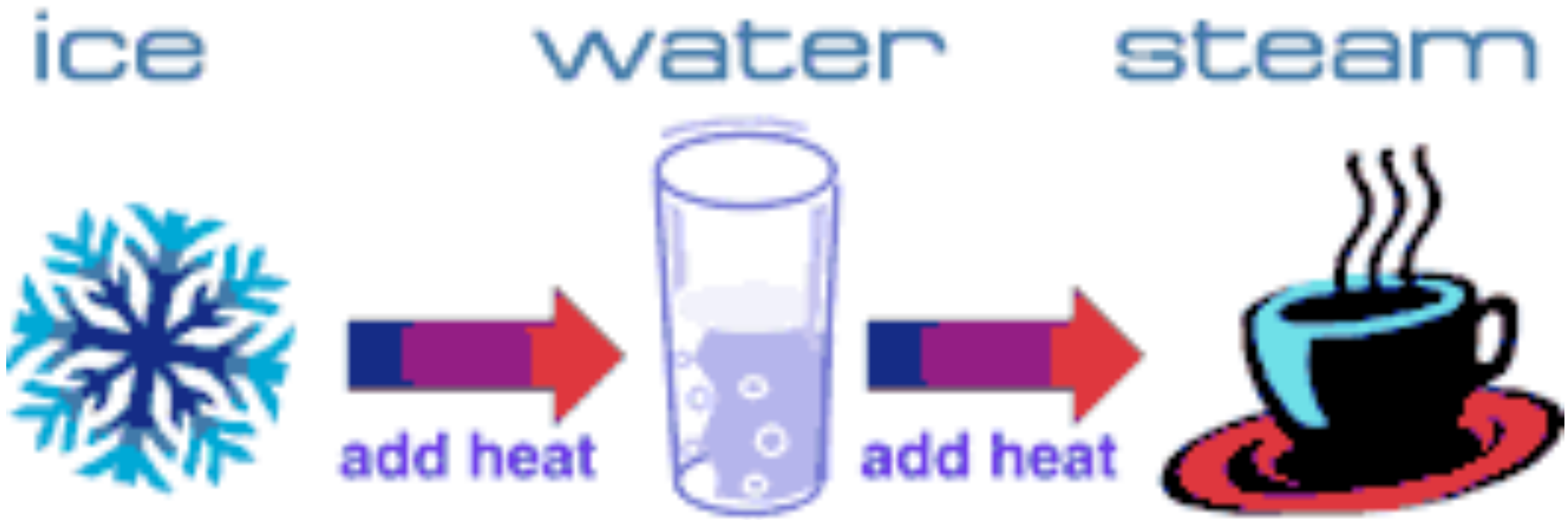
# Quantum Criticality and Black Holes

*Subir Sachdev*

Talk online at http://sachdev.physics.harvard.edu

# What is a "phase transition" ?



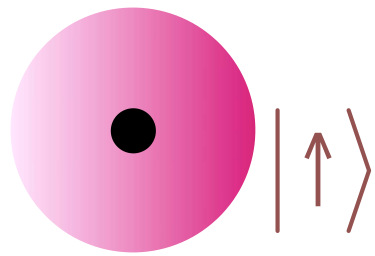

A change in the collective properties of a macroscopic number of atoms

# What is a “*quantum phase transition*” ?

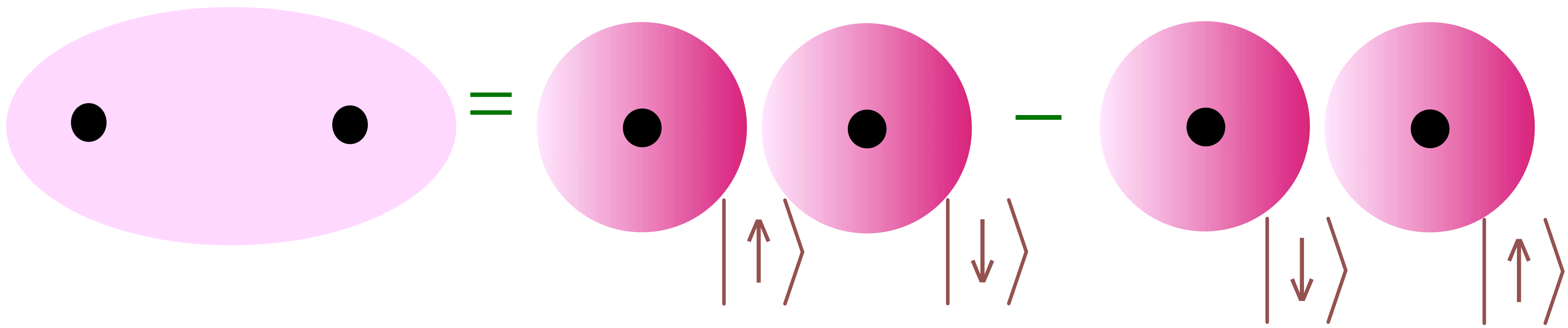
Change in the nature of entanglement in a macroscopic quantum system.

# Entanglement

Hydrogen atom:




Hydrogen molecule:




$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Superposition of two electron states leads to non-local correlations between spins

# Outline

1. Superfluid-insulator quantum transitions

   *Experiments on ultracold atoms*

2. Theory of quantum-critical transport

   *Collisionless-t0-hydrodynamic crossover of conformal field theories*

3. Entanglement of valence bonds

   *Deconfined criticality in antiferromagnets*

4. Nernst effect in the cuprate superconductors

   *Quantum criticality and dyonic black holes*

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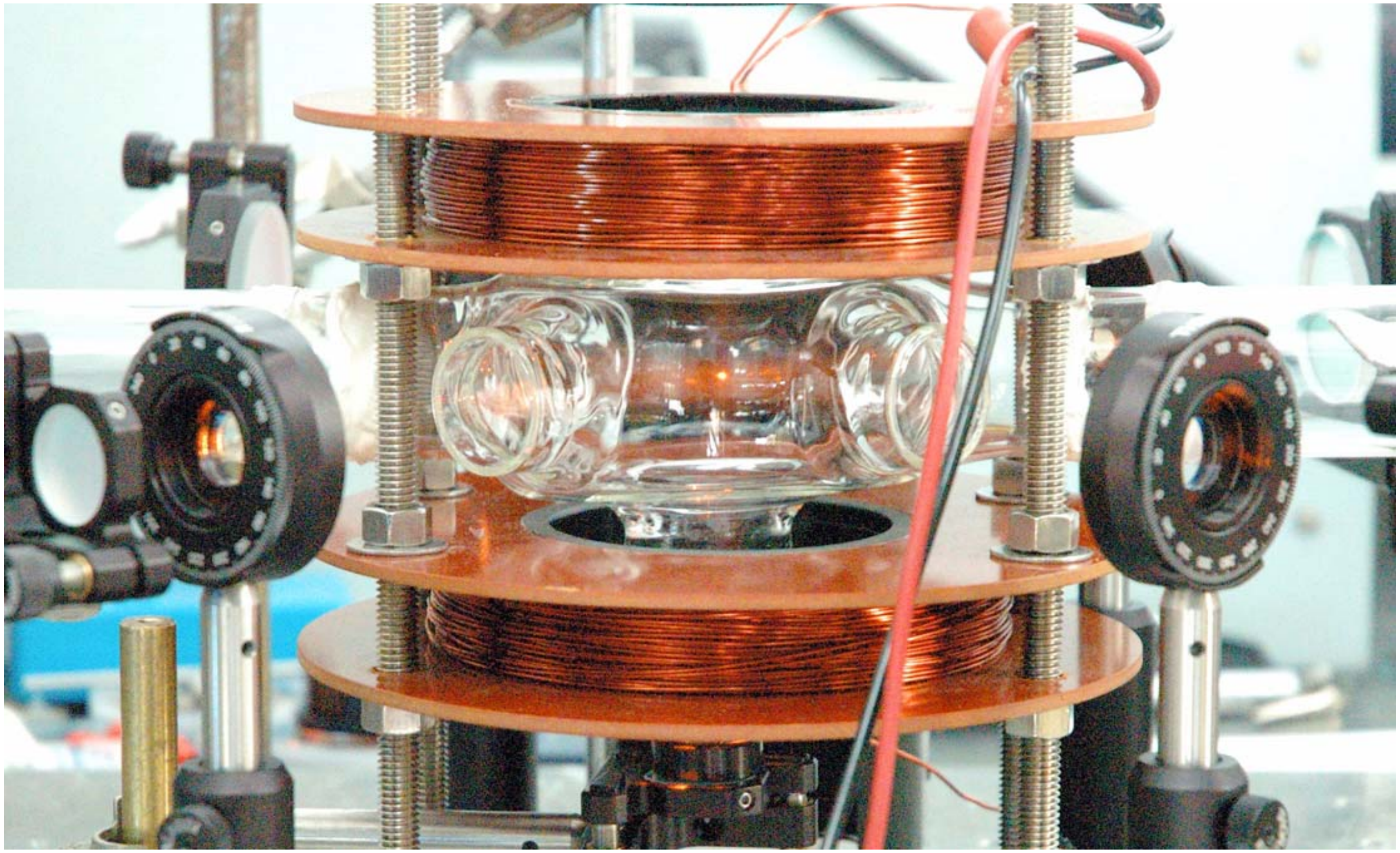
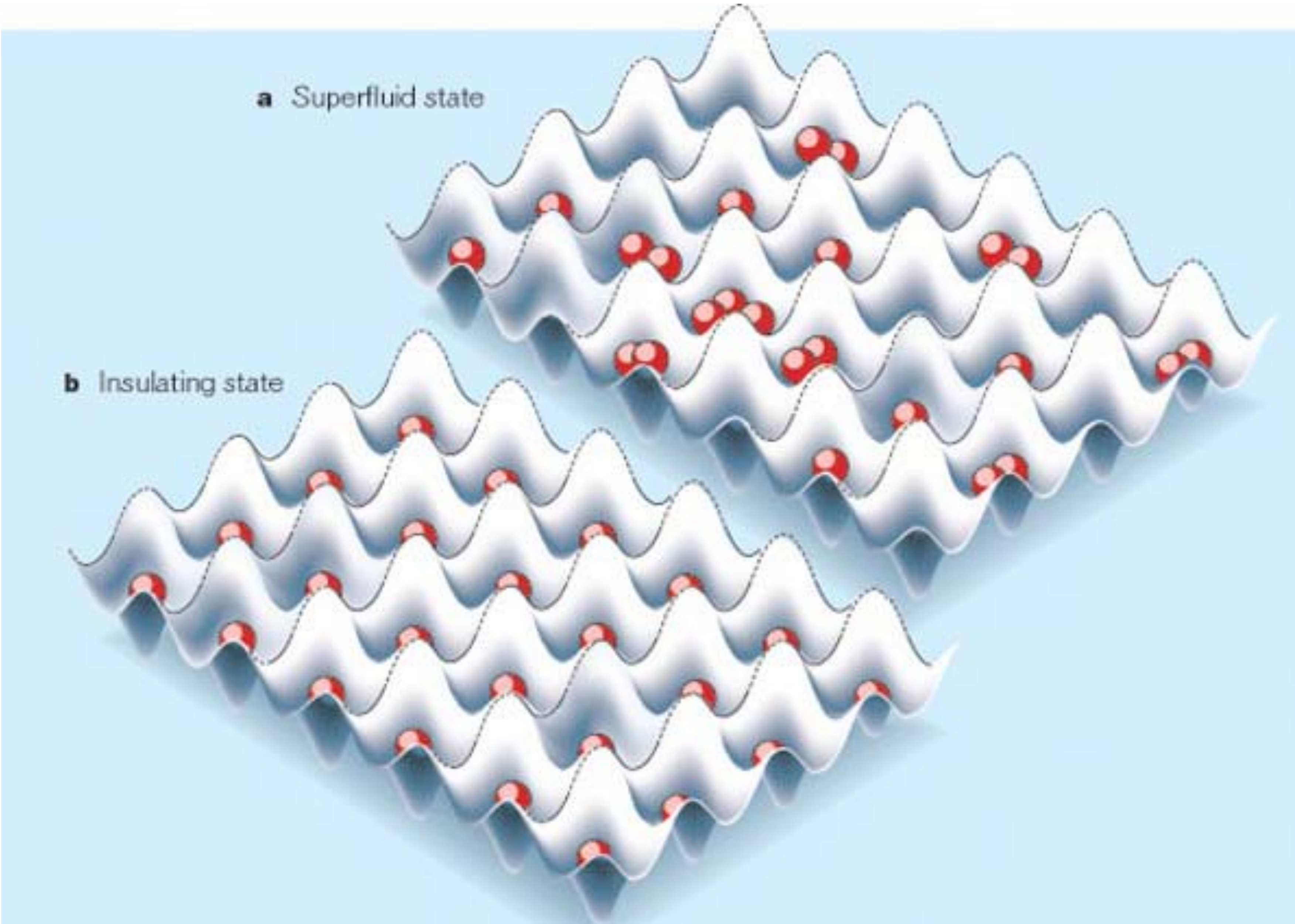
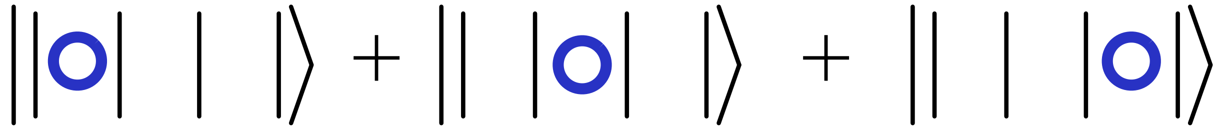


Trap for ultracold $^{87}$Rb atoms



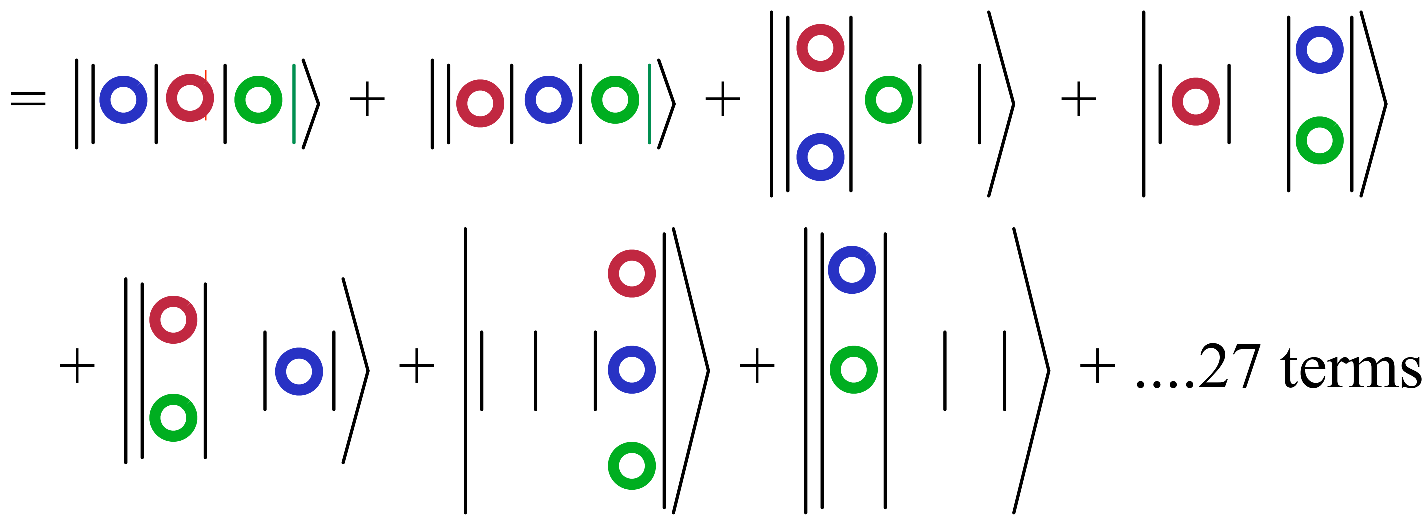

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

# The Bose-Einstein condensate in a periodic potential

$|G\rangle =$ 

## Lowest energy state for many atoms

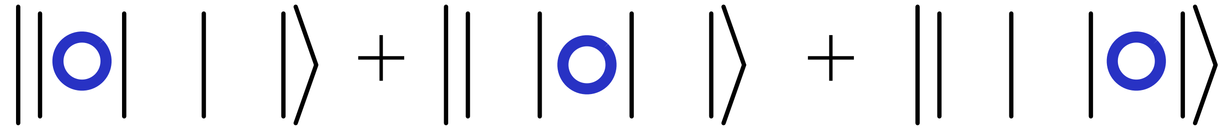
$$|BEC\rangle = |G\rangle|G\rangle|G\rangle$$




Large fluctuations in number of atoms in each potential well
– *superfluidity* (atoms can “flow” without dissipation)

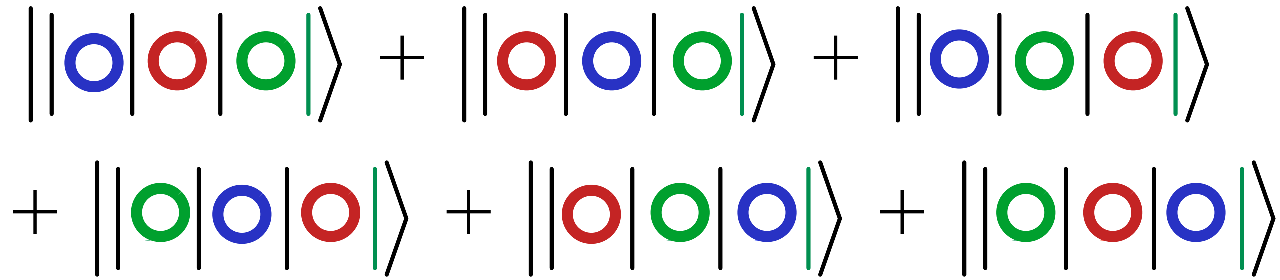
# Breaking up the Bose-Einstein condensate

$|G\rangle =$



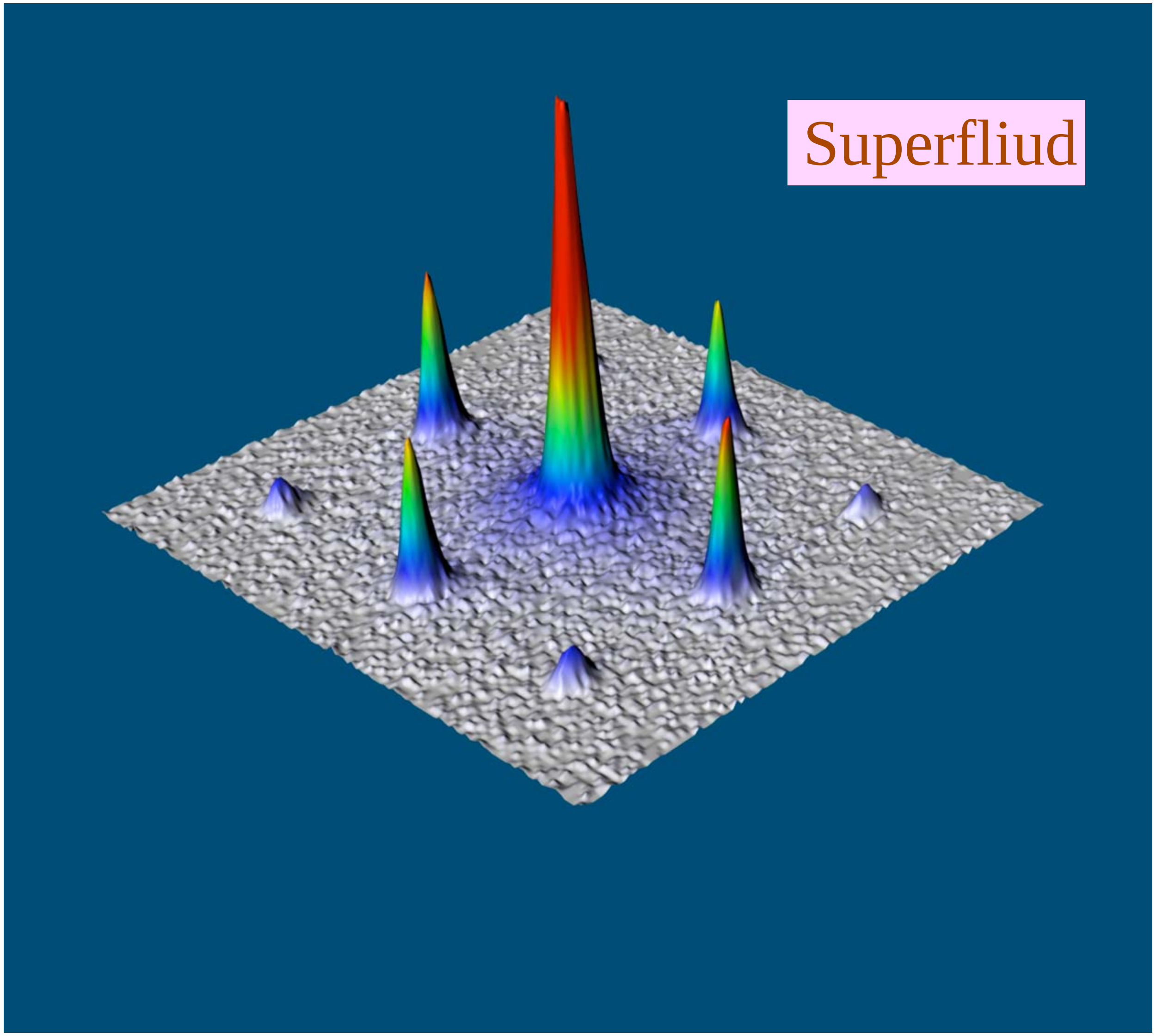
Lowest energy state for many atoms

$|MI\rangle =$



By tuning repulsive interactions between the atoms, states with multiple atoms in a potential well can be suppressed. The lowest energy state is then a *Mott insulator* – it has negligible number fluctuations, and atoms cannot “flow”

# Velocity distribution of $^{87}$Rb atoms

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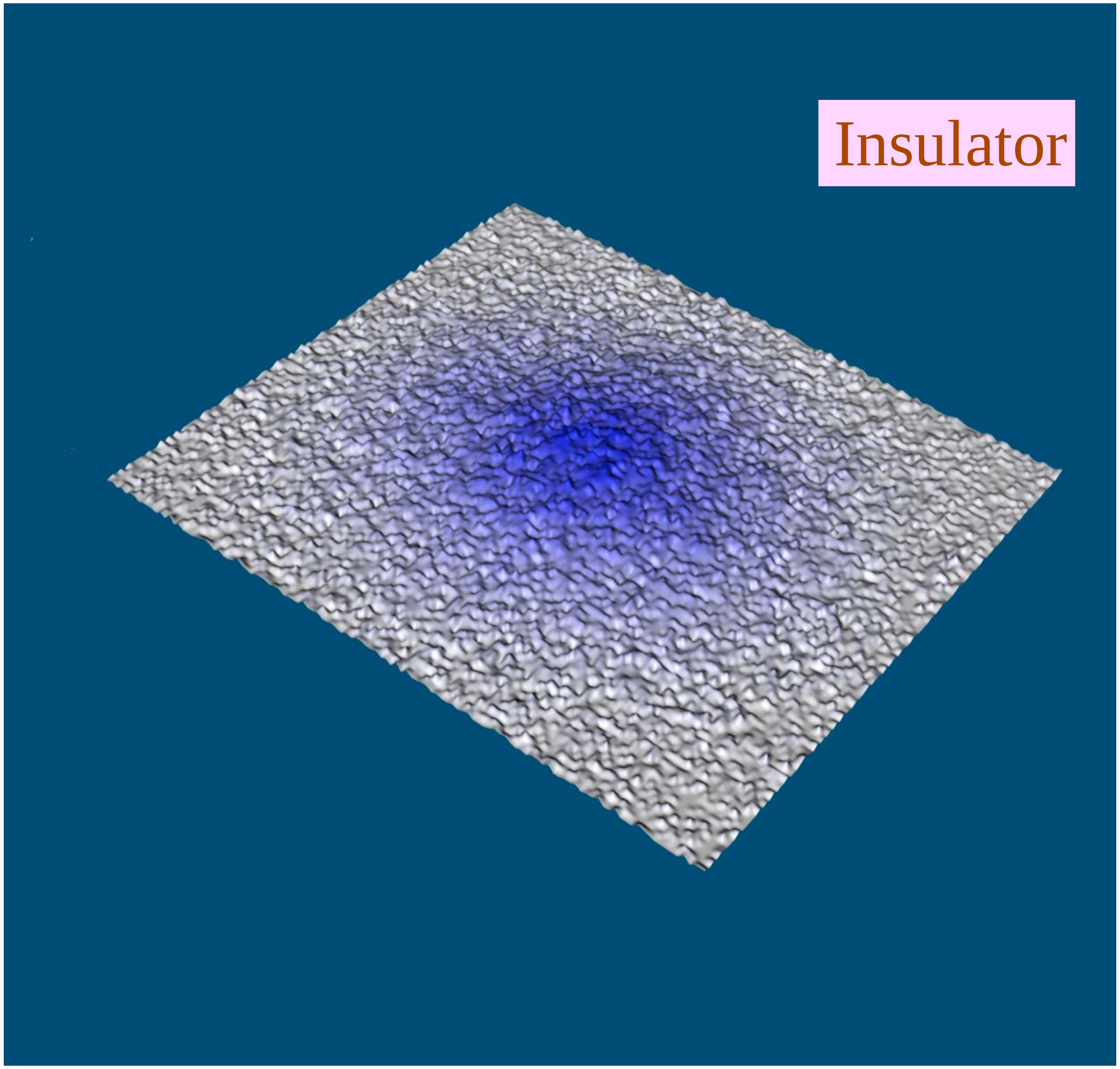


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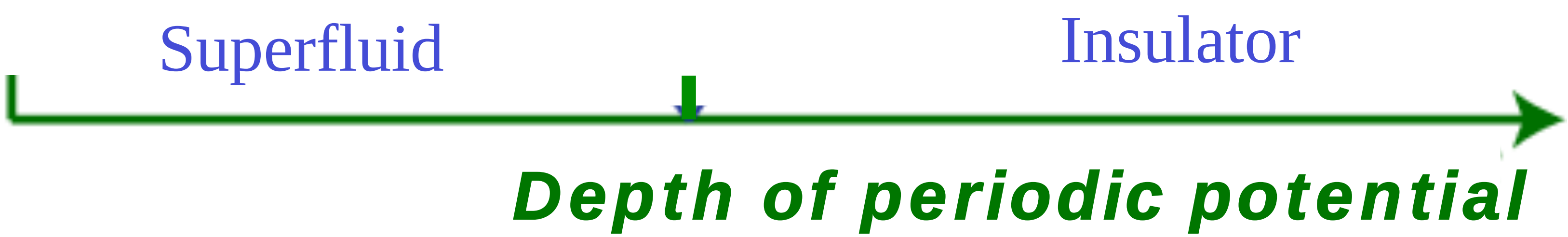
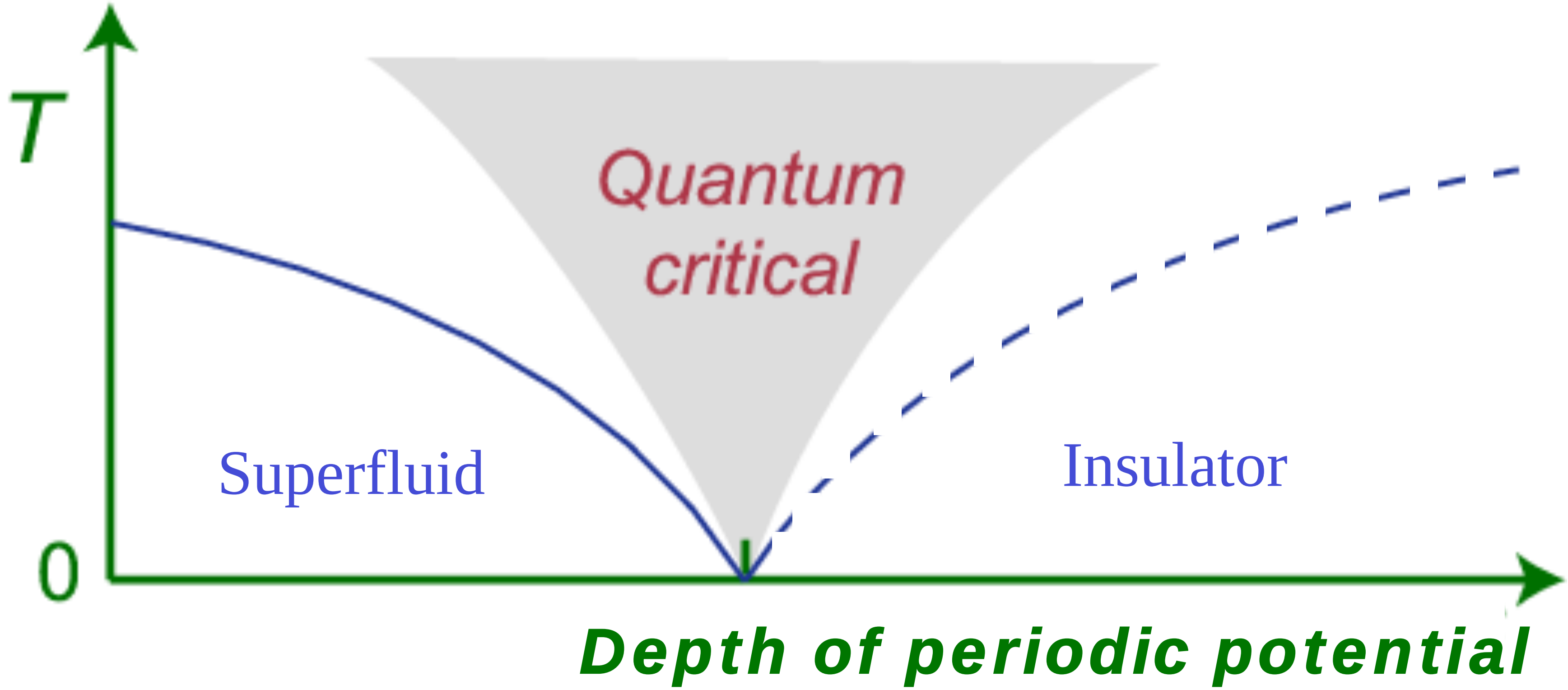

