

# The quantum phases of matter and gauge-gravity duality

Frontiers in Condensed Matter Science,  
Fortaleza, Brazil, April 10, 2013

Subir Sachdev

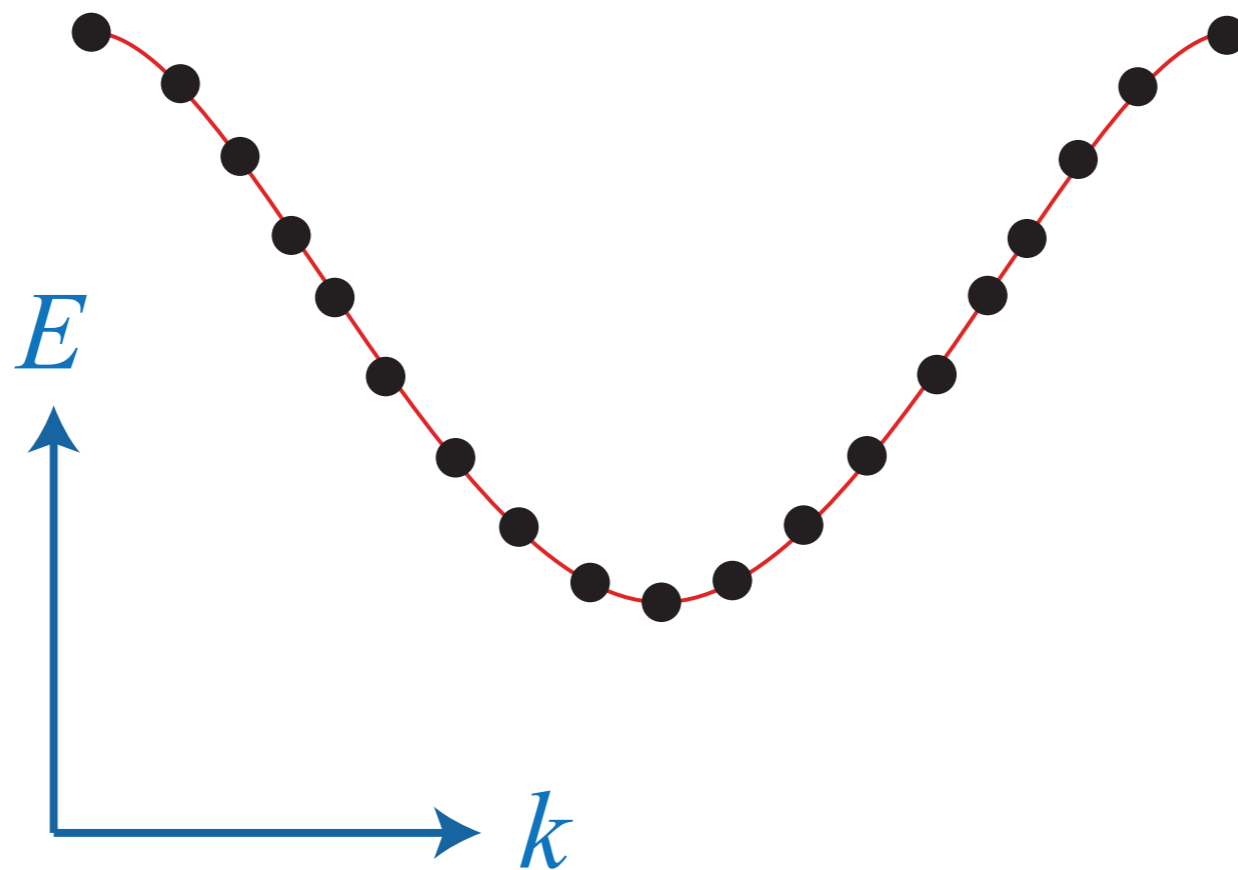
SCIENTIFIC AMERICAN 308, 44 (JANUARY 2013)



Sommerfeld-Bloch theory of  
metals, insulators, and superconductors:  
many-electron quantum states are adiabatically  
connected to independent electron states

Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states

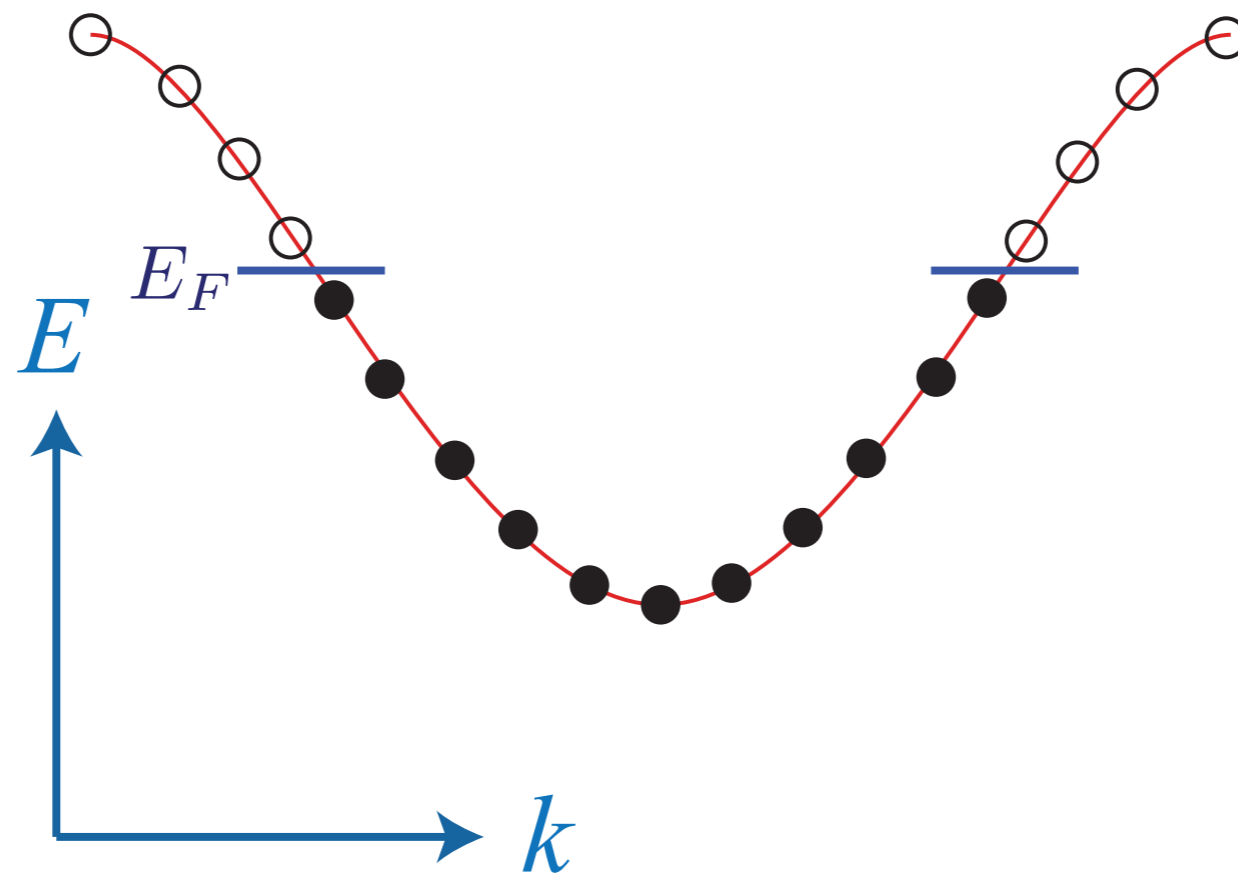
## Band insulators



An even number of electrons per unit cell

Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states

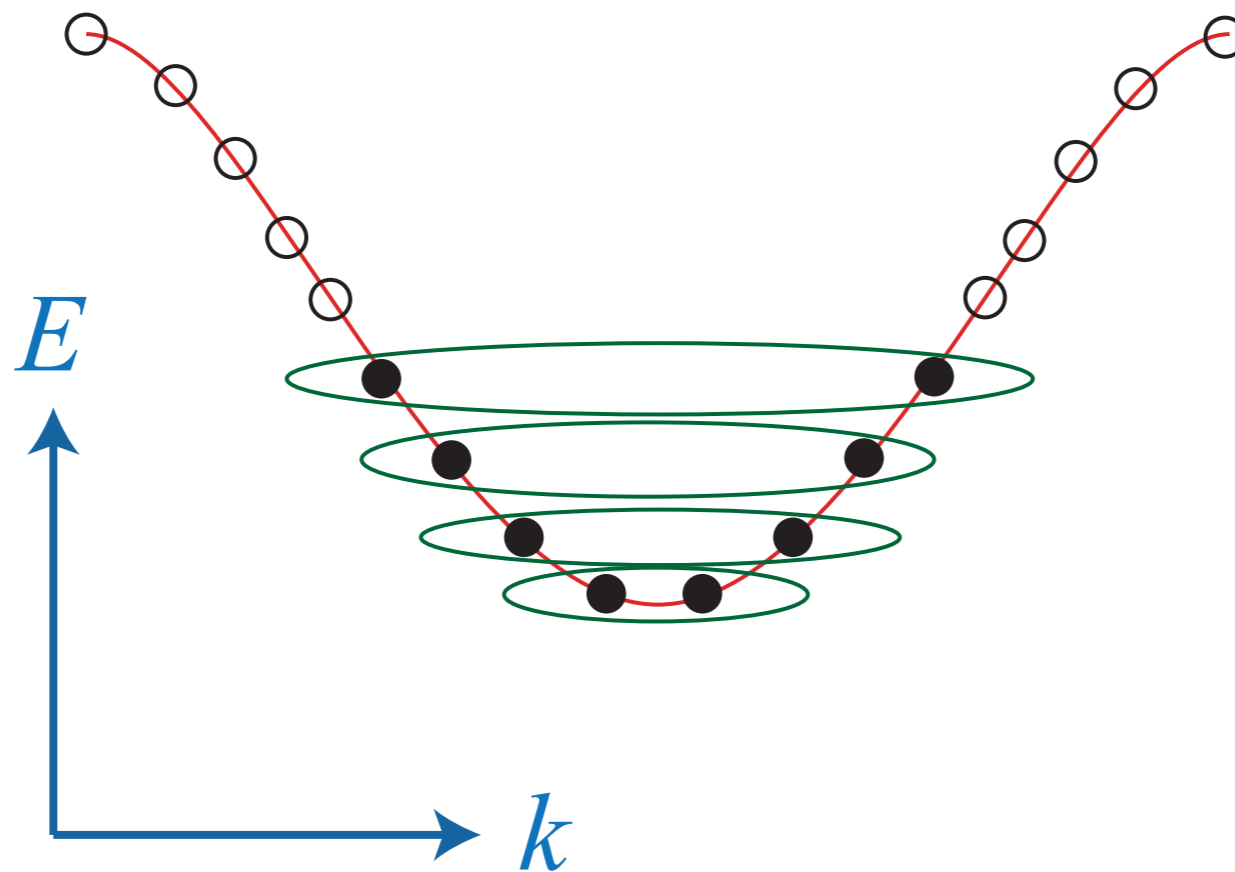
## Metals



An odd number of electrons per unit cell

Sommerfeld-Bloch theory of  
metals, insulators, and superconductors:  
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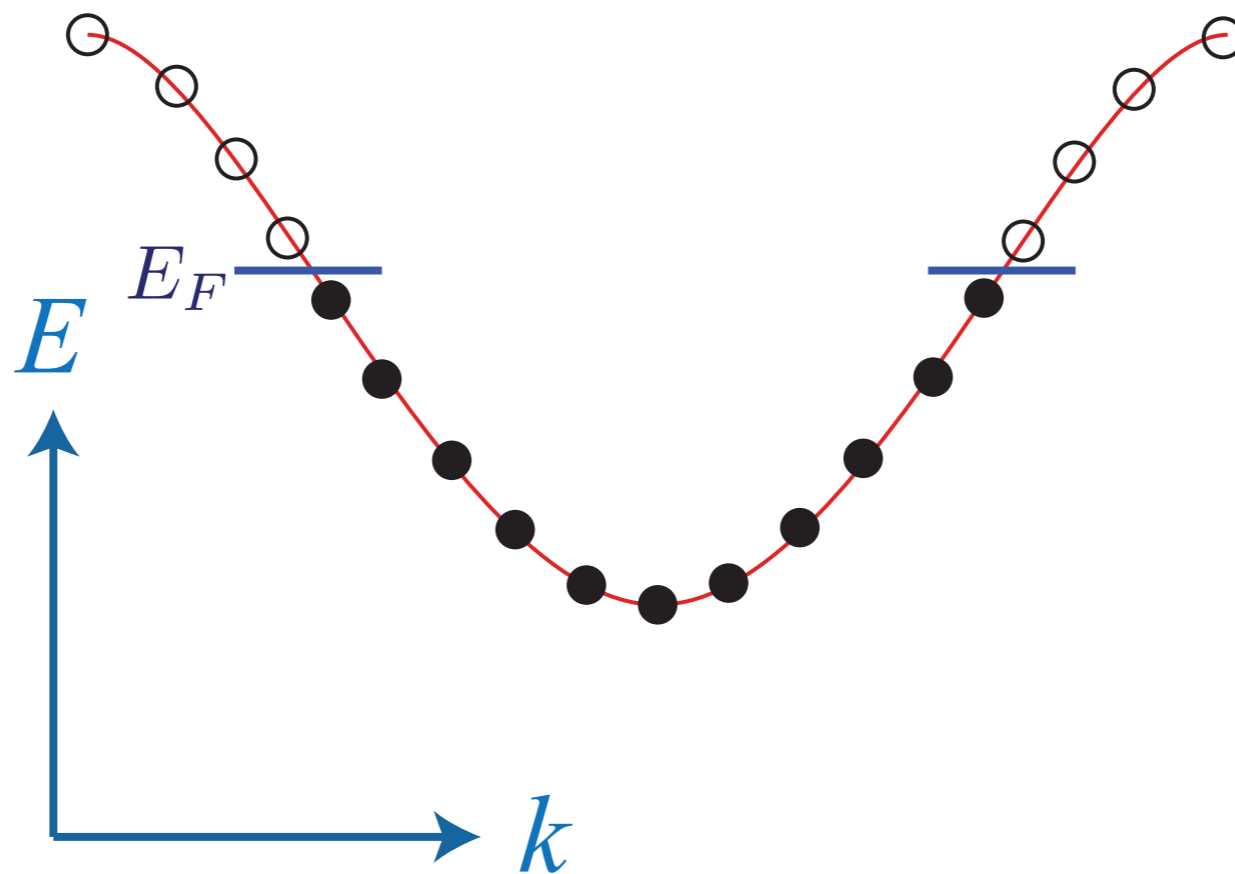
## Superconductors



# Boltzmann-Landau theory of dynamics of metals:

Long-lived **quasiparticles** (and **quasiholes**) have weak interactions which can be described by a Boltzmann equation

## Metals

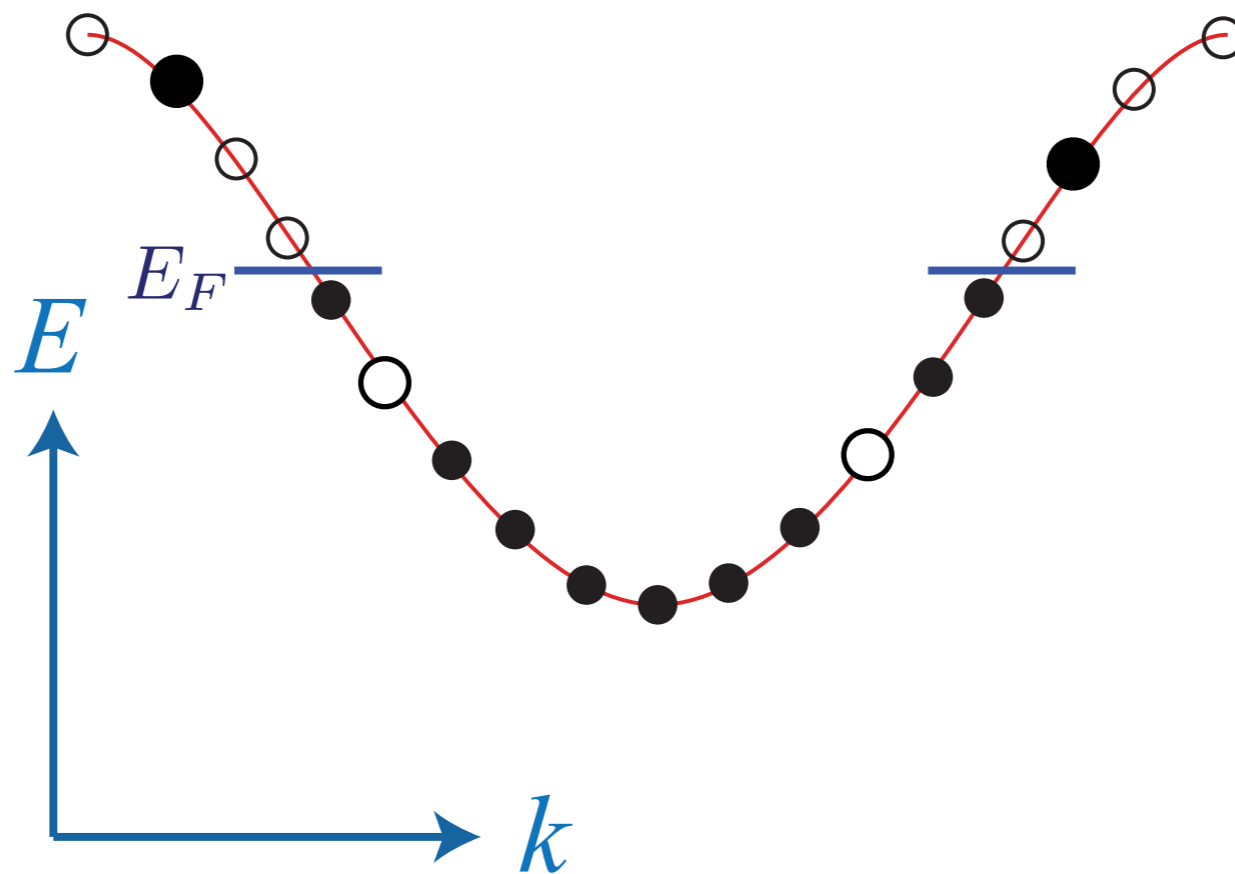


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# Boltzmann-Landau theory of dynamics of metals:

Long-lived **quasiparticles** (and **quasiholes**) have weak interactions which can be described by a Boltzmann equation

## Metals



An odd number of electrons per unit cell

**Modern phases of quantum matter**  
Not adiabatically connected  
to independent electron states:

# Modern phases of quantum matter

Not adiabatically connected  
to independent electron states:

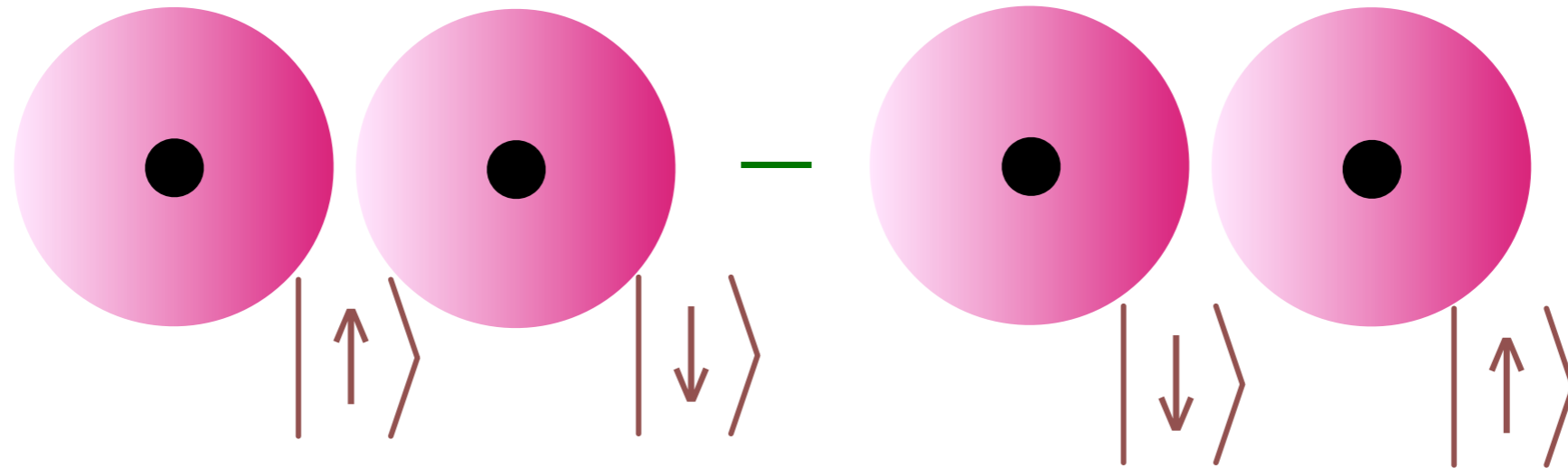
*many-particle*

*quantum entanglement,*

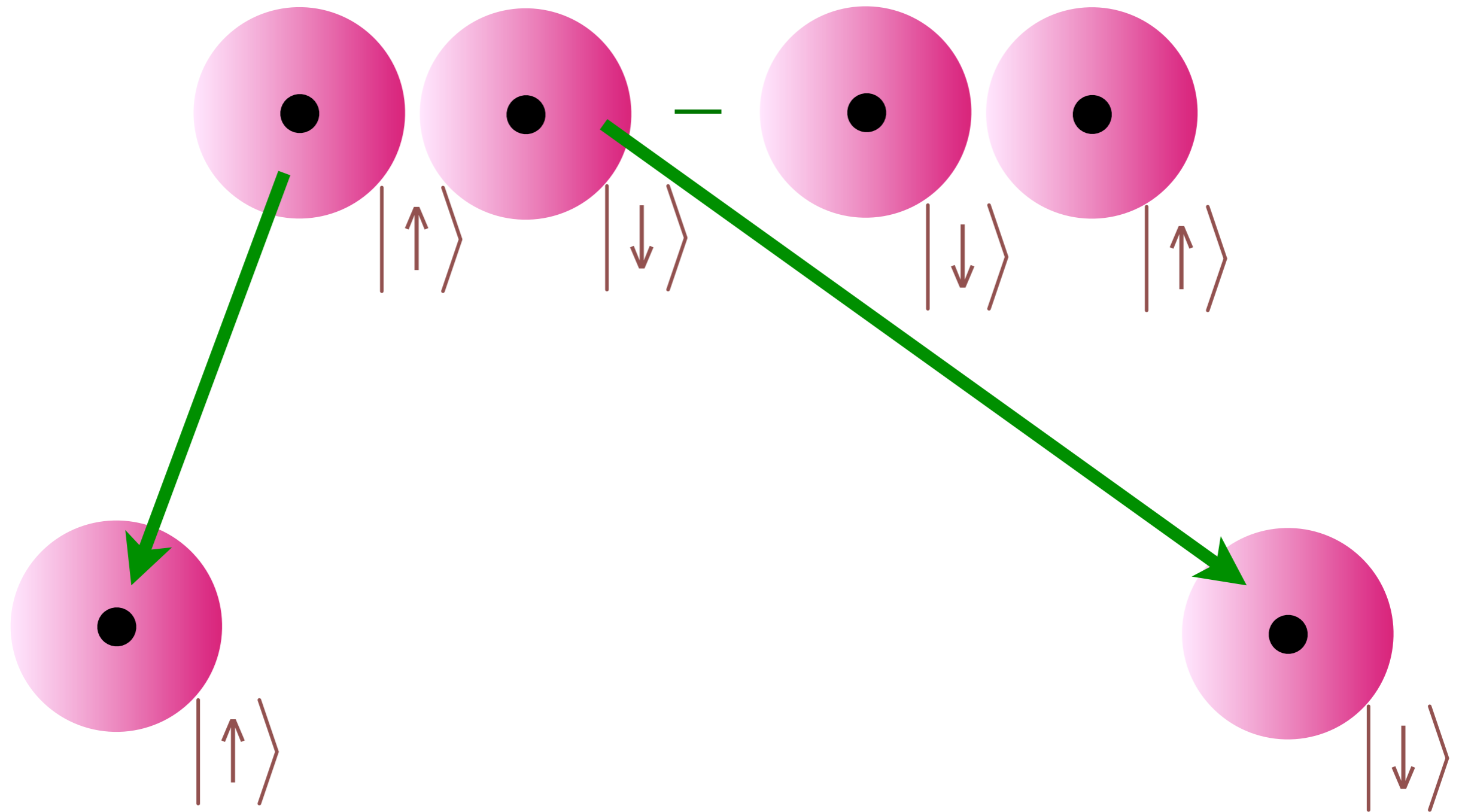
***and no quasiparticles***

# Quantum Entanglement: quantum superposition

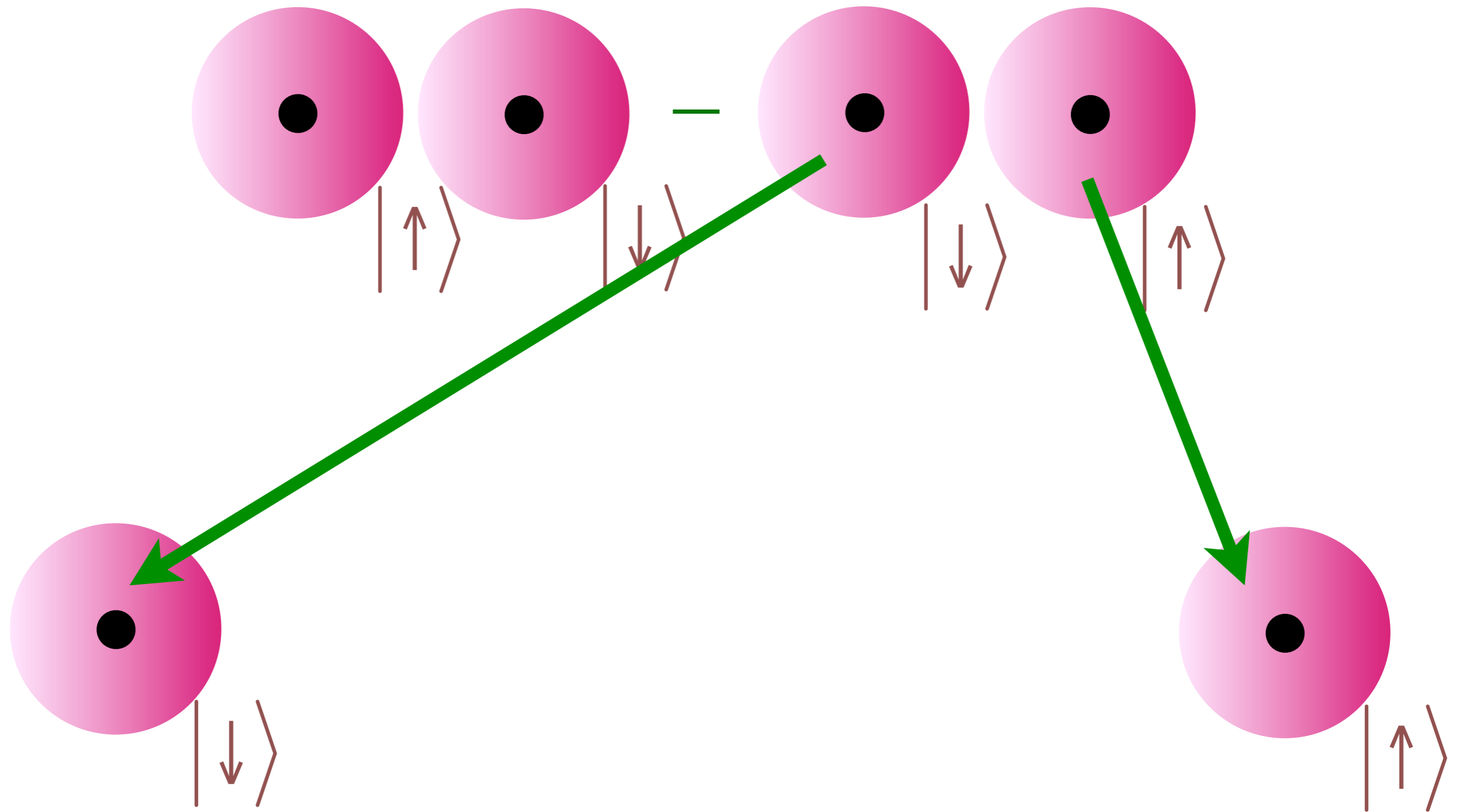
Hydrogen molecule: with more than one particle



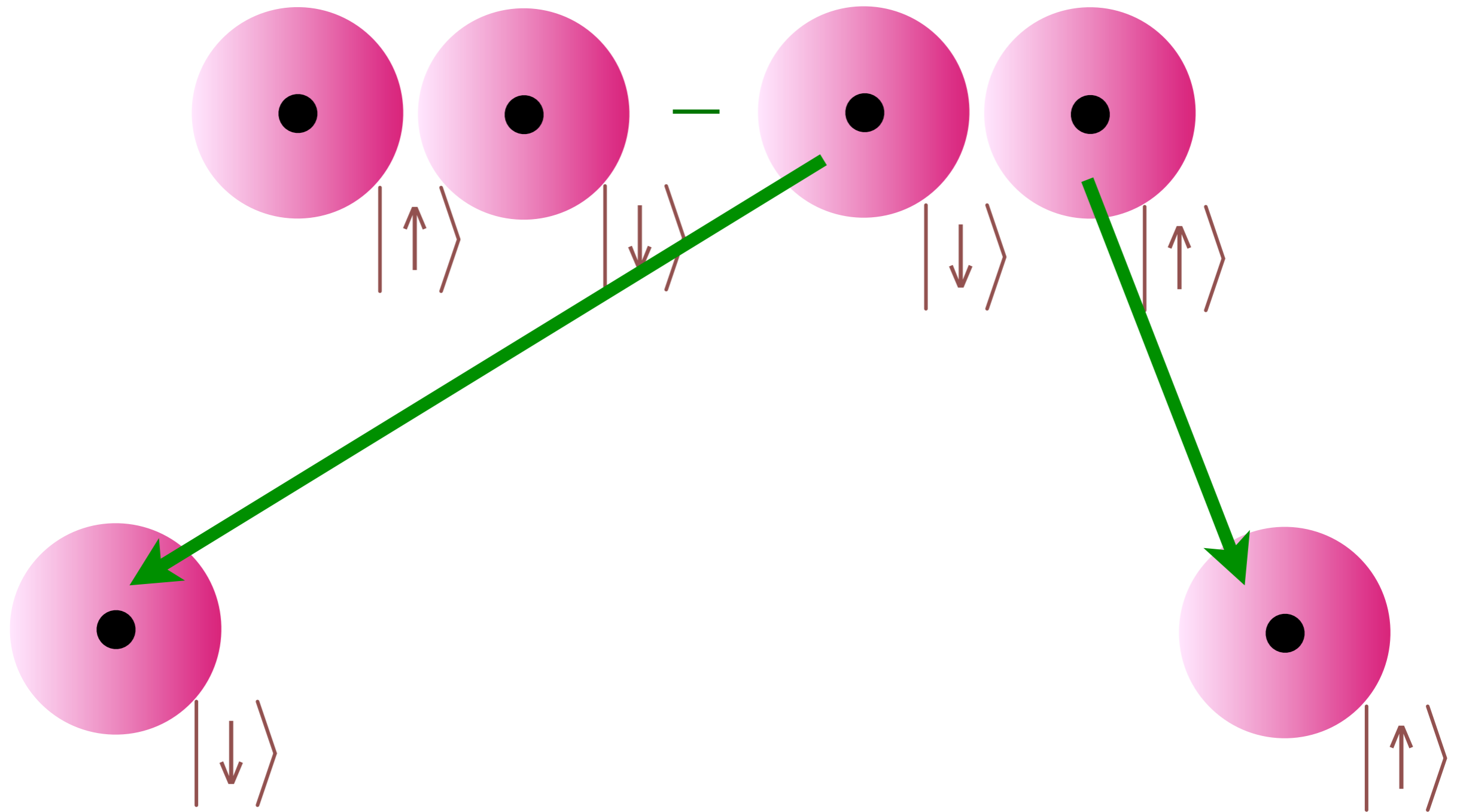
# Quantum Entanglement: quantum superposition with more than one particle



