Quantum spin liquids and the phases of the cuprates

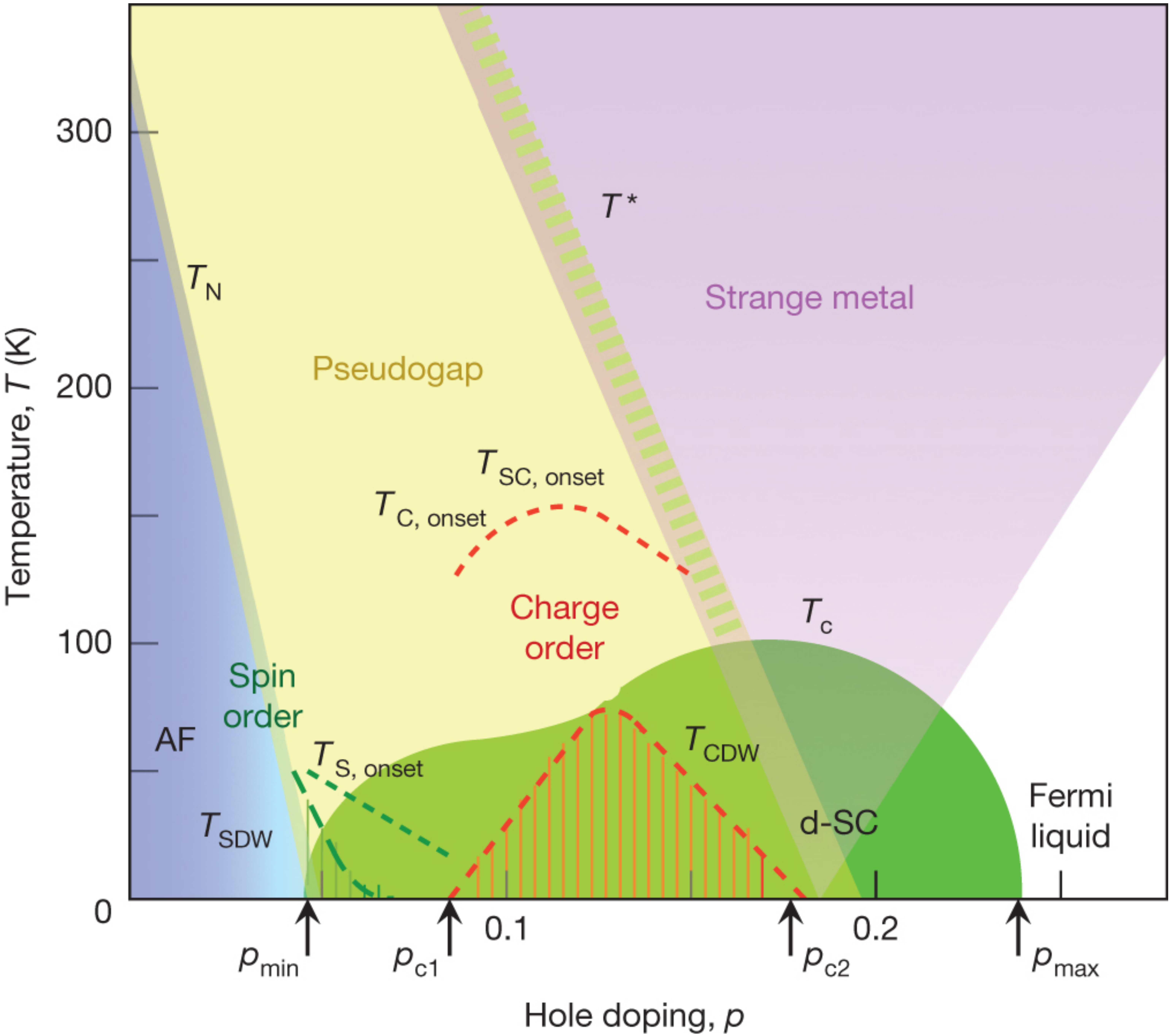
EPiQS Investigator Symposium
Rancho Bernado Inn, San Diego CA
August 3, 2023
Subir Sachdev

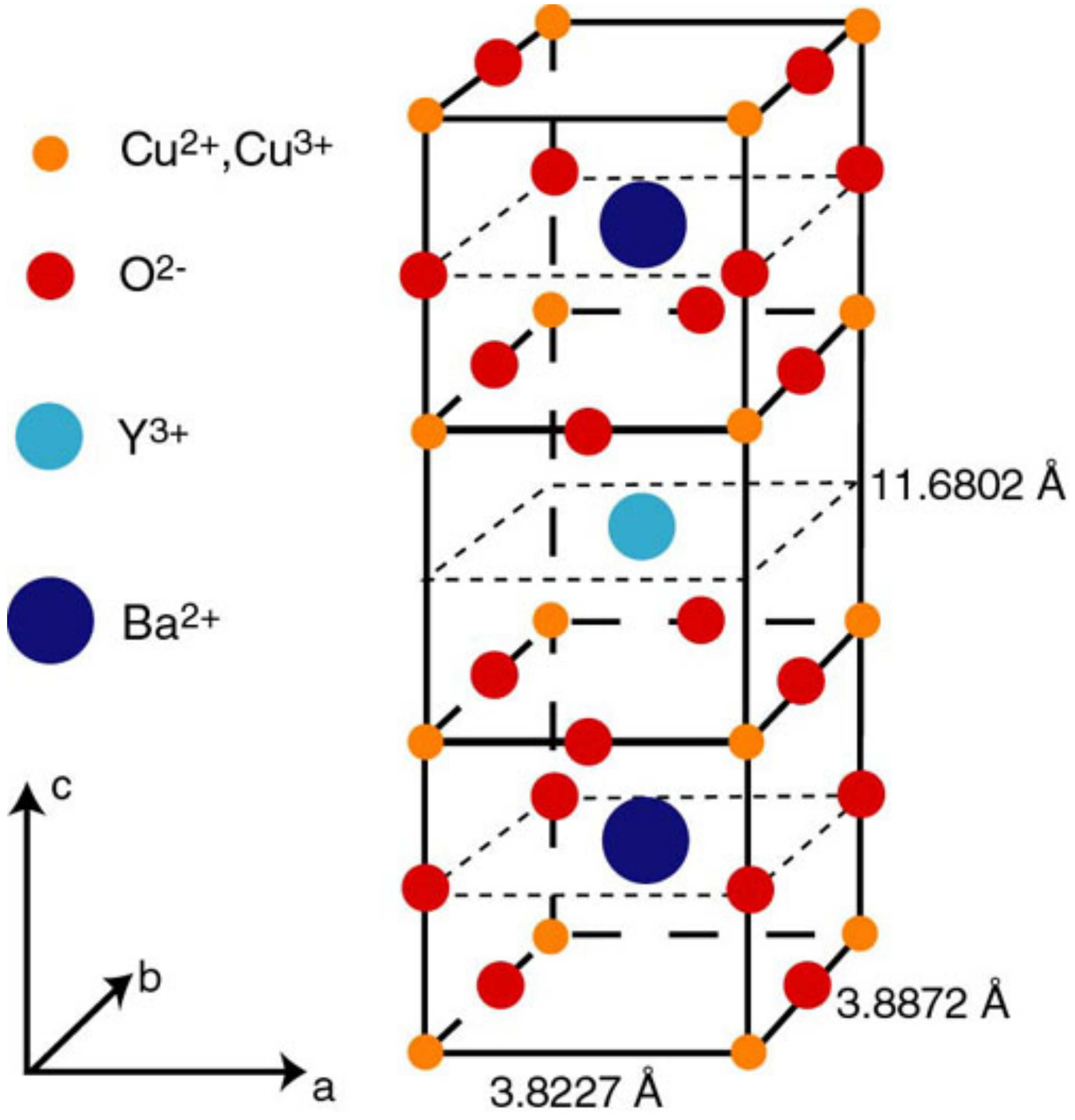
Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang, Mathias Scheurer, and S. S., PNAS **120**, e2302701120 (2023)
Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi, and S.S., PRB **108**, 045123 (2023)
Maine Christos and S.S., to appear

Talk online: sachdev.physics.harvard.edu

PHYSICS
HARVARD

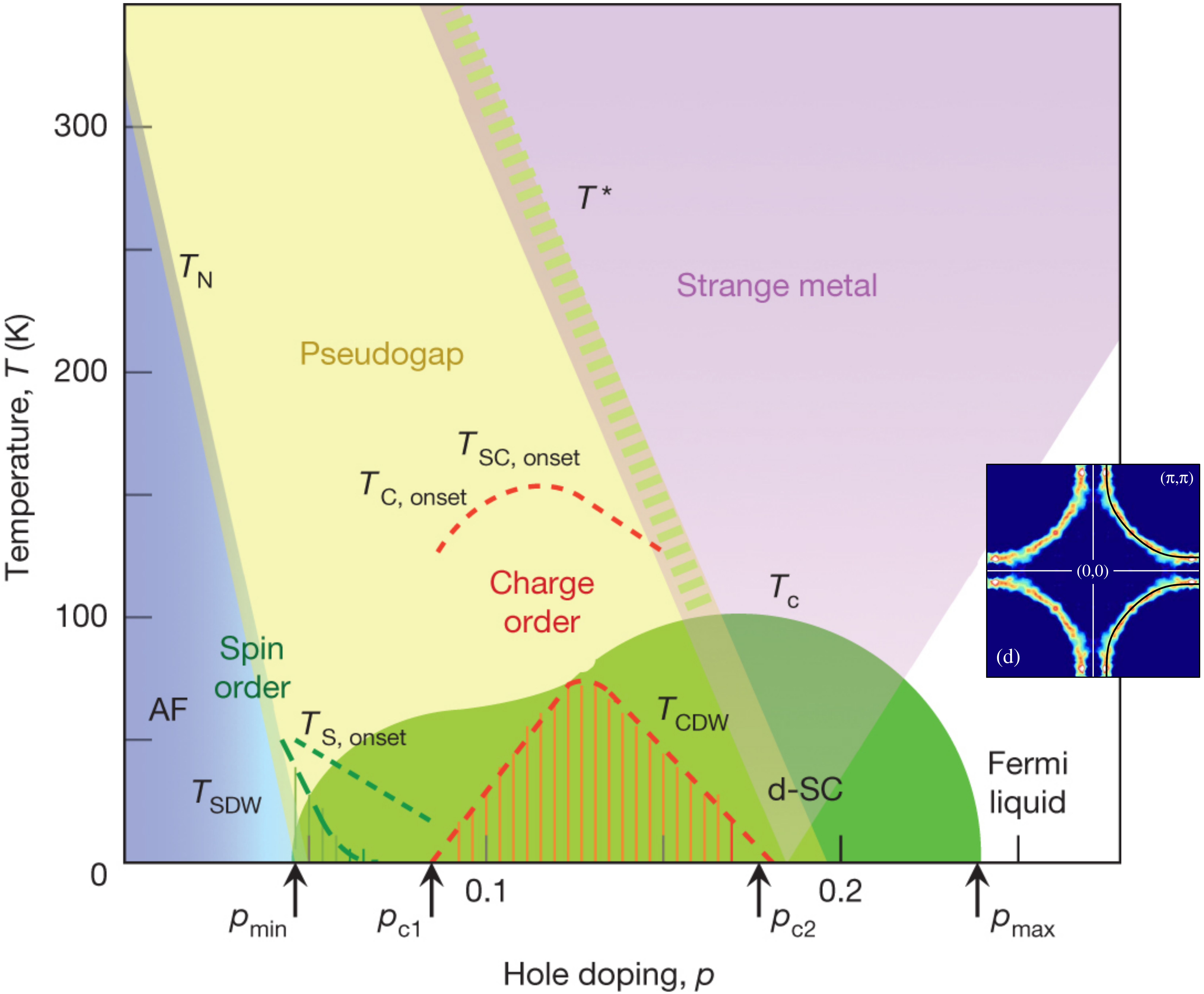
B Keimer, S.A. Kivelson, M.R. Norman, S. Uchida, J. Zaanen, Nature **518**, 179 (2015)





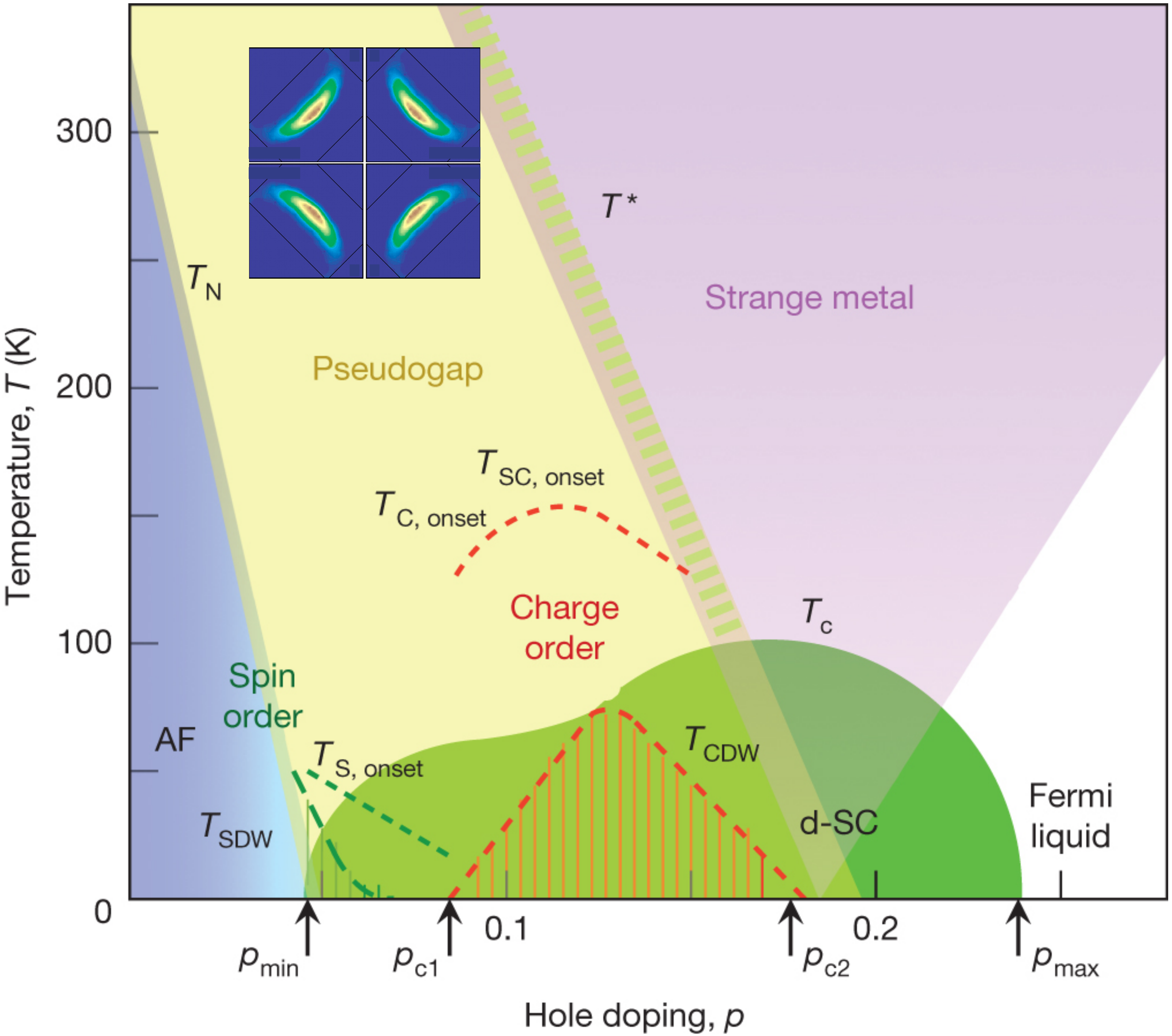
$YBa_2Cu_3O_{6+x}$

B Keimer, S.A. Kivelson, M.R. Norman, S. Uchida, J. Zaanen, Nature **518**, 179 (2015)



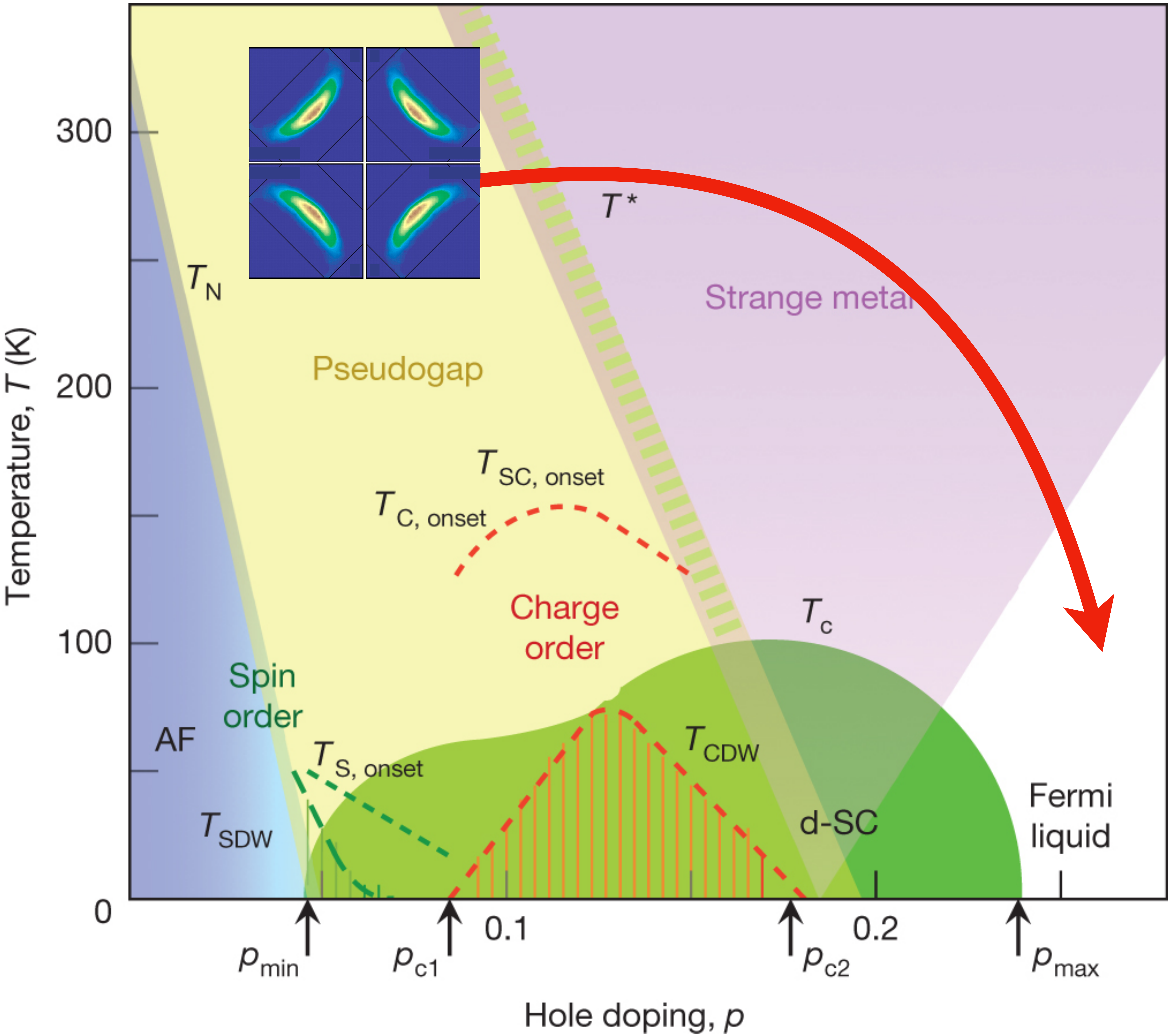
Fermi liquid in the overdoped metal

B Keimer, S.A. Kivelson, M.R. Norman, S. Uchida, J. Zaanen, Nature **518**, 179 (2015)



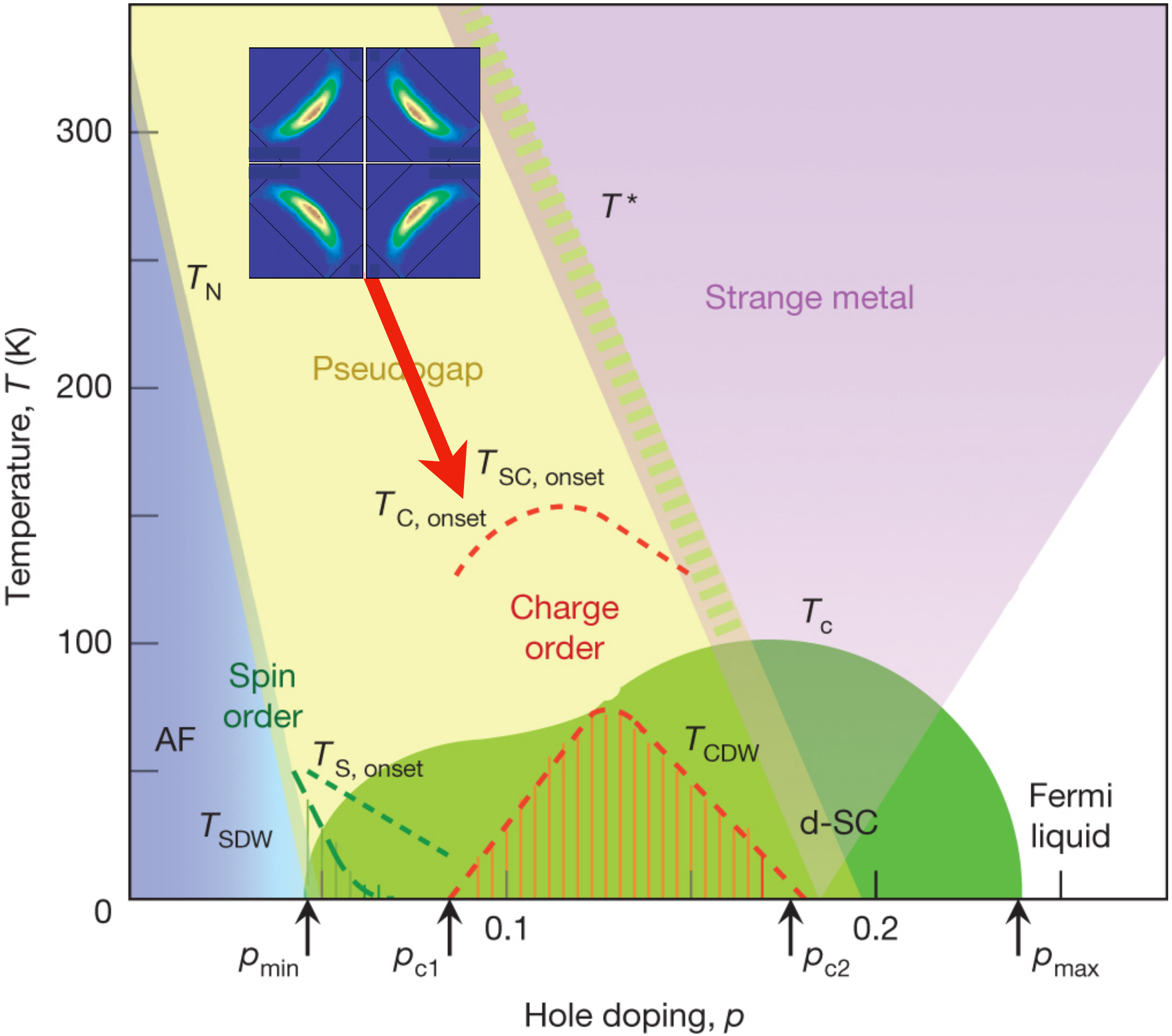
Theory for "pseudogap metal" with "Fermi arcs"?

B Keimer, S.A. Kivelson, M.R. Norman, S. Uchida, J. Zaanen, Nature **518**, 179 (2015)



Strange metal from quantum criticality between pseudogap and Fermi liquid?

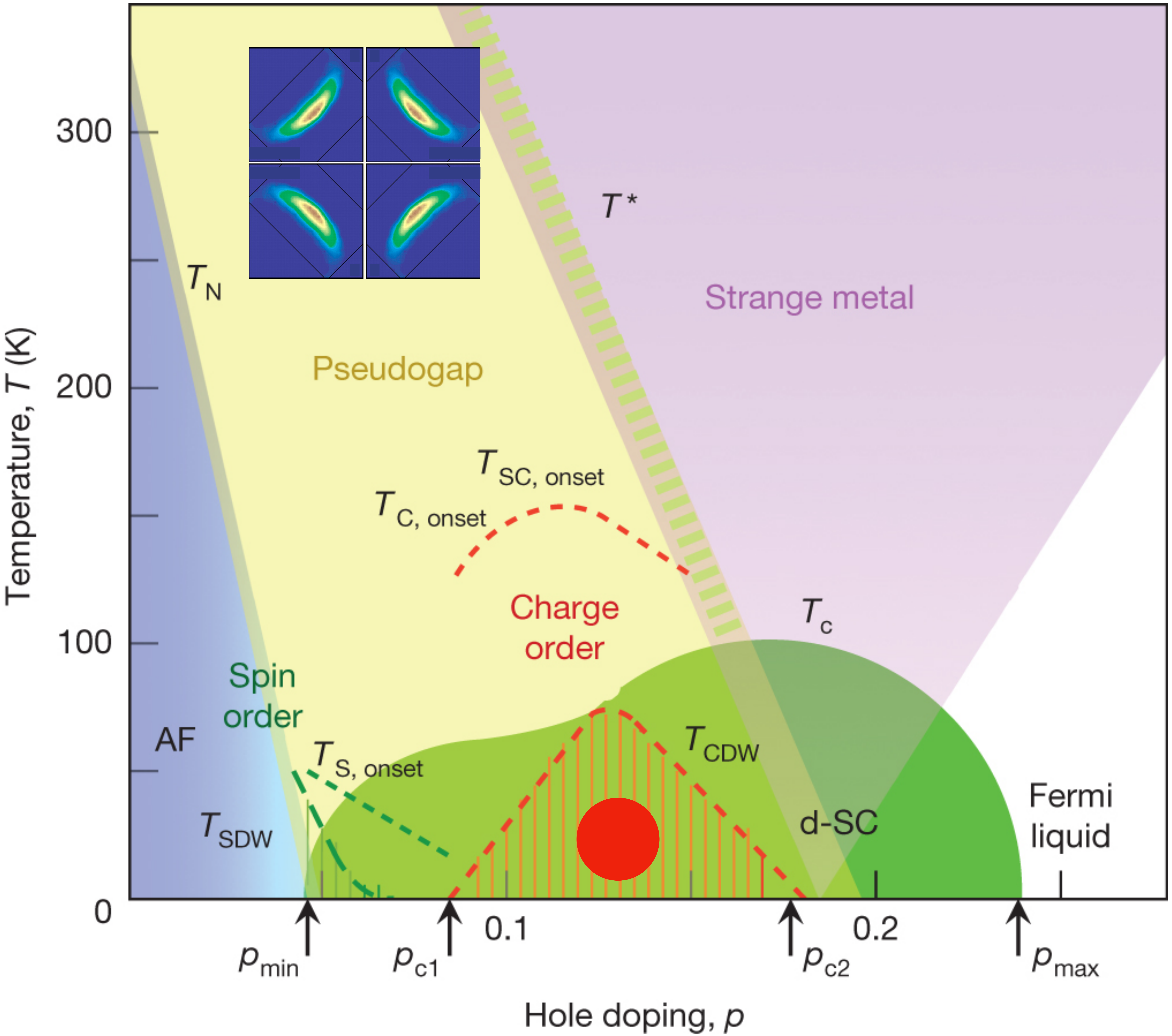
B Keimer, S.A. Kivelson, M.R. Norman, S. Uchida, J. Zaanen, Nature **518**, 179 (2015)



A theory for the onset of charge order and
d-wave superconductivity from the pseudogap metal.

Why are T_c and $T_{\rm CDW}$ about the same?

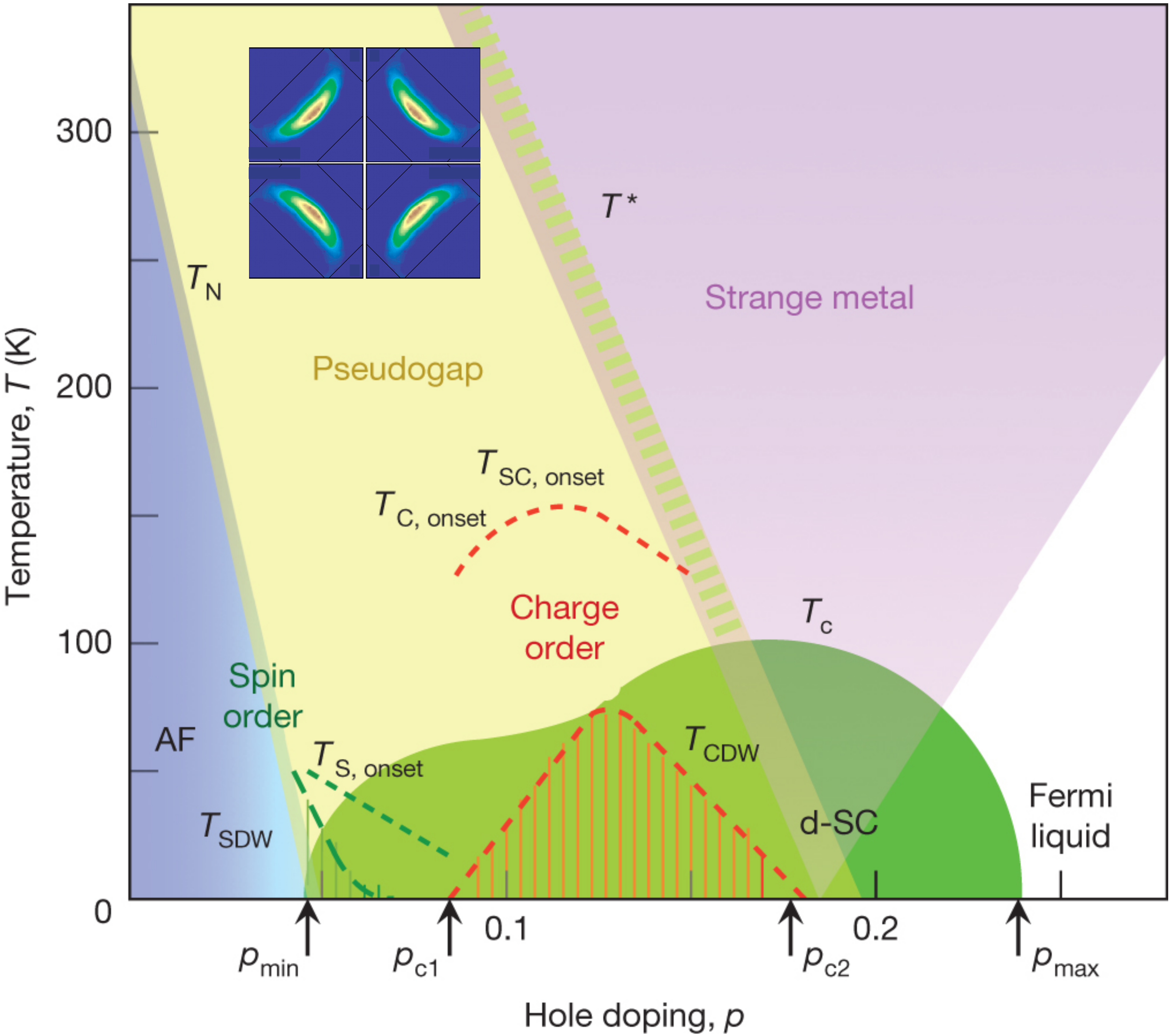
B Keimer, S.A. Kivelson, M.R. Norman, S. Uchida, J. Zaanen, Nature **518**, 179 (2015)



Quantum oscillations in the CDW phase at low T show only a single electron pocket.

This cannot be obtained as the CDW instability of a Fermi liquid

B Keimer, S.A. Kivelson, M.R. Norman, S. Uchida, J. Zaanen, Nature **518**, 179 (2015)



Build a theory for the phase diagram from a theory of the pseudogap metal

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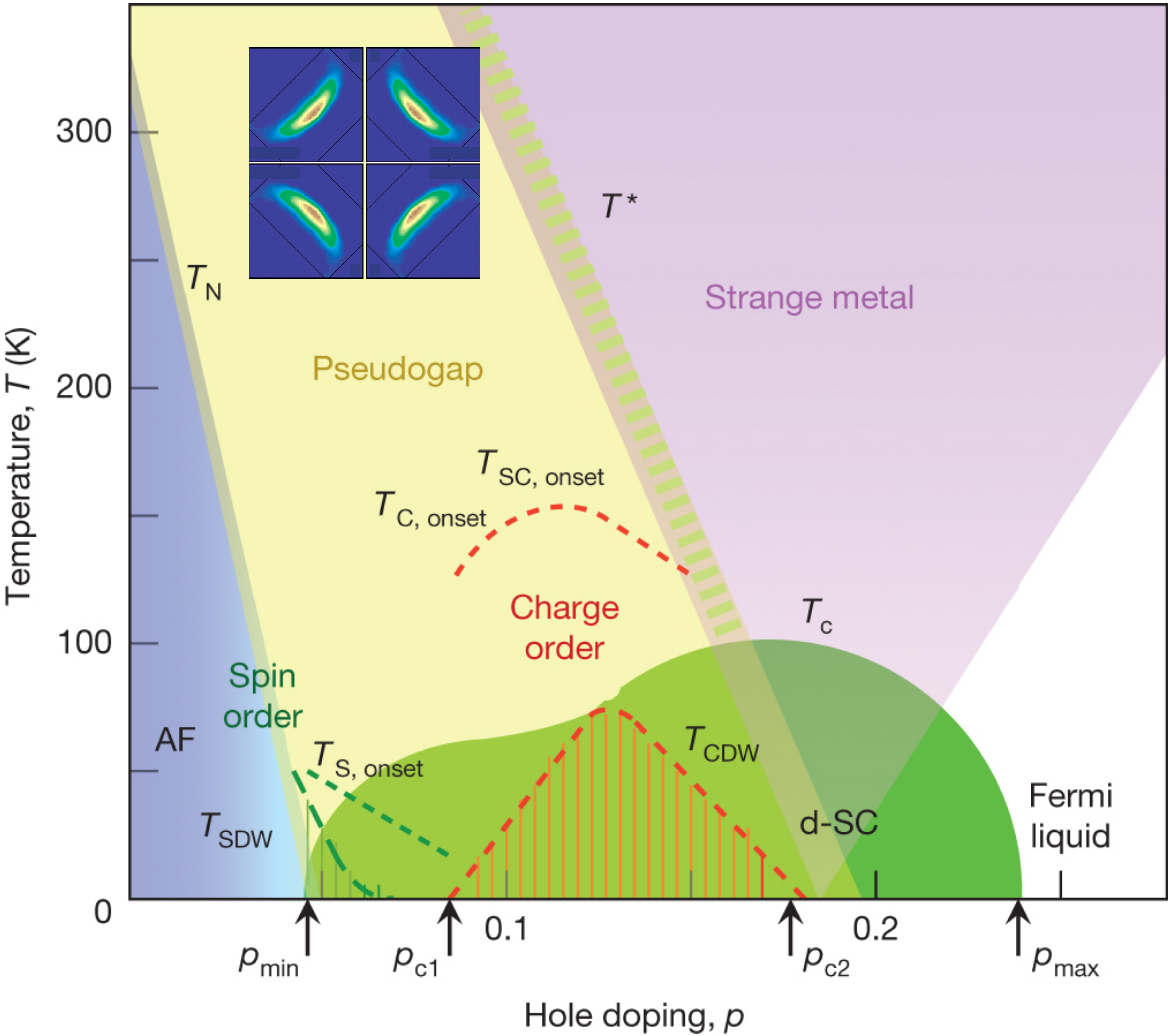
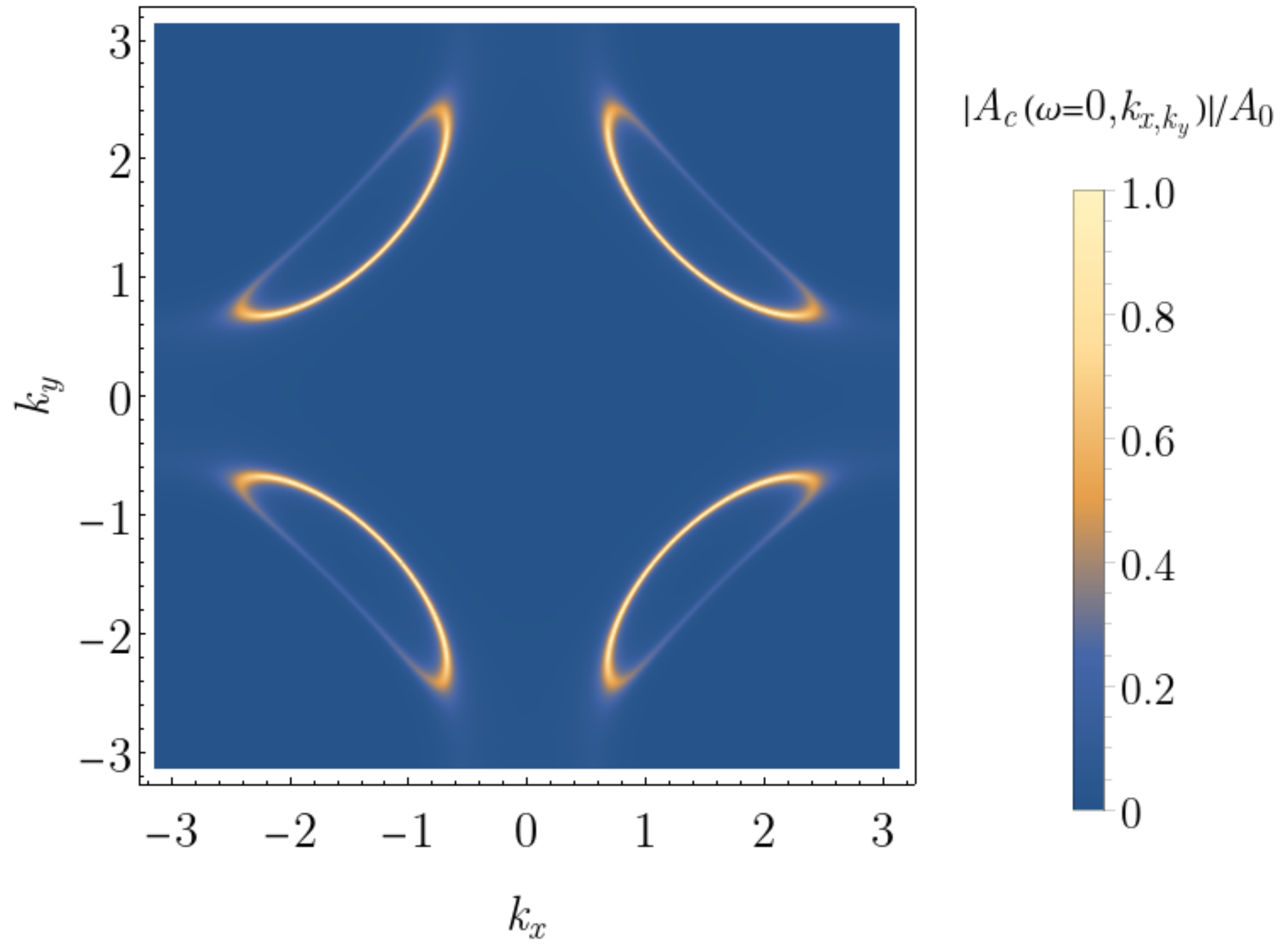




E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

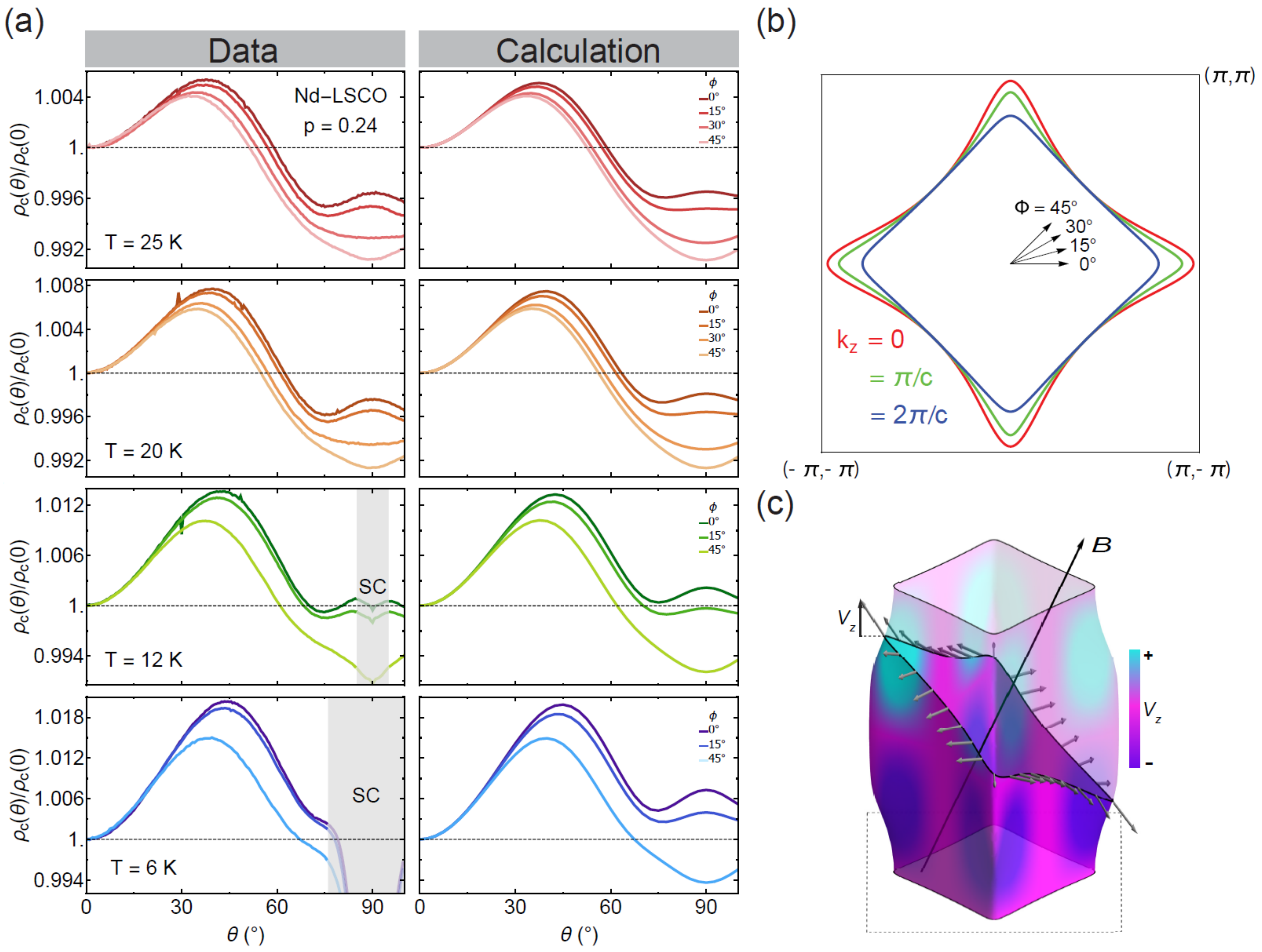
Hole pocket Fermi surfaces of size p with charge e, spin-$1/2$ quasiparticles

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006).
T. D. Stanescu and G. Kotliar, PRB **74**, 125110 (2006).
C. Berthod, T. Giamarchi, S. Biermann, and A. Georges, PRL **97**, 136401 (2006).
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009).
J. Skolimowski and M. Fabrizio, PRB **106**, 045109 (2022).
Jinchao Zhao, Gabriele La Nave, Philip Phillips, arXiv:2304.04787.