Superfluid
Insulator
Depth of periodic potential

# Non-zero temperature phase diagram



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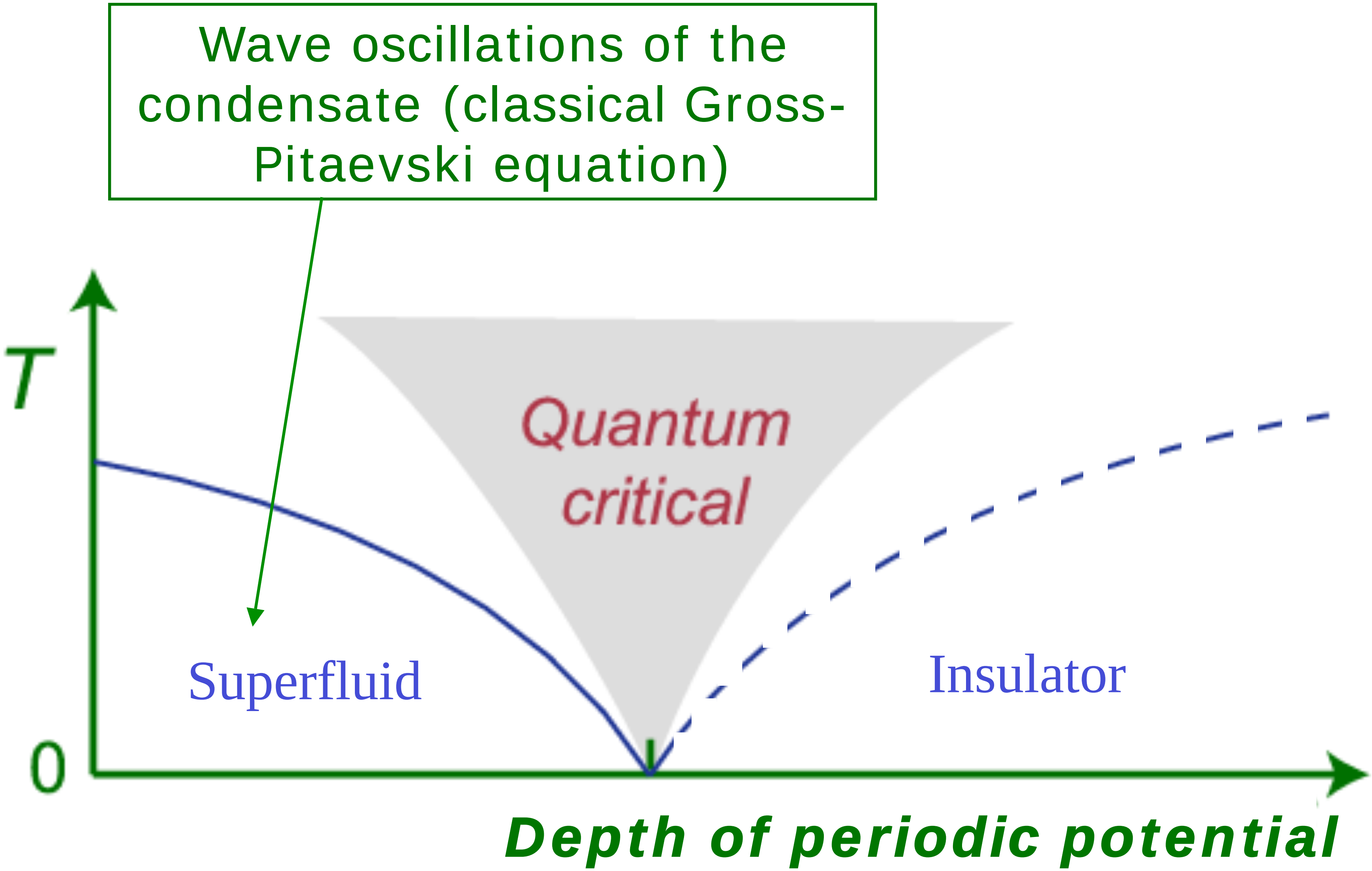

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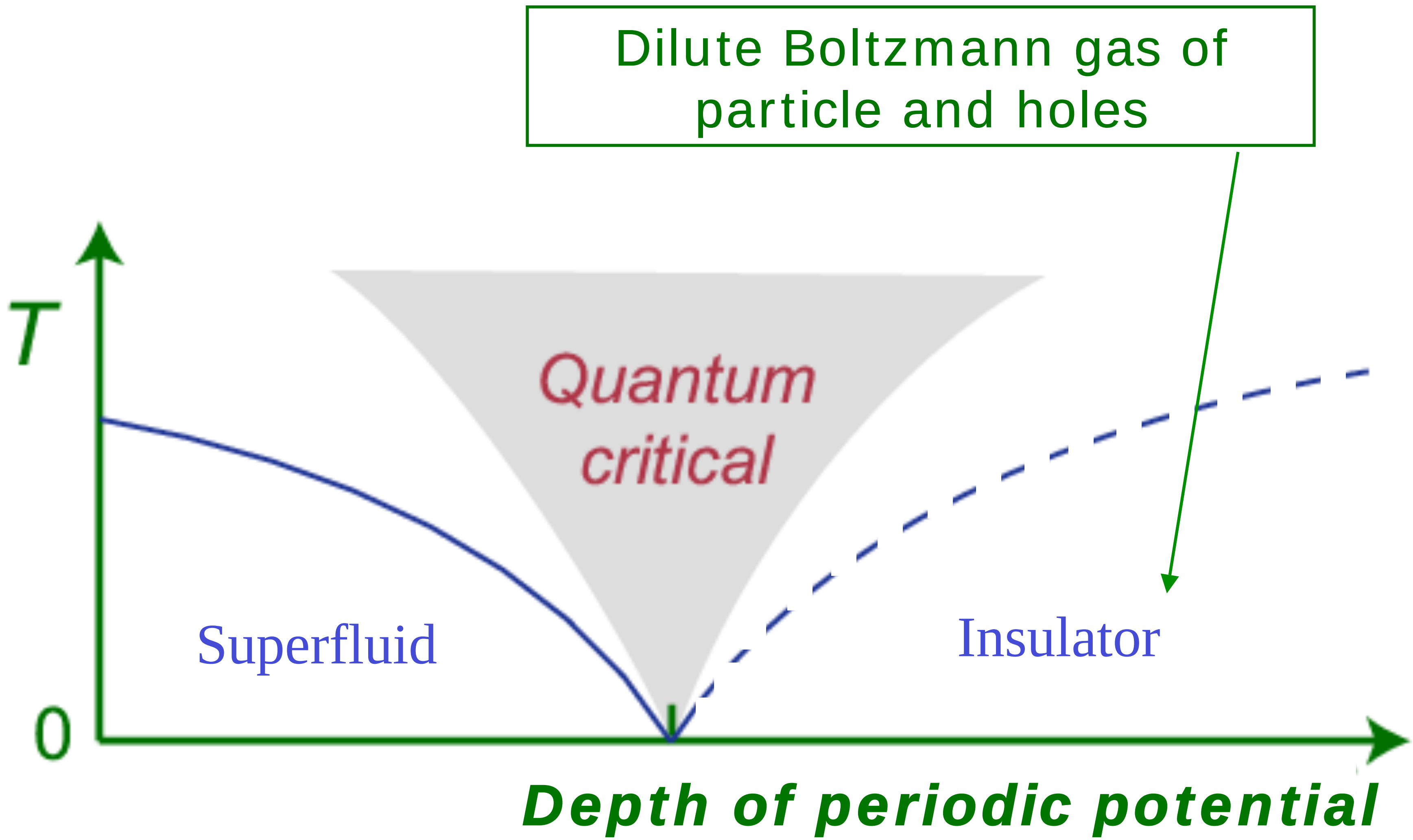

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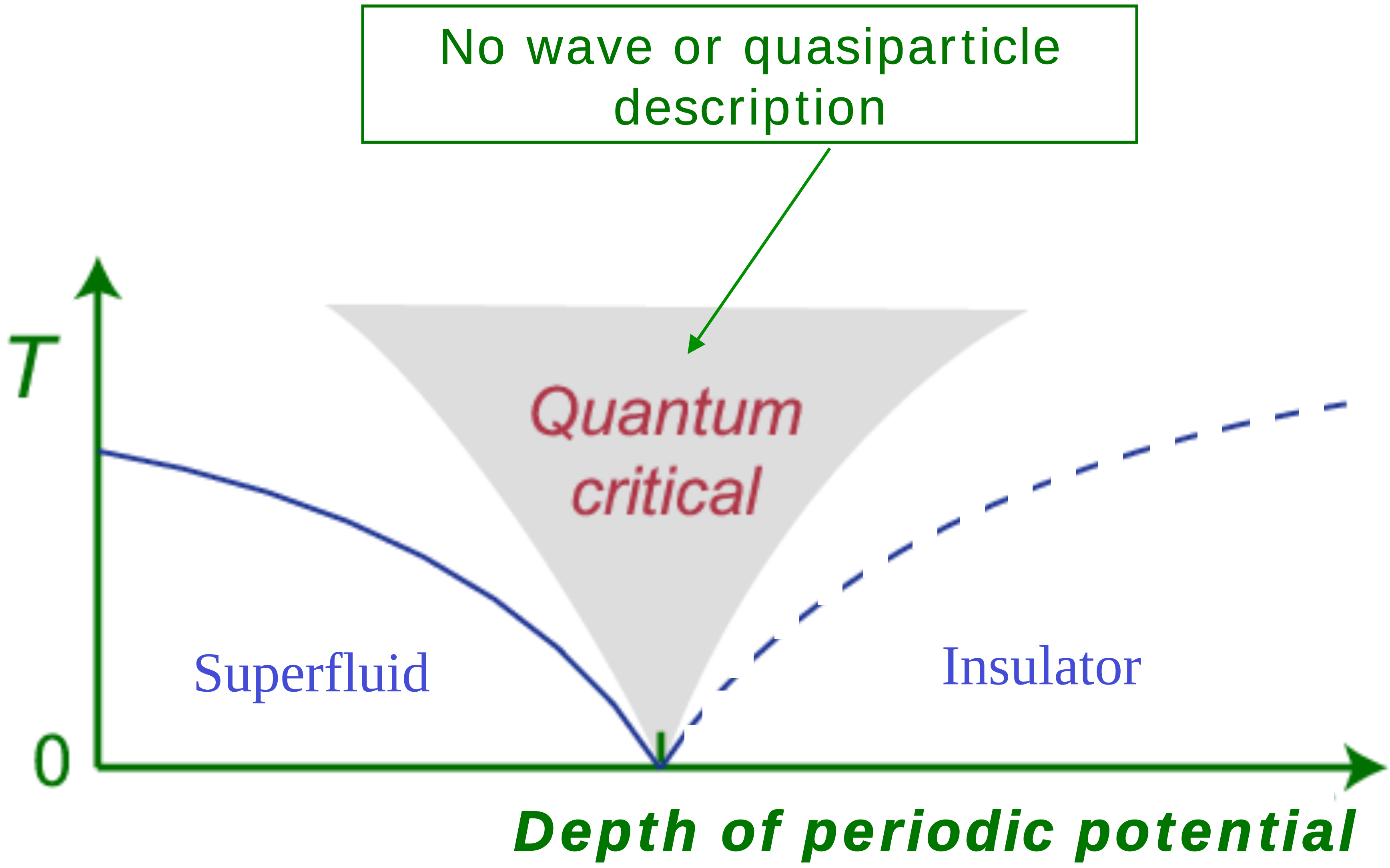

# Resistivity of Bi films

Conductivity $\sigma$

$$\begin{aligned}
\sigma_{\text{Superconductor}}(T \to 0) &= \infty \\
\sigma_{\text{Insulator}}(T \to 0) &= 0 \\
\sigma_{\text{Quantum critical point}}(T \to 0) &\approx \frac{4e^2}{h}
\end{aligned}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

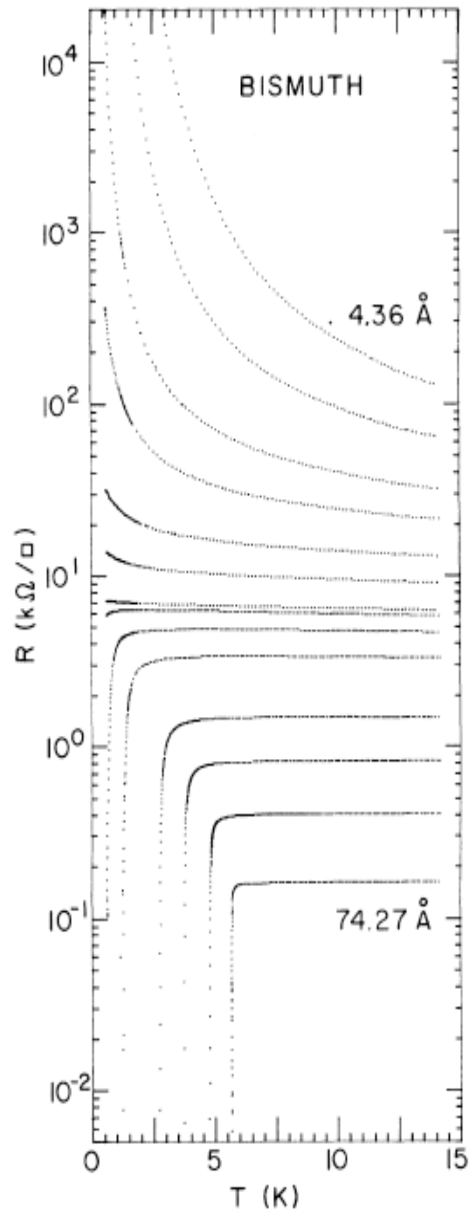


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

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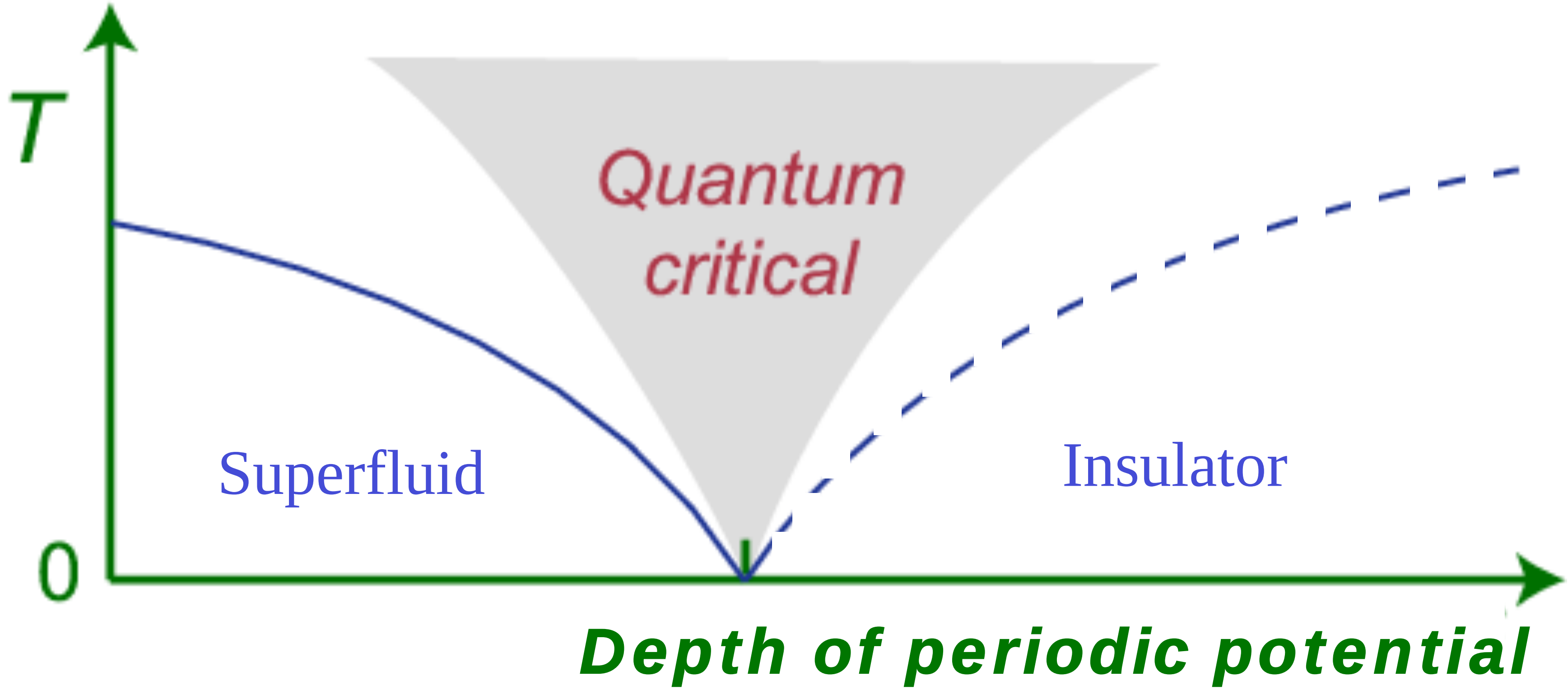

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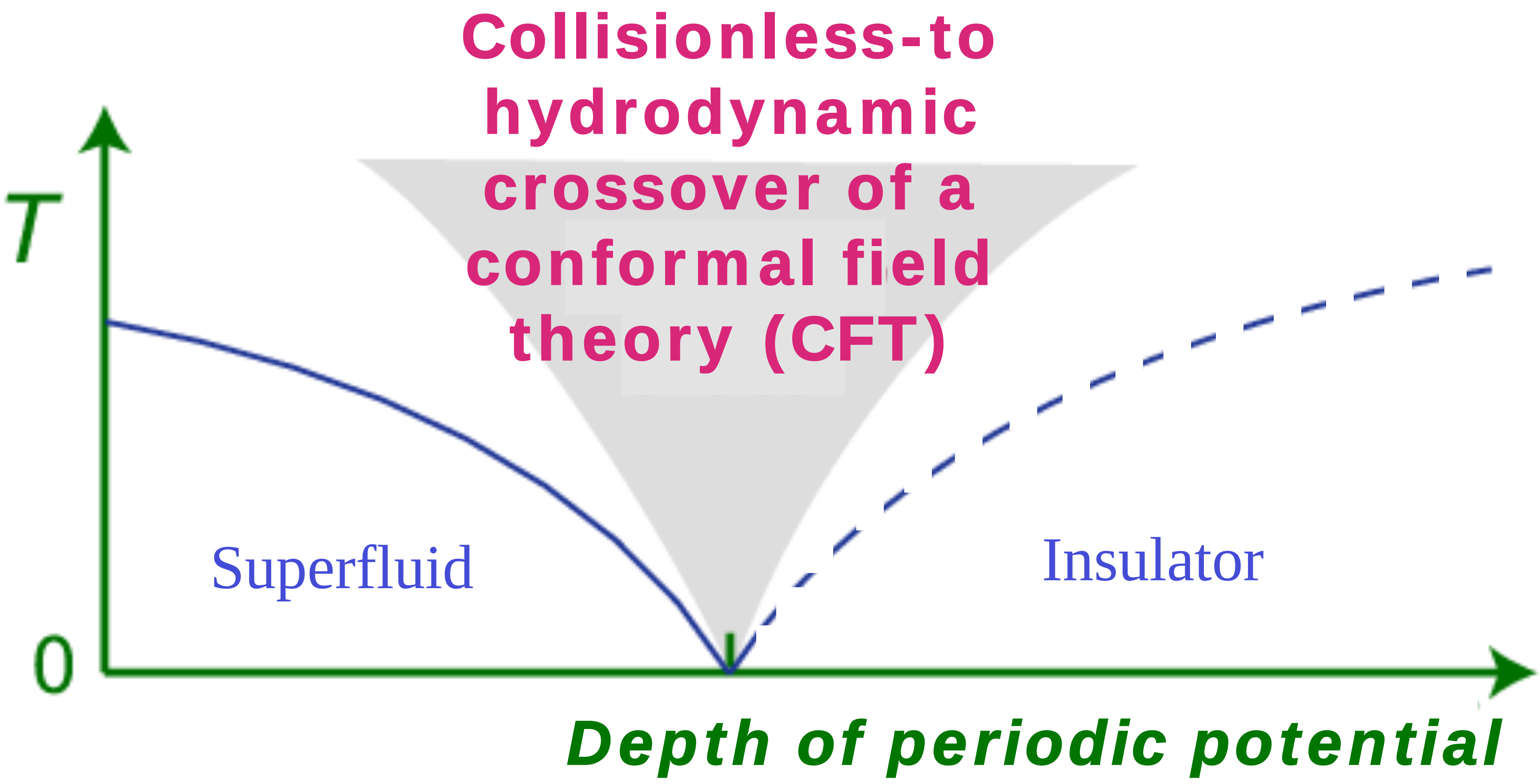


K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

## Hydrodynamics of a conformal field theory (CFT)

The scattering cross-section of the thermal excitations is universal and so transport co-efficients are universally determined by $k_B T$

Charge diffusion constant $$D_c = \Theta \frac{c^2}{k_B T}$$

Conductivity $$\sigma_Q = D_c \chi = \overline{\Theta} \frac{4e^2}{h}$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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The AdS/CFT correspondence (Maldacena, Polyakov) relates the hydrodynamics of CFTs to the quantum gravity theory of the horizon of a black hole in Anti-de Sitter space.

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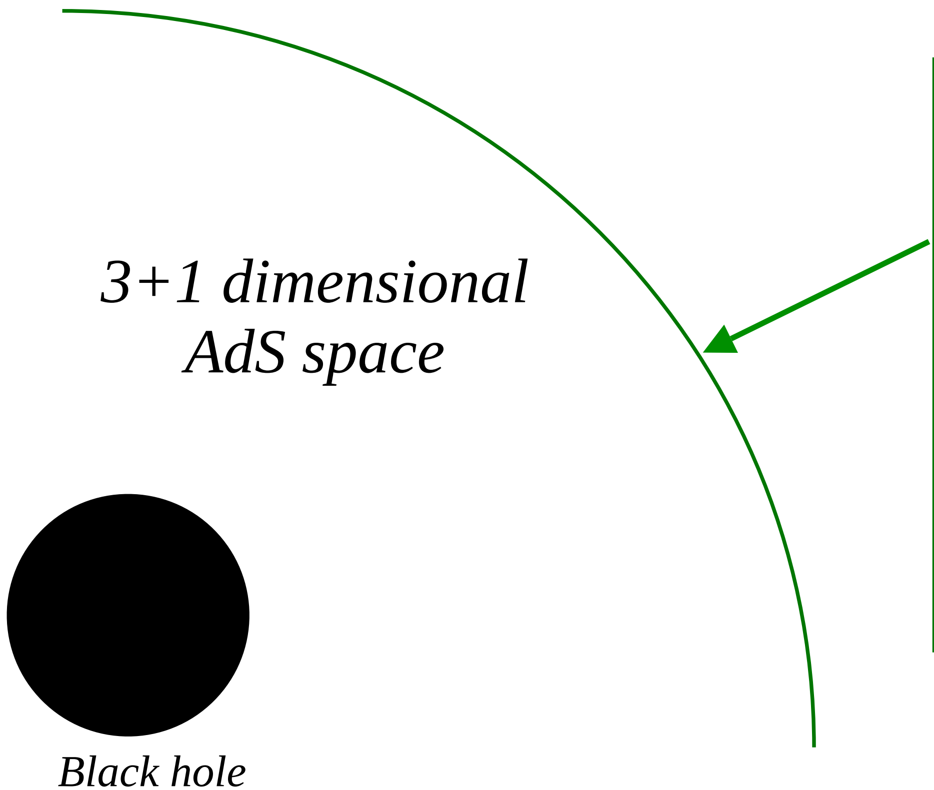
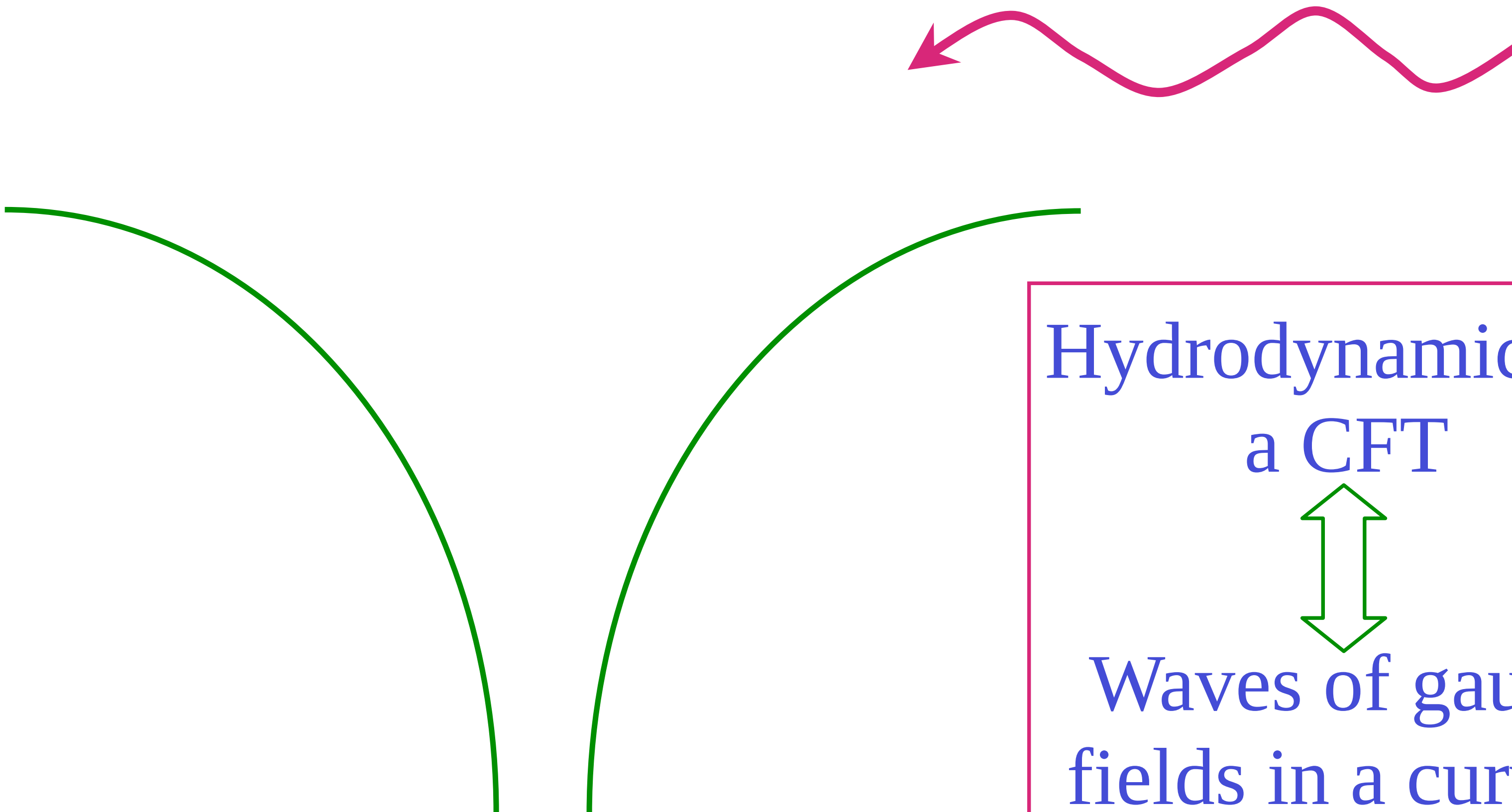



Holographic representation of black hole physics in a 2+1 dimensional CFT at a temperature equal to the Hawking temperature of the black hole.

# Hydrodynamics of a conformal field theory (CFT)



Hydrodynamics of a CFT

$\Updownarrow$

Waves of gauge fields in a curved background

## Hydrodynamics of a conformal field theory (CFT)

For the (unique) CFT with a SU($N$) gauge field and 16 supercharges, we know the exact diffusion constant associated with a global SO(8) symmetry:

Spin diffusion constant $$D_c = \frac{3}{4\pi}\frac{c^2}{k_B T}$$

Spin conductivity $$\sigma_Q = \frac{N^{3/2}}{3\sqrt{2}\pi}$$

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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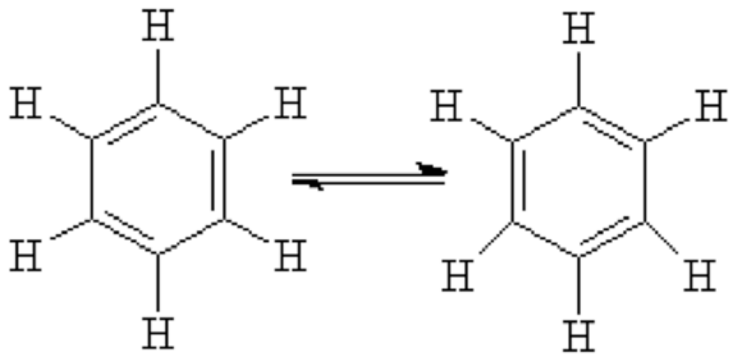
# Valence bonds in benzene




Resonance in benzene leads to a symmetric configuration of valence bonds (*F. Kekulé, L. Pauling*)

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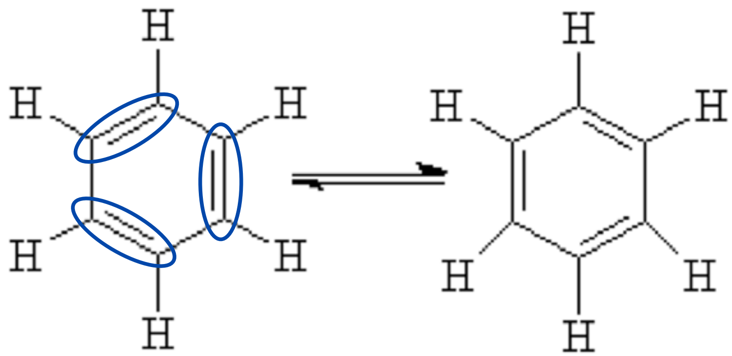


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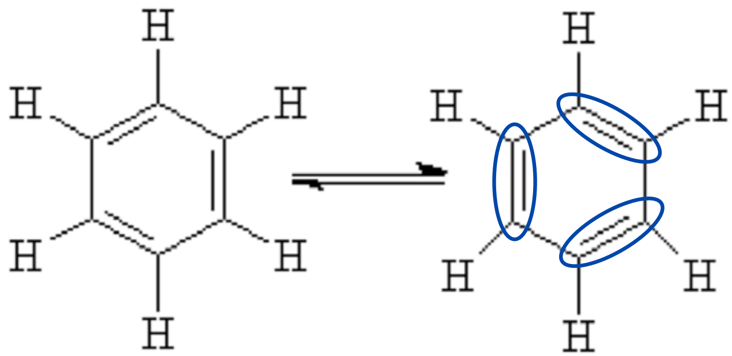