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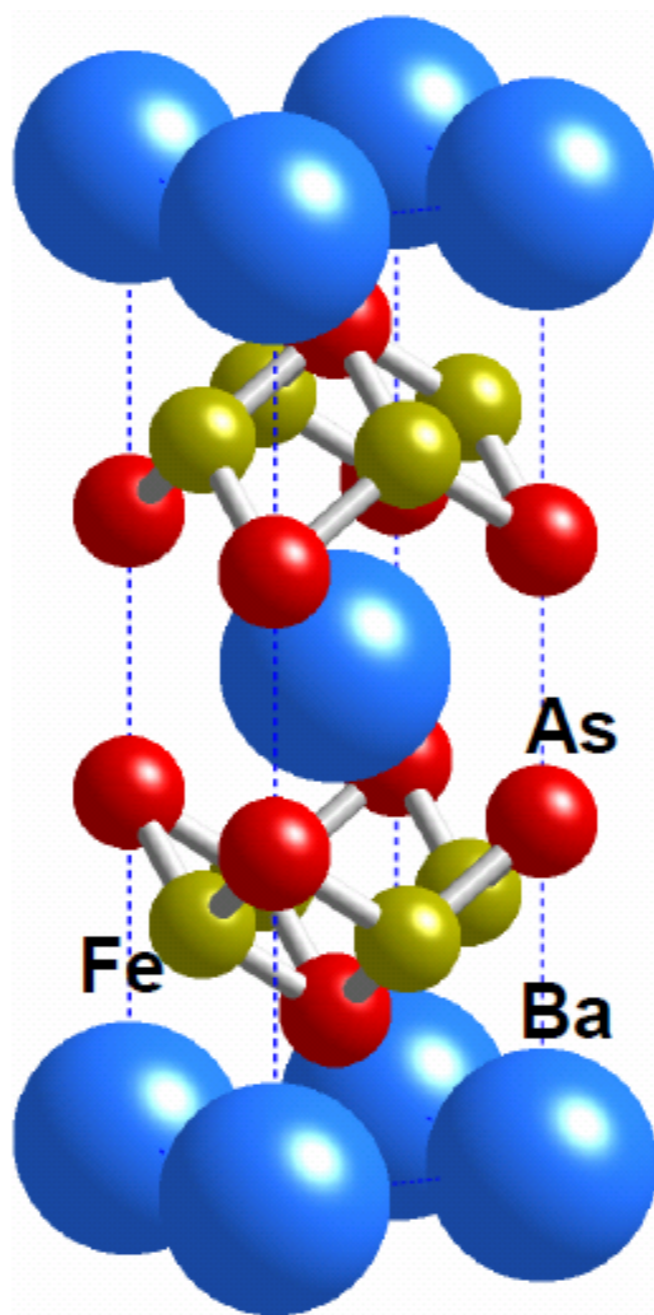
# Quantum Entanglement: quantum superposition with more than one particle



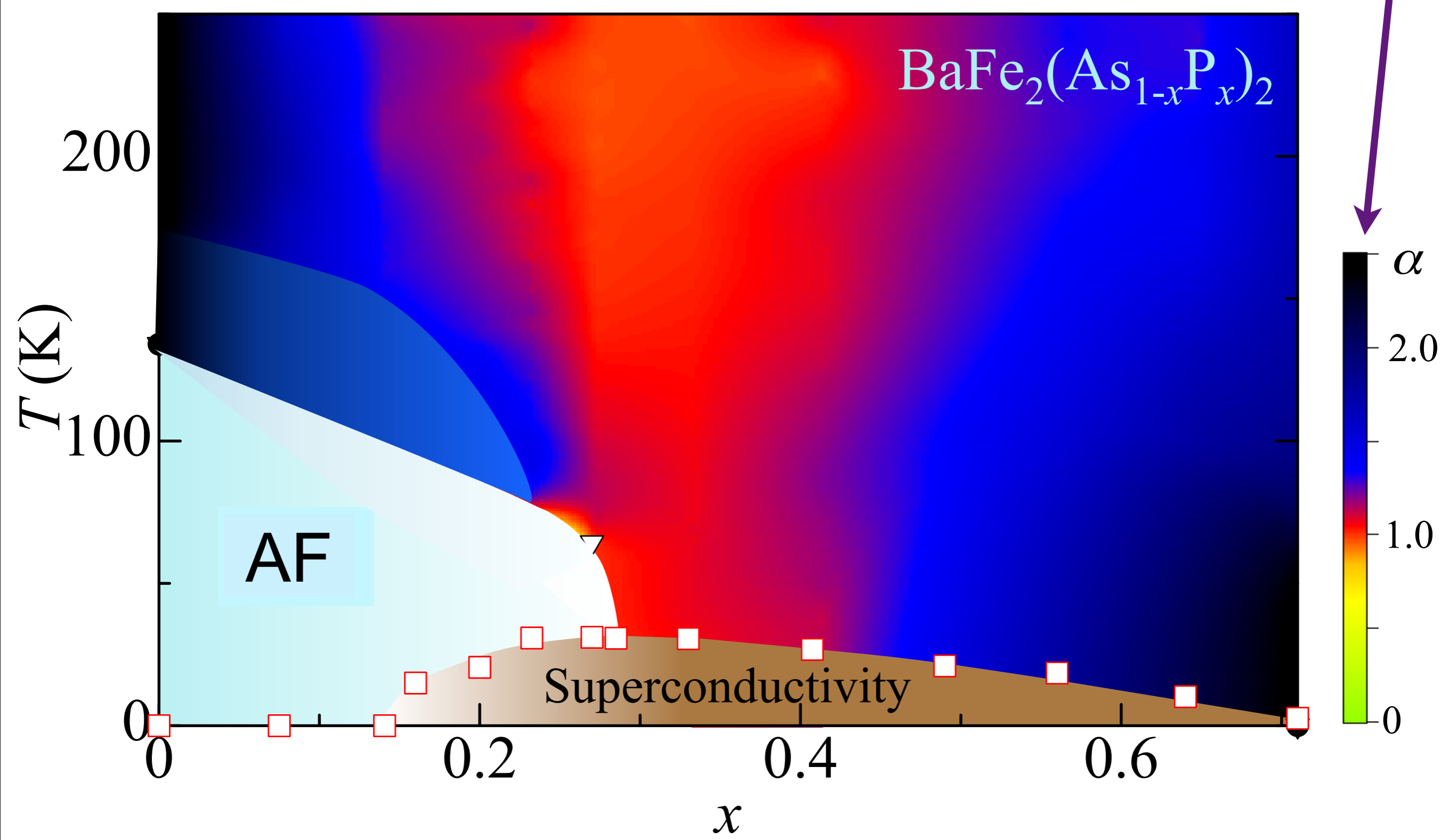
Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart

# Iron pnictides:

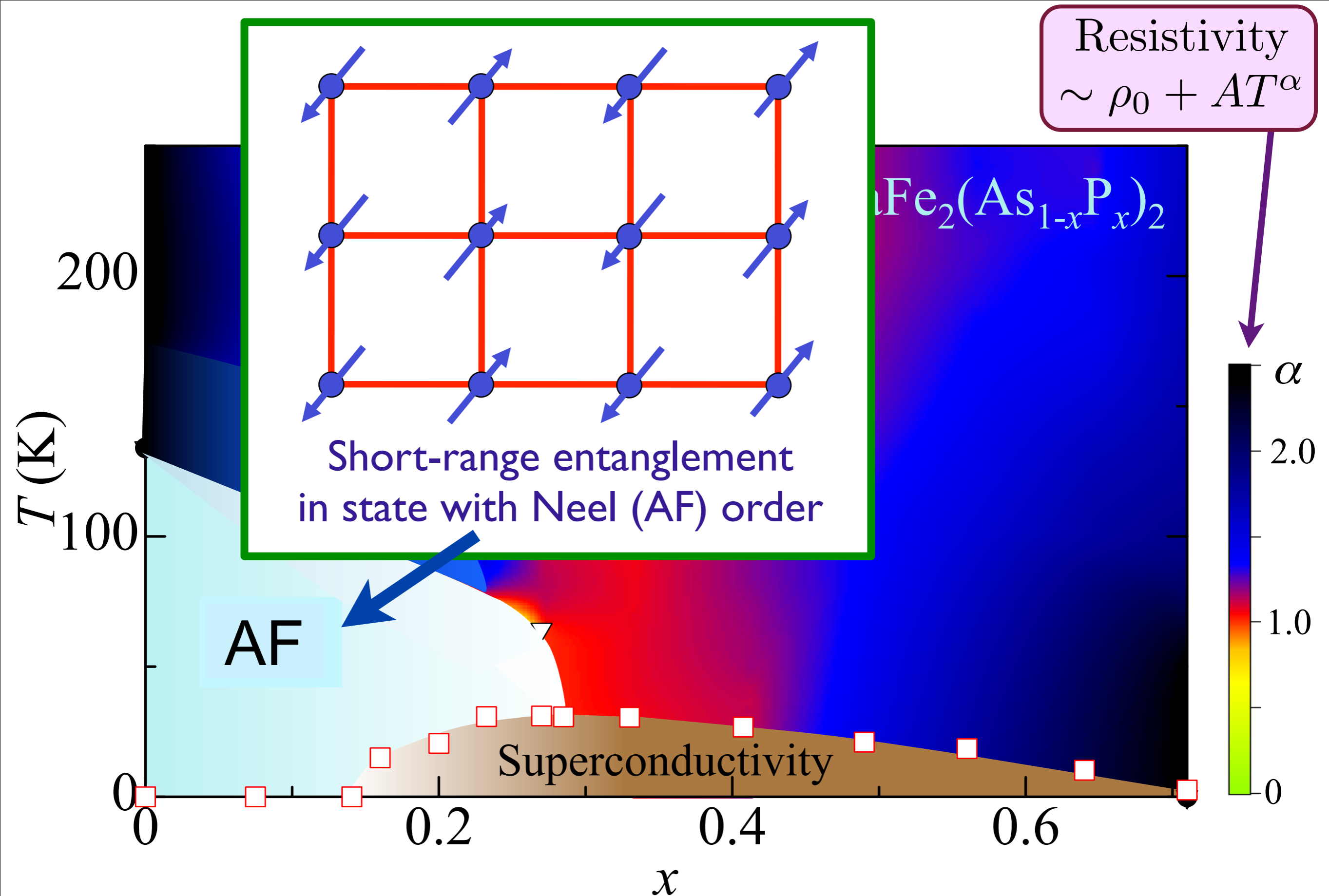
a new class of high temperature superconductors



Resistivity  
 $\sim \rho_0 + AT^\alpha$

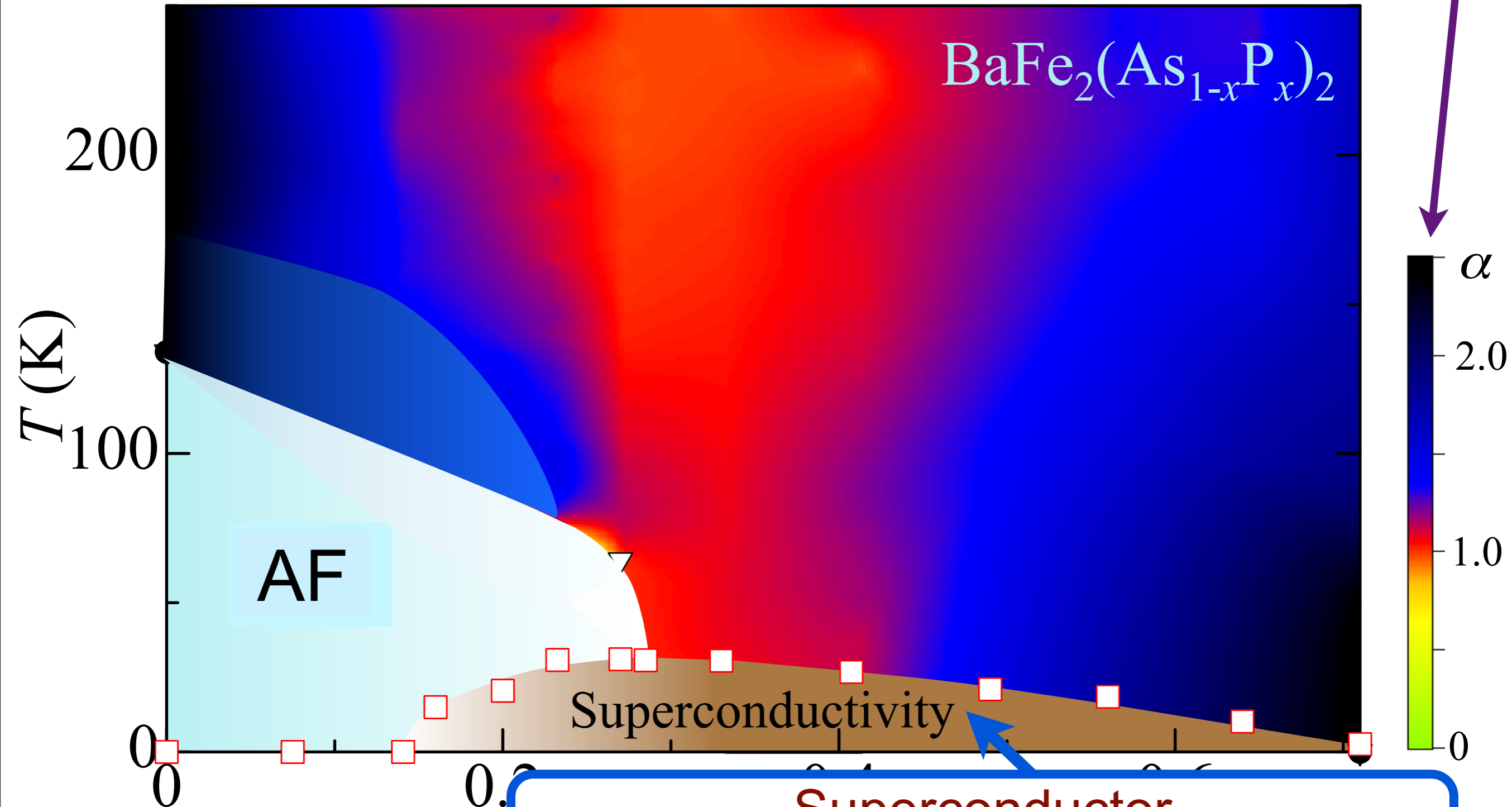


S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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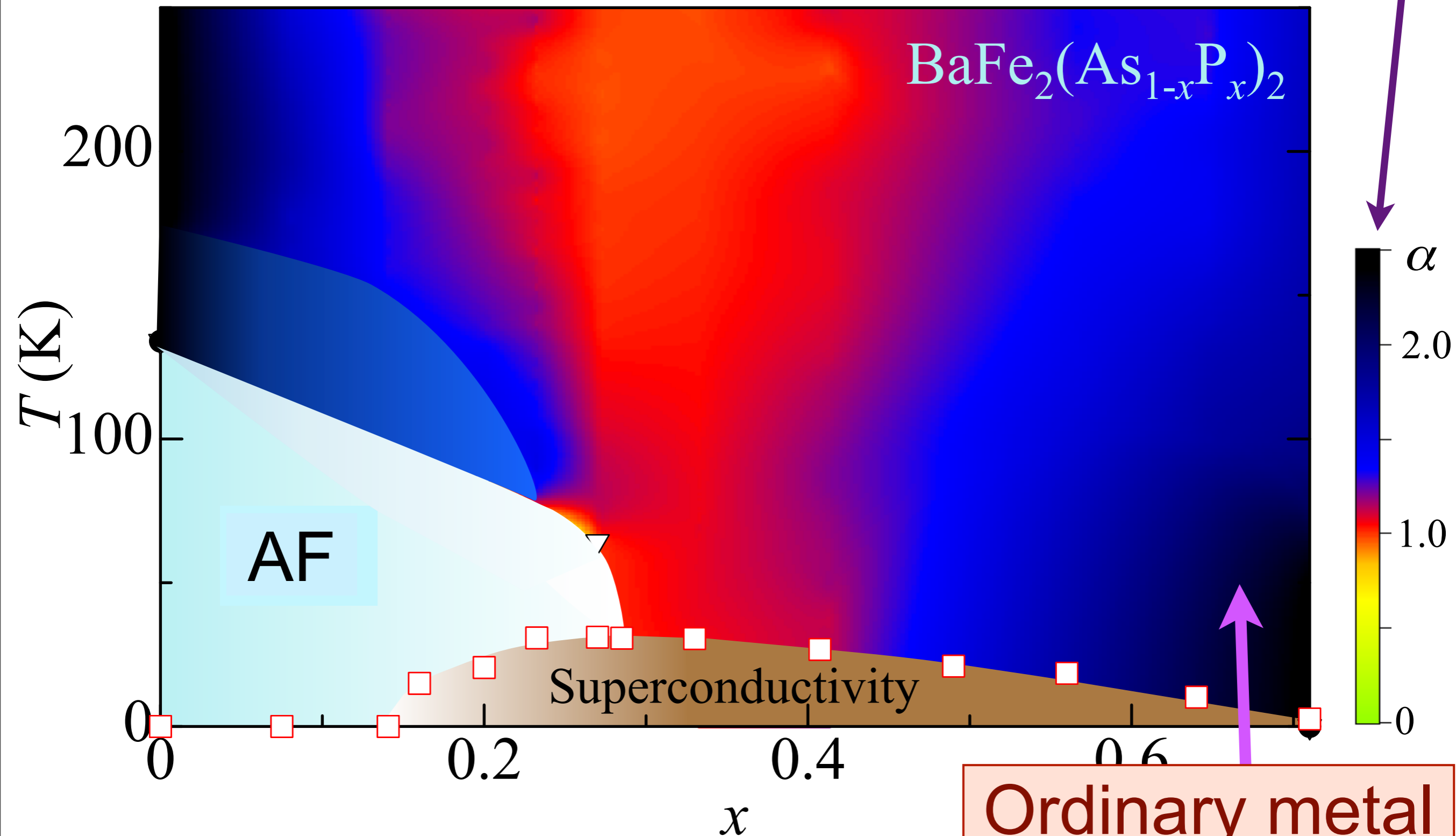
Resistivity  
 $\sim \rho_0 + AT^\alpha$



**Superconductor**  
Bose condensate of pairs of electrons  
Short-range entanglement

S. Kasahara, T. Shiba  
H. Ike

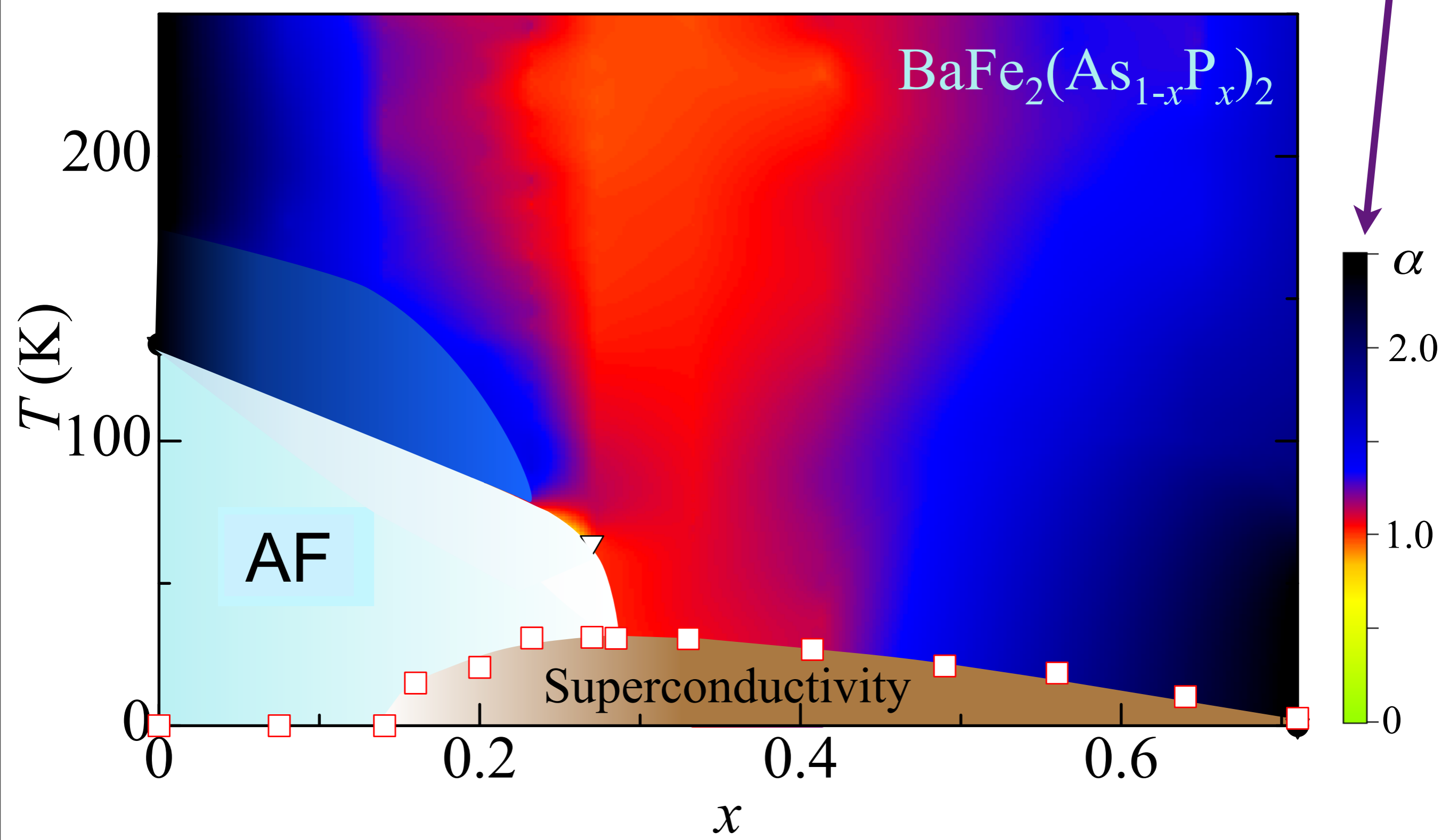
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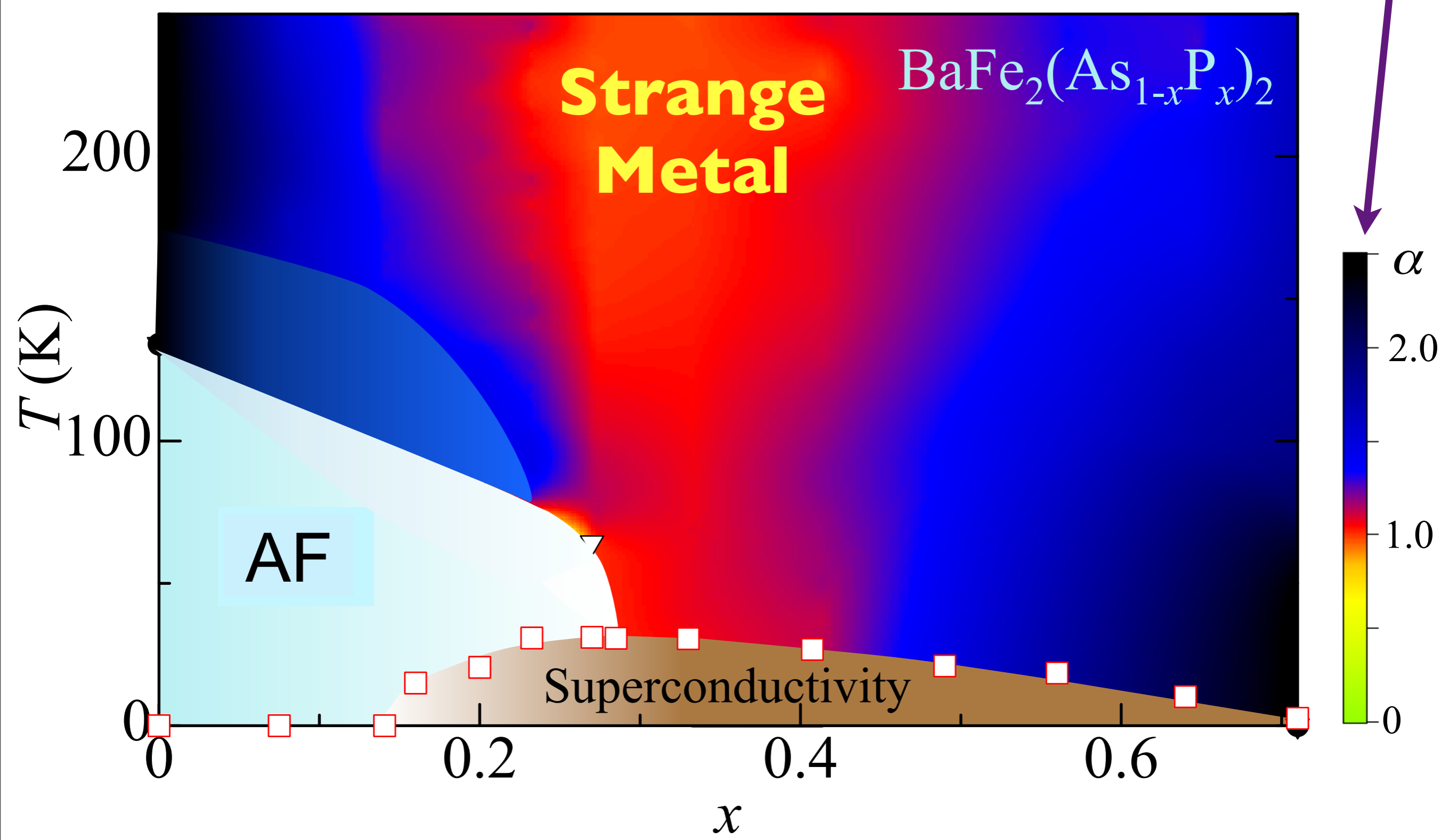
Ordinary metal  
(Fermi liquid)

Resistivity  
 $\sim \rho_0 + AT^\alpha$



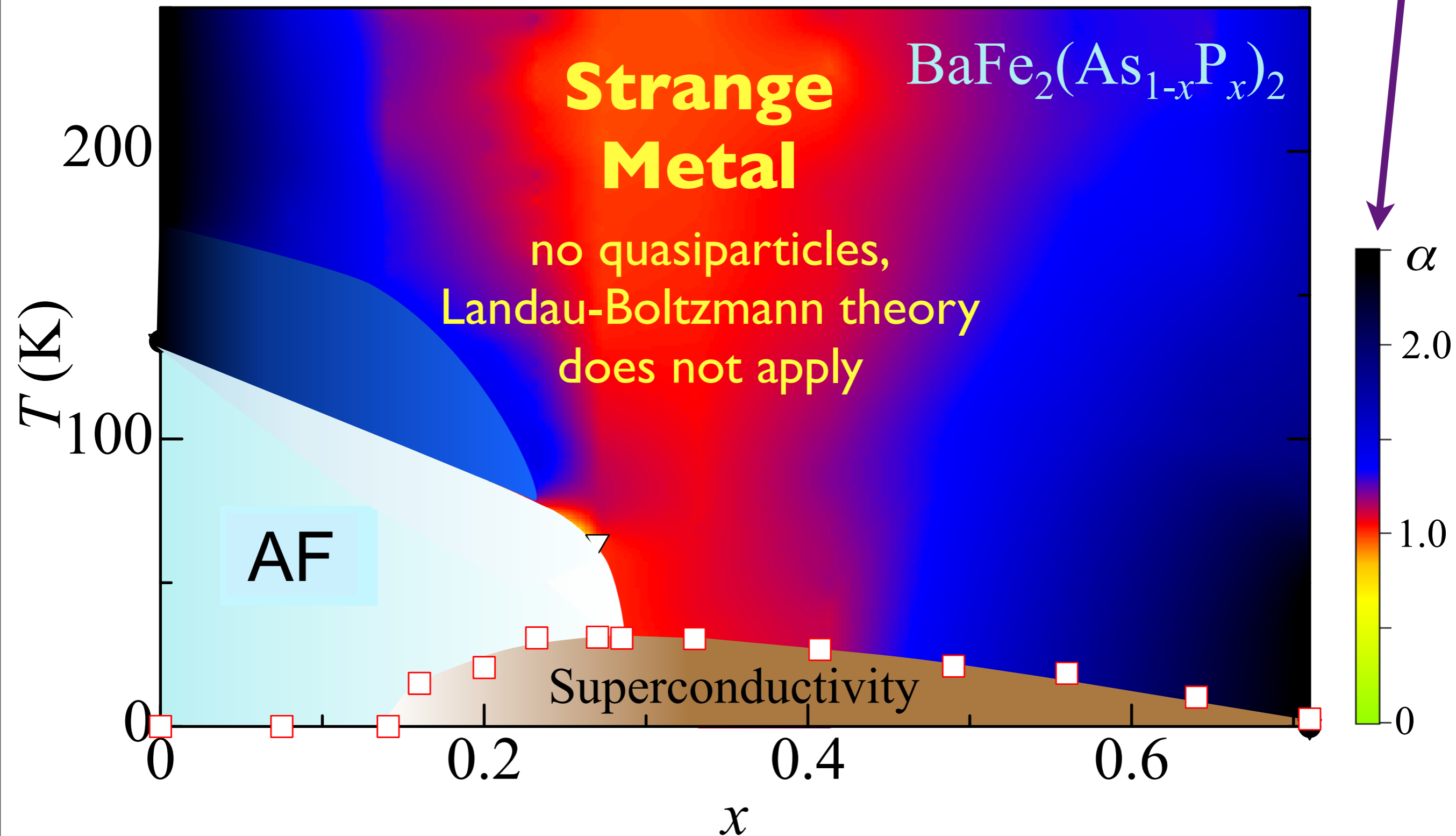
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

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# Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:

*Quantum criticality and conformal field theories*

2. Gauge-gravity duality

3. Black-hole horizons and quasi-normal modes

4. Strange metals:

*What lies beyond the horizon ?*

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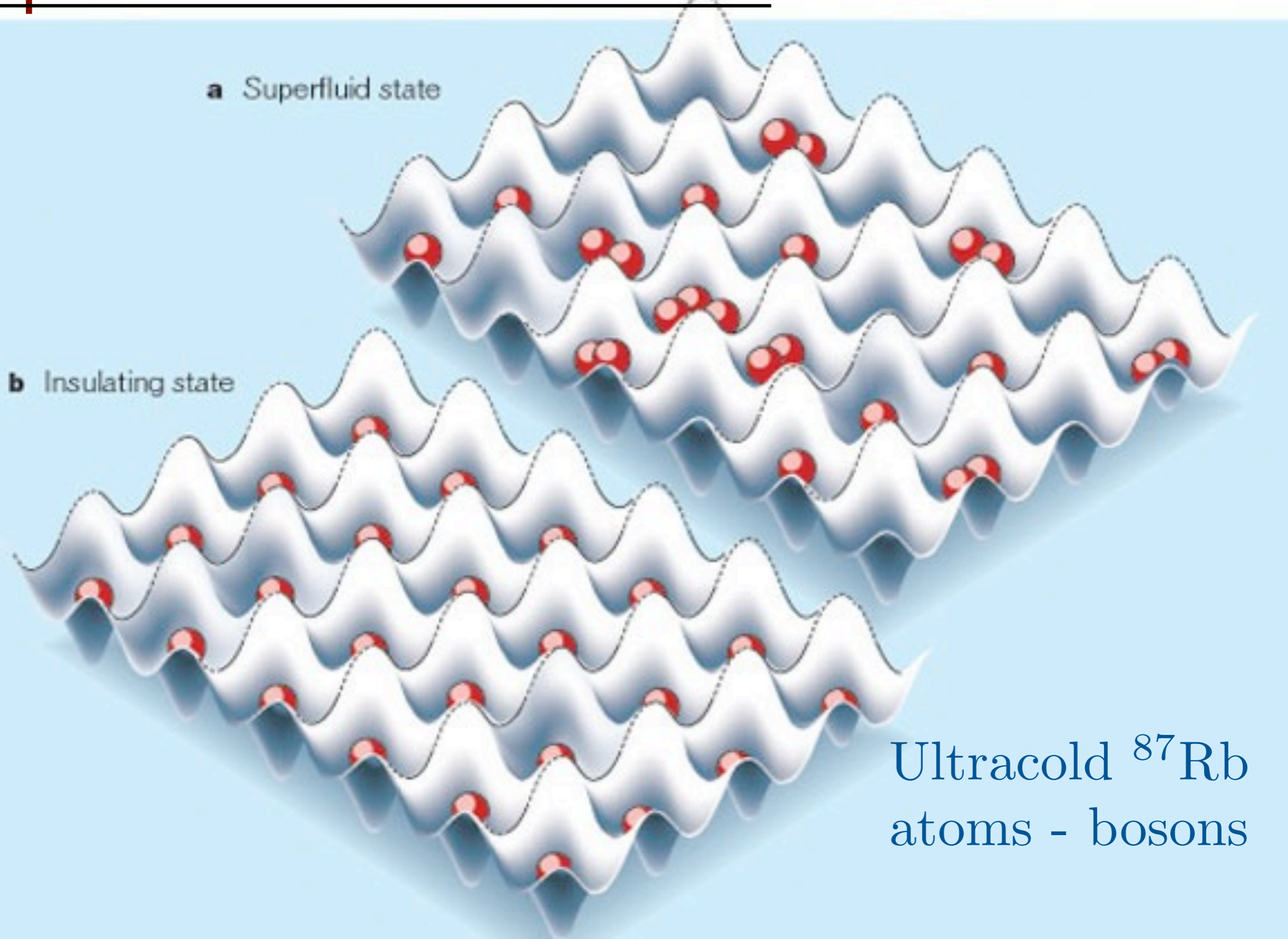
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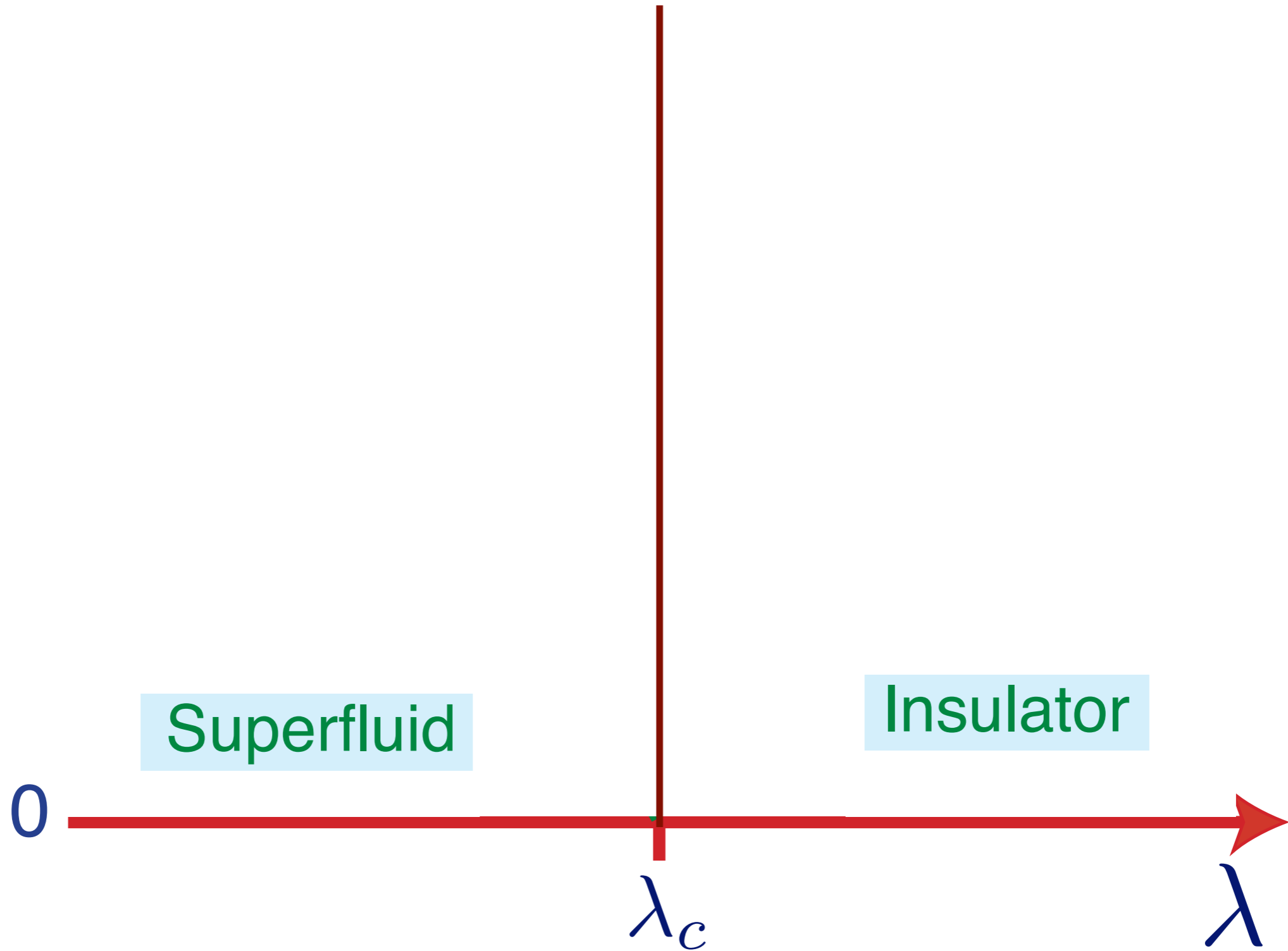
*What lies beyond the horizon ?*

