Enhanced thermal Hall effect in the square lattice Neel state

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Lake Tahoe, August 4-7, 2019

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PHYSICS
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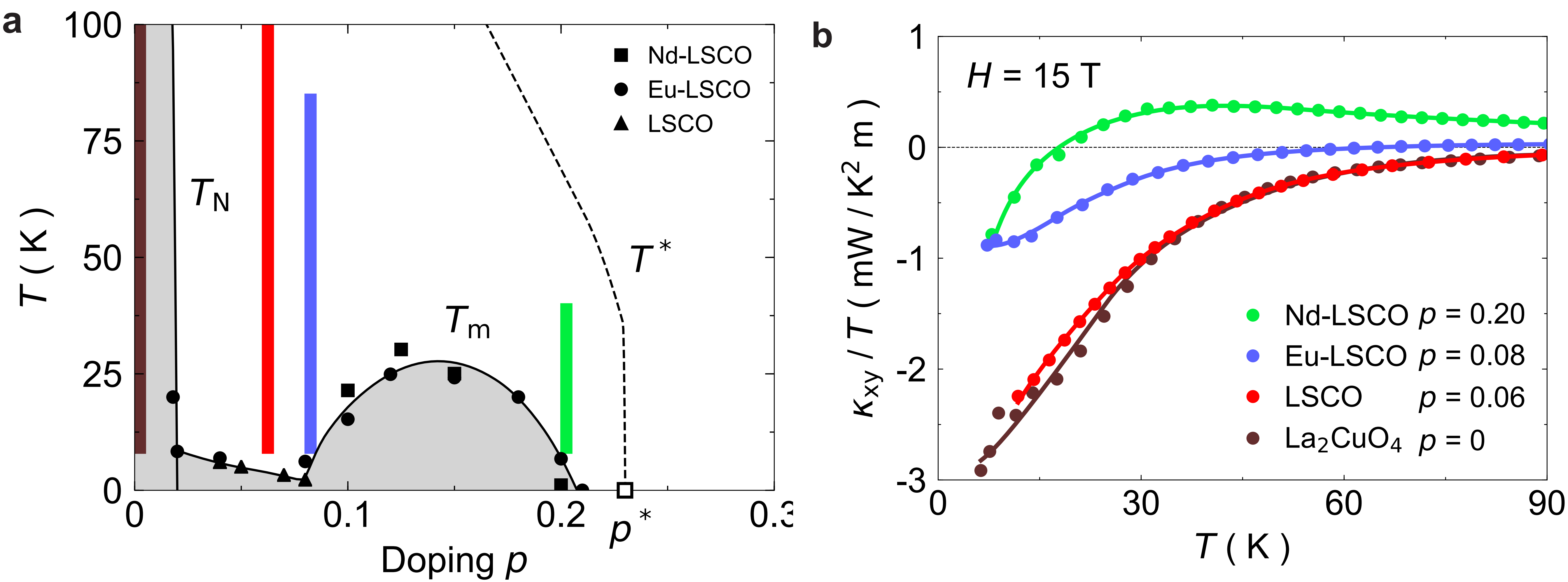
Nature **571**, 376 (2019)



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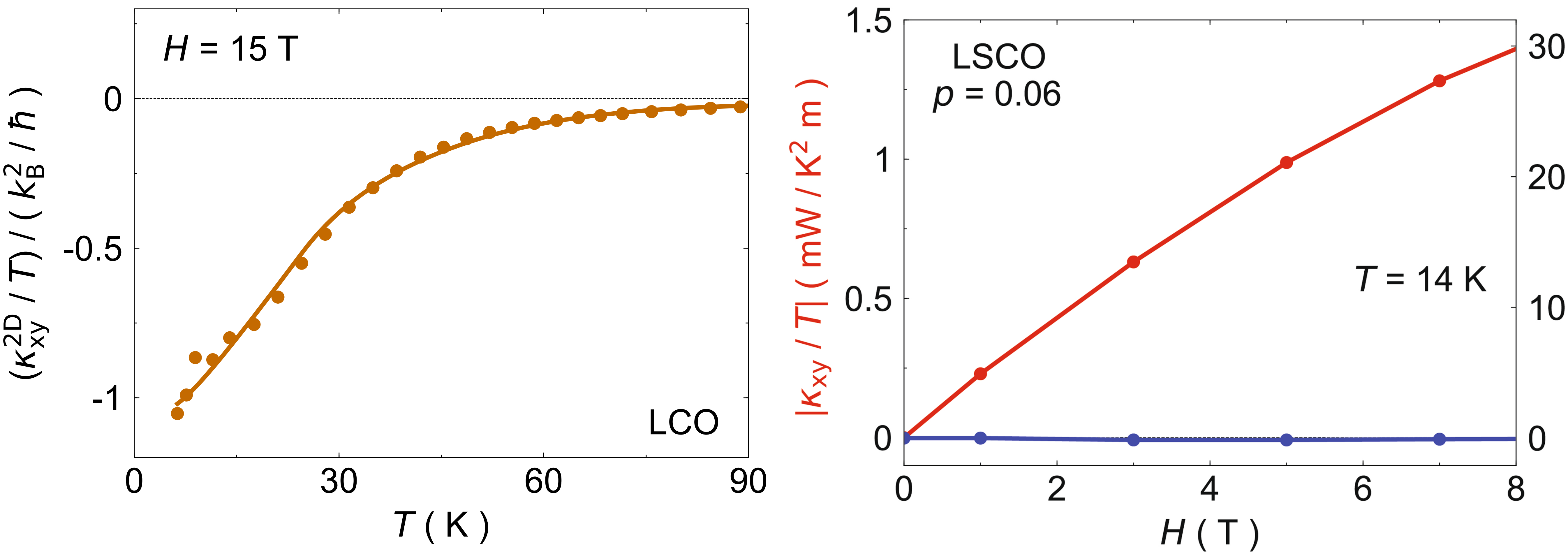

The Néel state of square lattice antiferromagnets described by

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

has zero transverse thermal conductivity $\kappa_{xy}/T = 0$.

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In the presence of a magnetic field

$$H_B = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$

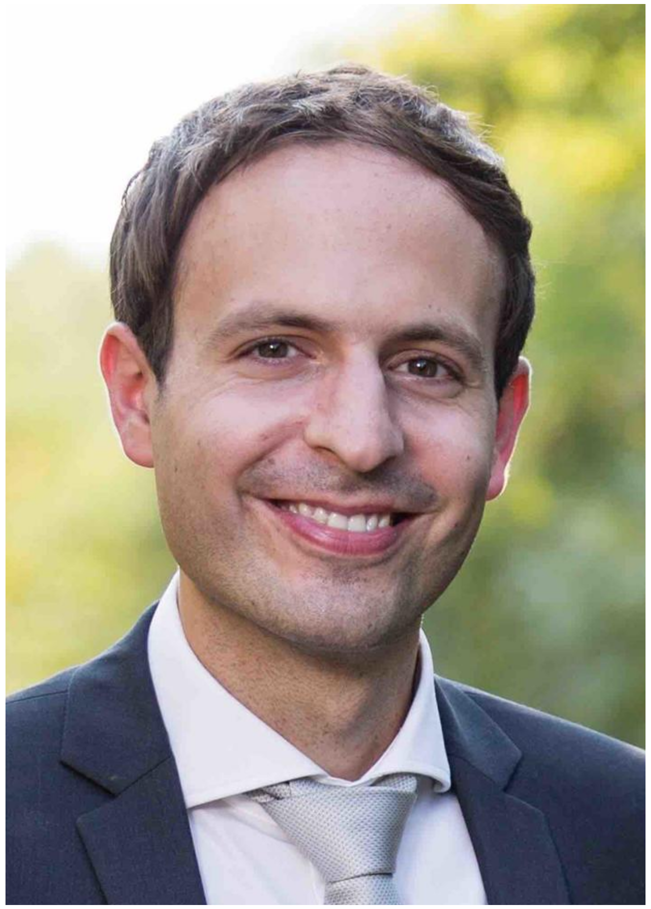
The *orbital coupling* $J_\chi \propto \boldsymbol{B}_\perp$ induces a non-zero Berry curvature in the spin-wave dispersion, which induces a non-zero κ_{xy}/T.

However, this Berry curvature is small at long wavelengths, and consequently the thermal Hall conductivity is very small $|\kappa_{xy}/T| \ll k_B^2/\hbar$, and vanishes rapidly as $T \to 0$.

Enhanced thermal Hall effect in the square-lattice Néel state

arXiv:1903.01992, Nature Physics, to appear

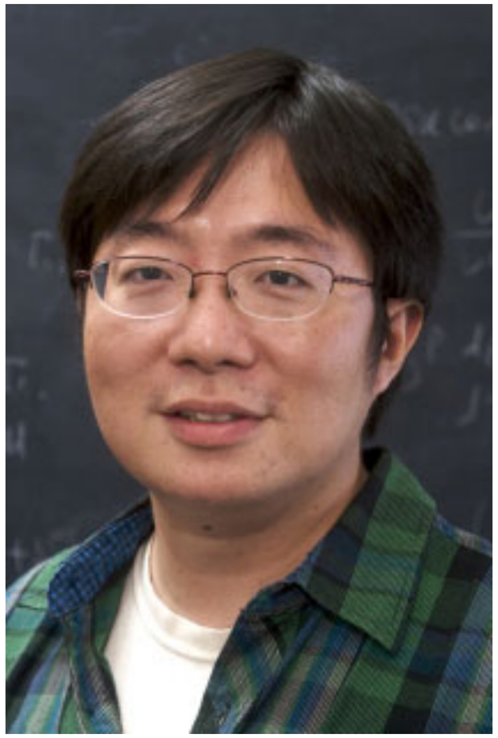
Rhine Samajdar



Mathias Scheurer

Shubhayu Chatterjee

Haoyu Guo



Cenke Xu

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Recent experiments on several cuprate compounds have identified an enhanced thermal Hall response in the pseudogap phase. Most strikingly, this enhancement persists even in the undoped system, which challenges our understanding of the insulating parent compounds. To explain these surprising observations, we study the quantum phase transition of a square-lattice antiferromagnet from a confining Néel state to a state with coexisting Néel and semion topological order. The transition is driven by an applied magnetic field and involves no change in the symmetry of the state. The critical point is described by a strongly-coupled conformal field theory with an emergent global $SO(3)$ symmetry. The field theory has four different formulations in terms of $SU(2)$ or $U(1)$ gauge theories, which are all related by dualities; we relate all four theories to the lattice degrees of freedom. We show how proximity of the confining Néel state to the critical point can explain the enhanced thermal Hall effect seen in experiment.

Enhanced thermal Hall effect in the square-lattice Néel state

arXiv:1903.01992, Nature Physics, to appear

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1. Frustrated square lattice antiferromagnets in zero magnetic field: Neel-VBS criticality

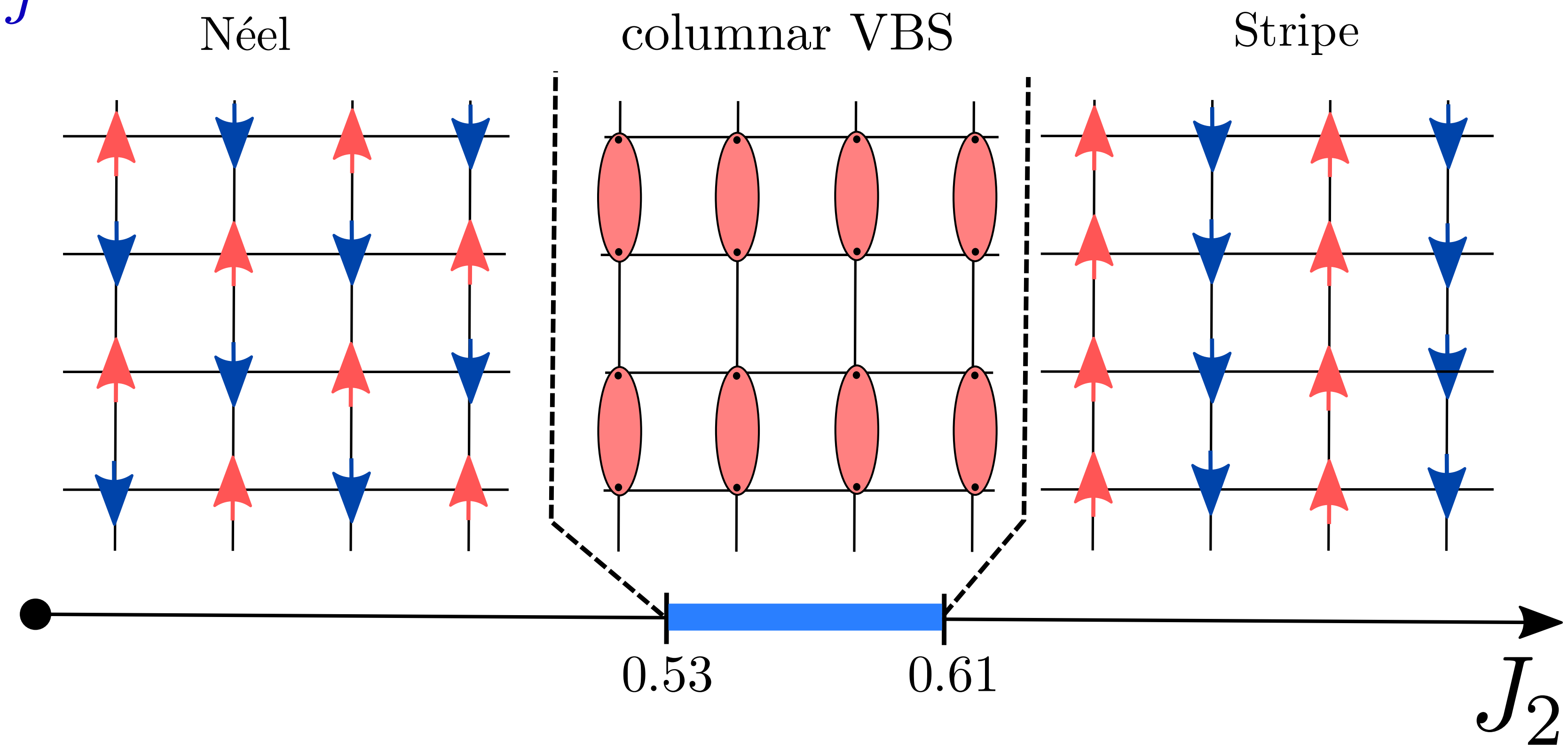
2. Proximity to field-induced semion topological order

3. Semion topological order on other lattices

4. Quantum criticality and more non-Abelian dualities

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Nearest ($J_1 = 1$) and next-nearest (J_2) neighbor interactions



$U(1)$-symmetric infinite projected entangled-pair states study of the spin-1/2 square J_1-J_2 Heisenberg model

PHYSICAL REVIEW B **97**, 174408 (2018)

R. Haghshenas and D. N. Sheng

By studying the finite-D scaling of the magnetically order parameter, we find a Néel phase for $J_2/J_1 < 0.53$. For $0.53 < J_2/J_1 < 0.61$, a nonmagnetic columnar valence bond solid (VBS) state is established as observed by the pattern of local bond energy. The divergent behavior of correlation length $\xi \sim D^{1.2}$ and vanishing order parameters are consistent with a deconfined Néel-to-VBS transition at $J_2^{c1}/J_1 = 0.530(5)$, where estimated critical anomalous exponents are $\eta_s \sim 0.6$ and $\eta_d \sim 1.9$ for spin and dimer correlations, respectively.

