The phase diagrams of the high temperature superconductors

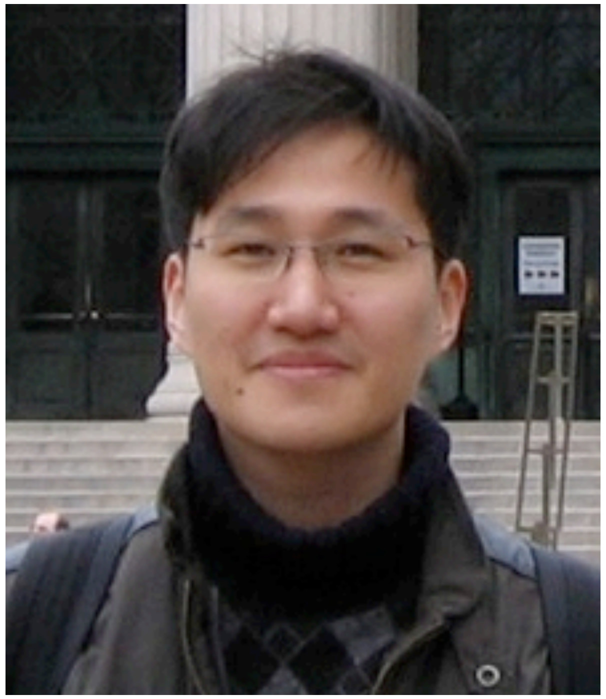
Talk online: sachdev.physics.harvard.edu

PHYSICS

VE RI TAS

HARVARD

Max Metlitski, Harvard



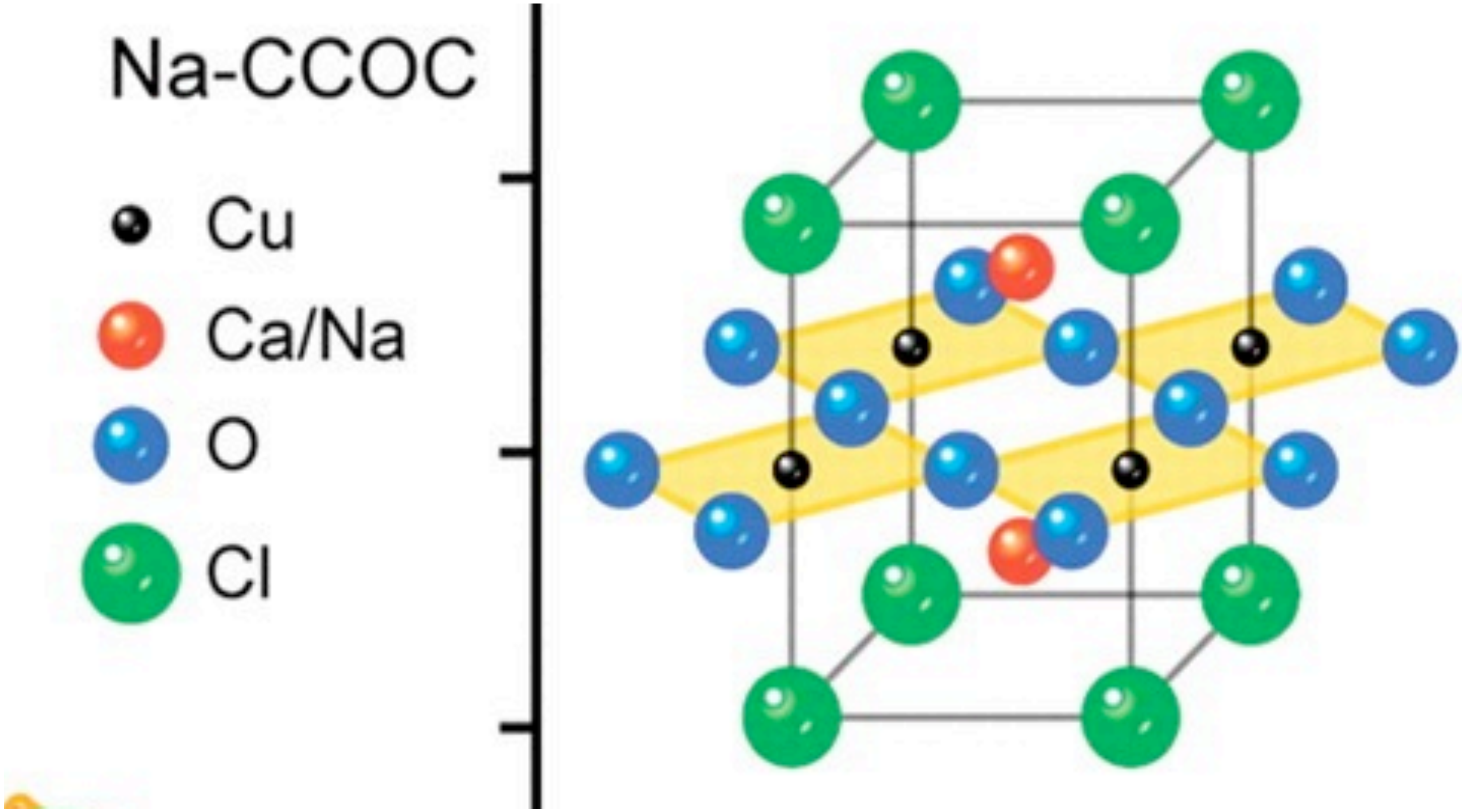
Eun Gook Moon, Harvard

PHYSICS

VE RI TAS

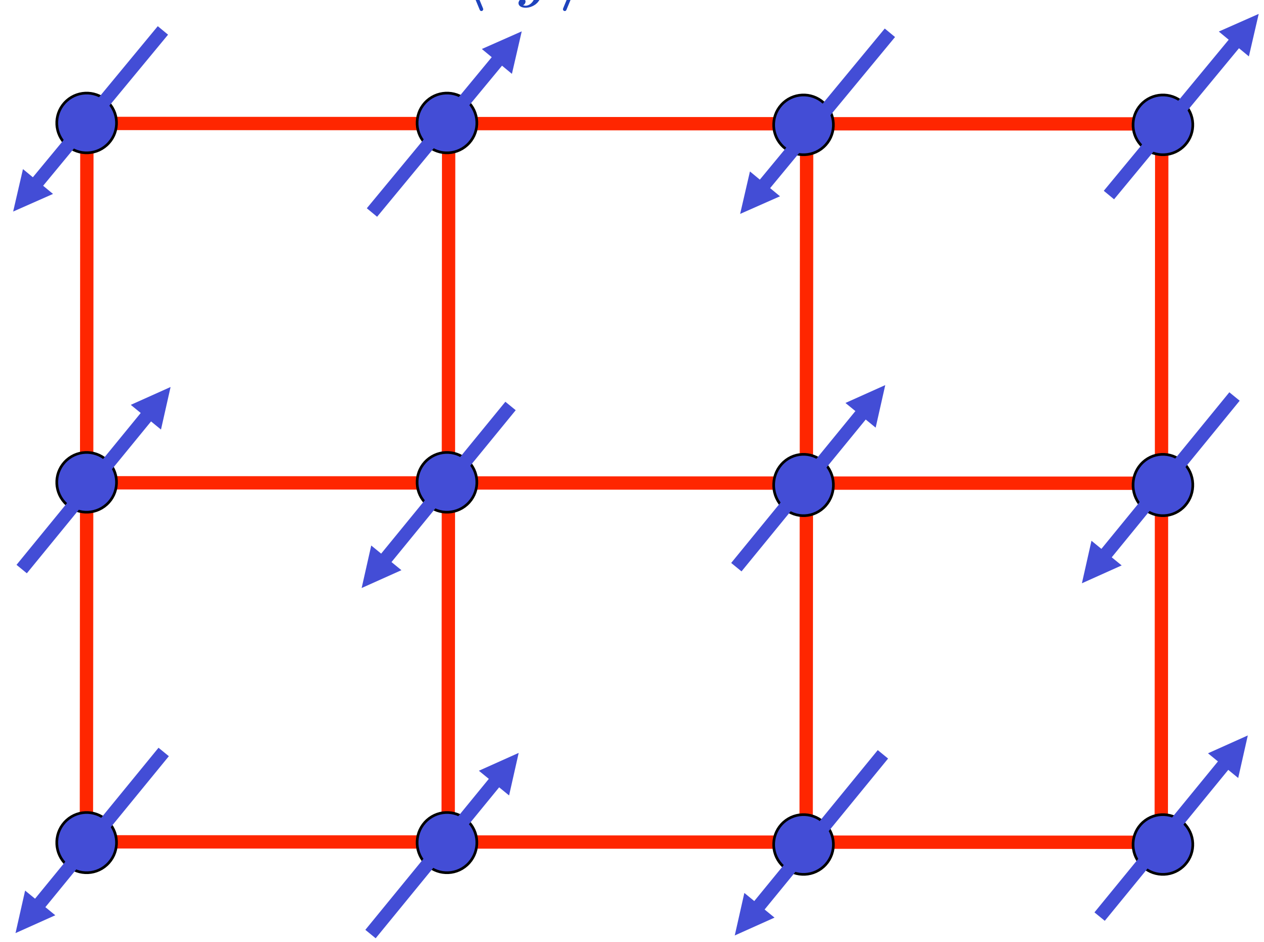
HARVARD

The cuprate superconductors



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



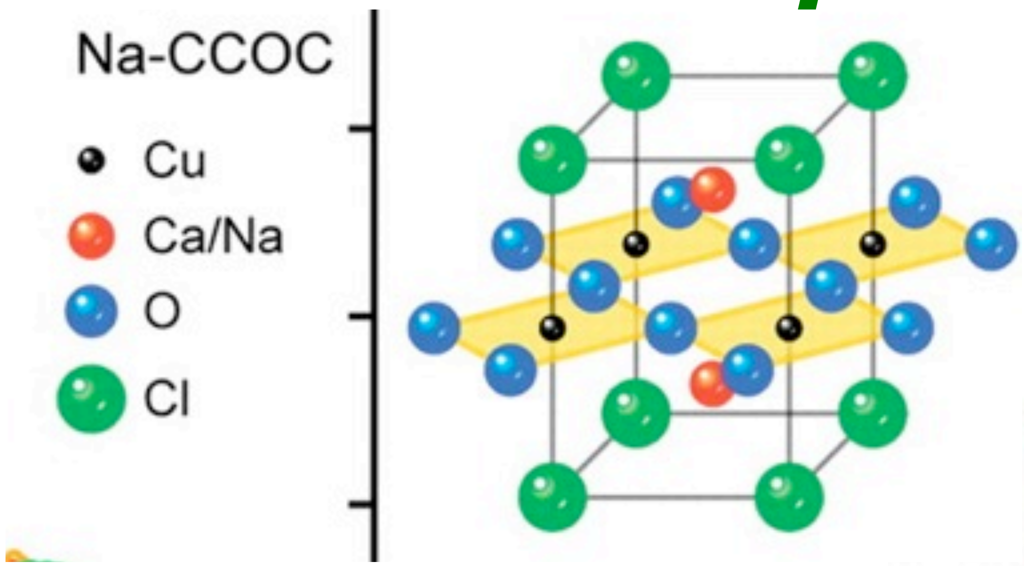
Ground state has long-range Néel order

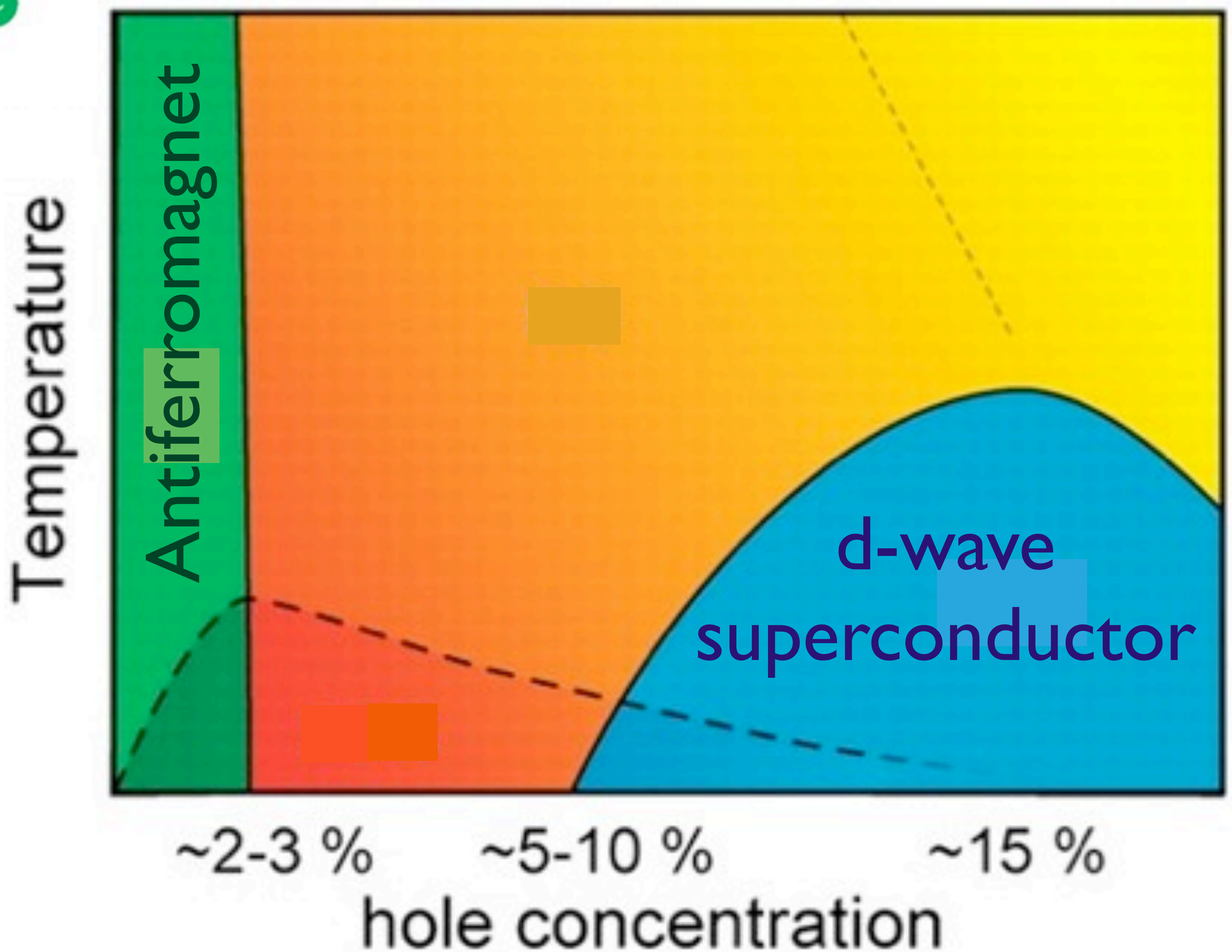
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

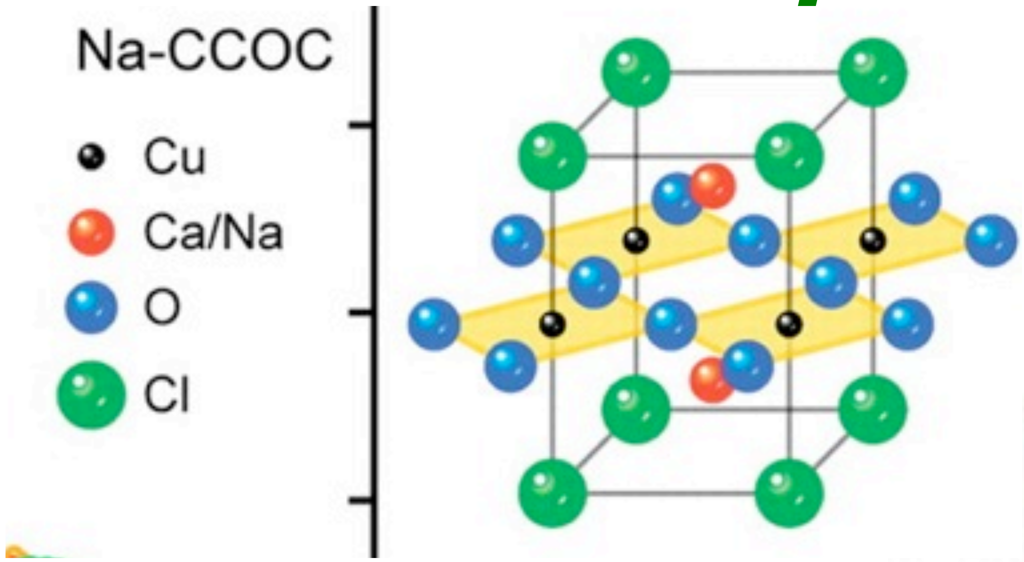
$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

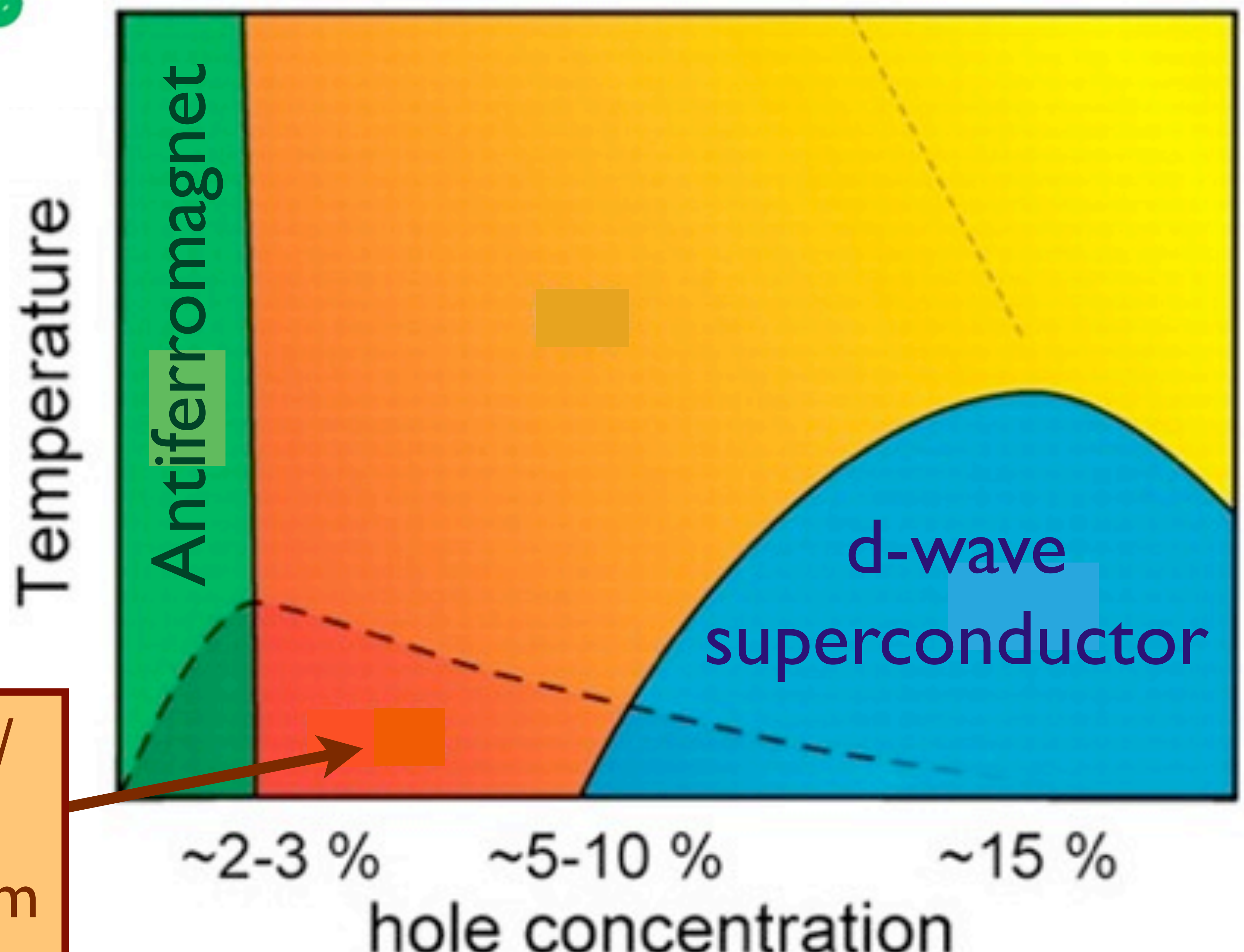
The cuprate superconductors





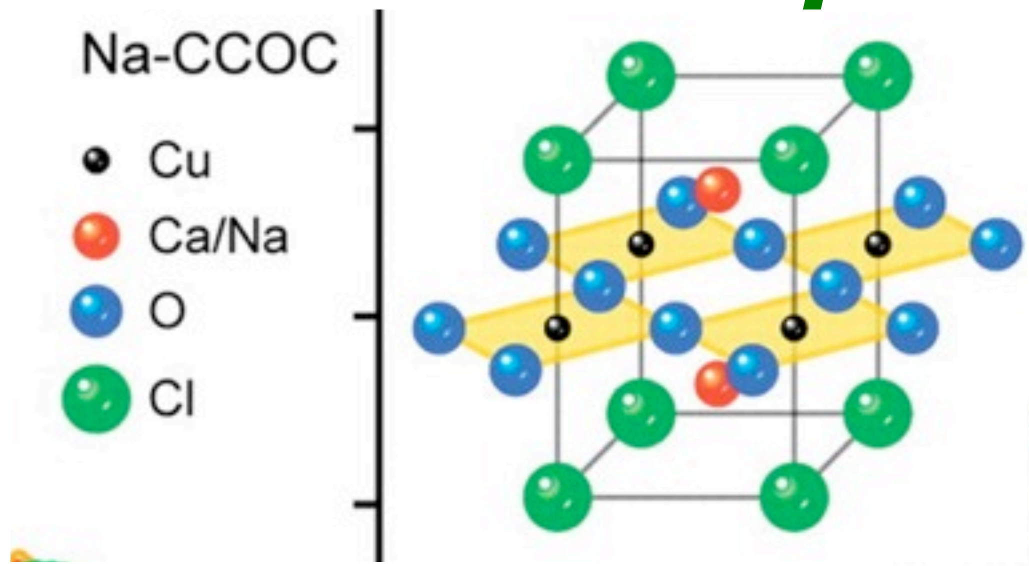
The cuprate superconductors

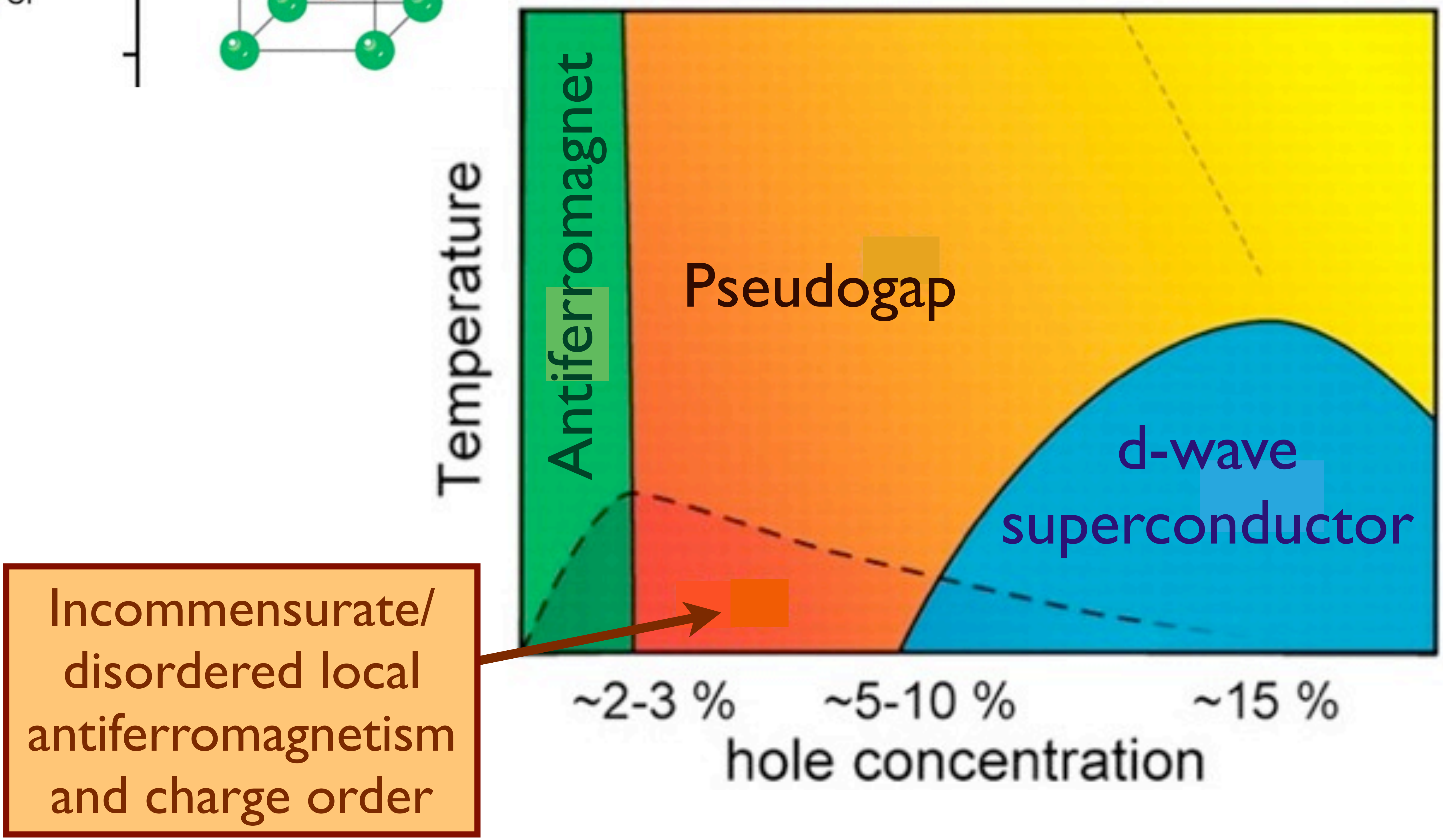




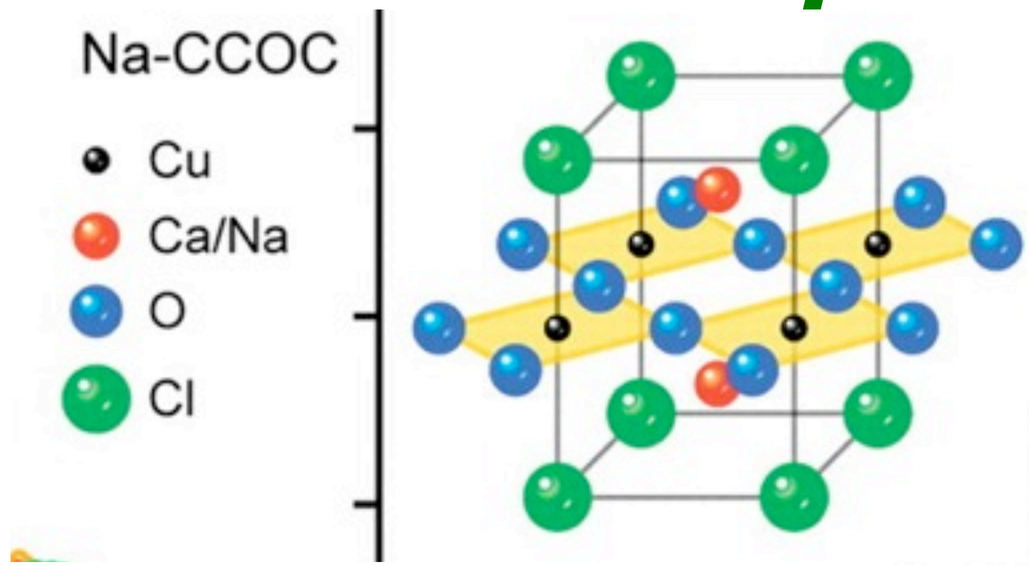
Incommensurate/
disordered
antiferromagnetism
and charge order

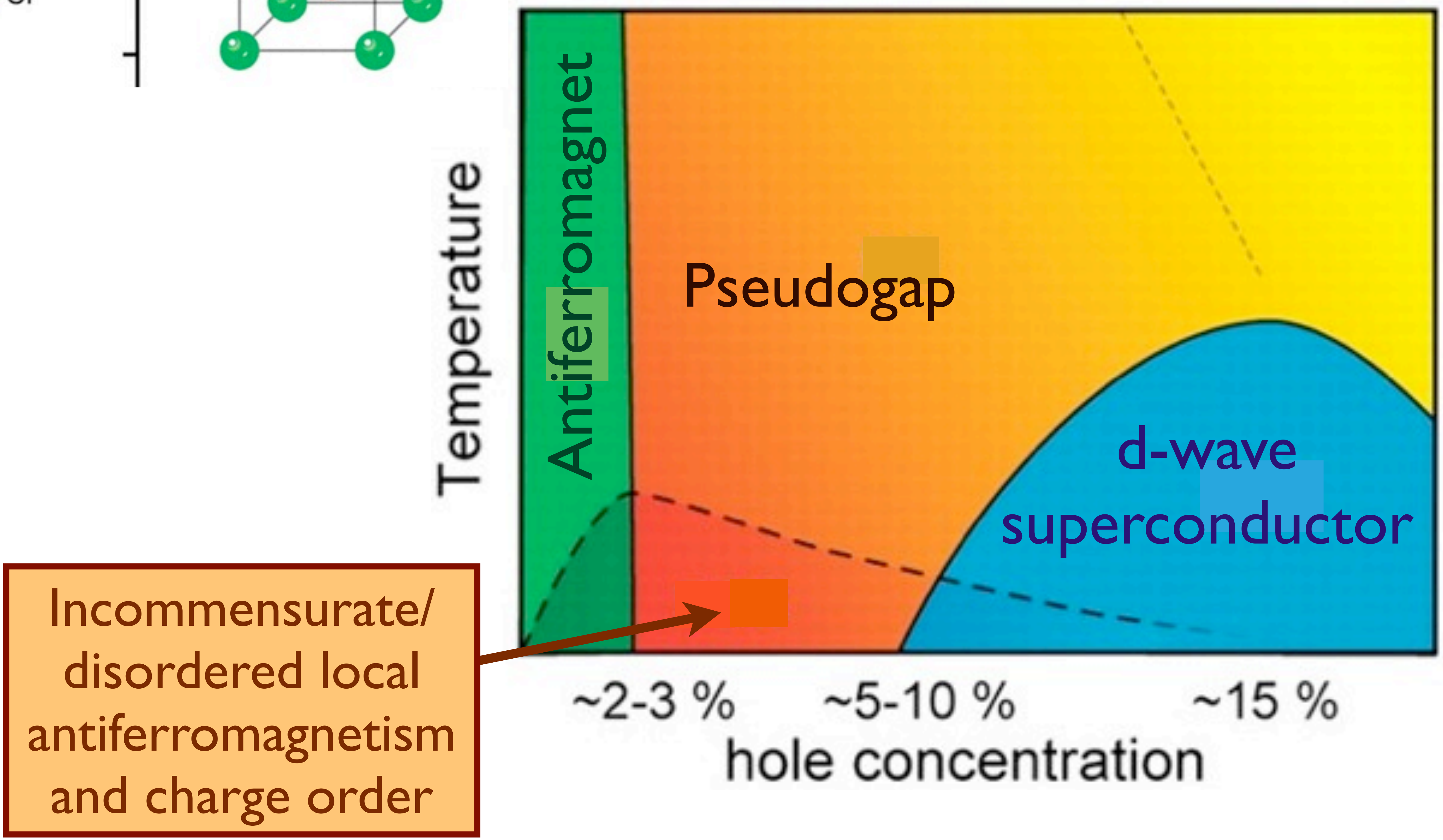
The cuprate superconductors



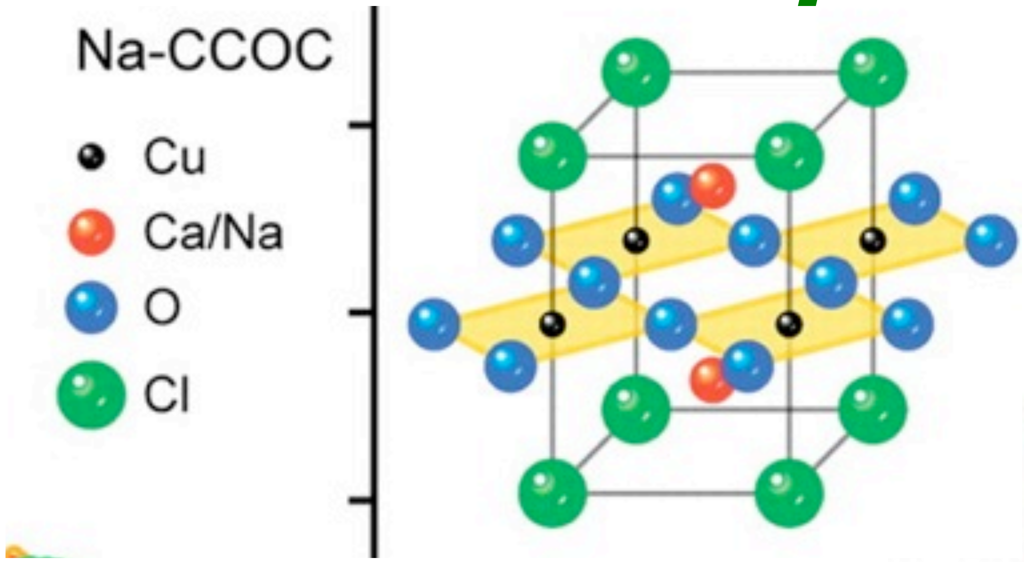


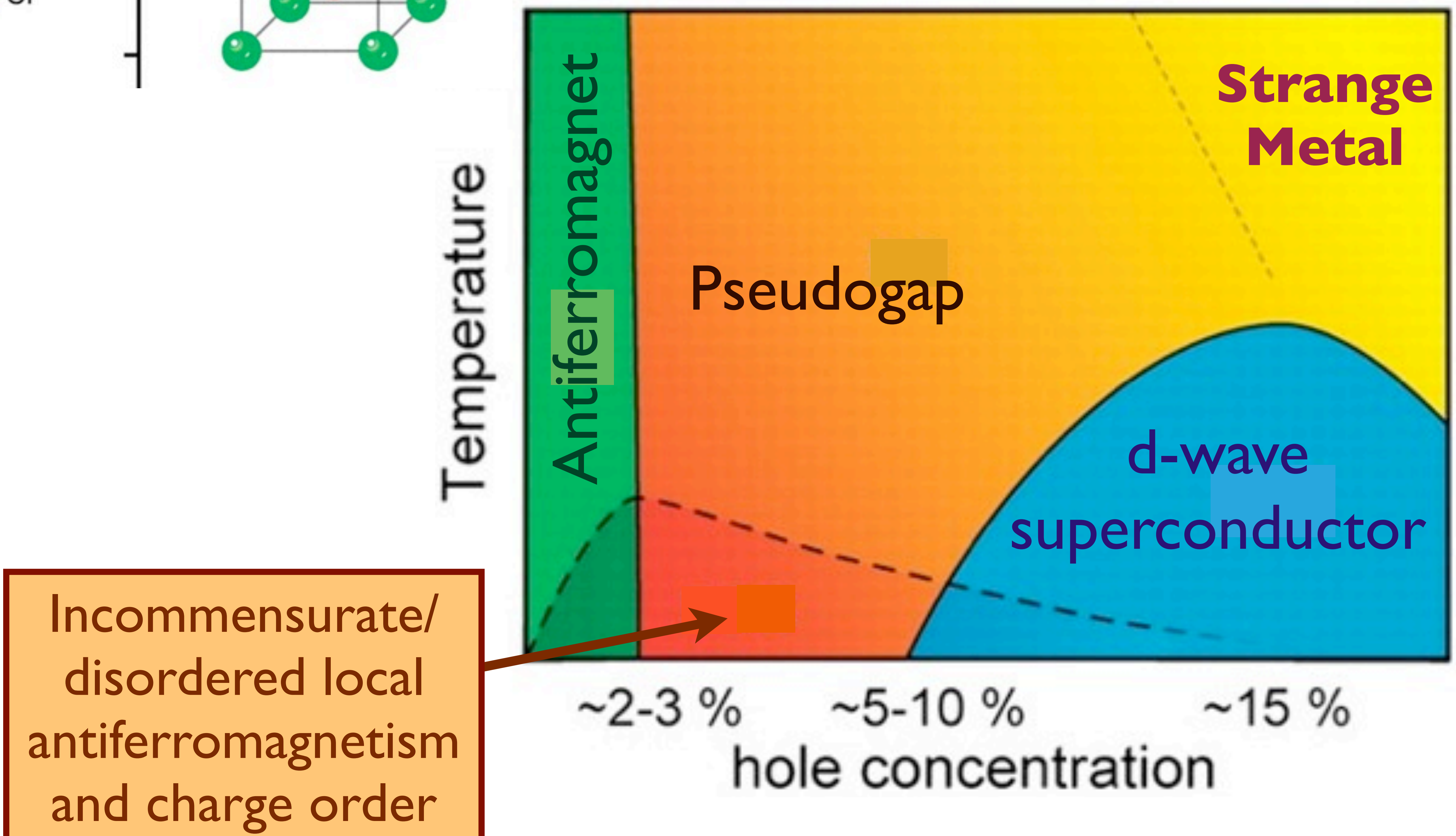
The cuprate superconductors



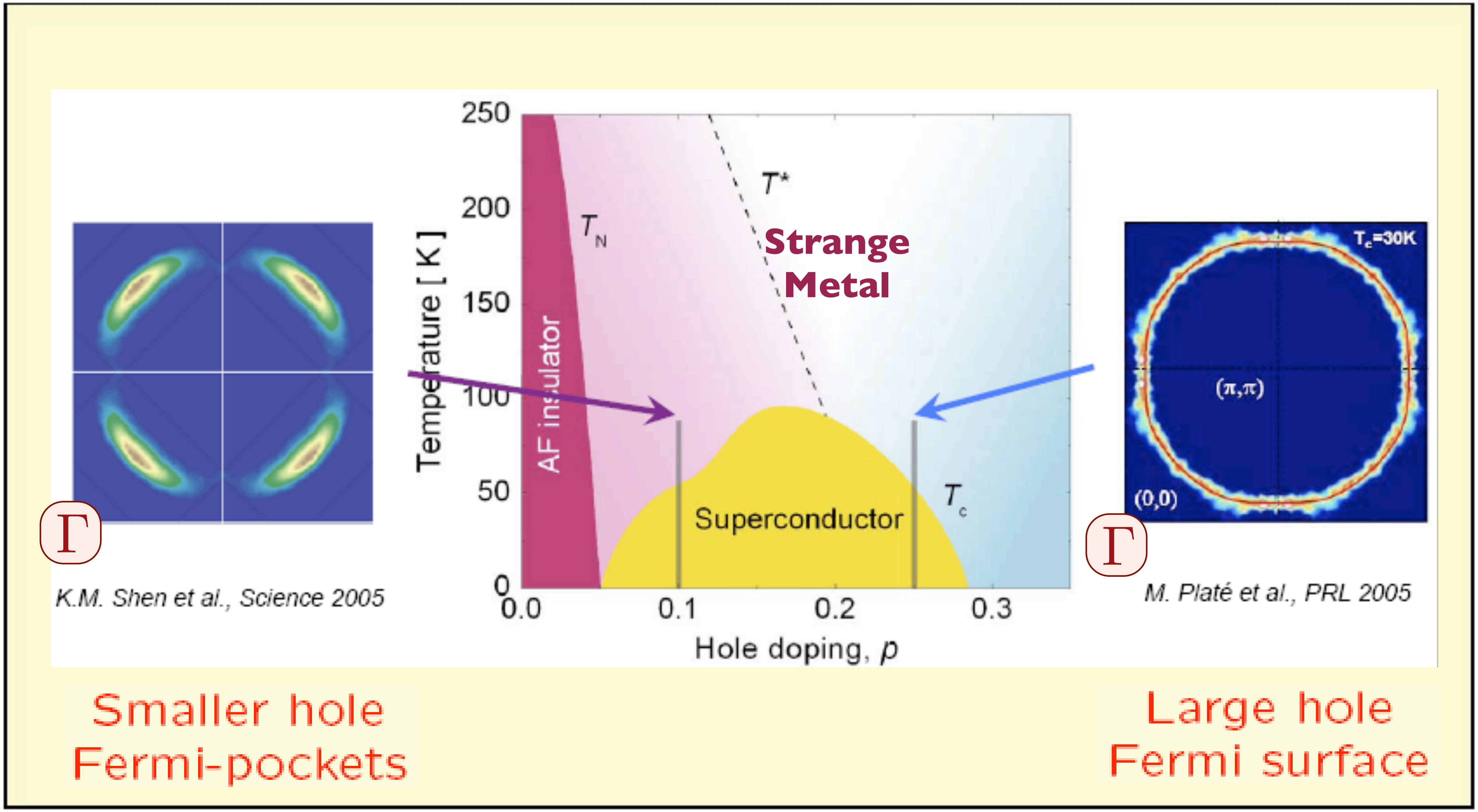


The cuprate superconductors



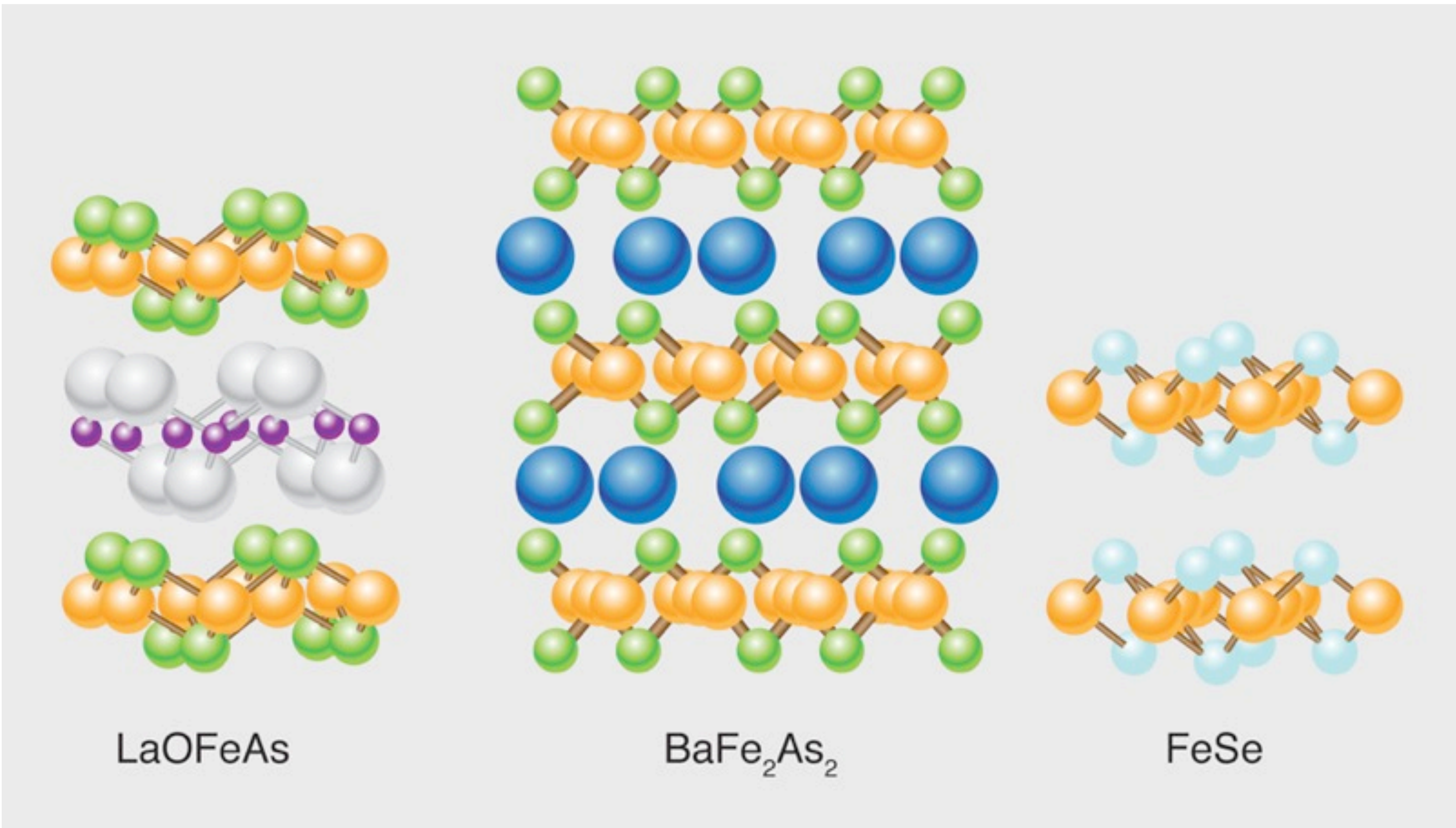


Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



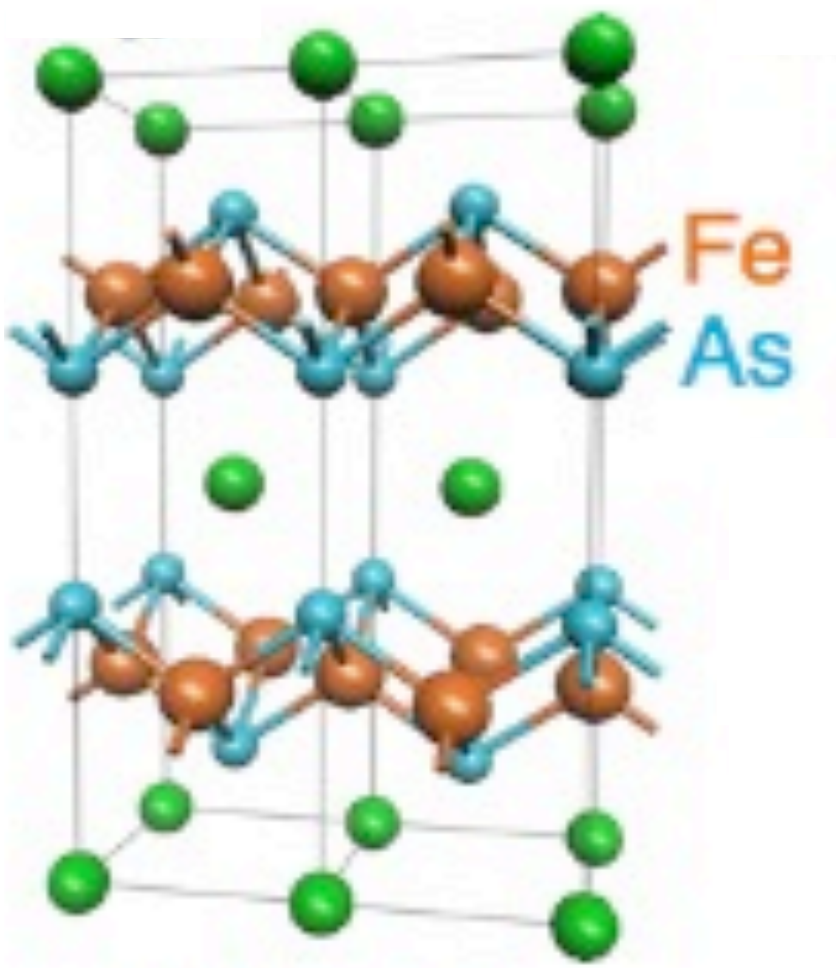
Iron pnictides:

a new class of high temperature superconductors

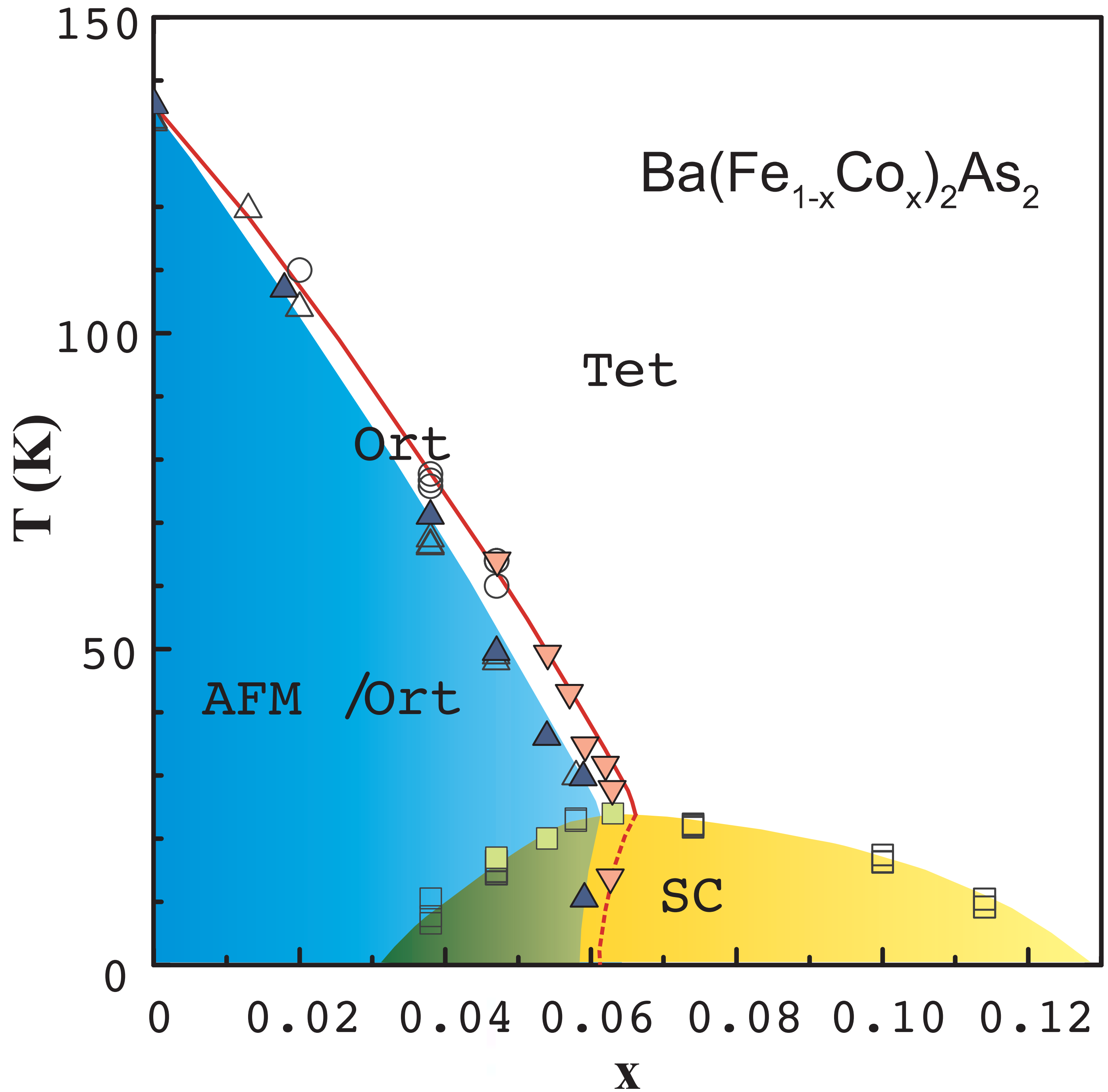


Iron pnictides:

a new class of high temperature superconductors

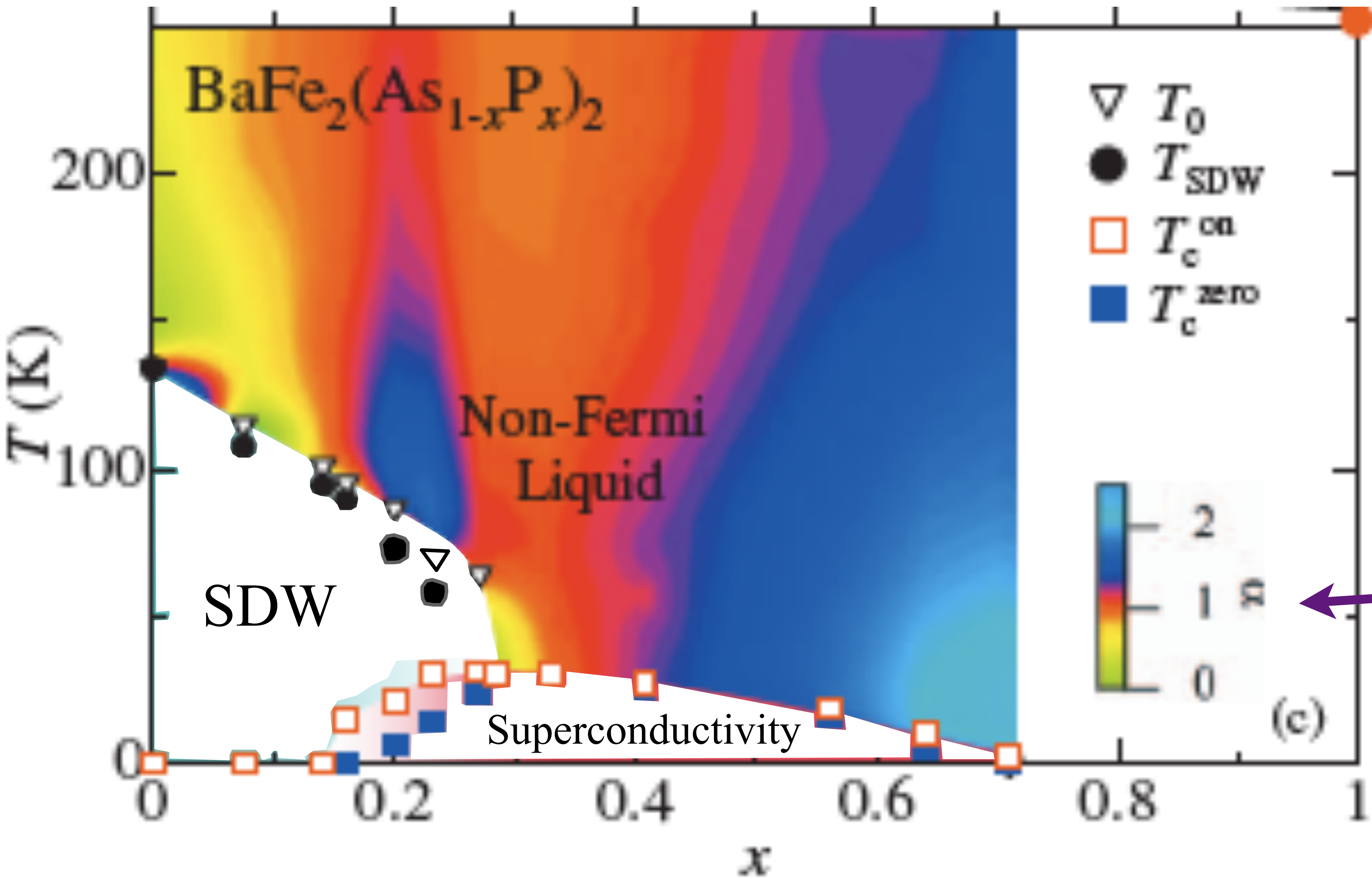


$BaFe_2As_2$
Unit cell



S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

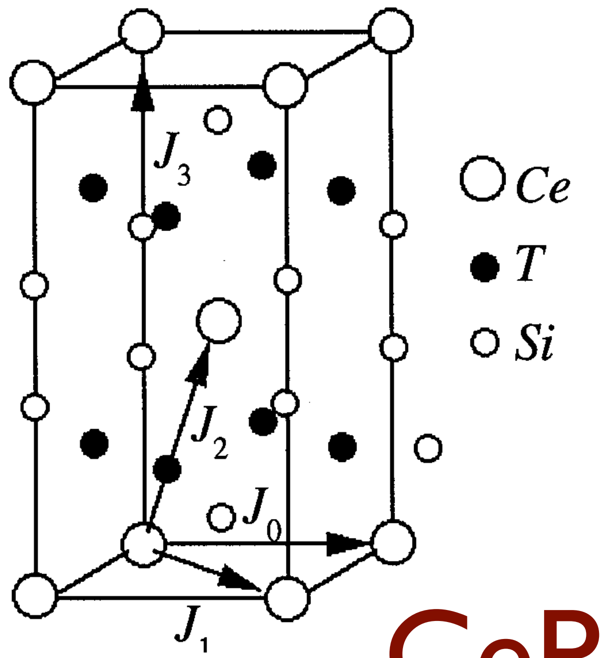
Temperature-doping phase diagram of the iron pnictides:



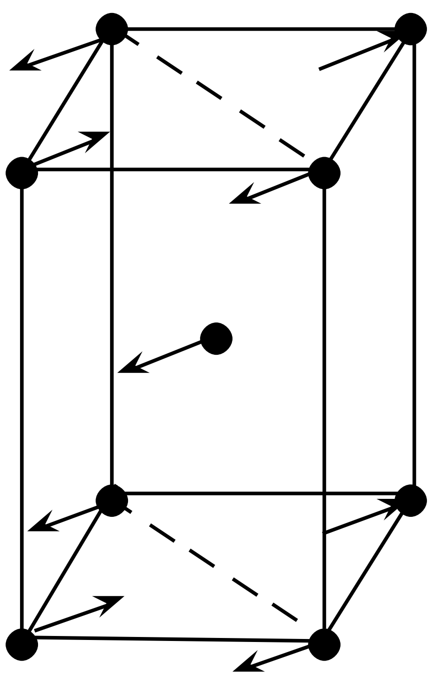
Resistivity $\sim \rho_0 + AT^\alpha$

S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

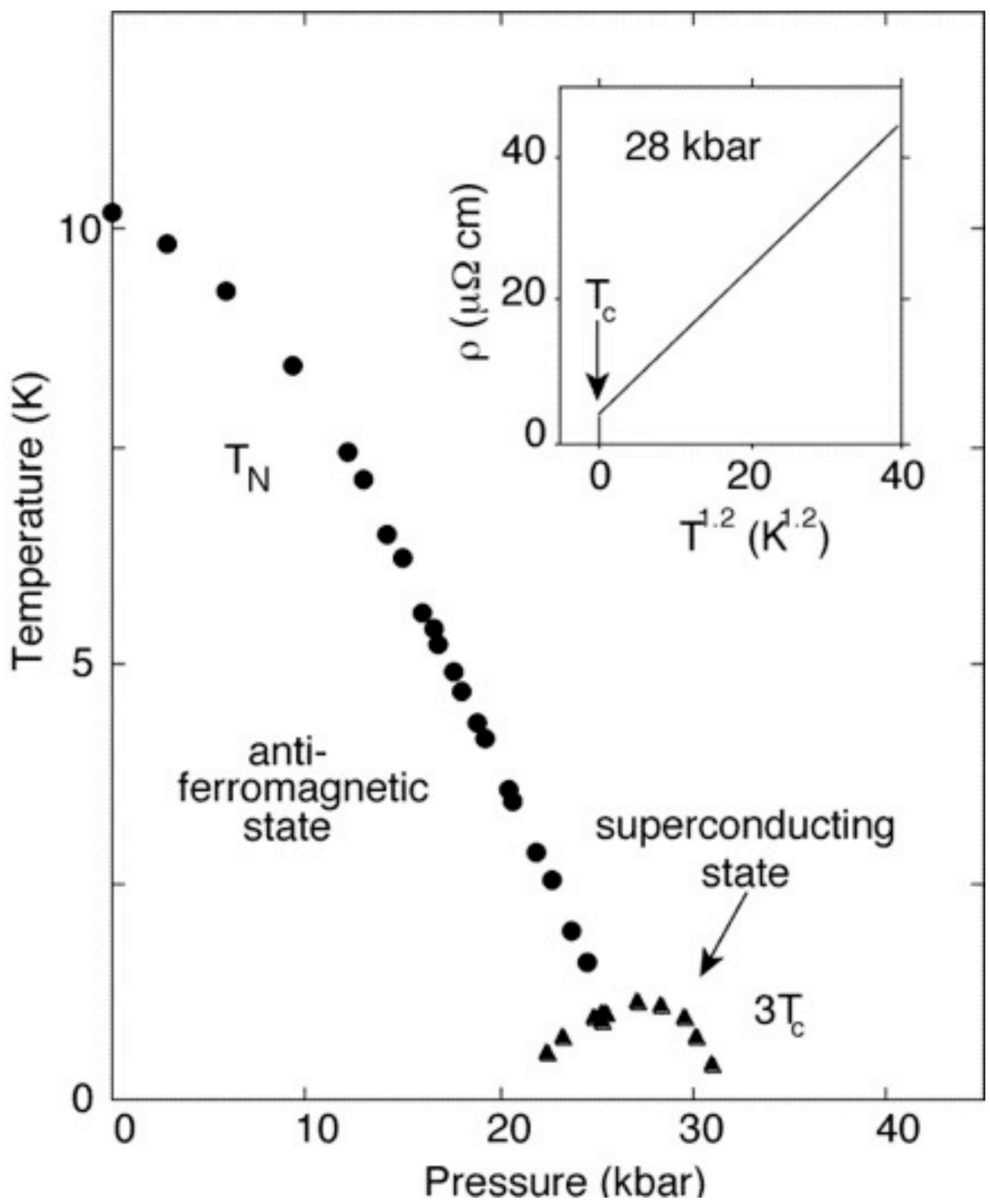
Lower T_c superconductivity in the heavy fermion compounds



a

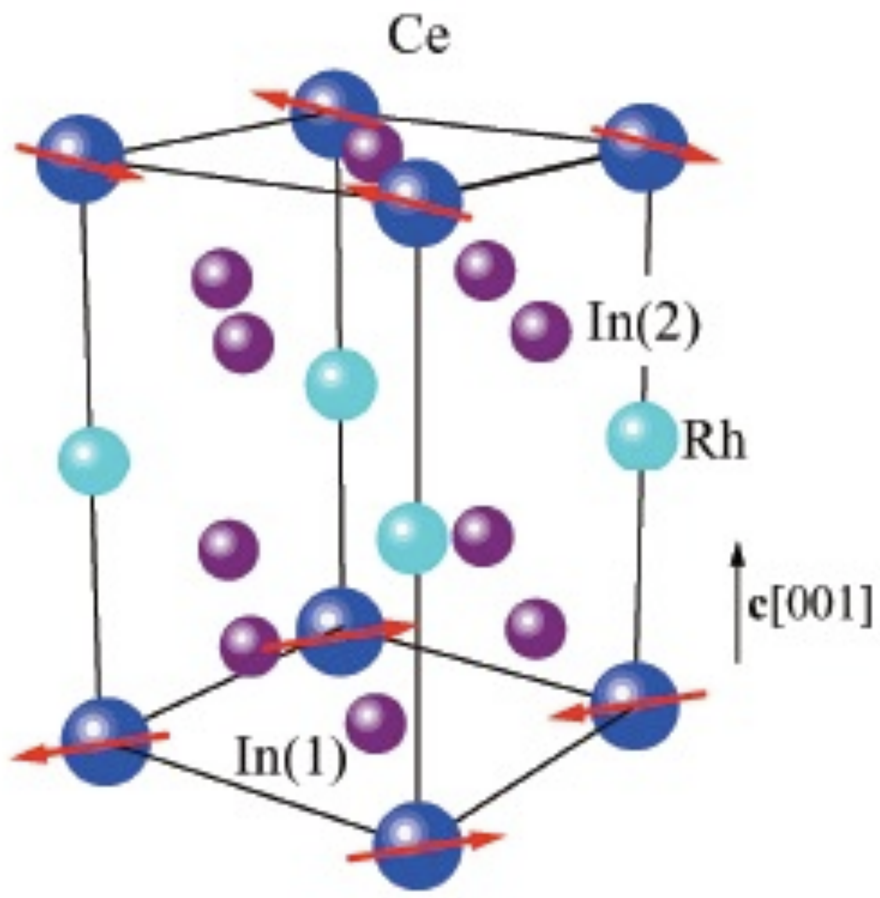


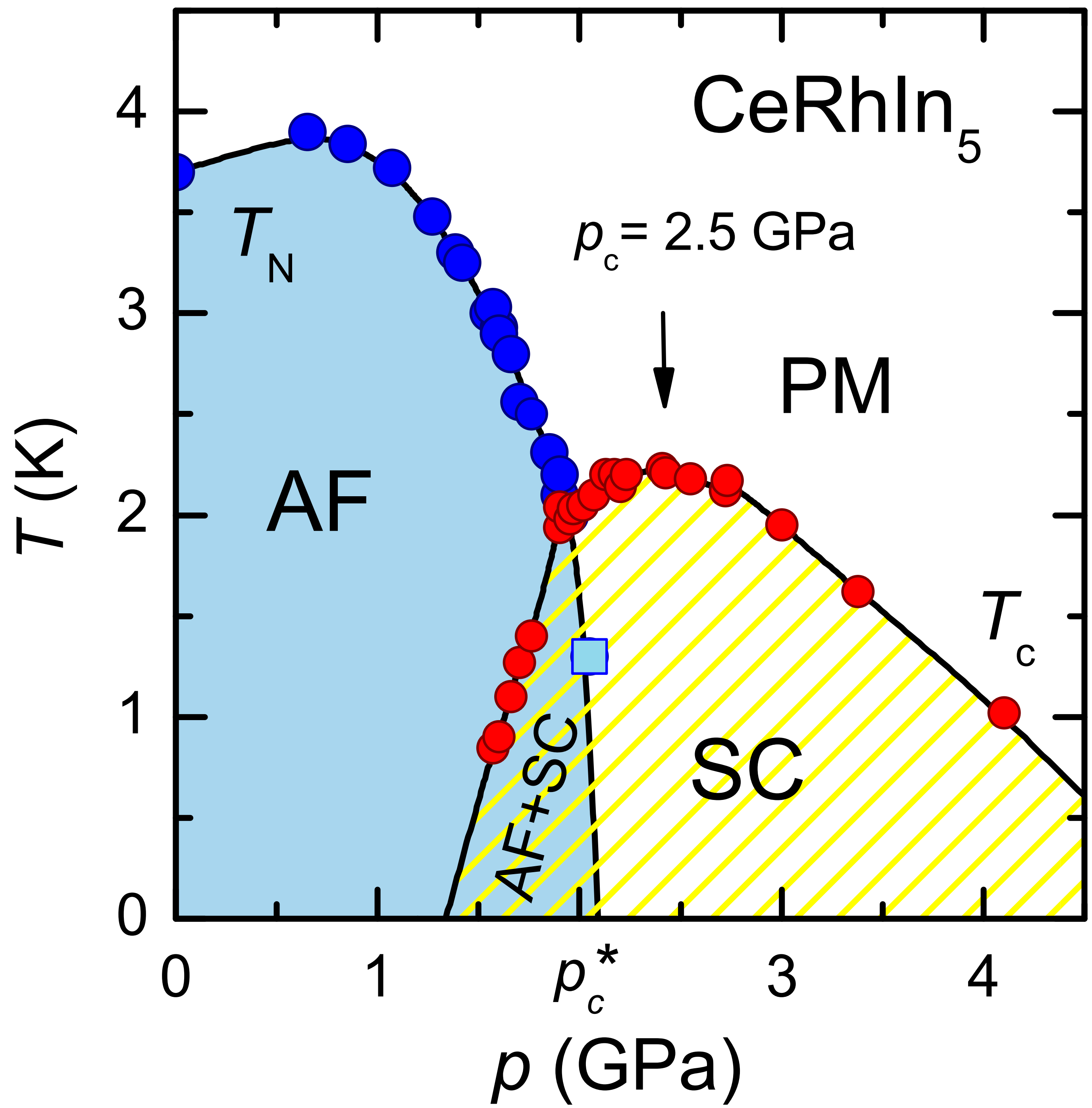
$CePd_2Si_2$



N. D. Mathur, F. M. Grosche, S. R. Julian, I. R. Walker, D. M. Freye, R. K. W. Haselwimmer, and G. G. Lonzarich, Nature **394**, 39 (1998)

Lower T_c superconductivity in the heavy fermion compounds





G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

- *If so, why is there no antiferromagnetism in the cuprates near the point where the superconductivity is strongest ?*

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

- *If so, why is there no antiferromagnetism in the cuprates near the point where the superconductivity is strongest ?*

- *What is the physics of the strange metal ?*

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

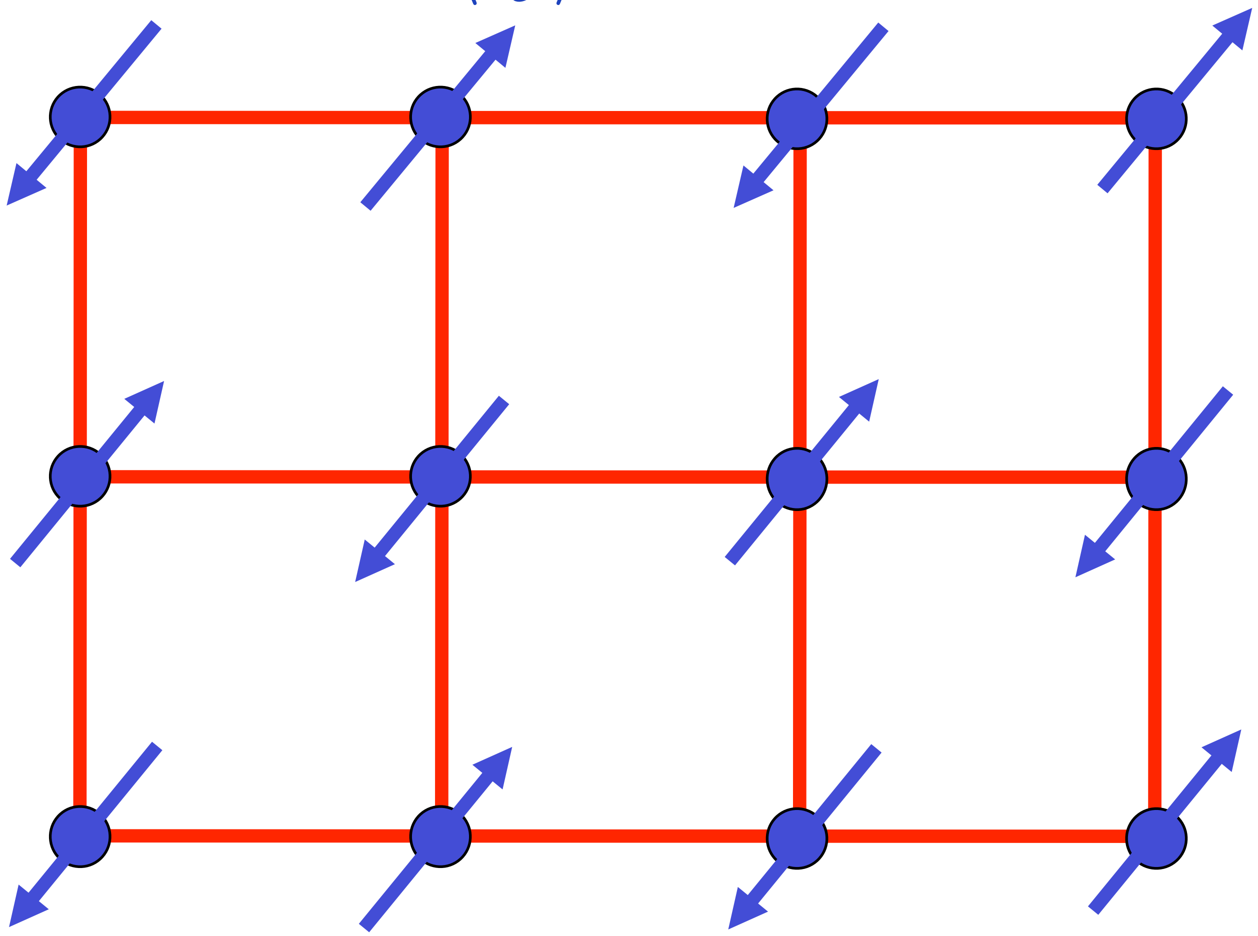
3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

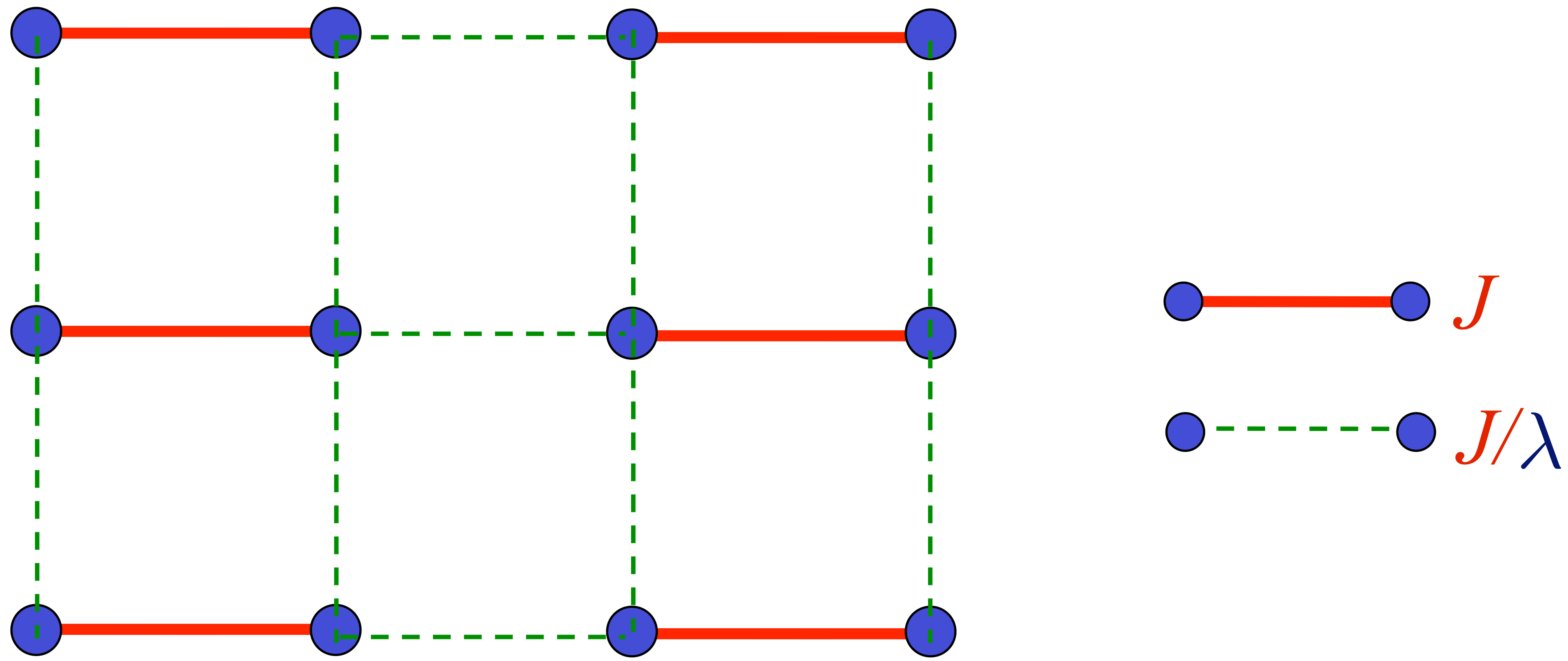
Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

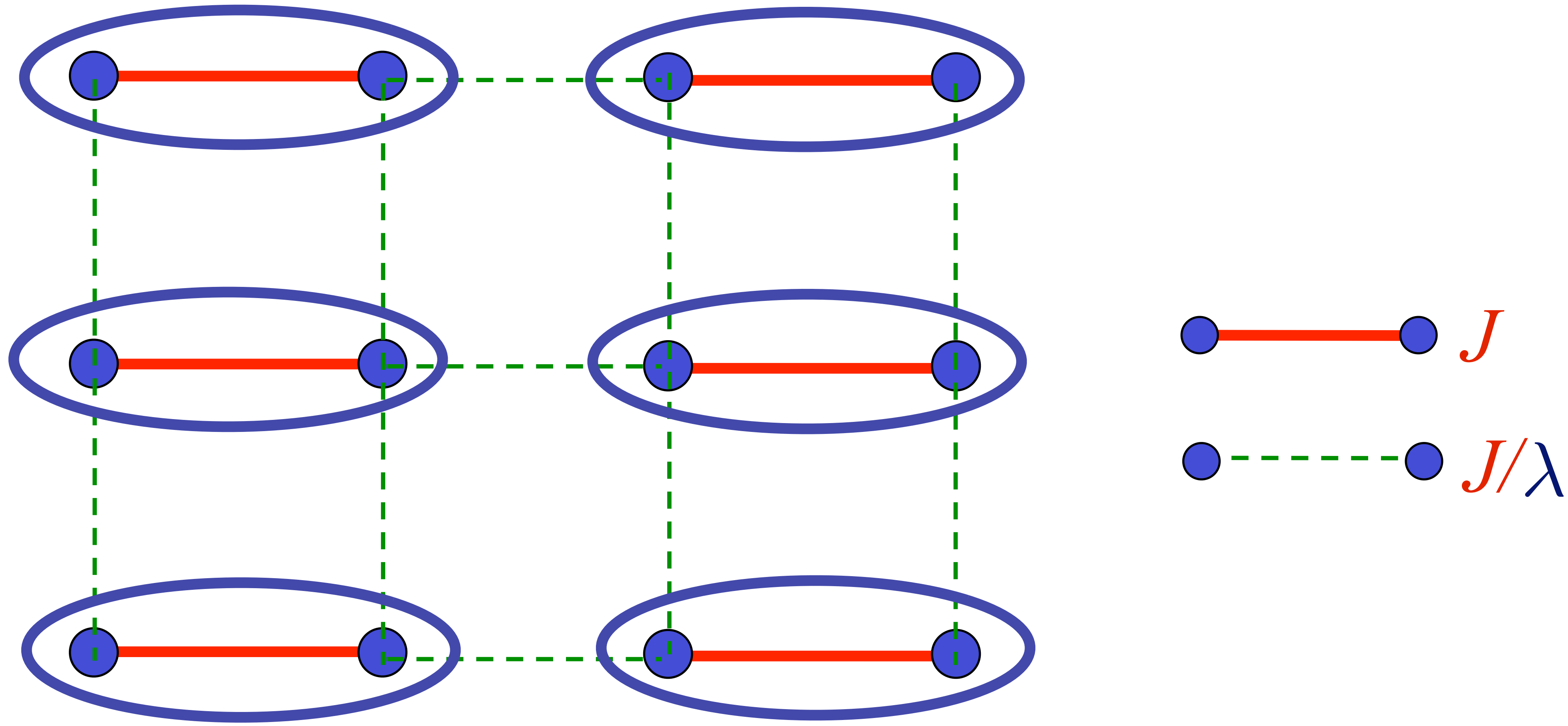
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



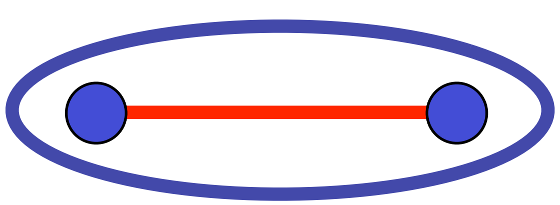
Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

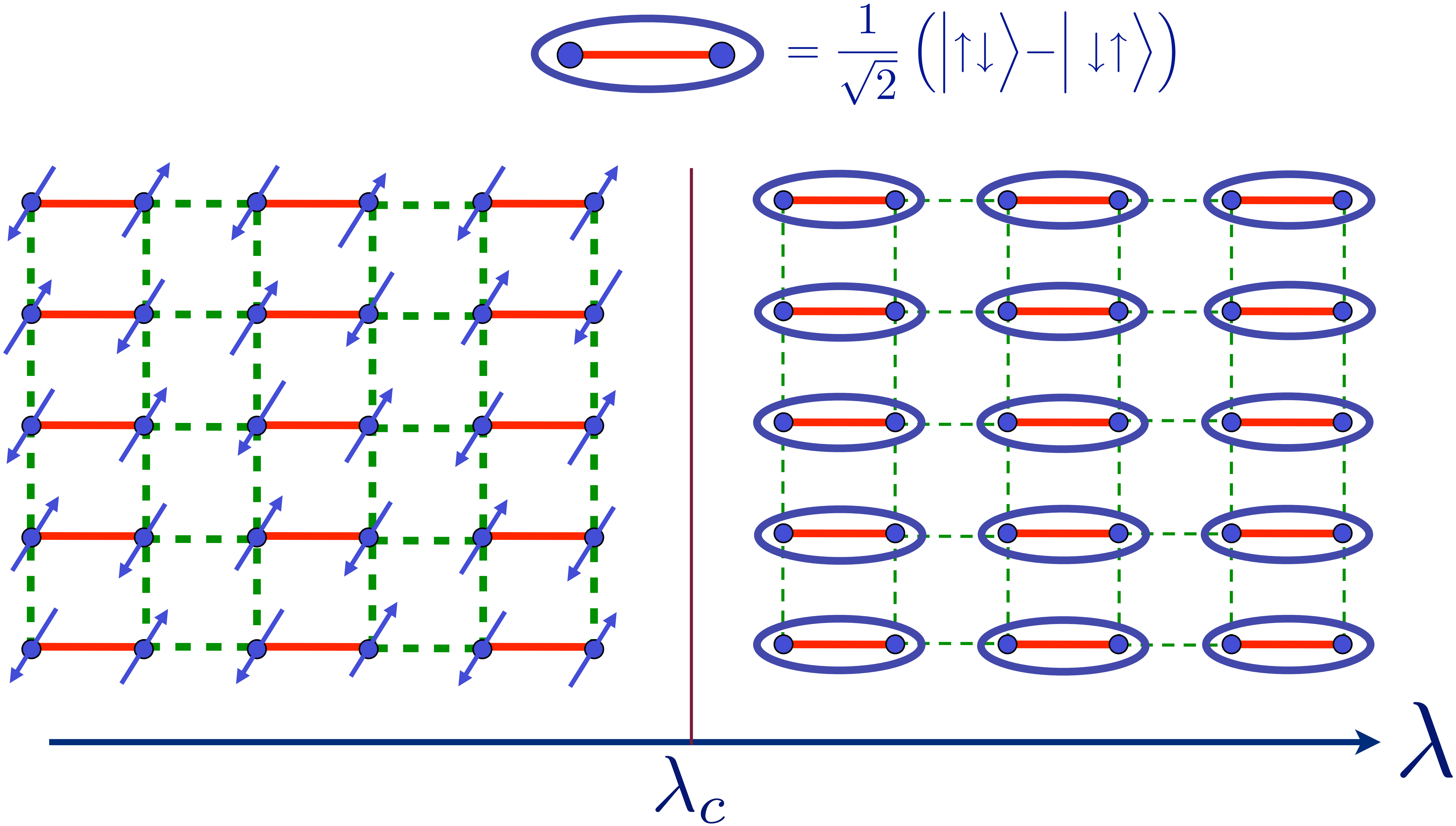
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



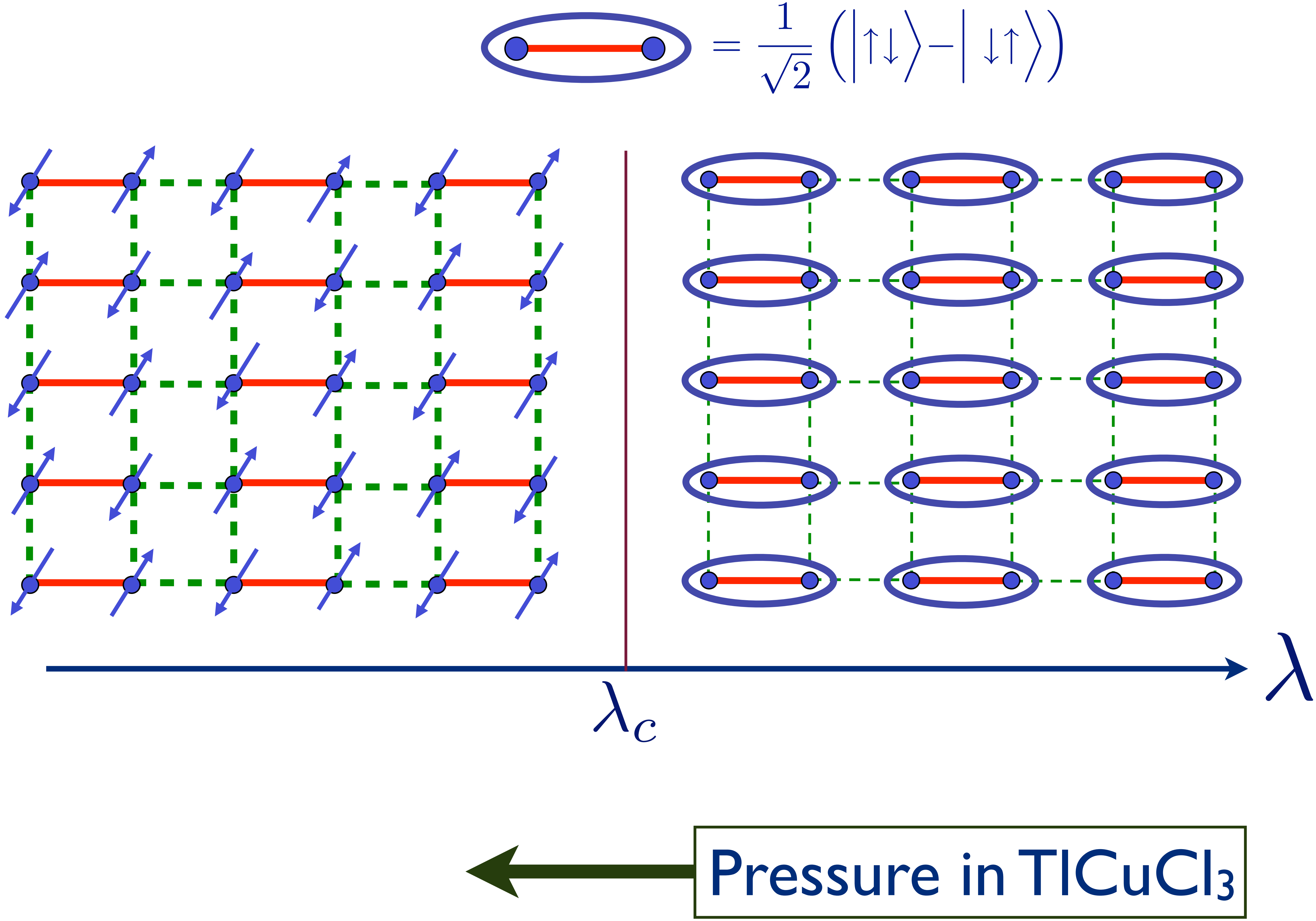
Ground state is a "quantum paramagnet" with spins locked in valence bond singlets