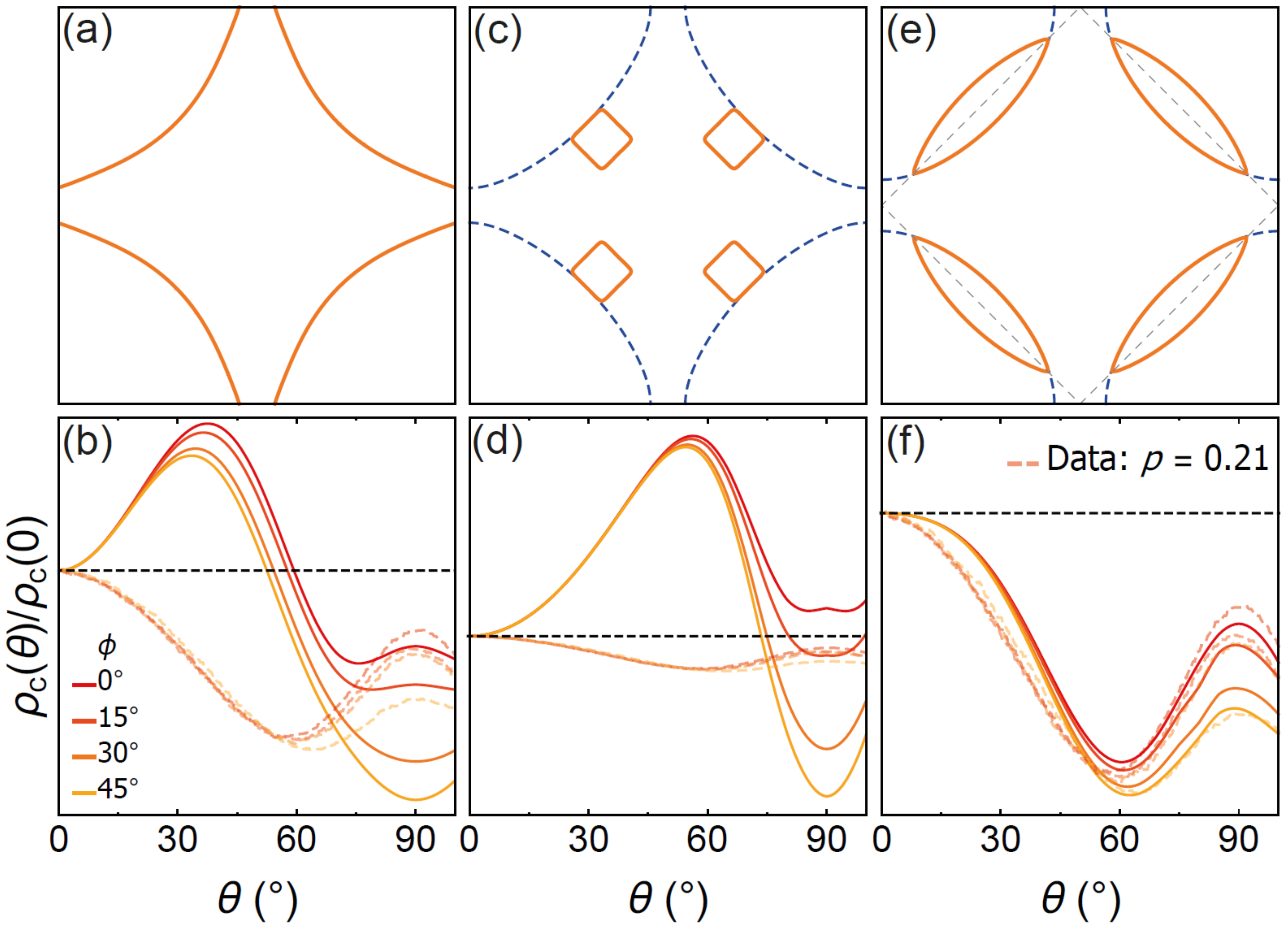
Green function zeros….

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei,
Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, Nature Physics **18**, 558 (2022)



$p > p_c$ Large Fermi surface



$p < p_c$ Reconstructed Fermi surface

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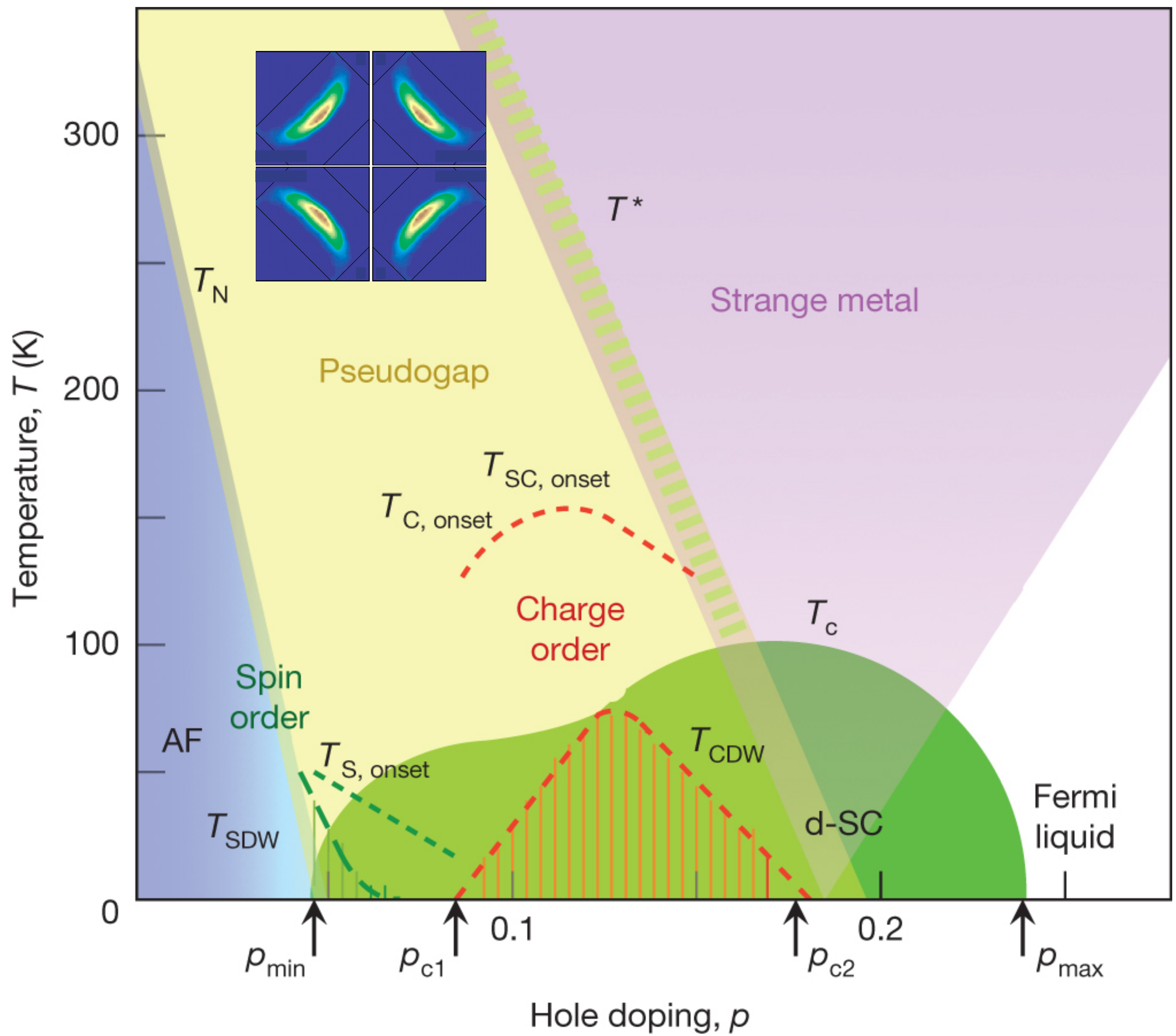

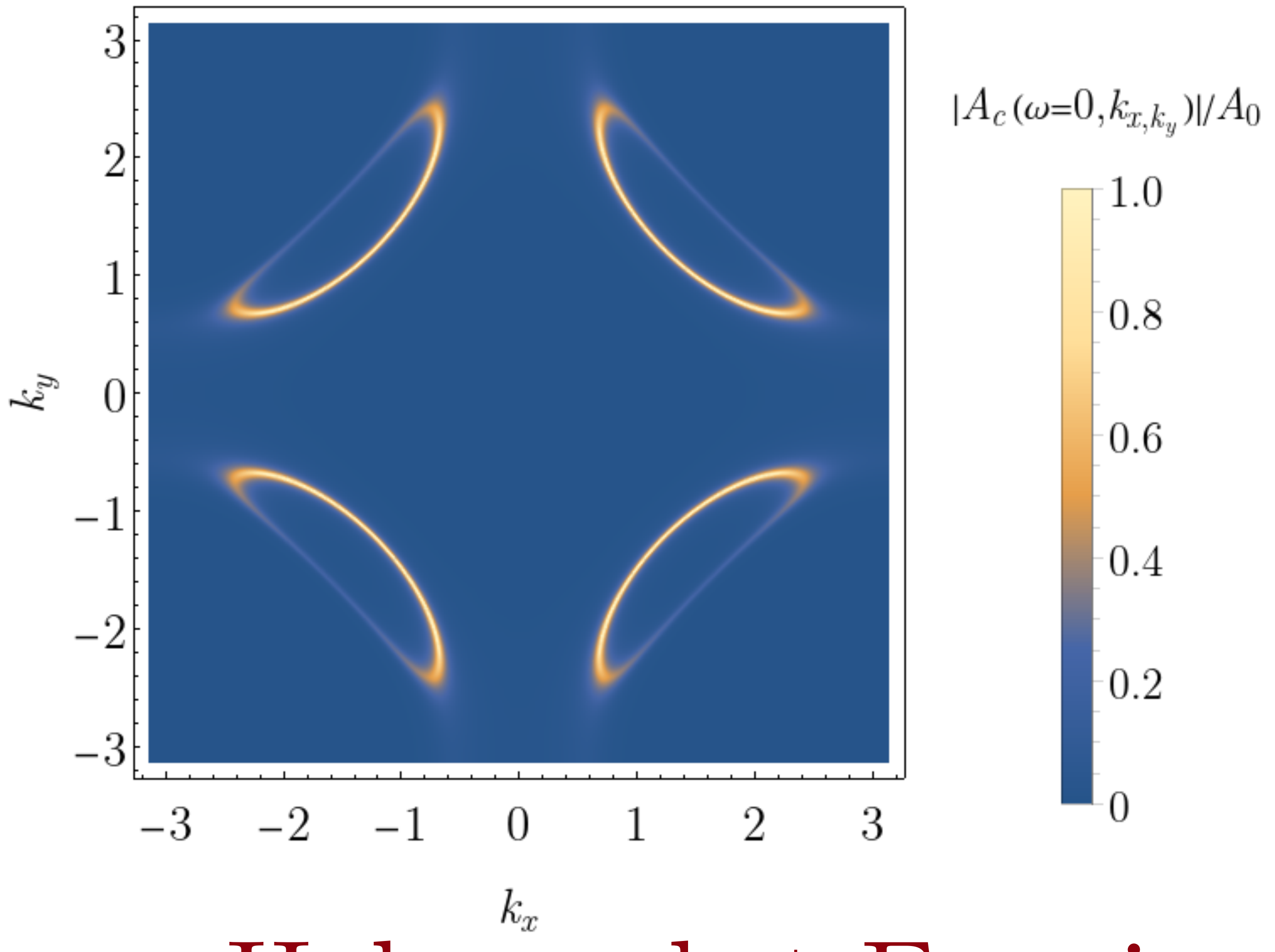

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075146 (2022)

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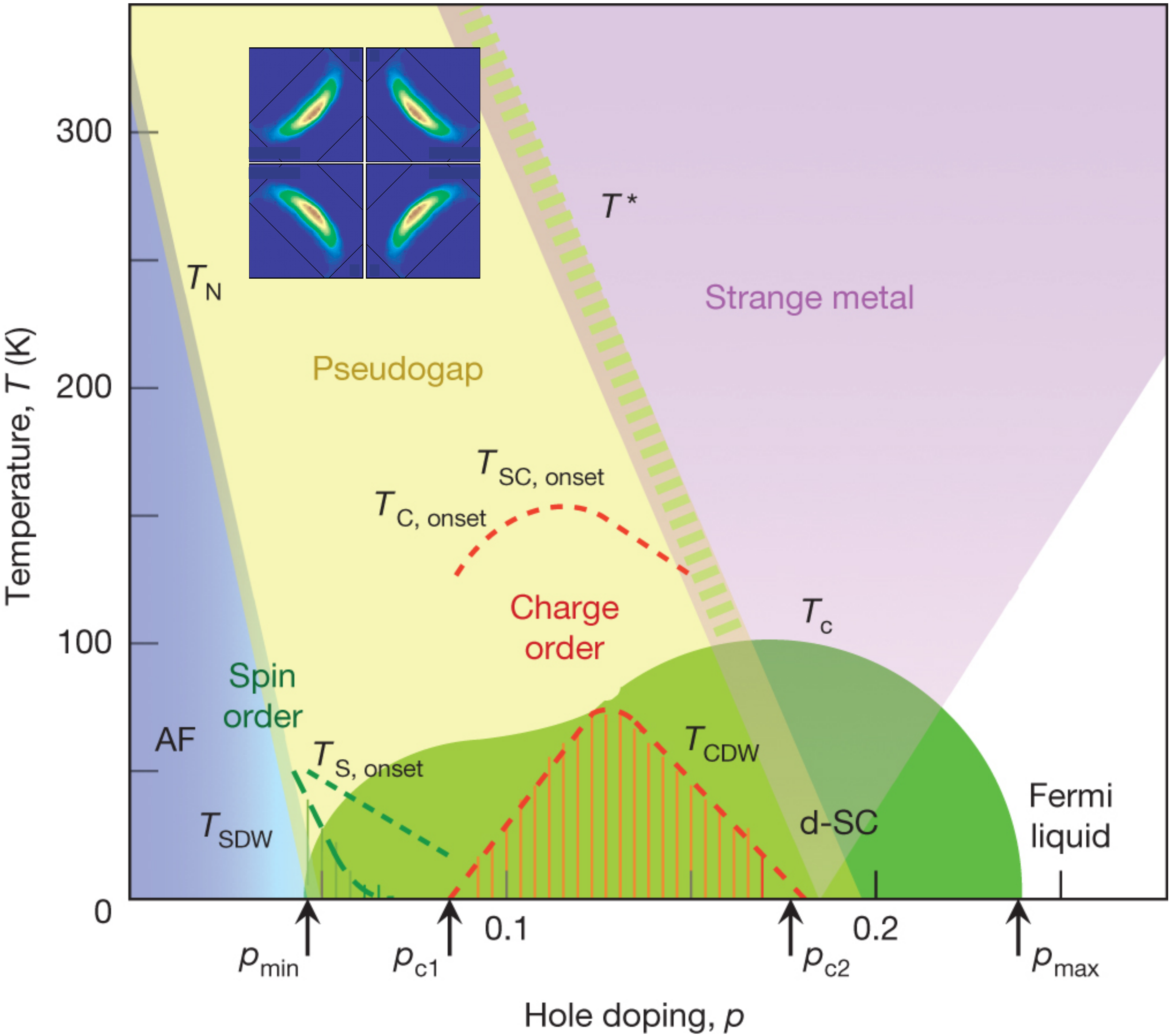

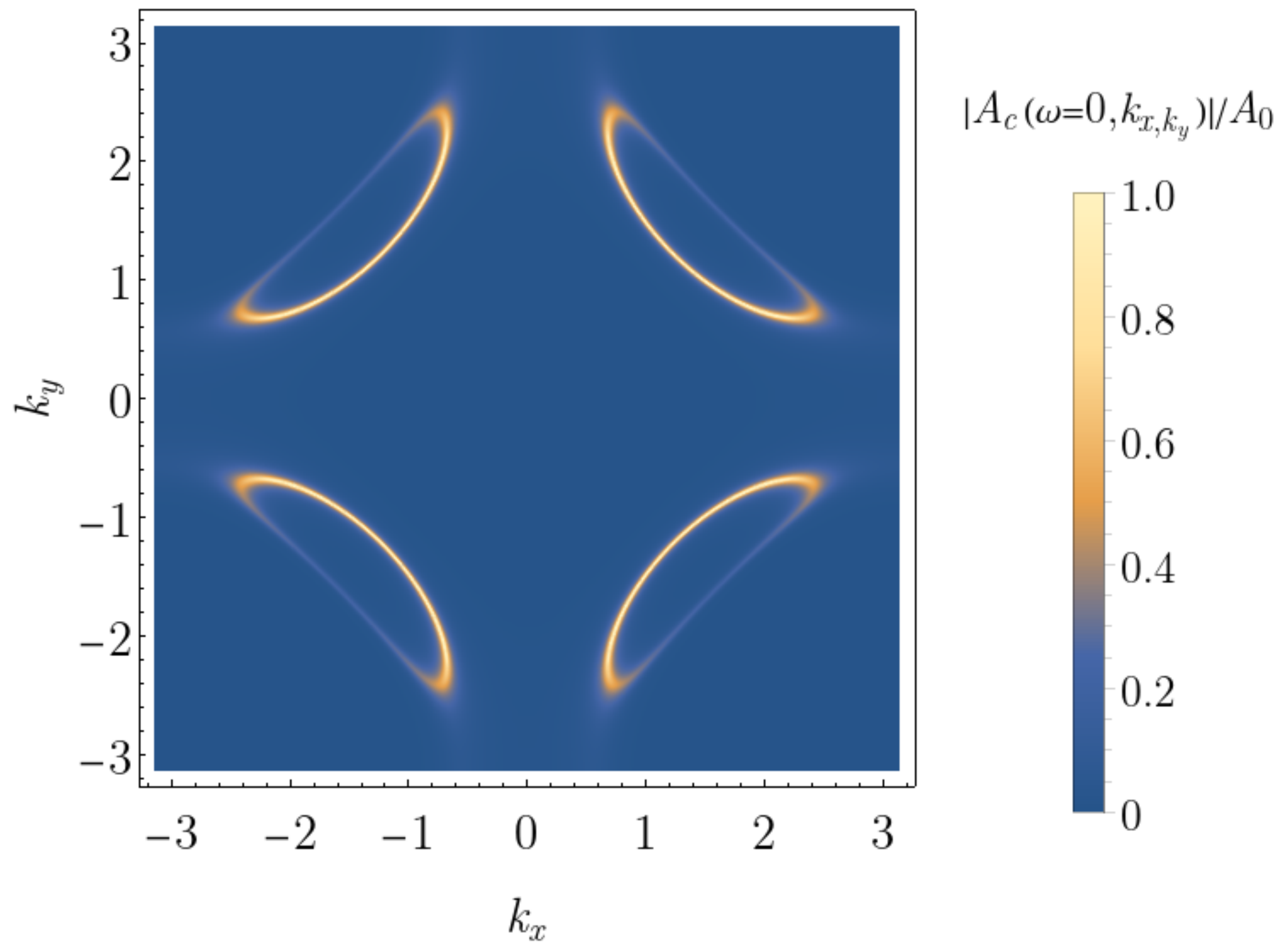

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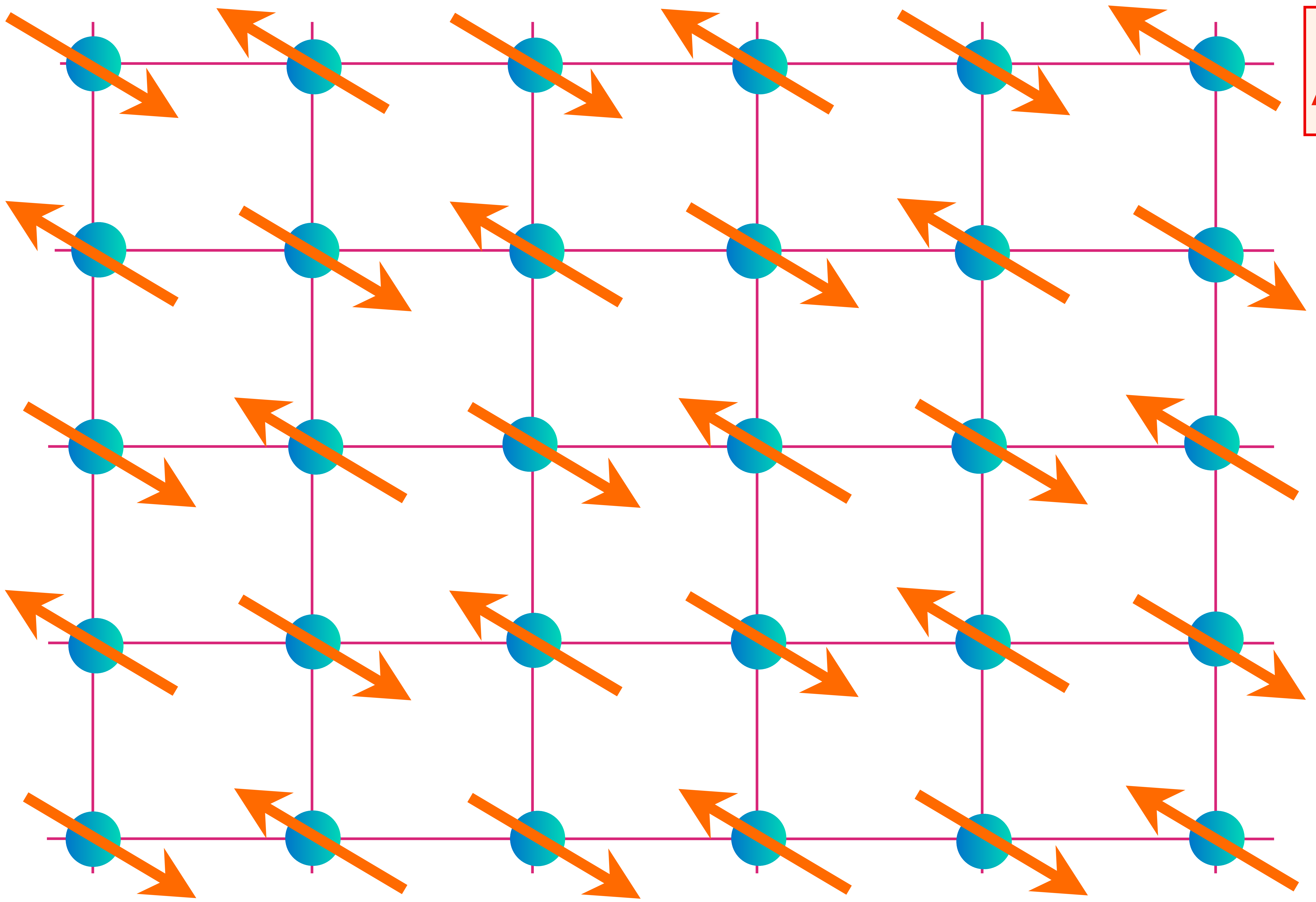
Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles
+
'spectator' square lattice spin liquid *at half-filling*.

FL*: Spin liquid is *required* because the Fermi surface does not enclose the Luttinger volume $(1+p)$.

T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Intro to
spin liquids
and
doped spin liquids (FL*)

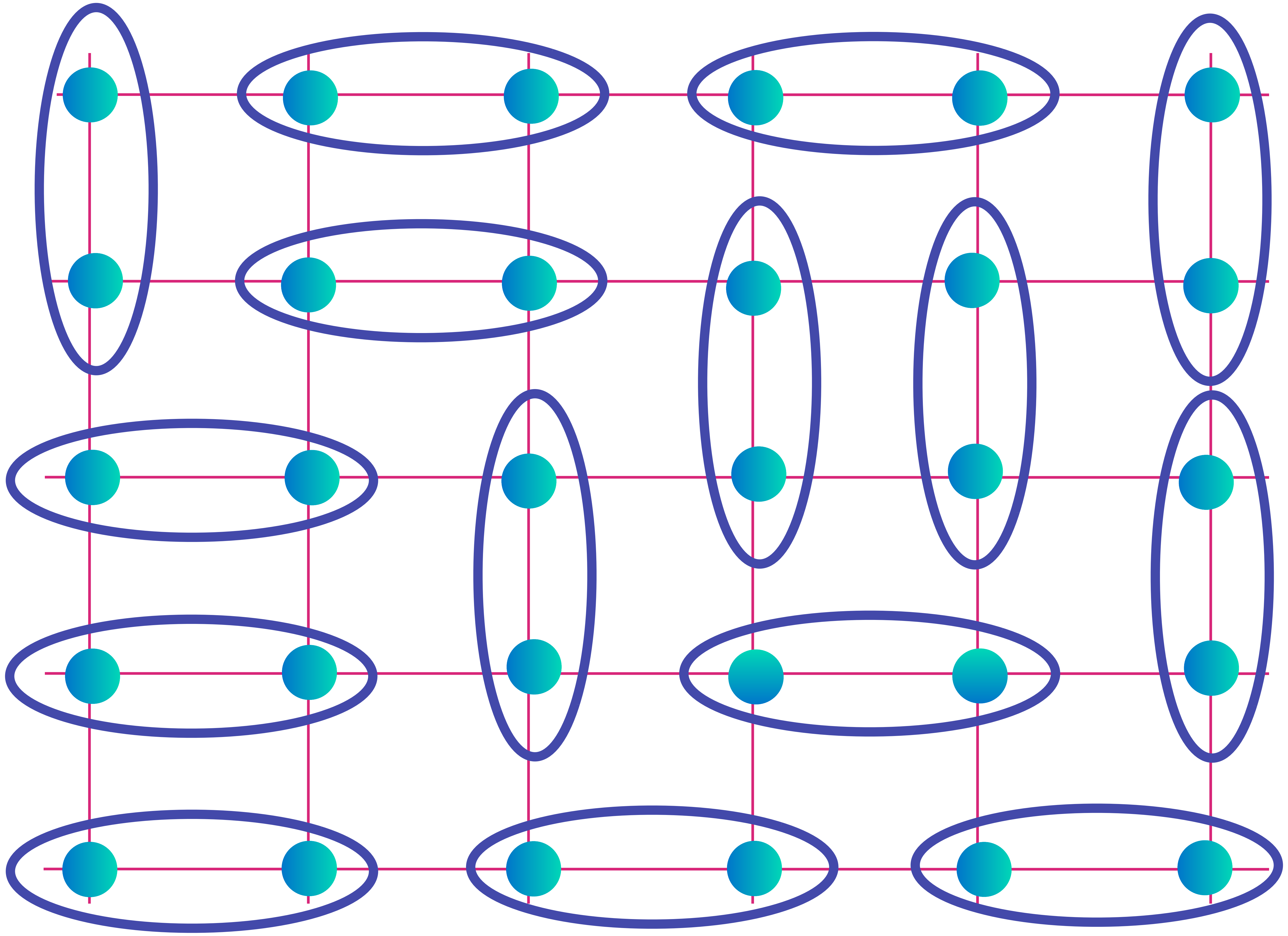
The dance of electrons on Cu atoms in YBCO



Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins

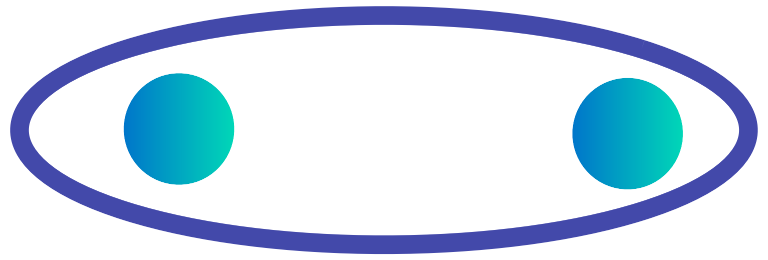
The dance of electrons on Cu atoms in YBCO



P.W. Anderson (1973)

Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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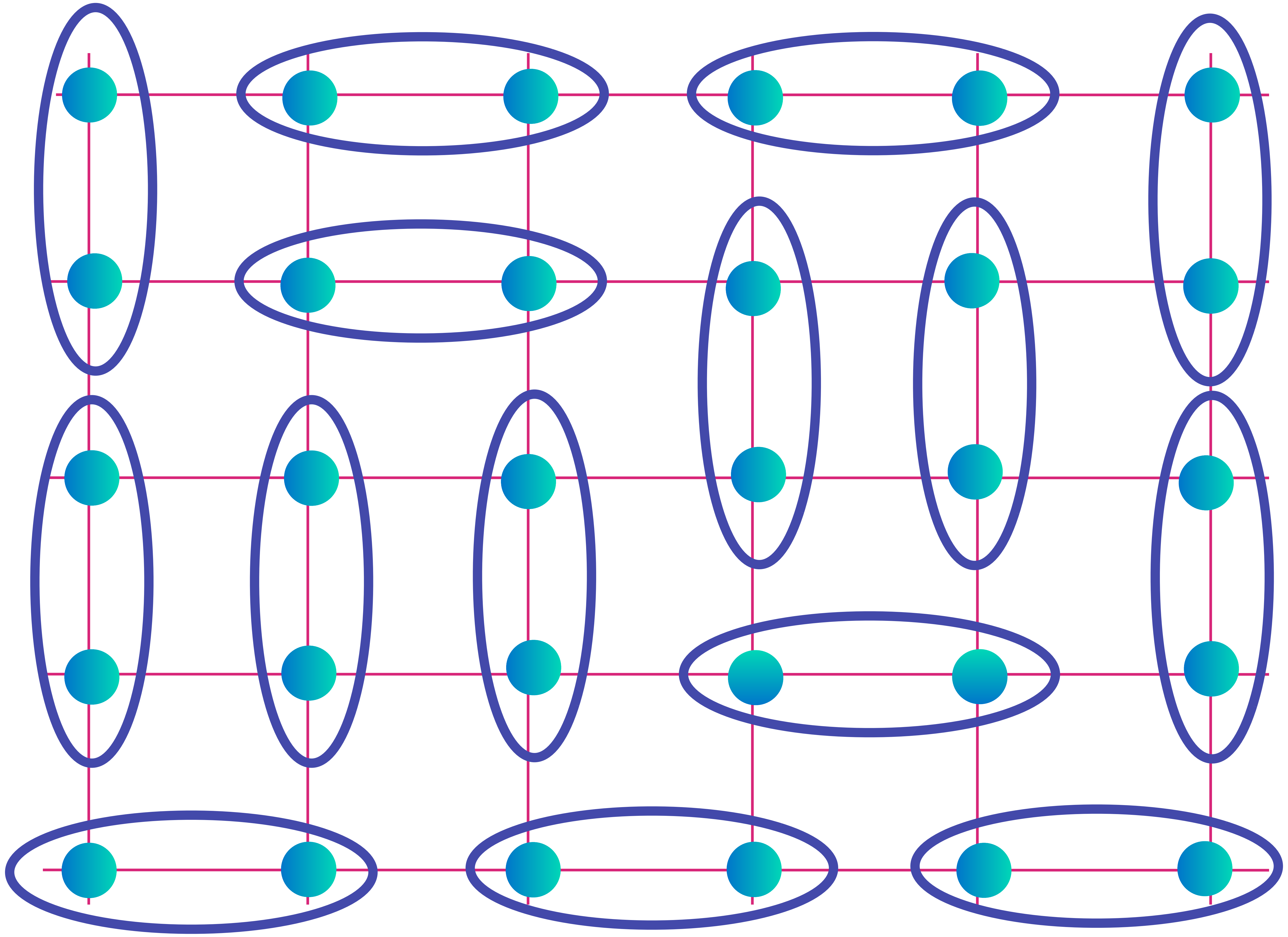

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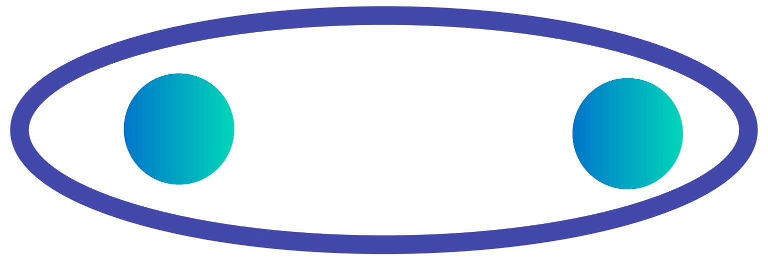 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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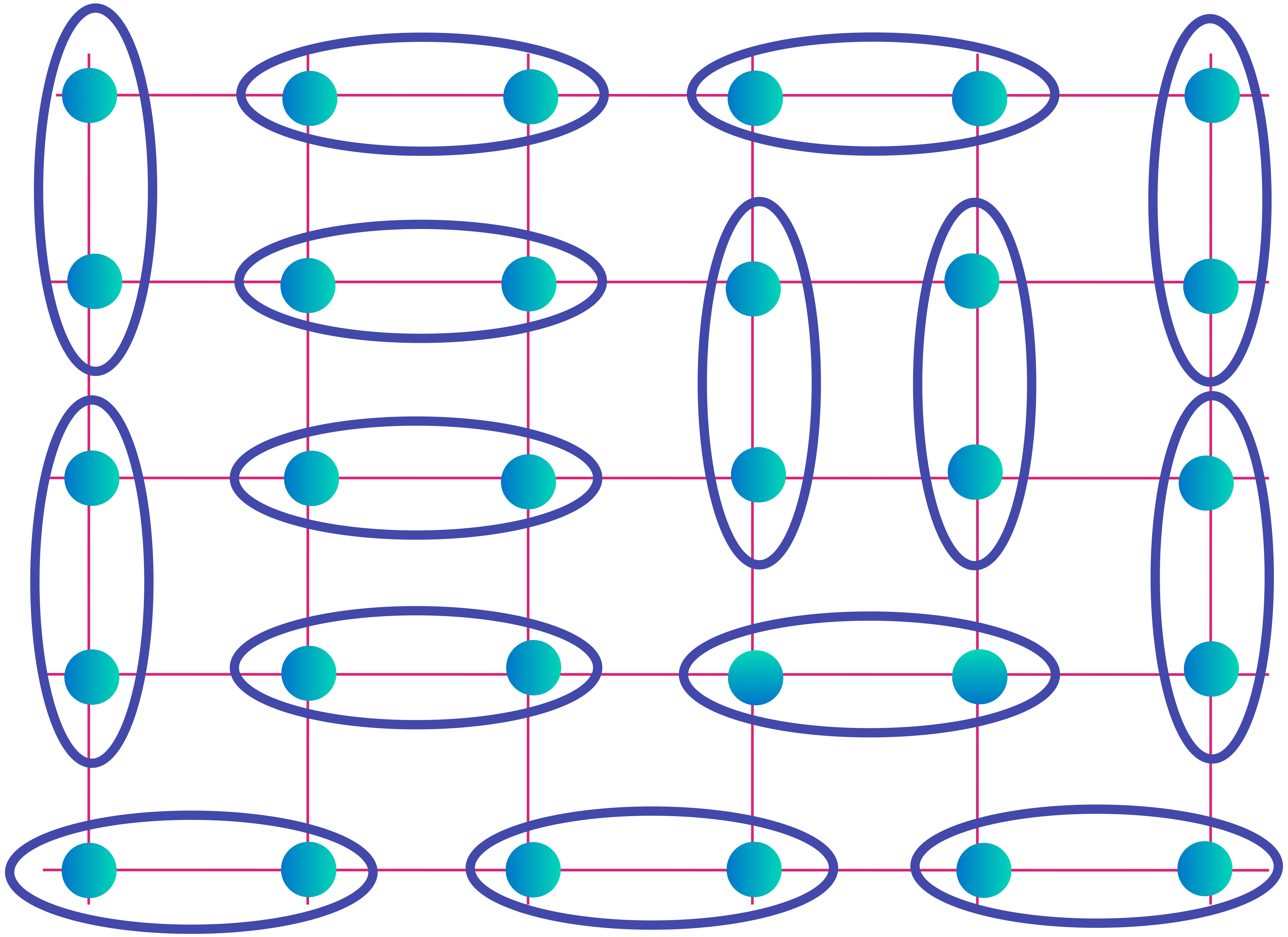

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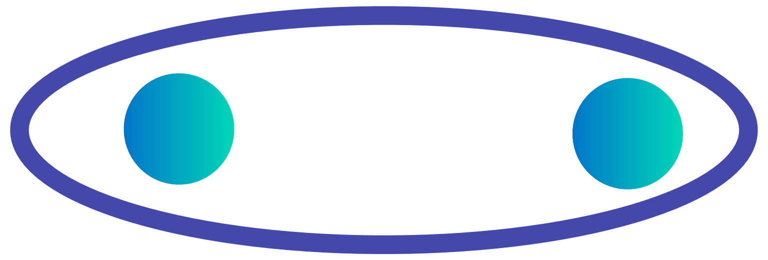

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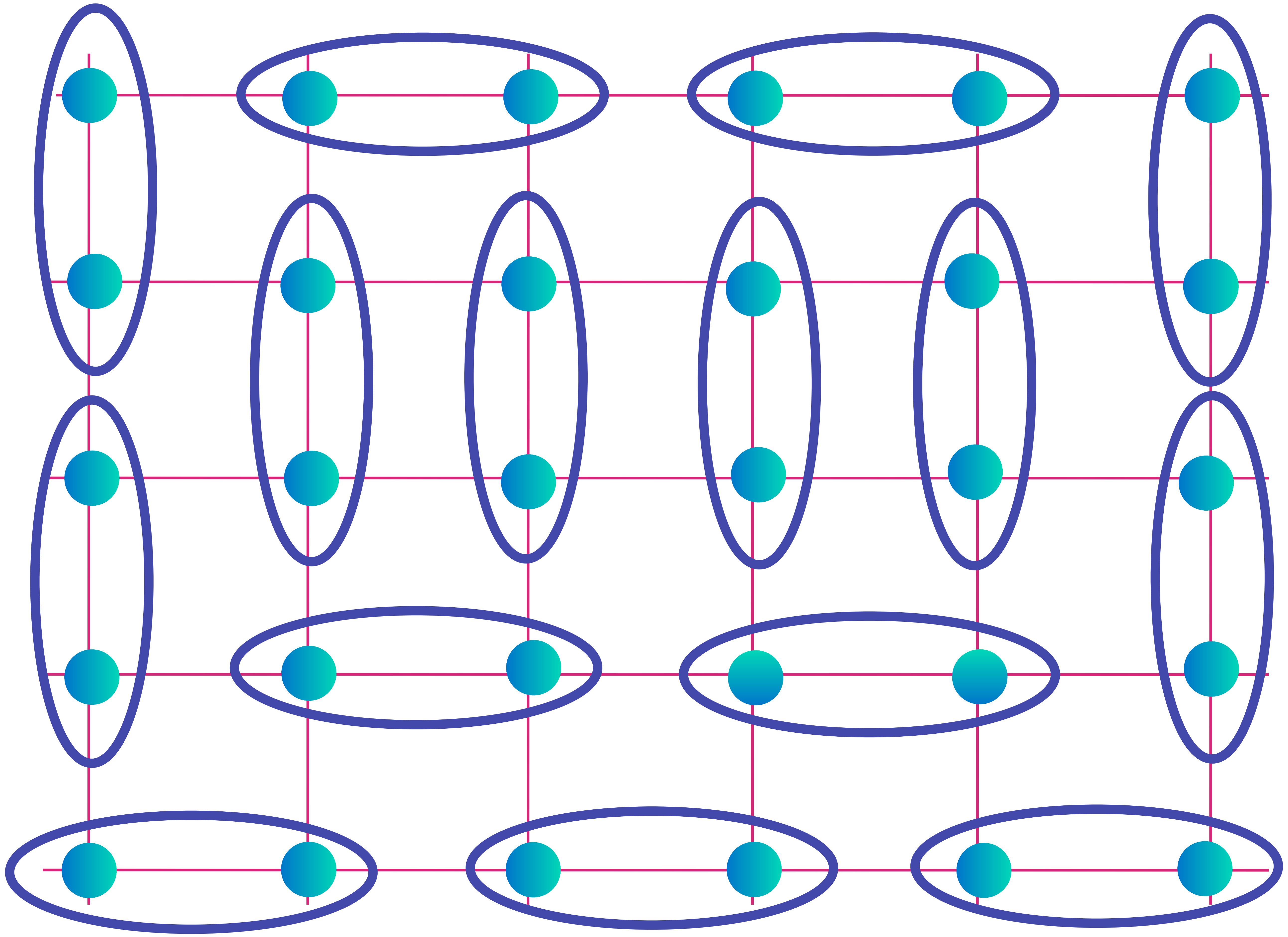

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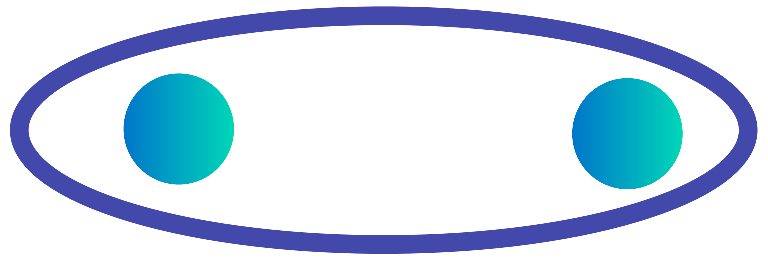

