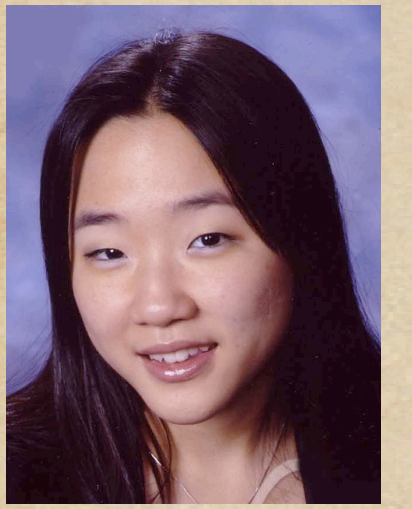
Onset of nematic order in d-wave superconductors



Yejin Huh
Harvard

Y. Huh and S. Sachdev, arXiv:0806.002.

E-A. Kim, M. Lawler, P. Oreto, S. Sachdev, E. Fradkin and S. Kivelson, Phys. Rev. B 77, 184154 (2008).

A. Pelissetto, S. Sachdev, and E. Vicari, arXiv:0802.0199.

Outline

1. Nematic order in YBCO

   Broken lattice symmetry but no spin order

2. Theory of the onset of nematic order in a d-wave superconductor

   Infinite anisotropy fixed point

3. SDW order in LSCO

   Emergent O(4) symmetry

4. Nodal quasiparticles at the O(4) critical point

   Unique selection of quasiparticle coupling to (composite) nematic order

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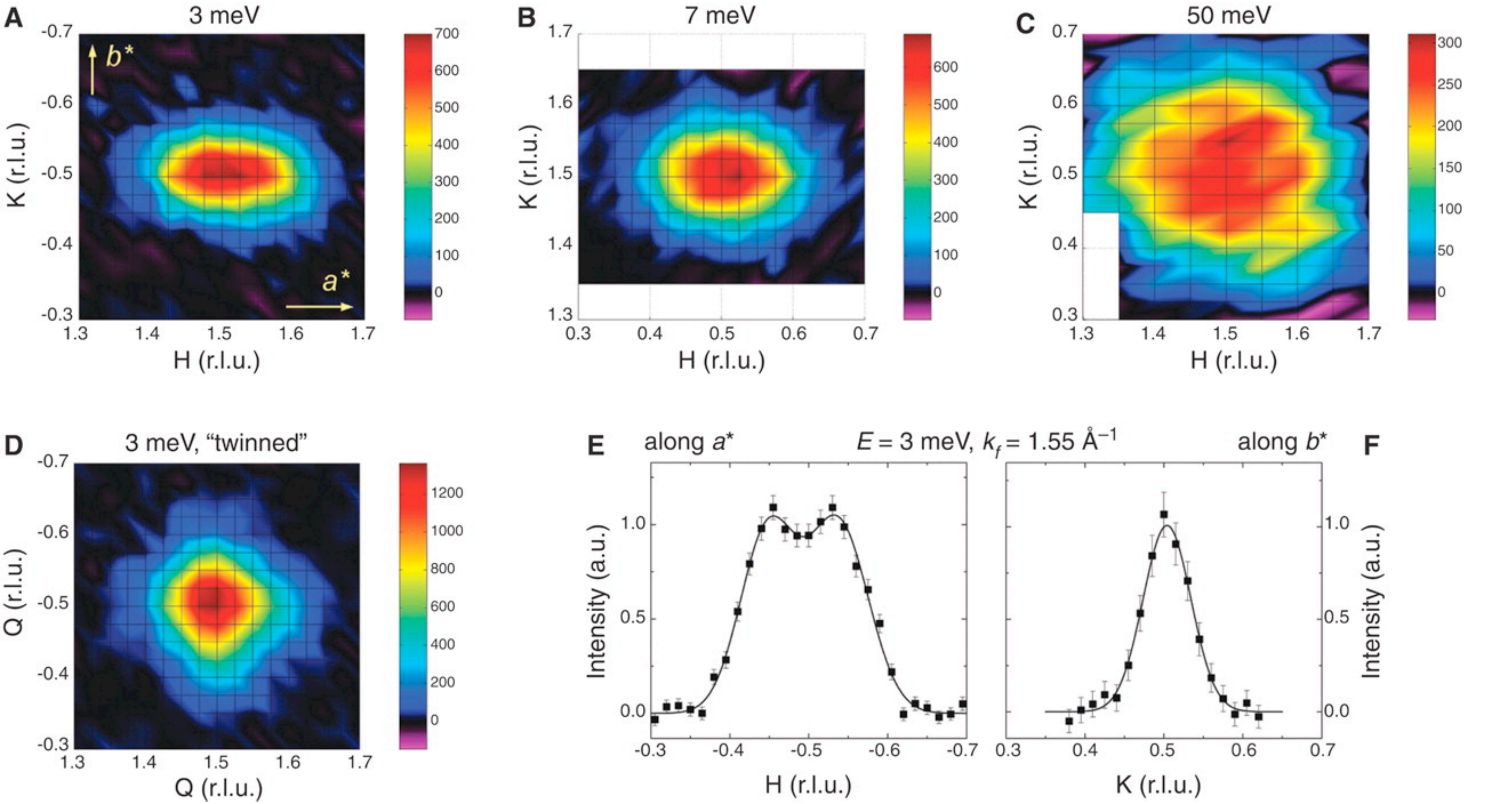
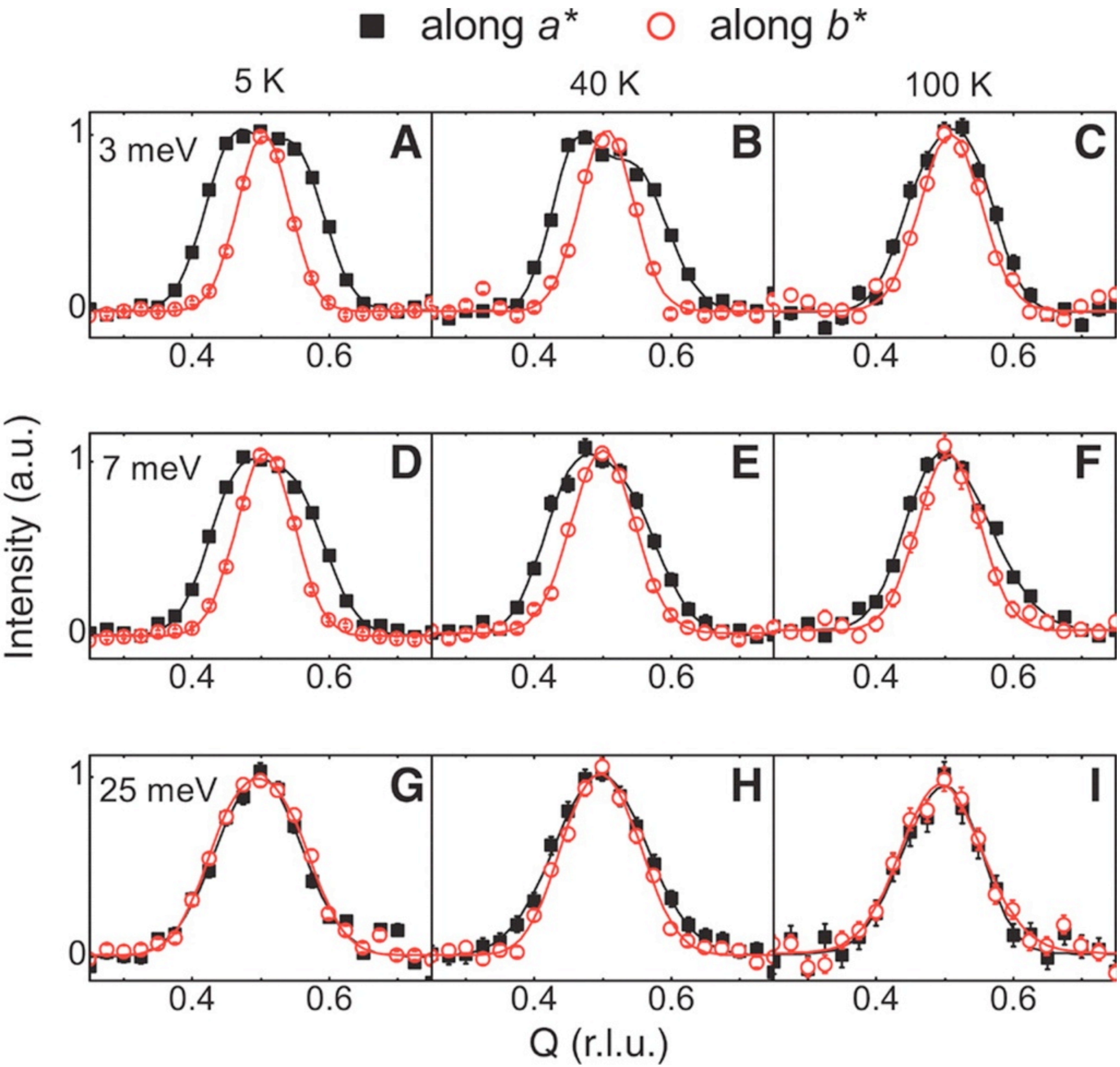


Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)