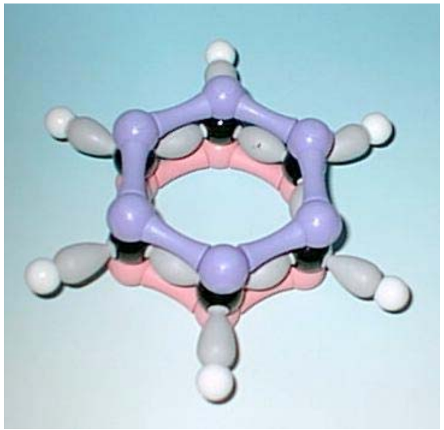

Resonance in benzene leads to a symmetric configuration of valence bonds
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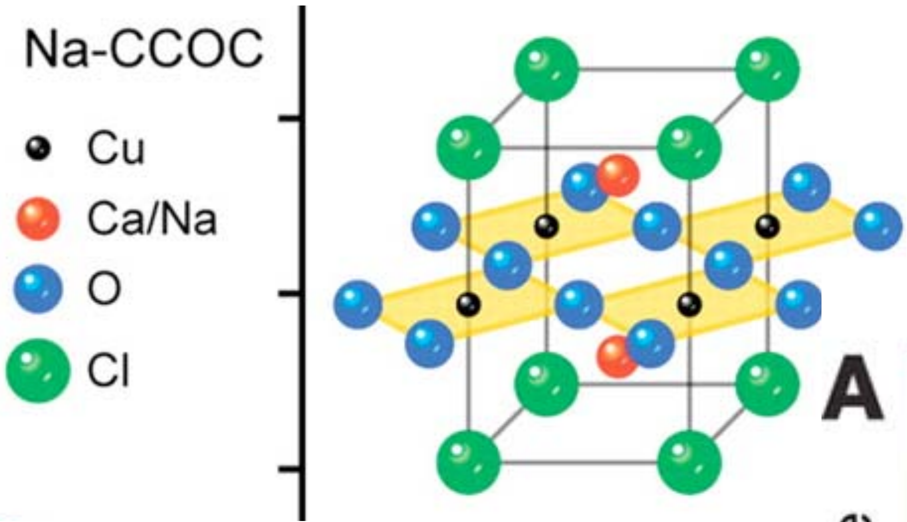
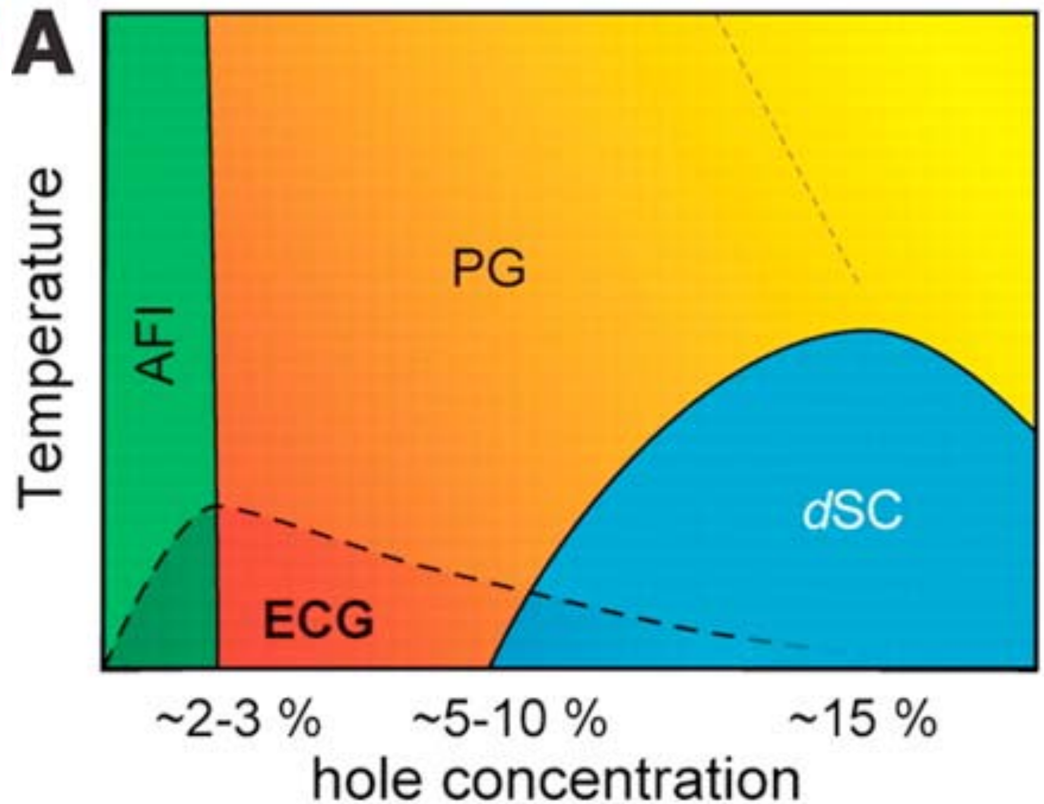
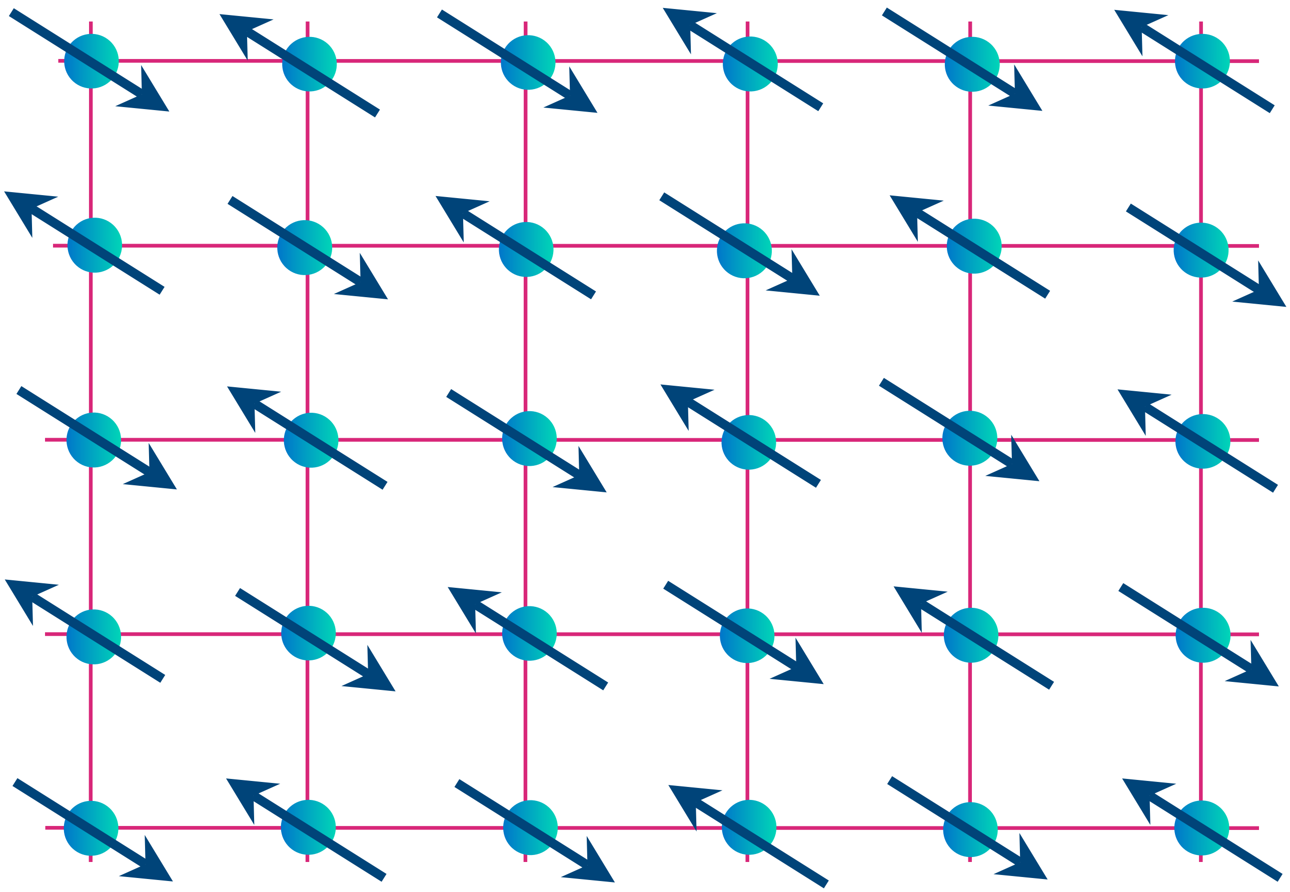
# Temperature-doping phase diagram of the cuprate superconductors



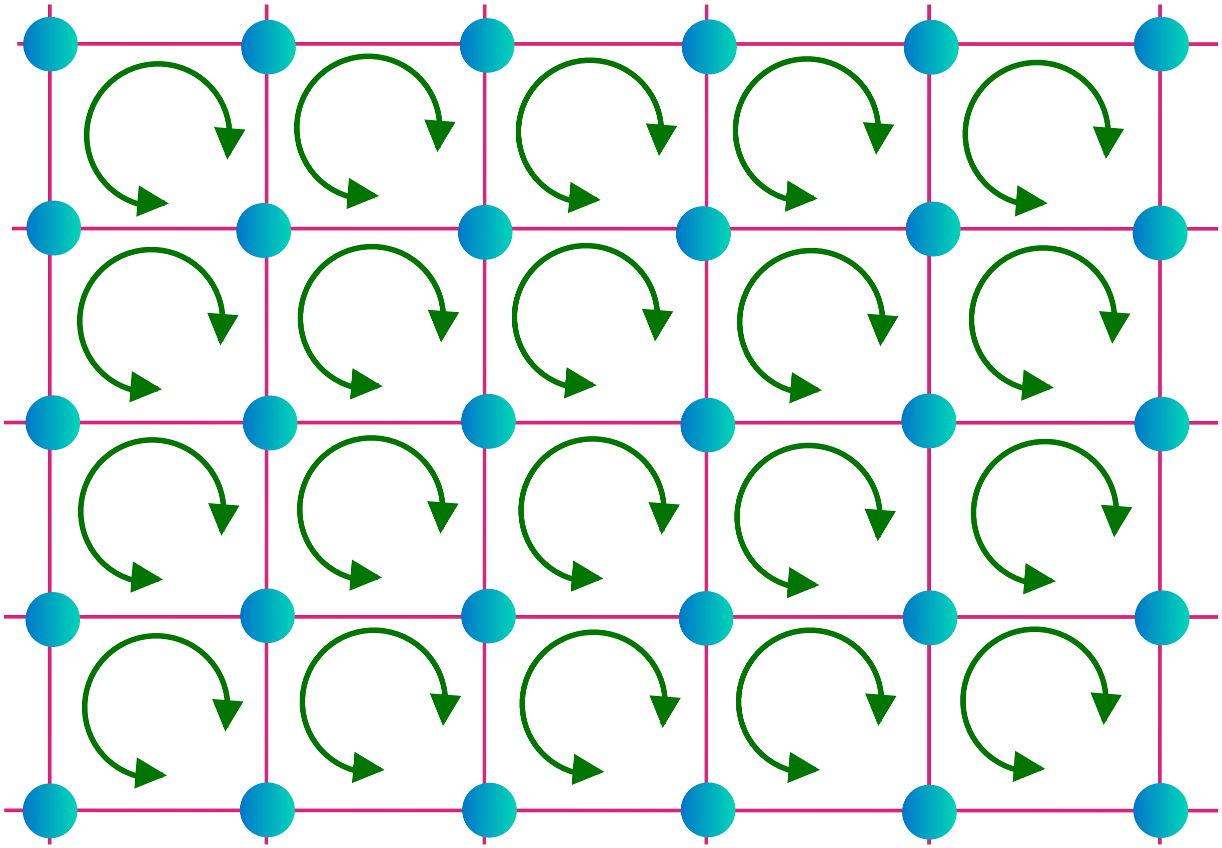



# Antiferromagnetic (Neel) order in the insulator



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \quad ; \quad \vec{S}_i \;\Rightarrow\; \text{spin operator with } S = 1/2$$

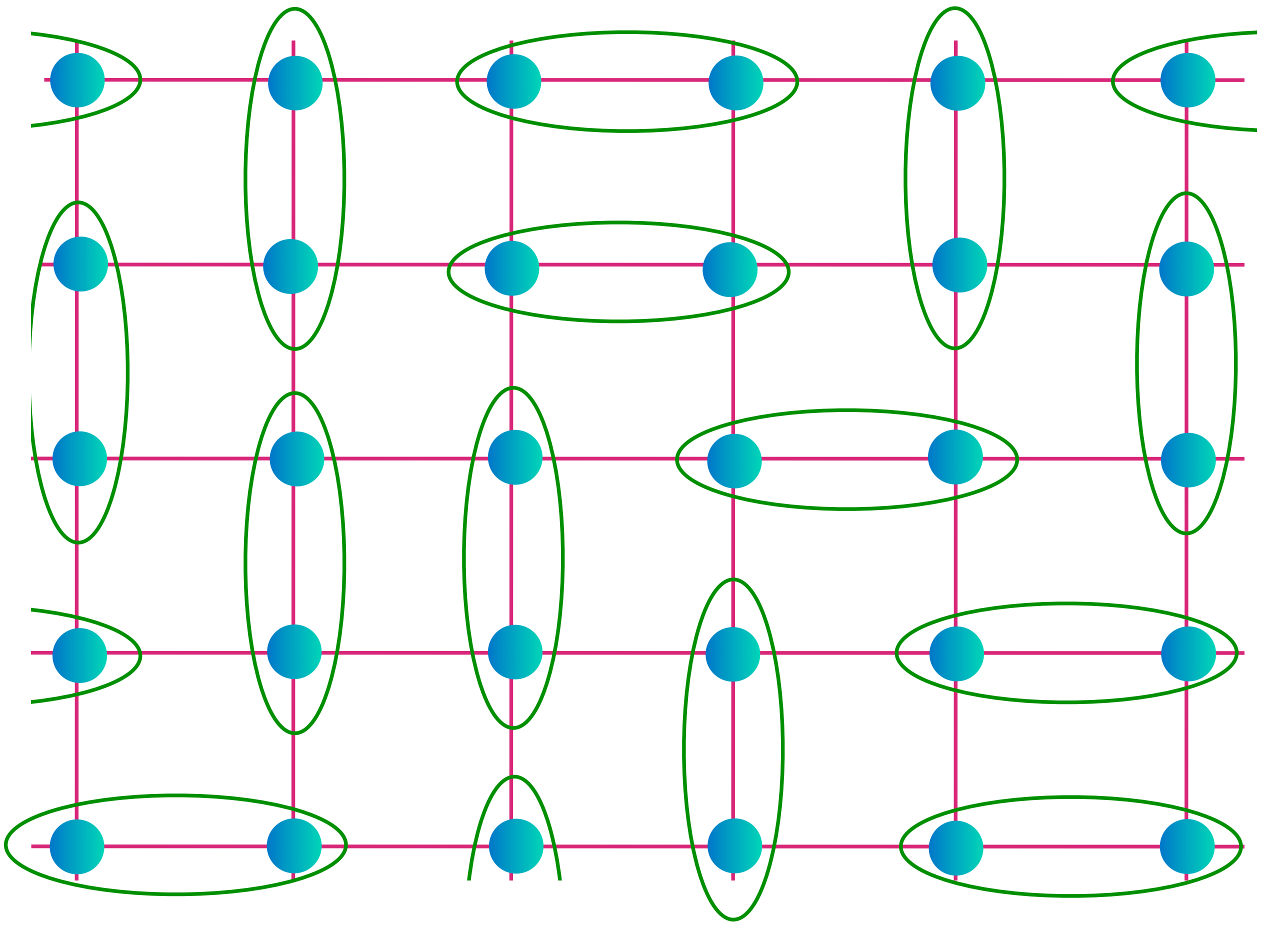
Induce formation of valence bonds by *e.g.* ring-exchange interactions

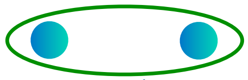


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{four spin exchange}$$

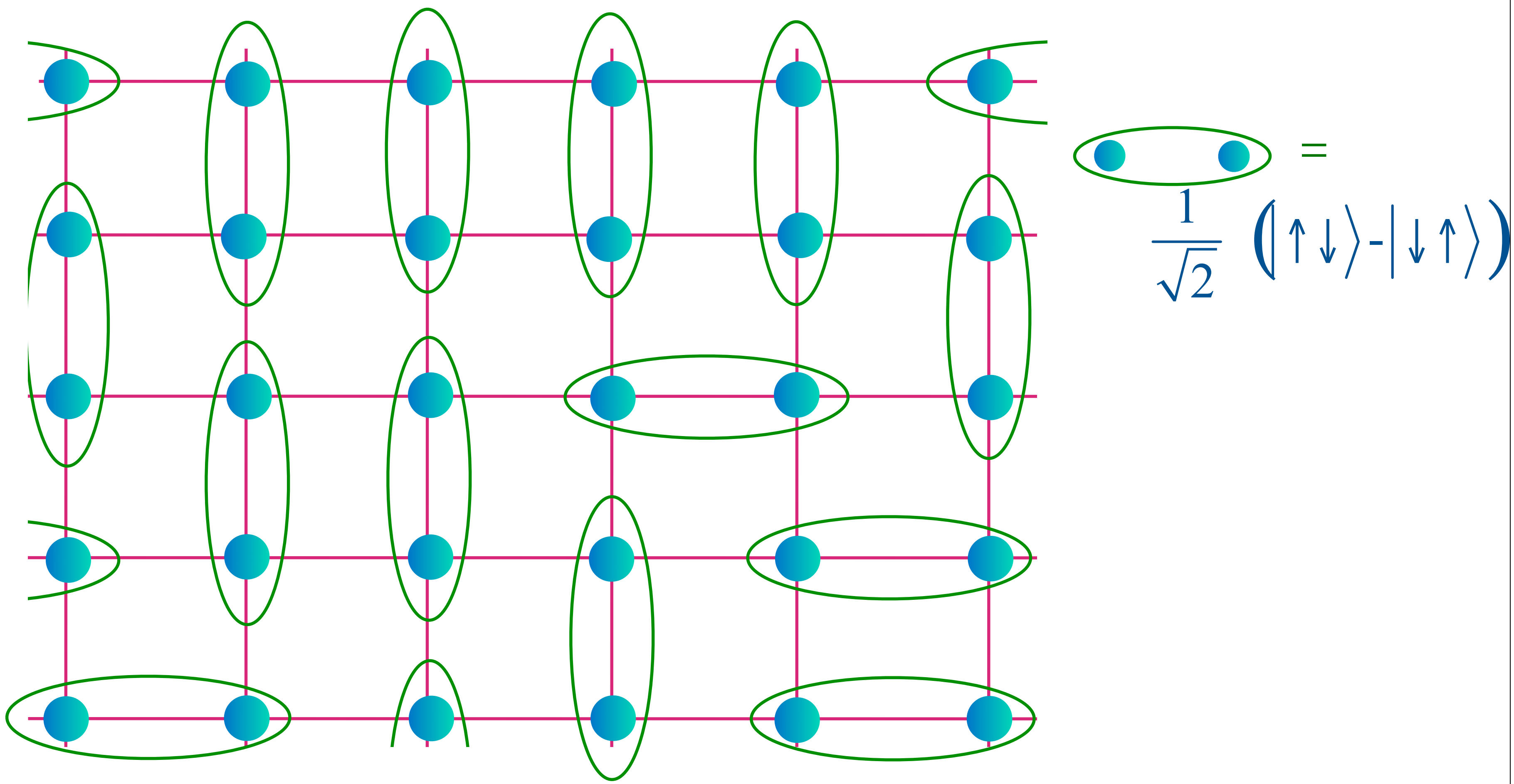
A. W. Sandvik, cond-mat/0611343

As in $H_2$ and benzene, each electron wants to pair up with another electron and form a valence bond





$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$



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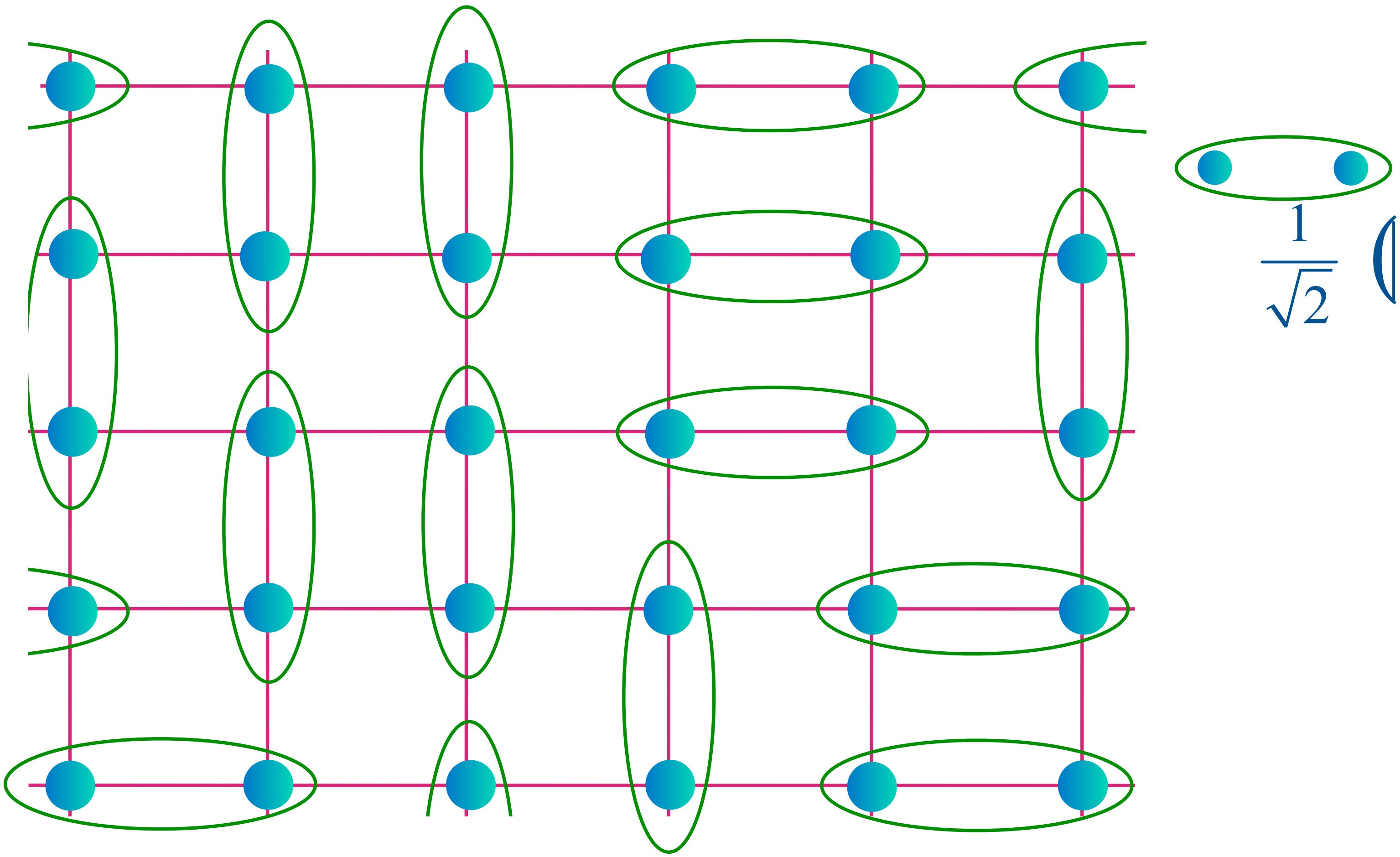

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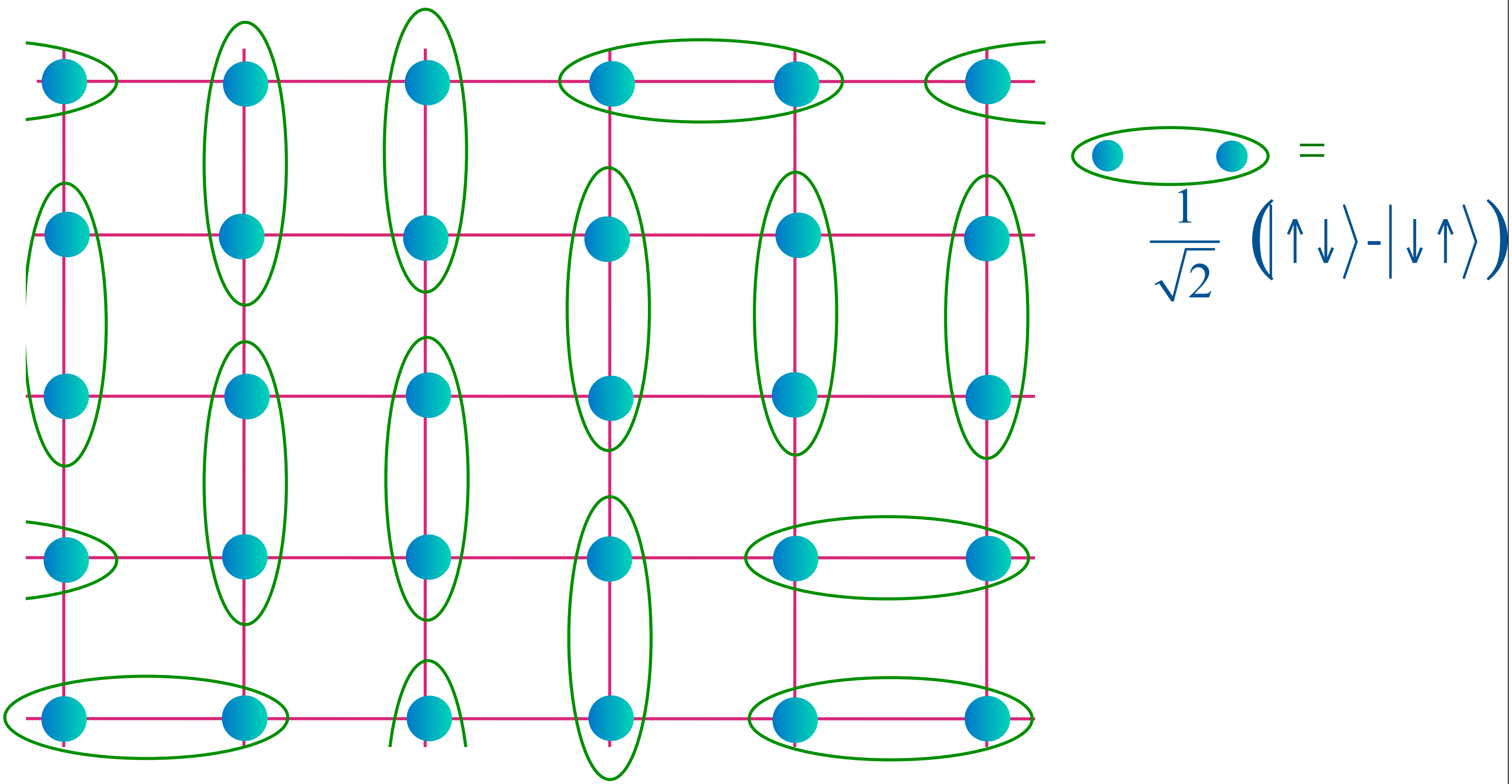
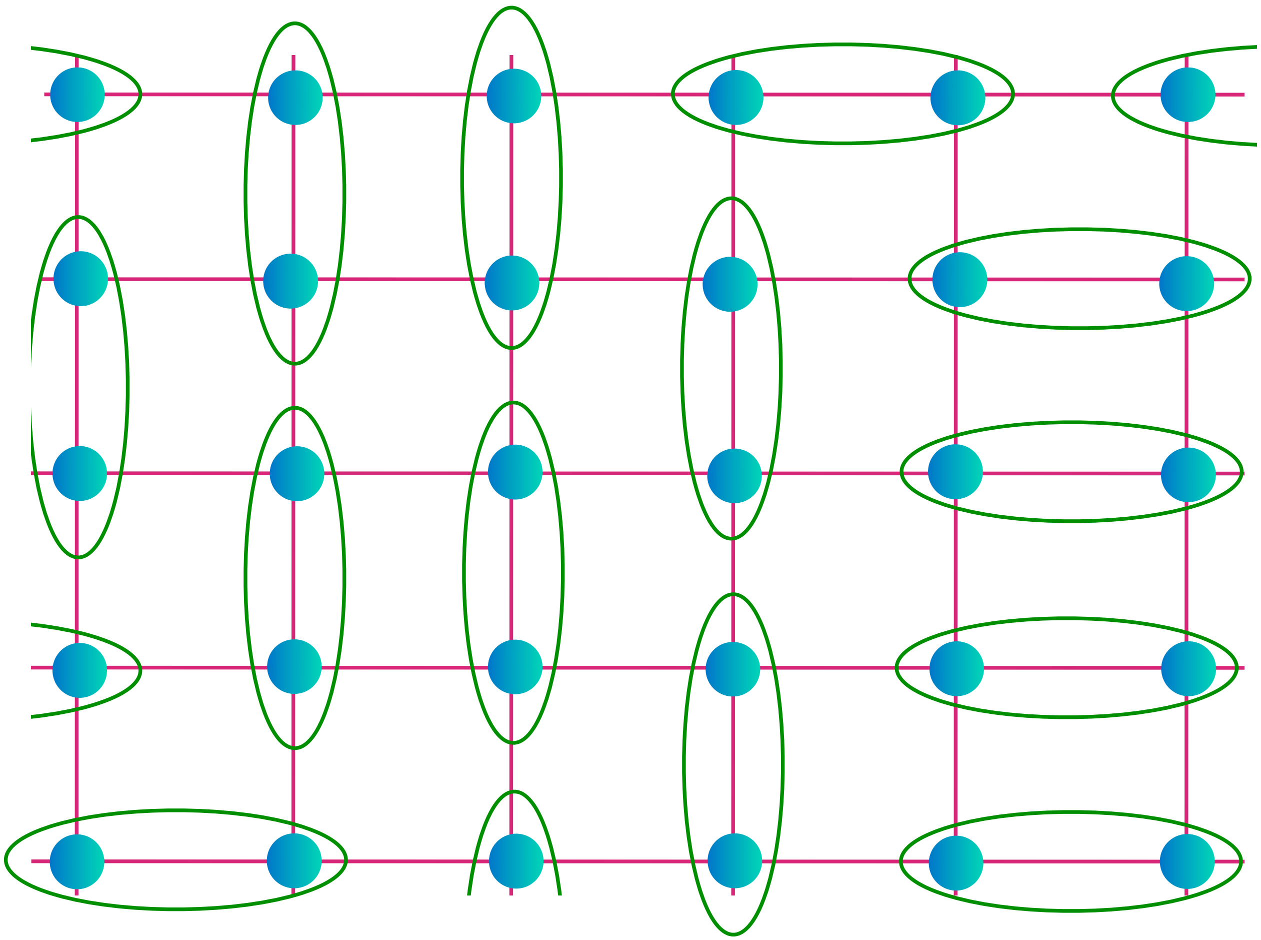
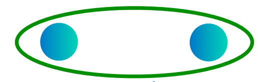


