Theory of a Planckian metal

Aavishkar Patel and Subir Sachdev

PRL **123**, 066601 (2019)

Quantum Criticality and Topology in Correlated Electron Systems
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Talk online: sachdev.physics.harvard.edu

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Ordinary metals and quasiparticles

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\text{eq}} \sim \frac{\hbar E_F^3}{U^2 (k_B T)^2} \quad , \quad \text{as } T \to 0,$$

  where U is the strength of interactions, and E_F is the Fermi energy.

- Similarly, a quasiparticle model implies a resistivity

$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau} \sim U^2 T^2 \quad \text{with} \;\; \tau \sim \tau_{\text{eq}}$$

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- Similarly, a quasiparticle model implies a resistivity

$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau} \sim U^2 T^2 \quad \text{with} \quad \tau \sim \tau_{\rm eq}$$

- These times are much longer than the 'Planckian time' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

$$\tau \sim \tau_{\rm eq} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

(While there are arguments for a lower bound on τ, it is hard to see why there should be a corresponding bound on ρ.)

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2}\,\frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar} \,,$$

independent of the strength of interactions!

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity is associated with a universal scattering time $\approx \hbar/(k_B T)$.

Universal *T*-linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142–147

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1], B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5], N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6*] and C. Proust[3,6*]

arXiv:1902.01034

Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,[1,2] Tristin Metz,[2] Christopher Eckberg,[2] Kevin Kirshenbaum,[2] Alex Hughes,[2] Renxiong Wang,[2] Limin Wang,[2] Shanta R. Saha,[2] I-Lin Liu,[2,3,4] Nicholas P. Butch,[2,4] Zhonghao Liu,[5,6] Sergey V. Borisenko,[5] Peter Y. Zavalij,[7] and Johnpierre Paglione[2,8]

Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,[1,*] Debanjan Chowdhury,[1,*] Daniel Rodan-Legrain,[1] Oriol Rubies-Bigordà,[1] Kenji Watanabe,[2] Takashi Taniguchi,[2] T. Senthil,[1,†] and Pablo Jarillo-Herrero[1,†]

arXiv:1901.03710

Bad metallic transport in a cold atom Fermi-Hubbard system

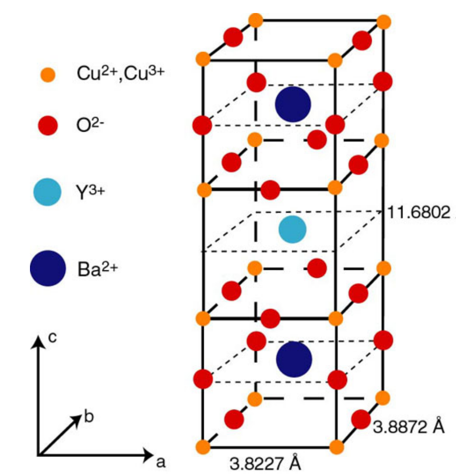
Science **363**, 379–382 (2019)

Peter T. Brown[1], Debayan Mitra[1], Elmer Guardado-Sanchez[1], Reza Nourafkan[2], Alexis Reymbaut[2], Charles-David Hébert[2], Simon Bergeron[2], A.-M. S. Tremblay[2,3], Jure Kokalj[4,5], David A. Huse[1], Peter Schauß[1*], Waseem S. Bakr[1]†

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$



A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

1. The complex SYK model

2. Quantum matter without quasiparticles:
lattice SYK models
and Planckian metals

The complex SYK model

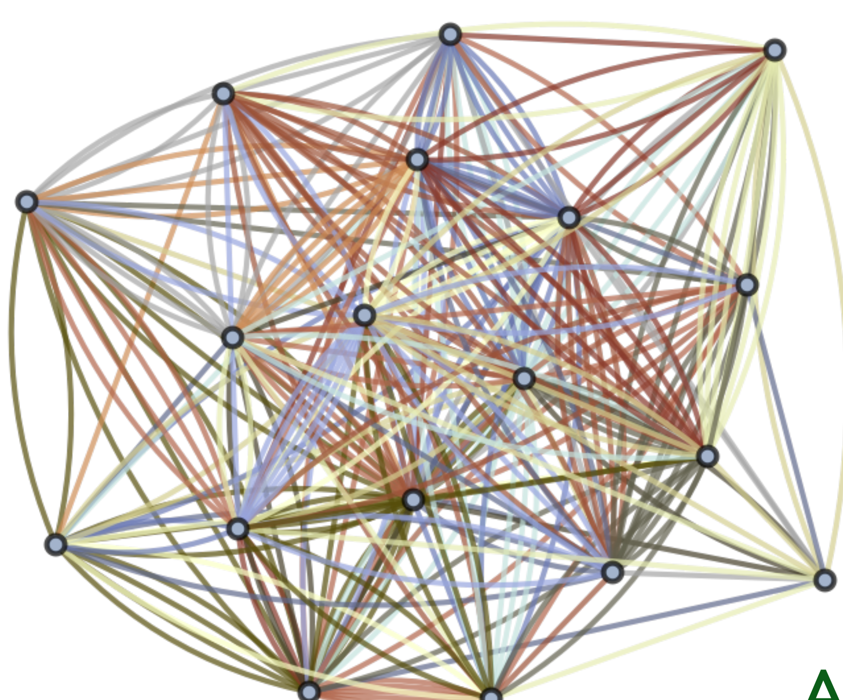
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

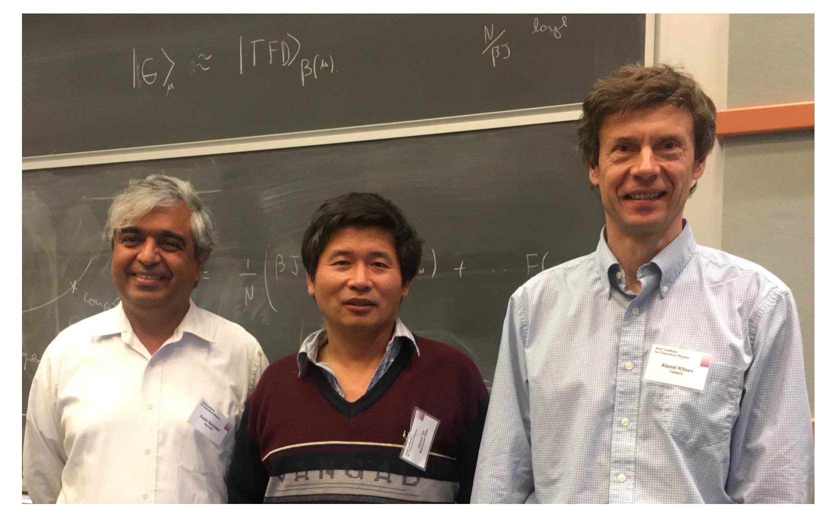
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta + \epsilon \sum_\alpha c_\alpha^\dagger c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.





S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

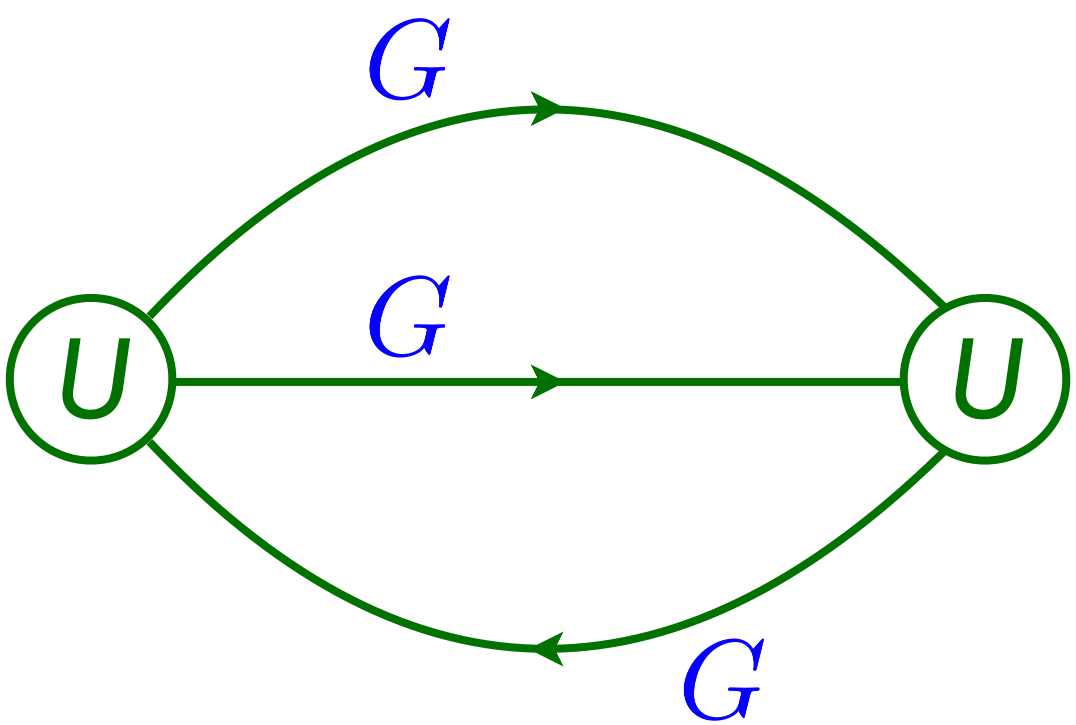
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega - \epsilon - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$ 

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

The complex SYK model

There is a one-parameter family of critical solutions with varying $\mathcal{Q}$, characterized by a dimensionless parameter $\mathcal{E}$.

