The quantum phases of matter

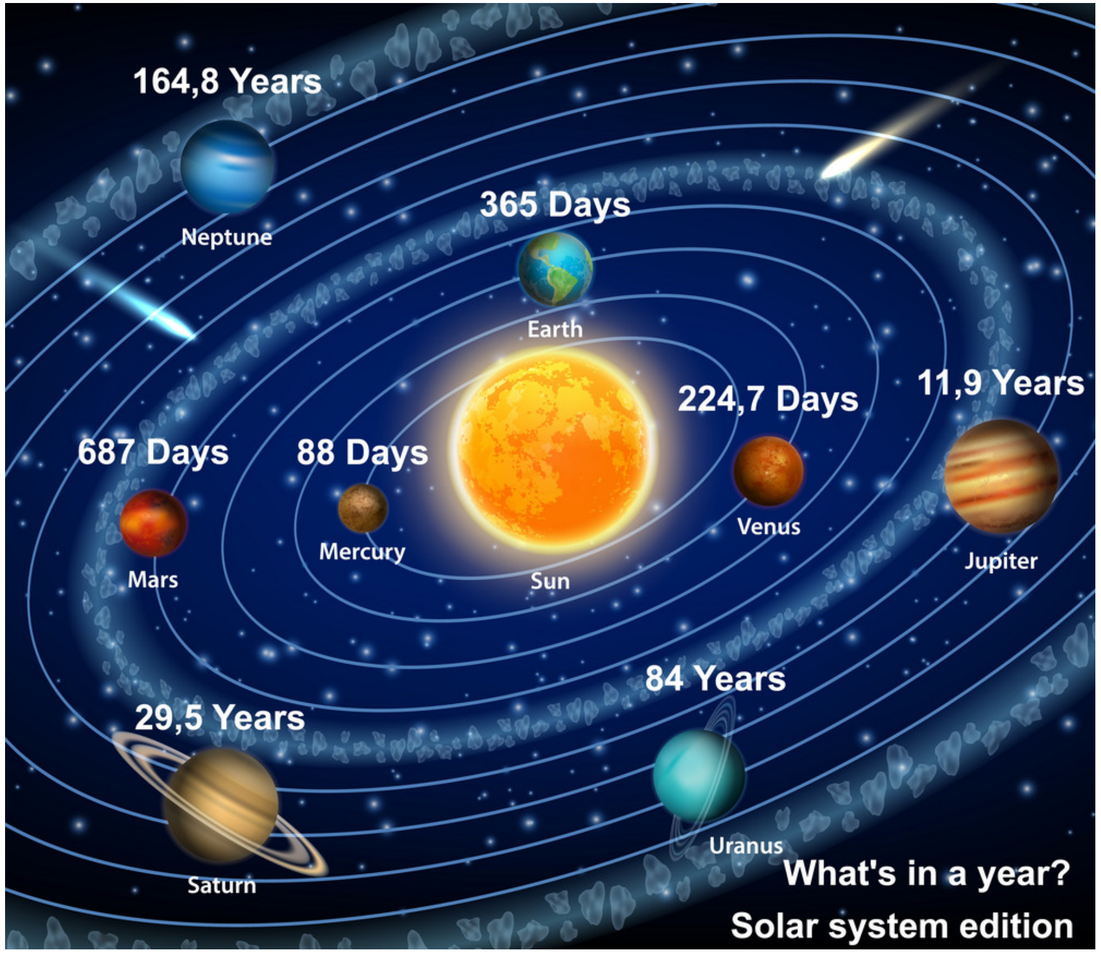
The Delhi Public School Society
August 16, 2021

Subir Sachdev

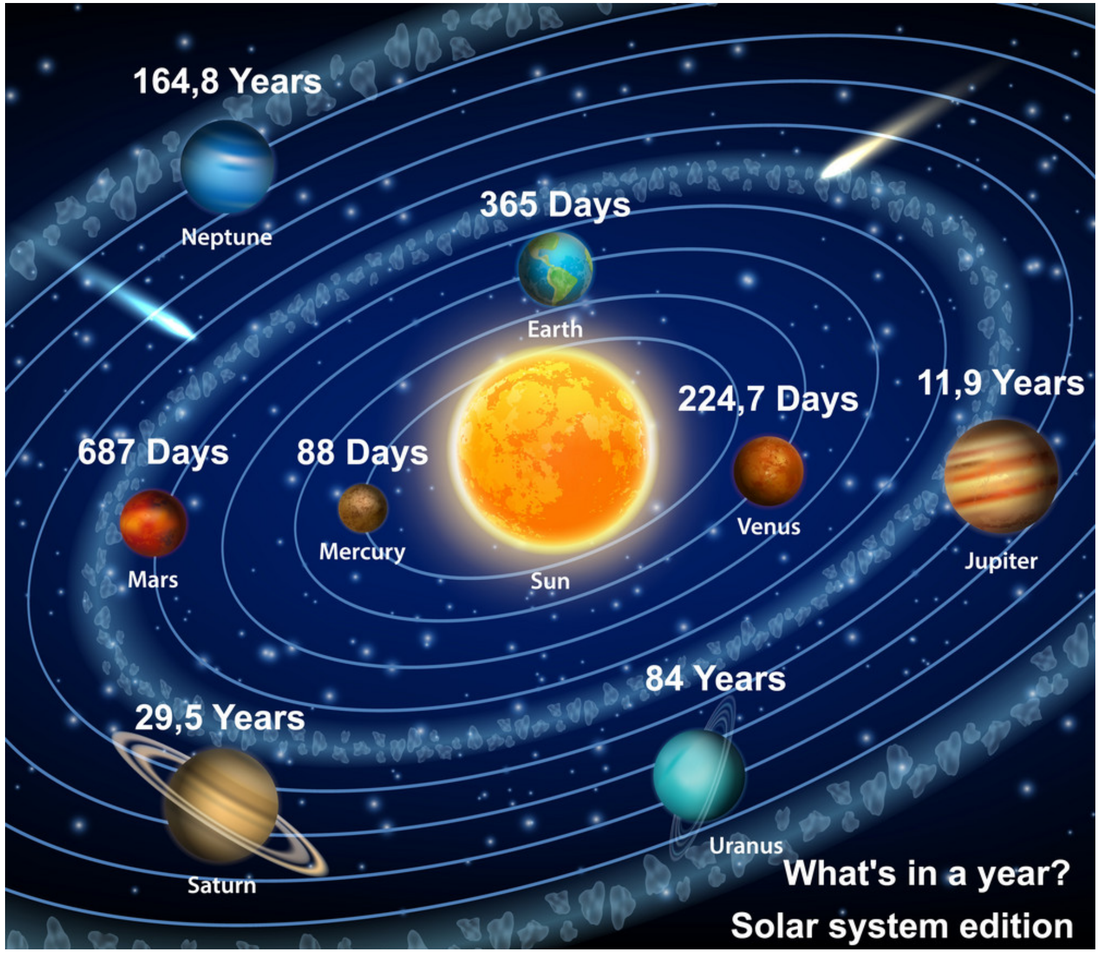
Talk online: sachdev.physics.harvard.edu

INSTITUTE FOR
ADVANCED STUDY

HARVARD



Newton showed (1687) that the same laws of motion applied on planetary length scales ($\sim$ 1 trillion meters) and the length scale of an apple tree (1 meter).



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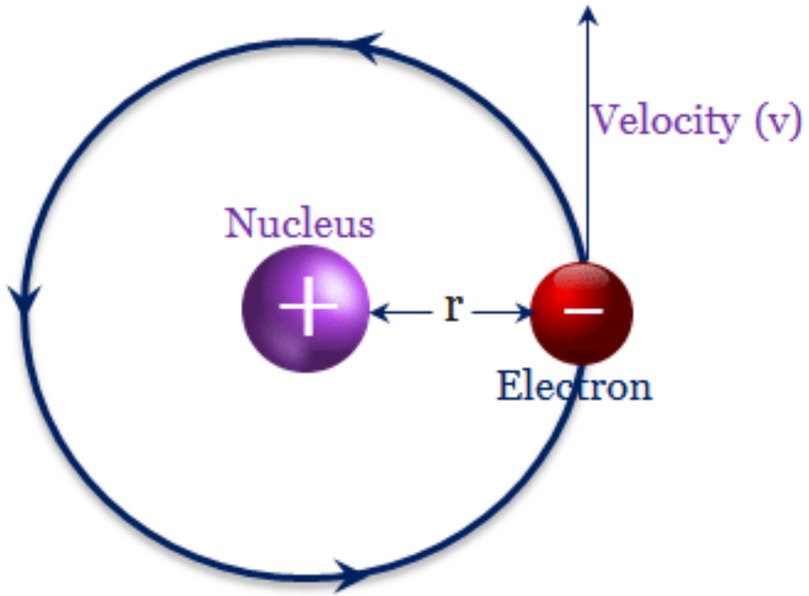
What happens on smaller distances ?

