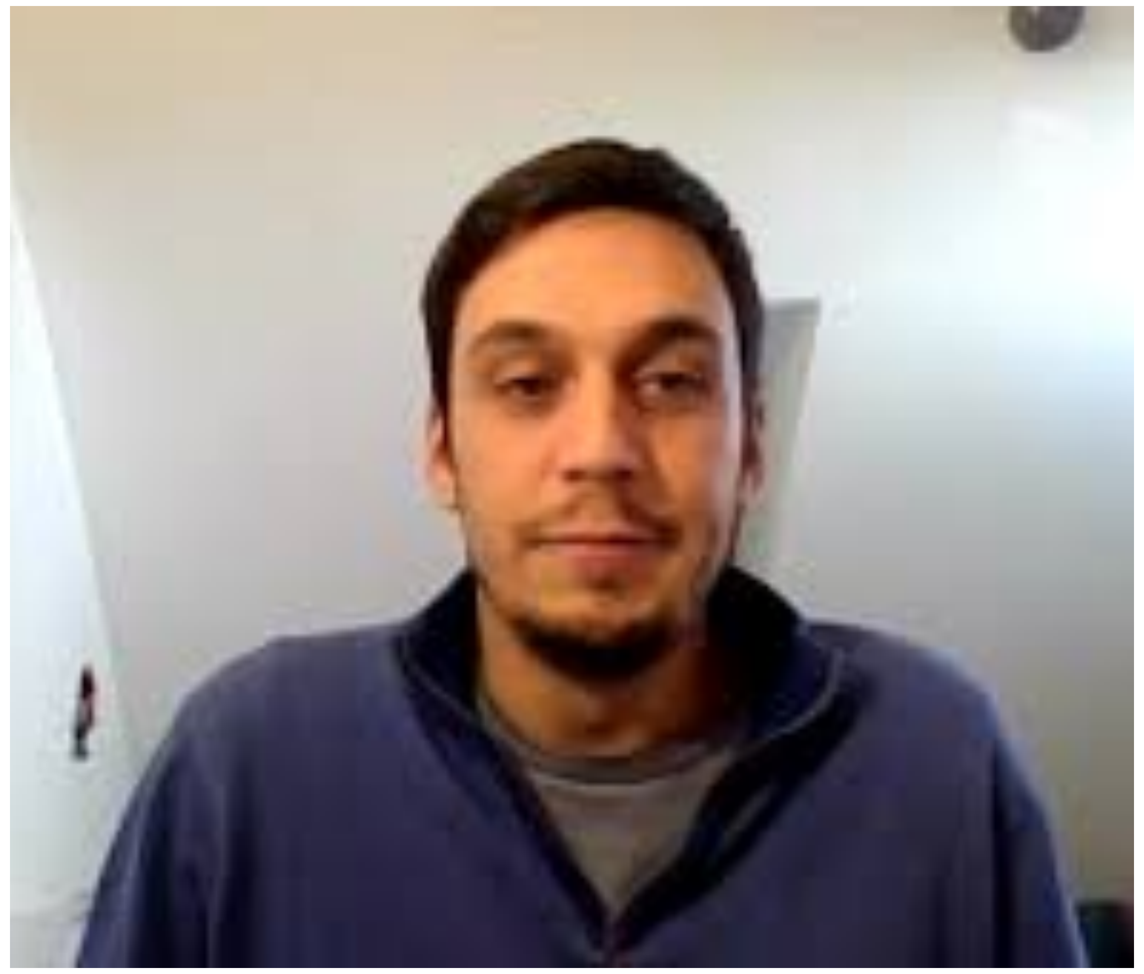
Fractionalized Fermi Liquid (FL*) theory of the cuprate pseudogap and its connections to the cuprate phase diagram

Quantum Cafe
Center for Computational Quantum Physics
Flatiron Institute, New York
September 16, 2025

Subir Sachdev

PHYSICS
HARVARD

Maine Christos
Caltech



Pietro Bonetti

Thermal SU(2) lattice gauge theory of the cuprate pseudogap: reconciling Fermi arcs and hole pockets

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

Harshit Pandey

Ravi Shanker

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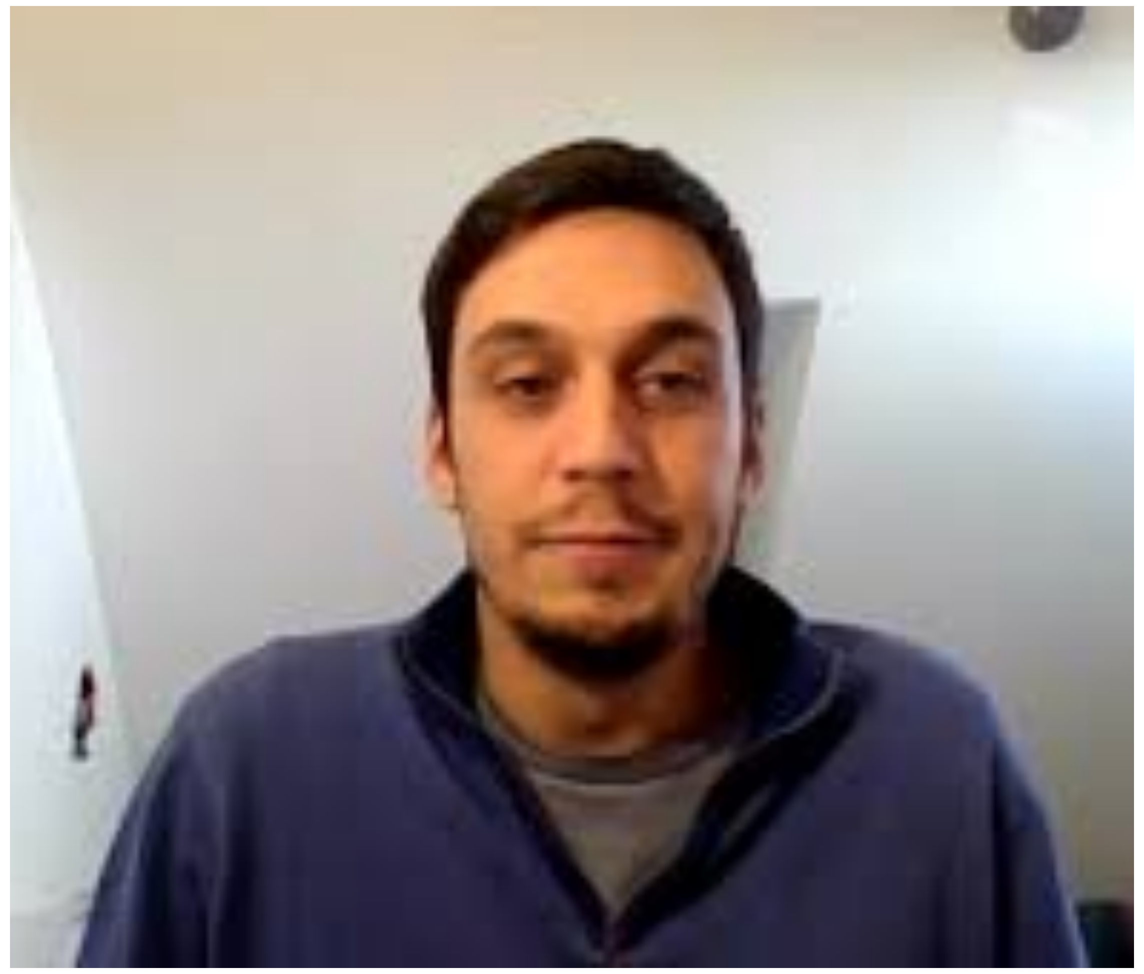


Pietro Bonetti

Alexander Nikolaenko

Aavishkar Patel
ICTS, Bengaluru

arXiv > cond-mat > arXiv:2508.20164

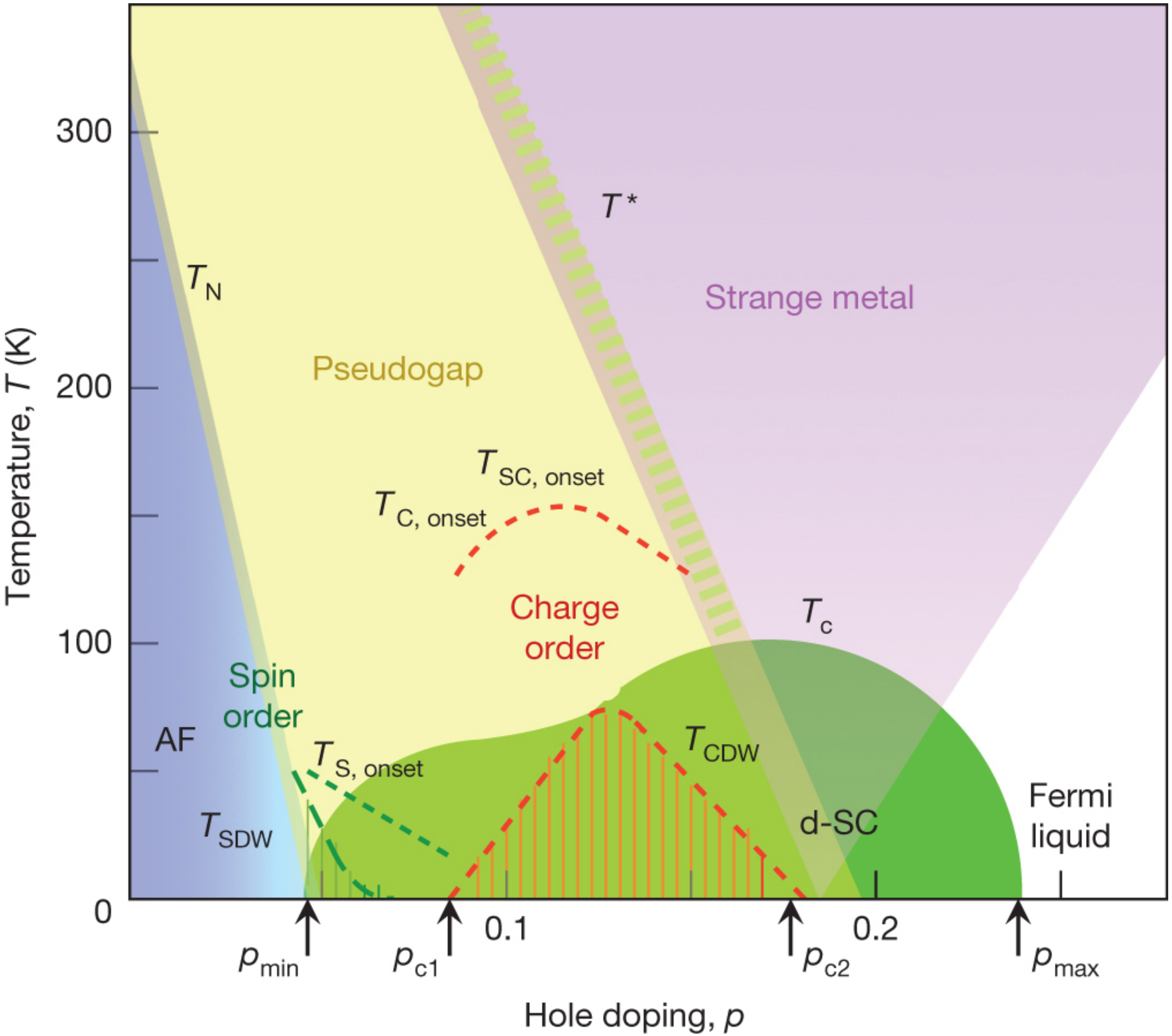
Condensed Matter > Strongly Correlated Electrons

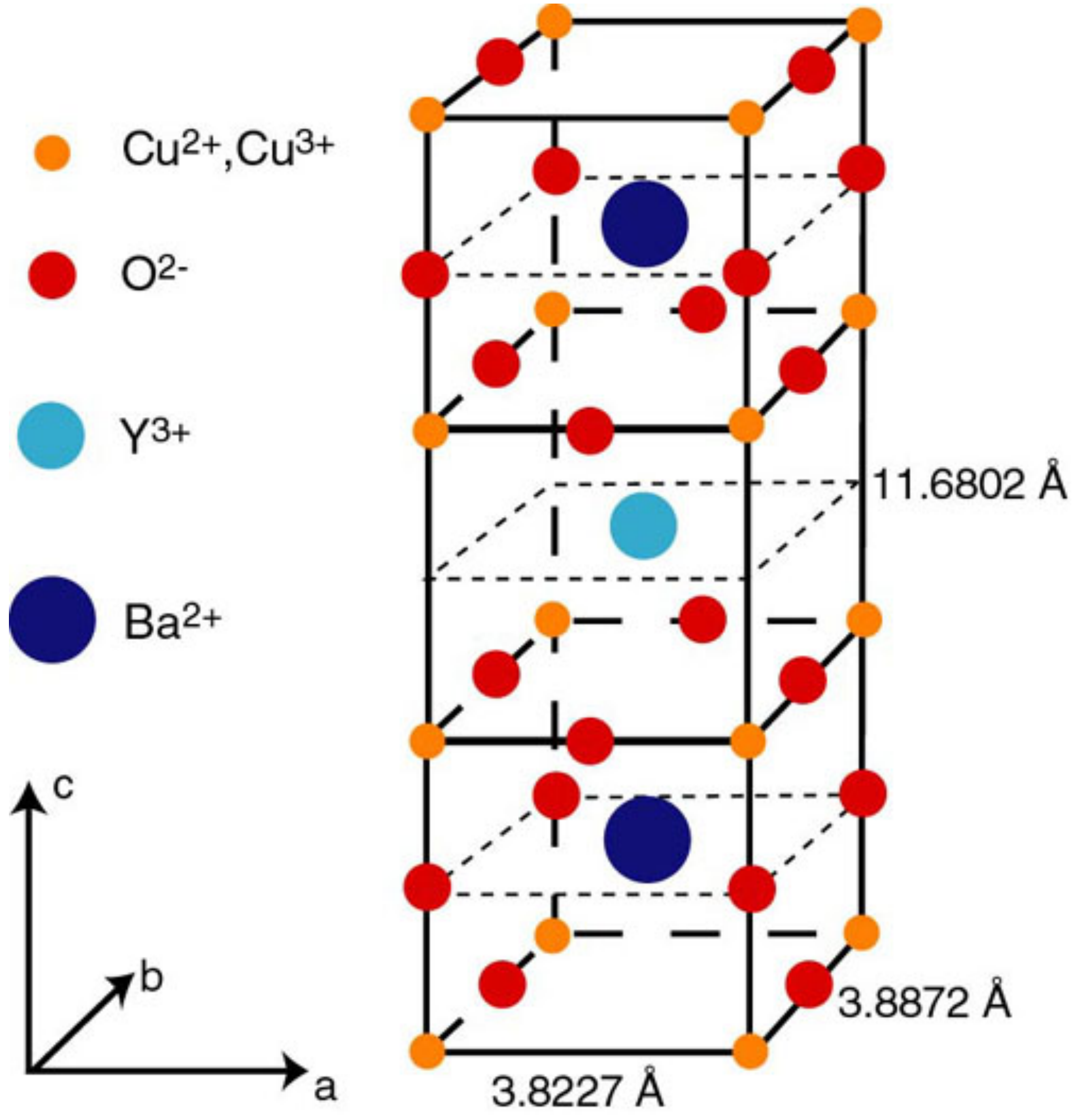
[Submitted on 27 Aug 2025]

Critical quantum liquids and the cuprate high temperature superconductors

Experiments on the cuprate pseudogap phase

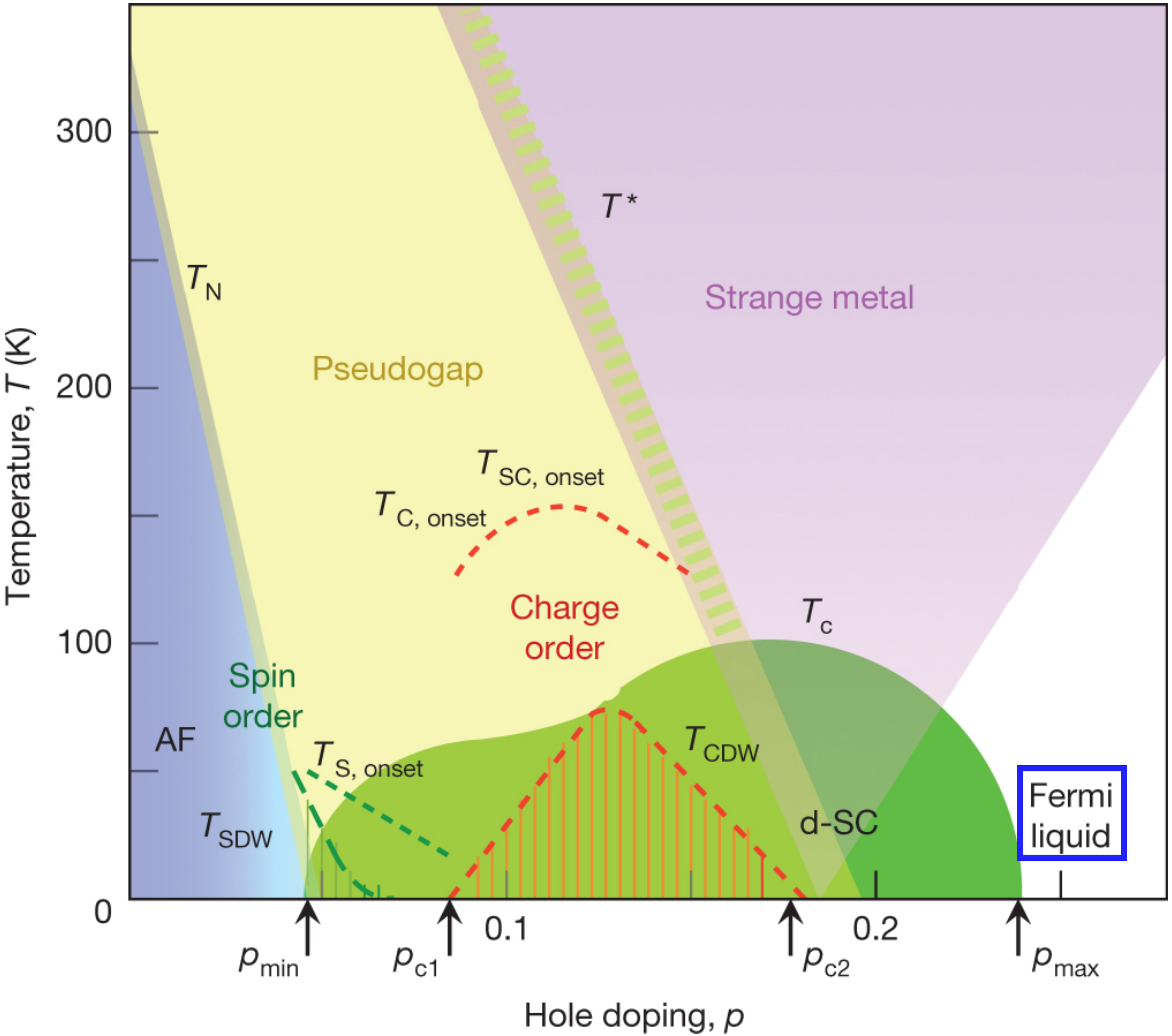
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

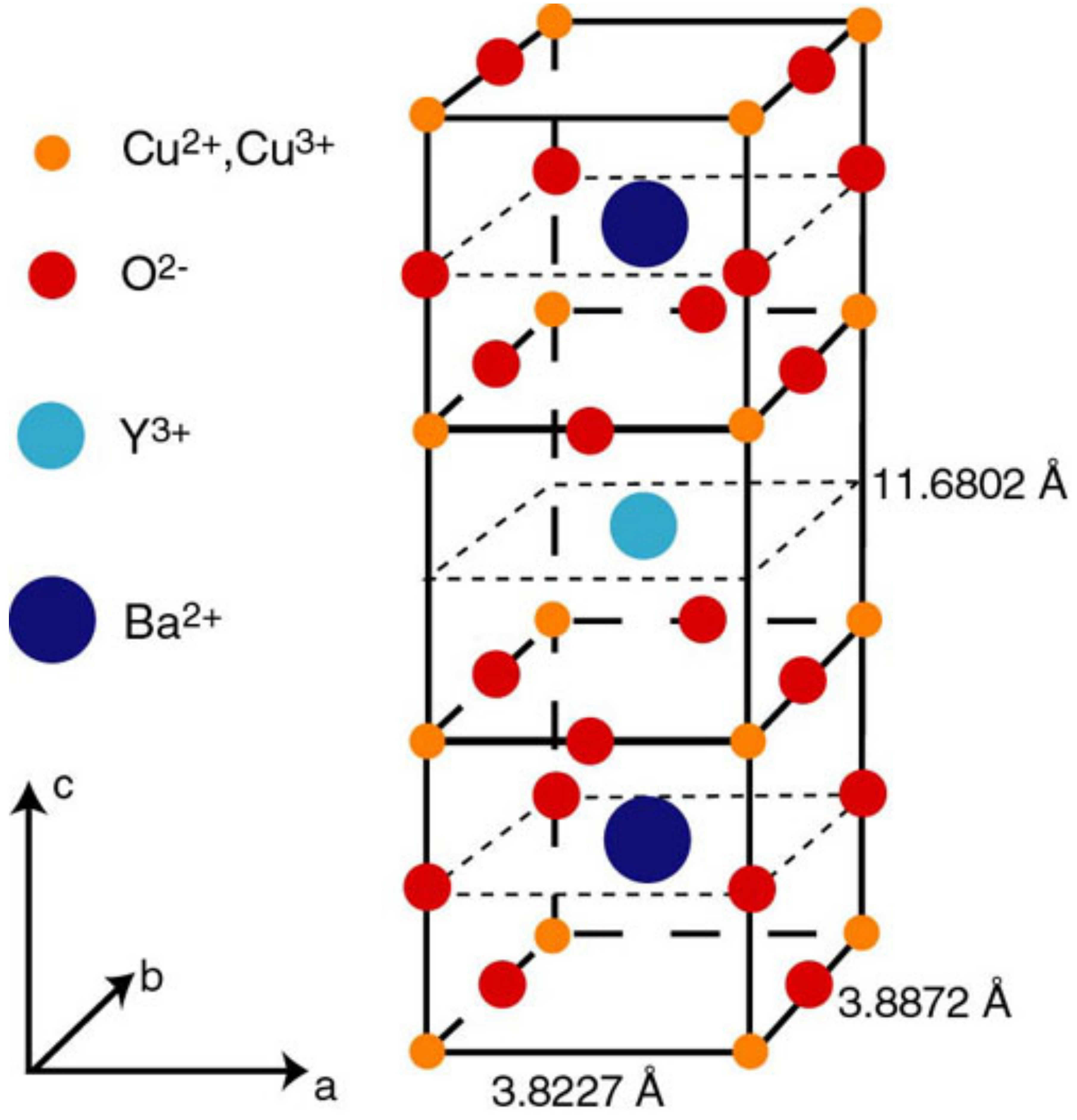




$YBa_2Cu_3O_{6+x}$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

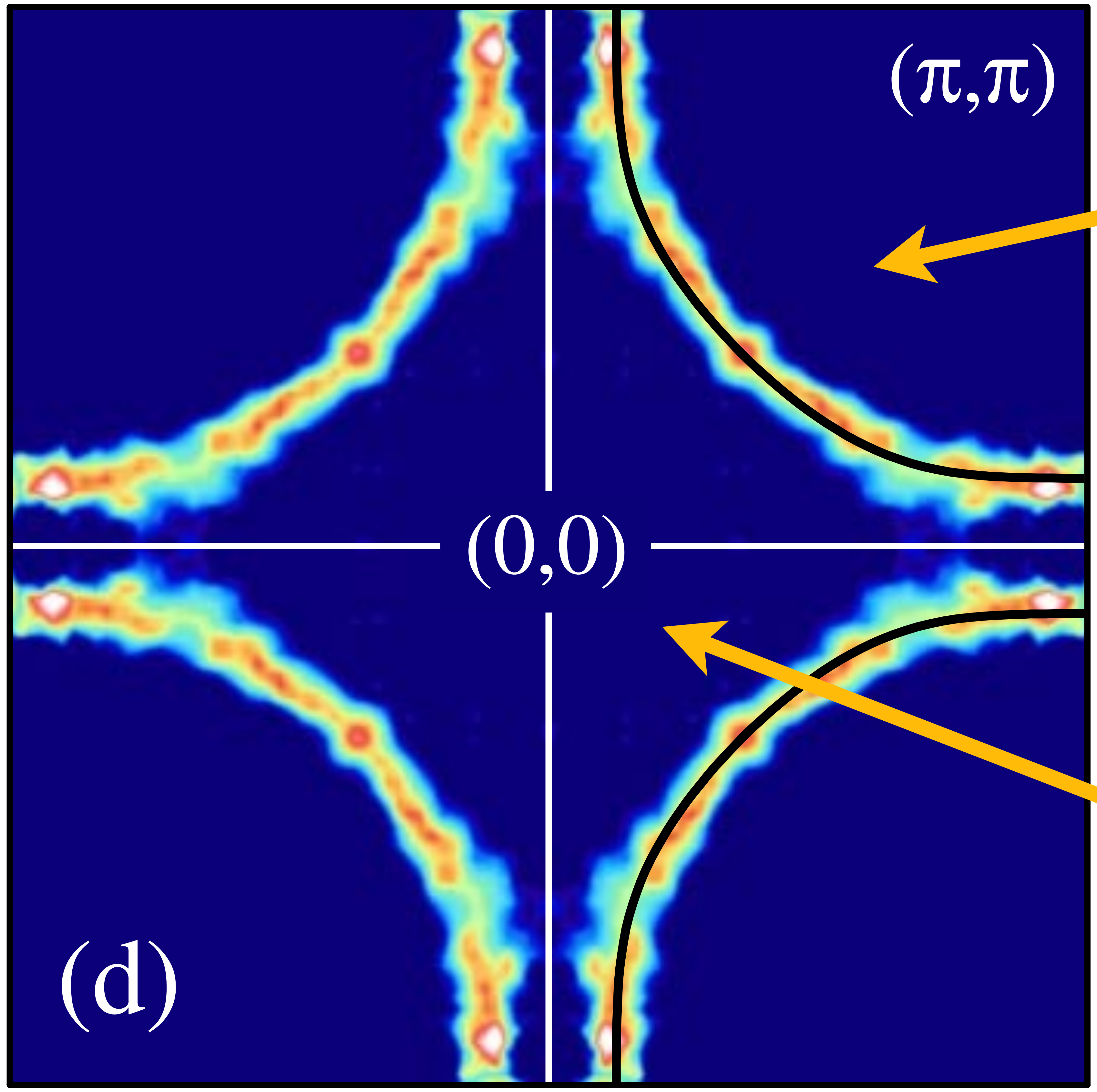




$YBa_2Cu_3O_{6+x}$

Photoemission at large p

Area enclosed by the Fermi surface is the same as that for free fermions with the same symmetry.
Luttinger, 1960 - perturbative



$1+p$ holes

1-p electrons

Overdoped $Tl_2Ba_2CuO_{6+\delta}$
$T_c = 30K$

$1+p$ mobile holes in a filled band

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

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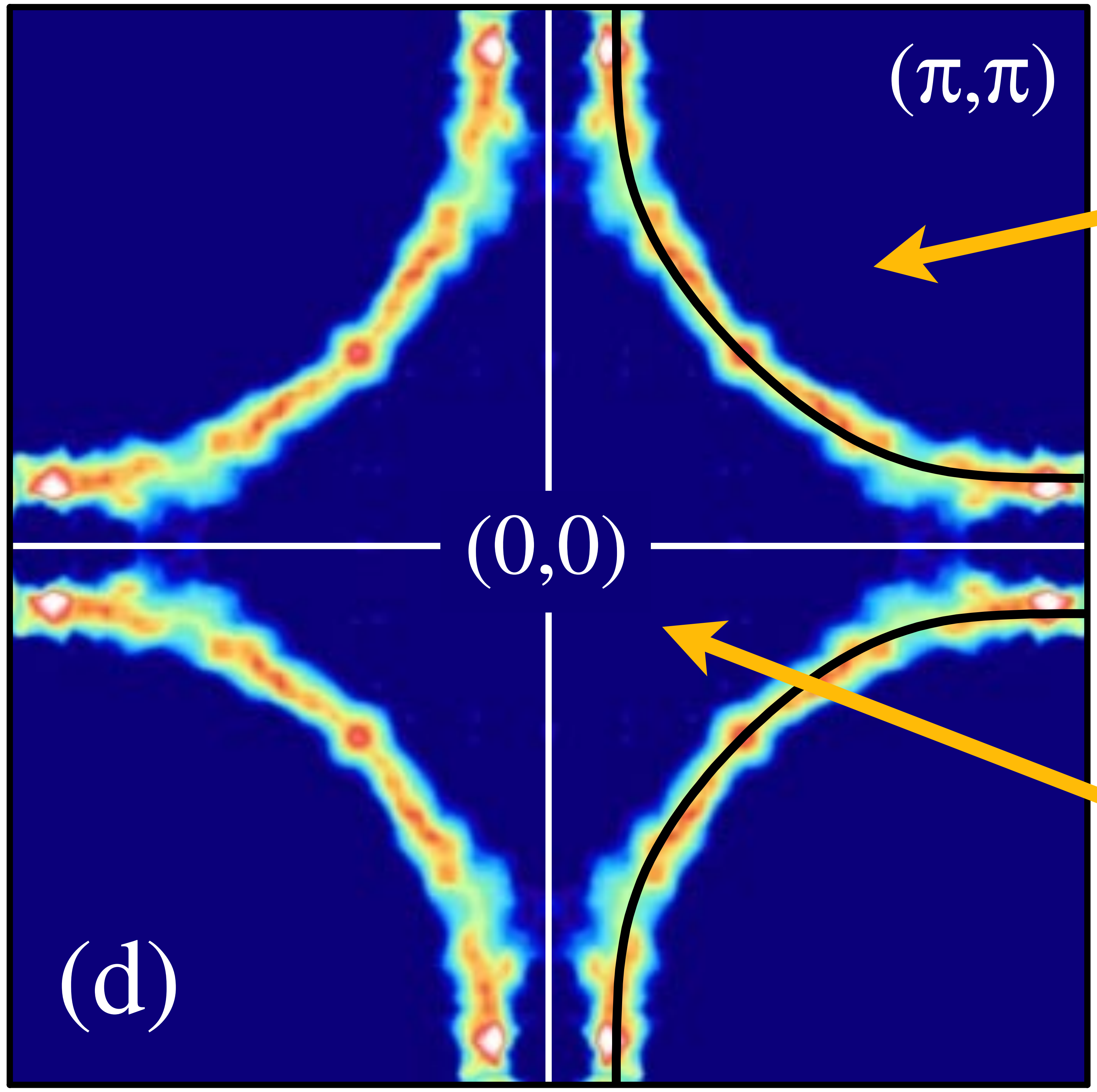

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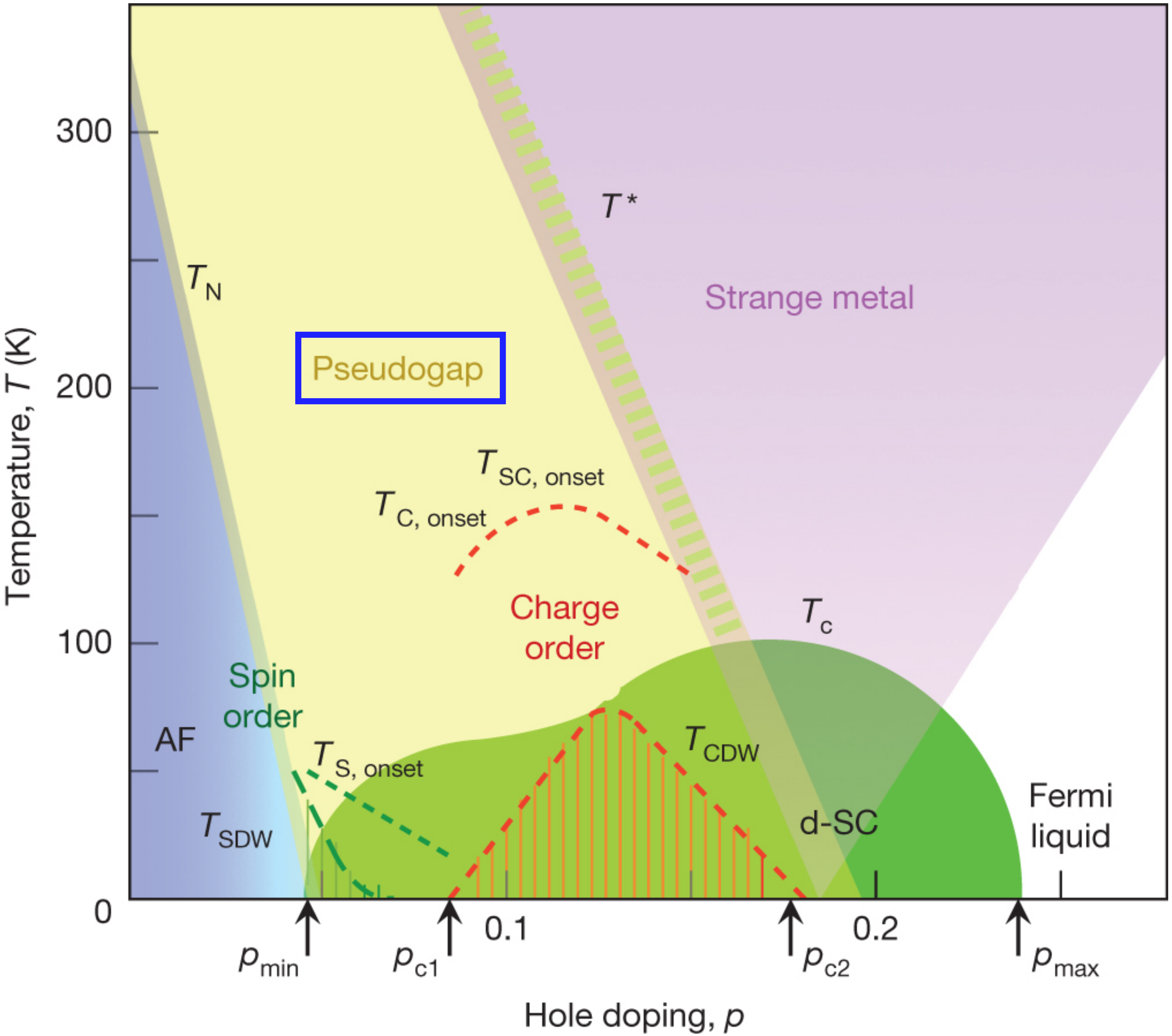
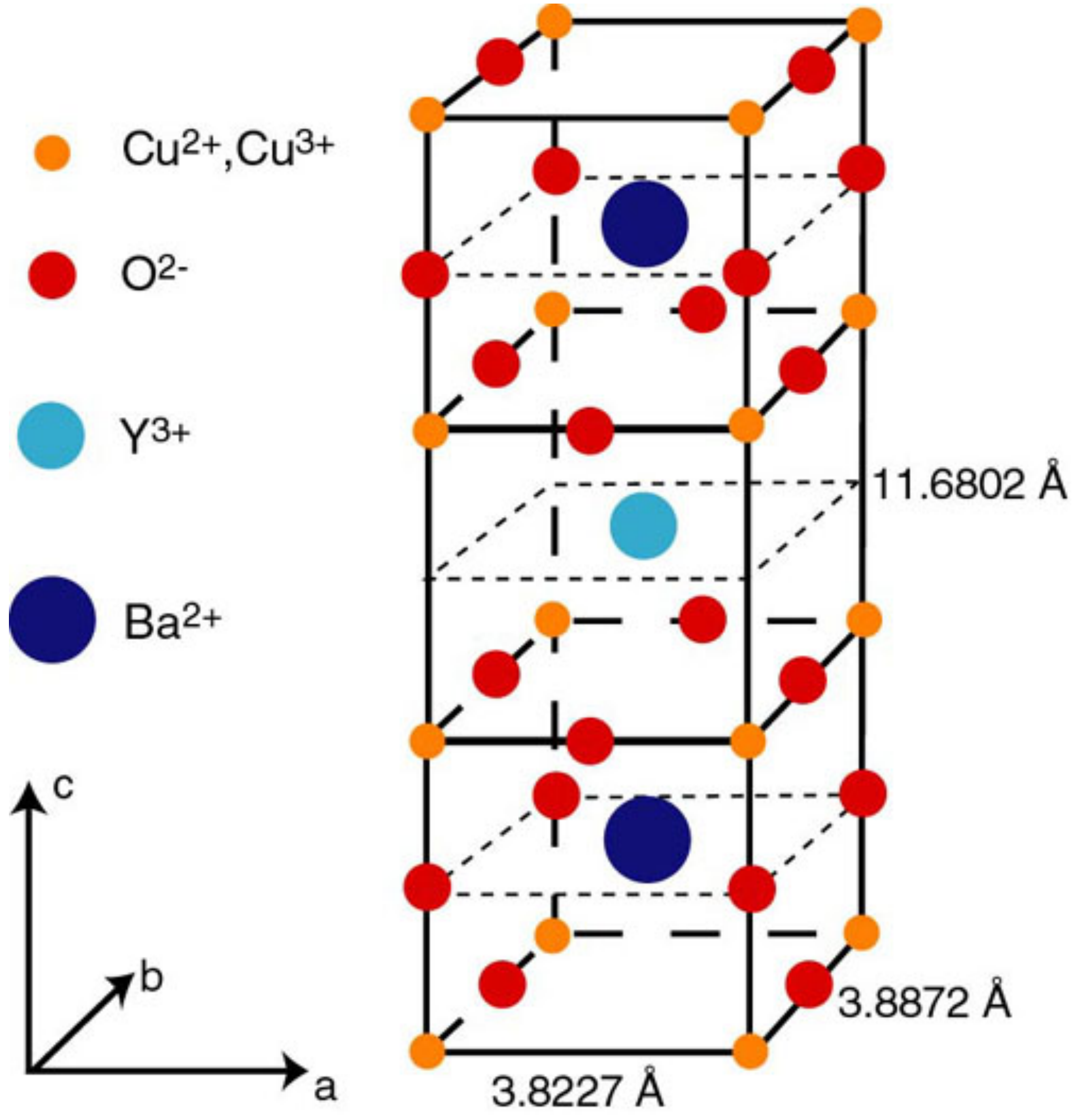
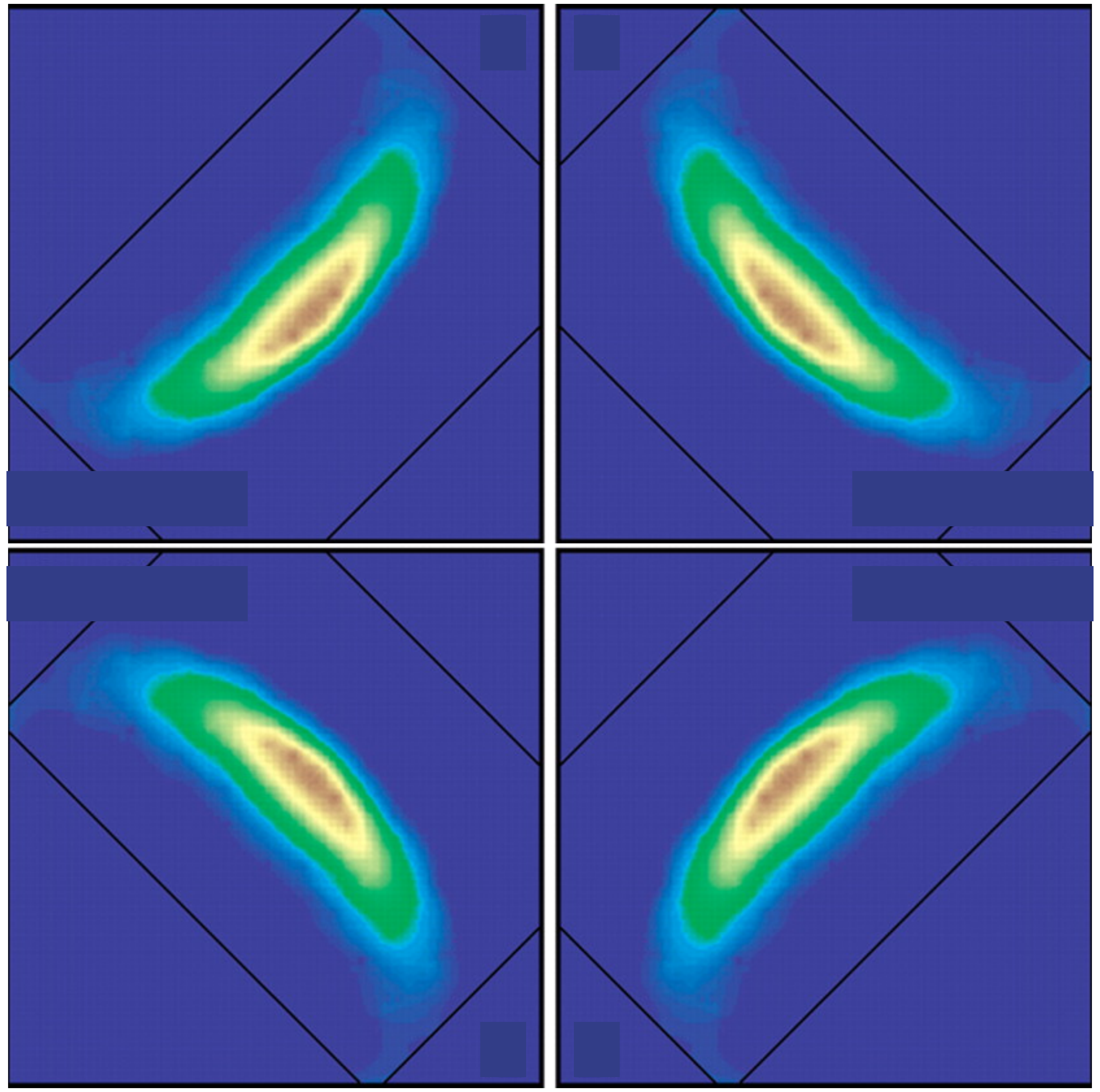




$YBa_2Cu_3O_{6+x}$

Photoemission at small p



$Ca_{2-x}Na_xCuO_2Cl_2$
at $x = 0.10$

"Fermi arcs"

Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

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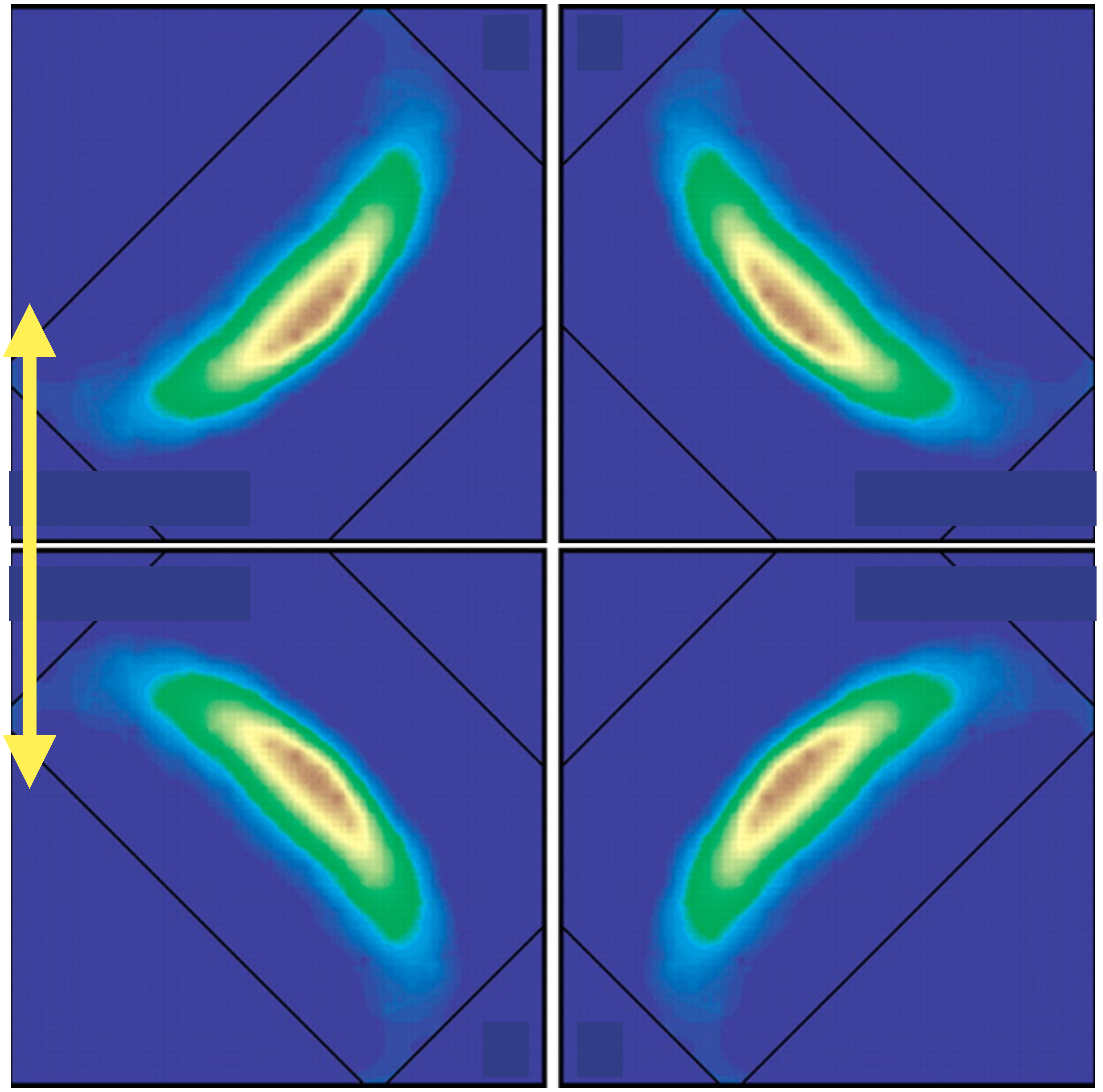

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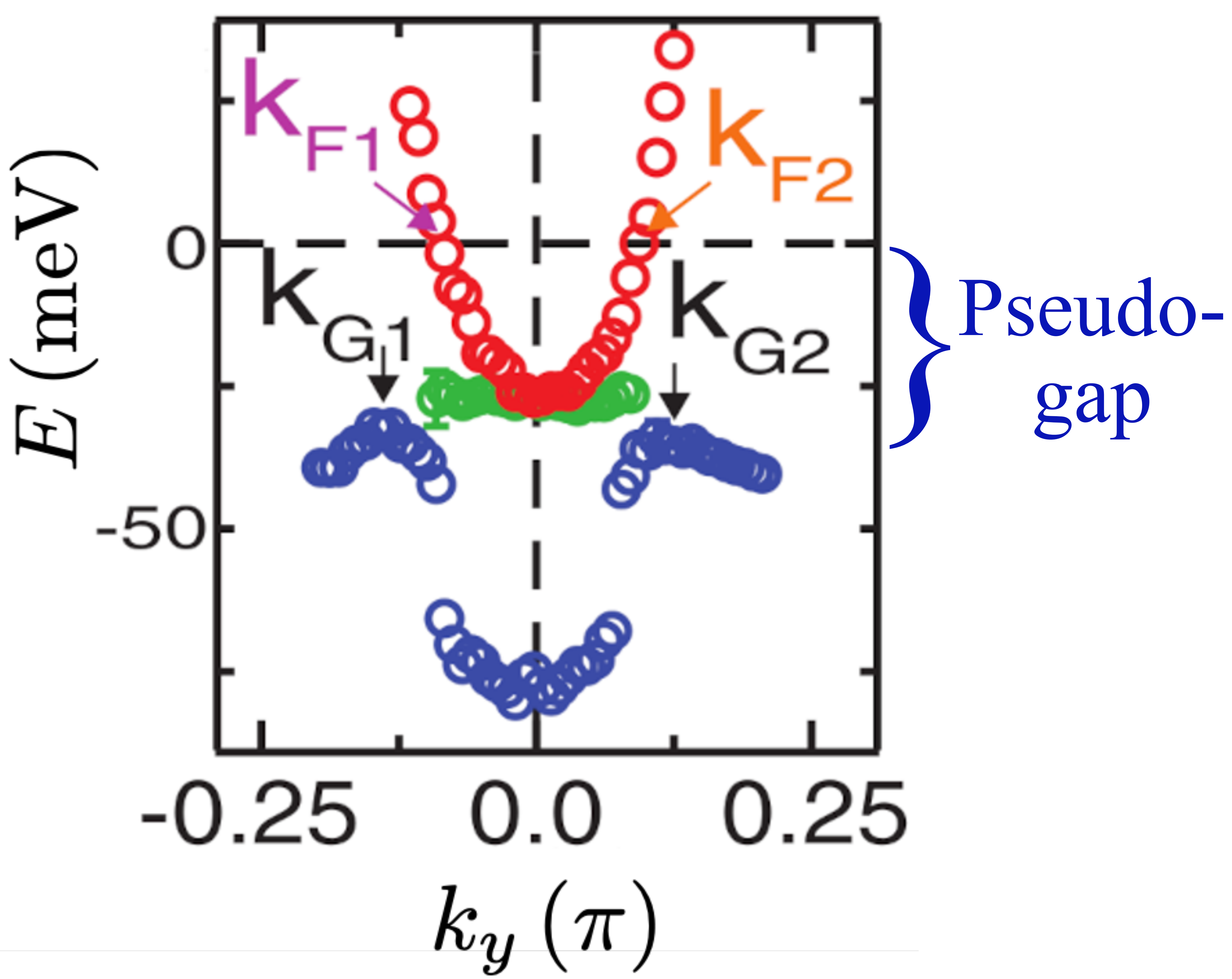
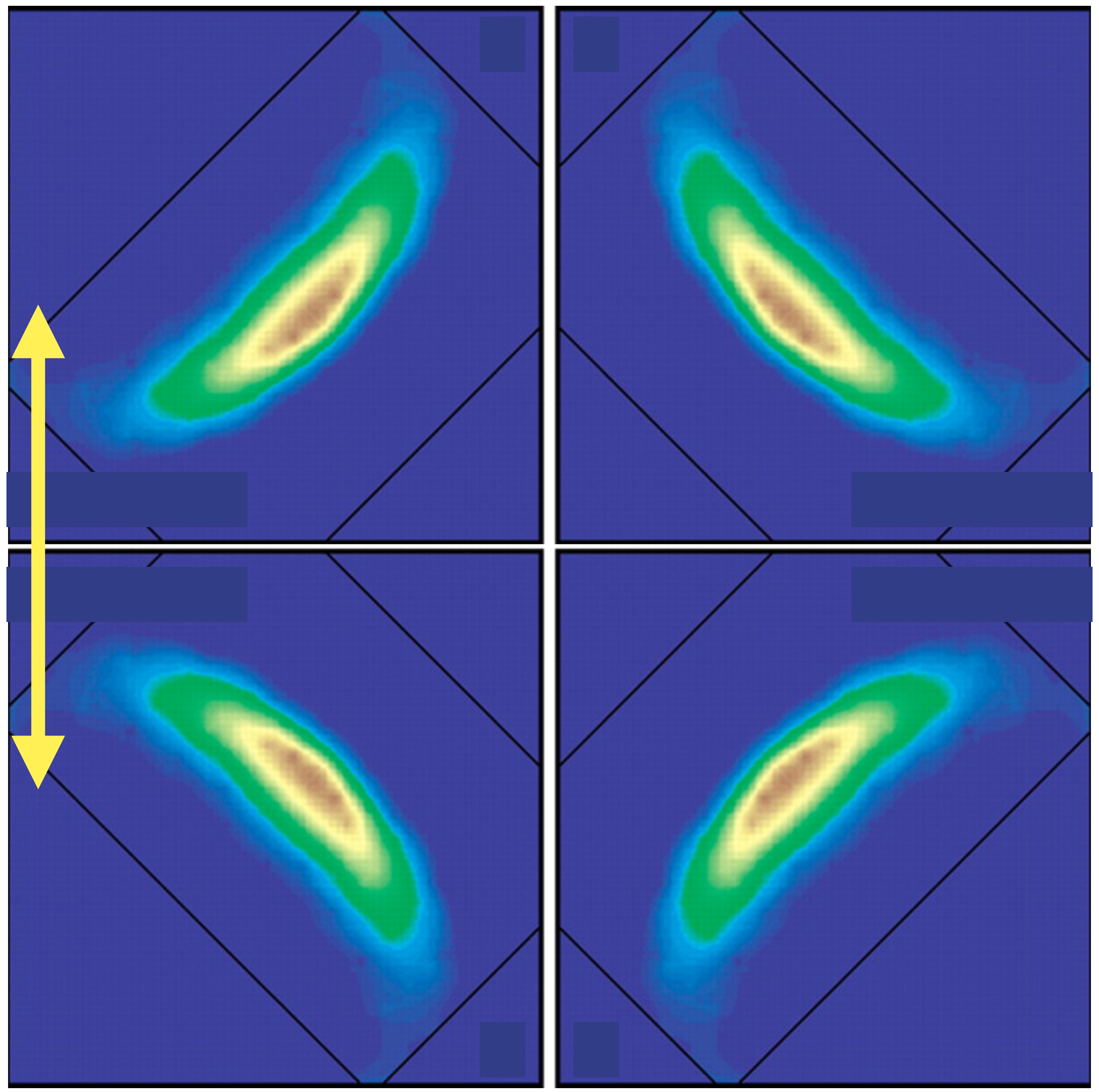


R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)



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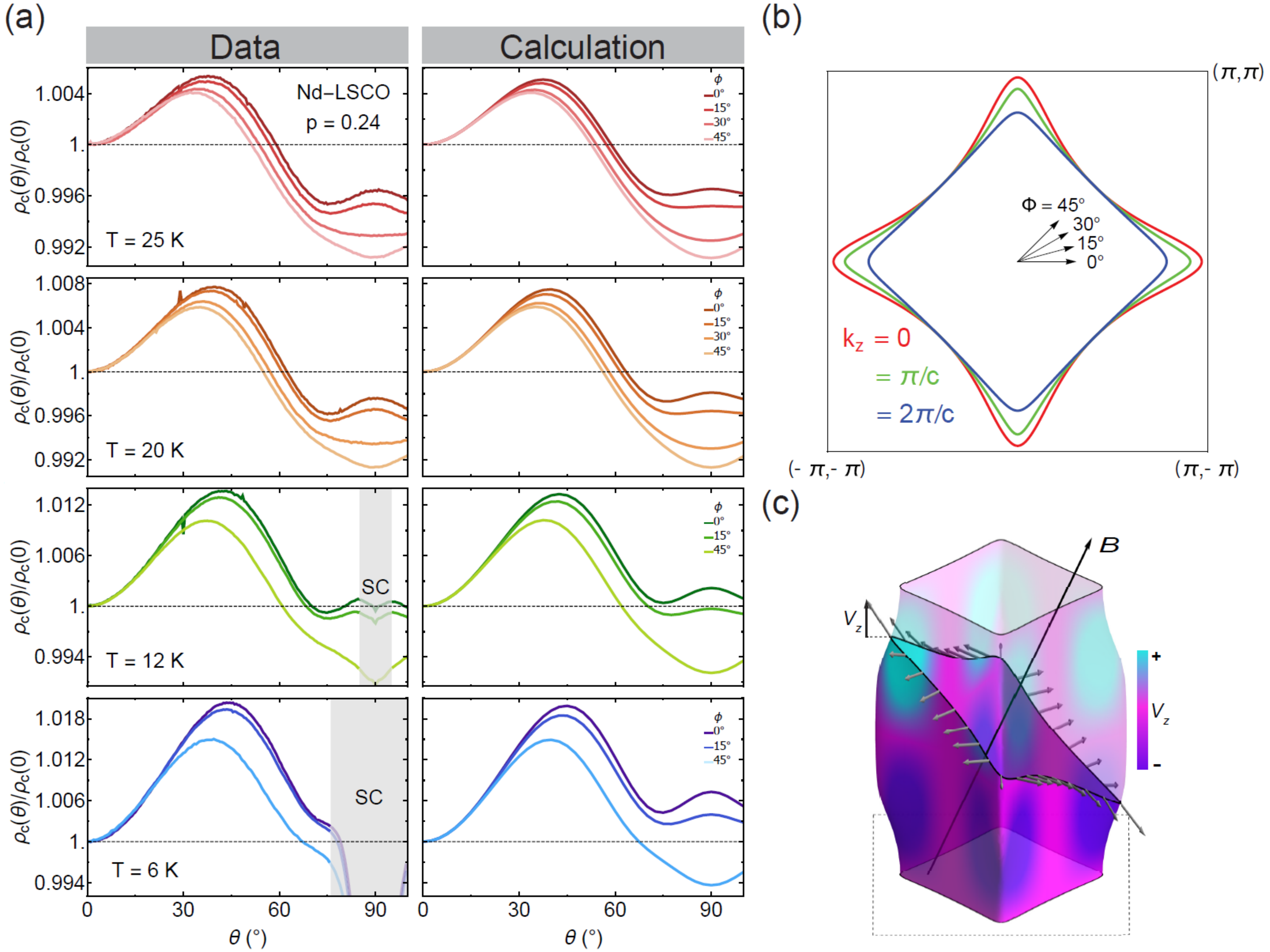
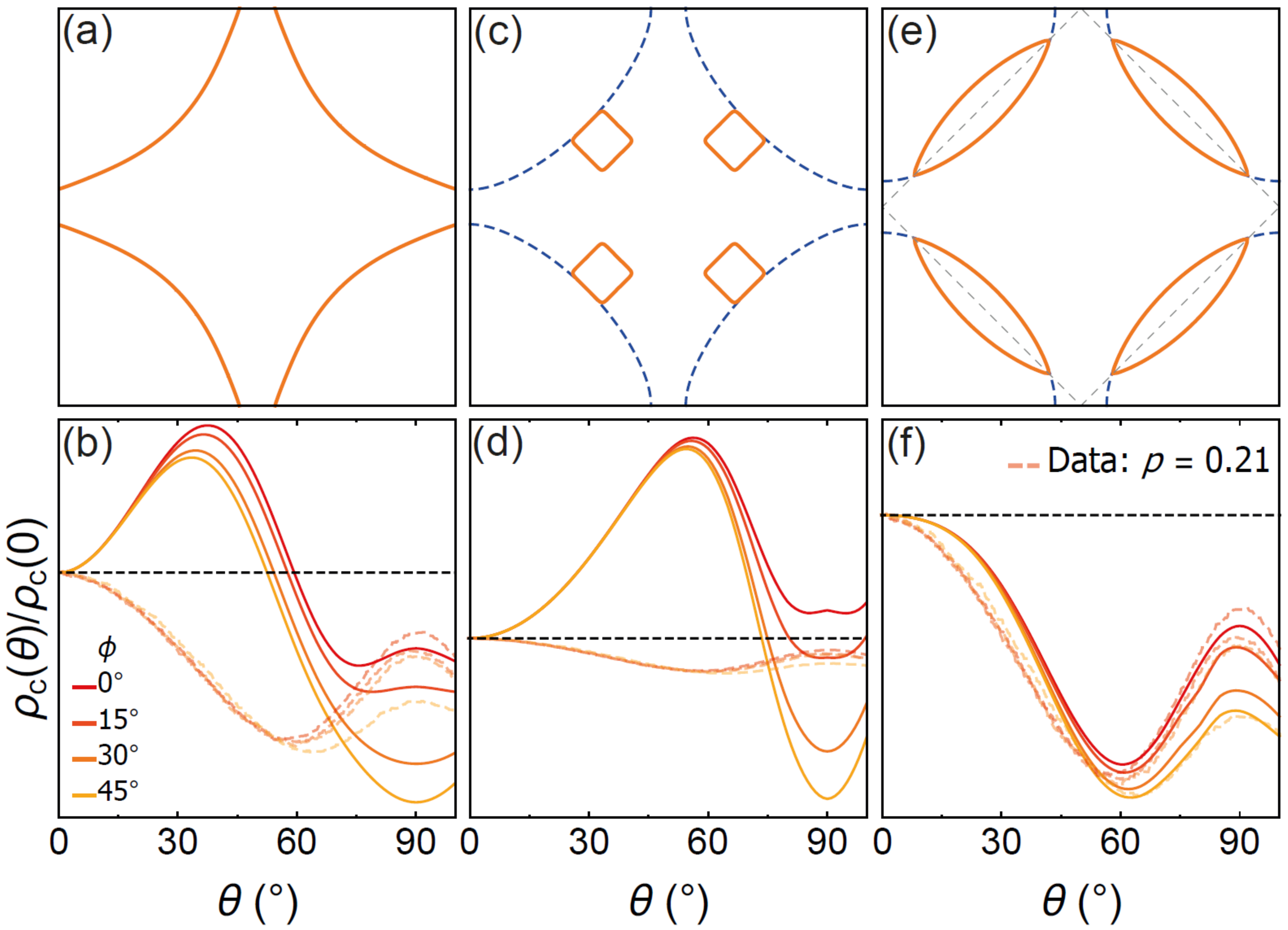
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei,
Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, *Nature Physics* **18**, 558 (2022)

Angle-dependent magnetoresistance (ADMR) of $La_{1.6-x}Nd_{0.4}Sr_xCuO_4$



$p > p_c$ Large Fermi surface

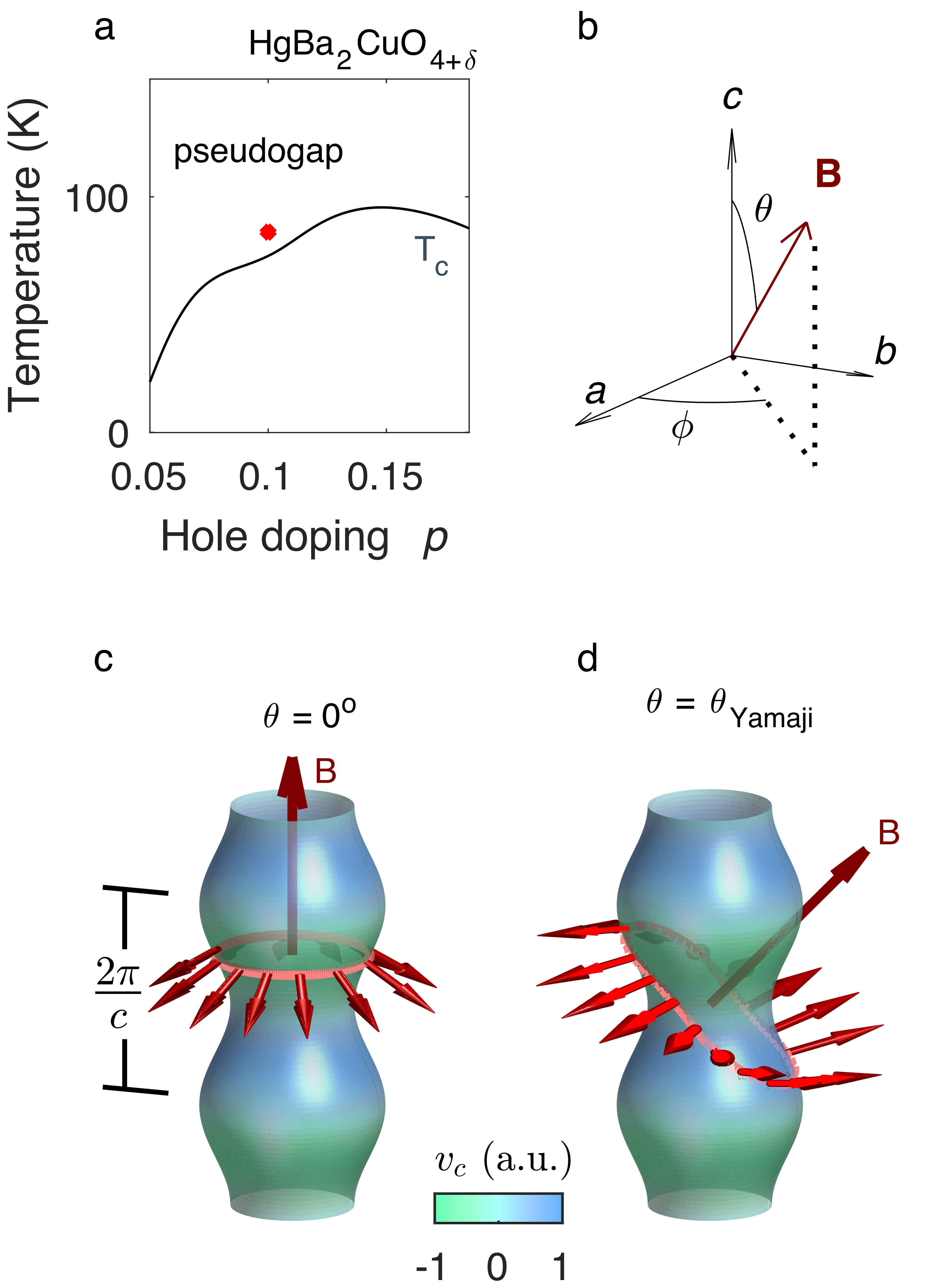


$p < p_c$ Reconstructed Fermi surface

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan,[1,*] Katherine A. Schreiber,[1] Oscar E. Ayala-Valenzuela,[1]
Eric D. Bauer,[2] Arkady Shekhter,[1] and Neil Harrison[1]

arXiv:2411.10631



At the Yamaji angle, the orbits in the plane orthogonal to $\boldsymbol{B}$ have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

K. Yamaji JPSJ **58**, 1520 (1989)

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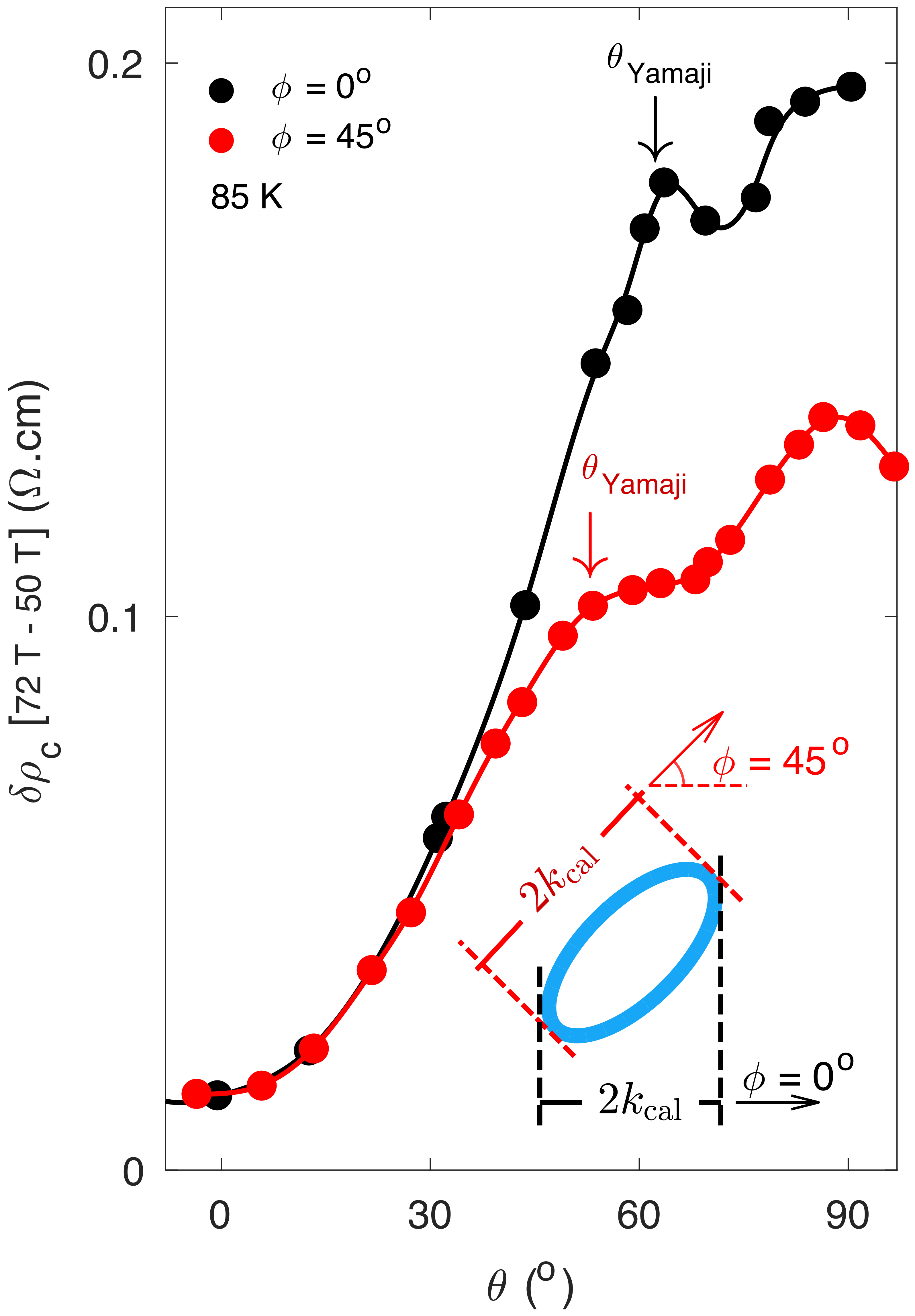

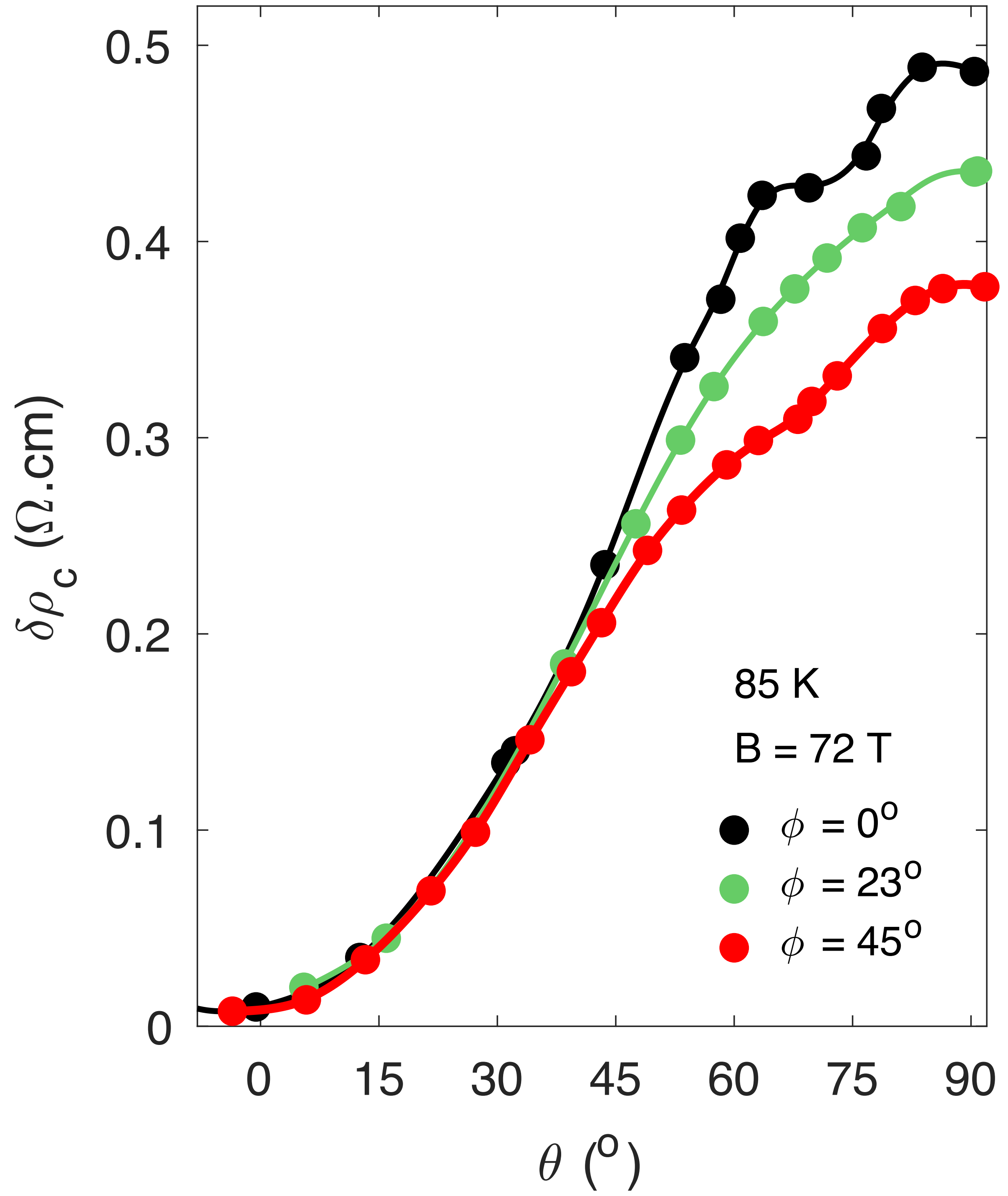

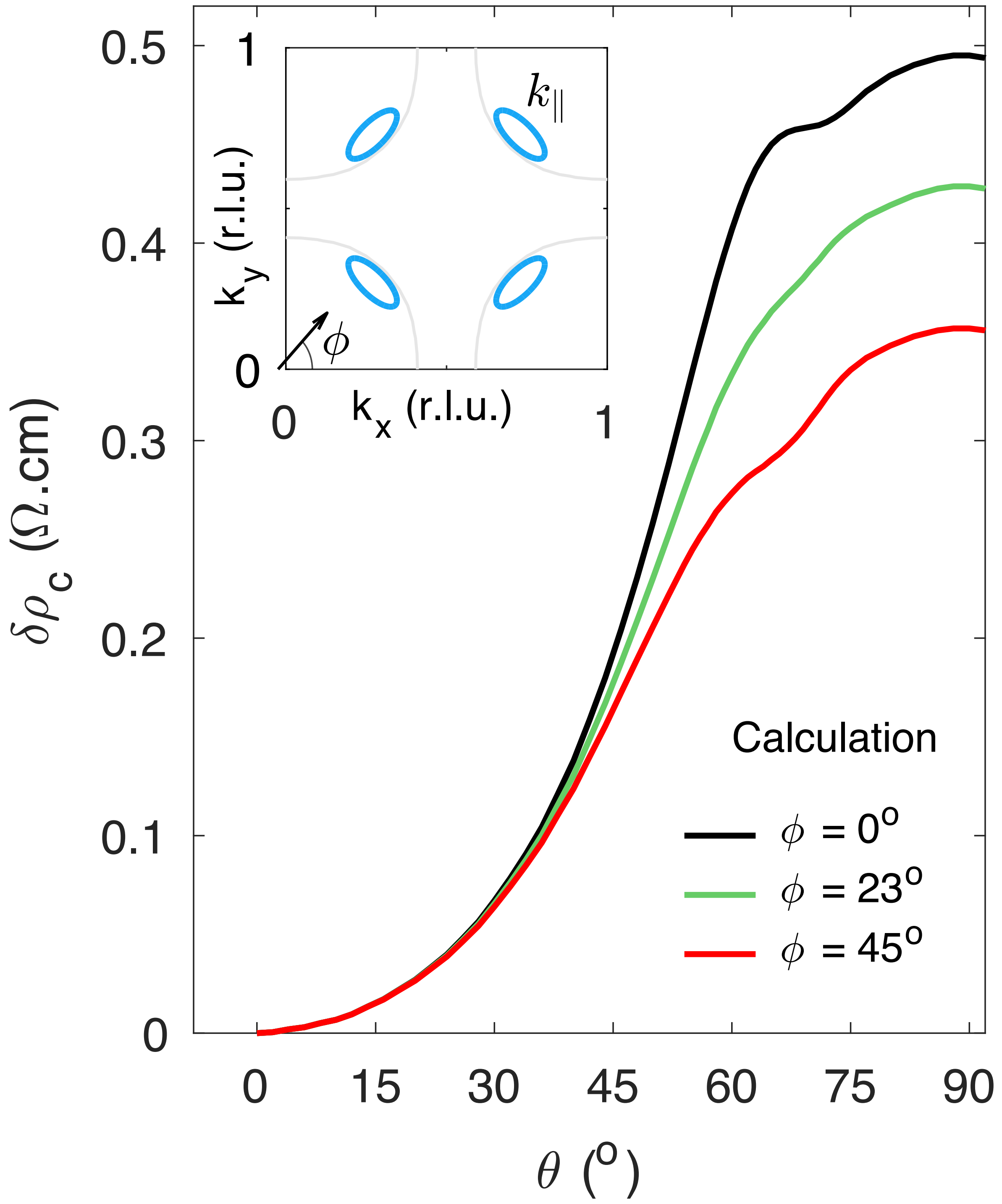

Doping
$p = 0.1$

"The small size of the pockets determined from the Yamaji effect is . . .
approximately 1.3% of the Brillouin zone area"

Experiments on the cuprate pseudogap phase

"Fermi arcs" or hole pockets??

