Influence of Harris disorder on strange metals and quantum-critical superconductivity

Fluctuations, Disorder, and Strong Correlations

Celebrating Thomas Vojta's 60th

MPIPKS, Dresden
June 22, 2025

Subir Sachdev

PHYSICS
HARVARD

Quantum Griffiths effects in itinerant Heisenberg magnets

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PHYSICAL REVIEW B **72**, 045438 (2005)

We study the influence of quenched disorder on quantum phase transitions in itinerant magnets with Heisenberg spin symmetry, paying particular attention to rare disorder fluctuations. In contrast to the Ising case where the Landau damping of the spin fluctuations suppresses the tunneling of the rare regions, the Heisenberg system displays strong power-law quantum Griffiths singularities in the vicinity of the quantum critical point. We discuss these phenomena based on general scaling arguments, and we illustrate them by an explicit calculation for $O(N)$ spin symmetry in the large-N limit. We also discuss broad implications for the classification of quantum phase transitions in the presence of quenched disorder.

Effects of Dissipation on a Quantum Critical Point with Disorder

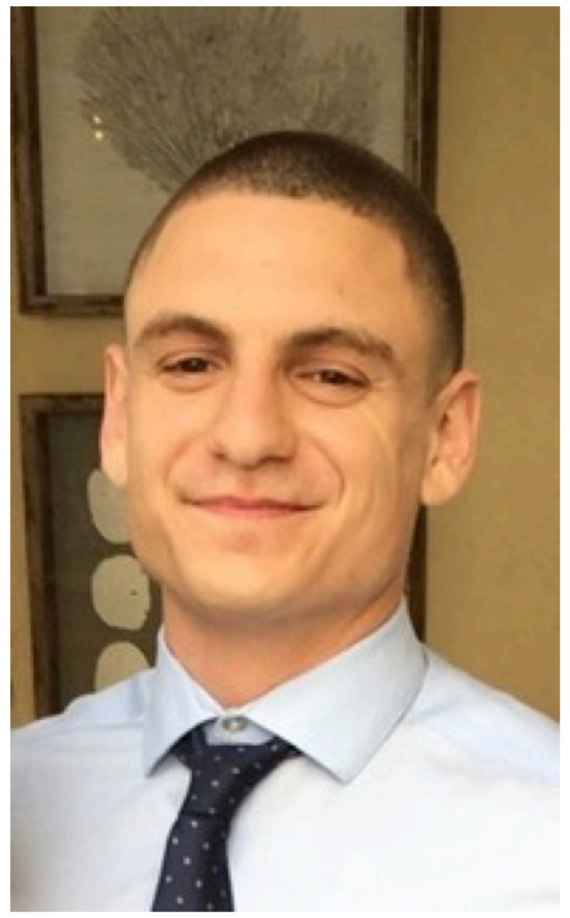
José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

(Received 19 May 2007; published 4 December 2007)

We study the effects of dissipation on a disordered quantum phase transition with $O(N)$ order-parameter symmetry by applying a strong-disorder renormalization group to the Landau-Ginzburg-Wilson field theory of the problem. We find that Ohmic dissipation results in a nonperturbative infinite-randomness critical point

PHYSICAL REVIEW LETTERS **99**, 230601 (2007)



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Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)
Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)
Davide Valentinis, Jorg Schmalian, S. S., and Aaavishkar A. Patel, Physical Review Research **8**, 013299 (2026)

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Influence of Harris disorder on quantum-critical superconductivity
Serhii Kryhin, Peter Lunts, Aavishkar A. Patel, S.S., Pavel A. Nosov
arXiv:2606.23582

1. Universal 2D-YSYK theory of strange metals

2. Theory of the "foot":
   quantum Griffiths SDW phase

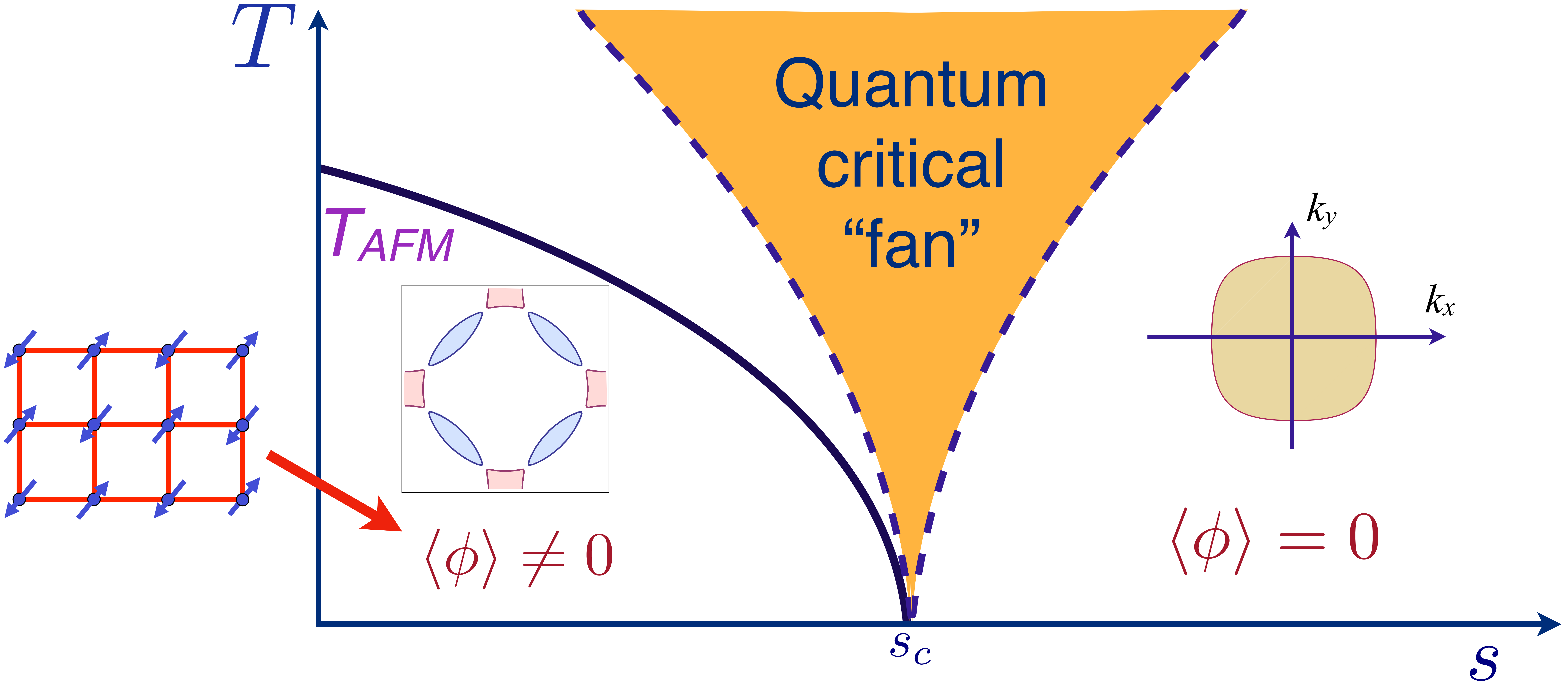
3. Quantum-critical superconductivity

1. Universal 2D-YSYK theory of strange metals

2. Theory of the "foot":
   quantum Griffiths SDW phase

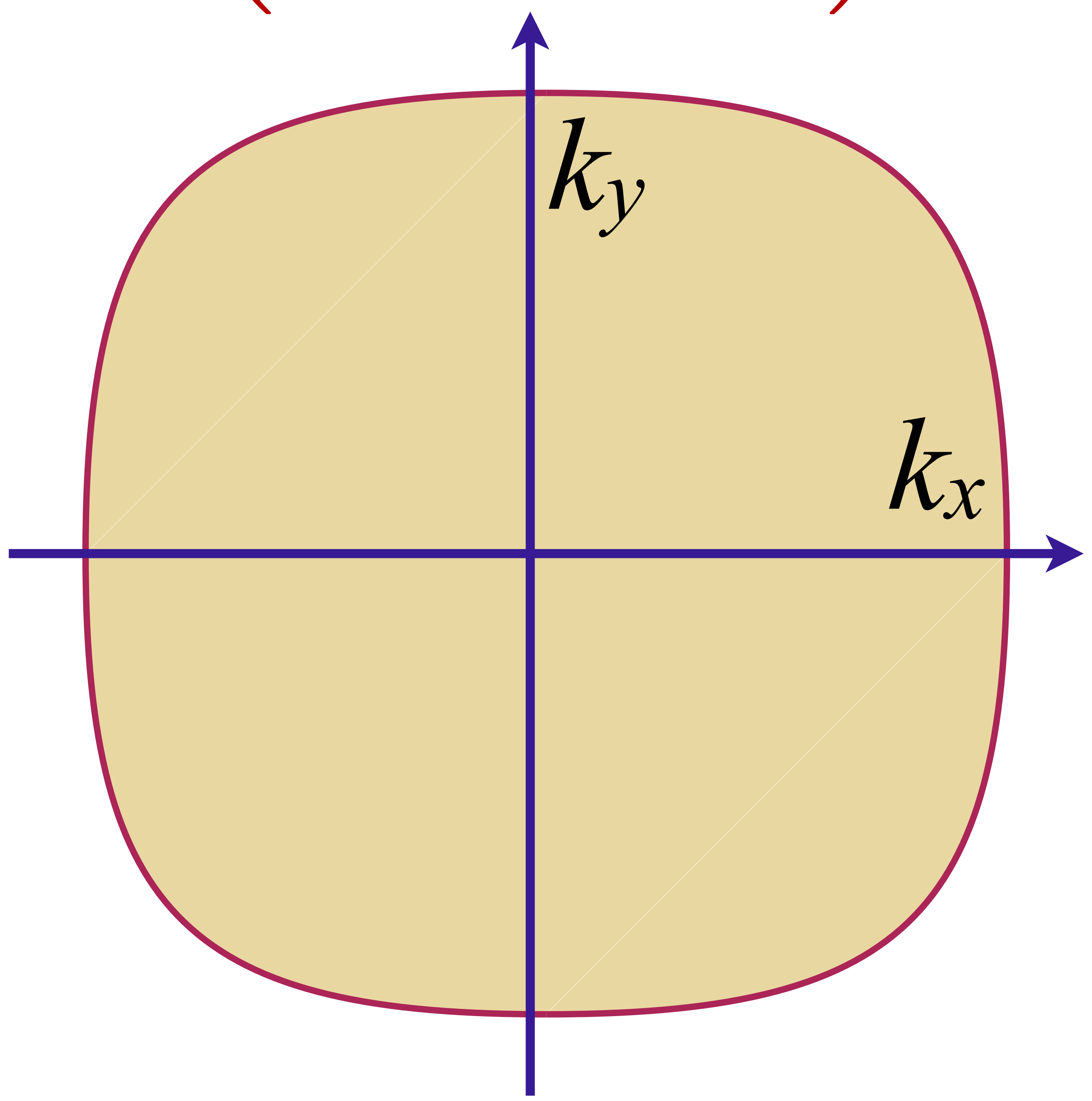
3. Quantum-critical superconductivity

Fermi surface reconstruction from spin density wave (SDW) order



Fermi surface + critical boson with no spatial disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$

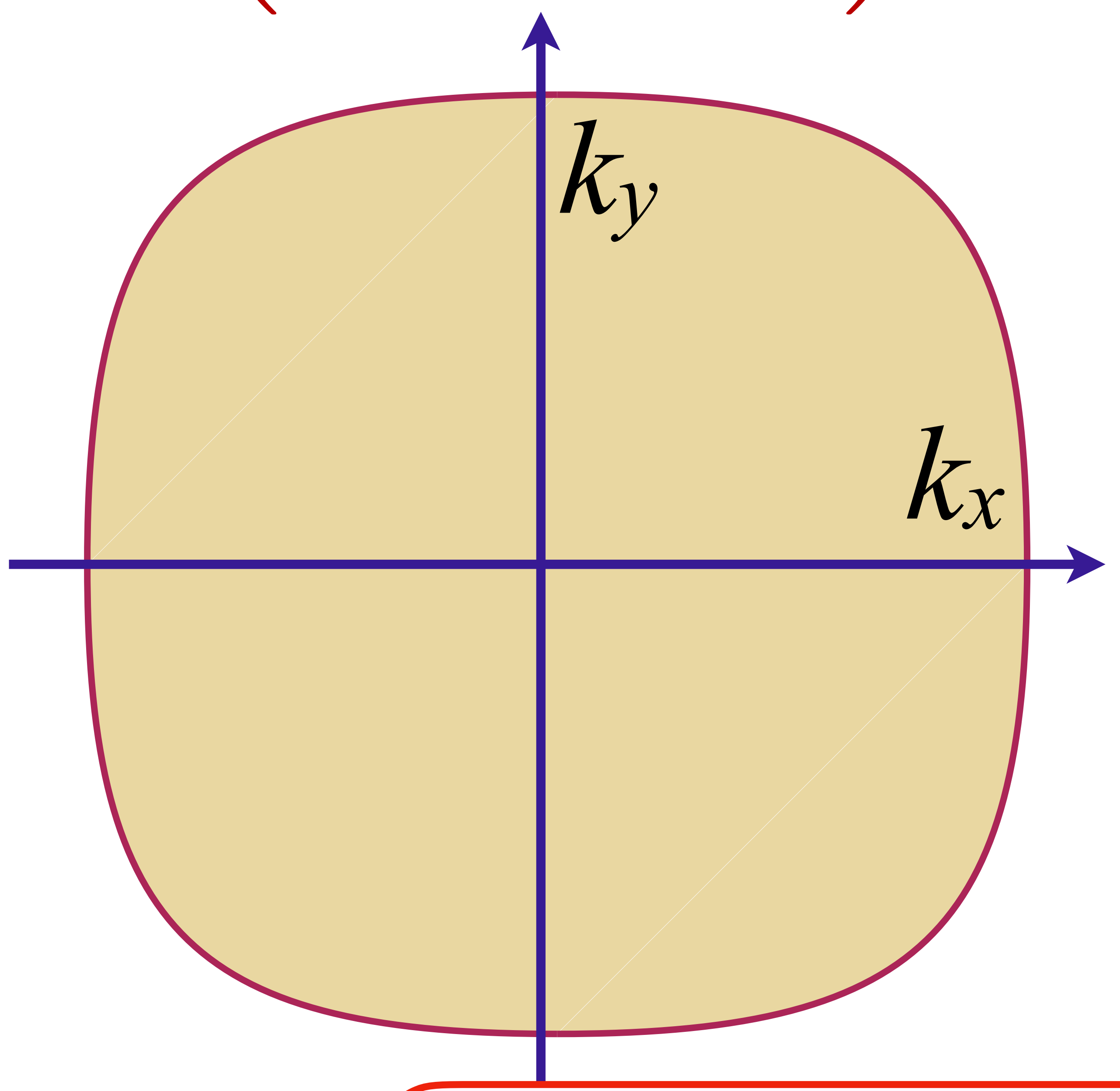


$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4$$

Fermi surface + critical boson with no spatial disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4$$

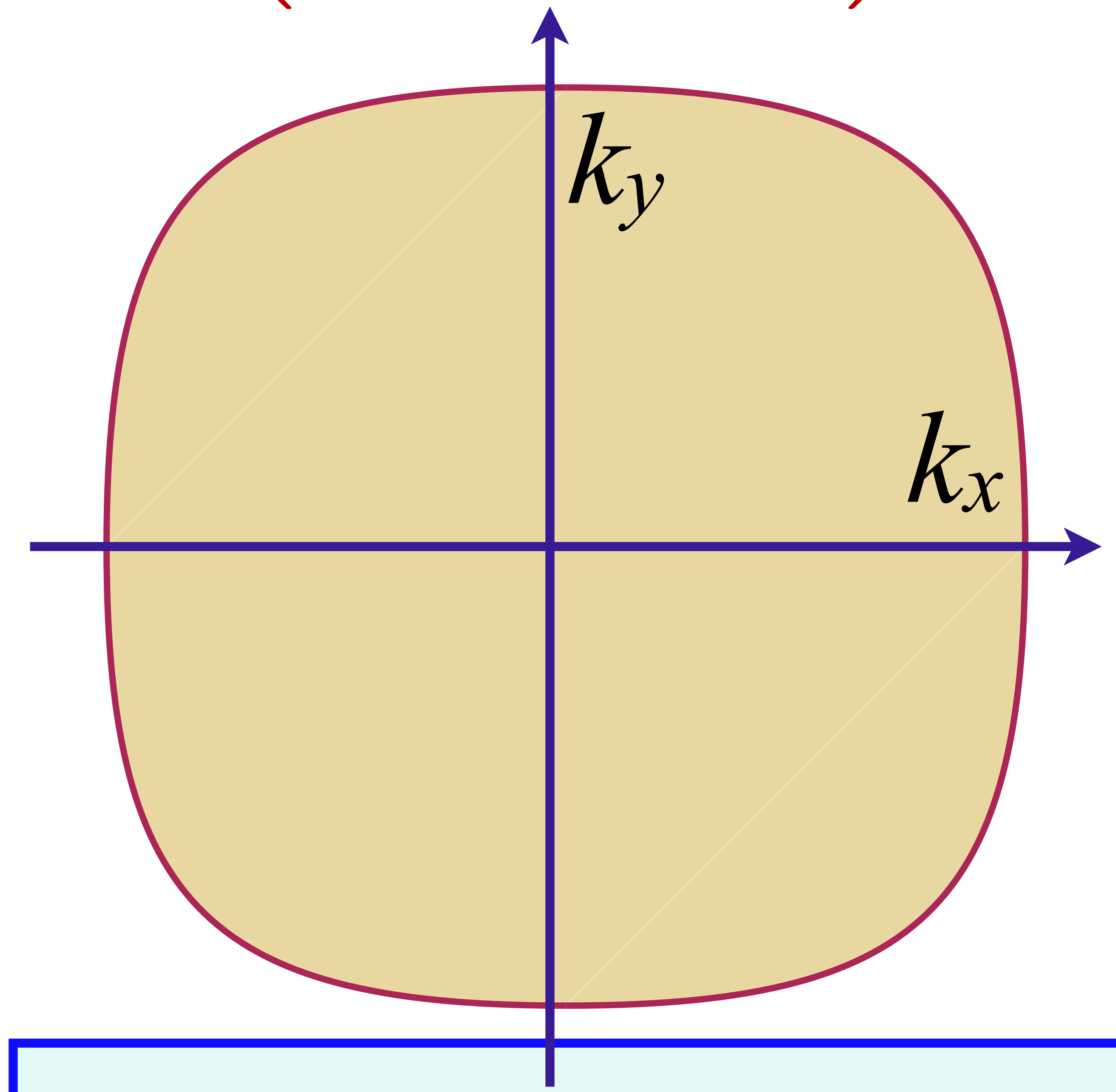
A. A. Patel and S. S.,
PRB **90**, 165146 (2014)

Not a strange metal, despite strongly coupled quantum criticality.

Extreme drag: the fermions c "drag" the bosons ϕ as they move, and so electrical current does not relax, even though strong c-ϕ scattering leads to absence of c quasiparticles.

Fermi surface + critical boson with potential disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+s\,[\phi(\boldsymbol{r})]^2 \qquad +g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

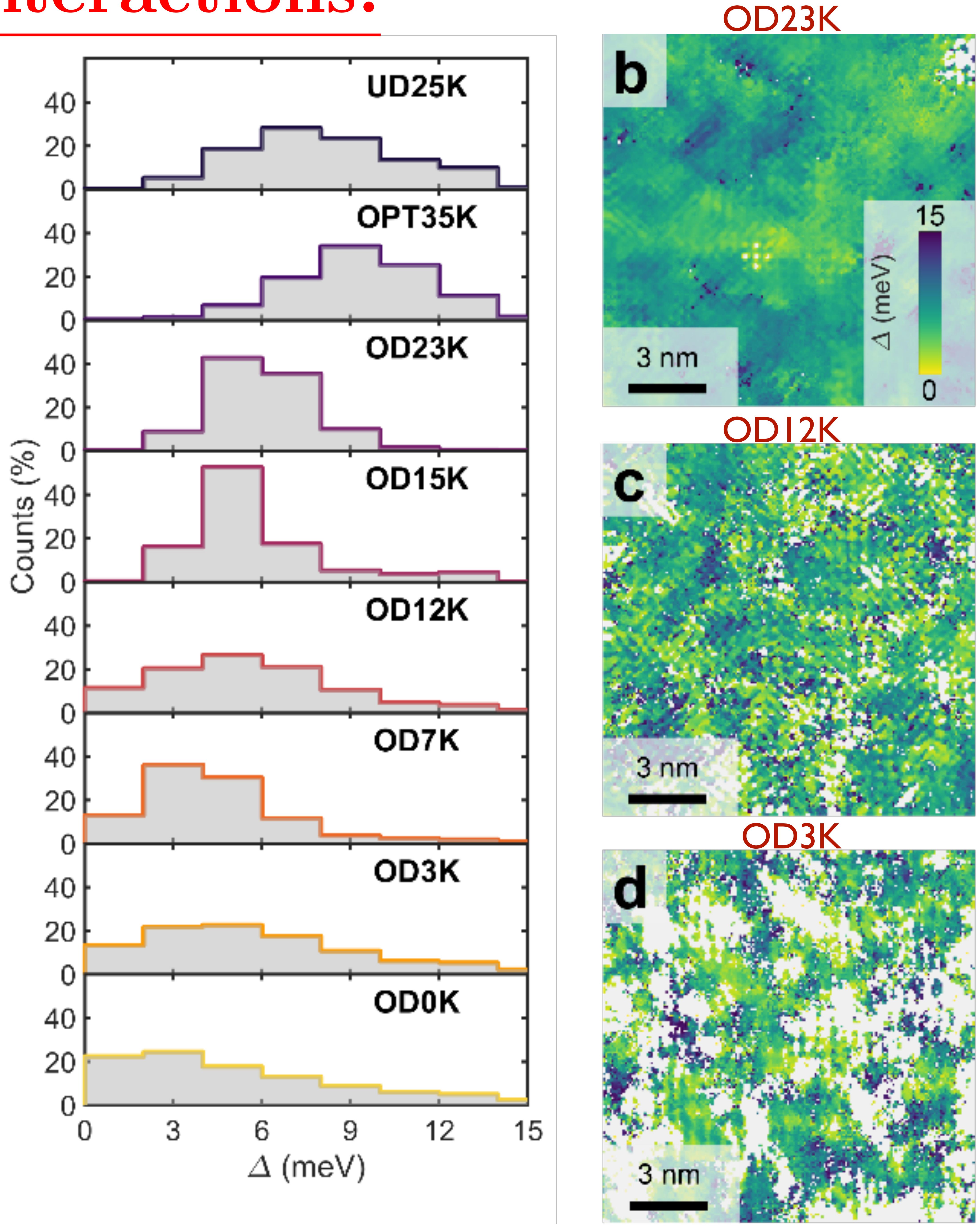
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0,\ \overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random interactions!

Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

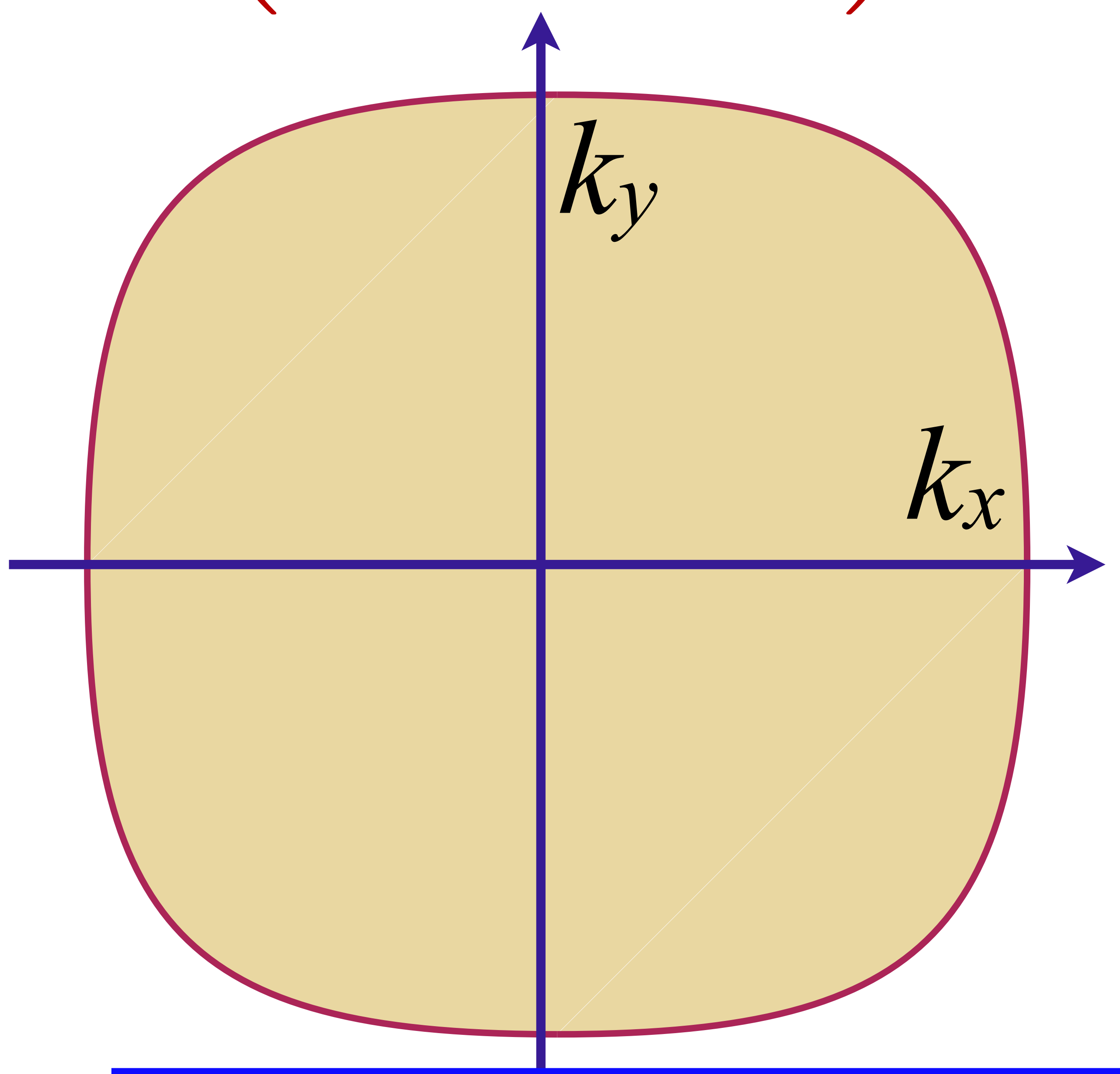
Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix



Nature Materials **22**, 703 (2023)