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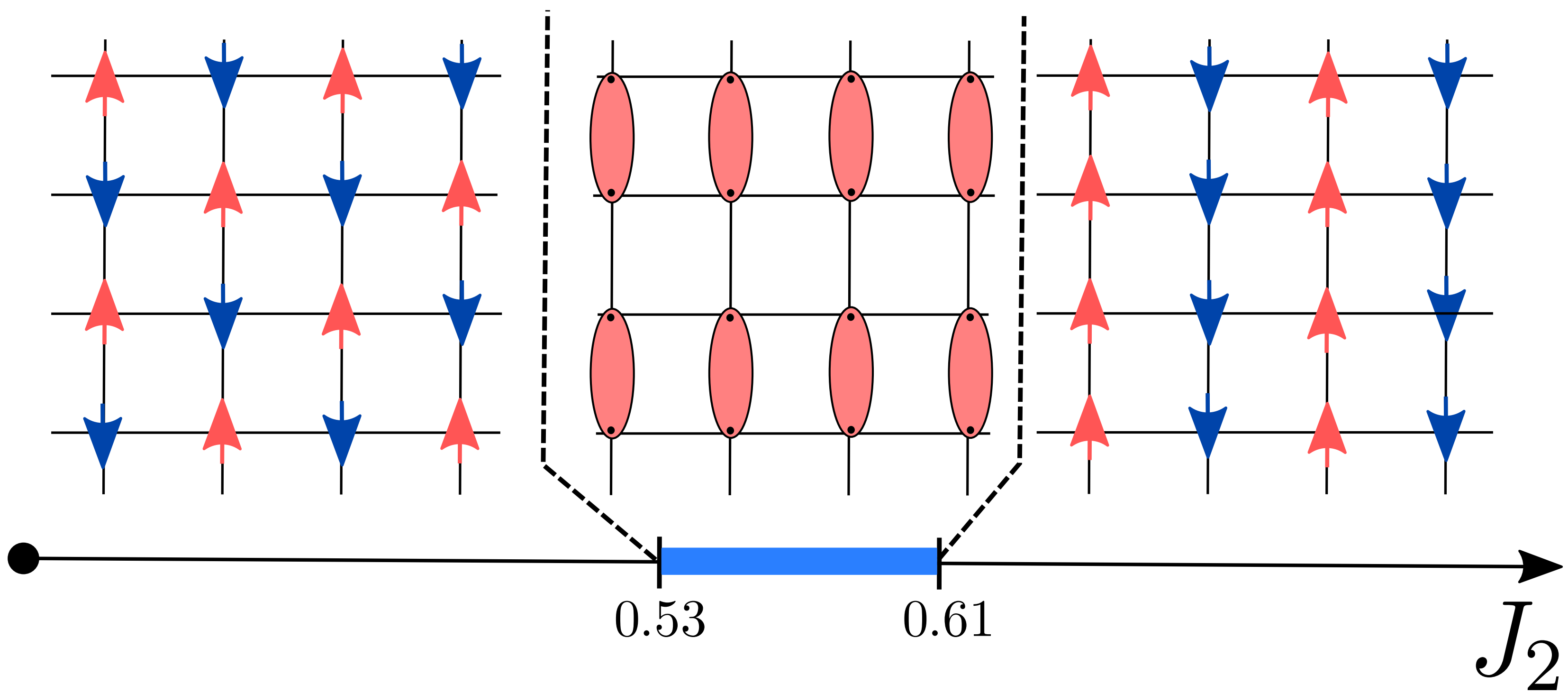
Nearest ($J_1 = 1$) and next-nearest (J_2) neighbor interactions



Critical Level Crossings and Gapless Spin Liquid in the Square-Lattice Spin-1/2 $J_1 - J_2$ Heisenberg Antiferromagnet

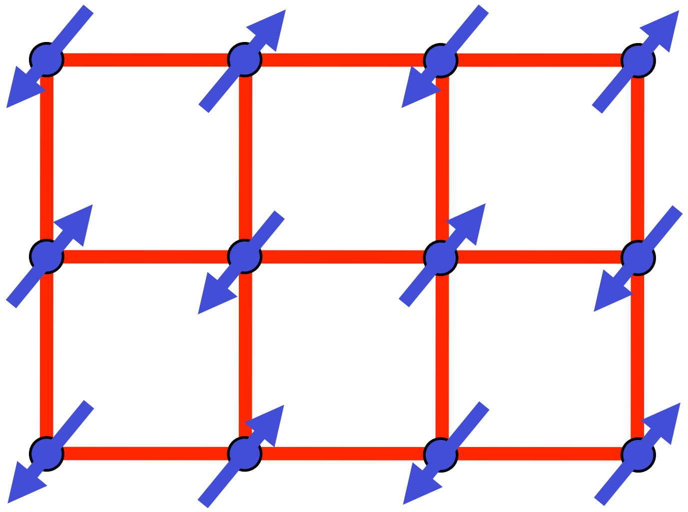
PHYSICAL REVIEW LETTERS **121,** 107202 (2018)

Ling Wang[1,*] and Anders W. Sandvik[2,1,3,†]

The lowest singlet-triplet and singlet-quintuplet crossings converge rapidly (with corrections $\propto L^{-2}$) to different g values, and we argue that these correspond to ground-state transitions between the Néel antiferromagnet and a gapless spin liquid, at $g_{c1} \approx 0.46$, and between the spin liquid and a valence-bond solid at $g_{c2} \approx 0.52$.

Quantum criticality in a frustrated square lattice antiferromagnet

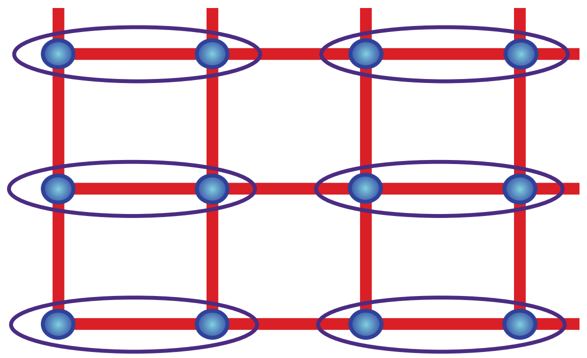
N. Read and S. Sachdev, PRL **62**, 1694 (1989)



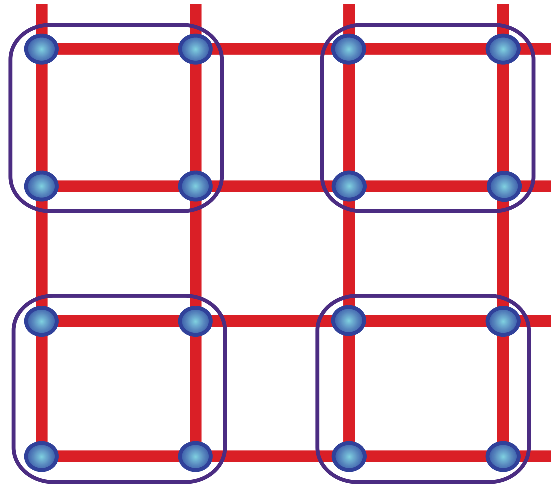
$\langle z_\alpha \rangle \neq 0$

Néel state

$\vec{N} = z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta$



or



$\langle z_\alpha \rangle = 0$

Valence bond solid (VBS) state, V_x, V_y
with a nearly gapless, emergent "photon"

s_c s

Critical $\mathbb{CP}^1$ theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

A non-Abelian duality

Critical U(1) gauge (a_μ) theory of $N_b = 2$ relativistic bosons
is dual to
SU(2) gauge (A_μ) theory of $N_f = 2$ Dirac fermions.

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right]$$

$$\mathcal{S}_f = \int d^2 r d\tau \left[\overline{f} \gamma^\mu (\partial_\mu - i A_\mu) f \right]$$

Chong Wang, A. Nahum, M. A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

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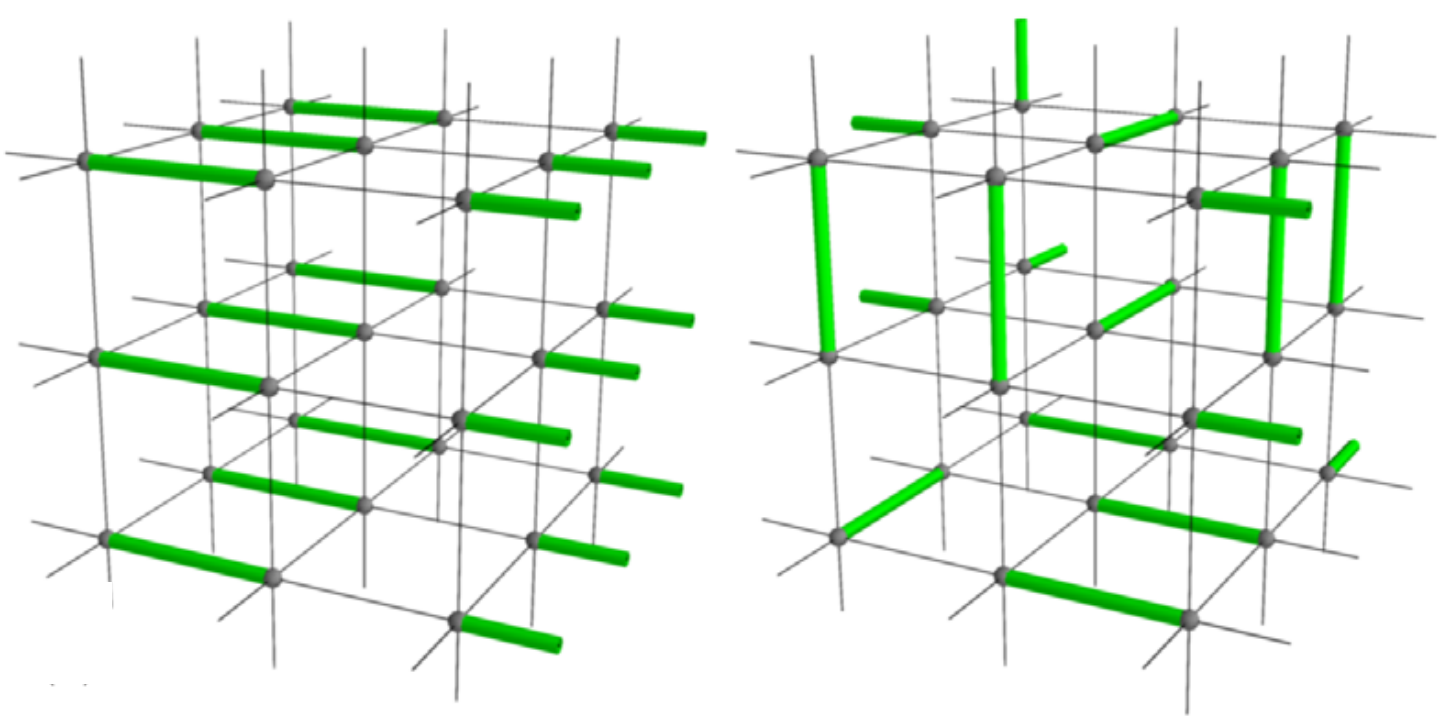
The fermion theory has a SO(5) global flavor symmetry, and the gauge-invariant fermion bilinears form a SO(5) vector which transforms as the Néel and VBS order parameters!

$$(N_x, N_y, N_z, V_x, V_y)$$

Akihiro Tanaka and Xiao Hu, PRL. **95**, 036402 (2005).
T. Senthil and M.P.A. Fisher, PRB **74**, 064405 (2006)
Chong Wang, A. Nahum, M.A. Metlitski, Cenke Xu, and T. Senthil, PRX **7**, 031051 (2017)

Emergent SO(5) Symmetry at the Columnar Ordering Transition in the Classical Cubic Dimer Model

"Studying linear system sizes up to L=96, we find that this symmetry applies with an excellent precision, consistently improving with system size over this range. It is remarkable that SO(5) emerges in a system as basic as the cubic dimer model, with only simple discrete degrees of freedom. Our results are important evidence for the generality of the SO(5) symmetry that has been proposed for the noncompact CP^1 field theory. We describe an interpretation for these results in terms of a consistent hypothesis for the renormalization-group flow structure, allowing for the possibility that SO(5) may ultimately be a near-symmetry rather than exact."