Introduction to FL* theory of the pseudogap

Ordinary metals

Area enclosed by the Fermi surface is the same as that for free fermions with the same symmetry

Luttinger, 1960 - perturbative;

Oshikawa, 2000 - non-perturbative anomaly matching

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Fractionalized Fermi liquids (FL*)

There can be metals with Fermi surface area not equal to the Luttinger value provided the fractionalized excitations of a spin liquid are also present.
The sum of the Fermi surface and spin liquid anomalies equals the Oshikawa anomaly.

(Anomaly → flux piercing on a torus)

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, S.S., PRB **69**, 035111 (2004)

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The FL* state Fermi surfaces enclose area $(\rho - 1)$ mod 2, where ρ is the total electron density.

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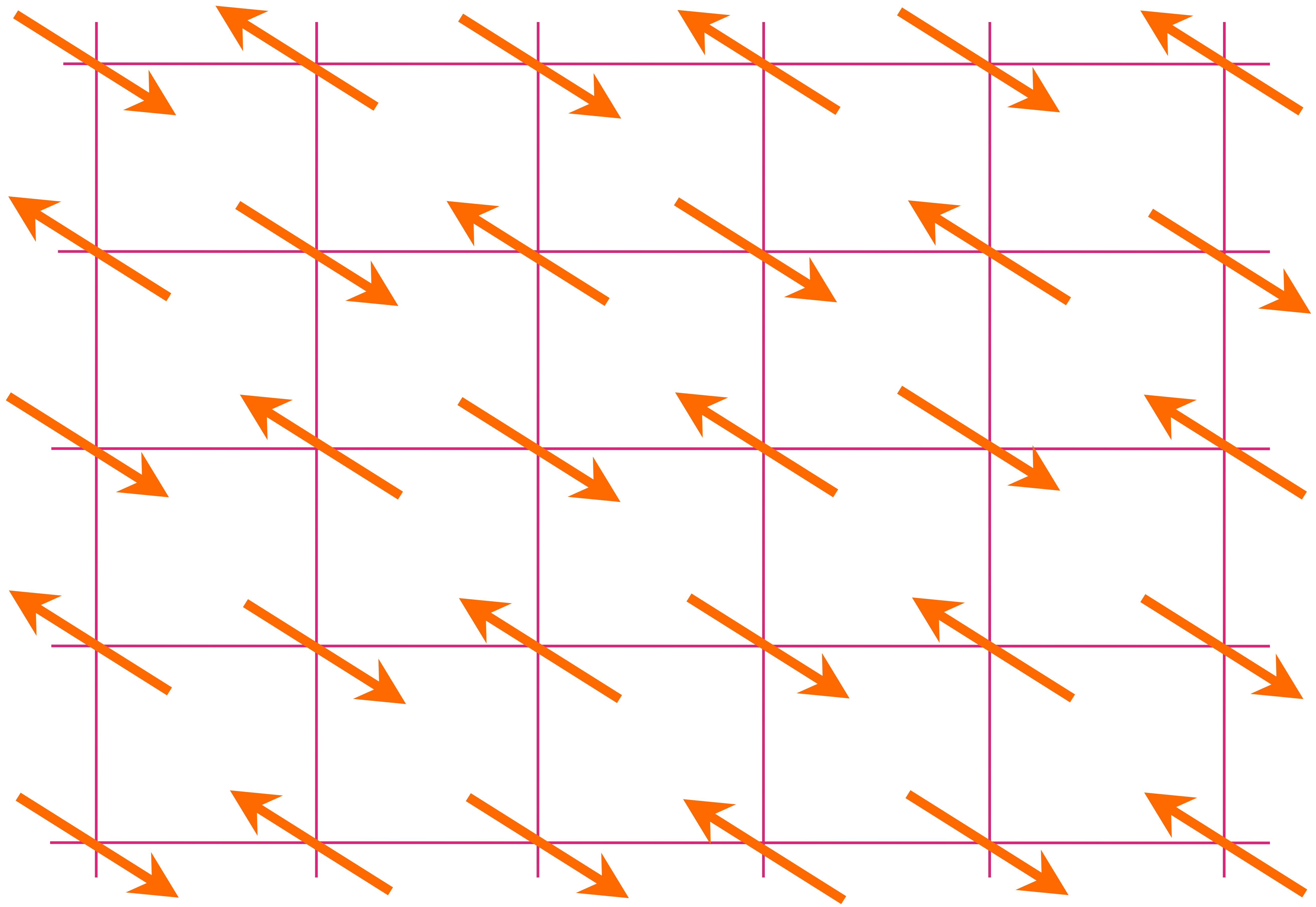
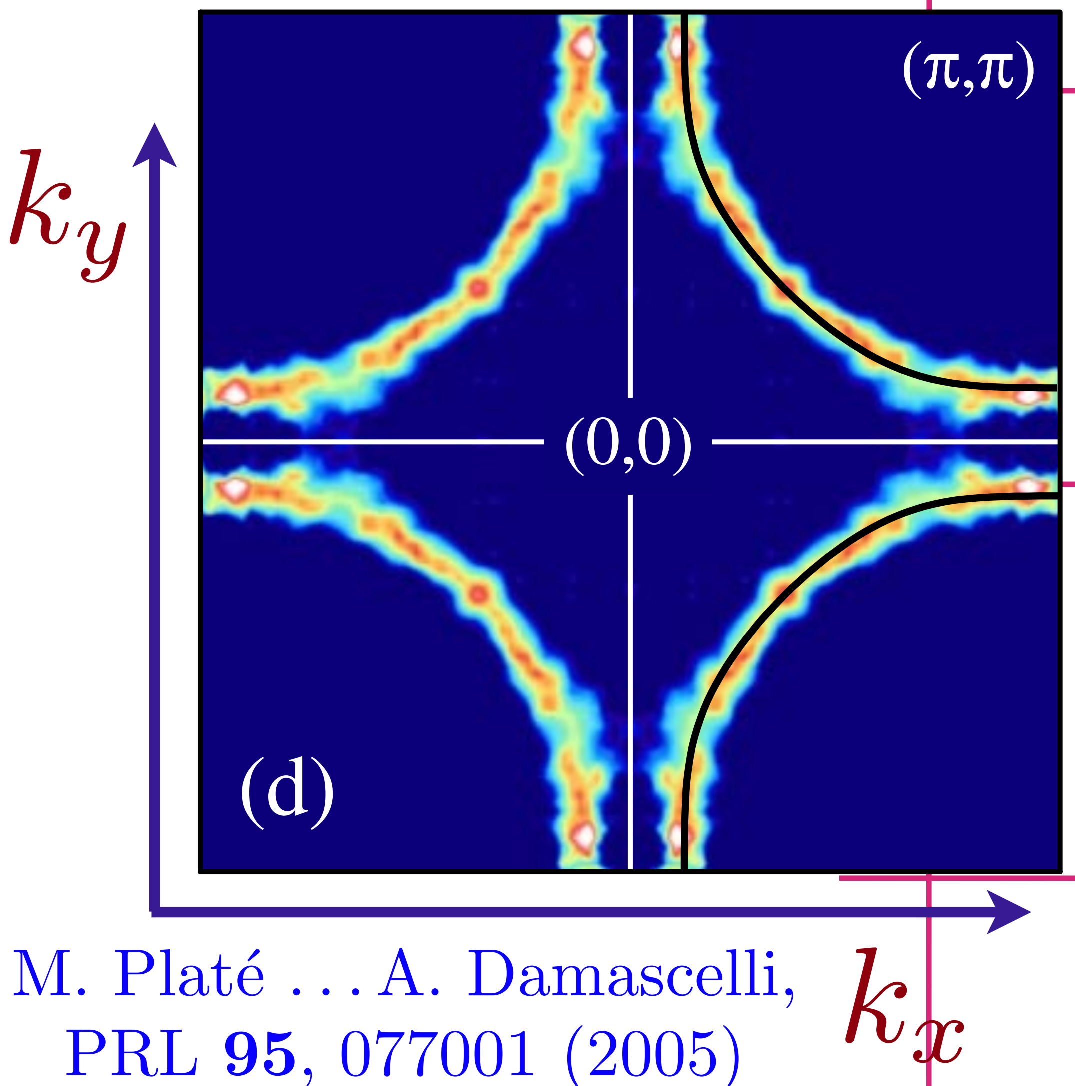
Insulating antiferromagnet

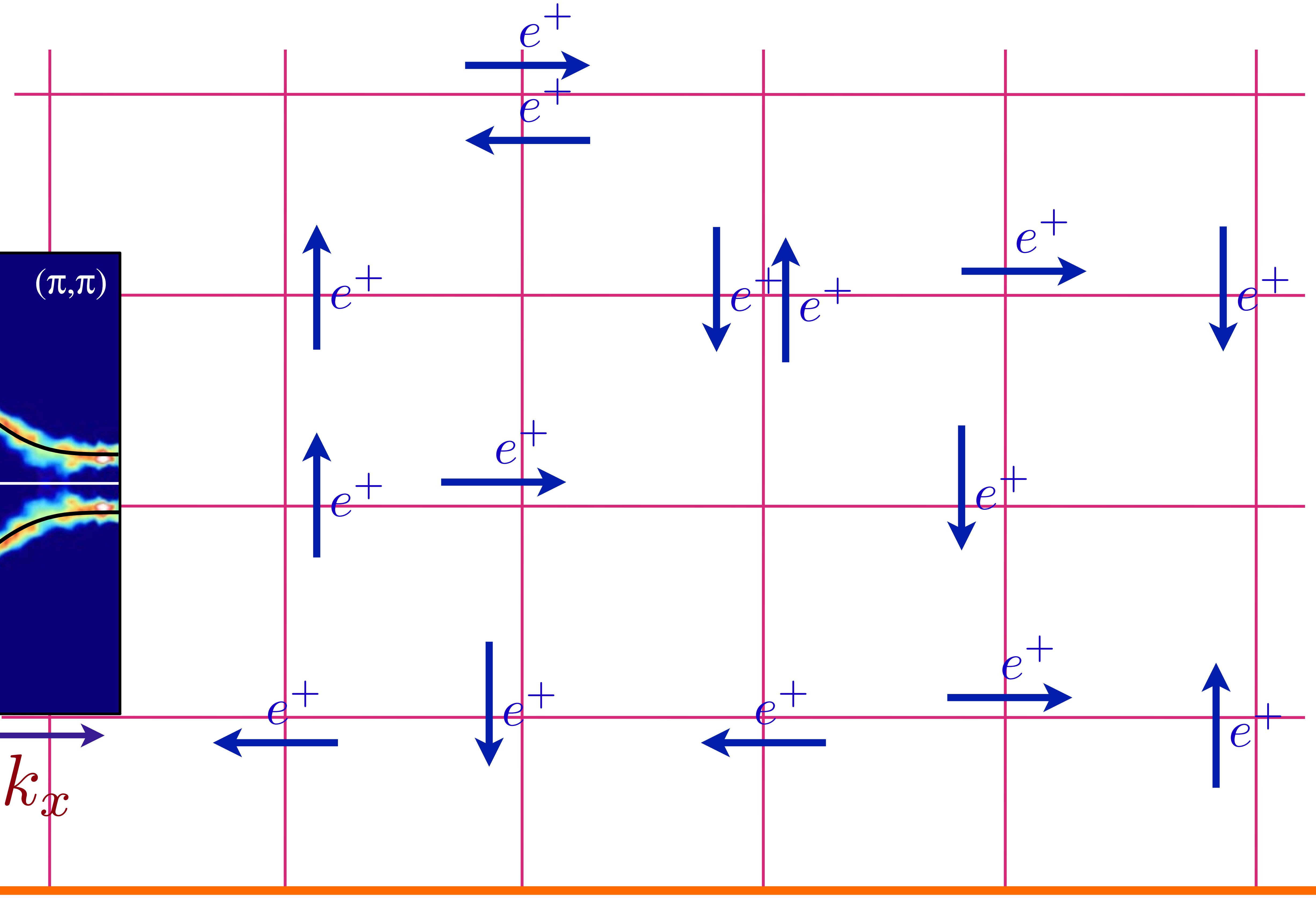


Doping an insulating antiferromagnet with holes of density p

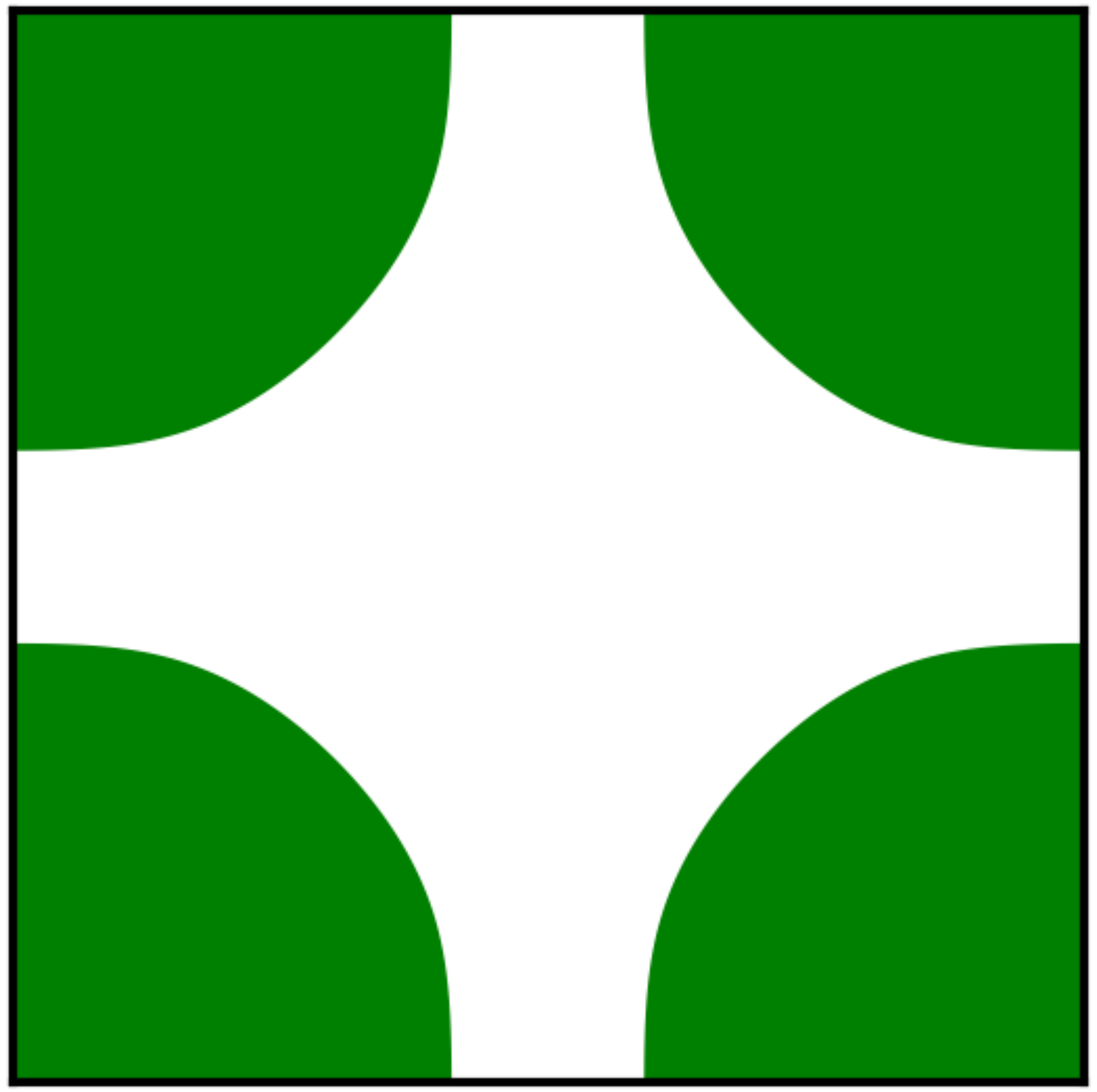
Ordinary metal



M. Platé ... A. Damascelli, PRL **95**, 077001 (2005)



Luttinger area.
No broken symmetry

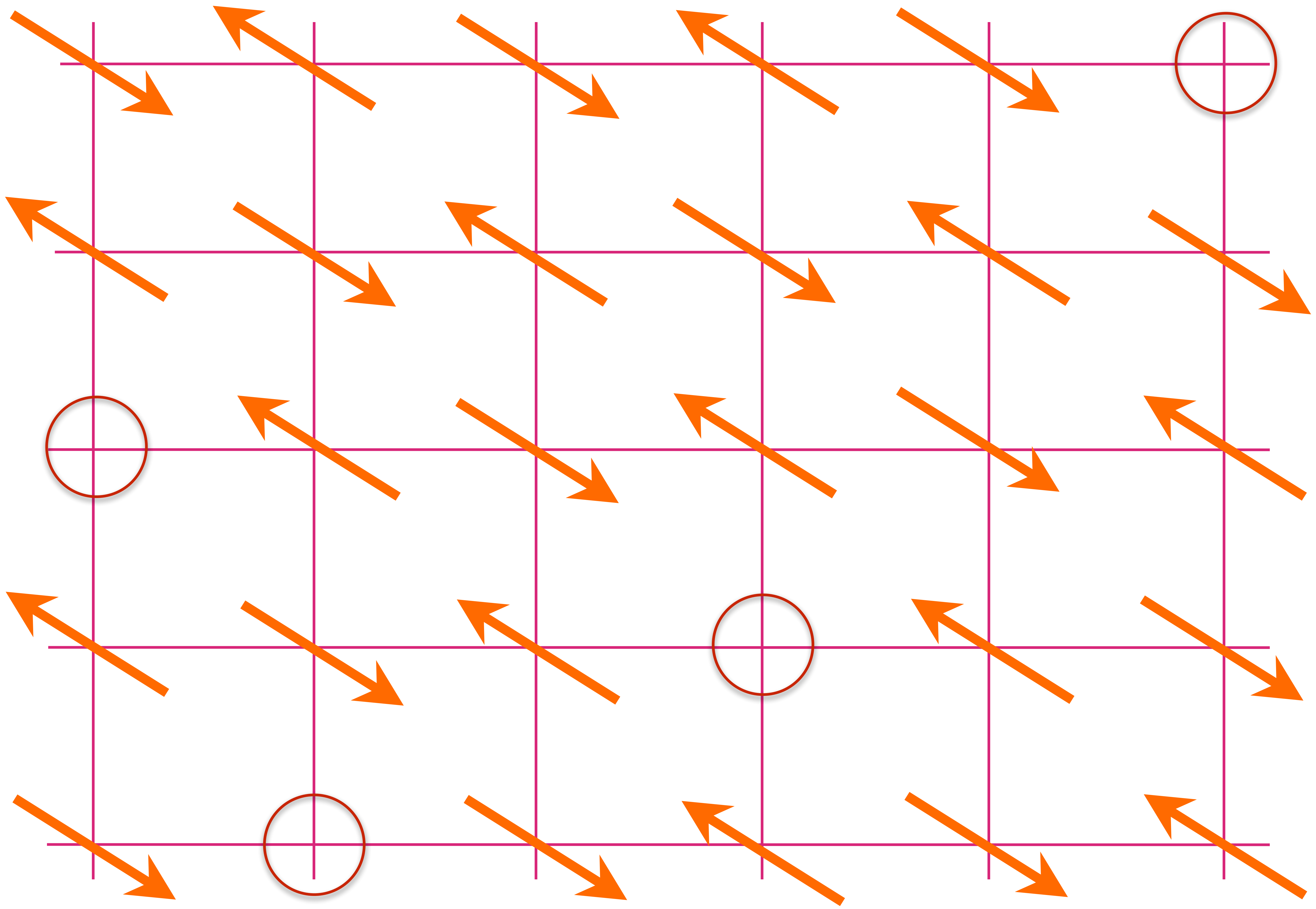


Area $(1 + p)/2$

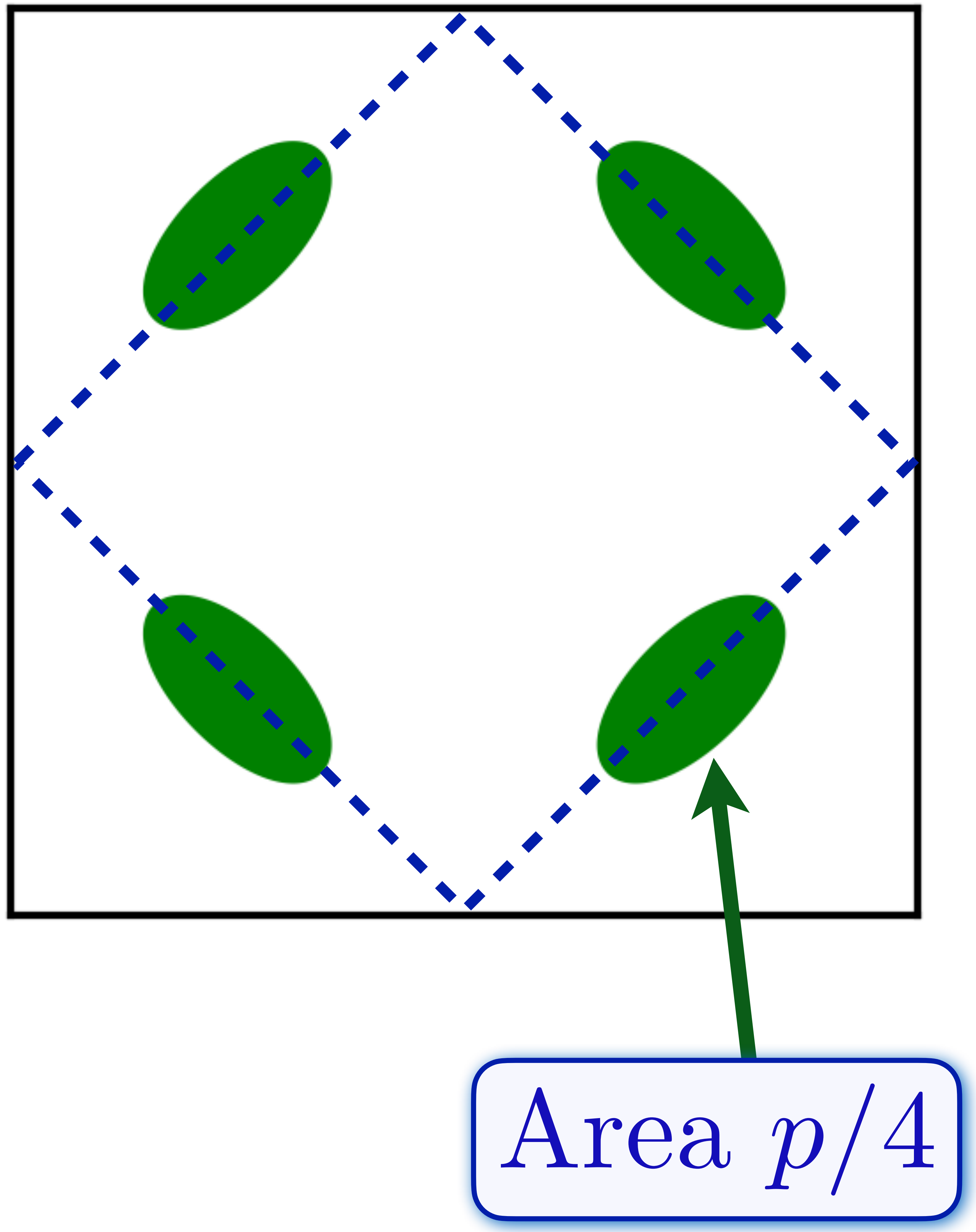
At large p, we obtain a gas of nearly free fermionic holes of density $1+p$ (relative to the filled band with 2 electrons per site)

Doping an insulating antiferromagnet with holes of density p

AF metal



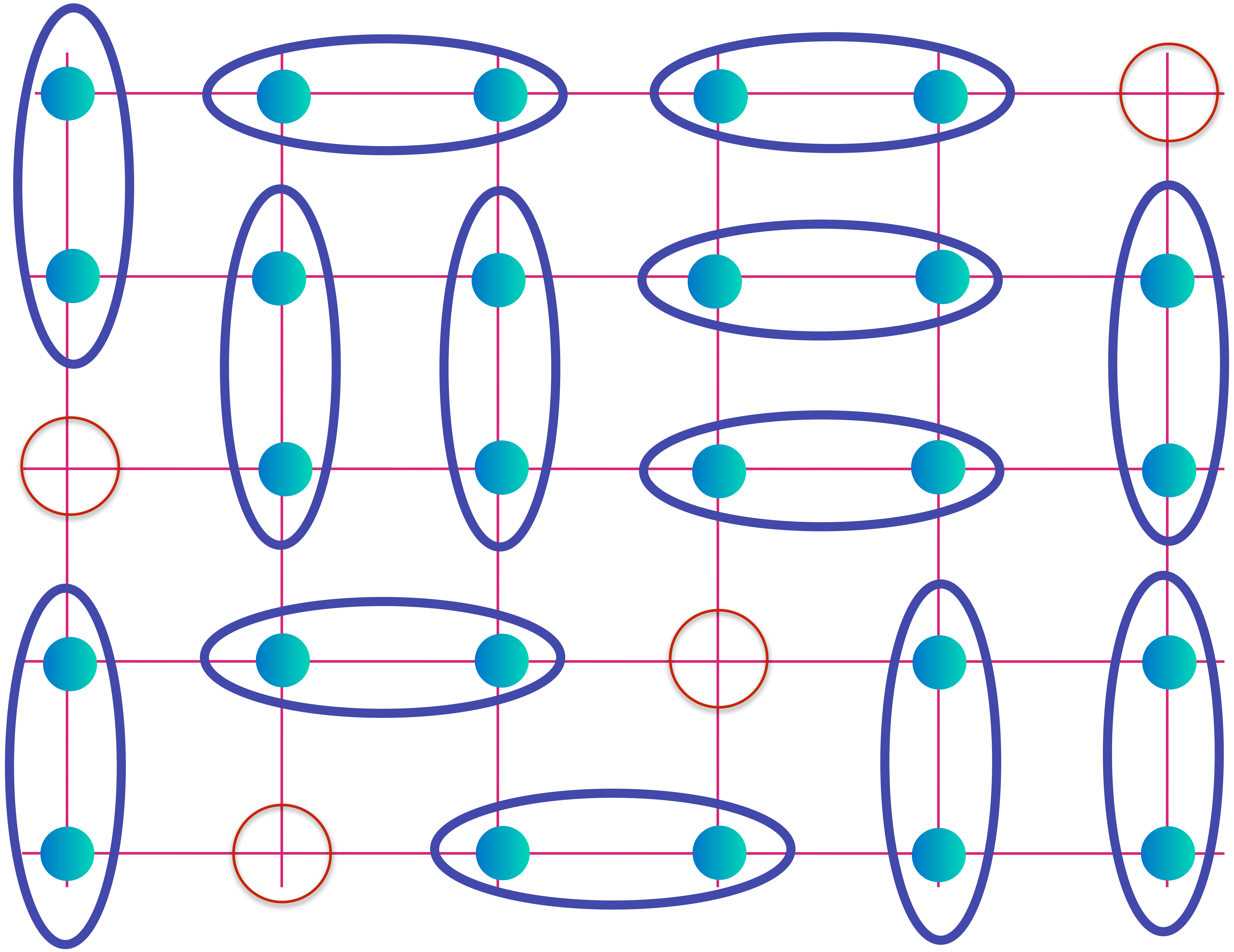
Luttinger area.
Broken symmetry

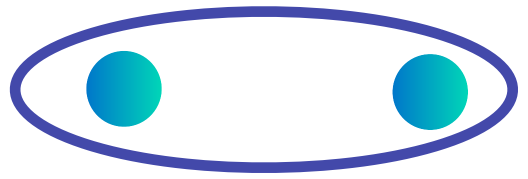


Doping an insulating antiferromagnet with holes of density p

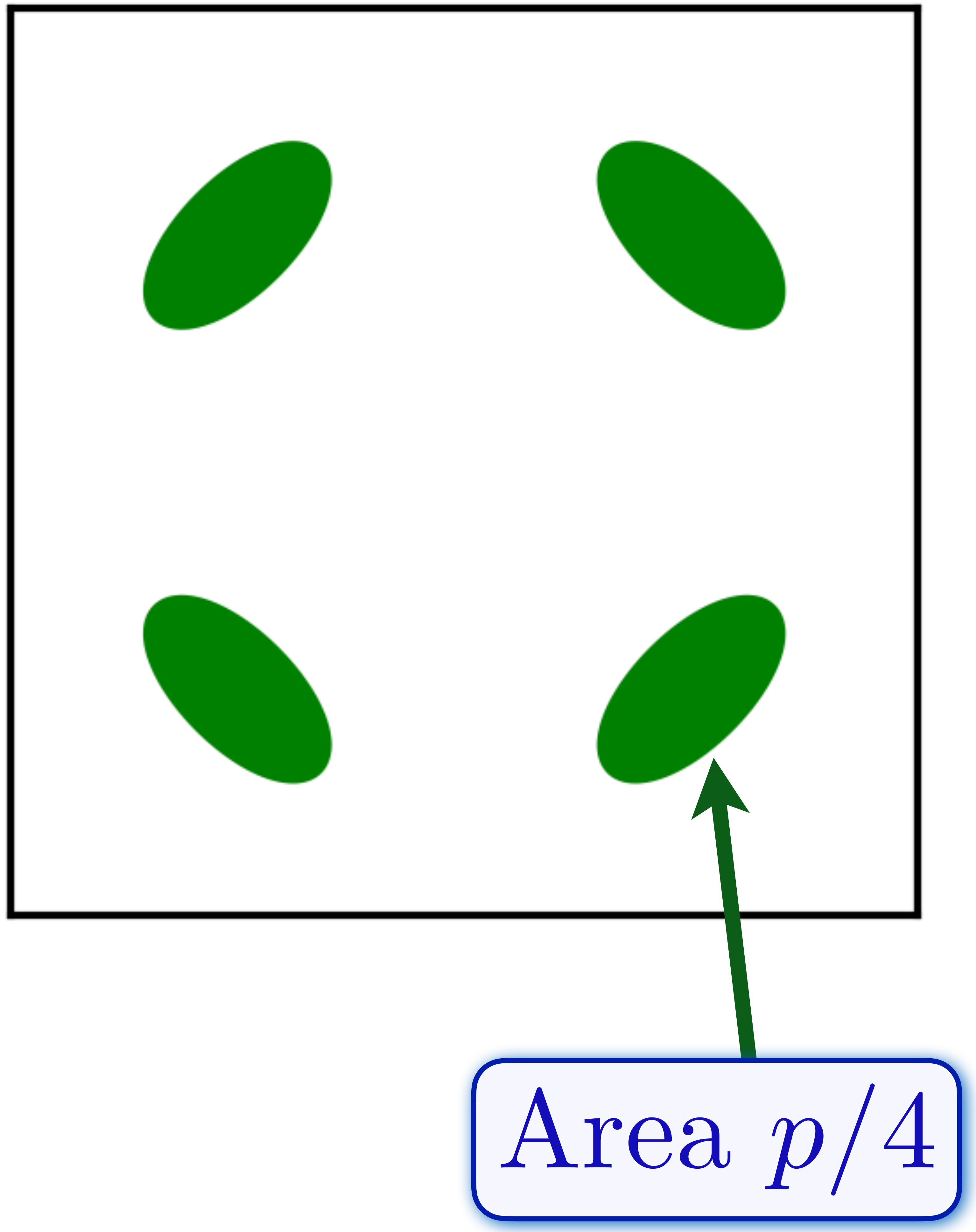
Holon metal

Spin liquid with density p of spinless, charge +e "holons".



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

non-Luttinger area.
Spin liquid



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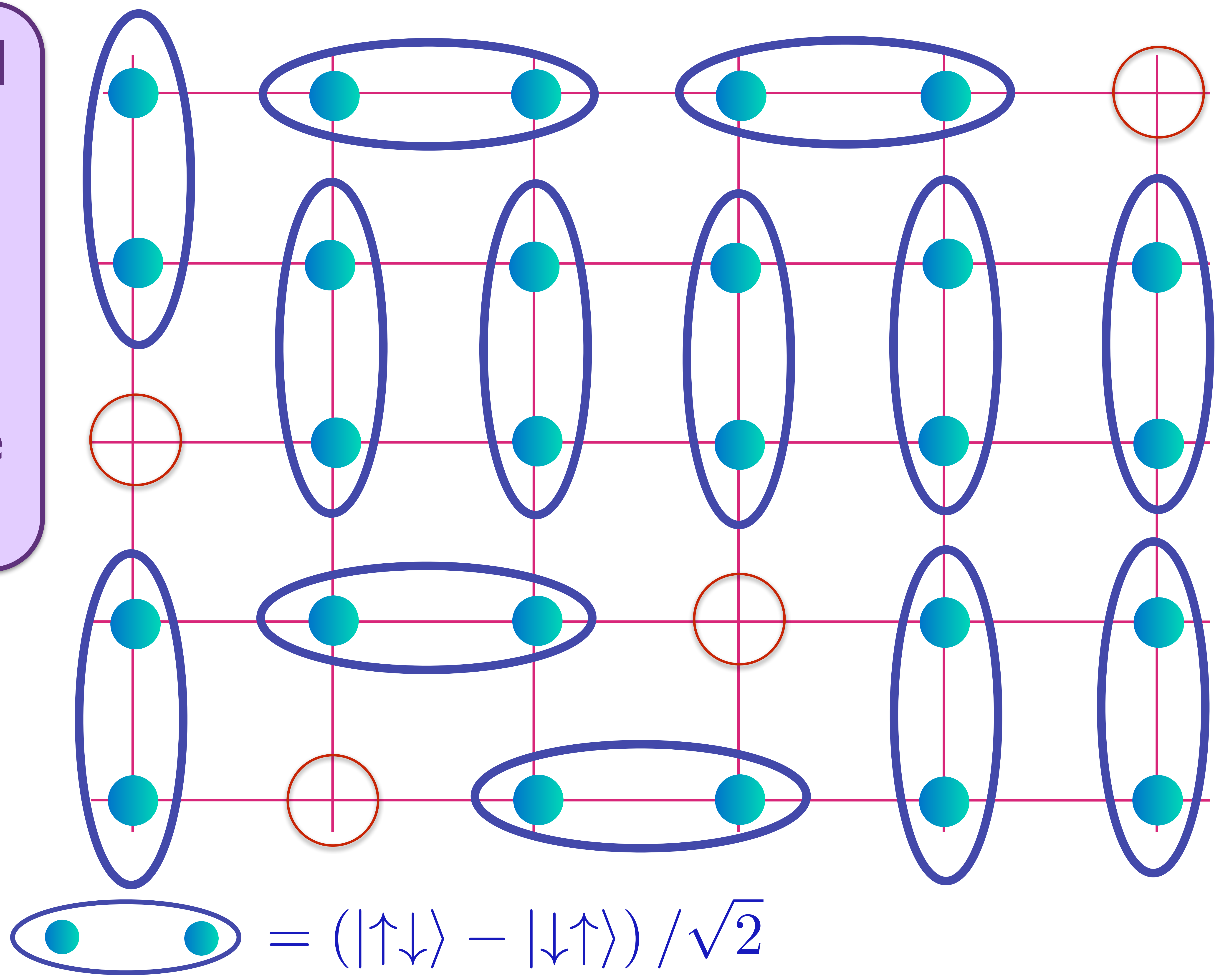

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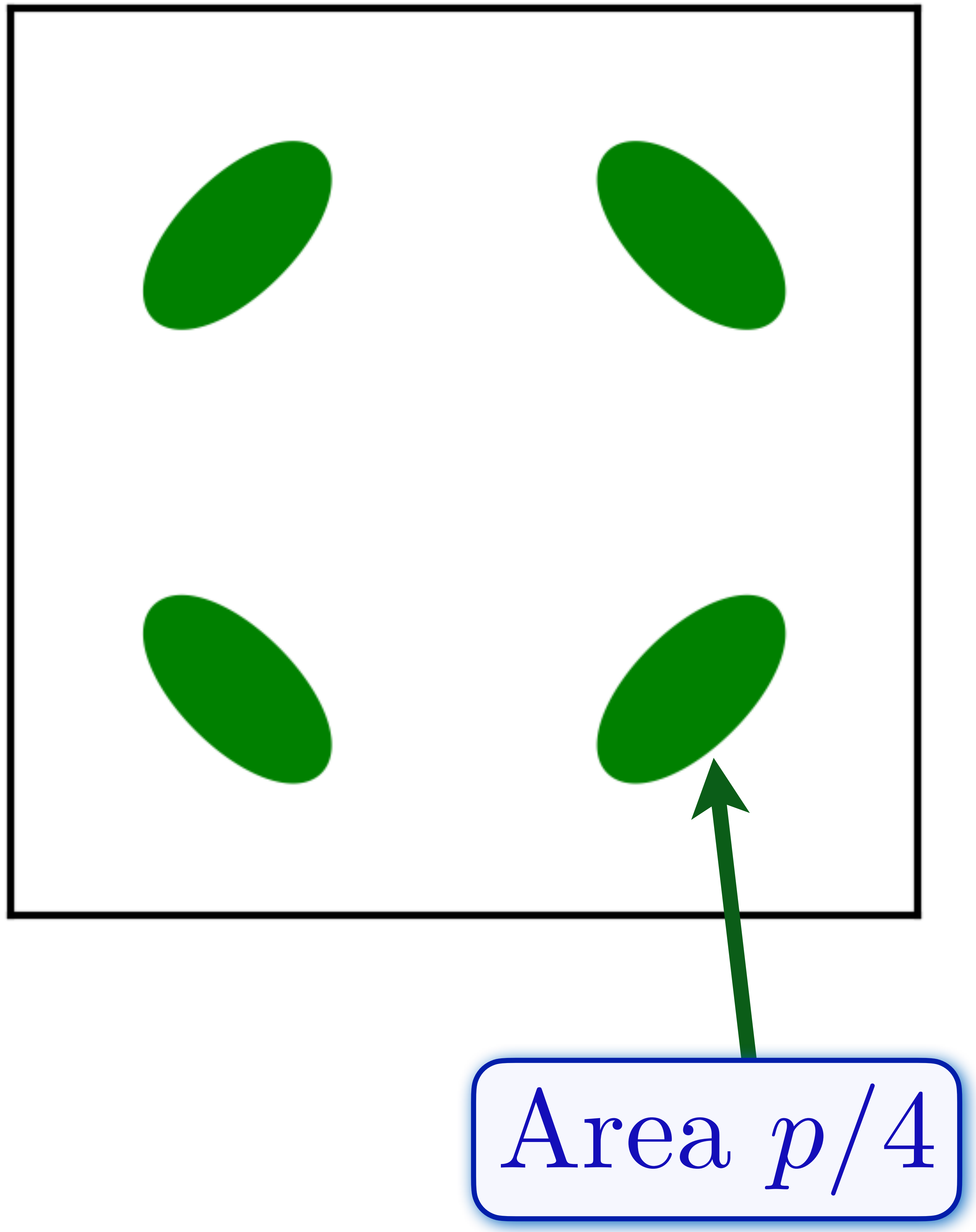

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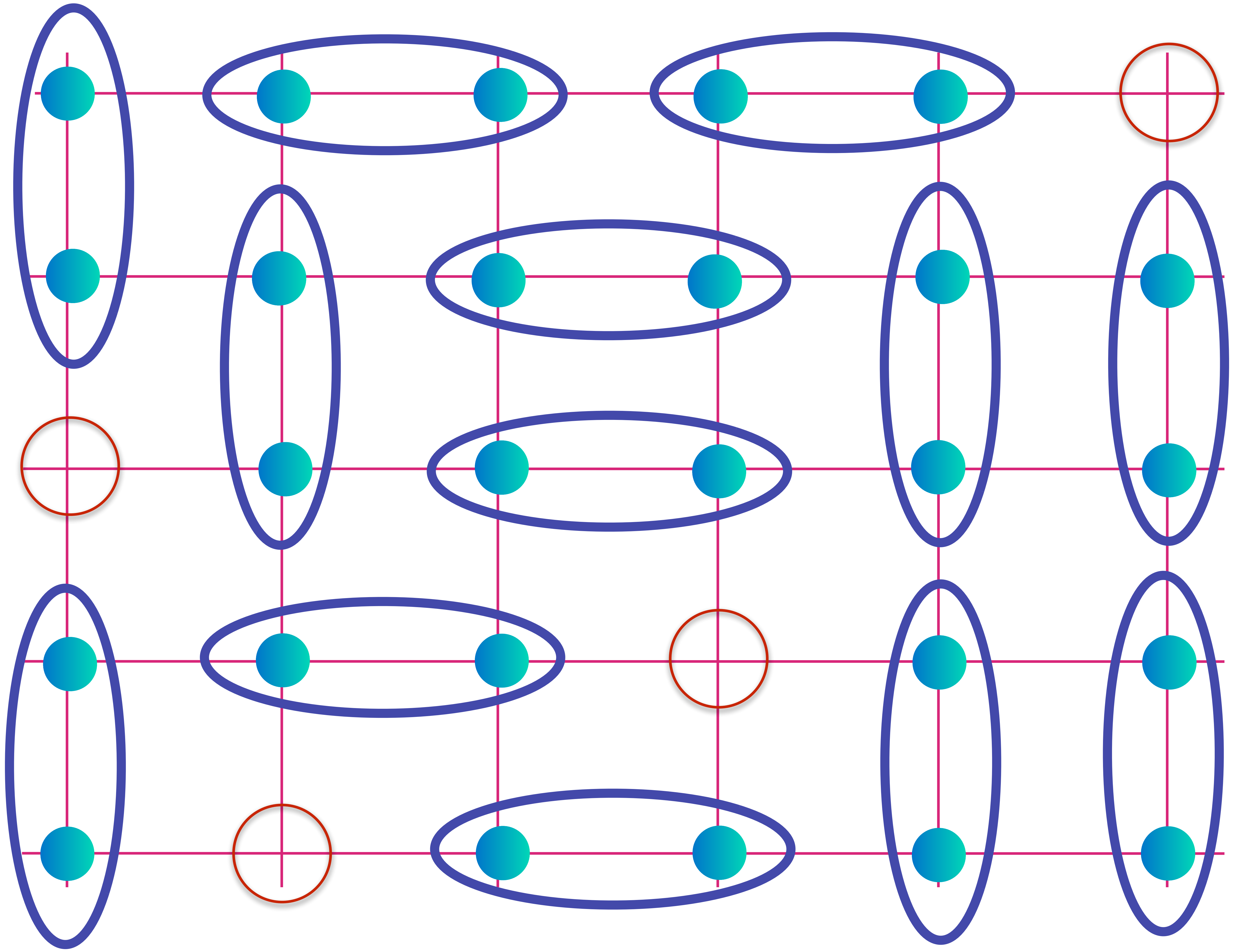

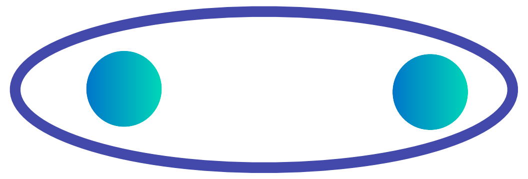 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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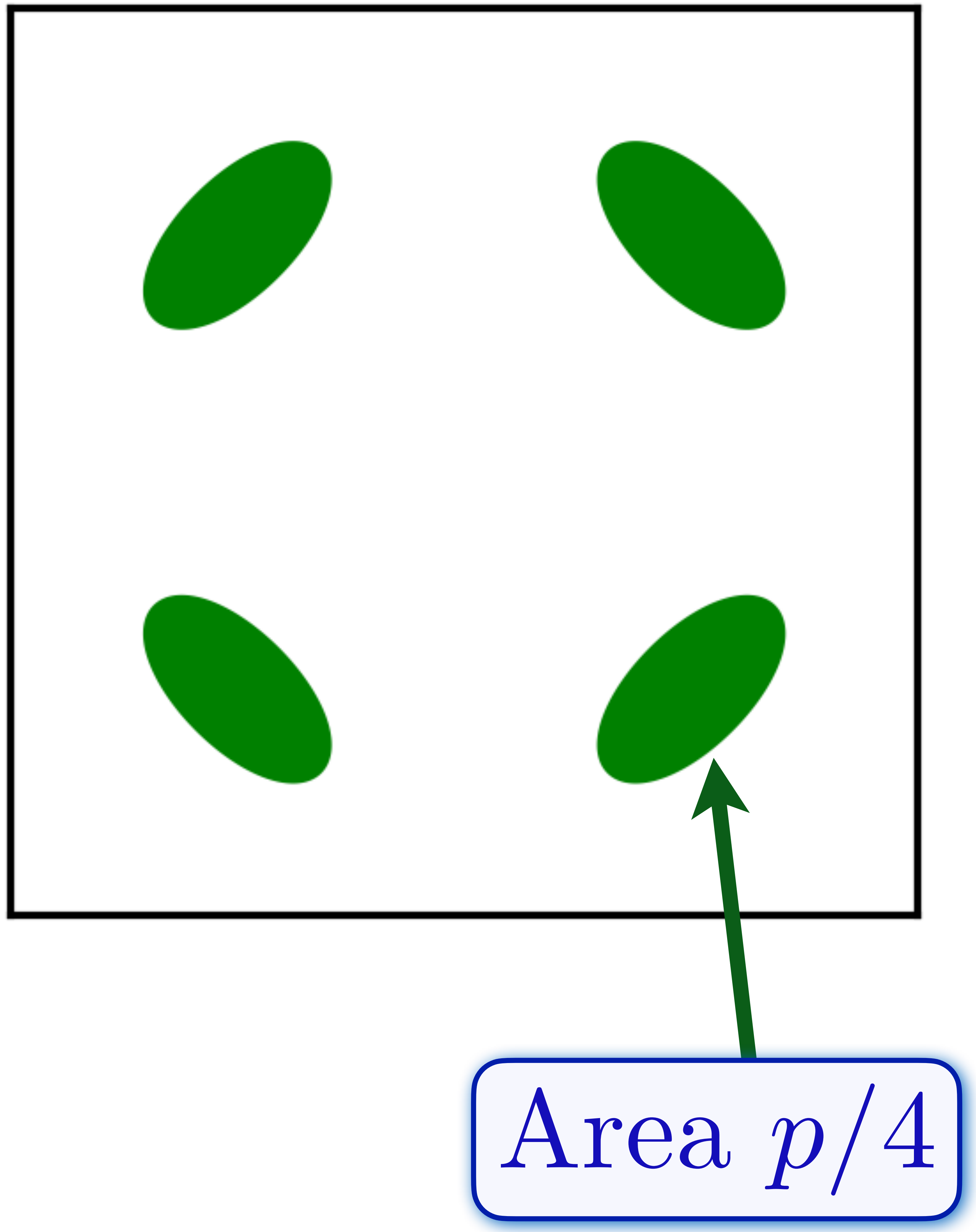

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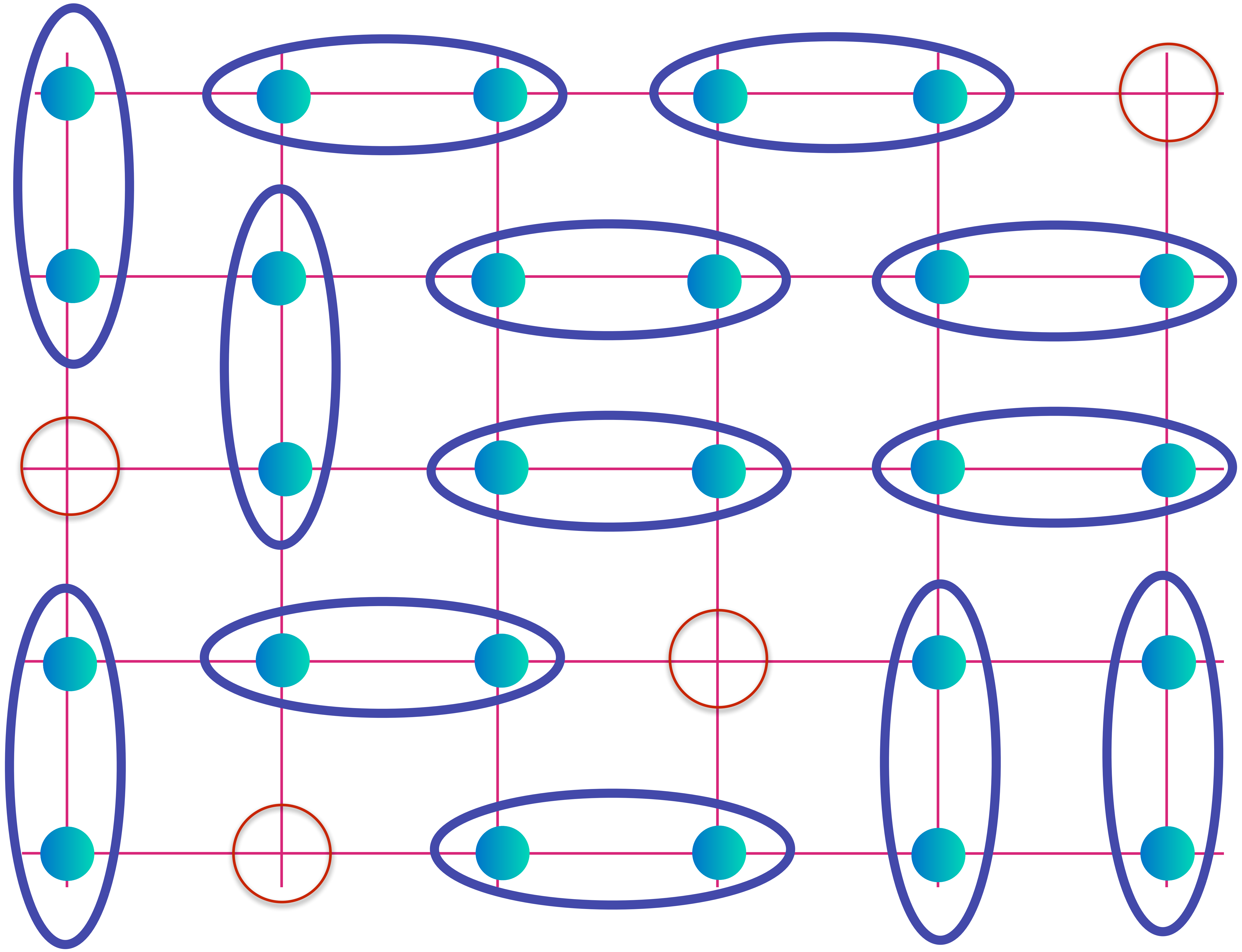

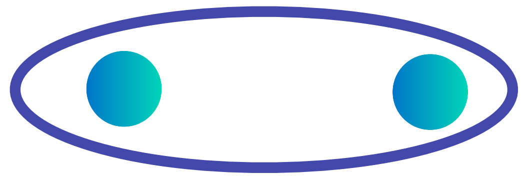 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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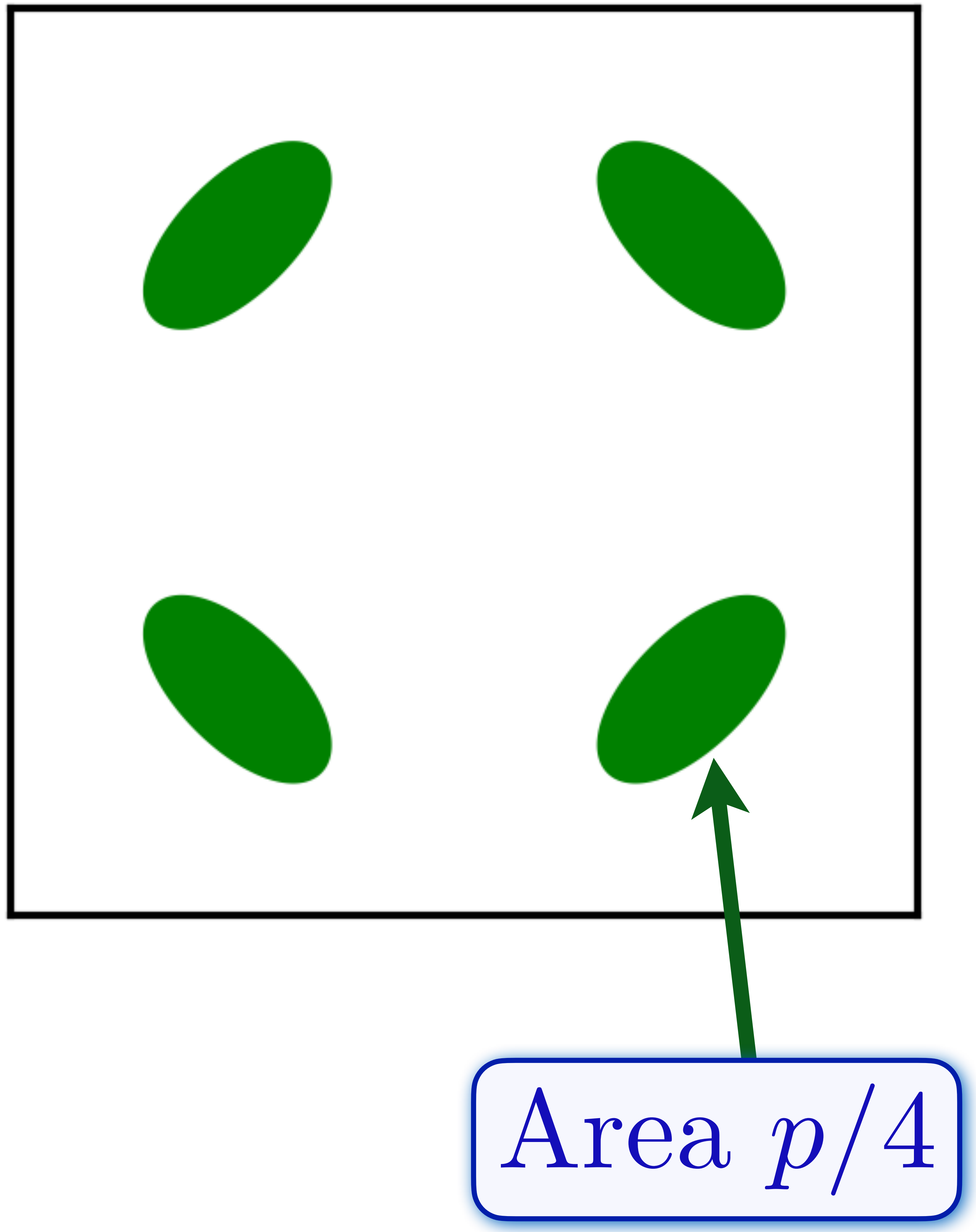

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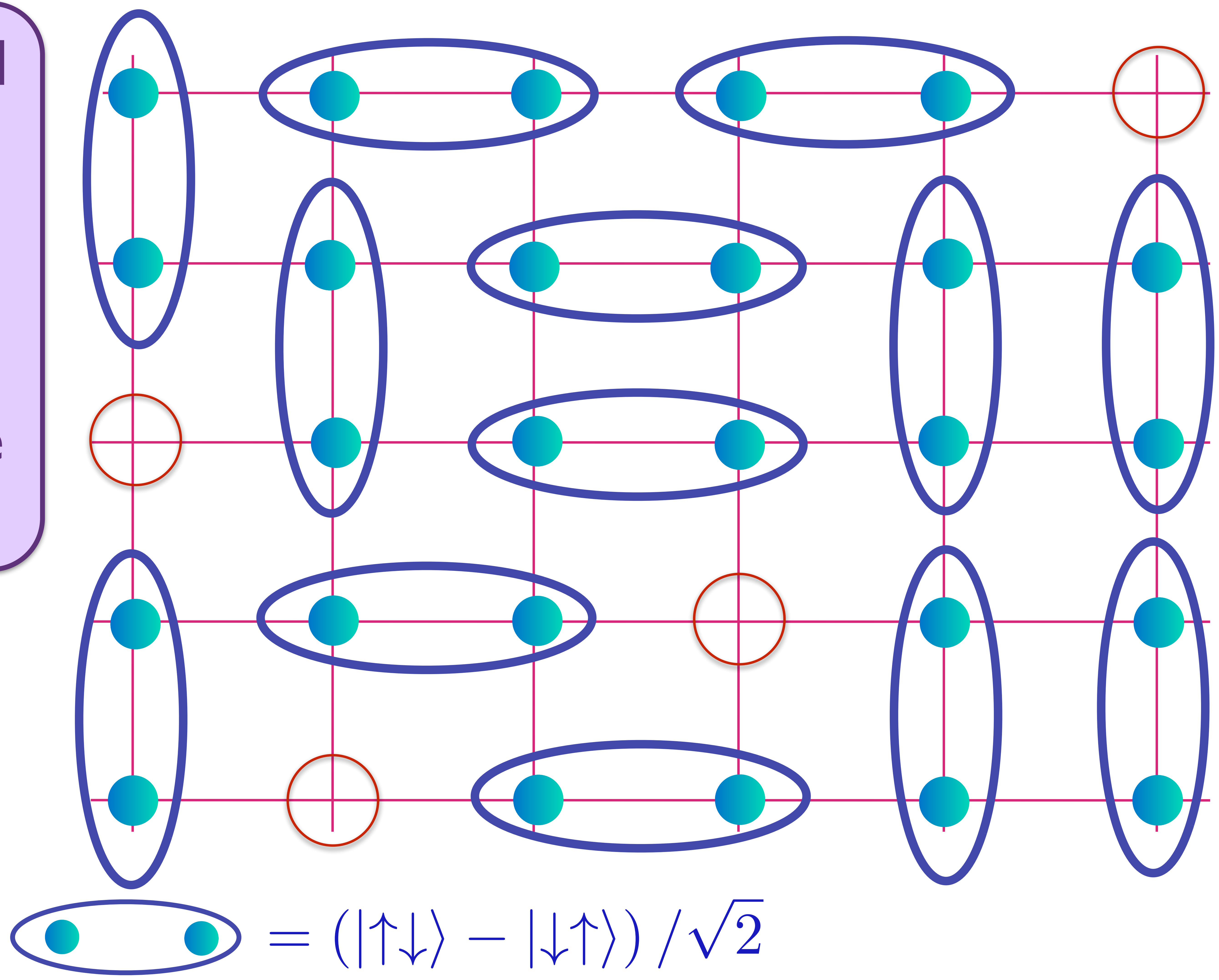
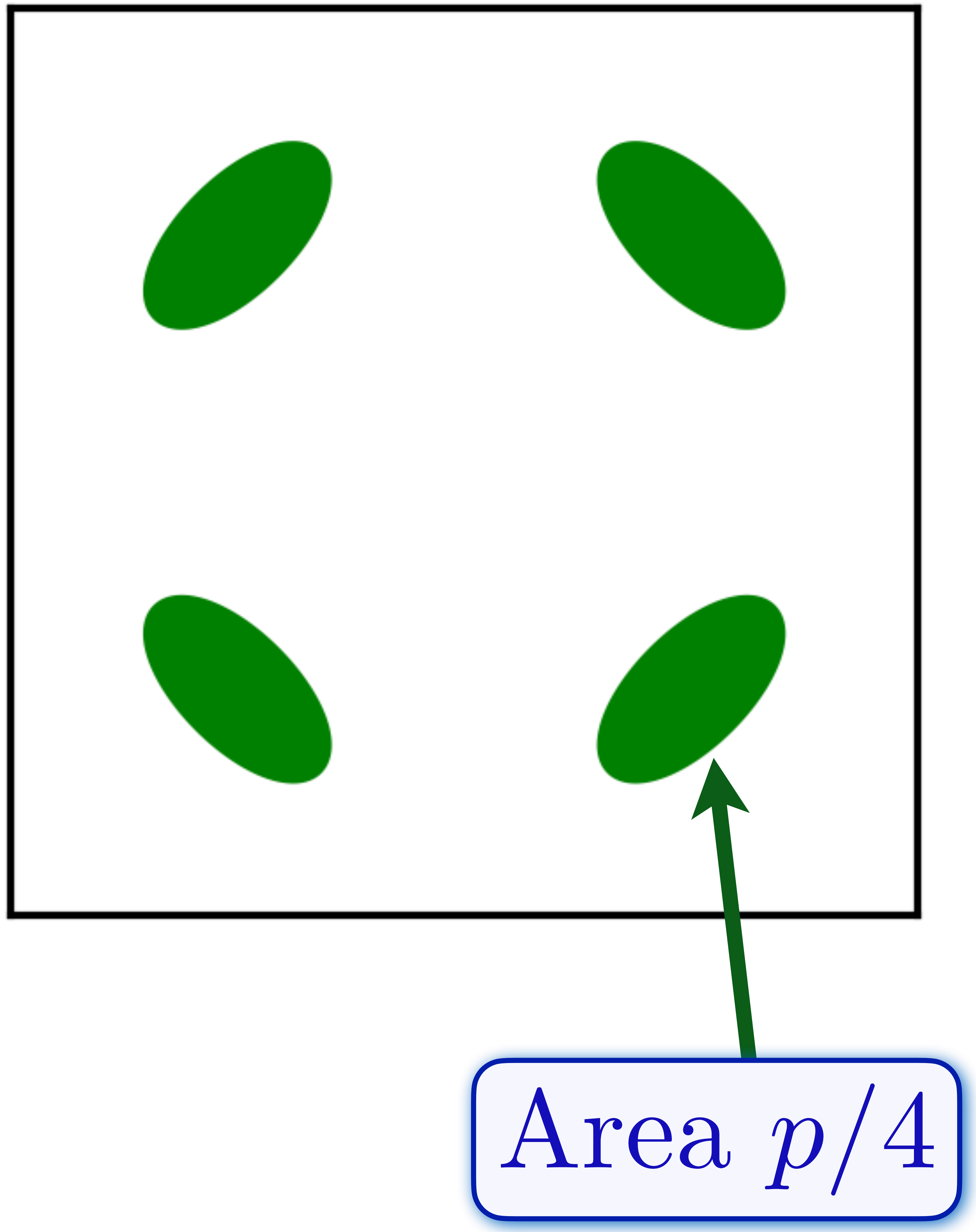


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

non-Luttinger area. Spin liquid



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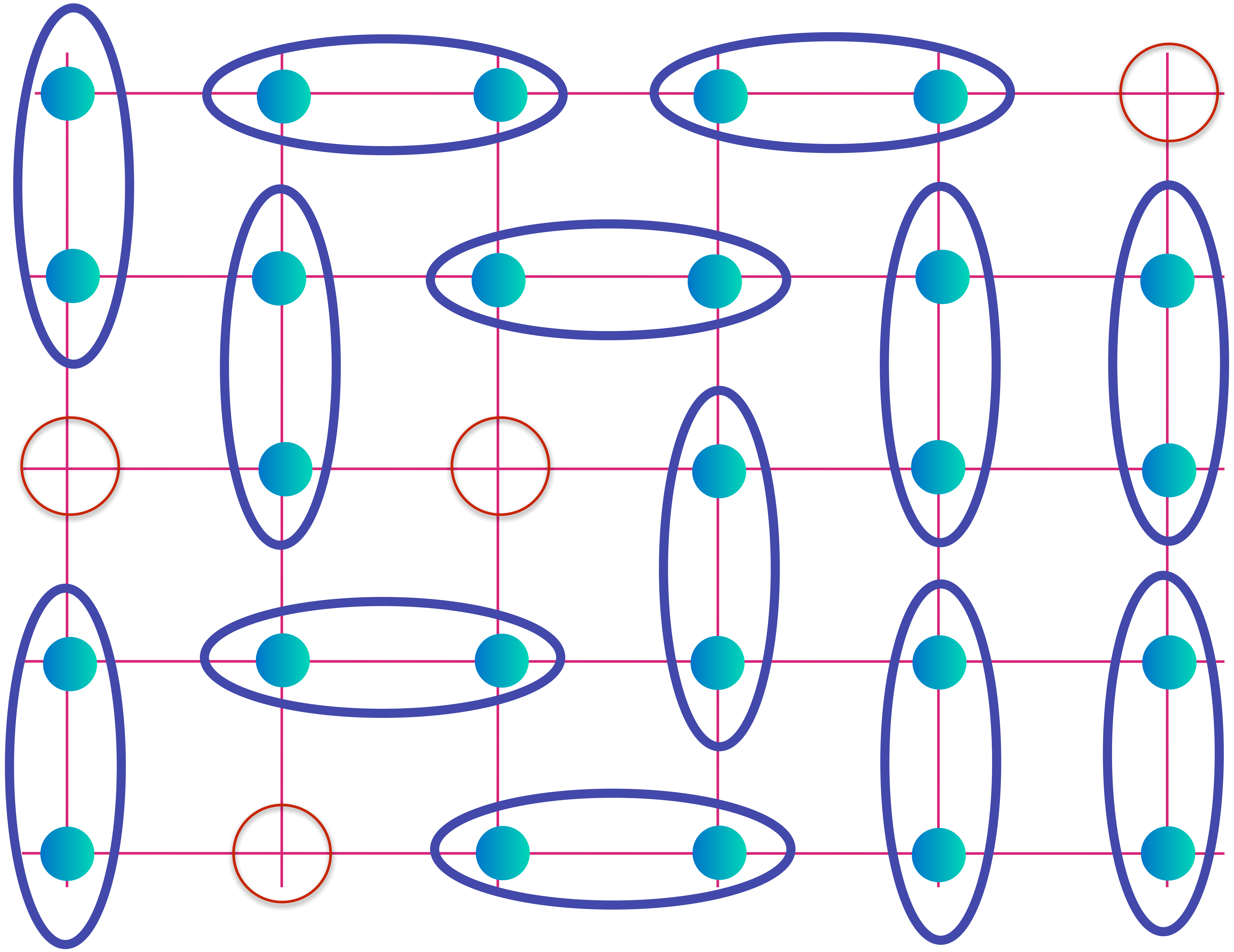

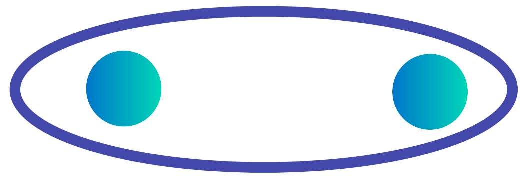 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

non-Luttinger area.
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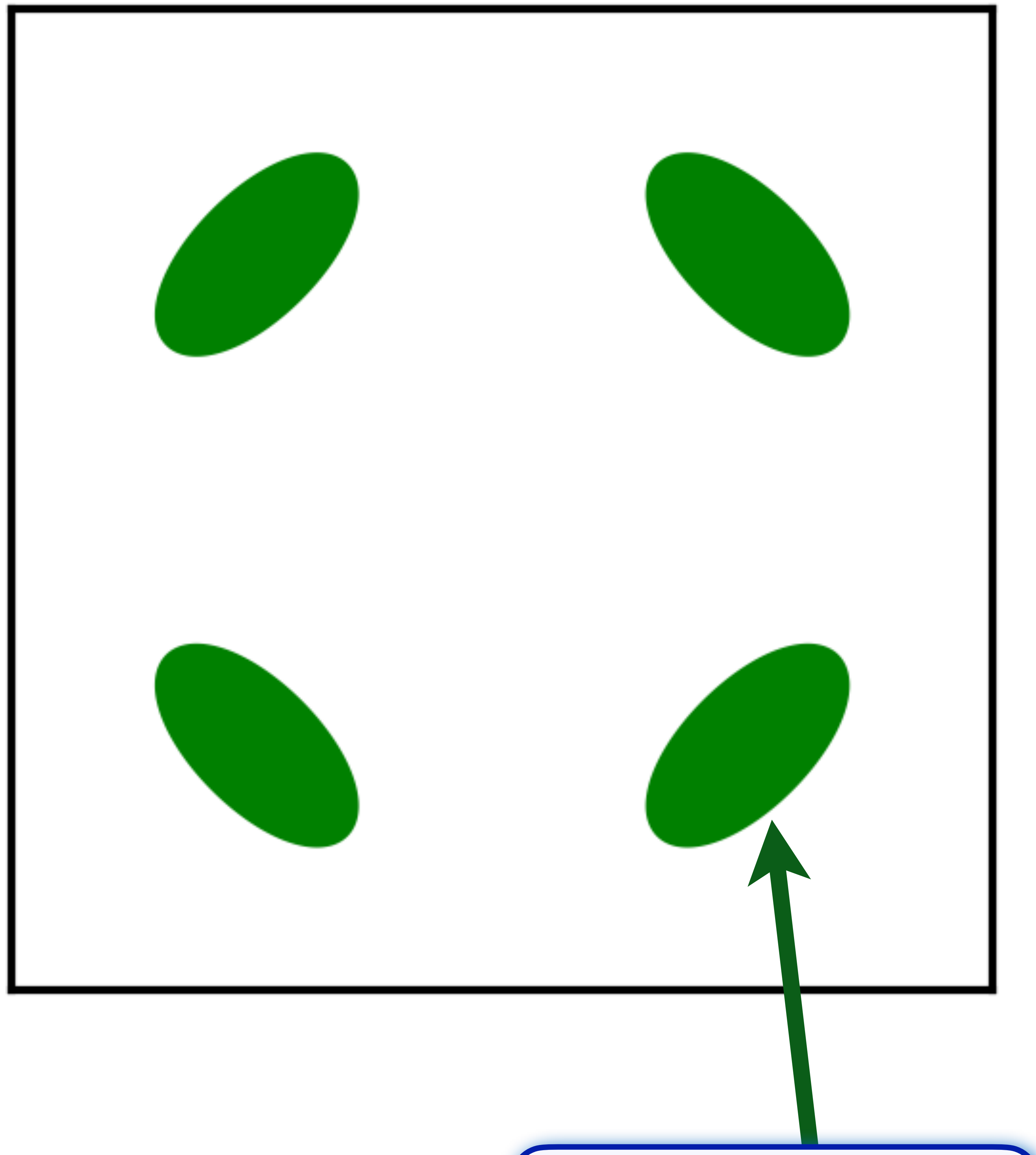

Area $p/4$

Doping an insulating antiferromagnet with holes of density p

Holon metal

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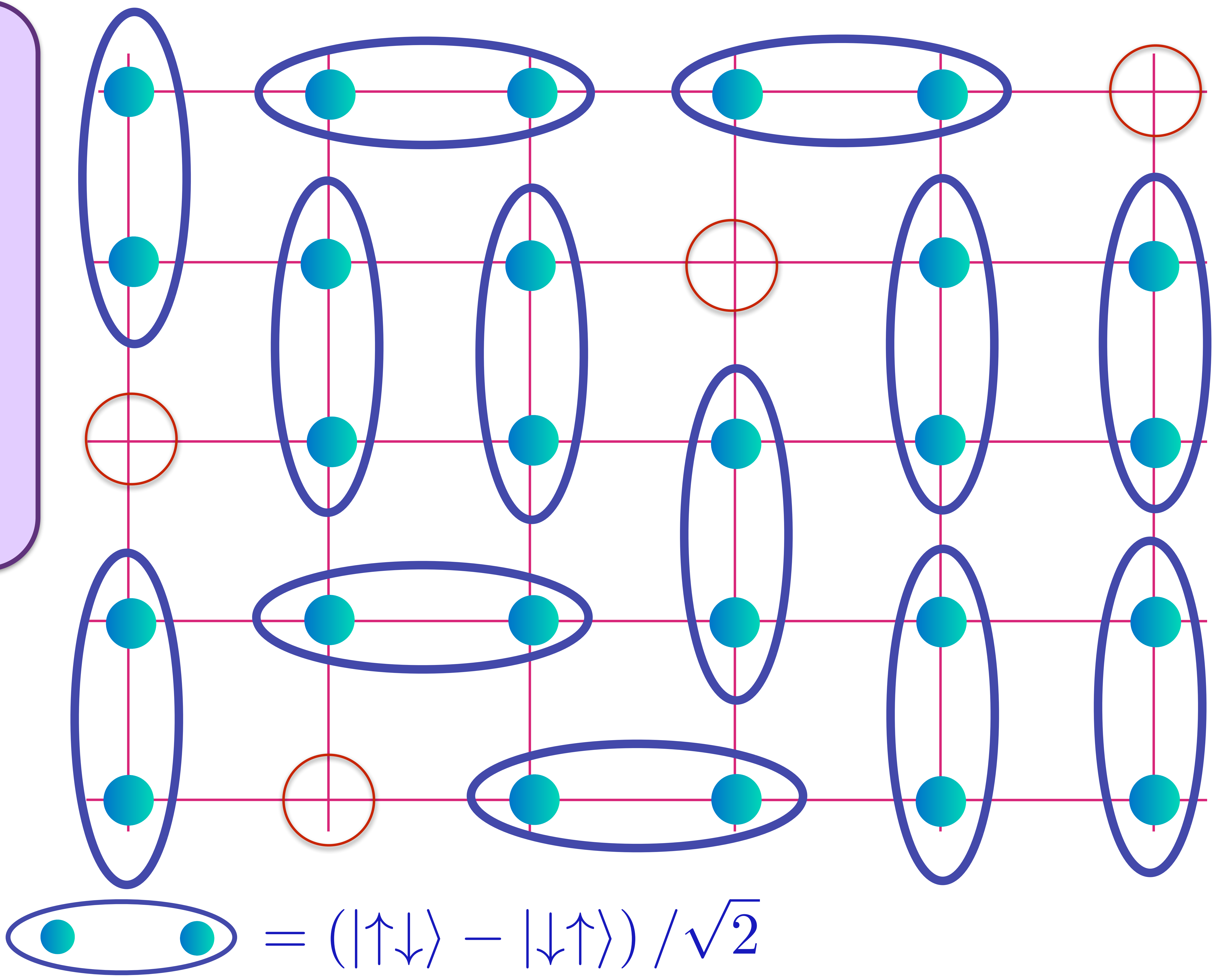