Fermi surface + critical boson with potential and mass disorder

$$c_{\boldsymbol{k}\sigma}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) c_{\boldsymbol{k}\sigma}$$



$$+ [s + s'(\boldsymbol{r})] [\phi(\boldsymbol{r})]^2 + g\, c_\sigma^\dagger(\boldsymbol{r}) \tau^a_{\sigma\sigma'} c_{\sigma'}(\boldsymbol{r})\, \phi_a(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+ K [\nabla_{\boldsymbol{r}} \phi(\boldsymbol{r})]^2 + u [\phi(\boldsymbol{r})]^4 + v(\boldsymbol{r}) c_\sigma^\dagger(\boldsymbol{r}) c_\sigma(\boldsymbol{r})$$

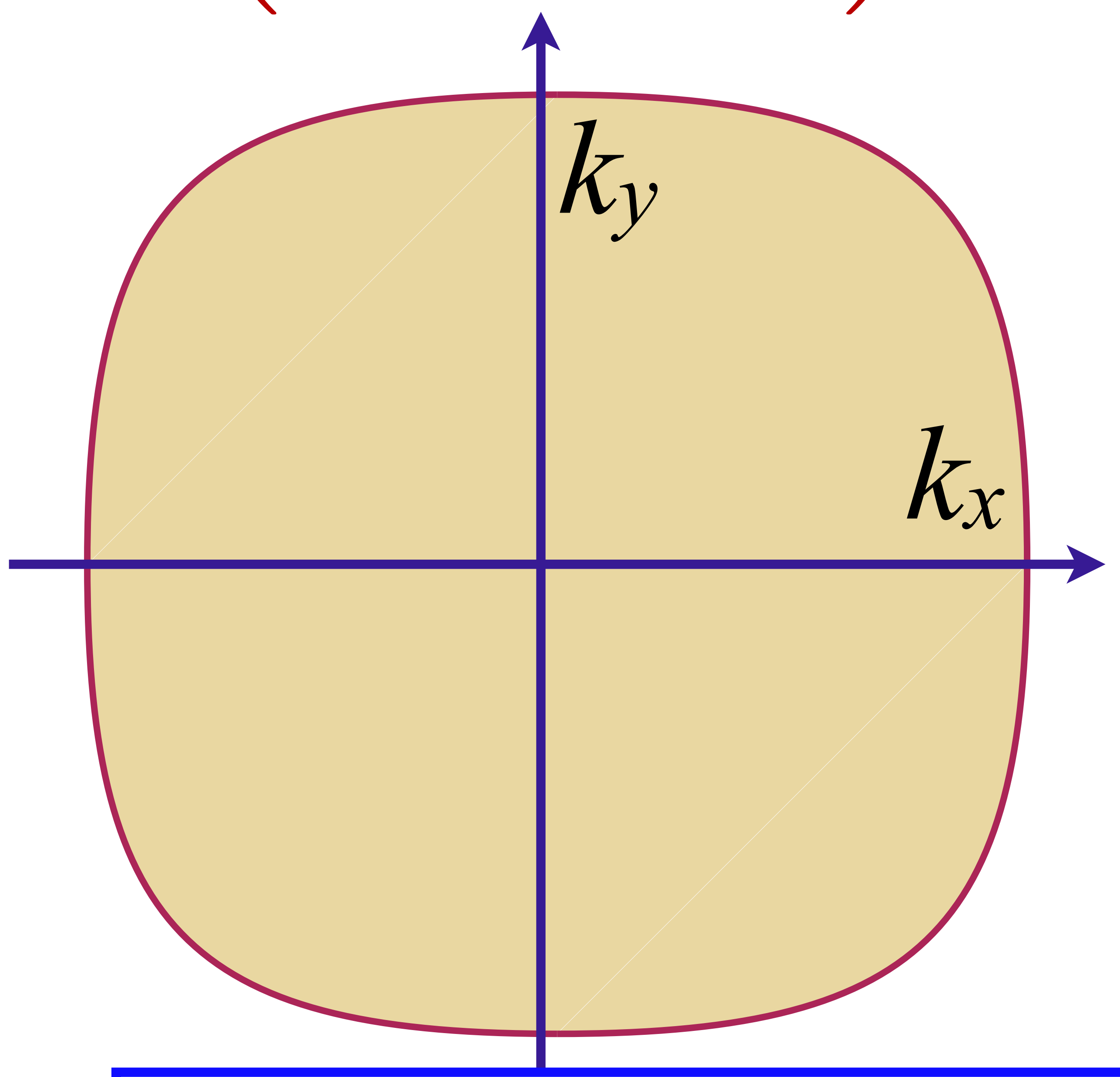
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random 'mass' $s'(\boldsymbol{r})$ with $\overline{s'(\boldsymbol{r})} = 0$, $\overline{s'(\boldsymbol{r})s'(\boldsymbol{r}')} = s'^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

$s'(\boldsymbol{r})$ is strongly relevant Harris disorder.
Not amenable to disorder averaged SYK-type analysis

Fermi surface + critical boson with potential and mass disorder

$$c^\dagger_{\boldsymbol{k}\sigma}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+\left[s+s'(\boldsymbol{r})\right]\left[\phi(\boldsymbol{r})\right]^2+g\,c^\dagger_\sigma(\boldsymbol{r})\tau^a_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\left[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})\right]^2+u\left[\phi(\boldsymbol{r})\right]^4+v(\boldsymbol{r})c^\dagger_\sigma(\boldsymbol{r})c_\sigma(\boldsymbol{r})$$

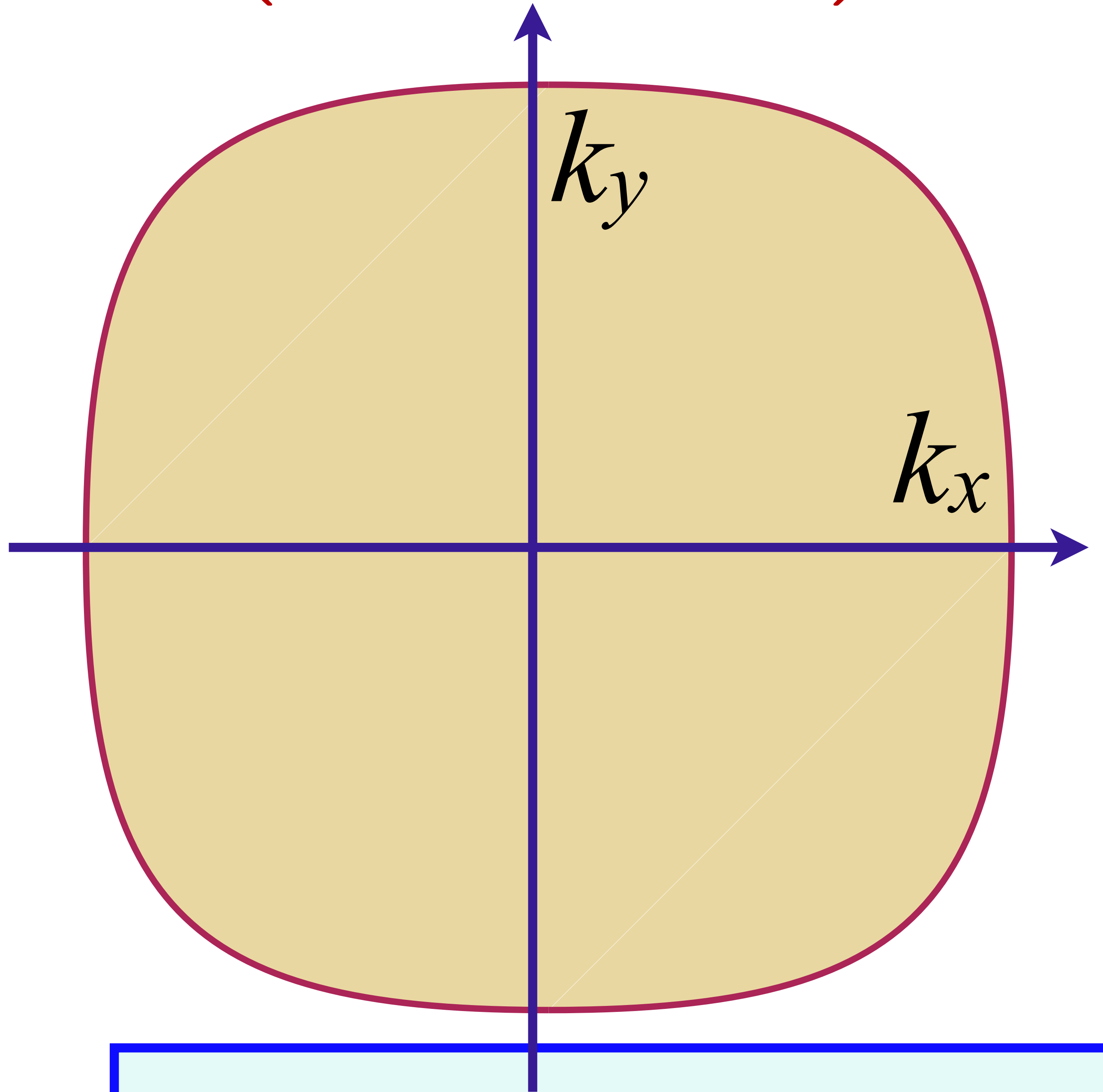
Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random 'mass' $s'(\boldsymbol{r})$ with $\overline{s'(\boldsymbol{r})}=0$, $\overline{s'(\boldsymbol{r})s'(\boldsymbol{r}')}=s'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

$s'(\boldsymbol{r})$ is strongly relevant Harris disorder.
Not amenable to disorder averaged SYK-type analysis

Fermi surface + critical boson with potential and coupling disorder

$$c_{\boldsymbol{k}\sigma}^{\dagger}\left(\frac{\partial}{\partial\tau}+\varepsilon(\boldsymbol{k})\right)c_{\boldsymbol{k}\sigma}$$



$$+s\,[\phi(\boldsymbol{r})]^2 \quad +[g+g'(\boldsymbol{r})]c_{\sigma}^{\dagger}(\boldsymbol{r})\tau^{a}_{\sigma\sigma'}c_{\sigma'}(\boldsymbol{r})\,\phi_a(\boldsymbol{r})e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$$

$$+K\,[\nabla_{\boldsymbol{r}}\phi(\boldsymbol{r})]^2+u\,[\phi(\boldsymbol{r})]^4+v(\boldsymbol{r})c_{\sigma}^{\dagger}(\boldsymbol{r})c_{\sigma}(\boldsymbol{r})$$

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})}=0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')}=v^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

Spatially random coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})}=0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')}=g'^2\delta(\boldsymbol{r}-\boldsymbol{r}')$

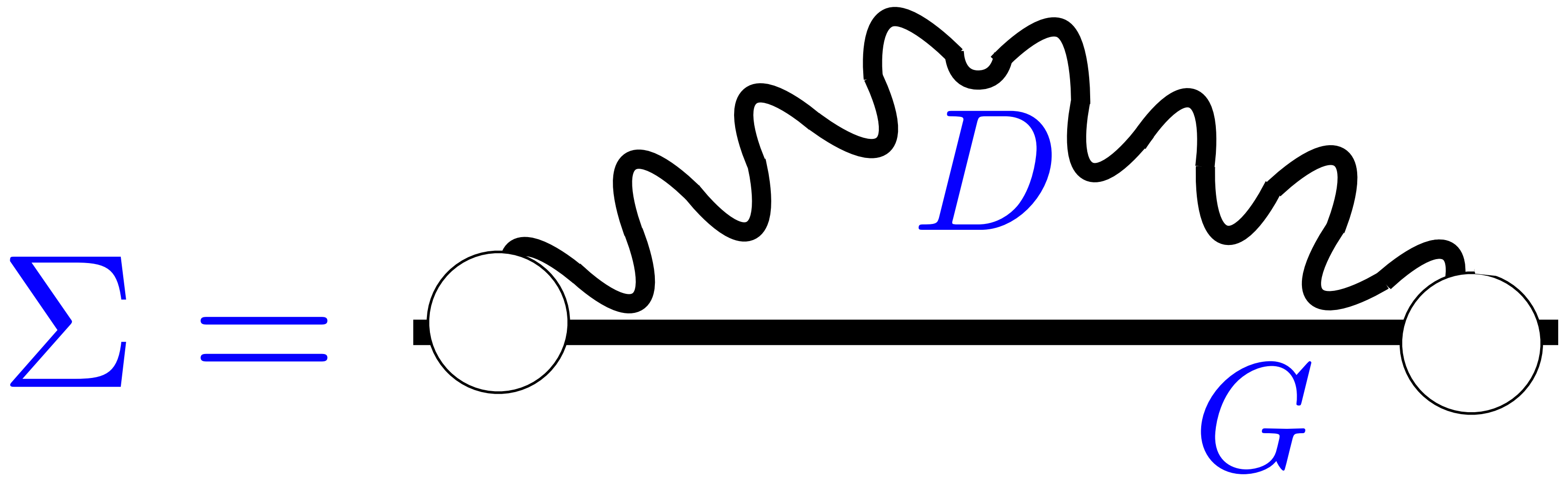
Rescale $\phi(\boldsymbol{r})$ to transfer disorder to random coupling $g+g'(\boldsymbol{r})$.
$g'(\boldsymbol{r})$ amenable to a SYK analysis, provided $\phi(\boldsymbol{r})$ eigenmodes are extended.

2D-YSYK model: Fermi surface + critical boson with interaction disorder

Eqns. for the simpler case of ordering at $\boldsymbol{K} = 0$. Similar results apply for ordering at $\boldsymbol{K} \neq 0$, and also for topological FL-FL* transitions.

All results are obtained from the large N saddle-point and response functions of this G-Σ-D-Π theory:

$$\mathcal{Z} = \int \mathcal{D}G\, \mathcal{D}\Sigma\, \mathcal{D}D\, \mathcal{D}\Pi \exp(-N S_{\text{all}})$$



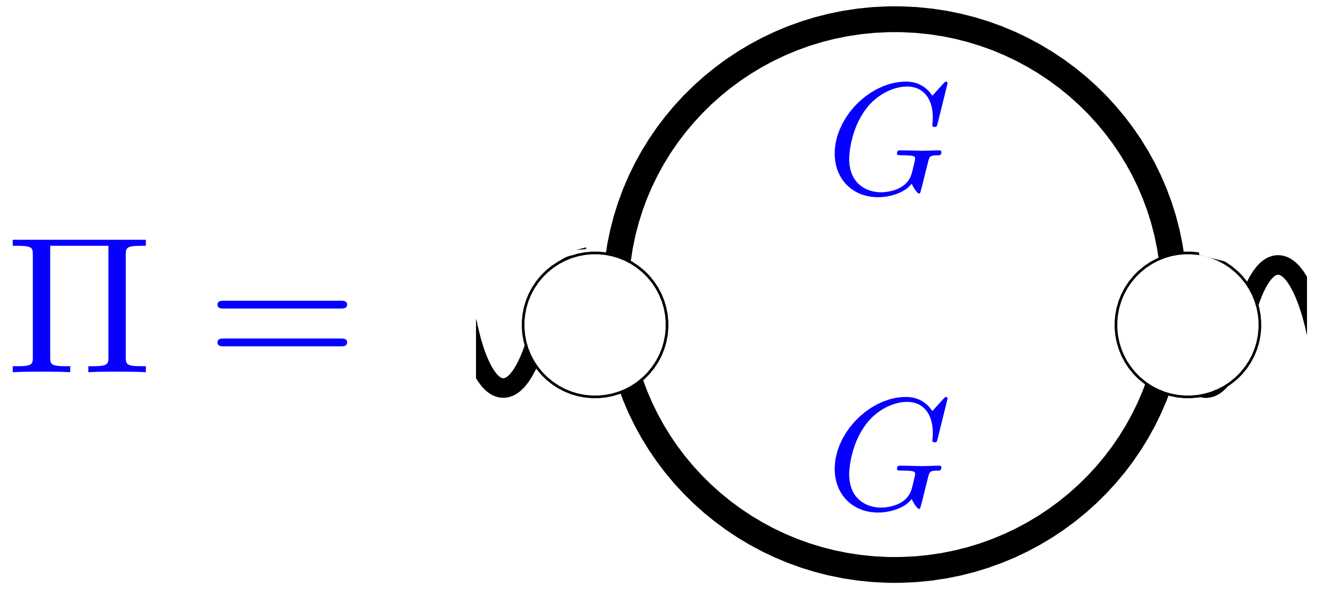
Saddle-point equations

$$\Sigma(\tau,\mathbf{r}) = g^2 D(\tau,\mathbf{r}) G(\tau,\mathbf{r}) + v^2 G(\tau,\mathbf{r})\delta^2(\mathbf{r}) + {g'}^2 G(\tau,\mathbf{r}) D(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$

$$\Pi(\tau,\mathbf{r}) = -g^2 G(-\tau,-\mathbf{r}) G(\tau,\mathbf{r}) - {g'}^2 G(-\tau,\mathbf{r}) G(\tau,\mathbf{r})\delta^2(\mathbf{r}),$$

$$G(i\omega,\mathbf{k}) = \frac{1}{i\omega - \varepsilon(\mathbf{k}) + \mu - \Sigma(i\omega,\mathbf{k})},$$

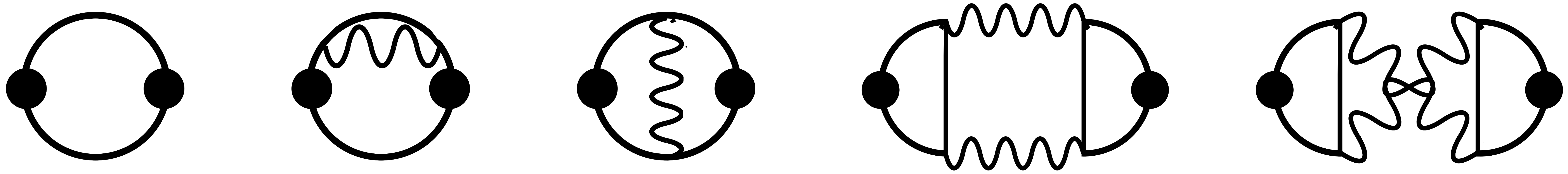
$$D(i\Omega,\mathbf{q}) = \frac{1}{\Omega^2 + \mathbf{q}^2 + m_b^2 - \Pi(i\Omega,\mathbf{q})}.$$



Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

2D-YSYK model: Fermi surface + critical boson with interaction disorder

Optical conductivity—Diagrams



2D-YSYK model: Fermi surface + critical boson with interaction disorder

Electron Green's function: $G(\omega) \sim \dfrac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\text{in}}(\omega)}\right)\text{sgn}(\omega)}$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\text{in}}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Conductivity: $\sigma(\omega) \sim \dfrac{1}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*_{\text{trans}}(\omega)}{m}}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*_{\text{trans}}(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

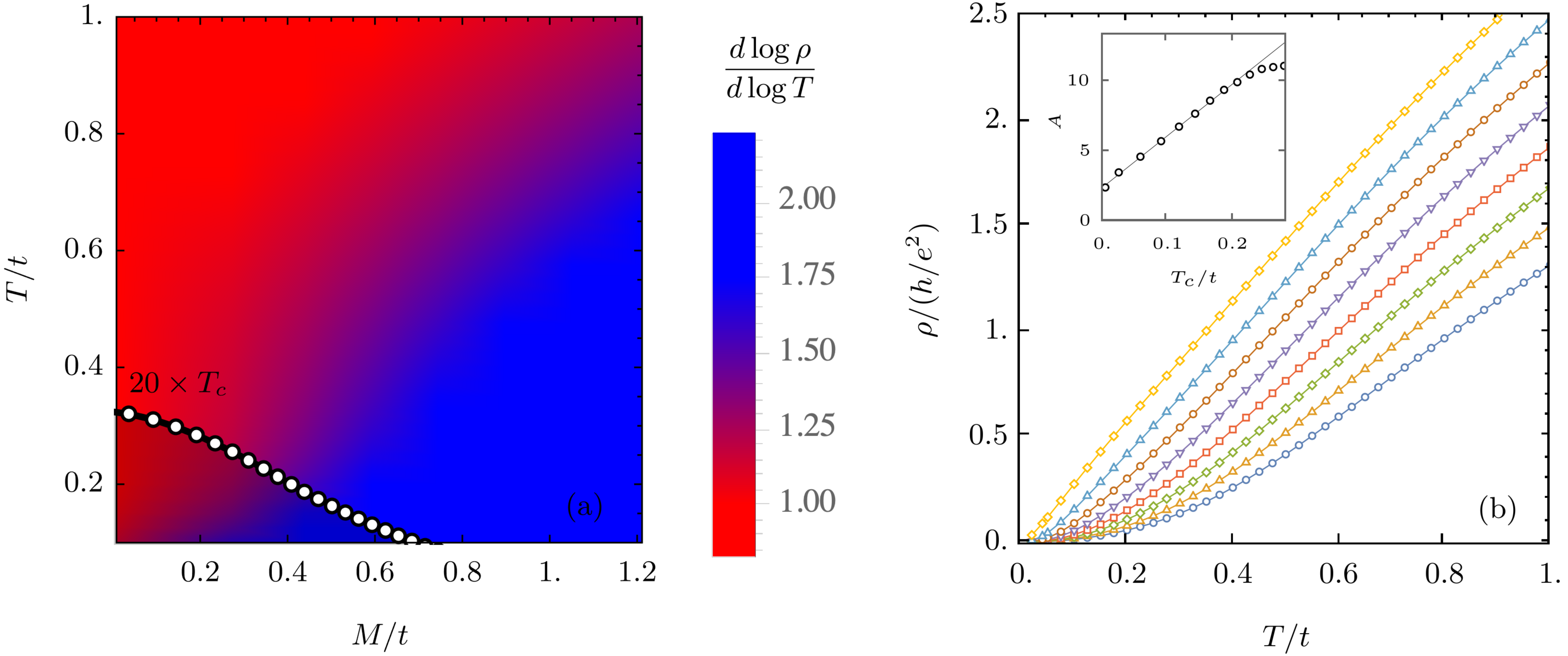
Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g; Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Strange metal and superconductor in the two-dimensional Yukawa-Sachdev-Ye-Kitaev model

$g = 0$

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

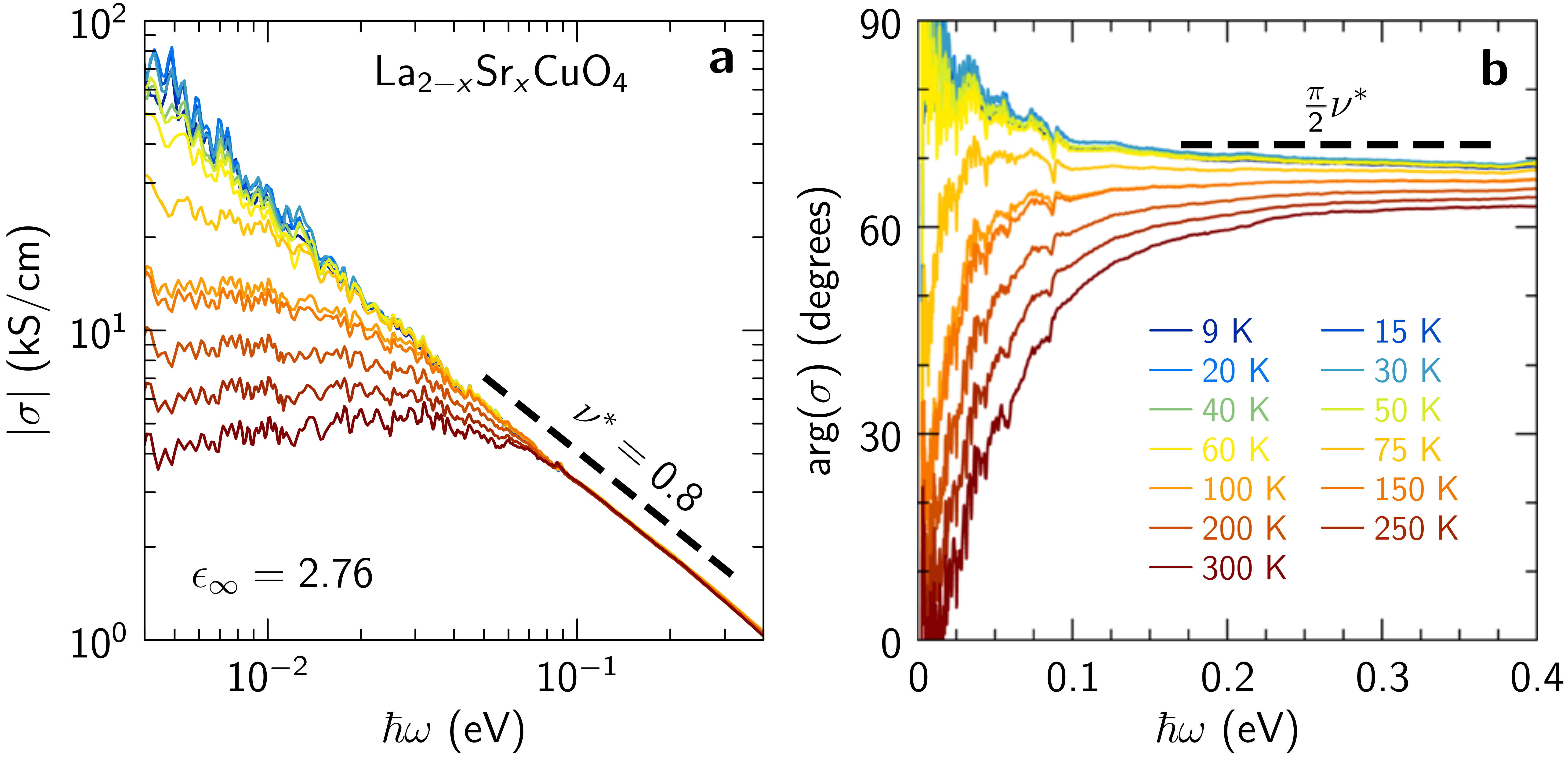


Reconciling scaling of the optical conductivity of cuprate superconductors with Planckian resistivity and specific heat

B. Michon, C. Berthod, C. W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges

Nature Communications **14**, Article number: 3033 (2023)

