Quantum entanglement and the phases of matter

Colloquium Ehrenfestii
Leiden University
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Liza Huijse


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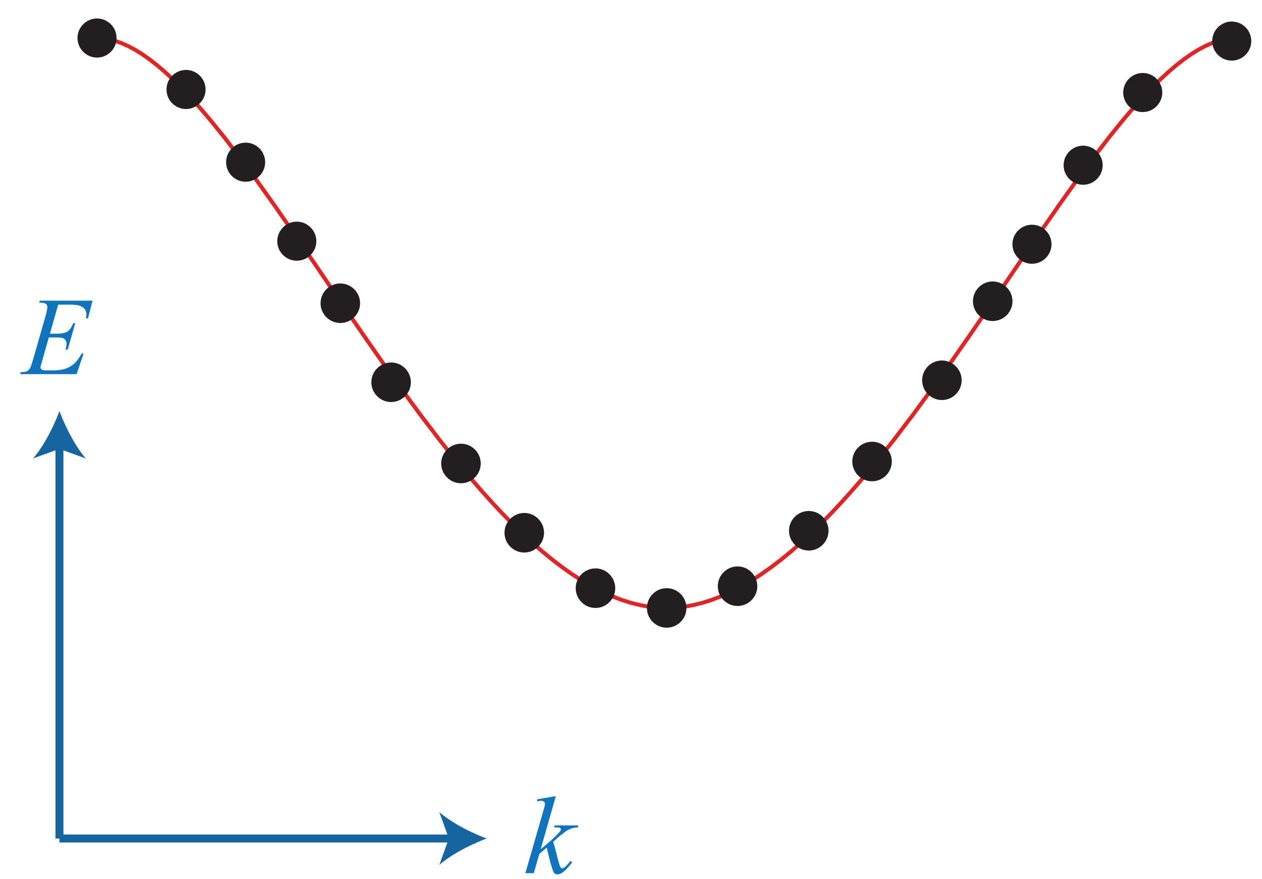
Sommerfeld-Bloch theory of metals, insulators, and superconductors:

many-electron quantum states are adiabatically connected to independent electron states

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

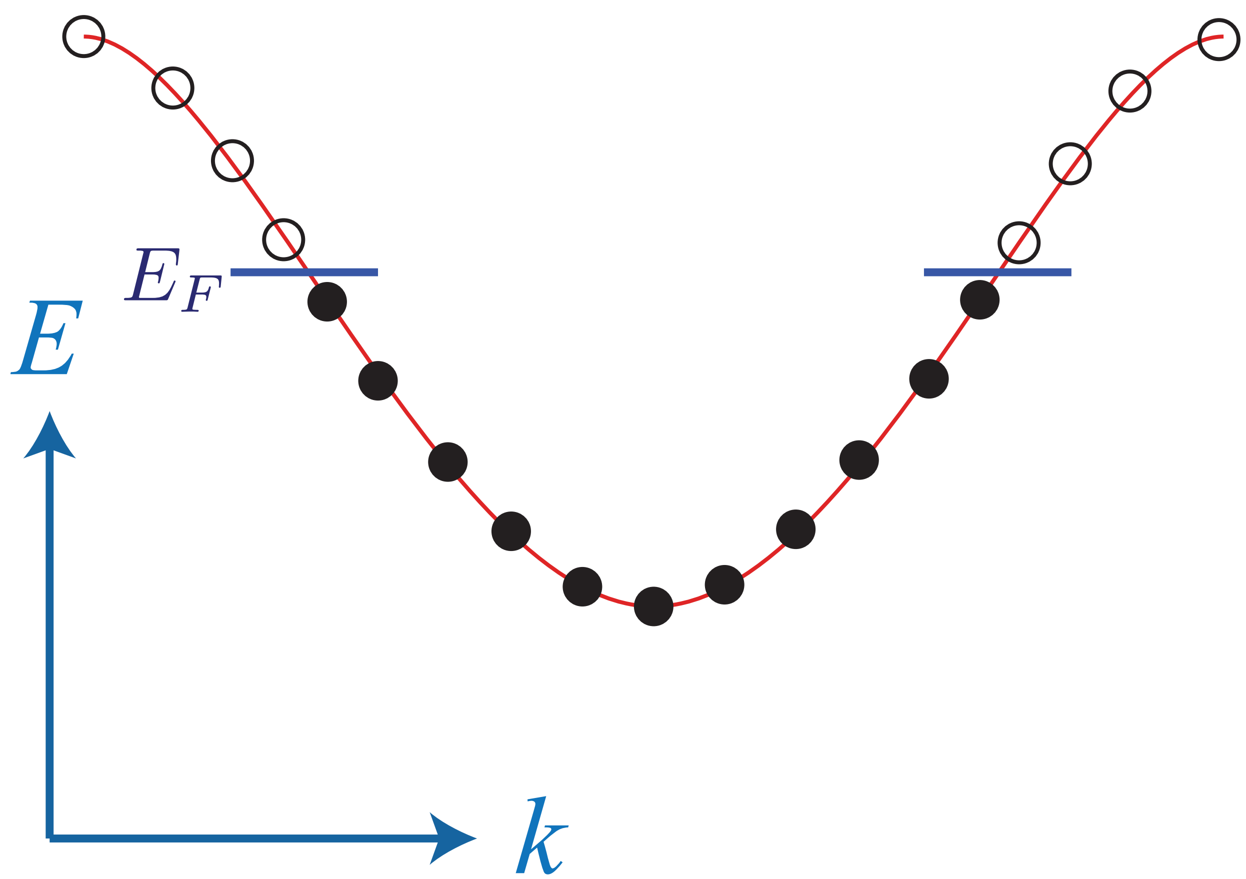
Band insulators



An even number of electrons per unit cell

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

Metals



An odd number of electrons per unit cell

Sommerfeld-Bloch theory of
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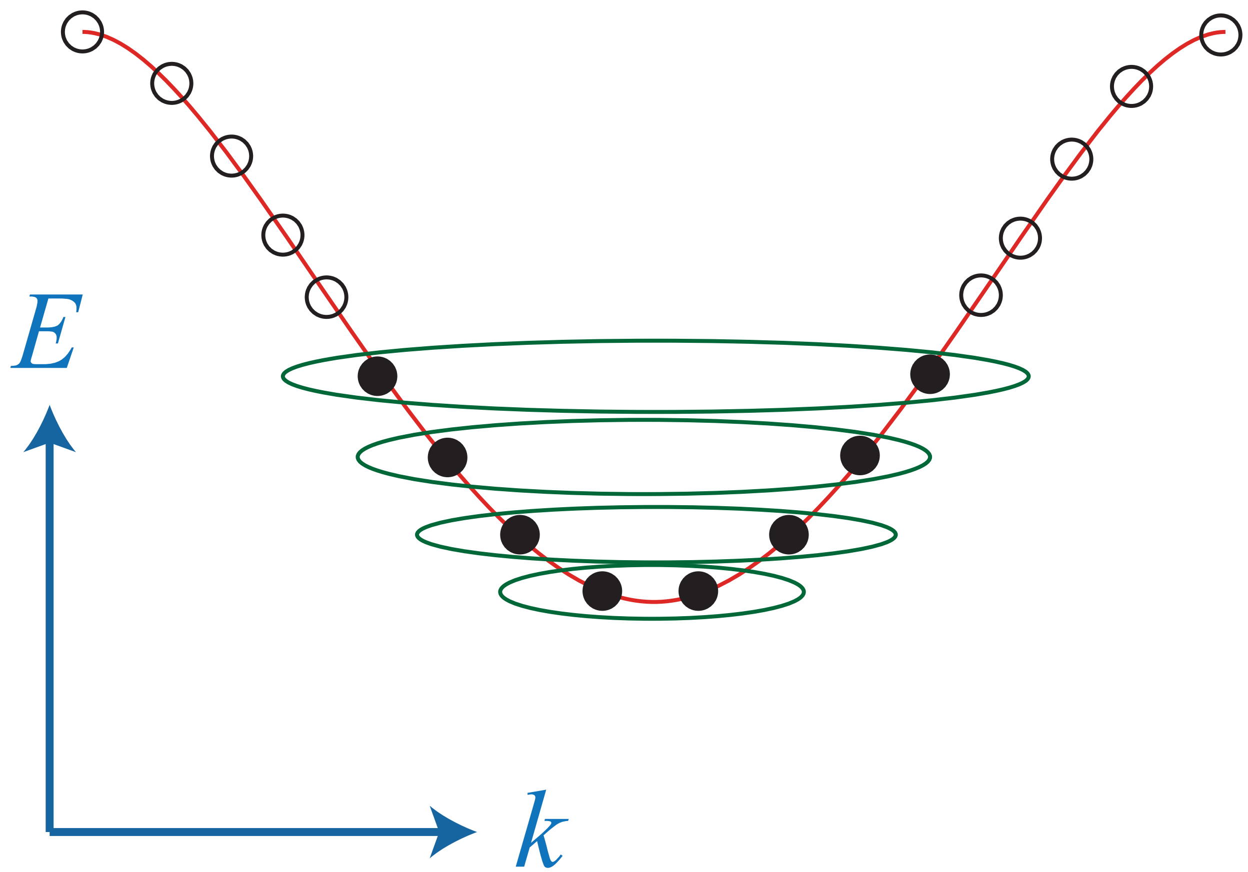
Superconductors



Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

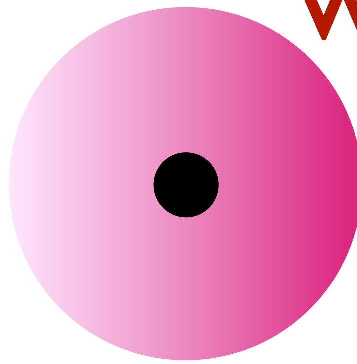
many-particle, long-range
quantum entanglement

Quantum superposition and entanglement

Quantum Entanglement: quantum superposition with more than one particle

Quantum Entanglement: quantum superposition with more than one particle

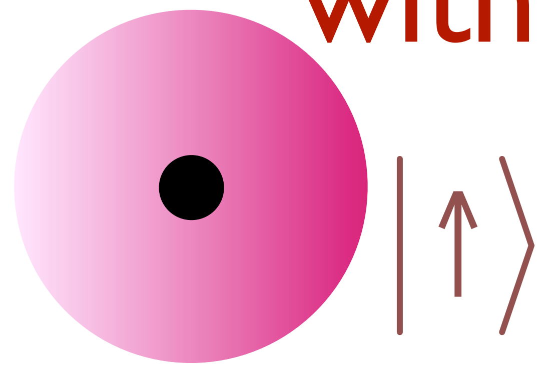
Hydrogen atom:



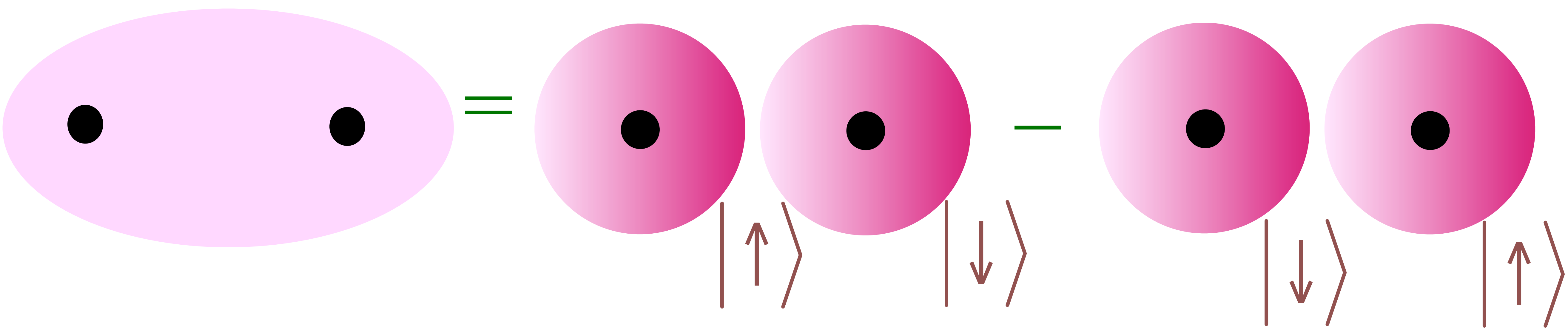
$$|\uparrow\rangle$$

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:



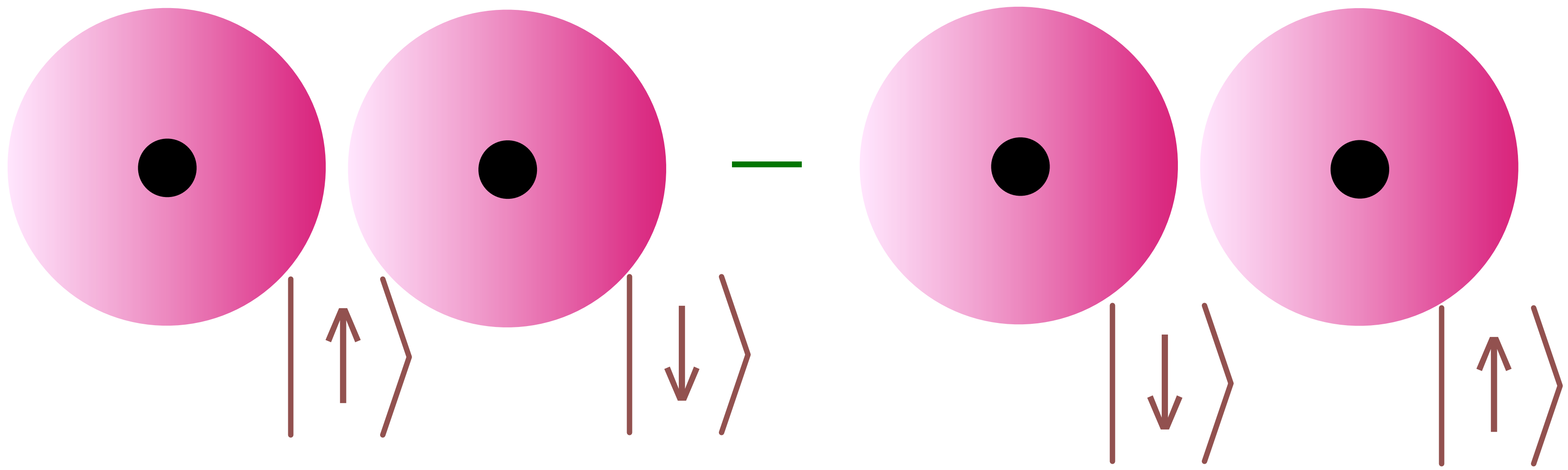
Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Superposition of two electron states leads to non-local correlations between spins

Quantum Entanglement: quantum superposition with more than one particle



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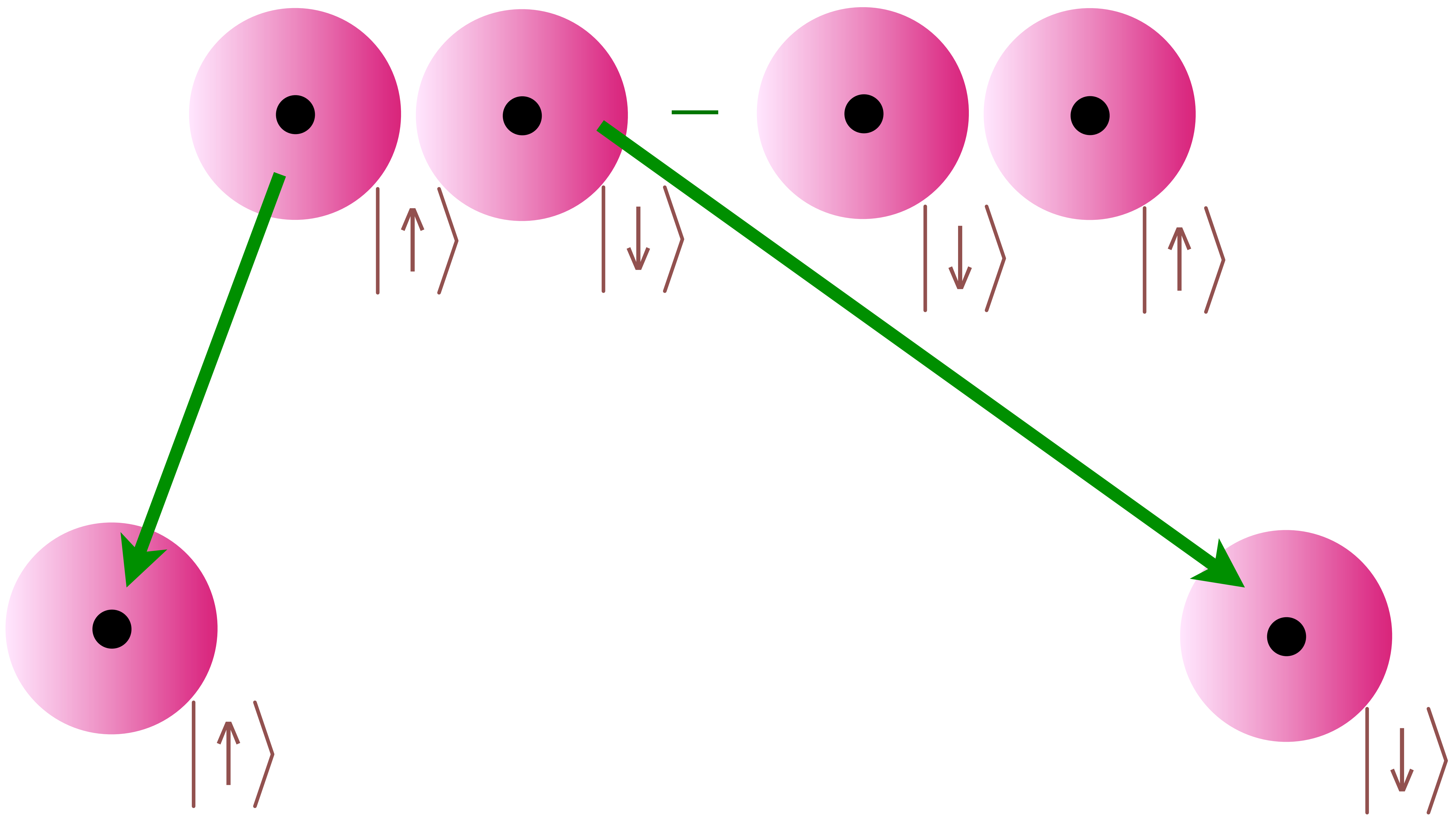

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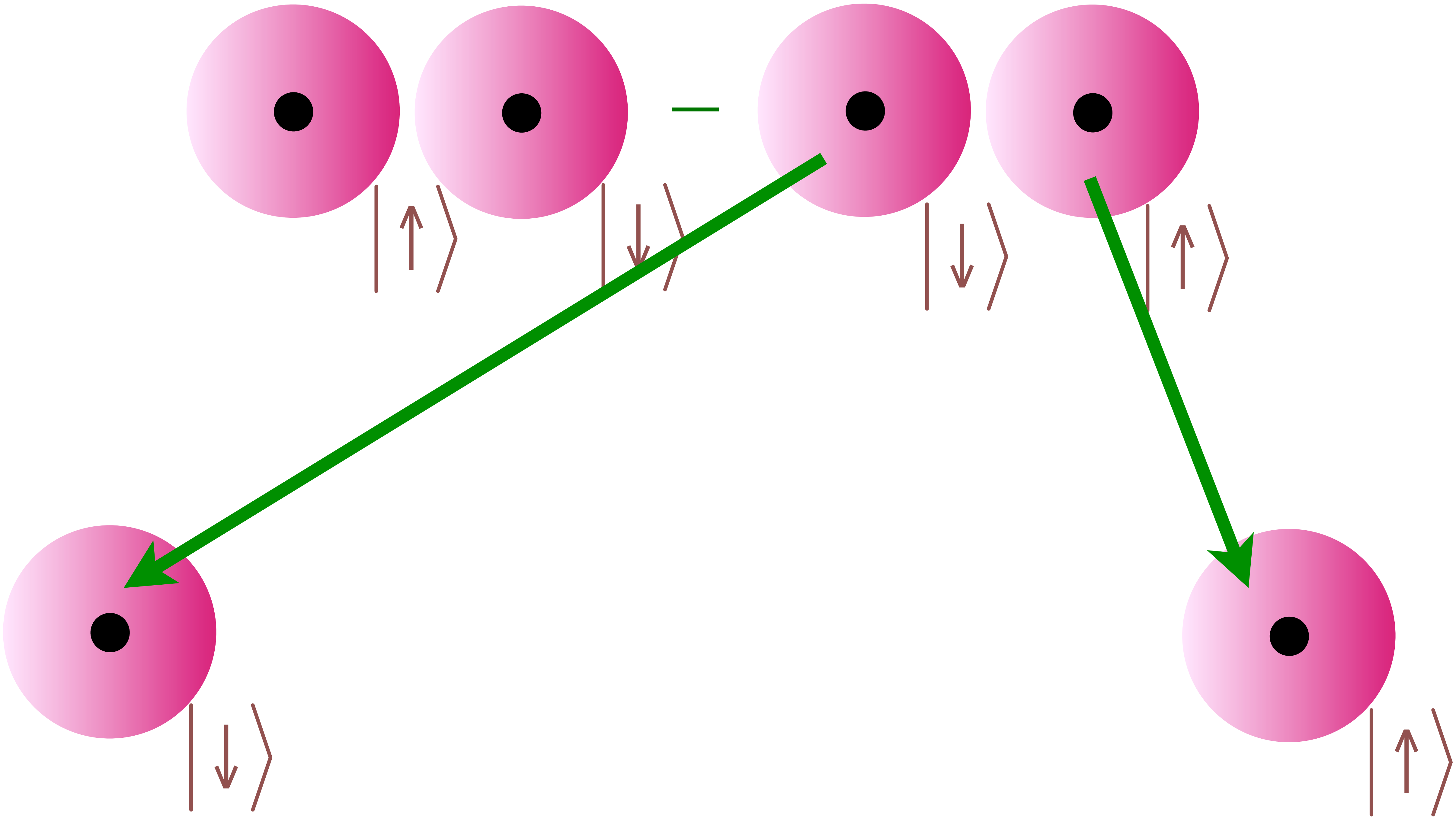
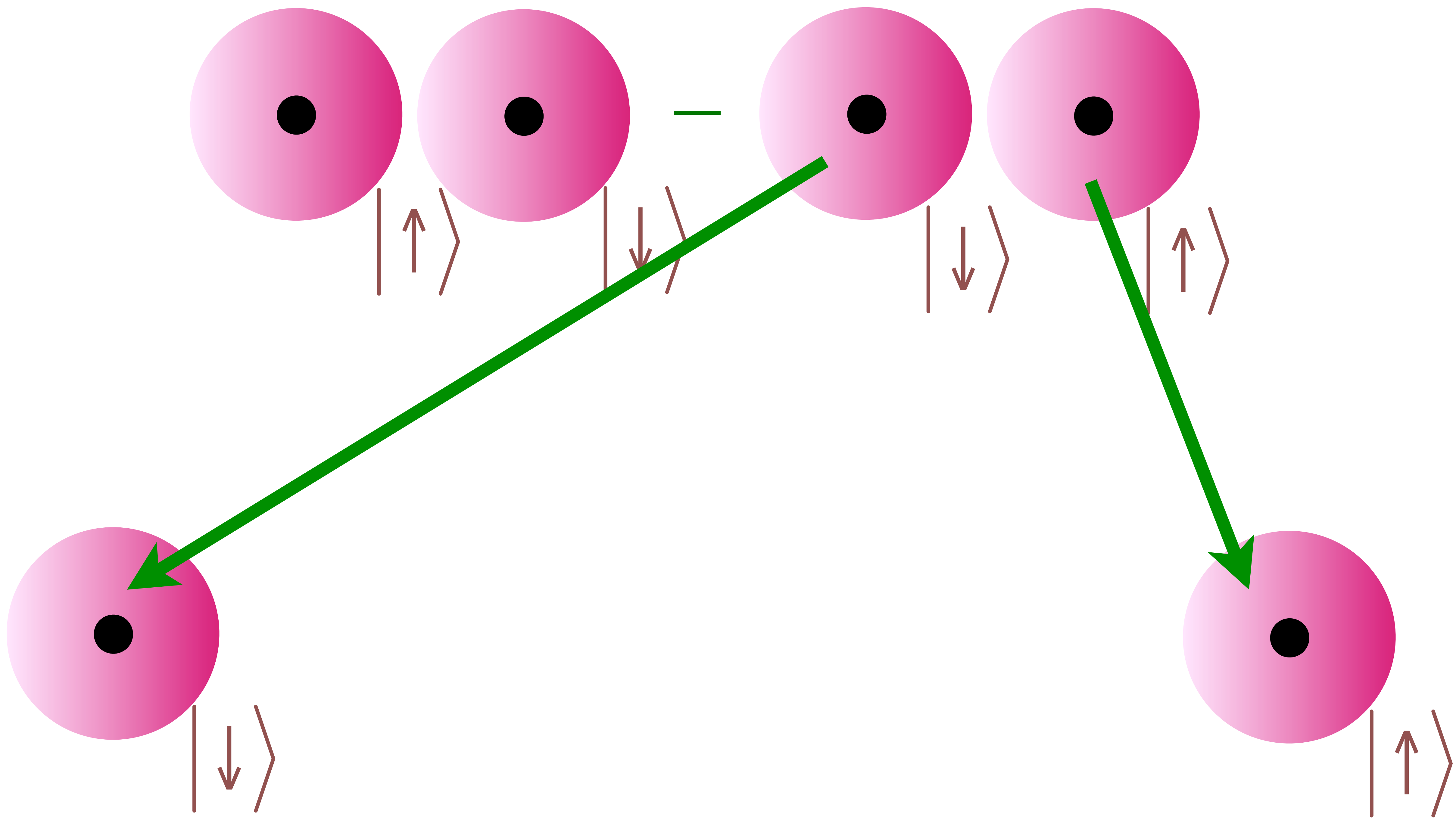


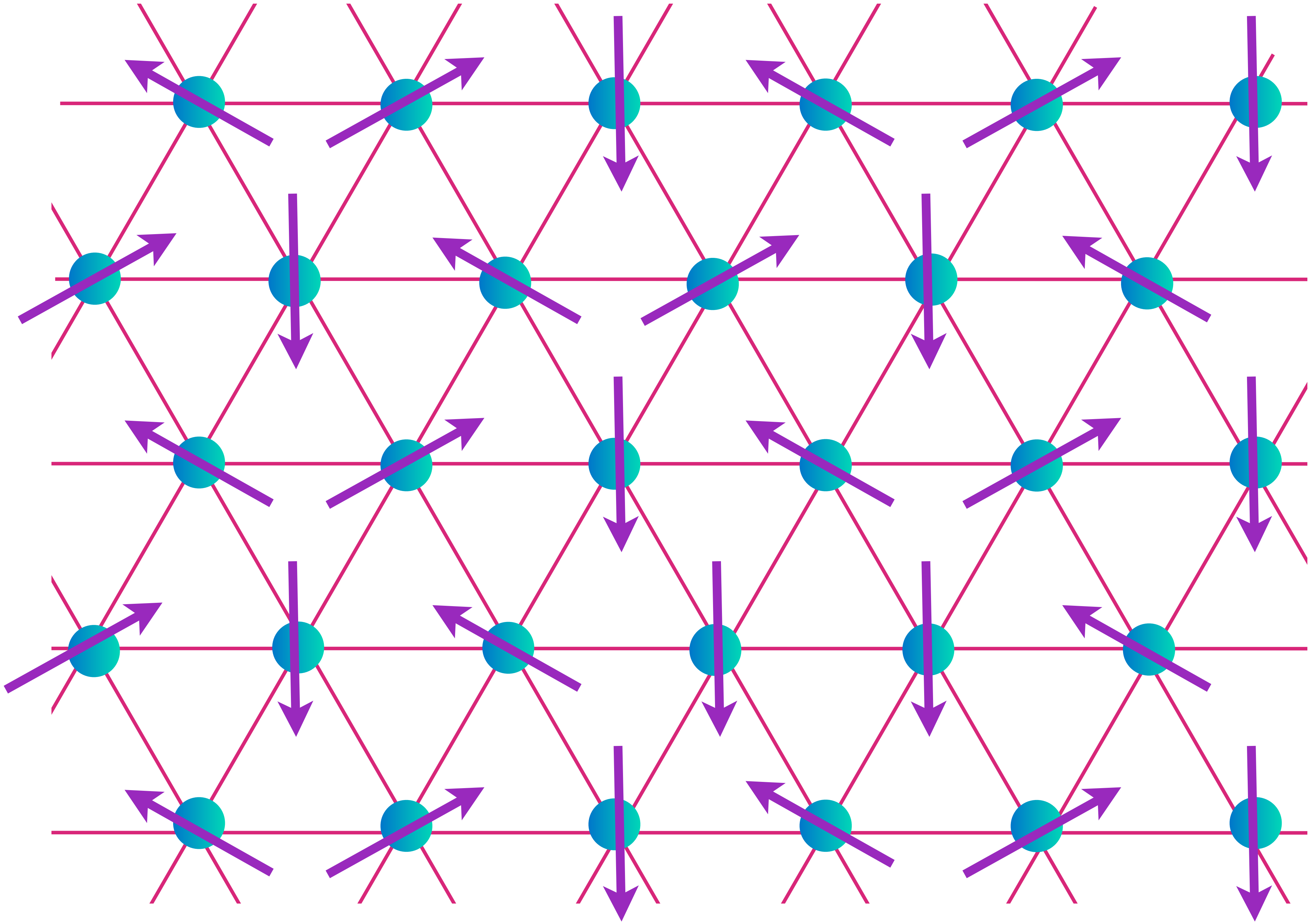
Quantum Entanglement: quantum superposition with more than one particle



Einstein-Podolsky-Rosen "paradox": Non-local correlations between observations arbitrarily far apart

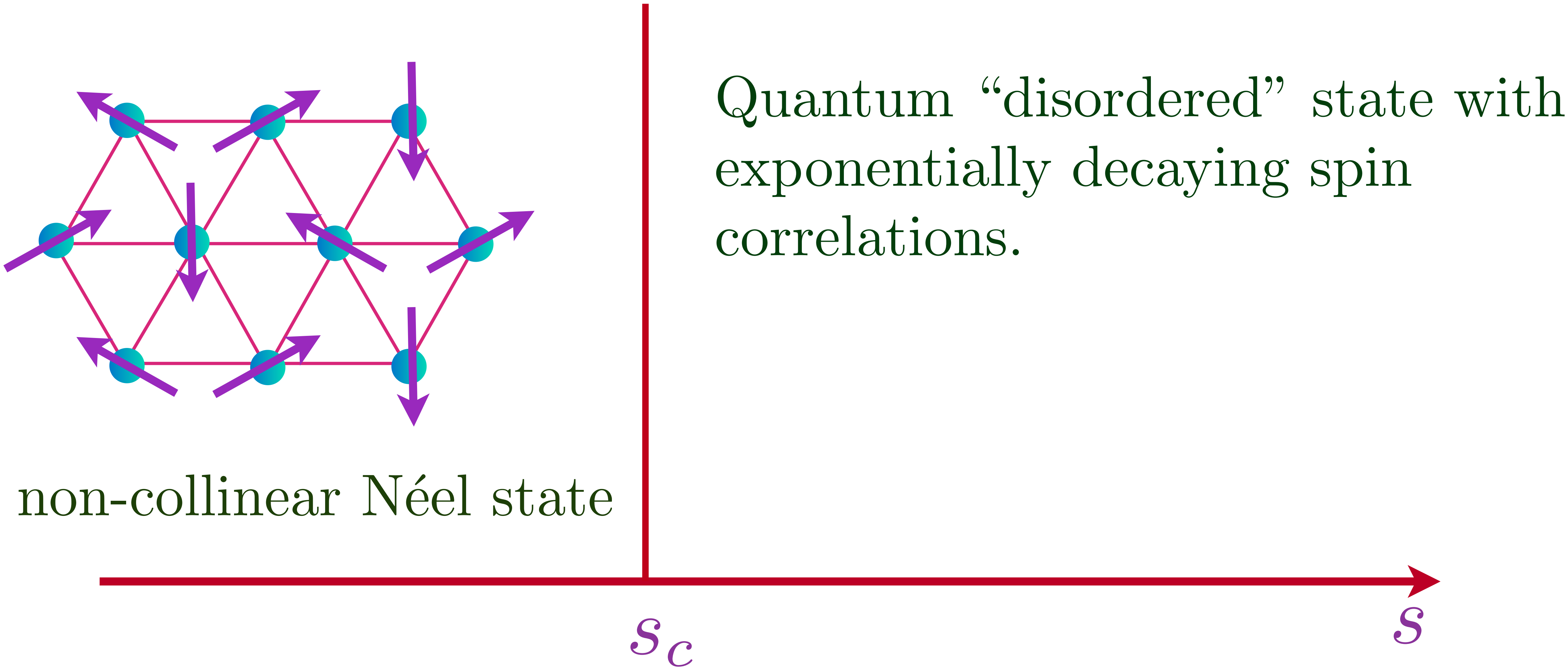
Mott insulator: Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

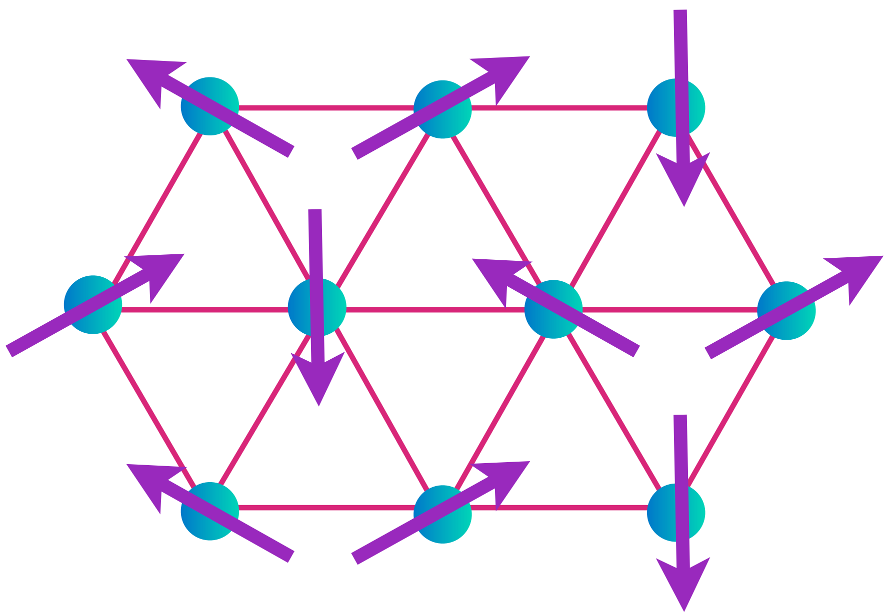


Nearest-neighbor model has non-collinear Neel order

Mott insulator: Triangular lattice antiferromagnet



Mott insulator: Triangular lattice antiferromagnet



non-collinear Néel state

Quantum "disordered" state with exponentially decaying spin correlations.

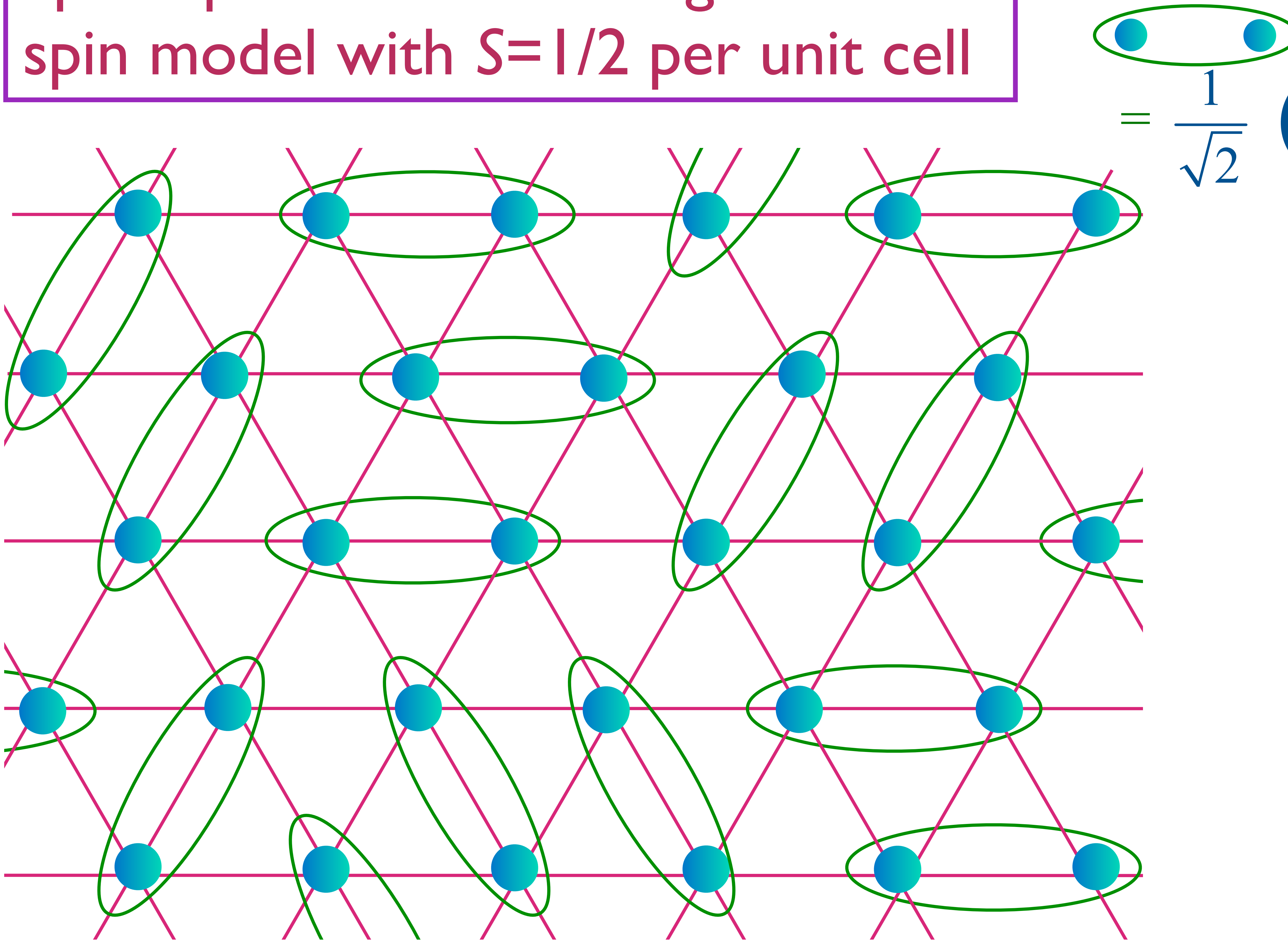
Z_2 spin liquid
with *long-range* entanglement.

s_c $\quad$ s

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Mott insulator: Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with S=1/2 per unit cell



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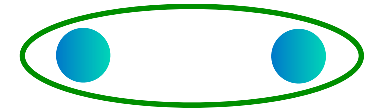

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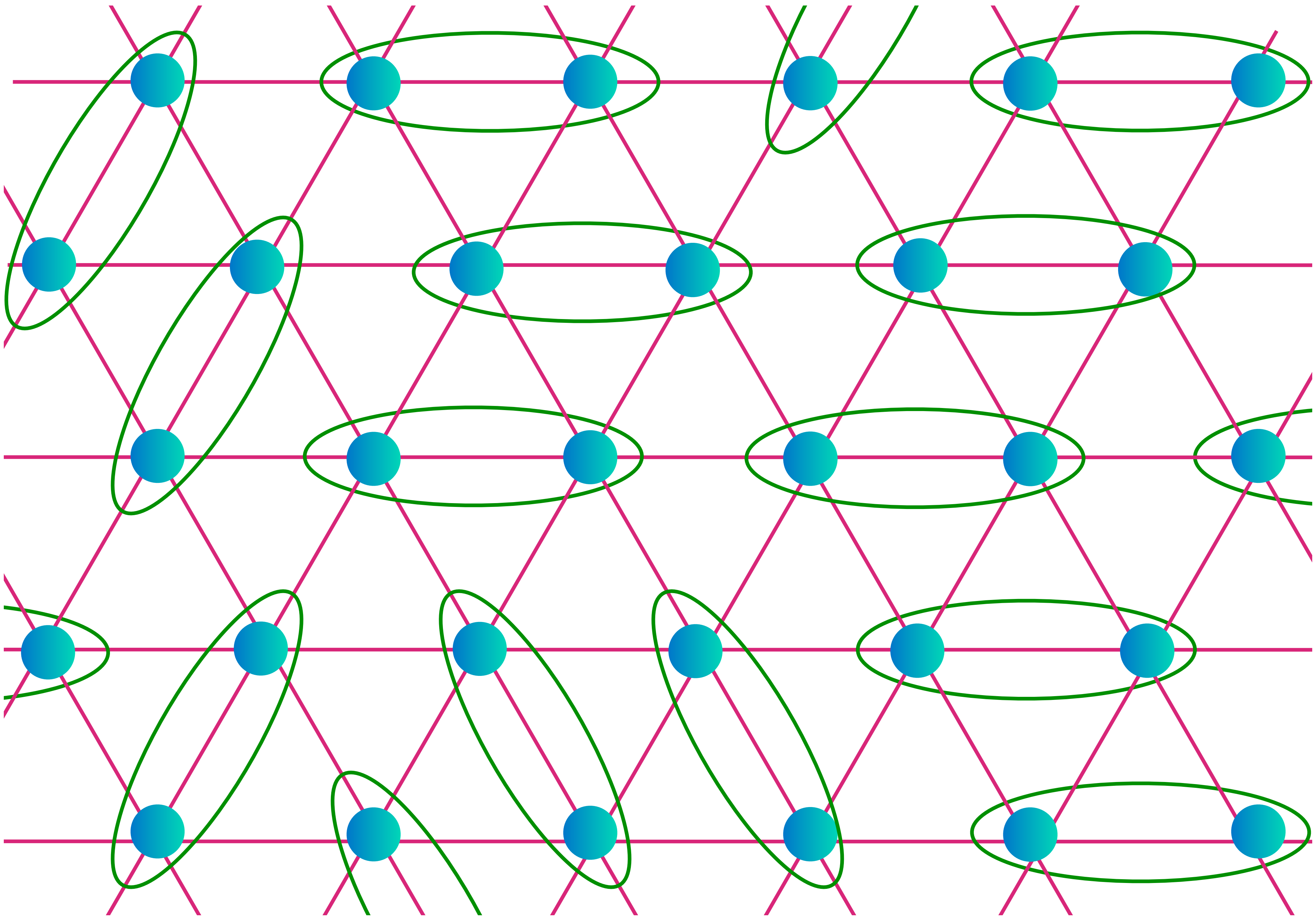

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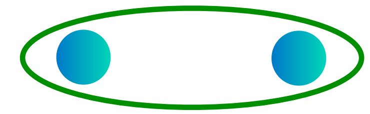

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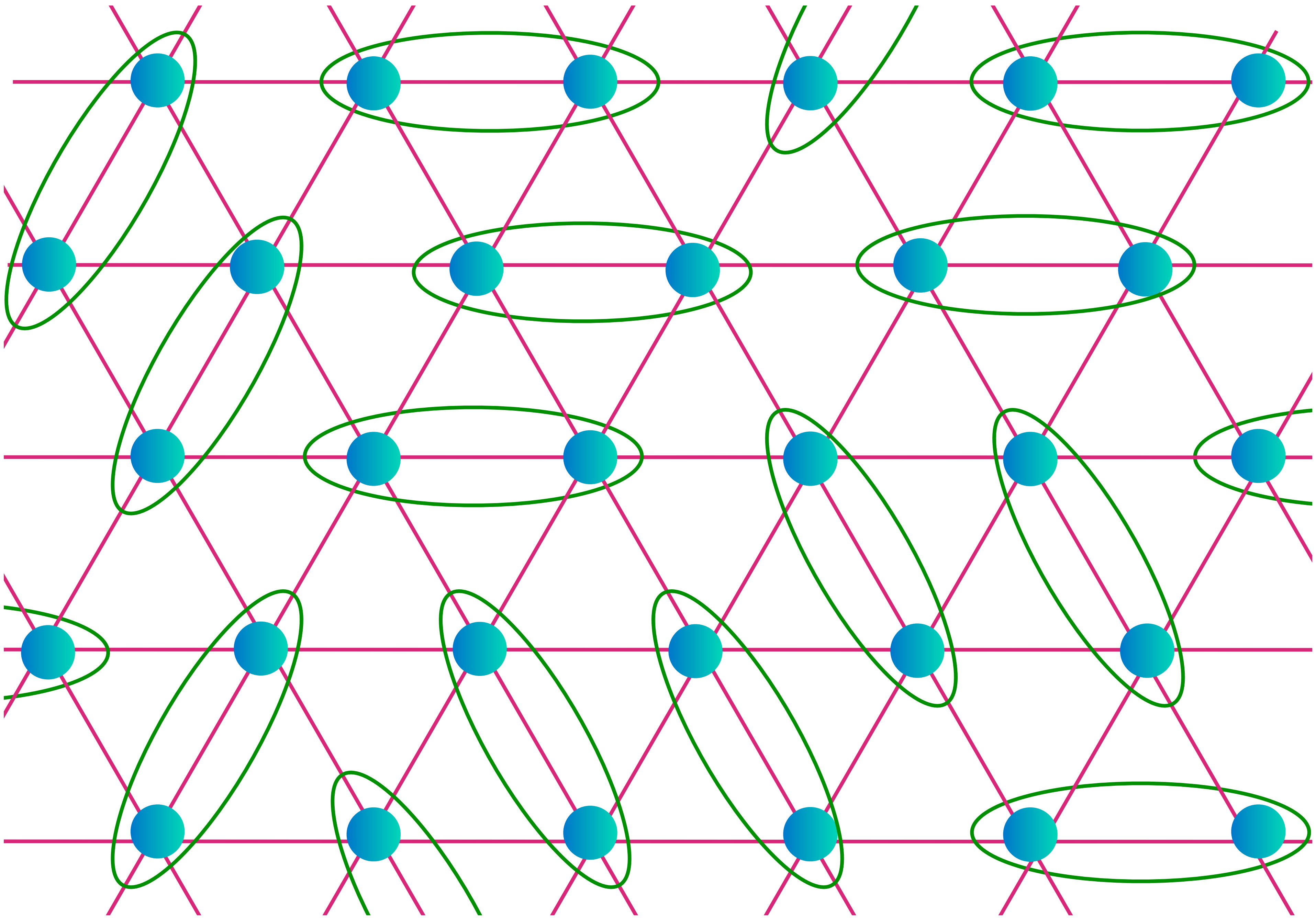

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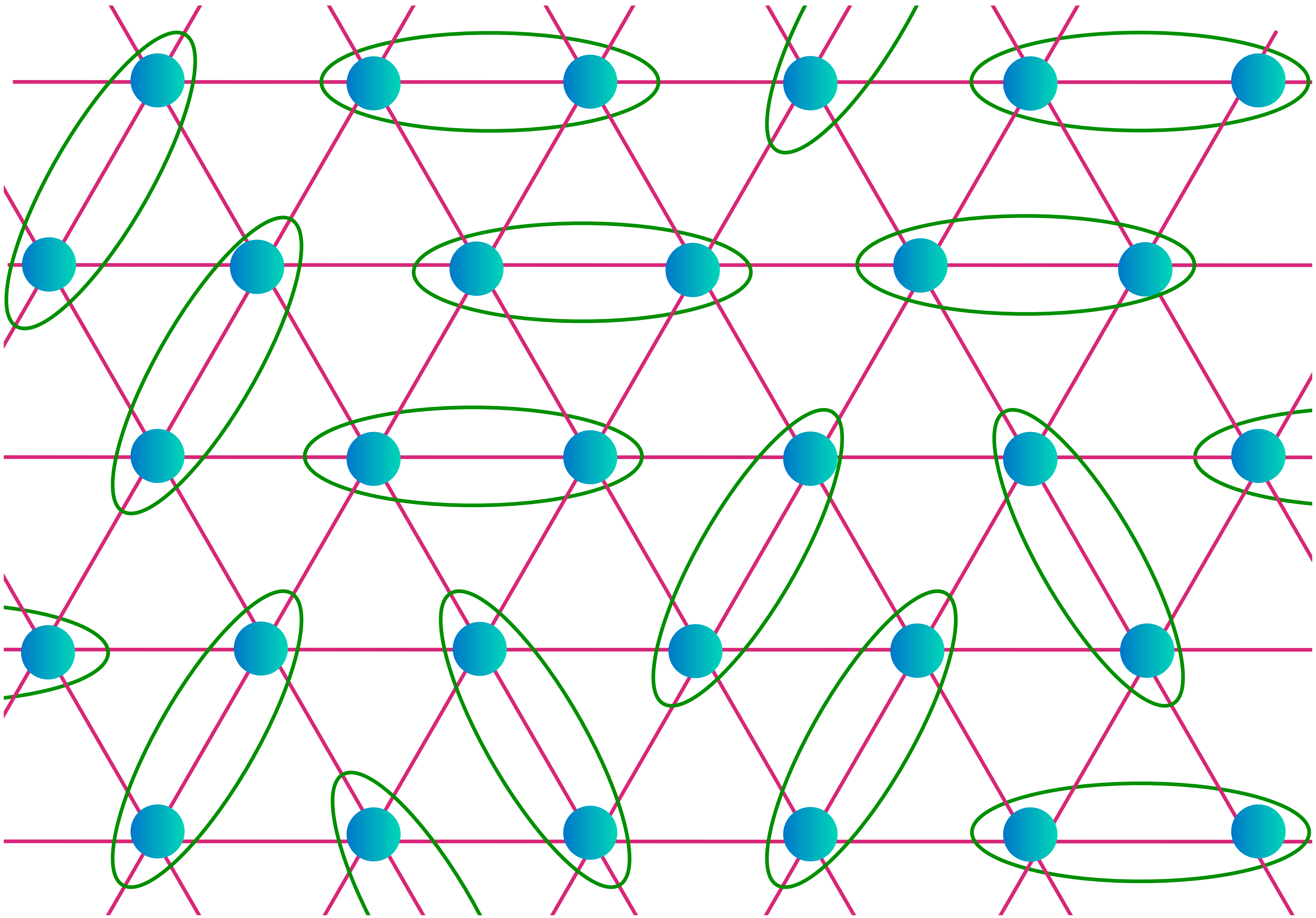