$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

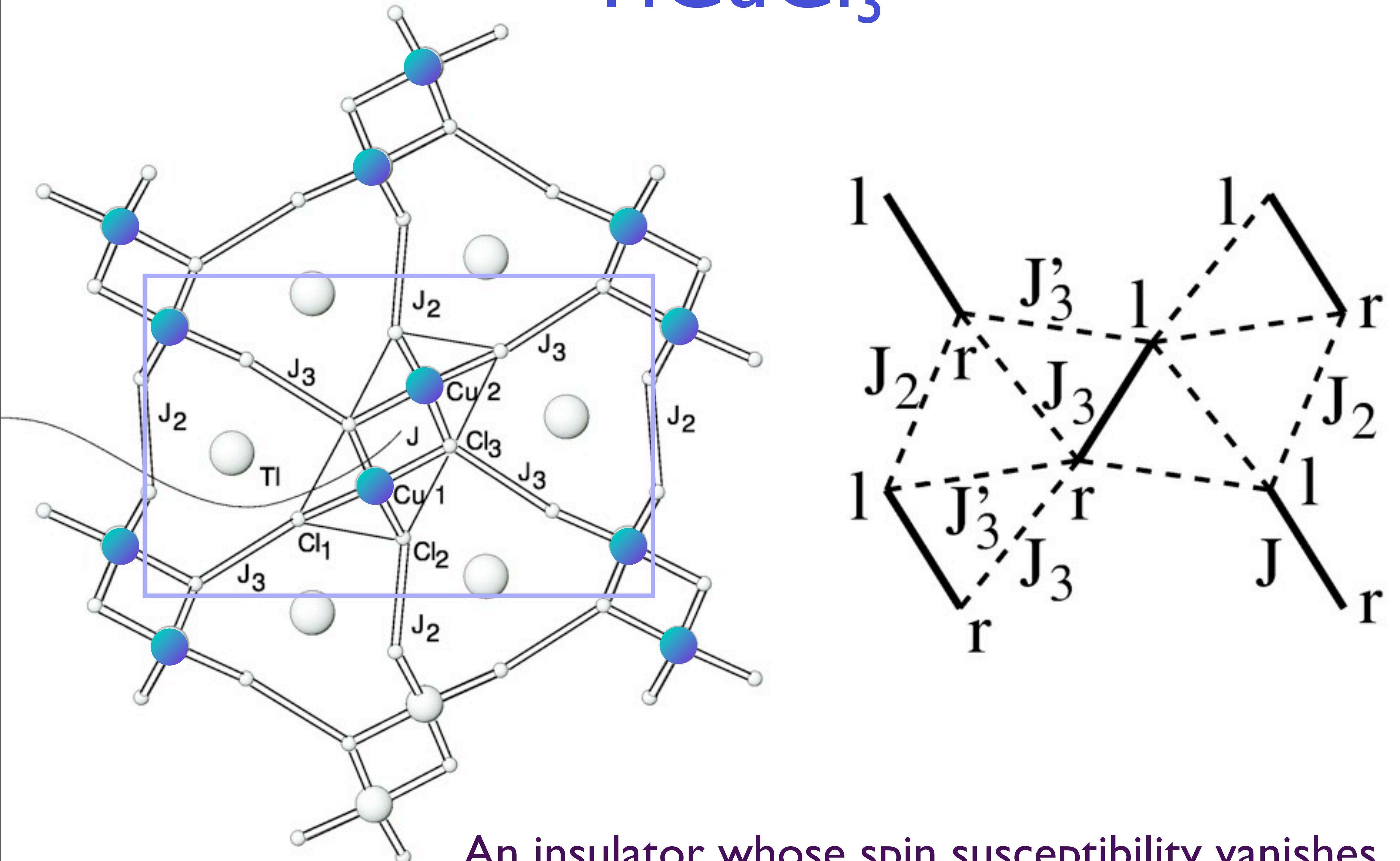


$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$
λ_c
λ



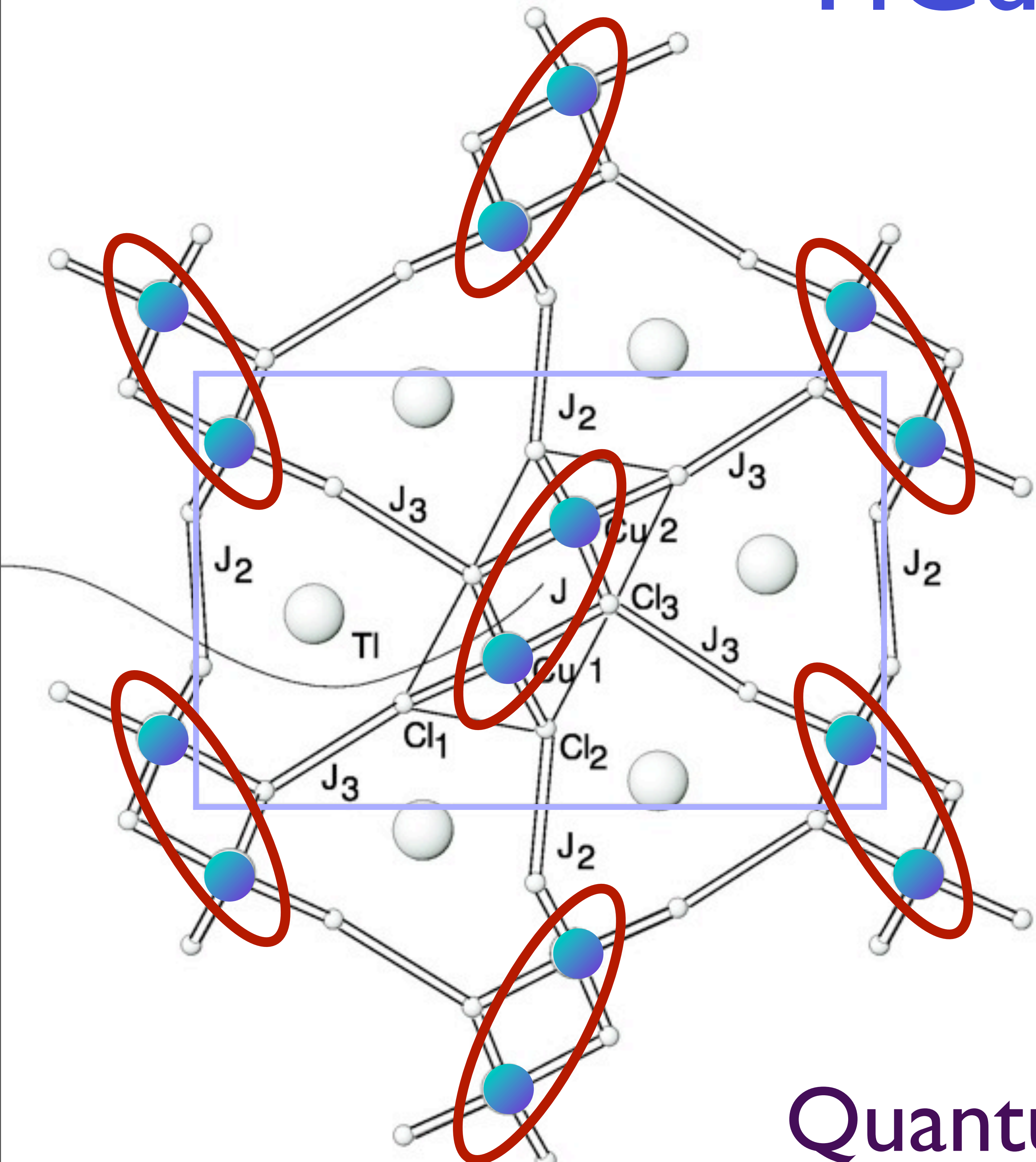
A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).

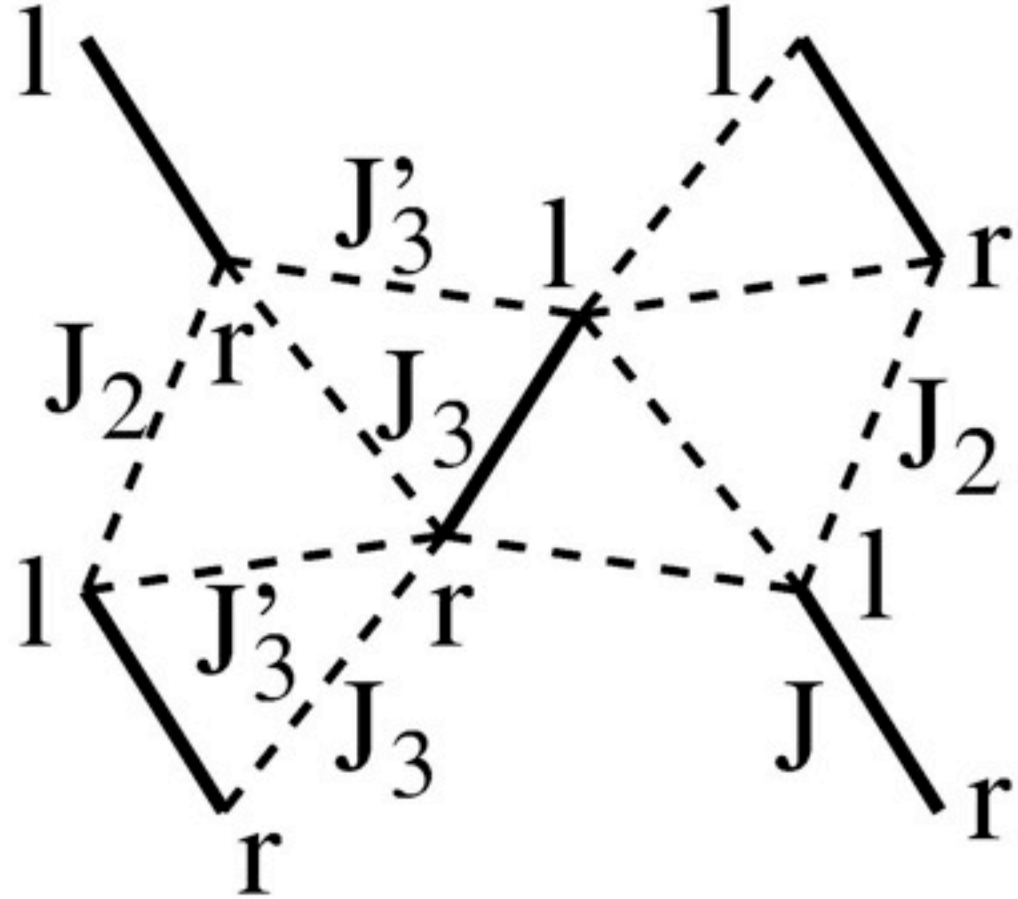
$TlCuCl_3$



An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

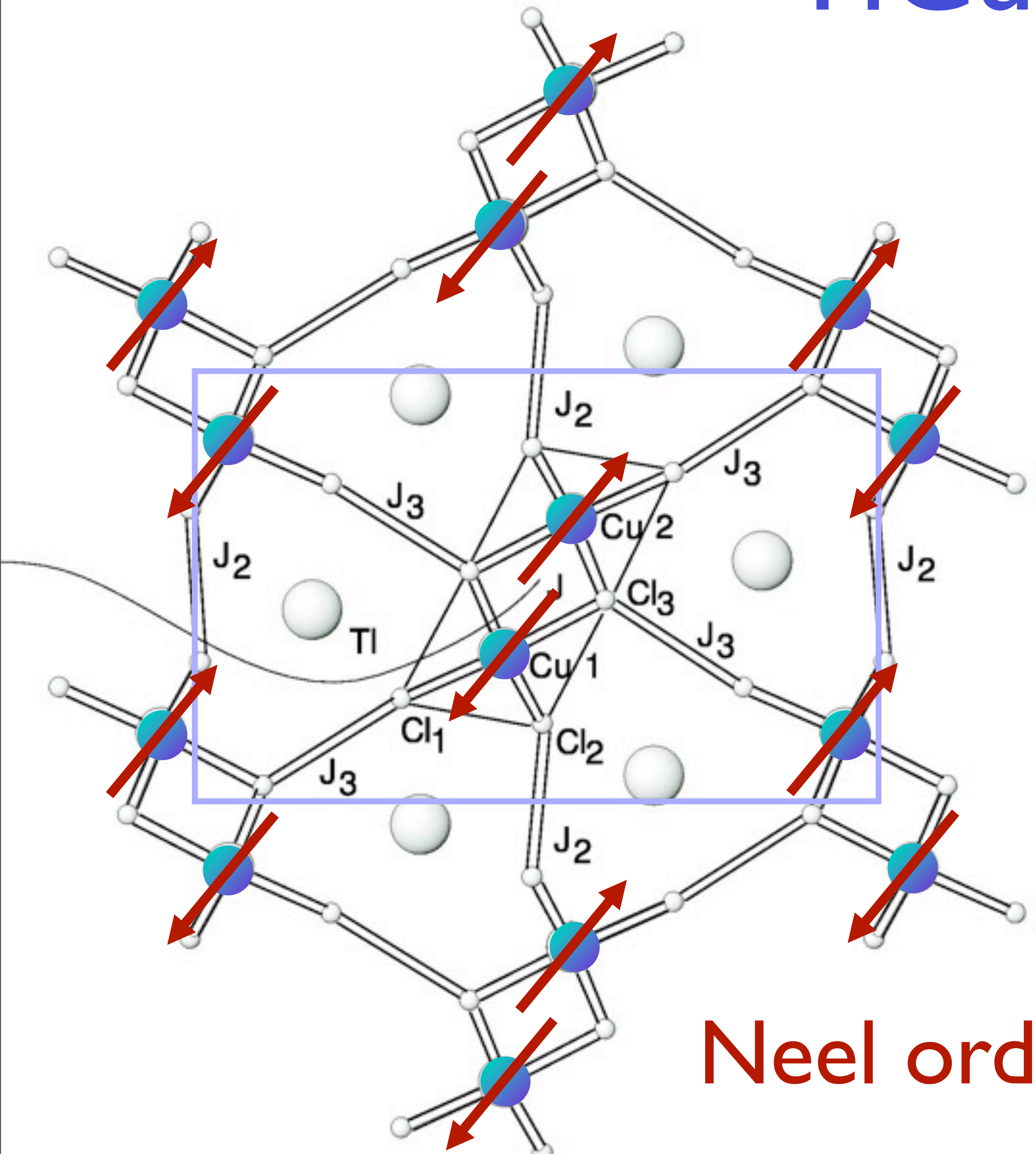
$TlCuCl_3$

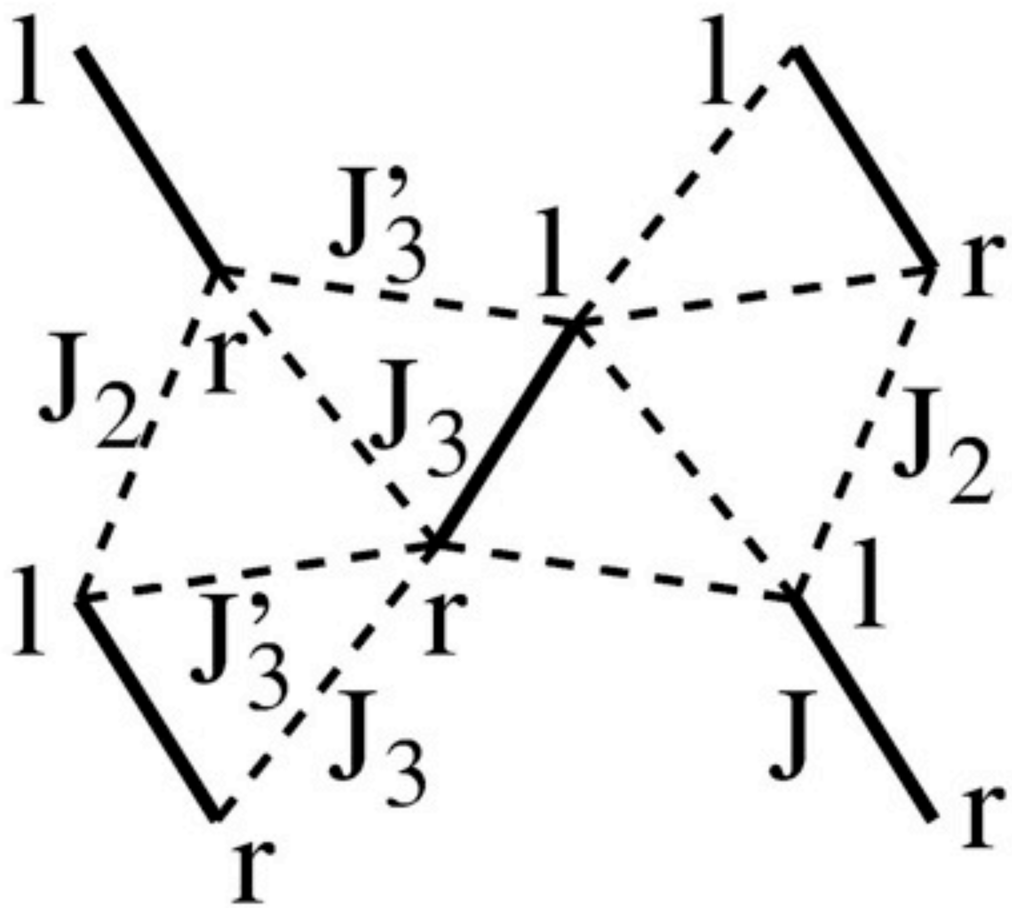




Quantum paramagnet at ambient pressure

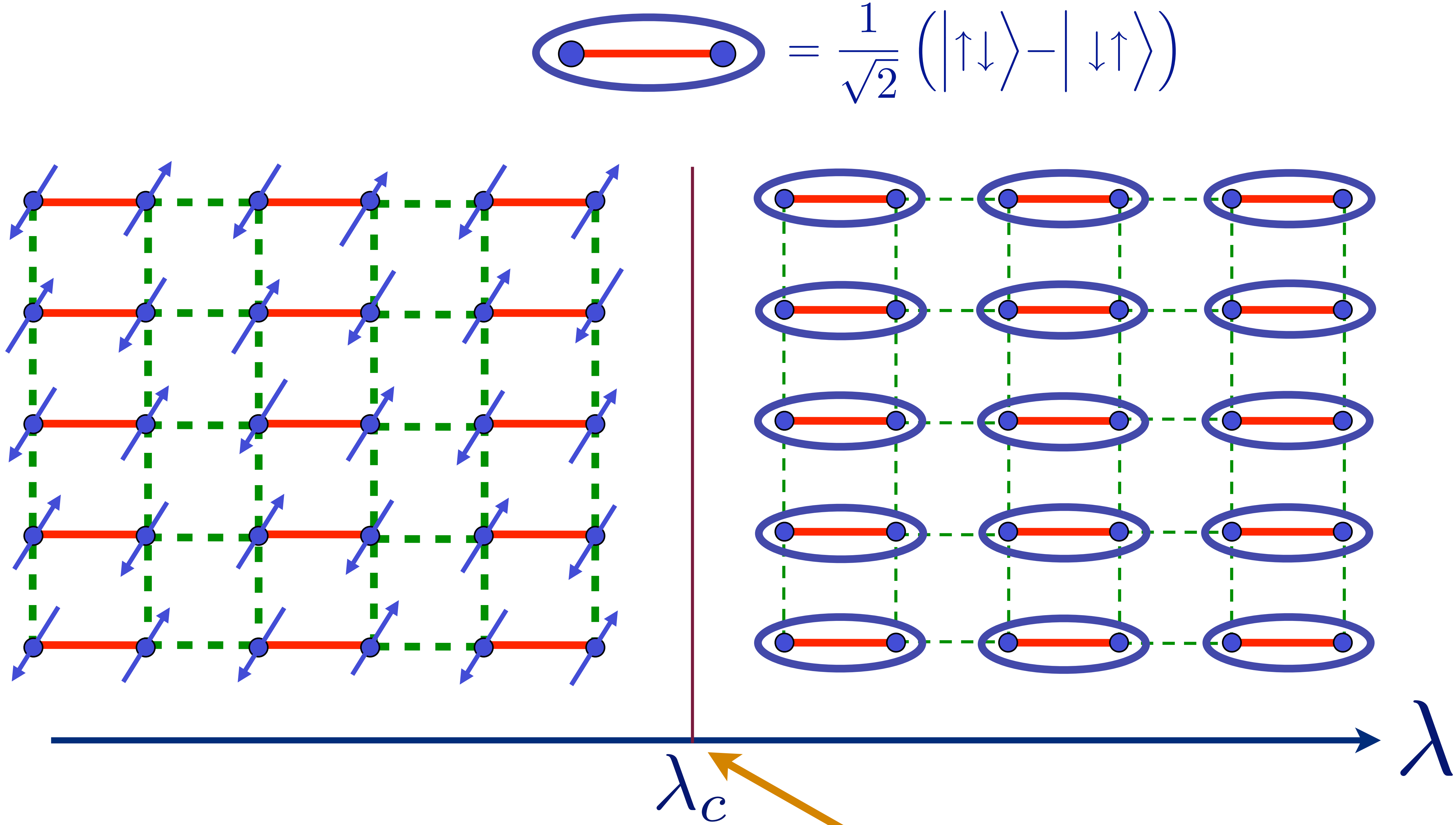
$TlCuCl_3$





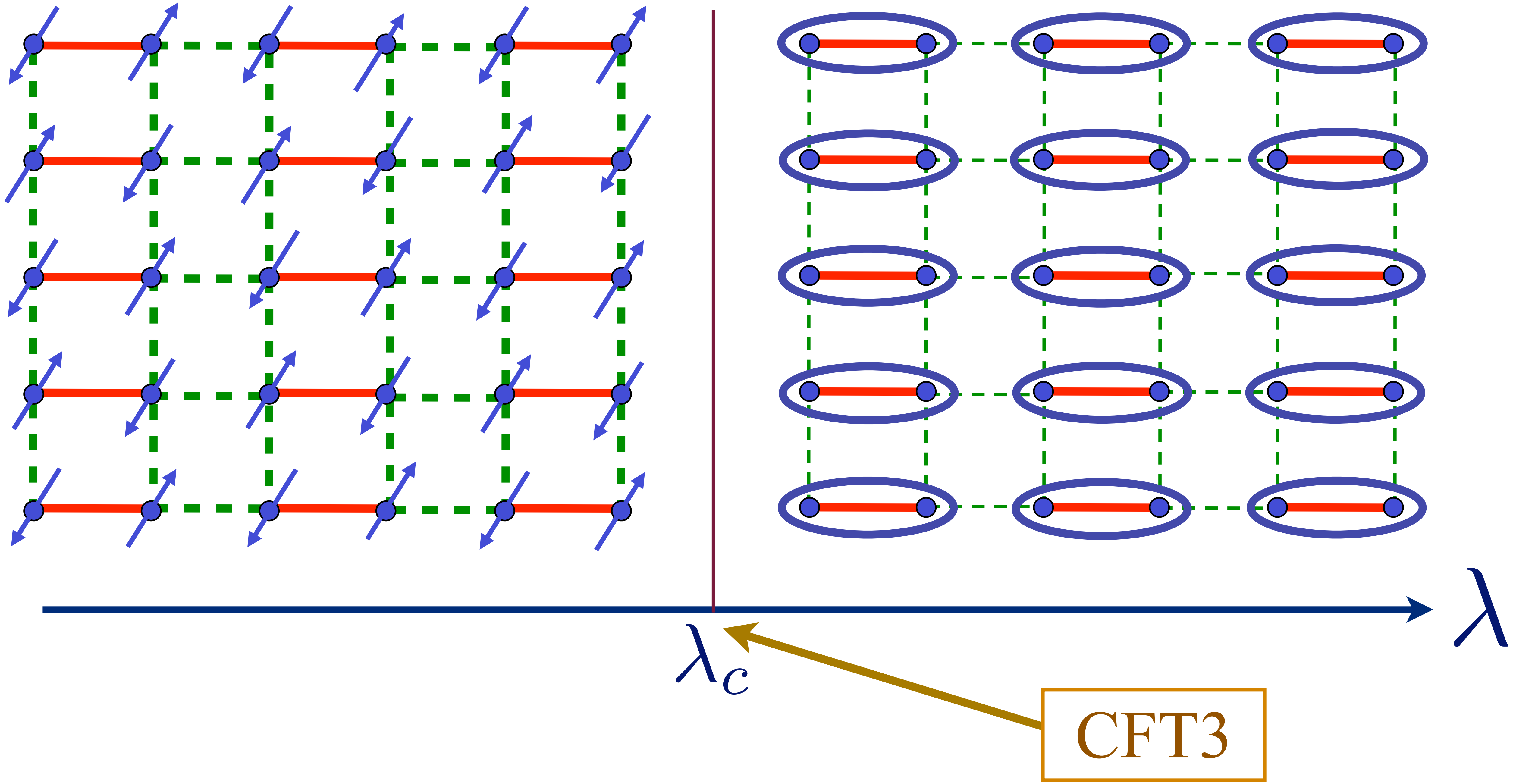
Neel order under pressure

A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, *Journal of the Physical Society of Japan*, **73**, 1446 (2004).



Quantum critical point with non-local entanglement in spin wavefunction

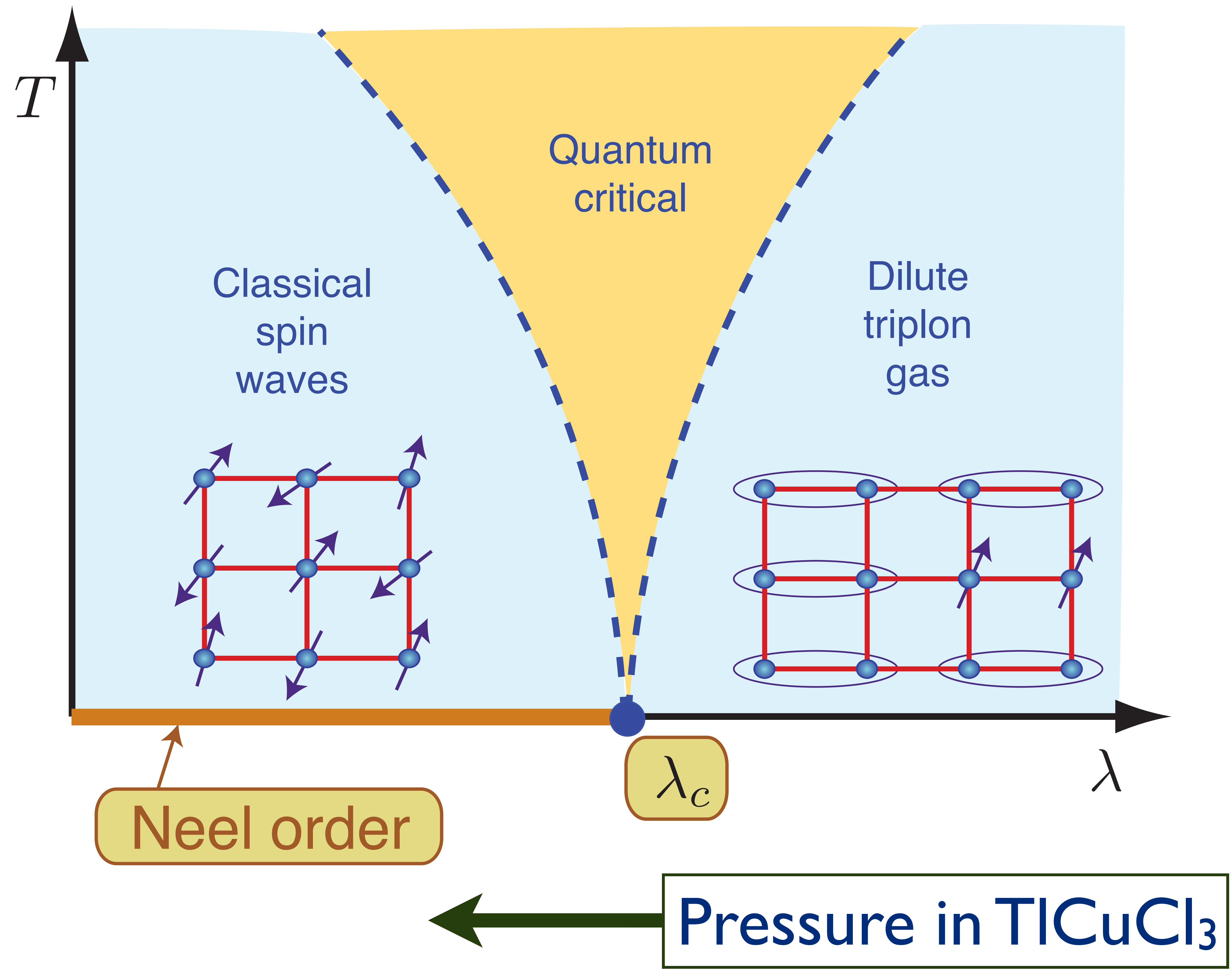
Description using Landau-Ginzburg field theory



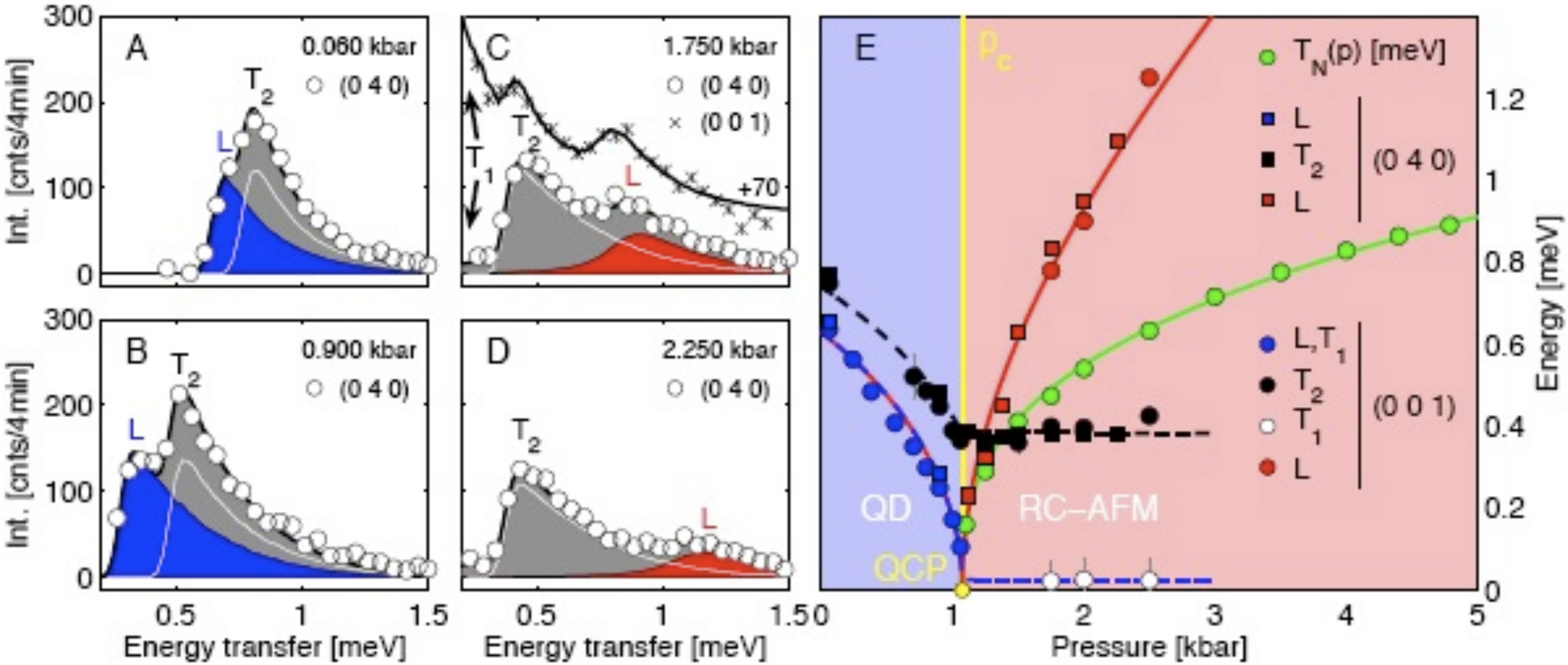
O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c)\vec{\varphi}^2 + u \left(\vec{\varphi}^2\right)^2 \right]$$

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).



$TlCuCl_3$ with varying pressure



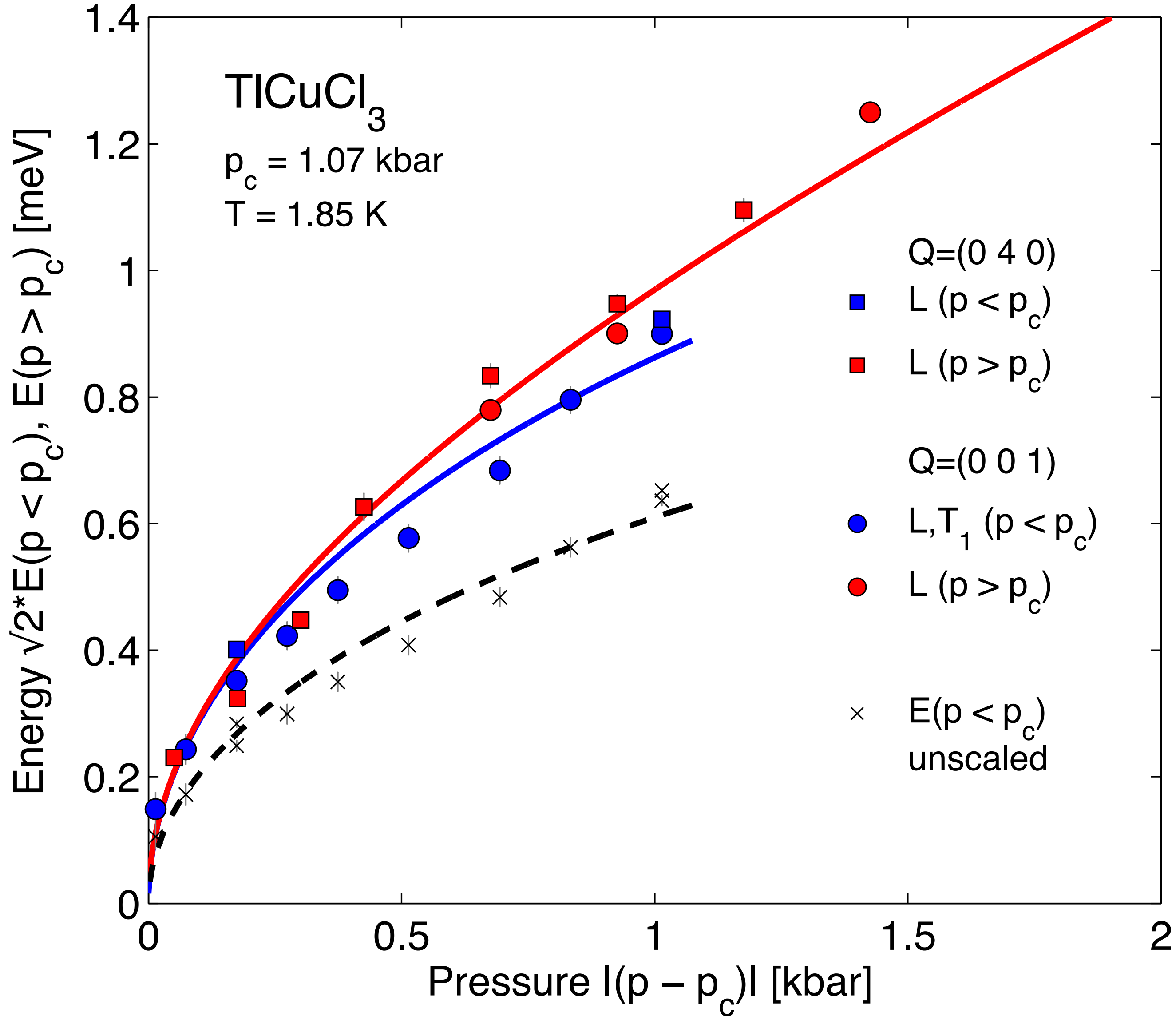
Observation of 3 → 2 low energy modes,
emergence of new Higgs particle in the Néel phase,
and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

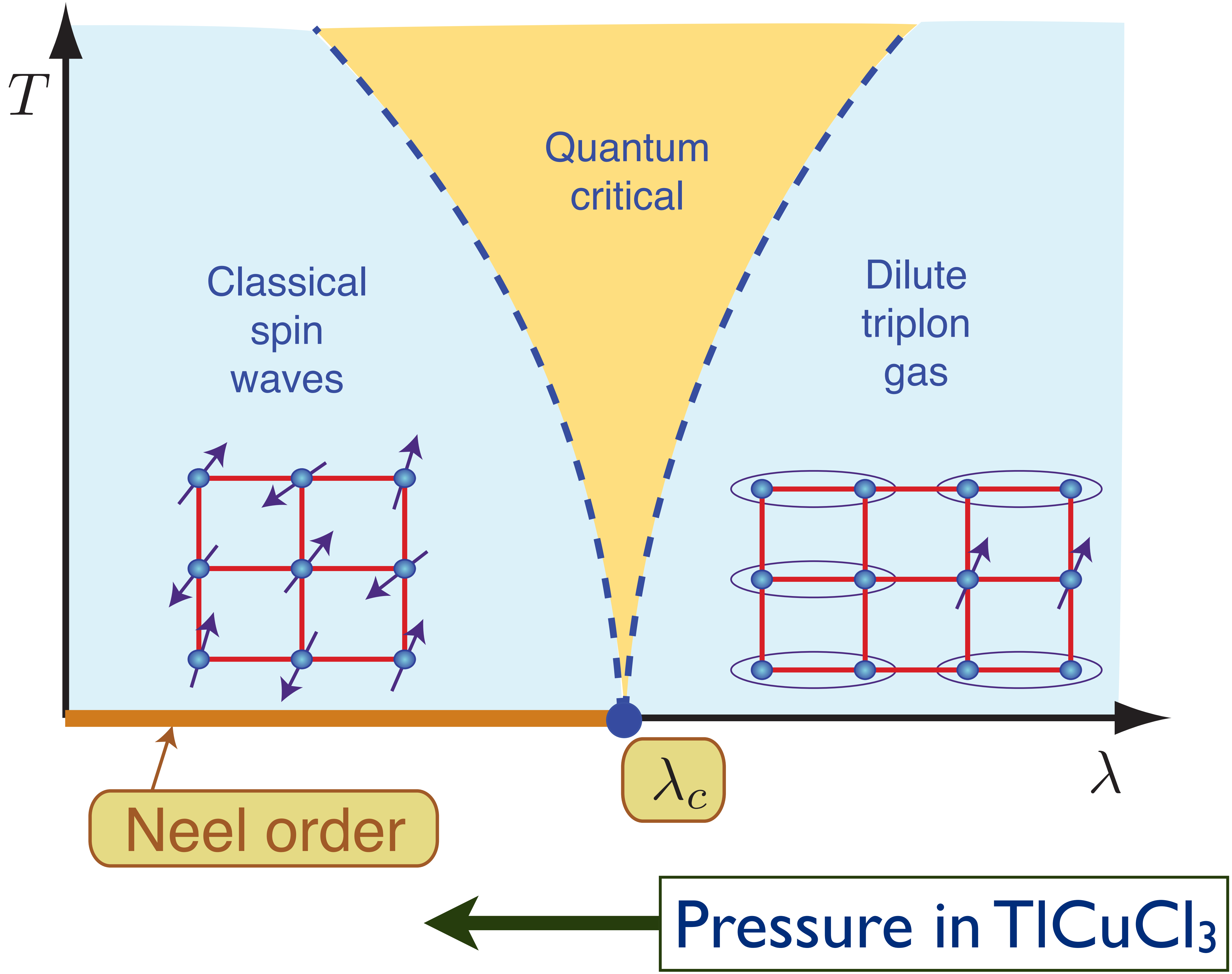
$$\frac{\text{Energy of Higgs particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$



