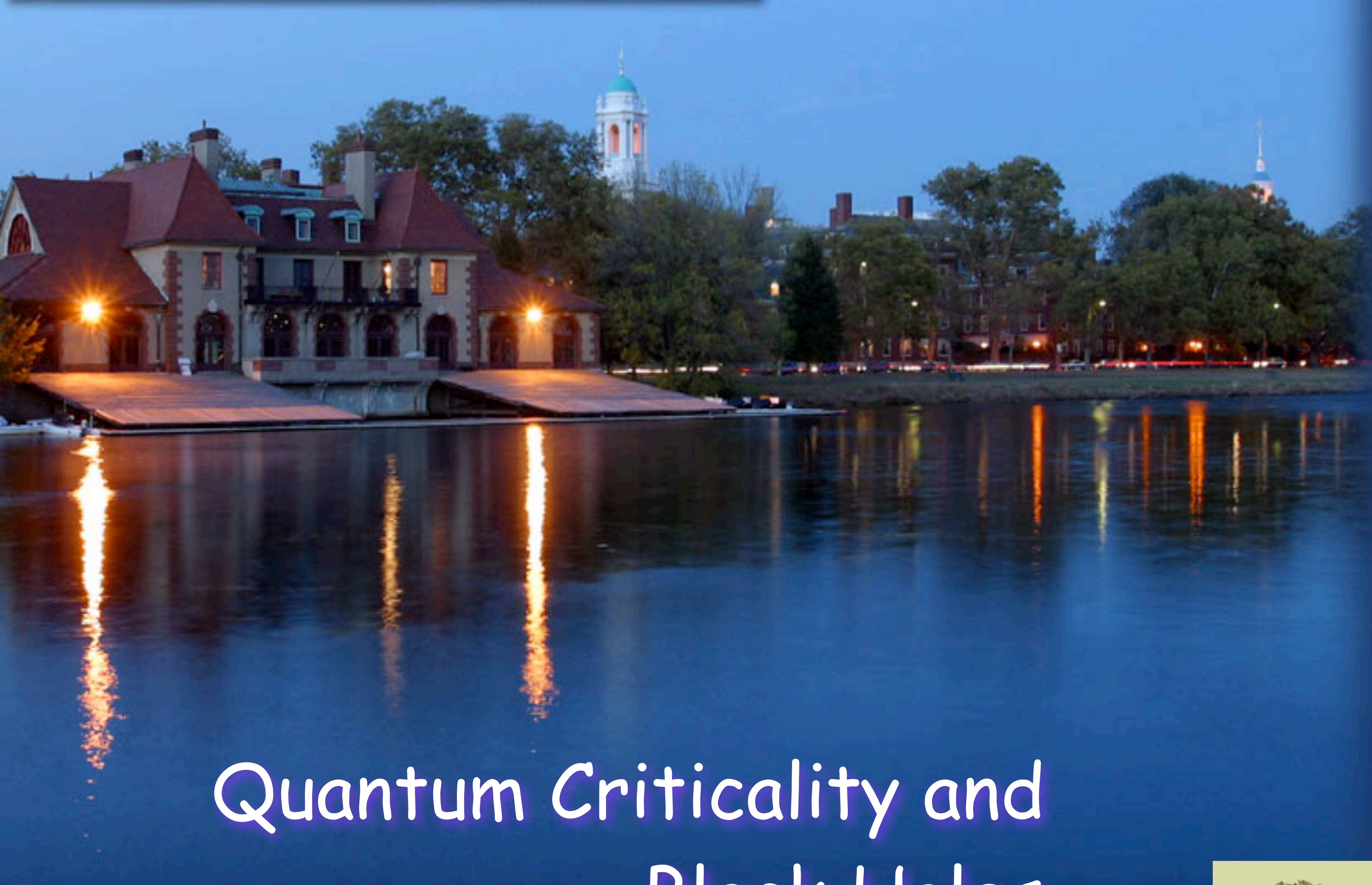


# Quantum Criticality and Black Holes

Talk online: sachdev.physics.harvard.edu

Particle theorists

Sean Hartnoll, KITP
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington

Condensed matter theorists

Markus Mueller, Harvard
Lars Fritz, Harvard
Subir Sachdev, Harvard

# Three foci of modern physics

Quantum phase transitions

# Three foci of modern physics

Quantum phase transitions

Many QPTs of correlated electrons in 2+1 dimensions are described by conformal field theories (CFTs)

# Three foci of modern physics

Quantum phase transitions

Black holes

# Three foci of modern physics

Quantum phase transitions

Black holes

Bekenstein and Hawking originated the quantum theory, which has found fruition in string theory

# Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Black holes

# Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Universal description of fluids based upon conservation laws and positivity of entropy production

Black holes

# Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Black holes

# Three foci of modern physics

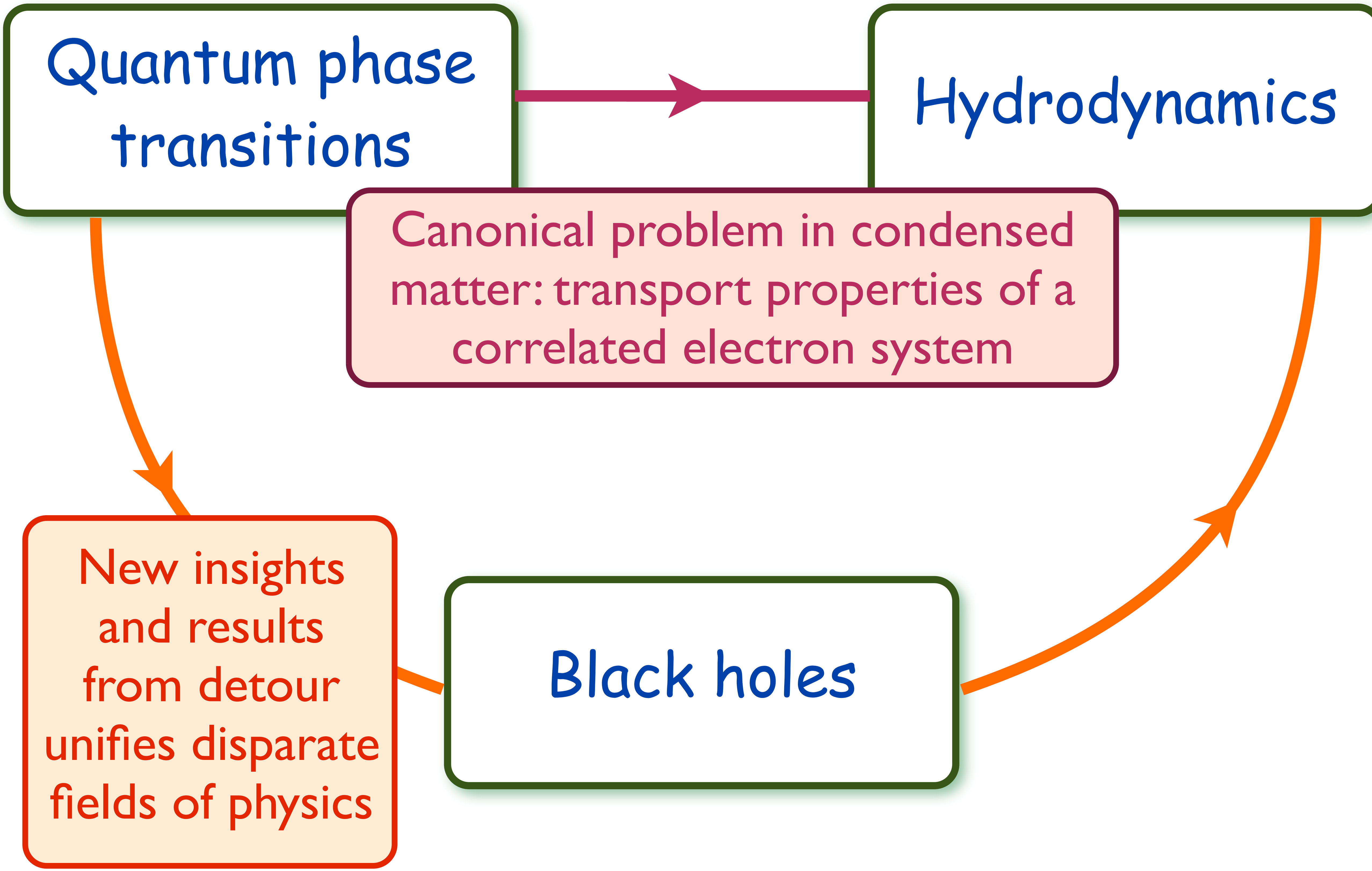
Quantum phase transitions

Hydrodynamics

Canonical problem in condensed matter: transport properties of a correlated electron system

Black holes

# Three foci of modern physics



# Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Black holes

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Quantum phase transitions

Many QPTs of correlated electrons in 2+1 dimensions are described by conformal field theories (CFTs)

Hydrodynamics

Black holes

# Three foci of modern physics

Quantum phase transitions

Many QPTs of correlated electrons in 2+1 dimensions are described by conformal field theories (CFTs)

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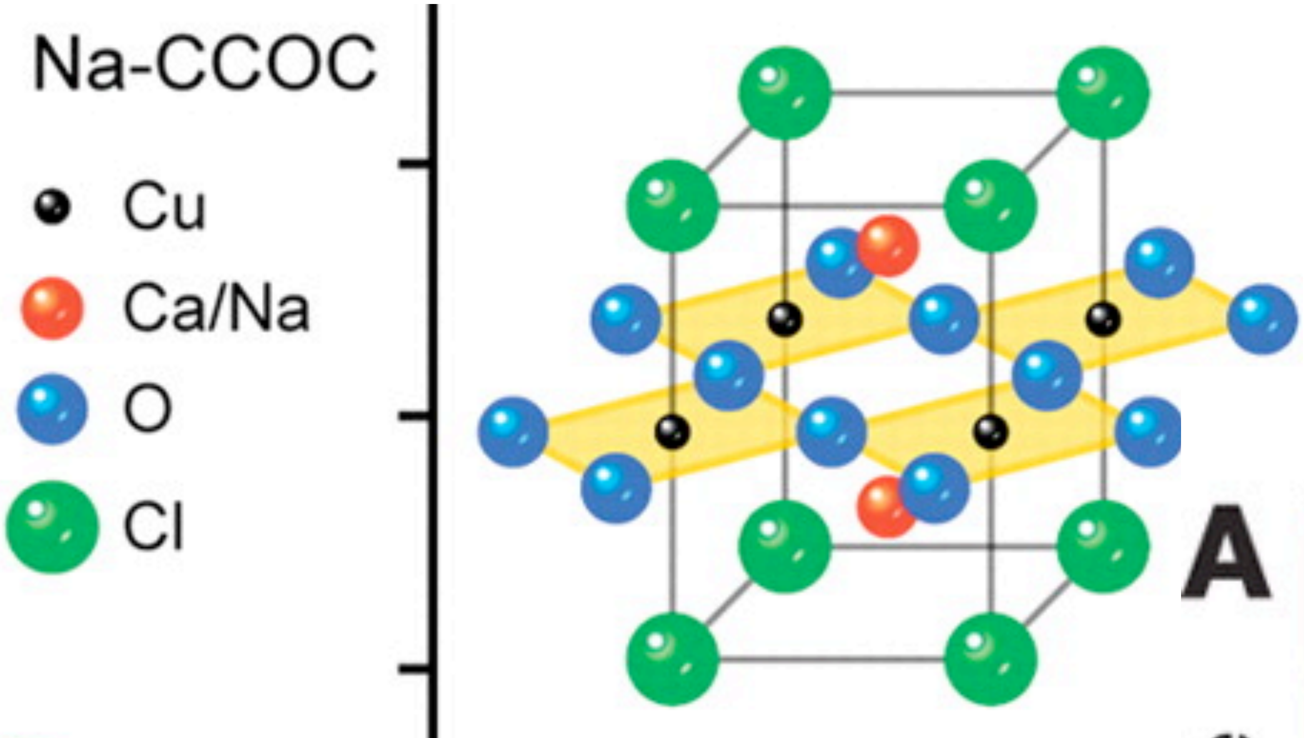
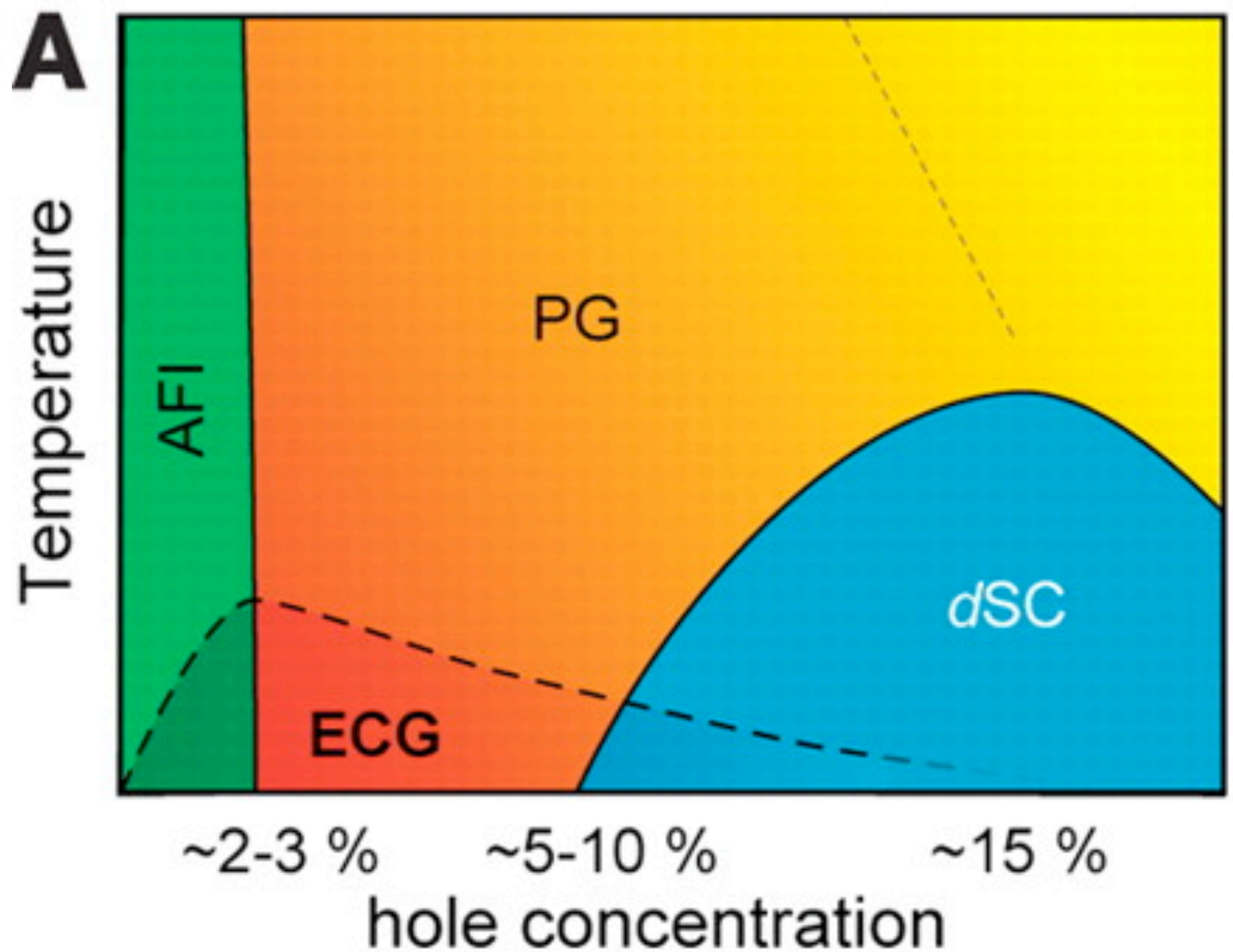
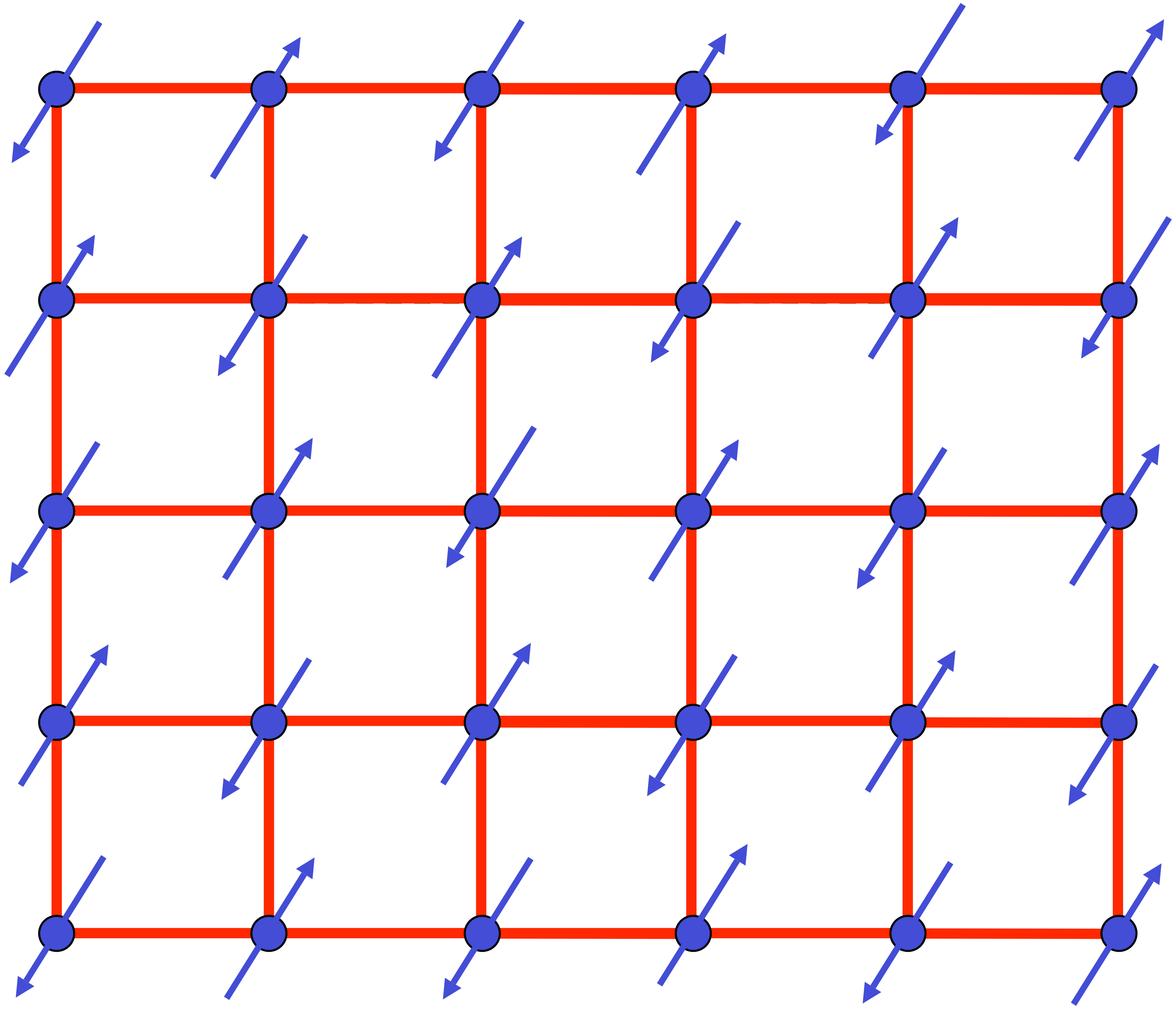
# The cuprate superconductors



# Antiferromagnetic (Neel) order in the insulator

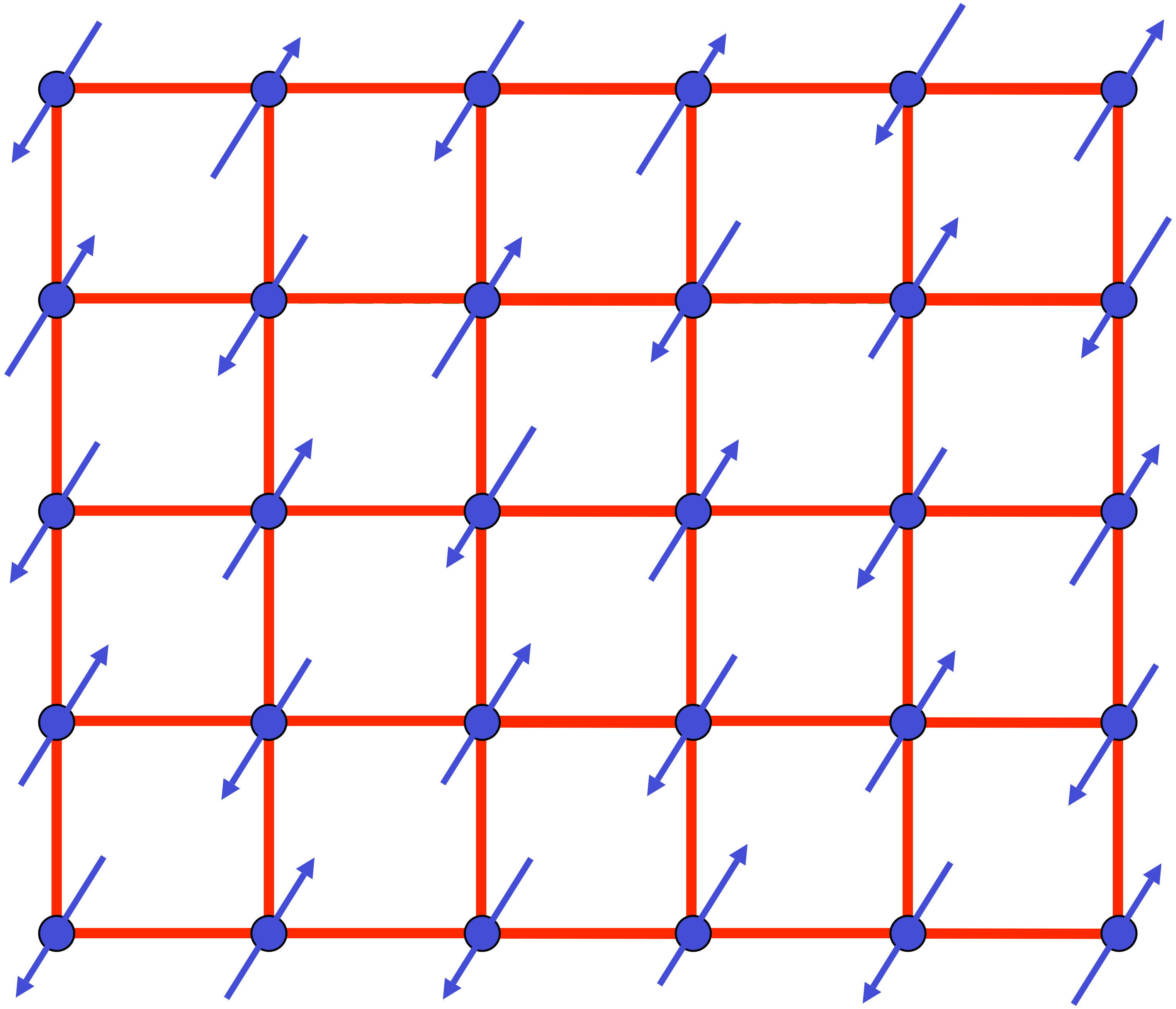


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$

No entanglement of spins

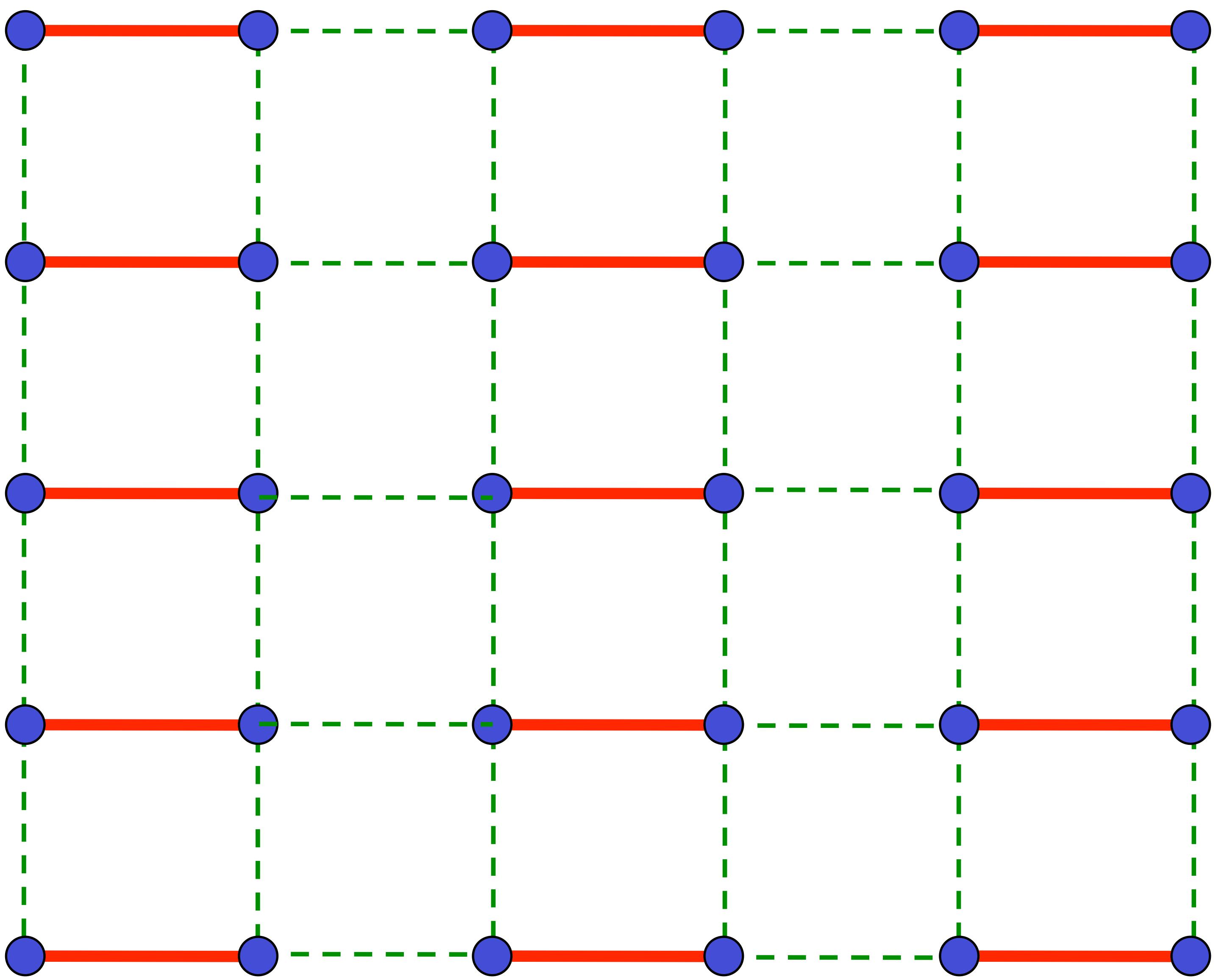
# Antiferromagnetic (Neel) order in the insulator



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$

Excitations: 2 spin waves (Goldstone modes)



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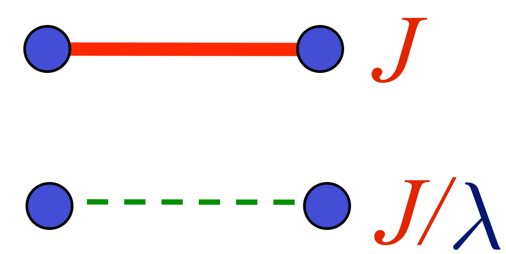
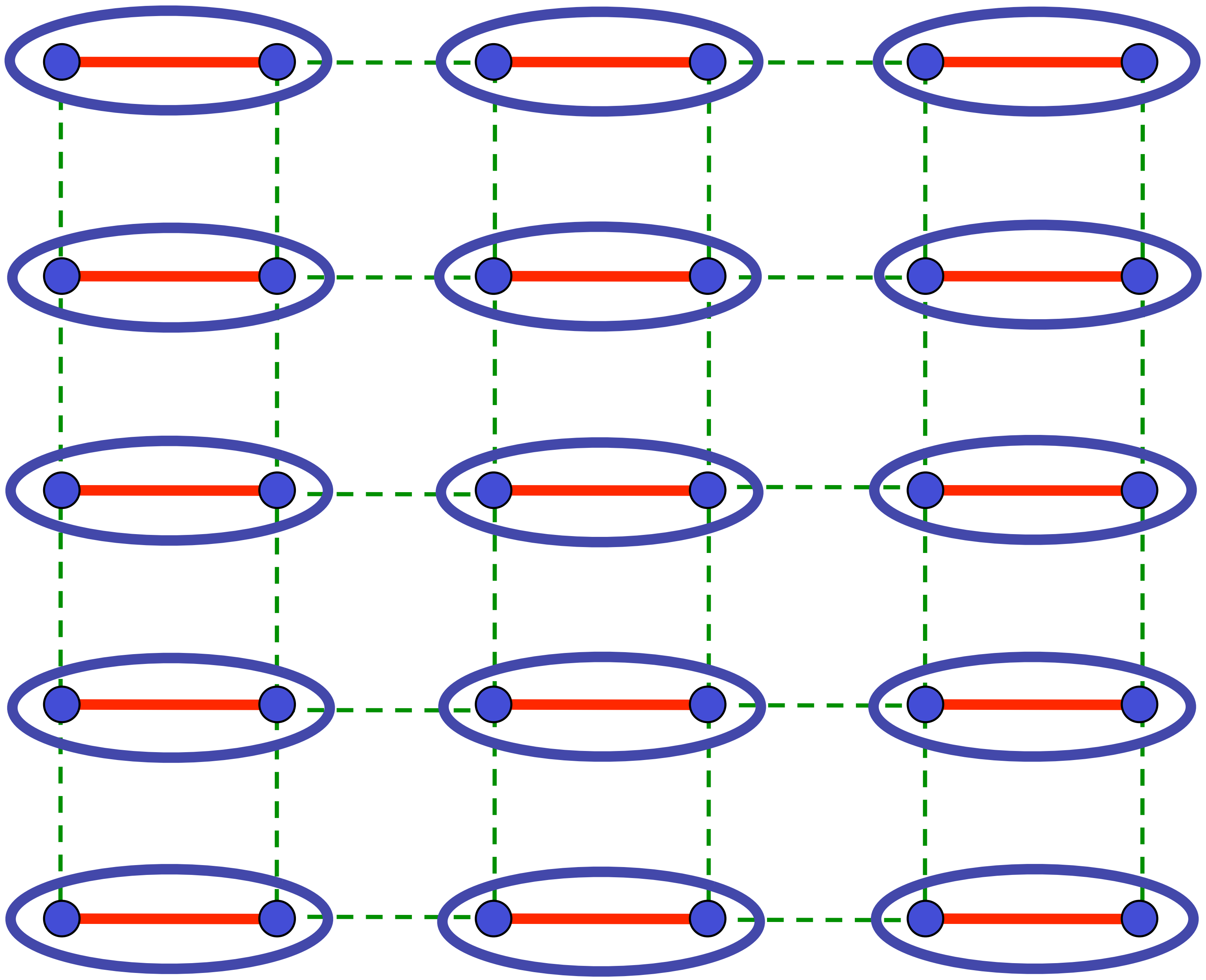



Weaken some bonds to induce spin entanglement in a new quantum phase

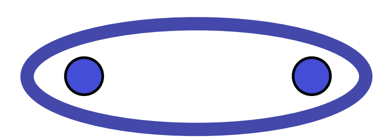


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$

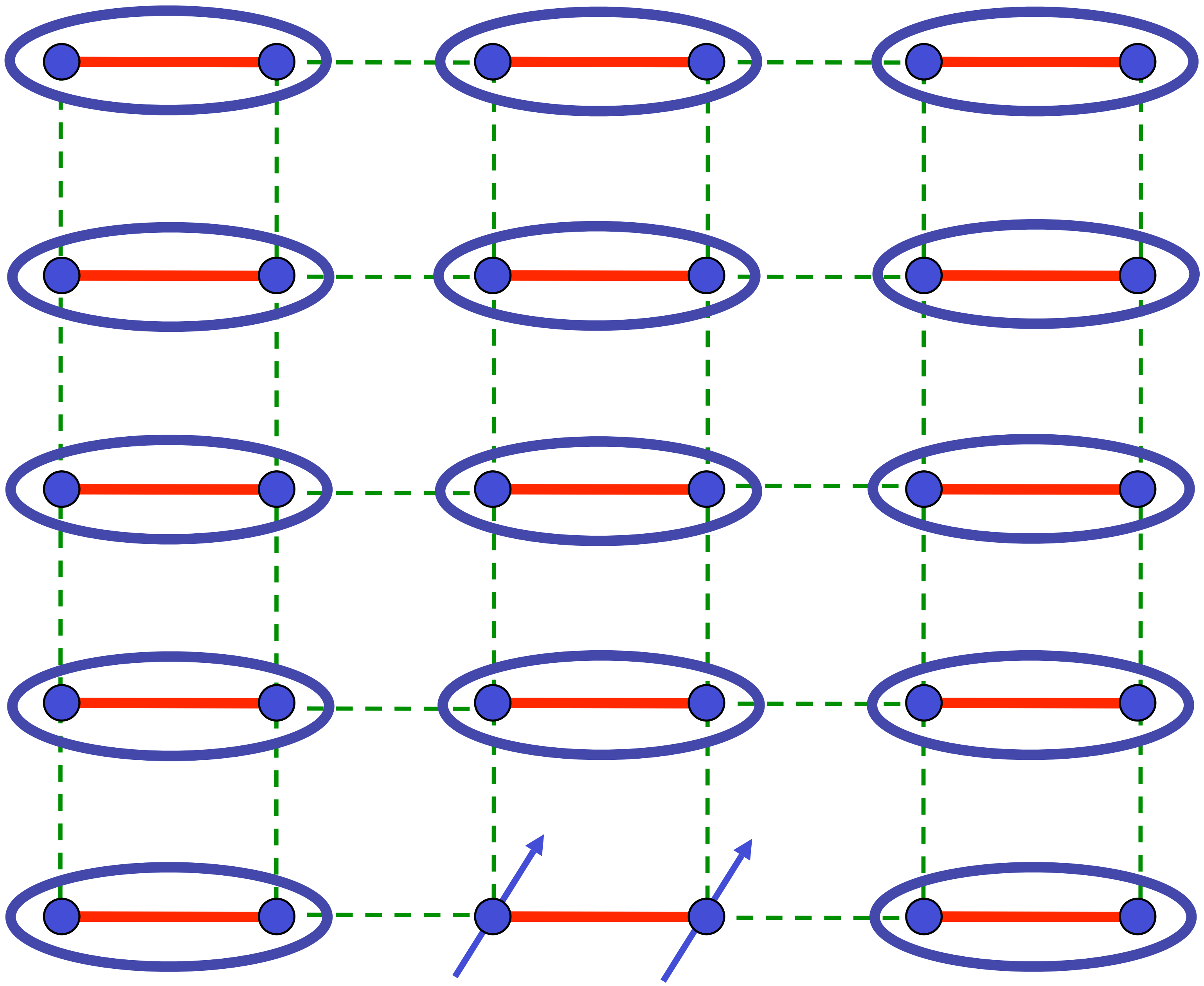
 $J$

 $J/\lambda$



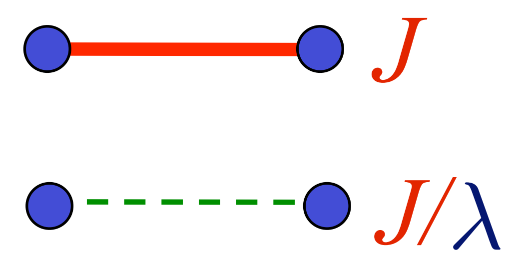
$$= \frac{1}{\sqrt{2}}\left( \left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

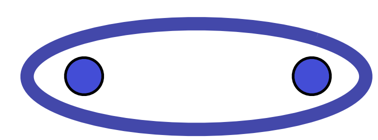
Ground state is a product of pairs of entangled spins.