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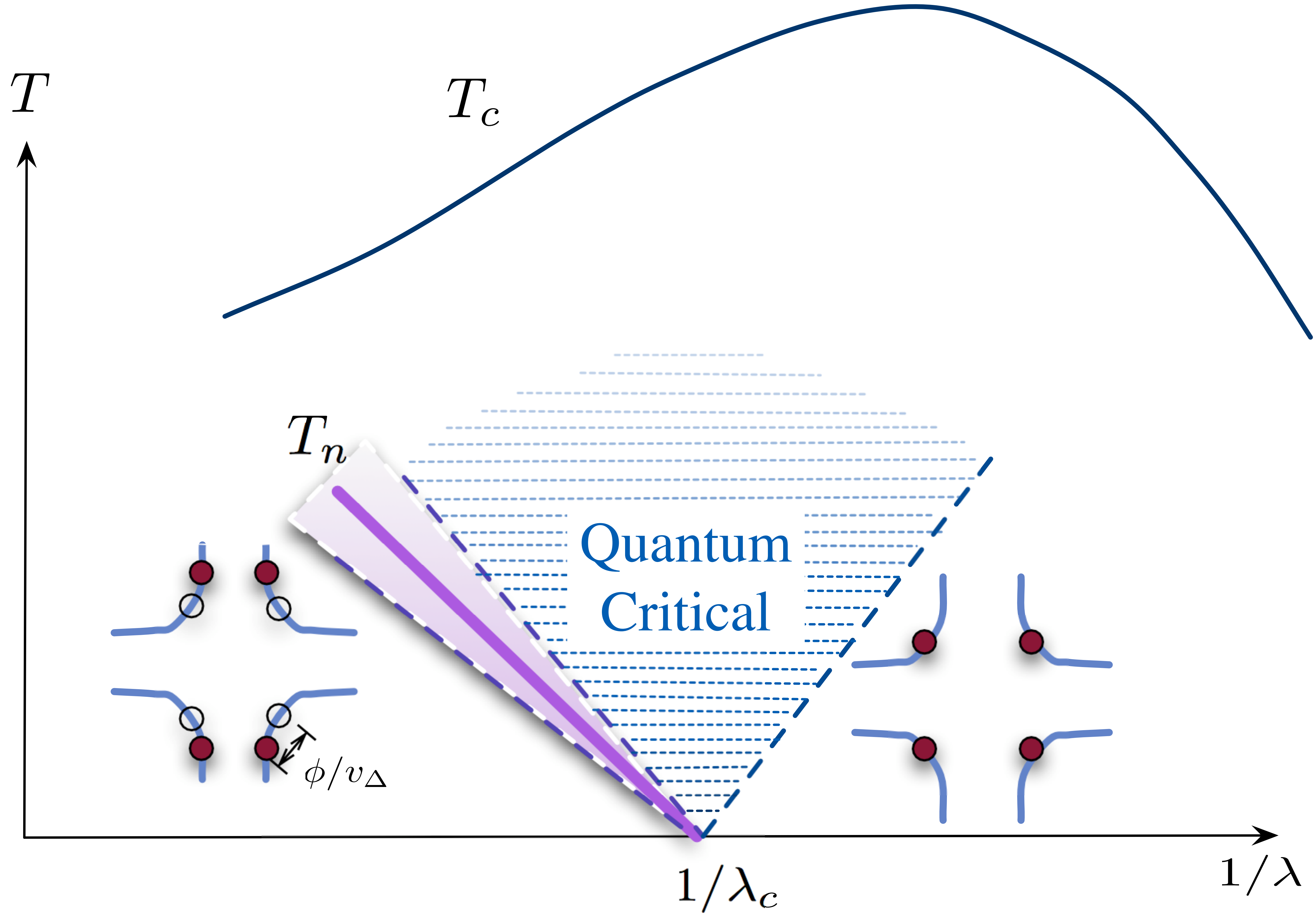
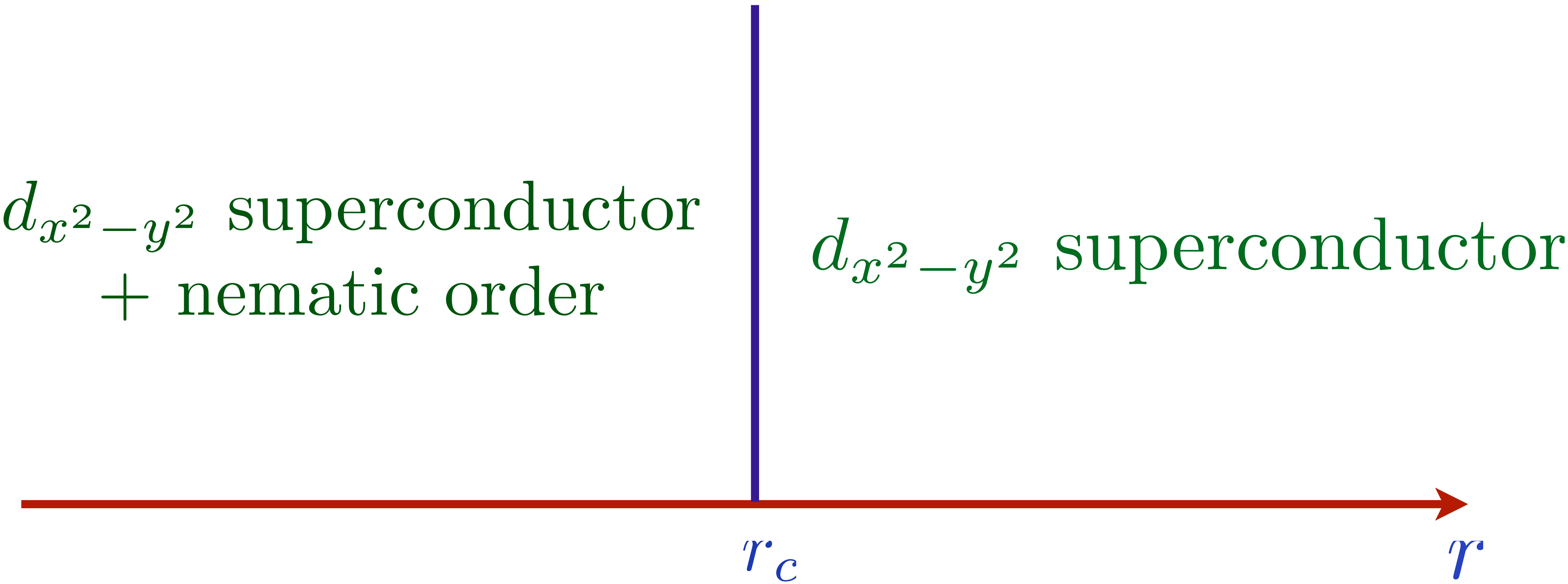
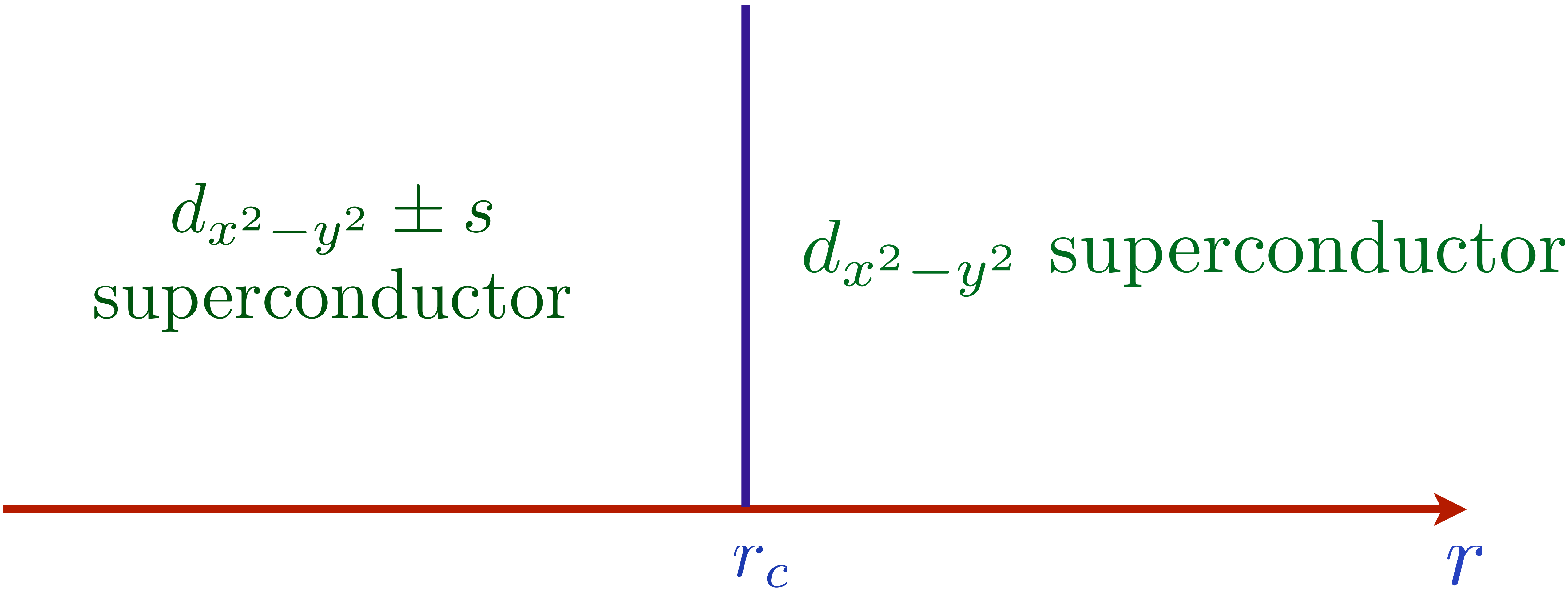


M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000)
E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, arXiv:0705.4099

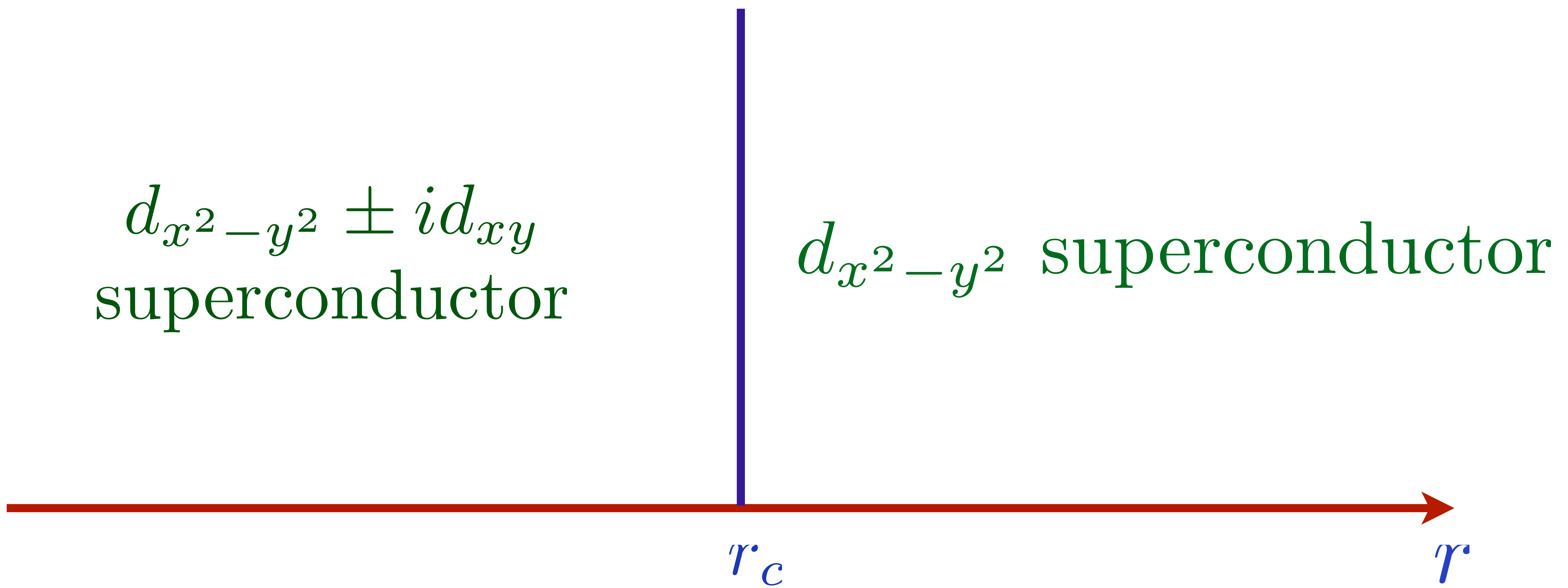

$d_{x^2-y^2}$ superconductor
+ nematic order
$d_{x^2-y^2}$ superconductor
r_c
r

Nematic ordering is equivalent to the appearance of subsidiary s pairing



Order parameter - s pairing amplitude $\sim \phi$

Also consider in parallel another simpler, and previously understood case



Order parameter - d_{xy} pairing amplitude $\sim \phi$

Field theory for dSC to dSC+nematic transition

$$S_\phi^0 = \int d^2x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{u_0}{24}\phi^4 \right]$$

Ising theory for nematic ordering

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Field theory for dSC to dSC+nematic transition

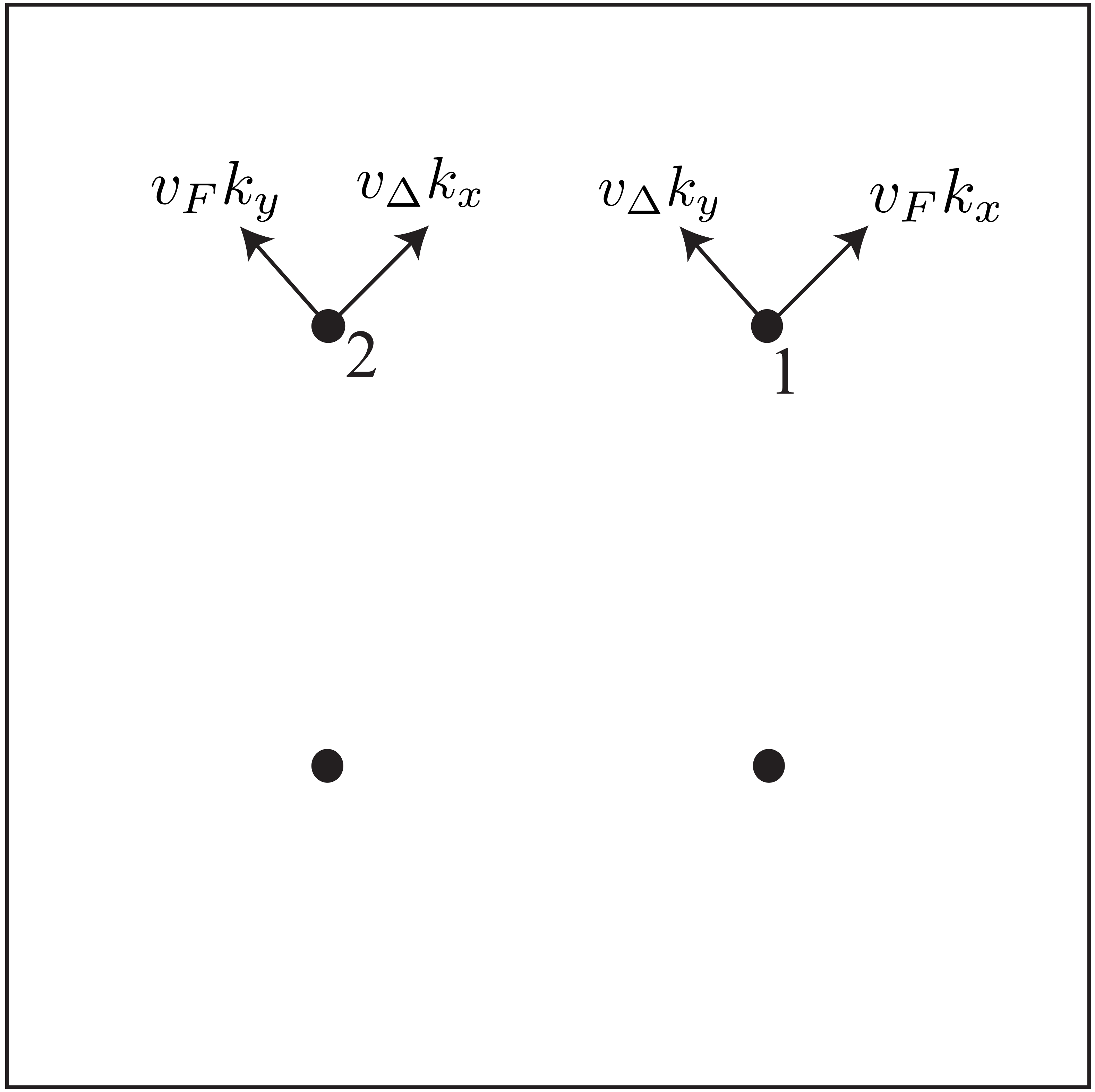
$$S_\phi^0 = \int d^2x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{u_0}{24}\phi^4 \right]$$

Ising theory for nematic ordering

$$\begin{aligned} S_\Psi &= \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{1a}^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x \right) \Psi_{1a} \\ &+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{2a}^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x \right) \Psi_{2a}. \end{aligned}$$

Free nodal quasiparticles

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)



$v_F k_y$
$v_\Delta k_x$
$v_\Delta k_y$
$v_F k_x$
2
1

Field theory for dSC to dSC+nematic transition

$$S_{\Psi\phi} = \int d^2 x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^{\dagger} \tau^x \Psi_{1a} + \Psi_{2a}^{\dagger} \tau^x \Psi_{2a} \right) \right],$$

Yukawa coupling is strongly relevant

RG analysis close to 3 dimensions
yields runaway flow to strong coupling

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Field theory for dSC to dSC+id$_{xy}$ transition

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^{\dagger}\tau^y\Psi_{1a} + \Psi_{2a}^{\dagger}\tau^y\Psi_{2a}\right)\right]$$

Yukawa coupling is strongly relevant

RG analysis close to 3 dimensions yields a relativistically invariant fixed point with all velocities equal

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the nematic order parameter

$$S_\phi = \frac{N_f}{v_\Delta v_F} \Gamma \left[\lambda_0 \phi(x,\tau); \frac{v_\Delta}{v_F} \right] + \frac{N_f}{2} \int d^2 x d\tau \Big(r \phi^2(x,\tau) \Big) + \text{irrelevant terms}$$

where Γ is a non-local and non-analytic functional of ϕ.