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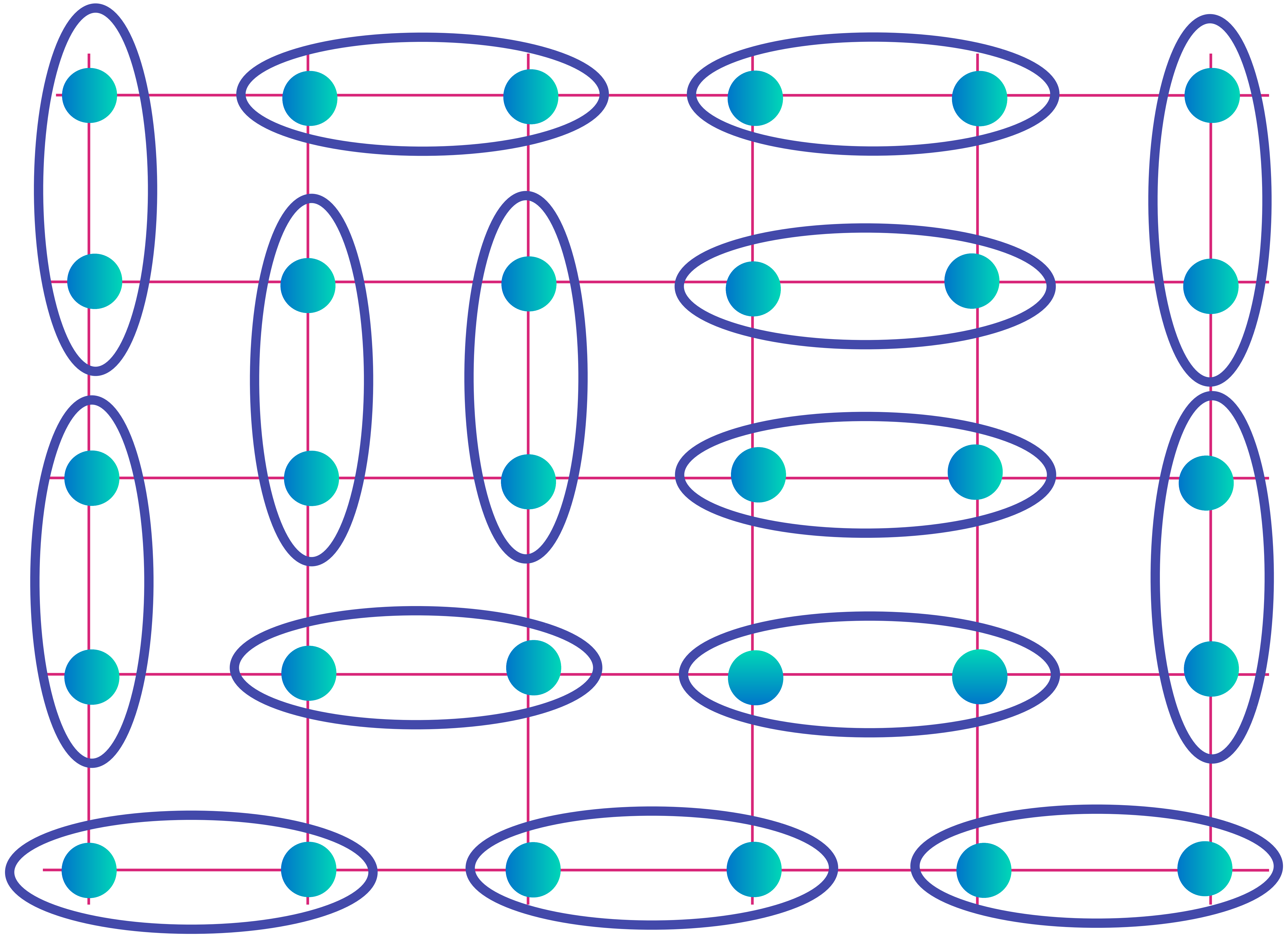

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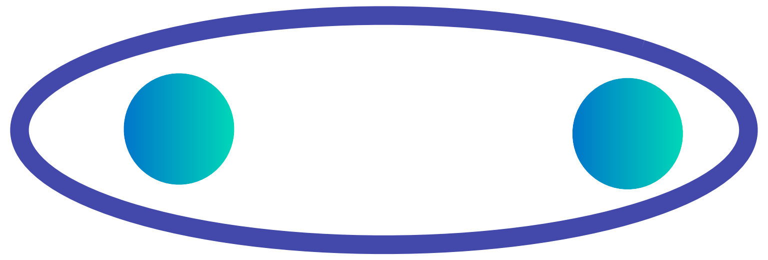

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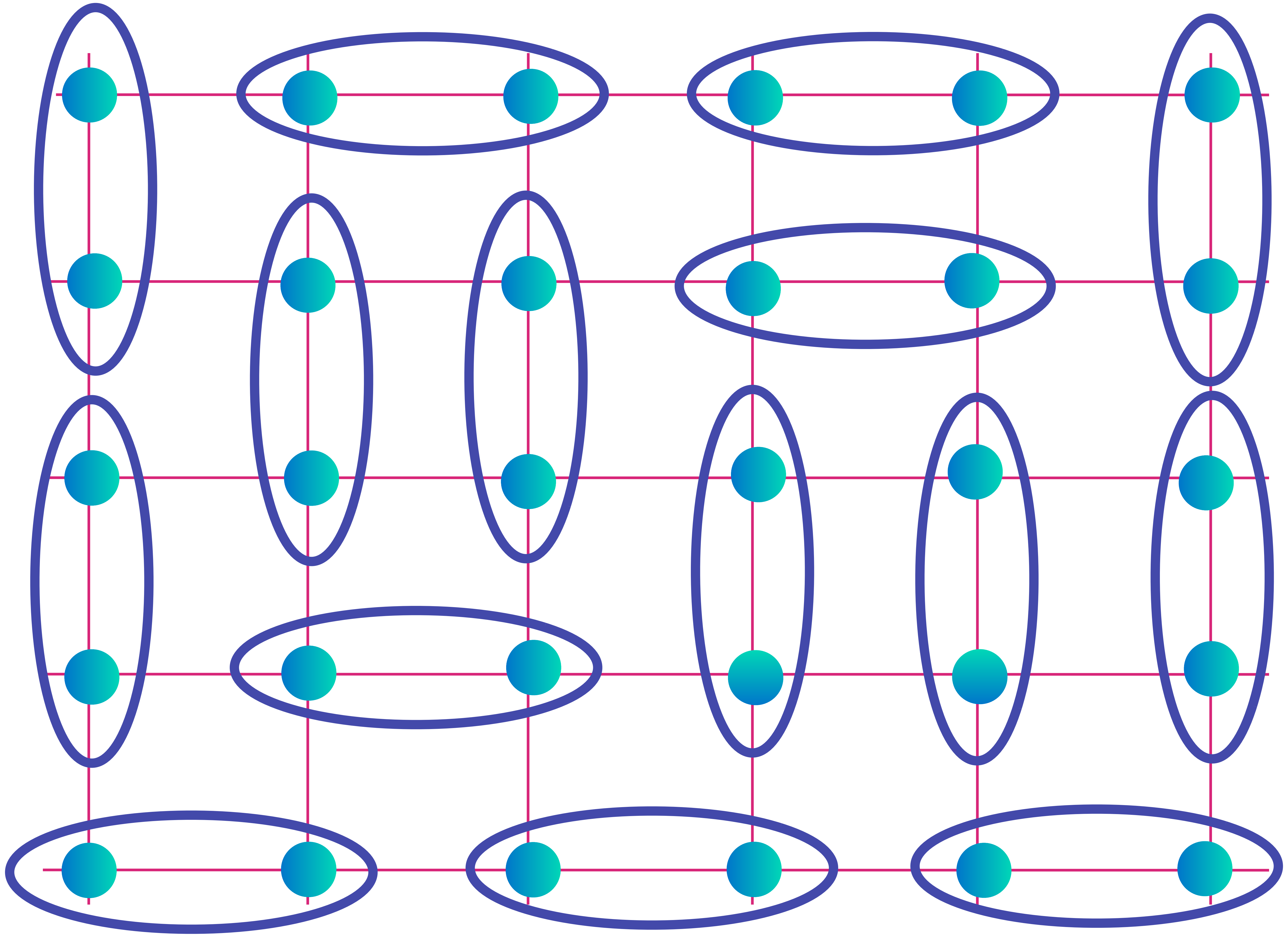

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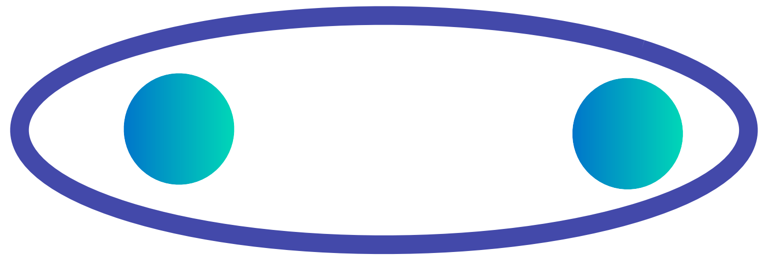
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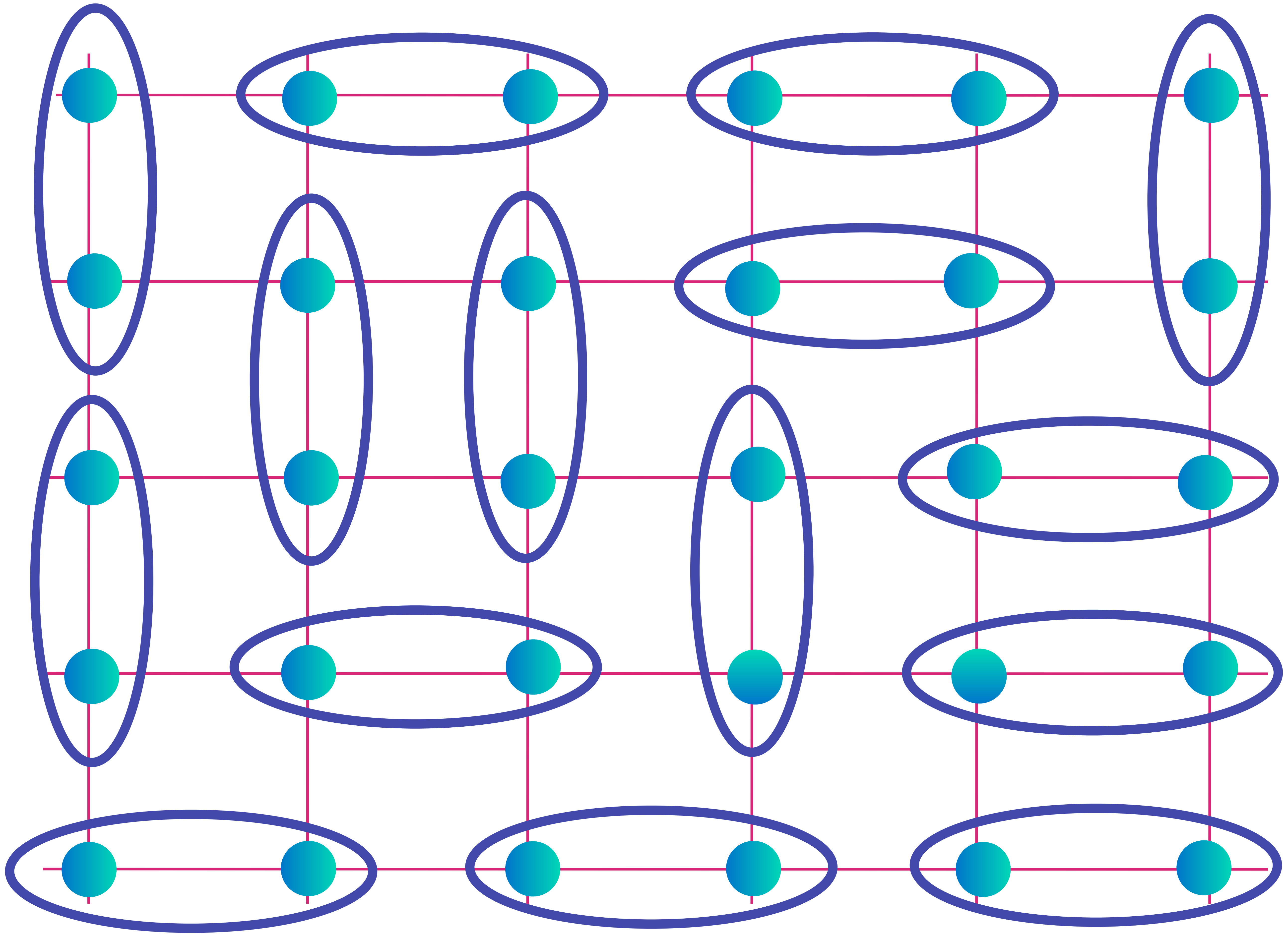

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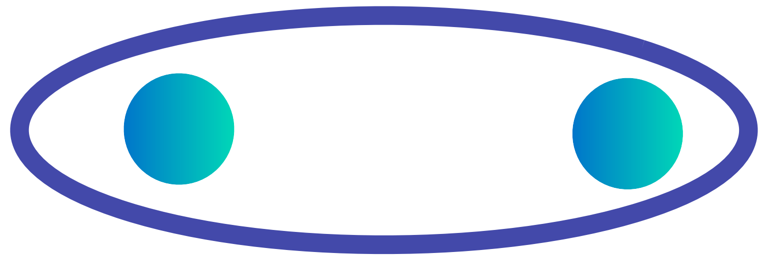 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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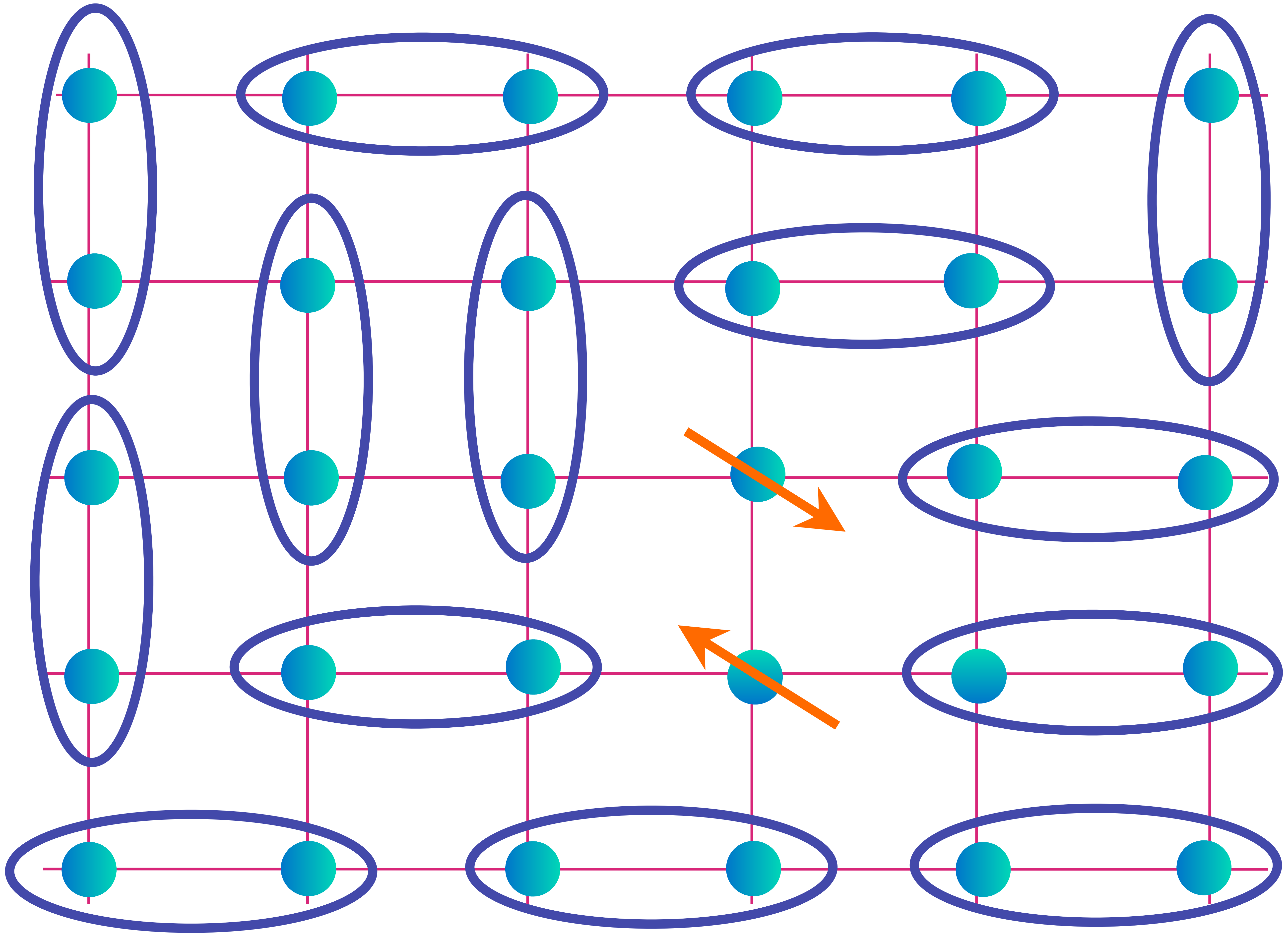
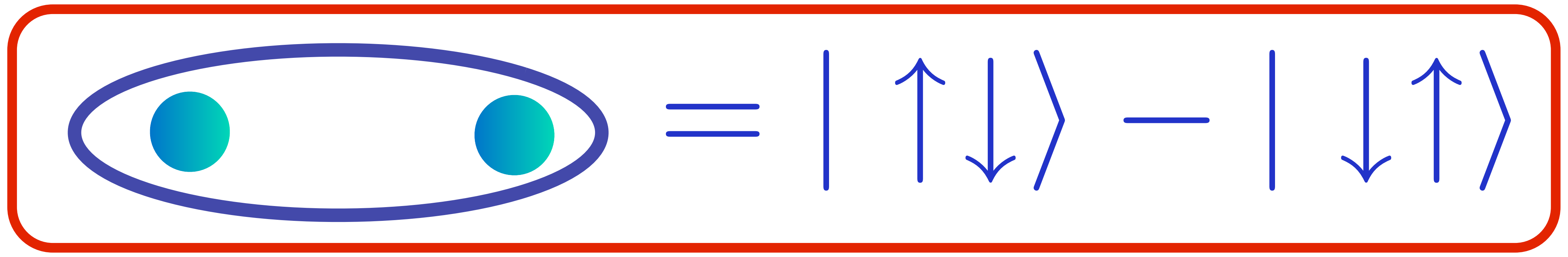


Spin liquid

Fractionalized spinon excitations with spin S=1/2 and charge 0.



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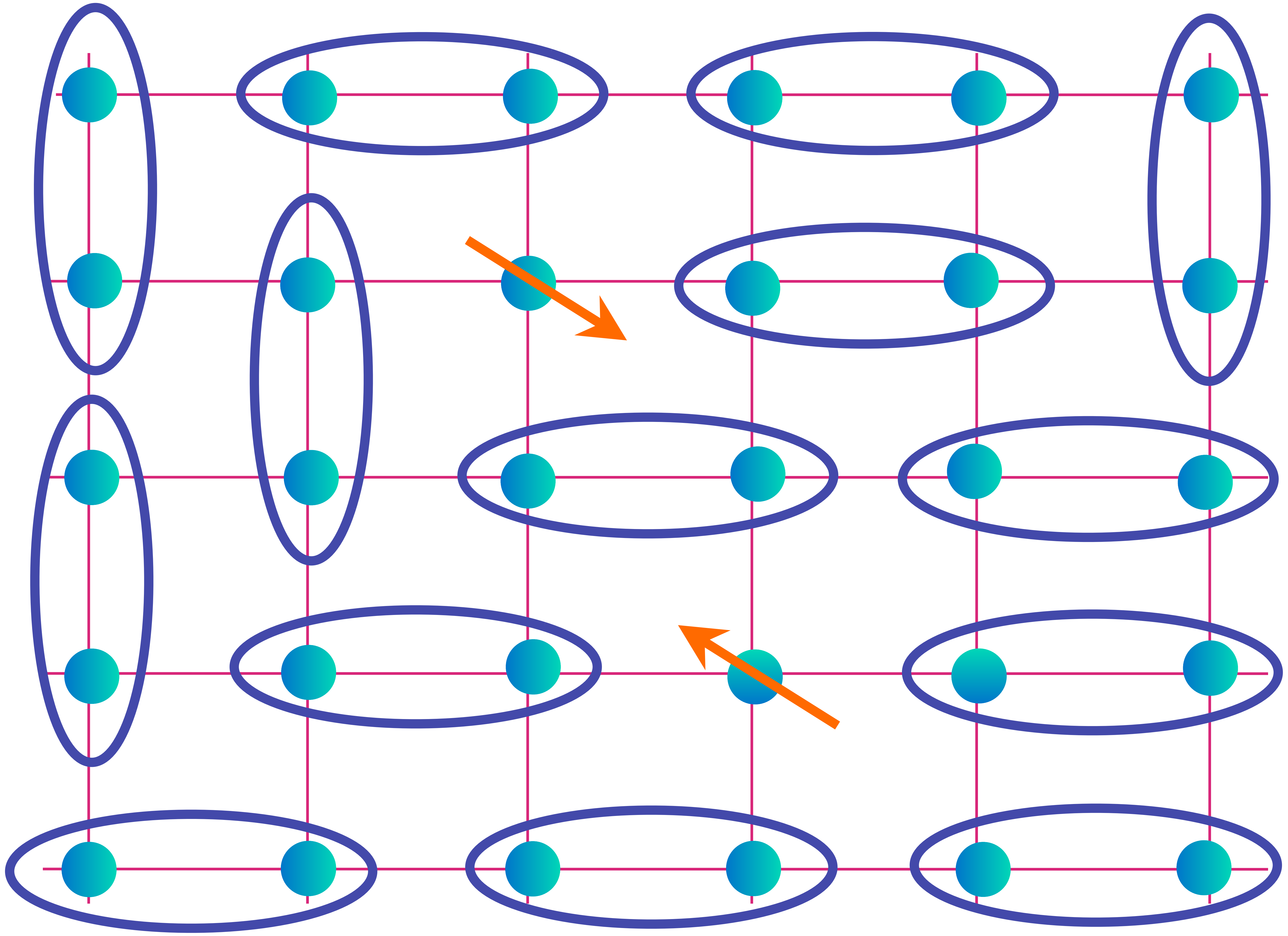

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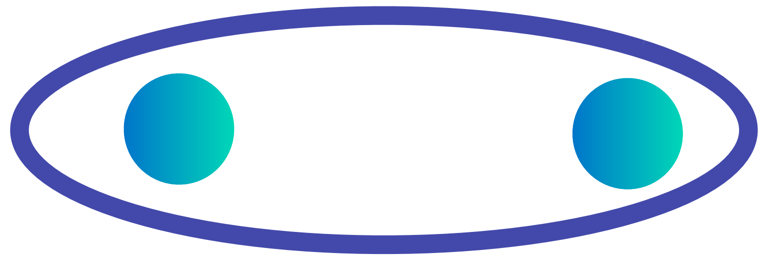 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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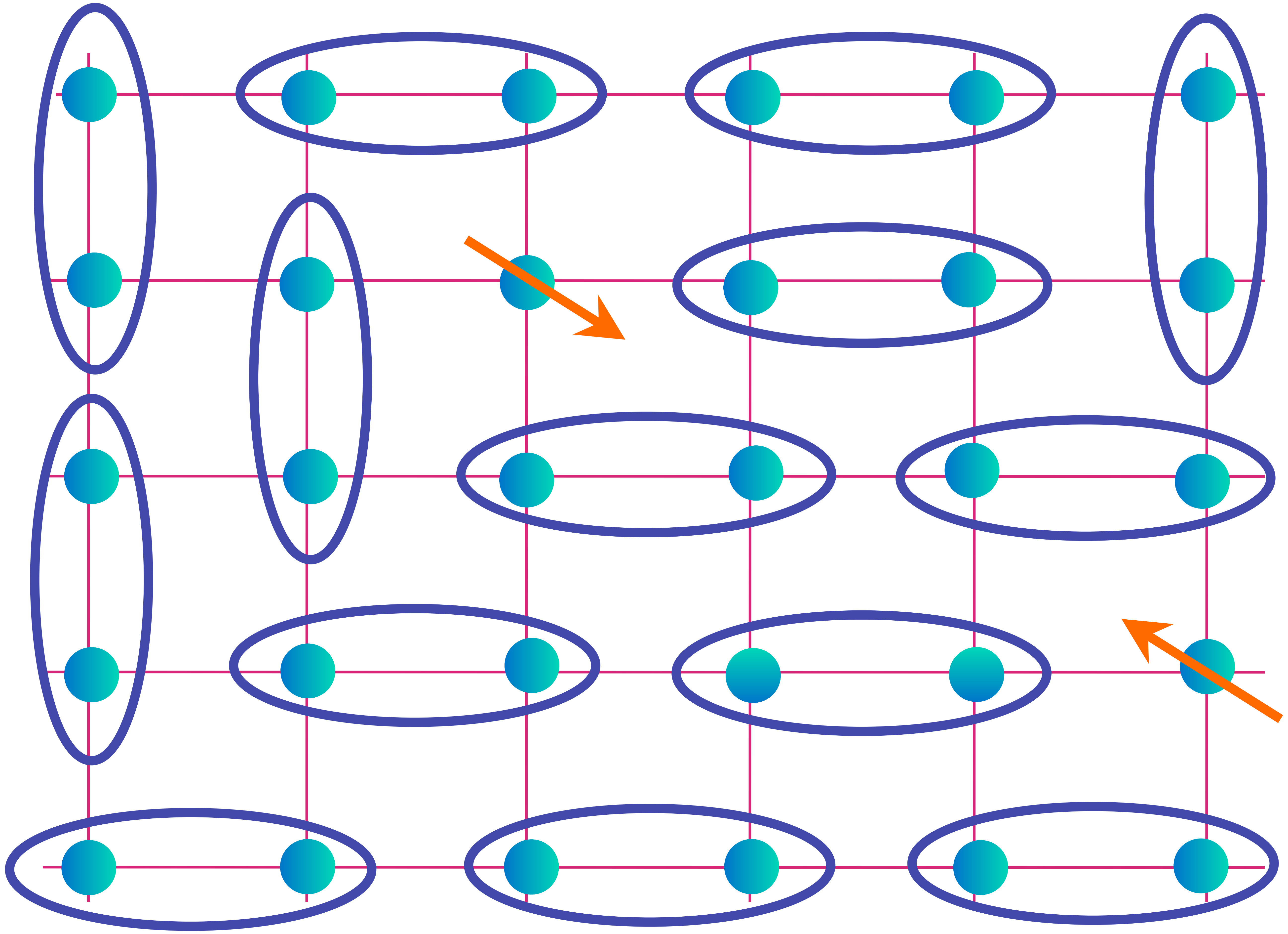

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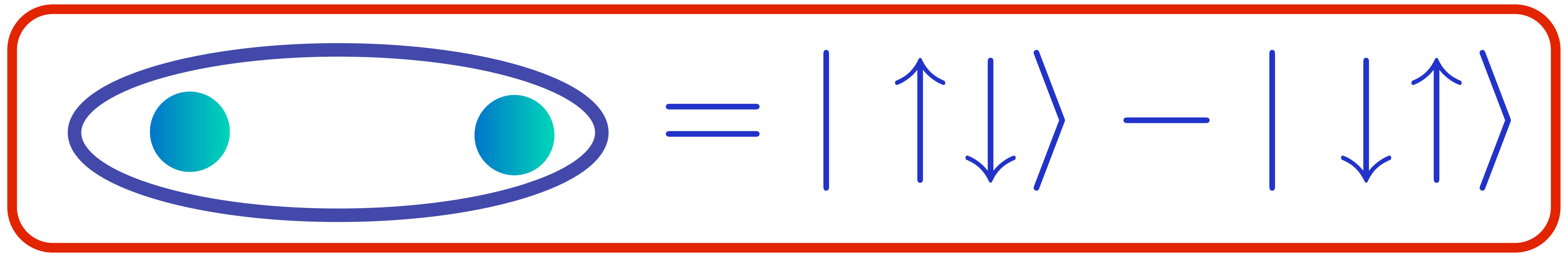

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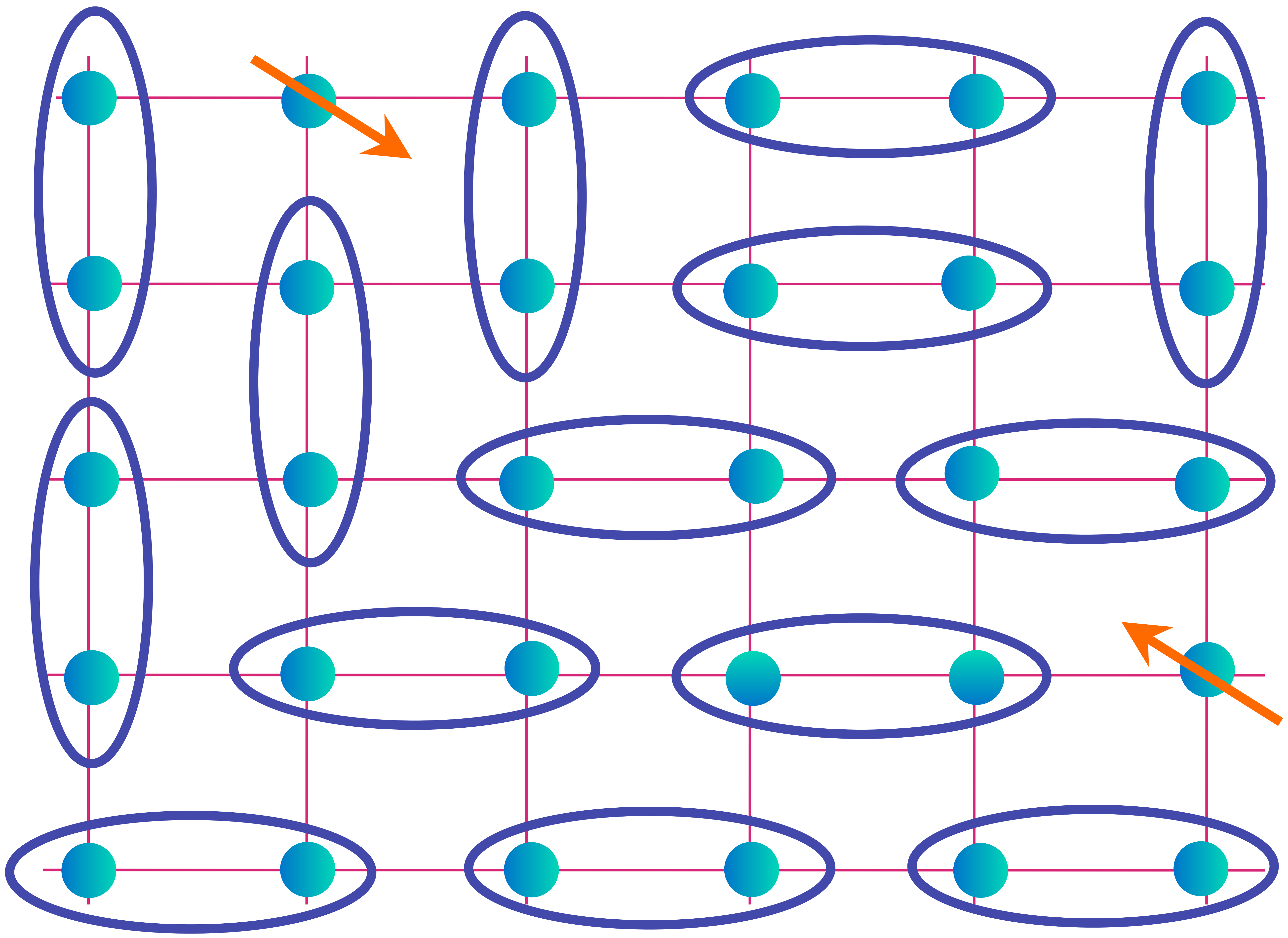

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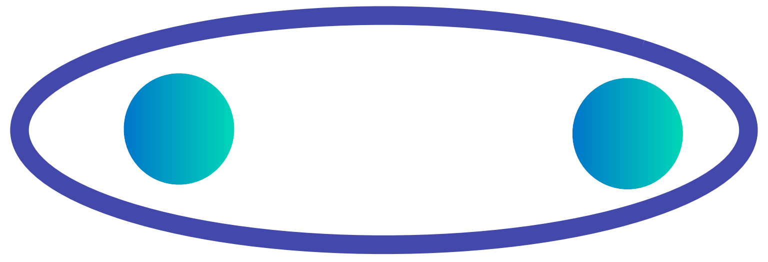 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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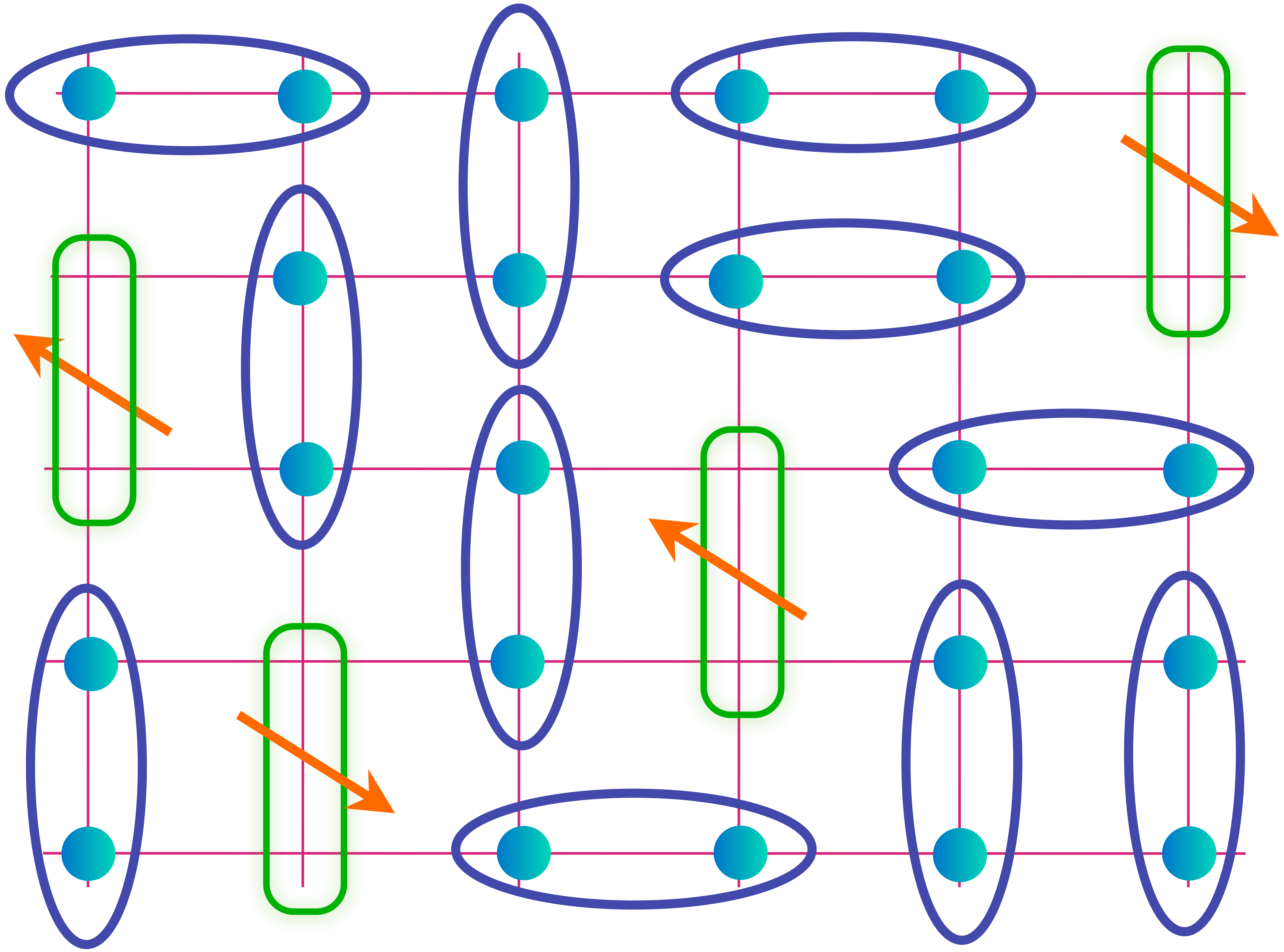
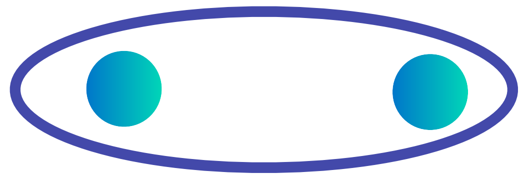
FL* in a ***one-band*** model

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

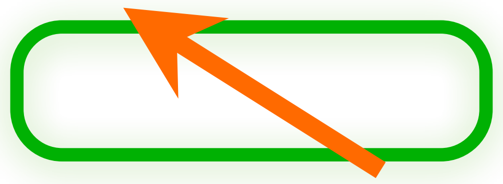
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a ***one-band*** model

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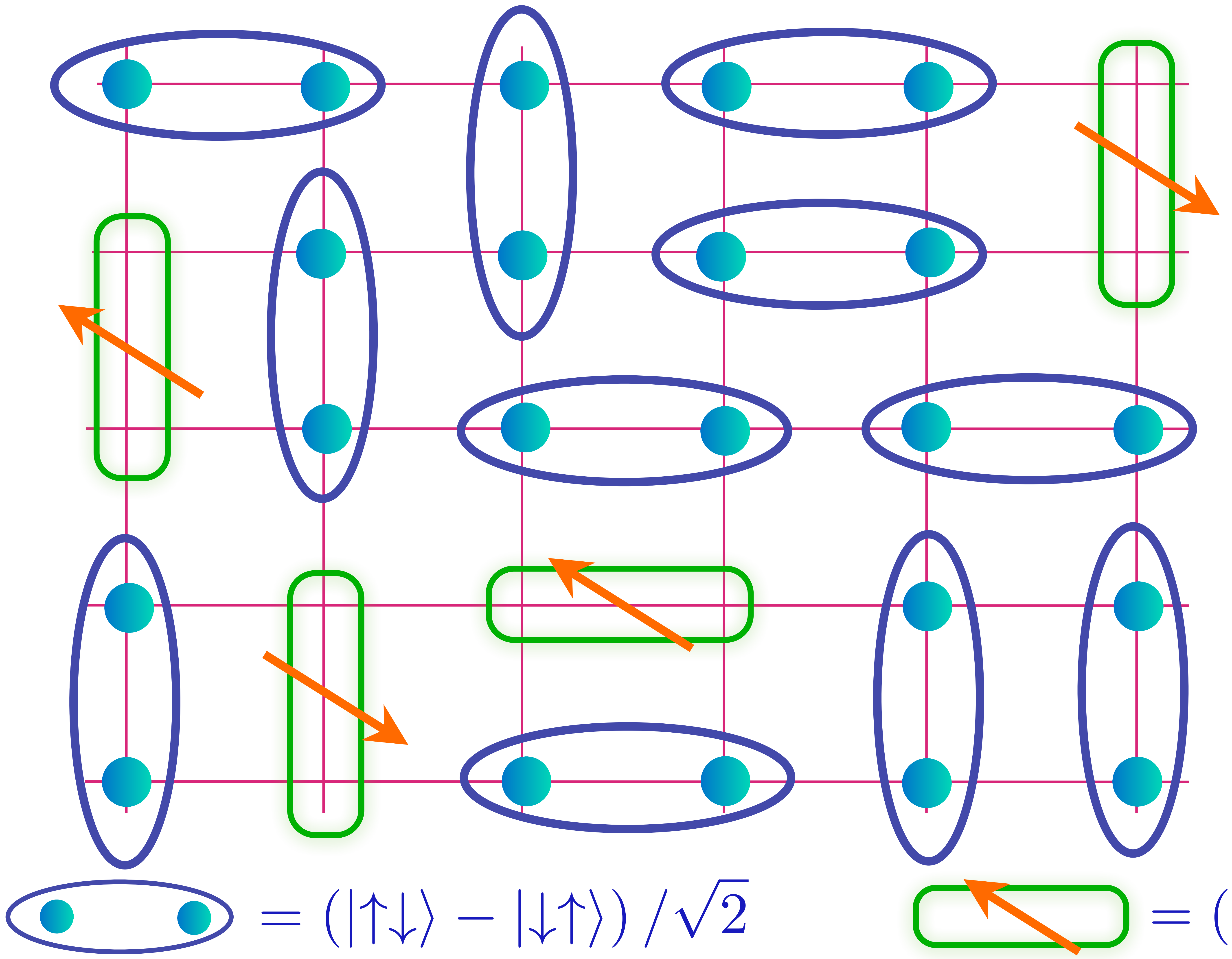
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$

$= (|\uparrow \circ\rangle + |\circ \uparrow\rangle) / \sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* in a ***one-band*** model

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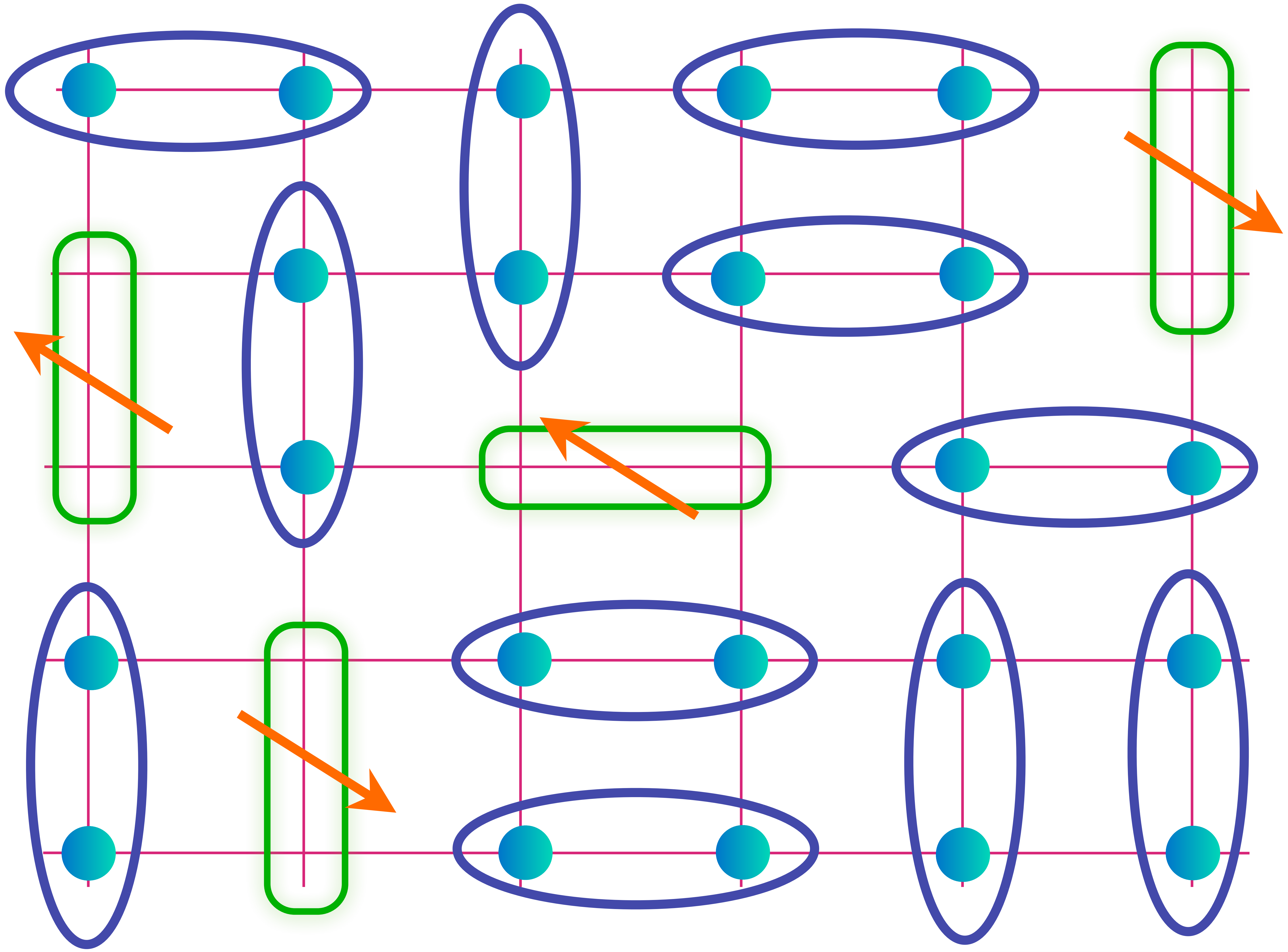