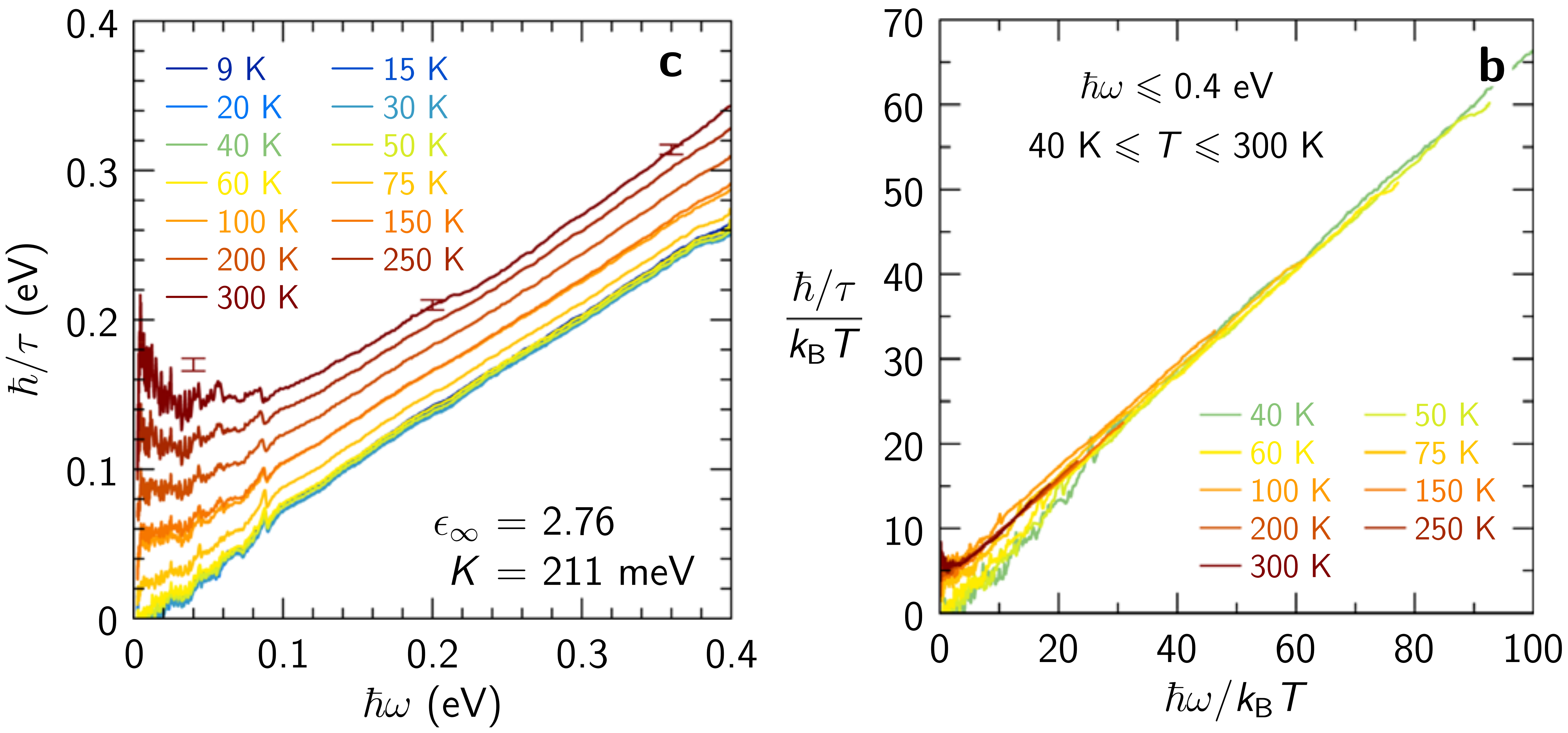
$$\sigma(\omega) = i\frac{e^2K/(\hbar d_c)}{\hbar\omega\frac{m^*(\omega)}{m} + i\frac{\hbar}{\tau(\omega)}}$$



La$_{2-x}$Sr$_x$CuO$_4$

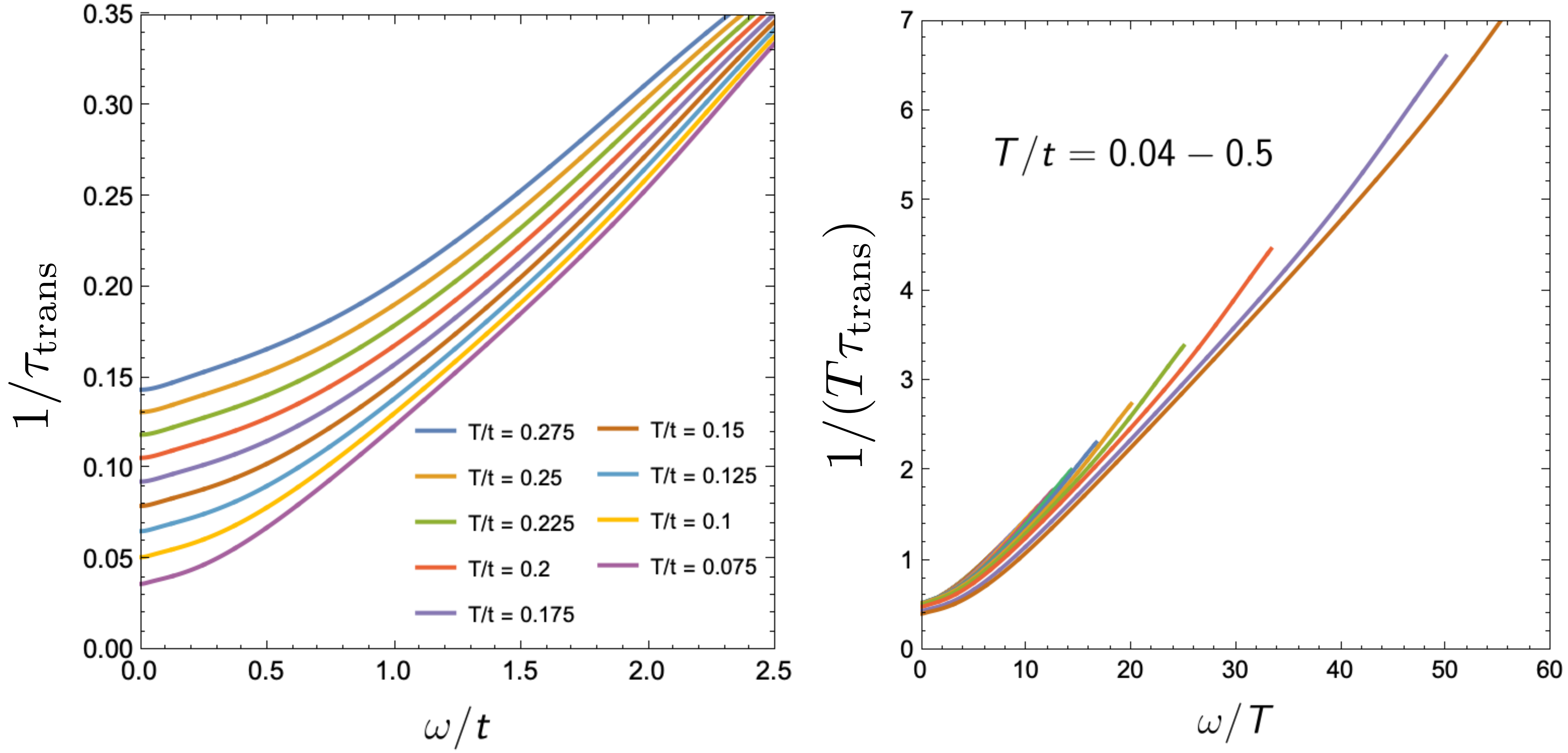
$p = 0.24$

$T_c = 19$ K



From optical conductivity data of Michon et al. (2023)

$$\frac{\hbar}{\tau} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T} \right)$$



2d-YSYK theory

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

1. Universal 2D-YSYK theory of strange metals

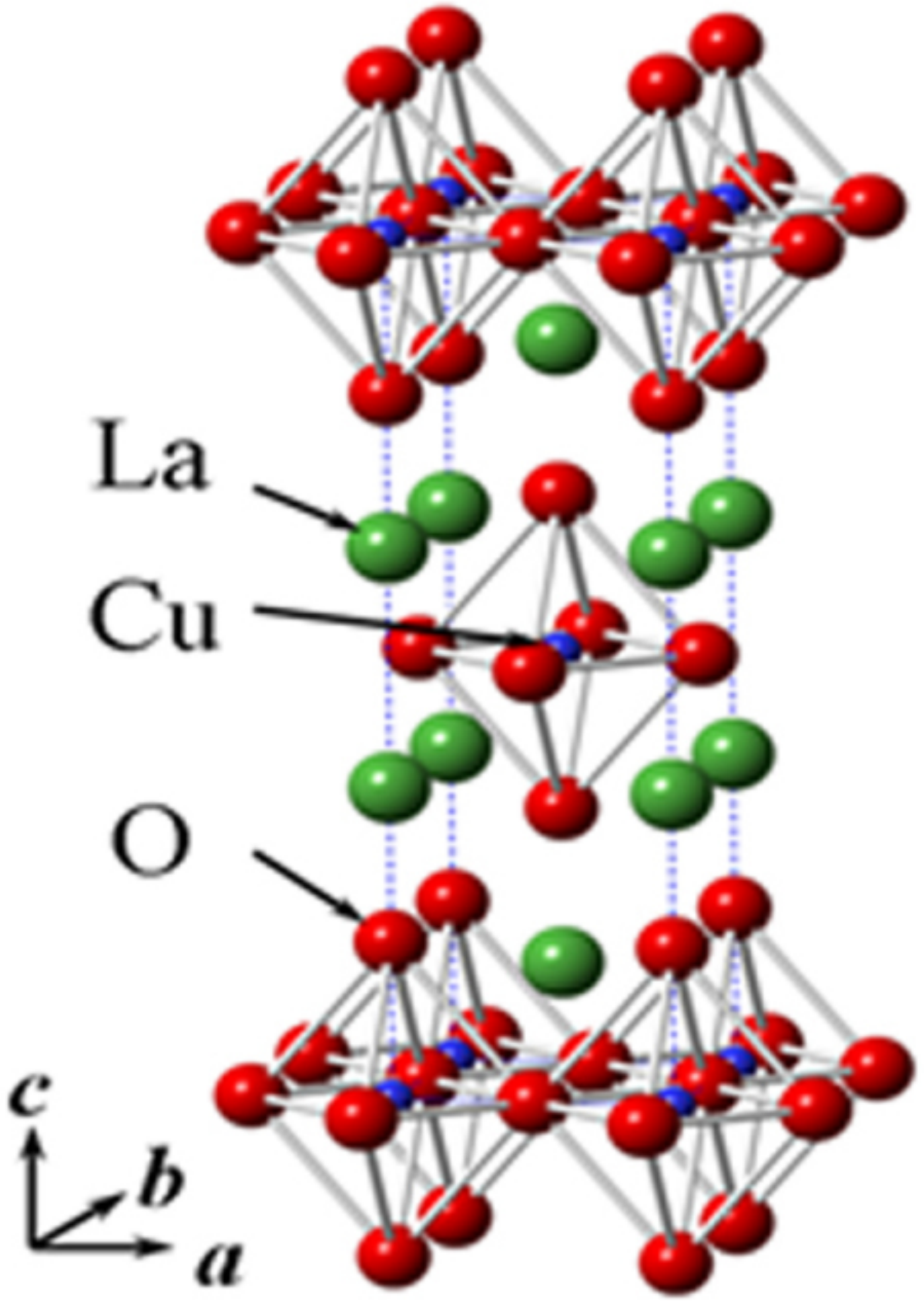
2. Theory of the "foot":
   quantum Griffiths SDW phase

3. Quantum-critical superconductivity

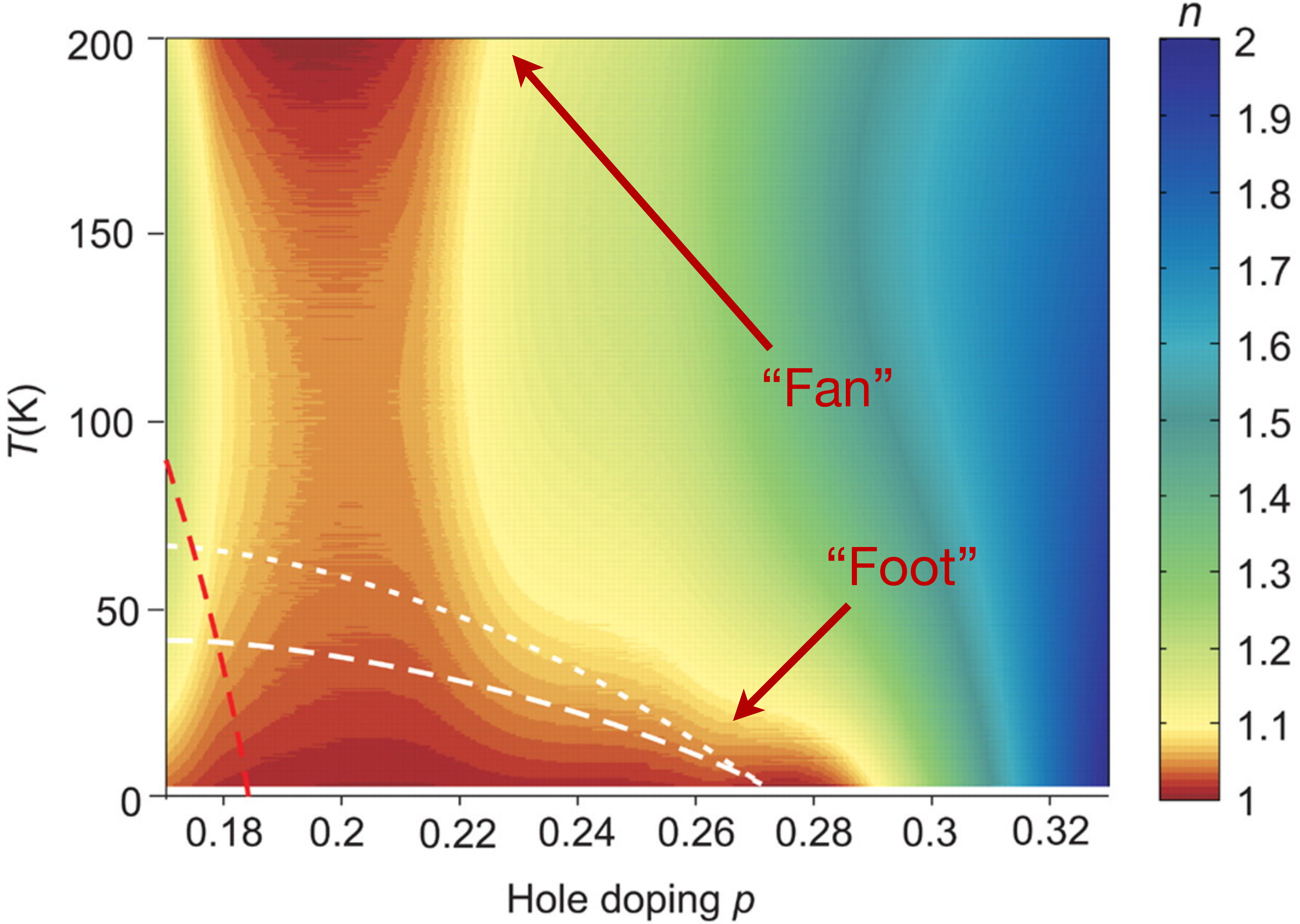
Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009



Resistivity $\rho \sim T^n$



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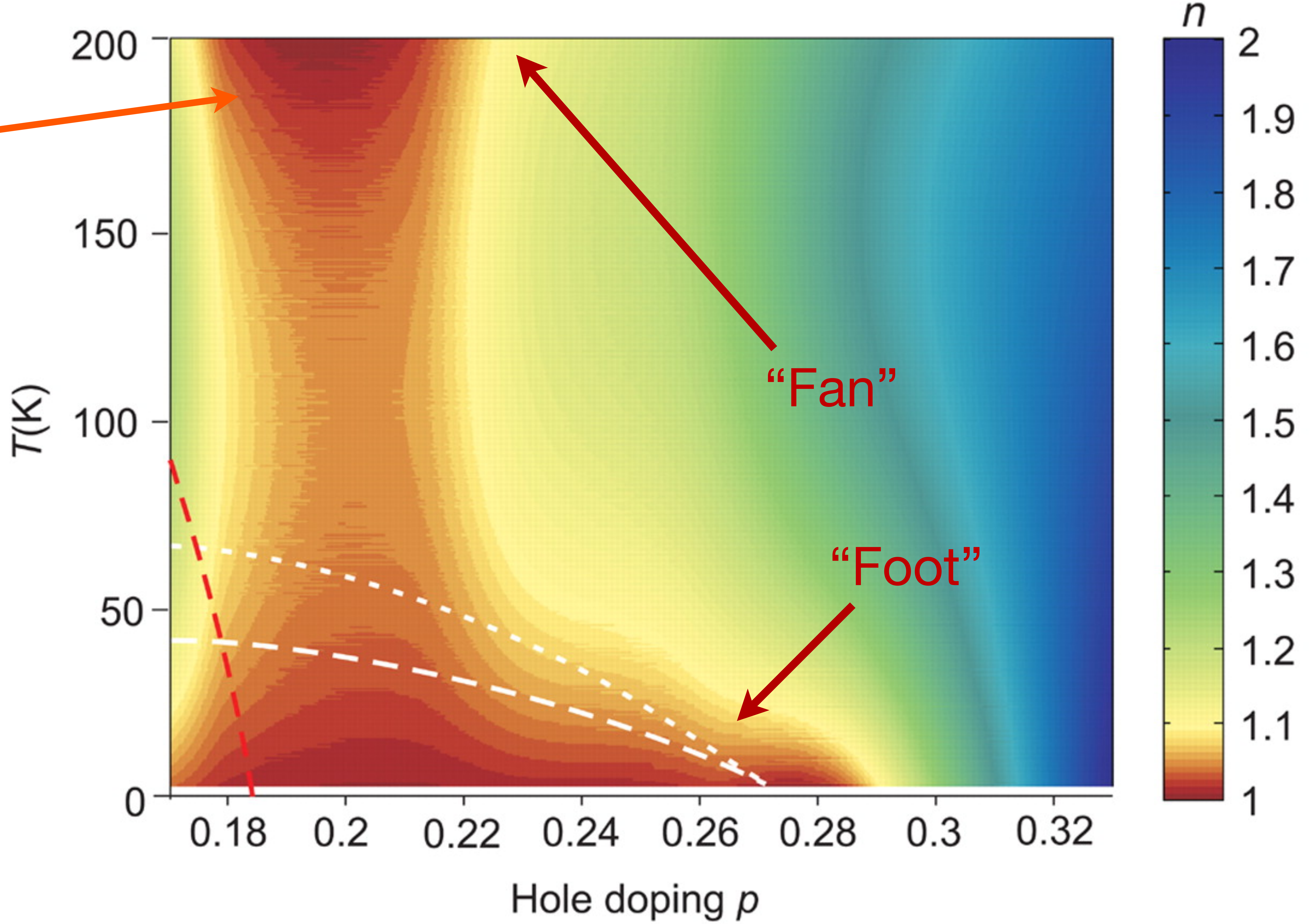
SCIENCE VOL 323 603 2009

Resistivity $\rho \sim T^n$

Extended fermions and bosons: 2D-YSYK theory of strange metal



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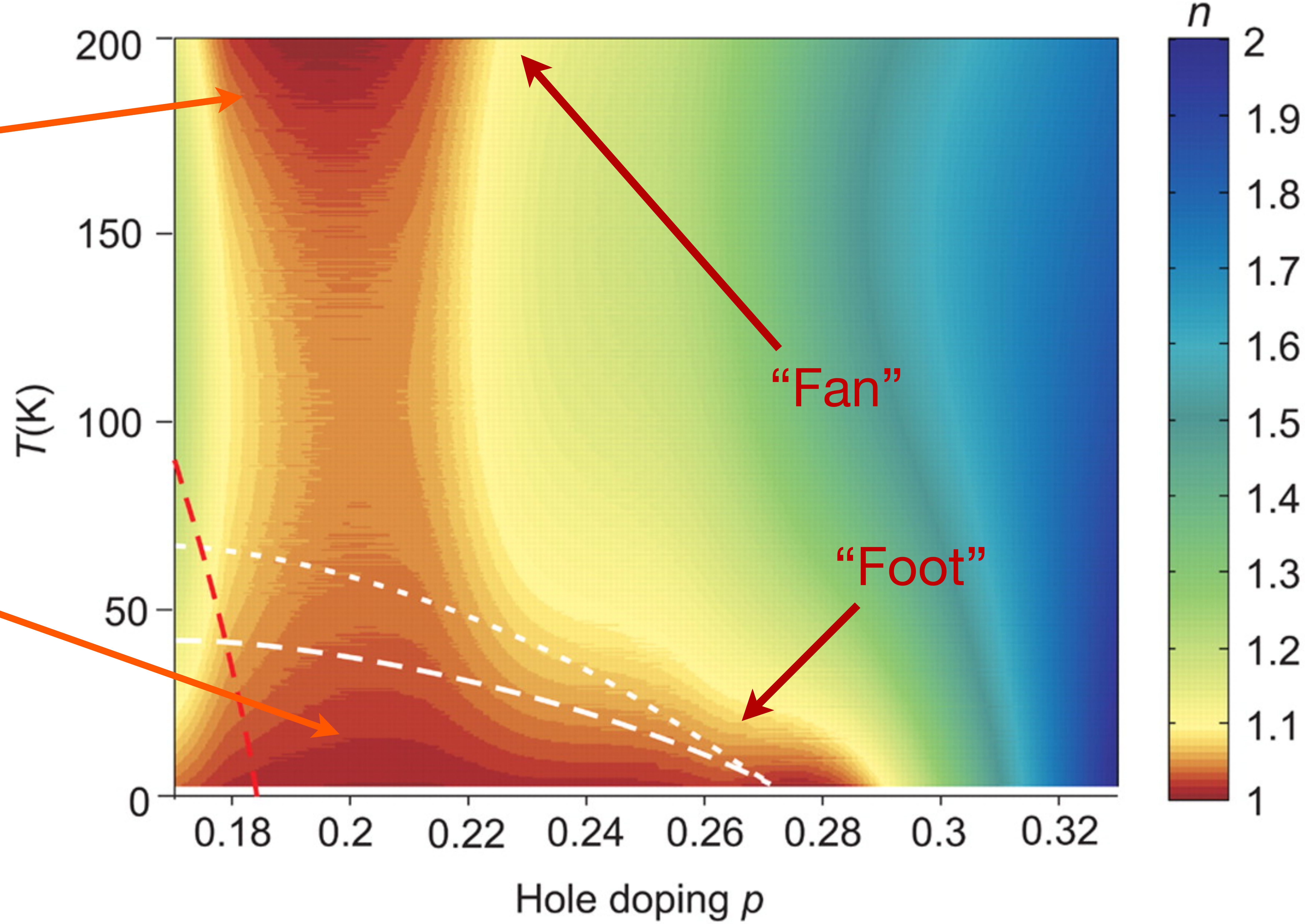
SCIENCE VOL 323 603 2009

Resistivity $\rho \sim T^n$

Extended fermions and bosons: 2D-YSYK theory of strange metal

Localized overdamped SDW bosons, but extended fermions: Griffths strange metal



Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_b = \int d\tau \left(- \sum_{\langle ij \rangle} J_{ij} \phi_{ia} \phi_{ja} + \sum_j \left[\frac{s_j}{2} \phi_{ja}^2 + \frac{u}{4M} \left(\phi_{ja}^2 \right)^2 \right] \right) + \frac{T\gamma}{2} \sum_{\omega_n} \sum_j |\omega_n| |\phi_{ja}(\omega_n)|^2$$

where $a = 1 \ldots M$ is a flavor index for an order parameter with $\mathrm{O}(M)$ symmetry.

Strong disorder RG identical to that for the RTFIM (D.S. Fisher)

$$\widetilde{J}_{ij} = J_{ij} + \frac{J_{i2} J_{2j}}{s_2}$$

$$\widetilde{s}_2 = 2 \frac{s_2 s_3}{J_{23}}$$

$$H_{\mathrm{RTFIM}} = - \sum_{\langle ij \rangle} J_{ij} Z_i Z_j - \sum_j s_j X_j$$

J. A. Hoyos, Chetan Kotabage, Thomas Vojta
Phys. Rev. Lett. **99**, 230601 (2007)

T. Vojta and J. Schmalian, PRB **72**, 045438 (2005)

PRL **99**, 230601 (2007) PHYSICAL REVIEW LETTERS

Effects of Dissipation on a Quantum Critical Point with Disorder

José A. Hoyos, Chetan Kotabage, and Thomas Vojta

Department of Physics, University of Missouri-Rolla, Rolla, Missouri 65409, USA

(Received 19 May 2007; published 4 December 2007)

We study the effects of dissipation on a disordered quantum phase transition with $O(N)$ order-parameter symmetry by applying a strong-disorder renormalization group to the Landau-Ginzburg-Wilson field theory of the problem. We find that Ohmic dissipation results in a nonperturbative infinite-randomness critical point

- Each rare region is described by a one-dimensional classical O(M) model with a long-range $1/\tau^2$ interaction.
- For $M \geq 2$, the classical model has an exponentially long correlation time at weak coupling (low ‘temperature’) - Kosterlitz, 1976.
- This is similar to the classical Ising chain with short-range interactions.

Bosonic eigenmodes in random mass Hertz theory

Integrate out the fermions (assuming fermionic eigenmodes remain extended), and considering the Landau-damped Hertz theory for the boson alone, in the presence of a random mass.

$$\mathcal{S}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \Big(\frac{s + \delta s_j}{2} \phi_{ja}^2 + \frac{u}{4M} \left(\phi_{ja}^2\right)^2 \Big) \Big]$$

$$\mathcal{S}_{\phi d} = \frac{T}{2} \sum_{\Omega} \sum_j \left(\gamma |\Omega| + \Omega^2 / c^2 \right) |\phi_{ja}(i\Omega)|^2 \,,$$

where $a = 1 \ldots M$ is a flavor index for an order parameter with $\mathrm{O}(M)$ symmetry. Analyze in a self-consistent quadratic theory, treating disorder numerically exactly

$$\overline{\mathcal{S}}_\phi = \int d\tau \Big[\frac{J}{2} \sum_{\langle ij \rangle} (\phi_{ia} - \phi_{ja})^2 + \sum_j \frac{\widetilde{\delta s_j}}{2} \phi_{ja}^2 \Big]$$

Similar analysis in $d = 1$ works very well
A. Del Maestro, B. Rosenow, M. Müller and S. Sachdev,
Phys. Rev. Lett. **101**, 035701 (2008).