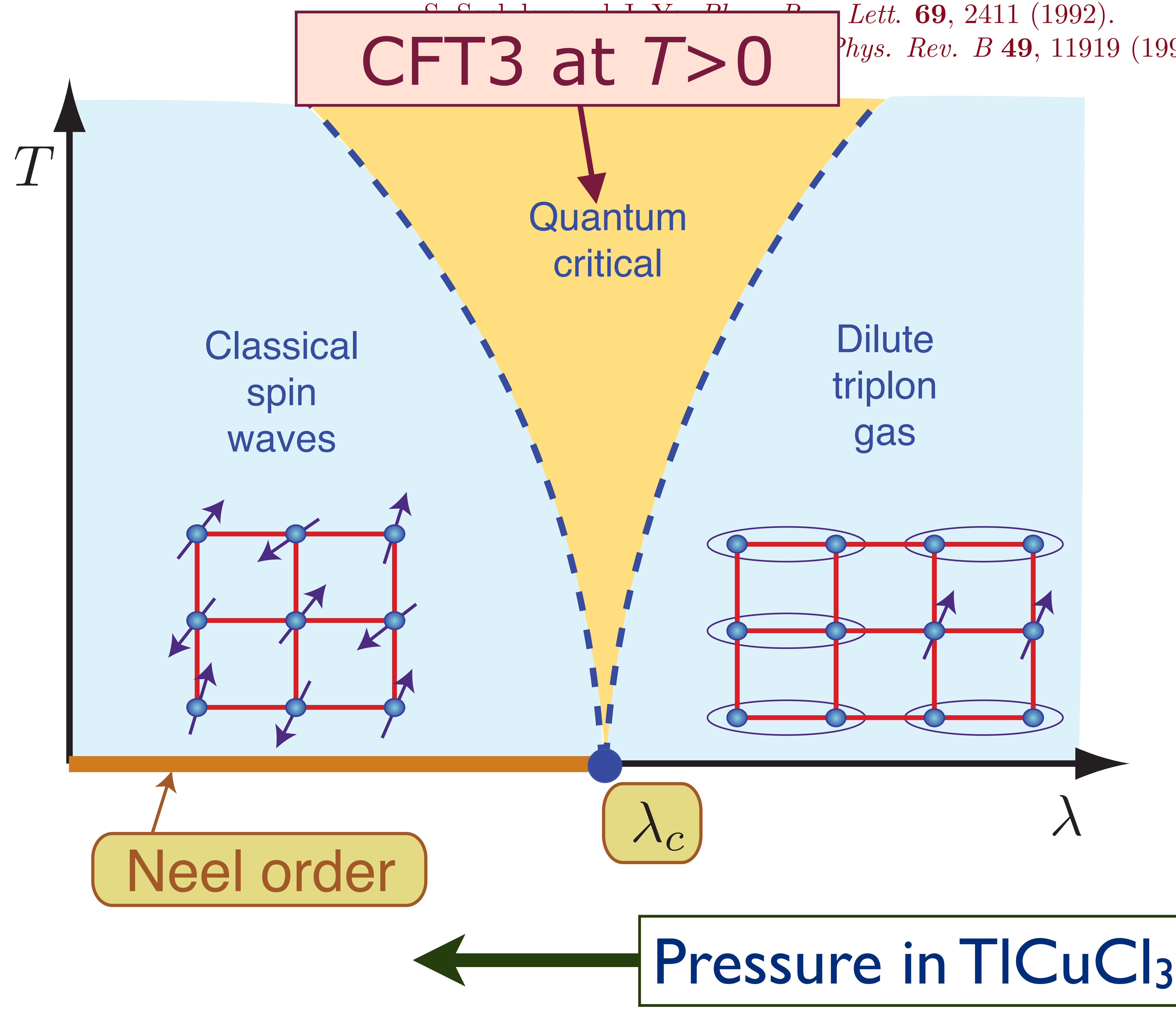
S. Sachdev, arXiv:0901.4103

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

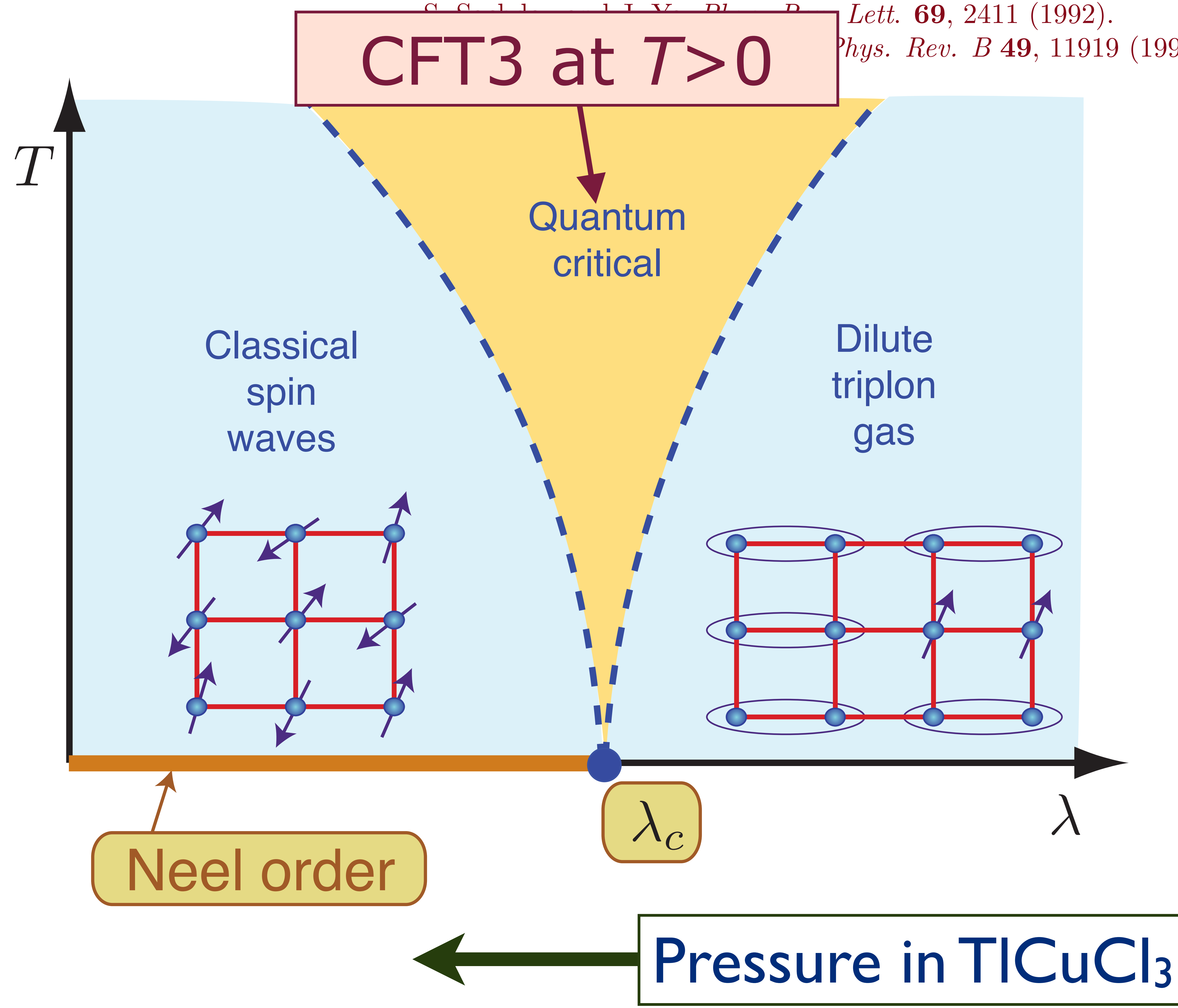


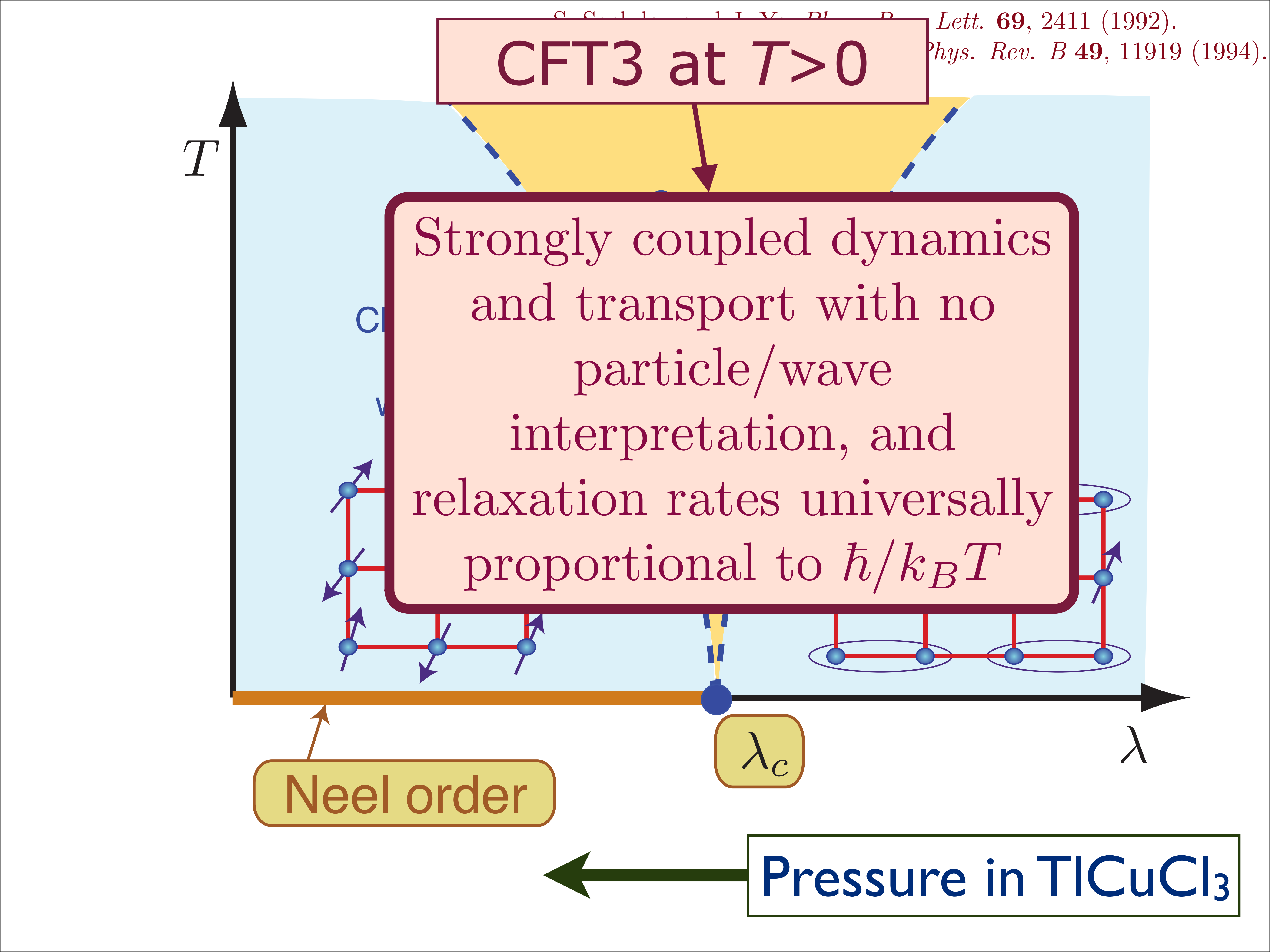
Lett. **69**, 2411 (1992).

Phys. Rev. B **49**, 11919 (1994).



Lett. **69**, 2411 (1992).
hys. Rev. B **49**, 11919 (1994).




Lett. 69, 2411 (1992).
Phys. Rev. B 49, 11919 (1994).
CFT3 at $T>0$
T
Strongly coupled dynamics and transport with no particle/wave interpretation, and relaxation rates universally proportional to $\hbar/k_B T$
λ_c
λ
Neel order
Pressure in $TlCuCl_3$

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

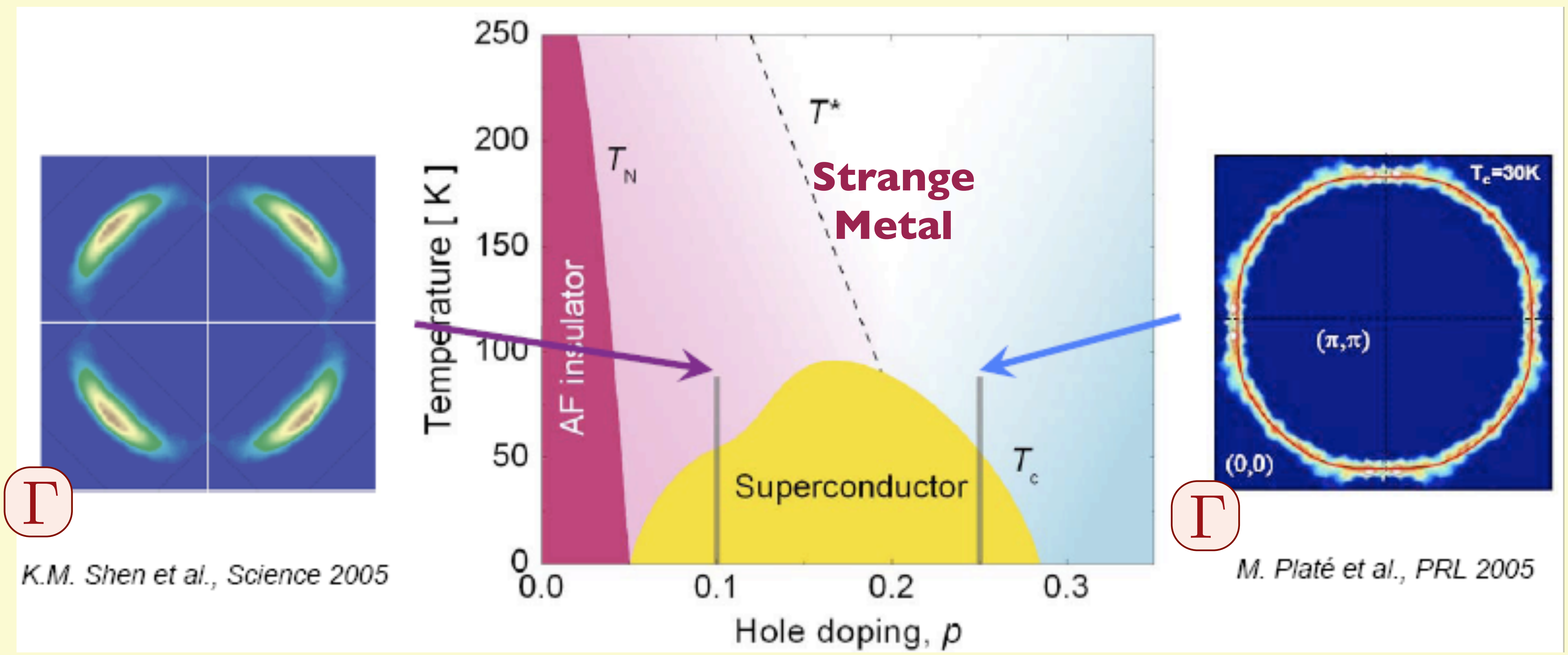
2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface

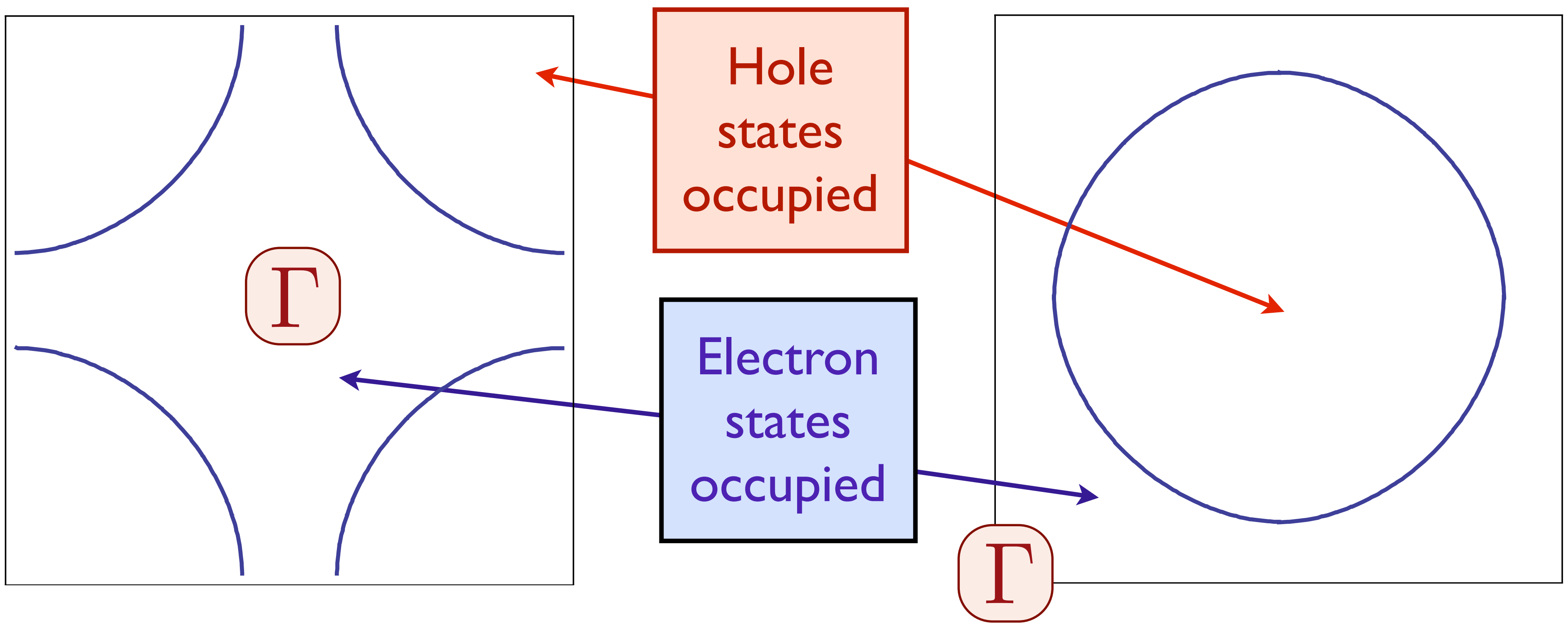


Smaller hole Fermi-pockets

Large hole Fermi surface

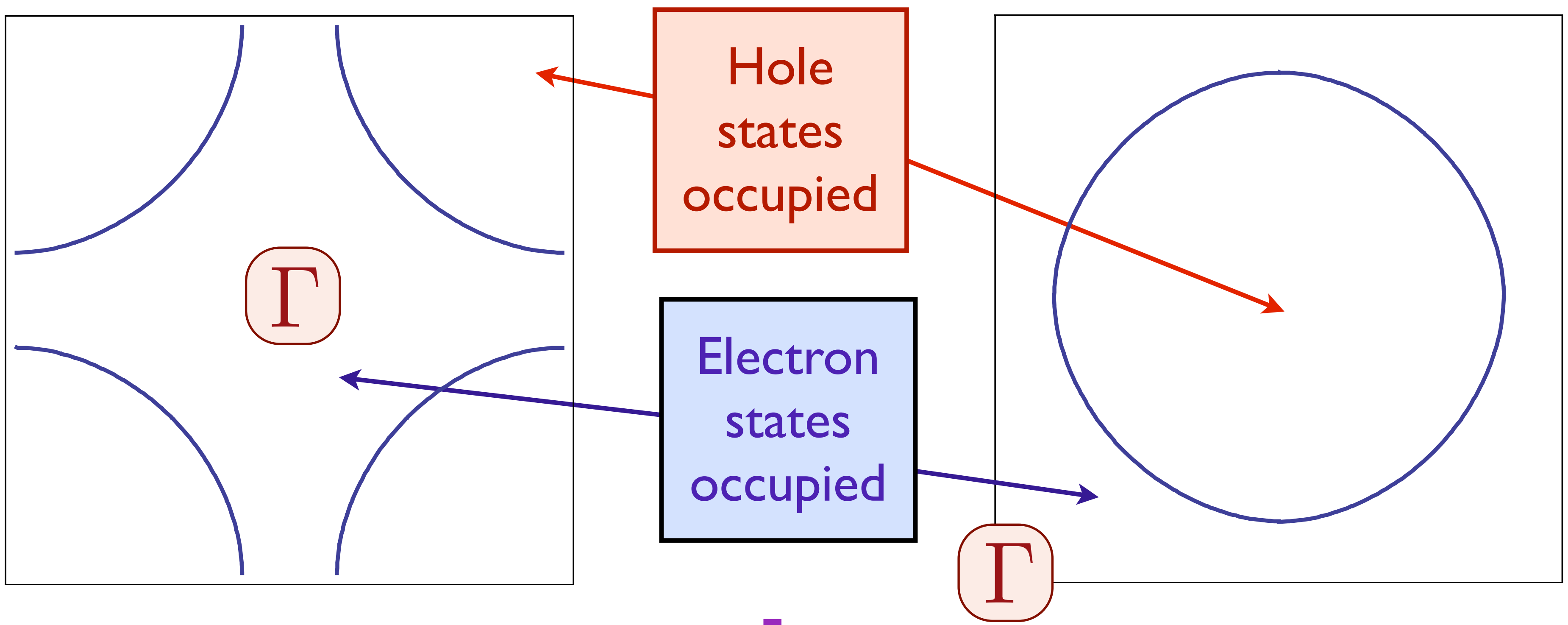
Fermi surface+antiferromagnetism

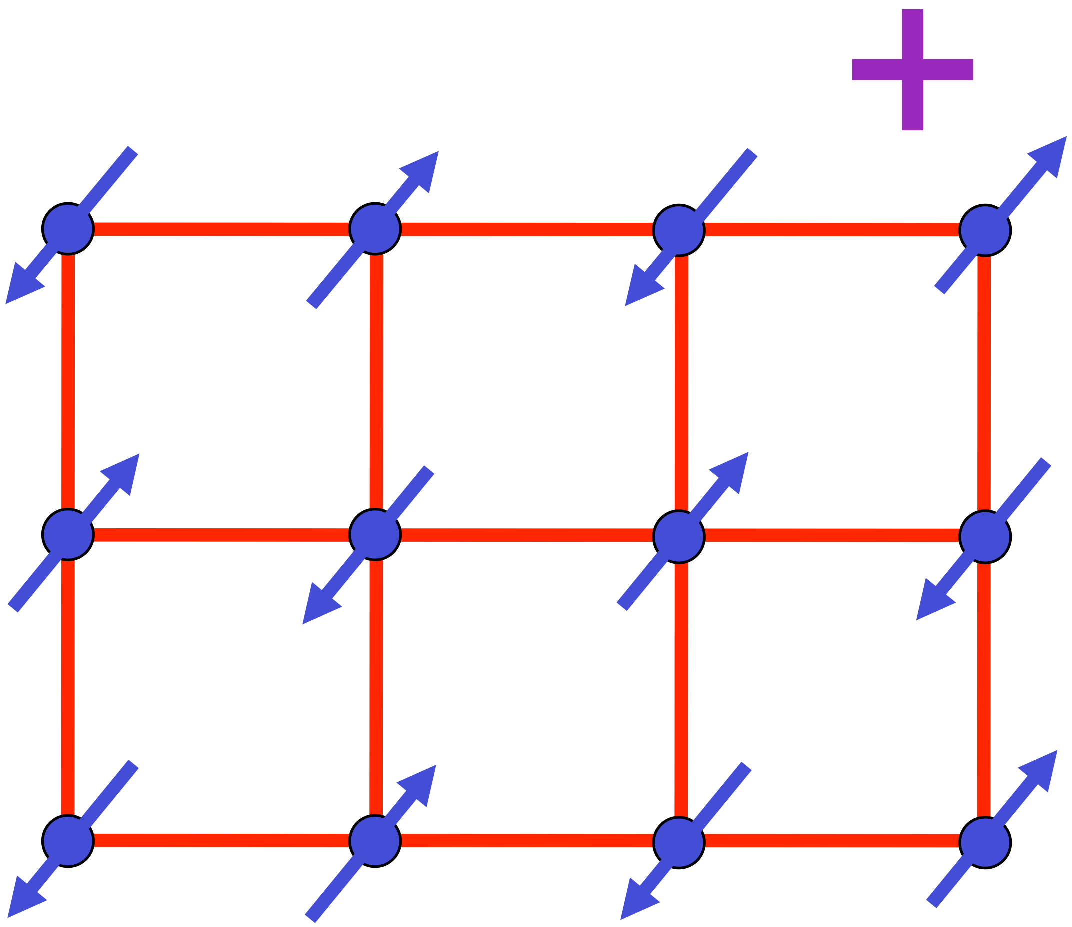
$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha}$$
$$\equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$



Fermi surface+antiferromagnetism

$$H_0 = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{j\alpha}$$
$$\equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$





The electron spin polarization obeys

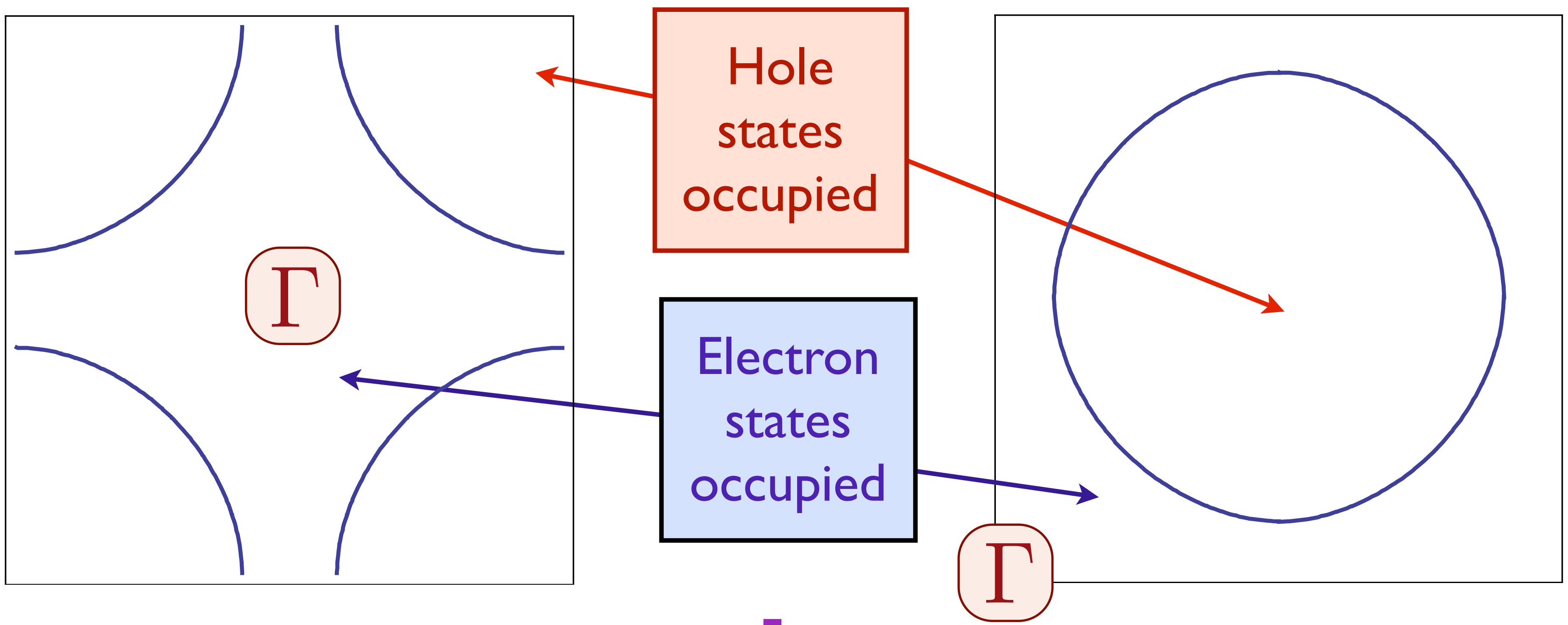
$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

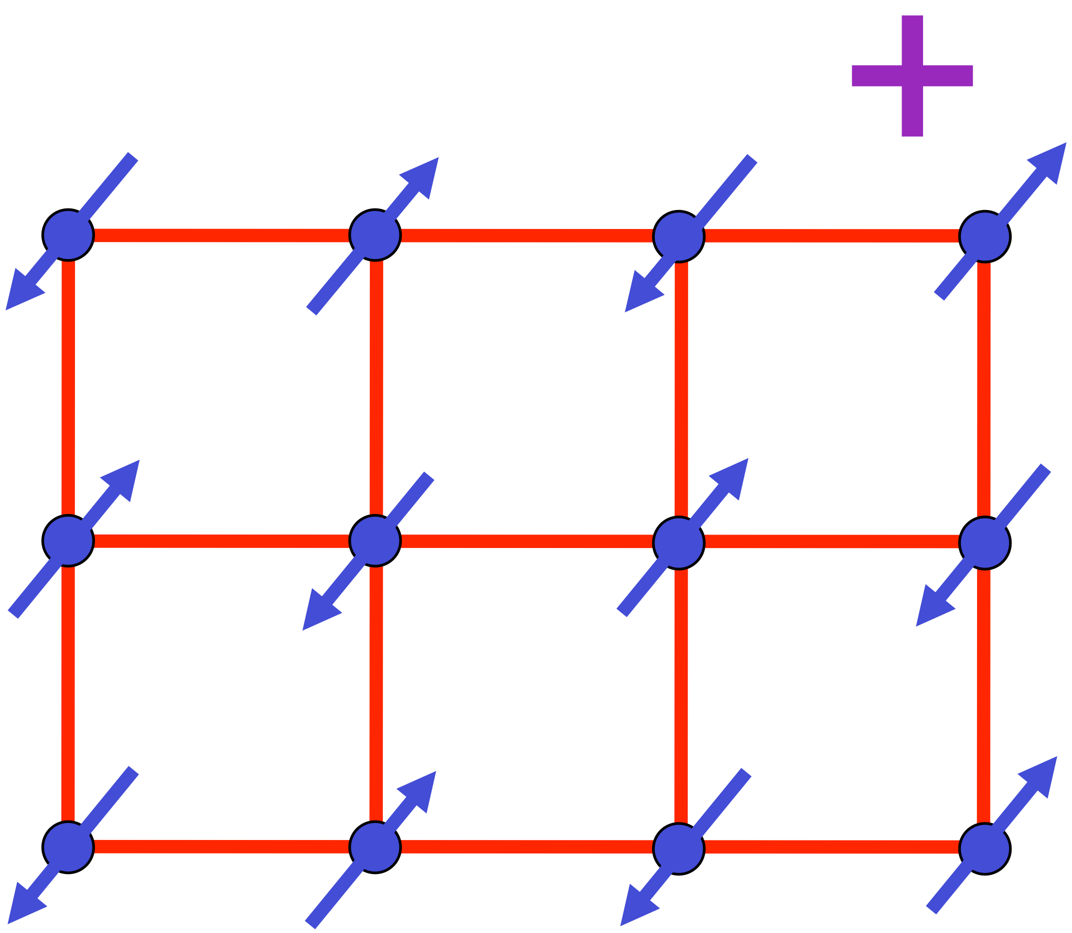
where $\mathbf{K}$ is the ordering wavevector.

Fermi surface+antiferromagnetism

$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha}$$

$$\equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

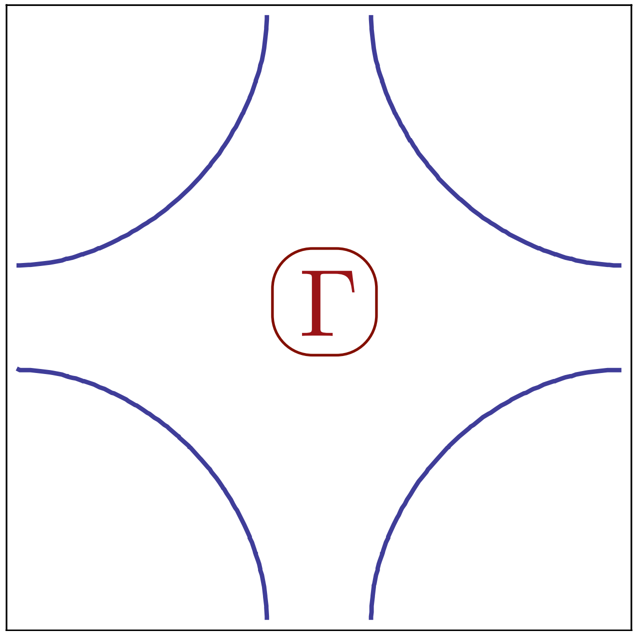




$$H_{\rm sdw} = \sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c_{\mathbf{k}+\mathbf{q},\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

Hole-doped cuprates

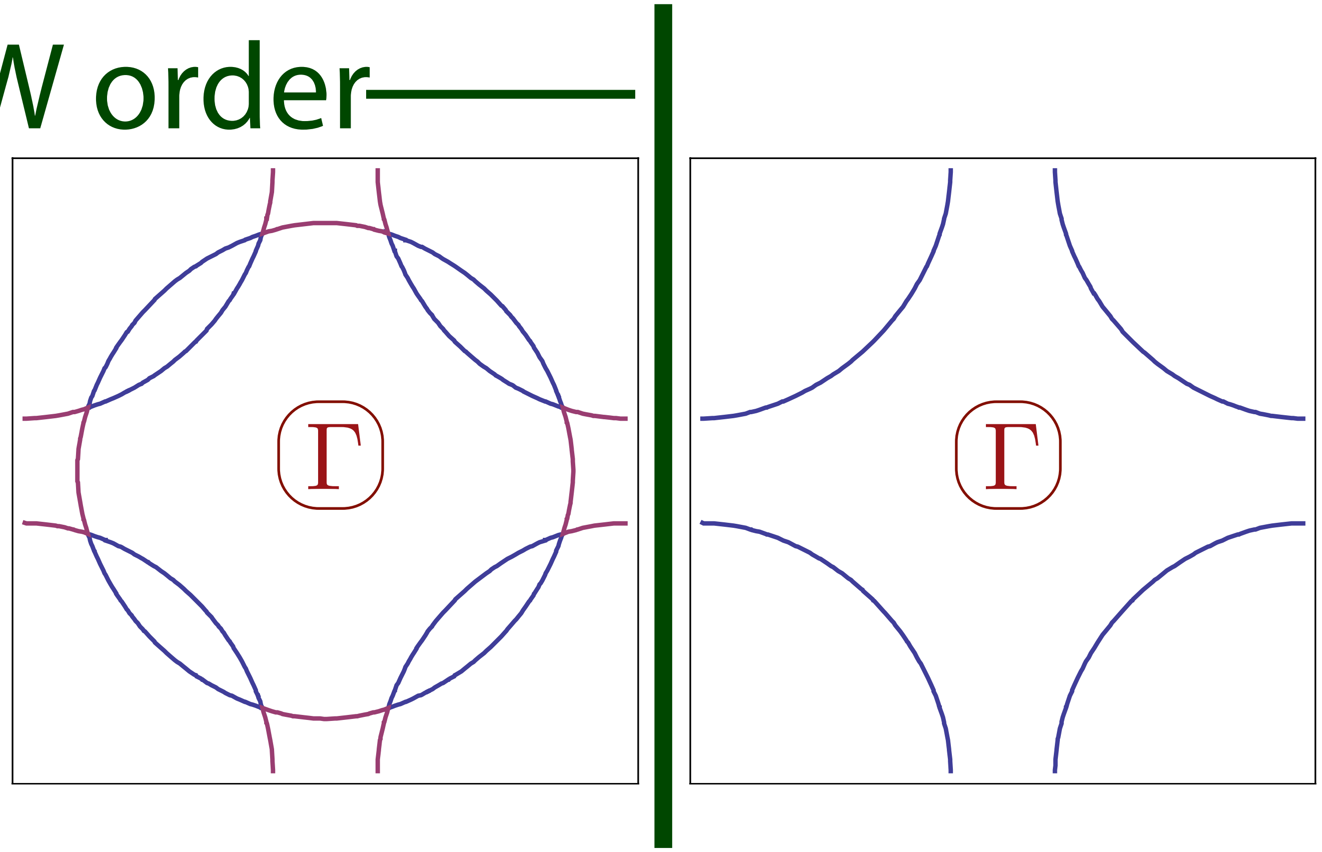
Increasing SDW order



S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

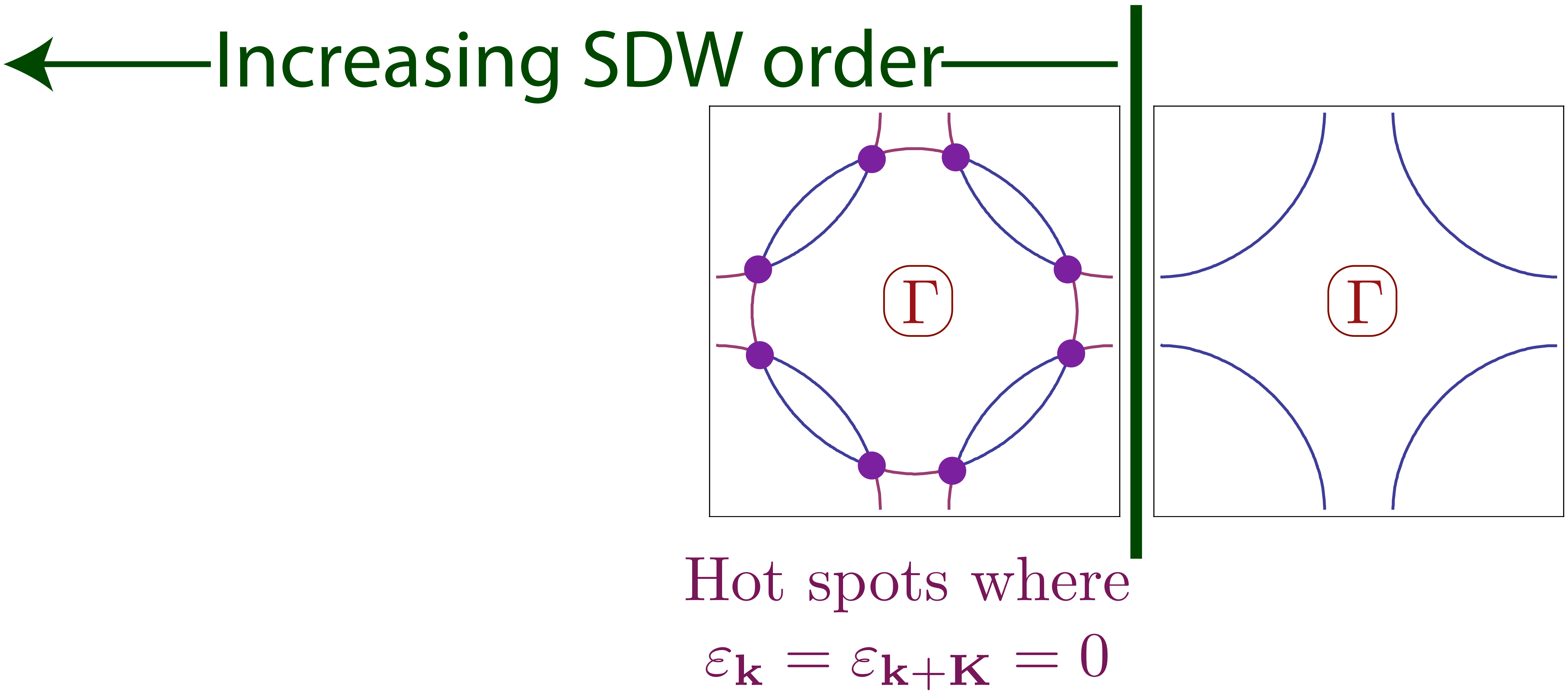
Hole-doped cuprates

Increasing SDW order



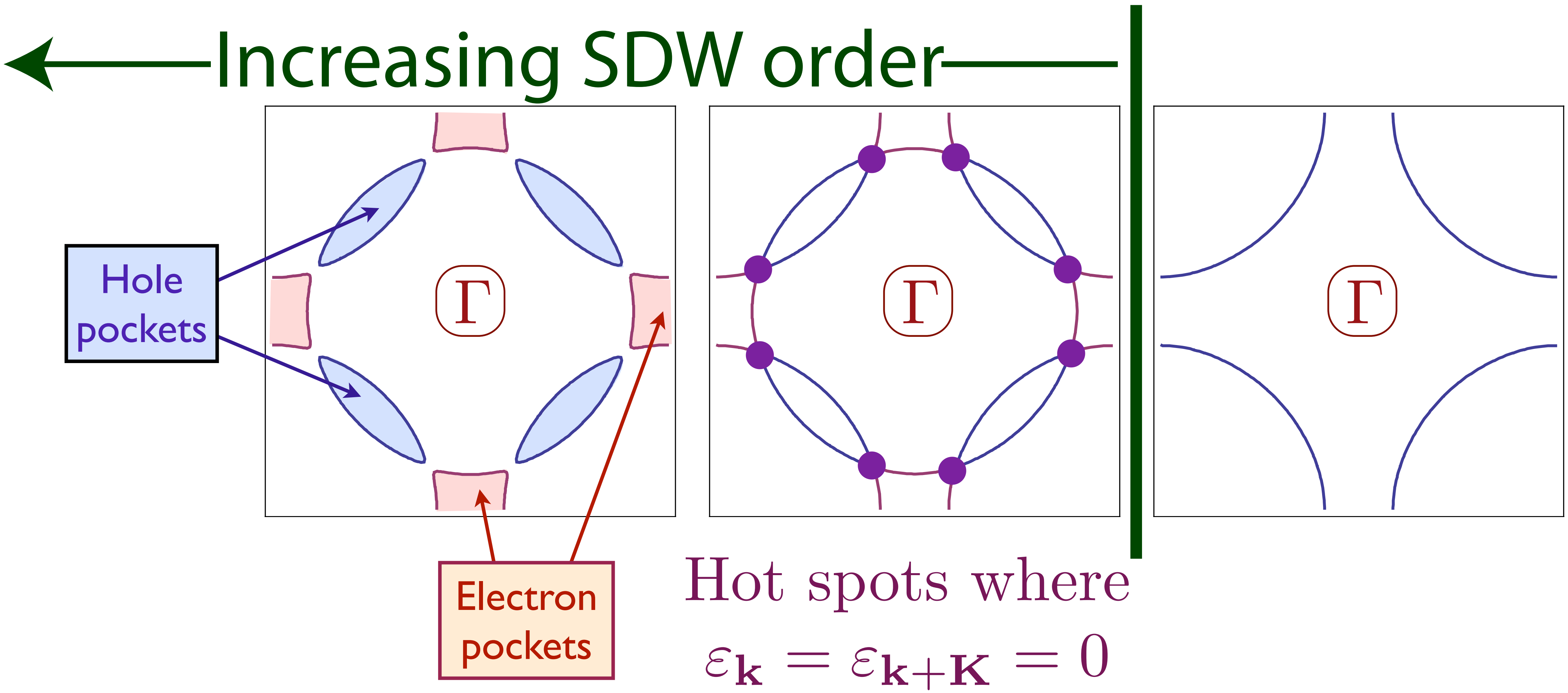
S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