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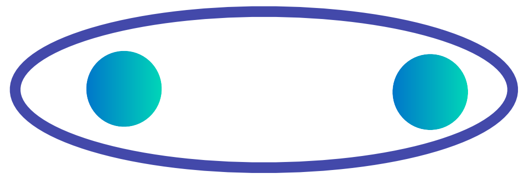
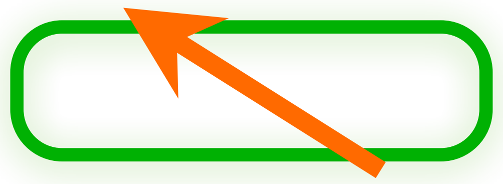
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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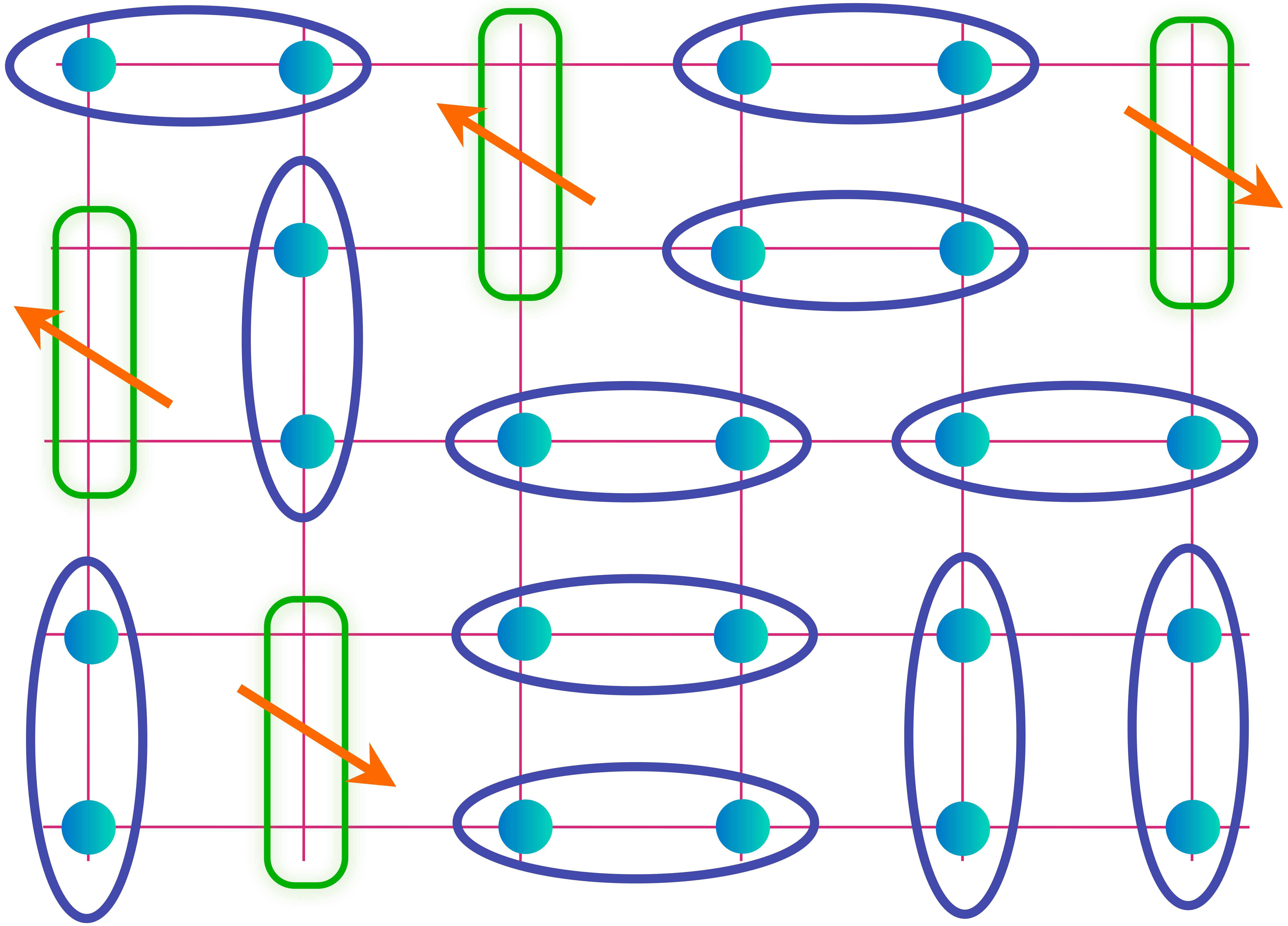
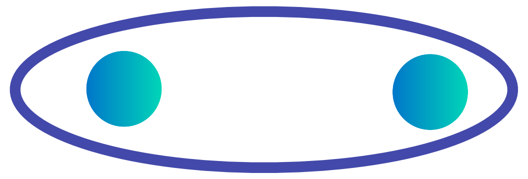
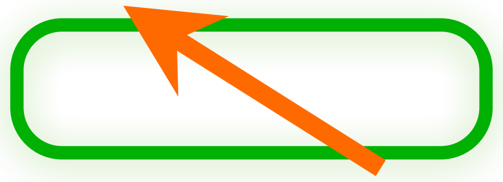


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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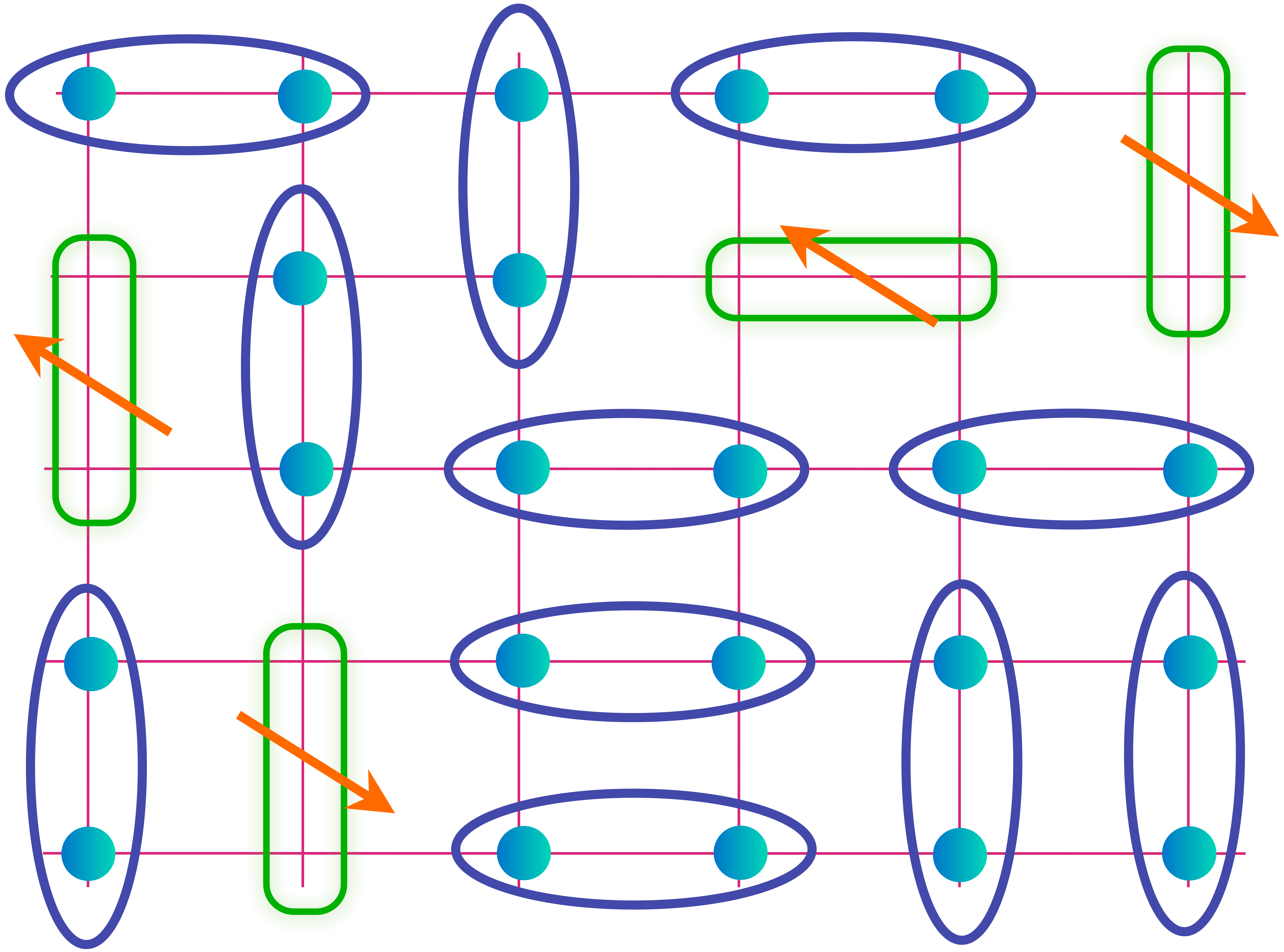
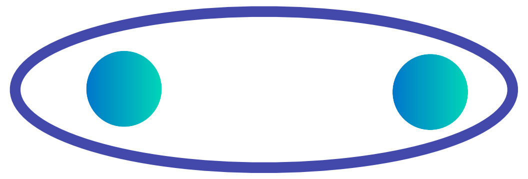
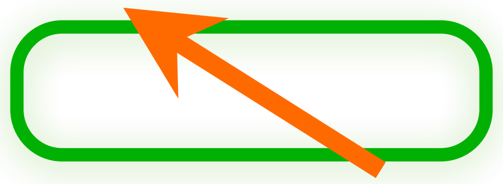


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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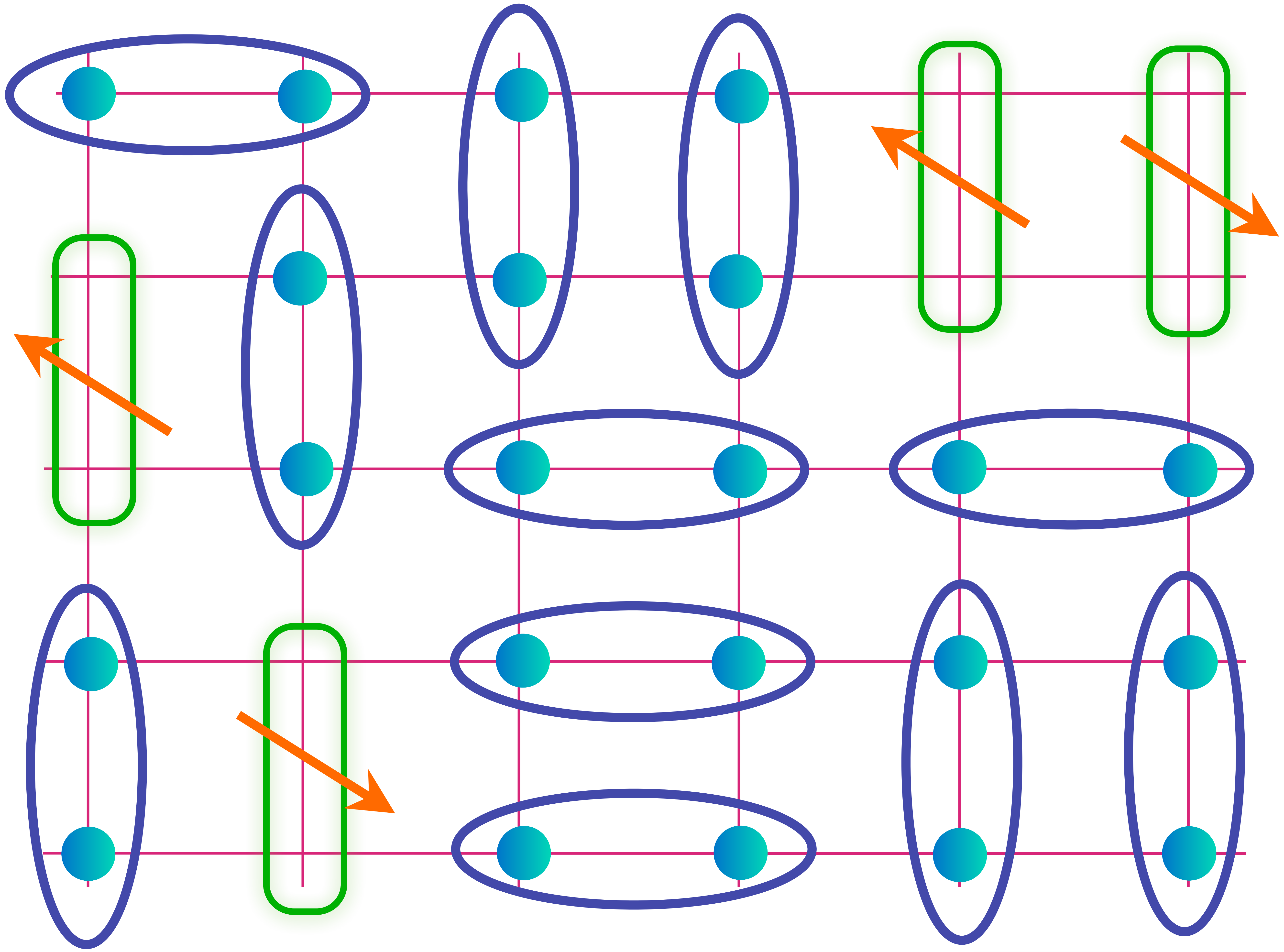
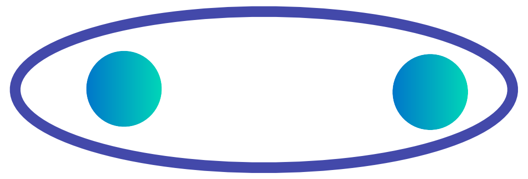
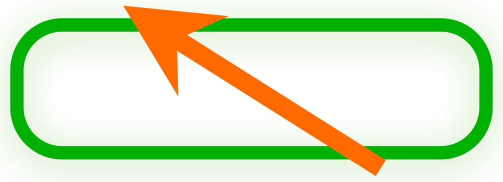


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

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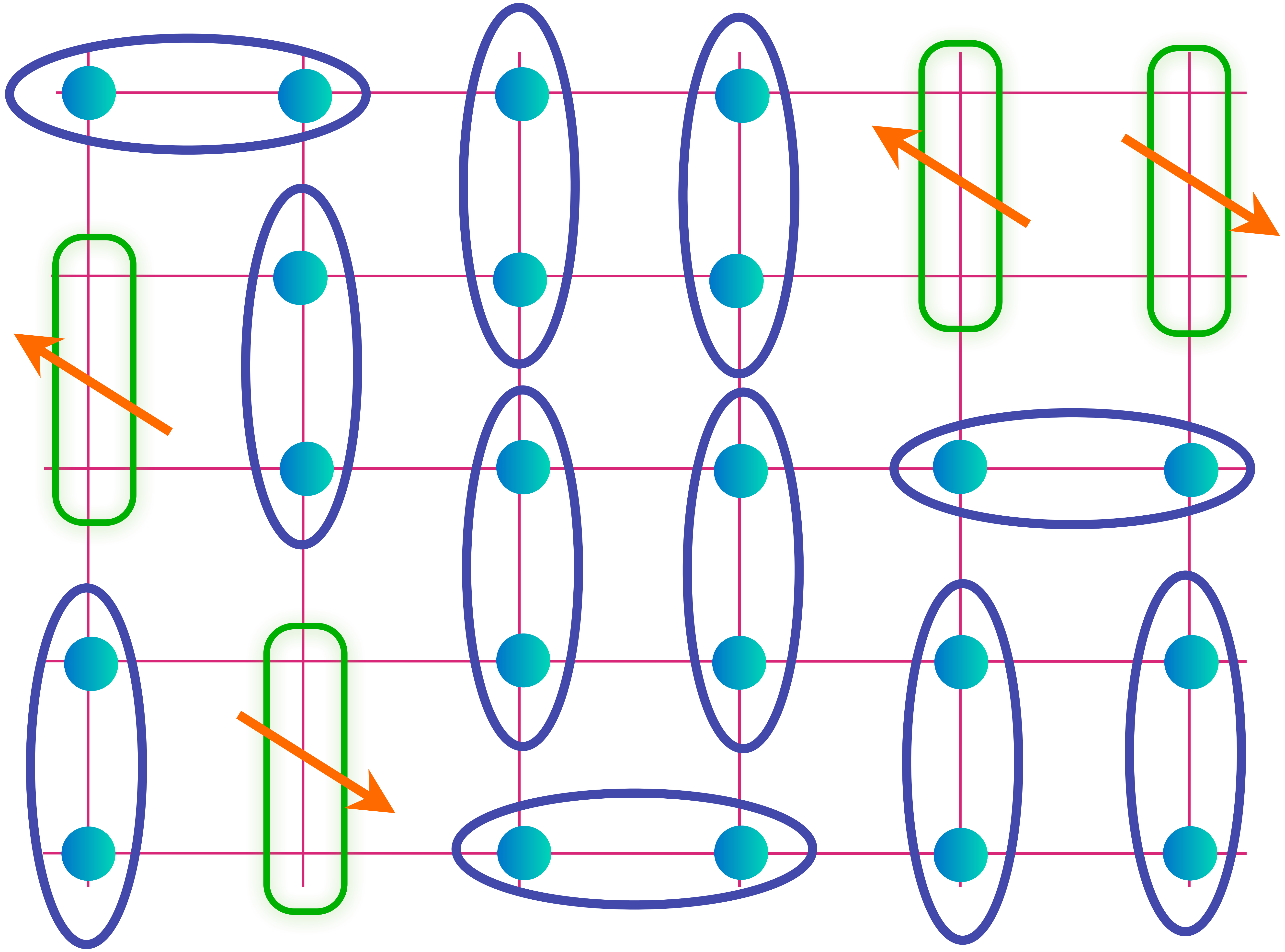
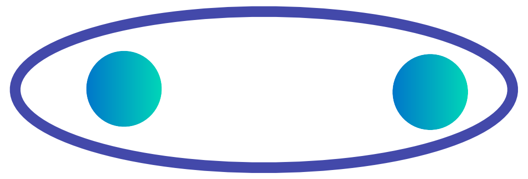
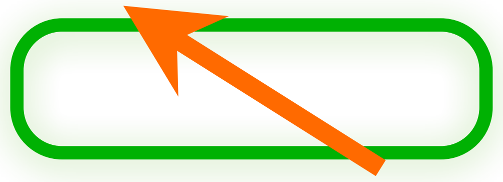


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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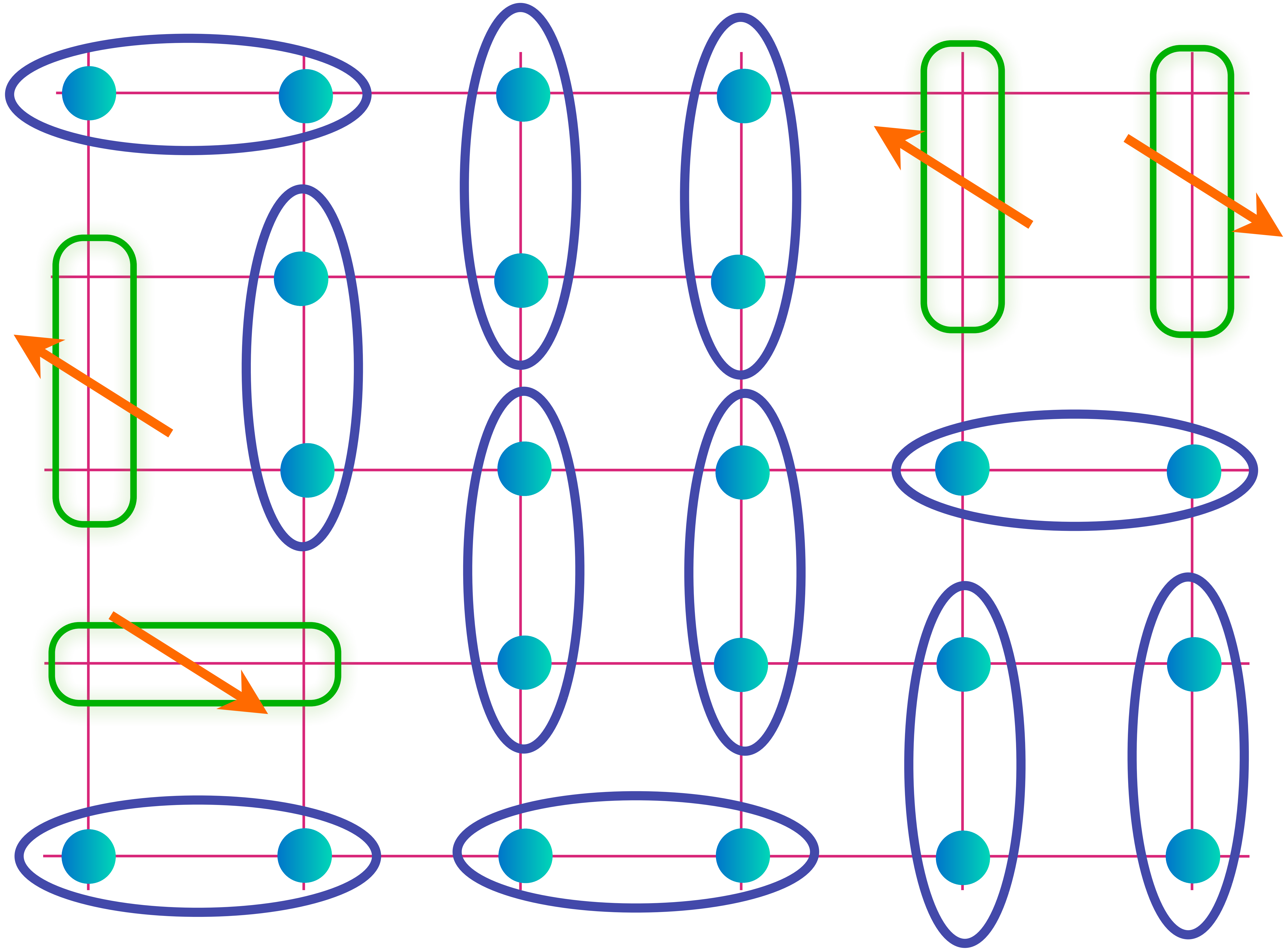
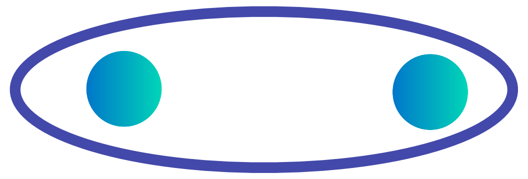
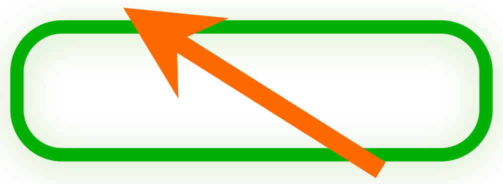


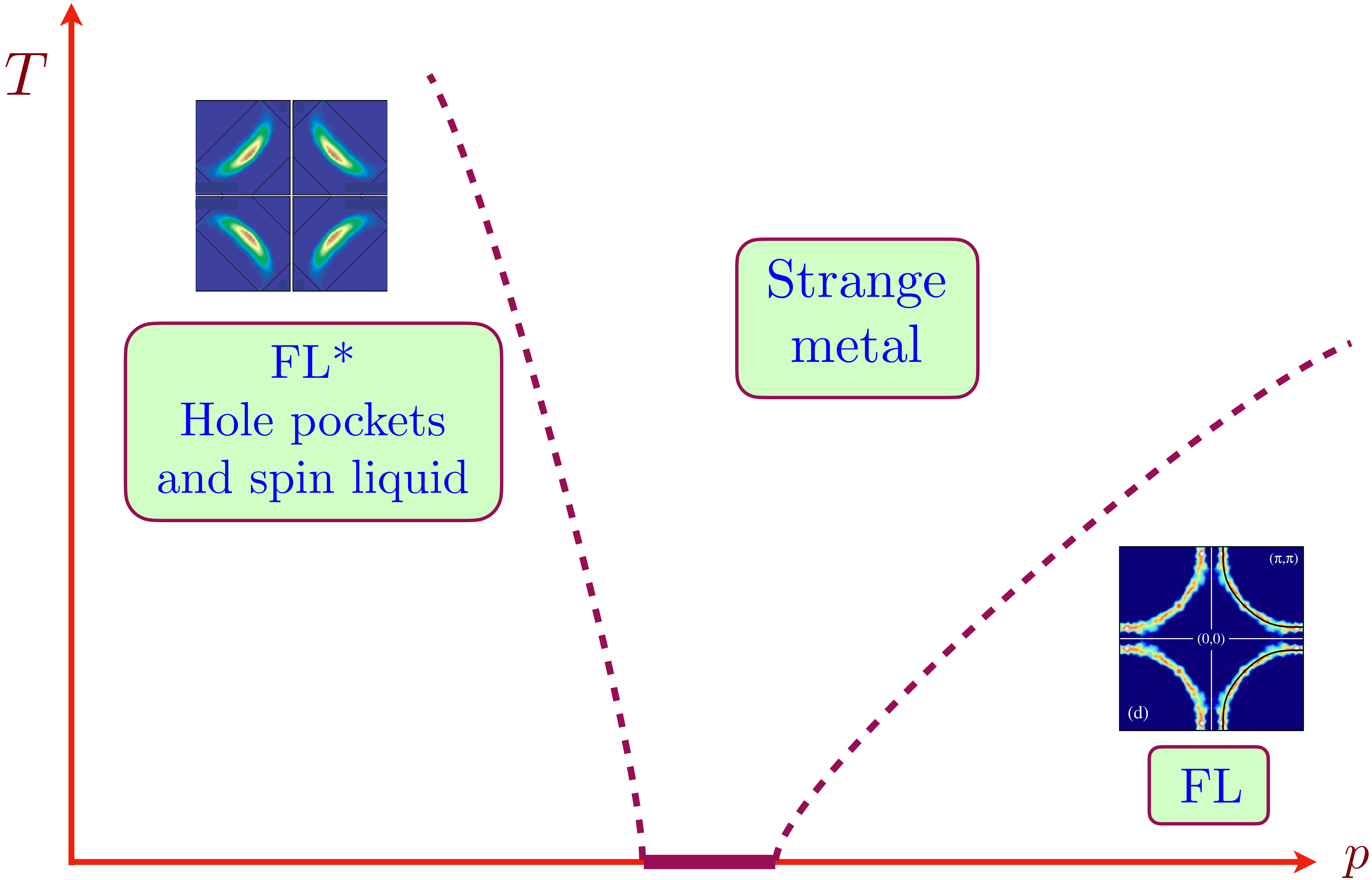
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

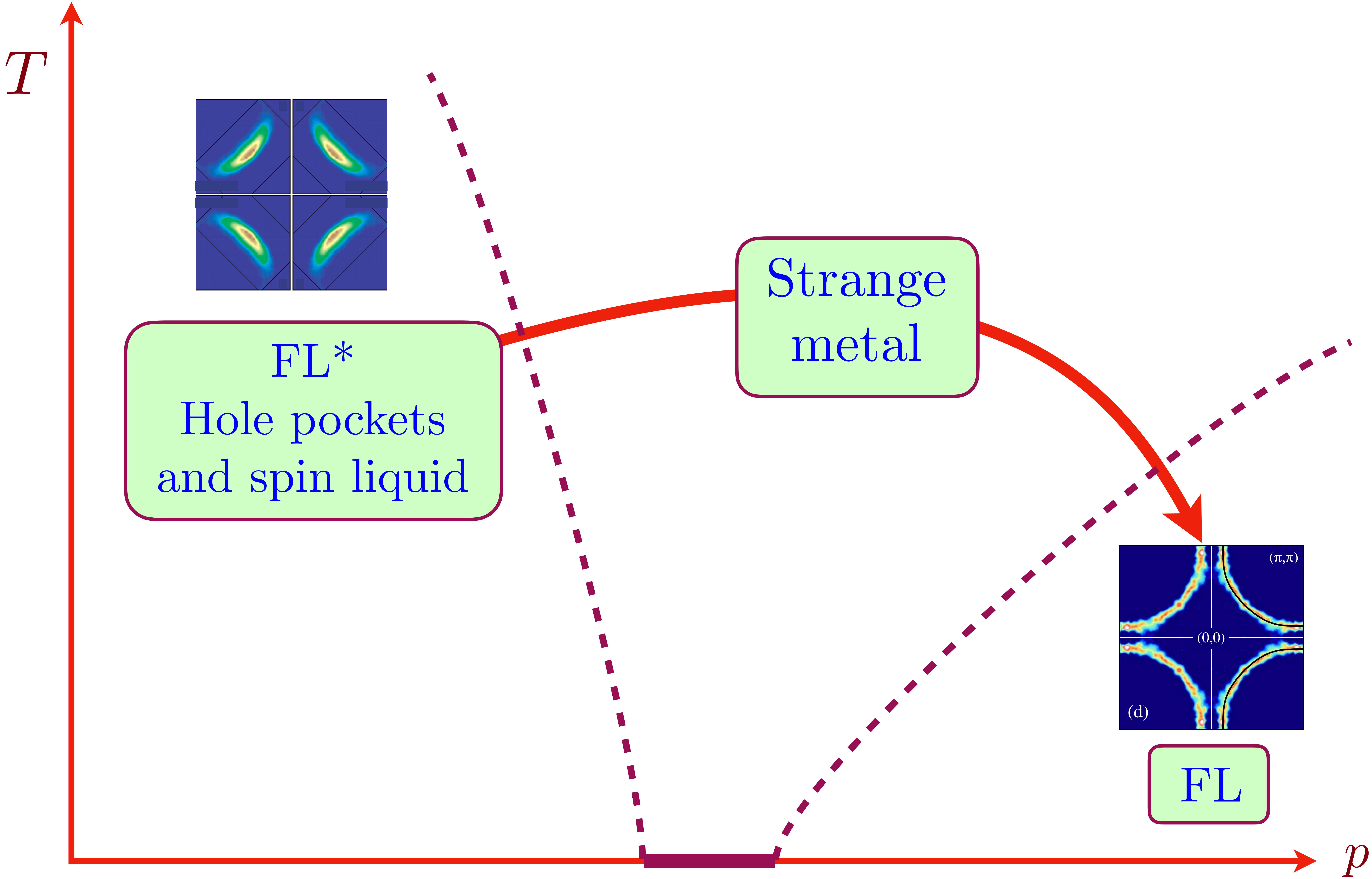
 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

E. G. Moon and S. Sachdev, PRB **83**, 224508 (2011); M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

From FL*
to
a cuprate phase diagram



T
FL*
Hole pockets
and spin liquid
Strange
metal
(π,π)
(0,0)
(d)
FL
p



T
FL*
Hole pockets
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Strange
metal
(π,π)
(0,0)
(d)
FL
p

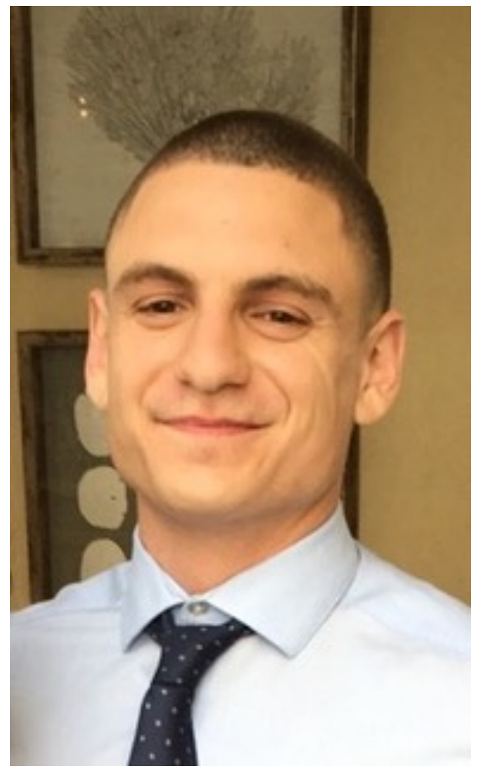
PHYSICS

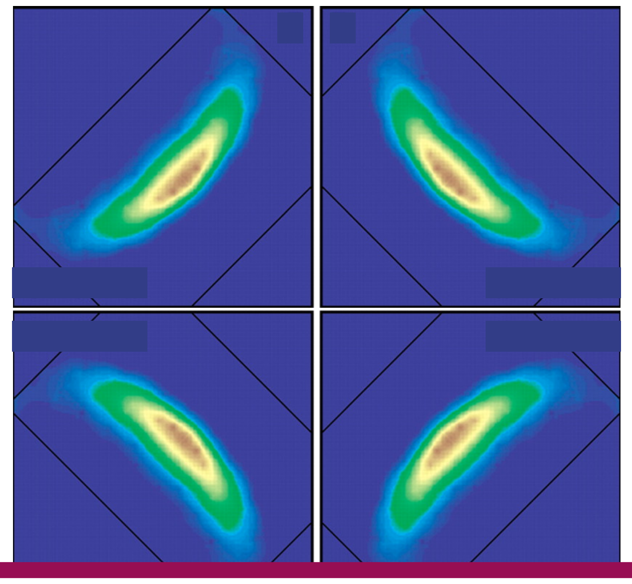
arXiv:2203.04990, Science, Aug 18, 2023

Universal theory of strange metals from spatially random interactions

Aavishkar A. Patel[1,2], Haoyu Guo[3,4,5], Ilya Esterlis[4,6], Subir Sachdev[4,7]*

T

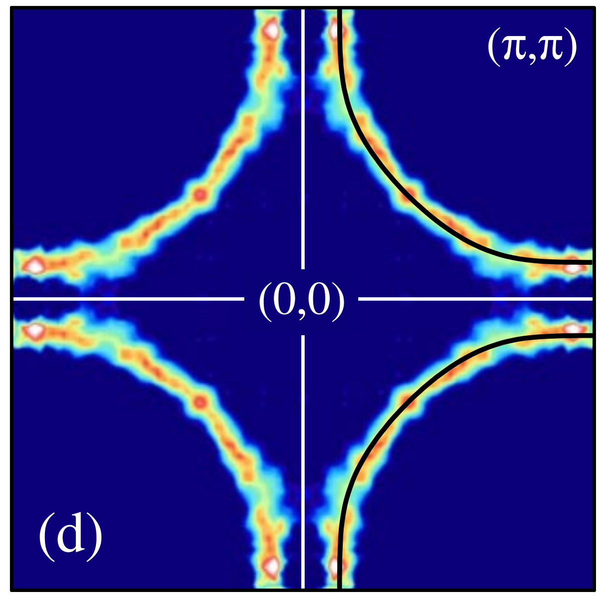




we consider two-dimensional metals of fermions coupled to quantum critical scalars, the latter representing order parameters or fractionalized particles. We show that at low temperatures (T), such metals generically exhibit strange metal behavior with a T-linear resistivity arising from spatially random fluctuations in the fermion-scalar Yukawa couplings about a nonzero spatial average.

FL*
Hole pockets
and spin liquid

Strange
metal



FL

p

See also E.E. Aldape, T. Cookmeyer, A.A. Patel, E. Altman PRB **105**, 235111 (2023)

Spatially random interactions!

