Insulators and superconductors with topological order

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Outline

1. Quantum "disordering" magnetic order

   Collinear order and confinement

2. Z_2 spin liquids

   Noncollinear order and fractionalization

3. Gapless U(1) spin liquids

   Deconfined criticality

4. Doped spin liquids

   Superconductors with topological order

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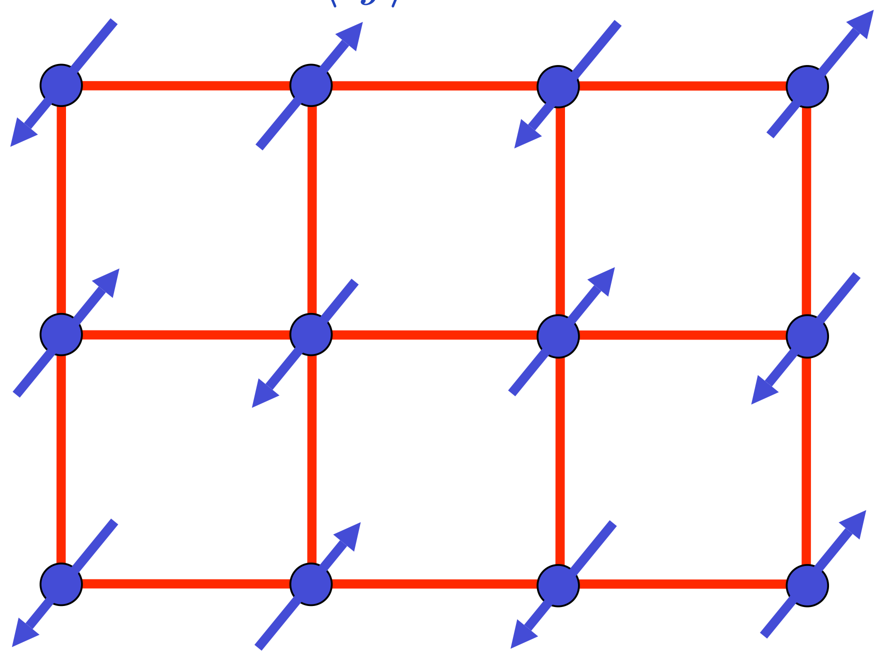
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

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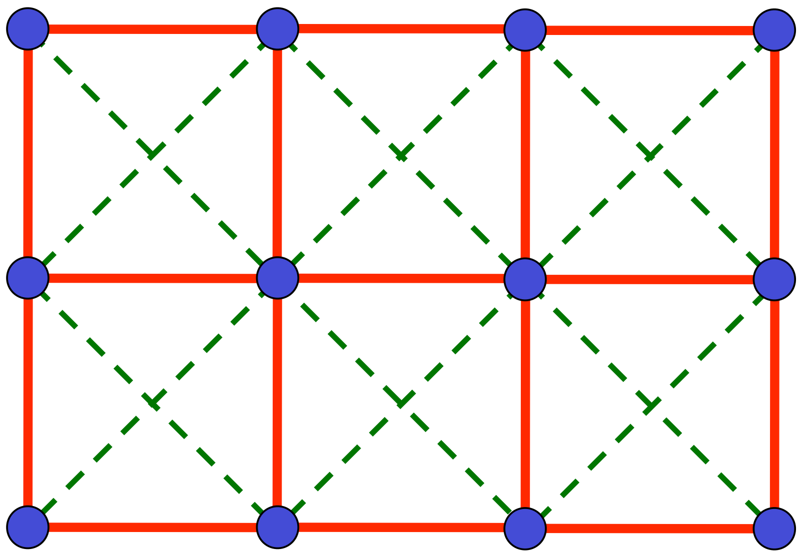

Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What is the state with $\langle \vec{\varphi} \rangle = 0$?

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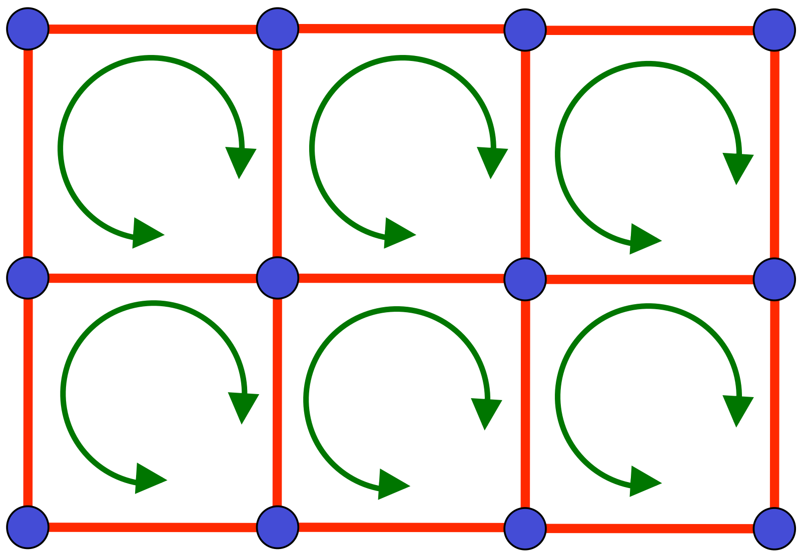

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LGW theory for quantum criticality

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$\mathcal{S}_\varphi = \int d^2x d\tau \left[\frac{1}{2} \left(c^2 \left(\nabla_x \vec{\varphi} \right)^2 + \left(\partial_\tau \vec{\varphi} \right)^2 + s \vec{\varphi}^2 \right) + u \left(\vec{\varphi}^2 \right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

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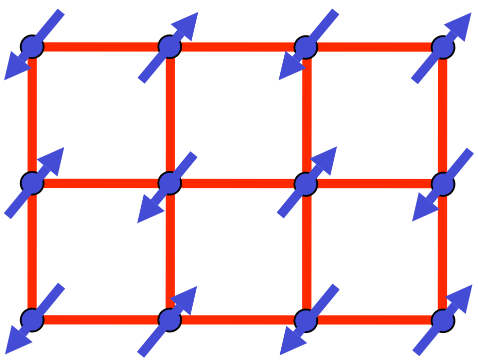

$\langle \vec{\varphi} \rangle \neq 0$

Néel state

State with no broken symmetries. Fluctuations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ realize a *stable* $S = 1$ quasiparticle with energy $\varepsilon_k = \sqrt{s + c^2 k^2}$

$\langle \vec{\varphi} \rangle = 0$

s_c s

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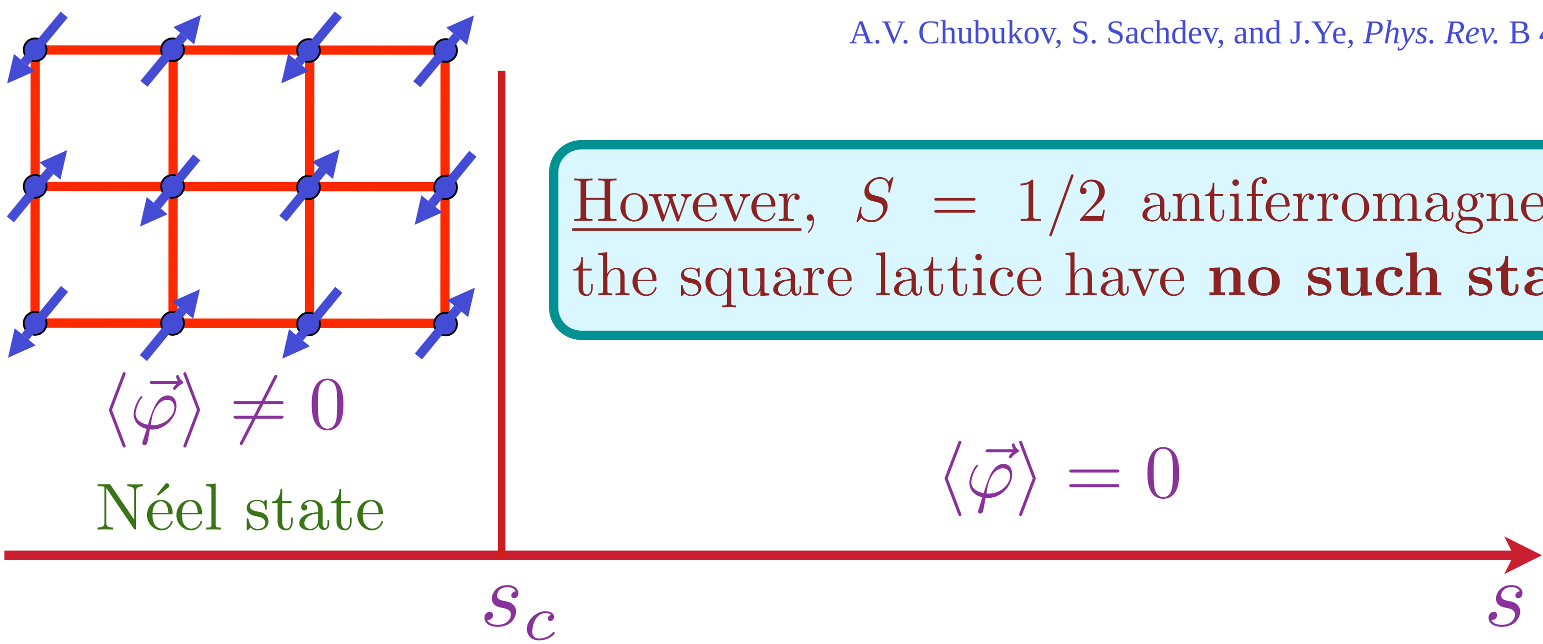

However, $S = 1/2$ antiferromagnets on the square lattice have **no such state**.

There is no state with a gapped, stable S=1 quasiparticle and no broken symmetries

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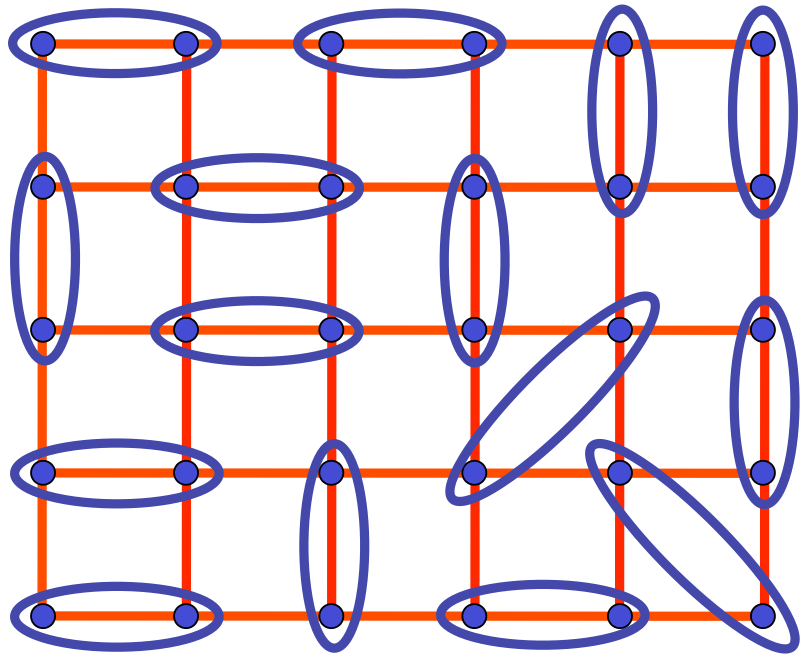

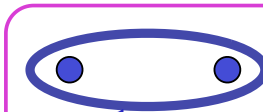

$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

"Liquid" of valence bonds has fractionalized *S=1/2* excitations

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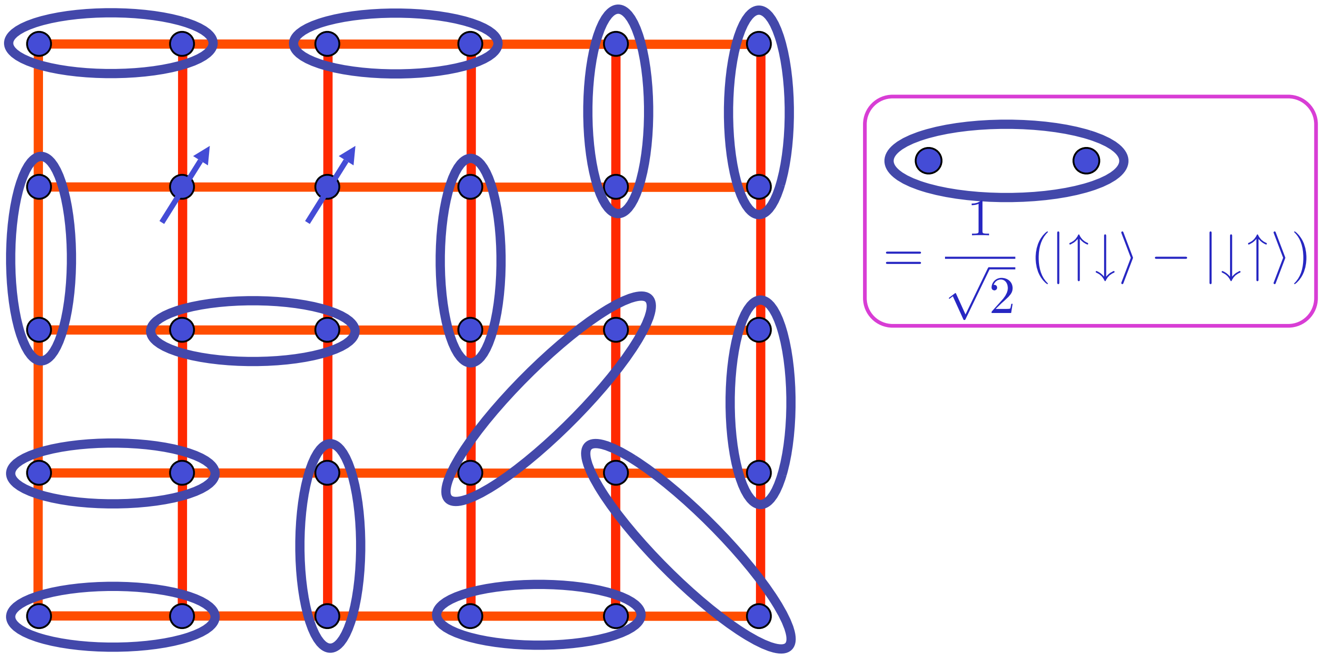

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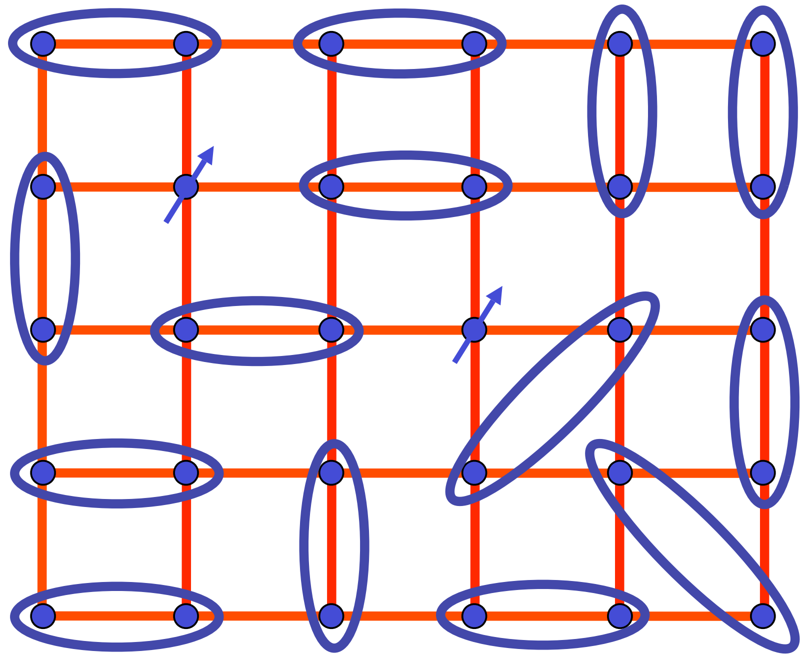

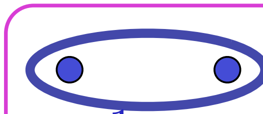
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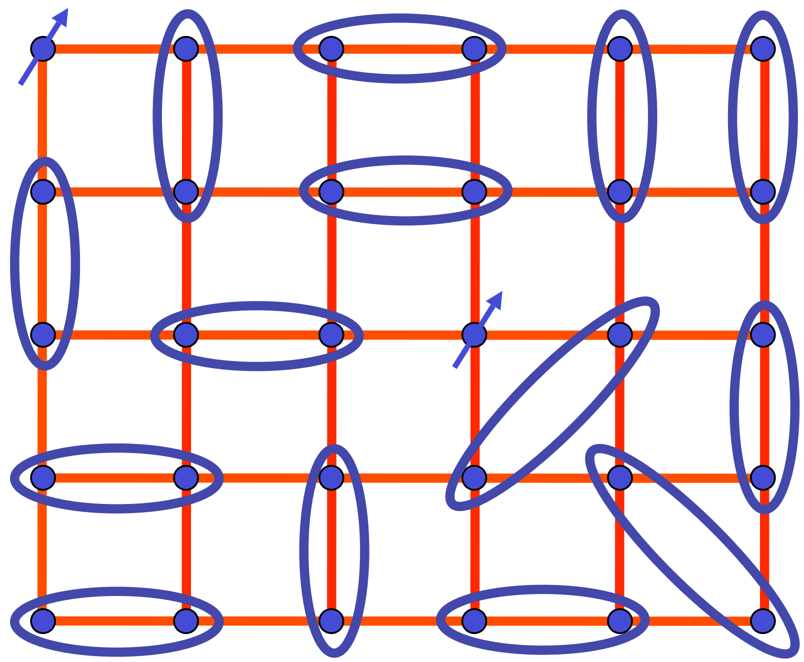

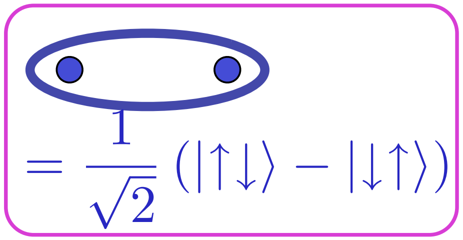

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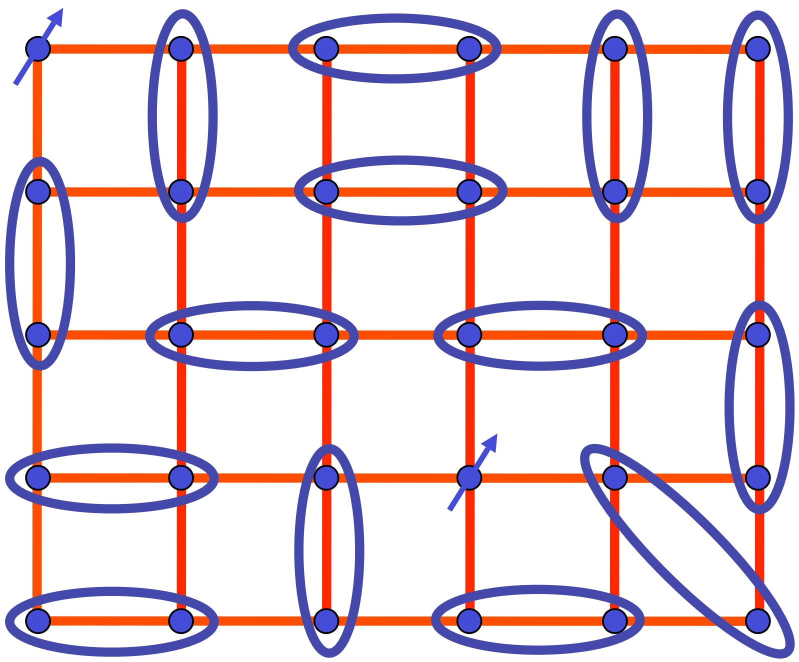

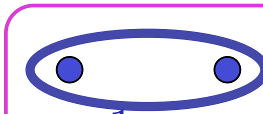
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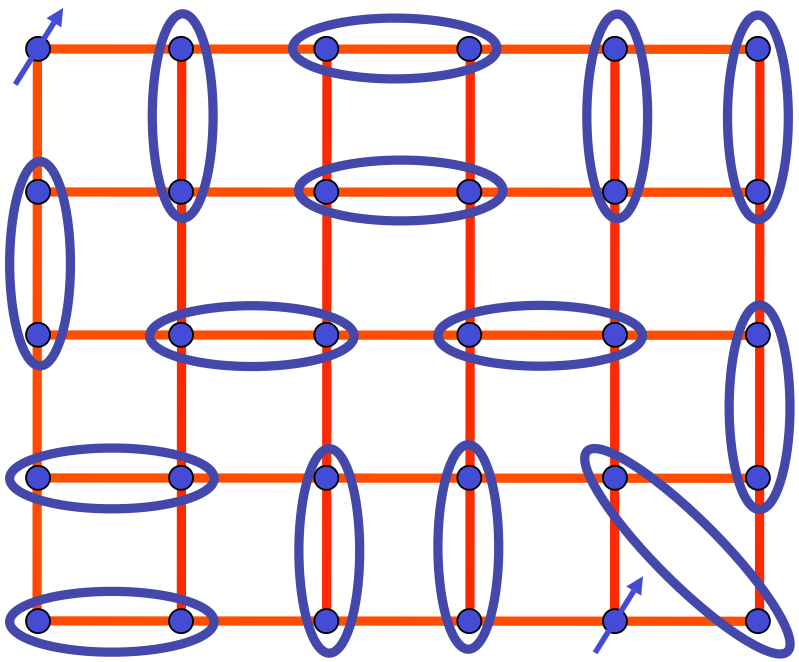

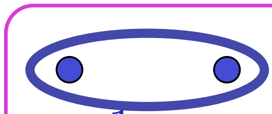

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Possible theory for fractionalization and topological order

Decompose the Néel order parameter into *spinors*

$$\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

where $\vec{\sigma}$ are Pauli matrices, and z_α are complex spinors which carry spin $S = 1/2$.