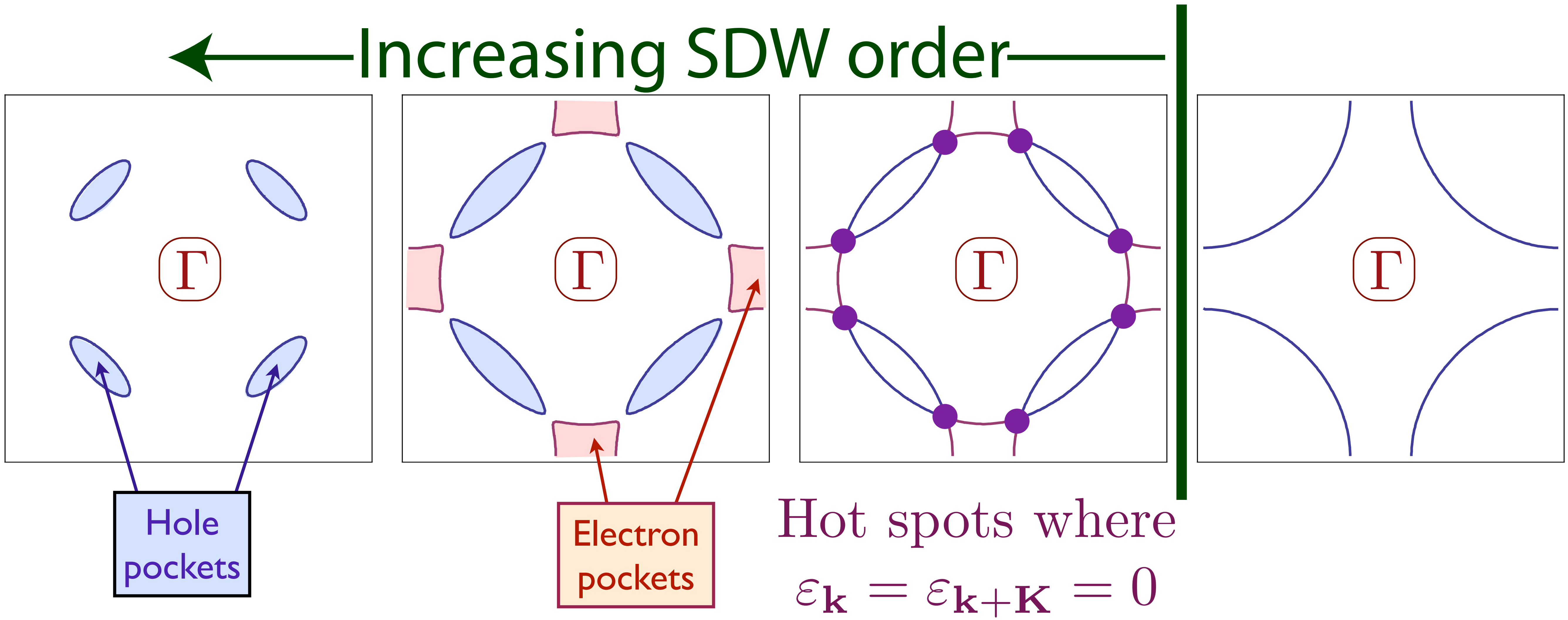
Hole-doped cuprates



Fermi surface breaks up at hot spots into electron and hole "pockets"

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



Fermi surface breaks up at hot spots into electron and hole "pockets"

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Evidence for small Fermi pockets

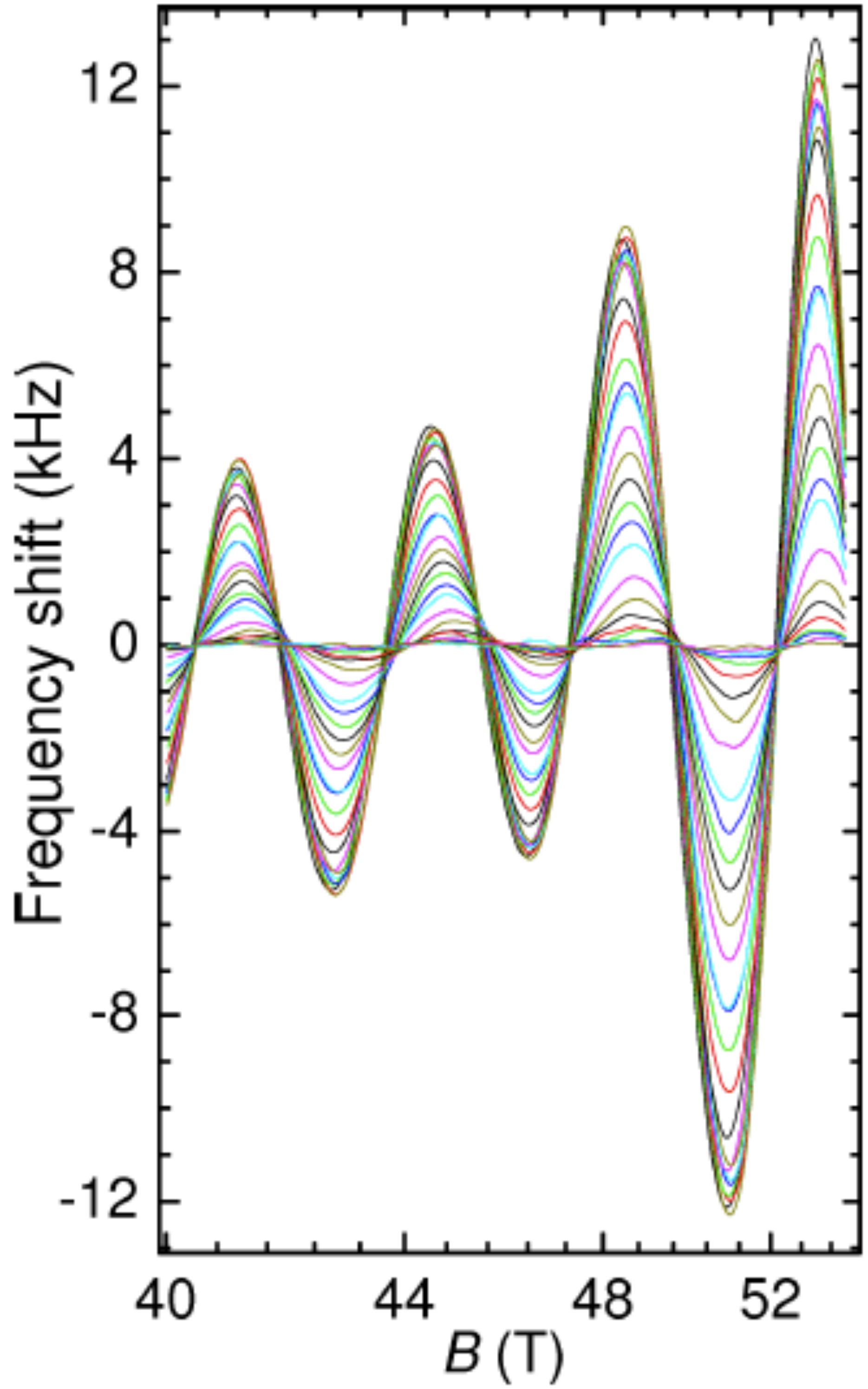
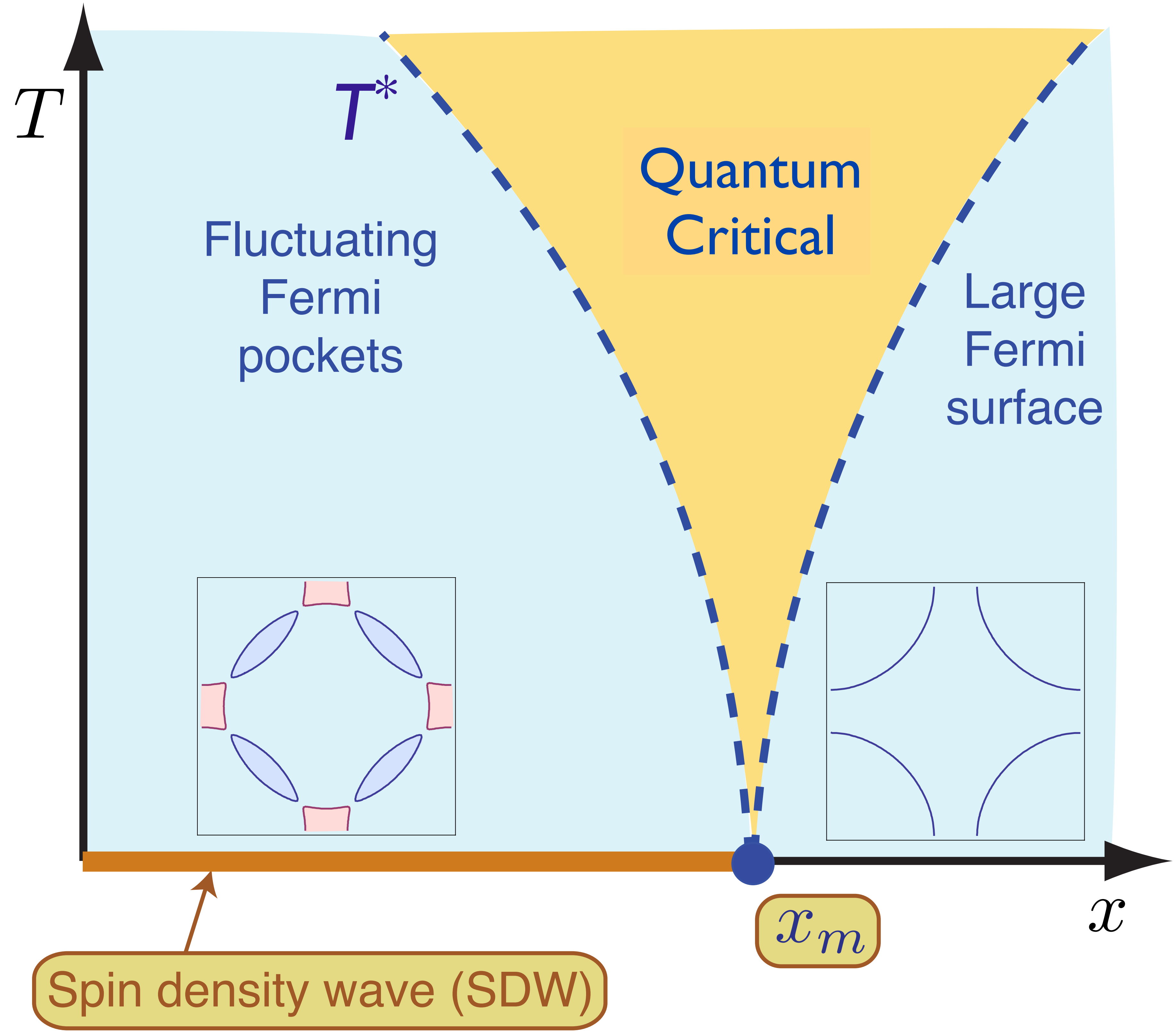


FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of $\sim$ 50 dB between T = 1 and 18 K. The actual T values are provided in Fig. 3.

Suchitra E. Sebastian, N. Harrison, M. M. Altarawneh, Ruixing Liang, D. A. Bonn, W. N. Hardy, and G. G. Lonzarich
Physical Review B **81**, 140505(R) (2010)

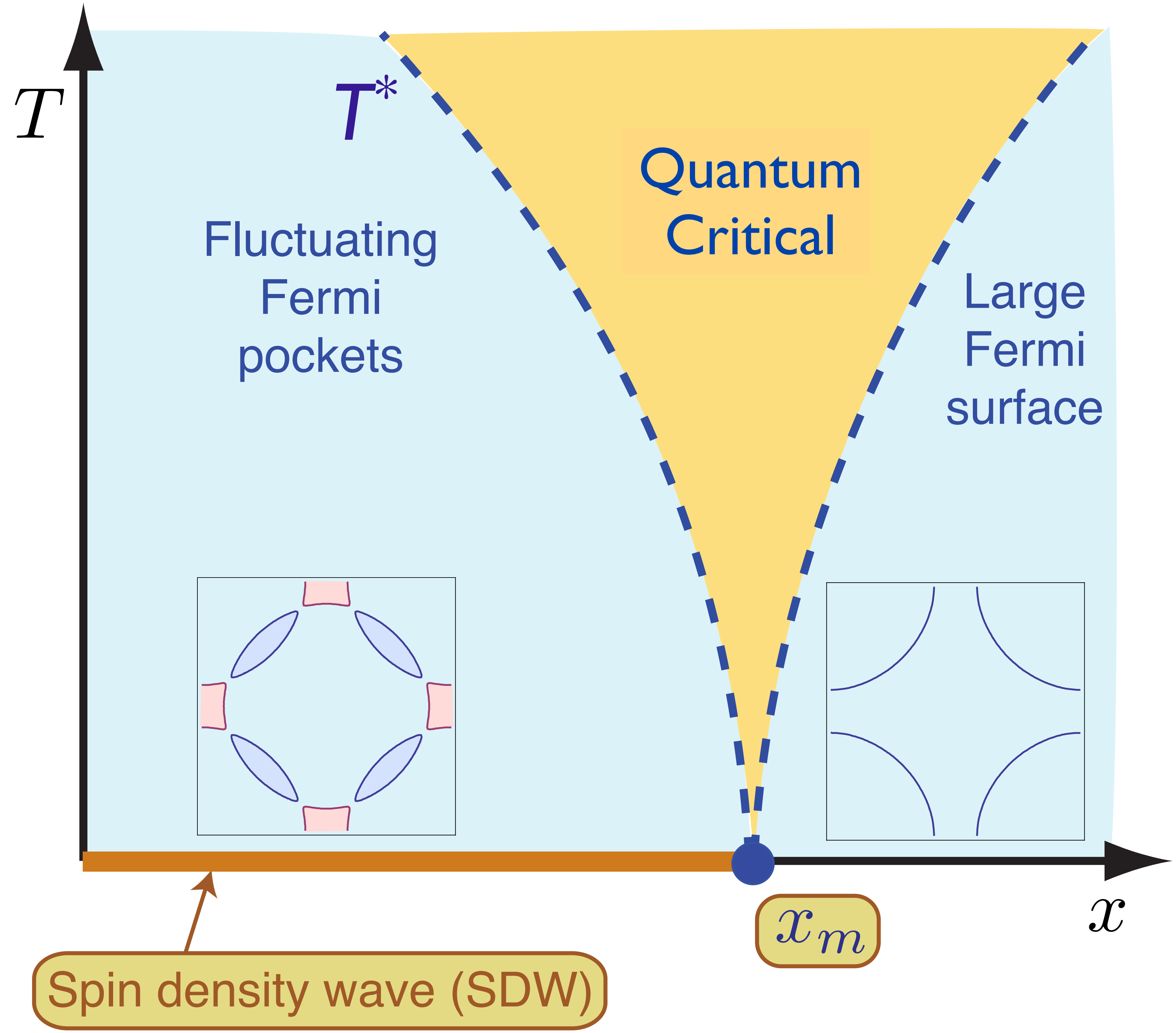
Original observation:
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer,
Nature **447**, 565 (2007)

Theory of quantum criticality in the cuprates



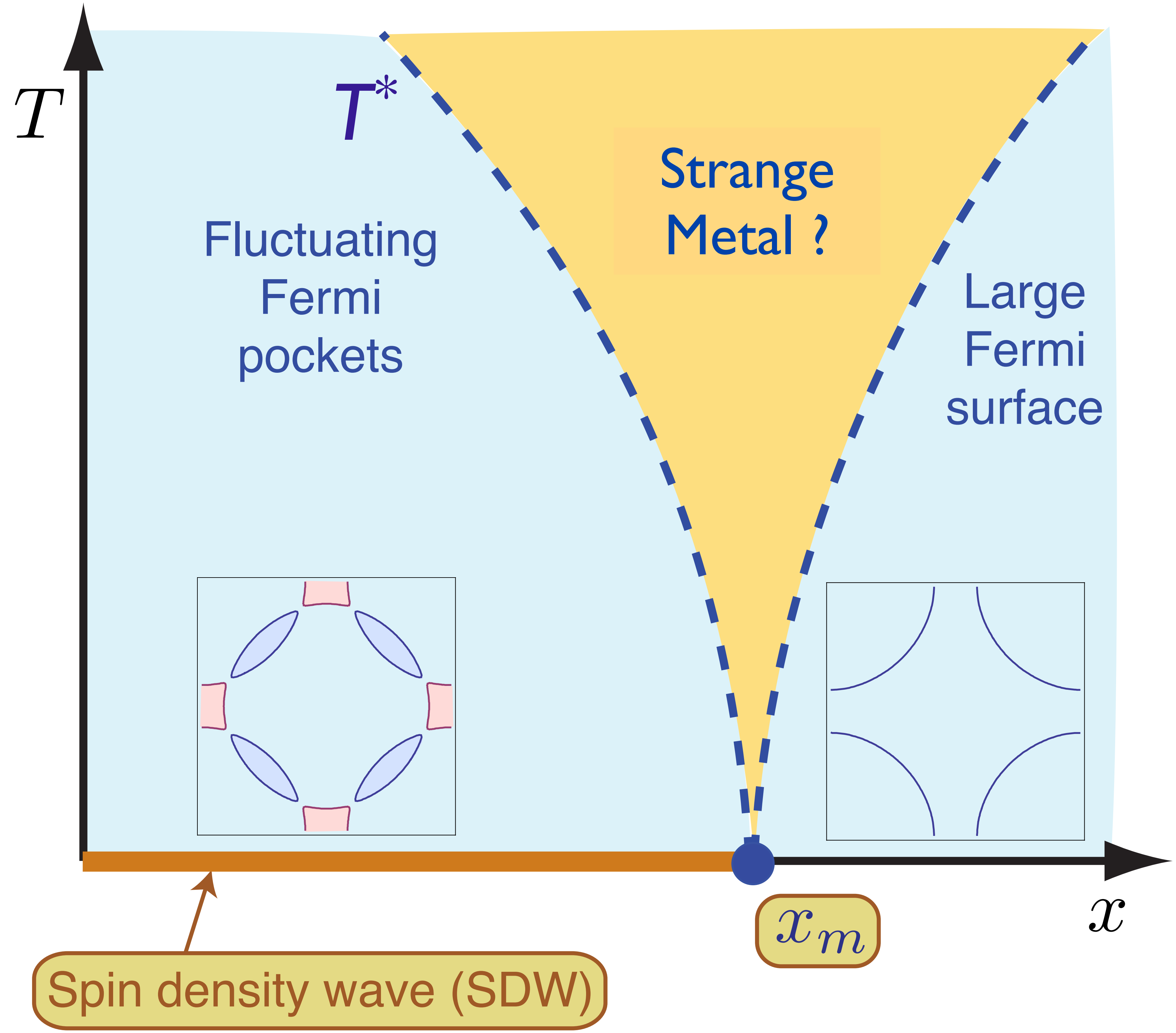
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

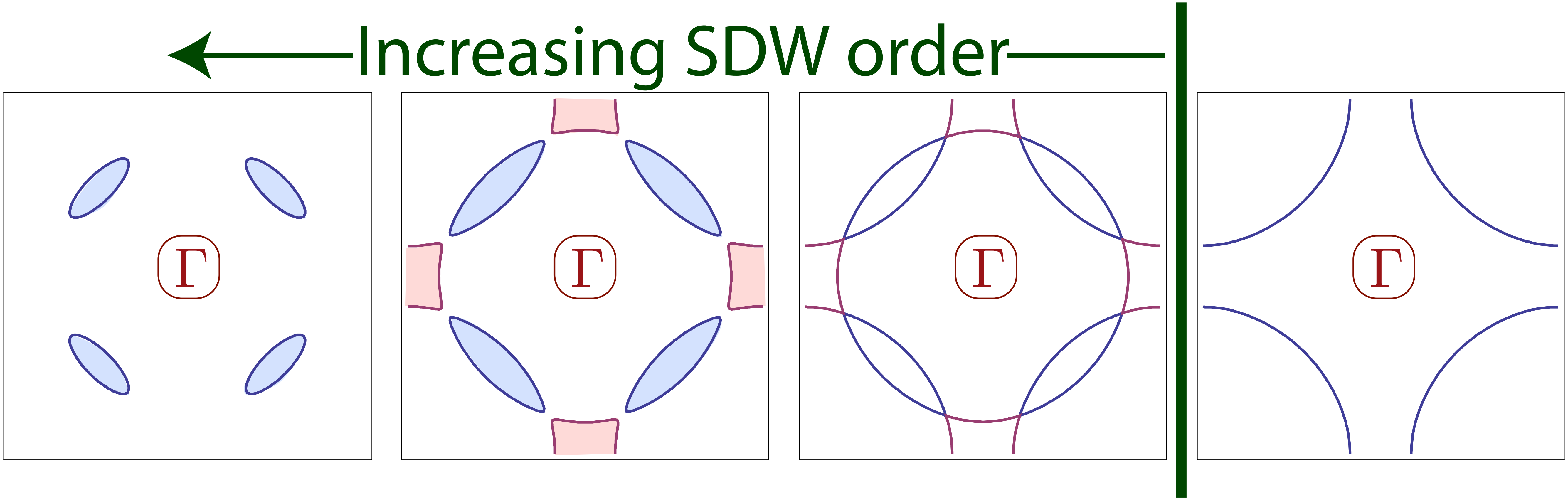
d-wave pairing near a spin-density-wave instability

D. J. Scalapino, E. Loh, Jr.,* and J. E. Hirsch†
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 23 June 1986)

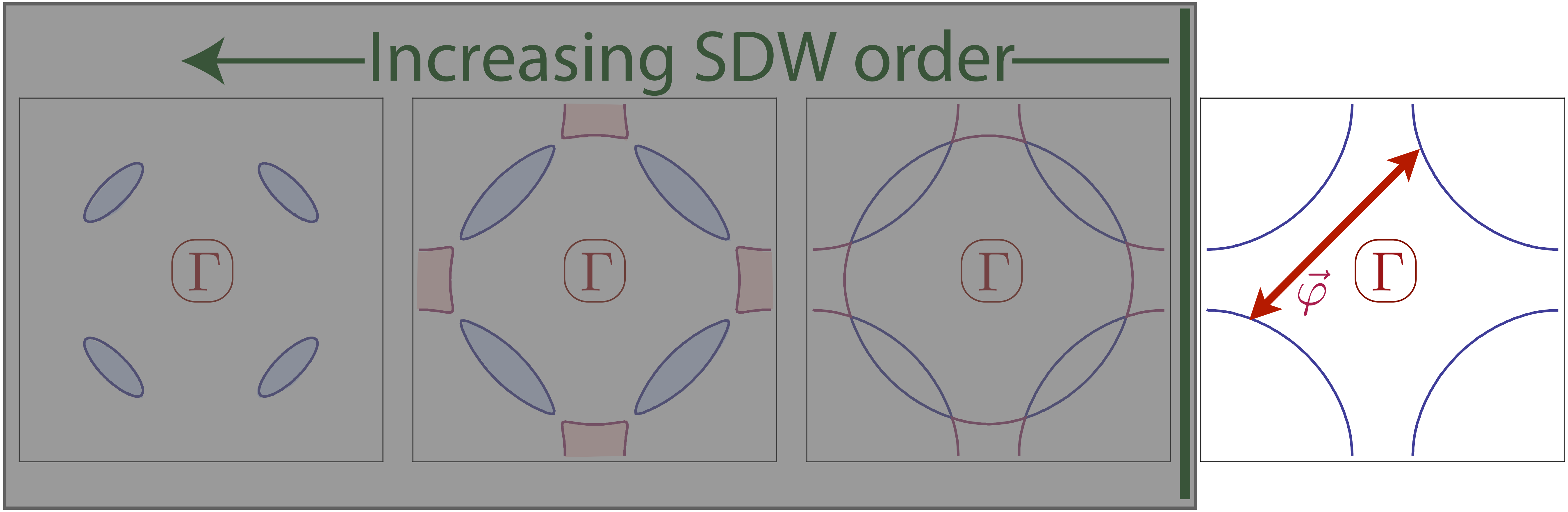
We investigate the three-dimensional Hubbard model and show that paramagnon exchange near a spin-density-wave instability gives rise to a strong singlet d-wave pairing interaction. For a cubic band the singlet ($d_{x^2-y^2}$ and $d_{3z^2-r^2}$) channels are enhanced while the singlet (d_{xy}, d_{xz}, d_{yz}) and triplet p-wave channels are suppressed. A unique feature of this pairing mechanism is its sensitivity to band structure and band filling.

Physical Review B **34**, 8190 (1986)

Spin density wave theory in hole-doped cuprates



Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\text{sdw}} = -\sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^{\dagger}_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q}-\mathbf{K})^2},$$

with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

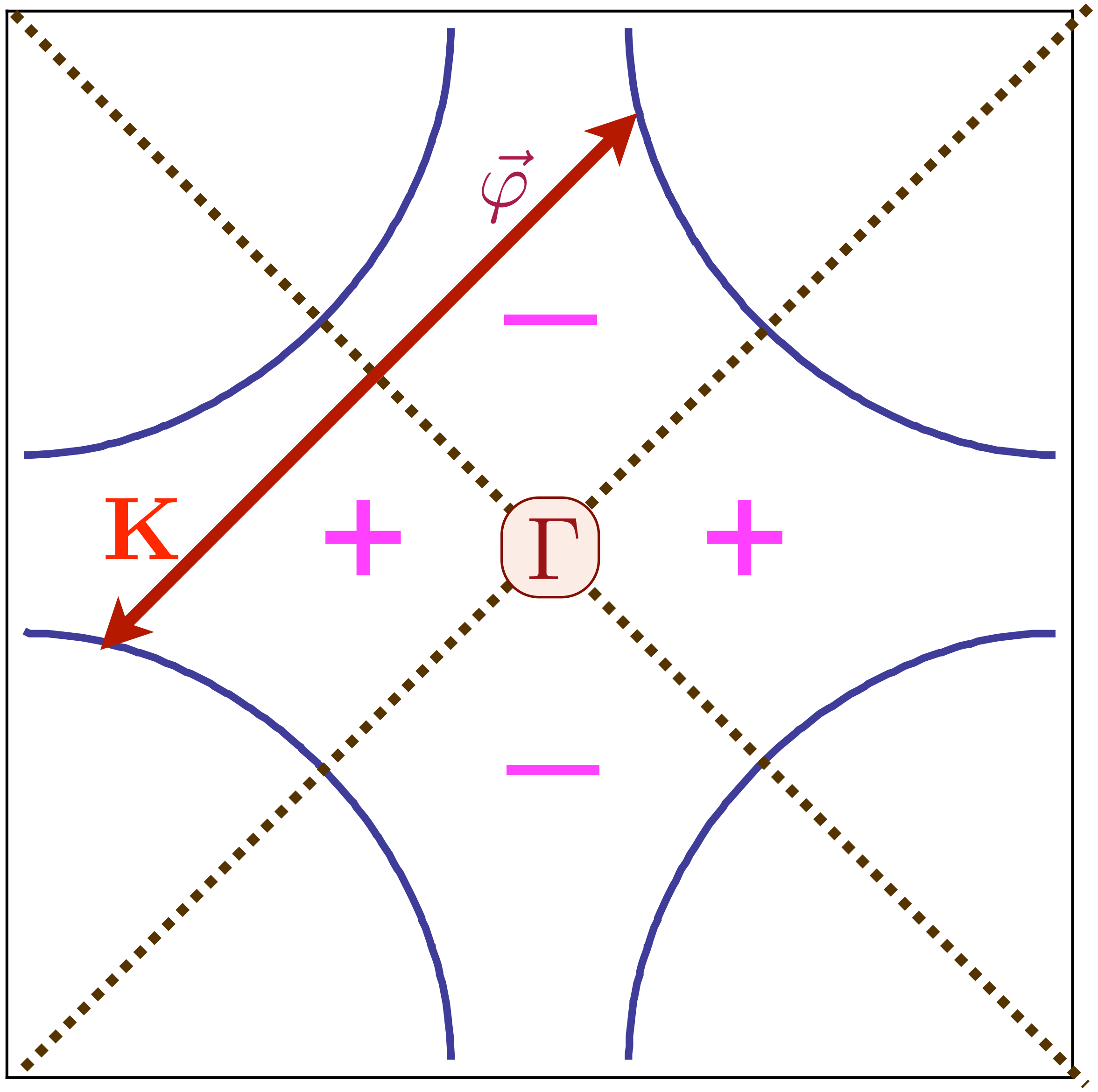
BCS Gap equation

In BCS theory, this interaction leads to the 'gap equation' for the pairing gap $\Delta_{\mathbf{k}} \propto \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$.

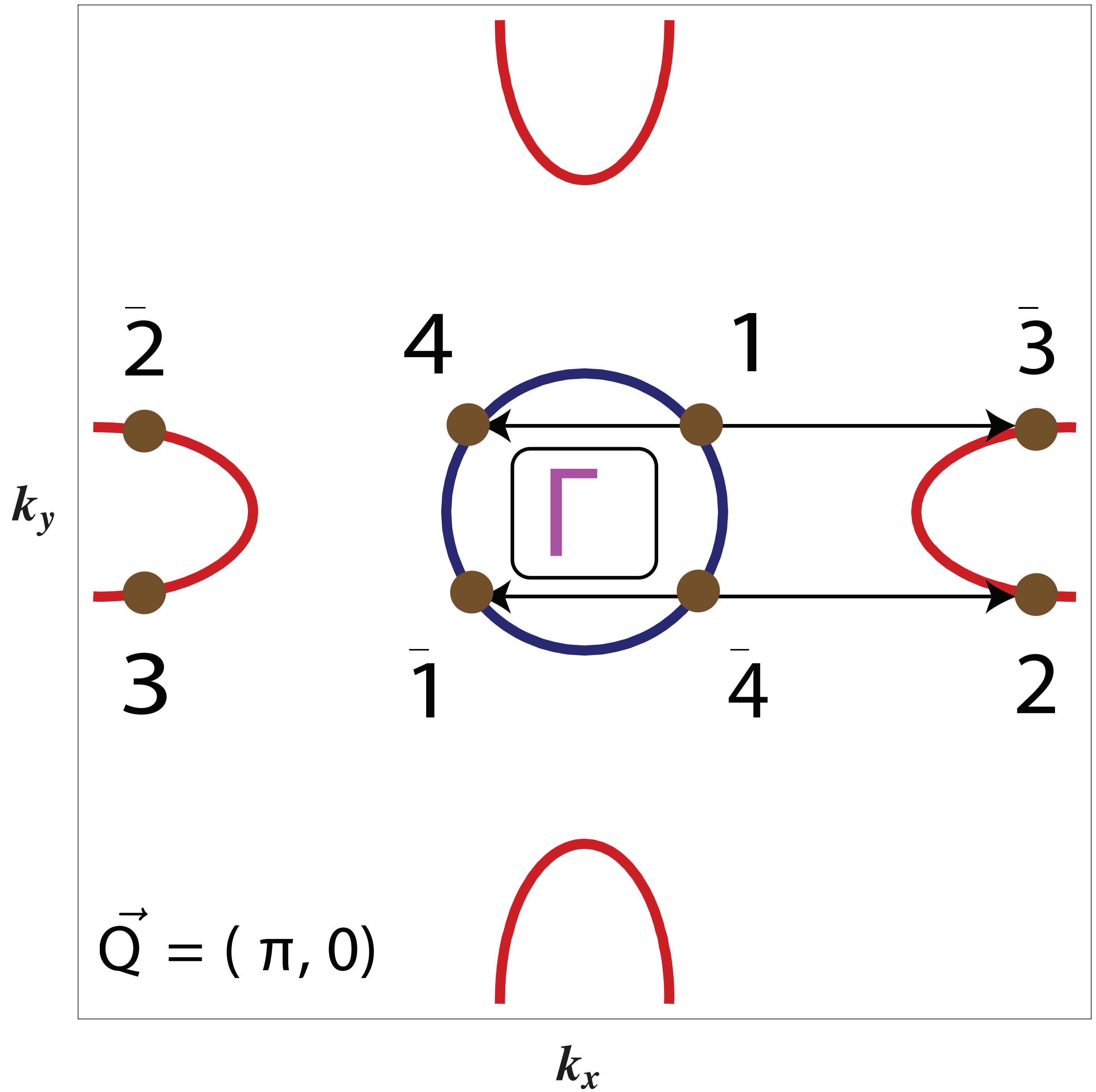
$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{p}} \left(\frac{3\chi_0}{\xi^{-2} + (\mathbf{p} - \mathbf{k} - \mathbf{K})^2} \right) \frac{\Delta_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{p}}^2 + \Delta_{\mathbf{p}}^2}}$$

Non-zero solutions of this equation require that $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{p}}$ have opposite signs when $\mathbf{p} - \mathbf{k} \approx \mathbf{K}$.

d-wave pairing of the large Fermi surface



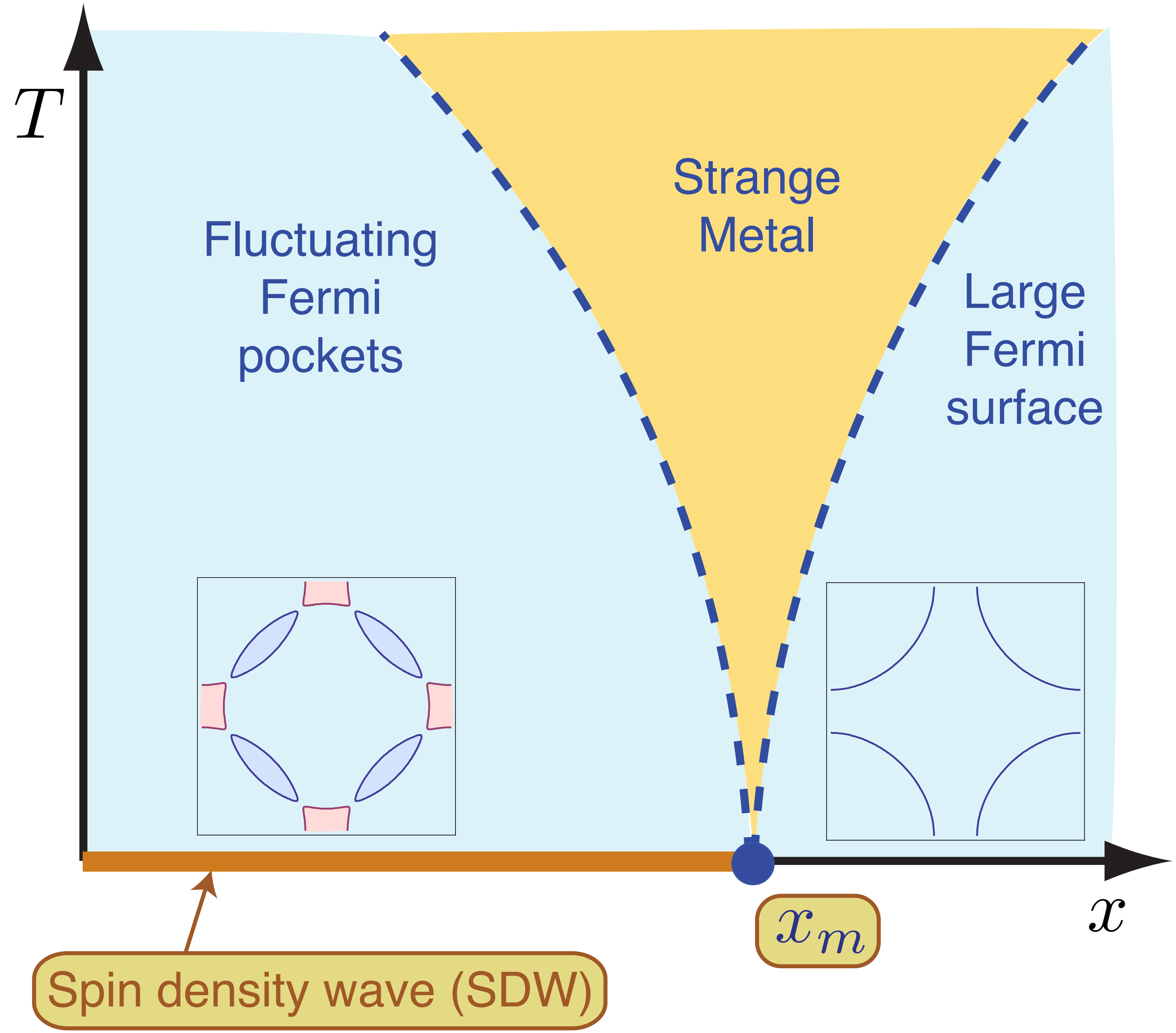
$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$



Similar theory applies to the pnictides, and leads to $s_{\pm}$ pairing.

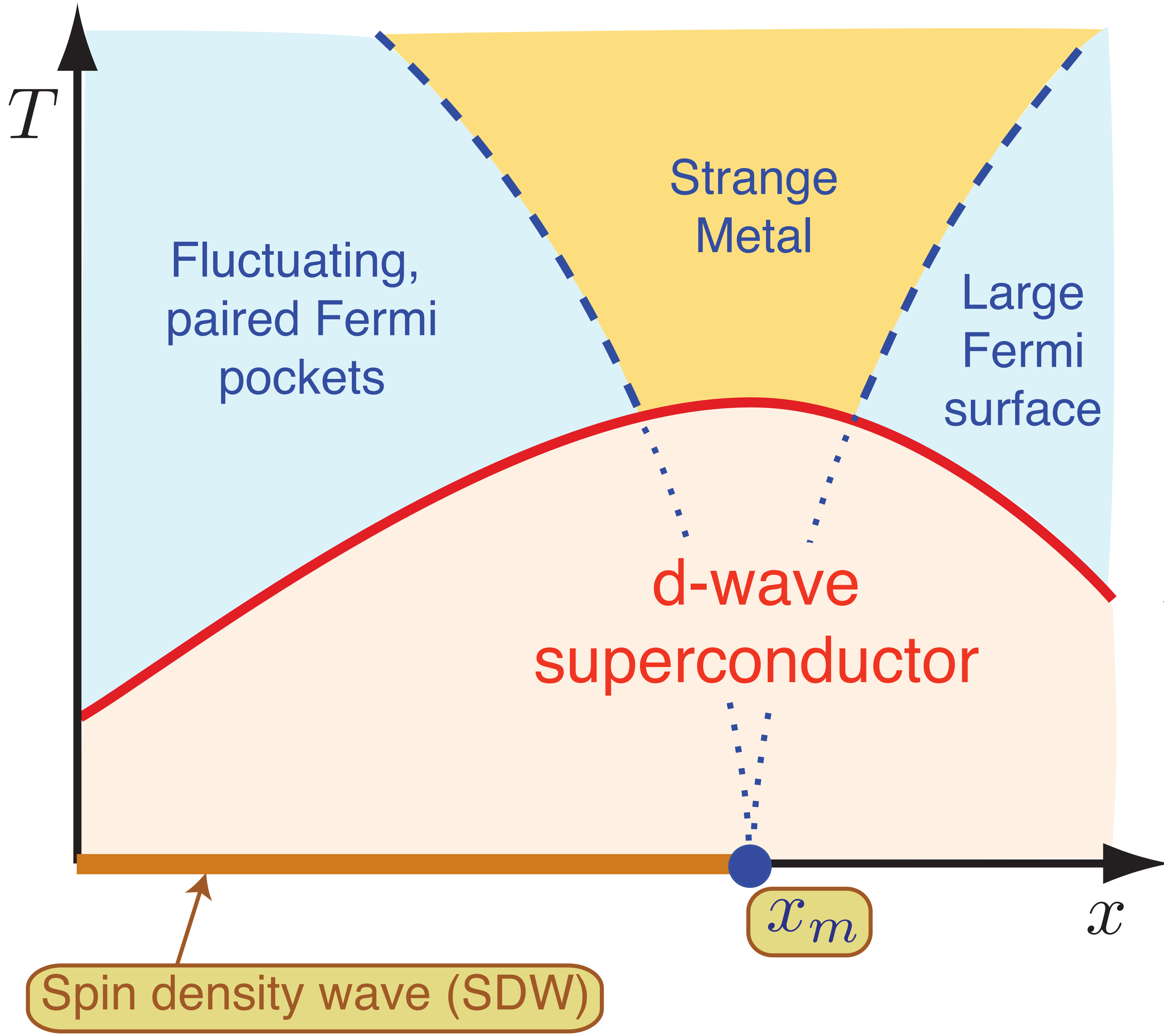
I. I. Mazin, D. J. Singh, M. D. Johannes, and M.-H. Du, Phys. Rev. Lett. **101**, 057003 (2008).
Kangjun Seo, B. A. Bernevig, and Jiangping Hu, Phys. Rev. Lett. **101**, 206404 (2008).