$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$





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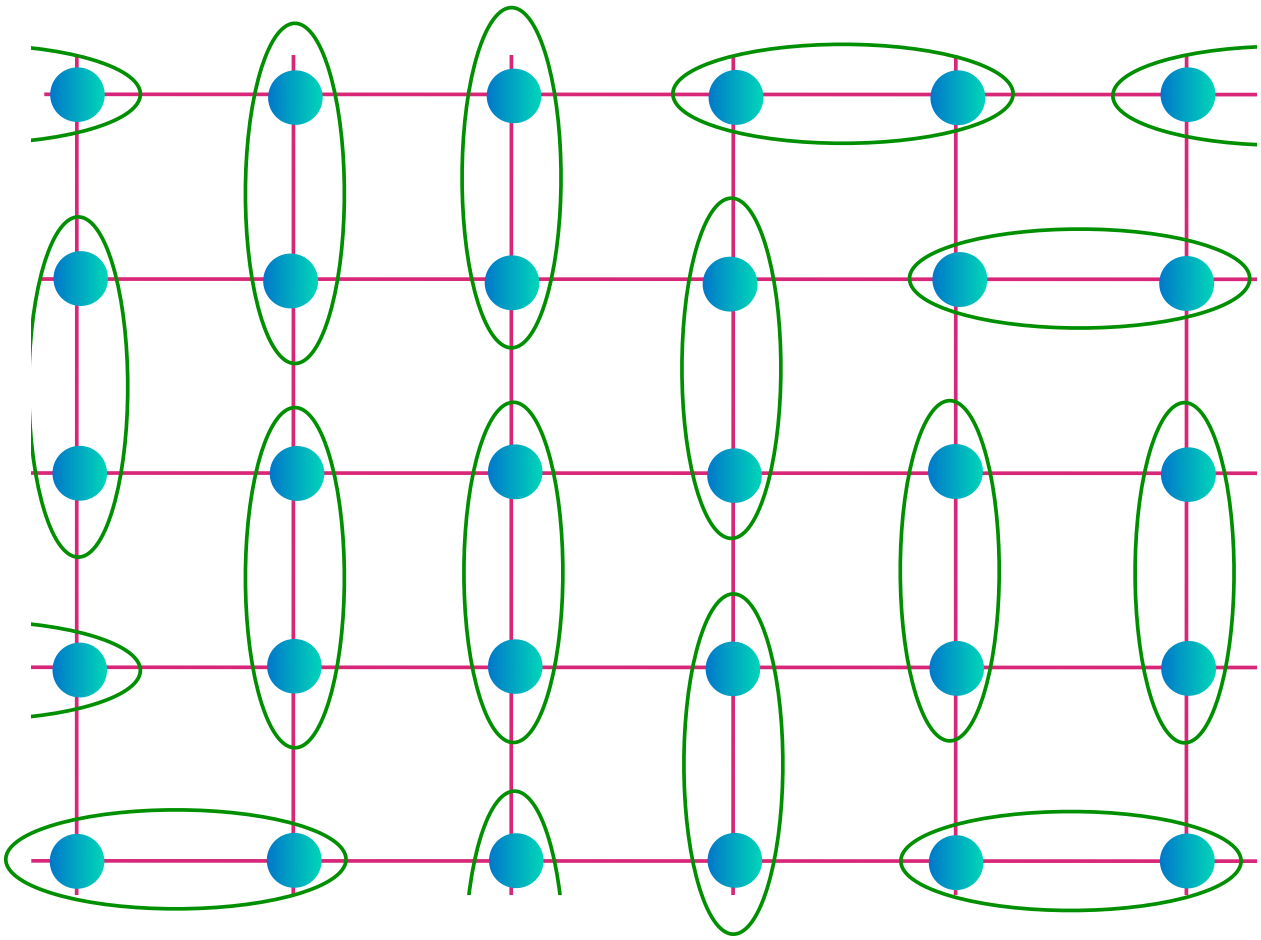
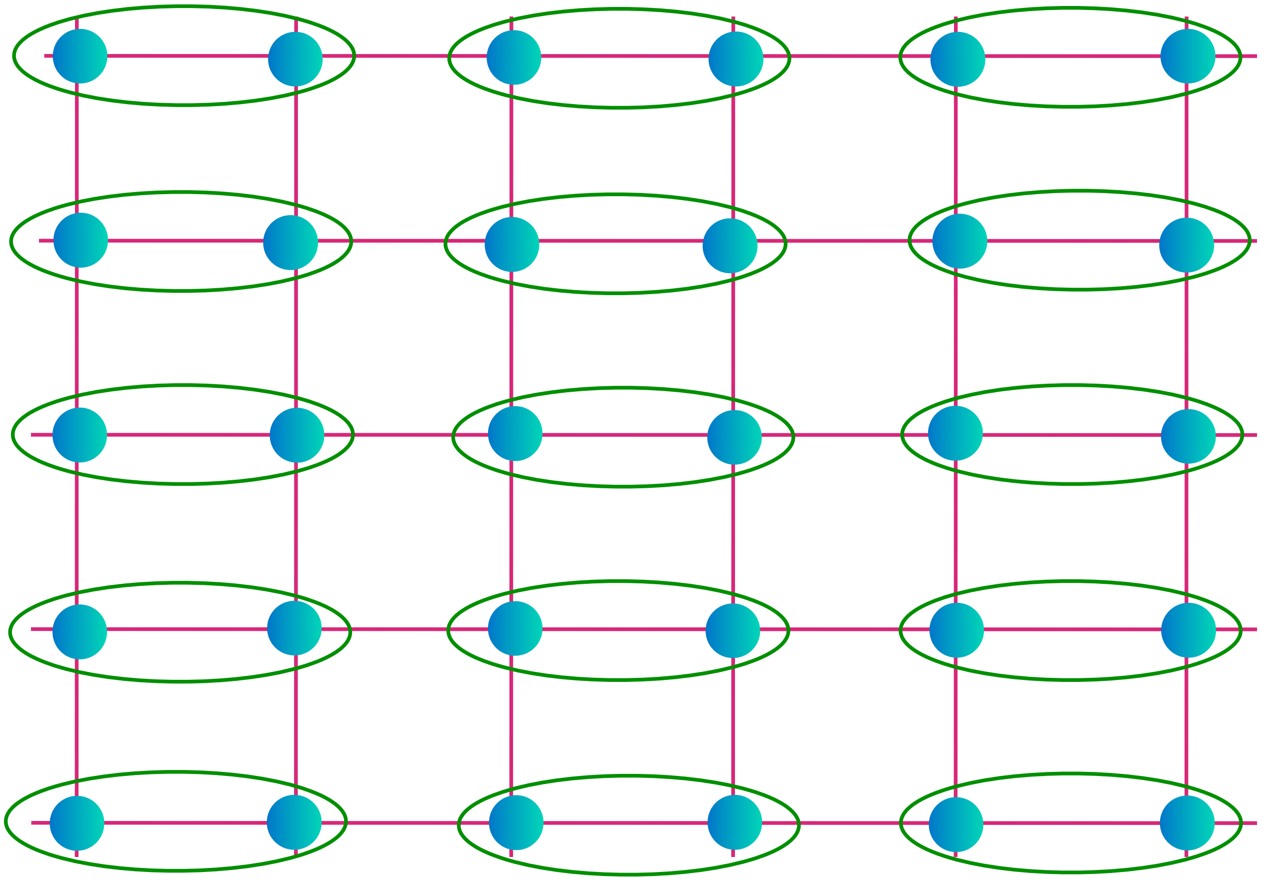
# Entangled liquid of valence bonds (Resonating valence bonds – RVB)

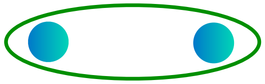


$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

# Valence bond solid (VBS)





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N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
R. Moessner and S. L. Sondhi, *Phys. Rev*. B **63**, 224401 (2001).

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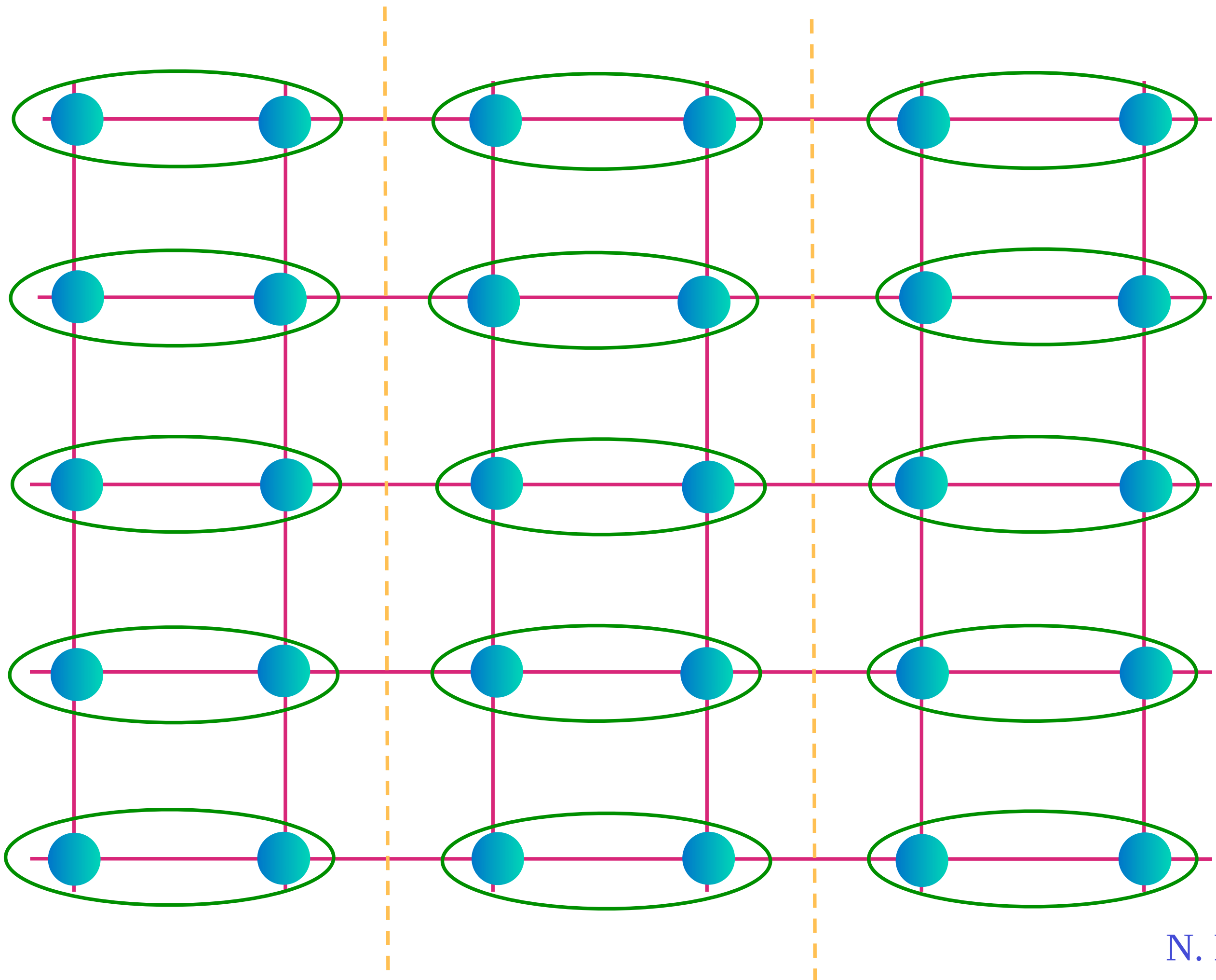

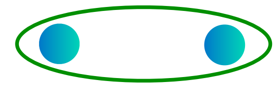 =

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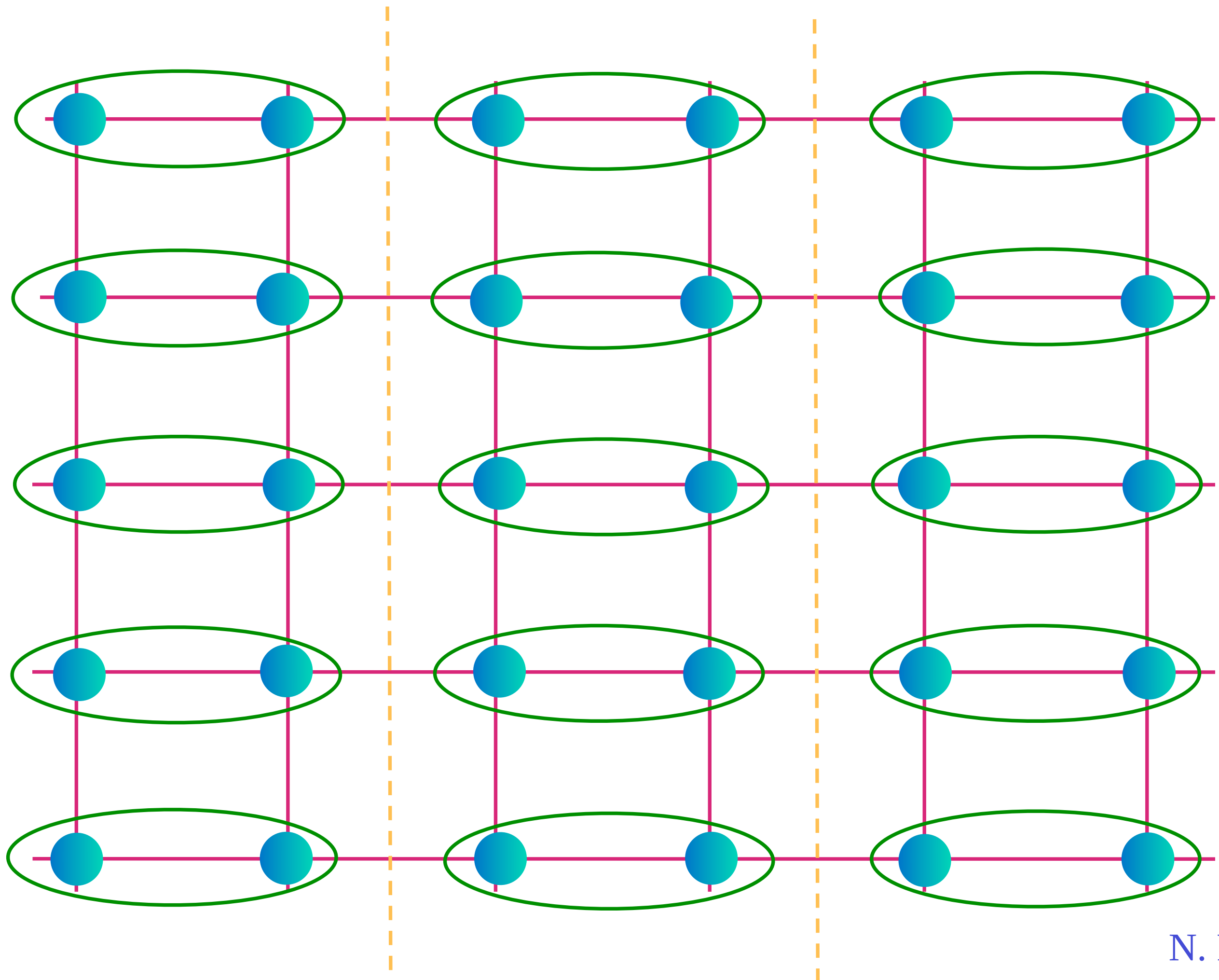
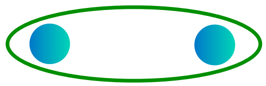
## More possibilities for entanglement with nearby states



 =

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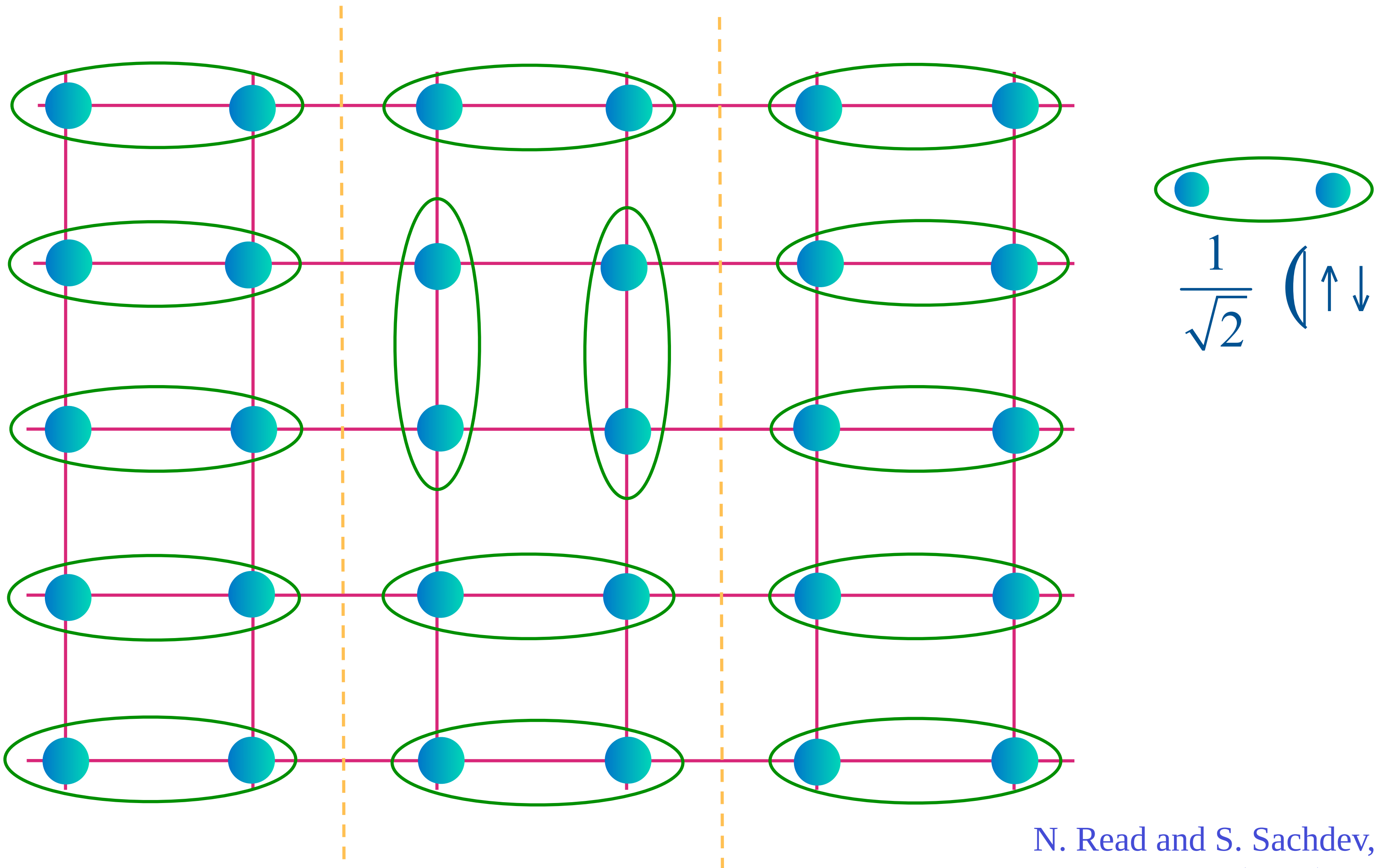

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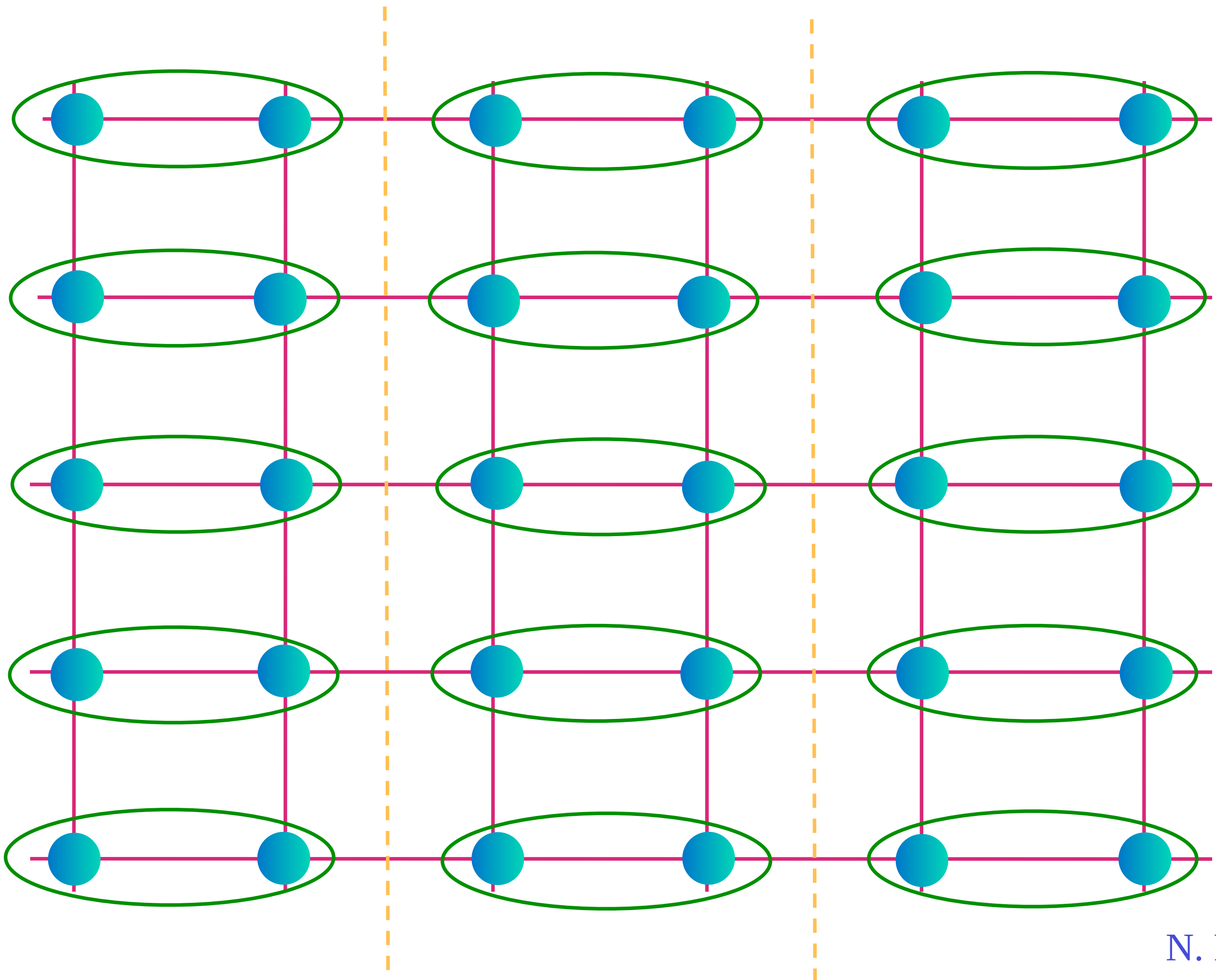

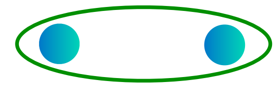 =

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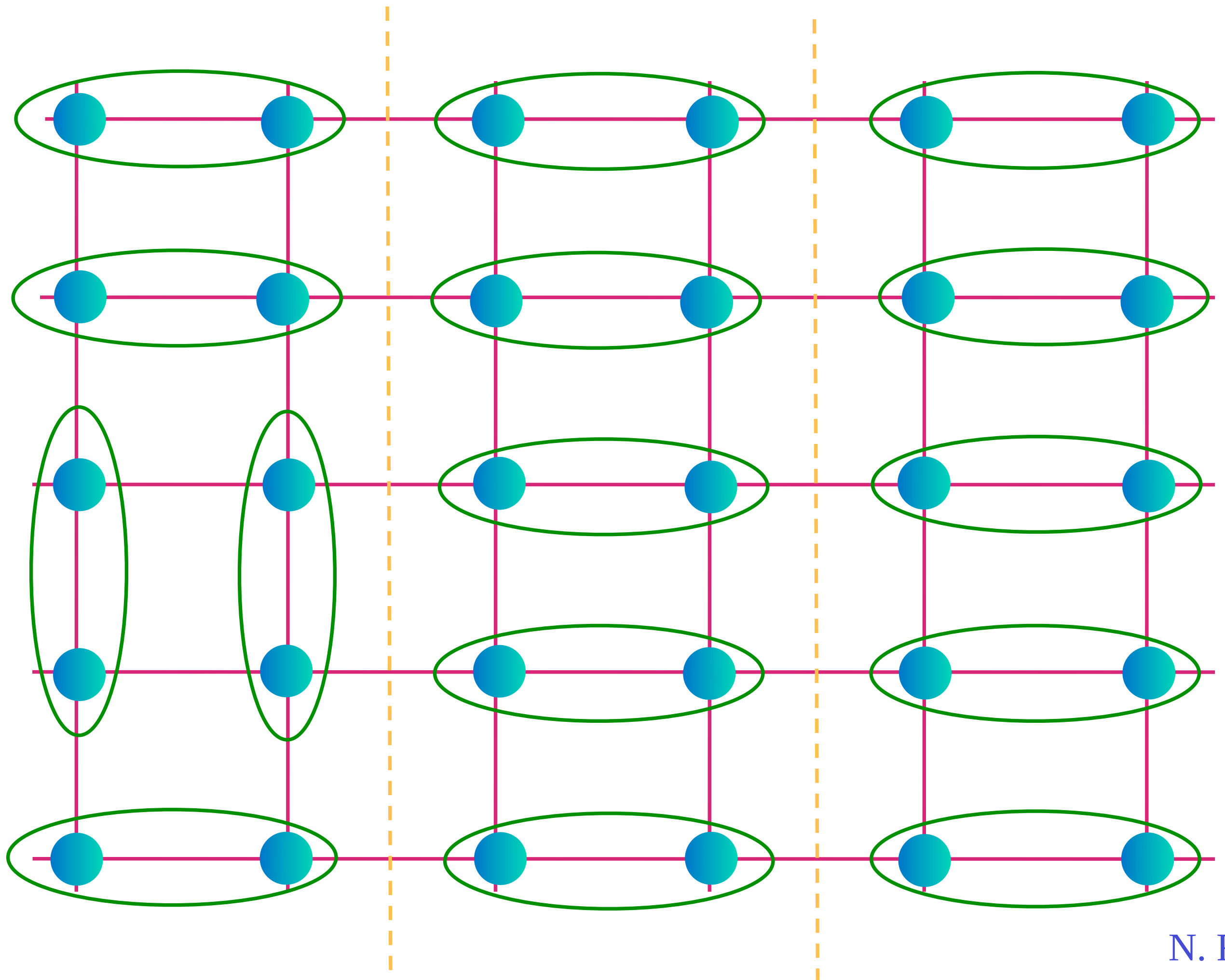

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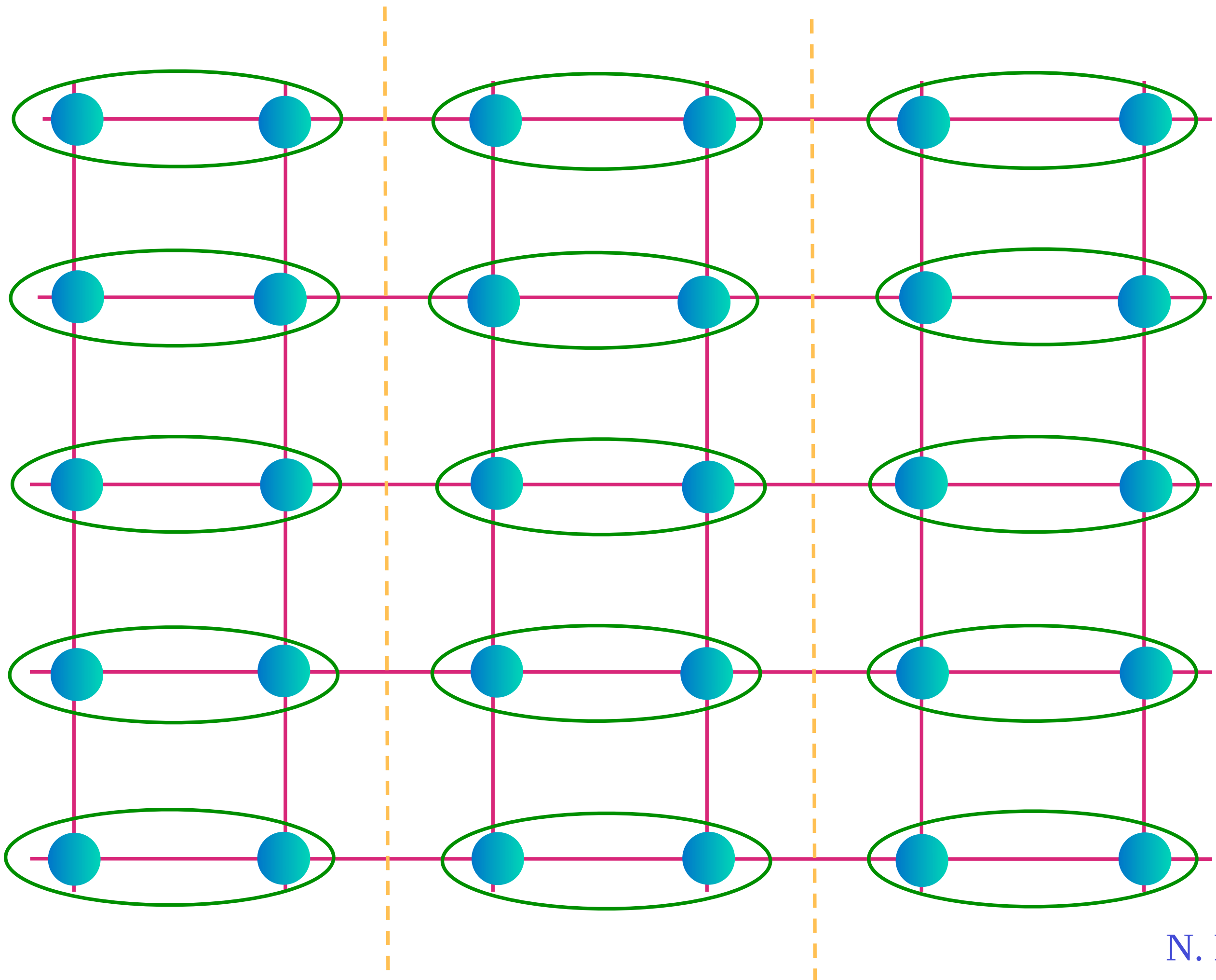
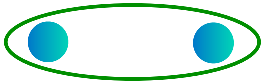




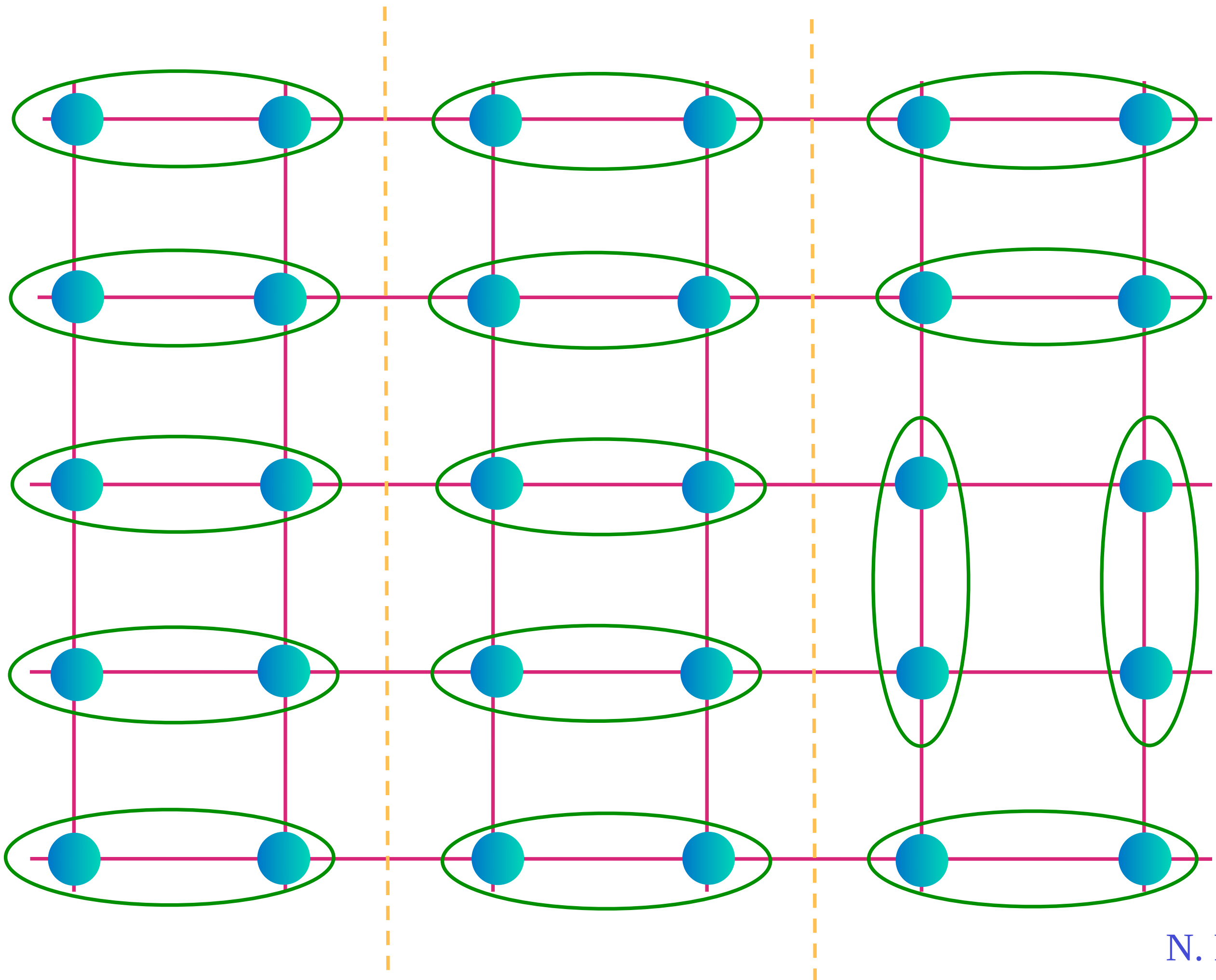
$=$

$$\frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big)$$


N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
R. Moessner and S. L. Sondhi, *Phys. Rev*. B **63**, 224401 (2001).