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Key question: Can the z_α become the needed $S = 1/2$ excitations of a fractionalized phase ?

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Effective theory for spinons must be invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\theta} z_\alpha$$

Possible theory for fractionalization and topological order

Naive expectation: Low energy spinon theory for "quantum disordering" a Néel state is

$$\mathcal{S}_z = \int d^2x d\tau \Big[c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s\left|z_\alpha\right|^2 + u\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda\right)^2 \Big]$$

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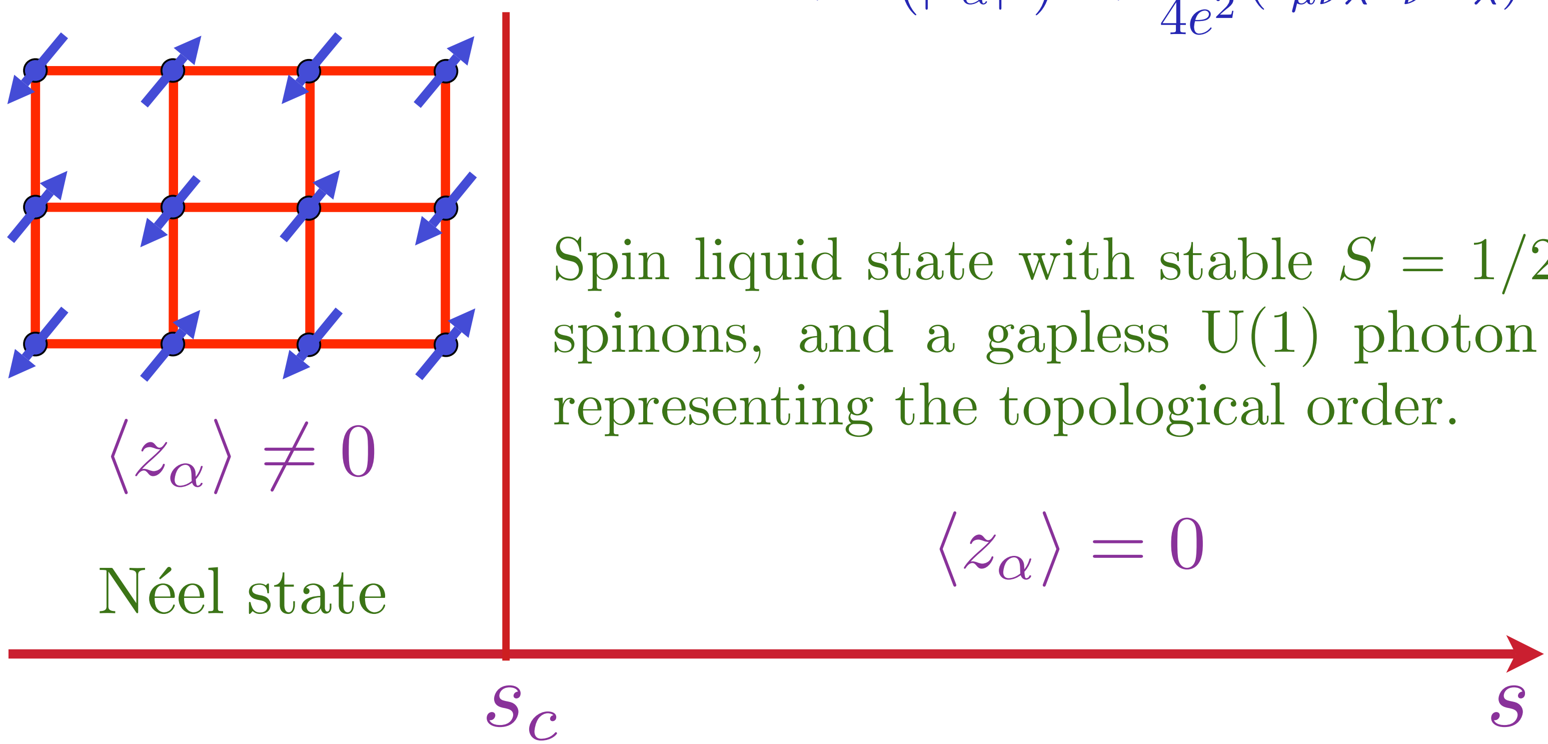

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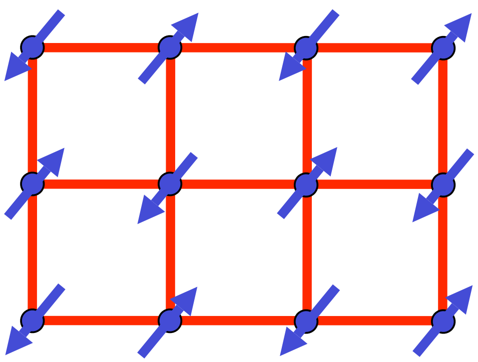

$\langle z_\alpha \rangle \neq 0$

Néel state

However, **monopoles** in the A_μ field will proliferate because of the gap to z_α excitations, and lead to **confinement** of z_α.

$\langle z_\alpha \rangle = 0$

s_c

s

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- Find a collective excitation Φ with the gauge transformation

$$\Phi \to e^{2i\theta}\Phi$$

- Higgs state with $\langle \Phi \rangle \neq 0$ is described by the fractionalized phase of a Z_2 gauge theory in the which the spinons z_α carry Z_2 gauge charges (E. Fradkin and S. Shenker, Phys. Rev. D **19**, 3682 (1979)).

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- *What is Φ in the antiferromagnet* ? Its physical interpretation becomes clear from its allowed coupling to the spinons:

$$\mathcal{S}_{z,\Phi} = \int d^2 r d\tau \left[\lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.}\right]$$

From this coupling it follows that the states with $\langle\Phi\rangle \neq 0$ have **coplanar spin correlations**.

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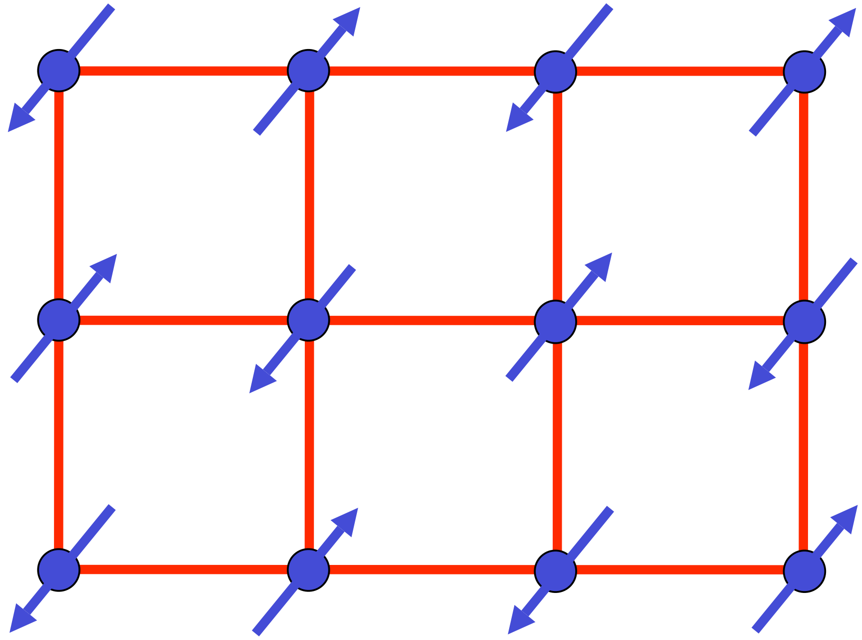
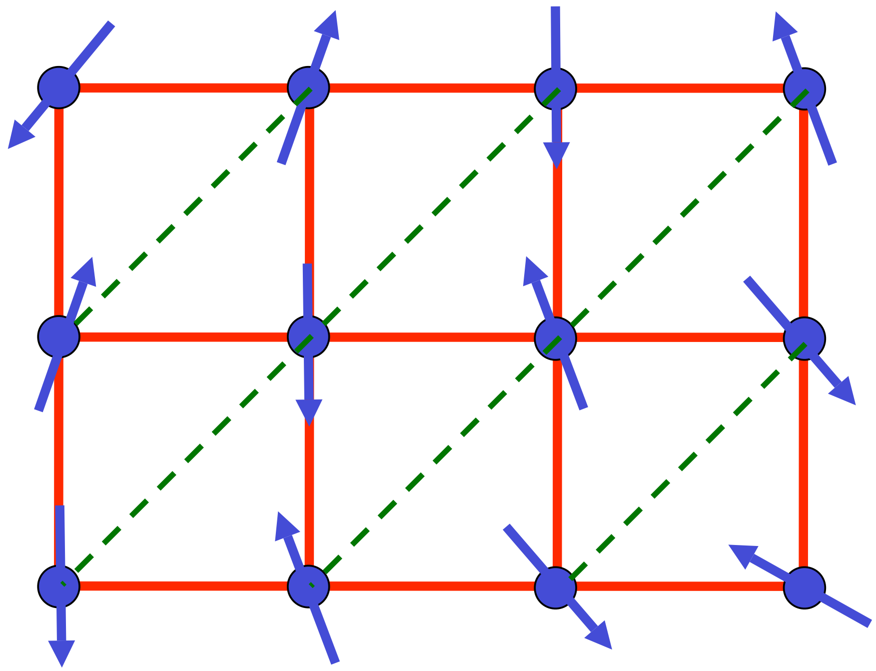


Collinear magnetic order with $\langle \Phi \rangle = 0$.

A spin density wave:

$$\langle \vec{S}_i \rangle \propto (\cos(\mathbf{K} \cdot \mathbf{r}_i), \sin(\mathbf{K} \cdot \mathbf{r}_i), 0)$$

$$\mathbf{K} = (\pi, \pi).$$



Coplanar magnetic order with $\langle \Phi \rangle \neq 0$.

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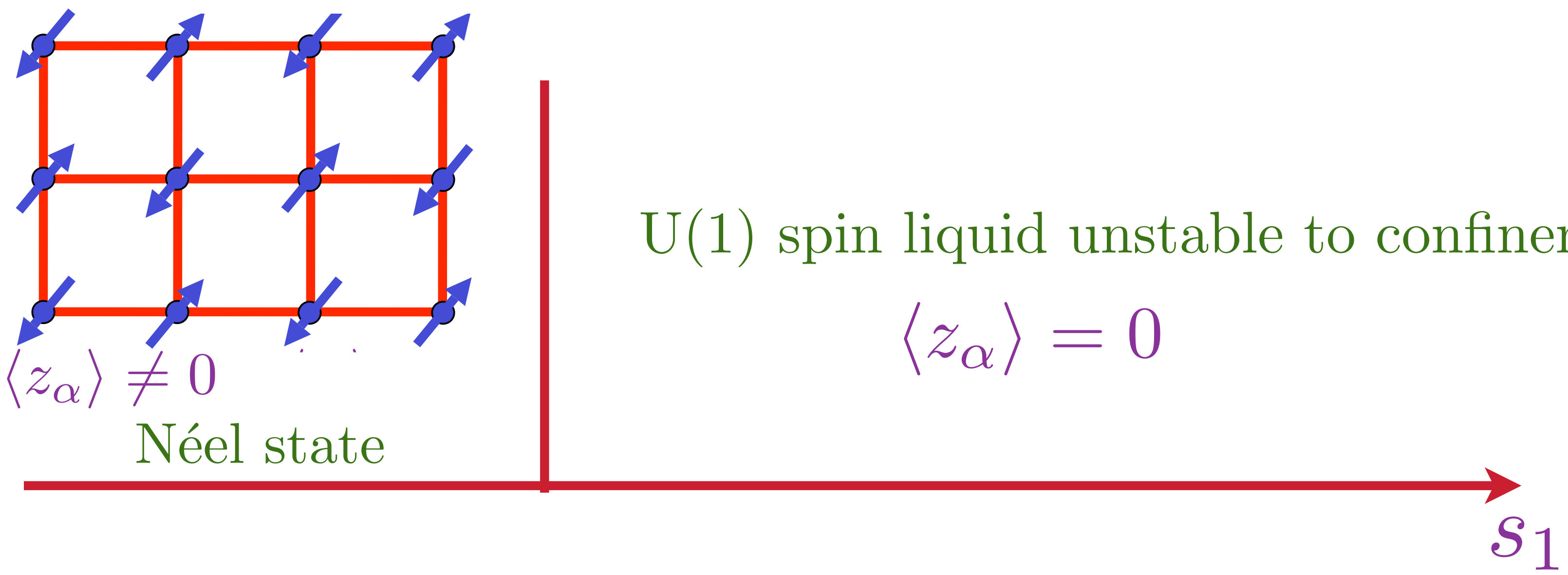
with

$$\mathbf{K} = (\pi + \langle \Phi \rangle, \pi + \langle \Phi \rangle).$$

Experimental realization: $CsCuCl_3$

Phase diagram of gauge theory of spinons

$$\mathcal{S}_z = \int d^2x d\tau \left[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + s_1 \left|z_\alpha\right|^2 + u \left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right.$$



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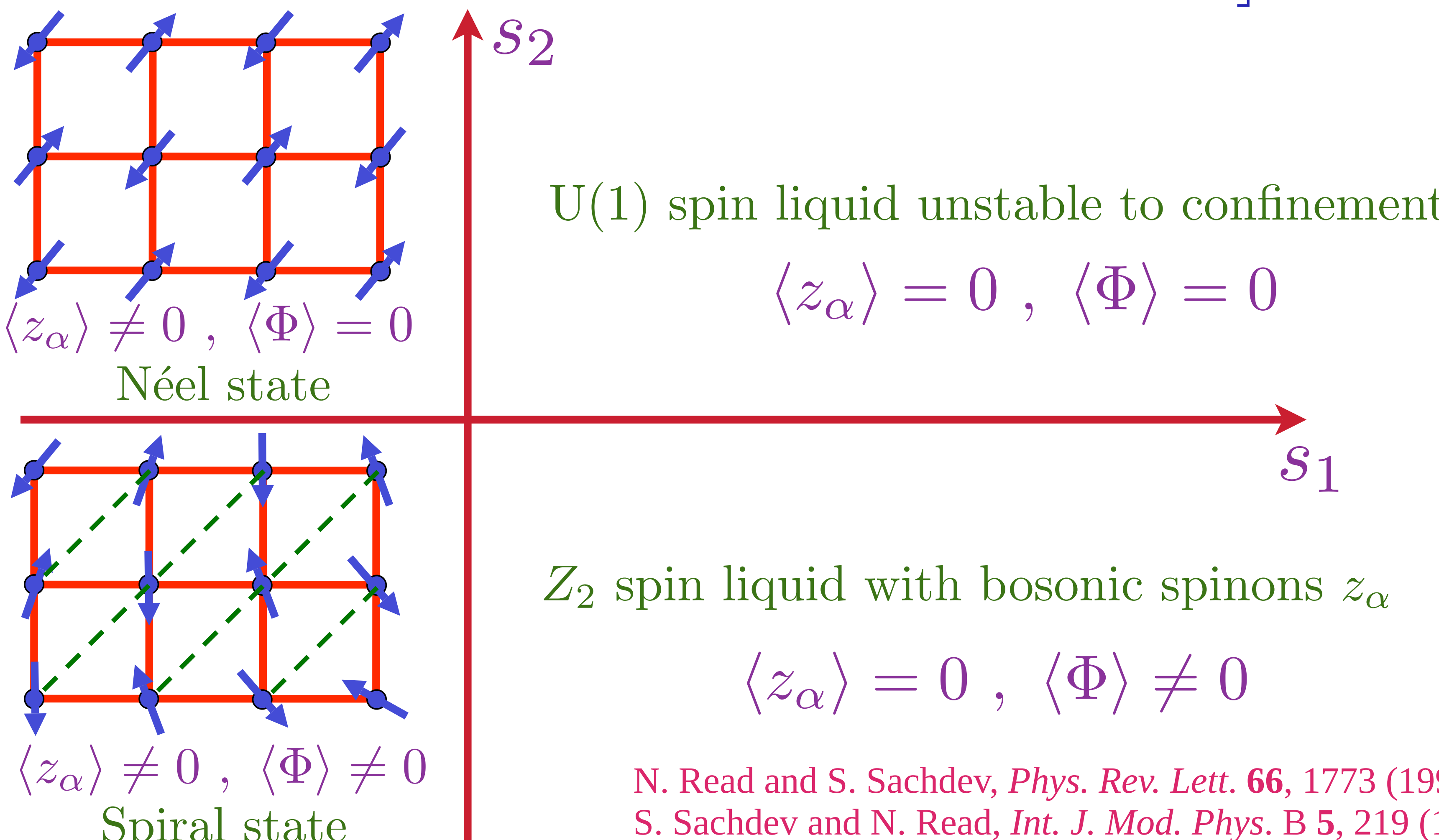

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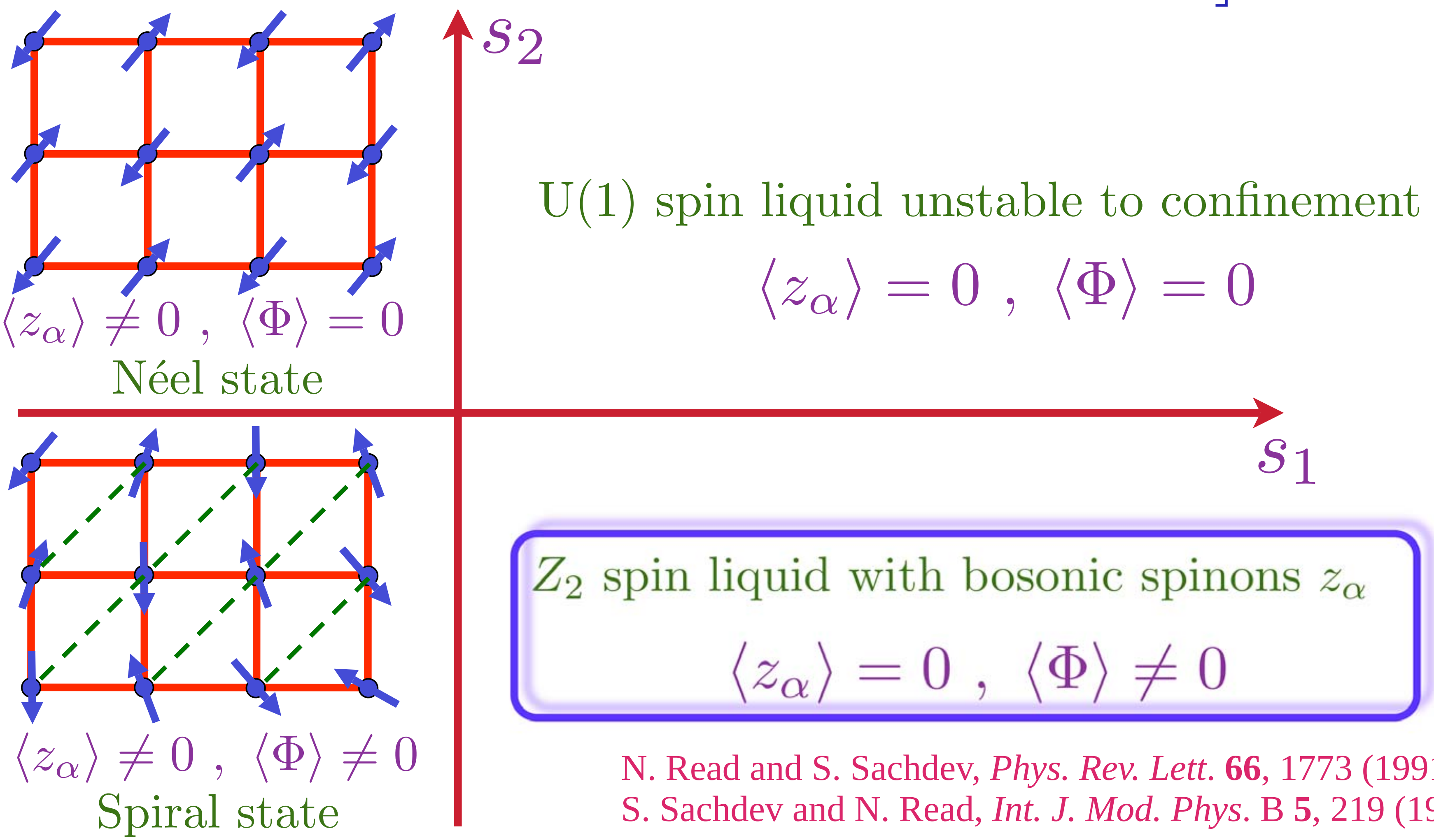

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Characteristics of Z_2 spin liquid

- Two classes of gapped excitations:
  - Bosonic spinons z_α which carry Z_2 gauge charge
  - Z_2 vortex associated with $2\pi n$ winding in phase of Φ. This vortex appears as a π flux-tubes to spinons

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- Same states (without spinons) and Z_2 gauge theories found to describe liquid phases of quantum dimer models (R. Moessner and S. L. Sondhi, Phys. Rev. Lett. **86**, 1881 (2001).

