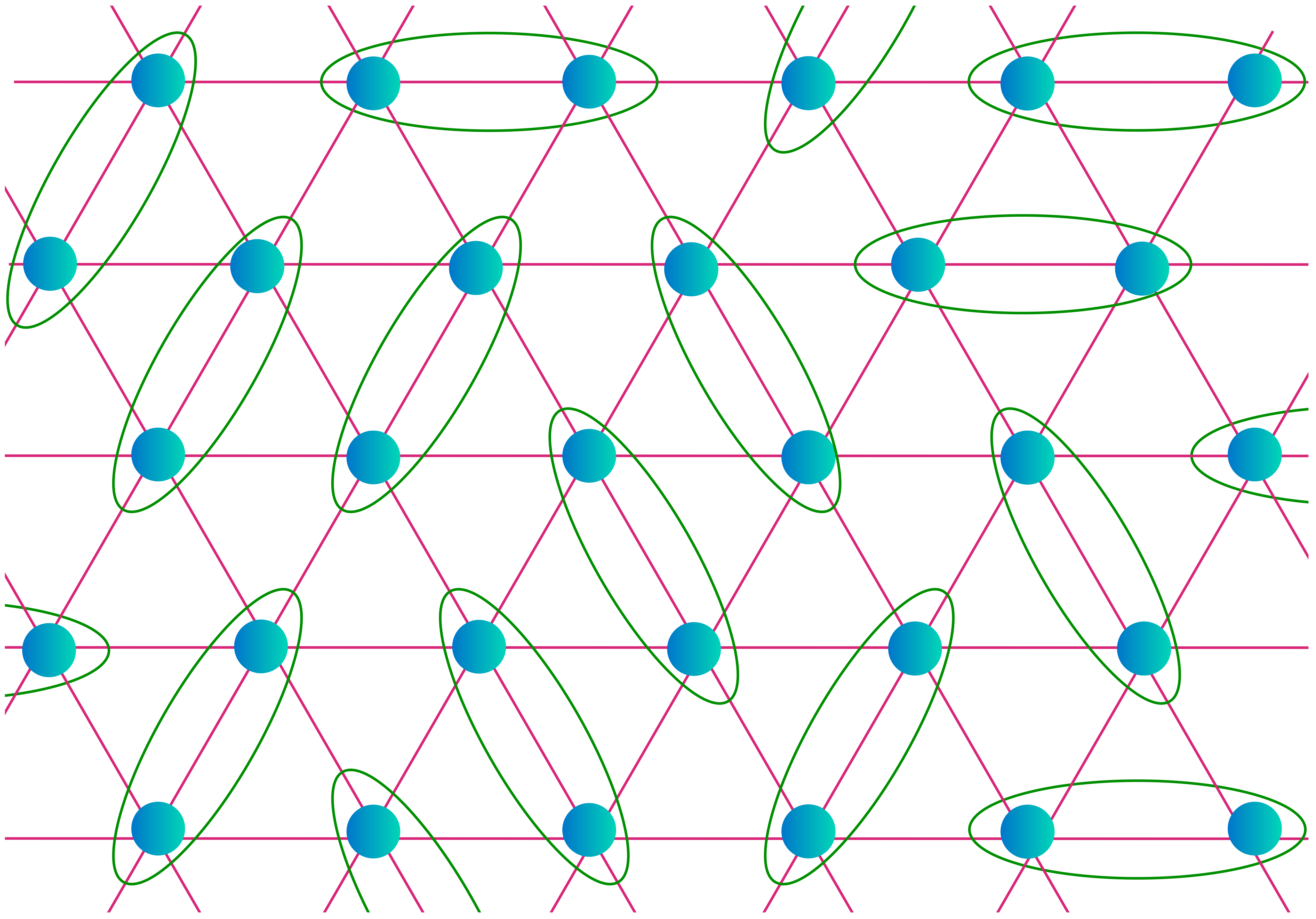

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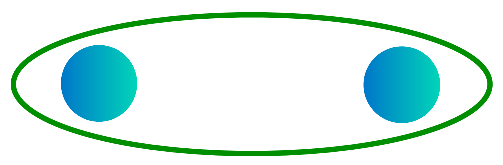
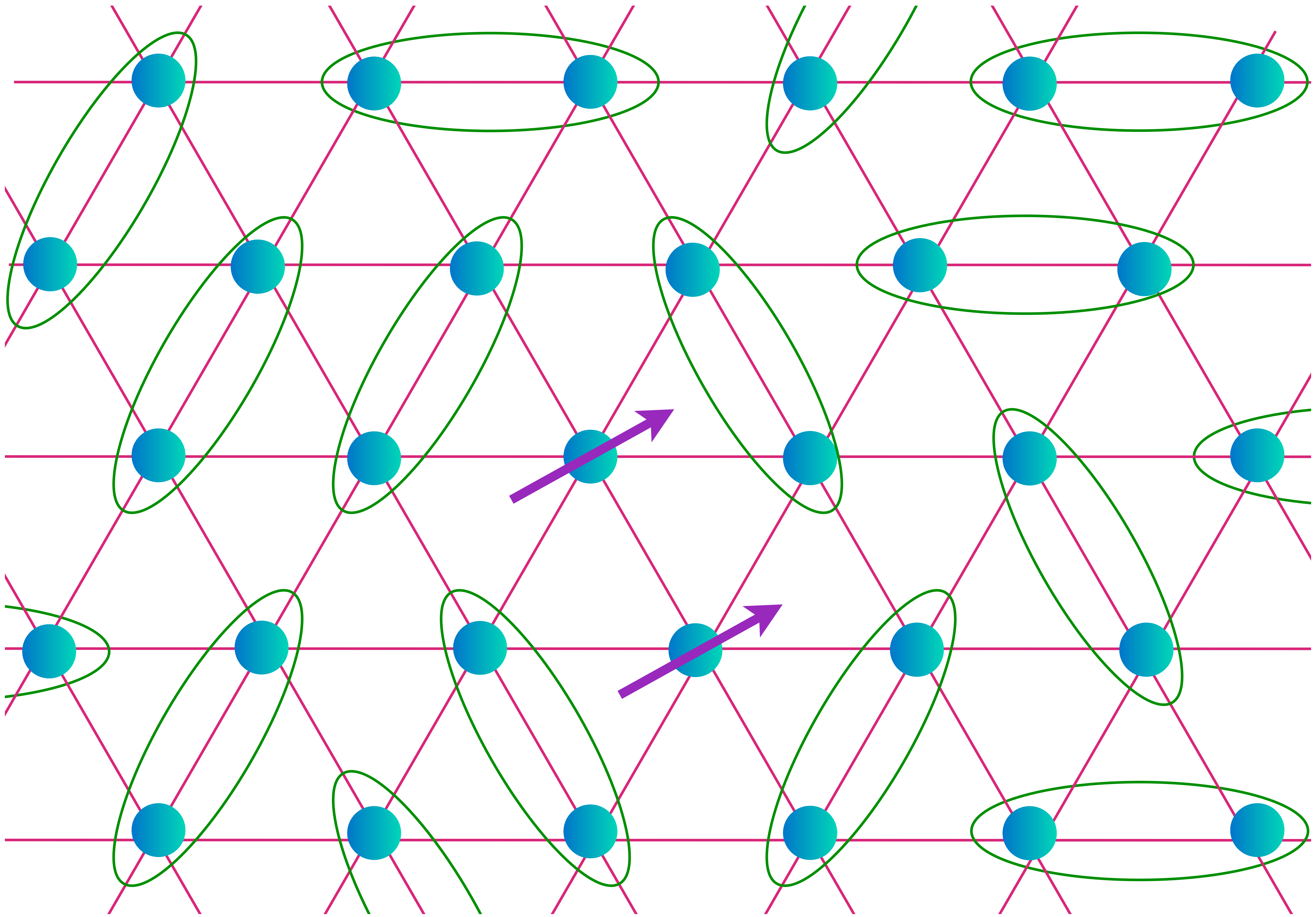
 $= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$



RVB: Z_2 spin liquid

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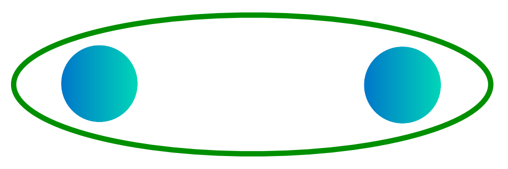

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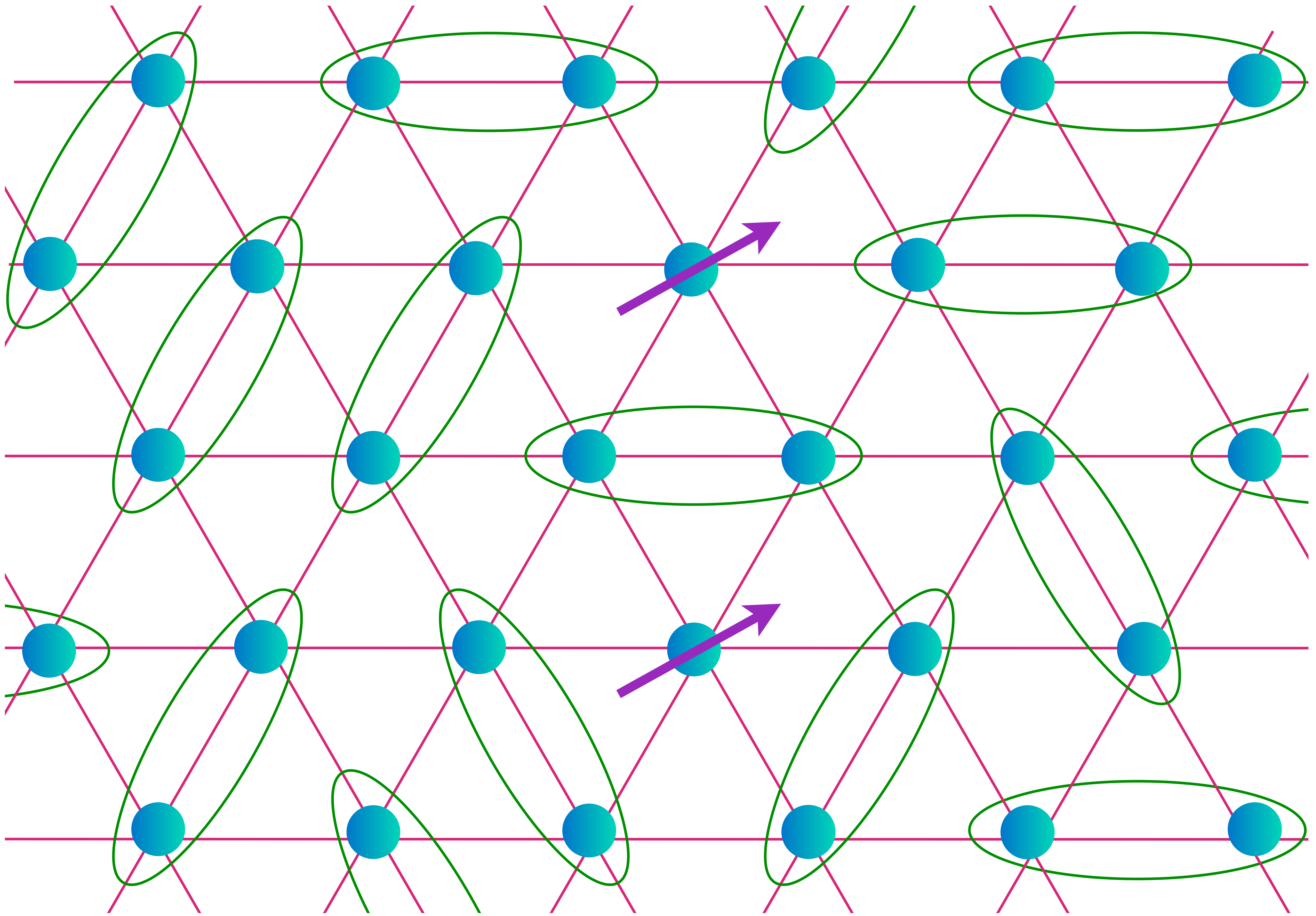
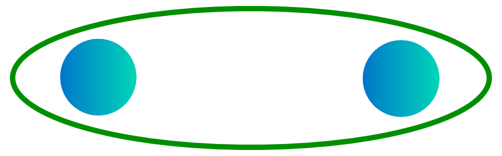
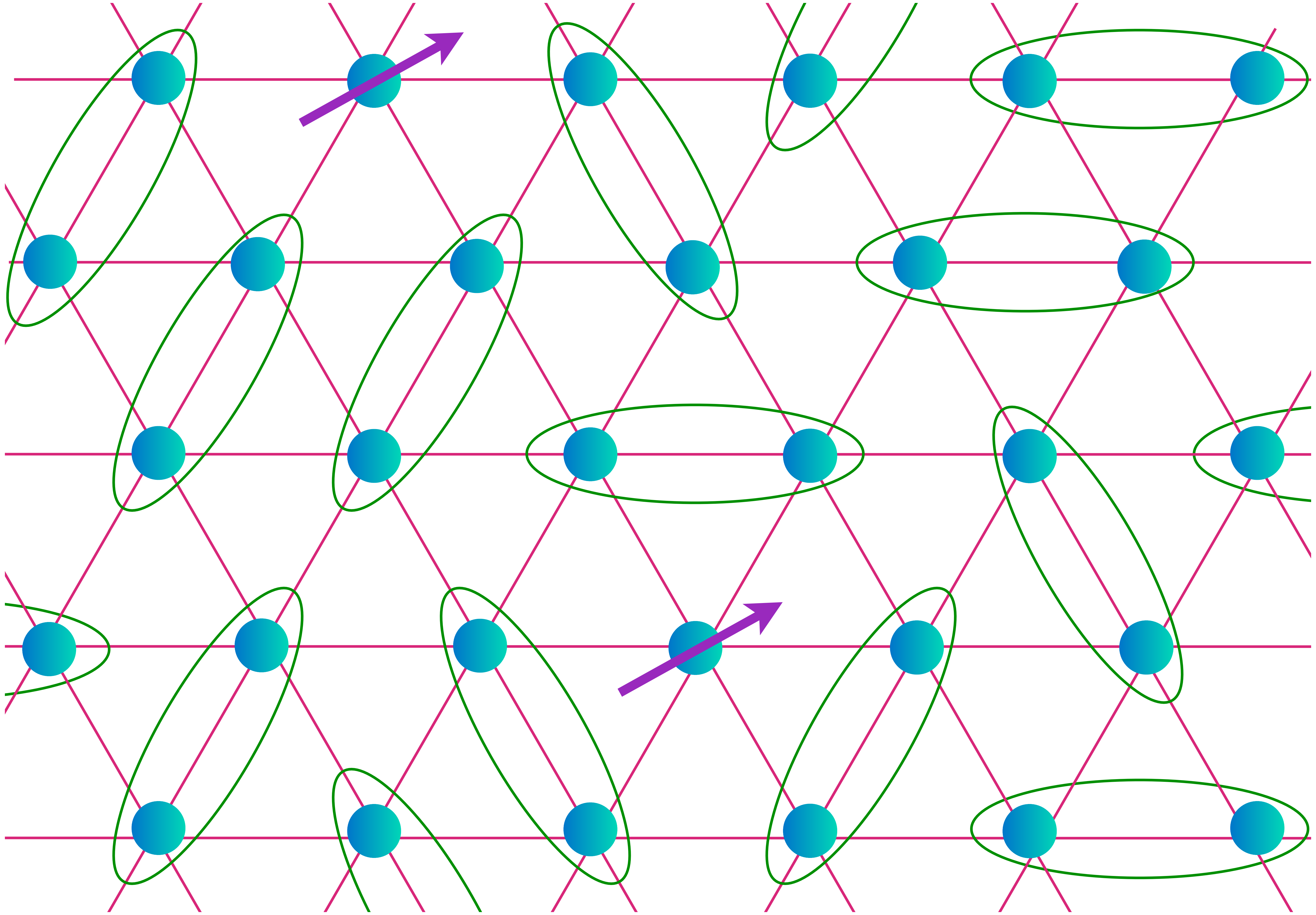


Kivelson, Baskaran…..

RVB: Z_2 spin liquid

Fractionalized excitations: a "spinon" with spin S=1/2

 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



Kivelson, Baskaran…..

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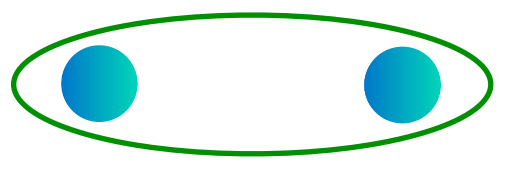

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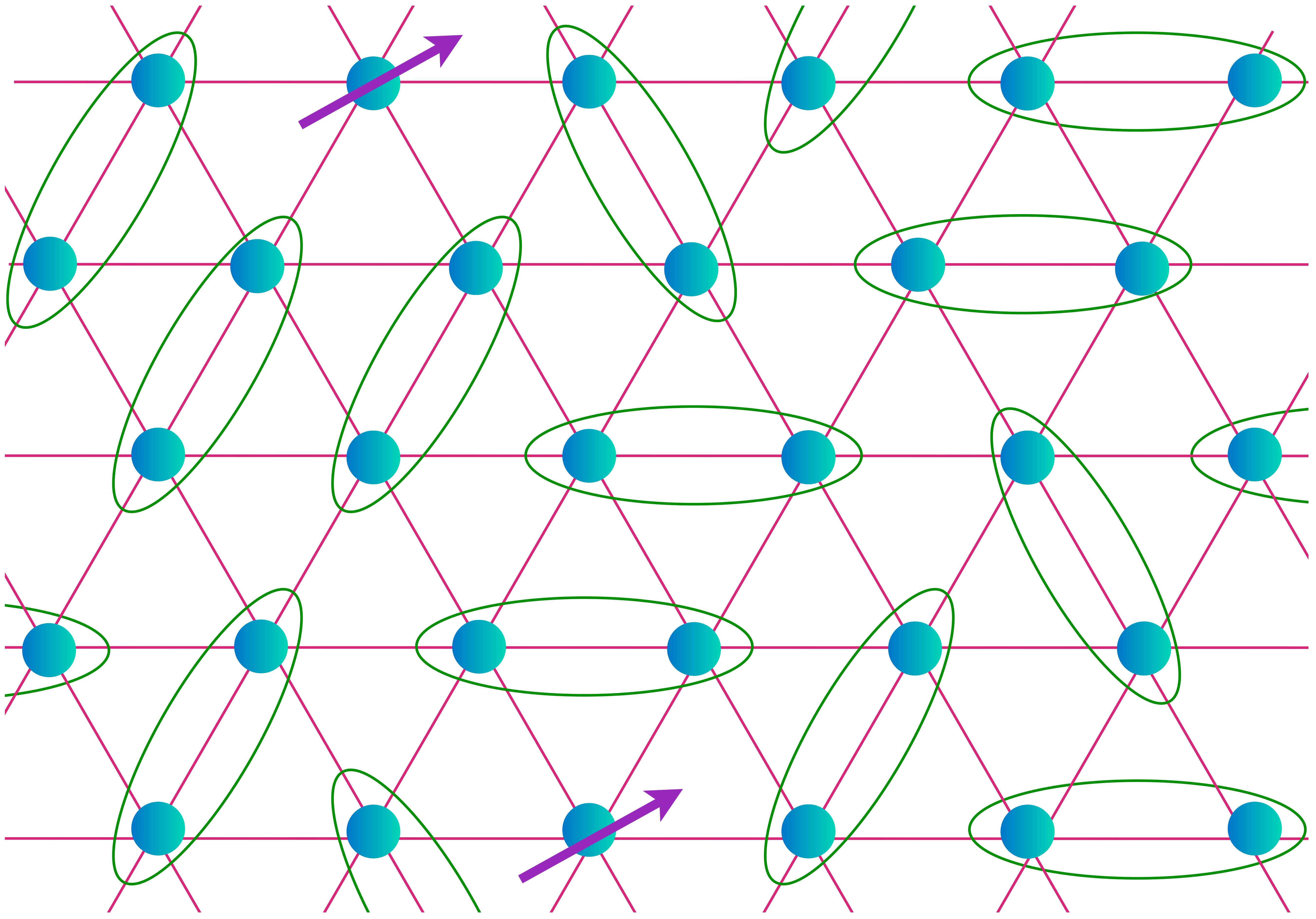
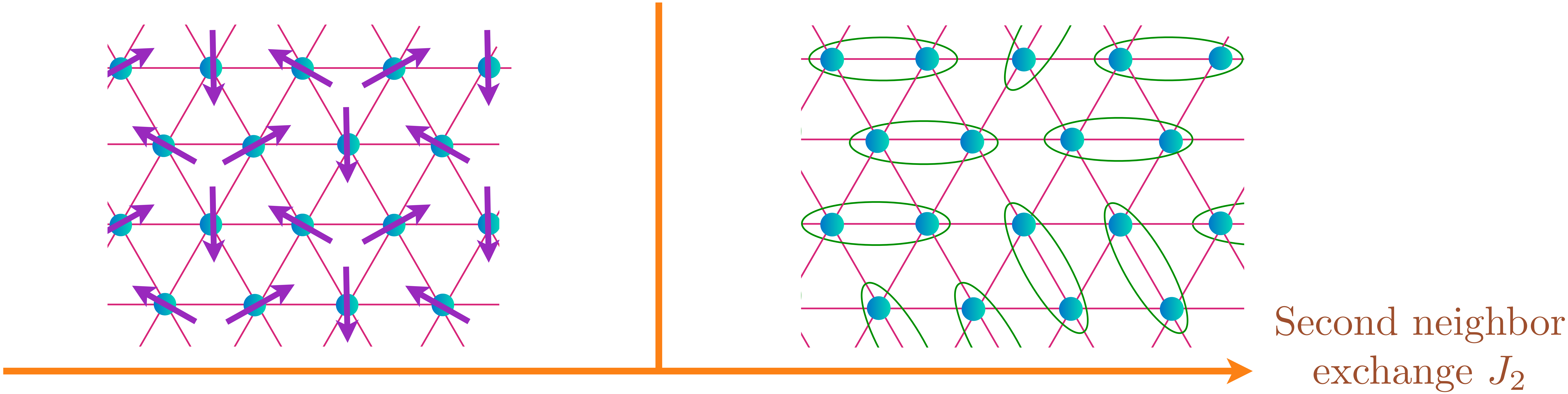


Kivelson, Baskaran…..

Quantum phase transition from ordered antiferromagnet to Z_2 spin liquid

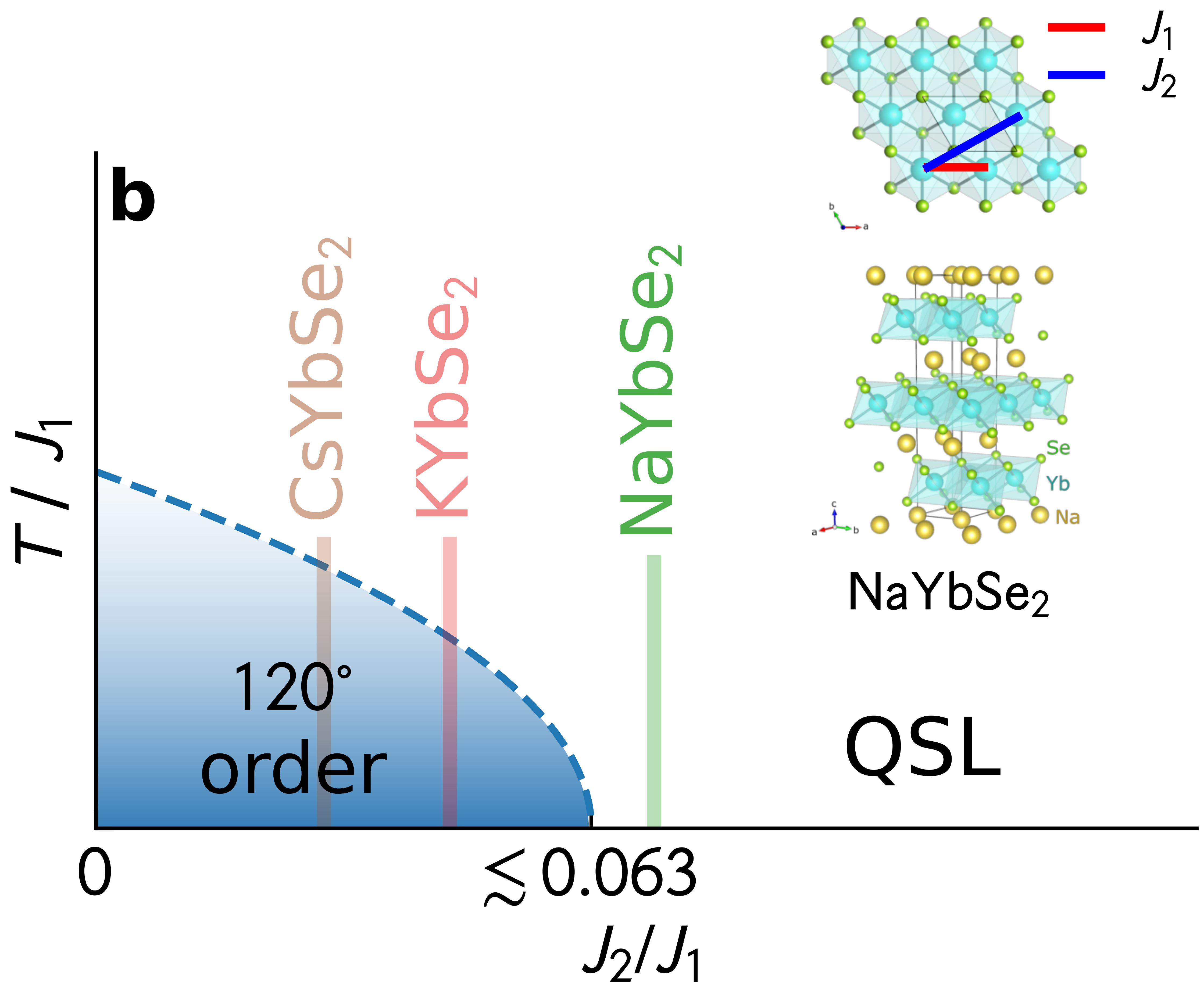


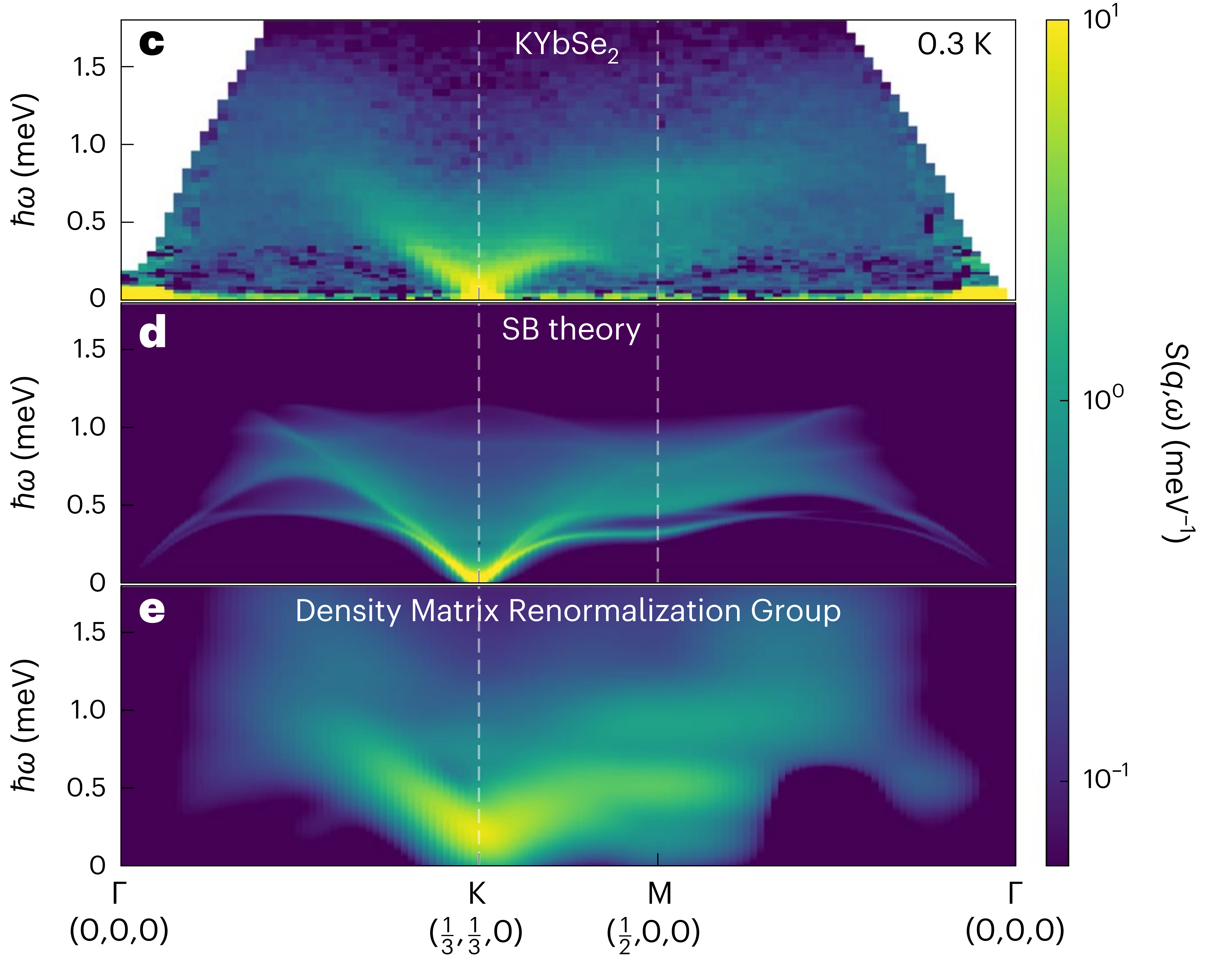
Read and Sachdev (1990):
The theory of this transition establishes the stability of the simplest spin liquid (which need not break time-reversal) as the deconfined phase of a $\mathbb{Z}_2$ gauge theory. The spinons carry unit $\mathbb{Z}_2$ electric charges, and 'vison' excitations which carry $\mathbb{Z}_2$ magnetic flux.
The structure ("unitary modular tensor category") is the same as that found in Kitaev's toric code (1997).

Proximate spin liquid and fractionalization in the triangular antiferromagnet $KYbSe_2$

A. O. Scheie[1], E. A. Ghioldi[2,3], J. Xing[4], J. A. M. Paddison[4], N. E. Sherman[5,6], M. Dupont[5,6], L. D. Sanjeewa[7,8], Sangyun Lee[9], A. J. Woods[9], D. Abernathy[1], D. M. Pajerowski[1], T. J. Williams[1], Shang-Shun Zhang[10], L. O. Manuel[3], A. E. Trumper[3], C. D. Pemmaraju[11], A. S. Sefat[4], D. S. Parker[4], T. P. Devereaux[11,12], R. Movshovich[9], J. E. Moore[5,6], C. D. Batista[2,13] & D. A. Tennant[2,13]

Nature Physics **20**, 74 (2024)

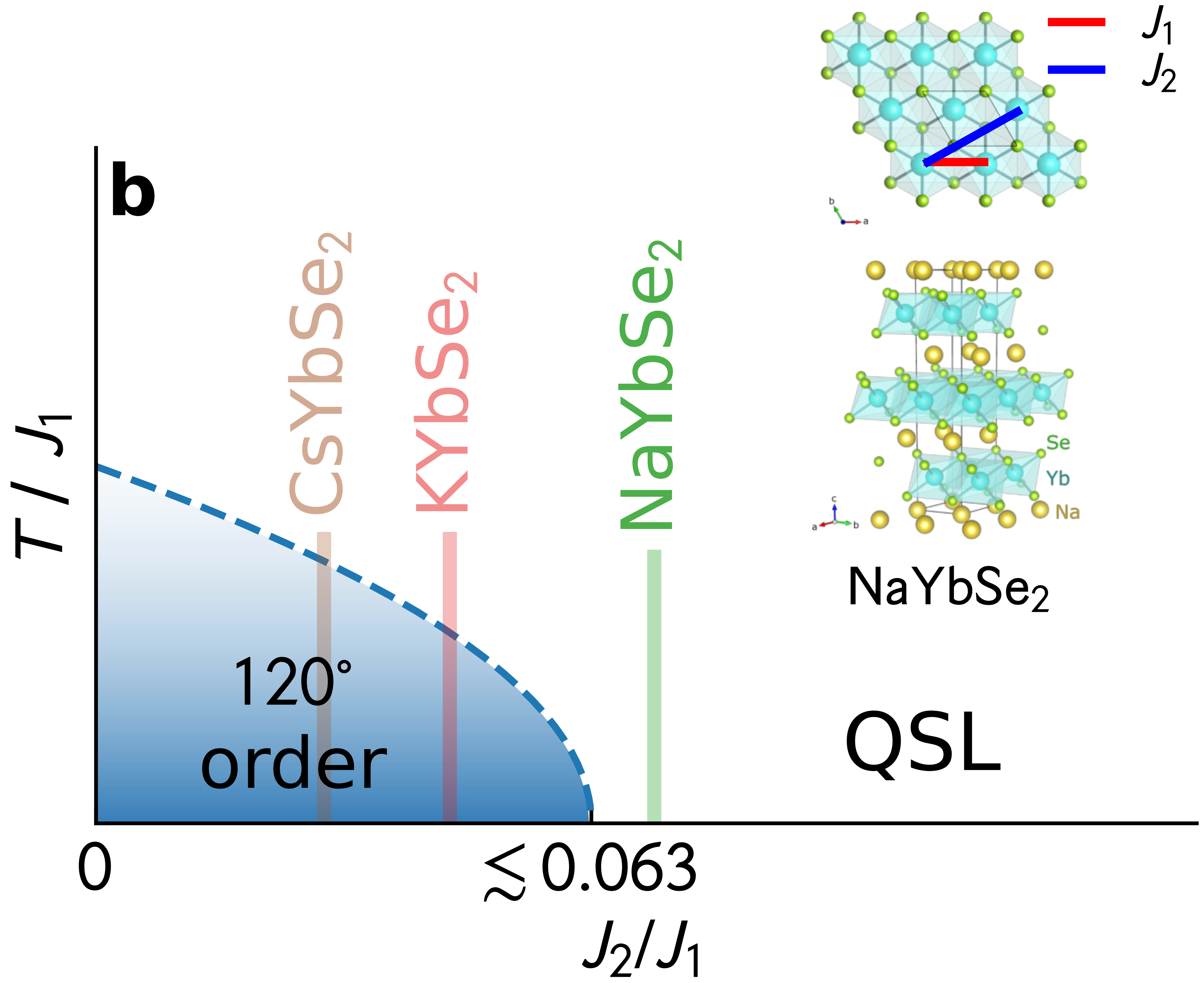




Spectrum and low-energy gap in triangular quantum spin liquid $NaYbSe_2$

A. O. Scheie,[1,*] Minseong Lee,[2,†] Kevin Wang,[3] P. Laurell,[4] E. S. Choi,[5] D. Pajerowski,[6] Qingming Zhang,[7] Jie Ma,[8] H. D. Zhou,[4] Sangyun Lee,[2] S. M. Thomas,[1] M. O. Ajeesh,[1] P. F. S. Rosa,[1] Ao Chen,[9] Vivien S. Zapf,[2] M. Heyl,[9] C. D. Batista,[4,6] E. Dagotto,[4,10] J. E. Moore,[3,‡] and D. Alan Tennant[4,11,§]

arXiv:2406.17773



We observe a continuum of (neutron) scattering, which is reproduced by matrix product simulations, and no phase transition is detected in any bulk measurements ...