# Valence bond solid (VBS)

## More possibilities for entanglement with nearby states



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
R. Moessner and S. L. Sondhi, *Phys. Rev*. B **63**, 224401 (2001).

# Valence bond solid (VBS)

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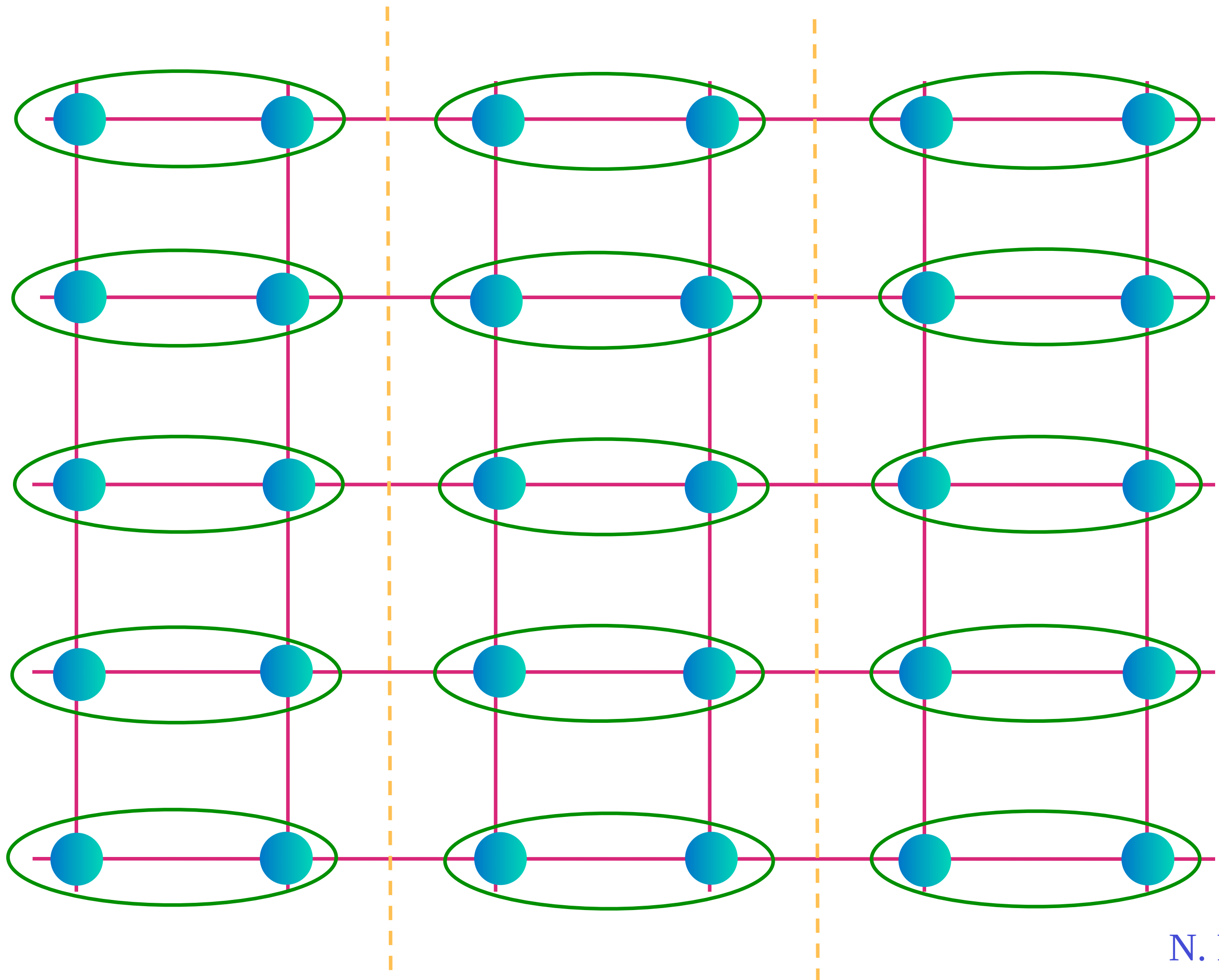
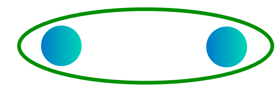
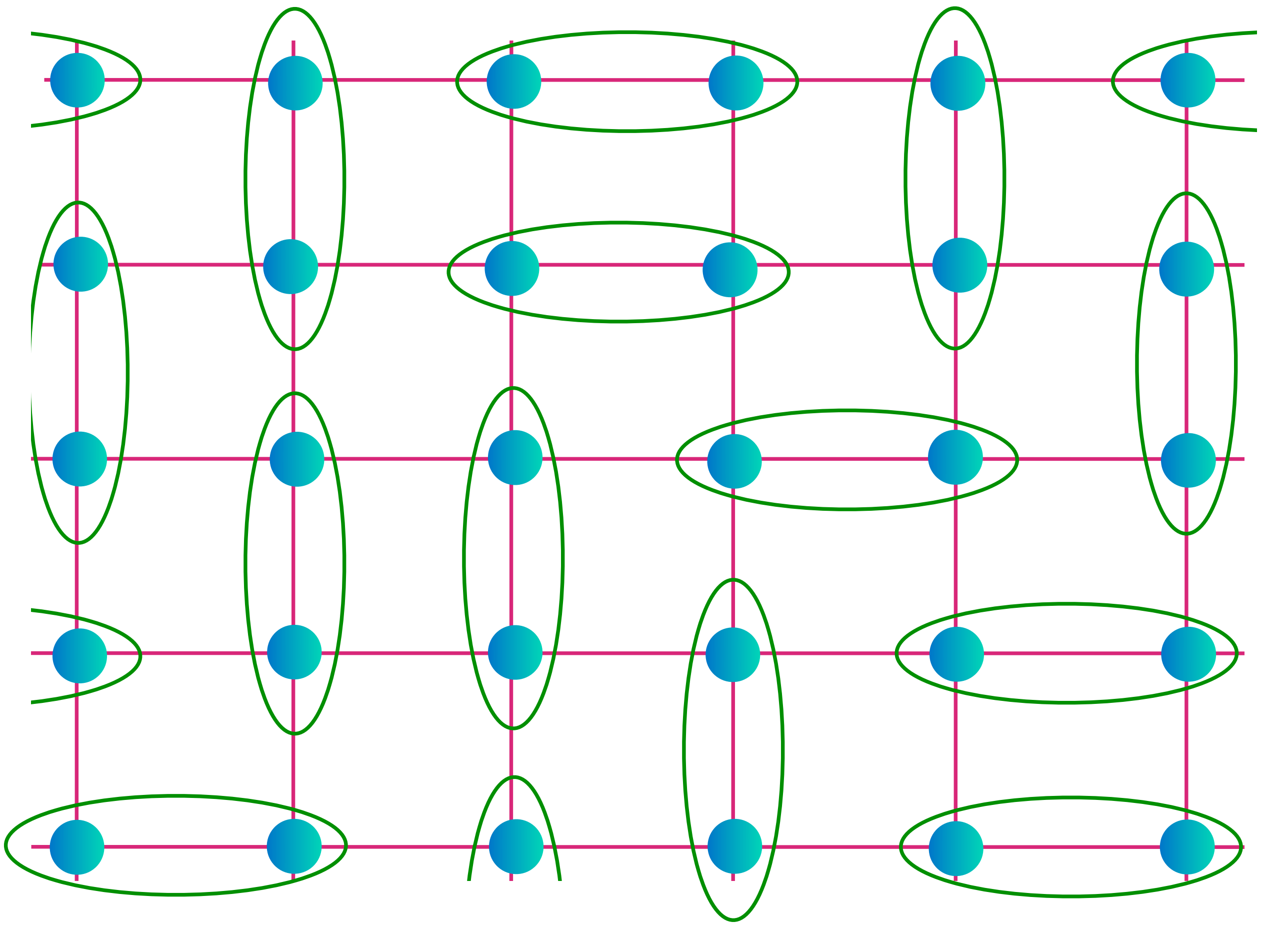


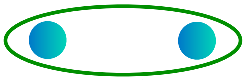

=

$$\frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
R. Moessner and S. L. Sondhi, *Phys. Rev.* B **63**, 224401 (2001).

# Excitations of the RVB liquid





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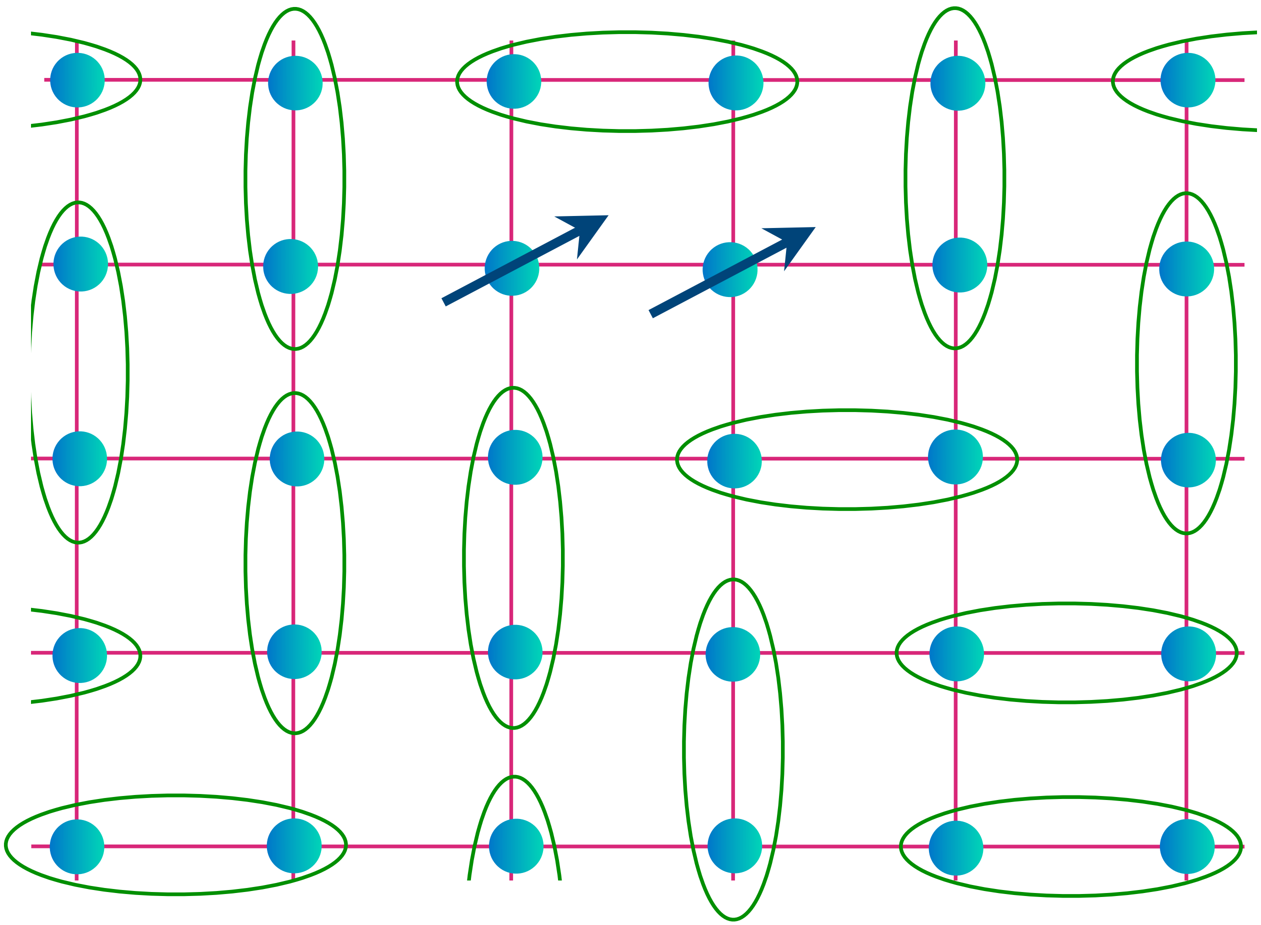

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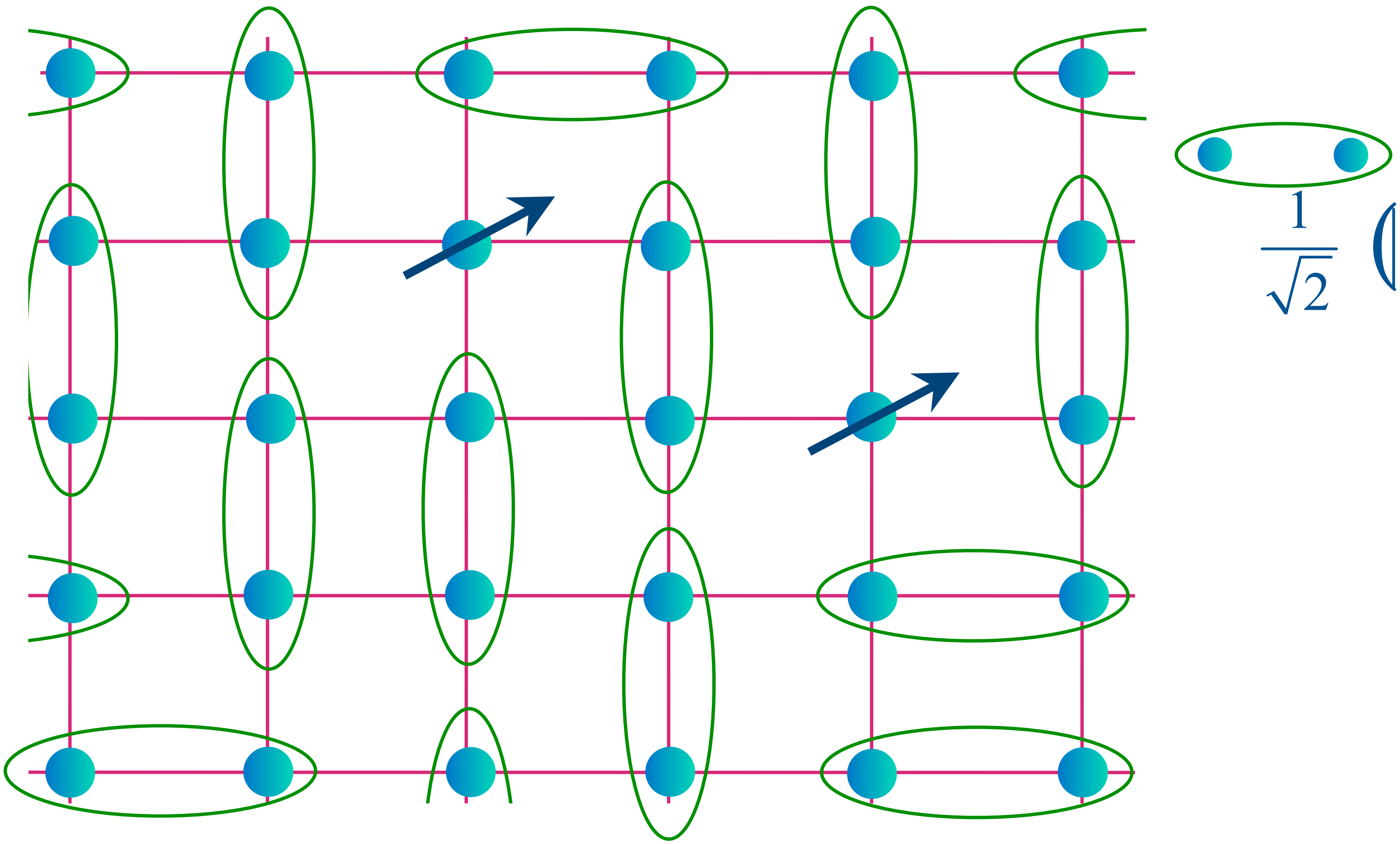
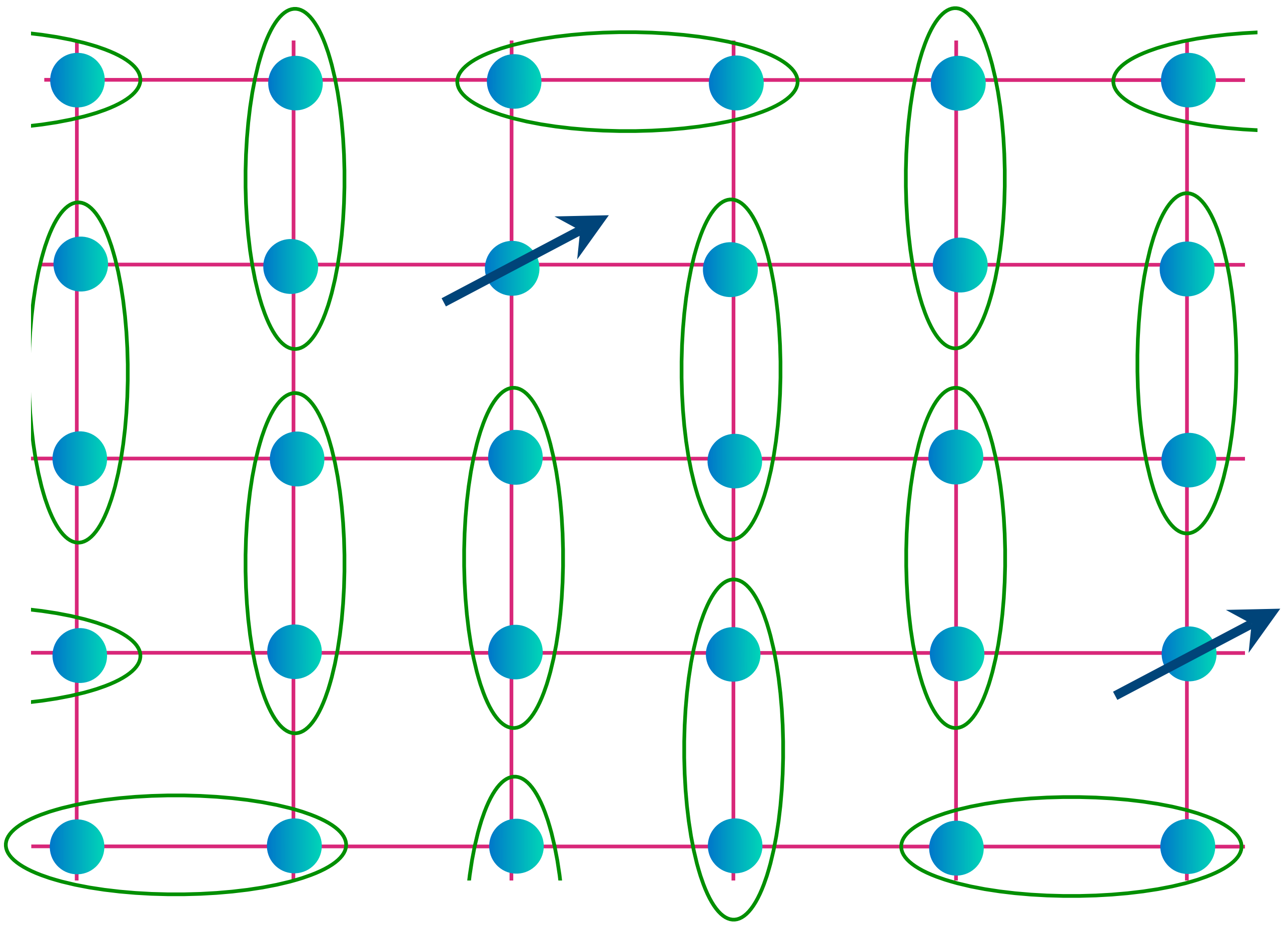
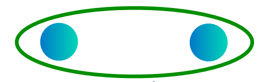
# Excitations of the RVB liquid



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

# Excitations of the RVB liquid





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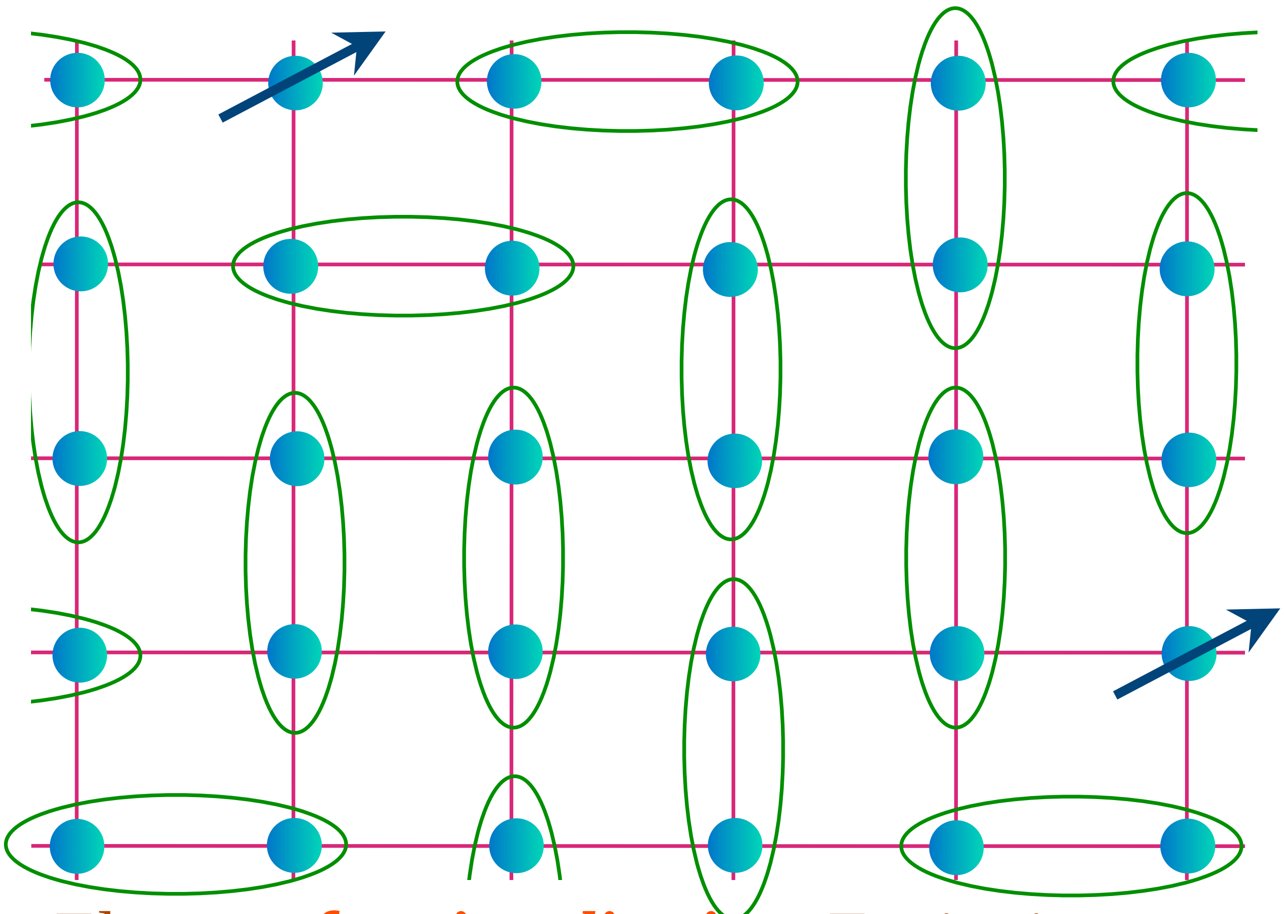
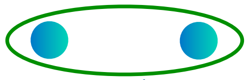
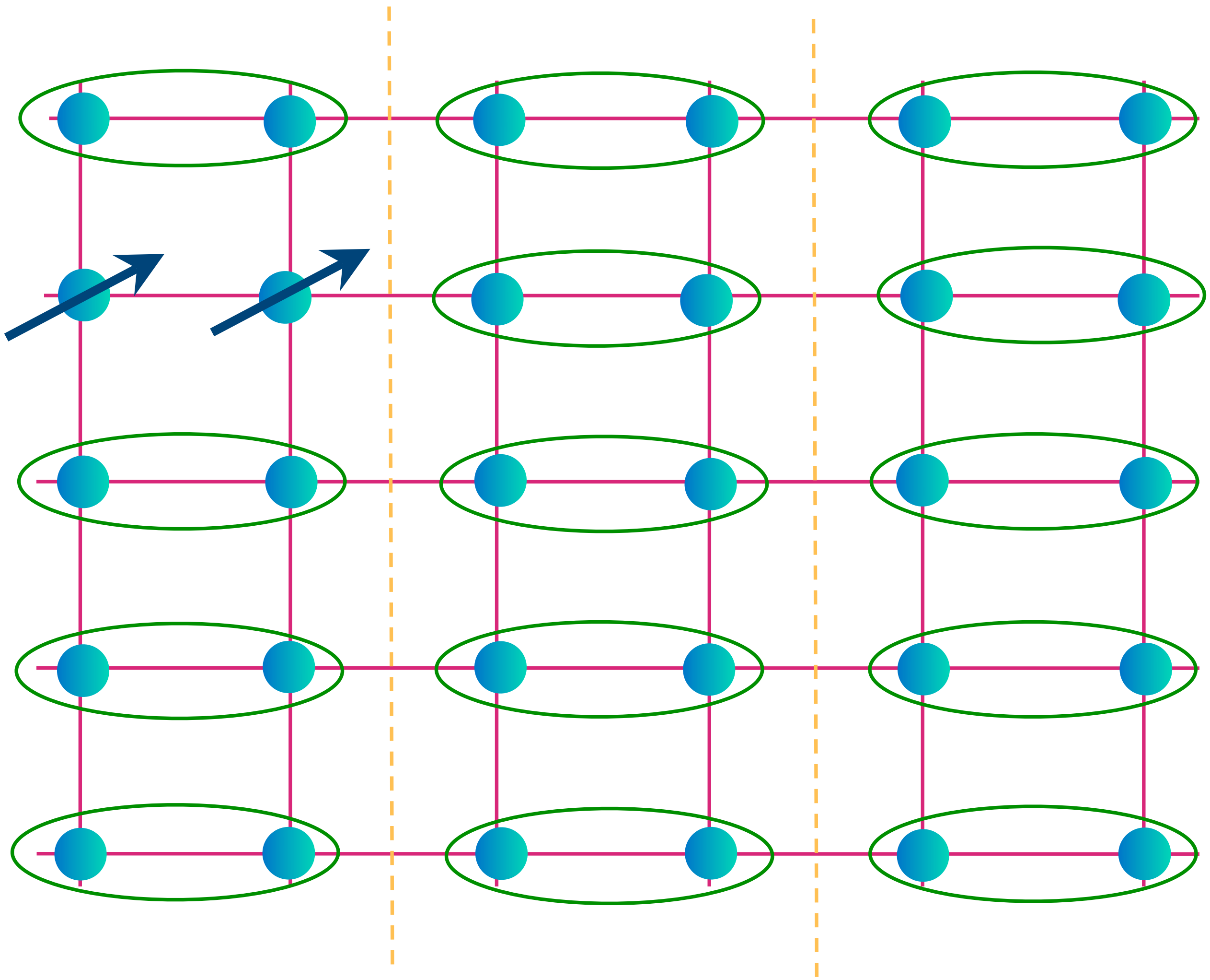


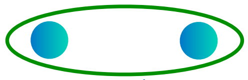


$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Electron ***fractionalization***: Excitations carry spin $S$=1/2 but no charge

# Excitations of the VBS





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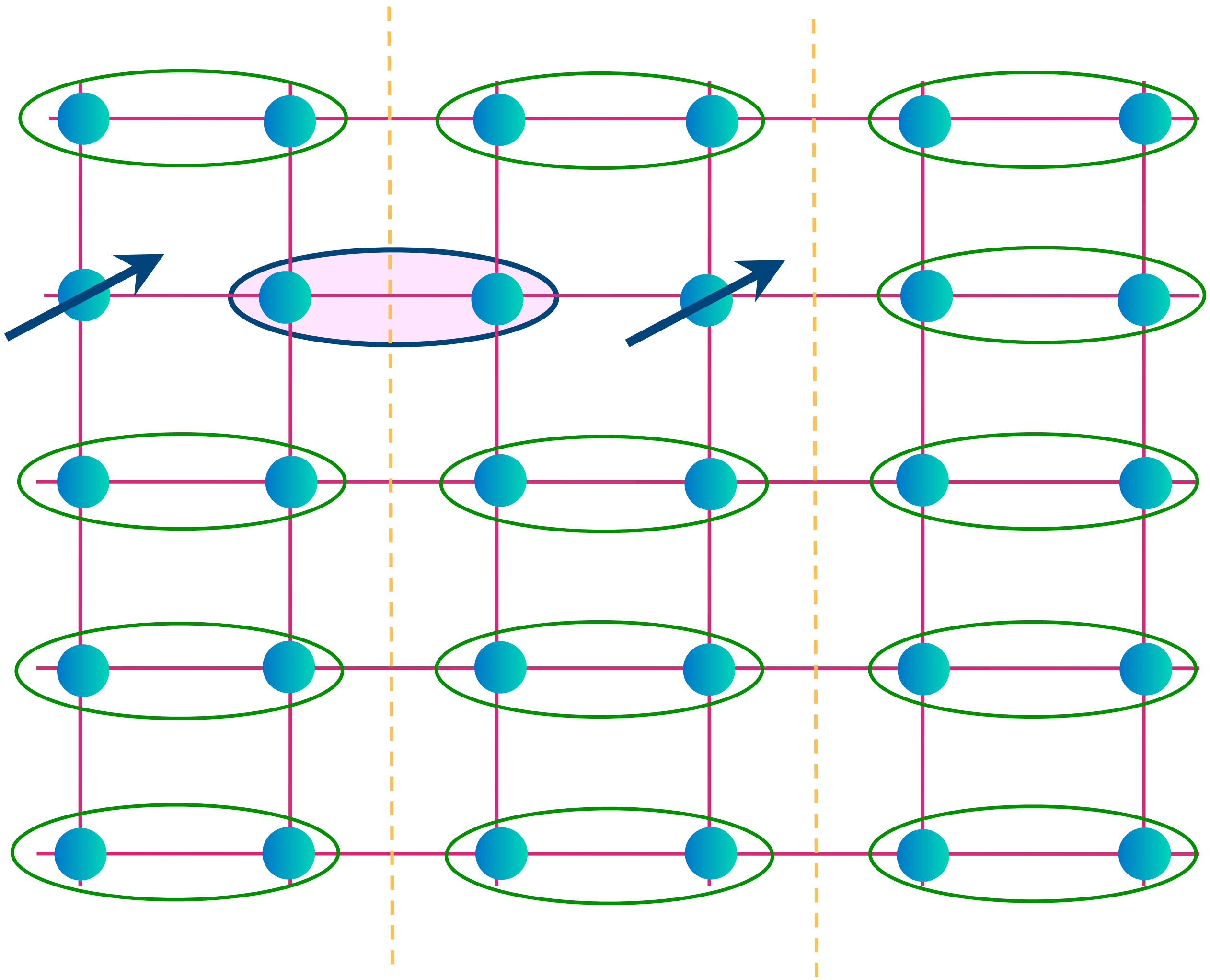