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

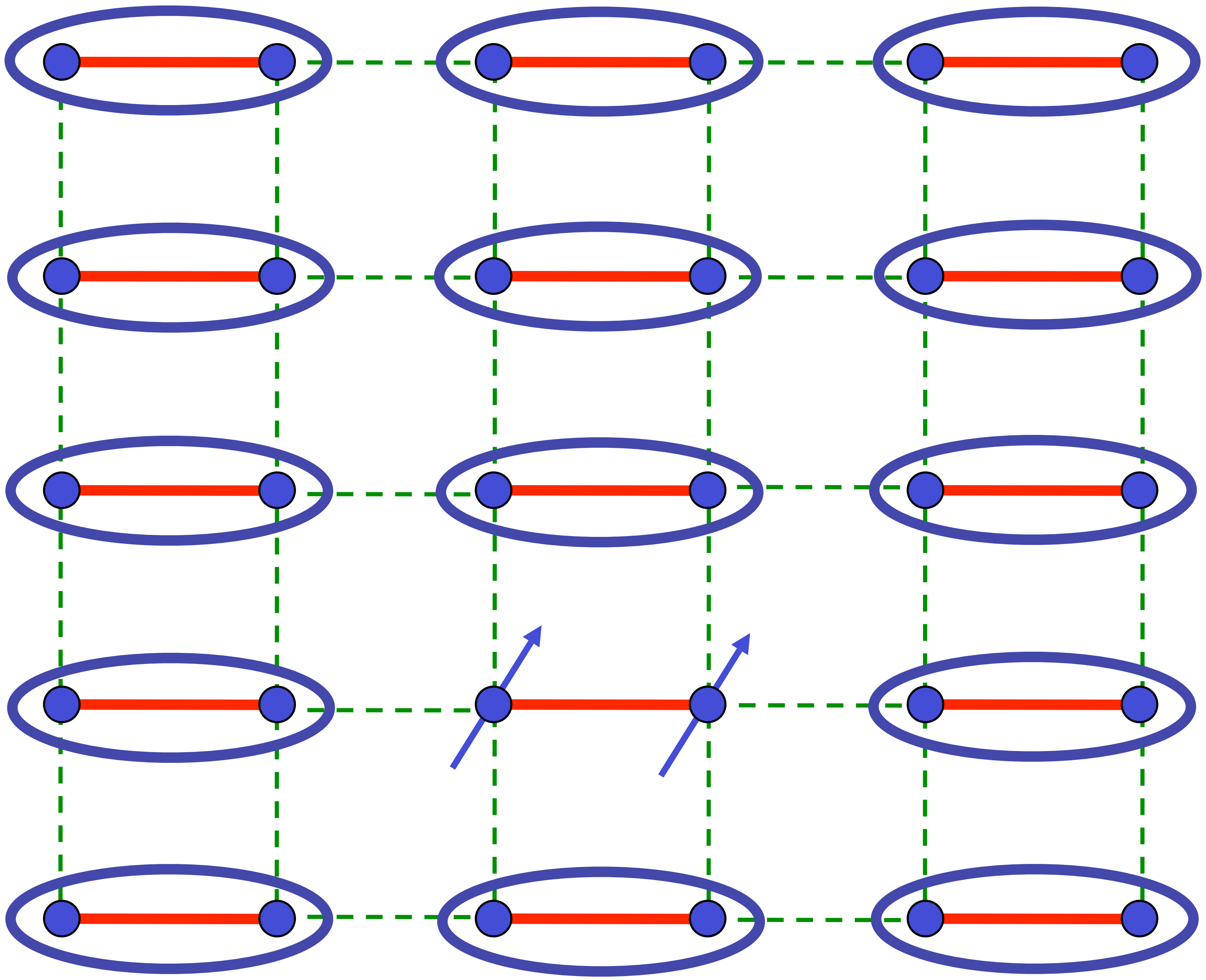
$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$



$$= \frac{1}{\sqrt{2}}\left( \left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

# Excitations: 3 *S=1* triplons



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

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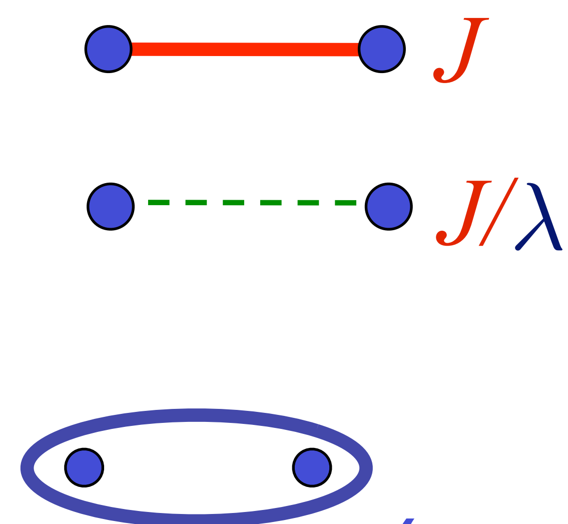
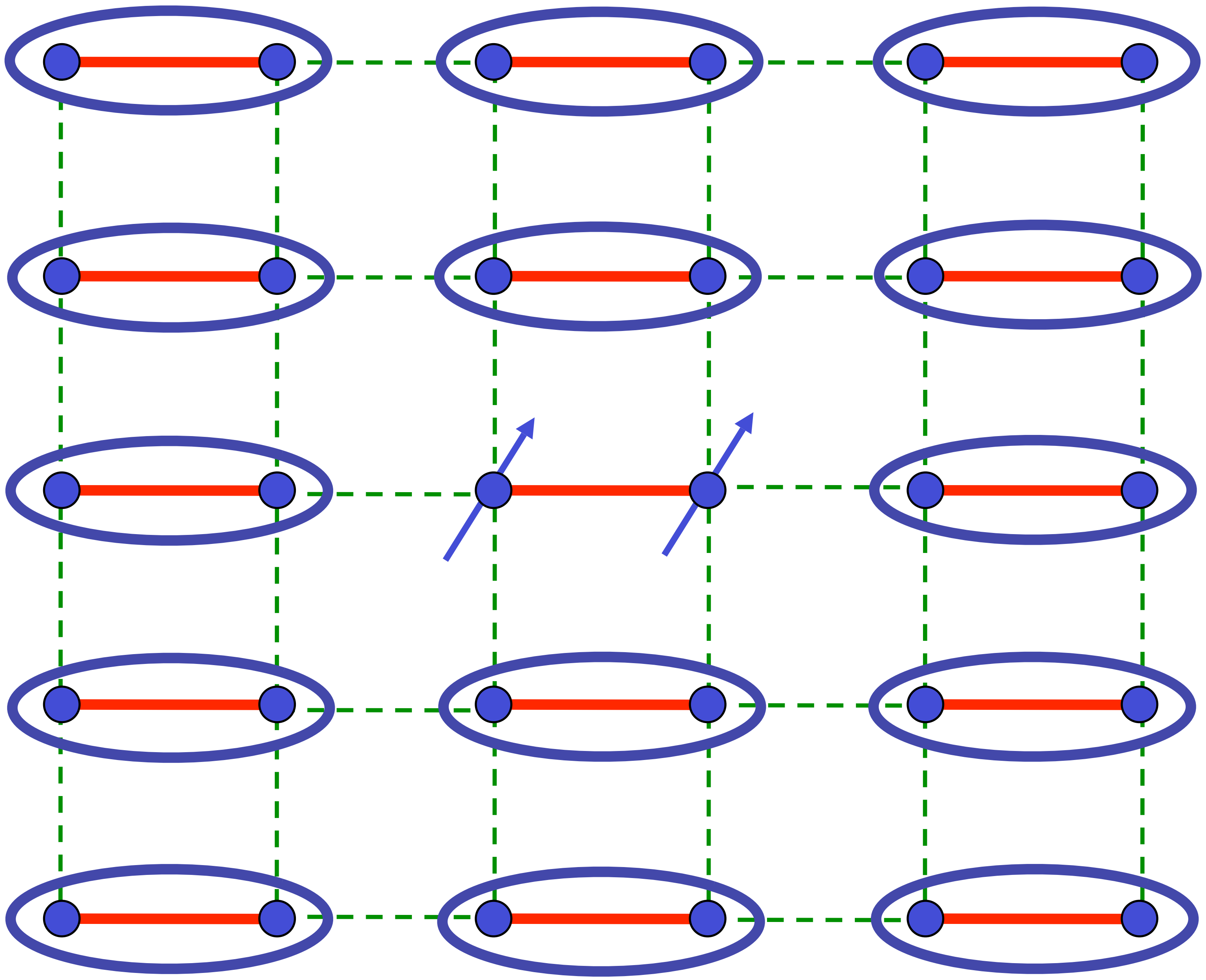



$$= \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

# Excitations: 3 *S=1* triplons



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

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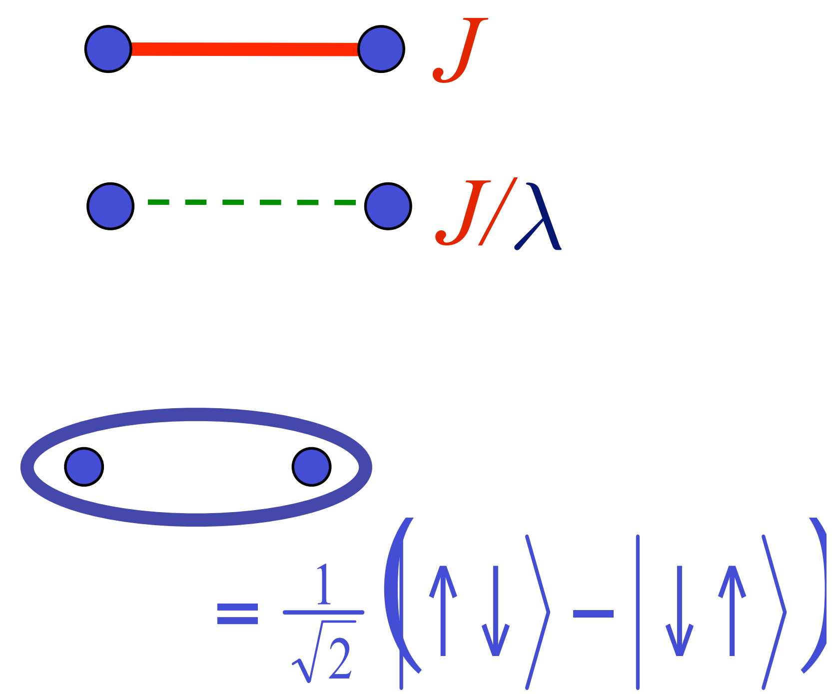
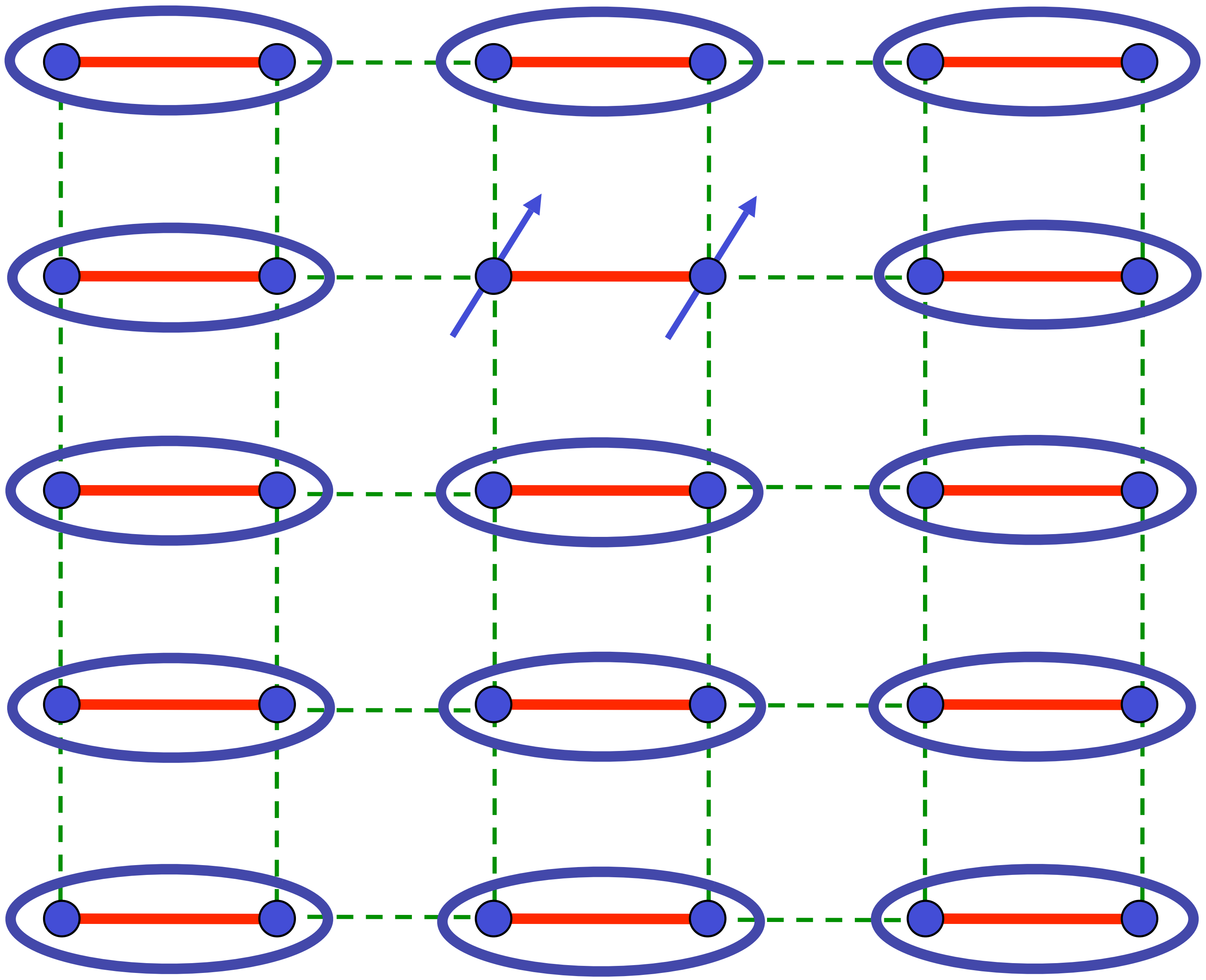



# Excitations: 3 $S=1$ triplons



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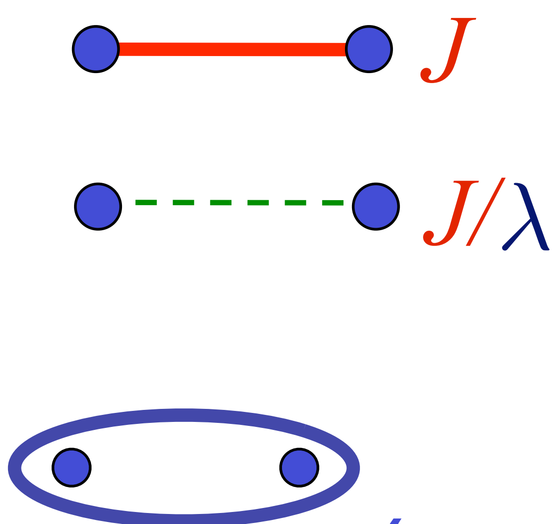
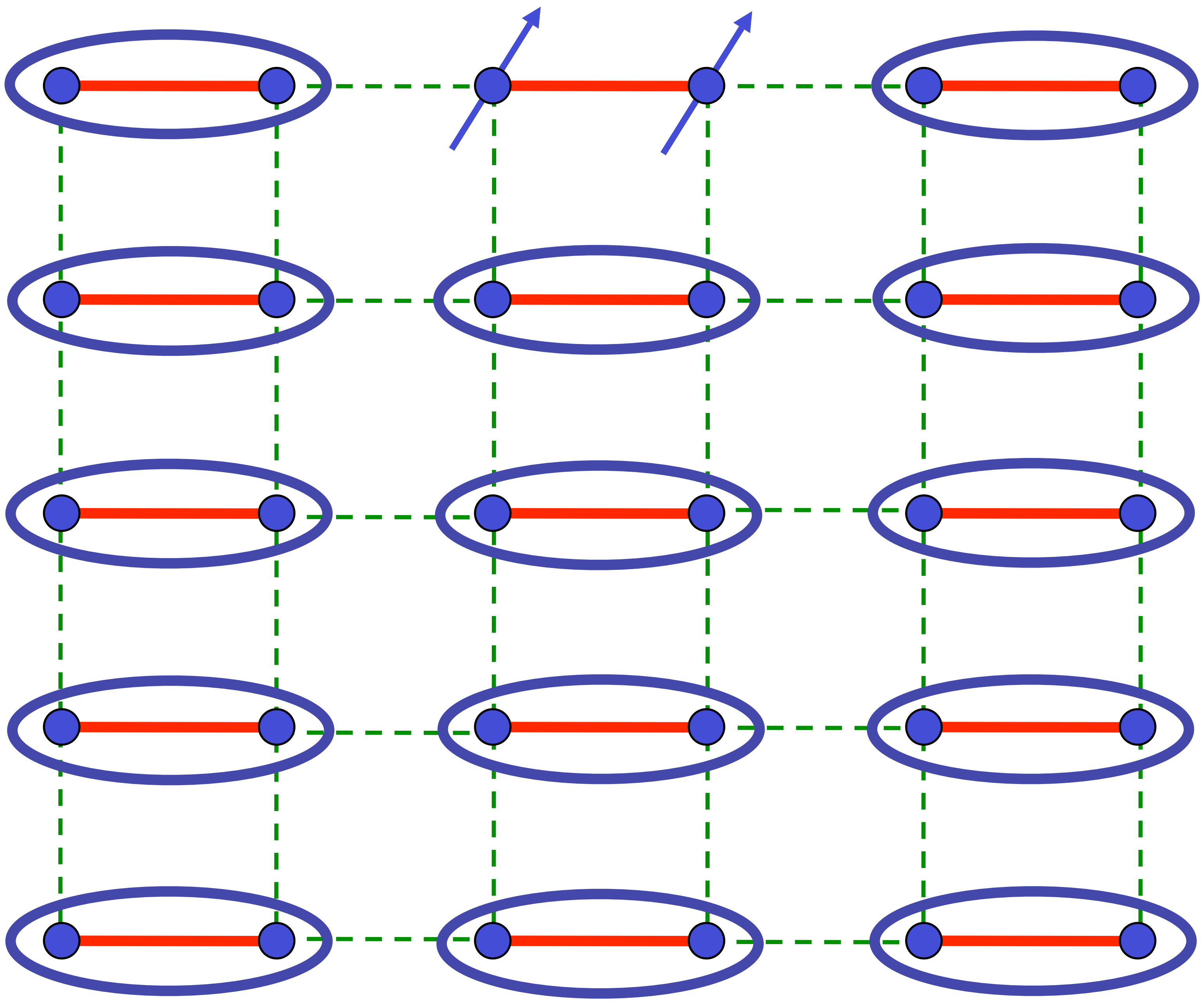
$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$



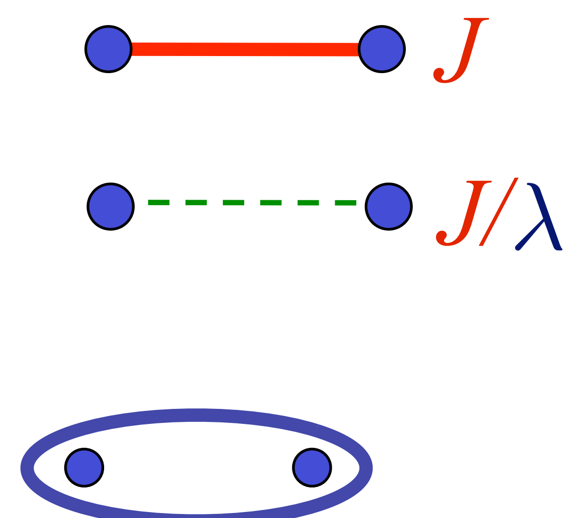

$$= \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

# Excitations: 3 *S=1* triplons



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

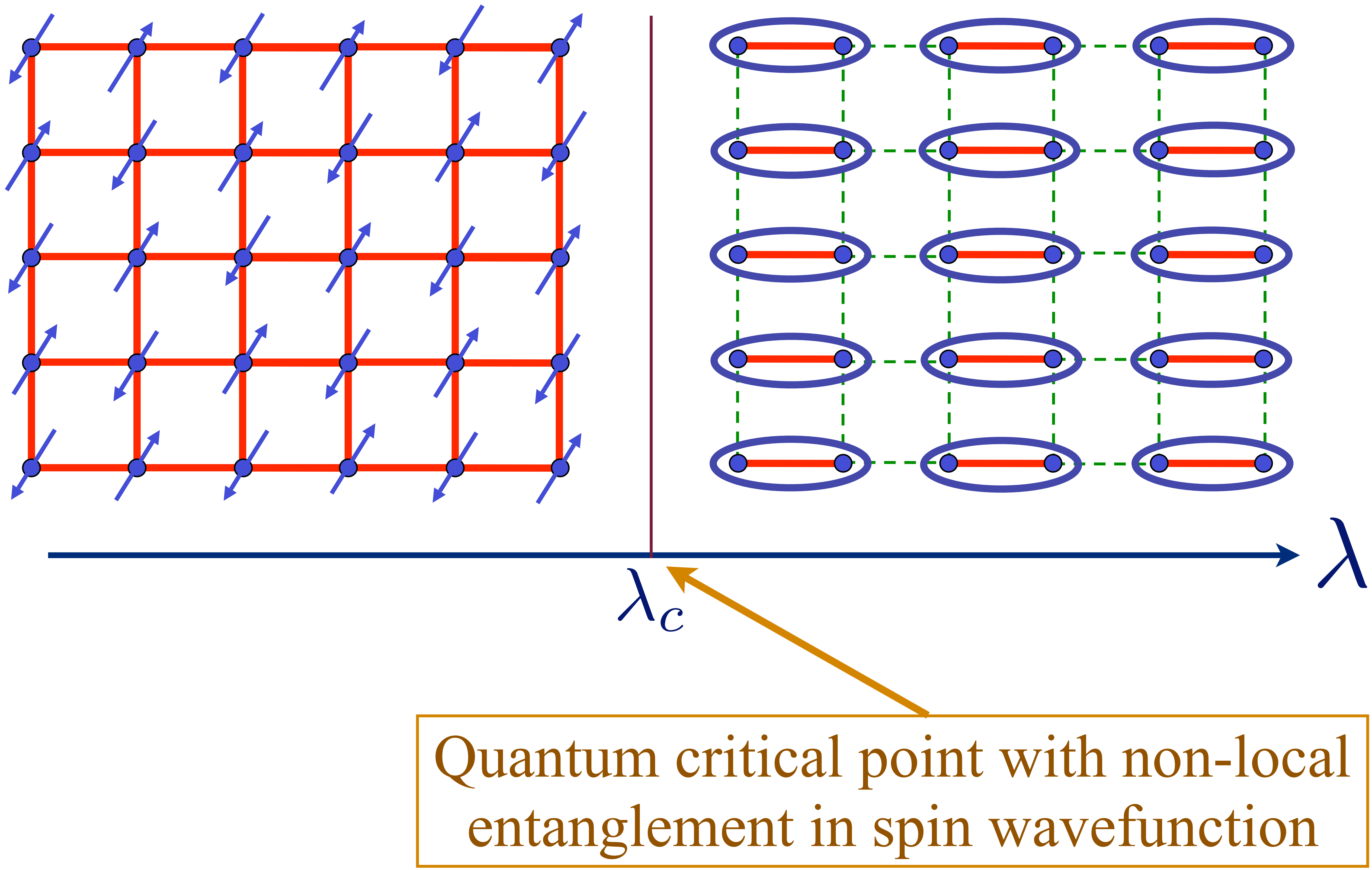
$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$




$$= \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

# Excitations: 3 $S=1$ triplons

# Phase diagram as a function of the ratio of exchange interactions, $\lambda$




M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

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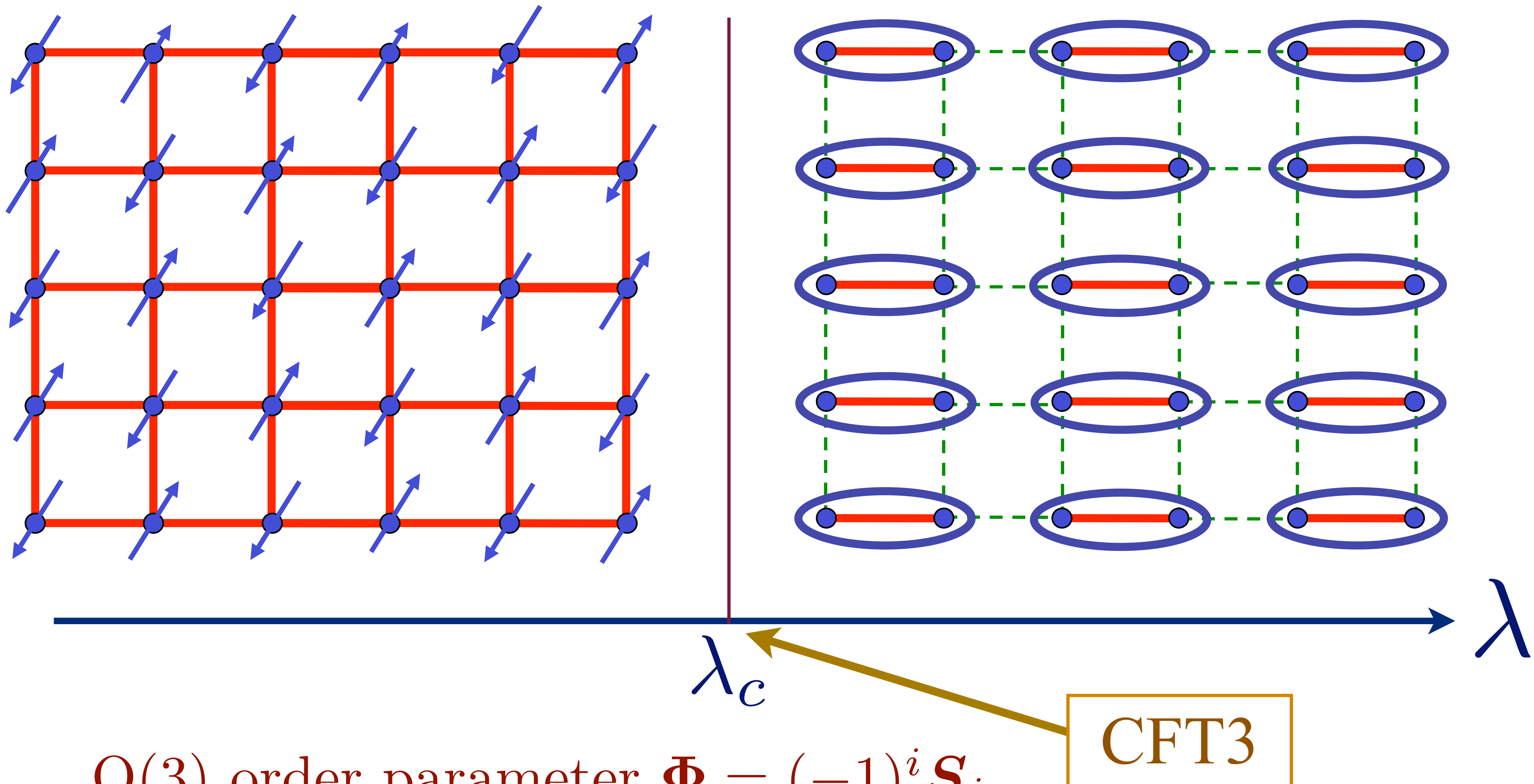
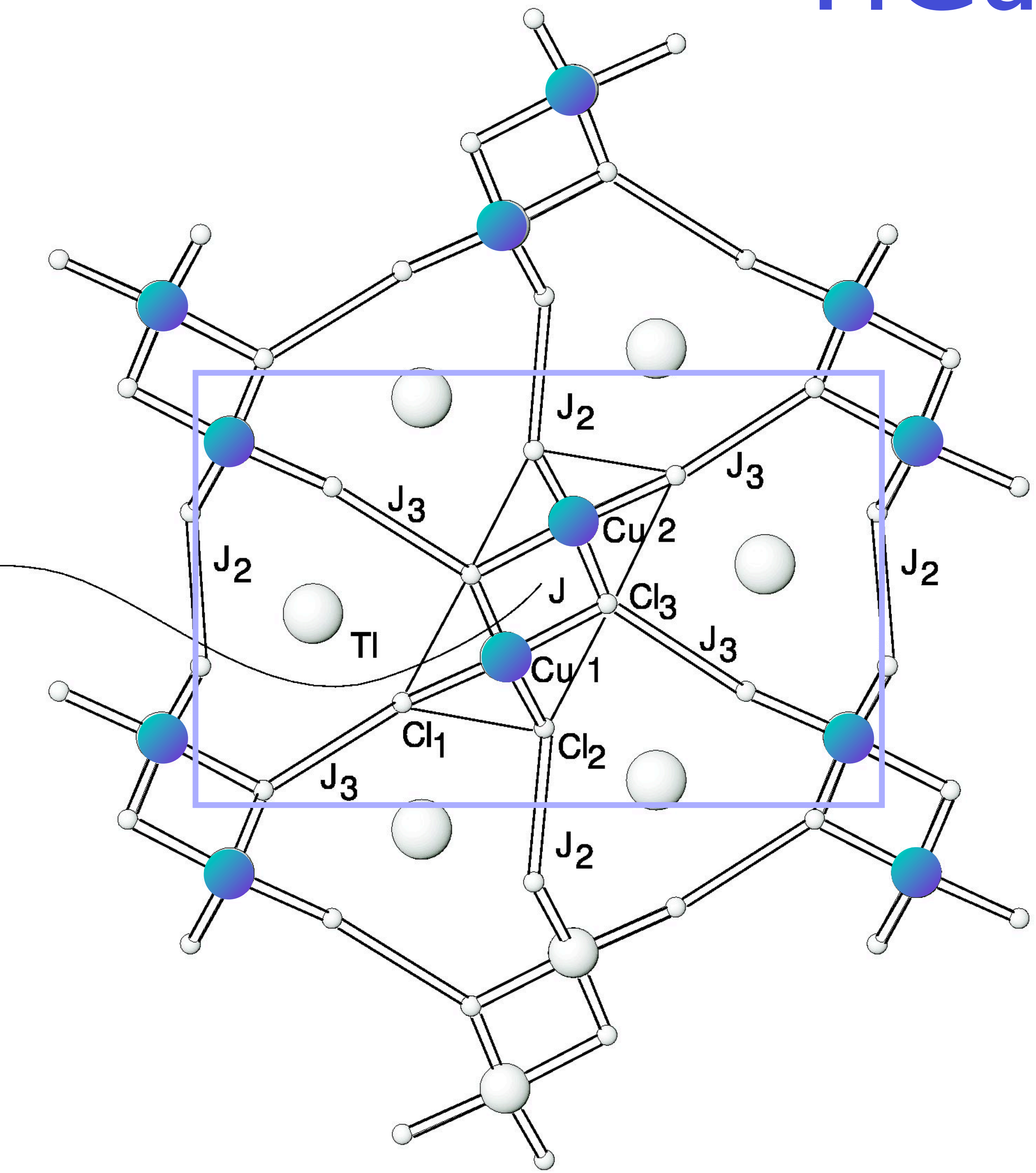


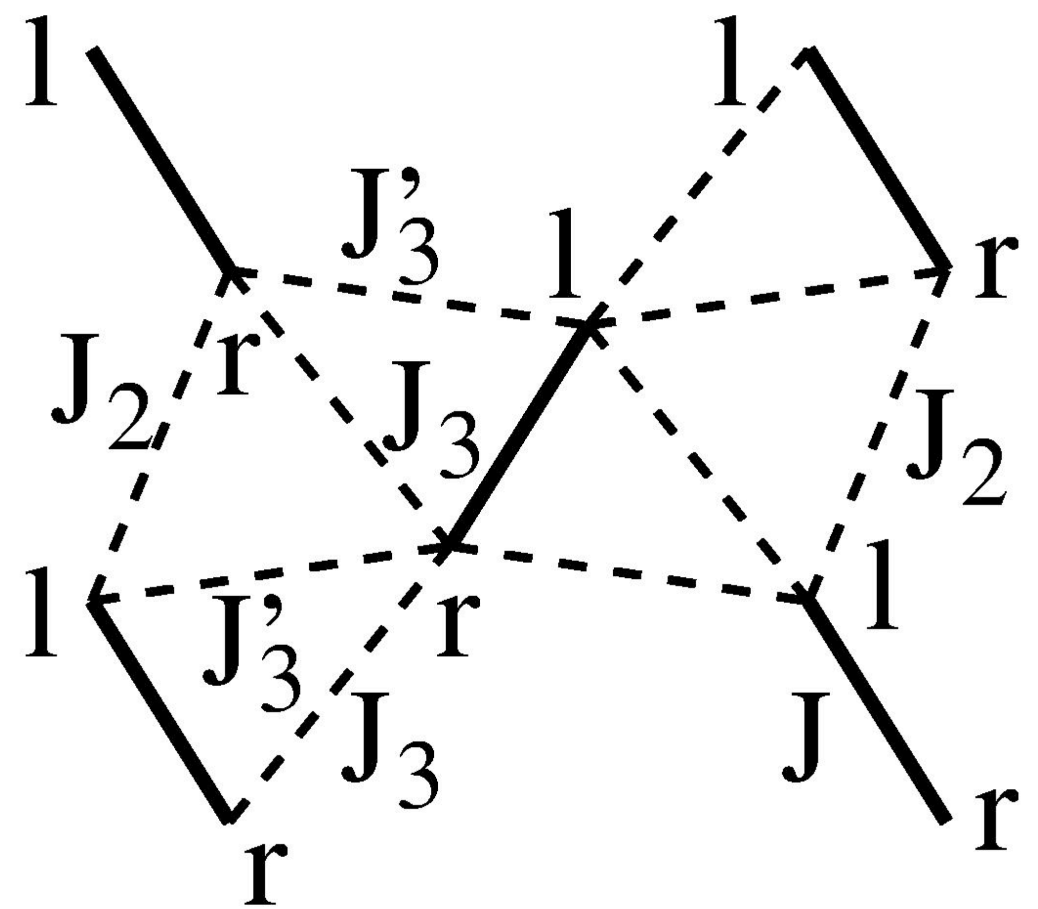

O(3) order parameter $\boldsymbol{\Phi} = (-1)^i \boldsymbol{S}_i$

$$\mathcal{S} = \int d^2 r d\tau \left[ (\partial_\tau \boldsymbol{\Phi})^2 + c^2 (\vec{\nabla} \boldsymbol{\Phi})^2 + s \boldsymbol{\Phi}^2 + u \left( \boldsymbol{\Phi}^2 \right)^2 \right]$$