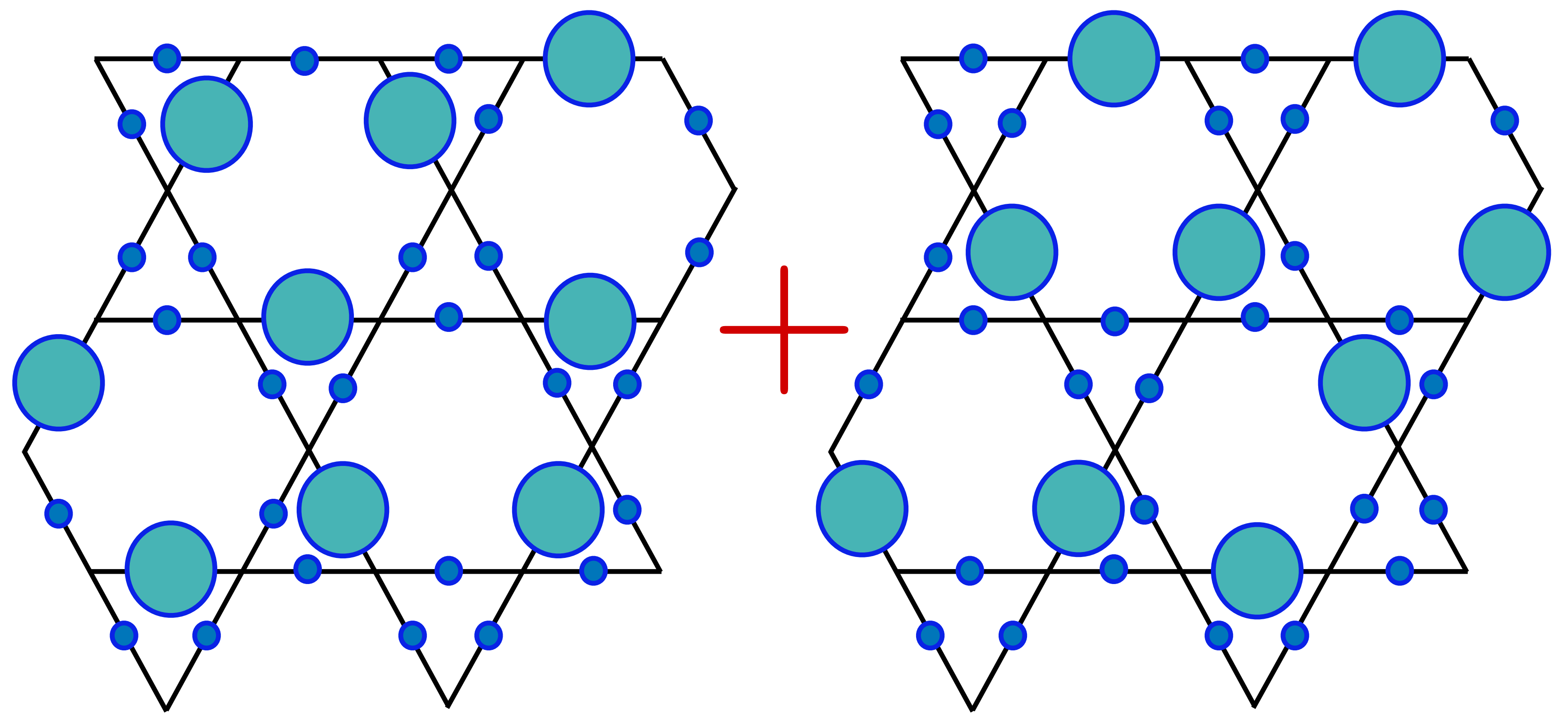
AC susceptibility shows a significant 23 mK downturn, indicating a gap in the magnetic spectrum ...

$NaYbSe_2$ is within the quantum spin liquid phase ... with a gapped $\mathbb{Z}_2$ liquid the most natural explanation.

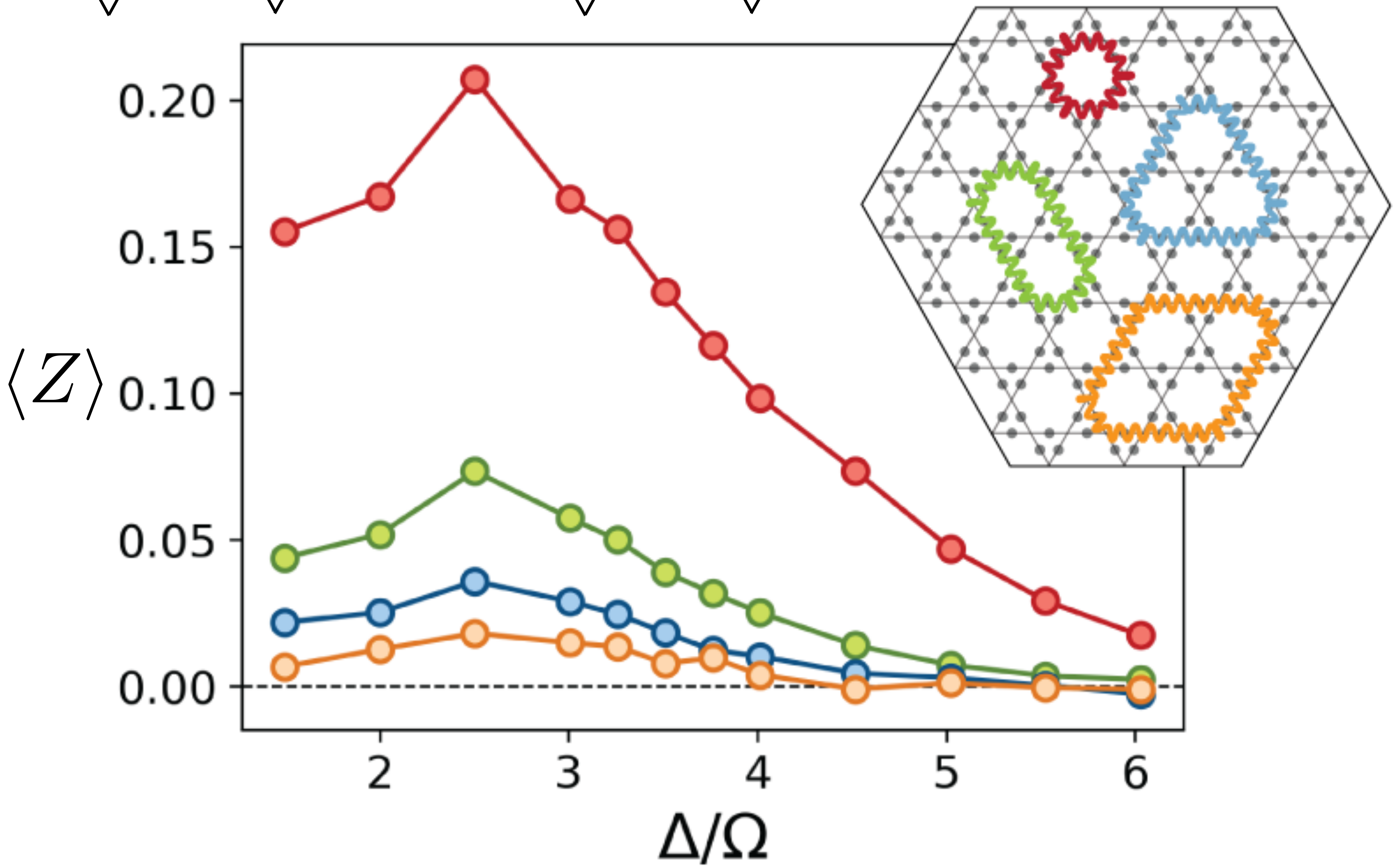
Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T. T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, Science **374**, 1242 (2021).

Rydberg atoms on the link-kagome lattice: experiment

$|\Psi\rangle =$  $+ \ldots$

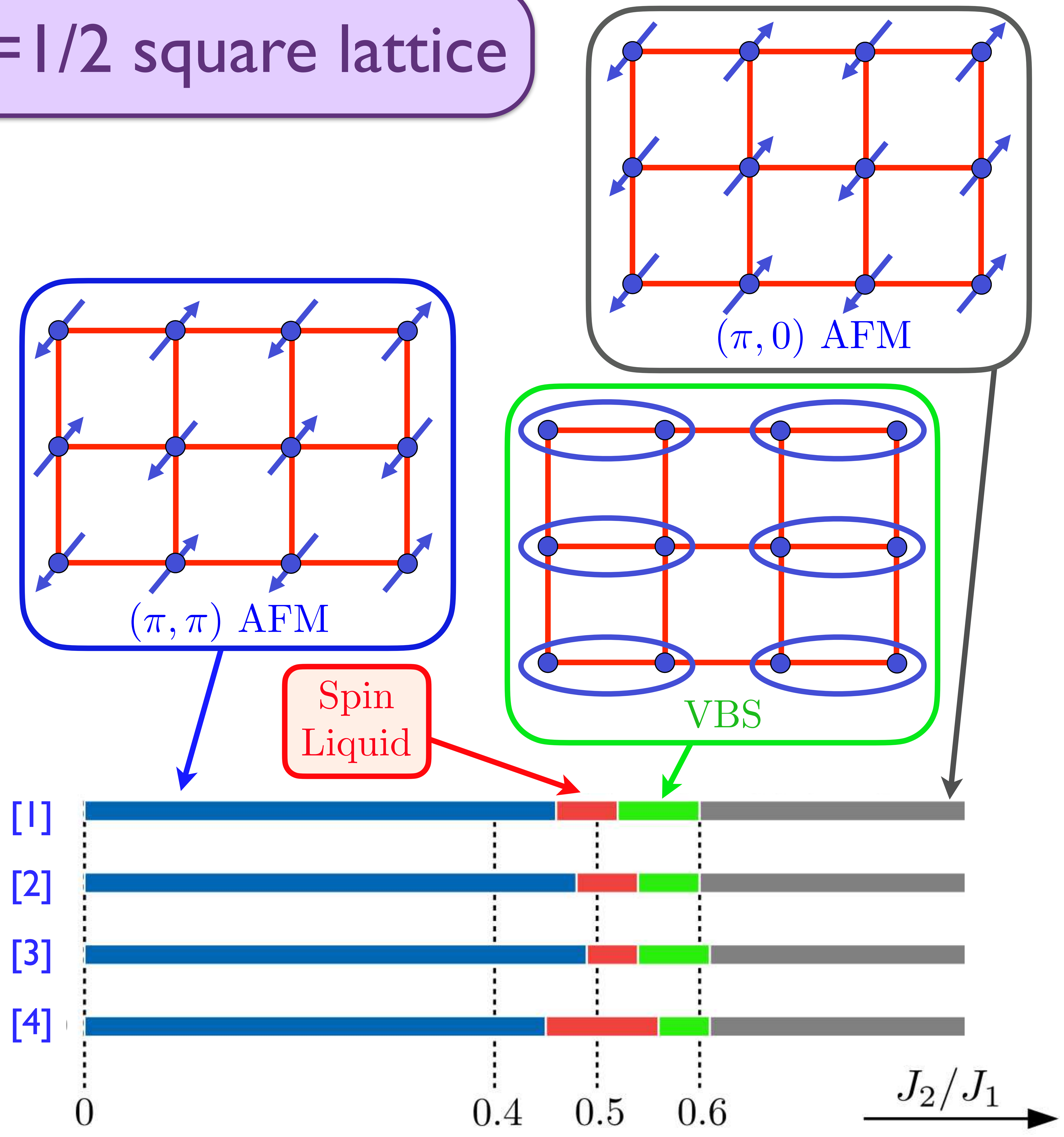
Evidence for $\mathbb{Z}_2$ spin liquid correlations

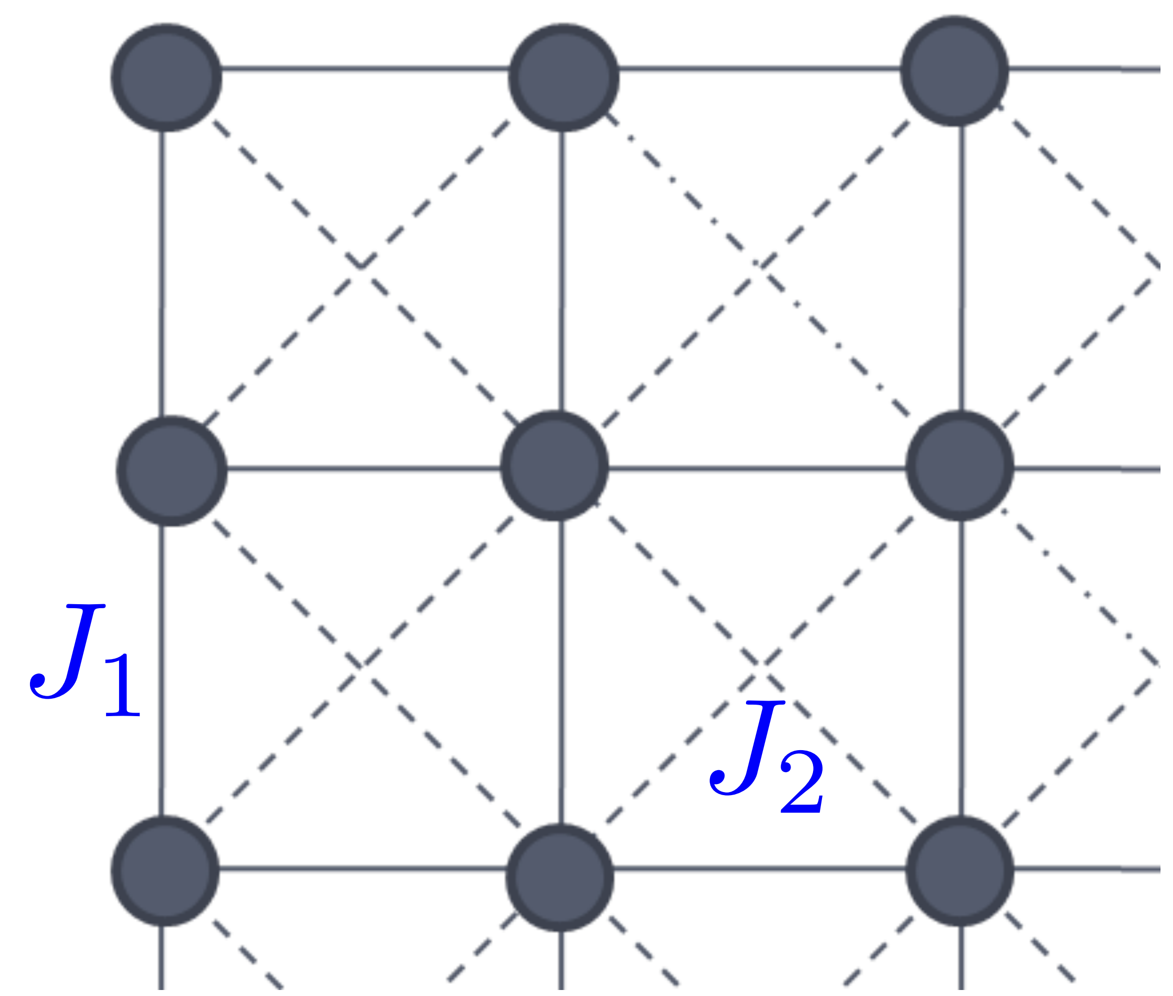


Square lattice spin liquid and the cuprate pseudogap metal

S=1/2 square lattice

$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



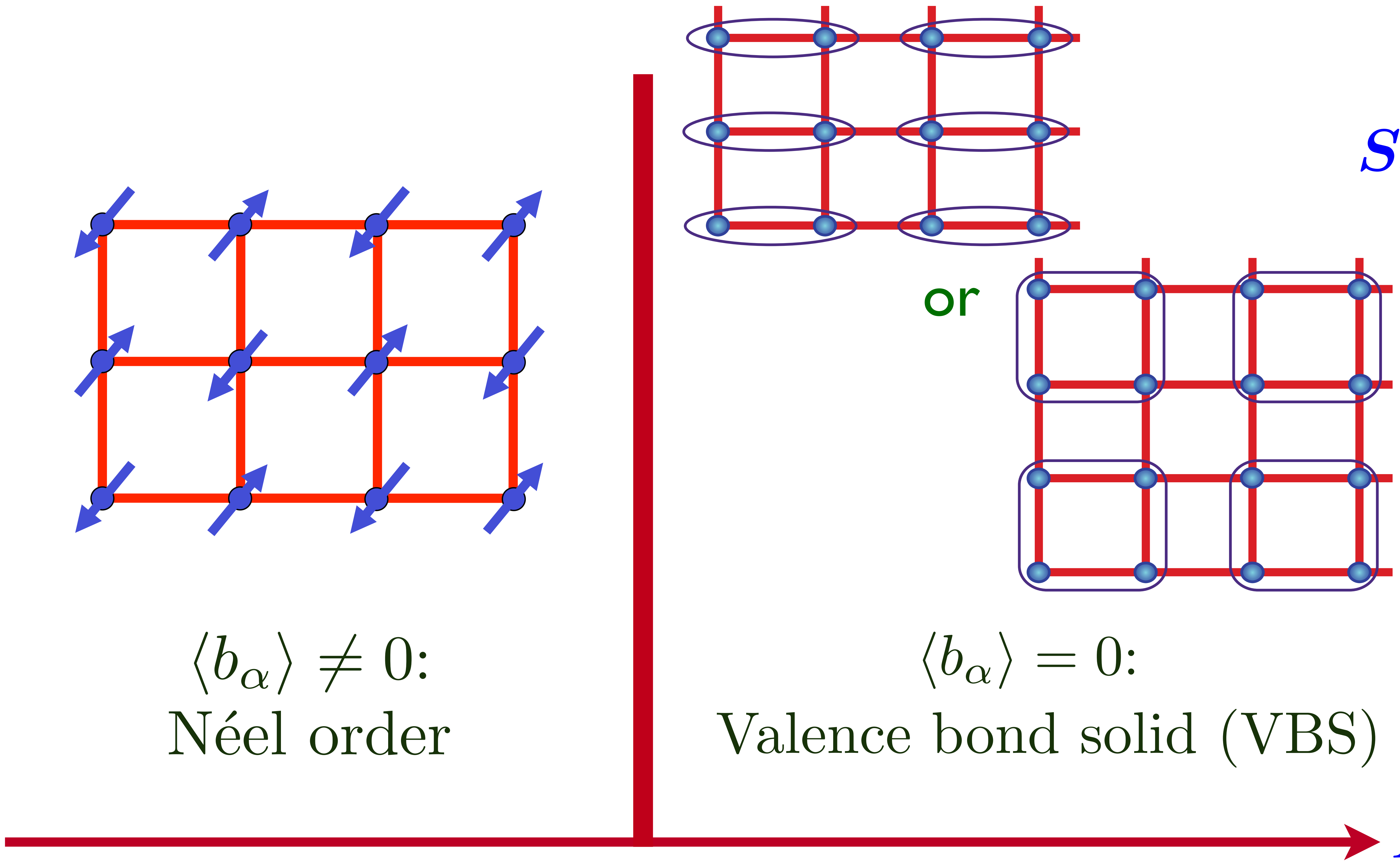


1. L. Wang and A. W. Sandvik, *Phys. Rev. Lett.* **121**, 107202 (2018)
2. F. Ferrari and F. Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Y. Nomura and M. Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, *Science Bulletin* **67,** 1034 (2022)

S=1/2 square lattice

$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$\boldsymbol{S}_i = \frac{1}{2} b^\dagger_{i\alpha} \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}\,, \qquad \sum_{\alpha=1}^{N=2} b^\dagger_{i\alpha} b_{i\alpha} = n_b = 2S$$



N/n_b

Parton representation with $S = 1/2$ bosons $b_{i\alpha}$.

Low energy: $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b^\dagger_{B\beta}$$

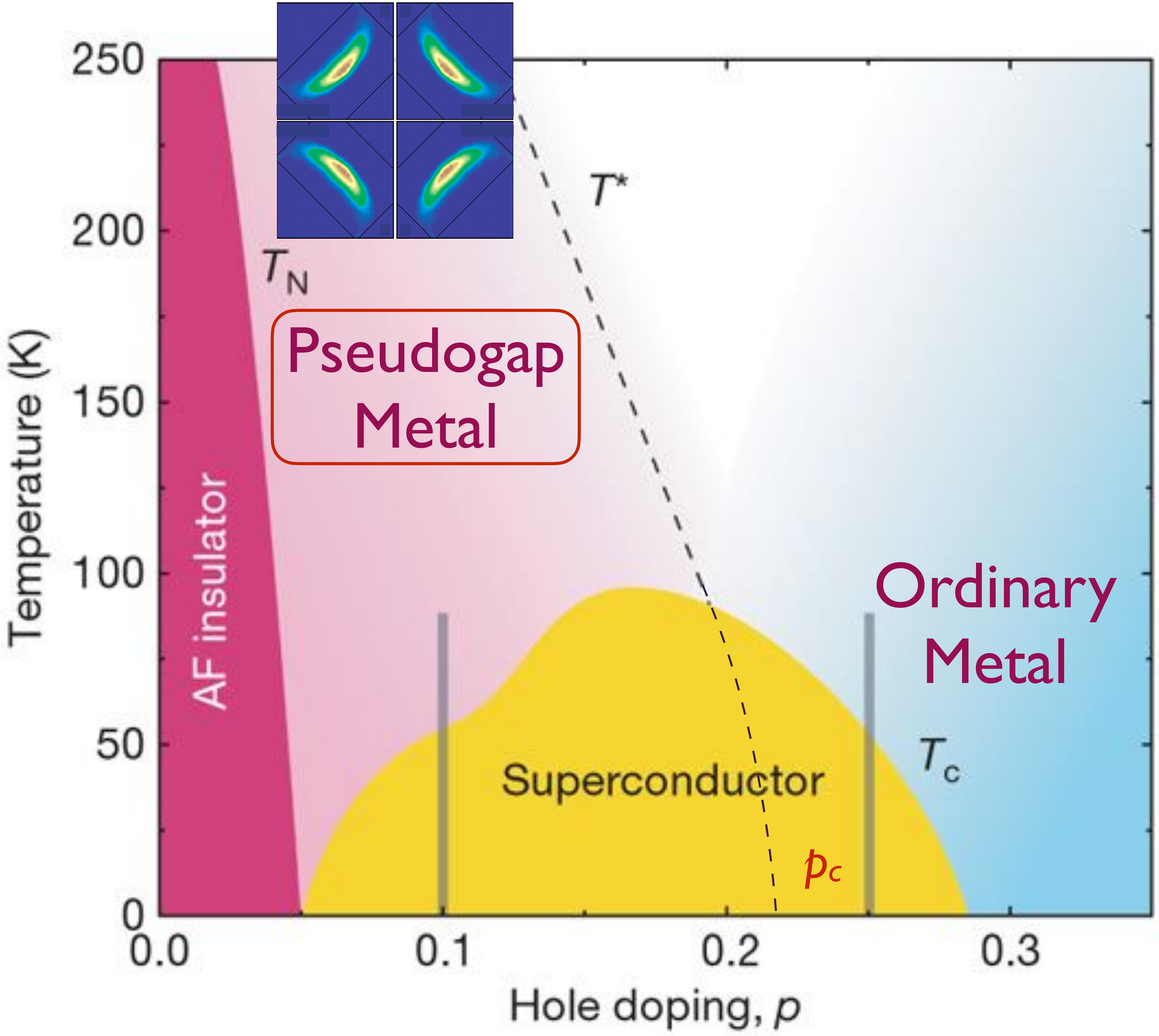
$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\rm monopole}$$

Gapless spin-liquid near a Neel-VBS transition

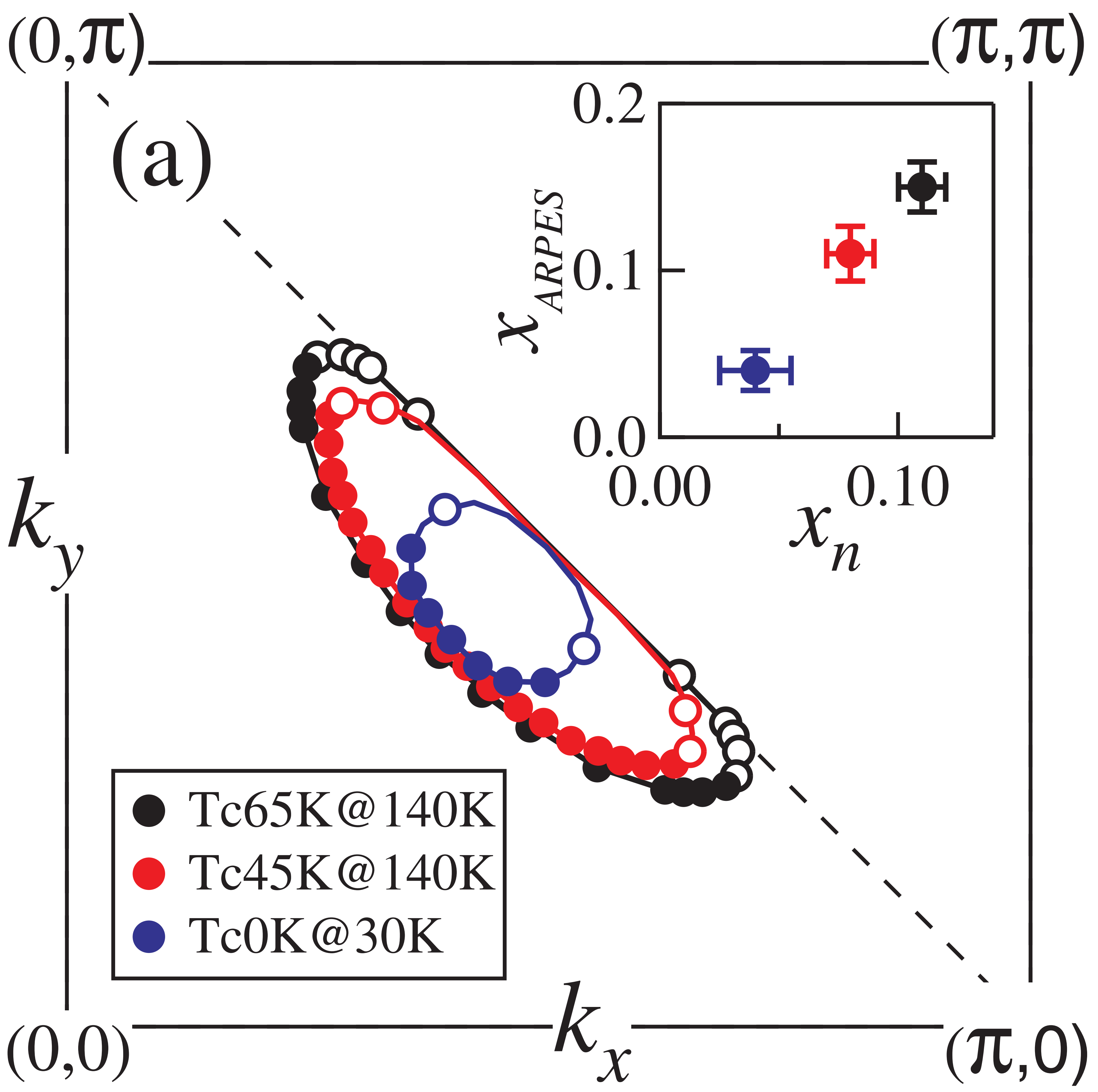
N. Read and S. S., PRL **62**, 1694 (1989); PRB **42**, 4568 (1990)

Pseudogap metal:

many-particle entanglement similar to that in a spin liquid



Photoemission expts in cuprates in pseudogap metal



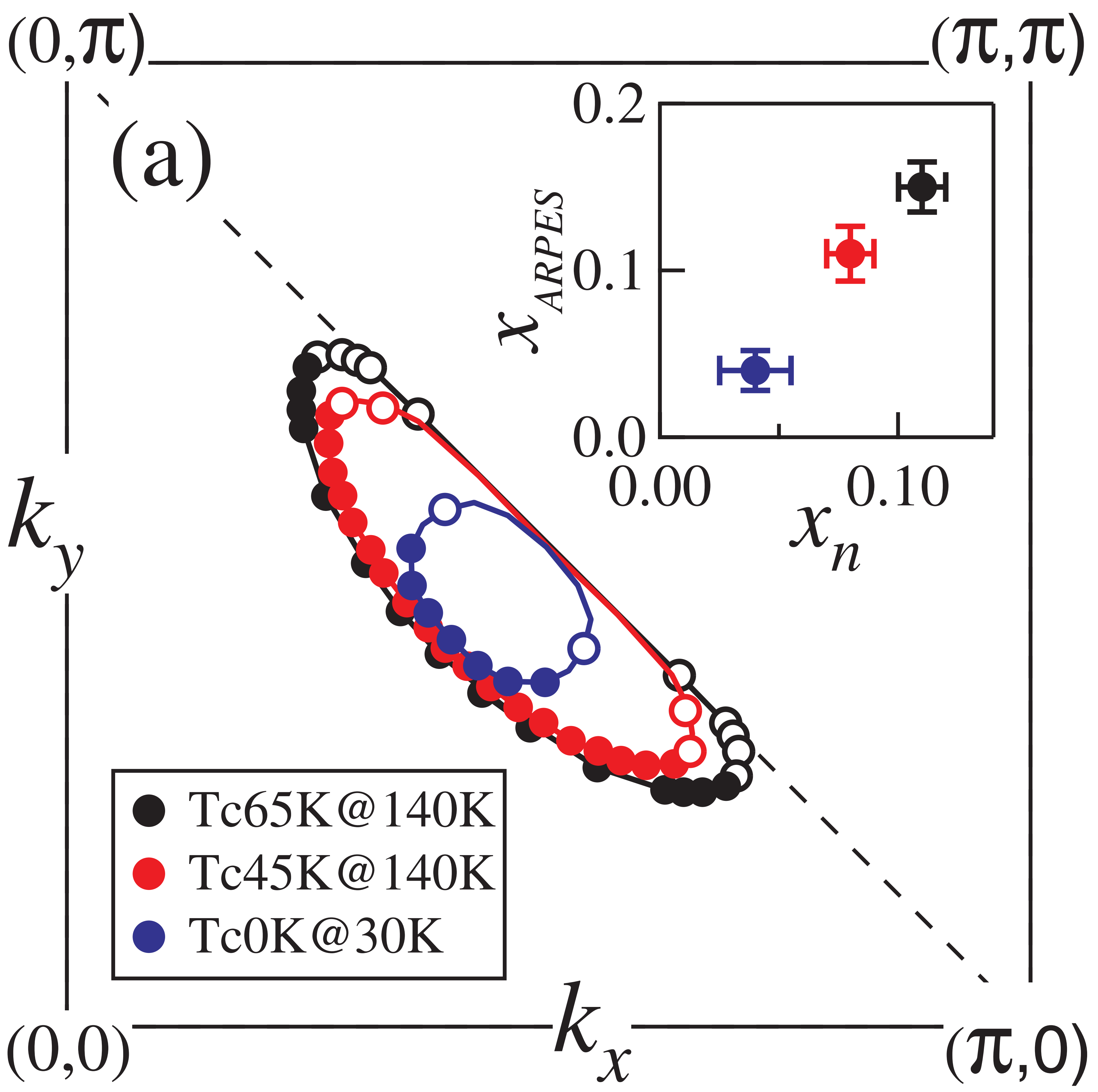
Reconstructed Fermi Surface of Underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ Cuprate Superconductors, H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

Non-Luttinger volume Fermi surface

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S. *PRB* **105**, 075146 (2022); Y. Qi and S. S. *PRB* **81**, 115129 (2010)

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, *PRB* **73**, 174501 (2006); T. D. Stanescu and G. Kotliar, *PRB* **74**, 125110 (2006). C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, *PRL* **97**, 136401 (2006). S. Sakai, Y. Motome, M. Imada, *PRL* **102**, 056404 (2009).

Photoemission expts in cuprates in pseudogap metal



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Non-Luttinger volume Fermi surface

The missing electrons are in a spin liquid!