The theory has only 2 couplings constants: r and v_Δ / v_F.

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the nematic order parameter

$$S_\phi = \frac{N_f}{2}\int_{k,\omega} |\phi(k,\omega)|^2 \Bigg[r + \frac{\lambda_0^2}{8 v_F v_\Delta}\left(\frac{\omega^2 + v_F^2 k_x^2}{\sqrt{\omega^2 + v_F^2 k_x^2 + v_\Delta^2 k_y^2}} + (x \leftrightarrow y) \right) \Bigg]$$

$$+\text{higher order terms which cannot be neglected}$$

E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, arXiv:0705.4099

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the nematic order parameter

Because the order parameter couples to a fermion current, a constant ϕ can be gauged away, and the effective potential is *unrenormalized*

$$V(\phi) = \frac{r}{2}\phi^2 + \frac{u}{4}\phi^4 + \ldots \tag{1}$$

The order parameter critical exponent $\gamma = 1$.

Y. Huh and S. Sachdev, arXiv:0806.0002

Renormalization group analysis

Couplings are local in the fermion action, so perform RG on fermion self energy

- The fermion self energy determines the wavefunction renormalization of the the fermions (η_f) and the renormalization of the velocities v_F and v_Δ.
- The wavefunction renormalization of ϕ (η_b) is set by the requirement that the Yukawa coupling $\phi \Psi^\dagger \tau^x \Psi$ have unit magnitude.
- The non-renormalization of the effective potential yields the correlation length exponent $\nu = 1/(2 - \eta_f)$.

Y. Huh and S. Sachdev, arXiv:0806.0002

Renormalization group analysis

Couplings are local in the fermion action, so perform RG on fermion self energy

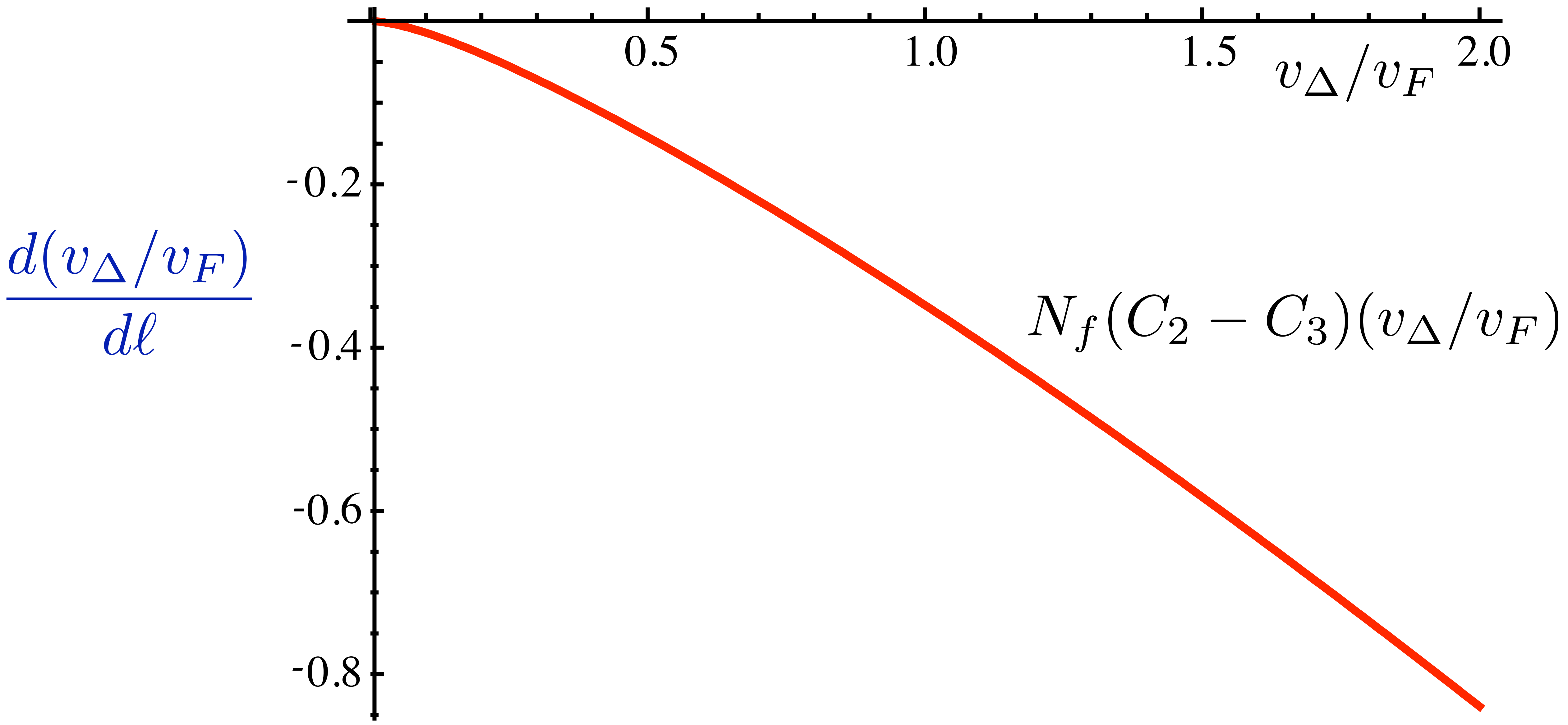
The $1/N_f$ expansion has only one coupling constant at criticality: v_Δ/v_F.

The RG has the structure:

$$\begin{aligned} \text{dynamic critical exponent}: \quad z &= 1 + \frac{1}{N_f} F_1(v_\Delta/v_F) \\ \text{fermion anomalous dimension}: \quad \eta_f &= \frac{1}{N_f} F_2(v_\Delta/v_F) \\ \text{RG flow equation}: \quad \frac{d(v_\Delta/v_F)}{d\ell} &= \frac{1}{N_f} F_3(v_\Delta/v_F) \end{aligned}$$

where we have computed the functions $F_{1,2,3}(v_\Delta/v_F)$.

Renormalization group analysis



Renormalization group analysis

The RG flow is to $v_\Delta / v_F \to 0$ with

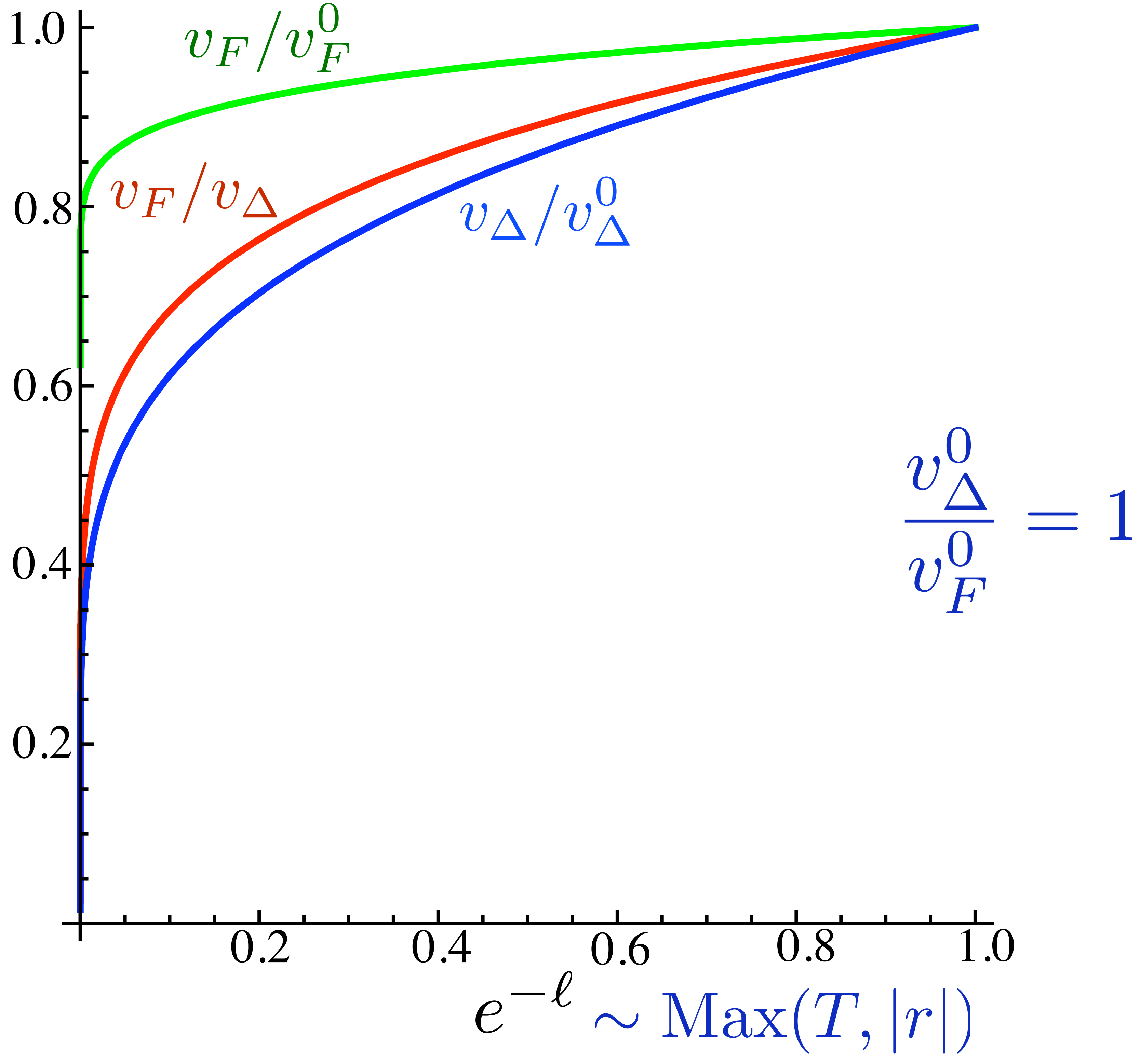
$$\frac{d(v_\Delta / v_F)}{d\ell} = -\frac{8}{\pi^2 N_f} (v_\Delta / v_F)^2 \ln \left(\frac{0.4699}{(v_\Delta / v_F)} \right)$$