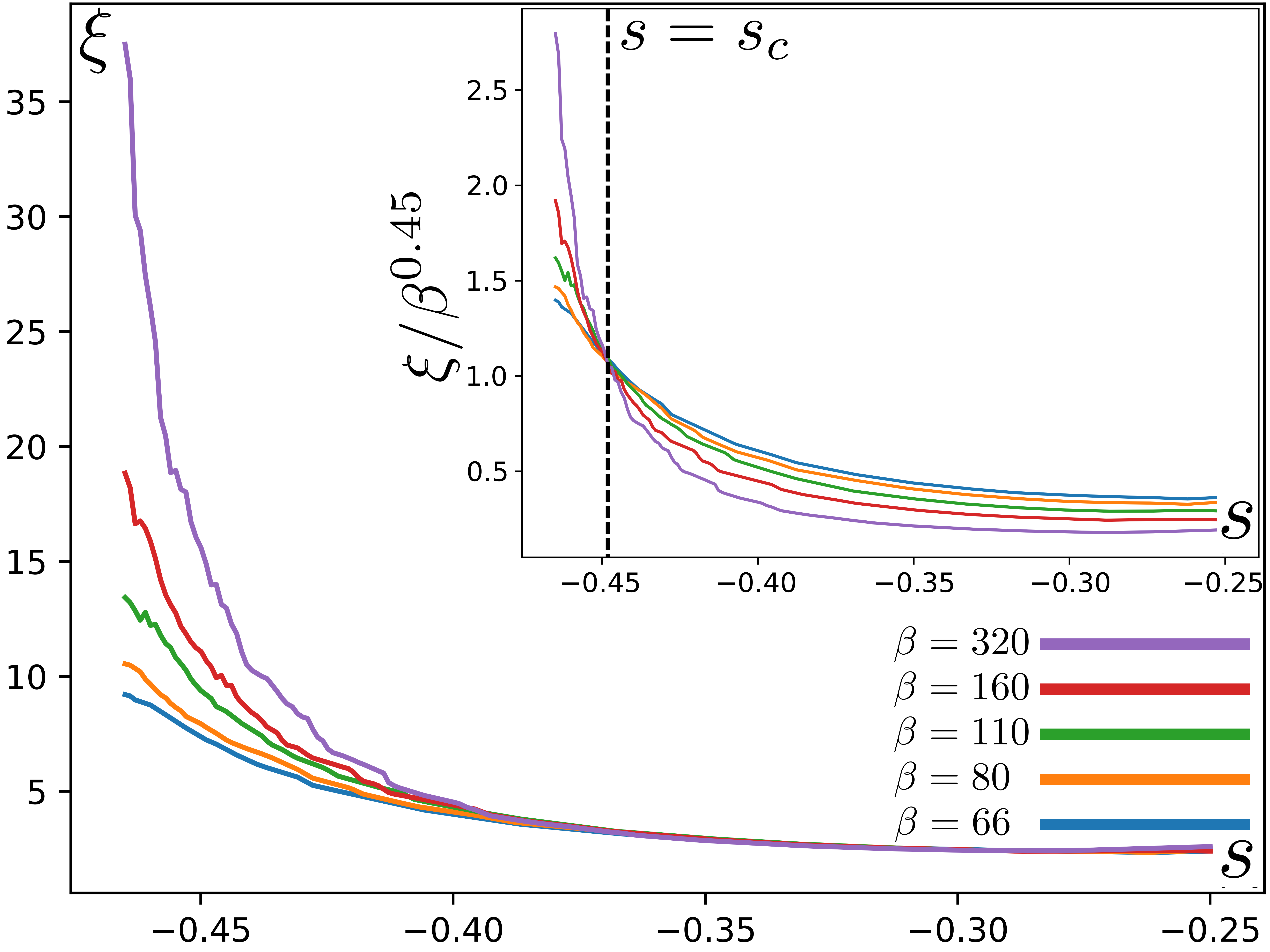
$$\widetilde{\delta s_j} = s + \delta s_j + \frac{u}{M} \sum_a \langle \phi_{ja}^2 \rangle_{\overline{\mathcal{S}}_\phi + \mathcal{S}_{\phi d}} = s + \delta s_j + uT \sum_{\Omega} \sum_{\alpha} \frac{\psi_{\alpha i} \psi_{\alpha j}}{\gamma |\Omega| + \Omega^2/c^2 + e_\alpha}$$

where e_α and $\psi_{\alpha j}$ are eigenvalues and eigenfunctions of the ϕ quadratic form in $\overline{\mathcal{S}}_\phi$, labeled by the index $\alpha = 1 \ldots L^2$ for a $L \times L$ sample.

Aavishkar A. Patel, Peter Lunts, S.S., PNAS **121**, e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

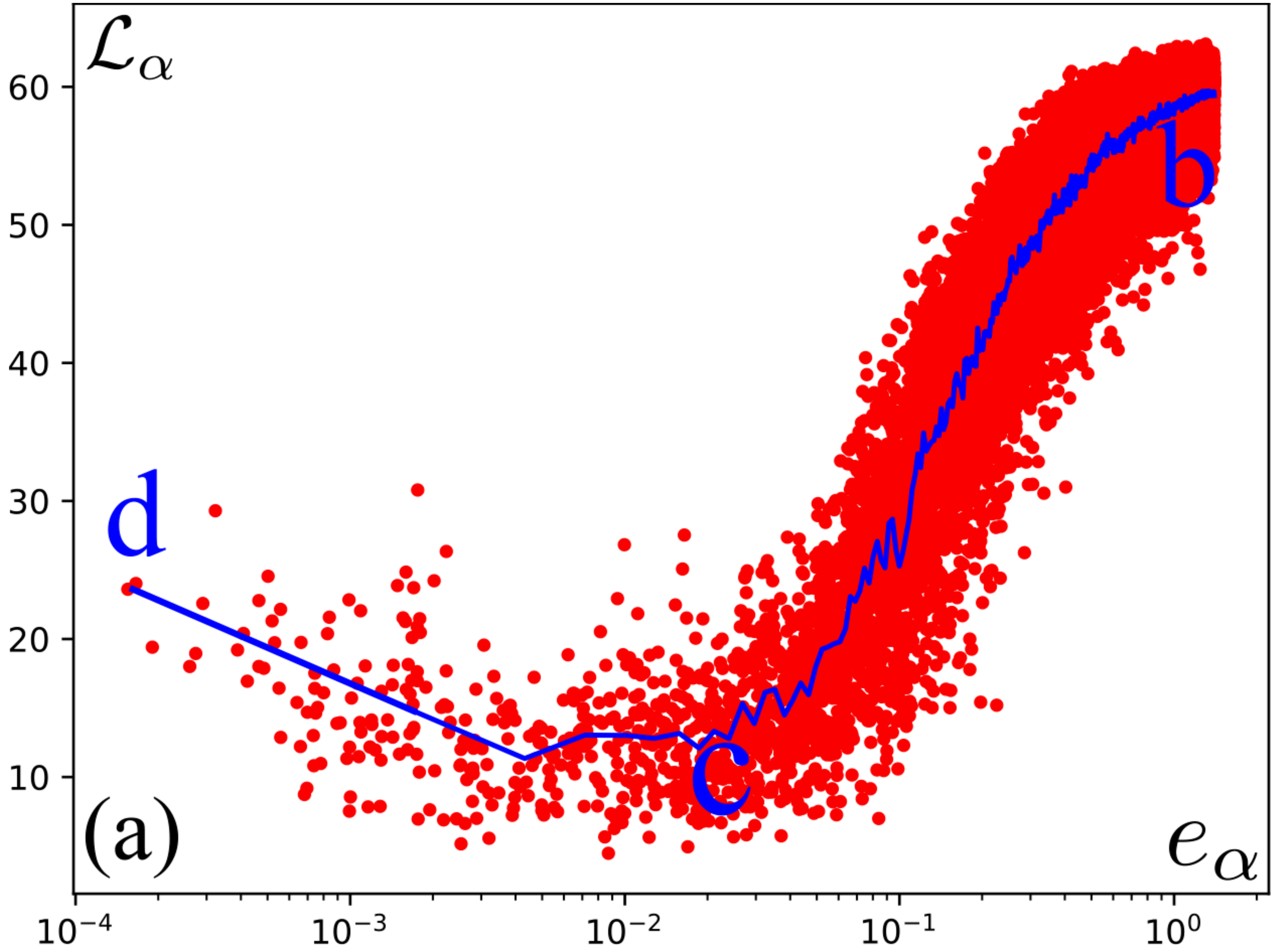
ϕ correlation length ξ



Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$



Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

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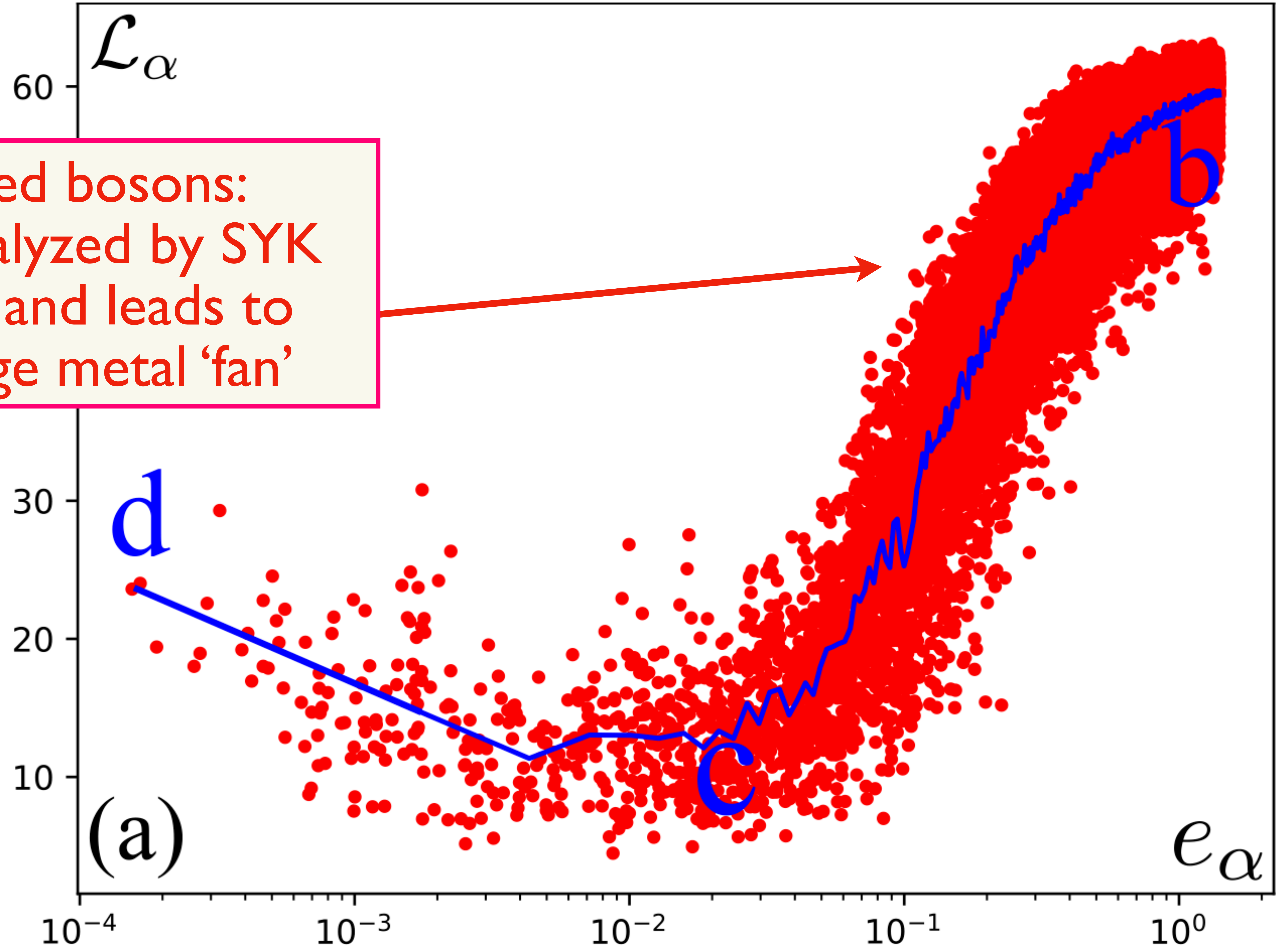
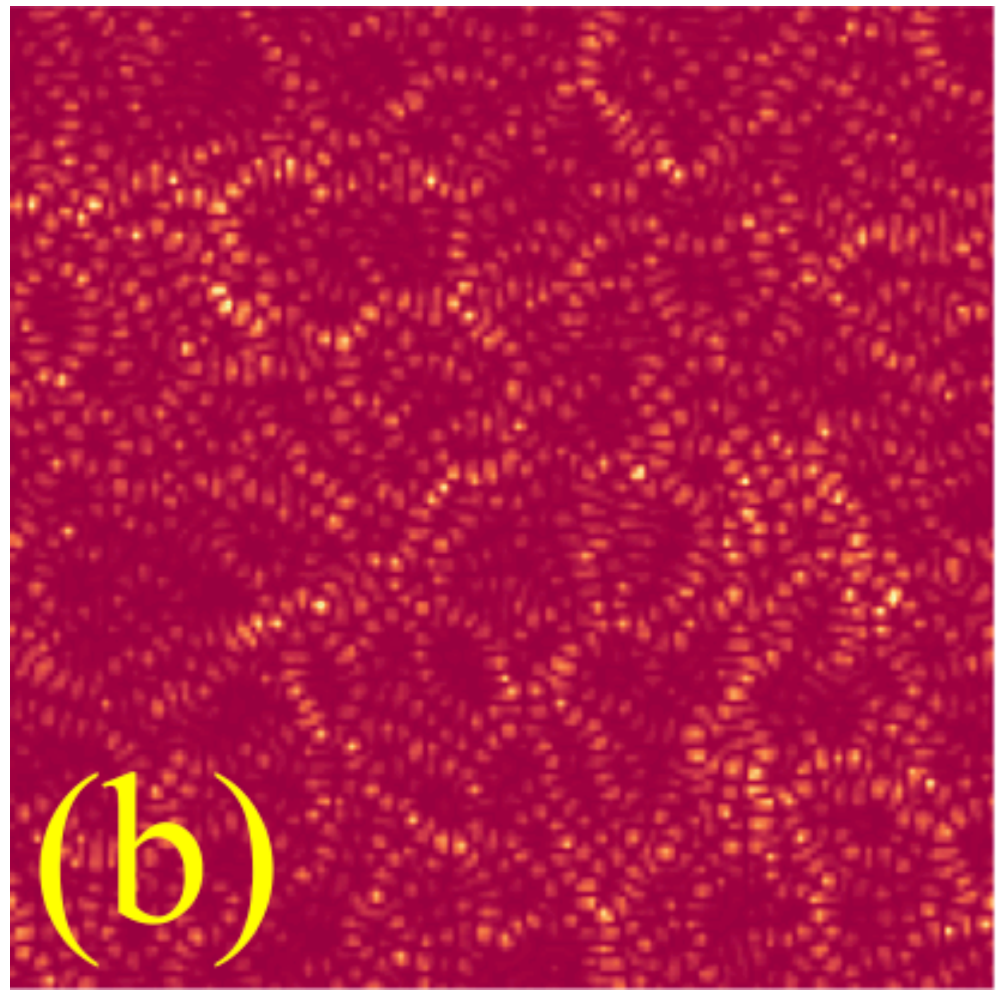


Extended bosons:
Can be analyzed by SYK methods, and leads to the strange metal 'fan'



Aavishkar A. Patel, Peter Lunts, S.S., *PNAS* **121**, e2402052121 (2024)

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Bosonic eigenmodes in random mass Hertz theory

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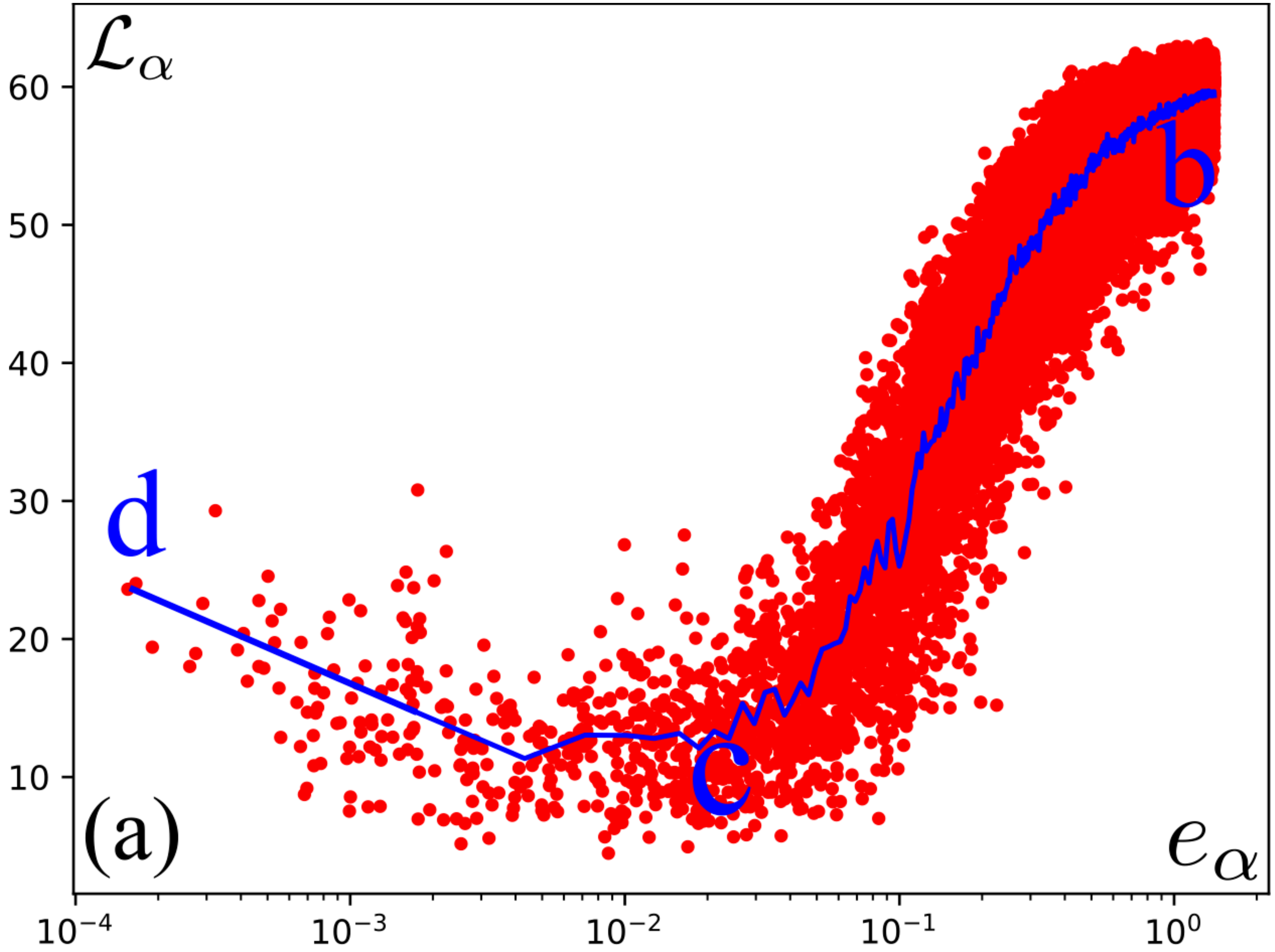

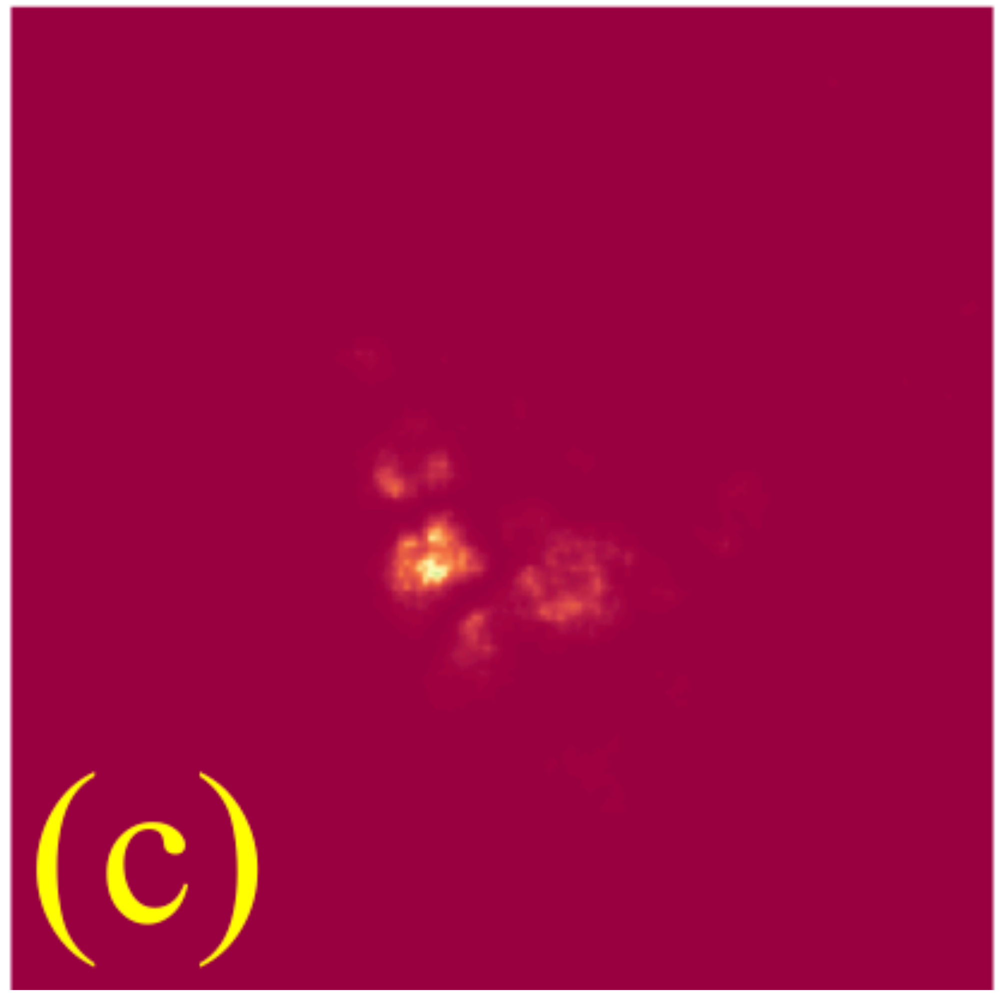

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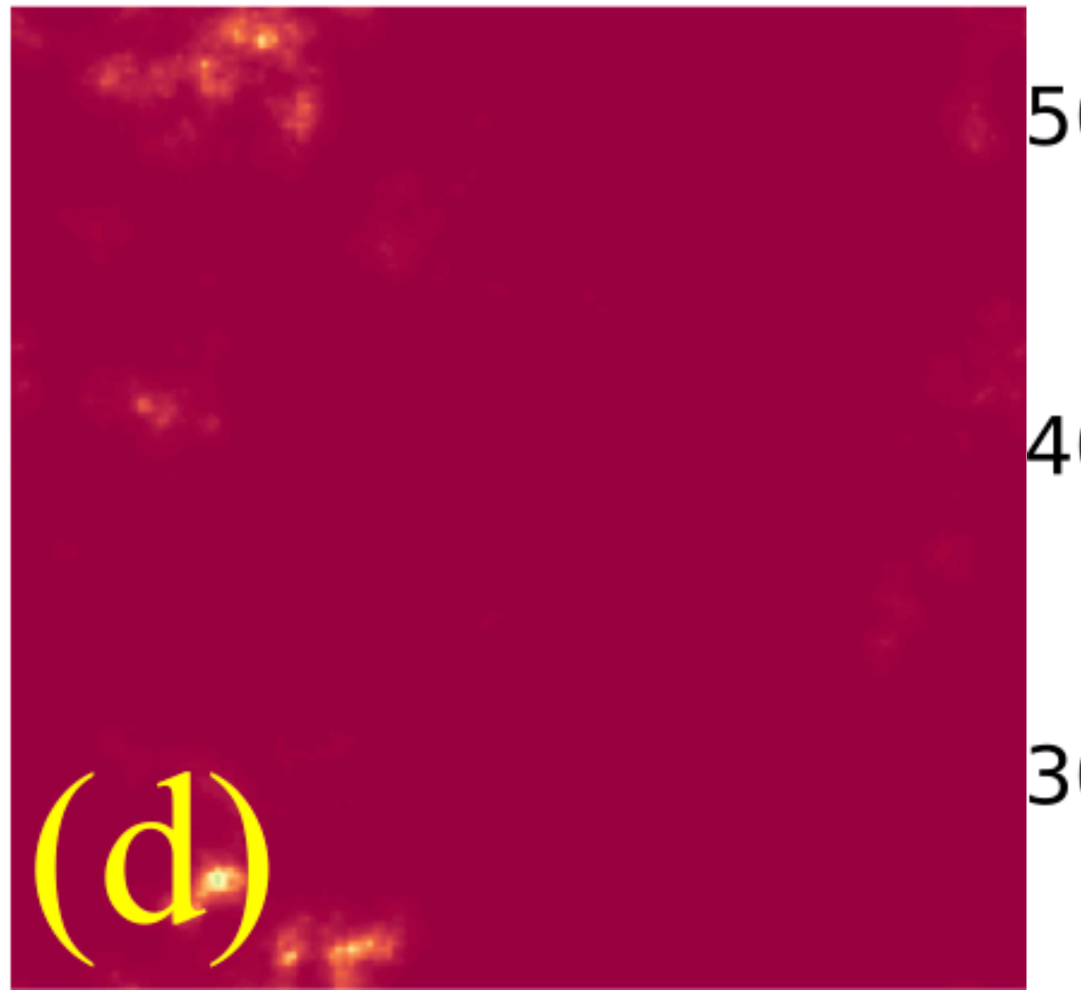
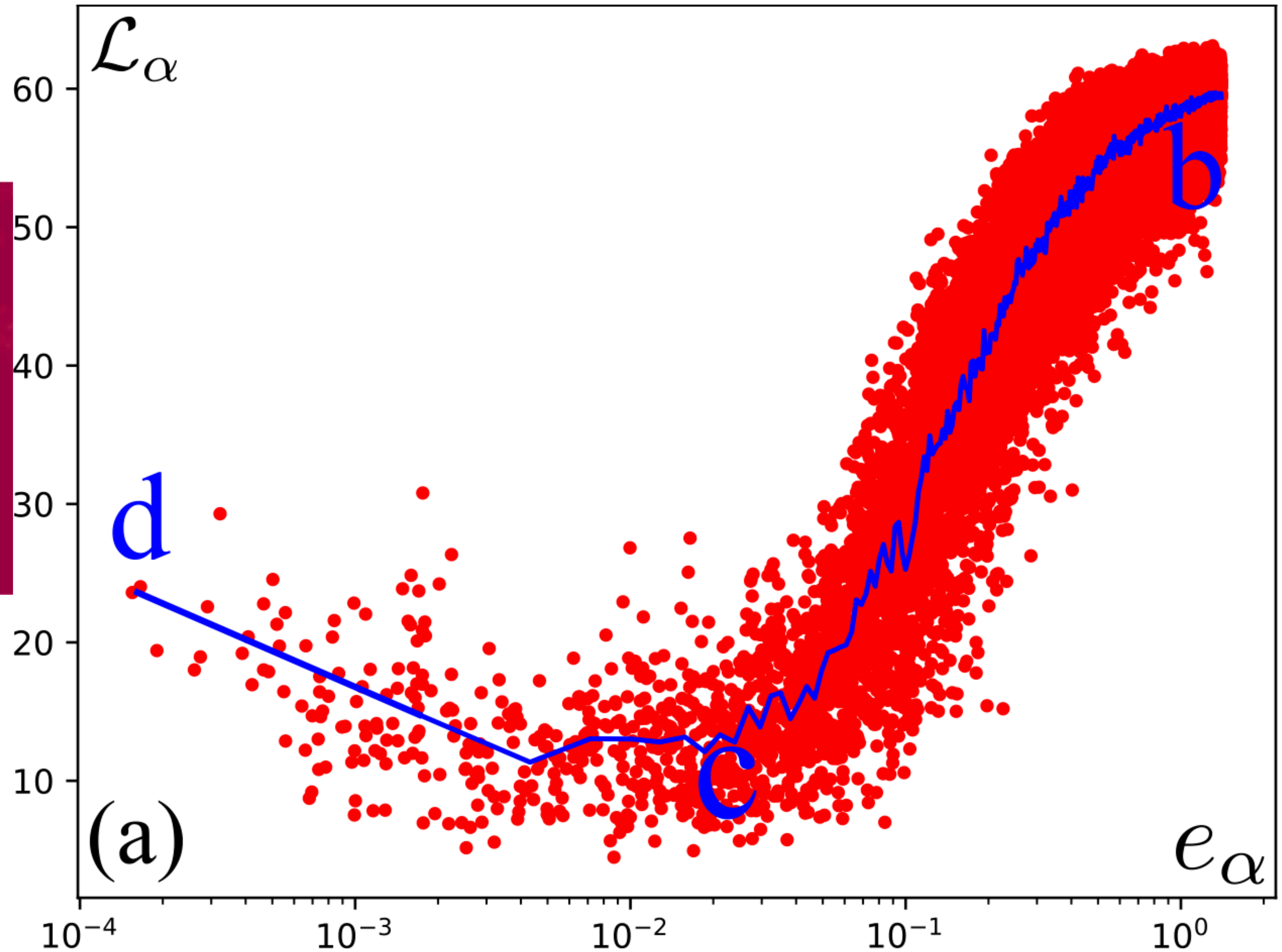
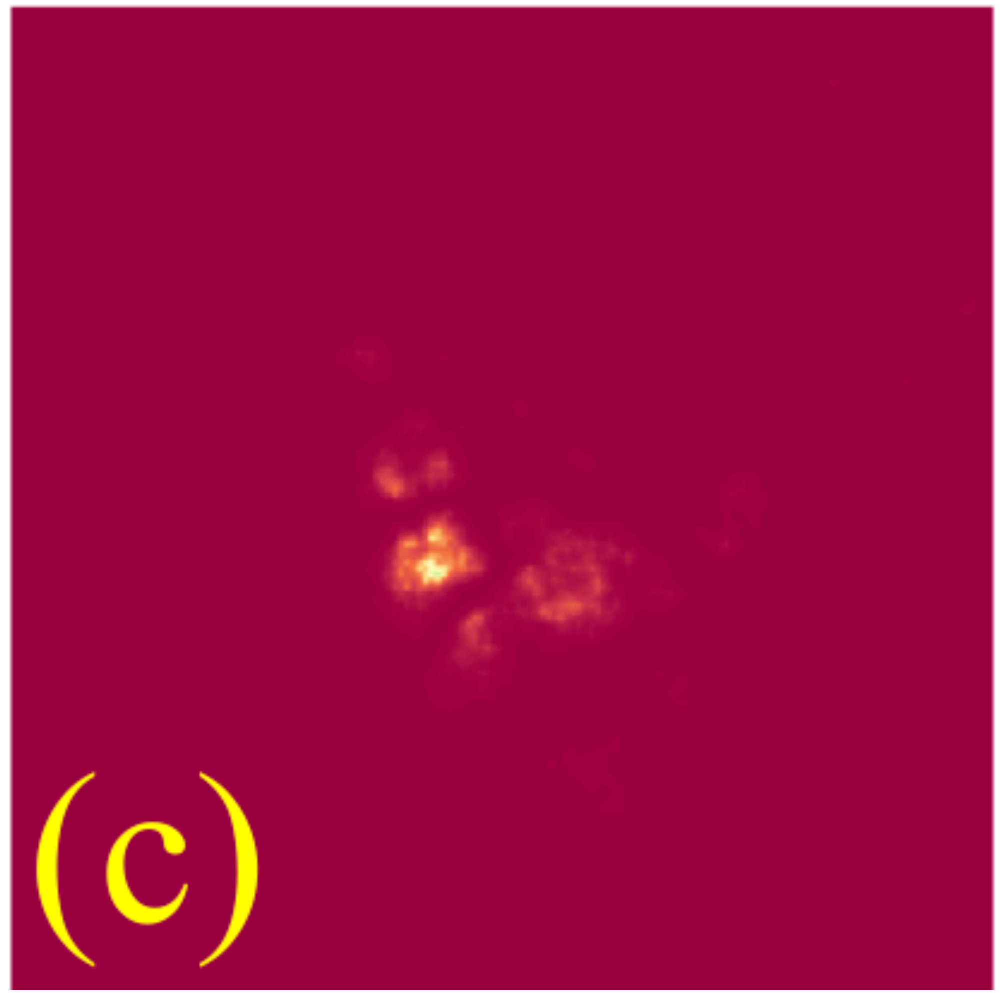