Oshikawa's topological Luttinger argument implies that non-Luttinger Fermi surfaces must be accompanied by a background of fractionalized spinon excitations of a spin liquid

T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)
R. K. Kaul, A. Kolezhuk, M. Levin, S. S., T. Senthil, PRB **75** , 235122 (2007)
Y. Qi, S. S., PRB **81**, 115129 (2010)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Anisotropic damping and wave vector dependent susceptibility of the spin fluctuations in $La_{2-x}Sr_xCuO_4$ studied by resonant inelastic x-ray scattering

H. C. Robarts, M. Barthélemy, K. Kummer, M. García-Fernández, J. Li, A. Nag, A. C. Walters, K. J. Zhou, and S. M. Hayden

PHYSICAL REVIEW B **100**, 214510 (2019)

- Difficult to have intense paramagnons from a small Fermi surface.
- Spin waves only present at low energies in the presence of antiferromagnetic order
- Most natural interpretation is a spinon continuum, similar to that observed on the triangular lattice in $KYbSe_2$

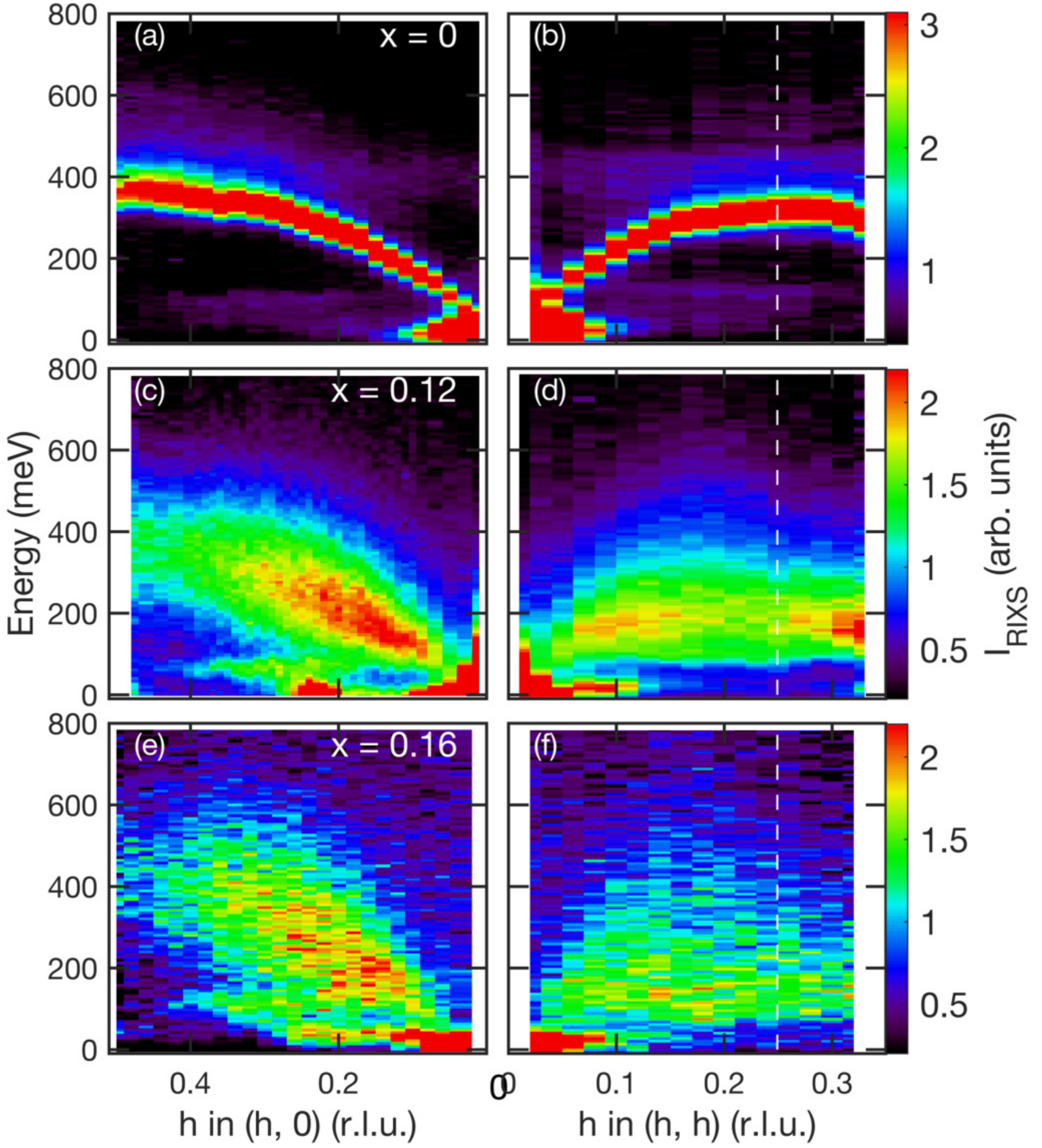
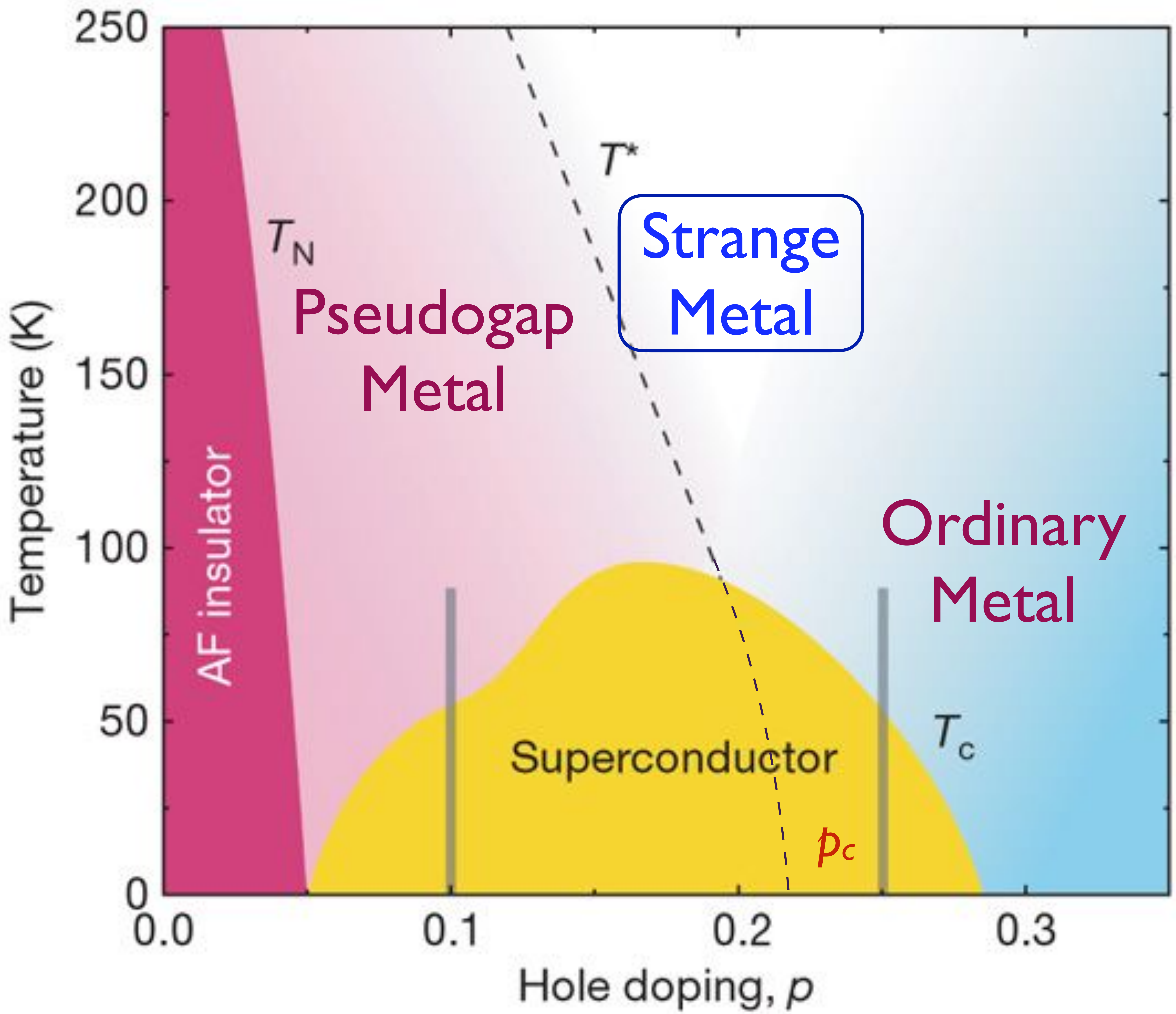


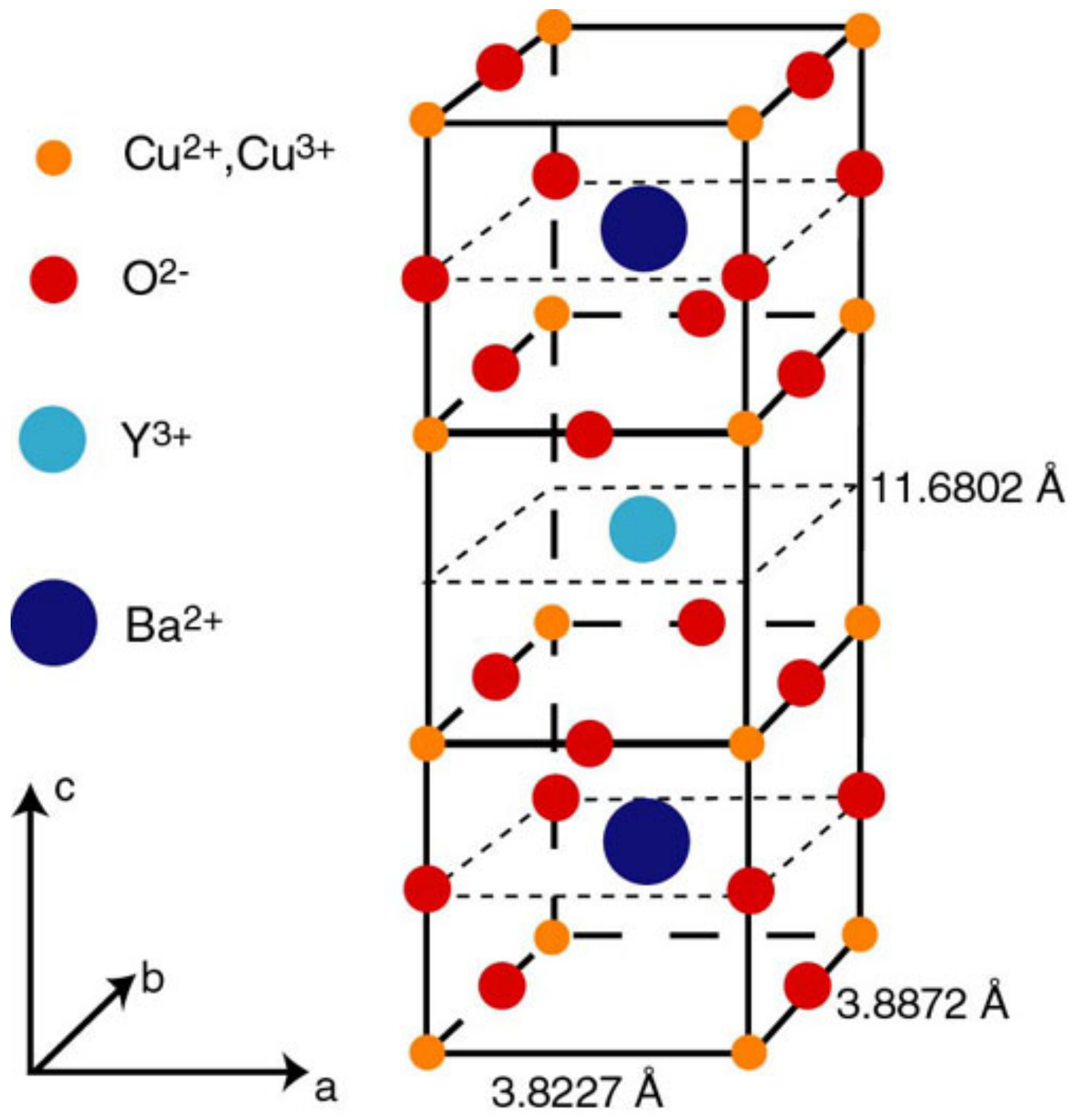
FIG. 2. I_{RIXS} intensity maps as a function of **Q** in LSCO $x = 0$ ($T \approx 20$ K), 0.12, and 0.16 ($T \approx 30$ K).

Pietro Bonetti, Maine Christos and S.S. (BCS), PNAS **121**, e2418633121 (2024)

Entanglement of mobile electrons.

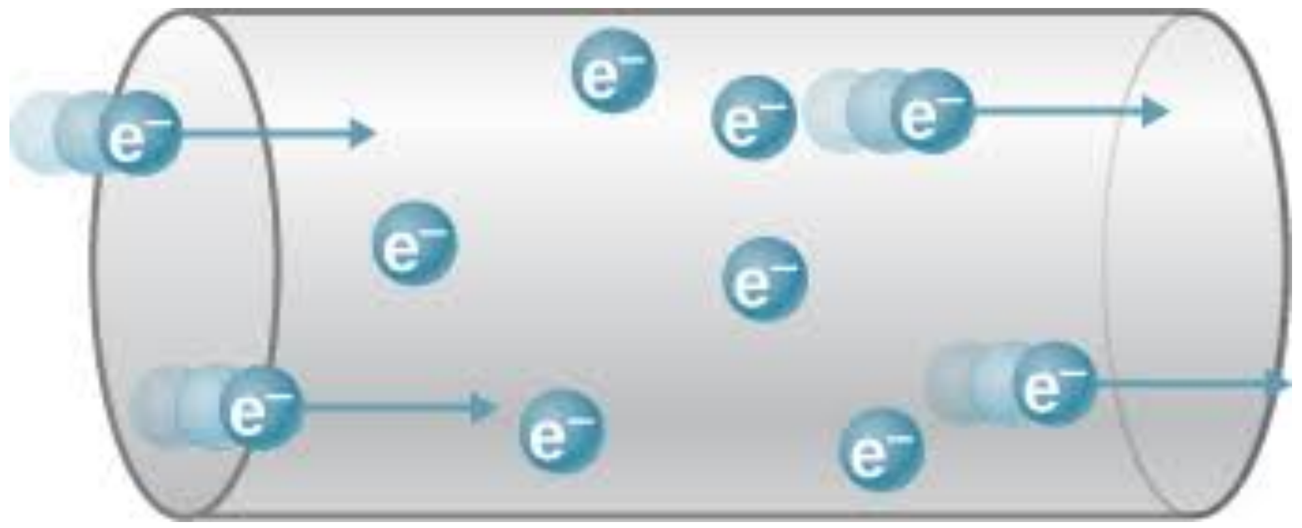
Metals without quasiparticles: the SYK model


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
Pseudogap
Metal
T*
Strange
Metal
Ordinary
Metal
Superconductor
T_c
p_c


Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$

Current flow with electrons in ordinary metals



Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/(UT)^2$ (U is the strength of interactions),
resistivity $\rho(T) = \rho(0) + AT^2$

The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

The motion of electrons is 'ballistic' or 'integrable'
up to the long time τ, after which it is chaotic.

Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

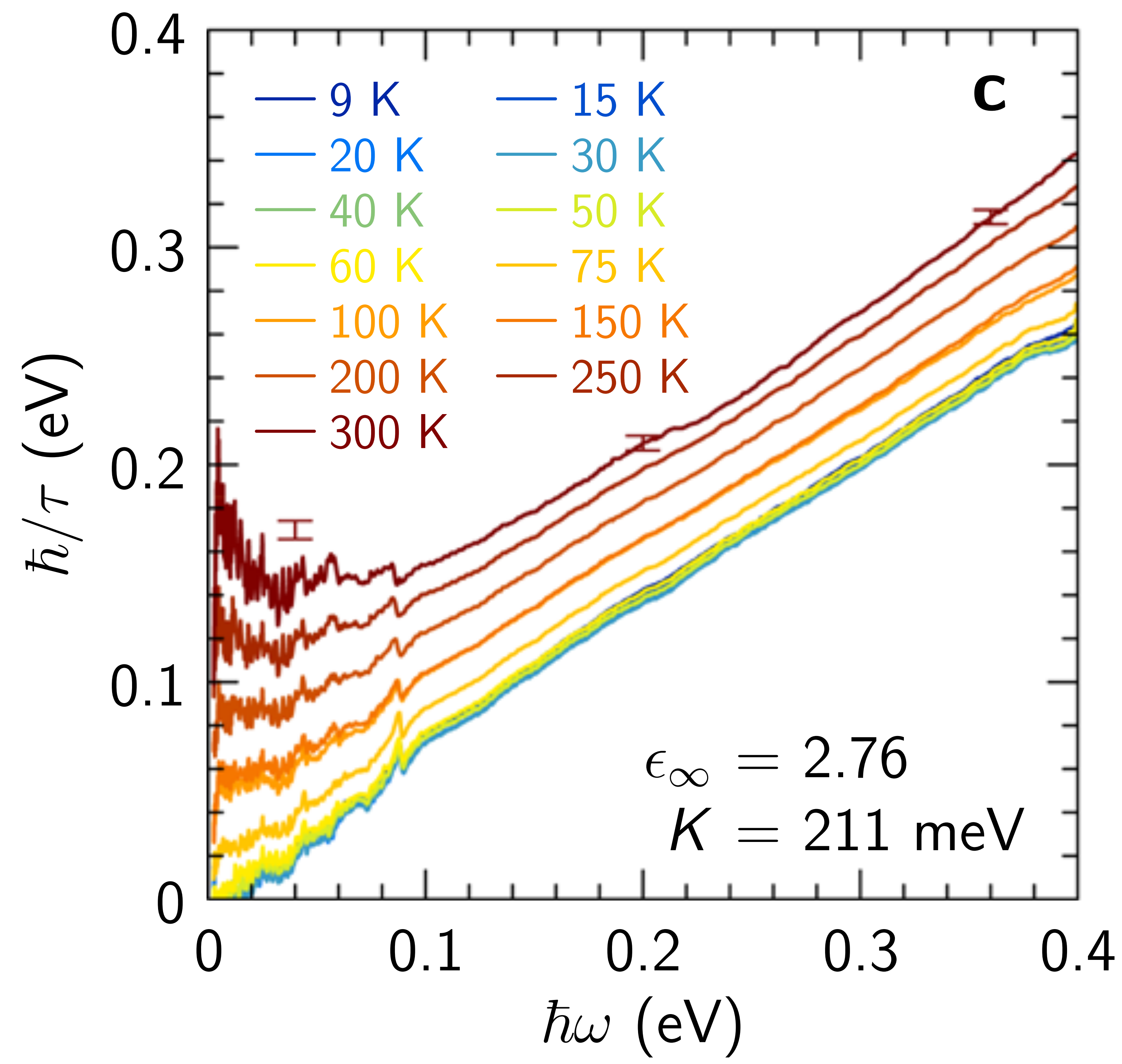
B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

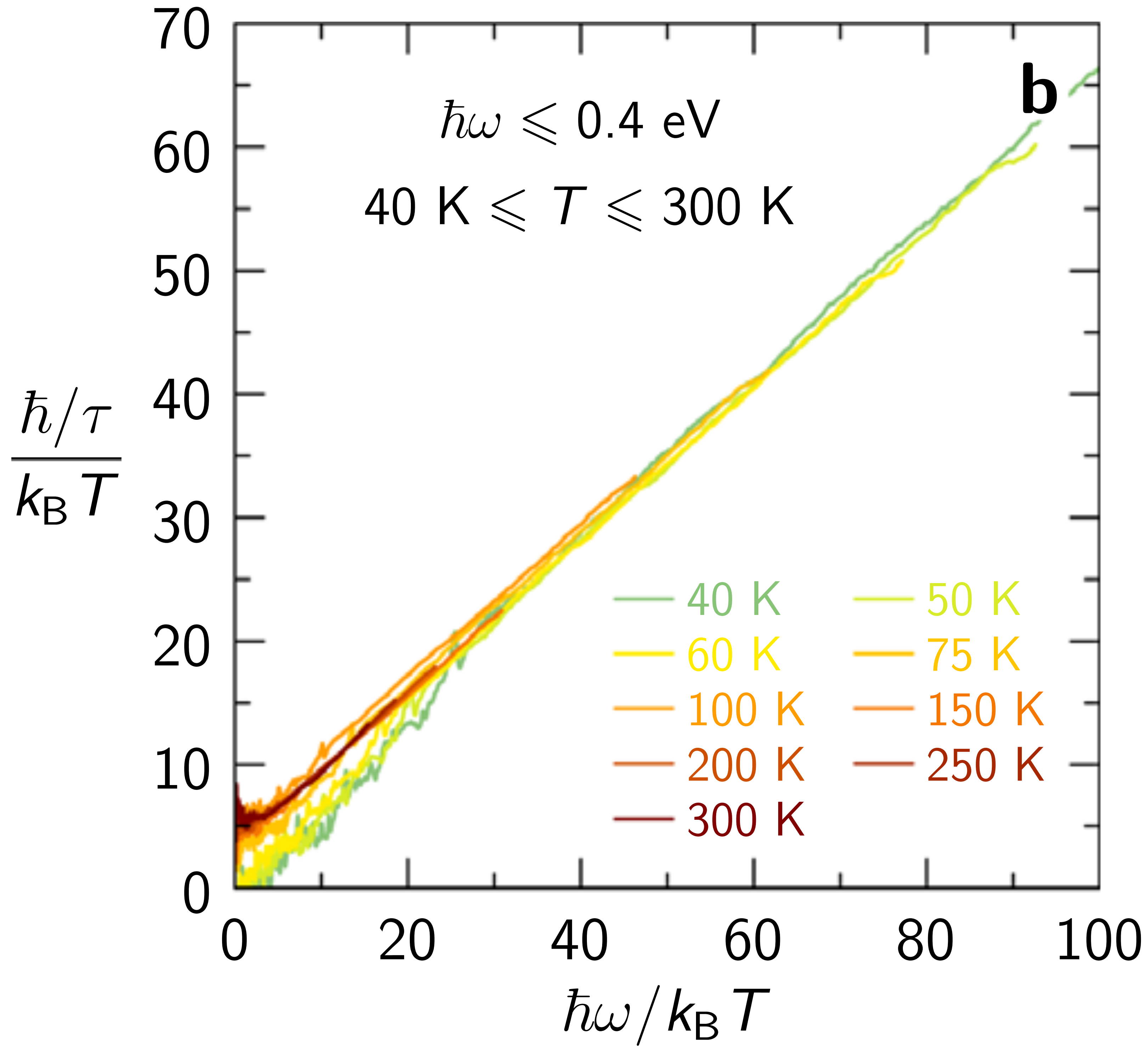
Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \frac{m^*(\omega)}{m} + i \frac{\hbar}{\tau(\omega)}}$$

Planckian dynamics!

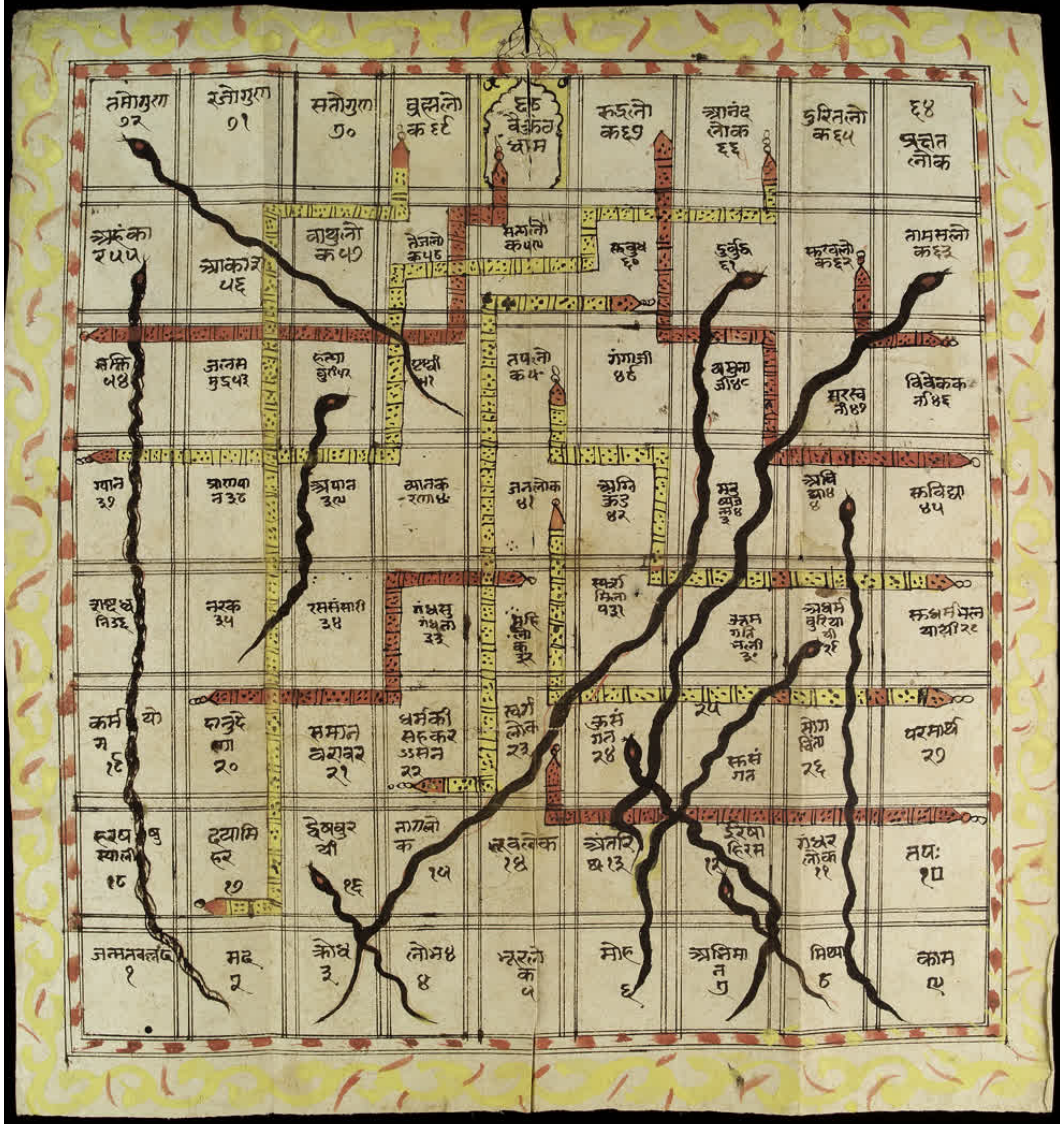
$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$





Needed:
a solvable model of
quantum entanglement
in a metal

The Sachdev-Ye-Kitaev model of many-particle entanglement



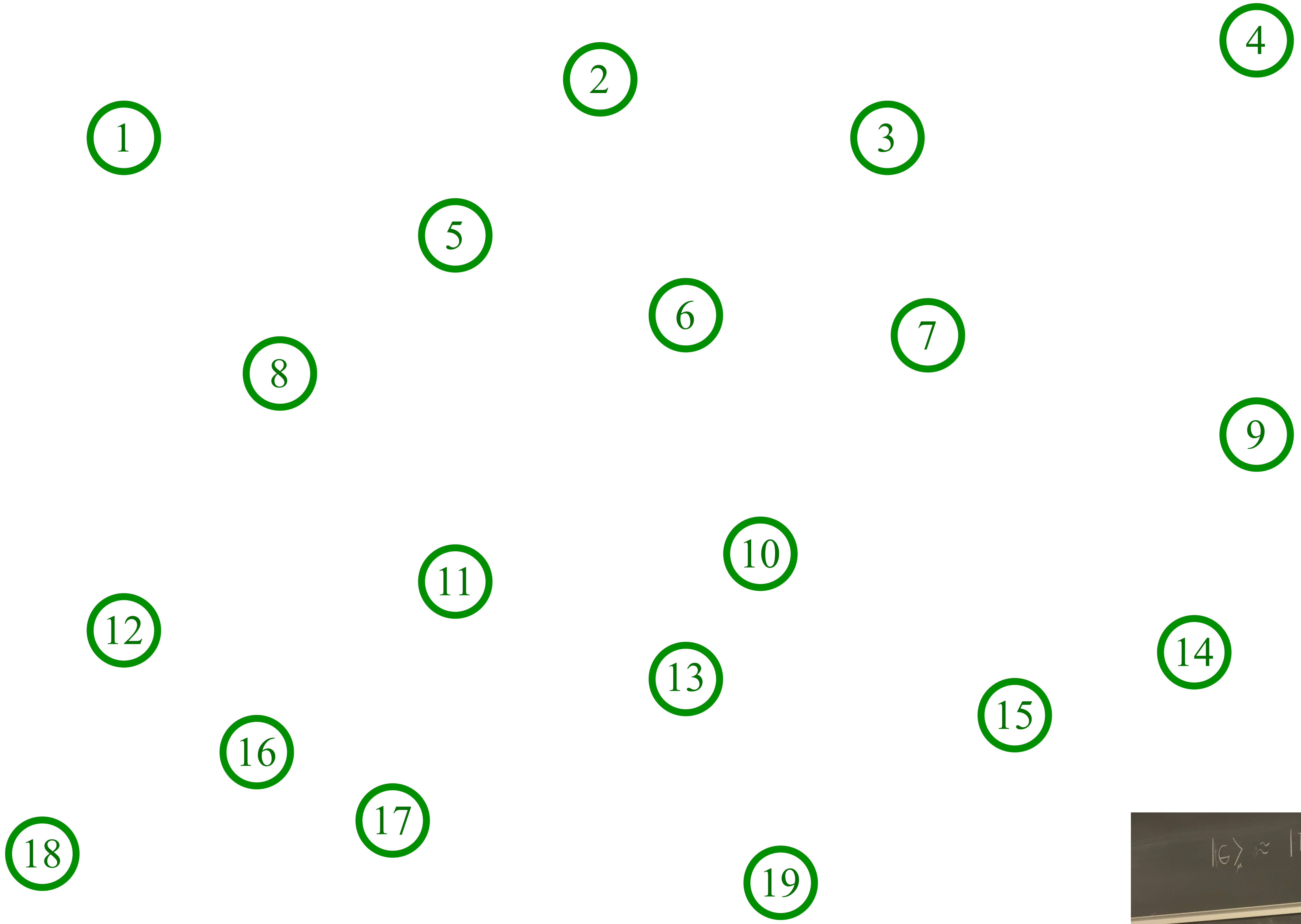
My spooky dream (1992)*

Ancient Indian game of Snakes and Ladders

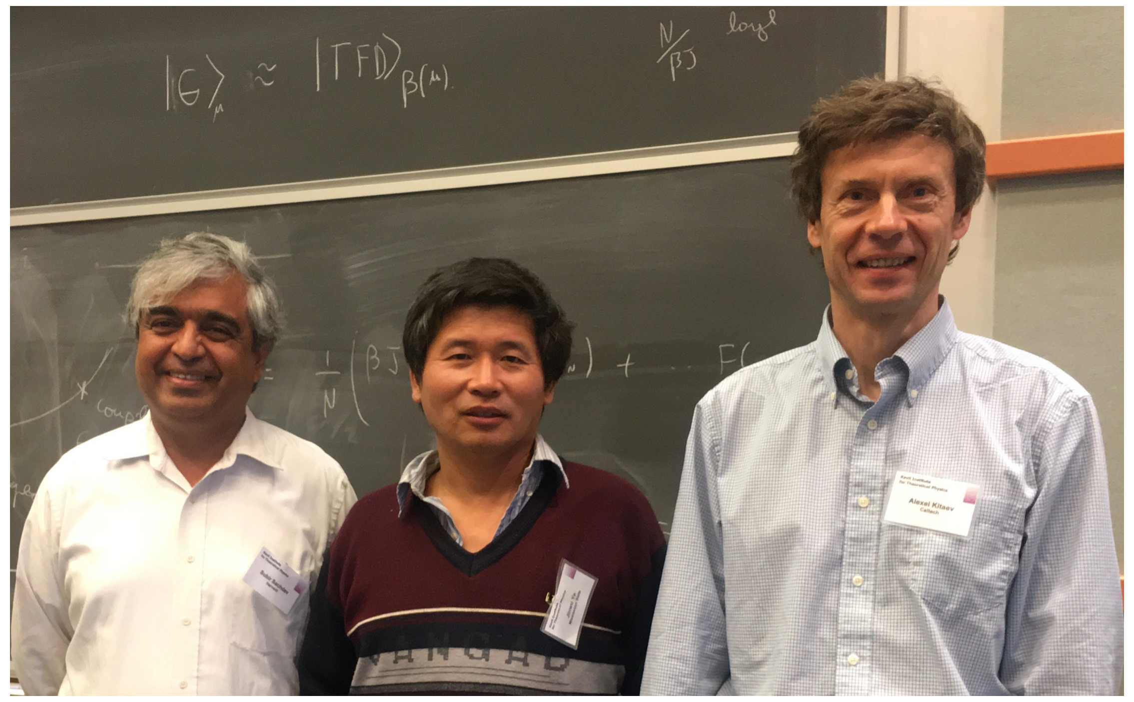
*Not true

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

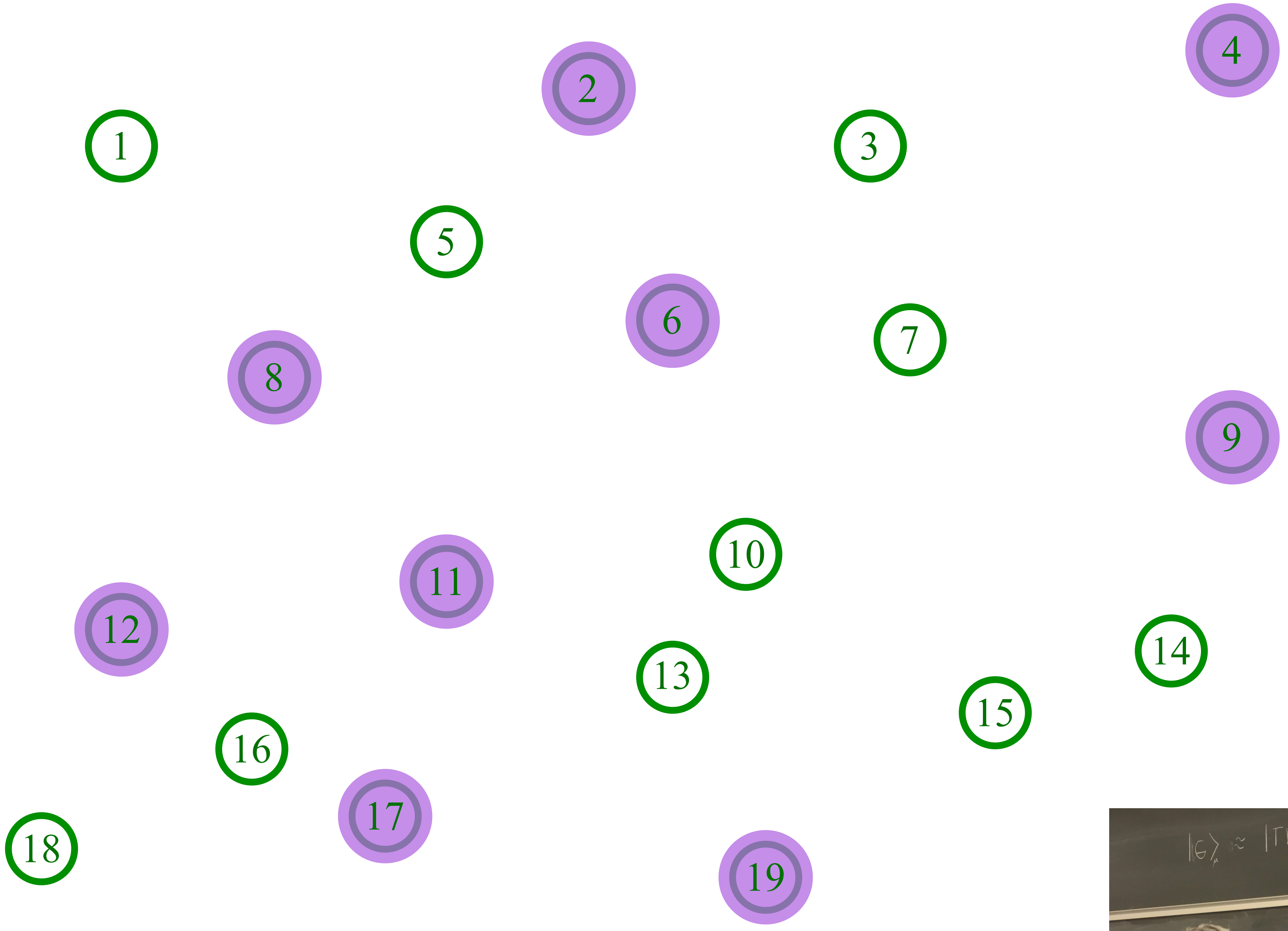


Pick a set of random positions

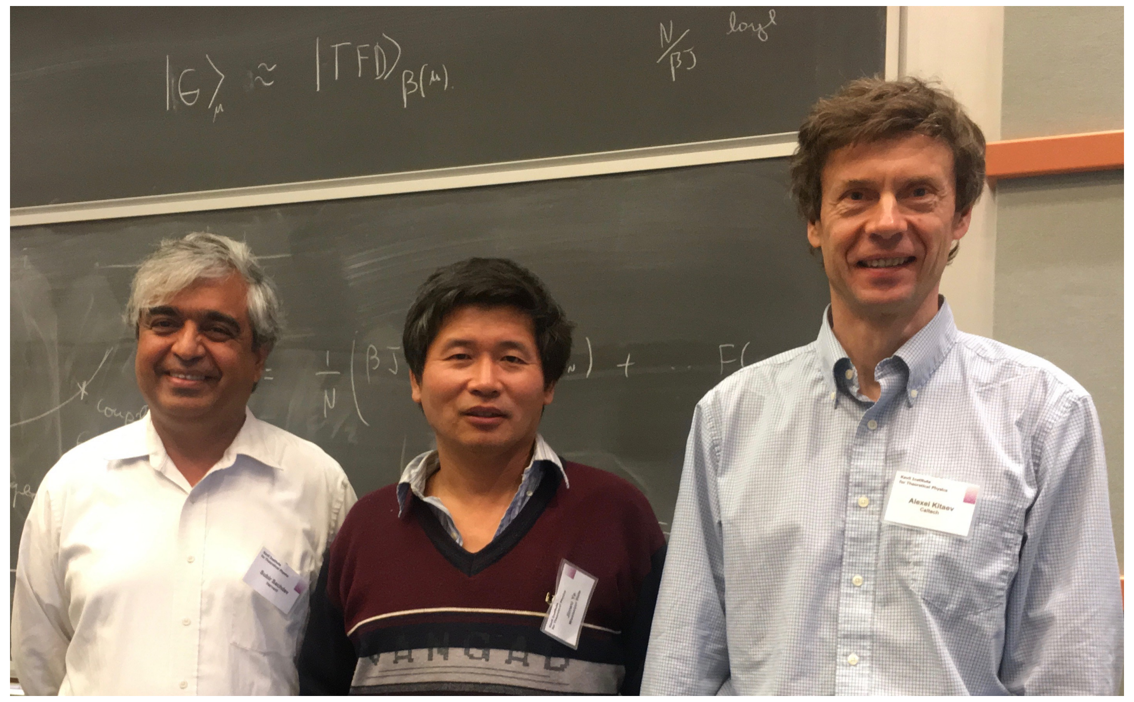


The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)