This implies that as we approach the critical point, $r \to 0$, $T \to 0$,

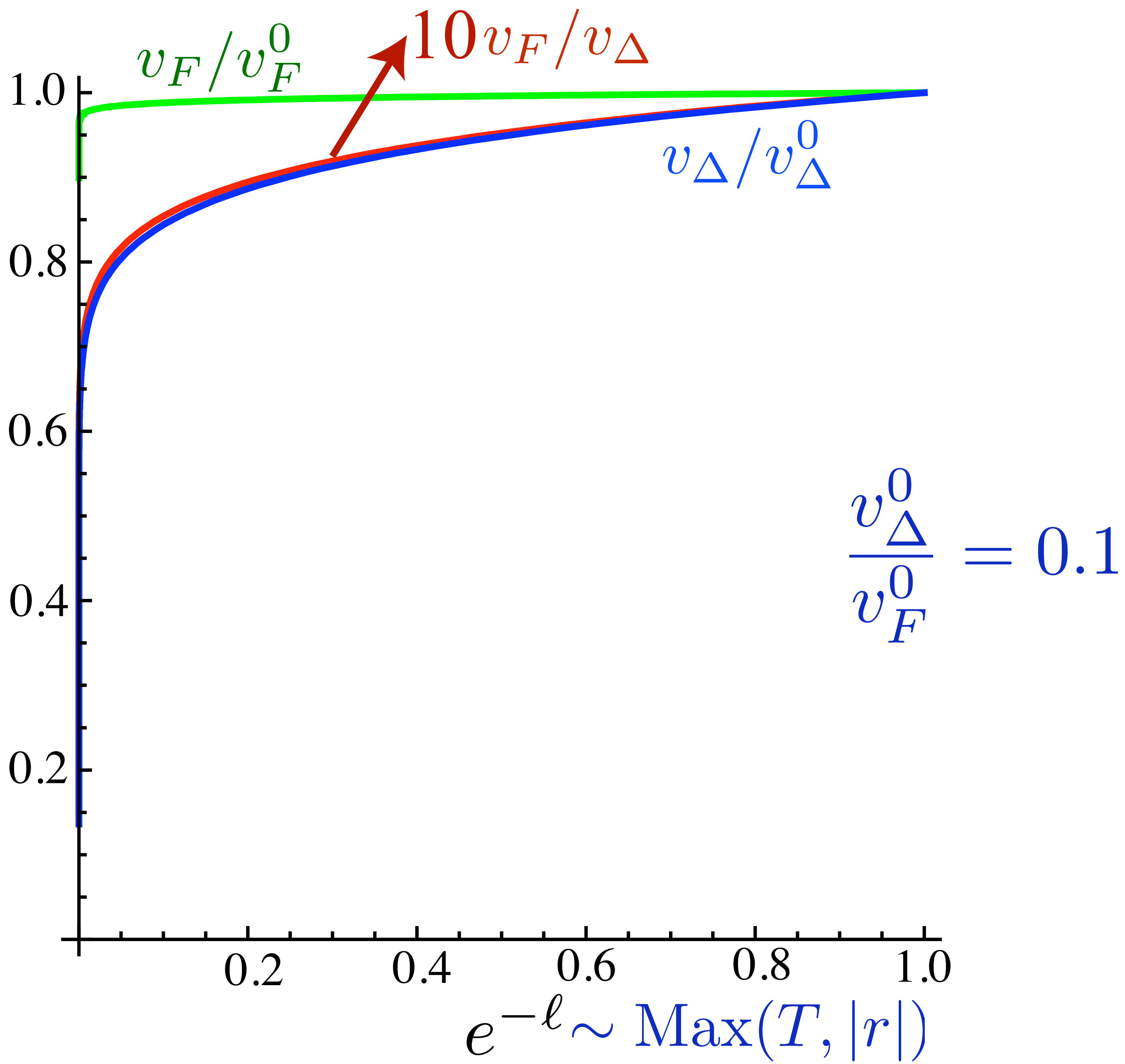
$$\frac{v_\Delta}{v_F} = \frac{\pi^2 N_f}{8} \frac{1}{\ln \left(\frac{\Lambda}{\text{Max}(|r|, T)} \right) \ln \left[\frac{0.3809}{N_f} \ln \left(\frac{\Lambda}{\text{Max}(|r|, T)} \right) \right]}$$

So v_Δ / v_F has a minimum as a function of r at the quantum critical point. More precise results are obtained by a numerical integration of the RG equation.

Renormalization group analysis



Renormalization group analysis



Renormalization group analysis

In the limit $v_\Delta / v_F \to 0$, the effective action becomes

$$
\begin{aligned}
S_\phi \quad = \quad & \frac{N_f}{v_\Delta v_F} \Gamma \left[\lambda_0 \phi(x,\tau); \frac{v_\Delta}{v_F} \right] \\
& + \frac{N_f}{2} \int d^2x d\tau \Big(r\phi^2(x,\tau) \Big) \\
& + \text{irrelevant terms}
\end{aligned}
$$

Renormalization group analysis

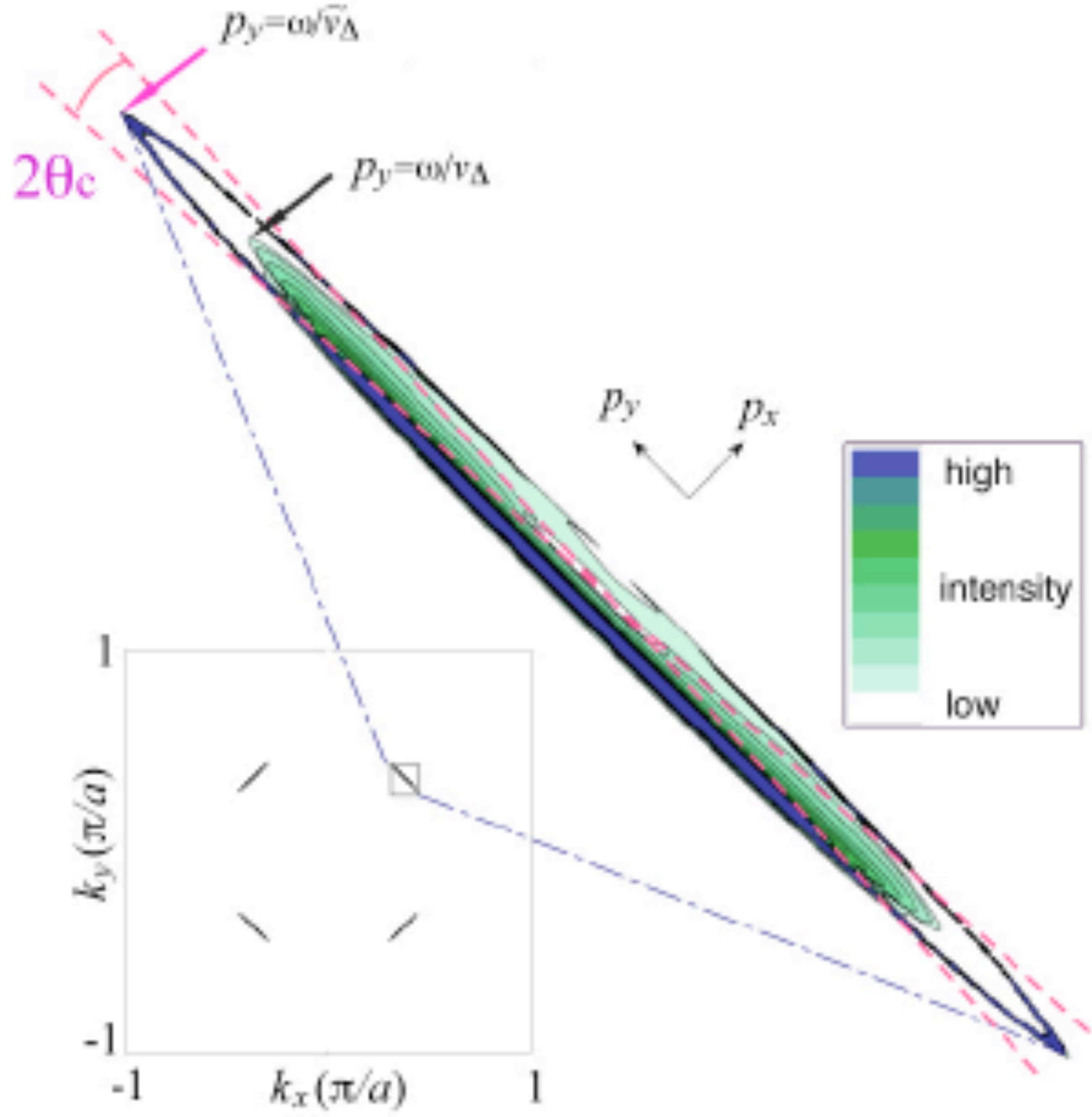
In the limit $v_\Delta / v_F \to 0$, the effective action becomes

$$S_\phi \sim \frac{N}{v_\Delta} \Big[\text{terms that have at most a divergence which is a power of} \ln(v_F / v_\Delta) \Big]$$

$\Rightarrow$ The theory is controlled by the expansion parameter v_Δ / N_f, and so results are asymptotically exact even for $N_f = 2$.

Fermion spectral functions

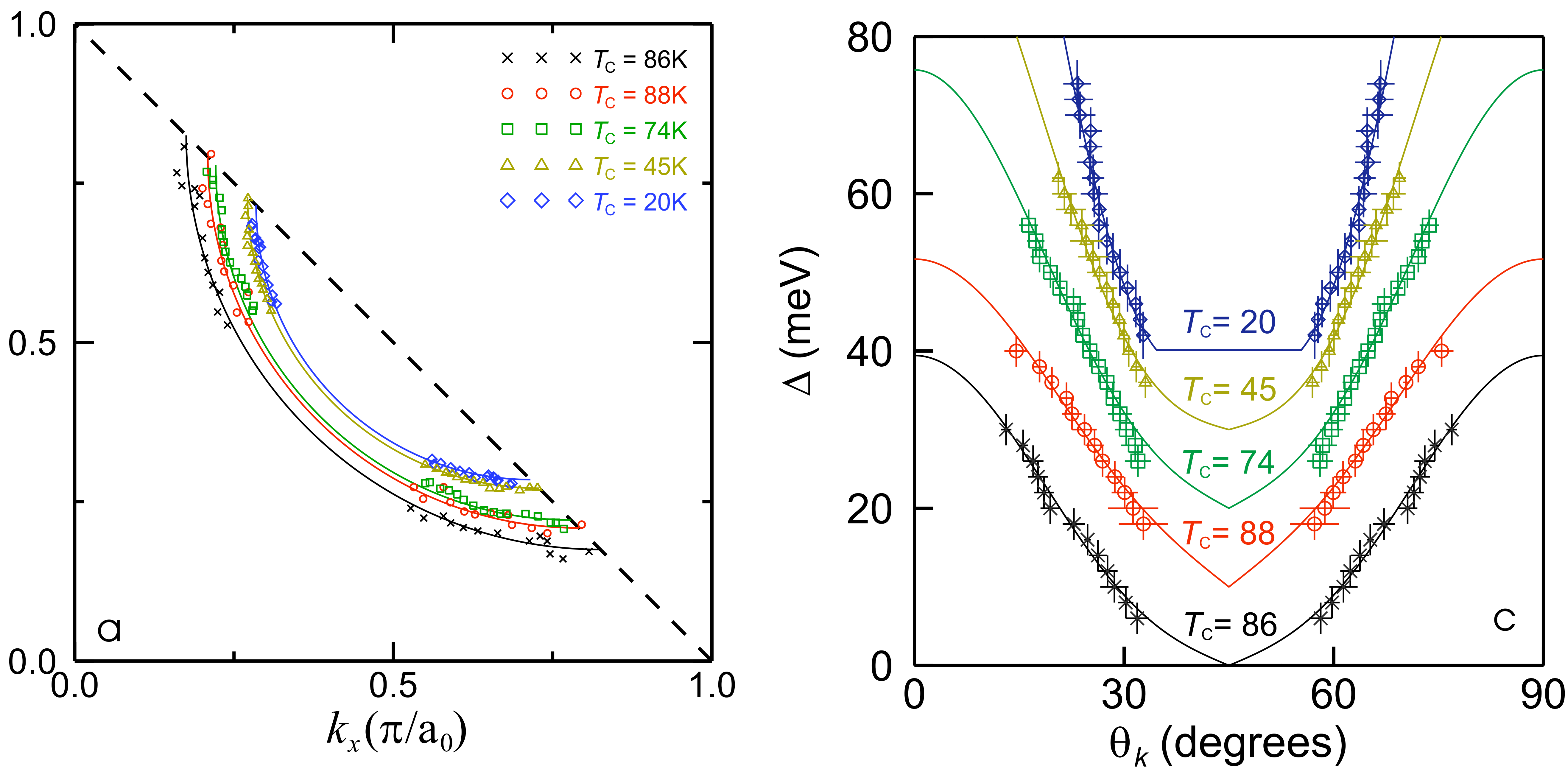
ϕ fluctuations broaden the fermion spectral functions except in a wedge near the nodal points



(a)$A(\vec{p}, -9meV)$

E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, arXiv:0705.4099

Quasiparticle spectra from STM on BSCCO



Y. Kohsaka, C. Taylor, P. Wahl, A. Schmidt, Jhinhwan Lee, K. Fujita, J. Alldredge, Jinho Lee, K. McElroy, H. Eisaki, S. Uchida, D.-H. Lee, & J.C. Davis, preprint