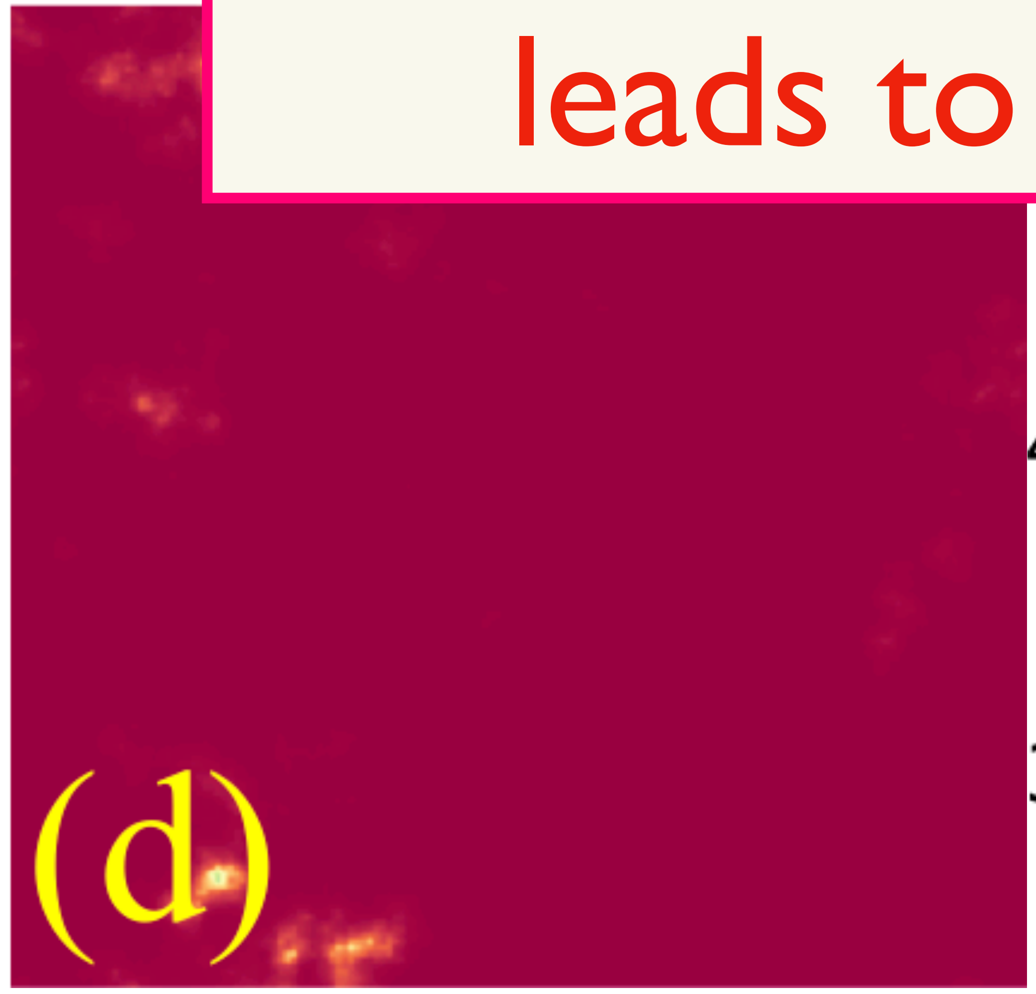


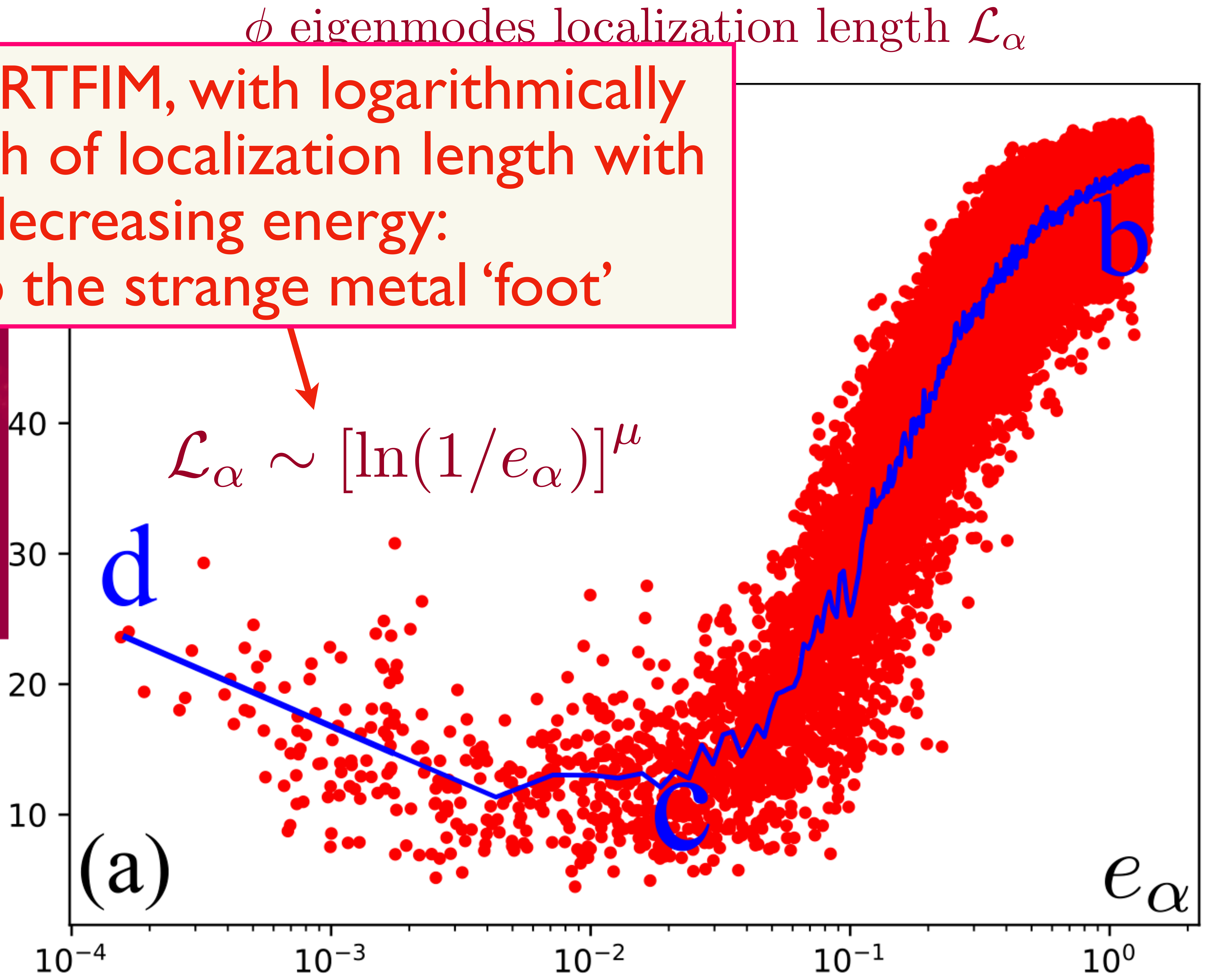


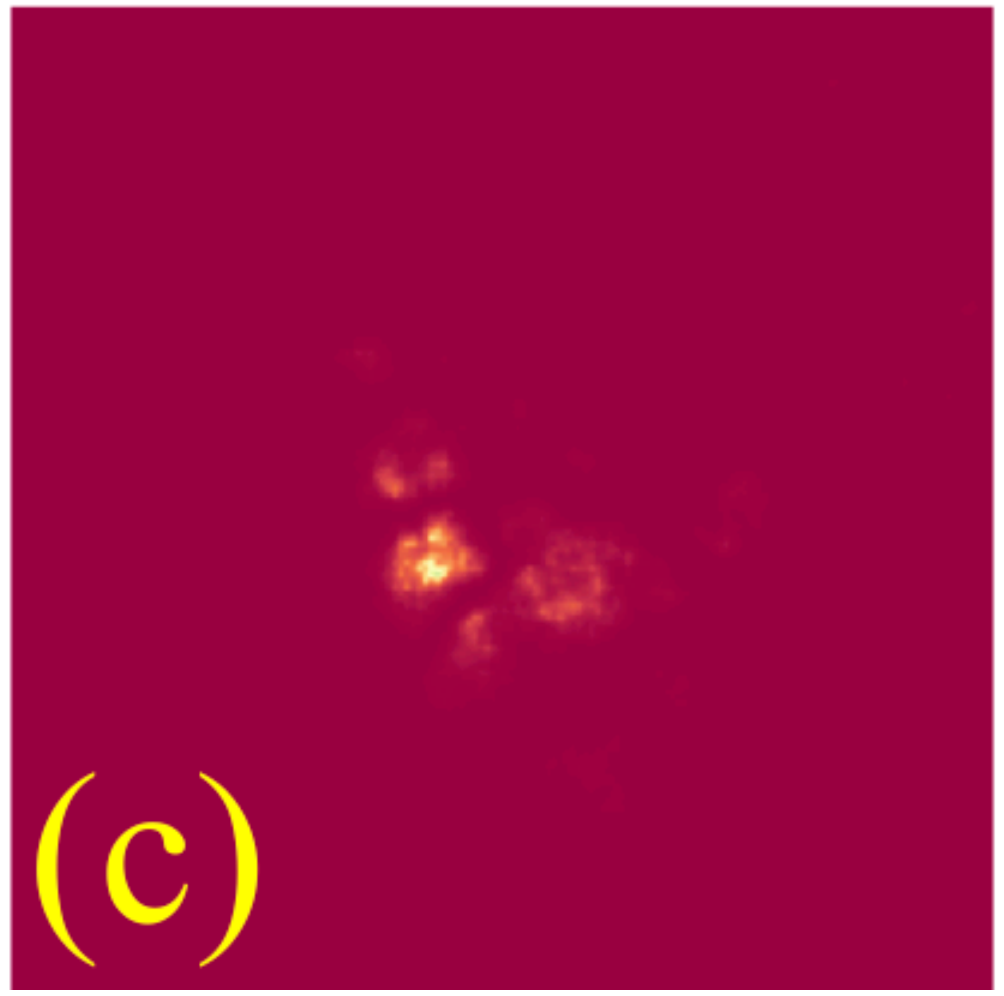
Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Bosonic eigenmodes in random mass Hertz theory

Physics of RTFIM, with logarithmically slow growth of localization length with decreasing energy: leads to the strange metal 'foot'







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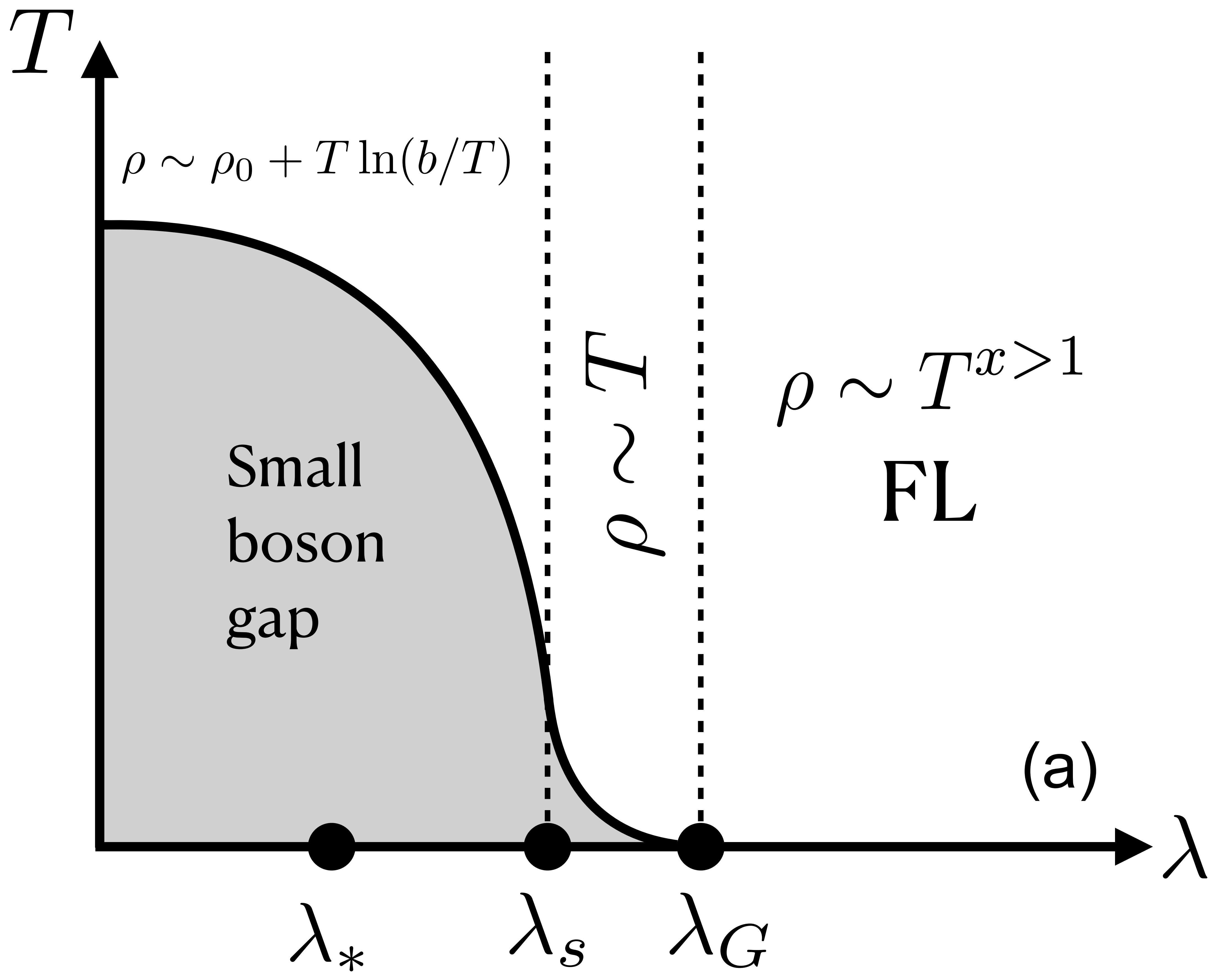
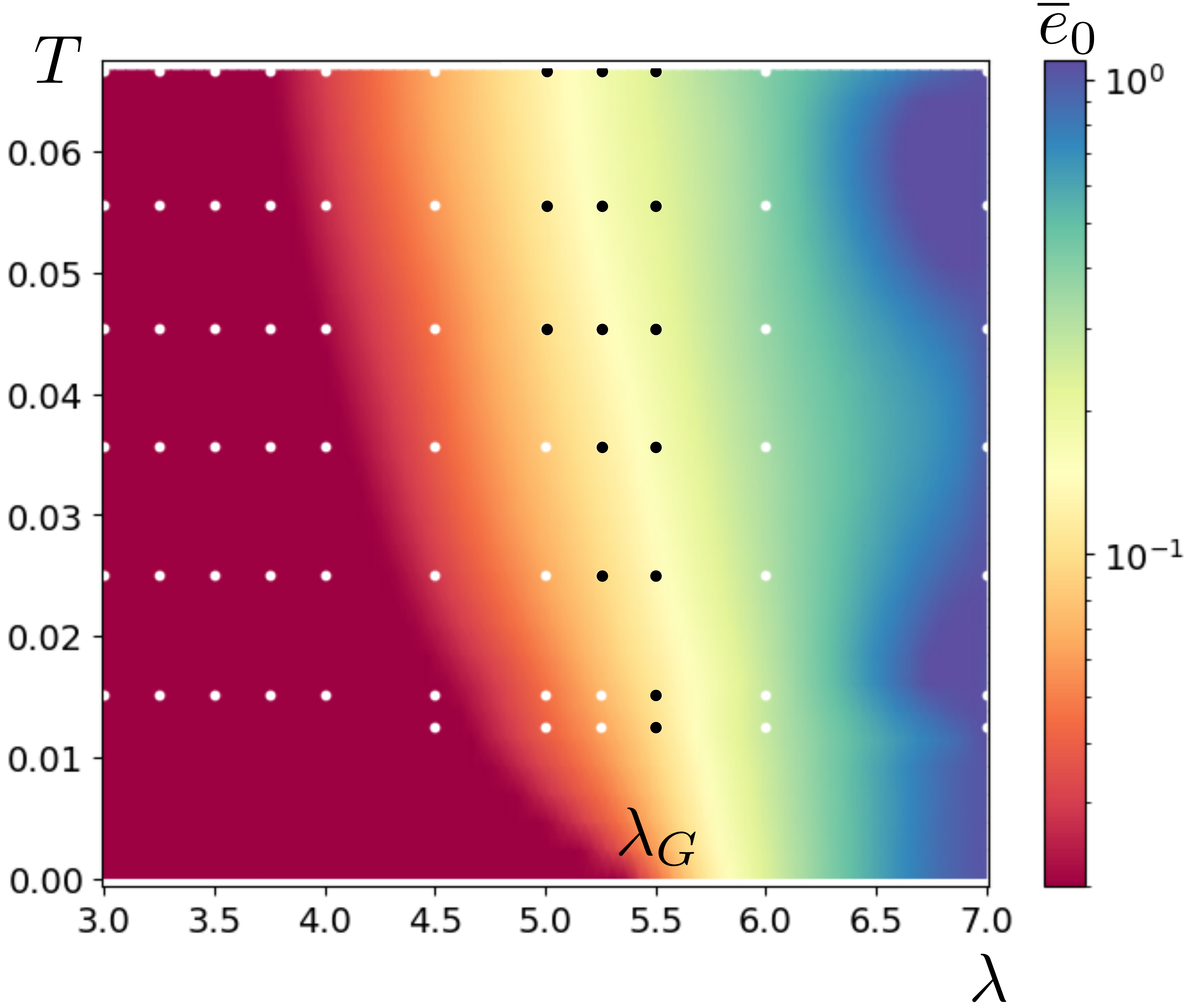
PNAS **121**, e2402052121 (2024)

QMC of SDW bosons and electrons

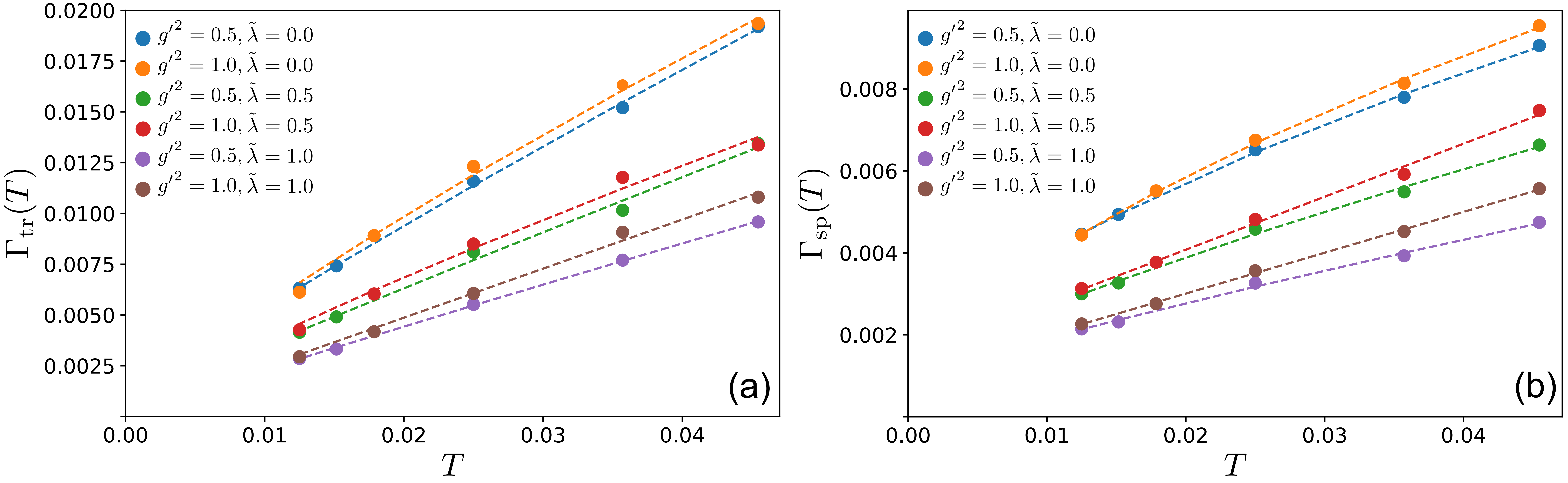




Smallest eigenvalue $\overline{e}_0$ of susceptibility $\chi_{ij}(\omega_n = 0)$

Strange Metals and Planckian Transport in a Gapless Phase from Spatially Random Interactions,
Aavishkar A. Patel, Peter Lunts, Michael S. Albergo, PRX **15**, 031064 (2025)

QMC of SDW bosons and electrons



(a) The dc transport scattering rate Γ_{tr} in the strange-metal phase for different values of g'. For the same value of $\tilde{\lambda}$, the scattering rates for different values of g' are nearly identical.
(b) The equivalent analysis for the single-particle scattering rate Γ_{sp}.