For long times $\tau > 0$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = e^{\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$$\left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = e^{-\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$\mathcal{E}$ determines the particle-hole asymmetry, and $A(\mathcal{E})$ is a known function. $\mathcal{E}$ is determined by ϵ/U.

In a Fermi liquid,

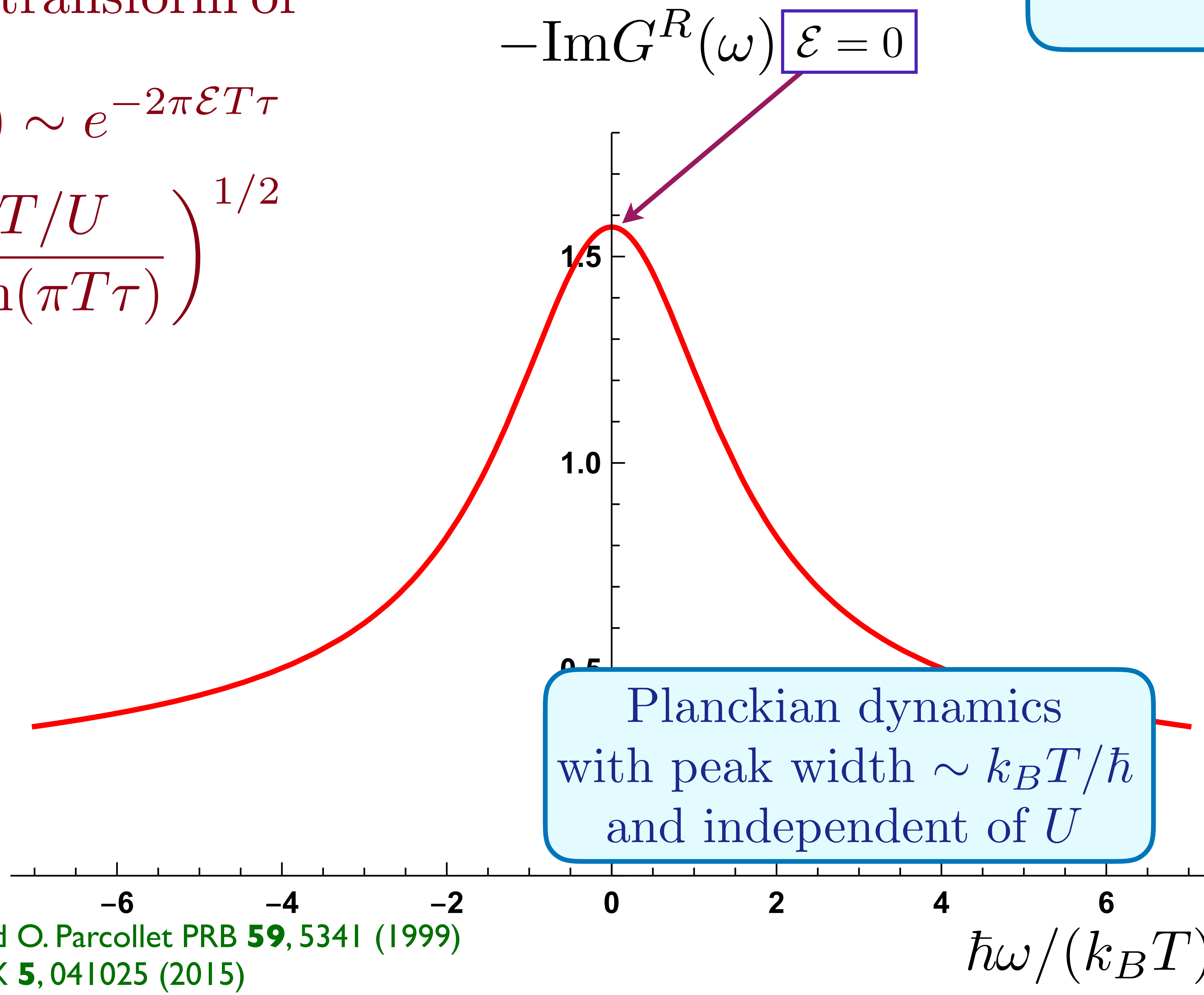
$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = \left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = \widetilde{A}/\tau$$

The complex SYK model

$G^R(\epsilon, \omega)$ is the Fourier transform of

$$G(\epsilon, \tau) \sim e^{-2\pi \mathcal{E} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

$$\mathcal{E} = \mathbb{C}\frac{\epsilon}{U}$$



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

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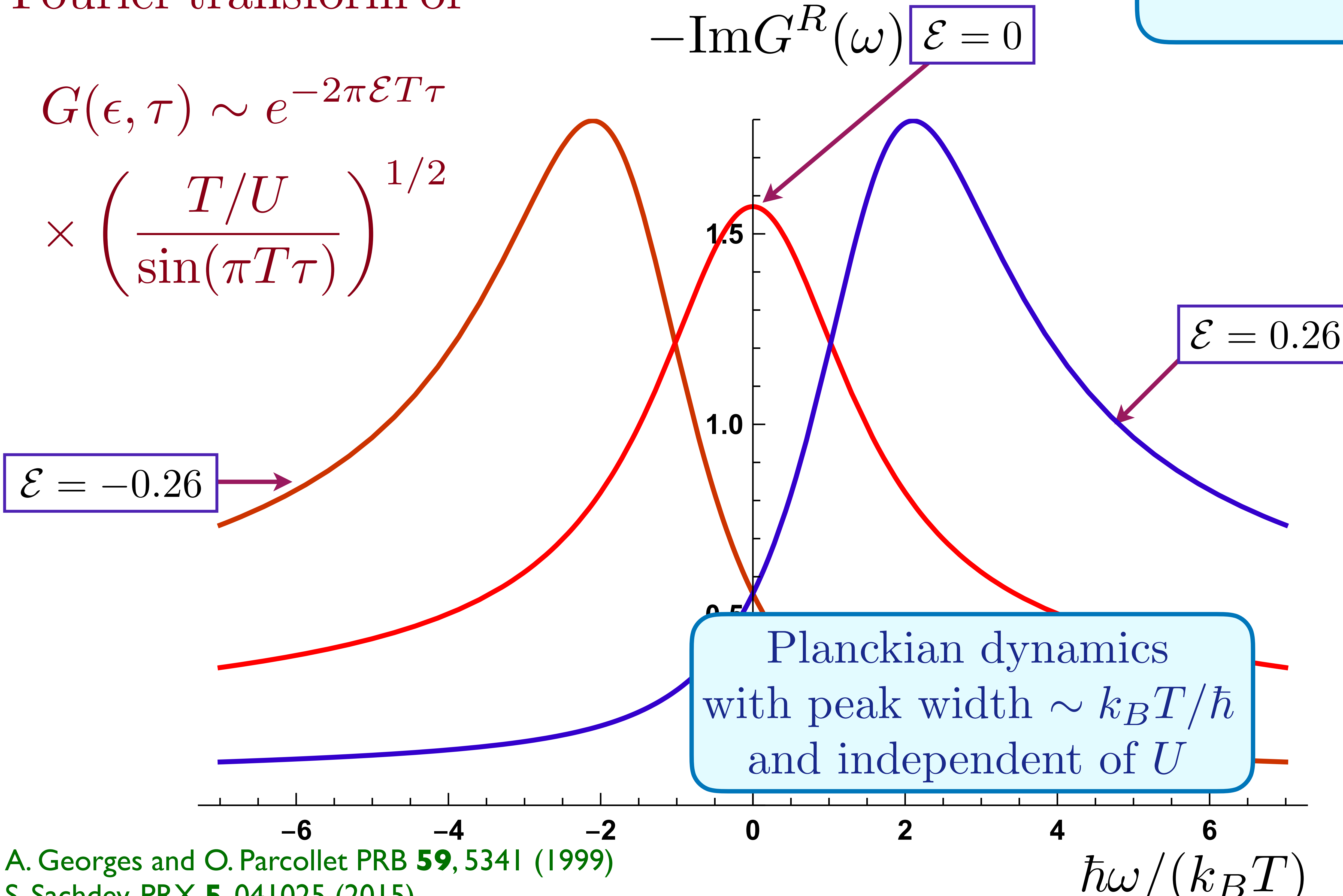