# $TlCuCl_3$

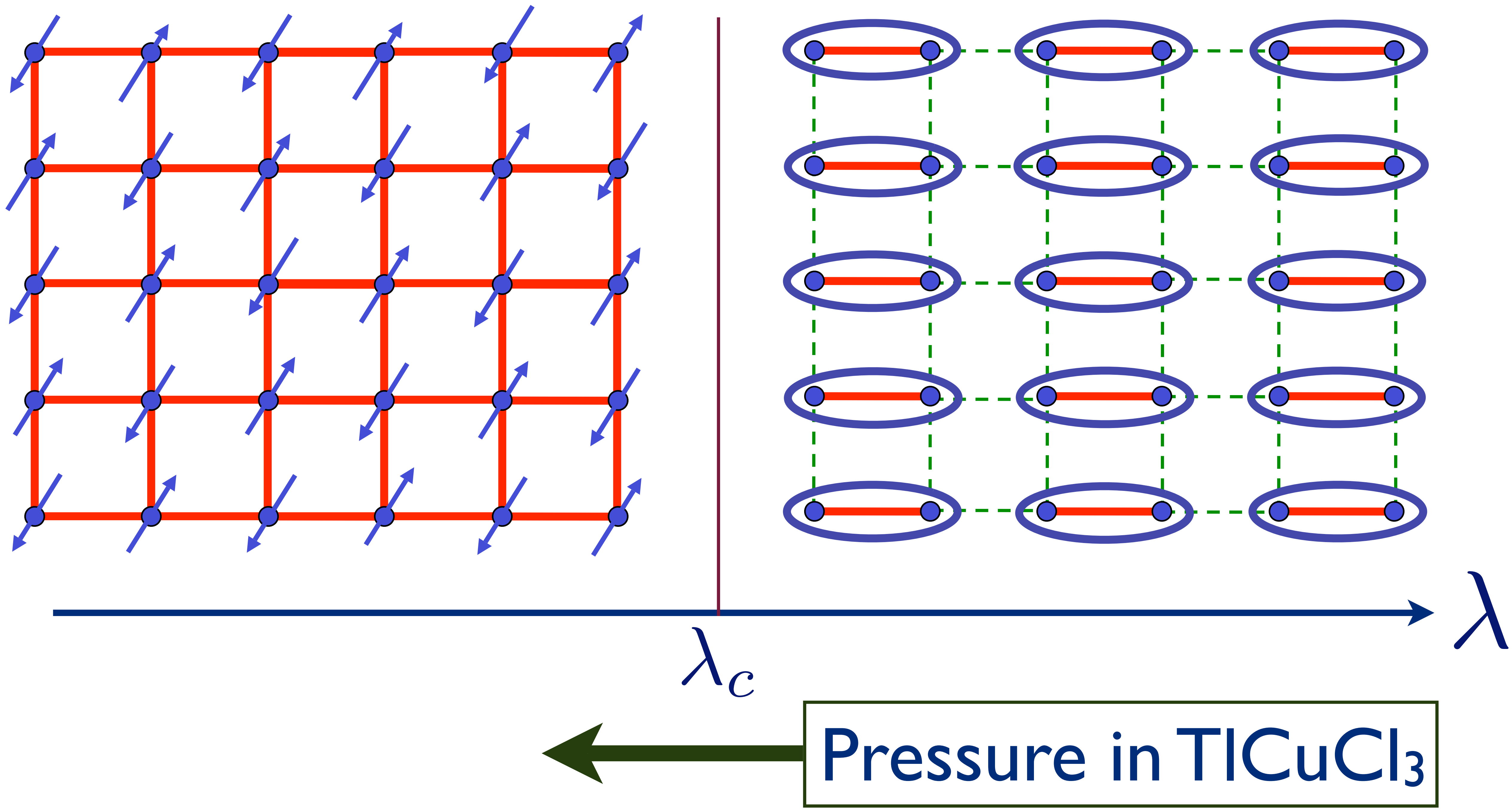


J2
J3
J3
Cu 2
J2
J2
J
Cl3
Tl
J3
Cu 1
Cl1
Cl2
J3
J2

l
l
J'3
l
r
r
J2
J3
J2
r
l
l
J'3
r
J3
J
r
r

# Phase diagram as a function of the ratio of exchange interactions, $\lambda$



# $TlCuCl_3$ at ambient pressure

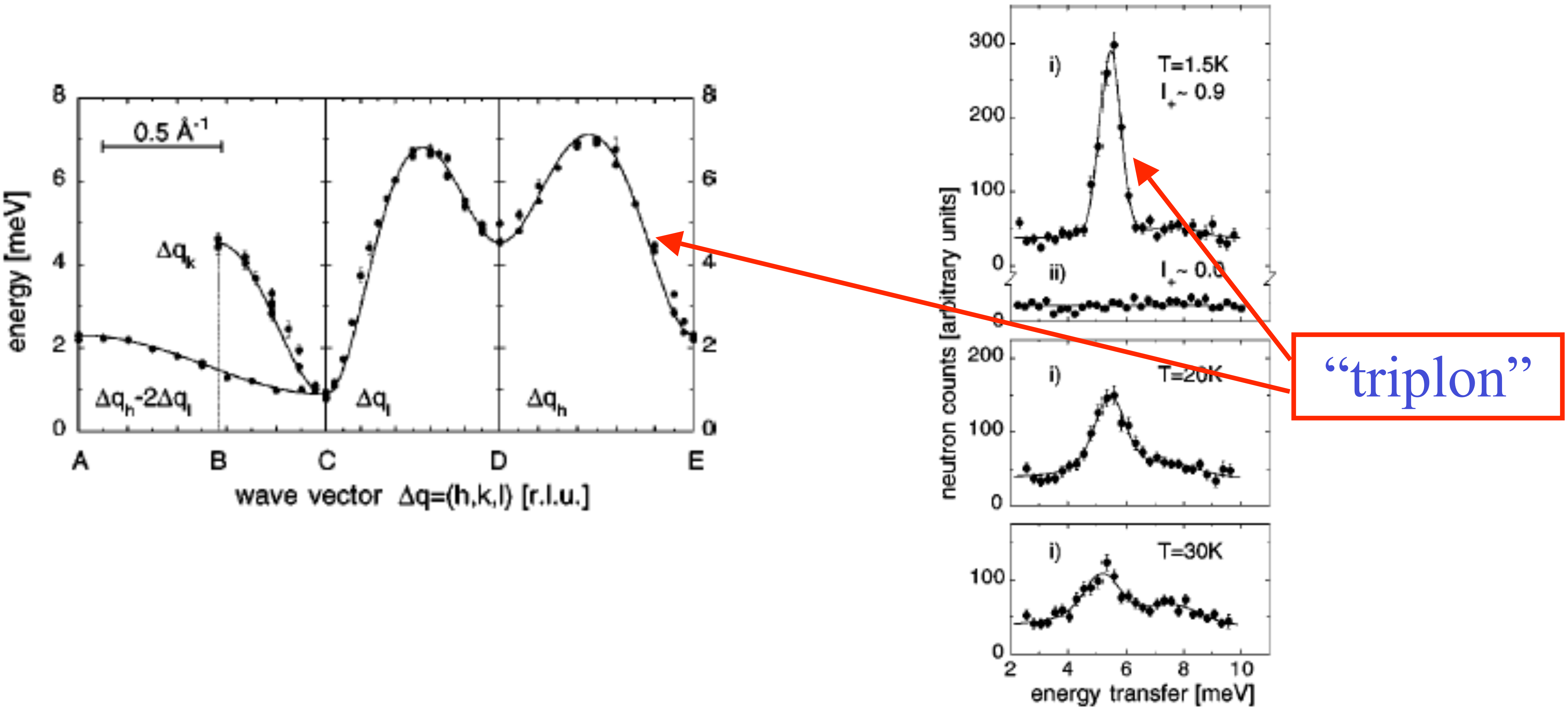
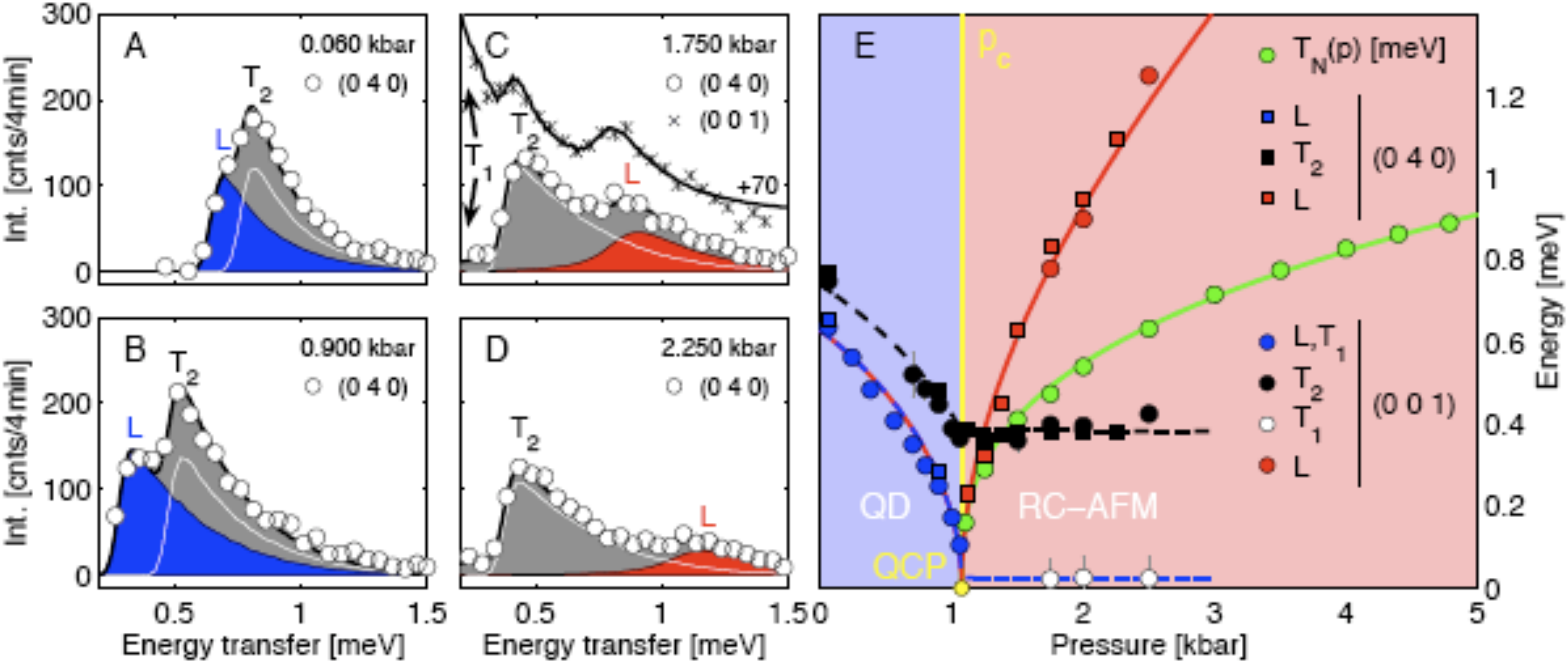



FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

# $TlCuCl_3$ with varying pressure




Observation of 3 → 2 low energy modes, emergence of new longitudinal mode in Néel phase, and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

# S=1/2 insulator on the square lattice

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left( \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \right) \left( \mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right)$$

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007).
R.G. Melko and R.K. Kaul, *Phys. Rev. Lett.* **100**, 017203 (2008).

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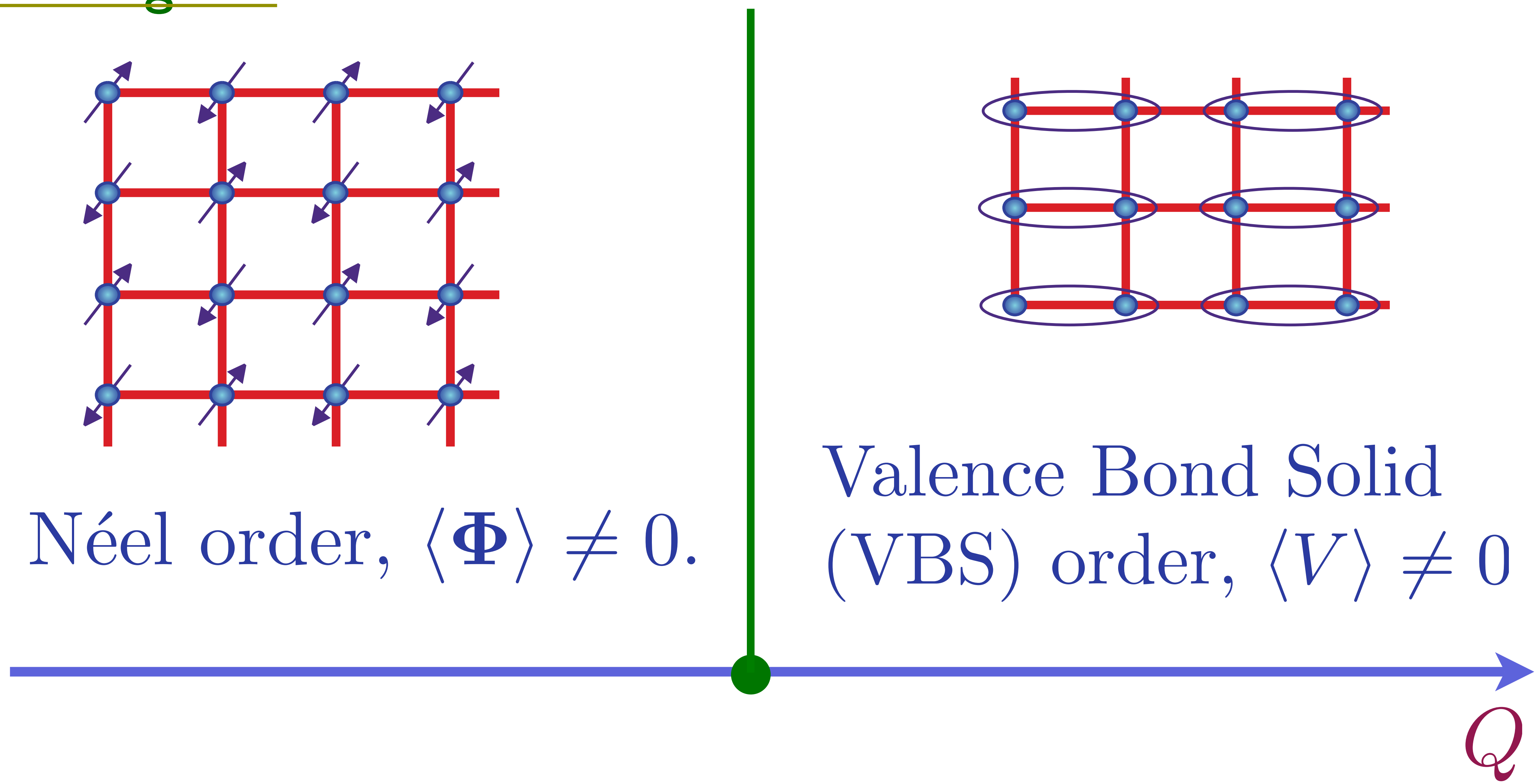
## Phase diagram





A.W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007).
R.G. Melko and R.K. Kaul, *Phys. Rev. Lett.* **100**, 017203 (2008).
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

# Theory for loss of Neel order

Decompose $\mathbf{\Phi}$ in terms of a complex scalar (a spinon) $z_\alpha$, $\alpha = \uparrow, \downarrow$:

$$\mathbf{\Phi} = z_\alpha^\dagger \vec{\sigma}_{\alpha\beta} z_\beta$$

where $\vec{\sigma}$ are Pauli matrices. Theory must be invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\theta} z_\alpha$$

Low energy spinon theory for loss of Néel order is the $\mathrm{CP}^1$ model

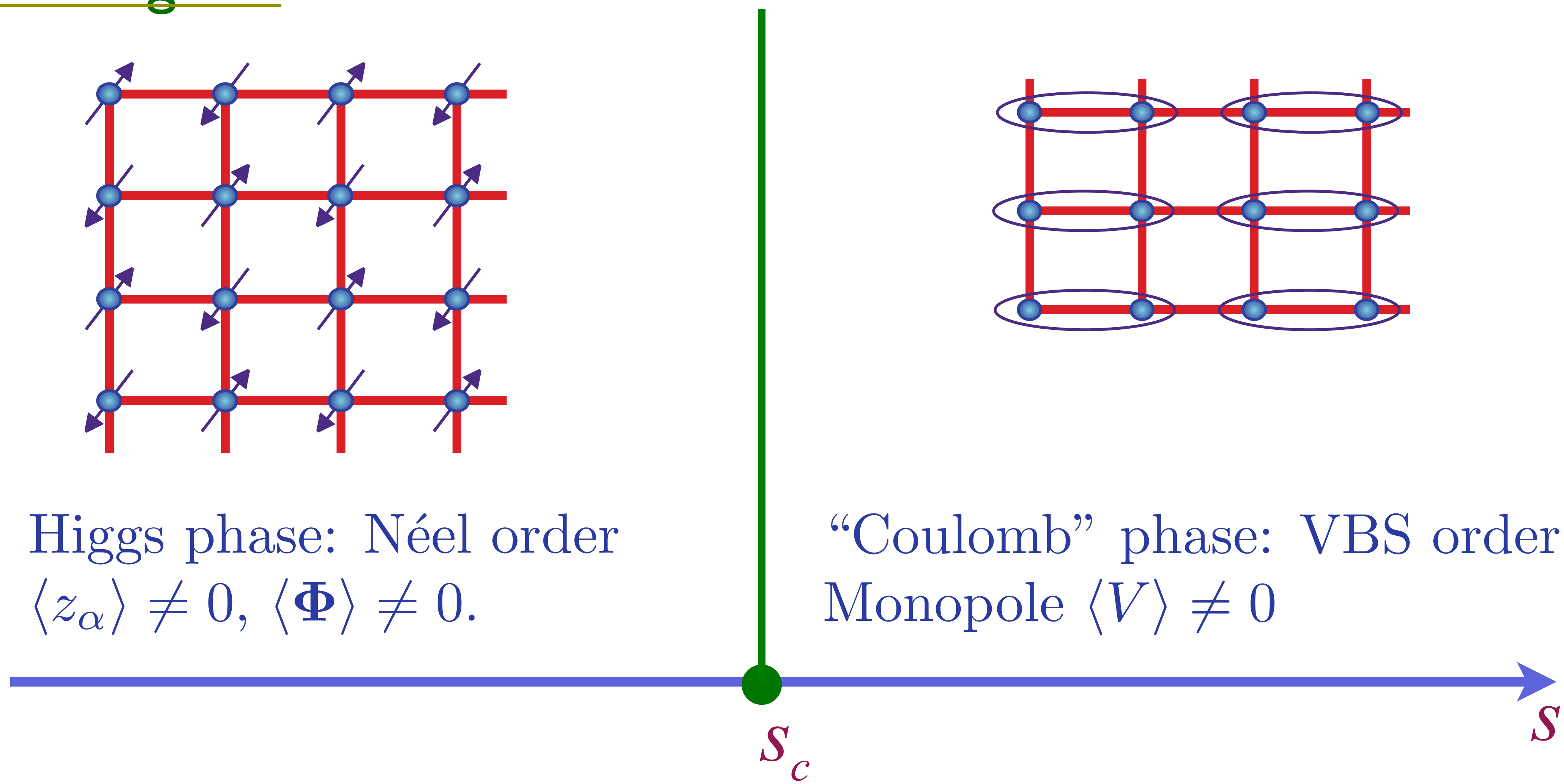
$$\mathcal{S}_z = \int d^2x d\tau \Big[ c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s\left|z_\alpha\right|^2 + u\left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$

where $A_\mu$ is an emergent U(1) gauge field.
**The monopole creation operator $V$ is identical to the VBS order parameter (the global "shift" symmetry of the dual photon is an enlargement of the lattice rotation symmetry).**

# S=1/2 insulator on the square lattice

$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right)\left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$

## Phase diagram

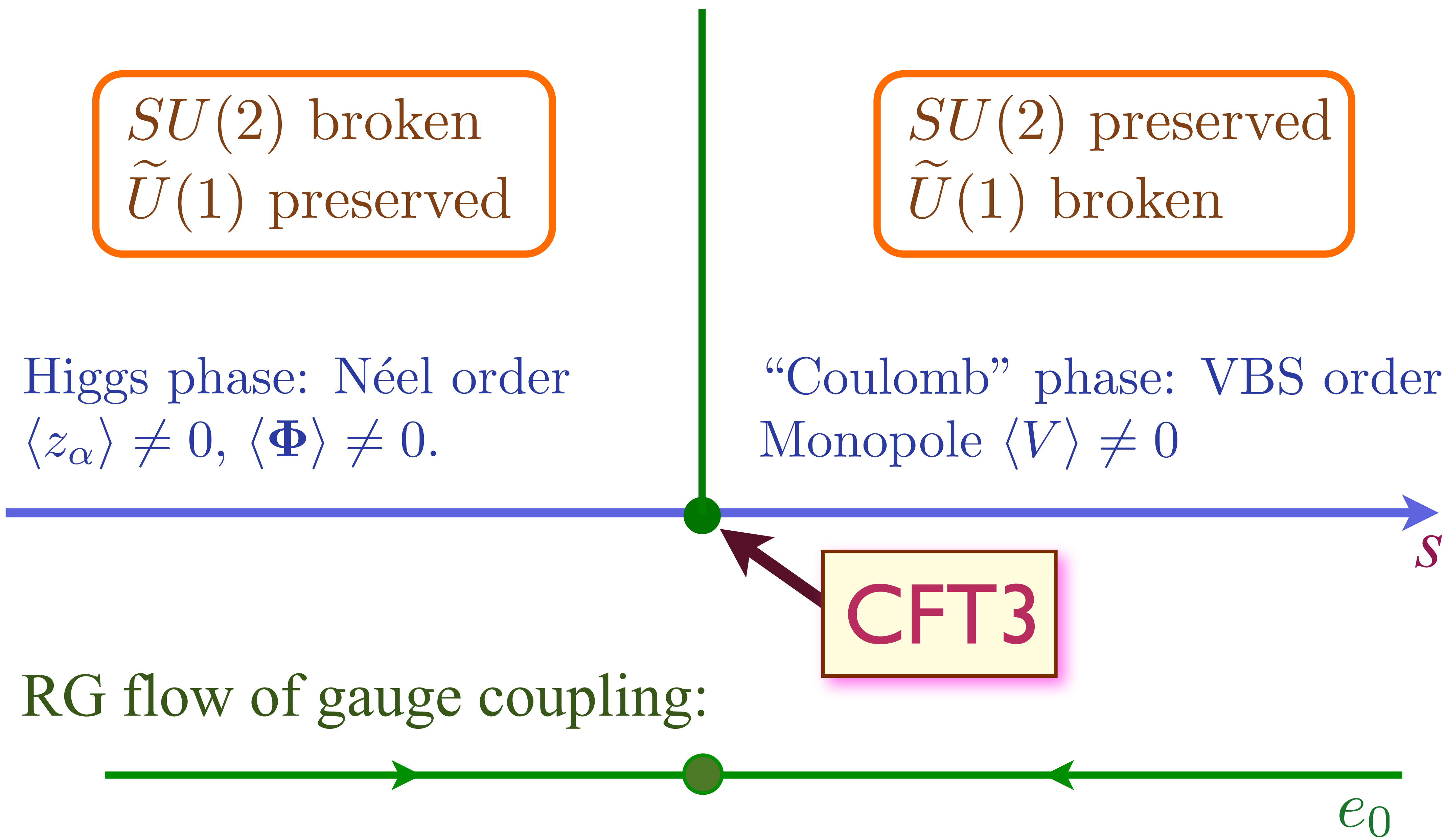



$$\mathcal{S}_z = \int d^2 r d\tau \left[ |(\partial_\mu - i A_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

# S=1/2 insulator on the square lattice

$$\mathcal{S}_z = \int d^2 r d\tau \left[ |(\partial_\mu - iA_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

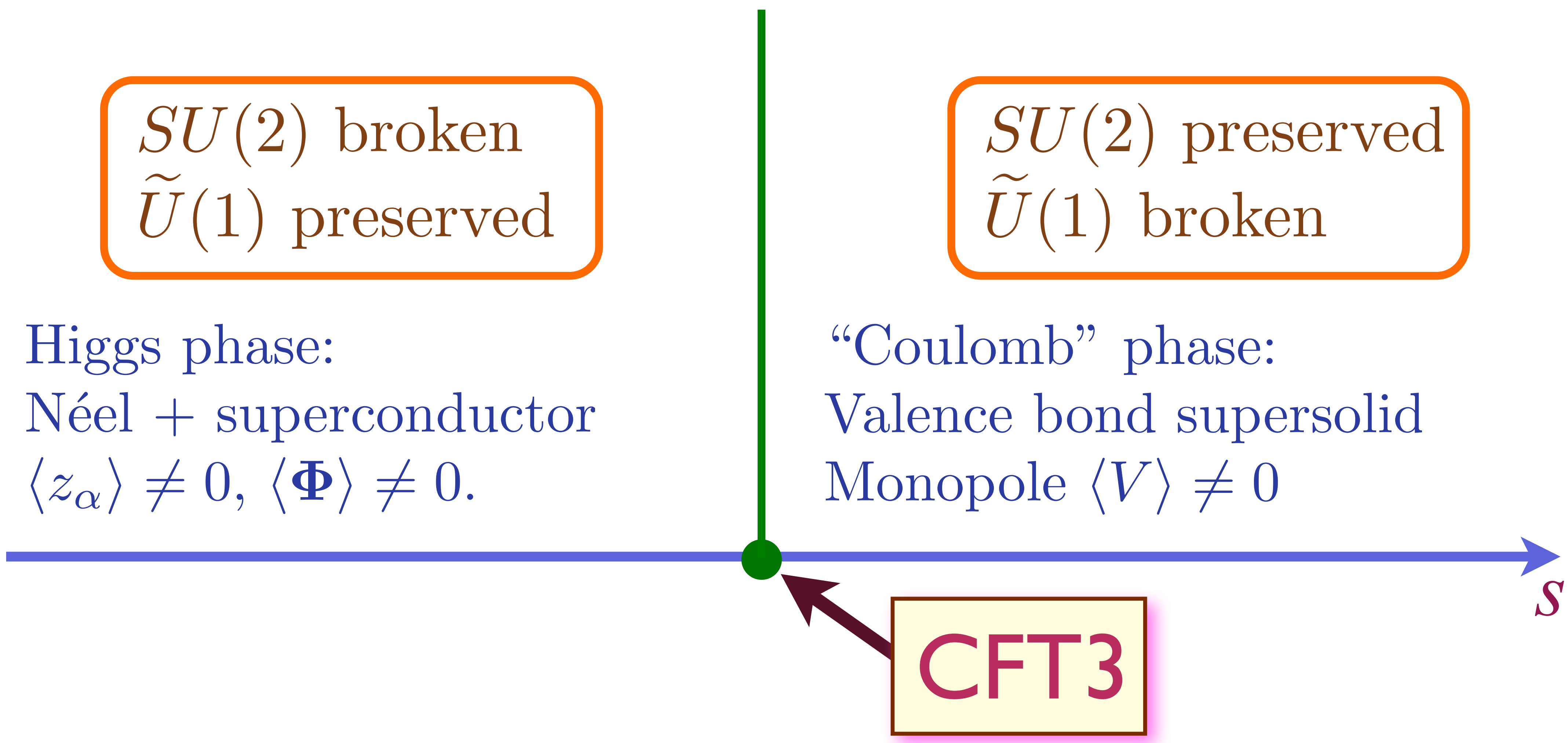
Global symmetry: $SU(2) \times \widetilde{U}(1)$ (the $\widetilde{U}(1)$ is the dual photon shift)



# *d*-wave superconductor on the square lattice

$$\mathcal{S}_z = \int d^2 r d\tau \Big[ |(\partial_\mu - i A_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 + \overline{\psi}_a \gamma^\mu \partial_\mu \psi_a \Big]$$

Global symmetry: $SU(2) \times \widetilde{U}(1)$ (the $\widetilde{U}(1)$ is the dual photon shift)



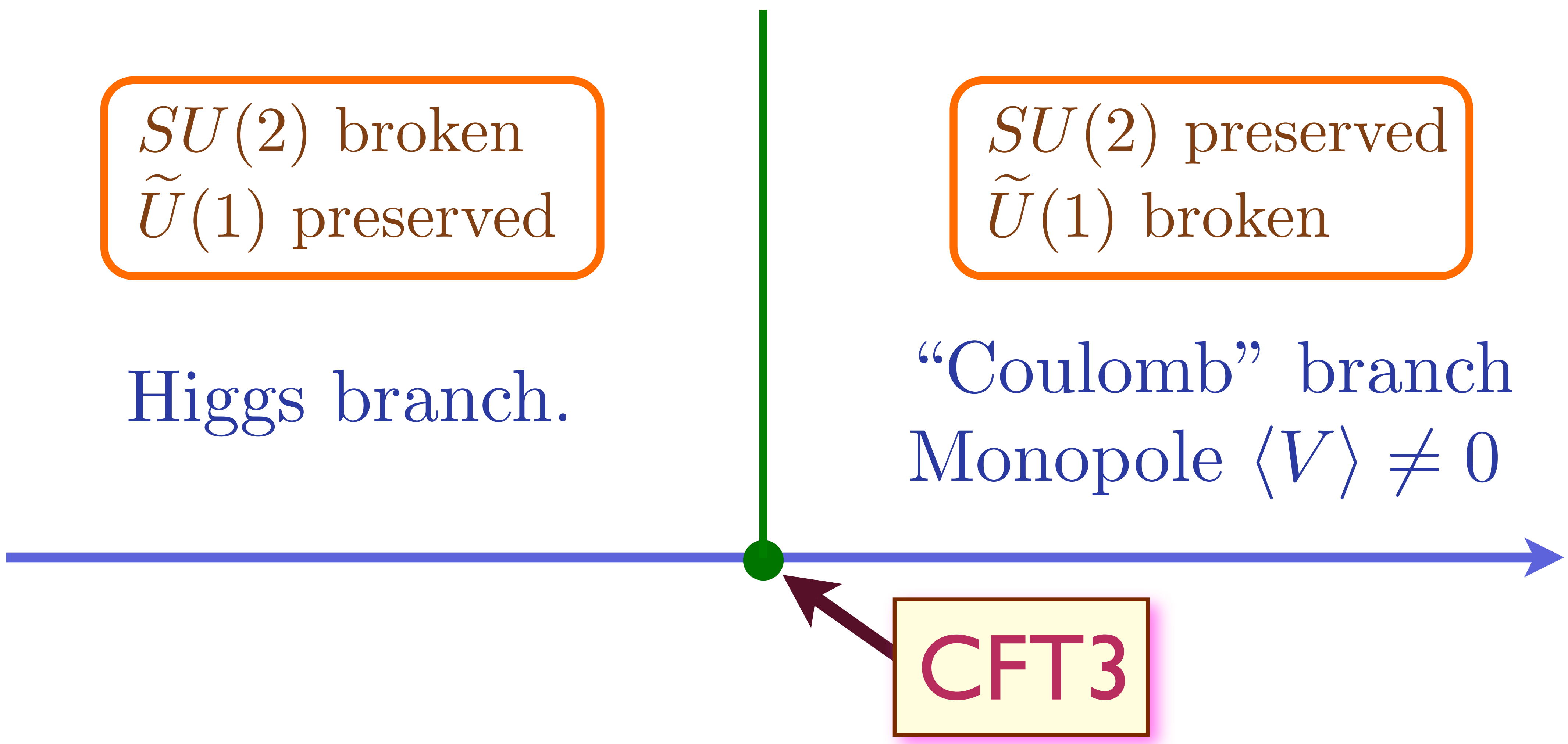


R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, *Nature Physics* **4**, 28 (2008)
R. K. Kaul, M.A. Metlitski, S. Sachdev, and C. Xu, arXiv:0804.1794

# *U(1) gauge theory with $\mathcal{N}$=4 supersymmetry*

Theory with a U(1) vector multiplet $\mathcal{V}$ and 2 hypermultiplets $\mathcal{Q}_i$.

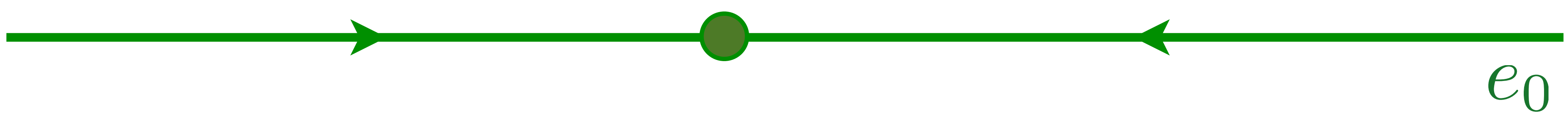
Global symmetry: $SU(2) \times \widetilde{U}(1) \times SO(4)_R$




N. Seiberg and E. Witten, hep-th/9607163
K.A. Intriligator and N. Seiberg, hep-th/9607207
A. Kapustin and M. J. Strassler, hep-th/9902033

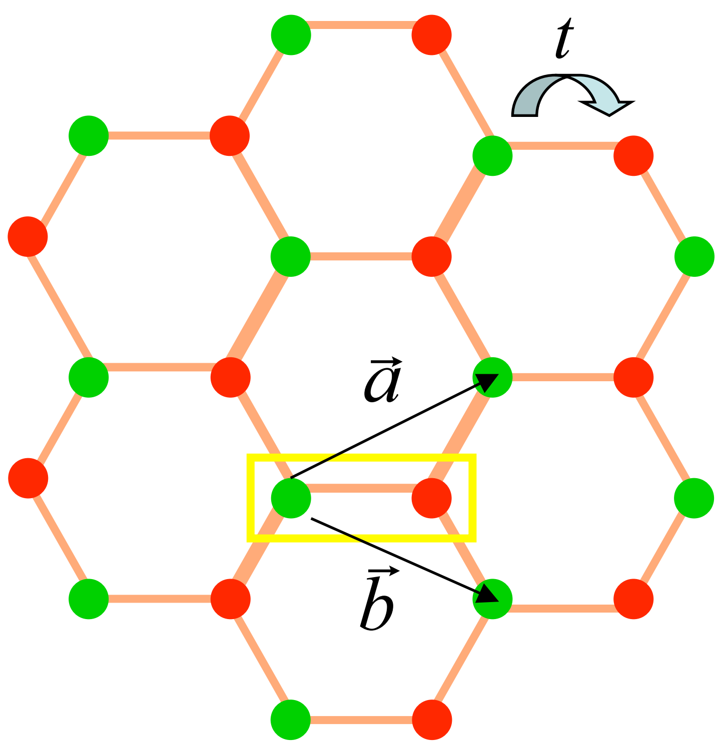
## SU(N) gauge theory with $\mathcal{N}$=8 supersymmetry (SYM3)

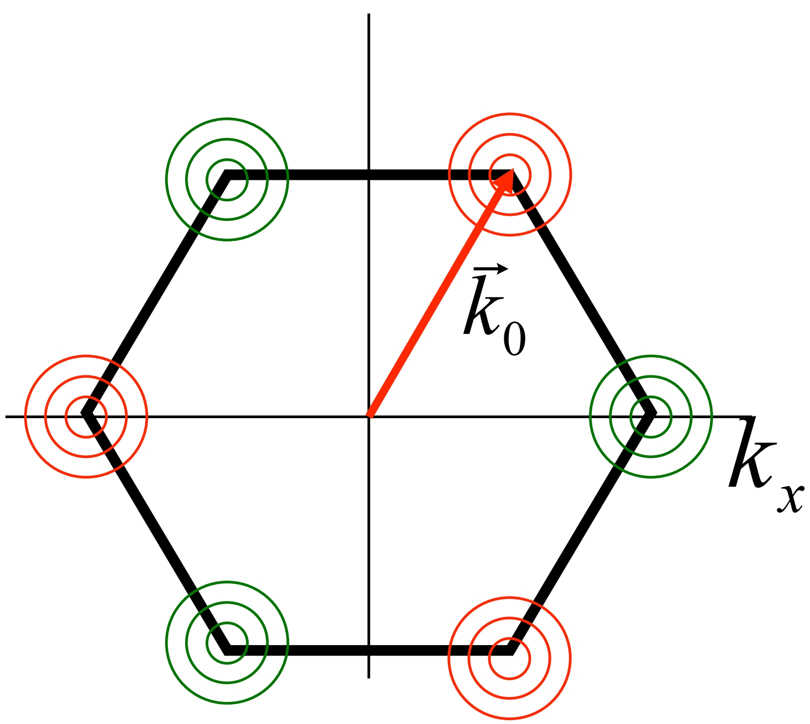
Unique theory with a single gauge coupling constant $e_0$.