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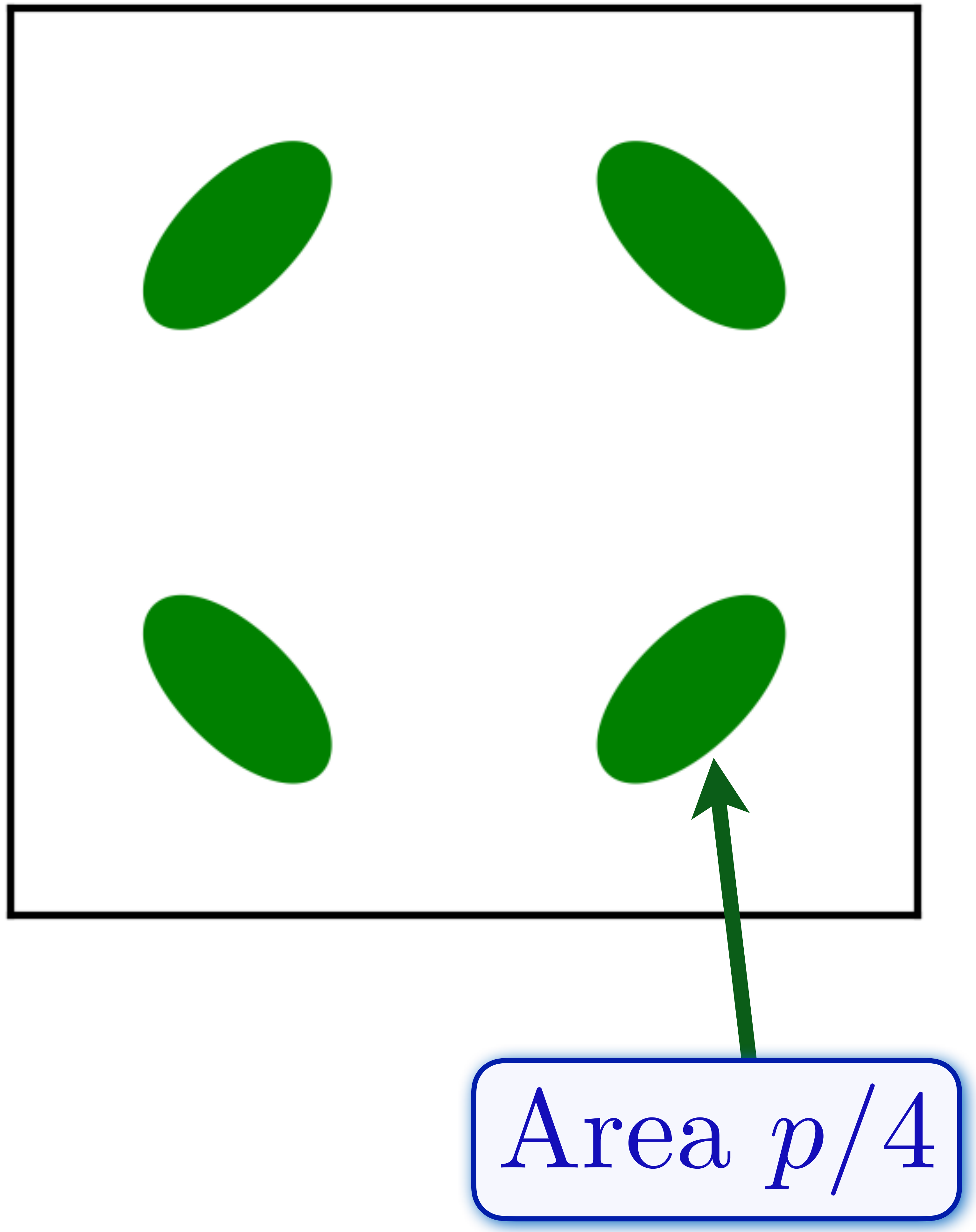
non-Luttinger area.
Spin liquid



Doping an insulating antiferromagnet with holes of density p

Holon metal

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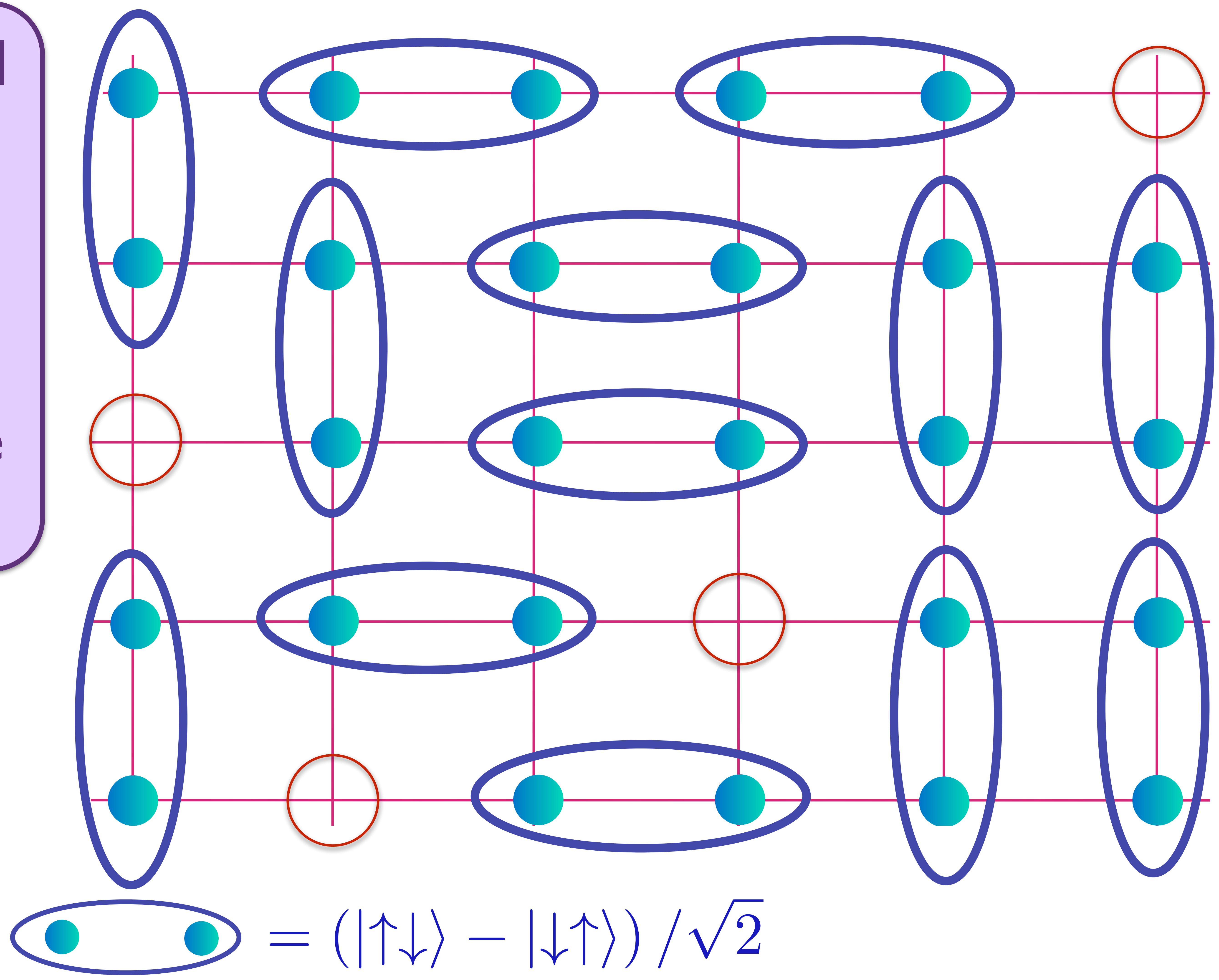

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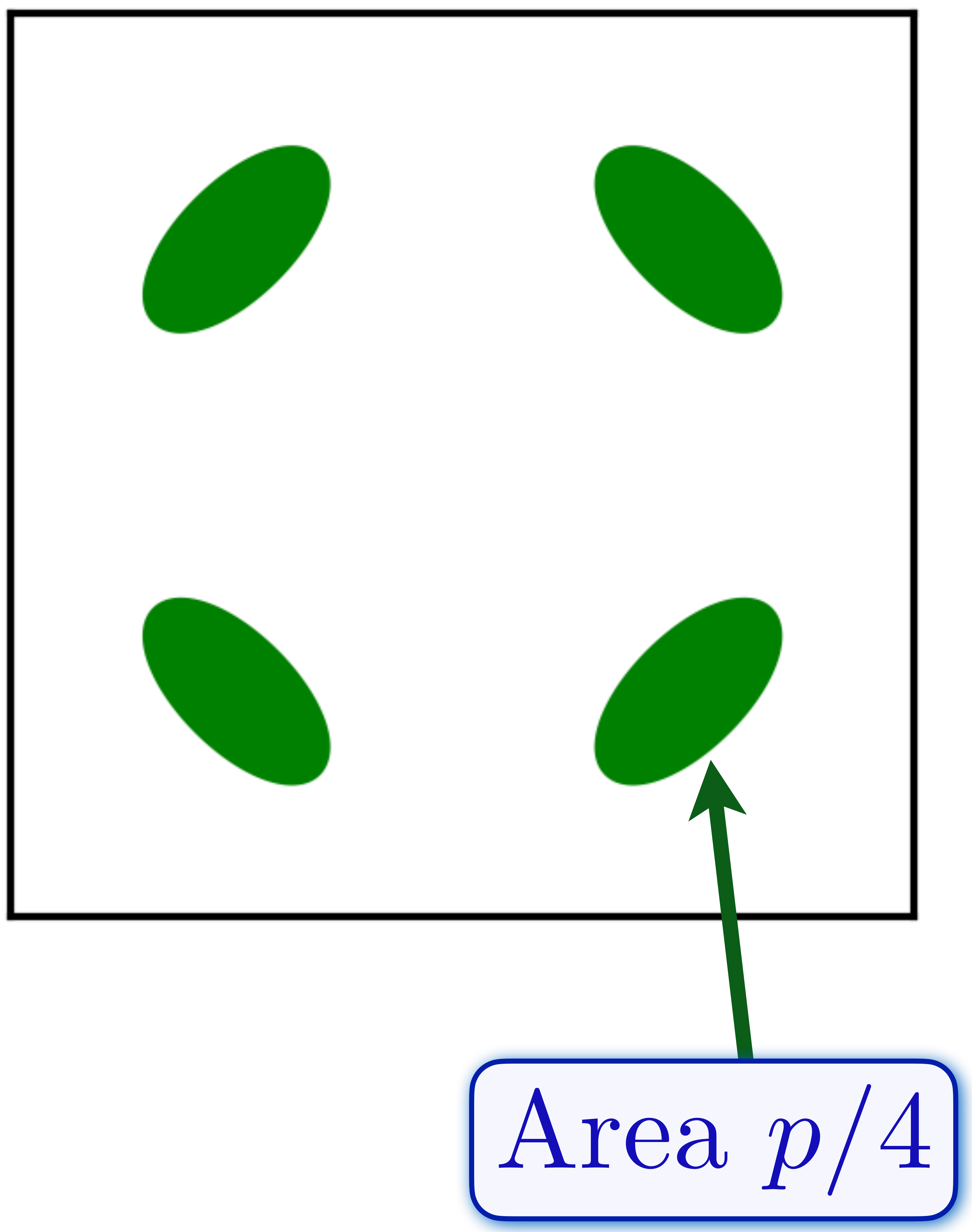
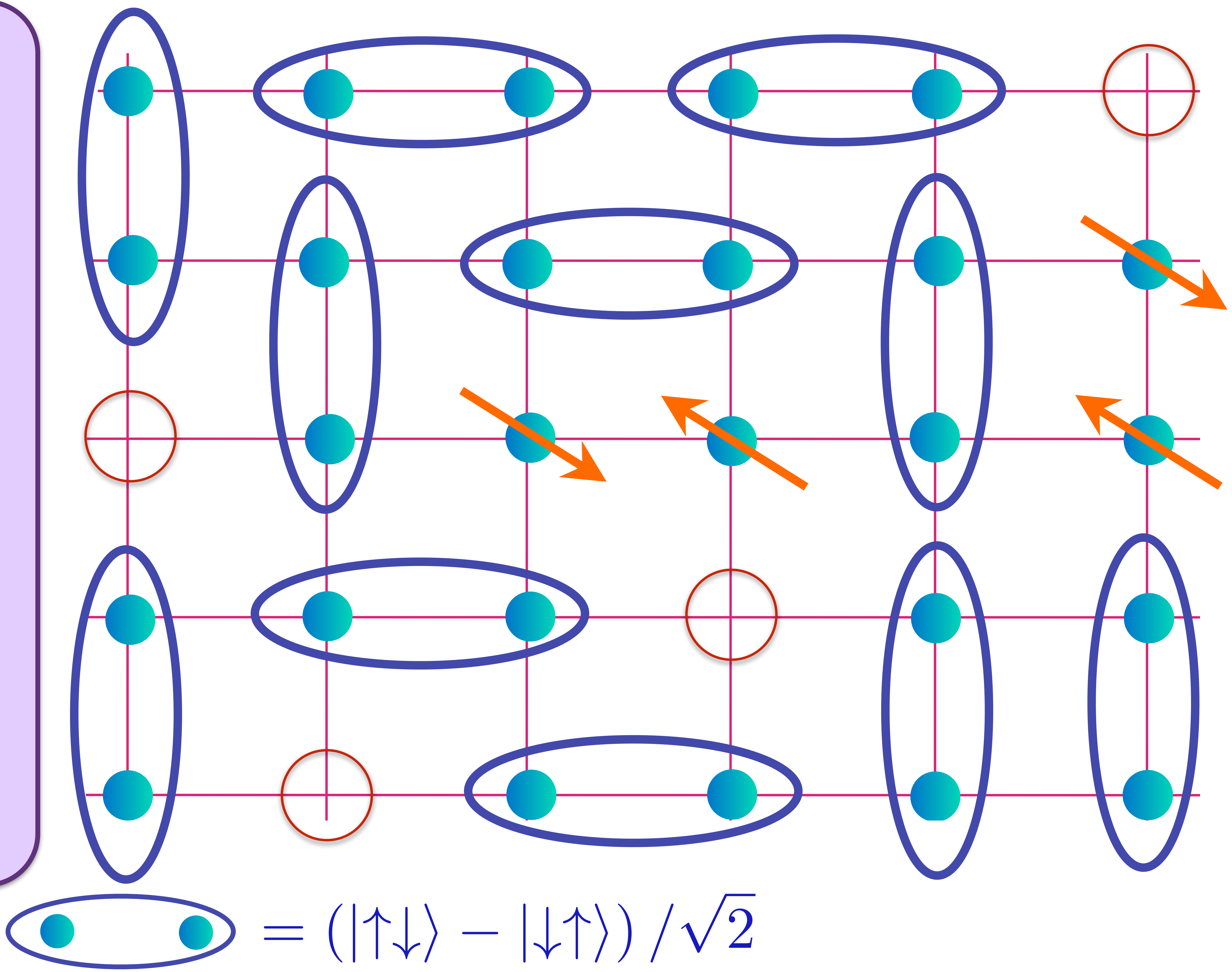


Doping an insulating antiferromagnet with holes of density p

Holon metal

Spin liquid with density p of spinless, charge +e "holons" and charge 0 spin-1/2 "spinons".



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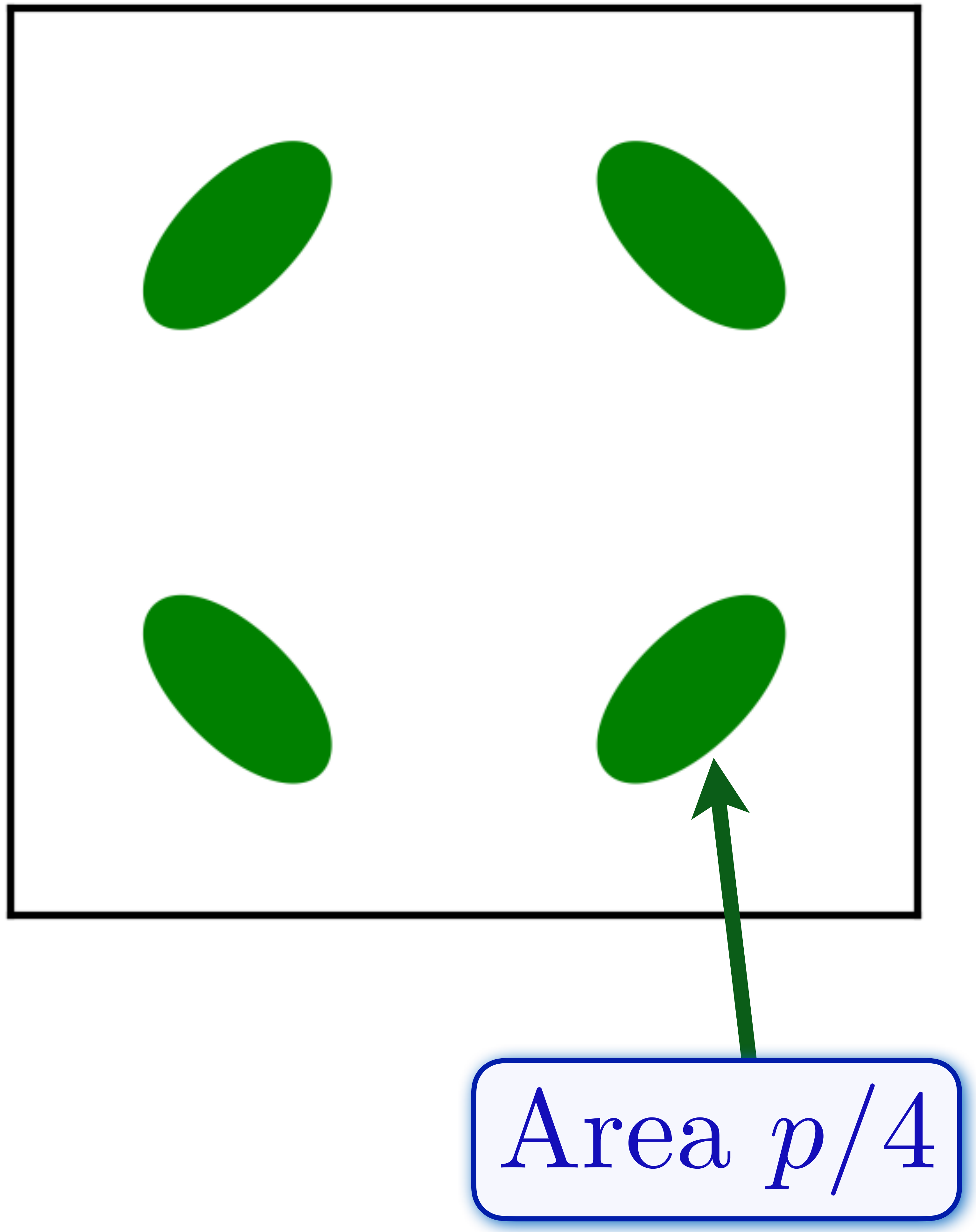

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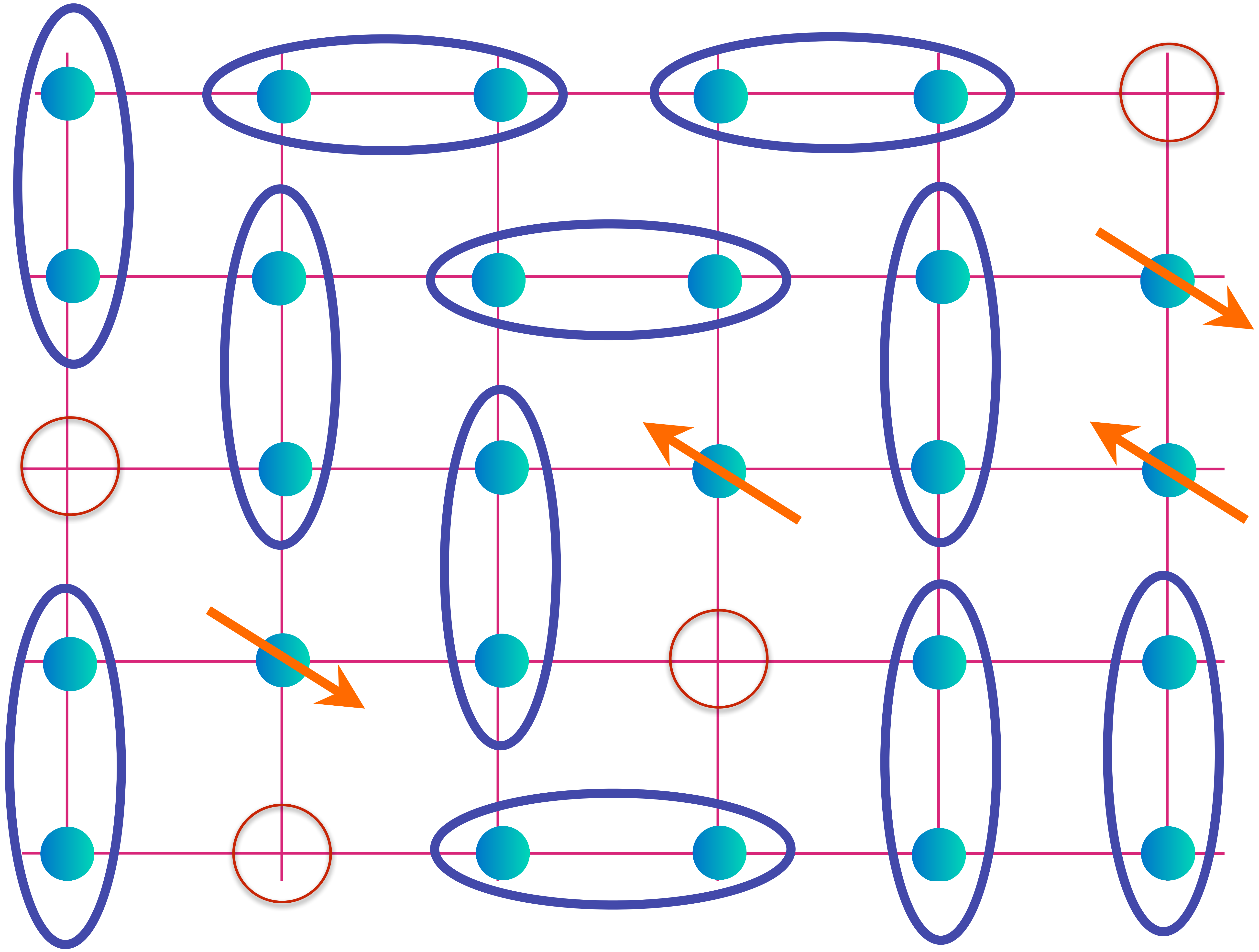
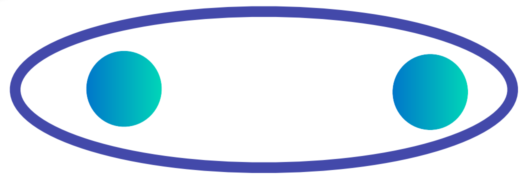
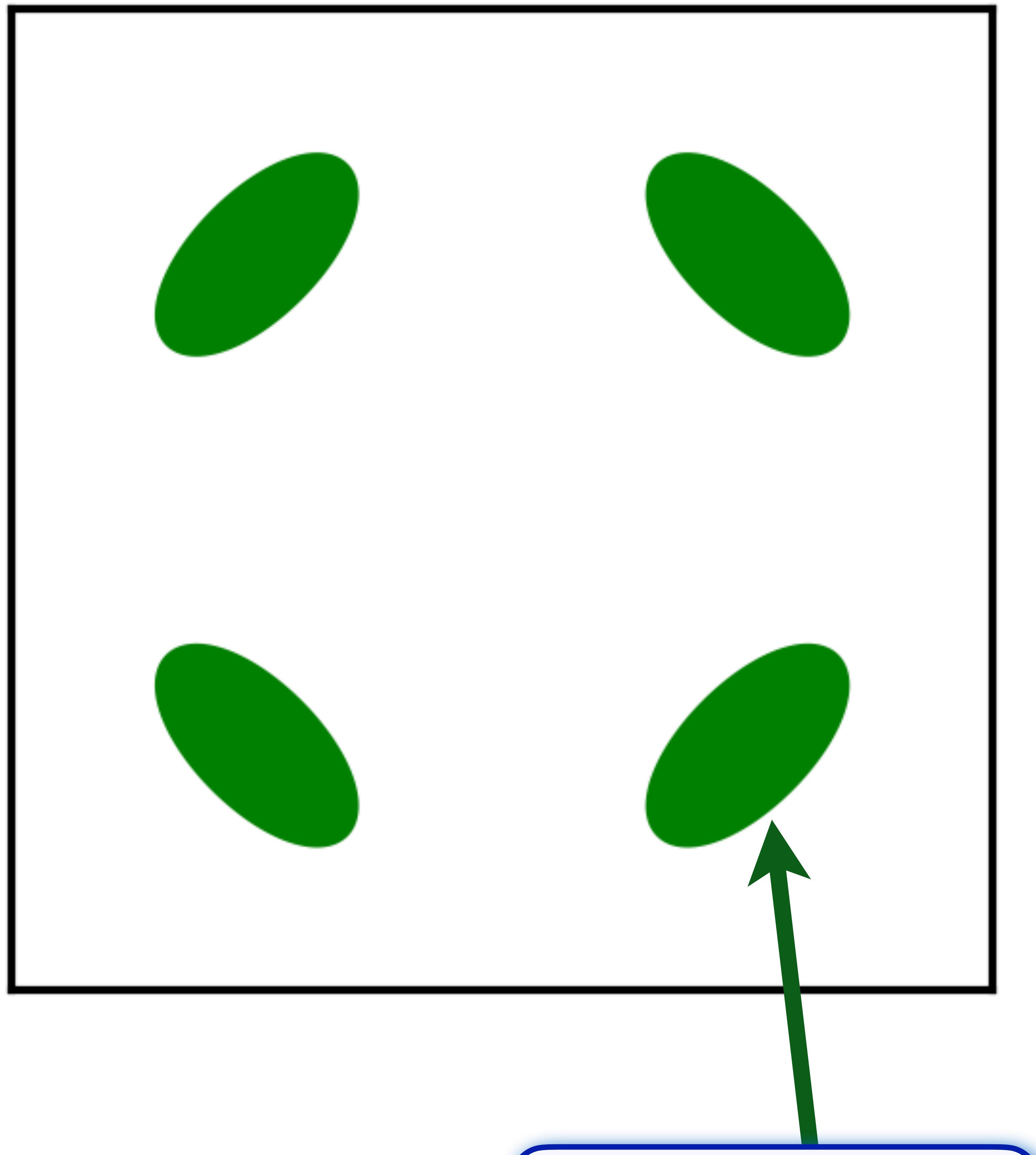
Holon metal

Spin liquid with density p of spinless, charge +e "holons" and charge 0 spin-1/2 "spinons".



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

non-Luttinger area.
Spin liquid

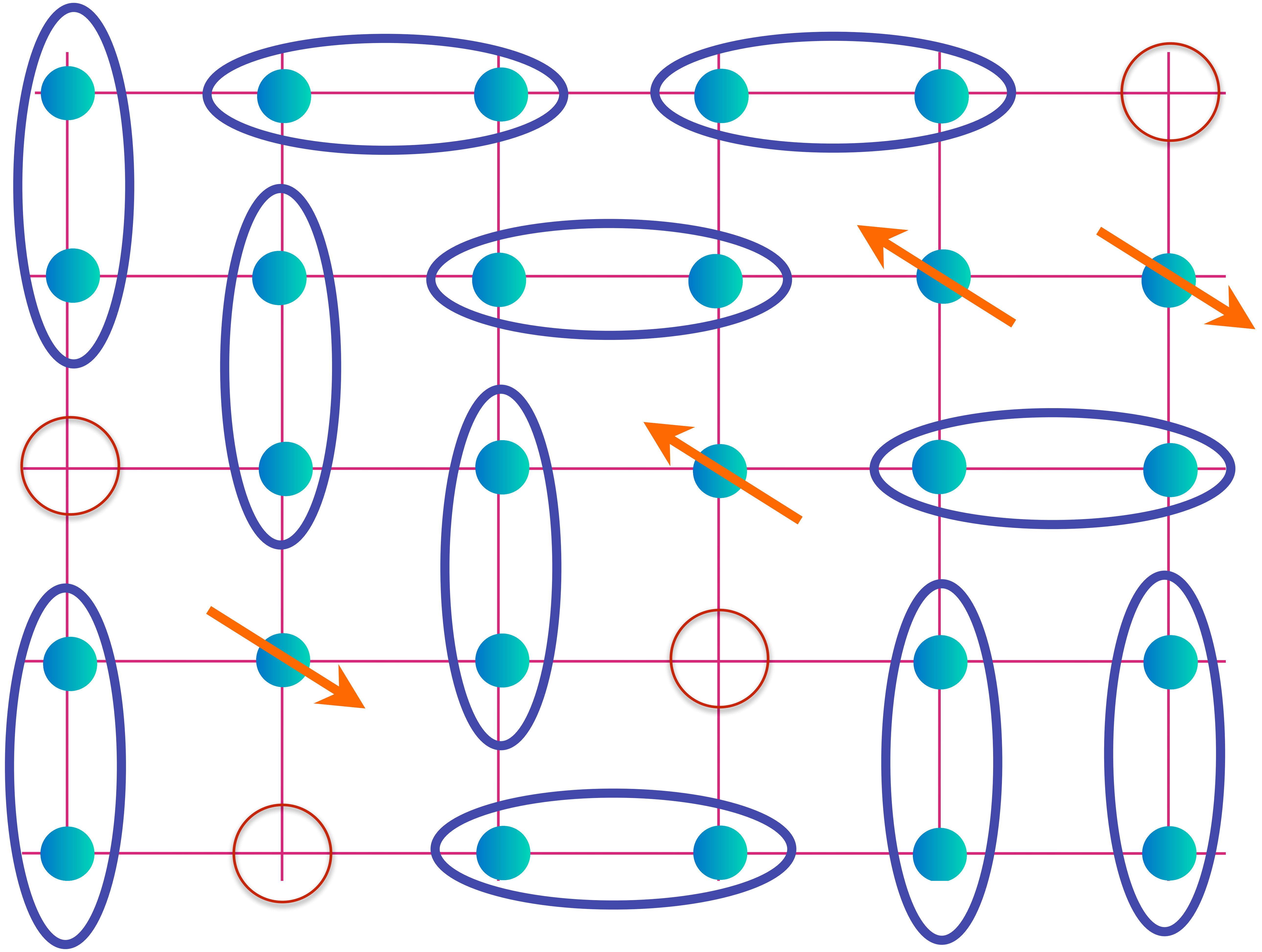


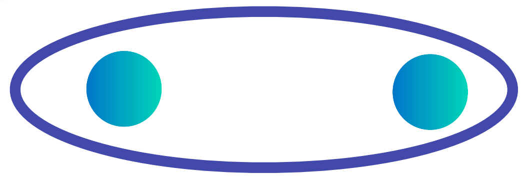
Area $p/4$

Doping an insulating antiferromagnet with holes of density p

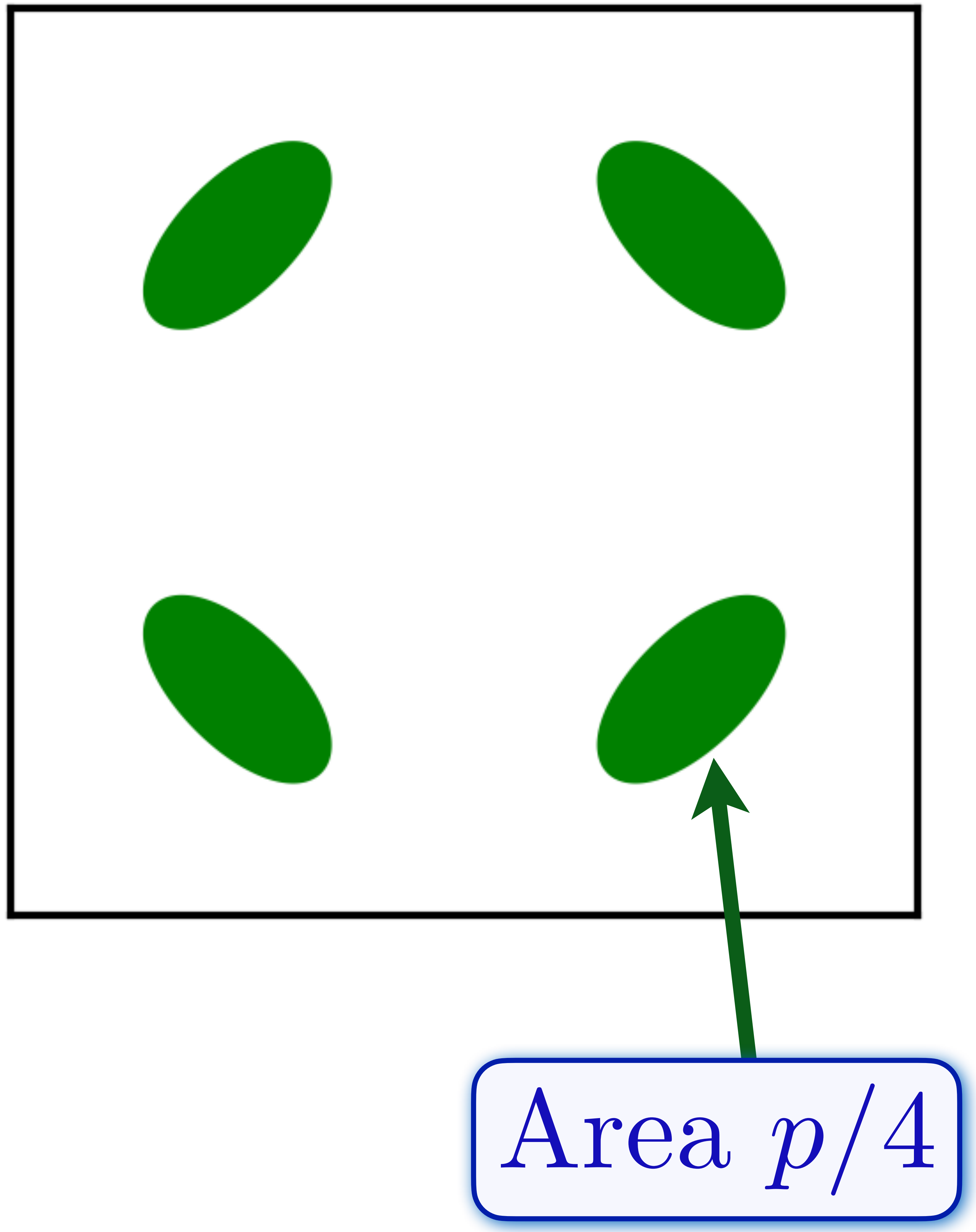
Holon metal

Spin liquid with density p of spinless, charge +e “holons” and charge 0 spin-1/2 “spinons”.



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

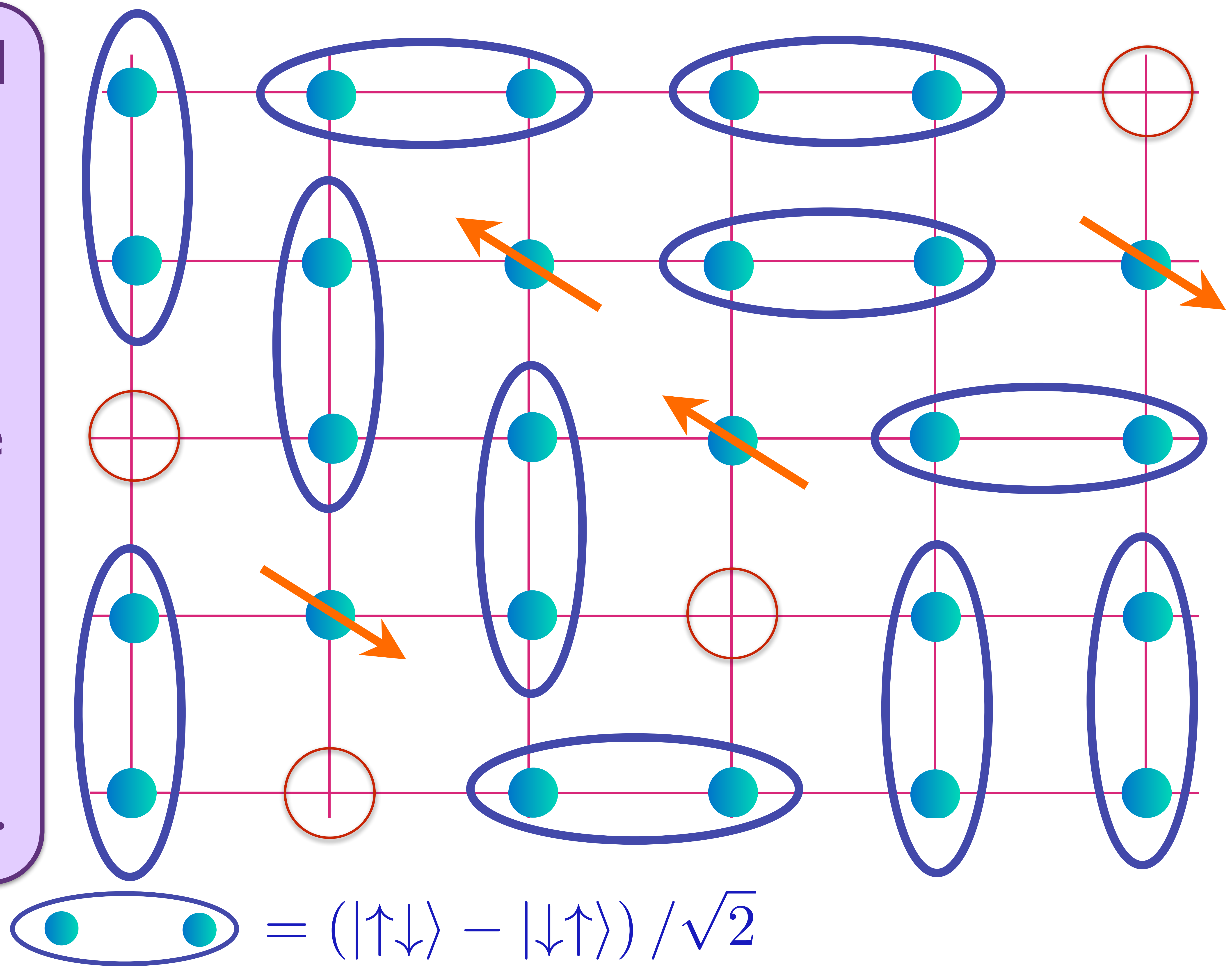
non-Luttinger area.
Spin liquid



Doping an insulating antiferromagnet with holes of density p

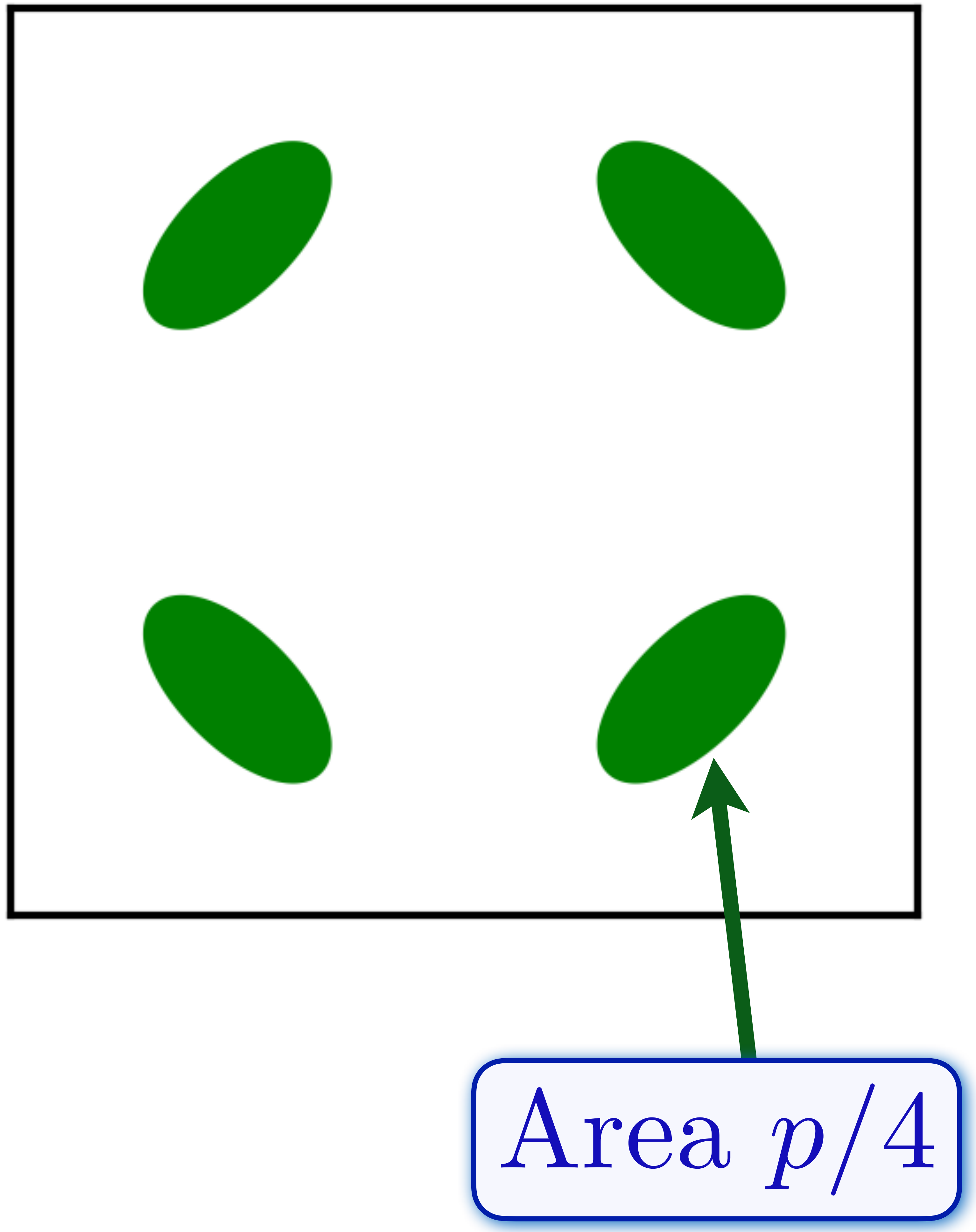
Holon metal

Spin liquid with density p of spinless, charge +e "holons" and charge 0 spin-1/2 "spinons".



$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$

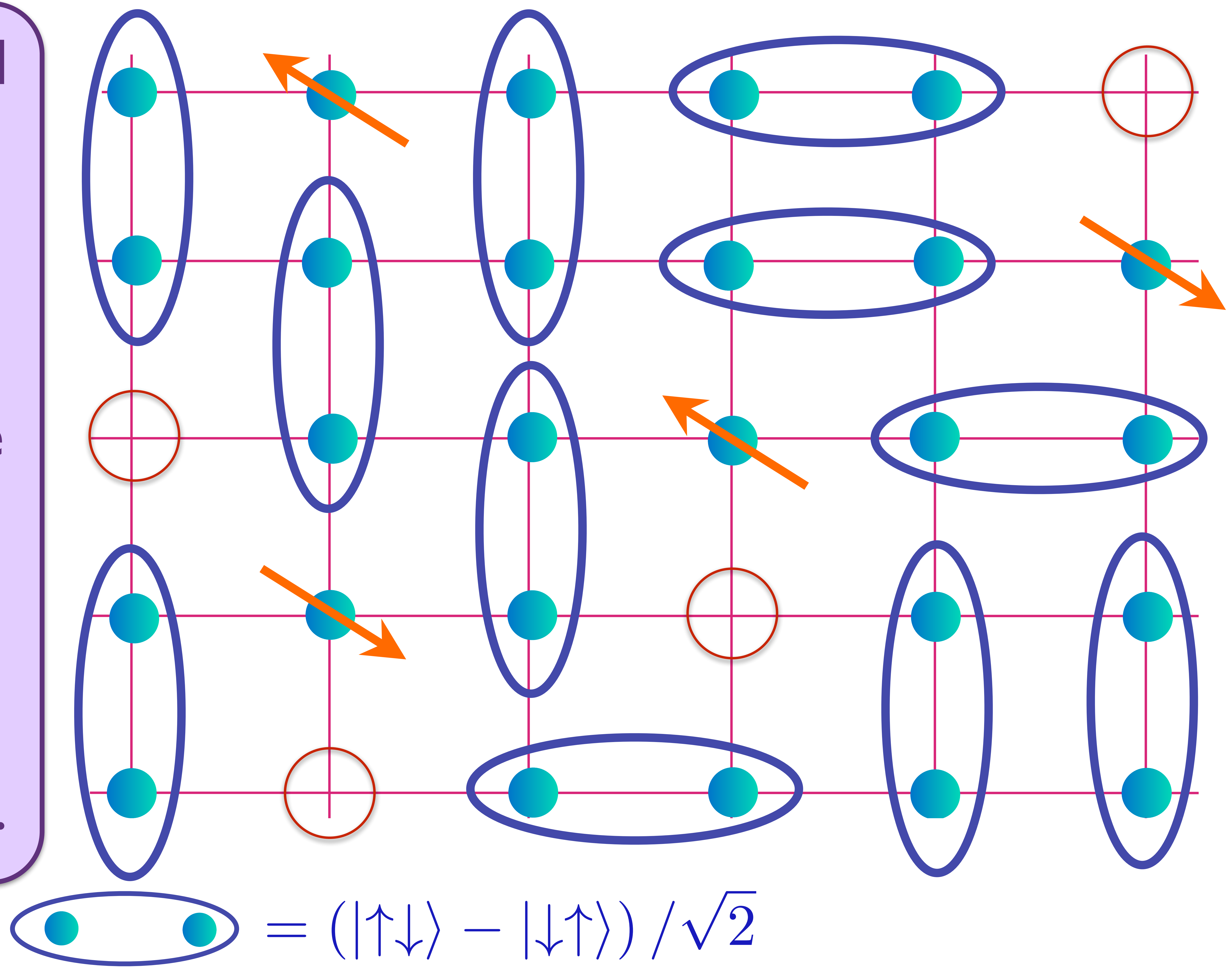
non-Luttinger area. Spin liquid



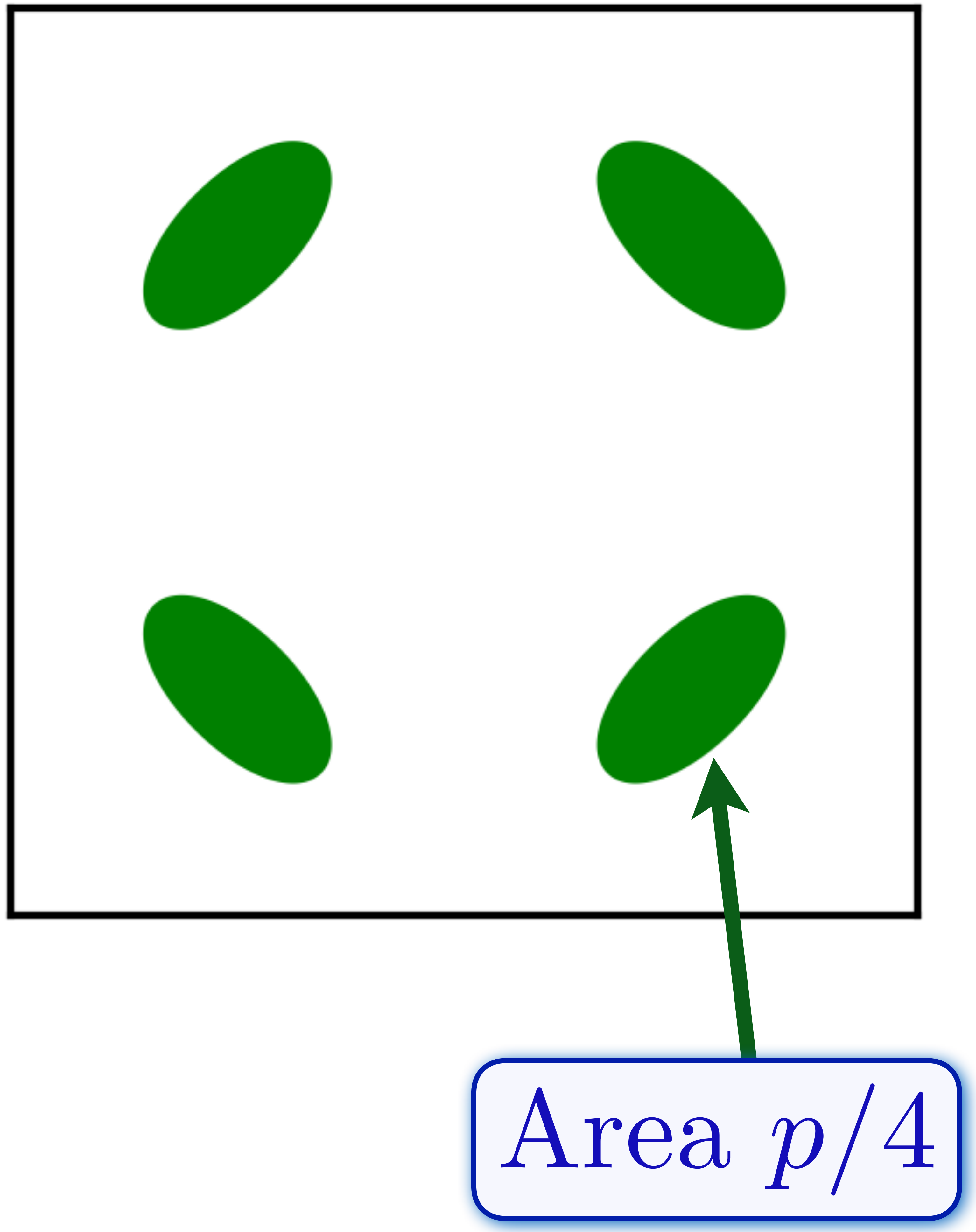
Doping an insulating antiferromagnet with holes of density p

Holon metal

Spin liquid with density p of spinless, charge +e "holons" and charge 0 spin-1/2 "spinons".



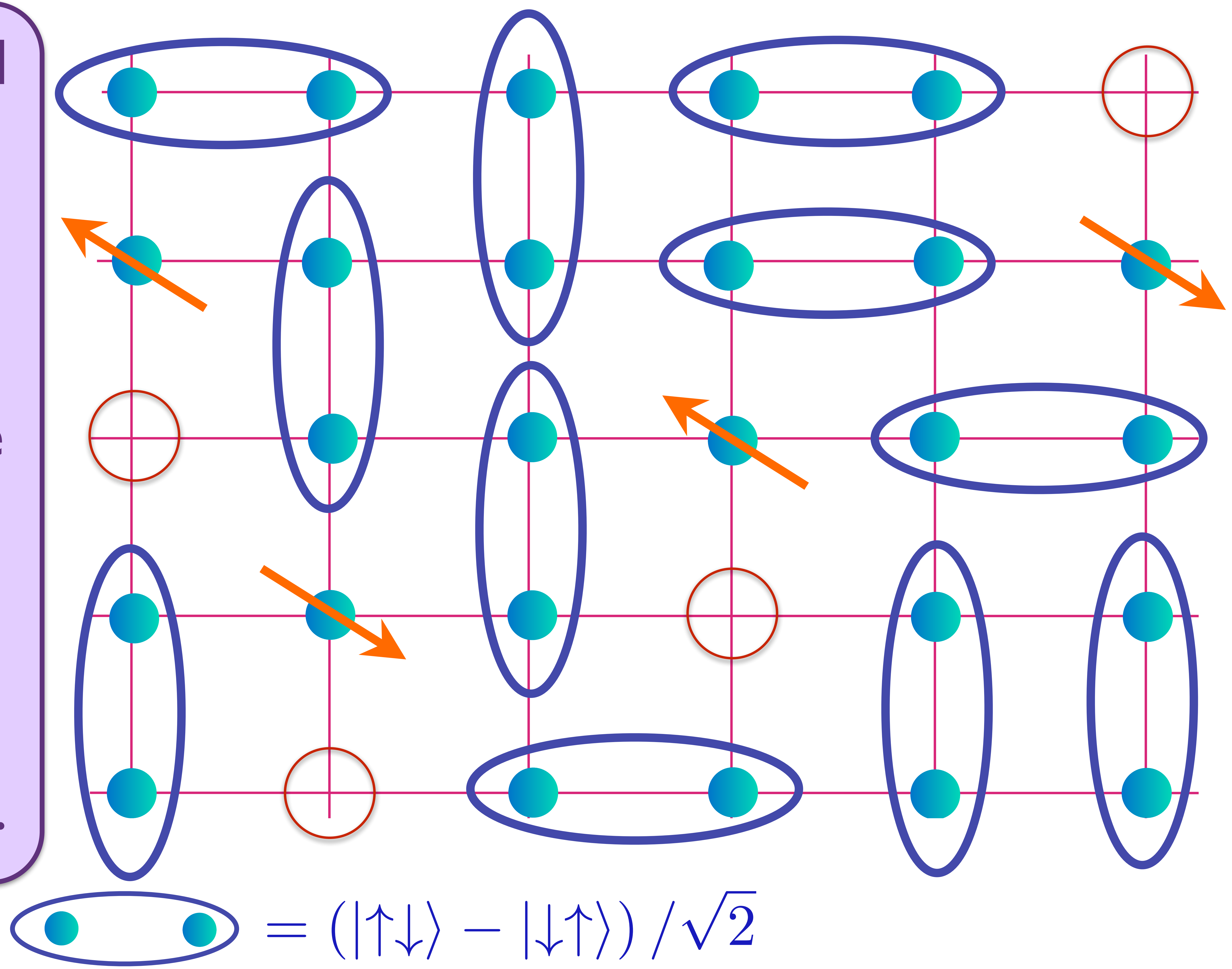
non-Luttinger area.
Spin liquid



Doping an insulating antiferromagnet with holes of density p

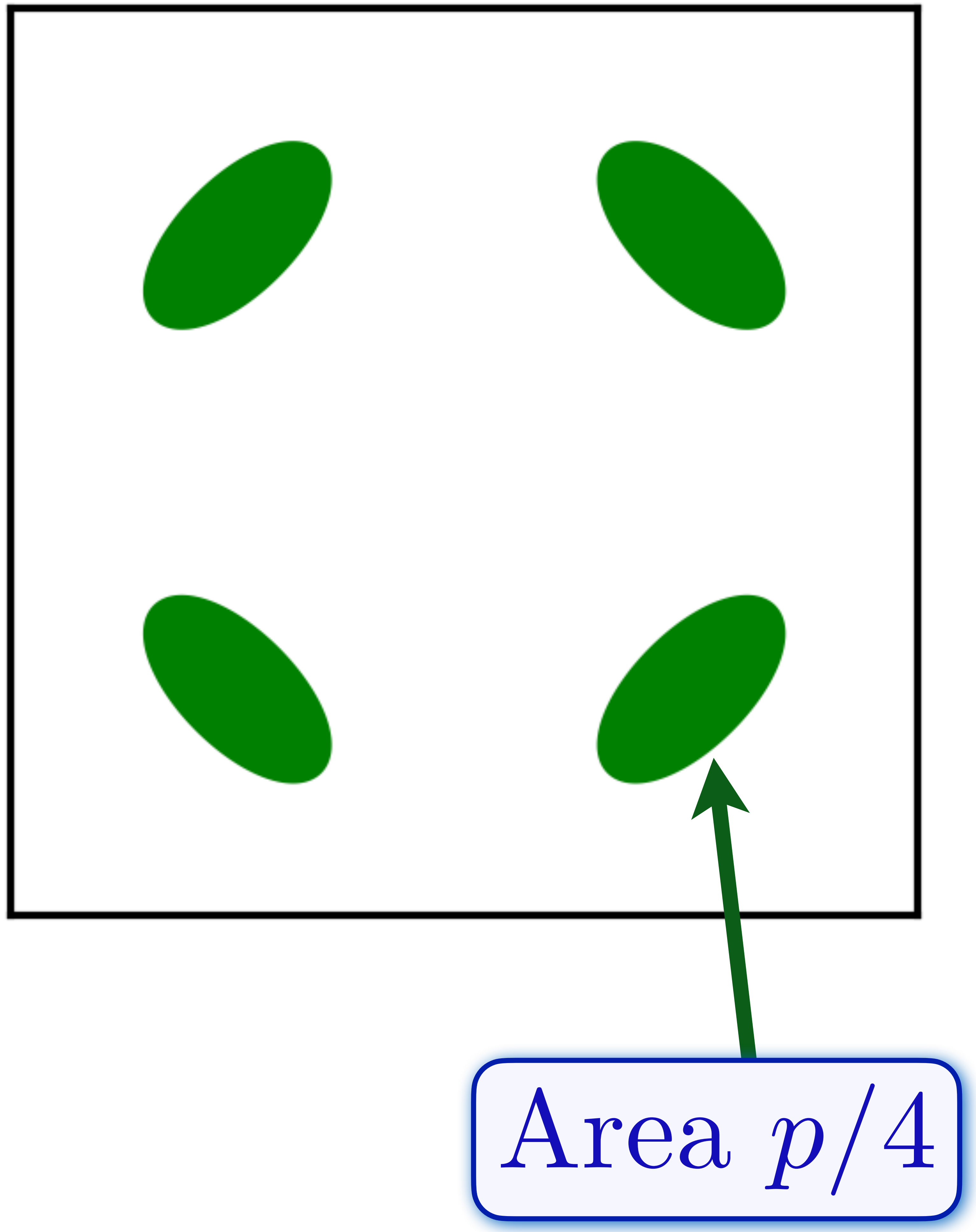
Holon metal

Spin liquid with density p of spinless, charge +e "holons" and charge 0 spin-1/2 "spinons".



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

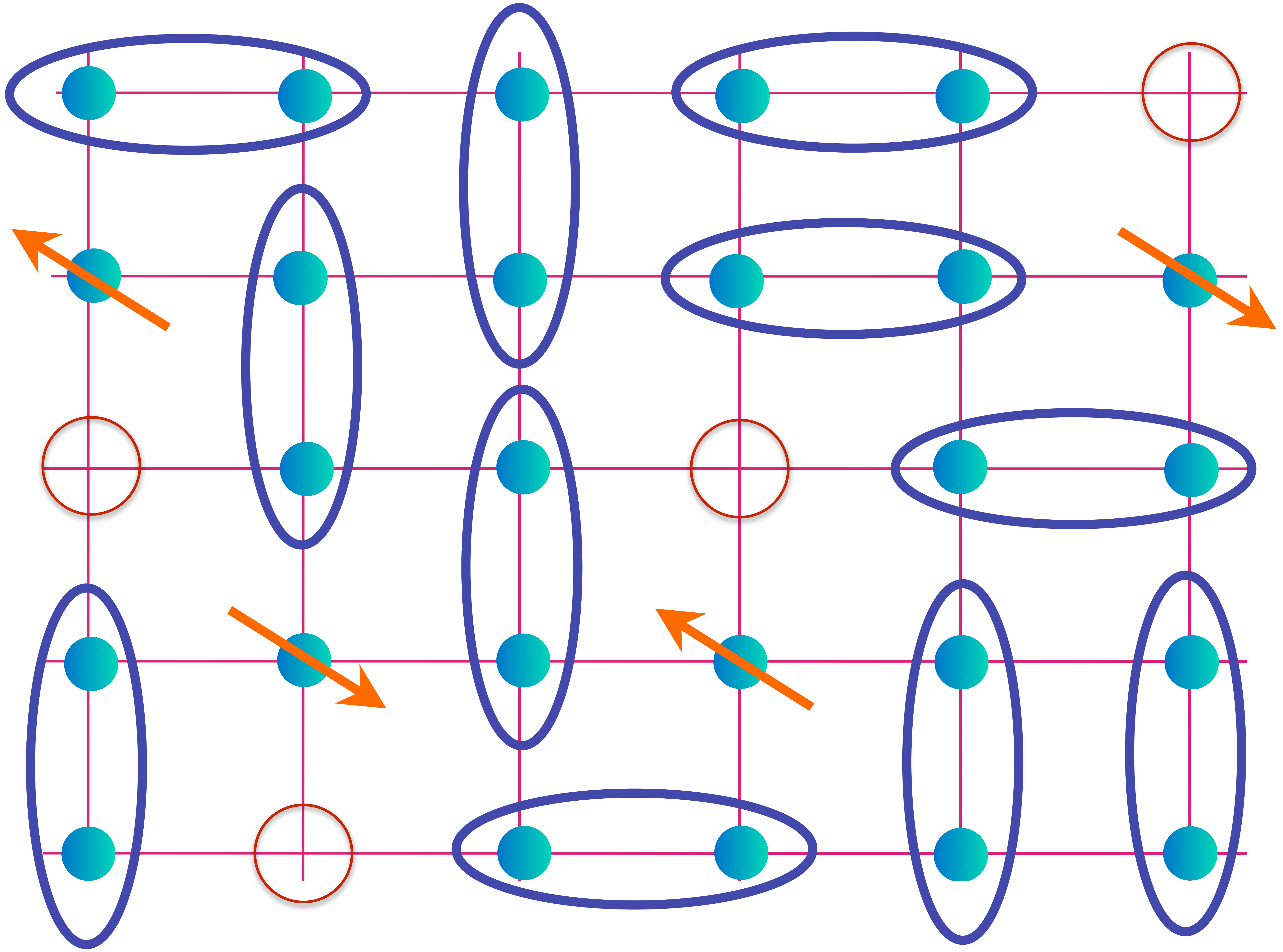
non-Luttinger area. Spin liquid

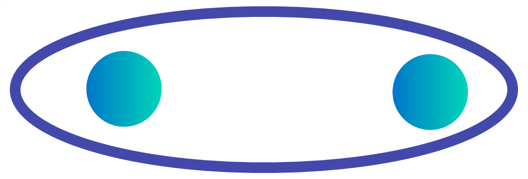


Doping an insulating antiferromagnet with holes of density p

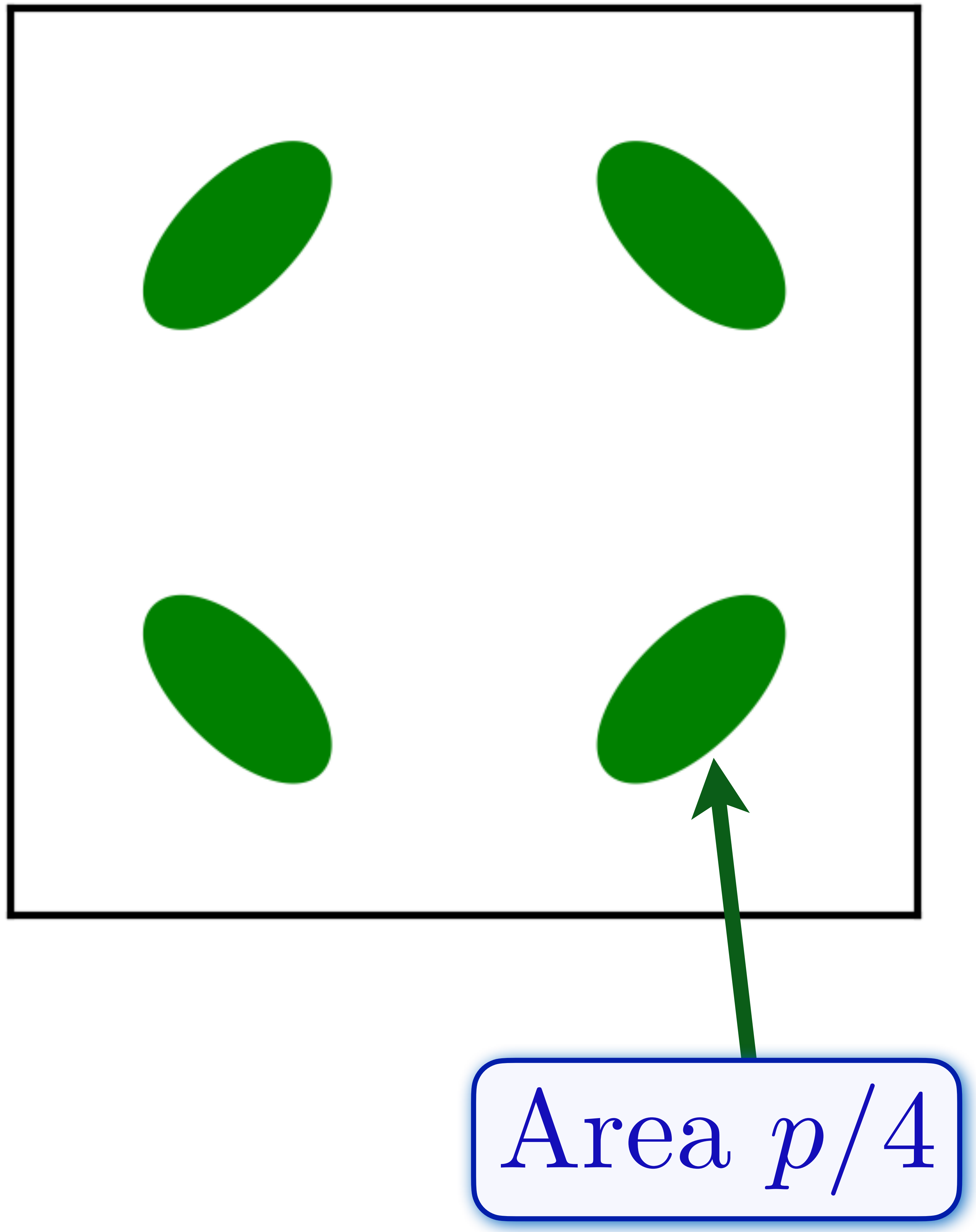
Holon metal

Spin liquid with density p of spinless, charge +e "holons" and charge 0 spin-1/2 "spinons".



 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

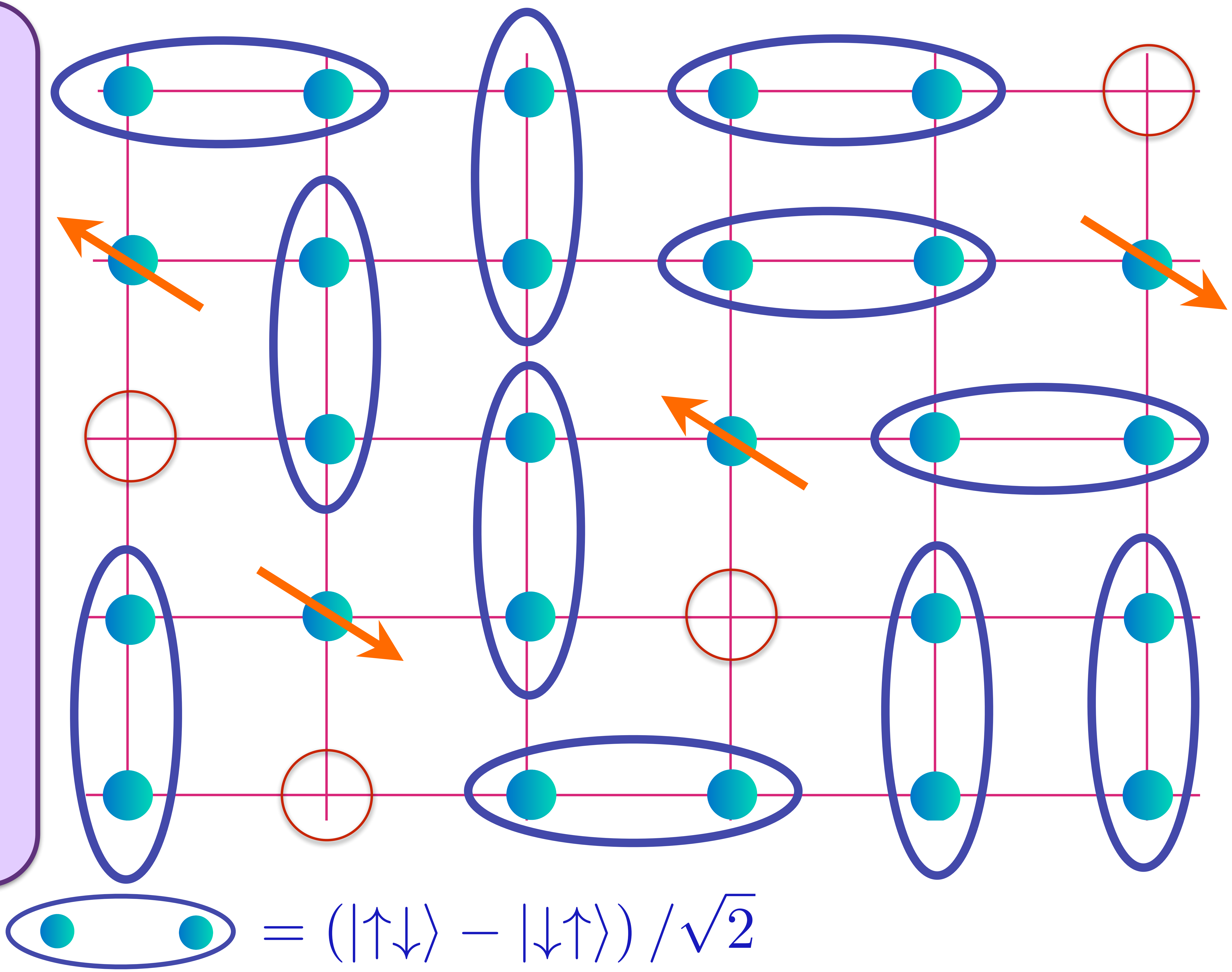
non-Luttinger area.
Spin liquid



Doping an insulating antiferromagnet with holes of density p

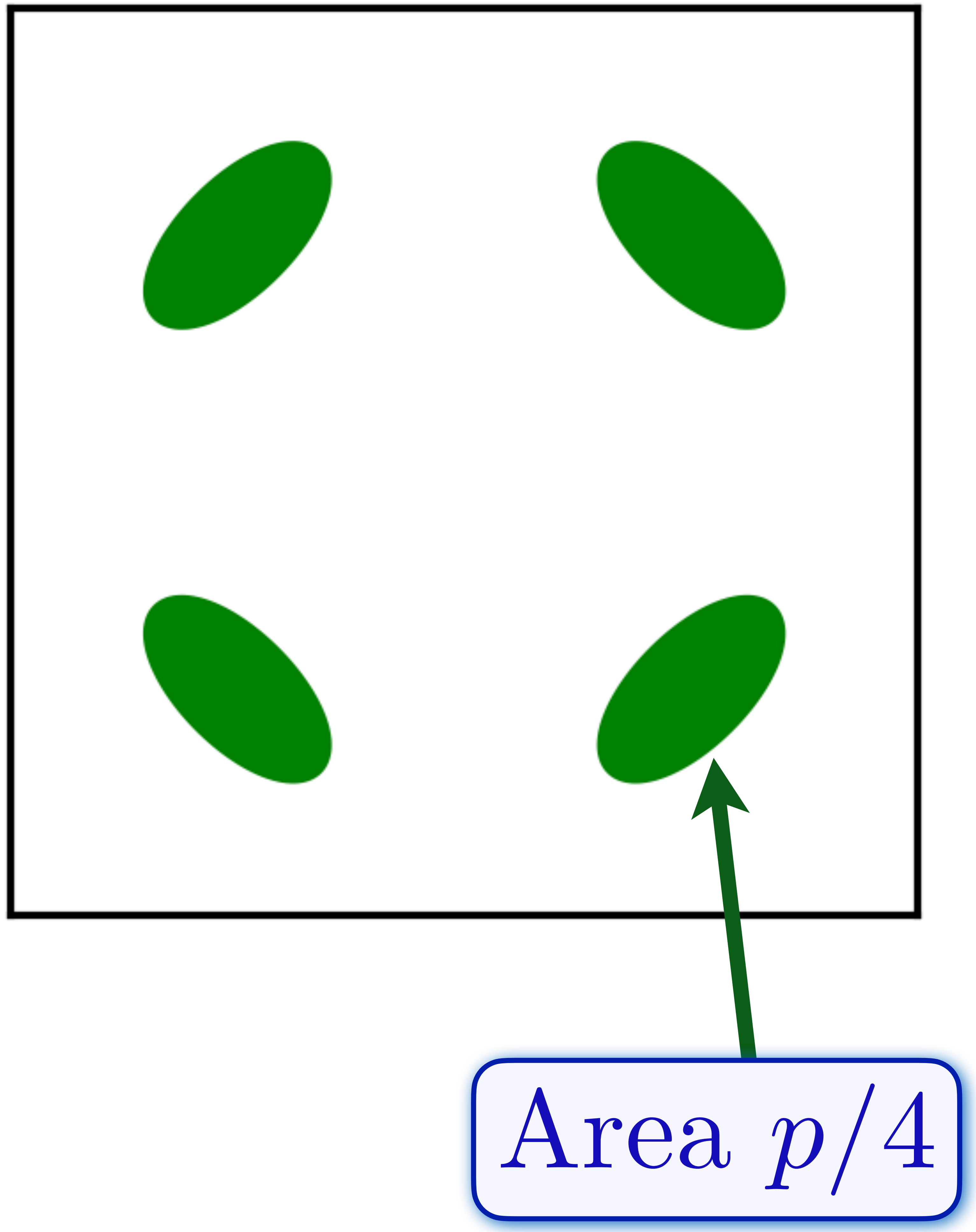
Holon metal

Spin liquid with density p of spinless, charge +e “holons” and charge 0 spin-1/2 “spinons”.