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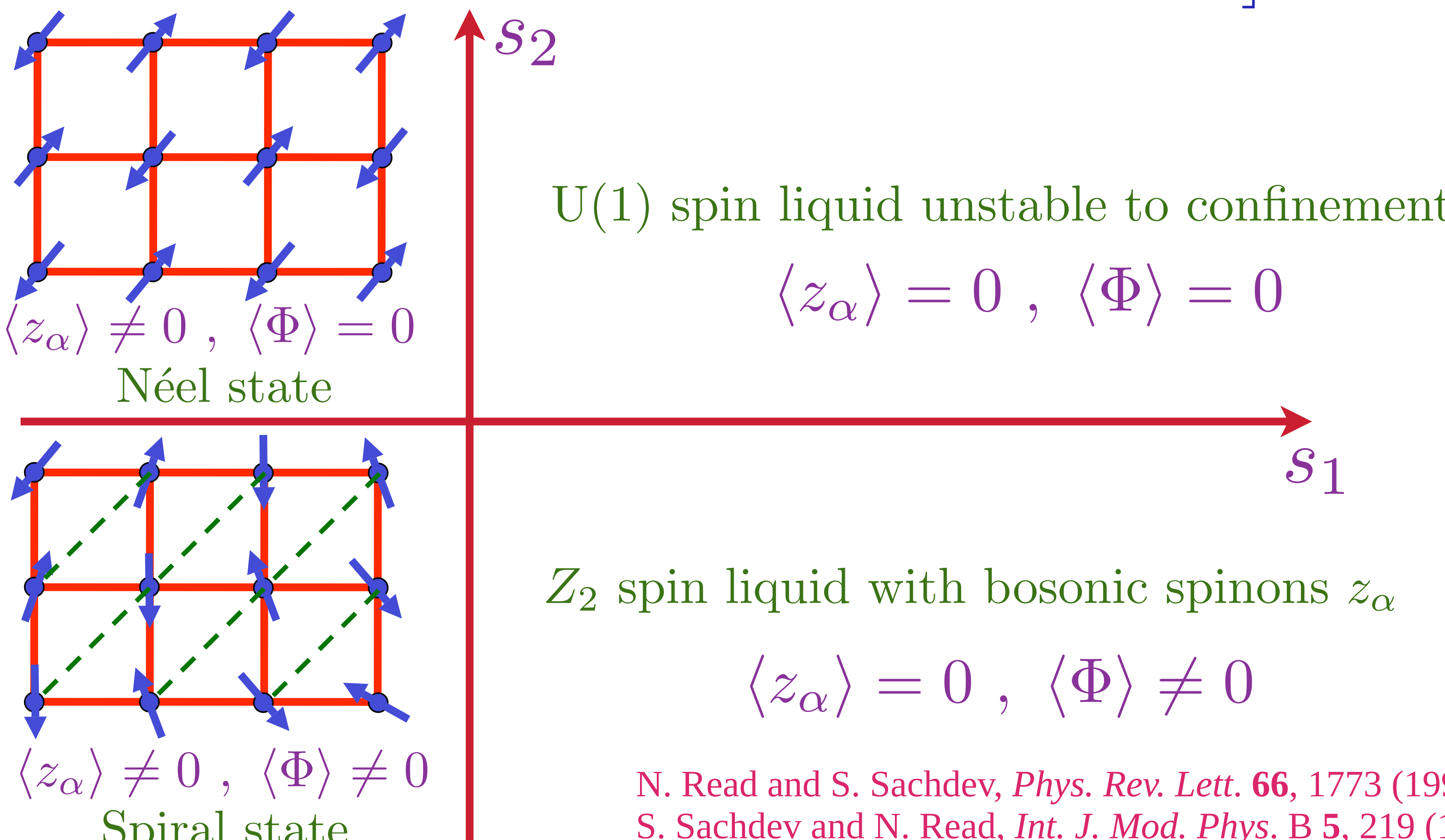

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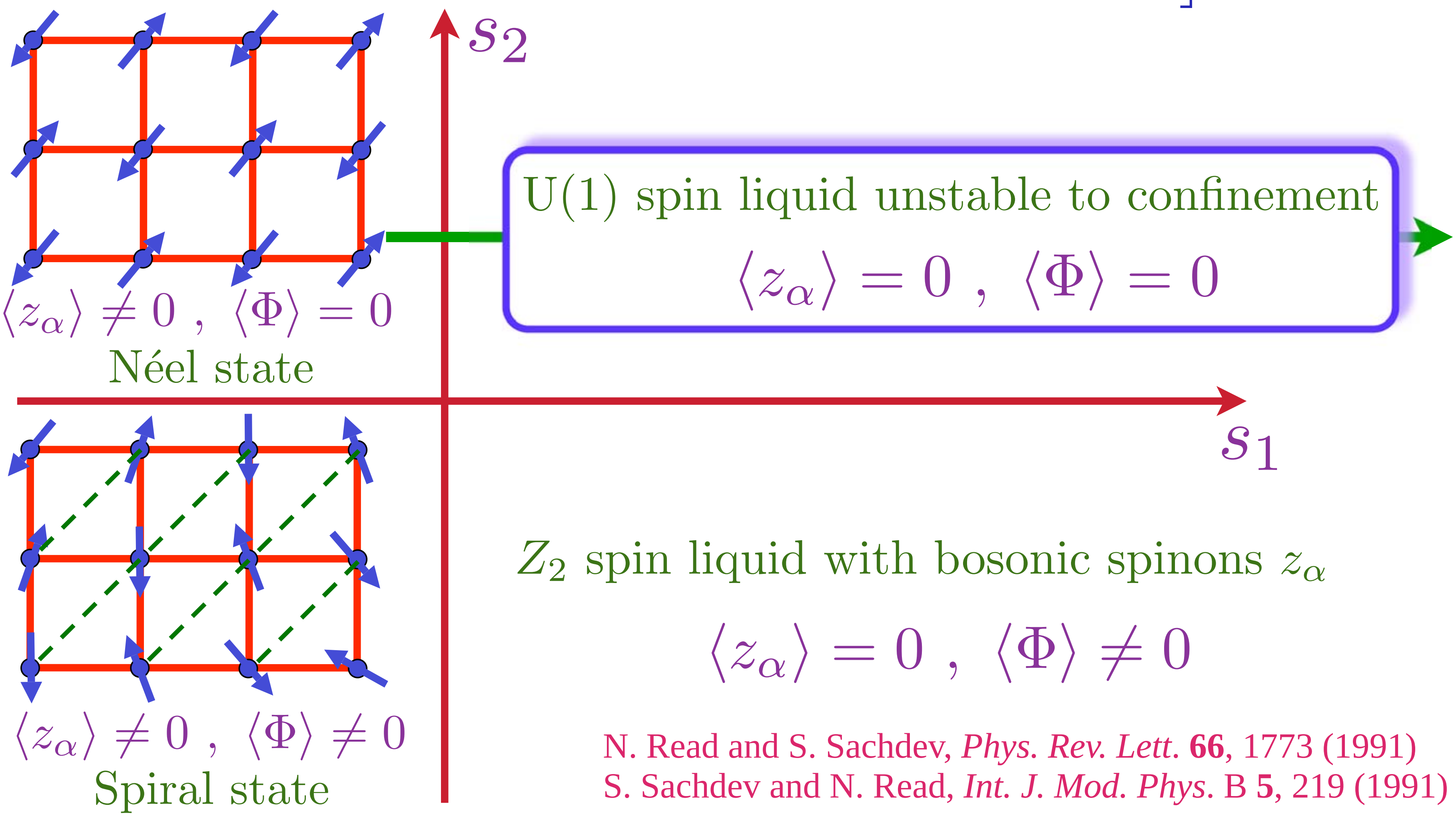


N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991)

Quantum theory for destruction of Neel order

Partition function on cubic lattice in spacetime

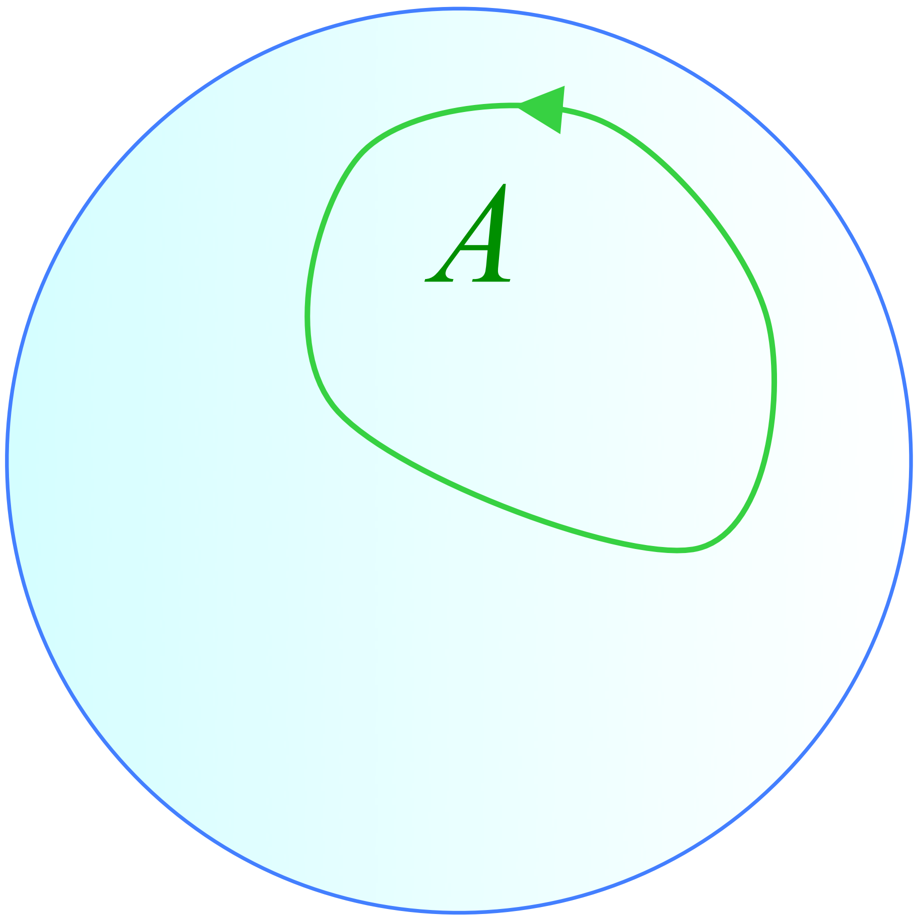
$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu}\right)$$

LGW theory: weights in partition function are those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Missing ingredient: Spin Berry Phases



$$e^{iA/2}$$

Quantum theory for destruction of Neel order

Partition function on cubic lattice in spacetime

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu}\right)$$

LGW theory: weights in partition function are those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Quantum theory for destruction of Neel order

Coherent state path integral on cubic lattice in spacetime

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i\mathcal{S}_{\mathrm{Berry}}\right)$$

Modulus of weights in partition function: those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Berry phases lead to large cancellations between different time histories

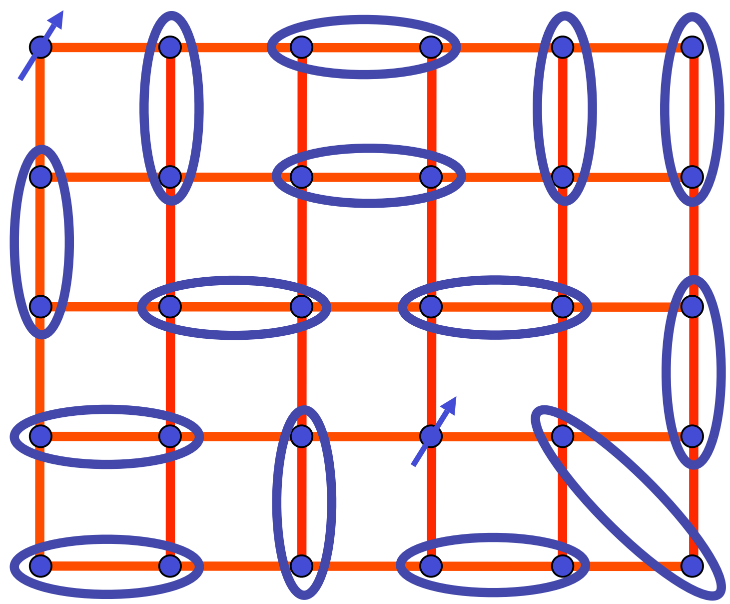
Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i\mathcal{S}_{\mathrm{Berry}}\right)$$

Rewrite partition function in terms of spinors $z_{a\alpha}$, with $\alpha = \uparrow, \downarrow$ and

$$\vec{\varphi}_a = z_{a\alpha}^* \vec{\sigma}_{\alpha\beta} z_{a\beta}$$



S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i\mathcal{S}_{\text{Berry}}\right)$$

Partition function expressed as a gauge theory of spinor degrees of freedom

$$\mathcal{Z} = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(\sum_\alpha |z_{a\alpha}|^2 - 1\right)$$
$$\times \exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i\sum_a \eta_a A_{a\tau}\right)$$

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Large g effective action for the $A_{a\mu}$ after integrating $z_{\alpha\mu}$

$$\mathcal{Z} = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right) + i \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

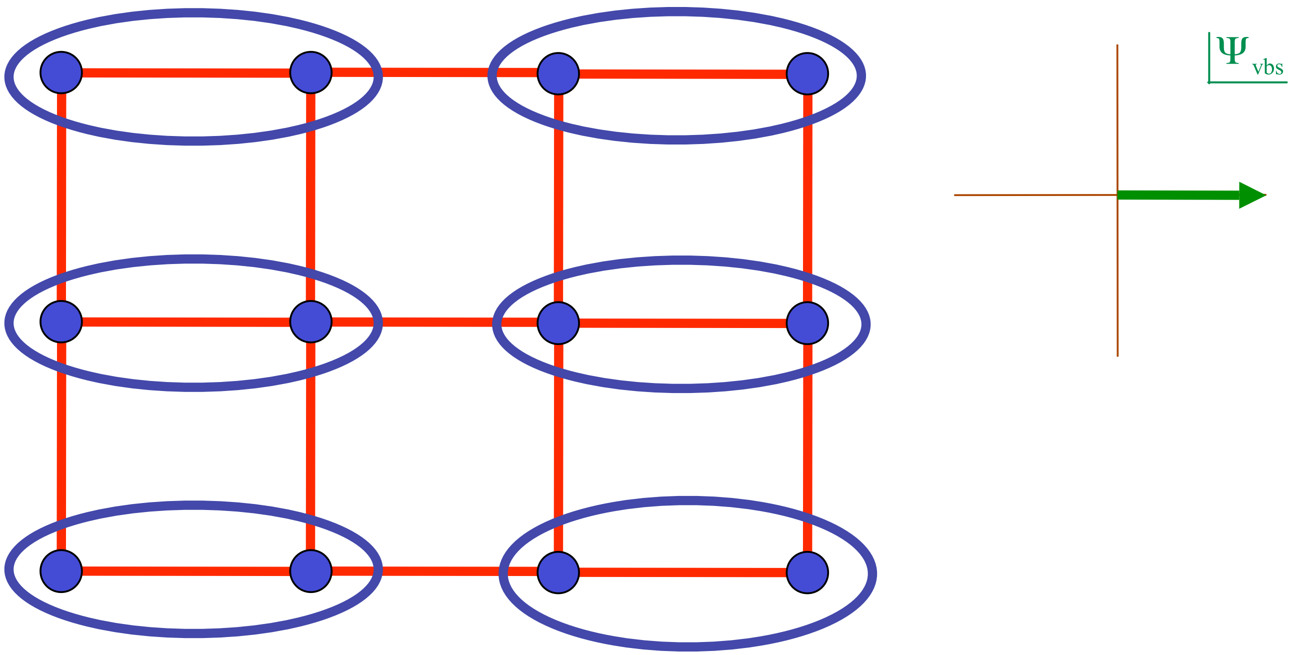
This is compact QED in 3 spacetime dimensions with static charges ± 1 on two sublattices.

This theory can be reliably analyzed by a duality mapping.

The gauge theory is in a ***confining*** phase, and there is VBS order in the ground state. (Proliferation of monopoles in the presence of Berry phases).

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev.* B **65**, 220405 (2002).

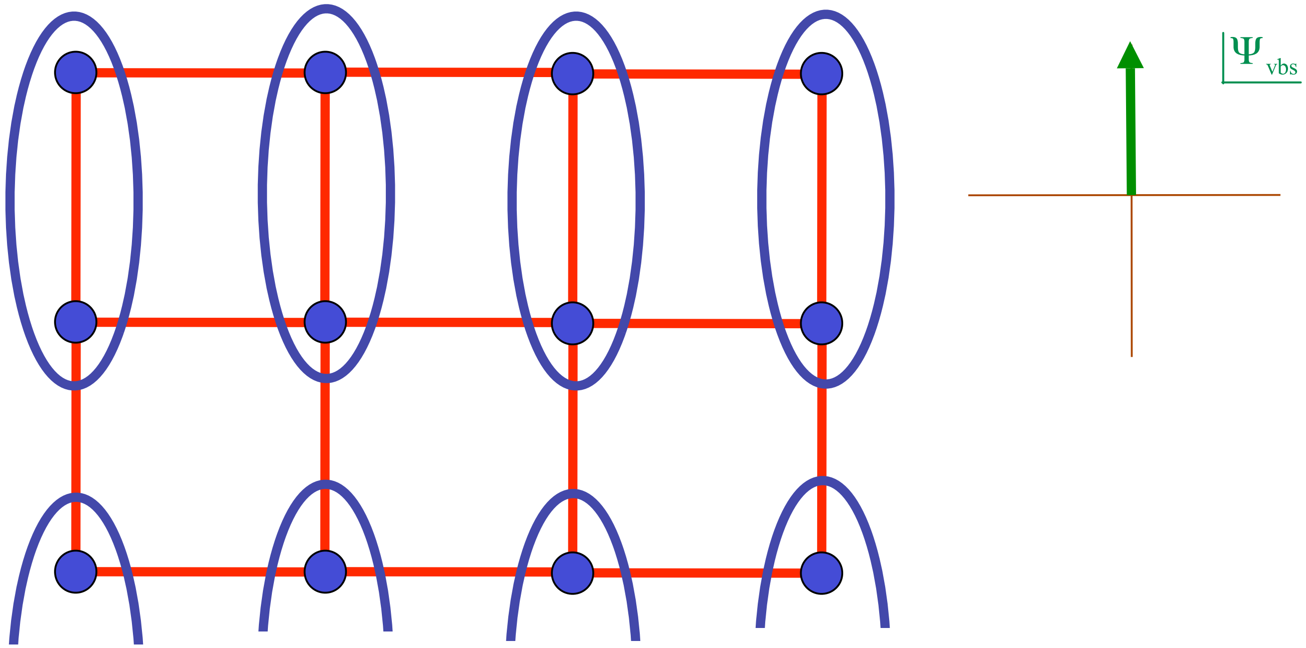
Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$



Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle \Psi_{\text{vbs}} \rangle \neq 0$, where Ψ_{vbs} is the ***VBS order parameter***

$$\Psi_{\text{vbs}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$



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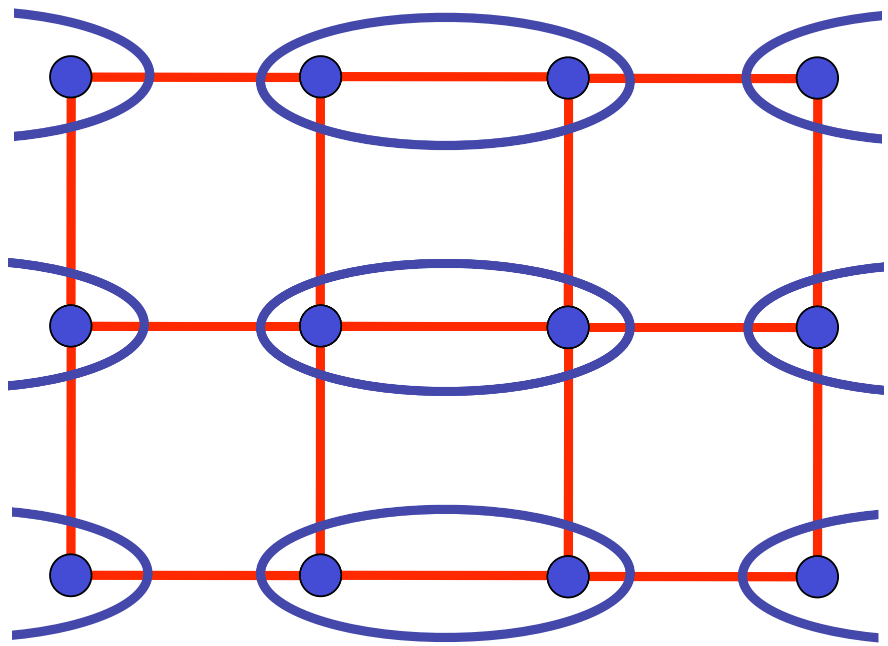
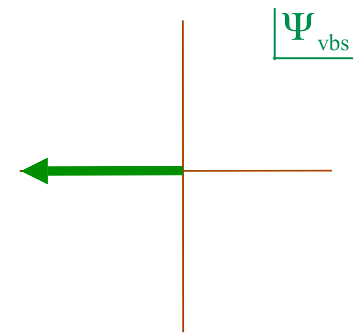
Characterization of VBS state with $\langle\vec{\varphi}\rangle = 0$