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The complex SYK model

We now examine the behavior of the chemical potential, μ, as $T \to 0$ at fixed $\mathcal{Q}$. For this we relate the long-time 'conformal' Greens function, (valid for $\tau \gg 1/U$) to its short-time behavior. In particular at $|\omega_n| \gg U$ we have

$$G(i\omega_n) = \frac{1}{i\omega_n} - \frac{\mu}{(i\omega_n)^2} + \ldots$$

which implies for the spectral density of the Green's function, $\rho(\Omega)$

$$\mu = -\int_{-\infty}^{\infty} \frac{d\Omega}{\pi} \, \Omega \rho(\Omega),$$

which makes it evident that μ depends only upon the particle-hole asymmetric part of the spectral density. Next, we can relate the Ω integrals to the derivative of the imaginary time correlator

$$\mu = -\partial_\tau G(\tau = 0^+) - \partial_\tau G(\tau = (1/T)^-).$$

The complex SYK model

We pull out an explicitly particle-hole asymmetric part of $G(\tau)$ by defining

$$G(\tau) \equiv e^{-2\pi \mathcal{E} T \tau} G_c(\tau) \quad , \quad 0 < \sigma < \frac{1}{T}.$$

where G_c will be given by a particle-hole symmetric conformal form at low T and low ω. Then we obtain

$$\begin{aligned} \mu &= 2\pi \mathcal{E} T \left[G(\tau = 0^+) + G(\tau = (1/T)^-) \right] \\ &\qquad\qquad + \text{terms dependent on } G_c \\ &= -2\pi \mathcal{E} T + \text{terms dependent on } G_c \end{aligned}$$

It can be shown that all the terms dependent upon G_c have a T dependence that is weaker than linear in T provided $\mathcal{Q}$ is held fixed. Hence we have

$$\mu = \mu_0 - 2\pi \mathcal{E} T + \text{terms vanishing as } T^p \text{ with } p > 1$$

with μ_0 a non-universal constant. From this relation we obtain

The complex SYK model

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$$\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = -2\pi\mathcal{E} \quad , \quad T \to 0,$$

Using a Maxwell relation we then have

$$\frac{1}{N}\left(\frac{\partial S}{\partial \mathcal{Q}}\right)_T = 2\pi\mathcal{E} \neq 0 \ \text{ as } T \to 0.$$

The complex SYK model

Solution of these equations and corresponding evaluation of the free energy yields the following universal results (*i.e.* all results are *quantitatively* unchanged by adding additional higher q fermion terms):

- There is a non-vanishing entropy in the zero temperature limit

$$S(T \to 0) = N s_0 + \ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

The complex SYK model

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- There is a non-vanishing entropy in the zero temperature limit

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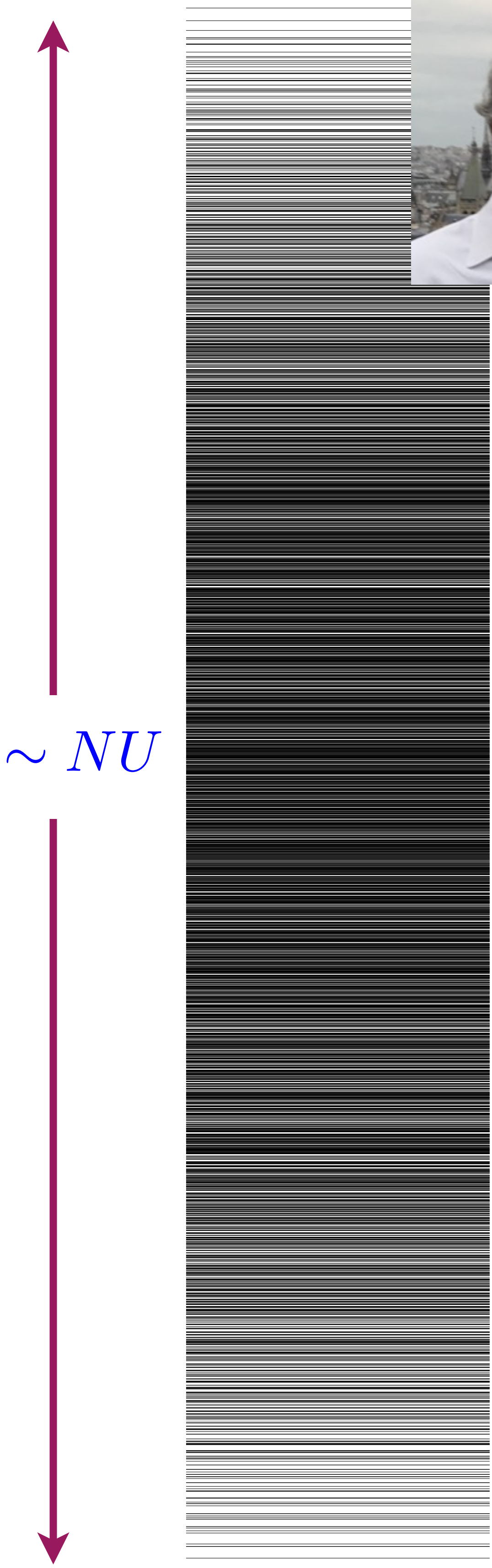
- The saddle point equations imply the relation

$$\boxed{\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}}$$

Integrating this relation from $s_0 = 0$, $\mathcal{Q} = 0$, allows us to compute s_0 as a function of $\mathcal{Q}$.

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

The complex SYK model



GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

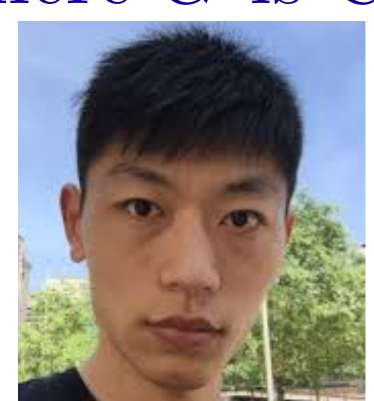
There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{d\mathcal{Q}} = 2\pi\mathcal{E}\,.$$

At $\mathcal{Q} = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.



W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

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The complex SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta + \epsilon \sum_\alpha c^\dagger_\alpha c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

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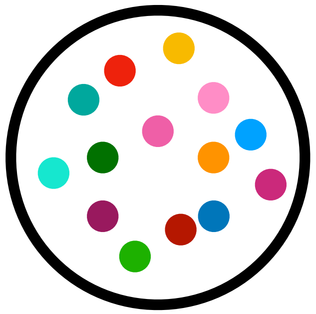
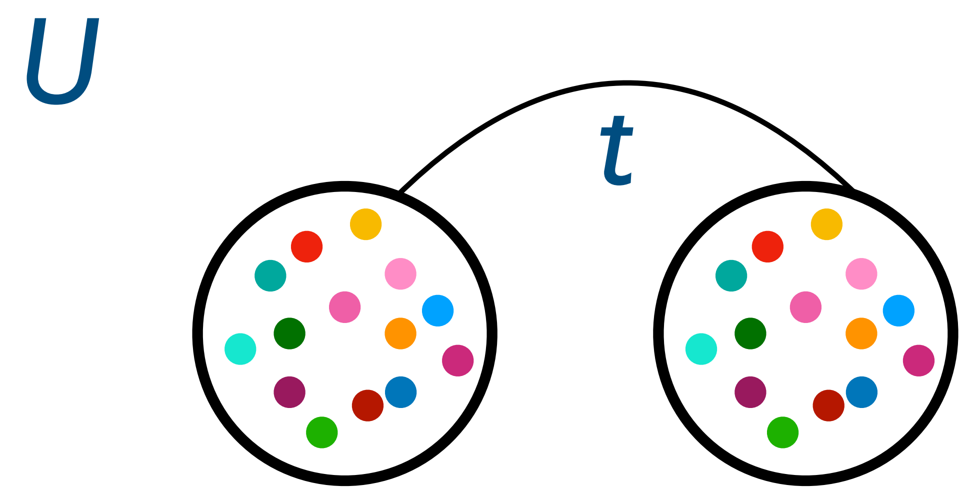