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

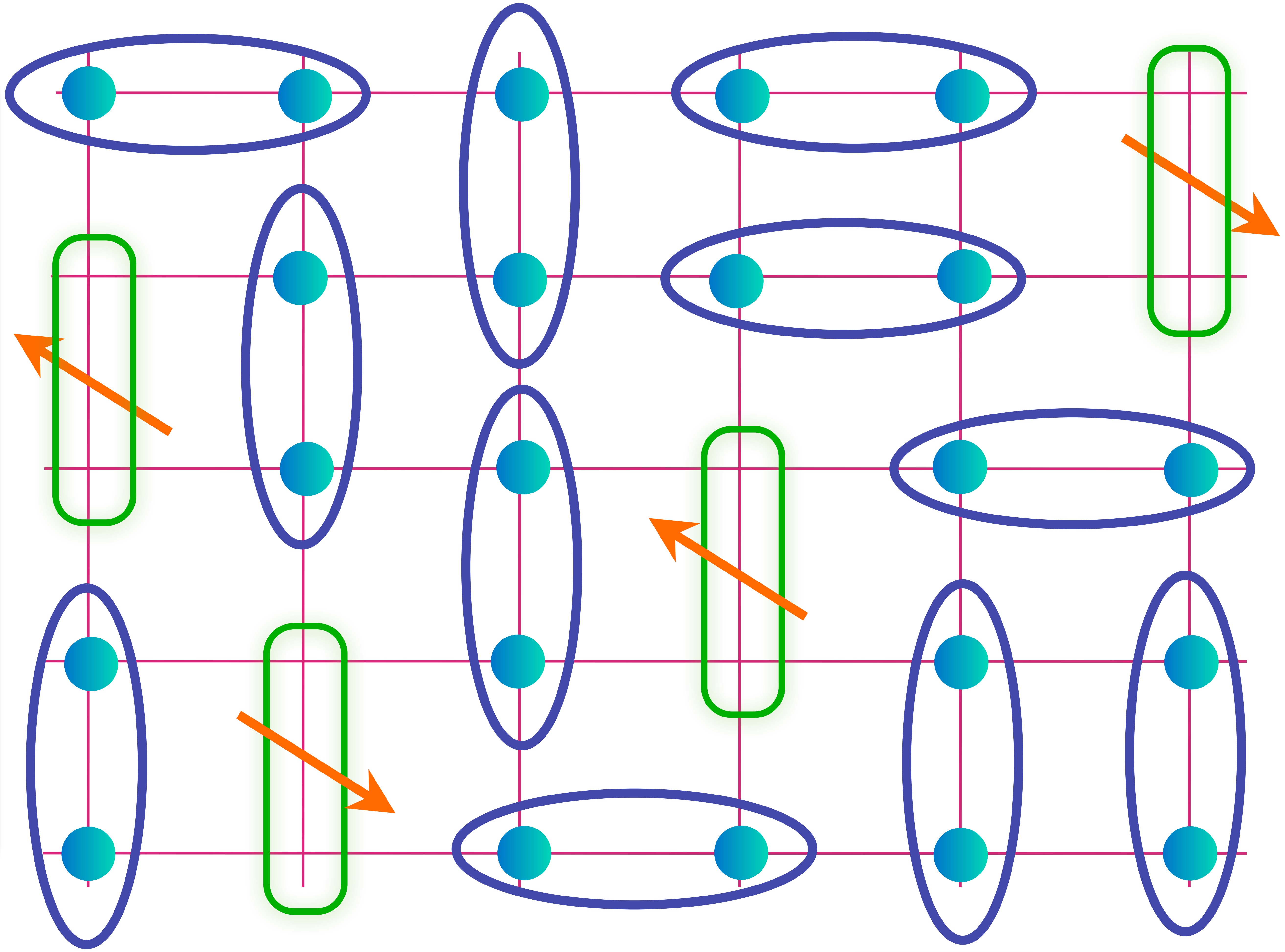
non-Luttinger area.
Spin liquid



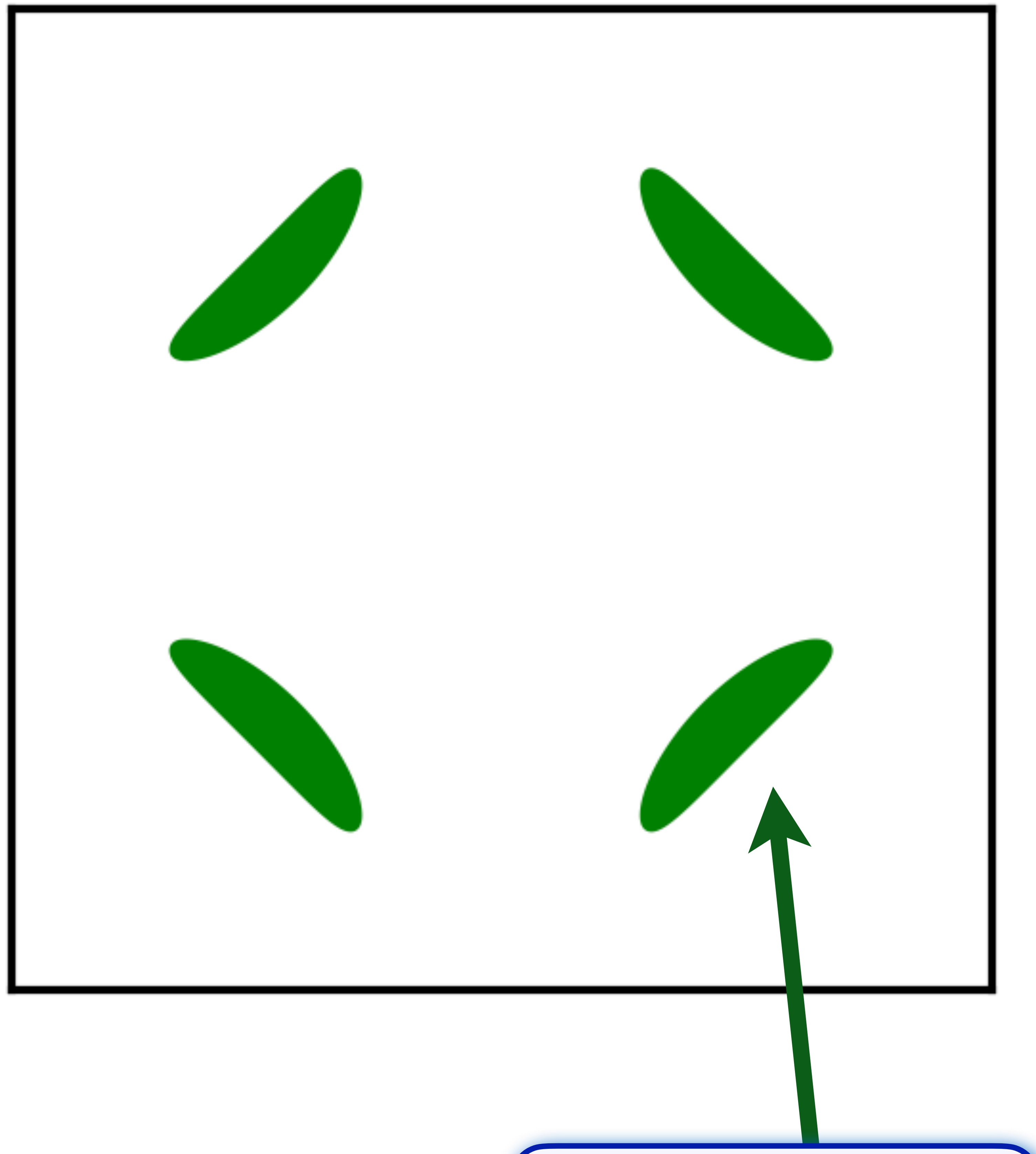
Doping an insulating antiferromagnet with holes of density p

FL*

Each green "dimer" is a bound state (a "magnetic polaron") of a vacancy and a free spin



non-Luttinger area. Spin liquid



Area $p/8$

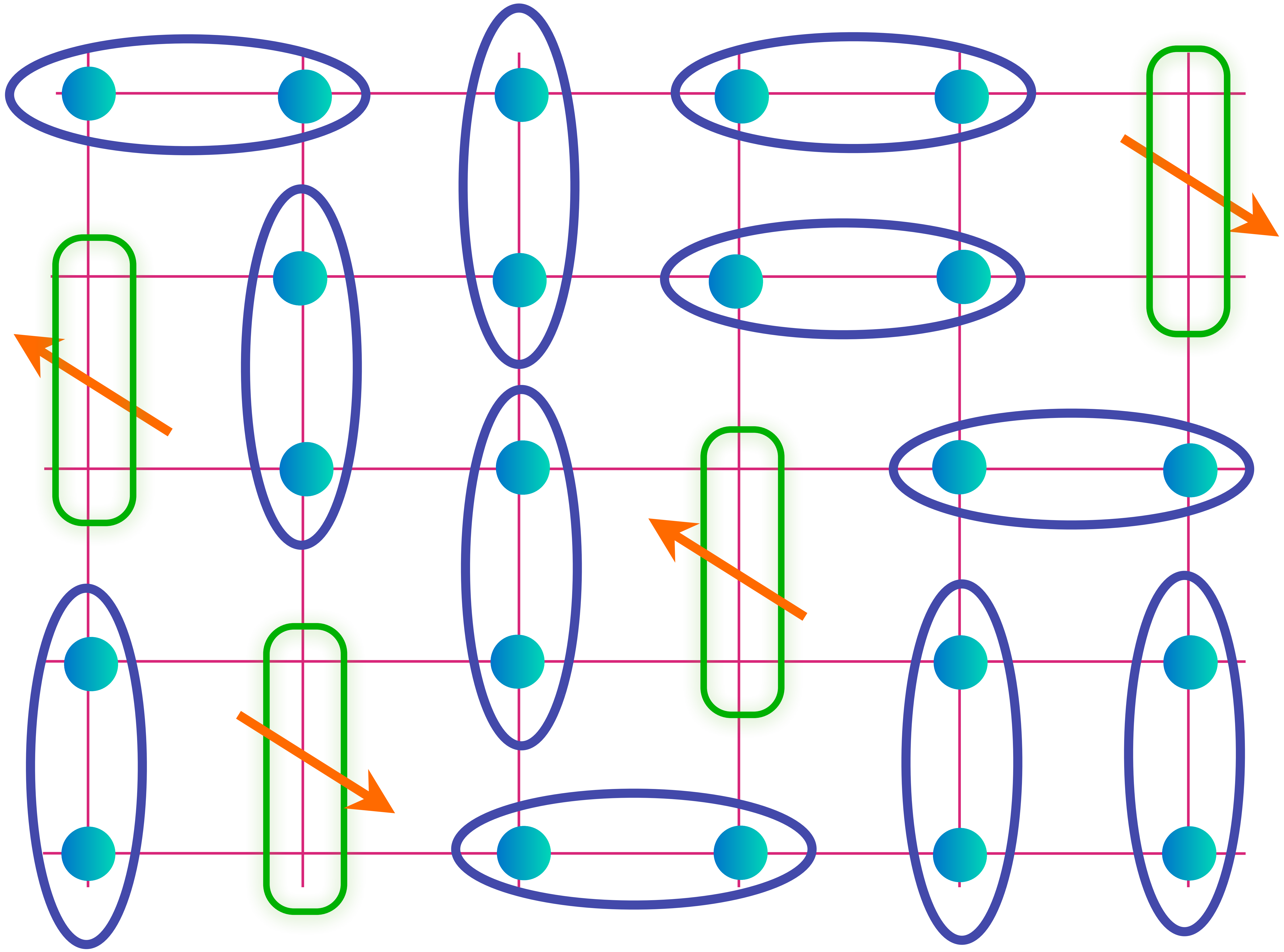
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$ $\quad$ $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

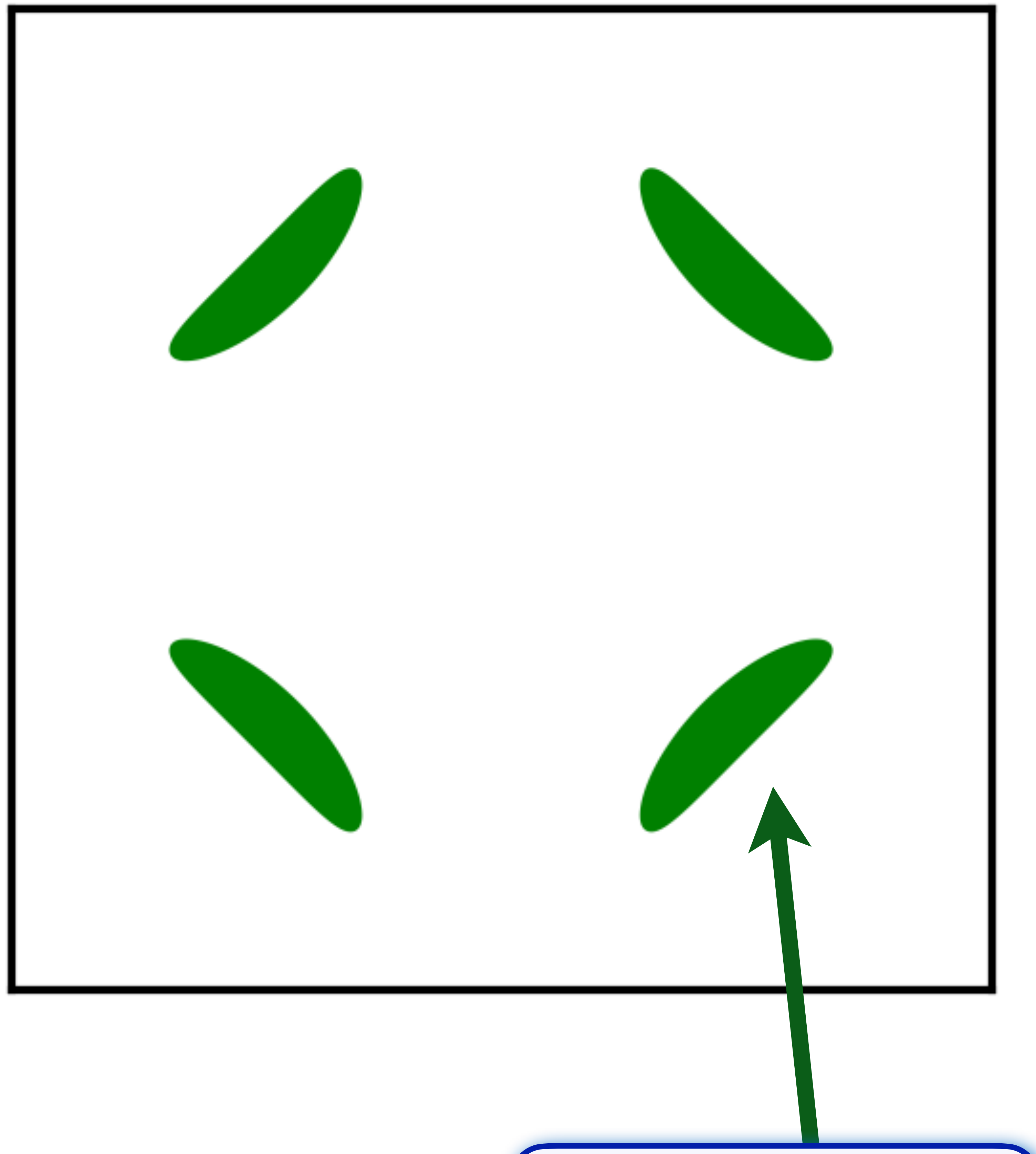
Doping an insulating antiferromagnet with holes of density p

FL*

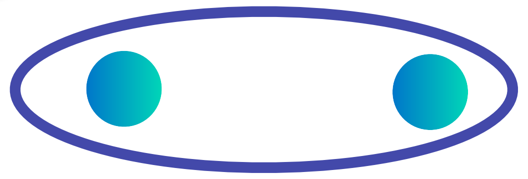
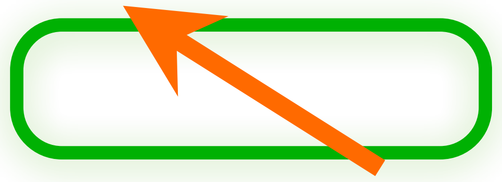
Metal with density p of spin-1/2, charge +e "holes" (or "magnetic polarons") and charge 0 spin-1/2 "spinons".



non-Luttinger area.
Spin liquid



Area $p/8$

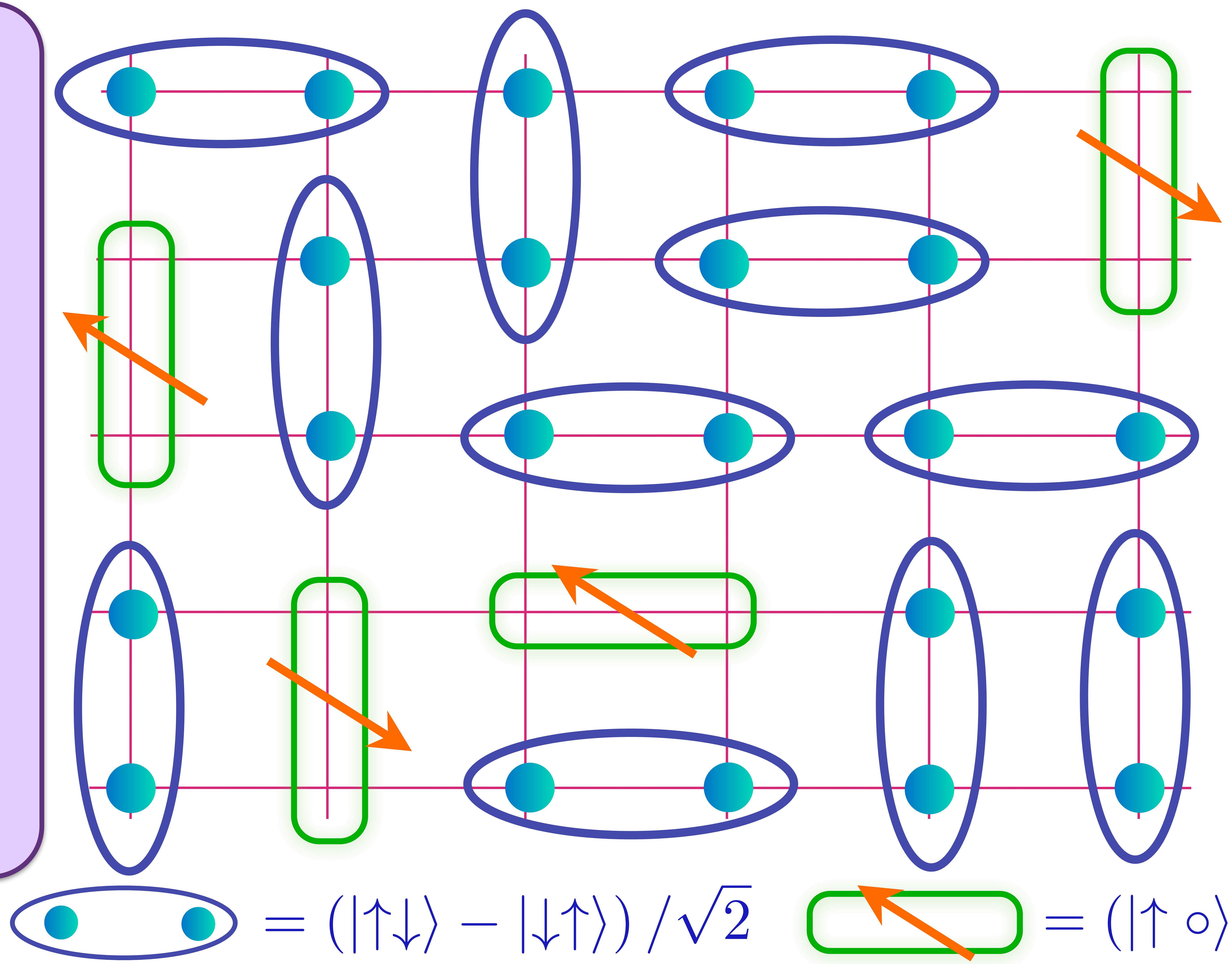
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

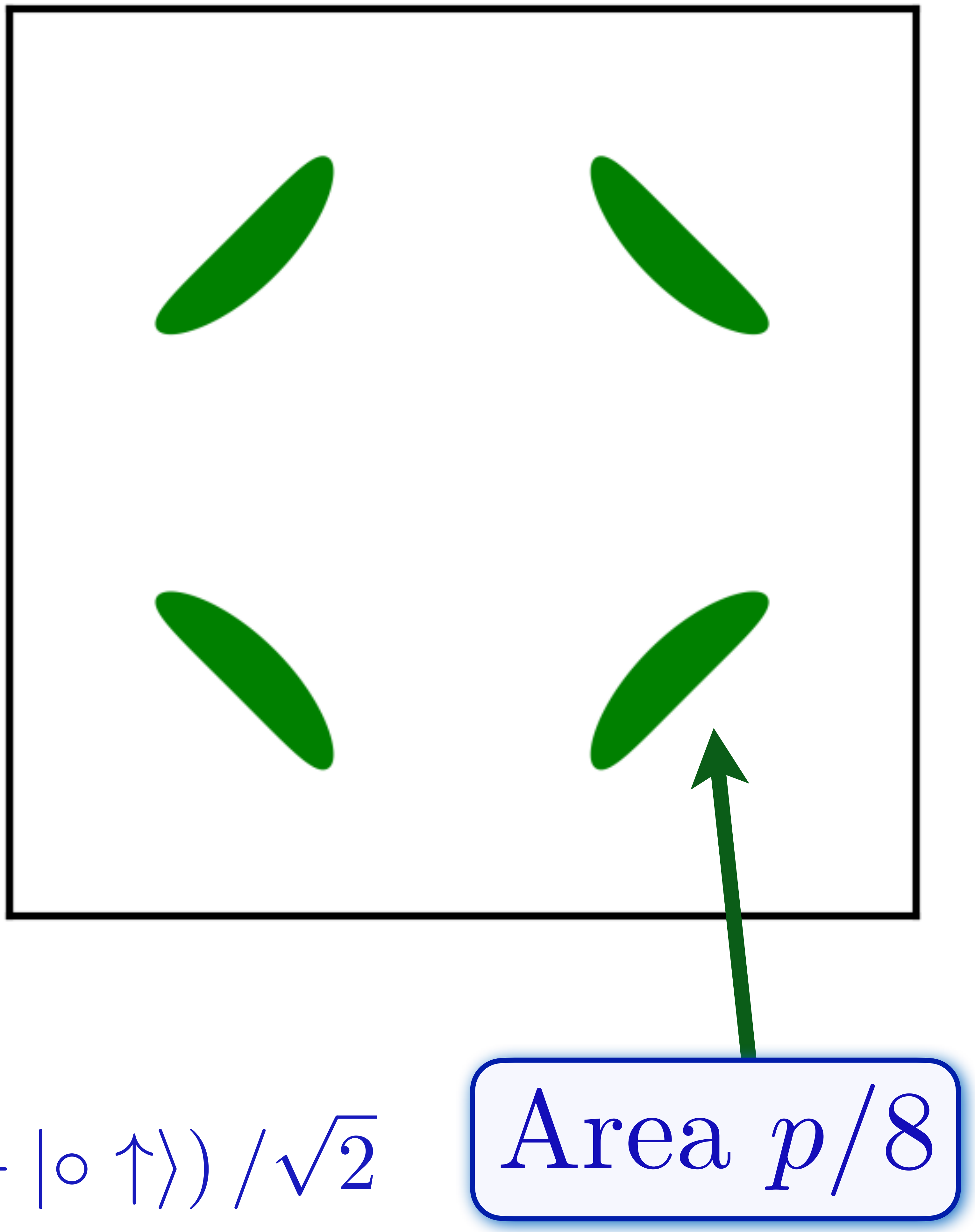
Doping an insulating antiferromagnet with holes of density p

FL*

Metal with density p of spin-1/2, charge +e "holes" (or "magnetic polarons") and charge 0 spin-1/2 "spinons".



non-Luttinger area.
Spin liquid



T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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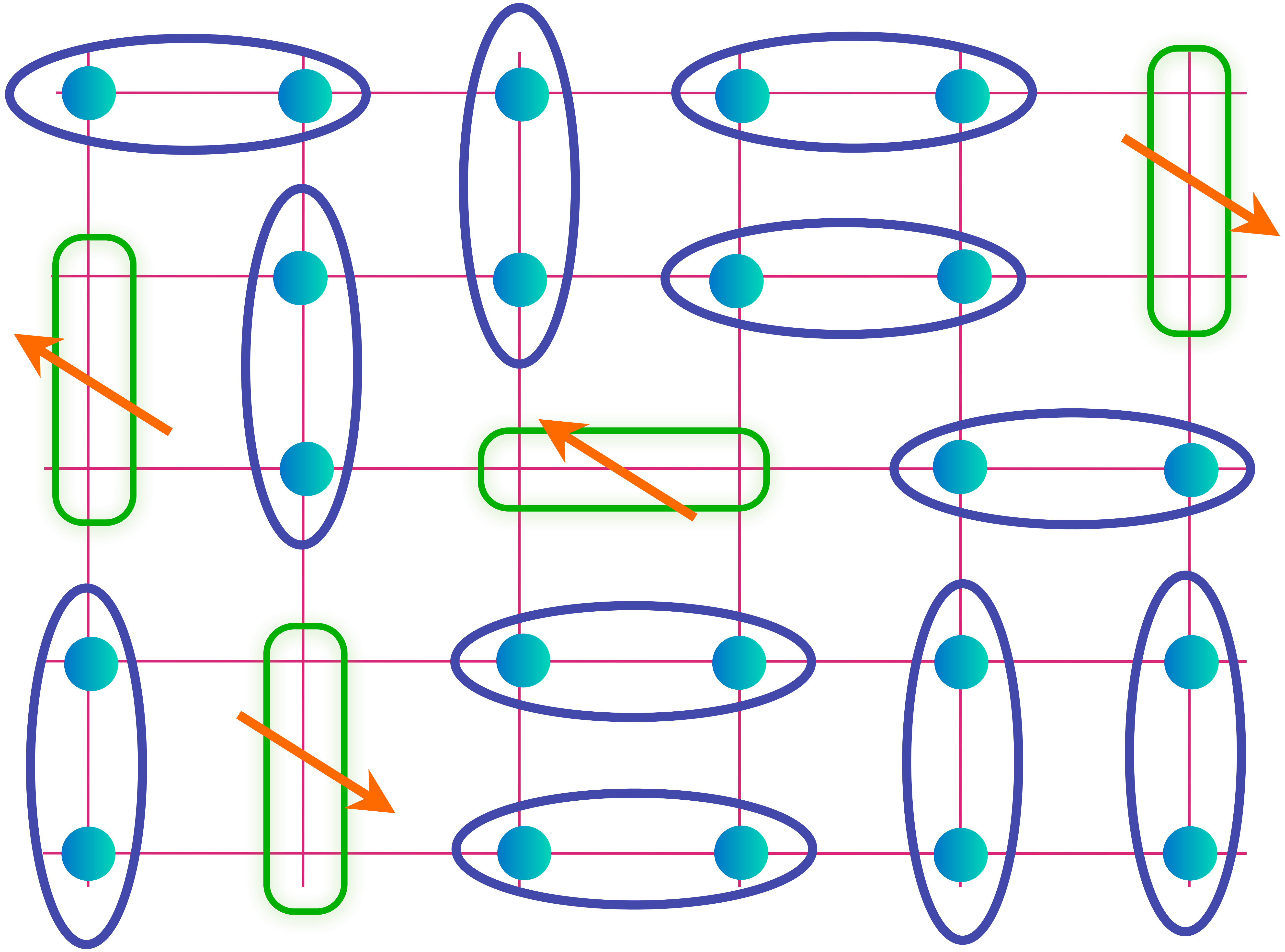
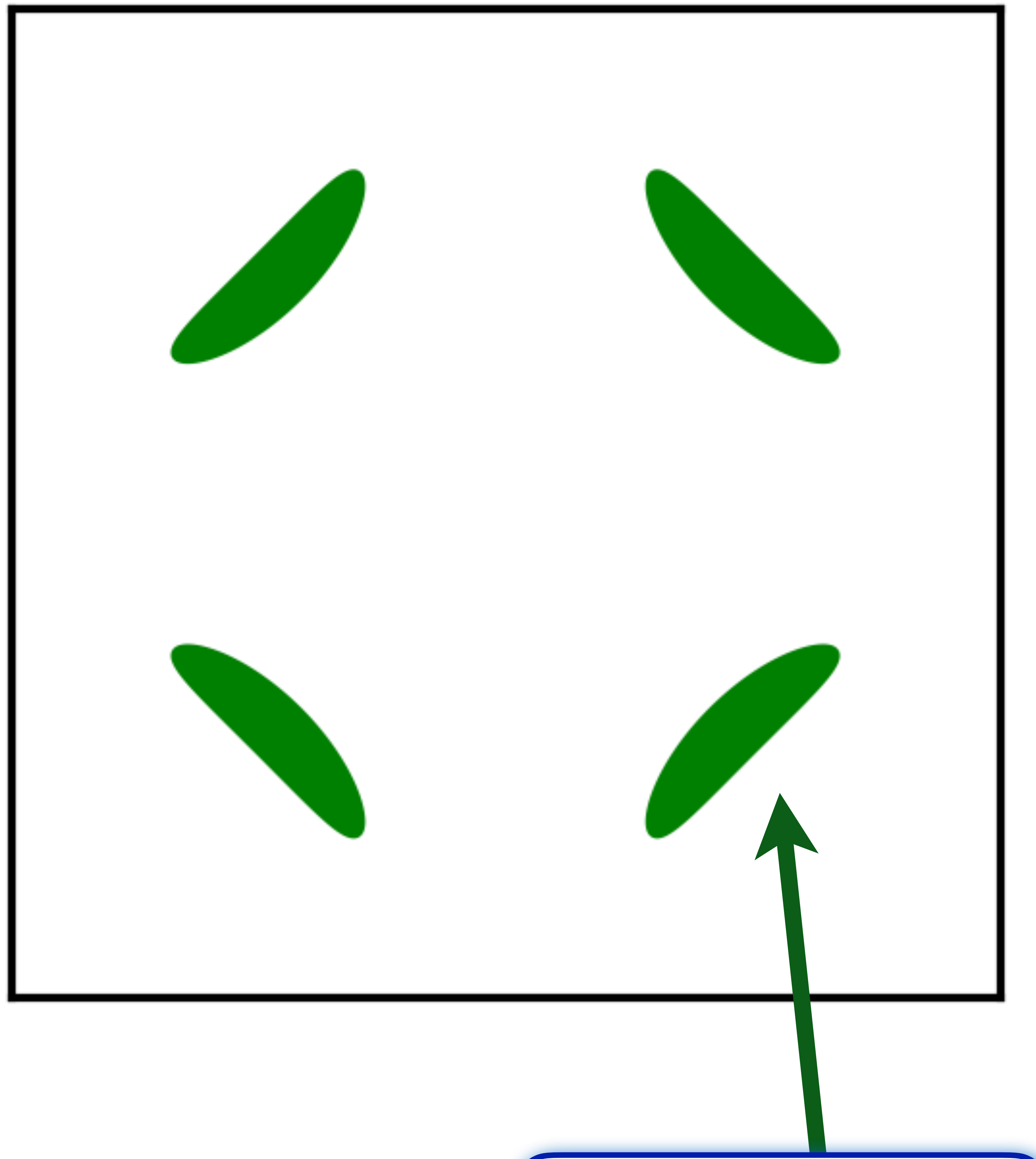
Doping an insulating antiferromagnet with holes of density p

FL*

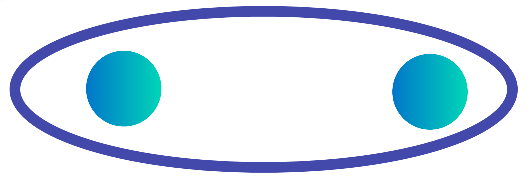
Metal with density p of spin-1/2, charge +e "holes" (or "magnetic polarons") and charge 0 spin-1/2 "spinons".

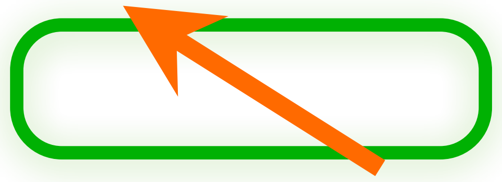


non-Luttinger area.
Spin liquid



Area $p/8$

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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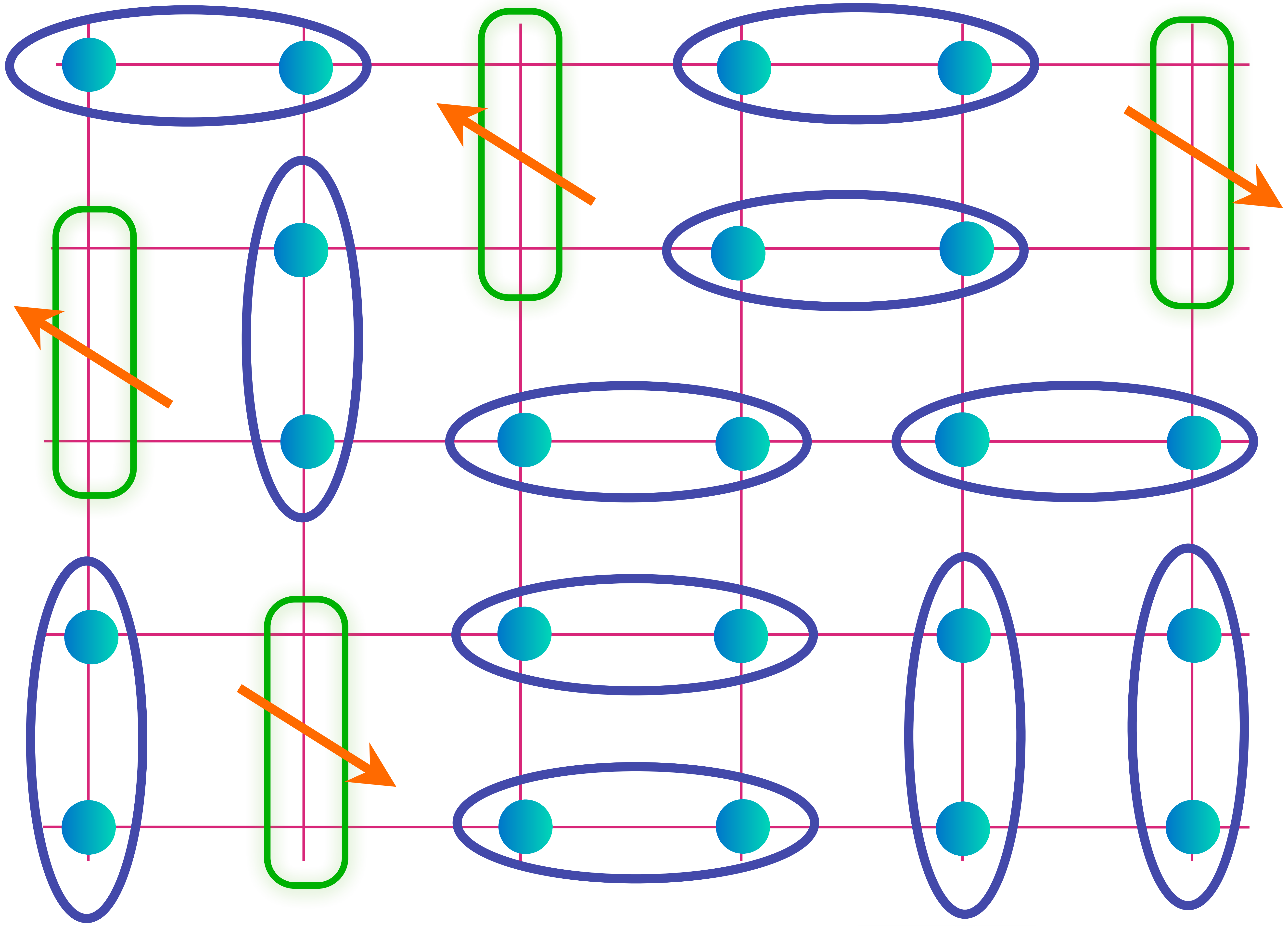
Doping an insulating antiferromagnet with holes of density p

FL*

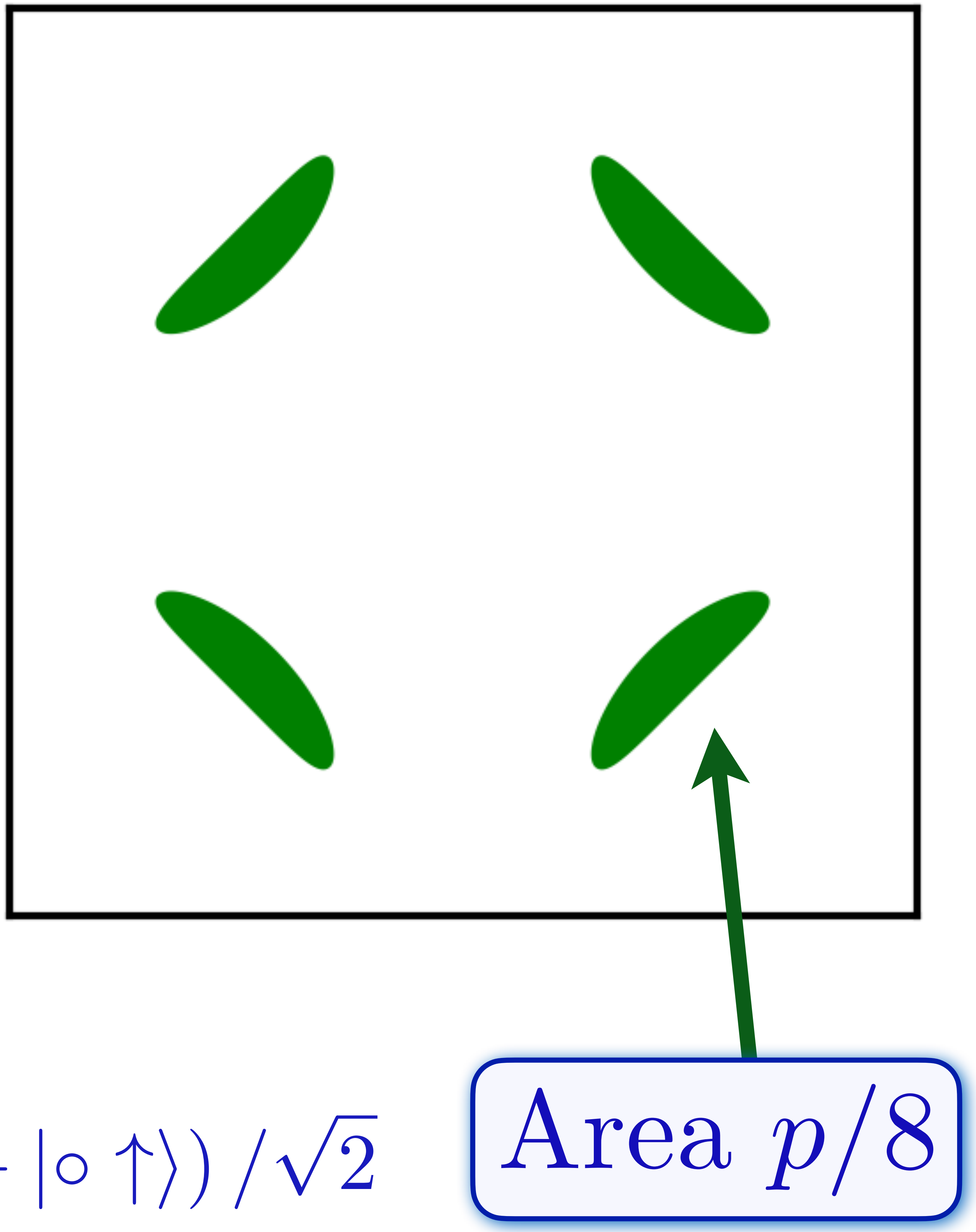
Metal with density p of spin-1/2, charge +e "holes" (or "magnetic polarons") and charge 0 spin-1/2 "spinons".



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

non-Luttinger area.
Spin liquid



T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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Doping an insulating antiferromagnet with holes of density p

FL*

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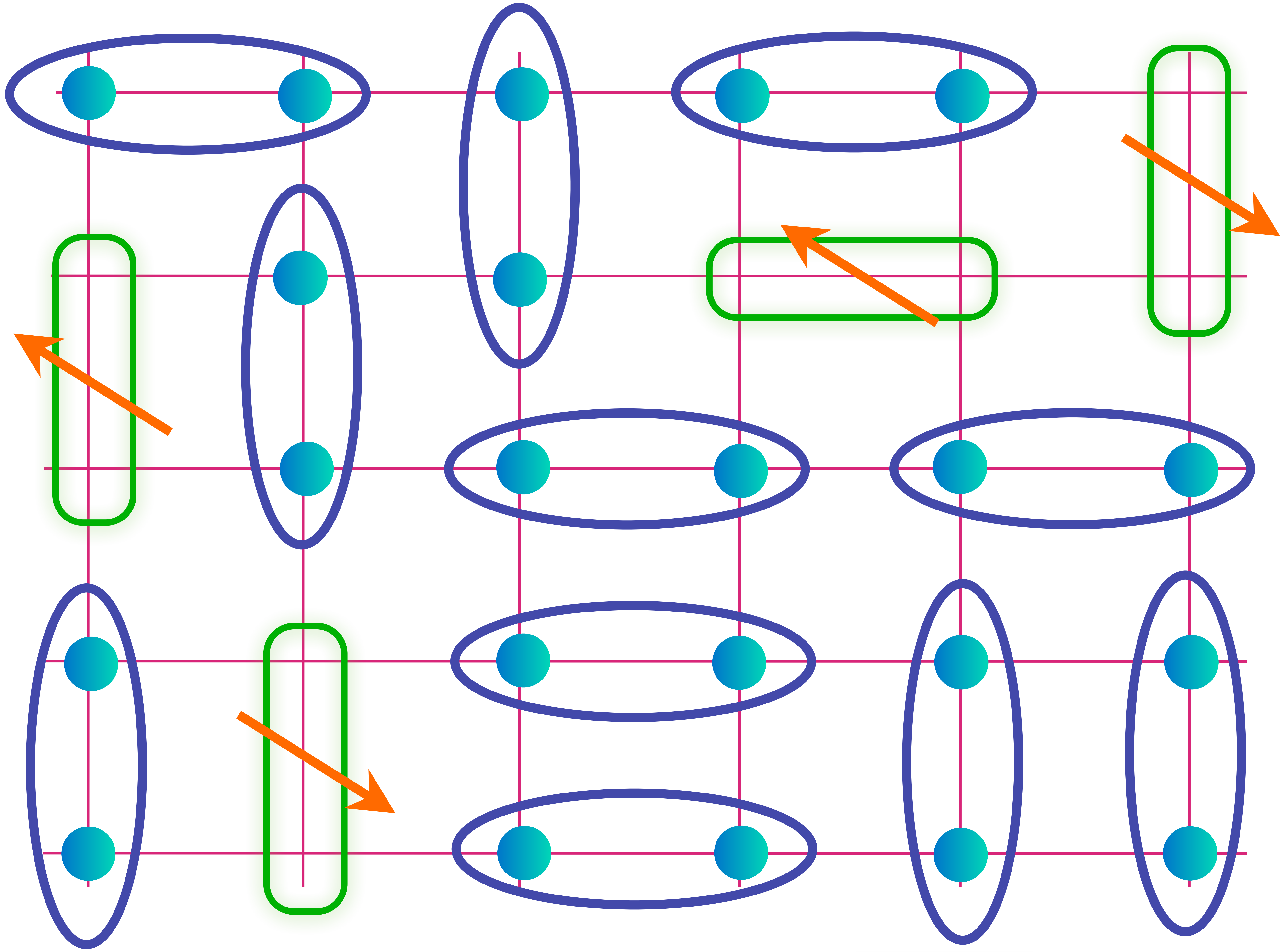
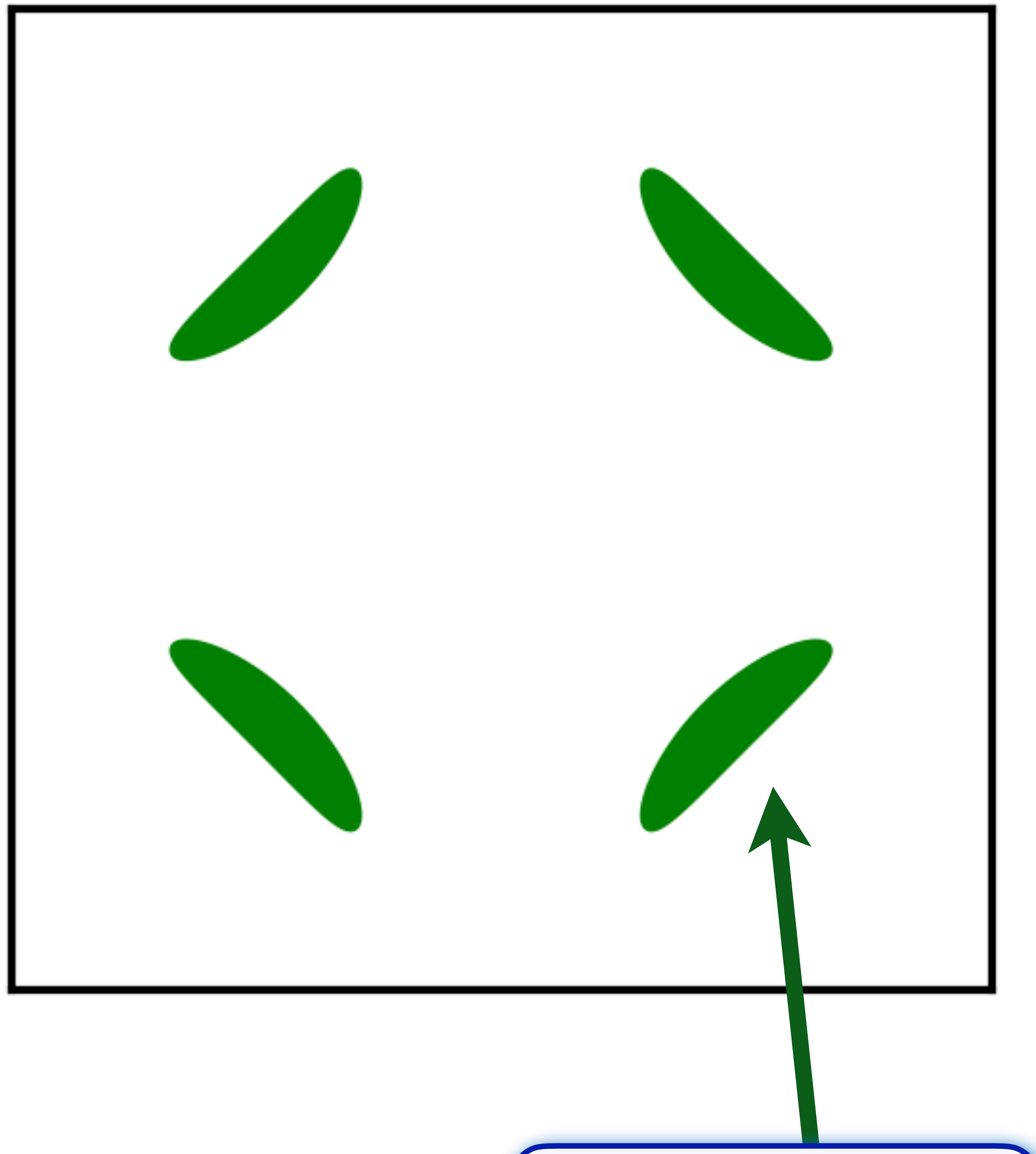
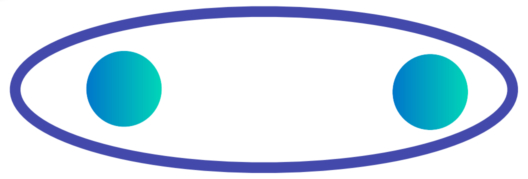
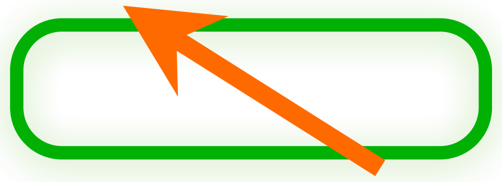


non-Luttinger area.
Spin liquid



Area $p/8$

 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$  $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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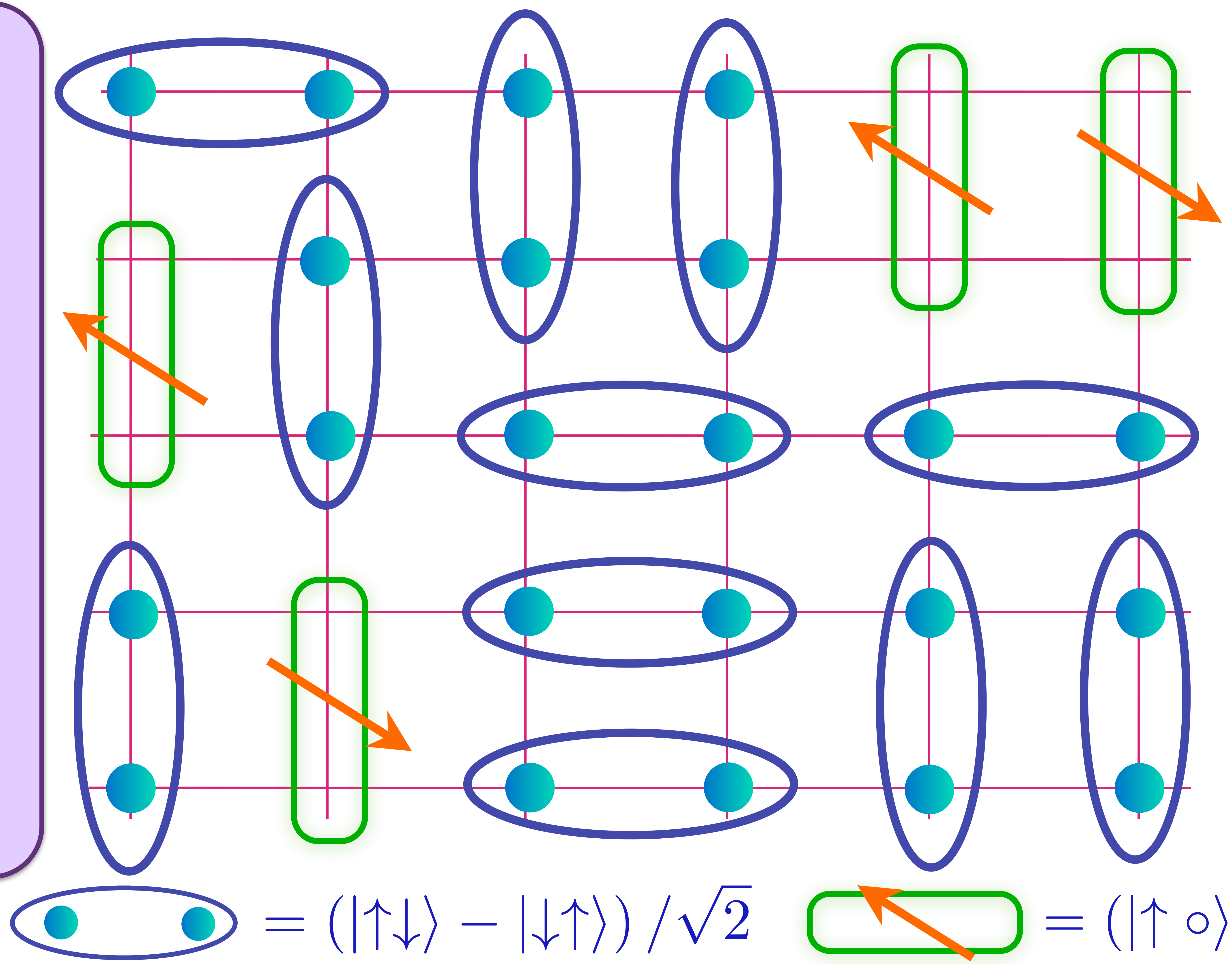
Doping an insulating antiferromagnet with holes of density p

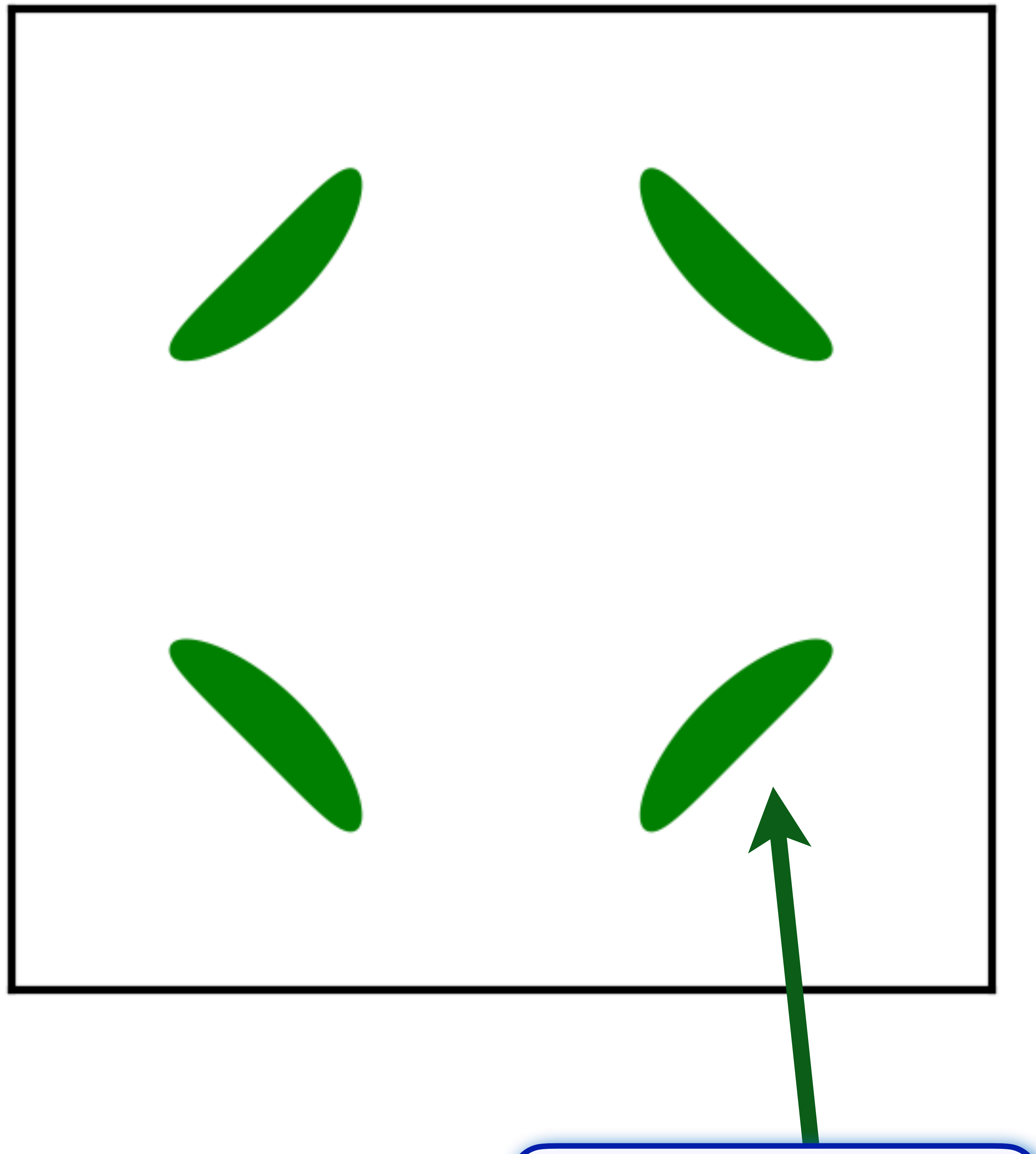
FL*

Metal with density p of spin-1/2, charge +e "holes" (or "magnetic polarons") and charge 0 spin-1/2 "spinons".



$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

non-Luttinger area.
Spin liquid



Area $p/8$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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Doping an insulating antiferromagnet with holes of density p

FL*

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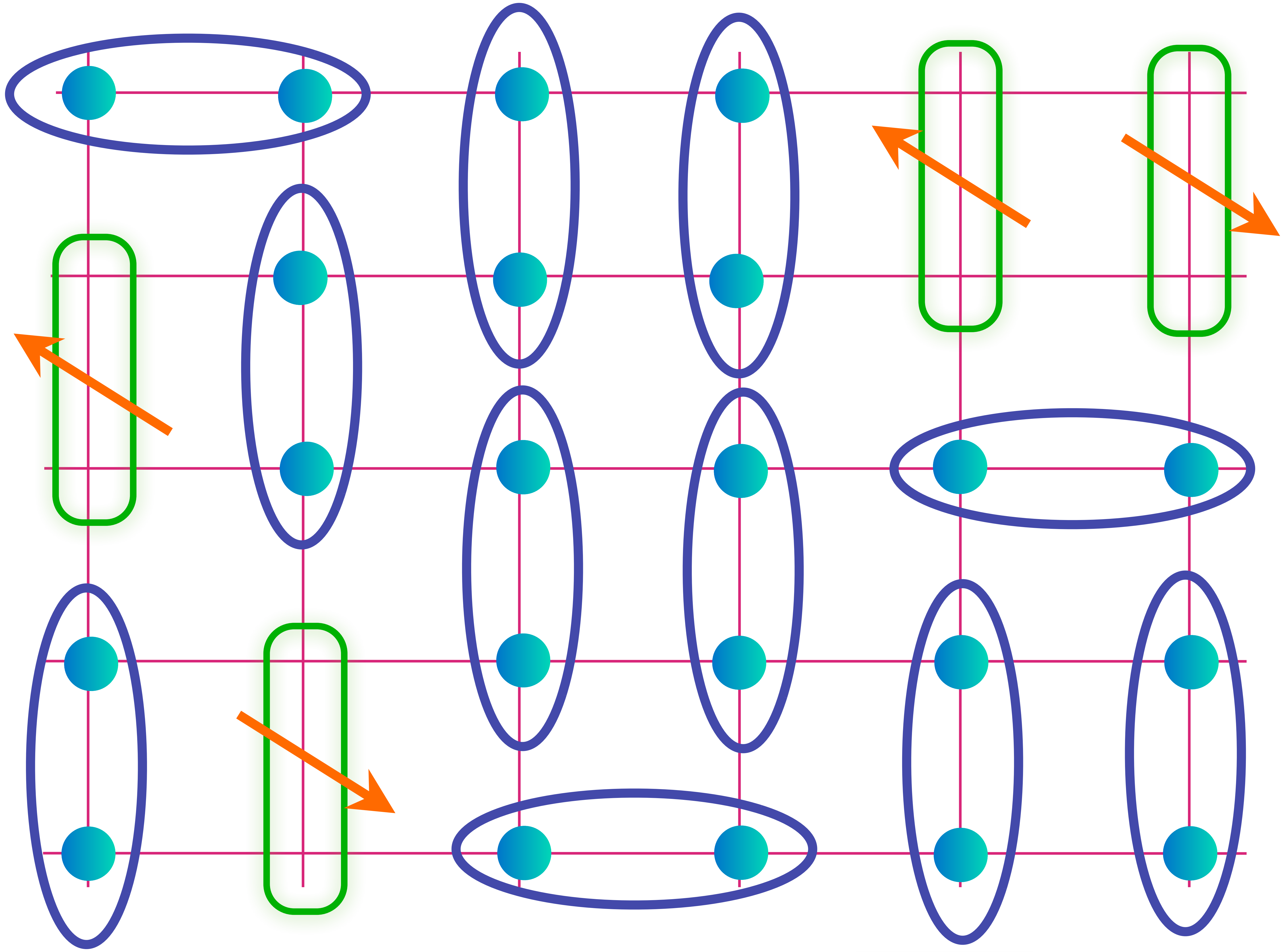
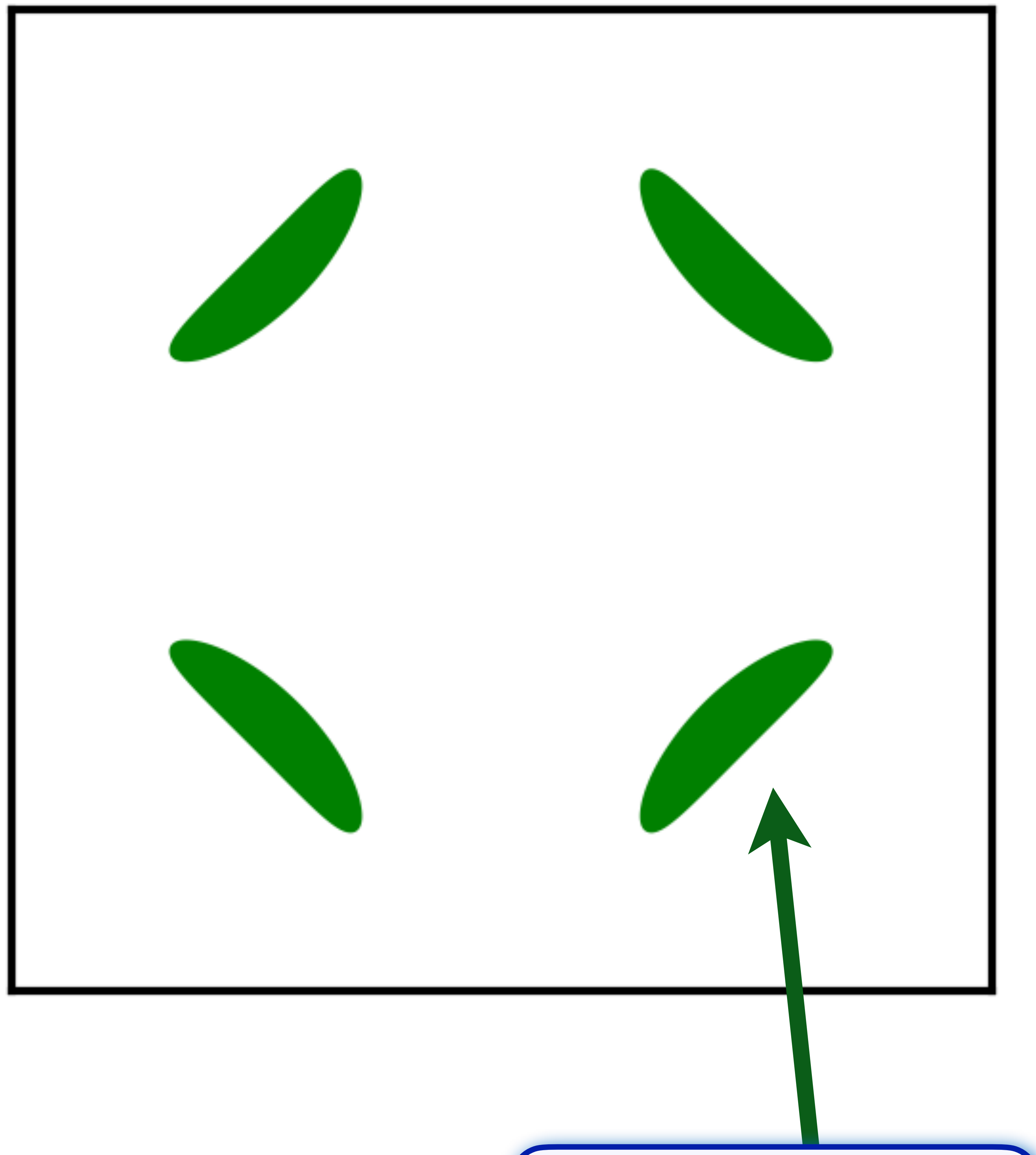


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$ $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

non-Luttinger area.
Spin liquid



Area $p/8$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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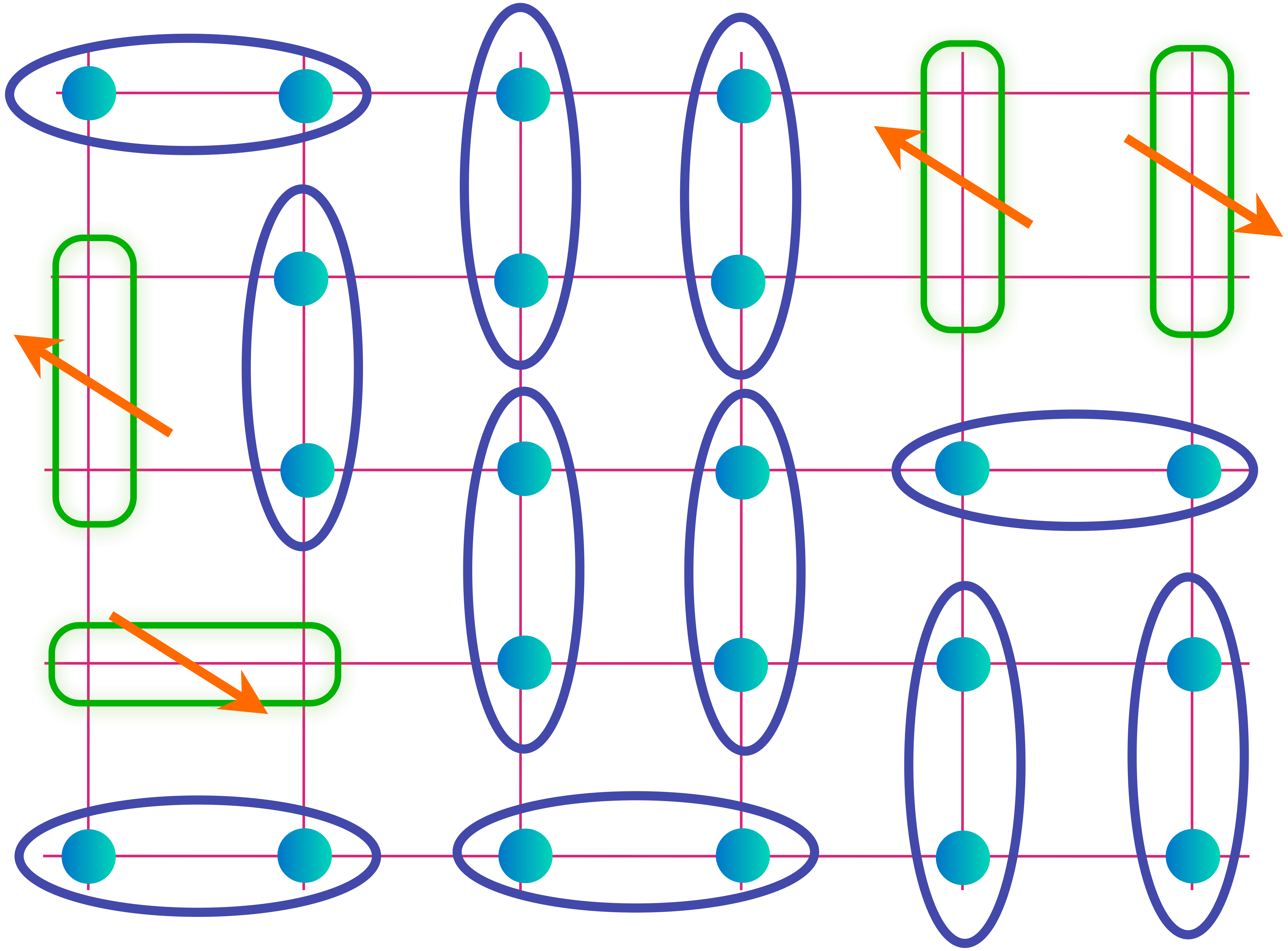
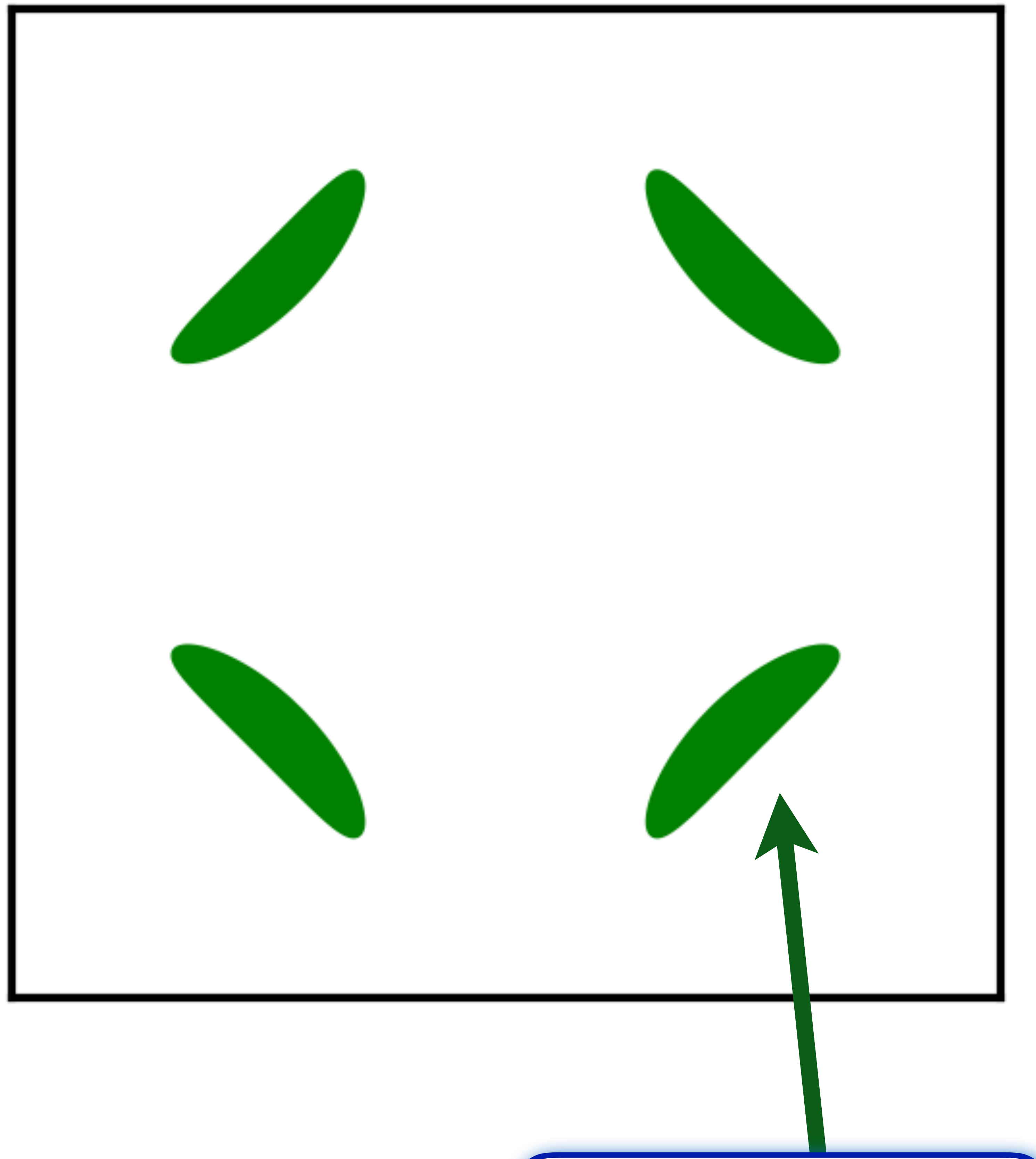
Doping an insulating antiferromagnet with holes of density p

FL*

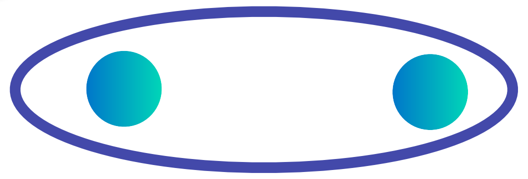
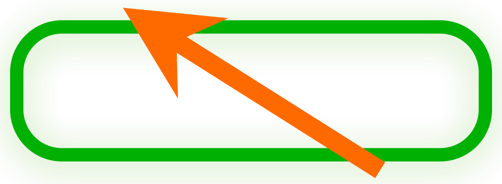
Metal with density p of spin-1/2, charge +e "holes" (or "magnetic polarons") and charge 0 spin-1/2 "spinons".



non-Luttinger area.
Spin liquid



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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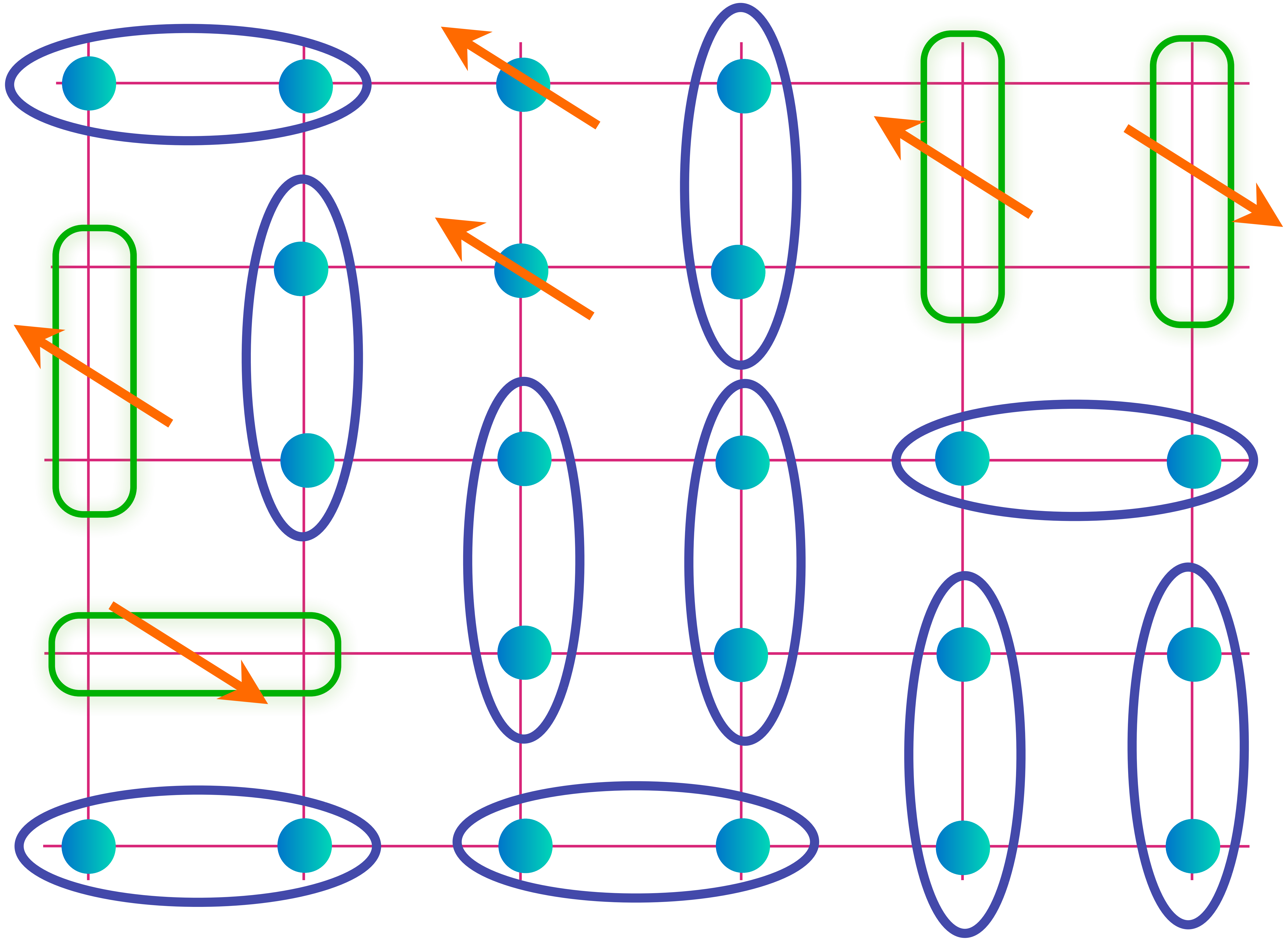
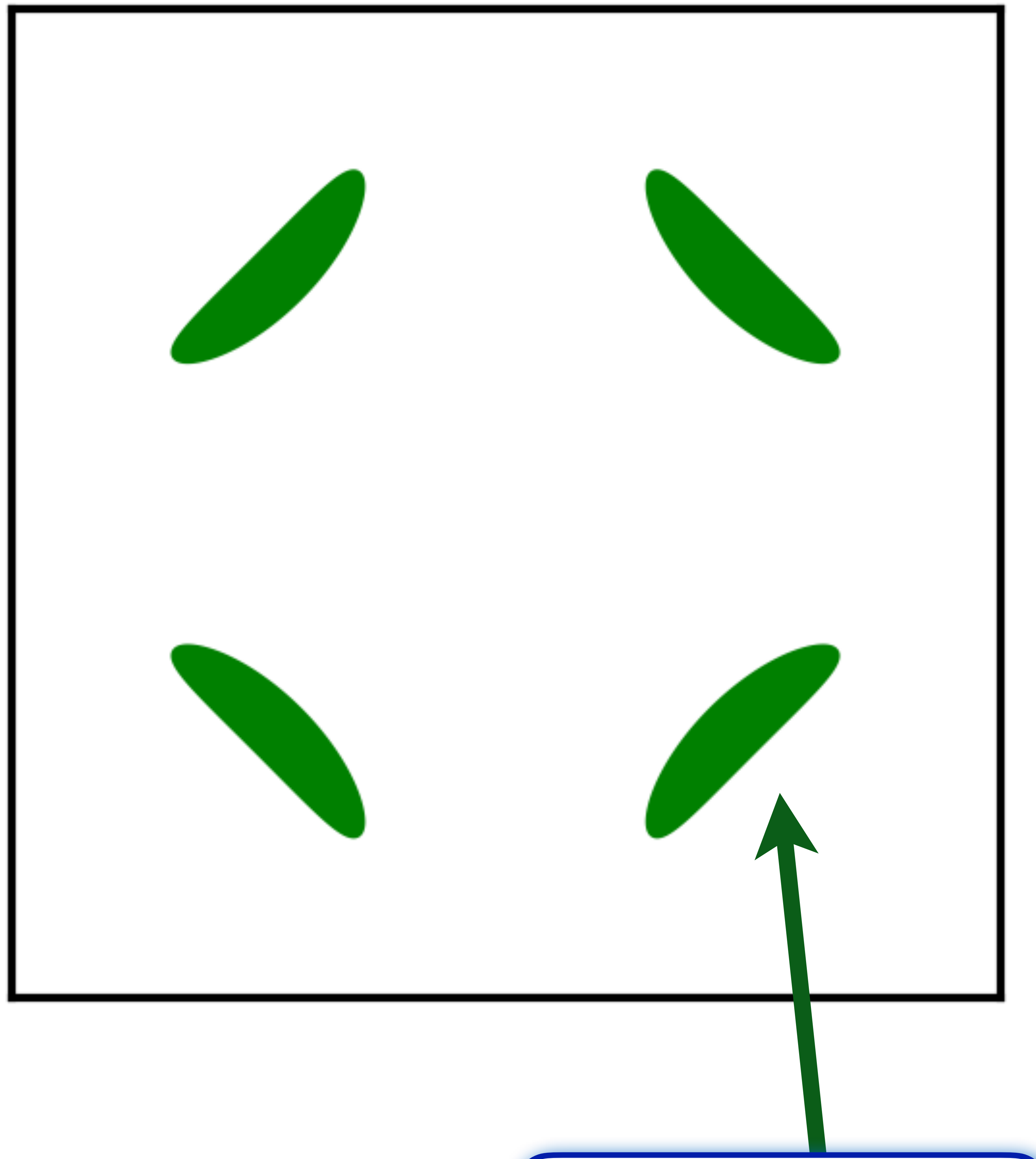
Doping an insulating antiferromagnet with holes of density p

FL*

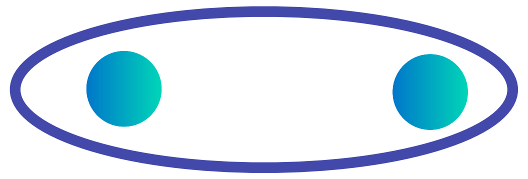
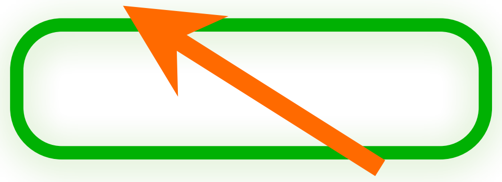
non-Luttinger area.
Spin liquid