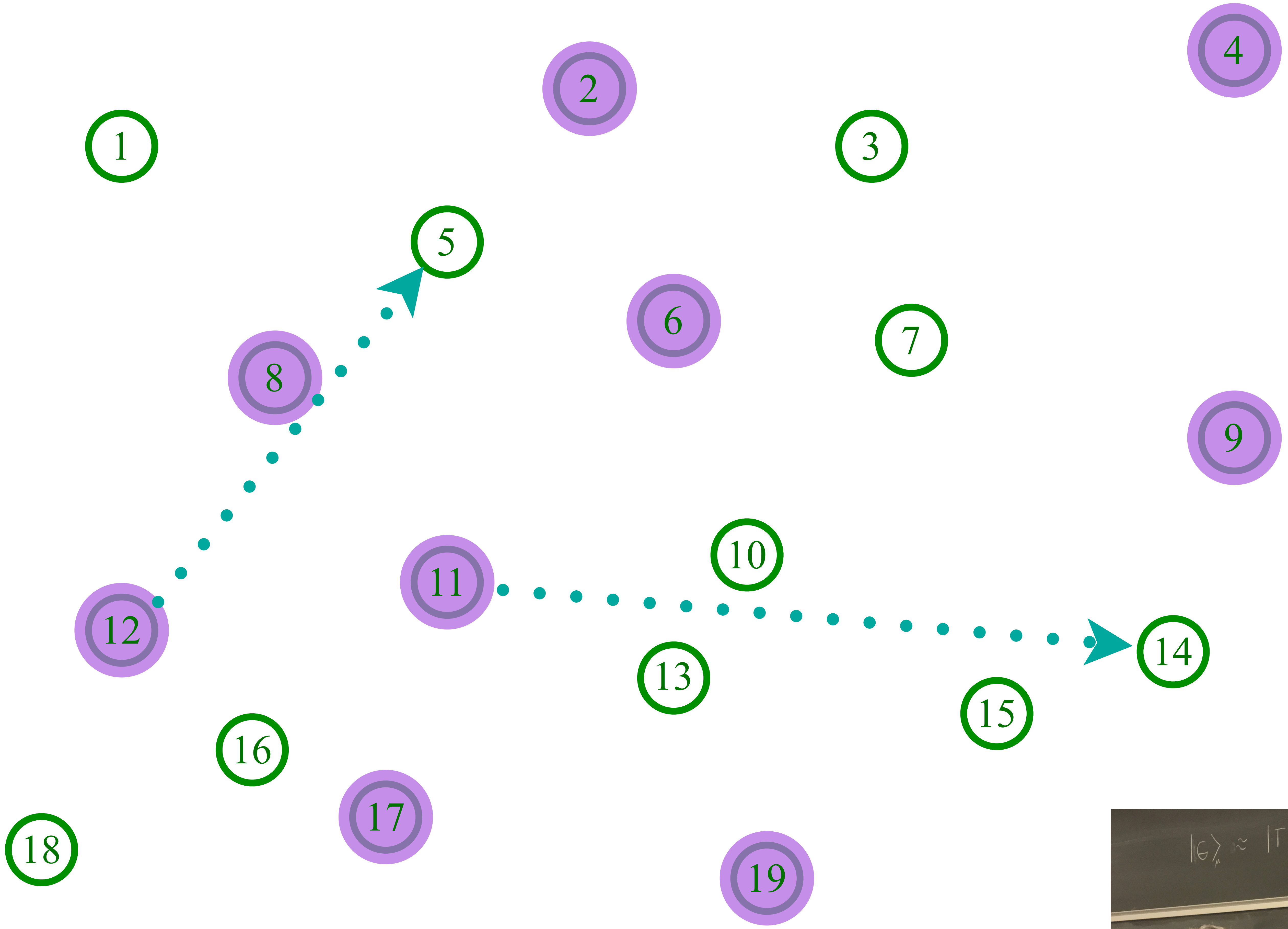
Place electrons randomly on some sites



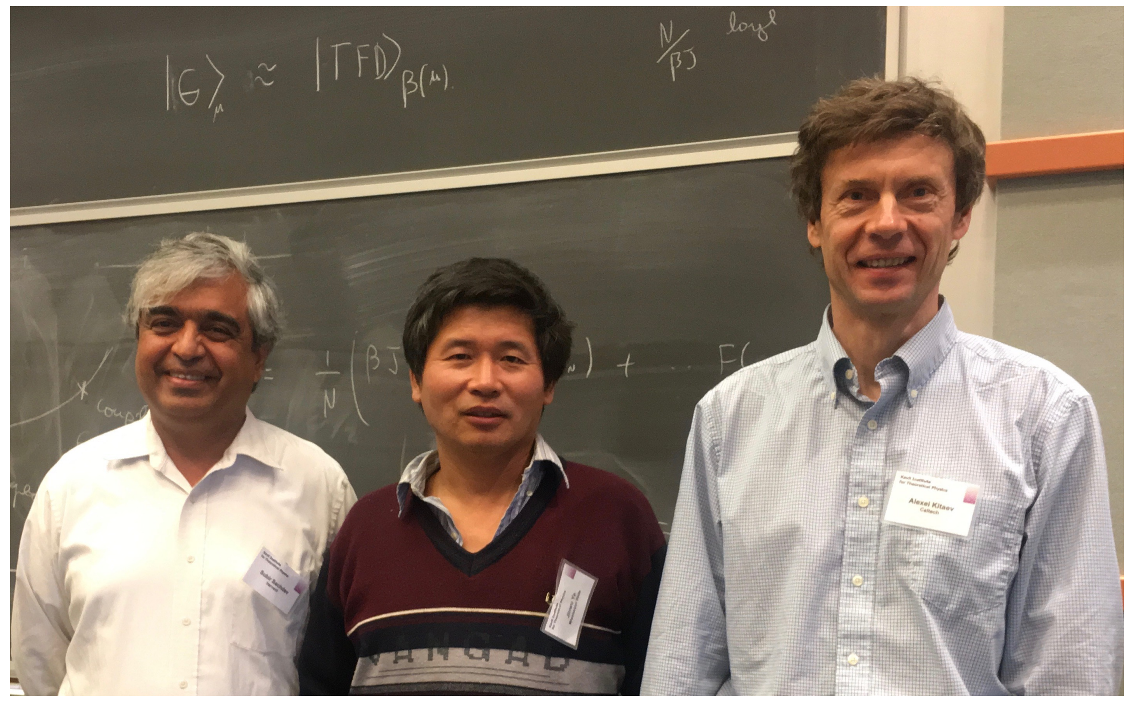
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{11,12;5,14}$$



Place electrons randomly on some sites



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

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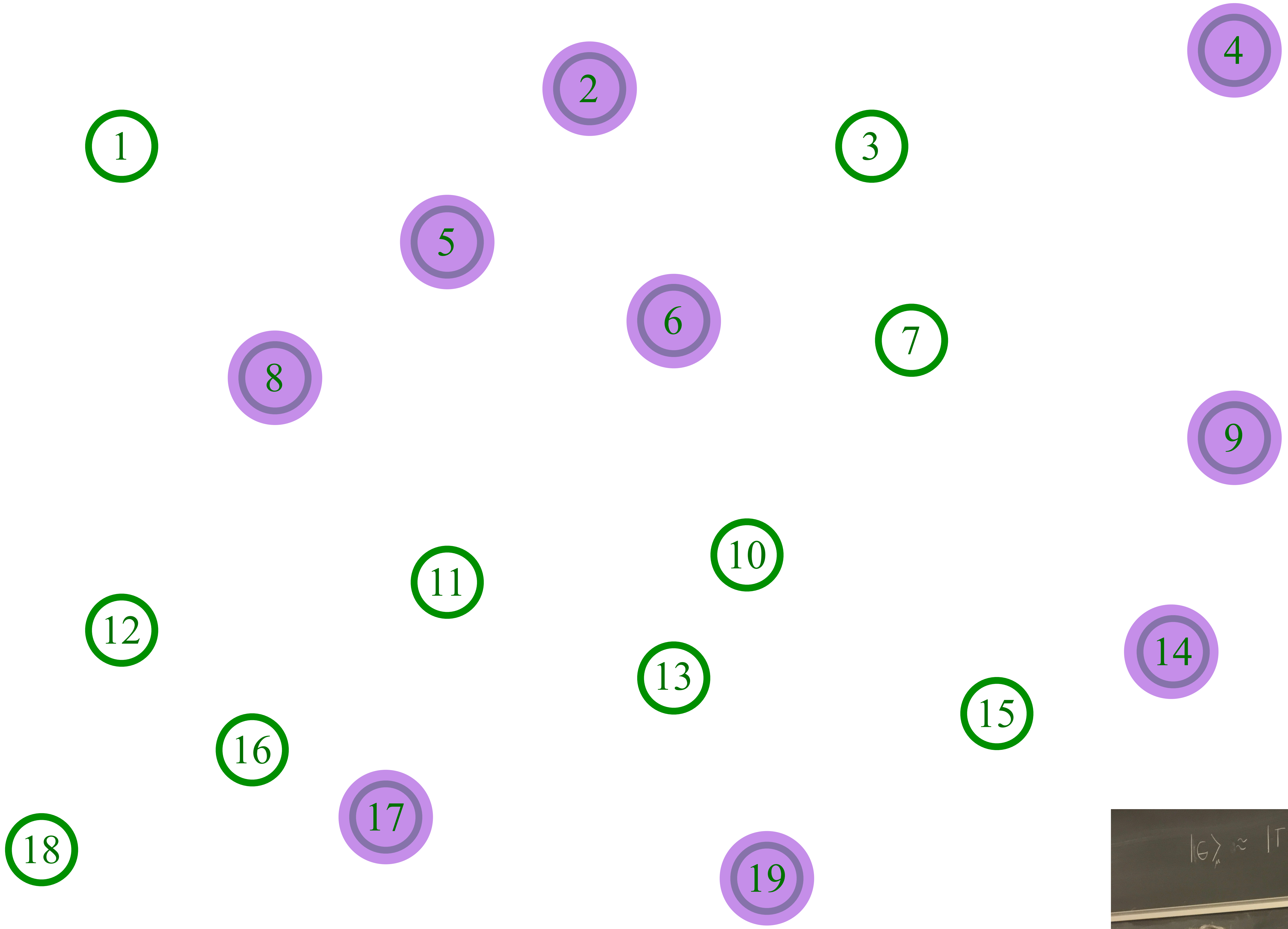
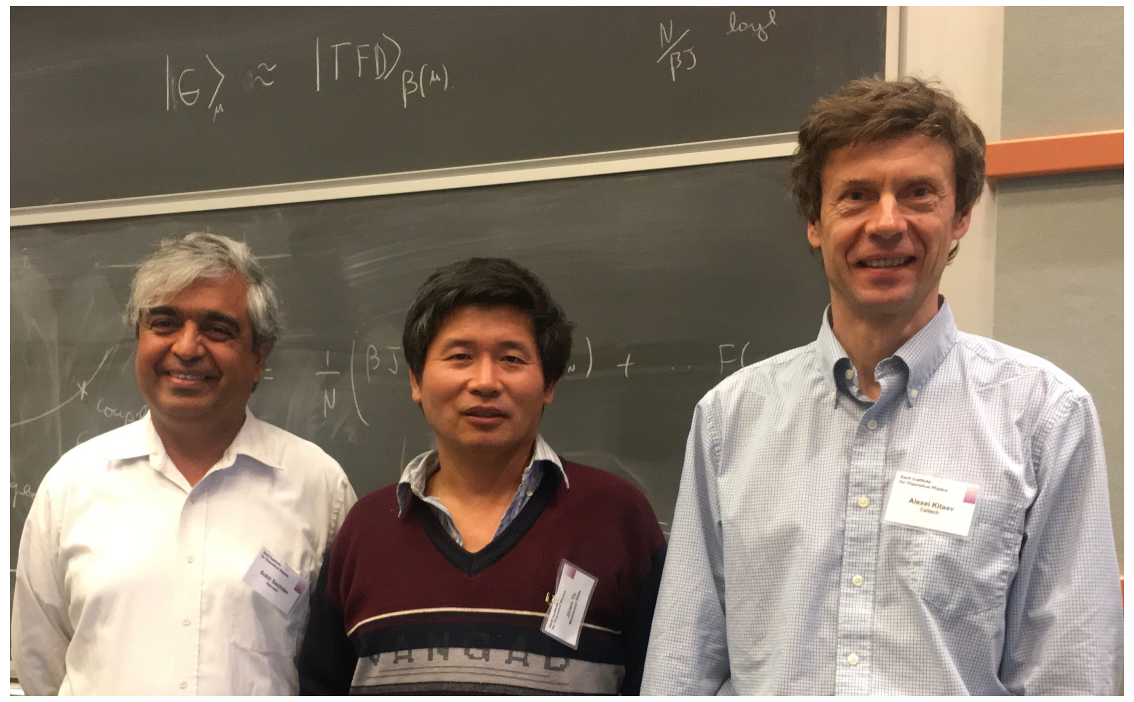


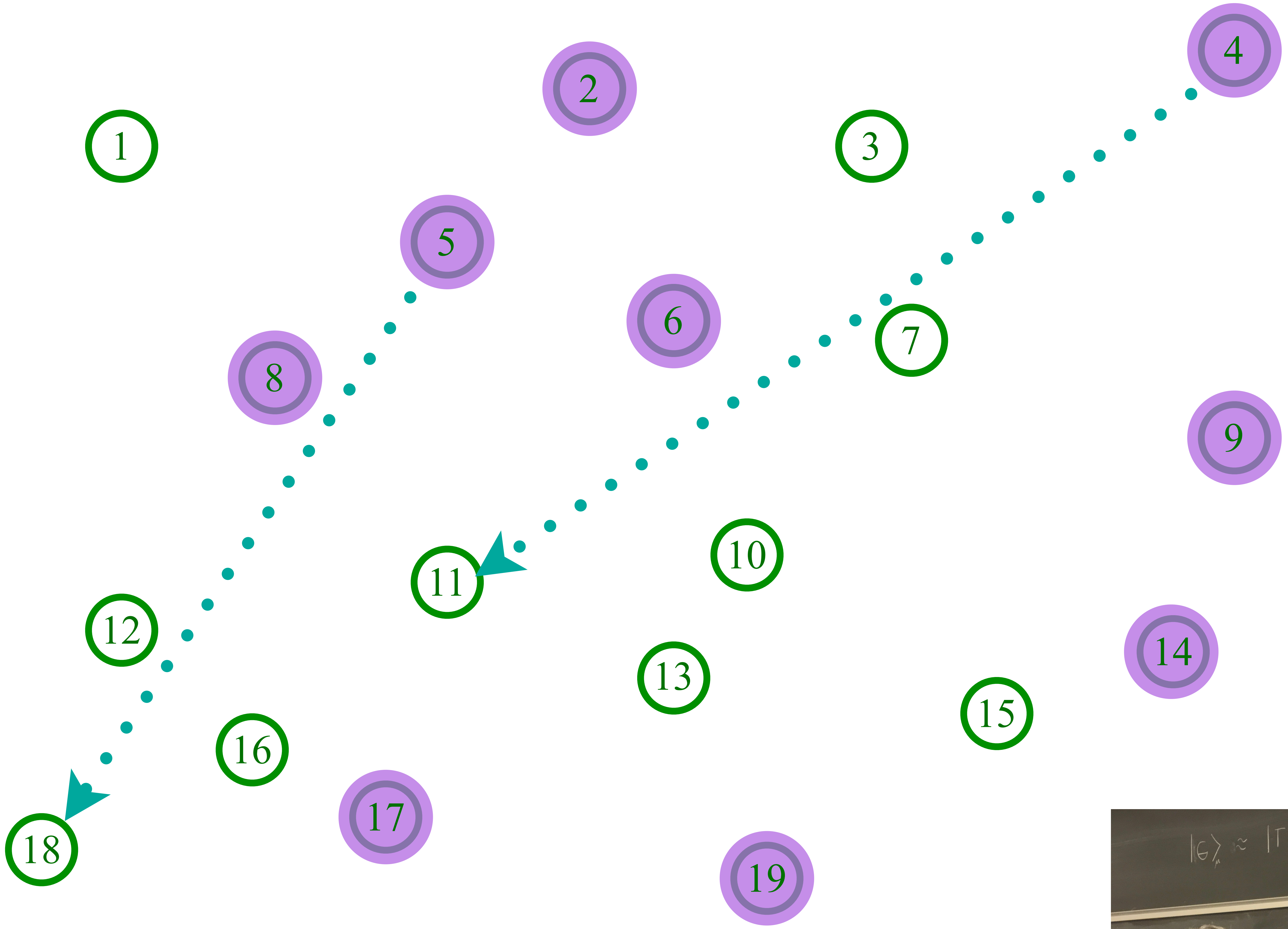
Entangle electrons pairwise randomly



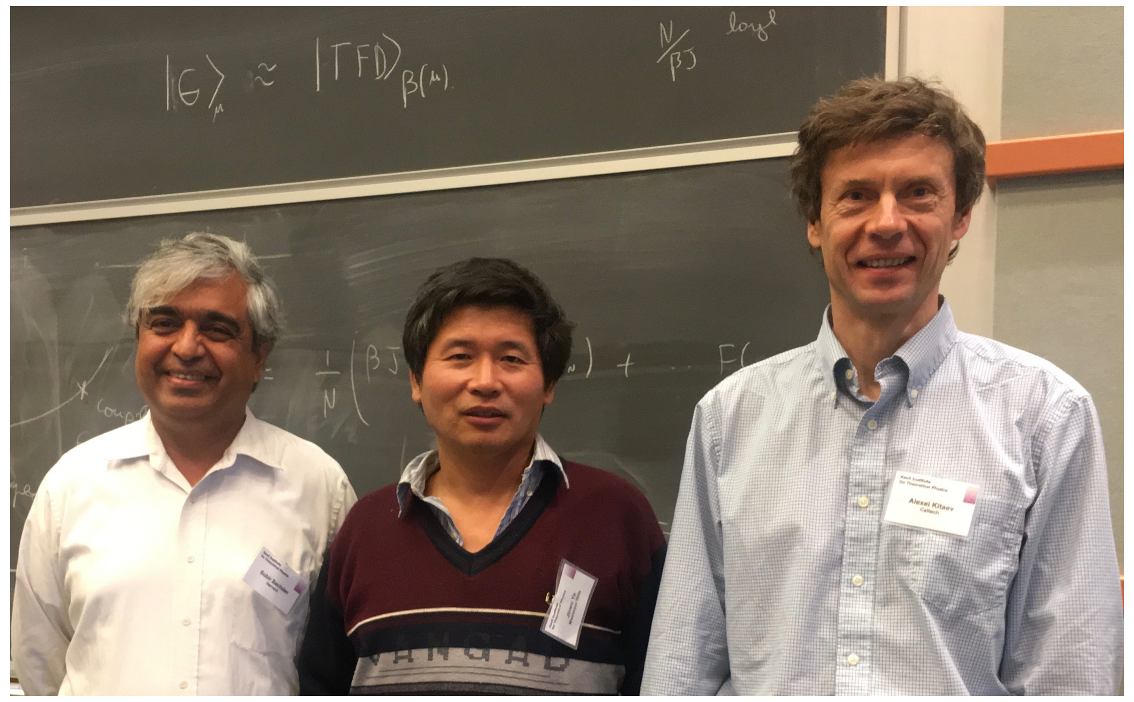
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{4,5;11,18}$$



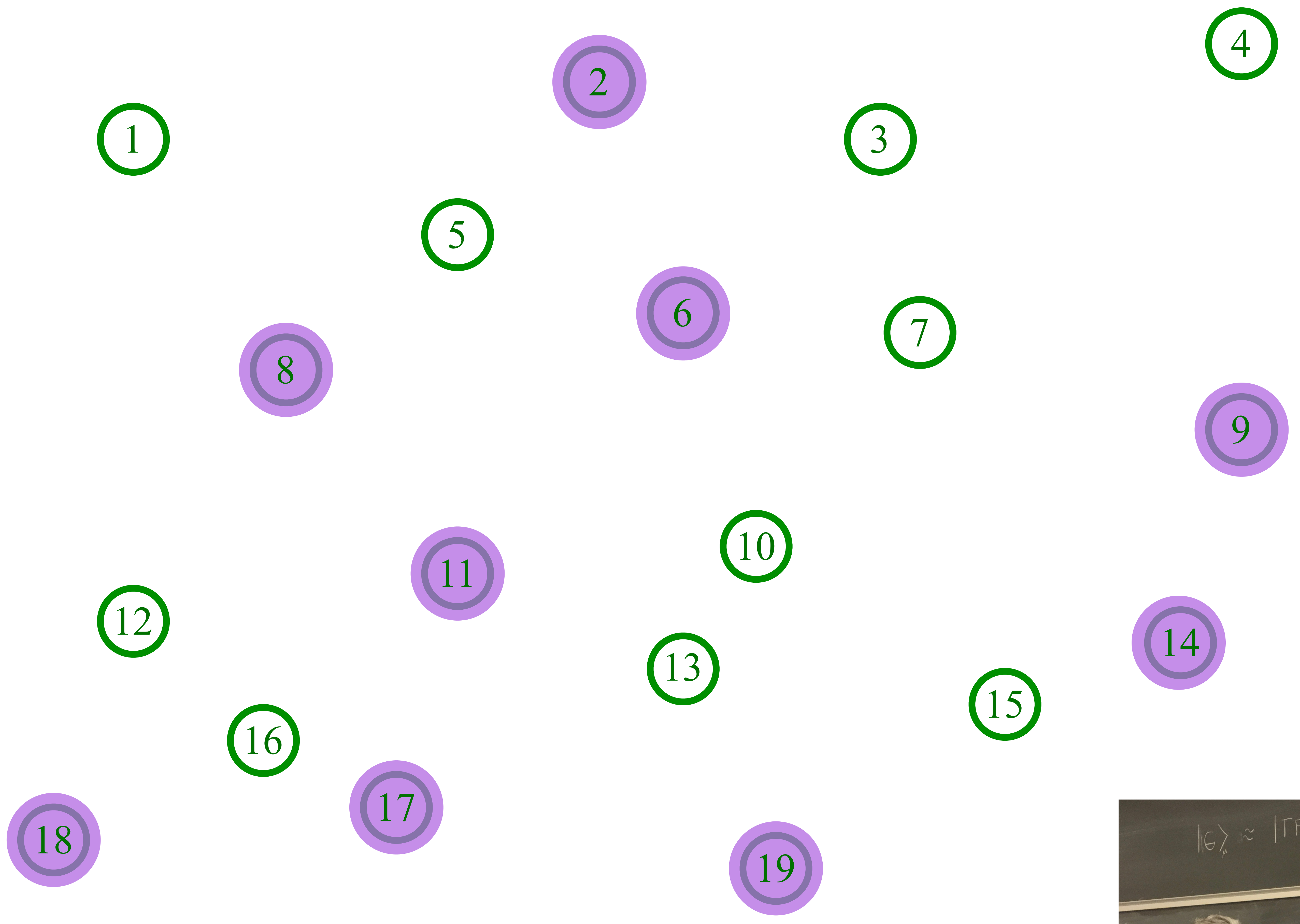
Entangle electrons pairwise randomly



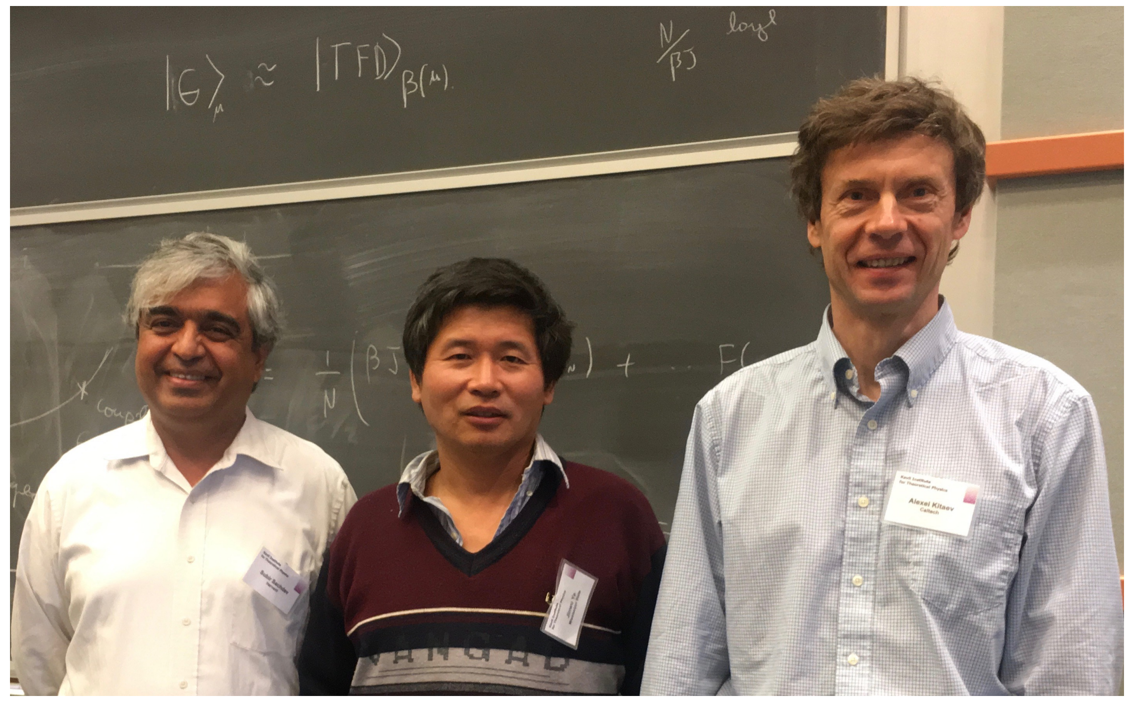
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$U_{4,5;11,18}$



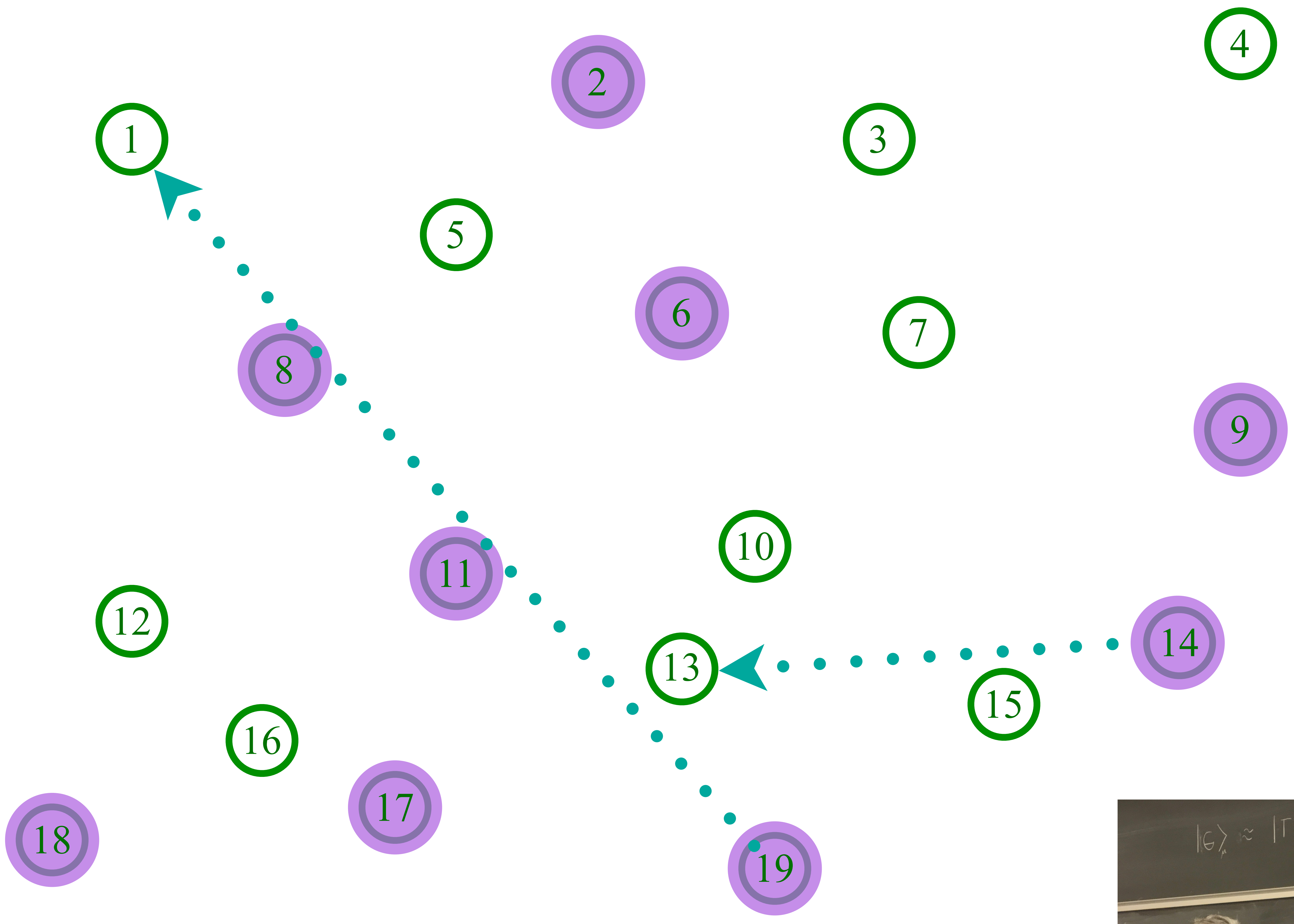
Entangle electrons pairwise randomly



The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



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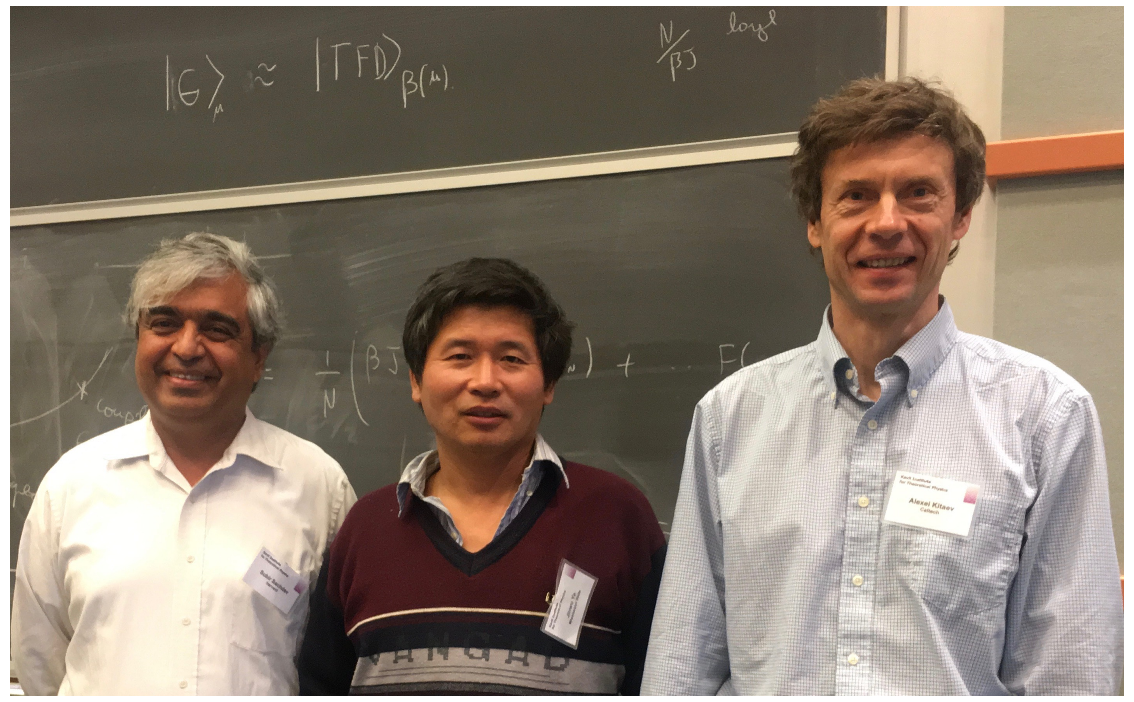
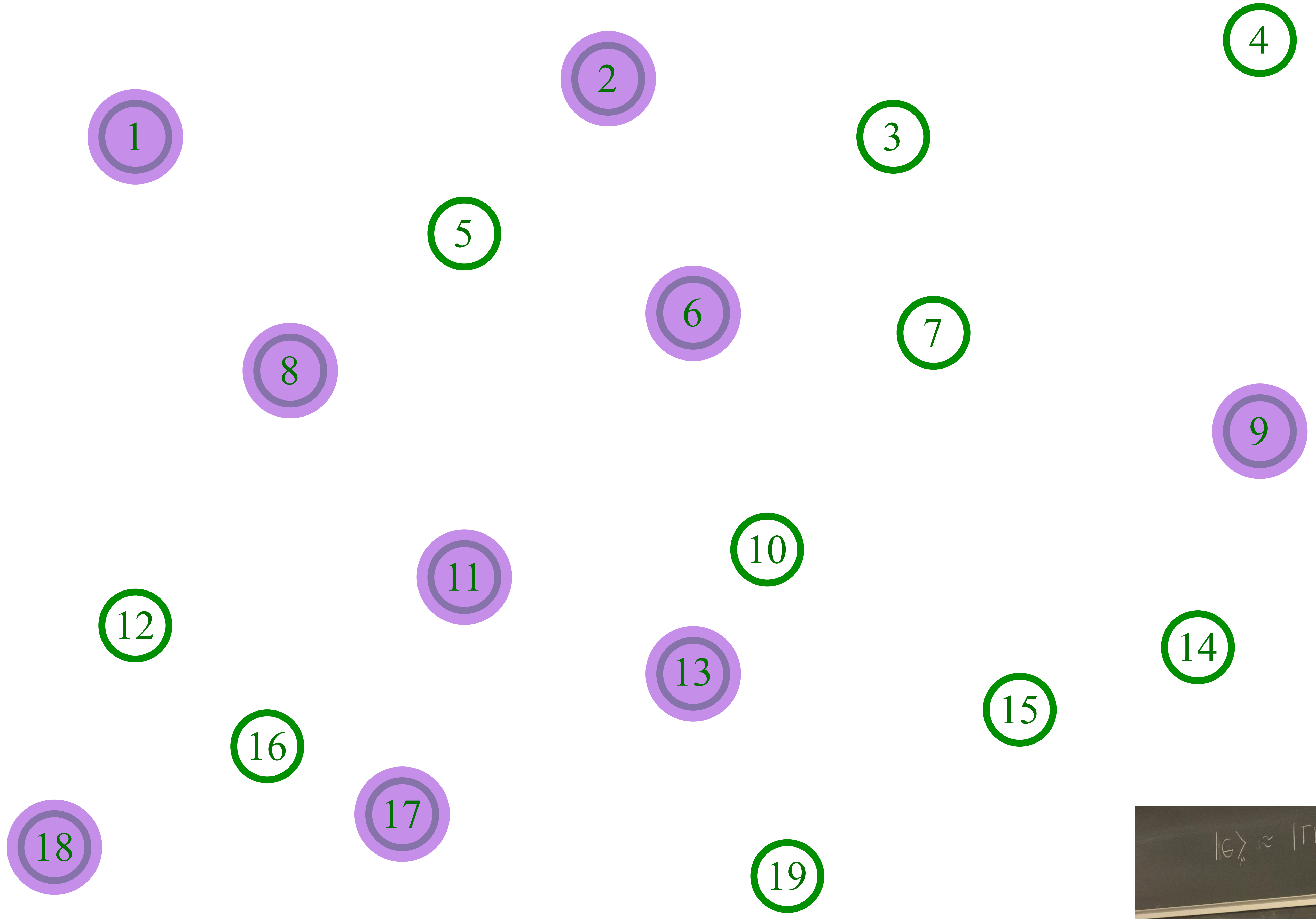