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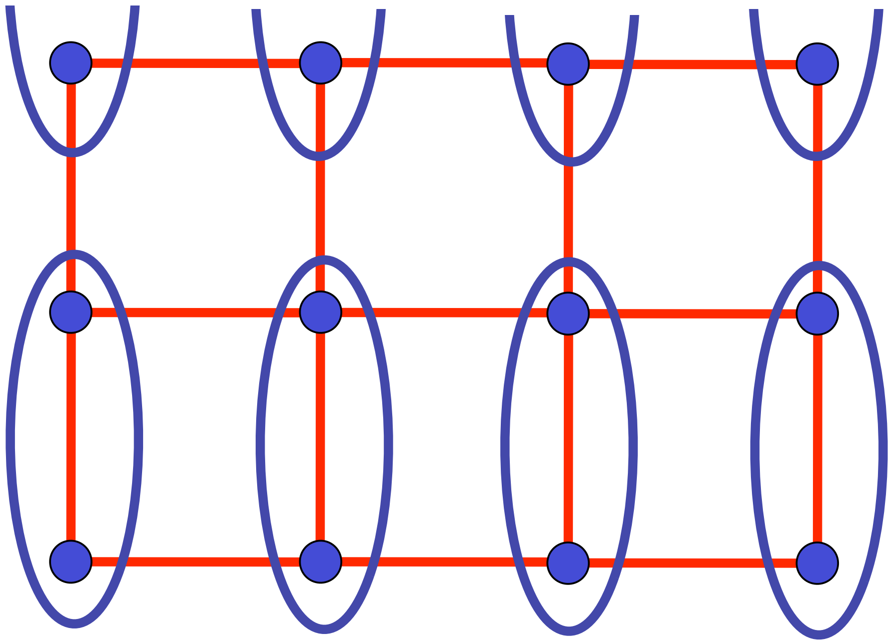
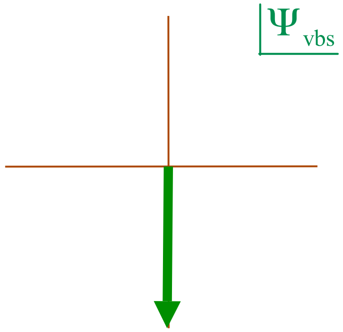
Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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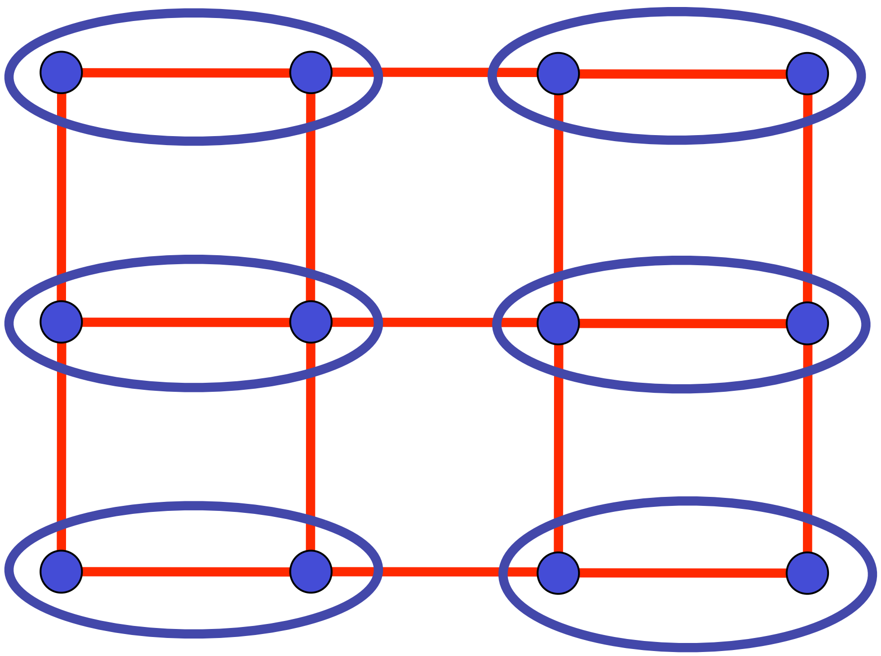
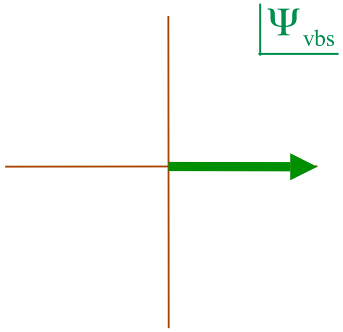
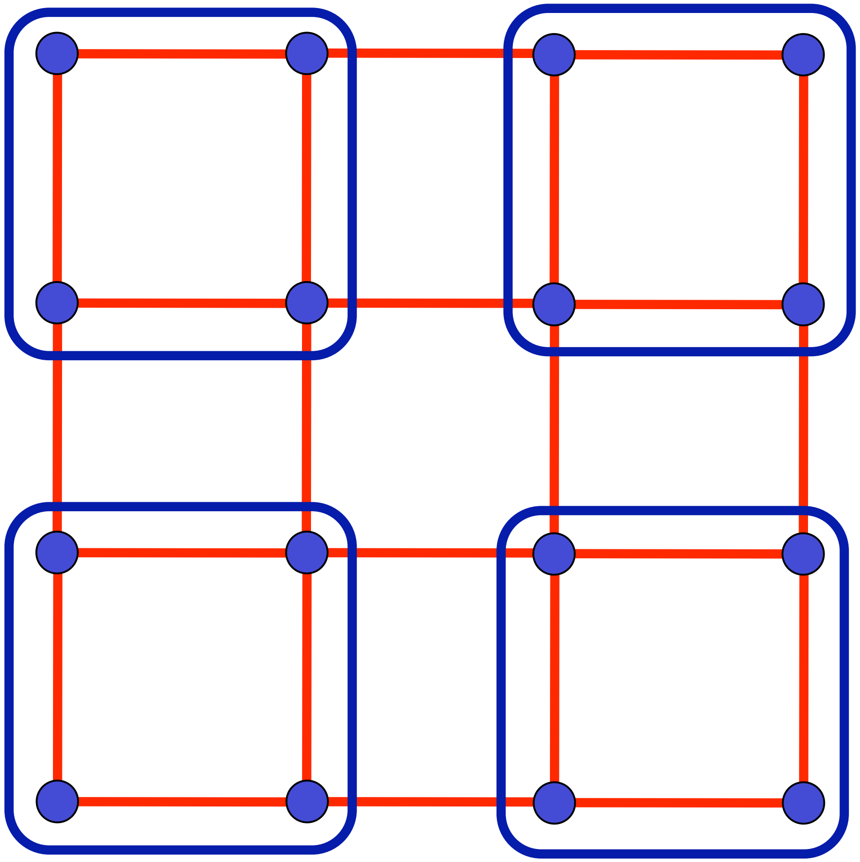


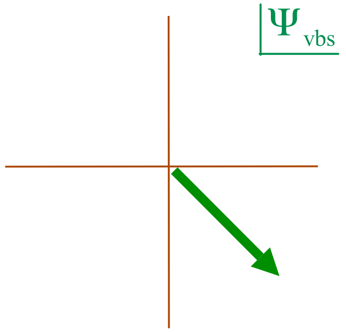


Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle\Psi_{\text{vbs}}\rangle \neq 0$, where Ψ_{vbs} is the ***VBS order parameter***

$$\Psi_{\text{vbs}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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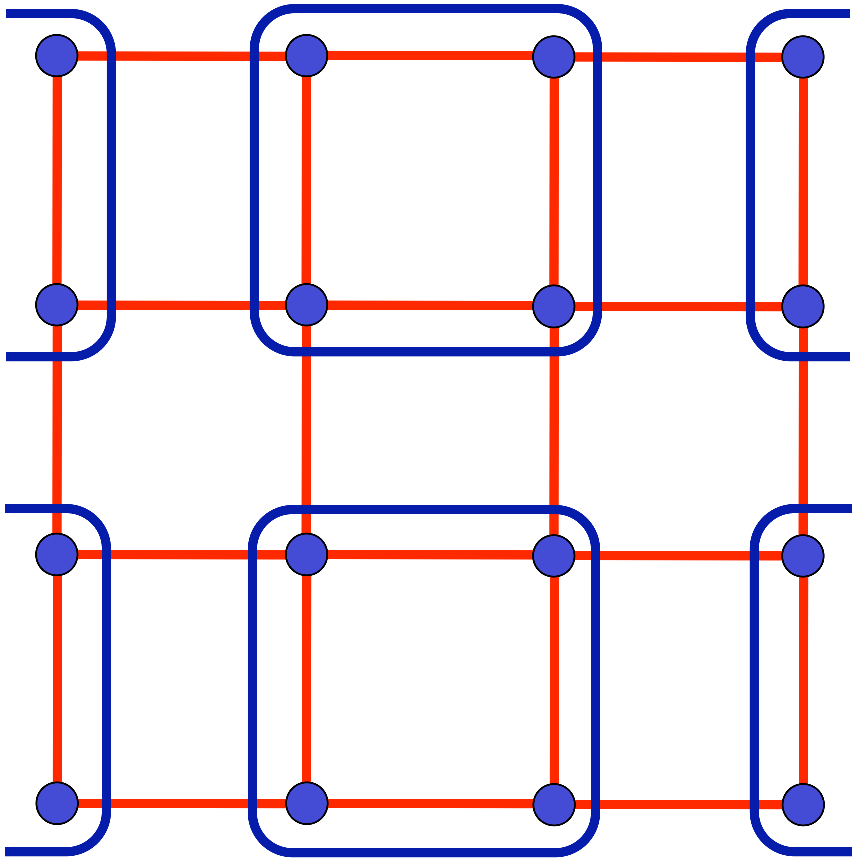
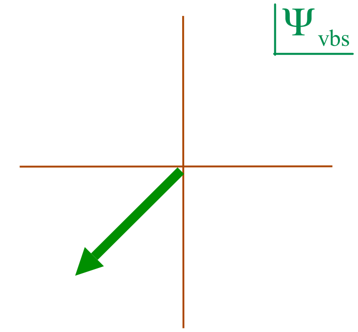
Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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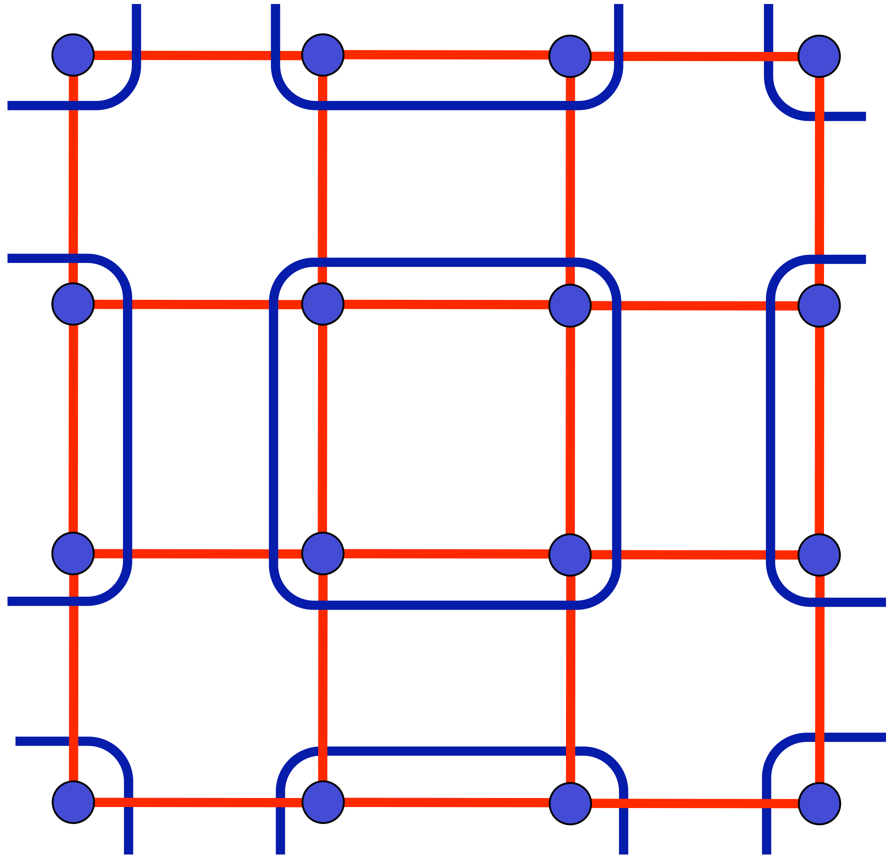
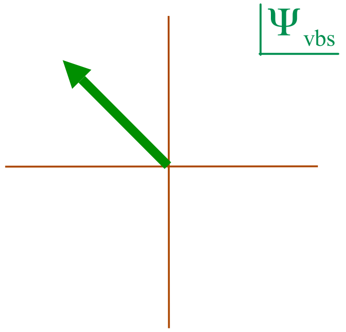
Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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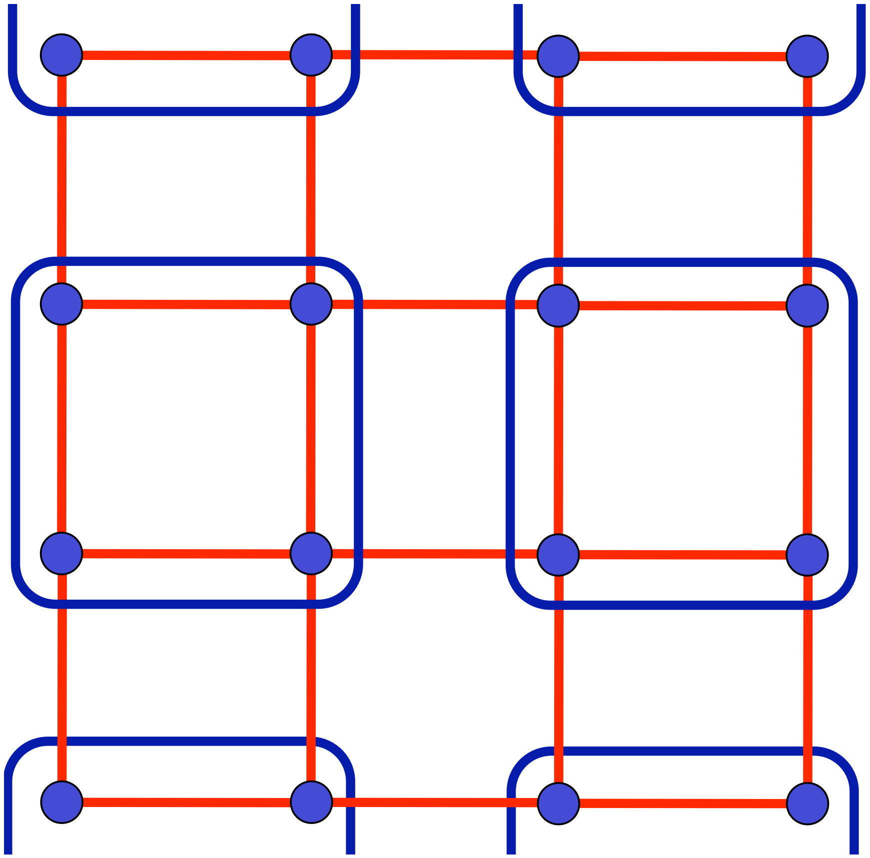
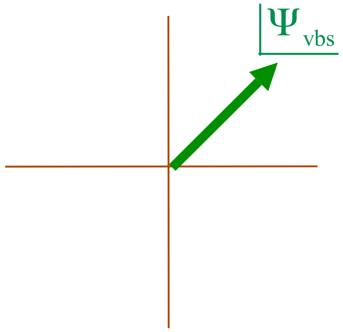
Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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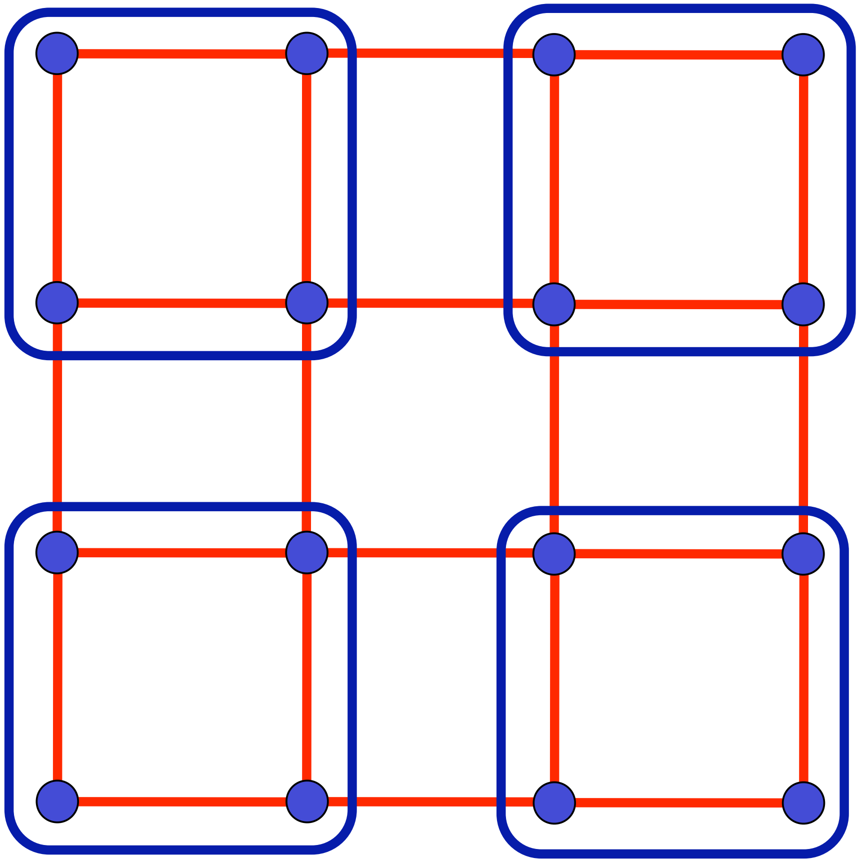
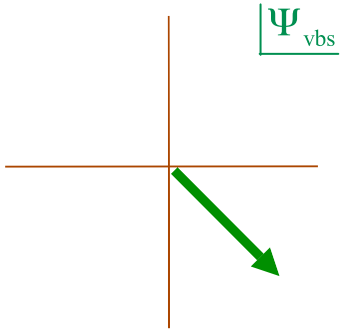
Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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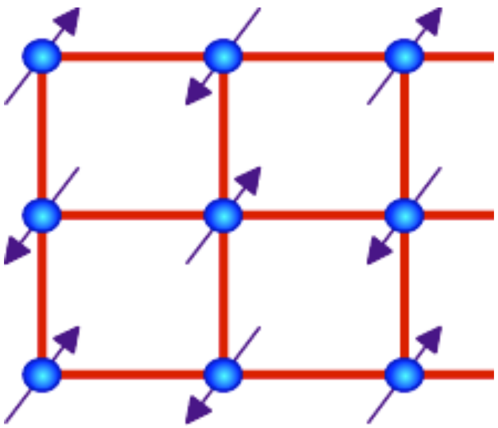
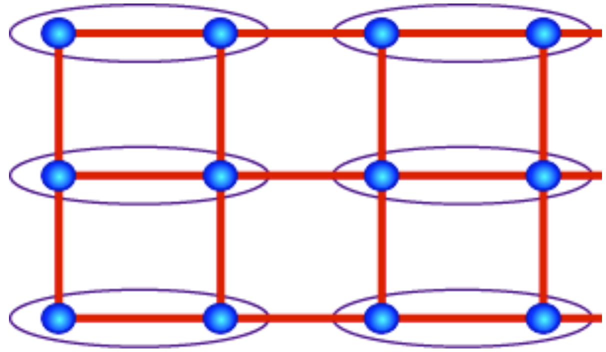
$$\mathcal{Z} = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(\sum_\alpha |z_{a\alpha}|^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau}\right)$$



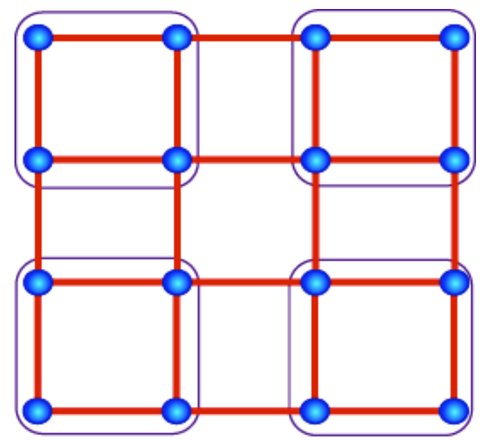
Neel order

$\langle \vec{\varphi} \rangle \neq 0$

?