The FL* state retains the spinon excitations





Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$  $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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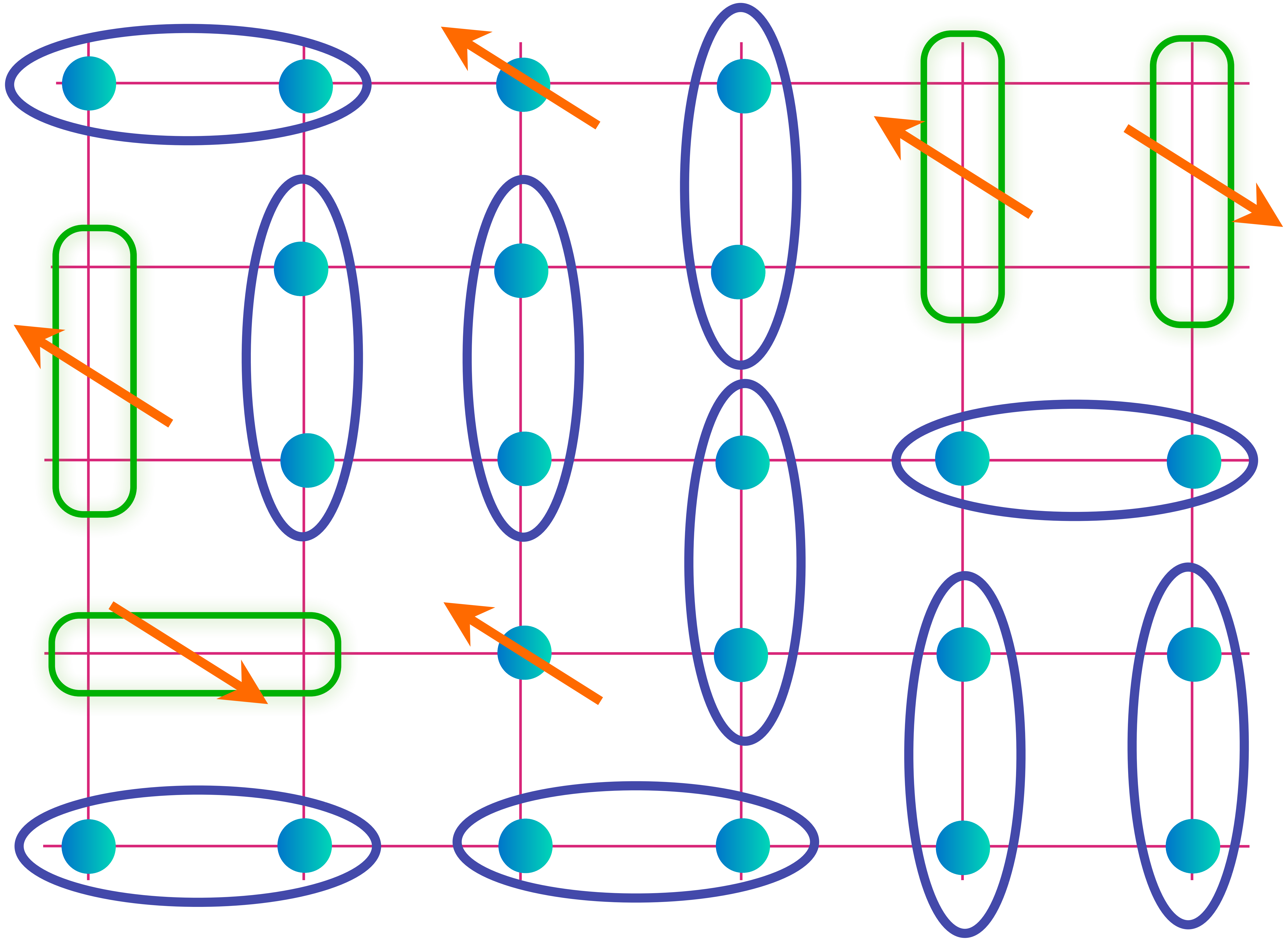
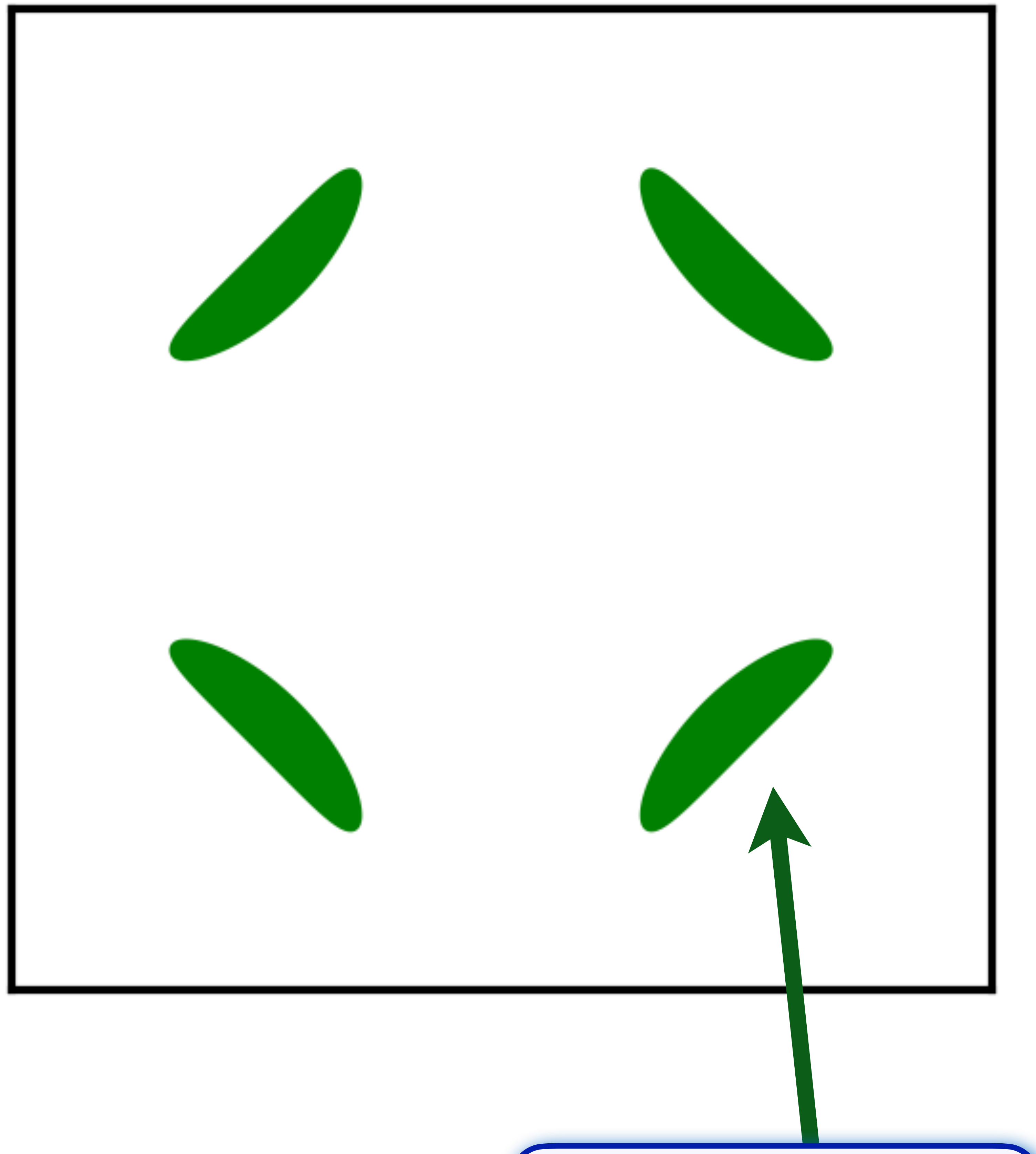
Doping an insulating antiferromagnet with holes of density p

FL*

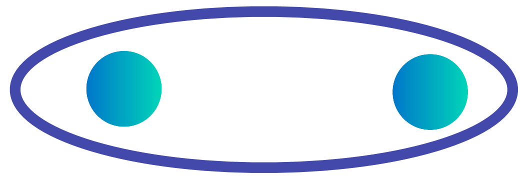
The FL* state retains the spinon excitations

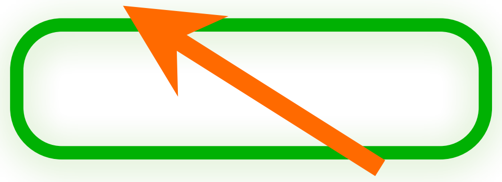


non-Luttinger area.
Spin liquid



Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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Doping an insulating antiferromagnet with holes of density p

$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$

$$= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
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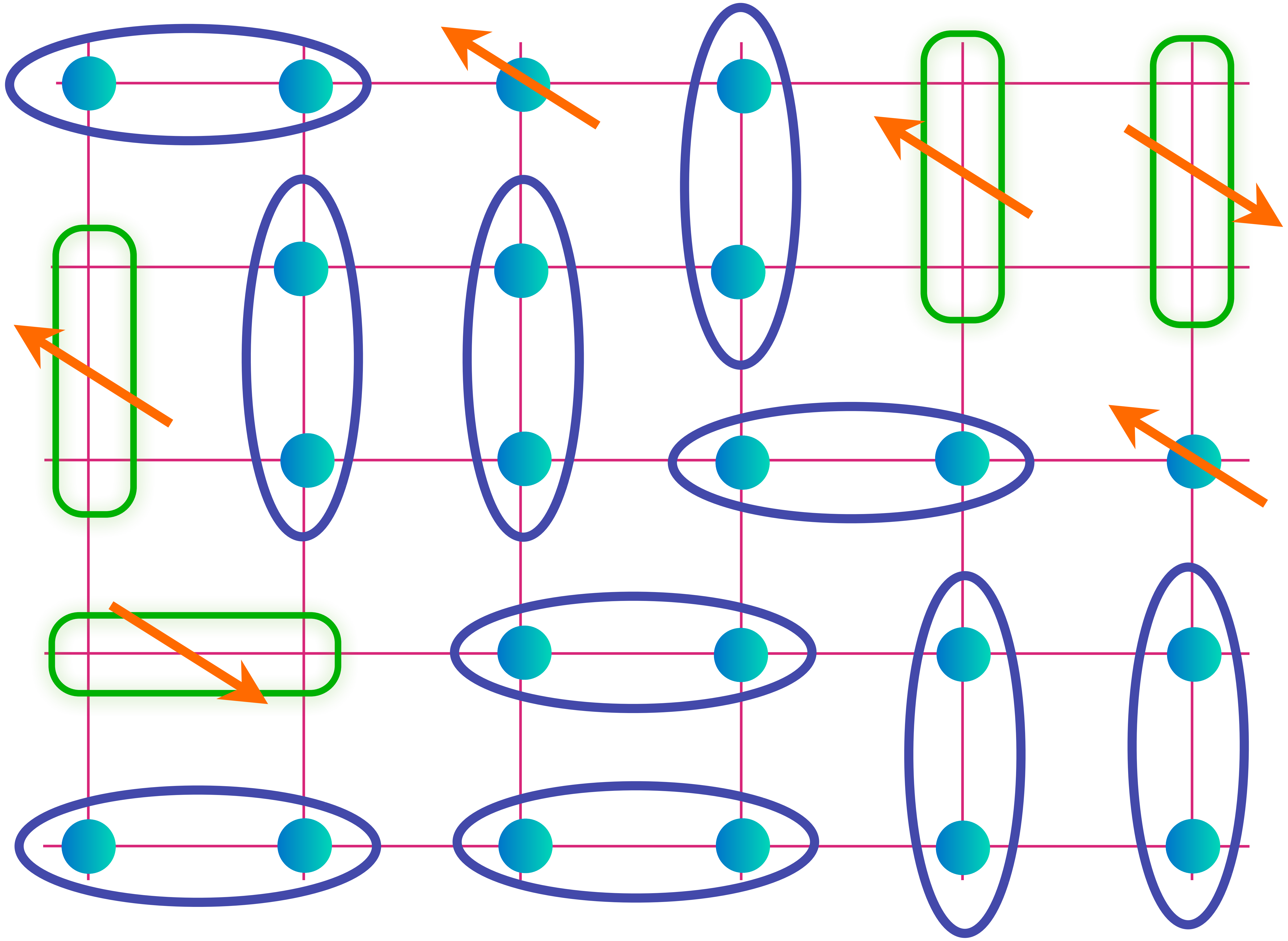
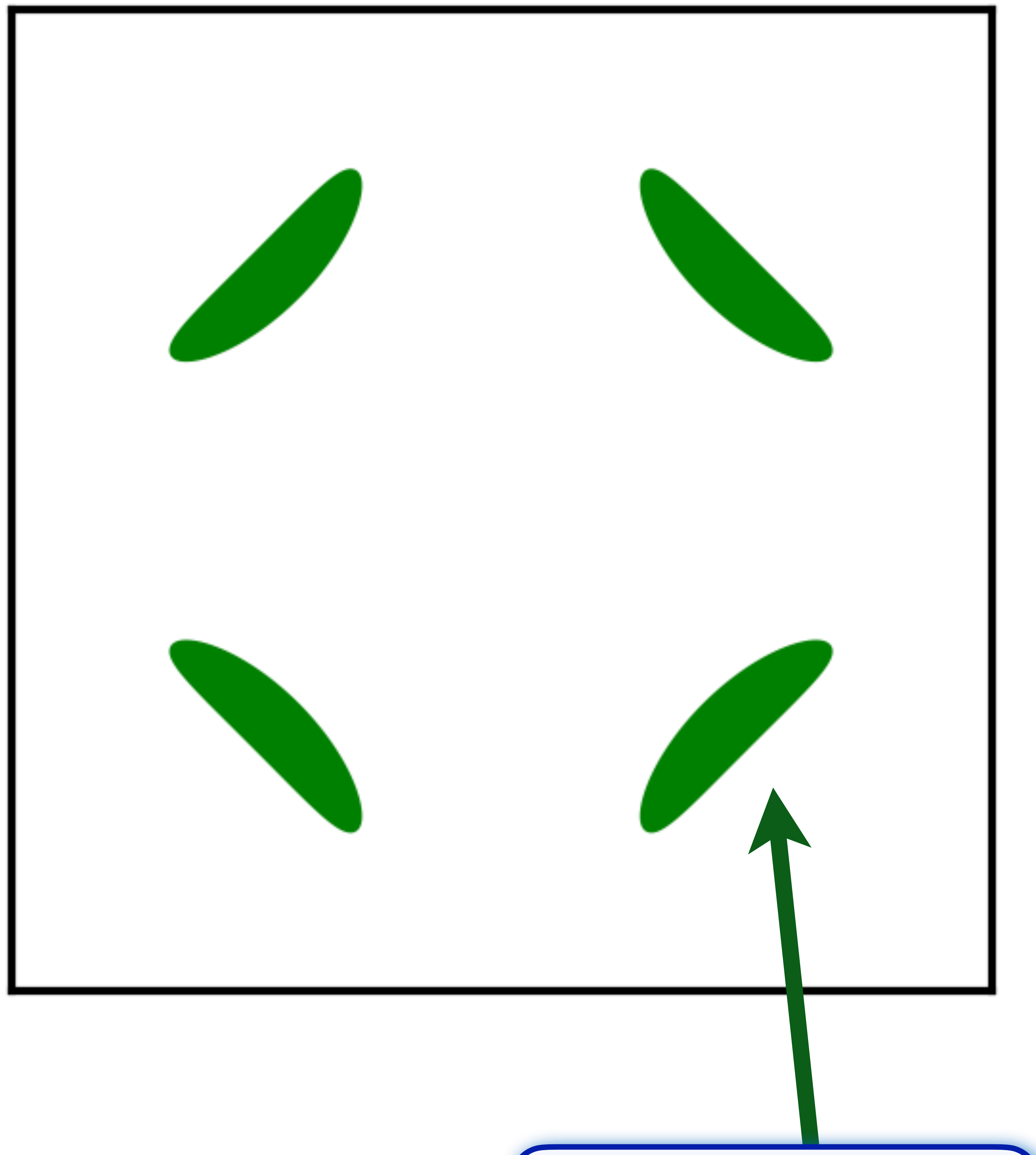
Doping an insulating antiferromagnet with holes of density p

FL*

The FL* state retains the spinon excitations



non-Luttinger area.
Spin liquid



Area $p/8$

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2} \qquad = \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$$

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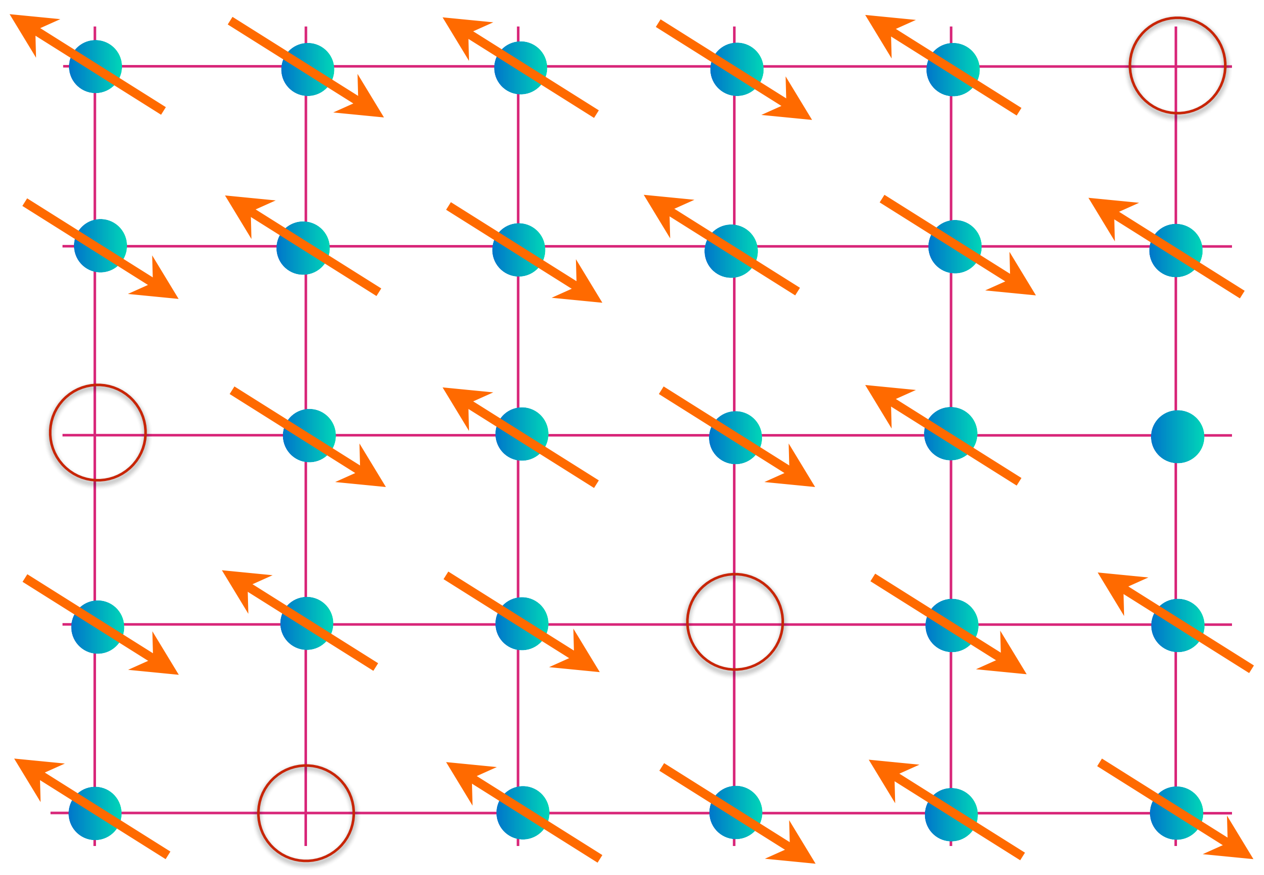
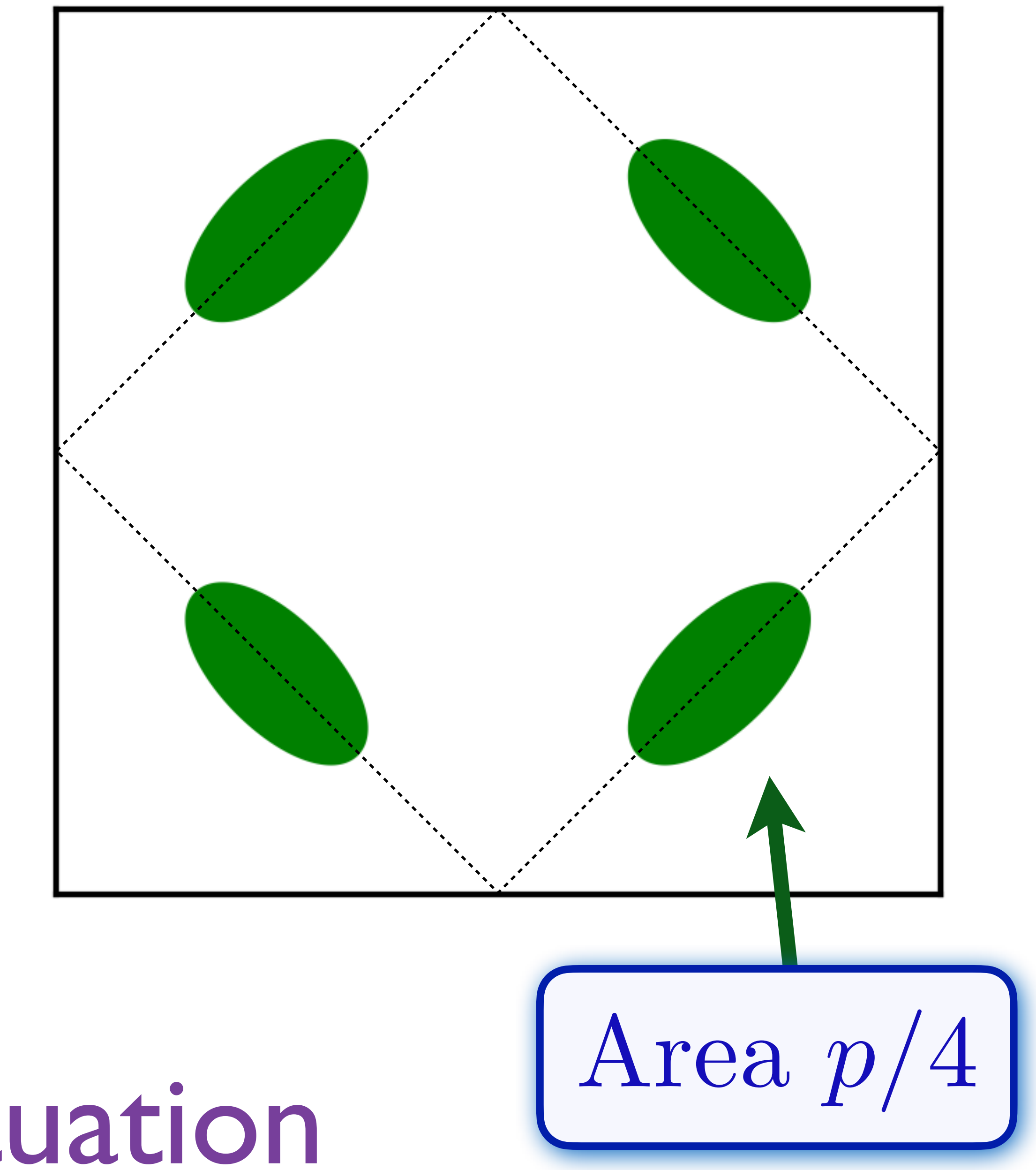
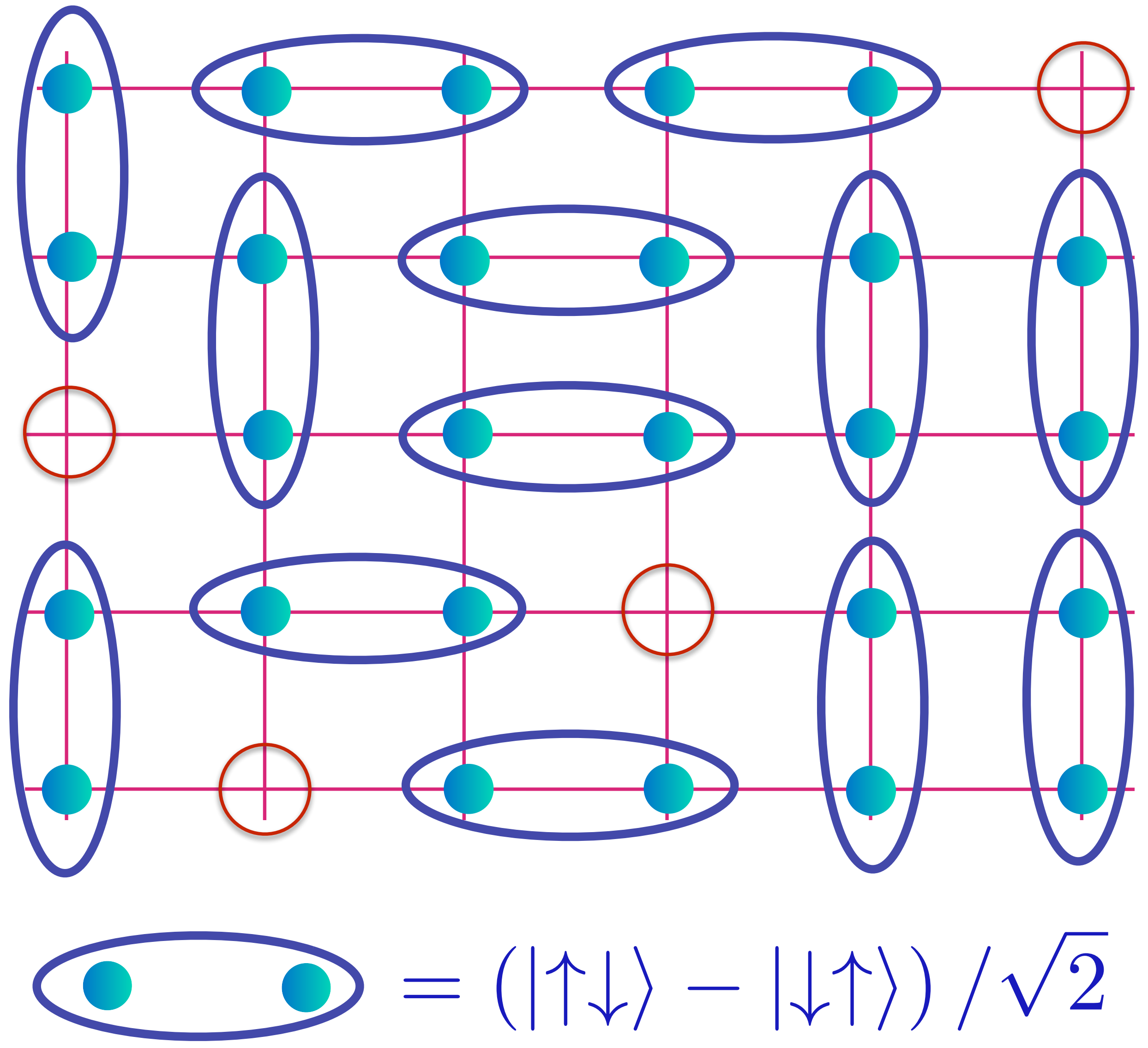
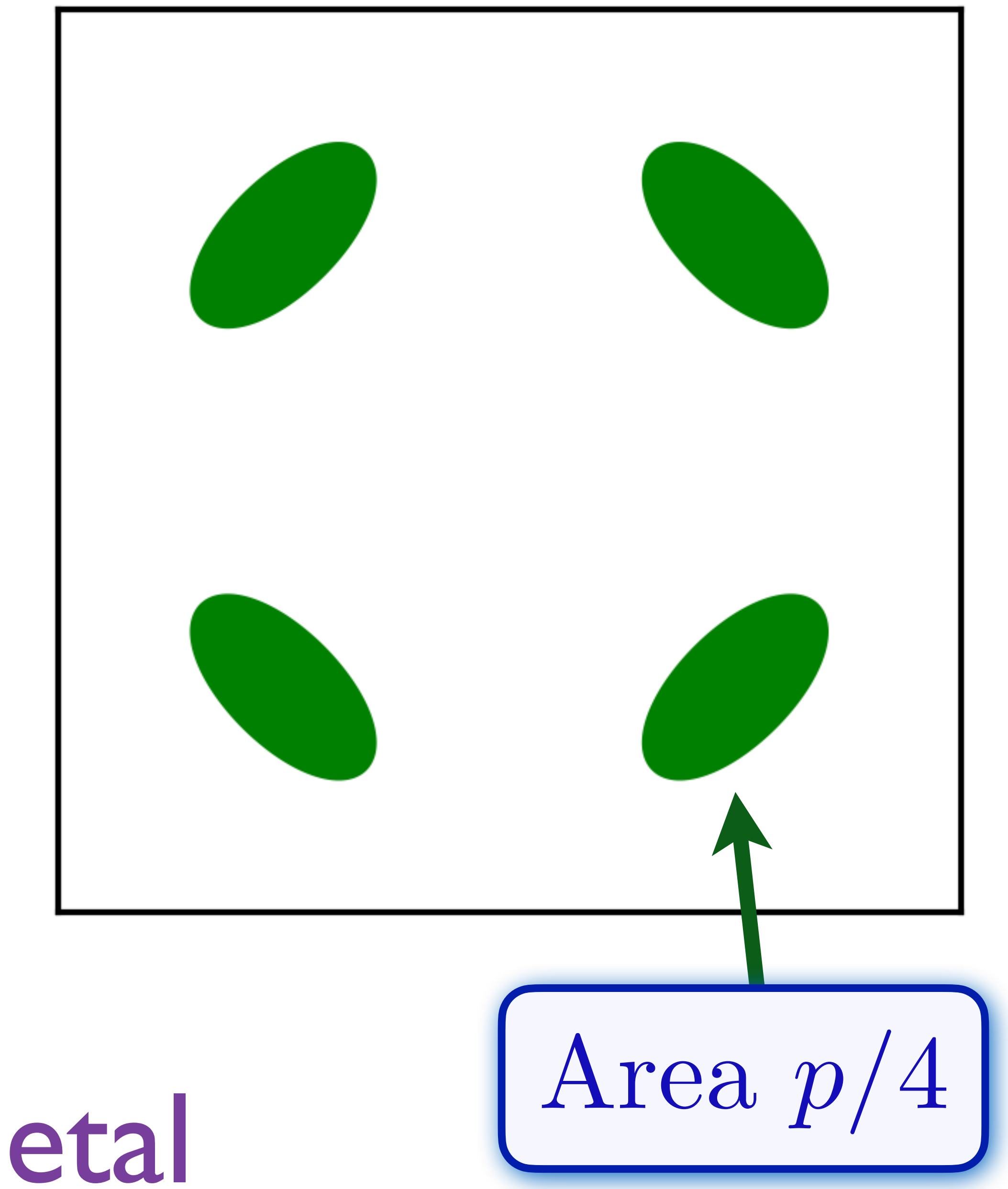




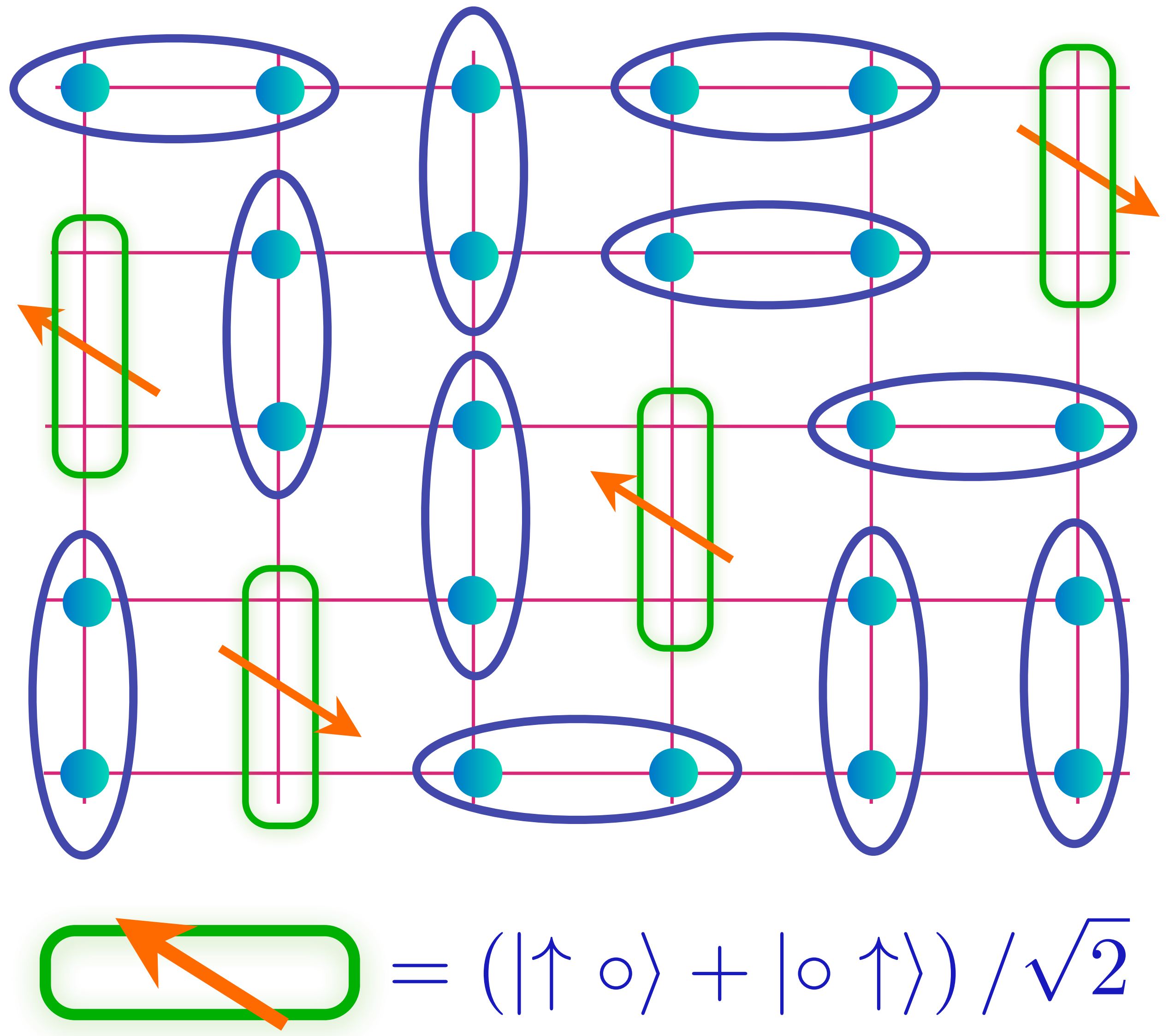
AF metal and SDW fluctuation



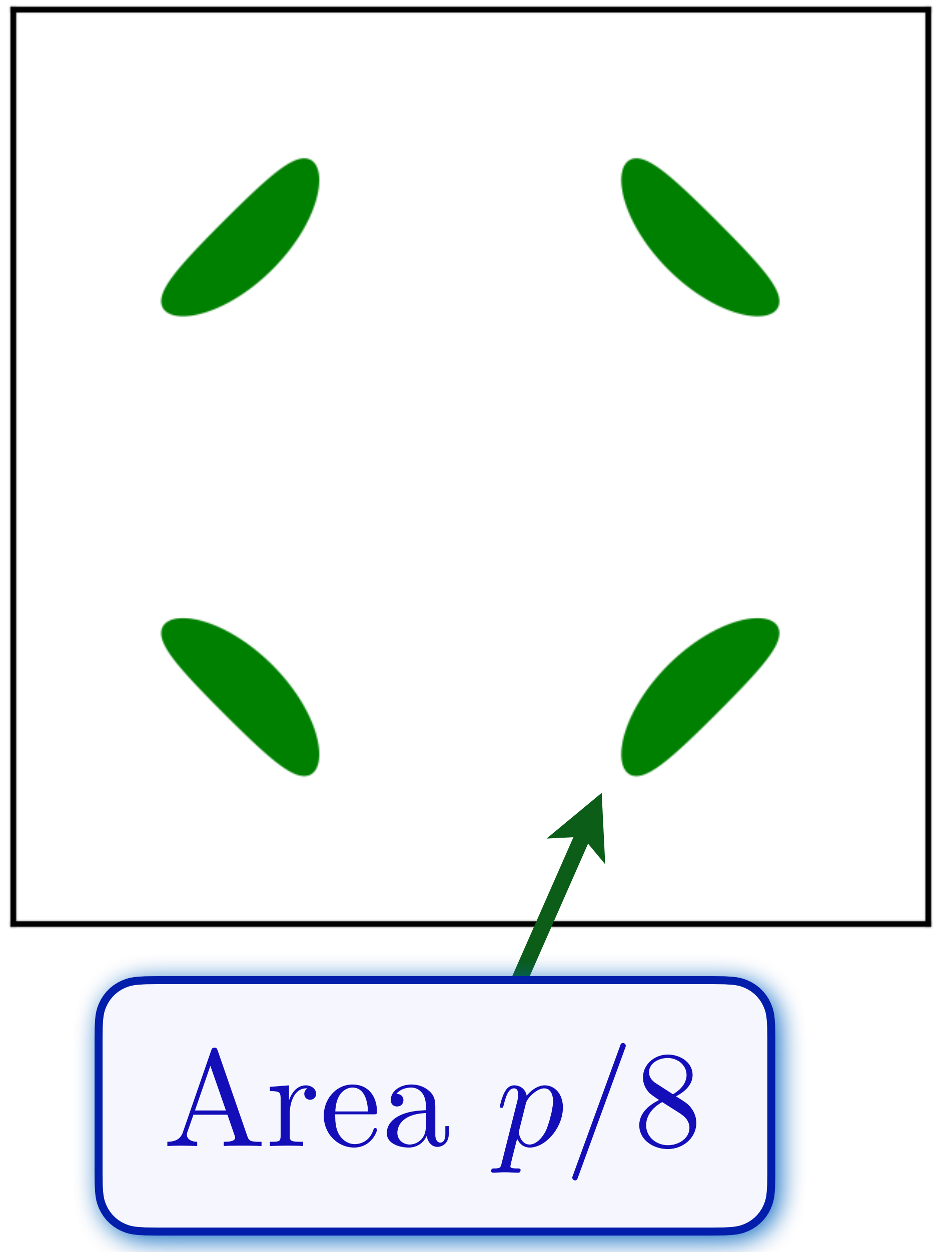


Holon metal

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003);
R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. S., PRB **105**, 075146 (2022)



FL*

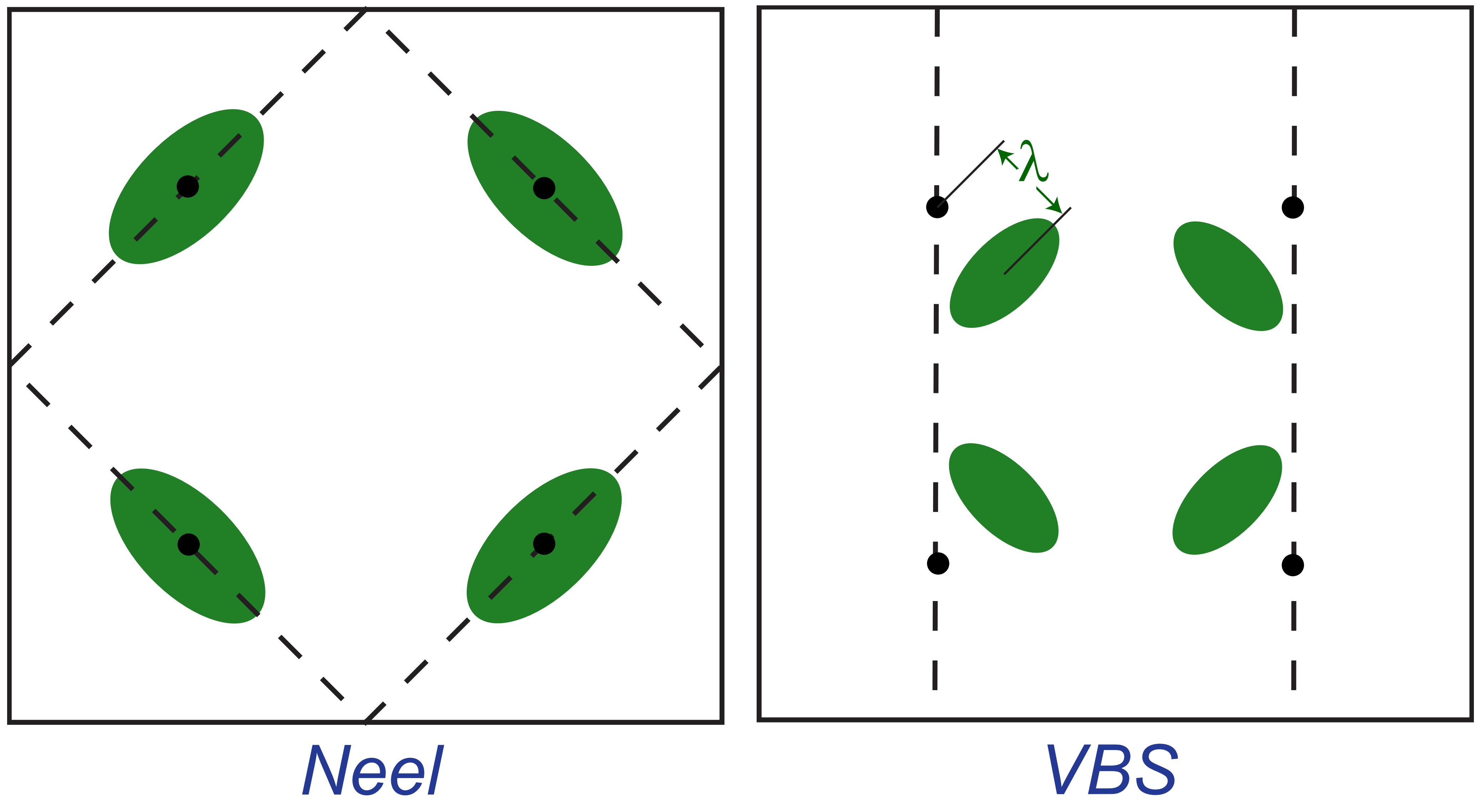


Fermi surface areas are *quantized* and robust to all corrections!
Factor of 2 between SDW fluctuation and FL*

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



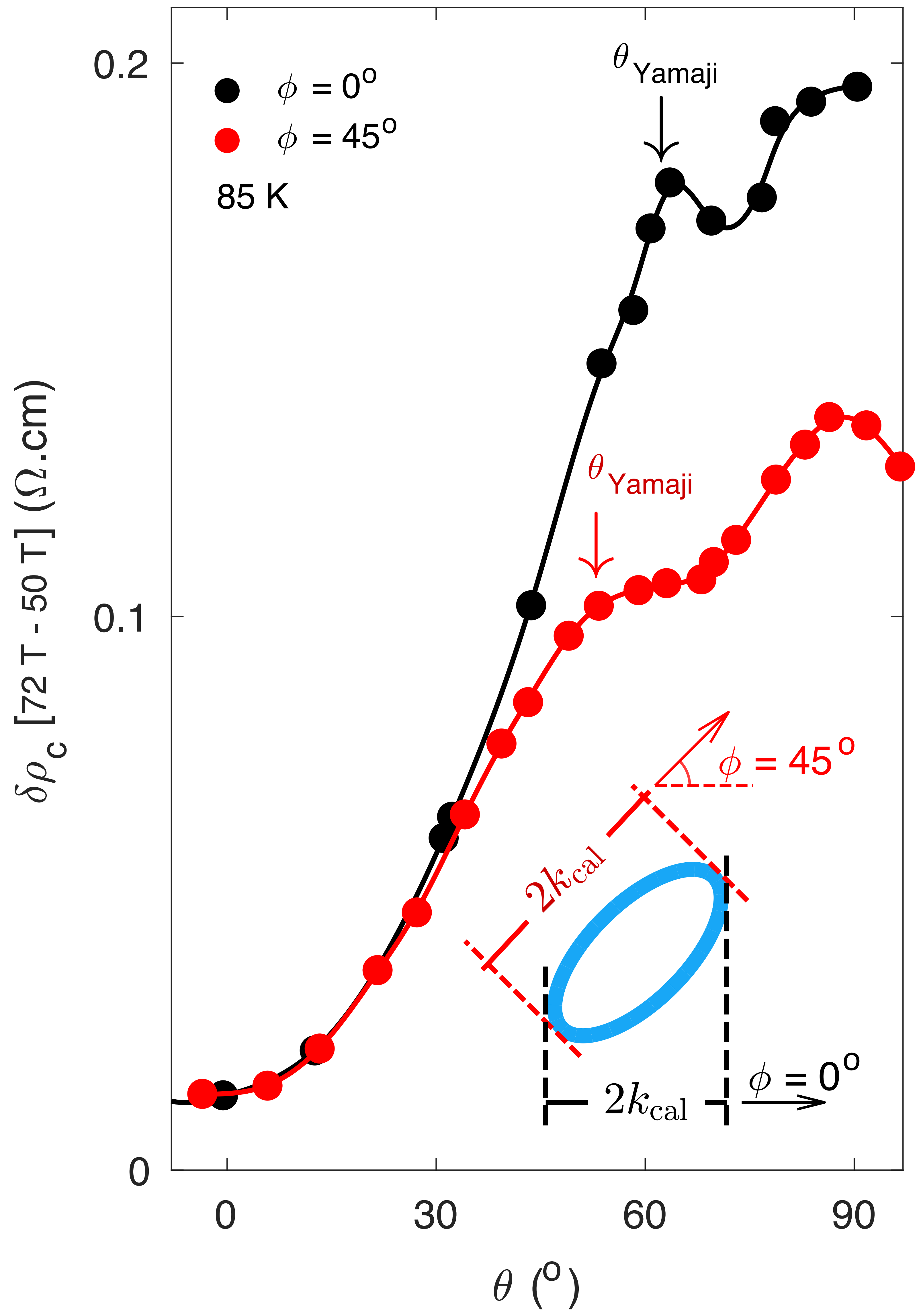
The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/8$.

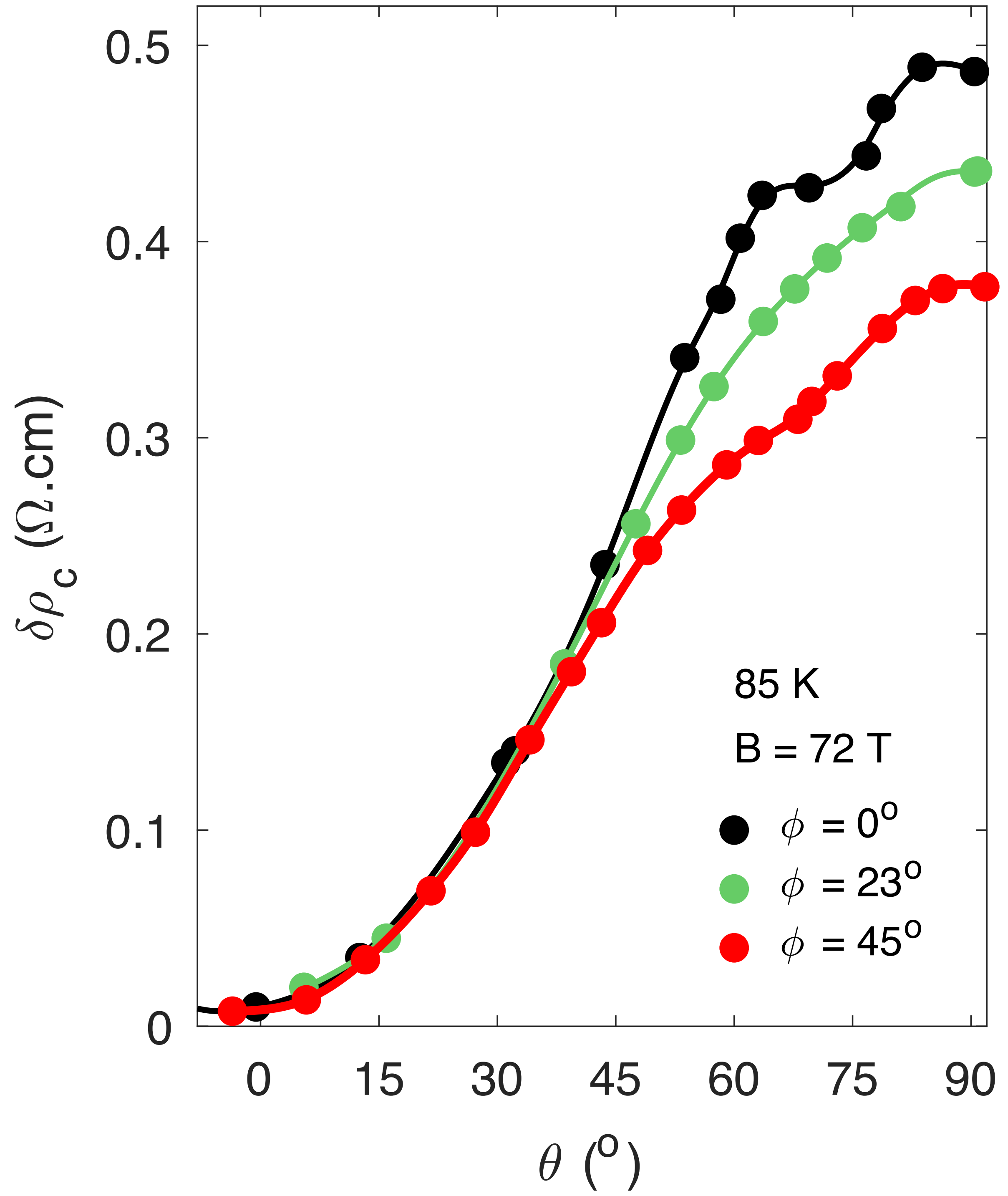
Fermi surface areas are *quantized* and robust to all corrections! Factor of 2 between SDW fluctuation and FL*

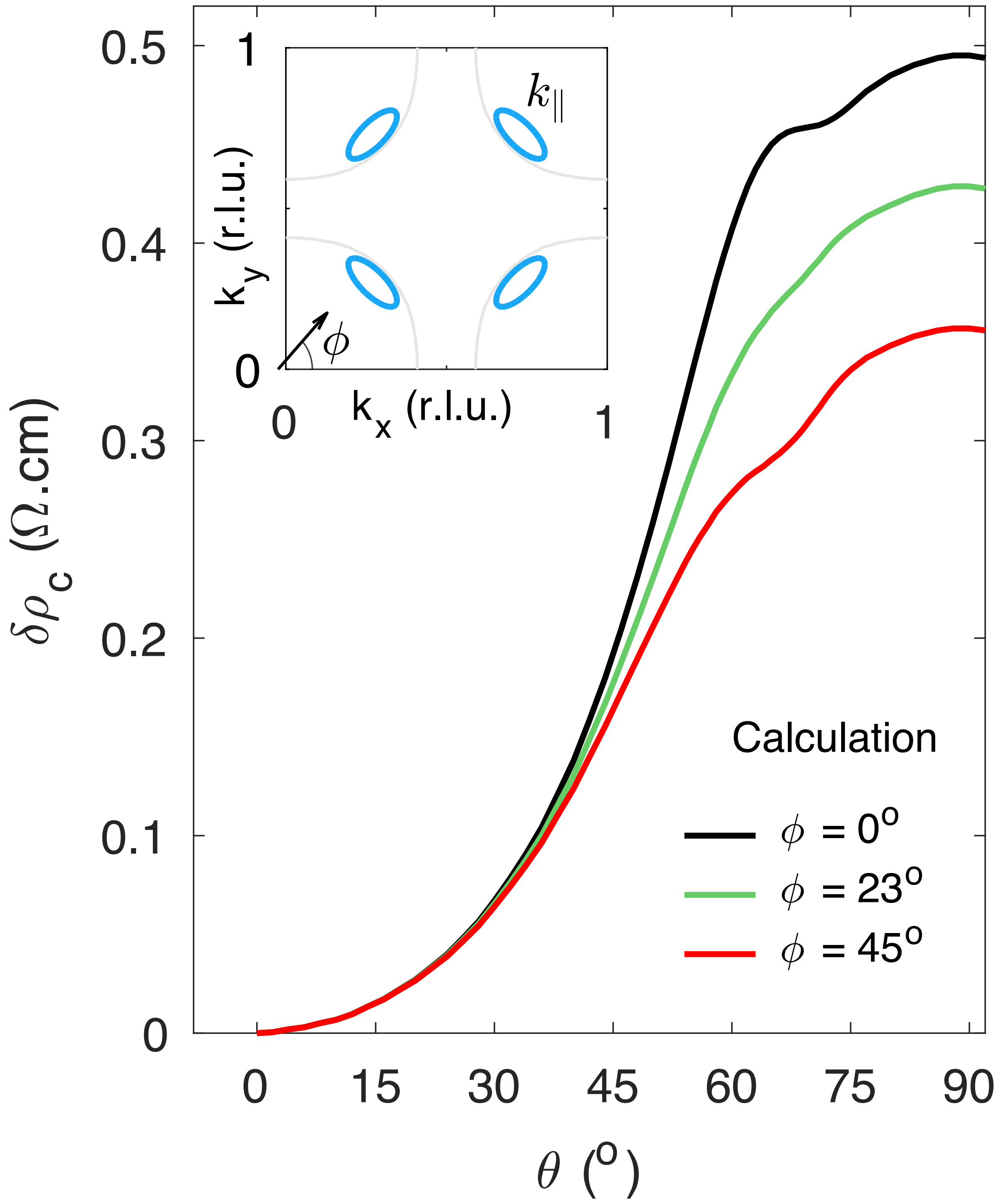
Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan,[1,*] Katherine A. Schreiber,[1] Oscar E. Ayala-Valenzuela,[1]
Eric D. Bauer,[2] Arkady Shekhter,[1] and Neil Harrison[1]

arXiv:2411.10631







Doping
$p = 0.1$

"The small size of the pockets determined from the Yamaji effect is ... approximately 1.3% of the Brillouin zone area"

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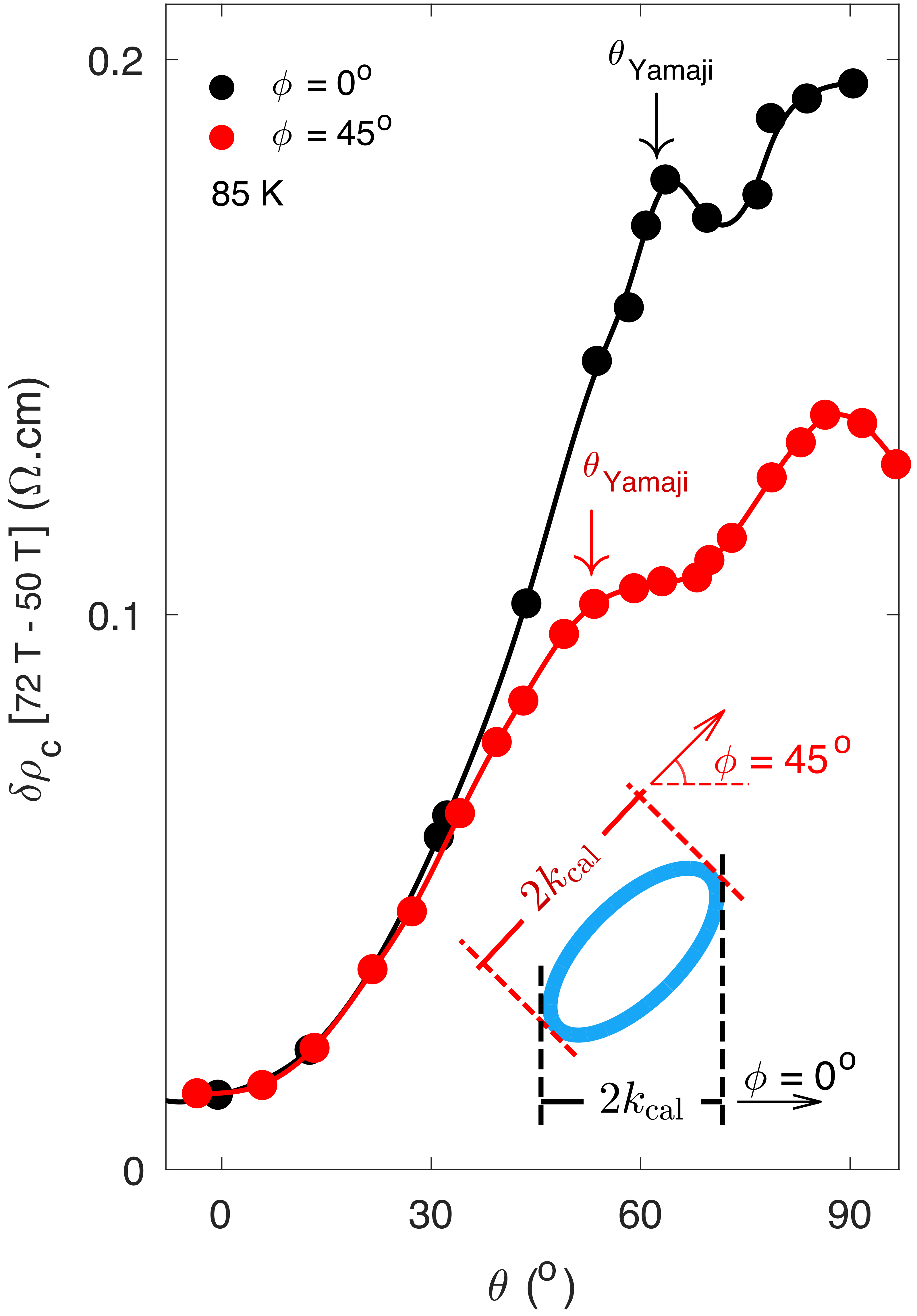

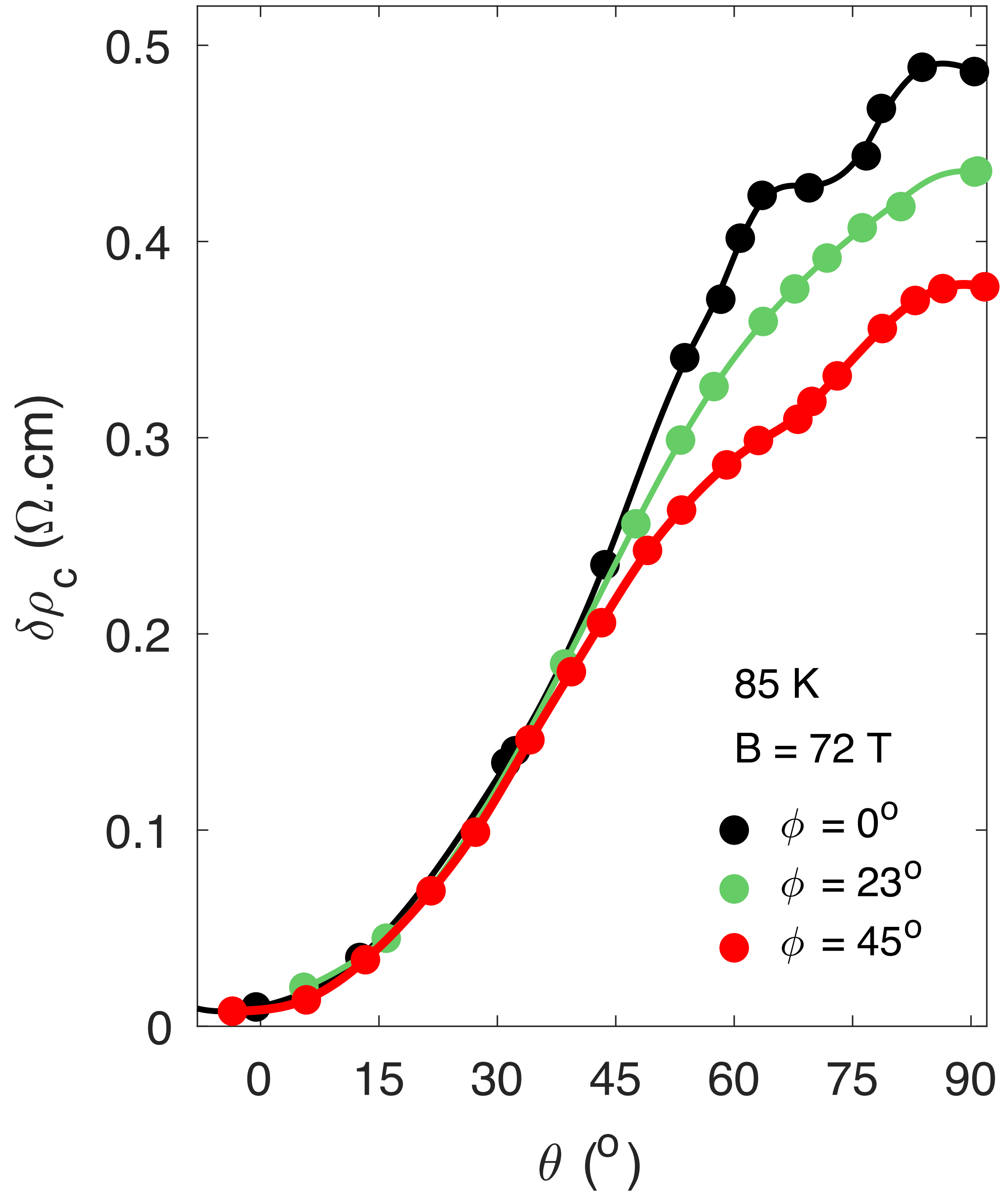

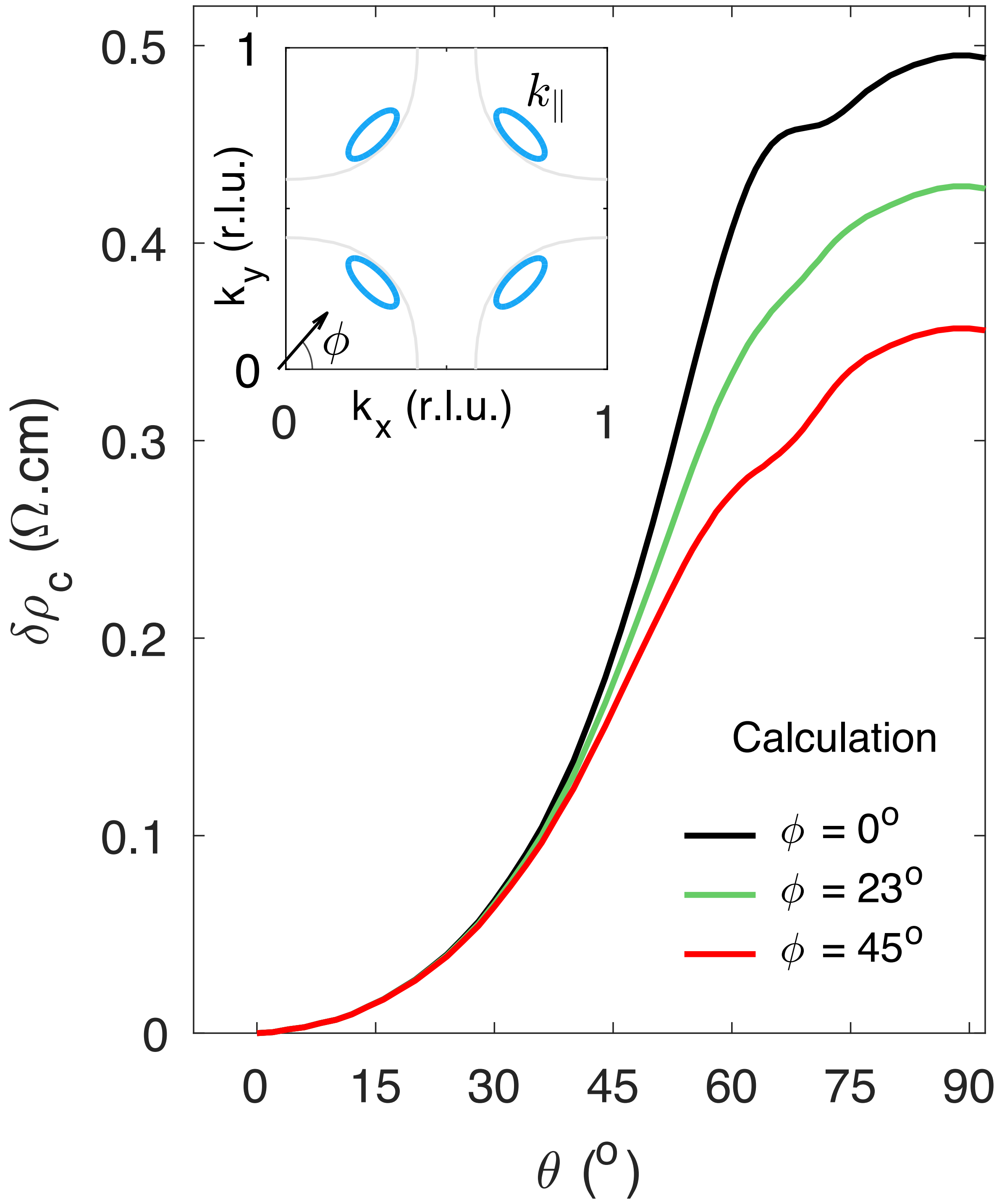

Doping
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FL* pocket fraction $= p/8 = 1.25\%$!
Fluctuating AF metal fraction $= p/4 = 2.5\%$.

(p/8 also in Yang-Rice-Zhang ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar?)

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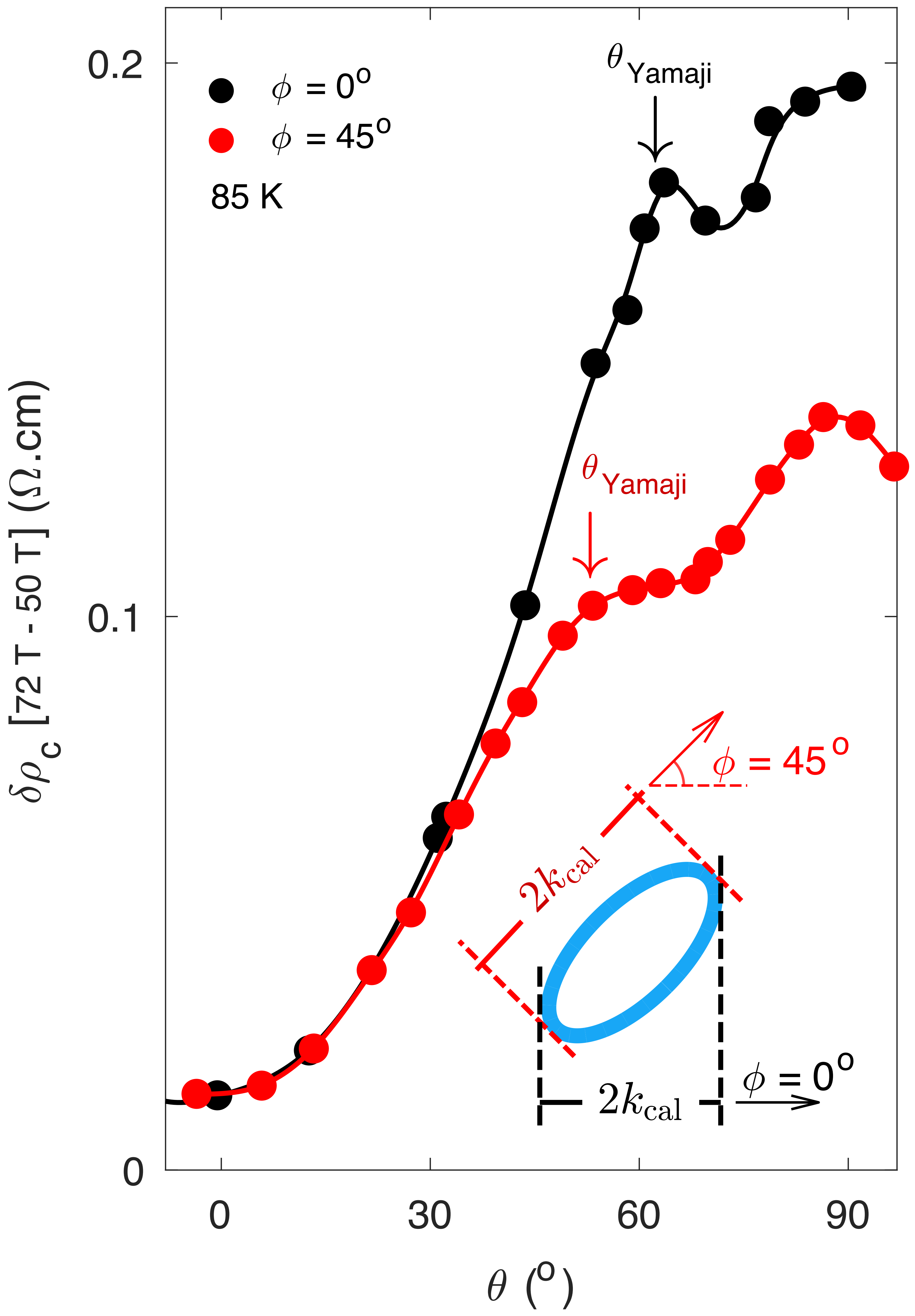

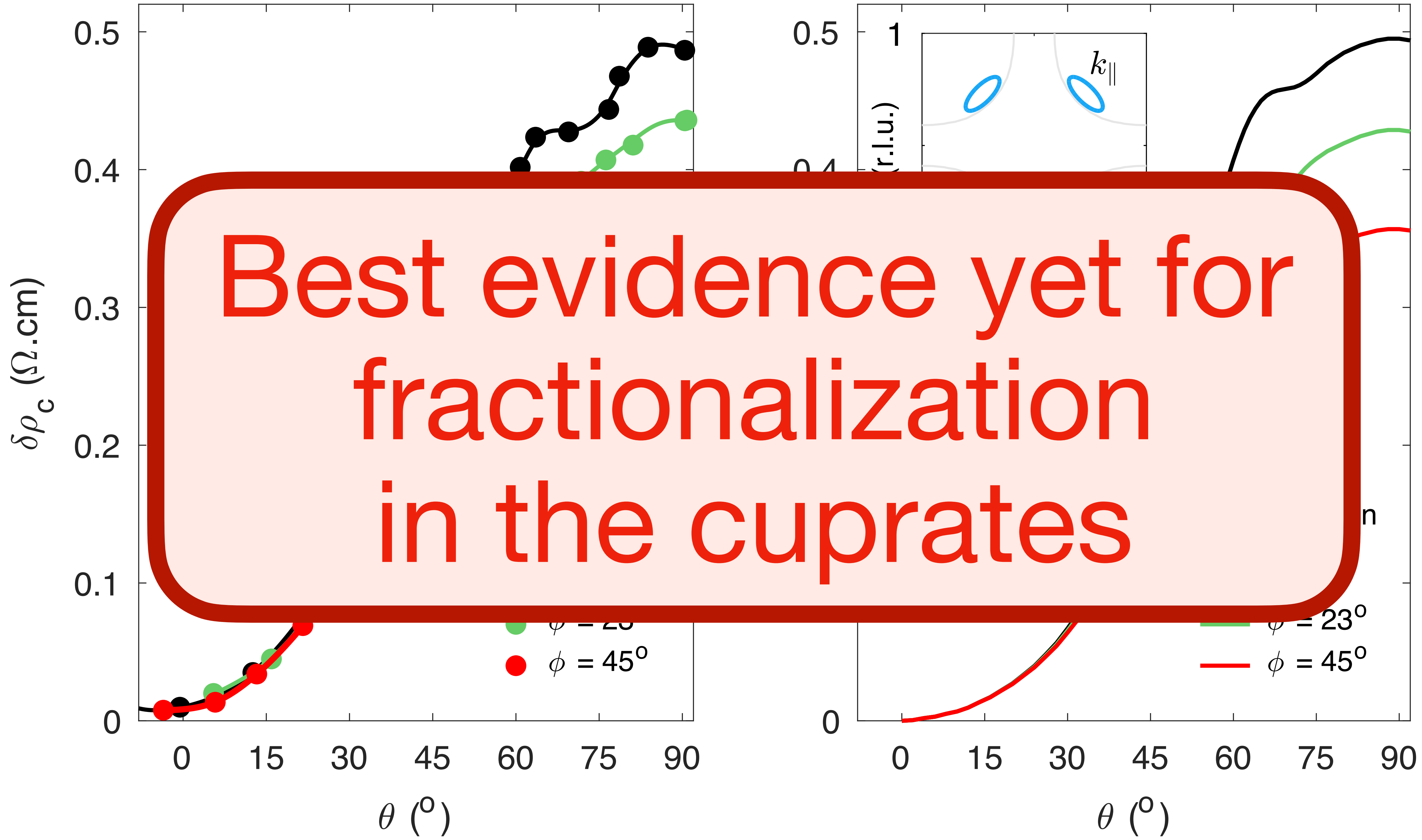

Best evidence yet for fractionalization in the cuprates

Doping
$p = 0.1$

"The small size of the pockets determined from the Yamaji effect is . . .
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FL* pocket fraction $= p/8 = 1.25\%$!
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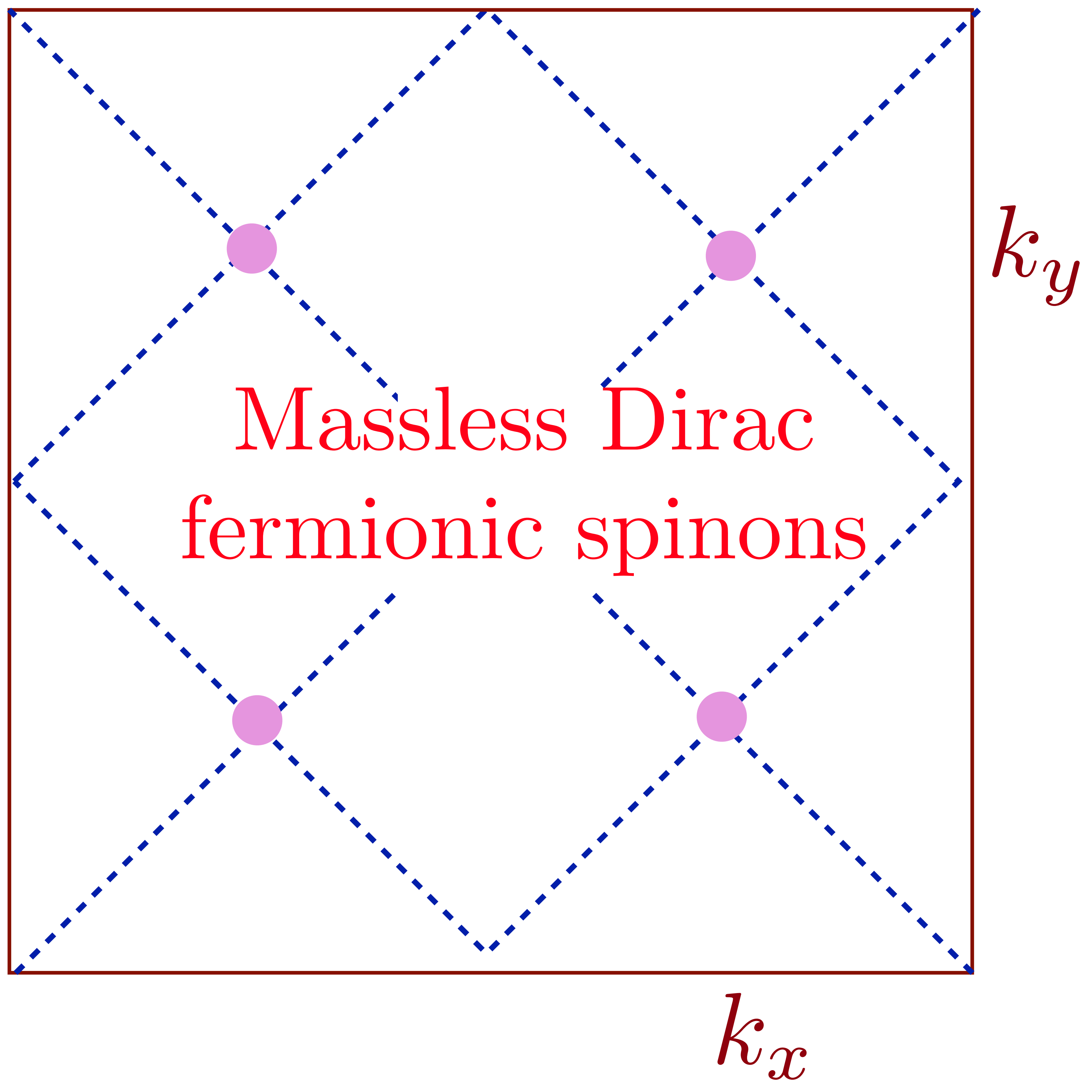
(p/8 also in YRZ ansatz, Peter Johnson photoemission,
and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar?)