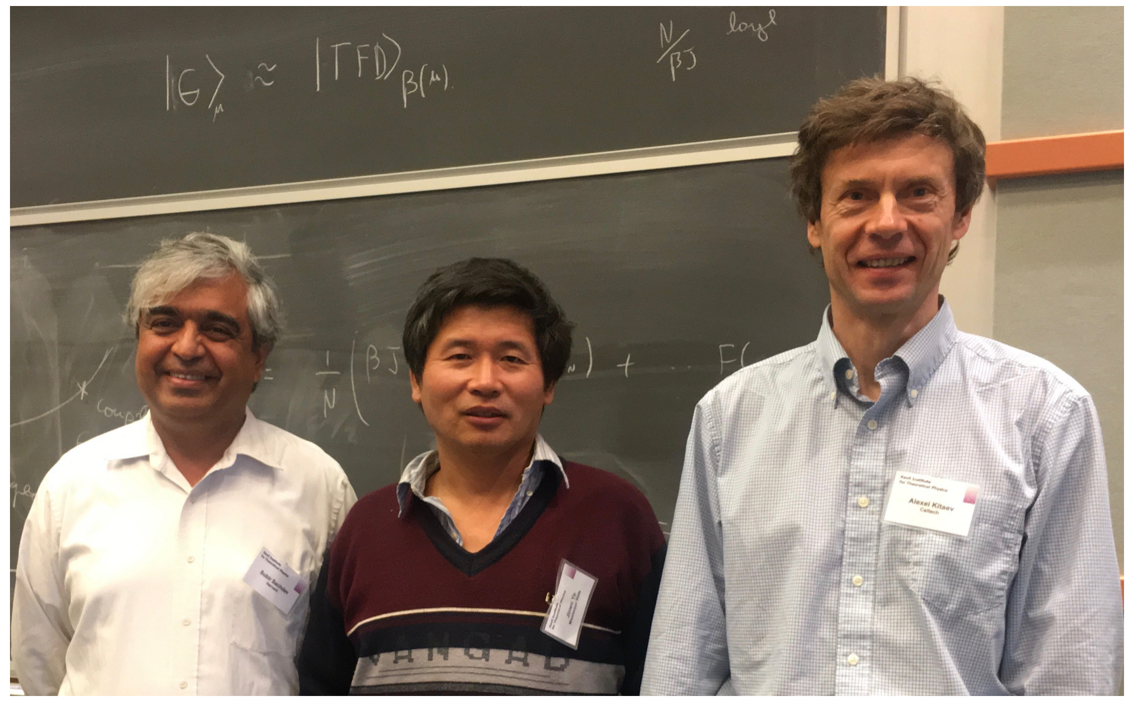
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{14,19;1,13}$$



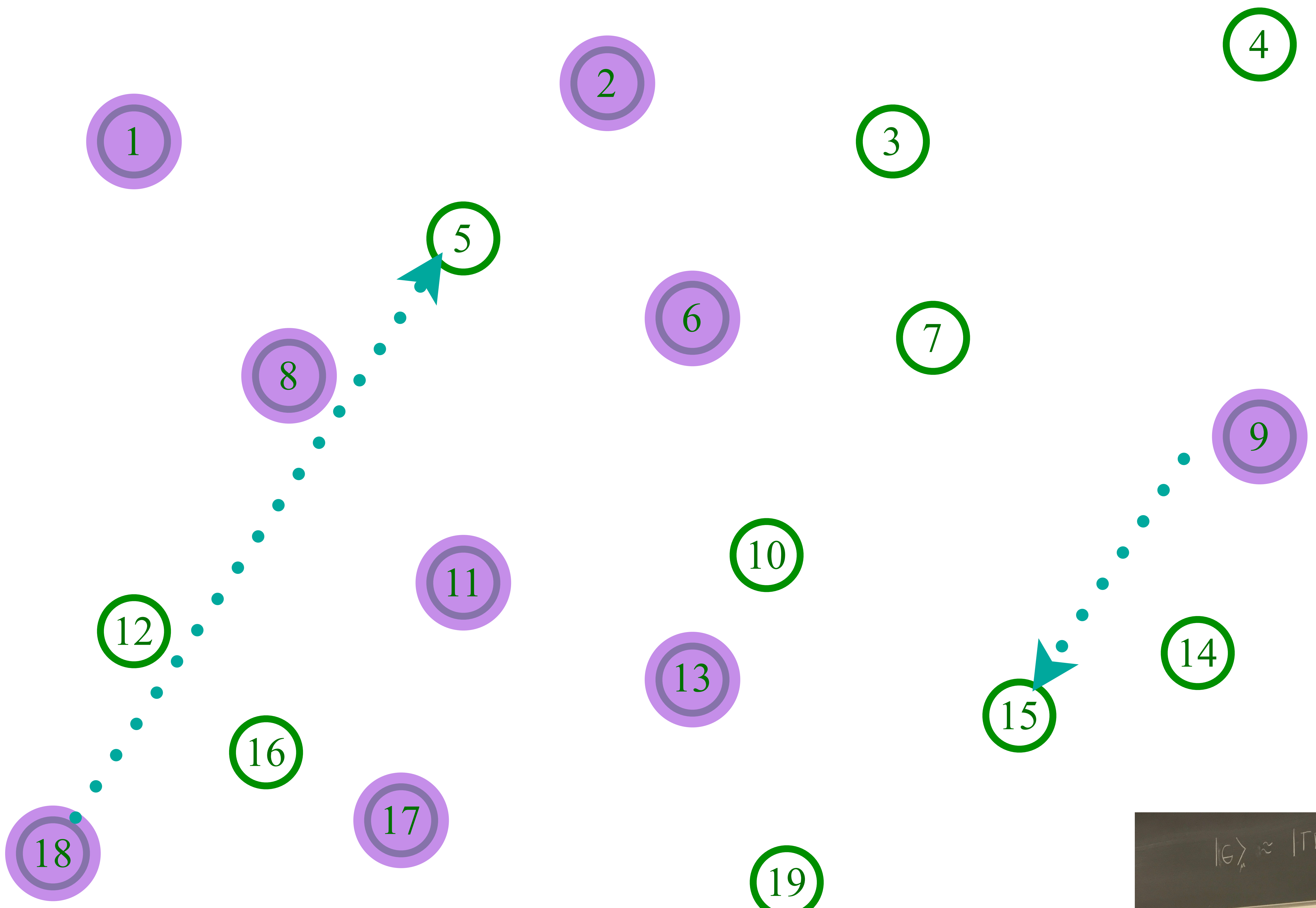
Entangle electrons pairwise randomly



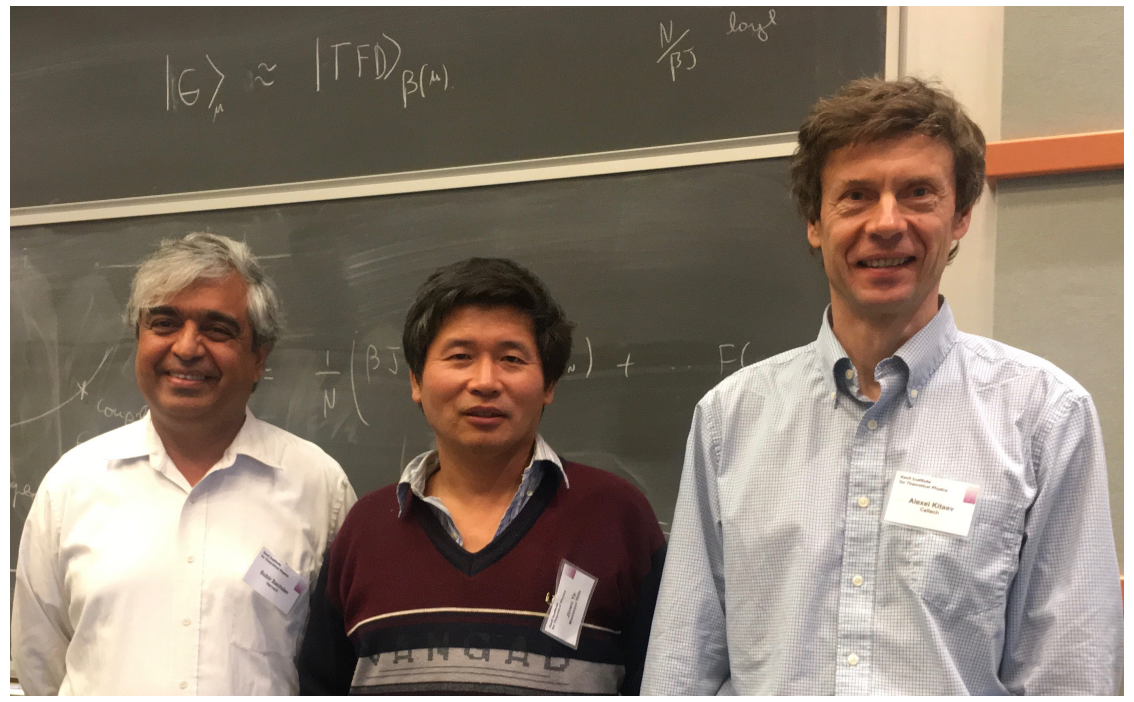
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



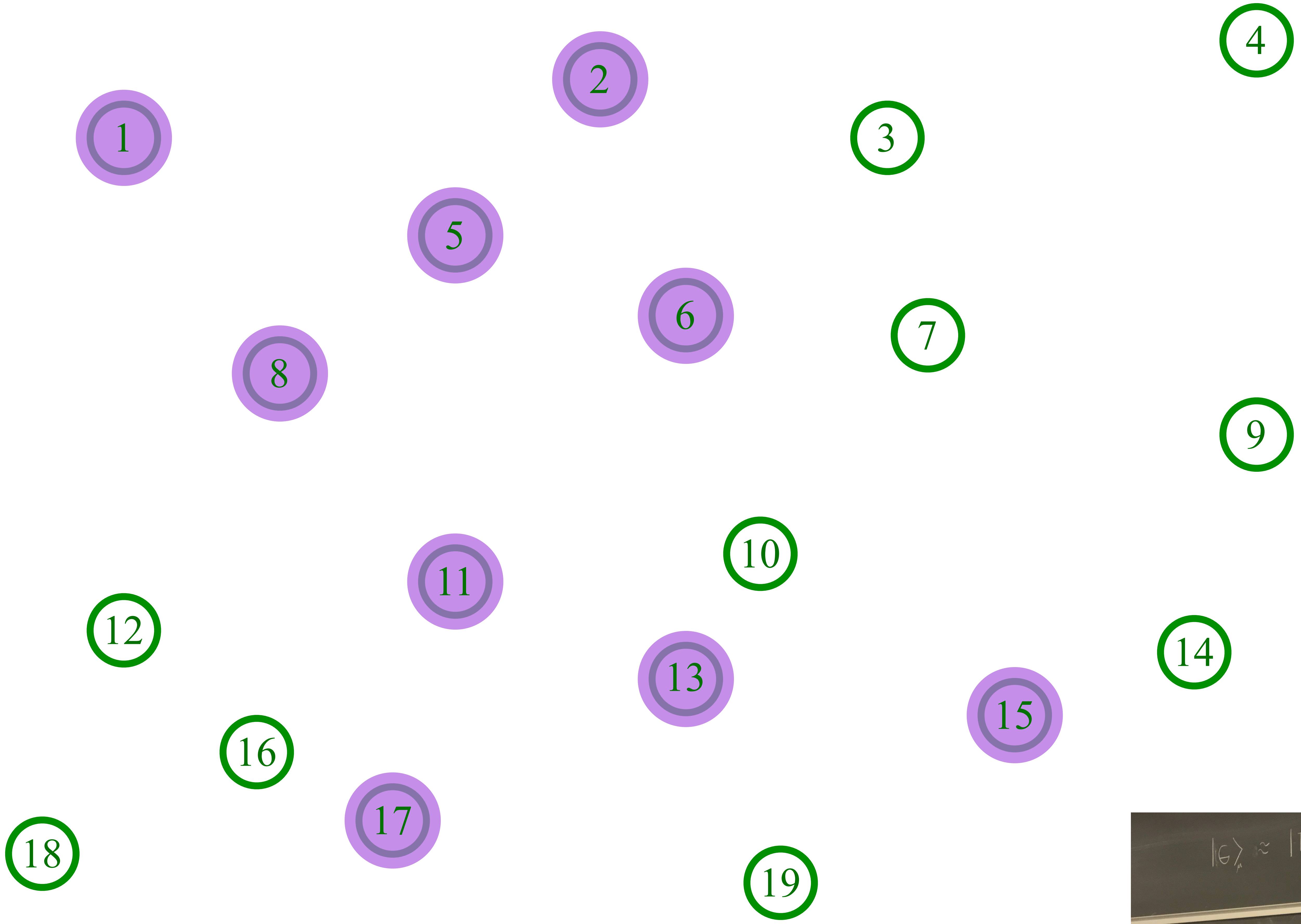
Entangle electrons pairwise randomly



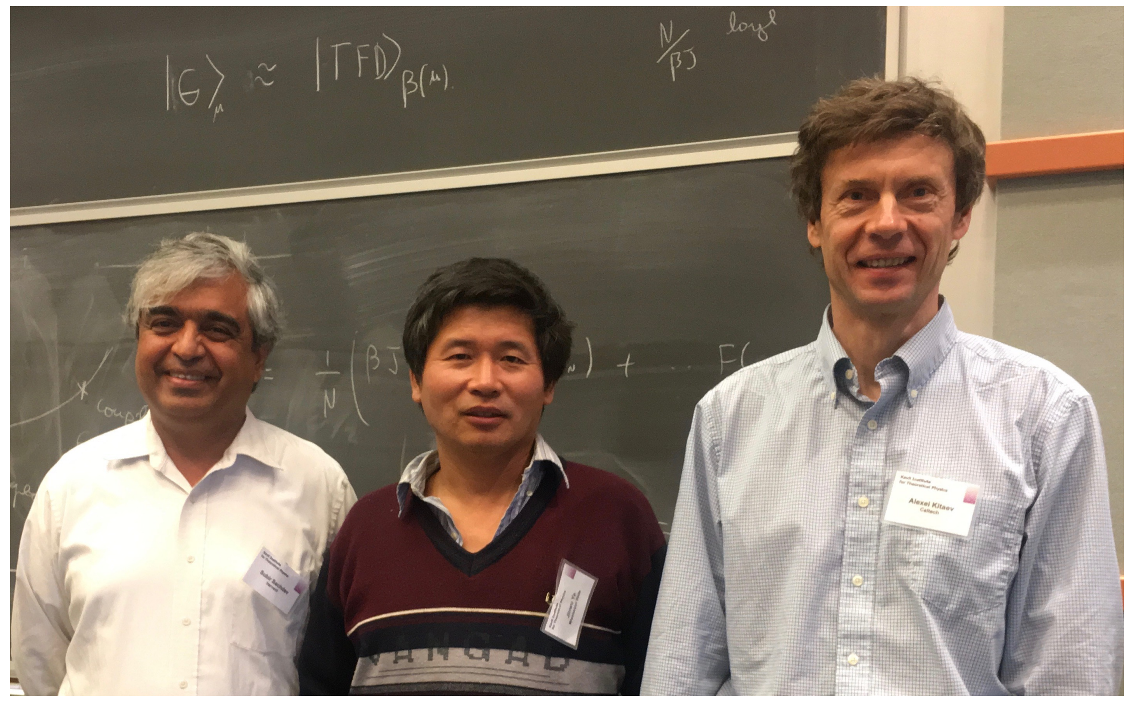
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{9,18;5,15}$$



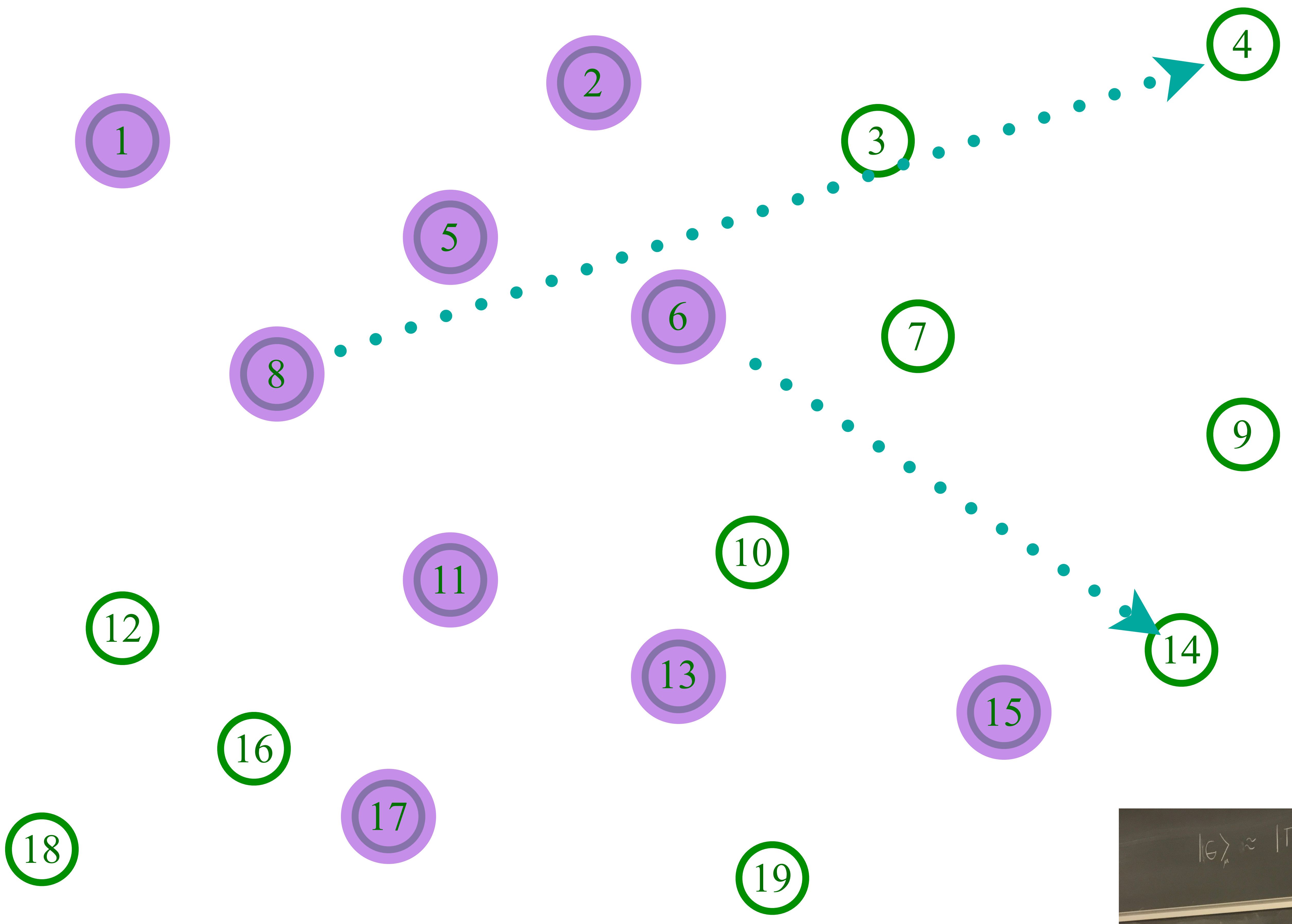
Entangle electrons pairwise randomly



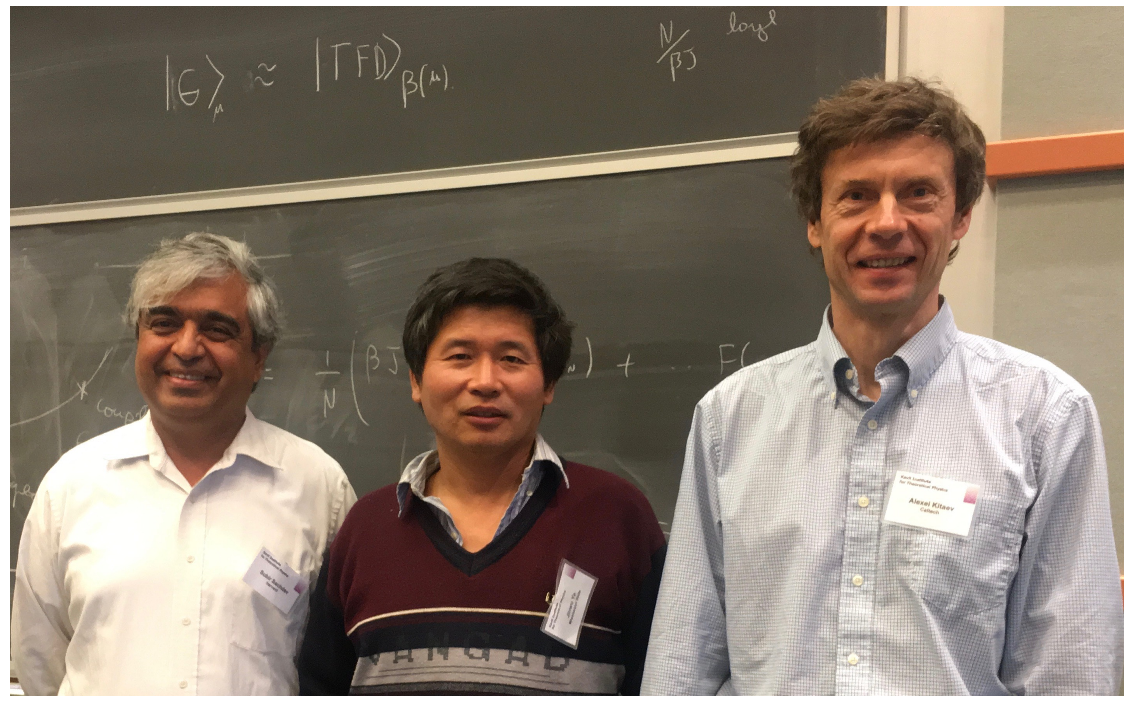
The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



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Sachdev, Ye (1993); Kitaev (2015)

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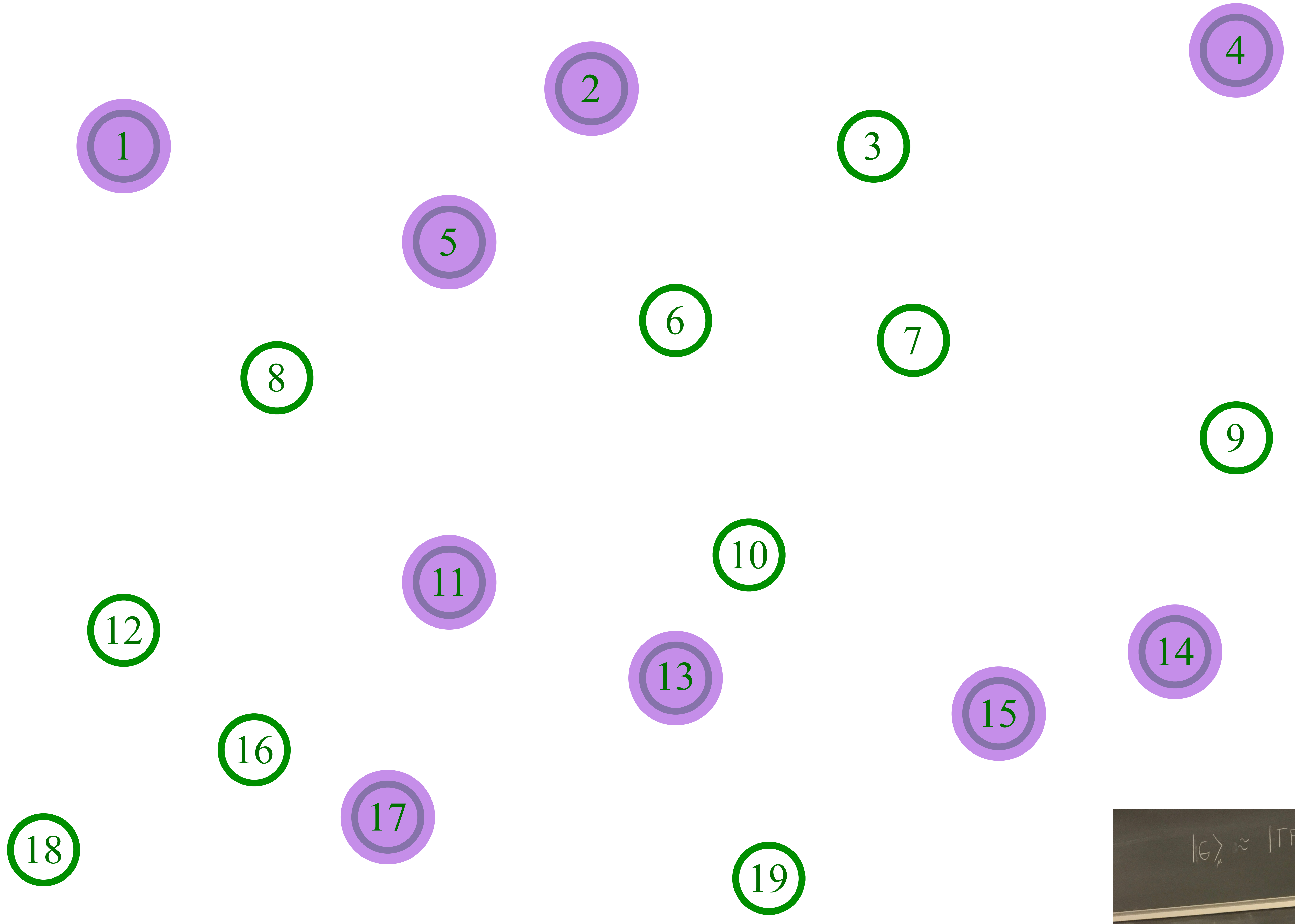
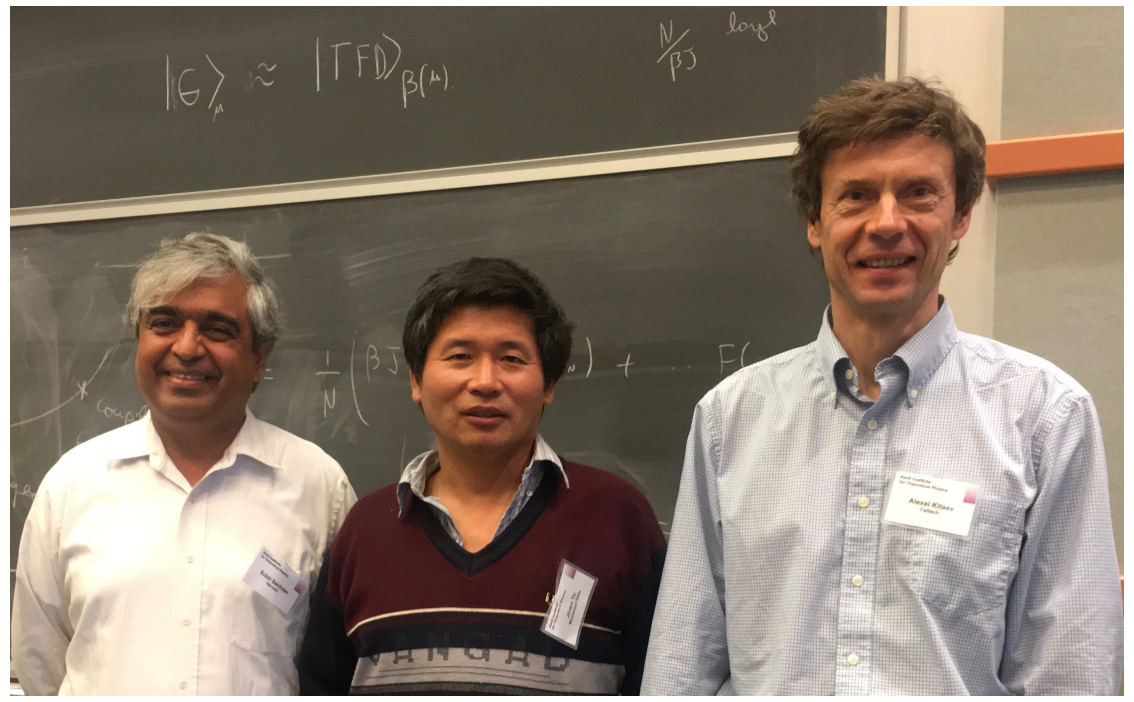




Entangle electrons pairwise randomly

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

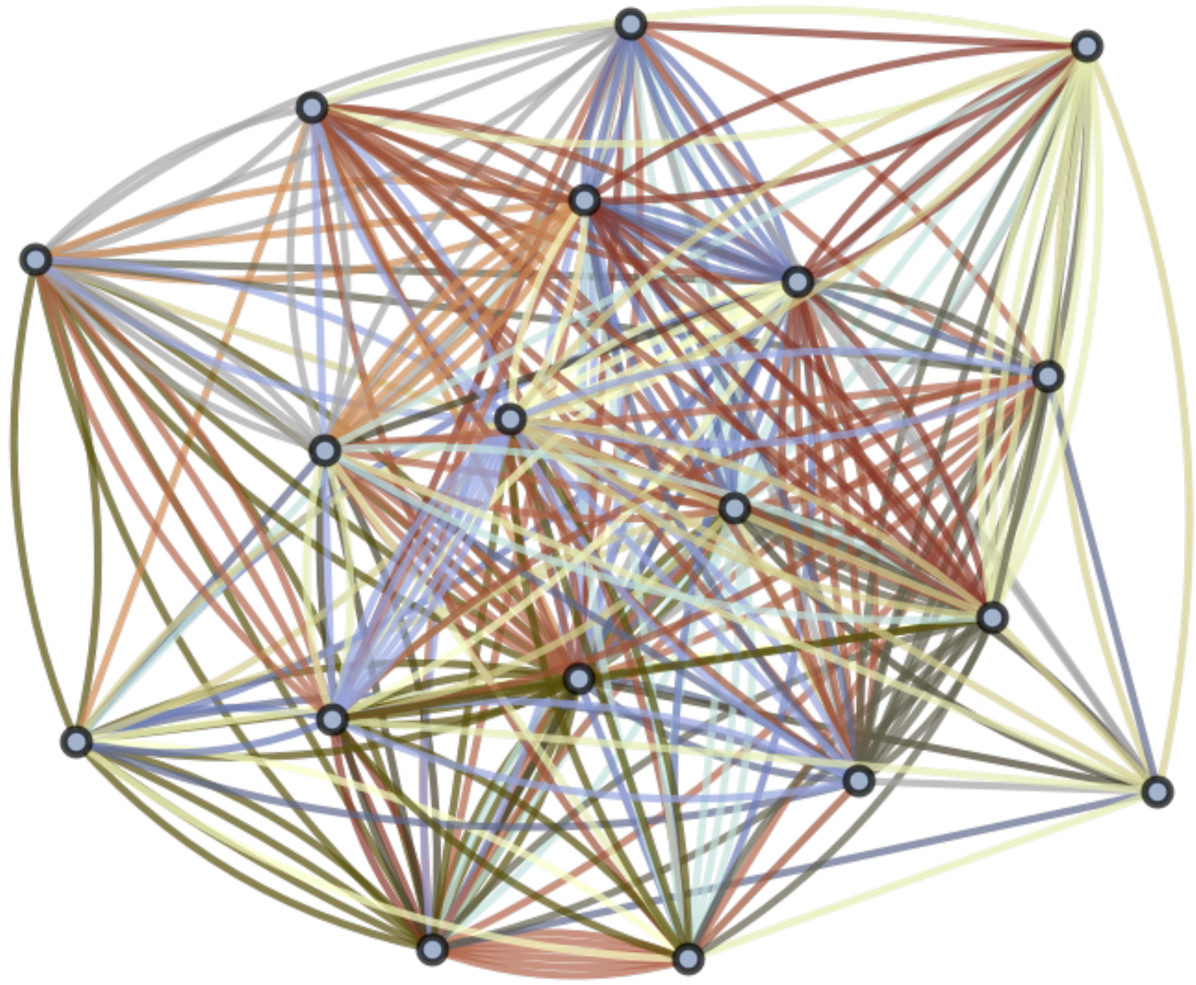
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha \,; \quad [\mathcal{H}, \mathcal{Q}] = 0\,; \quad 0 \leq \mathcal{Q} \leq 1$$

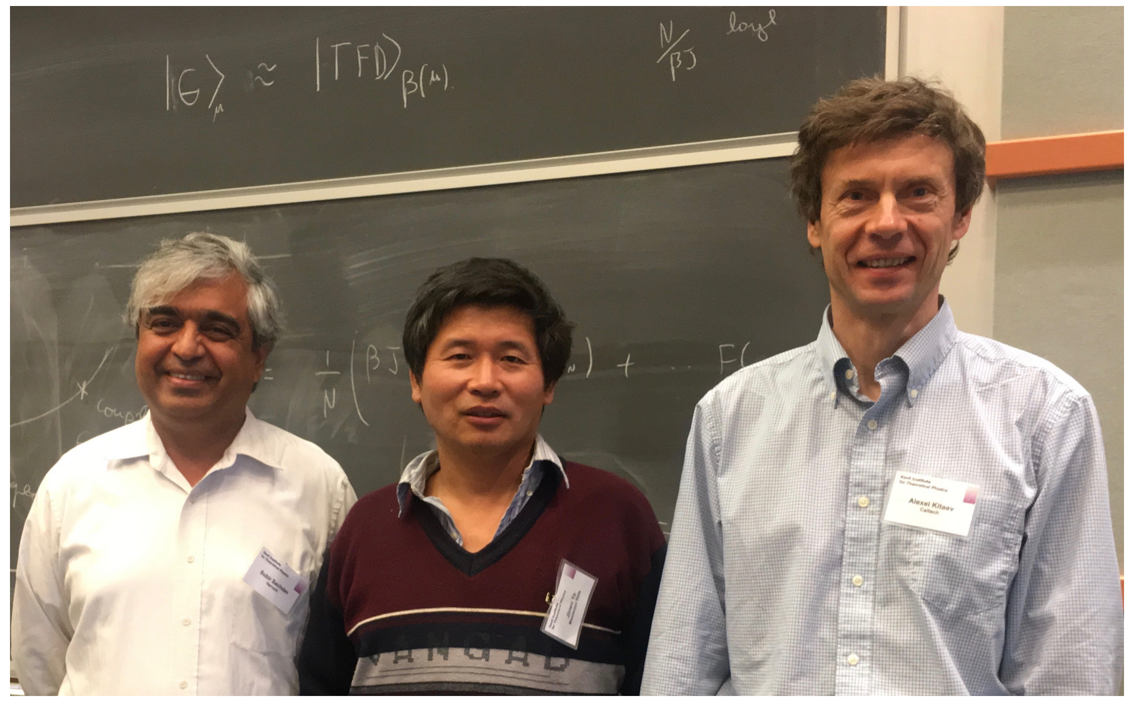
$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

A solvable model of multi-particle quantum entanglement.

No quasiparticles: yields a metal in which current is carried not by individual electrons, but by an entangled "quantum soup"

The SYK model

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

1. Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right) \text{ independent of } U.$$

No bosons, fermions, anyons ...

S. Sachdev and J. Ye, PRL **70**, 3339 (1993); A. Georges and O. Parcollet PRB **59**, 5341 (1999)

The SYK model

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No bosons, fermions, anyons ...