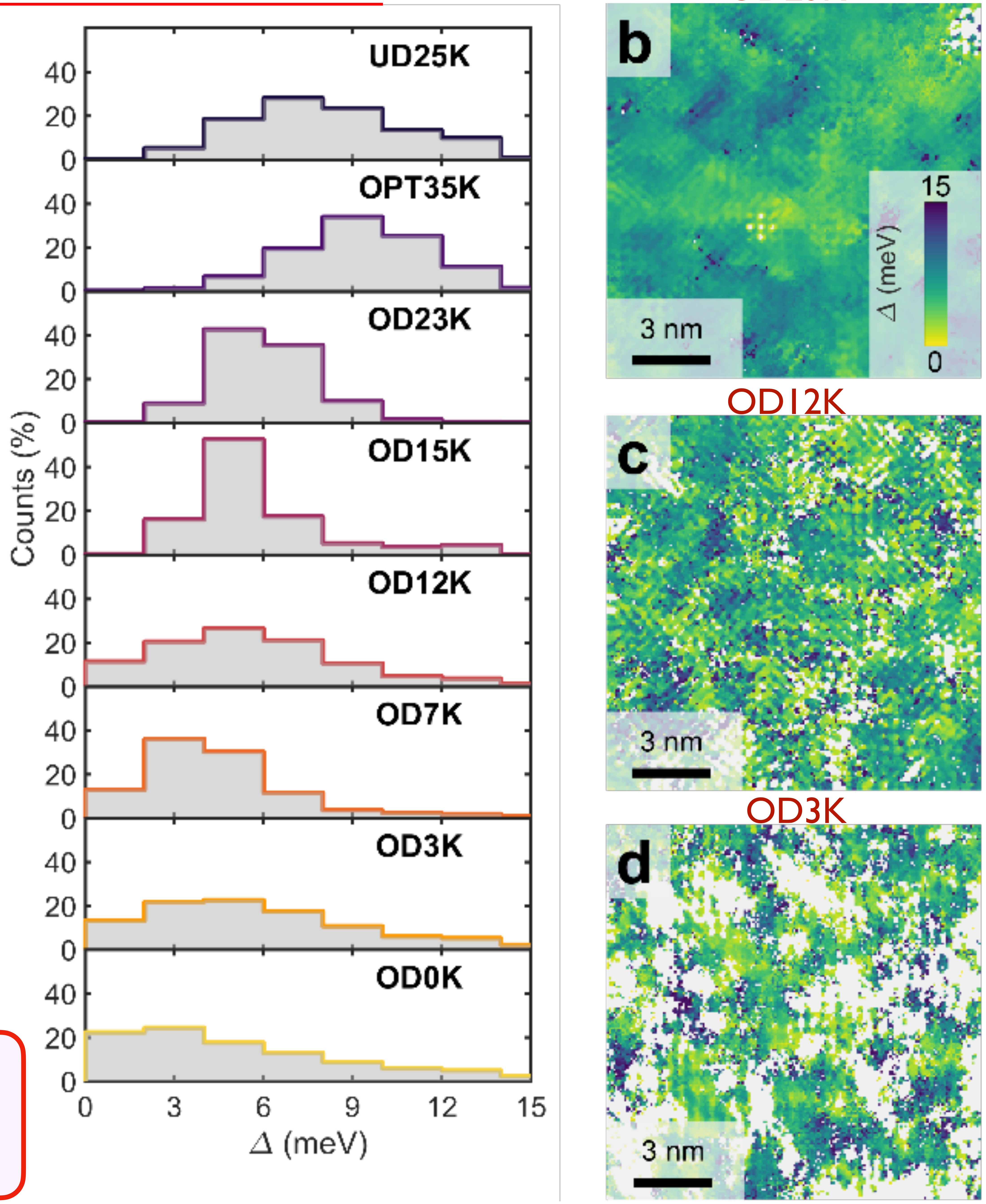
Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

Nature Materials **22**, 703 (2023)

Disorder in t_{ij} induces disorder in $J_{ij} = \dfrac{t_{ij}^2}{U}$

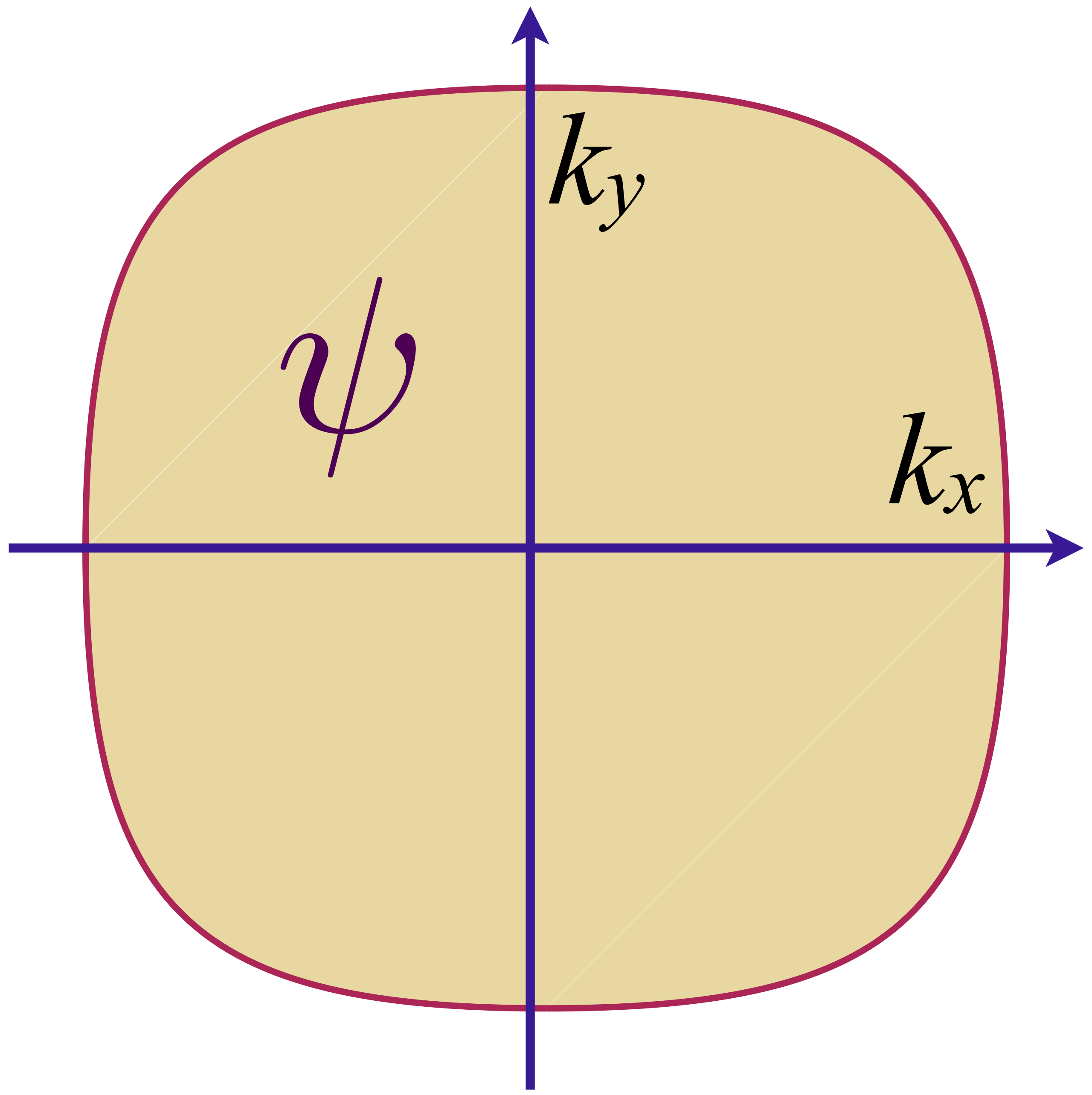


Fermi surface + critical boson with potential and interaction disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. gauge-charged boson or Ising-nematic order



$$+ [\phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})] \, \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r}) \, \phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

ϕ^2 “mass” disorder is strongly relevant;
rescale ϕ to move disorder to the Yukawa coupling

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Transport properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon......D. van der Marel, A. Georges, Nat. Comm. **14**, 3033 (2023)

Electronic properties of a marginal Fermi liquid:

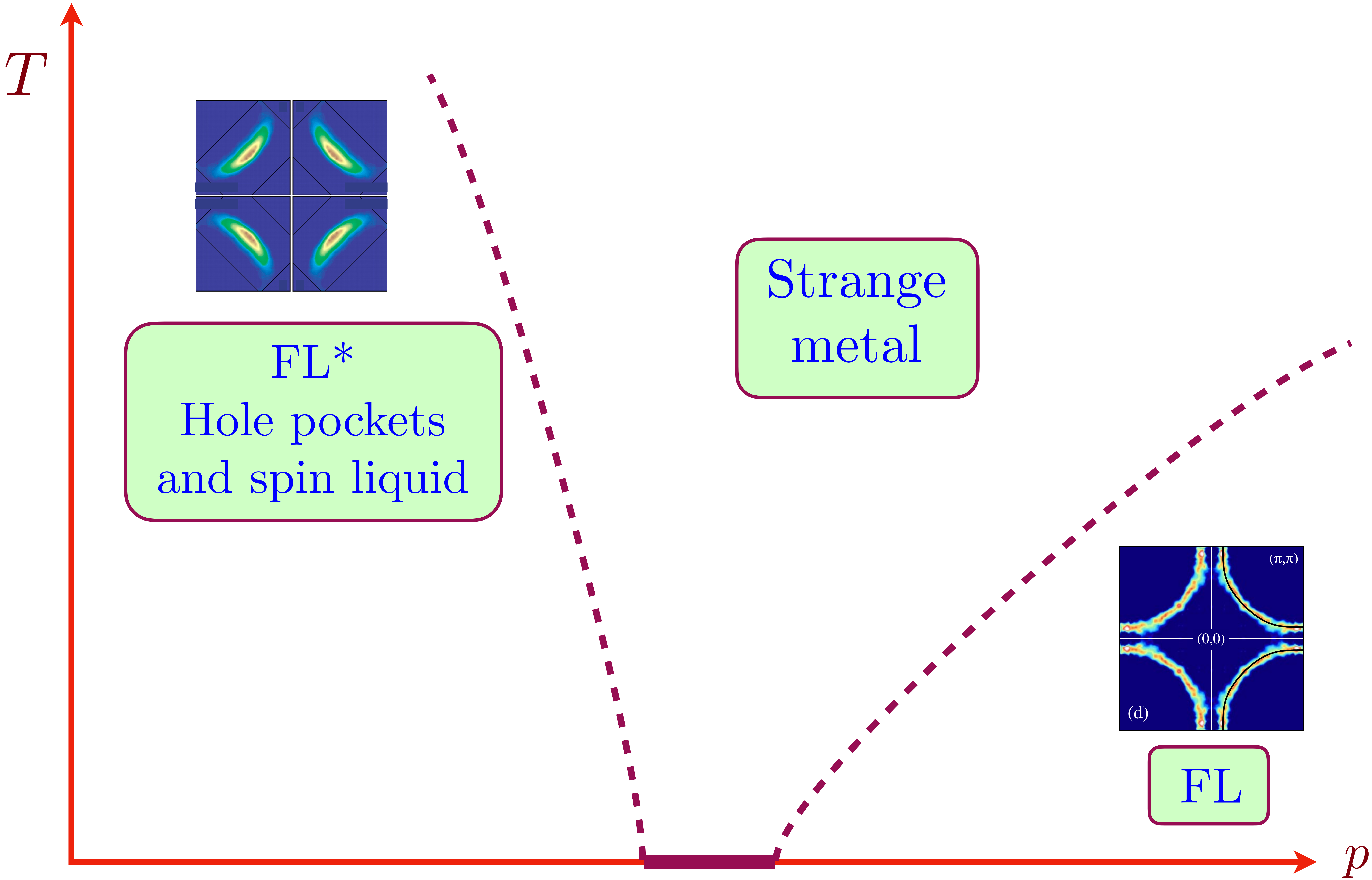
1. Photoemission: nearly marginal Fermi liquid electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

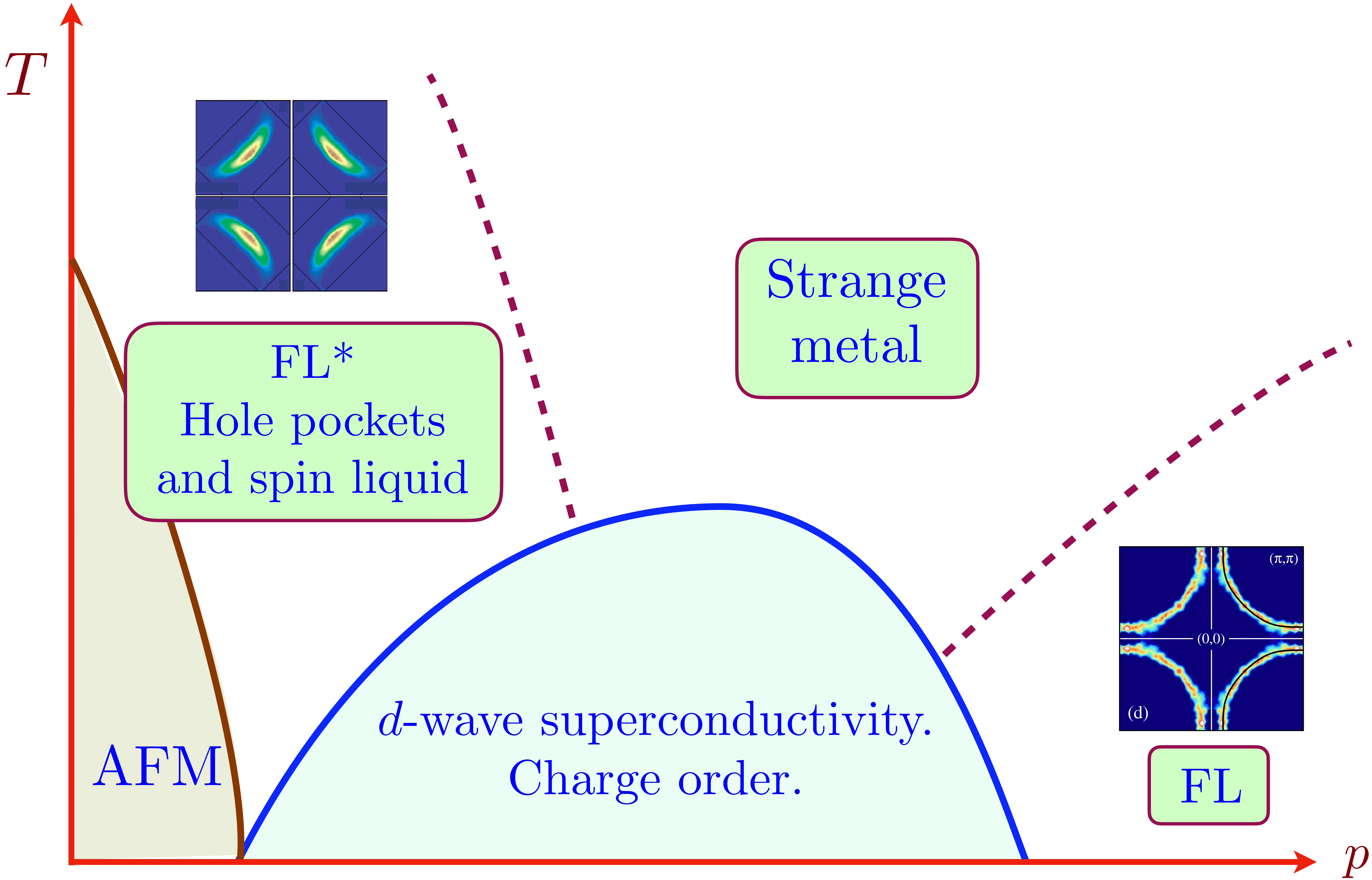
T.J. Reber....D. Dessau, Nat. Comm. **10**, 5737 (2019)

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, RMP (2022)

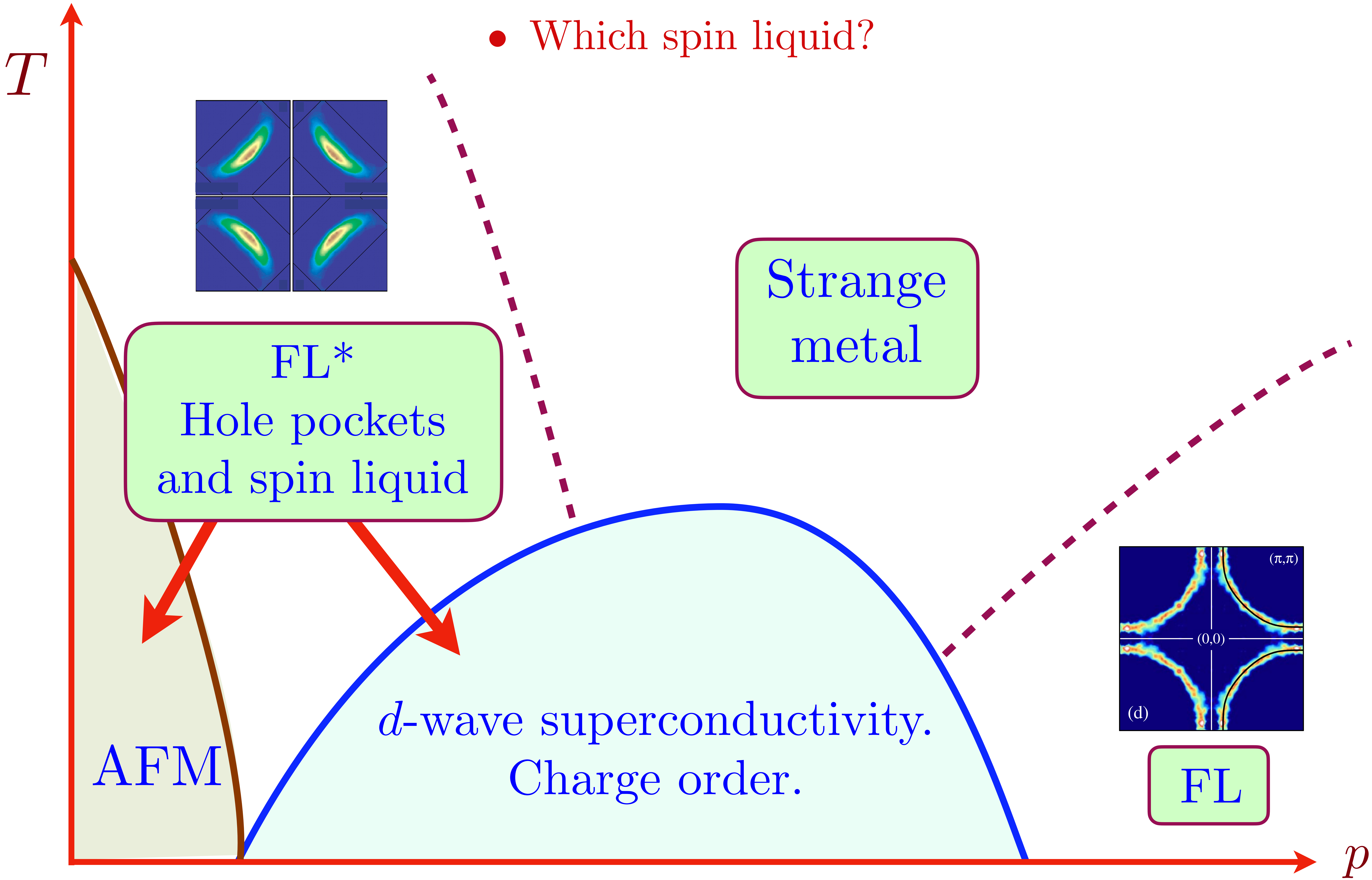


T
FL*
Hole pockets
and spin liquid
Strange
metal
(π,π)
(0,0)
(d)
FL
p

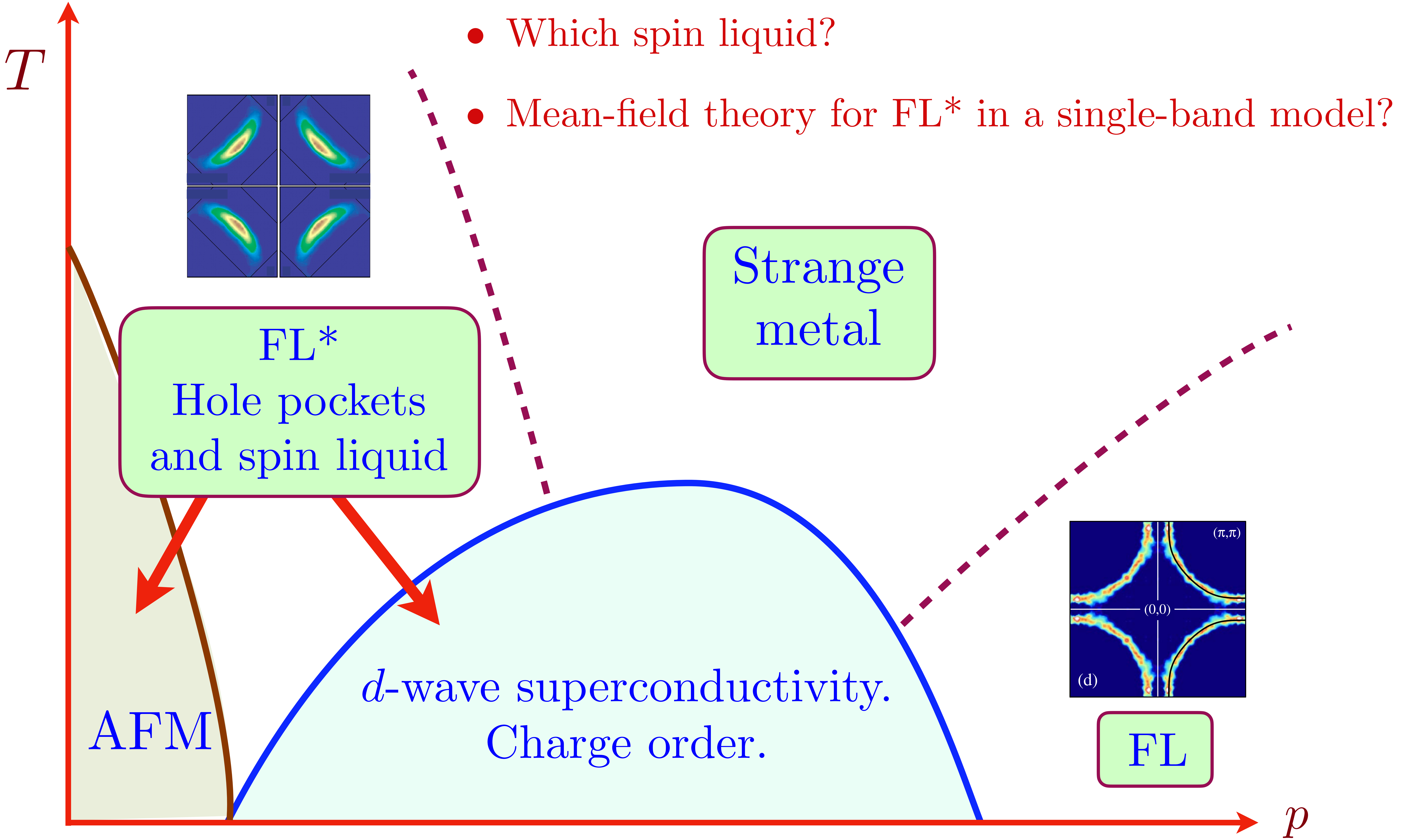


T
FL*
Hole pockets
and spin liquid
Strange
metal
(π,π)
(0,0)
(d)
FL
AFM
d-wave superconductivity.
Charge order.
p


The onset of conventional order is a *confinement transition* for the gauge theory describing the fractionalized excitations of the FL* spin liquid
T
p
FL*
Hole pockets
and spin liquid
Strange
metal
AFM
d-wave superconductivity.
Charge order.
(π,π)
(0,0)
(d)
FL



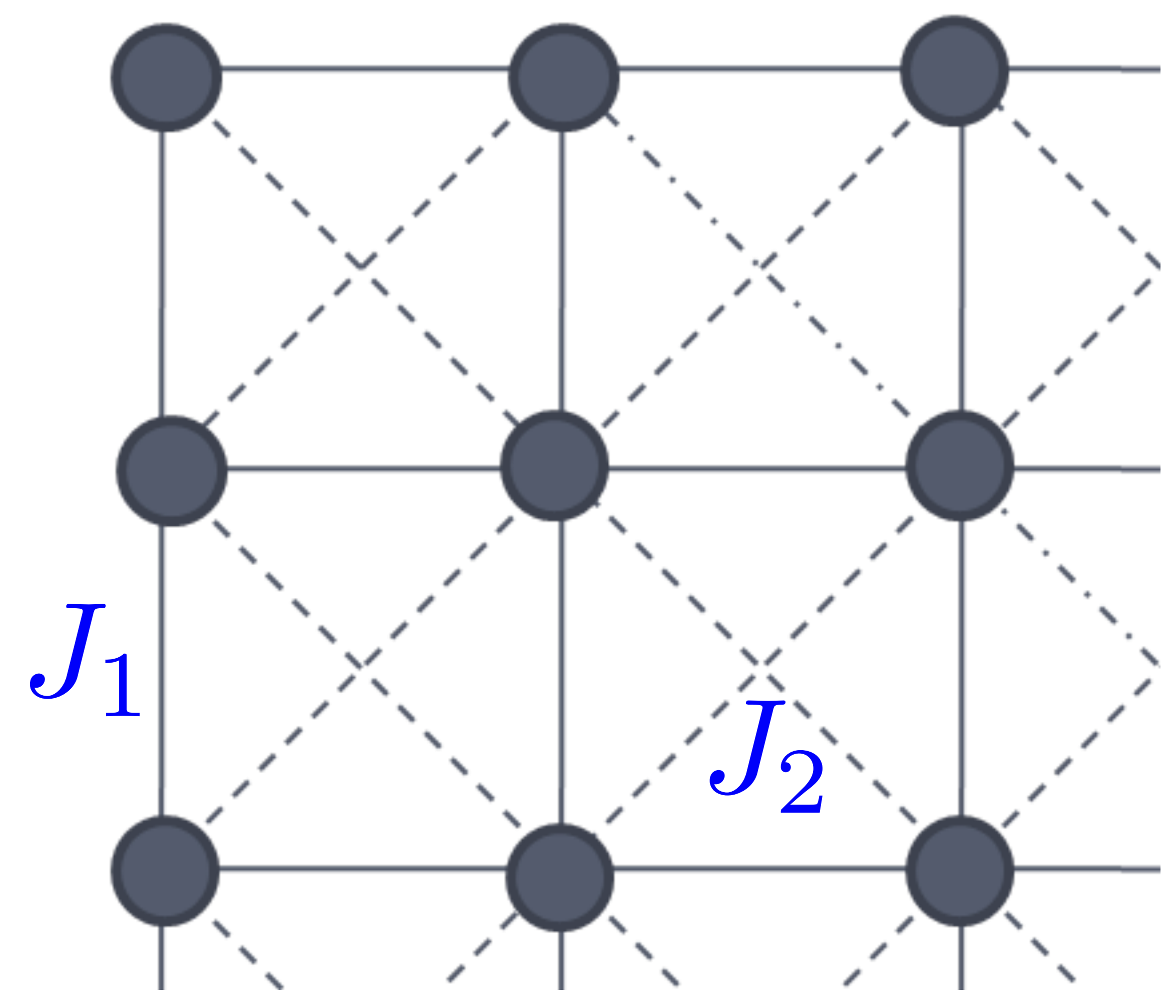
• Which spin liquid?
T
p
FL*
Hole pockets
and spin liquid
Strange
metal
AFM
d-wave superconductivity.
Charge order.
(π,π)
(0,0)
(d)
FL

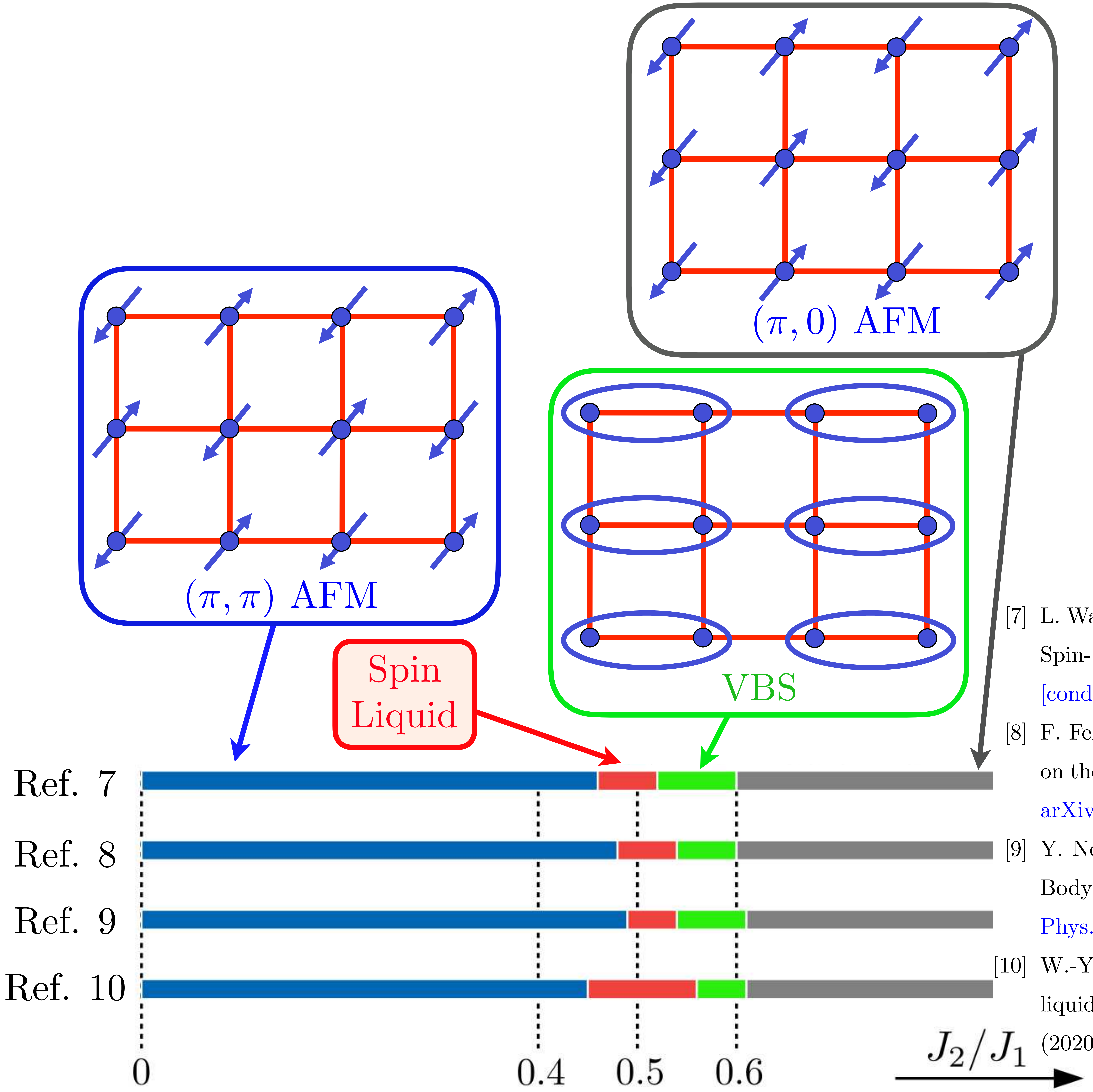


• Which spin liquid?
• Mean-field theory for FL* in a single-band model?
T
FL*
Hole pockets
and spin liquid
Strange
metal
AFM
d-wave superconductivity.
Charge order.
(π,π)
(0,0)
(d)
FL
p

The CP^1/π-flux spin liquid

$$H = J_1 \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





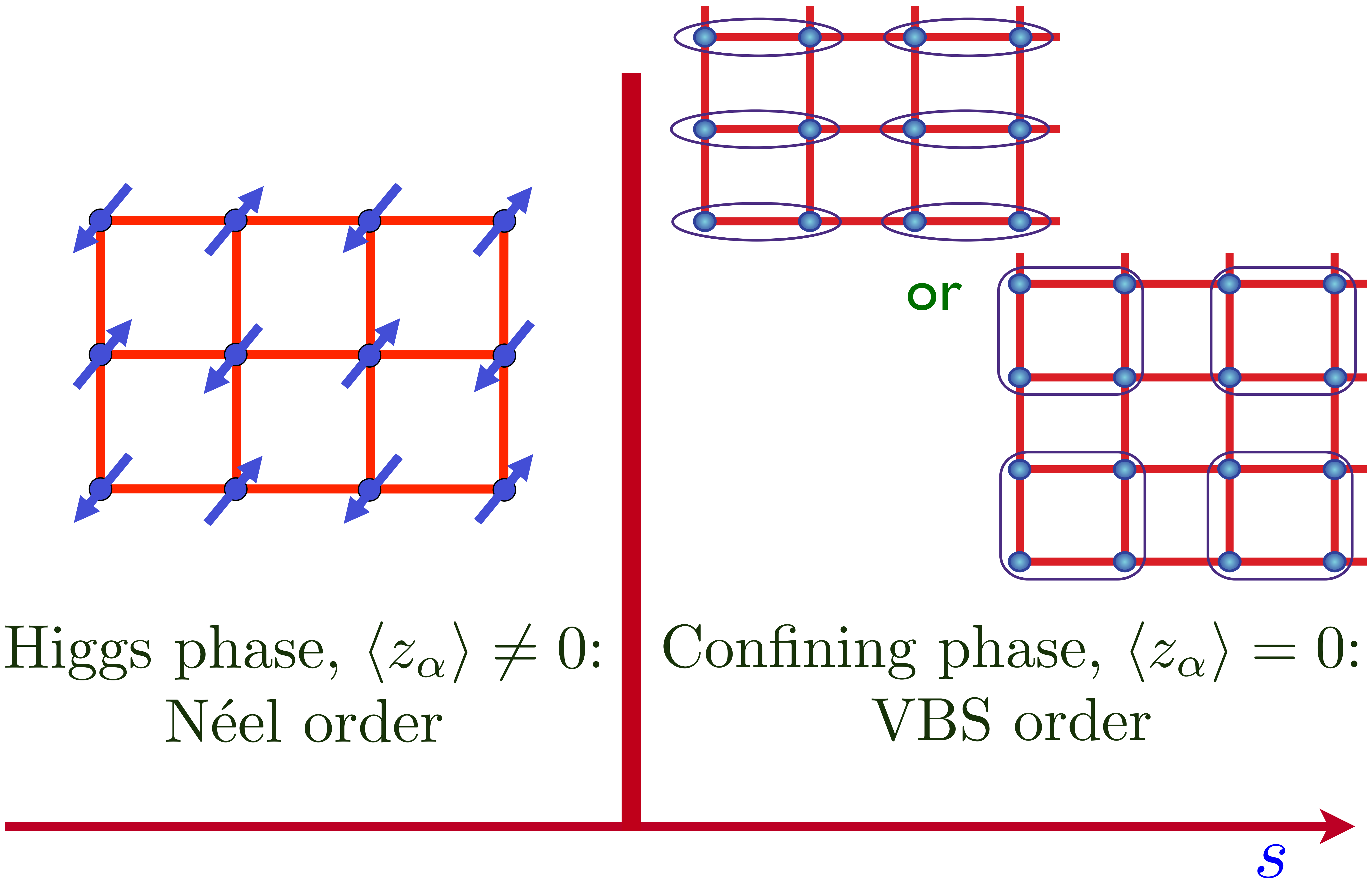
[7] L. Wang and A. W. Sandvik, "Critical Level Crossings and Gapless Spin Liquid in the Square-Lattice Spin-1 /2 J_1-J_2 Heisenberg Antiferromagnet," Phys. Rev. Lett. **121**, 107202 (2018), arXiv:1702.08197 [cond-mat.str-el].

[8] F. Ferrari and F. Becca, "Gapless spin liquid and valence-bond solid in the J_1-J_2 Heisenberg model on the square lattice: Insights from singlet and triplet excitations," Phys. Rev. B **102**, 014417 (2020), arXiv:2005.12941 [cond-mat.str-el].

[9] Y. Nomura and M. Imada, "Dirac-Type Nodal Spin Liquid Revealed by Refined Quantum Many-Body Solver Using Neural-Network Wave Function, Correlation Ratio, and Level Spectroscopy," Phys. Rev. X **11**, 031034 (2021), arXiv:2005.14142 [cond-mat.str-el].

[10] W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, "Gapless quantum spin liquid and global phase diagram of the spin-1/2 J_1-J_2 square antiferromagnetic Heisenberg model," (2020), arXiv:2009.01821 [cond-mat.str-el].

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \qquad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

D.P. Arovas and A. Auerbach, PRB **38**, 316 (1988)

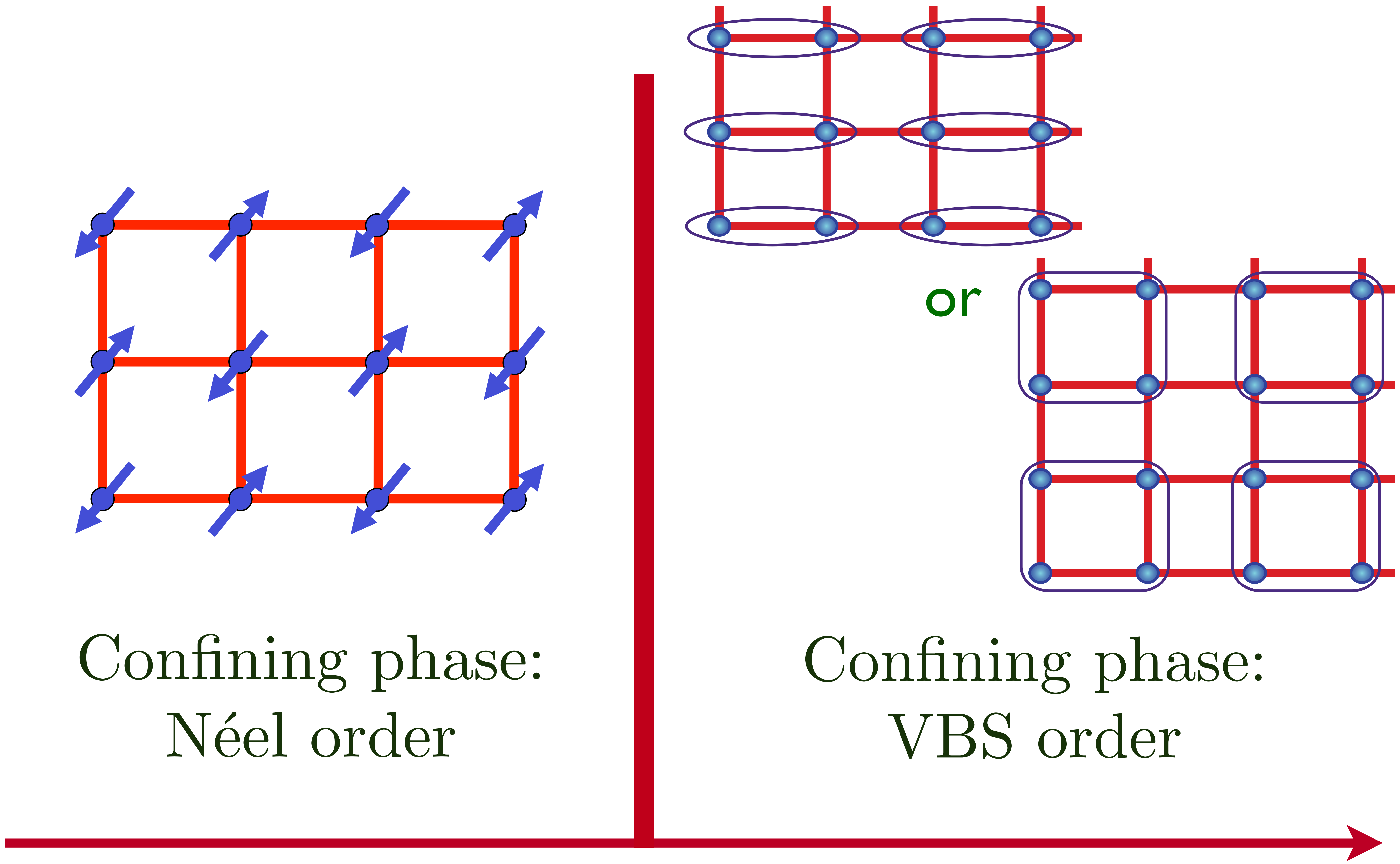
Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. S., PRL **62**, 1694 (1989)

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger fermions

I. Affleck and J.B. Marston, PRB **37**, 3774 (1988)

$$\boldsymbol{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} f_{i\alpha}^\dagger f_{i\alpha} = 1$$

π-flux mean-field theory
with gapless spinons at 2 Dirac points.
Low energy theory of $N_f = 2$
Dirac fermions Ψ_s coupled to
an emergent $\mathrm{SU}(2)_N$ gauge field.
Confining order parameters
are Néel and VBS states,
with a global $\mathrm{SO}(5)_f$ symmetry!

$$\mathcal{L} = i\overline{\Psi}_s \gamma_\mu D_\mu \Psi_s + \ldots$$

Dual to $\mathbb{CP}^1$ U(1) gauge theory.

A. Tanaka and X. Hu, PRL **95**, 036402 (2005); T. Senthil and M.P.A. Fisher PRB **74**, 064405 (2006); Ying Ran and X.-G. Wen, cond-mat/0609620
C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, Physical Review X **7**, 031051 (2017)

arXiv > cond-mat > arXiv:2306.16435

Condensed Matter > Strongly Correlated Electrons

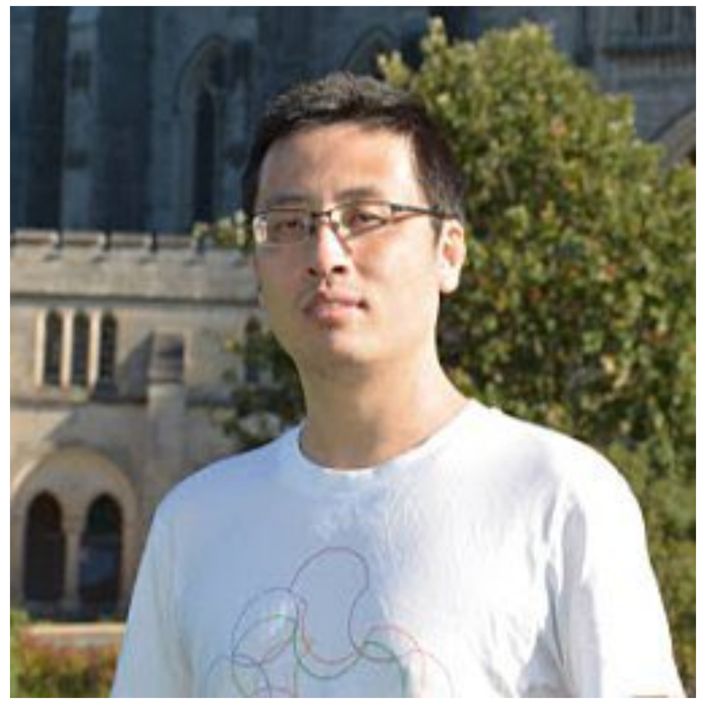
[Submitted on 28 Jun 2023]

The $\mathrm{SO}(5)$ Deconfined Phase Transition under the Fuzzy Sphere Microscope: Approximate Conformal Symmetry, Pseudo-Criticality, and Operator Spectrum

Zheng Zhou, Liangdong Hu, W. Zhu, Yin-Chen He

The deconfined quantum critical point (DQCP) is an example of phase transitions beyond the Landau symmetry breaking paradigm that attracts wide interest. However, its nature has not been settled after decades of study. In this paper, we apply the recently proposed fuzzy sphere regularization to study the $\mathrm{SO}(5)$ non-linear sigma model (NLσM) with a topological Wess-Zumino-Witten term, which serves as a dual description of the DQCP with an exact $\mathrm{SO}(5)$ symmetry. We demonstrate that the fuzzy sphere functions as a powerful microscope, magnifying and revealing a wealth of crucial information about the DQCP, ultimately paving the way towards its final answer. In particular, through exact diagonalization, we provide clear evidence that the DQCP exhibits approximate conformal symmetry. The evidence includes the existence of a conserved $\mathrm{SO}(5)$ symmetry

Ancilla theory of FL* in a single-band model



Ya-Hui Zhang

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\sigma}^\dagger c_{i\sigma}$$

We use the operator equation (valid on each site i):

$$U \left(n_\uparrow - \frac{1}{2} \right) \left(n_\downarrow - \frac{1}{2} \right) = -\frac{2U}{3} \boldsymbol{S}^2 + \frac{U}{4}$$

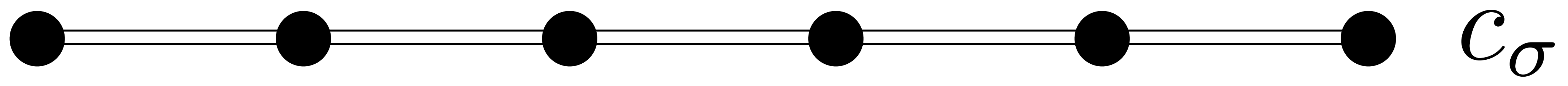
Then we decouple the interaction via

$$\exp\left(\frac{2U}{3} \sum_i \int d\tau \boldsymbol{S}_i^2 \right) = \int \mathcal{D}\boldsymbol{\Phi}_i(\tau) \exp\left(-\sum_i \int d\tau \left[\frac{3}{8U} \boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \right] \right)$$

This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

$\boldsymbol{\Phi}_i$ is the creation/annihilation operator for charge 0, spin $S = 1$ particle.