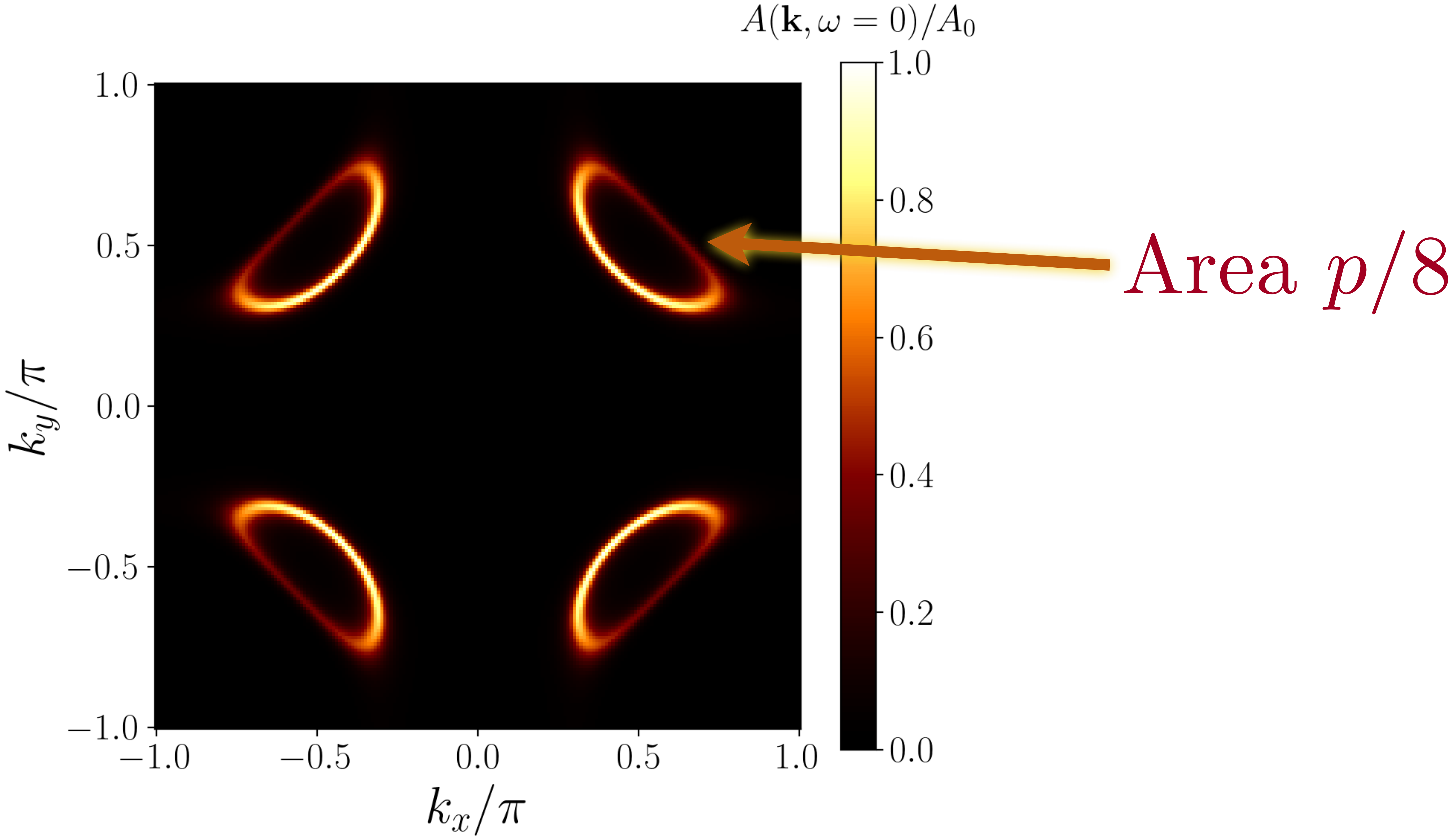
Plan:

- Identification of spin liquid: critical spin liquid without quasiparticles. One description is a SU(2) gauge theory with $N_f = 2$ massless Dirac spinons.



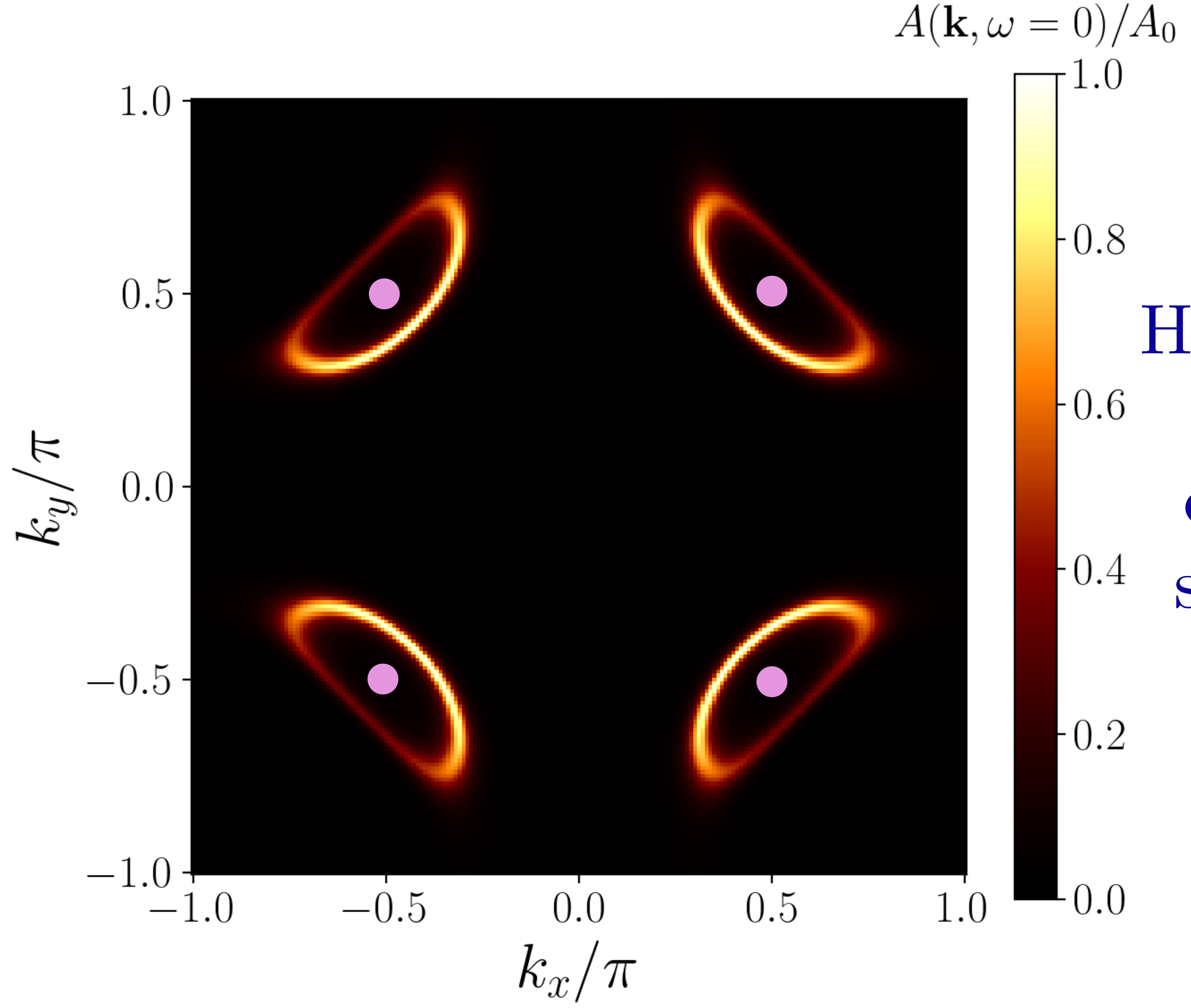
Plan:

- Dope spin liquid with *holes*, not holons.
  The Ancilla Layer Model (ALM) enables a theory of FL* hole pockets for a general spin liquid.



Plan:

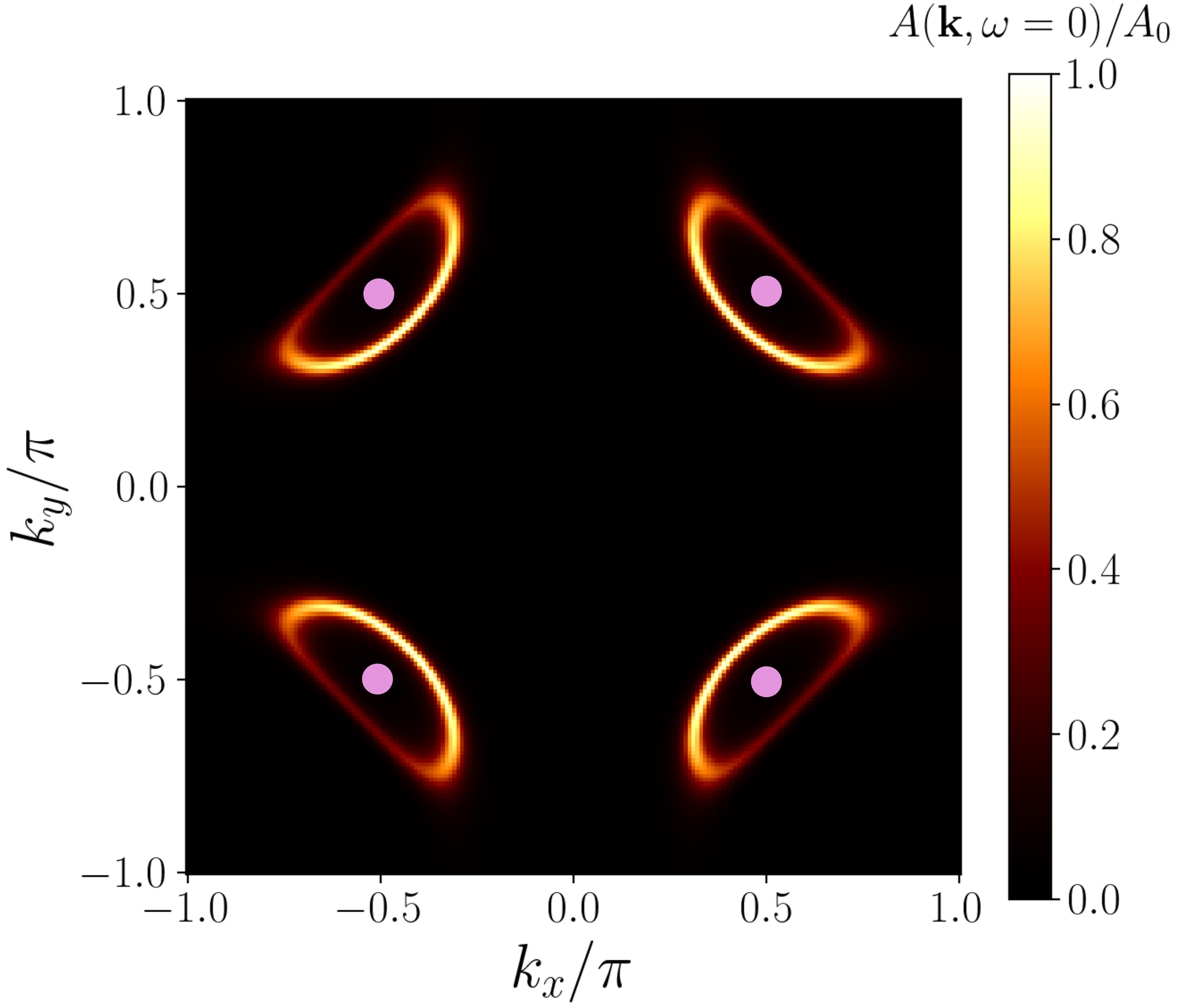
- Theory of pseudogap (and its low T instabilities): Hole pockets coupled to critical spin liquid.

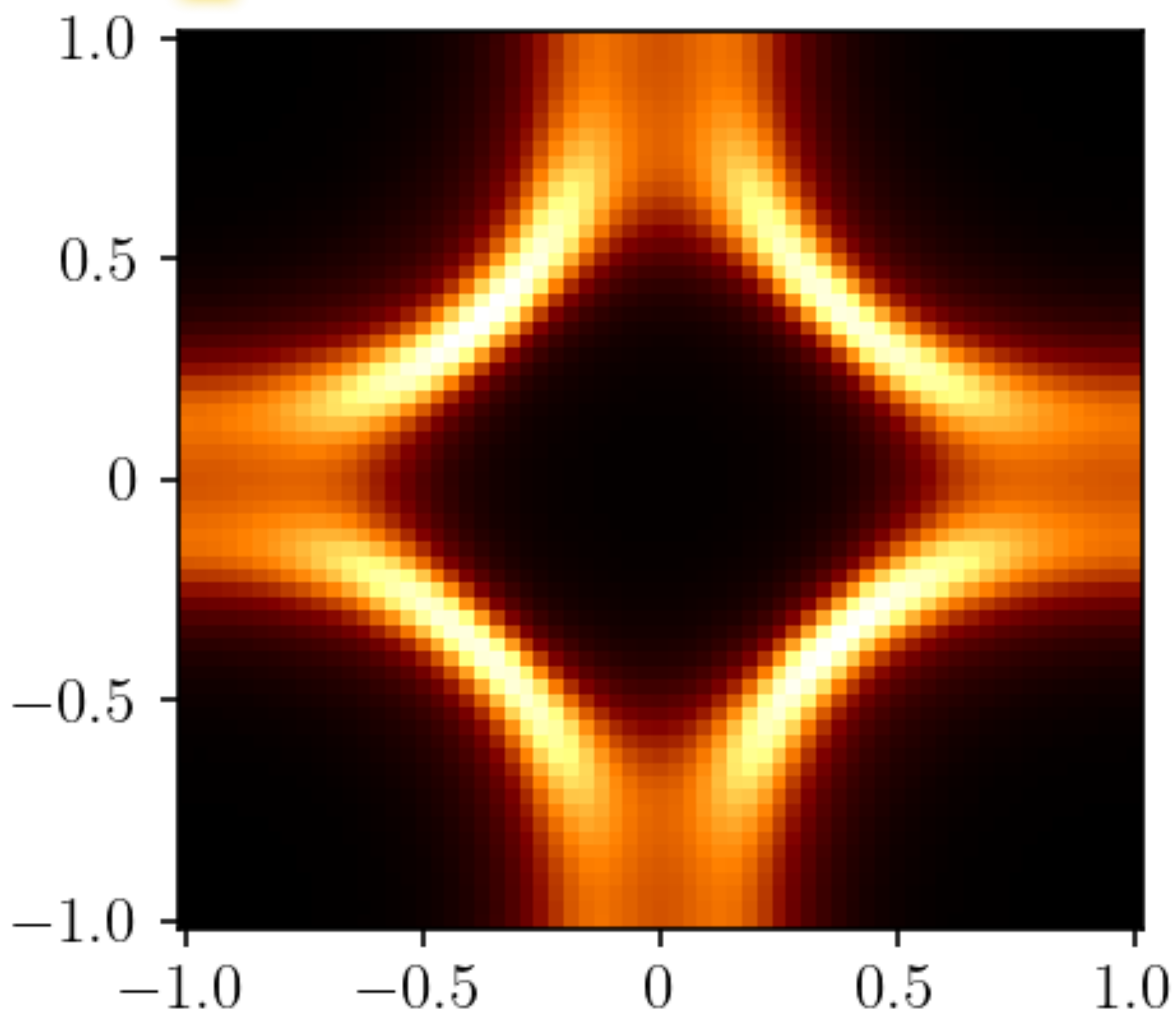


Hole pockets
and
decoupled
spin liquid

Plan:

- Theory of pseudogap (and its low T instabilities): Hole pockets coupled to critical spin liquid.





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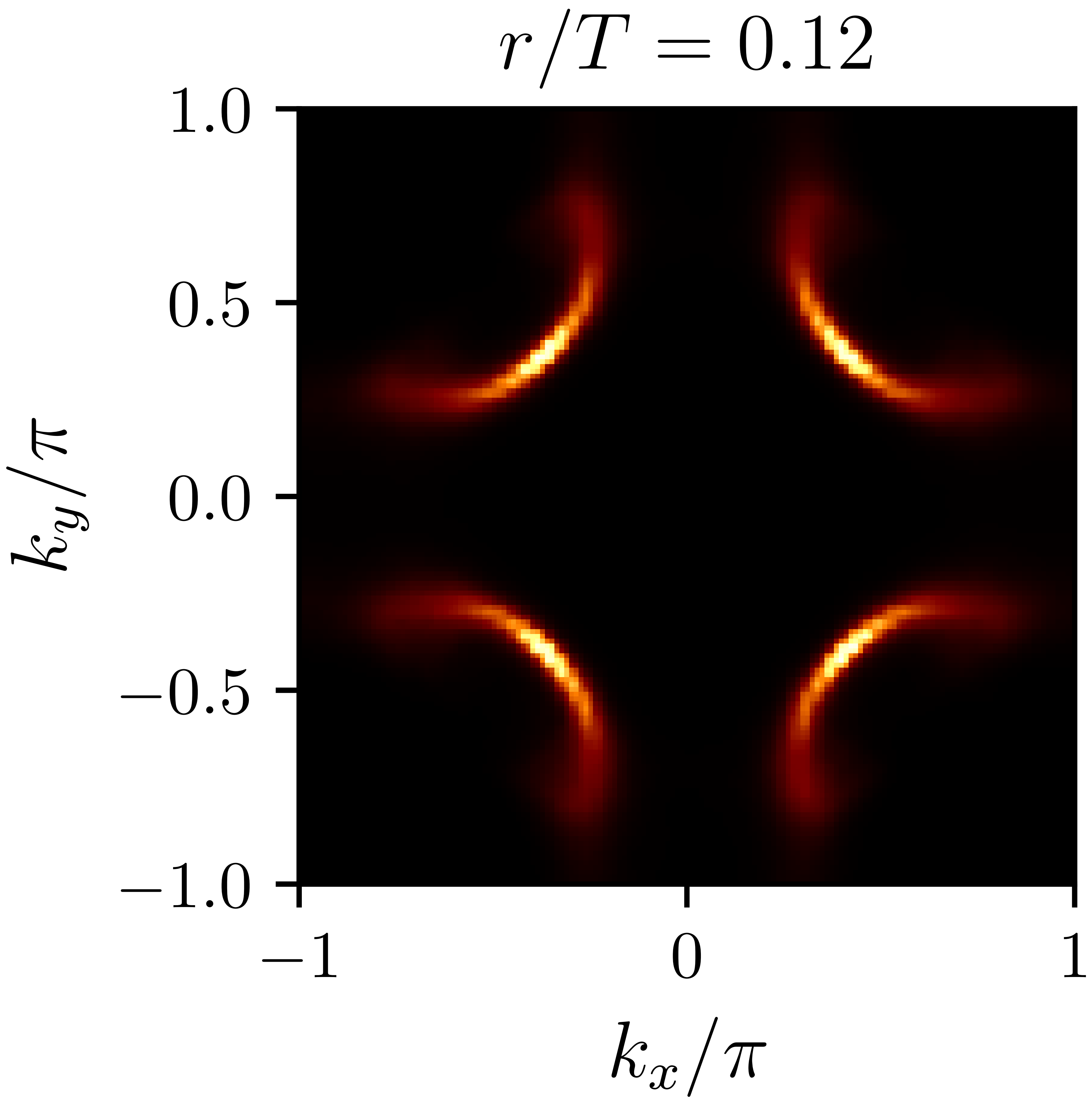
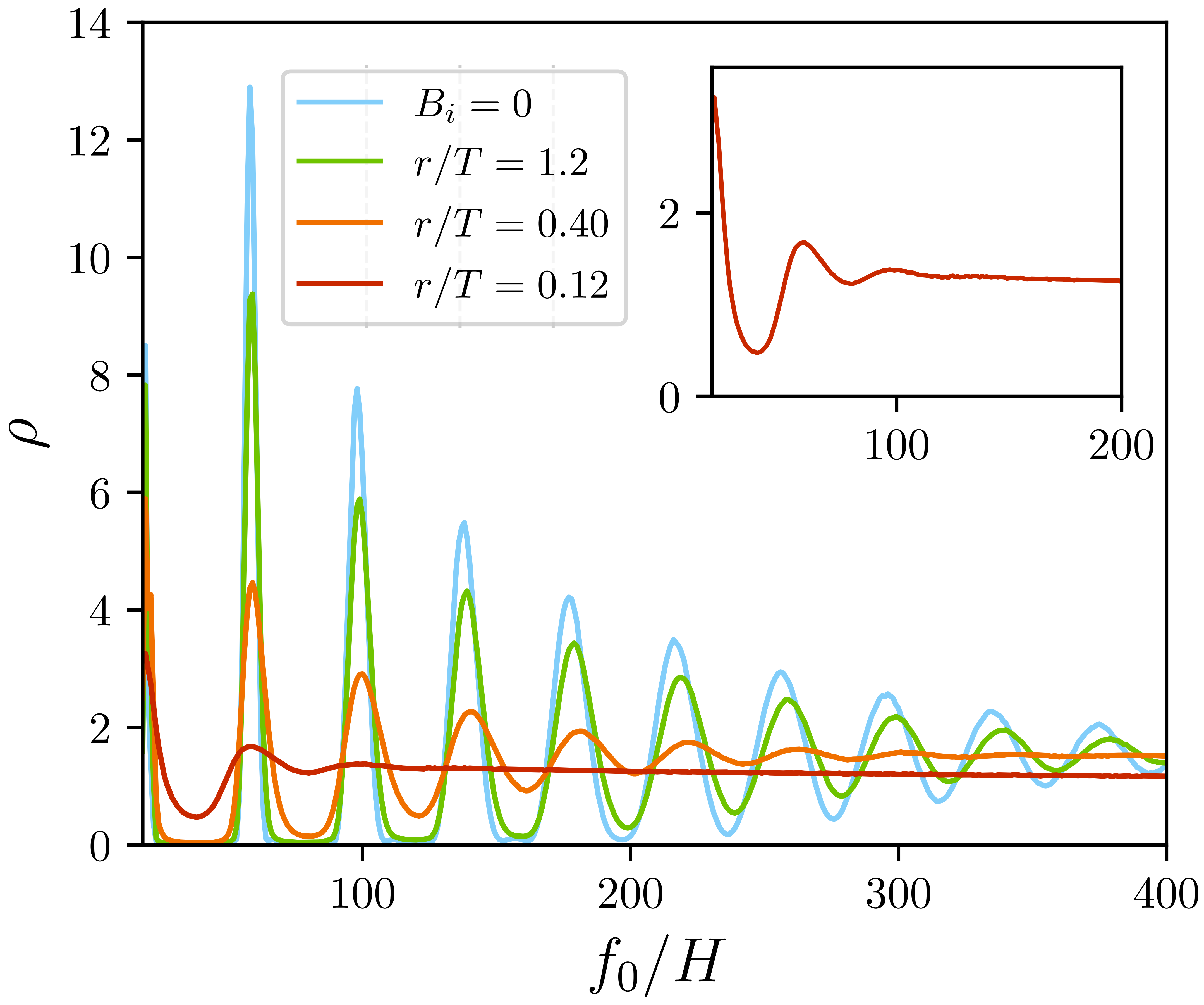




Fermi arcs co-exist with quantum oscillations of hole pockets of area $p/8$.

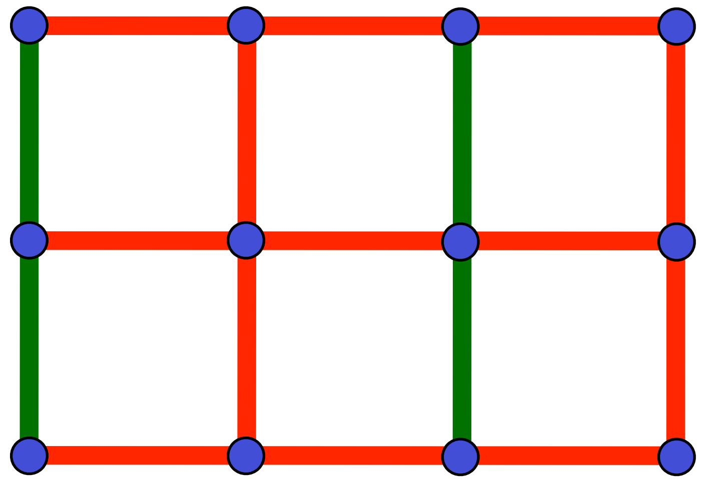
Critical quantum spin liquid on the square lattice

S=1/2 square lattice

Represent spins in terms of
$S = 1/2$ fermionic spinons $\boldsymbol{S} \sim f_\alpha^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

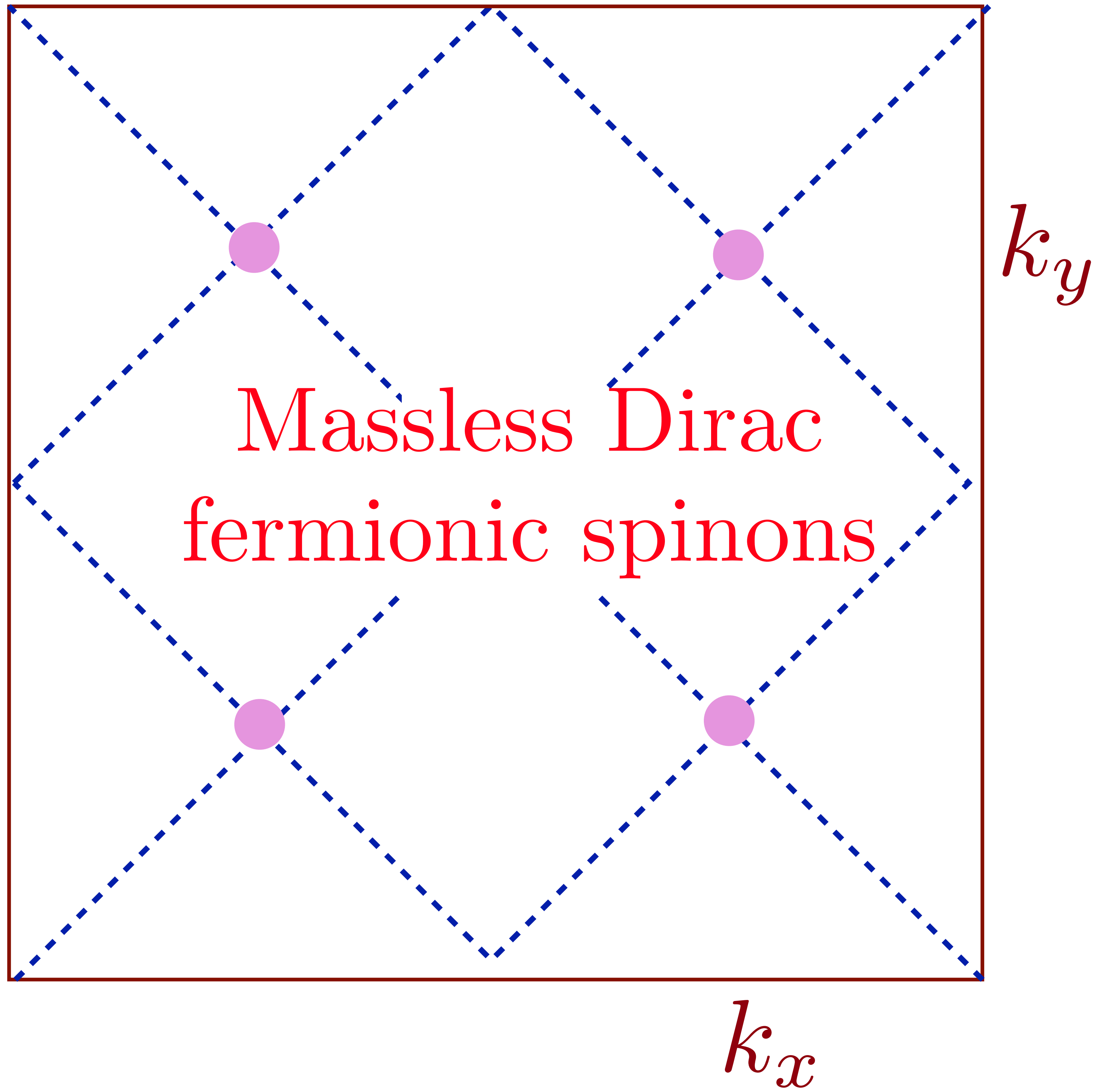
$$H_{\text{spin liquid}} = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right) ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$



$e_{ij} = 1$

$e_{ij} = -1$

$$\mathcal{L} = i\overline{\psi}\gamma_\mu D_\mu \psi.$$



$N_f = 2$ SU(2) QCD

S=1/2 square lattice

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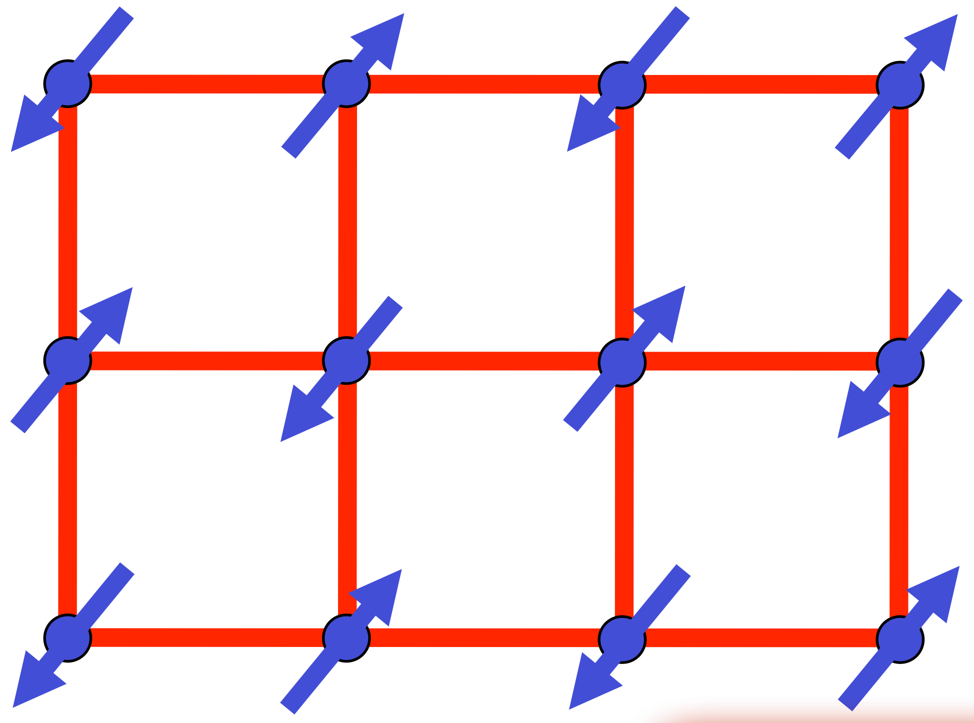
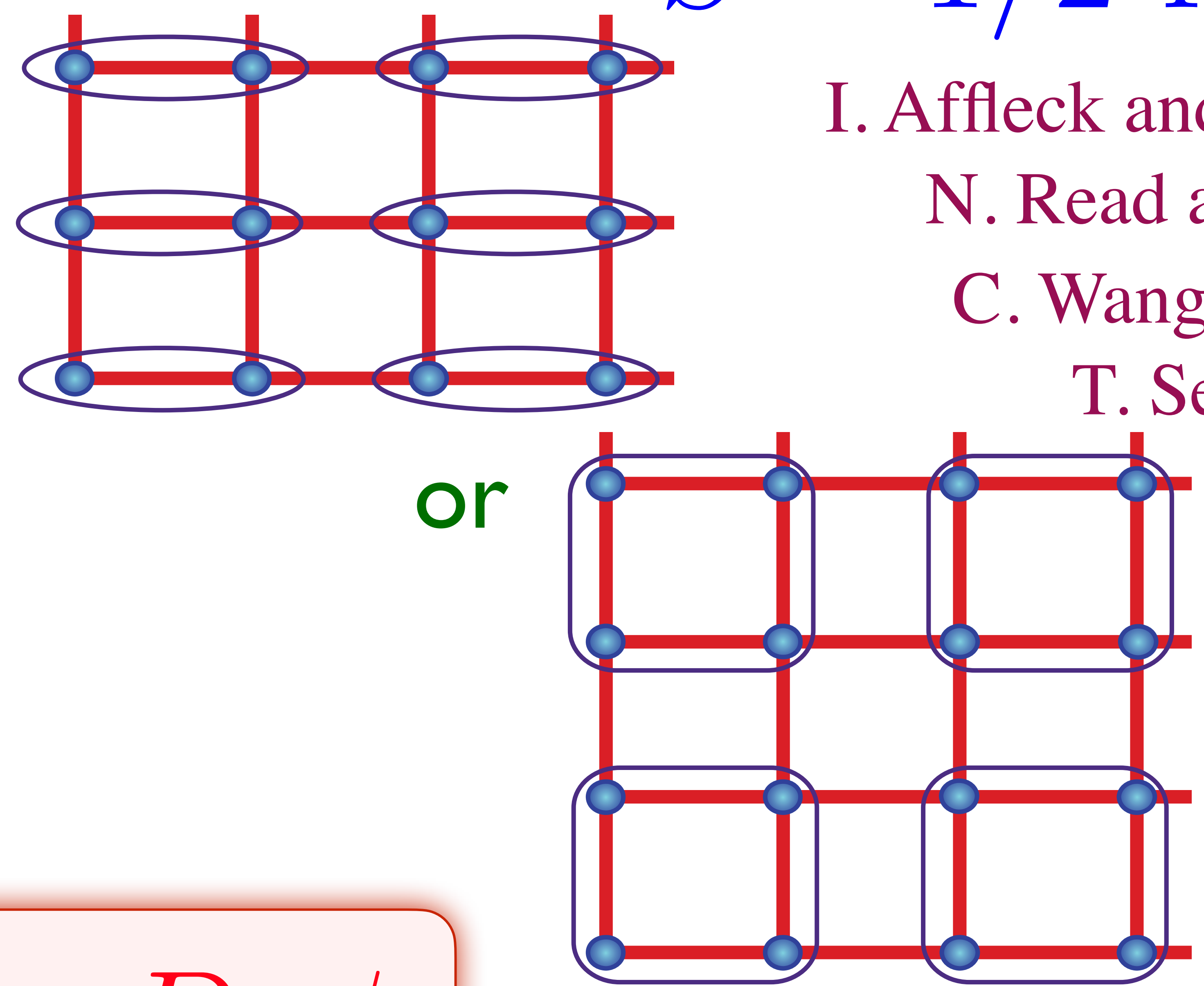
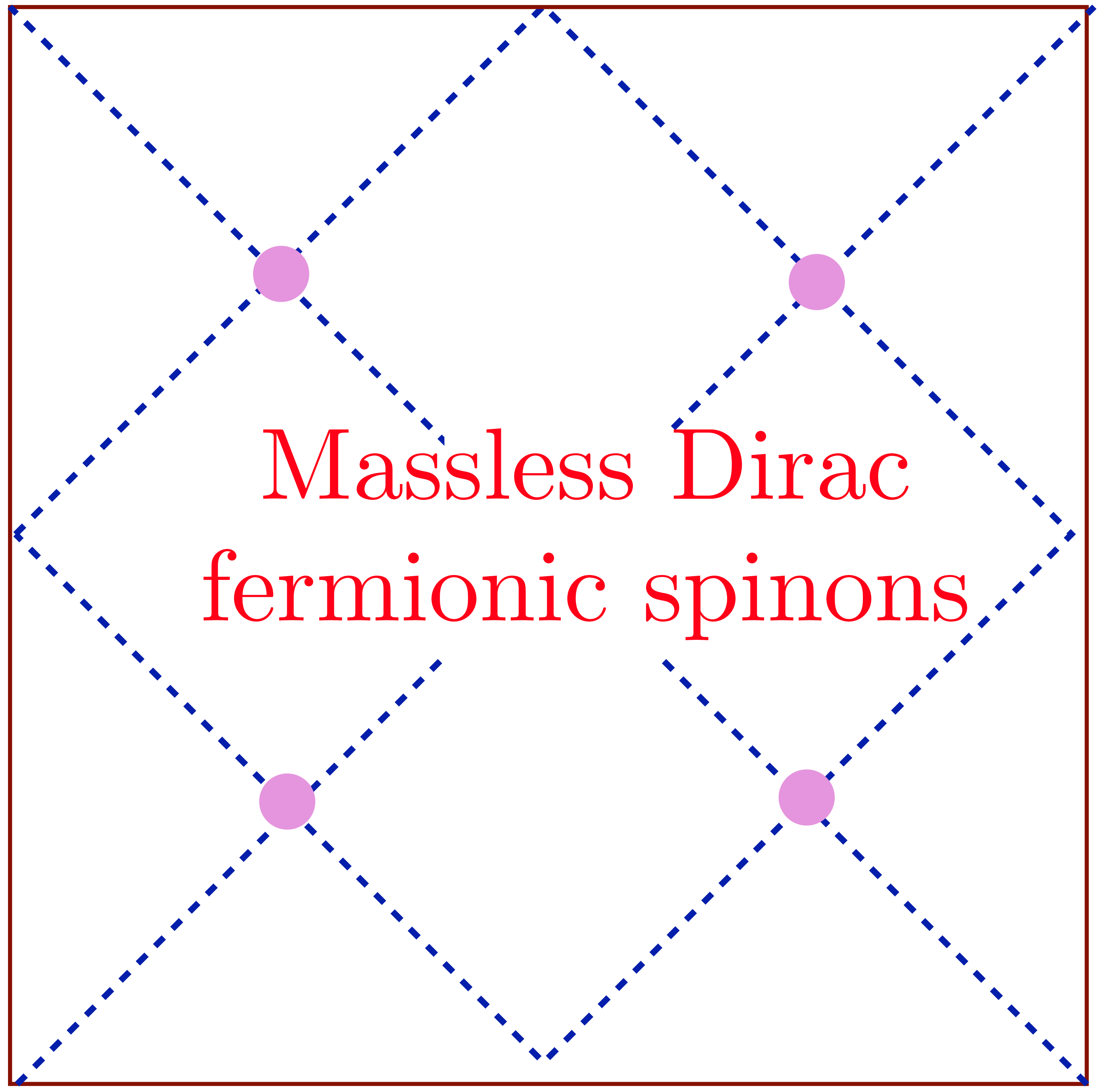
I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)
N. Read and S. Sachdev, PRL **62**, 1694 (1989)
C. Wang, A. Nahum, M. A. Metlitski, C. Xu, T. Senthil, *Phys. Rev. X* **7**, 031051 (2017)

or





$$\mathcal{L} = i\overline{\psi}\gamma_\mu D_\mu \psi.$$

Néel order

Valence bond solid (VBS)

J_2/J_1

Critical spin liquid without quasiparticles?

$N_f = 2$ SU(2) QCD

Confining instability to Néel and VBS, as in $\mathbb{CP}^1$ theory of Read+SS

S=1/2 square lattice

Bosonic spinons:
$\mathbb{CP}^1$ U(1) gauge theory

N. Read and S. Sachdev, PRL **62**, 1694 (1989)

Nearly-critical S=1/2 square lattice antiferromagnet without quasiparticles

SU(2) gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

Obtained from a saddle-point of fermionic spinons moving in π-flux.

SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

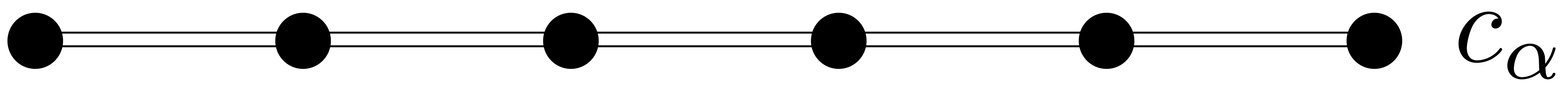
Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

A. Tanaka and X. Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); T. Senthil and M.P.A. Fisher *Phys. Rev. B* **74**, 064405 (2006); C. Wang, A. Nahum, M. A. Metlitski, C. Xu, T. Senthil, *Phys. Rev. X* **7**, 031051 (2017); Zheng Zhou, Liangdong Hu, Wei Zhu, Yin-Chen He, PRX **14**, 021044 (2024); S. M. Chester N. Su, PRL **132**, 111601 (2024). B.-B. Chen, X. Zhang, Y. Wang, K. Sun, Z. Y. Meng, PRL **132**, 246503 (2024); J. Takahashi, H. Shao, B. Zhao, W. Guo, A. W. Sandvik, arXiv:2405.06607.

FL* pseudogap metal
in single-band models
using the
Ancilla Layer Model (ALM)

Ancilla Layer Model of the Hubbard model

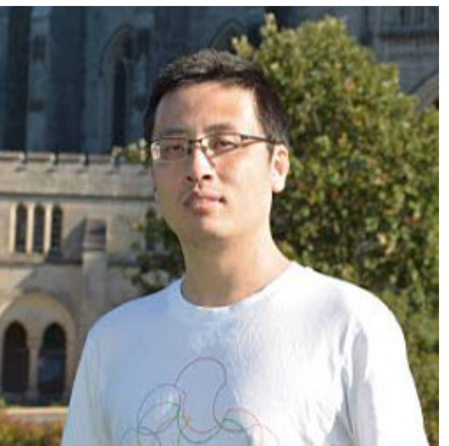
(Foolproof method to satisfy the Oshikawa anomaly)



Hubbard model of hole density 1+p

$$\mathcal{H}_{\text{Hubbard}} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{i}\boldsymbol{j}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} + U \sum_{\boldsymbol{i}} (c^\dagger_{\boldsymbol{i}\uparrow} c_{\boldsymbol{i}\uparrow})(c^\dagger_{\boldsymbol{i}\downarrow} c_{\boldsymbol{i}\downarrow})$$

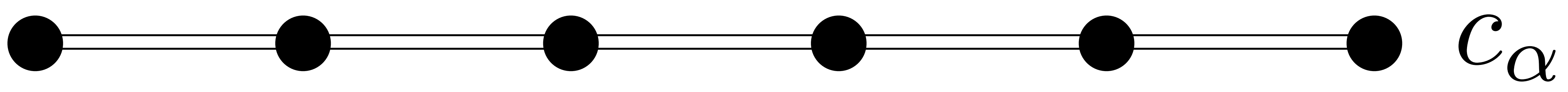
Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

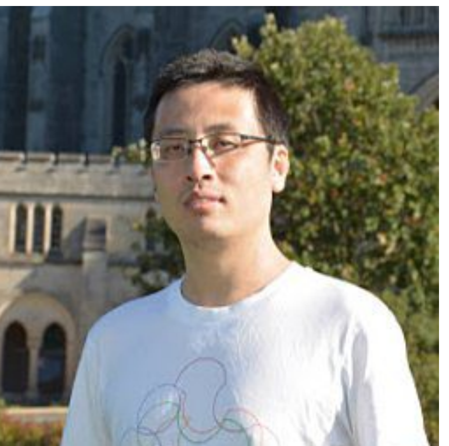


Hubbard model of hole density 1+p

$$\mathcal{H}_{\text{Hubbard}} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{ij}} c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \left[\frac{3U}{8} \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}^2 + U \boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \cdot c_{\boldsymbol{i}\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{\boldsymbol{i}\beta} \right] .$$

$$\boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \quad \Rightarrow \quad \text{Paramagnon}$$

Ya-Hui Zhang



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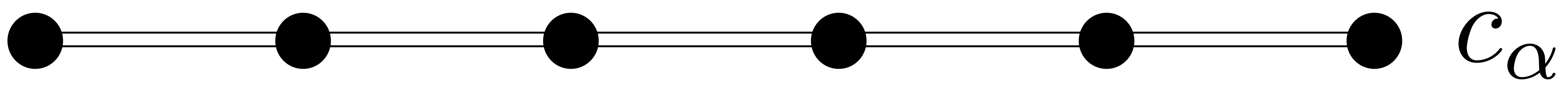

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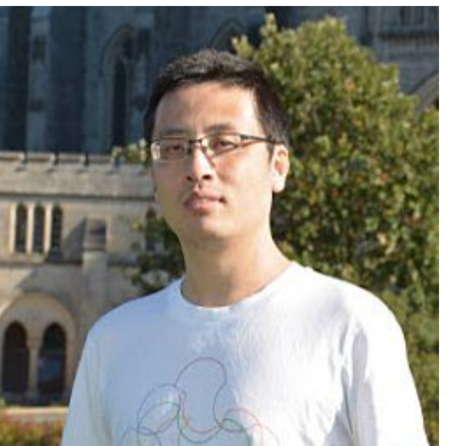

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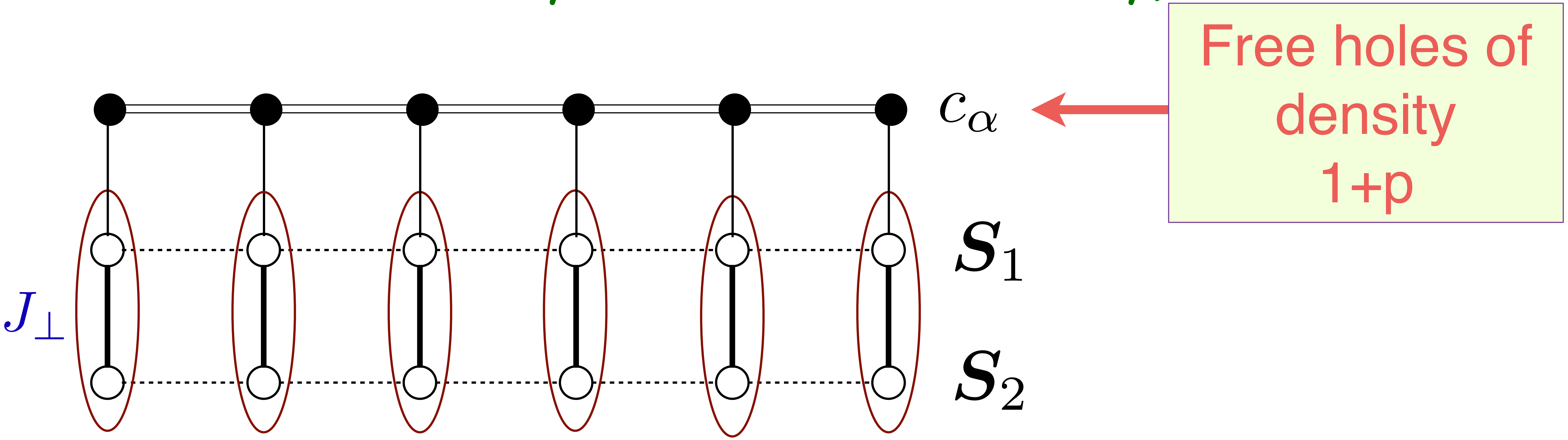


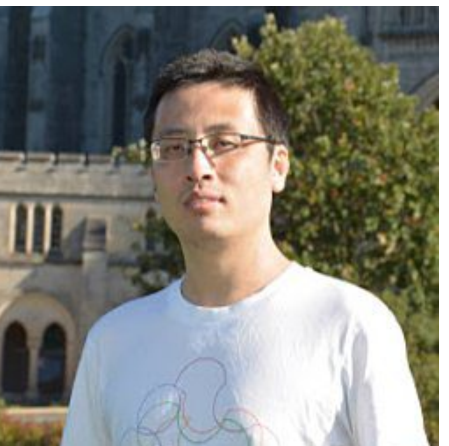
$$\mathcal{H}_{\rm Hubbard} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{i}\boldsymbol{j}} c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \left[\frac{m_{\boldsymbol{\mathcal{P}}}}{2} \left(\partial_\tau \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}\right)^2 + \frac{3U}{8} \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}^2 + U \boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \cdot c_{\boldsymbol{i}\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{\boldsymbol{i}\beta} \right] .$$

$3\ \boldsymbol{\mathcal{P}}$ oscillators' states: $|0,0,0\rangle,\ |1,0,0\rangle,\ |0,1,0\rangle,\ |0,0,1\rangle$

$\boldsymbol{S}_{1,2}$ ancilla qubits states : $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2},\ |\uparrow\uparrow\rangle,\ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2},\ |\downarrow\downarrow\rangle$

$$\boldsymbol{\mathcal{P}} \sim \boldsymbol{S}_1 - \boldsymbol{S}_2$$

Ya-Hui Zhang



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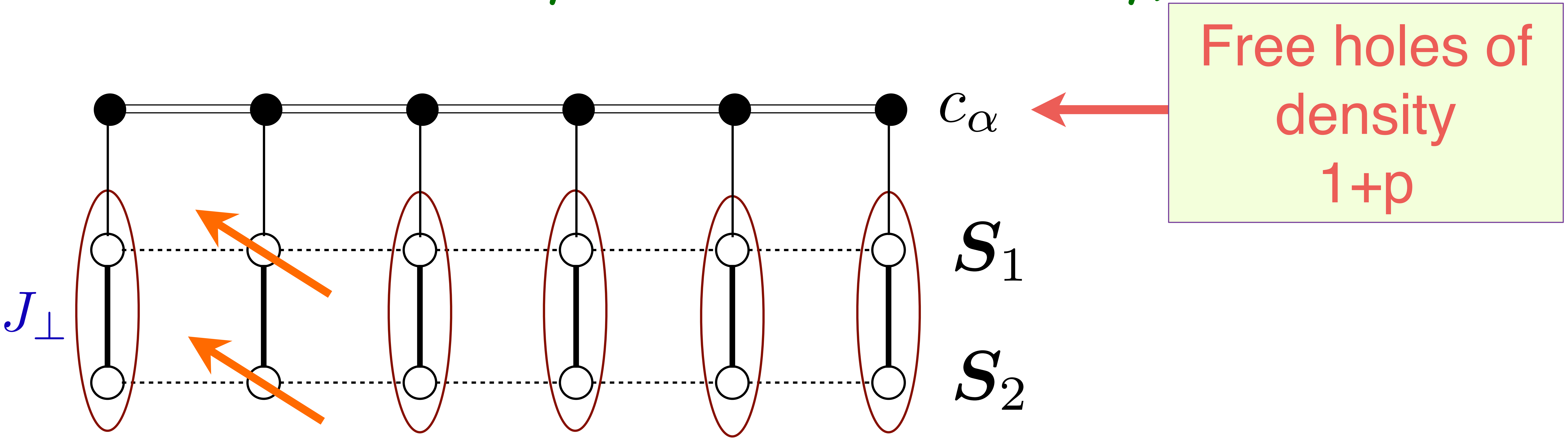

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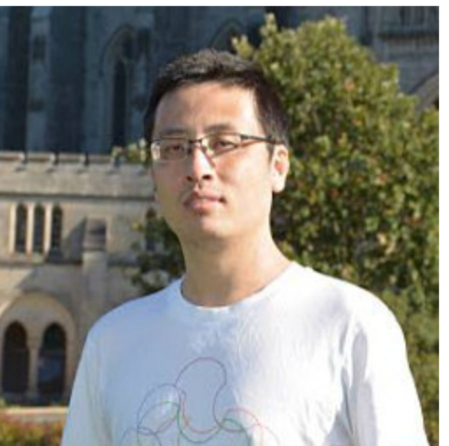
Ya-Hui Zhang



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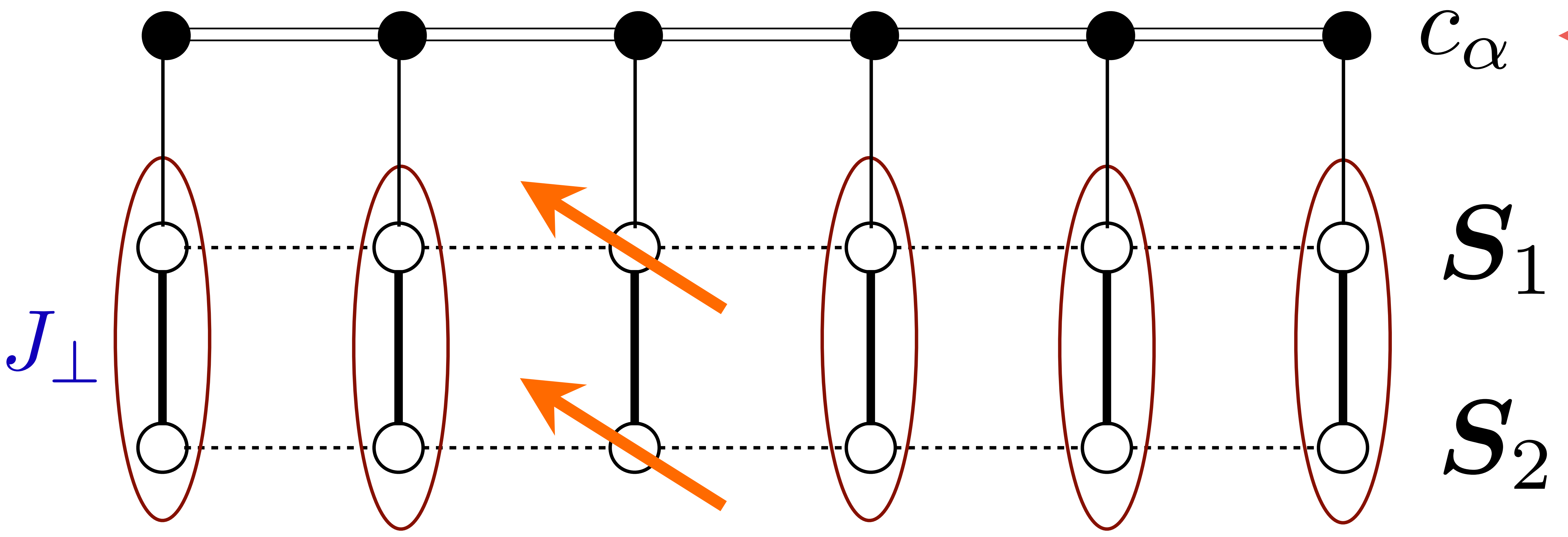
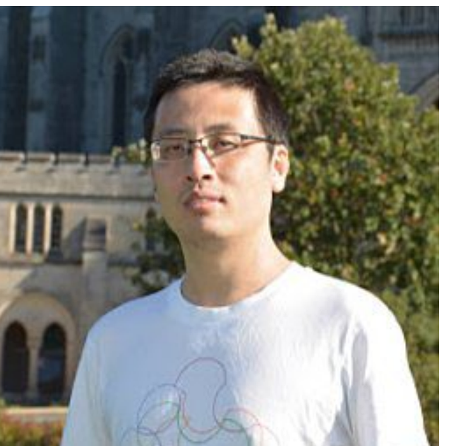
Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)



$$\mathcal{H}_{\rm Hubbard} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{i}\boldsymbol{j}} c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \left[\frac{m_{\mathcal{P}}}{2} \left(\partial_\tau \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}\right)^2 + \frac{3U}{8} \boldsymbol{\mathcal{P}}_{\boldsymbol{i}}^2 + U \boldsymbol{\mathcal{P}}_{\boldsymbol{i}} \cdot c_{\boldsymbol{i}\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\beta}}{2} c_{\boldsymbol{i}\beta} \right] .$$

Ya-Hui Zhang



$$3\ \boldsymbol{\mathcal{P}} \text{ oscillators' states: } |0,0,0\rangle,\ |1,0,0\rangle,\ |0,1,0\rangle,\ |0,0,1\rangle$$

$$\boldsymbol{S}_{1,2} \text{ ancilla qubits states : } (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2},\ |\uparrow\uparrow\rangle,\ (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2},\ |\downarrow\downarrow\rangle$$

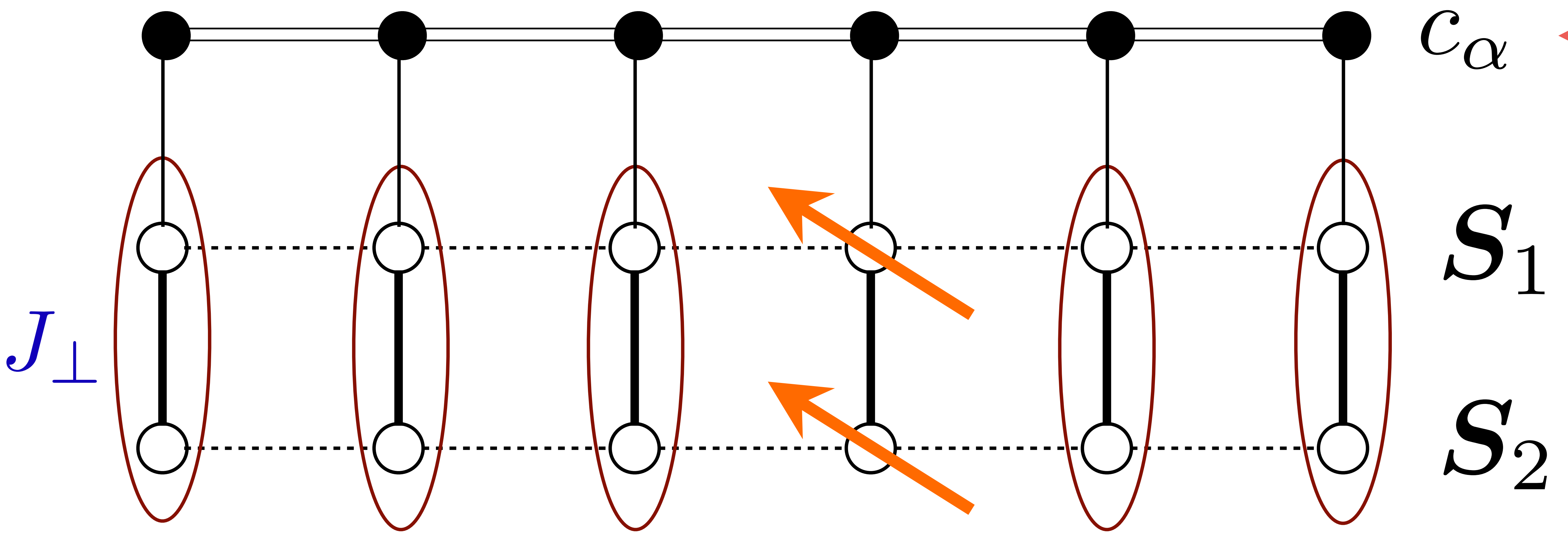
$$\boldsymbol{\mathcal{P}} \sim \boldsymbol{S}_1 - \boldsymbol{S}_2$$

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

Free holes of density 1+p



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$$3\ \boldsymbol{\mathcal{P}} \text{ oscillators' states: } |0,0,0\rangle,\ |1,0,0\rangle,\ |0,1,0\rangle,\ |0,0,1\rangle$$

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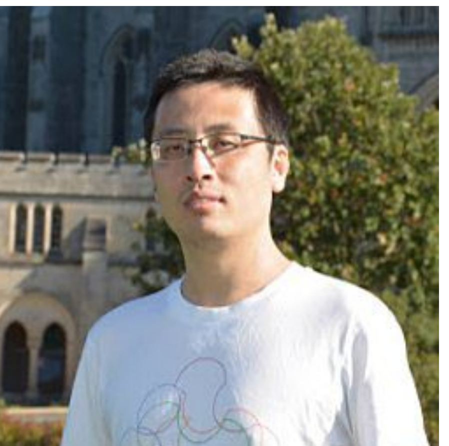
Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

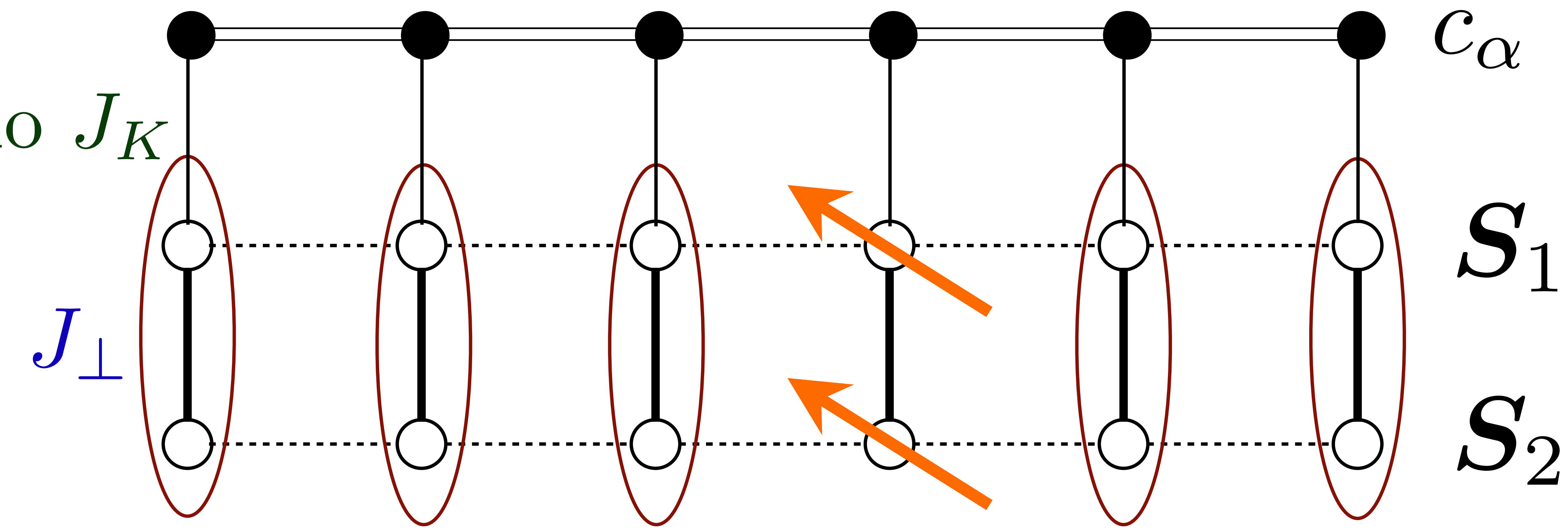
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

Ancilla Layer Model of the Hubbard model

(Foolproof method to satisfy the Oshikawa anomaly)

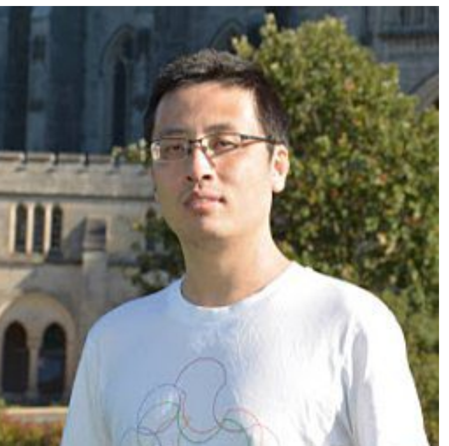
Free holes of density 1+p

Antiferromagnetic Kondo J_K



$$\mathcal{H}_{\rm ALM} = -\sum_{\boldsymbol{i},\boldsymbol{j}} t_{\boldsymbol{i}\boldsymbol{j}} c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + \sum_{\boldsymbol{i}} \frac{J_K}{2} \boldsymbol{S}_{1i} \cdot c_{\boldsymbol{i}\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} c_{\boldsymbol{i}\beta} + J_\perp \sum_{\boldsymbol{i}} \boldsymbol{S}_{1\boldsymbol{i}} \cdot \boldsymbol{S}_{2\boldsymbol{i}} \,.$$

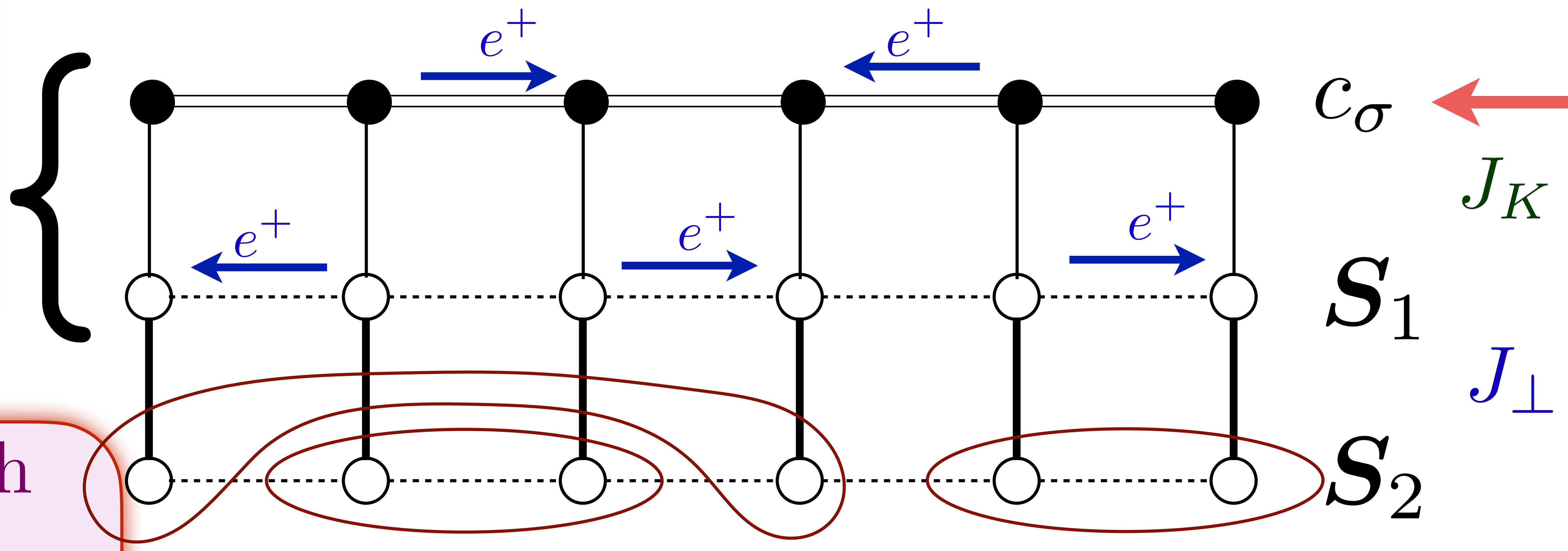
Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

ALM of FL* of Hubbard model

Kondo lattice heavy Fermi liquid.
Area $(1 + p + 1)/2$ $= p/2$ (mod 1).
Small Fermi surface!