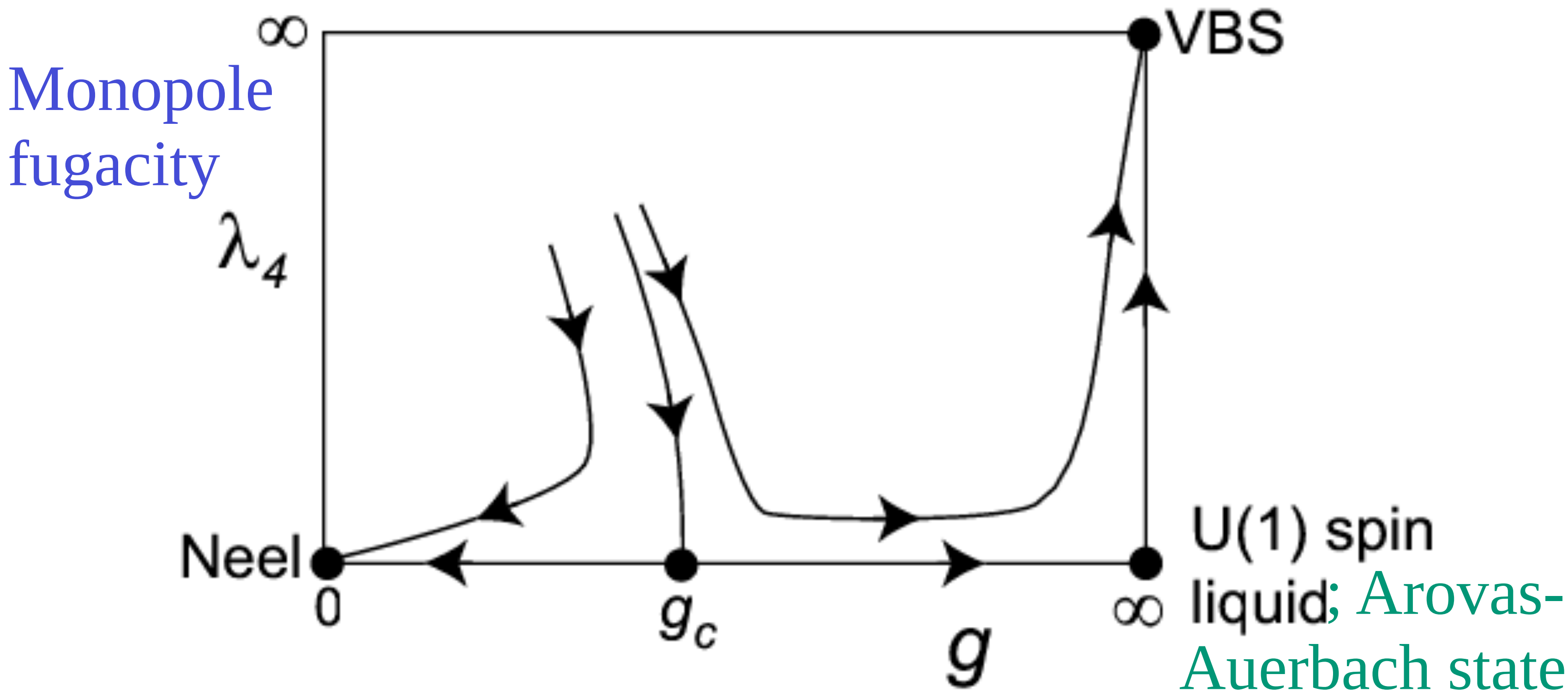
or



VBS order

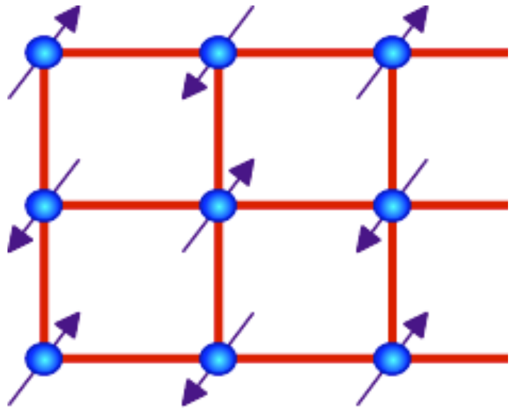
$\langle \Psi_{\text{vbs}} \rangle \neq 0$

Not present in LGW theory of $\vec{\varphi}$ order



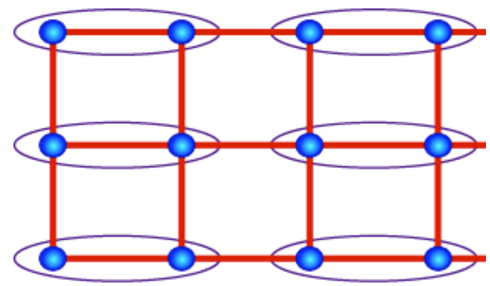
Monopole
fugacity
λ_4
∞
VBS
Neel
0
g_c
g
∞
U(1) spin
liquid; Arovas-
Auerbach state

Phase diagram of S=1/2 square lattice antiferromagnet

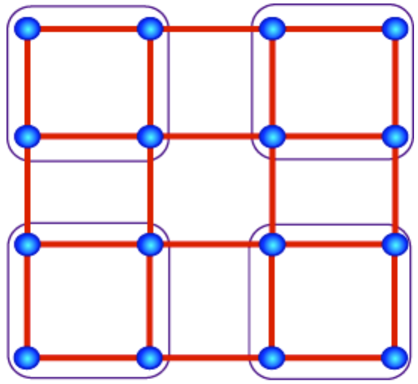


Neel order

$\langle \vec{\varphi} \rangle \sim \langle z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$



or



VBS order $\langle \Psi_{vbs} \rangle \neq 0$

(associated with condensation of monopoles in A_μ),

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

g

Second-order critical point described by

$$\mathcal{S}_z = \int d^2 x d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + r|z_\alpha|^2 + \frac{u}{2} \left(|z_\alpha|^2 \right)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

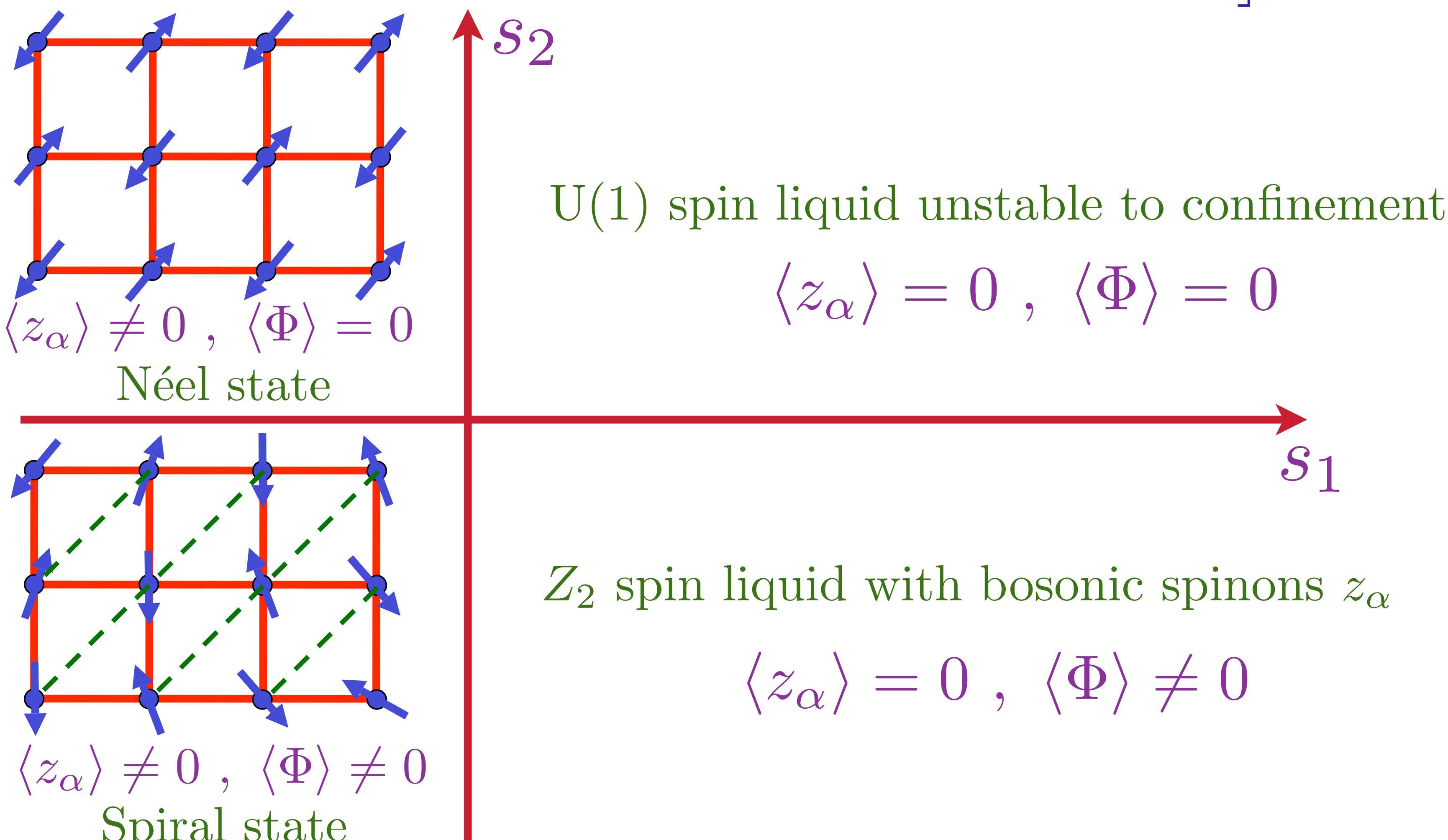
at its critical point $r = r_c$ where A_μ is *non-compact*.

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Phase diagram of gauge theory of spinons

$$\mathcal{S}_{z,\Phi} = \int d^2x d\tau \Big[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + s_1 \left|z_\alpha\right|^2 + u \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2$$

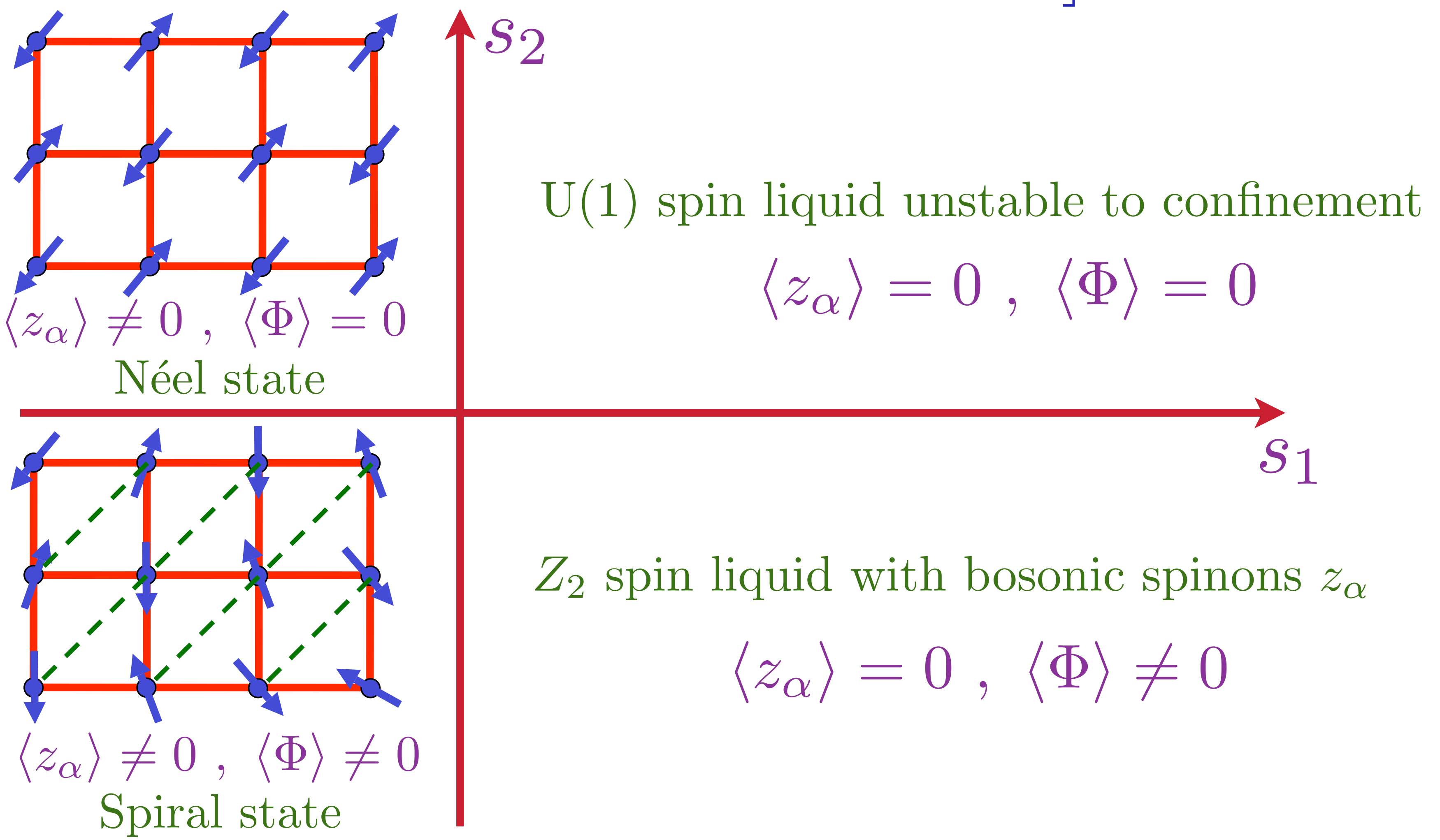
$$+ |(\partial_\mu - 2iA_\mu)\Phi|^2 + s_2|\Phi|^2 + \tilde{u}|\Phi|^4 + \lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.} \Big]$$



Phase diagram of gauge theory of spinons

$$\mathcal{S}_{z,\Phi} = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + s_1 |z_\alpha|^2 + u \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right.$$

$$\left. + |(\partial_\mu - 2iA_\mu)\Phi|^2 + s_2|\Phi|^2 + \tilde{u}|\Phi|^4 + \lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.} \right] + \text{monopoles} + \mathcal{S}_{\text{Berry}}$$



Phase diagram of gauge theory of spinons

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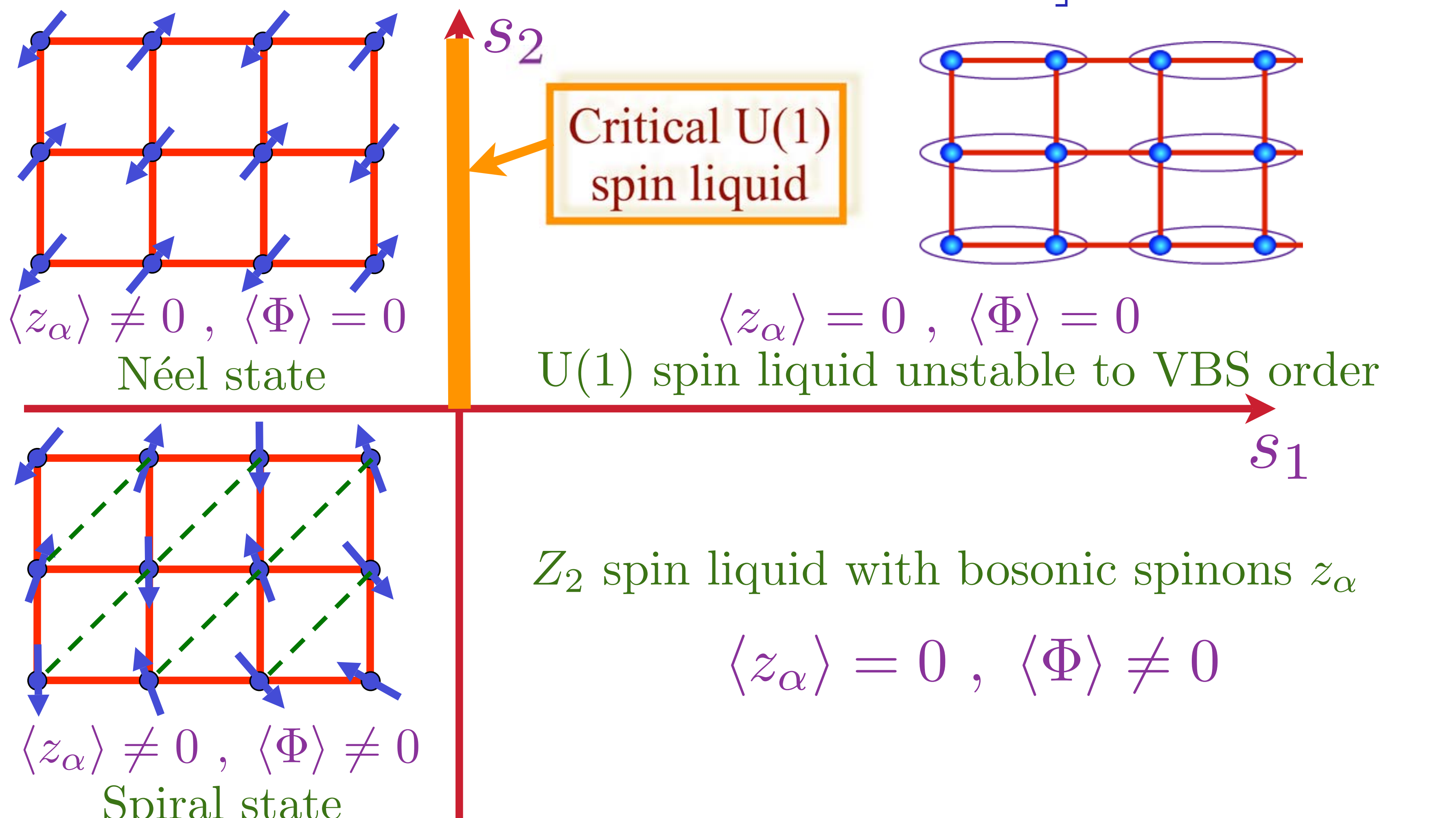
$$+ |(\partial_\mu - 2iA_\mu)\Phi|^2 + s_2 |\Phi|^2 + \tilde{u}|\Phi|^4 + \lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.} \Big] + \text{monopoles} + \mathcal{S}_{\text{Berry}}$$



Large scale Quantum Monte Carlo studies

Easy-plane model:

$$\mathcal{H}_{\mathrm{XY}} = 2J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle} \left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$

A.W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002)
A.W. Sandvik and R.G. Melko, *Phys. Rev.* E **72**, 026702 (2005).

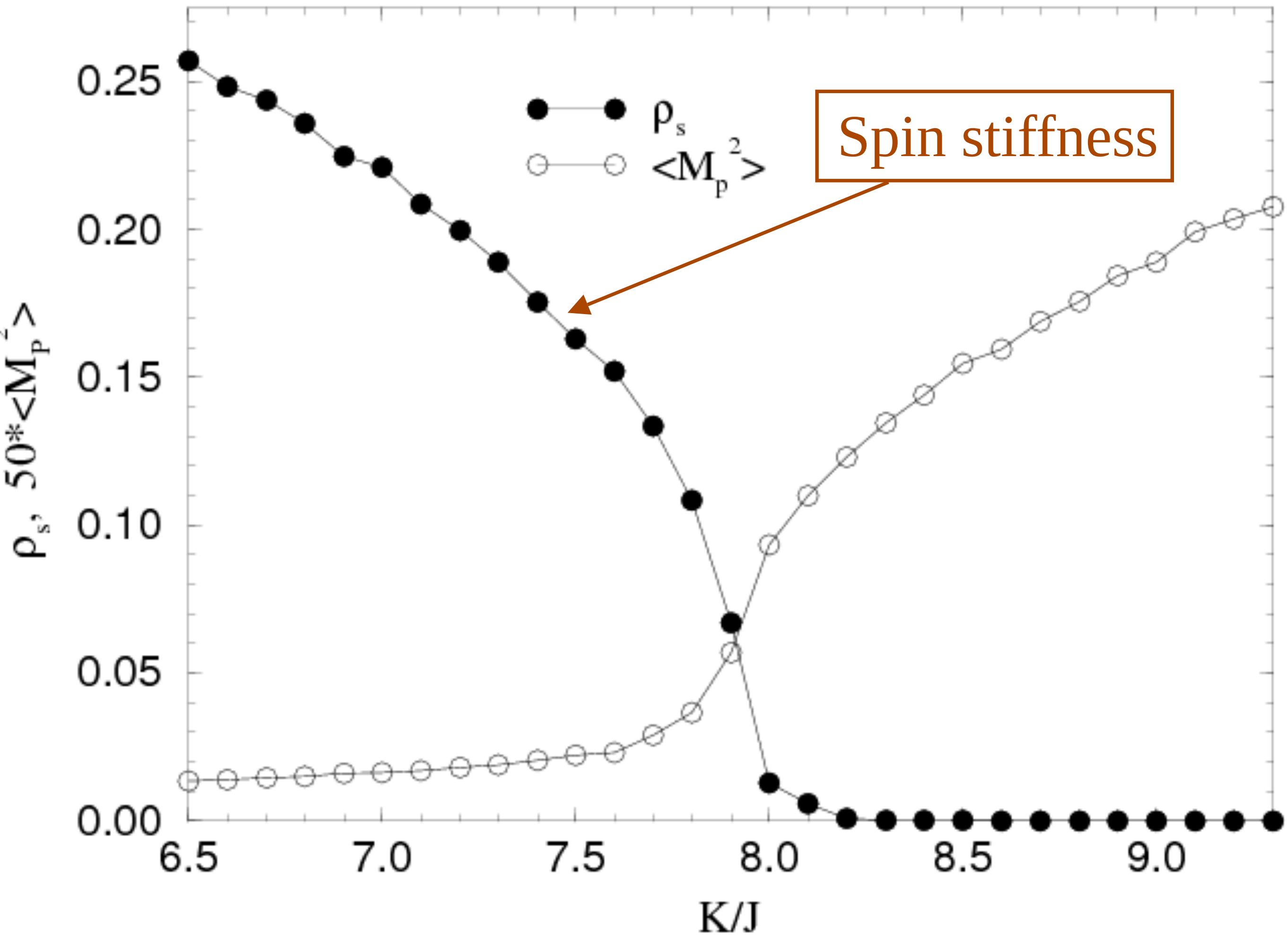
SU(2)-invariant model:

$$\mathcal{H}_{\mathrm{SU(2)}} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4} \right)$$