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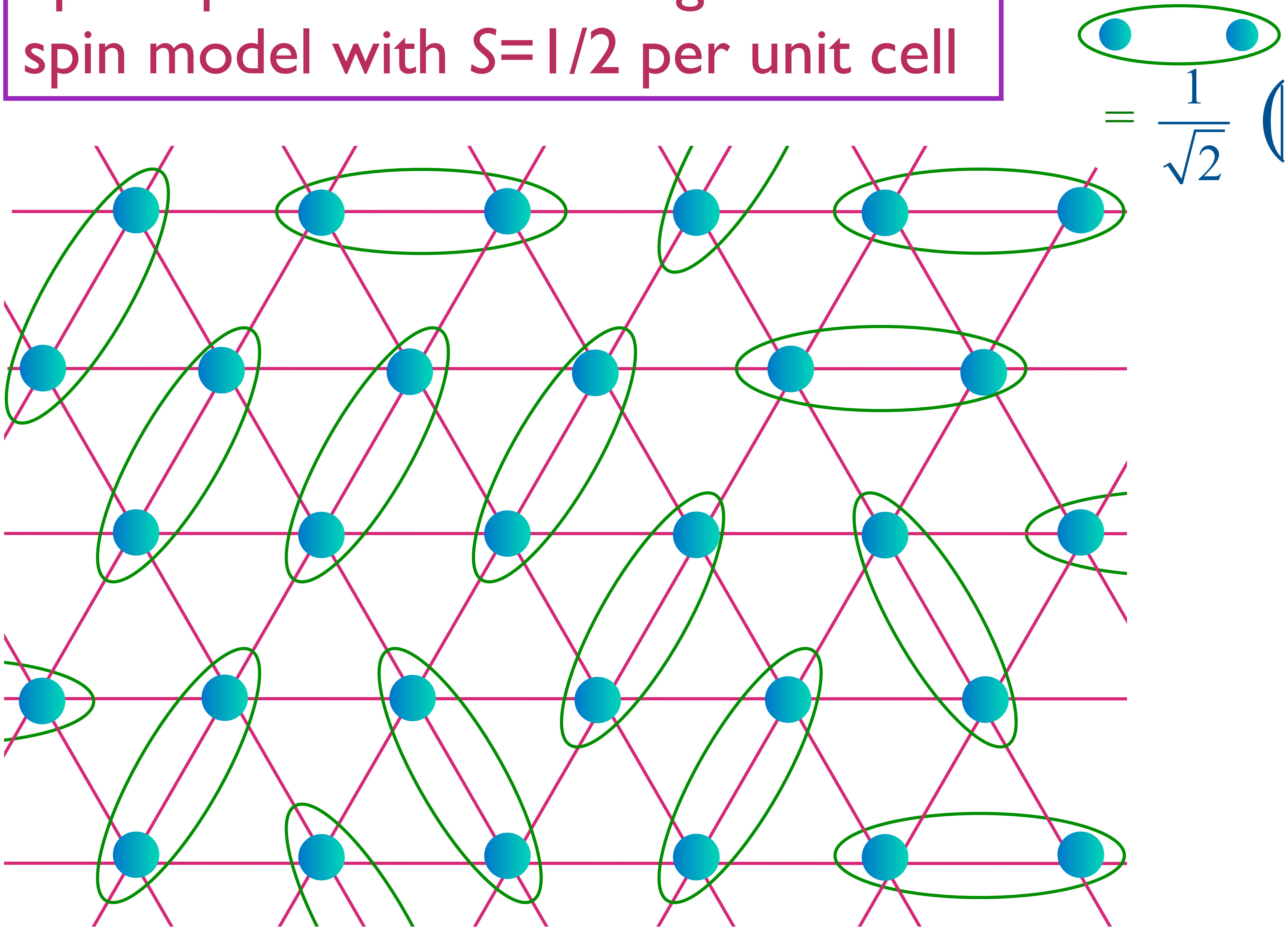
Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



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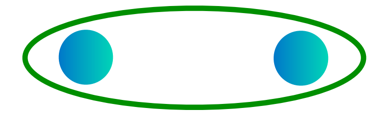

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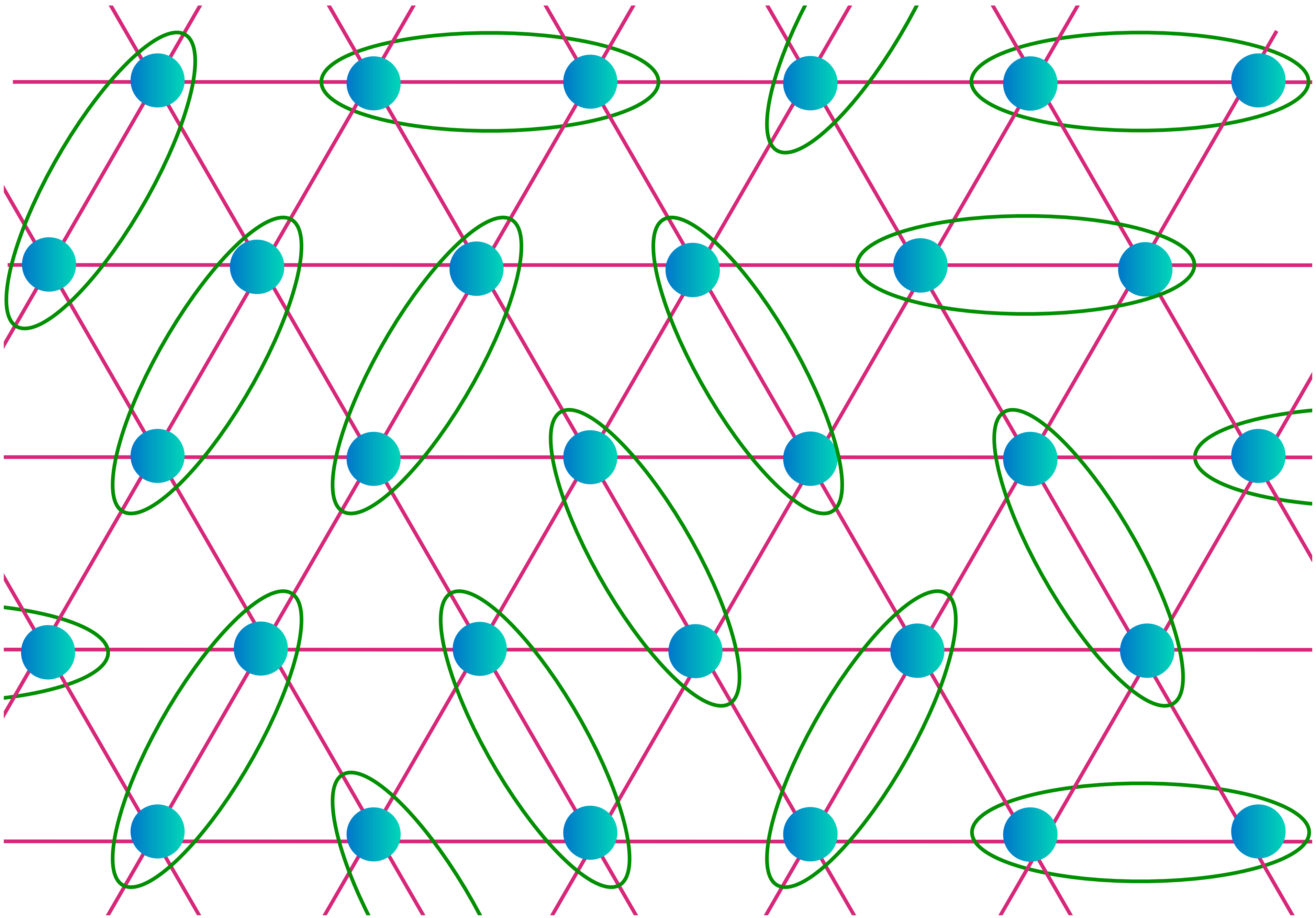

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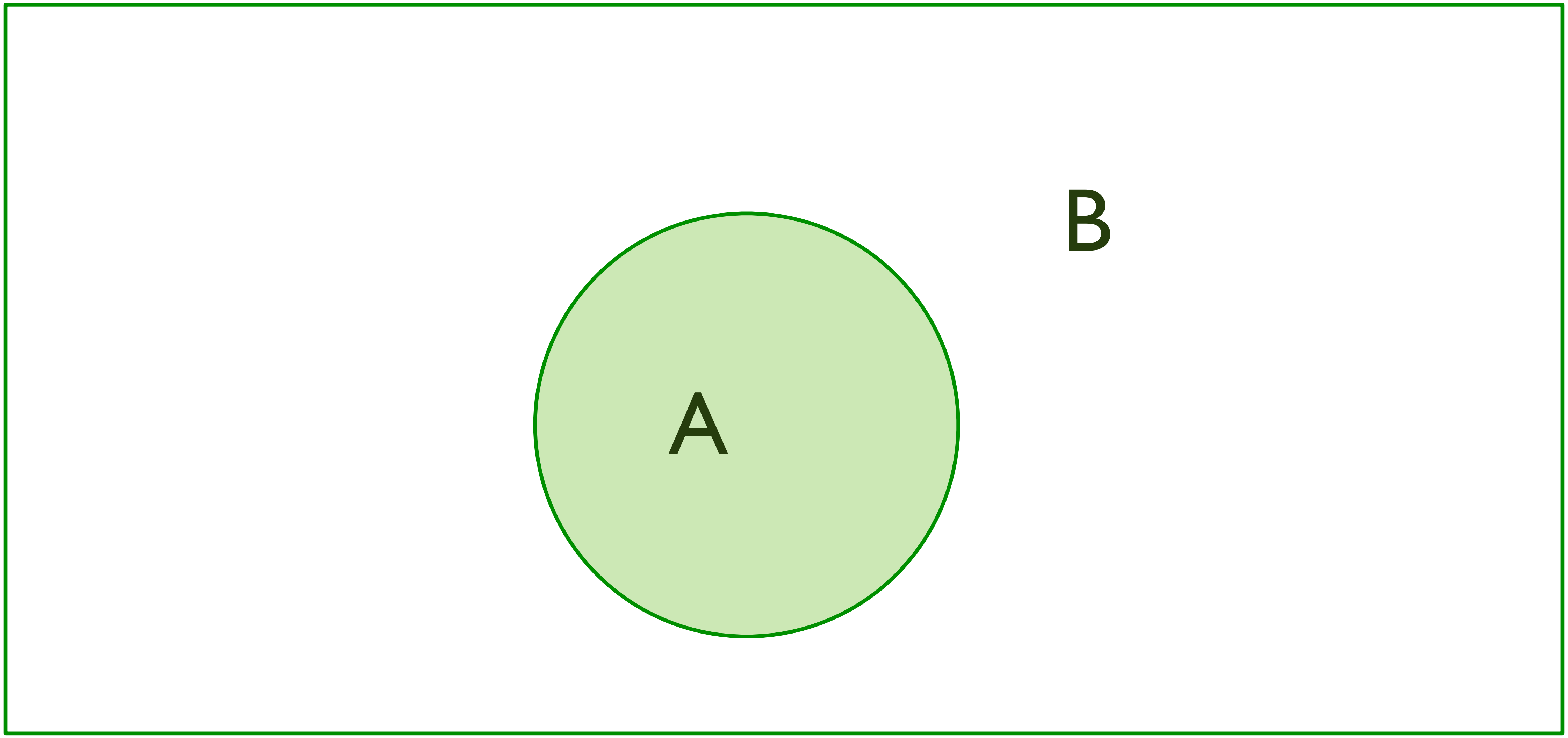
Topological order in the Z_2 spin liquid ground state

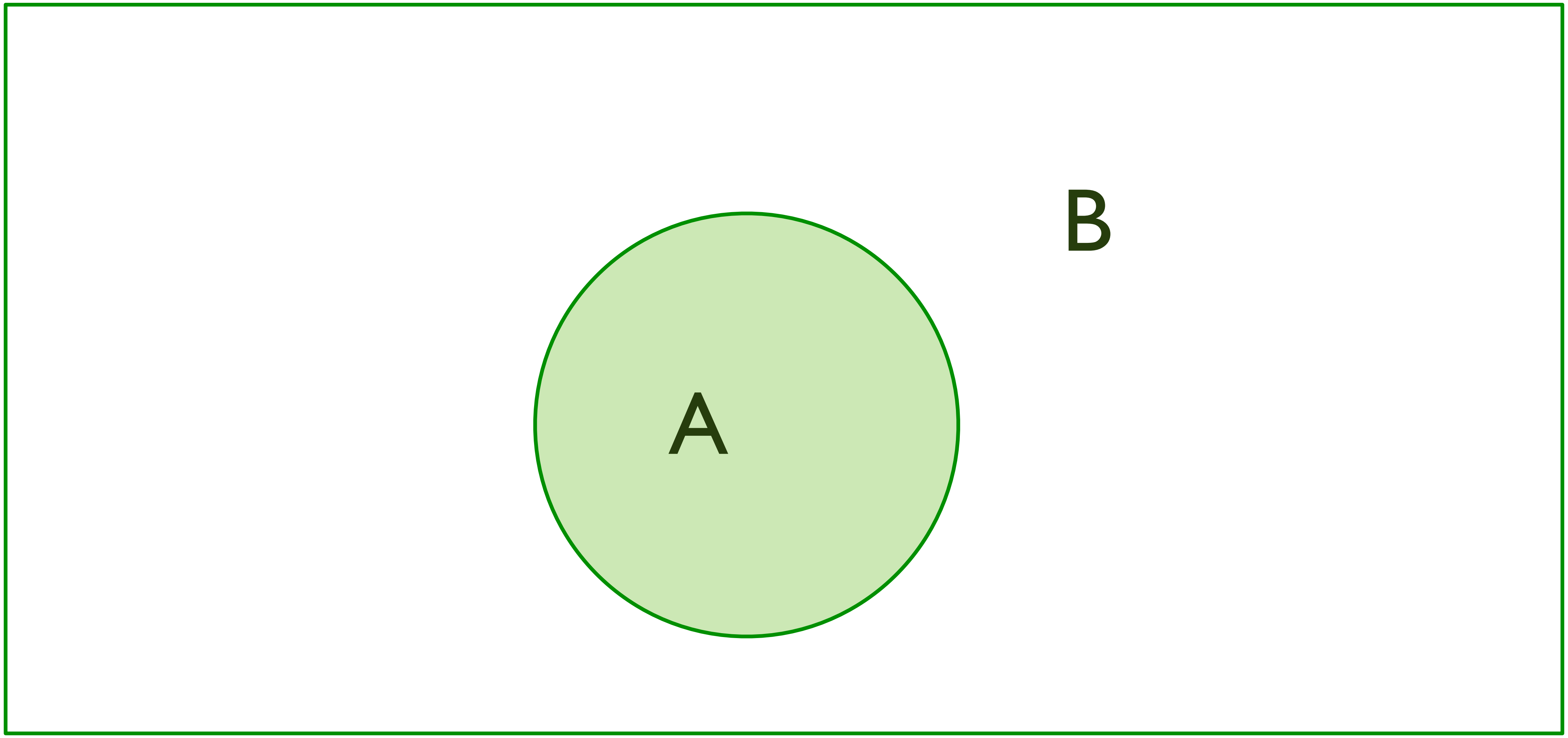


$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Topological order in the Z_2 spin liquid ground state

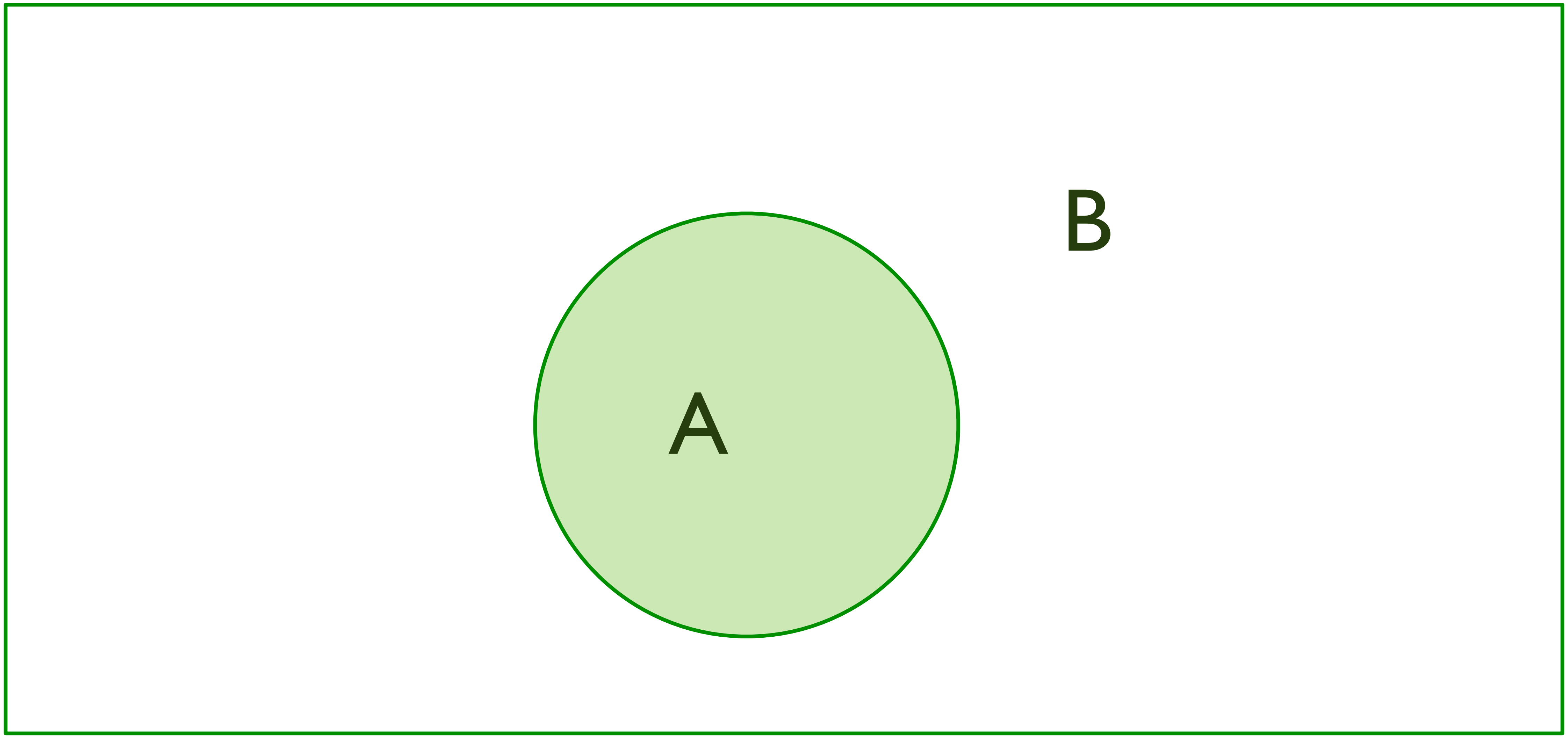


Entanglement entropy of a band insulator:

$$S_{EE} = aL - exp(-bL)$$

where L is the perimeter of the boundary between A and B.

Topological order in the Z_2 spin liquid ground state



Entanglement entropy of a Z_2 spin liquid:

$$S_{EE} = aL - \ln(2)$$

where L is the perimeter of the boundary between A and B.
The $\ln(2)$ is a universal characteristic of the Z_2 spin liquid, and implies *long-range* quantum entanglement.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

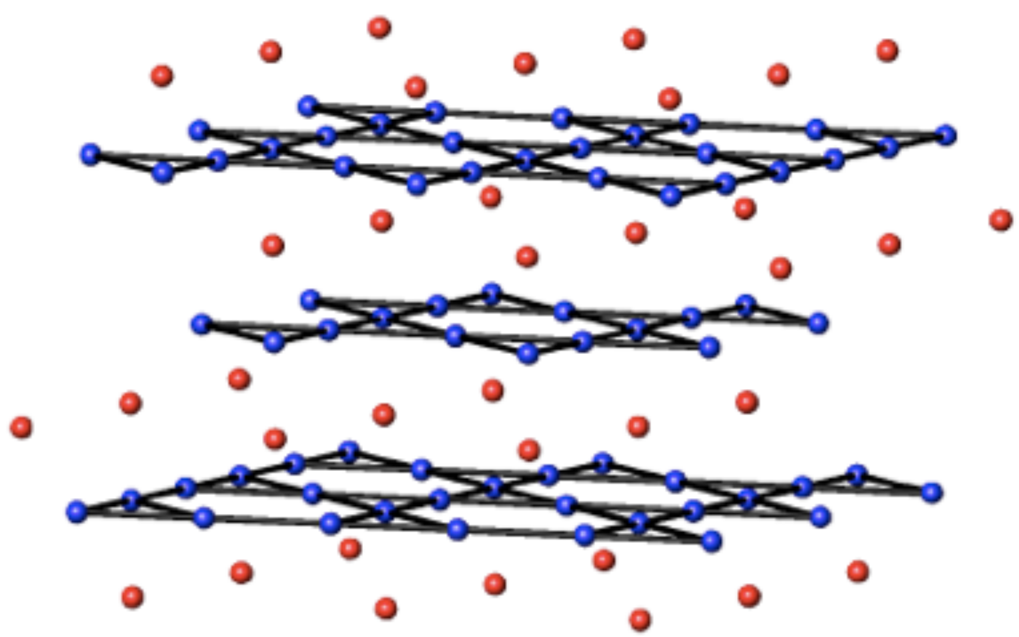
Promising candidate: the kagome antiferromagnet

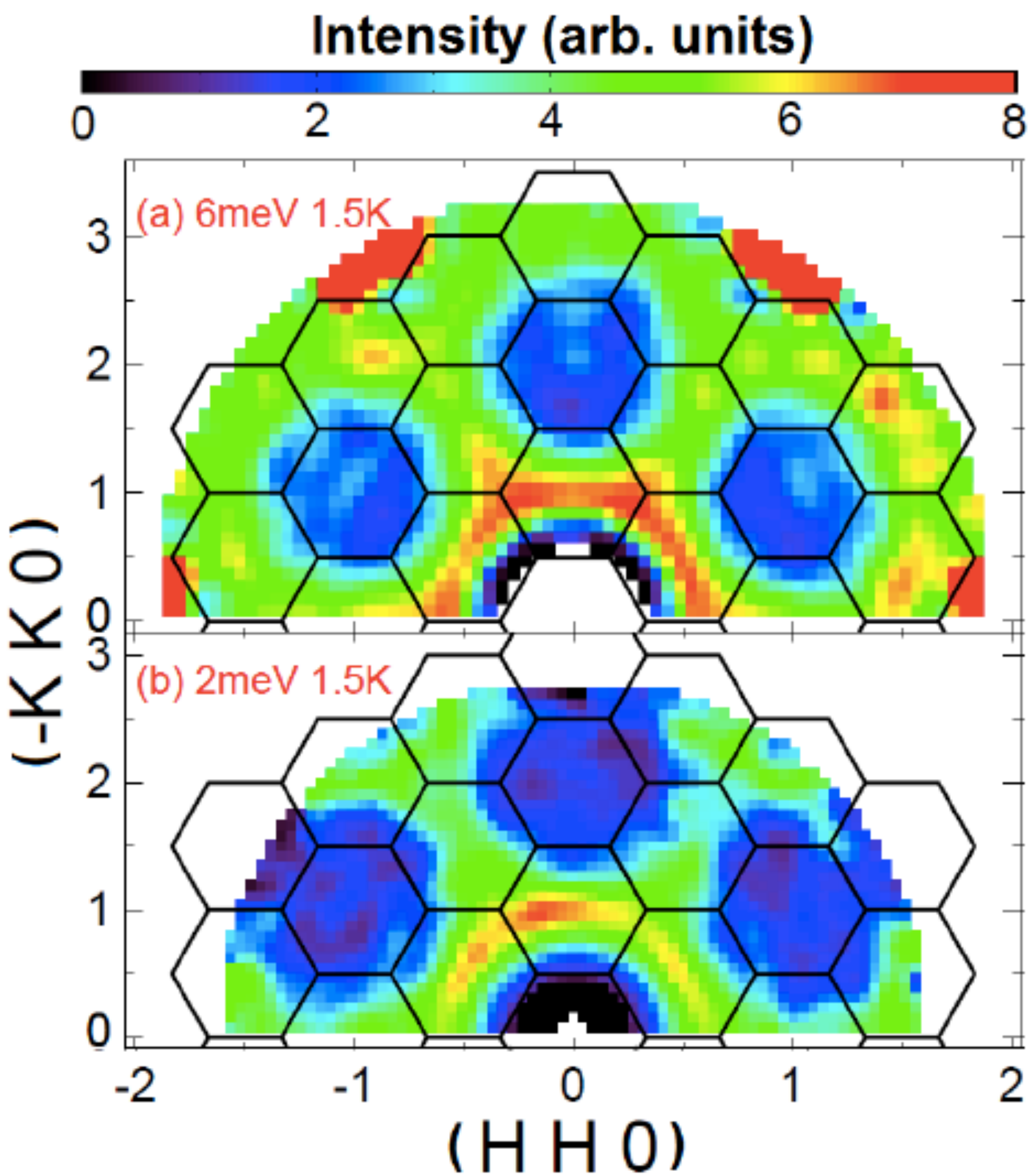
Numerical evidence for a gapped spin liquid:

Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).

Young Lee,
APS meeting, March 2012

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





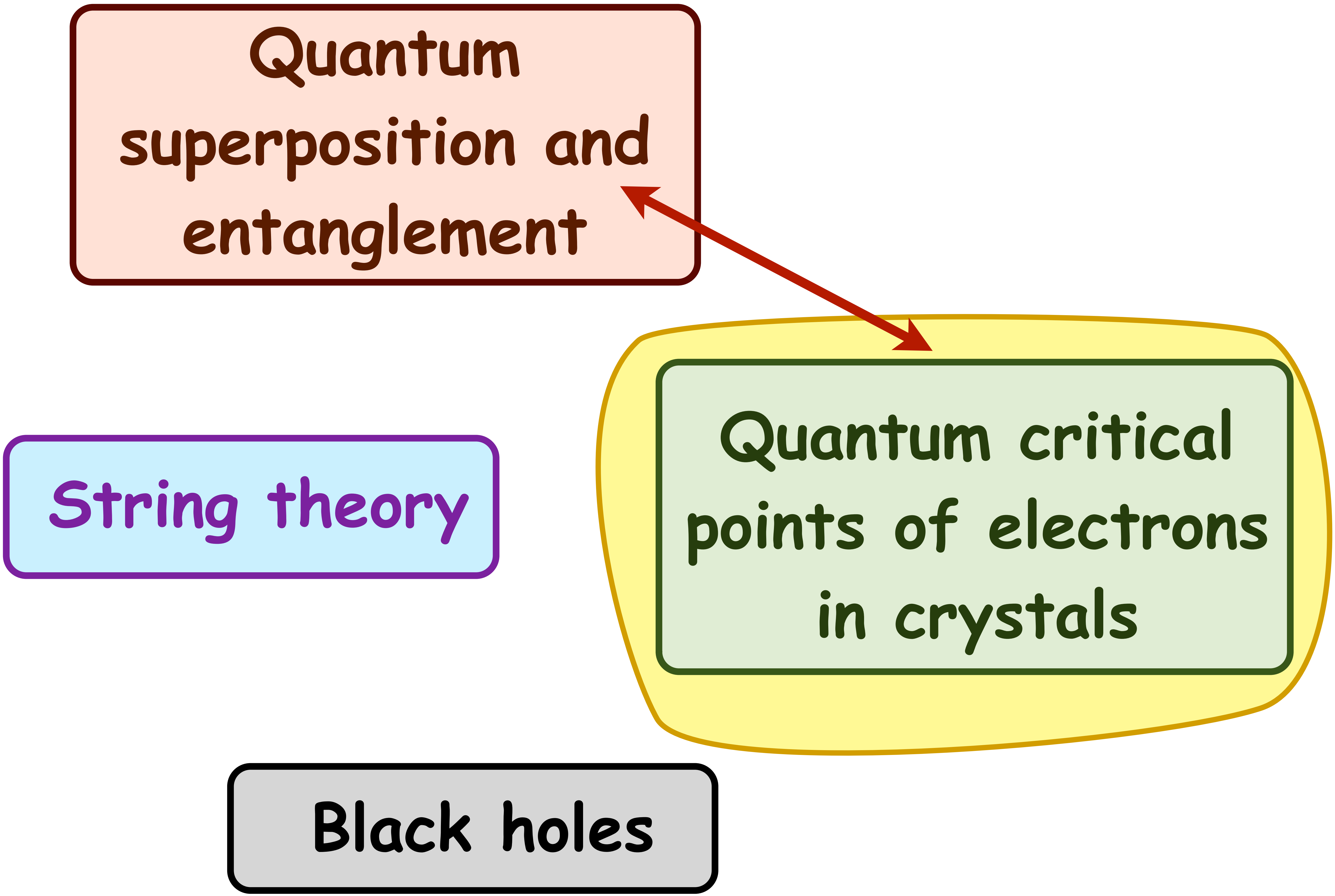
Quantum superposition and entanglement

Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

Black holes


Quantum superposition and entanglement
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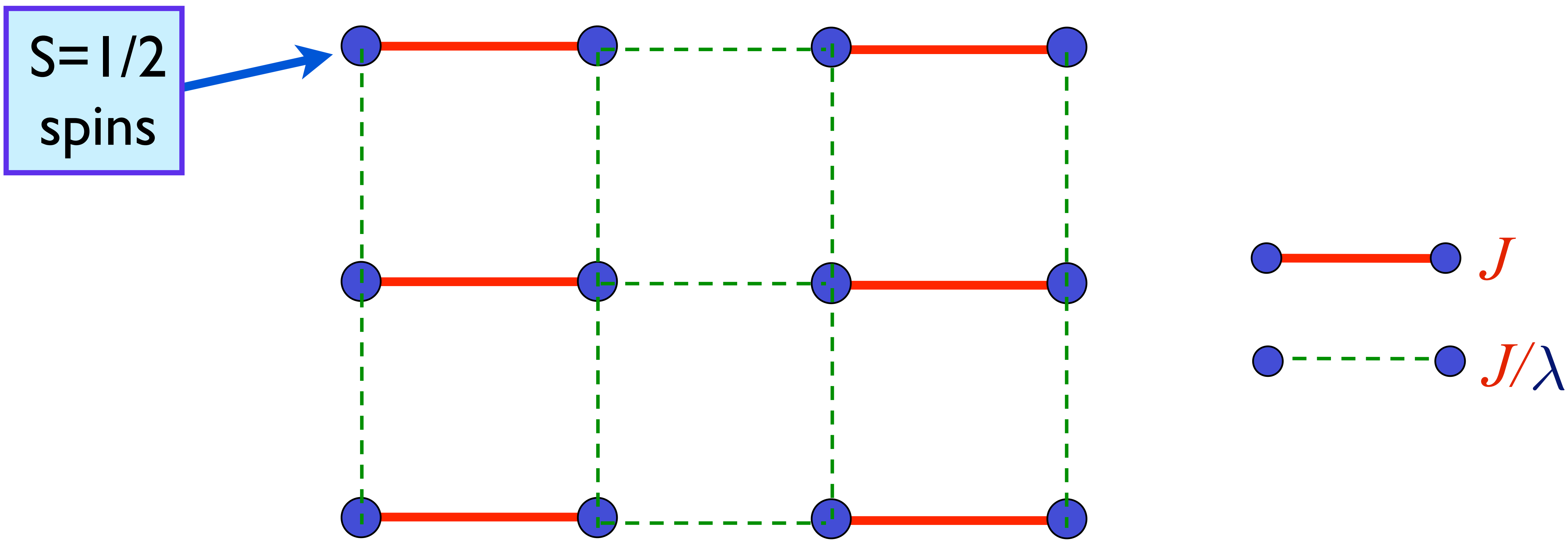
Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

Spinning electrons localized on a square lattice

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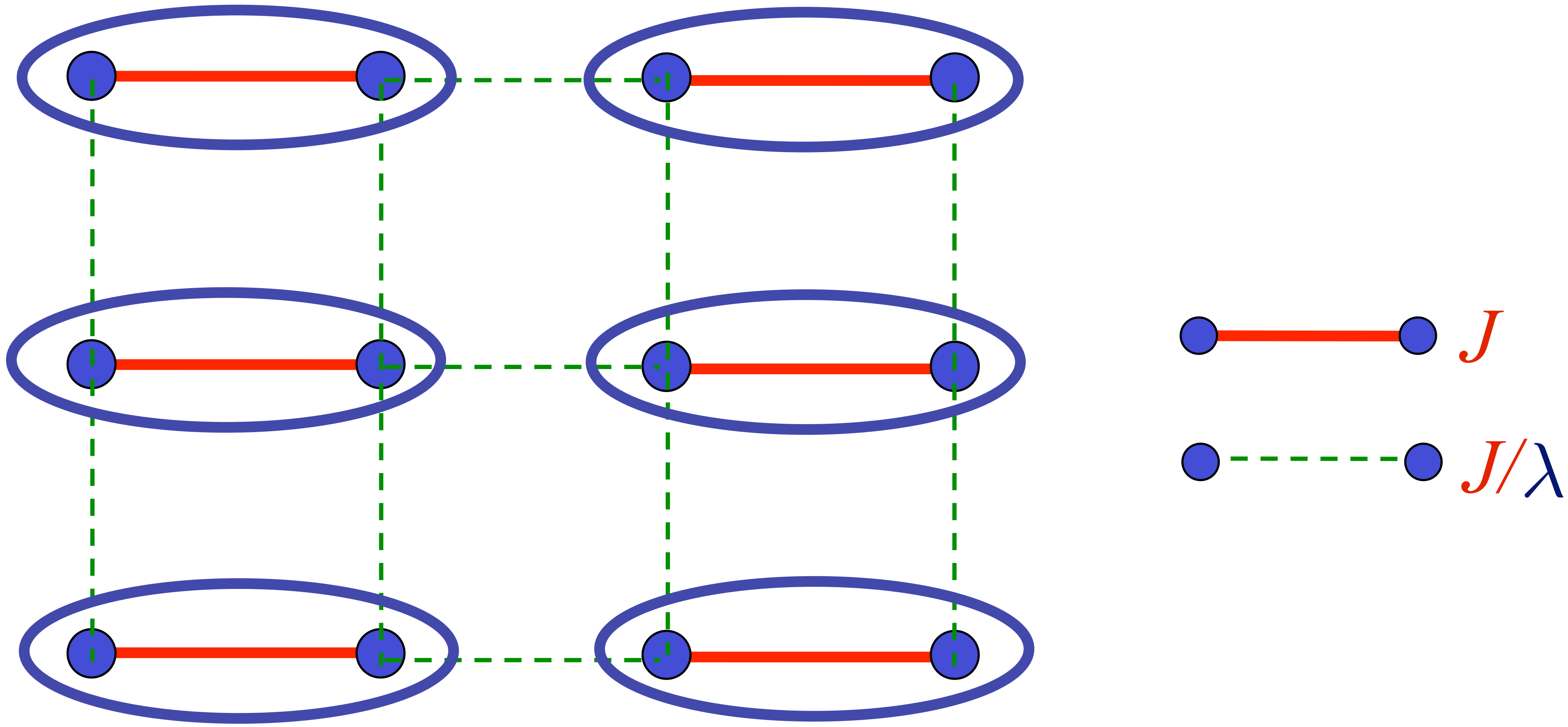

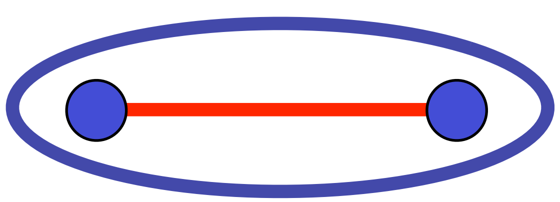

$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

Spinning electrons localized on a square lattice

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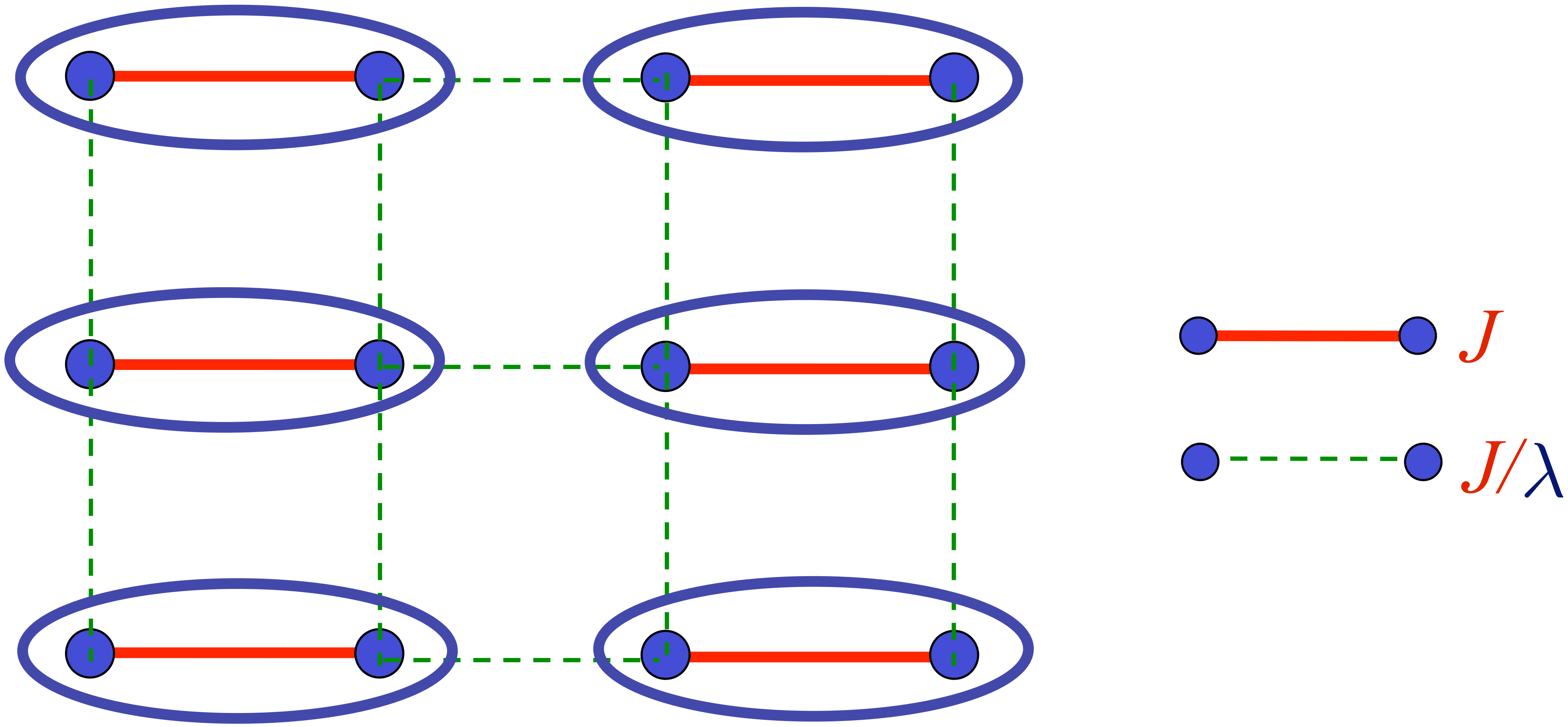
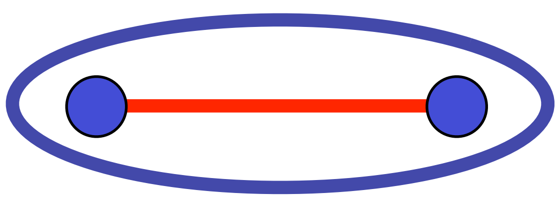


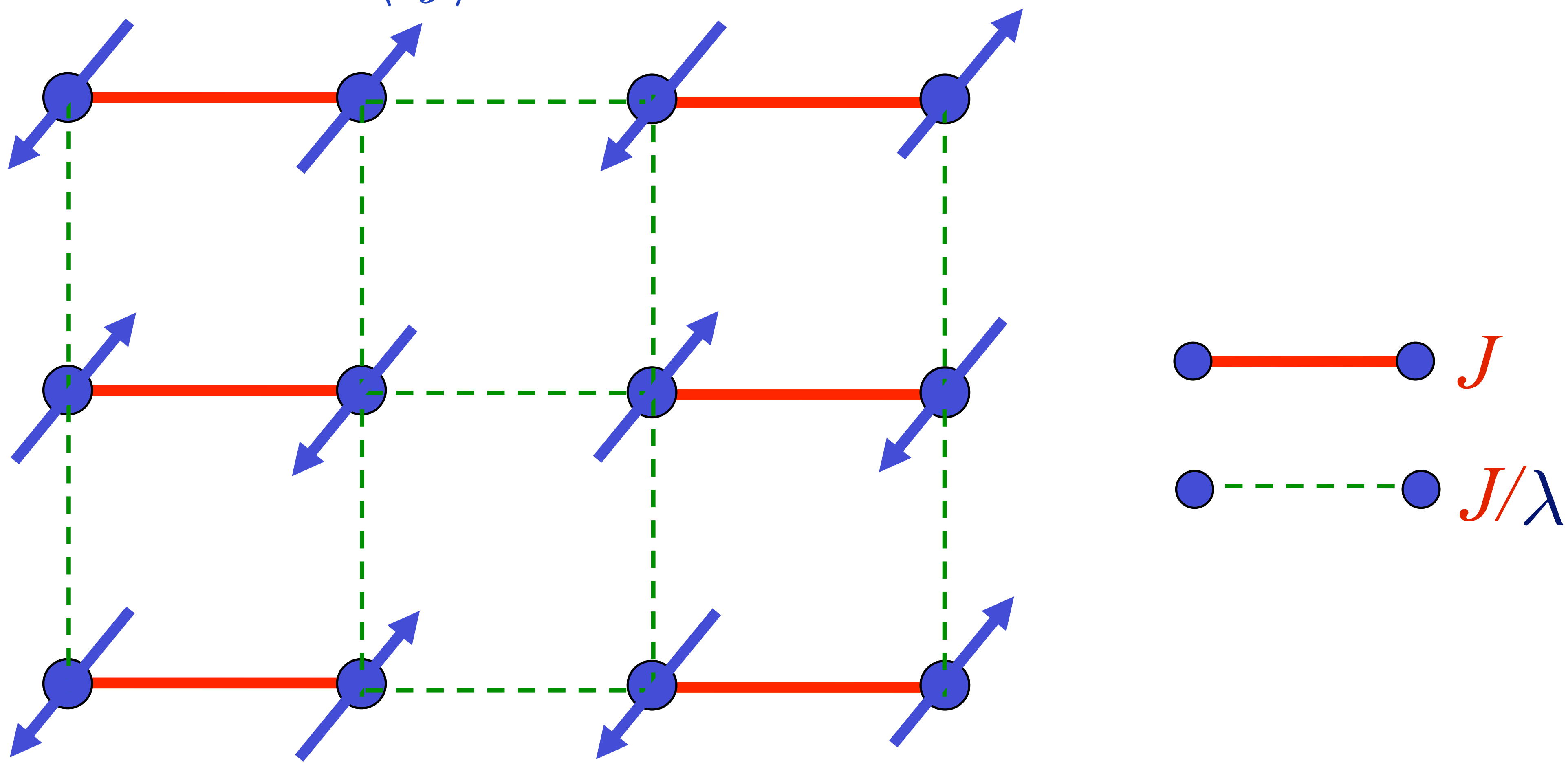


$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Nearest-neighor spins are "entangled" with each other.
Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern

Spinning electrons localized on a square lattice

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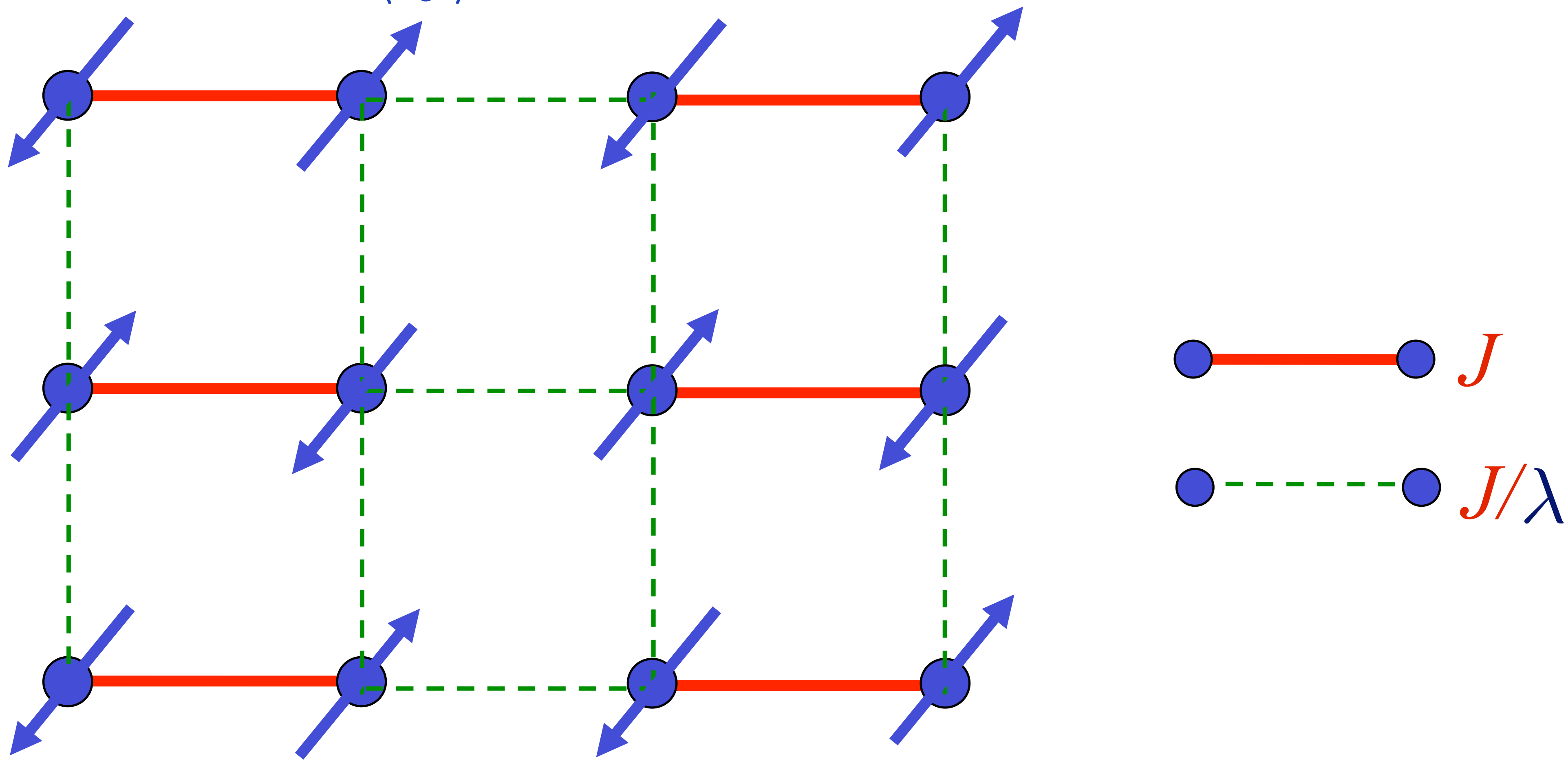