2. Thermodynamics ...

S. Sachdev and J. Ye, PRL **70**, 3339 (1993); A. Georges and O. Parcollet PRB **59**, 5341 (1999)

A simple model of a metal with quasiparticles

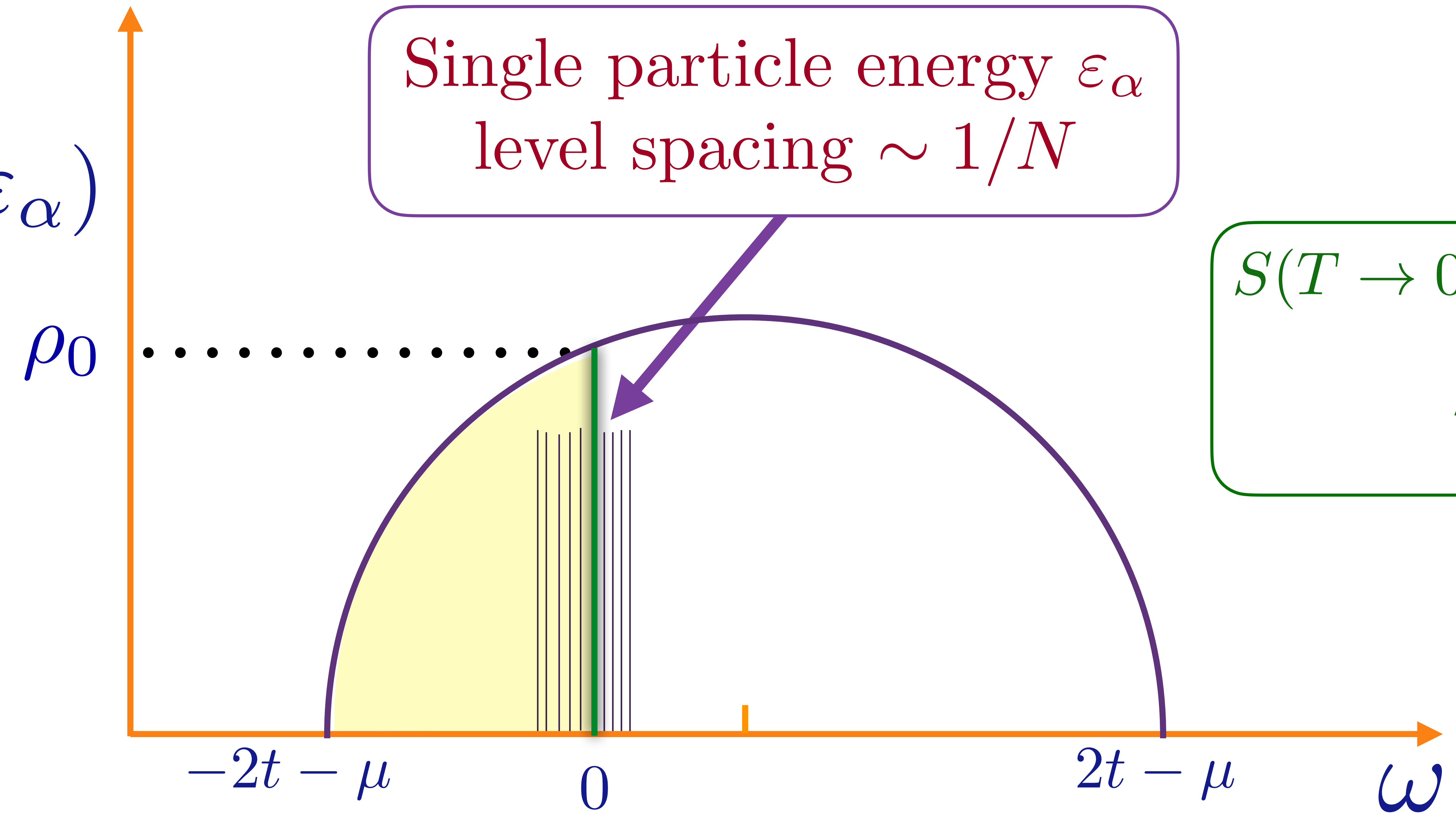
$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

$\rho(\omega) = \frac{1}{N} \sum_\alpha \delta(\omega - \varepsilon_\alpha)$



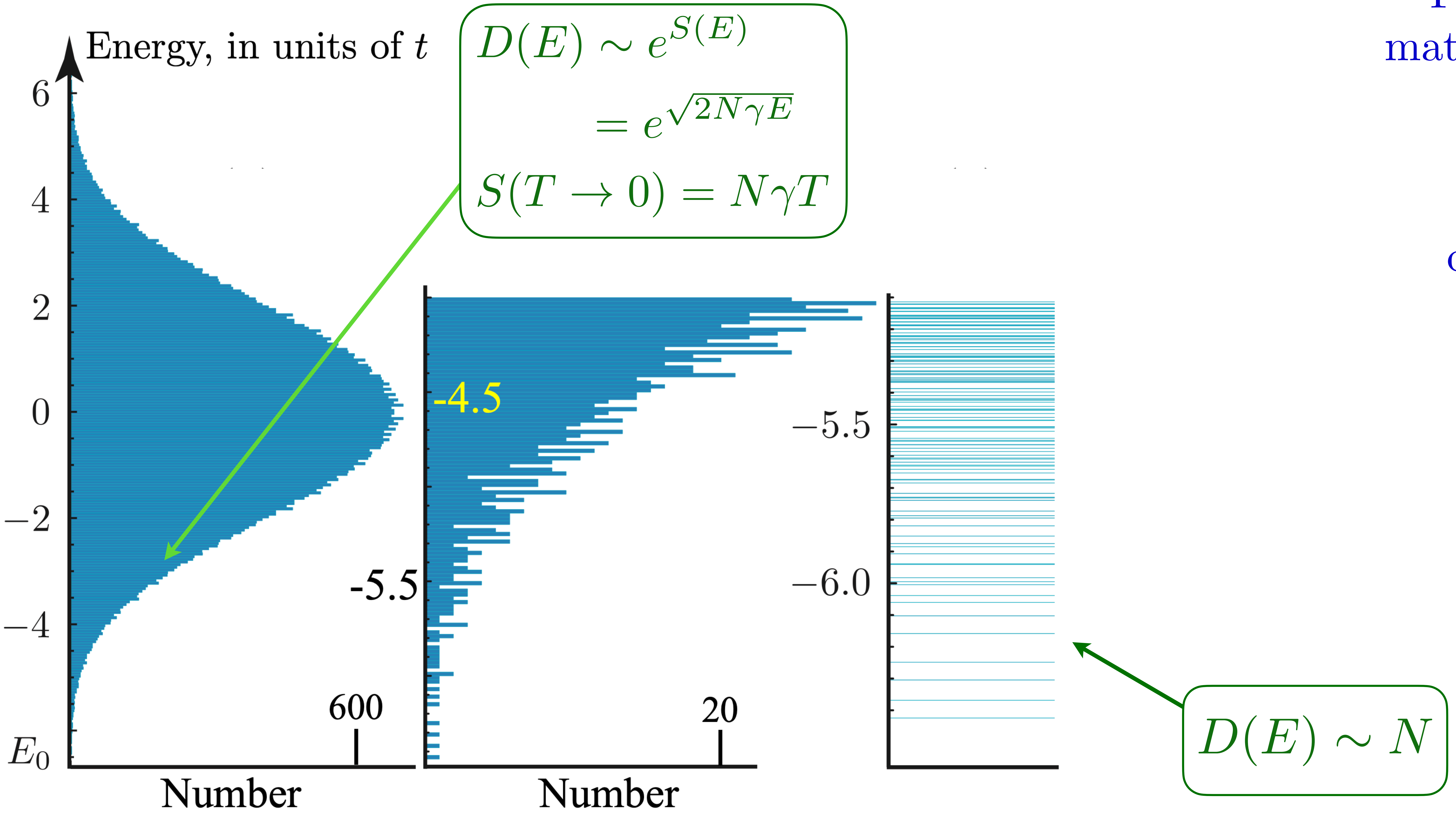
$$S(T \to 0) = N\gamma T$$

$$\gamma = \frac{\pi^2}{3}\rho_0$$

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

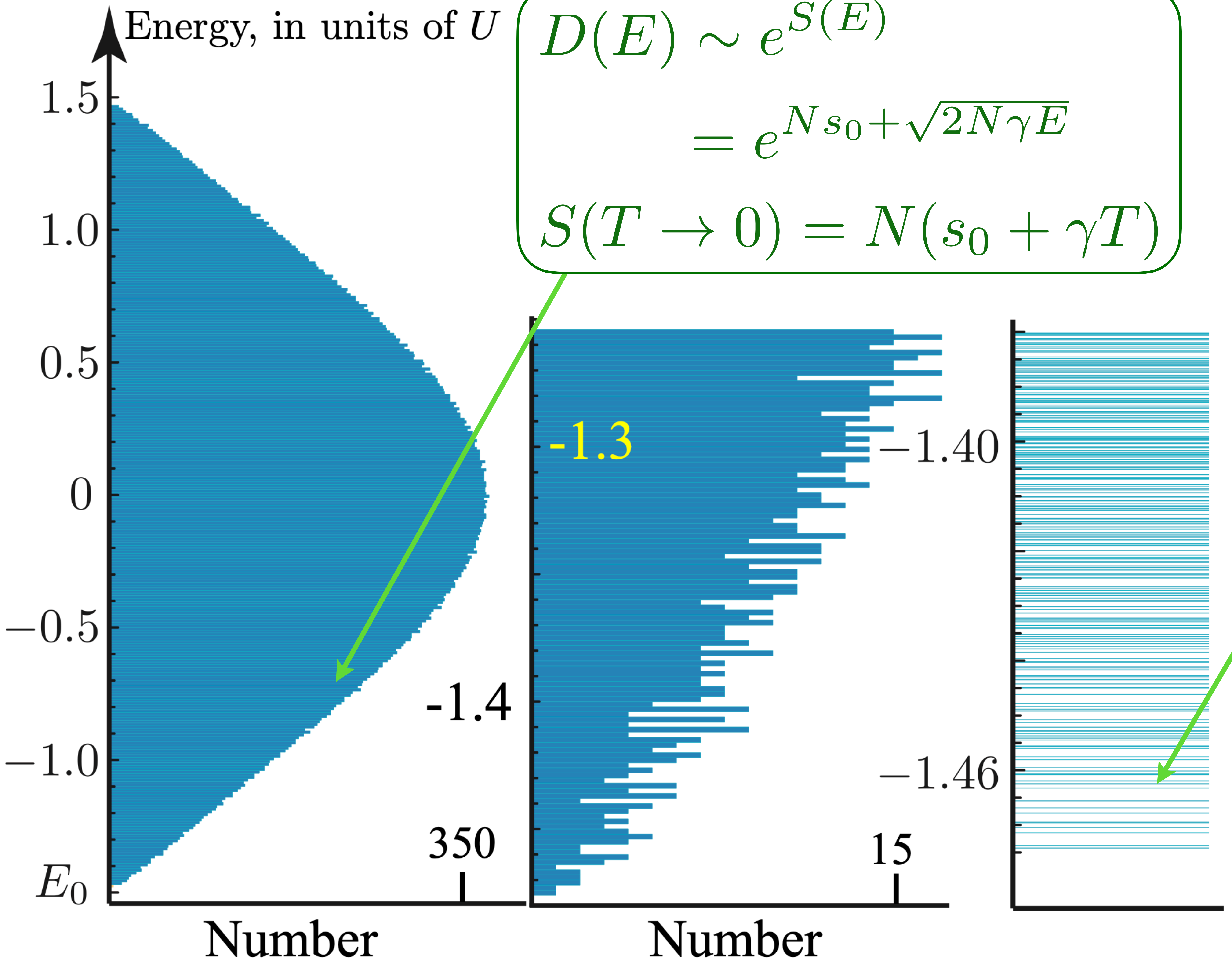
For random matrix model: $E_0 + E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$ $n_\alpha = 0, 1,$ occupation number



Random matrix model

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



$$D(E) \sim e^{S(E)}$$
$$= e^{N s_0 + \sqrt{2N\gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim 2\, e^{N s_0} \sqrt{2N\gamma E}$$

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Complex SYK model

The SYK model

Consequences of emergent time-reparameterization and conformal symmetries in low-energy theory in 0+1 spacetime dimensions:

1. Planckian dynamics!

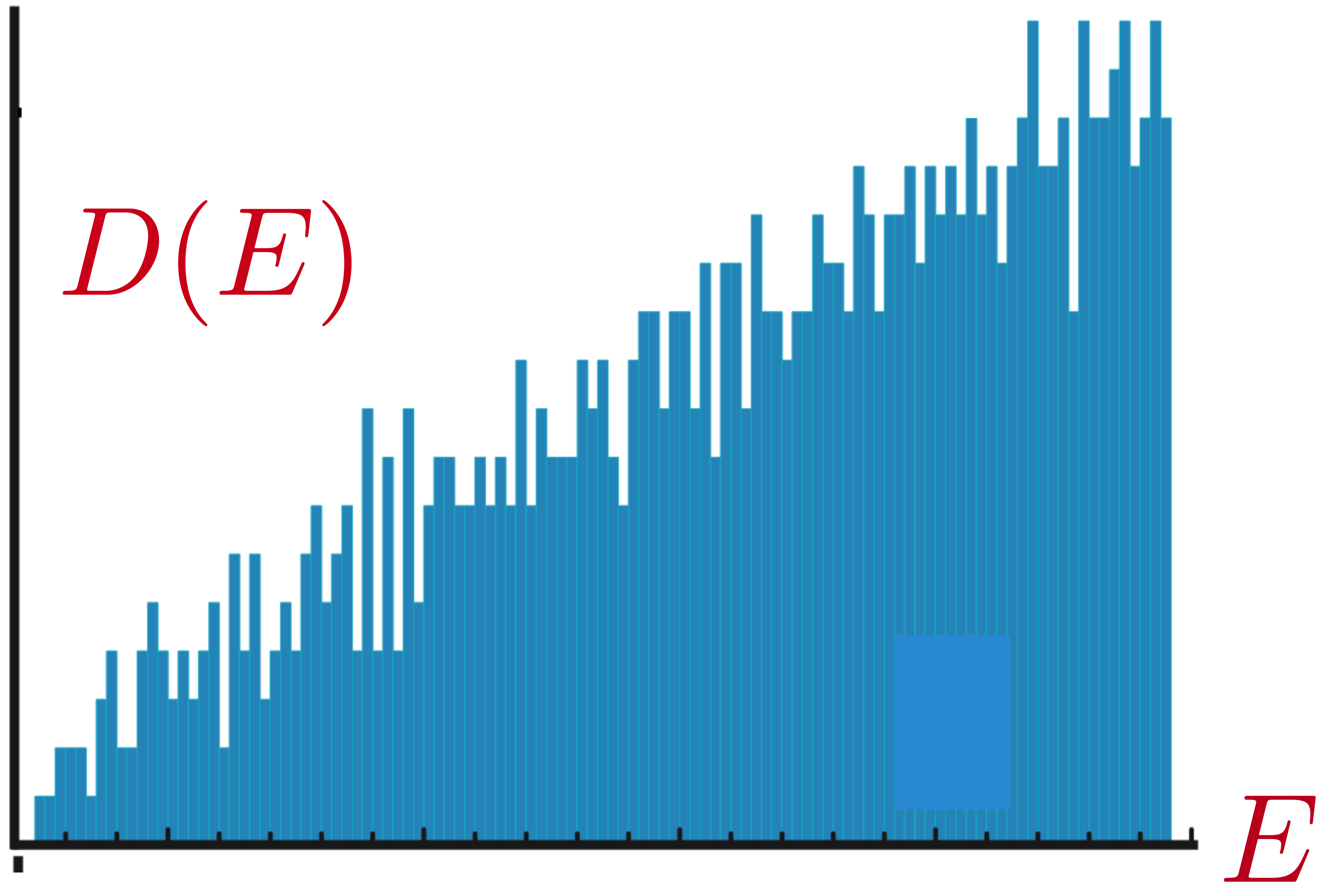
$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right) \text{ independent of } U.$$

No bosons, fermions, anyons . . .

2. Zero temperature entropy without exponential ground state degeneracy!

$$\lim_{T\to 0}\lim_{N\to\infty}\frac{1}{N}S(T) = s_0 \quad , \quad D(E\to 0) = e^{Ns_0}\sinh(\sqrt{2N\gamma E})$$

$$s_0 = 0.4648476991708051074\ldots \text{ for } \mathcal{Q} = 1/2.$$

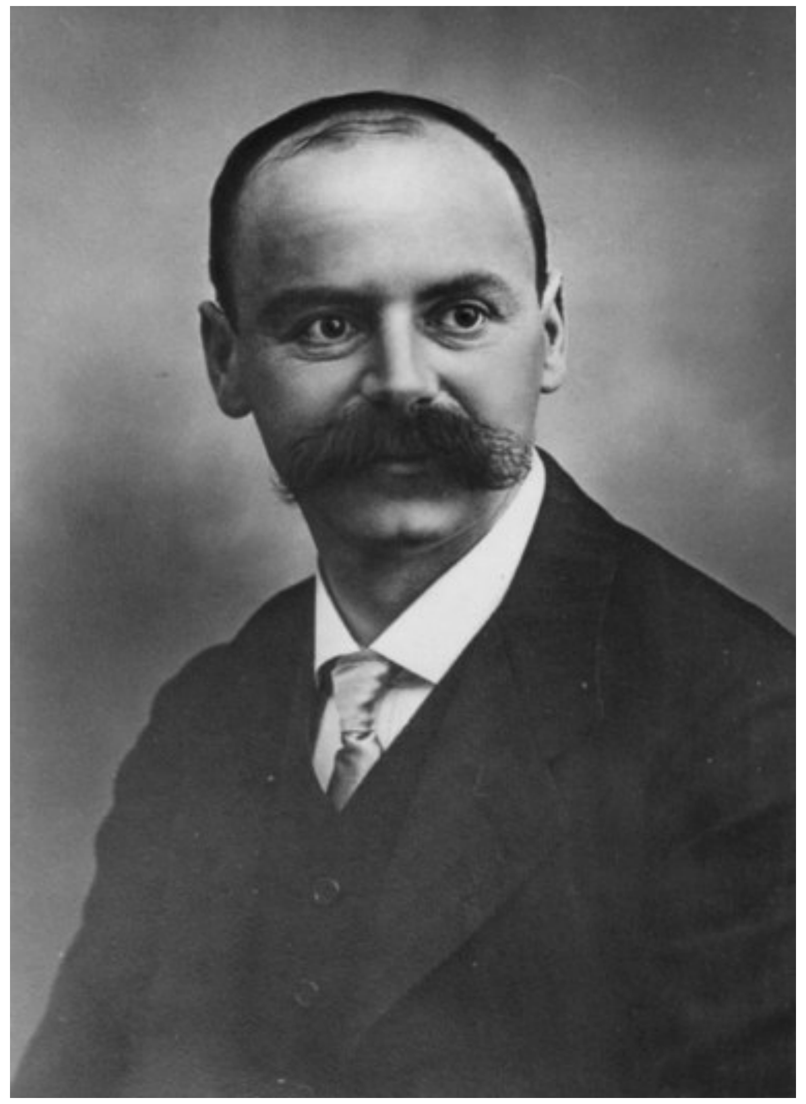


A. Georges, O. Parcollet, and S. Sachdev (**GPS**), Physical Review B **63**, 134406 (2001)

From the SYK model to black holes

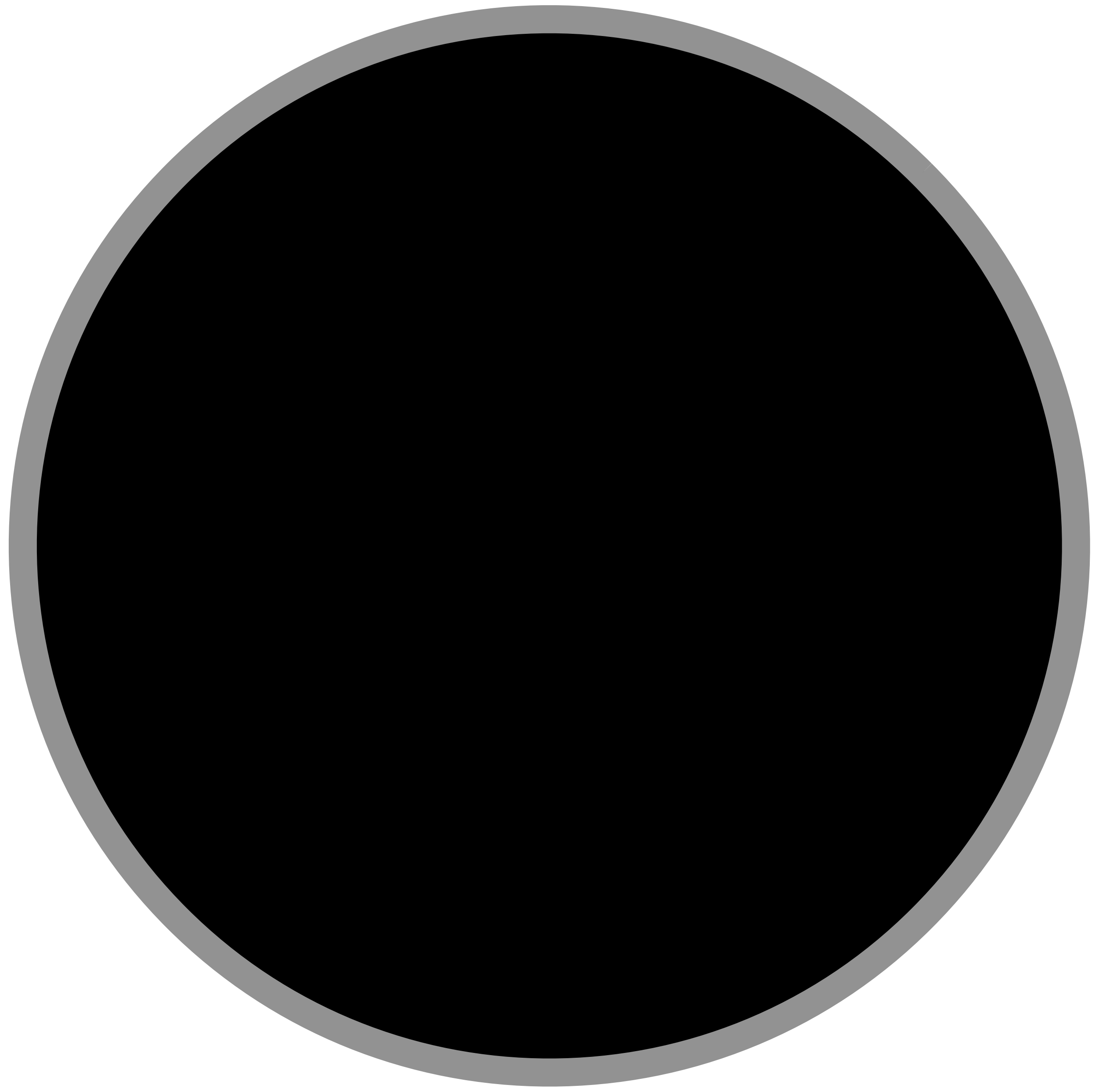
Black Holes

Objects so dense that light is gravitationally bound to them.



Karl Schwarzschild (1916)

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

Event Horizon Telescope

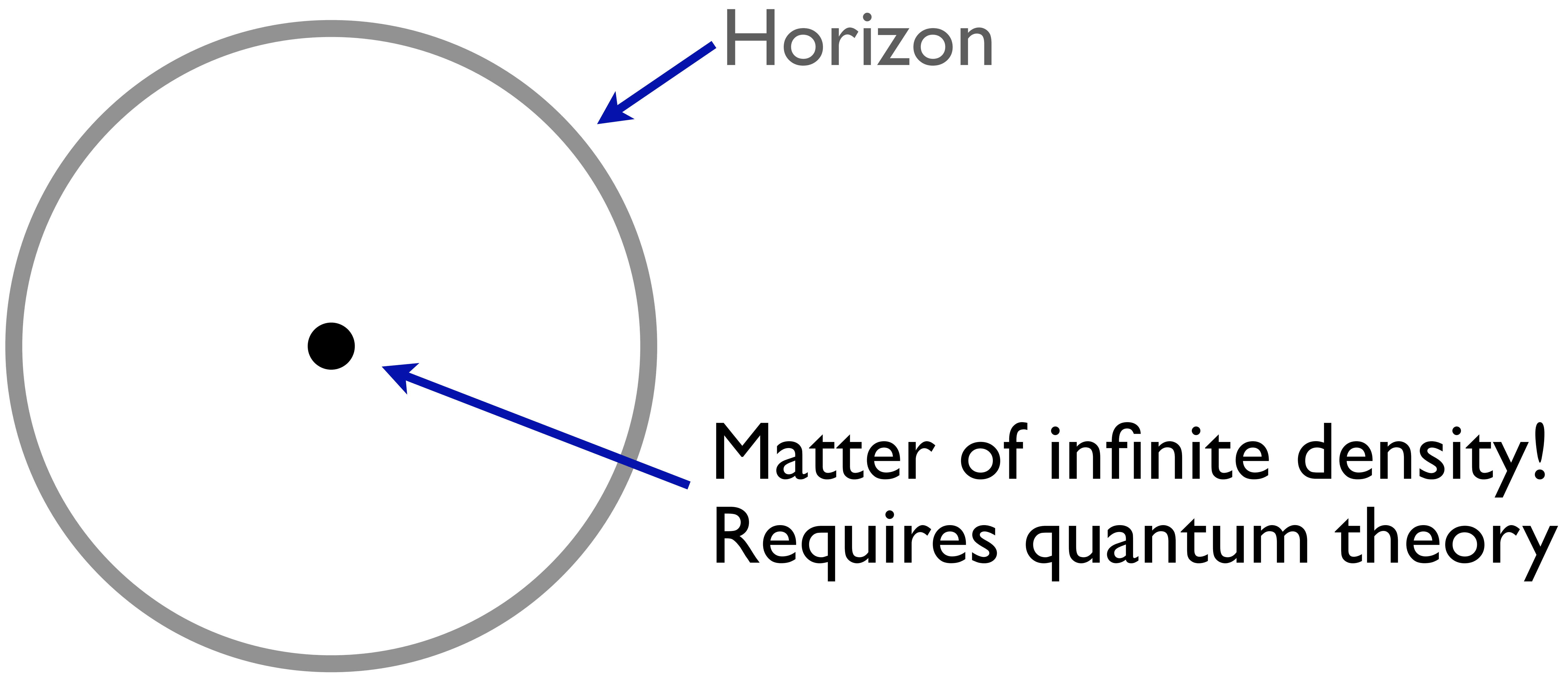
The supermassive black hole at the center of the M87 galaxy contains about 6.5 billion solar masses.

It is rotating at about 90% of the maximal spin

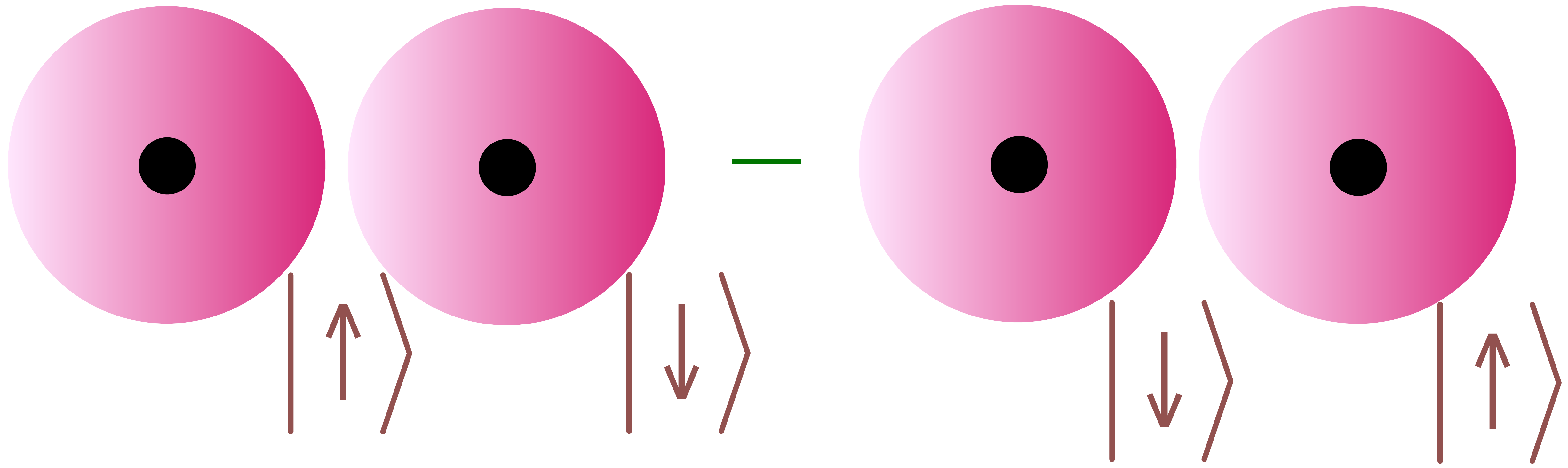
$$R = 1.8 \times 10^{13}\text{m}$$
$$\approx \text{solar system size}$$

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



Quantum Entanglement across a black hole horizon



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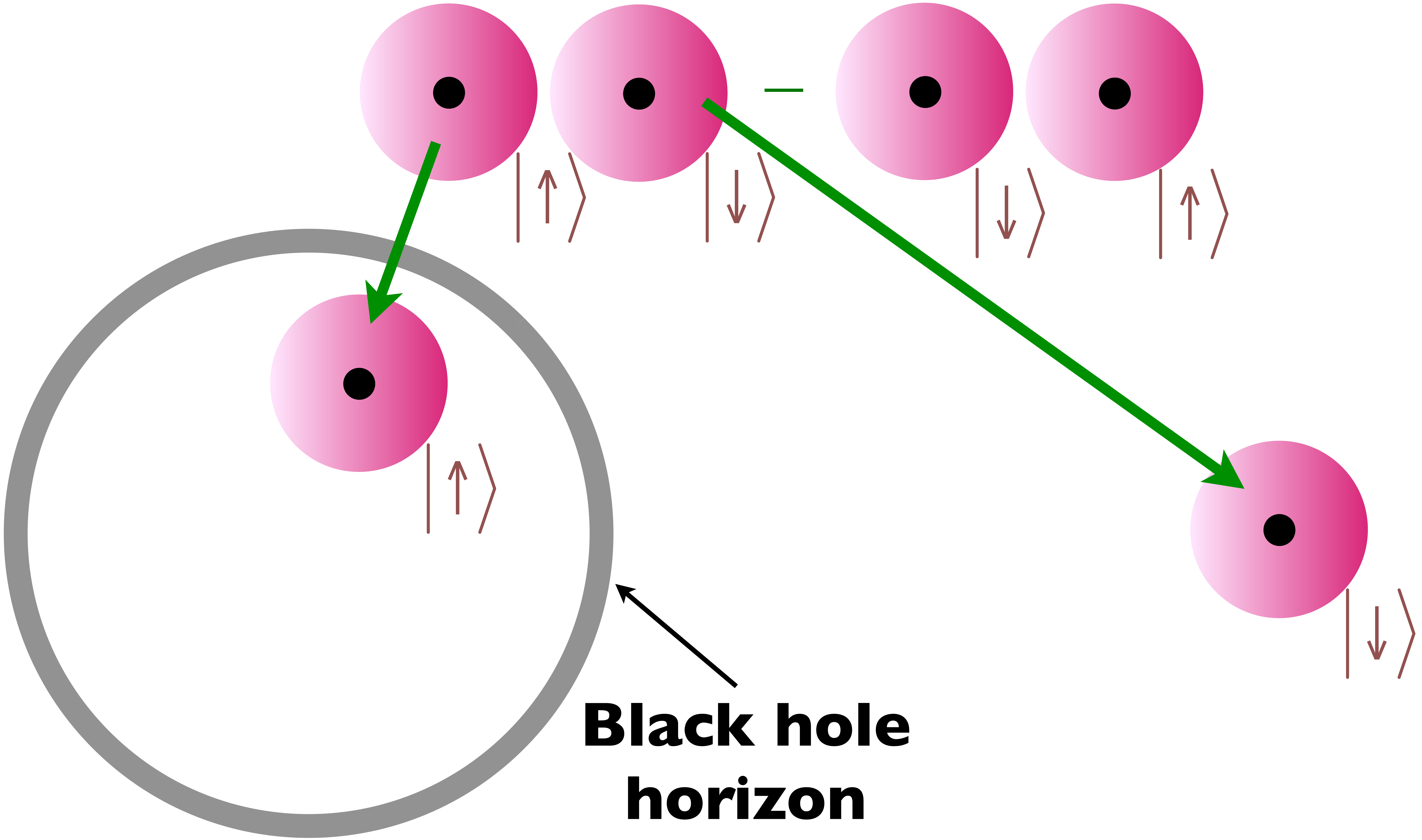
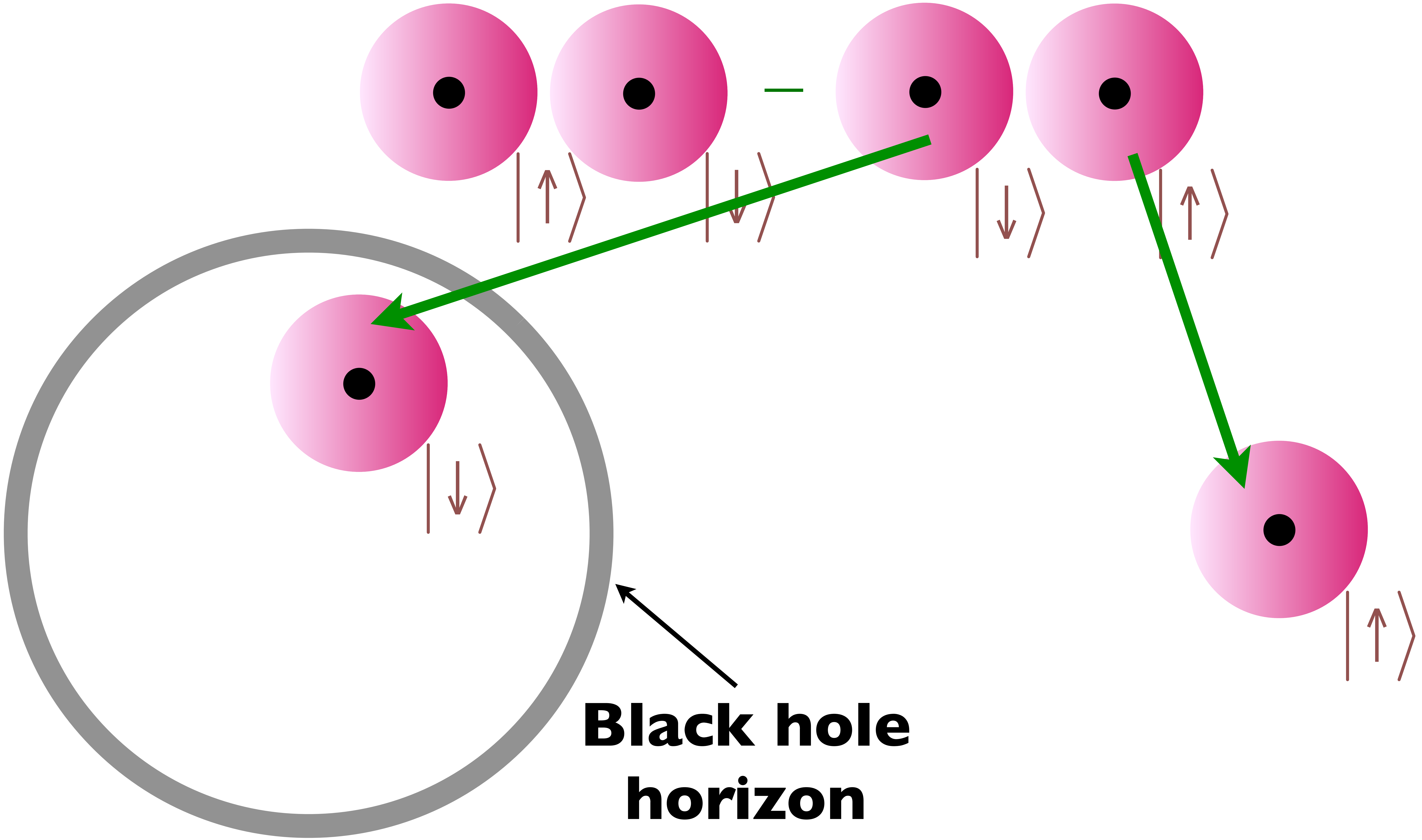


Quantum Entanglement across a black hole horizon

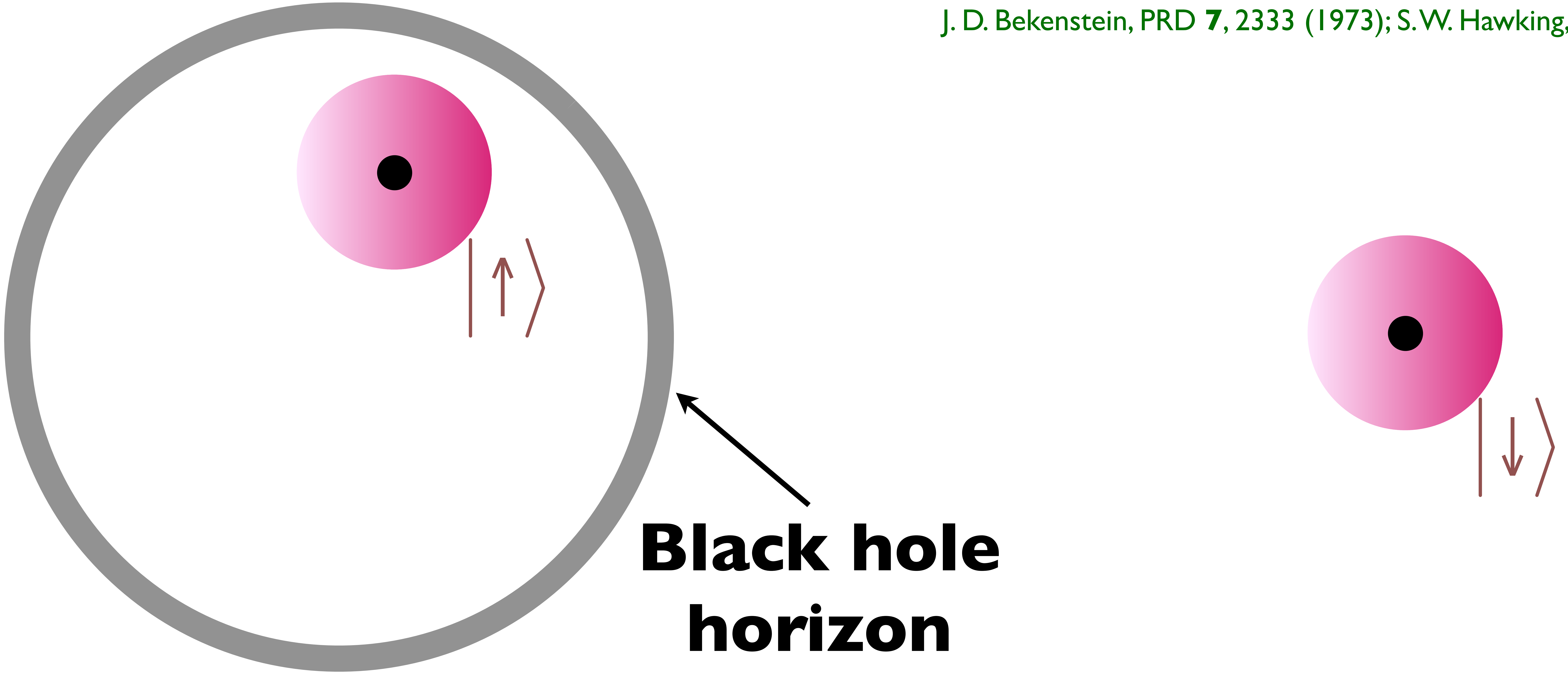


Quantum Entanglement across a black hole horizon

Bekenstein, Hawking: Black holes have a temperature and an entropy!