# Superfluid-insulator transition

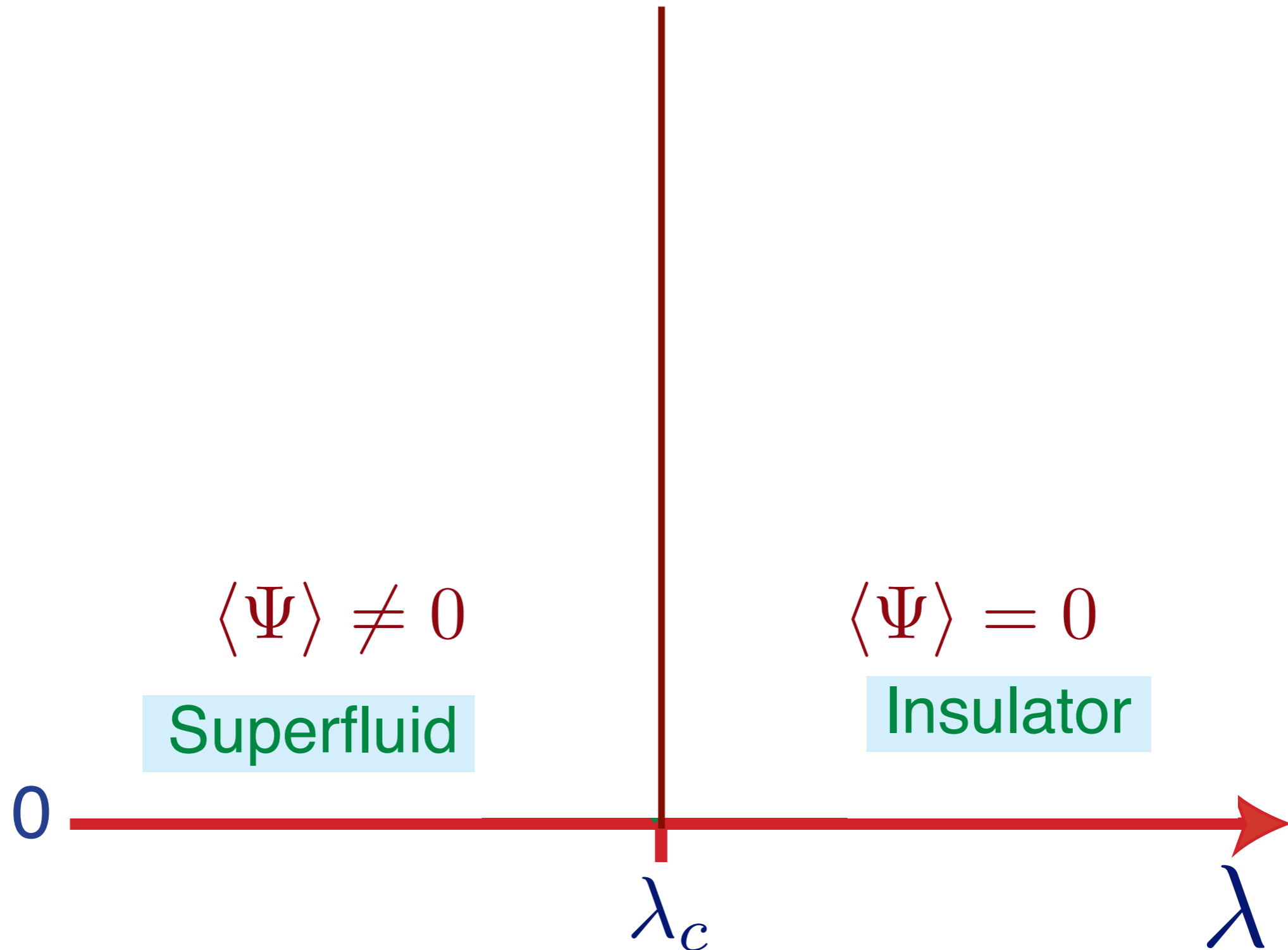


Ultracold  $^{87}\text{Rb}$   
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

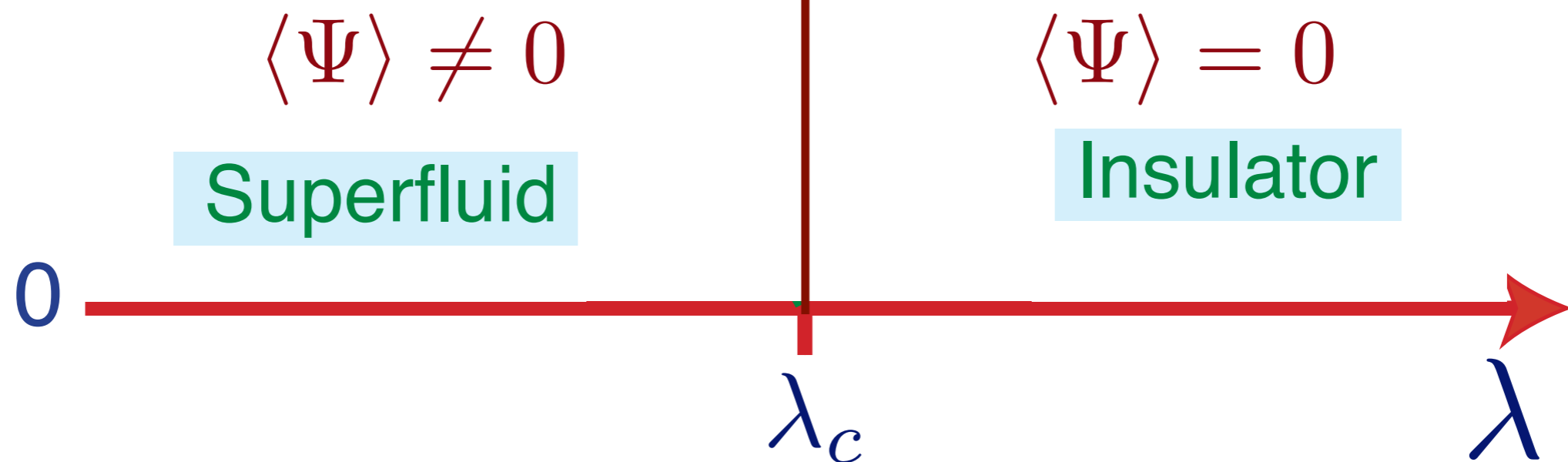


$\Psi \rightarrow$  a complex field representing the Bose-Einstein condensate of the superfluid



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

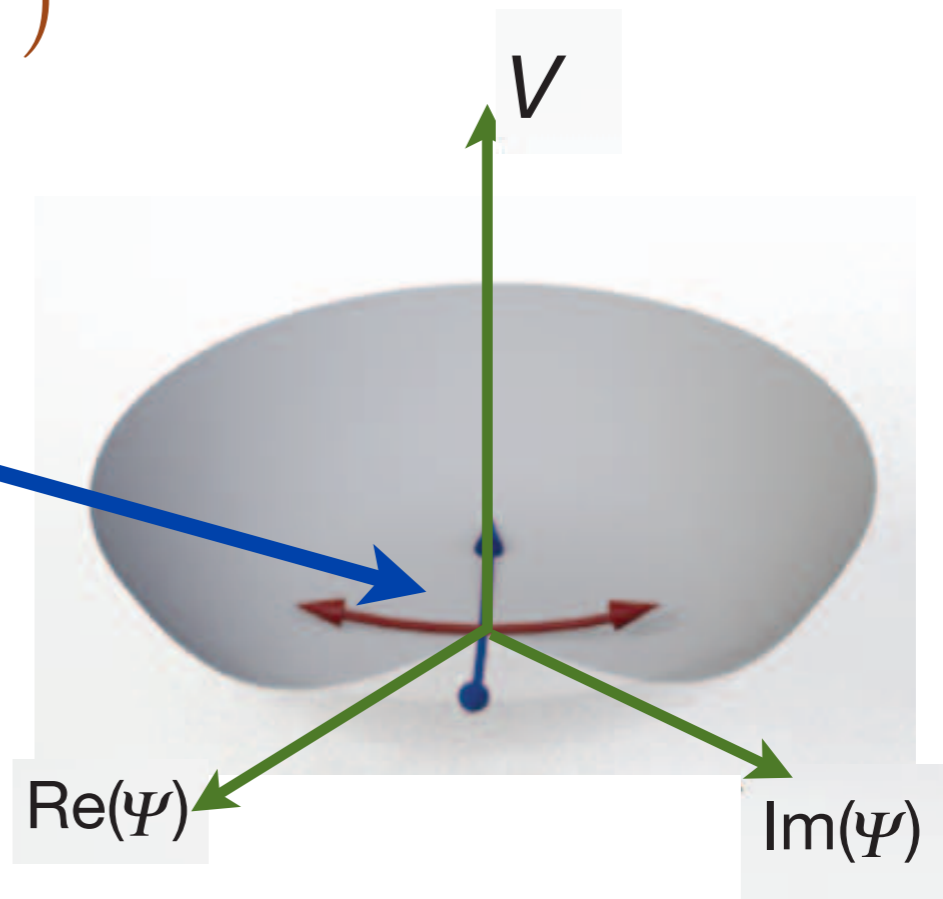
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$



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Particles and holes correspond to the 2 normal modes in the oscillation of  $\Psi$  about  $\Psi = 0$ .

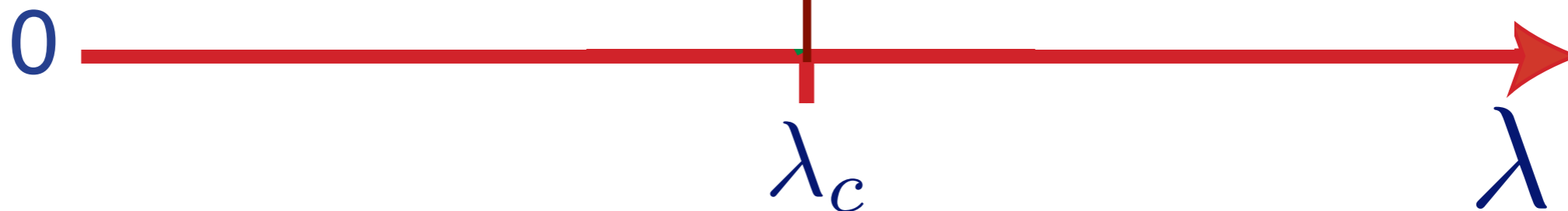


$$\langle \Psi \rangle \neq 0$$

Superfluid

$$\langle \Psi \rangle = 0$$

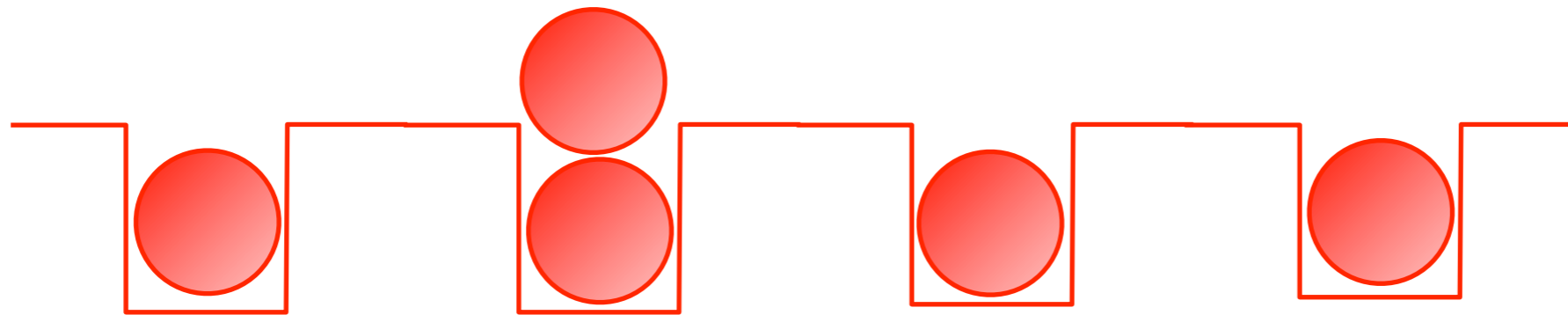
Insulator





Insulator (the vacuum)  
at large repulsion between bosons

# Excitations of the insulator:



Particles  $\sim \Psi^\dagger$

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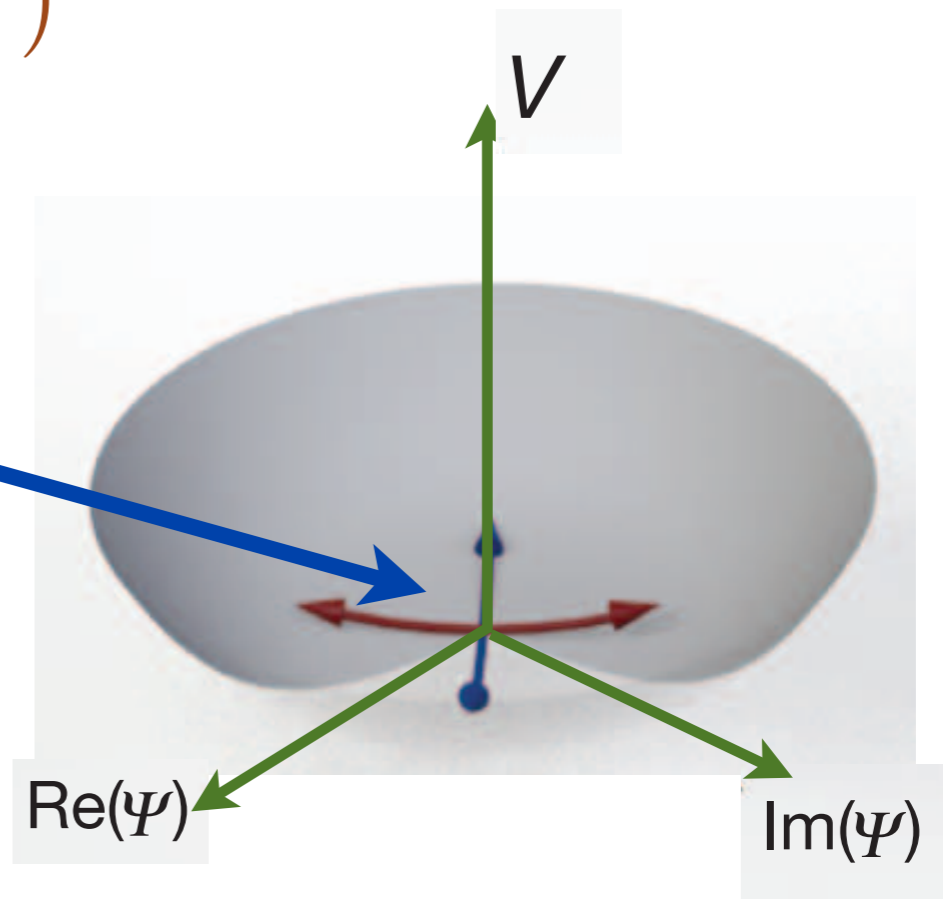


Holes  $\sim \Psi$

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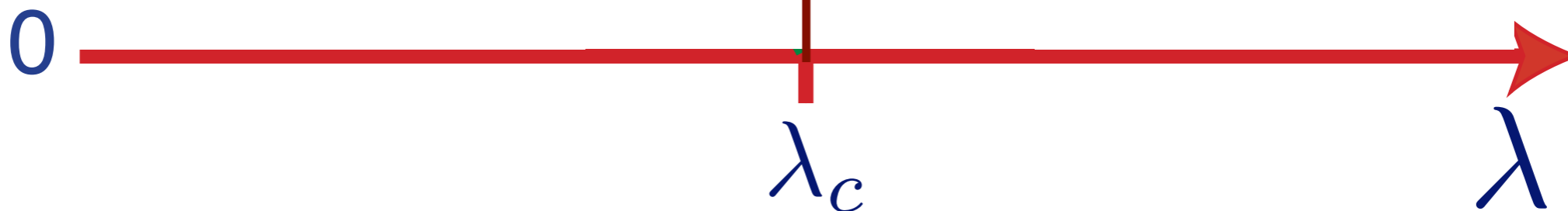


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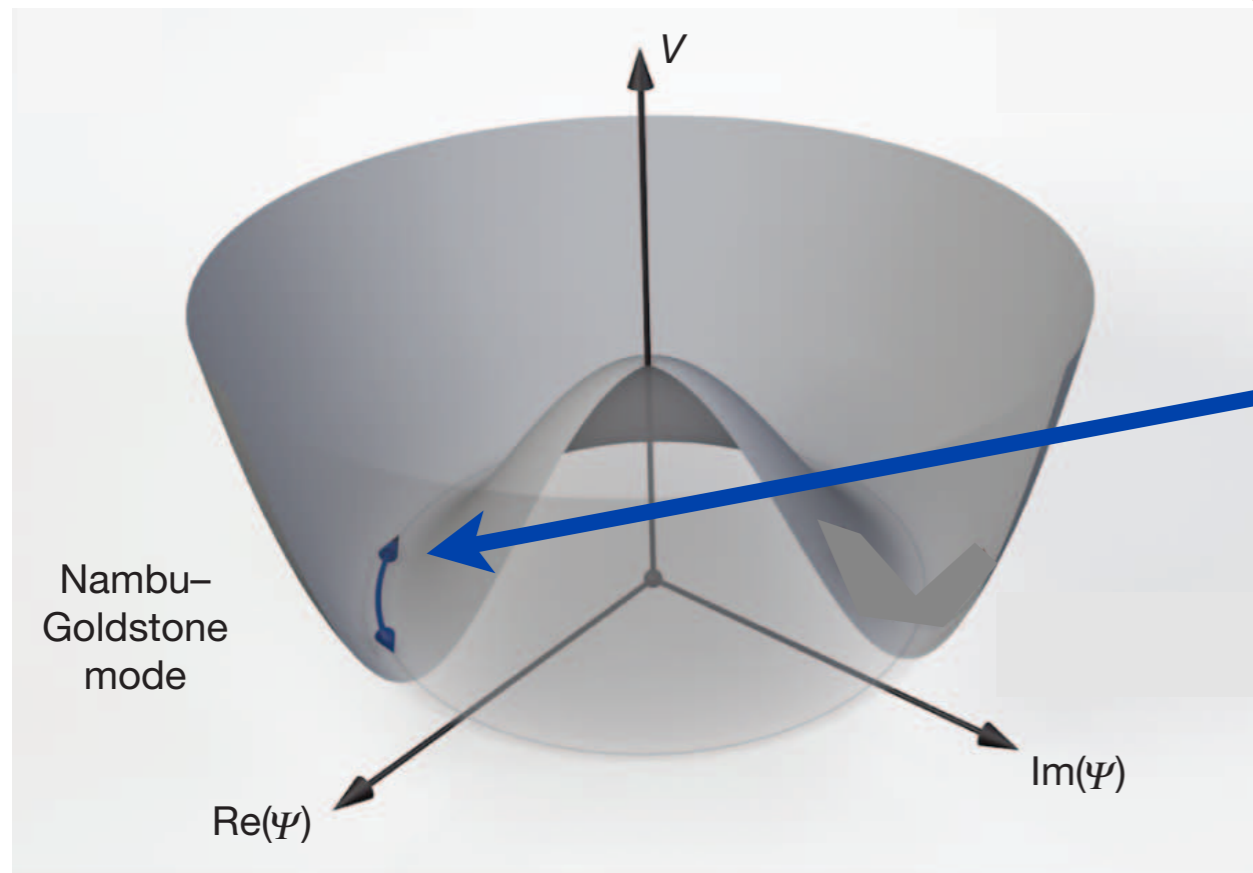
$$\langle \Psi \rangle = 0$$

Insulator

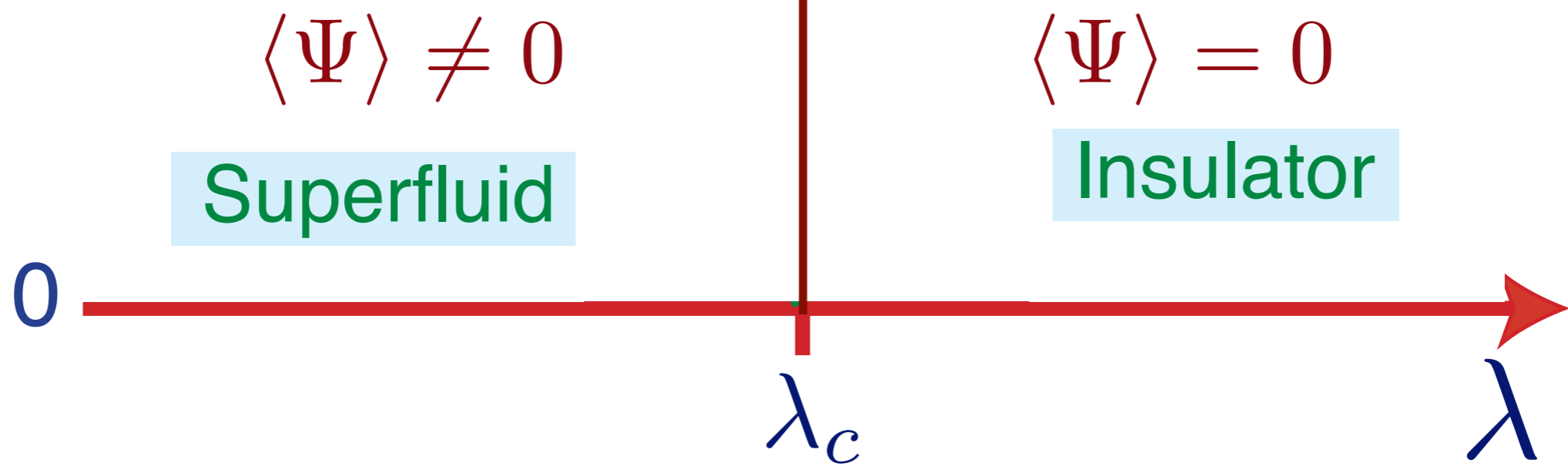


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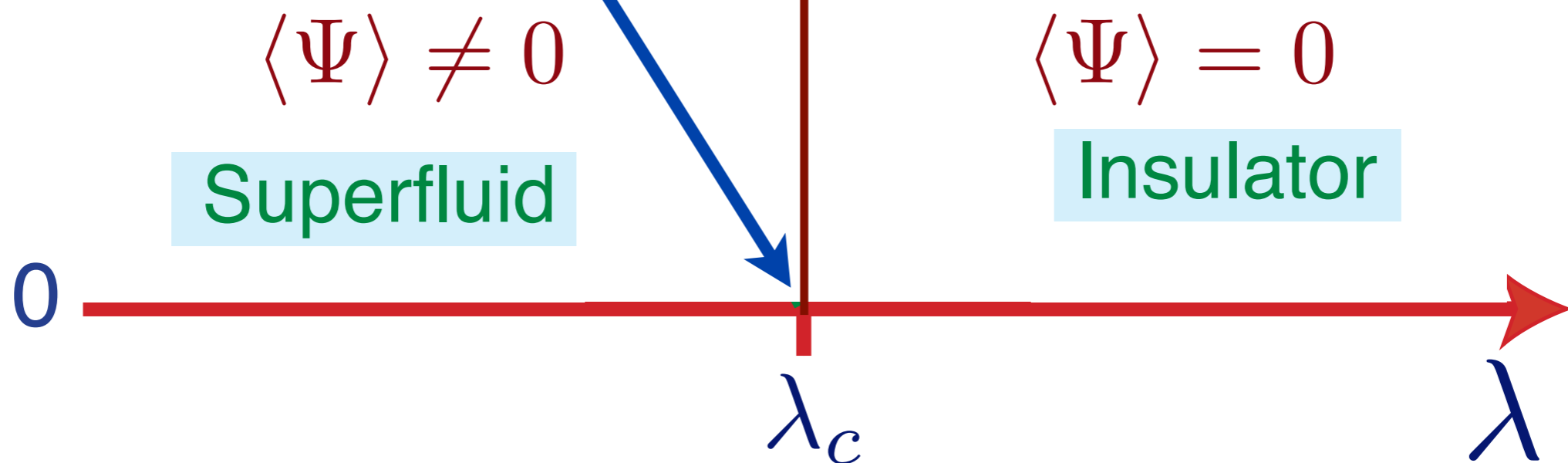
Nambu-Goldstone mode is the oscillation in the phase  $\Psi$  at a constant non-zero  $|\Psi|$ .



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

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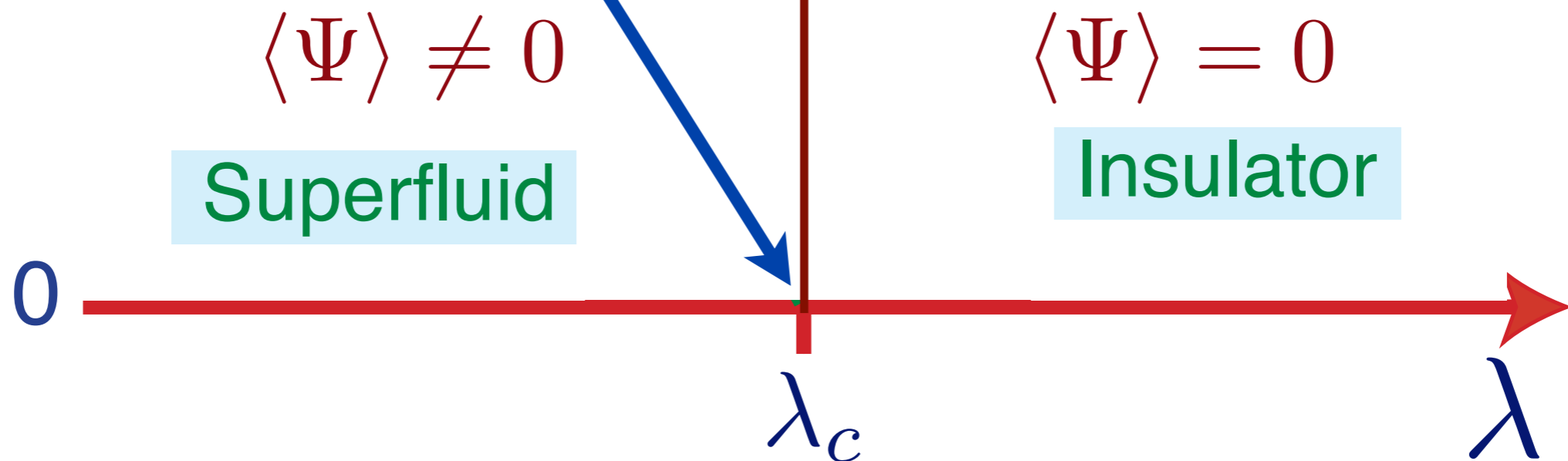
A conformal field theory  
in 2+1 spacetime dimensions:  
a CFT3



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

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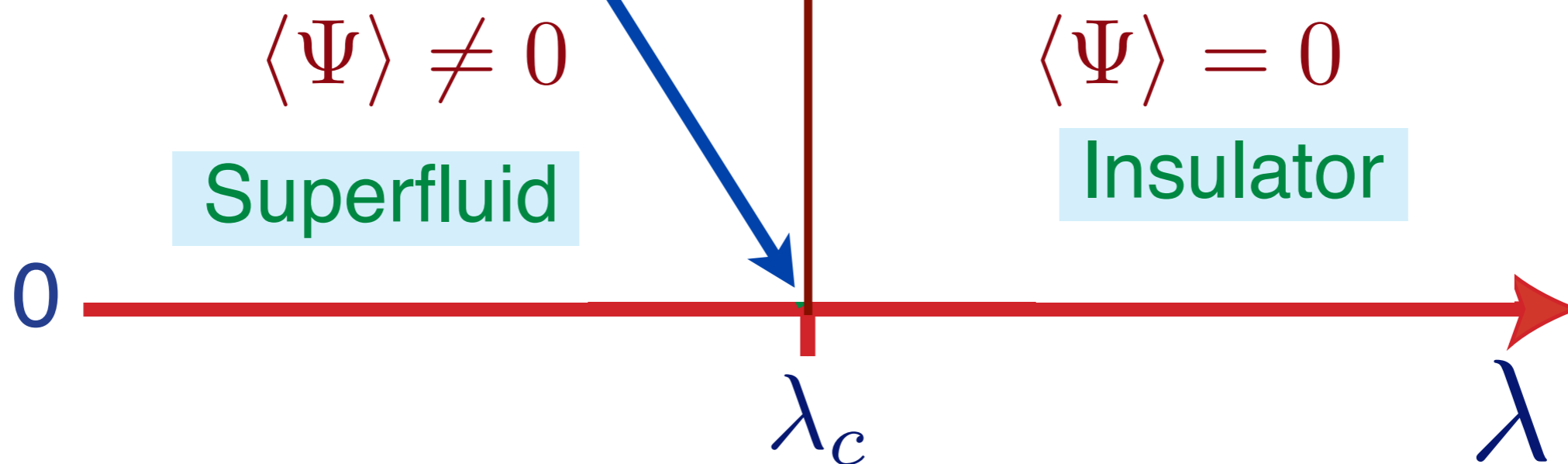
Quantum state with  
complex, many-body,  
“long-range” quantum entanglement