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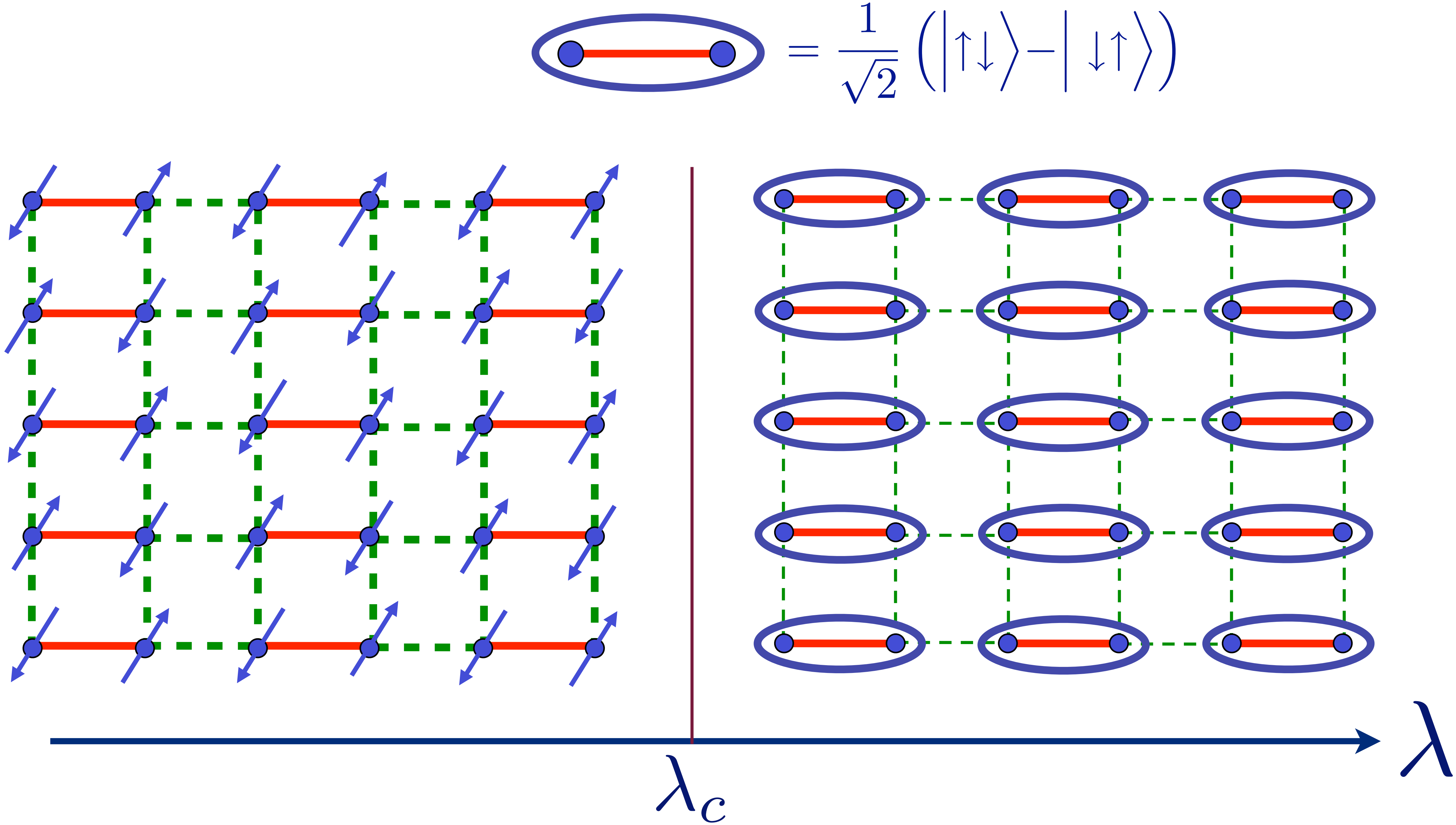
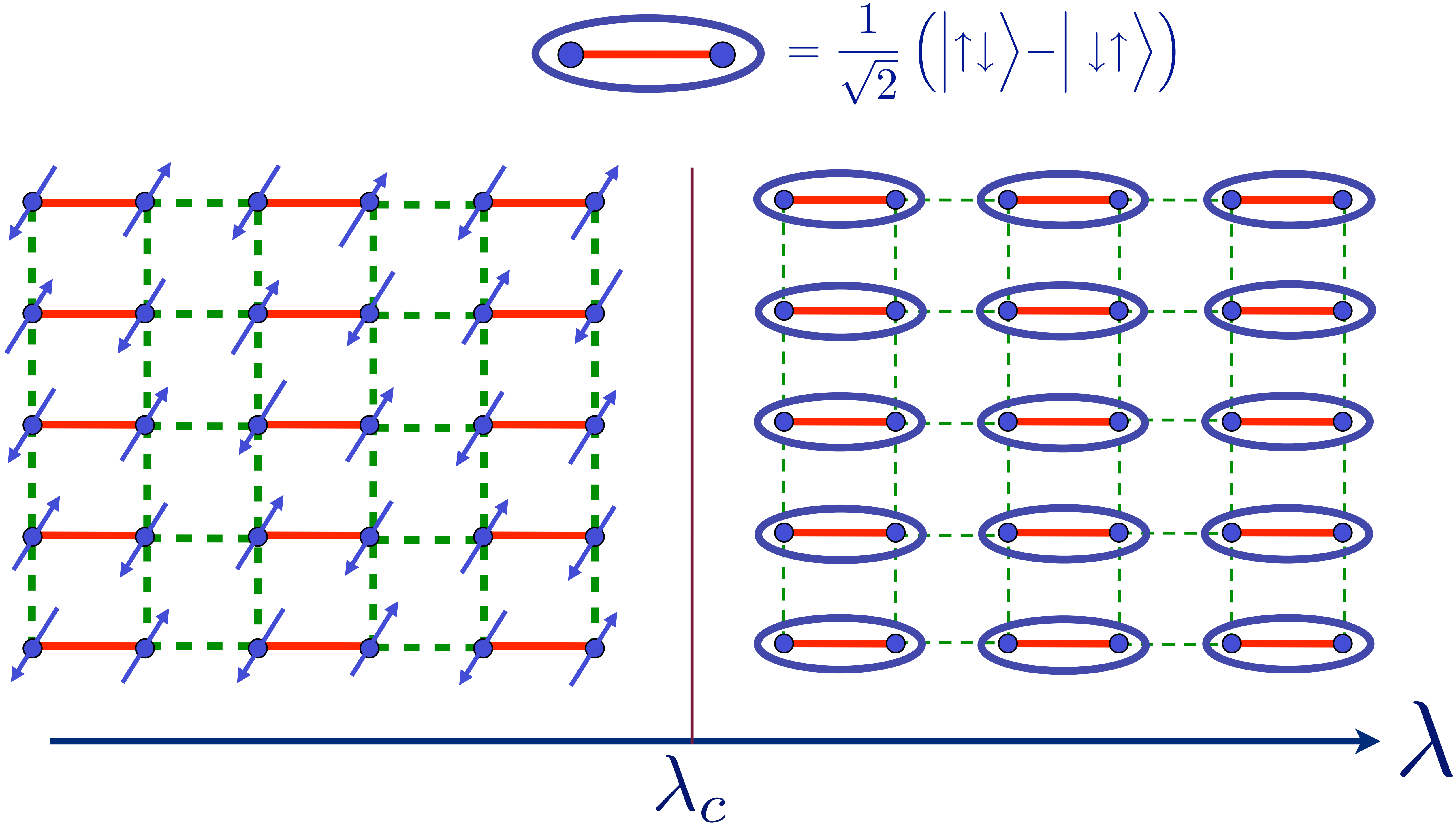
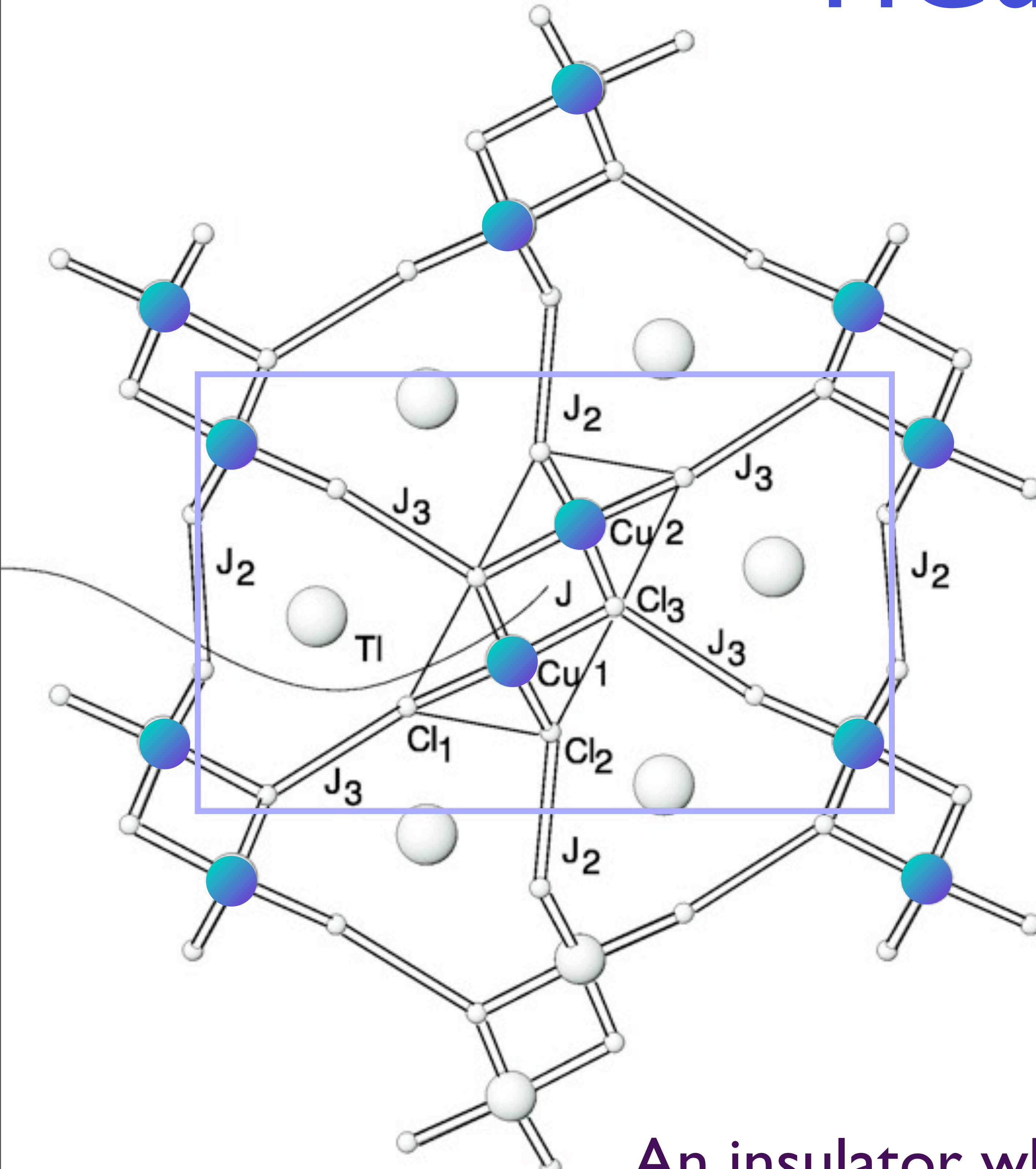
No EPR pairs

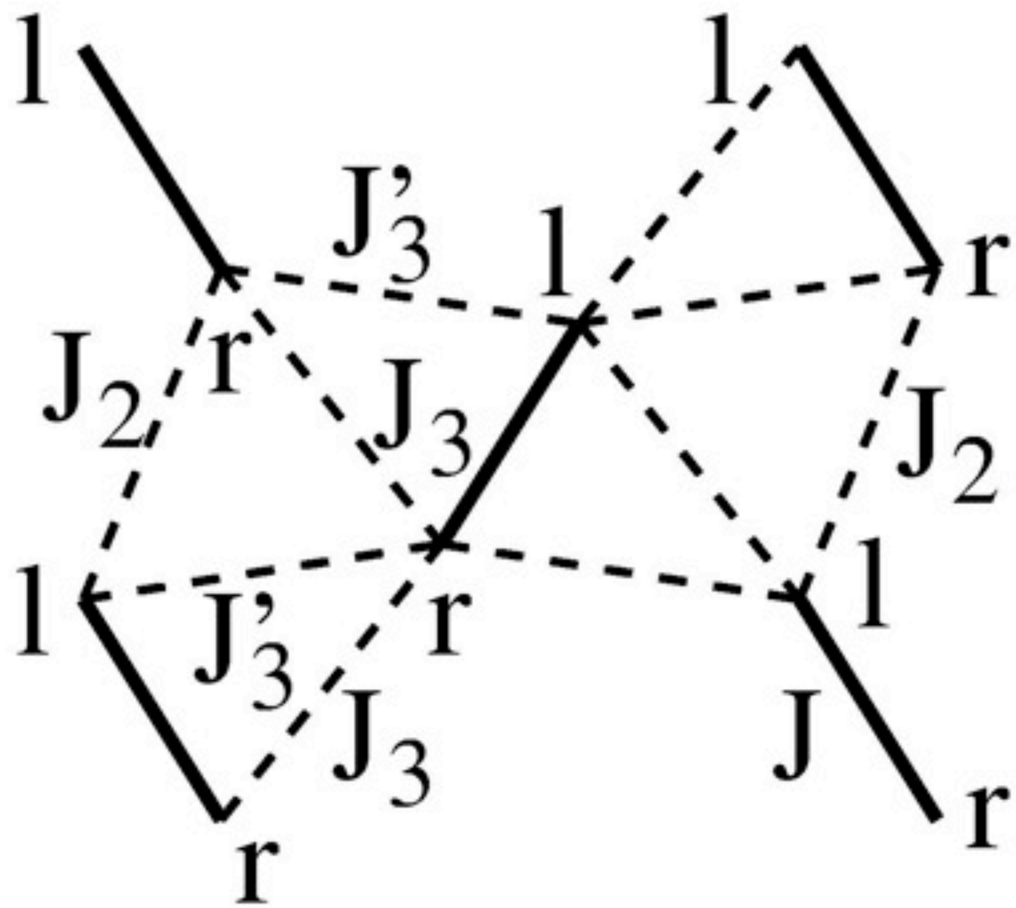

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ



A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

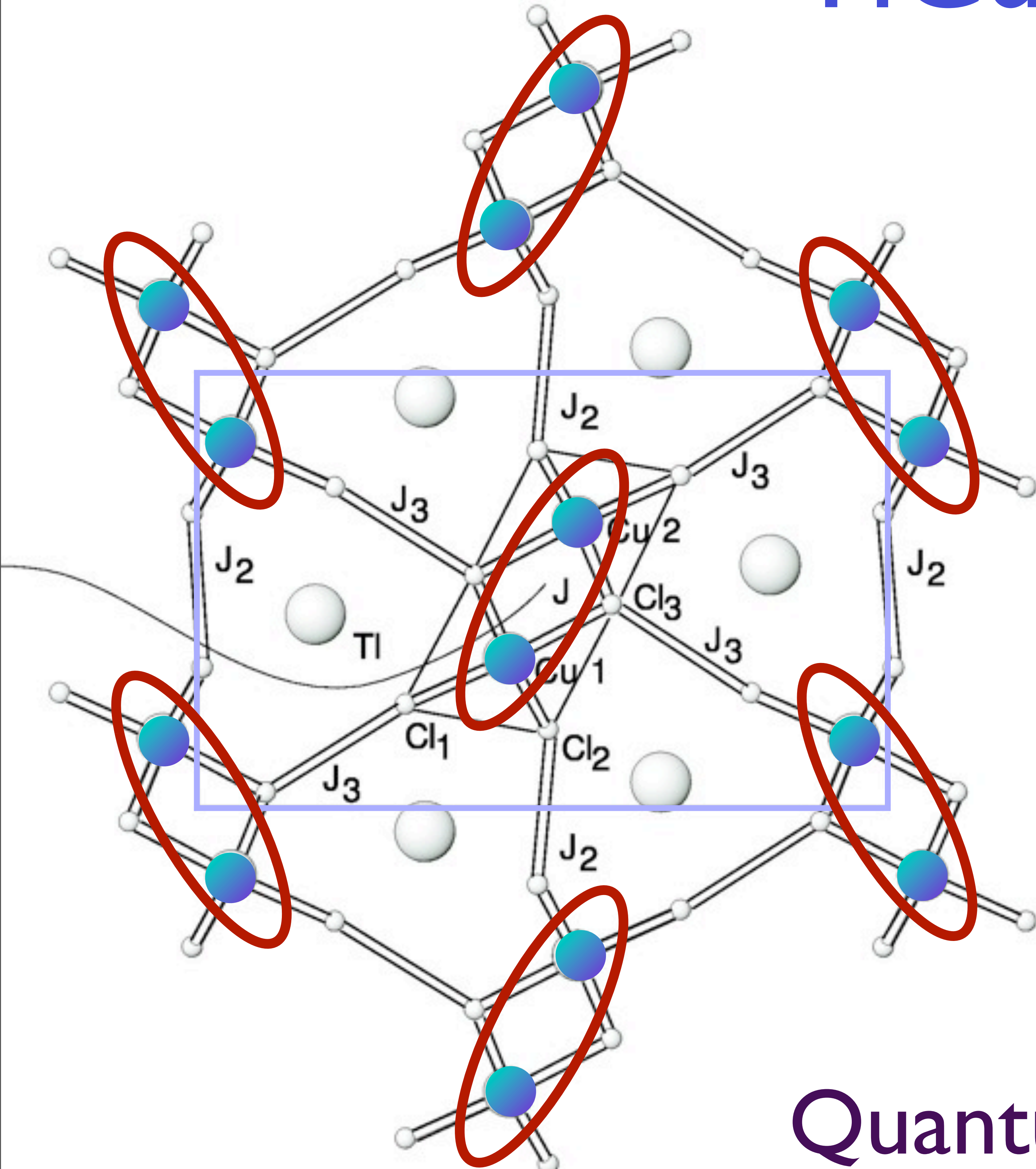
$TlCuCl_3$

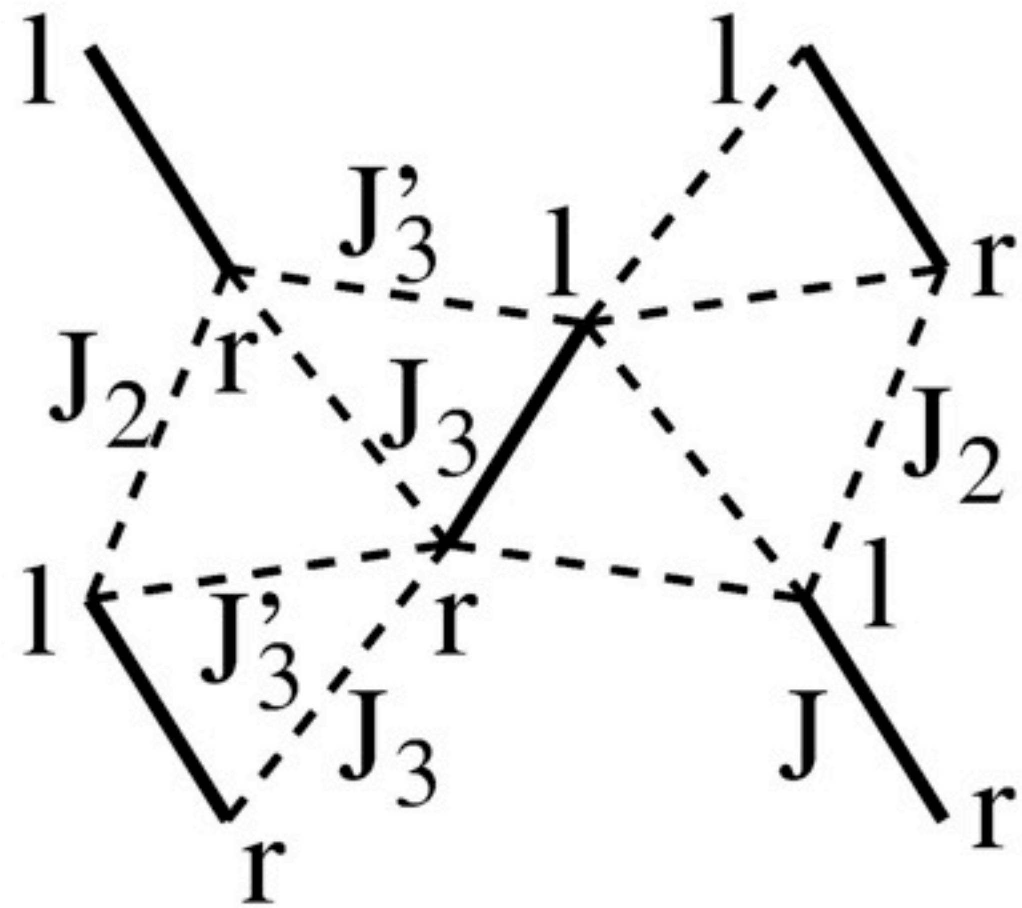




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

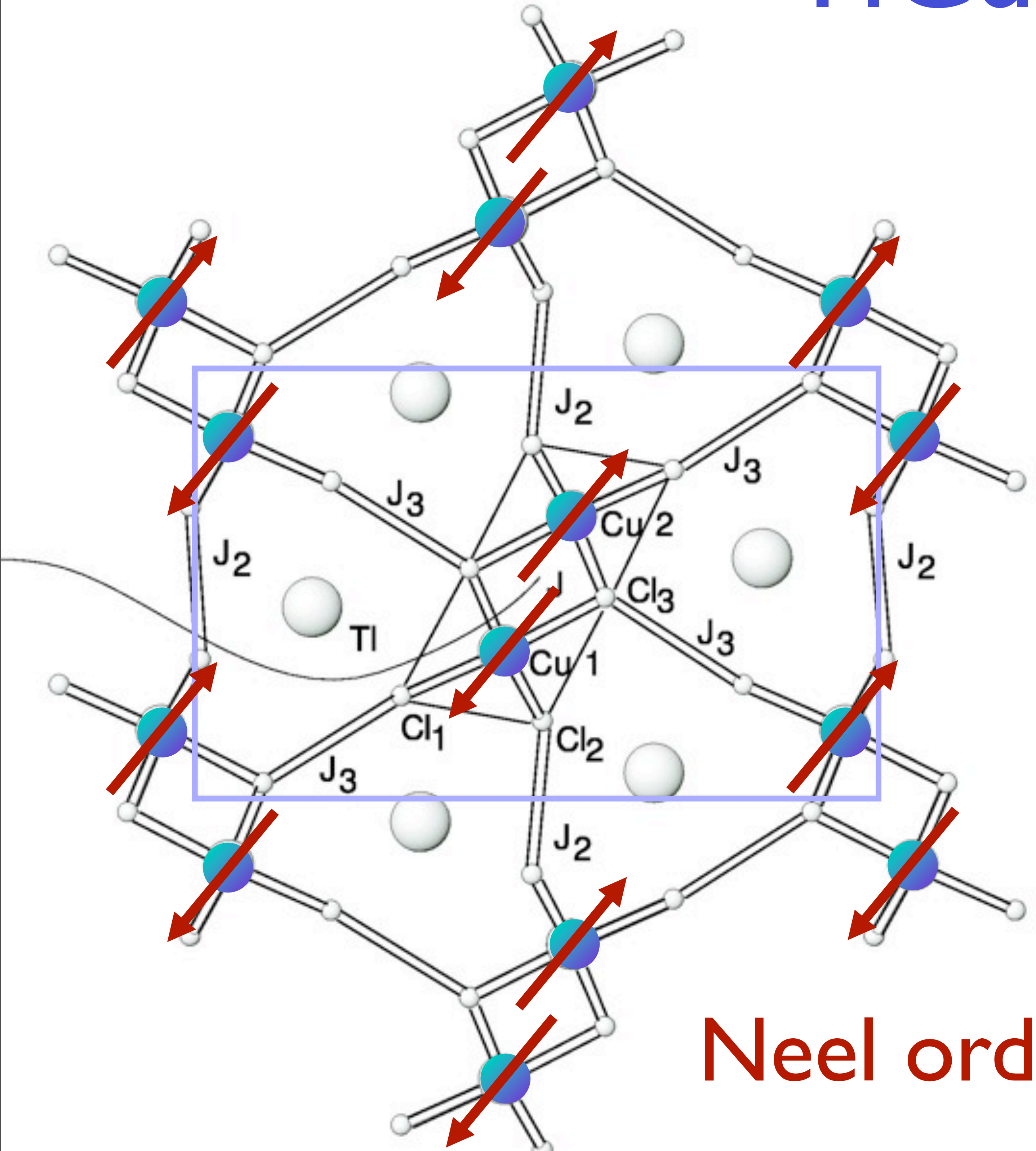
$TlCuCl_3$

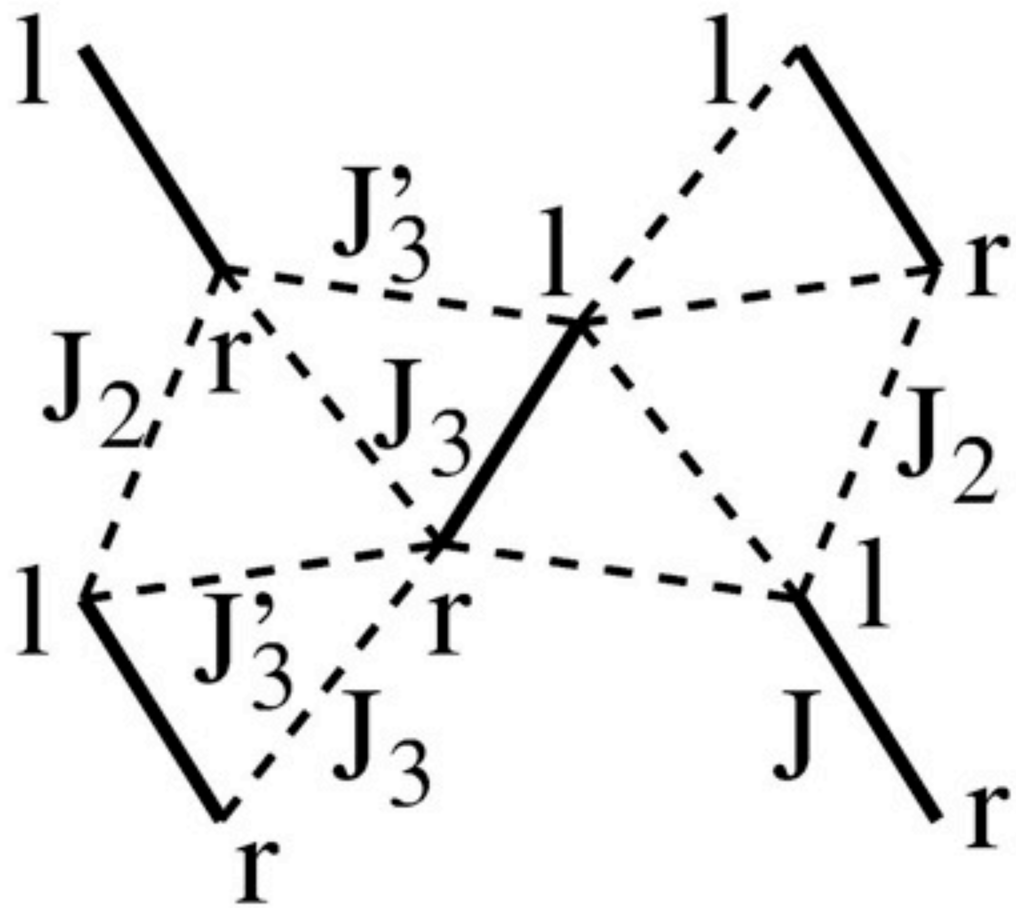




Quantum paramagnet at ambient pressure

$TlCuCl_3$

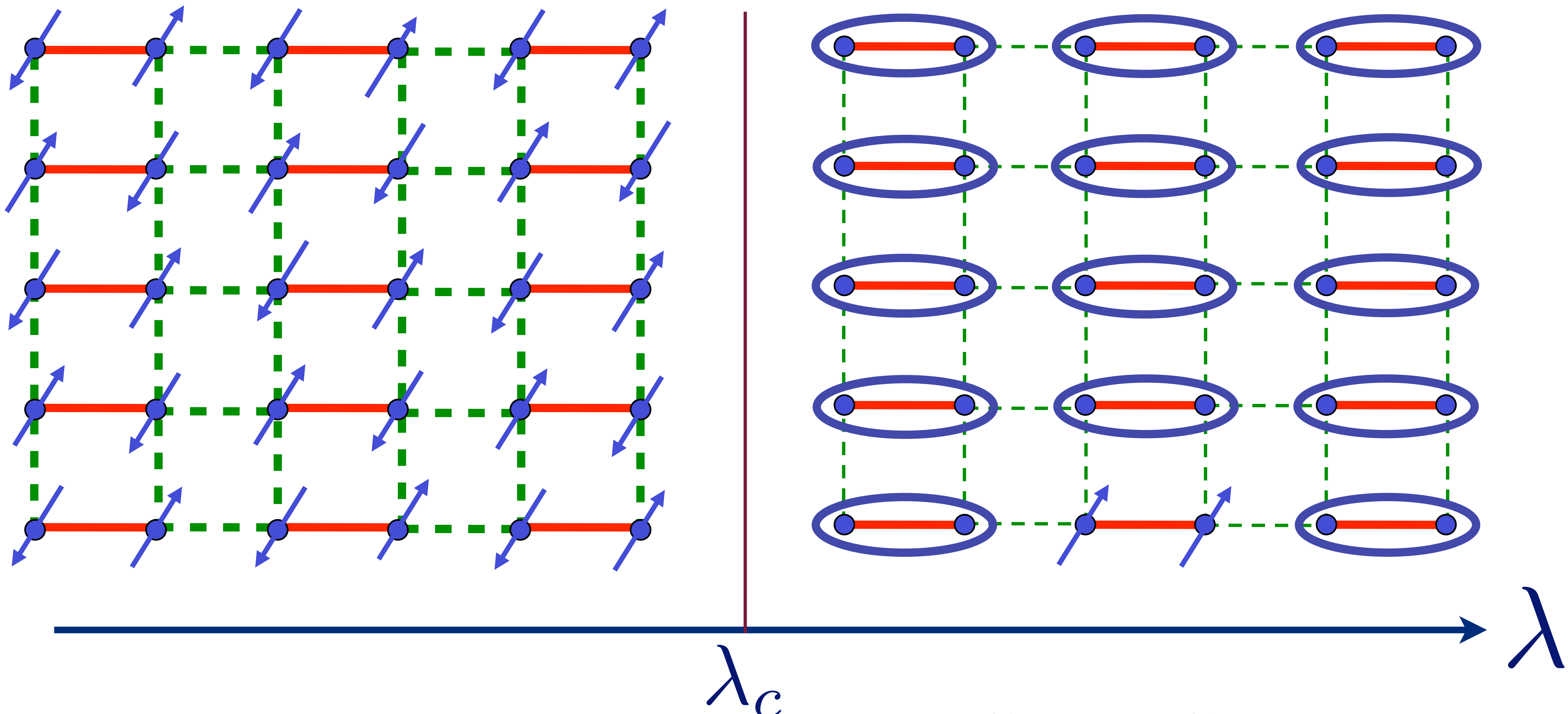




Neel order under pressure

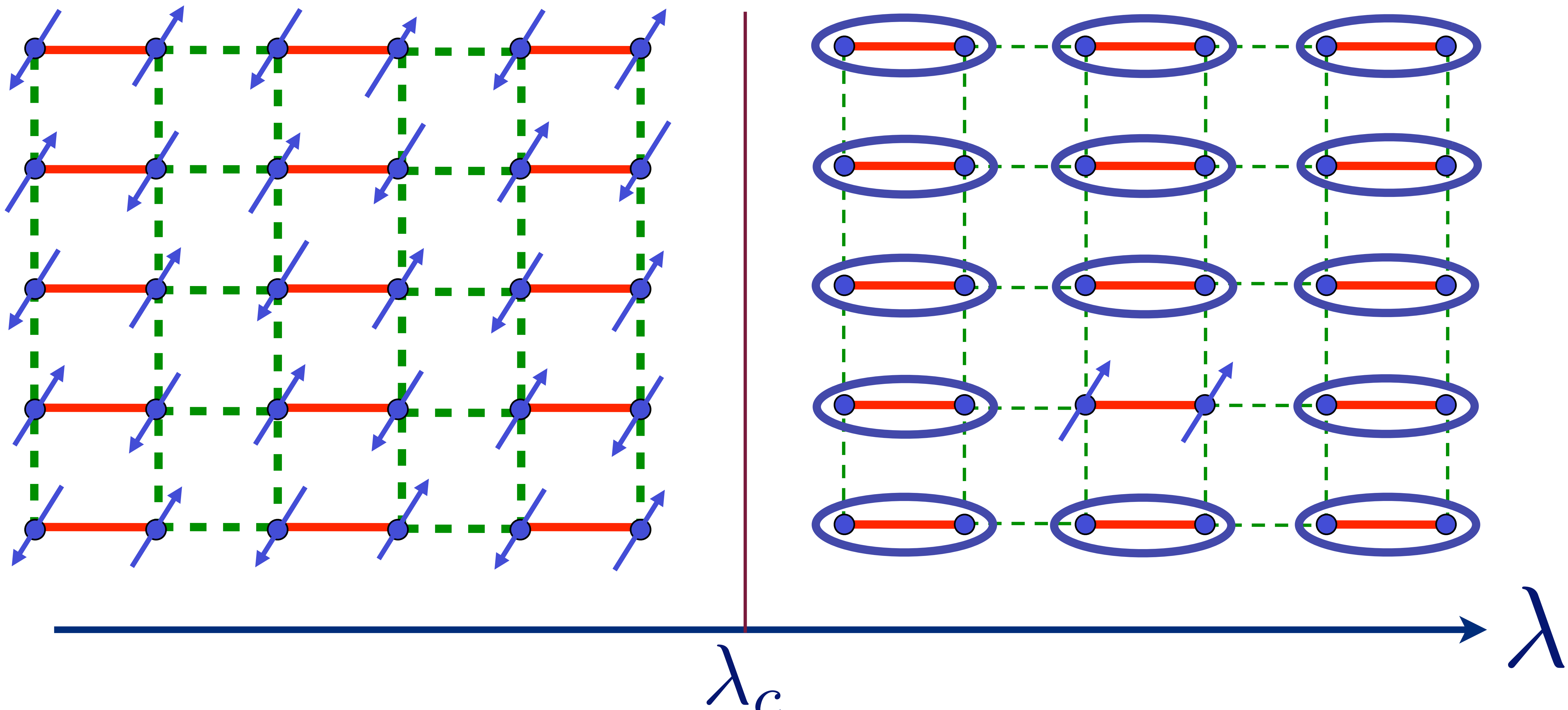
A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

Excitation spectrum in the paramagnetic phase



Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



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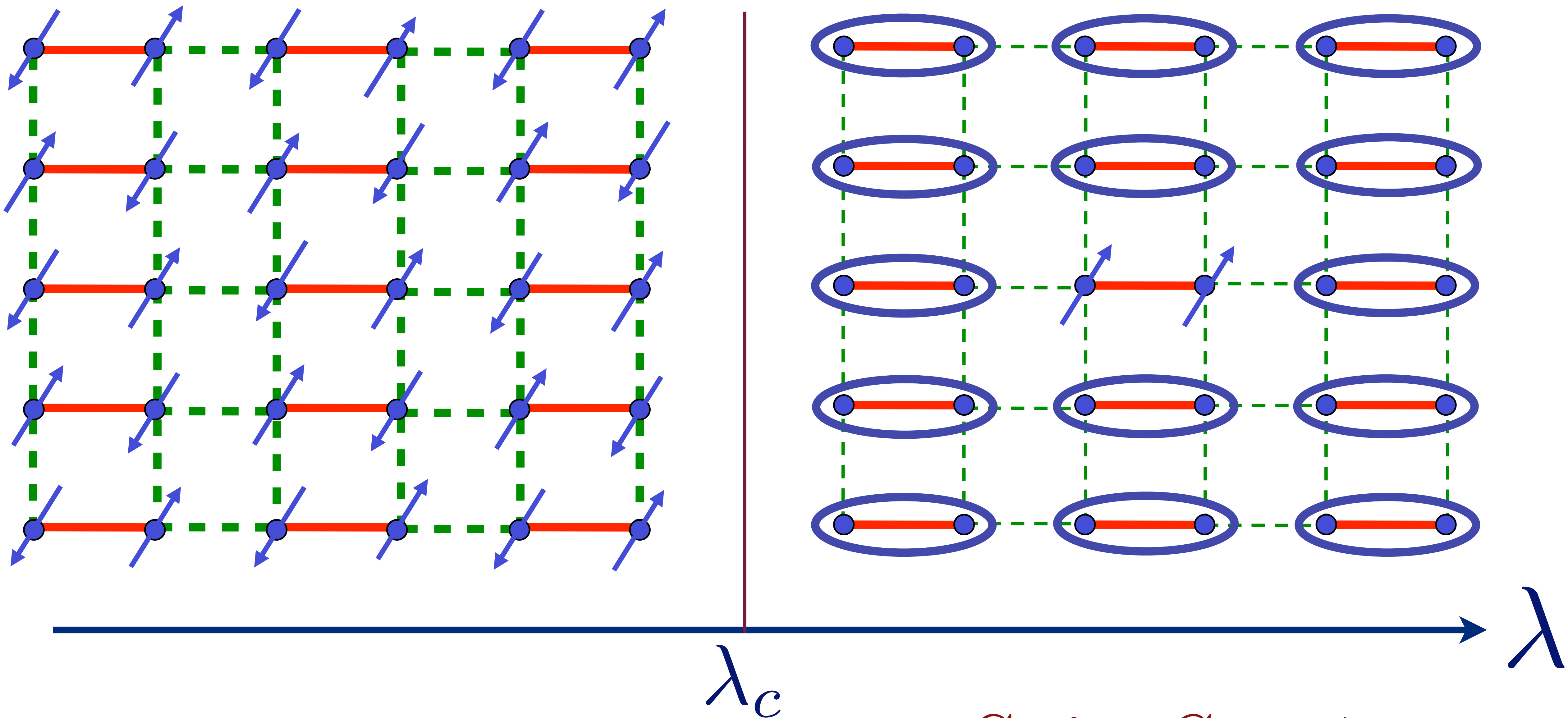

Spin $S = 1$
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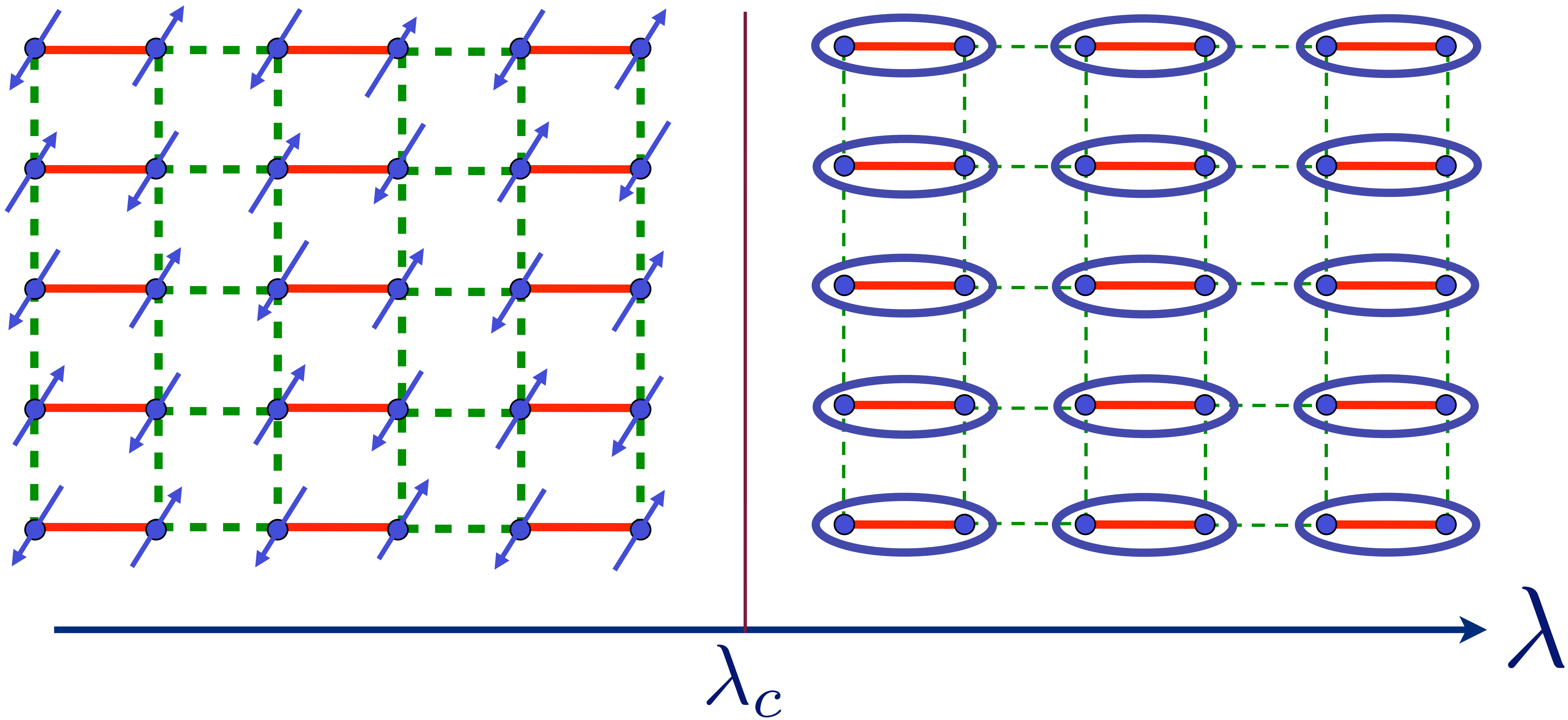
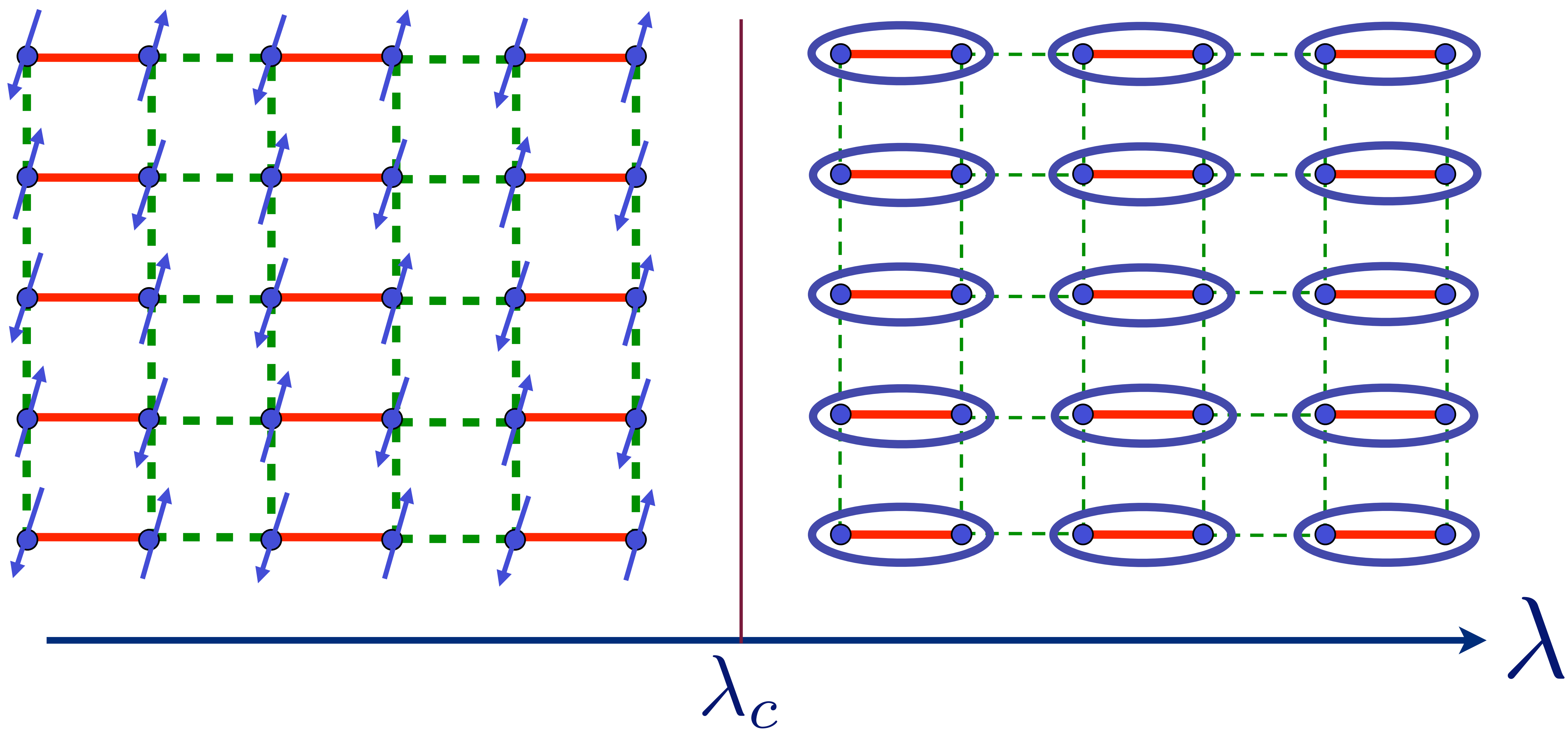
Excitation spectrum in the Néel phase



Spin waves

Excitation spectrum in the Néel phase



Spin waves

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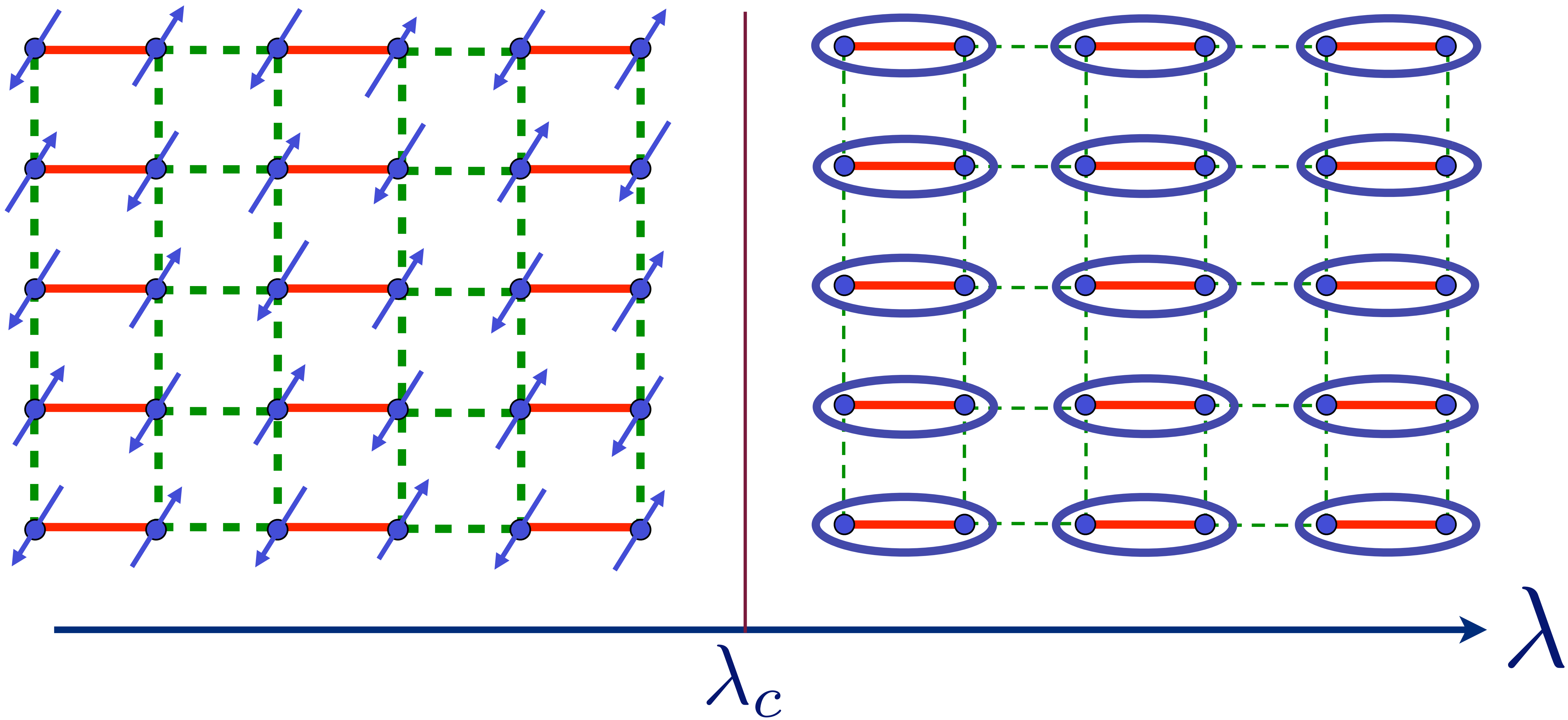
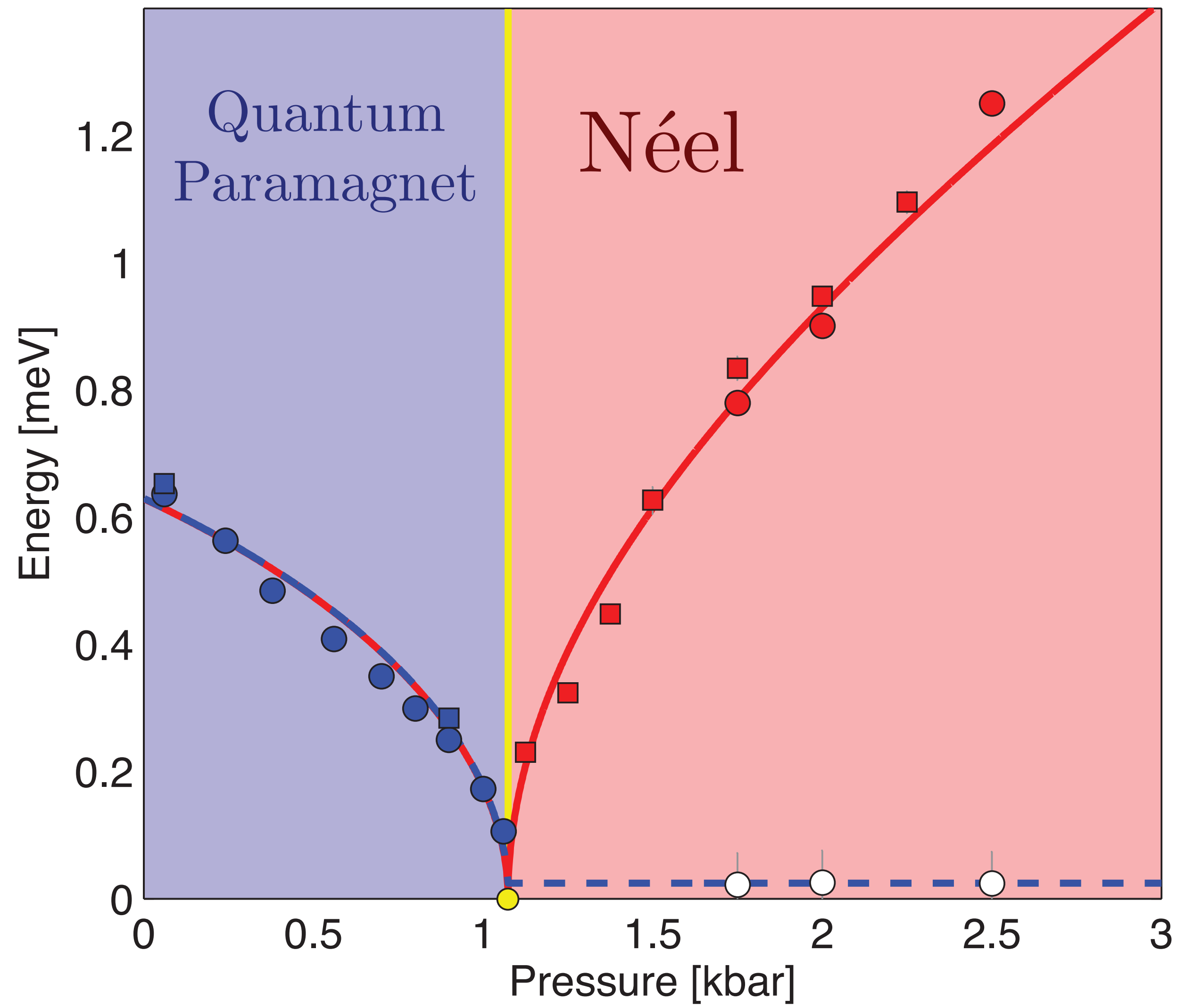


Spin waves

Excitations of $TlCuCl_3$ with varying pressure



Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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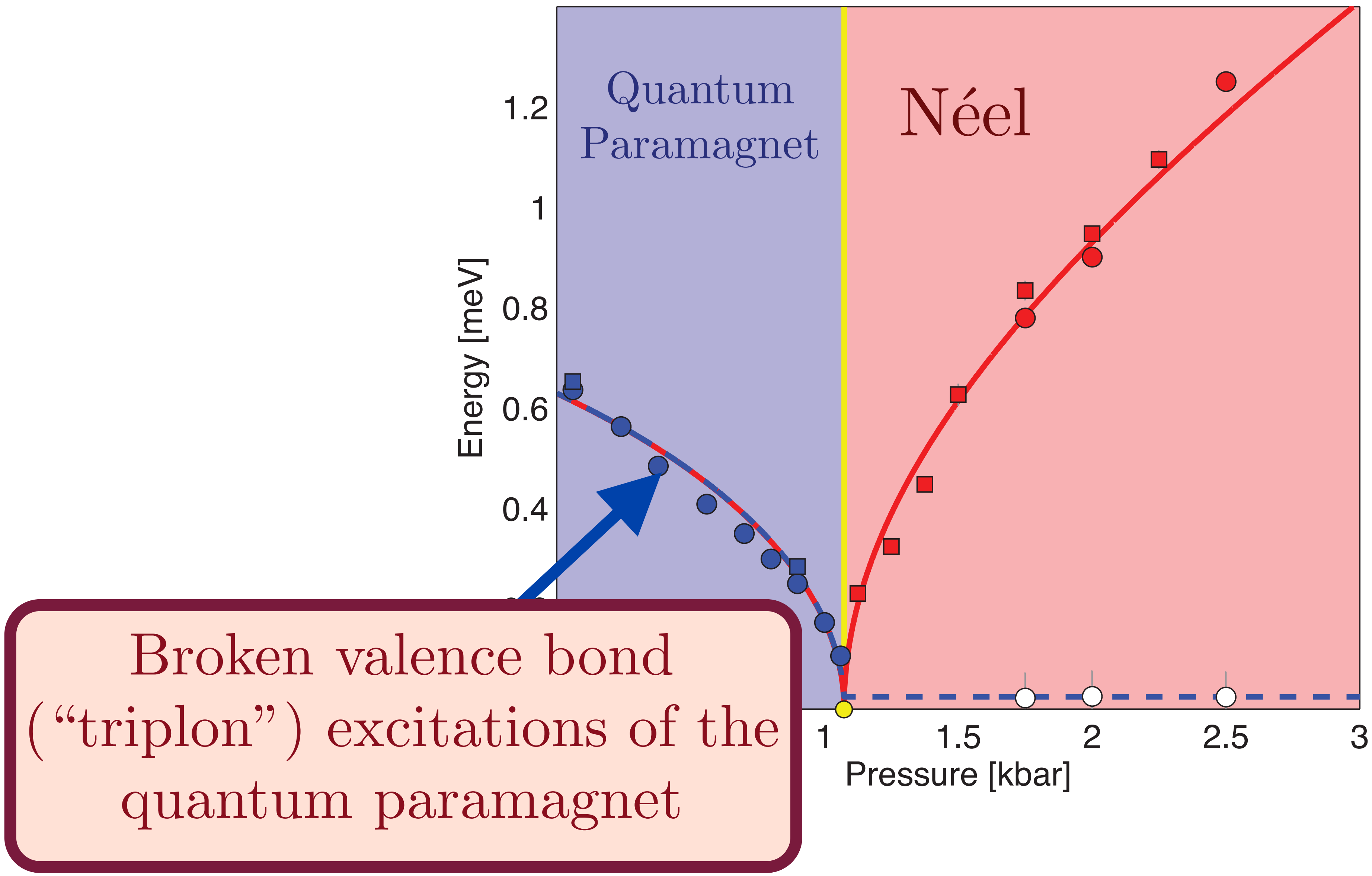