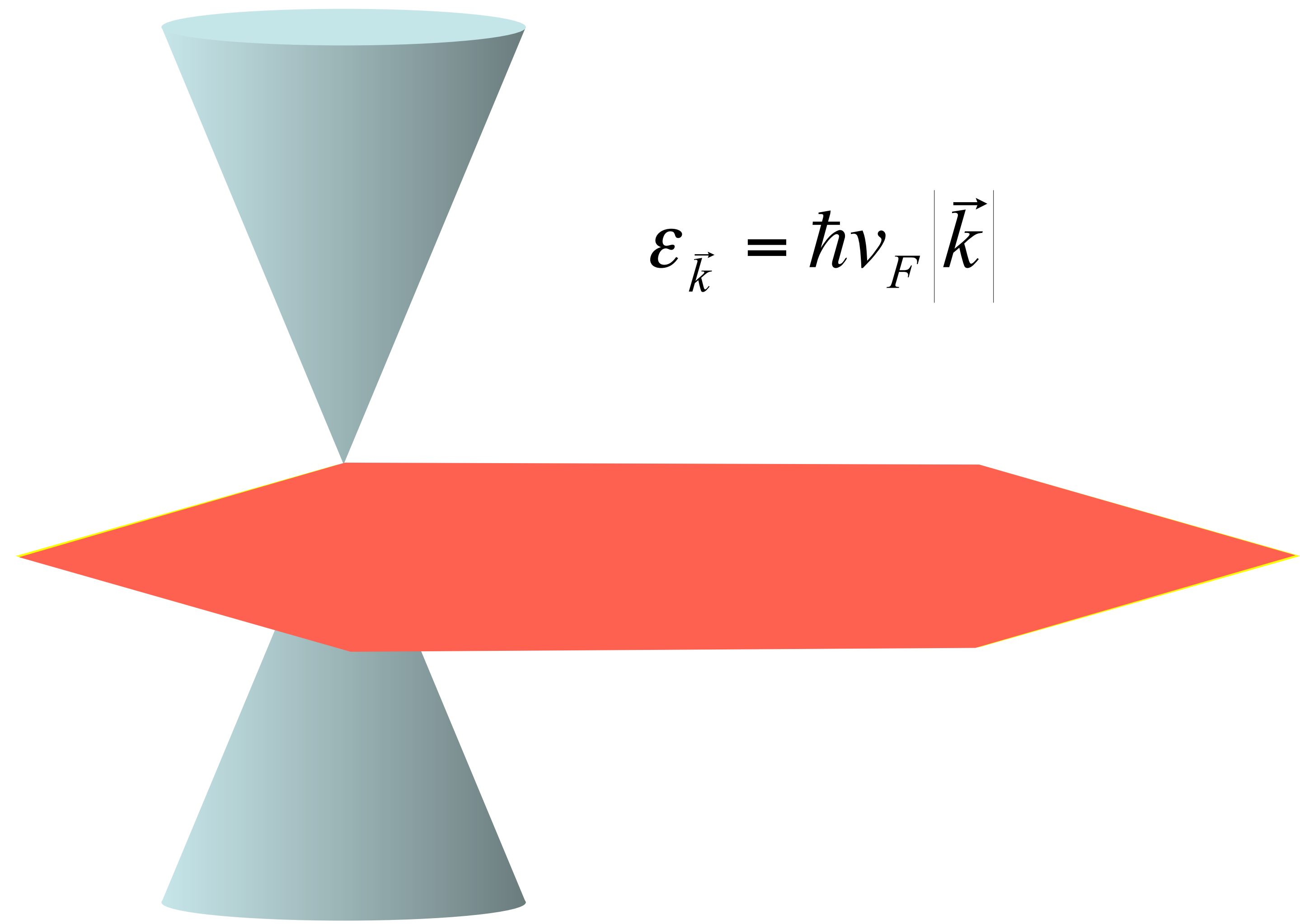



RG flow to an attractive fixed point

# Graphene



$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$

# Graphene

Low energy theory has 4 two-component Dirac fermions, $\psi_\alpha$, $\alpha = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$\begin{aligned} \mathcal{S} &= \int d^2r d\tau \psi_\alpha^\dagger \left( \partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\alpha \\ &\quad + \frac{e^2}{2} \int d^2r d^2r' d\tau \psi_\alpha^\dagger \psi_\alpha(r) \frac{1}{|r-r'|} \psi_\beta^\dagger \psi_\beta(r') \end{aligned}$$

Dimensionless "fine-structure" constant $\alpha = e^2/(4\hbar v_F)$.
RG flow of $\alpha$:

$$\frac{d\alpha}{d\ell} = -\alpha^2 + \ldots$$

**Behavior is similar to a CFT3 with $\alpha \sim 1/\ln(\mathbf{scale})$.**

# Three foci of modern physics

Quantum phase transitions

Hydrodynamics

Black holes

# Three foci of modern physics

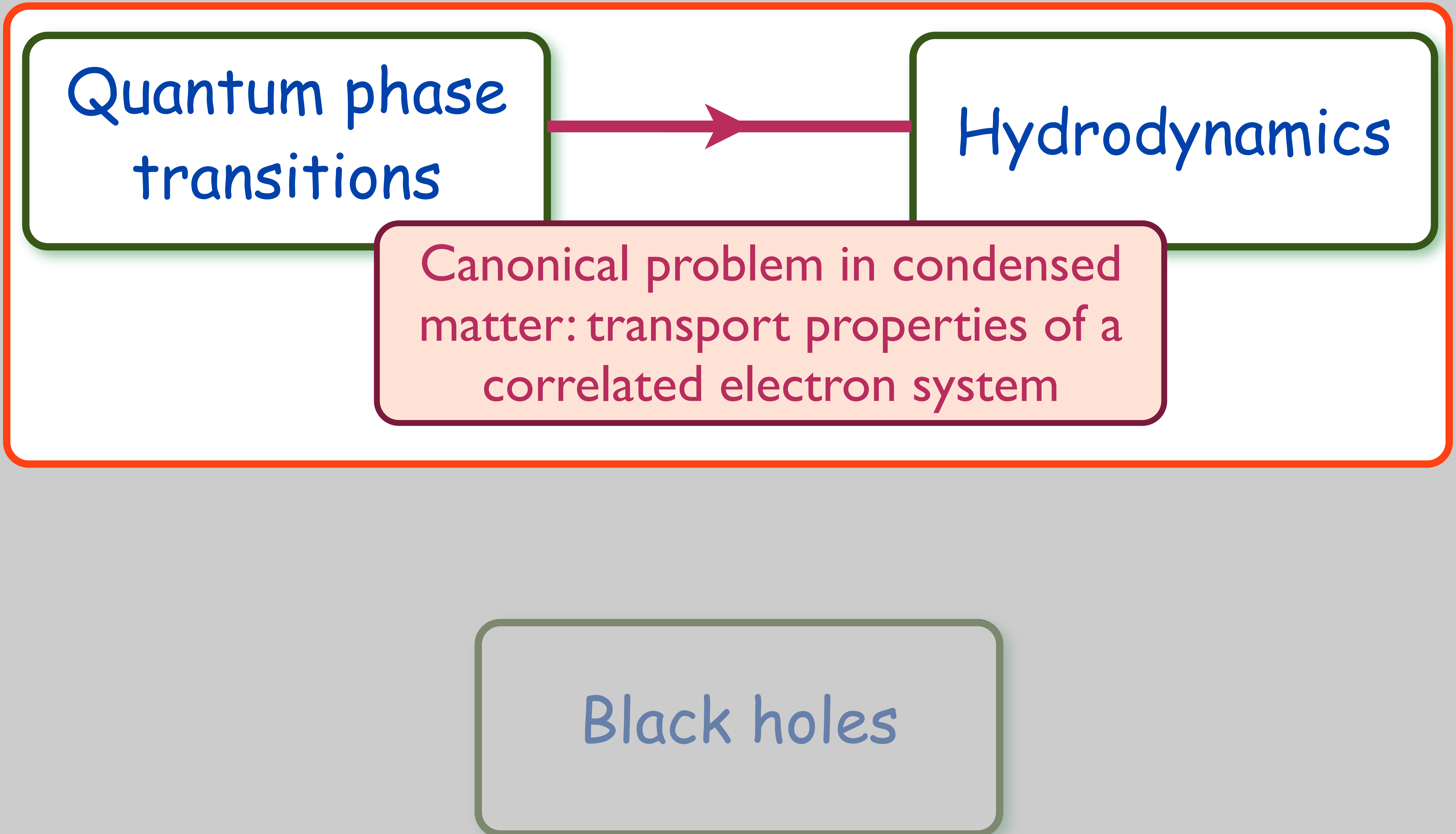
Quantum phase transitions

Hydrodynamics

Canonical problem in condensed matter: transport properties of a correlated electron system

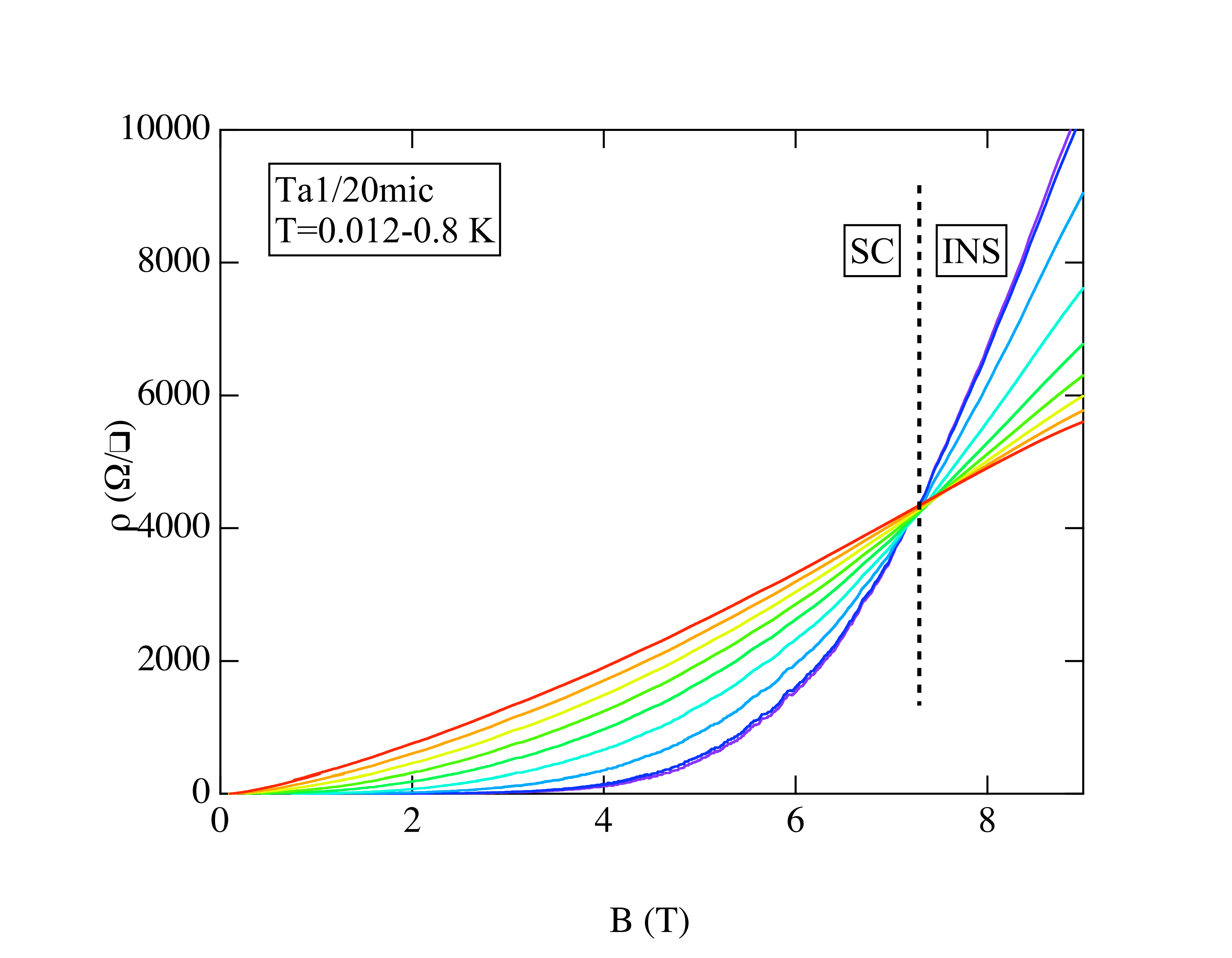
Black holes

# Three foci of modern physics



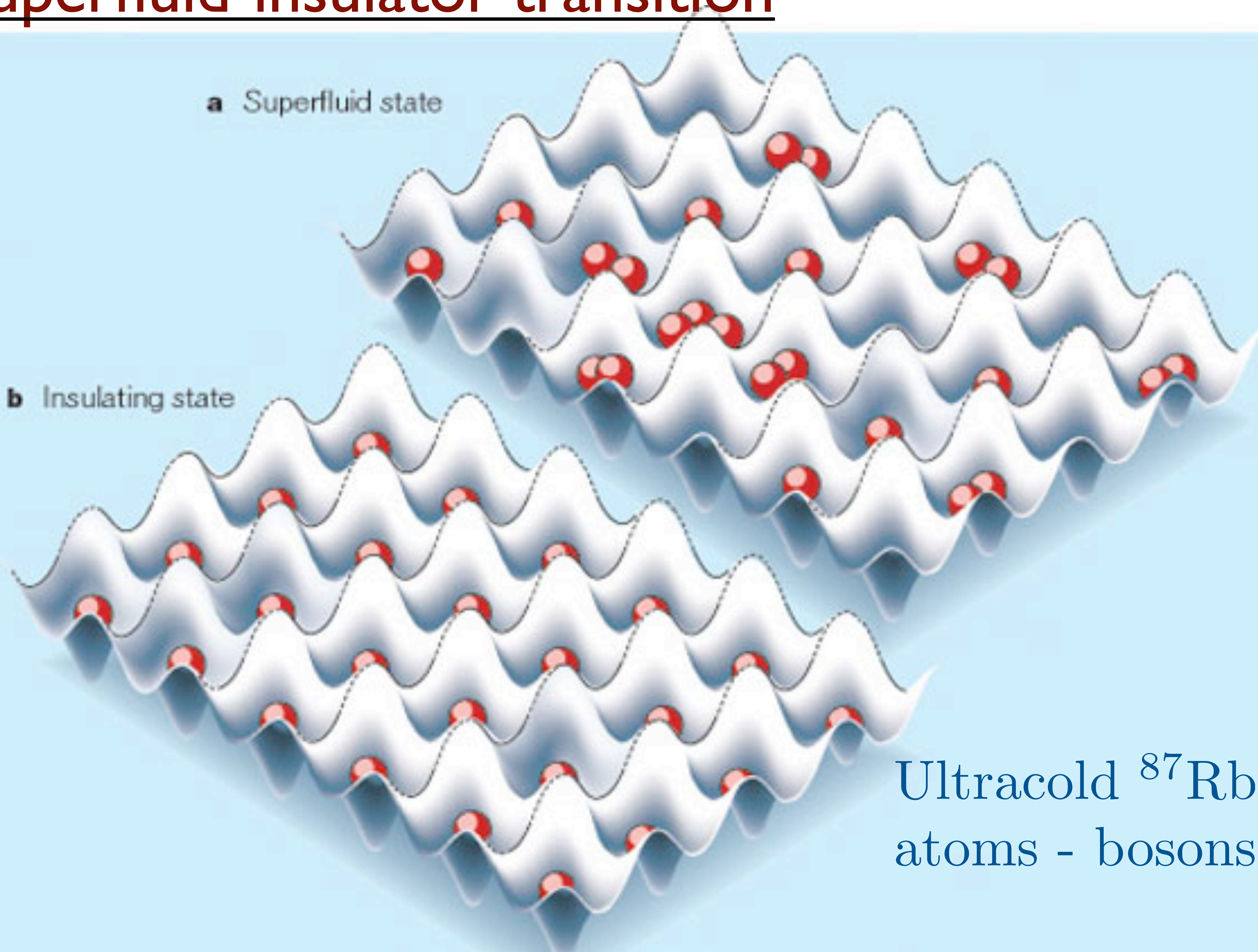
# Superfluid-insulator transition

## Indium Oxide films



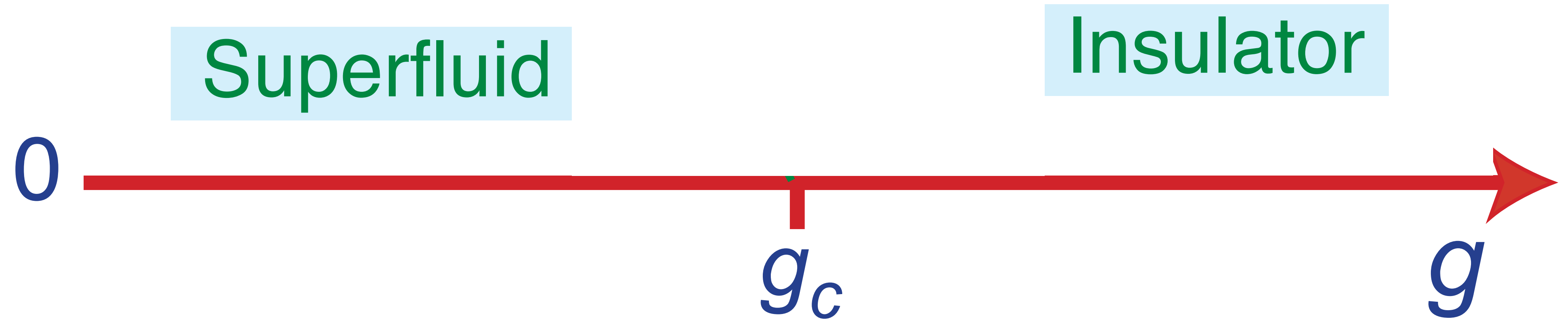

G. Sambandamurthy, A. Johansson, E. Peled, D. Shahar, P. G. Bjornsson, and K. A. Moler, *Europhys. Lett.* **75**, 611 (2006).

# Superfluid-insulator transition

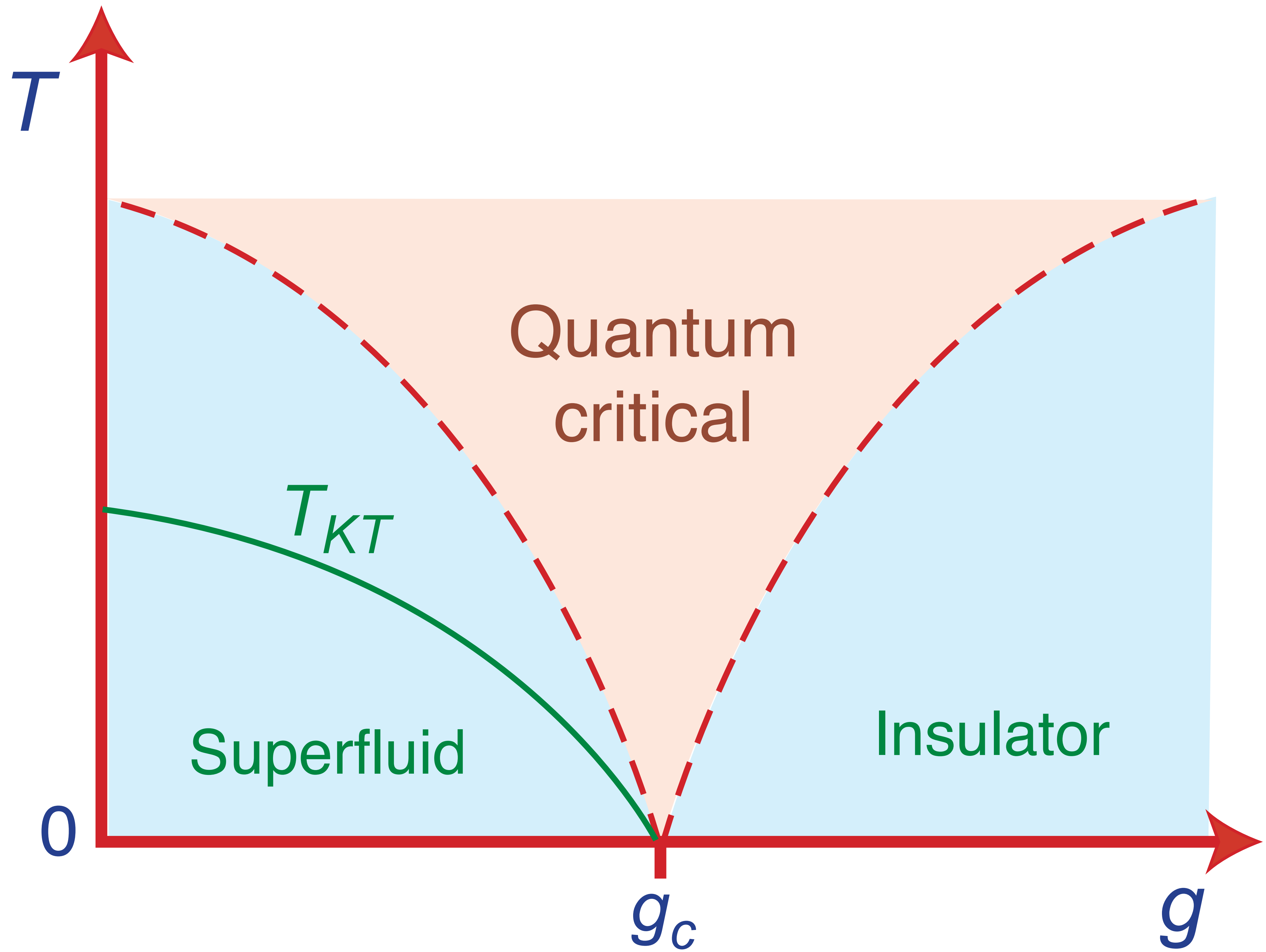



Ultracold $^{87}$Rb atoms - bosons

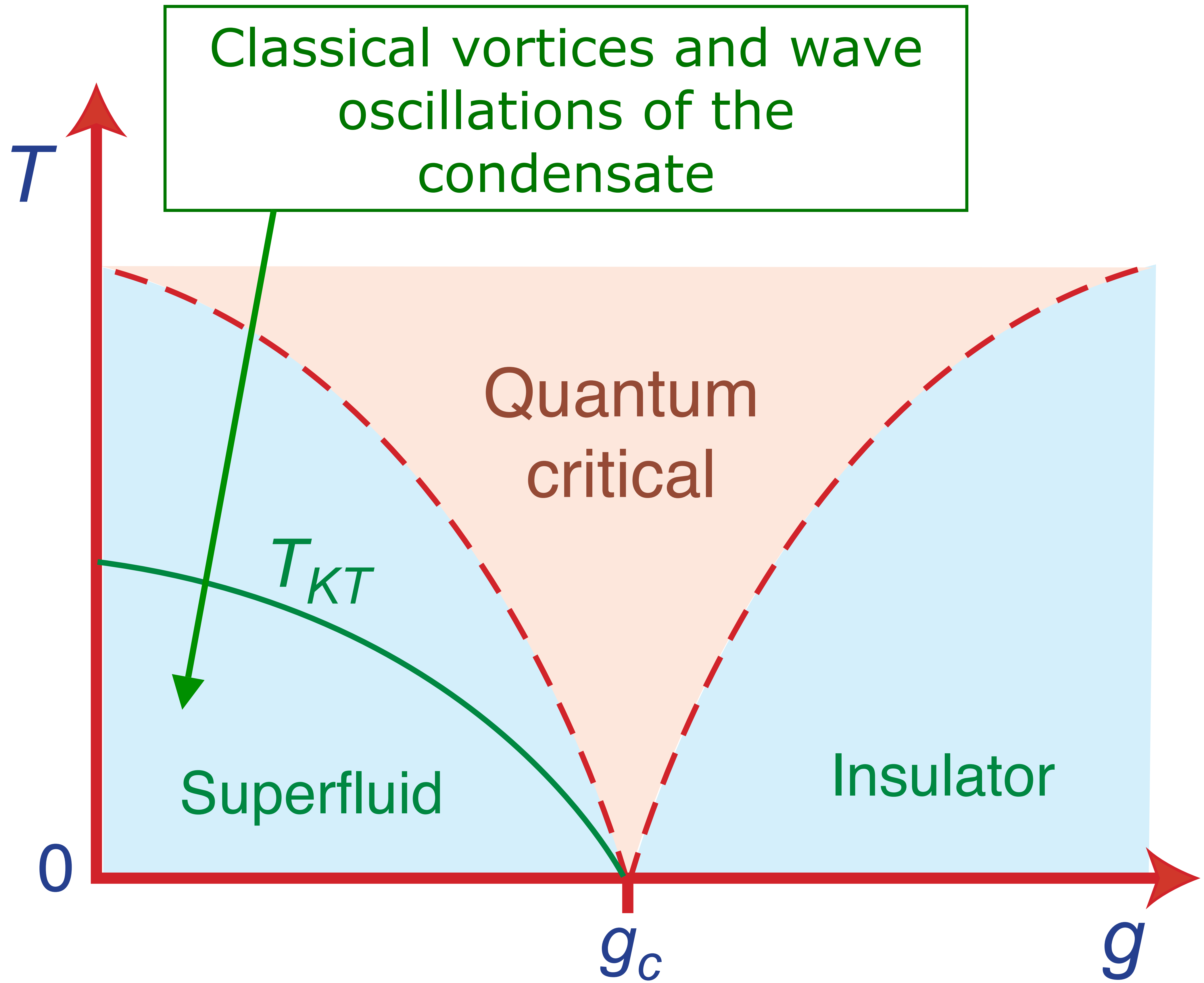
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



Superfluid
Insulator
0
$g_c$
$g$

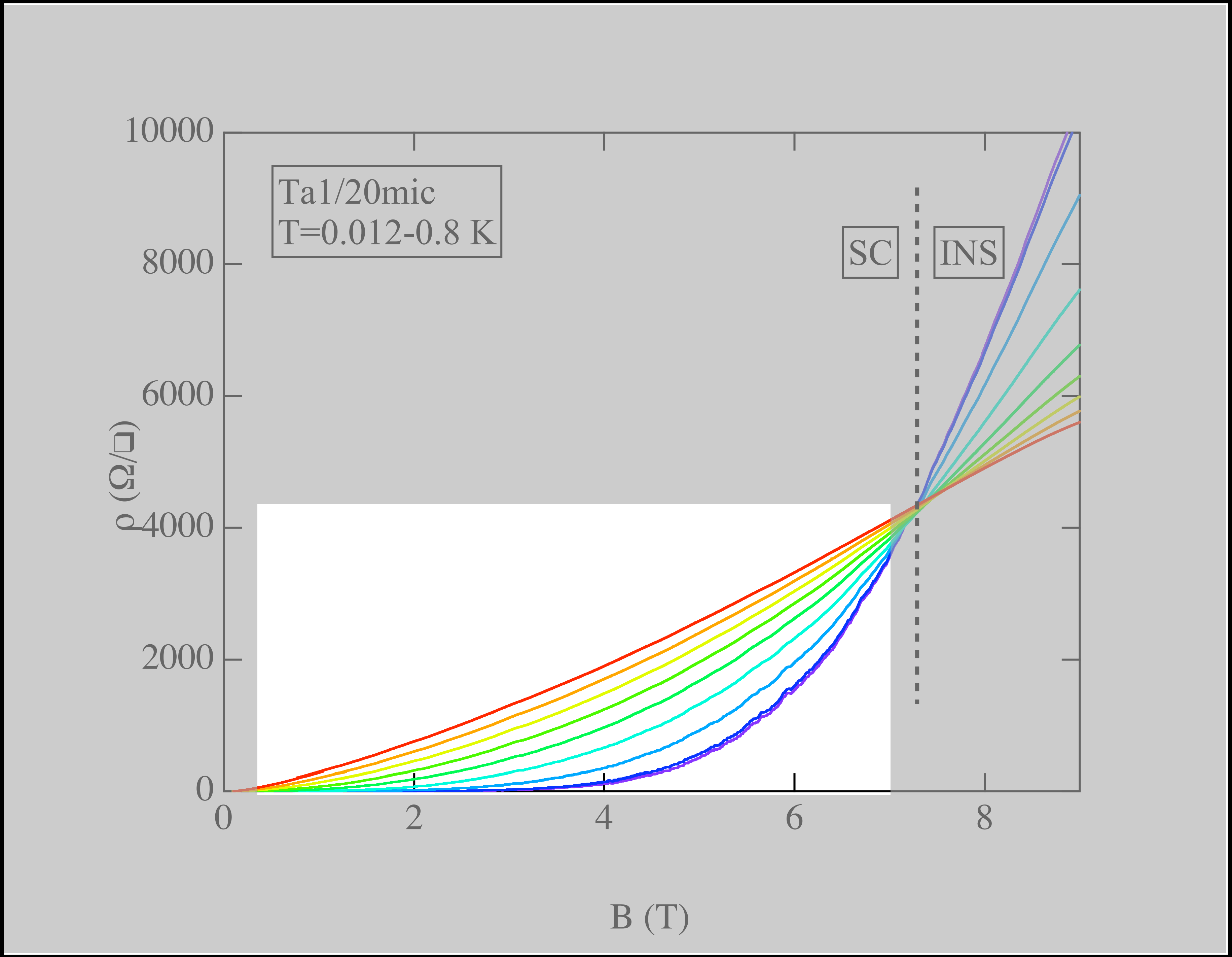


T
Quantum
critical
$T_{KT}$
Superfluid
Insulator
0
$g_c$
g

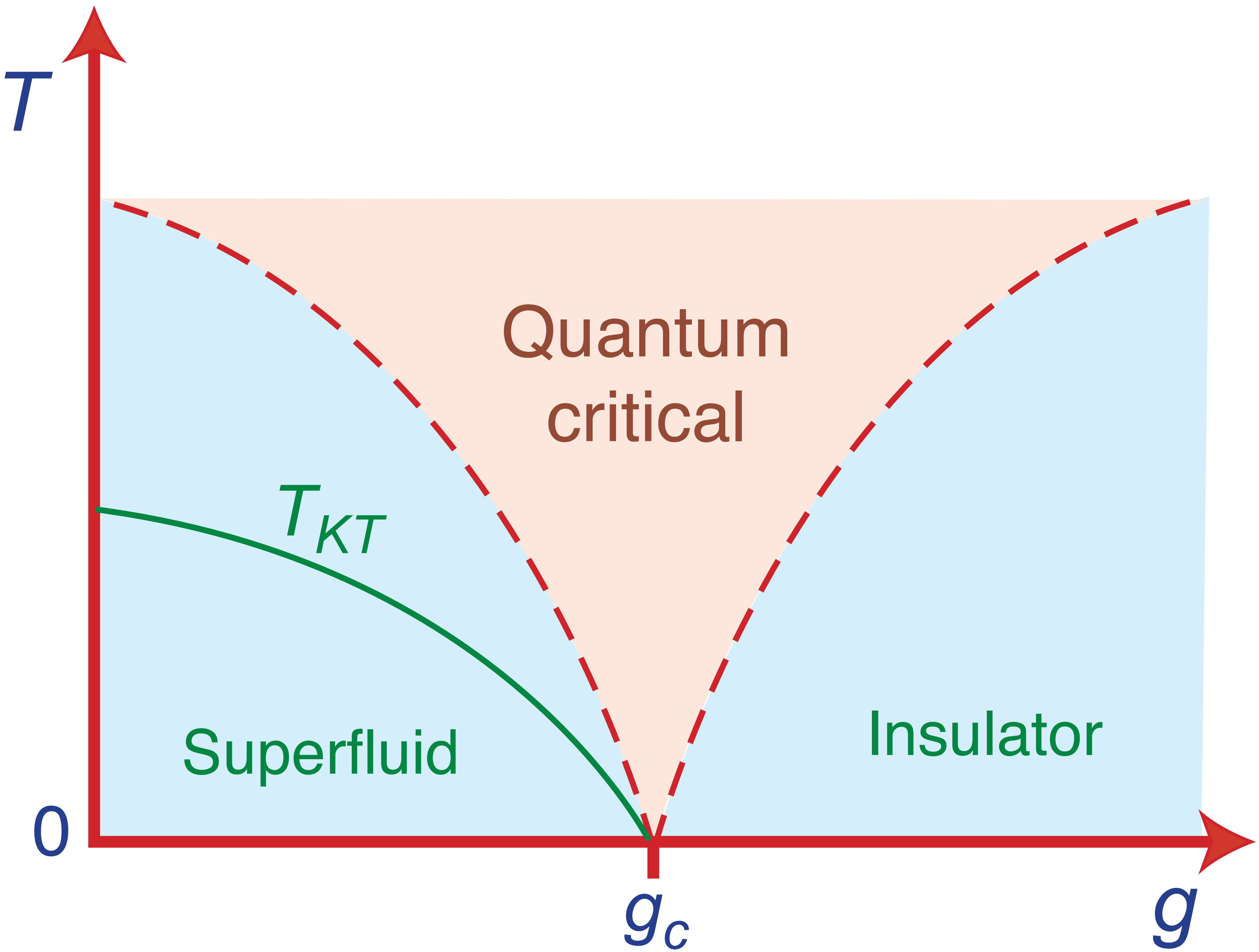

Classical vortices and wave oscillations of the condensate
T
Quantum critical
$T_{KT}$
Superfluid
Insulator
0
$g_c$
g

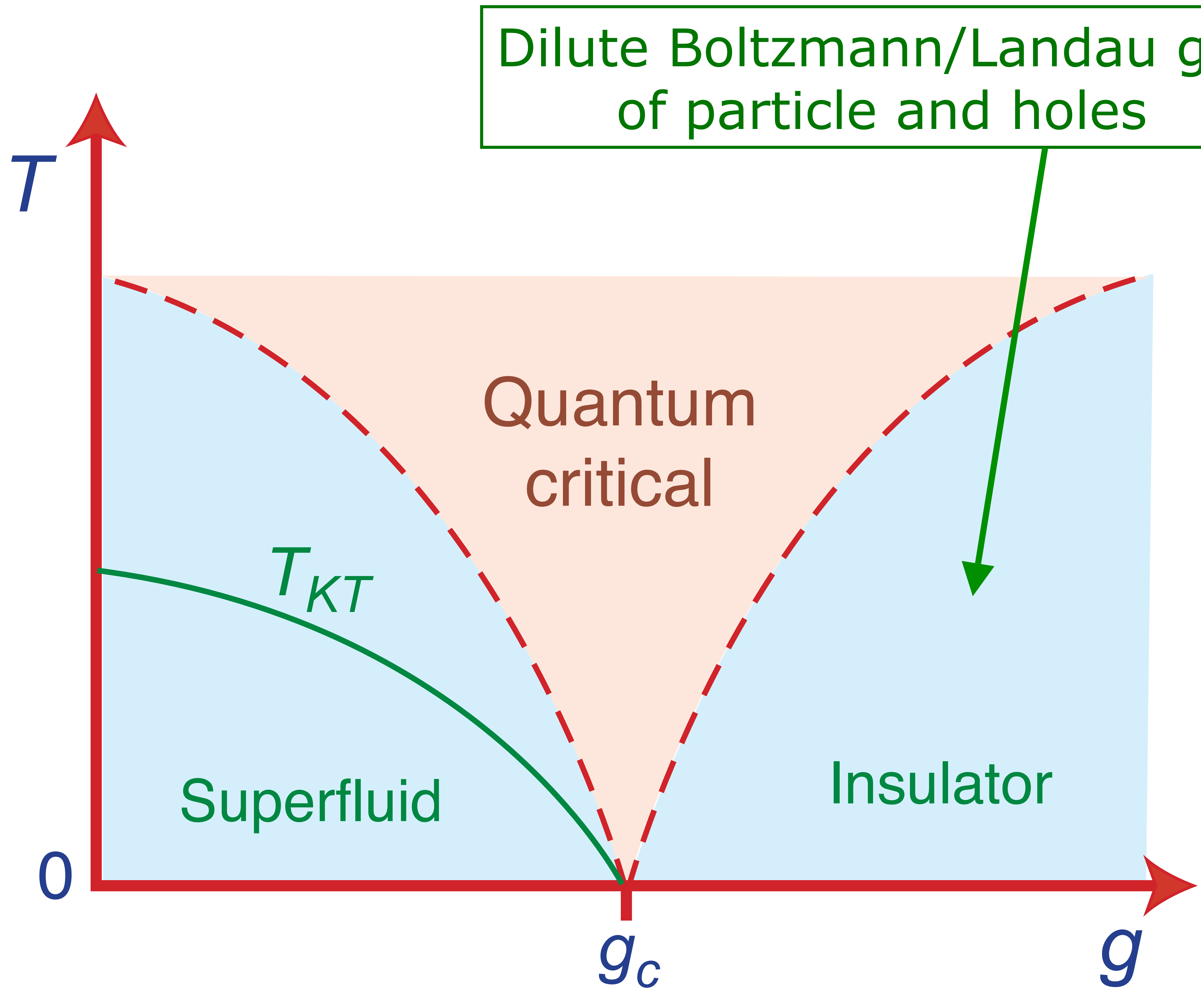
# Superfluid-insulator transition

## Indium Oxide films




G. Sambandamurthy, A. Johansson, E. Peled, D. Shahar, P. G. Bjornsson, and K. A. Moler, *Europhys. Lett.* **75**, 611 (2006).