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

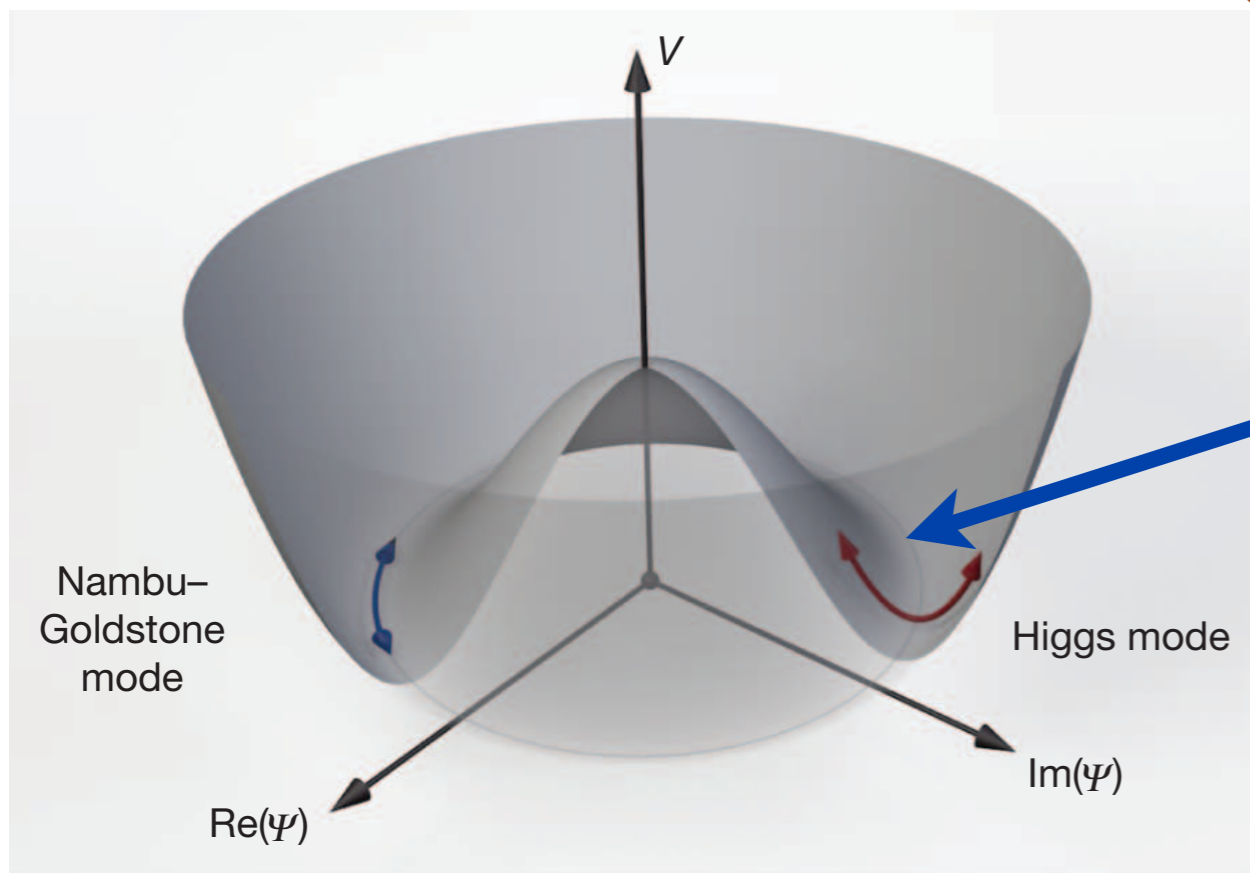
$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$

No well-defined normal modes,  
or particle-like excitations



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$



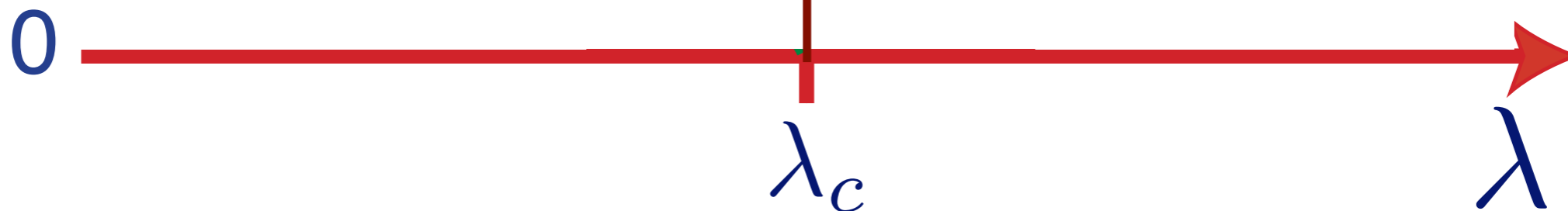
Higgs mode is the oscillation in the amplitude  $|\Psi|$ . This decays rapidly by emitting pairs of Nambu-Goldstone modes.

$$\langle \Psi \rangle \neq 0$$

Superfluid

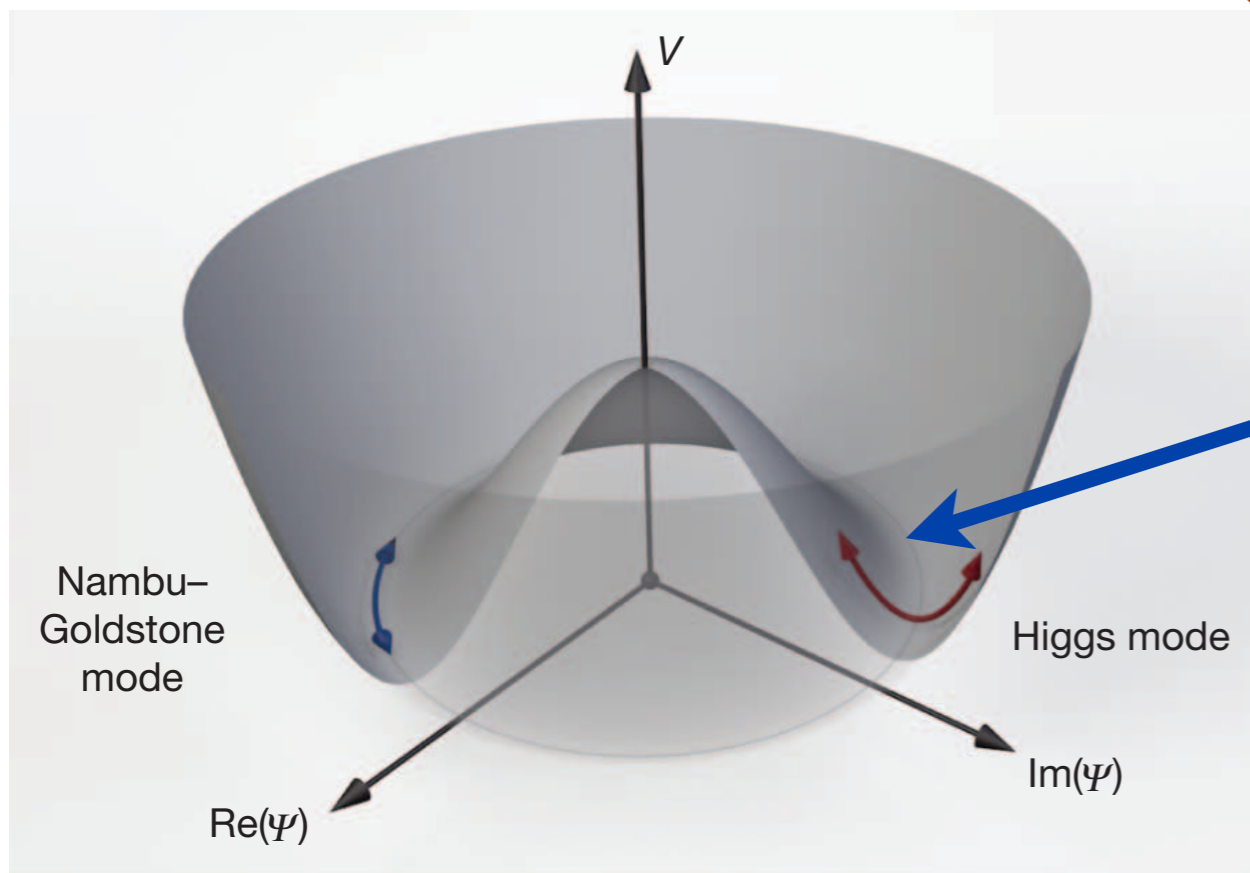
$$\langle \Psi \rangle = 0$$

Insulator



$$\mathcal{S} = \int d^2r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

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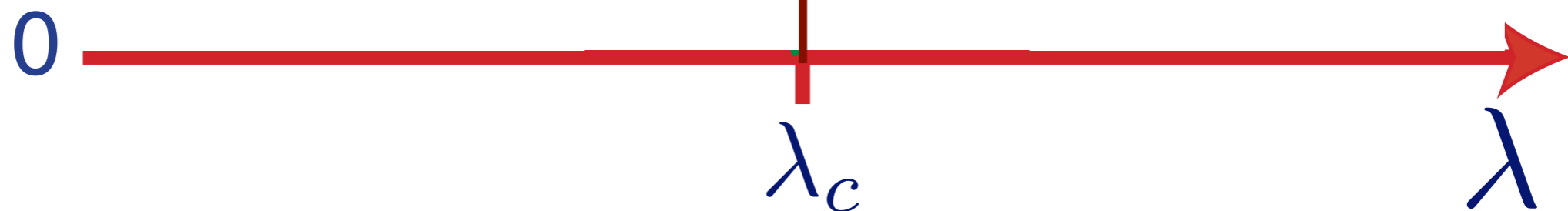
Despite rapid decay, there is a well-defined Higgs “quasi-normal mode”. This is associated with a pole in the lower-half of the complex frequency plane.

$$\langle \Psi \rangle \neq 0$$

Superfluid

$$\langle \Psi \rangle = 0$$

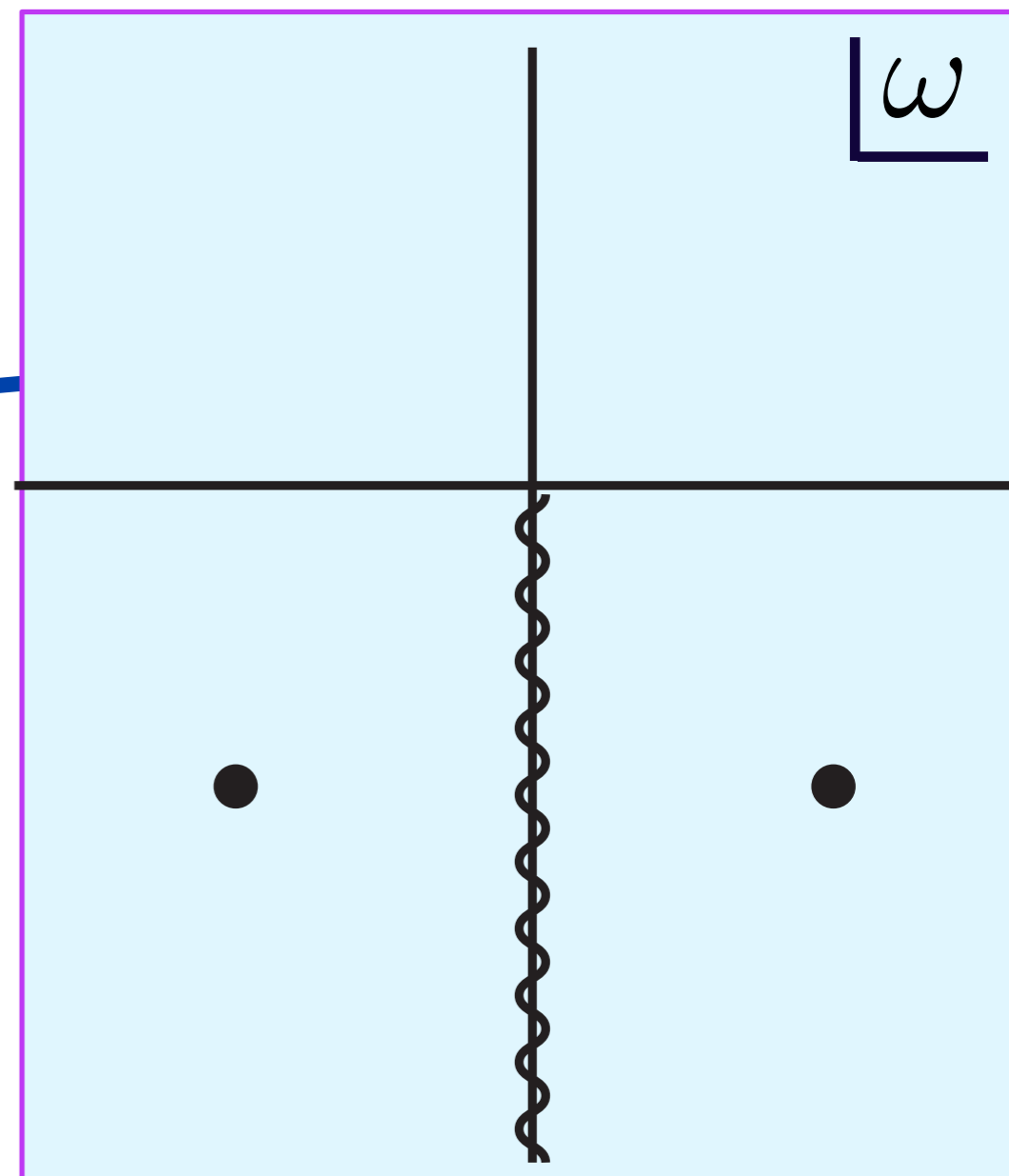
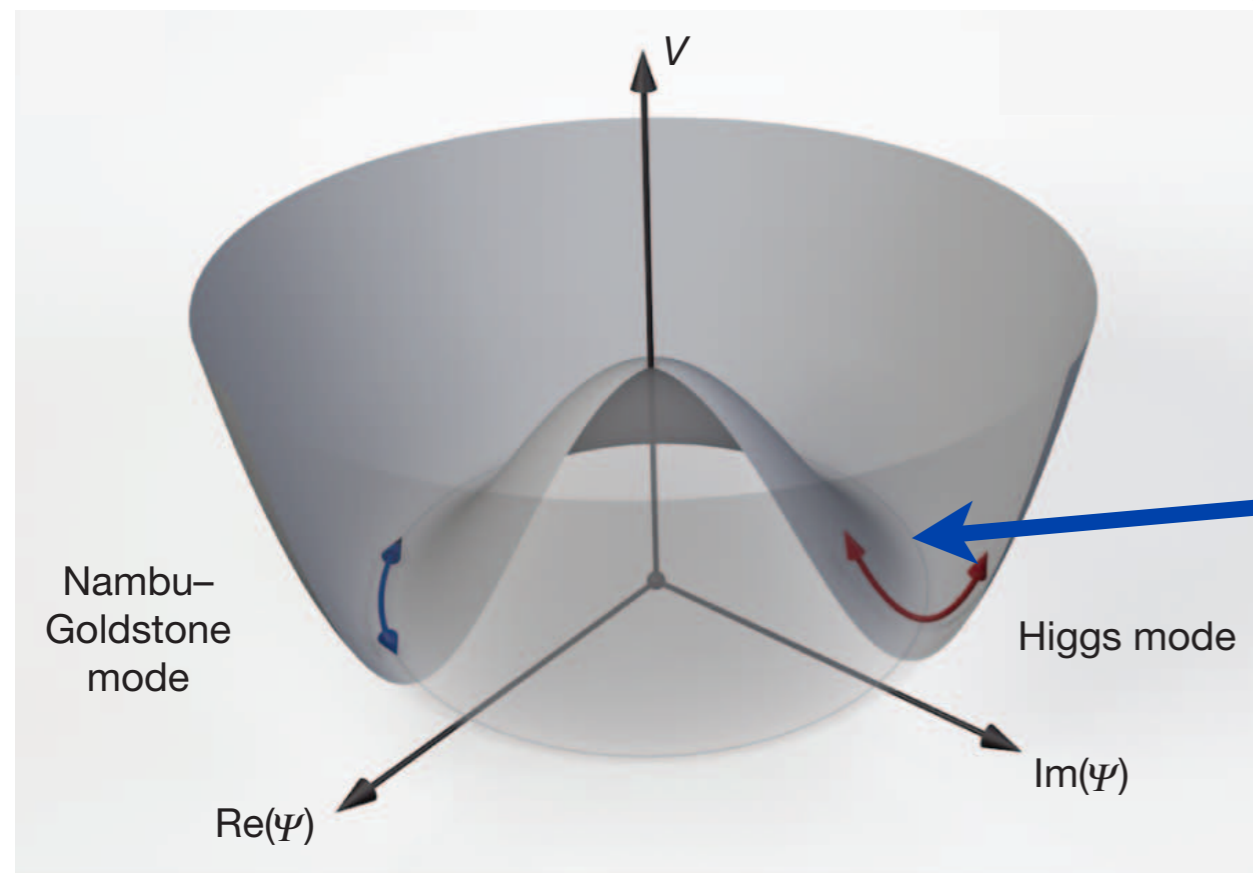
Insulator



D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

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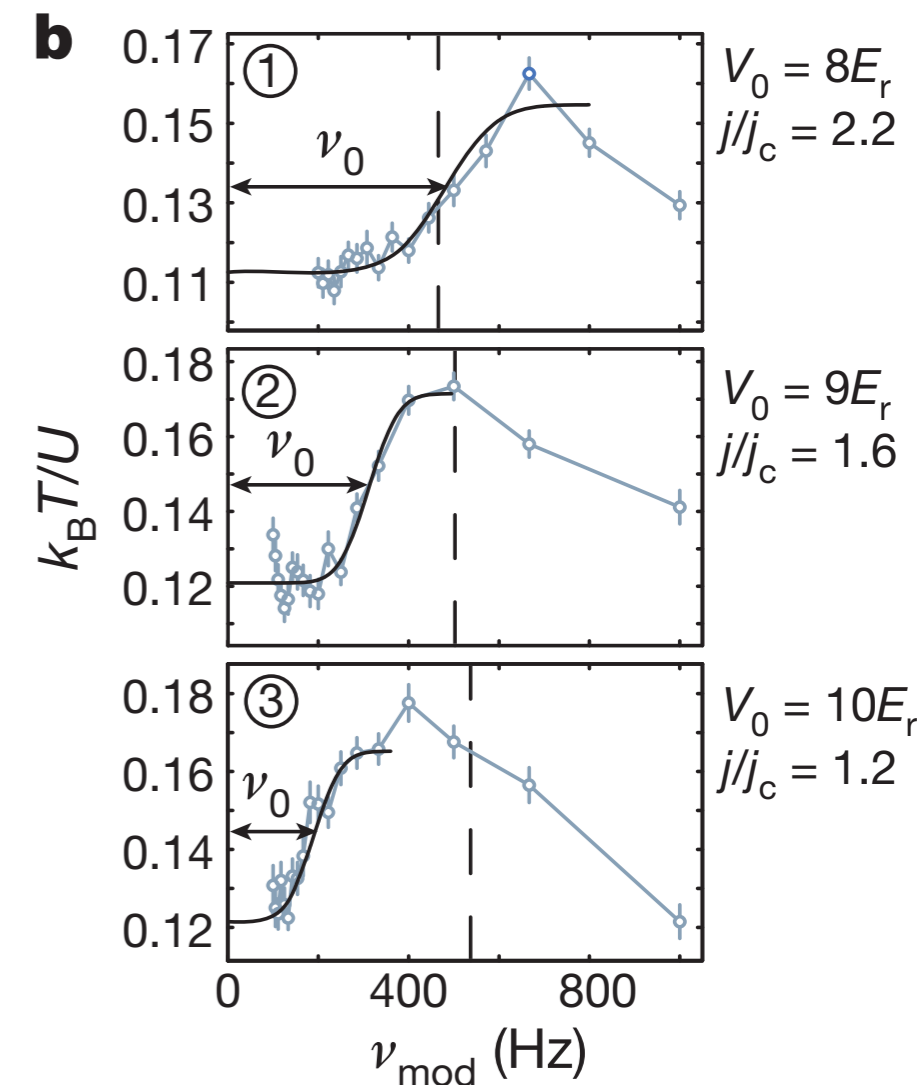
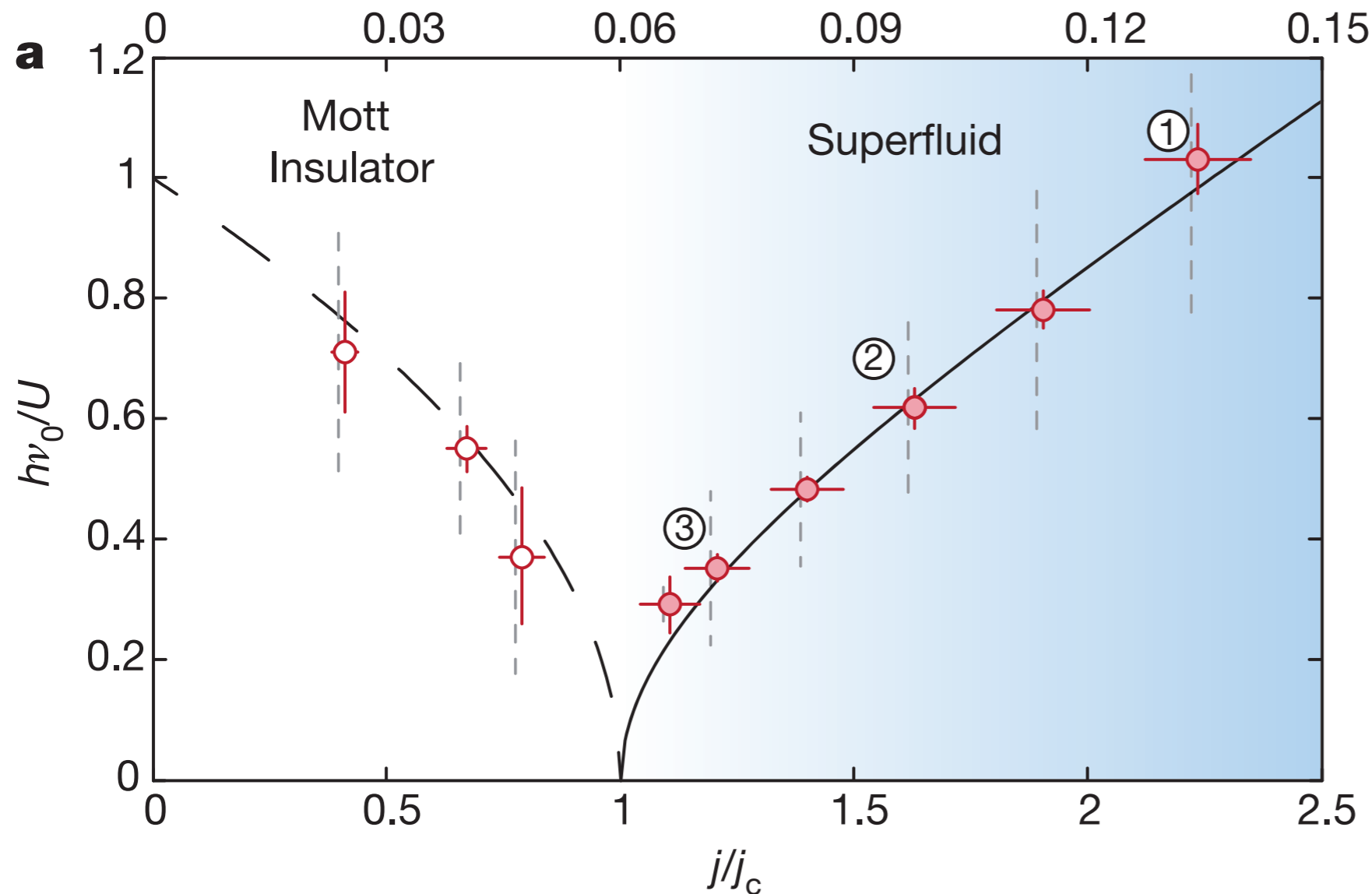
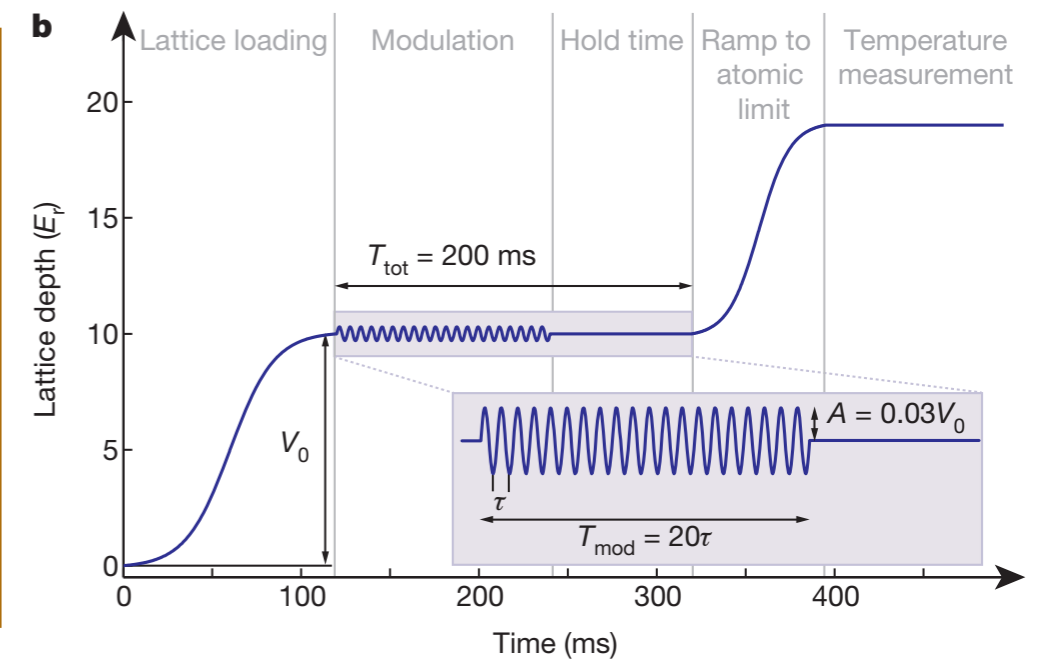
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D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

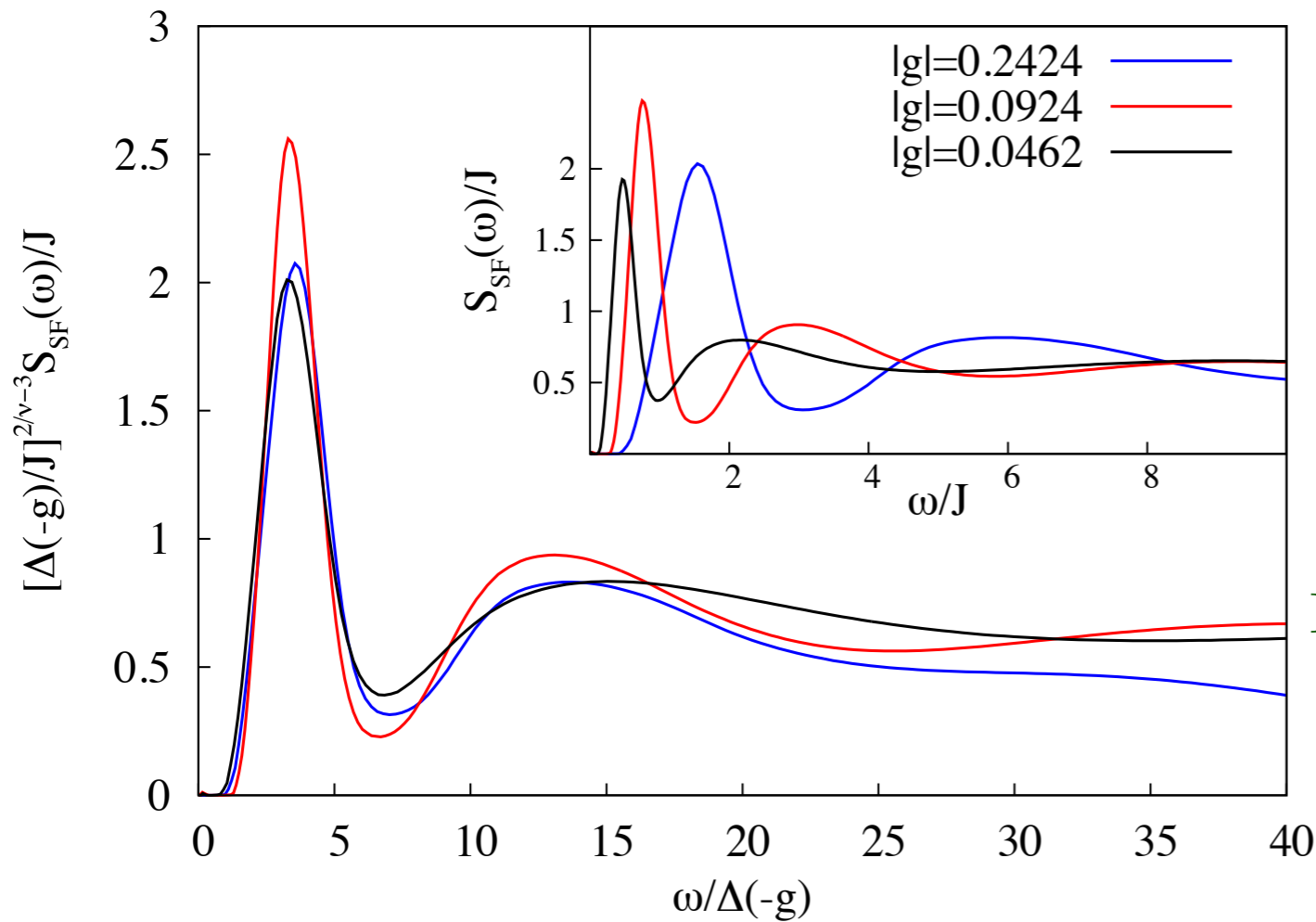
Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole



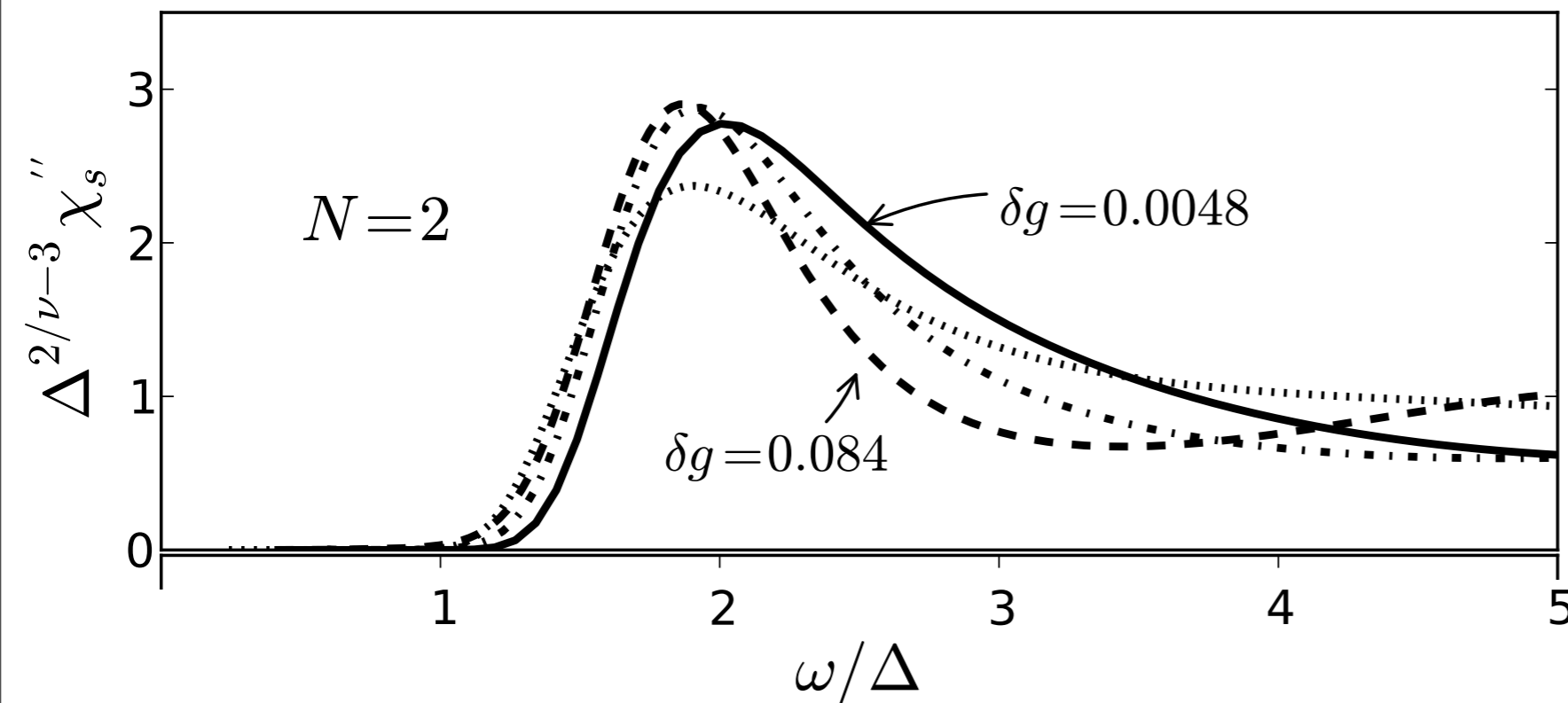
Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* **487**, 454 (2012).

# Observation of Higgs quasi-normal mode in quantum Monte Carlo



Scaling of spectral response functions predicted in D. Podolsky and S. Sachdev, Phys. Rev. B **86**, 054508 (2012).