Quantum theory of electrons, one at a time: metals and insulators

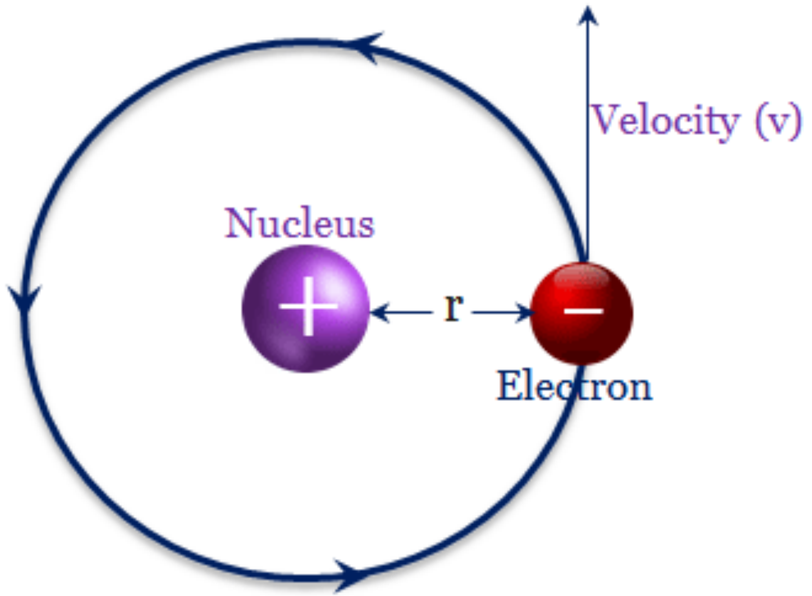
Hydrogen atom



$\Rightarrow 10^{-10}$ meters $\Leftarrow$

The motion of the electron around the proton is *not* described by the same theory as the motion of the planets around the sun.

Hydrogen atom



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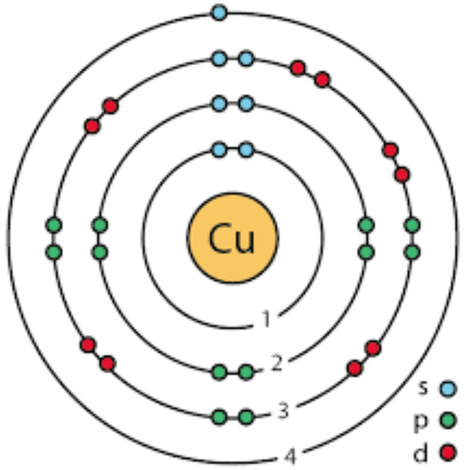
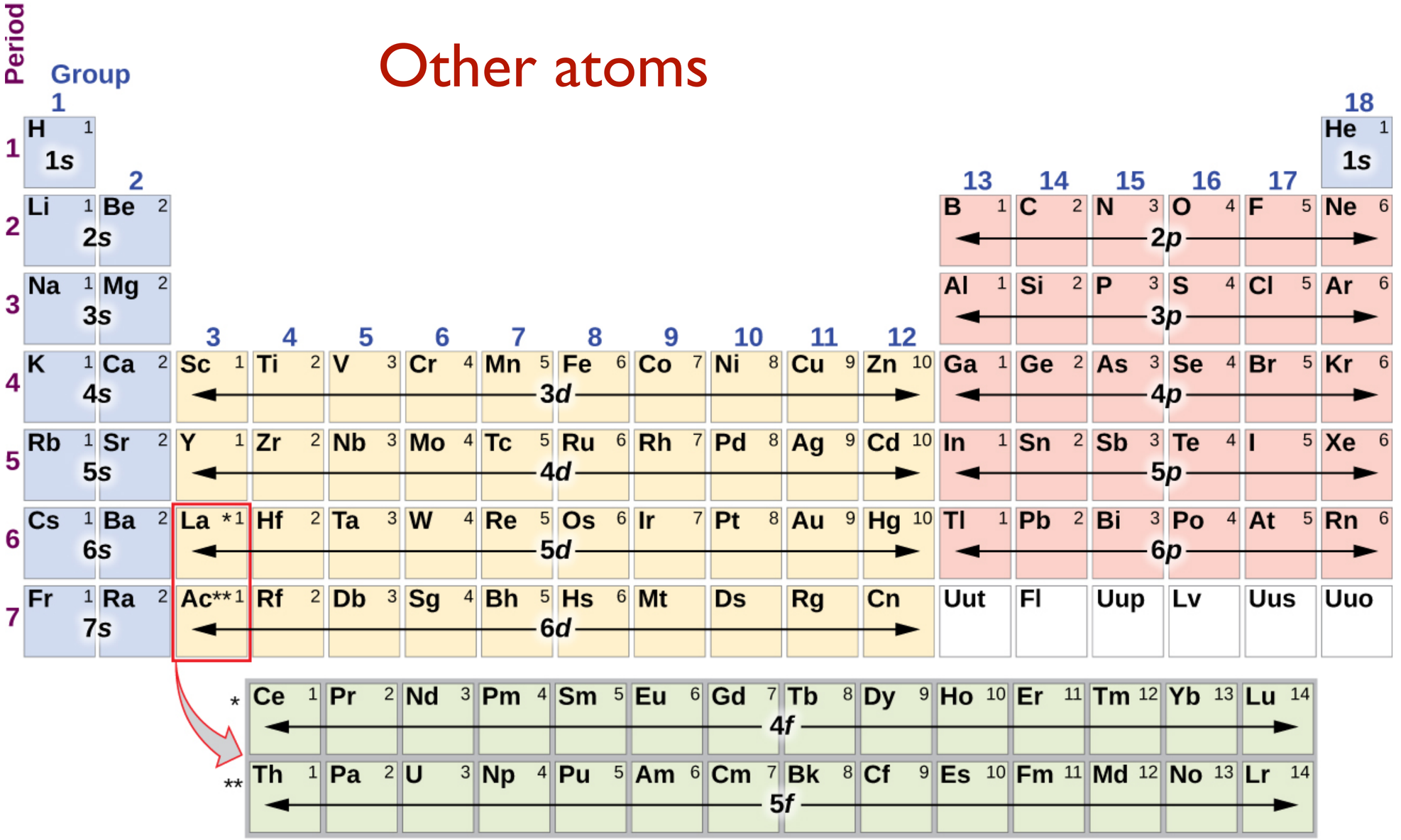
It is described by the quantum theory of Schrödinger and Heisenberg (1925).

Other atoms



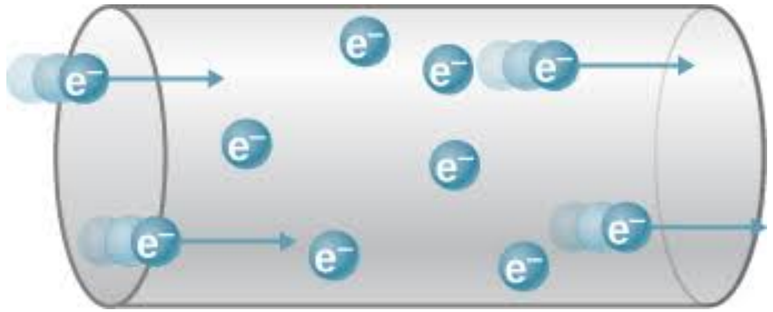


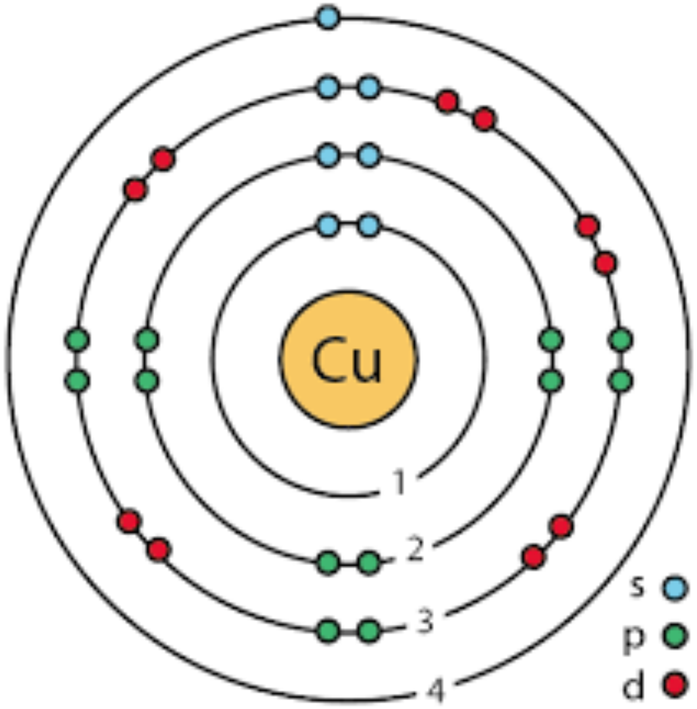
The periodic table follows from (i) the exclusion principle, and (ii) each electron has 2 spin states $|\uparrow\rangle$, and $|\downarrow\rangle$.