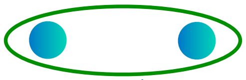

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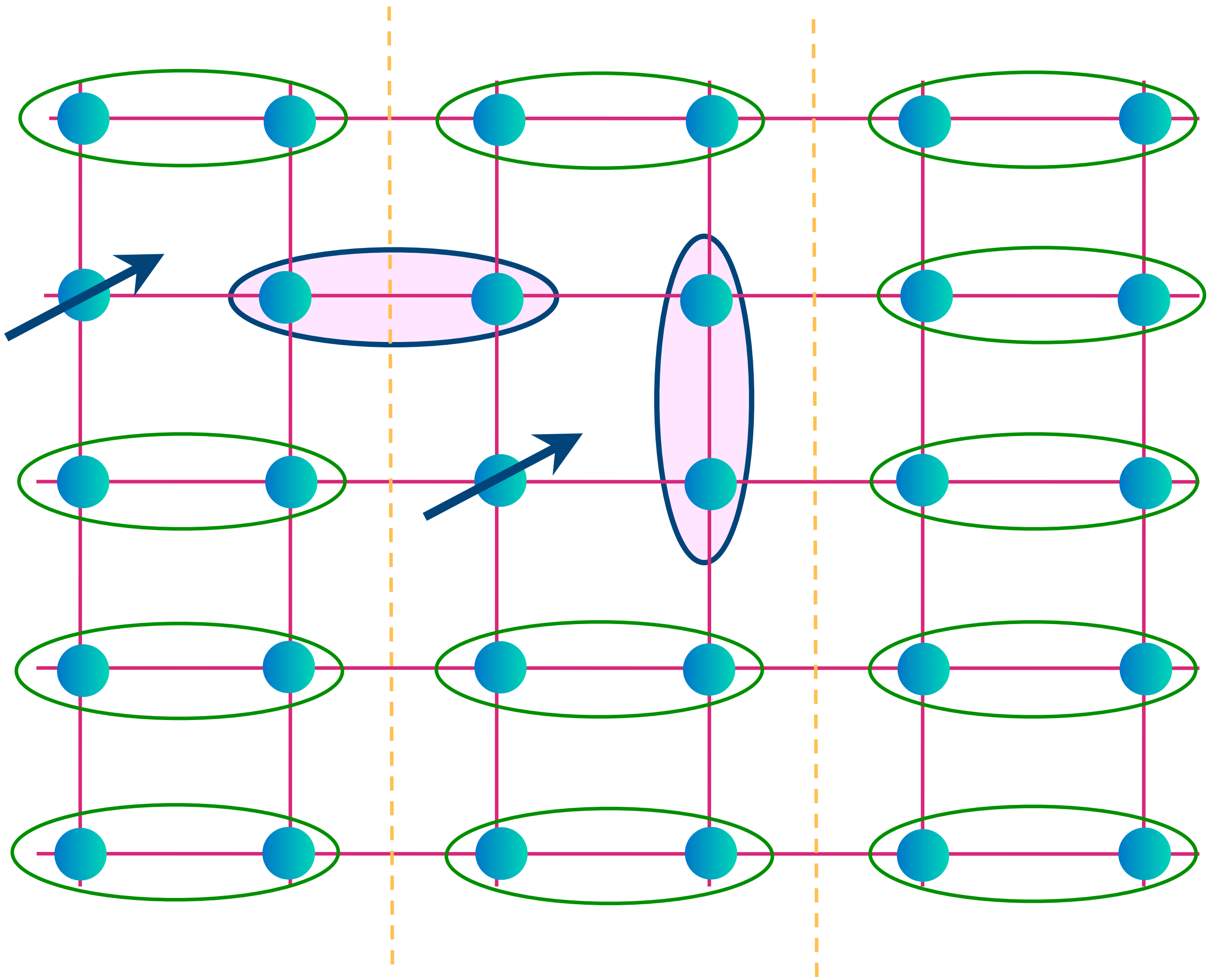
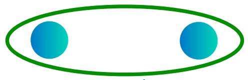
# Excitations of the VBS





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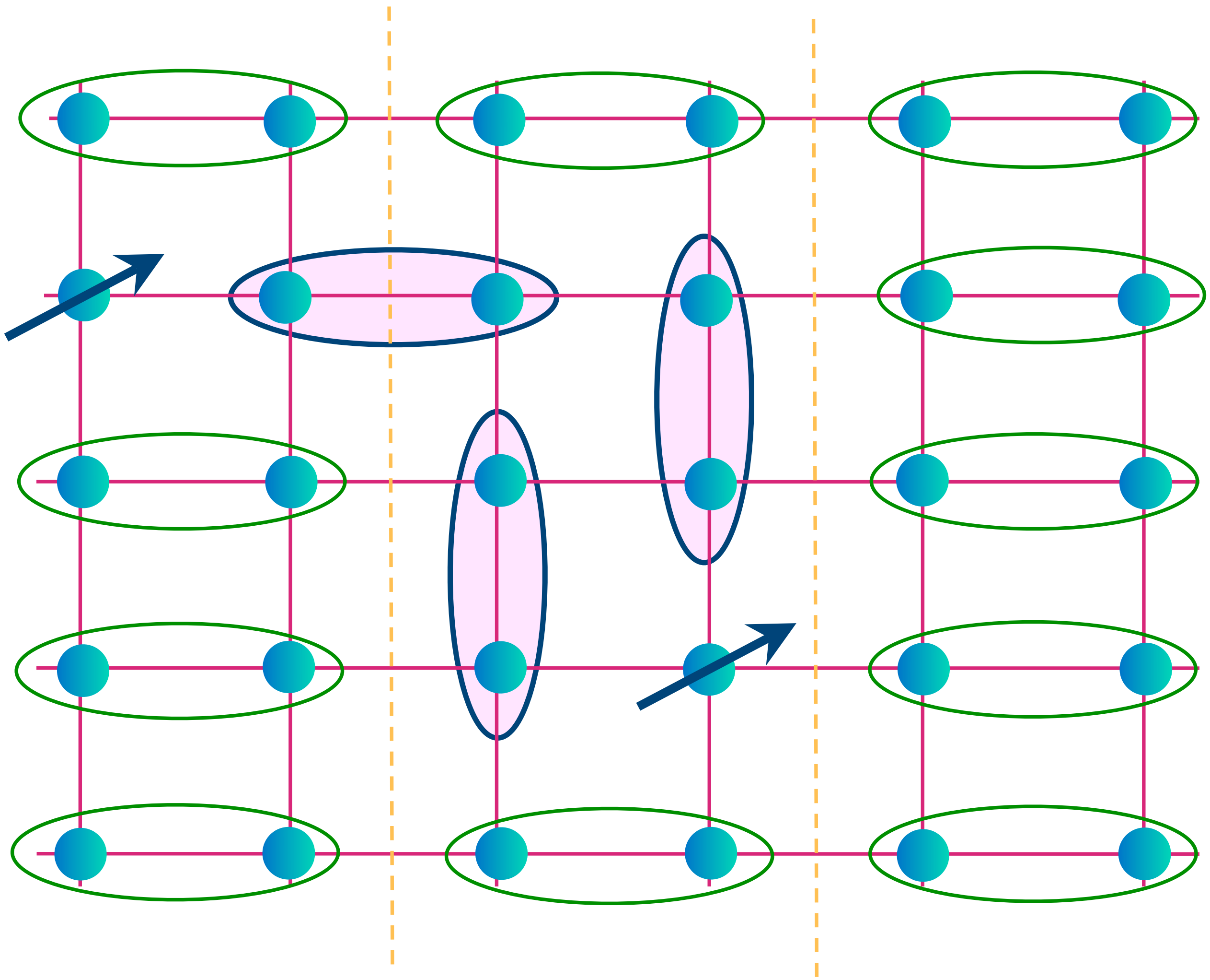

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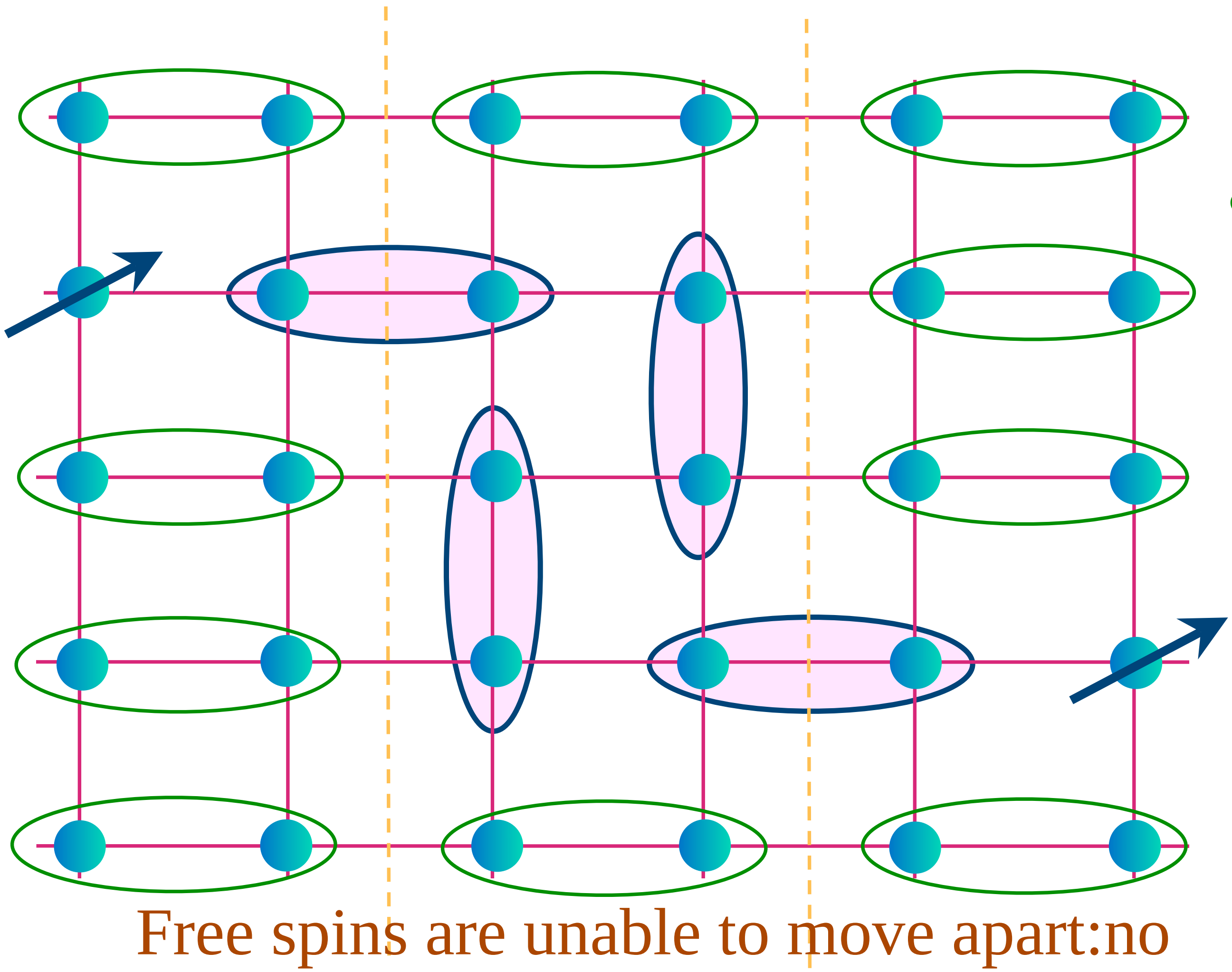
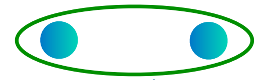
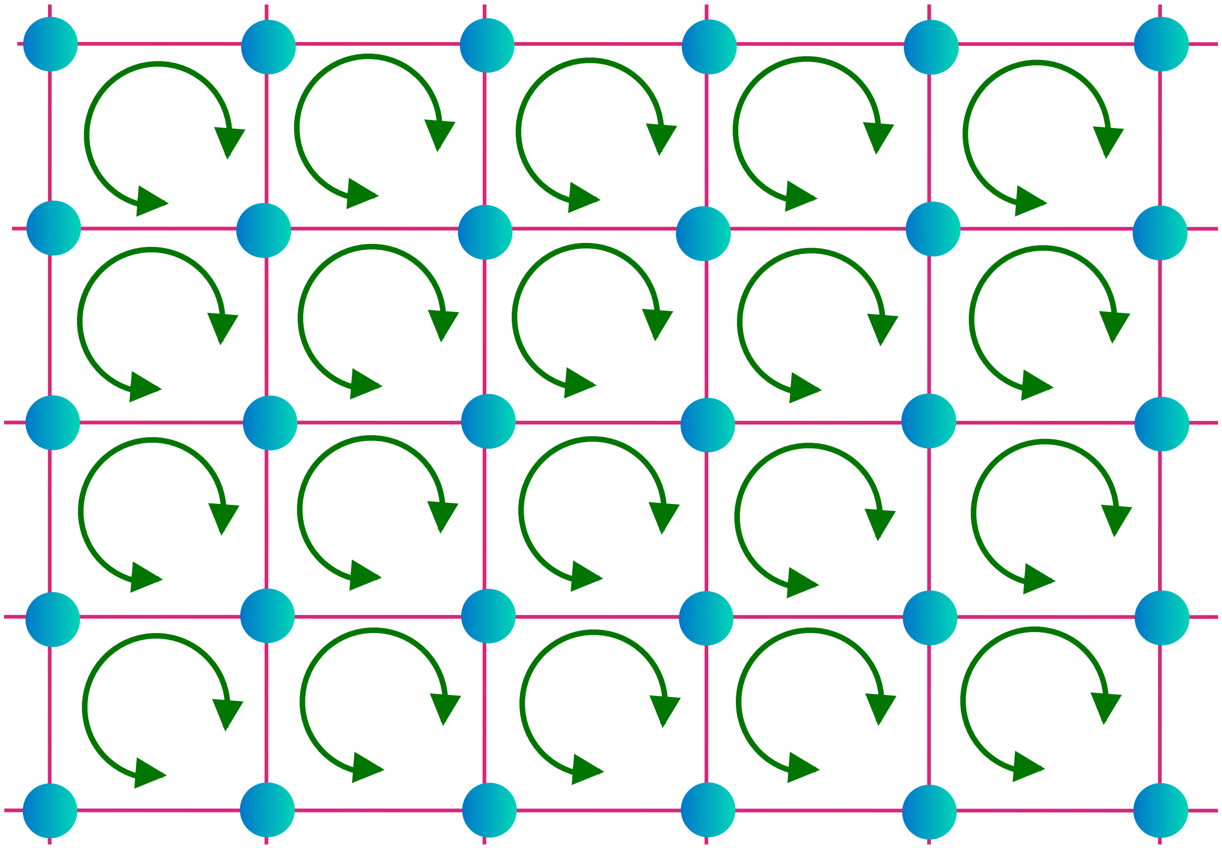



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Free spins are unable to move apart:no fractionalization, but ***confinement***

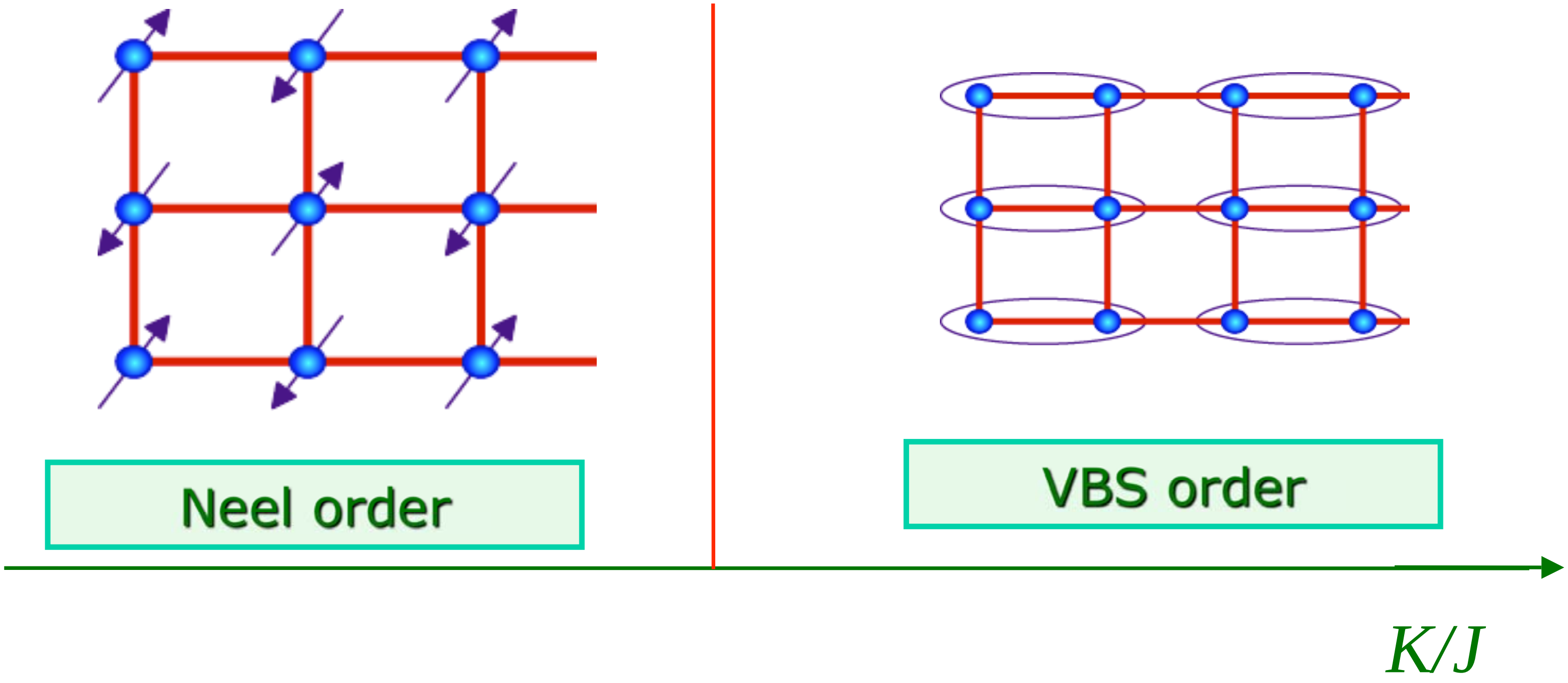
# Phase diagram of square lattice antiferromagnet



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{four spin exchange}$$

A. W. Sandvik, cond-mat/0611343

# Phase diagram of square lattice antiferromagnet

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T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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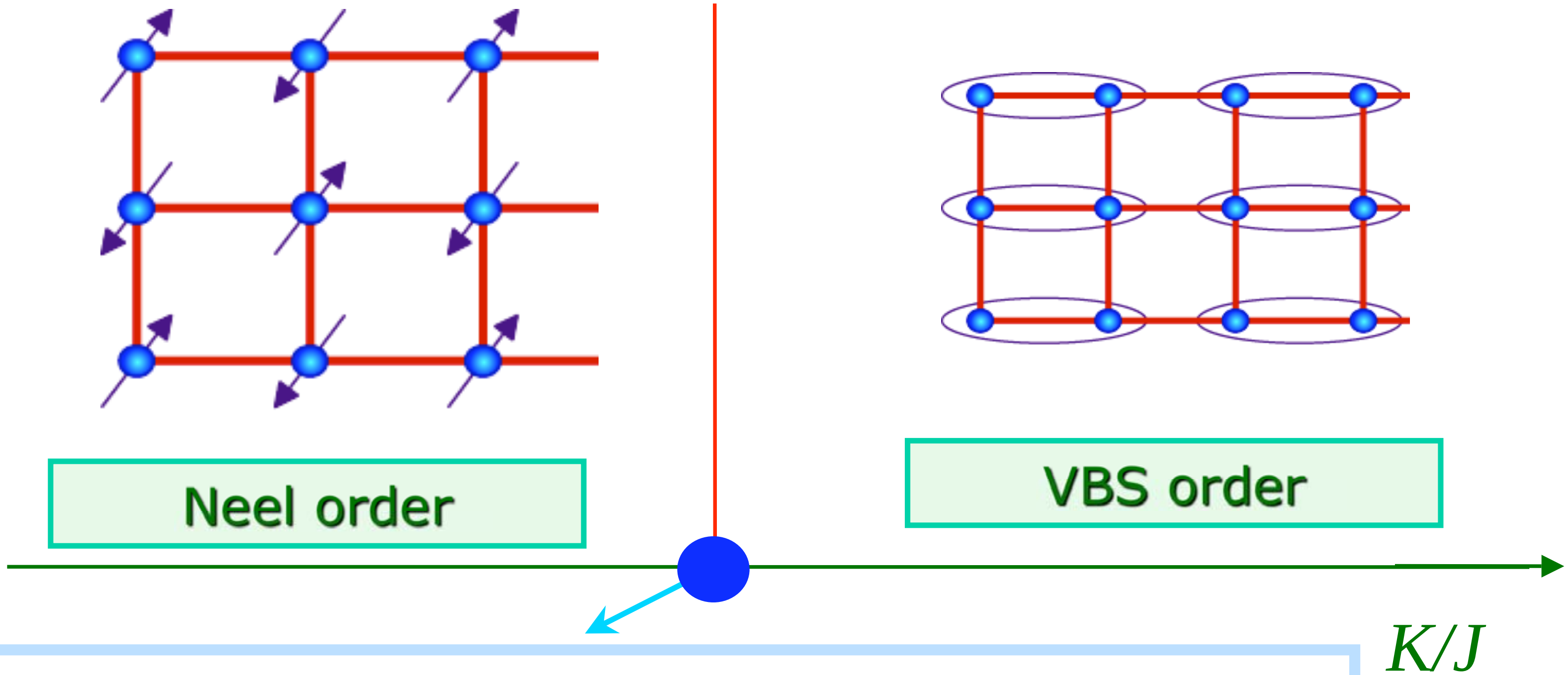


RVB physics appears at the quantum critical point which has fractionalized excitations: "*deconfined criticality*"

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\Box} \text{four spin exchange}$$

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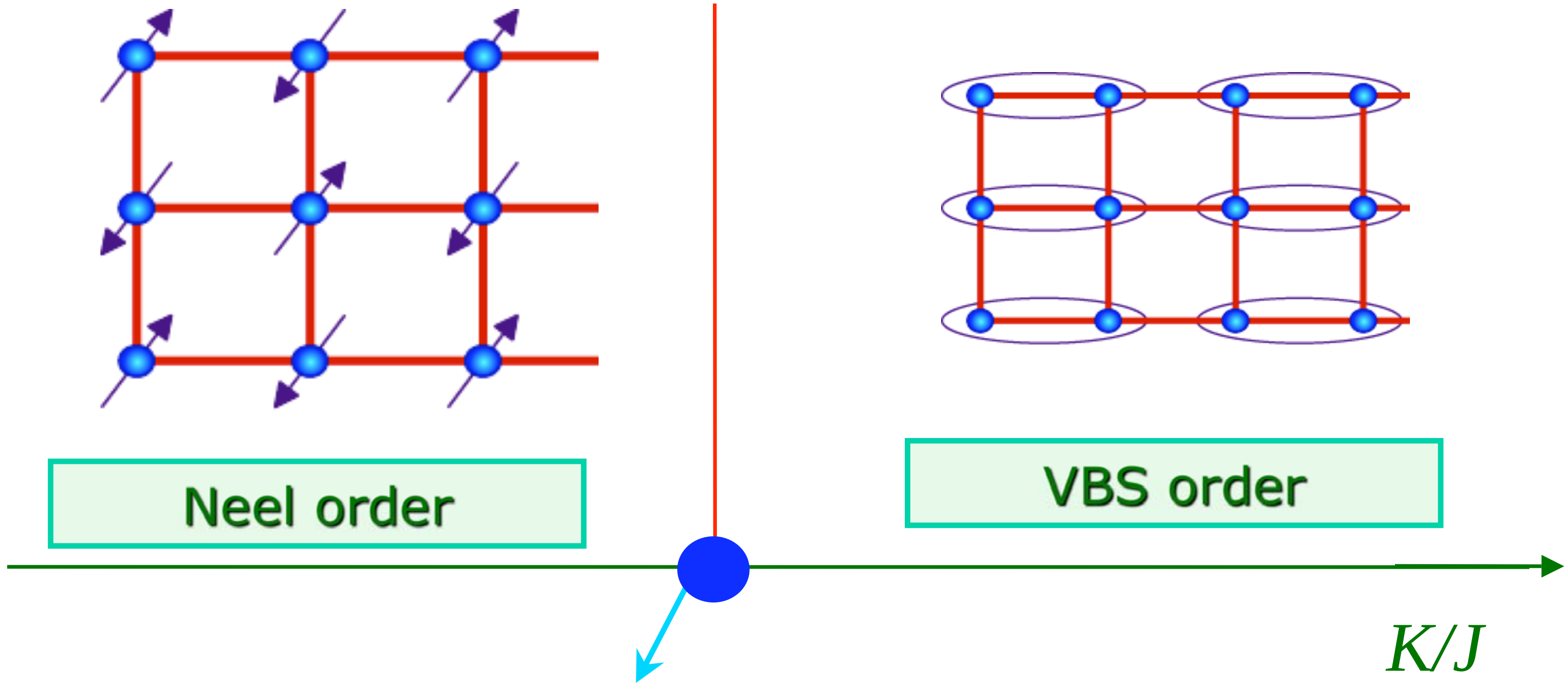
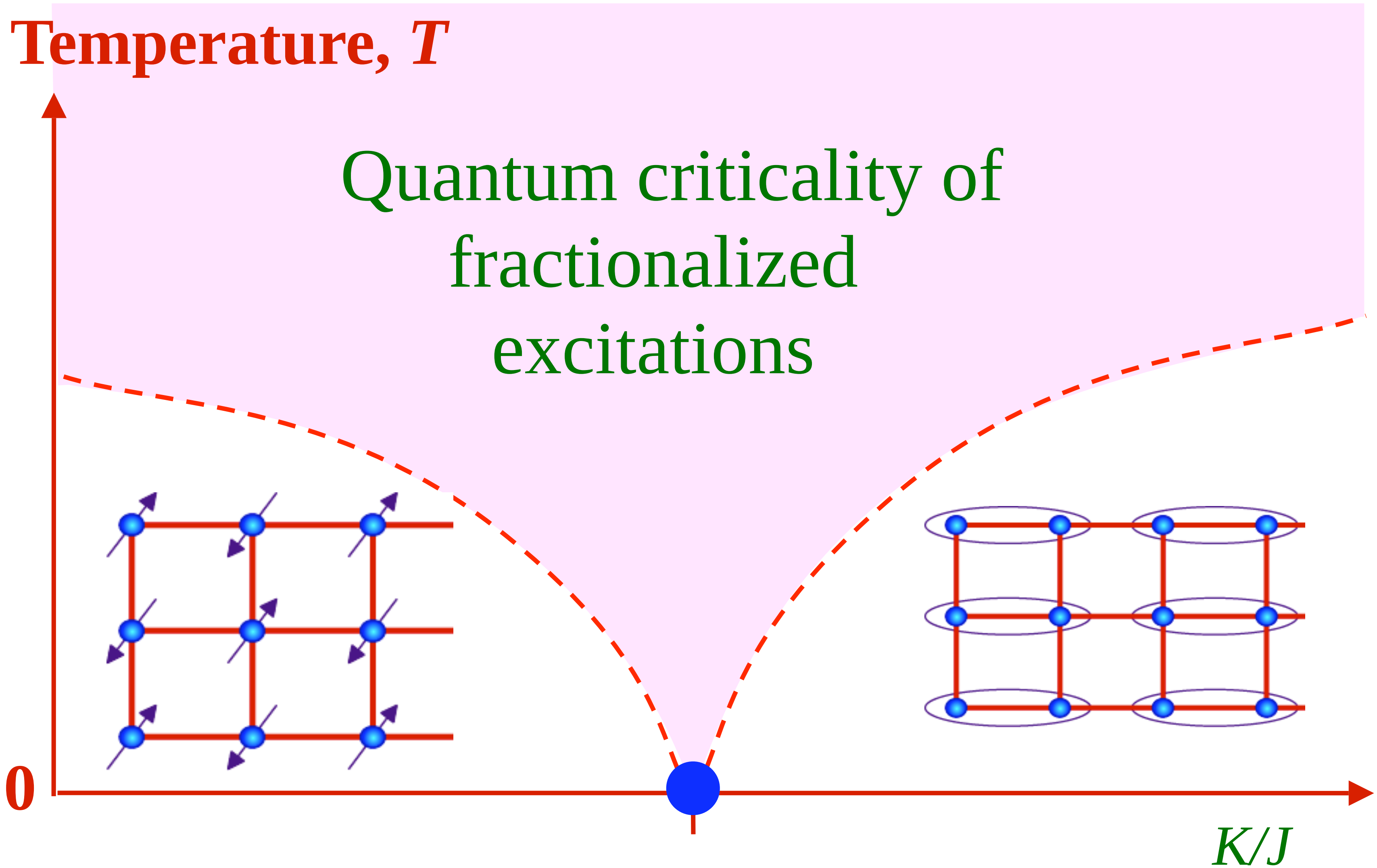



Second-order critical point described by

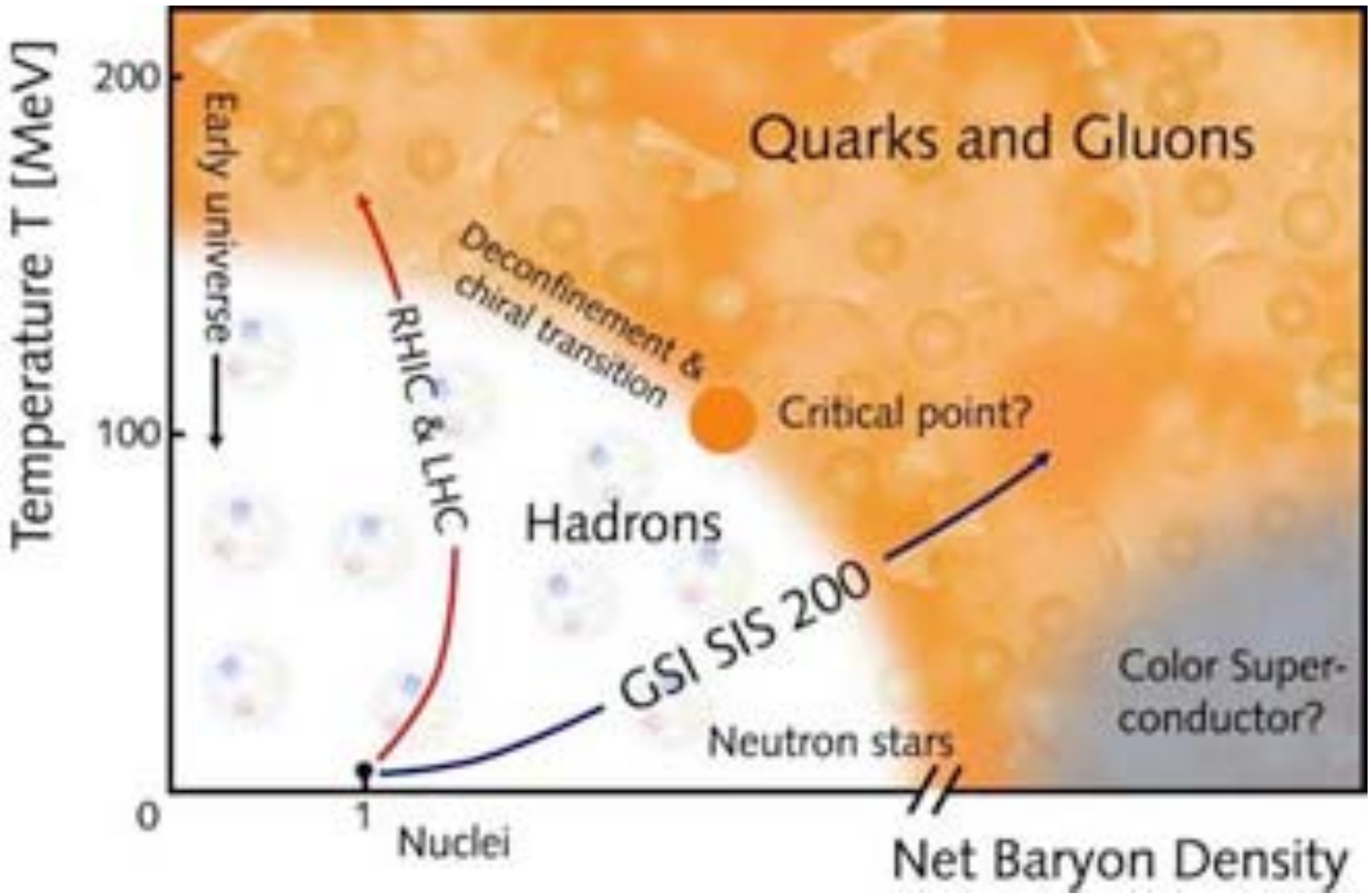
$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[ \left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + r\left|z_\alpha\right|^2 + \frac{u}{2}\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

at its critical point $r = r_c$, where $z_\alpha$ are the neutral $S = 1/2$ spinons and $A_\mu$ is a *non-compact* U(1) gauge field.