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, and Nikolay Prokof'ev, arXiv:1301.3139

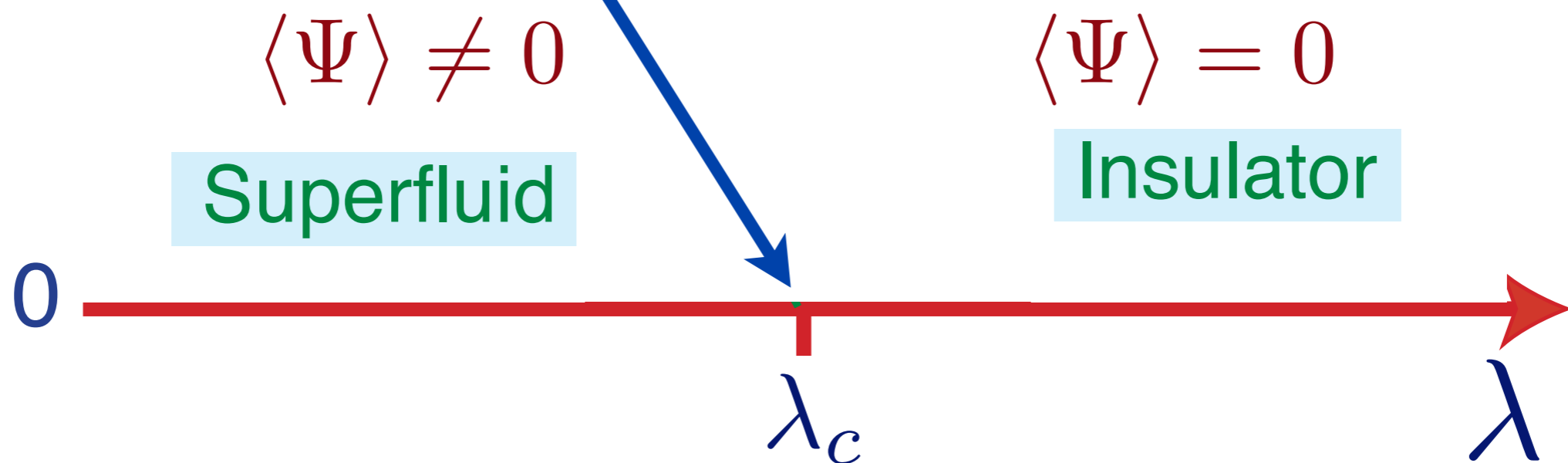


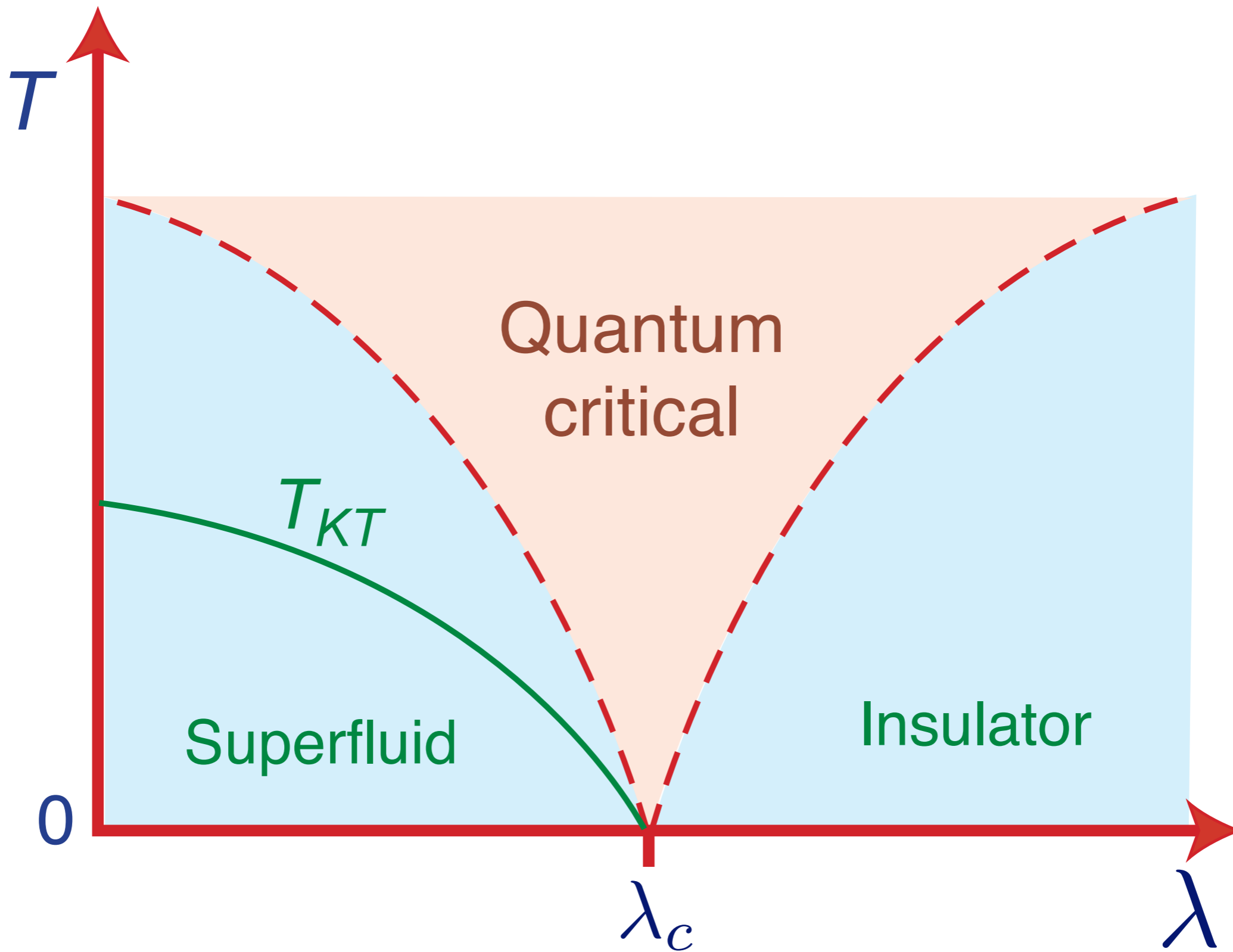
Snir Gazit, Daniel Podolsky, and Assa Auerbach, arXiv:1212.3759

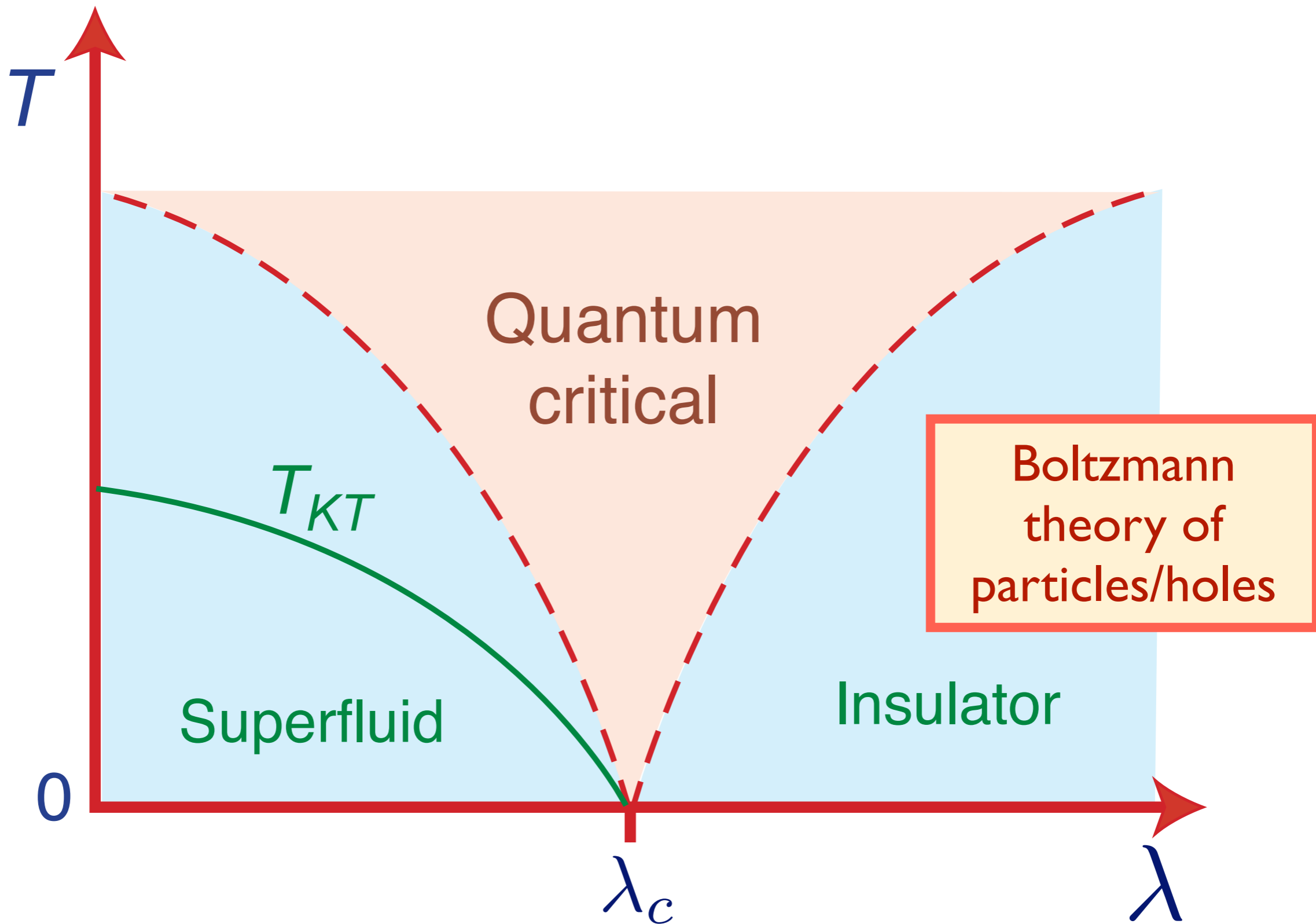
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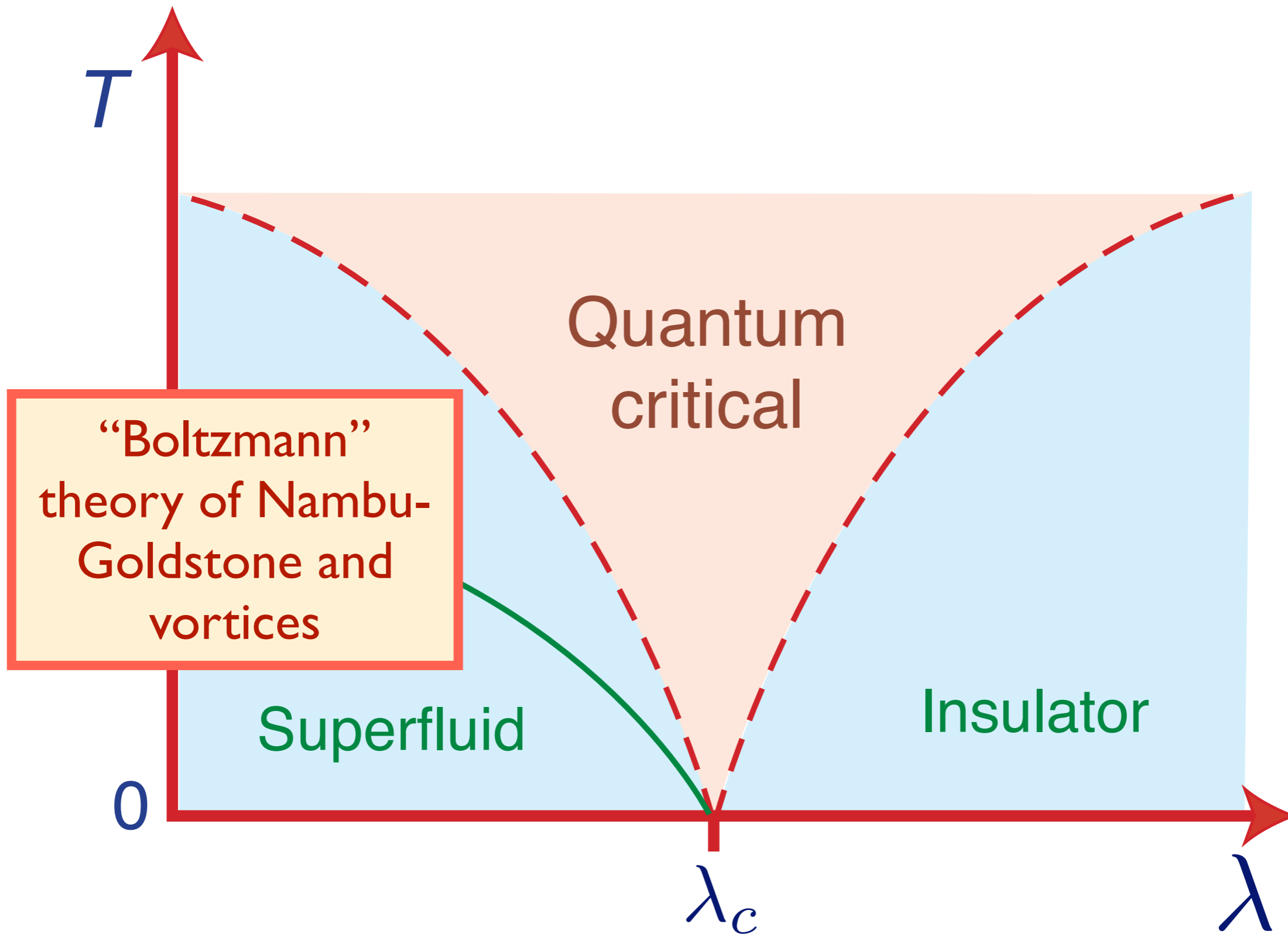
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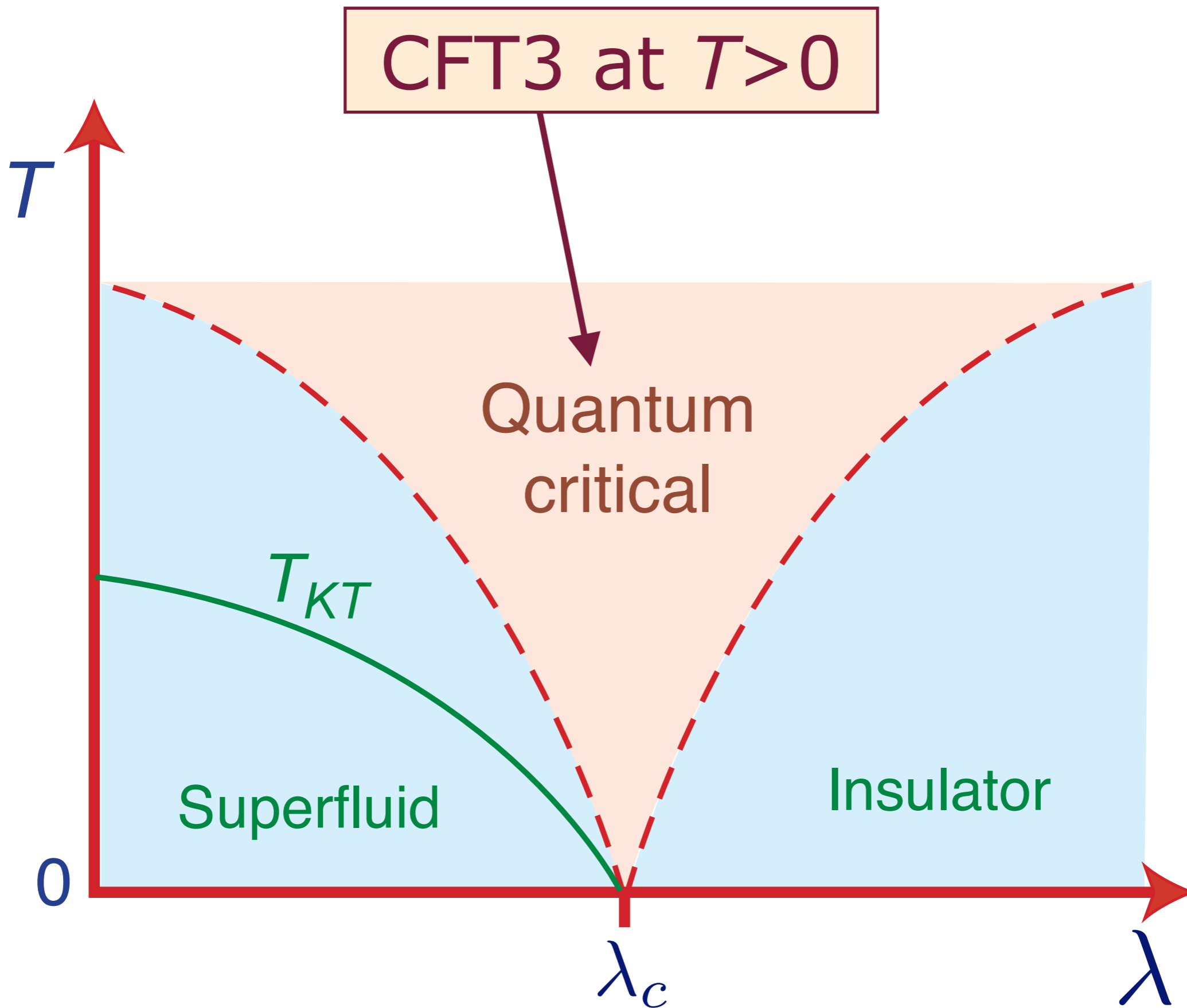
A conformal field theory  
in 2+1 spacetime dimensions:  
a CFT3

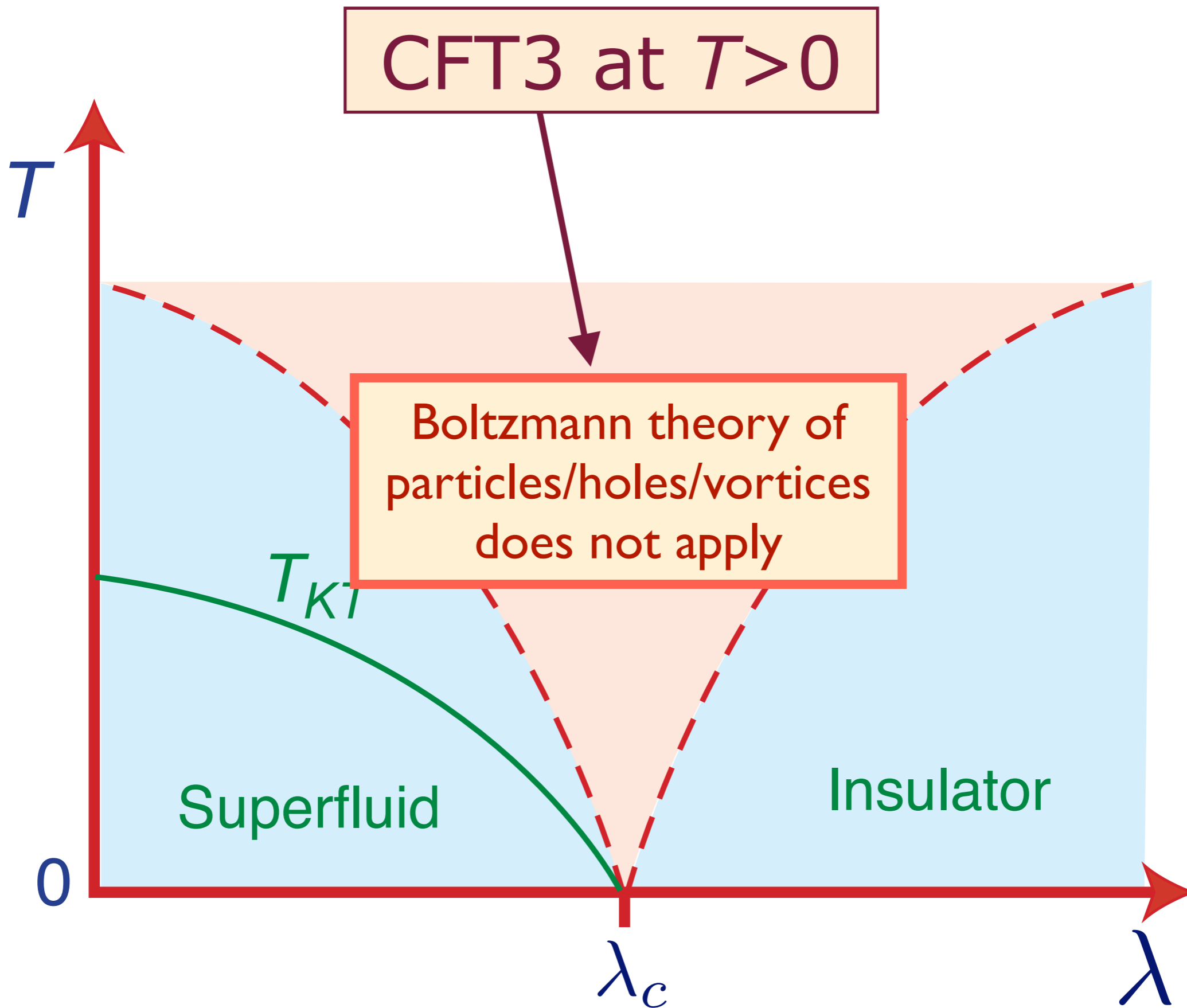












CFT3 at  $T > 0$

Boltzmann theory of particles/holes/vortices does not apply

Superfluid

Insulator

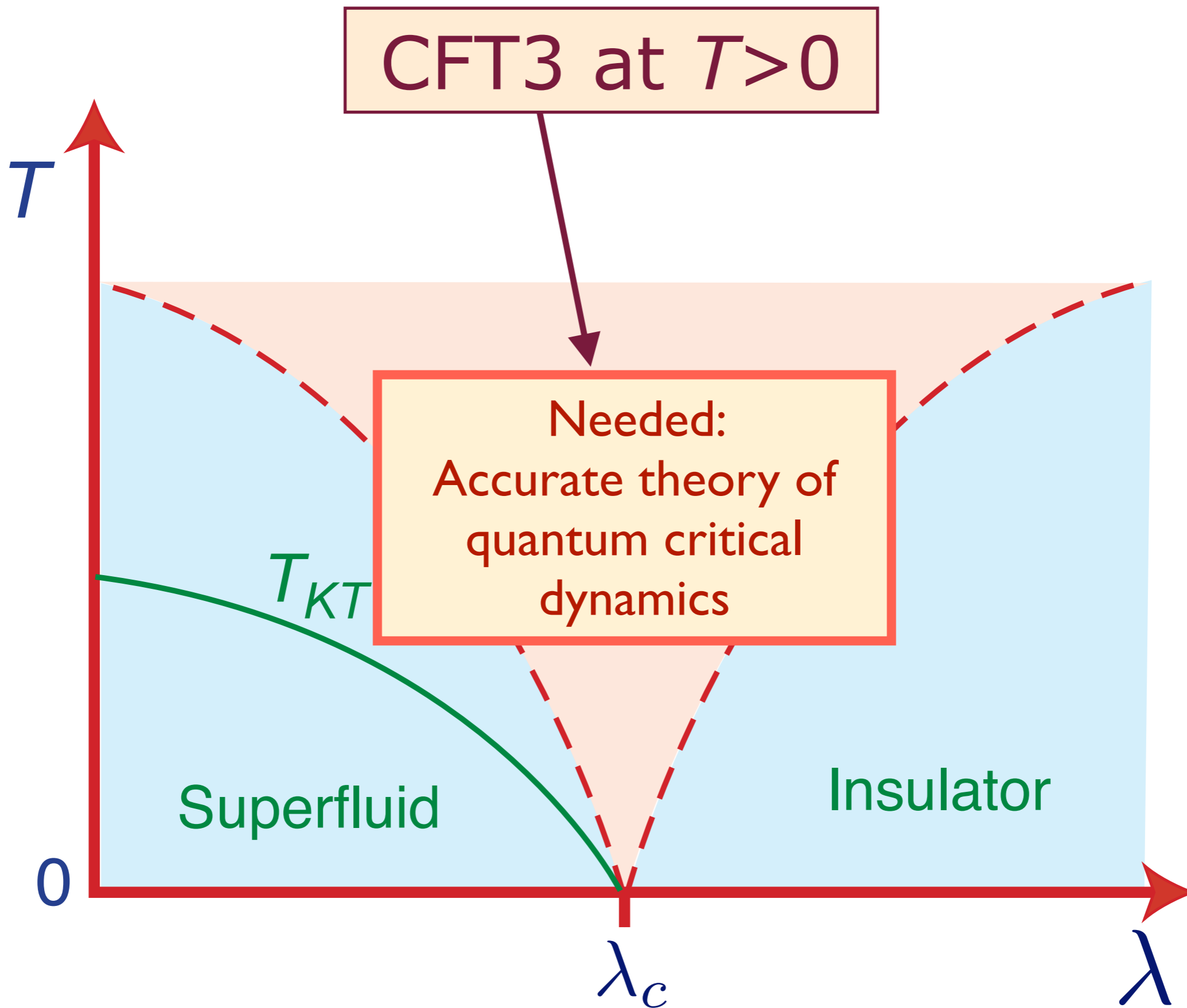
$T_{KT}$

$\lambda_c$

$\lambda$

$T$

0



# Quantum critical dynamics

Quantum “*nearly perfect fluid*”  
with shortest possible *local* equilibration time,  $\tau_{\text{eq}}$

$$\tau_{\text{eq}} = \mathcal{C} \frac{\hbar}{k_B T}$$

where  $\mathcal{C}$  is a *universal* constant.

Response functions are characterized by poles in LHP  
with  $\omega \sim k_B T / \hbar$ .

These poles (quasi-normal modes) appear naturally in  
the holographic theory.

(Analog of Higgs quasi-normal mode.)

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999).

# Quantum critical dynamics

Transport co-efficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

## Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)]$$

( $Q$  is the “charge” of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett.* **64**, 587 (1990)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

# Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:  
*Quantum criticality and conformal field theories*
2. Gauge-gravity duality
3. Black-hole horizons and quasi-normal modes
4. Strange metals:  
*What lies beyond the horizon ?*

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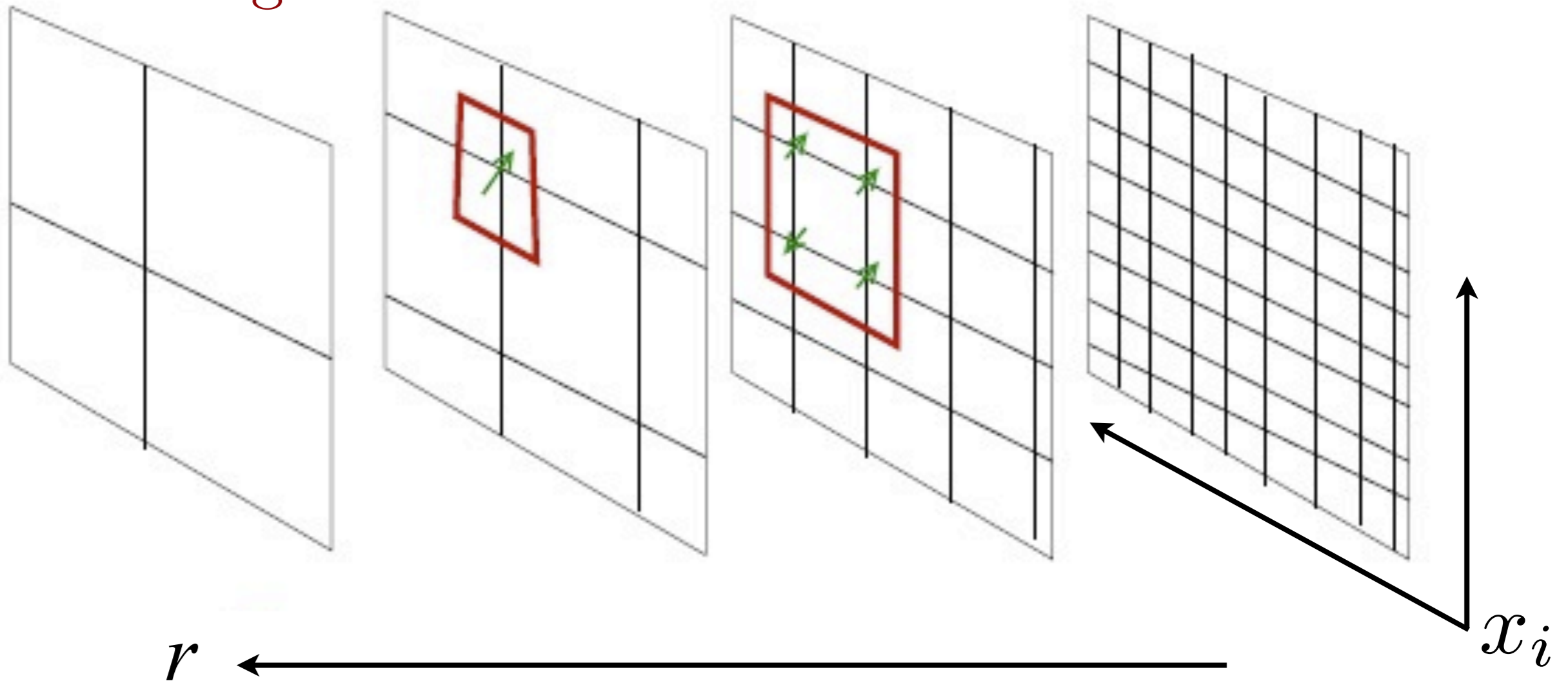
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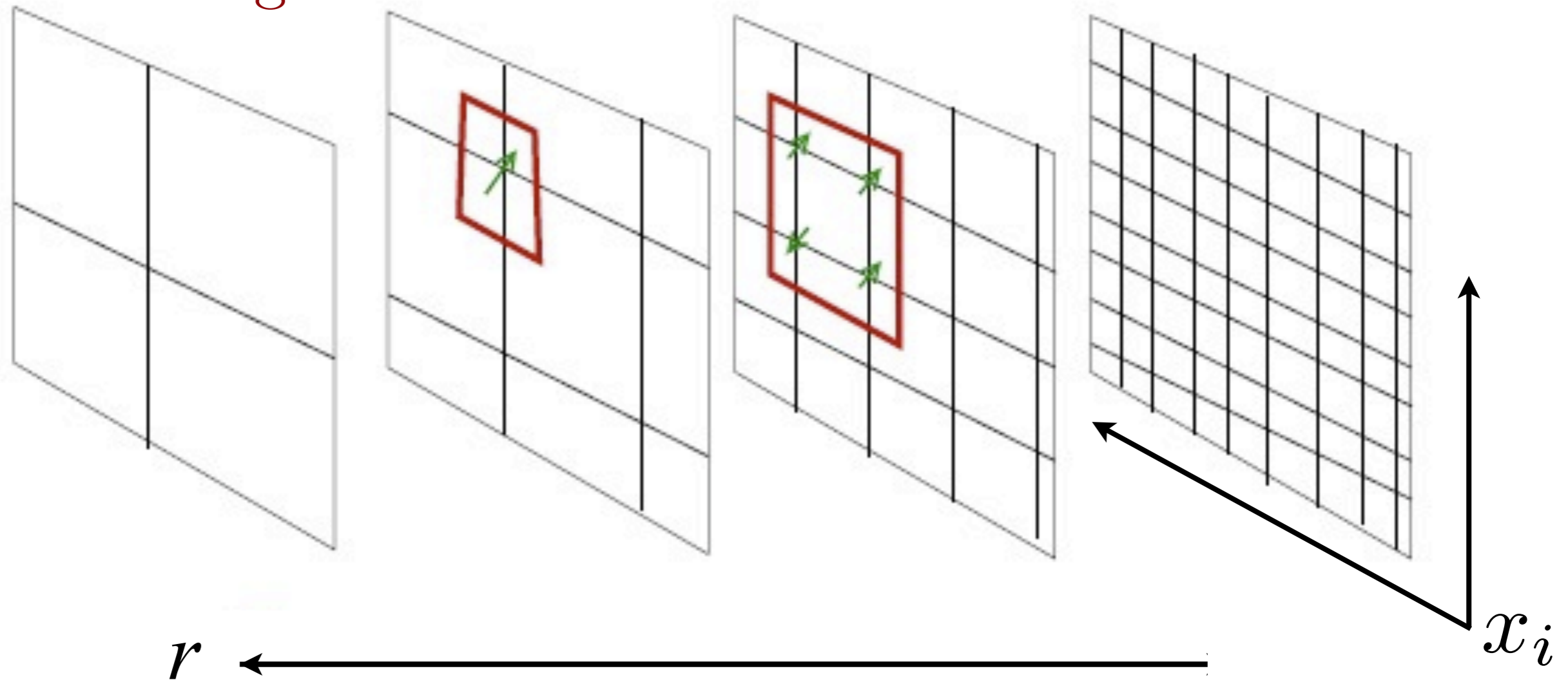
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**Renormalization group:**  $\Rightarrow$  Follow coupling constants of quantum many body theory as a function of length scale  $r$



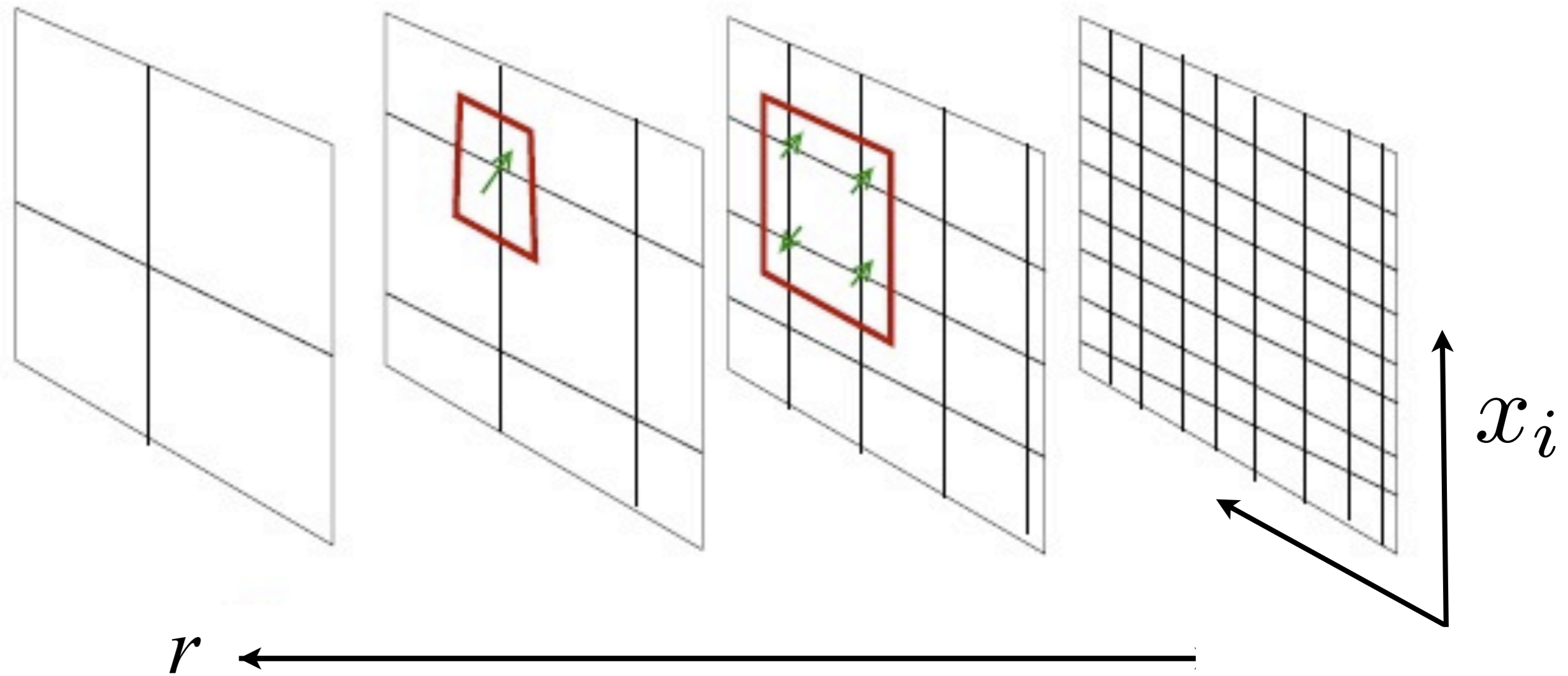
**Renormalization group:**  $\Rightarrow$  Follow coupling constants of quantum many body theory as a function of length scale  $r$



**Key idea:**  $\Rightarrow$  Implement  $r$  as an extra dimension, and map to a local theory in  $d + 2$  spacetime dimensions.