Ordinary metals

Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal

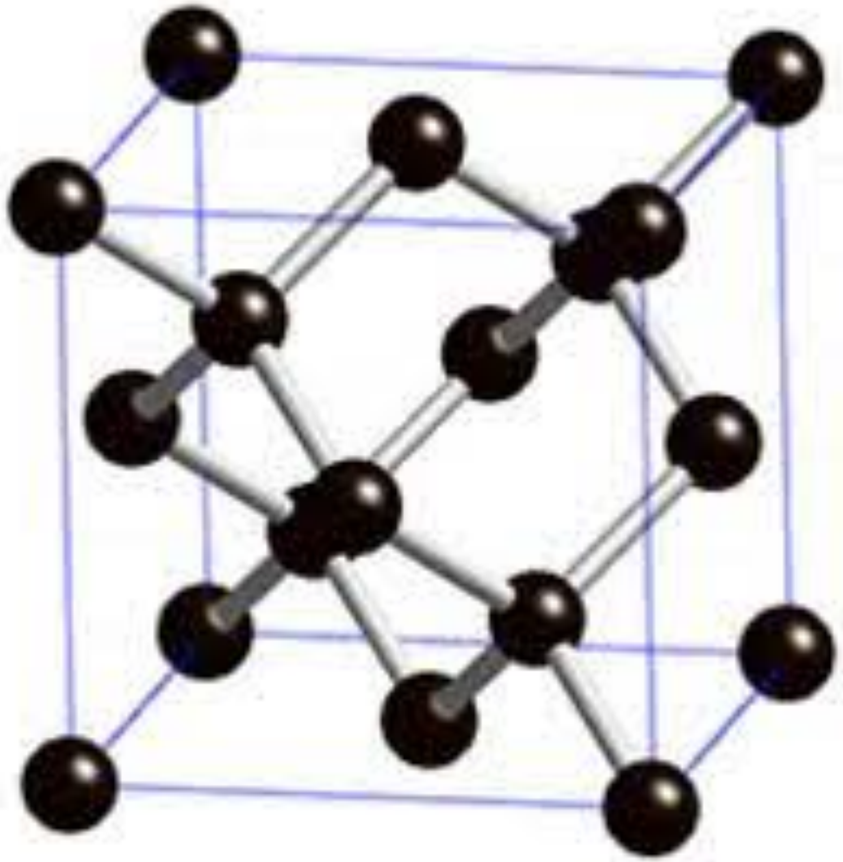
Copper

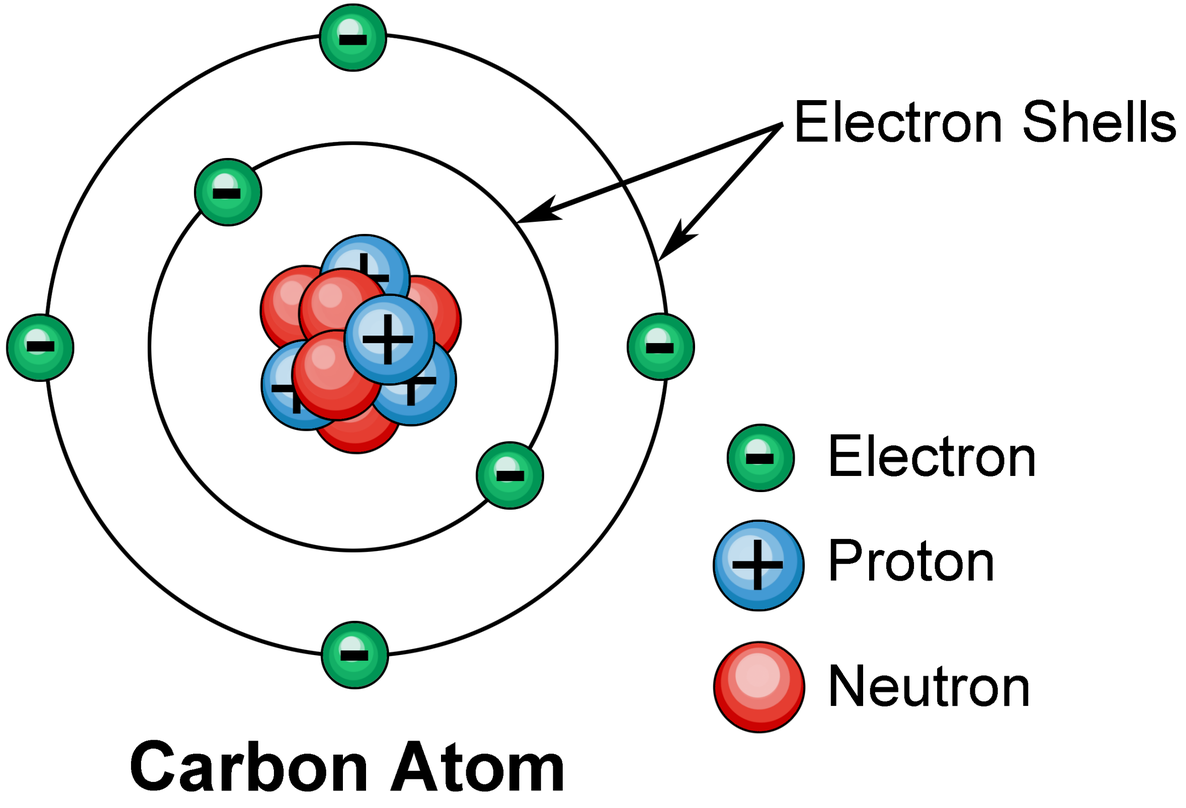




Each copper atom donates its outermost electron
These electrons move freely throughout the crystal and carry current

Diamond - a very good insulator





Each carbon atom donates 4 electrons

These electrons occupy filled "bands" and are not able to carry current

Quantum entanglement of electron pairs: superconductivity

The most remarkable new idea in the quantum theory is the
principle of superposition:
a physical system can be in a
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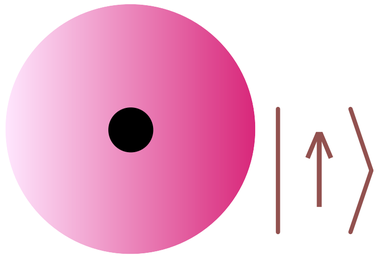
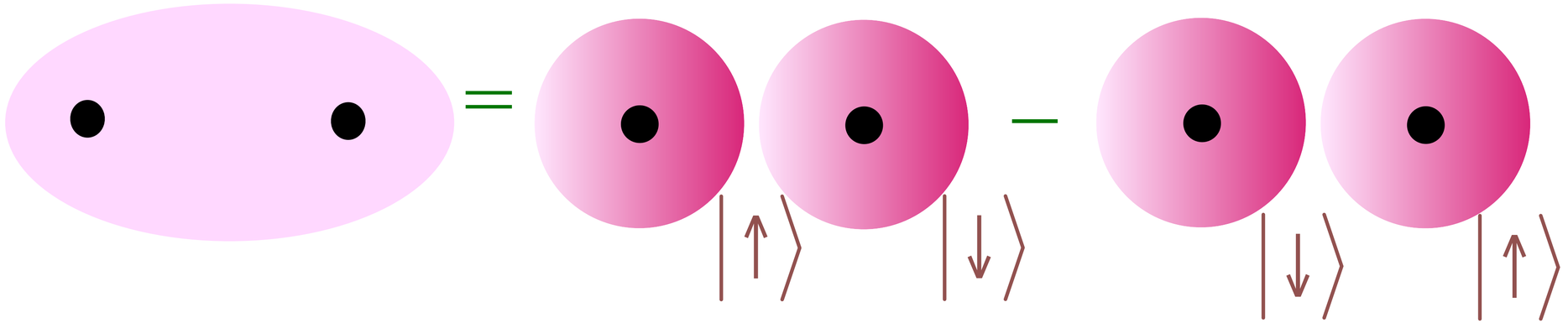
Molecules

Hydrogen atom:



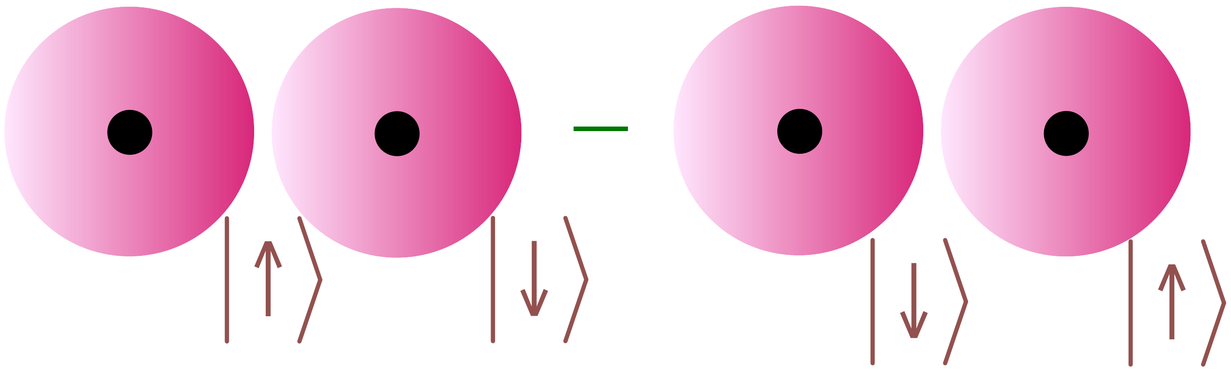
Covalent bond

Hydrogen molecule:



Quantum Entanglement

Einstein, Podolsky, Rosen (1935)



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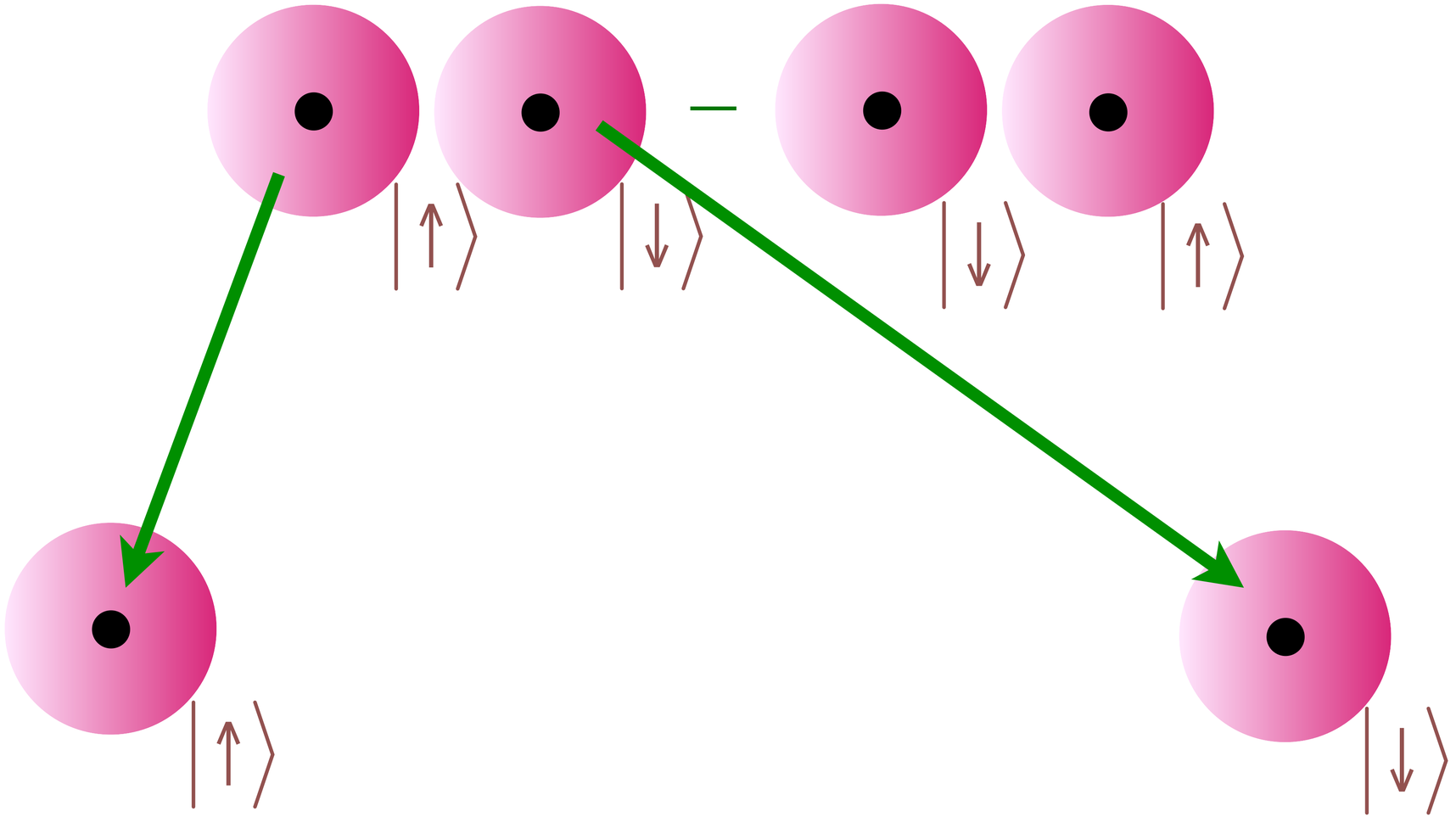

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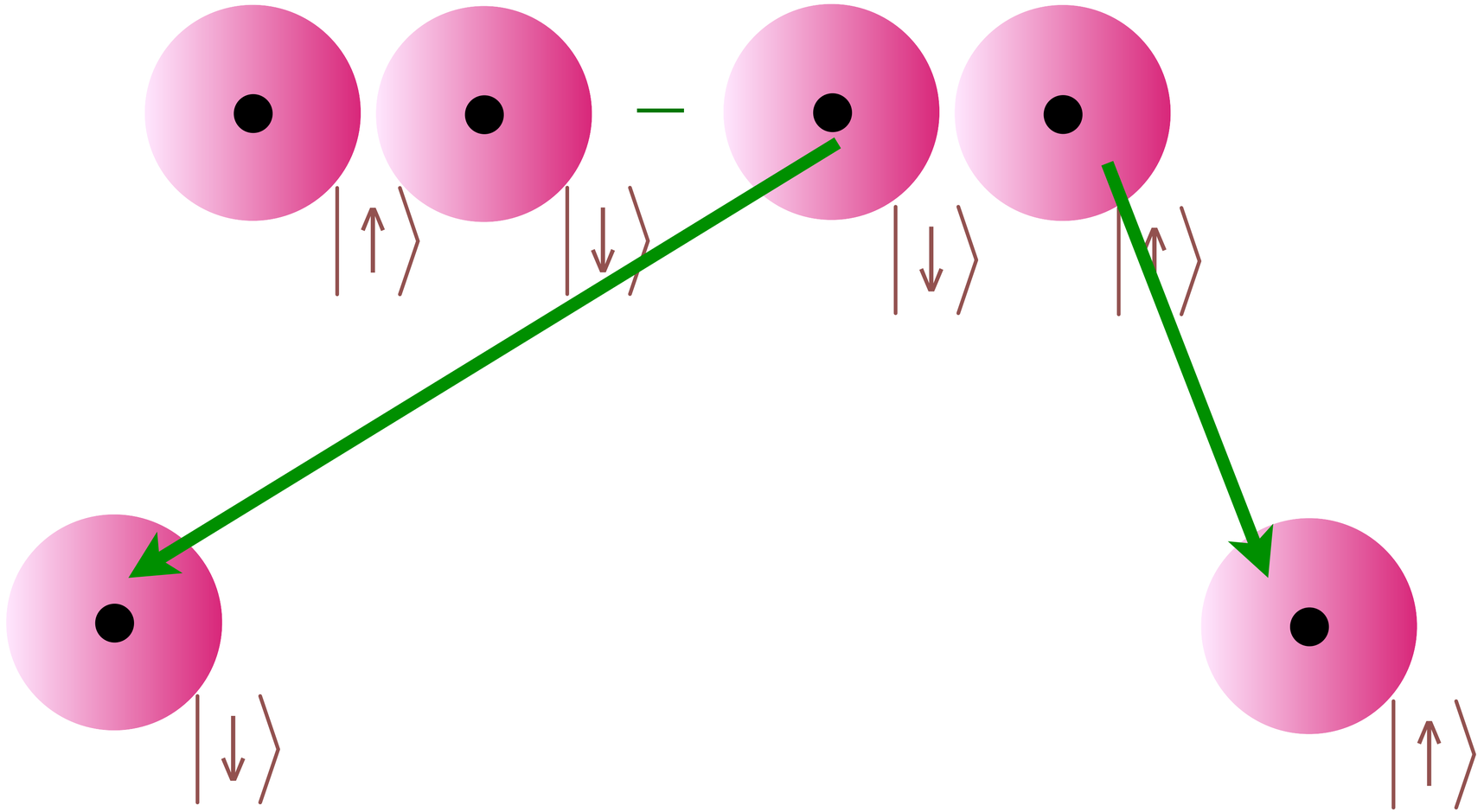