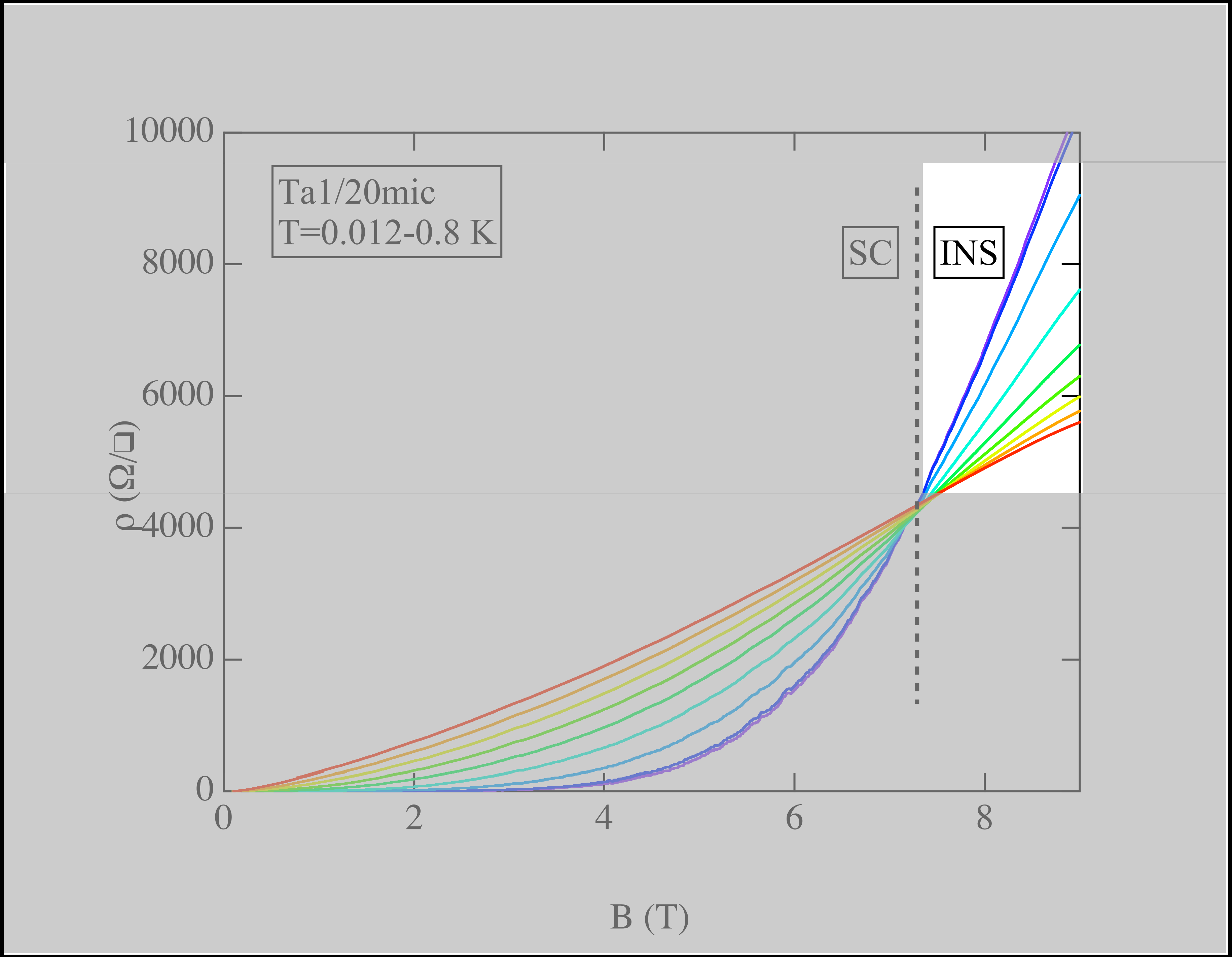

T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g



Dilute Boltzmann/Landau gas
of particle and holes
T
Quantum
critical
$T_{KT}$
Superfluid
Insulator
0
$g_c$
g

# Superfluid-insulator transition

## Indium Oxide films

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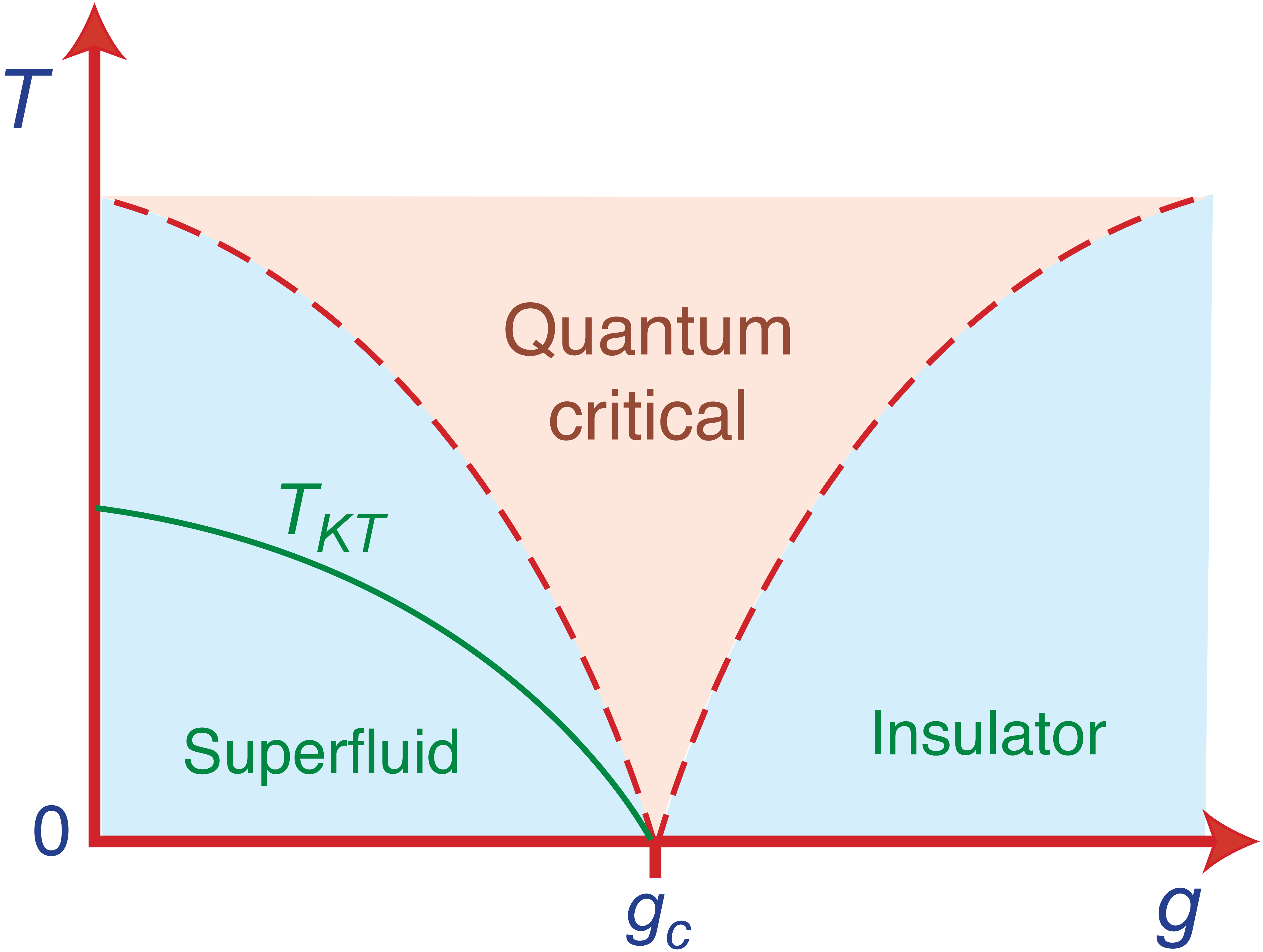
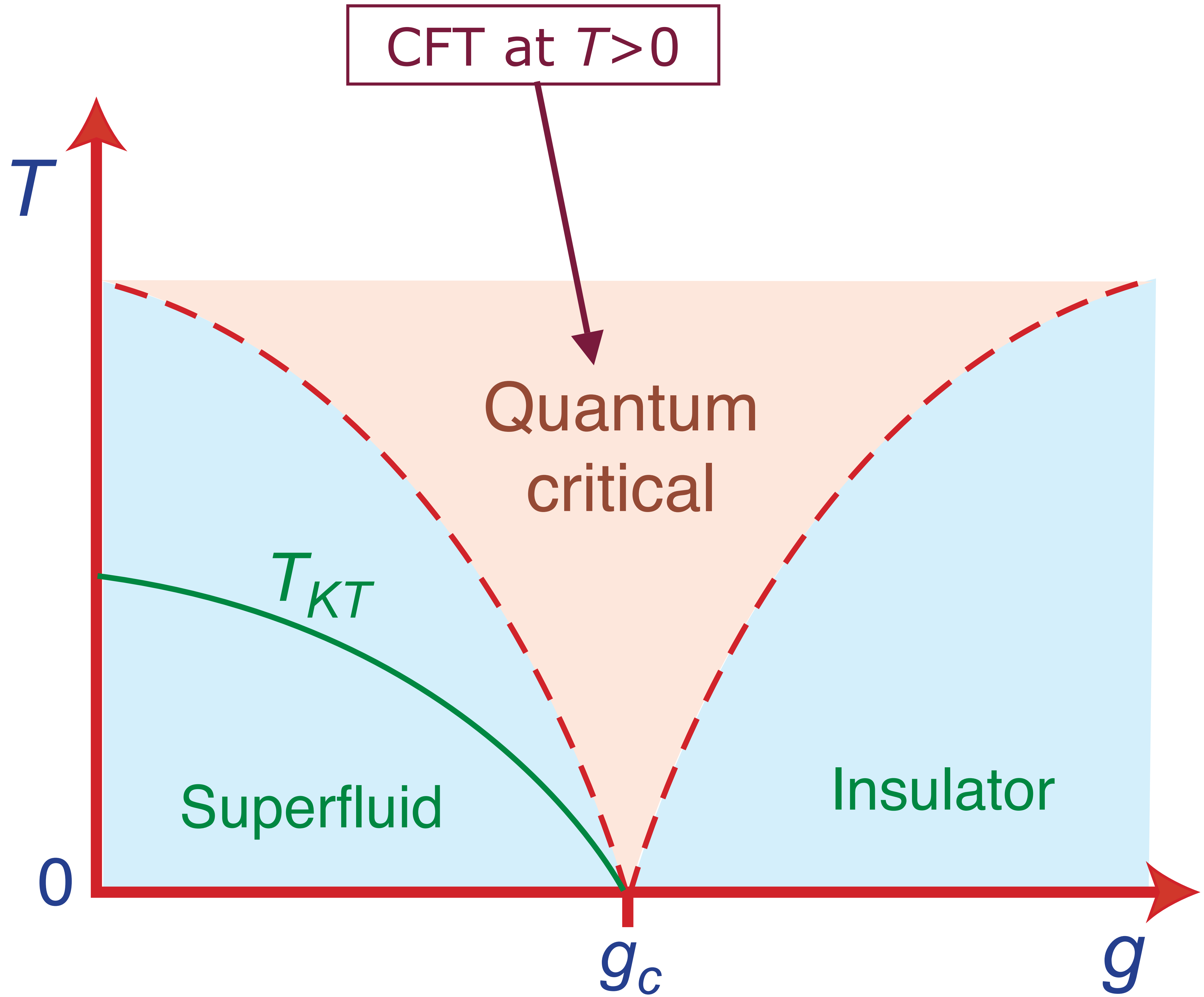

T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g



CFT at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

# Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, $\tau_R$

$$\tau_R \gtrsim \frac{\hbar}{k_B T}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

# Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

## Electrical conductivity

$$\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

# Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

## Momentum transport

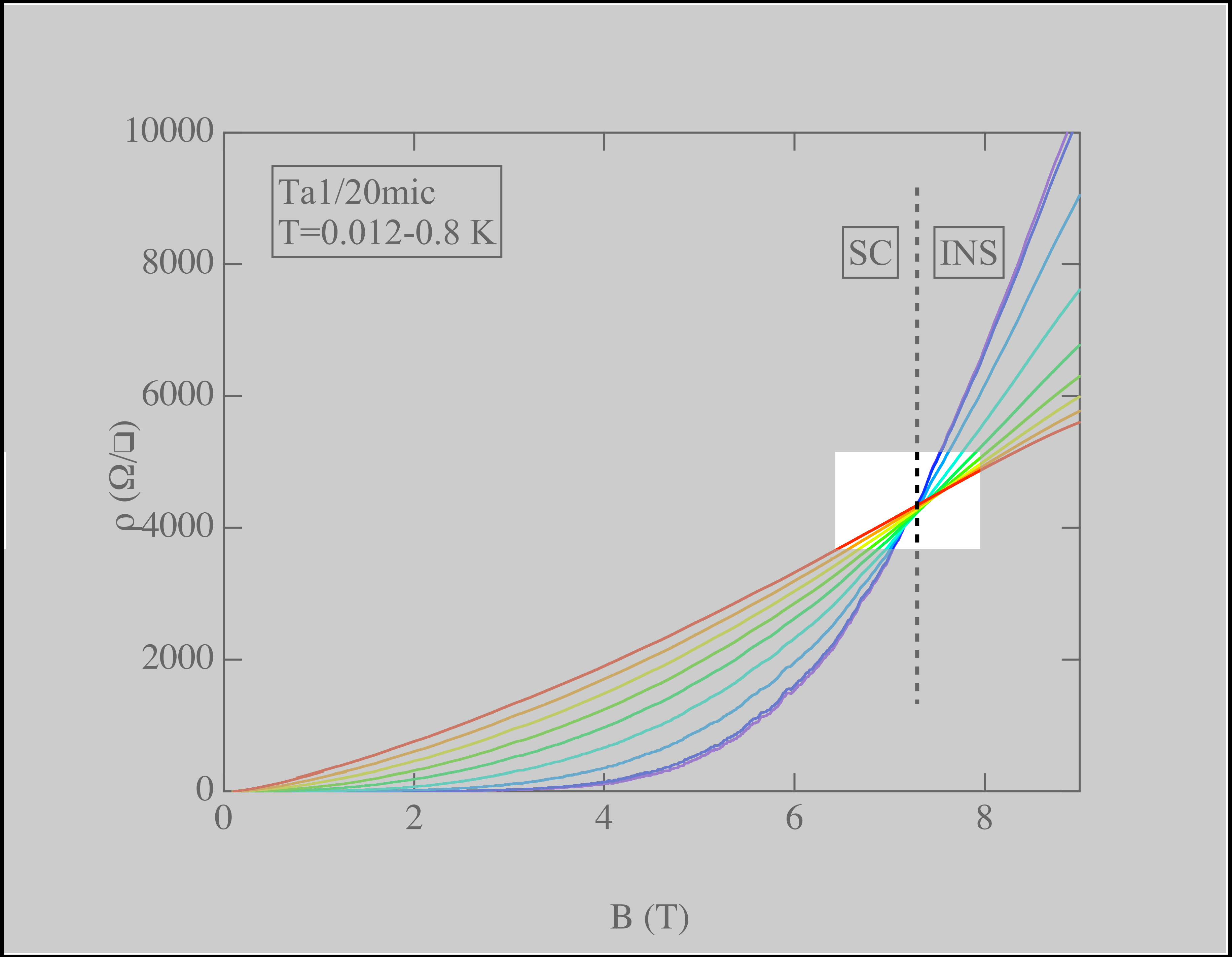
$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

# Superfluid-insulator transition

## Indium Oxide films




G. Sambandamurthy, A. Johansson, E. Peled, D. Shahar, P. G. Bjornsson, and K. A. Moler, *Europhys. Lett.* **75**, 611 (2006).

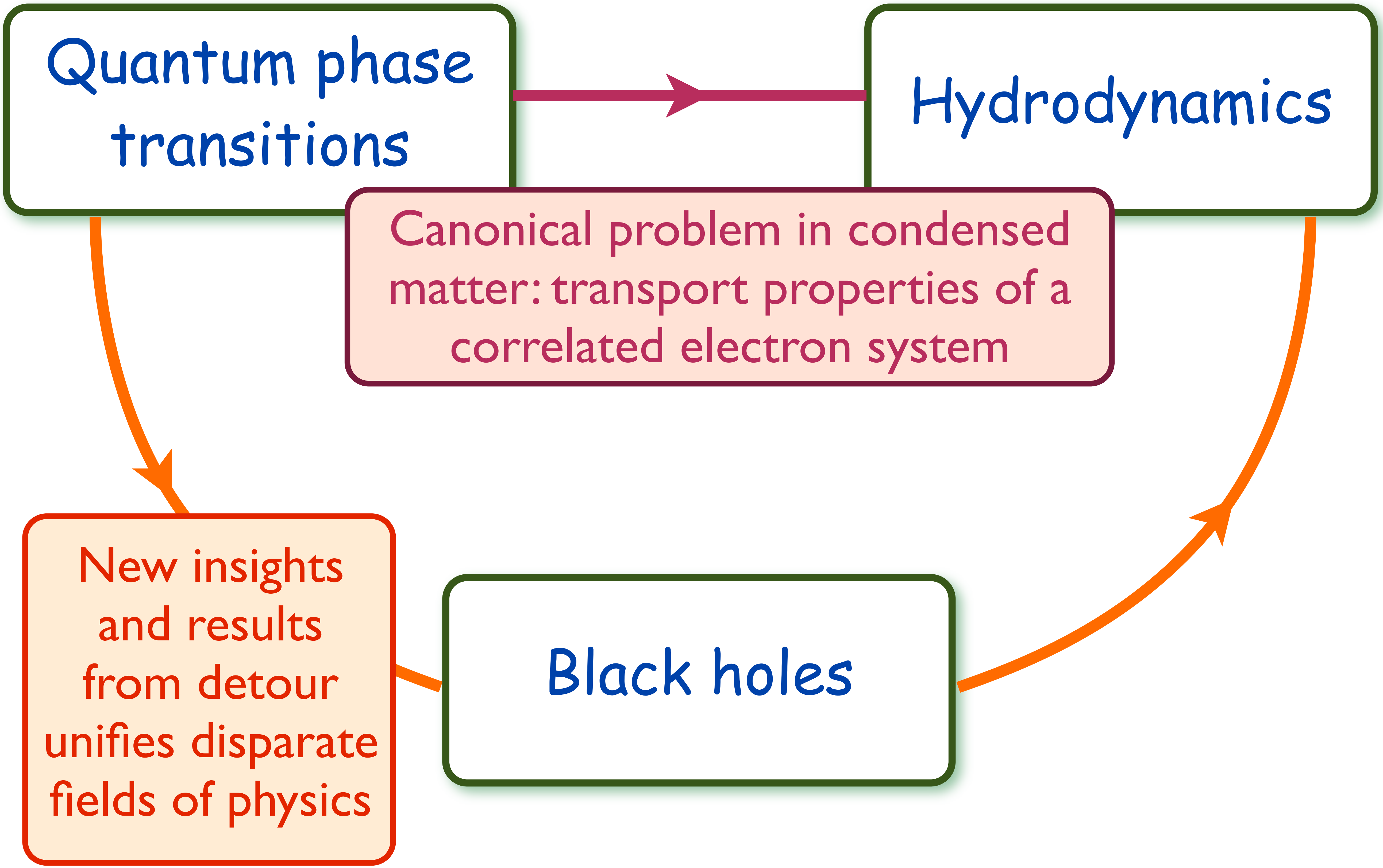
# Three foci of modern physics

Quantum phase transitions

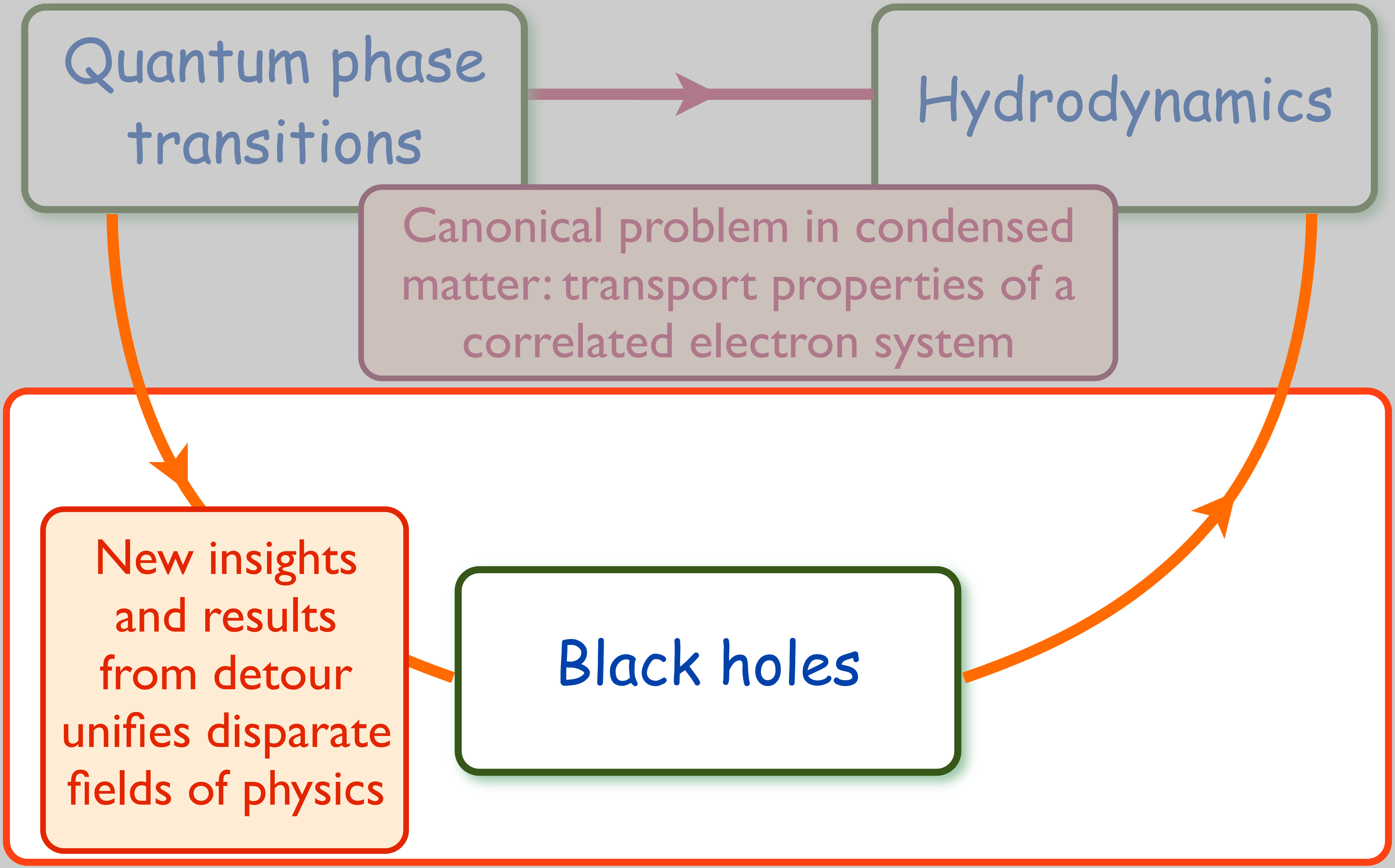
Hydrodynamics

Black holes

# Three foci of modern physics



# Three foci of modern physics



# Black Holes

Objects so massive that light is gravitationally bound to them.

# Black Holes

Objects so massive that light is gravitationally bound to them.

The region inside the black hole horizon is causally disconnected from the rest of the universe.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

# Black Hole Thermodynamics

**Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics**

$$\text{Entropy of a black hole } S = \frac{k_B A}{4\ell_P^2}$$

where $A$ is the area of the horizon, and

$$\ell_P = \sqrt{\frac{G\hbar}{c^3}} \text{ is the Planck length.}$$

## The Second Law: $dA \geq 0$

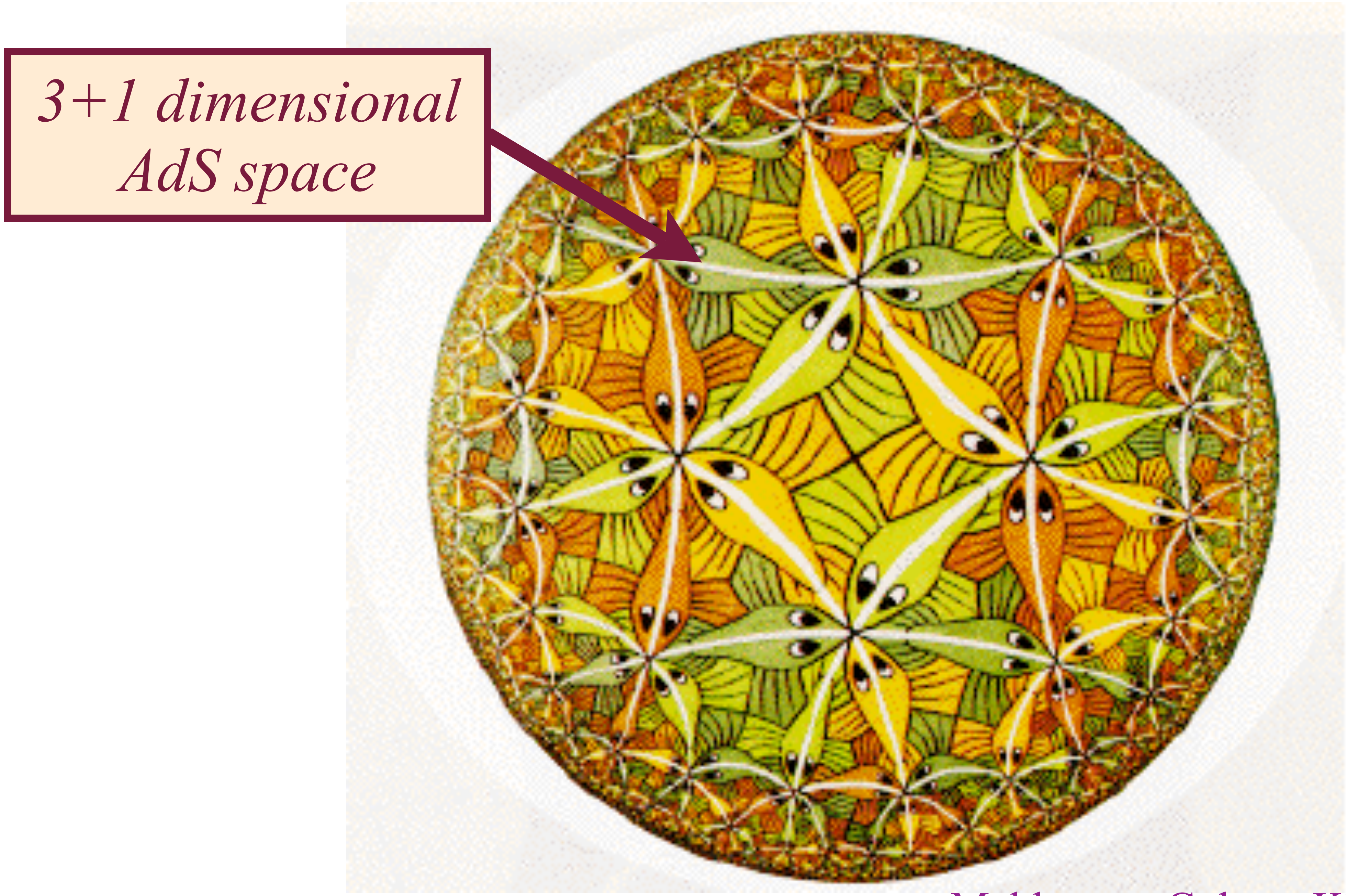
# Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

$$\text{Horizon temperature: } 4\pi k_B T = \frac{\hbar^2}{2M\ell_P^2}$$

# AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions




Maldacena, Gubser, Klebanov, Polyakov, Witten

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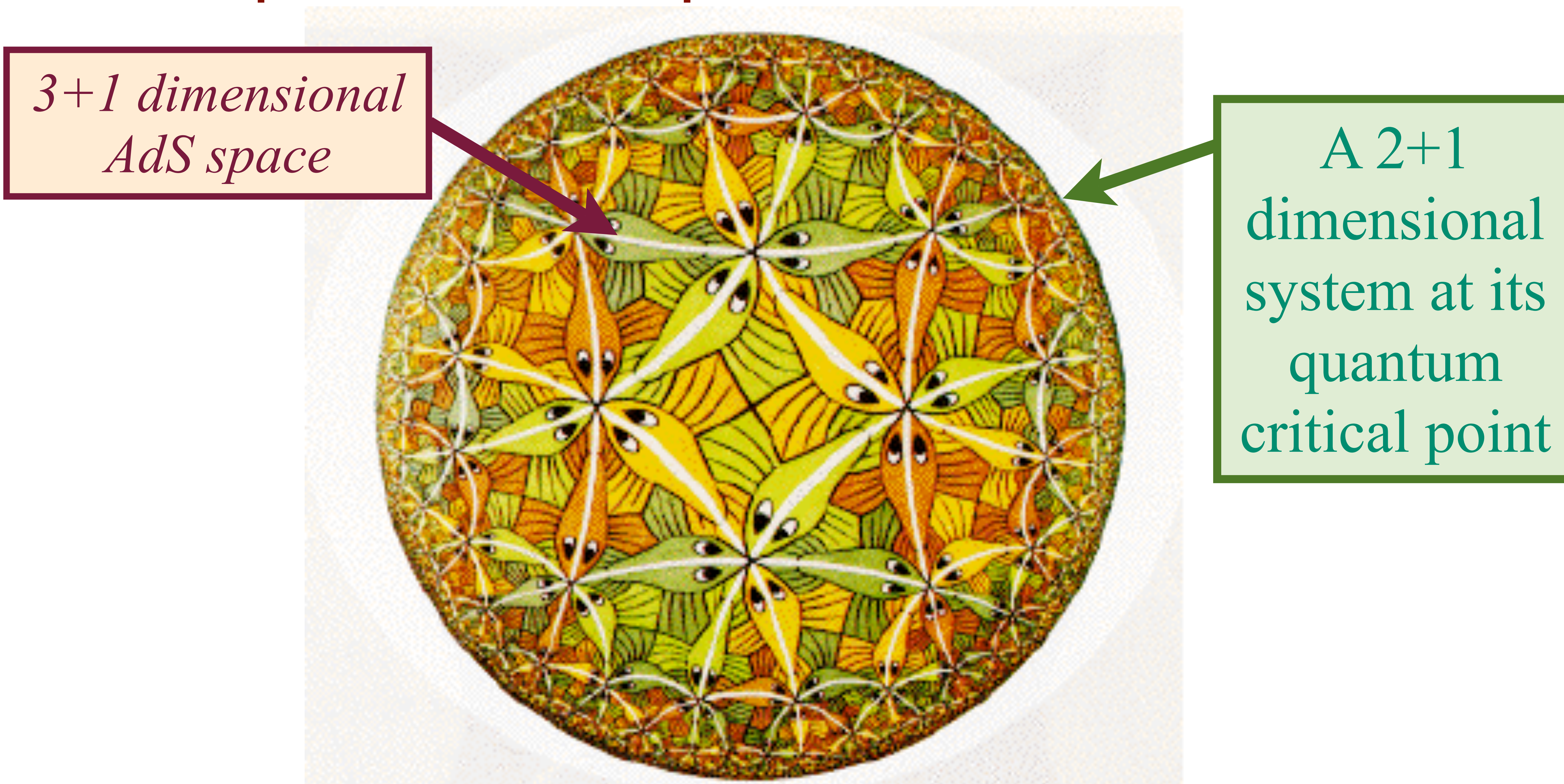


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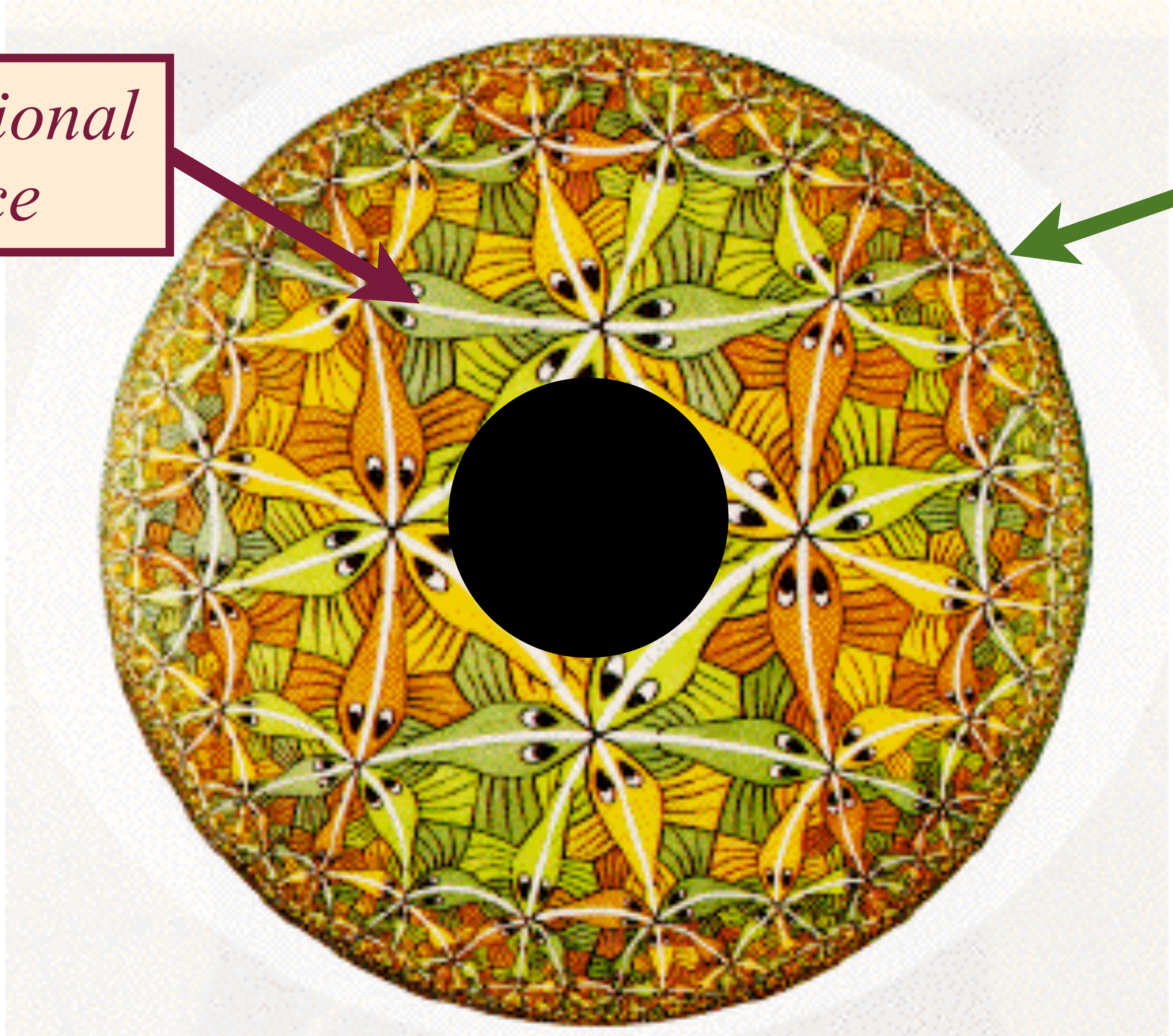

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Maldacena, Gubser, Klebanov, Polyakov, Witten

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*3+1 dimensional AdS space*

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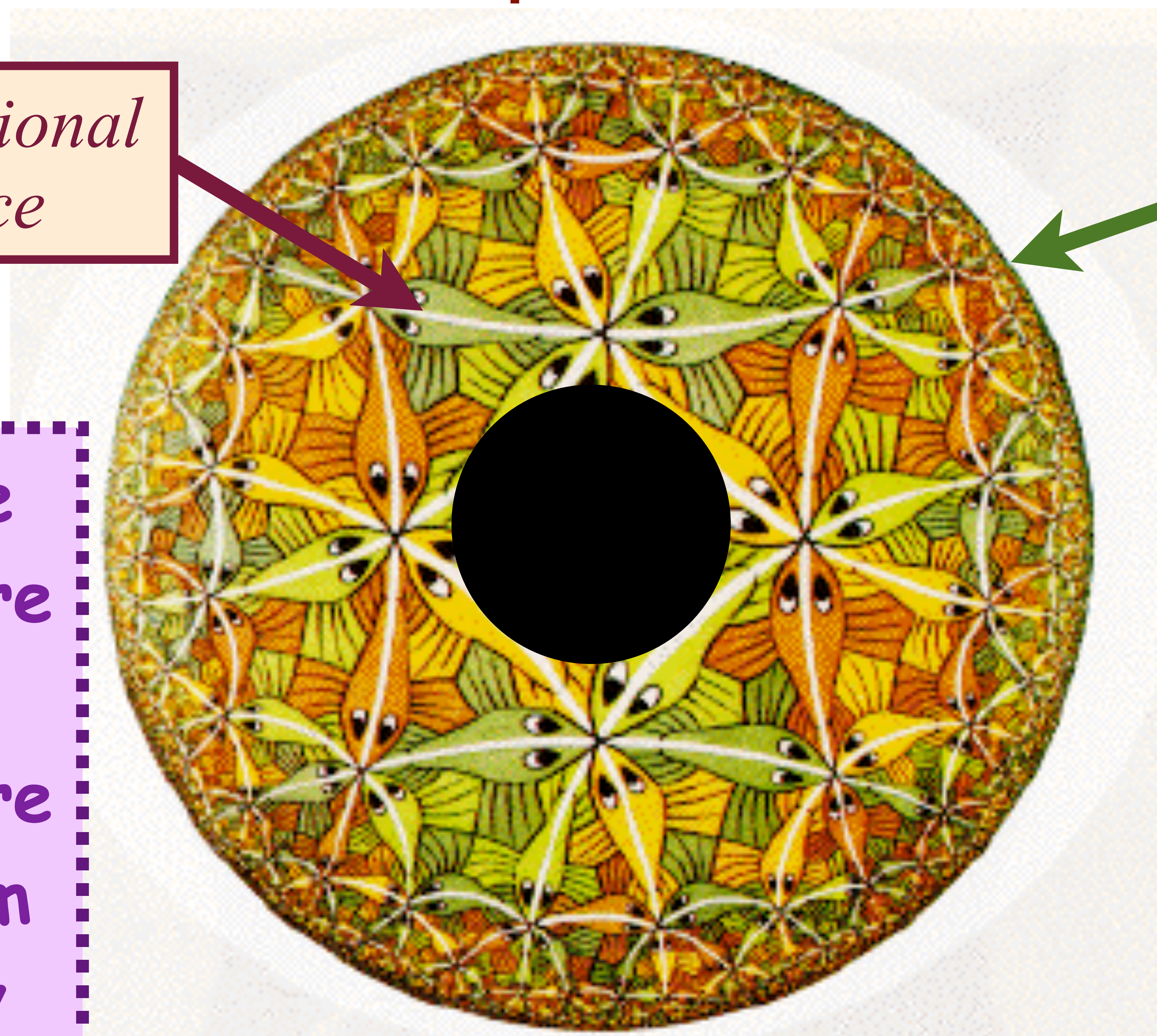