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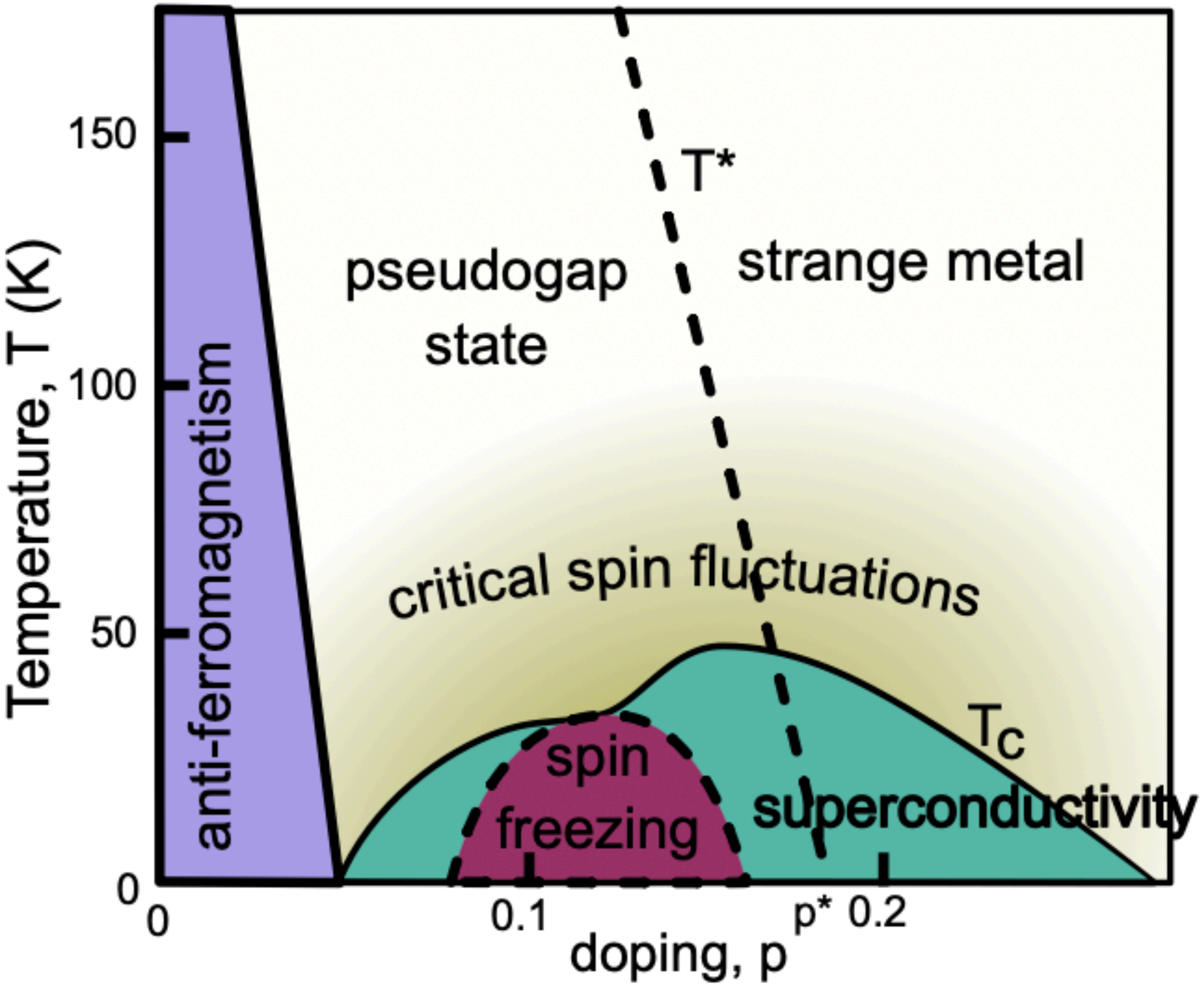
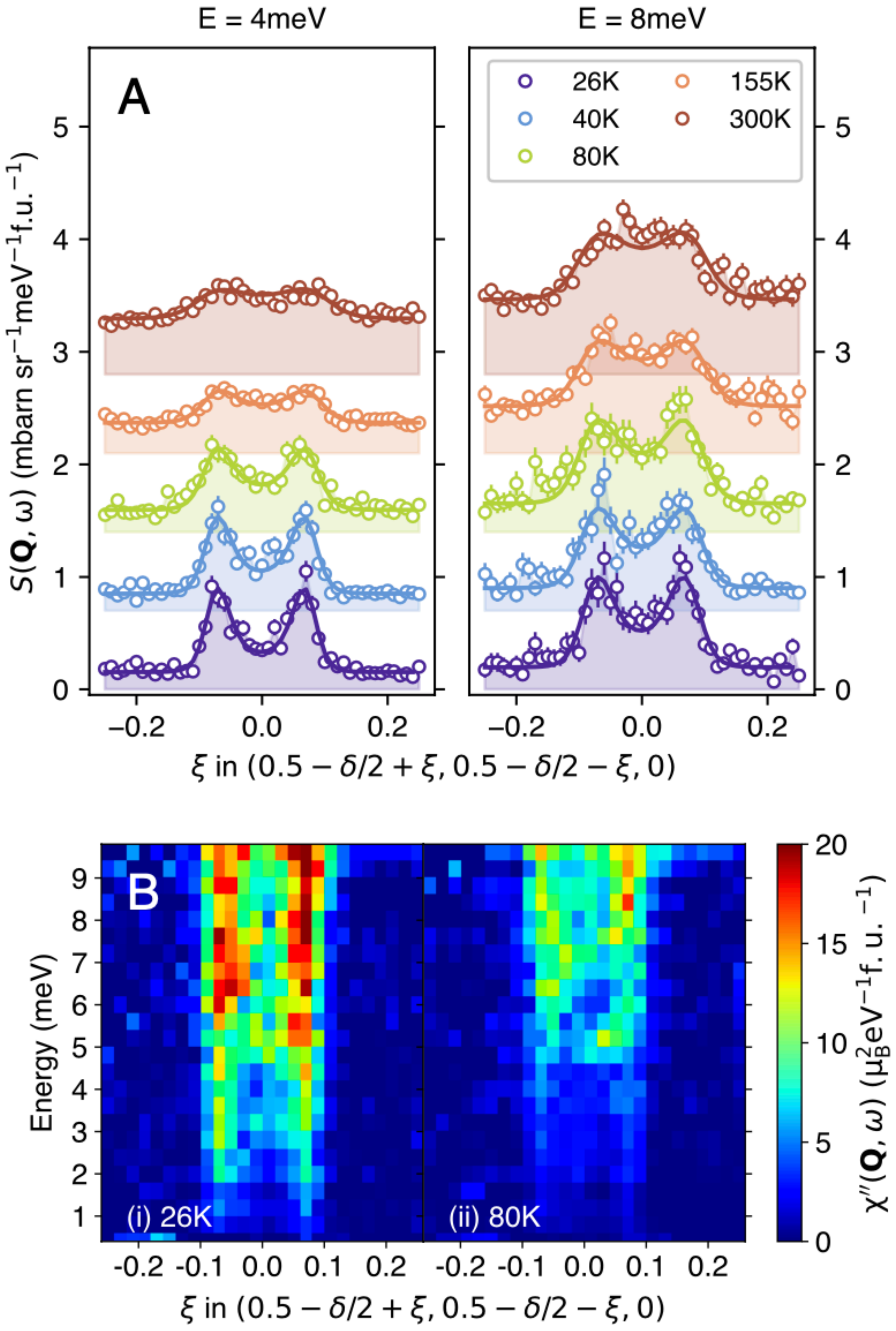
Critical spin fluctuations across the superconducting dome in $La_{2-x}Sr_xCuO_4$,

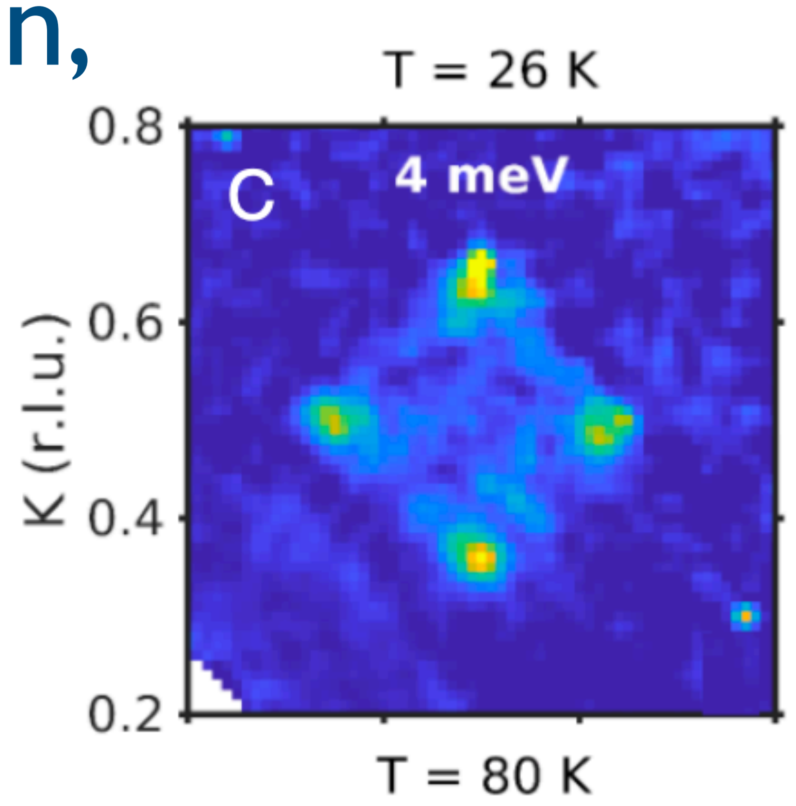
x=0.22

J. Radaelli, A.A. Patel, O.J. Lipscombe, Mengze Zhu, J.R. Stewart, S. S. and S.M. Hayden,

Nature Comm.
17, 4564 (2026)







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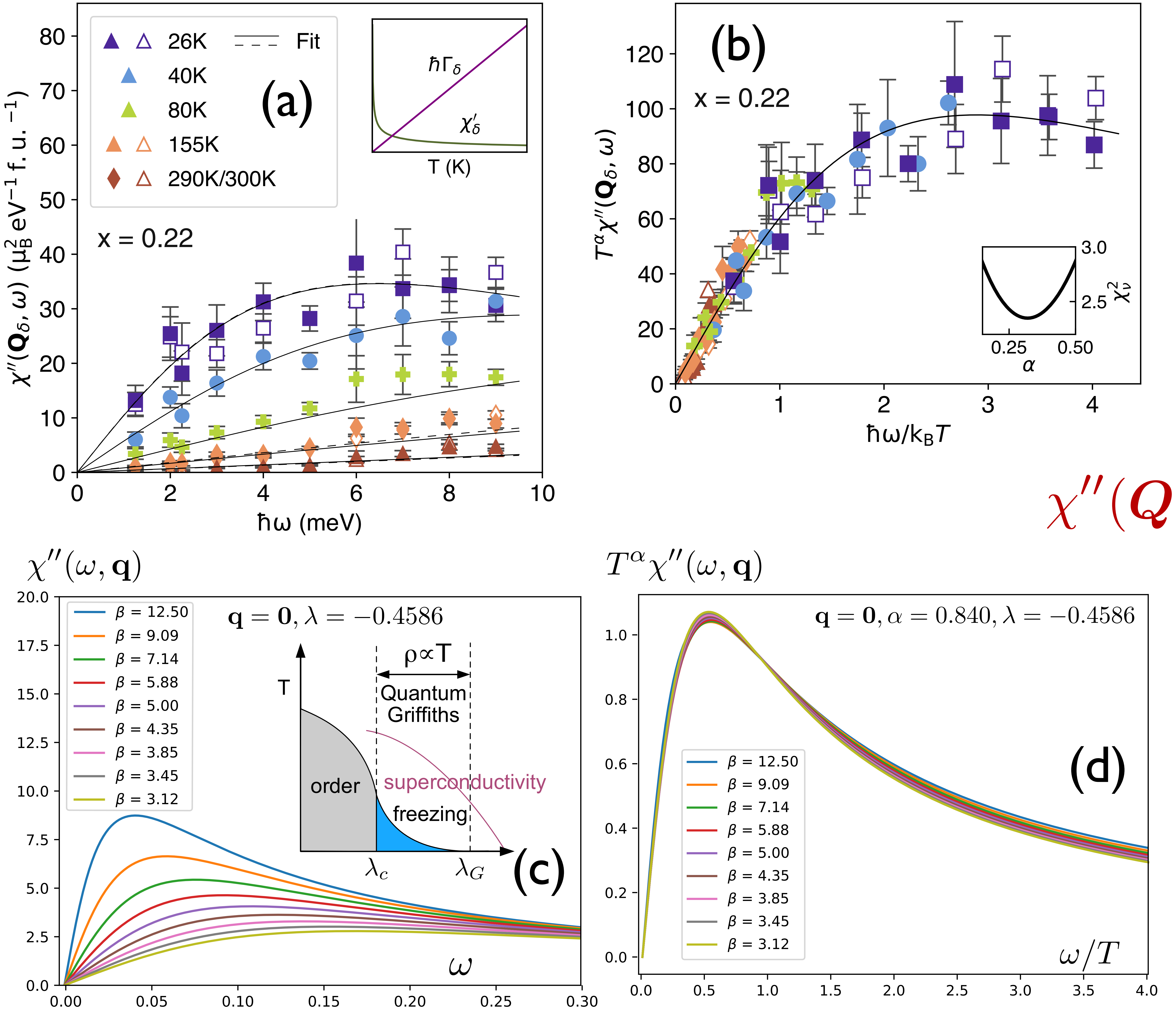
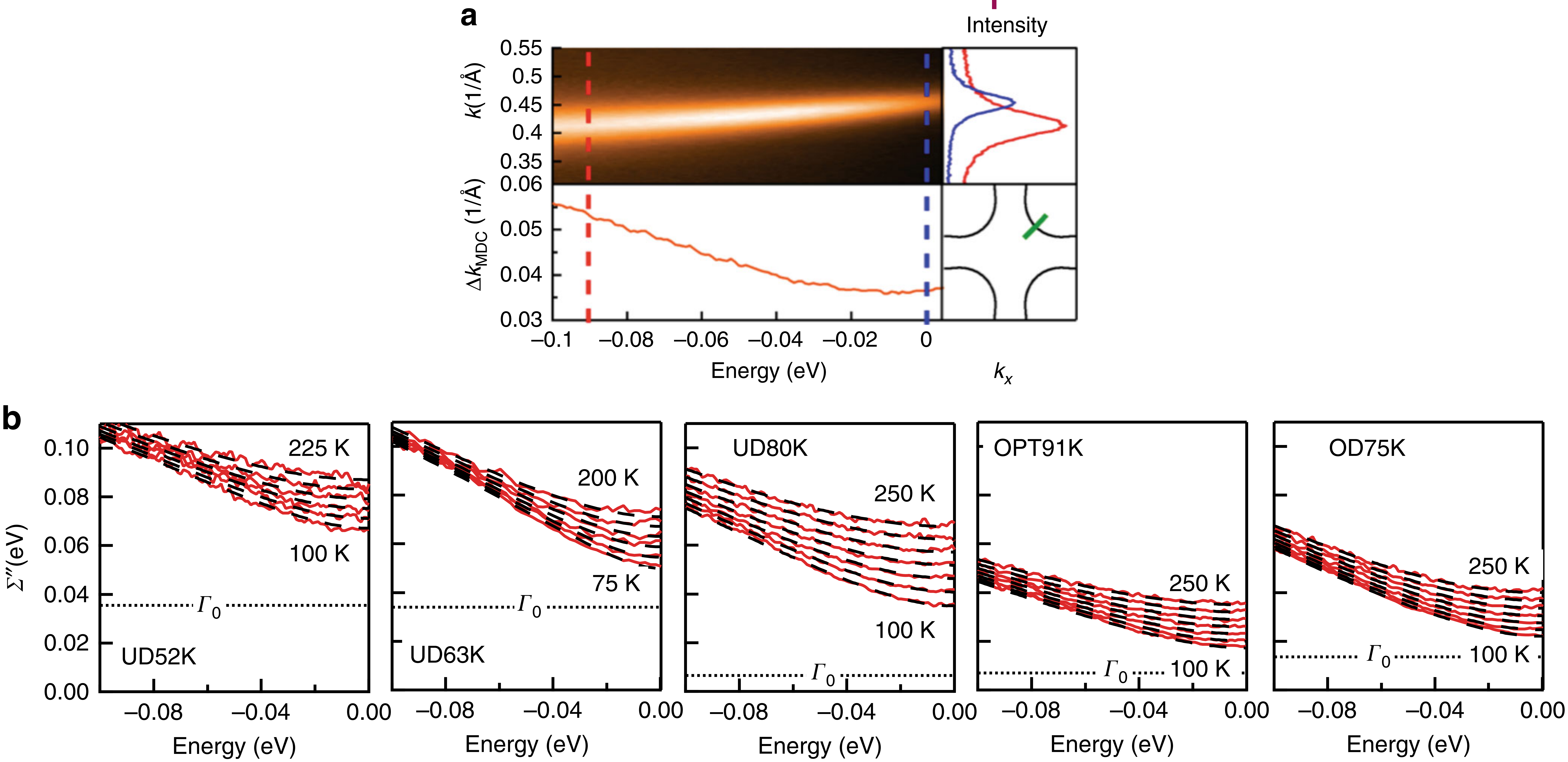


Neutron scattering observations

$$\chi''(\boldsymbol{Q}_\delta, \omega) \sim T^{-\alpha} \Phi_\chi \left(\frac{\hbar\omega}{k_B T} \right)$$

Large-M
random "mass"
SDW Hertz theory

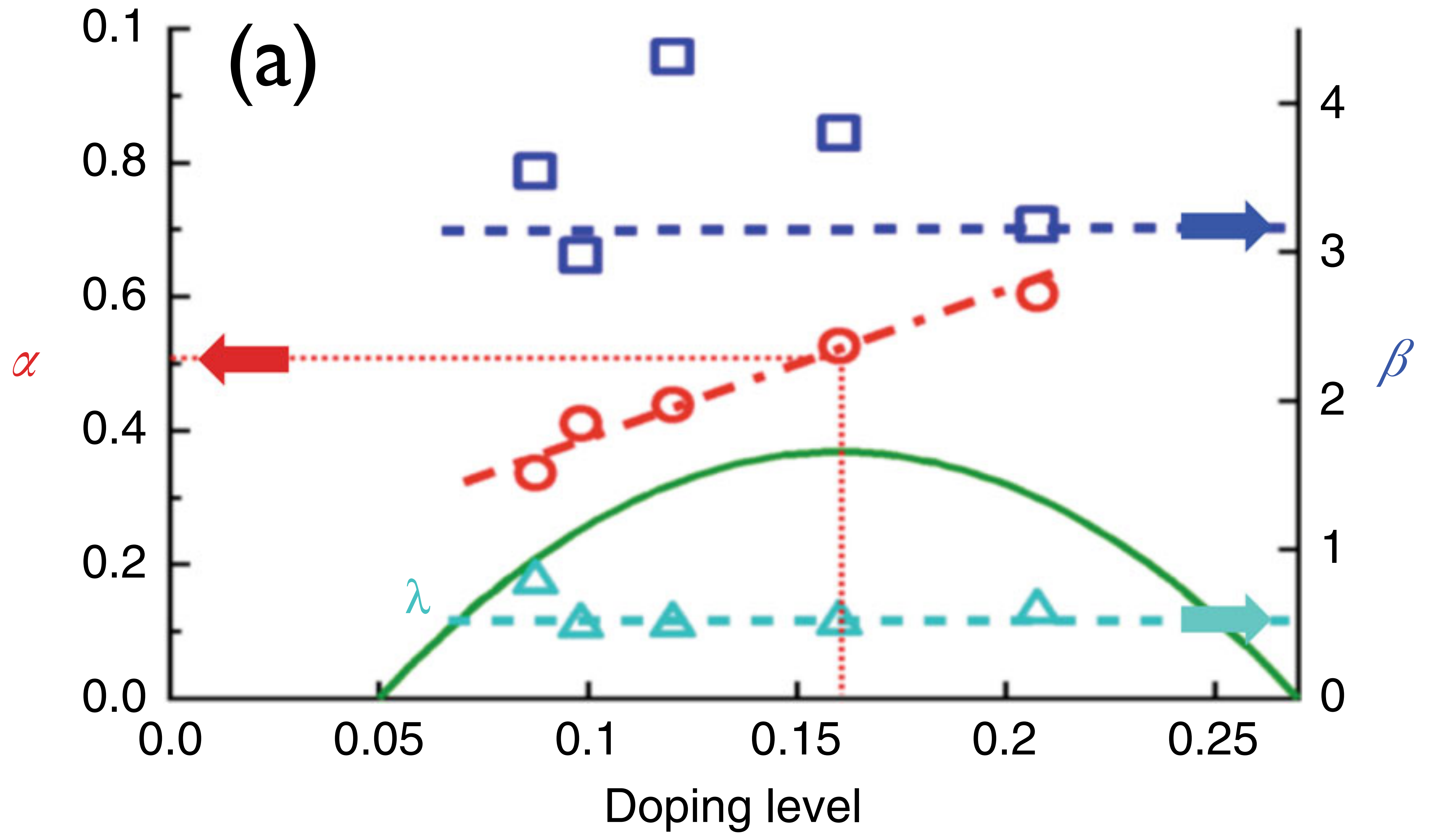
Photoemission on cuprates

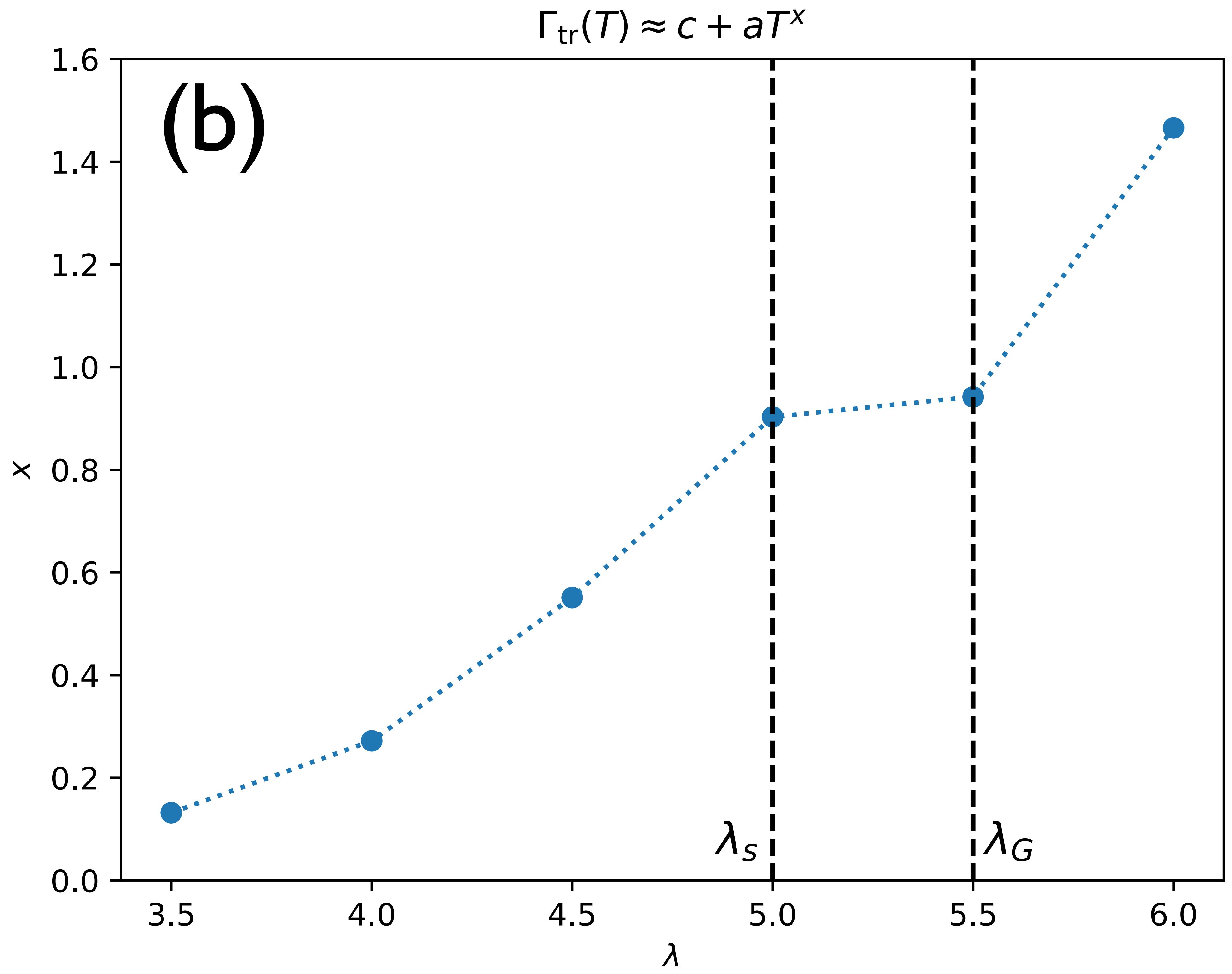


A unified form of low-energy nodal electronic interactions in hole-doped cuprate superconductors
T.J. Reber, X. Zhou, N.C. Plumb, S. Parham, J.A. Waugh, Y. Cao, Z. Sun, H. Li,
Q. Wang, J.S. Wen, Z.J. Xu, G. Gu, Y. Yoshida, H. Eisaki, G.B. Arnold, D.S. Dessau, Nature Comm. **10** (2019) 5737.