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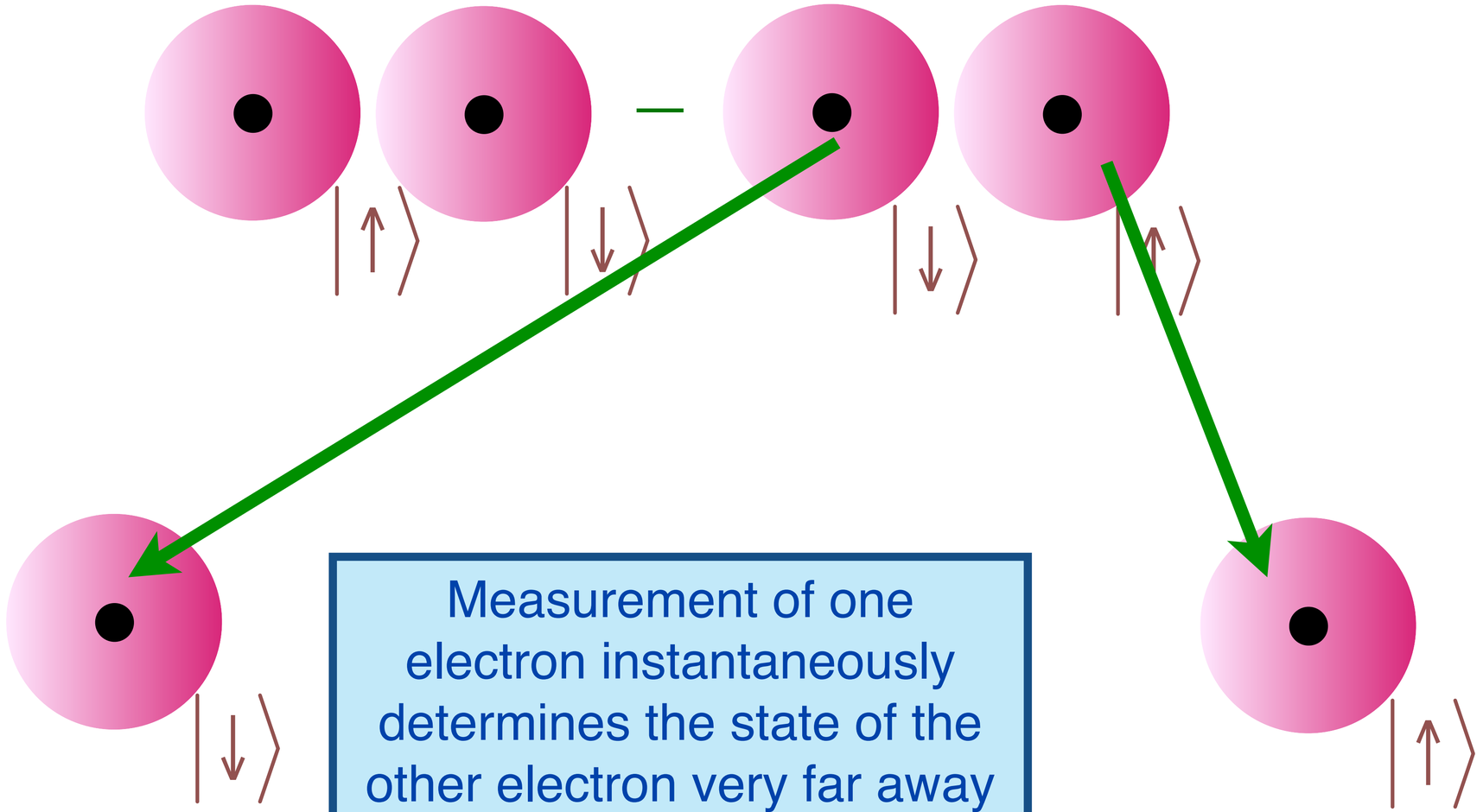

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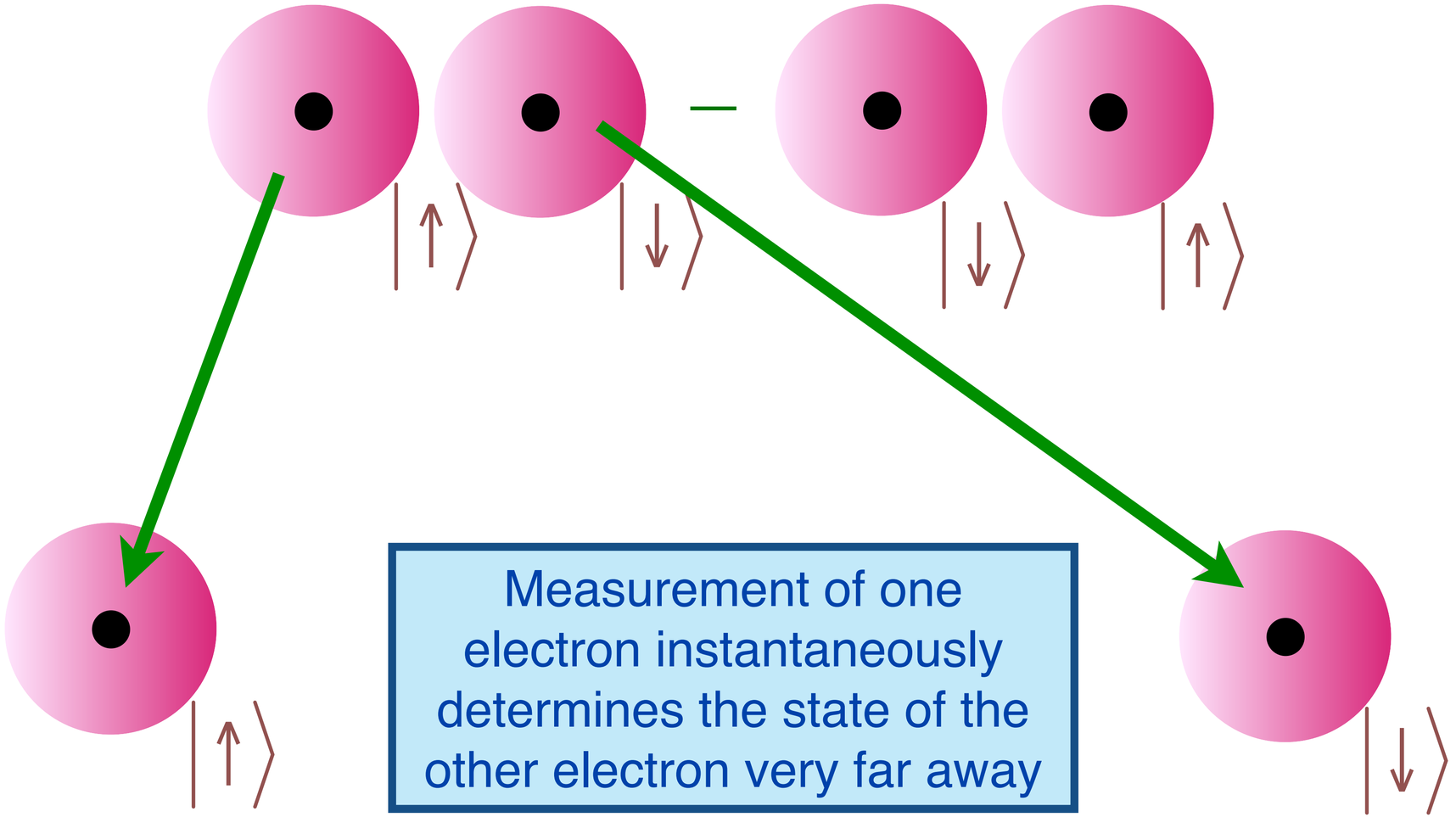
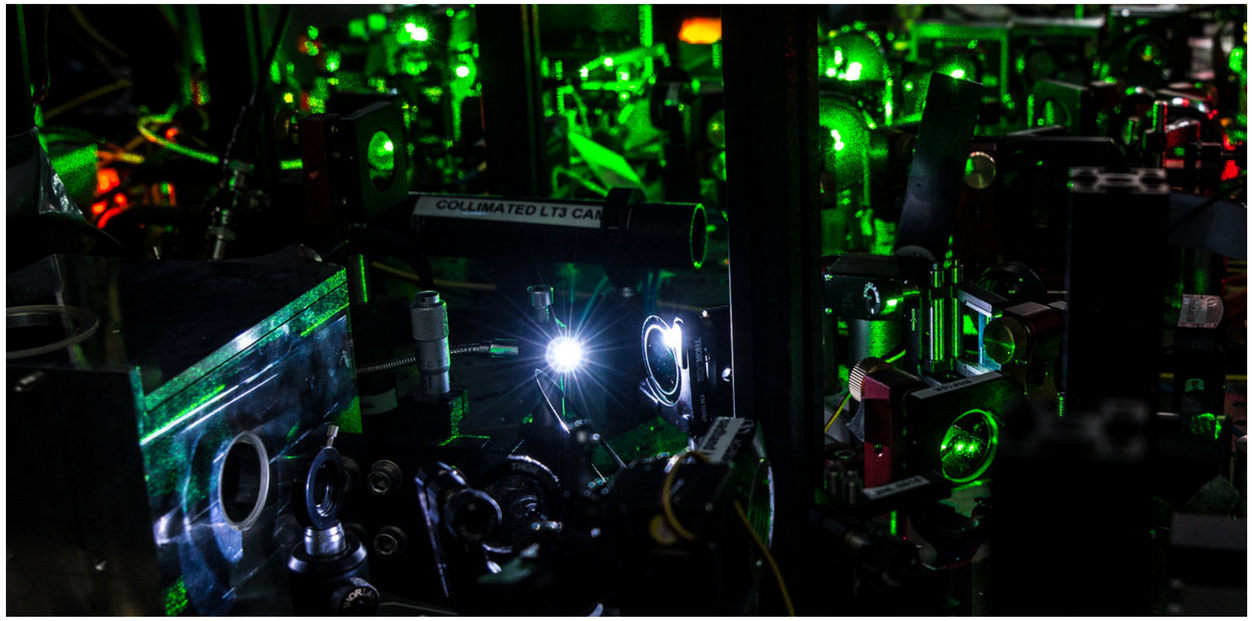


The New York Times

Sorry, Einstein. Quantum Study Suggests ‘Spooky Action’ Is Real.