J. McGreevy, arXiv0909.0518

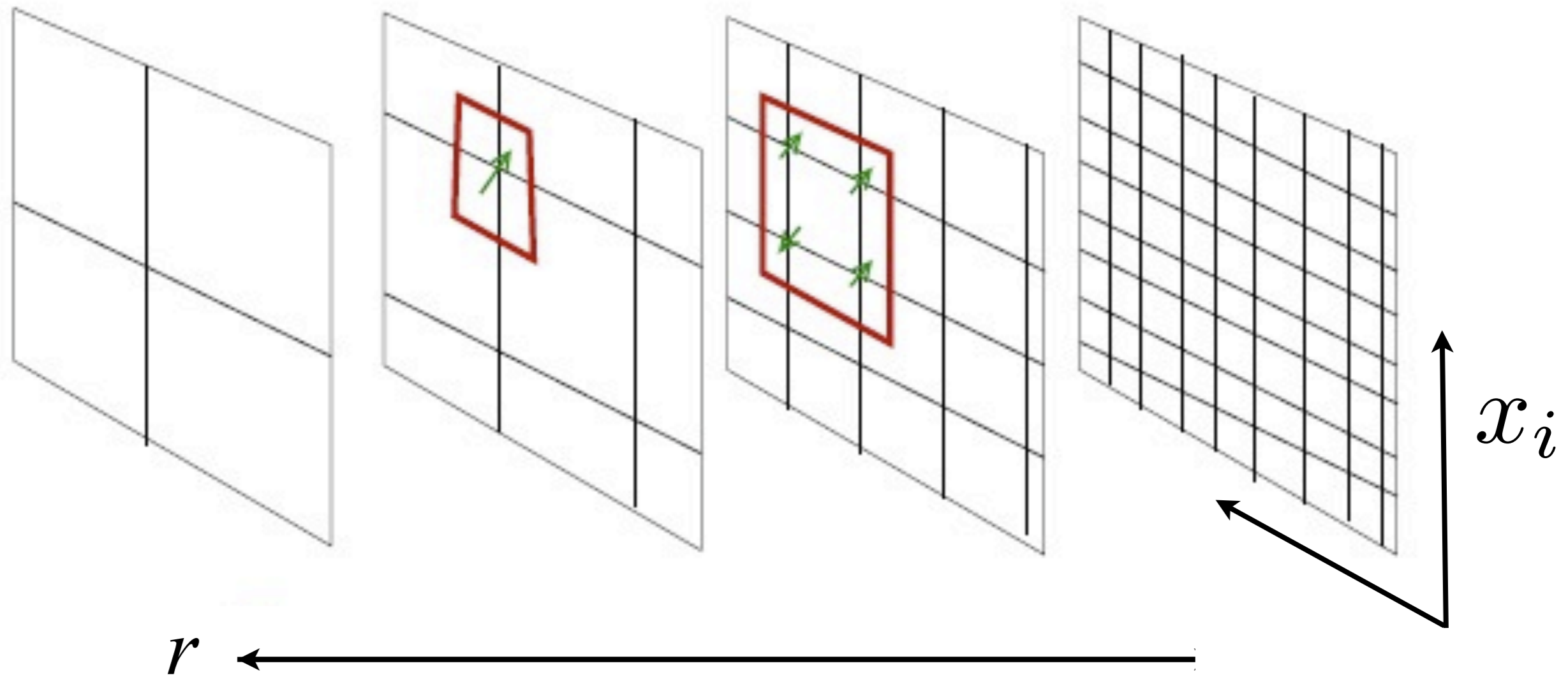
# Holography



For a relativistic CFT in  $d$  spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ( $i = 1 \dots d$ )

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$

# Holography

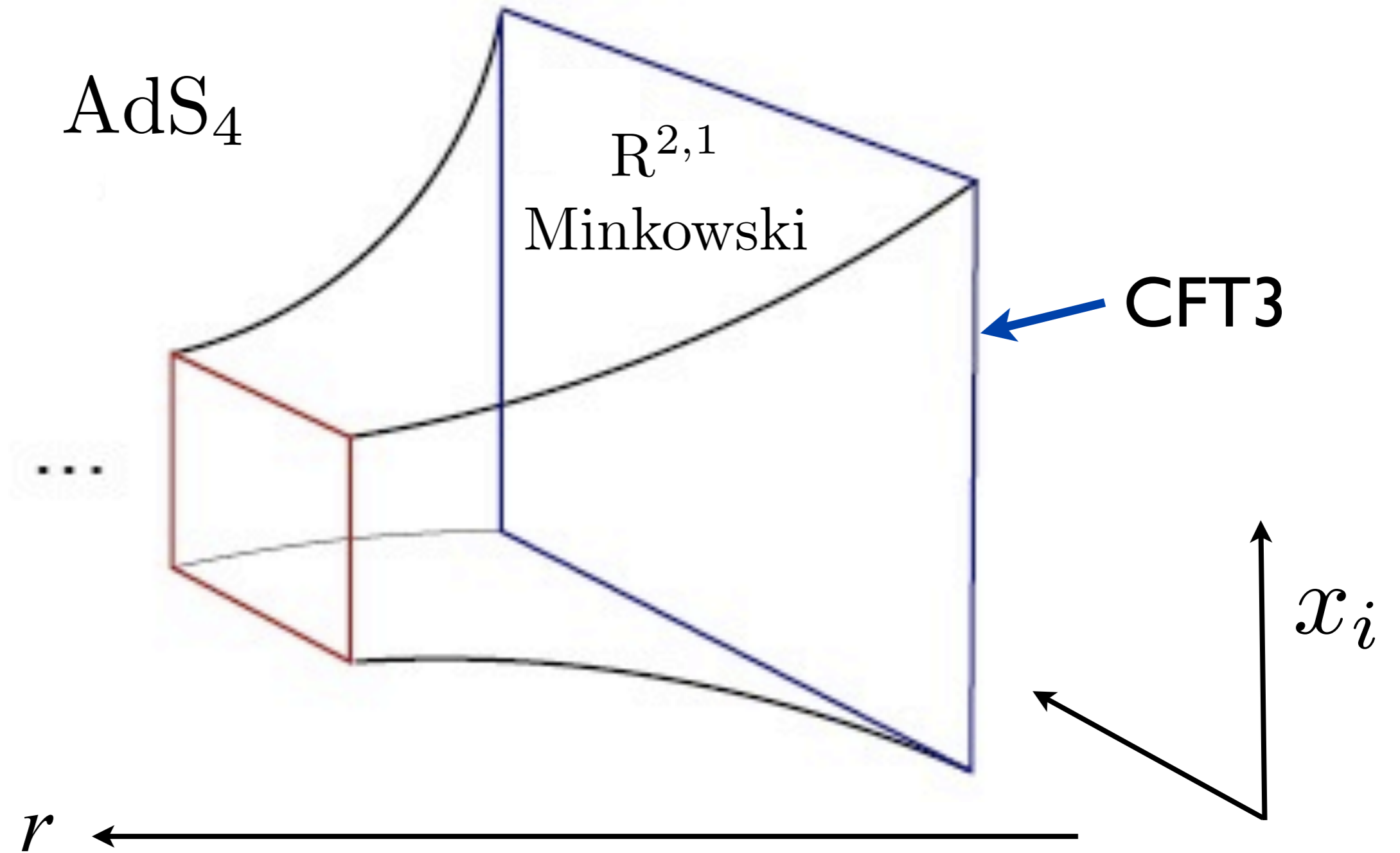


This gives the unique metric

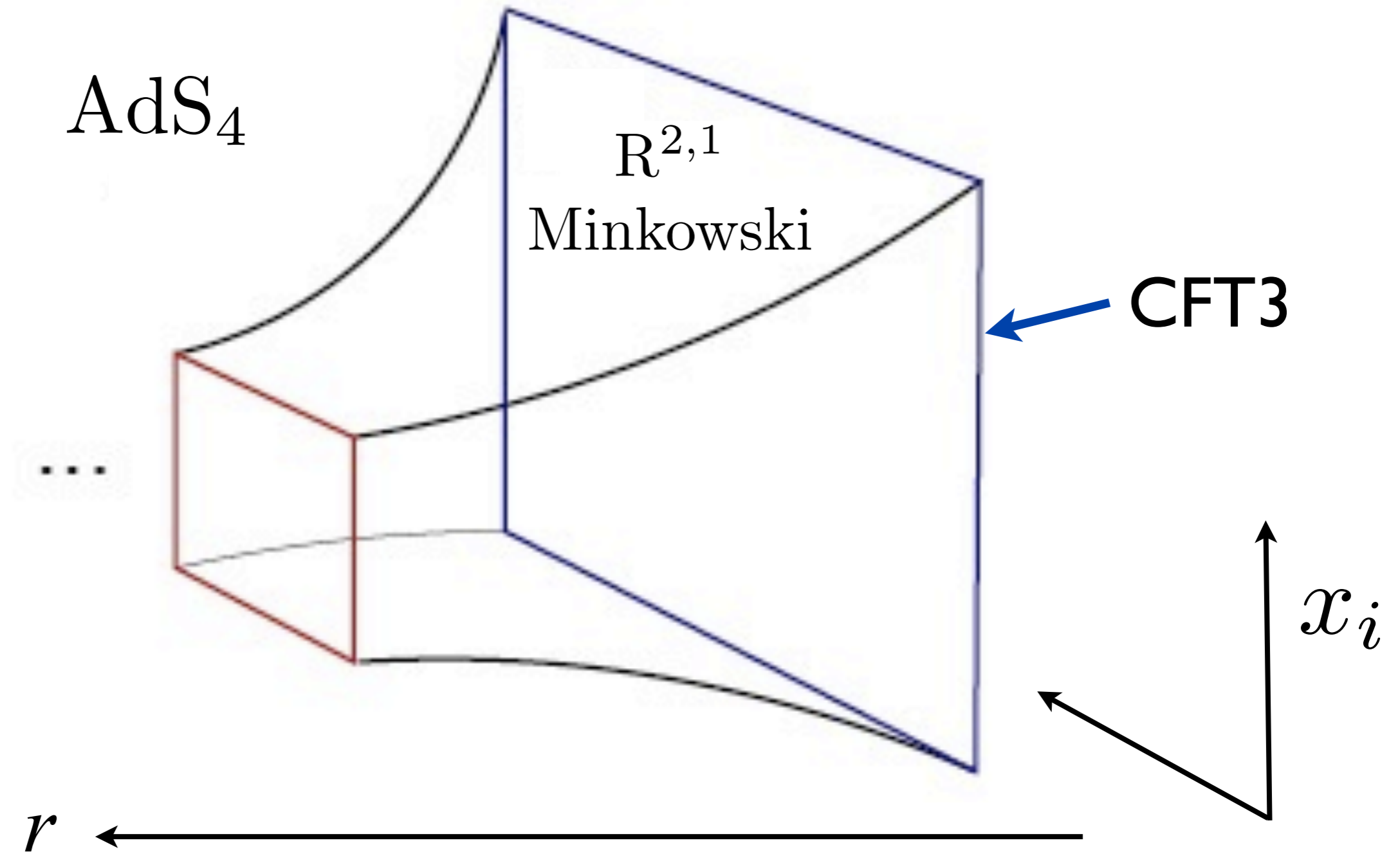
$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

This is the metric of anti-de Sitter space  $\text{AdS}_{d+2}$ .

# AdS/CFT correspondence



# AdS/CFT correspondence



This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S}_E = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$$

# Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:

*Quantum criticality and conformal field theories*

2. Gauge-gravity duality

3. Black-hole horizons and quasi-normal modes

4. Strange metals:

*What lies beyond the horizon ?*

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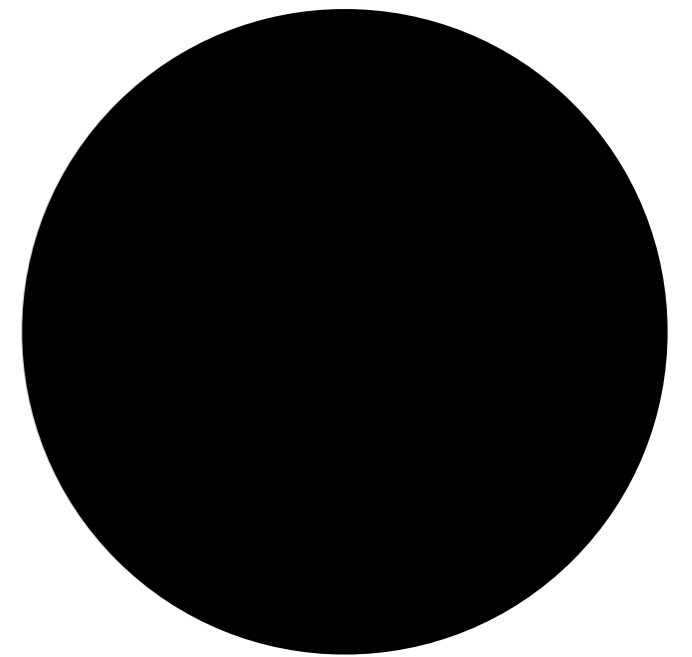
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# Black Holes

Objects so massive that light is gravitationally bound to them.

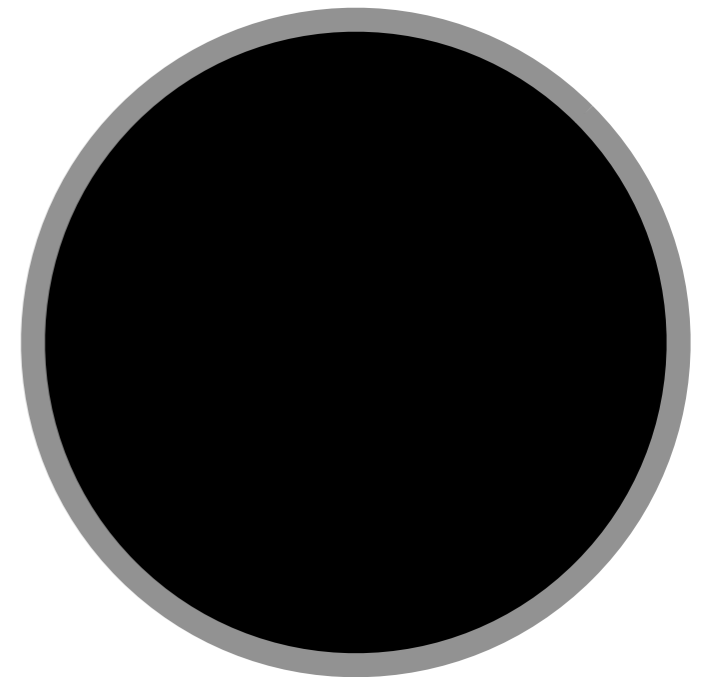


# Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

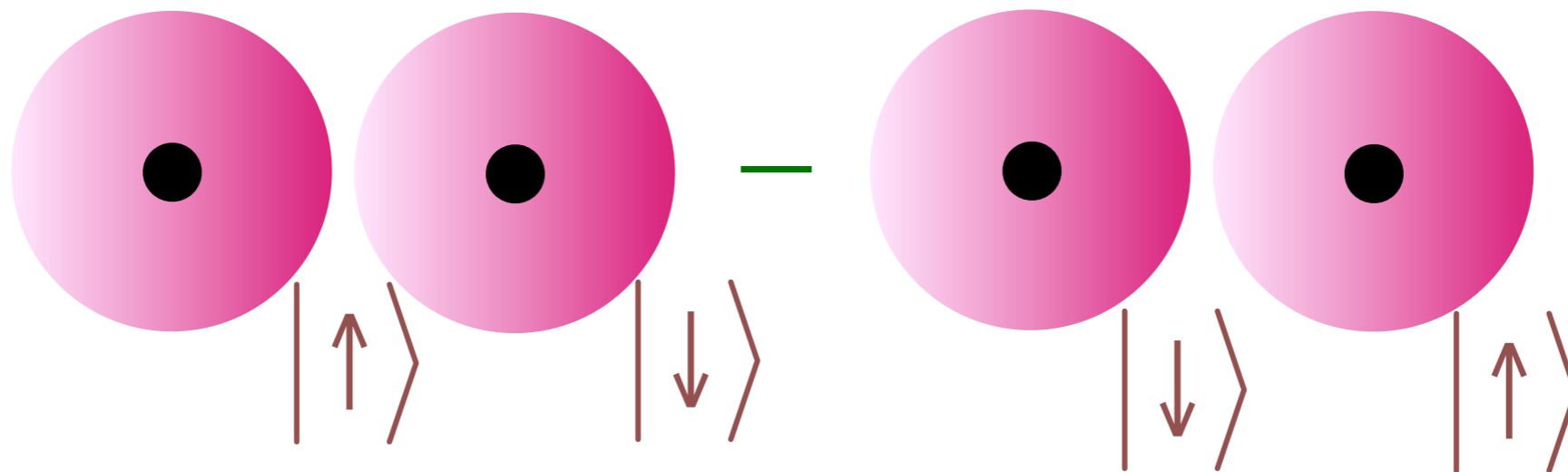
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



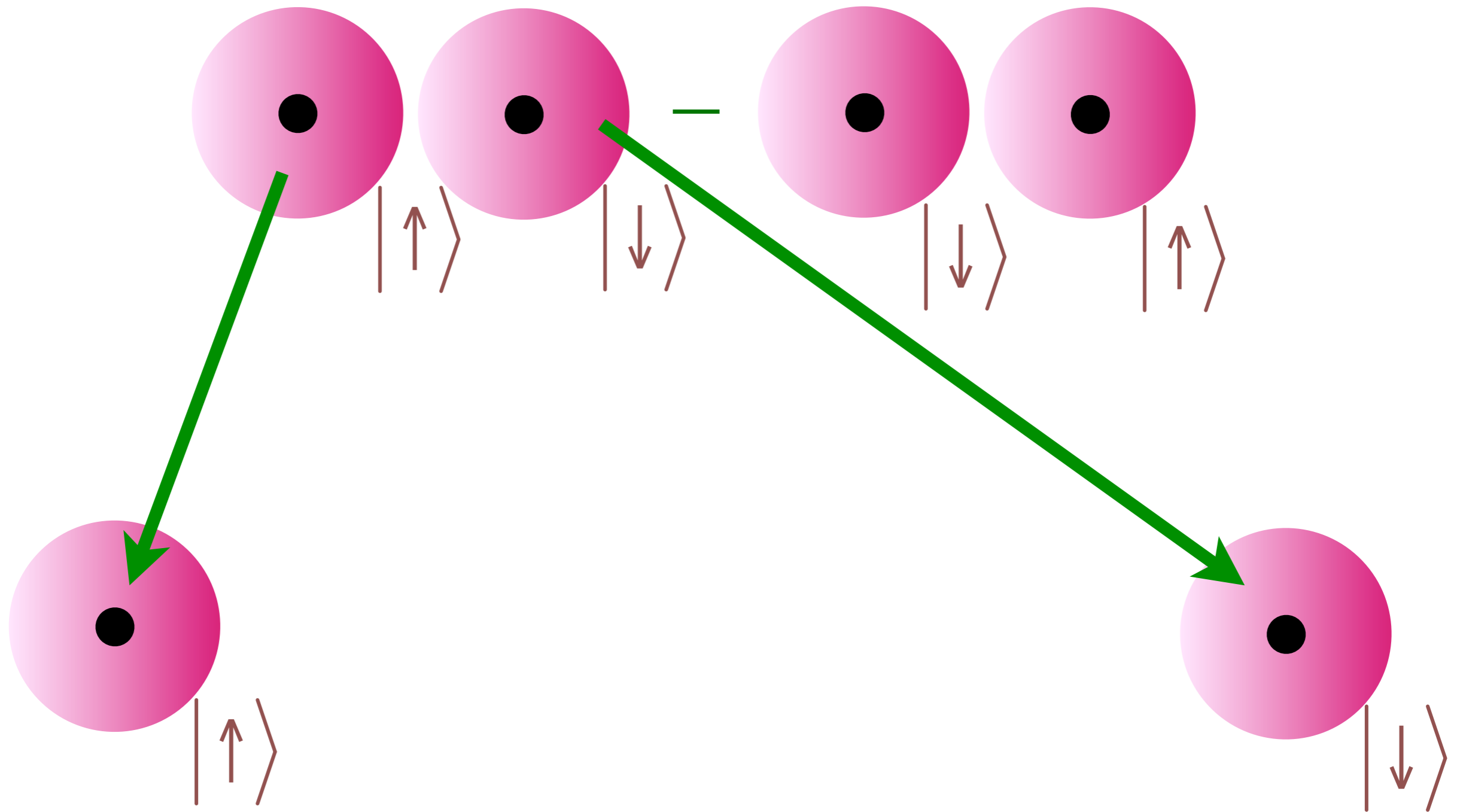
# Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

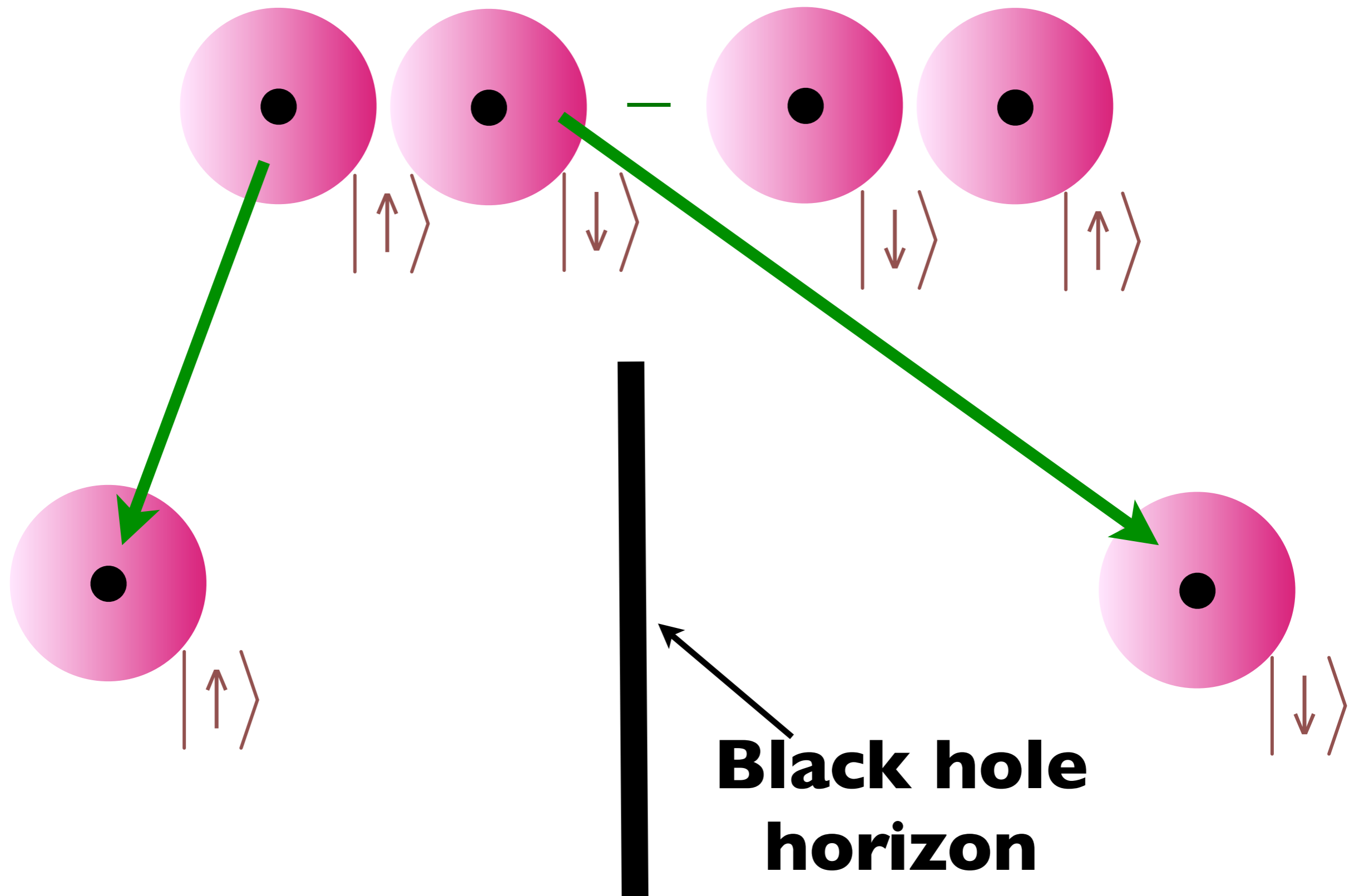
# Quantum Entanglement across a black hole horizon



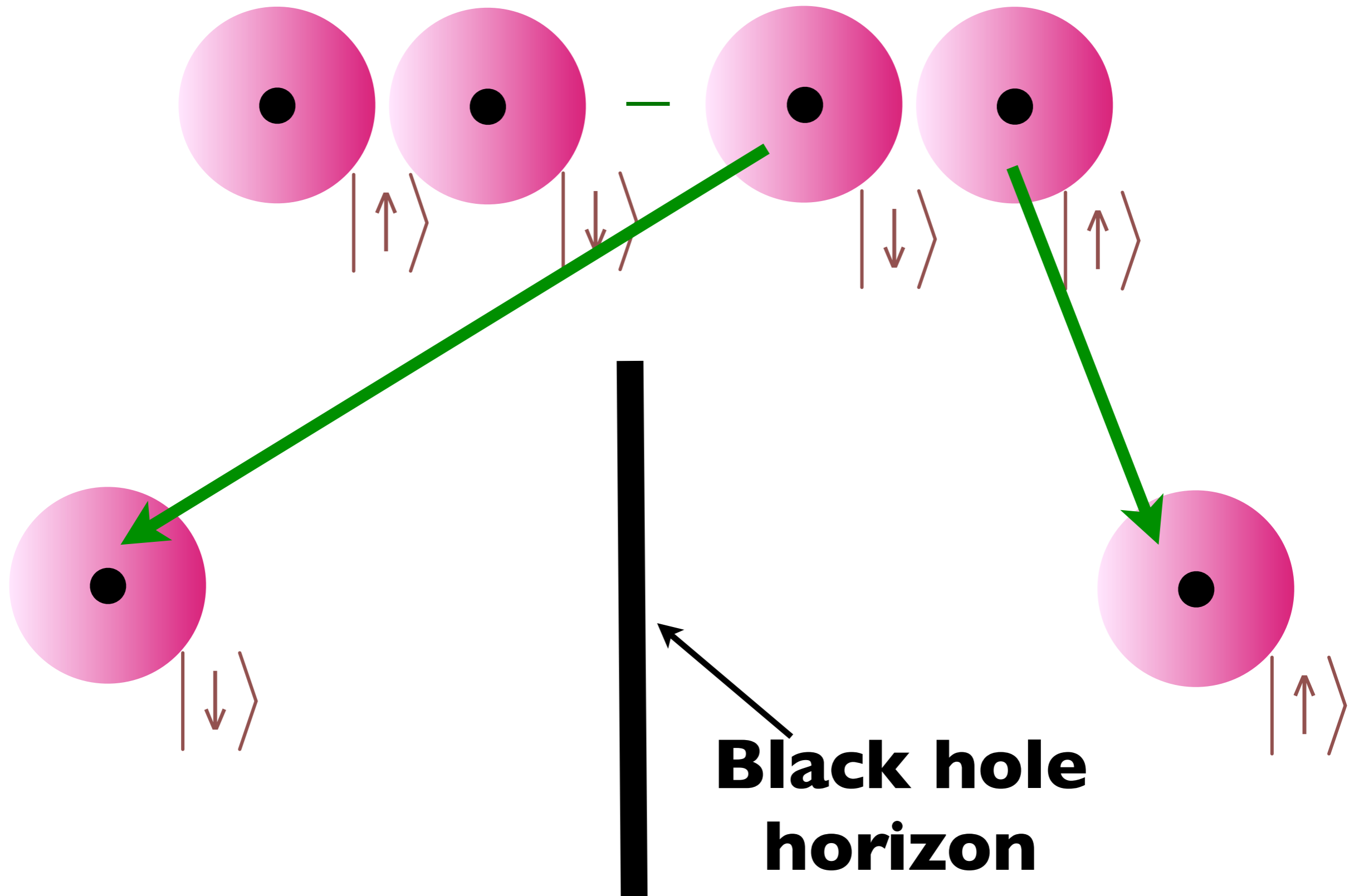
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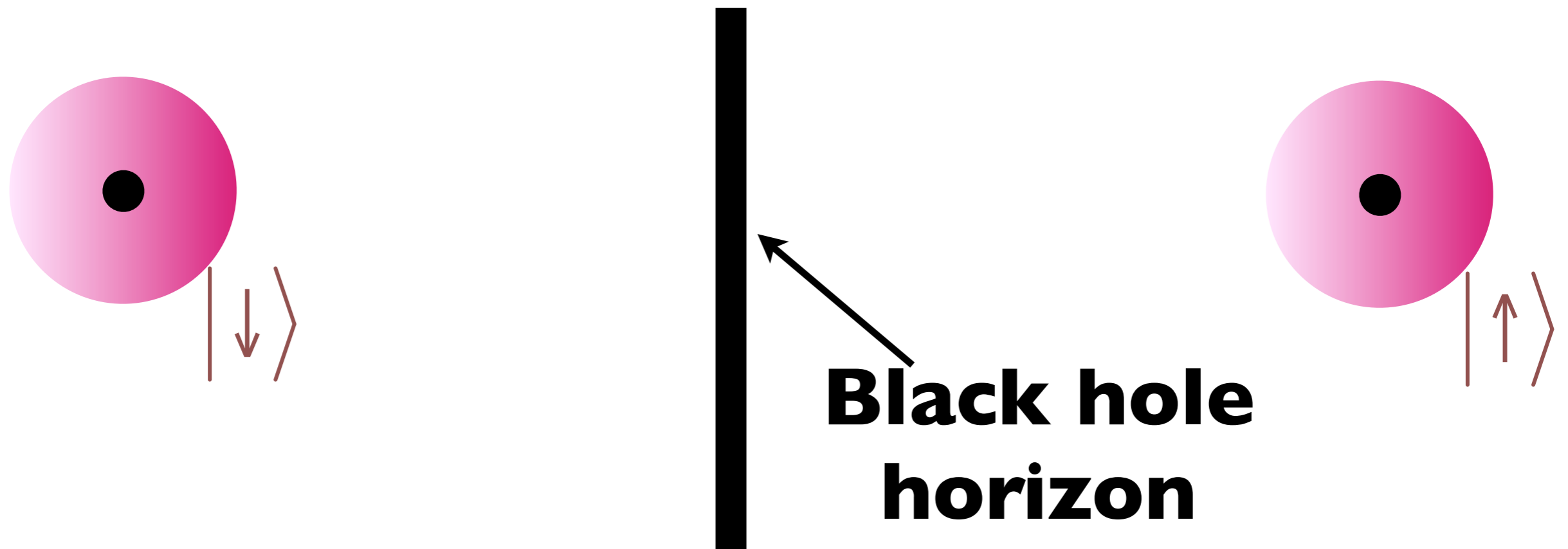