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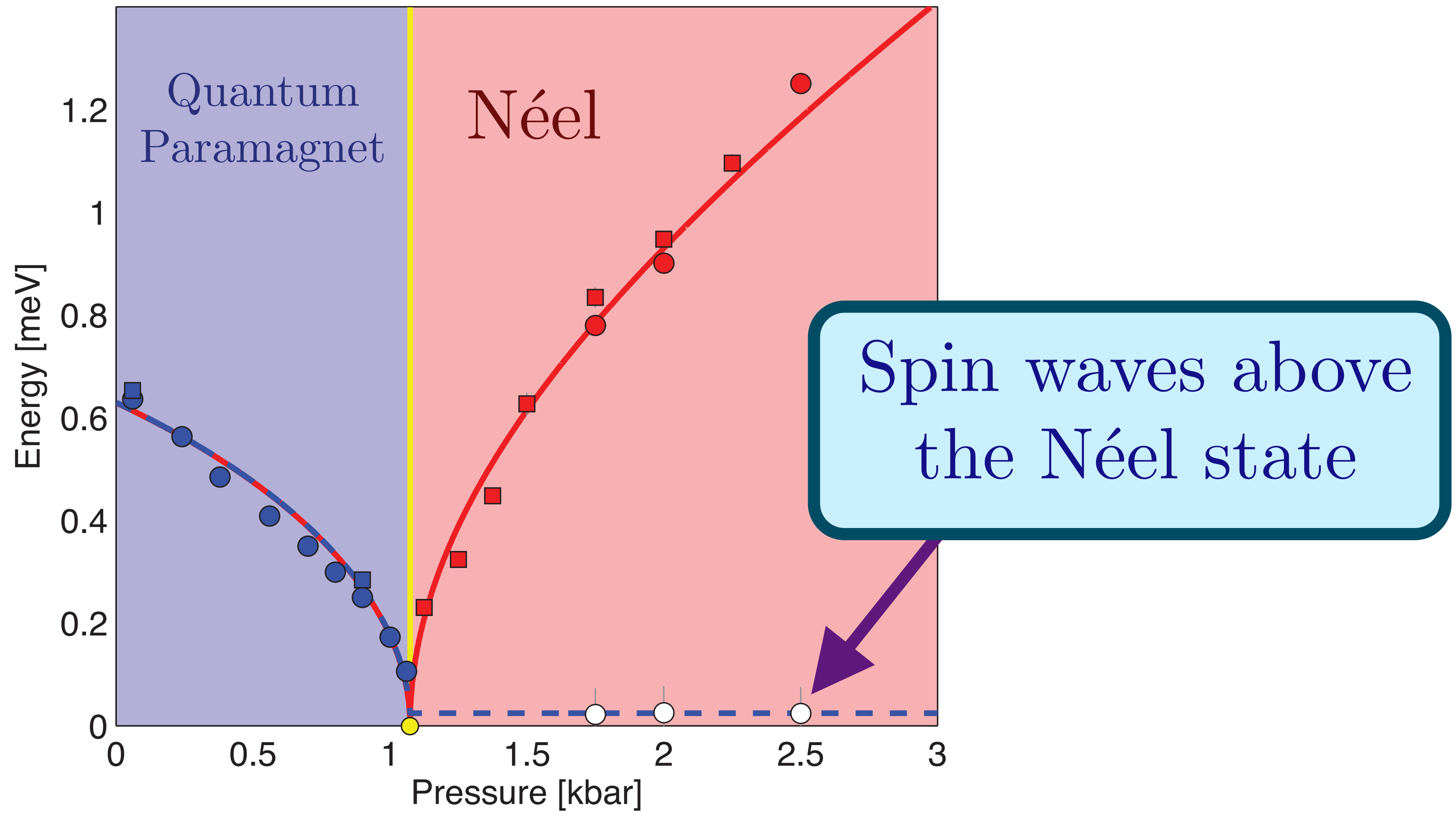
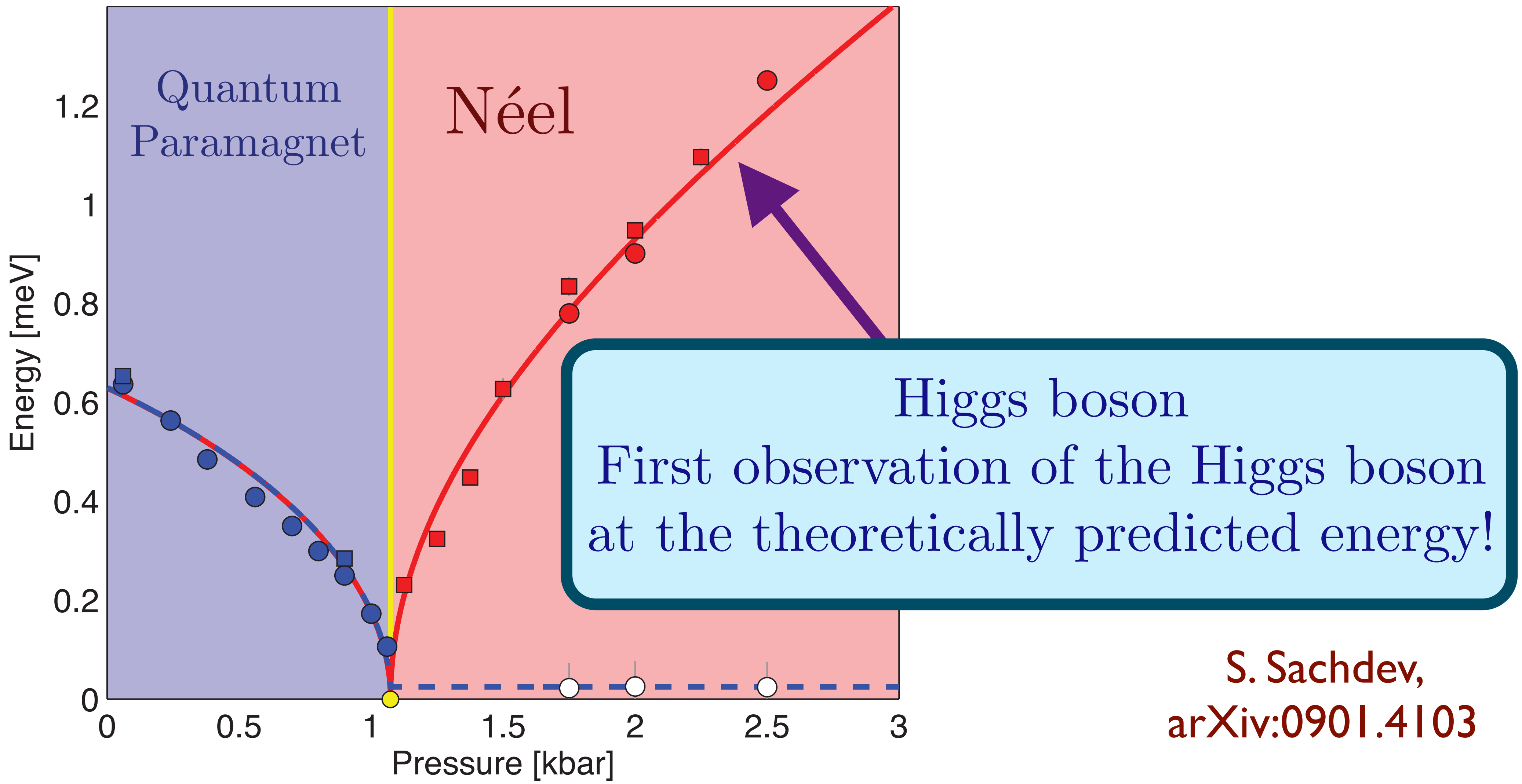


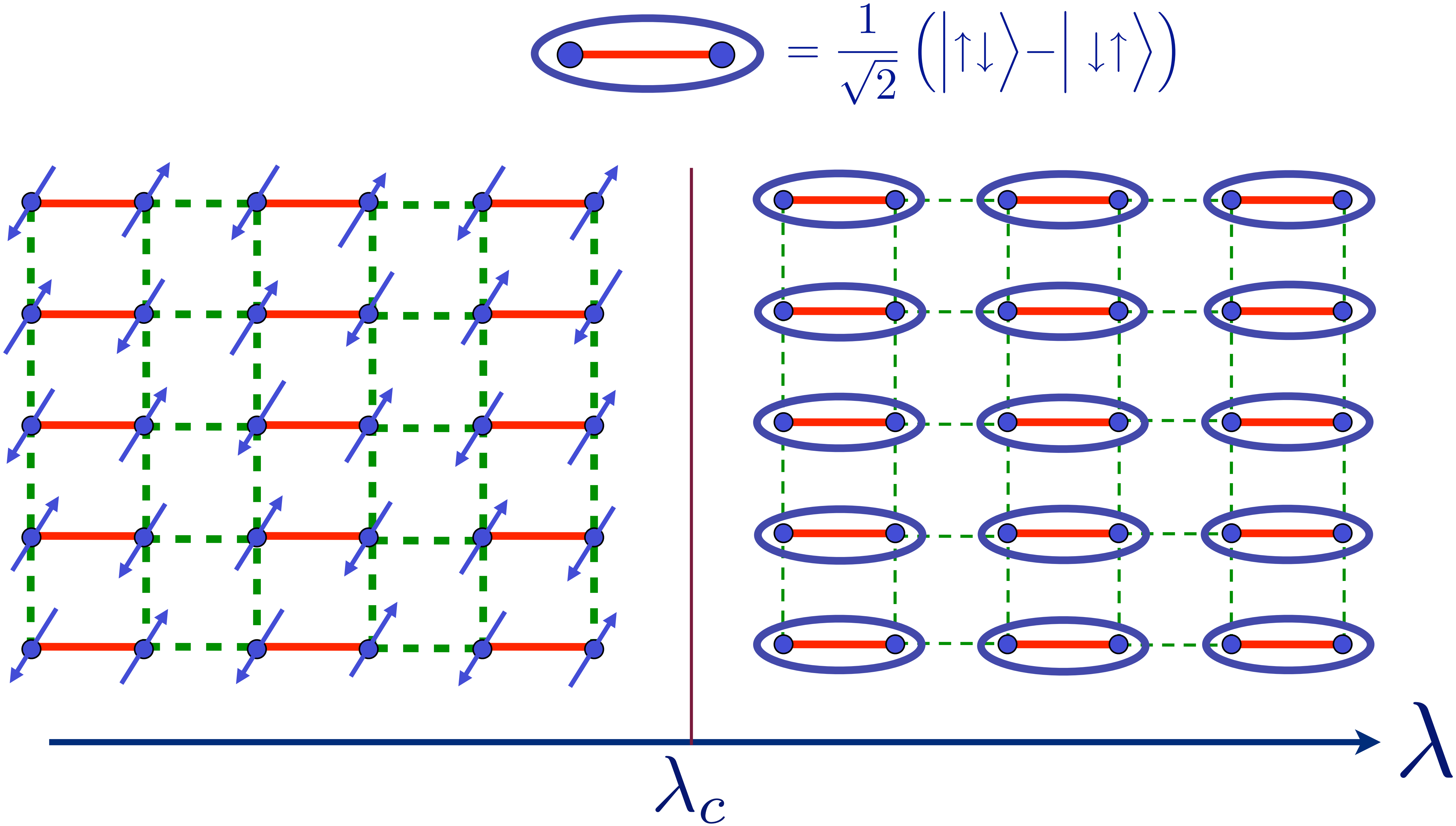
Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans–Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

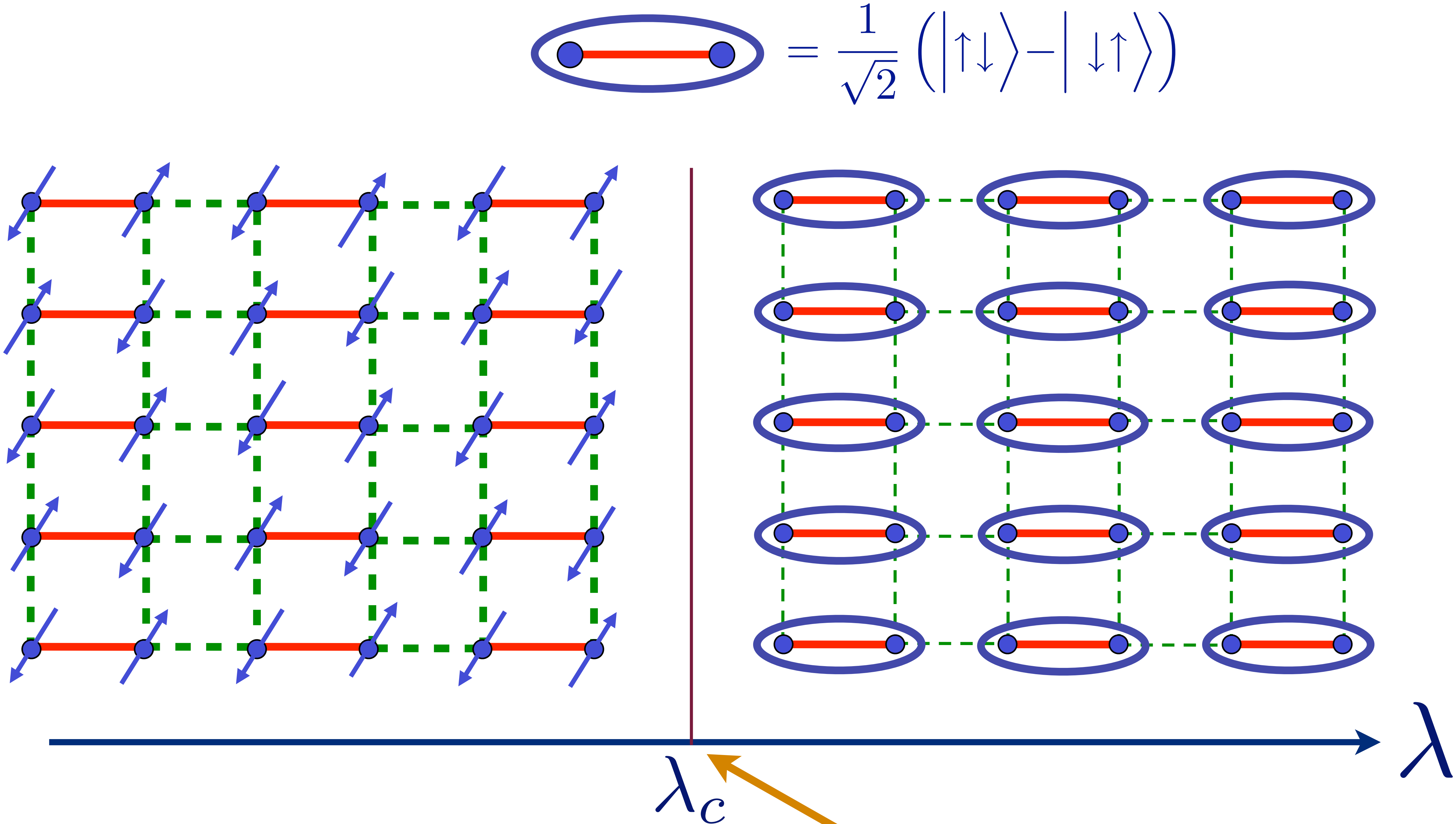
Excitations of $TlCuCl_3$ with varying pressure



S. Sachdev,
arXiv:0901.4103

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

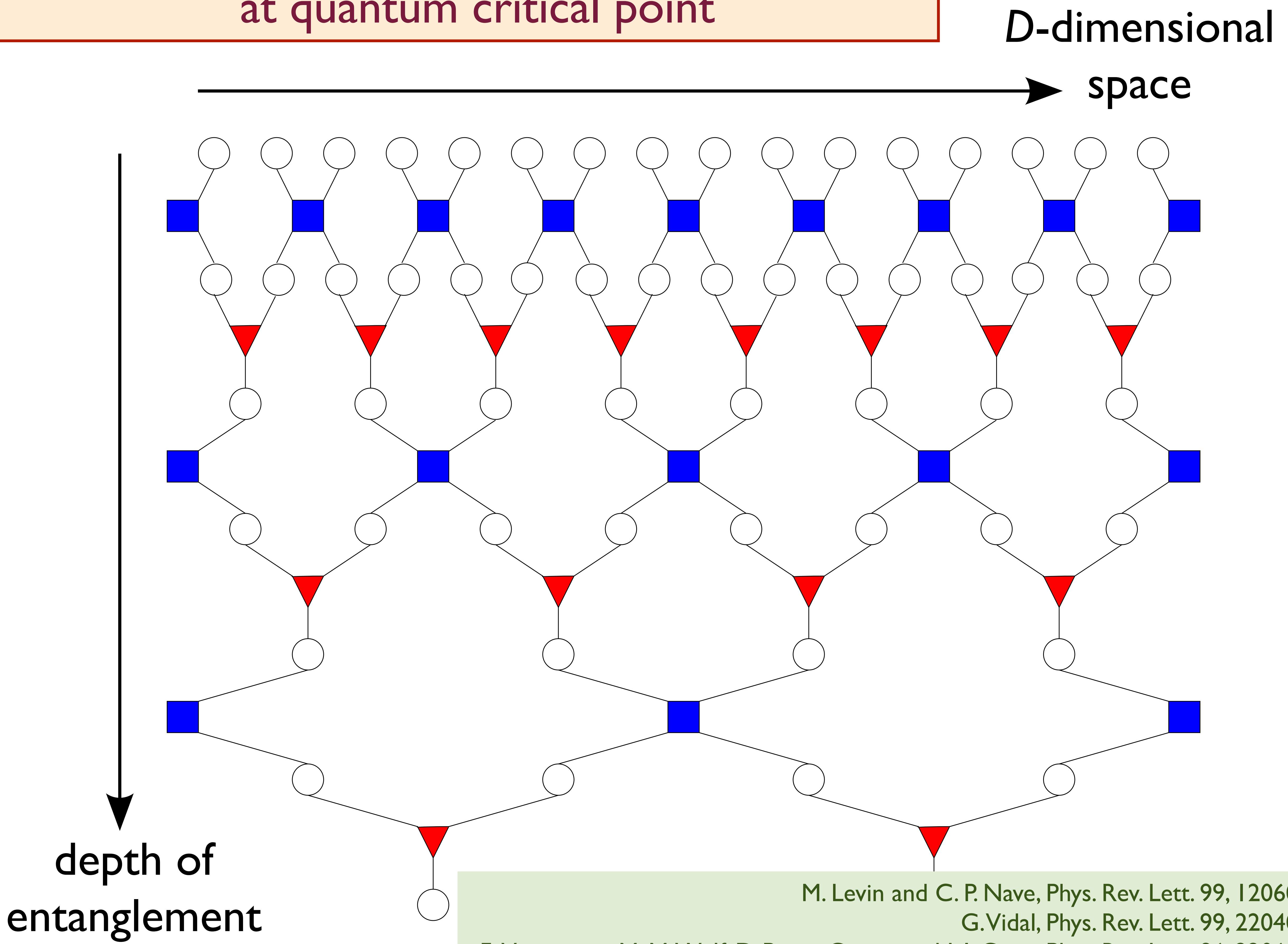

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ
λ_c



Quantum critical point with non-local entanglement in spin wavefunction

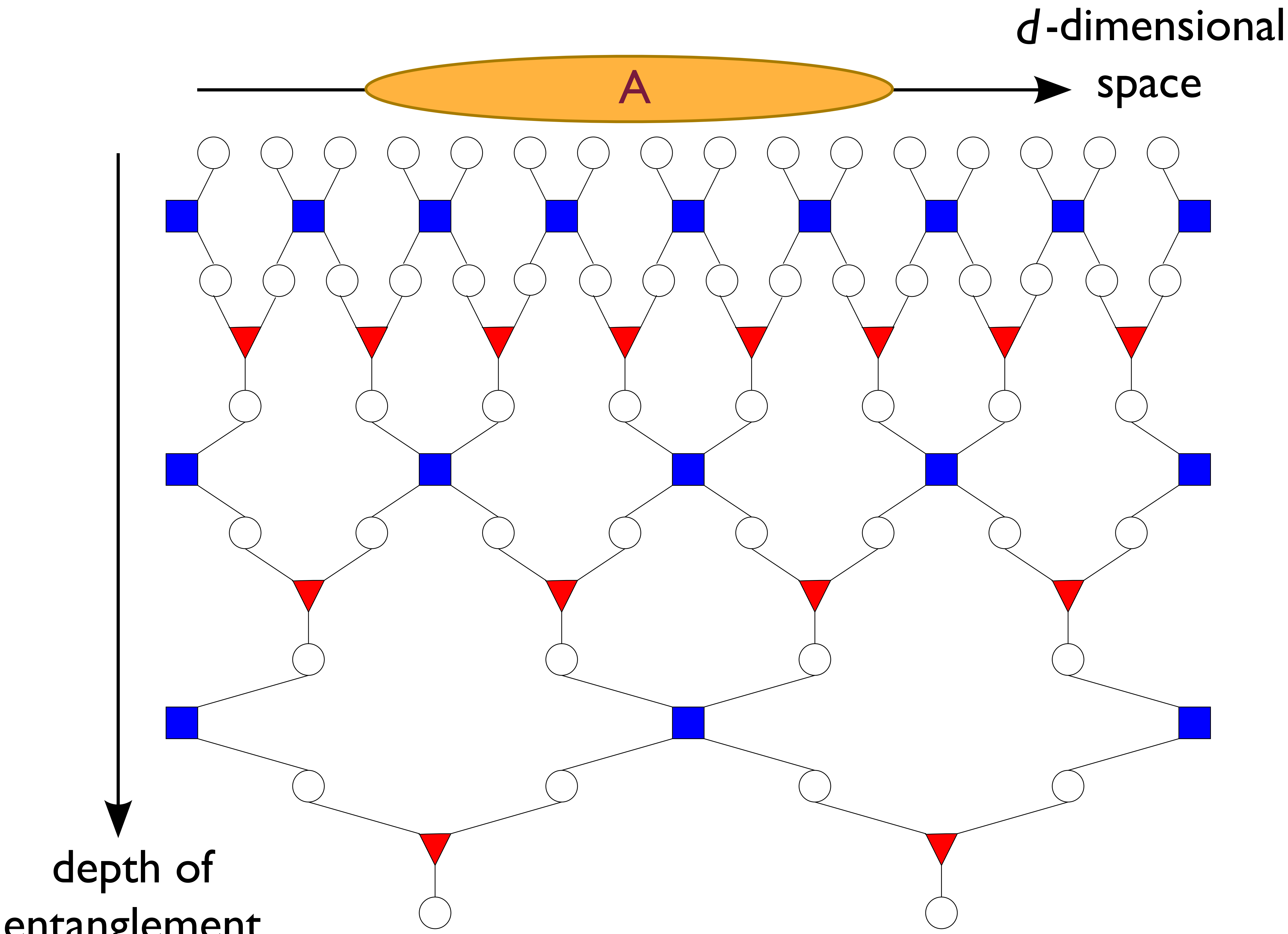
A. W. Sandvik and D. J. Scalapino, Phys. Rev. Lett. **72**, 2777 (1994).

Tensor network representation of entanglement at quantum critical point



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
G. Vidal, Phys. Rev. Lett. 99, 220405 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

Entanglement entropy



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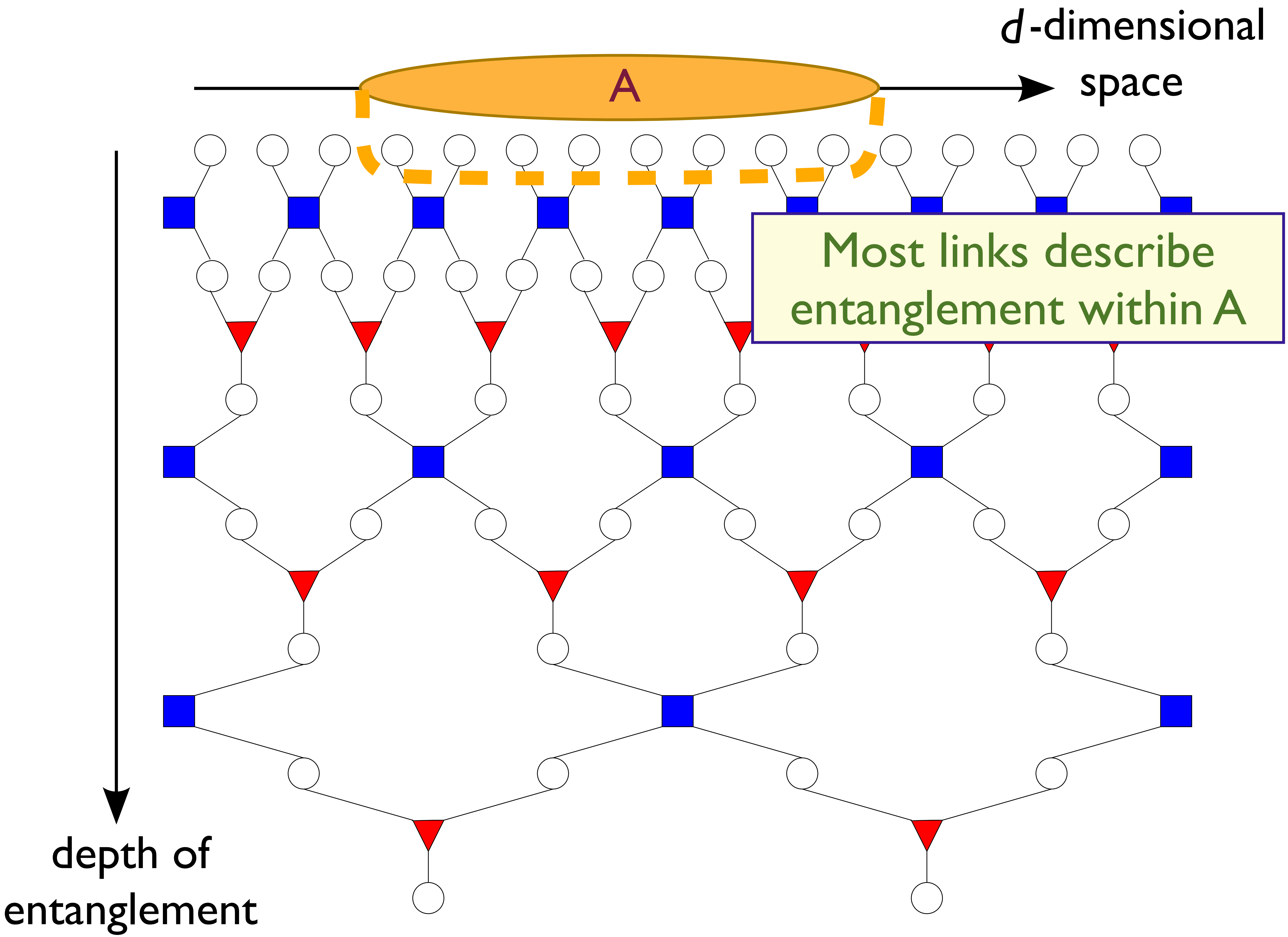

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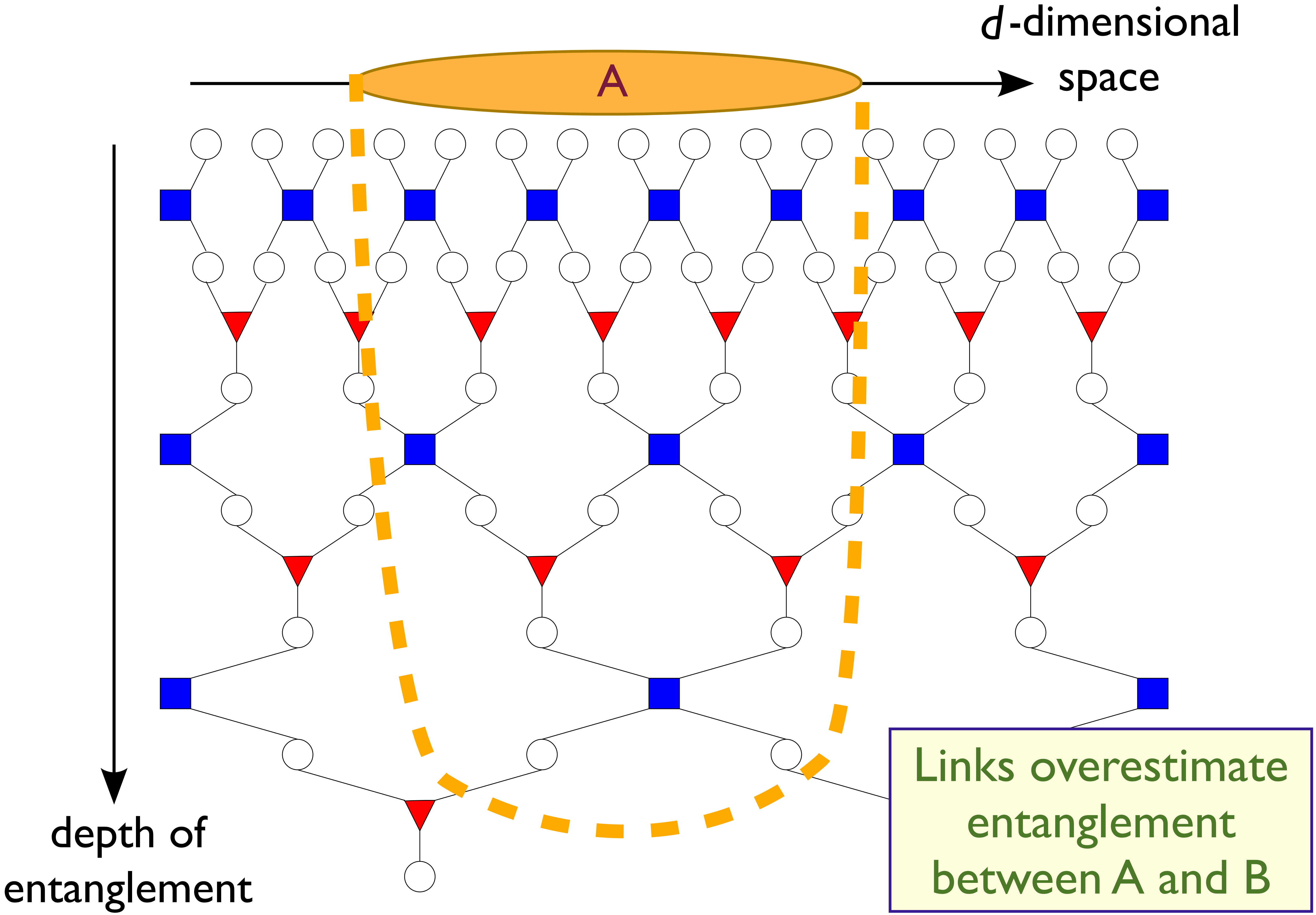
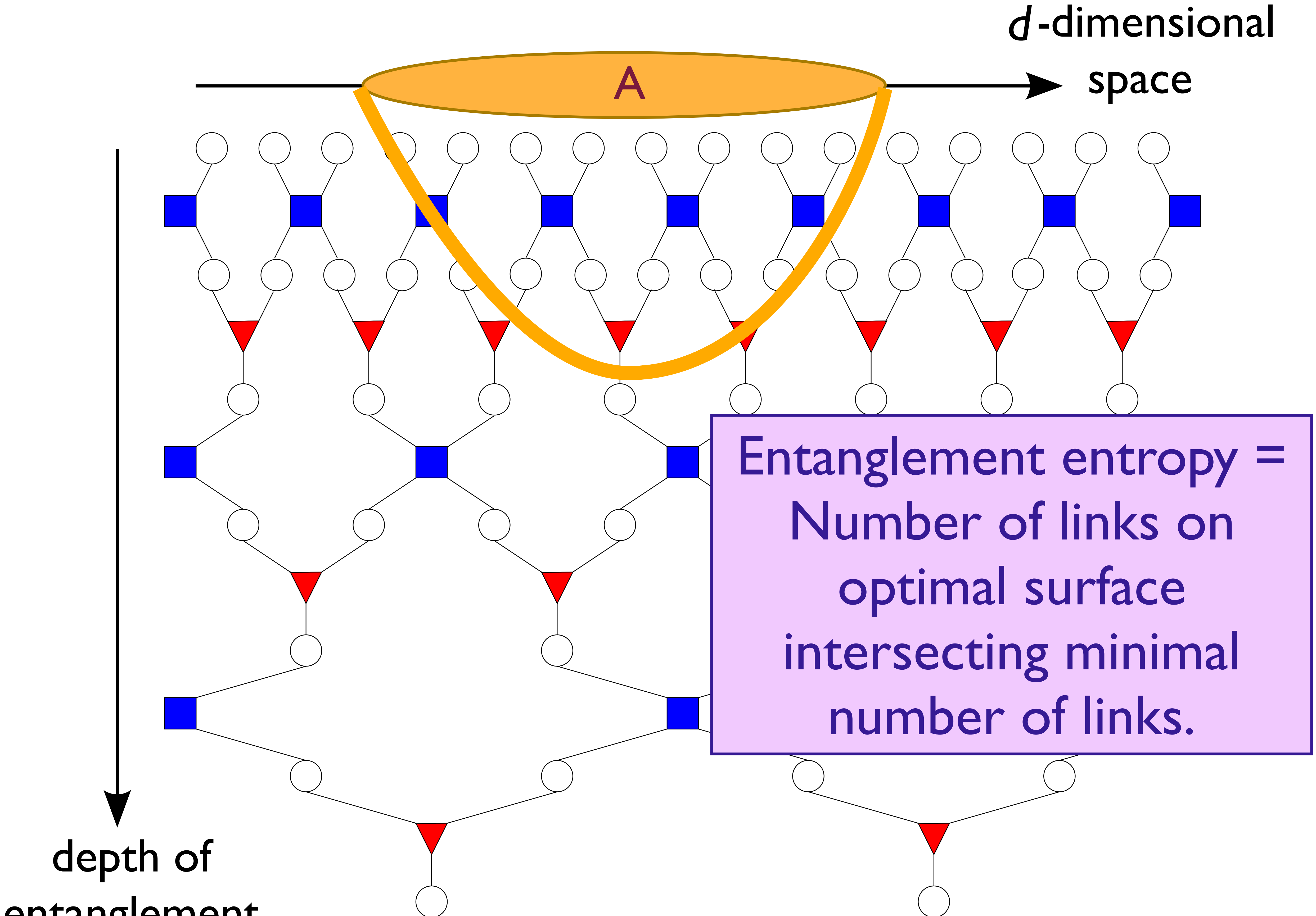


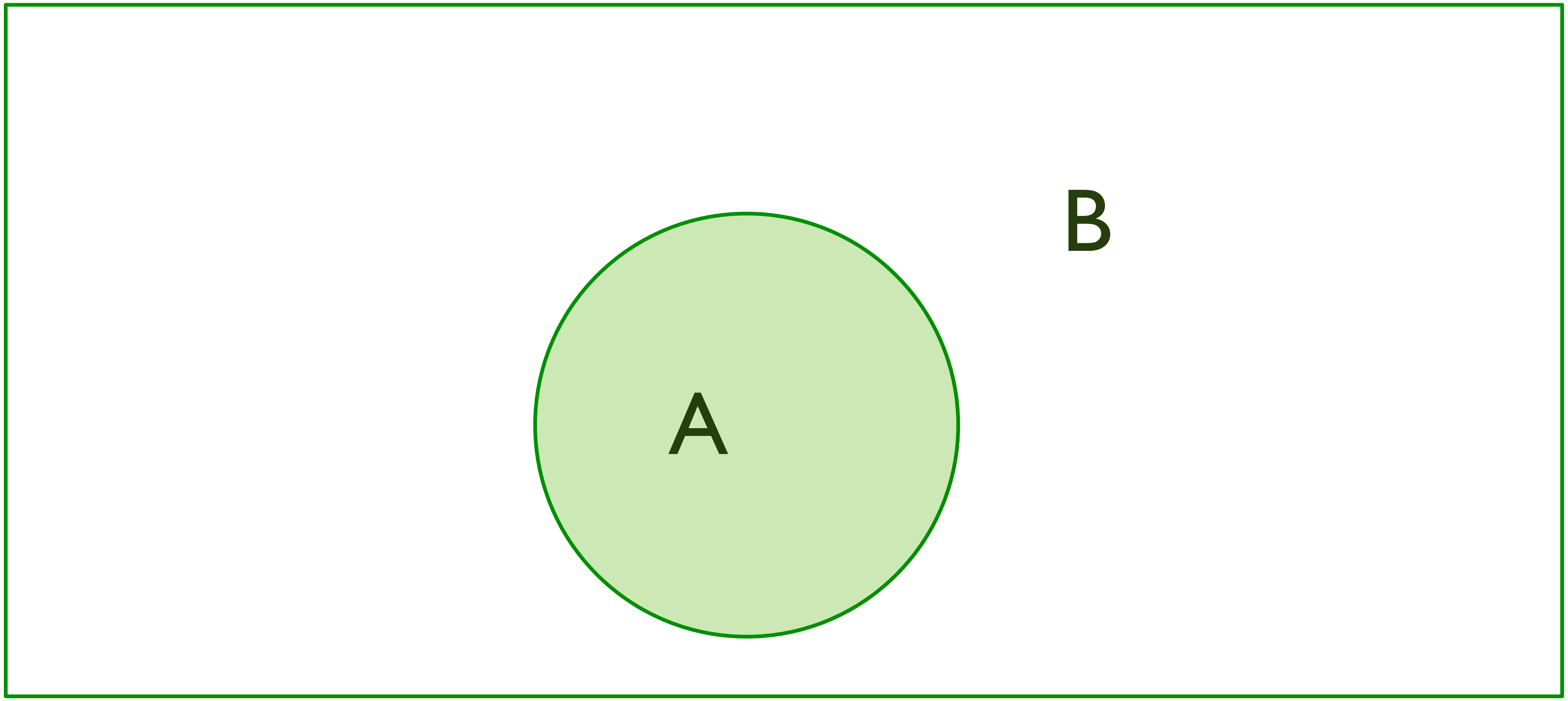
Entanglement entropy



Entanglement entropy = Number of links on optimal surface intersecting minimal number of links.

Long-range entanglement in a CFT3

- Long-range entanglement: entanglement entropy obeys $S_{EE} = aL - \gamma$, where γ is a universal number associated with the quantum critical point.



M. A. Metlitski, C. A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

Characteristics of quantum critical point

- Long-range entanglement

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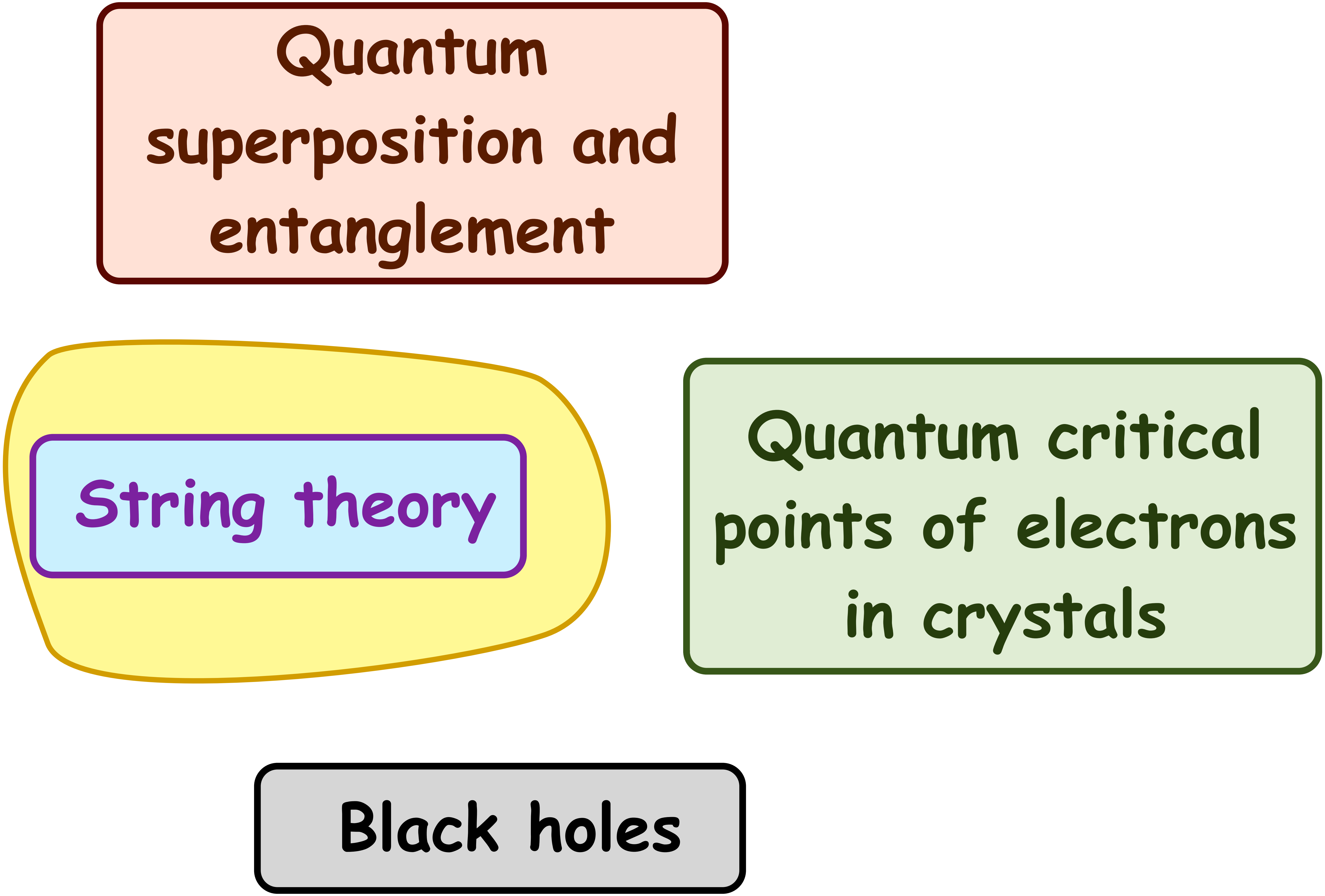
Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

Black holes


Quantum superposition and entanglement
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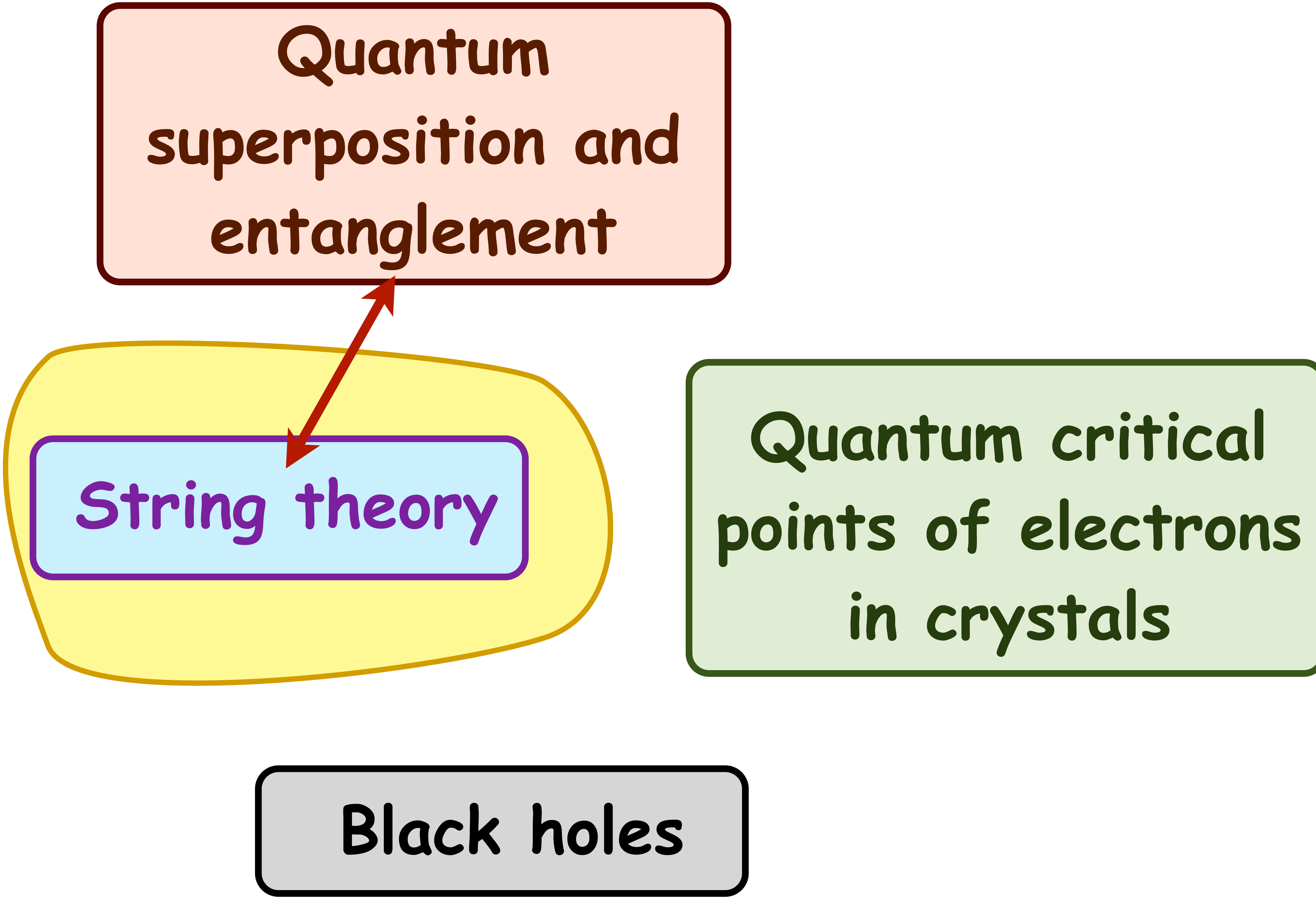
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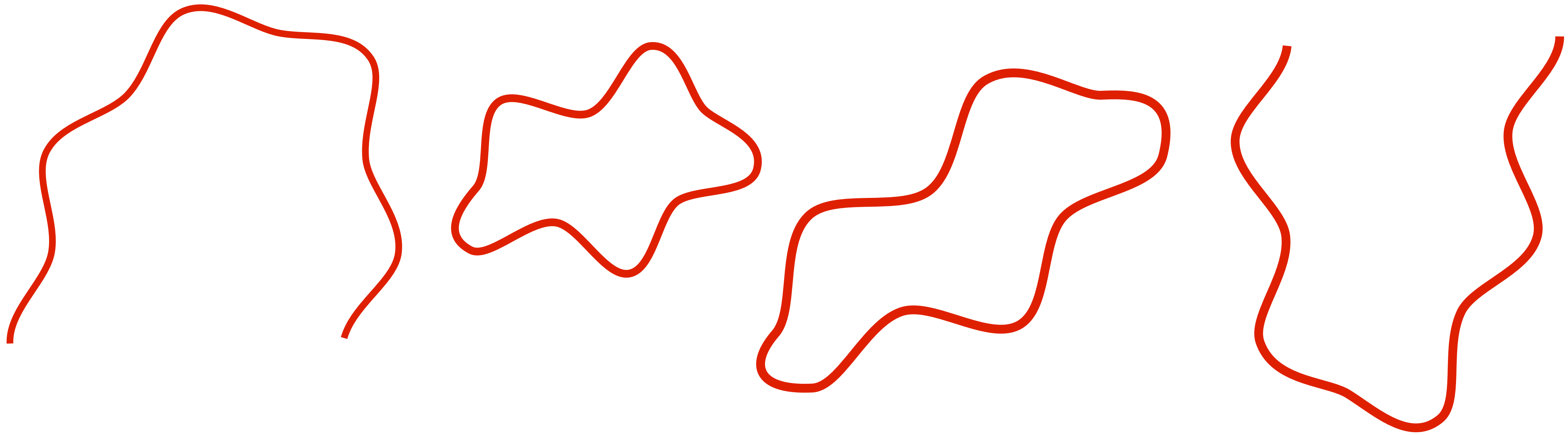
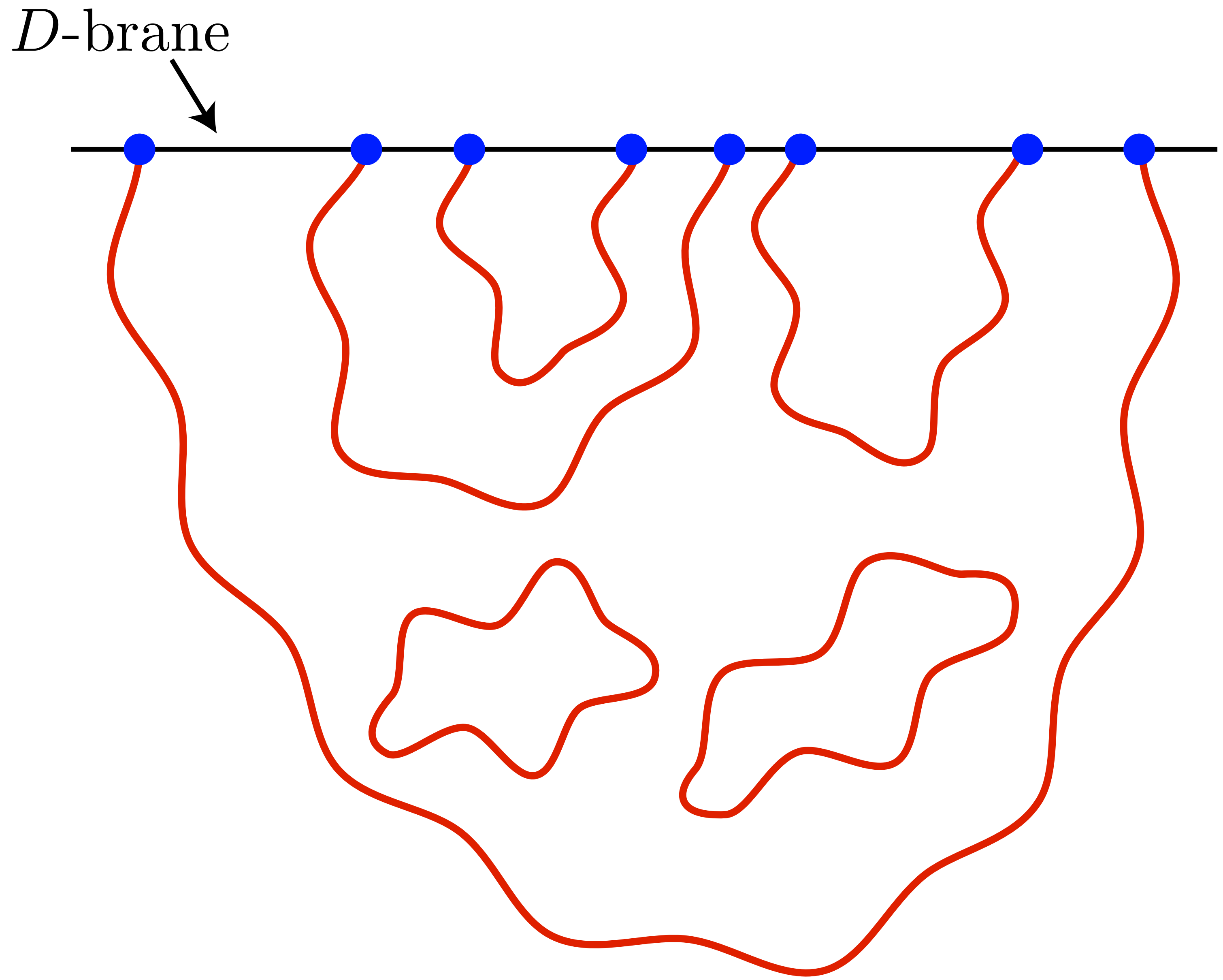
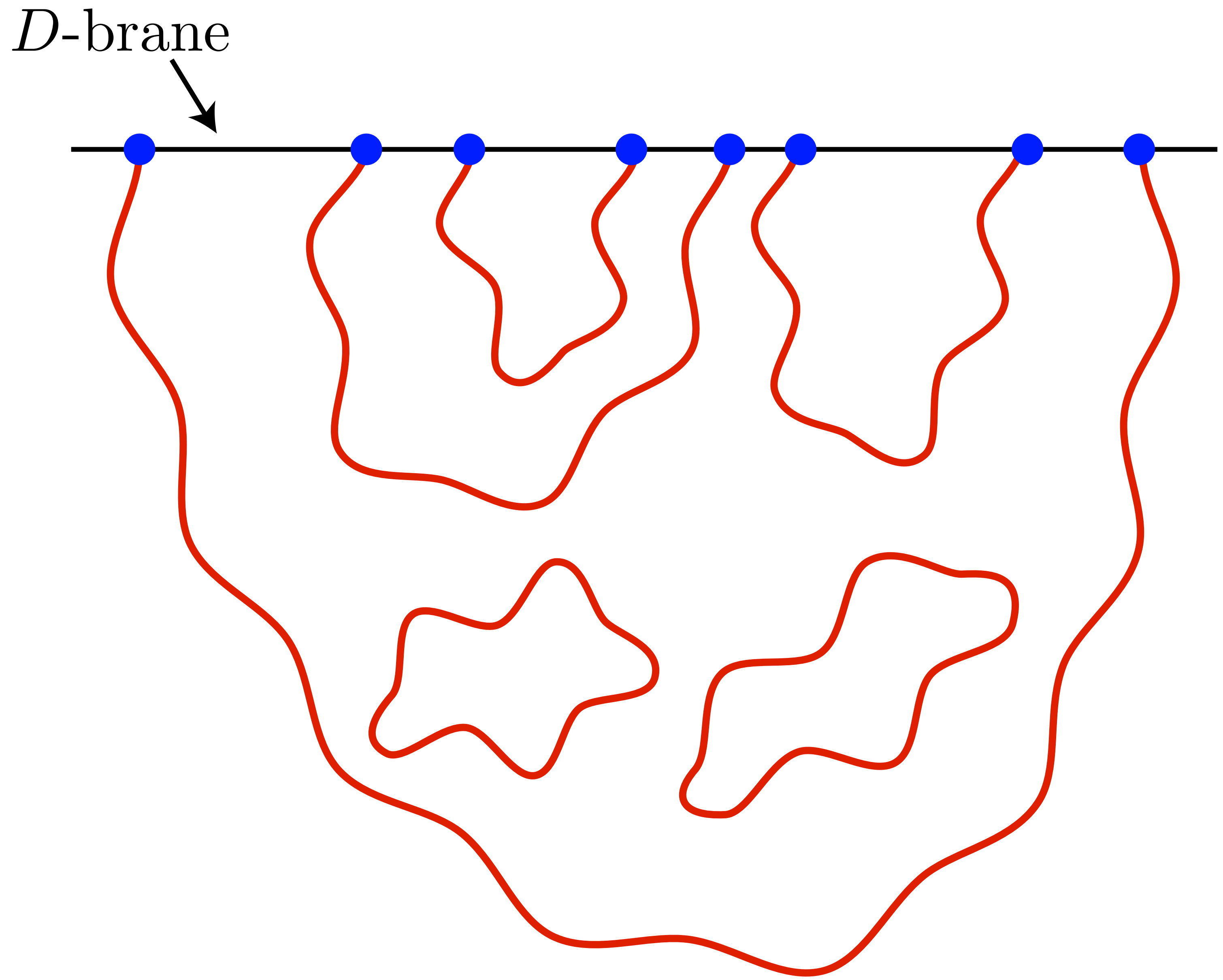
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks ...