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).
R.G. Melko and R.K. Kaul, arXiv:0707.2961

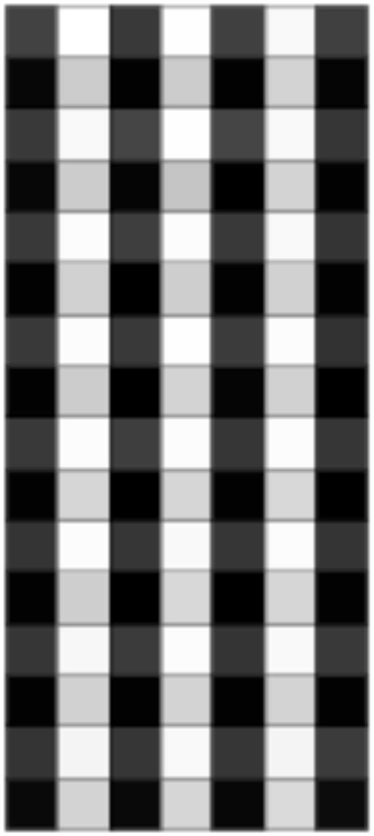
Easy-plane model

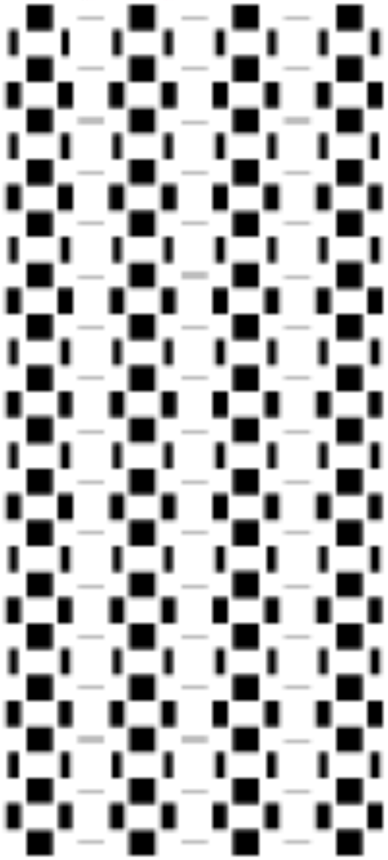
$$\mathcal{H}_{\mathrm{XY}} = 2J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle} \left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$



Easy-plane model

$$\mathcal{H}_{\rm XY} = 2J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle} \left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$

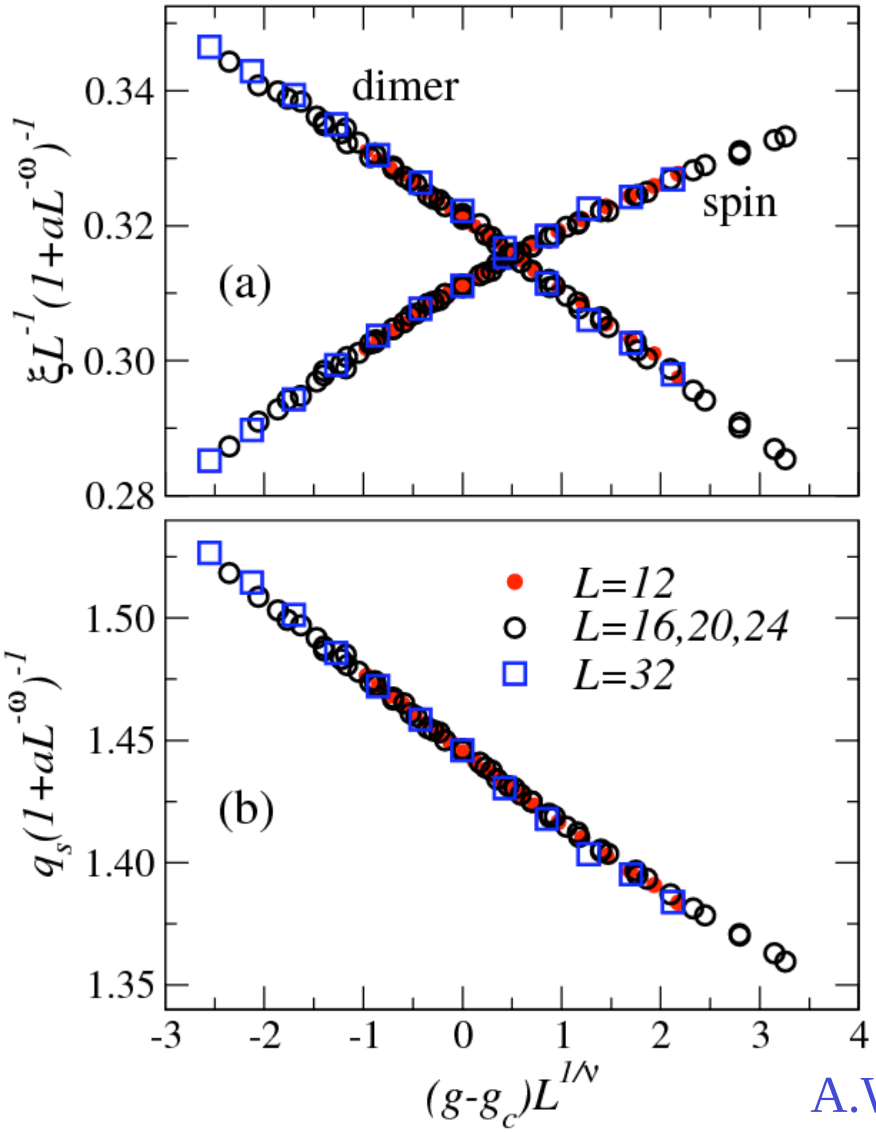




Valence bond solid (VBS) order in expectation values of plaquette and exchange terms

SU(2) invariant model

$$\mathcal{H}_{\mathrm{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right)\left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$



Strong evidence for a continuous "deconfined" quantum critical point

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

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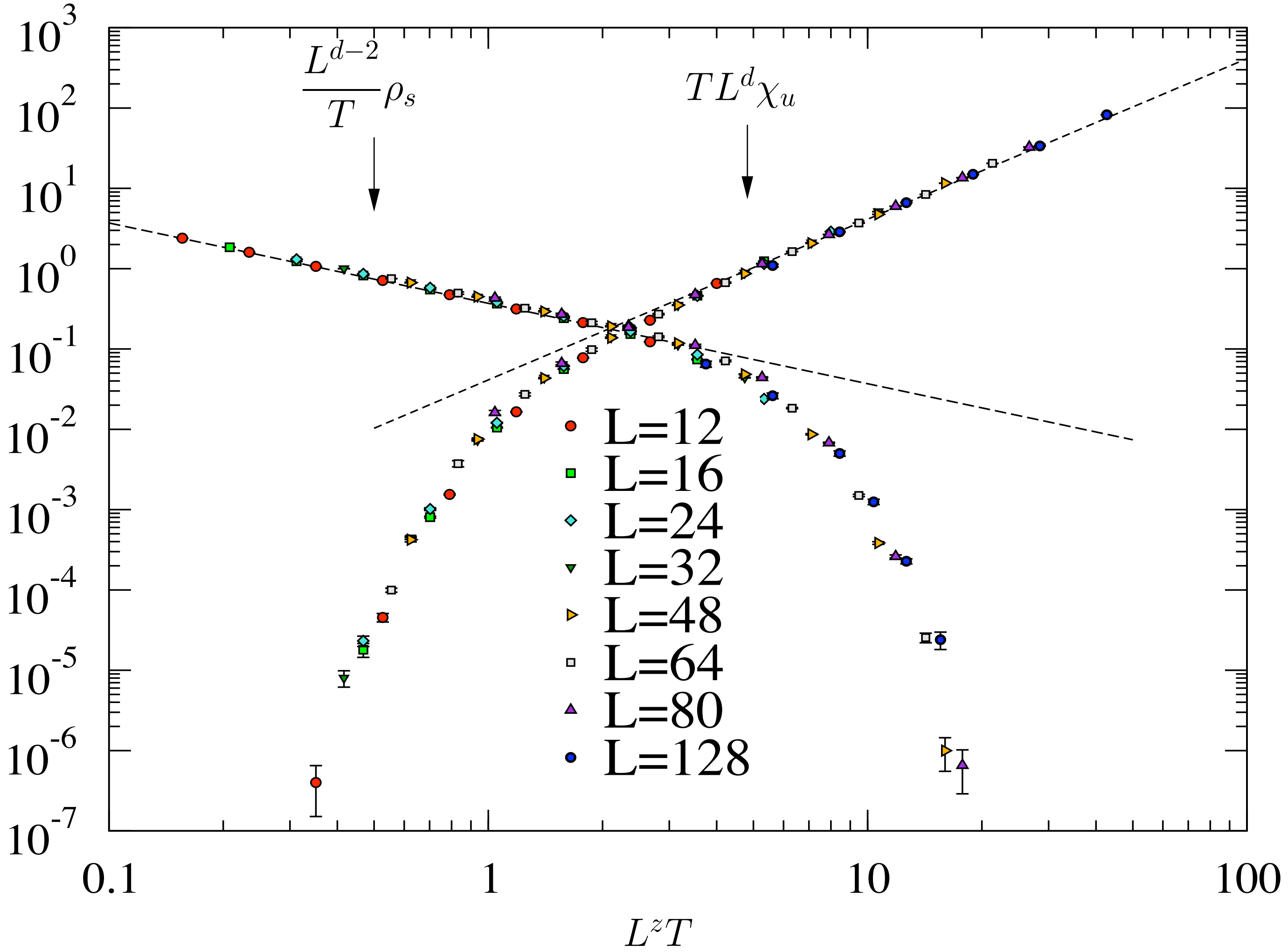

R.G. Melko and R.K. Kaul, arXiv:0707.2961

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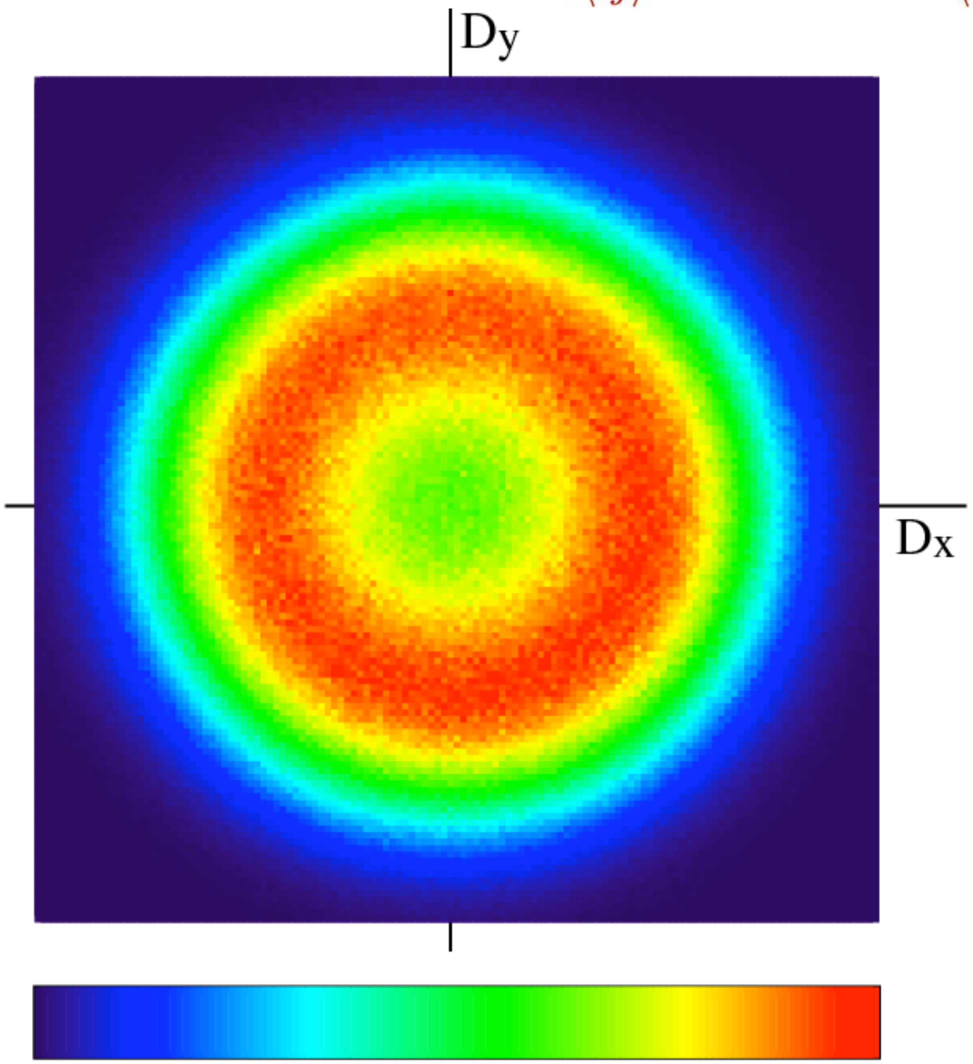

Probability distribution of VBS order Ψ at quantum critical point

Emergent circular symmetry is evidence for U(1) photon and topological order

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

Outline

1. Quantum "disordering" magnetic order
   Collinear order and confinement

2. Z_2 spin liquids
   Noncollinear order and fractionalization

3. Gapless U(1) spin liquids
   Deconfined criticality

4. Doped spin liquids
   Superconductors with topological order

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4. Doped spin liquids

   Superconductors with topological order

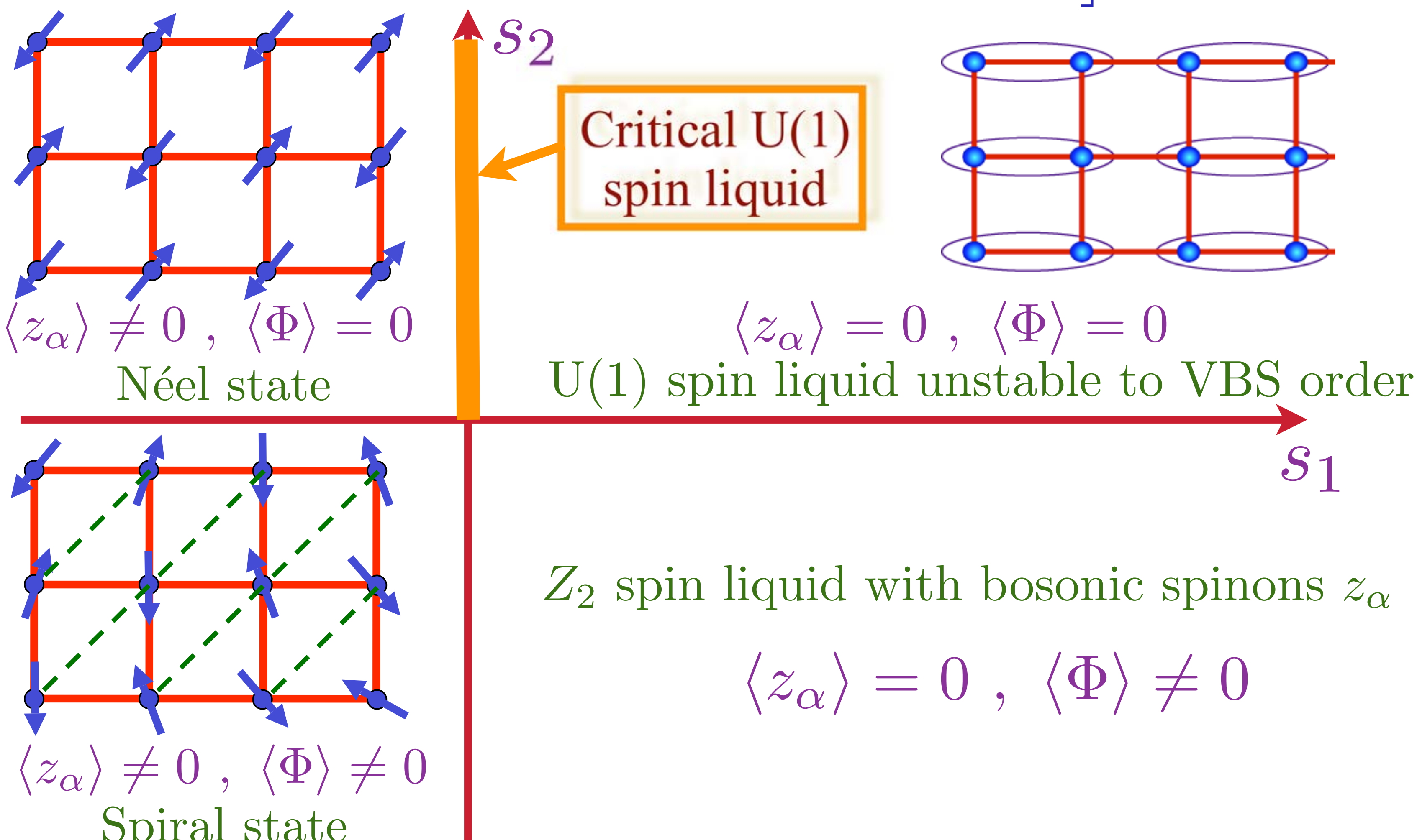
Hole dynamics in an antiferromagnet across a deconfined quantum critical point,
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil,
Physical Review B **75** , 235122 (2007)

Algebraic charge liquids and the underdoped cuprates,
R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil,
arXiv:0706.2187

Phase diagram of gauge theory of spinons

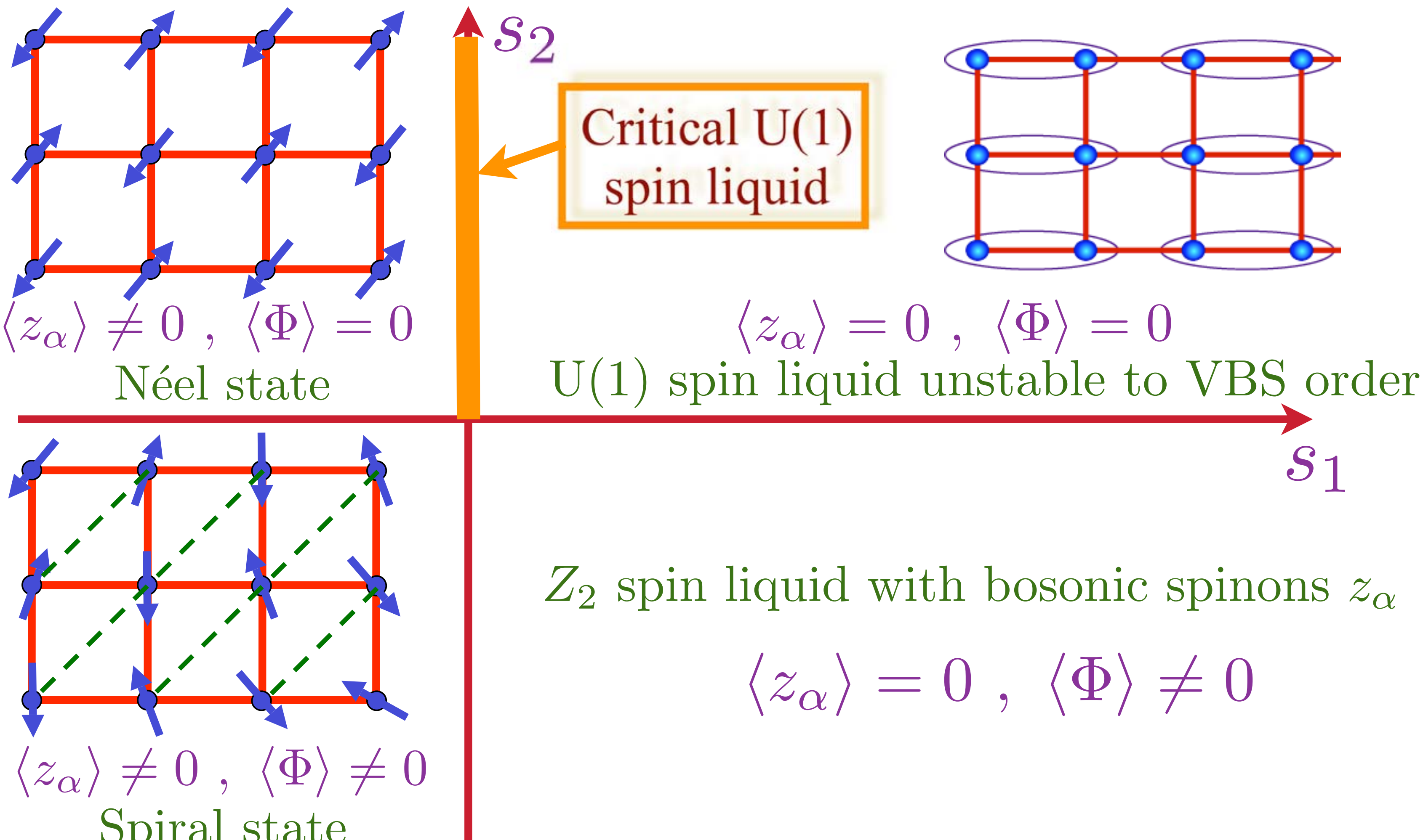
$$\mathcal{S}_{z,\Phi} = \int d^2x d\tau \Big[|(\partial_\mu - iA_\mu) z_\alpha|^2 + s_1 |z_\alpha|^2 + u \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2$$

$$+|(\partial_\mu - 2iA_\mu)\Phi|^2 + s_2|\Phi|^2 + \tilde{u}|\Phi|^4 + \lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.} \Big] + \text{monopoles} + \mathcal{S}_{\text{Berry}}$$