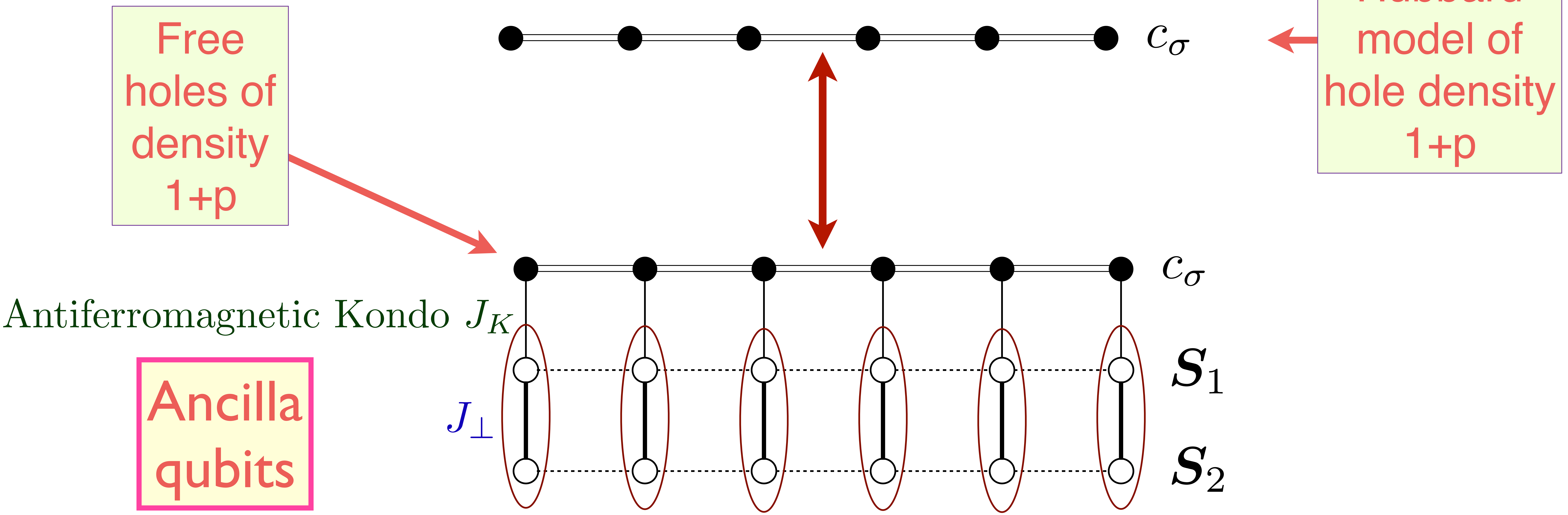
Ancilla theory of the Hubbard model



Hubbard model of hole density 1+p

Ancilla theory of the Hubbard model

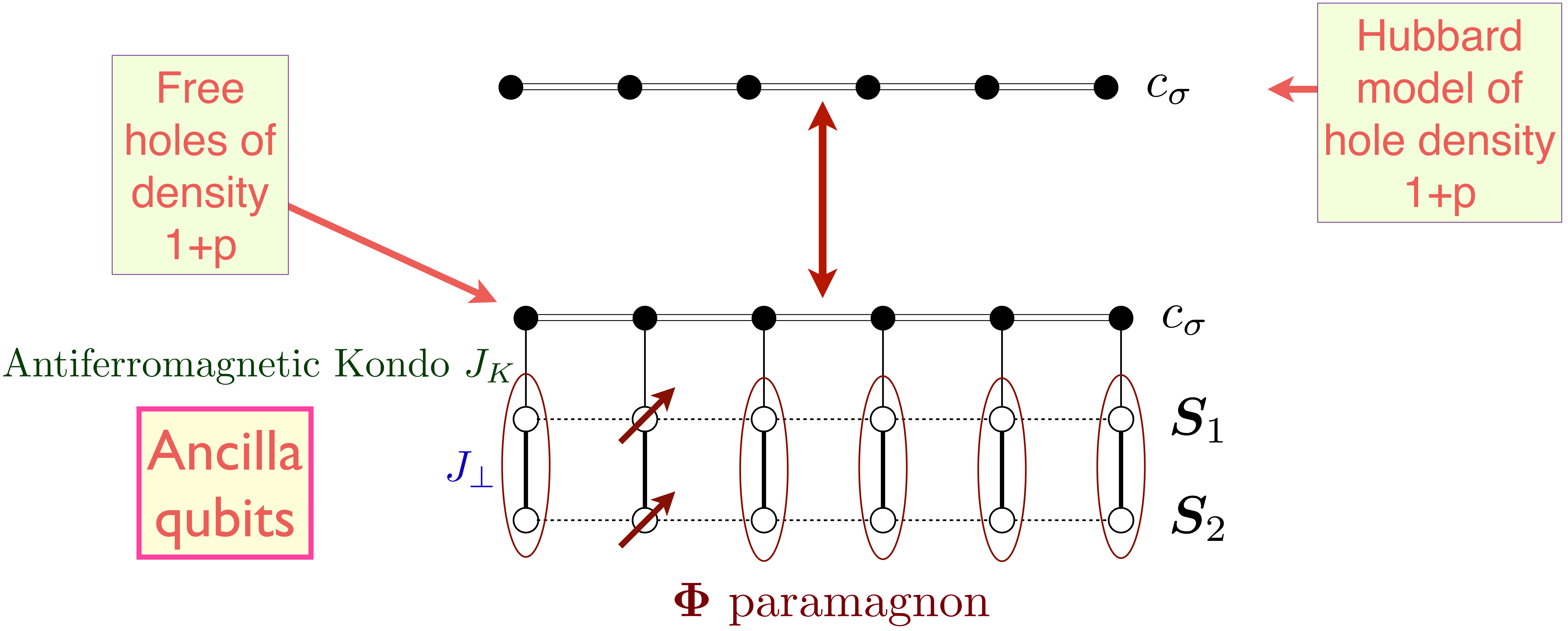
Ya-Hui Zhang and S. S.,
PRR **2**, 023172 (2020)



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + J_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

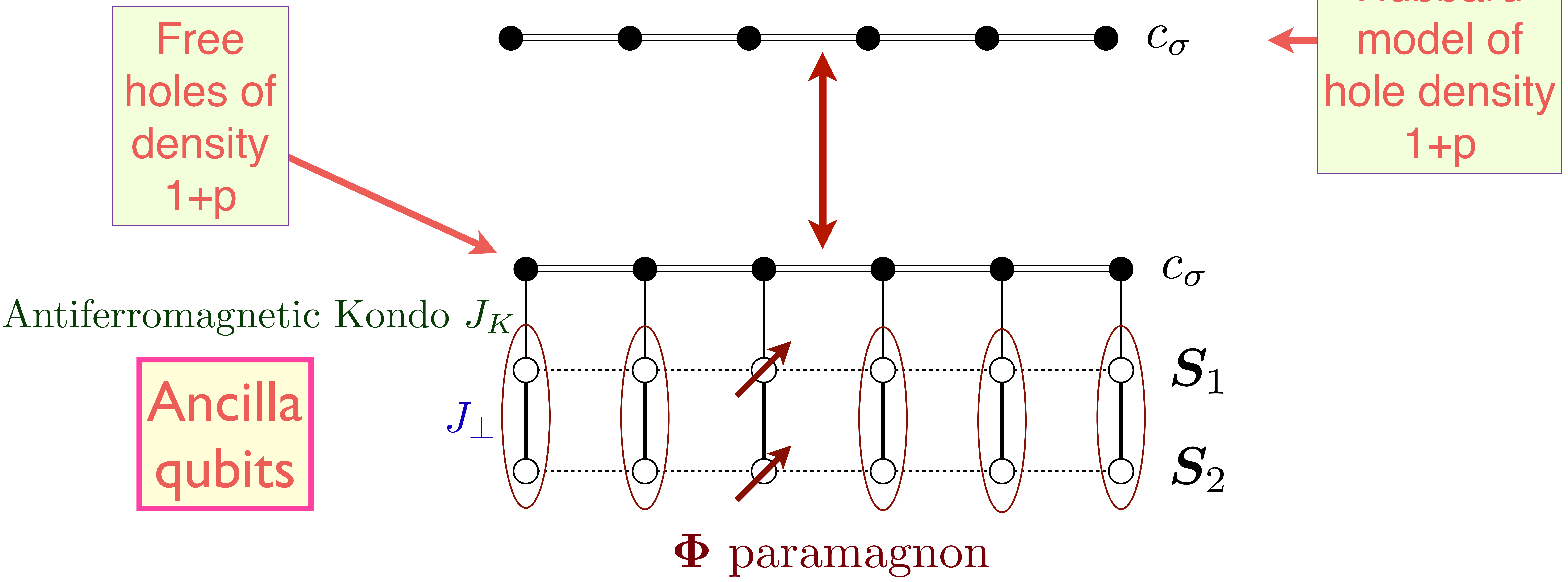
Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\alpha}^\dagger c_{\mathbf{p}\alpha} + J_K \sum_i c_{i\alpha}^\dagger \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



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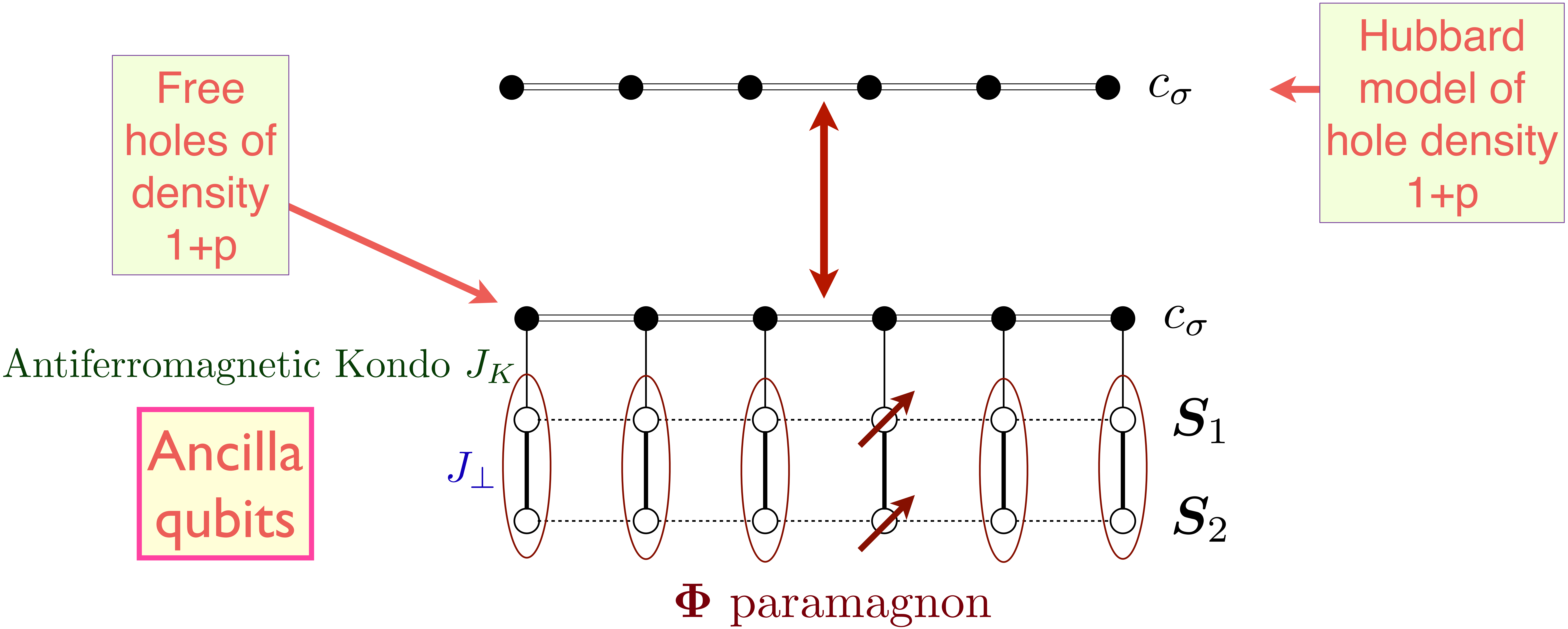
Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



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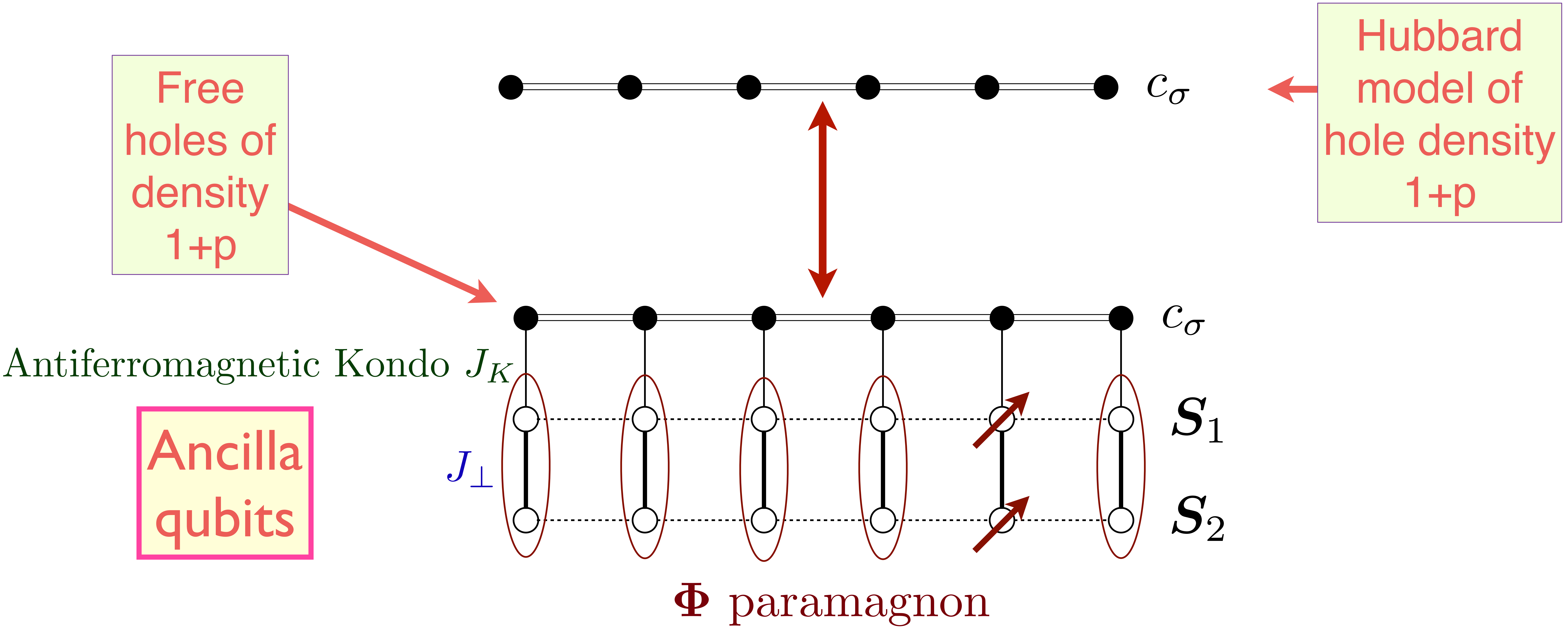
Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



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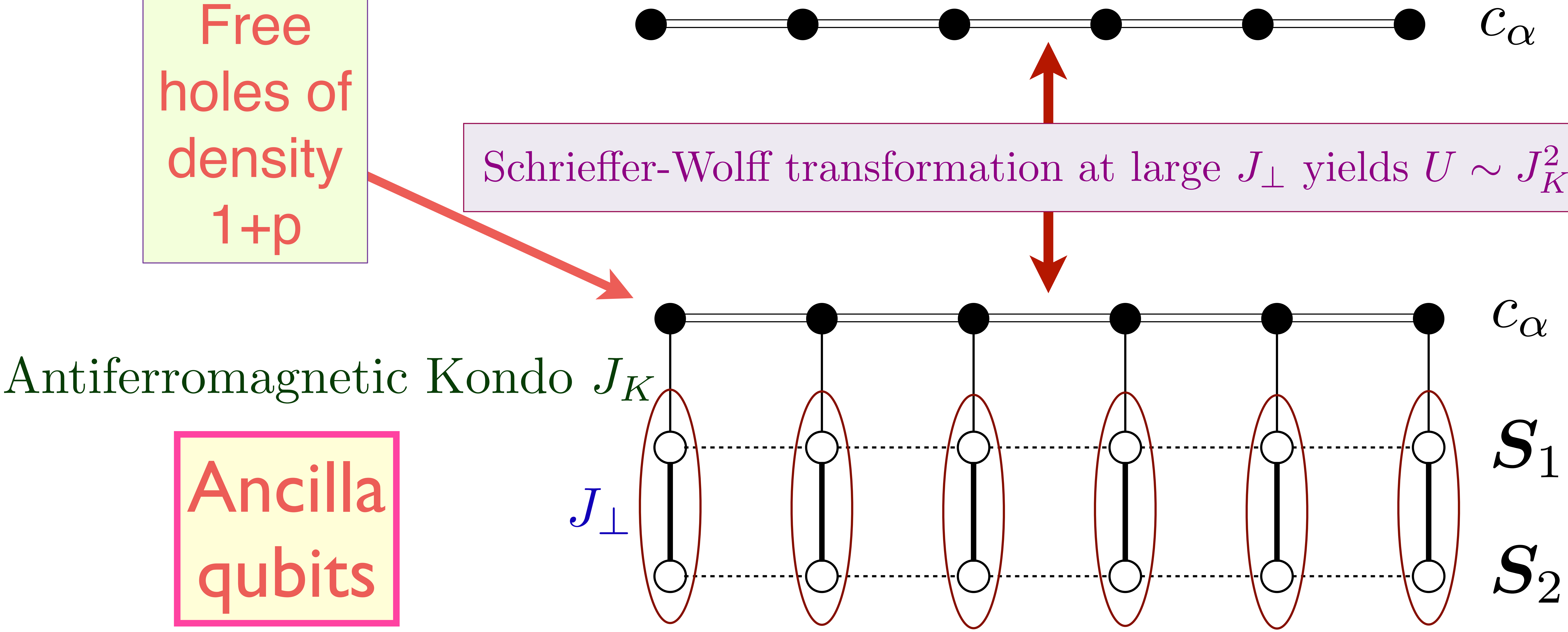
Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Free holes of density 1+p

Hubbard model of hole density 1+p

Schrieffer-Wolff transformation at large $J_\perp$ yields $U \sim J_K^2/J_\perp$

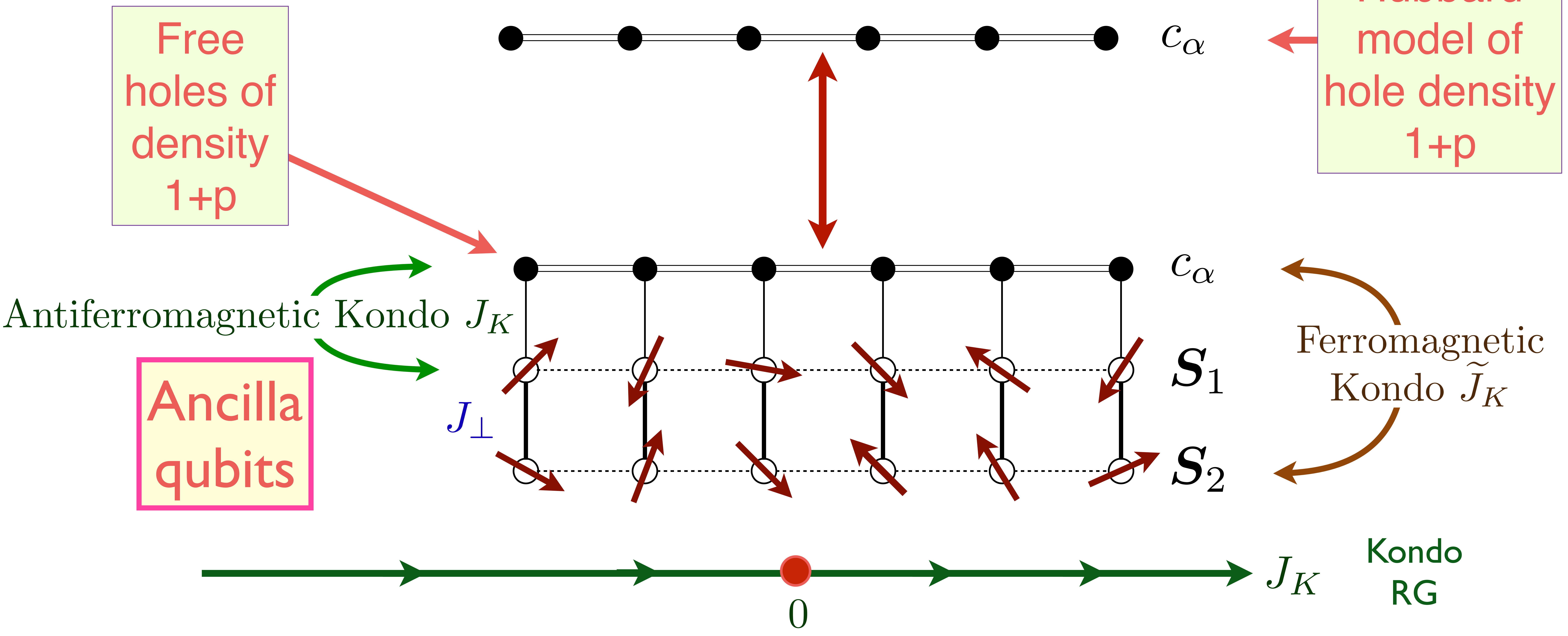


Ancilla qubits

$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\alpha} c_{\mathbf{p}\alpha} + J_K \sum_i c^\dagger_{i\alpha} \frac{\boldsymbol{\sigma}_{\alpha\alpha'}}{2} c_{i\alpha'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



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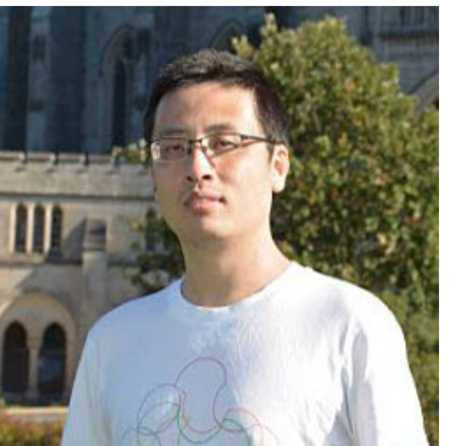
Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

Ya-Hui Zhang

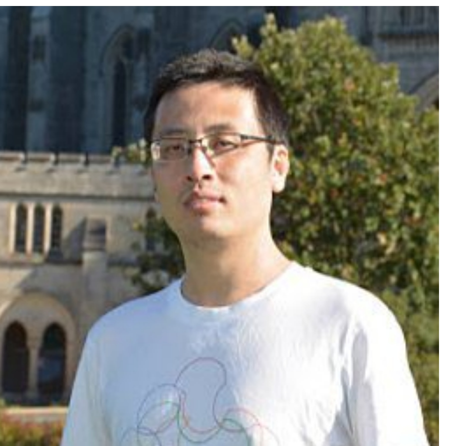


Ancilla theory of the Hubbard model

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

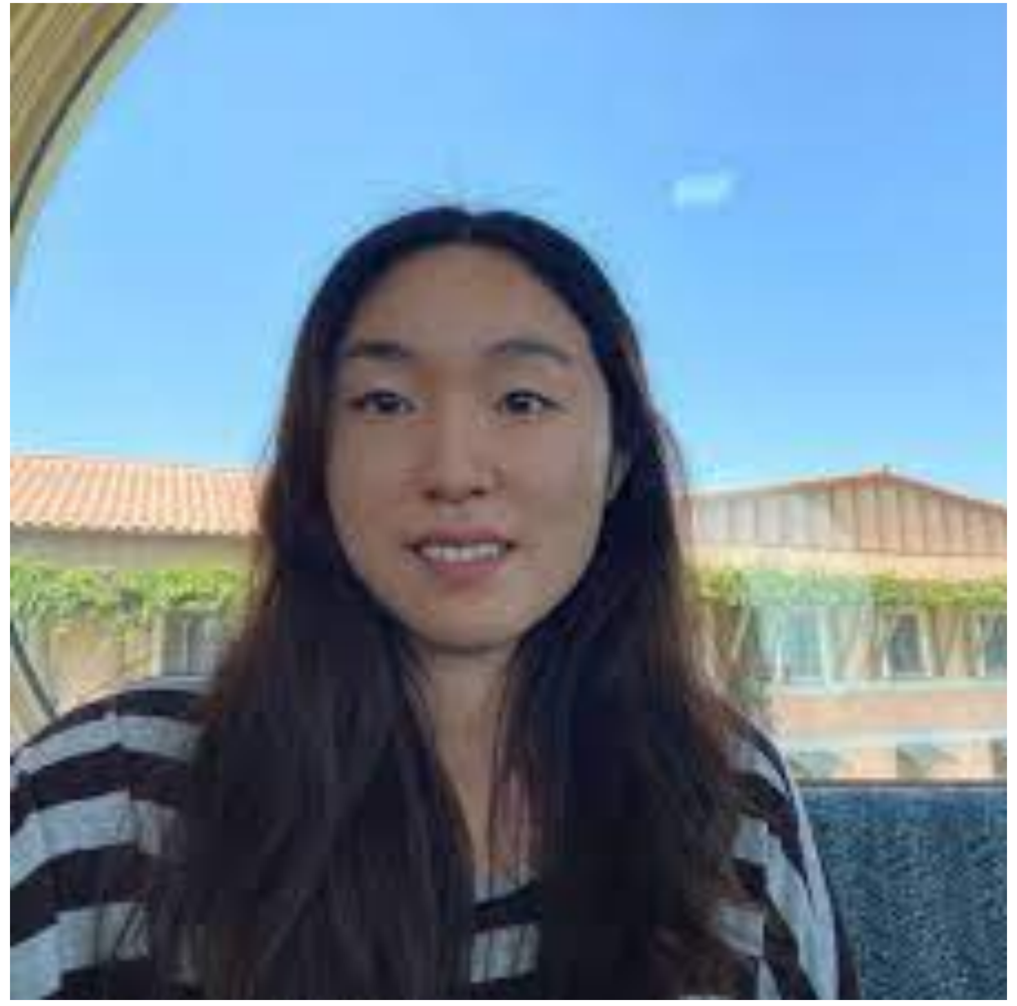
Pseudogap metal =
Kondo Lattice Heavy Fermi Liquid
$\oplus$
Spin Liquid

Ya-Hui Zhang



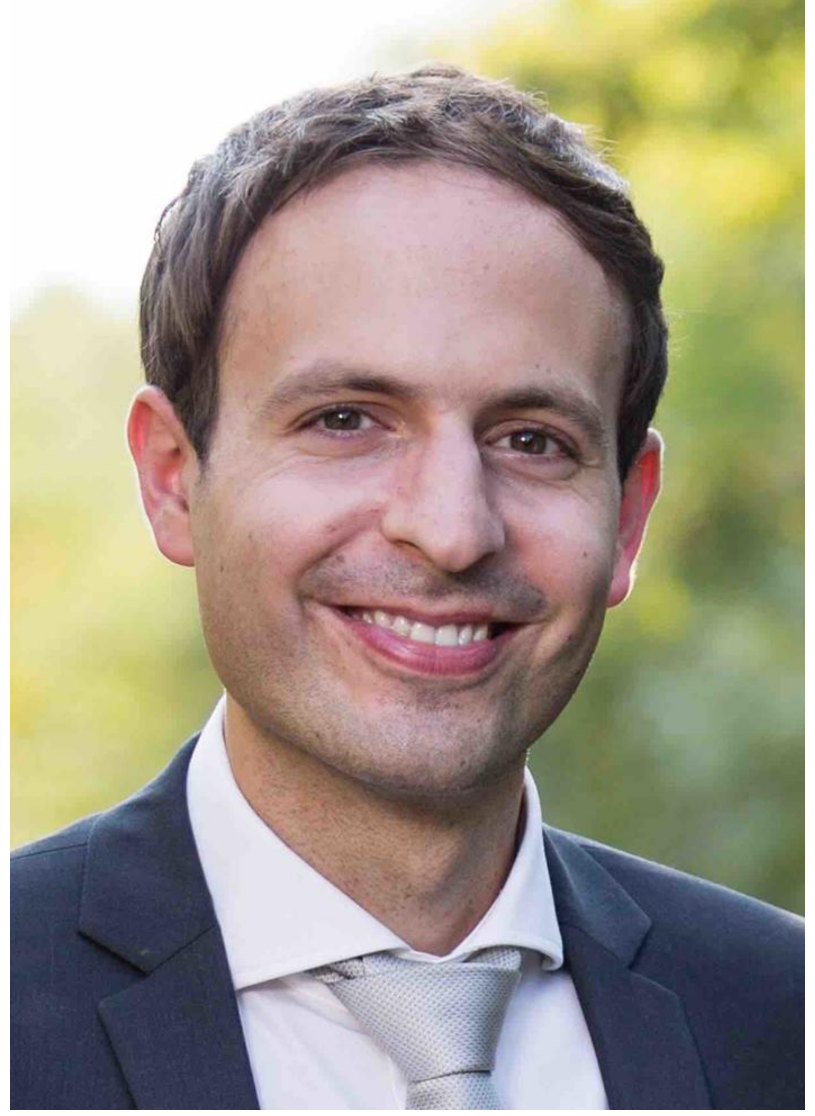
From CP^1/π-flux FL*
to
d-wave superconductivity

Maine Christos

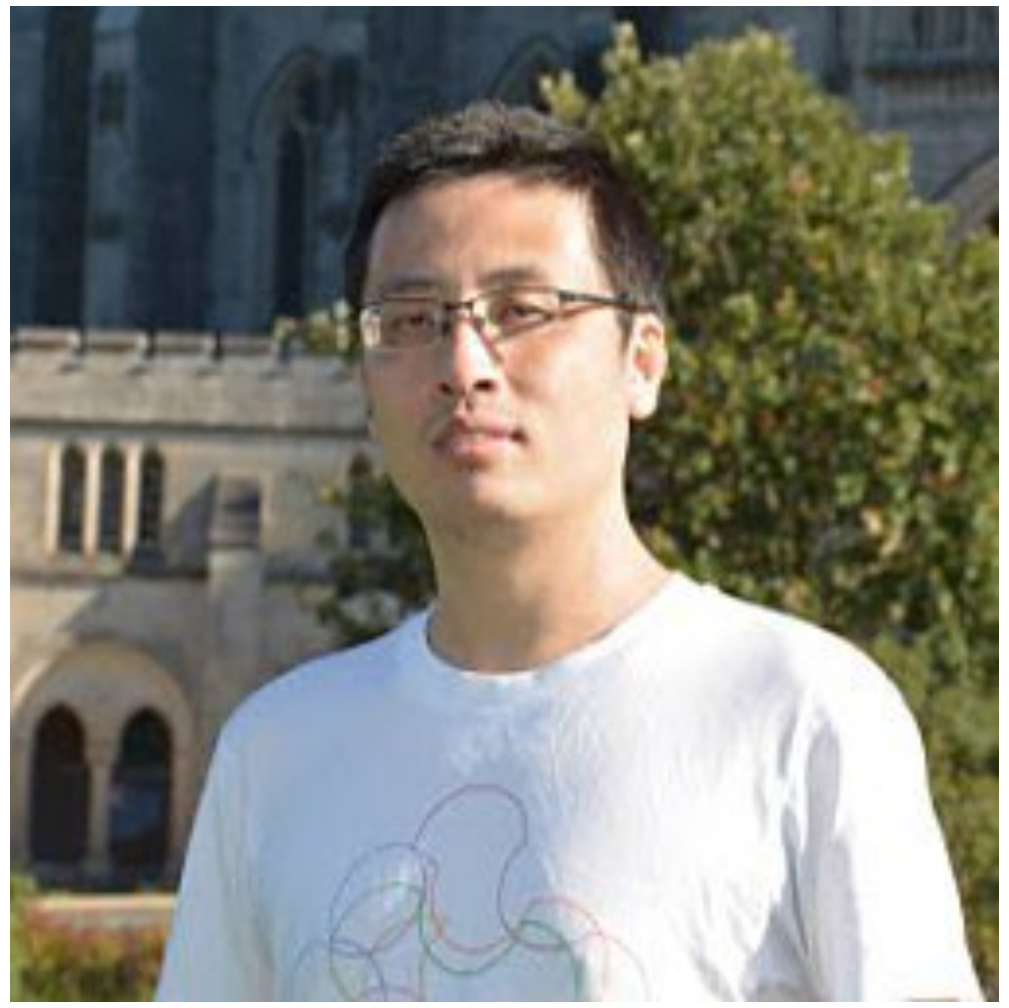


Zhu-Xi Luo
→GA Tech

Henry
Shackleton



Mathias Scheurer
Innsbruck→Stuttgart

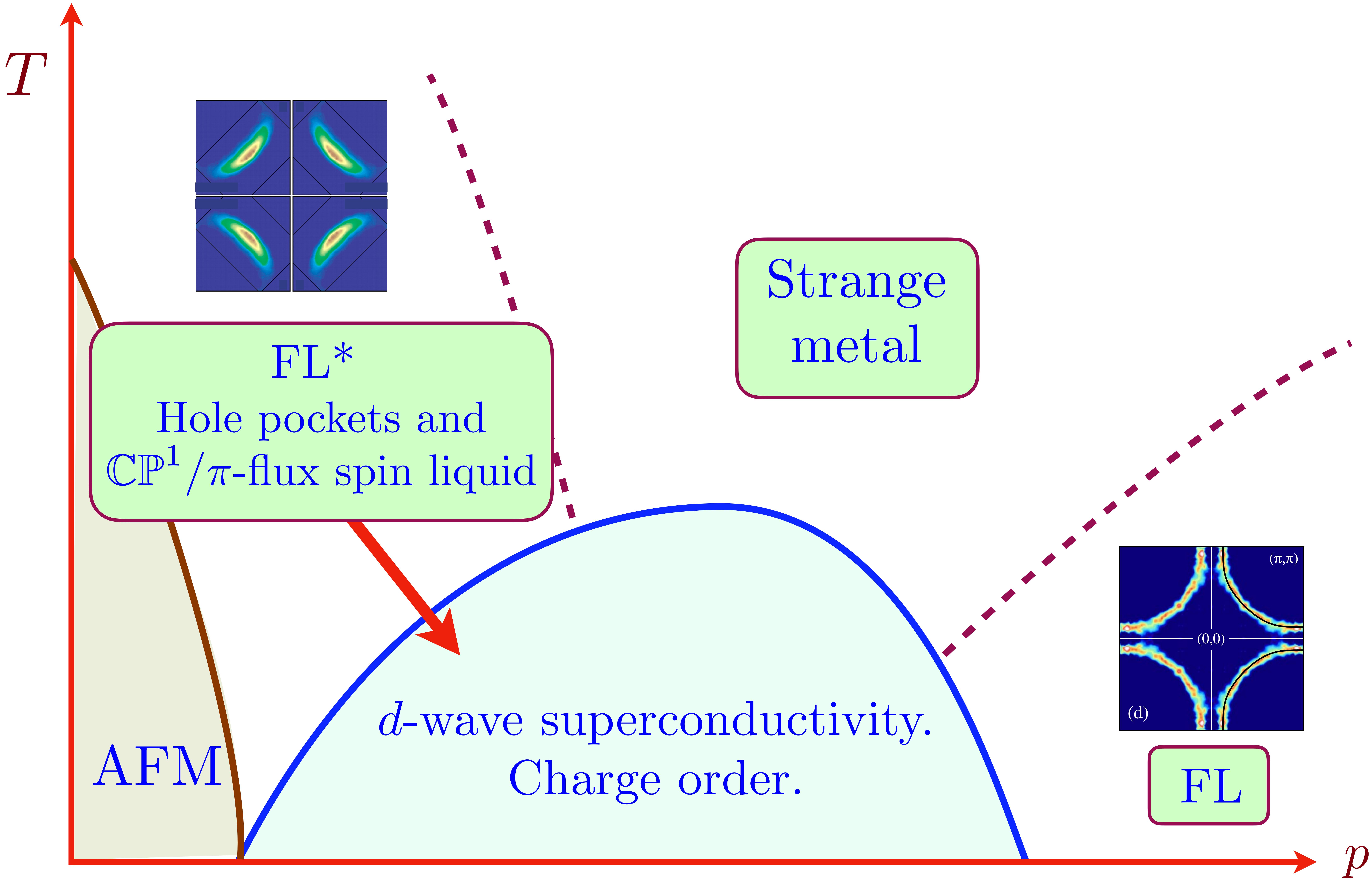


Ya-Hui Zhang
Johns Hopkins

Alexander
Nikolaenko

Darshan Joshi
TIFR Hyderabad

Jonas von Milczewski



T
FL*
Hole pockets and
$\mathbb{CP}^1/\pi$-flux spin liquid
Strange
metal
(π,π)
(0,0)
(d)
FL
AFM
d-wave superconductivity.
Charge order.
p

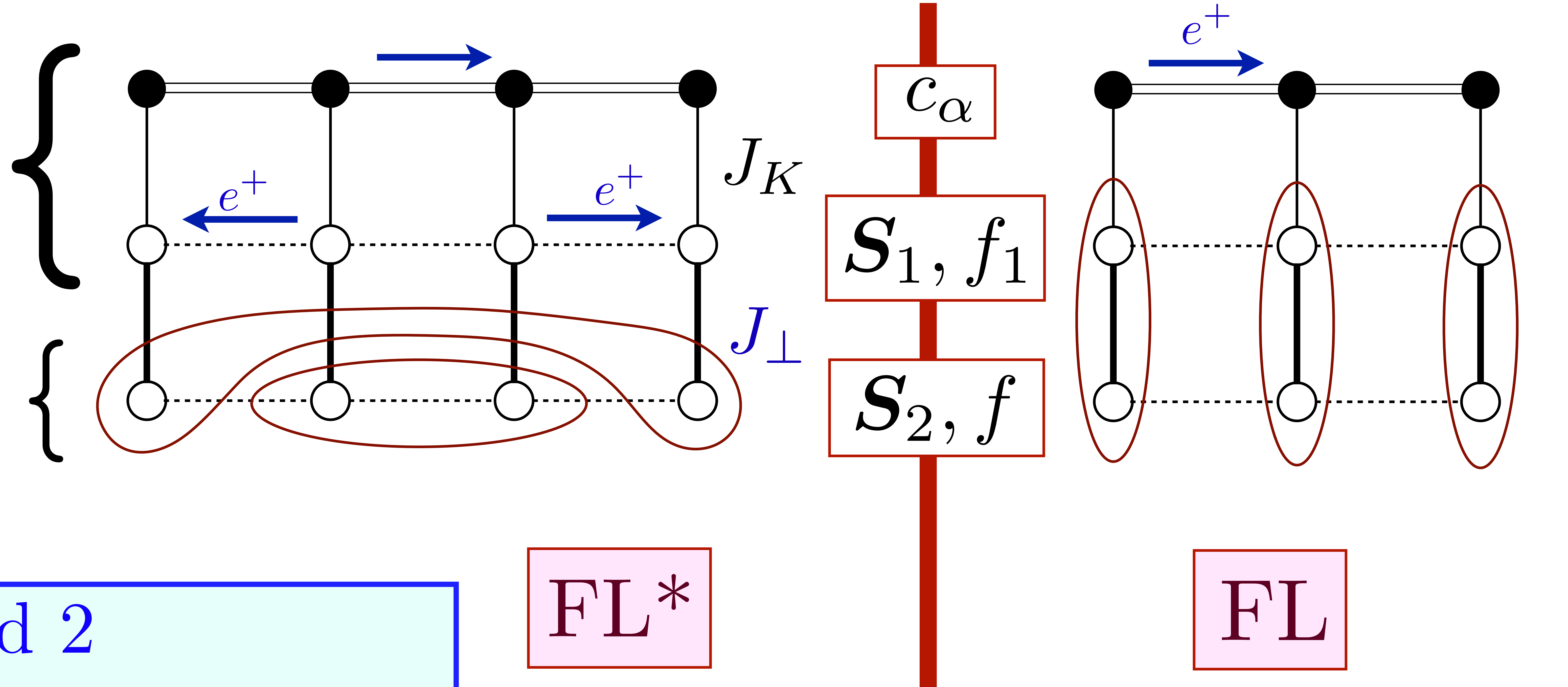
Ancilla theory of the Hubbard model

B has same quantum numbers as boson in X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996)

Ancilla theory of the Hubbard model

Higgs field 1
$\Phi \sim c_\alpha^\dagger f_{1\alpha}$
$\langle \Phi \rangle \neq 0$

π-flux spin liquid
of f_α with $\mathrm{SU}(2)_N$
gauge field



Higgs field 2
Charge e, $\mathrm{SU}(2)_N$ fundamental

$$B \sim \begin{pmatrix} f_{1\alpha}^\dagger f_\alpha \\ \varepsilon_{\alpha\beta} f_{1\alpha}^\dagger f_\beta^\dagger \end{pmatrix}$$

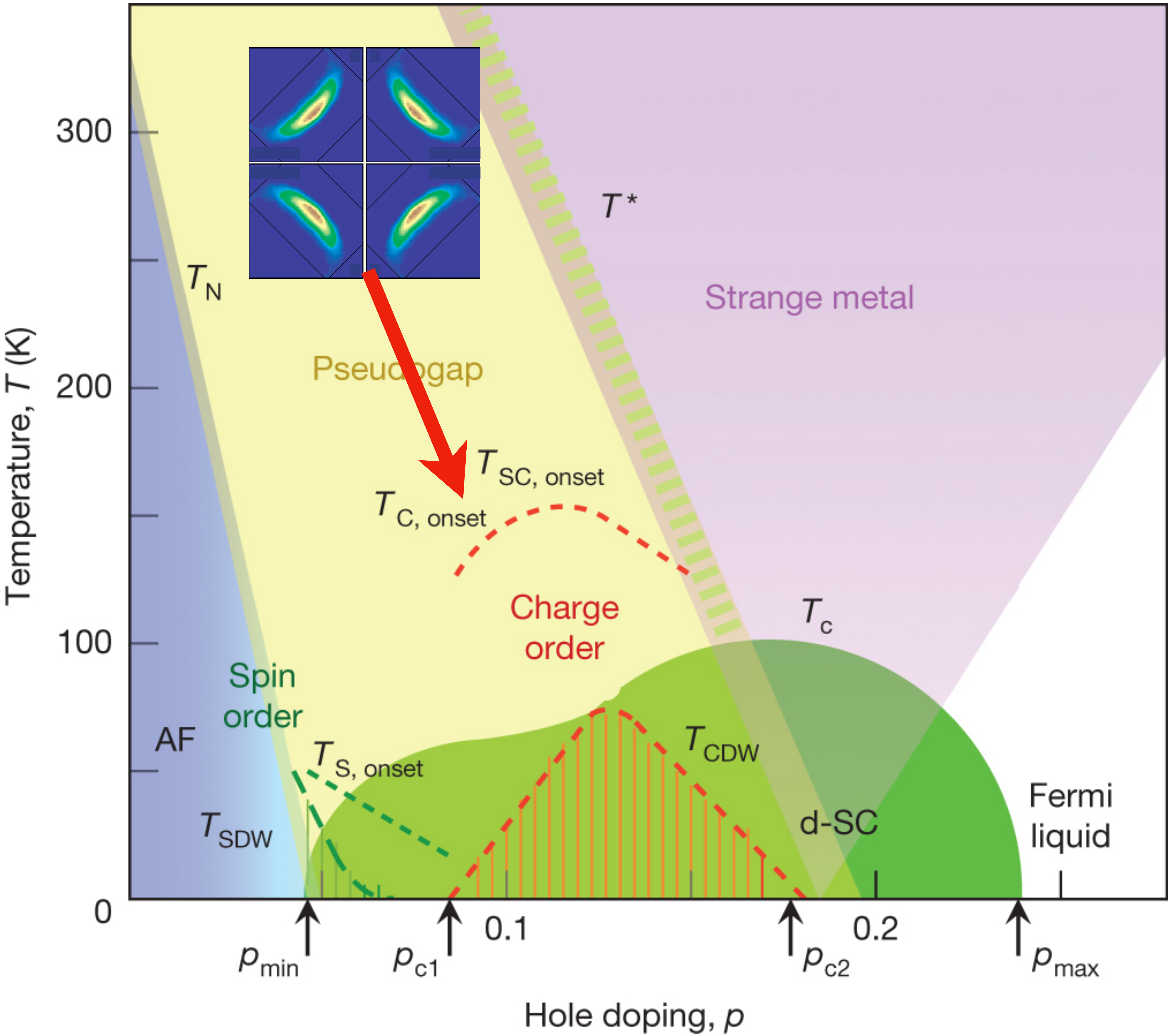
J_K

doping p

Higgs field Φ drives FL*-strange metal-FL, Higgs field B drives FL*-dSC-CDW

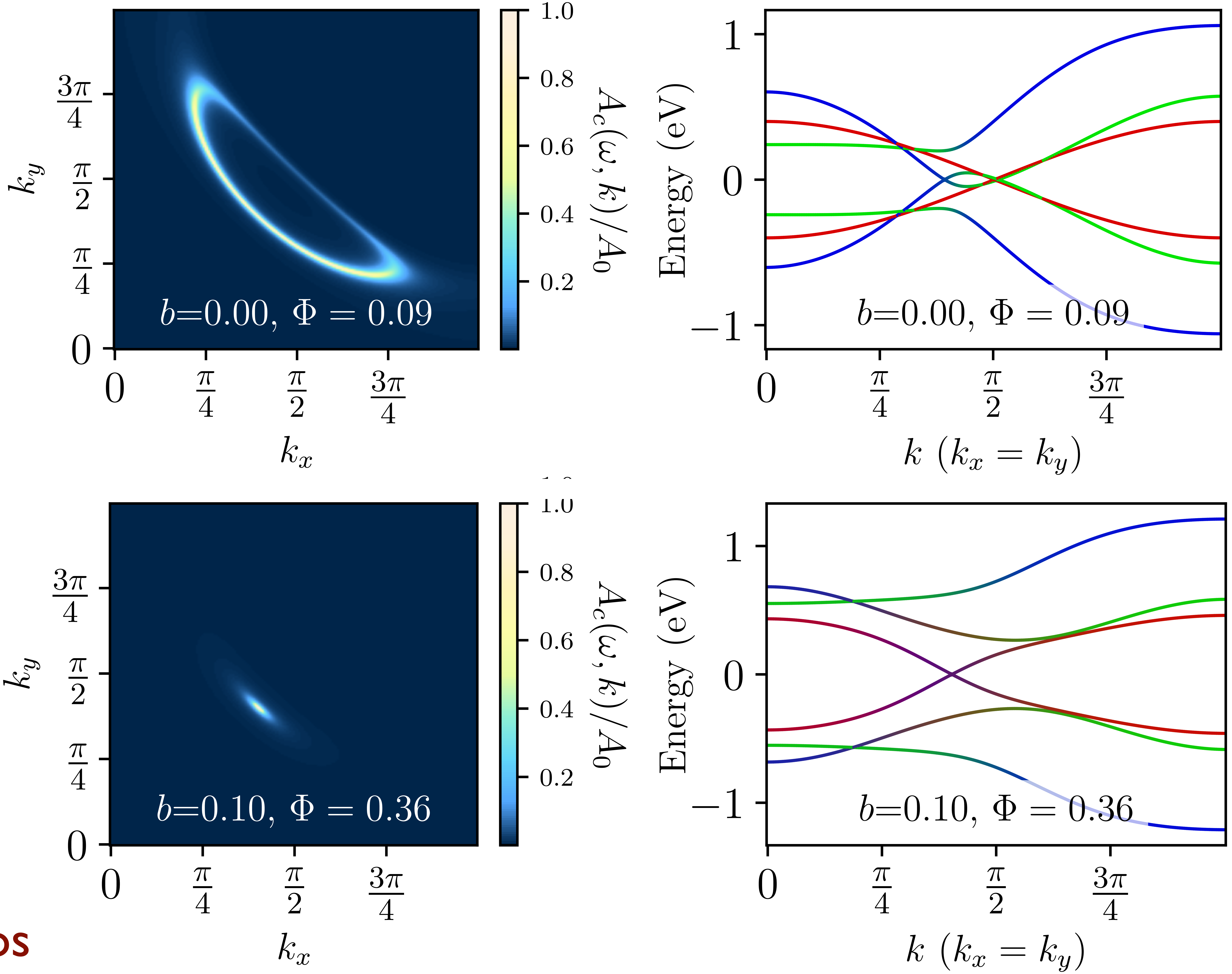
Yields nearly degenerate instabilities to
dSC with 4 nodal points
and charge order.
T
FL*
Hole pockets and
$\mathbb{CP}^1/\pi$-flux spin liquid
Strange
metal
(π,π)
(0,0)
(d)
FL
AFM
d-wave superconductivity.
Charge order.
p

B Keimer, S.A. Kivelson, M.R. Norman, S. Uchida, J. Zaanen, Nature **518**, 179 (2015)



The onset of conventional order is a *confinement transition* for the emergent gauge theory describing the fractionalized excitations of the spin liquid in the FL* state.