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).


Temperature, *T*
Quantum criticality of fractionalized excitations
0
*K/J*

# Phases of nuclear matter



# Outline

1. Superfluid-insulator quantum transitions

   *Experiments on ultracold atoms*

2. Theory of quantum-critical transport

   *Collisionless-t0-hydrodynamic crossover of conformal field theories*

3. Entanglement of valence bonds

   *Deconfined criticality in antiferromagnets*

4. Nernst effect in the cuprate superconductors

   *Quantum criticality and dyonic black holes*

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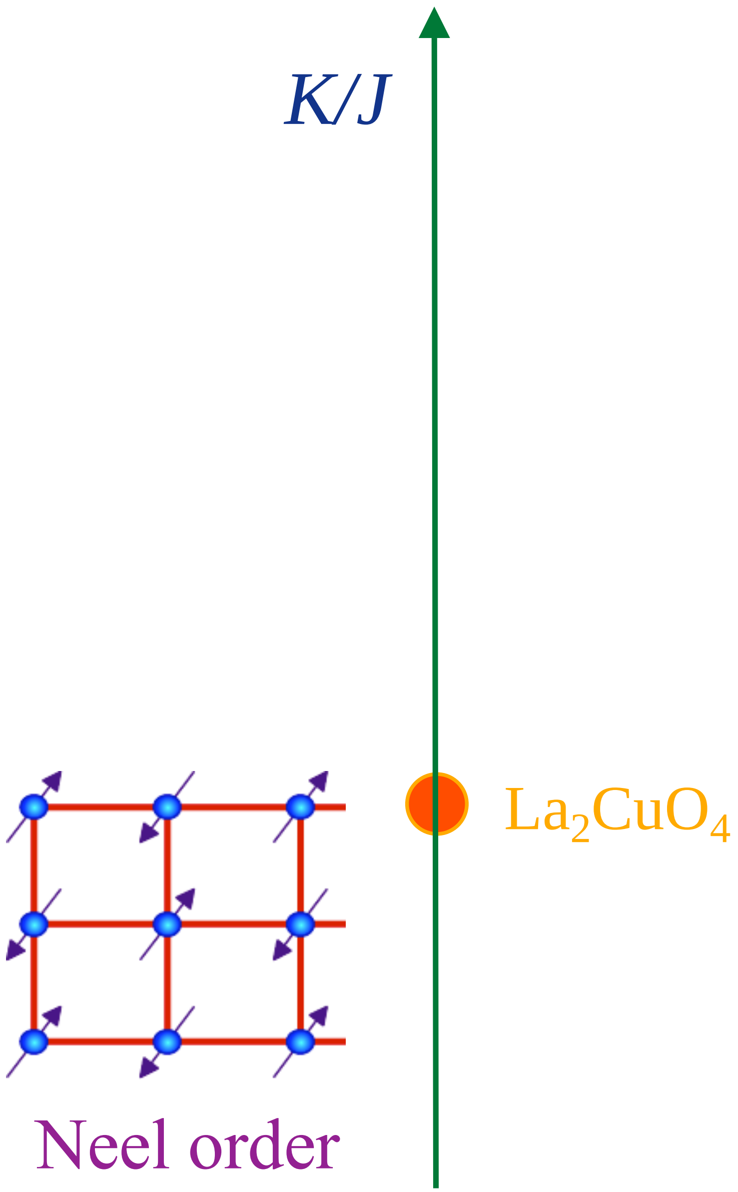
# Phase diagram of doped antiferromagnets



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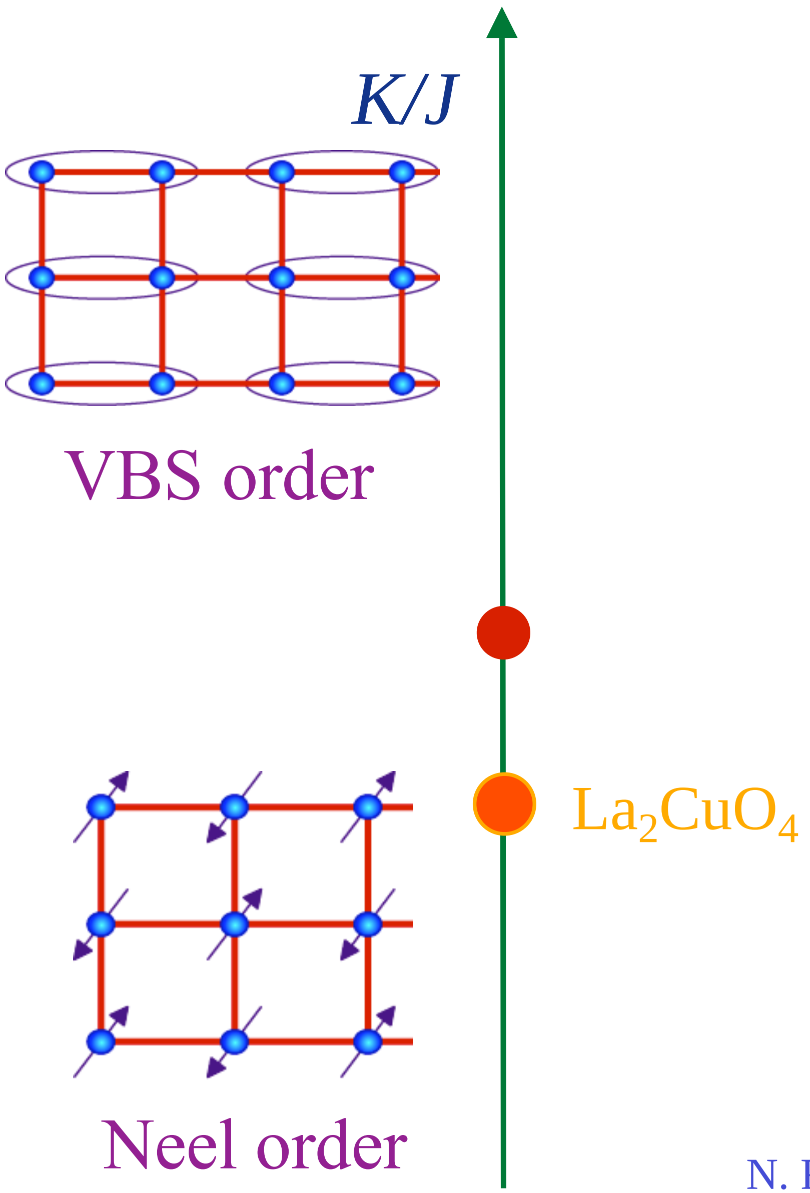


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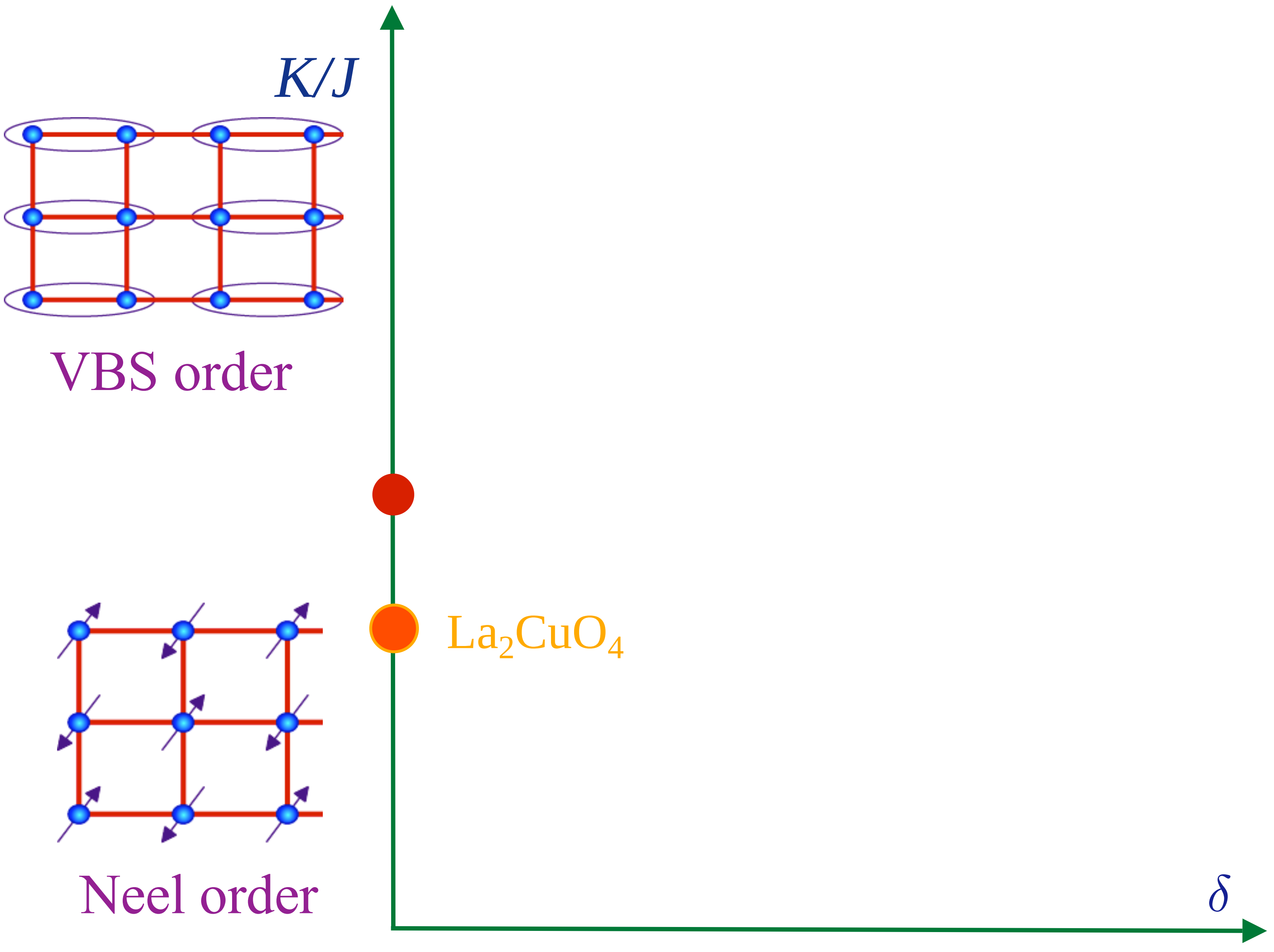
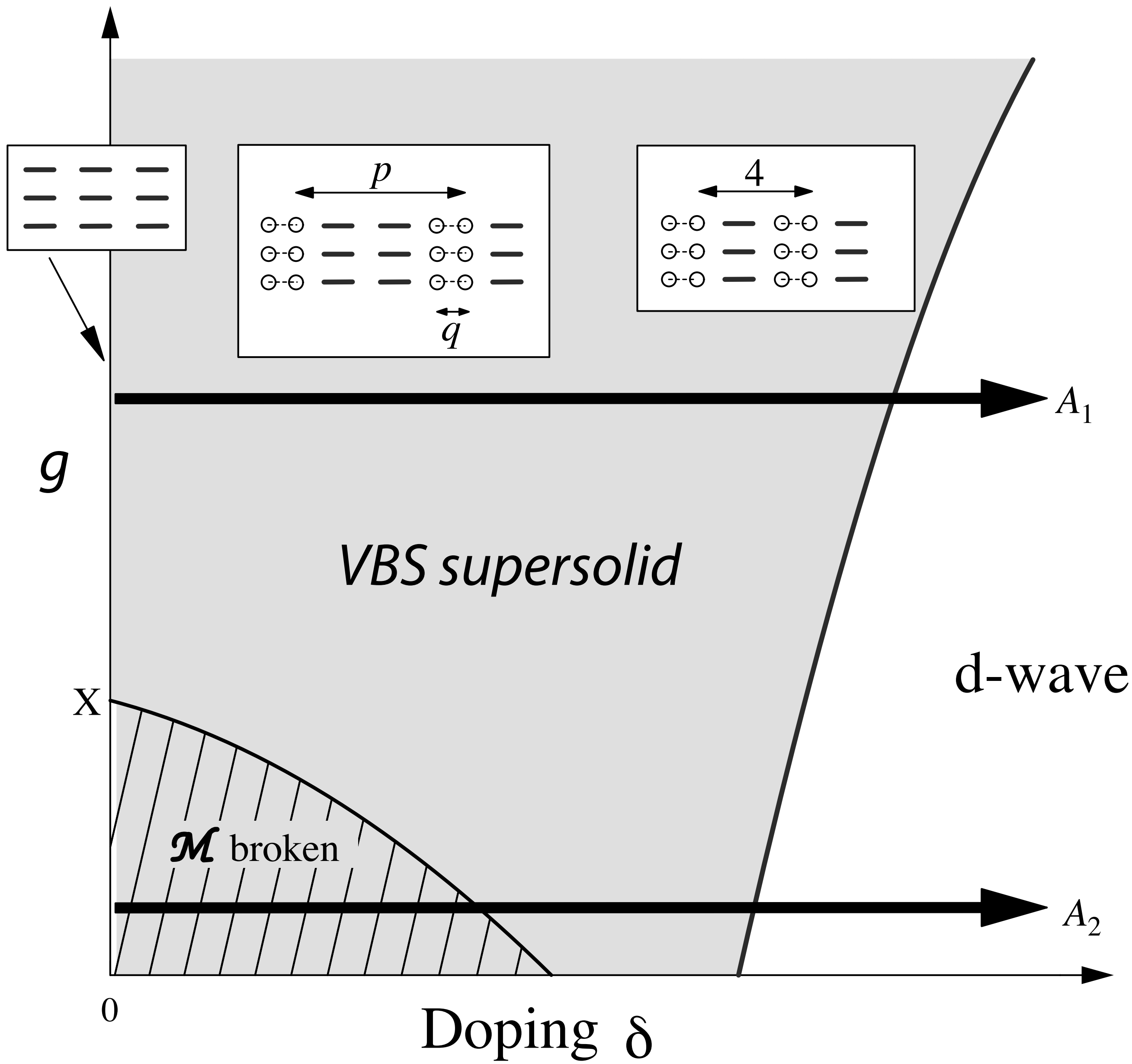
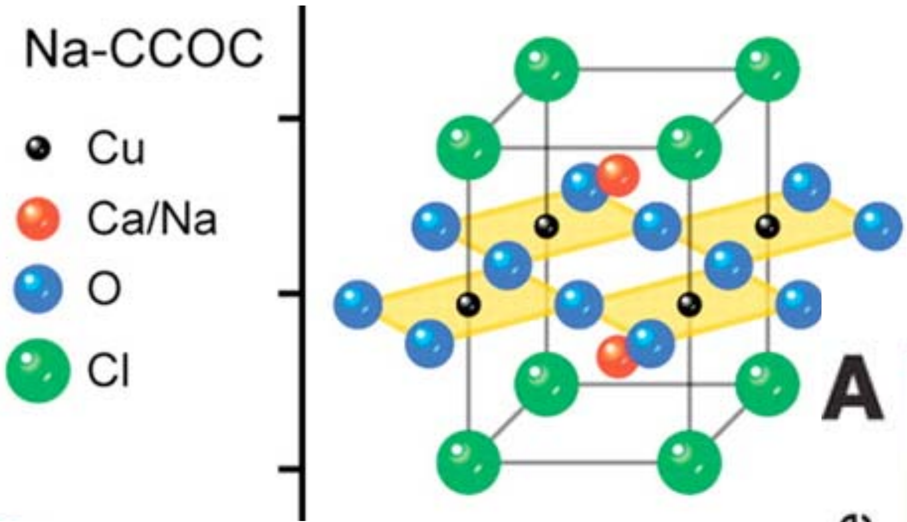
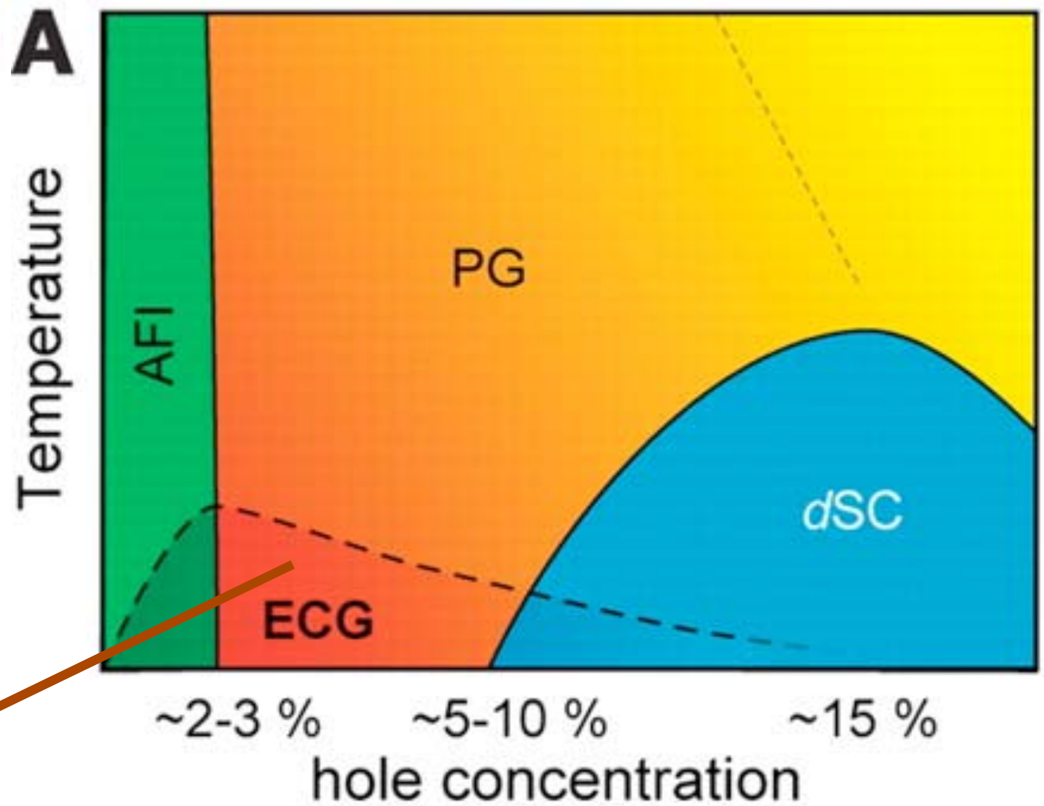




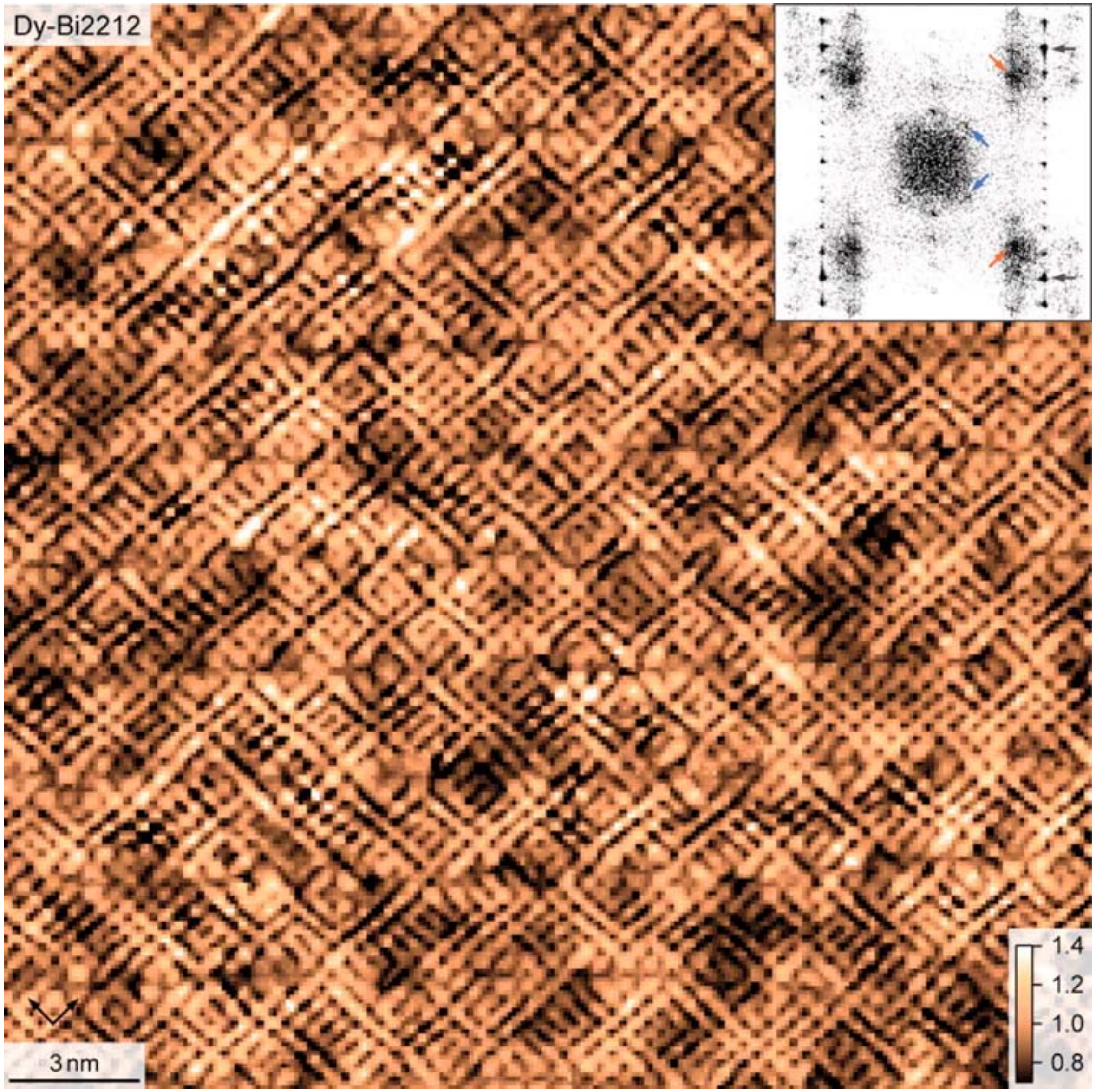

M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999)

# Temperature-doping phase diagram of the cuprate superconductors

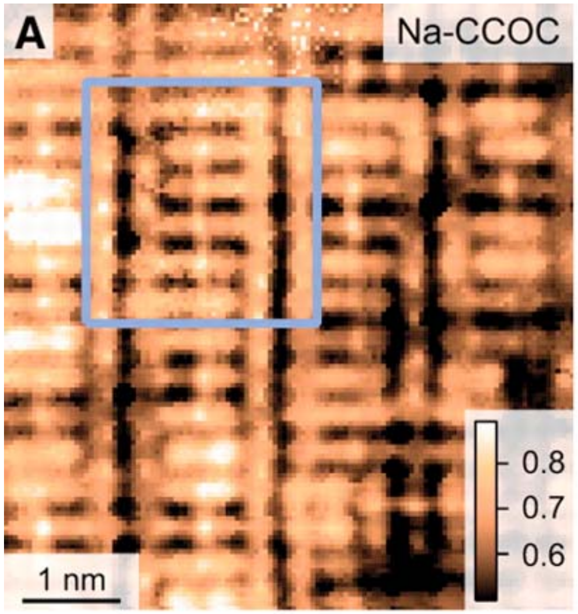



STM in zero field




Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)

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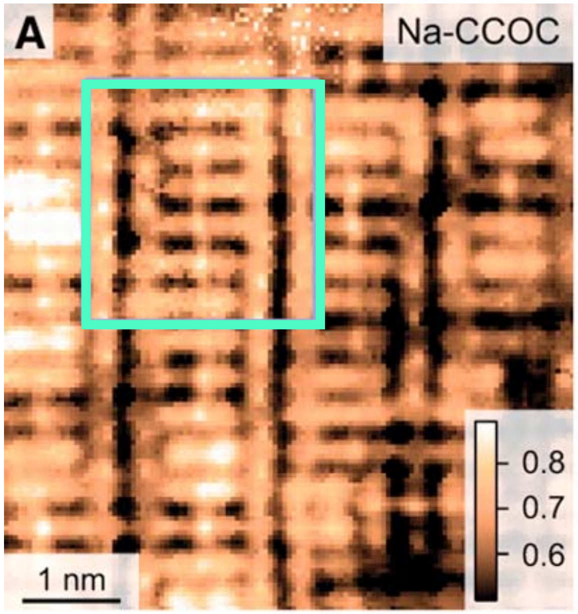


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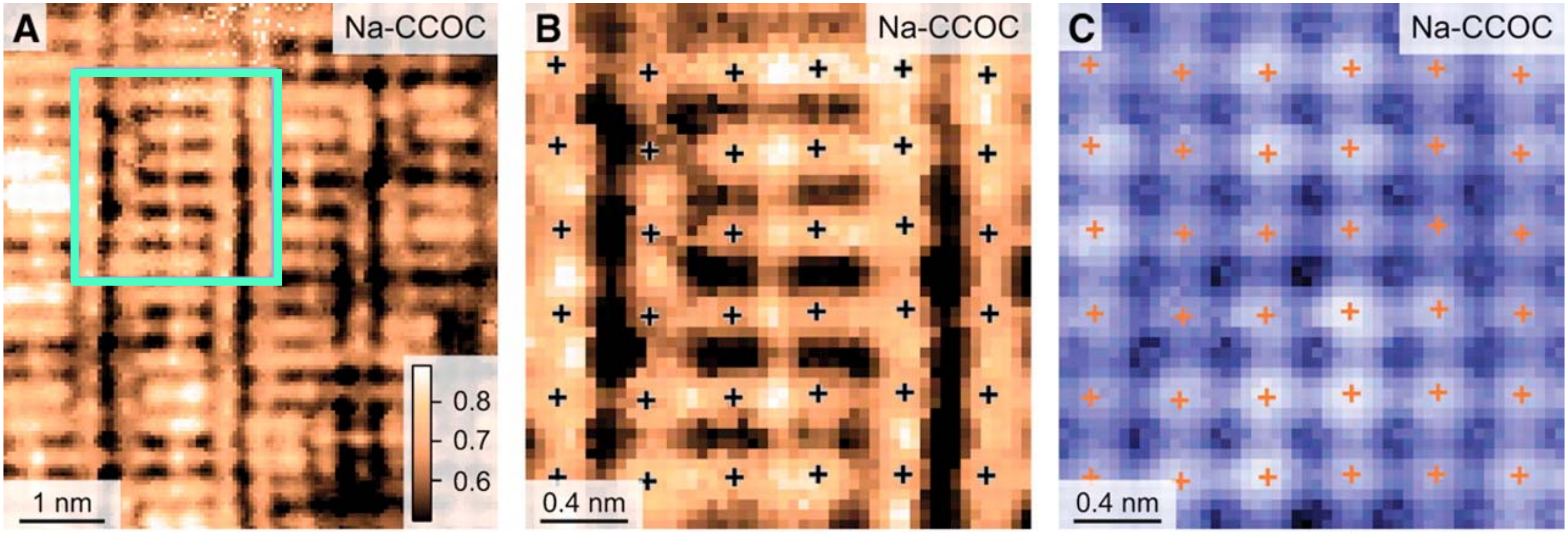


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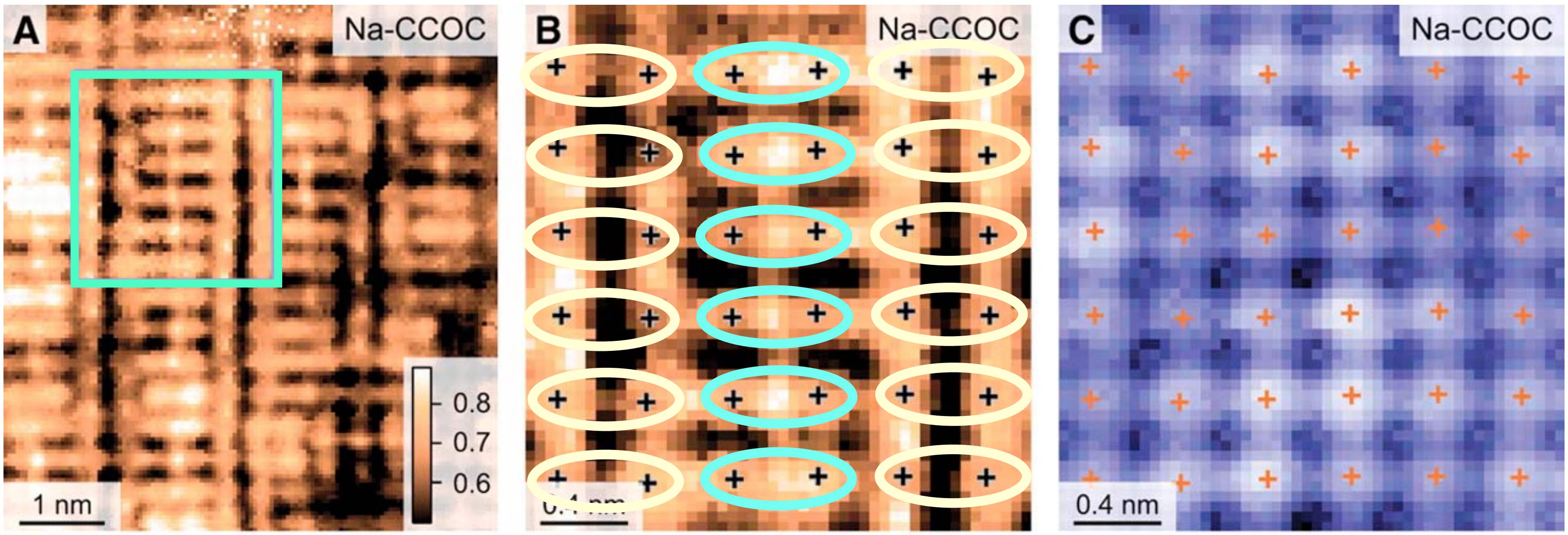


“Glassy” Valence Bond Solid (VBS)

Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)