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



Ar. Abanov, A.V. Chubukov, and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Onset of d-wave superconductivity hides the critical point $x = x_m$

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Begin with the Landau-Ginzburg field theory for quantum fluctuations of the antiferromagnetism ($\vec{\varphi}$).

$$\mathcal{S} = \int d^2 r d\tau \left[\frac{1}{2} (\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2} (\nabla_x \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \left(\vec{\varphi}^2 \right)^2 \right.$$

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Begin with the Landau-Ginzburg field theory for quantum fluctuations of the antiferromagnetism ($\vec{\varphi}$). Include the Landau-Ginzburg mean-field action for superconductivity in an applied magnetic field $H = \nabla \times \mathcal{A}$:

$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 \right.$$

$$\left. + \int d^2r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\Delta|^2 - |\Delta|^2 + \frac{|\Delta|^4}{2} \right] \right.$$

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Begin with the Landau-Ginzburg field theory for quantum fluctuations of the antiferromagnetism ($\vec{\varphi}$). Include the Landau-Ginzburg mean-field action for superconductivity in an applied magnetic field $H = \nabla \times \mathcal{A}$:

$$
\begin{aligned}
\mathcal{S} &= \int d^2r d\tau \Bigg[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 \\
&\qquad\qquad + \kappa\, \vec{\varphi}^2 \left|\Delta\right|^2 \Bigg] \\
&+ \int d^2r \left[\left|(\nabla_x - i(2e/\hbar c)\mathcal{A})\Delta\right|^2 - \left|\Delta\right|^2 + \frac{\left|\Delta\right|^4}{2} \right]
\end{aligned}
$$

Include the simplest allowed coupling between the two orders, $\kappa > 0$, with a positive sign implying repulsion or competition between them.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Begin with the Landau-Ginzburg field theory for quantum fluctuations of the antiferromagnetism ($\vec{\varphi}$). Include the Landau-Ginzburg mean-field action for superconductivity in an applied magnetic field $H = \nabla \times \mathcal{A}$:

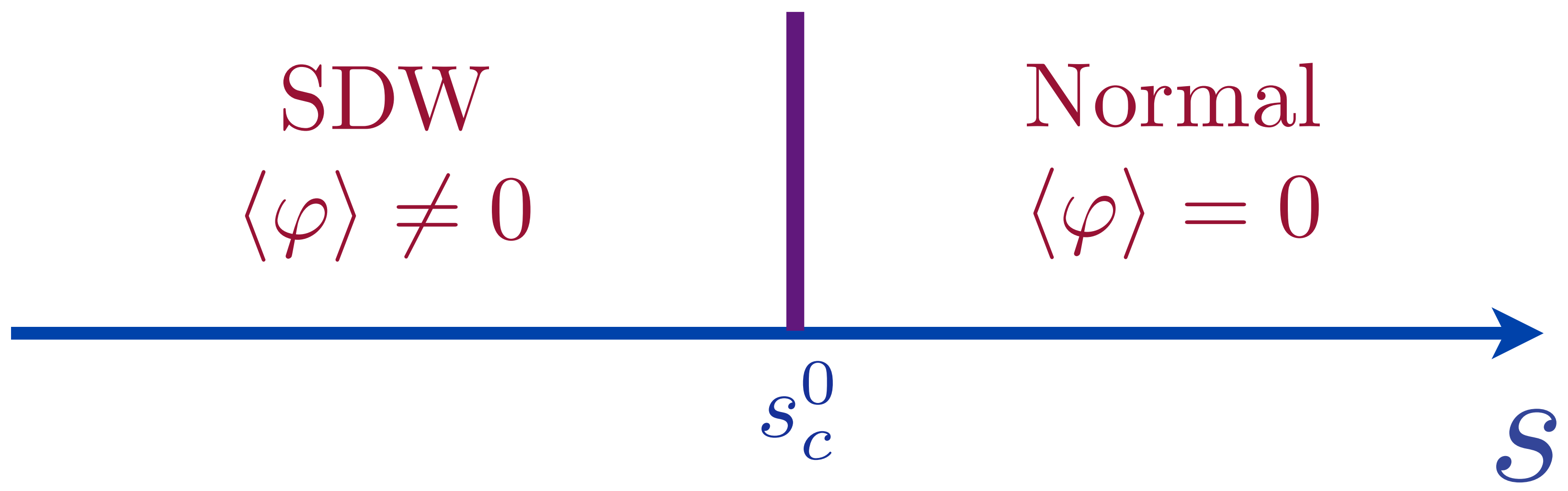
$$\mathcal{S} = \int d^2r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 + \kappa\, \vec{\varphi}^2 \left|\Delta\right|^2 \right] + \int d^2r \left[\left|(\nabla_x - i(2e/\hbar c)\mathcal{A})\Delta\right|^2 - \left|\Delta\right|^2 + \frac{\left|\Delta\right|^4}{2} \right]$$

Include the simplest allowed coupling between the two orders, $\kappa > 0$, with a positive sign implying repulsion or competition between them.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

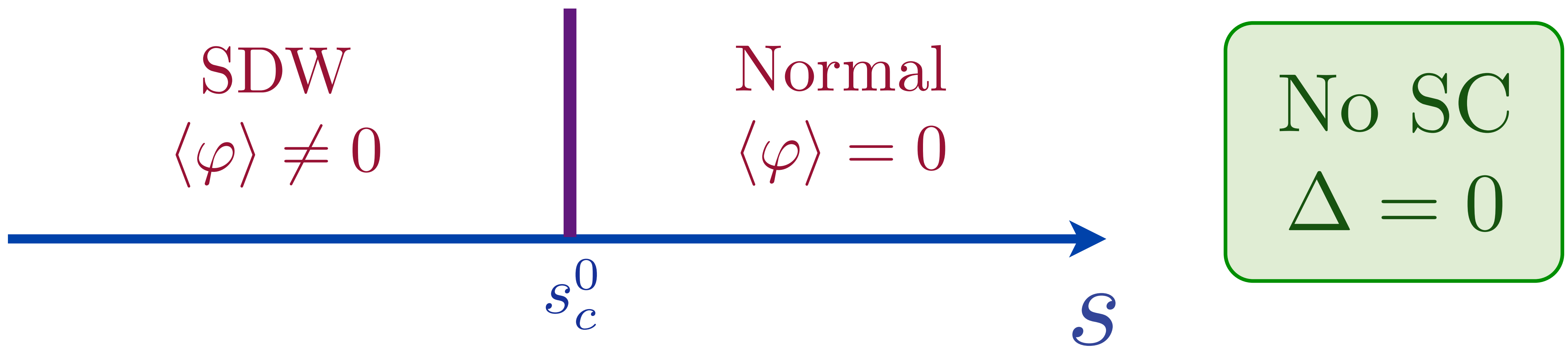


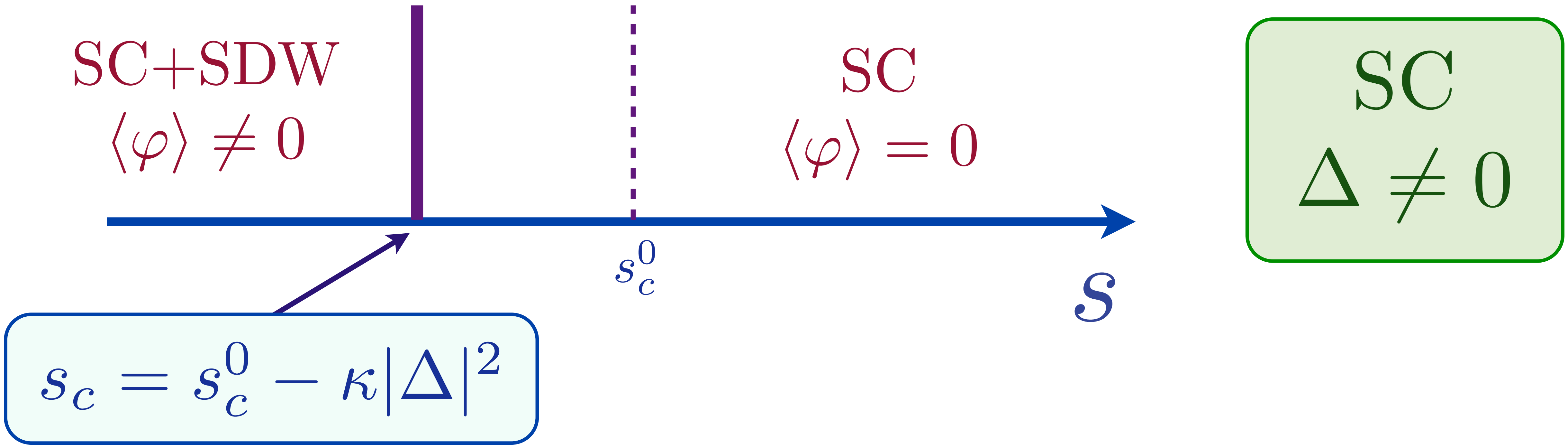
No SC

$\Delta = 0$

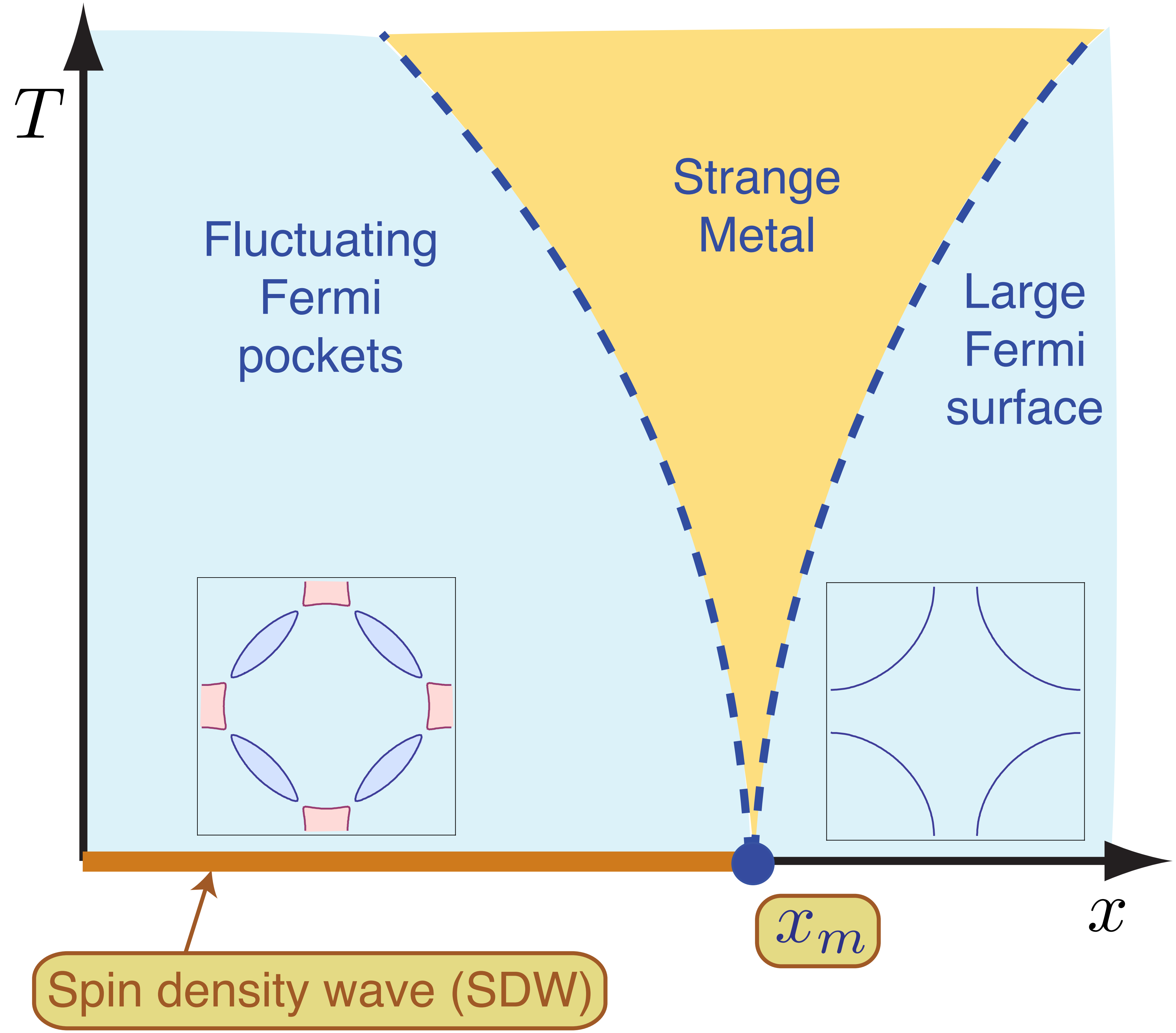
Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order



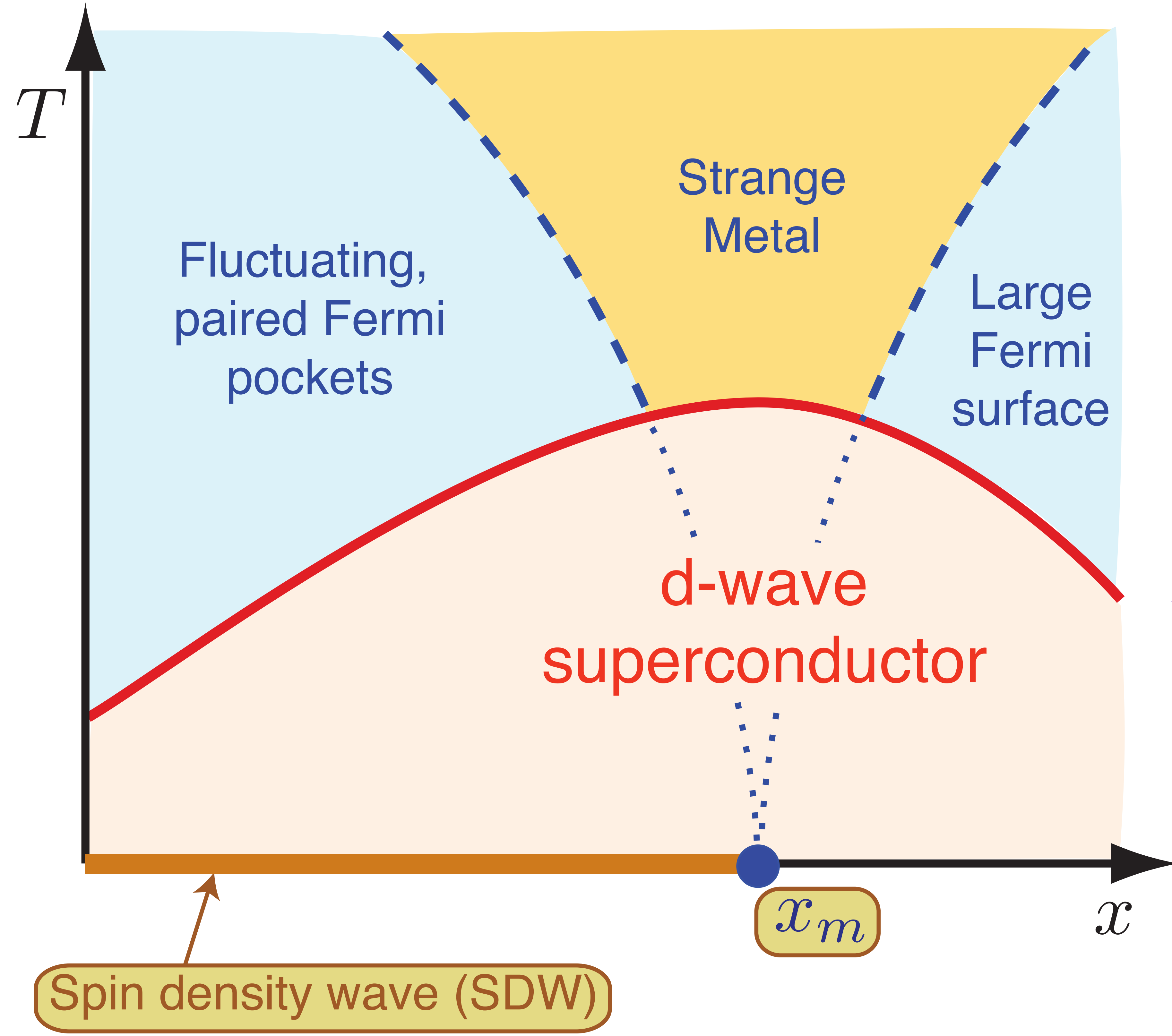


Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

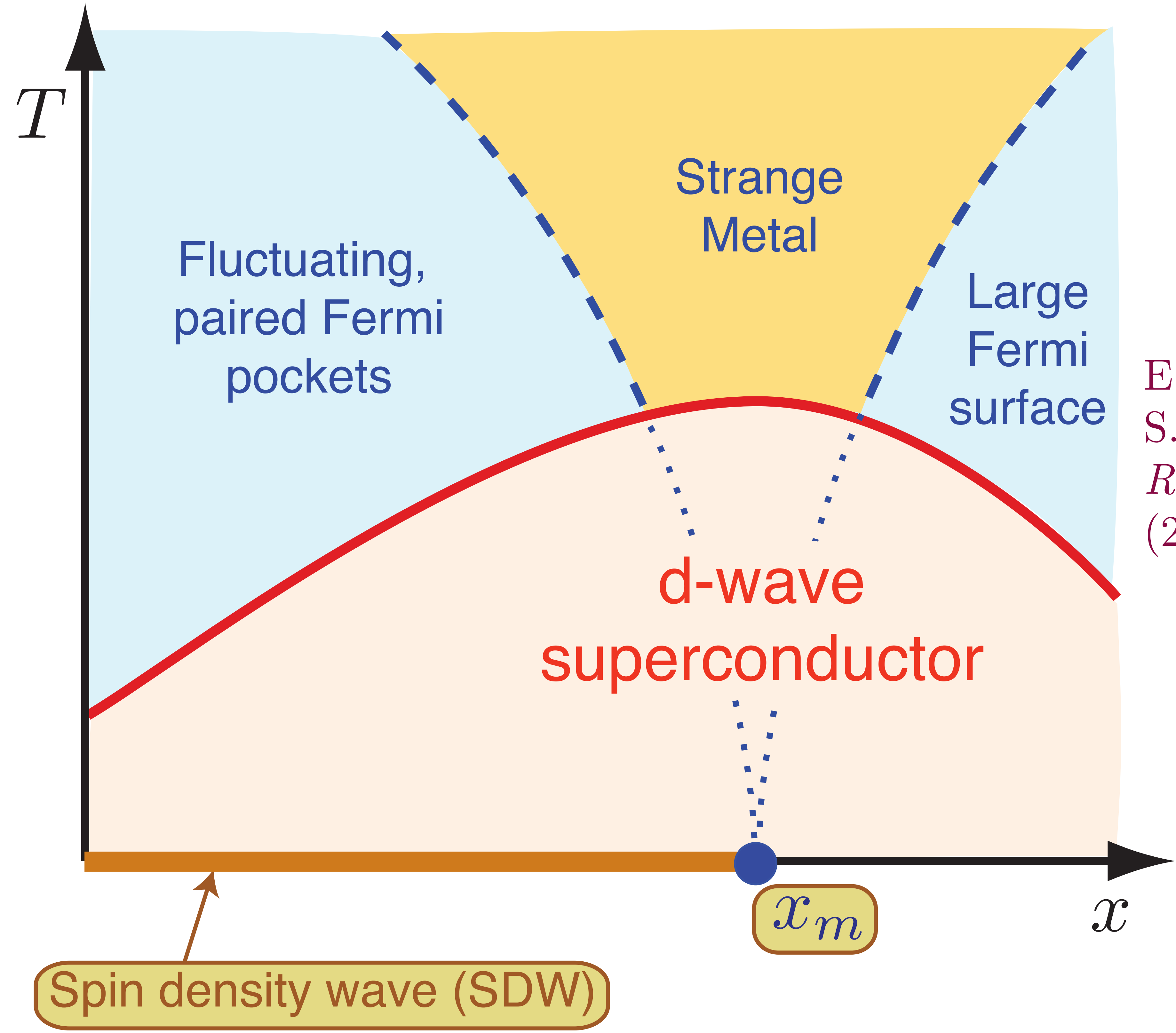
Theory of quantum criticality in the cuprates



Ar. Abanov, A.V. Chubukov, and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Onset of d-wave superconductivity hides the critical point $x = x_m$

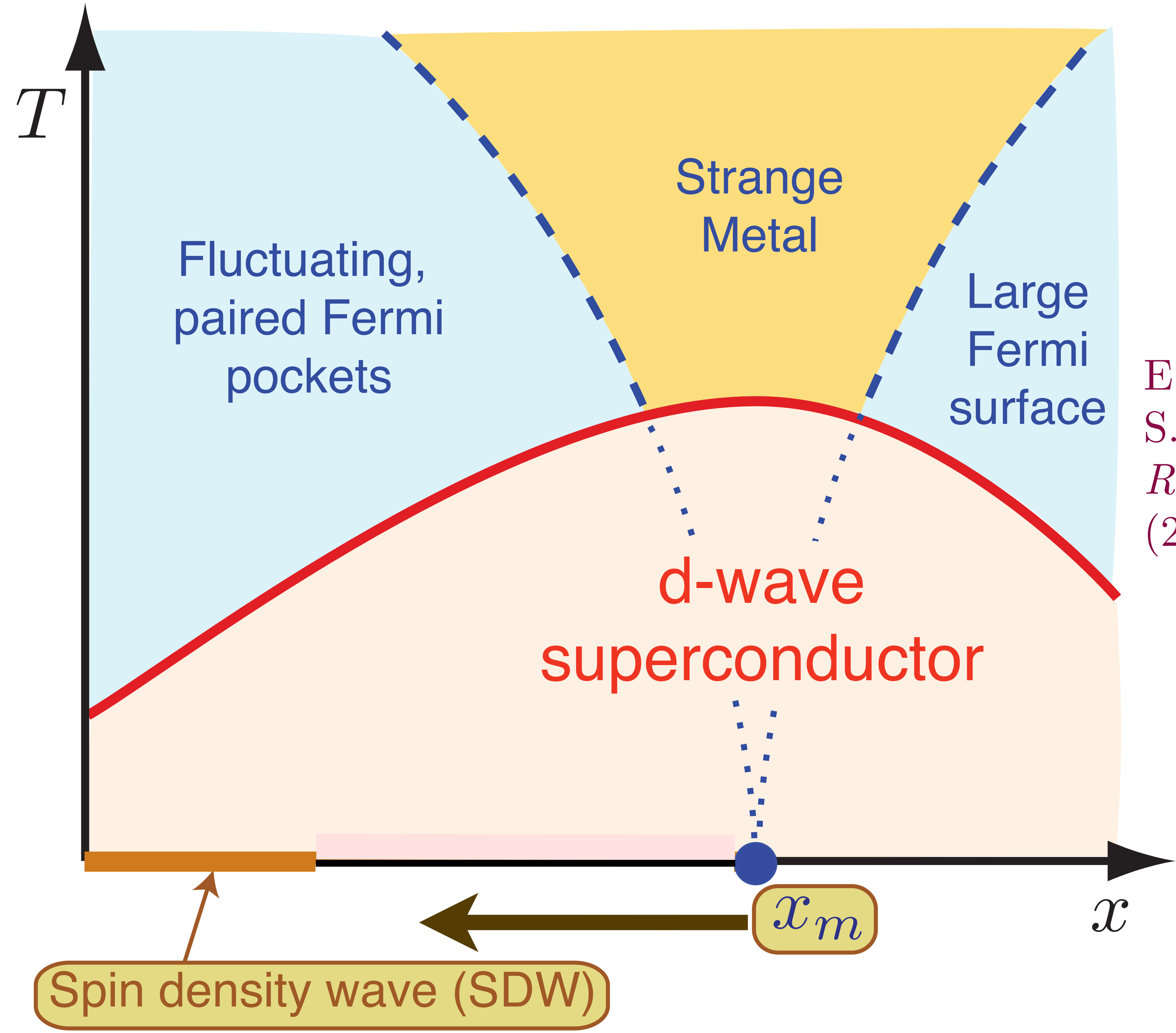
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

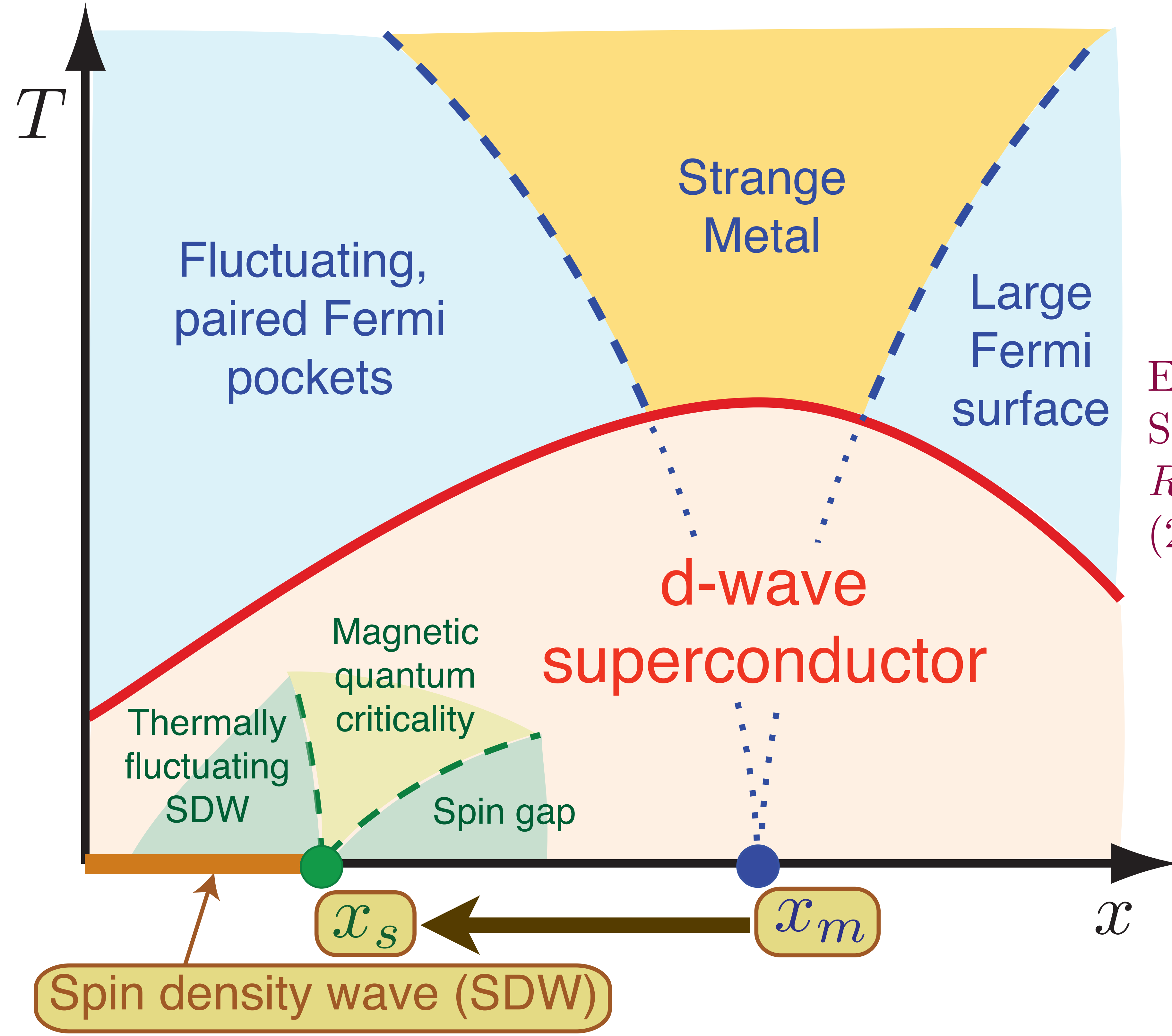
Theory of quantum criticality in the cuprates



E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

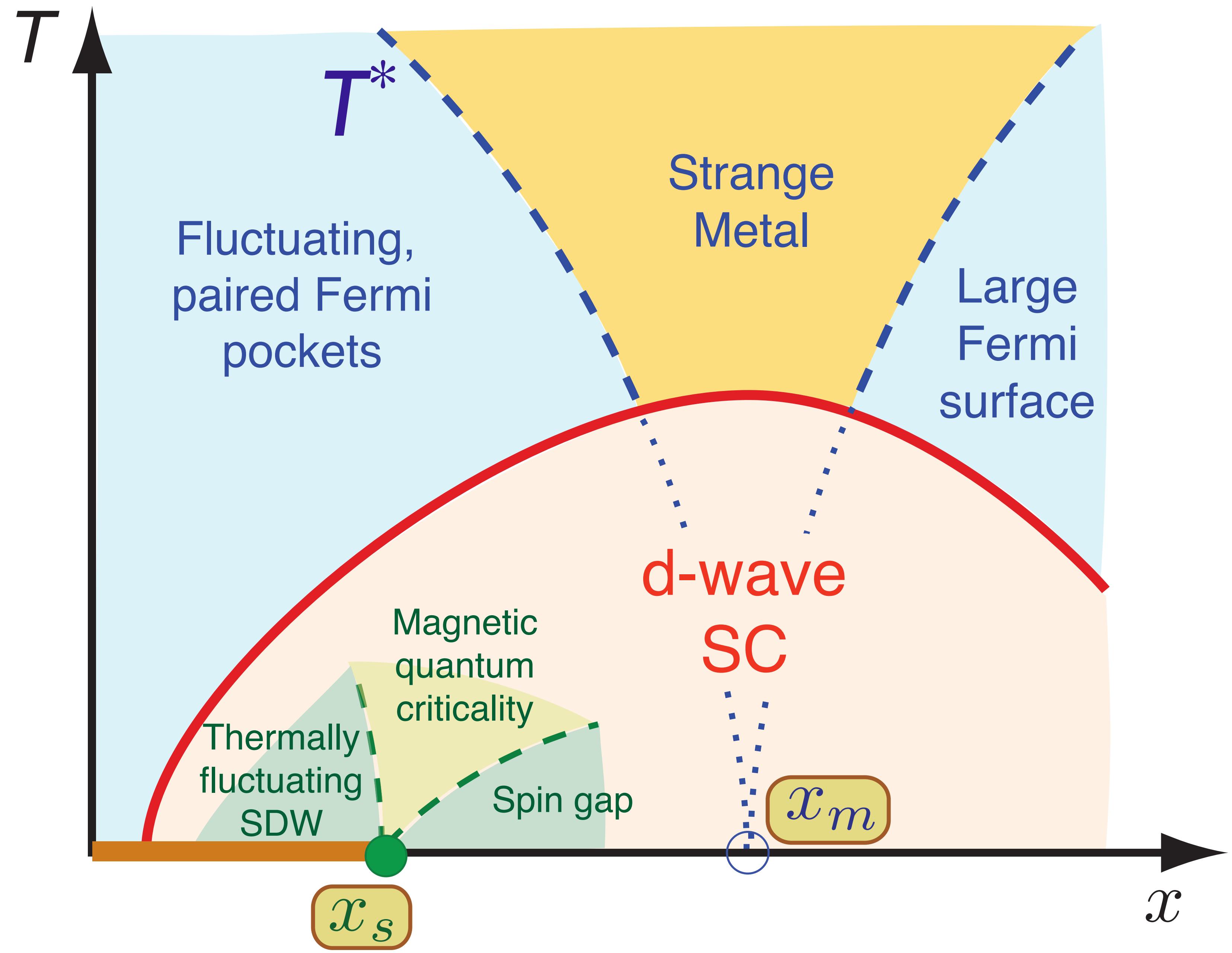
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates

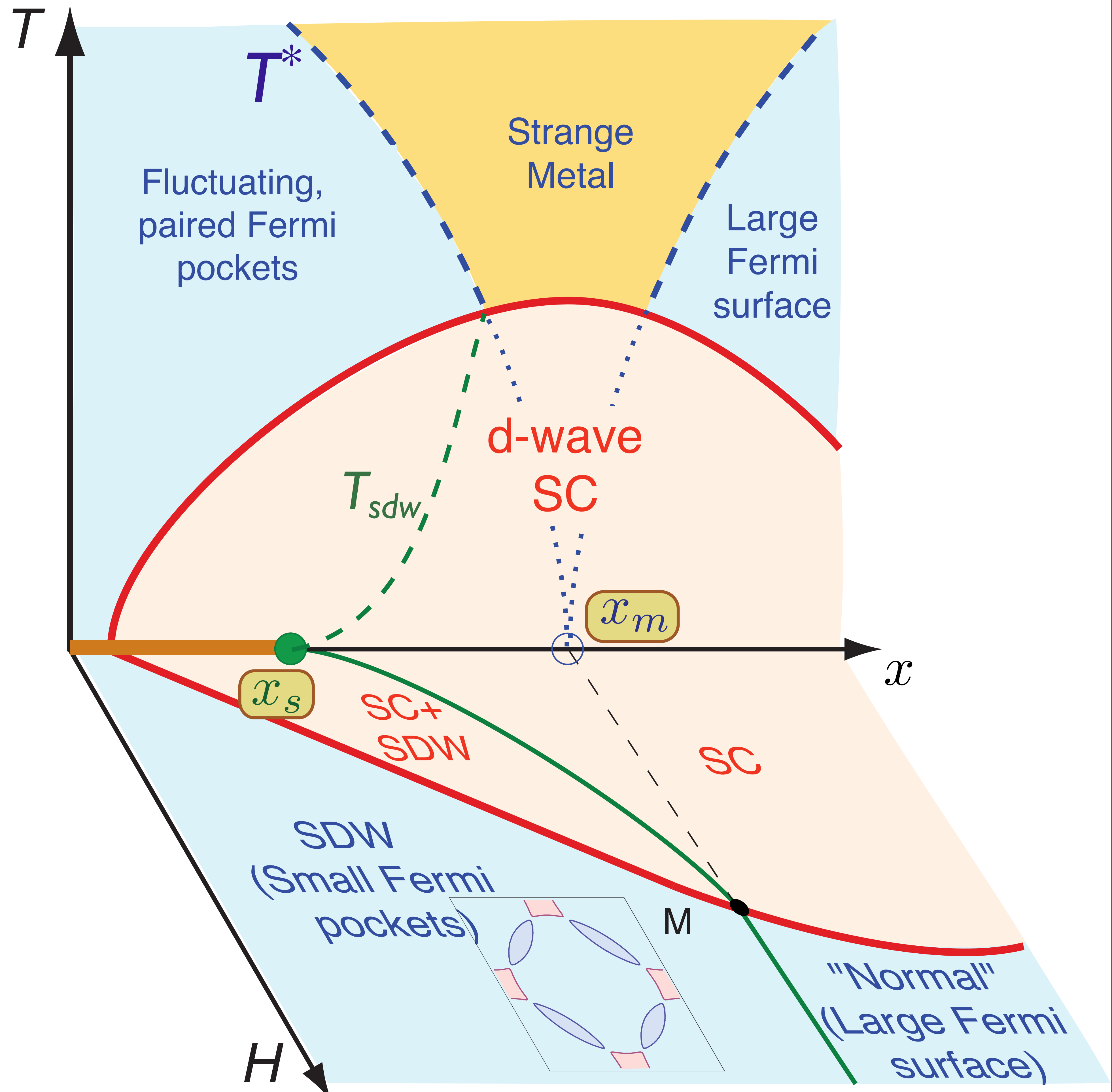


E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

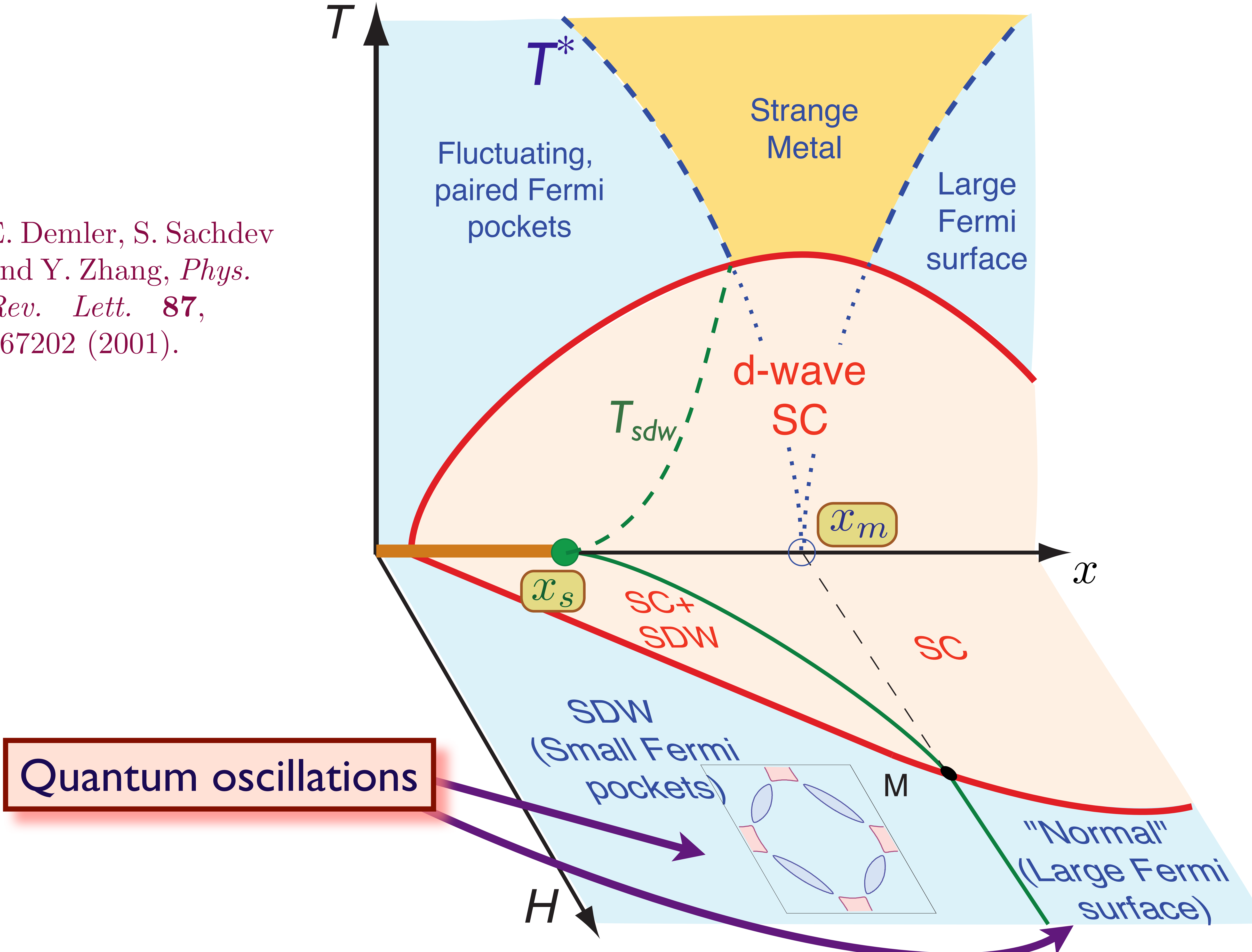
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.