Electron spectral density in hole-doped cuprates

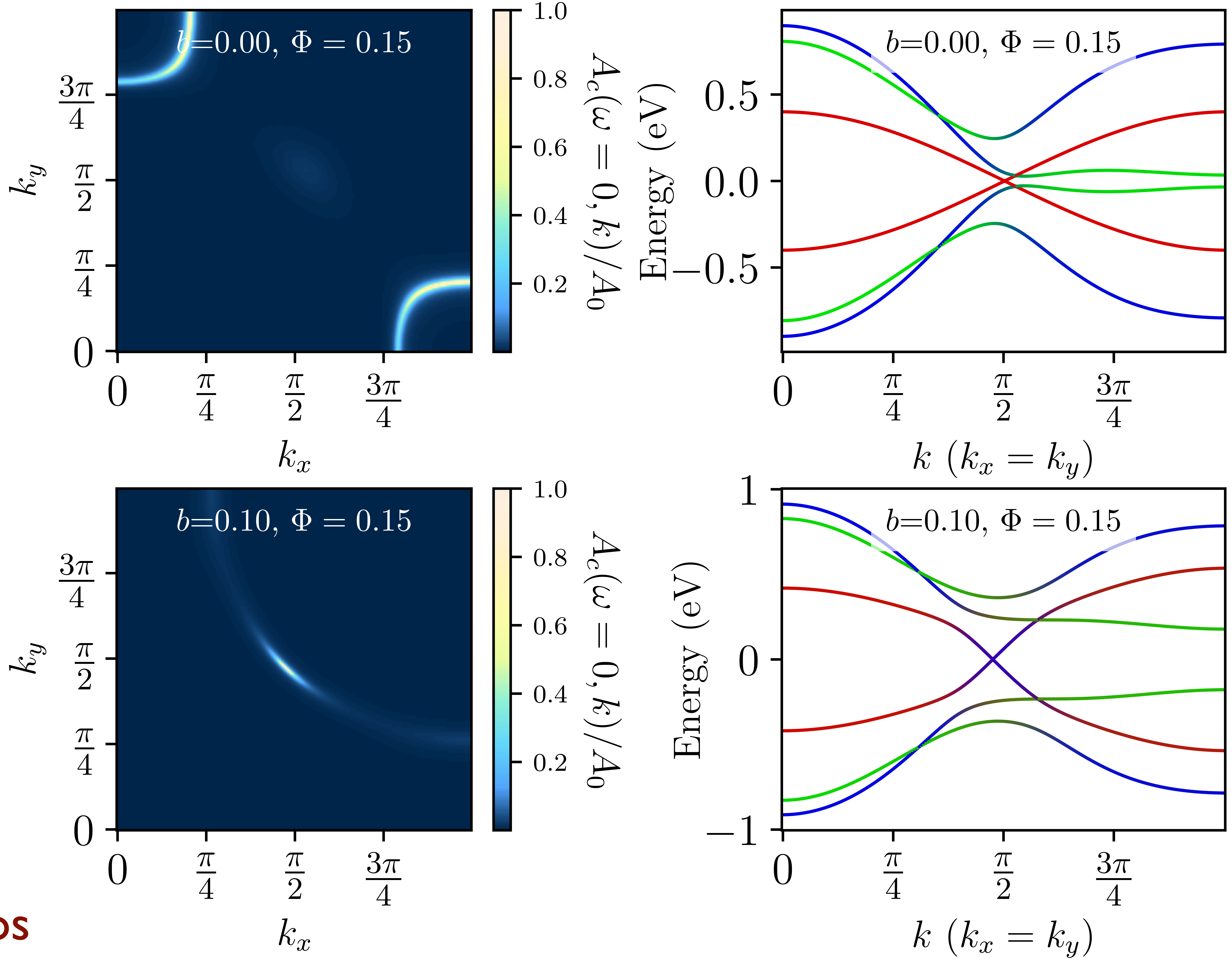


FL*

dSC

Maine Christos

Electron spectral density in electron-doped cuprates

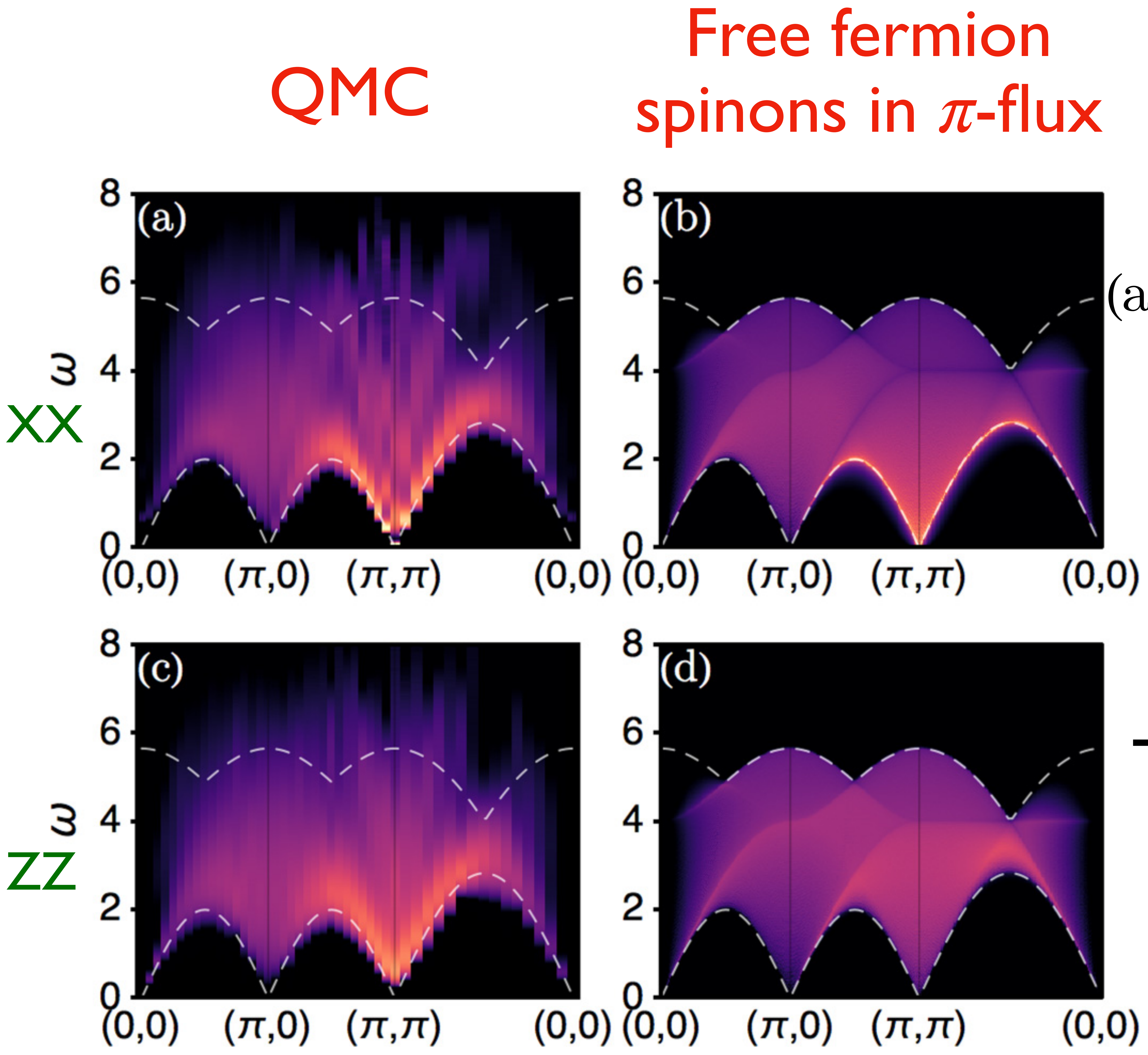


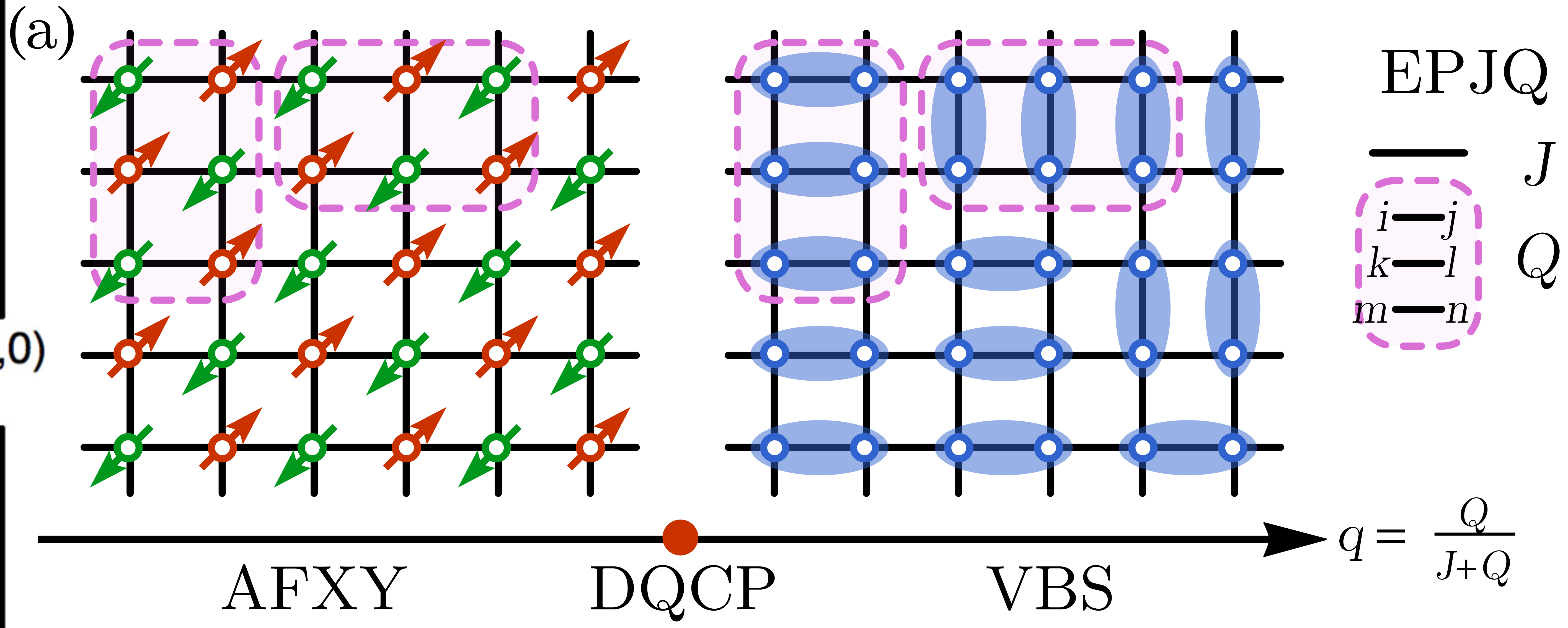
FL*

dSC

Maine Christos

Observable by neutron scattering in pseudogap ?



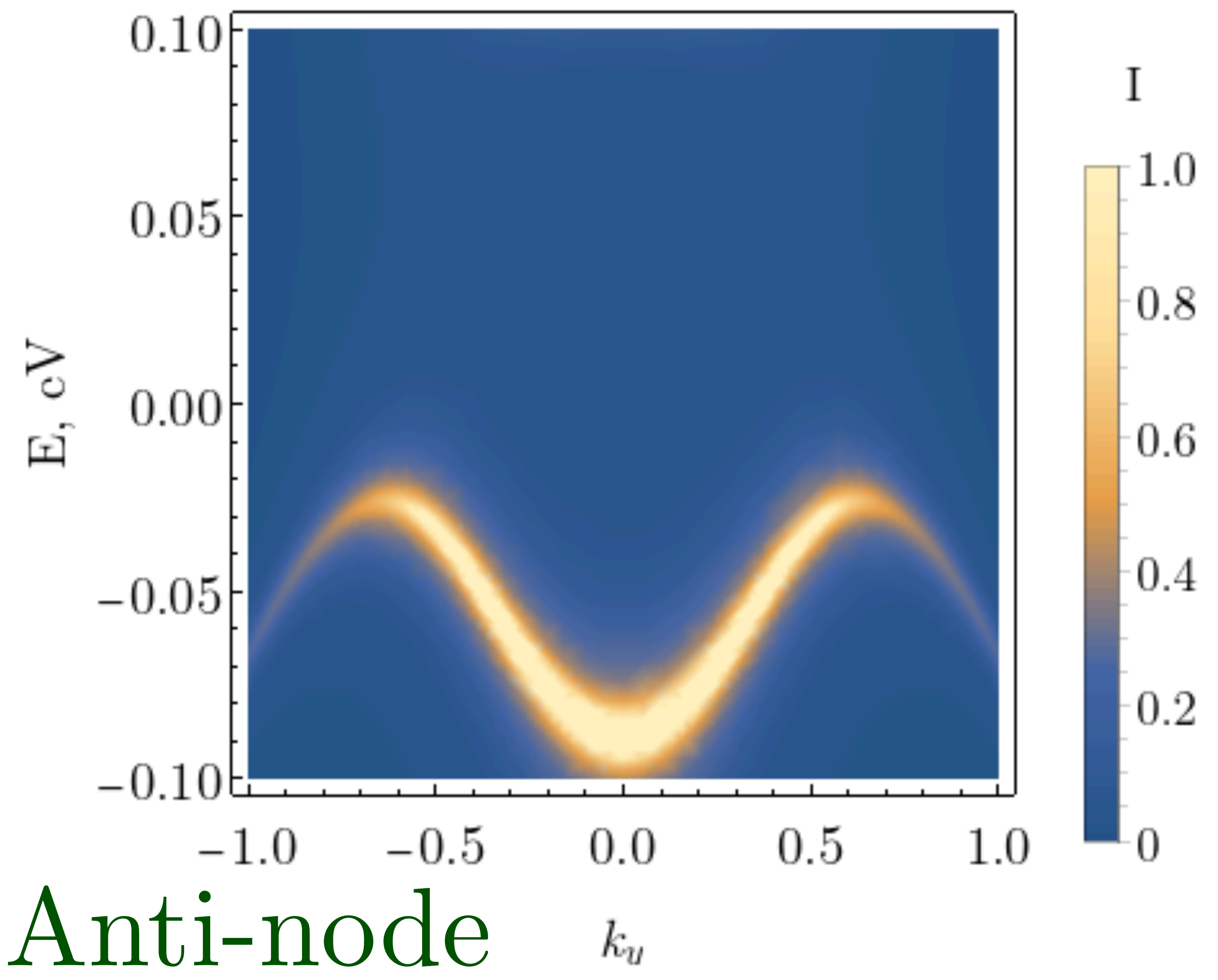


Nvsen Ma, Guang-Yu Sun, Yi-Zhuang You, Cenke Xu, Ashvin Vishwanath, Anders W. Sandvik, and Zi Yang Meng, PRB **98**, 174421 (2018)

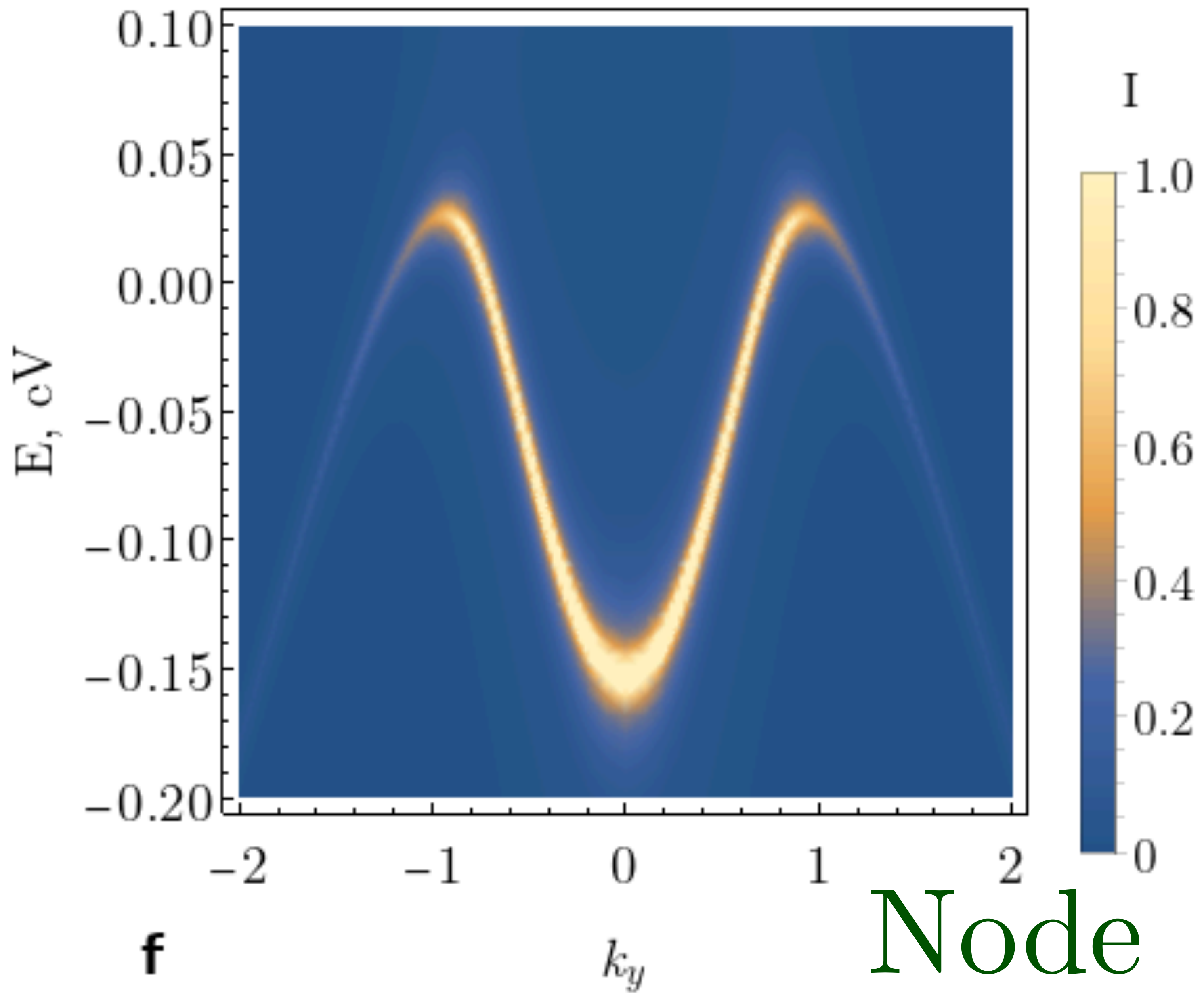
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, and S. S., PRB **105**, 075146 (2022)

FL* in a ***one-band*** model

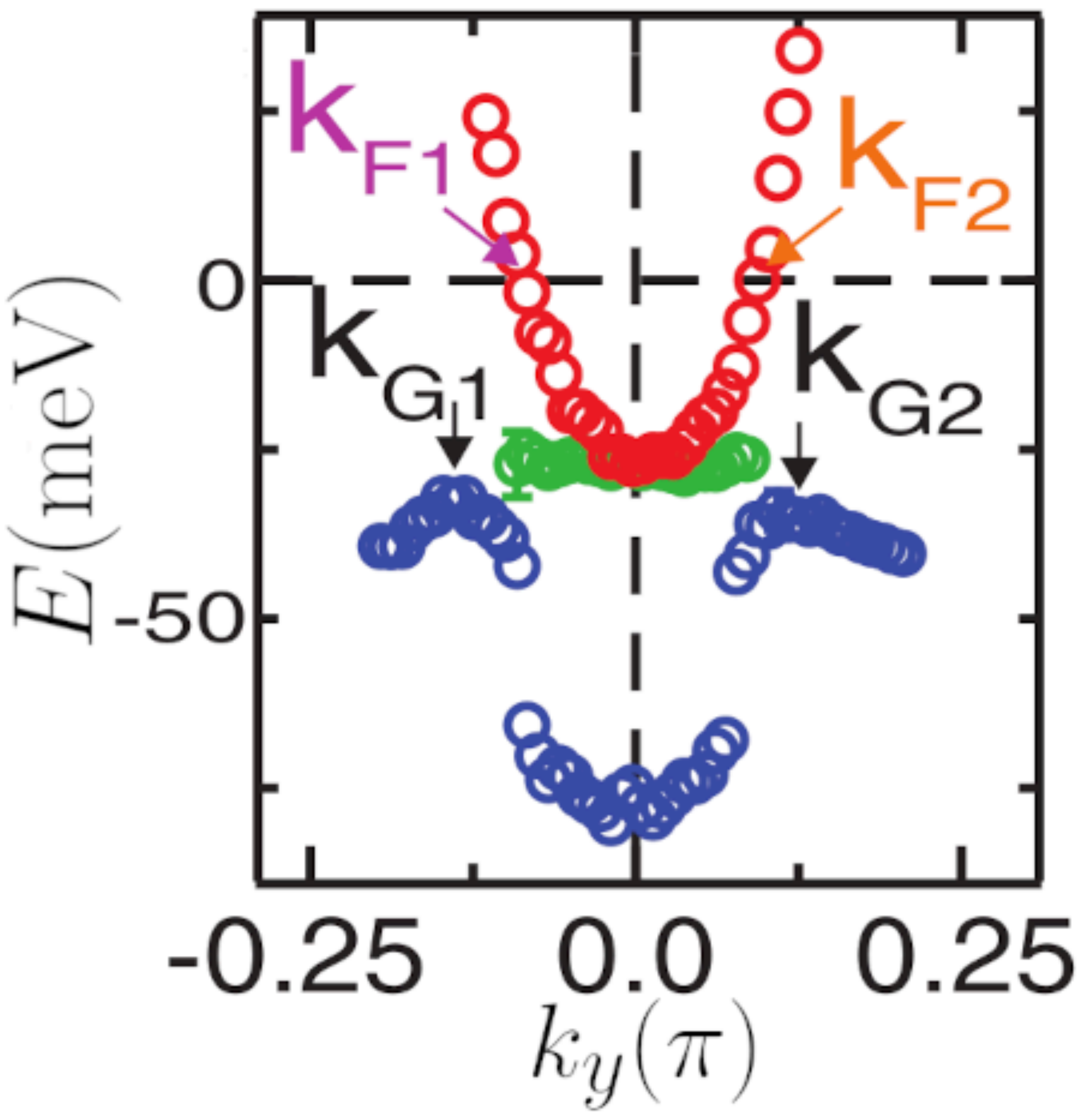
Second ancilla layer is needed to describe MDC and EDC

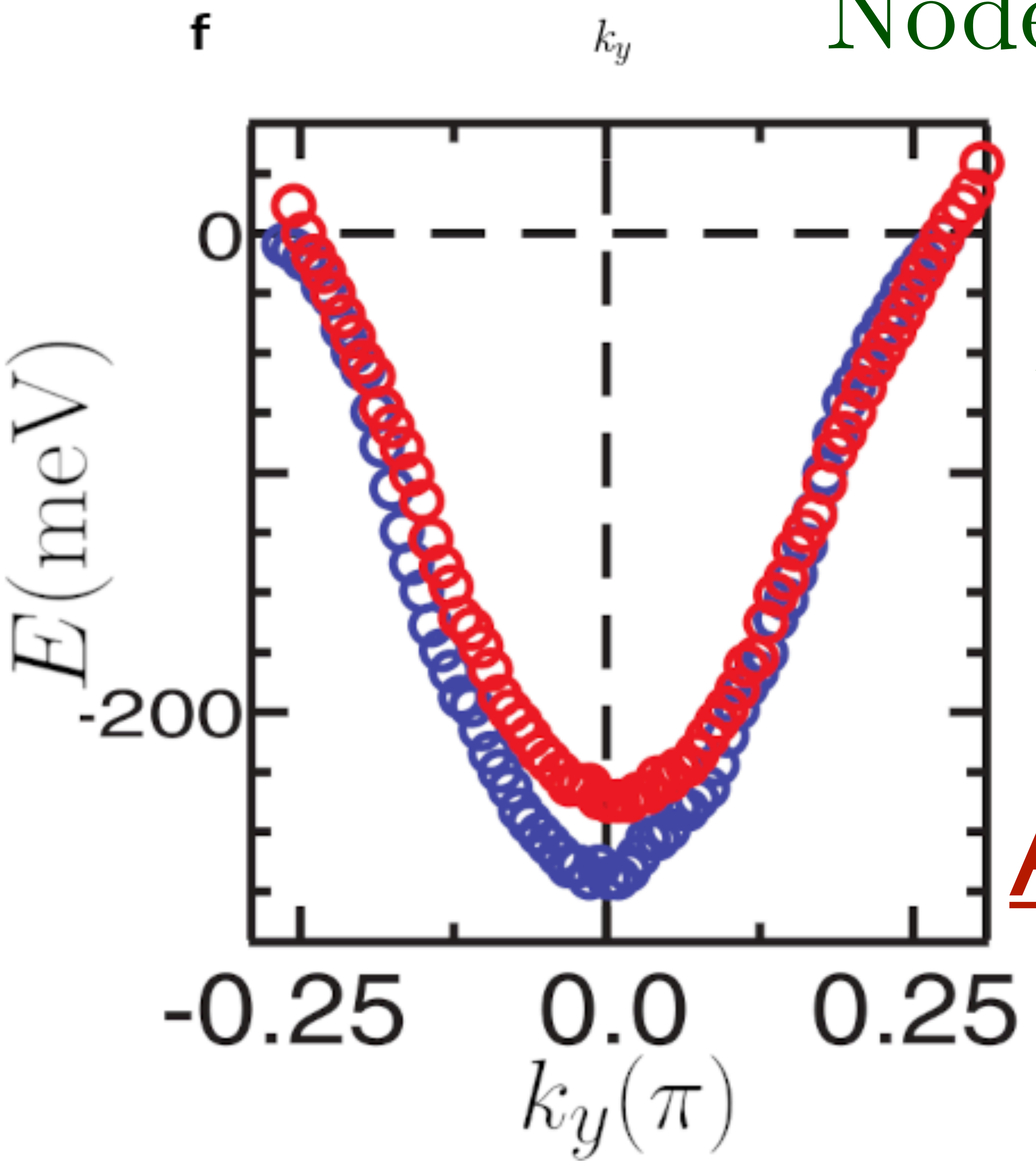


Anti-node



Node



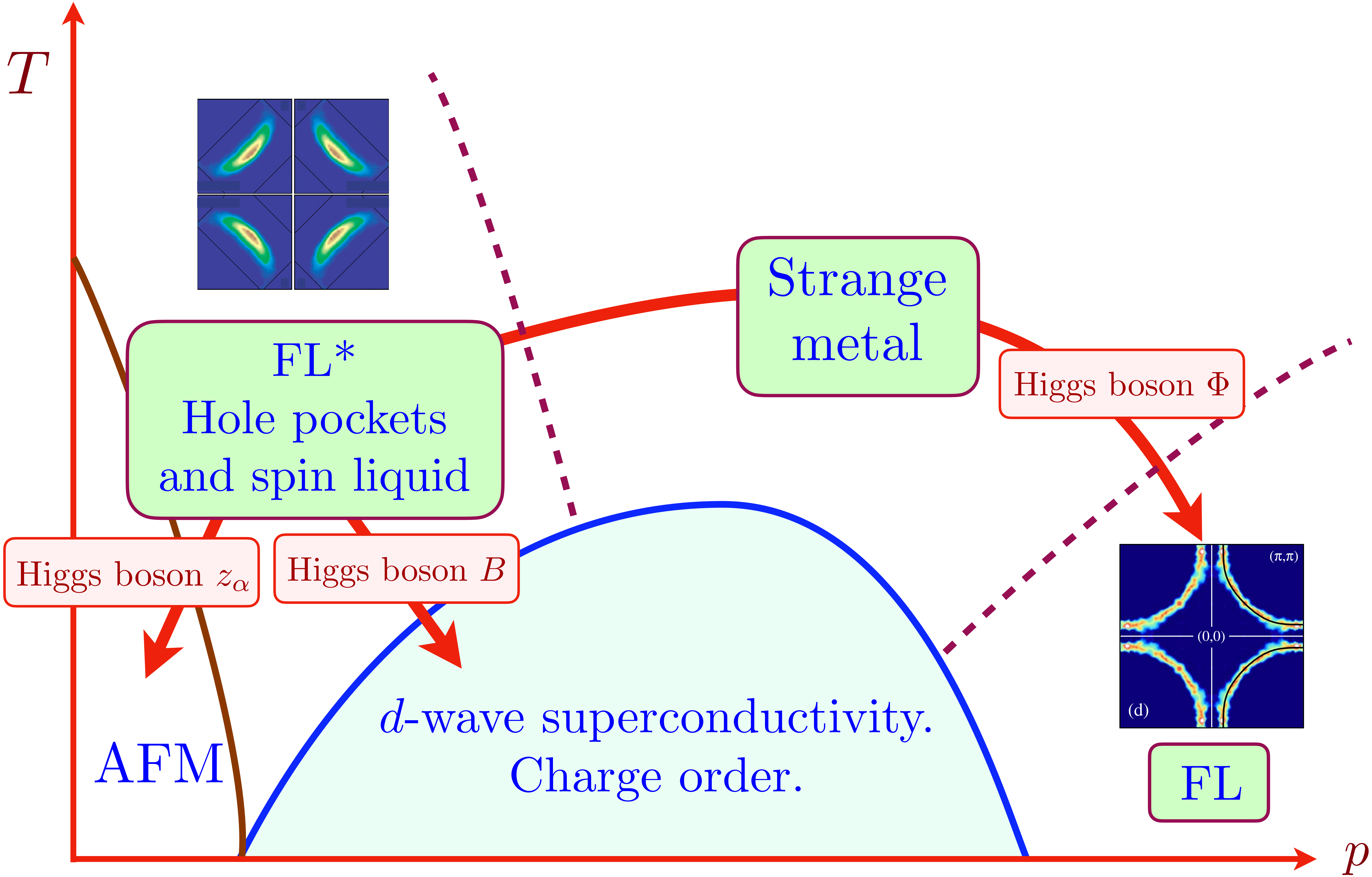


R.-H. He, M. Hashimoto, H. Karapetyan, J. D. Koralek, J. P. Hinton, J. P. Testaud, V. Nathan, Y. Yoshida, H. Yao, K. Tanaka, W. Meevasana, R. G. Moore, D. H. Lu, S. K. Mo, M. Ishikado, H. Eisaki, Z. Hussain, T. P. Devereaux, S. A. Kivelson, J. Orenstein, A. Kapitulnik, and Z.-X. Shen, *Science* **331**, 1579 (2011)

ARPES on Bi2201

Similarities to theories, which do not have explicit reference to spin liquid on second ancilla layer

Kai-Yu Yang, T. M. Rice, Fu-Chun Zhang, PRB **73**, 174501 (2006)
S. Sakai, Y. Motome, M. Imada, PRL **102**, 056404 (2009)



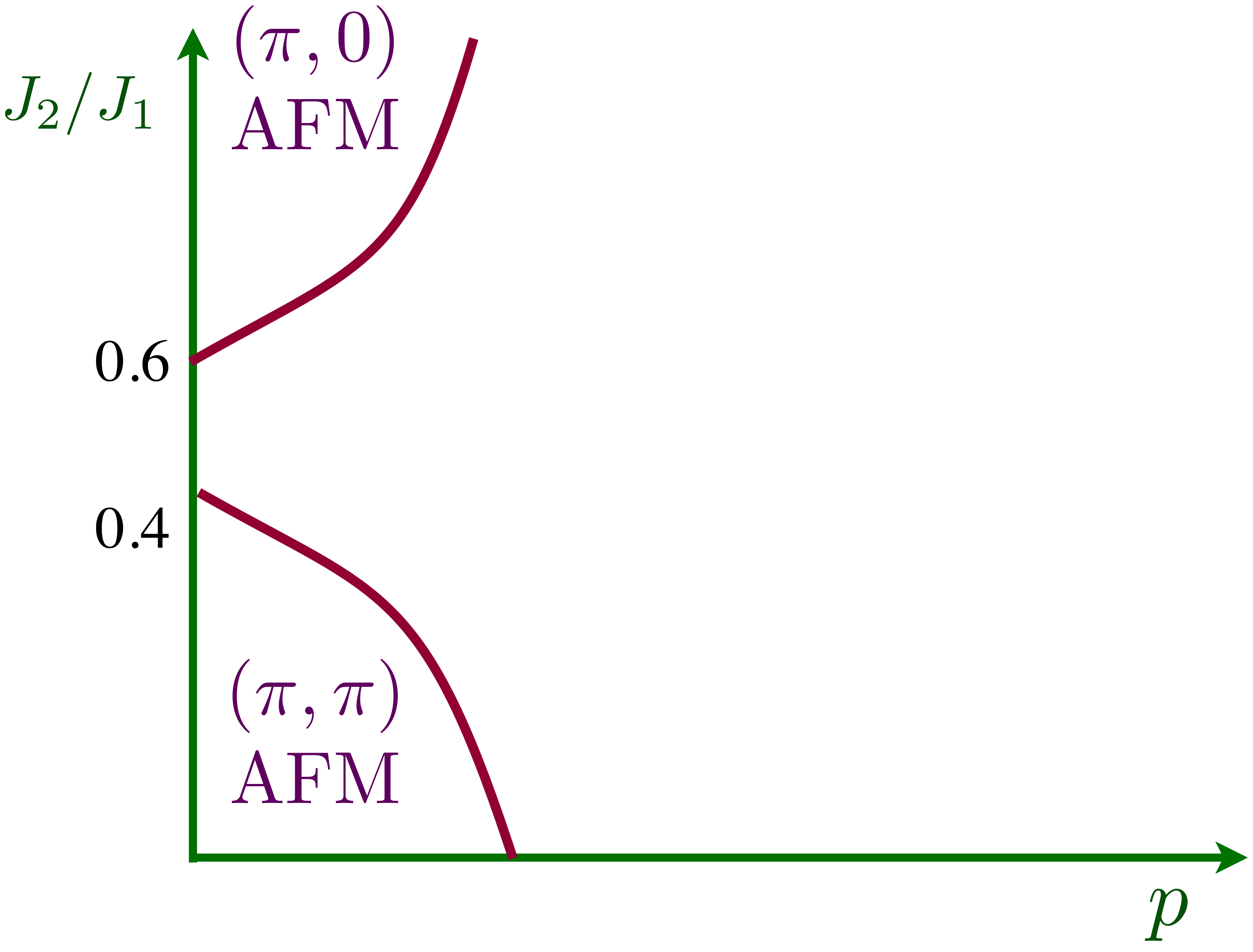
T
FL*
Hole pockets
and spin liquid
Strange
metal
Higgs boson Φ
Higgs boson z_α
Higgs boson B
d-wave superconductivity.
Charge order.
AFM
FL
(π,π)
(0,0)
(d)
p

Extra slides

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang[1,*] and Steven A. Kivelson[2]

PHYSICAL REVIEW LETTERS **127**, 097002 (2021)



Superconducting valence bond fluid in lightly doped 8-leg *t-J* cylinders
Hong-Chen Jiang, Steven A. Kivelson, and Dung-Hai Lee, arXiv:2302.11633

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any "competing" charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

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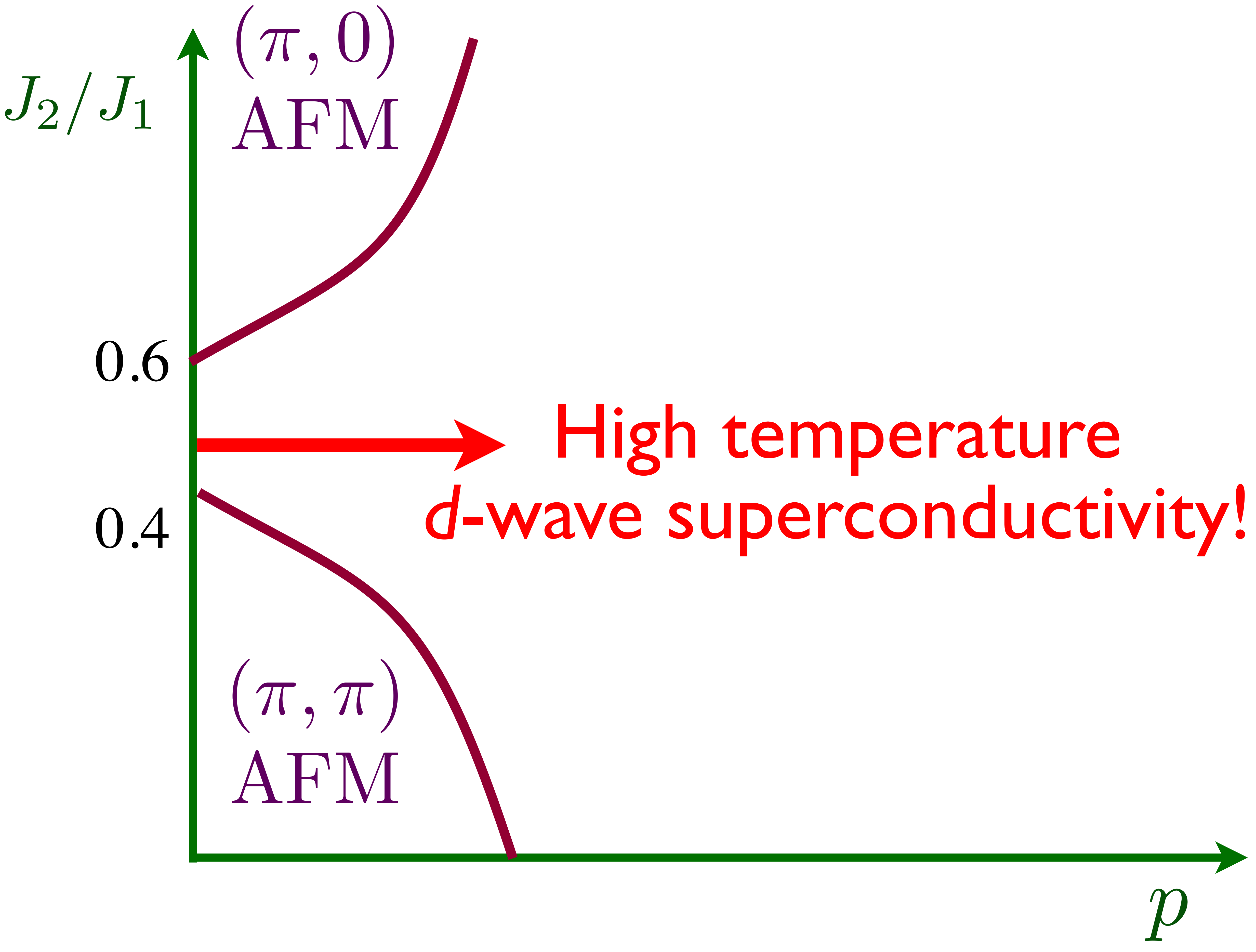
PHYSICAL REVIEW LETTERS **127**, 097002 (2021)



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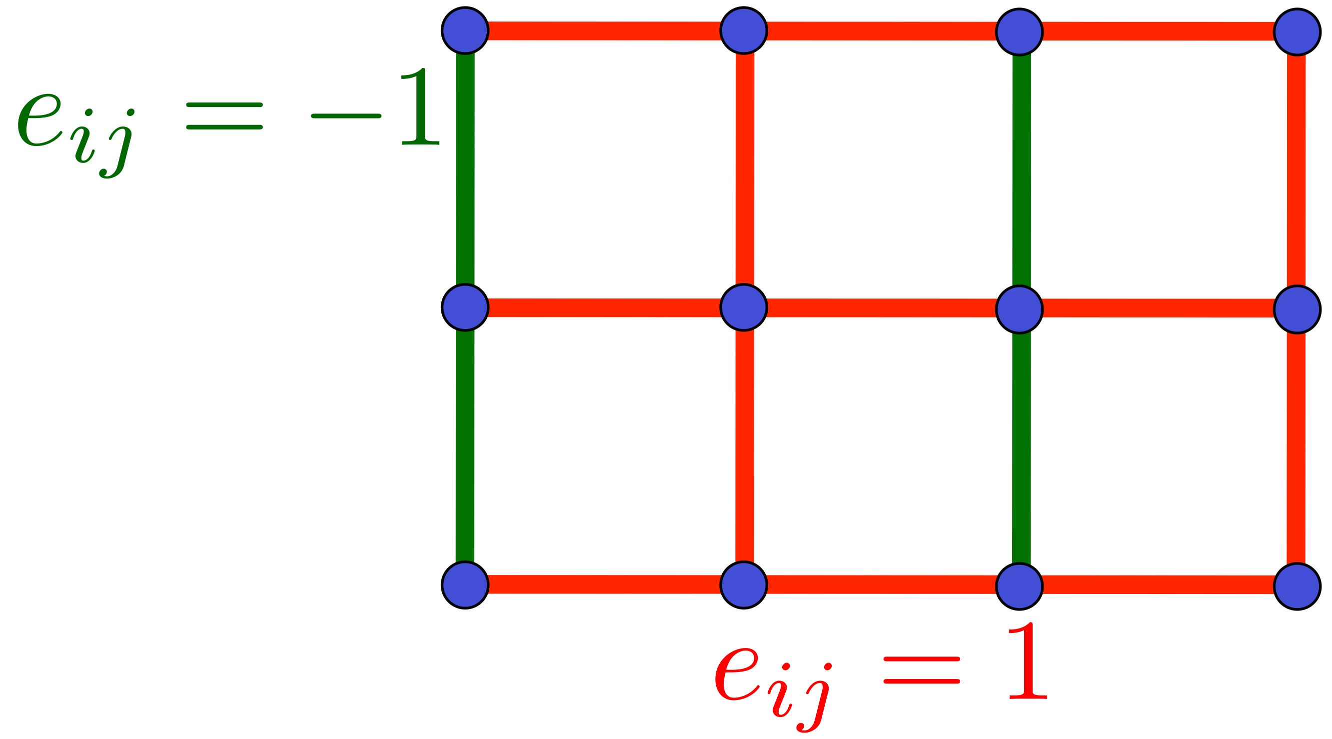
Confinement of SU(2)$_N$ gauge theory by charge fluctuations

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right) = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right) ; \quad \Psi_i = \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

H_f is invariant under SU(2) rotations in spin and SU(2)$_N$ rotations in Nambu space; U_{ij} is the SU(2)$_N$ gauge field.