Scanning tunneling microscopy of BSCCO

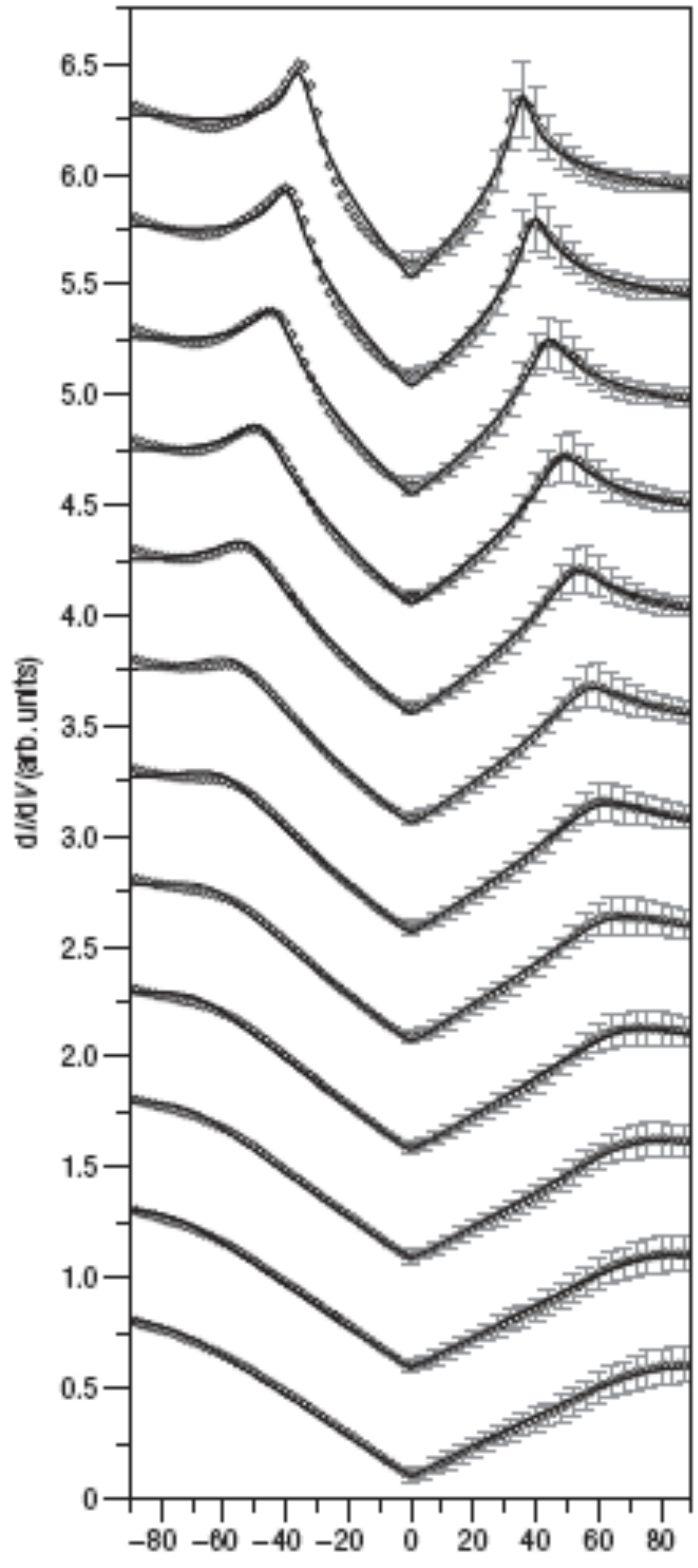
$$N(E,\Gamma_2) = A \times \mathrm{Re}\left(\left\langle \frac{E + i\Gamma_2(E)}{\sqrt{(E + i\Gamma_2(E))^2 - \Delta(k)^2}} \right\rangle_{fs}\right) + I$$

Good fit with

$\Gamma_2(E) = \alpha\, E$

J. W. Alldredge, Jinho Lee,
K. McElroy, M. Wang,
K. Fujita, Y. Kohsaka,
C. Taylor, H. Eisaki,
S. Uchida, P. J. Hirschfeld,
and J. C. Davis

Nature Physics **4**, 319 (2008)



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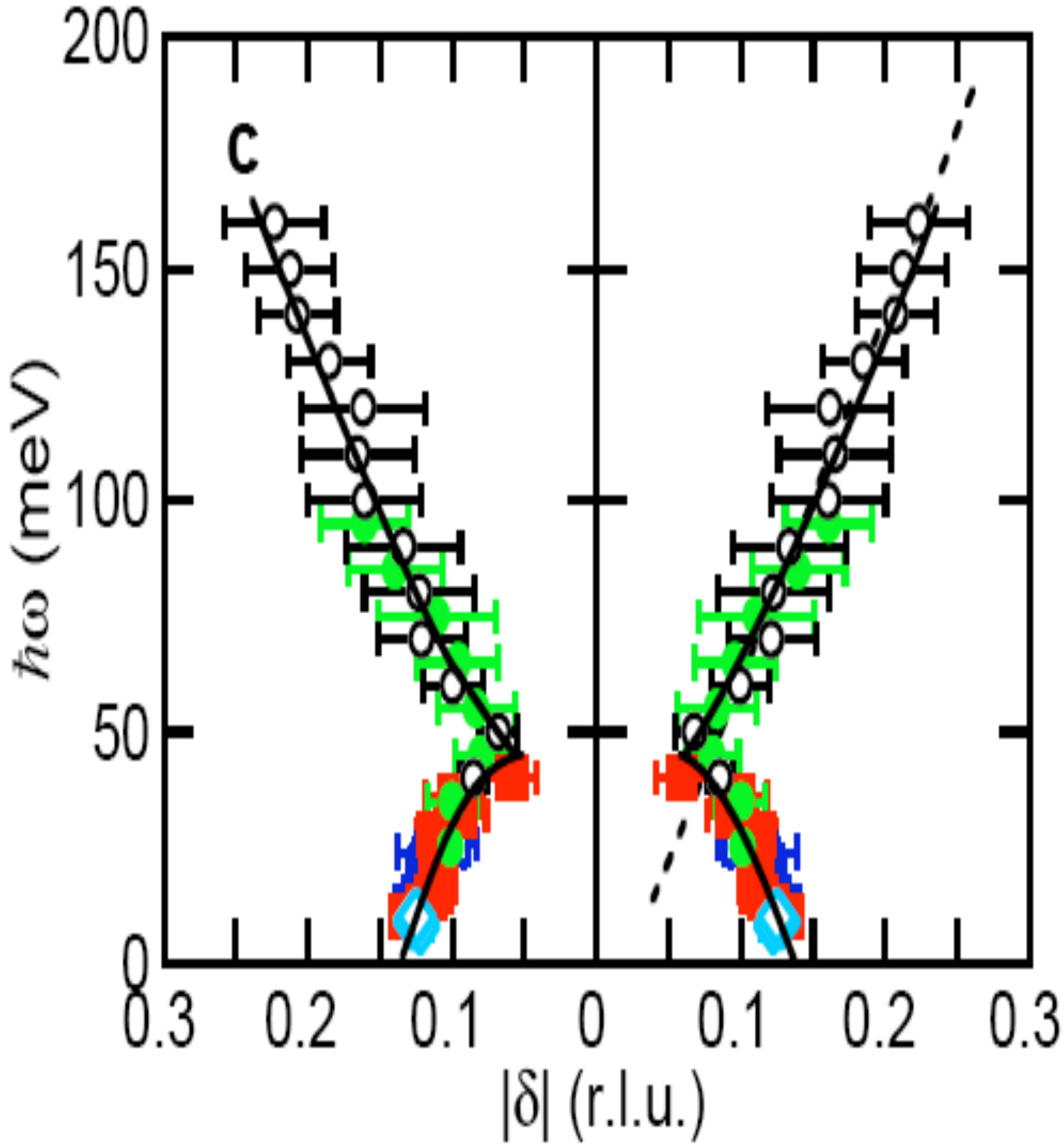
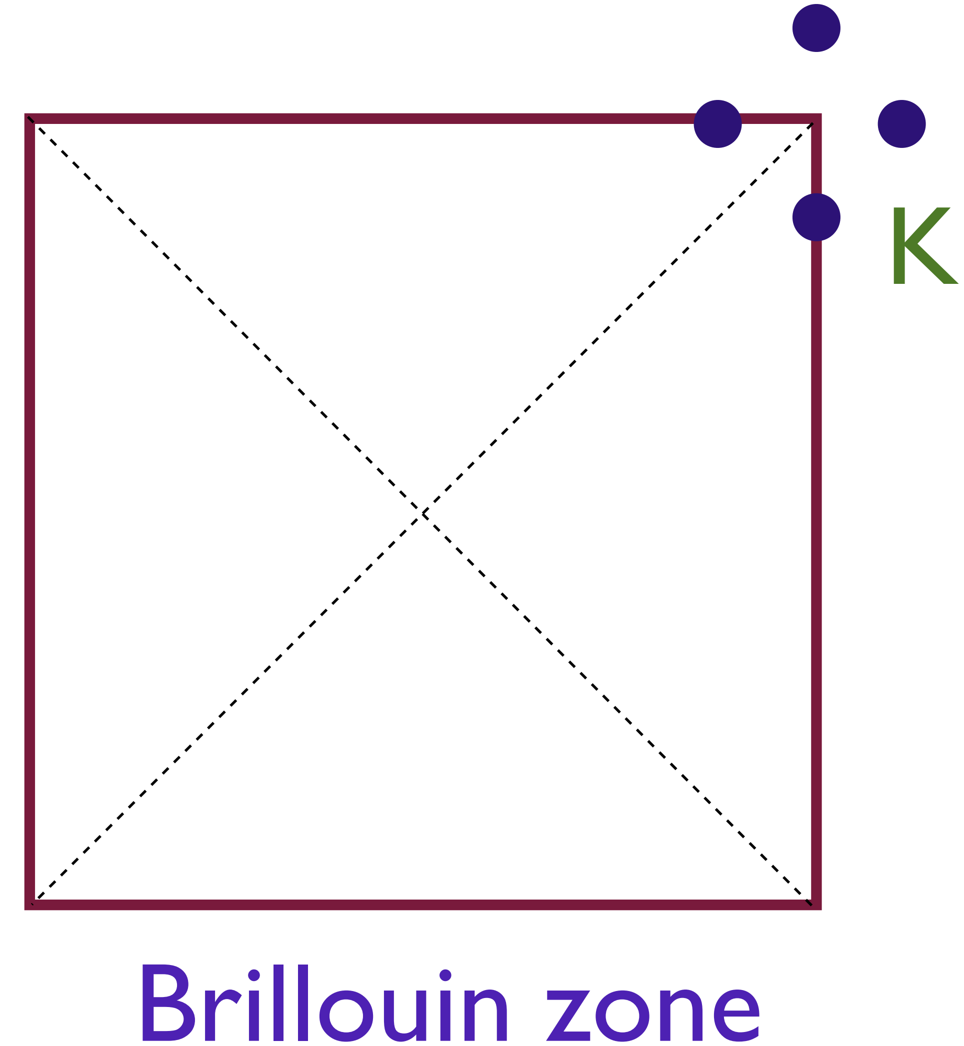
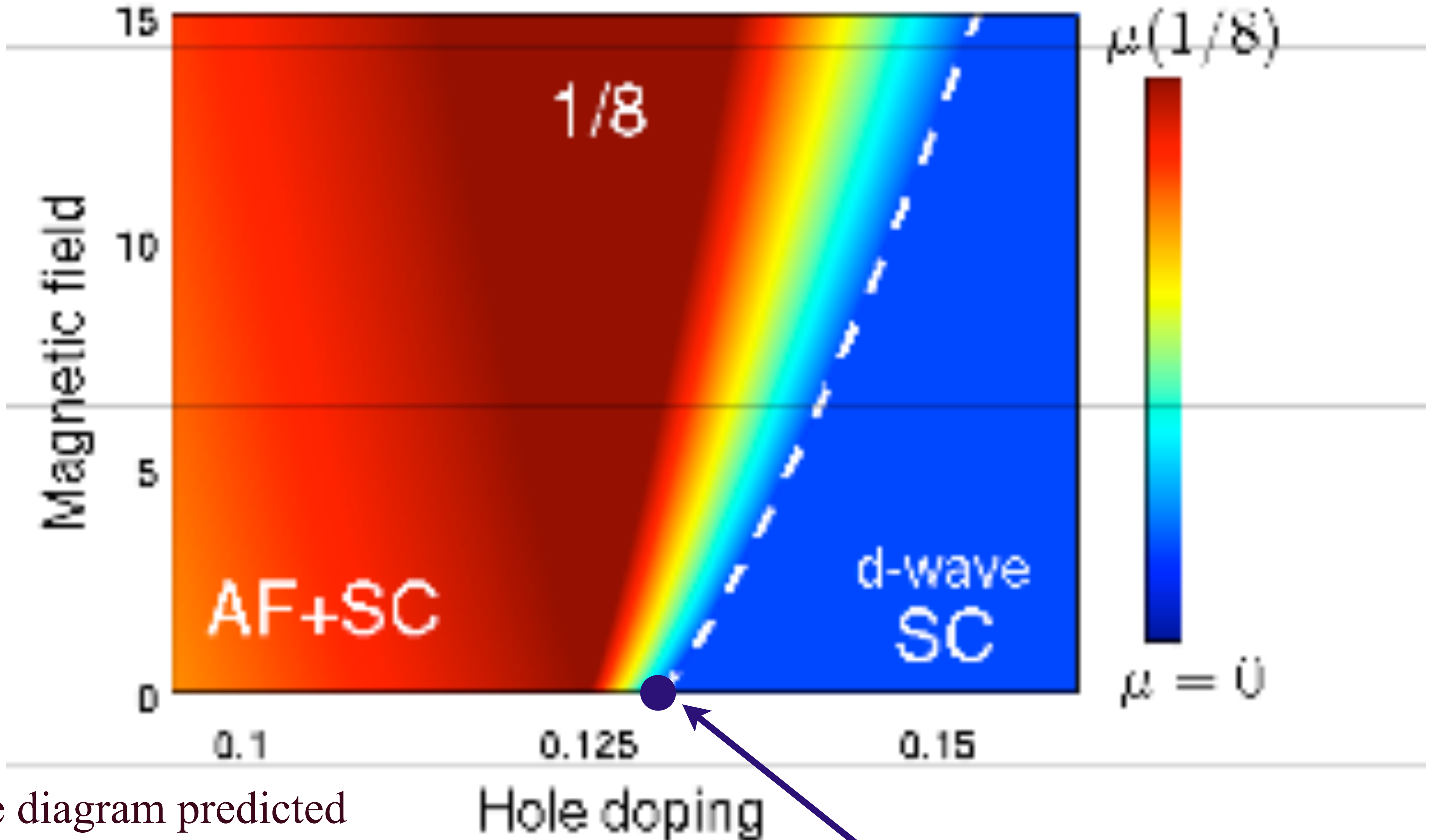
Neutron Scattering-LSCO