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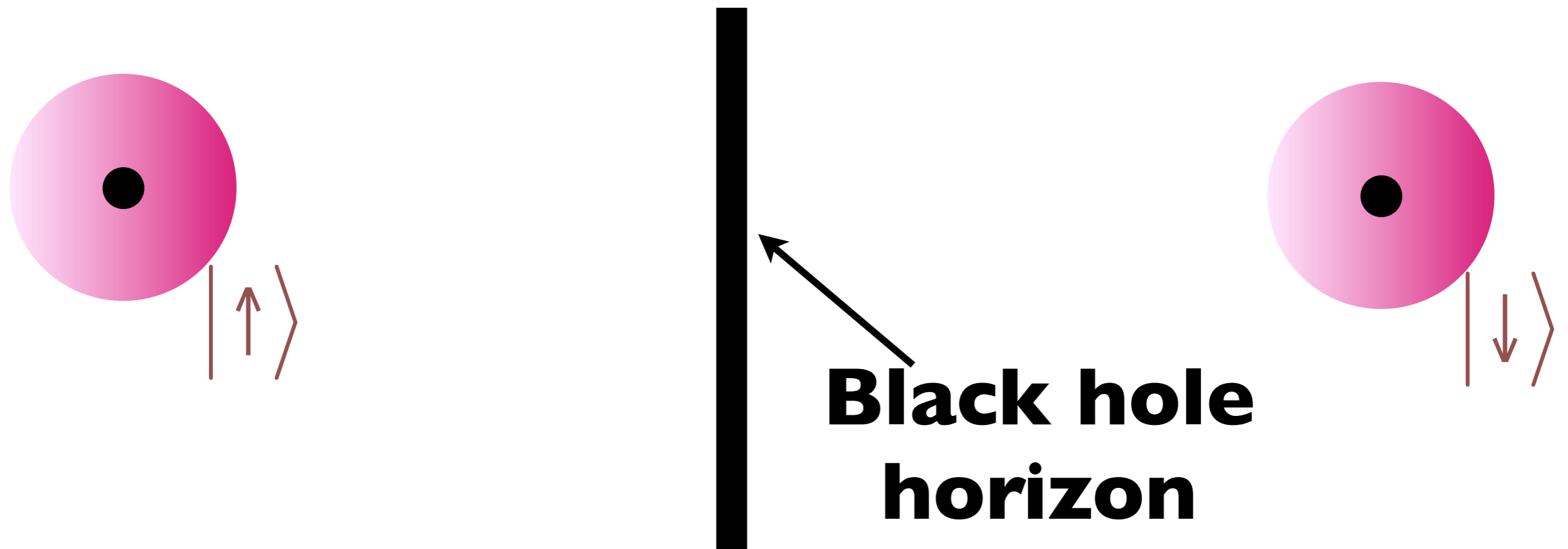
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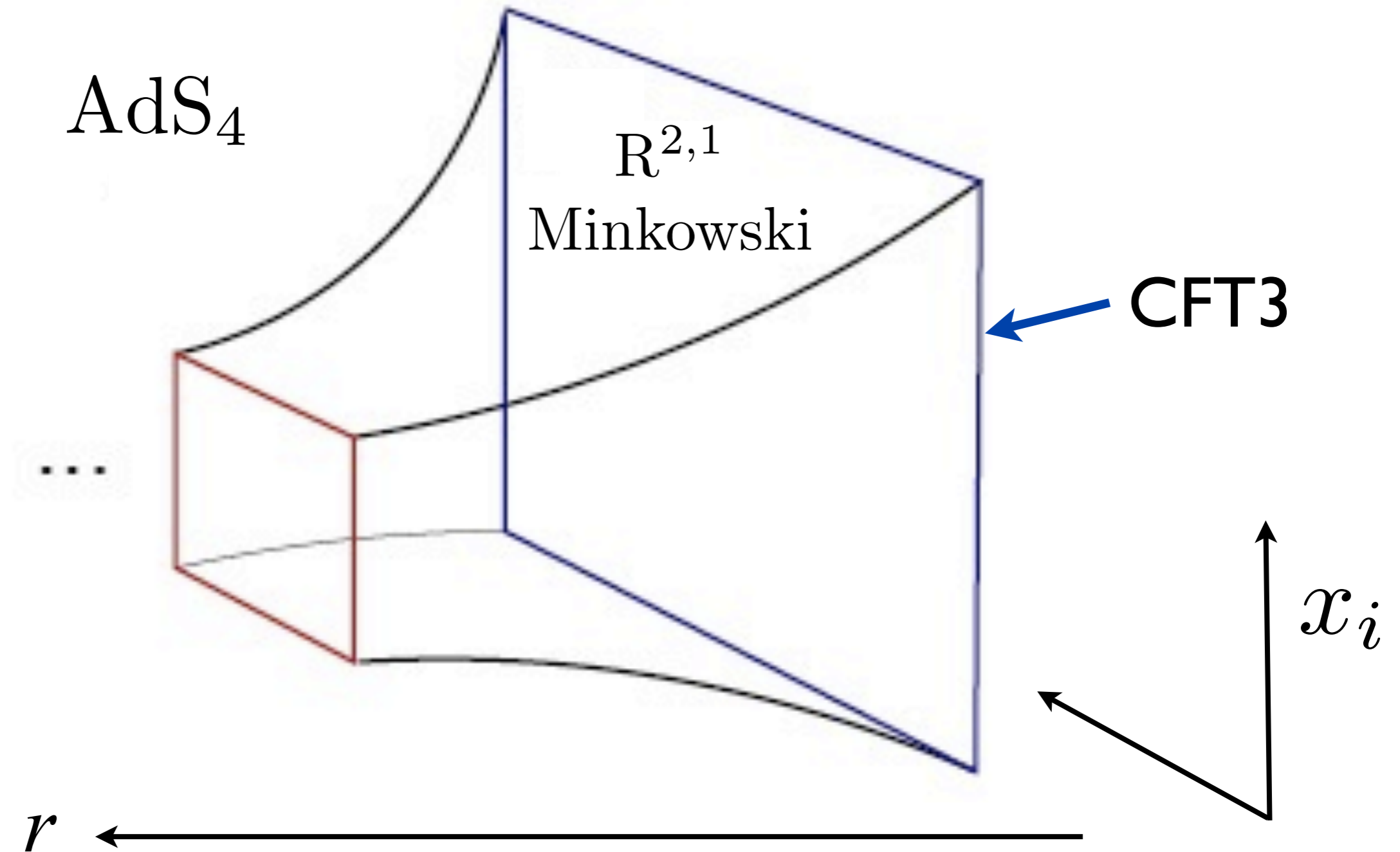


# Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

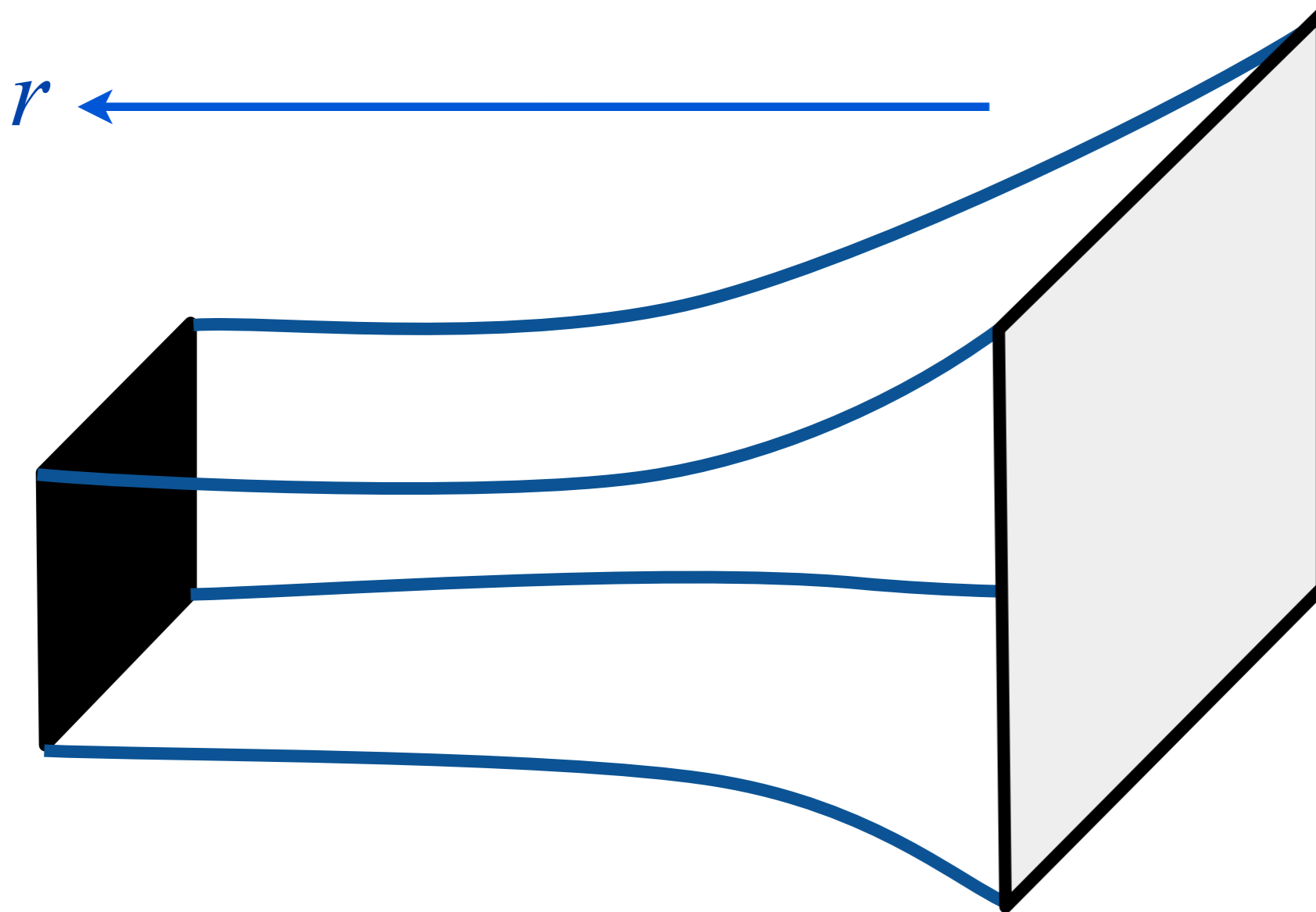
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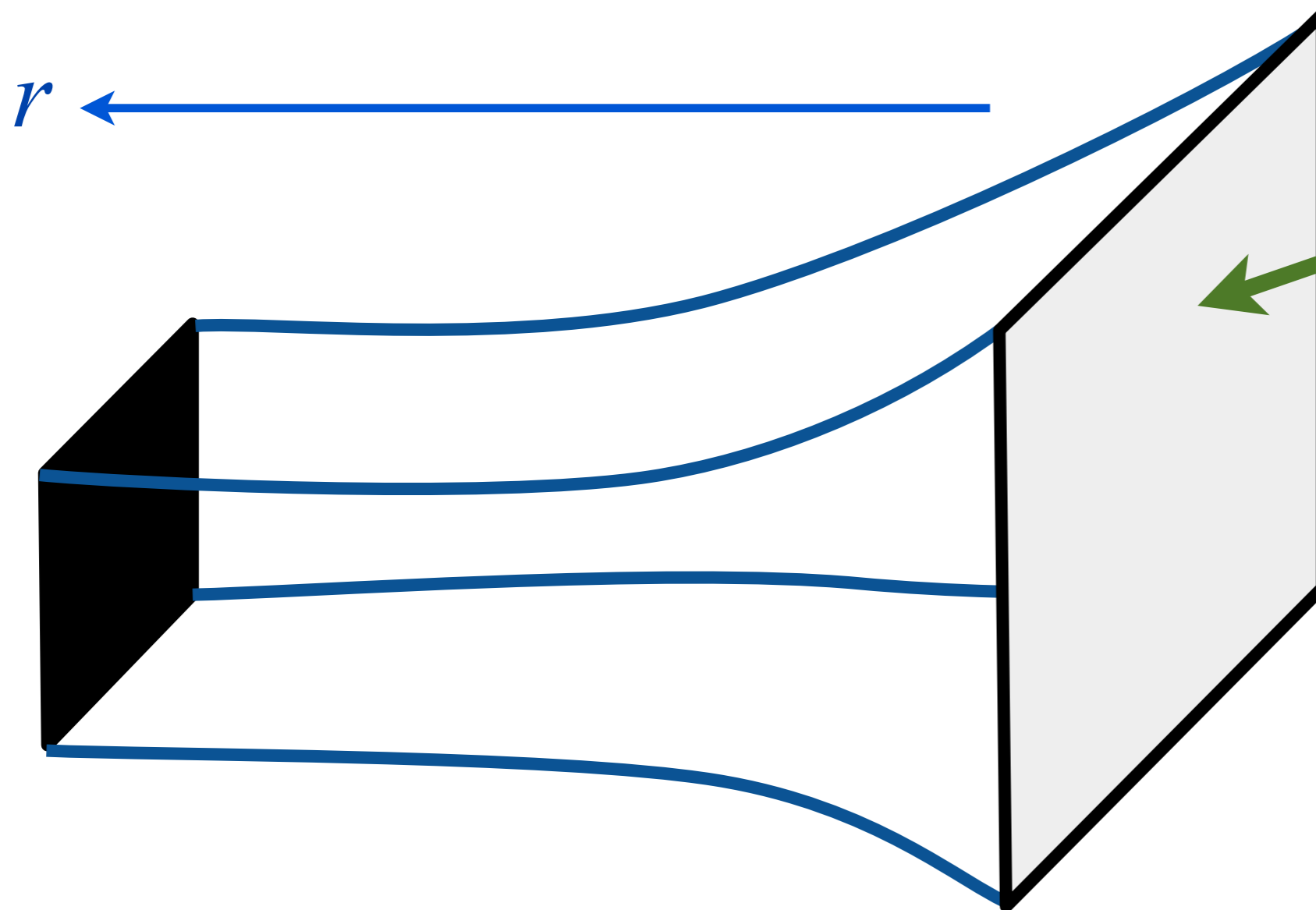
# Gauge-gravity duality at non-zero temperatures



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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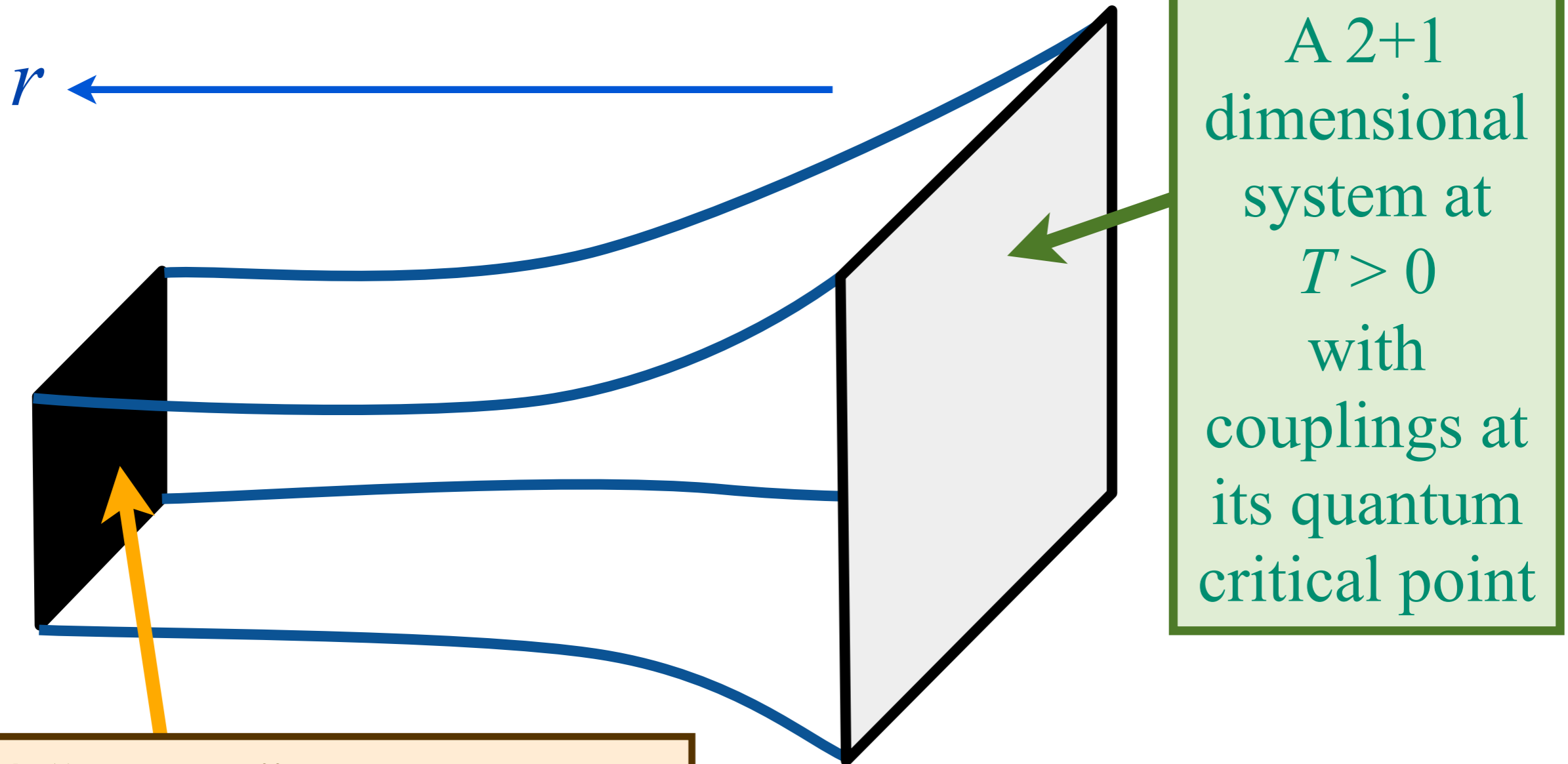
# Gauge-gravity duality at non-zero temperatures



A 2+1  
dimensional  
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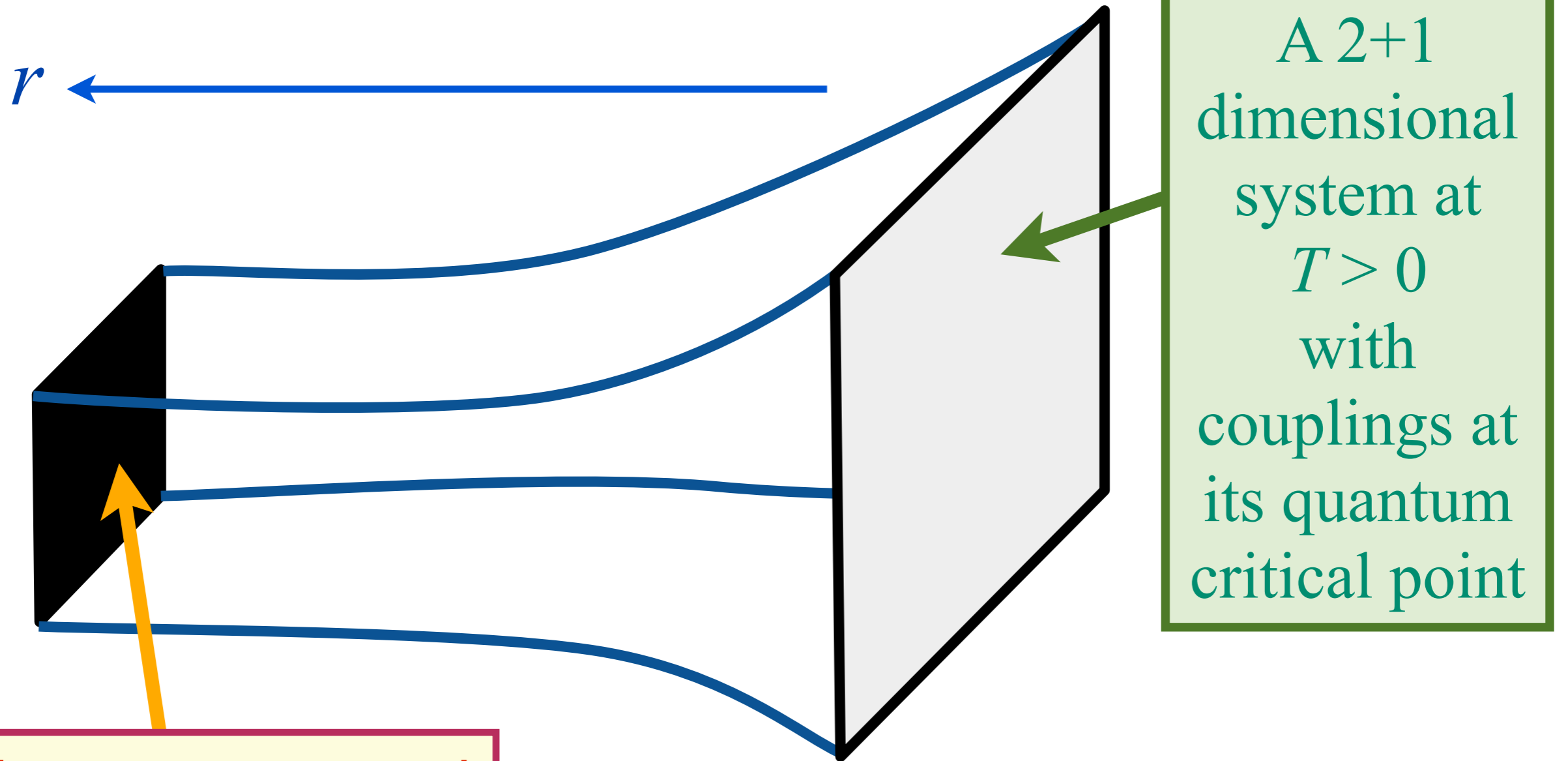
# Gauge-gravity duality at non-zero temperatures



A “horizon”, similar to the surface of a black hole !

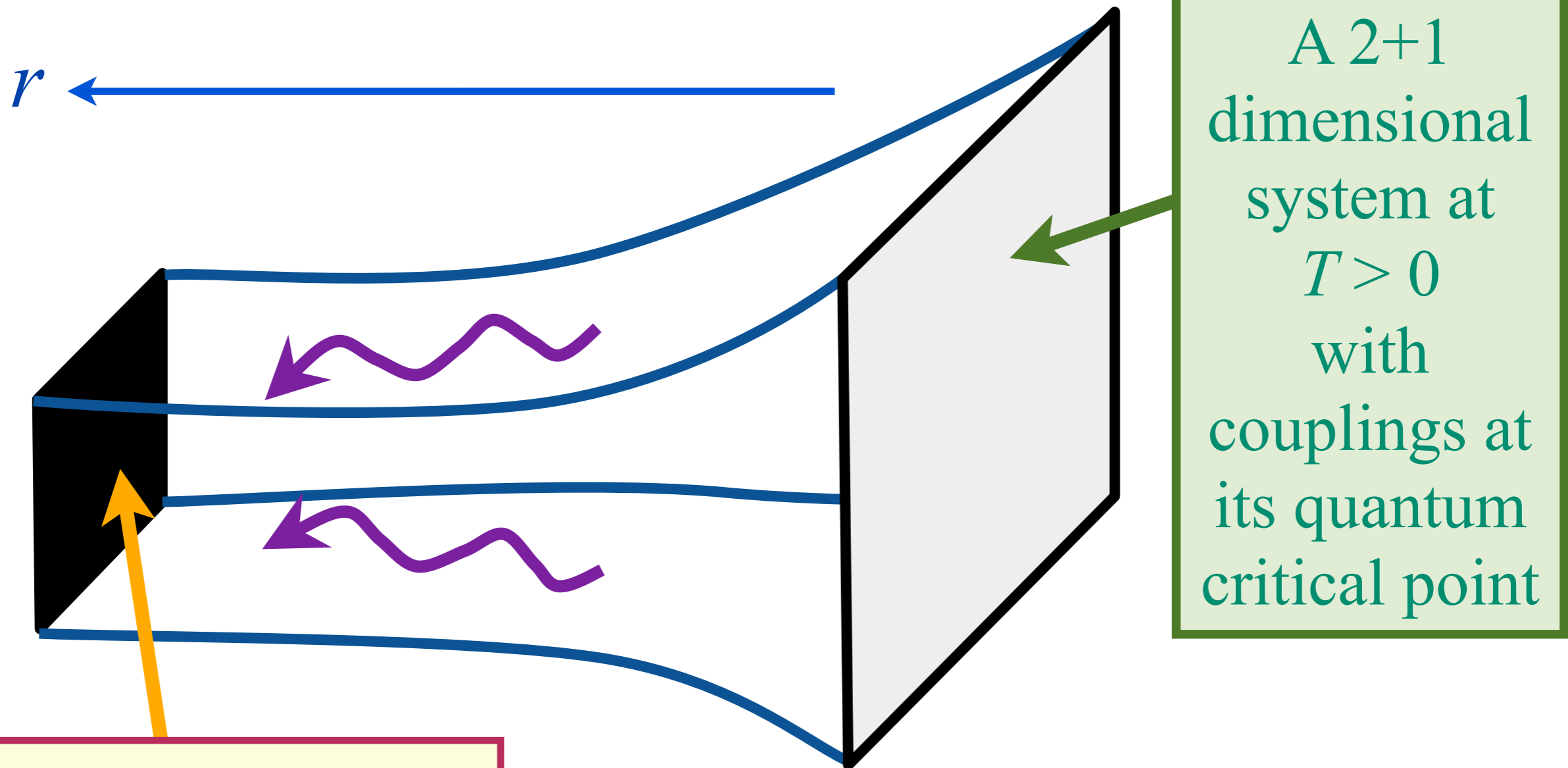
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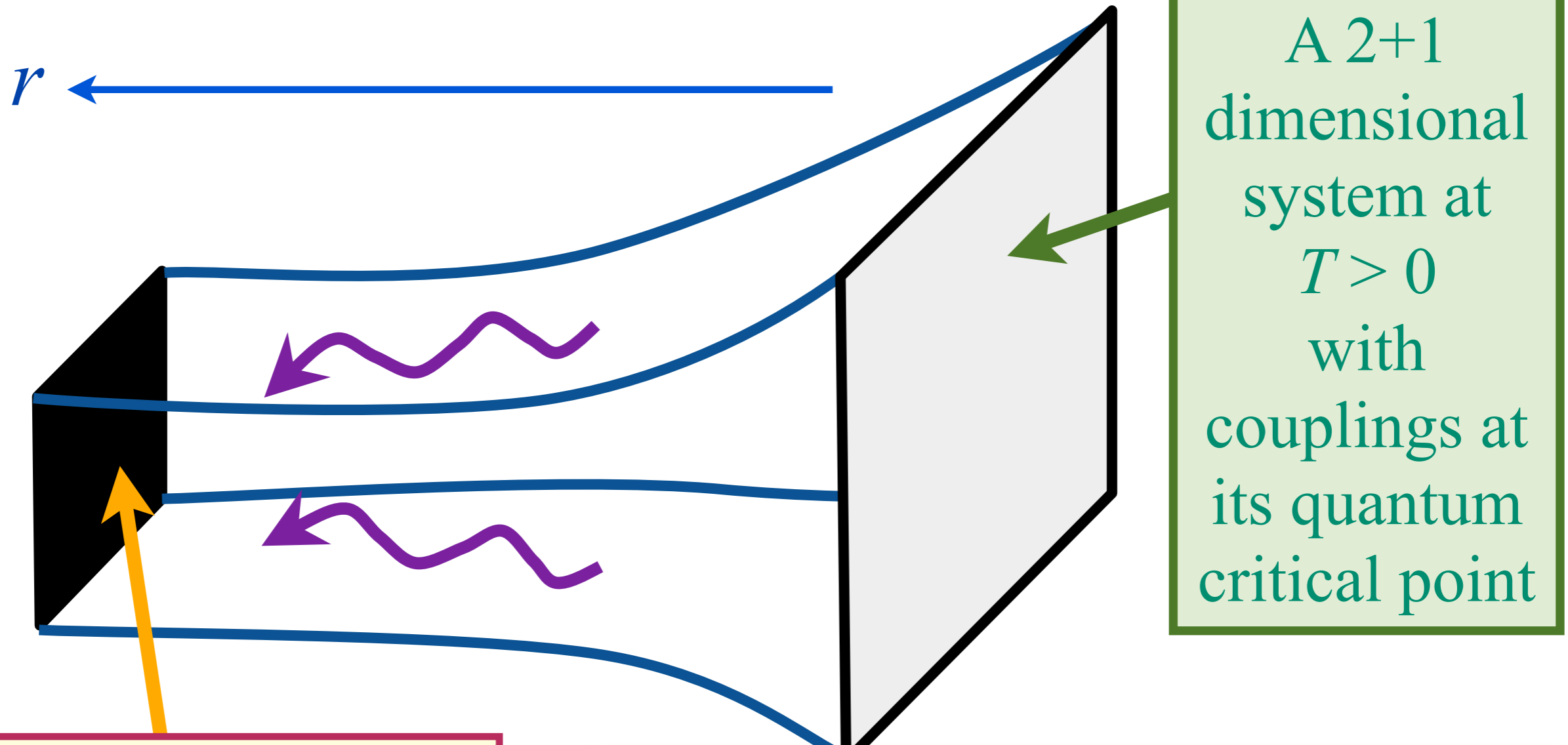


A 2+1 dimensional system at  $T > 0$  with couplings at its quantum critical point

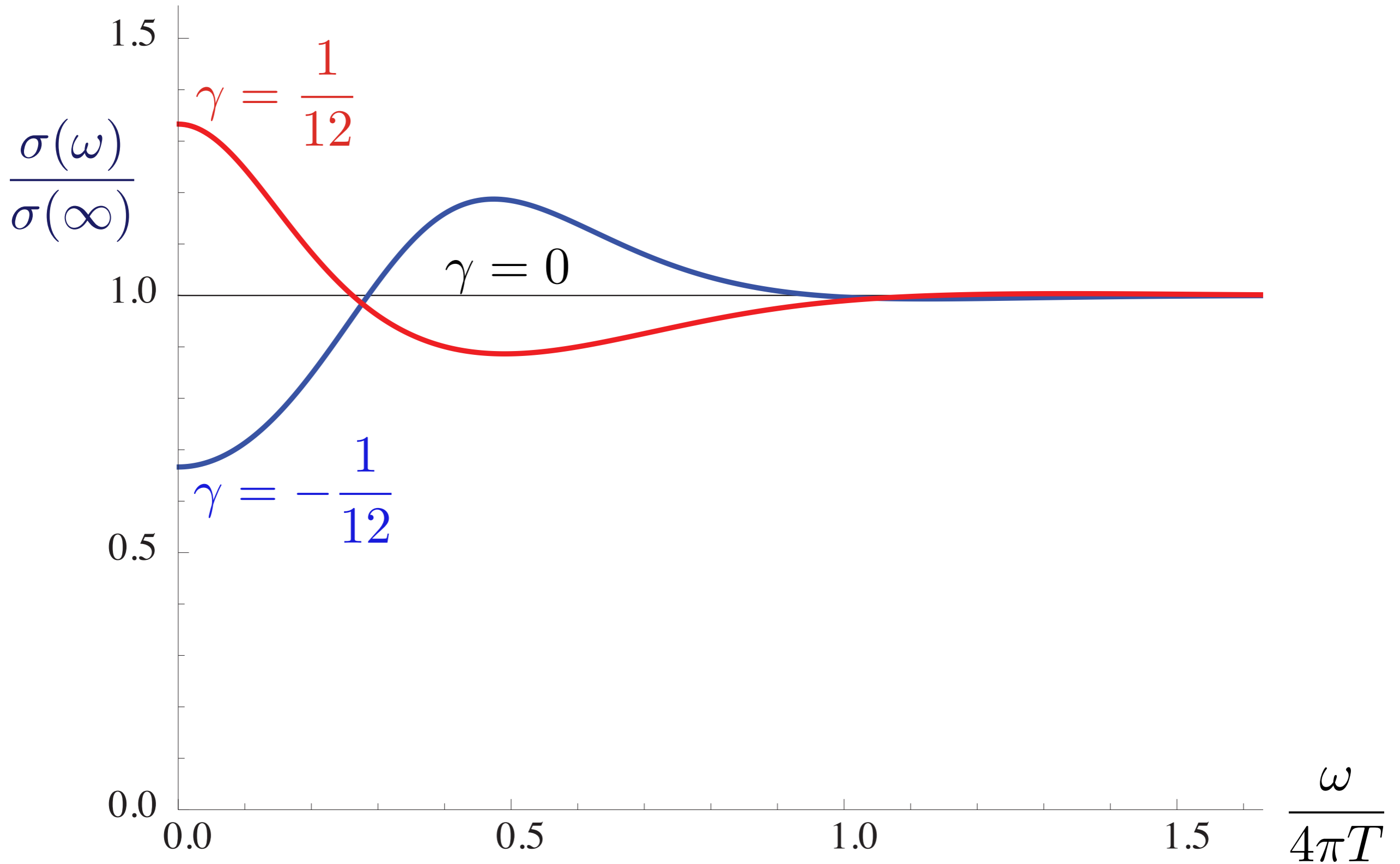
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Quasi-normal modes of quantum criticality = waves falling into black hole