G.J. Sreejith, Stephen Powell, and Adam Nahum
PRL **122**, 080601 (2019)

Stephen Powell and John T. Chalker,
PRB **80**, 134413 (2009)

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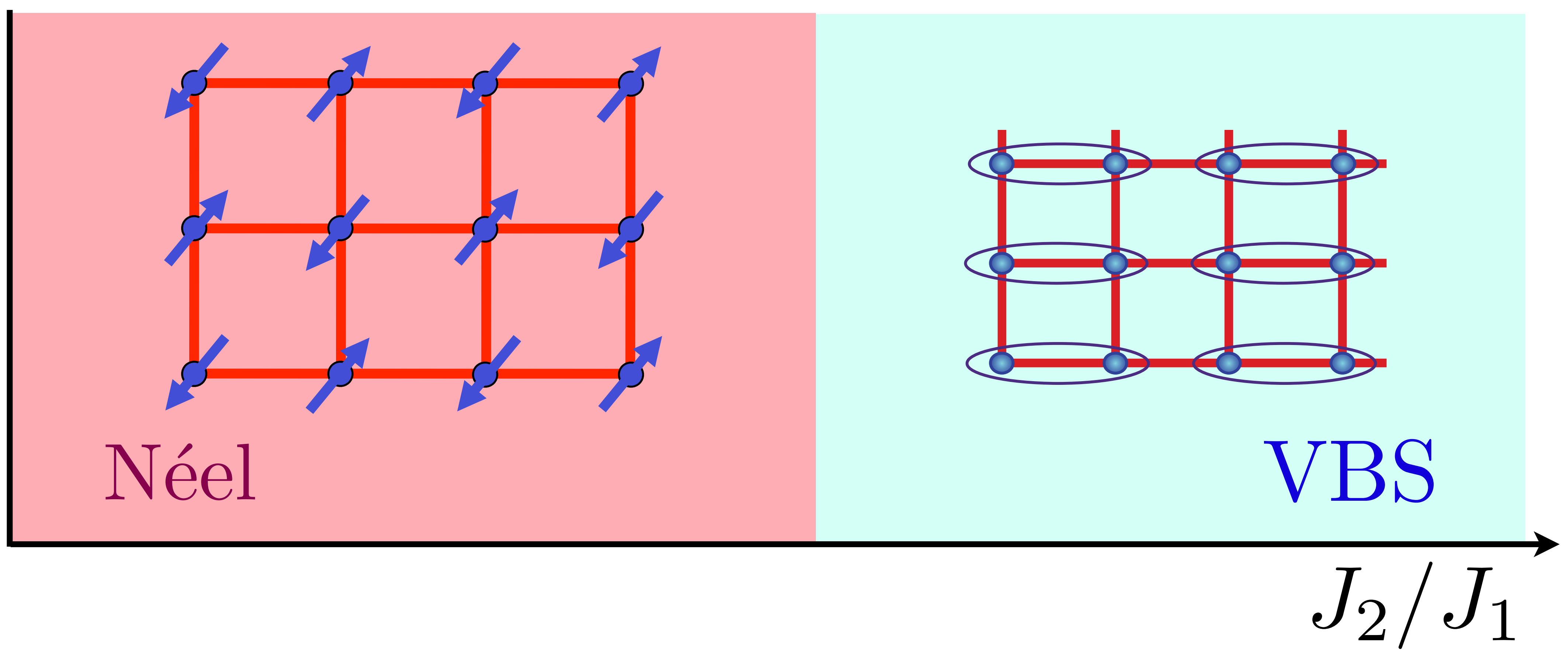

$$\mathcal{S}_f = \int d^2 r d\tau \left[\overline{f} \gamma^\mu (\partial_\mu - i A_\mu) f \right]$$

Quantum critical theory

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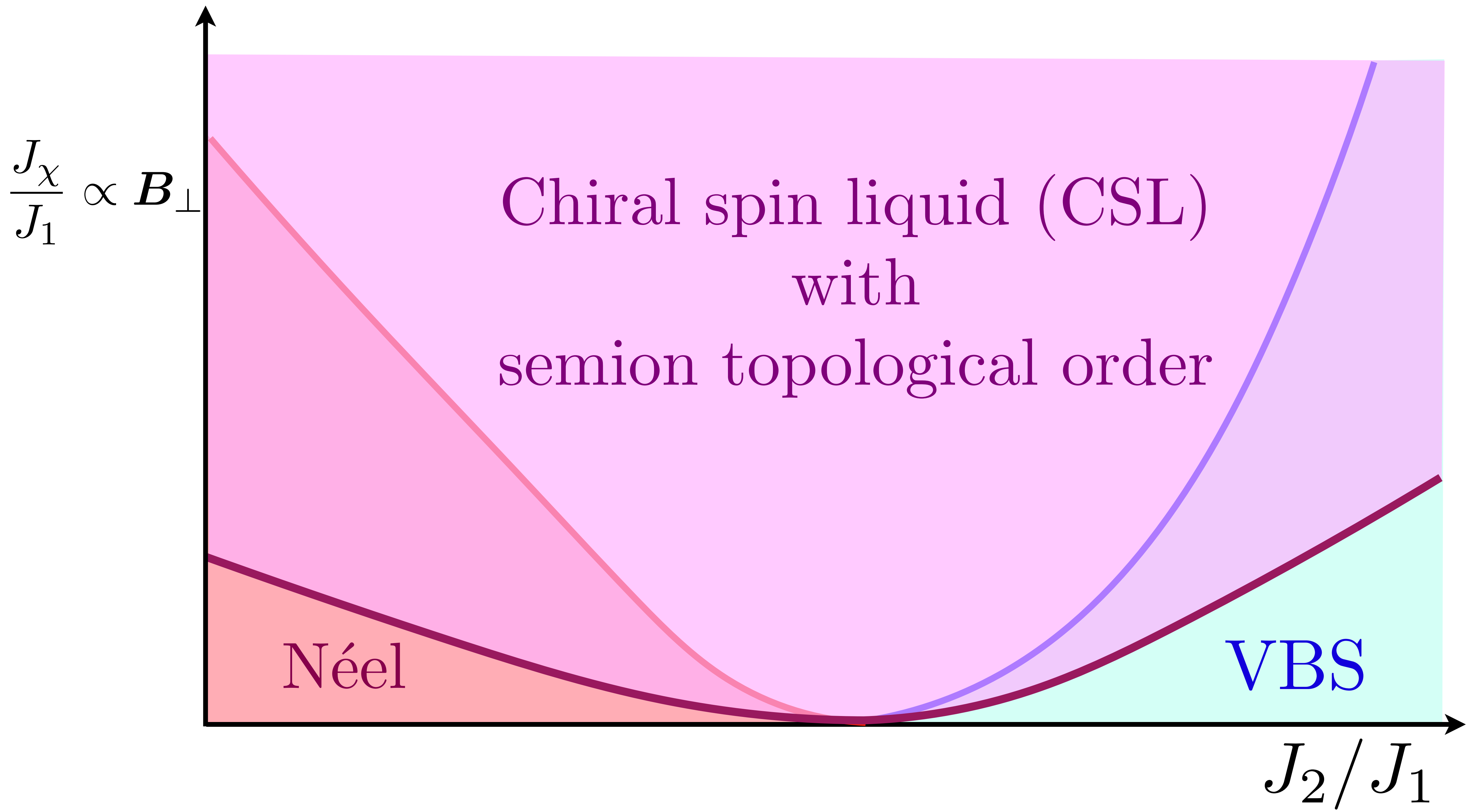

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Quantum critical theory + J_χ

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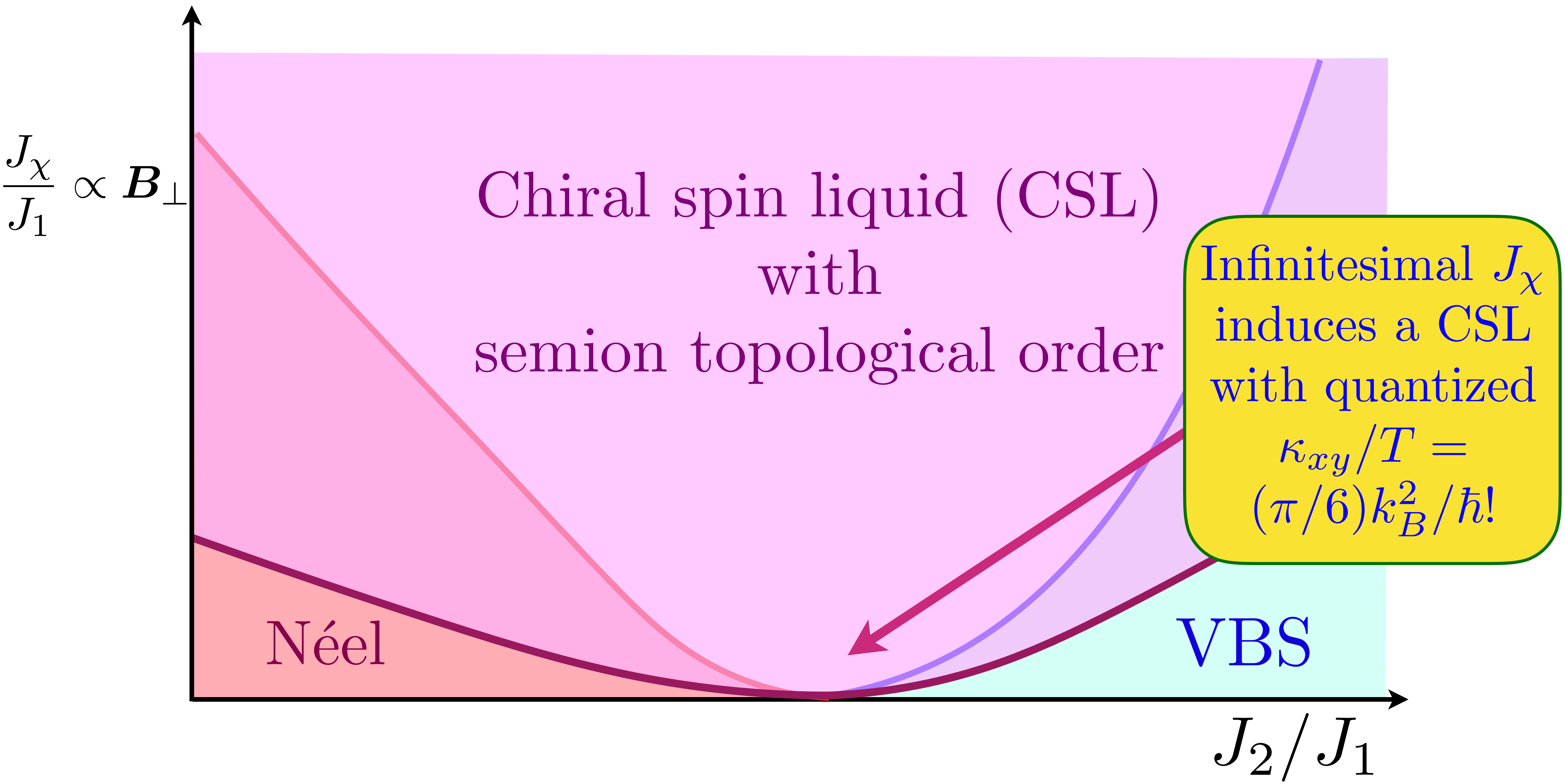

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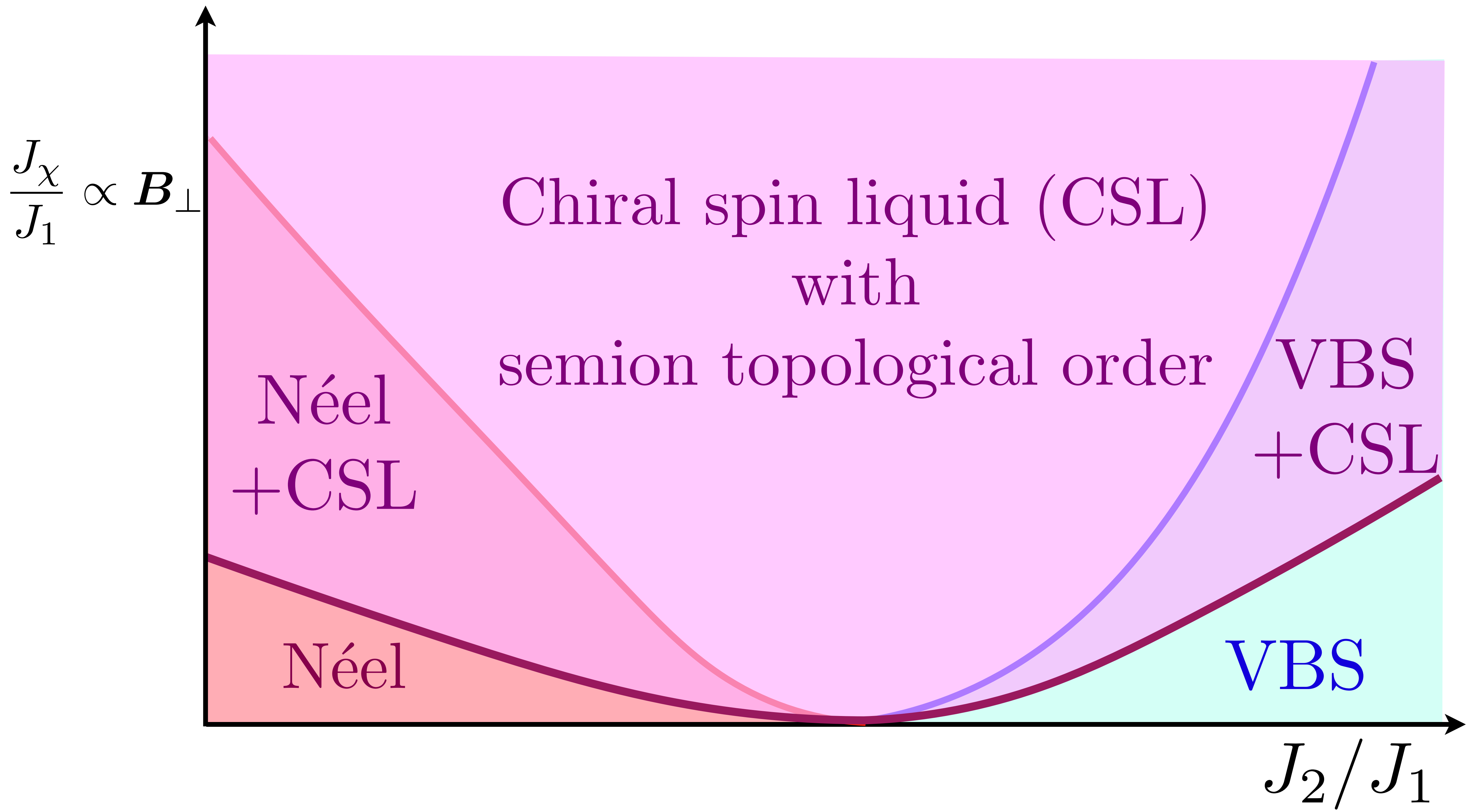

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Quantum critical theory + J_χ + Néel order