Vignolle *et al*., Nature Phys. 07
Christensen *et al*., PRL 04
Hayden *et al*., Nature 04
Tranquada *et al*., Nature 04



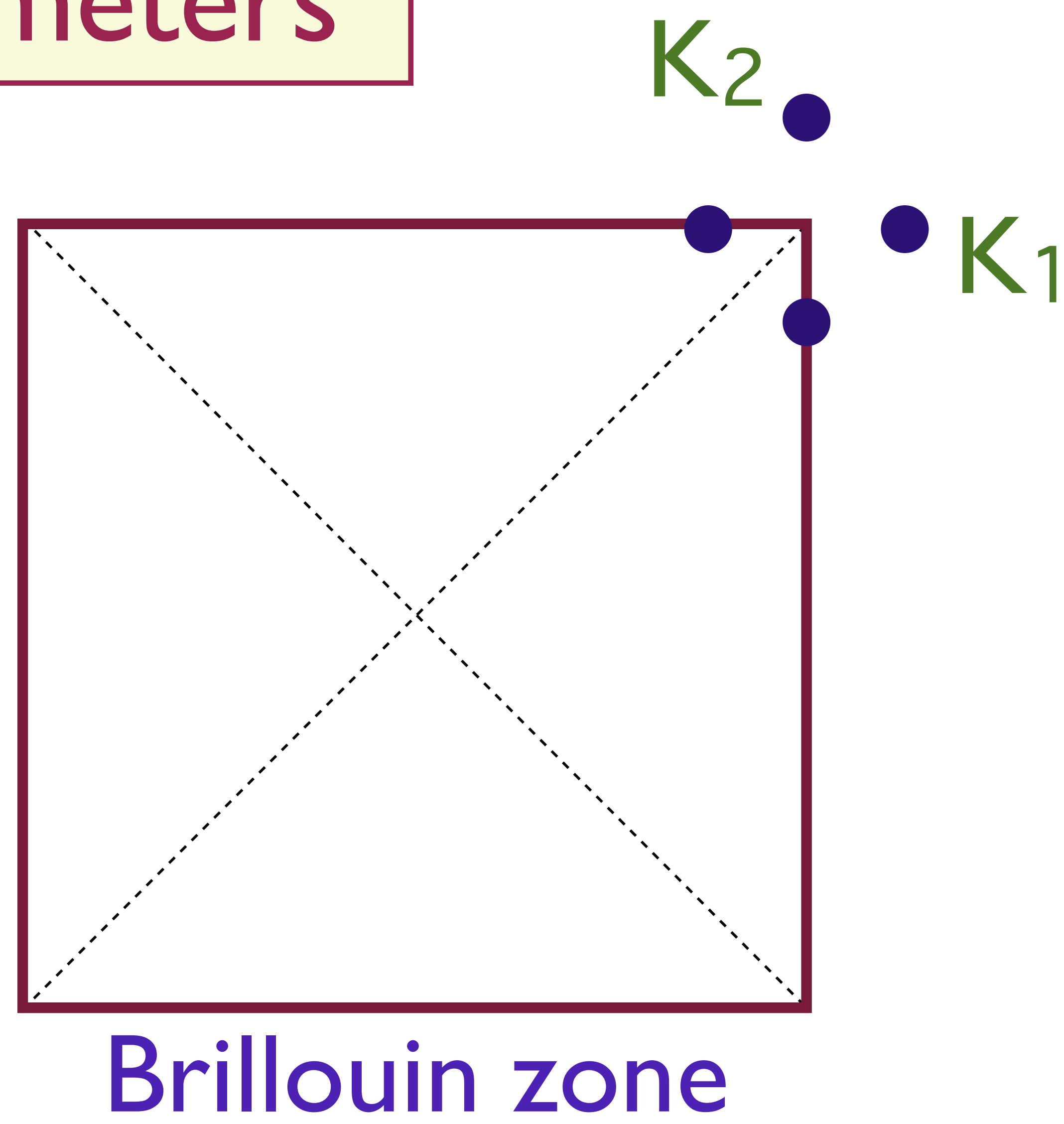
J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, arXiv:0712.2181



Phase diagram predicted by E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

SC to SC+SDW quantum critical point

SDW order parameters



$$S_i(\mathbf{r},\tau) = \text{Re}\left[e^{i\mathbf{K}_1\cdot\mathbf{r}}\,\Phi_{1i}(\mathbf{r},\tau) + e^{i\mathbf{K}_2\cdot\mathbf{r}}\,\Phi_{2i}(\mathbf{r},\tau)\right].$$

$$\mathbf{K}_1 = \left(\frac{2\pi}{a}\right)\left(\frac{1}{2}-\vartheta,\frac{1}{2}\right) \quad , \quad \mathbf{K}_2 = \left(\frac{2\pi}{a}\right)\left(\frac{1}{2},\frac{1}{2}-\vartheta\right),$$

SDW field theory

$$\mathcal{L}_\Phi = |\partial_\tau \Phi_1|^2 + v_1^2 |\partial_x \Phi_1|^2 + v_2^2 |\partial_y \Phi_1|^2$$
$$+|\partial_\tau \Phi_2|^2 + v_2^2 |\partial_x \Phi_2|^2 + v_1^2 |\partial_y \Phi_2|^2 + r(|\Phi_1|^2 + |\Phi_2|^2)$$
$$+\frac{u_1}{2}(|\Phi_1|^4 + |\Phi_2|^4) + \frac{u_2}{2}(|\Phi_1^2|^2 + |\Phi_2^2|^2)$$
$$+w_1 |\Phi_1|^2 |\Phi_2|^2 + w_2 |\Phi_1 \cdot \Phi_2|^2 + w_3 |\Phi_1^* \cdot \Phi_2|^2$$

Most general theory invariant under spin rotation, square lattice space group, and time-reversal symmetries

	T_x	T_y	R	I	$\mathcal{T}$
Φ_{1i}	$-e^{-i\vartheta}\Phi_{1i}$	$-\Phi_{1i}$	Φ_{2i}	Φ_{1i}^*	$-\Phi_{1i}$
Φ_{2i}	$-\Phi_{2i}$	$-e^{-i\vartheta}\Phi_{2i}$	Φ_{1i}^*	Φ_{2i}^*	$-\Phi_{2i}$

SDW field theory

$$
\begin{aligned}
\mathcal{L}_\Phi = & \, |\partial_\tau \Phi_1|^2 + v_1^2 |\partial_x \Phi_1|^2 + v_2^2 |\partial_y \Phi_1|^2 \\
& + |\partial_\tau \Phi_2|^2 + v_2^2 |\partial_x \Phi_2|^2 + v_1^2 |\partial_y \Phi_2|^2 + r(|\Phi_1|^2 + |\Phi_2|^2) \\
& + \frac{u_1}{2}(|\Phi_1|^4 + |\Phi_2|^4) + \frac{u_2}{2}(|\Phi_1^2|^2 + |\Phi_2^2|^2) \\
& + w_1 |\Phi_1|^2 |\Phi_2|^2 + w_2 |\Phi_1 \cdot \Phi_2|^2 + w_3 |\Phi_1^* \cdot \Phi_2|^2
\end{aligned}
$$

Symmetries:

$$\mathrm{U}(1) \otimes \mathrm{U}(1) \otimes Z_4 \otimes \mathrm{O}(3)$$

x-translations

y-translations

lattice rotations

spin rotations

SDW field theory

Stable fixed point in a 6-loop RG analysis:
$w_1^* = u_1^* - u_2^*$, $w_2^* = w_3^* = u_2^*$, $v_1^* = v_2^*$

O(4)⊗O(3) invariant theory for φ_{ai}, with $a = 1 \ldots 4$ an O(4) index, and $i = 1 \ldots 3$ an O(3) index, and

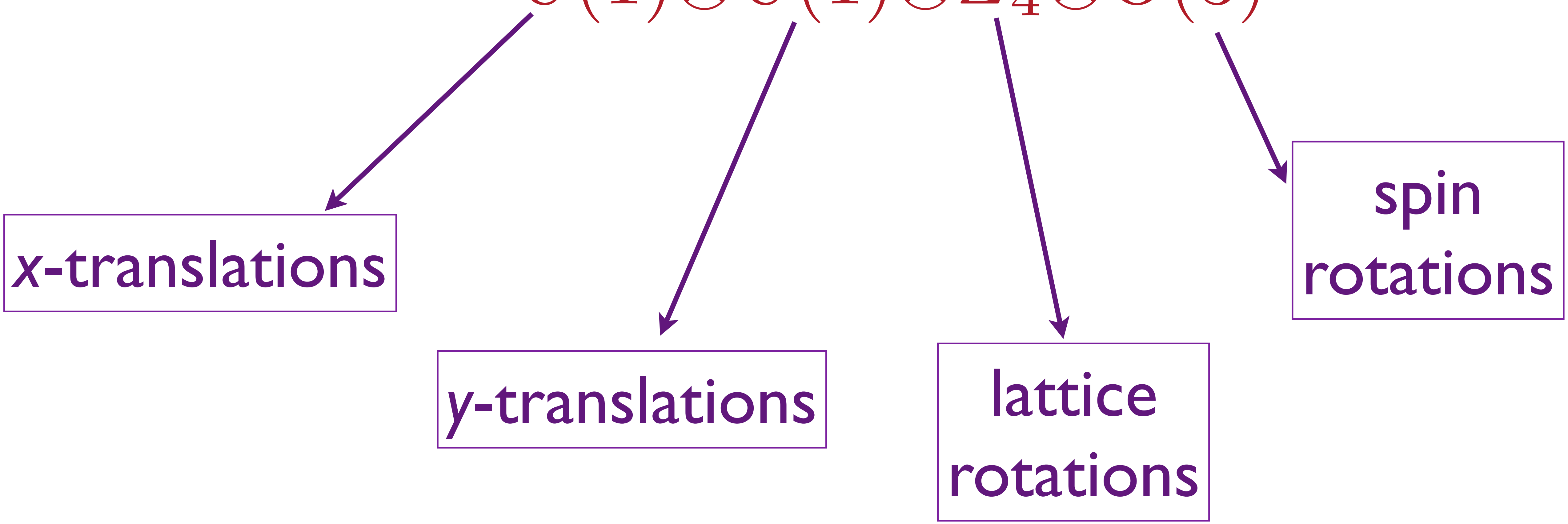
$$\Phi_{1i} = \varphi_{1i} + i\varphi_{2i}, \quad \Phi_{2i} = \varphi_{3i} + i\varphi_{4i}.$$

M. De Prato, A. Pelissetto, and E. Vicari
Phys. Rev. B **74**, 144507 (2006).

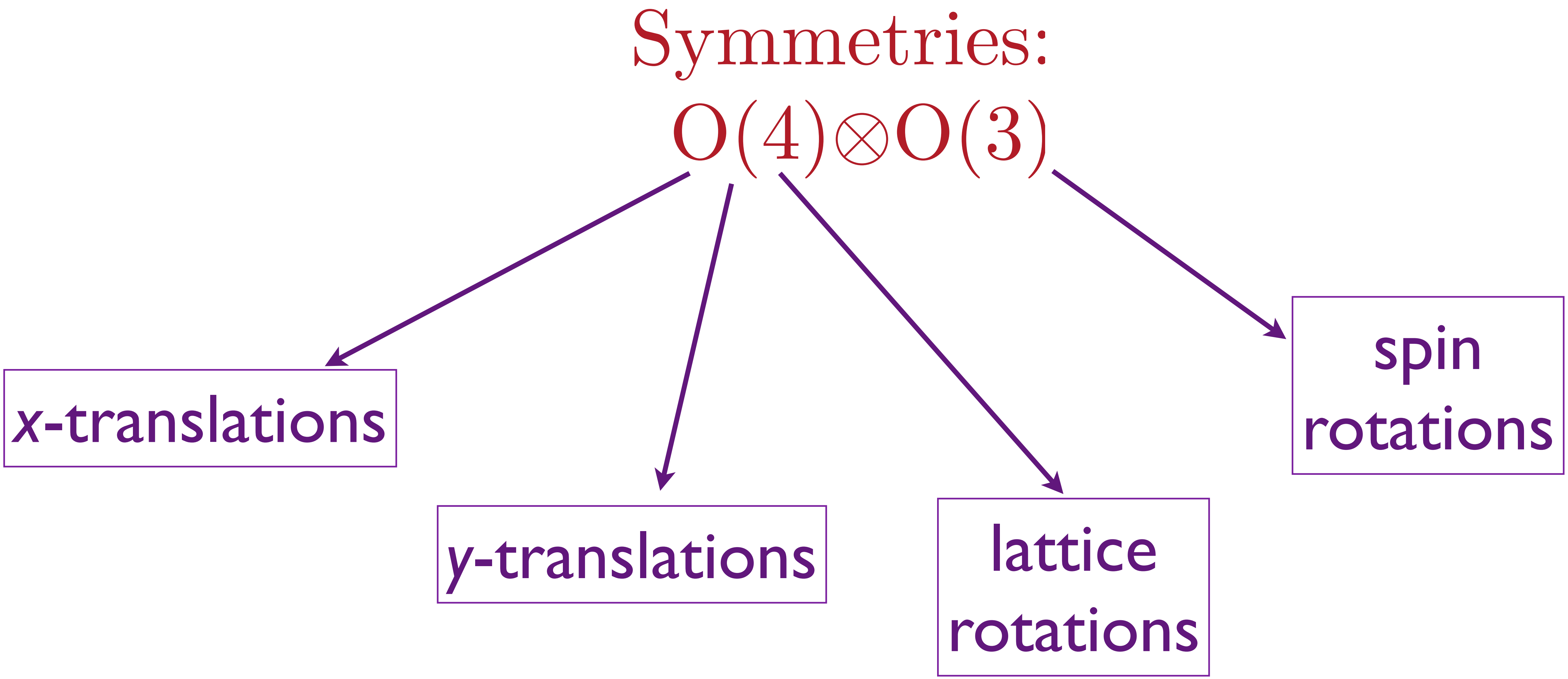
SDW field theory

Symmetries:

$$\mathrm{U}(1) \otimes \mathrm{U}(1) \otimes Z_4 \otimes \mathrm{O}(3)$$



SDW field theory



Properties of O(4)⊗O(3) fixed point:

The following 9 order parameters have divergent fluctuations at the spin-density wave ordering transition with the same exponent $\overline{\gamma}$:

- The real 'nematic' order parameter, $\phi \equiv \sum_i \left(|\Phi_{1i}|^2 - |\Phi_{2i}|^2 \right)$ which measures breaking of Z_4 symmetry
- Charge density waves at $2\mathbf{K}_1$ and $2\mathbf{K}_2$: $\sum_i \Phi_{1i}^2$ and $\sum_i \Phi_{2i}^2$
- Charge density waves at $\mathbf{K}_1 \pm \mathbf{K}_2$: $\sum_i \Phi_{1i}\Phi_{2i}$ and $\sum_i \Phi_{1i}^*\Phi_{2i}$

At the quantum critical point, the susceptibilities of these orders *all* diverge as $\chi \sim T^{-\overline{\gamma}}$, with

$$\overline{\gamma} = \begin{cases} 0.90(36) & \mathrm{MZM}, 6 \text{ loops} \\ 0.80(54) & d = 3\ \overline{\mathrm{MS}}, 5 \text{ loops} \end{cases}$$

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

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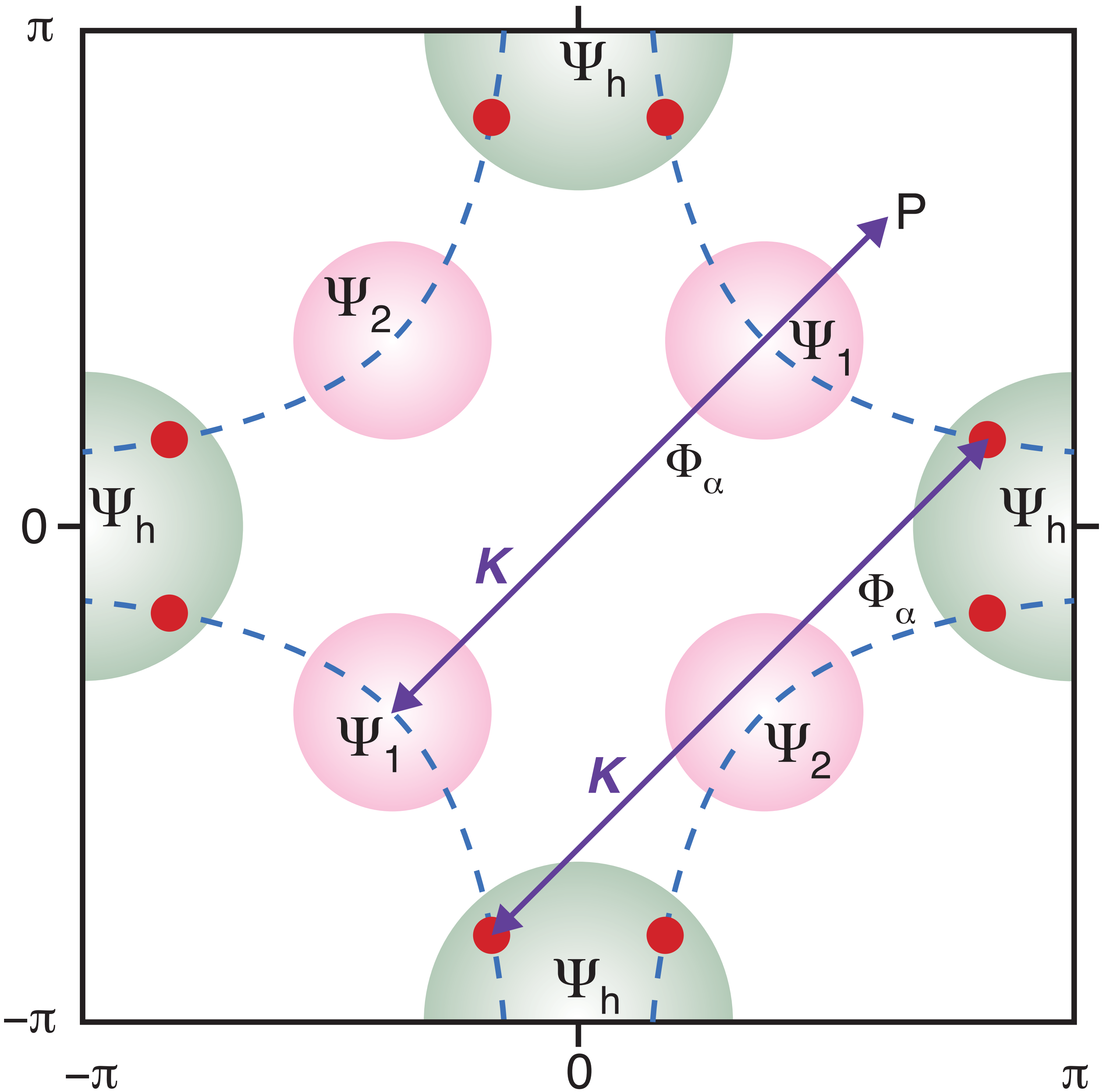
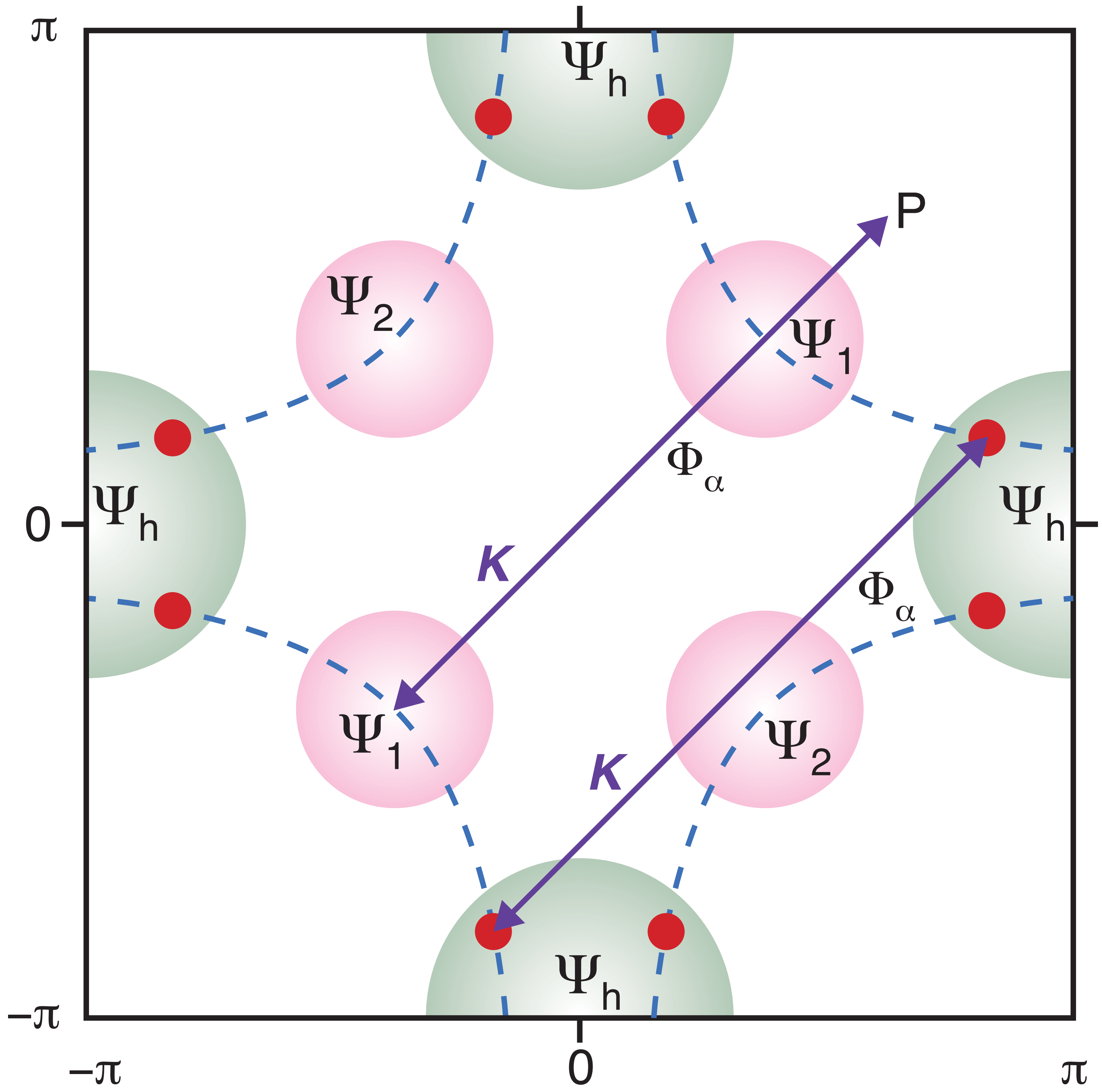
Coupling of quasiparticles to SDW order



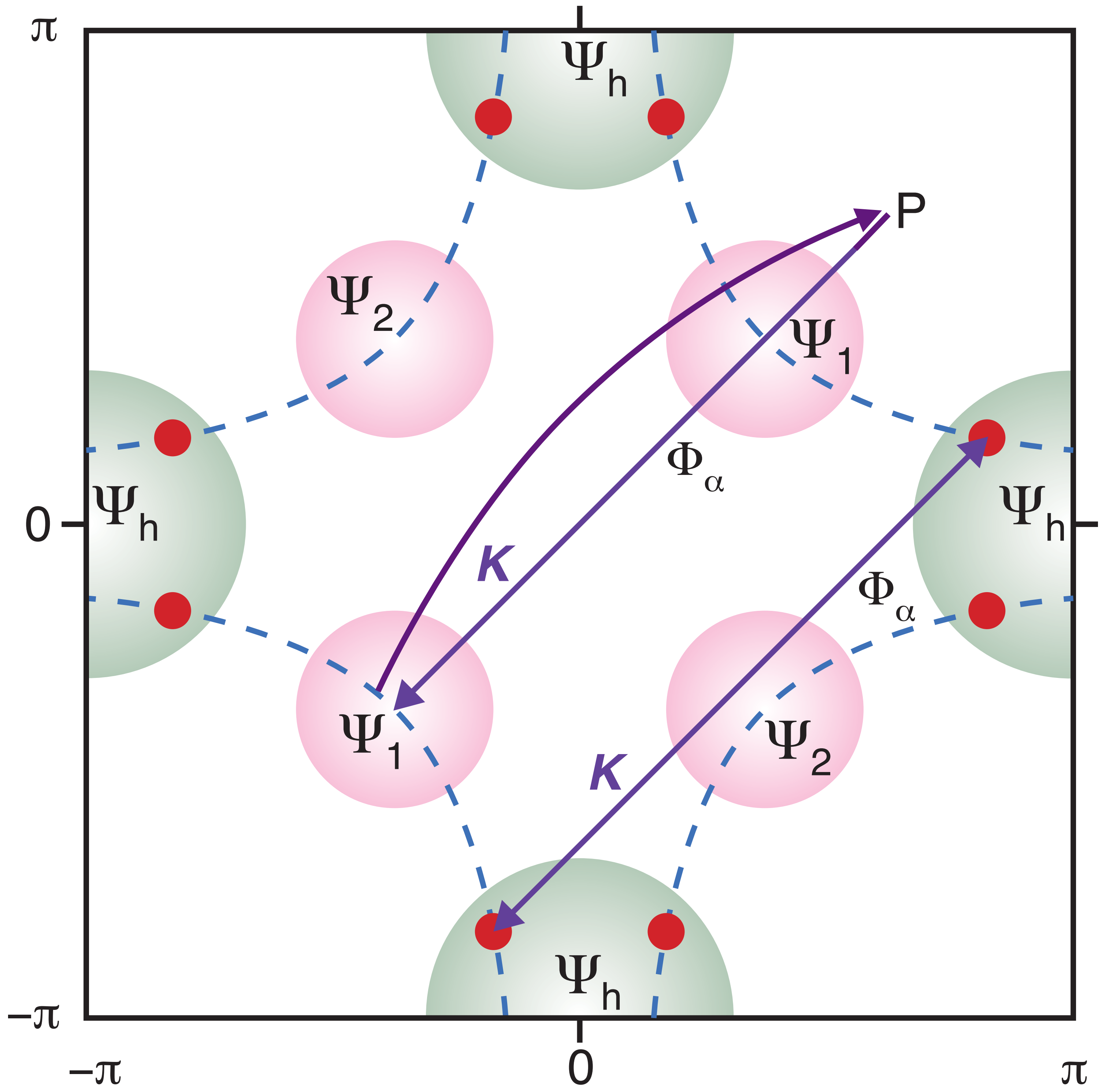
Wavevector mismatch suggests SDW order and nodal quasiparticles are decoupled