- The nearest-neighbor effective Hamiltonian for charge e, SU(2)$_N$ fundamental boson B_i is constrained by the fact that the composite of B_i and Ψ_i is an electron:

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \ldots$$

Confinement of SU(2)$_N$ gauge theory by charge fluctuations

$$\mathcal{L}(B) = H_B + \frac{u}{2}\sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}}^2 + V_1 \sum_{\boldsymbol{i}} \rho_{\boldsymbol{i}} \left(\rho_{\boldsymbol{i}+\hat{\boldsymbol{x}}} + \rho_{\boldsymbol{i}+\hat{\boldsymbol{y}}}\right) + g \sum_{\langle \boldsymbol{ij}\rangle} |\Delta_{\boldsymbol{ij}}|^2$$

$$+ J_1 \sum_{\langle \boldsymbol{ij}\rangle} Q_{\boldsymbol{ij}}^2 + K_1 \sum_{\langle \boldsymbol{ij}\rangle} J_{\boldsymbol{ij}}^2 .$$

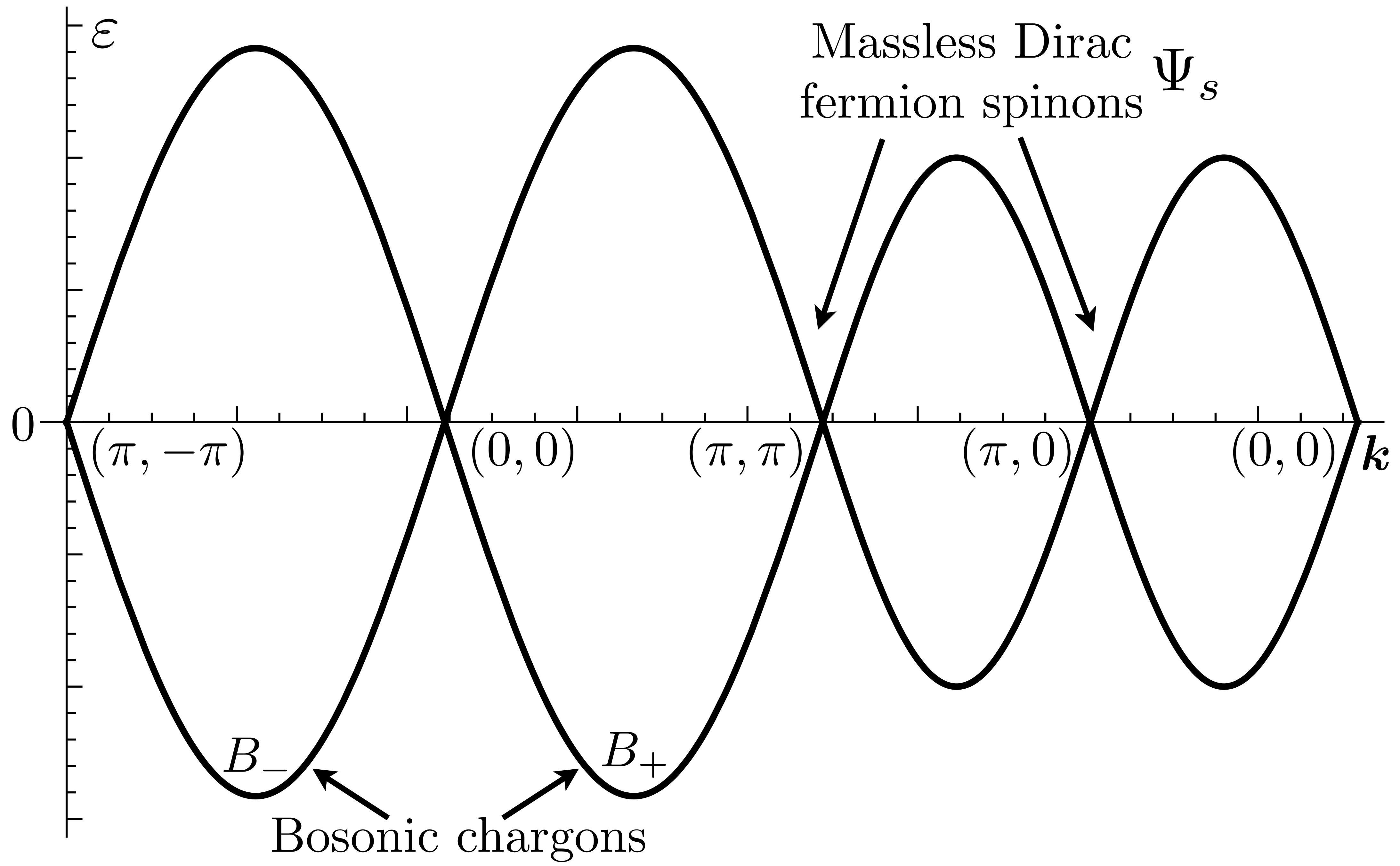
site charge density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim \rho_{\boldsymbol{i}} = B_{\boldsymbol{i}}^\dagger B_{\boldsymbol{i}}$

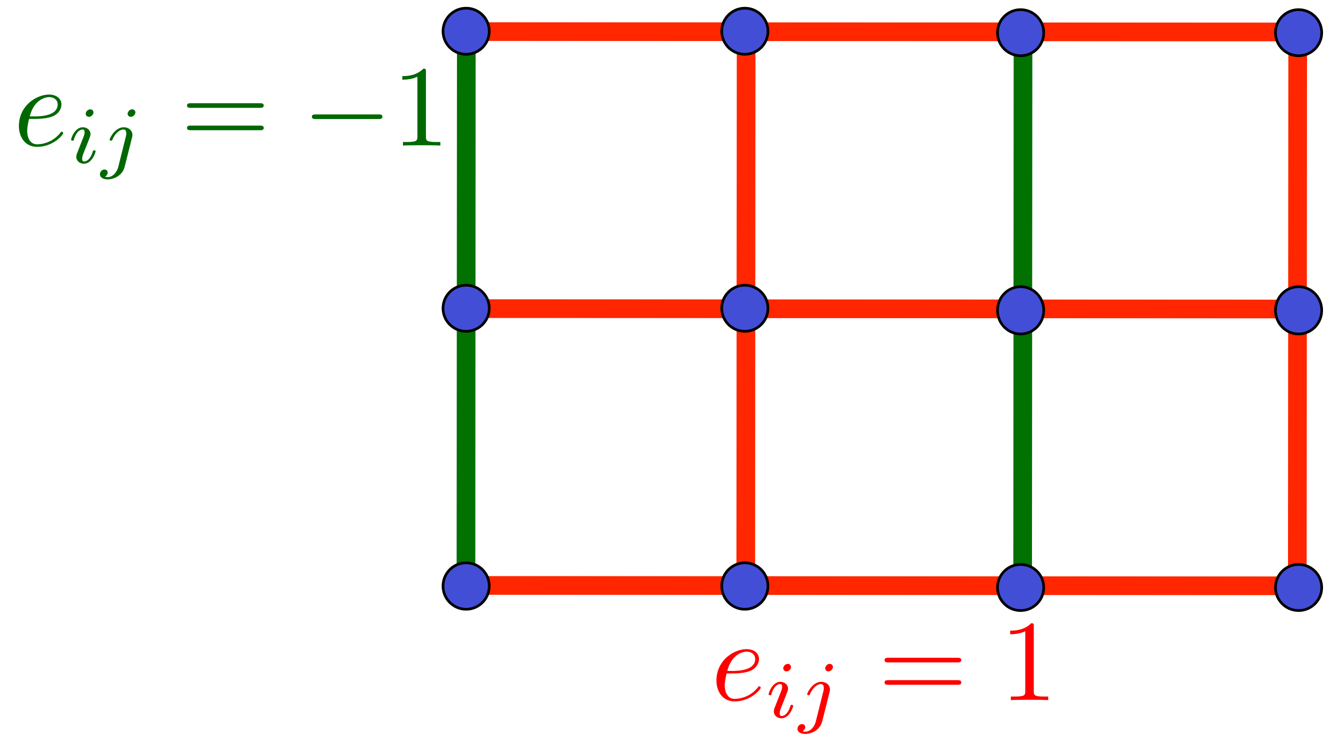
bond density: $\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} + c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim Q_{\boldsymbol{ij}} = Q_{\boldsymbol{ji}} = \mathrm{Im}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

bond current: $i\left\langle c_{\boldsymbol{i}\alpha}^\dagger c_{\boldsymbol{j}\alpha} - c_{\boldsymbol{j}\alpha}^\dagger c_{\boldsymbol{i}\alpha} \right\rangle \sim J_{\boldsymbol{ij}} = -J_{\boldsymbol{ji}} = \mathrm{Re}\left(B_{\boldsymbol{i}}^\dagger e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{\boldsymbol{j}}\right)$

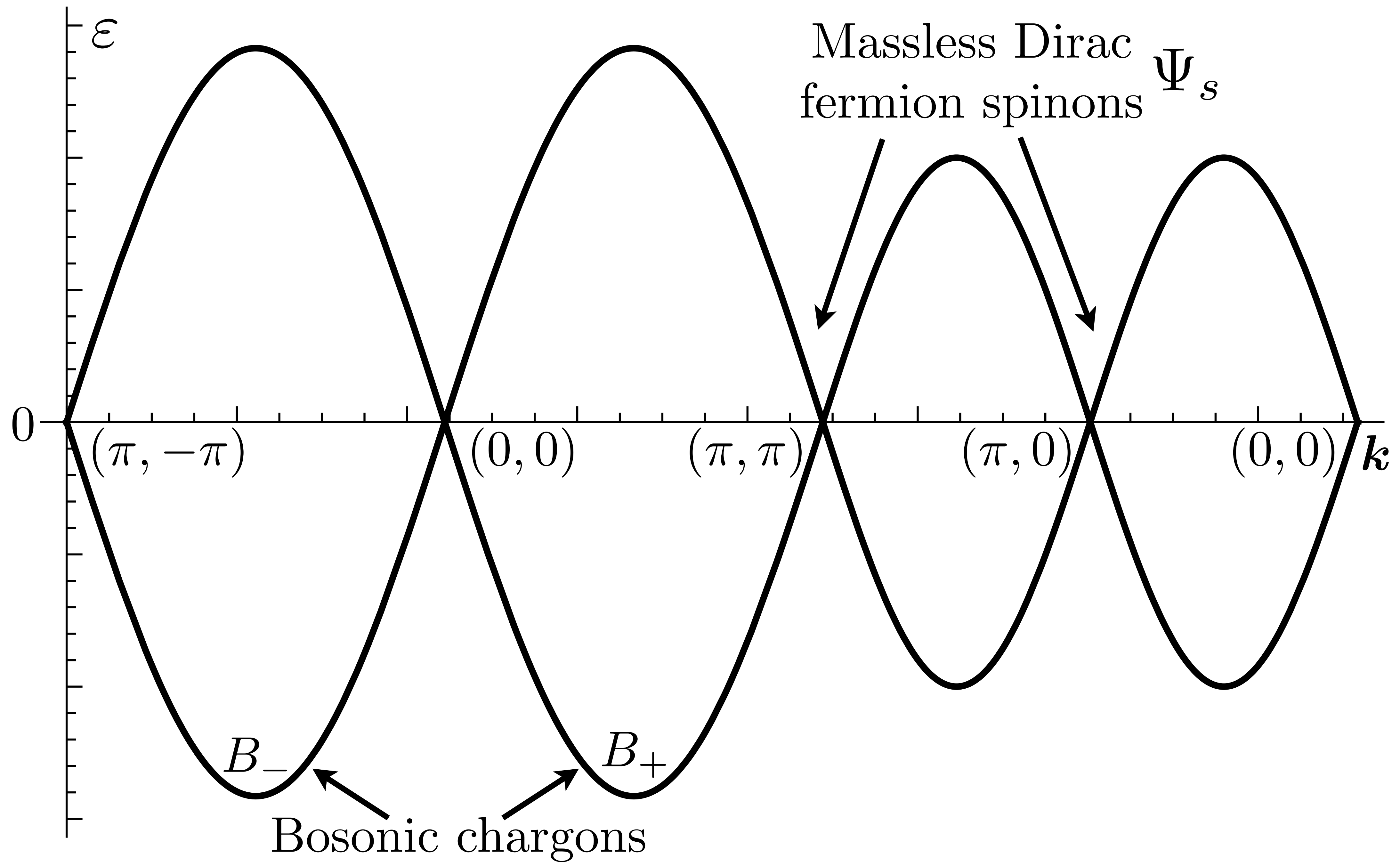
Pairing: $\langle \varepsilon_{\alpha\beta} c_{i\alpha} c_{j\beta} \rangle \sim \Delta_{\boldsymbol{ij}} = \Delta_{\boldsymbol{ji}} = \varepsilon_{ab} B_{a\boldsymbol{i}} e_{\boldsymbol{ij}} U_{\boldsymbol{ij}} B_{b\boldsymbol{j}}$.

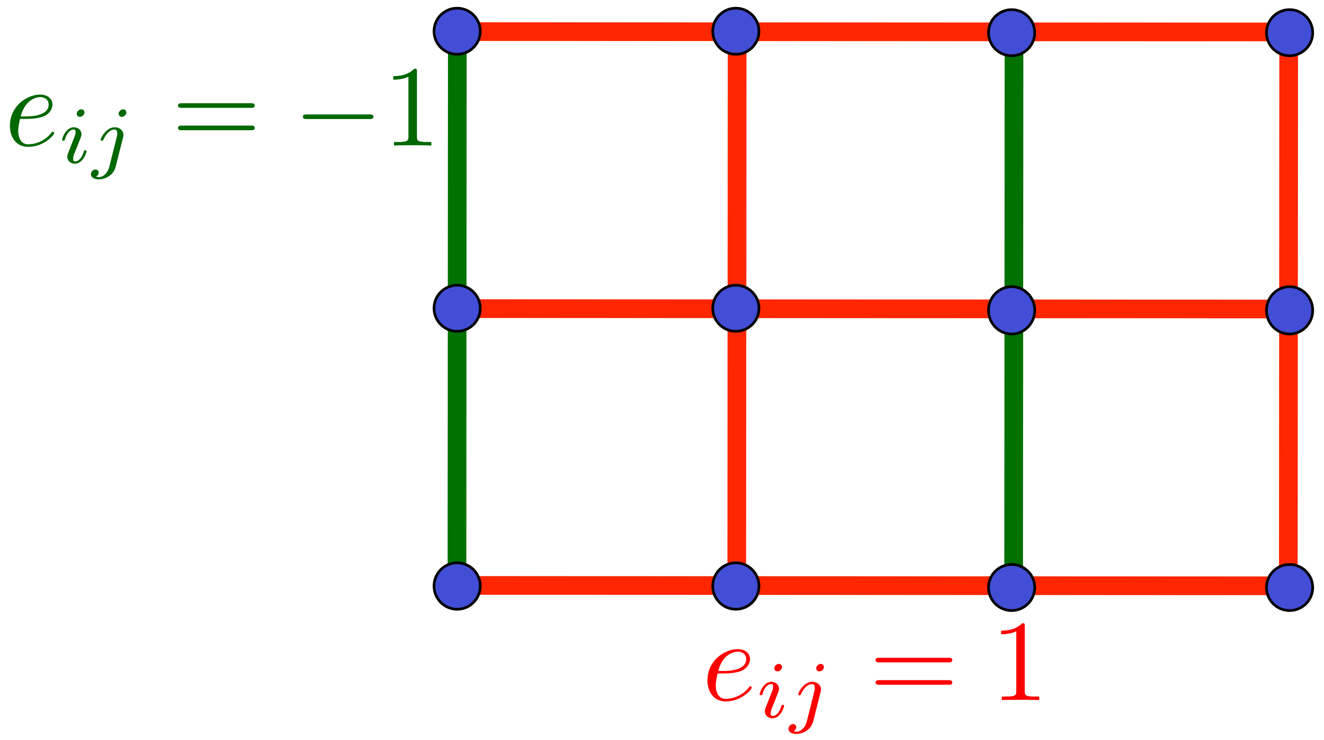
Confinement of SU(2)$_N$ gauge theory by charge fluctuations





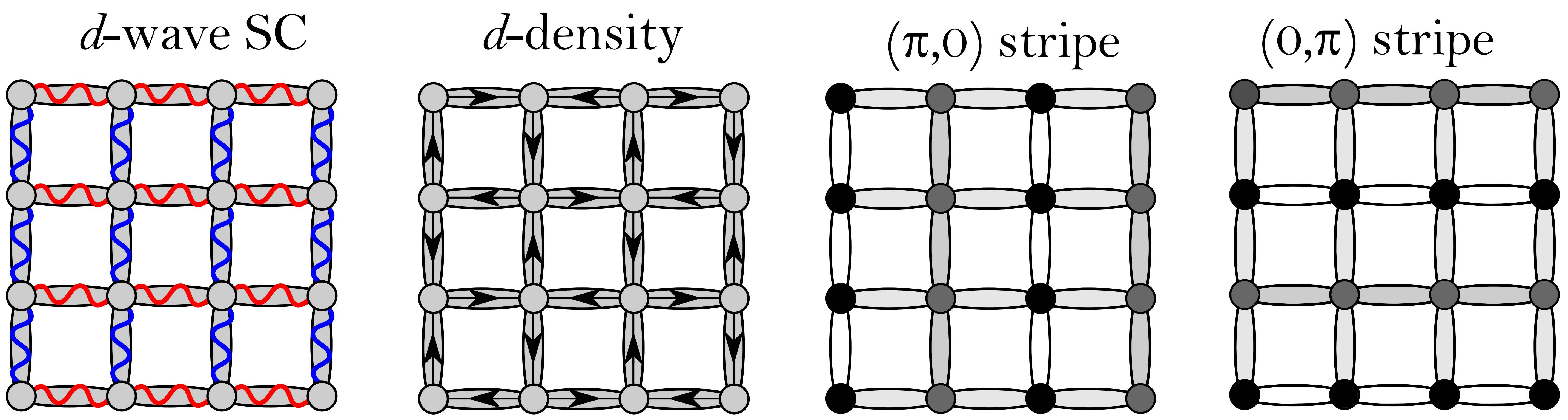
Confinement of $SU(2)_N$ gauge theory by charge fluctuations





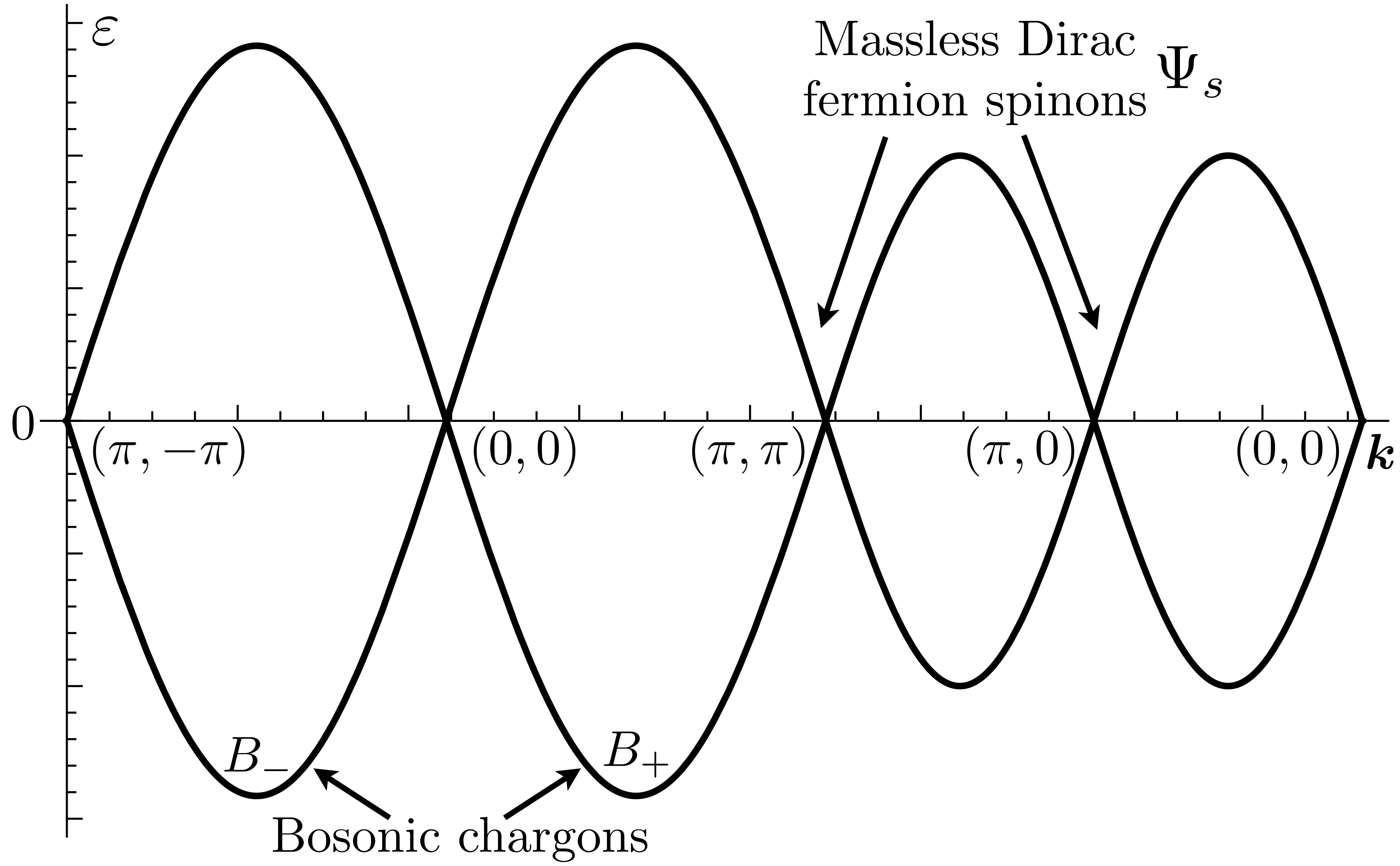
$SU(2)_N$ gauge-invariant and $SU(2)$ spin invariant order parameters of Higgs phases:

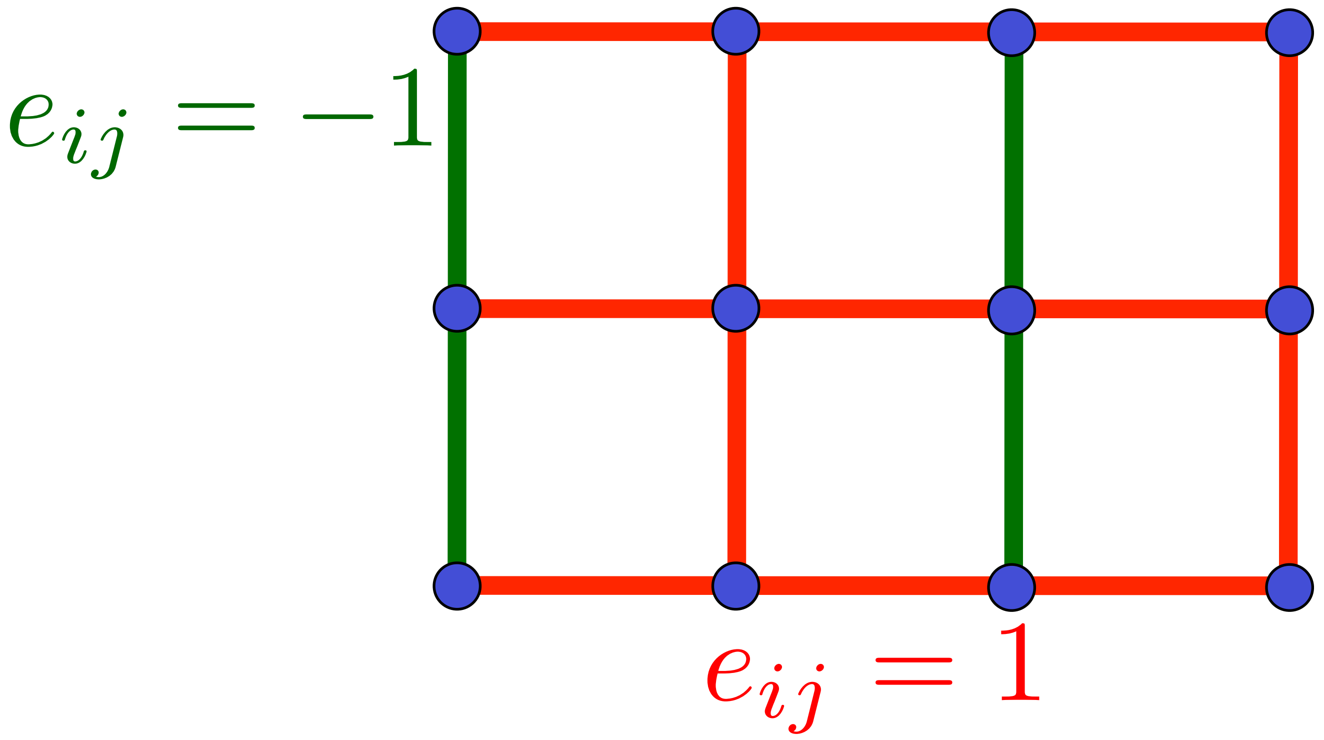
$$
\begin{aligned}
x\text{-CDW}: &\quad \rho_{(\pi,0)} = B^*_{a+}B_{a+} - B^*_{a-}B_{a-} \\
y\text{-CDW}: &\quad \rho_{(0,\pi)} = B^*_{a+}B_{a-} + B^*_{a-}B_{a+} \\
d\text{-density wave}: &\quad D = i\left(B^*_{a+}B_{a-} - B^*_{a-}B_{a+}\right) \\
d\text{-wave superconductor}: &\quad \Delta = \varepsilon_{ab}B_{a+}B_{b-}
\end{aligned}
$$



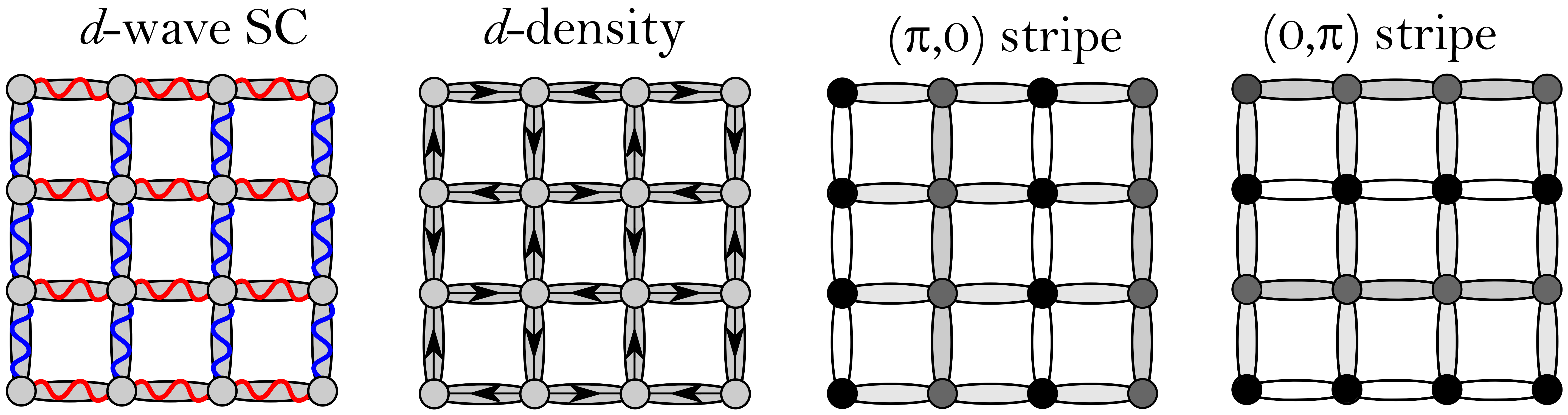
The $\mathcal{O}(B^2_{a\pm})$ terms in the energy have a $SO(5)_b$ rotation symmetry between these orders.

Confinement of $SU(2)_N$ gauge theory by charge fluctuations

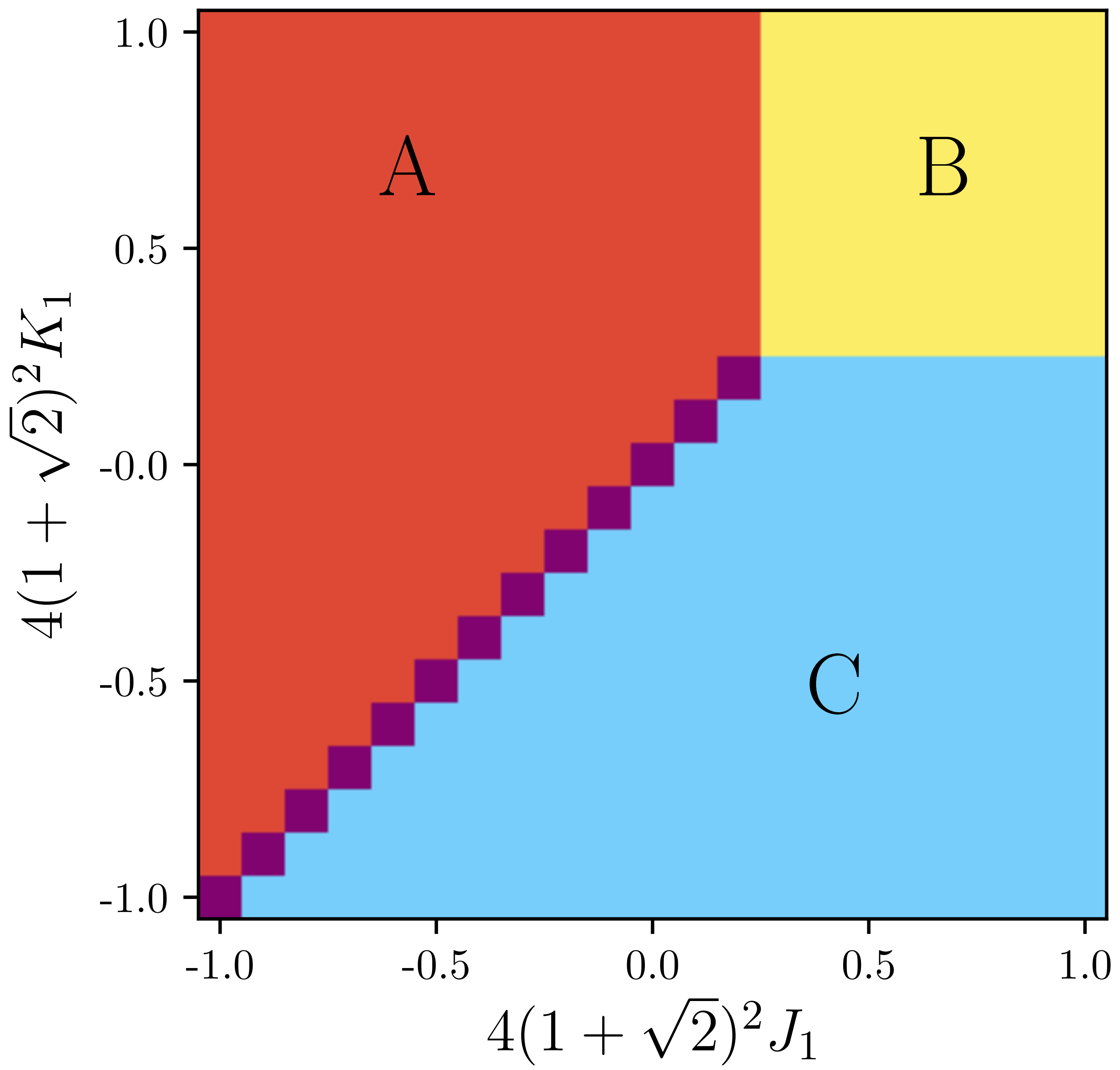




The B_{av}
($a \to SU(2)_N$ gauge, $v \to$ valley)
are the "square roots" of
conventional
d-wave superconductor,
charge density wave,
pair density wave
…

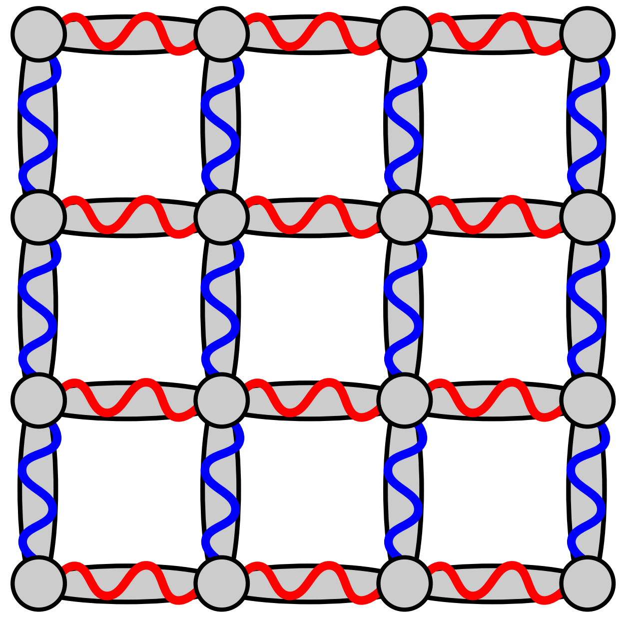


Confinement of $SU(2)_N$ gauge theory by charge fluctuations

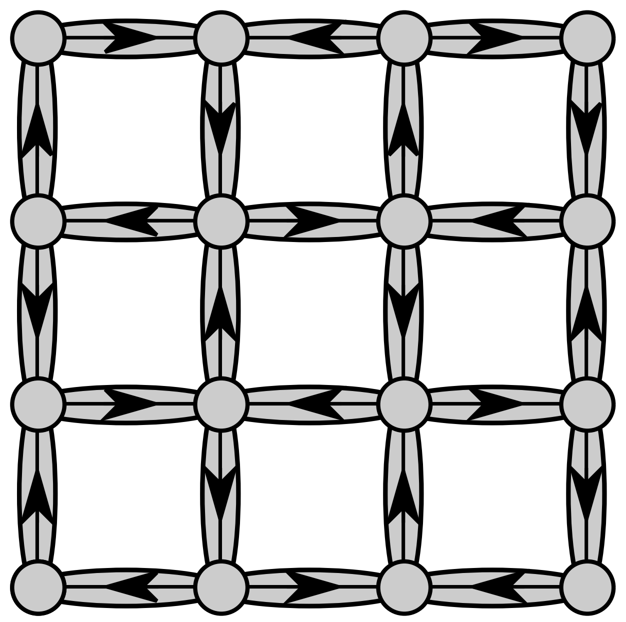


$$\langle B \rangle \neq 0$$

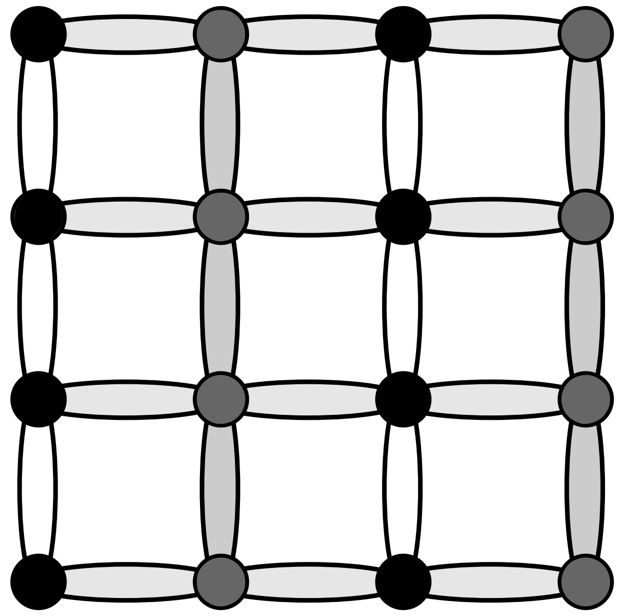
Phase B
d-wave SC



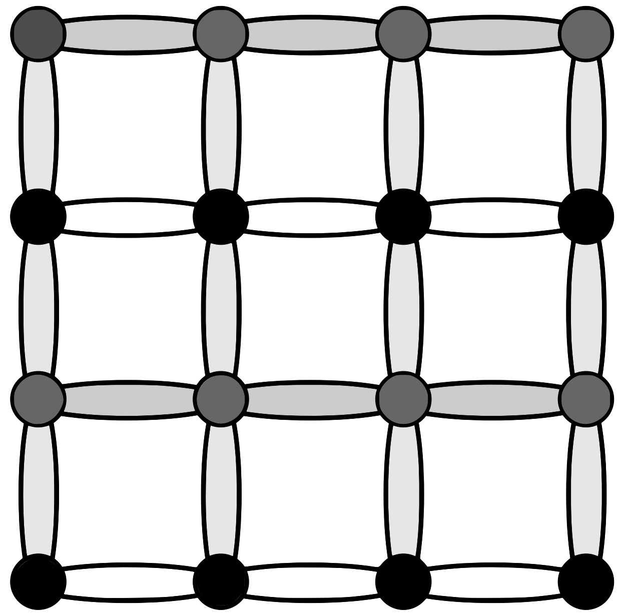
Phase C
d-density



Phase A
(π,0) stripe



Phase A
(0,π) stripe



Global phase diagram of $SU(2)_N$ gauge theory

Global phase diagram of SU(2)$_N$ gauge theory

Pseudogap Metal (FL*)
with hole pocket Fermi surfaces
and underlying π-flux spin liquid
T
$\mathbb{B}$
Metal with
(π, π) SDW
(Néel) order.
$\mathbb{A}$
Strange
Metal.
$\mathbb{C}$
(π,π)
(0,0)
Fermi
Liquid.
FL
d-wave superconductivity.
Charge order.
p
Arrow $\mathbb{A}$
Condensation of B
in $SU(2)_N$ gauge theory.
Arrow $\mathbb{B}$
Condensation of z_α in dual $\mathbb{CP}^1$
U(1) gauge theory.
Arrow $\mathbb{C}$
$SU(2)_s \times U(1)_a$ gauge theory
in ancilla model
with Higgs field $H_{\alpha a} \sim c_\alpha^\dagger f_{1a}$

Unified SU(2)xU(1) gauge theory of spinons, electrons and Higgs bosons:

uncanny similarities to the Salam-Weinberg-Glashow theory of weak interactions

- The electromagnetic U(1) is effectively global, because $\alpha \ll 1$.

- The fermionic spinons transform as a fundamental of gauge SU(2), with a massless Dirac spectrum

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(\Psi_i^\dagger U_{ij} \Psi_j - \Psi_j^\dagger U_{ji} \Psi_i \right) ,$$

  where U_{ij} is the (lattice) SU(2) gauge field. The spinons are the analog of the neutrinos

- The Higgs sector has a boson B_i which is fundamental of SU(2)

$$H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger U_{ij} B_j - B_j^\dagger U_{ji} B_i \right) + \mathcal{O}(B_i^4)$$

- The hole pockets in the nodal region of the Brillouin zone are described by electron $\bar{c}_{i\alpha}$ which have a Yukawa coupling to the spinons and the Higgs field $B_i = (B_{1i}, B_{2i})$:

$$H_Y = \sum_{ij} \bar{t}_{ij} \bar{c}_{i\alpha}^\dagger \bar{c}_{j\alpha} + i \sum_i \left(B_{1i} f_{i\alpha}^\dagger \bar{c}_{i\alpha} - B_{2i} \varepsilon_{\alpha\beta} f_{i\alpha} \bar{c}_{i\beta} \right) + \text{H.c.} .$$