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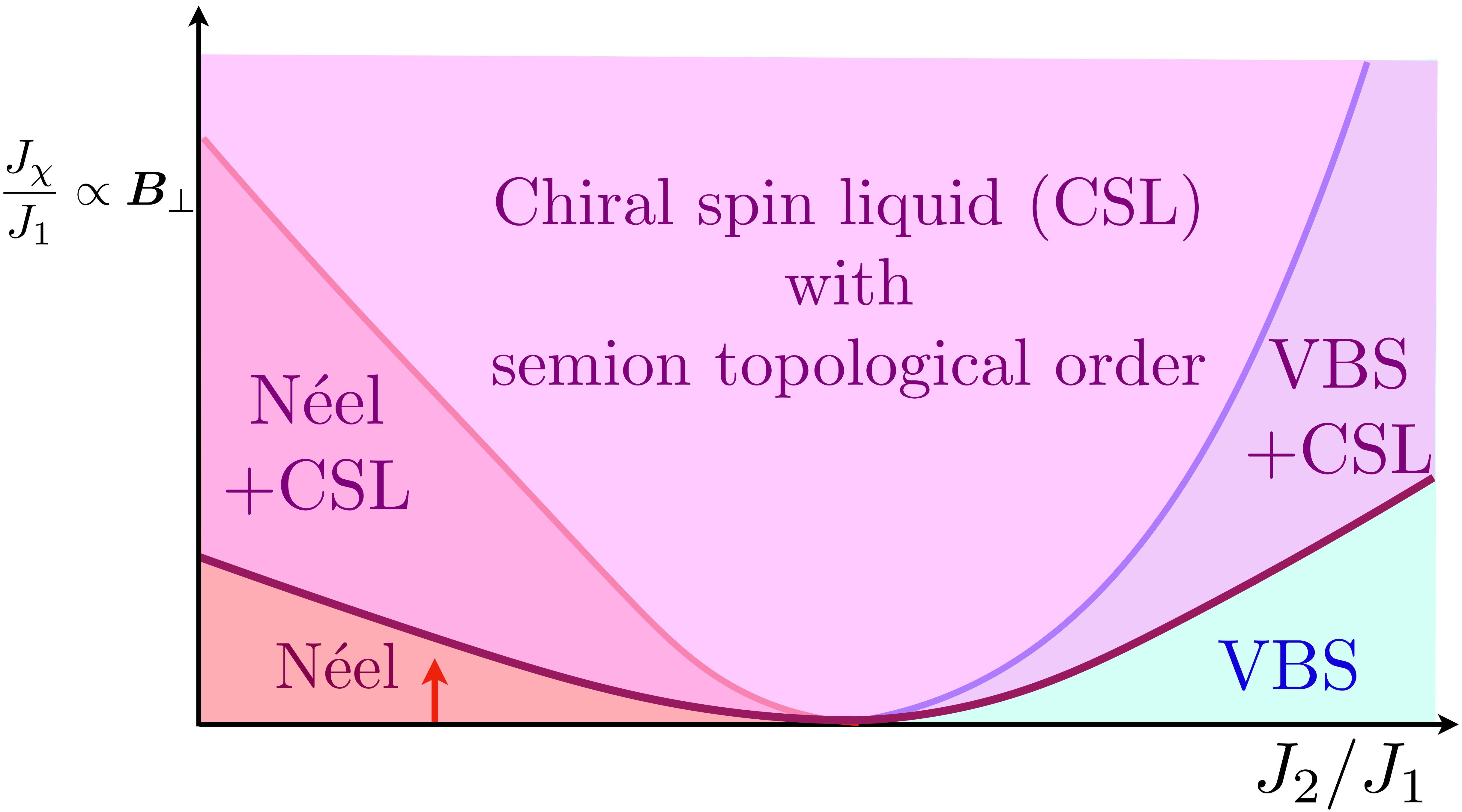

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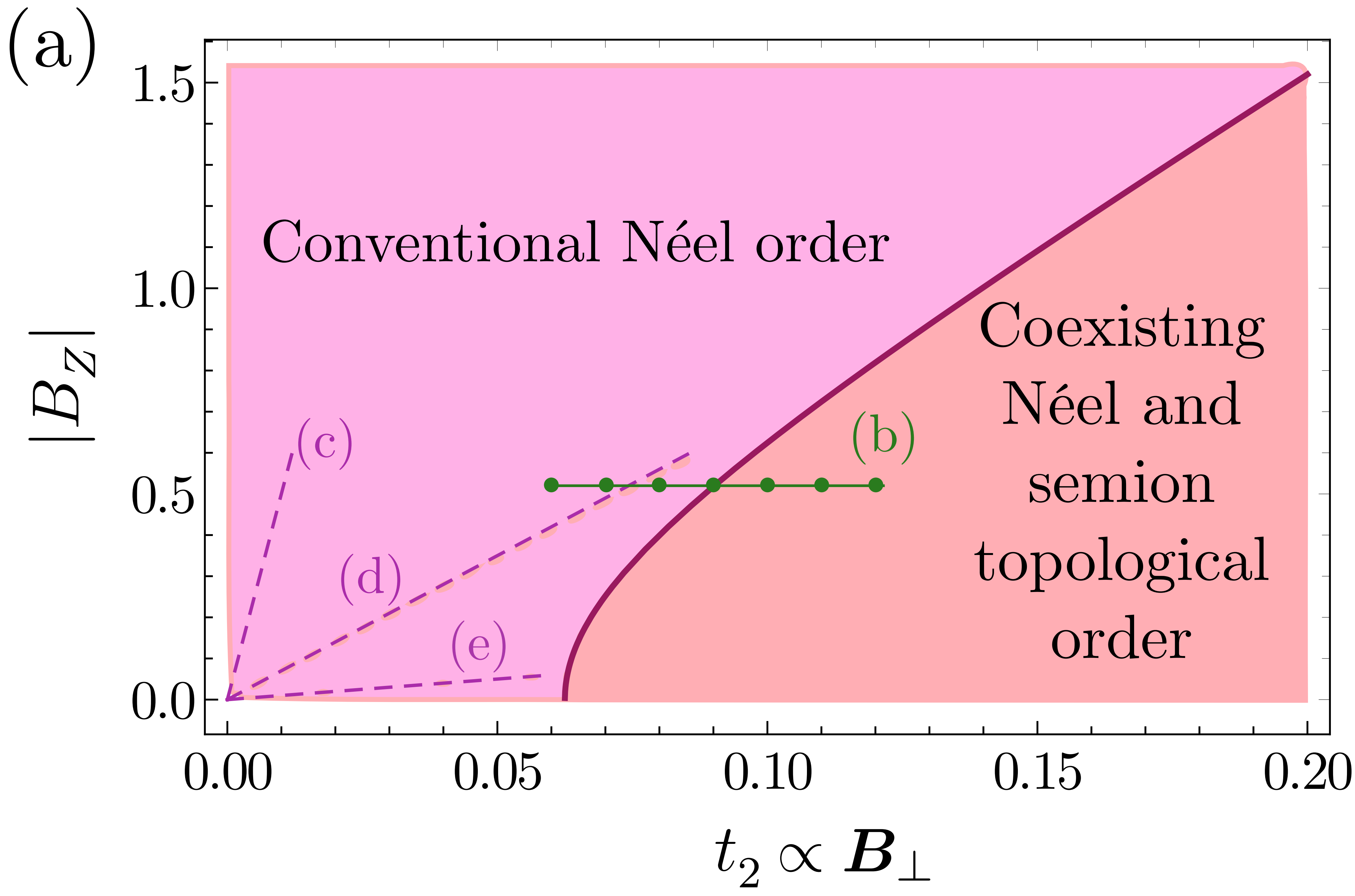

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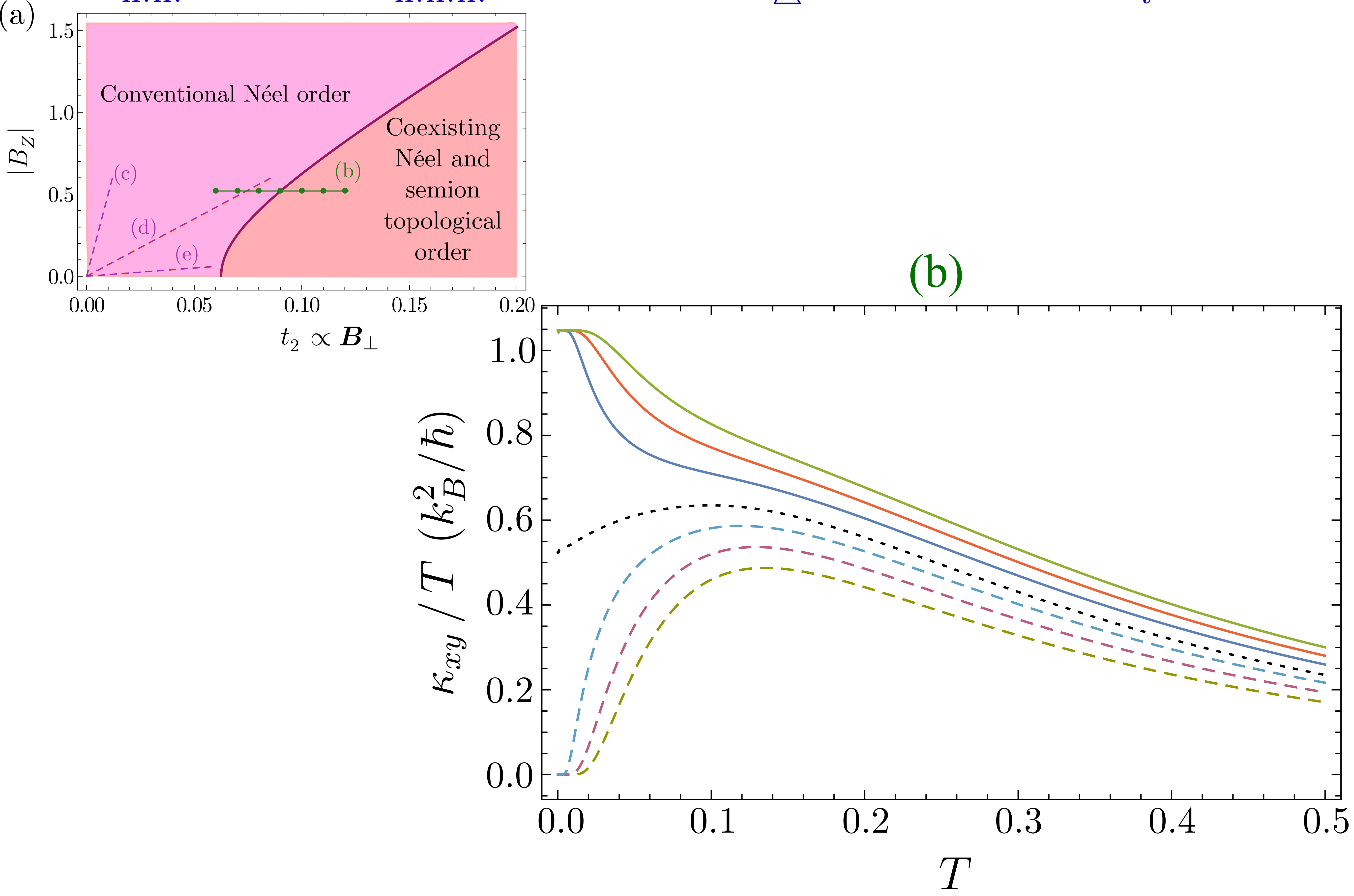

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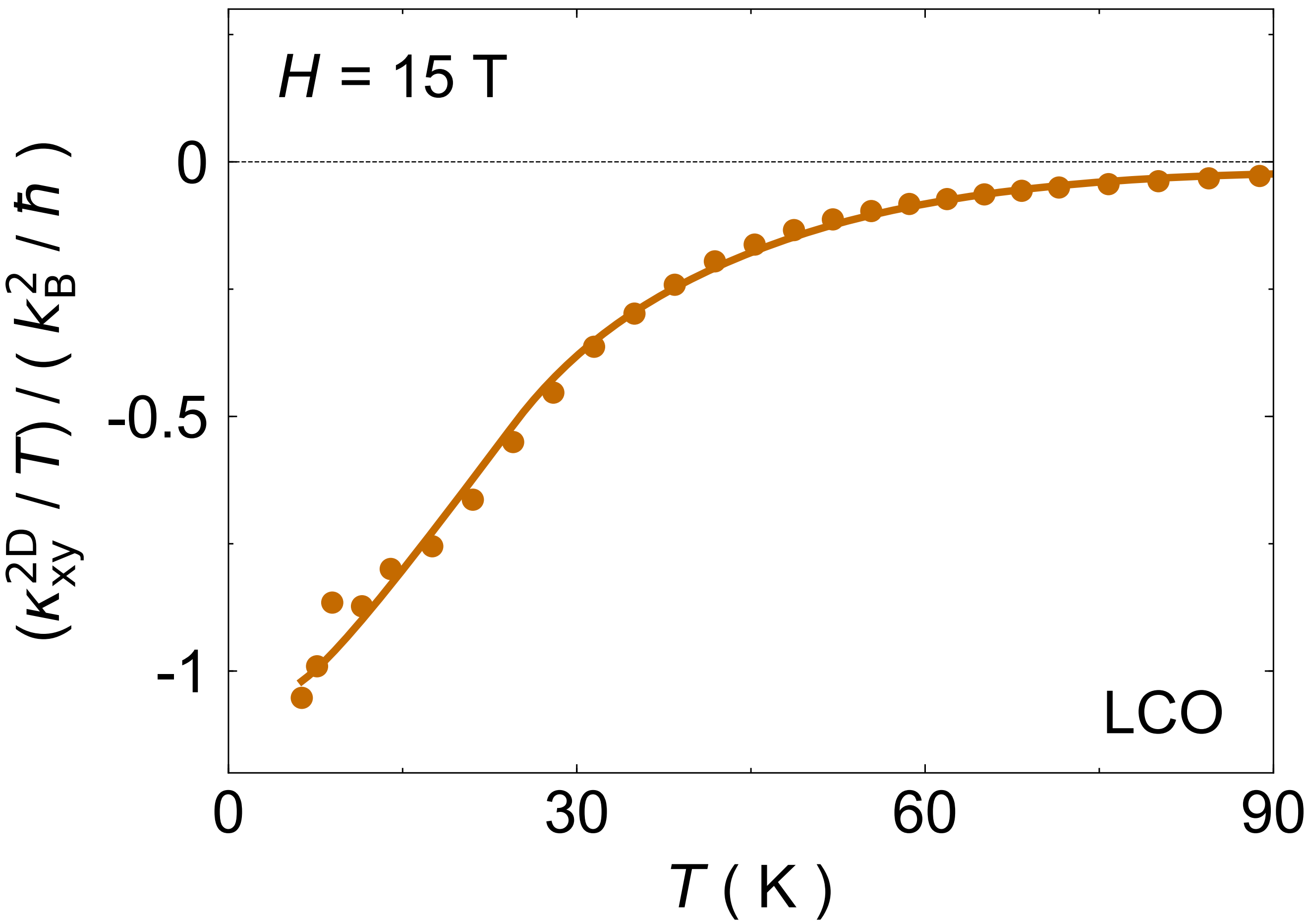