Photoemission on cuprates

$$\Sigma''_{\mathrm{PLL}}(\omega) = \Gamma_0 + \lambda \frac{\left[(\hbar\omega)^2 + (\beta k_B T)^2\right]^{\alpha}}{(\hbar\omega_N)^{2\alpha-1}}$$





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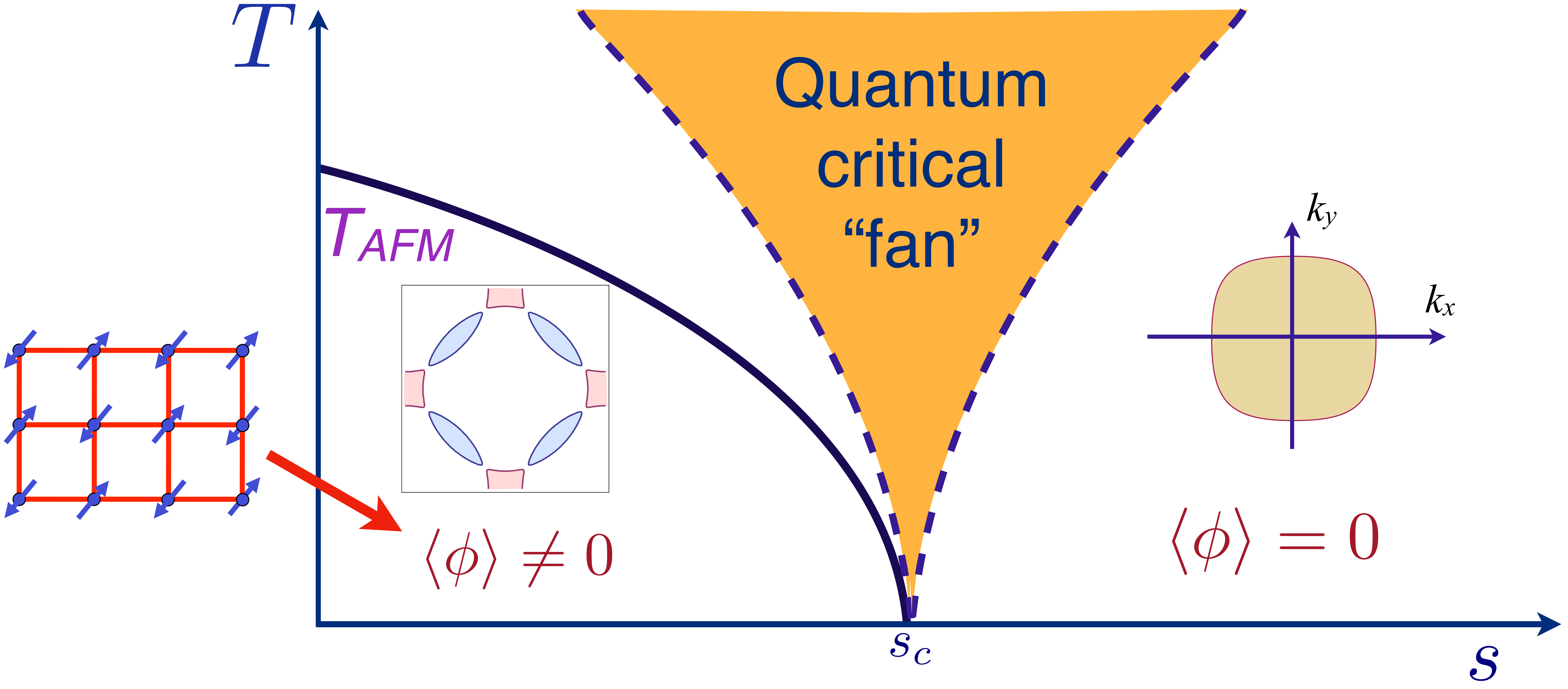
1. Universal 2D-YSYK theory of strange metals

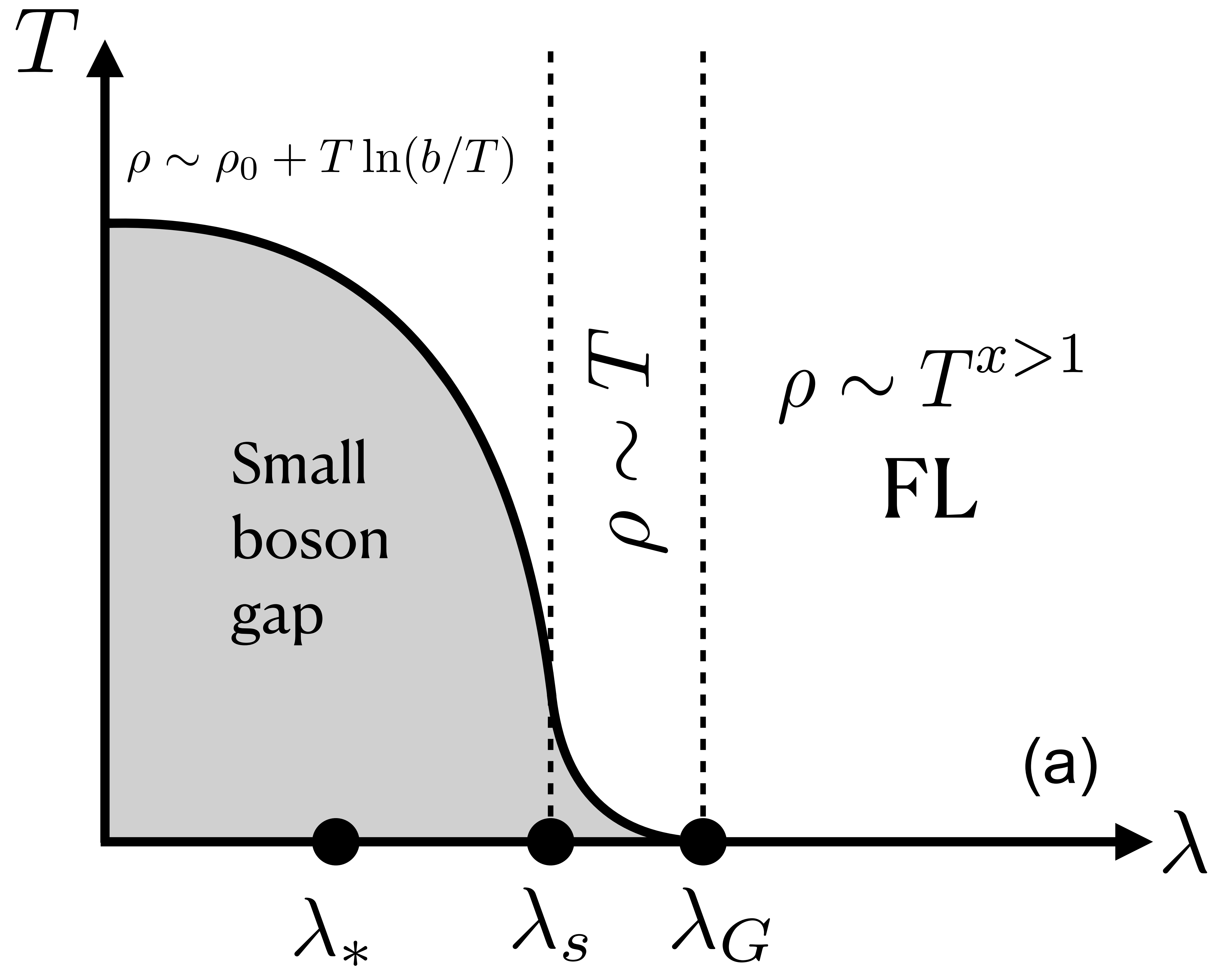
2. Theory of the "foot":
   quantum Griffiths SDW phase

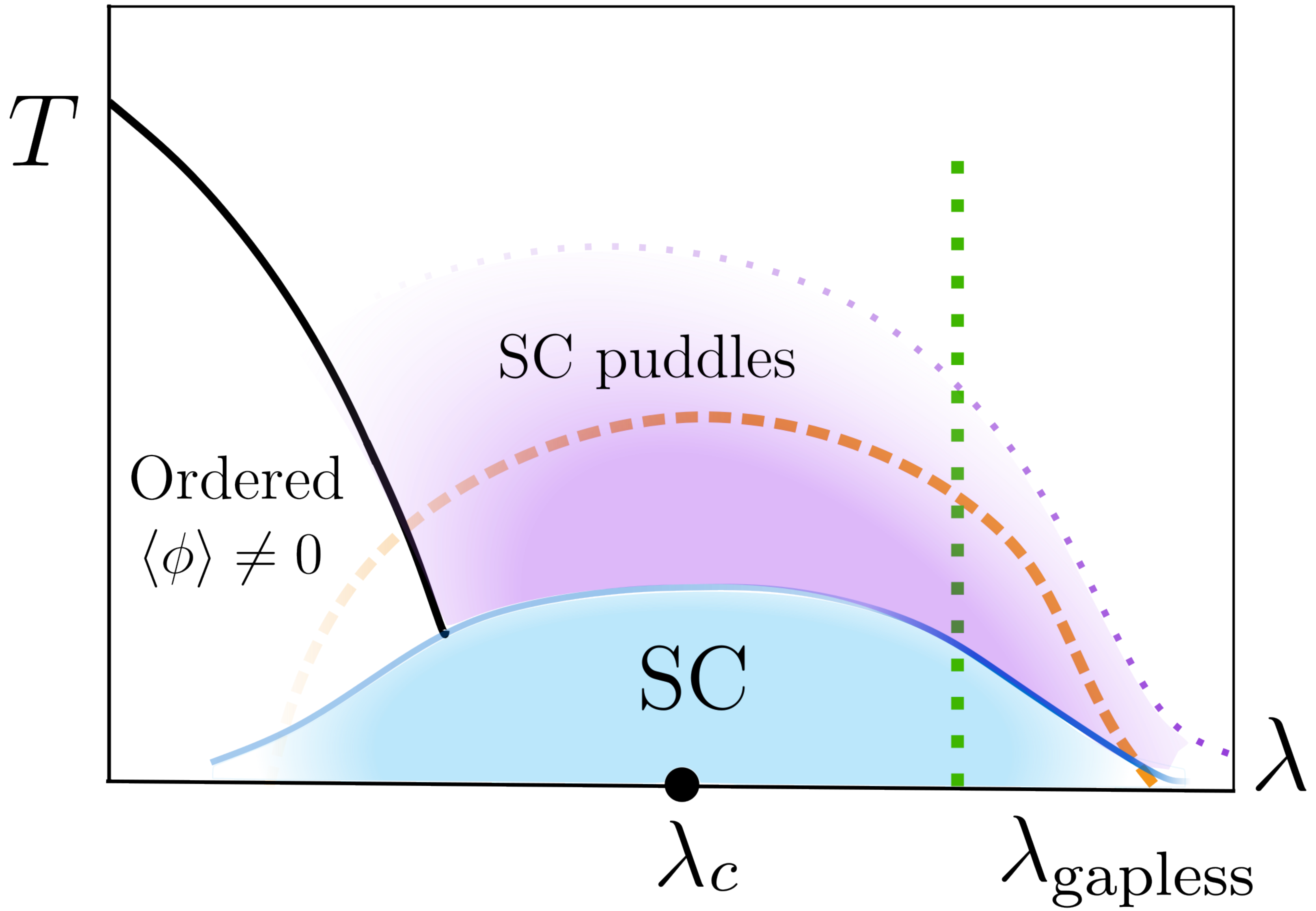
3. Quantum-critical superconductivity

Fermi surface reconstruction from spin density wave (SDW) order



Quantum-critical superconductivity





Quantum-critical superconductivity

Linearized Usadel equation for pairing eigenmodes Φ_α:

$$\Big[-D\nabla^2 + 2|\varepsilon_n| + 2\bar{\Sigma}(i\varepsilon_n, \vec{r}) \Big] \Phi_\alpha(i\varepsilon_n, \vec{r})$$
$$-2\bar{g}^2 T \sum_{\varepsilon_m} \bar{D}(i\varepsilon_m - i\varepsilon_n, \vec{r}) \Phi_\alpha(i\varepsilon_m, \vec{r}) = \epsilon_\alpha \Phi_\alpha(i\varepsilon_n, \vec{r}).$$

Input from normal state theory:

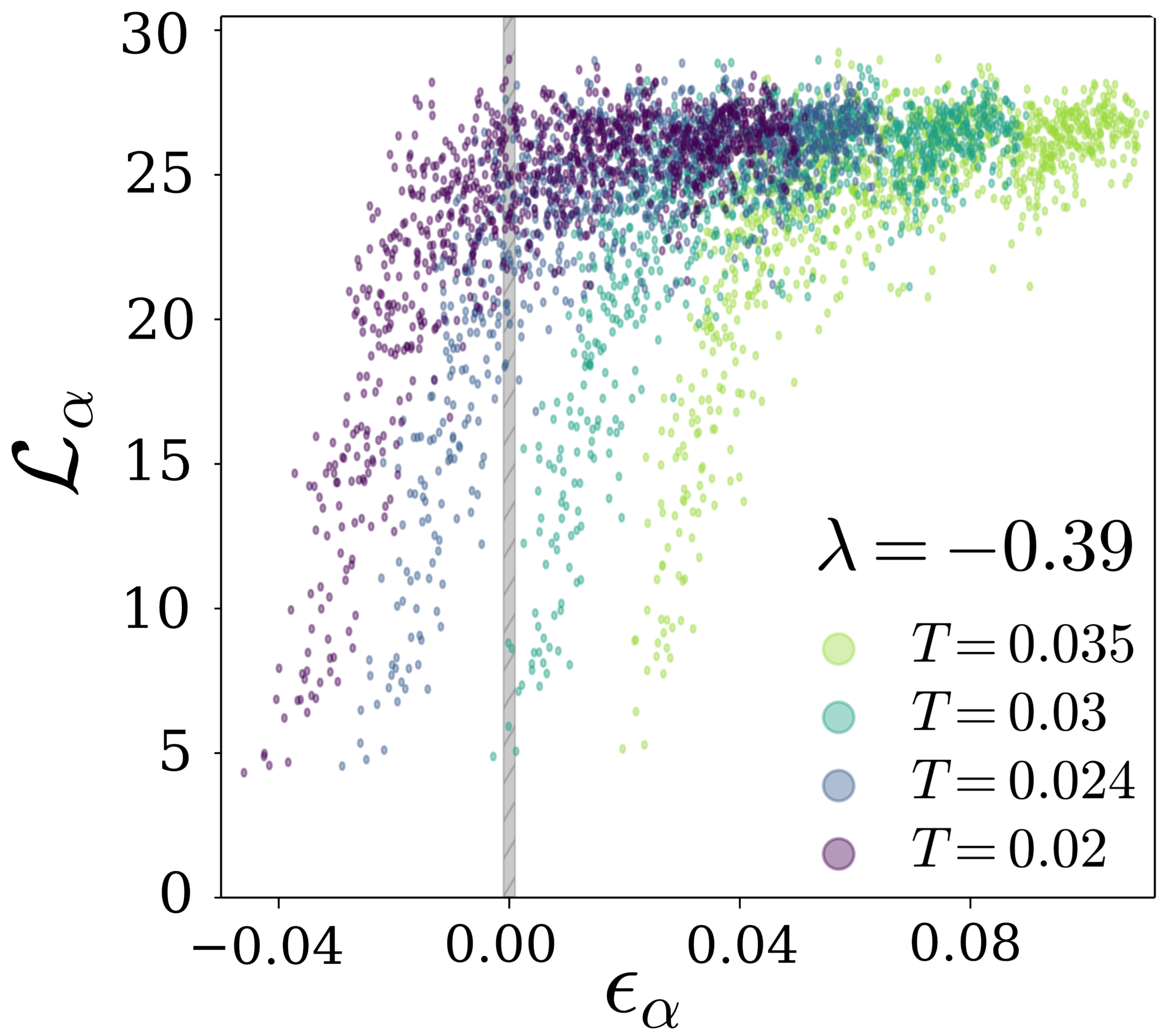
$$\bar{D}(i\omega_m, \vec{r}) = \sum_\alpha \frac{|\phi_\alpha(\vec{r})|^2}{\omega_m^2 + c_d|\omega_m| + e_\alpha}$$

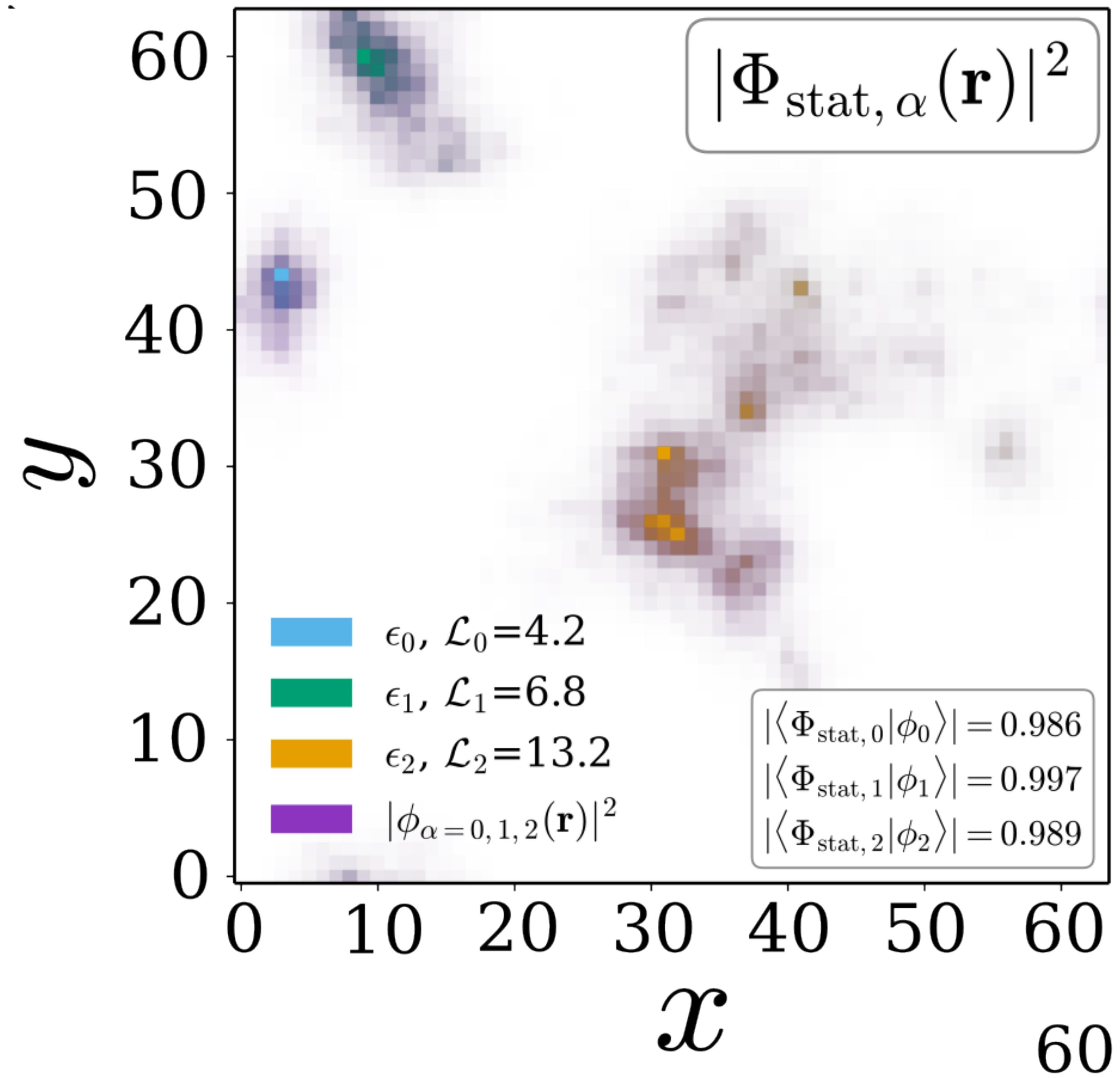
$$\bar{\Sigma}(i\varepsilon_n, \vec{r}) = \frac{\bar{g}^2}{\pi c_d} \sum_\alpha |\phi_\alpha(\vec{r})|^2 \log\left(\frac{c_d|\varepsilon_n| + e_\alpha}{\pi c_d T + e_\alpha} \right).$$



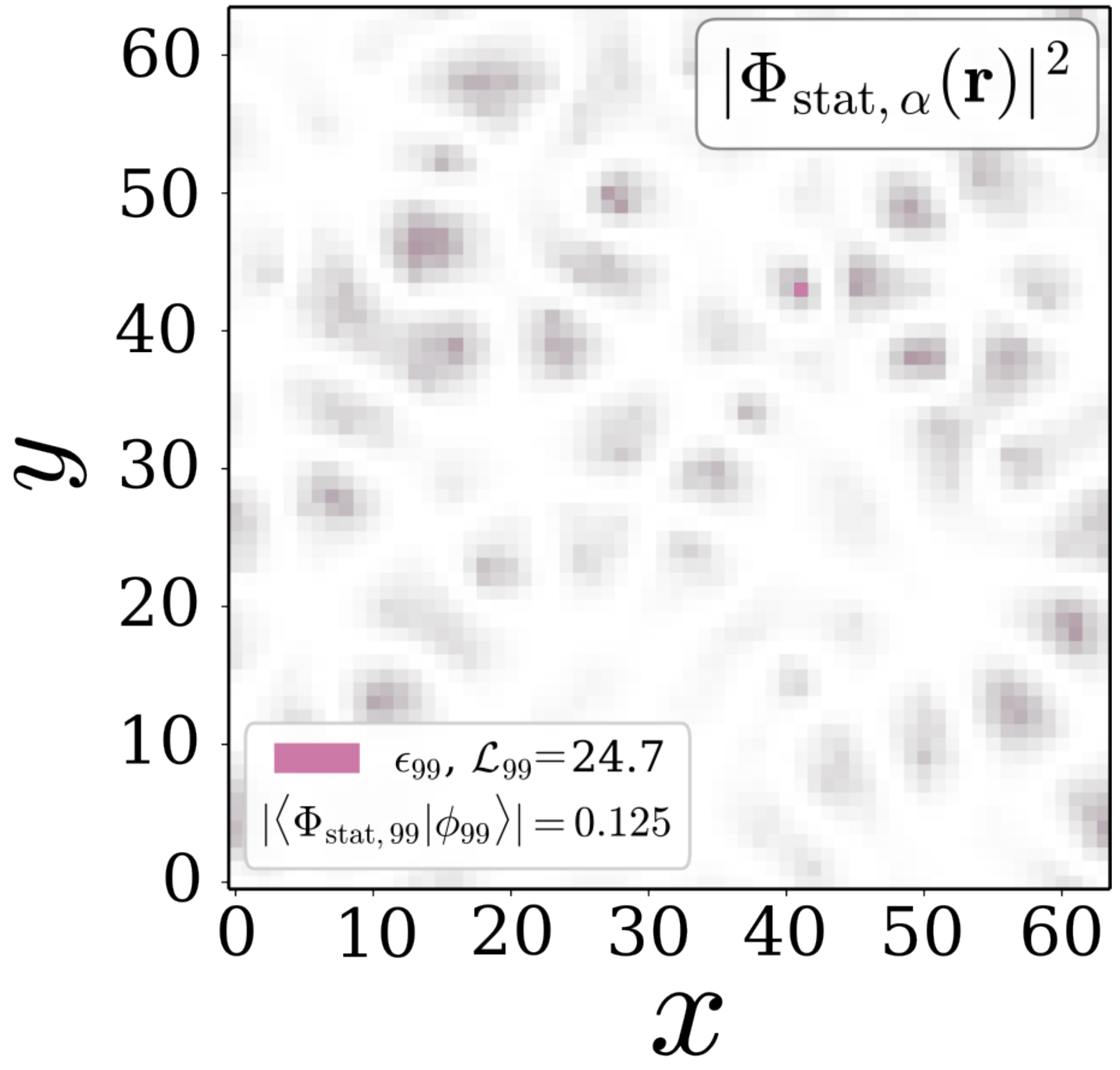
Influence of Harris disorder on quantum-critical superconductivity
Serhii Kryhin, Peter Lunts, Aavishkar A. Patel, S.S., Pavel A. Nosov, arXiv:2606.23582

Localization length of Φ_α





3 localized eigenmodes



1 extended eigenmode



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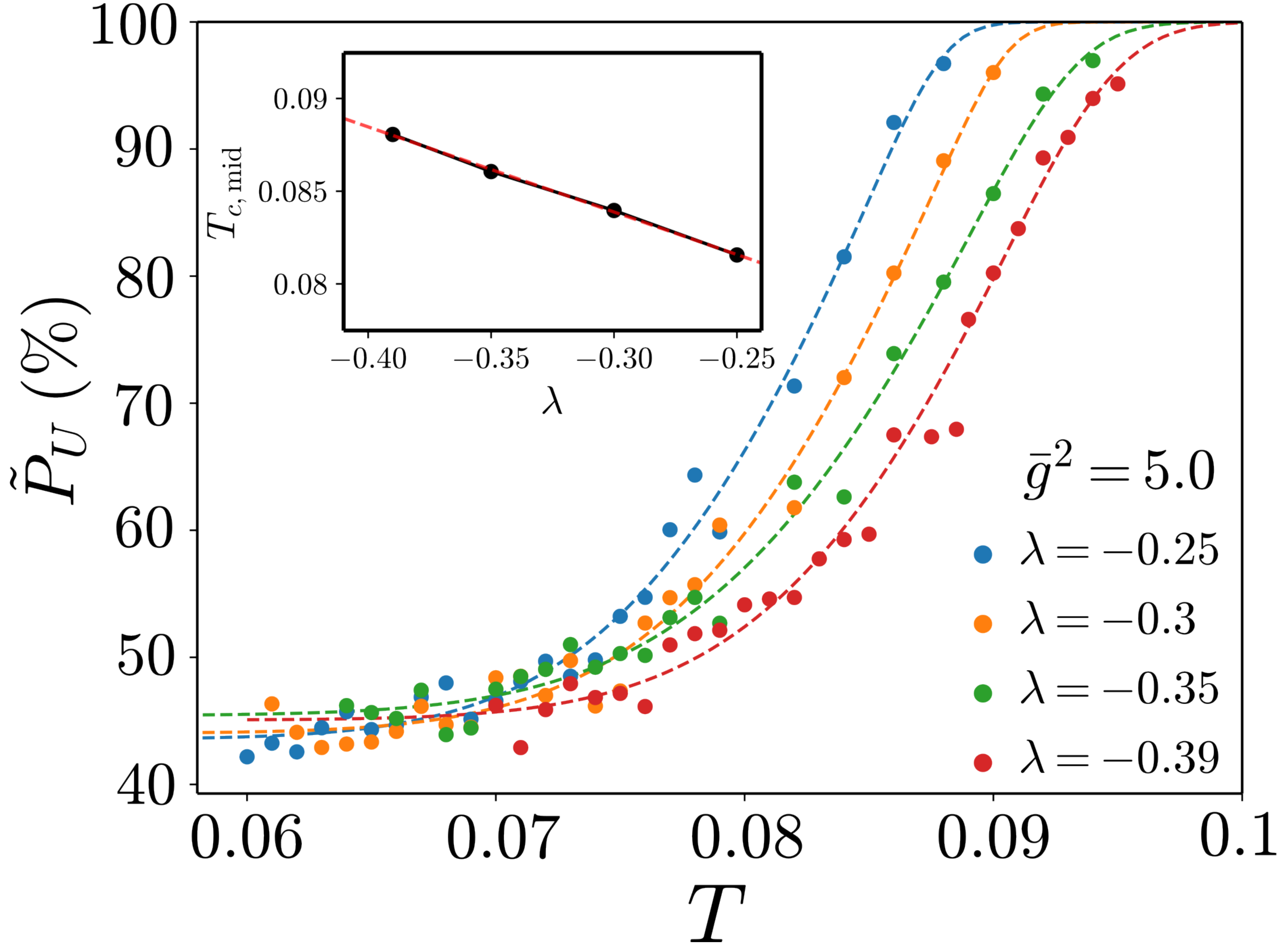
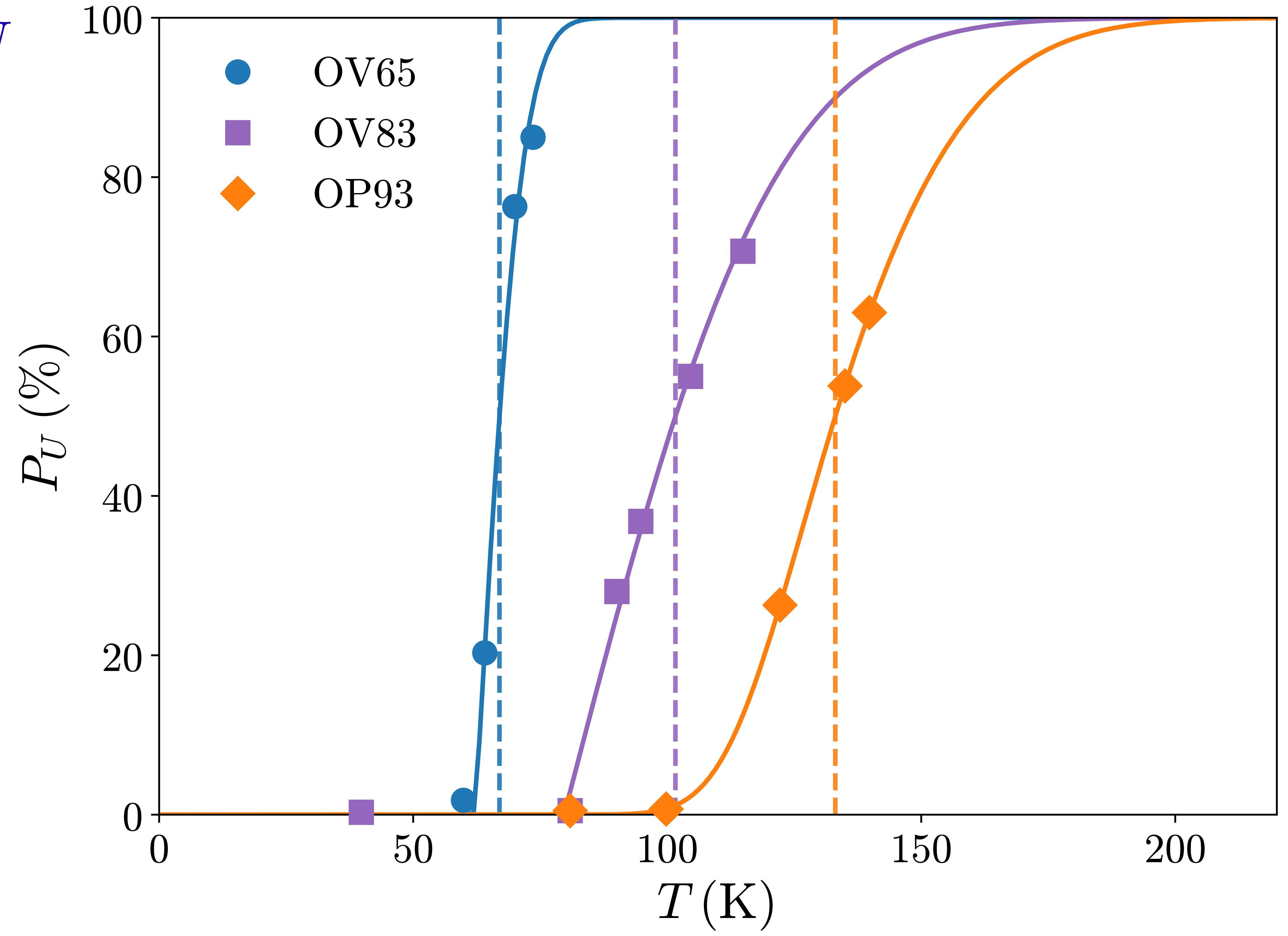
Measure of the percentage of ungapped area $\tilde{P}_U$