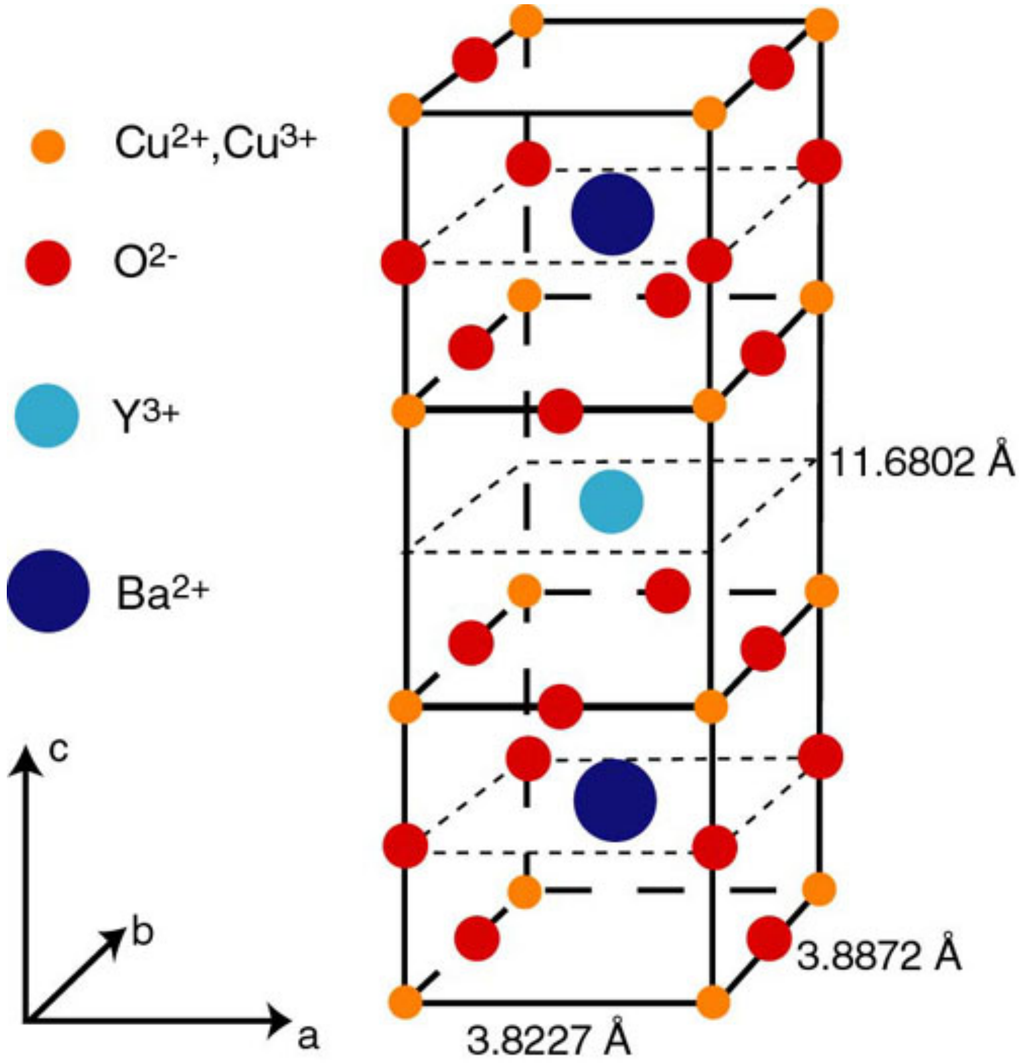
By **JOHN MARKOFF** OCT. 21, 2015

In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

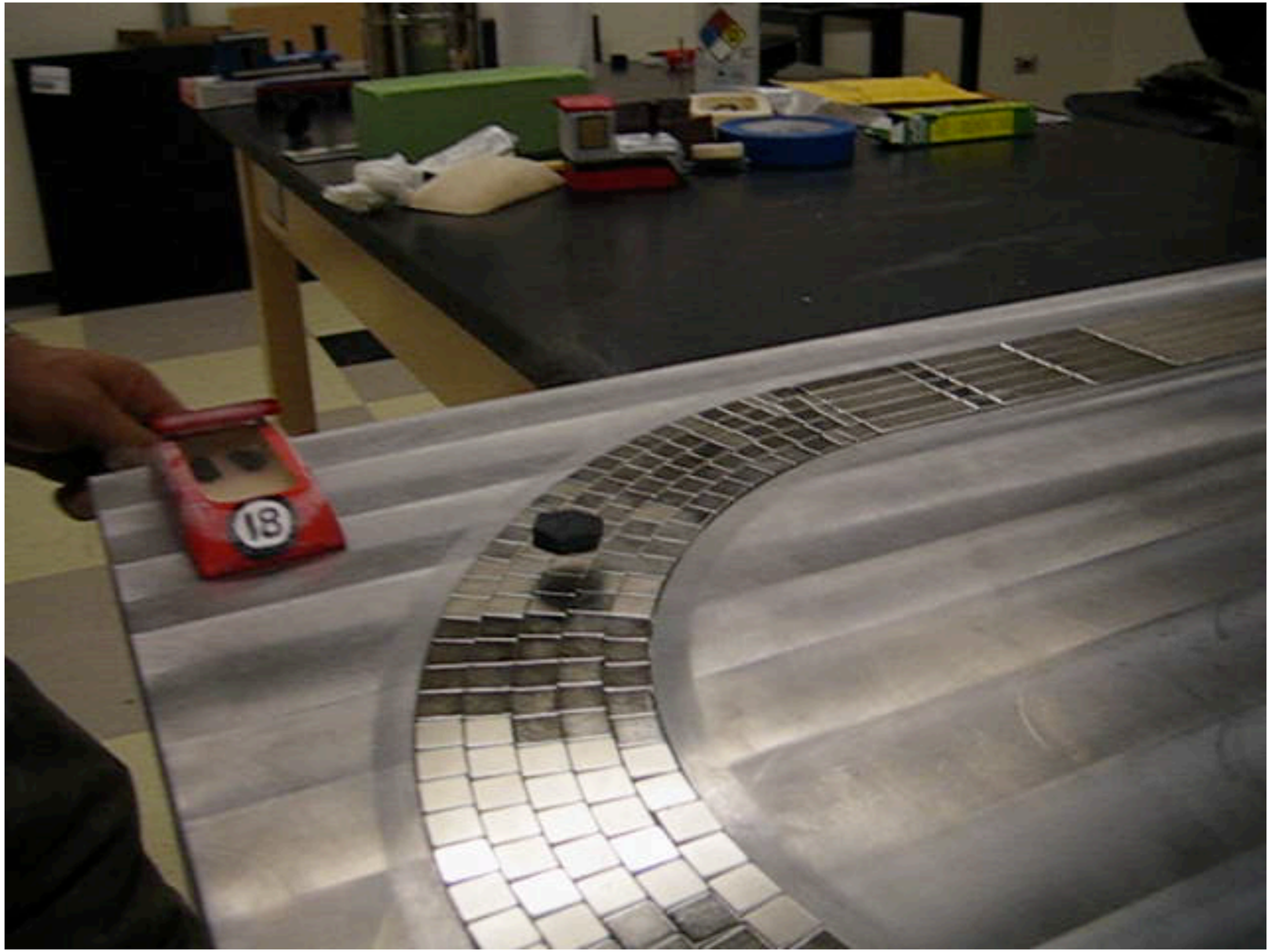


Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.