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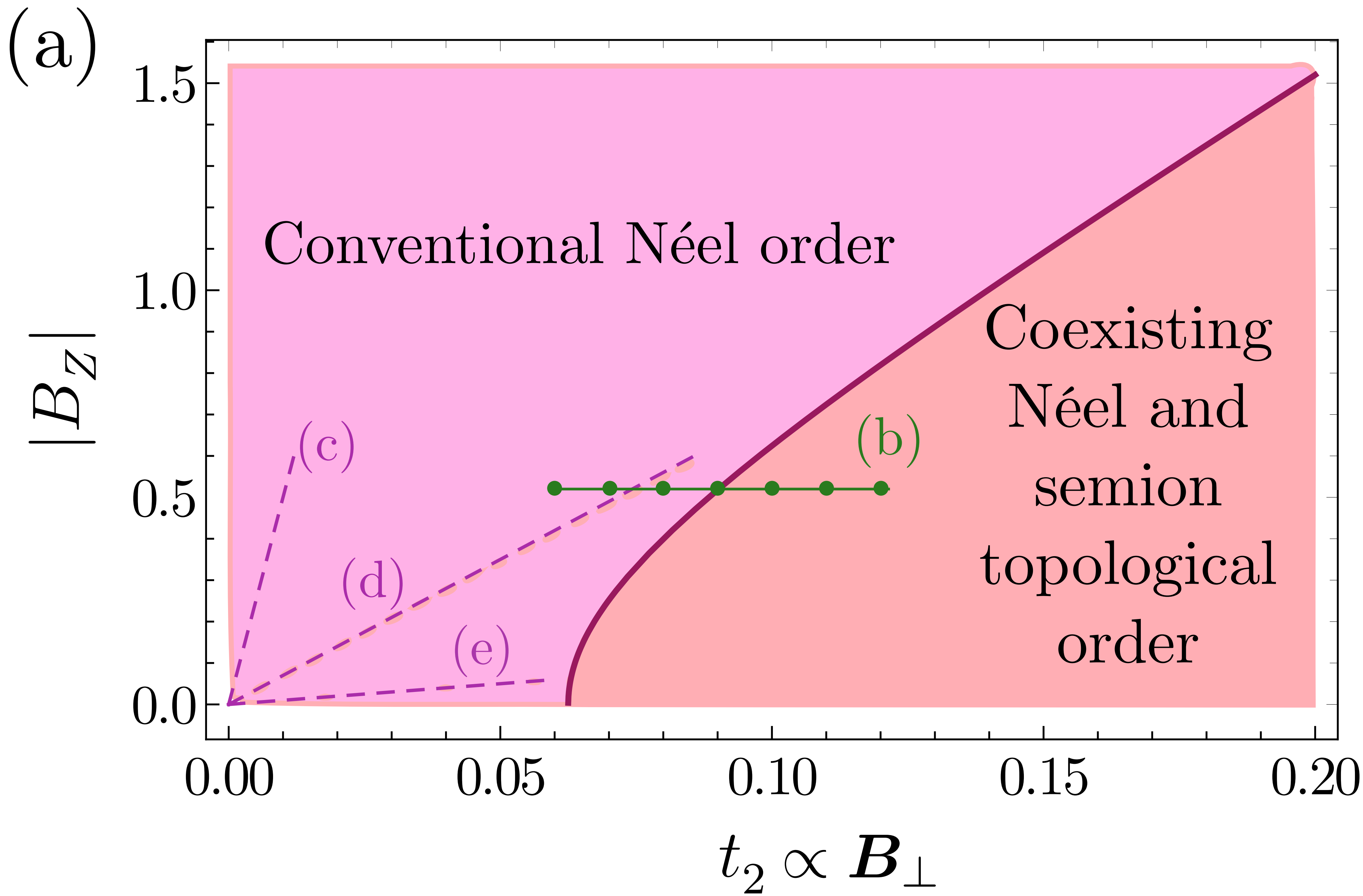

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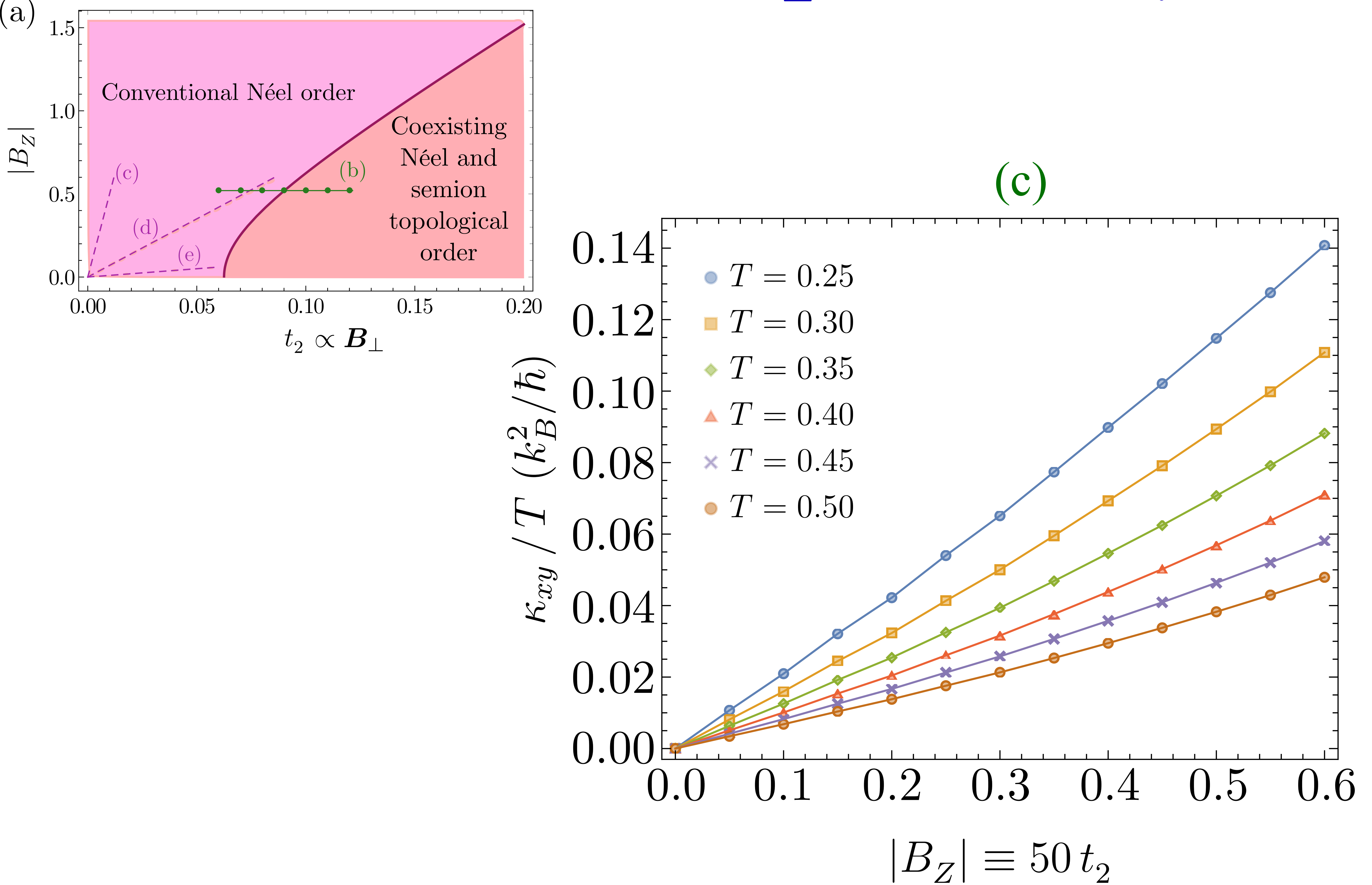

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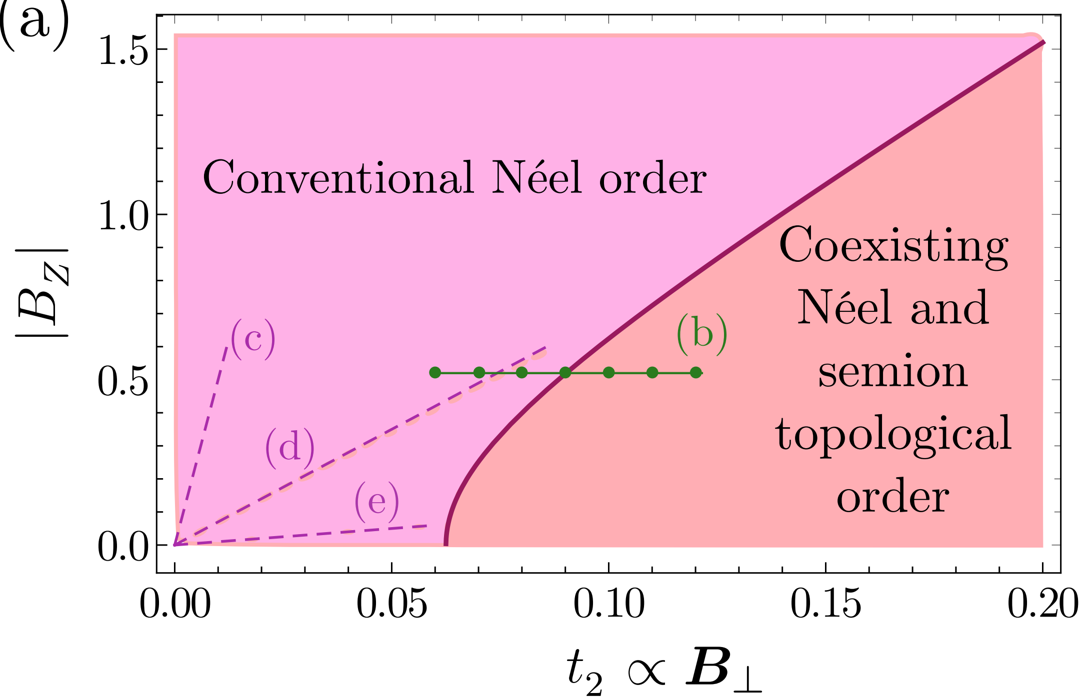
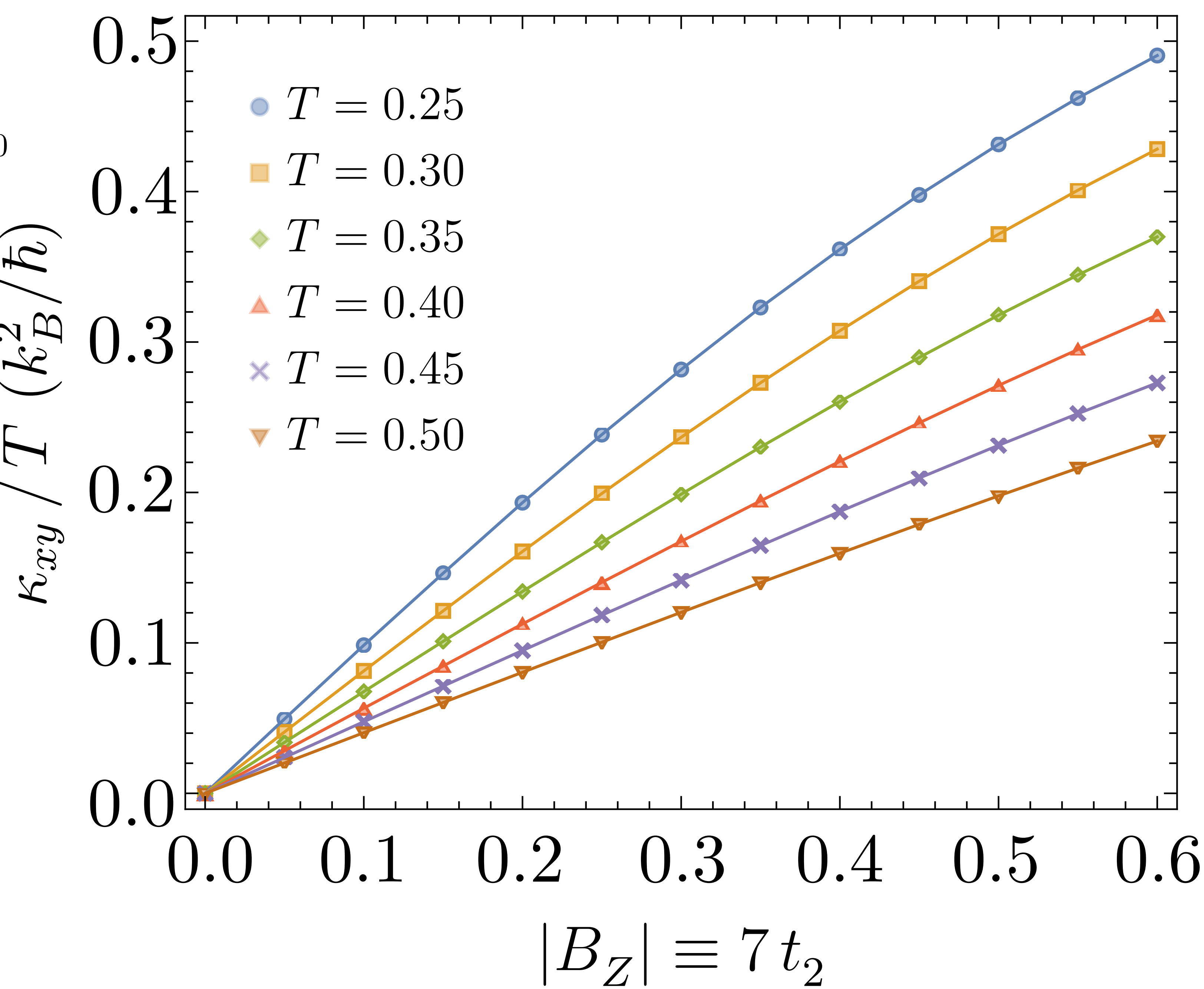
(a)



(d)