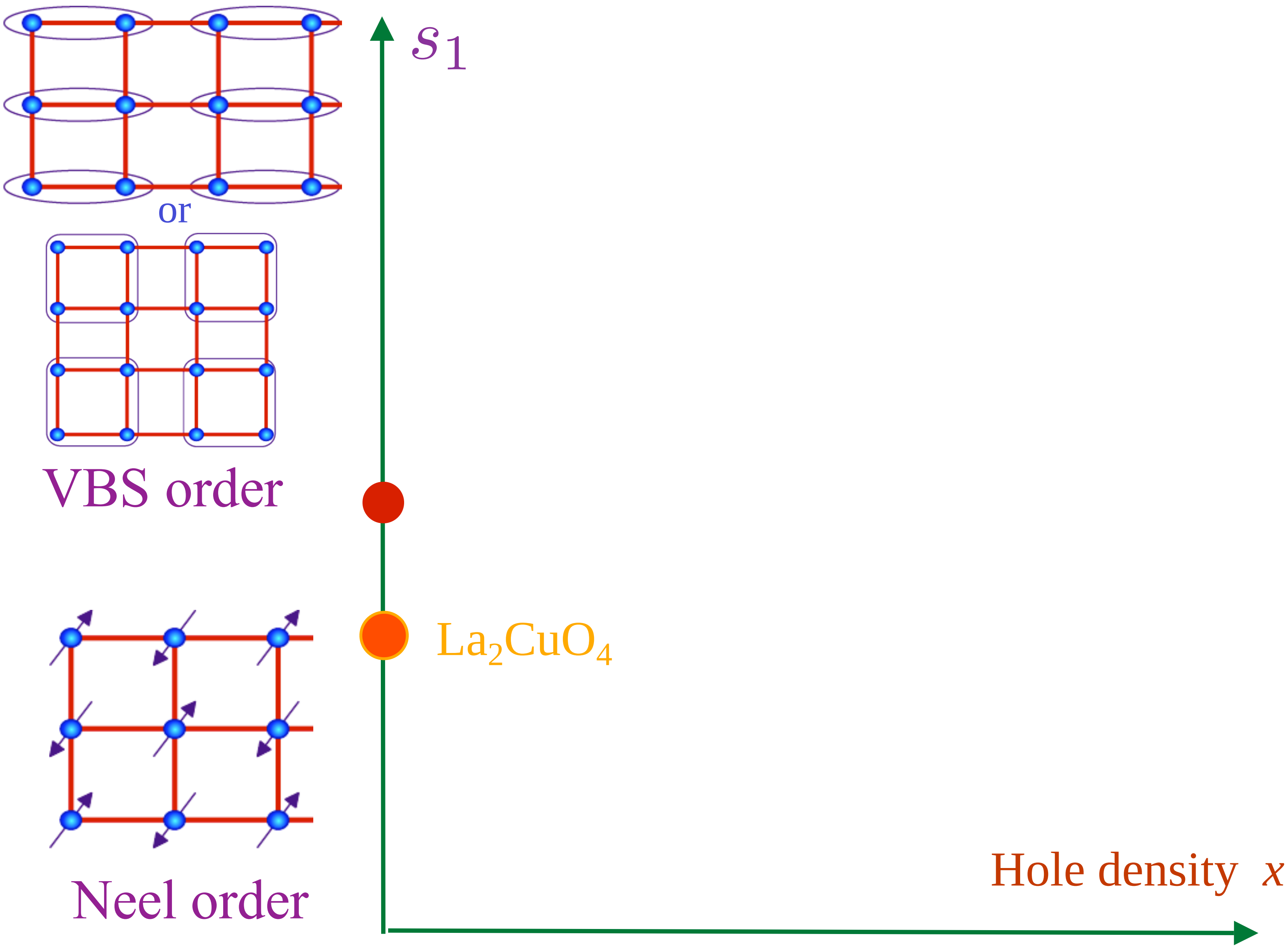


Phase diagram of gauge theory of spinons and holons

Add a finite concentration of charge carriers



Phase diagram of doped antiferromagnets



• Begin with the representation of the quantum antiferromagnet as the lattice CP^1 model:

$$\mathcal{S}_z = -\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i\sum_a \eta_a A_{a\tau}$$

• Write the electron operator at site r, $c_\alpha(r)$ in terms of **fermionic holon** operators $f_\pm$

$$c_\alpha(r) = \begin{cases} f_+^\dagger(r) z_{r\alpha} & \text{for } r \text{ on sublattice A} \\ \varepsilon_{\alpha\beta} f_-^\dagger(r) z^*_{r\beta} & \text{for } r \text{ on sublattice B} \end{cases}$$

Note that the holons f_s have charge s under the U(1) gauge field A_μ.

• Choose the dispersion, $\epsilon(\vec{k})$ of the $f_\pm$ in momentum space so that its minima are at $(\pm\pi/2, \pm\pi/2)$. *To avoid double-counting, these dispersions must be restricted to be within the diamond Brillouin zone.*

$$\mathcal{S}_f = \int d\tau \sum_{s=\pm} \int_\Diamond \frac{d^2k}{4\pi^2} f_s^\dagger(\vec{k}) \left(\partial_\tau - isA_\tau + \epsilon(\vec{k} - s\vec{A}) \right) f_s(\vec{k})$$

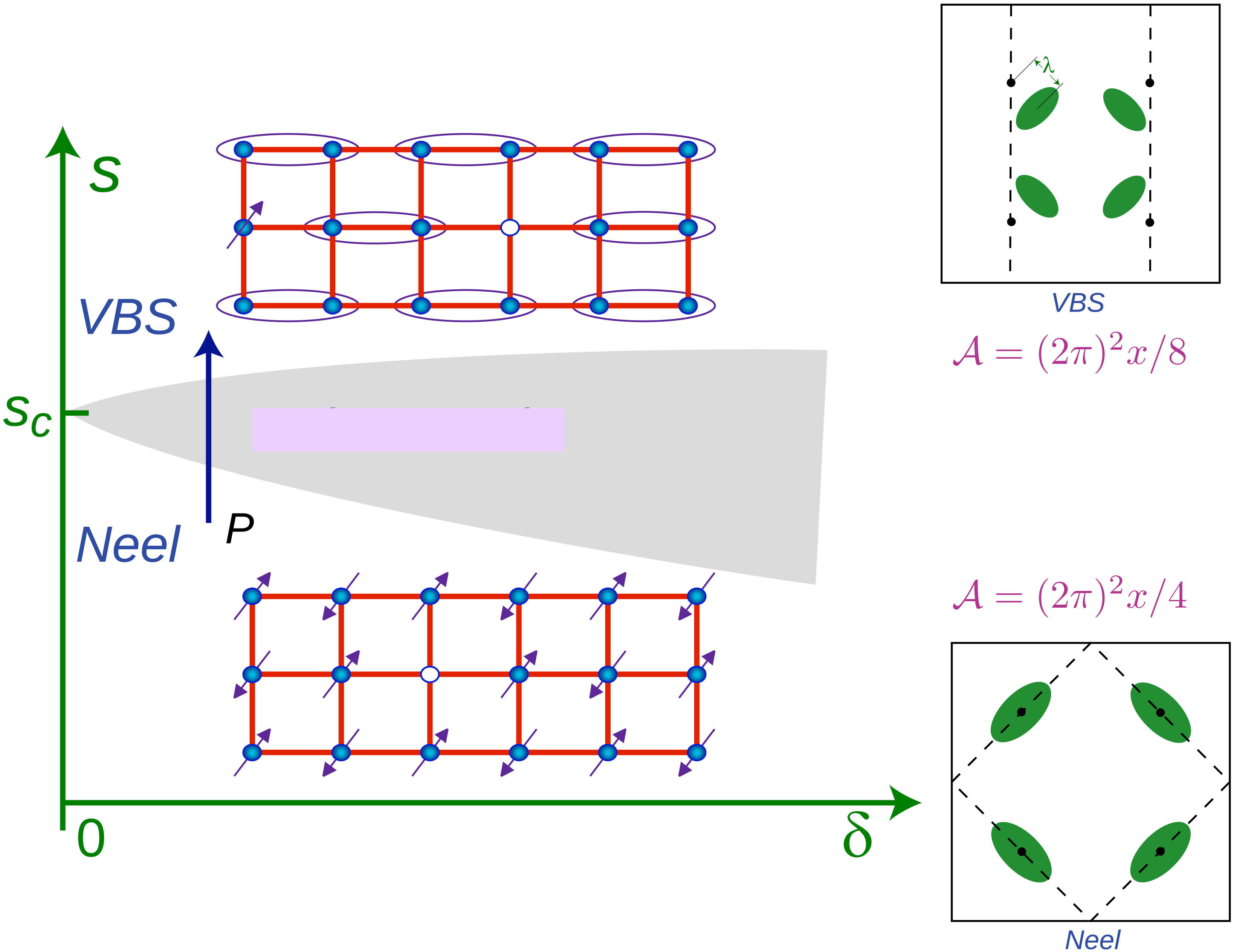
• Include the hopping between opposite sublattices (Shraiman-Siggia term):

$$\begin{aligned} \mathcal{S}_t &= -t \sum_{\langle rr' \rangle} c_\alpha^\dagger(r) c_\alpha(r') + \text{h.c.} \\ &= -t \sum_{\langle rr' \rangle} (f_+^\dagger(r) z_{r\alpha})^\dagger \varepsilon_{\alpha\beta} f_-^\dagger(r') z_{r\beta}^* \end{aligned}$$

• Complete theory for doped antiferromagnet:

$$\mathcal{S} = \mathcal{S}_z + \mathcal{S}_f + \mathcal{S}_t$$

Phase diagram of lightly doped antiferromagnet

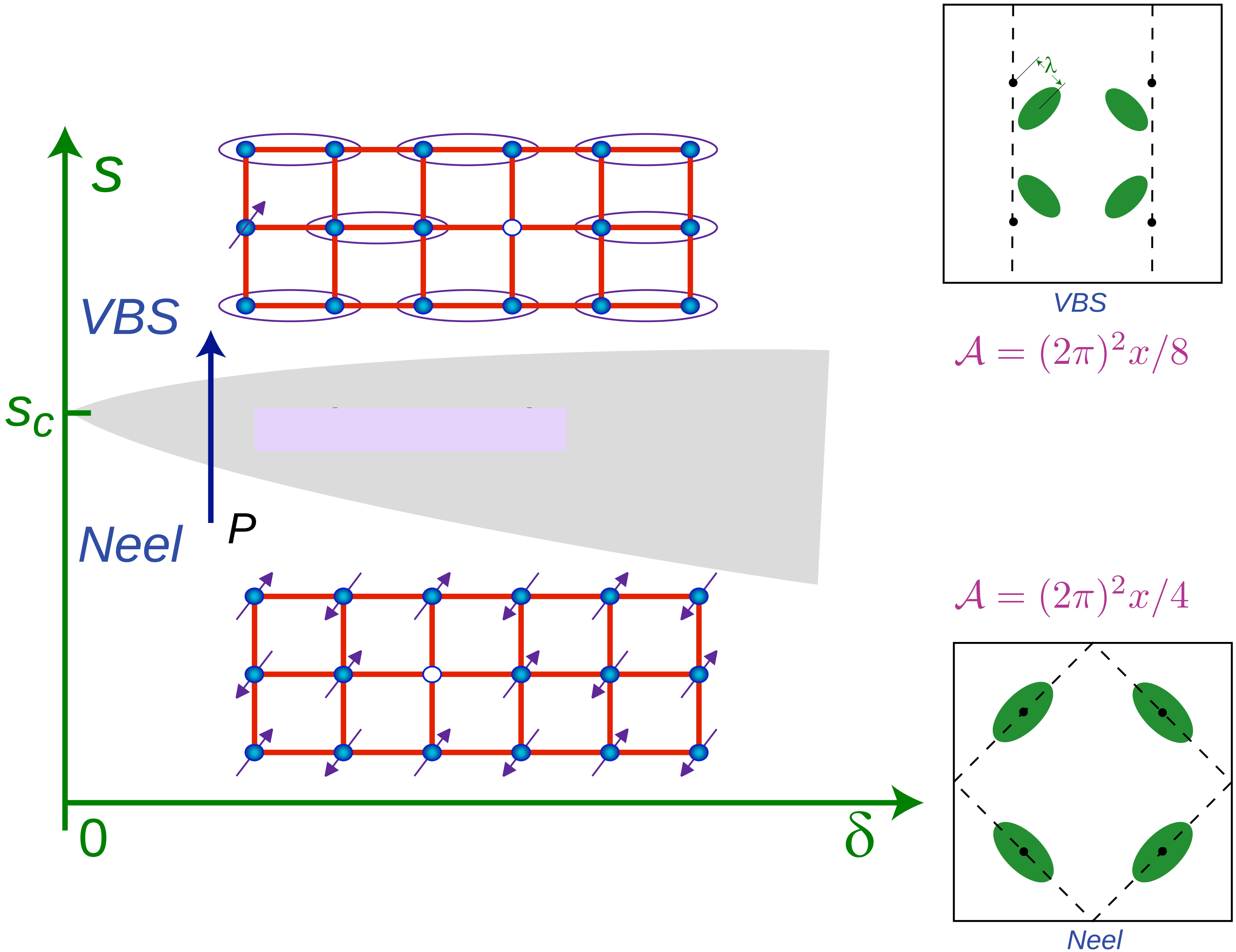


Pictorial explanation of factor of 2:

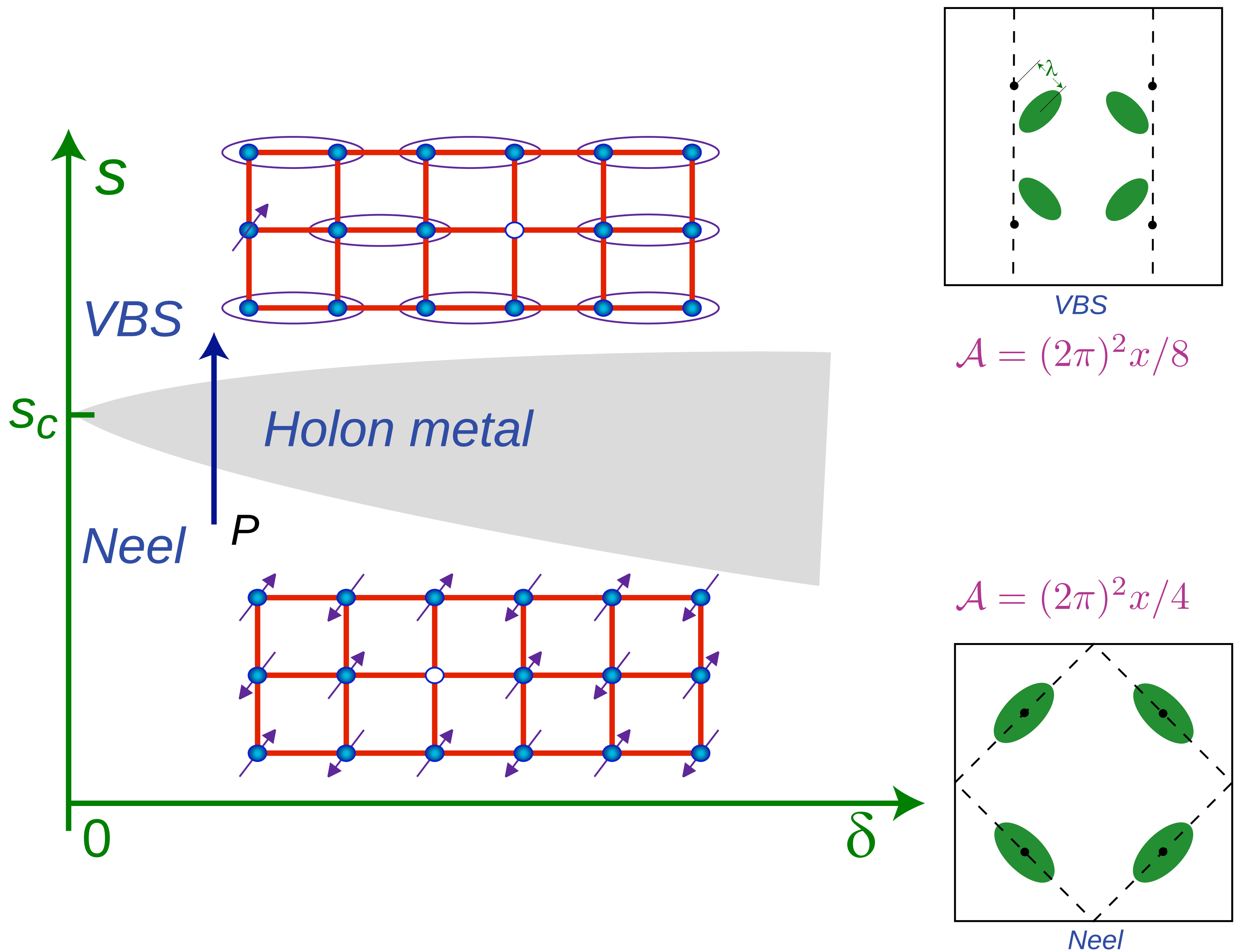
- In the Néel phase, sublattice index is identical to spin index. So for each valley and momentum, degeneracy of the hole state is 2.

- In the VBS state, the sublattice index and the spin index are distinct. So for each valley and momentum, degeneracy of the hole state is 4.

Phase diagram of lightly doped antiferromagnet

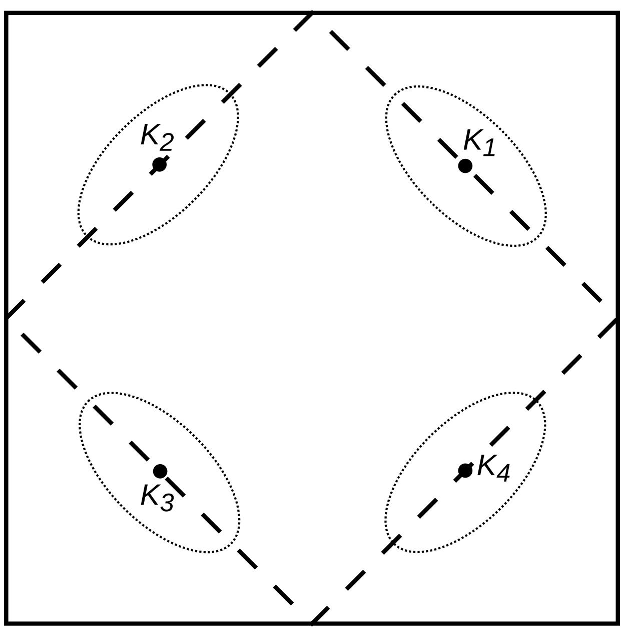


Phase diagram of lightly doped antiferromagnet



A new non-Fermi liquid phase:
The holon metal
An algebraic *charge* liquid.

- Ignore compactness in A_μ and Berry phase term.
- Neutral spinons z_α are gapped.
- Charge e fermions f_s form Fermi surfaces and carry charges $s = \pm 1$ under the U(1) gauge field A_μ.
- Quasi-long range order in a variety of VBS and pairing correlations.

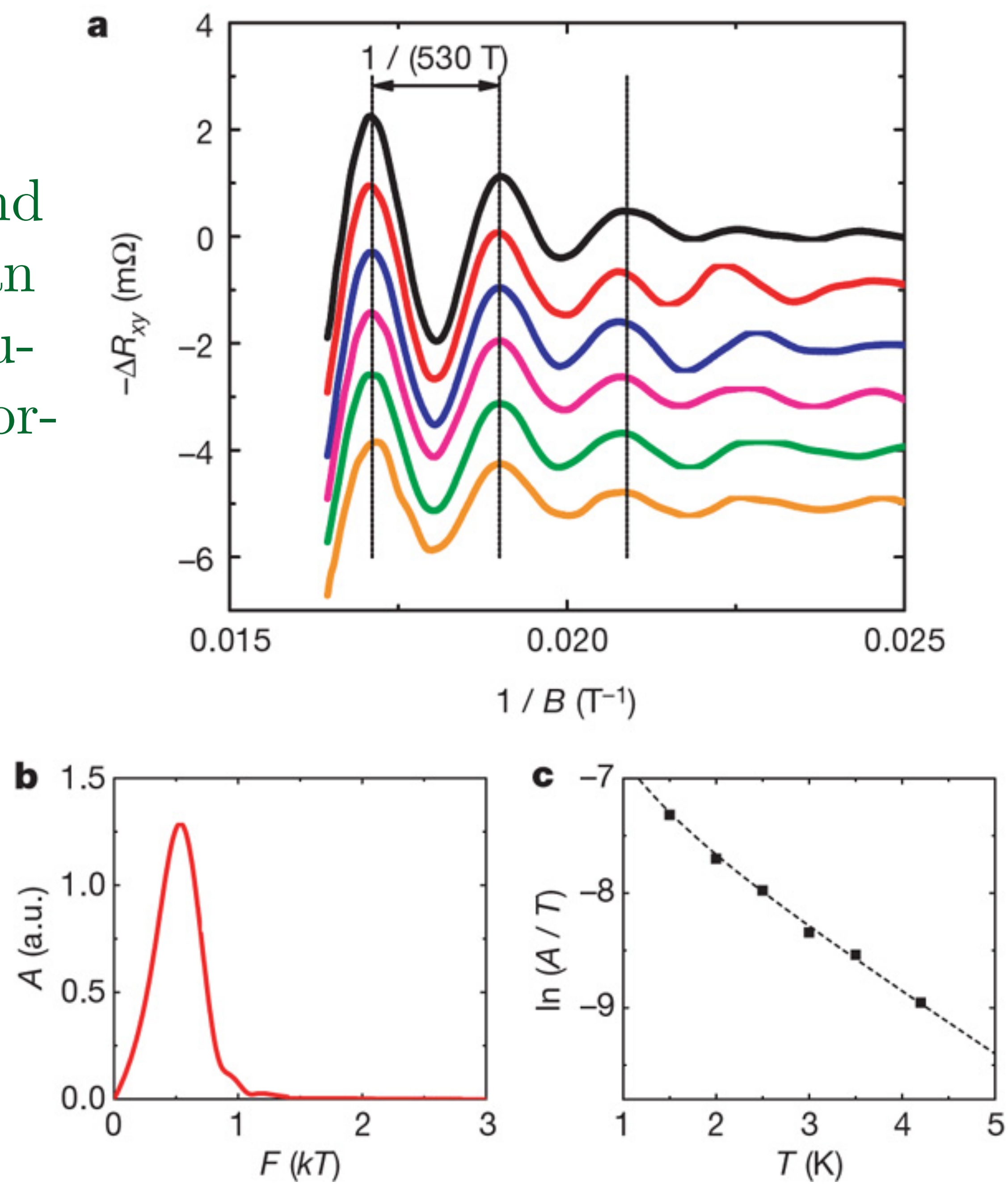


Area of each Fermi pocket,

$$\mathcal{A} = (2\pi)^2 x/4.$$

The Fermi pocket will show sharp magnetoresistance oscillations, but it is invisible to photoemission.