# Gauge-gravity duality at non-zero temperatures

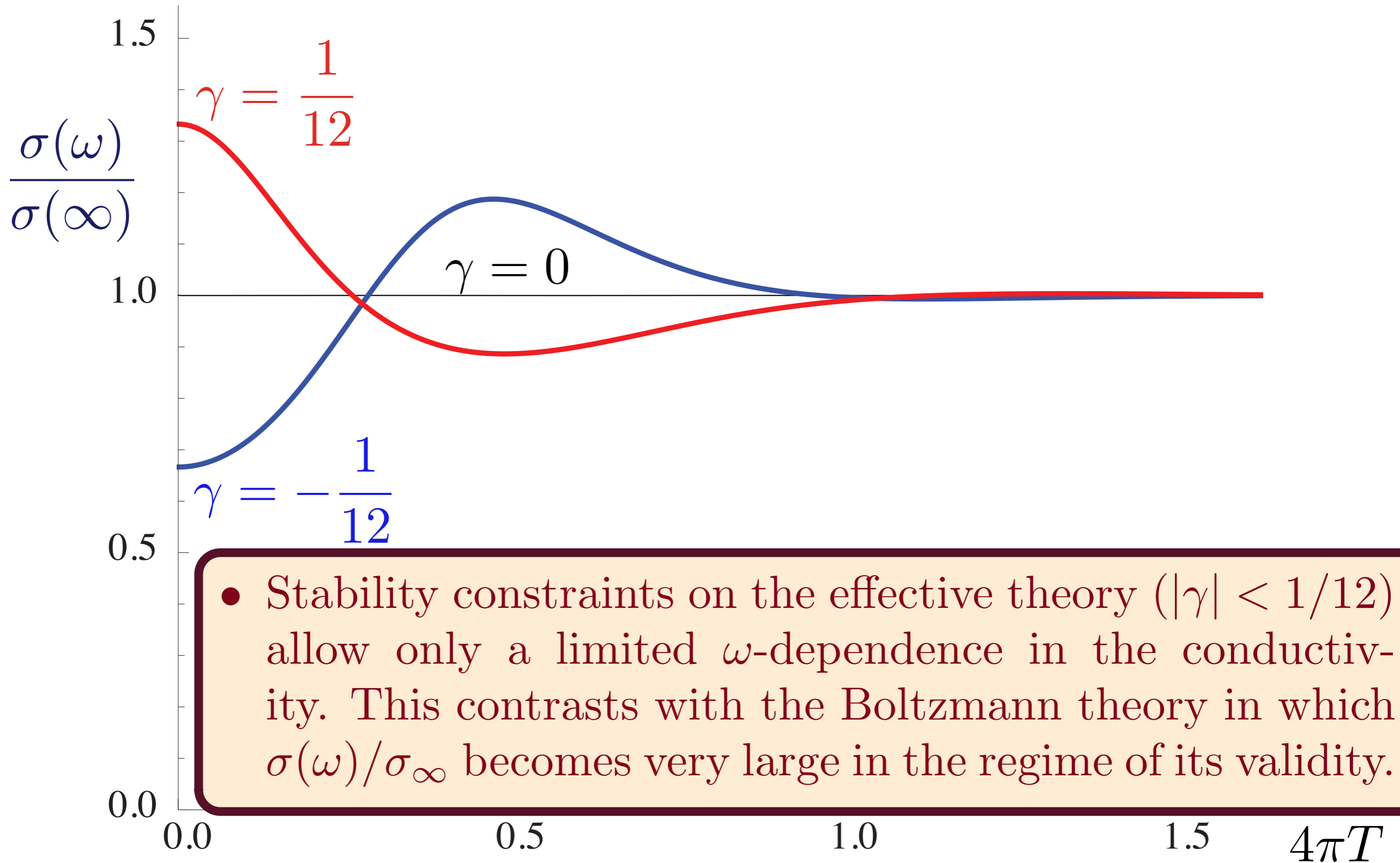


# AdS<sub>4</sub> theory of quantum criticality



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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PRL **95**, 180603 (2005)

PHYSICAL REVIEW LETTERS

week ending  
28 OCTOBER 2005

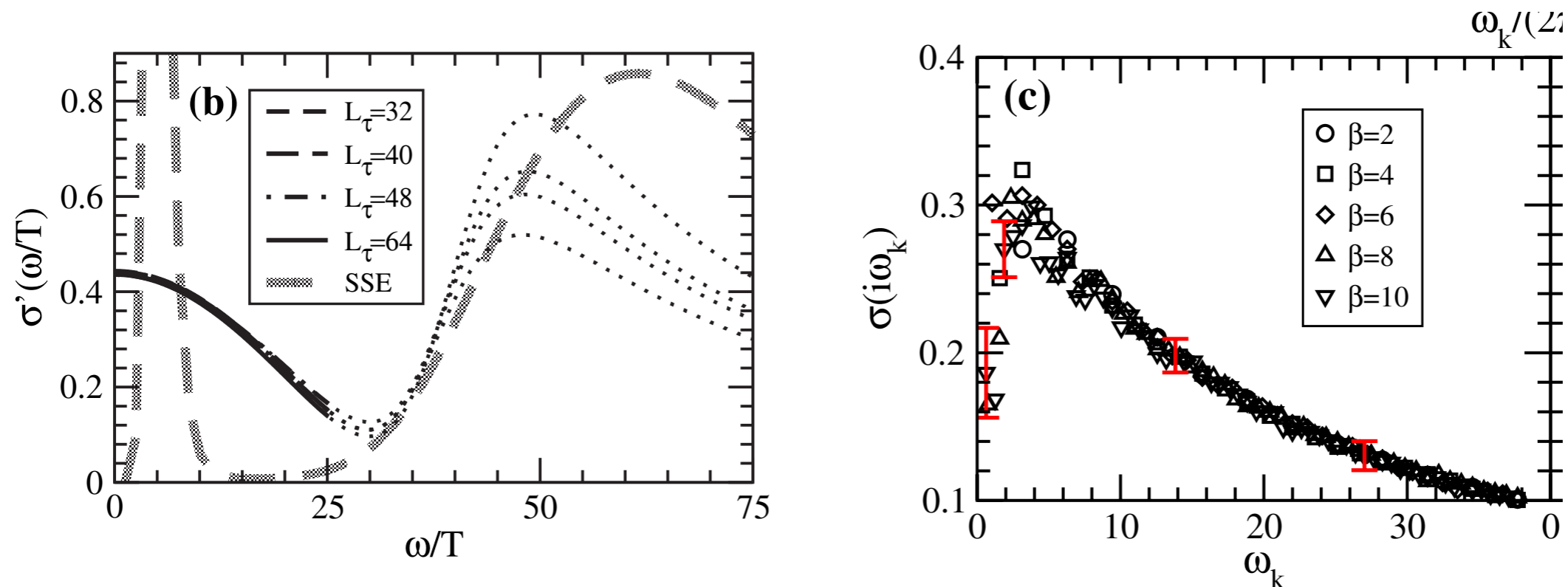
## Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

Jurij Šmakov and Erik Sørensen

*Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada*

(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature  $T$ . We find clear evidence for *deviations* from  $\omega_k$  scaling of the conductivity towards  $\omega_k/T$  scaling at low Matsubara frequencies  $\omega_k$ . By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with  $\omega/T$  at small frequencies and low temperatures. We estimate the universal dc conductivity to be  $\sigma^* = 0.45(5)Q^2/h$ , distinct from previous estimates in the  $T = 0$ ,  $\omega/T \gg 1$  limit.



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QMC yields  $\sigma(0)/\sigma_\infty \approx 1.36$

Holography yields  $\sigma(0)/\sigma_\infty = 1 + 4\gamma$  with  $|\gamma| \leq 1/12$ .

Maximum possible holographic value  $\sigma(0)/\sigma_\infty = 1.33$

W. Witzack-Krempa and S. Sachdev, arXiv:1302.0847

# Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

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- Solve Einstein-Maxwell equations. Dynamics of quasi-normal modes of black branes.

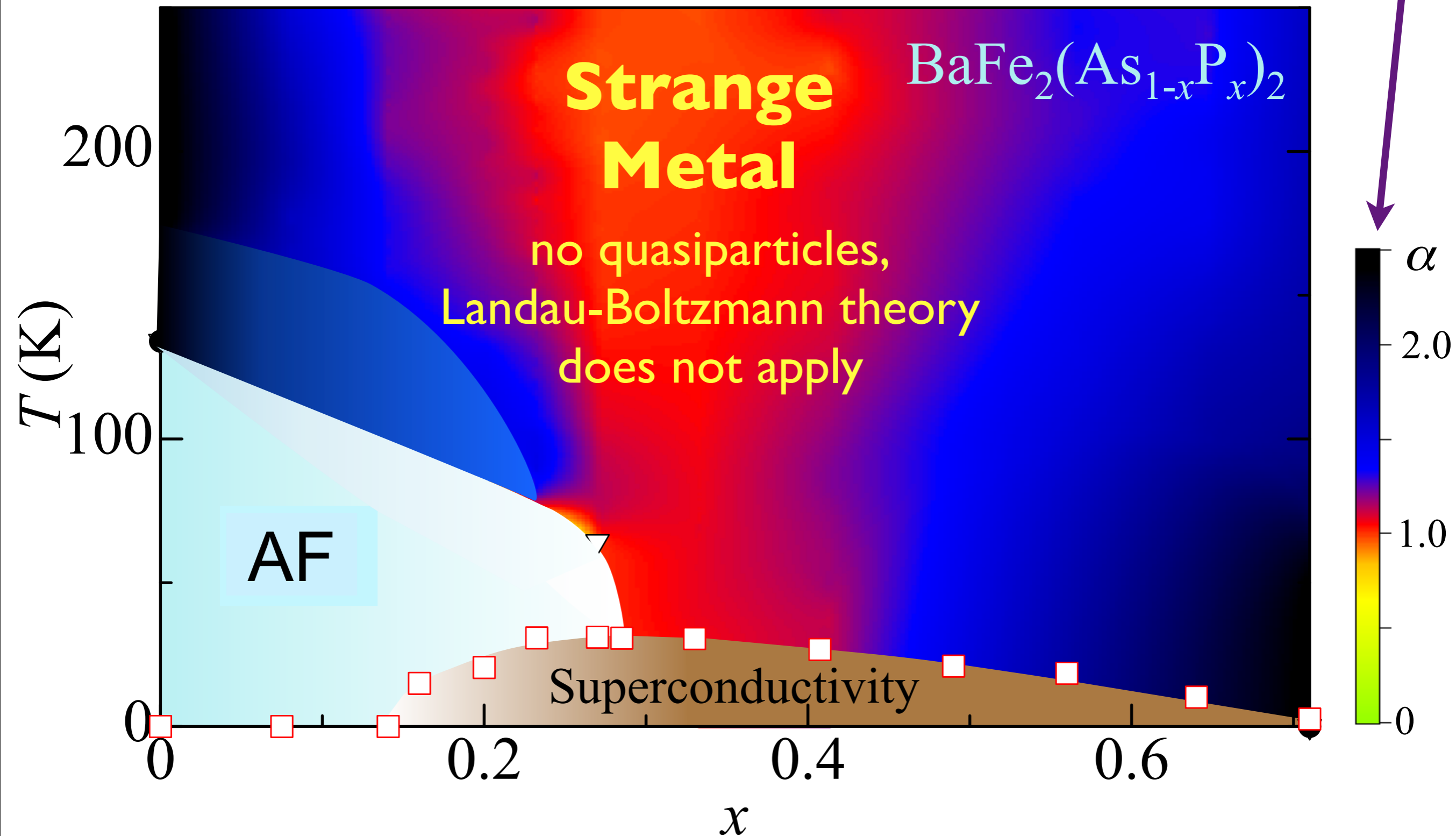
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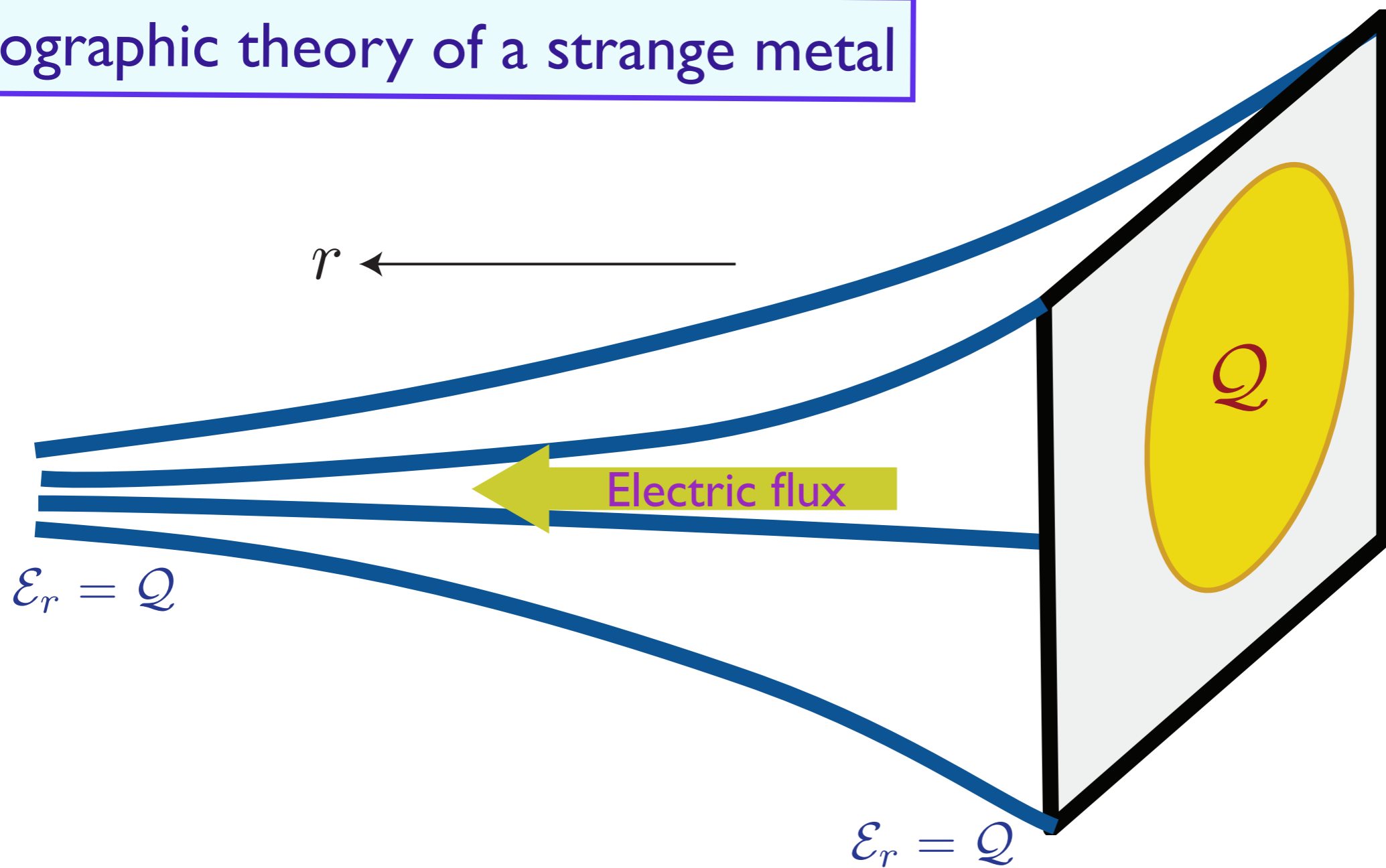
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Resistivity  
 $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido,  
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda,  
*Physical Review B* **81**, 184519 (2010)

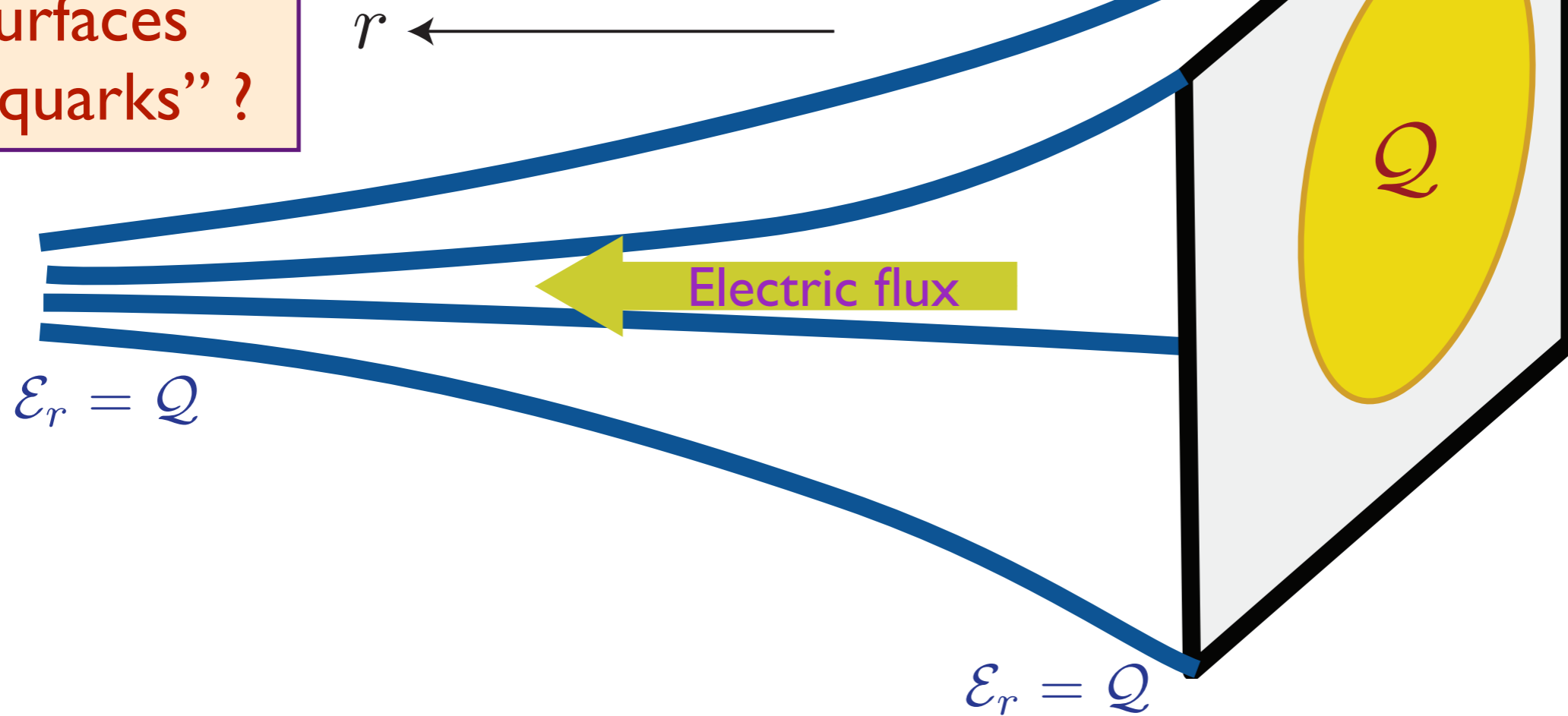
# Holographic theory of a strange metal



The density of particles  $Q$  creates an electric flux  $\mathcal{E}_r$  which modifies the metric of the emergent spacetime.

# Holographic theory of a strange metal

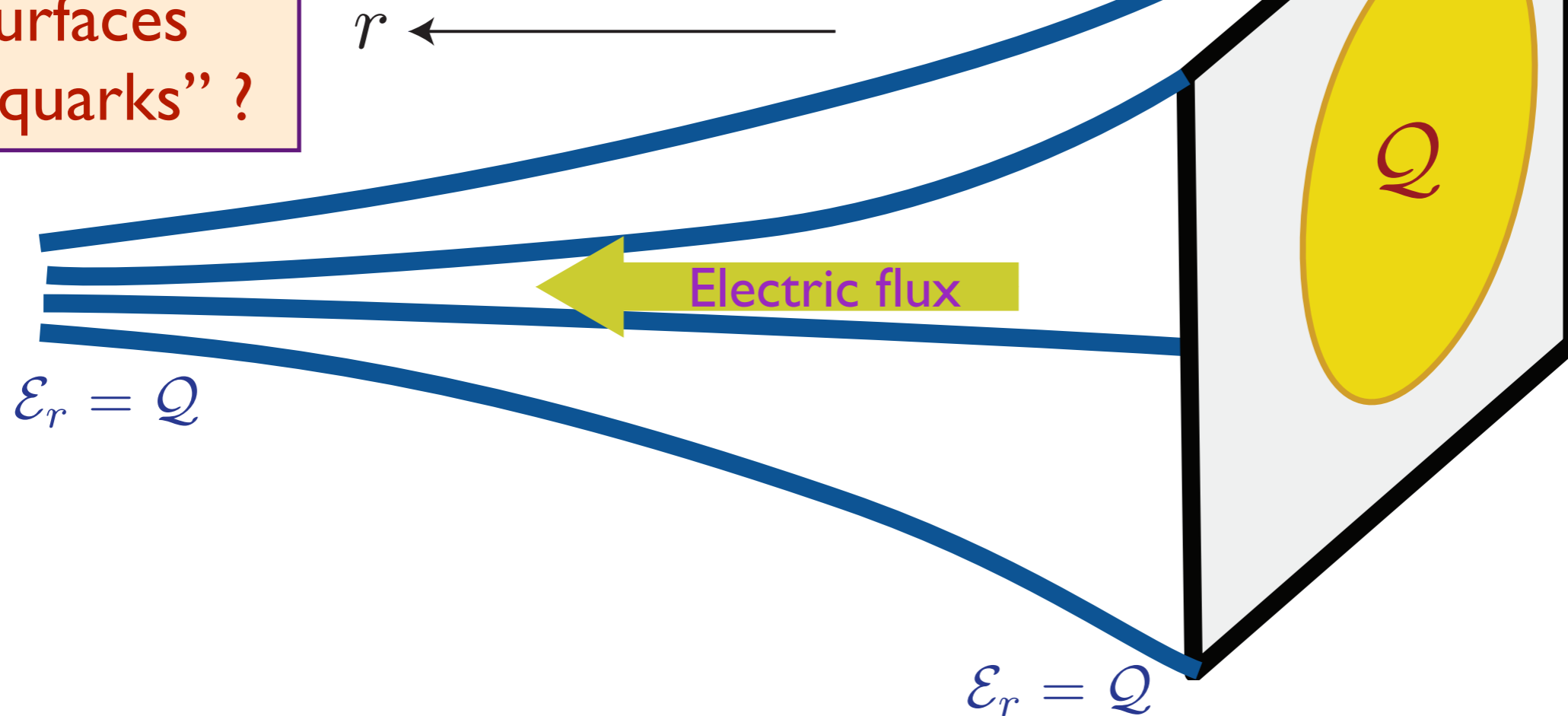
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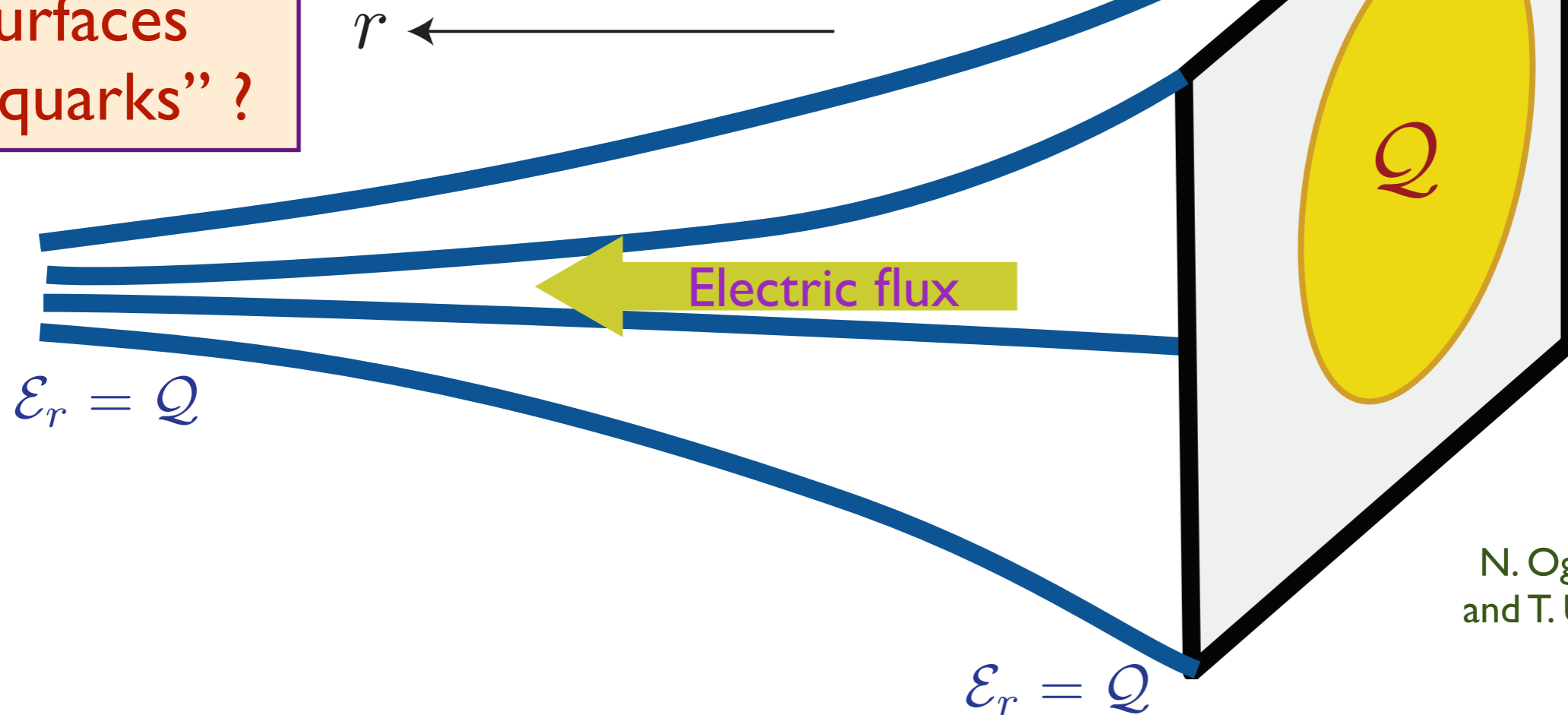
The general metric transforms under rescaling as

$$x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds.$$

Recall: conformal matter has  $\theta = 0$ ,  $z = 1$ , and the metric is anti-de Sitter

# Holographic theory of a strange metal

Hidden Fermi surfaces of “quarks” ?



N. Ogawa, T. Takayanagi, and T. Ugajin, JHEP **1201**, 125 (2012).

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

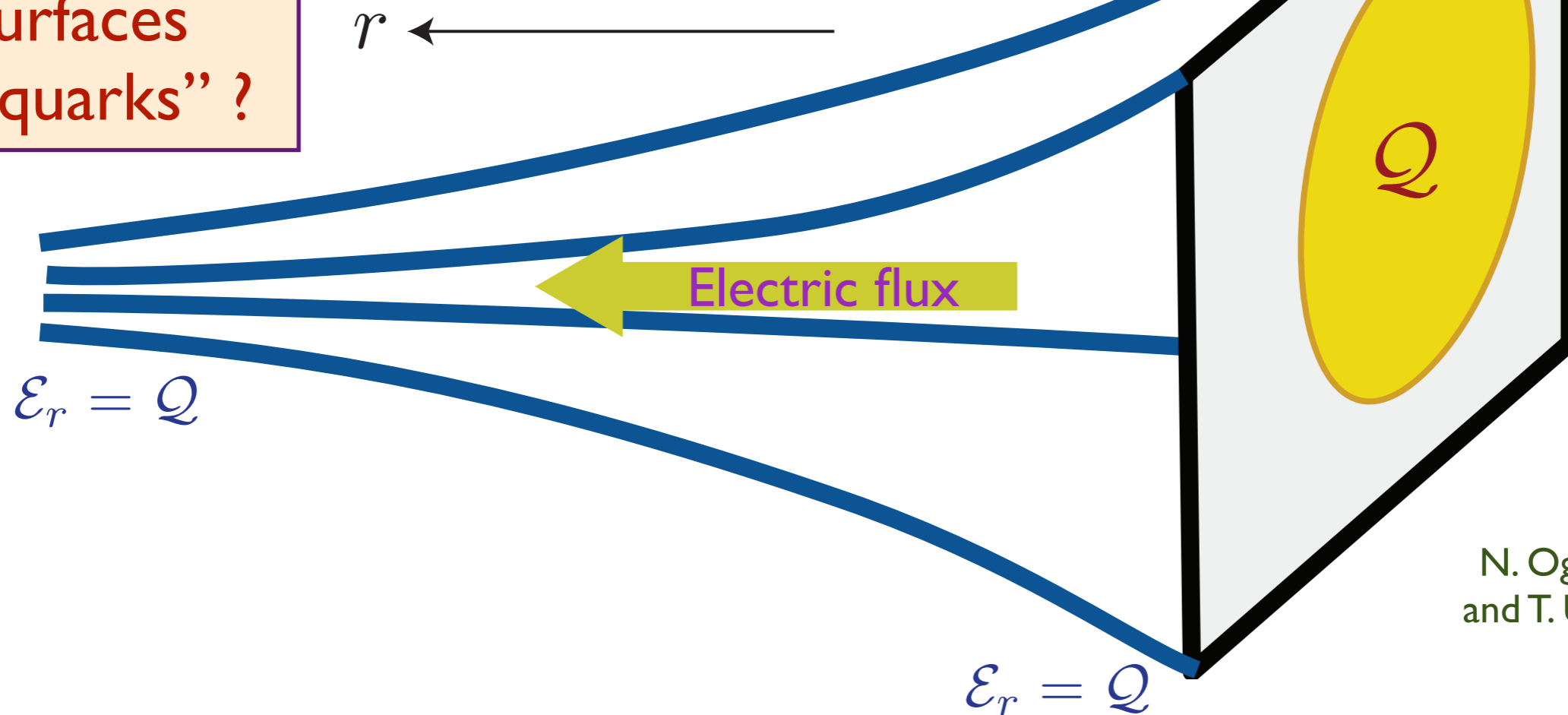
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The value  $\theta = d - 1$  reproduces *all* the essential characteristics of the **entropy** and **entanglement entropy** of a strange metal.

# Holographic theory of a strange metal

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The null-energy condition of gravity yields  $z \geq 1 + \theta/d$ . In  $d = 2$ , this leads to  $z \geq 3/2$ . Field theory on strange metal yields  $z = 3/2$  to 3 loops!

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

# Conclusions

## Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points
- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.
- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

# Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

# Conclusions

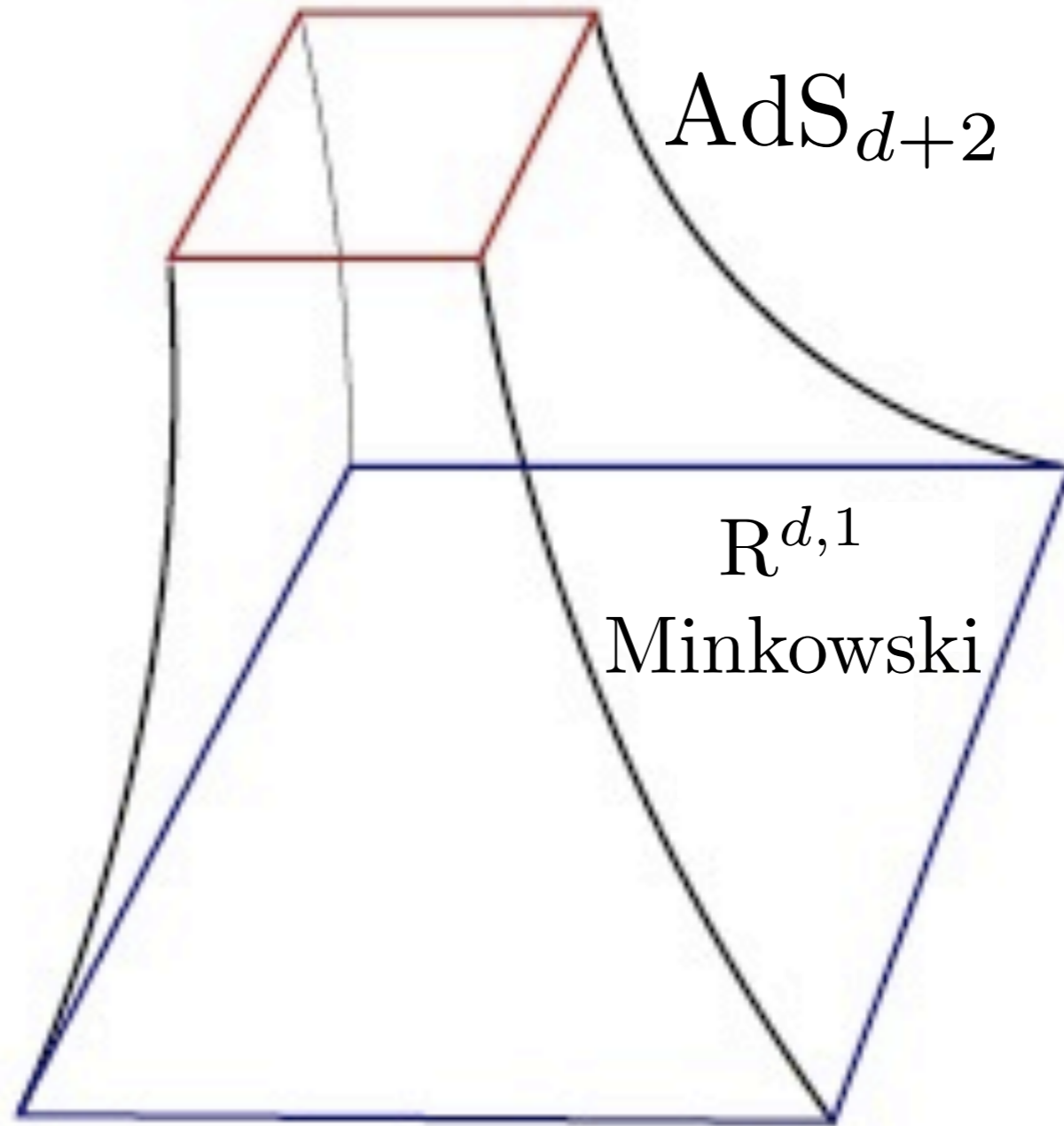
String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with many-particle quantum entanglement.

Much recent progress offers hope of a holographic description of “strange metals”

# anti-de Sitter space

Emergent holographic direction

$r$



# anti-de Sitter space