Analysis of STM data on $Bi_2Sr_2CaCu_2O_{8+\delta}$

Visualizing pair formation on the atomic scale in the high-Tc superconductor $Bi_2Sr_2CaCu_2O_{8+\delta}$,
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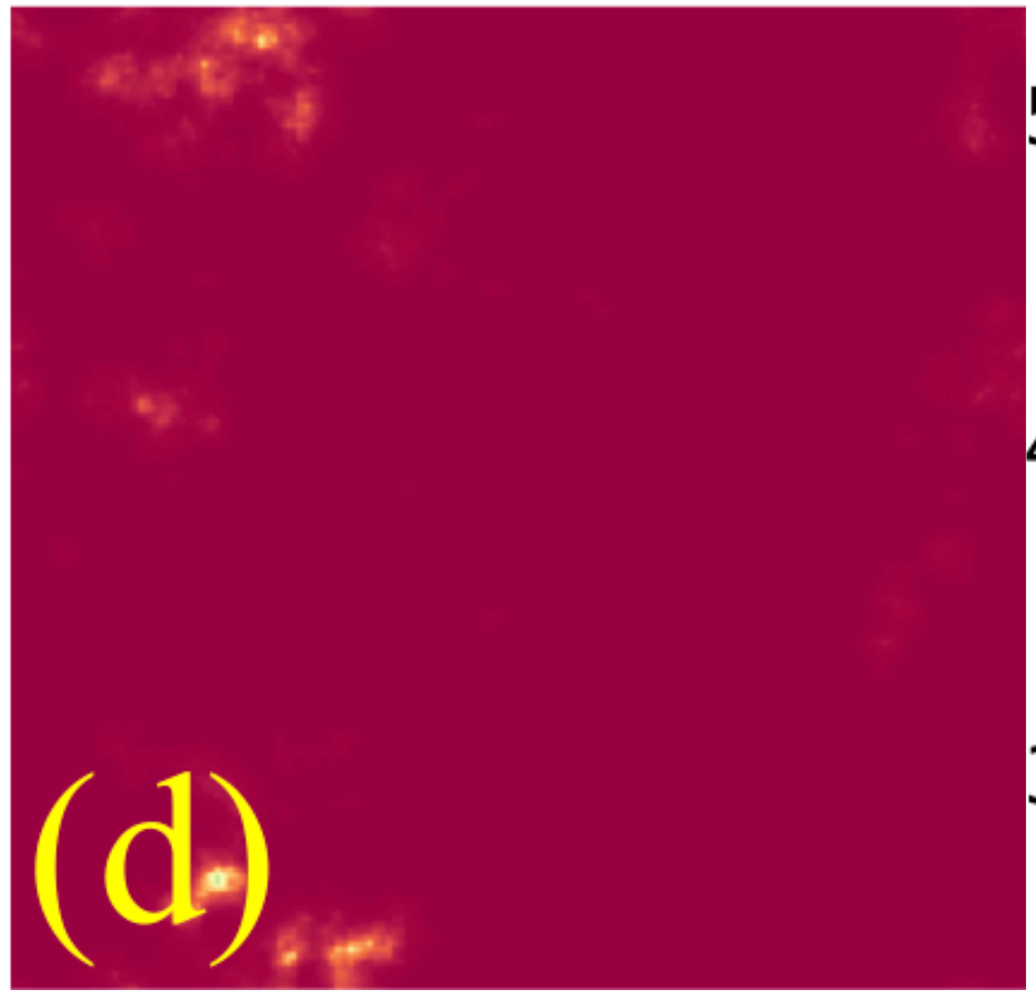
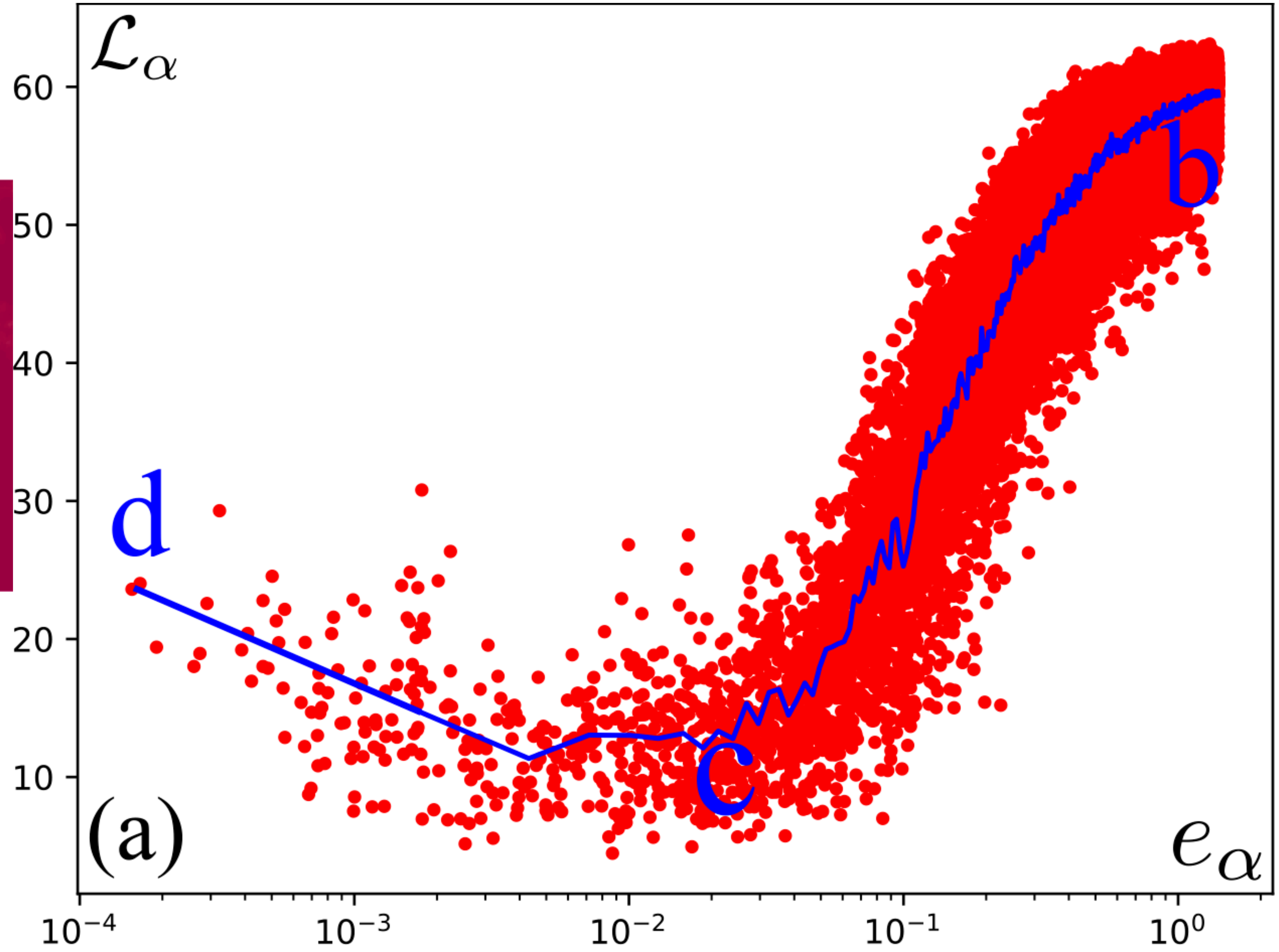
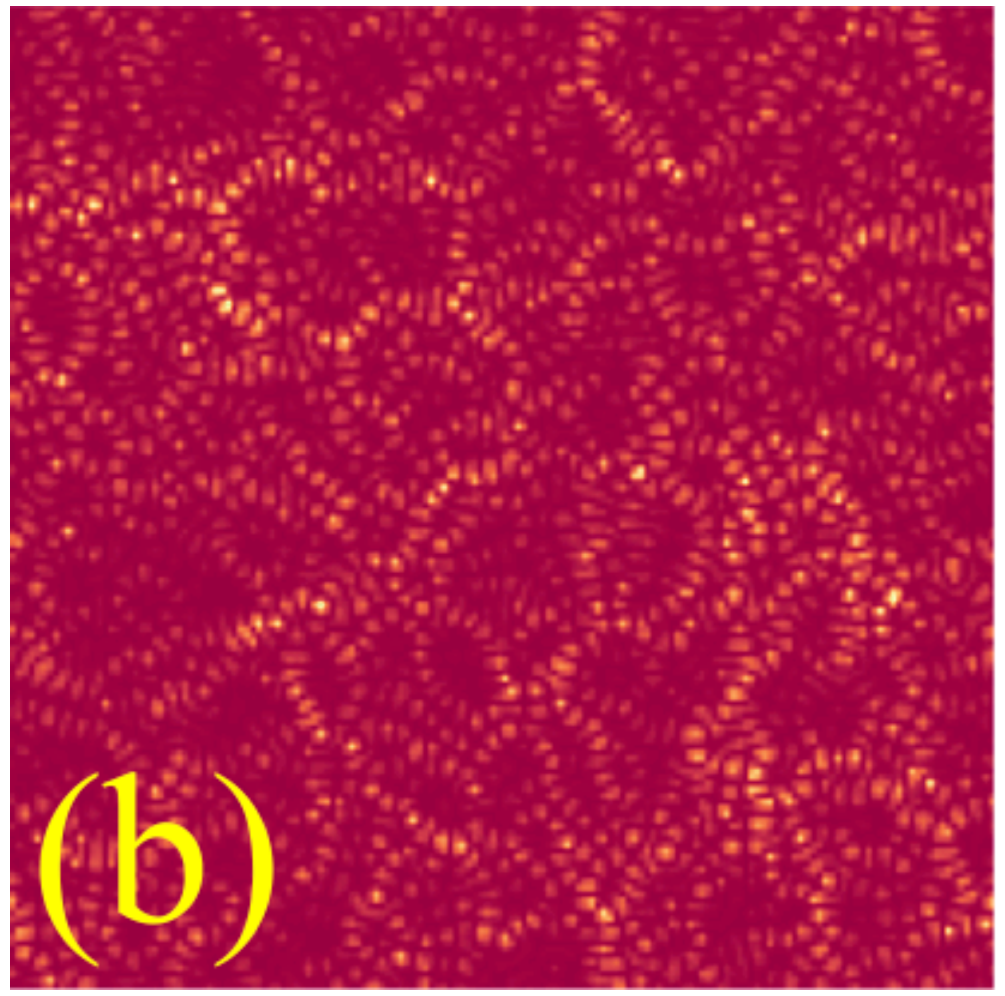
Summary

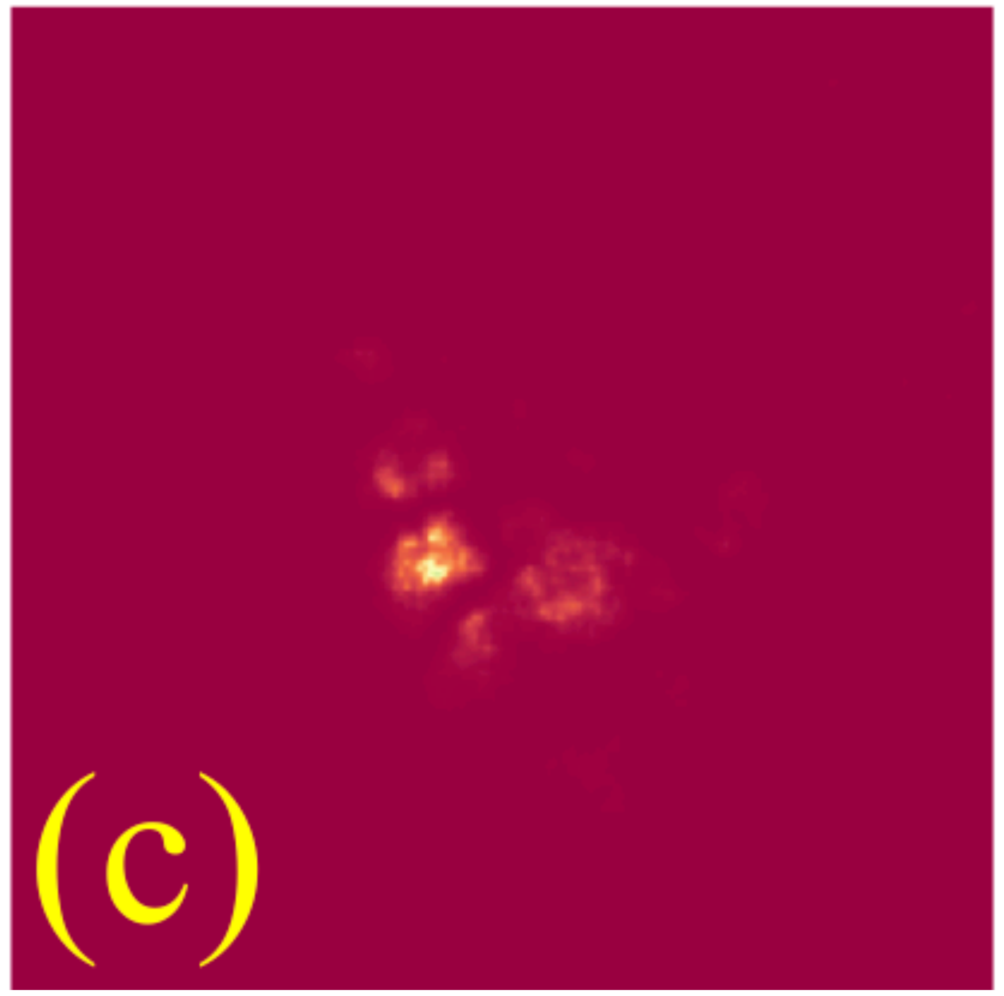
Bosonic eigenmodes in random mass Hertz theory

ϕ eigenmodes localization length $\mathcal{L}_\alpha$









Aavishkar A. Patel,
Peter Lunts, S.S.,
PNAS **121**,
e2402052121 (2024)

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

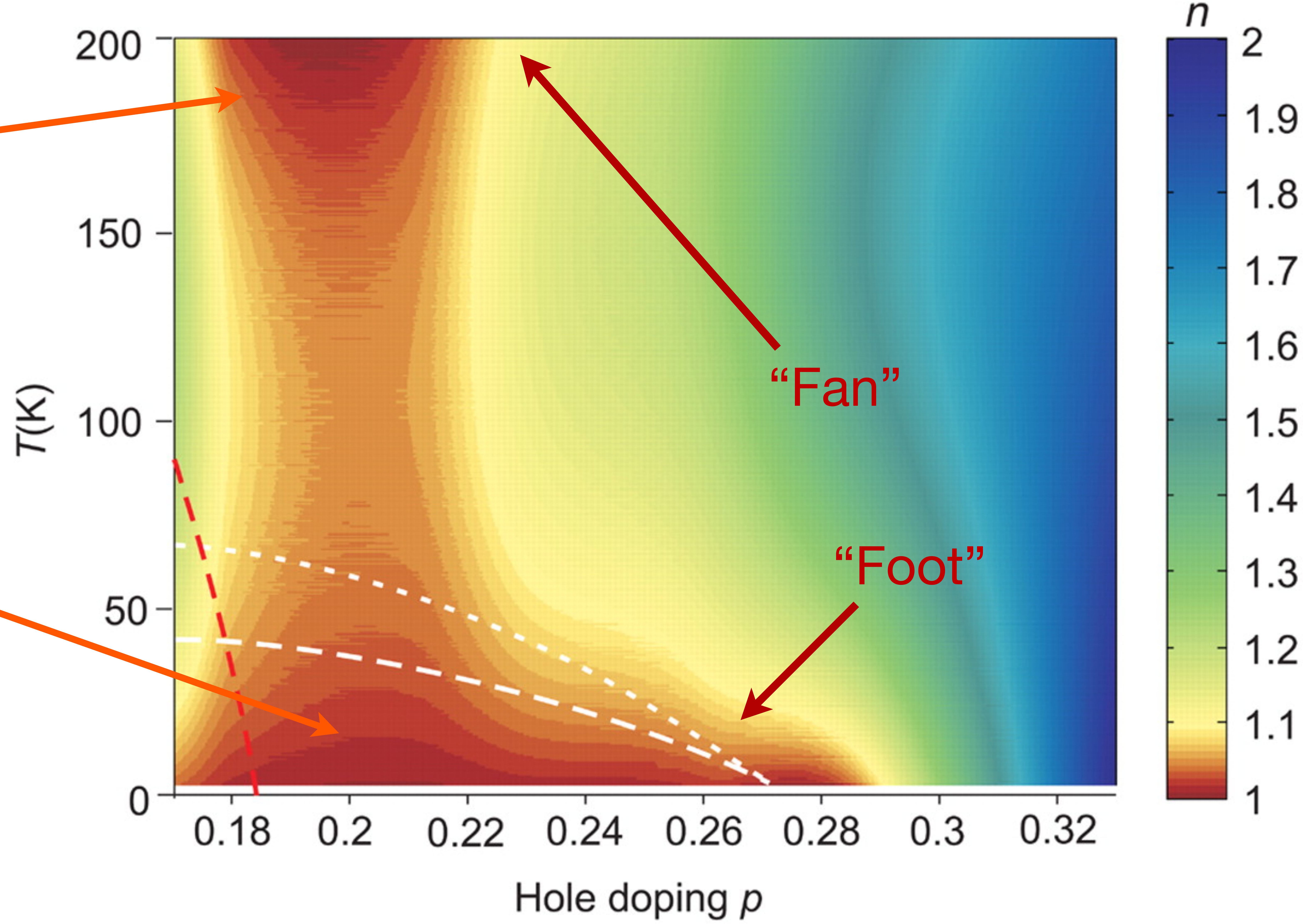
R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4*] H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†

SCIENCE VOL 323 603 2009

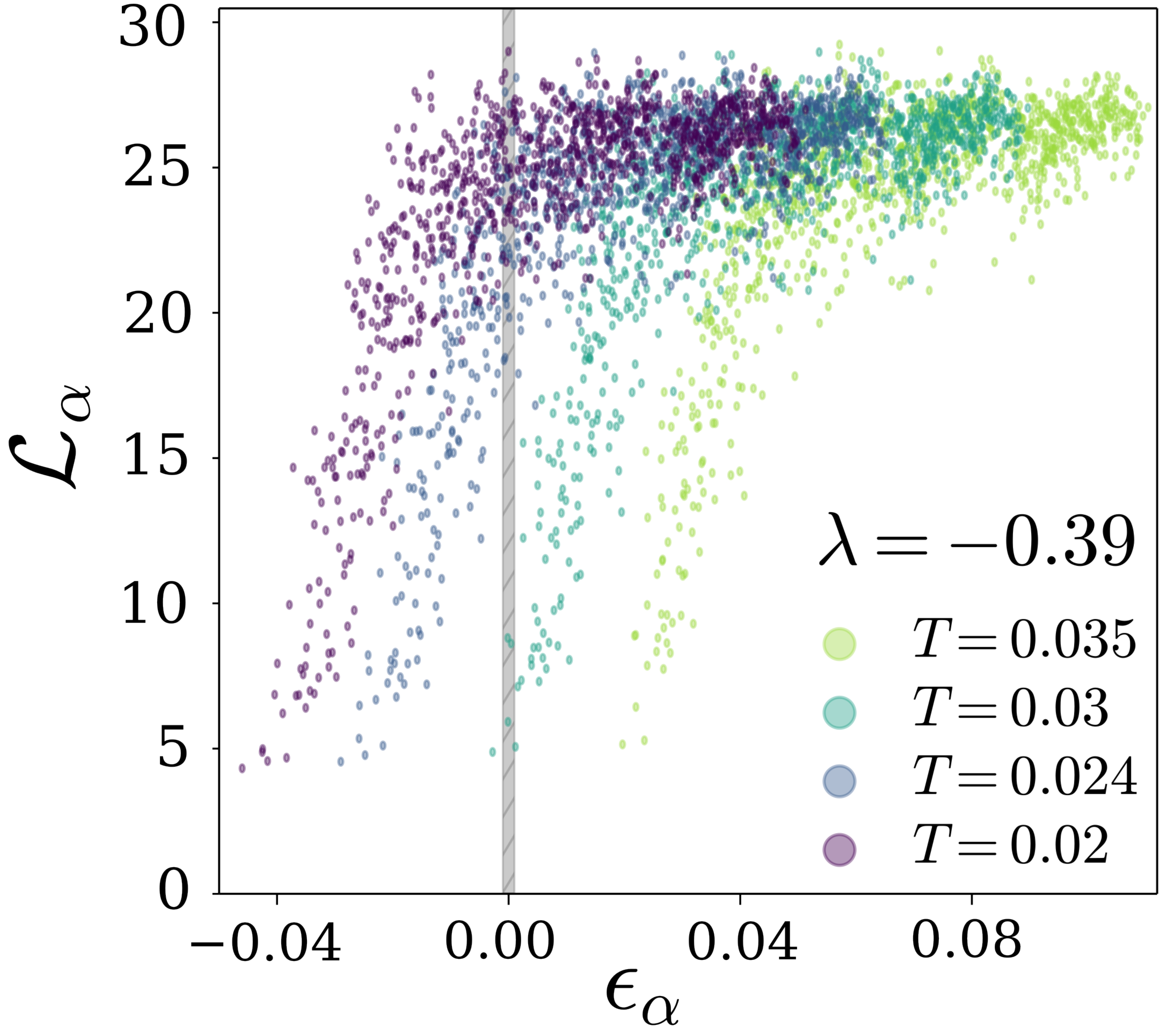
Resistivity $\rho \sim T^n$

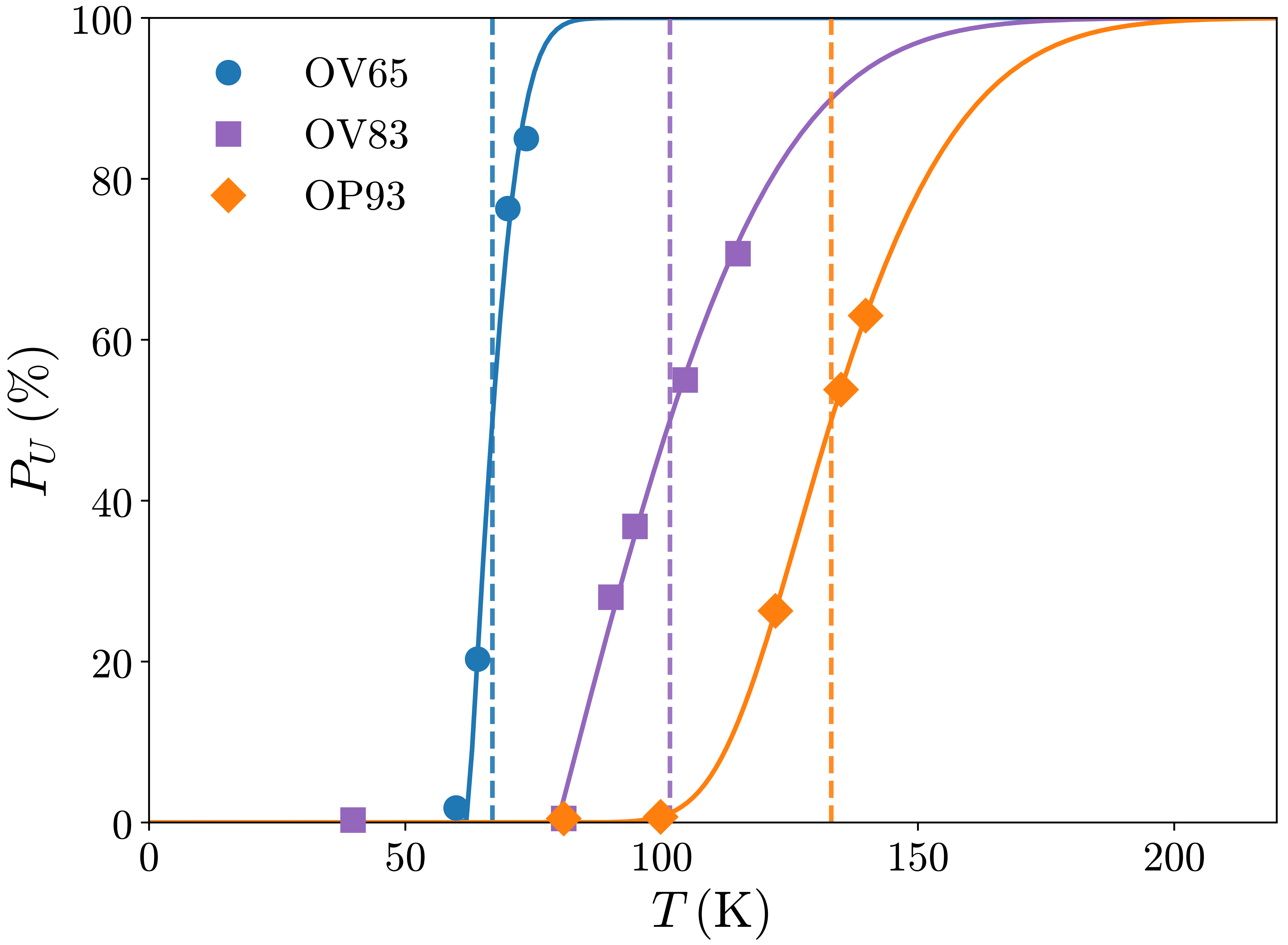
Extended fermions and bosons: 2D-YSYK theory of strange metal

Localized overdamped SDW bosons, but extended fermions: Griffths strange metal



Localization length of Φ_α





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Influence of Harris disorder on quantum-critical superconductivity
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Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

Nature Materials **22**, 703 (2023)