$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_\alpha c_{i\alpha}^\dagger c_{j\alpha}$$

Equivalent to an
"eternal traversable wormhole"
between two black holes with
AdS_2 horizons



J. Maldacena and Xiao-Liang Qi, arXiv:1804.00491

Generalized SYK models

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$

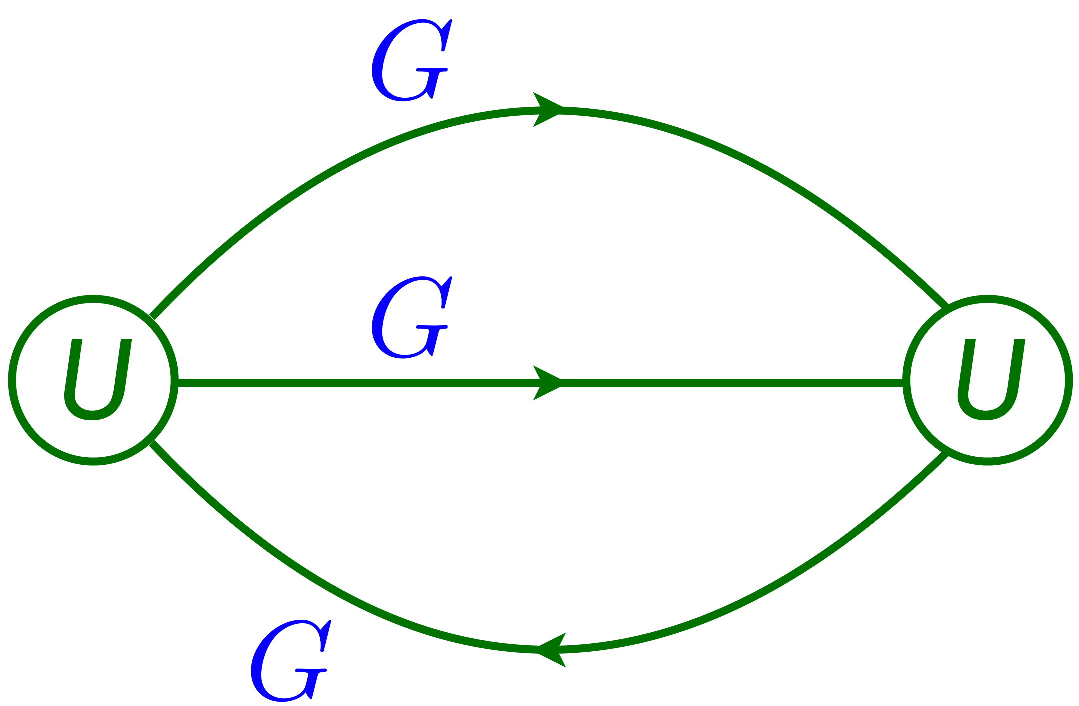
$$+ \sum_{k\alpha} \epsilon_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)

ϵ_k has a range of values of width W.

The large N limit is still given by the sum of "melon" diagrams.

$$G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)}$$

$\Sigma =$ 

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ϵ_k has a range of values of width W.

The large N limit is still given by the sum of "melon" diagrams.

For many generic models in this class with $U \gg W$, $\hbar\omega/(k_B T)$ scaling of SYK holds for $W^2/U \ll k_B T \ll U$, and Fermi liquid theory is recovered for $k_B T \ll W^2/U$.

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999); Yingfei Gu, Xiao-Liang Qi, D. Stanford, JHEP (2017) 125

A lattice SYK model

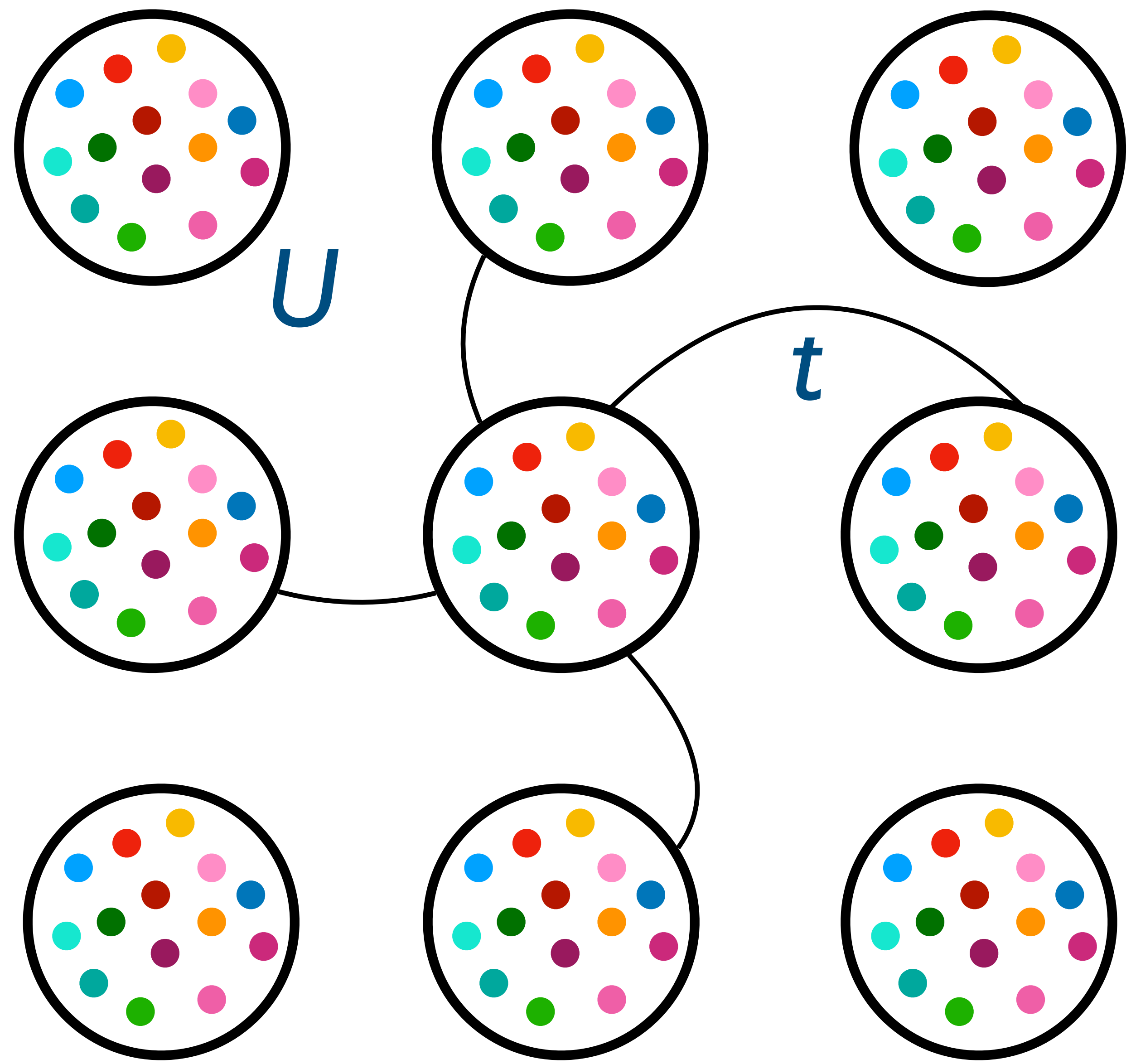
$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_{i\alpha} c^\dagger_{i\beta} c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_{\alpha} c^\dagger_{i\alpha} c_{j\alpha}$$

Choose $U \gg t$ on-site, and the same on all sites; yields 'incoherent metal' with no Fermi surface for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k},\omega) = G_{\rm SYK}(\epsilon, \hbar\omega/(k_B T))$$

independent of $\mathbf{k}$. There is linear-in-T resistivity but only with bad metal behavior with $\rho > h/e^2$, and co-efficient dependent upon U:

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



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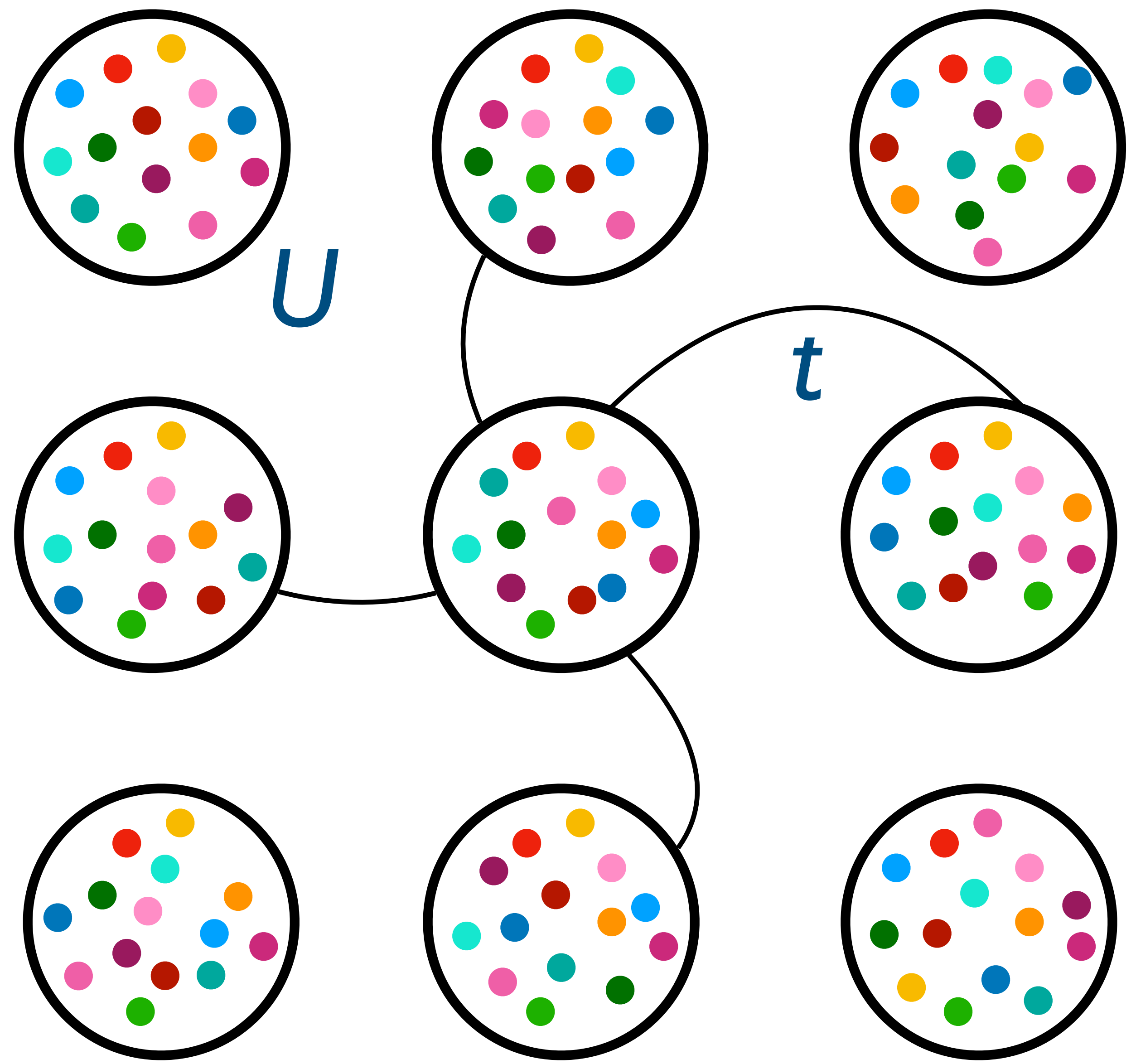

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017);
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ϵ_k has a range of values of width W.

The large N limit is still given by the sum of "melon" diagrams.

We will examine a model with weaker $U \lesssim W$.

After imposing a **resonance condition**

we find that $\hbar\omega/(k_B T)$ scaling holds for $k_B T \ll U$, down to $T \to 0$.

Moreover, the resistivity exhibits Planckian universality.

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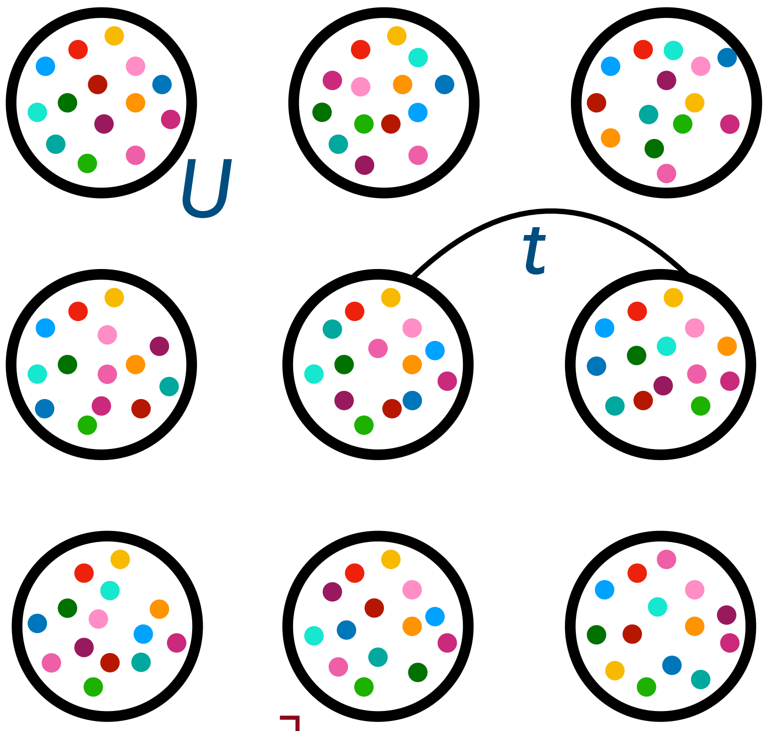
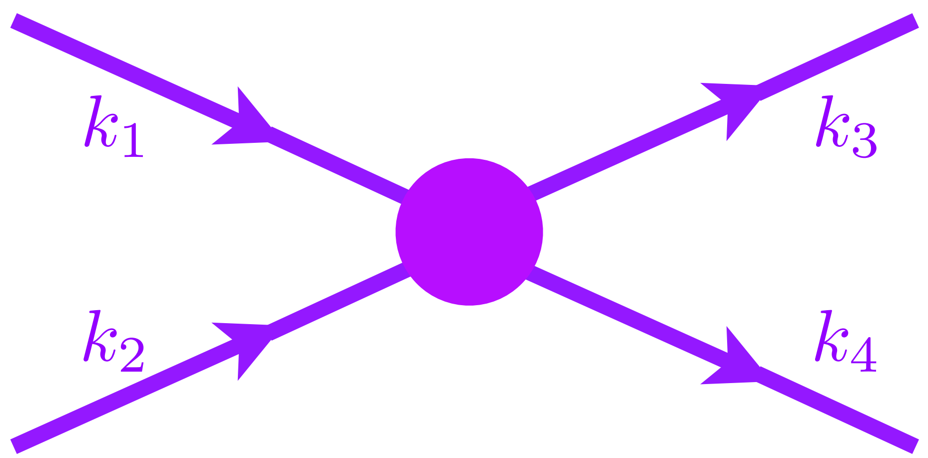


Rewriting of lattice model of incoherent and bad metal in momentum space

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} =$$
$$U^2 \left[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\right]$$

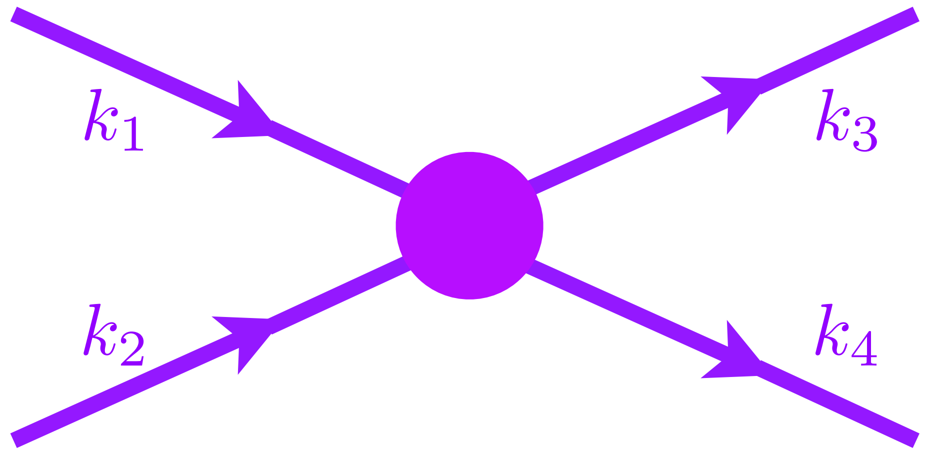
Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we "integrate these out" in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k, which we have already accounted for.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we "integrate these out" in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k, which we have already accounted for.