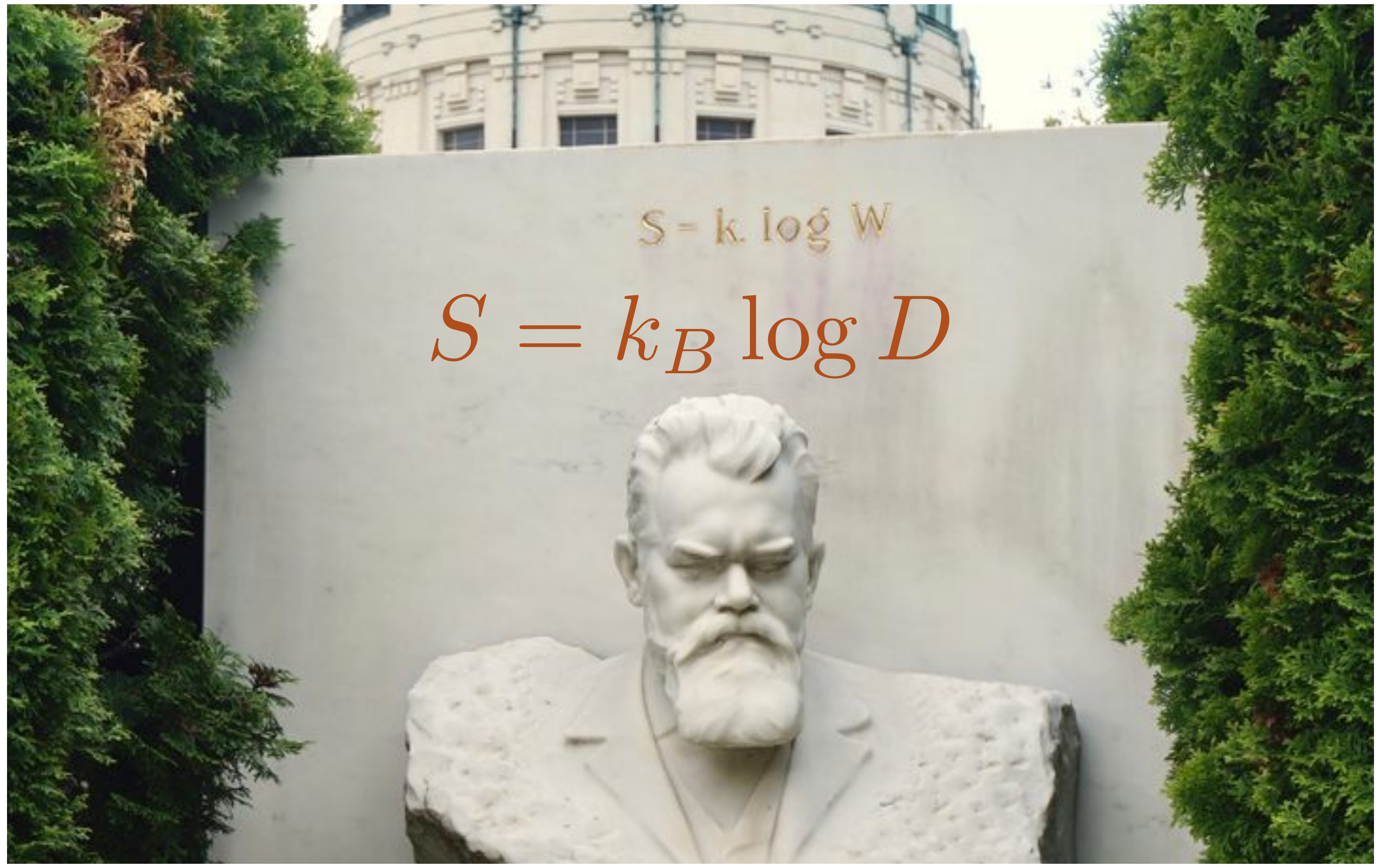
To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.

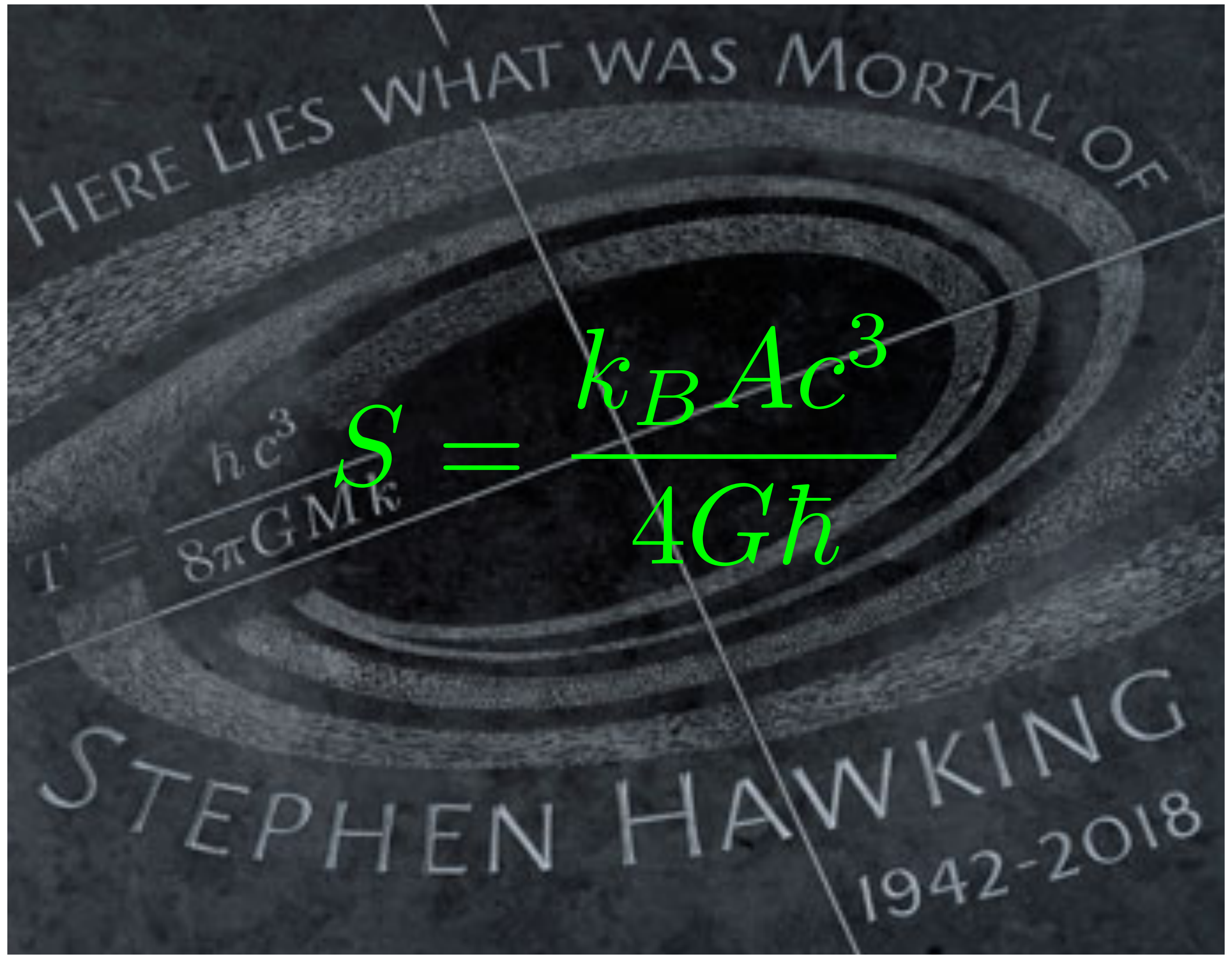
J. D. Bekenstein, PRD **7**, 2333 (1973); S.W. Hawking, Nature **248**, 30 (1974)



Quantum Black Holes

- Can we find a quantum theory for the collapsed matter at the center of the black hole, whose *density of quantum states* $D(E)$ [the quantum analog of Boltzmann's W] matches Bekenstein-Hawking entropy, in accordance with Boltzmann's principles of statistical mechanics, $S(E) = k_B \log D(E)$?





Connections between the SYK model and black holes

- Black hole 'ring-down' or 'quasinormal mode damping' or 'chaos' times are Planckian $\sim \hbar/(k_B T)$

C.V. Vishveshwara, Nature **227**, 936 (1970)

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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- Charged black holes have a non-zero Bekenstein-Hawking entropy in the limit $T \to 0$:

  $S_{BH} = A_0 c^3/(4\hbar G)$ where $A_0 = 2GQ^2/c^4$ is the area of the charged black hole horizon at $T = 0$.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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  Also applies to rotating neutral black holes.

U. Moitra, S.K. Sake, S.P. Trivedi and V. Vishal, JHEP 11 (2019) 047.
D. Kapec, A. Sheta, A. Strominger and C. Toldo, PRL 133 (2024) 021601
M. Kolanowski, D. Marolf, I. Rakic, M. Rangamani and G.J. Turiaci, arXiv:2409.16248

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

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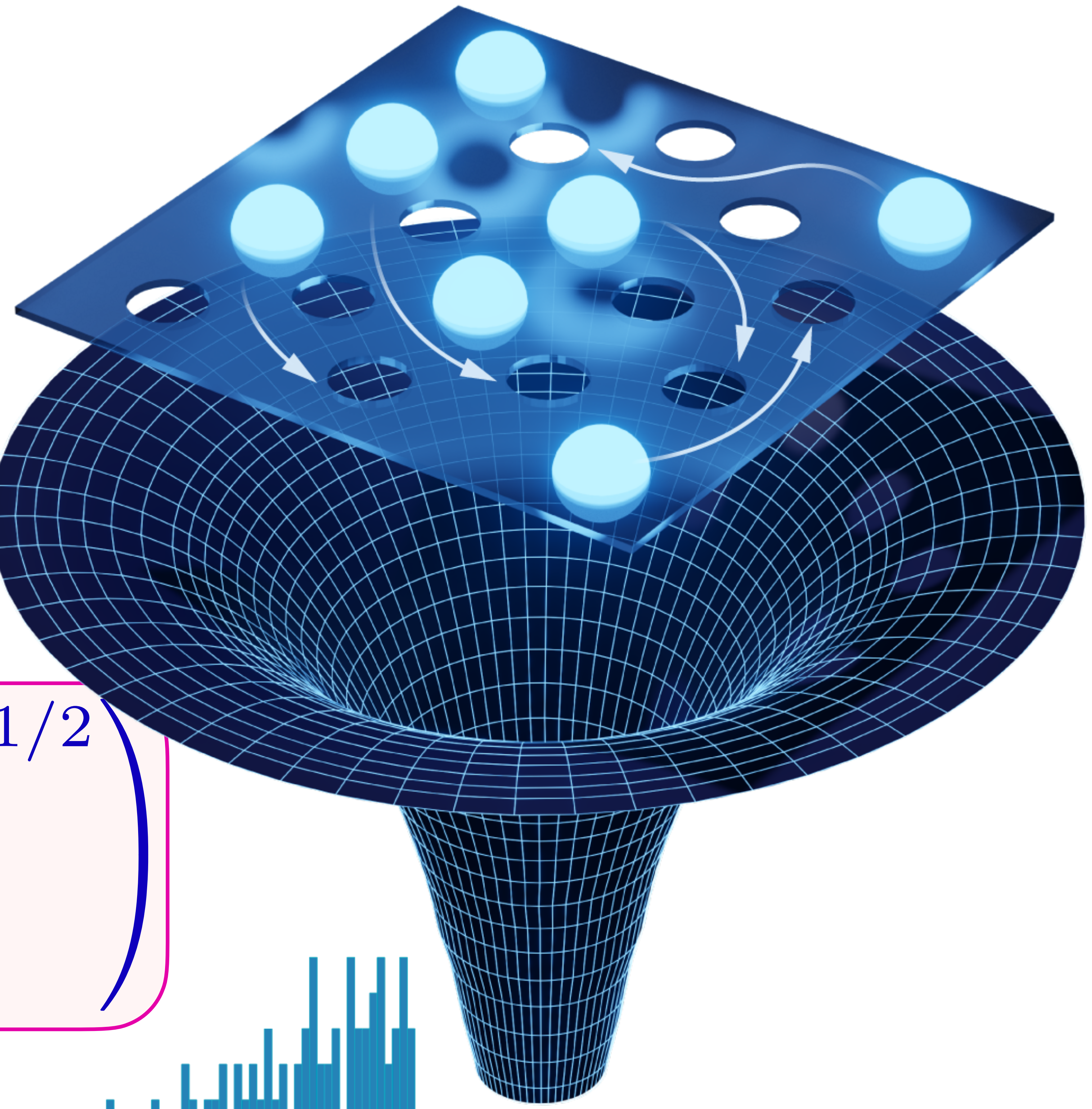
  Also applies to rotating neutral black holes.

- The example of the SYK model implies that S_{BH} is *not* realized by an exponentially large ground state degeneracy (as is the case in all earlier string-theoretic computations).

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

D(E) of charged black holes from the SYK model



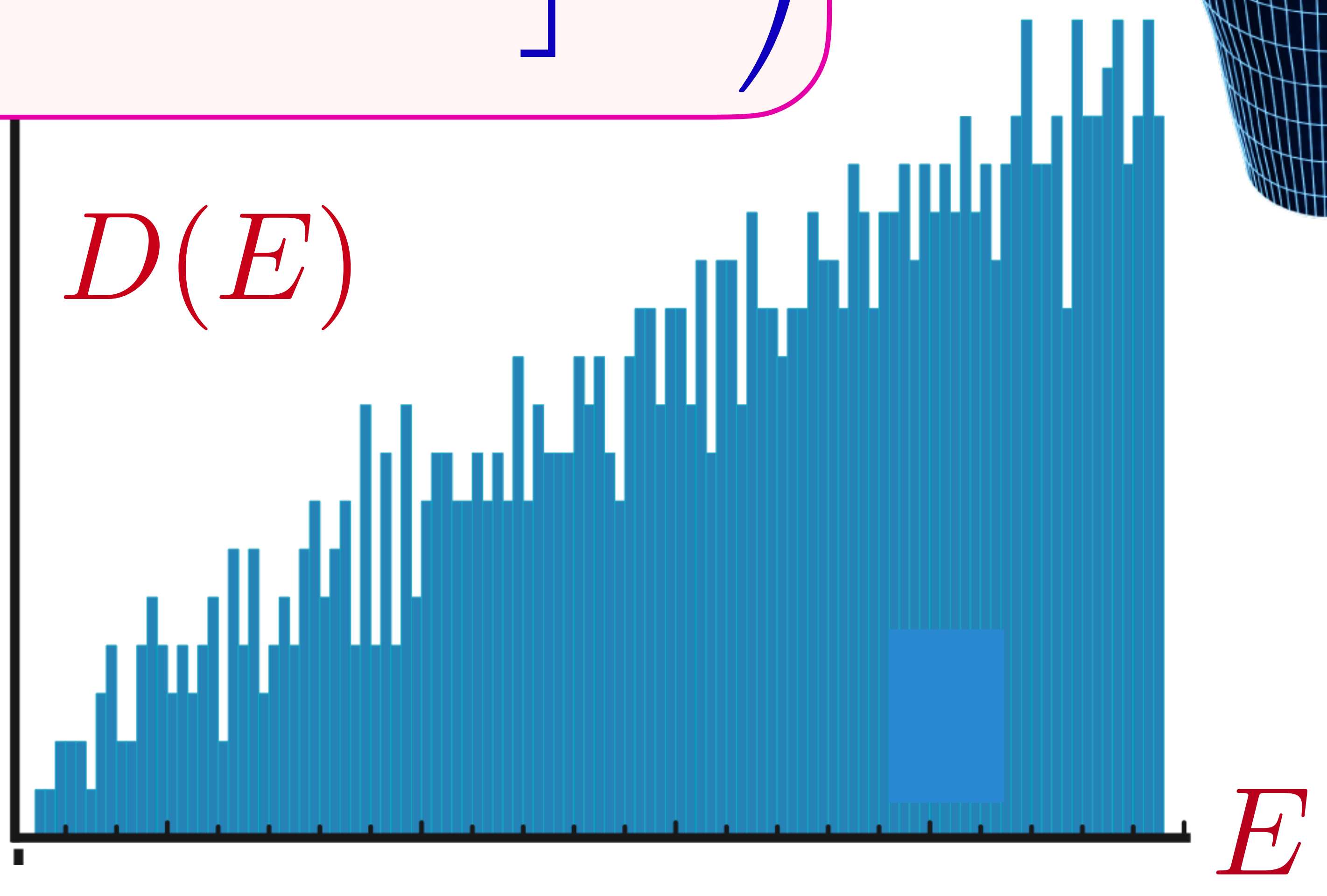
- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}\, {A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Bekenstein-Hawking

Iliesiu, Murthy, Turiaci (2022)

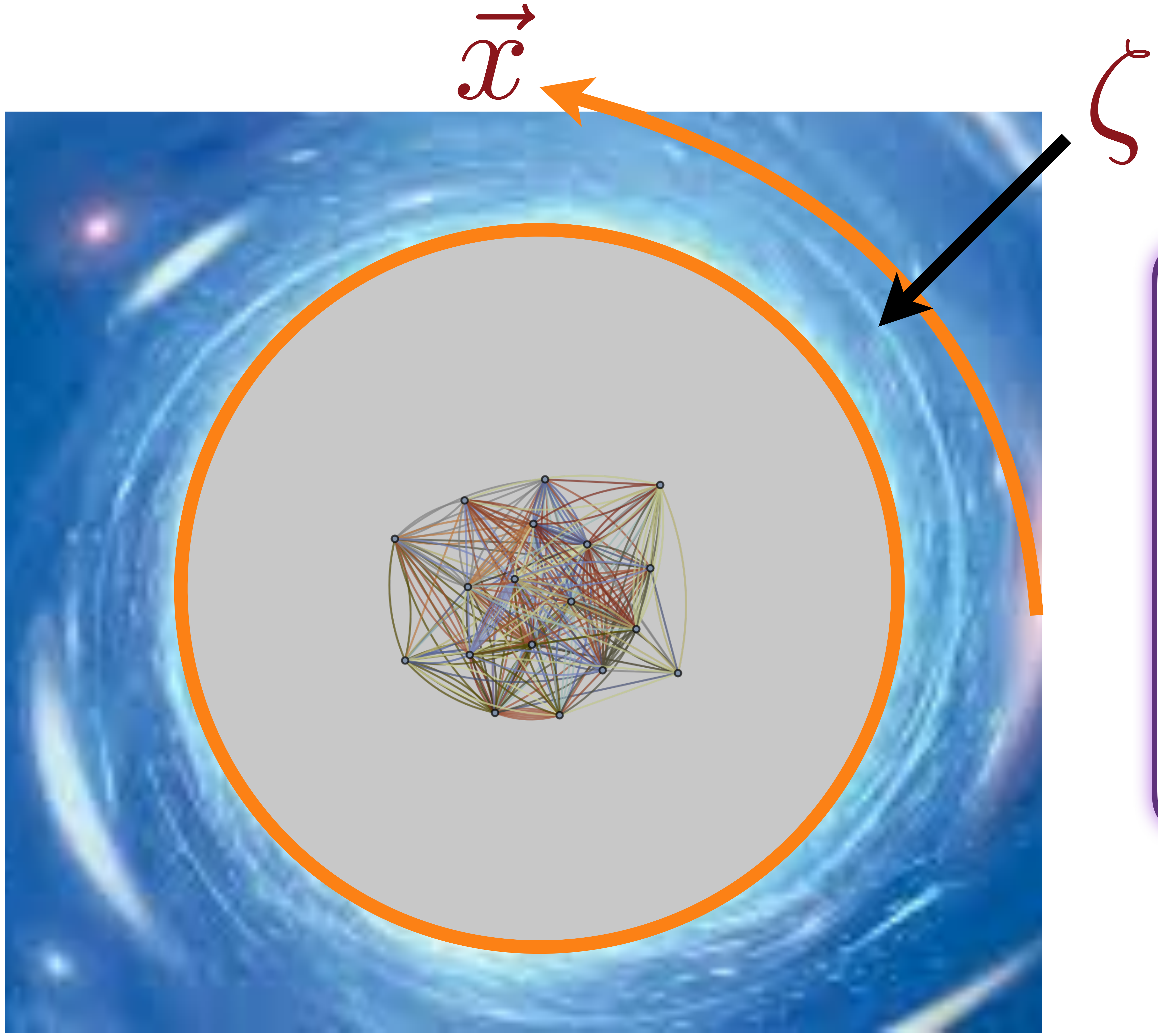
Developments from the SYK model



Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

Quantum simulation of charged black holes by the SYK model



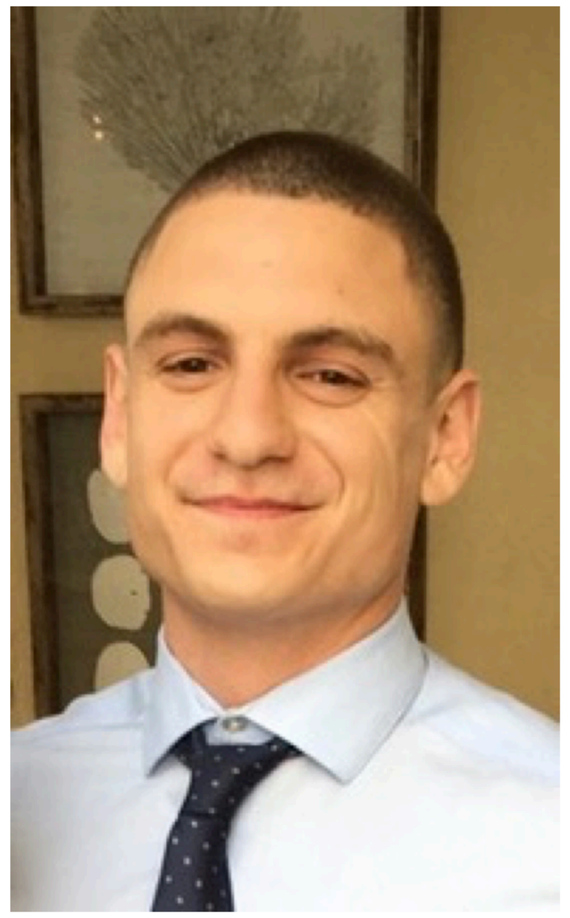
The SYK model simulates the low energy properties of the interior of the black hole for an outside observer in ζ-τ co-ordinates.

From the SYK model to the universal 2d-YSYK theory of strange metals

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

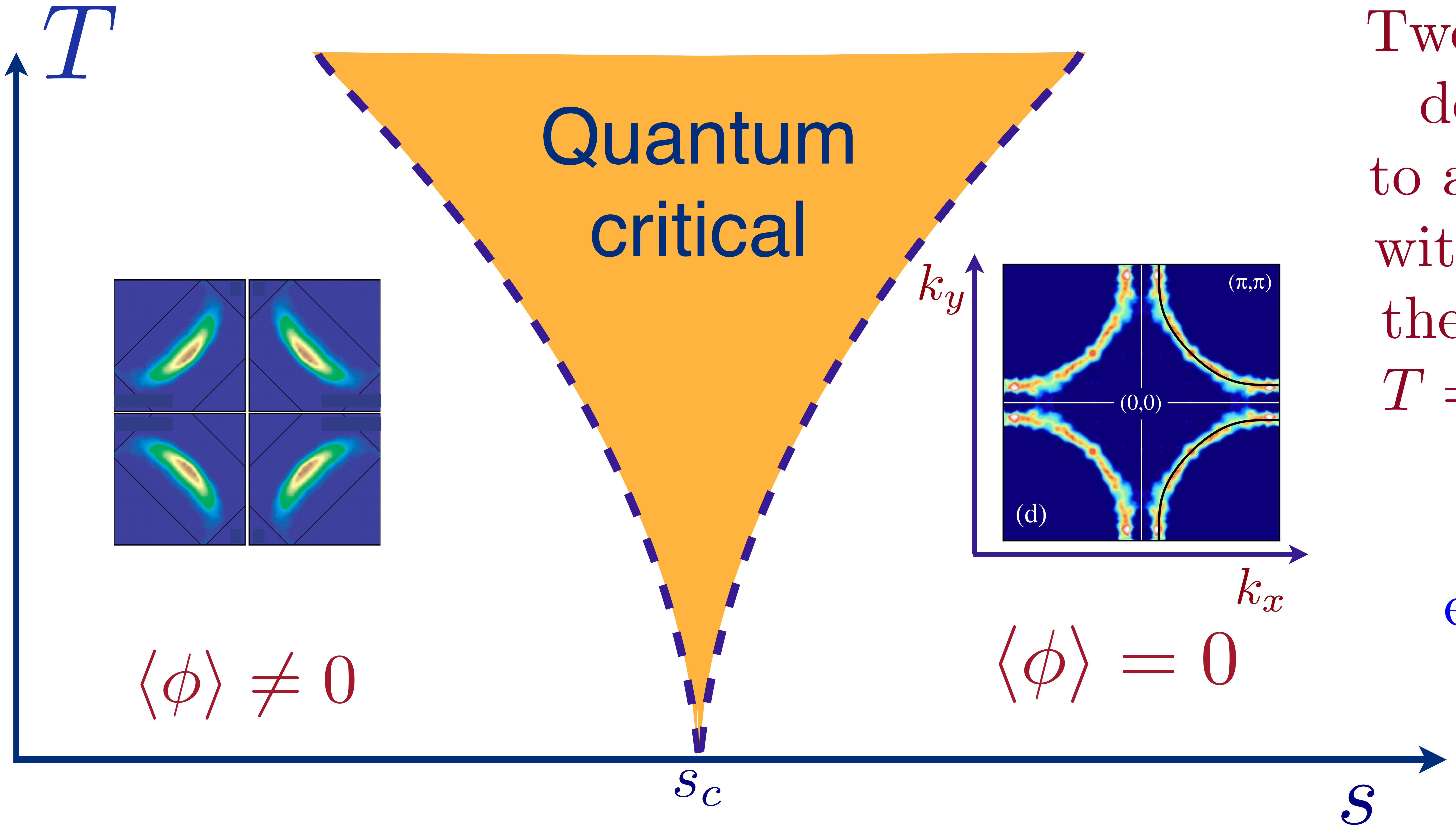
Aavishkar Patel
Flatiron

Haoyu Guo
Cornell



Ilya Esterlis
Wisconsin

Quantum phase transition of Fermi surface change



Two-dimensional YSYK model describes electrons coupled to a boson ϕ driving the QPT, with spatial randomness in s_c, the position of the underlying $T = 0$ quantum critical point.

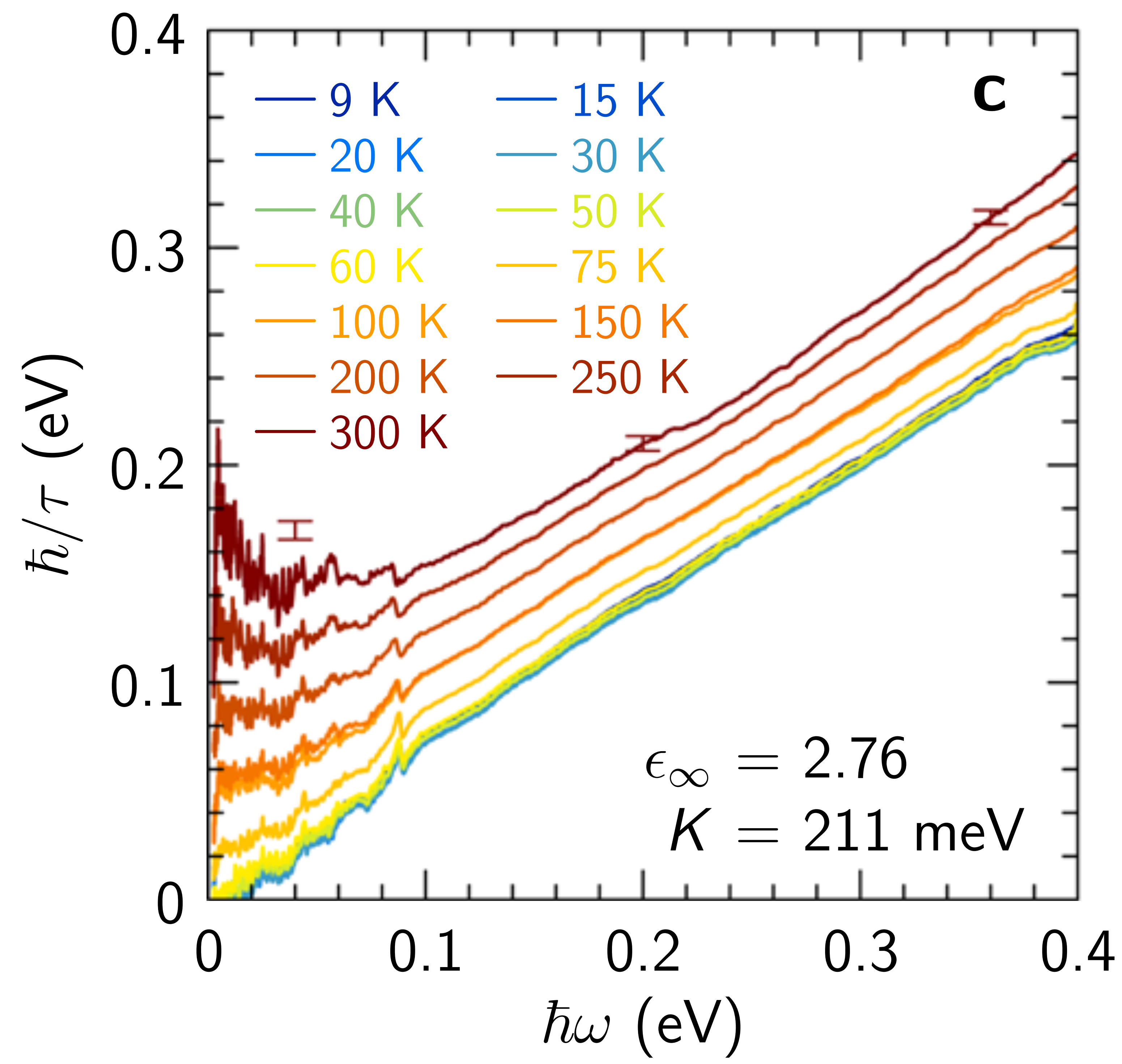
Analyze with methods exact for the SYK model

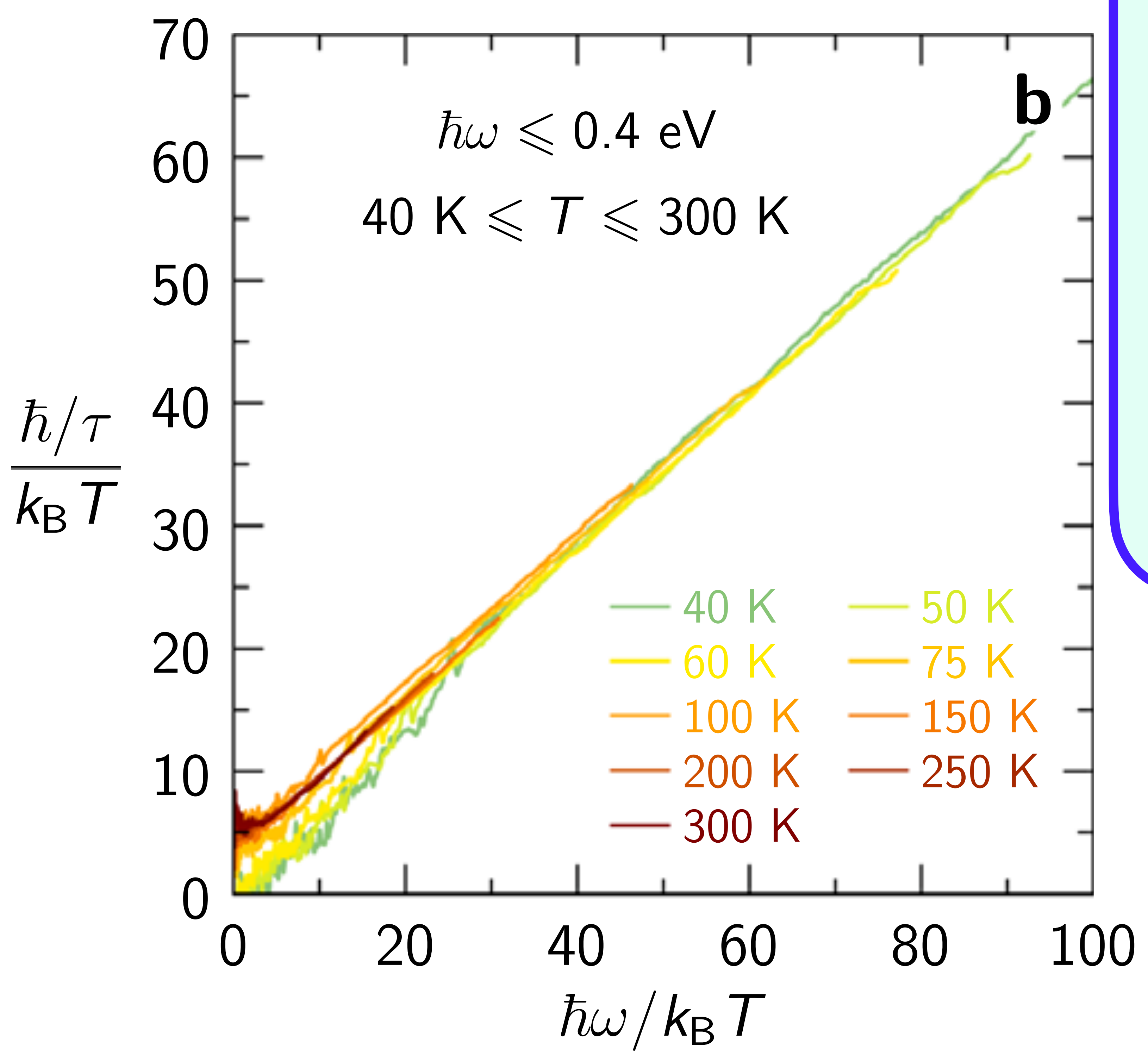
Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_BT}F\left(\frac{\hbar\omega}{k_BT}\right)$$

and entropy $S(T \to 0) \sim T\ln(1/T)$.