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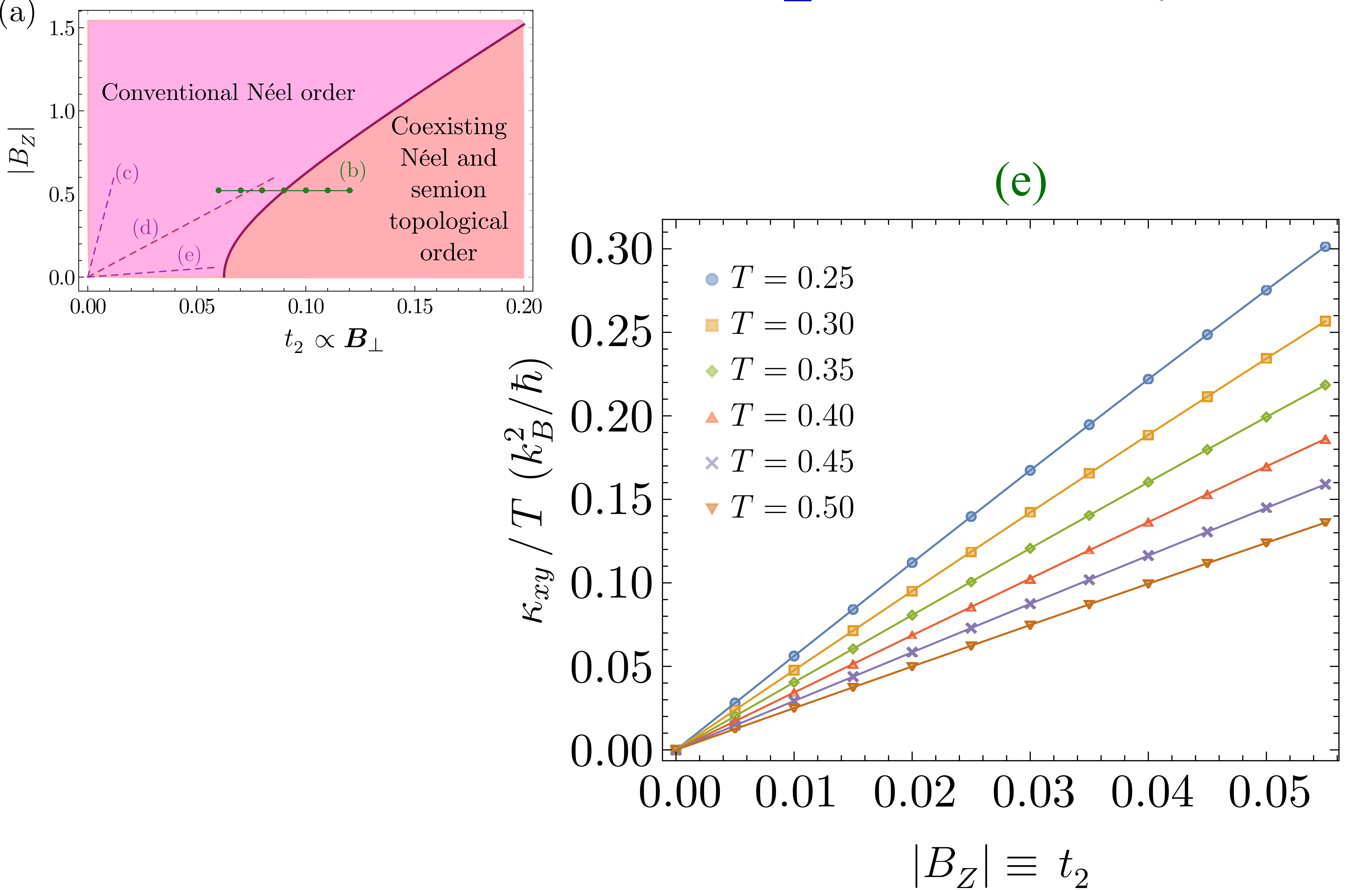

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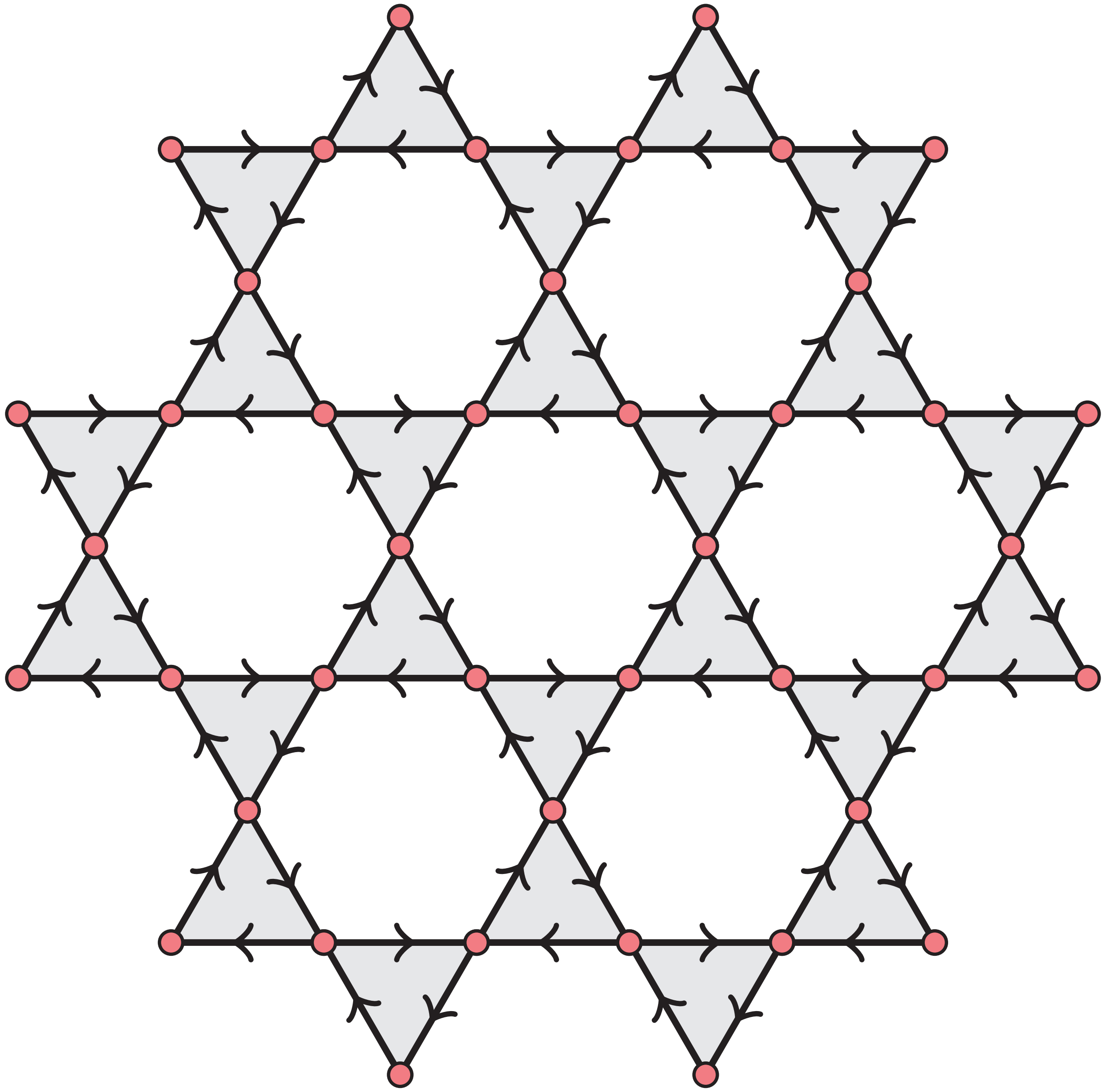


$$H = H_1 + H_\chi$$

$$H_1 = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \ldots$$

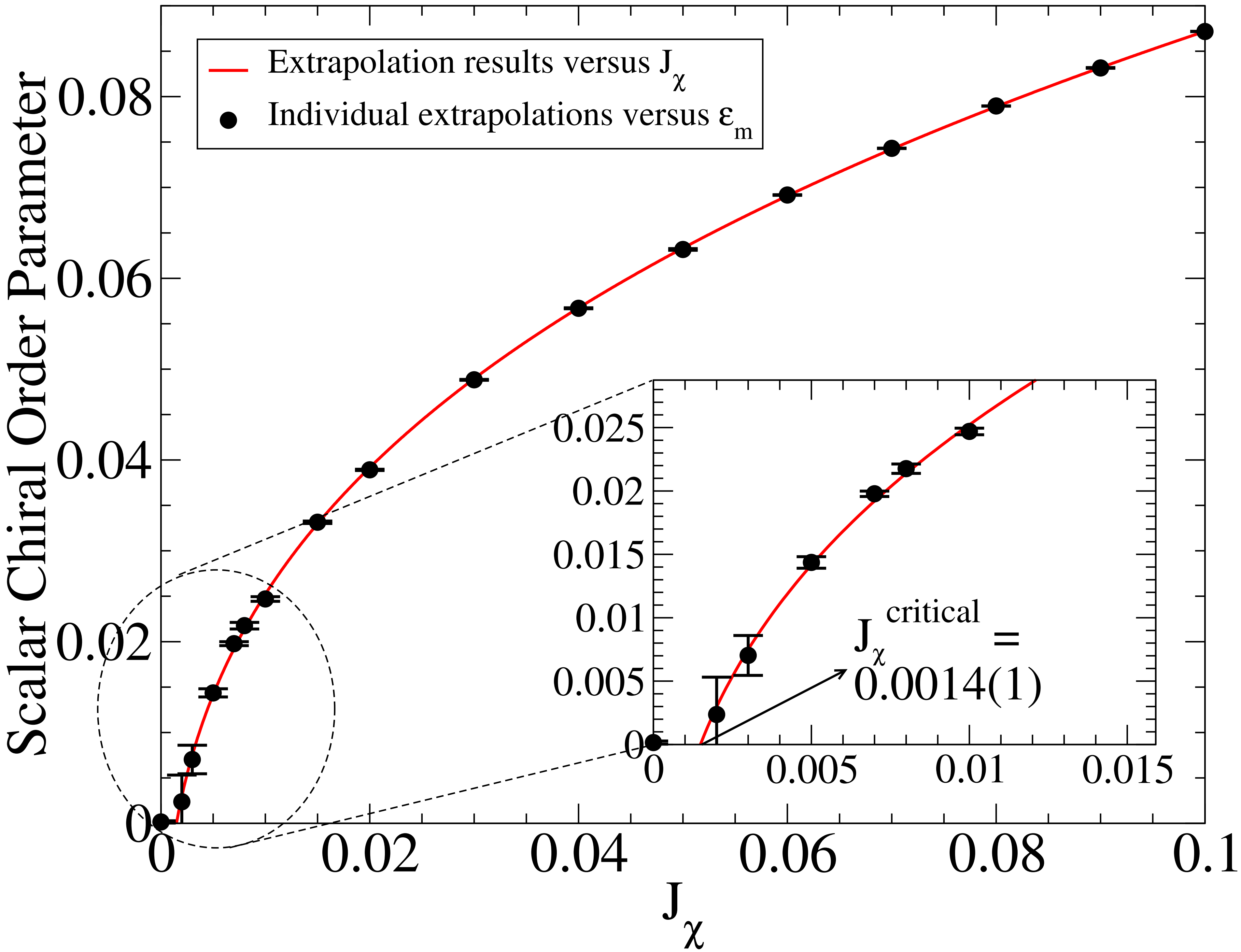
$$H_\chi = J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k)$$

B. Bauer, L. Cincio, B.P. Keller, M. Dolfi, G. Vidal, S. Trebst and A.W.W. Ludwig, Nature Communications **5**, 5137 (2014)

Semion topological order,
i.e. the Kalmeyer-Laughlin chiral spin liquid,
appears for $J_\chi / J > 0.01$.

R. Haghshenas, Shou-Shu Gong, and D.N. Sheng, arXiv:1812.11436

Triangular lattice antiferromagnet

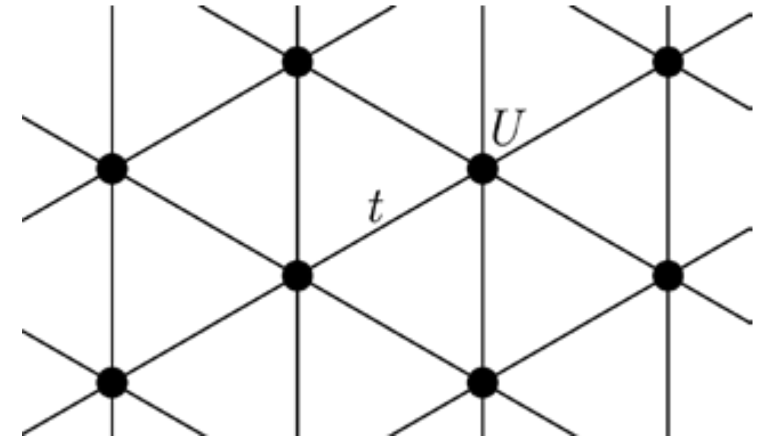


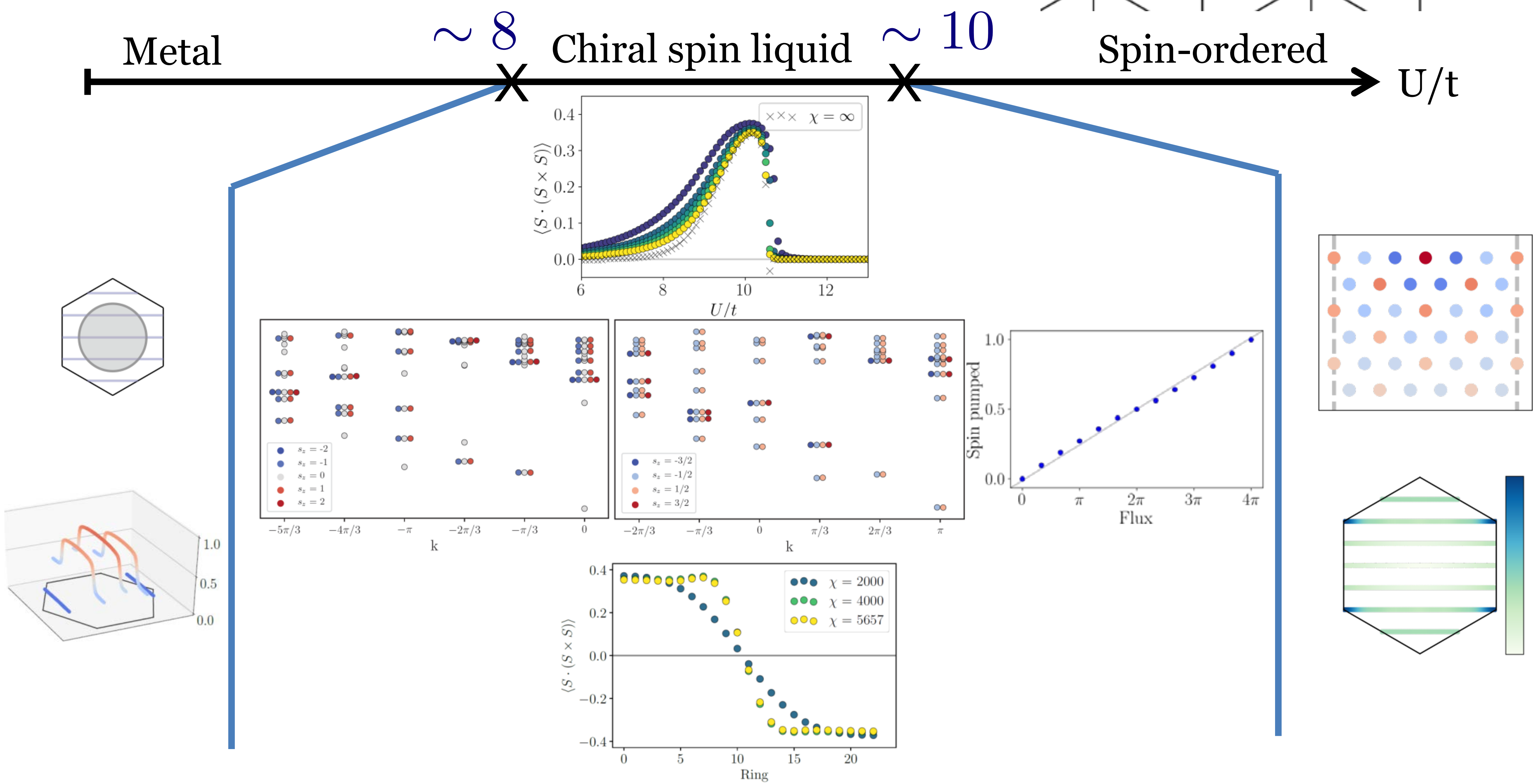
$$J_2/J_1 = 1/8;\ \text{critical}\ J_\chi = 0.0014$$