Keep only the **interactions resonant in the bare quasiparticle energy** with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

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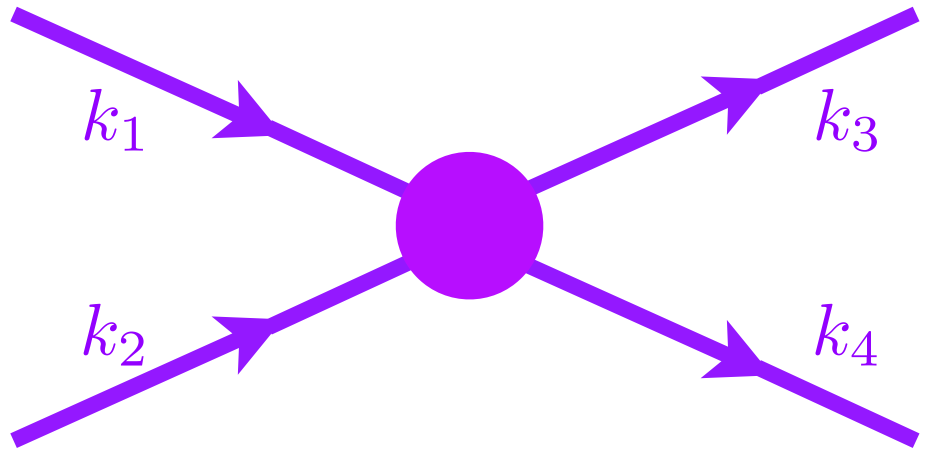

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Keep only the **interactions resonant in the bare quasiparticle energy** with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

This is precisely the effective Hamiltonian method, when low energy states are separated from high energy states by a gap; we are assuming it can also apply in a gapless system.

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Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$

$$+ \sum_{k\alpha} \epsilon_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)

The random k_i dependence of U allows only

interactions resonant in the bare quasiparticle energies

with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$.

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} =$$

$$U^2 \Big[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8)\Big]$$

$$\times \Big[\delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} + \epsilon_{k_6} - \epsilon_{k_7} - \epsilon_{k_8})\Big]$$

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This implies off-site interactions with correlations which decay with a power-law in space.

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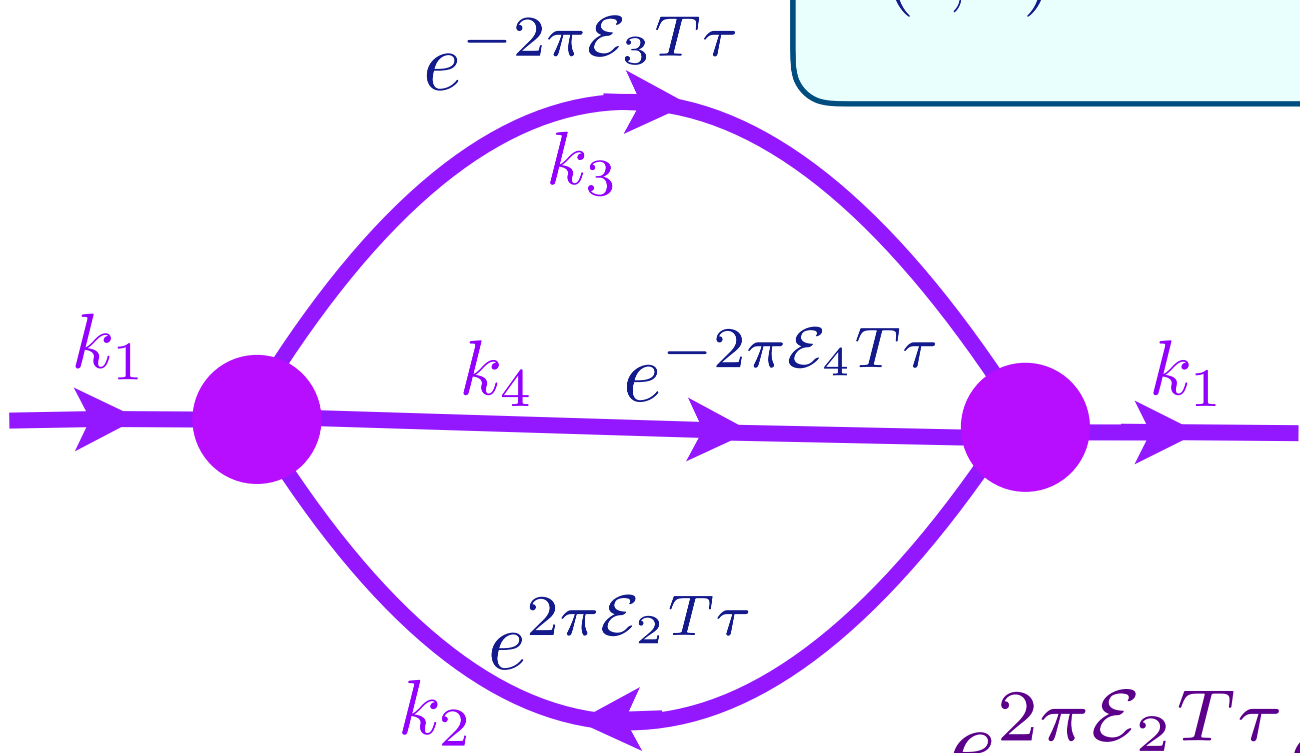
Resonant SYK model

Conformal Green's function at $T > 0$ must have the form

$$G(\epsilon, \tau) \sim e^{-2\pi \mathcal{E} T \tau} \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T \,.$$



$$e^{2\pi \mathcal{E}_2 T \tau} e^{-2\pi \mathcal{E}_3 T \tau} e^{-2\pi \mathcal{E}_4 T \tau} = e^{-2\pi \mathcal{E}_1 T \tau}$$

if

$$\mathcal{E}_a = \mathbb{C} \epsilon_a / U$$

and

$$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4$$

SYK behavior in a
Planckian metal as $T \to 0$
with a remnant Fermi surface:

$$G(k, \omega) = G_{\rm SYK}(\epsilon_k, \hbar\omega/(k_B T)),$$

with $\mathcal{E}_k = \mathbb{C}\, \epsilon_k / U$

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Incoherent metal

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathcal{E}} \frac{A(\mathcal{E})}{\sqrt{U\tau}}$$

The parameter $\mathcal{E}$ is independent of k, and determined by the total density

Planckian metal with remnant Fermi surface

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathbb{C} \epsilon_k / U} \frac{A(\epsilon_k/U)}{\sqrt{U\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathbb{C} \epsilon_k / U} \frac{A(\epsilon_k/U)}{\sqrt{U\tau}}$$