**Black hole temperature = temperature of quantum criticality**

Maldacena, Gubser, Klebanov, Polyakov, Witten

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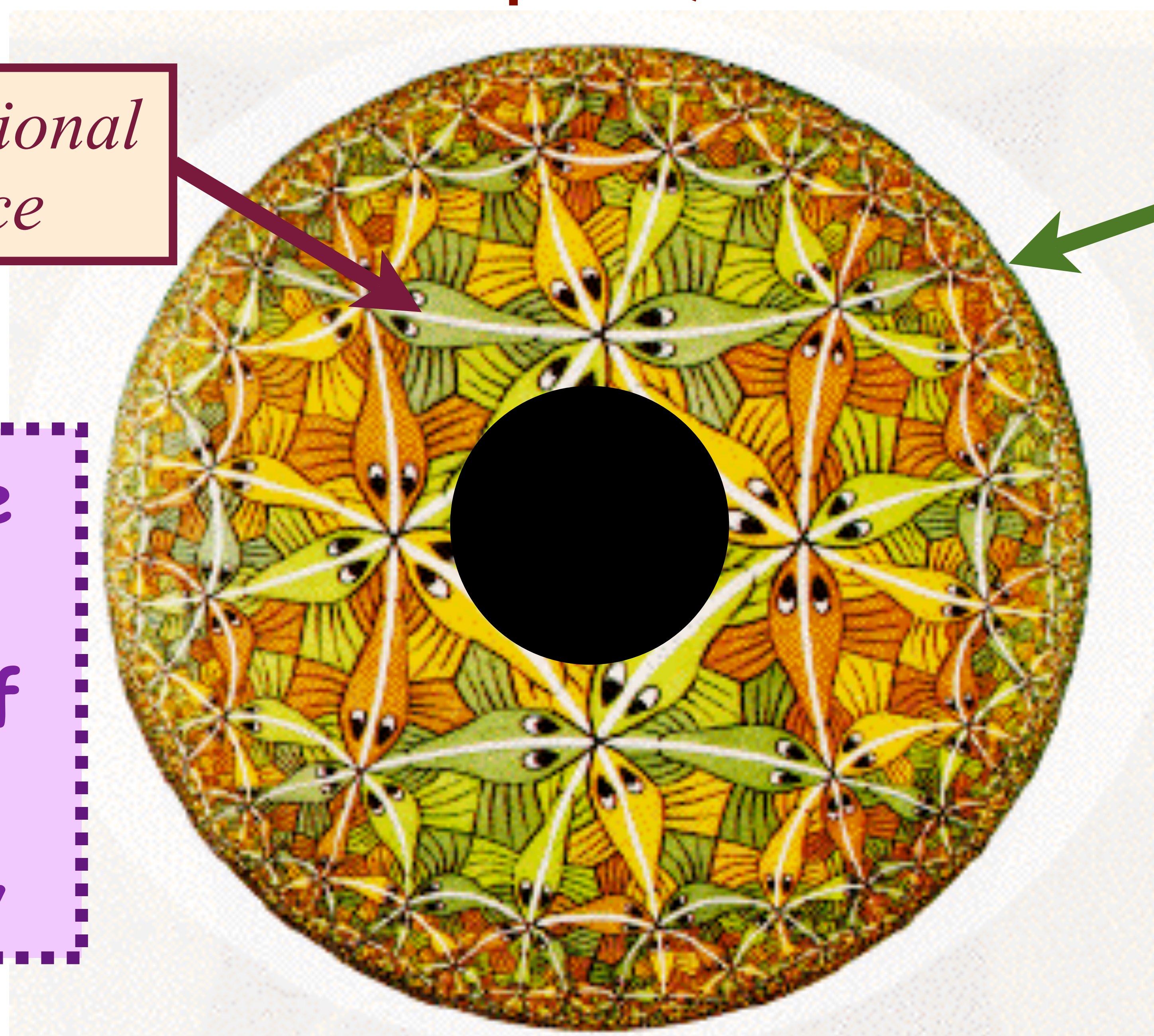

Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

# AdS/CFT correspondence

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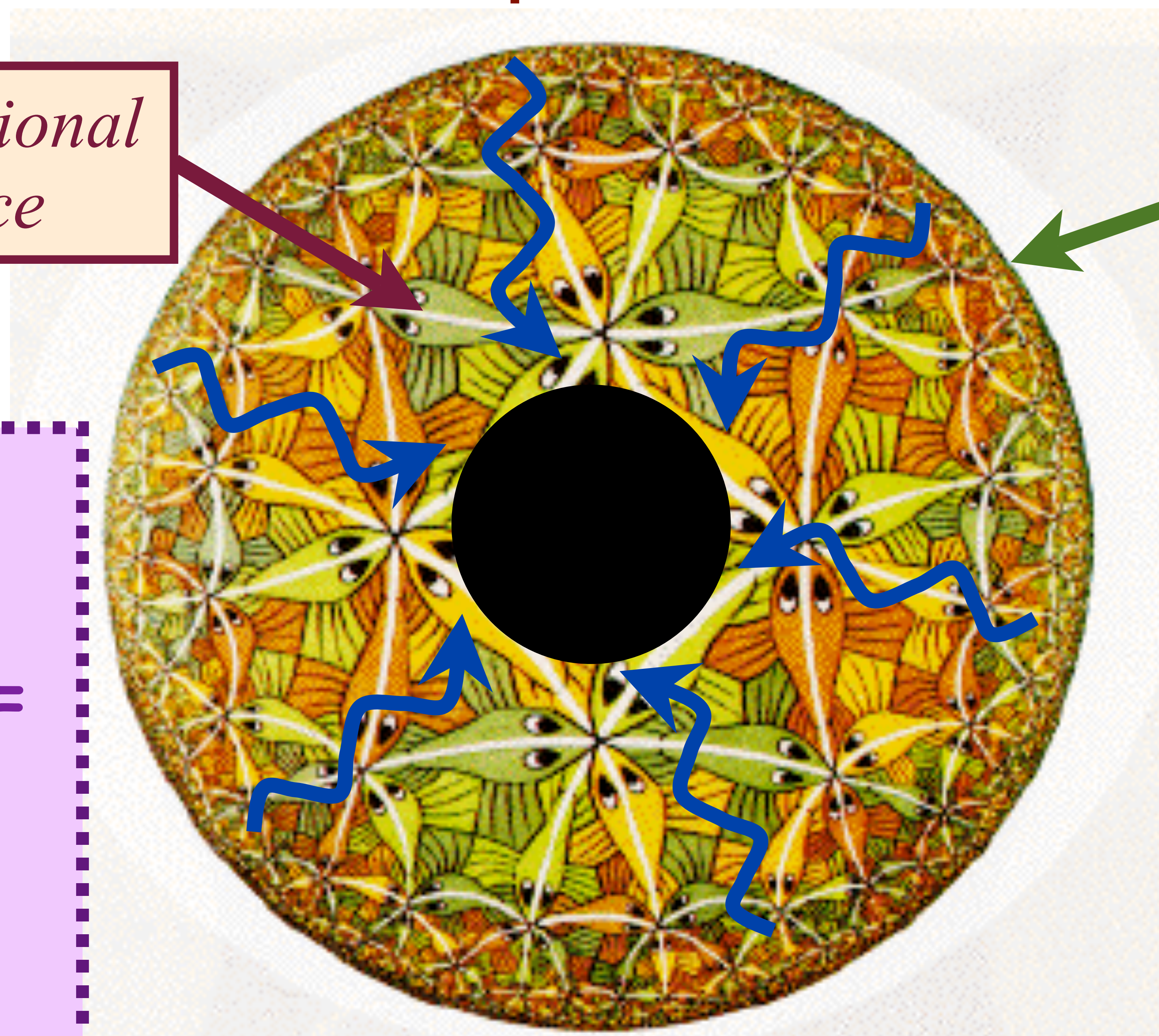


Maldacena, Gubser, Klebanov, Polyakov, Witten

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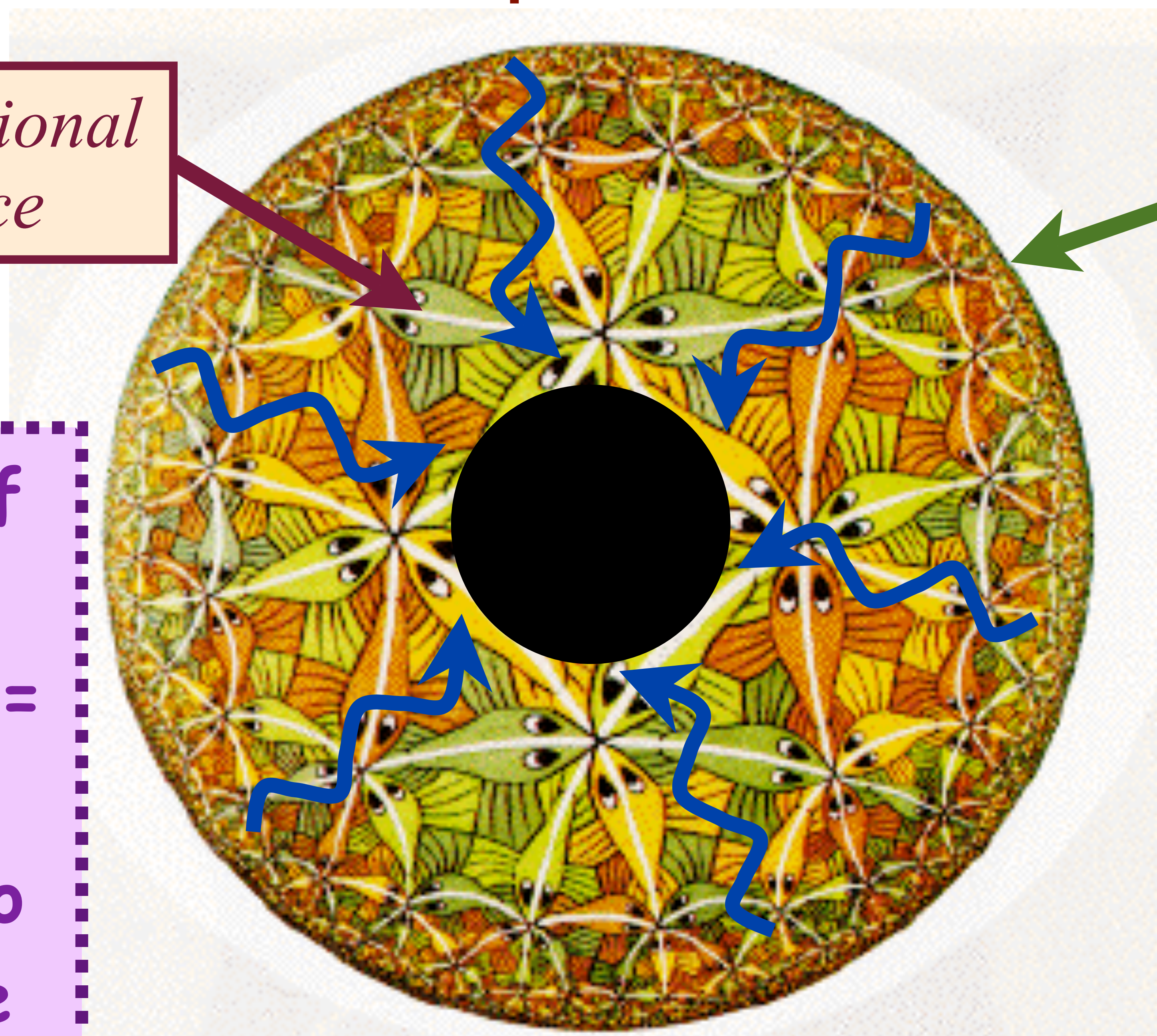


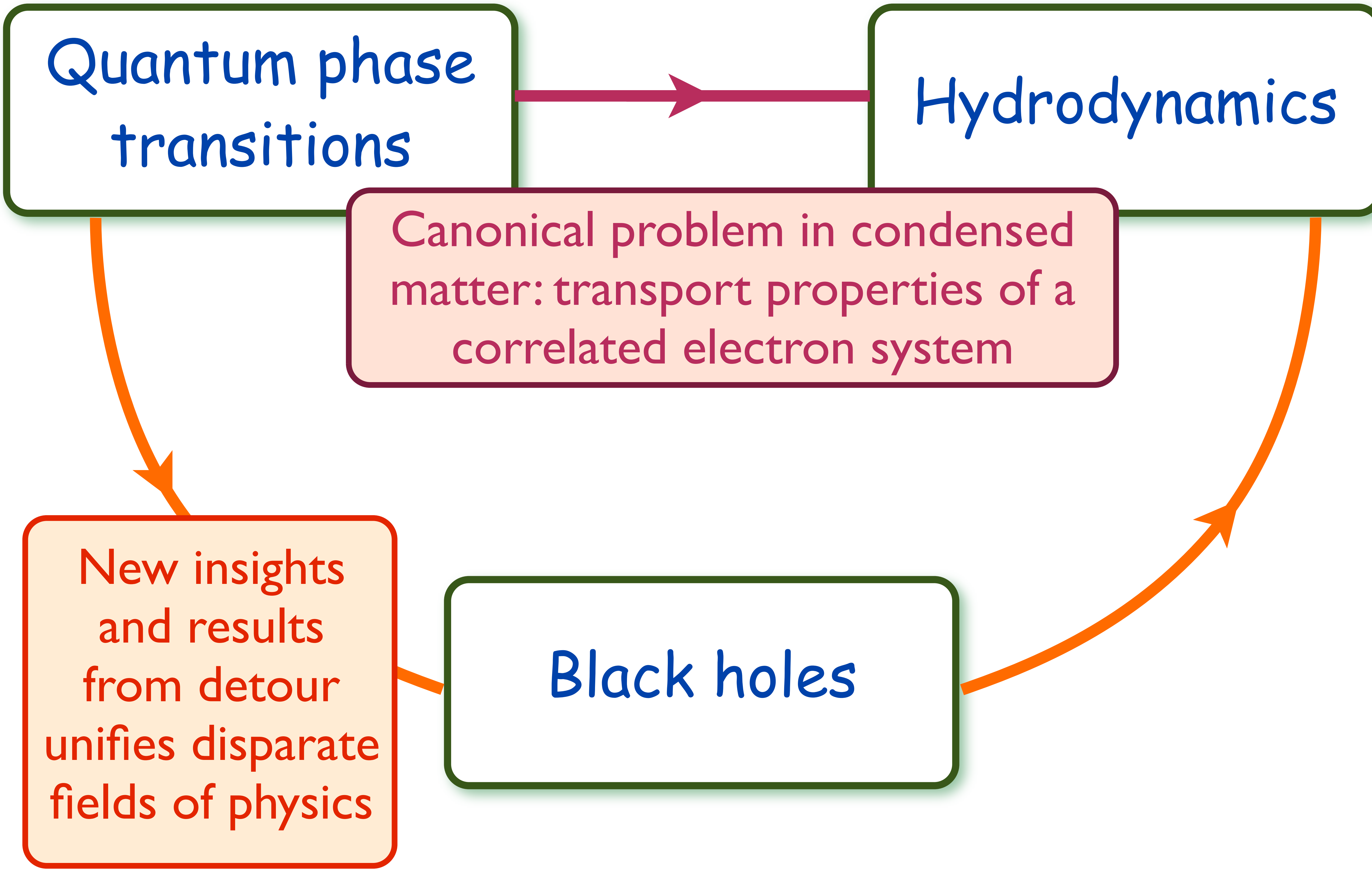
*3+1 dimensional AdS space*

Quantum criticality in 2+1 dimensions

**Friction of quantum criticality = waves falling into black hole**

Kovtun, Policastro, Son

# Three foci of modern physics



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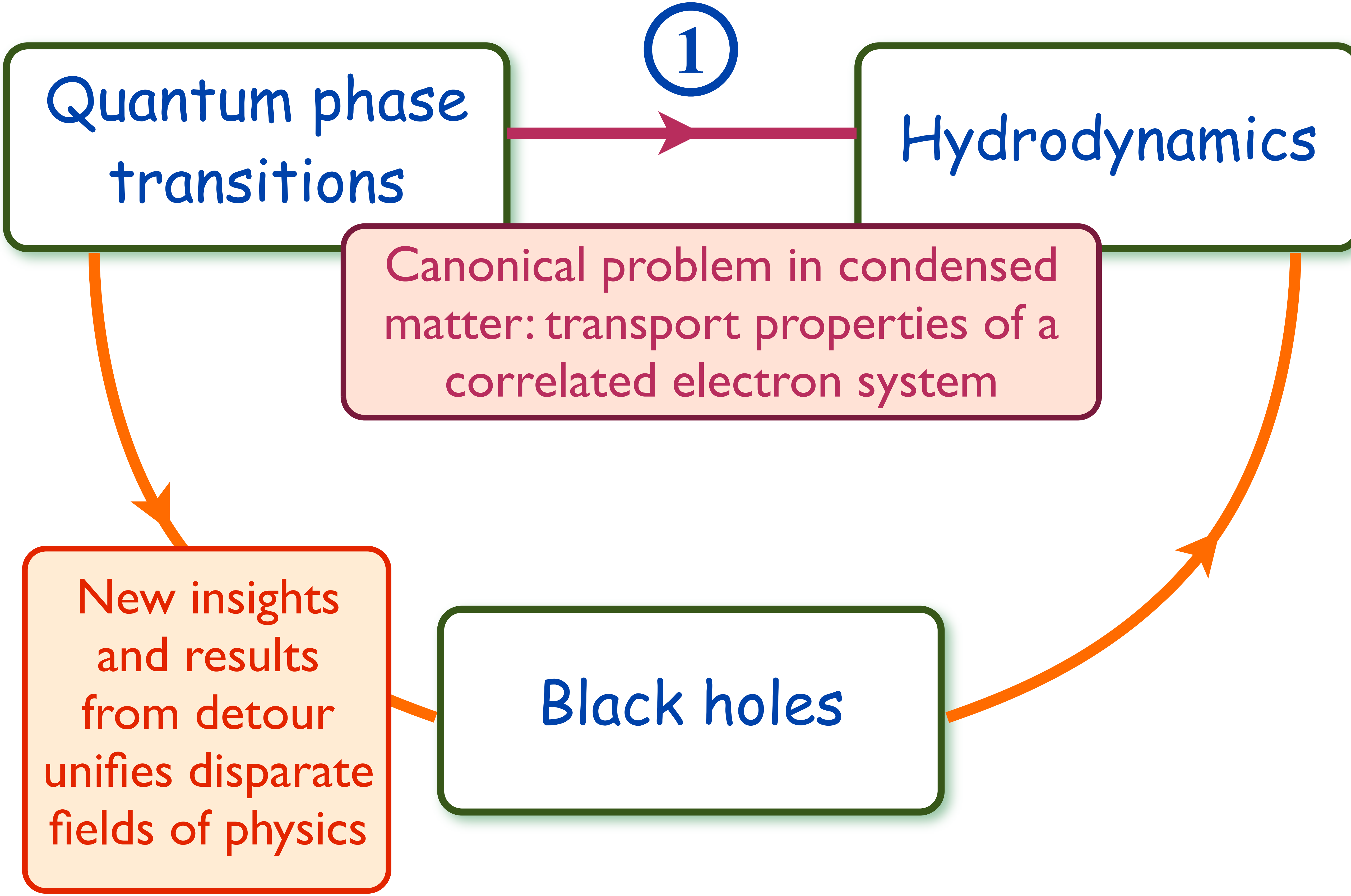
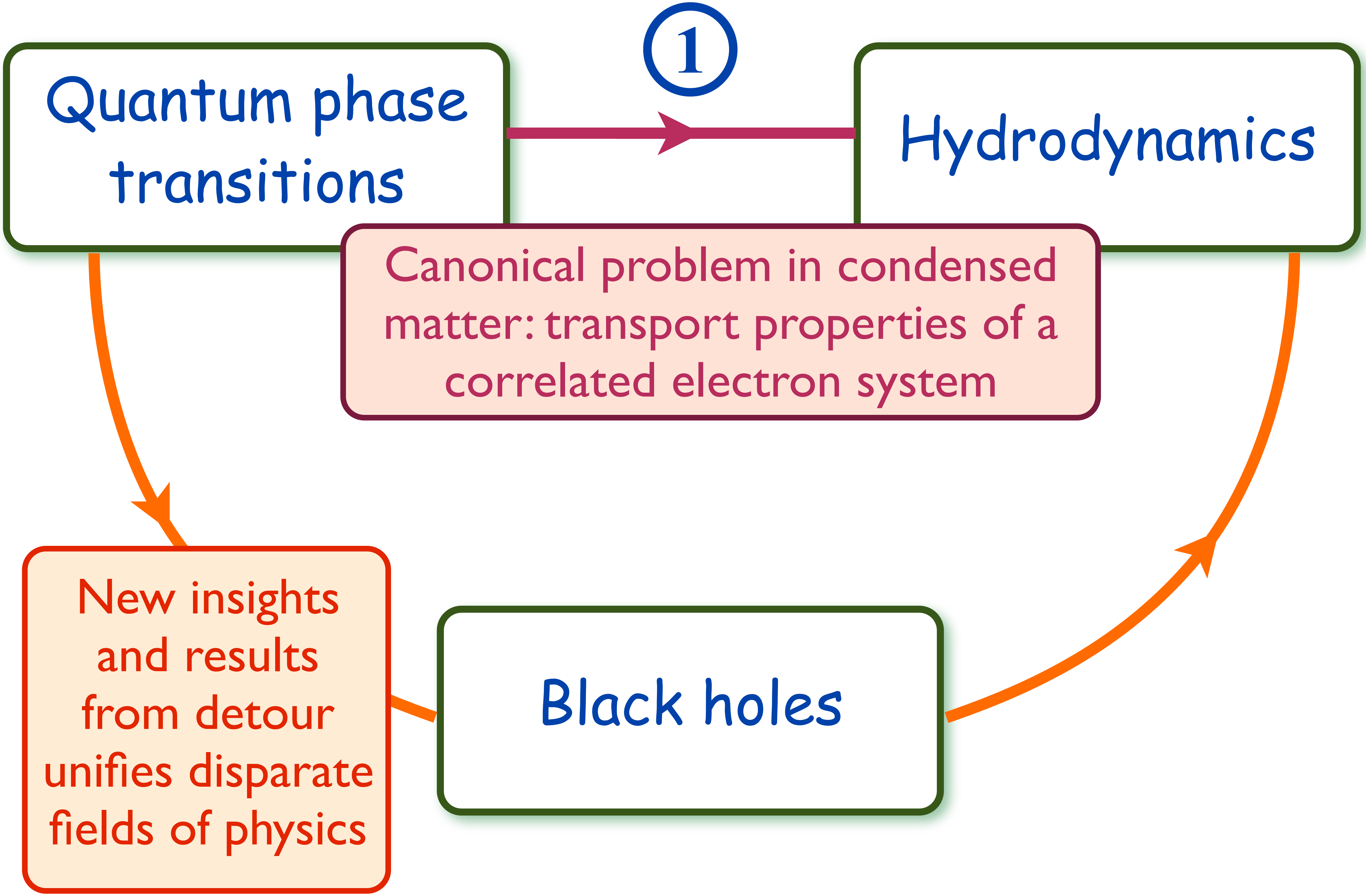


# Hydrodynamics of quantum critical systems

1. Use quantum field theory + quantum transport equations + classical hydrodynamics

   *Uses physical model but strong-coupling makes explicit solution difficult*

# Three foci of modern physics



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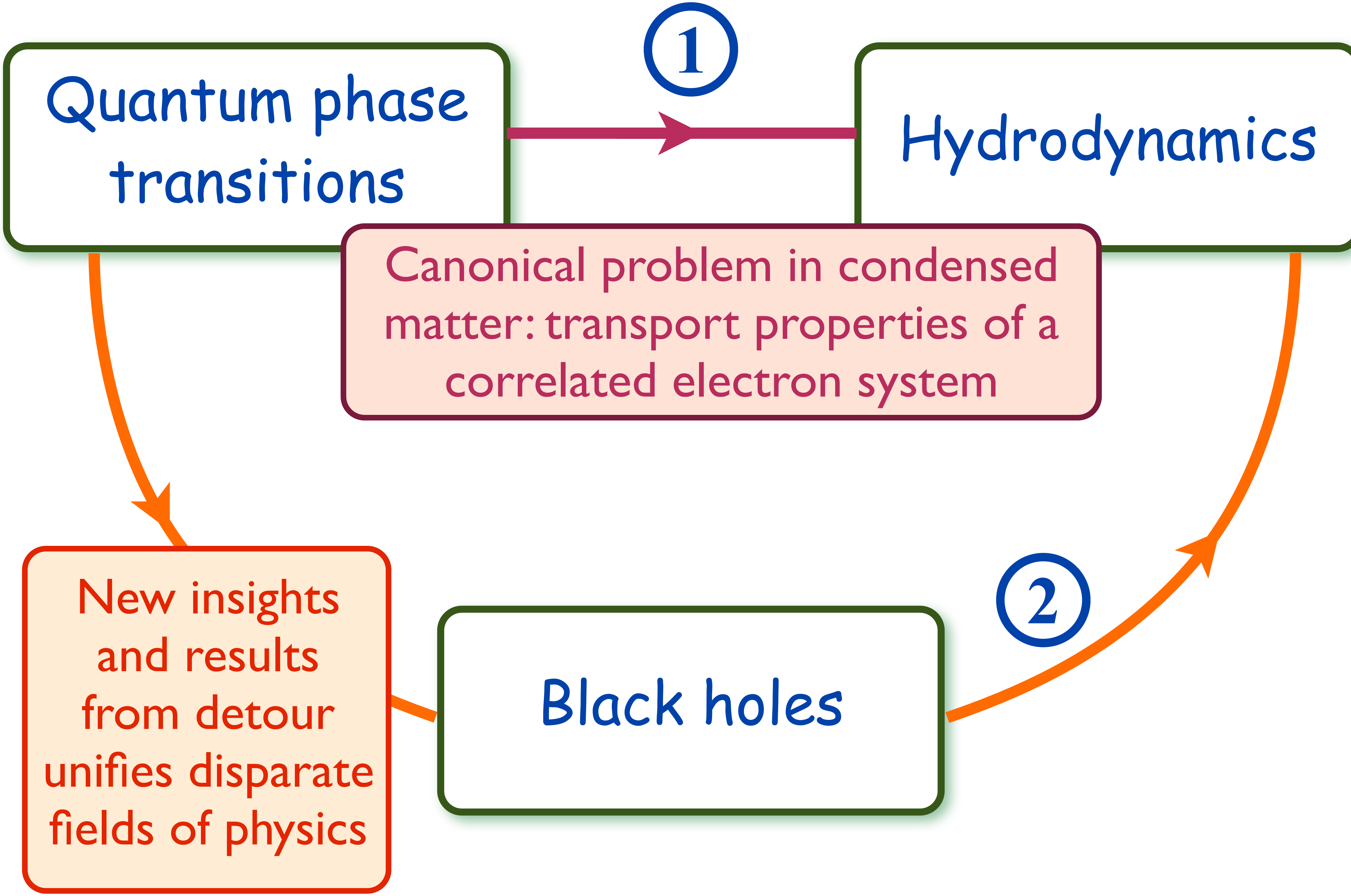

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Find perfect agreement between 1. and 2.
In some cases, results were obtained by 2. earlier !!

# Applications:

1. Magneto-thermo-electric transport in graphene and near the superconductor-insulator transition

   *Hydrodynamic cyclotron resonance*

   *Nernst effect*

2. Quark-gluon plasma

   *Low viscosity fluid*

3. Fermi gas at unitarity

   *Non-relativistic AdS/CFT*

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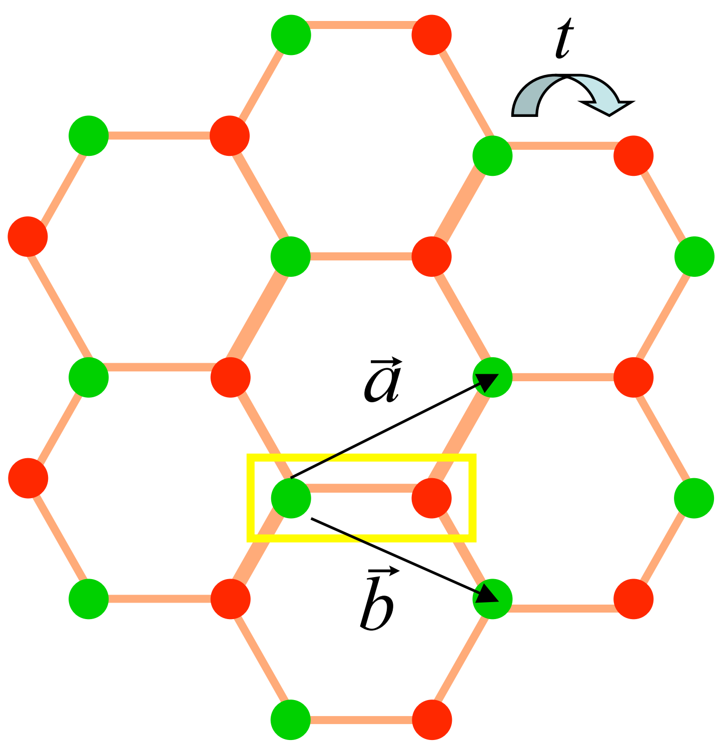
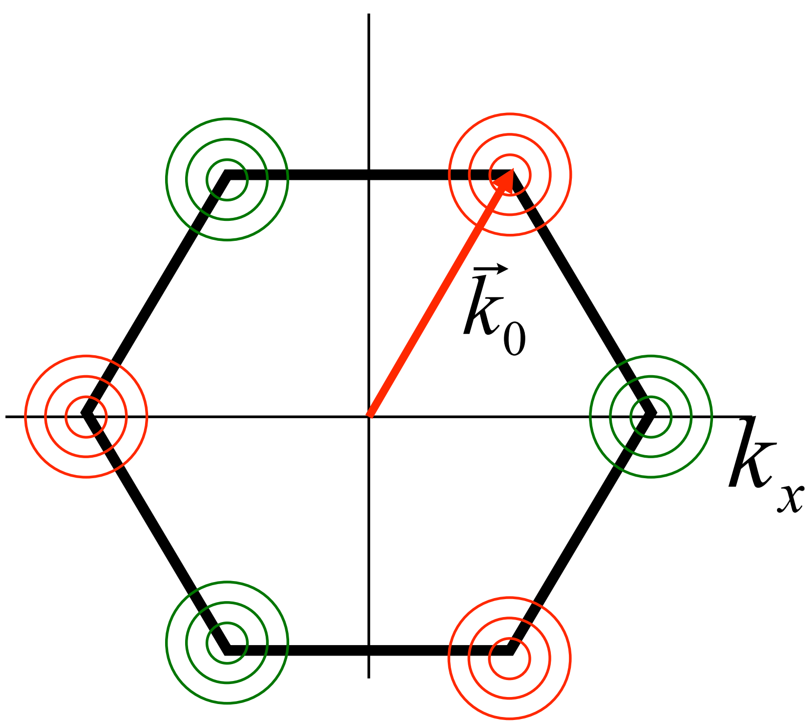
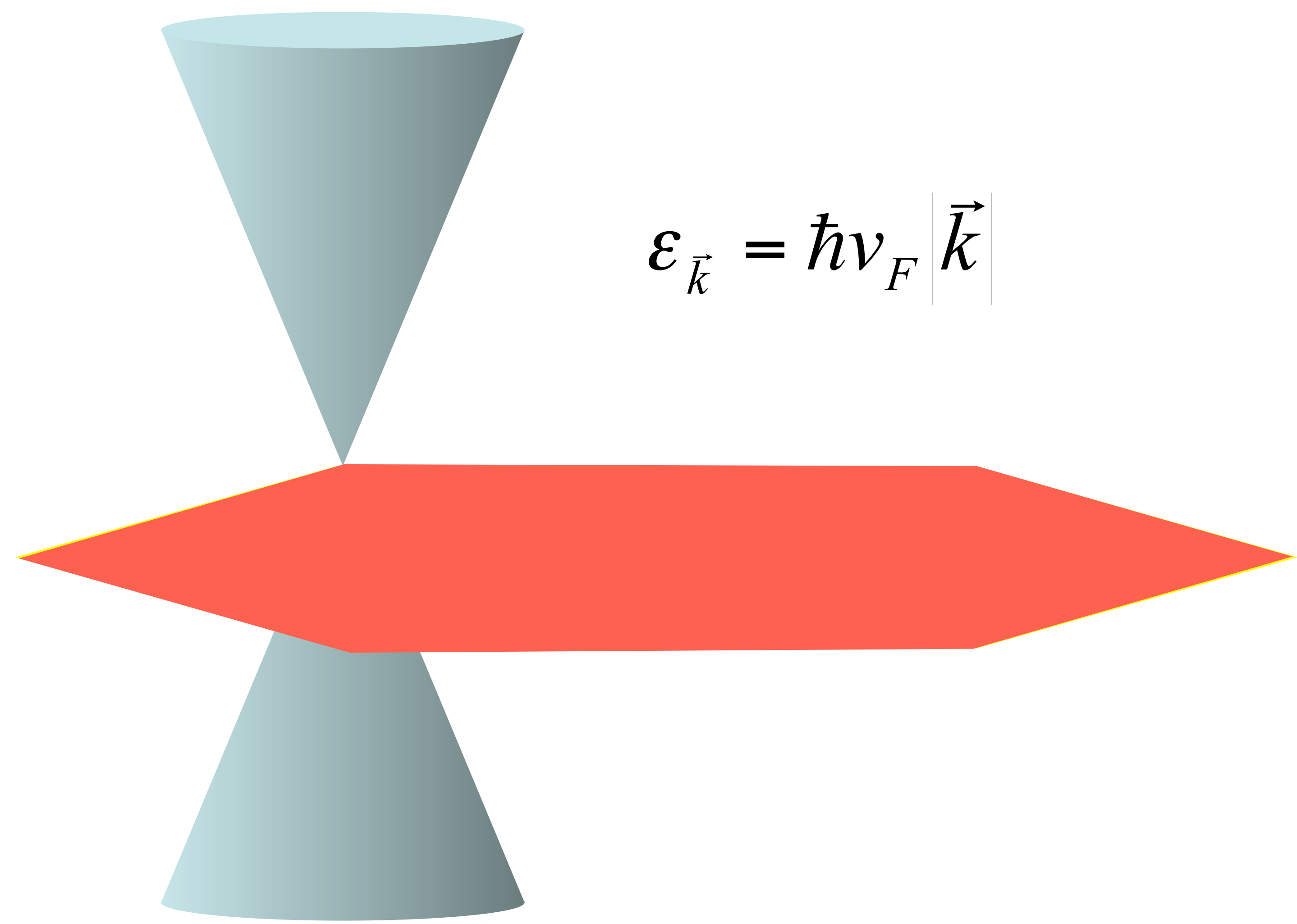
# Graphene