S. N. Saadatmand and I. P. McCulloch, PRB **96**, 075117 (2017)

Hubbard model:

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left(c^\dagger_{i\sigma} c_{j\sigma} + \text{h.c.} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$





A. Szasz, J. Motruk, M. P. Zaletel, and J. E. Moore, arXiv: 1808.00463

(from slides by Aaron Szasz)

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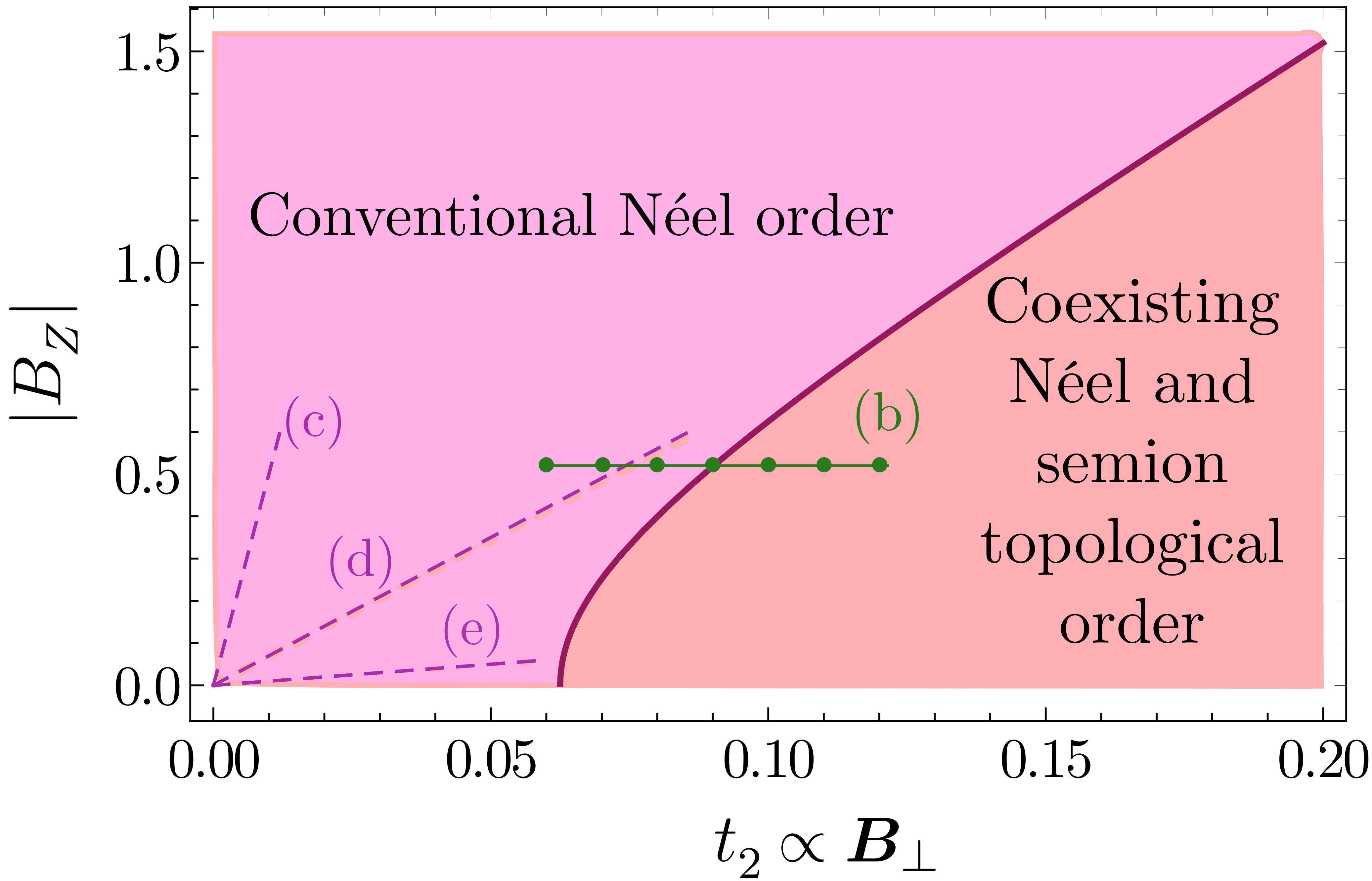
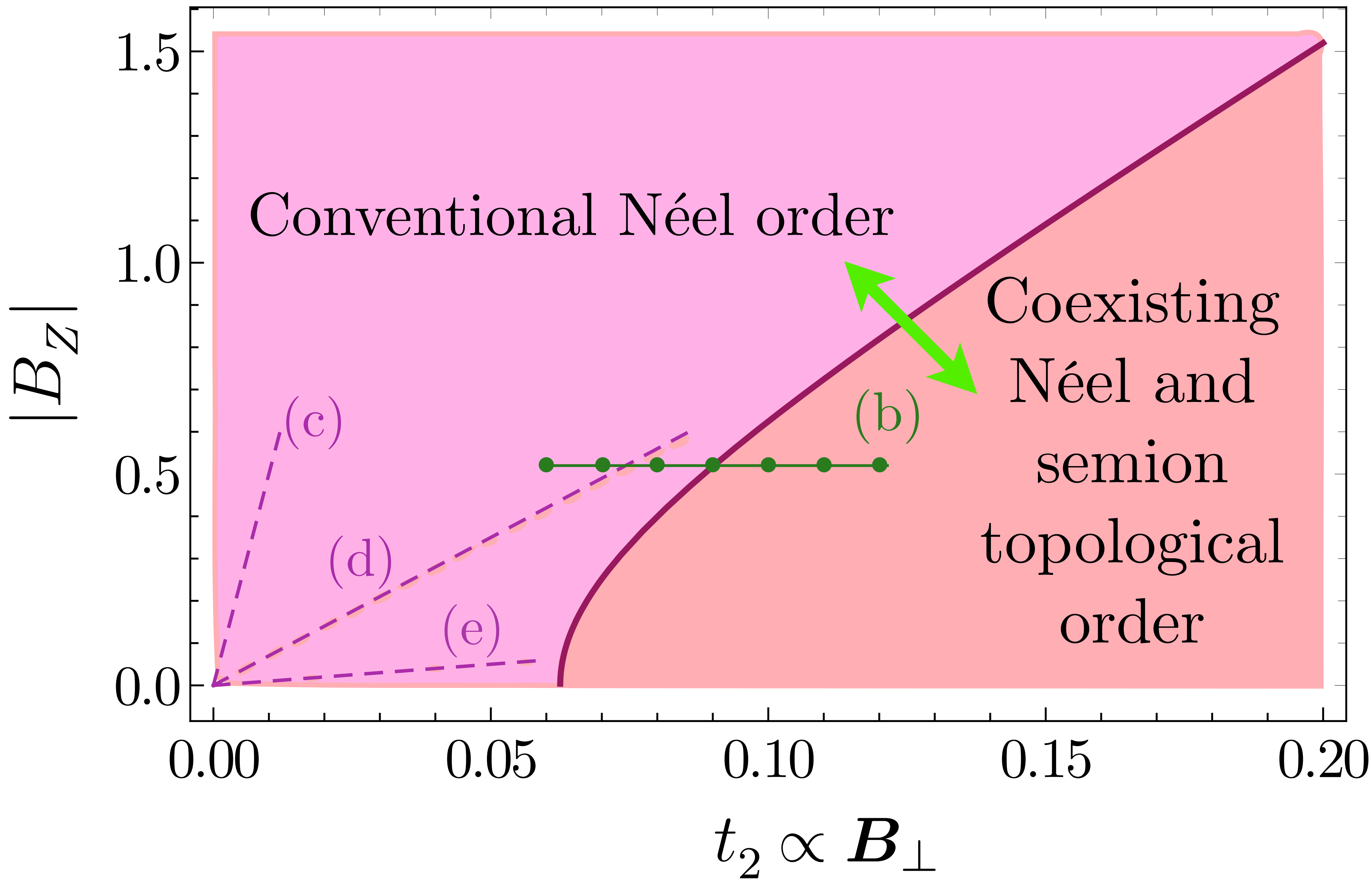
3. Semion topological order on other lattices

4. Quantum criticality and more non-Abelian dualities

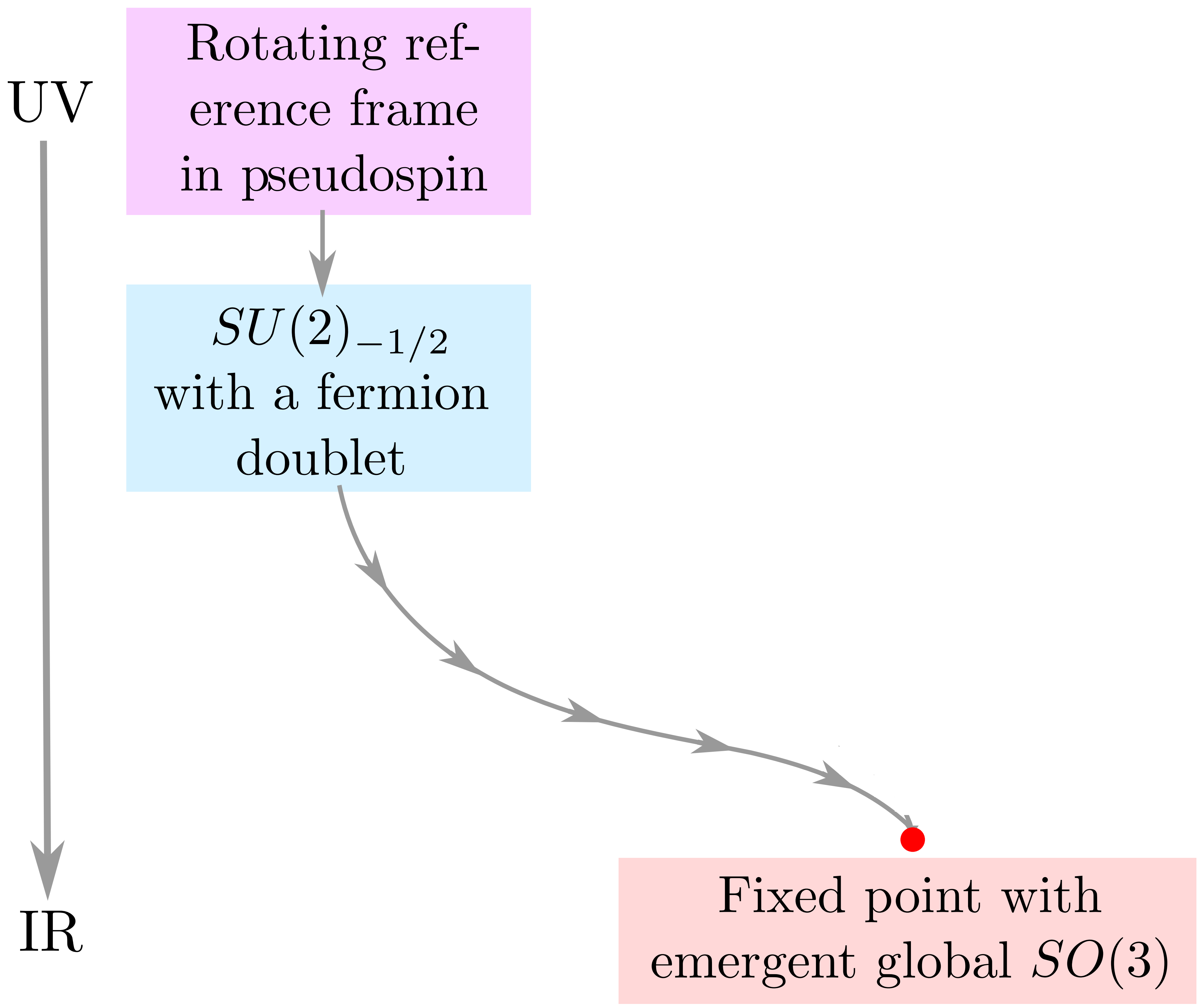
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \, .$$