High temperature superconductors



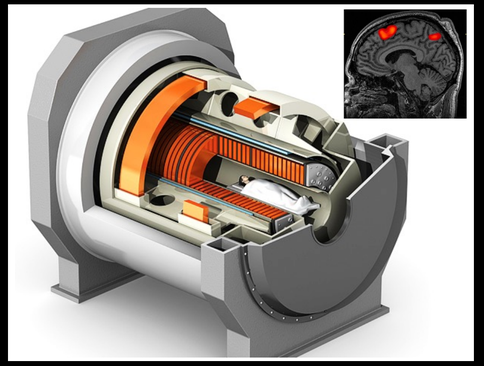
$$YBa_2Cu_3O_{6+x}$$



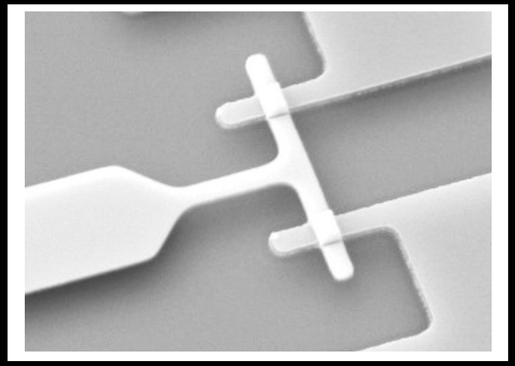
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University

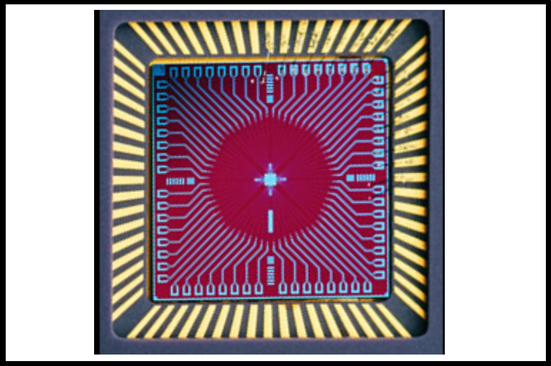
SUPERCONDUCTIVITY: SCIENTIFIC APPLICATIONS



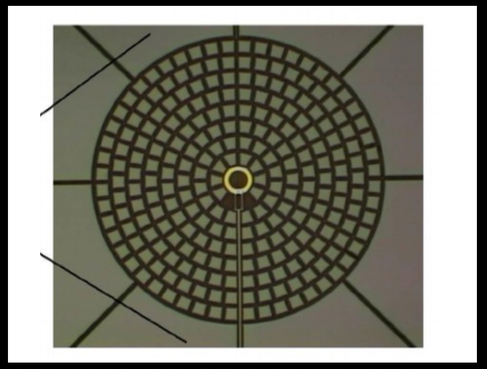
FUNCTIONAL MRI



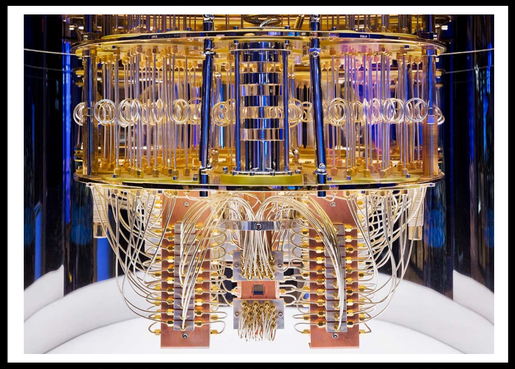
SQUID SENSORS



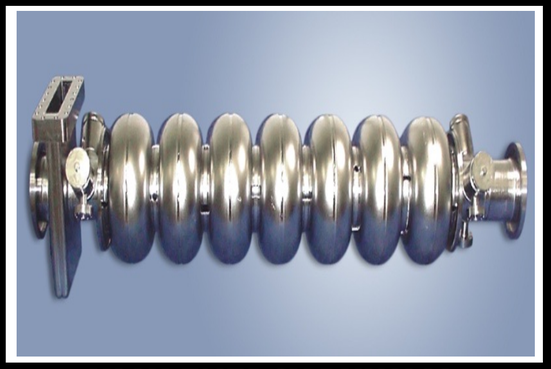
SINGLE PHOTON IMAGING



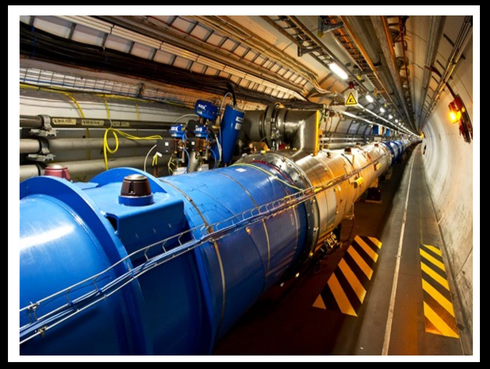
TRANS. EDGE BOLOMETER



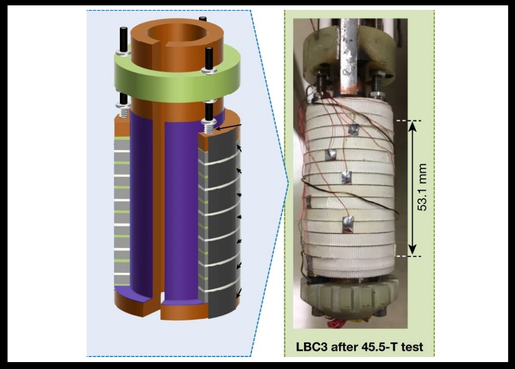
QUANTUM INFO. TECH.