- A D-brane is a d-dimensional surface on which strings can end.



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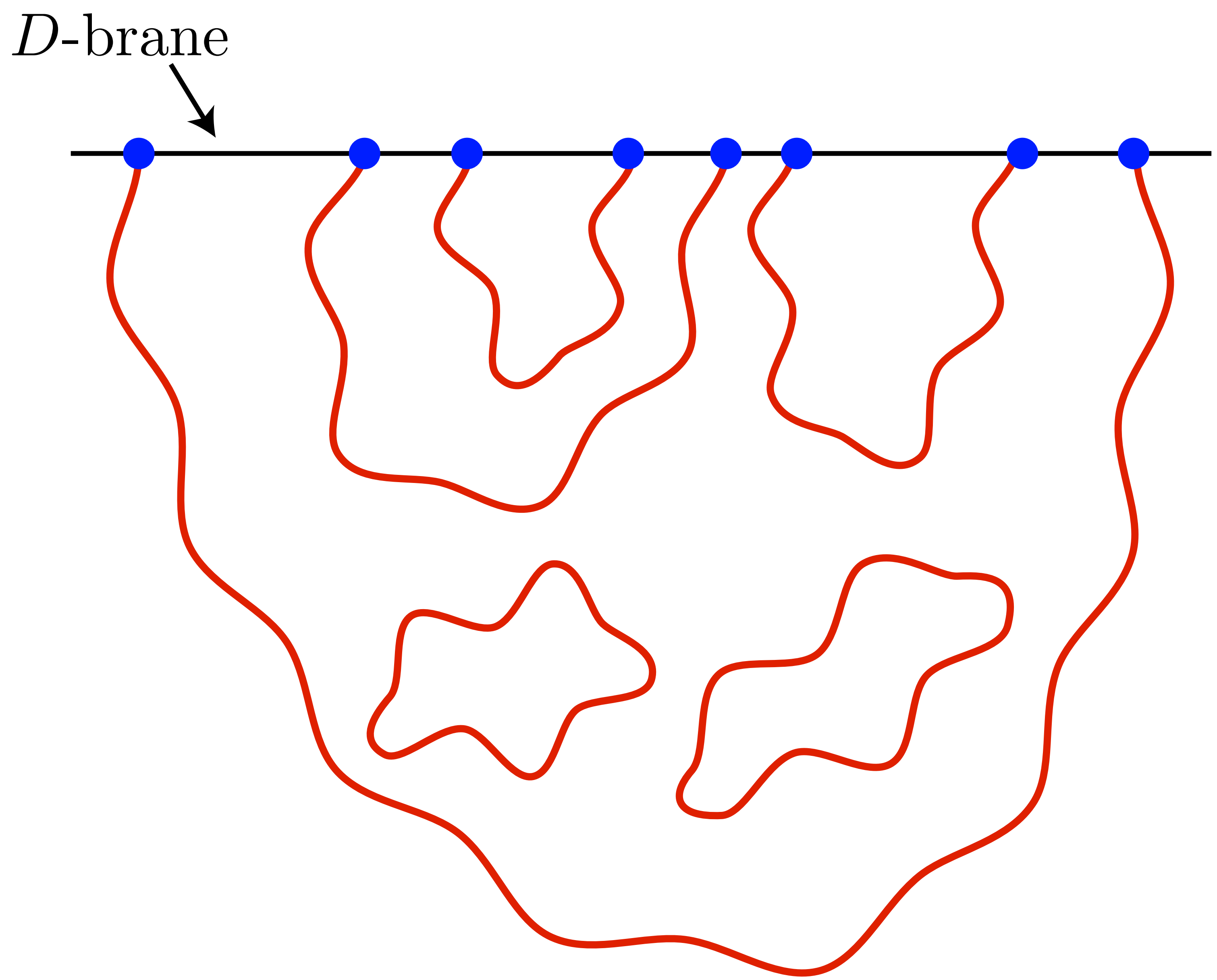

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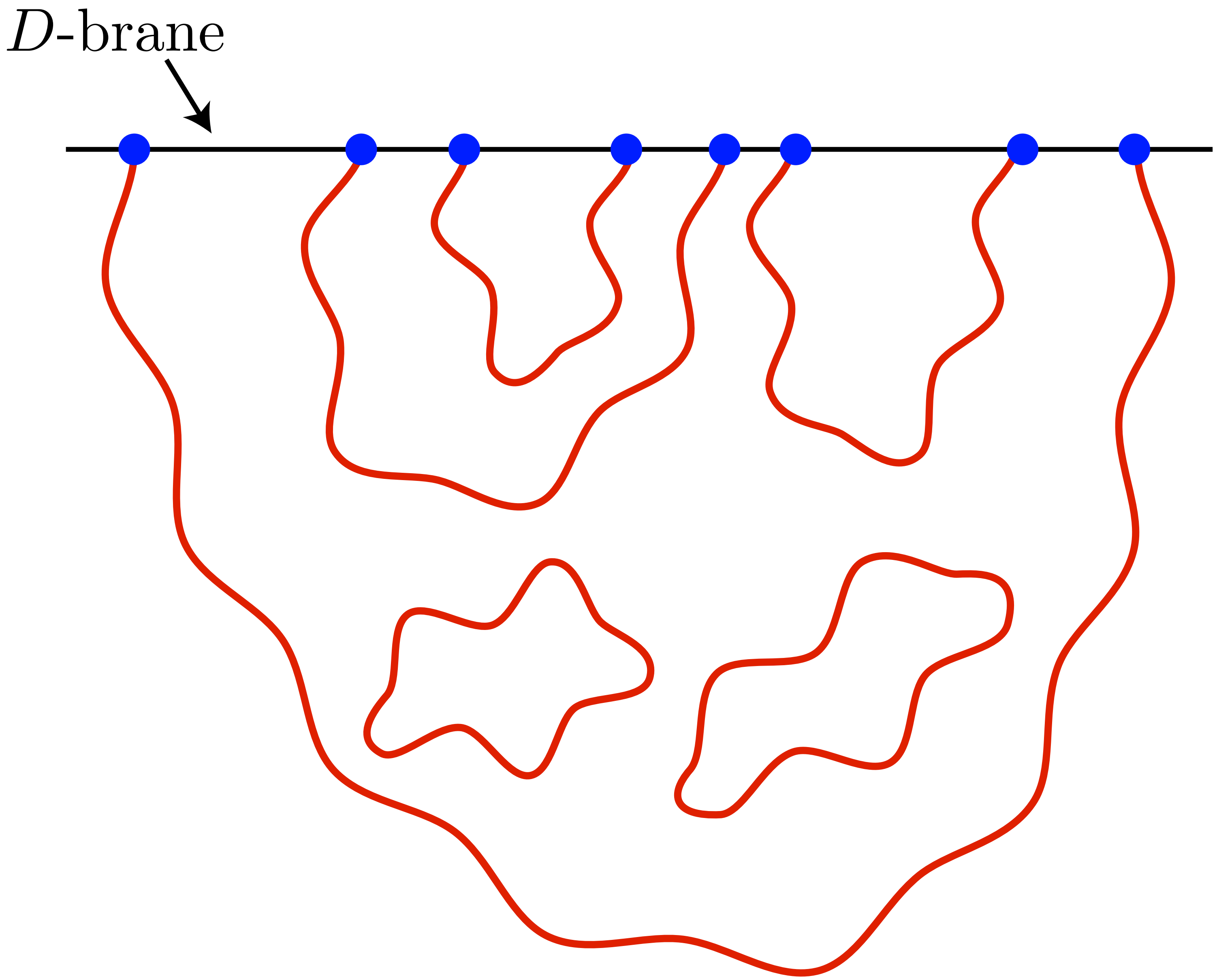

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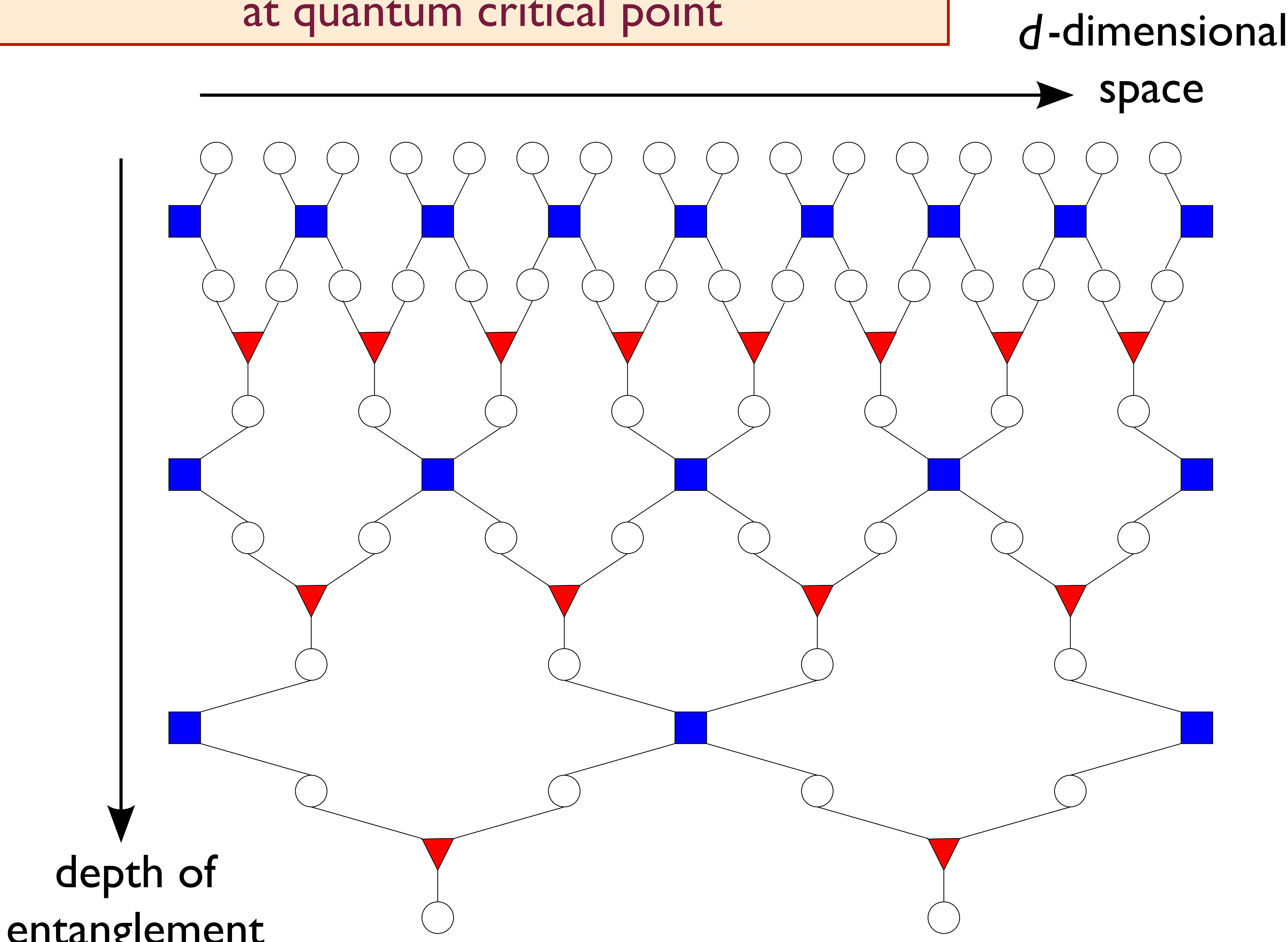
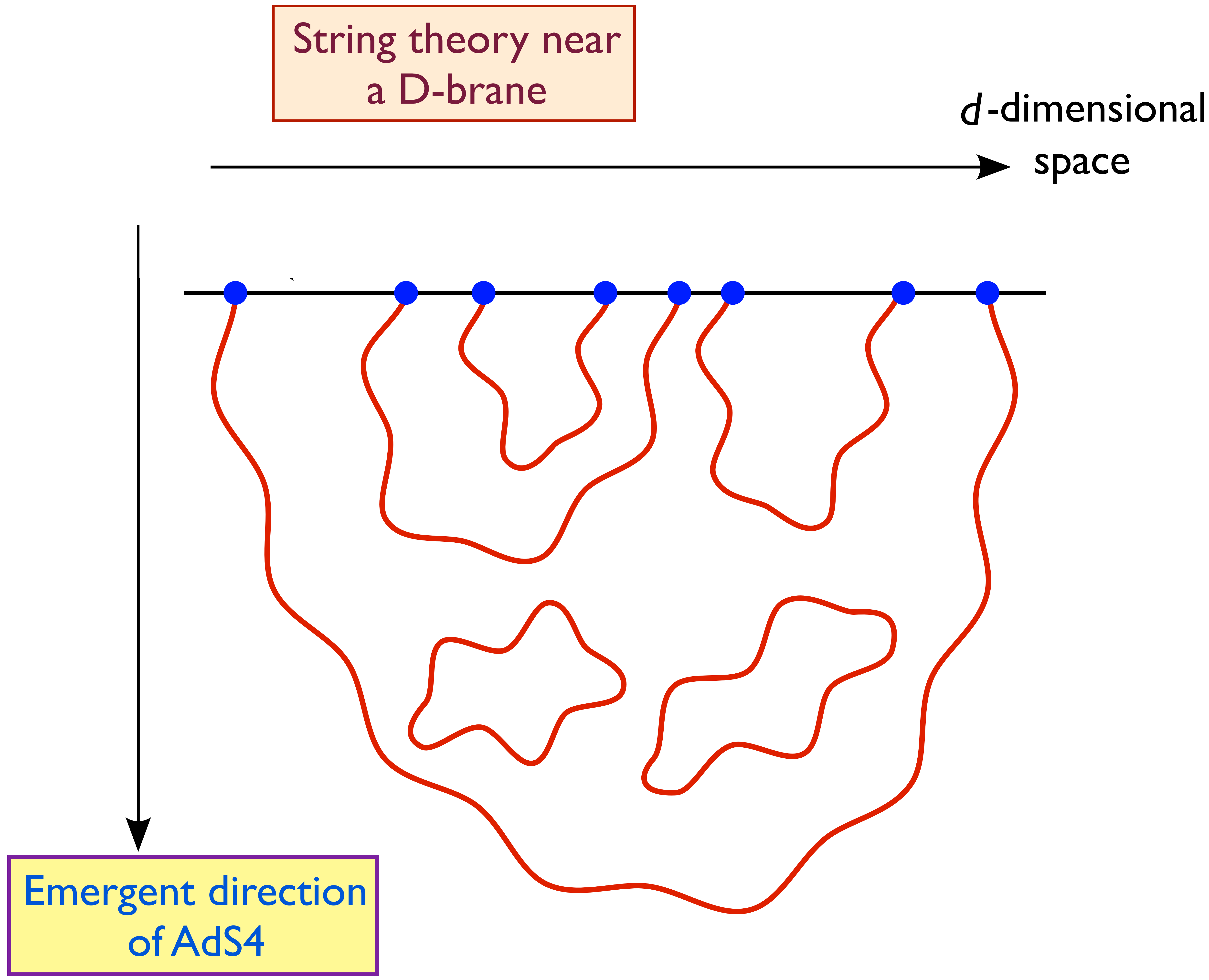
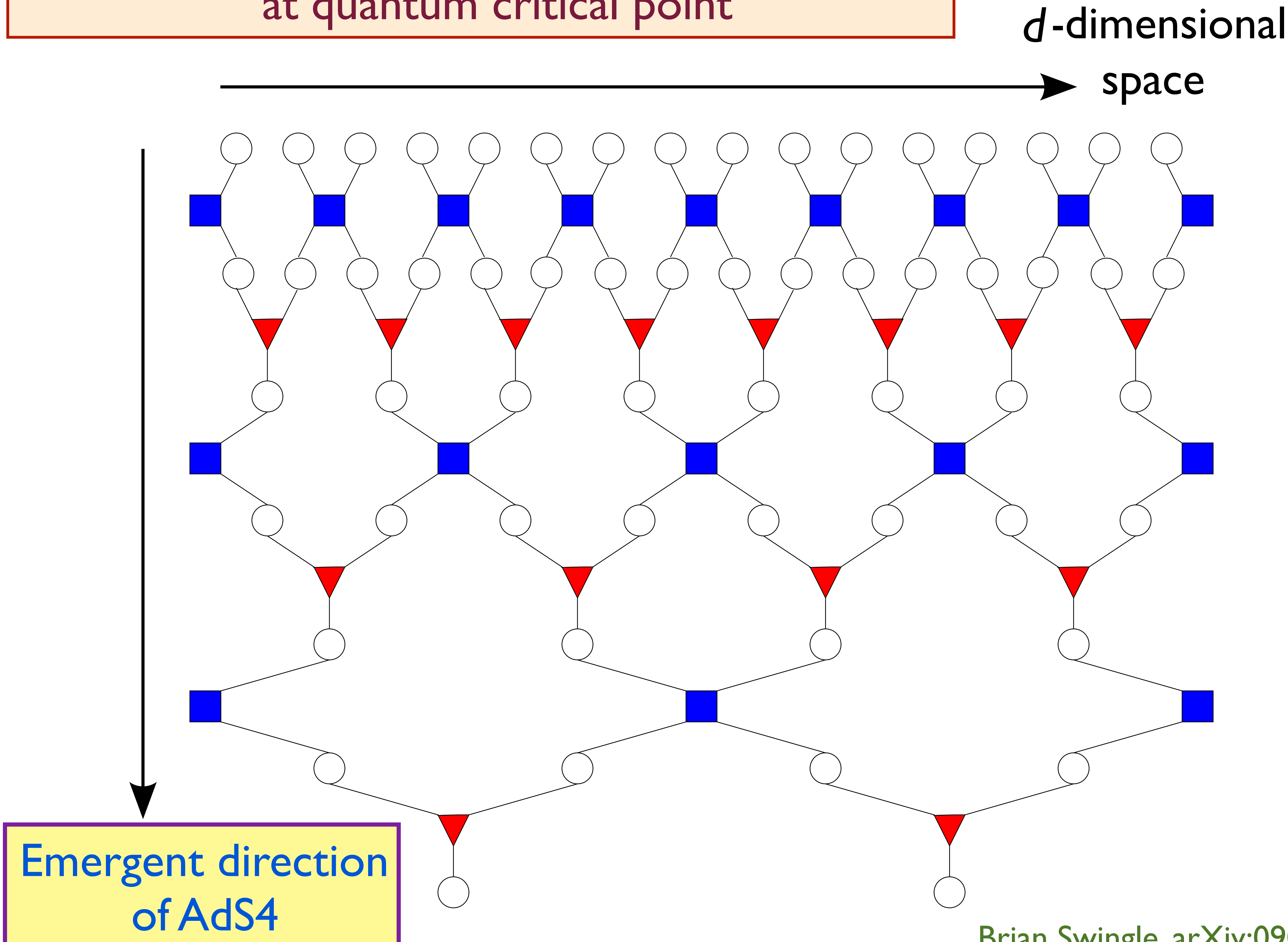
Tensor network representation of entanglement at quantum critical point



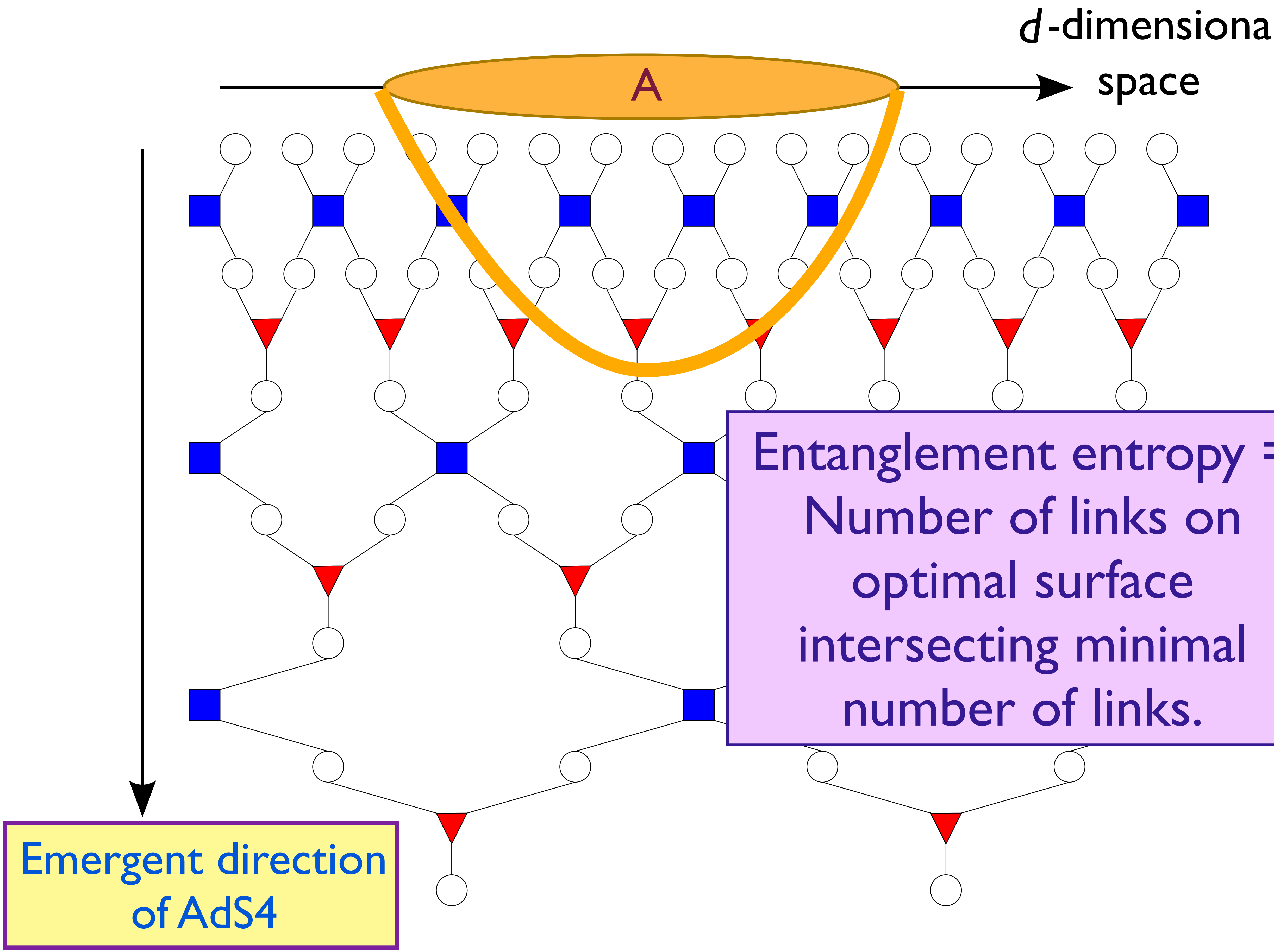

String theory near a D-brane
d-dimensional space
Emergent direction of AdS4

Tensor network representation of entanglement at quantum critical point



Brian Swingle, arXiv:0905.1317

Entanglement entropy



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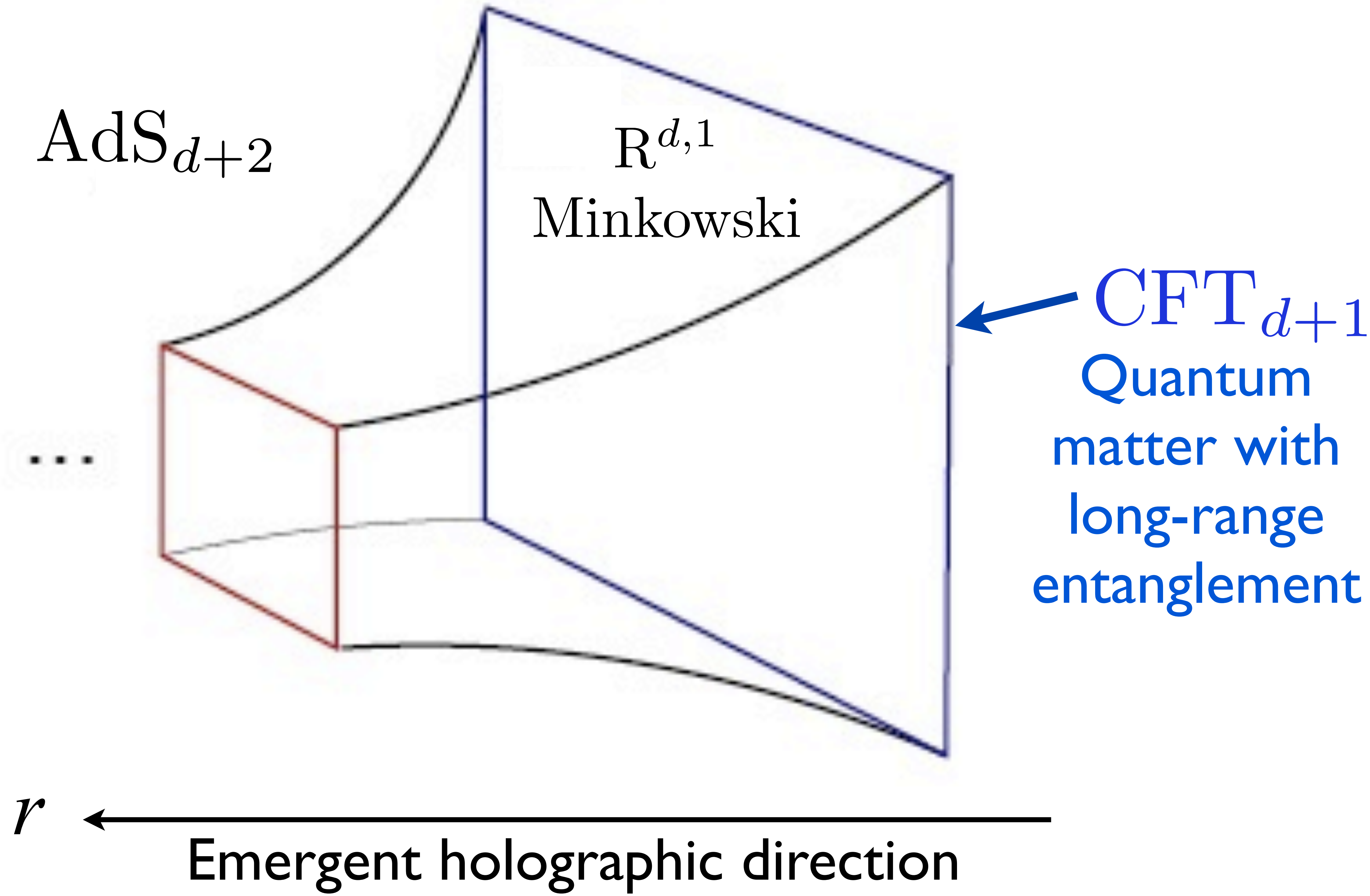

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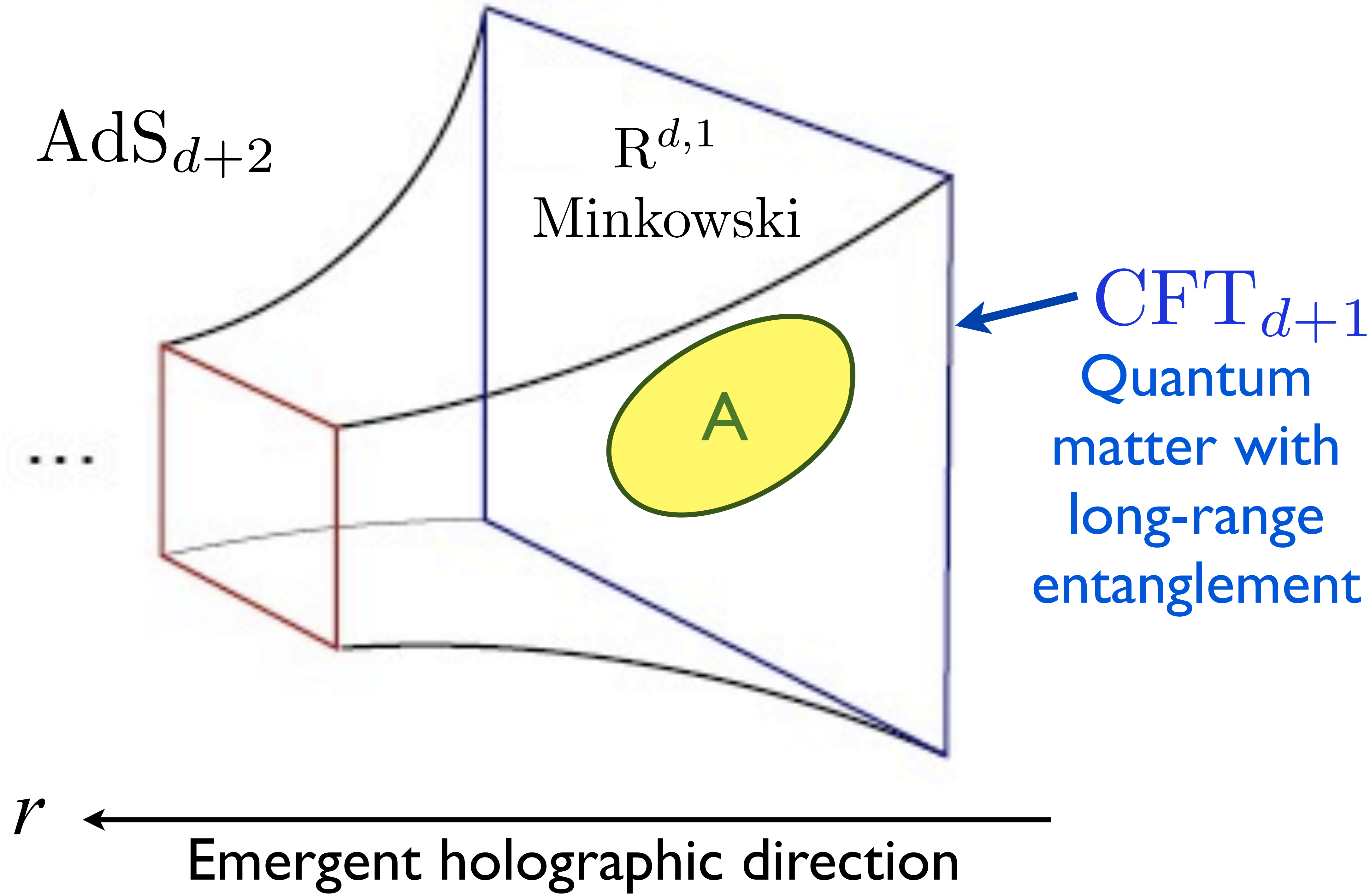
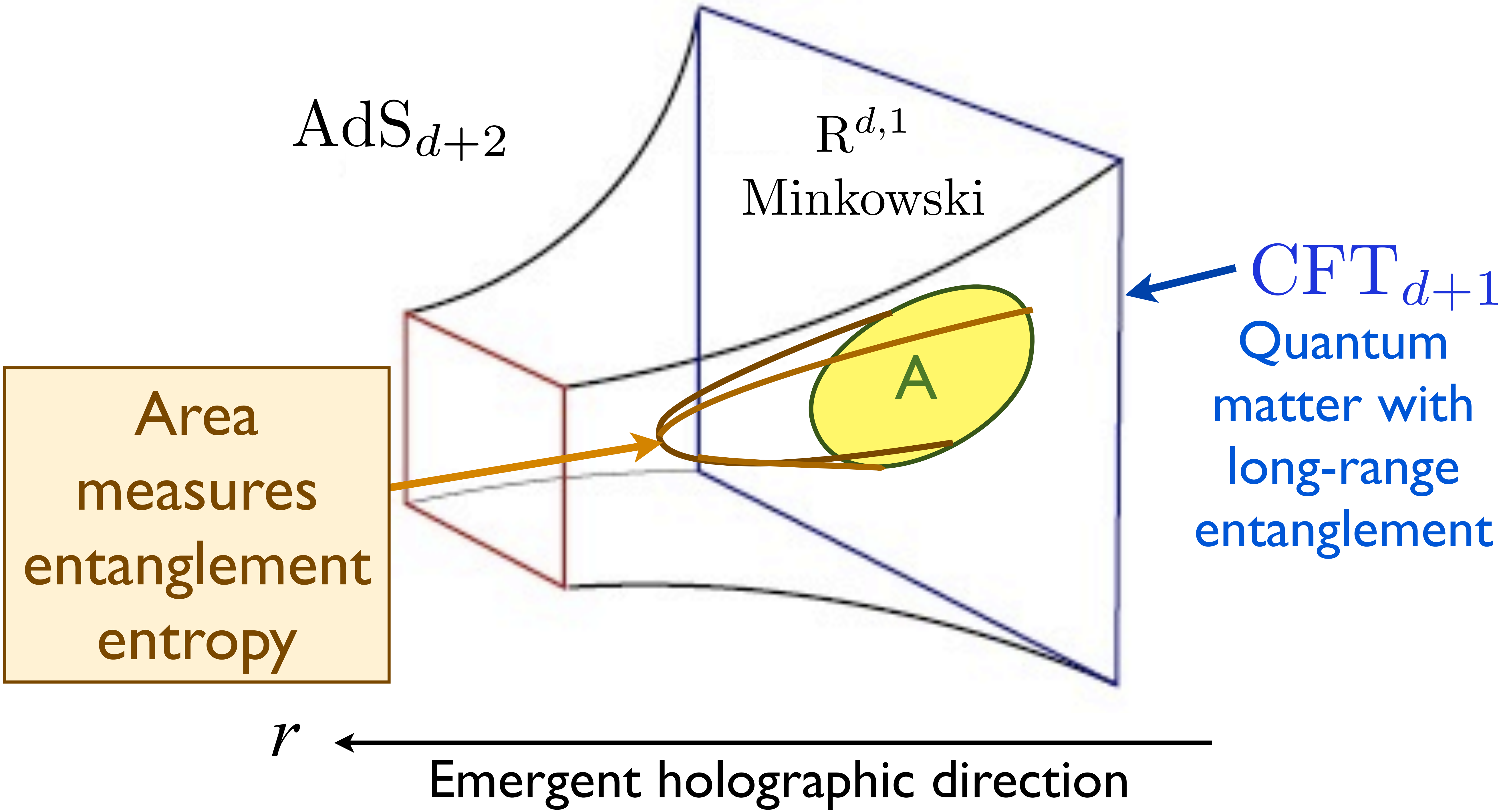

AdS_{d+2}
$R^{d,1}$
Minkowski
CFT_{d+1}
Quantum matter with long-range entanglement
A
...
r
Emergent holographic direction



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Quantum superposition and entanglement

String theory

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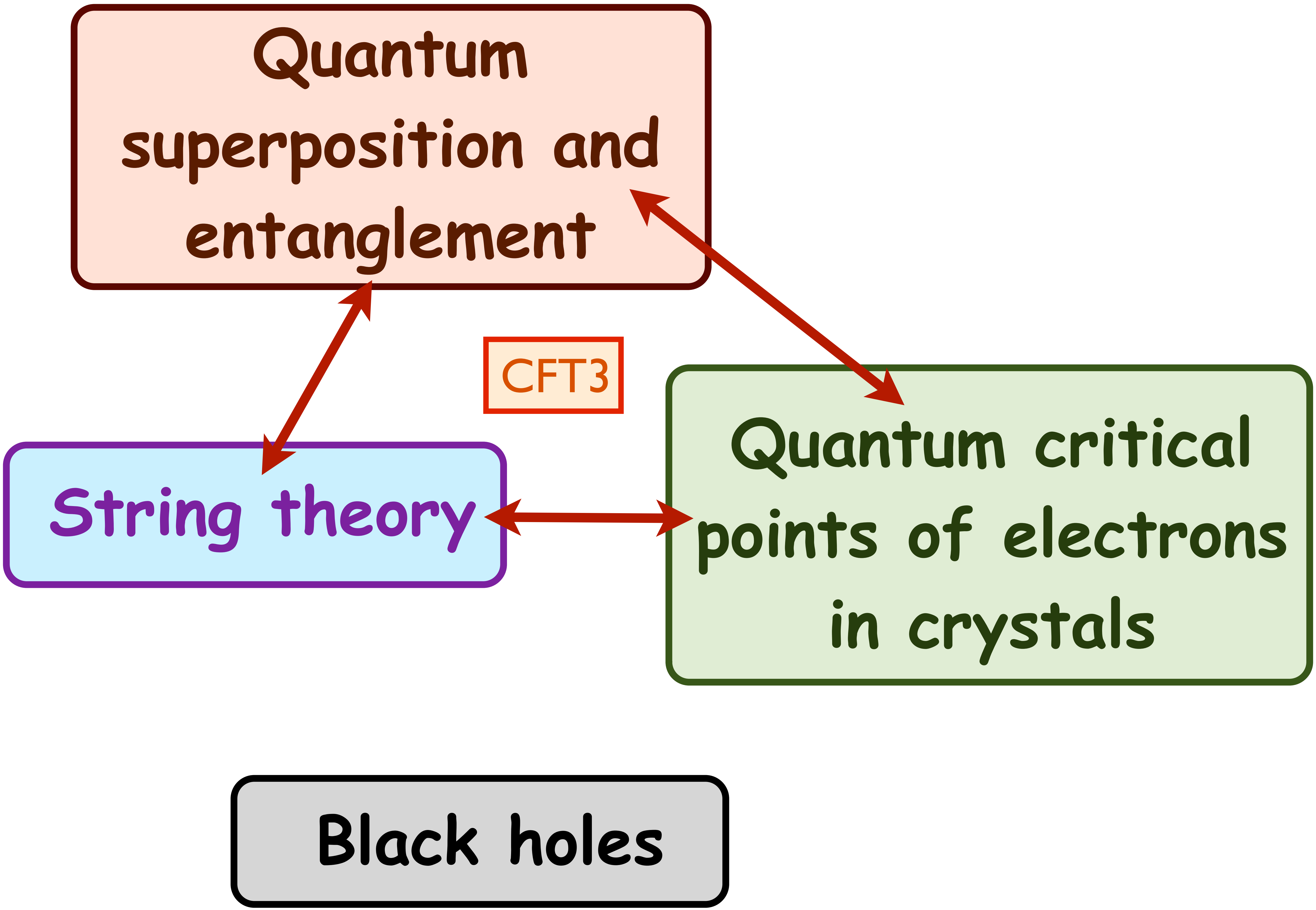
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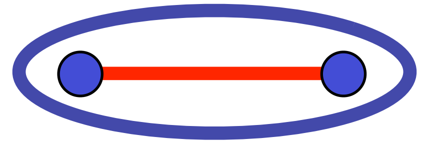
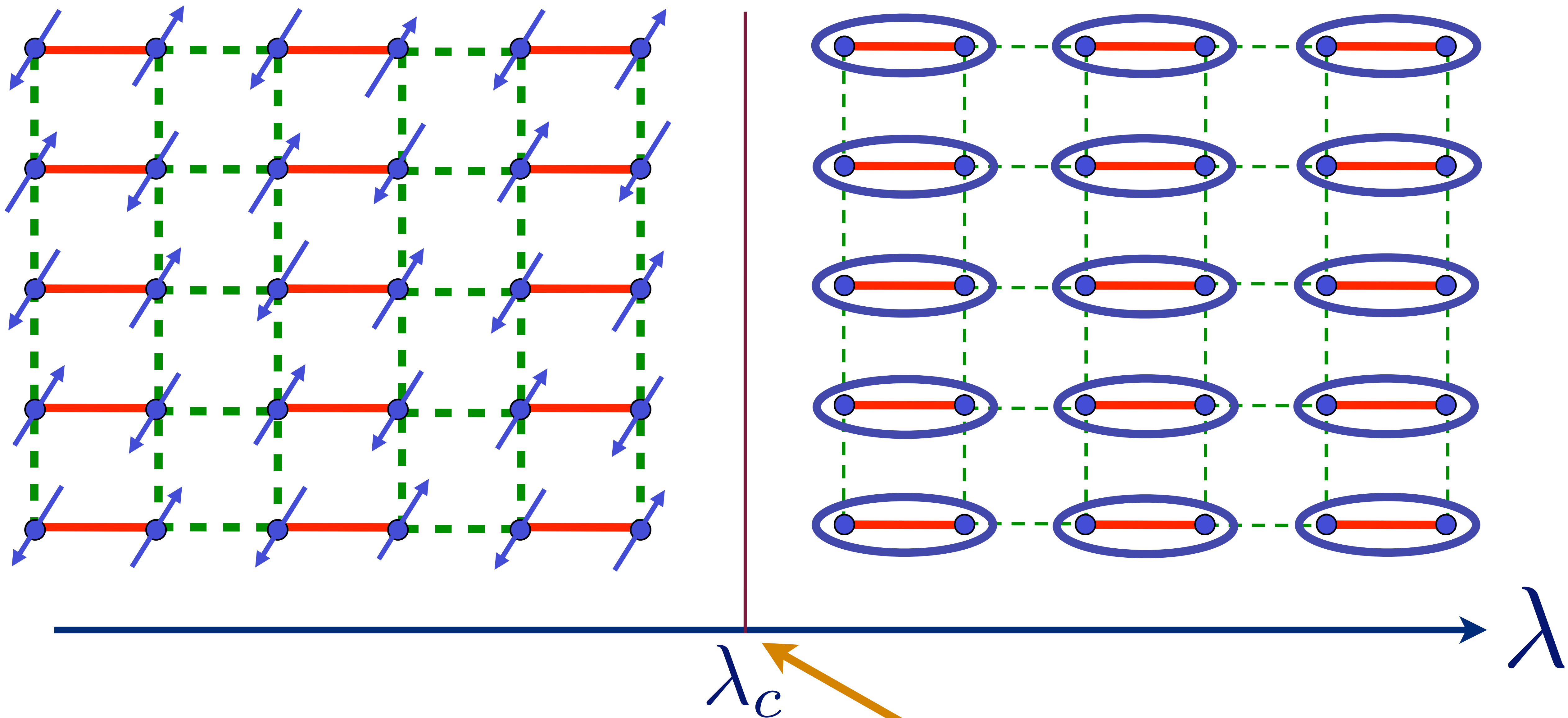


$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$



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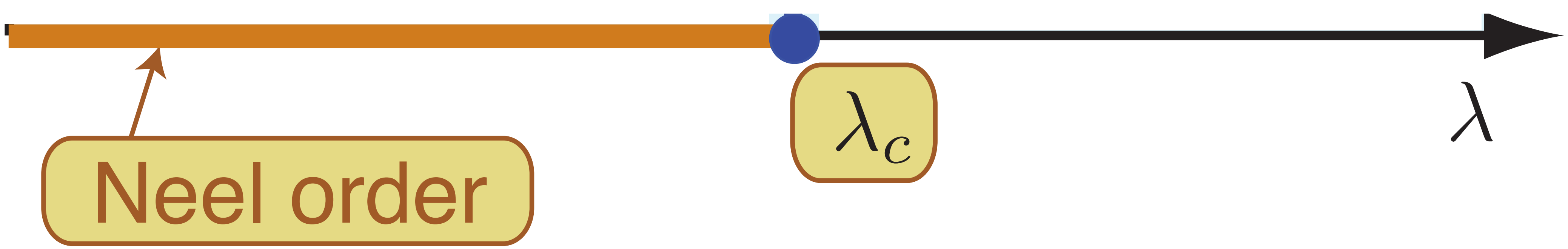
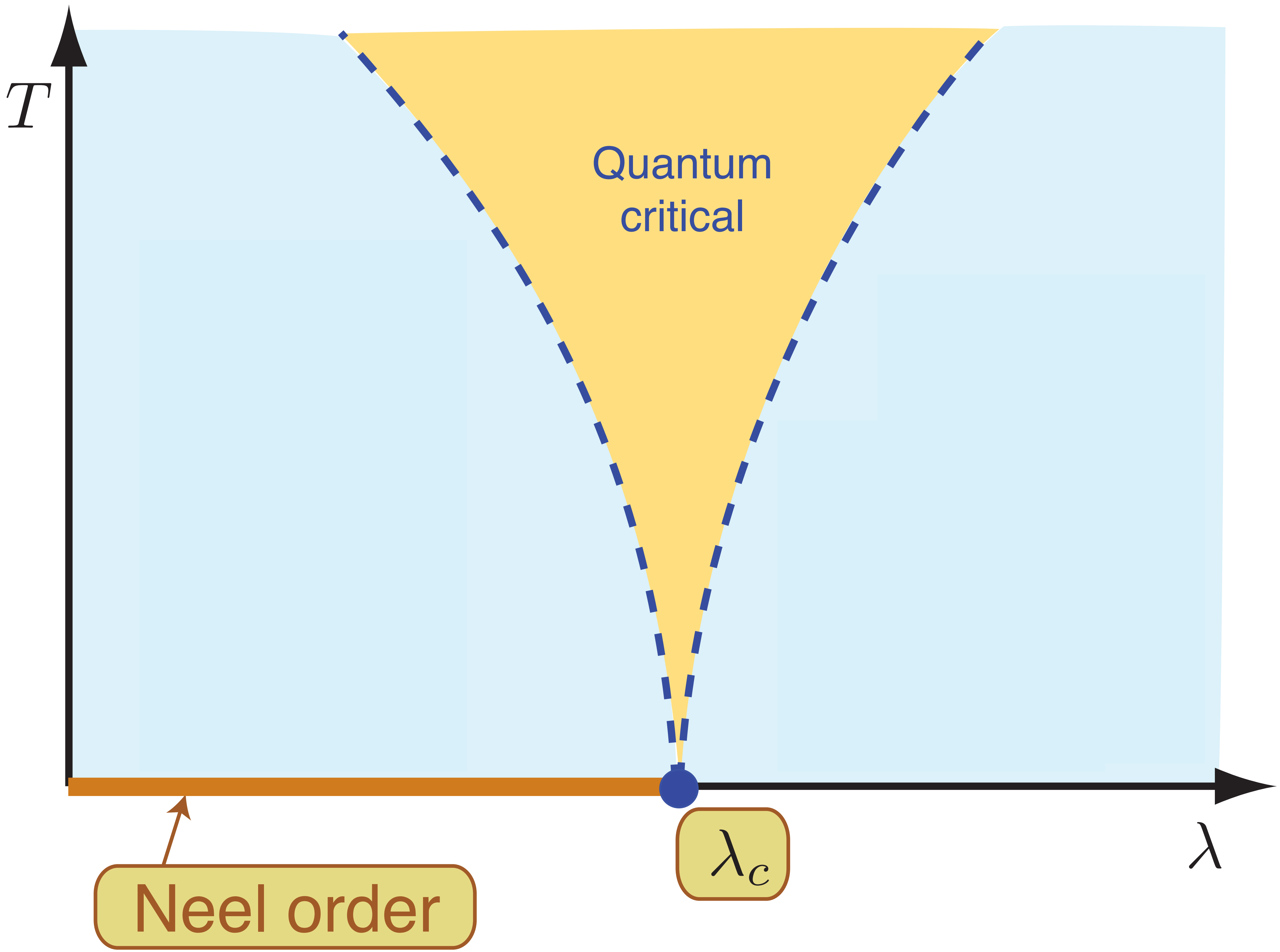

Neel order
λ_c
λ



T
Quantum
critical
Neel order
λ_c
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S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
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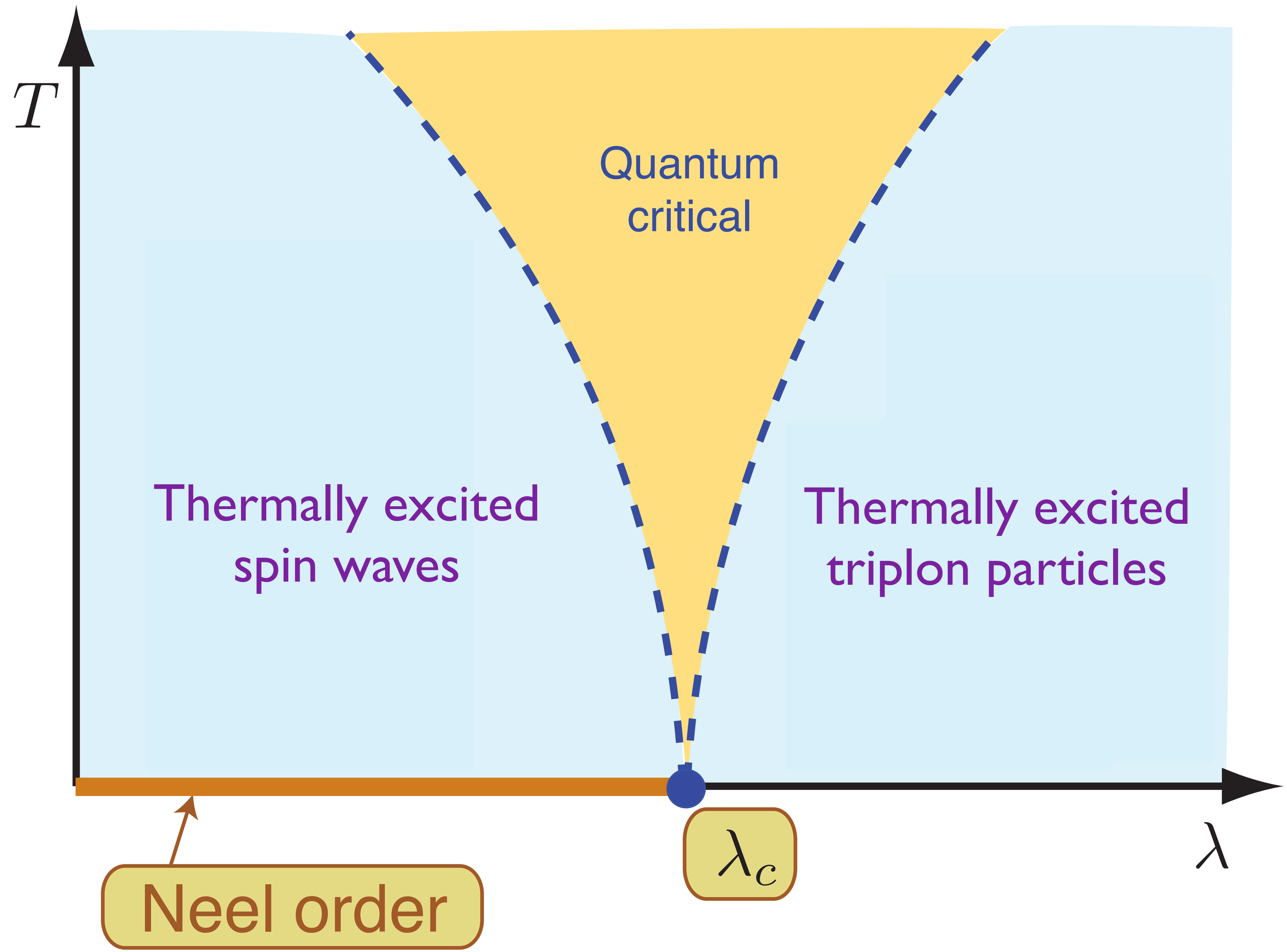