Coupling of quasiparticles to SDW order



No "Yukawa" coupling $\Phi\Psi^\dagger\Psi$

Coupling of quasiparticles to SDW order



Possible higher order coupling ~ $|\Phi|^2 \Psi^\dagger \Psi$

Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_1 = \lambda_1 \left(|\Phi_1|^2 + |\Phi_2|^2 \right) \left(\Psi_1^\dagger \tau^z \Psi_1 + \Psi_2^\dagger \tau^z \Psi_2 \right)$$

Energy-energy coupling

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_2 = \lambda_2 \left(|\Phi_1|^2 - |\Phi_2|^2 \right) \left(\Psi_1^\dagger \tau^x \Psi_1 + \Psi_2^\dagger \tau^x \Psi_2 \right)$$

Nematic coupling

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

Coupling of quasiparticles to SDW order

Higher - order couplings allowed by symmetry:

$$\mathcal{L}_3 = \epsilon_{ijk} \Big[\left(\Phi^*_{1j}\Phi_{1k} + \Phi^*_{2j}\Phi_{2k}\right)\left(-\lambda_3 \Psi_2^\dagger \tau^x \sigma^i \Psi_2 + \lambda_3' \Psi_1^\dagger \tau^z \sigma^i \Psi_1\right) + \left(\Phi^*_{1j}\Phi_{1k} - \Phi^*_{2j}\Phi_{2k}\right)\left(\lambda_3 \Psi_1^\dagger \tau^x \sigma^i \Psi_1 - \lambda_3' \Psi_2^\dagger \tau^z \sigma^i \Psi_2\right)\Big].$$

Spiral spin order coupling

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

Coupling of quasiparticles to SDW order

Scaling dimensions of these couplings:

$$\begin{aligned}
\dim[\lambda_1] &= \frac{1}{\nu} - 2 = -0.9(2) \\
\dim[\lambda_2] &= \frac{(\overline{\gamma} - 1)}{2} = \begin{cases} -0.05(18) & \text{MZM}, 6 \text{ loops} \\ -0.10(27) & d = 3\ \overline{\text{MS}}, 5 \text{ loops} \end{cases} \\
\dim[\lambda_3, \lambda_3'] &= \begin{cases} -0.84(8) & \text{MZM}, 6 \text{ loops} \\ -0.76(8) & d = 3\ \overline{\text{MS}}, 5 \text{ loops} \end{cases}
\end{aligned}$$

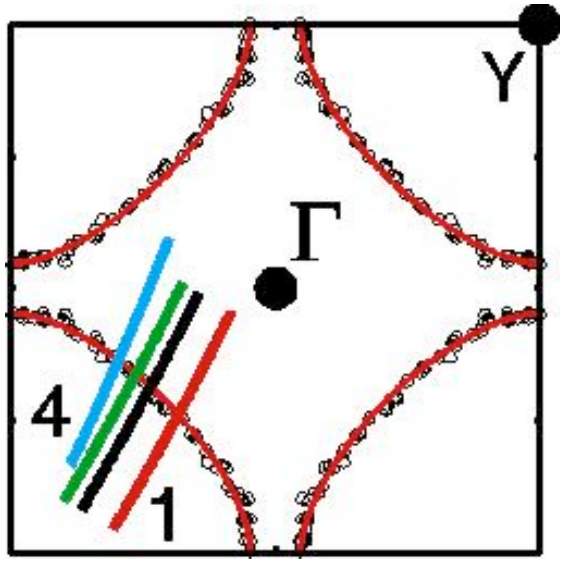
A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

Coupling of quasiparticles to SDW order

Coupling of nematic order is nearly marginal:

Quantum-critical features appear in fermion spectrum via coupling to nematic fluctuations of spin density wave order.

A. Pelissetto, S. Sachdev and E. Vicari, arXiv:0802.0199

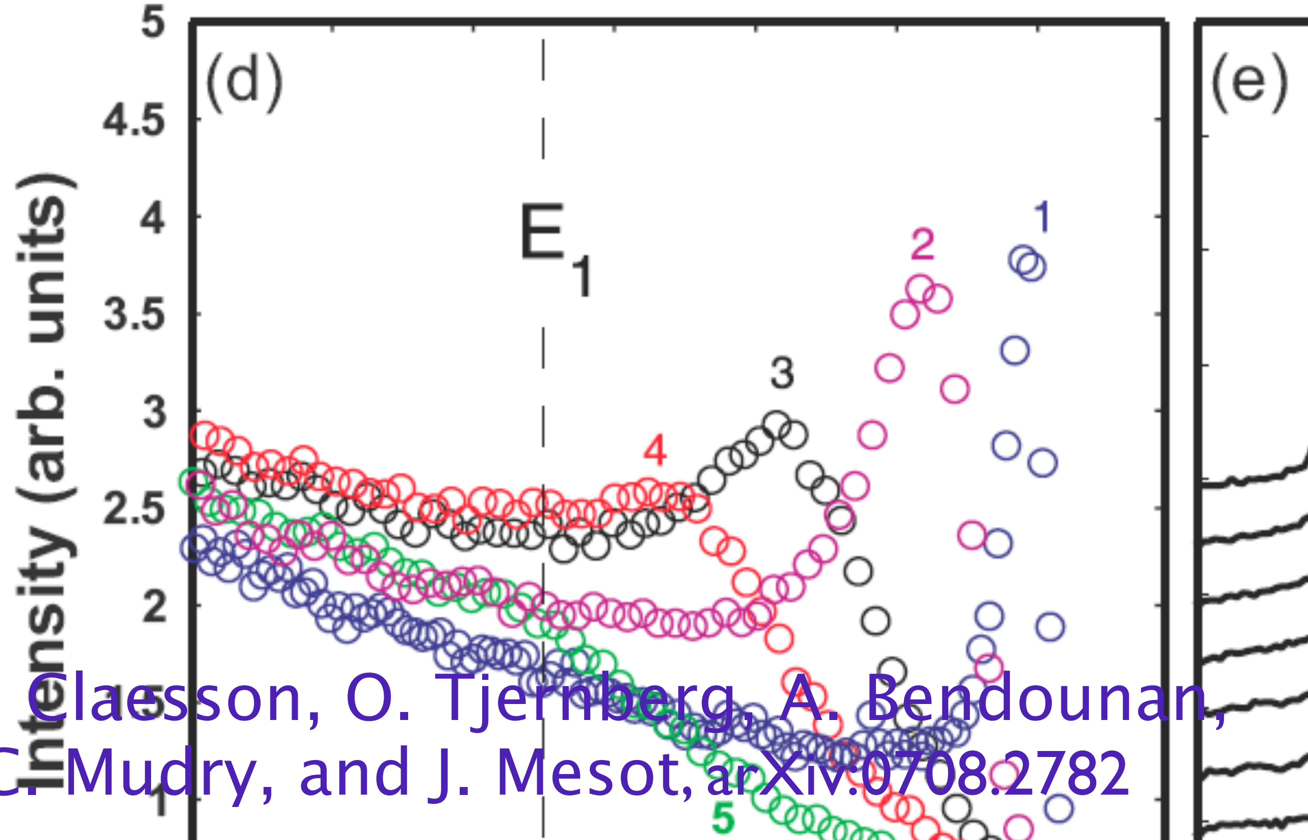


Photoemission spectra of $La_{2-x}Sr_xCuO_4$

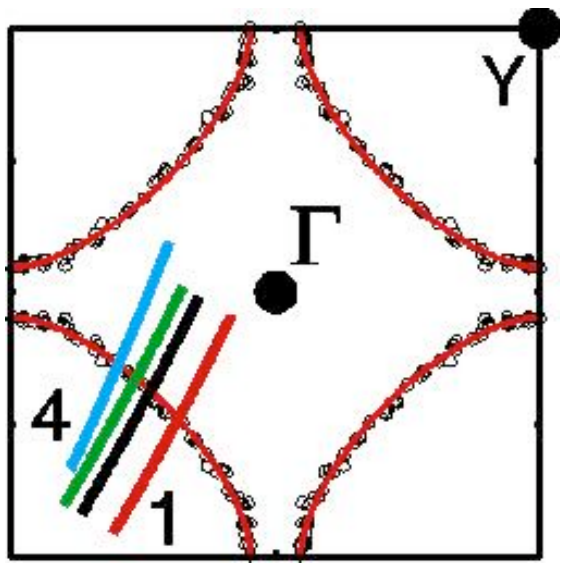
EDC

x=0.145

MDC



J. Chang, M. Shi, S. Pailhes, M. Maansson, T. Claesson, O. Tjernberg, A. Bendounan, L. Patthey, N. Momono, M. Oda, M. Ido, C. Mudry, and J. Mesot, arXiv:0708.2782



Photoemission spectra of $La_{2-x}Sr_xCuO_4$

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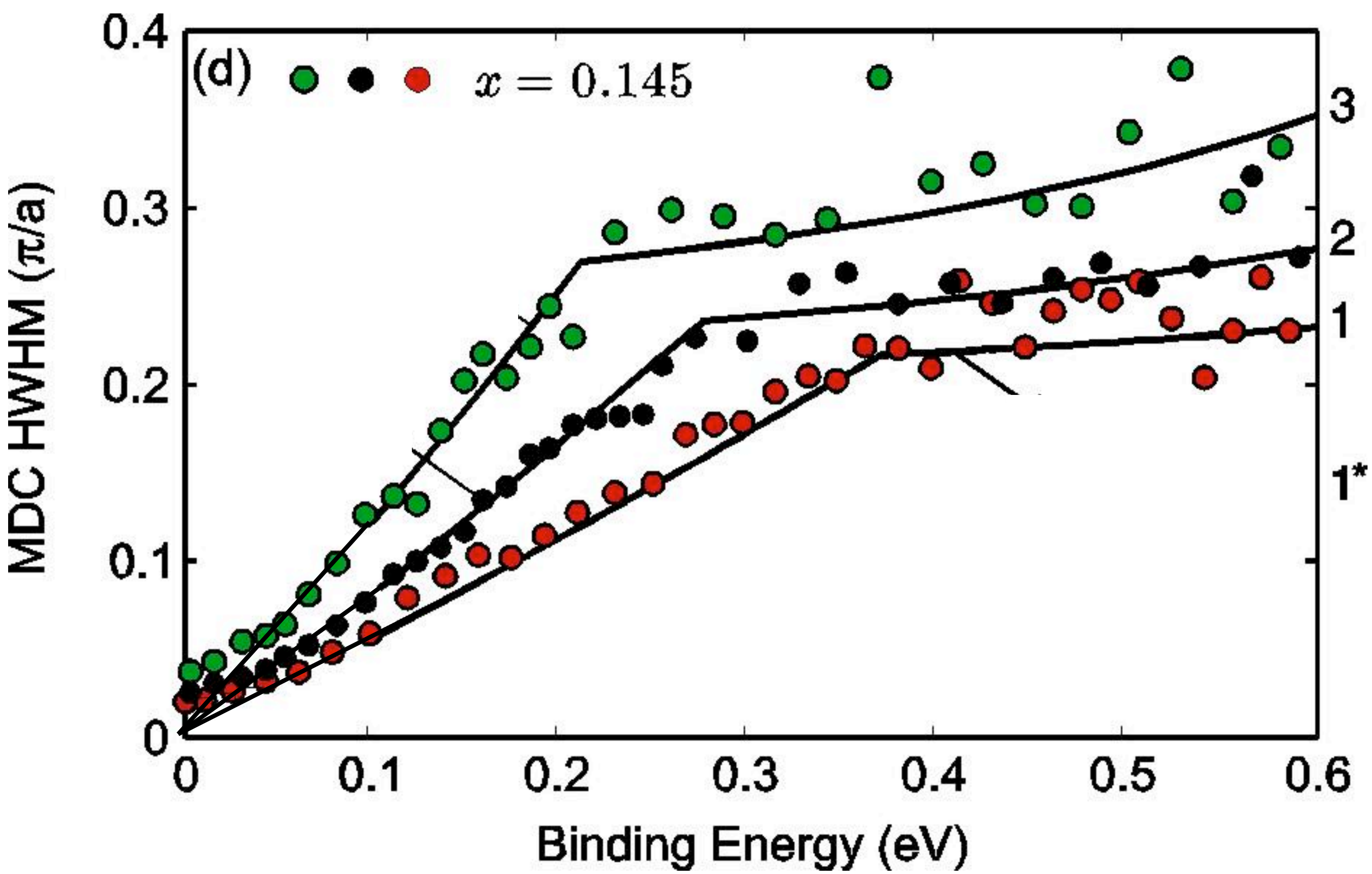

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Conclusions

1. Theories for damping of nodal quasiparticles in cuprates

2. Nematic theory for has a fixed point with v_Δ/v_F = 0 which is approached logarithmically. The theory is expressed as an expansion in v_Δ/v_F

3. Exact results for a strongly interacting fixed point with large fermion anomalous dimensions - leads to "Fermi arc" spectra at low temperatures.

4. Nematic order also emerges naturally from theory of SDW ordering, as the most efficient source of quasiparticle damping.