T
T^*
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x

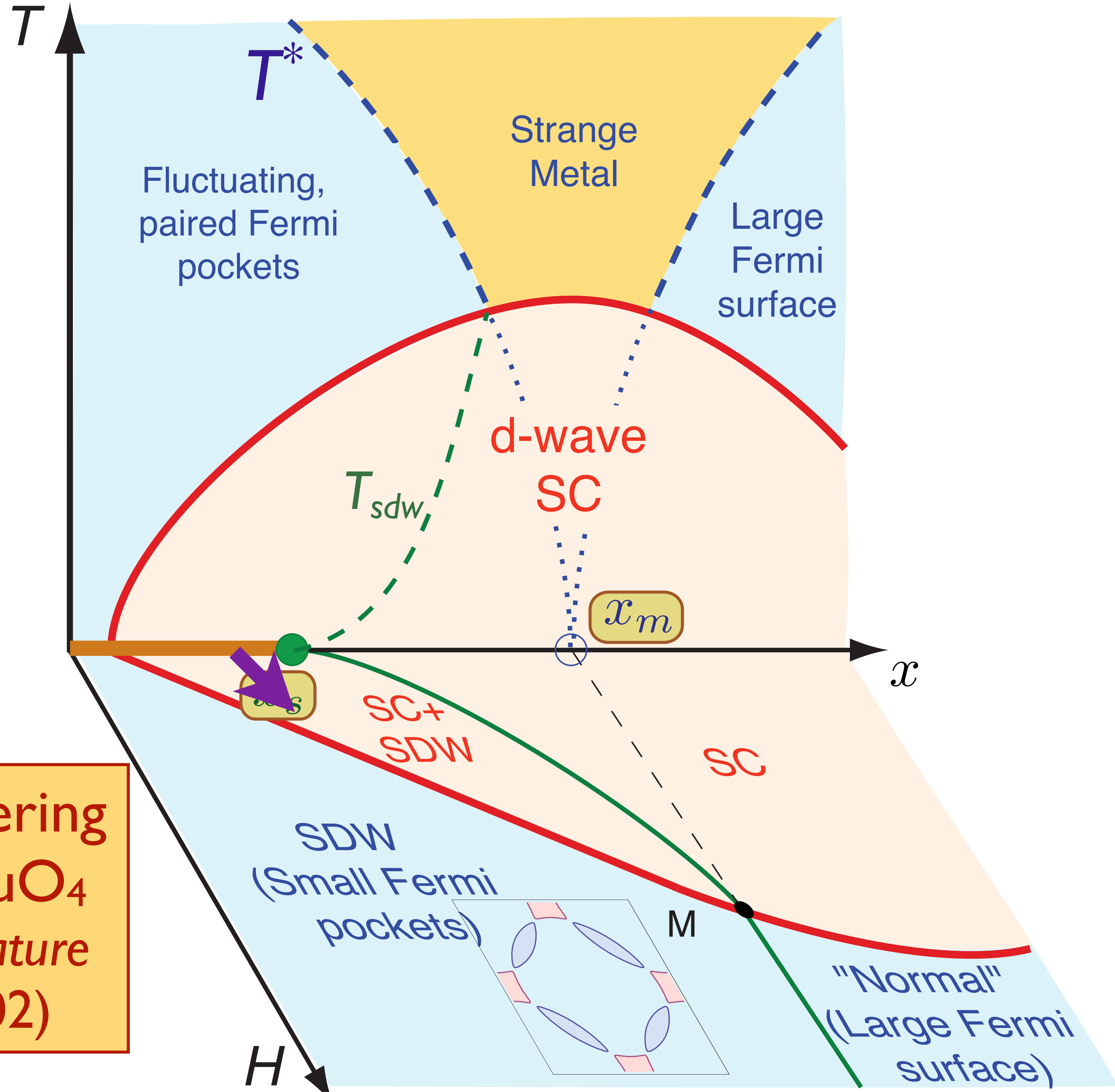
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

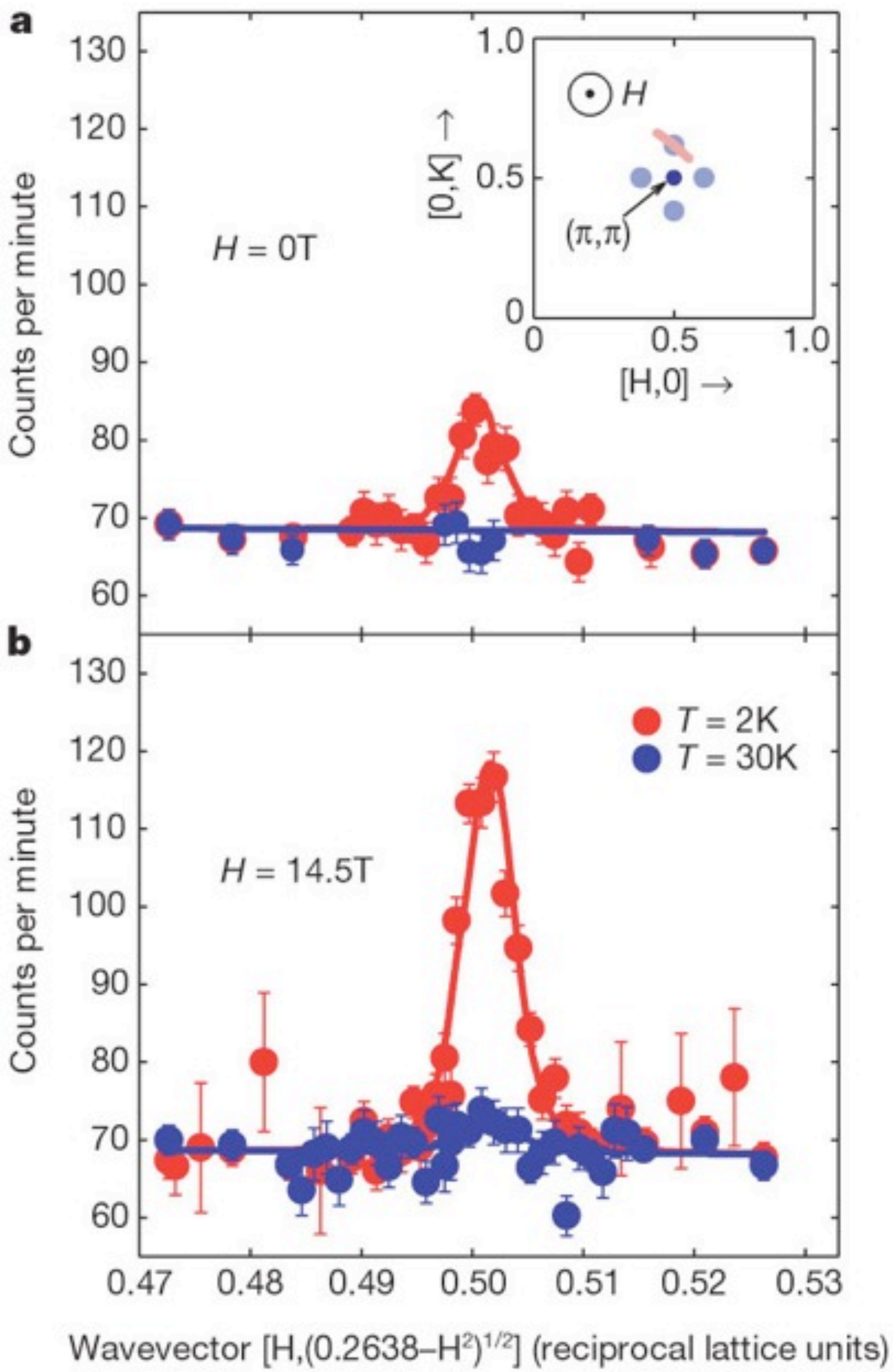


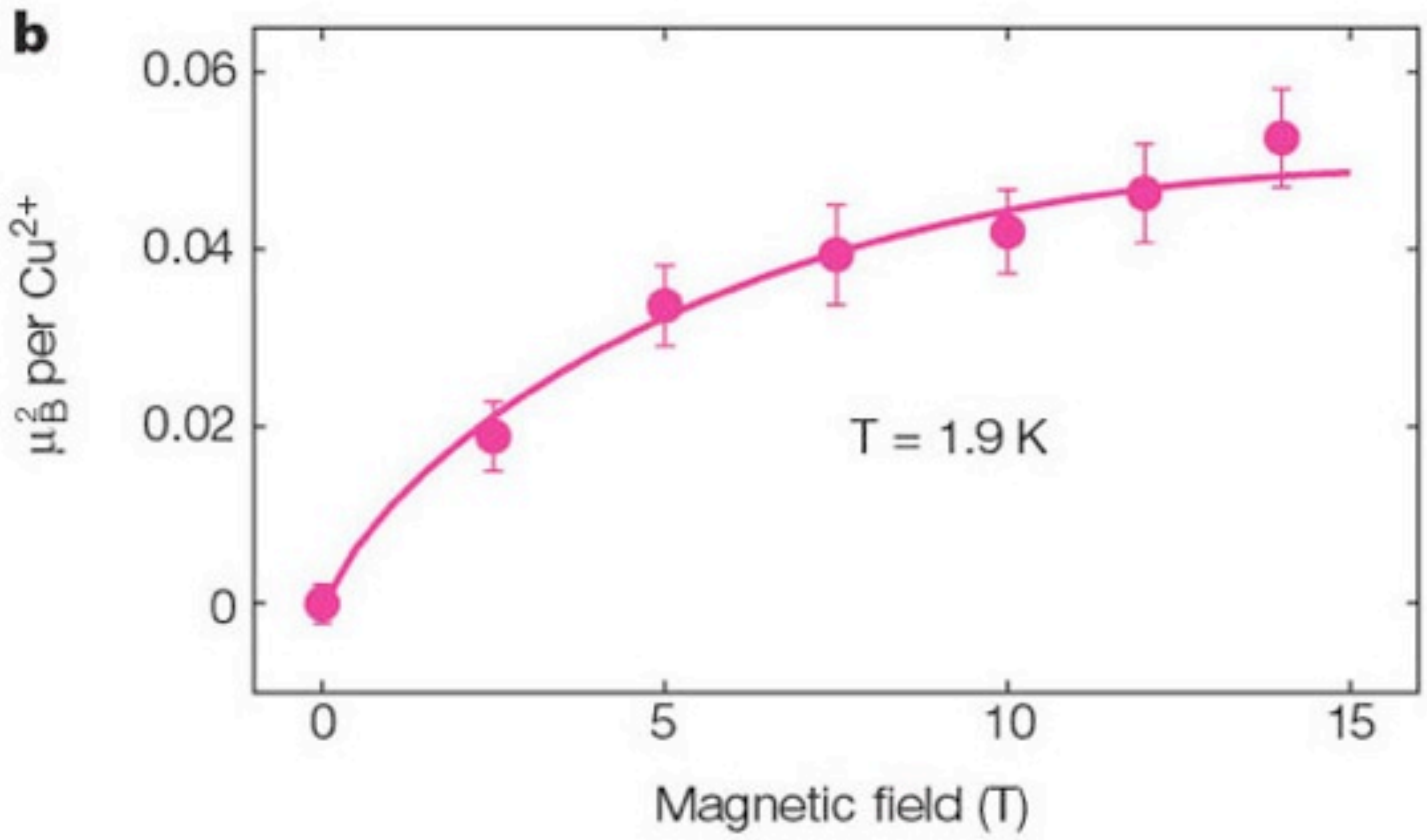
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



Neutron scattering on $La_{1.9}Sr_{0.1}CuO_4$

B. Lake *et al.*, *Nature* 415, 299 (2002)

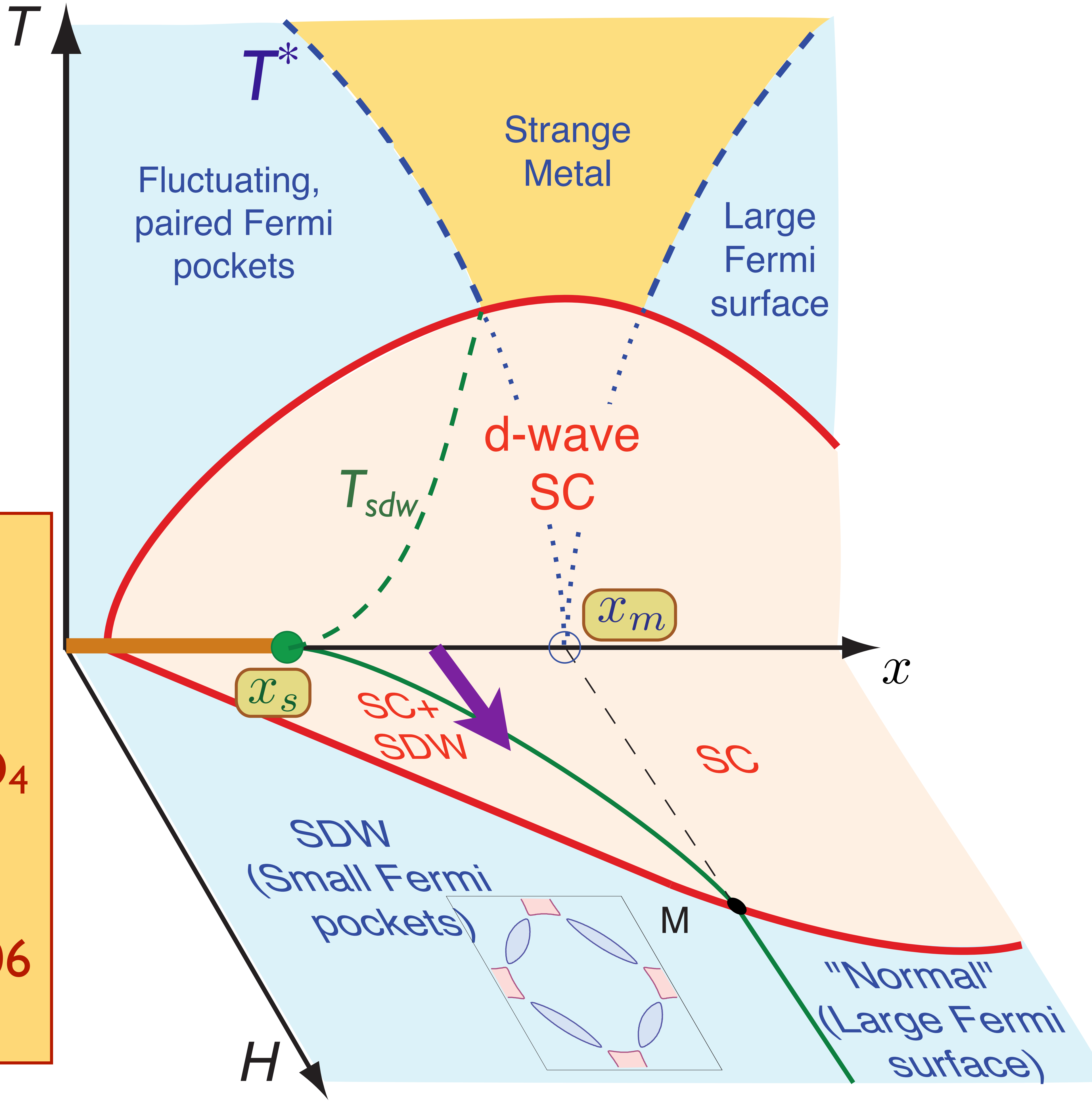




B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, and T. E. Mason, Nature **415**, 299 (2002)

B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder *Science* **291**, 1759 (2001).

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



Neutron scattering and μSR on $La_{1.855}Sr_{0.145}CuO_4$

J. Chang *et al.*, *Physical Review Letters* **102**, 177006 (2009)

PHYSICAL REVIEW B **71**, 220508(R) (2005)

Field-induced transition between magnetically disordered and ordered phases in underdoped $La_{2-x}Sr_xCuO_4$

B. Khaykovich,[1] S. Wakimoto,[2] R. J. Birgeneau,[3] M. A. Kastner,[1] Y. S. Lee,[1] P. Smeibidl,[4] P. Vorderwisch,[4] and K. Yamada[5]

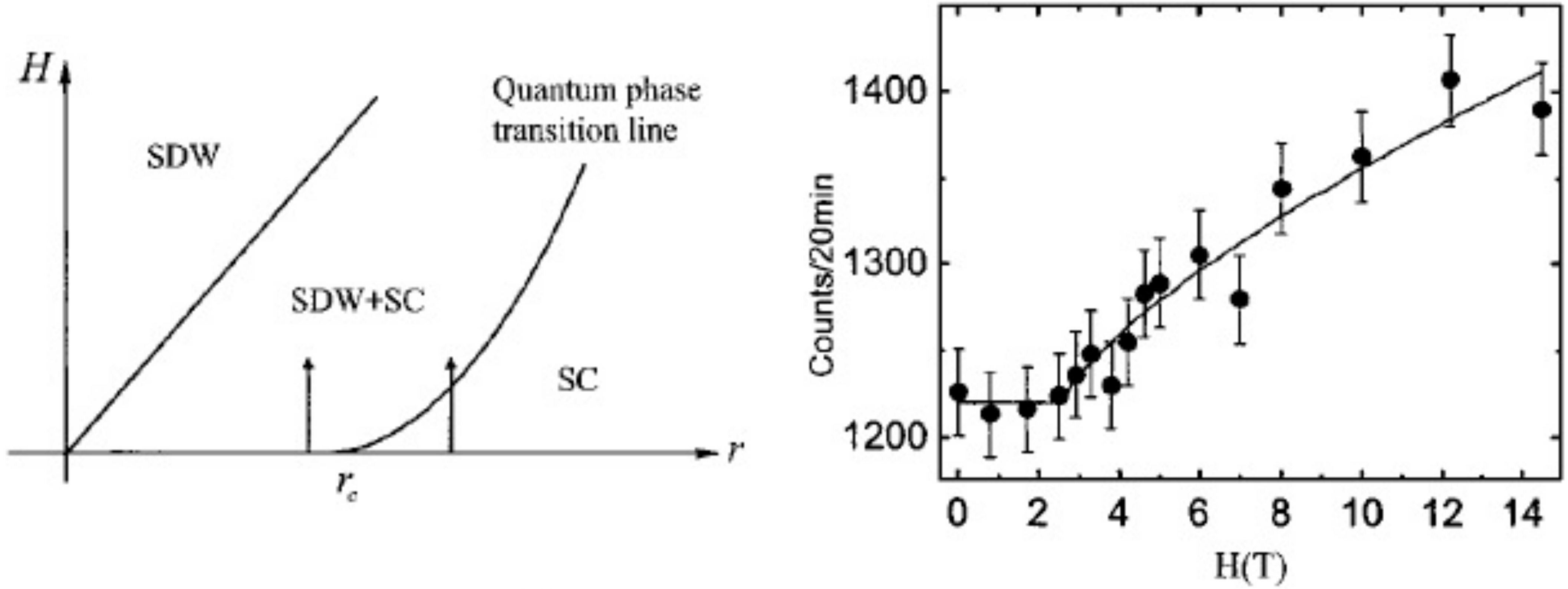
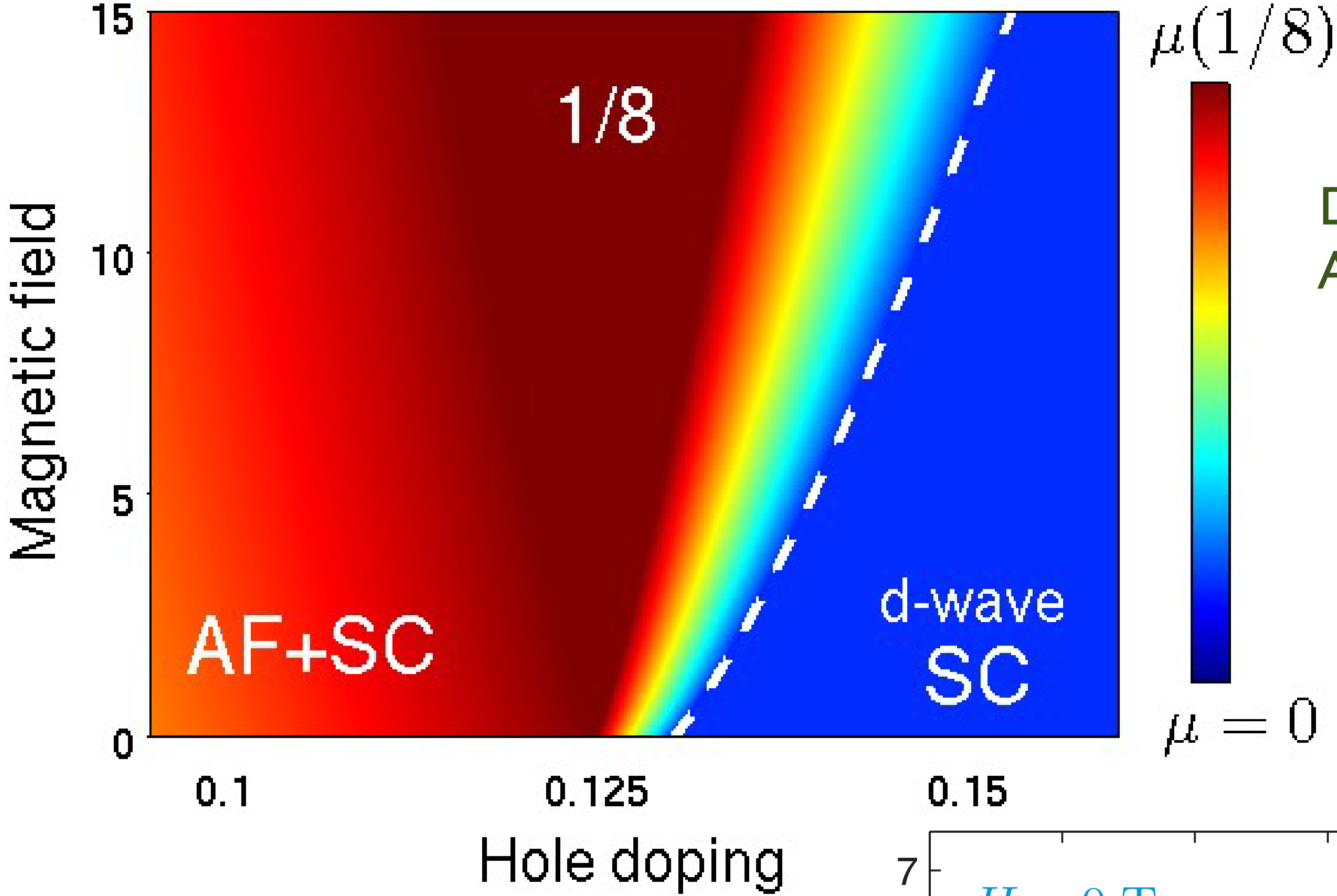
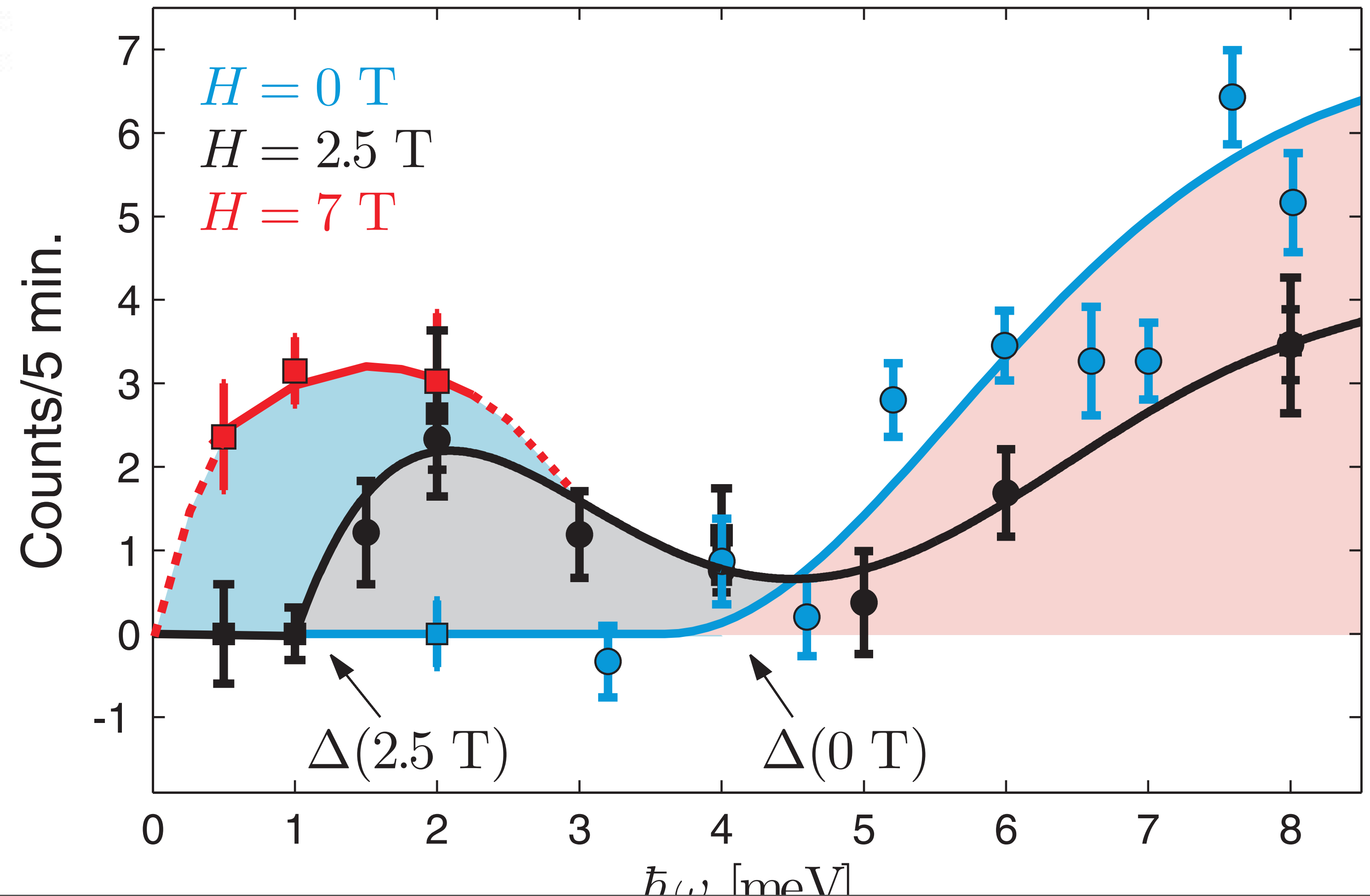


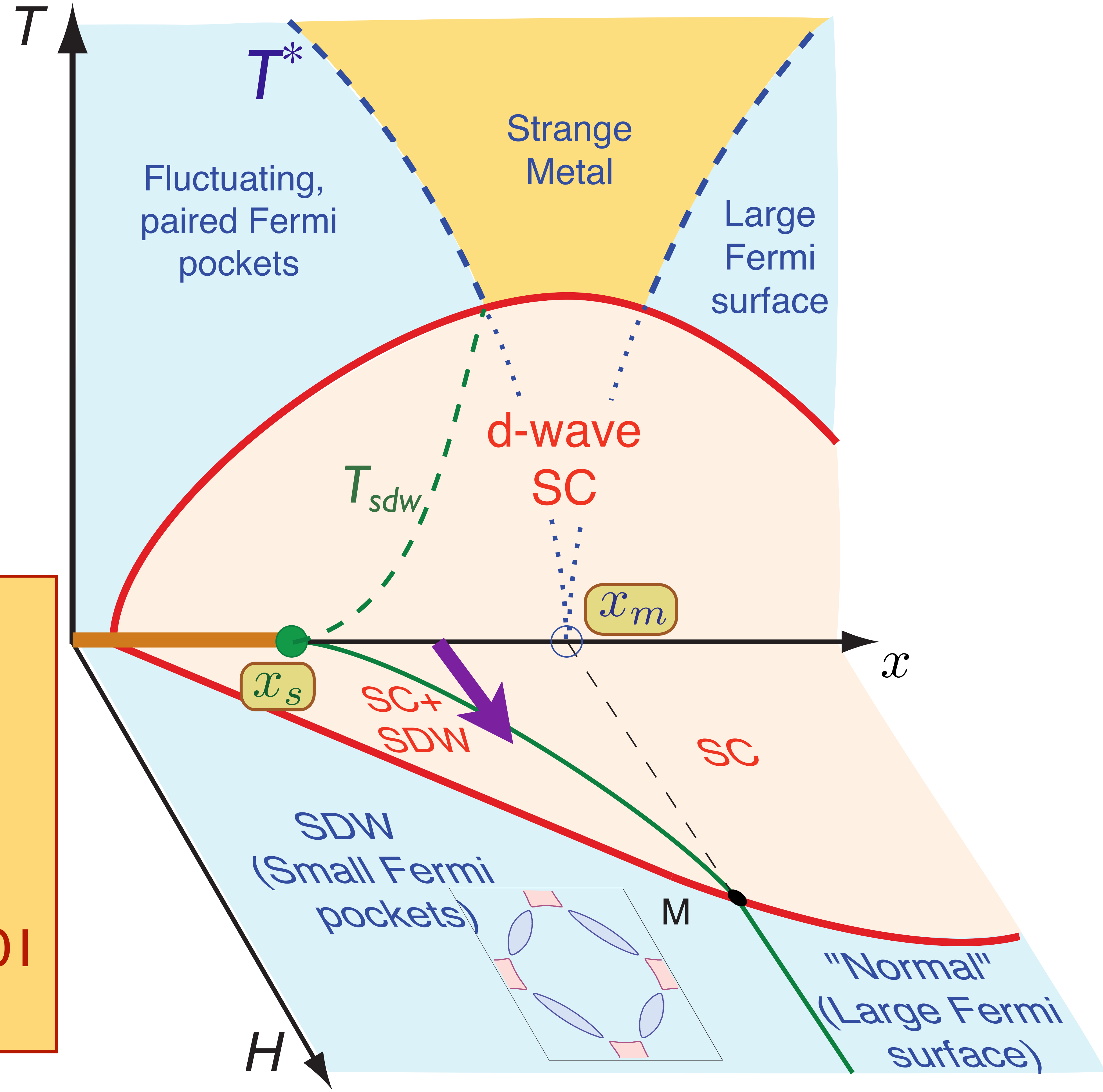
FIG. 1. (a) A fragment of the theoretical phase diagram, adopted from Refs. 4 and 20. The vertical axis is the magnetic field and the horizontal axis is the coupling strength between superconductivity and magnetic order. (b) Field dependence of the magnetic Bragg peak corresponding to the incommensurate SDW peak at $Q=(1.125,0.125,0)$. Every point is measured after field cooling at $T=1.5$ K. The data are fitted to $I=I_0+A|H-H_c|^{2\beta}$ above H_c as explained in the text. Spectrometer configuration: 45-60-Be—S—Be-60-open; cold Be filters were used before and after the sample to eliminate contamination from high-energy neutrons; $E=4$ meV.



J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).

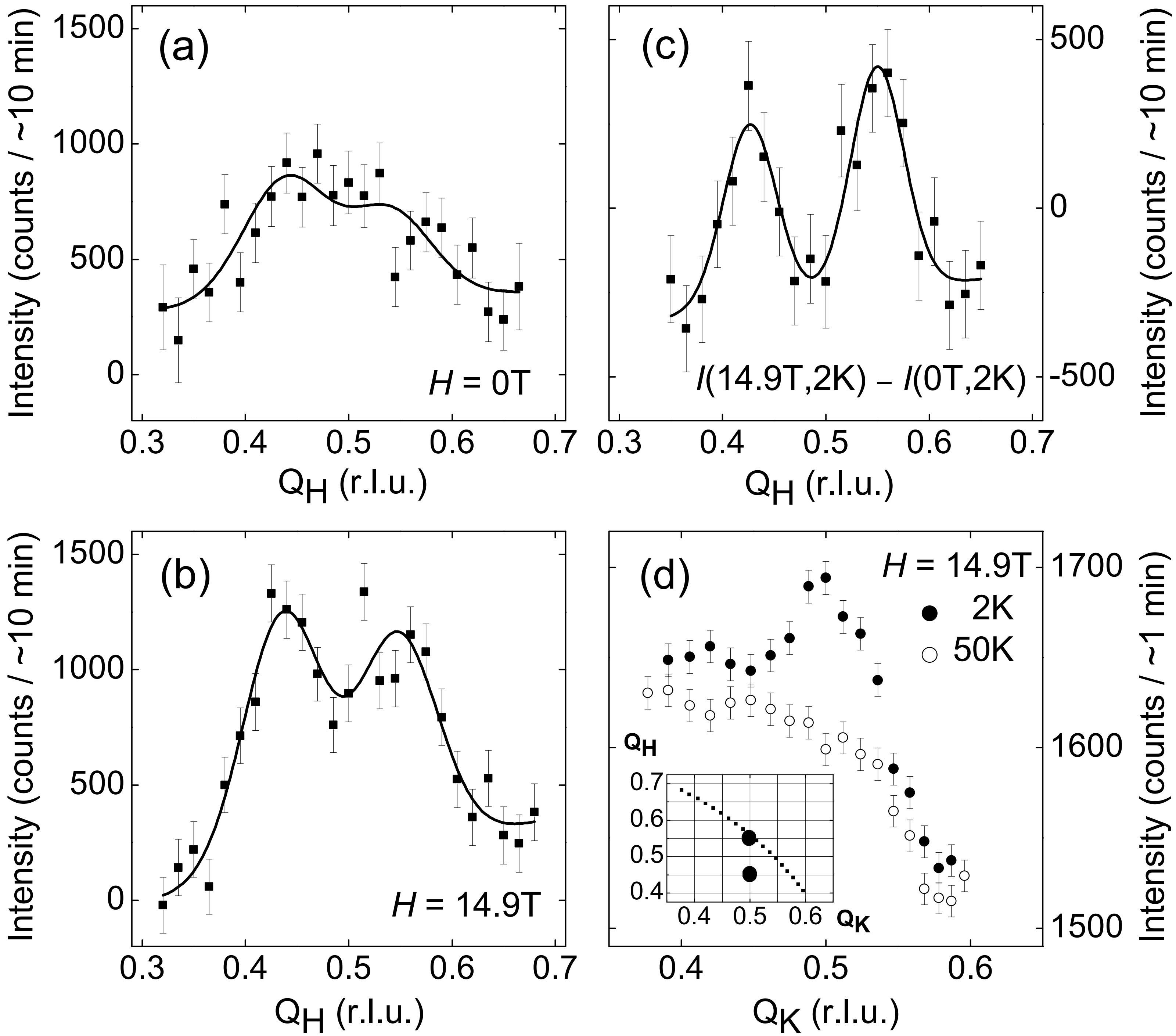


J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

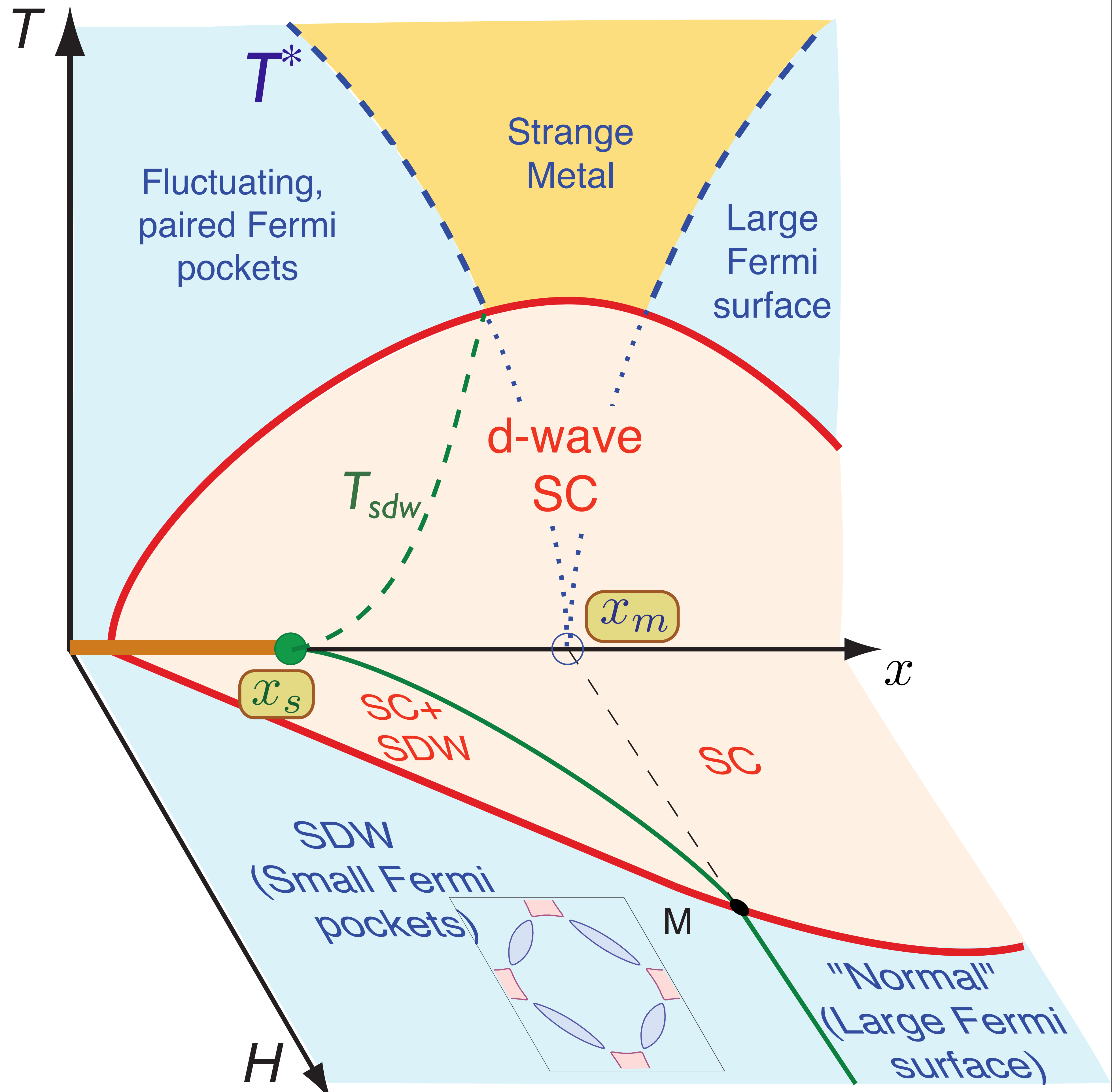
Neutron scattering on $YBa_2Cu_3O_{6.45}$
D. Haug *et al.*, *Physical Review Letters* **103**, 017001 (2009)



D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Physical Review Letters* **103**, 017001 (2009)

Phase diagram also applies to the electron-doped cuprates

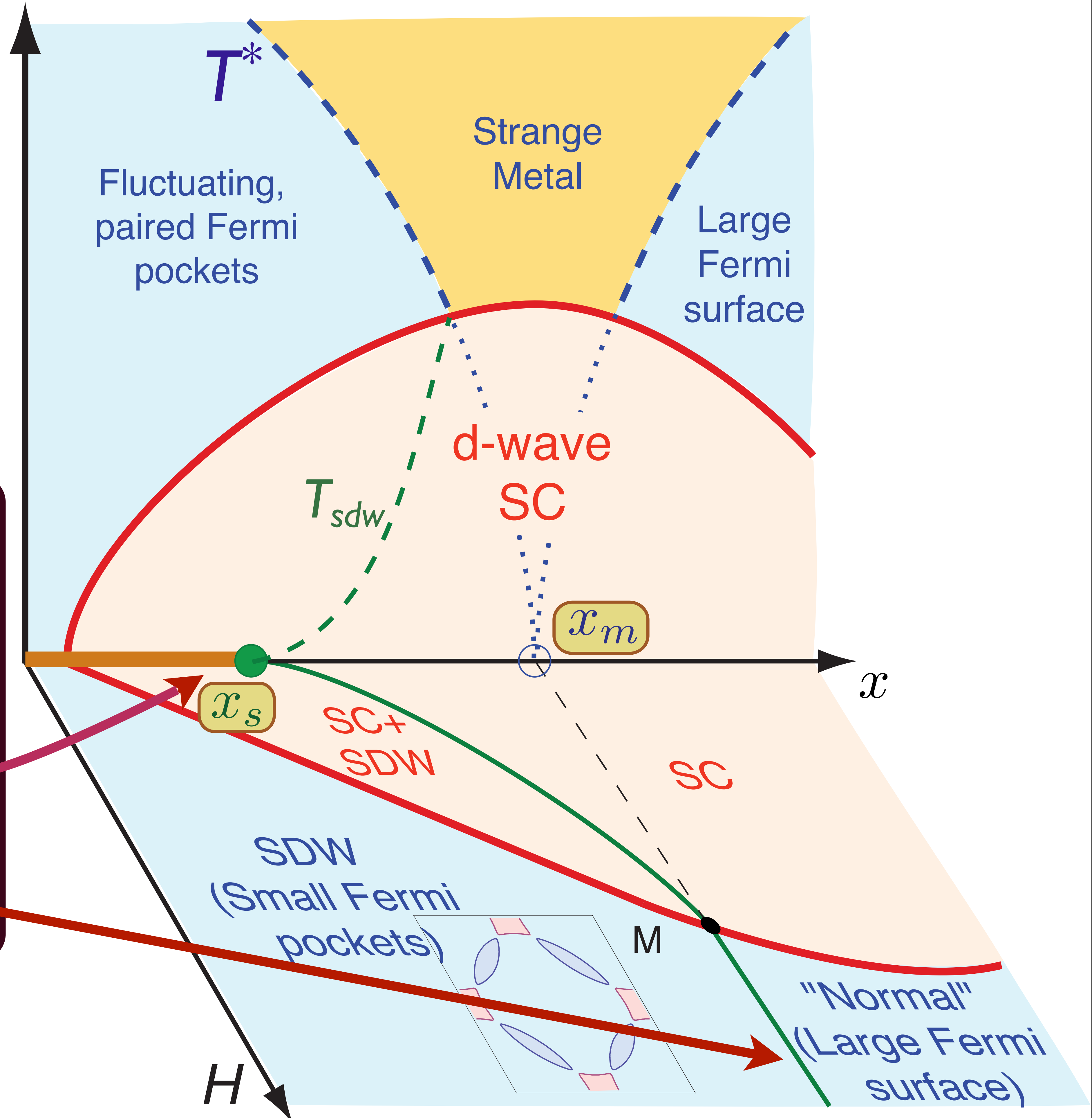
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

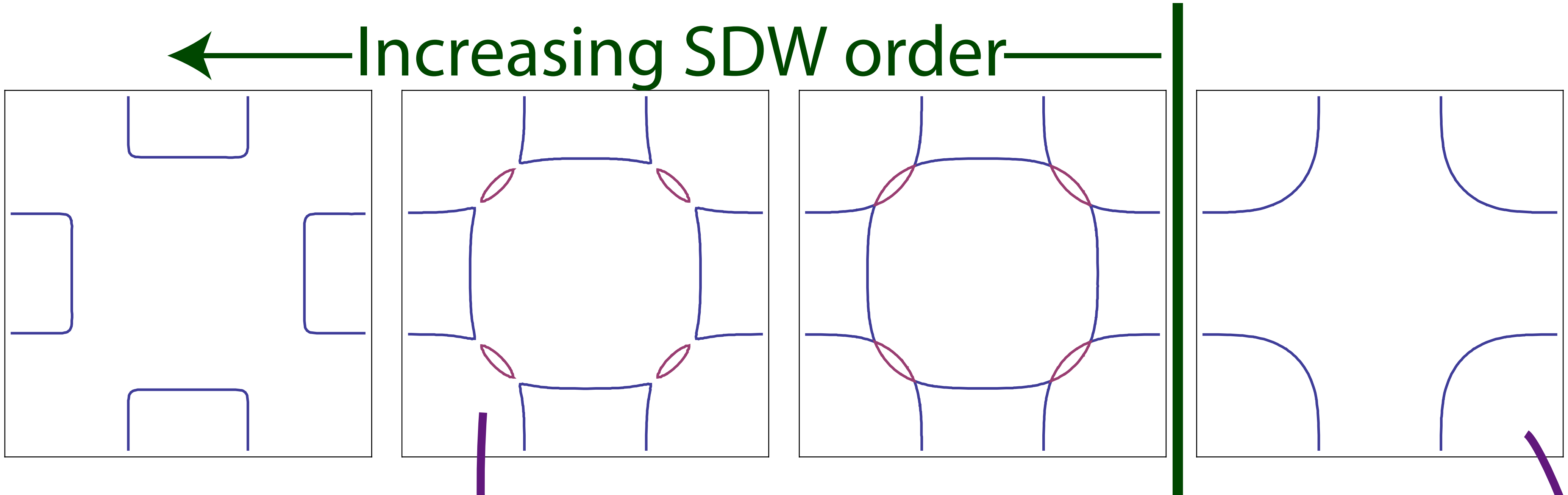


Phase diagram also applies to the electron-doped cuprates

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

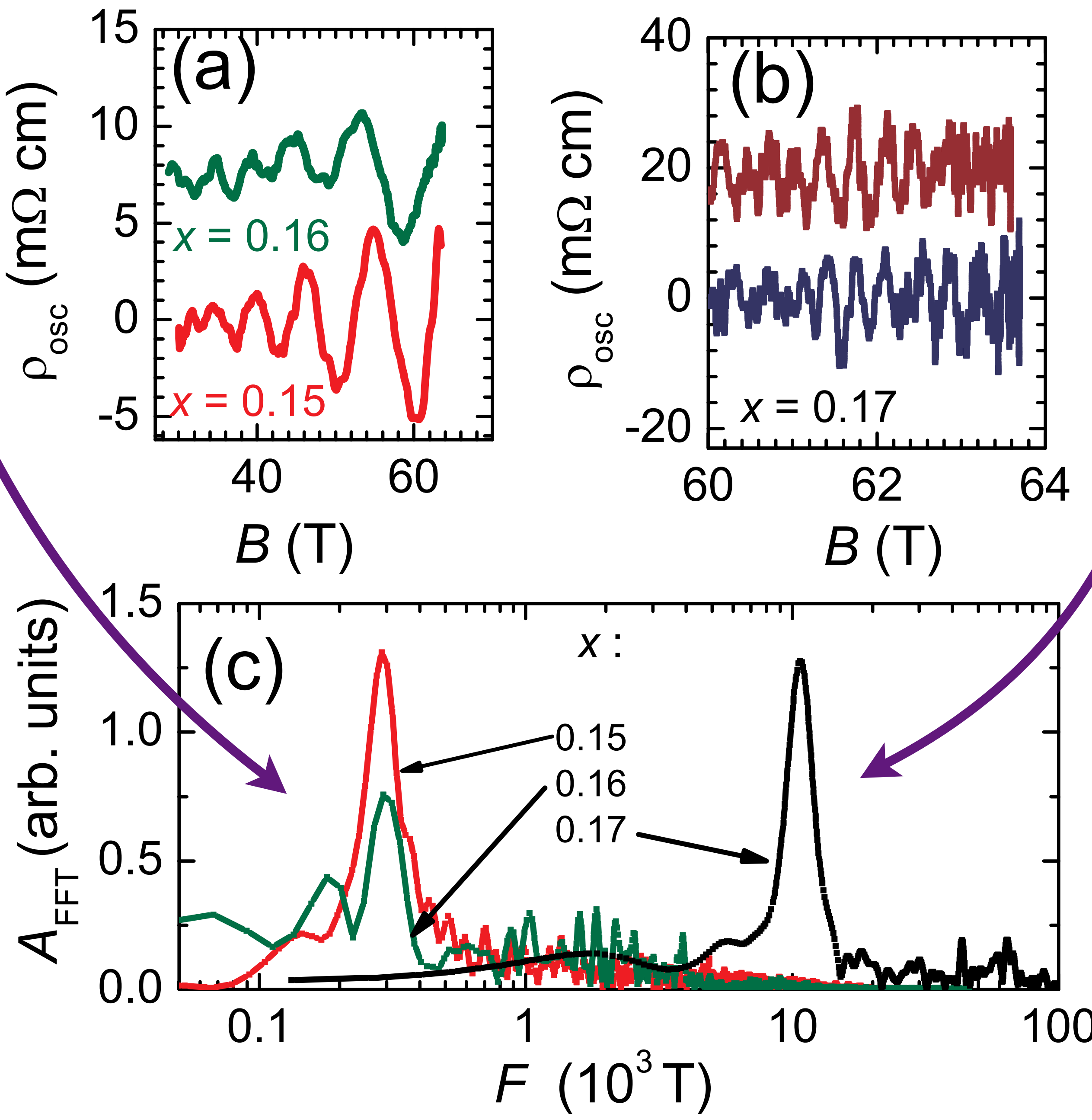
Neutron scattering experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.