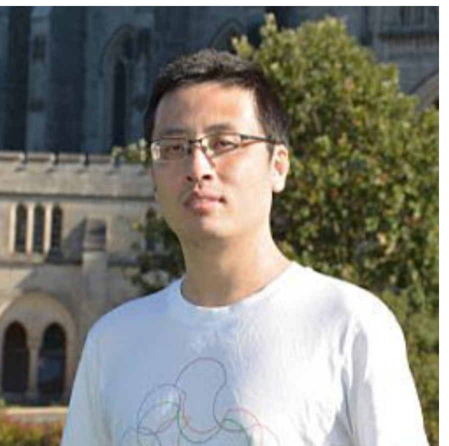


Free holes of density 1+p

π-flux spin liquid with SU(2) gauge field and massless Dirac spinons

$$\text{Pseudogap metal} = \begin{array}{c} \text{Kondo Lattice FL of } c_\alpha \text{ and } \boldsymbol{S}_1 \\ \oplus \\ \text{Spin Liquid of } \boldsymbol{S}_2 \end{array}$$

Ya-Hui Zhang



Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)
A. Nikolaenko, M. Tikhanovskaya, S. S., and Ya-Hui Zhang, PRB **103**, 235138 (2021)

ALM of FL* of Hubbard model

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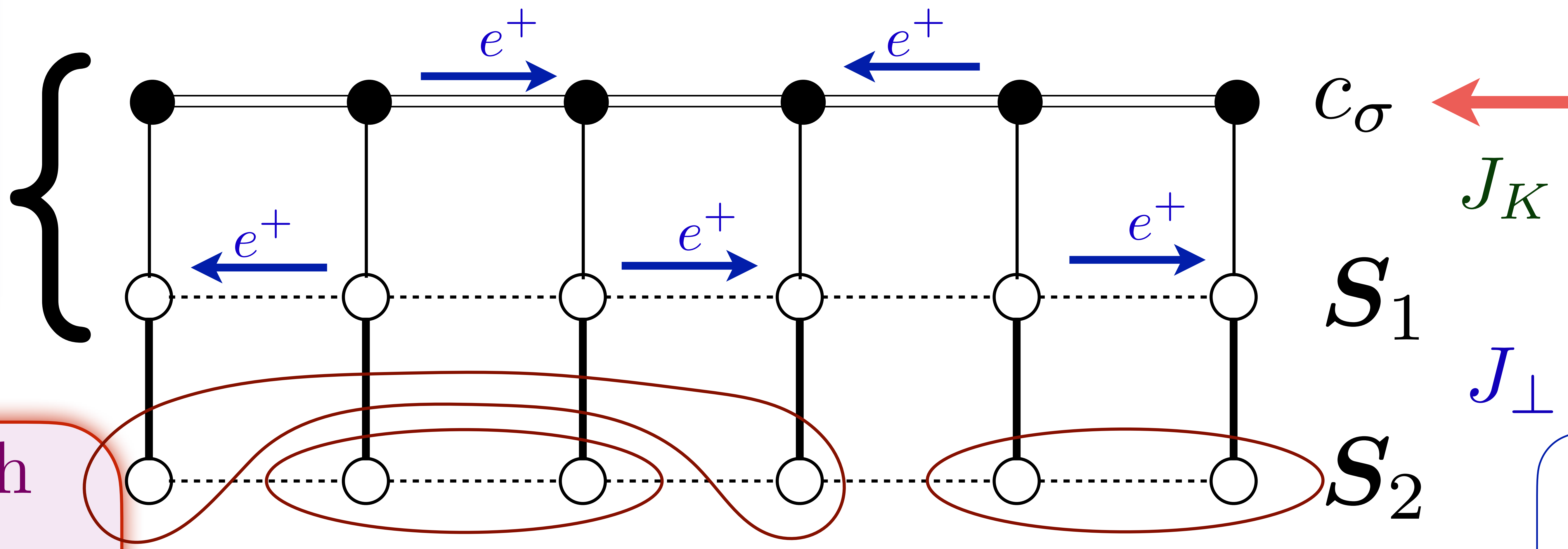


Free holes of density 1+p

π-flux spin liquid with SU(2) gauge field and massless Dirac spinons

$$\boldsymbol{S}_{1i} = \frac{1}{2} f_{1i\alpha}^{\dagger} \boldsymbol{\sigma}_{\alpha\beta} f_{1i\beta}$$

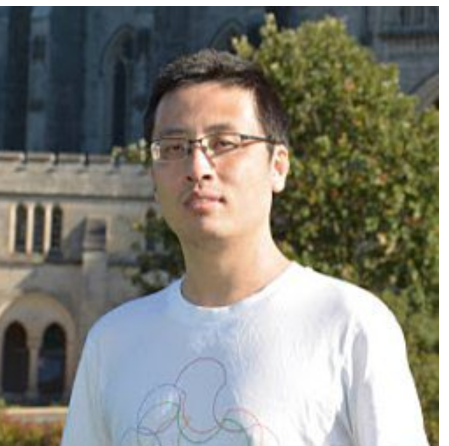
$$\boldsymbol{S}_{2i} = \frac{1}{2} f_{i\alpha}^{\dagger} \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta}$$

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

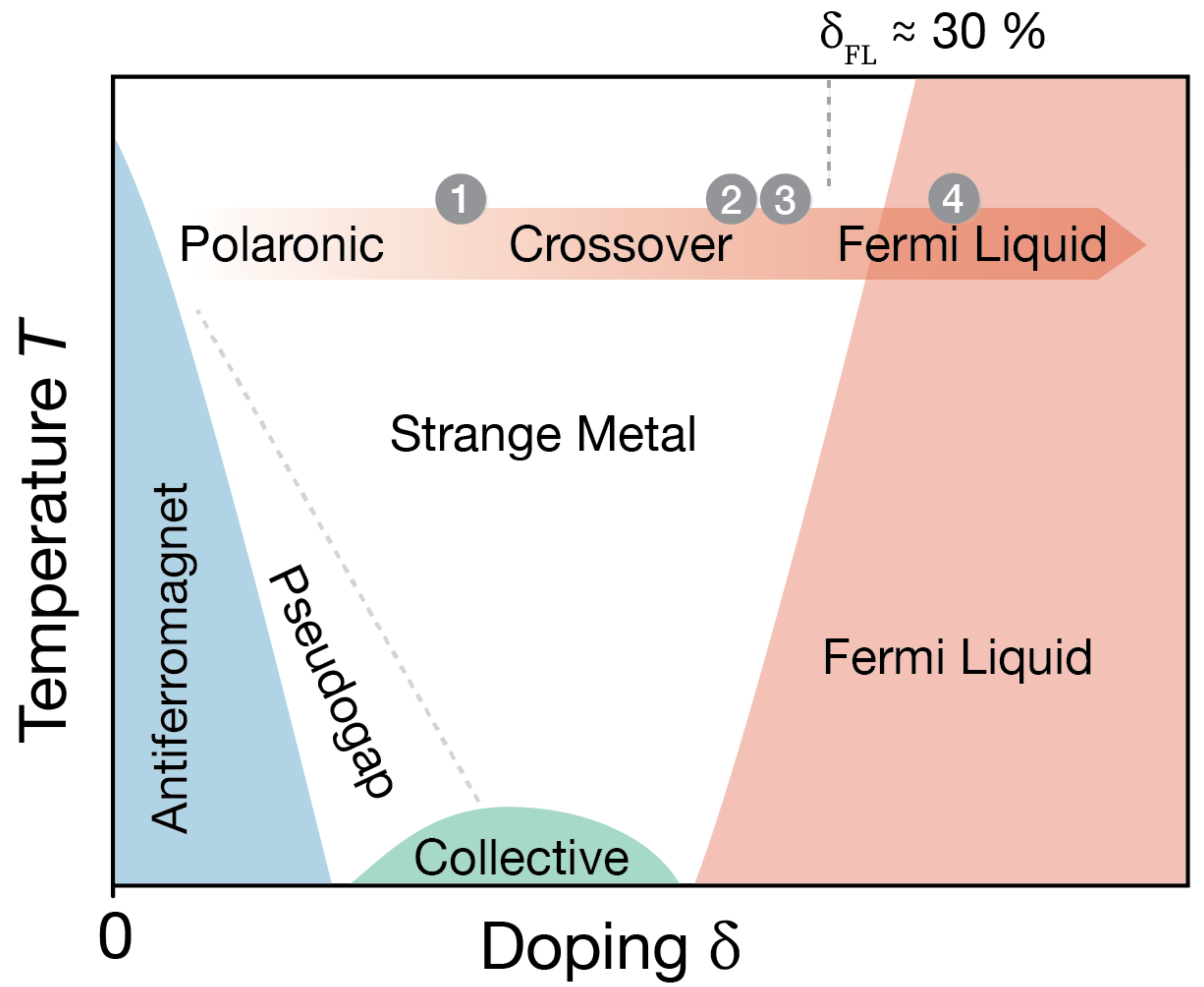
Pseudogap metal =
Kondo Lattice Heavy Fermi Liquid
$\oplus$
Spin Liquid

$|\mathrm{FL}*\rangle = [\text{Projection onto rung singlets of } \boldsymbol{S}_1, \boldsymbol{S}_2]$
$\bowtie |\text{Slater determinant of } (c, f_1)\rangle$
$\otimes |\text{Slater determinant of } f\rangle$

Ya-Hui Zhang



Replacement for "vanilla" Gutzwiller-projected Fermi liquid in the underdoped regime

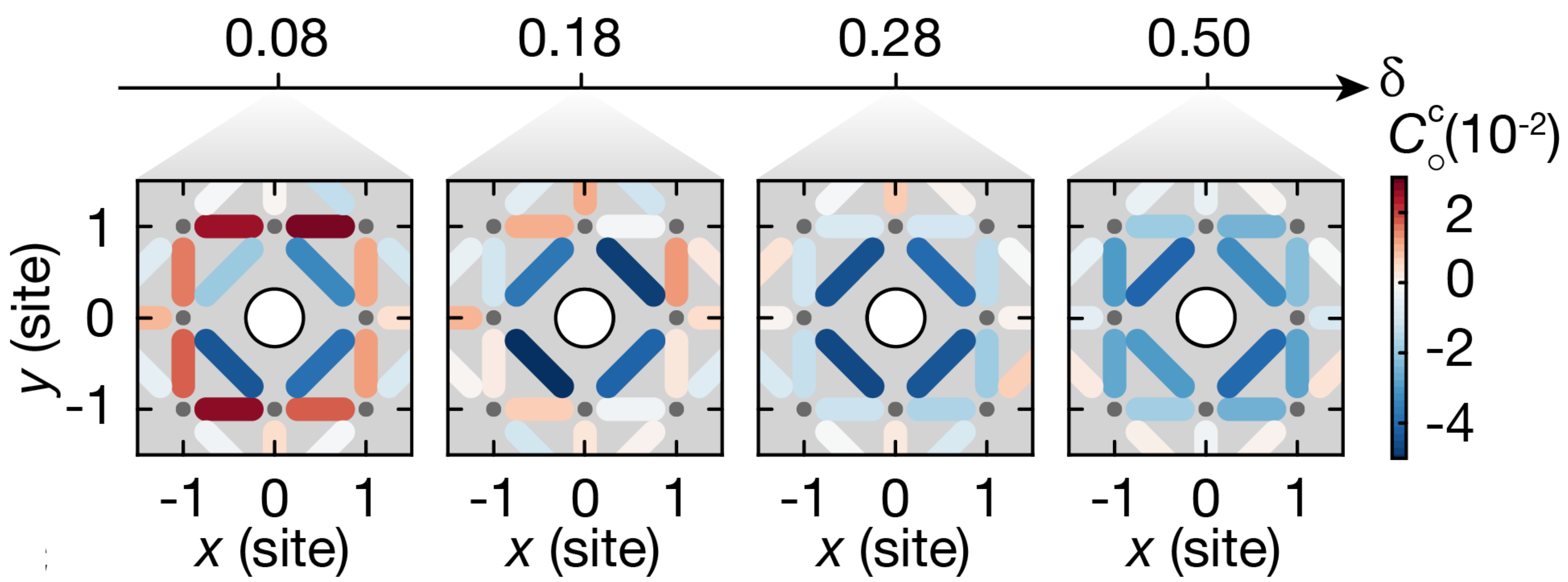


Ultracold fermionic atoms in optical lattices

Microscopic evolution of doped Mott insulators from polaronic metal to Fermi liquid

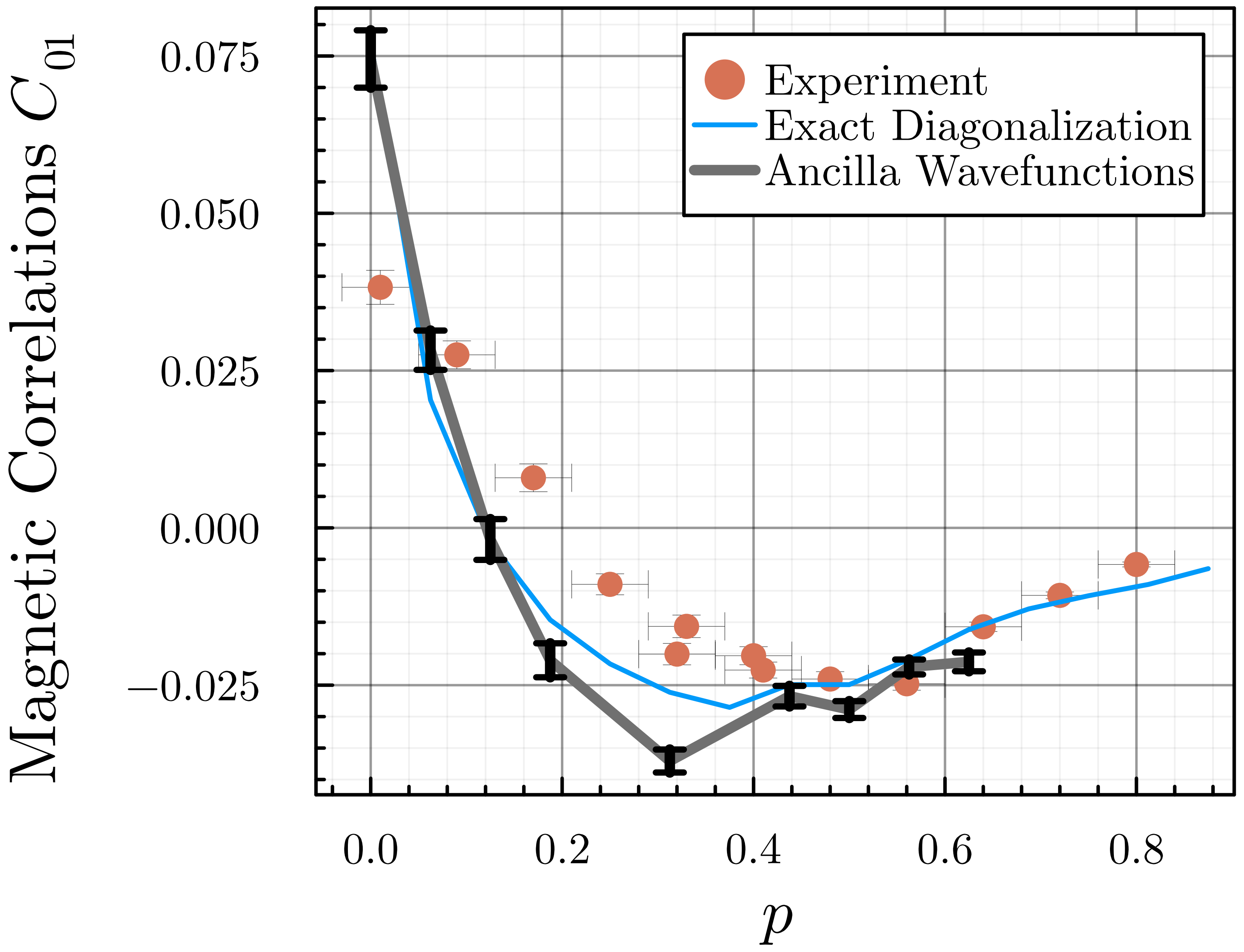
Joannis Koepsell, Dominik Bourgund, Pimonpan Sompet, Sarah Hirthe, Annabelle Bohrdt, Yao Wang, Fabian Grusdt, Eugene Demler, Guillaume Salomon, Christian Gross, Immanuel Bloch

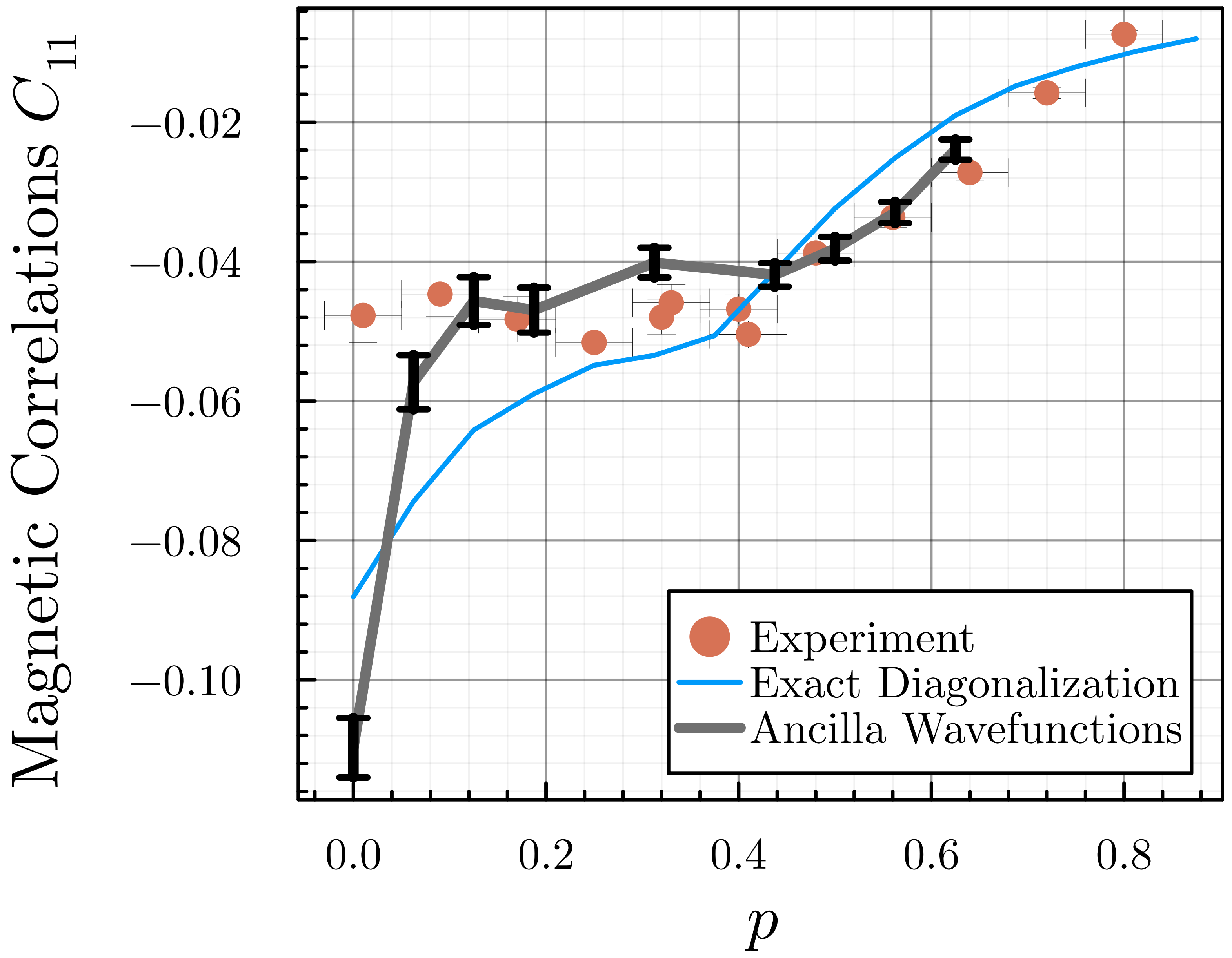
Science **374** (2021) 82

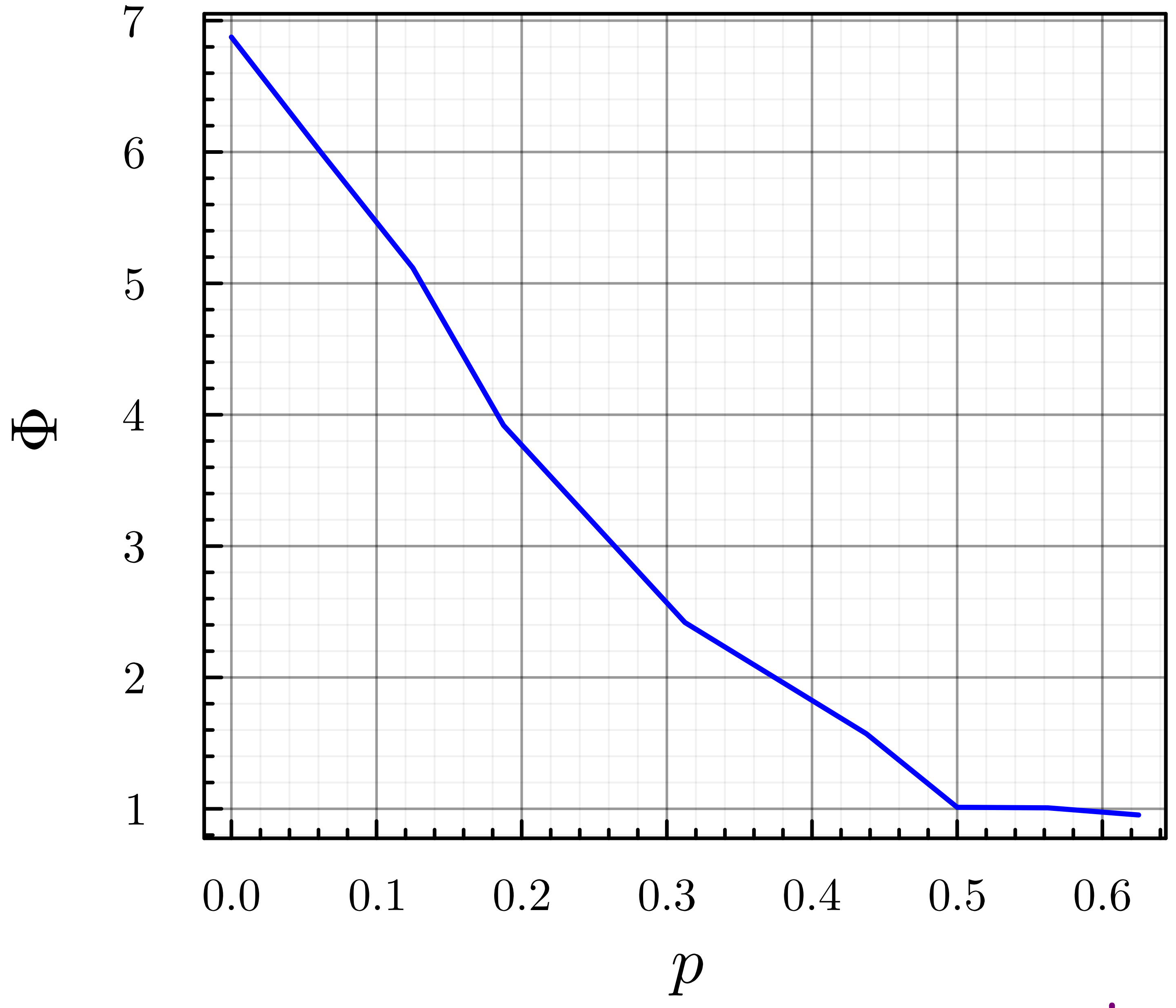


Max Planck Institute of Quantum Optics, Garching

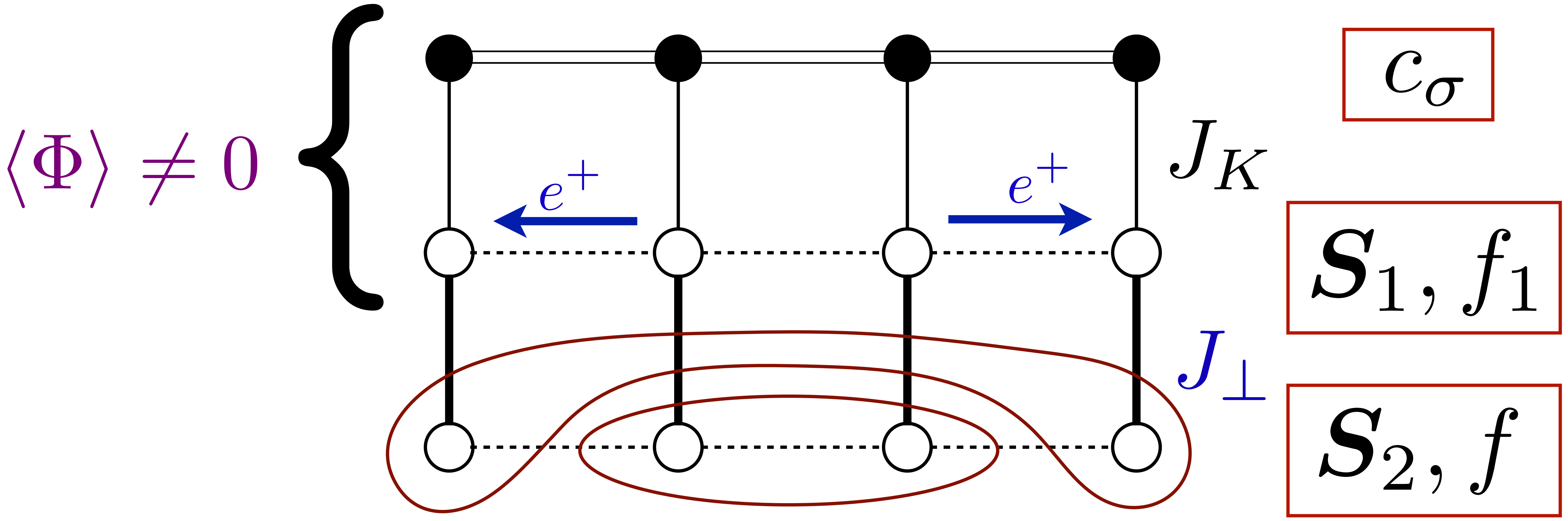
Ancilla Layer Model of the Hubbard model







Higgs boson $\Phi \sim f^{\dagger}_{1\sigma} c_{\sigma}$.



H. Shackleton and Shiwei Zhang, arXiv:2408.02190
(Tobias Müller, Yasir Iqbal, S.S., Ronny Thomale, arXiv:2408.01492)

Ancilla layer model of FL*

$$H_{\text{Kondo lattice}} = \sum_{\boldsymbol{i},\boldsymbol{j}} \left[-t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} - t_{1,\boldsymbol{ij}} f^\dagger_{1\boldsymbol{i}\alpha} f_{1\boldsymbol{j}\alpha} \right] - \sum_{\boldsymbol{i}} \Phi \left(c^\dagger_{\boldsymbol{i}\alpha} f_{1\boldsymbol{i}\alpha} + f^\dagger_{1\boldsymbol{i}\alpha} c_{\boldsymbol{i}\alpha} \right)$$

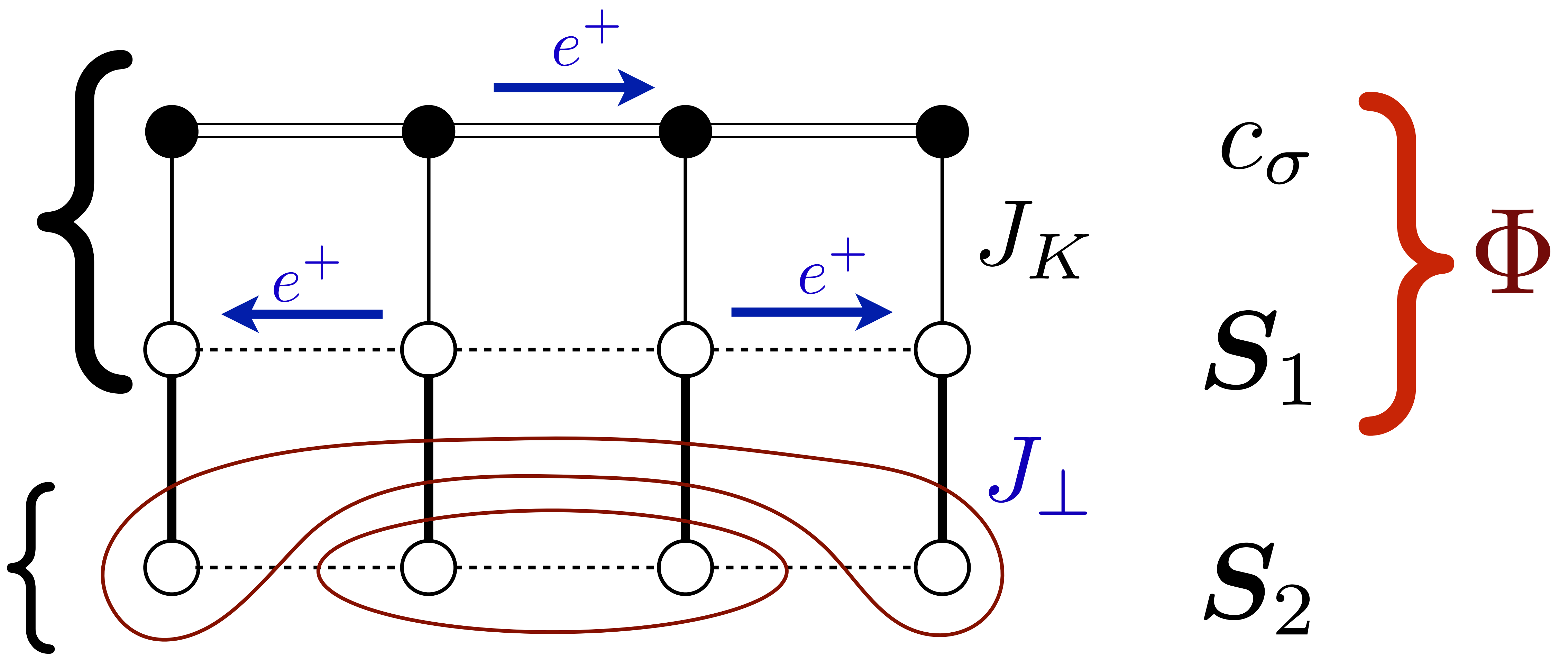
Heavy Fermi liquid of electrons c, f_1

$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p \pmod 2$.
Small Fermi surface!

Your favorite spin liquid



E. Mascot,
A. Nikolaenko,
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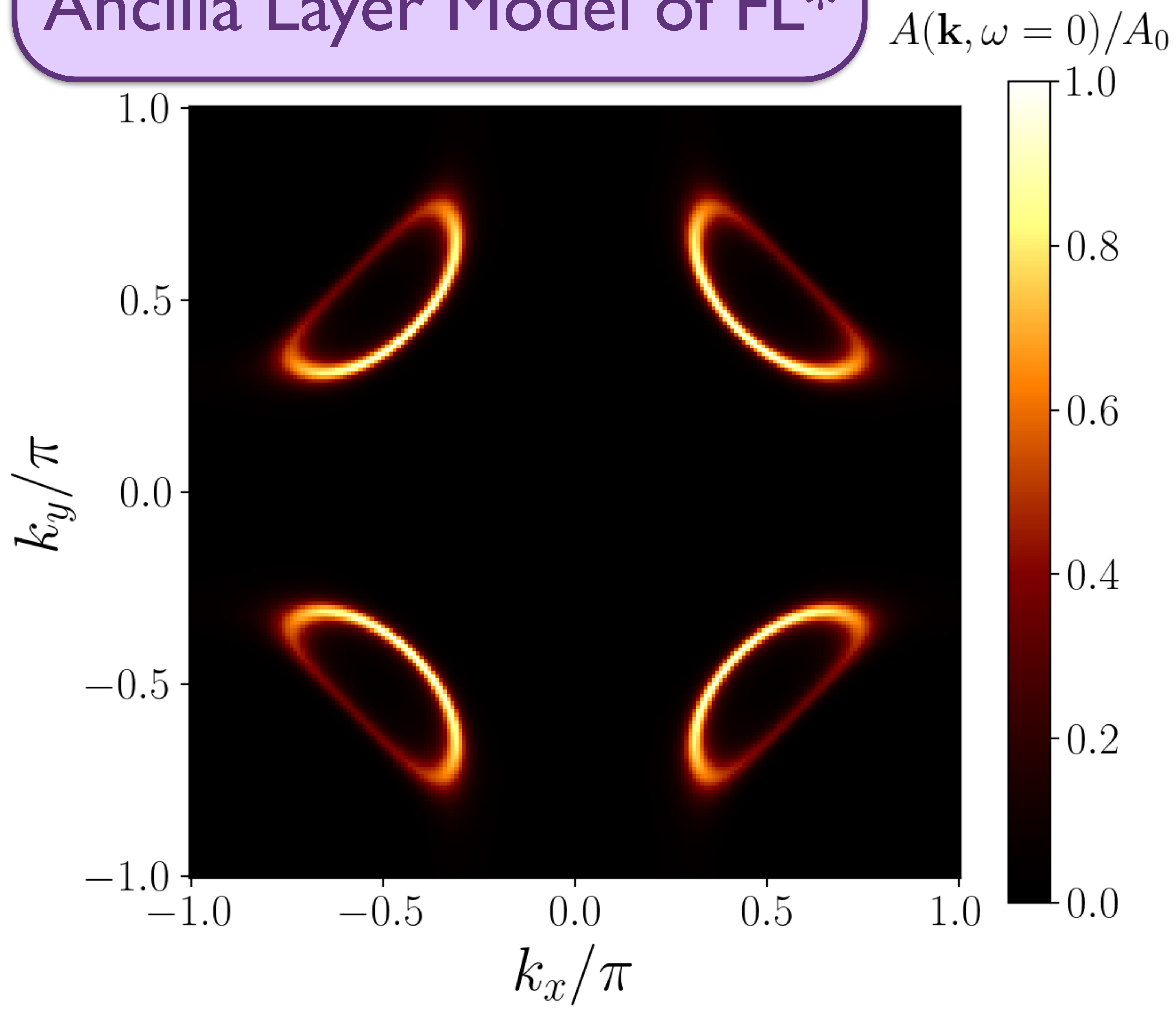
PRB **105**,
075146 (2022)

Ancilla Layer Model of FL*

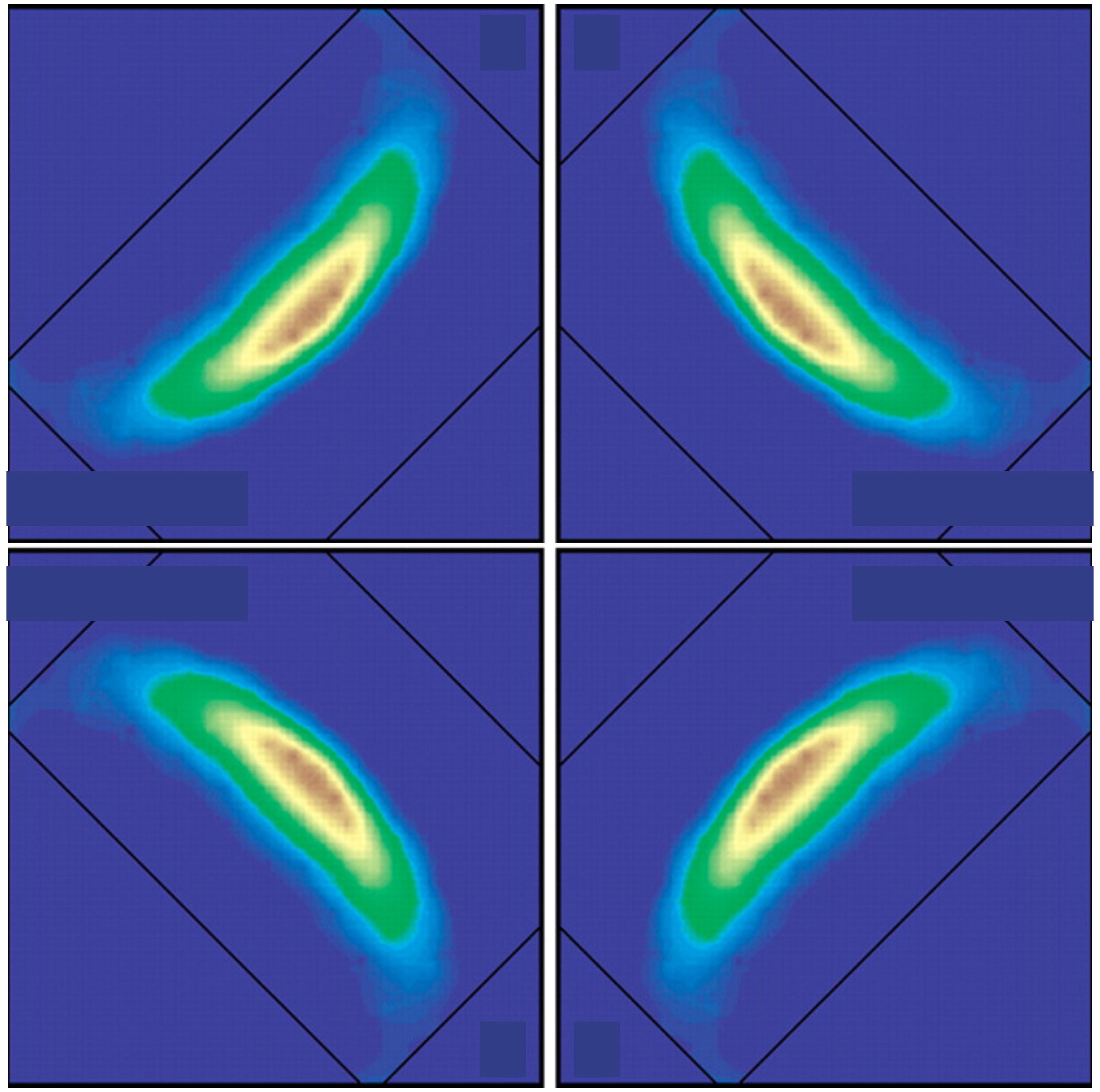


Decoupled Kondo lattice and spin liquid yields pockets of area $\underline{p/8}$.

Hybridization of Fermi surfaces of size $1 + p$ and 1: imposed by "rigidity" of bottom layer spin liquid. (SDW theory has 2 Fermi surfaces of size $1 + p$)

Photoemission expts

$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$



Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

E. Mascot,
A. Nikolaenko,
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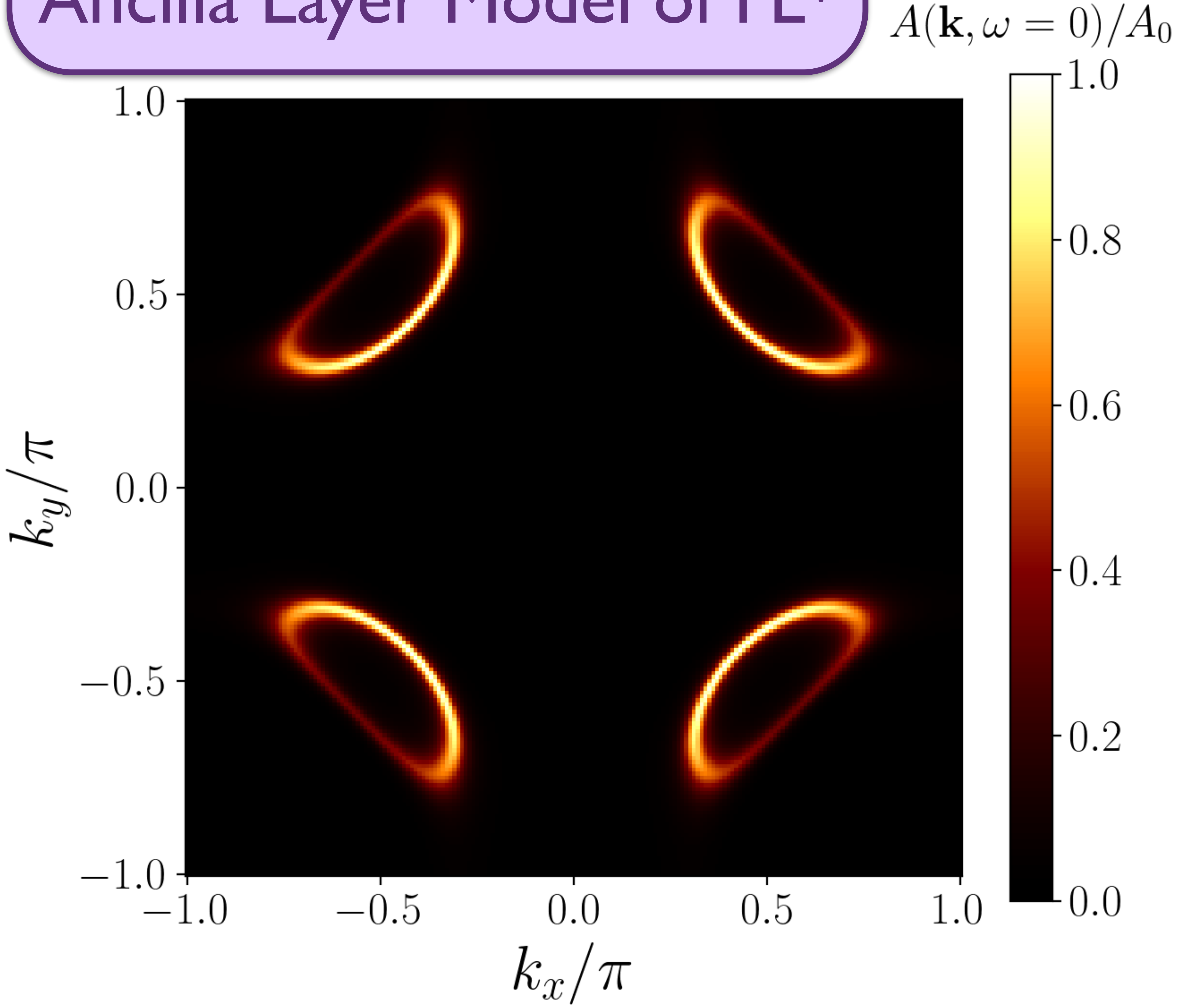
Ya-Hui Zhang,
D. K. Morr,
and S. S.,
PRB **105**,
075146 (2022)

Ancilla Layer Model of FL*

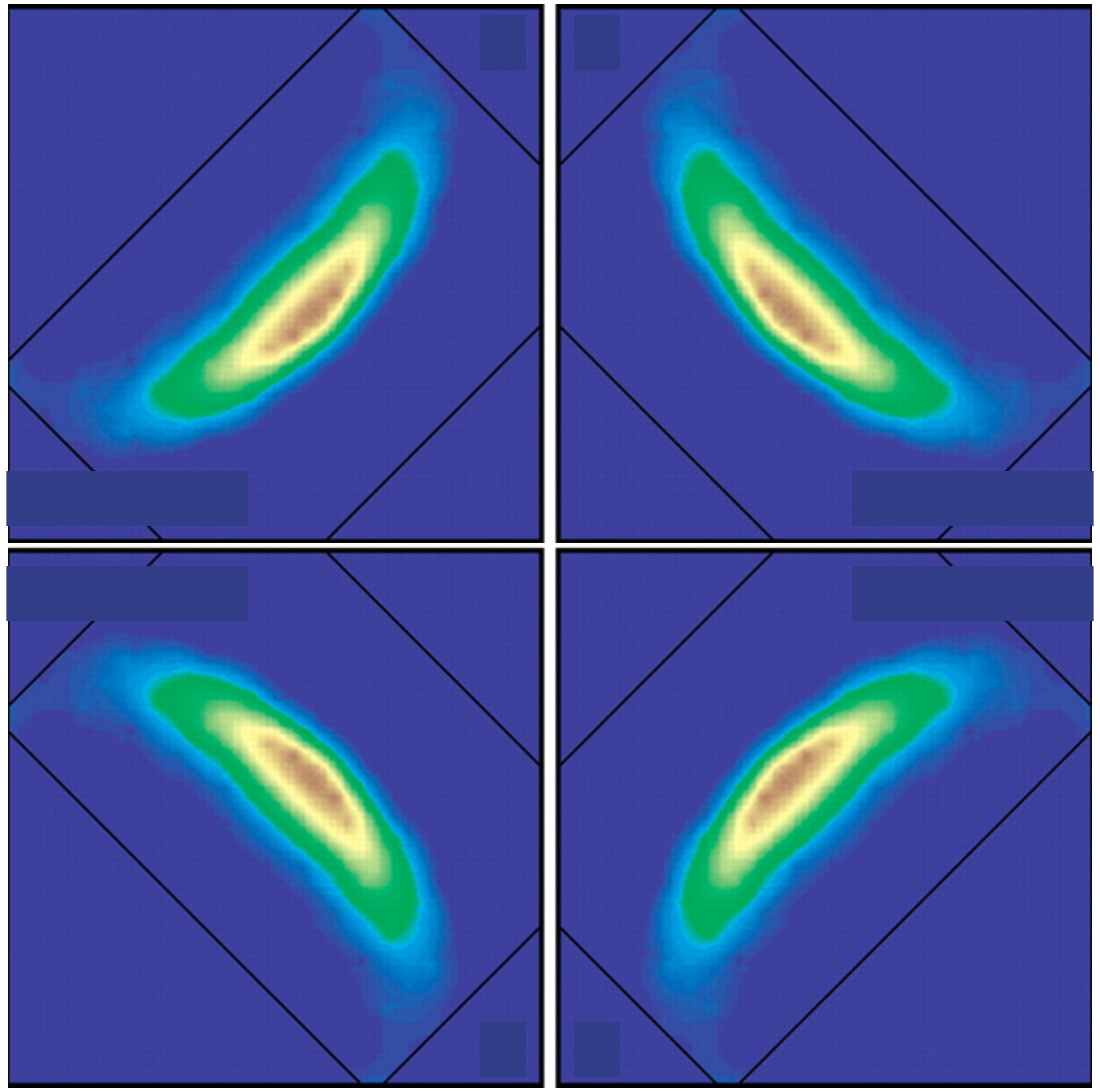


Decoupled Kondo lattice and spin liquid yields pockets of area $\underline{p/8}$.

But pocket backsides are not observed!

Photoemission expts

$Ca_{2-x}Na_xCuO_2Cl_2$ at $x = 0.10$



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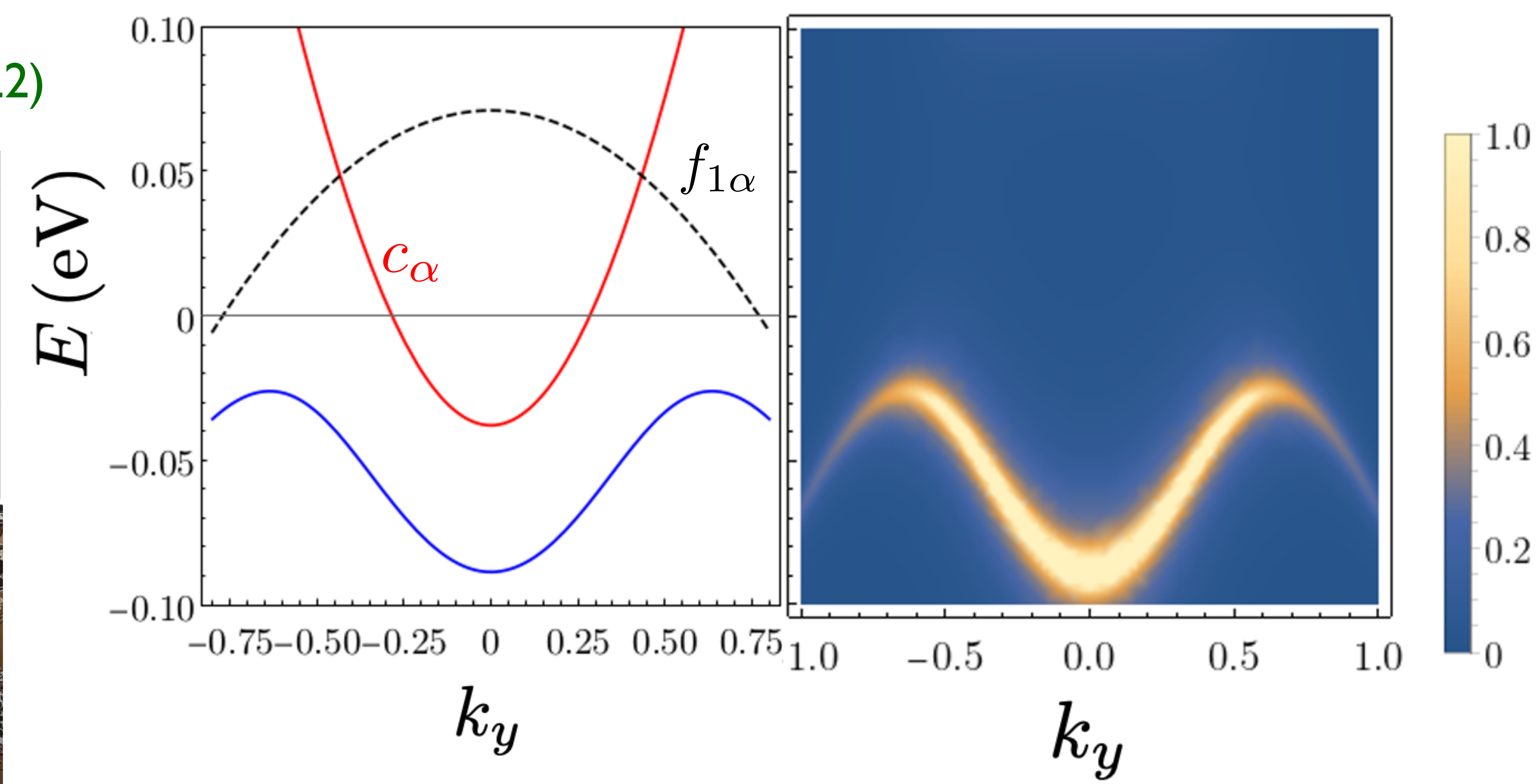
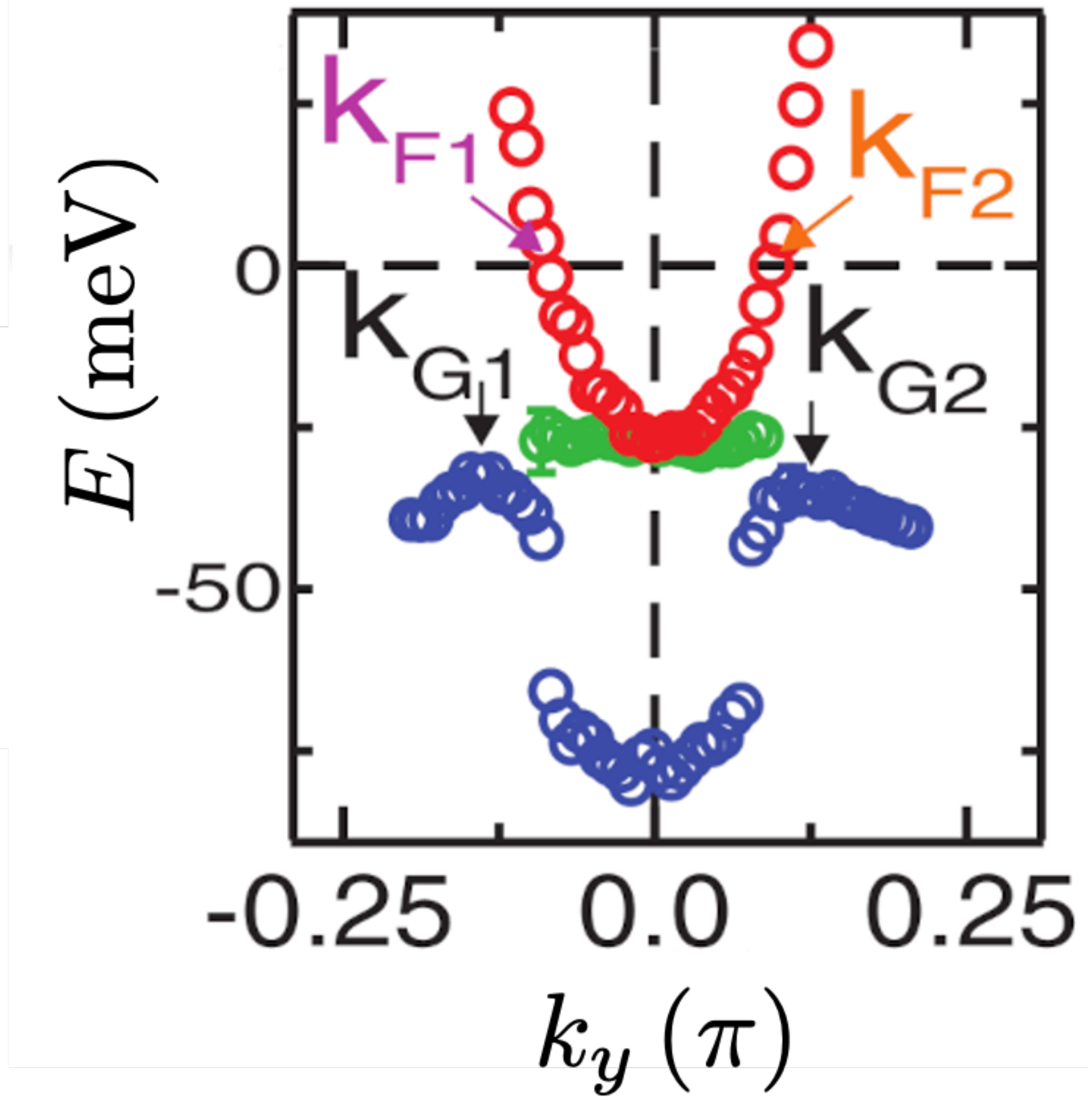
Ancilla Layer Model of FL*



Decoupled Kondo lattice and spin liquid

Shift in k_F also related to
Hybridization of Fermi surfaces of size $1 + p$ and 1:
(SDW theory has 2 Fermi surfaces of size $1 + p$)

Photoemission expts



Bi2201

R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

SU(2) gauge theory of FL* in the Ancilla Layer Model (ALM)

Ancilla Layer Model of FL*

$$H_{\text{Kondo lattice}} = \sum_{\boldsymbol{i},\boldsymbol{j}} \left[-t_{\boldsymbol{ij}} c^\dagger_{\boldsymbol{i}\alpha} c_{\boldsymbol{j}\alpha} - t_{1,\boldsymbol{ij}} f^\dagger_{1\boldsymbol{i}\alpha} f_{1\boldsymbol{j}\alpha} \right] - \sum_{\boldsymbol{i}} \Phi \left(c^\dagger_{\boldsymbol{i}\alpha} f_{1\boldsymbol{i}\alpha} + f^\dagger_{1\boldsymbol{i}\alpha} c_{\boldsymbol{i}\alpha} \right)$$

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E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

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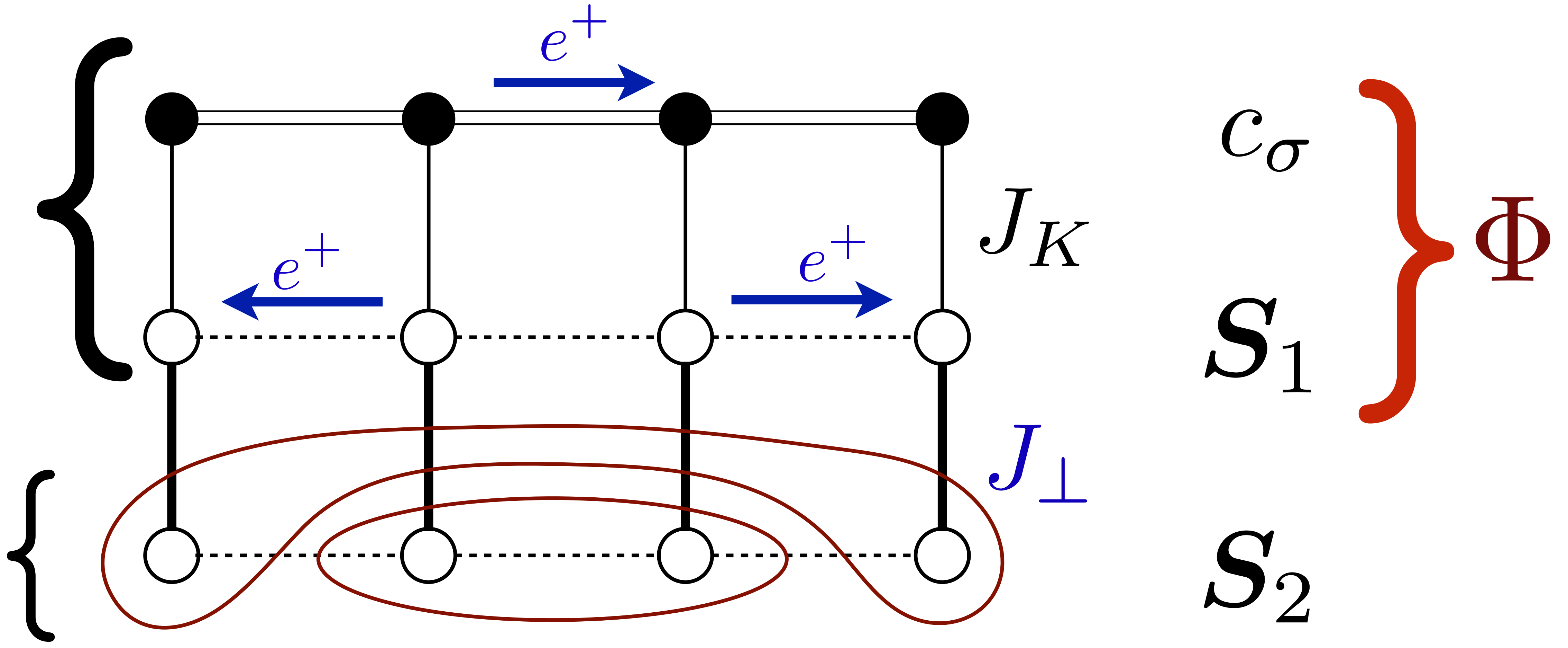
Size $1 + p + 1 = p$ (mod 2).
Small Fermi surface!

Your favorite spin liquid



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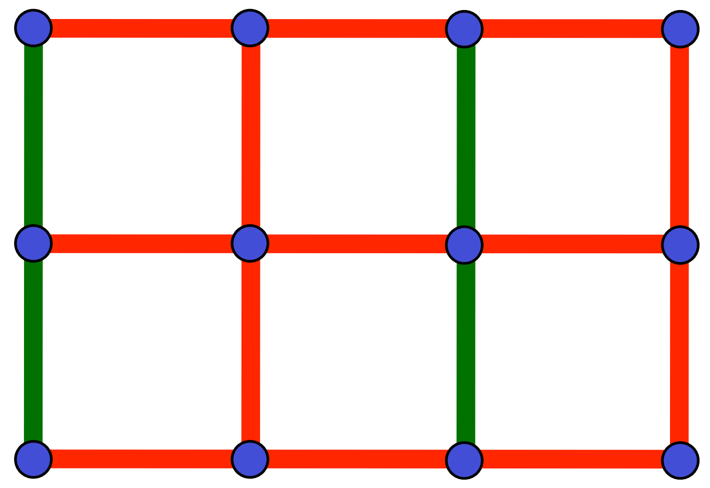
Heavy Fermi liquid of electrons c, f_1

$\boldsymbol{S}_1 \sim f^\dagger_{1\alpha} \boldsymbol{\sigma}_{\alpha\beta} f_{1\beta}$

$$H_{\text{spin liquid}} = iJ \sum_{\langle \boldsymbol{ij} \rangle} e_{\boldsymbol{ij}} \left(\Psi^\dagger_{\boldsymbol{i}} U_{\boldsymbol{ij}} \Psi_{\boldsymbol{j}} - \Psi^\dagger_{\boldsymbol{j}} U_{\boldsymbol{ji}} \Psi_{\boldsymbol{i}} \right) ; \quad \Psi_{\boldsymbol{i}} = \begin{pmatrix} f_{\boldsymbol{i}\uparrow} \\ f^\dagger_{\boldsymbol{i}\downarrow} \end{pmatrix}$$

π-flux $\boldsymbol{S}_2$ spin liquid.

$\boldsymbol{S}_2 = f^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta$



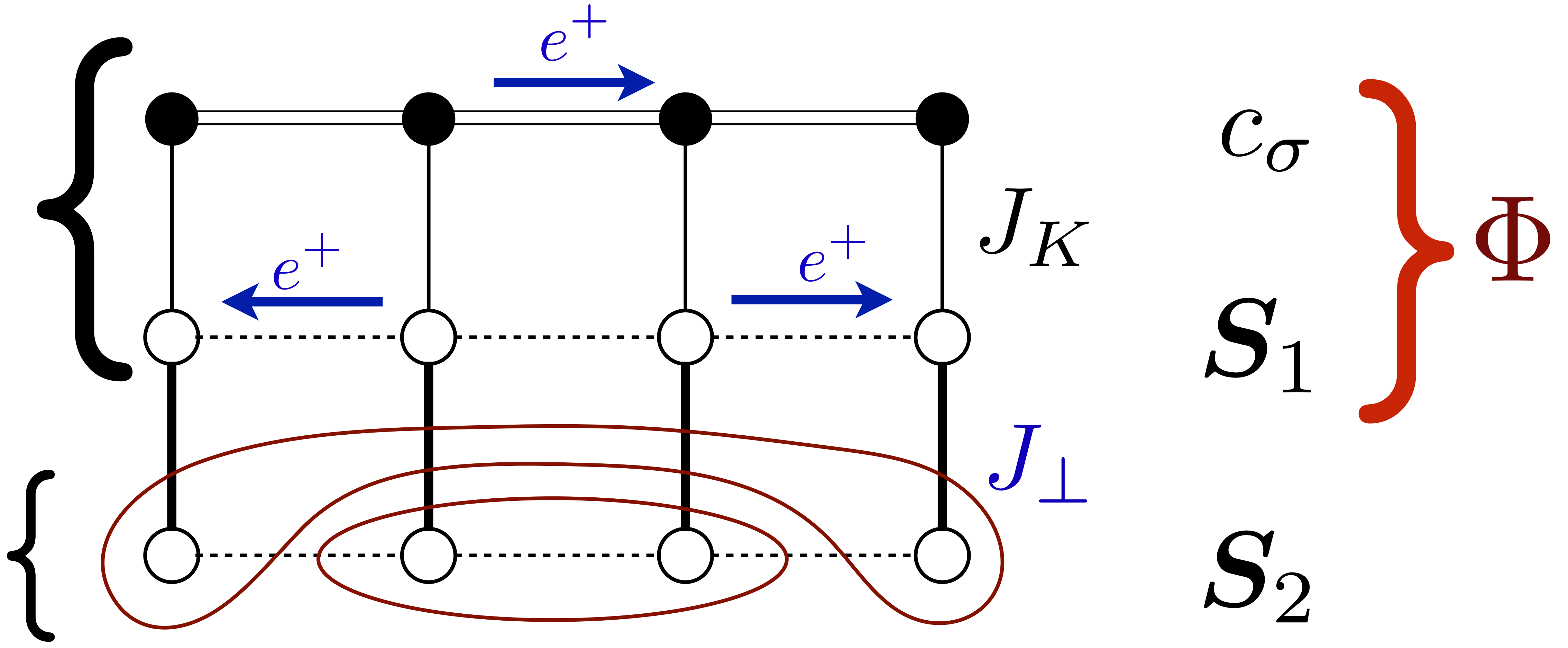
$e_{\boldsymbol{ij}} = 1$

$e_{\boldsymbol{ij}} = -1$

Fermionic spinons f moving in π-flux and an emergent SU(2) gauge field U

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p$ (mod 2).
Small Fermi surface!

π-flux spin liquid



Ancilla Layer Model of FL*

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π-flux $\boldsymbol{S}_2$ spin liquid.
$\boldsymbol{S}_2 = f^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} f_\beta$

$$H_{\text{coupling}} = \sum_{\boldsymbol{i}} \left(B^*_{1\boldsymbol{i}} f^\dagger_{1\boldsymbol{i}\alpha} f_{\boldsymbol{i}\alpha} + B^*_{2\boldsymbol{i}} \varepsilon_{\alpha\beta} f^\dagger_{1\boldsymbol{i}\alpha} f^\dagger_{\boldsymbol{i}\beta} + \text{H.c.} \right)$$

Couple Kondo lattice and spin liquid by charge e, SU(2) fundamental Higgs boson B

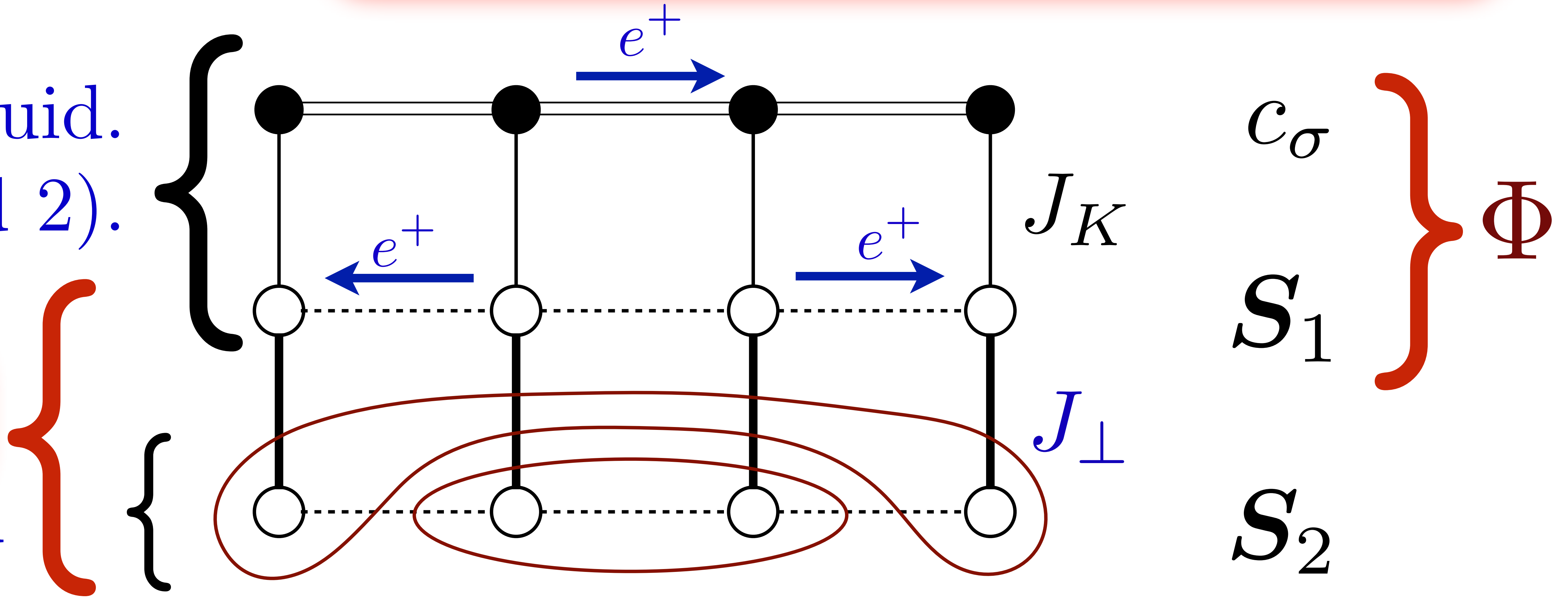
$$V_{\text{Higgs}} = \mathcal{E}_2[B,U] + \mathcal{E}_4[B,U]$$

V_{Higgs} dictated by symmetry of spin liquid

Kondo lattice heavy Fermi liquid.
Size $1 + p + 1 = p$ (mod 2).
Small Fermi surface!

B

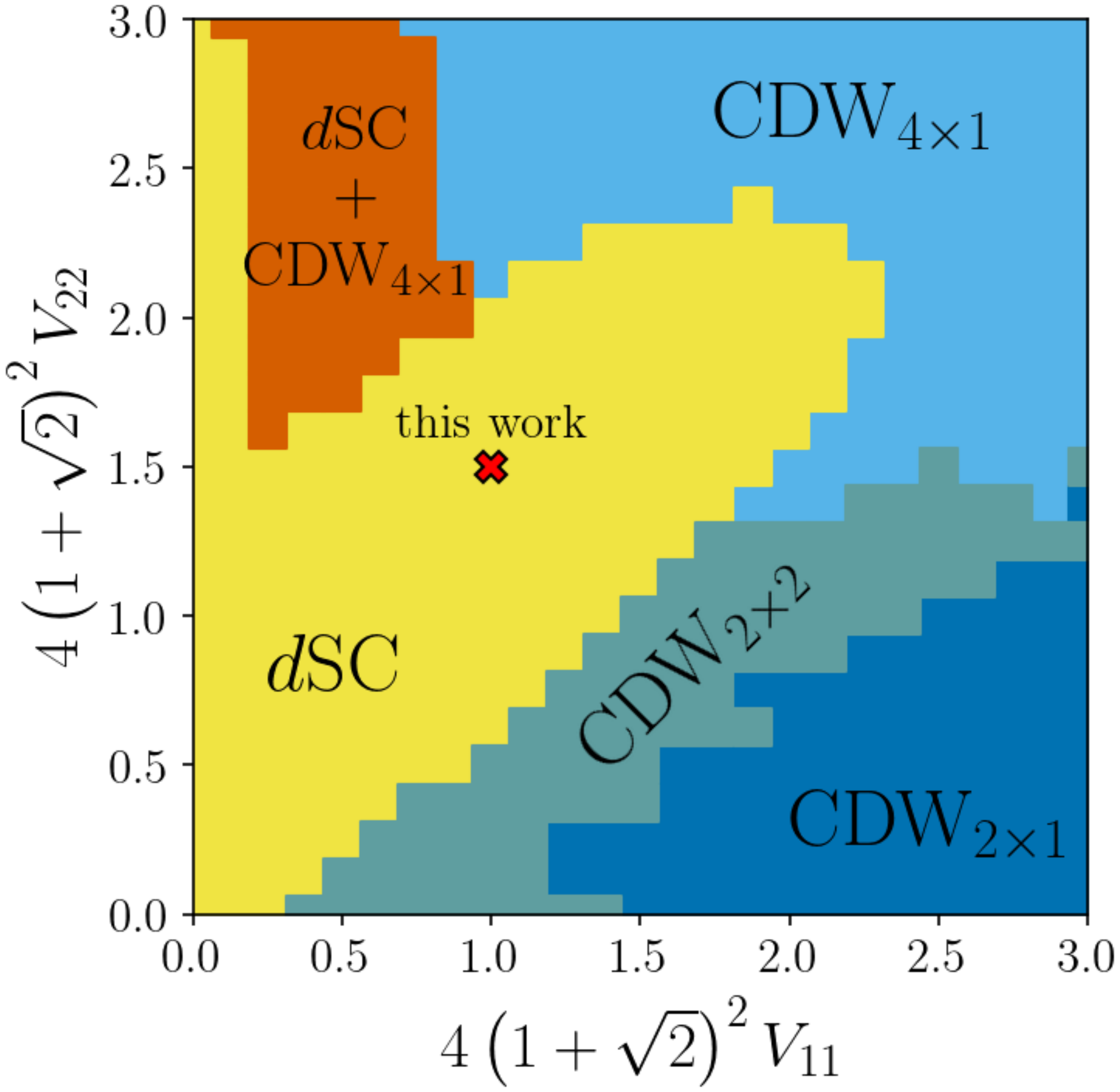
π-flux spin liquid



Born-Oppenheimer theory of FL* pseudogap

$T = 0$ phase diagram

V_{Higgs} chosen so that the ground state is a d-wave superconductor, and second best state is a period-4 stripe.



H. Pandey,
M. Christos,
P.M. Bonetti,
R. Shanker,
S. Sharma,
S.S.,

arXiv:2507.05336

Minimize V_{Higgs} w.r.t. $\left\langle \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix} \right\rangle$

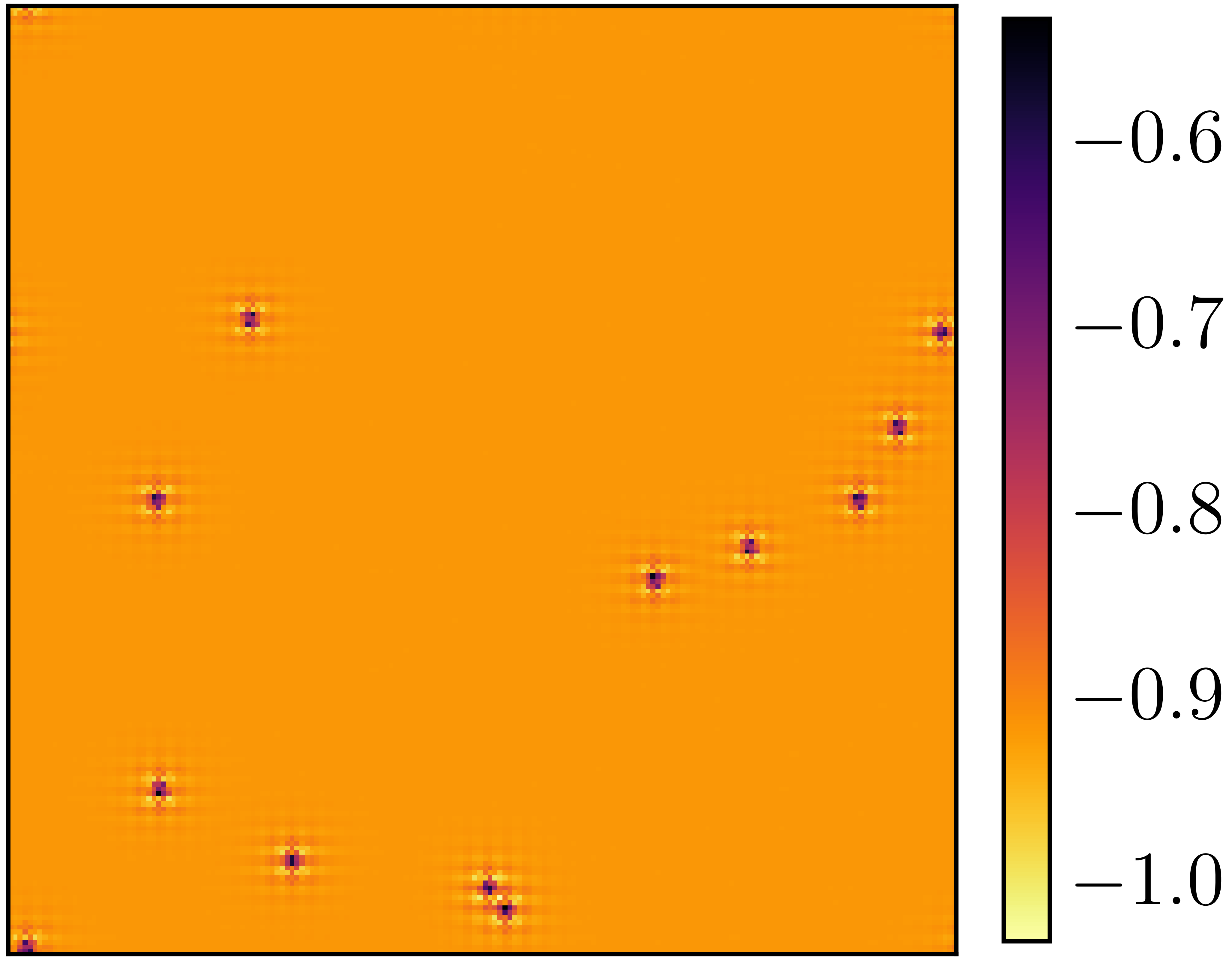
Set $U_{ij} = 1$.

Born-Oppenheimer theory of FL* pseudogap

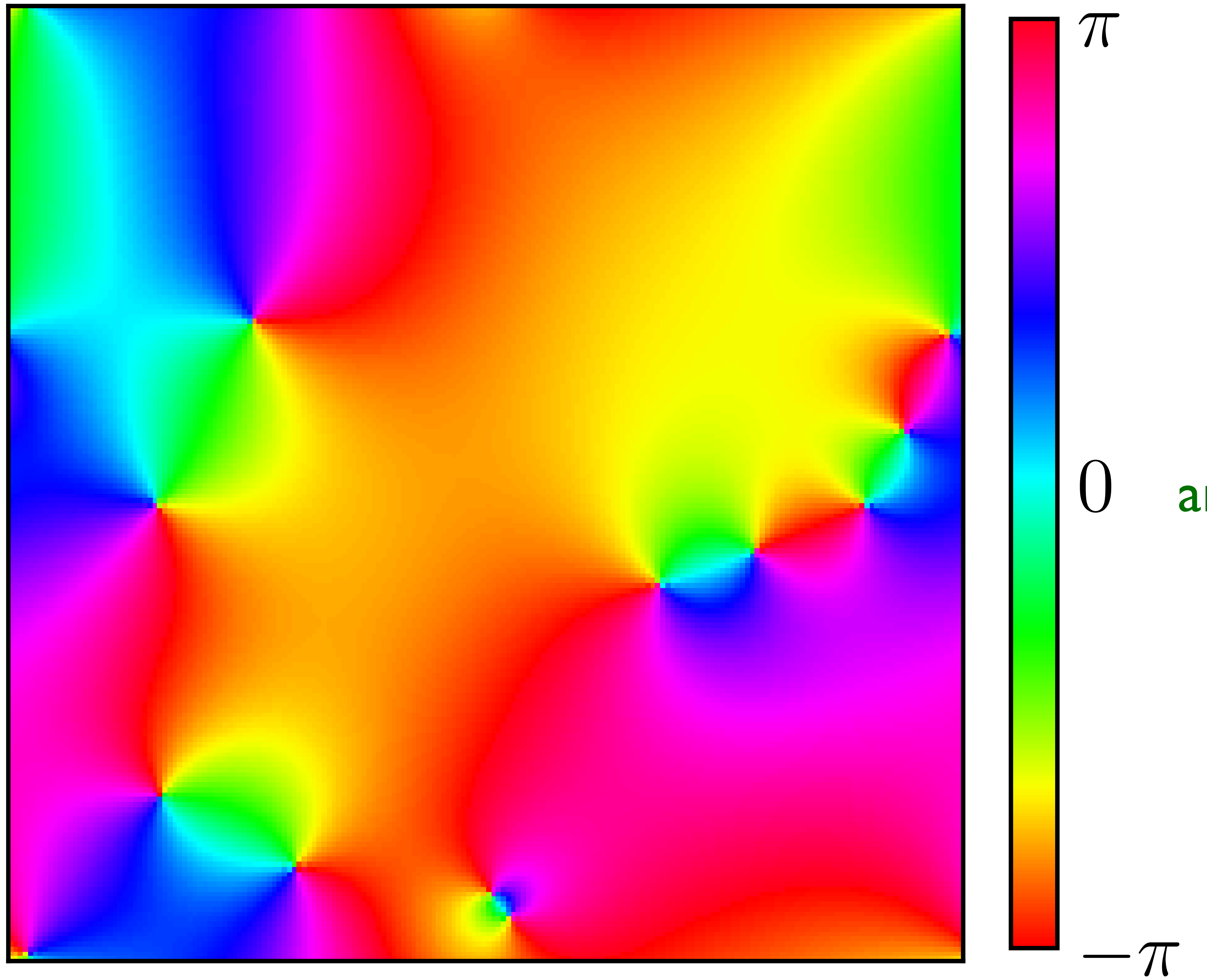
$$\mathcal{Z}_{2+0} = \int \prod_i \mathcal{D}B_i \int \prod_{\langle \boldsymbol{ij} \rangle} \mathcal{D}U_{\boldsymbol{ij}} \exp\left[- \left(V_{\rm Higgs}[B,U] - \kappa \sum_{\square} {\rm Re Tr} \prod_{\boldsymbol{ij} \in \square} U_{\boldsymbol{ij}} \right) /T \right]$$

- Simulation of classical, thermal theory for bosons B, U defined by $\mathcal{Z}_{2+0}$
- Diagonalize 3-layer fermion Hamiltonian for c, f_1, f for each snapshot of B, U, and average.