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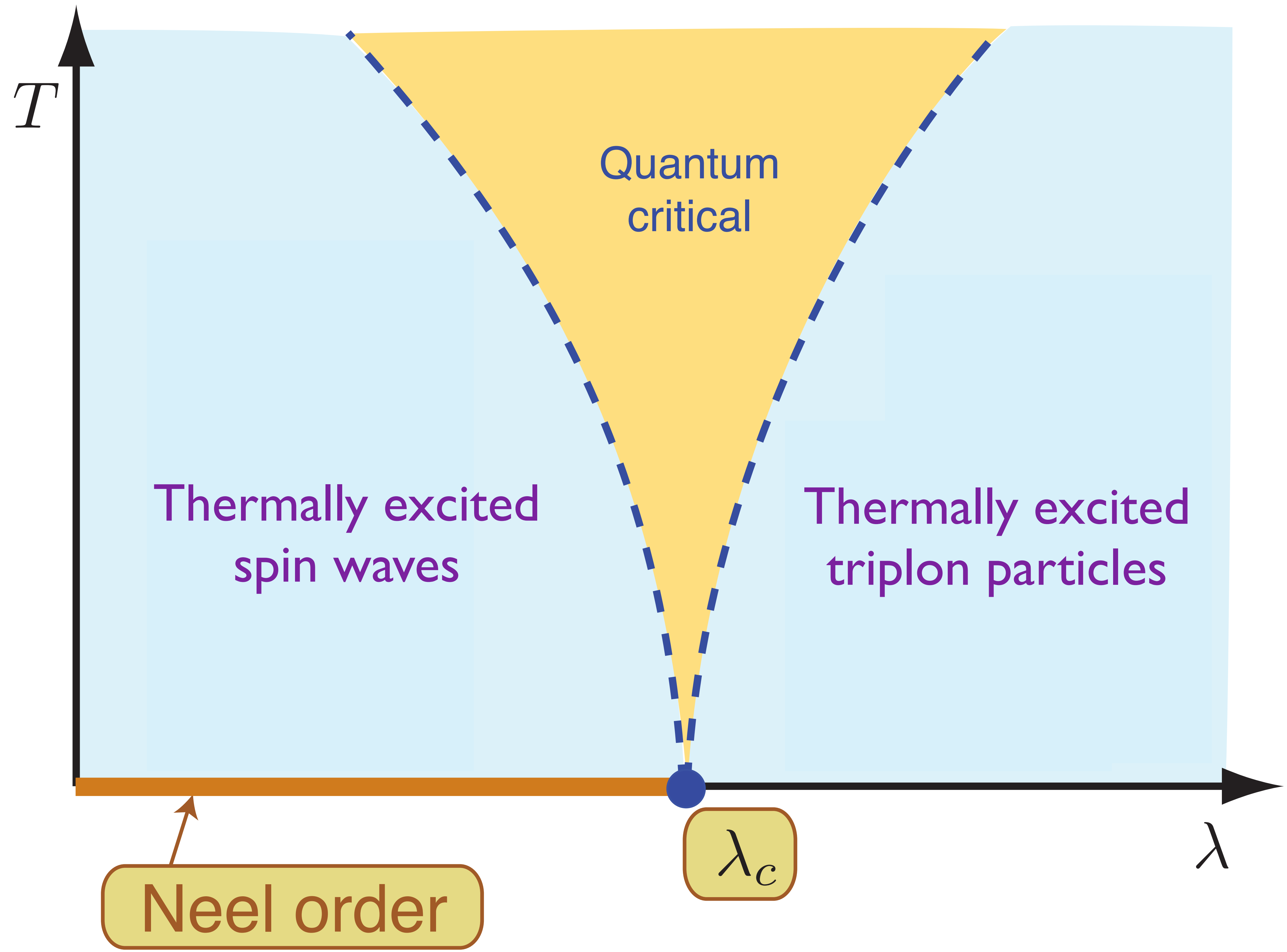

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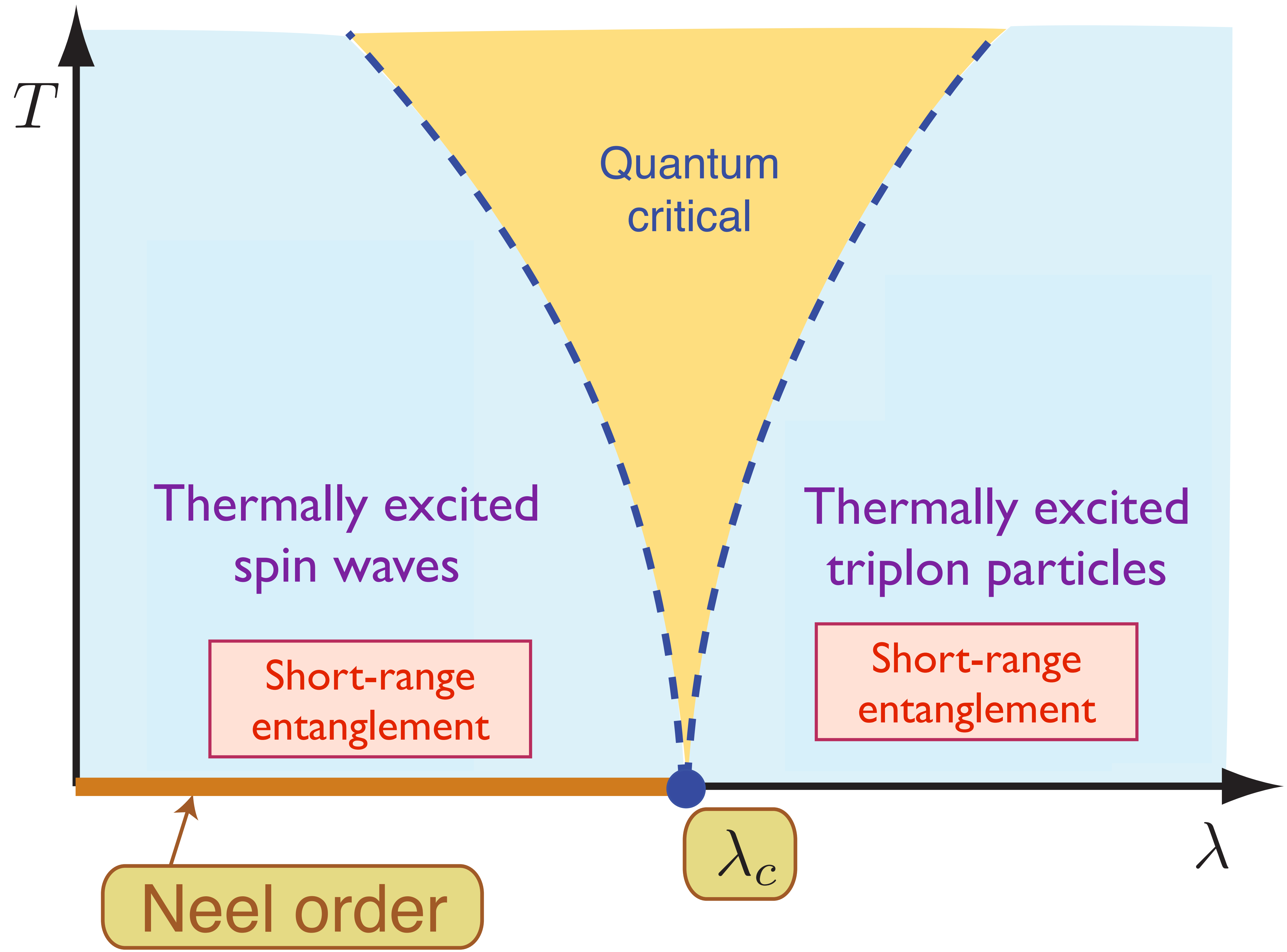

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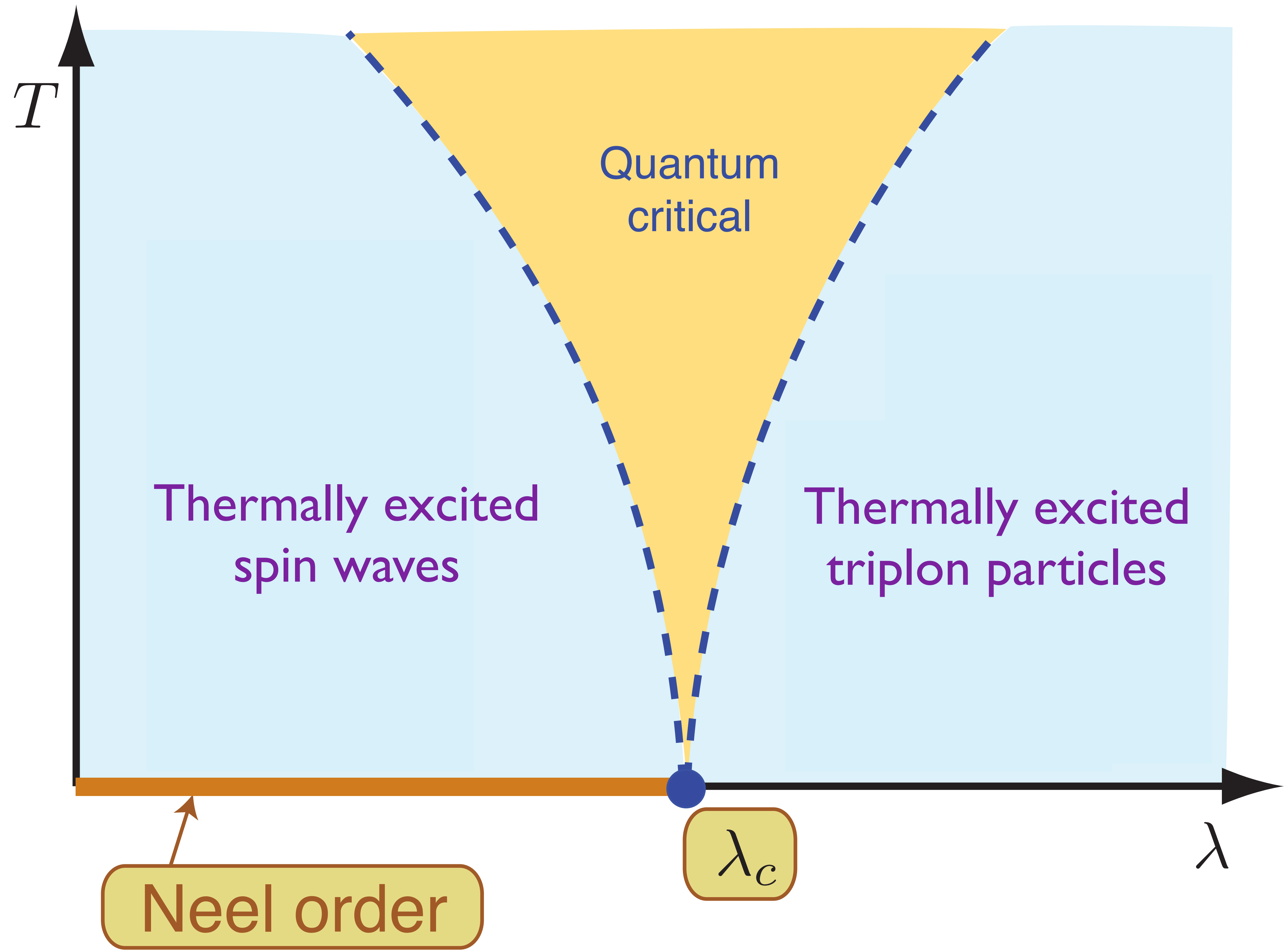

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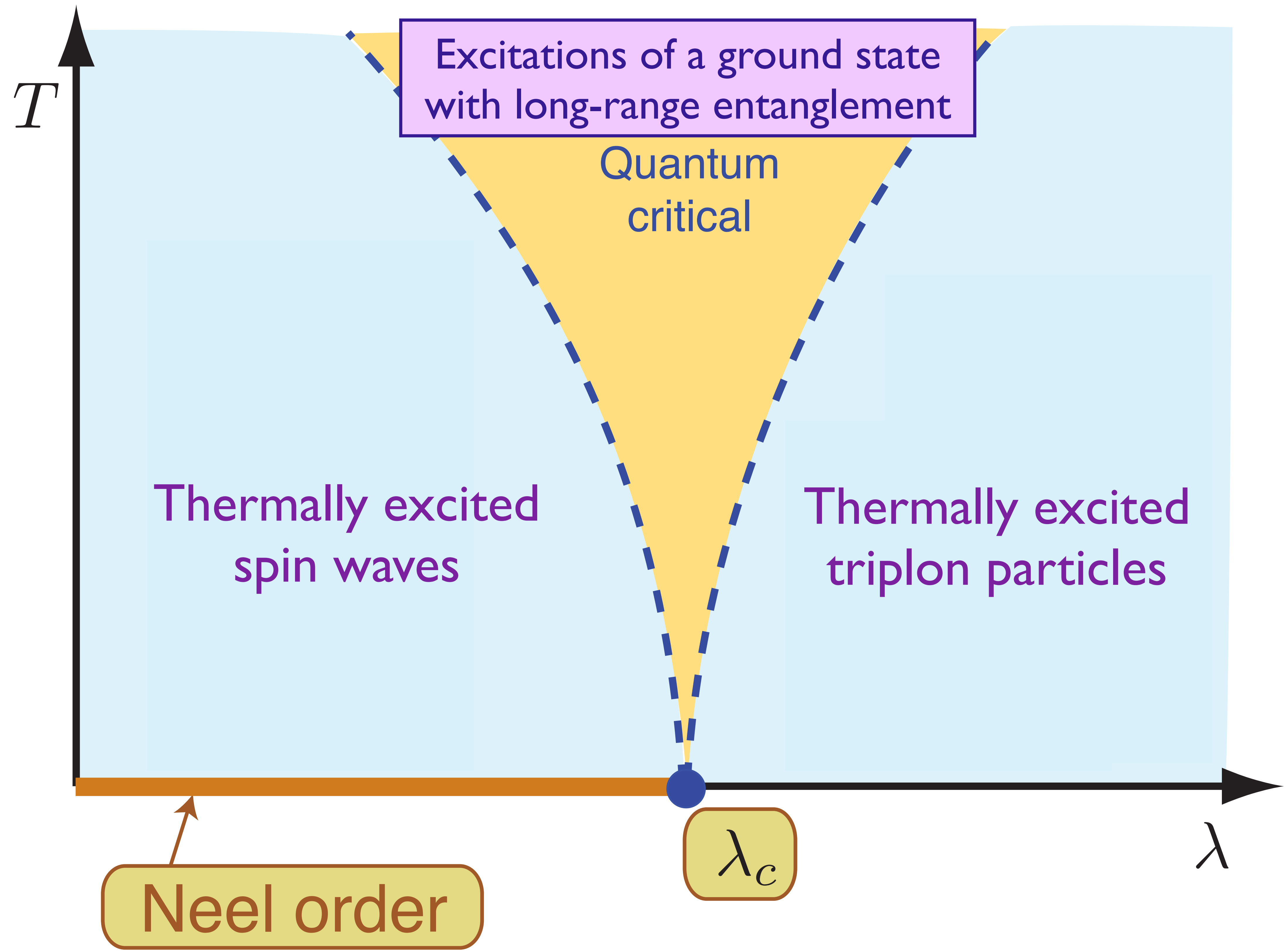

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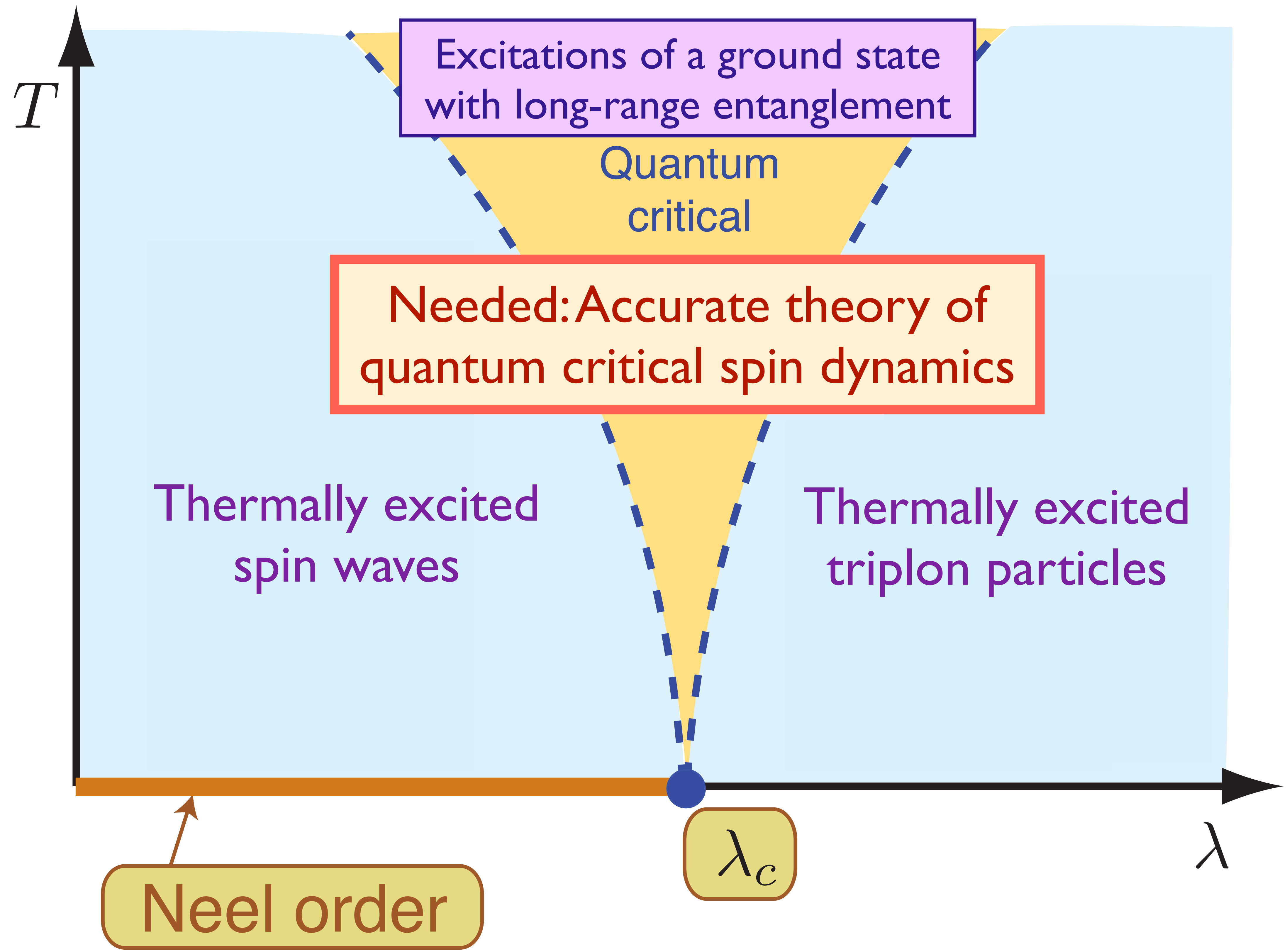
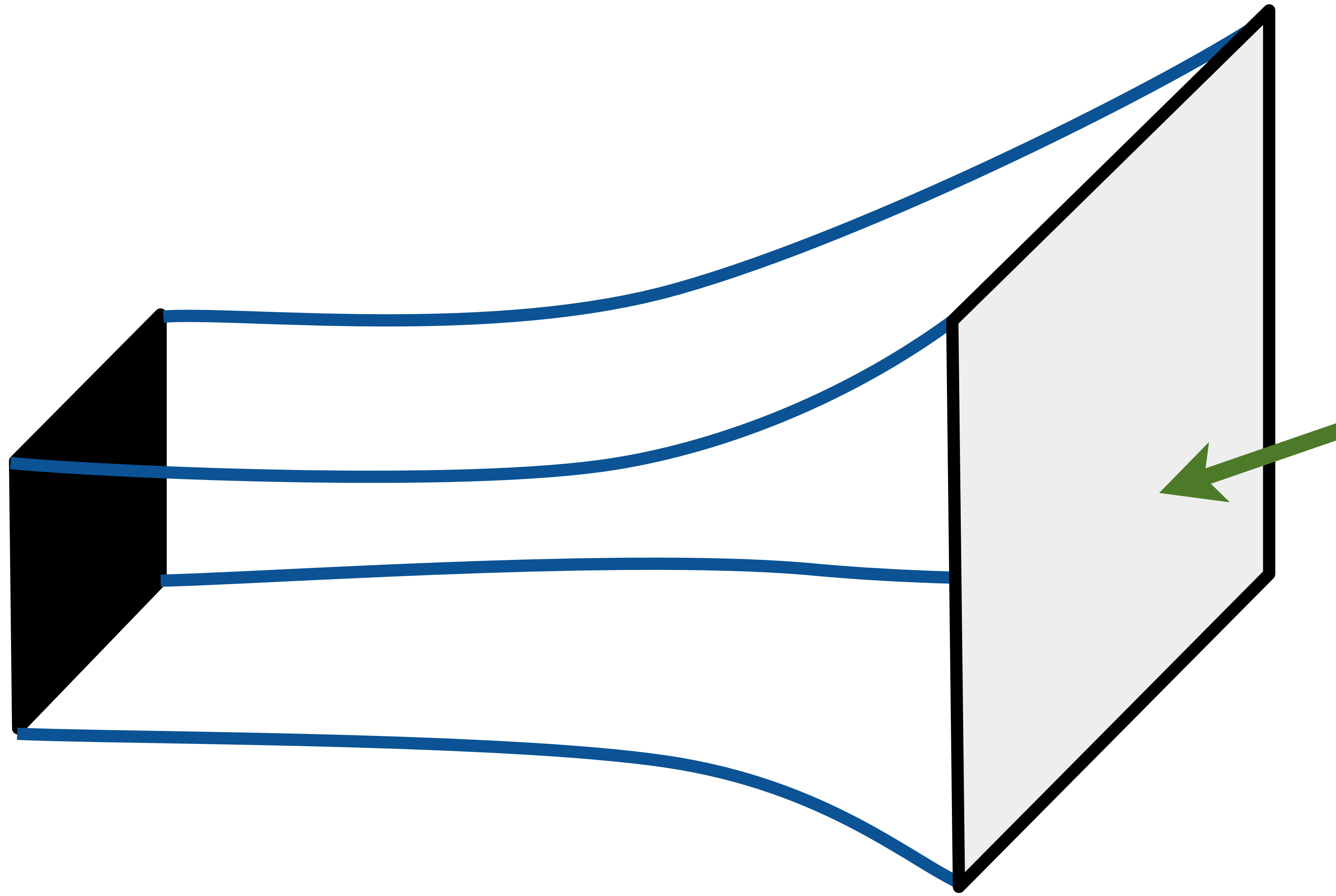


String theory at non-zero temperatures



A 2+1 dimensional system at its quantum critical point

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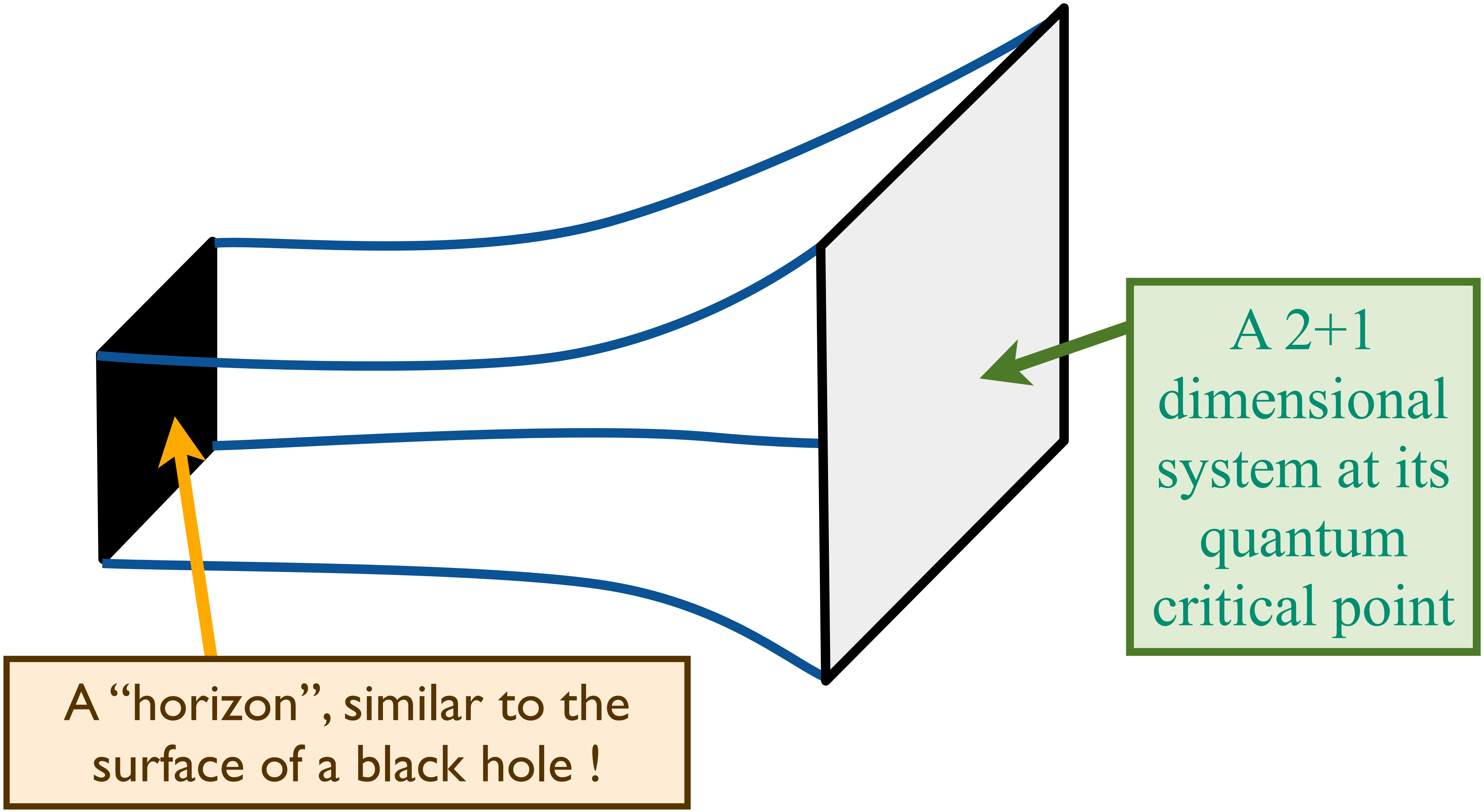
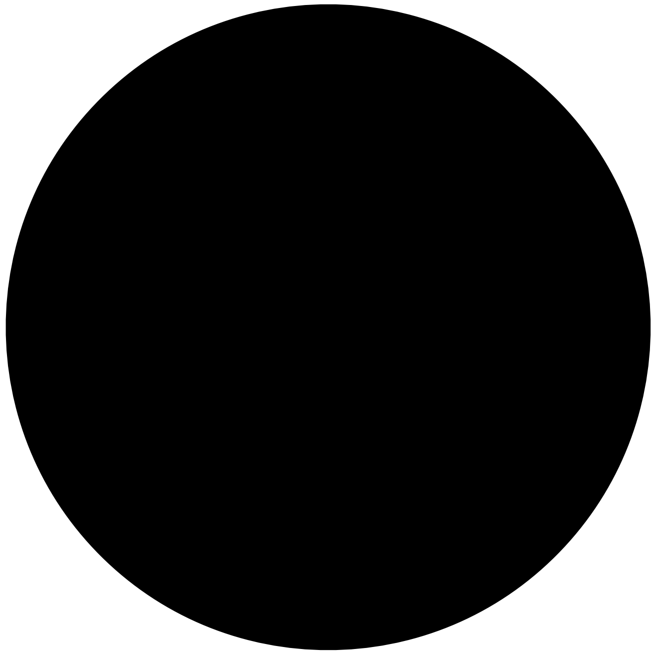


Black Holes

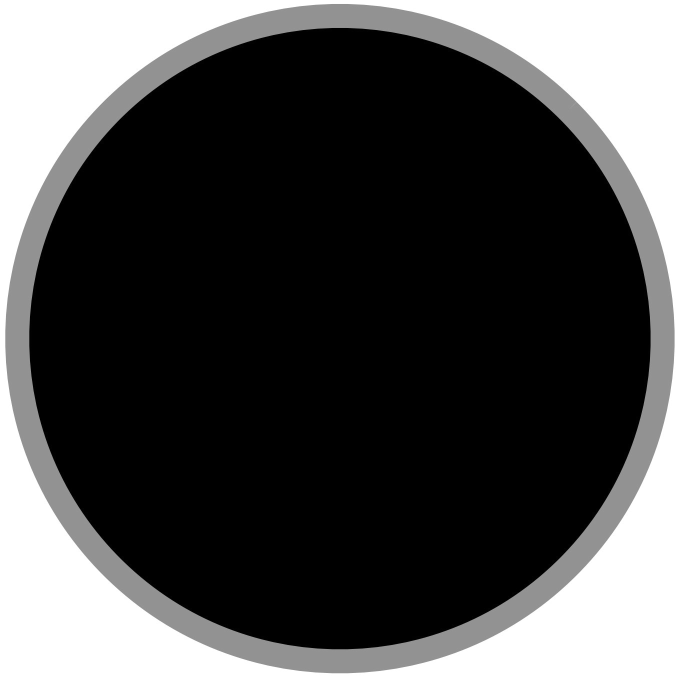
Objects so massive that light is gravitationally bound to them.



Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

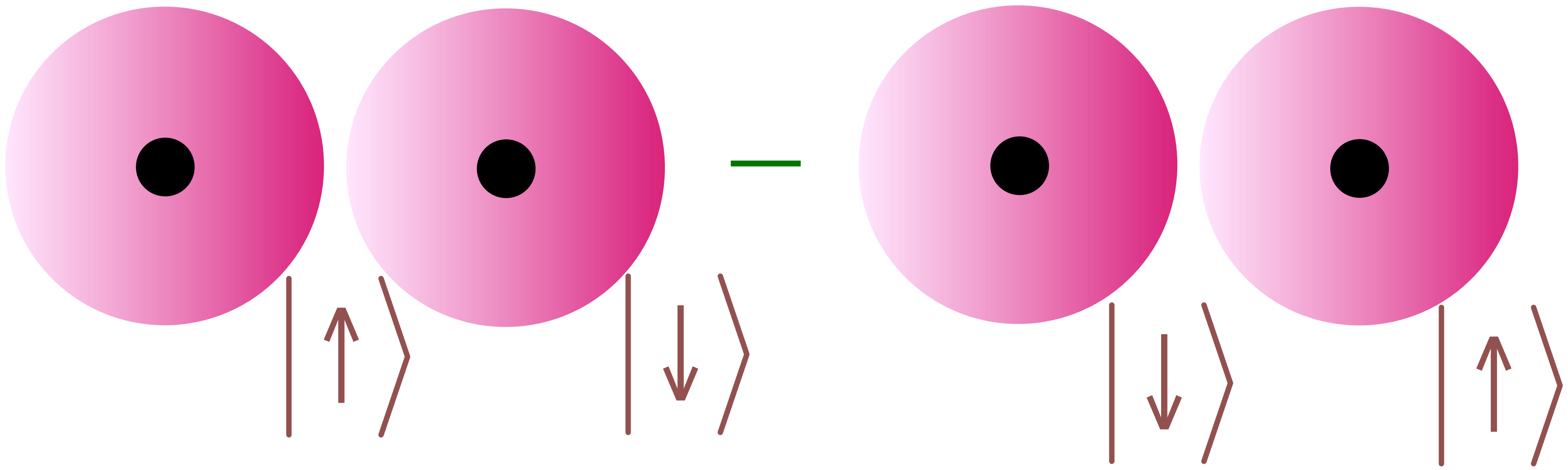


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Holes + Quantum theory

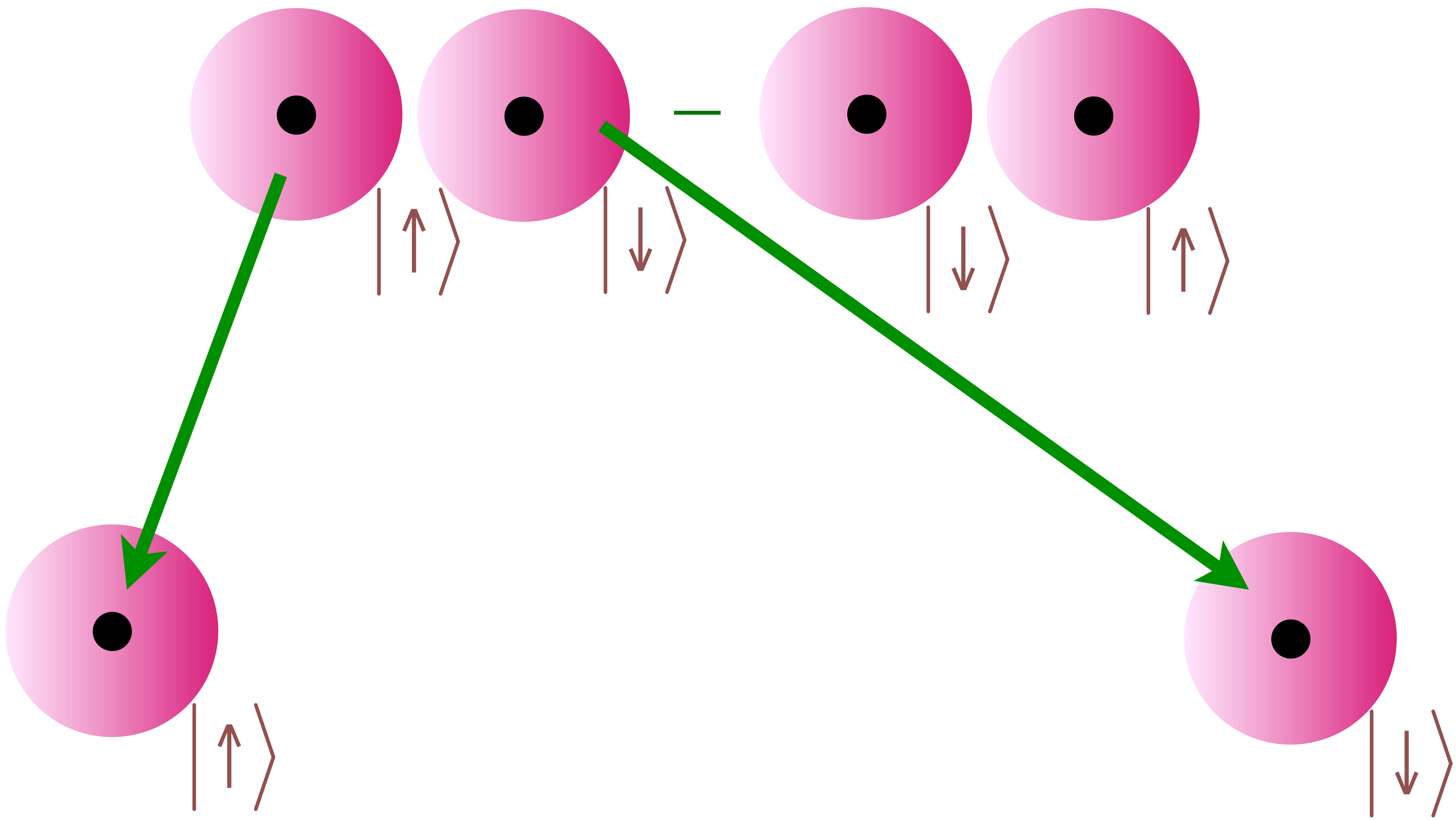
Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

Quantum Entanglement across a black hole horizon



Wednesday, May 9, 2012

Quantum Entanglement across a black hole horizon



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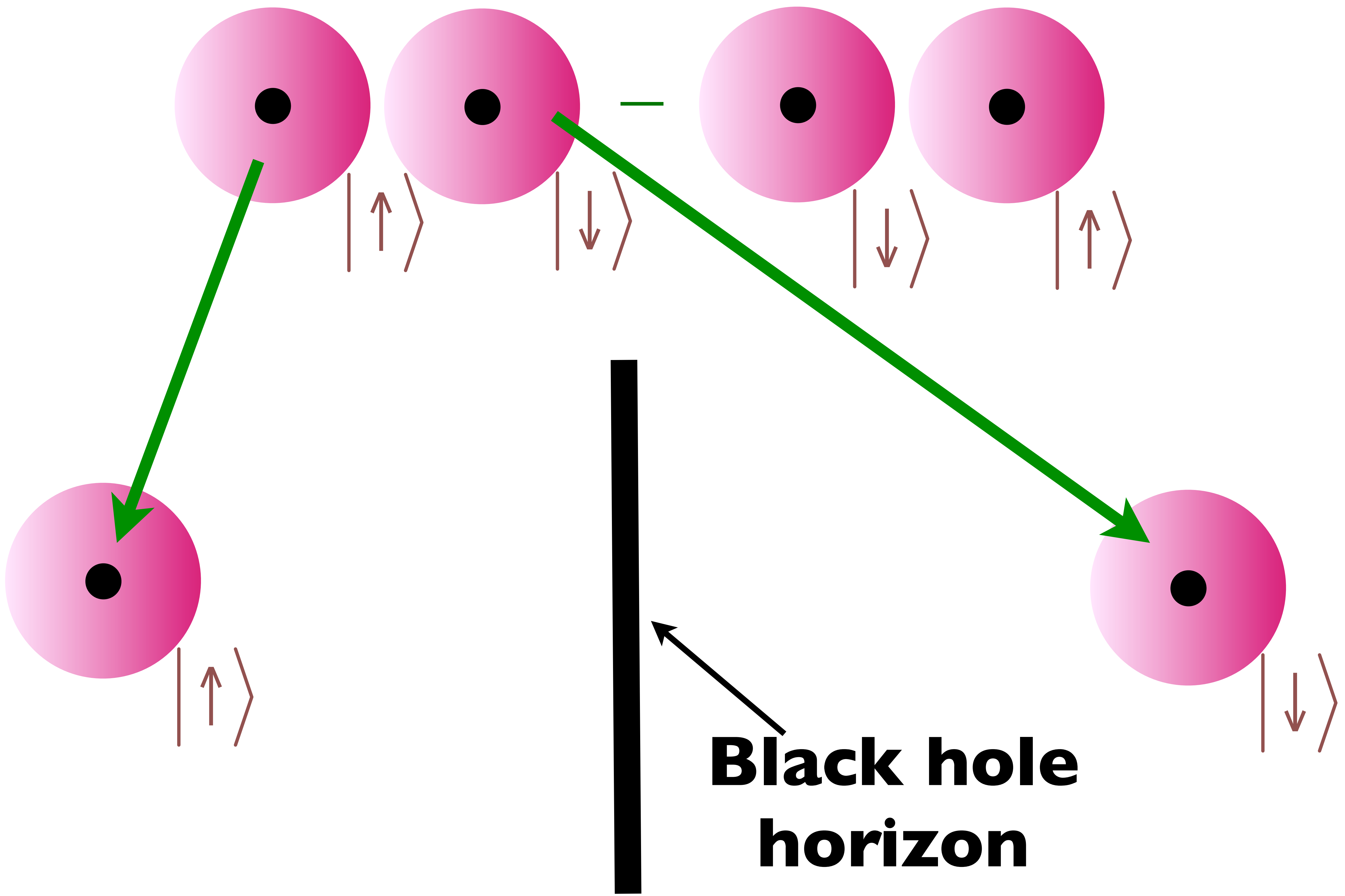
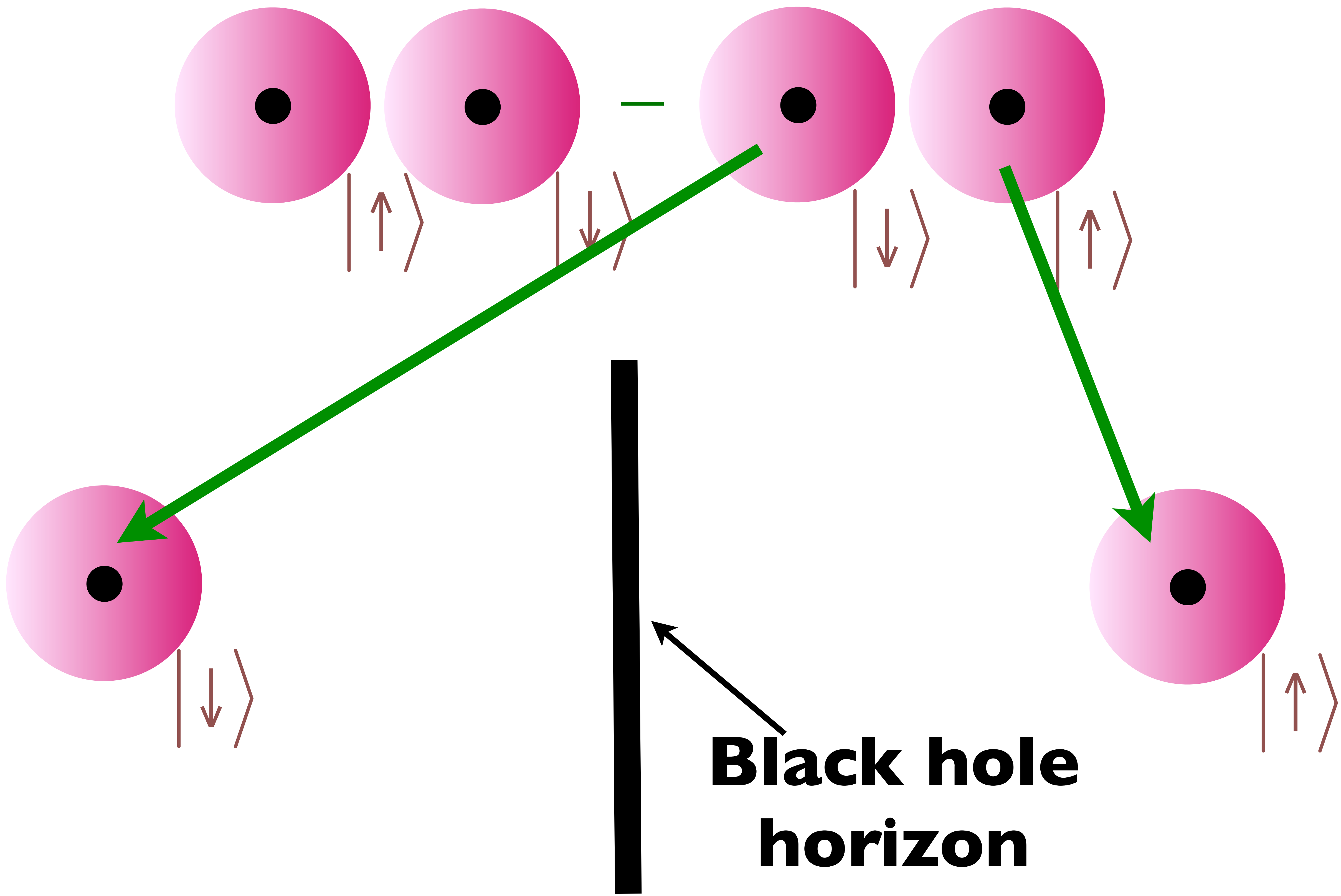


Quantum Entanglement across a black hole horizon



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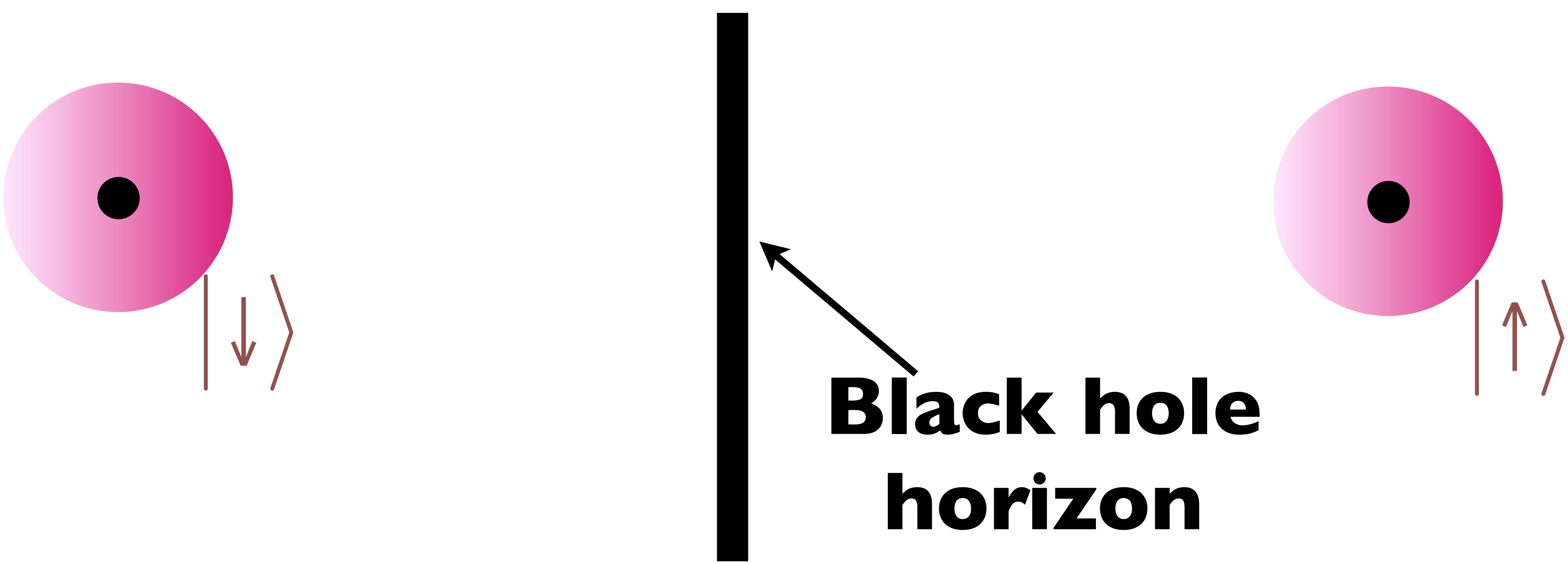
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



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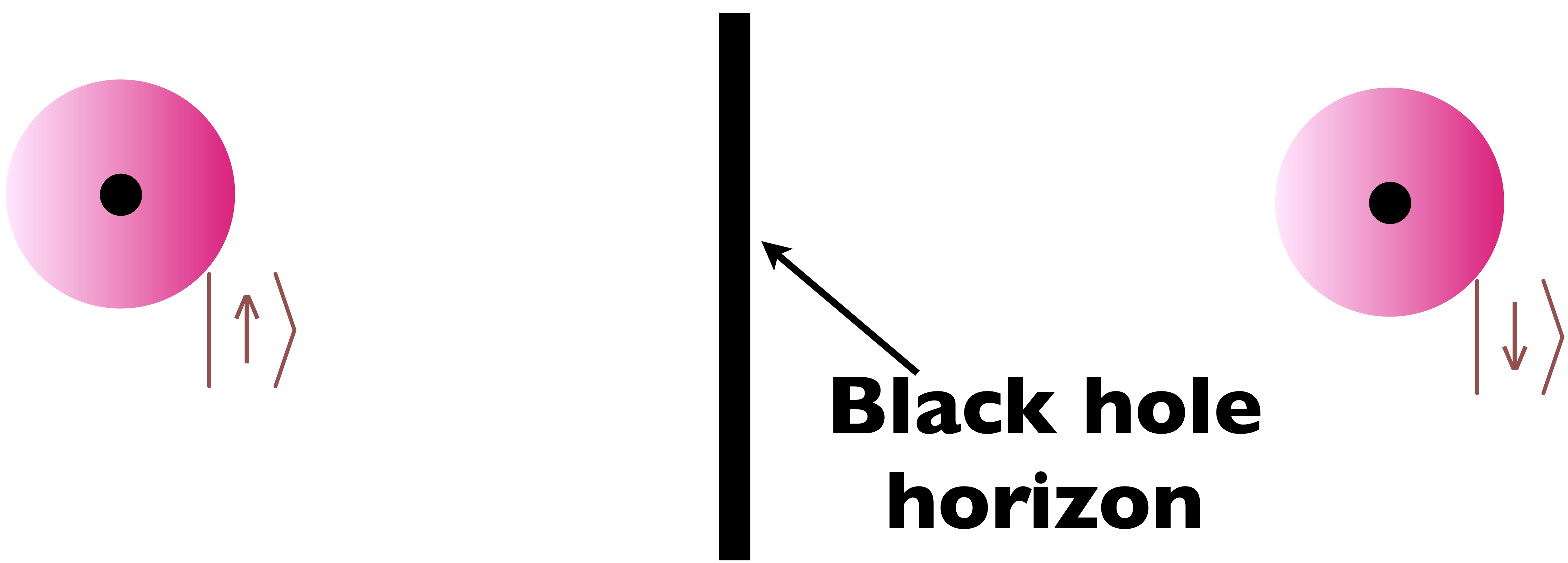
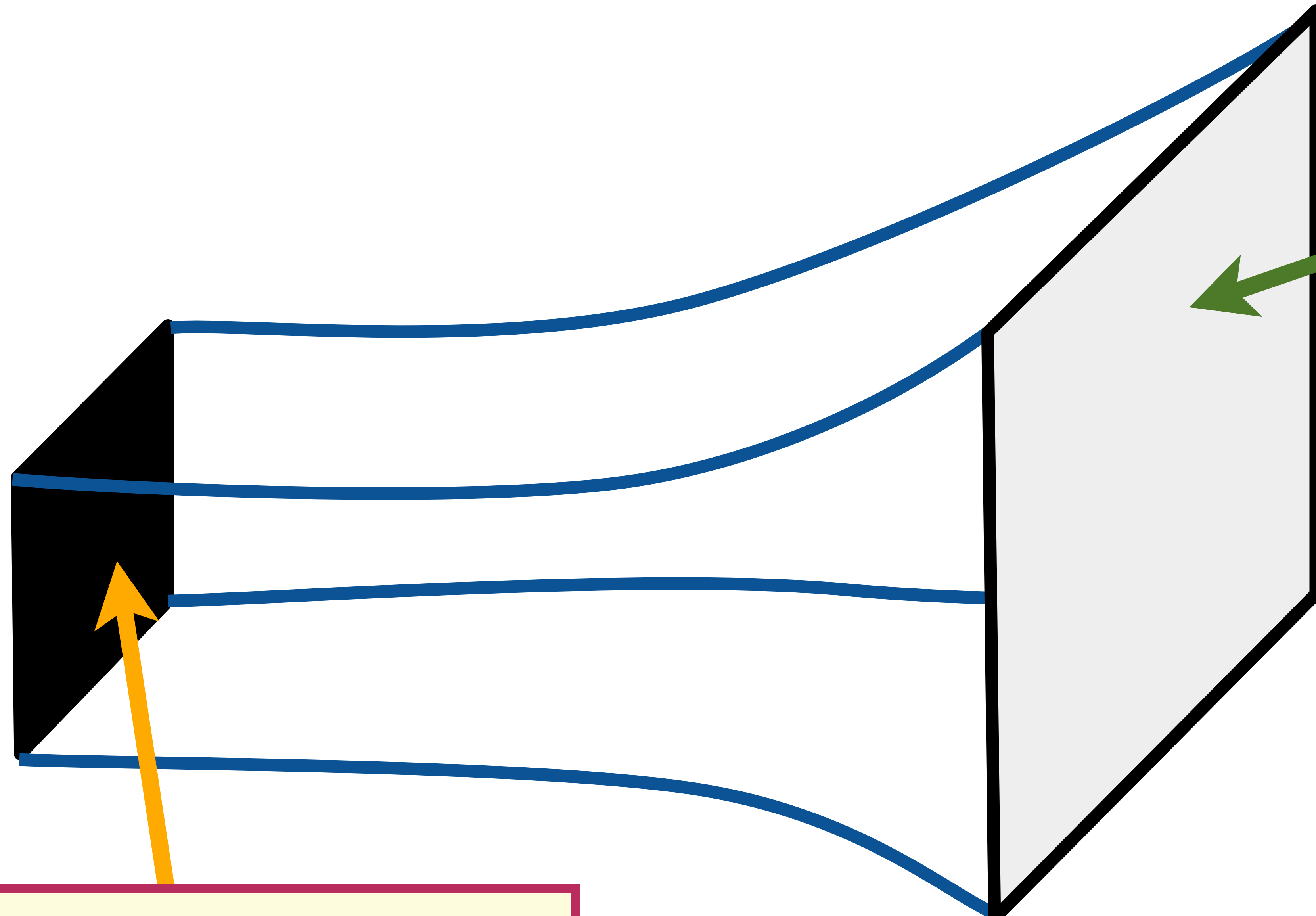


Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

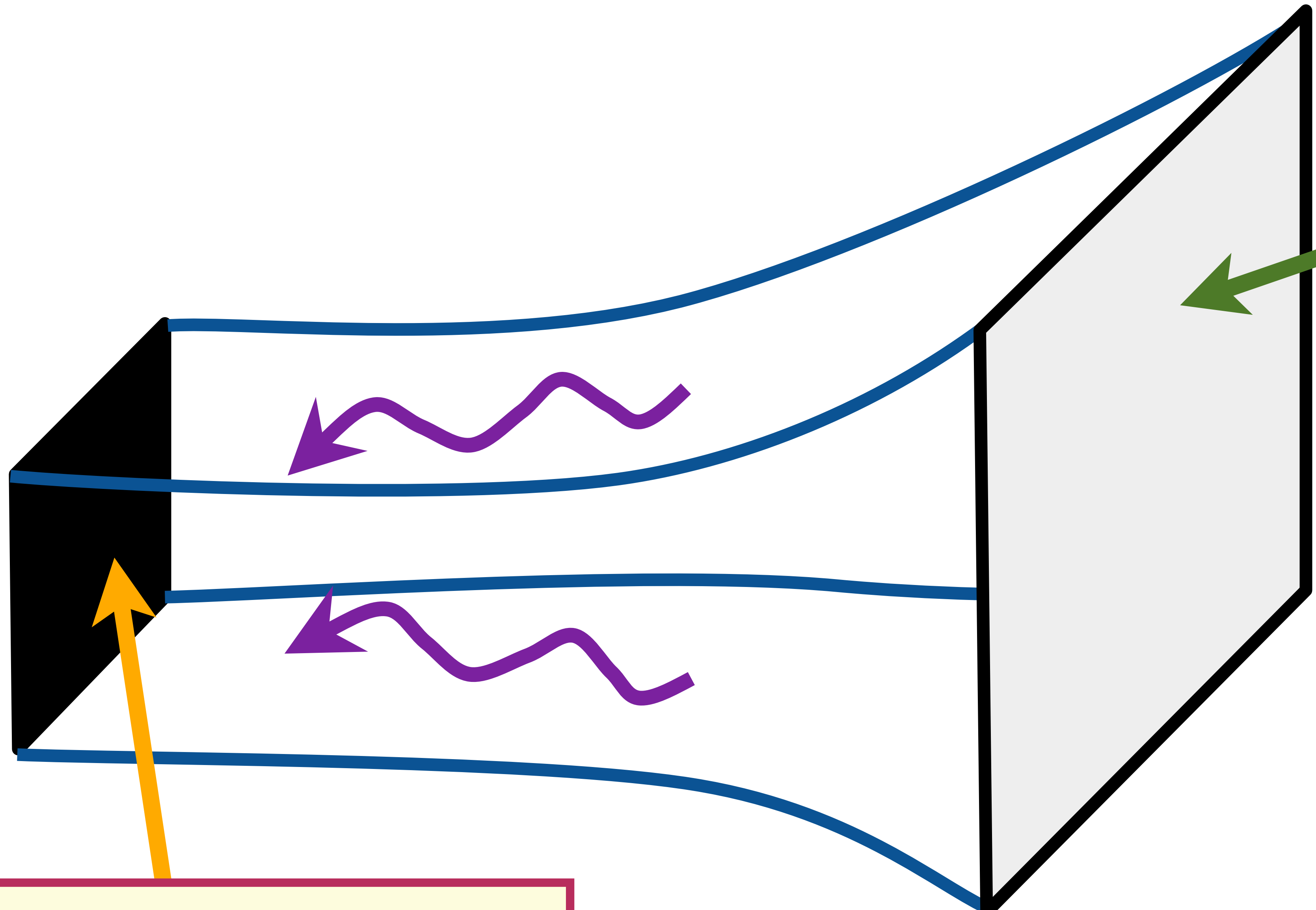
String theory at non-zero temperatures



A 2+1 dimensional system at its quantum critical point

A "horizon", whose temperature and entropy equal those of the quantum critical point

String theory at non-zero temperatures



A 2+1 dimensional system at its quantum critical point

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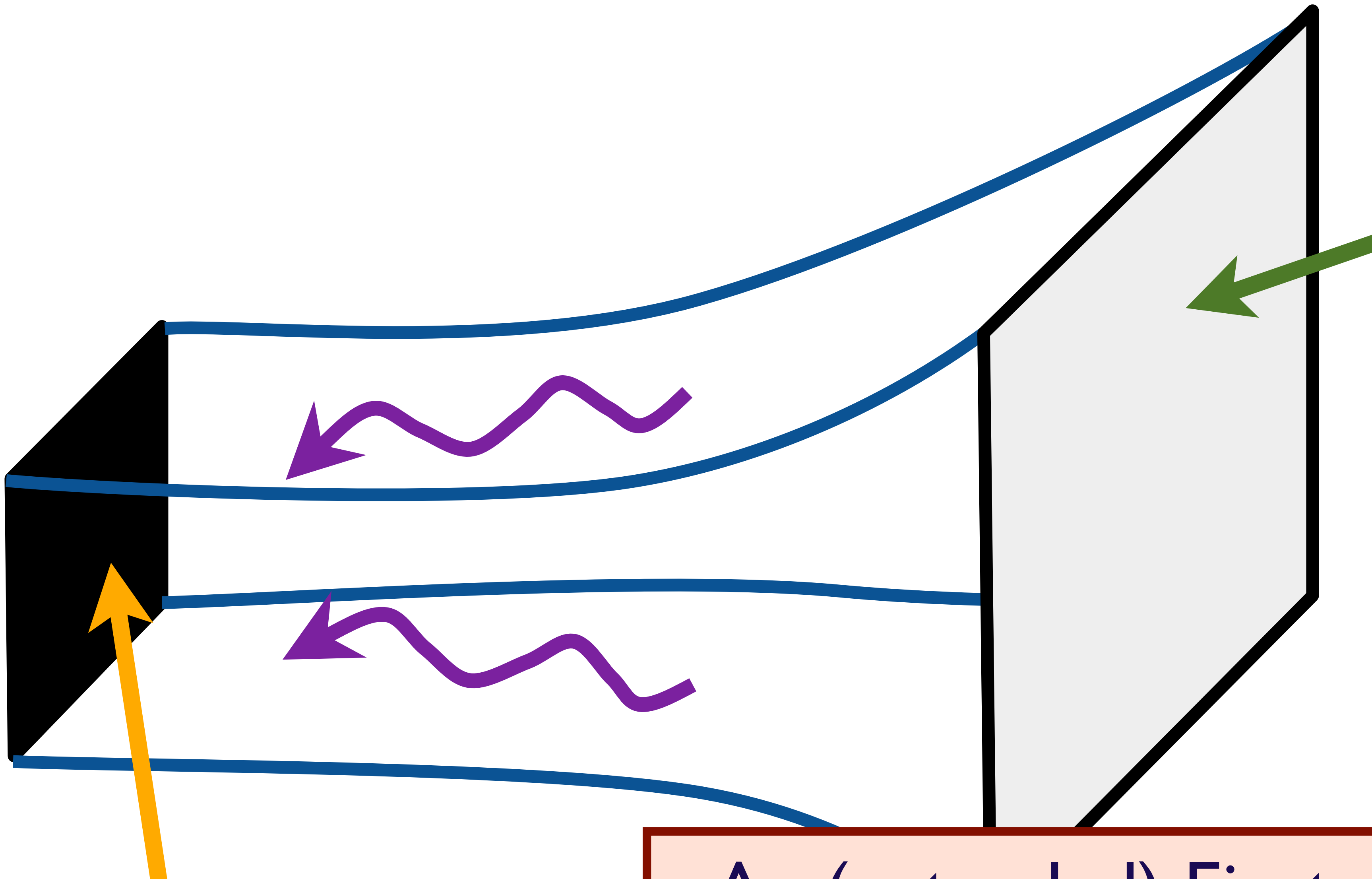
Friction of quantum criticality = waves falling into black brane

String theory at non-zero temperatures



A 2+1 dimensional system at its quantum critical point

A "horizon", whose temperature and entropy equal those of the quantum critical point

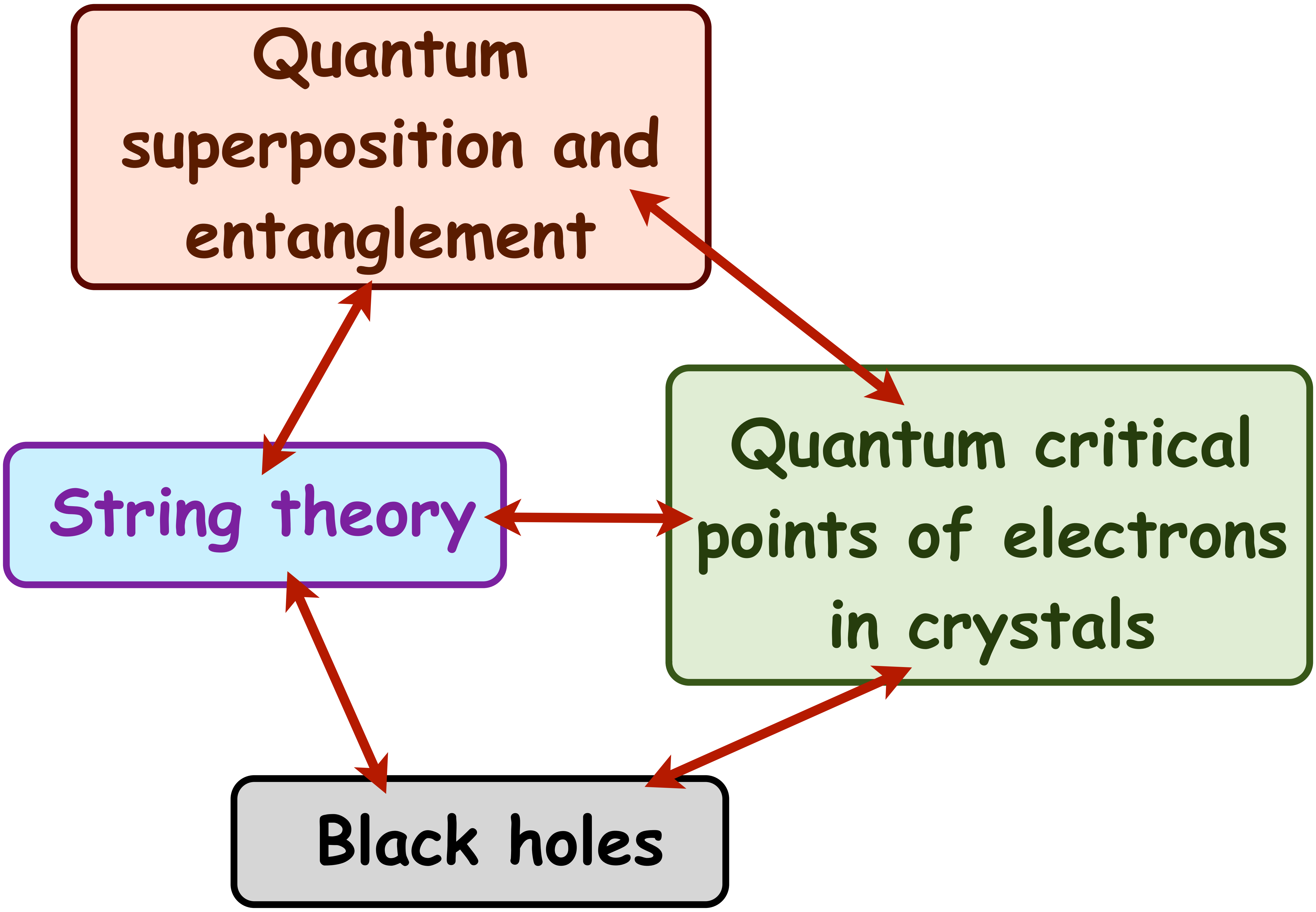
An (extended) Einstein-Maxwell provides successful description of dynamics of quantum critical points at non-zero temperatures (where no other methods apply)

Quantum superposition and entanglement

String theory

Quantum critical points of electrons in crystals

Black holes


Quantum superposition and entanglement
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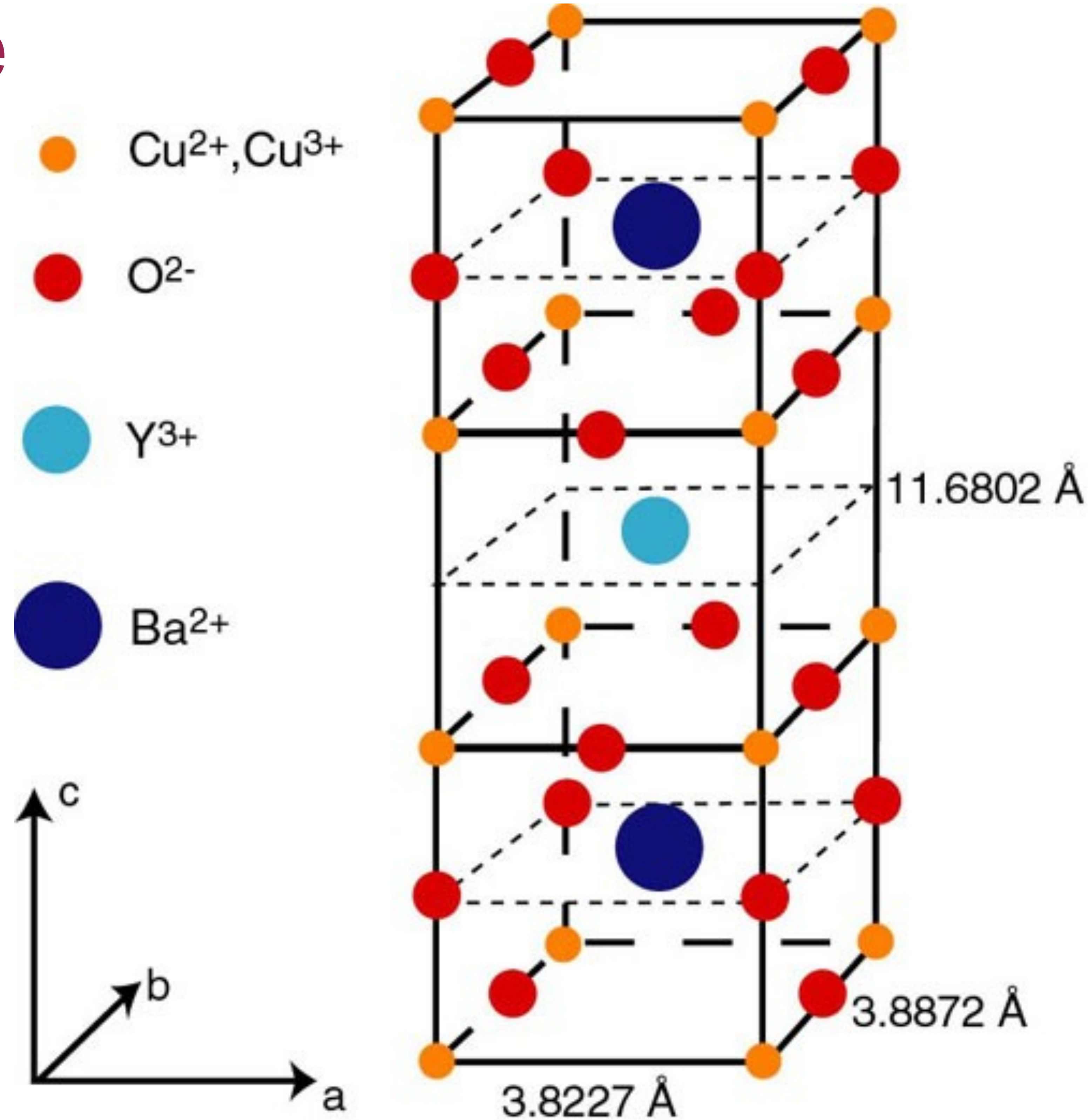
Metals, “strange metals”, and high temperature superconductors

Insights from gravitational “duals”

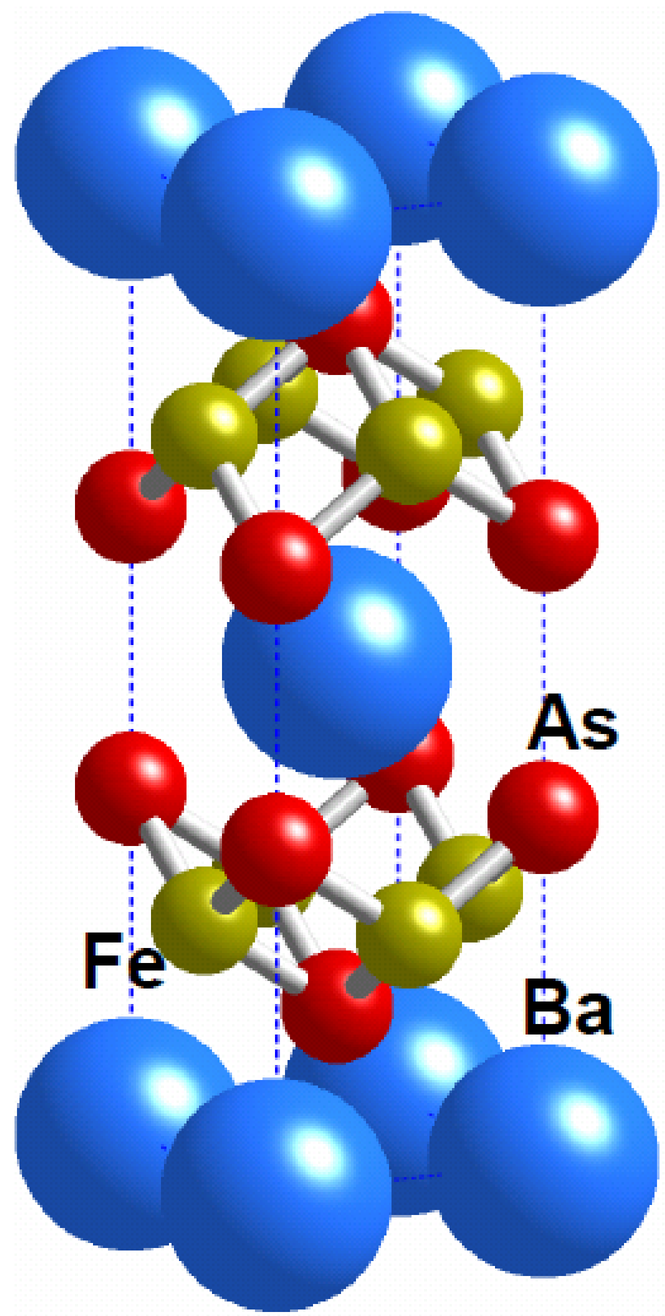
High temperature superconductors

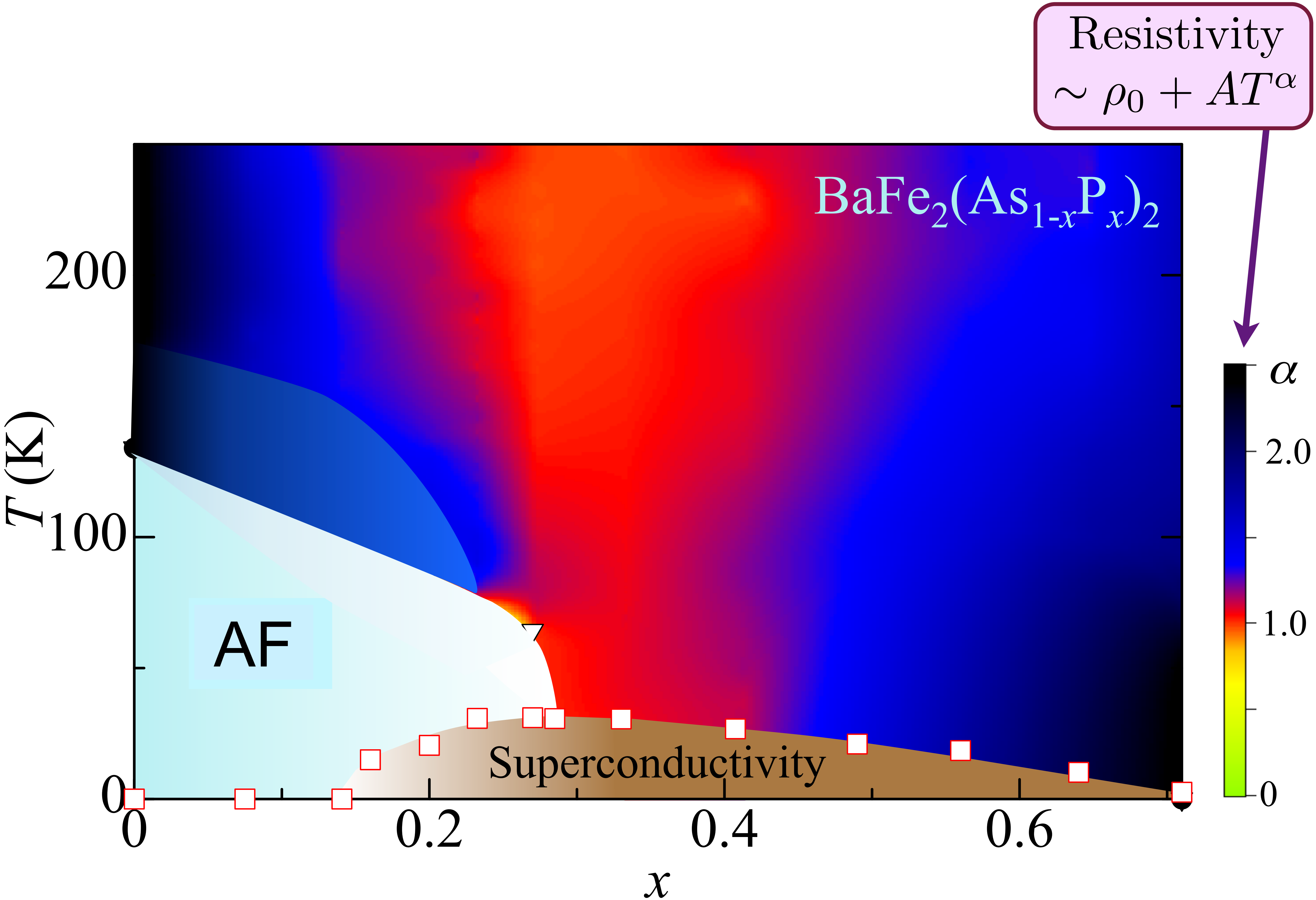


$$\mathrm{YBa_2Cu_3O_{6+x}}$$

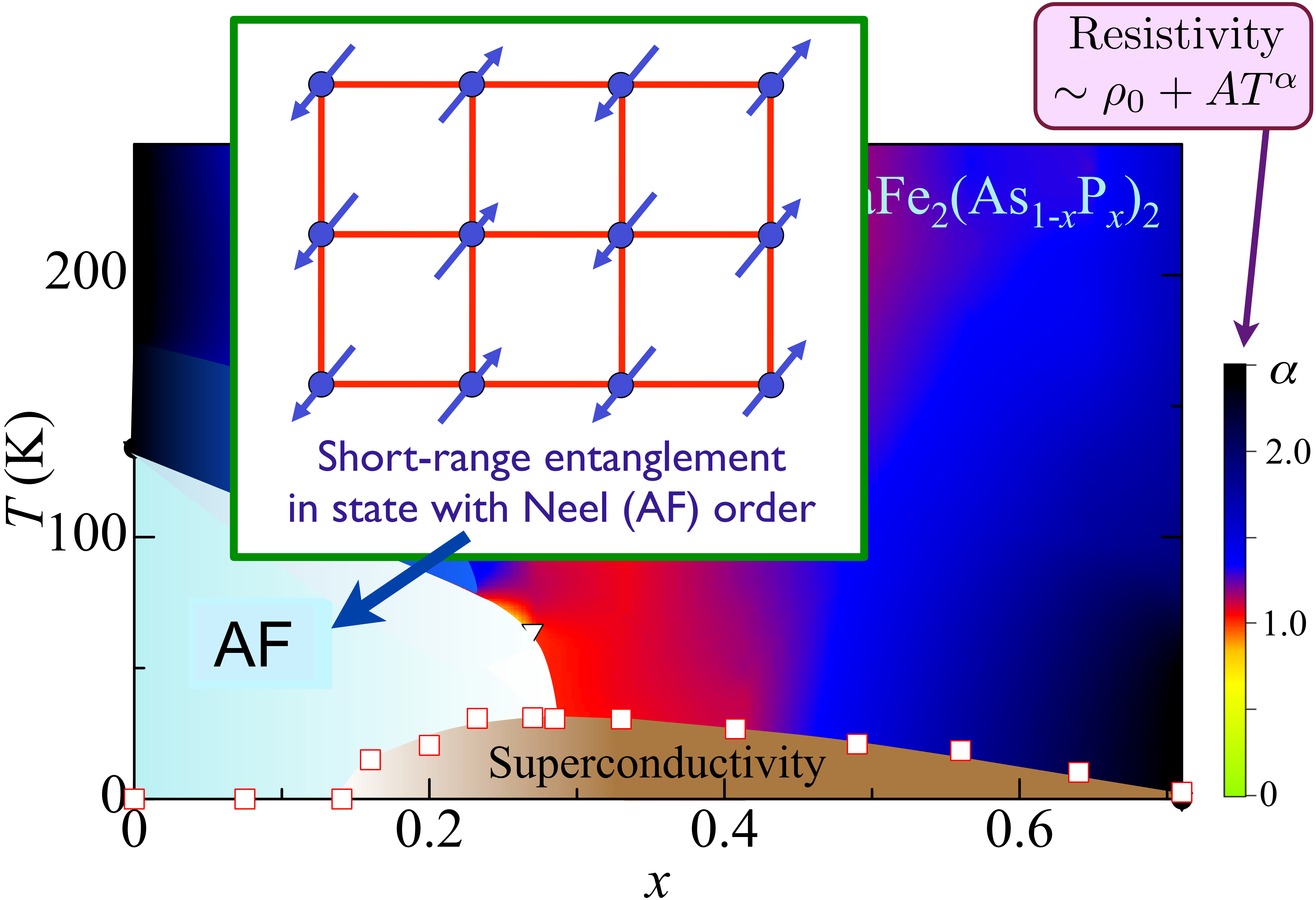
Iron pnictides:

a new class of high temperature superconductors





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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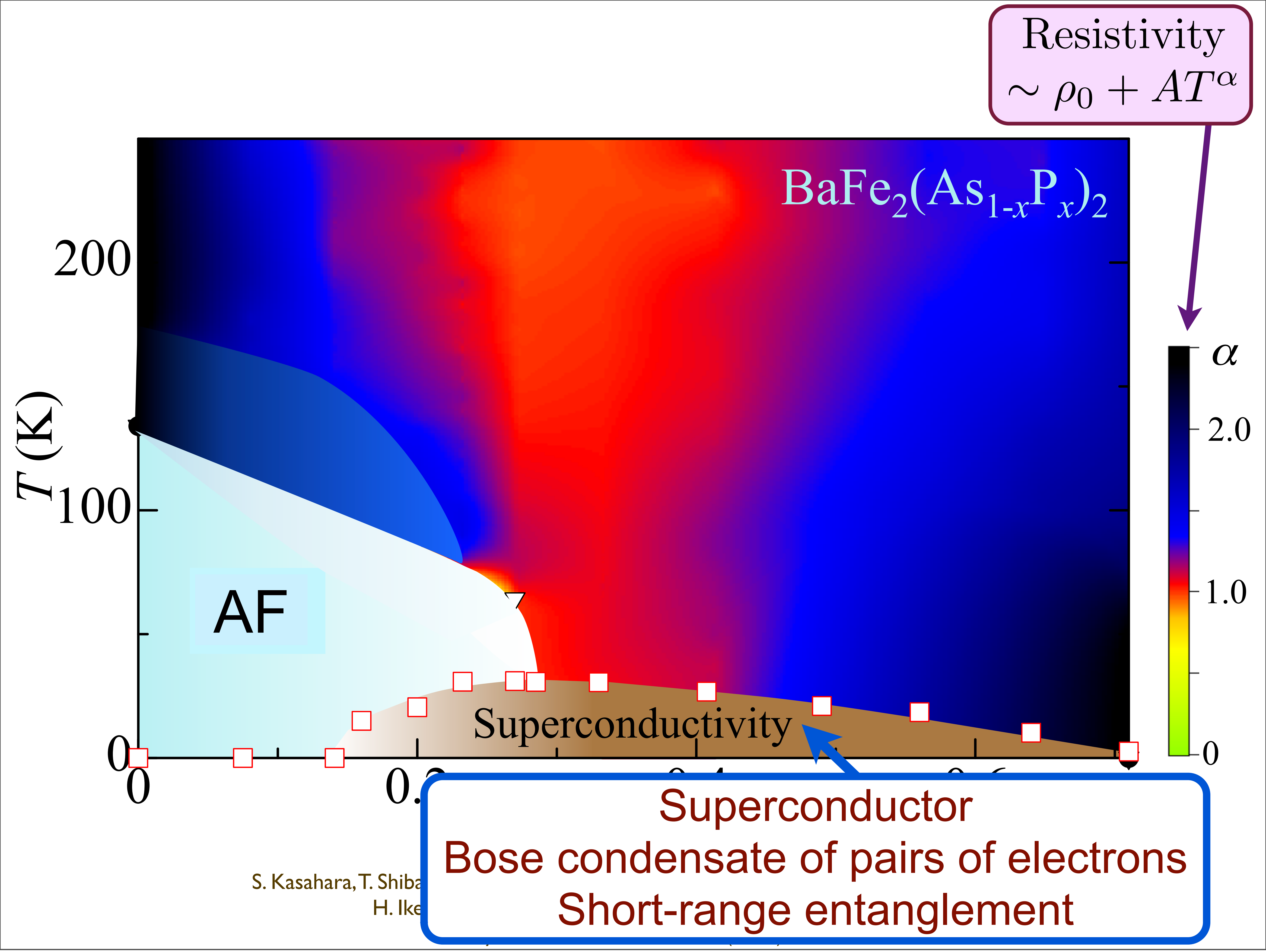
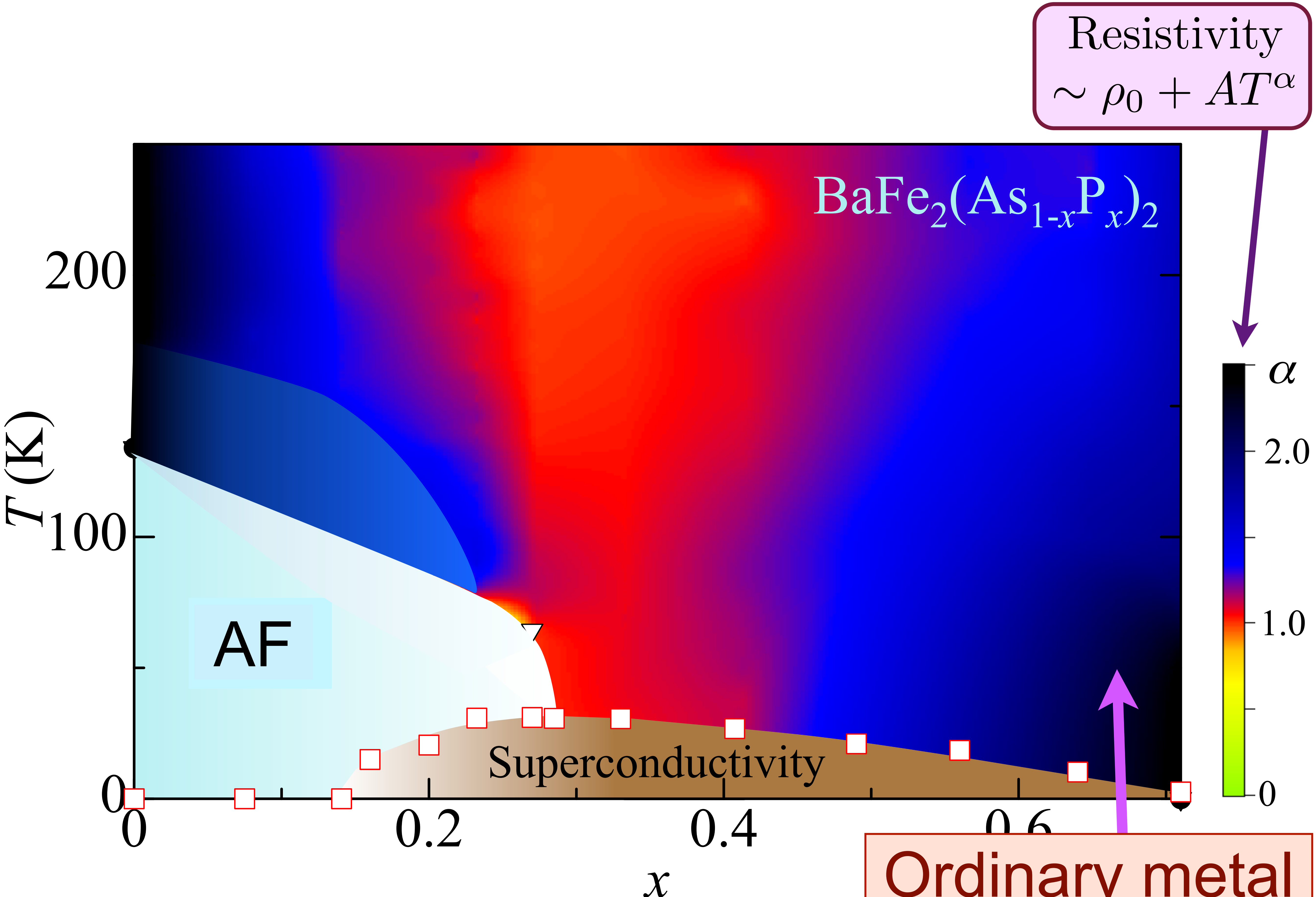
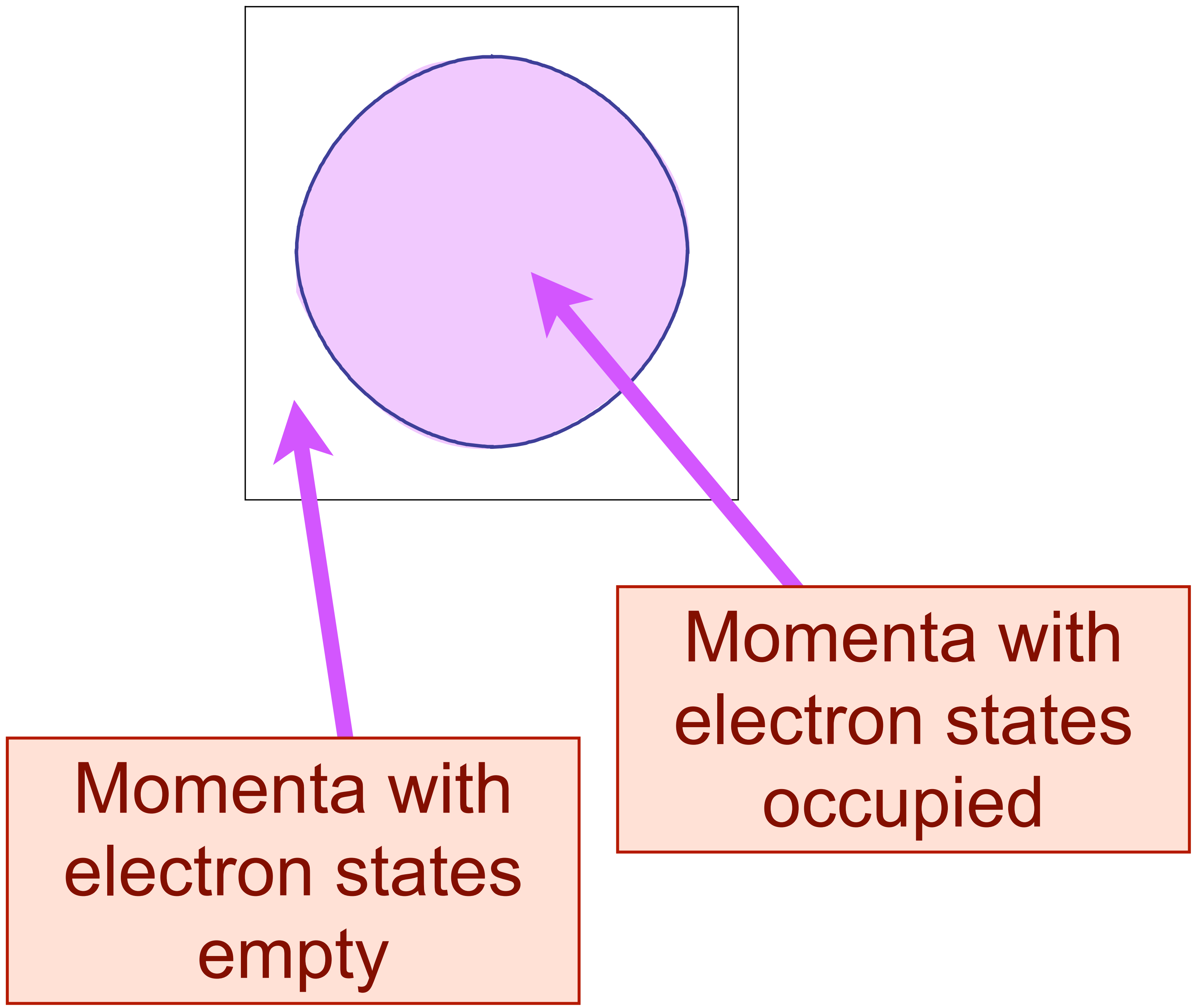

Resistivity
$\sim \rho_0 + AT^\alpha$
$BaFe_2(As_{1-x}P_x)_2$
T (K)
200
100
0
0
α
2.0
1.0
0
AF
Superconductivity
Superconductor
Bose condensate of pairs of electrons
Short-range entanglement



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikeda, S. Tonegawa, R. O
H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Mat
Physical Review B **81**, 184519 (2010)

Sommerfeld-Bloch theory of ordinary metals



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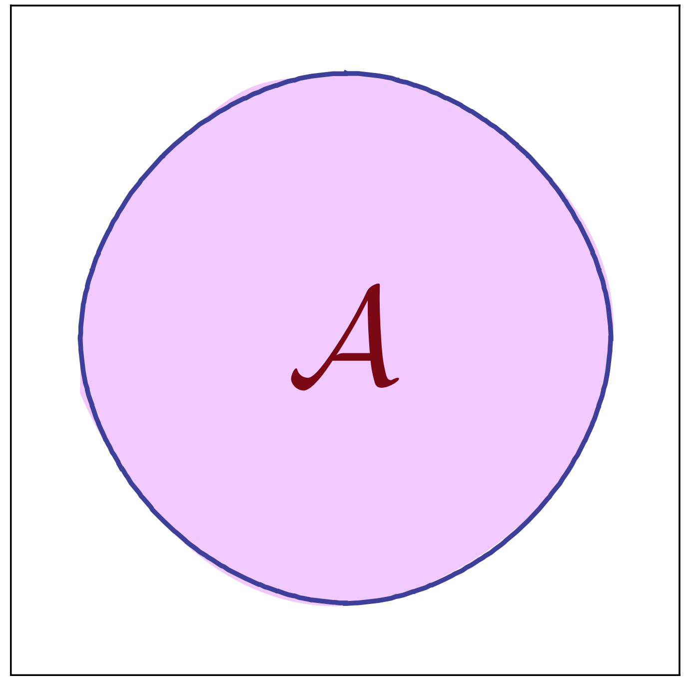
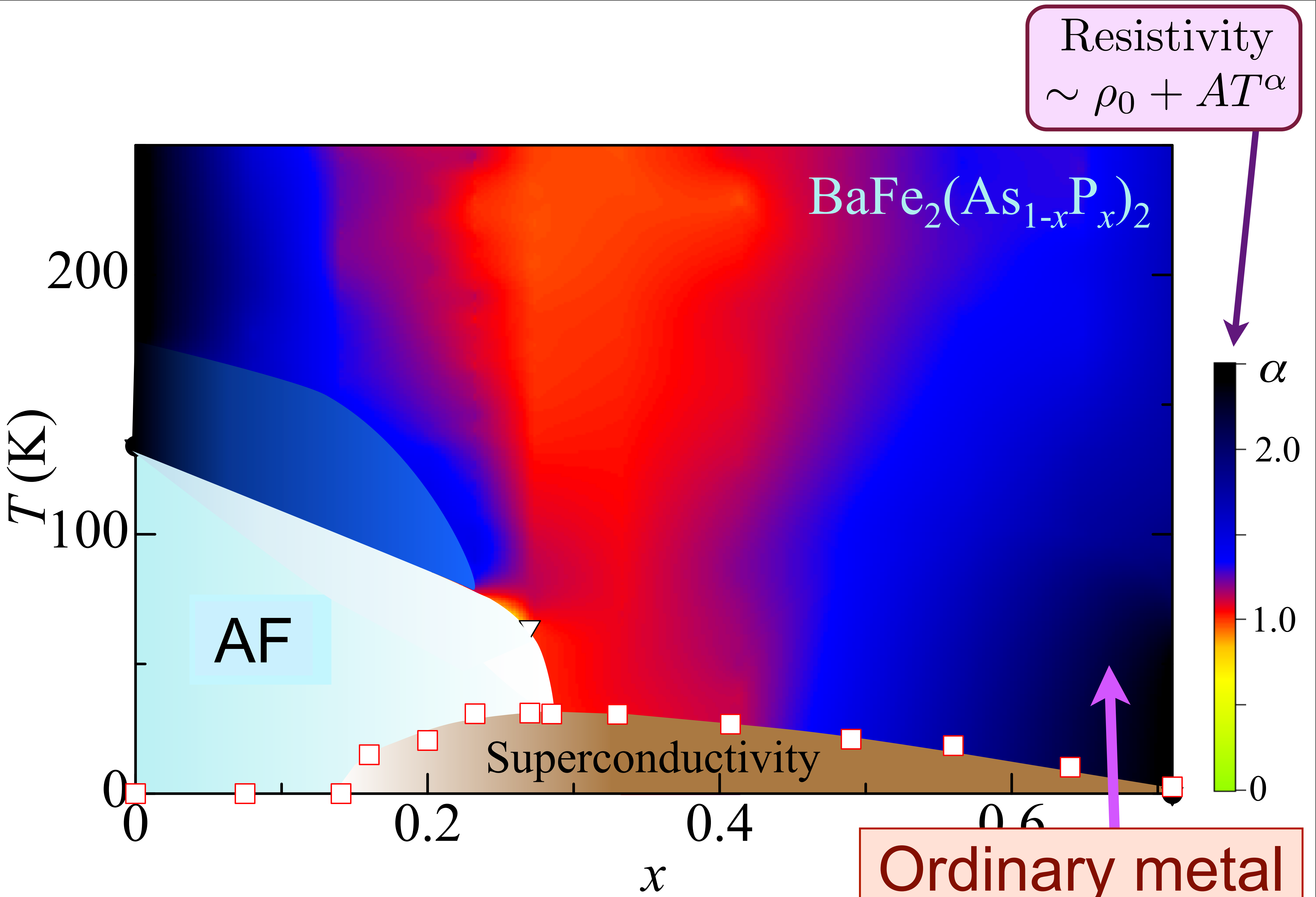


Key feature of the theory:
the **Fermi surface**

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the electron density
- Excitations near the Fermi surface are responsible for the familiar properties of ordinary metals, such as resistivity $\sim T^2$.