Quantum oscillations and the Fermi surface in an underdoped high T_c superconductor (ortho-II ordered $YBa_2Cu_3O_{6.5}$).



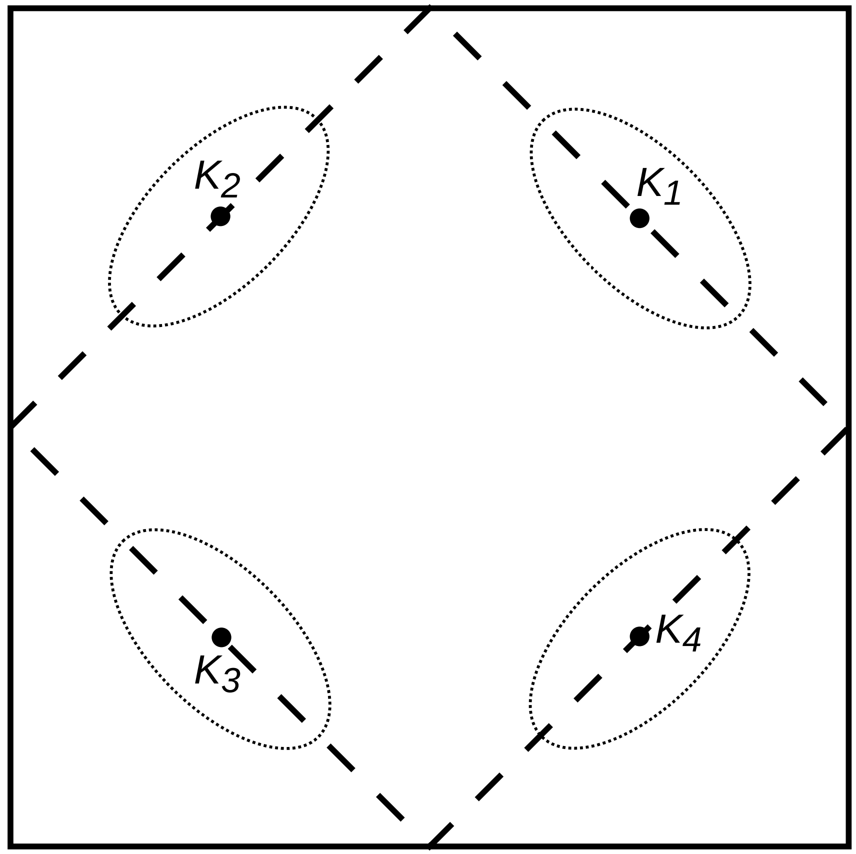
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

Holon pairing leading to *d*-wave superconductivity

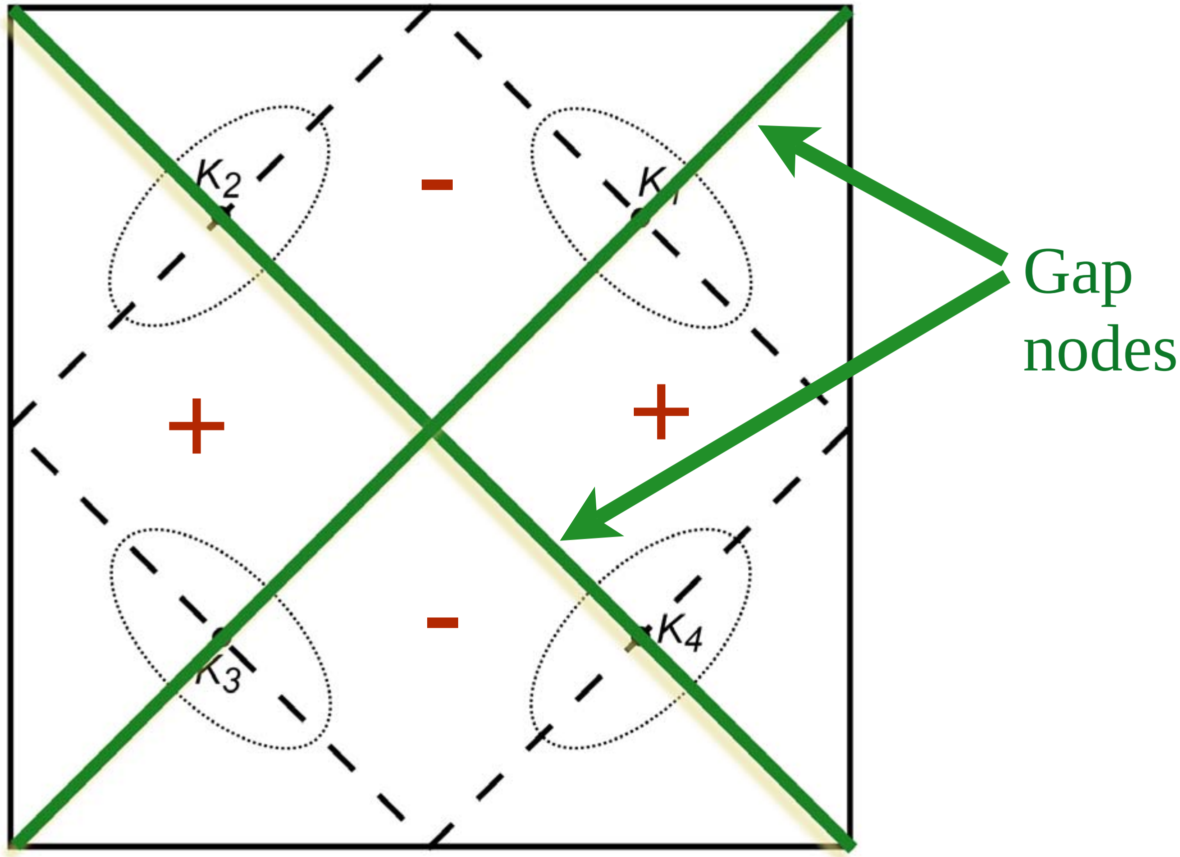
- First consider holon pairing in the Neel state, where holon=hole.

- This was studied in V. V. Flambaum, M. Yu. Kuchiev, and O. P. Sushkov, *Physica* C **227**, 267 (1994); V. I. Belincher *et al.*, *Phys. Rev.* B **51**, 6076 (1995). They found *p*-wave pairing of holons, induced by spin-wave exchange from the sublattice mixing term $\mathcal{S}_t$. This corresponds to *d*-wave pairing of physical electrons

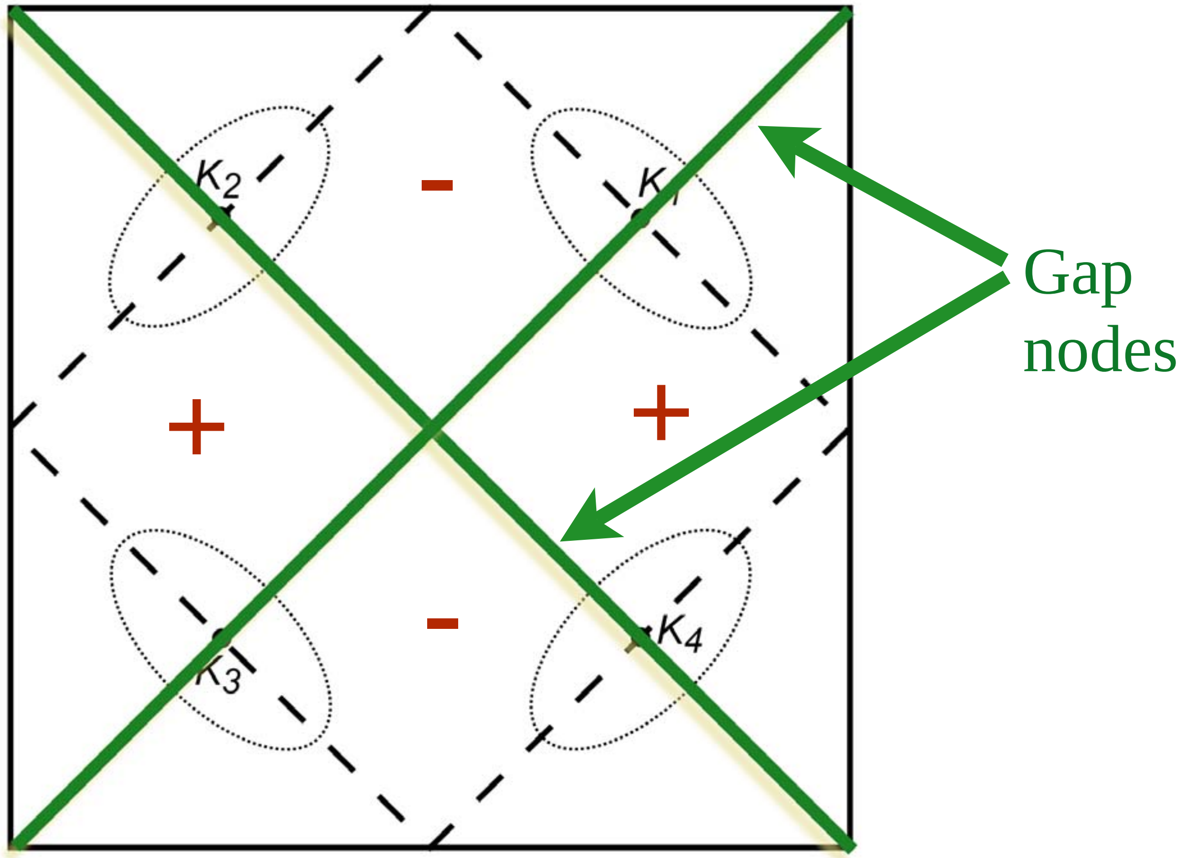
Holon pairing leading to *d*-wave superconductivity



Holon pairing leading to *d*-wave superconductivity



Holon pairing leading to *d*-wave superconductivity



We assume the same pairing holds across a transition involving loss of long-range Néel order. The resulting phase is another algebraic charge liquid - the *holon superconductor*. This superconductor has gapped spinons with no electrical charge, and spinless, nodal Bogoliubov-Dirac quasiparticles. The superconductivity does **not** gap the U(1) gauge field A_μ, because the Cooper pairs are gauge neutral.

Low energy theory of holon superconductor

4 two-component Dirac quasiparticles coupled to a U(1) gauge field

$$\mathcal{S}_{\text{holon superconductor}} = \int d\tau d^2 r \left[\frac{1}{2e_0^2} \left(\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda \right)^2 + \sum_{i=1}^{4} \psi_i^\dagger \left(D_\tau - i v_F (\partial_x - i A_x) \tau^x - i v_F (\partial_y - i A_y) \tau^y \right) \psi_i \right]$$

Low energy theory of holon superconductor

External vector potential $\vec{A}$ couples as

$$\mathcal{H}_A = \vec{j} \cdot \vec{A}$$

where

$$j_x = v_F \left(\psi_3^\dagger \psi_3 - \psi_1^\dagger \psi_1 \right) \quad , \quad j_x = v_F \left(\psi_4^\dagger \psi_4 - \psi_2^\dagger \psi_2 \right)$$

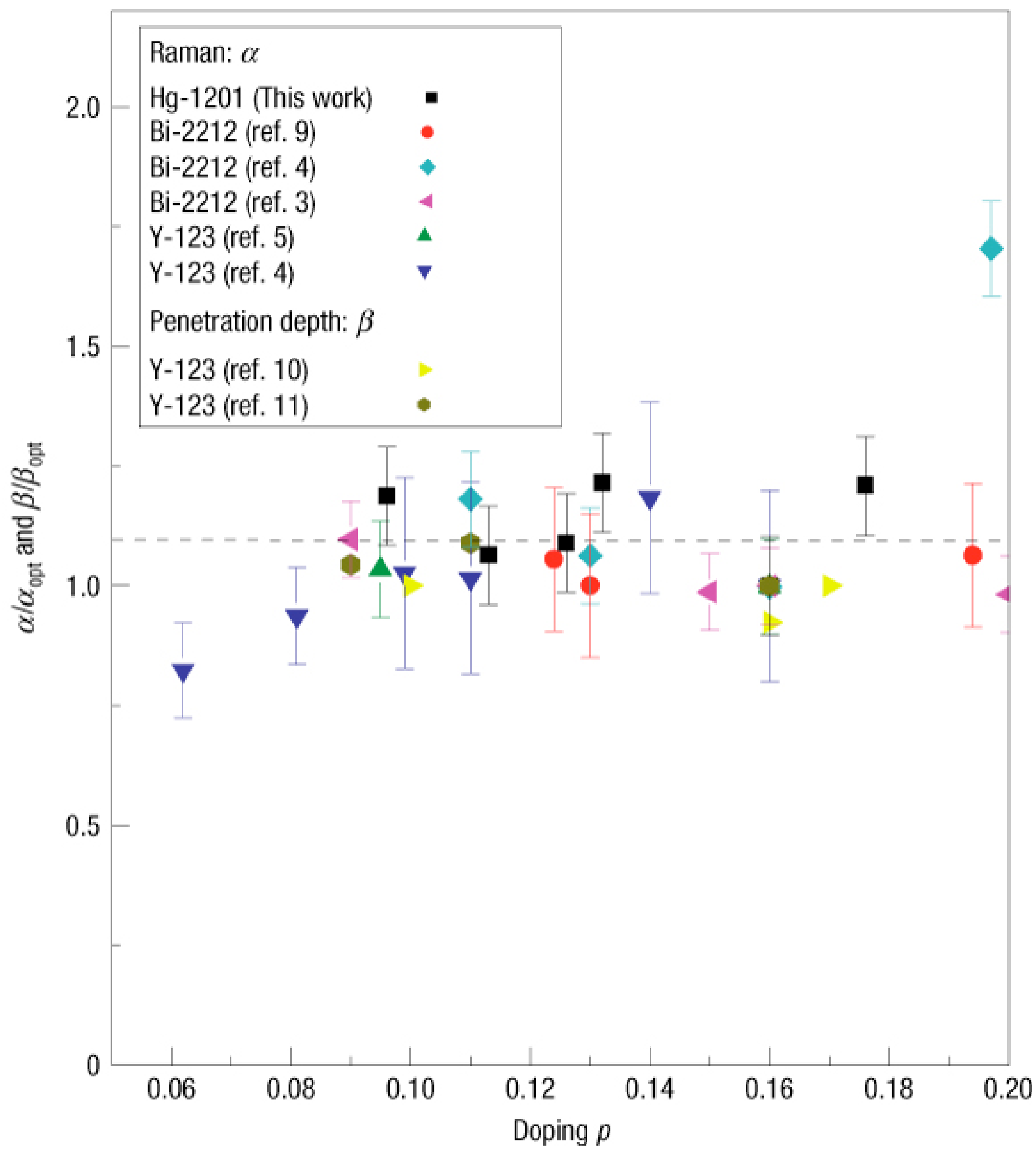
are conserve charges of $\mathcal{S}_{\text{holon superconductor}}$.

Fundamental property: The superfluid density, ρ_s, has the following x and T dependence:

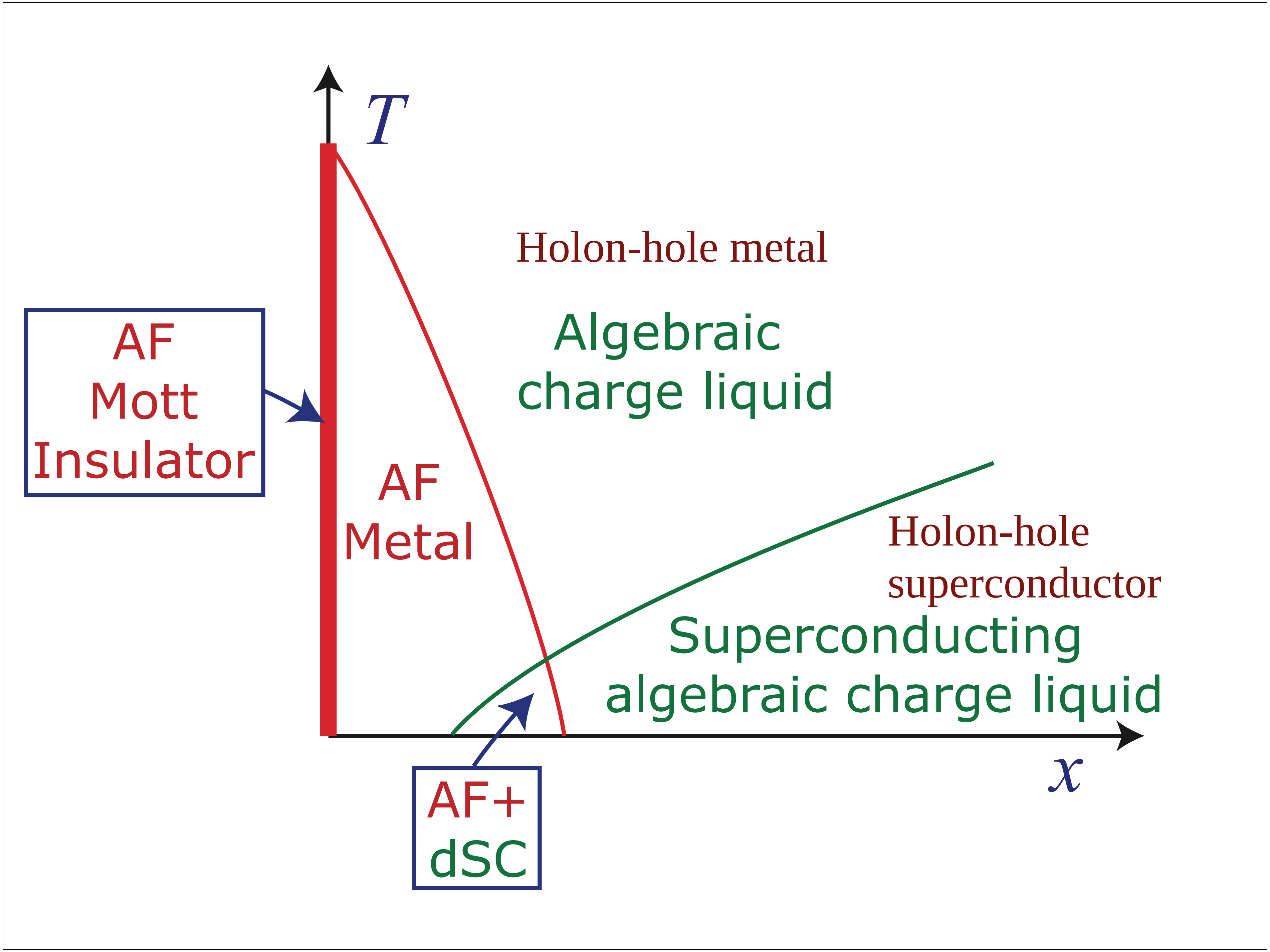
$$\rho_s(x, T) = cx - \mathcal{R} k_B T$$

where c is a non-universal constant and $\mathcal{R}$ is a universal constant obtained in a $1/N$ expansion ($N = 4$ is the number of Dirac fermions):

$$\mathcal{R} = 0.4412 + \frac{0.074}{N} + \ldots$$



Raman: α
Hg-1201 (This work)
Bi-2212 (ref. 9)
Bi-2212 (ref. 4)
Bi-2212 (ref. 3)
Y-123 (ref. 5)
Y-123 (ref. 4)
Penetration depth: β
Y-123 (ref. 10)
Y-123 (ref. 11)
α/α_{opt} and β/β_{opt}
0
0.5
1.0
1.5
2.0
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
Doping p



T
Holon-hole metal
Algebraic
charge liquid
AF
Mott
Insulator
AF
Metal
Holon-hole
superconductor
Superconducting
algebraic charge liquid
x
AF+
dSC

Conclusions

1. Theory for Z_2 and U(1) spin liquids in quantum antiferromagnets, and evidence for their realization in model spin systems.

2. *Algebraic charge liquids* appear naturally upon adding fermionic carriers to spin liquids with bosonic spinons. These are conducting states with topological order.

3. The holon metal/superconductor, obtained by doping a Neel-ordered insulator, matches several observed characteristics of the underdoped cuprates.