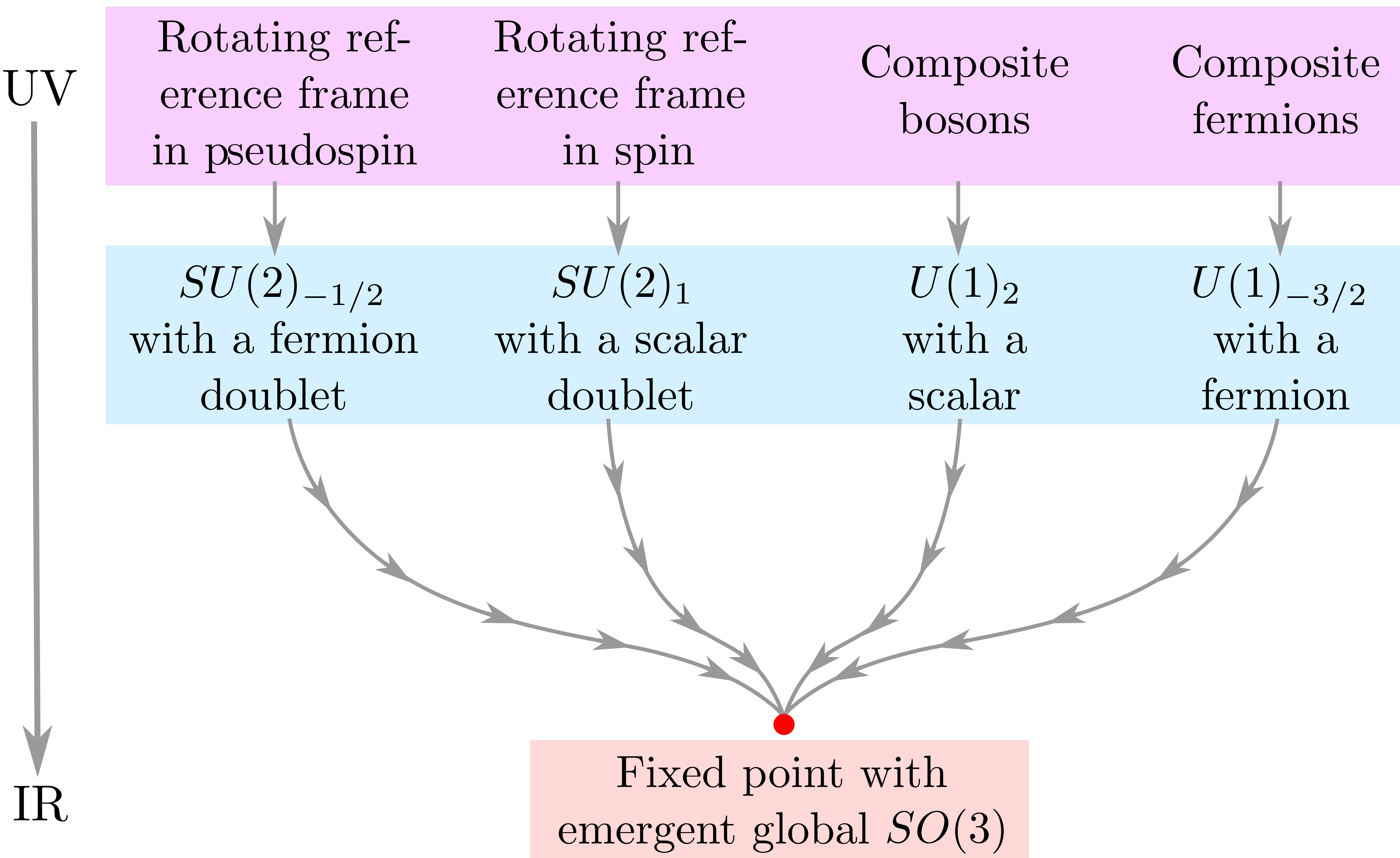


$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$



$$\mathcal{L}_f = \bar{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\bar{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$


UV
IR
Rotating reference frame in pseudospin
$SU(2)_{-1/2}$ with a fermion doublet
Fixed point with emergent global $SO(3)$



Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

A quadrilarity

$$\mathcal{L}_z = |(\partial_\mu - iA_\mu)z|^2 + s|z|^2 + u(|z|^2)^2 + \mathrm{CS}[A_\mu]$$

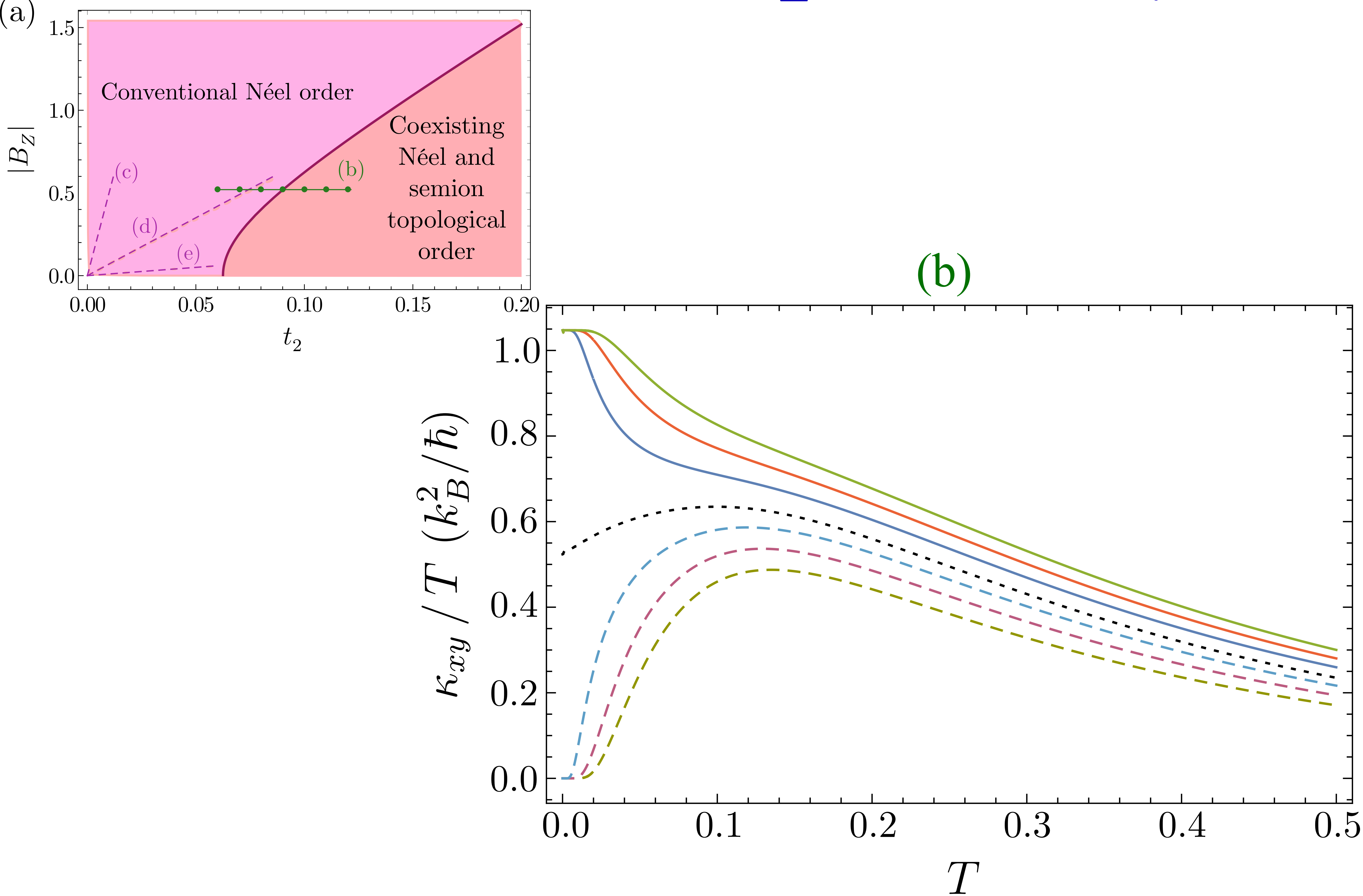
$$\mathcal{L}_f = \overline{f}\gamma^\mu(\partial_\mu - iA_\mu)f + m\overline{f}f - \frac{1}{2}\mathrm{CS}[A_\mu]$$

$$\mathcal{L}_\phi = |(\partial_\mu - ia_\mu)\phi|^2 + s|\phi|^2 + u(|\phi|^2)^2 + 2\mathrm{CS}[a_\mu]$$

$$\mathcal{L}_g = \overline{g}\gamma^\mu(\partial_\mu - ia_\mu)g + m\overline{g}g - \frac{3}{2}\mathrm{CS}[a_\mu]$$

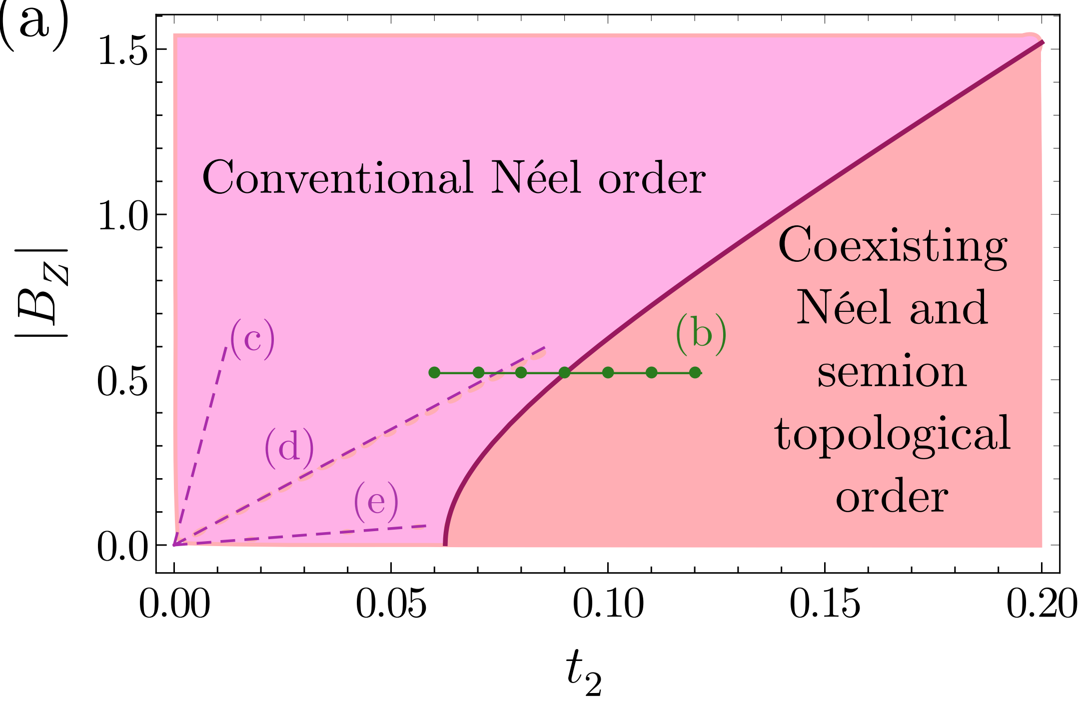
Francesco Benini, Po-Shen Hsin, and Nathan Seiberg, arXiv:1702.07035

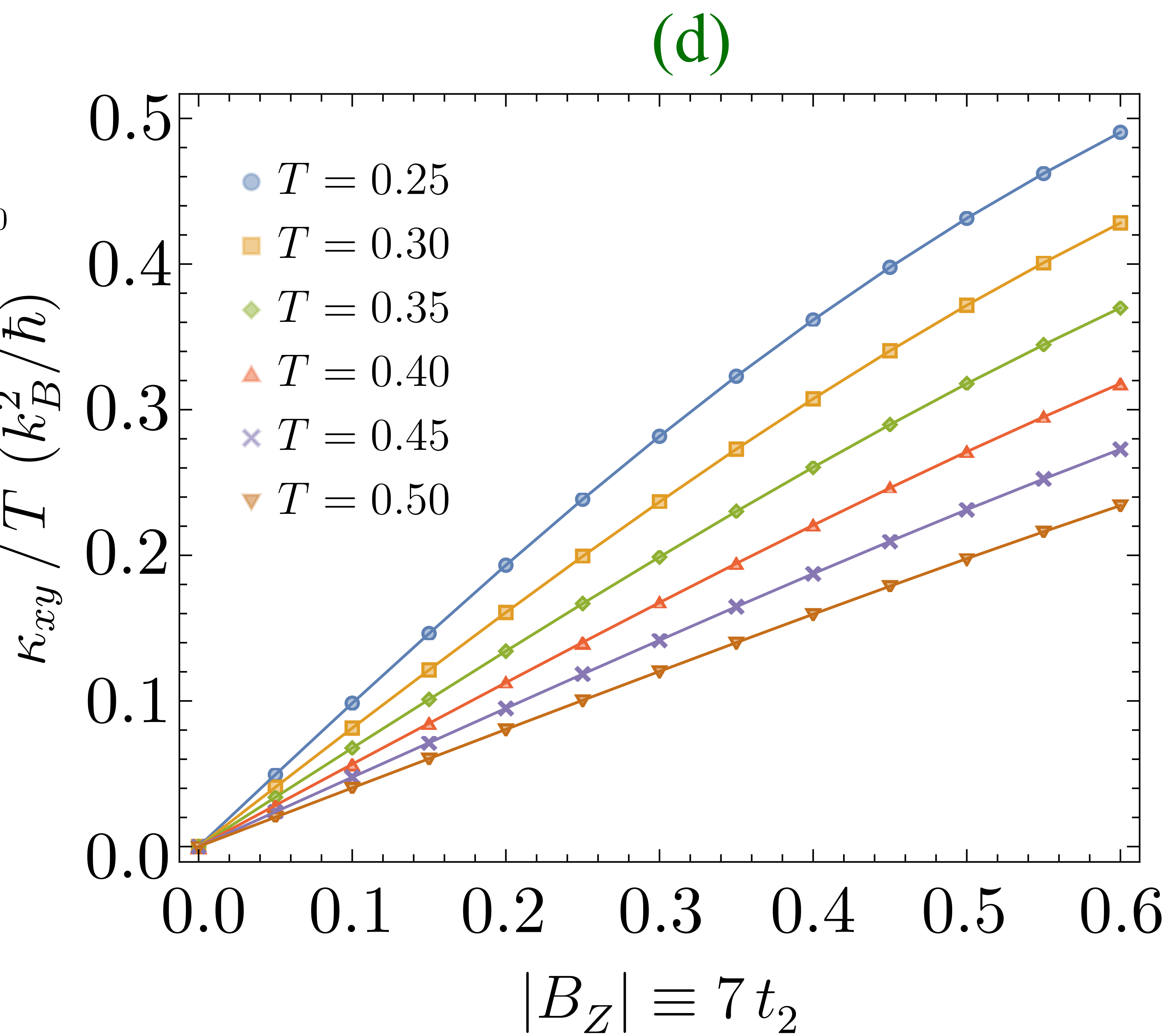
$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \, .$$



$$H = \sum_{\text{n.n.}} J_1 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + \sum_{\text{n.n.n.}} J_2 \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_\chi \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k) - \sum_i \boldsymbol{B}_Z \cdot \boldsymbol{S}_i \,.$$

(a)





Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors

G. Grissonnanche, A. Legros, S. Badoux, E. Lefrancois, V. Zatko,
M. Lizaire, F. Laliberte, A. Gourgout, J. Zhou, S. Pyon, T. Takayama,
H. Takagi, S. Ono, N. Doiron- Leyraud, and L. Taillefer,
Nature **571**, 376 (2019)