The particle-hole asymmetry changes as we cross the Fermi surface

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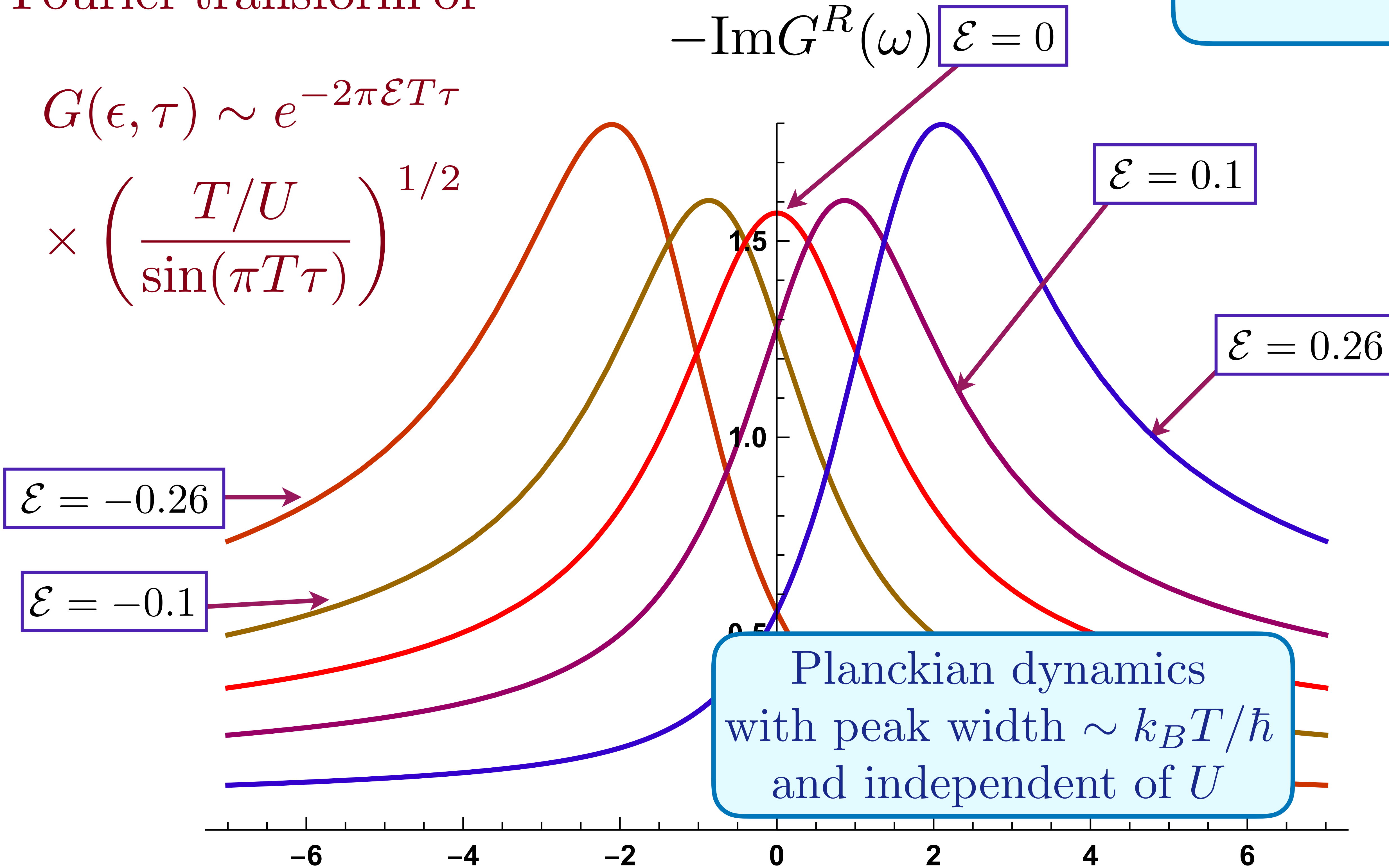
The complex SYK model

$G^R(\epsilon,\omega)$ is the Fourier transform of

$$G(\epsilon,\tau) \sim e^{-2\pi\mathcal{E}T\tau} \times \left(\frac{T/U}{\sin(\pi T\tau)}\right)^{1/2}$$

$$\mathcal{E} = \mathbb{C}\frac{\epsilon}{U}$$



Planckian dynamics with peak width $\sim k_B T/\hbar$ and independent of U

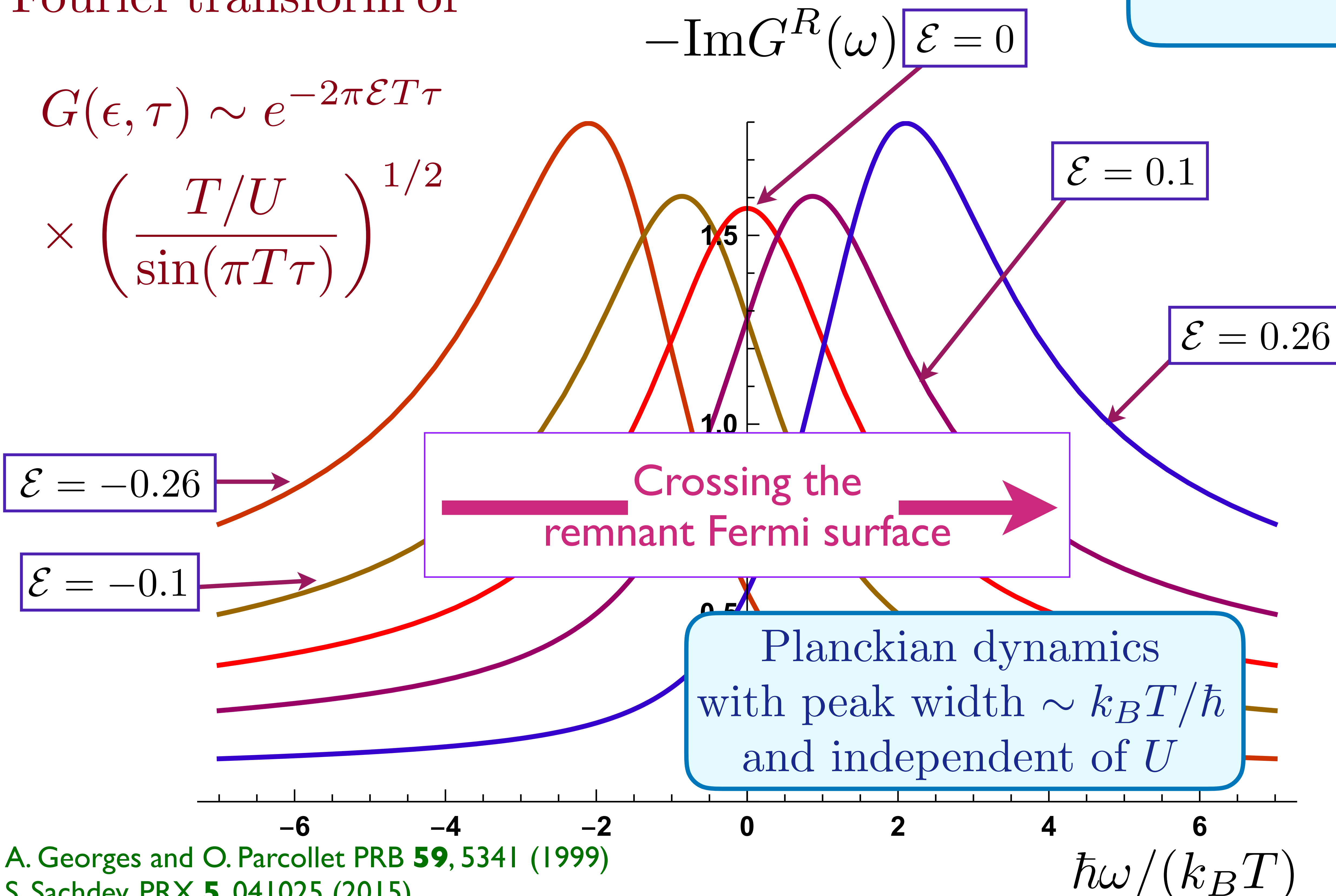
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

Resonant SYK model

$G^R(\epsilon, \omega)$ is the Fourier transform of

$$G(\epsilon, \tau) \sim e^{-2\pi \mathcal{E} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