$La_{2-x}Sr_xCuO_4$

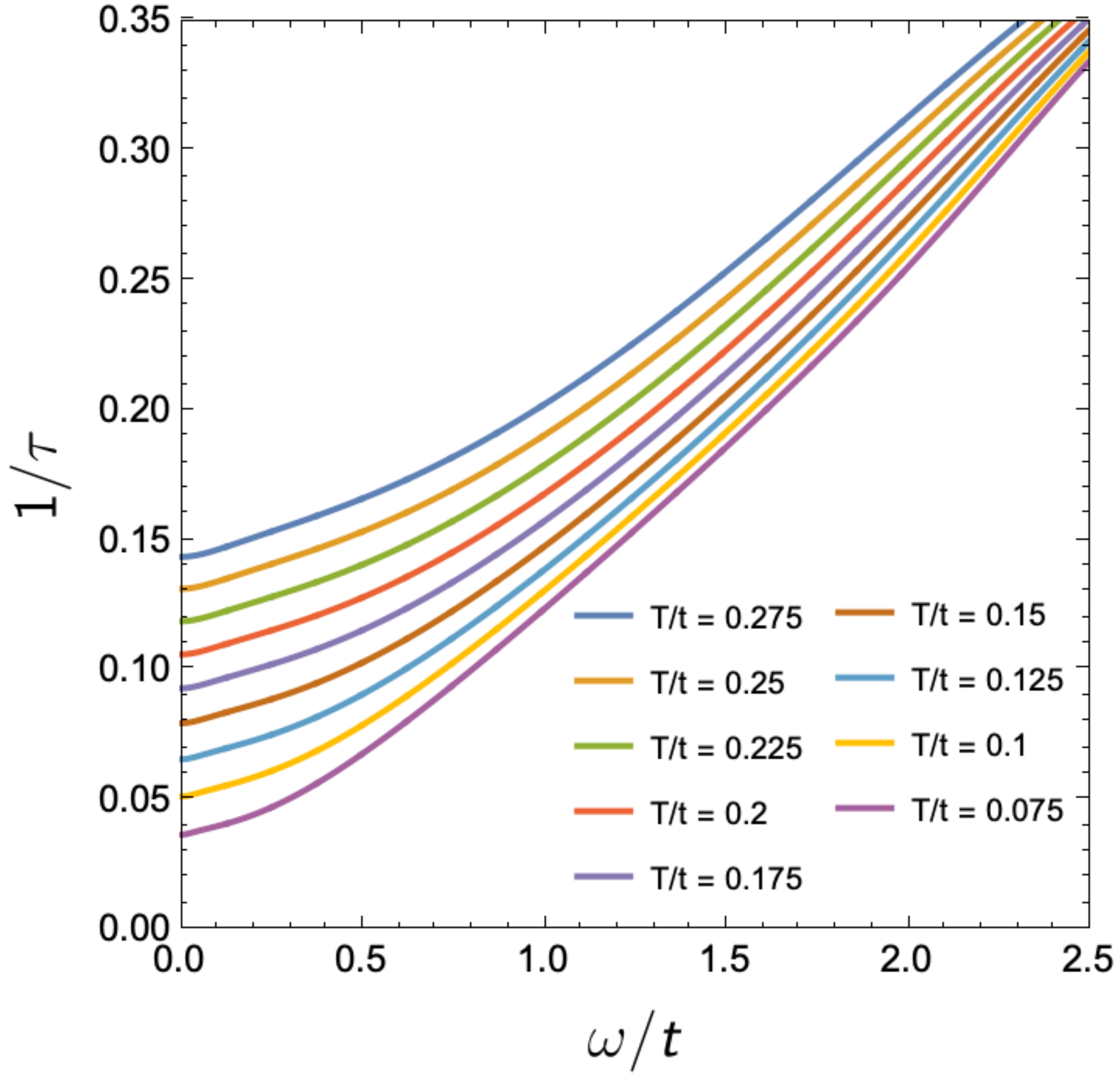
$p = 0.24$

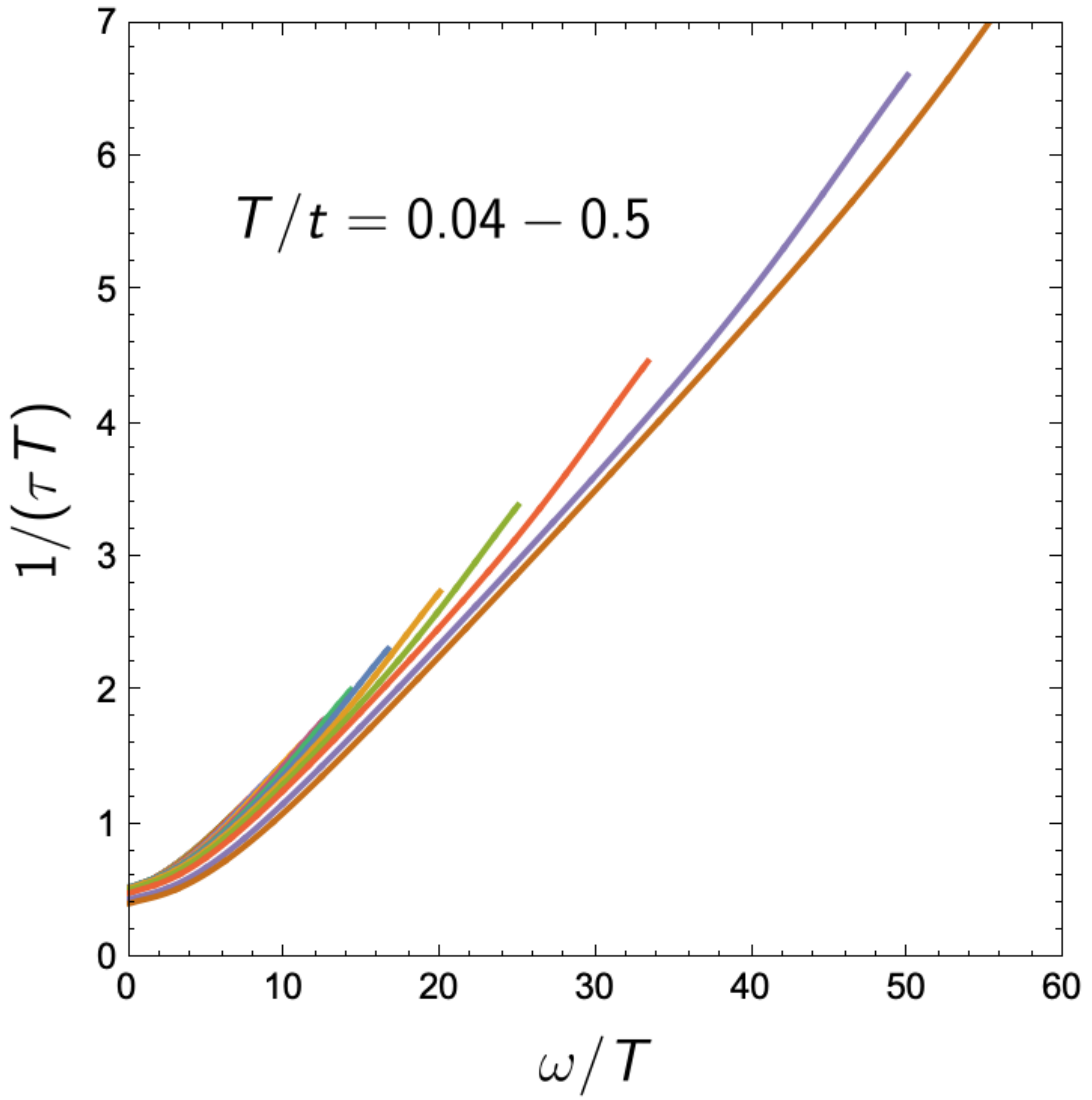
$T_c = 19$ K

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

$$\sigma(\omega) = i \frac{e^2 K/(\hbar d_c)}{\hbar\omega \dfrac{m^*(\omega)}{m} + i \dfrac{\hbar}{\tau(\omega)}}$$





Planckian dynamics!

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$

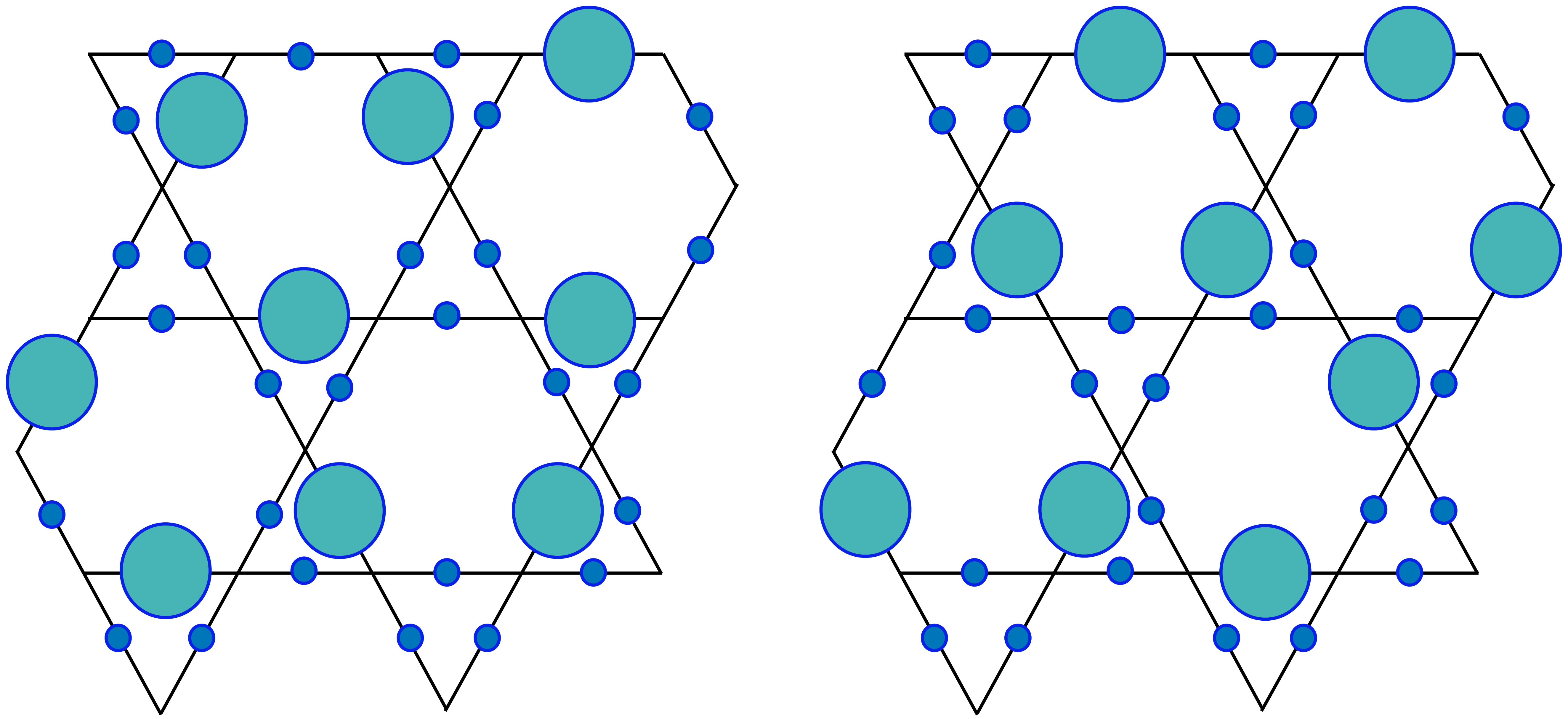
and entropy $S(T \to 0) \sim T \ln(1/T)$ in 2d-YSYK model (unlike zero temperature entropy in SYK model).

Recap

Entanglement of stationary electrons

Theory and experiments on Z_2 gapped spin liquids

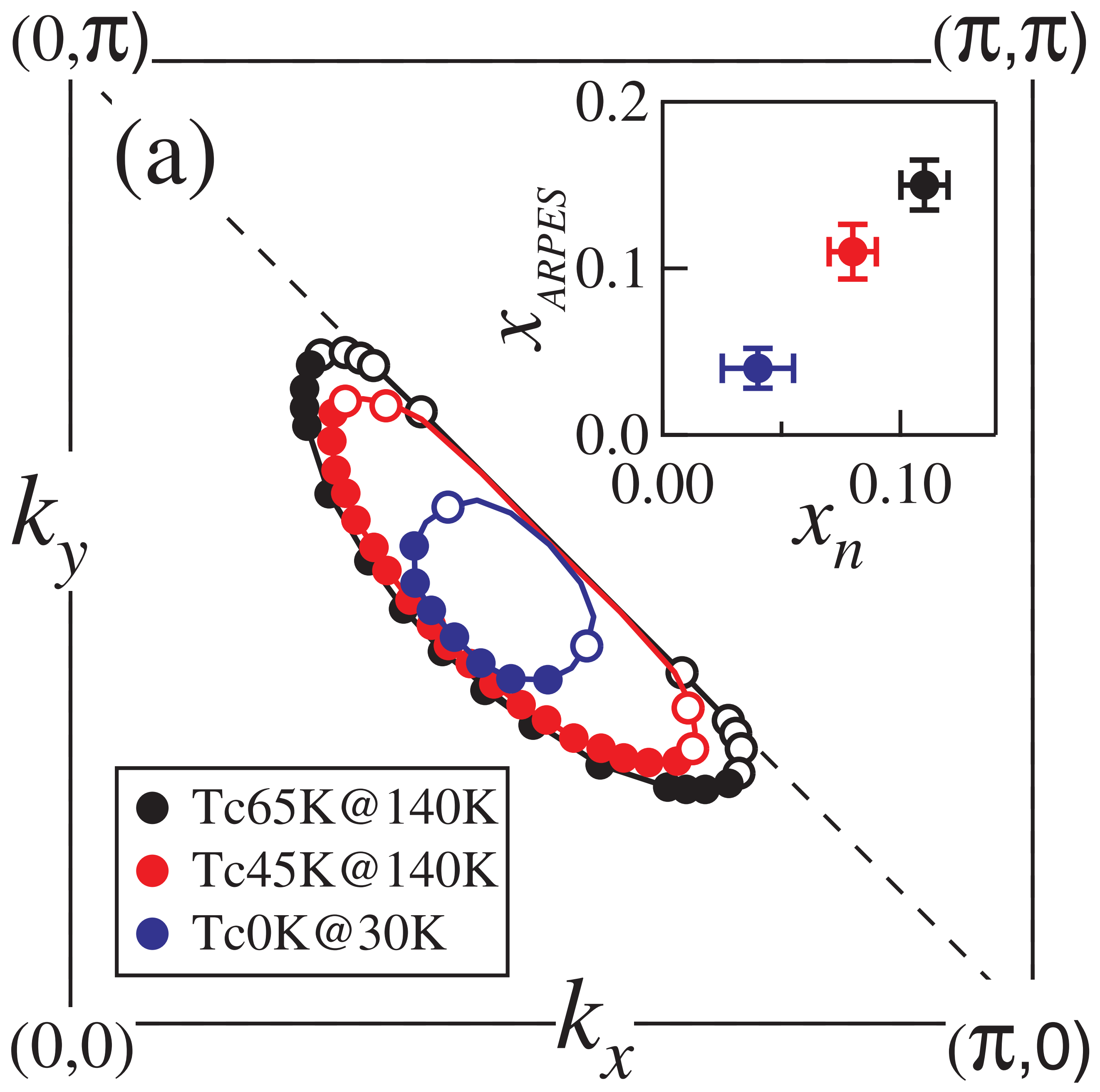
$$|\Psi\rangle =$$



$$+ \ldots$$

Entanglement of stationary electrons

Gapless spin liquid character of the pseudogap metal in the cuprates



H.-B. Yang, J. D. Rameau, Z.-H. Pan, G. D. Gu, P. D. Johnson, H. Claus, D. G. Hinks, and T. E. Kidd, PRL **107**, 047003 (2011).

H. C. Robarts…..S. M. Hayden
PRB **100**, 214510 (2019)

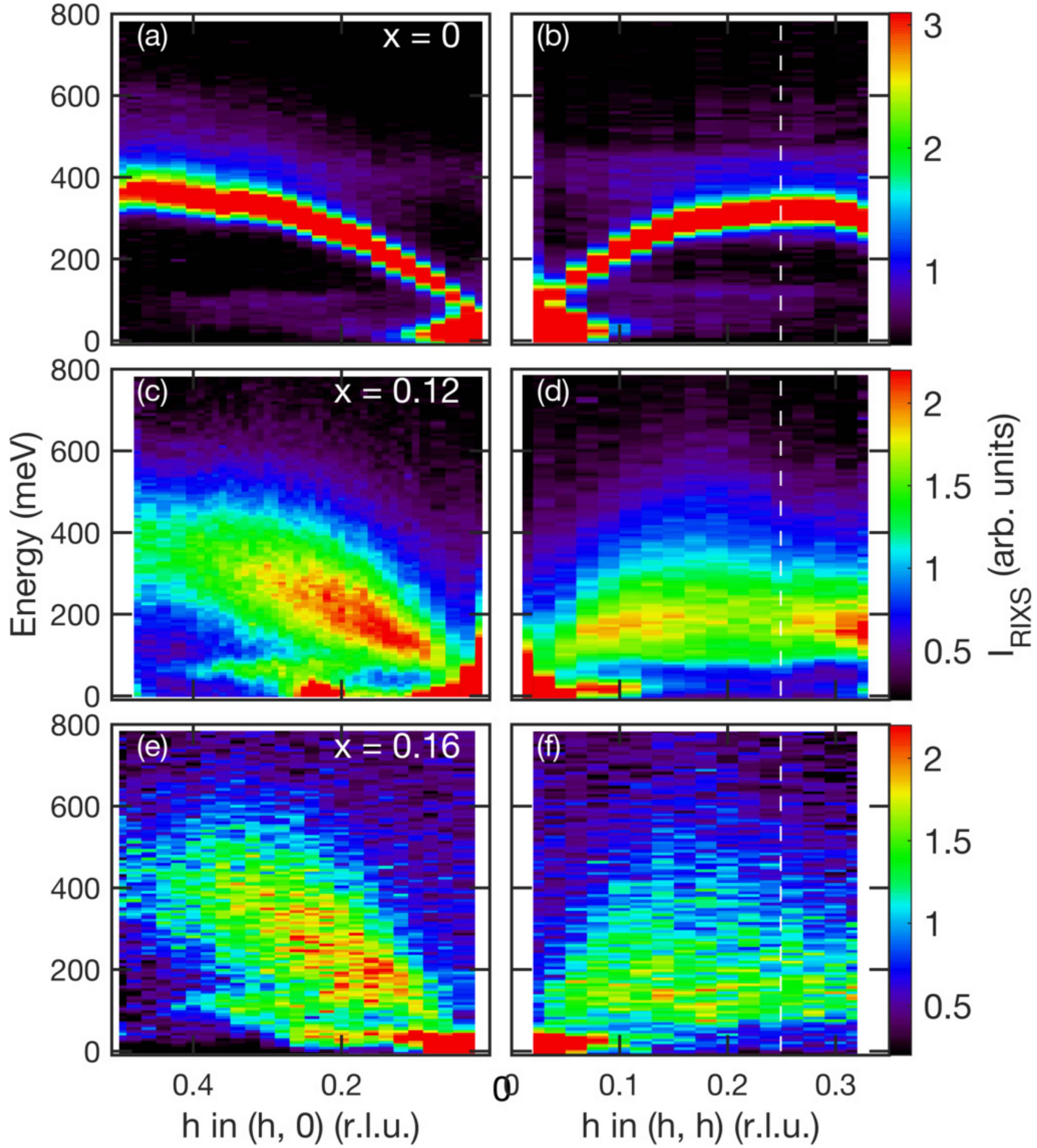
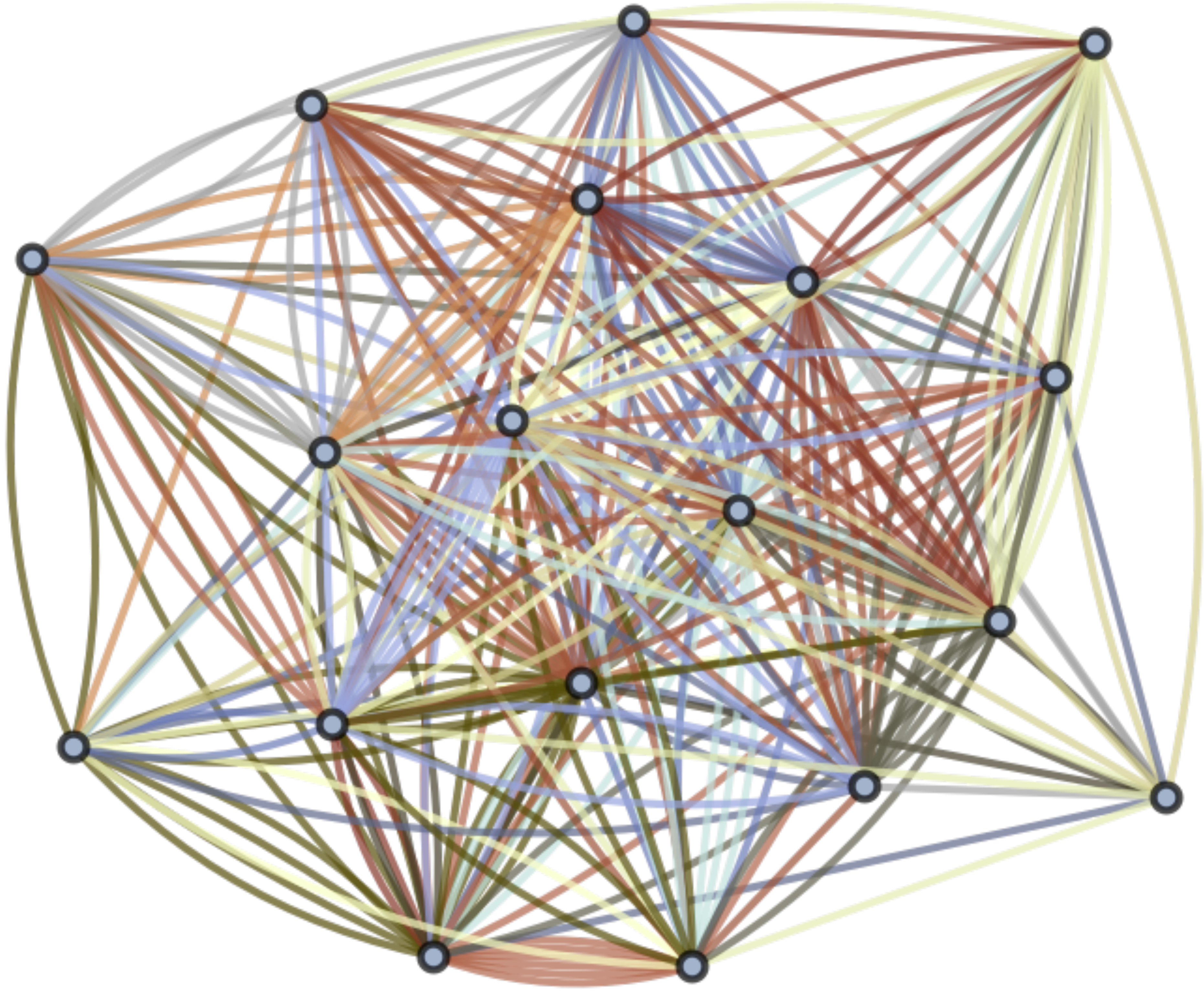


FIG. 2. I_{RIXS} intensity maps as a function of **Q** in LSCO $x = 0$ ($T \approx 20$ K), 0.12, and 0.16 ($T \approx 30$ K).

Entanglement of mobile electrons

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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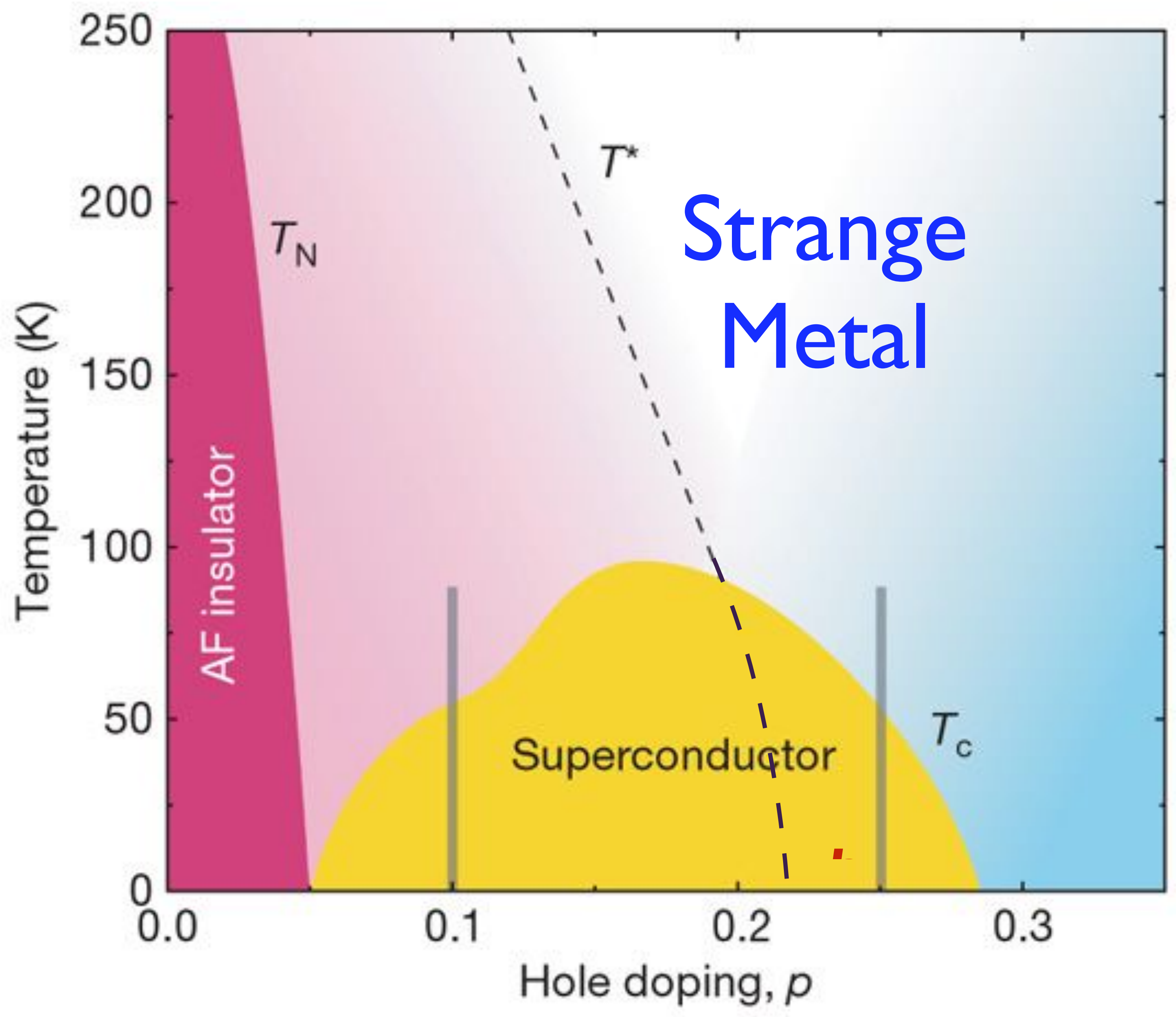
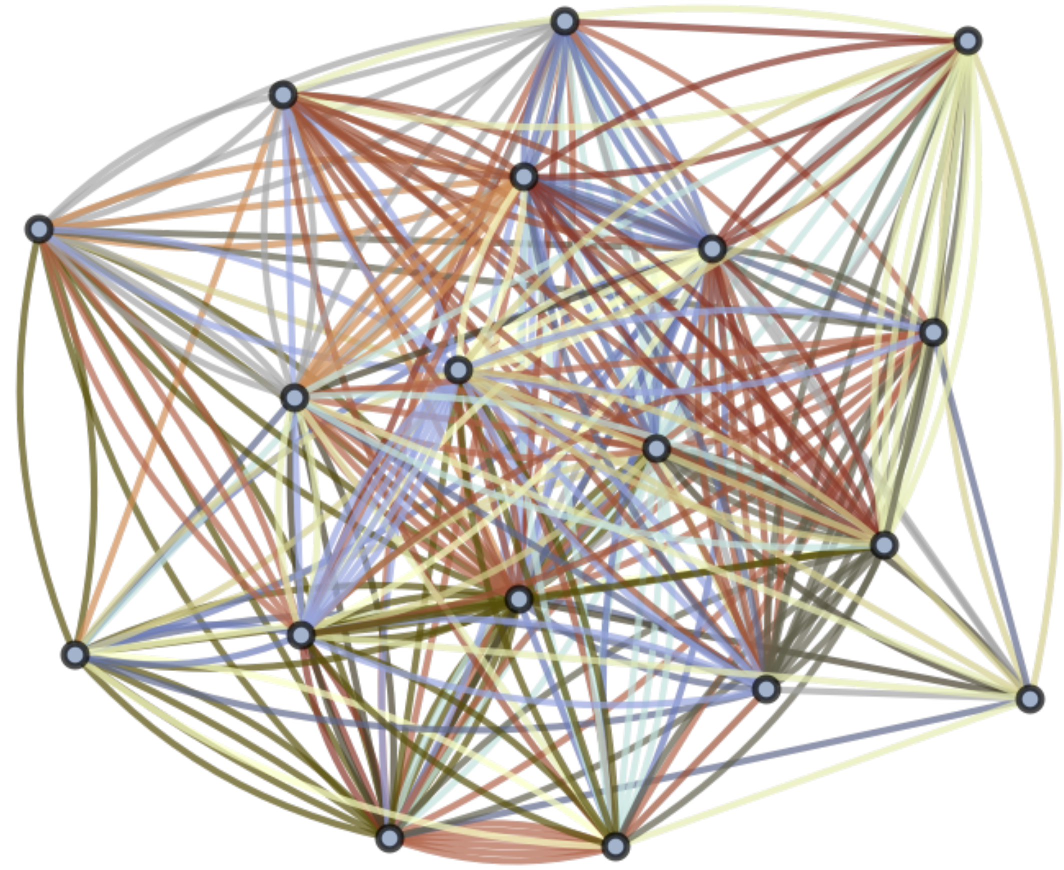


A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials



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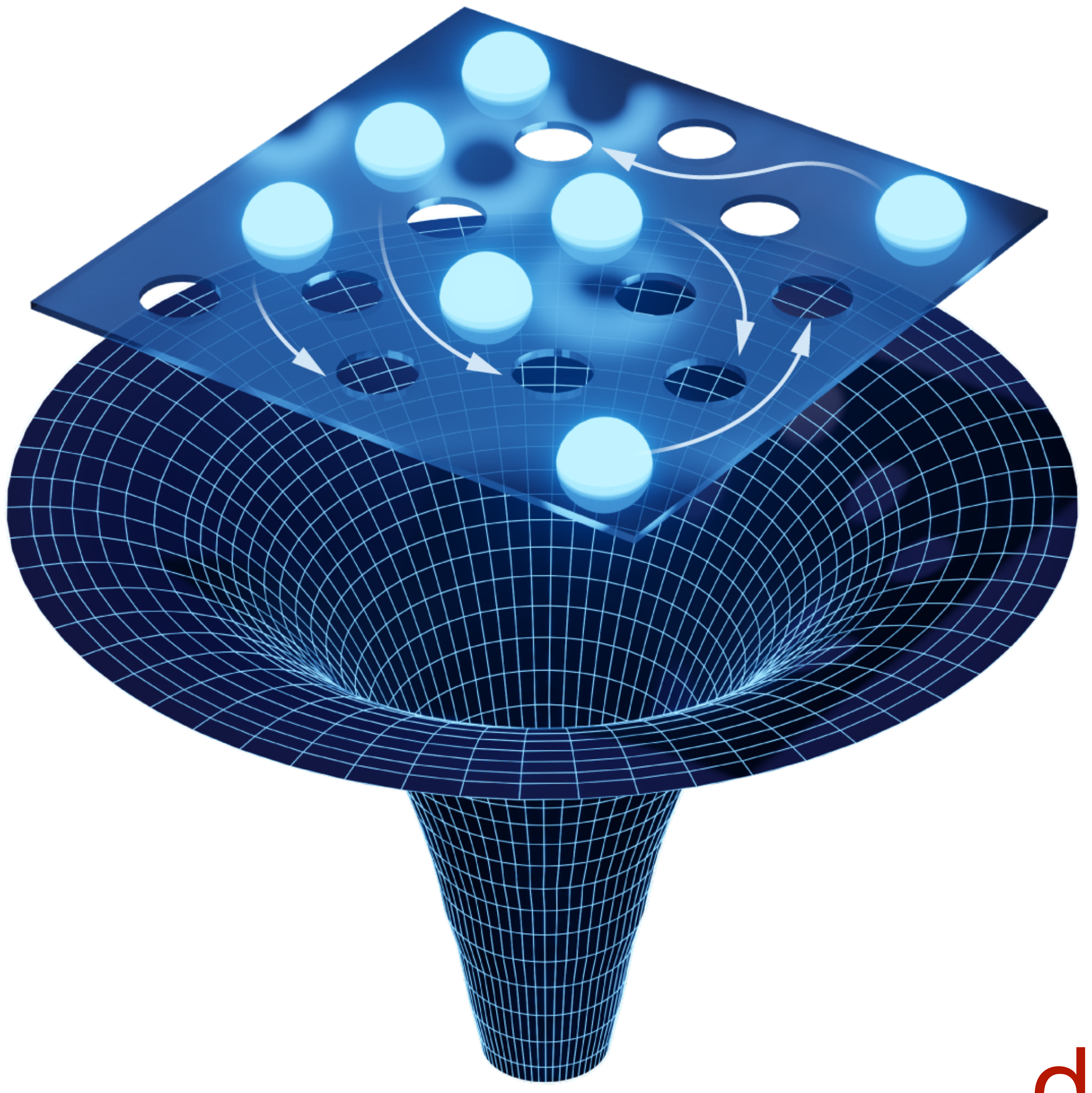

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles

A 2d-YSYK theory describes the **strange metal** behavior of numerous quantum materials

In a *dual* set of variables the SYK model has led to the computation of the low energy density of states of ***charged/rotating black holes***