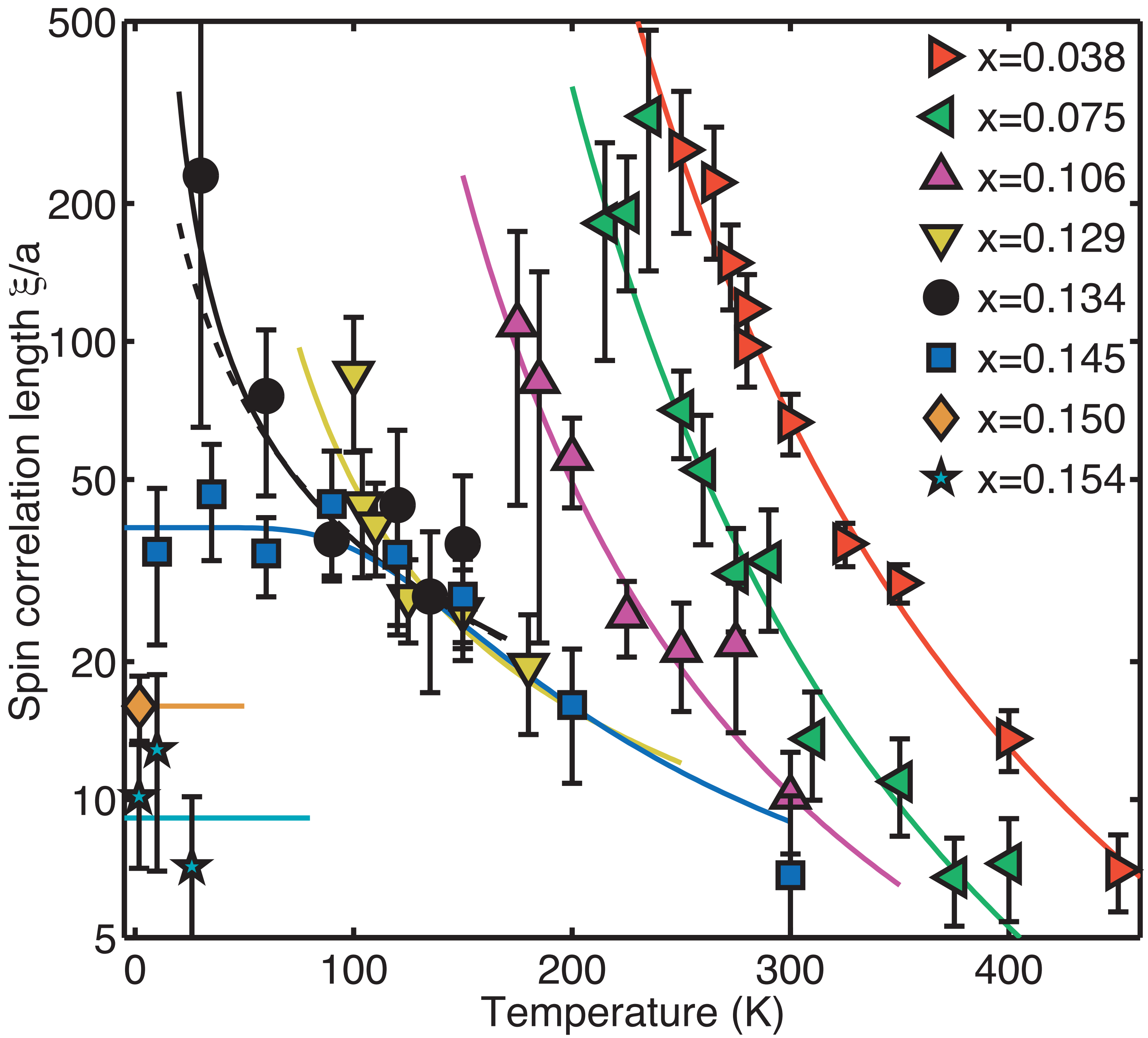
Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$



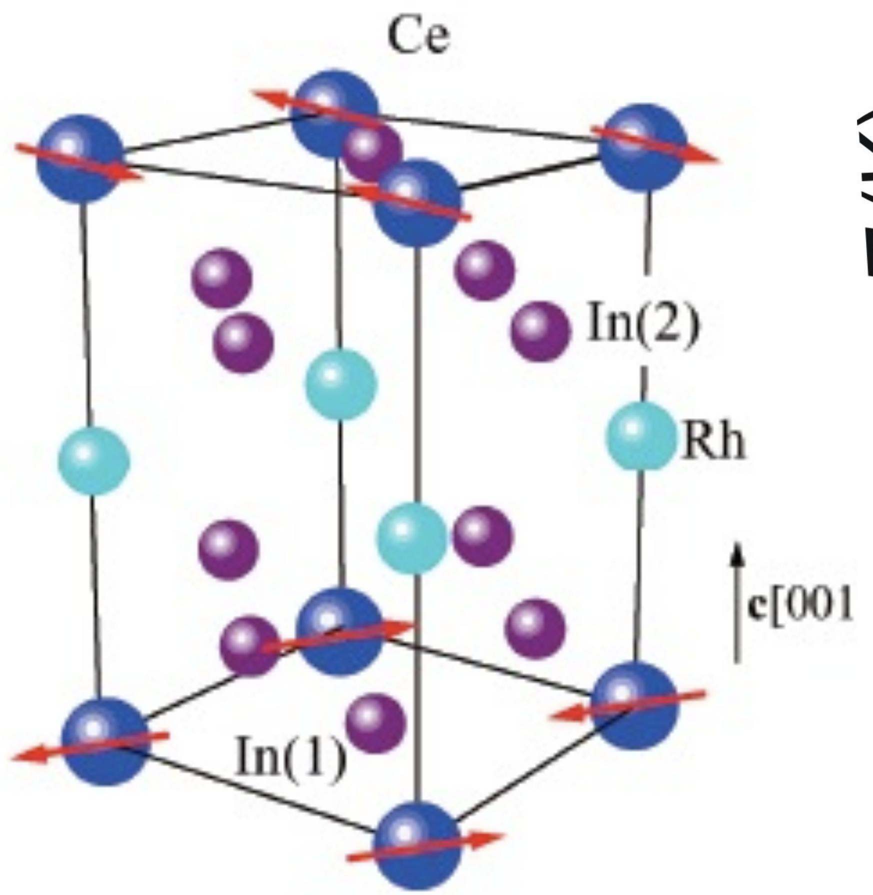
T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).

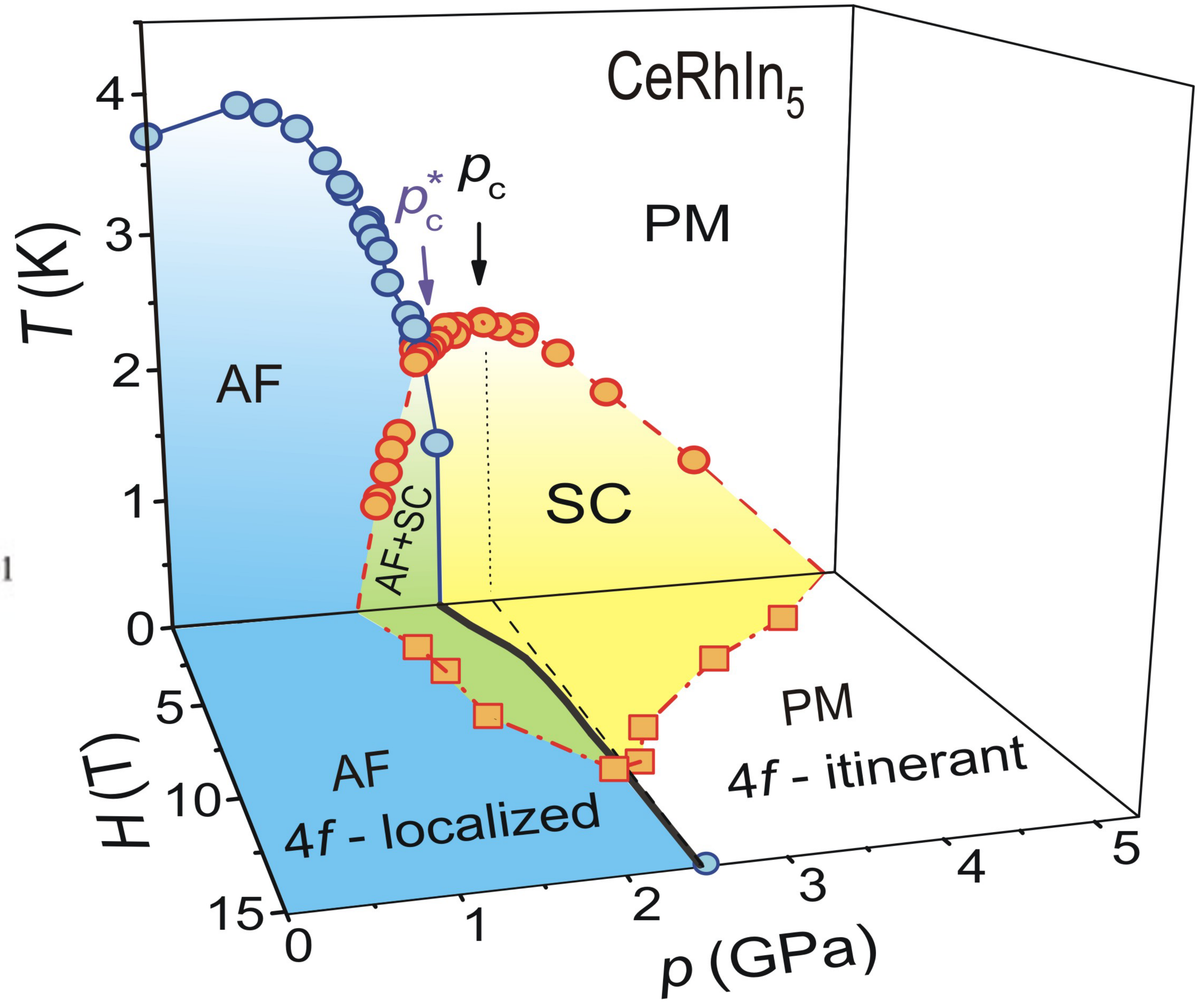
$Nd_{2-x}Ce_xCuO_4$



E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).

Similar phase diagram for $CeRhIn_5$





G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

Outline

1. Loss of antiferromagnetism in an insulator
   Coupled-dimer antiferromagnets and quantum criticality

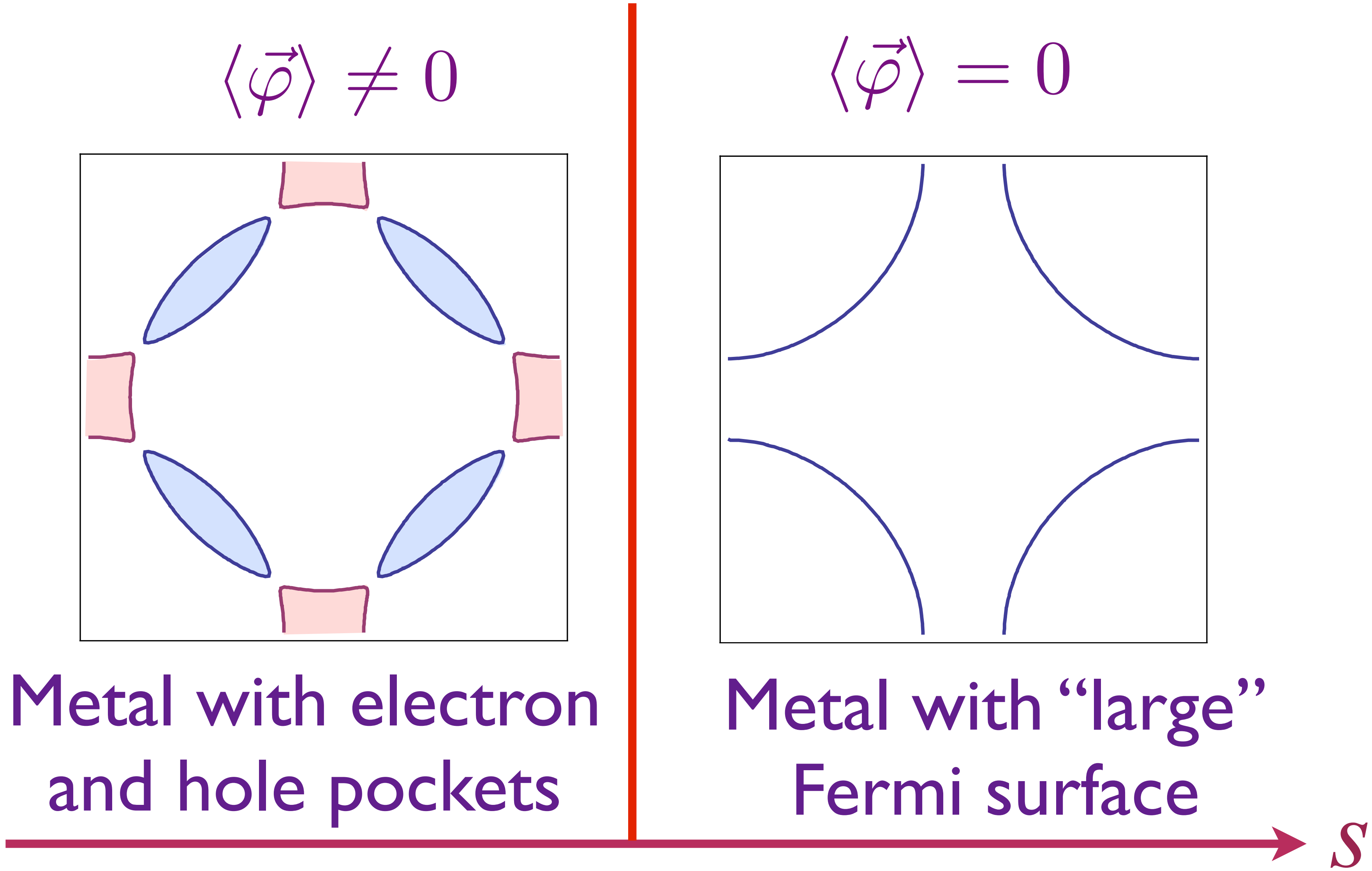
2. Onset of antiferromagnetism in a metal
   From large Fermi surfaces to Fermi pockets

3. Unconventional superconductivity
   Pairing from antiferromagnetic fluctuations

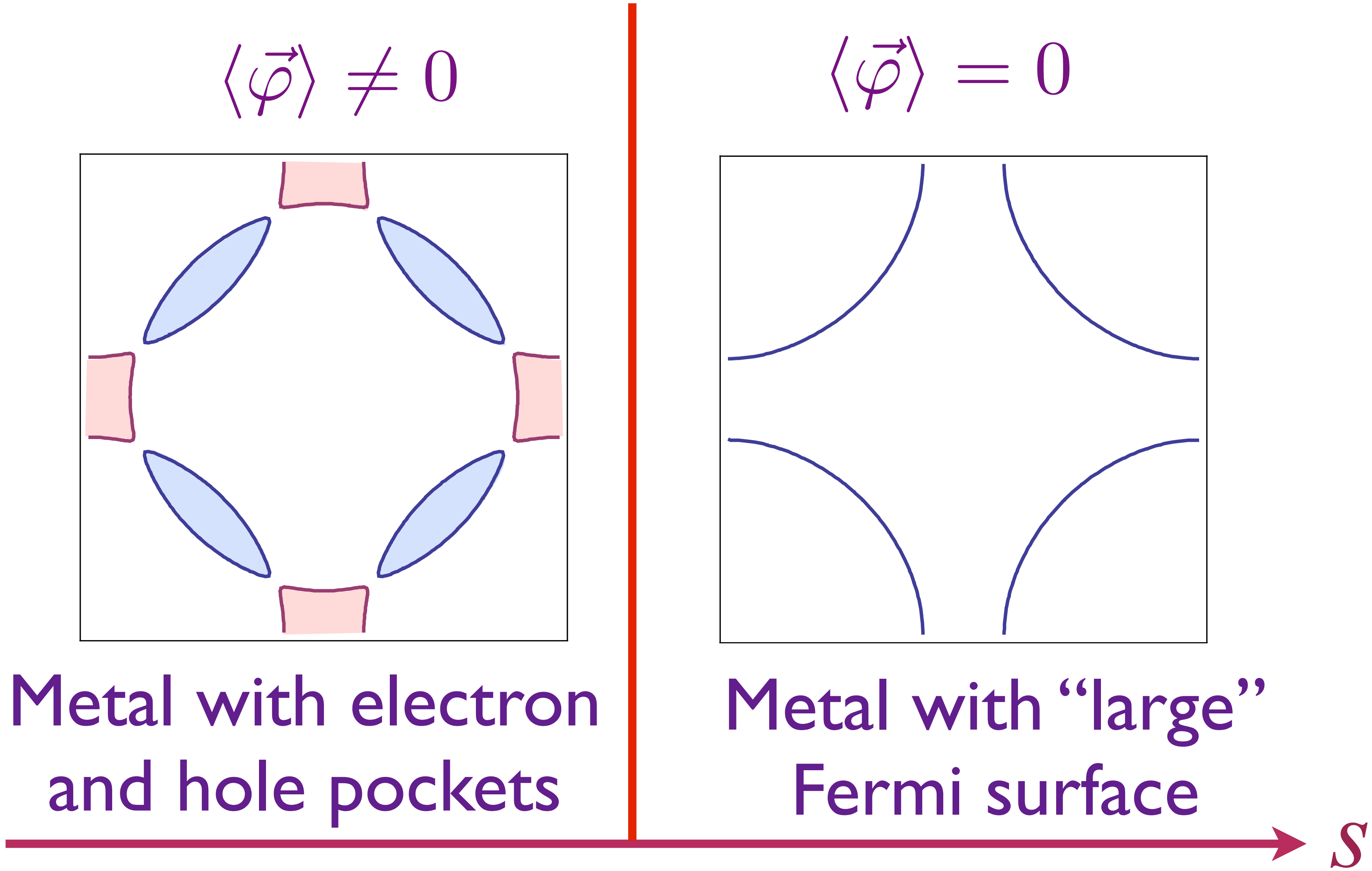
4. Competing orders
   Phase diagram in a magnetic field

5. Strongly-coupled quantum criticality in metals
   Fluctuating antiferromagnetism and Fermi surfaces

Quantum criticality of the onset of antiferromagnetism in a metal



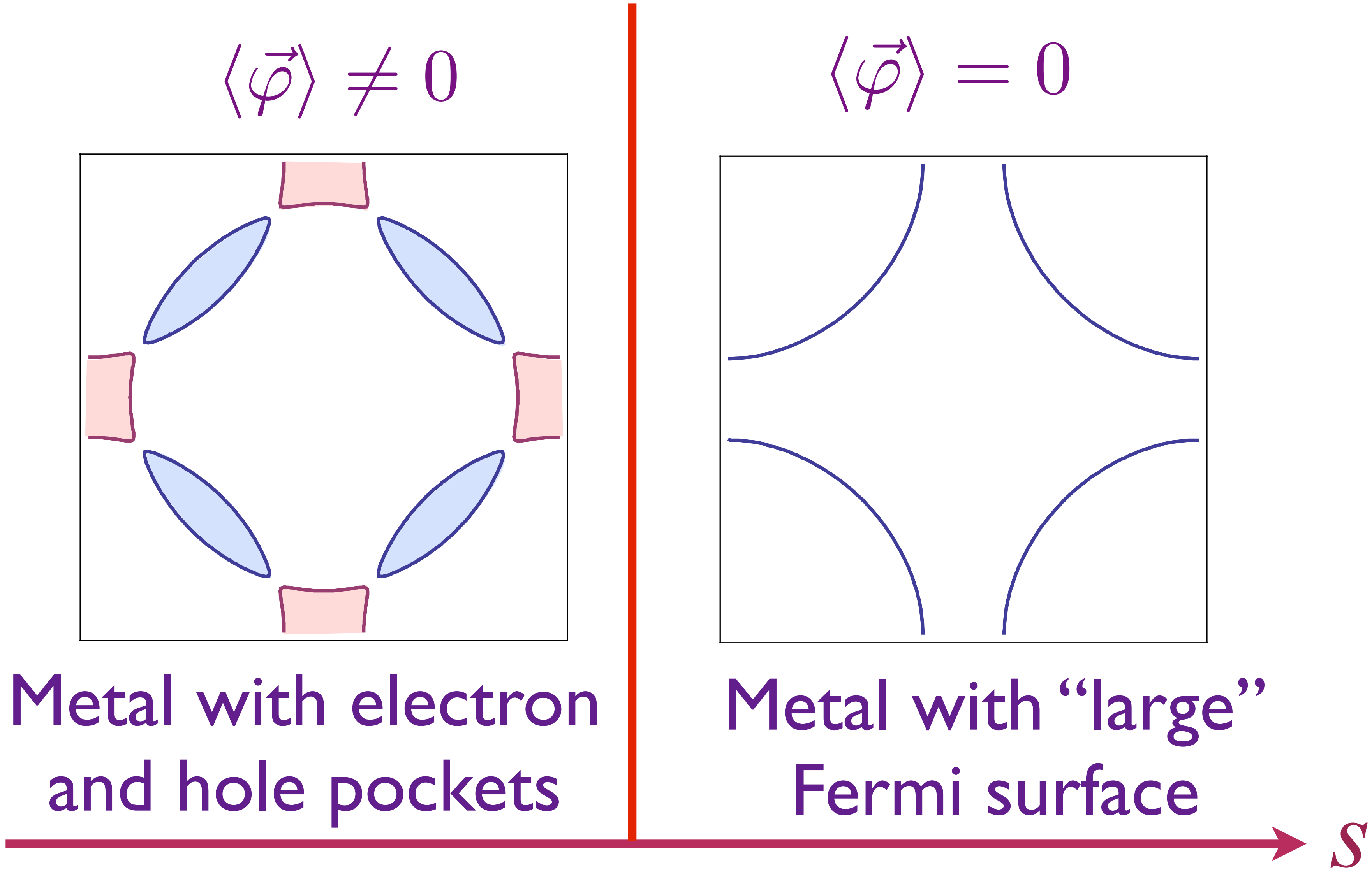
Quantum criticality of the onset of antiferromagnetism in a metal



- Theory with strong *amplitude* fluctuations of antiferromagnetic order parameter. Quantum critical theory is *strongly-coupled* in two (but not higher) spatial dimensions

M.A. Metlitski and S. Sachdev, *Physical Review B* **82**, 075128 (2010)

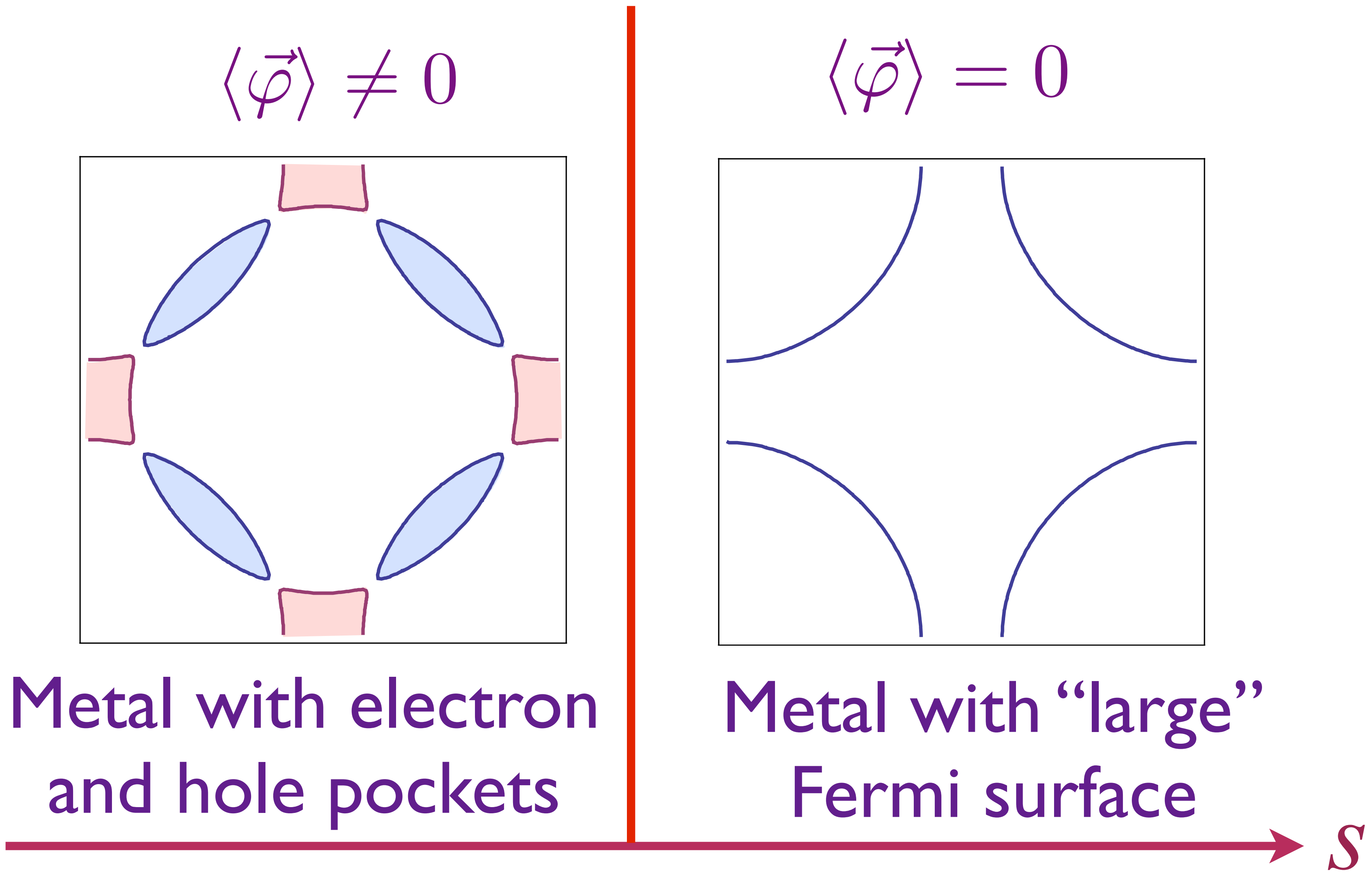
Quantum criticality of the onset of antiferromagnetism in a metal



- There is non-Fermi liquid behavior at the QCP not only at hotspots, but on entire Fermi surface.

M.A. Metlitski and S. Sachdev, *Physical Review B* **82**, 075128 (2010)

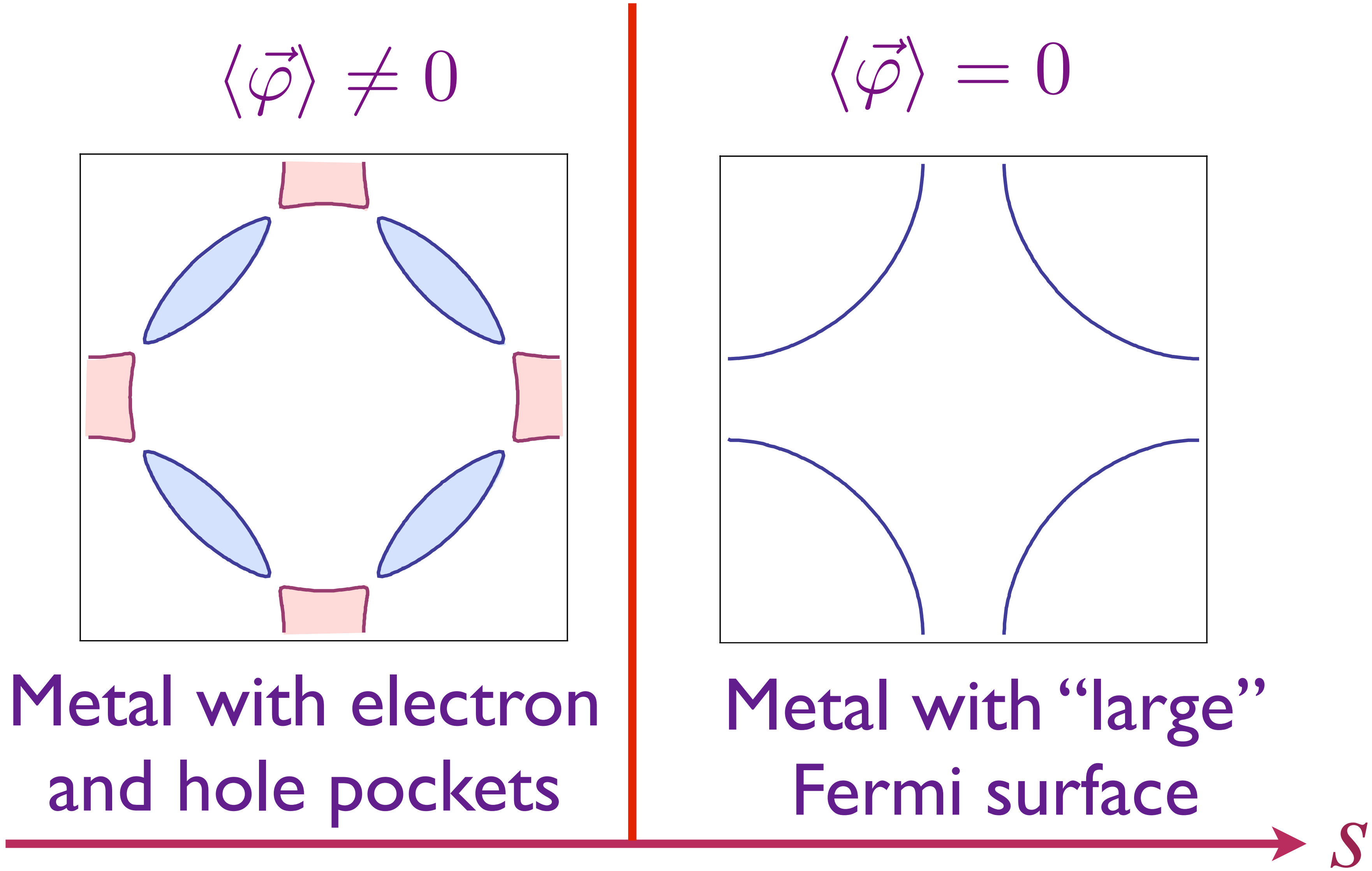
Quantum criticality of the onset of antiferromagnetism in a metal



- QCP has a strong log-square instability to the onset of unconventional superconductivity.

M.A. Metlitski and S. Sachdev, *Physical Review B* **82**, 075128 (2010)

Quantum criticality of the onset of antiferromagnetism in a metal

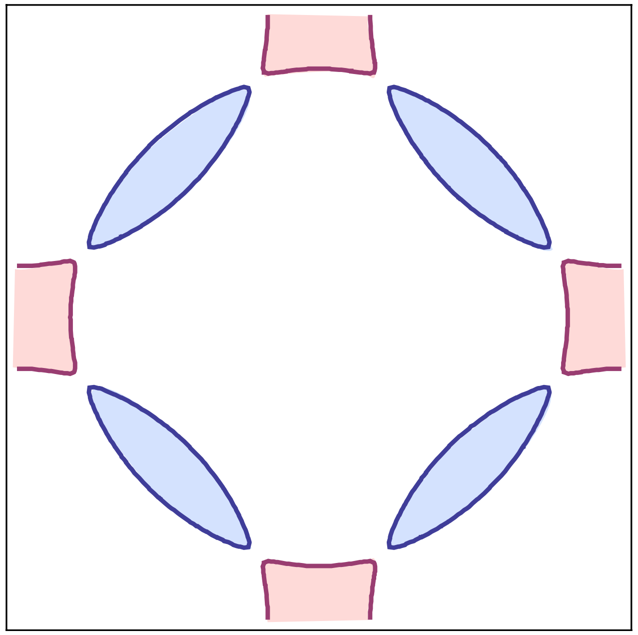


- On some portions of the Fermi surface, there is competition between antiferromagnetism and superconductivity, while on others there is attraction. The net effect depends on details and could be small.

E.G. Moon and S. Sachdev, arXiv:1005.3312

Quantum criticality of the onset of antiferromagnetism in a metal

$\langle \vec{\varphi} \rangle \neq 0$



Metal with electron and hole pockets

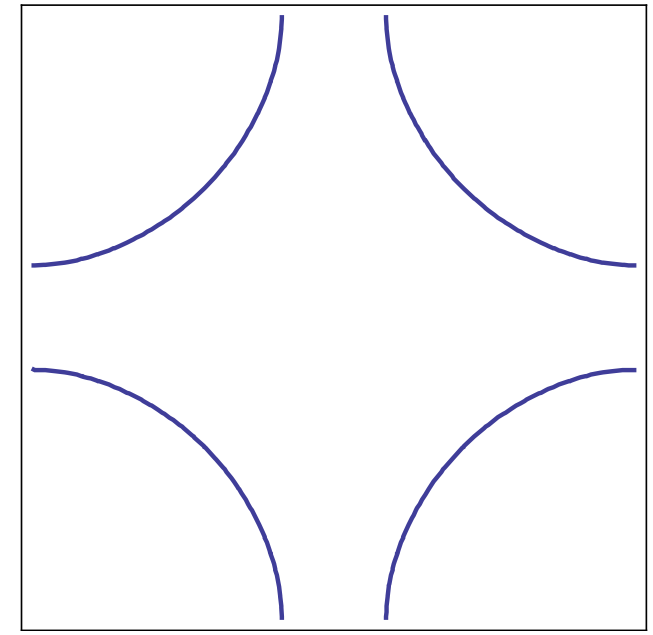
Non-Fermi liquid phase with fluctuating Fermi pockets and local antiferromagnetism

U(1) gauge theory

Non-Fermi liquid phase with fluctuating large Fermi surface local antiferromagnetism

SU(2) gauge theory

$\langle \vec{\varphi} \rangle = 0$



Metal with "large" Fermi surface

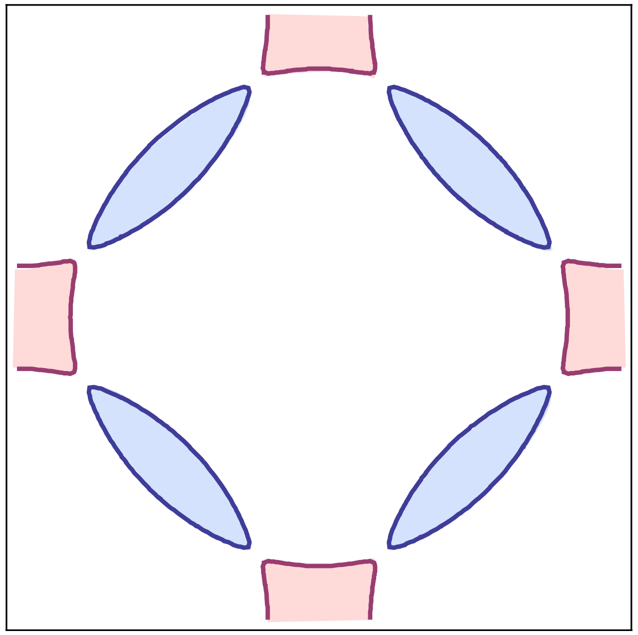
s

- Theory with important *angular* fluctuations ("hedgehogs" suppressed) of antiferromagnetic order parameter. Leads to intermediate phases with electron fractionalization and non-Fermi liquid behavior.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Phys. Rev. B* **80**, 155129 (2009).
Y. Qi and S. Sachdev, *Phys. Rev. B* **81**, 115129 (2010).

Quantum criticality of the onset of antiferromagnetism in a metal

$\langle \vec{\varphi} \rangle \neq 0$



Metal with electron and hole pockets

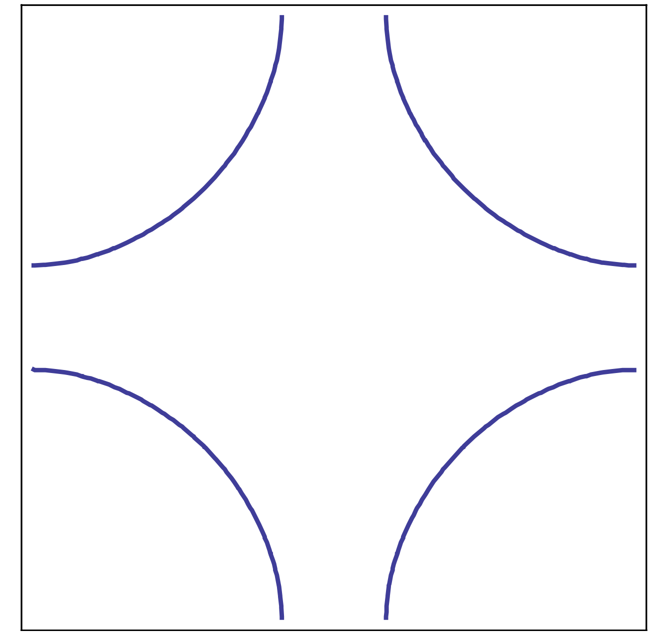
Non-Fermi liquid phase with fluctuating Fermi pockets and local antiferromagnetism

U(1) gauge theory

Non-Fermi liquid phase with fluctuating large Fermi surface local antiferromagnetism

SU(2) gauge theory

$\langle \vec{\varphi} \rangle = 0$



Metal with "large" Fermi surface

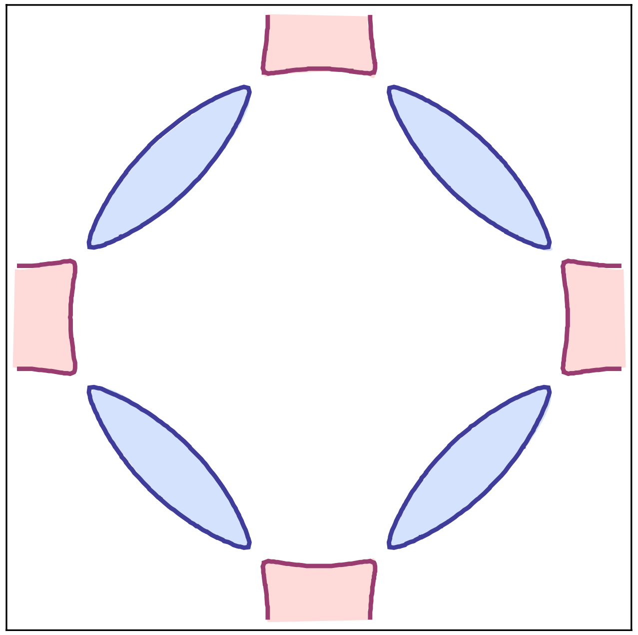
s

- Non-Fermi liquid phases are ultimately unstable to confinement and varieties of bond order

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Phys. Rev. B* **80**, 155129 (2009).
Y. Qi and S. Sachdev, *Phys. Rev. B* **81**, 115129 (2010).

Quantum criticality of the onset of antiferromagnetism in a metal

$\langle \vec{\varphi} \rangle \neq 0$



Metal with electron and hole pockets

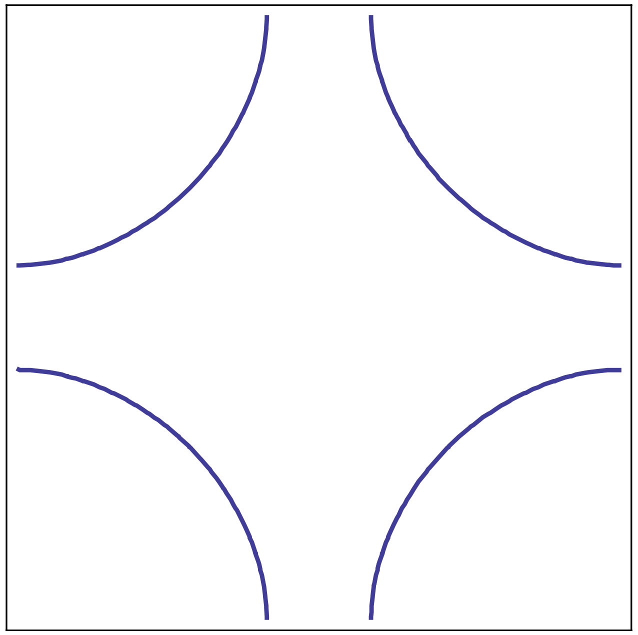
Non-Fermi liquid phase with fluctuating Fermi pockets and local antiferromagnetism

U(1) gauge theory

Non-Fermi liquid phase with fluctuating large Fermi surface local antiferromagnetism

SU(2) gauge theory

$\langle \vec{\varphi} \rangle = 0$



Metal with "large" Fermi surface

s

- This gauge-theoretic formulation generically leads to competition between antiferromagnetism and superconductivity

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Questions

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

- *If so, why is there no antiferromagnetism in the cuprates near the point where the superconductivity is strongest ?*

- *What is the physics of the strange metal ?*

Questions and answers

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

  Yes

- *If so, why is there no antiferromagnetism in the cuprates near the point where the superconductivity is strongest ?*

- *What is the physics of the strange metal ?*

Questions and answers

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

  Yes

- *If so, why is there no antiferromagnetism in the cuprates near the point where the superconductivity is strongest ?*

  Competition between antiferromagnetism and superconductivity has shifted the antiferromagnetic quantum-critical point (QCP), and shrunk the region of antiferromagnetism. This QCP shift is largest in the cuprates

- *What is the physics of the strange metal ?*

Questions and answers

- *Can quantum fluctuations near the loss of antiferromagnetism induce higher temperature superconductivity ?*

  Yes

- *If so, why is there no antiferromagnetism in the cuprates near the point where the superconductivity is strongest ?*

  Competition between antiferromagnetism and superconductivity has shifted the antiferromagnetic quantum-critical point (QCP), and shrunk the region of antiferromagnetism. This QCP shift is largest in the cuprates

- *What is the physics of the strange metal ?*

  Proposal: strongly-coupled quantum criticality of fluctuating antiferromagnetism in a metal