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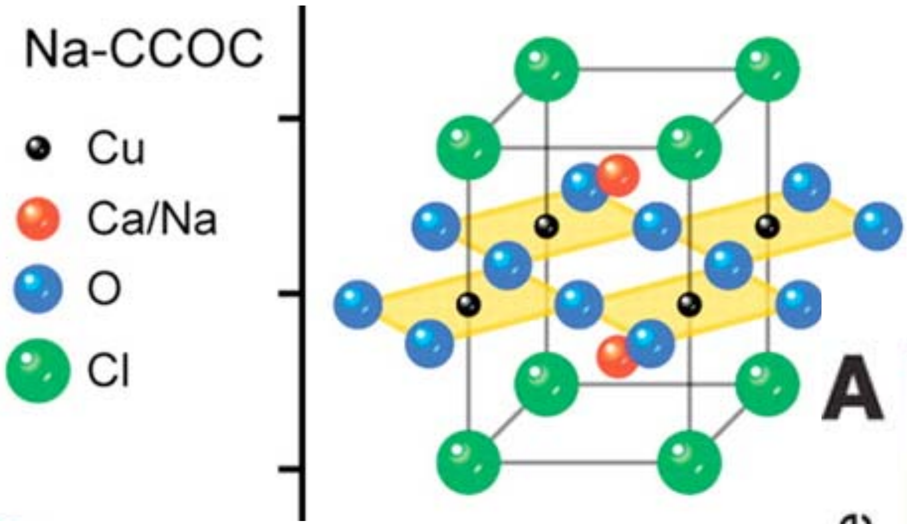
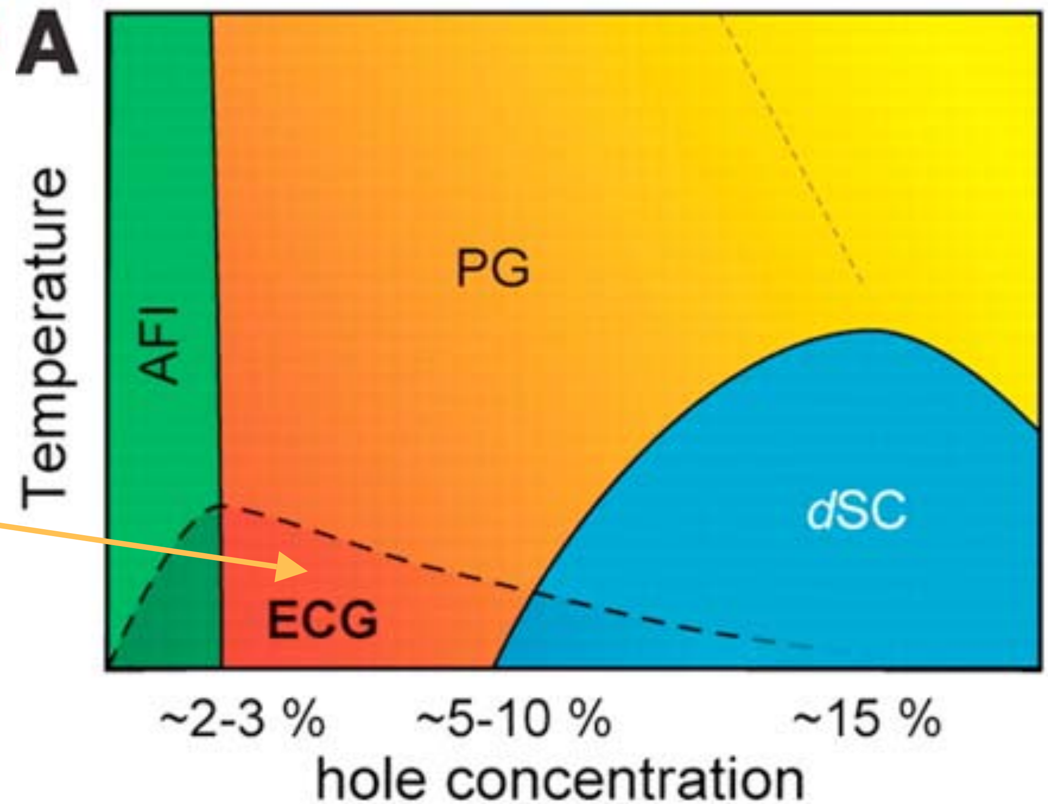
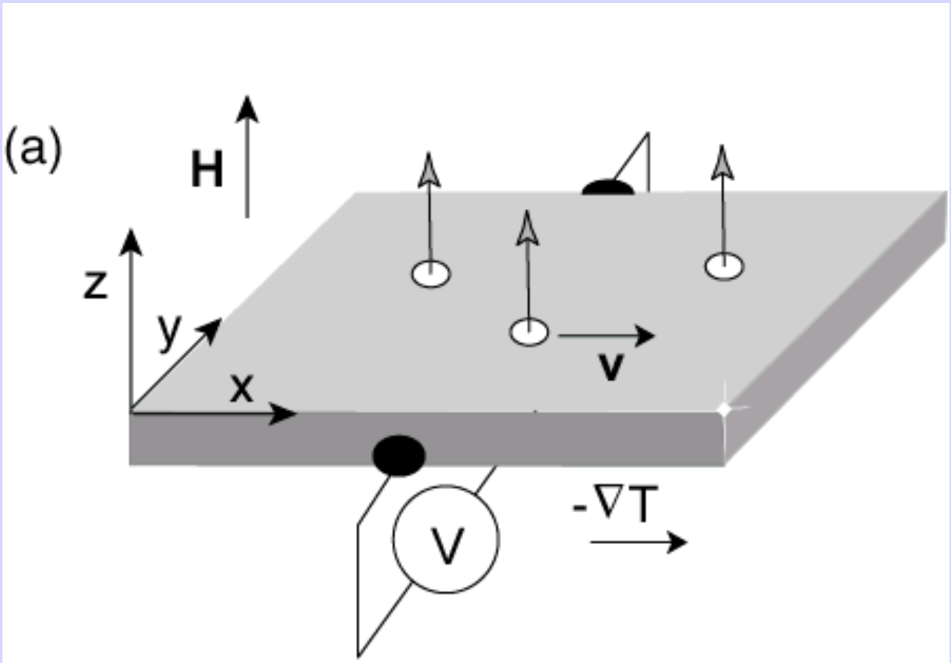
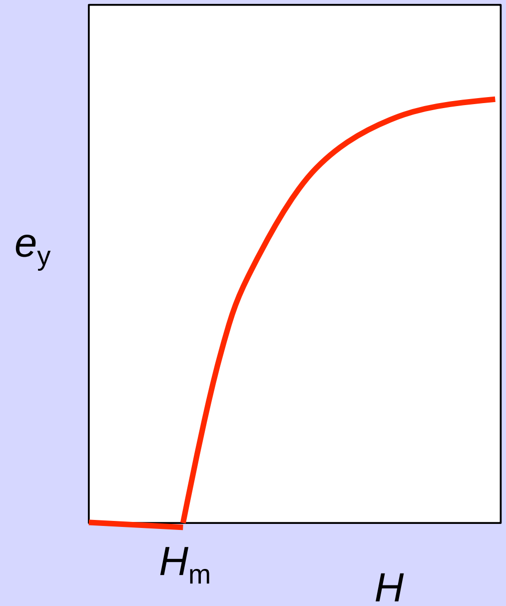


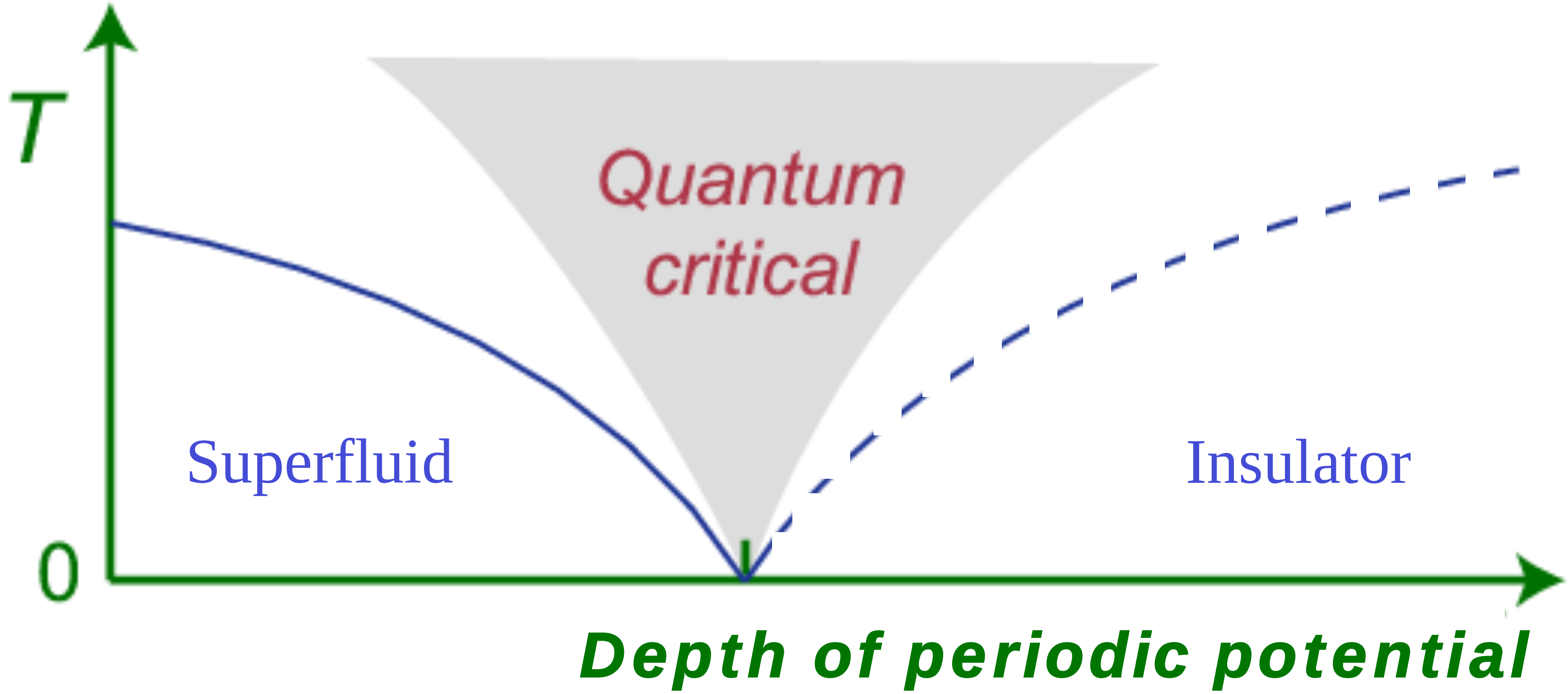

“Glassy” Valence Bond Solid (VBS)

# Nernst experiment



# Non-zero temperature phase diagram



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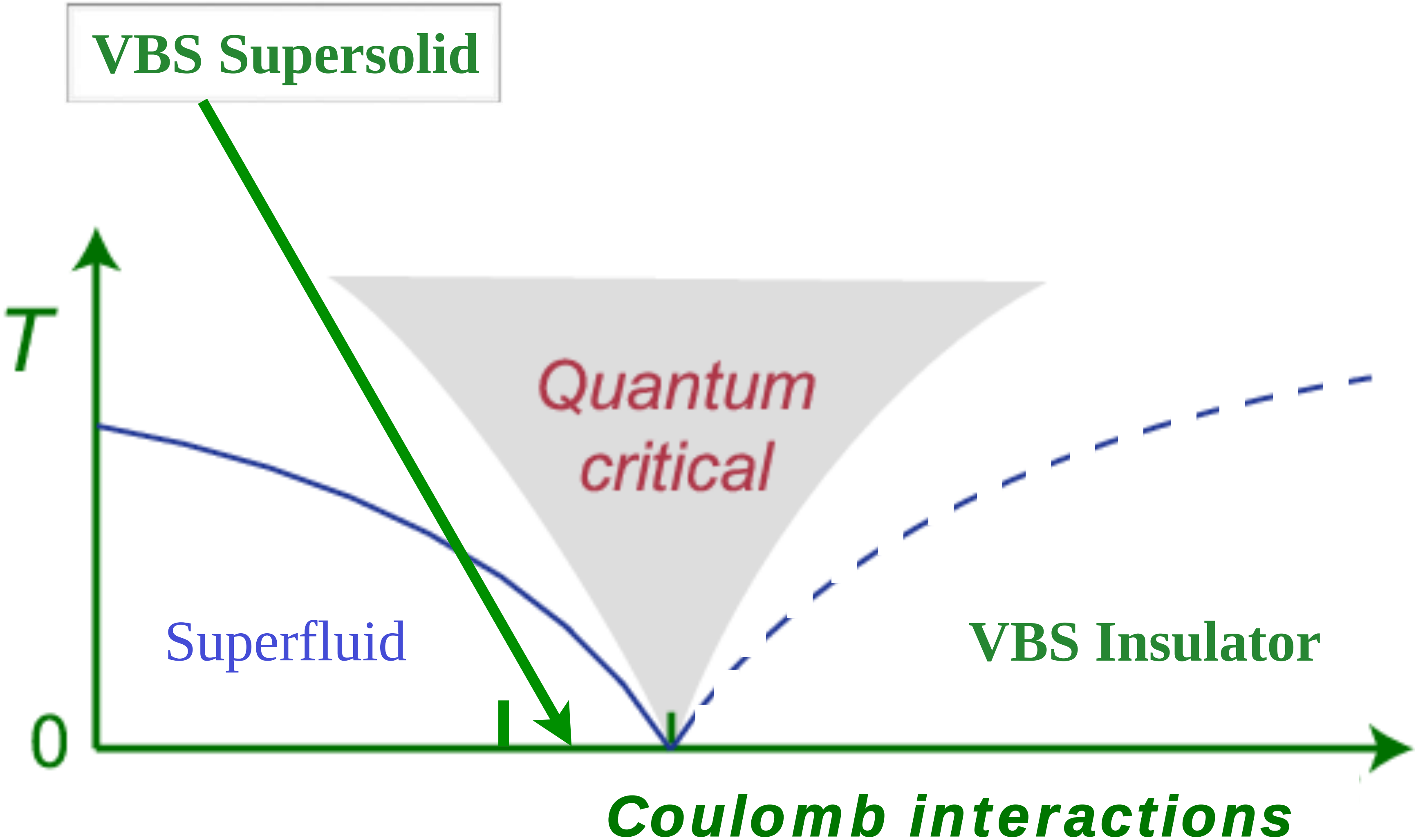

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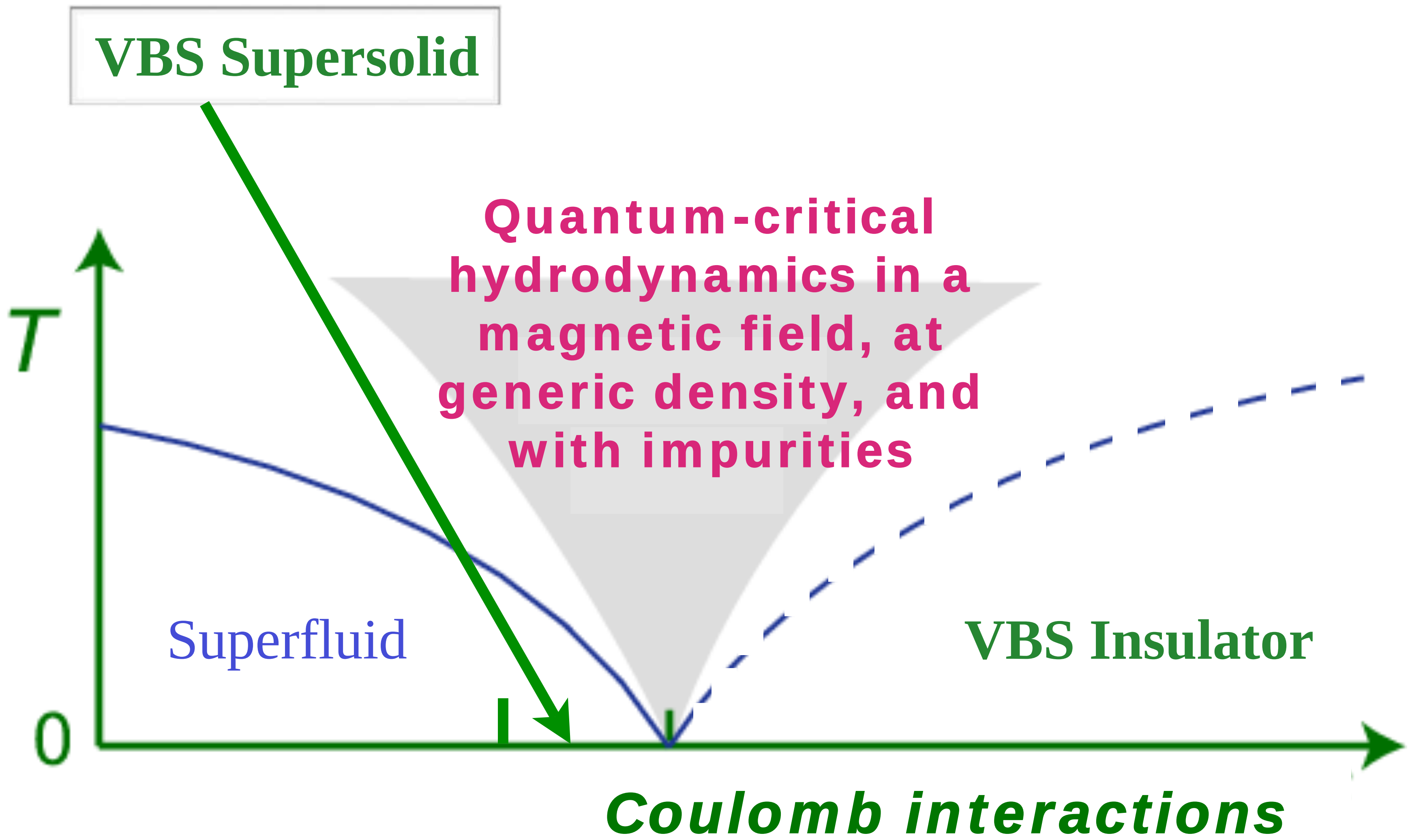
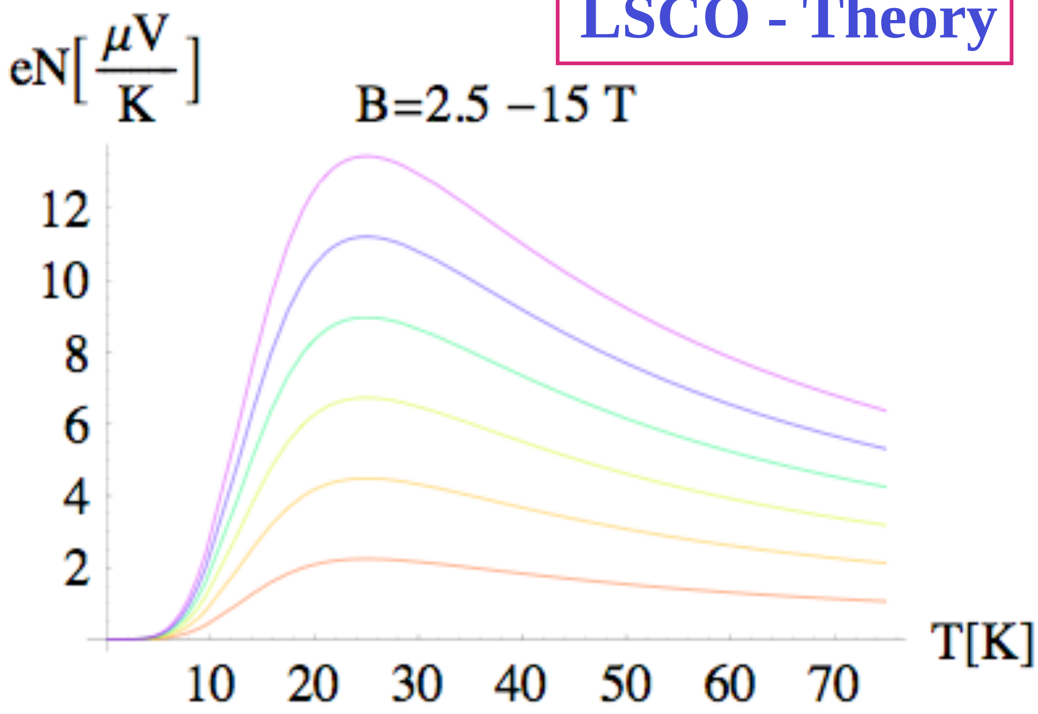
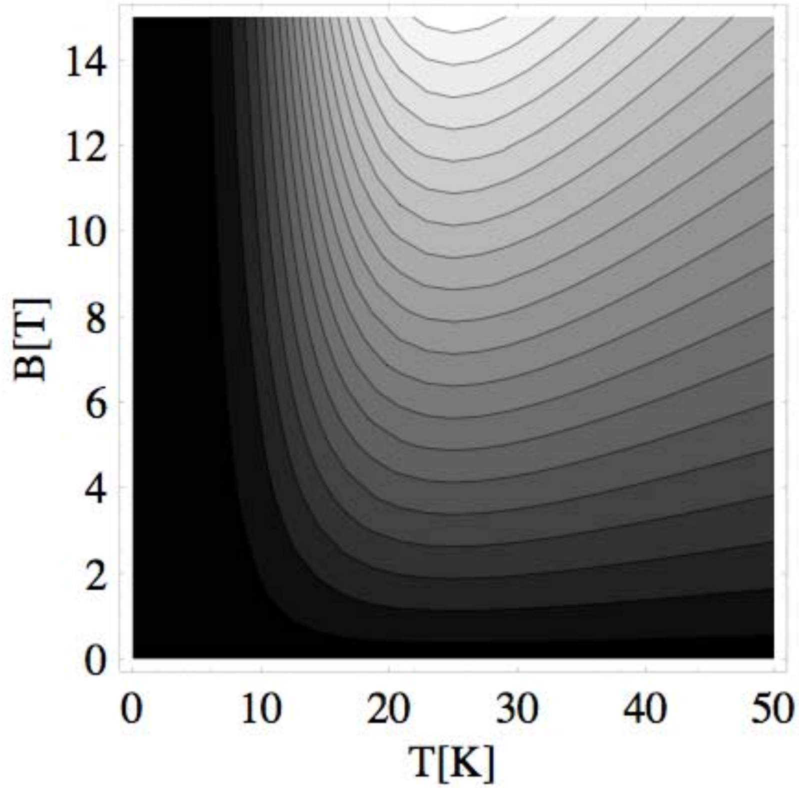


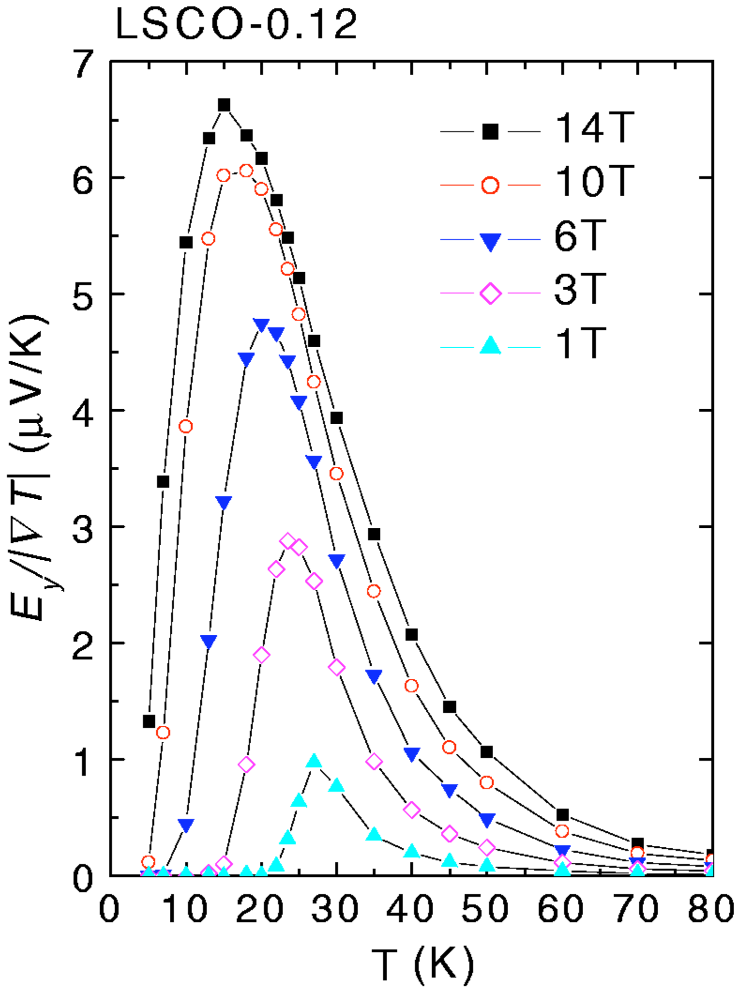
# LSCO - Theory

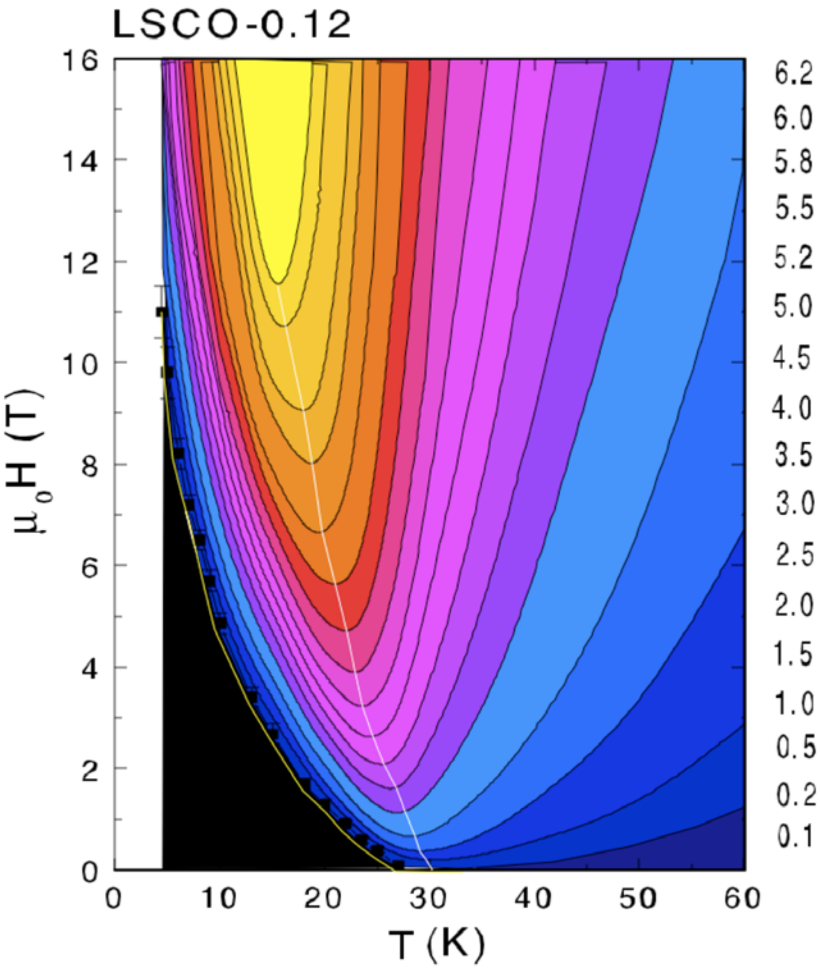



S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

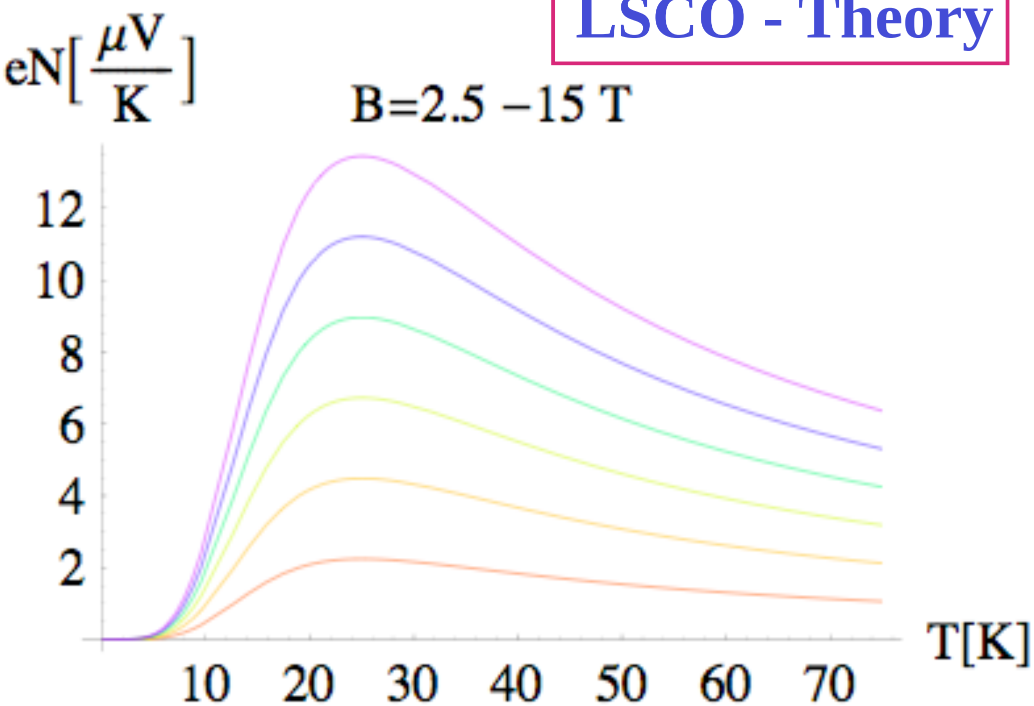
# LSCO - Experiments

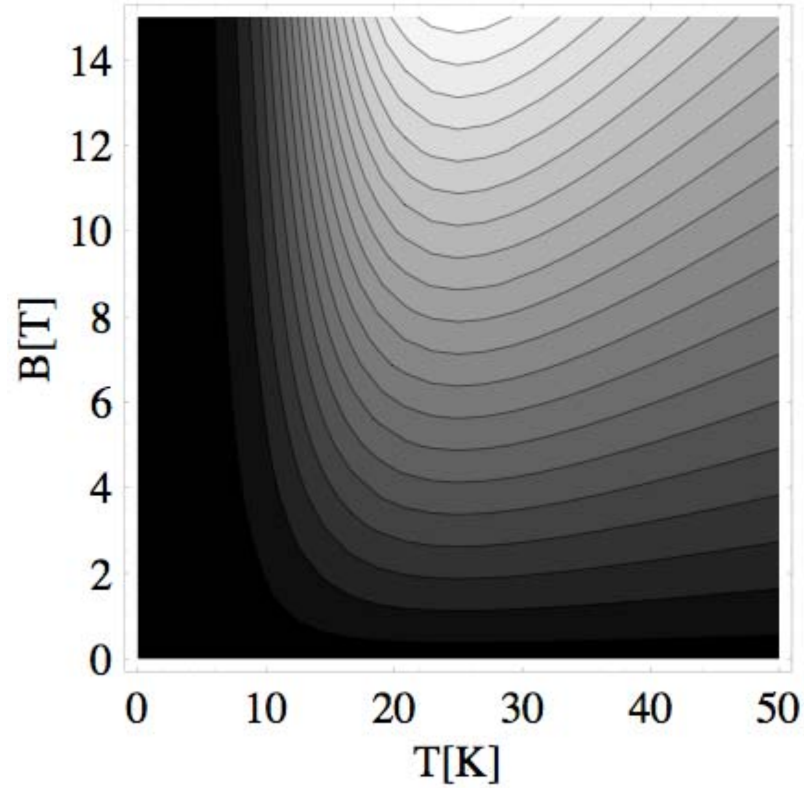



N. P. Ong *et al.*

## LSCO - Theory




Only input parameters

$$\hbar v = 47 \text{ meV Å}$$
$$\tau_{\text{imp}} \approx 10^{-12} \text{ s}$$

Output

$$\omega_c = 6.2\text{GHz} \cdot \frac{B}{1\text{T}} \left(\frac{35\text{K}}{T}\right)^3$$

Similar to velocity estimates by
A.V. Balatsky and Z-X. Shen, *Science* **284**, 1137 (1999).

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential $\mu$
- A magnetic field $B$

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

The exact results are found to be in <u>*precise*</u> accord with <u>*all*</u> hydrodynamic results presented earlier

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, arXiv:0706.3215

# *Conclusions*

- Studies of new materials and trapped ultracold atoms are yielding new quantum phases, with novel forms of quantum entanglement.
- Some materials are of technological importance: e.g. high temperature superconductors.
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of VBS order and Nernst effect in curpates.
- Tabletop “laboratories for the entire universe”: quantum mechanics of black holes, quark-gluon plasma, neutrons stars, and big-bang physics.