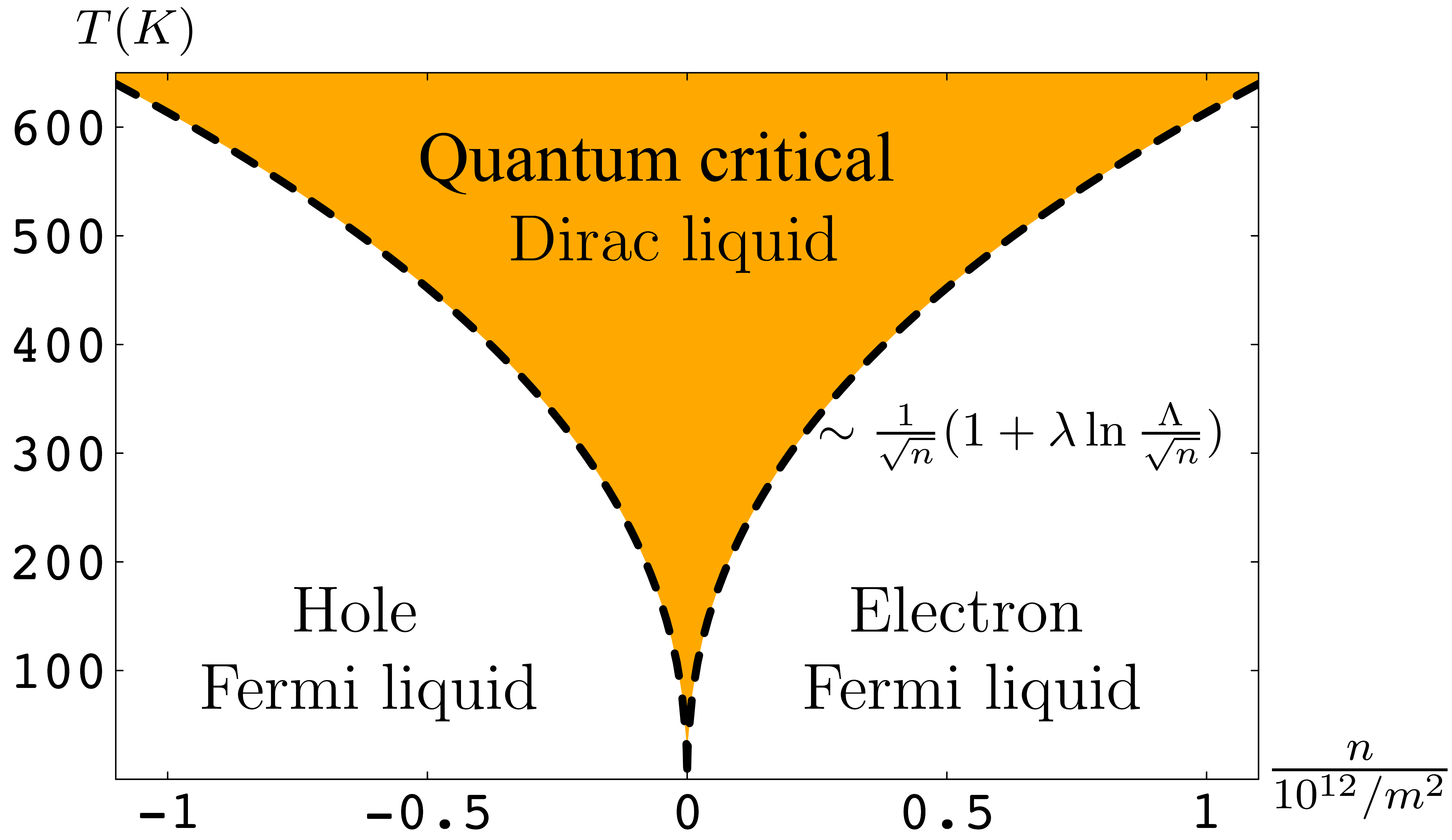


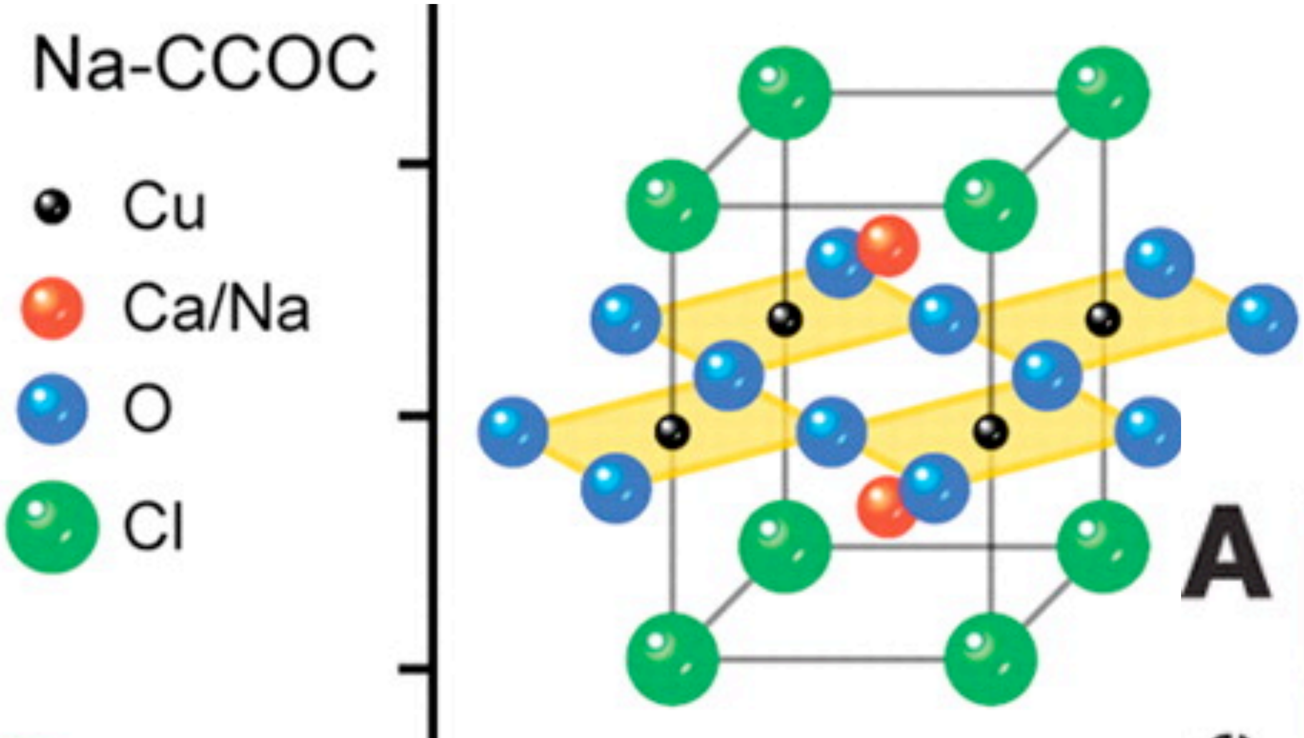
$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$

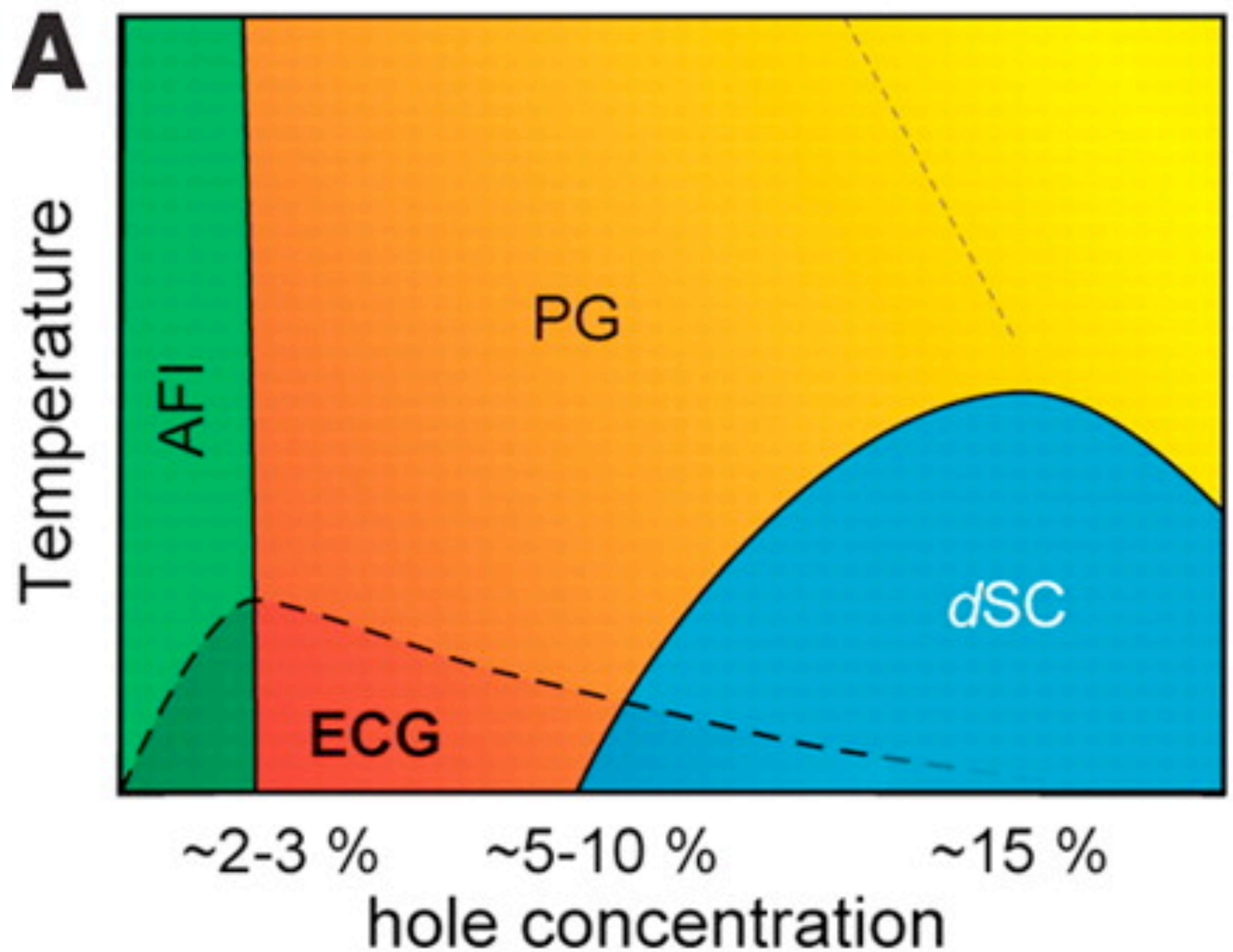
# Graphene



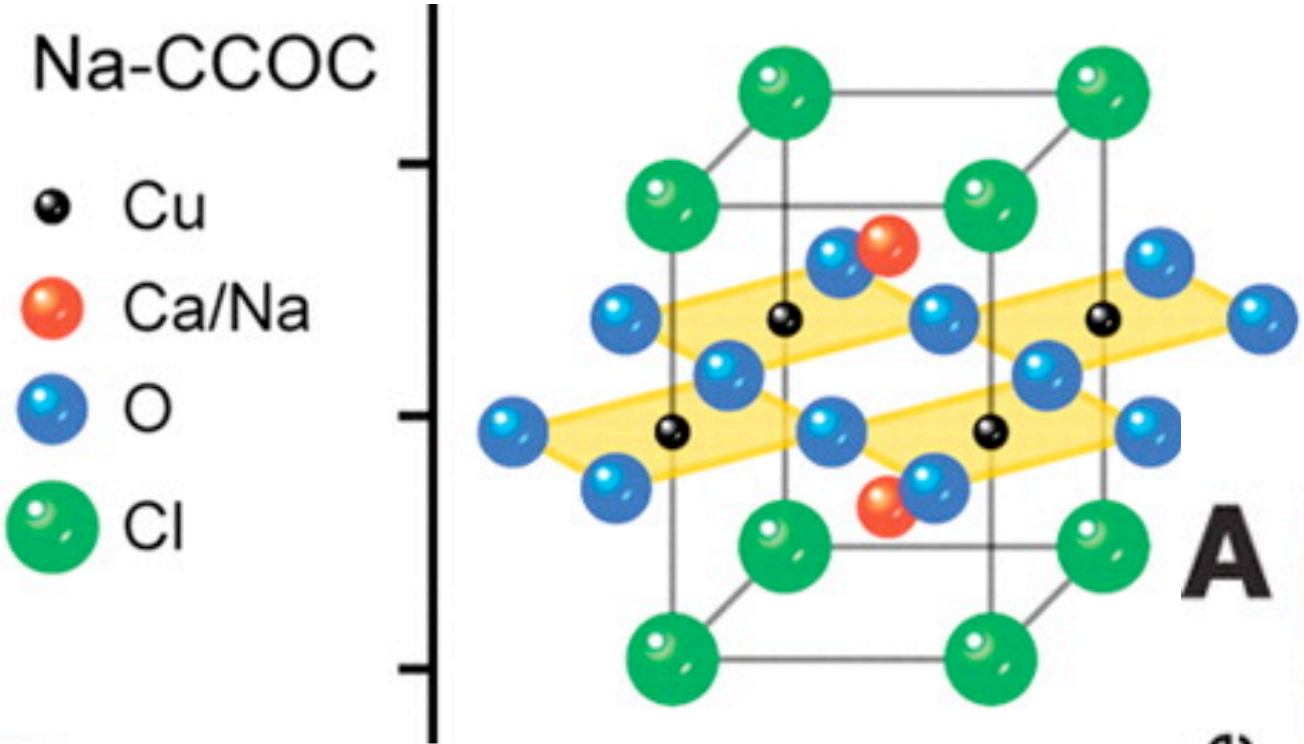
$T(K)$
600
500
400
300
200
100
Quantum critical
Dirac liquid
$\sim \frac{1}{\sqrt{n}}(1 + \lambda \ln \frac{\Lambda}{\sqrt{n}})$
Hole
Fermi liquid
Electron
Fermi liquid
-1
-0.5
0
0.5
1
$\frac{n}{10^{12}/m^2}$

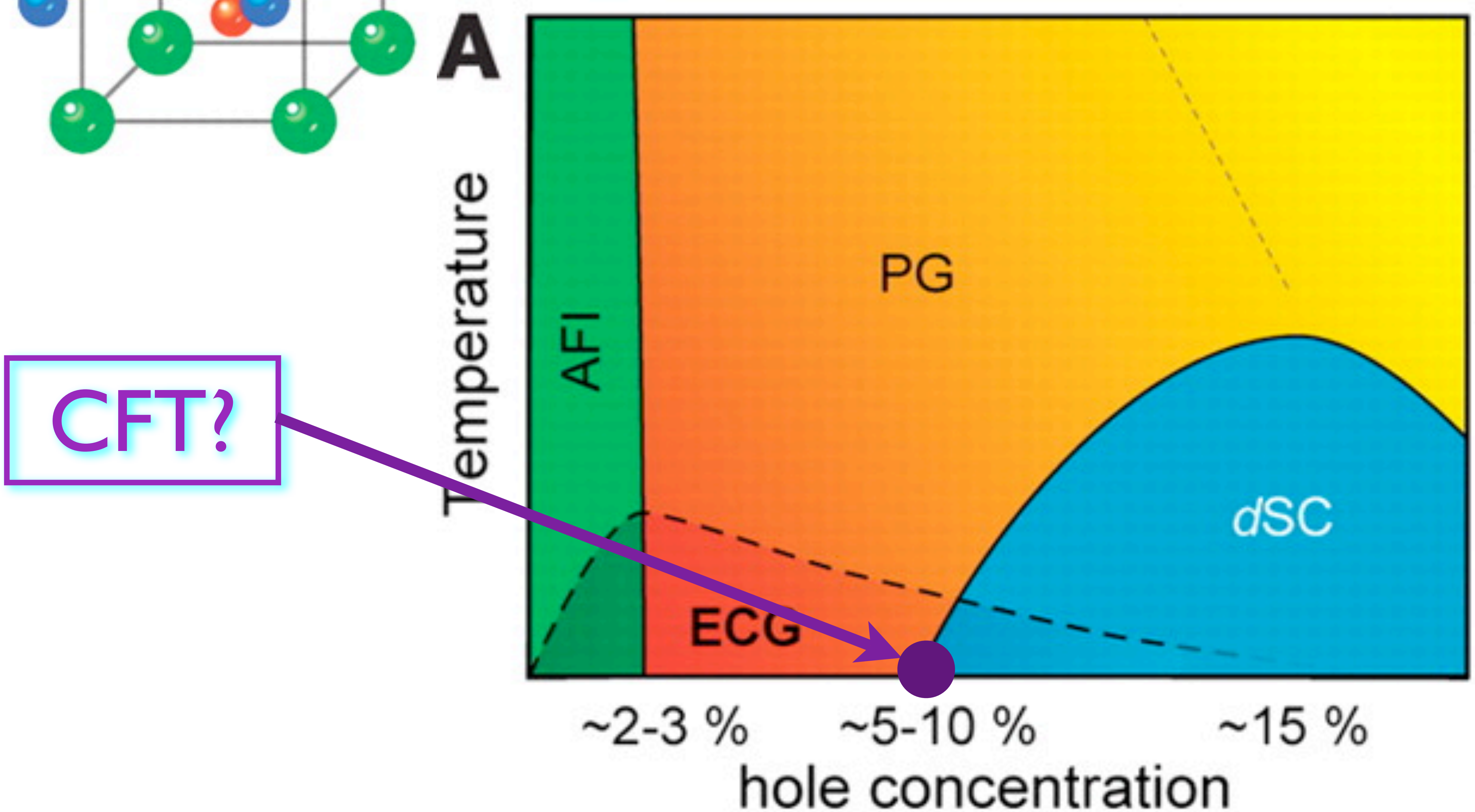
# The cuprate superconductors



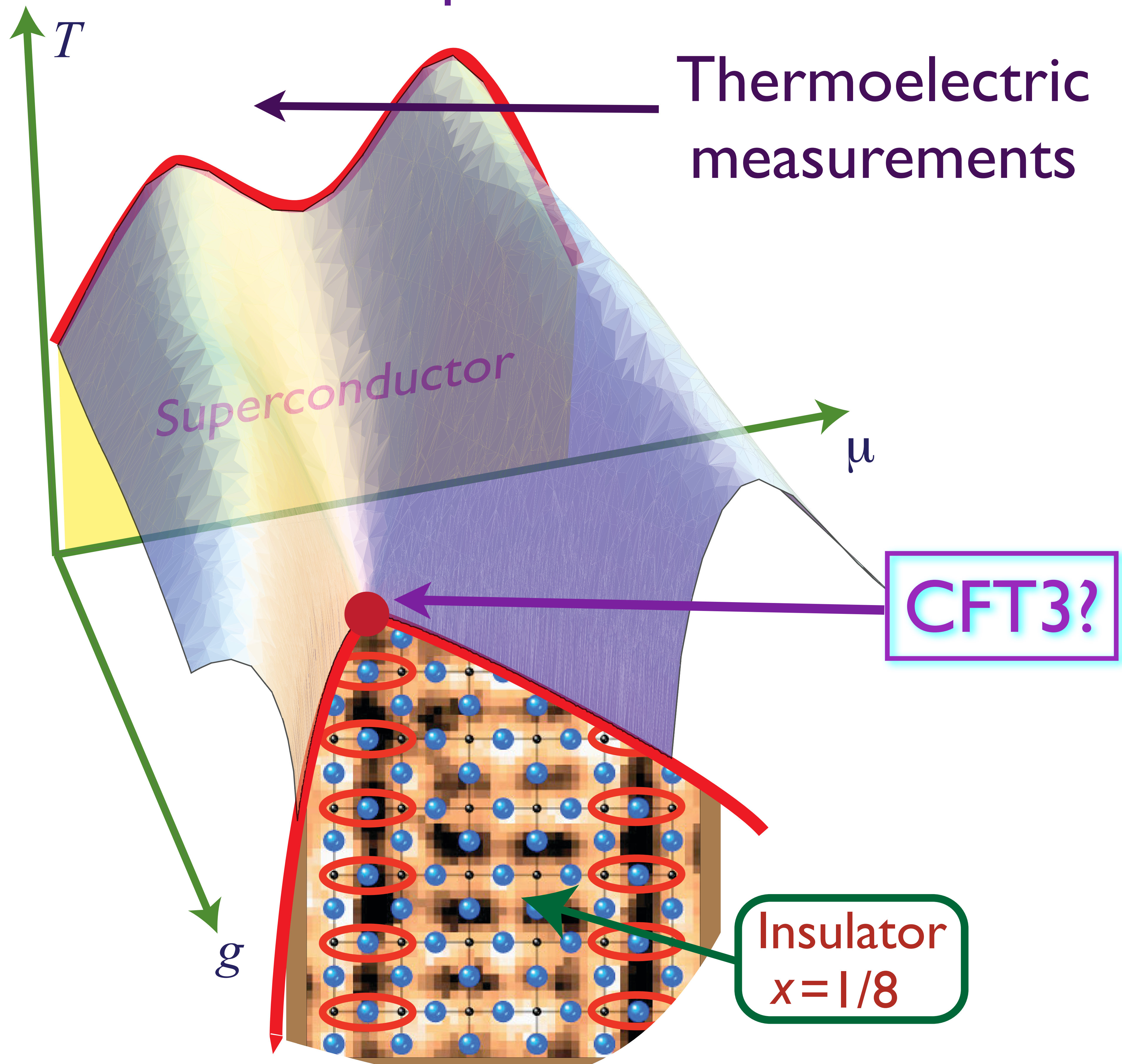



# *The cuprate superconductors*



# Cuprates



T
Thermoelectric measurements
Superconductor
μ
CFT3?
g
Insulator
x=1/8

Hydrodynamic cyclotron resonance at a frequency

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

and with width

$$\gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

where $B$ = magnetic field, $\rho$ = charge density away from density of CFT, $\varepsilon$ = energy density, $P$ = pressure, $v$ = velocity of "light" in CFT, and $\sigma_Q e^2/h$ is the universal conductivity of the CFT.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

"Wiedemann-Franz"-like relation for thermal conductivity, $\kappa$ at $B = 0$

$$\kappa = \sigma_Q \left( \frac{k_B^2 T}{e^{*2}} \right) \left( \frac{\varepsilon + P}{k_B T \rho} \right)^2 .$$

At $B \neq 0$ and $\rho = 0$ we have a "Wiedemann-Franz" relation for "vortices"

$$\kappa = \frac{1}{\sigma_Q} k_B^2 T \left( \frac{v(\varepsilon + P)}{k_B T B} \right)^2 .$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

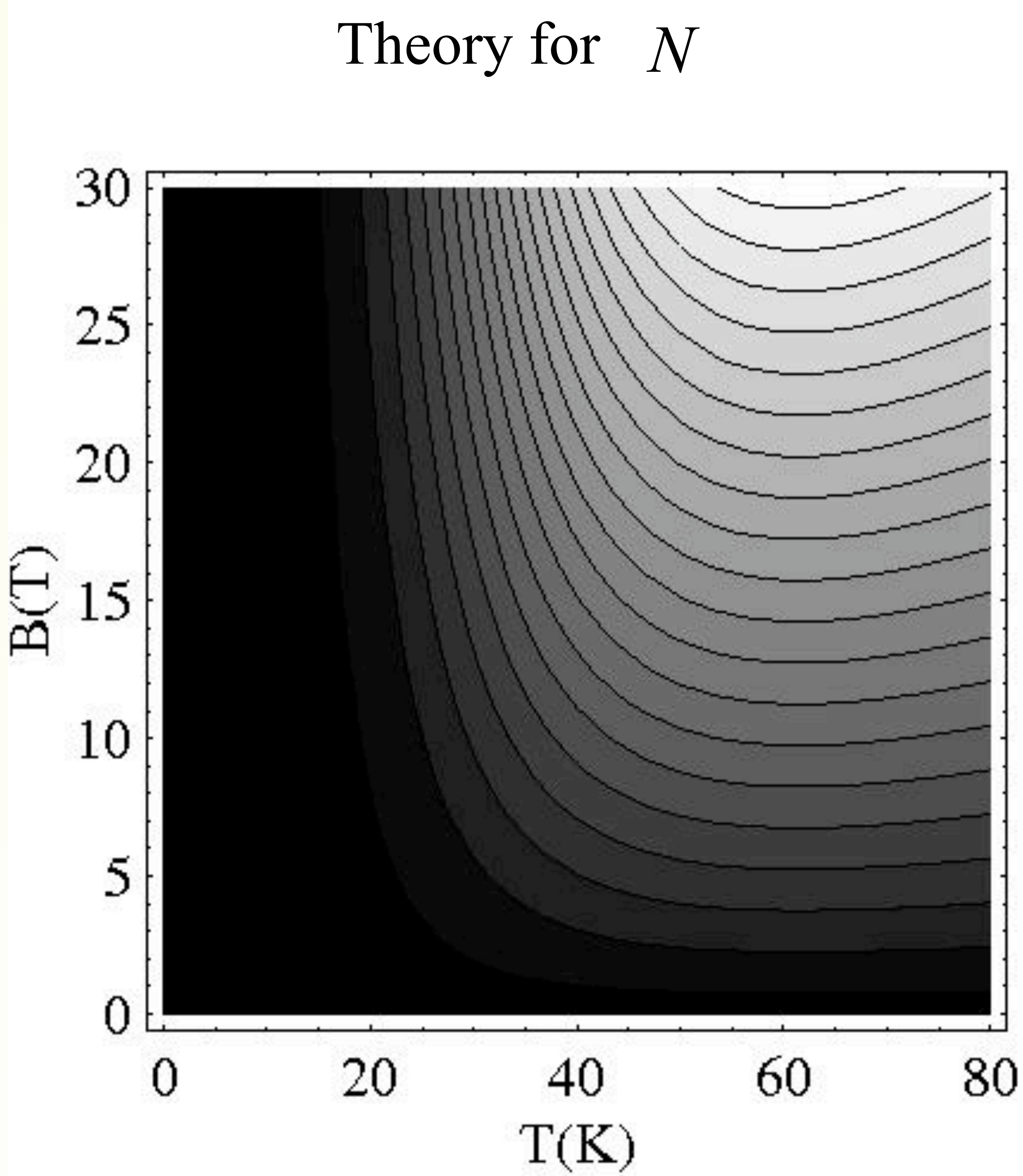
Nernst signal (transverse thermoelectric response)

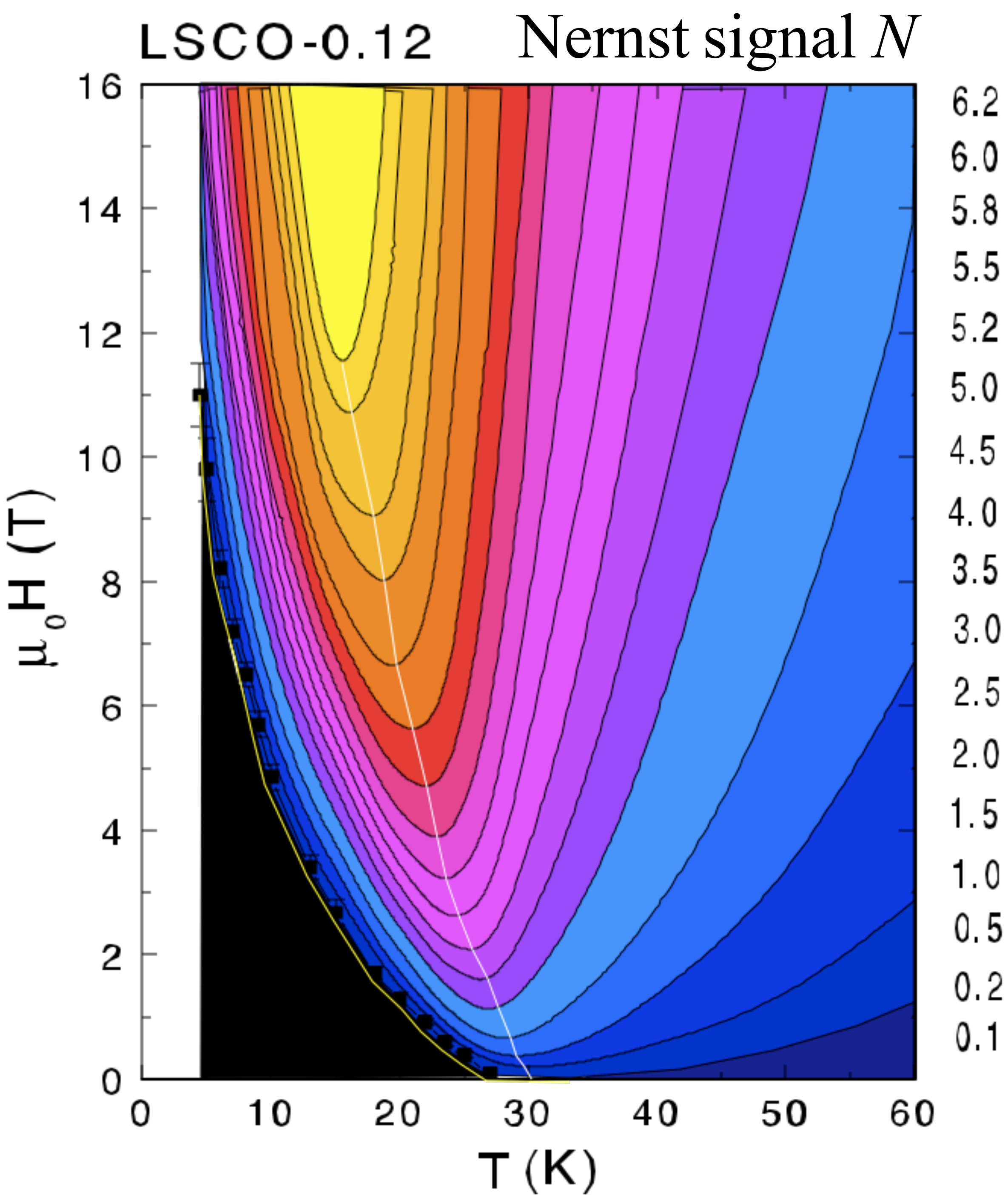
$$e_N = \left(\frac{k_B}{e^*}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\rm imp}}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

where $\tau_{\rm imp}$ is the momentum relaxation time due to impurities or umklapp scattering.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

# LSCO Experiments




*Y. Wang, L. Li, and N. P. Ong, Phys. Rev. B 73, 024510 (2006).*

$B$ and $T$ dependencies are in semi-quantitative agreement with observations on cuprates, with reasonable values for only 2 adjustable parameters, $\tau_{\mathrm{imp}}$ and $v$.

Specific quantitative predictions for THz experiments on graphene at room temperature.

# Applications:

1. Magneto-thermo-electric transport in graphene and near the superconductor-insulator transition

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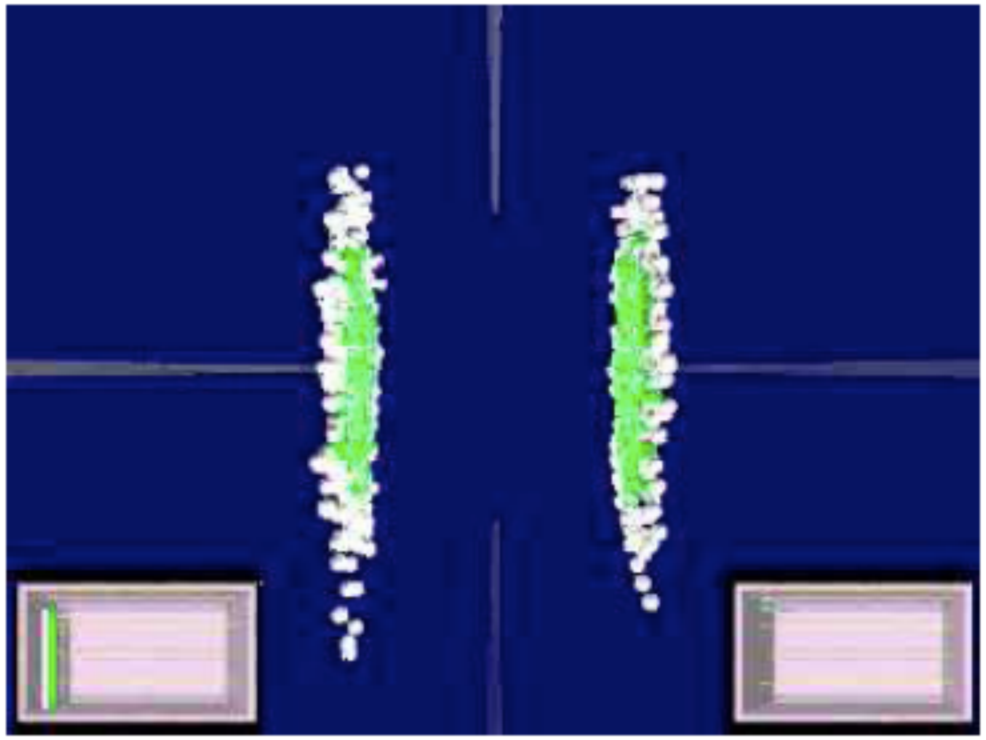
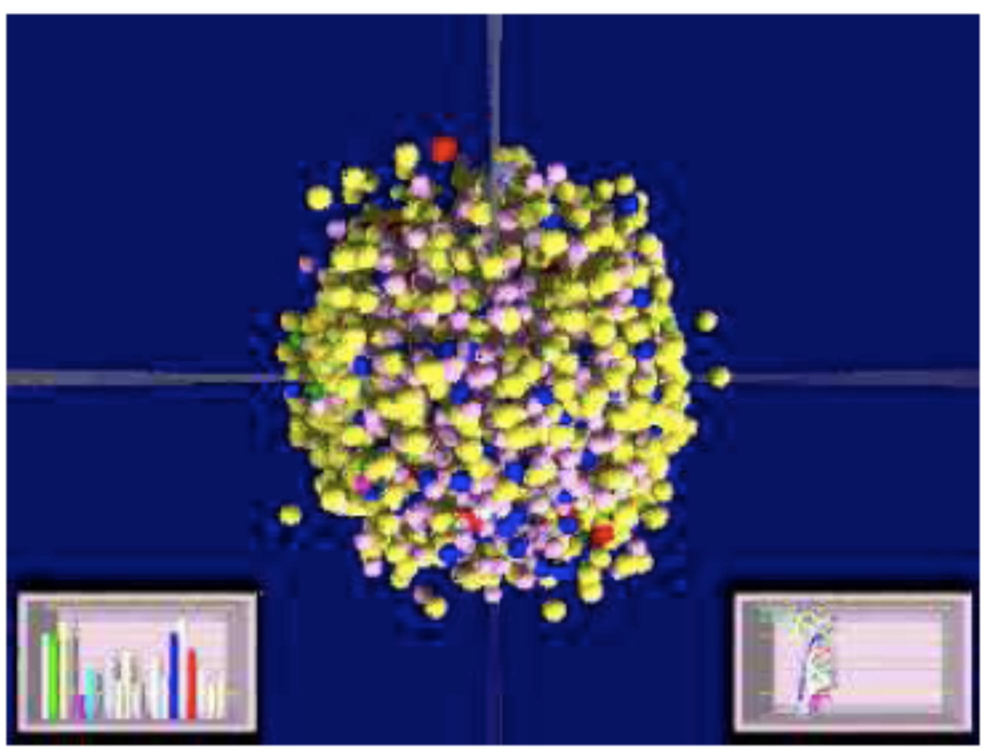
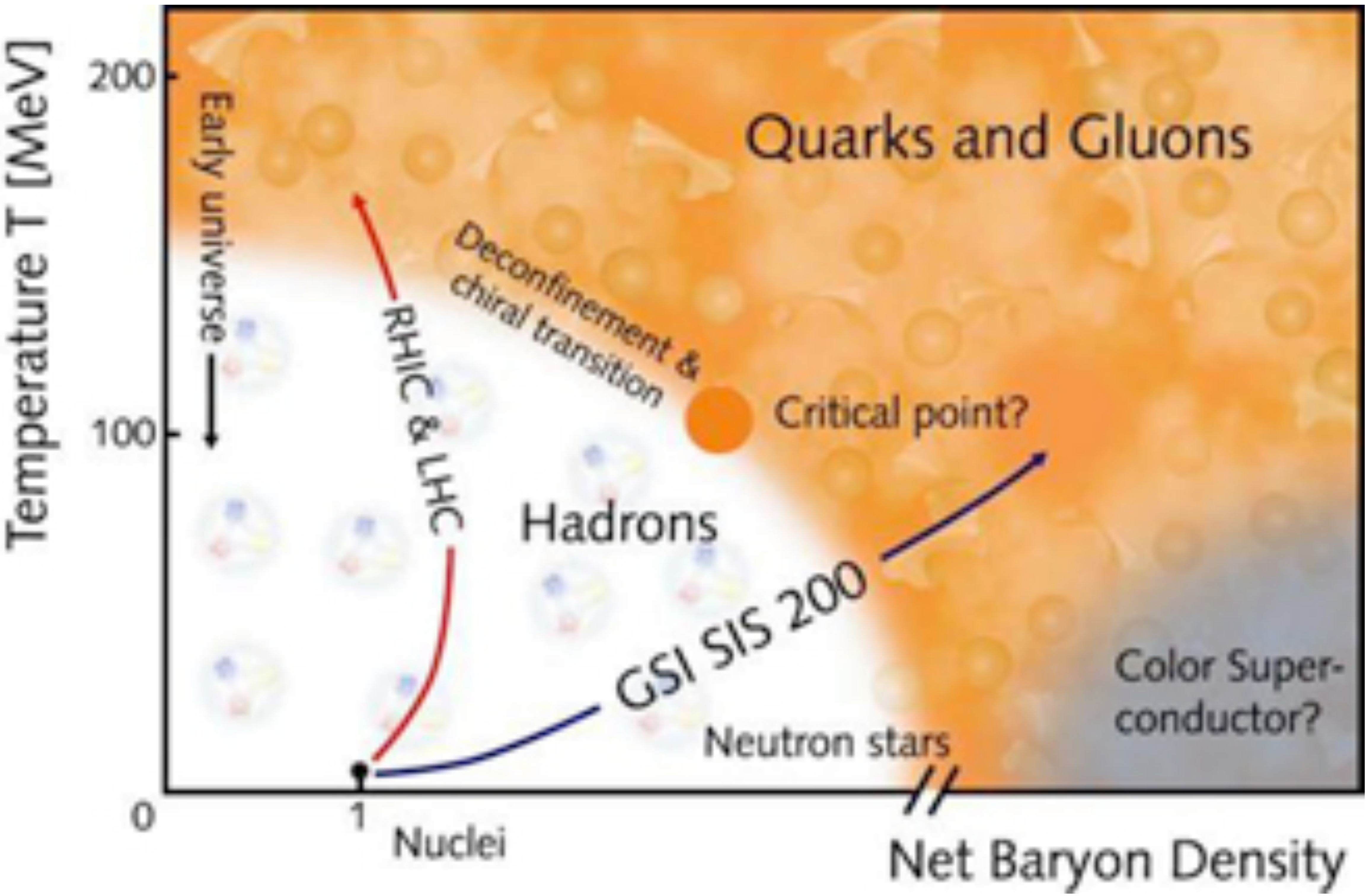
# Au+Au collisions at RHIC