Born-Oppenheimer theory of FL* pseudogap



Bond density

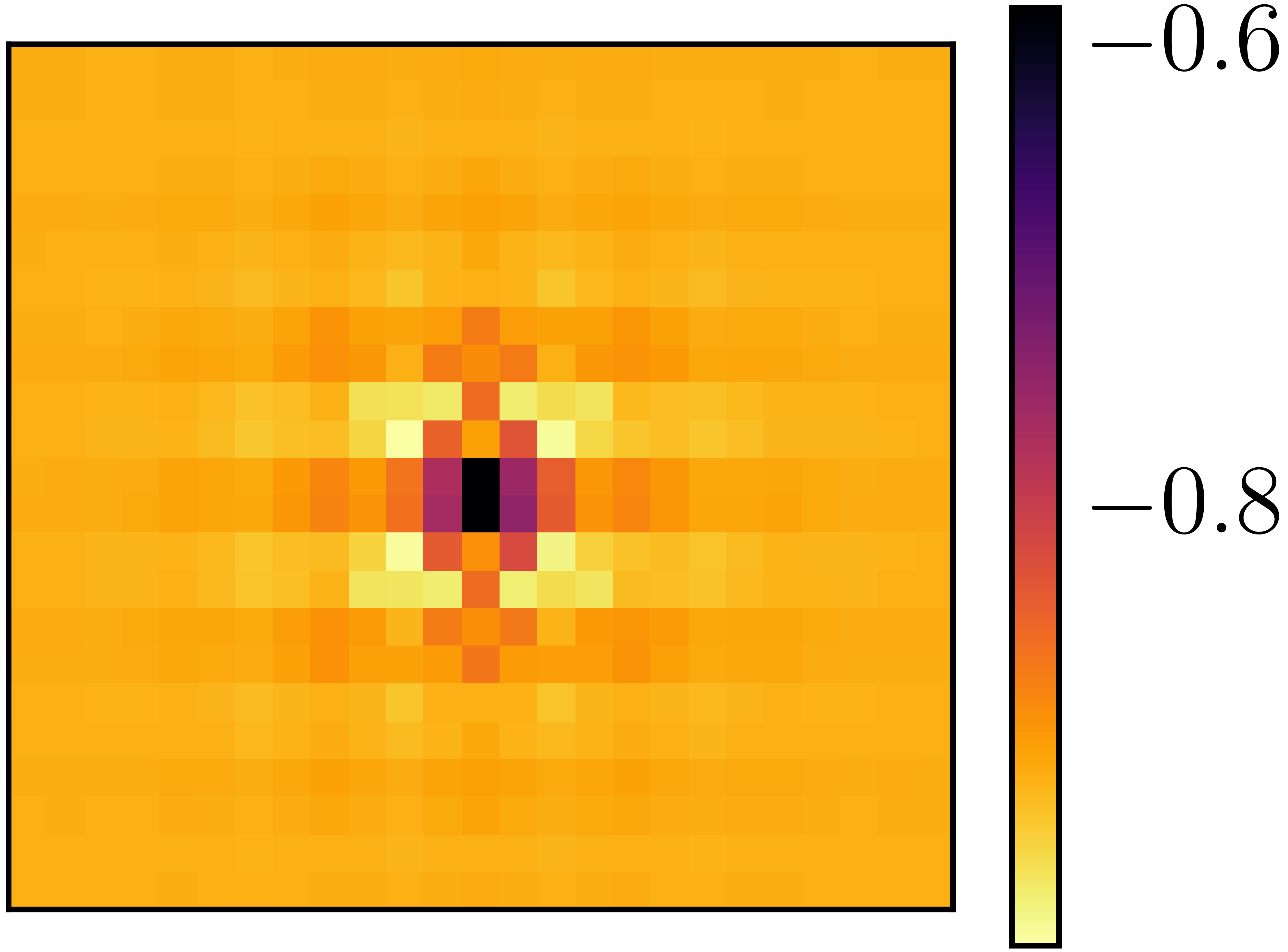


Phase of pairing amplitude

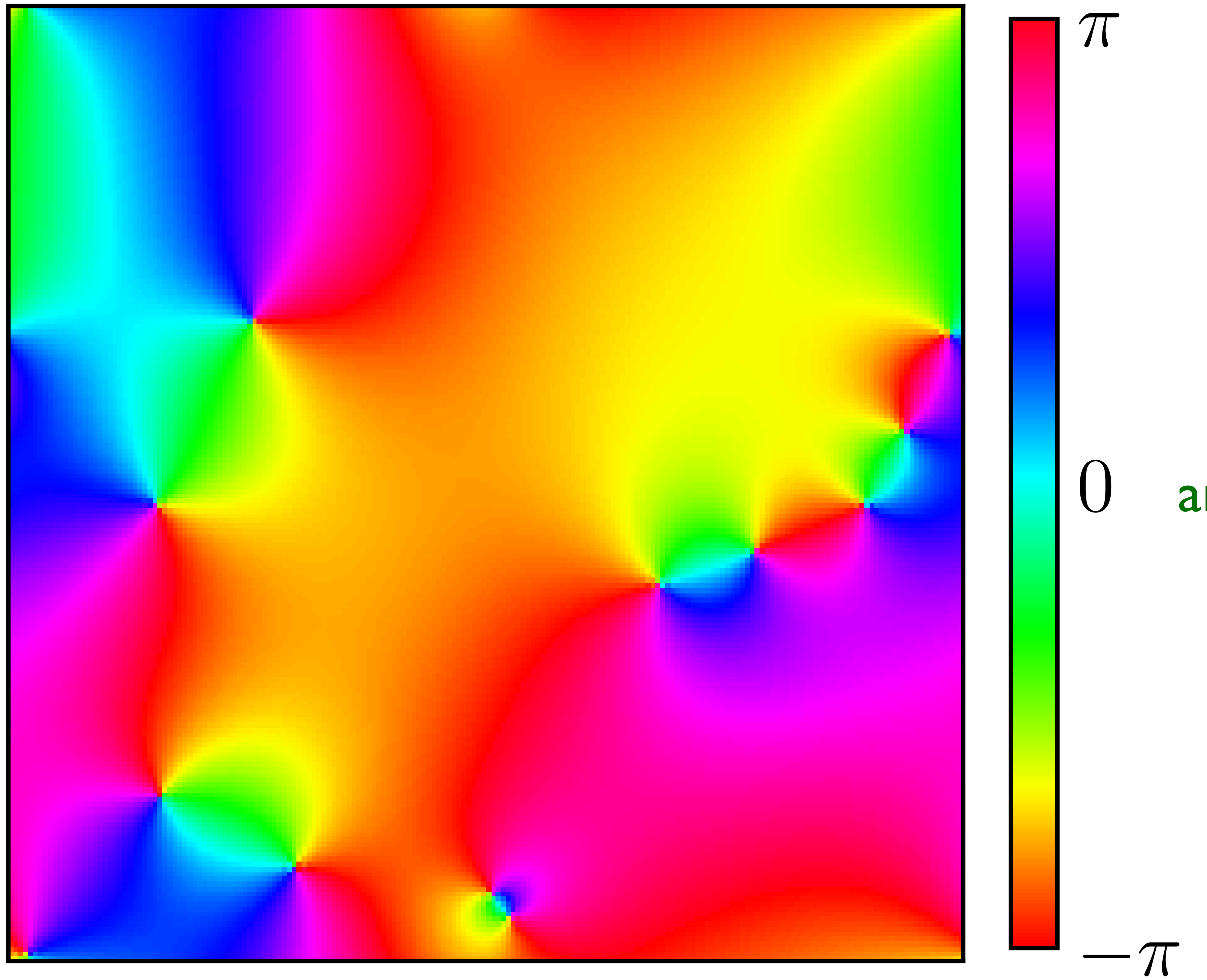
H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

See also Jia-Xin Zhang and S. S., PRB **110**, 235120 (2024)

Born-Oppenheimer theory of FL* pseudogap



Bond density



Phase of pairing amplitude

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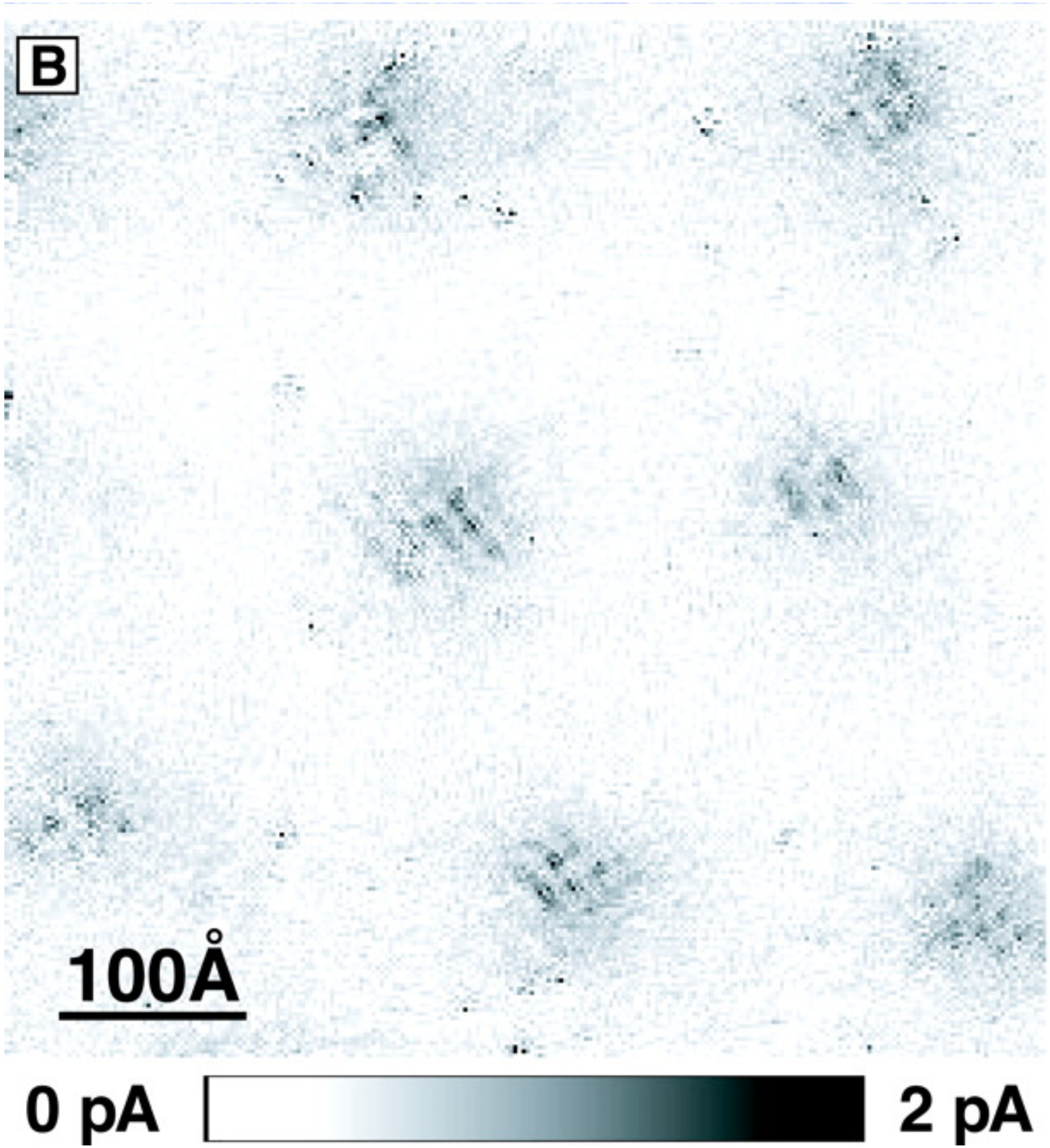
PRB **110**,
235120
(2024)

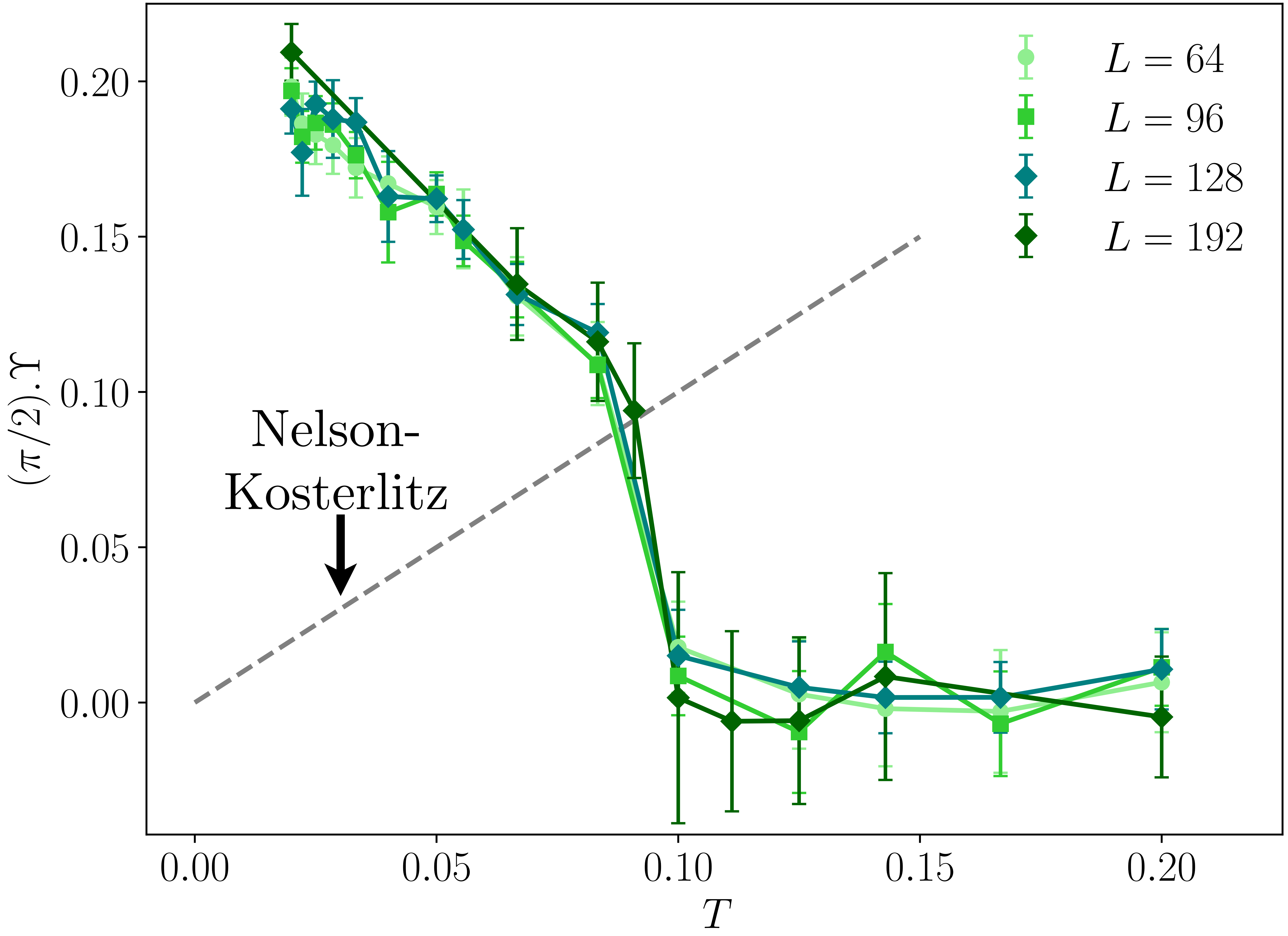


A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $Bi_2Sr_2CaCu_2O_{8+\delta}$

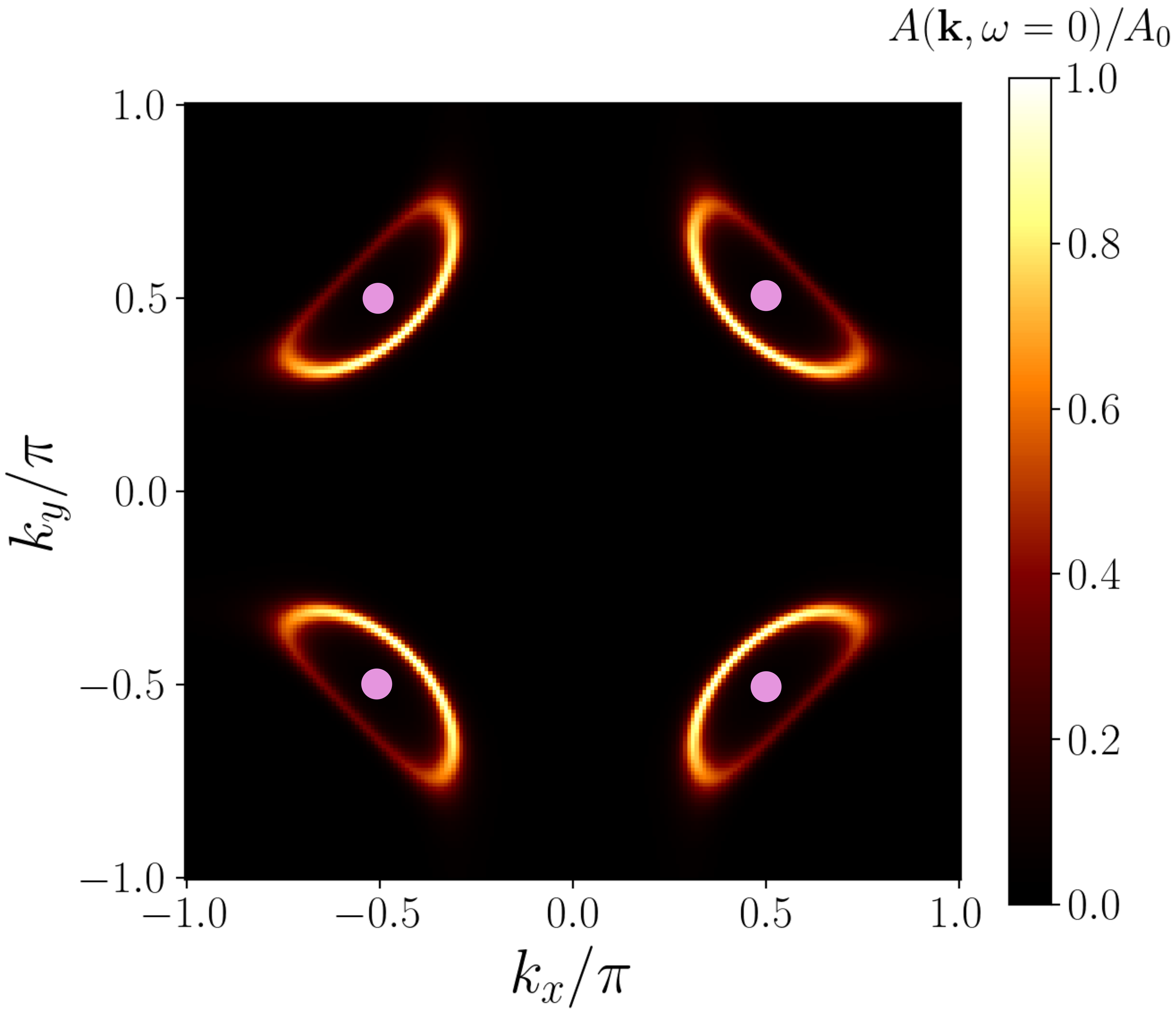
J. E. Hoffman, E. W. Hudson,
K. M. Lang, V. Madhavan,
H. Eisaki, S. Uchida, J.C. Davis
Science **295**, 466 (2002)

Born-Oppenheimer theory of FL* pseudogap

$\Upsilon =$ Helicity Modulus



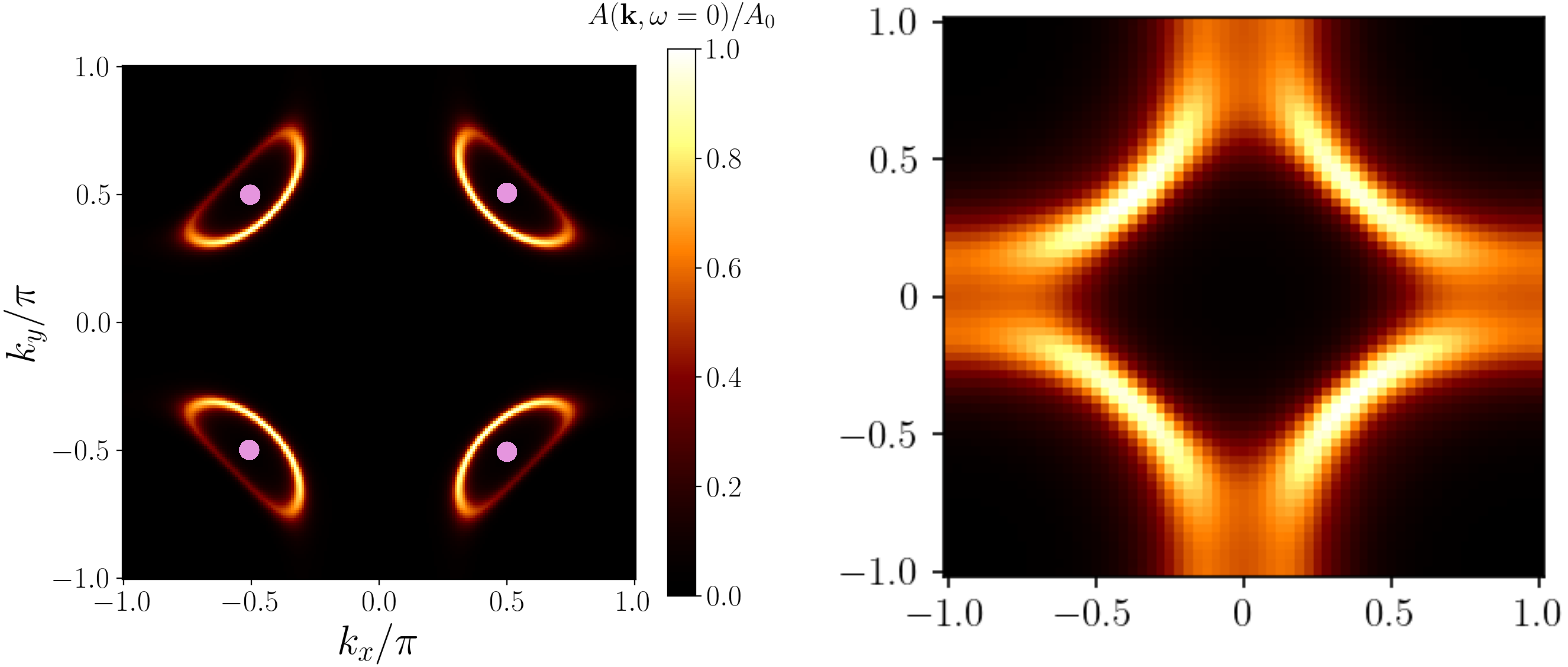
H. Pandey,
M. Christos,
P.M. Bonetti,
R. Shanker,
S. Sharma,
S.S.,
arXiv:2507.05336



FL* has 4 hole pockets
(consistent with Yamaji)
and 4 nodal spinons
of the π-flux spin liquid

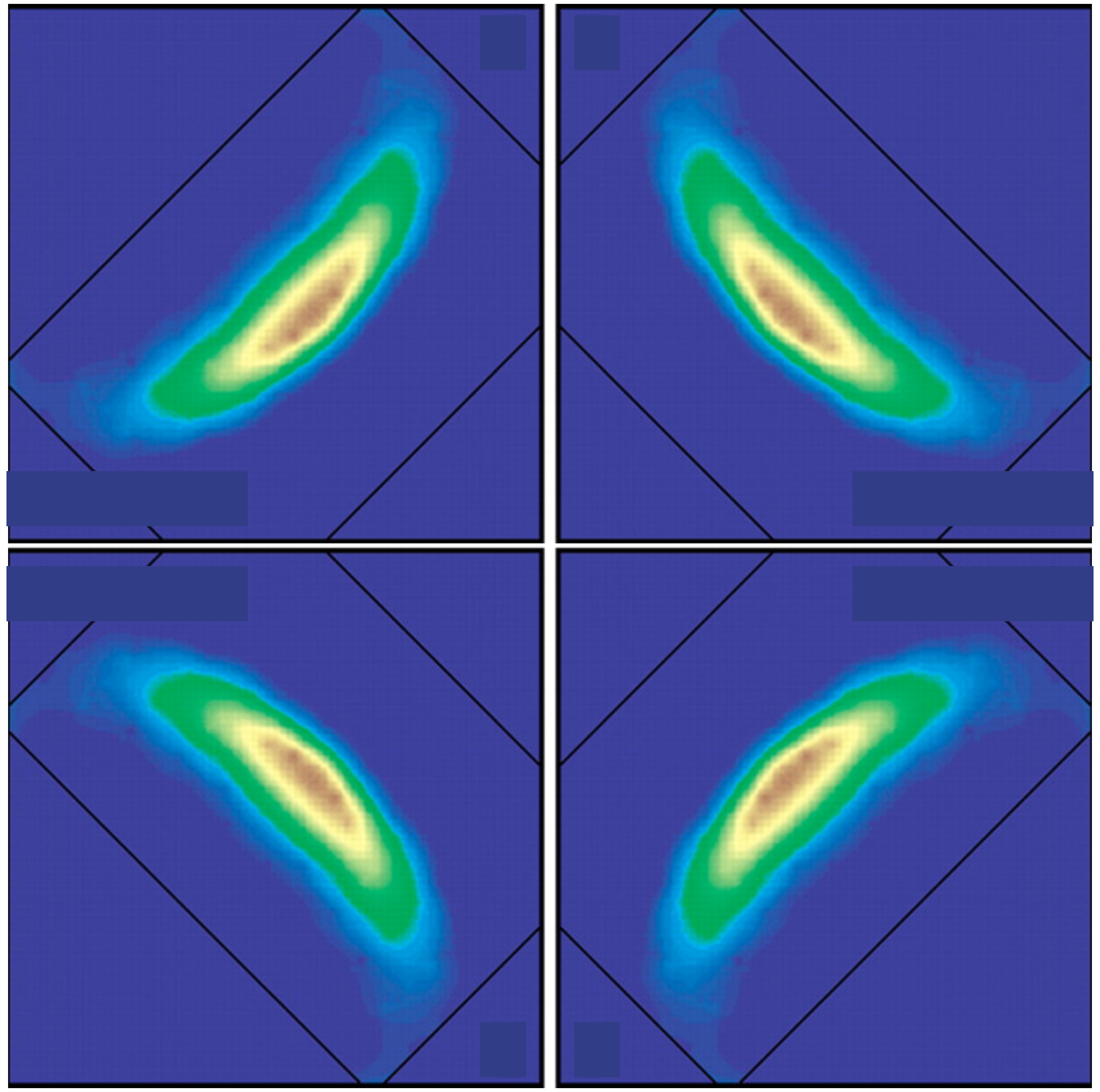
M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023)

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

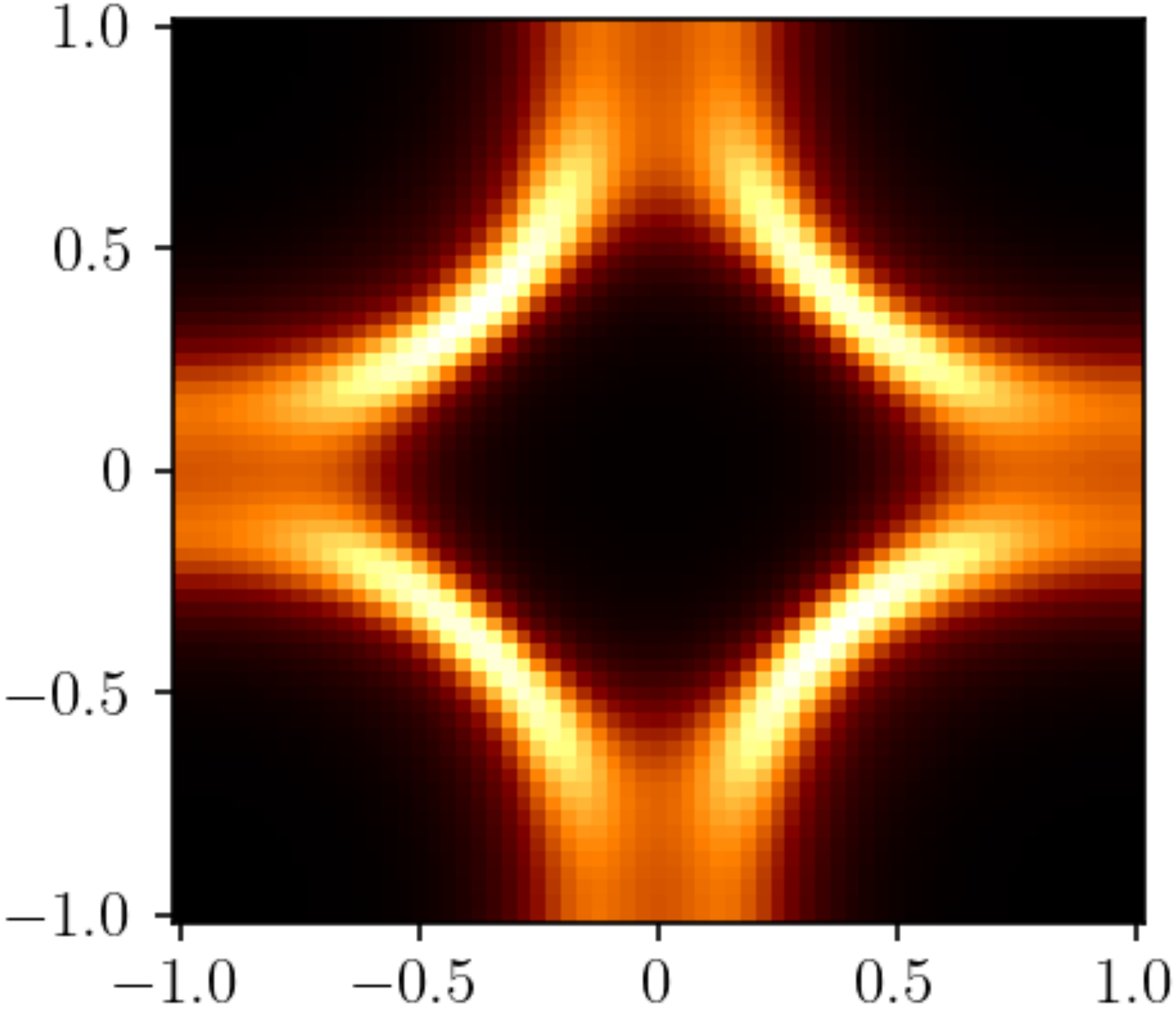


Thermal SU(2) gauge theory of mixing between holes and spinons
mediated by Yukawa couplings to a SU(2) fundamental, charge $+e$ Higgs boson B.
Condensation of B leads to a d-wave superconductor
with BCS-like nodal quasiparticles with anisotropic velocities

H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336



Kyle M. Shen, . . . Z.-X. Shen, Science **307**, 901 (2005)



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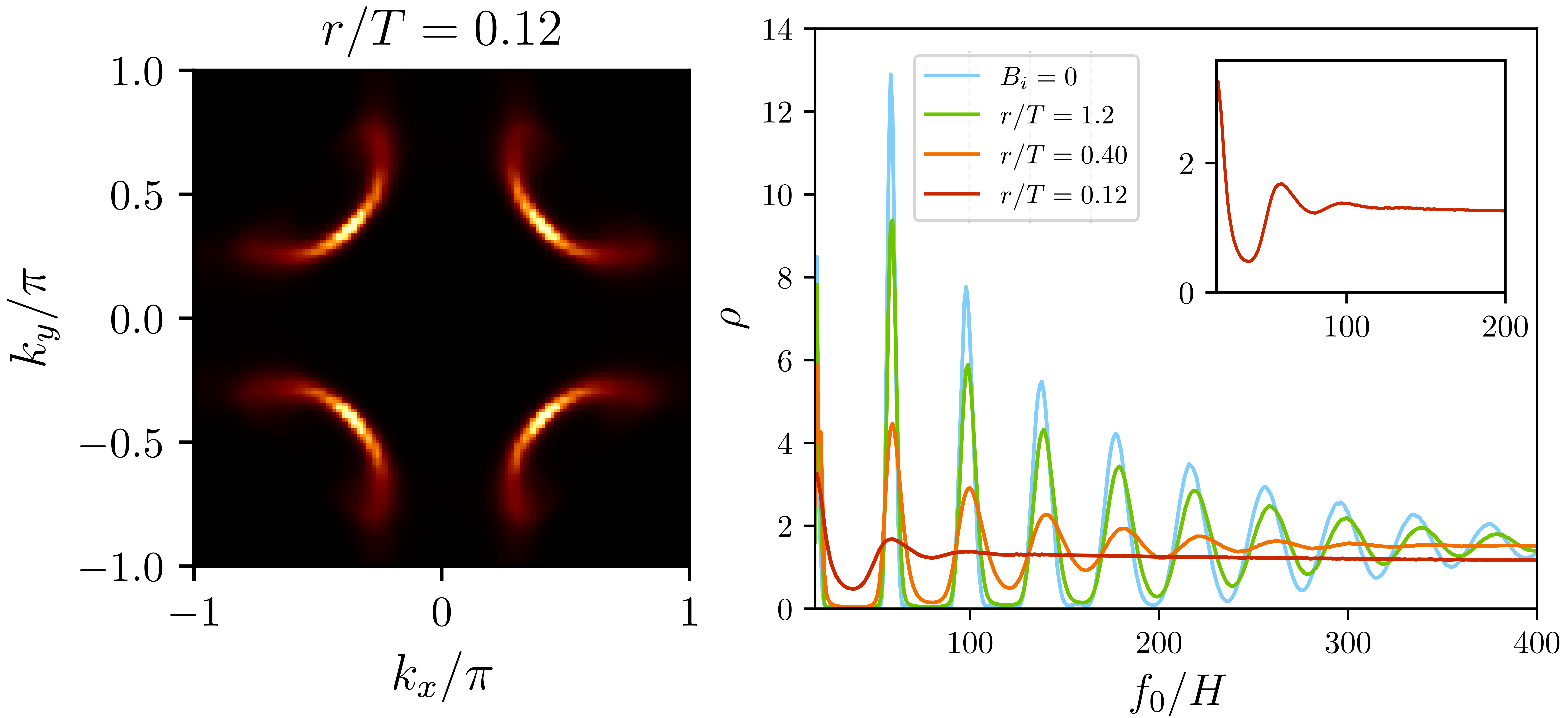
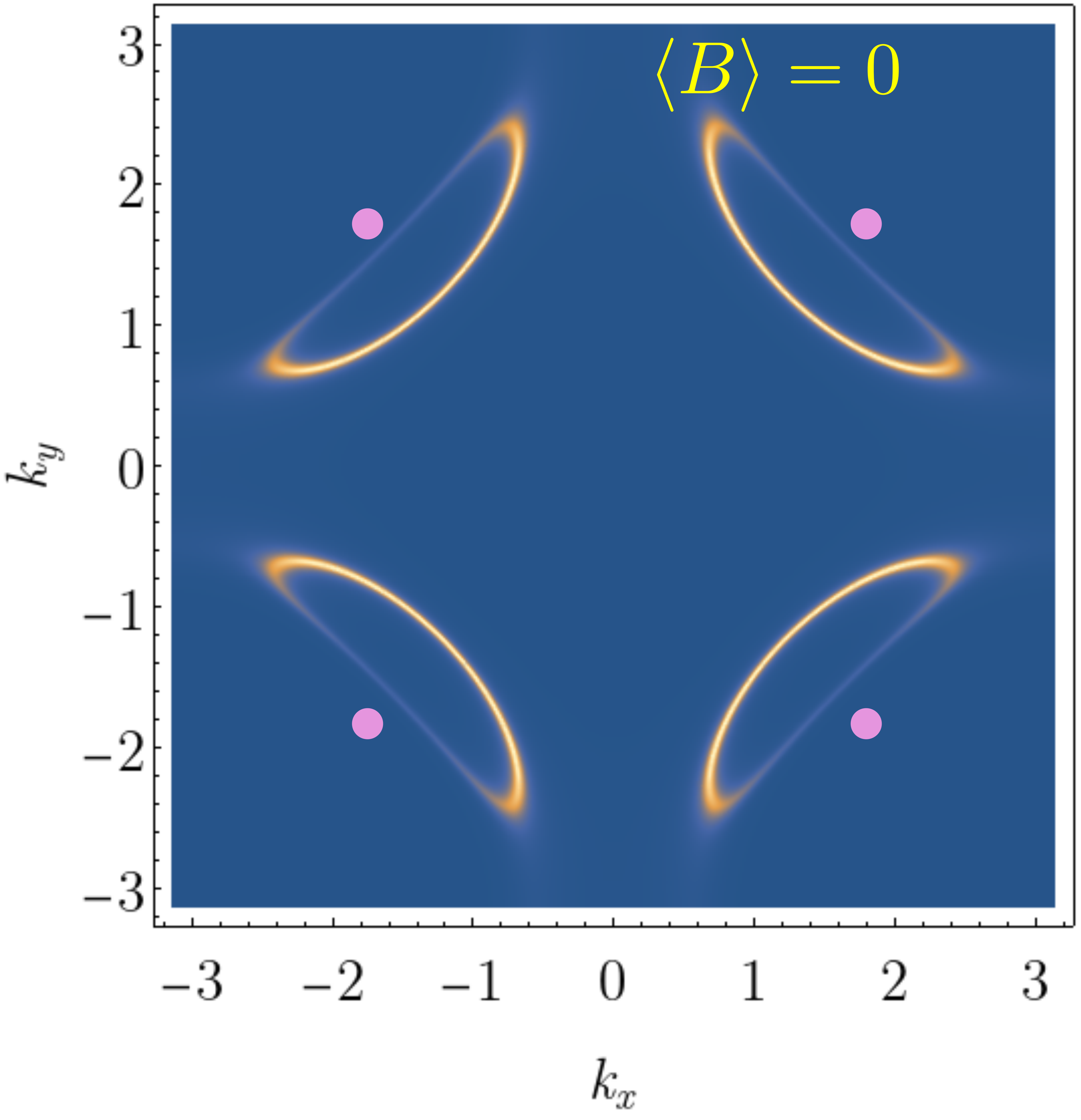
H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336



Thermal SU(2) gauge theory of mixing between holes and spinons mediated by Yukawa couplings to a SU(2) fundamental, charge $+e$ Higgs boson B.

Quantum oscillations survive even when pockets have turned to arcs in photoemission.

FL*

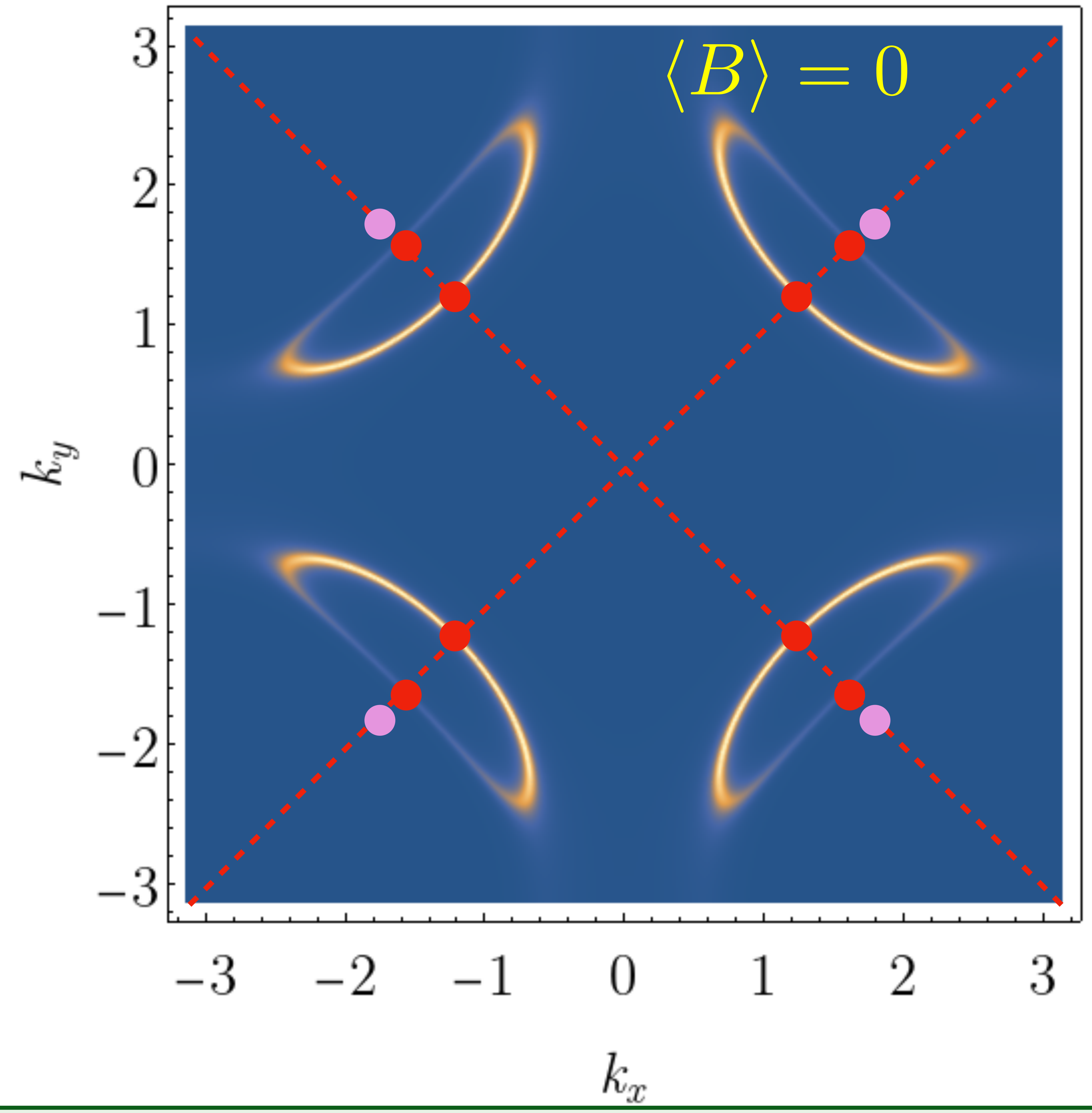


FL* ⇒ d-SC:

Cooper pairing of the Fermi surface?

FL* has 4 electron-like pockets
and 4 nodal spinons
of the π-flux spin liquid

FL*→d-SC*



FL* ⇒ d-SC:

Cooper pairing of the Fermi surface?

$$E_{\boldsymbol{k}} = \left(\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2\right)^{1/2}$$

$$\Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y\right)$$

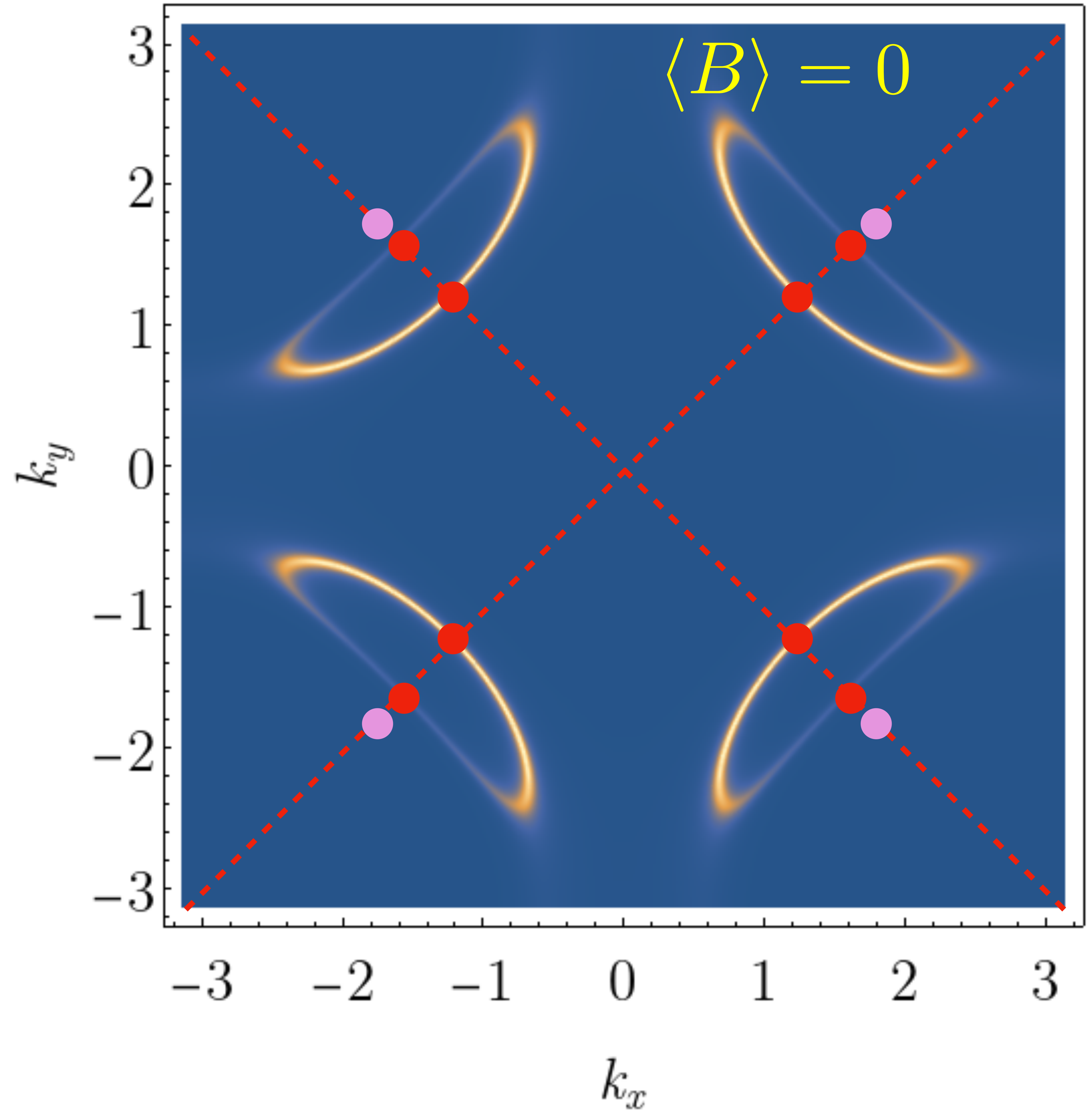
No!

Leads to 8 nodal points of Bogoliubov quasiparticles and 4 nodal spinons of π-flux spin liquid.

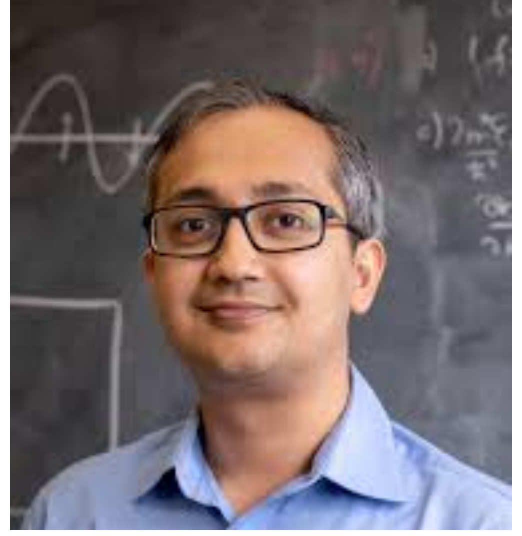
FL* ⇒ d-SC*

BCS mechanism applied to FL* pseudogap leads to non-BCS superconductor!

FL*→d-SC



Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)

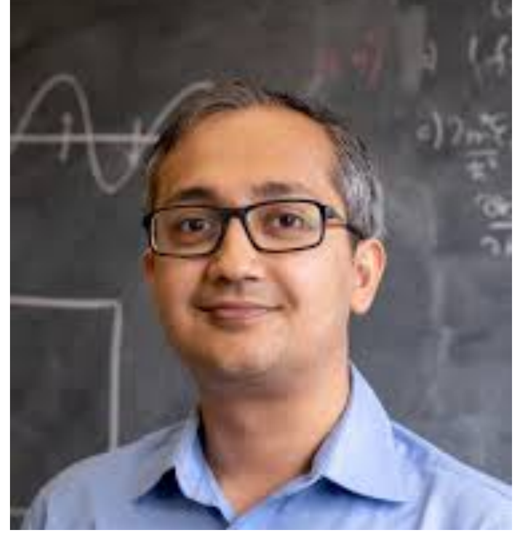


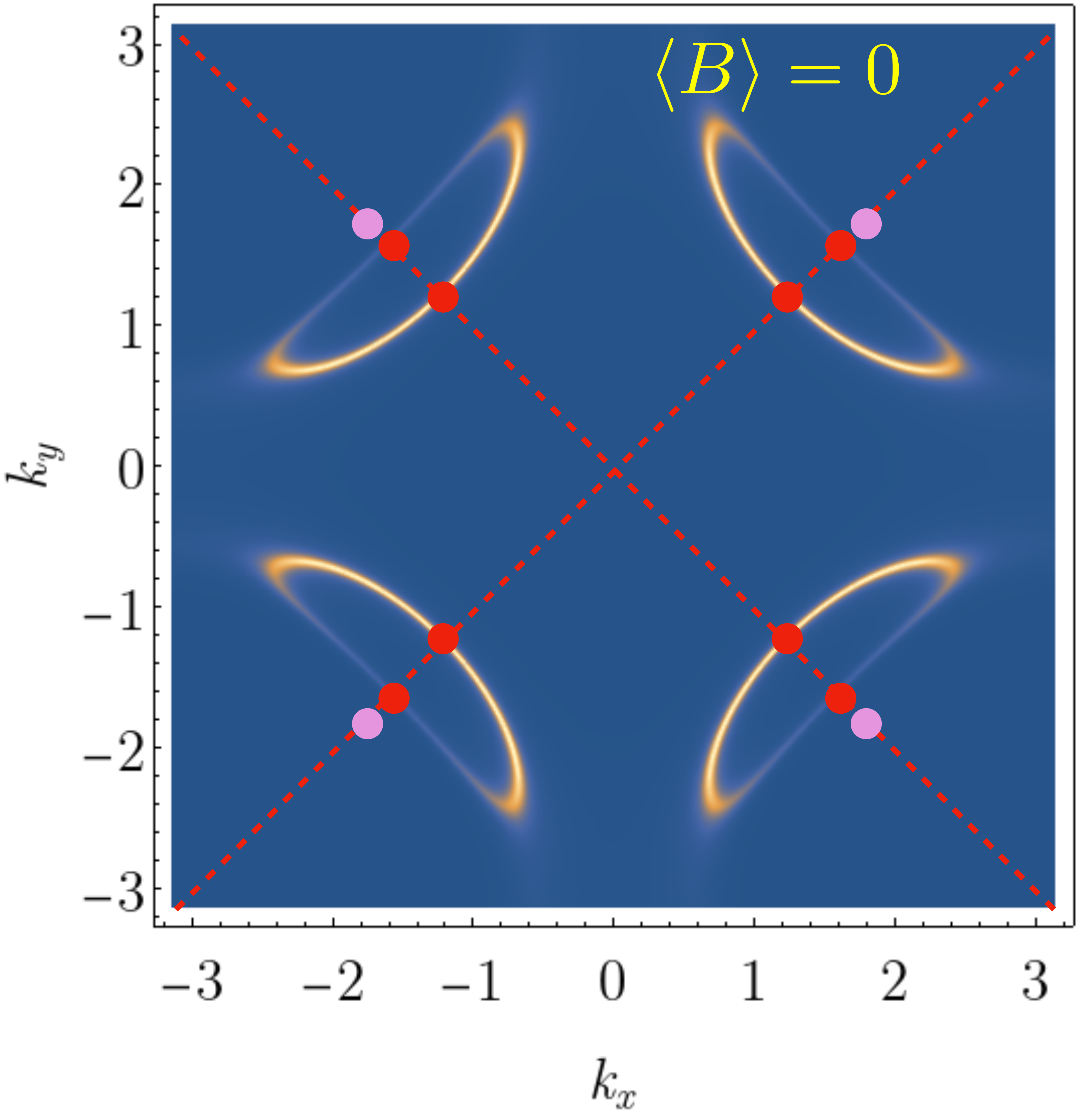
Alternative route to d-wave superconductivity:

Use the pre-existing pairing of the underlying spin liquid and confine the spin liquid!

FL*→d-SC

Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)





Alternative route to d-wave superconductivity:

Confine the π-flux spin-liquid by a condensate of B, $\langle B \rangle \neq 0$ for a suitable Higgs potential $\mathcal{E}_4(B)$. This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

FL*→d-SC

Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
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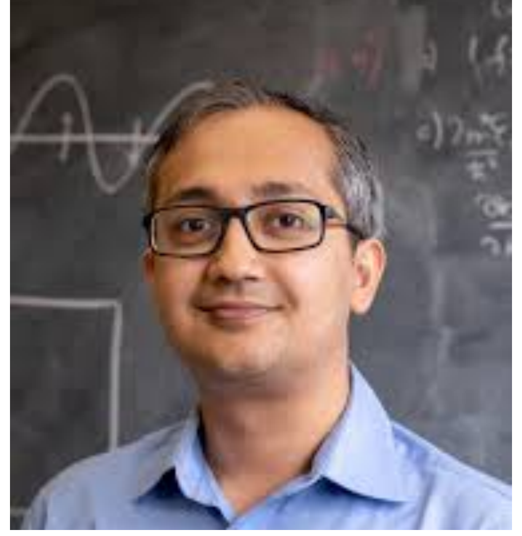

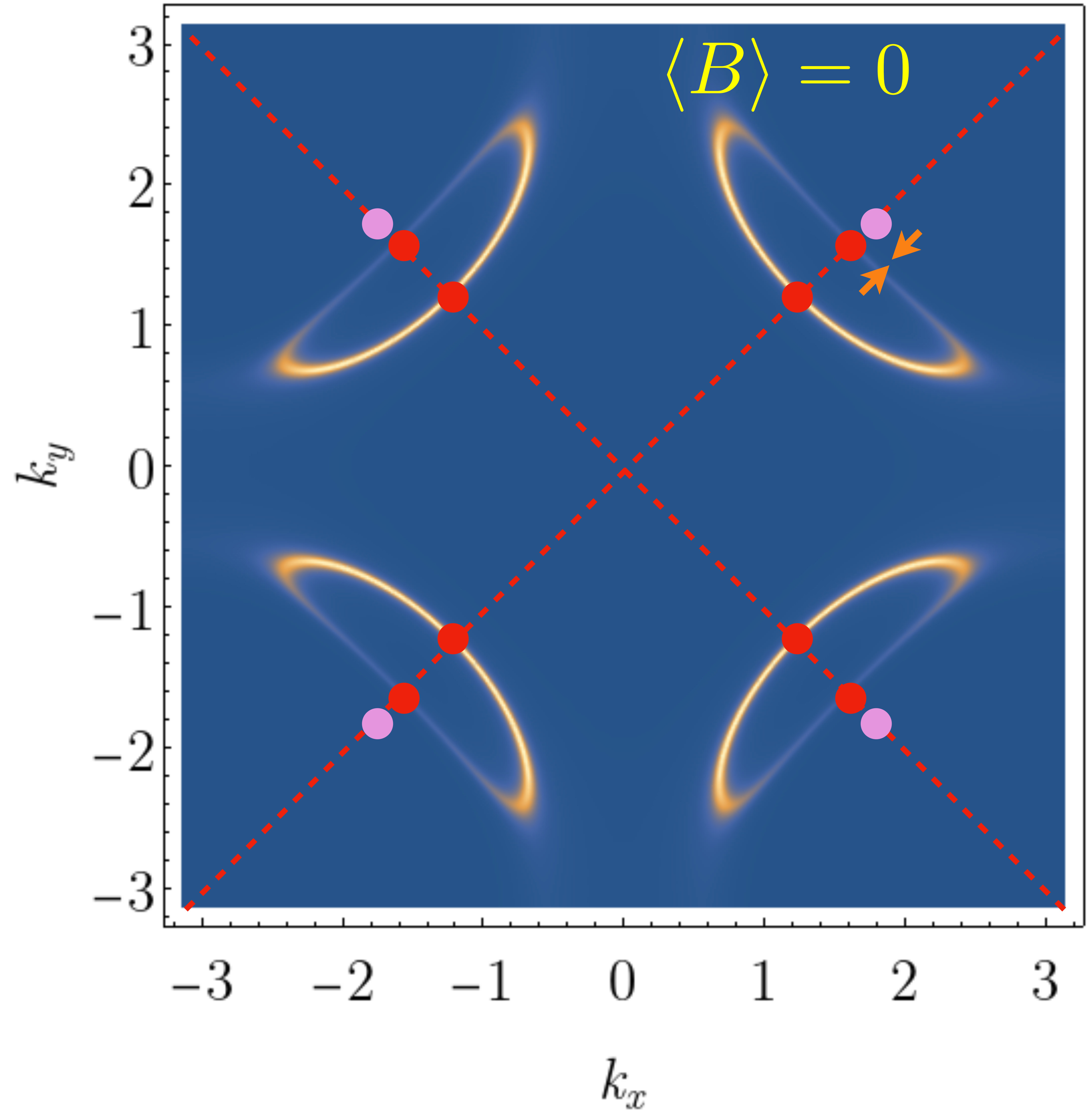

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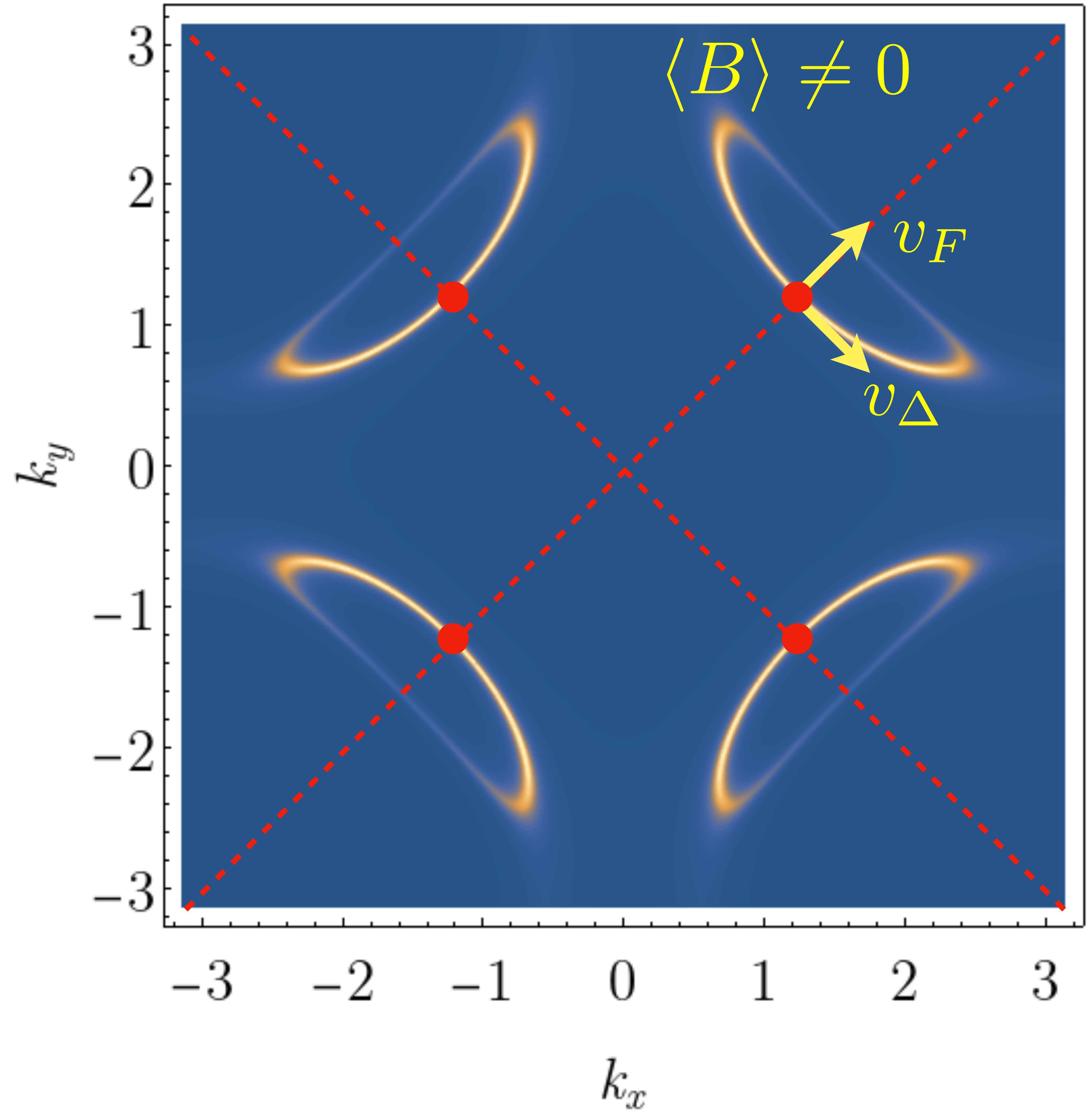
FL*→d-SC



Shubhayu Chatterjee and S. S.,
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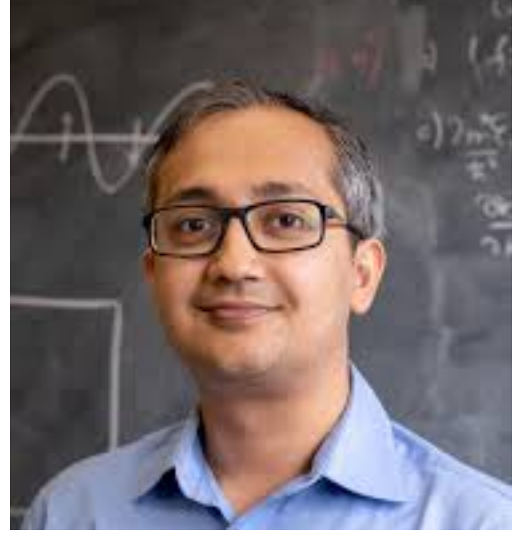
npj Quantum Materials **9**, 4 (2024)



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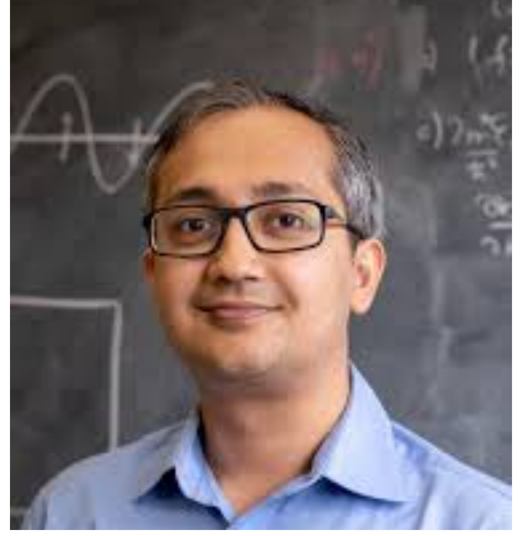
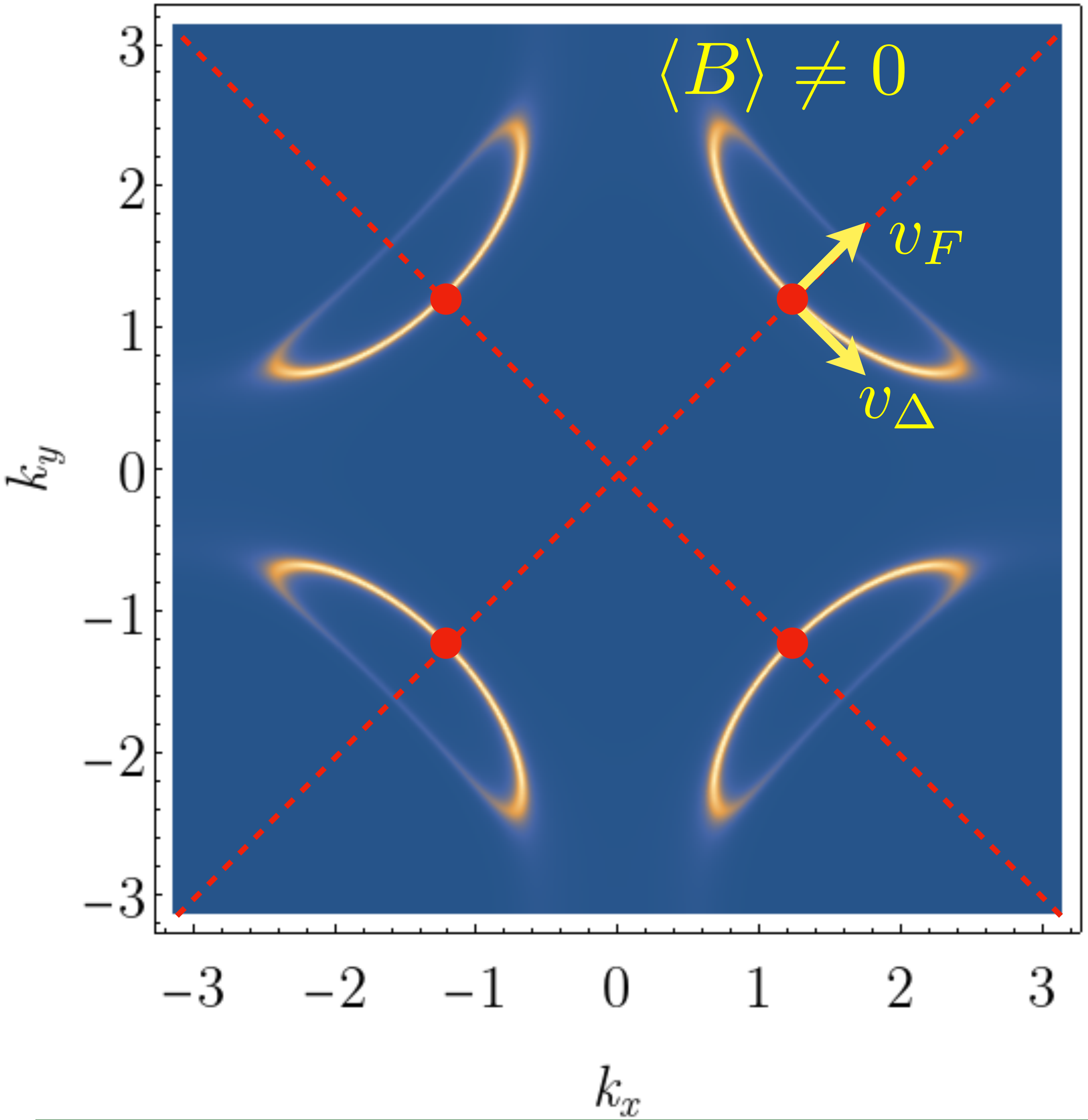
This leads to a d-wave superconductor with 4 nodal points and $v_F \gg v_\Delta$!

Resolves difficulty in early theories of d-SC from spin liquids (Zhang, Gros,Rice, Shiba (1988); Kotliar, Liu (1988)), in which spinons turned into Bogoliubov quasiparticles with $v_F \approx v_\Delta$.

FL* → d-SC

Shubhayu Chatterjee and S. S.,
PRB **94**, 205117 (2016)
Maine Christos and S.S.,
npj Quantum Materials **9**, 4 (2024)





Alternative route to d-wave superconductivity:

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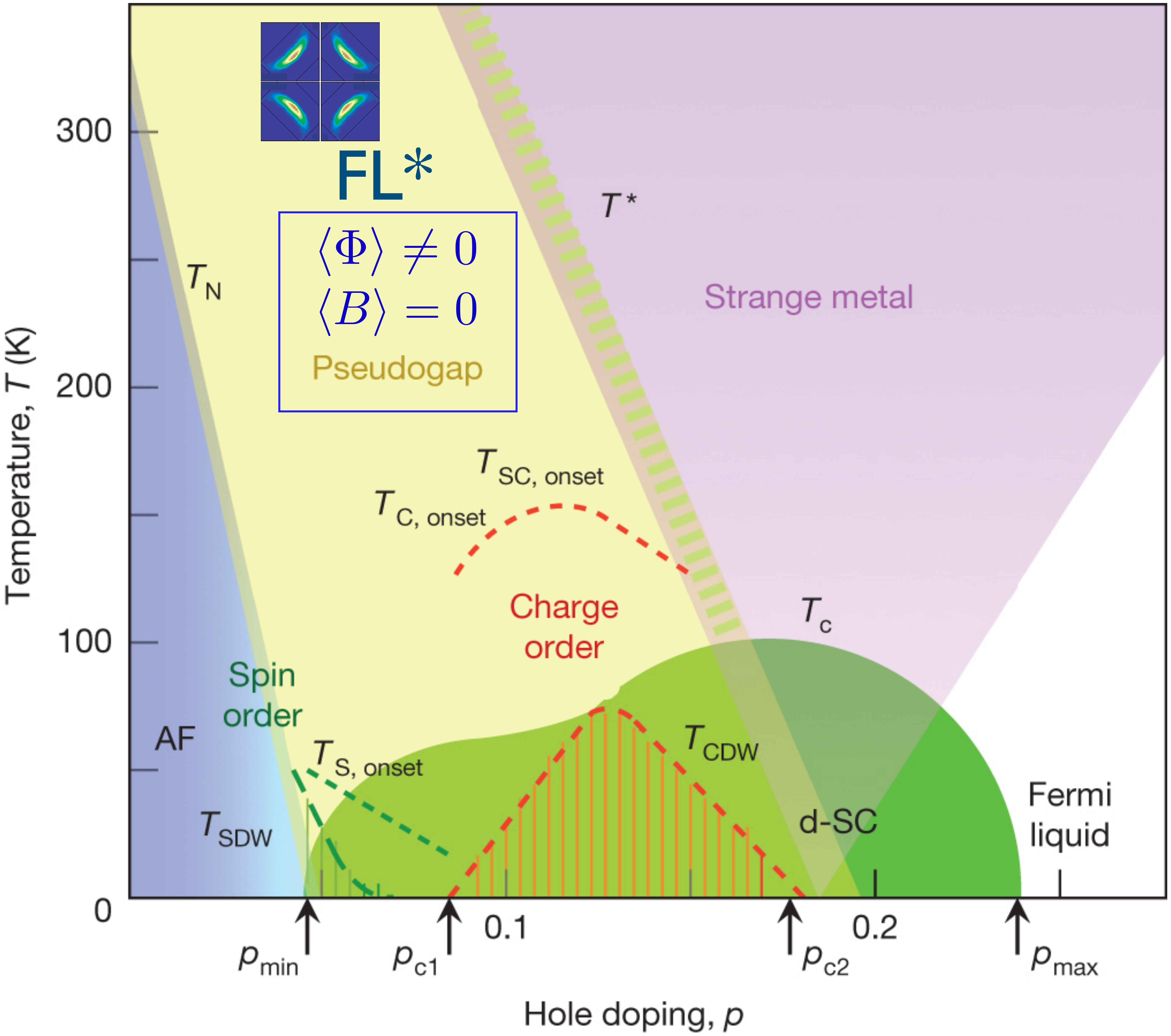
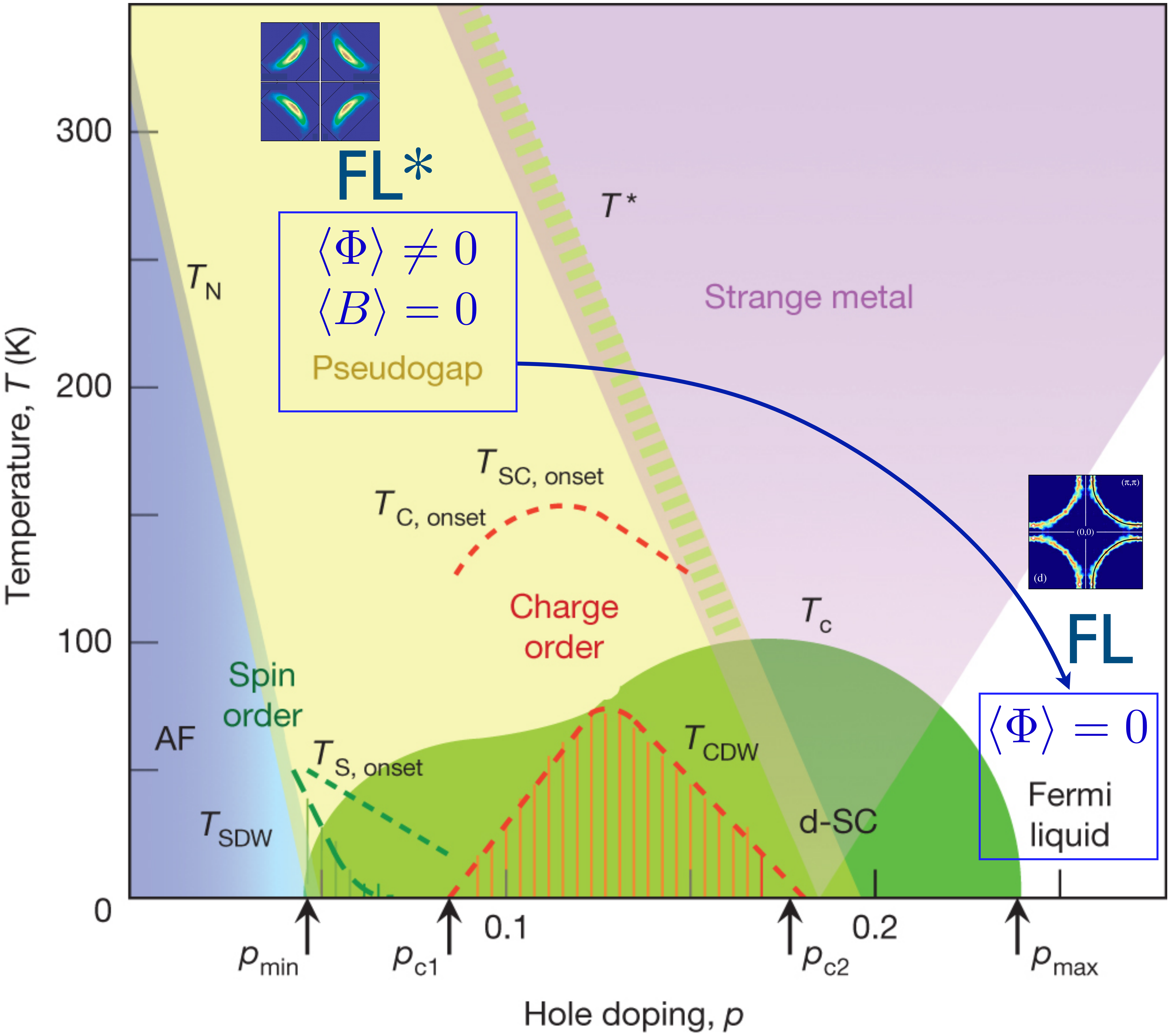
Non-BCS mechanism applied to pseudogap leads to BCS superconductor!



Pseudogap described by FL* with hole pockets of area $p/8$, a *critical quantum spin liquid* with a SU(2) gauge field, massless Dirac spinons, and charge e fundamental Higgs.

FL*
$\langle \Phi \rangle \neq 0$
$\langle B \rangle = 0$
Pseudogap
T^*
T_N
Strange metal
$T_{SC, onset}$
$T_{C, onset}$
Charge order
T_c
Spin order
AF
$T_{S, onset}$
$\langle B \rangle \neq 0$
T_{CDW}
T_{SDW}
d-SC
Fermi liquid
300
200
100
0
0.1
0.2
p_{min}
p_{c1}
p_{c2}
p_{max}
Temperature, T (K)
Hole doping, p
Kondo lattice heavy Fermi liquid when $\langle \Phi \rangle \neq 0$.
Area $(1 + p + 1)/2 = p/2$ (mod 1).
Small Fermi pockets!
e^+
c_σ
J_K
$\boldsymbol{S}_1$
Φ
B
$J_\perp$
$\boldsymbol{S}_2$
π-flux spin liquid with SU(2) gauge field U

FL*
$\langle \Phi \rangle \neq 0$
$\langle B \rangle = 0$
Pseudogap
T^*
T_N
Strange metal
$T_{SC, onset}$
$T_{C, onset}$
Charge order
Spin order
AF
$T_{S, onset}$
T_{CDW}
T_{SDW}
T_c
d-SC
FL
$\langle \Phi \rangle = 0$
Fermi liquid
Temperature, T (K)
300
200
100
0
0.1
0.2
p_{min}
p_{c1}
p_{c2}
p_{max}
Hole doping, p
Kondo lattice heavy Fermi liquid when $\langle \Phi \rangle \neq 0$.
Area $(1 + p + 1)/2 = p/2$ (mod 1).
Small Fermi pockets!
e^+
c_σ
J_K
$\boldsymbol{S}_1$
Φ
B
$J_\perp$
$\boldsymbol{S}_2$
π-flux spin liquid with SU(2) gauge field U



Quantum-criticality of FL*-FL transition described by 2d-Yukawa-SYK model with local, spatially random interactions.

Obeys equations similar to SYK in momentum and frequency space

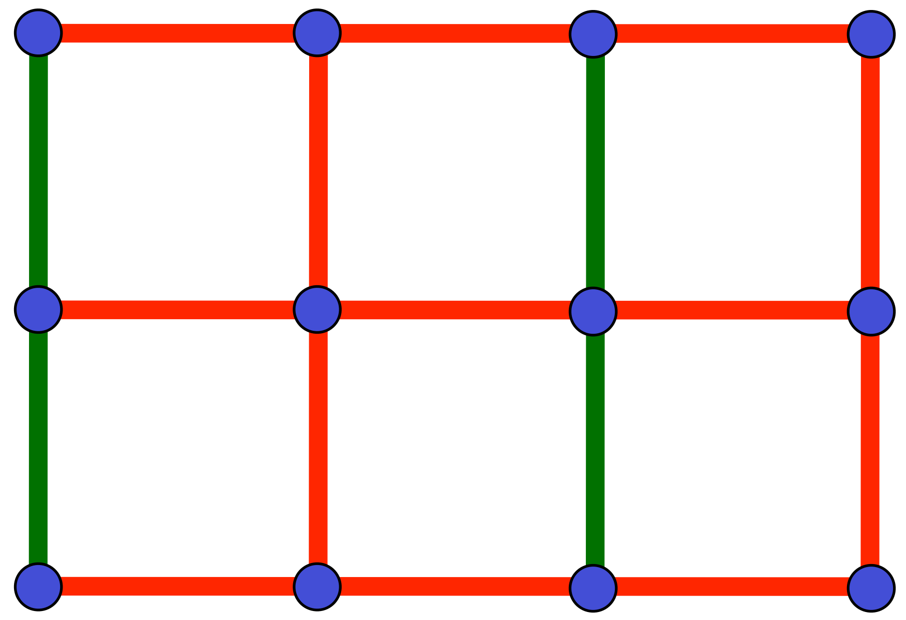
Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. Sachdev, *Science* **381**, 790 (2023)
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

Critical spin liquid
(a SU(2) gauge theory with
massless Dirac and Higgs matter)

and a **critical charge liquid**
(the SYK model)

which are gapless and
strongly interacting, with
no particle-like excitations

Adding charge fluctuations to the π-flux spin liquid



$$T_x T_y = -T_y T_x$$

- Introduce a charge e, SU(2) fundamental boson B_i such that the composite of B_i and Ψ_i is an electron. The projective symmetries require

$$\mathcal{E}_2[B,U] = r \sum_{i} B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right)$$

M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023)

Adding charge fluctuations to the π-flux spin liquid

$$
\mathcal{E}_4[B,U] = \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} |\Delta_{\boldsymbol{i}\boldsymbol{j}}|^2 + J_1 \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} Q_{\boldsymbol{i}\boldsymbol{j}}^2 + K_1 \sum_{\langle \boldsymbol{i}\boldsymbol{j}\rangle} J_{\boldsymbol{i}\boldsymbol{j}}^2
$$

$$
+ V_{11} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}+\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}-\hat{\boldsymbol{y}}}\right) + V_{22} \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}+2\hat{\boldsymbol{y}}} + \rho_{\boldsymbol{i}+2\hat{\boldsymbol{x}}-2\hat{\boldsymbol{y}}}\right)
$$

site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim Q_{\boldsymbol{i}\boldsymbol{j}} = Q_{\boldsymbol{j}\boldsymbol{i}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{i}\boldsymbol{j}} U_{\boldsymbol{i}\boldsymbol{j}} B_{\boldsymbol{j}}\right)$

bond current: $i \left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim J_{\boldsymbol{i}\boldsymbol{j}} = -J_{\boldsymbol{j}\boldsymbol{i}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{i}\boldsymbol{j}} U_{\boldsymbol{i}\boldsymbol{j}} B_{\boldsymbol{j}}\right)$

Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{\boldsymbol{i}\boldsymbol{j}} = \Delta_{\boldsymbol{j}\boldsymbol{i}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{i}\boldsymbol{j}} U_{\boldsymbol{i}\boldsymbol{j}} B_{b\boldsymbol{j}}\,.$

M. Christos, Zhu-Xi Luo, H. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023)