$$\mathcal{E} = \mathbb{C} \frac{\epsilon}{U}$$



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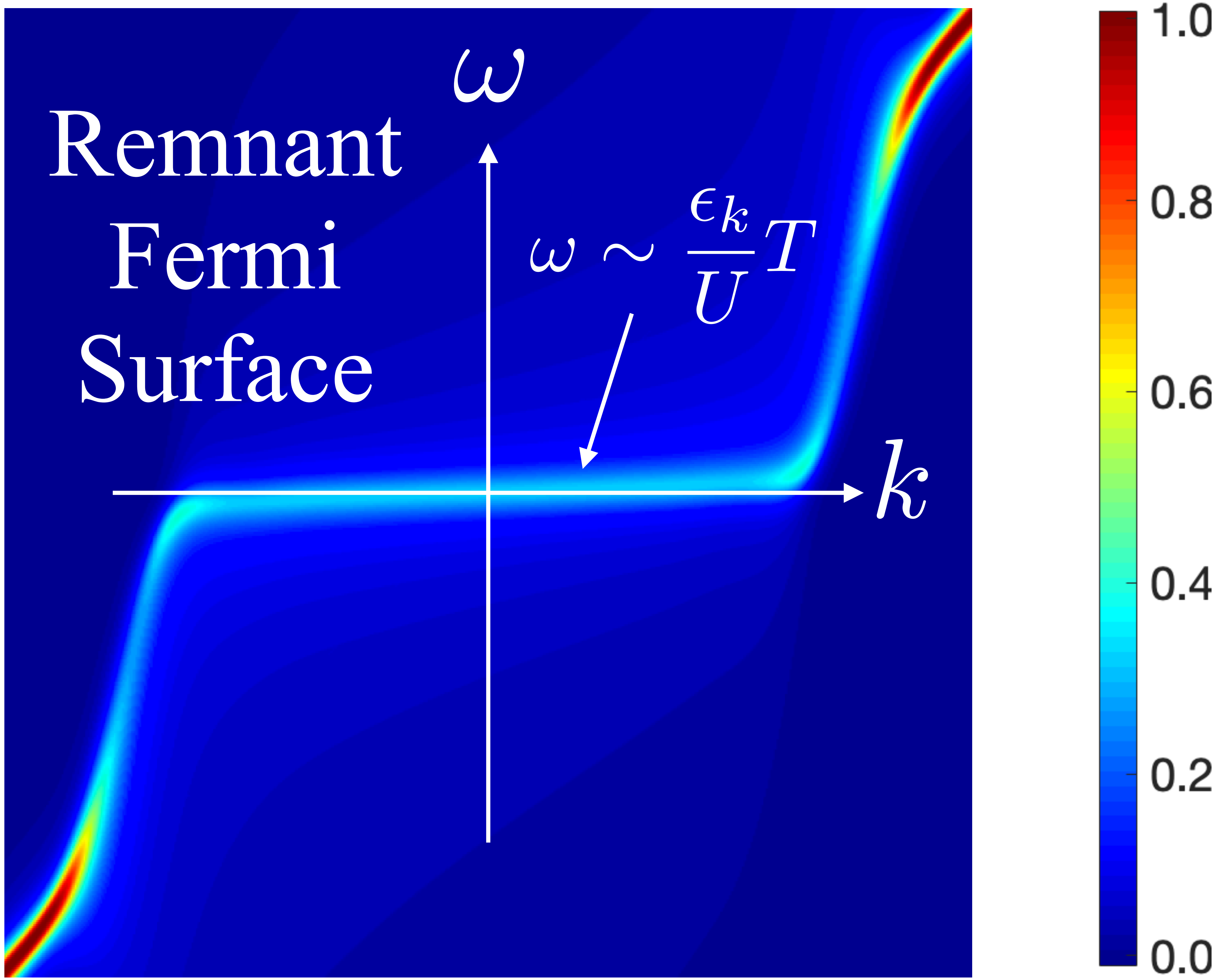

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Resonant SYK model

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)
The random k_i dependence of U allows only
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with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$.

Resistivity of a Planckian metal as $T \to 0$

From the Kubo formula, in the large N limit

$$\sigma = \frac{Ne^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[\mathrm{Im}\, G^R_{\rm SYK}\left(\epsilon, \frac{\omega}{T}\right)\right]^2 \mathrm{sech}^2\left(\frac{\omega}{2T}\right)$$

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$$\boxed{\rho = \frac{m^*}{ne^2} \, 2.71 \mathbb{C} \, \frac{k_B T}{\hbar}} \quad , \quad \text{using } \mathcal{E} = \mathbb{C}\epsilon/U \, ,$$

where

$$m^* = \frac{d\, V_{FS}}{\oint_{FS} |\boldsymbol{v}_F|} \, ,$$

where d is spatial dimensionality and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^*.

Resonant SYK model

Resistivity of a Planckian metal as $T \to 0$

$$\rho = \frac{m^*}{ne^2} \, 2.71 \mathbb{C} \, \frac{k_B T}{\hbar}$$

Note that all explicit dependence on U has cancelled out!

The number $\mathbb{C}$ is defined by $\mathcal{E}_k = \mathbb{C}\, \epsilon_k / U$ as $|\epsilon_k| \to 0$. This is determined by UV physics, and is very weakly dependent upon the ratio of the energy width of the interactions, W_U, to U.

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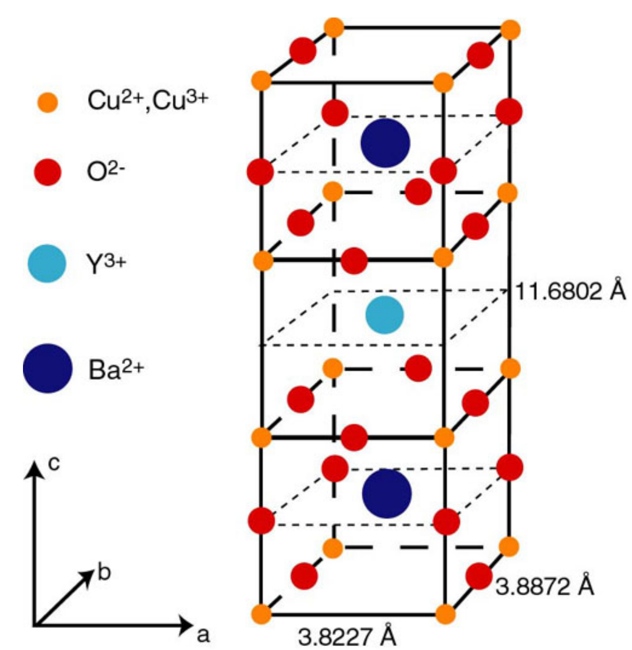

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Resonant SYK model

Take the independent momentum shell limit, $W_U/U \to 0$,

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2\Big[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\Big] \times \Big[\delta(\epsilon_{k_1}-\epsilon_{k_2})\delta(\epsilon_{k_2}-\epsilon_{k_3})\delta(\epsilon_{k_3}-\epsilon_{k_4}) + \delta(\epsilon_{k_5}-\epsilon_{k_6})\delta(\epsilon_{k_6}-\epsilon_{k_7})\delta(\epsilon_{k_7}-\epsilon_{k_8})\Big]$$

$\mathbb{C} = 0.41$ as in a single SYK model,
and we obtain a Planckian metal with

$$\rho = \frac{m^*}{ne^2}\, 1.11\, \frac{k_B T}{\hbar}$$

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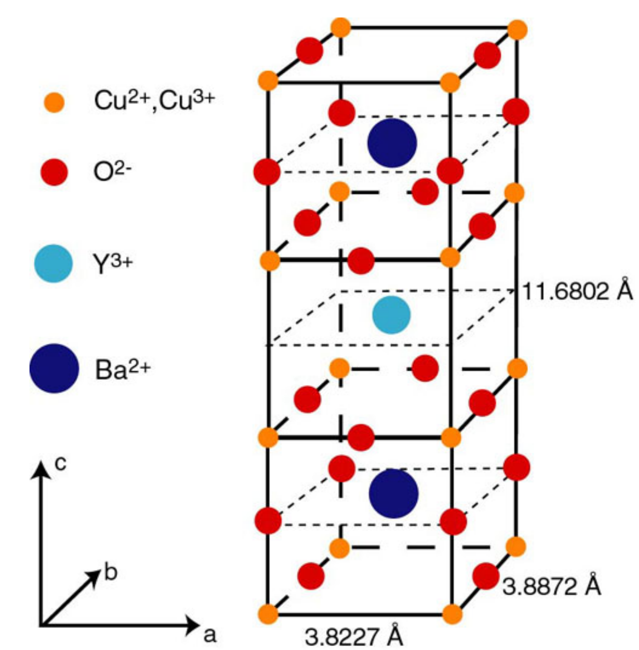

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Planckian metals with a remnant Fermi surface

- Resonant SYK models are compressible and dispersive quantum systems with $\hbar\omega/(k_B T)$ scaling as $T \to 0$.

- The resonance condition is supported by a RG argument: non-resonant interactions mainly renormalize the underlying quasi-particle dispersion ϵ_k, while resonant interactions have to be treated self-consistently.

- The resonance is a single 'fine-tuning' condition designed to obtain $\hbar\omega/(k_B T)$ scaling as $T \to 0$. However, then many other nice features follow: we obtain a Planckian metal with remnant large Fermi surface at $\epsilon_k = 0$, and an effective mass m^* defined by the dispersion of ϵ_k, with a resistivity $\rho \sim (m^*/(ne^2))k_B T/\hbar$ independent of the strength of interactions.

Aavishkar Patel (Harvard $\longrightarrow$ Miller Fellow at Berkeley)

A.A. Patel and S. Sachdev, PRL **123**, 066601 (2019)