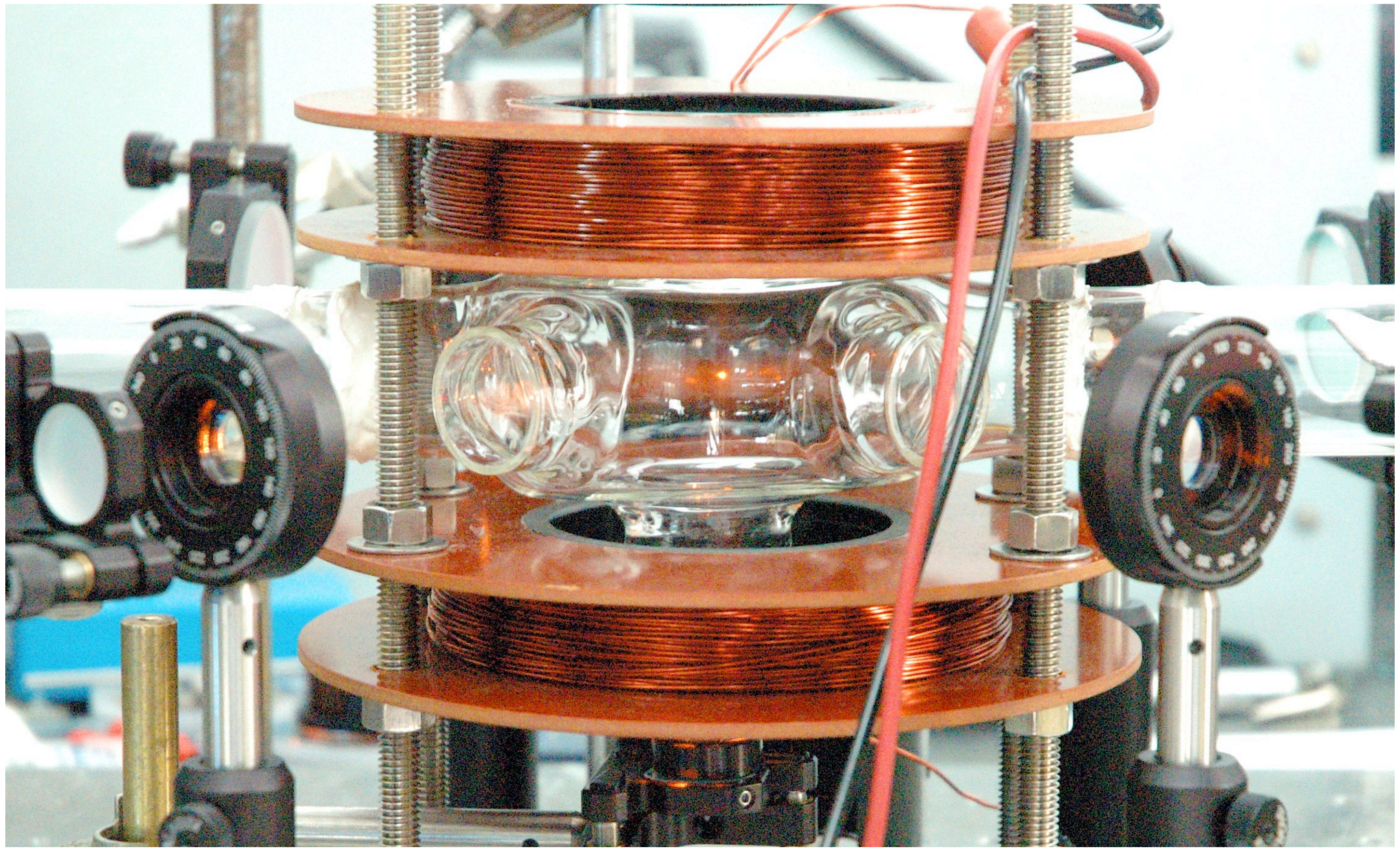




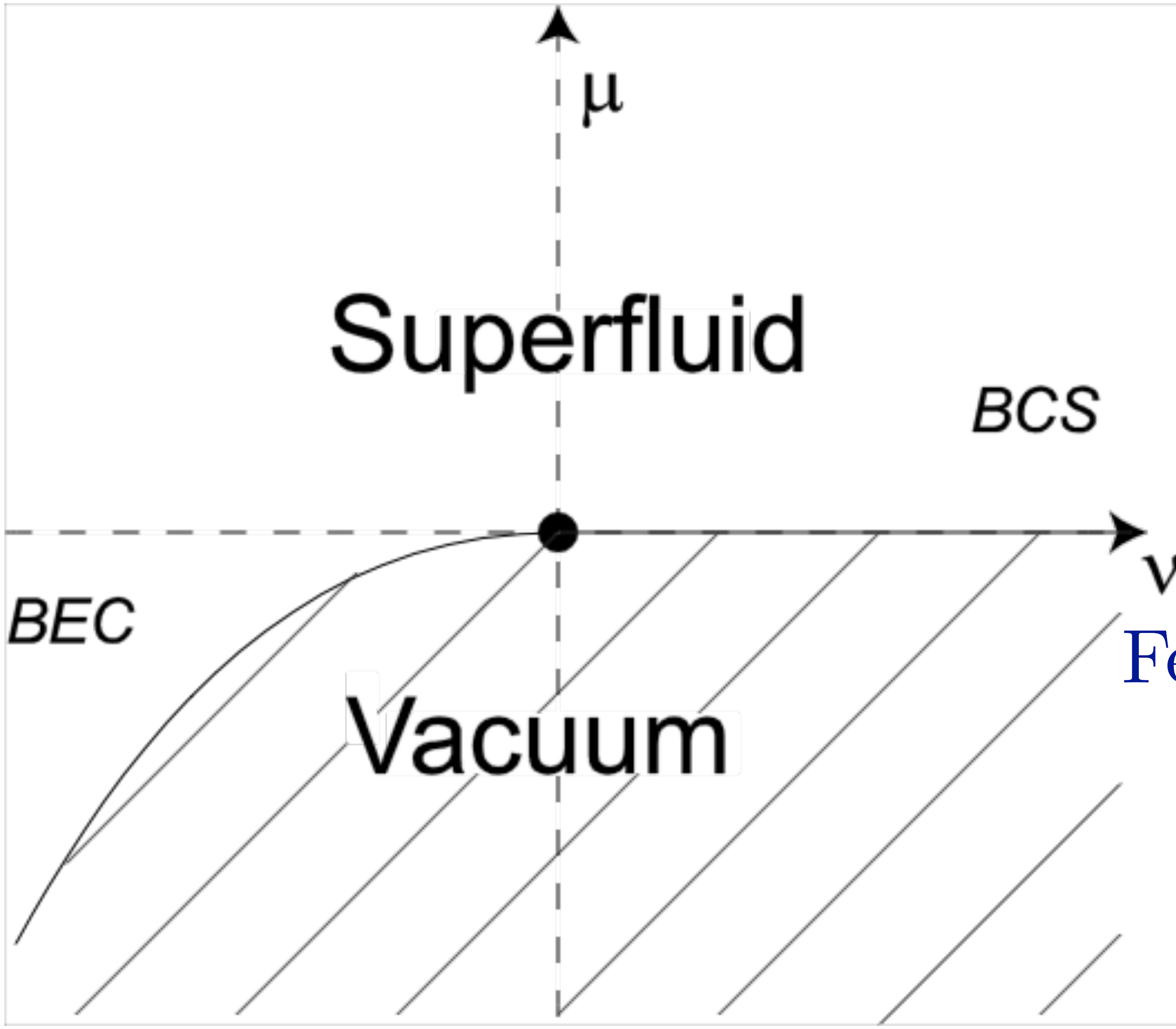
Quark-gluon plasma can be described as "quantum critical QCD"

# Phases of nuclear matter

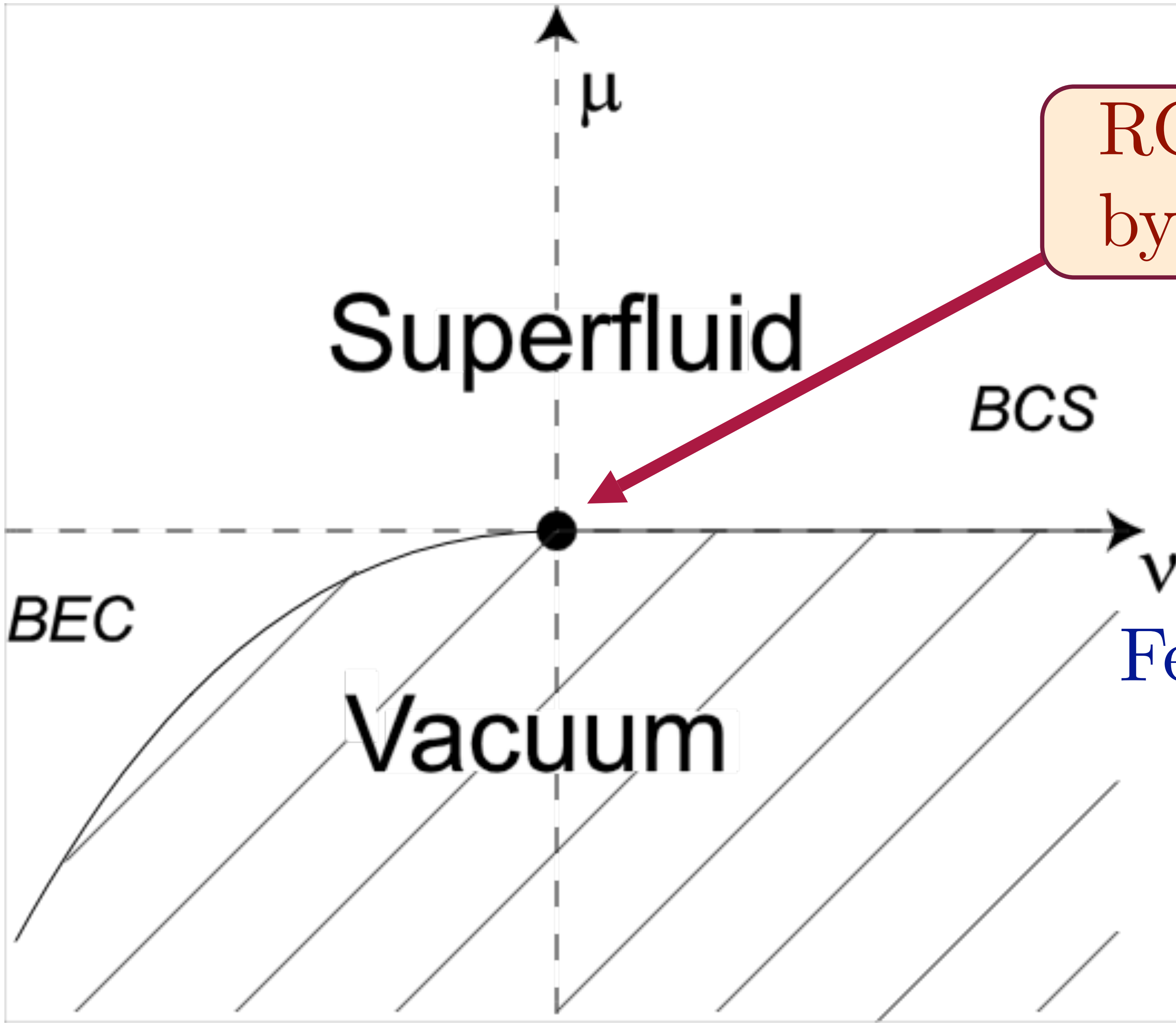




S=1/2 Fermi gas at a Feshbach resonance

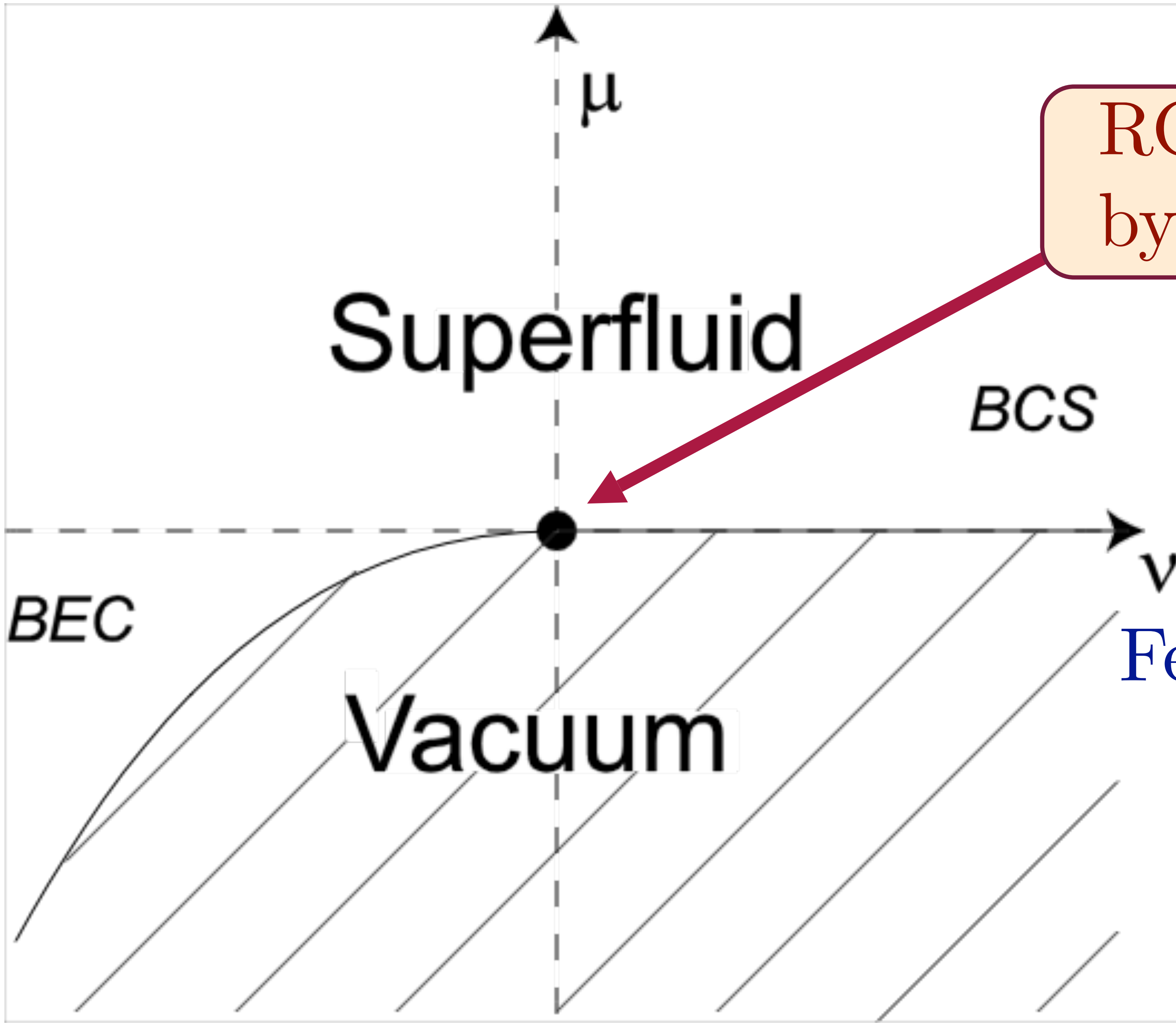



$= $ detuning from Feshbach resonance




RG fixed point described by a "non-relativistic" CFT

$\nu$ = detuning from Feshbach resonance

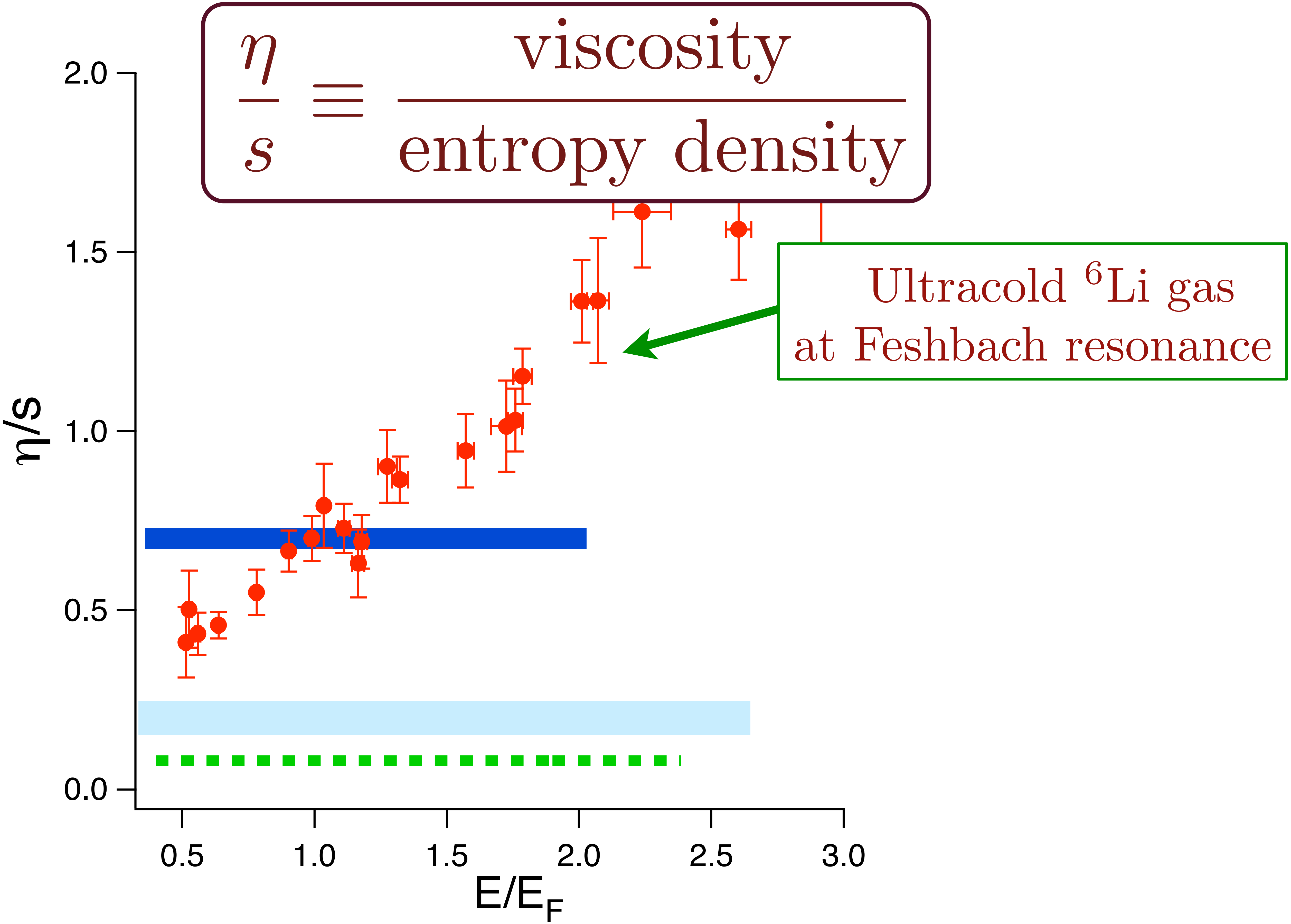



RG fixed point described by a "non-relativistic" CFT

$\nu$ = detuning from Feshbach resonance

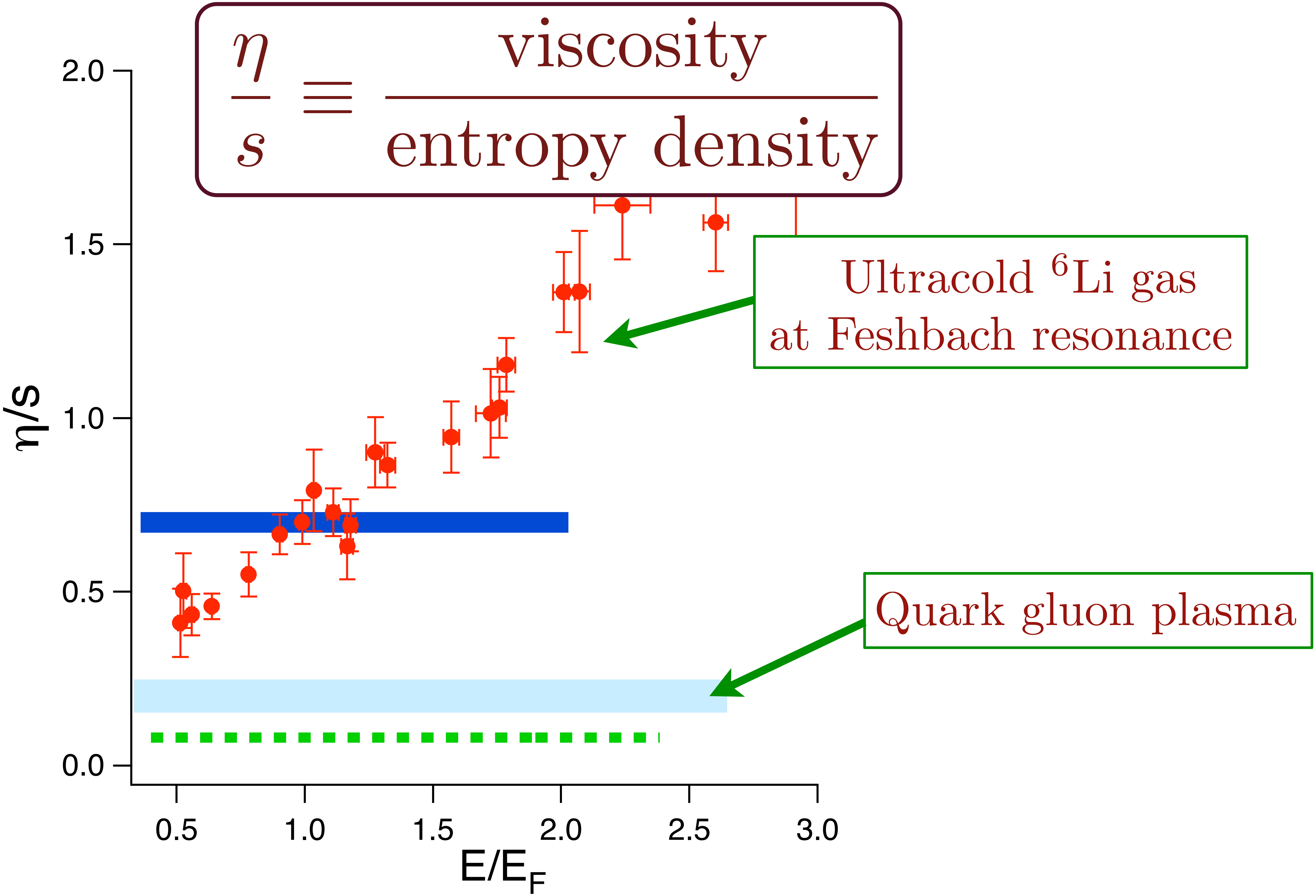
CFT is dual to quantum gravity models on AdS space. Explicit solutions of such gravity models with supersymmetry have been obtained

P. Nikolic and S. Sachdev, *Phys. Rev. A* **75**, 033608 (2007); D. T. Son, arXiv:0804.3972; K. Balasubramaniam and J. McGreevy, arXiv:0804.4053; W. D. Goldberger, arXiv:0806.2867; J. L. F. Barbón and C. A. Fuertes, arXiv:0806.3244; J. Maldacena, D. Martelli, and Y. Tachikawa, arXiv:0807.1100; A. Adams, K. Balasubramanian, and J. McGreevy, arXiv:0807.1111.




T. Schafer, *Phys. Rev. A* **76**, 063618 (2007).

A. Turlapov, J. Kinast, B. Clancy, Le Luo, J. Joseph, J. E. Thomas, *J. Low Temp. Physics* **150**, 567 (2008)

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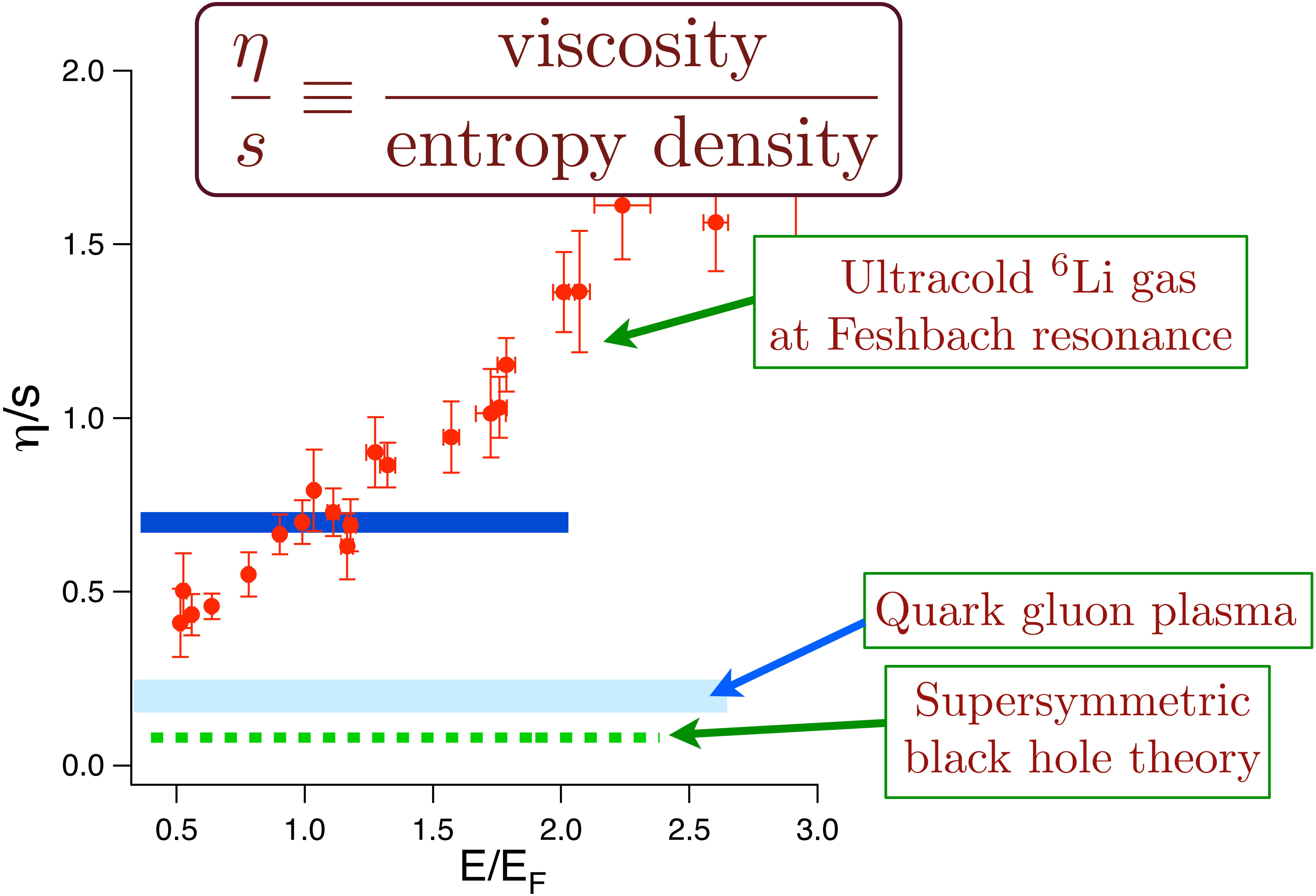


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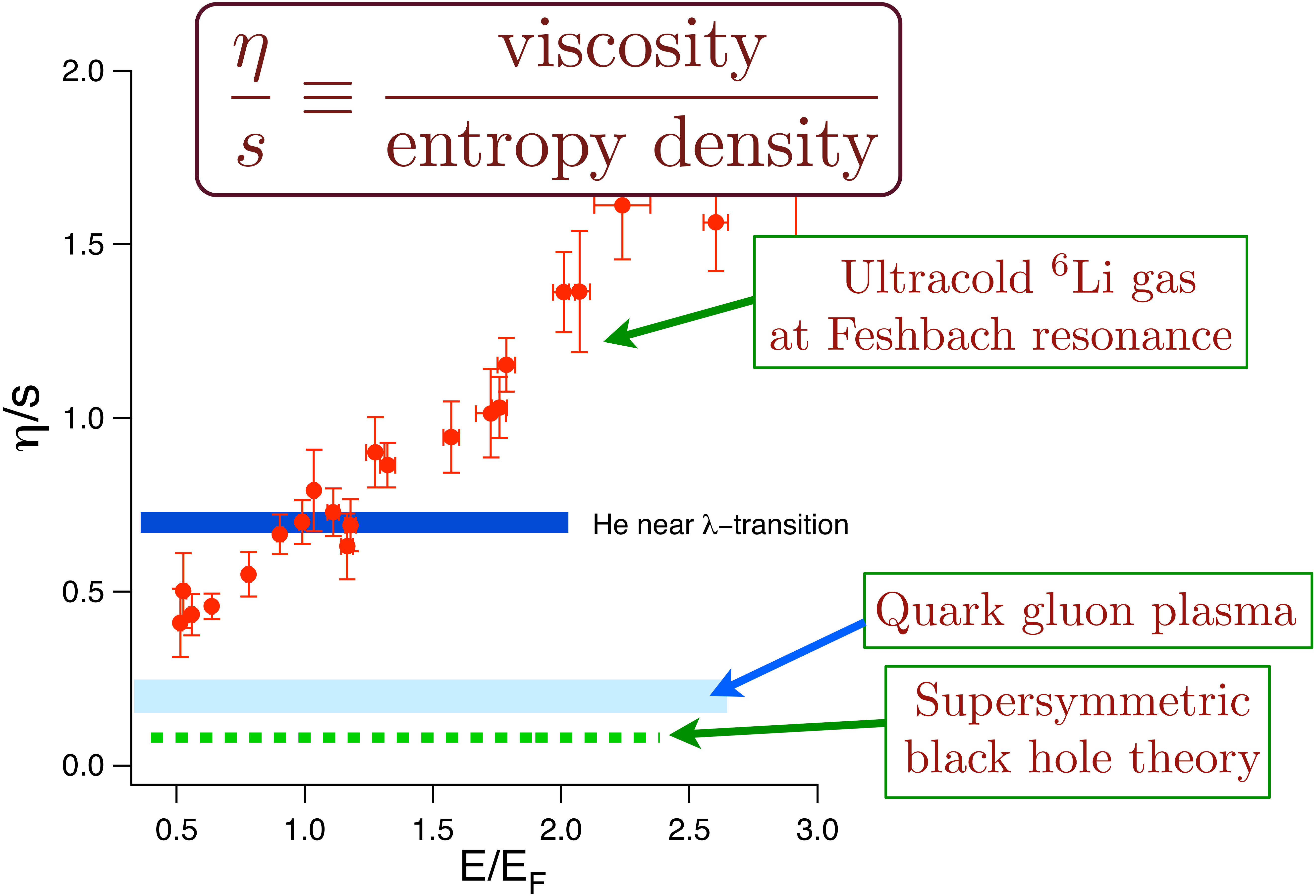


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THEORETICAL PHYSICS

# A black hole full of answers

Jan Zaanen

**A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.**

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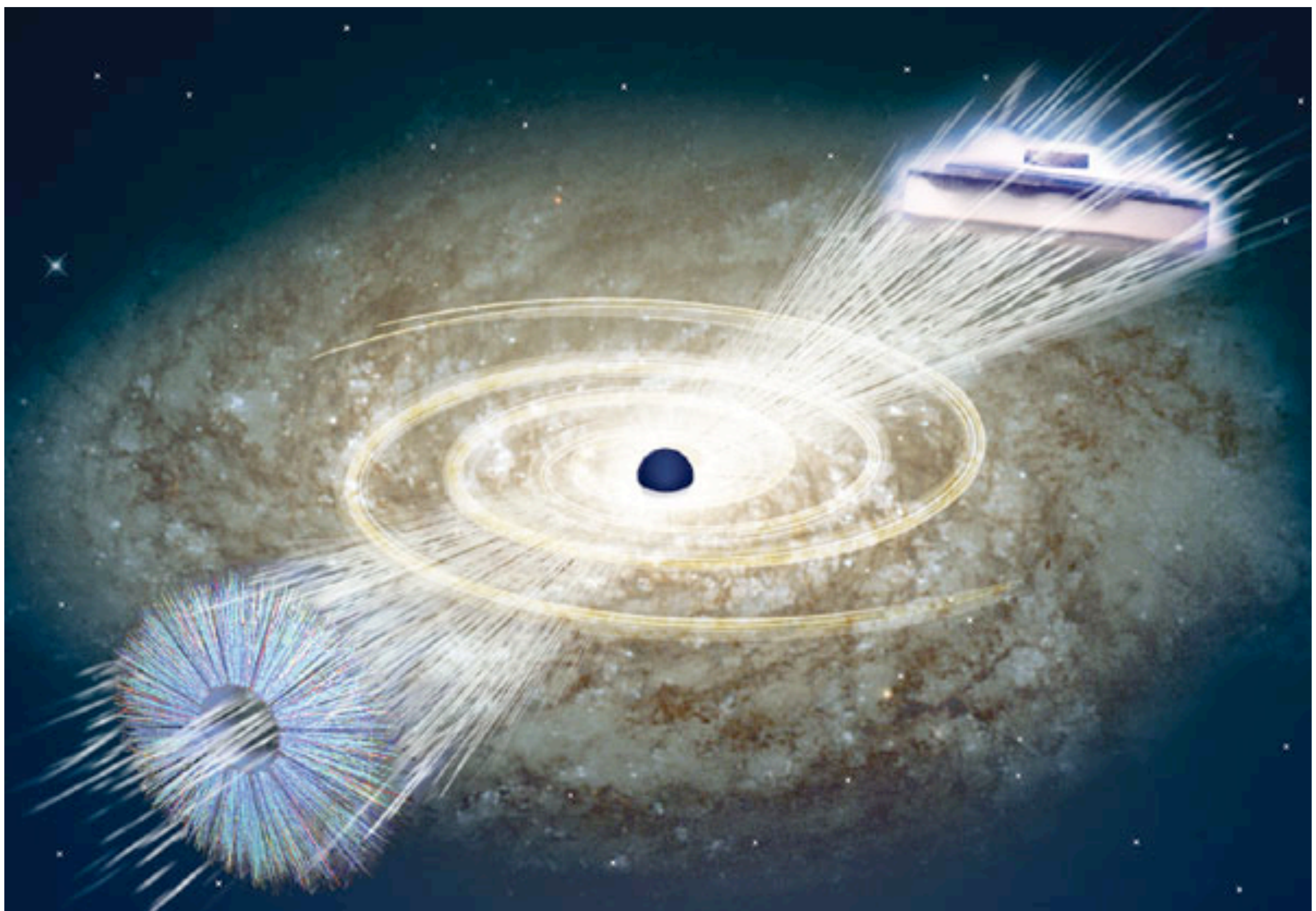

# Conclusions

- Theory for transport near quantum phase transitions in superfluids and antiferromagnets
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of  Nernst effect near the superfluid-insulator transition, and connection to cuprates.
- Quantum-critical magnetotransport in graphene.