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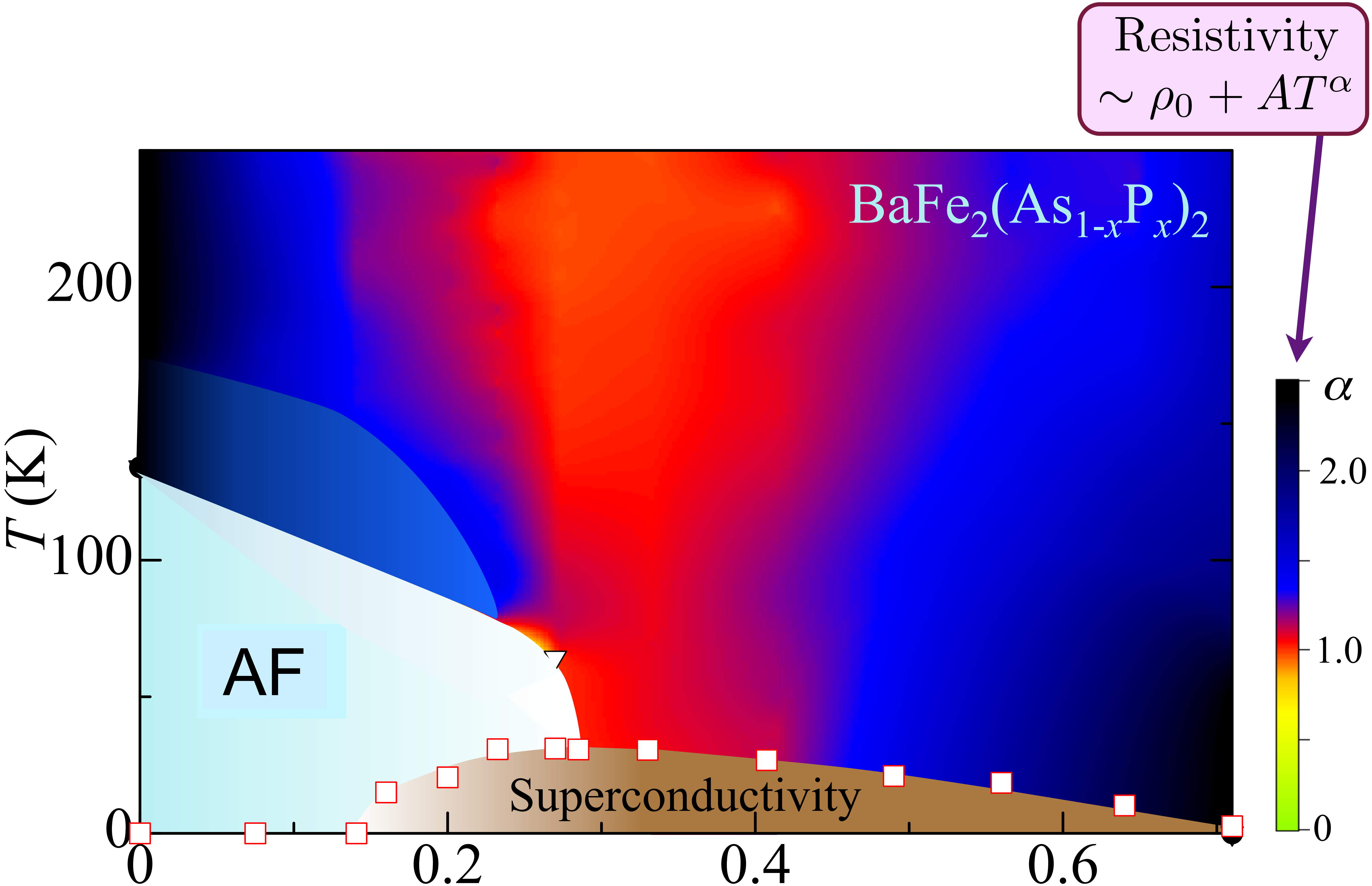
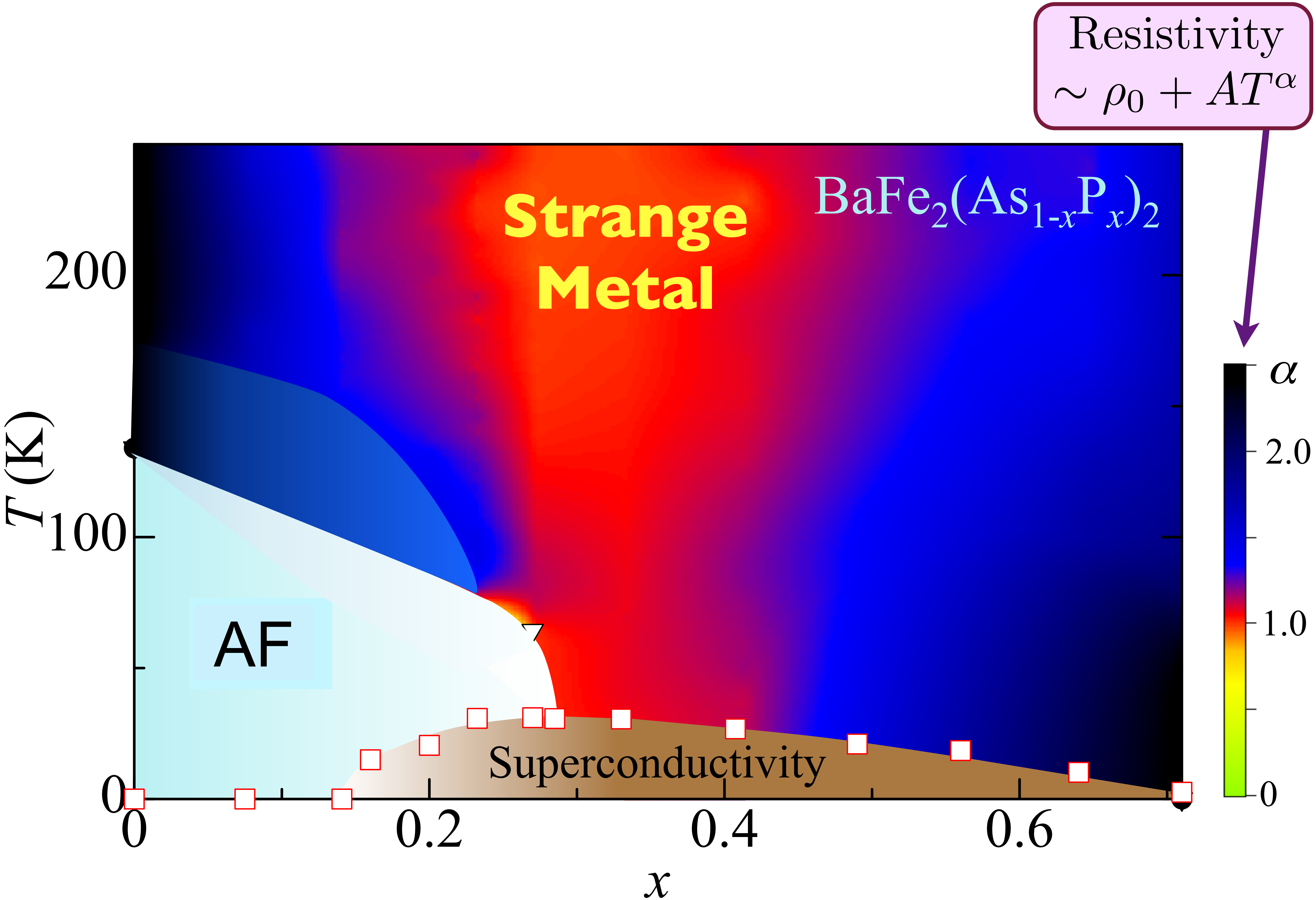
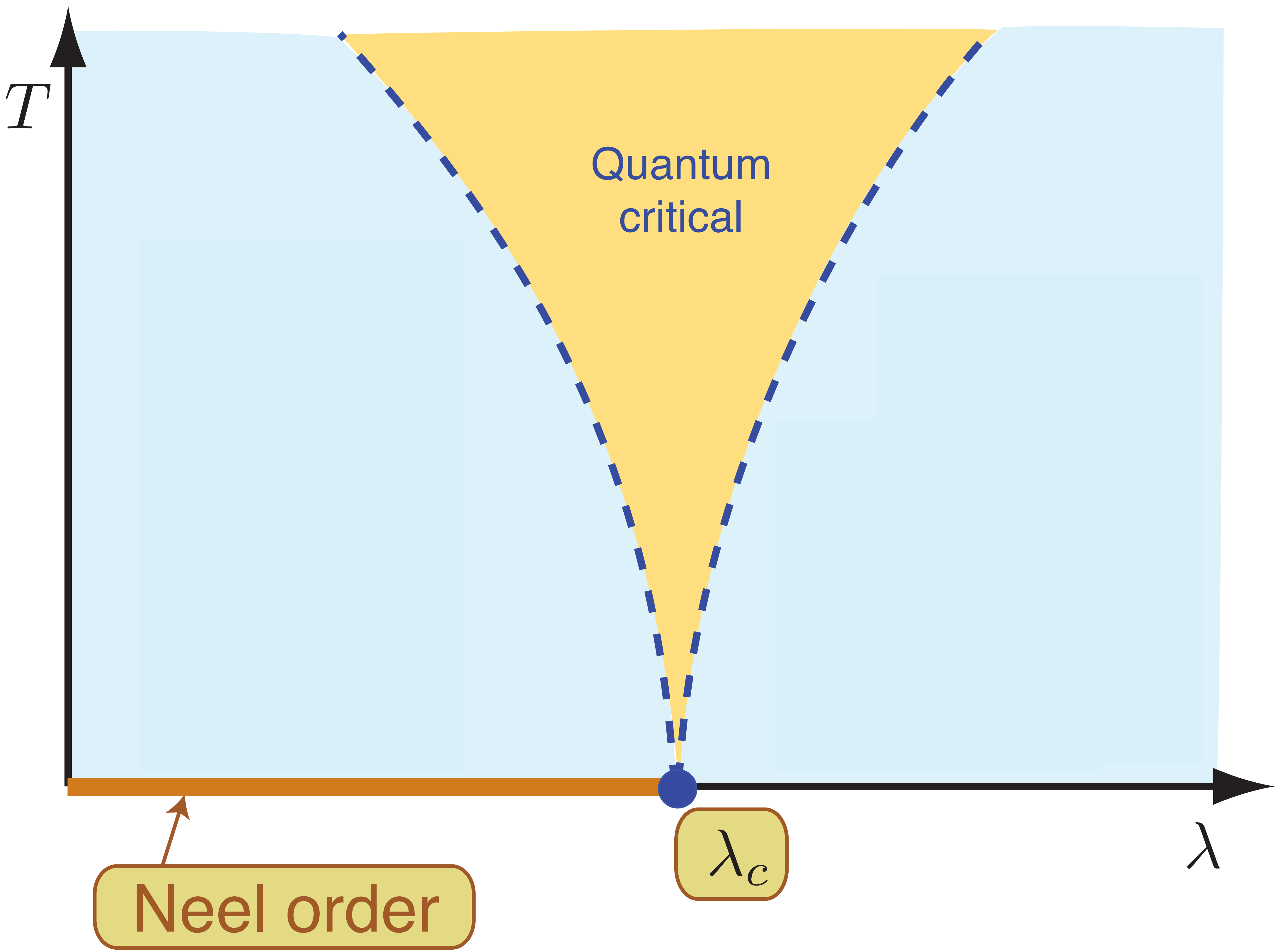


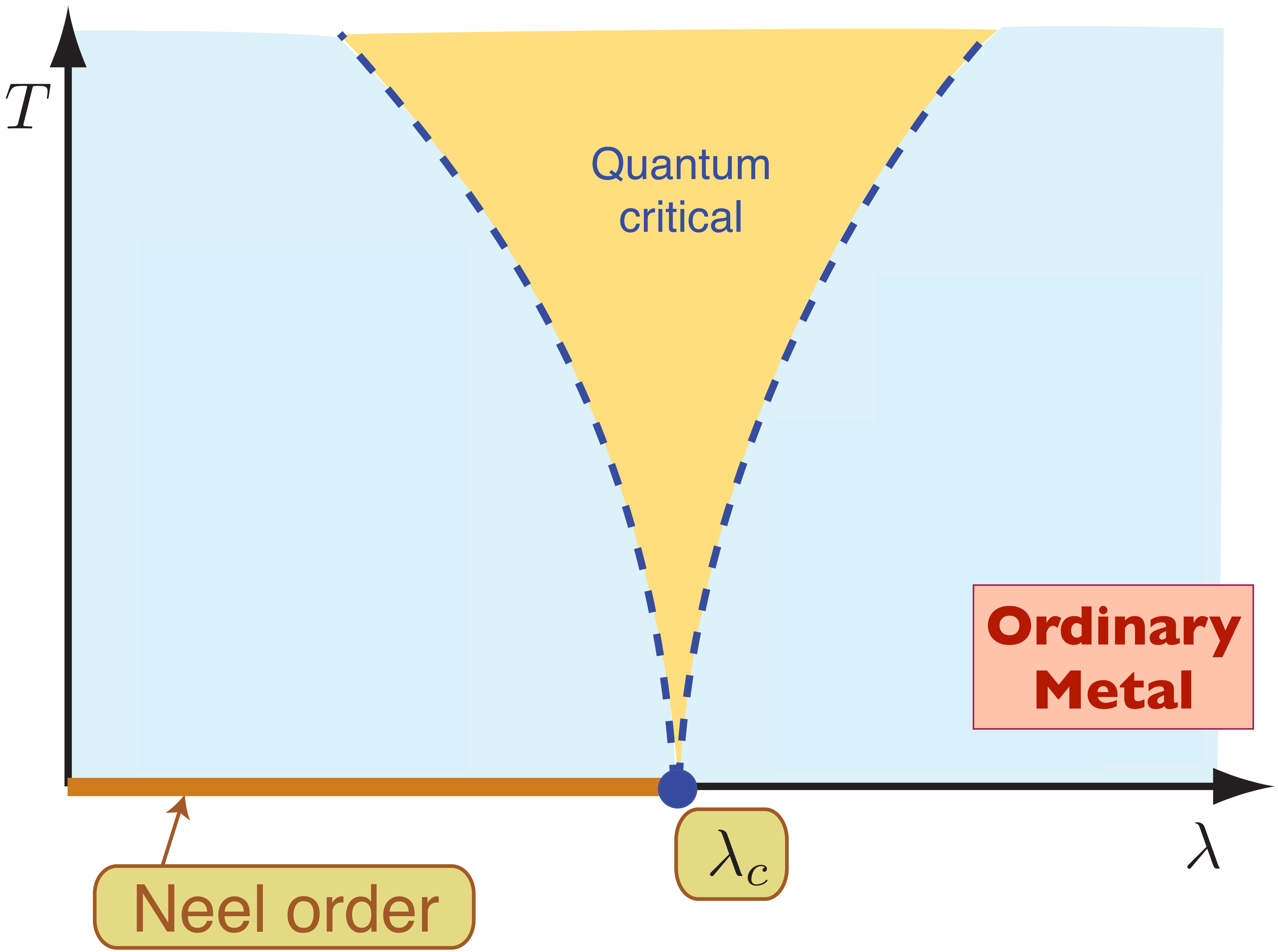
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

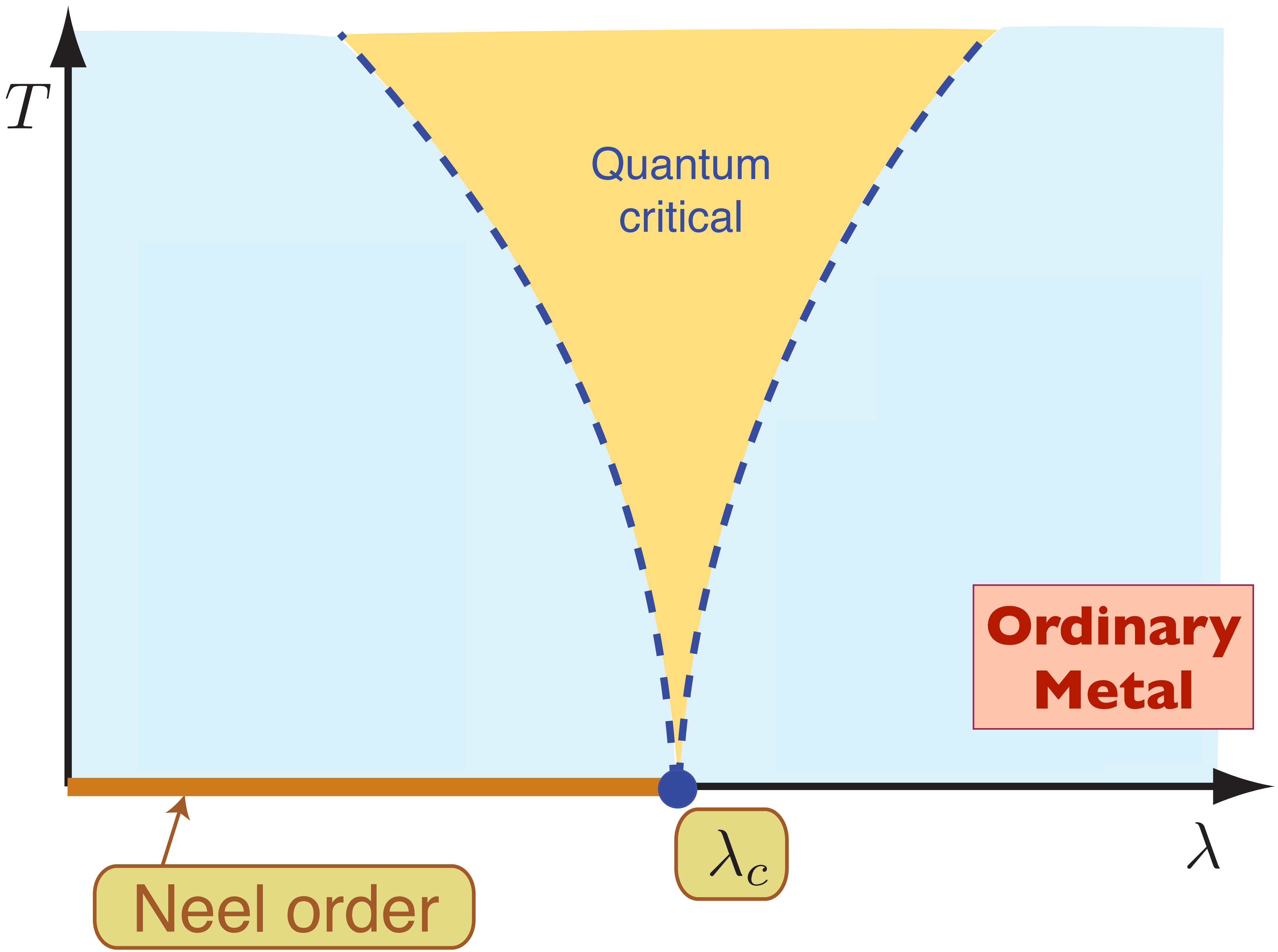


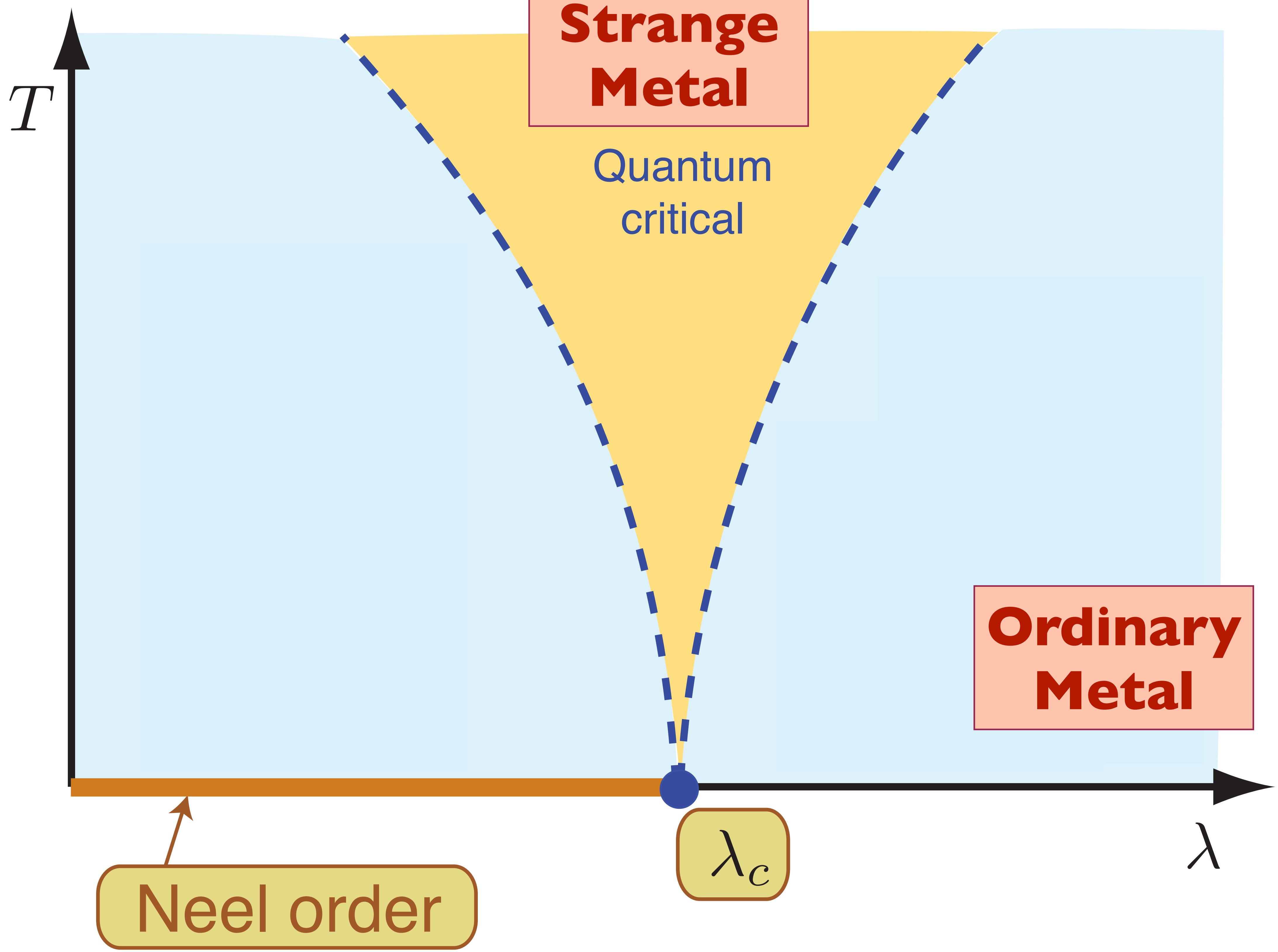
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

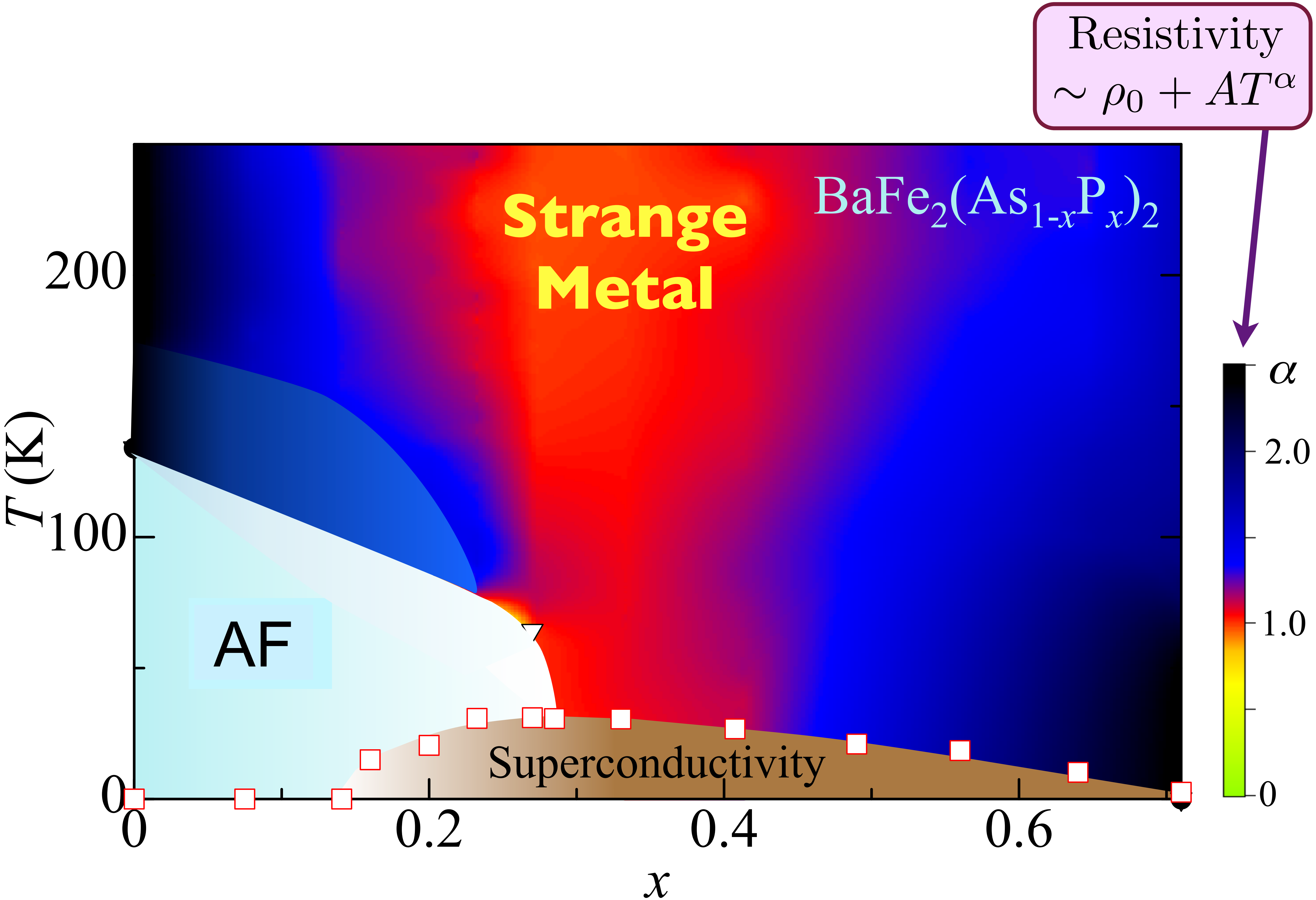



T
Quantum
critical
Ordinary
Metal
λ
Neel order
λ_c


T
Quantum critical
Ordinary Metal
Neel order
λ_c
λ



T
Strange Metal
Quantum critical
Ordinary Metal
λ
λ_c
Neel order



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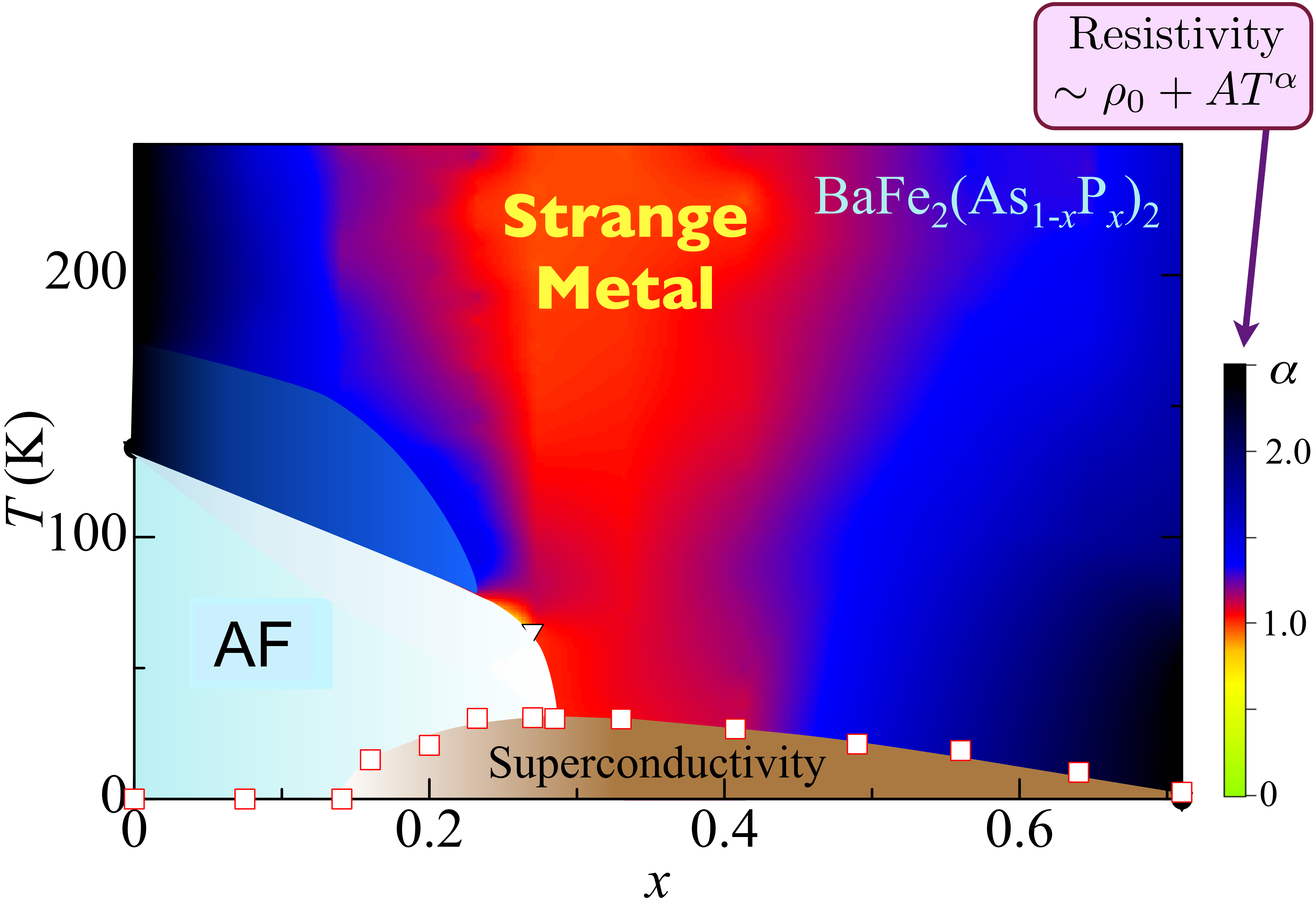

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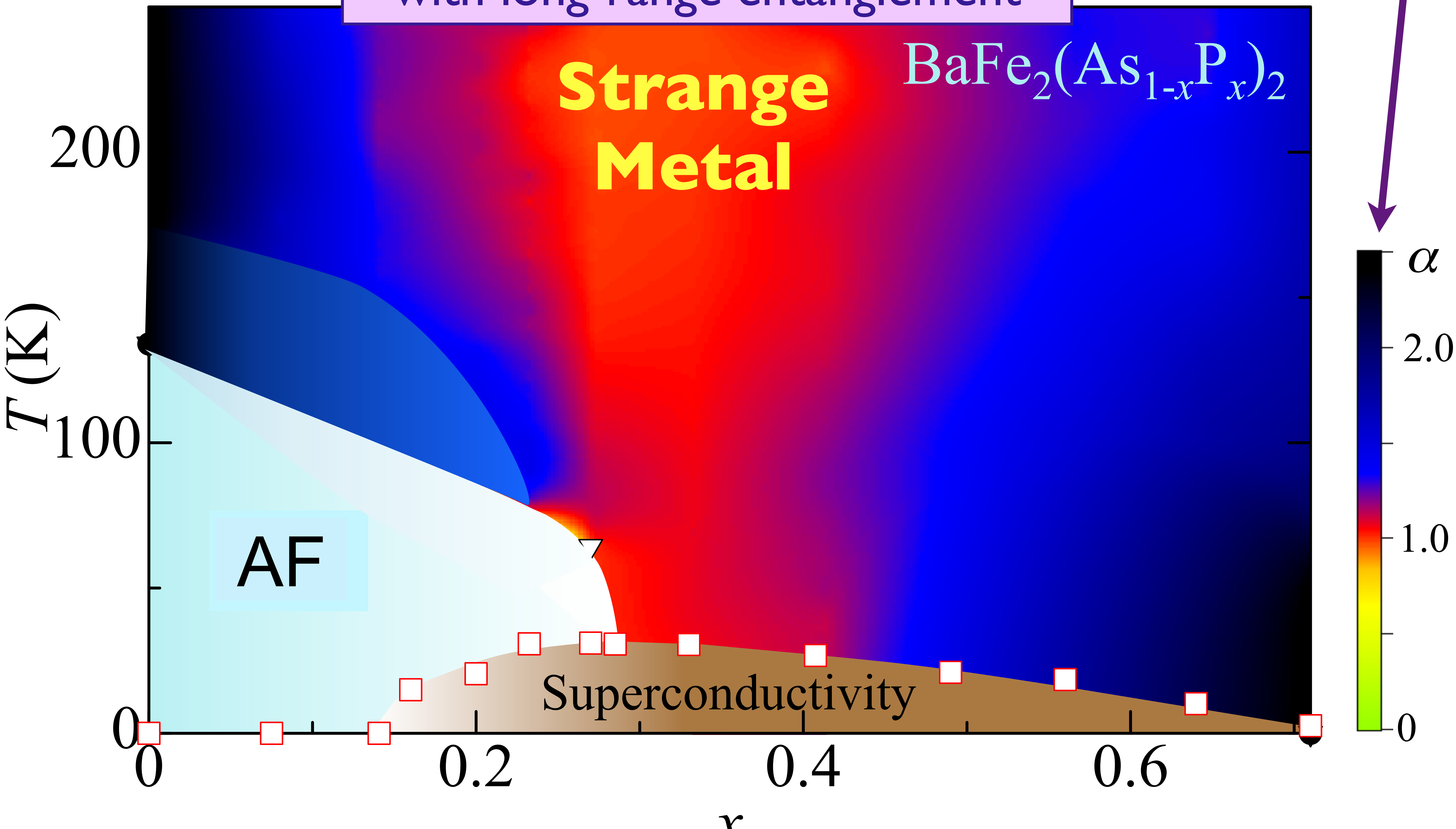
Excitations of a ground state with long-range entanglement

Resistivity $\sim \rho_0 + AT^\alpha$



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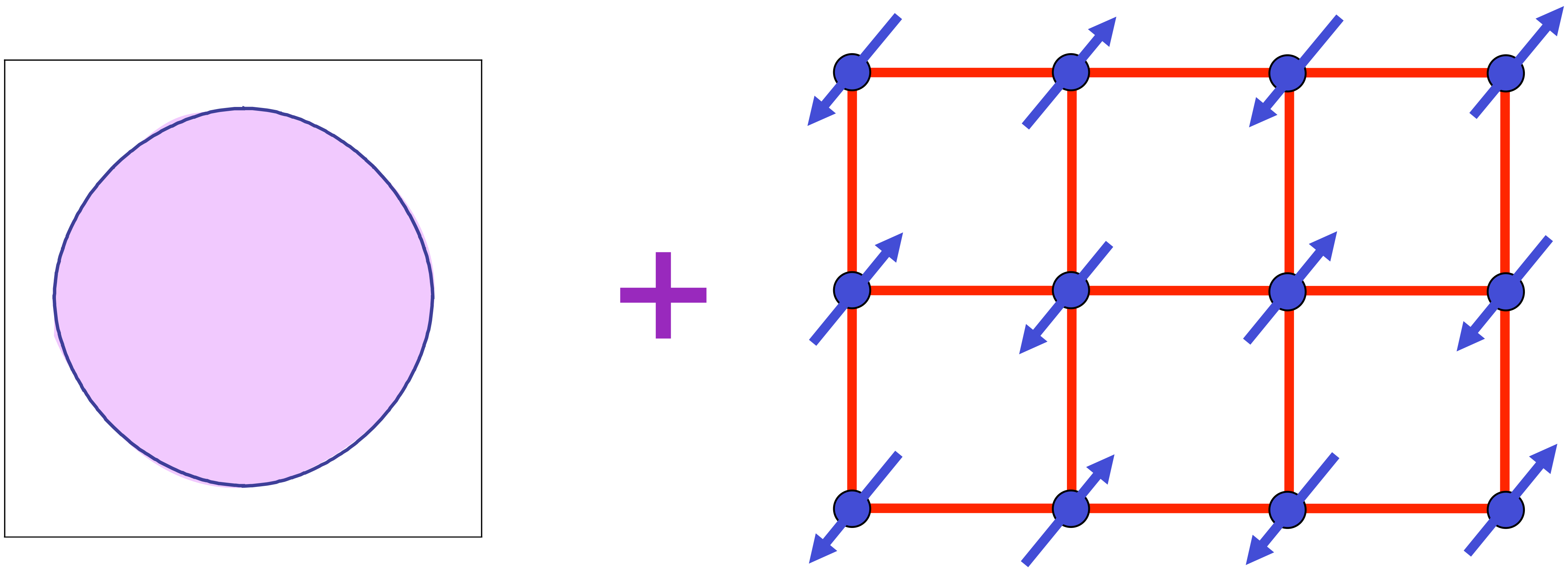
Key (difficult) problem:

Describe quantum critical points and phases of systems with Fermi surfaces leading to metals with novel types of long-range entanglement



Wednesday, May 9, 2012

Challenge to string theory:

Describe quantum critical points
and phases of metals

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Can we obtain gravitational theories of superconductors and ordinary Sommerfeld-Bloch metals ?

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Can we obtain gravitational theories of superconductors and ordinary Sommerfeld-Bloch metals ?

Yes

T. Nishioka, S. Ryu, and T. Takayanagi, JHEP **1003**, 131 (2010)
G. T. Horowitz and B. Way, JHEP **1011**, 011 (2010)
S. Sachdev, Physical Review D **84**, 066009 (2011)

Challenge to string theory:

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Do the "holographic" gravitational theories also yield metals distinct from ordinary Sommerfeld-Bloch metals ?

Challenge to string theory:

Describe quantum critical points and phases of metals

Do the "holographic" gravitational theories also yield metals distinct from ordinary Sommerfeld-Bloch metals ?

Yes, lots of them, with many "strange" properties !

S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694;
F. Denef, S. A. Hartnoll, and S. Sachdev, *Phys. Rev. D* **80**, 126016 (2009)

Challenge to string theory:

Describe quantum critical points
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How do we discard artifacts, and choose the
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Challenge to string theory:

Describe quantum critical points and phases of metals

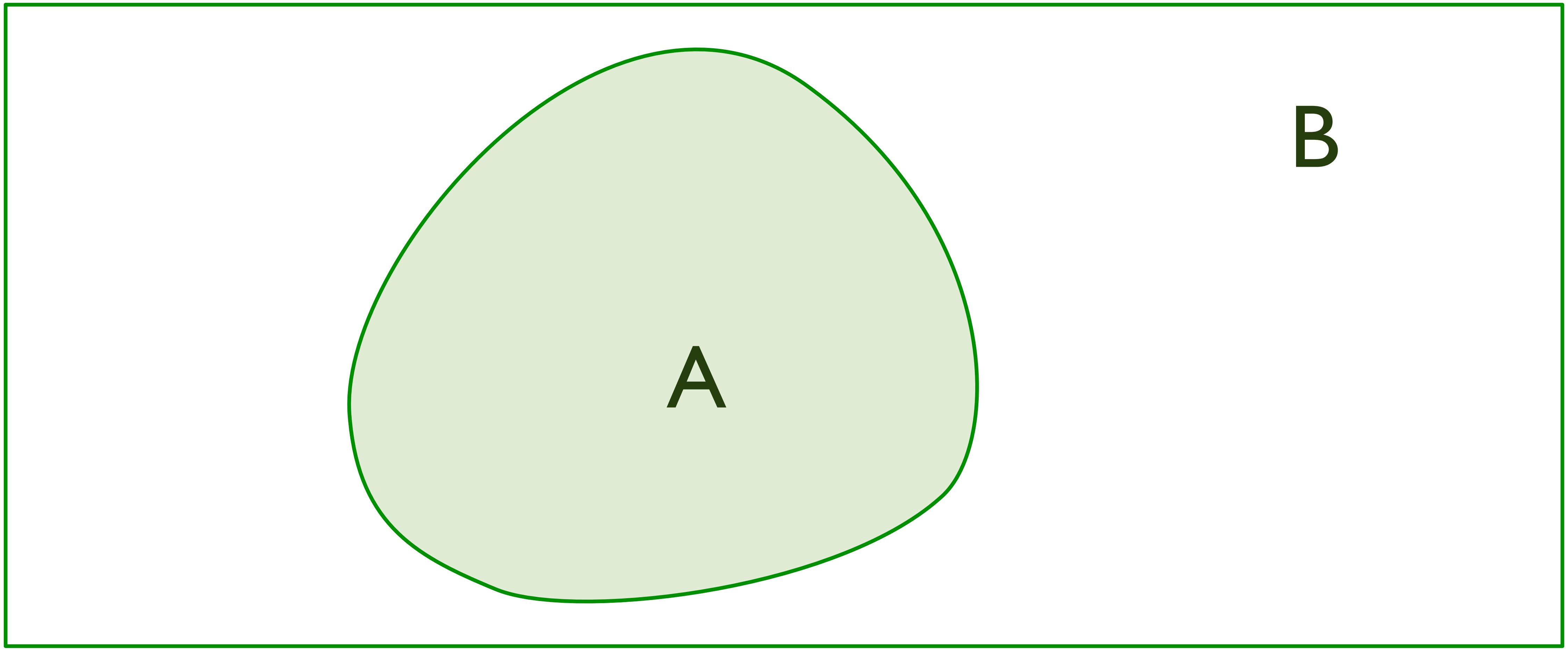
How do we discard artifacts, and choose the holographic theories applicable to condensed matter physics ?

Choose the theories with the proper entropy density

Checks: these theories also have the proper entanglement entropy and Fermi surface size !

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Entanglement entropy of Fermi surfaces

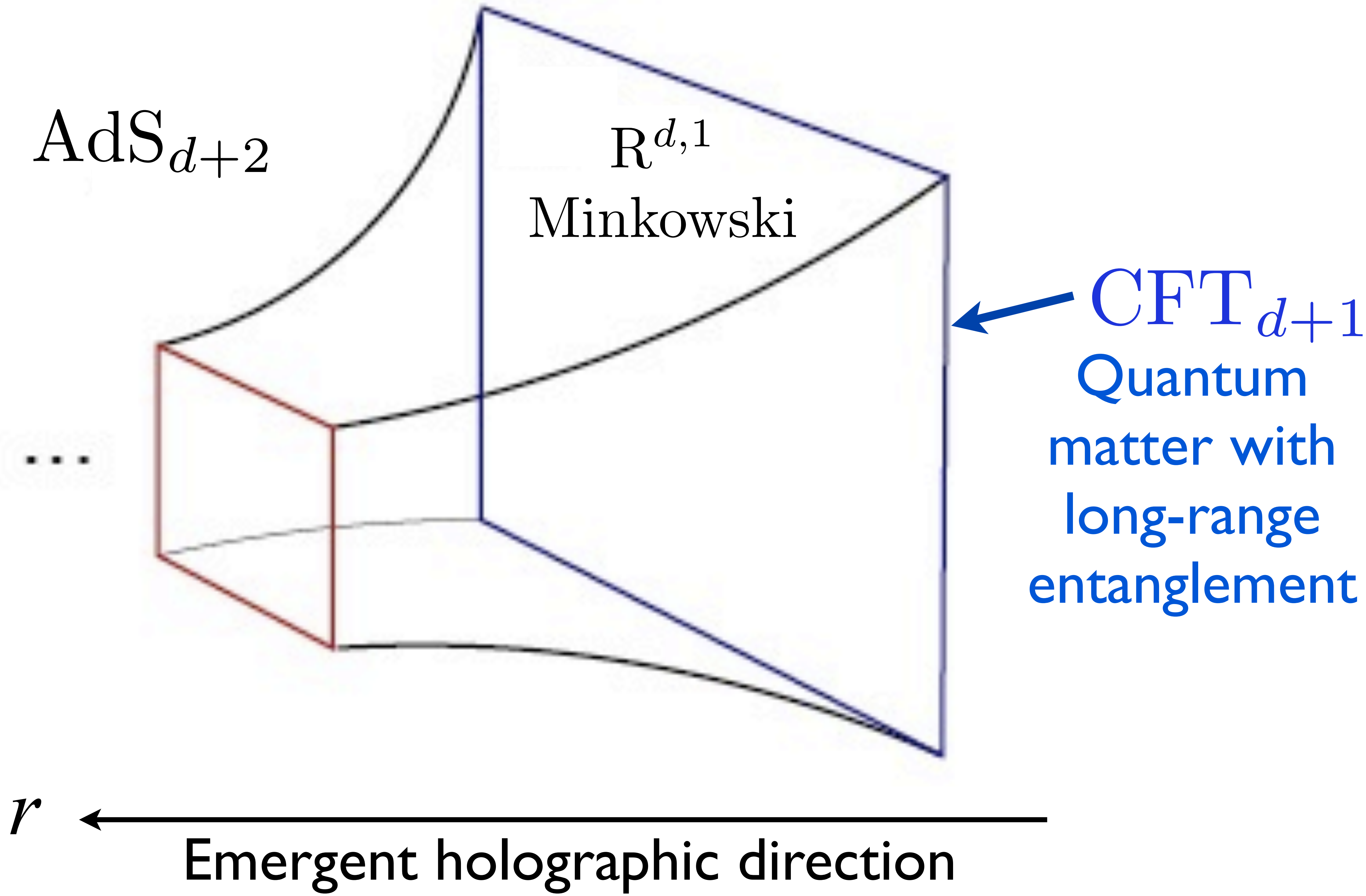


Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F L)\ln(k_F L)$

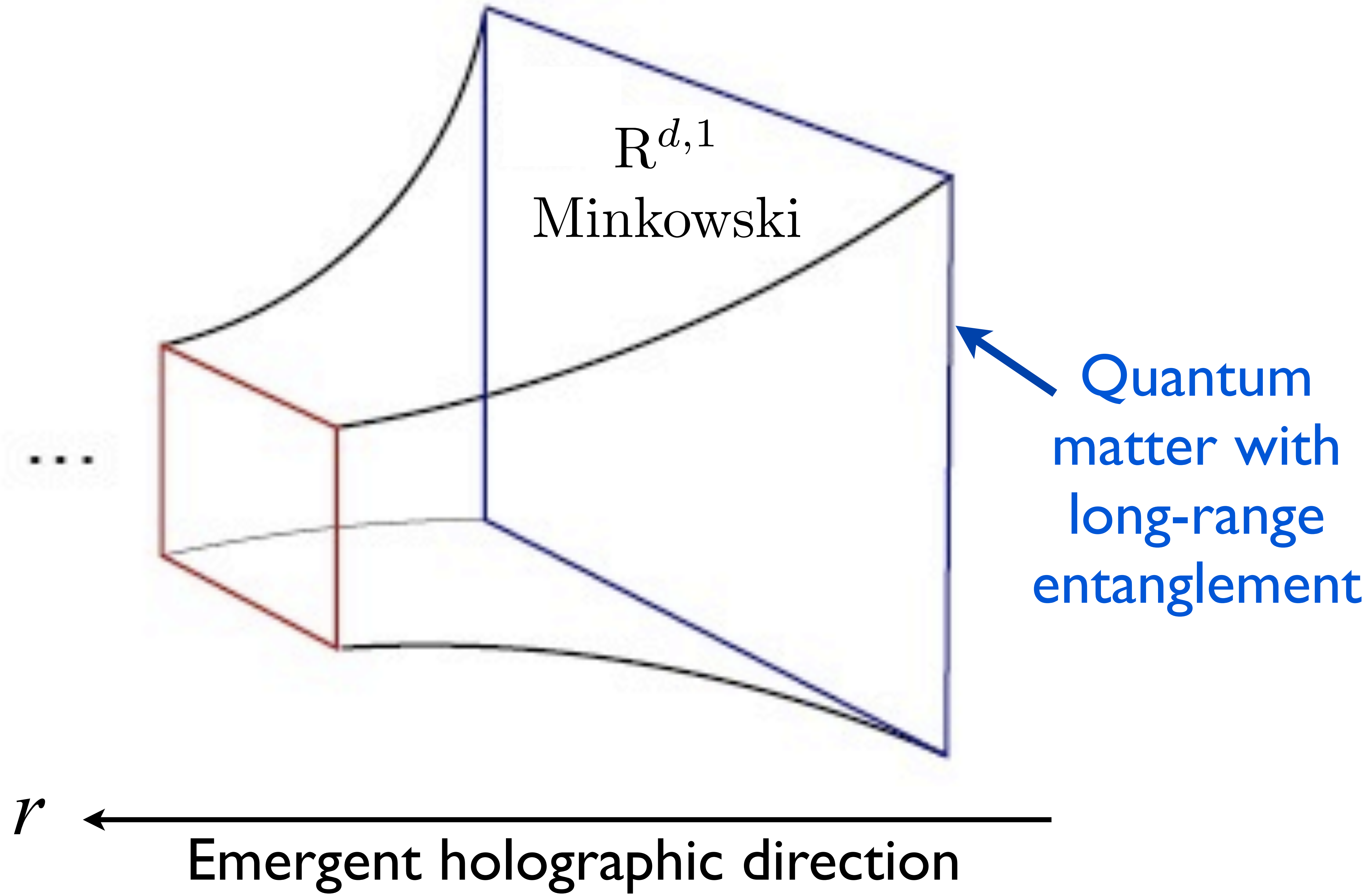
for a circular Fermi surface with Fermi momentum k_F,
where L is the perimeter of region A with an arbitrary smooth shape.

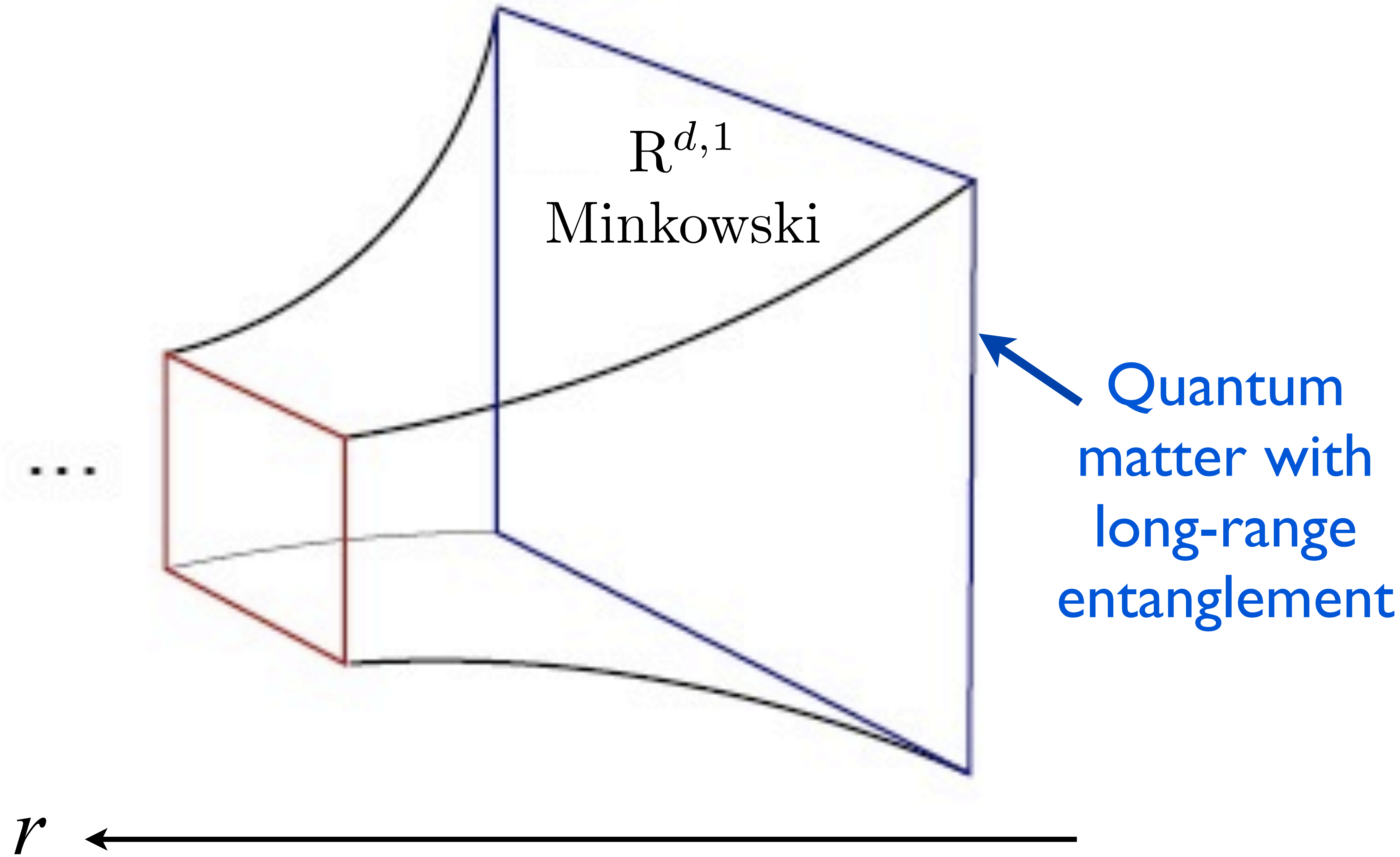
Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)



J. McGreevy, arXiv0909.0518


$R^{d,1}$
Minkowski
Quantum matter with long-range entanglement
r
Emergent holographic direction



Abandon conformal invariance, and only require scale invariance at long lengths and times.....

Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.

-

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

AdS_{d+2} corresponds to $\theta = 0$, $z = 1$. We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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- The thermal entropy density scales as (Stefan-Boltzman law or "hyperscaling")

$$S \sim T^{(d-\theta)/z}.$$

where $d - \theta$ is the effective dimension. A Fermi surface has excitations which disperse only in one direction, and so we require $\theta = d - 1$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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- The entanglement entropy, S_E, of an entangling region with boundary surface 'area' Σ scales as

$$S_E \sim \begin{cases} \Sigma & , \quad \text{for } \theta < d-1 \\ \Sigma \ln \Sigma & , \quad \text{for } \theta = d-1 \\ \Sigma^{\theta/(d-1)} & , \quad \text{for } \theta > d-1 \end{cases}$$

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023

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So we have log violation of area law for $\theta = d-1$, just as expected for a Fermi surface !

- The null energy condition implies $z \geq 1 + \dfrac{\theta}{d}$. This implies $z \geq 3/2$ in $d=2$. Remarkably the value $z = 3/2$ is obtained from a field theory of a Fermi surface coupled to an emergent gauge field !!

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.

Conclusions

Simplest examples of long-range entanglement are in insulating antiferromagnets: Z_2 spin liquids and quantum critical points

Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory

Conclusions

String theory and gravity in
emergent dimensions
offer a remarkable new approach
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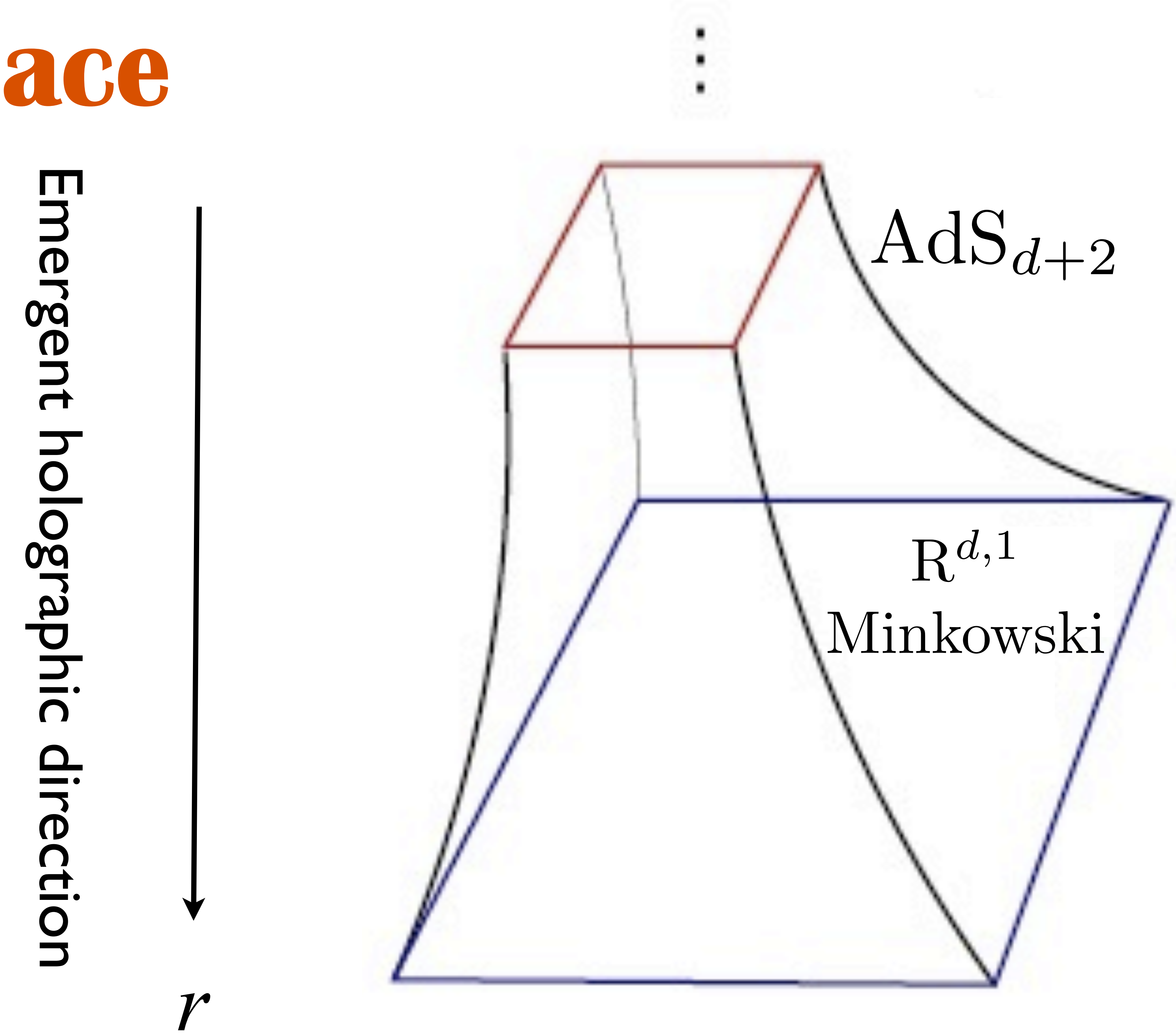
Much recent progress offers hope of a
holographic description of "strange metals"

anti-de Sitter space



anti-de Sitter space