HIGH ENERGY PHYSICS



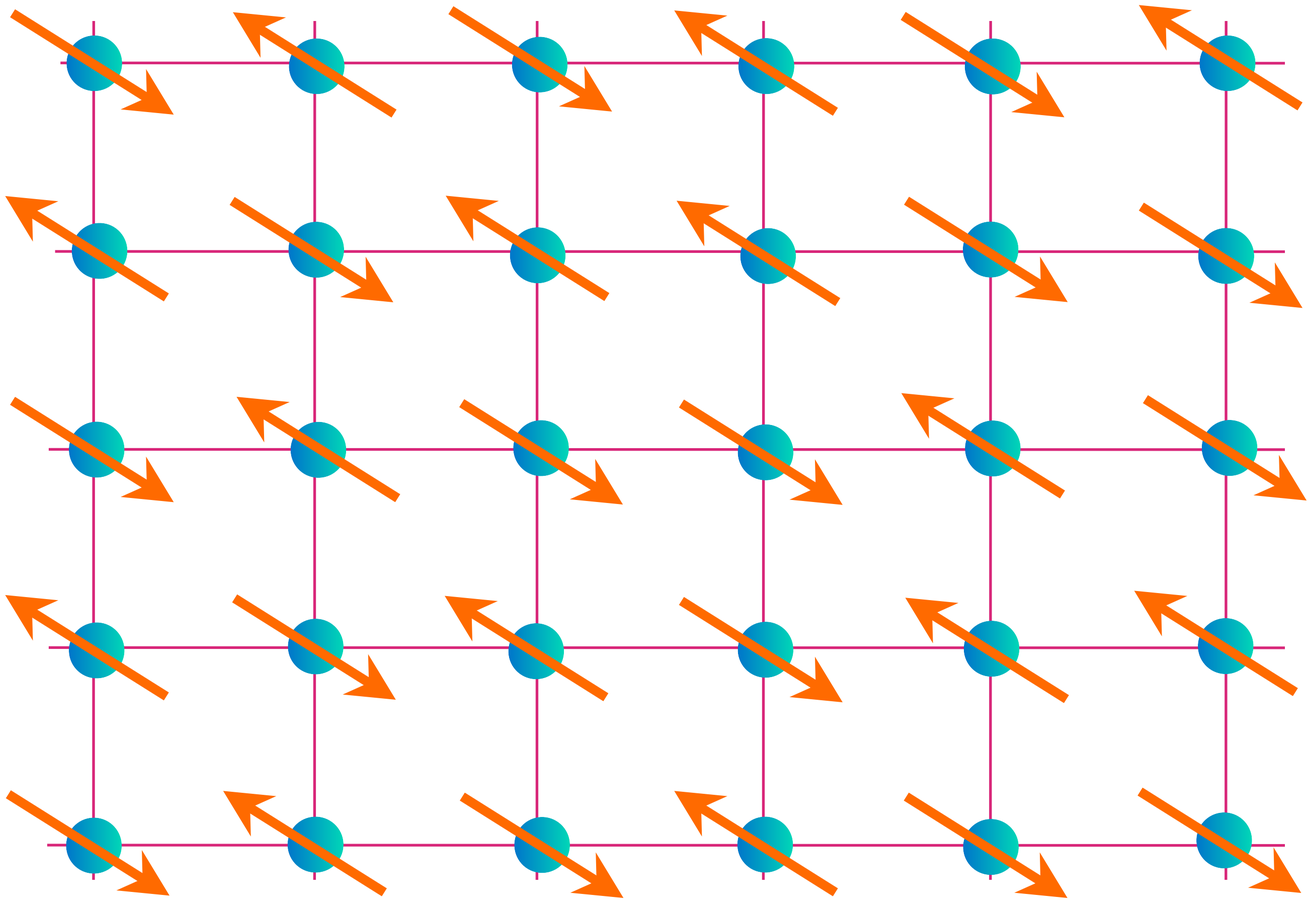
ACCELERATORS

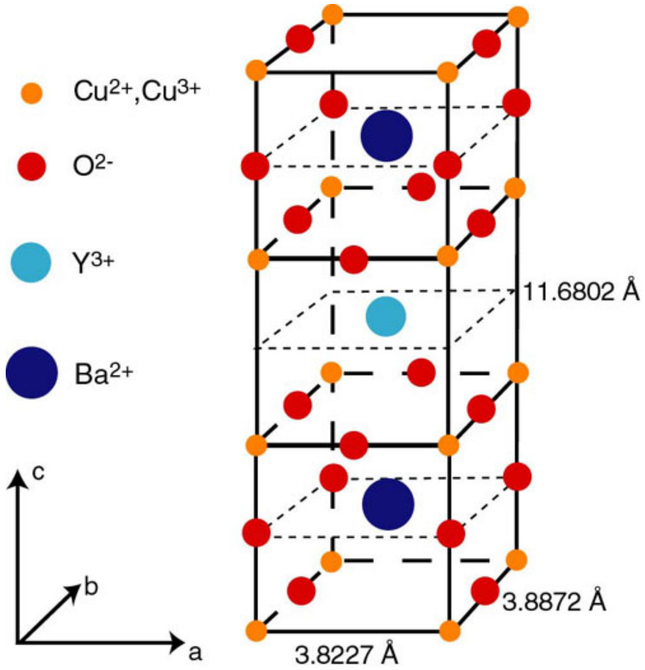


50+ TESLA MAGNETS

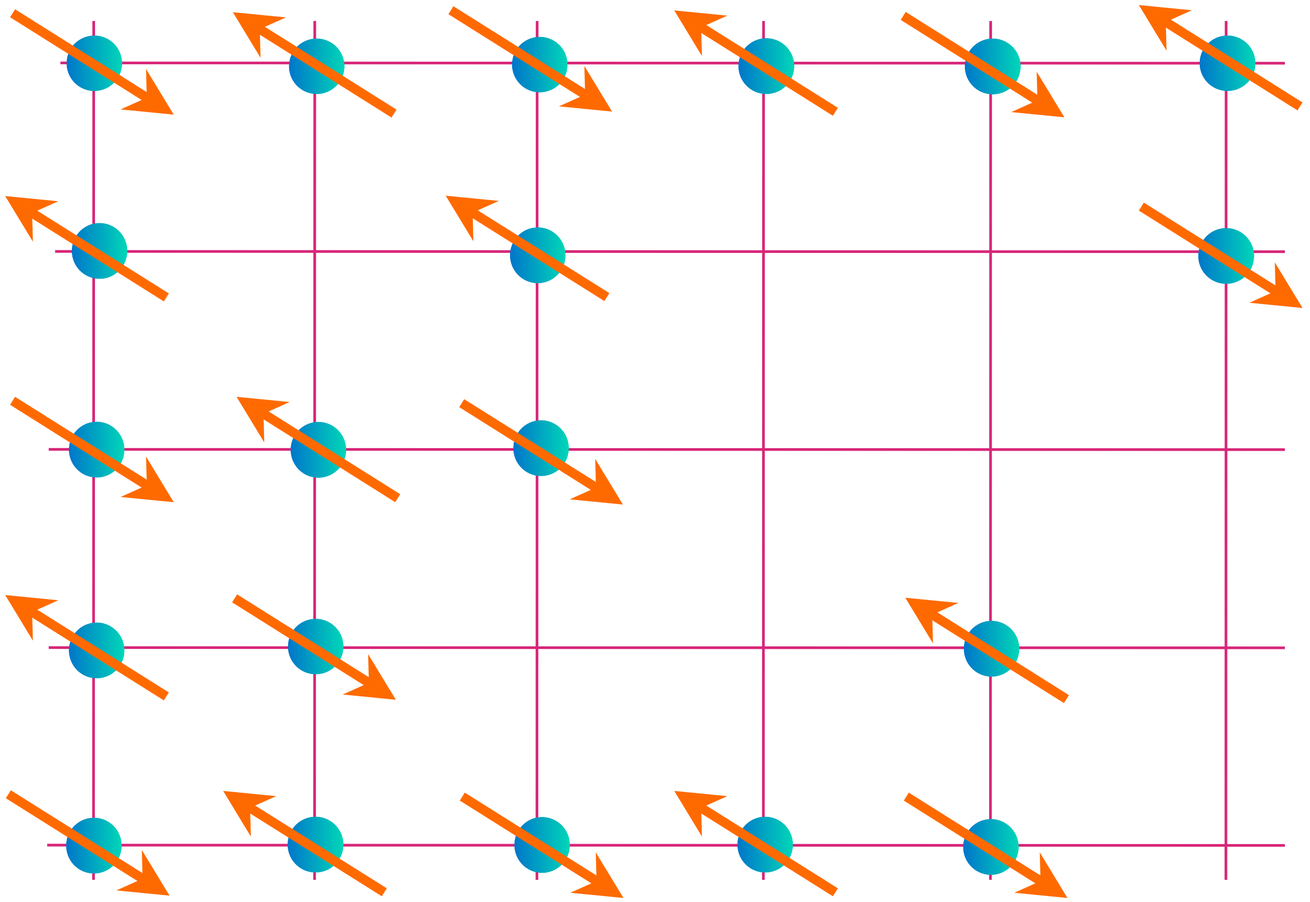
TOKOMAK FUSION

Insulating antiferromagnet



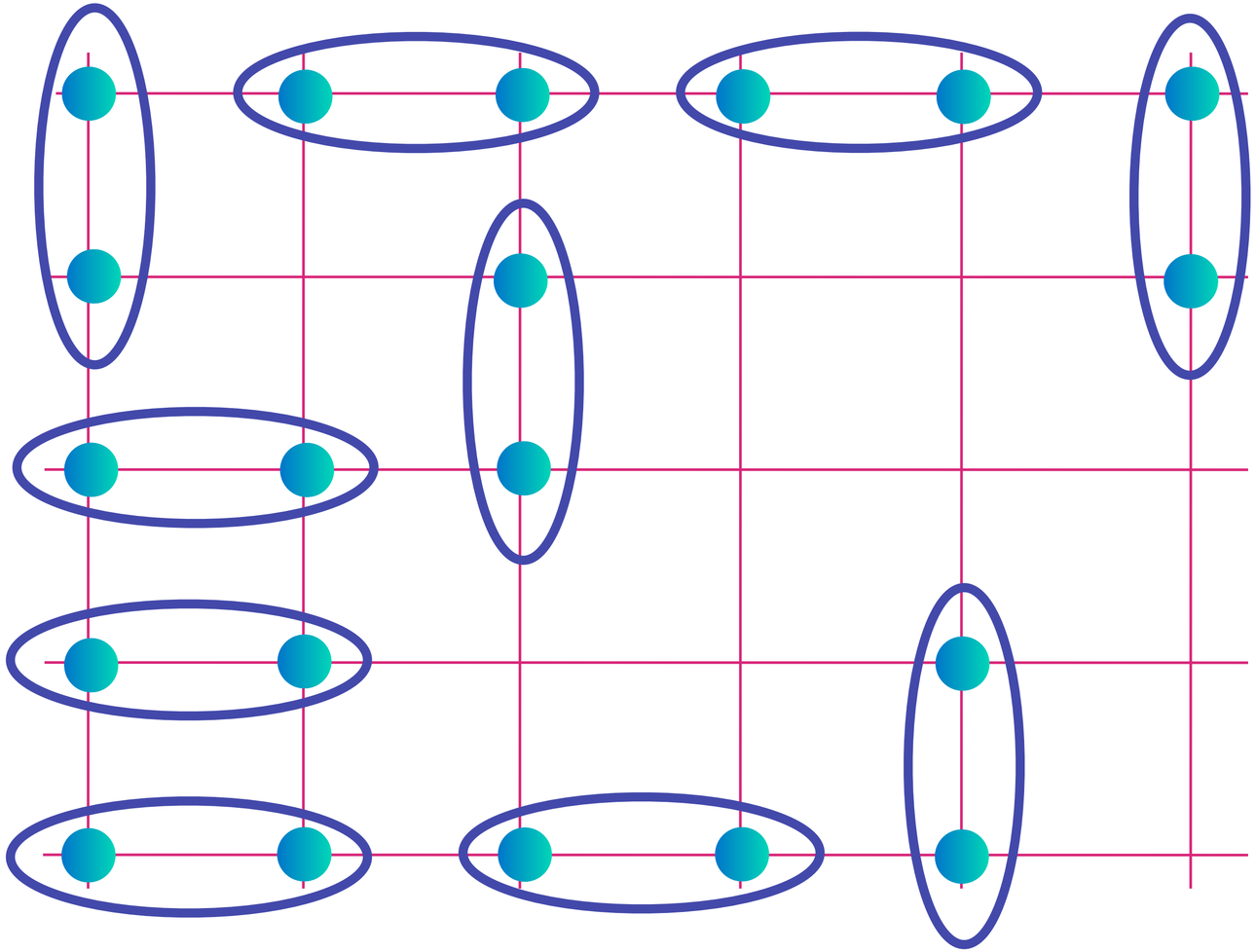


Antiferromagnet doped with hole density p



Remove fraction p electrons

Antiferromagnet doped with hole density p



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Motion of electron pairs leads to Bose-Einstein condensation and superconductivity

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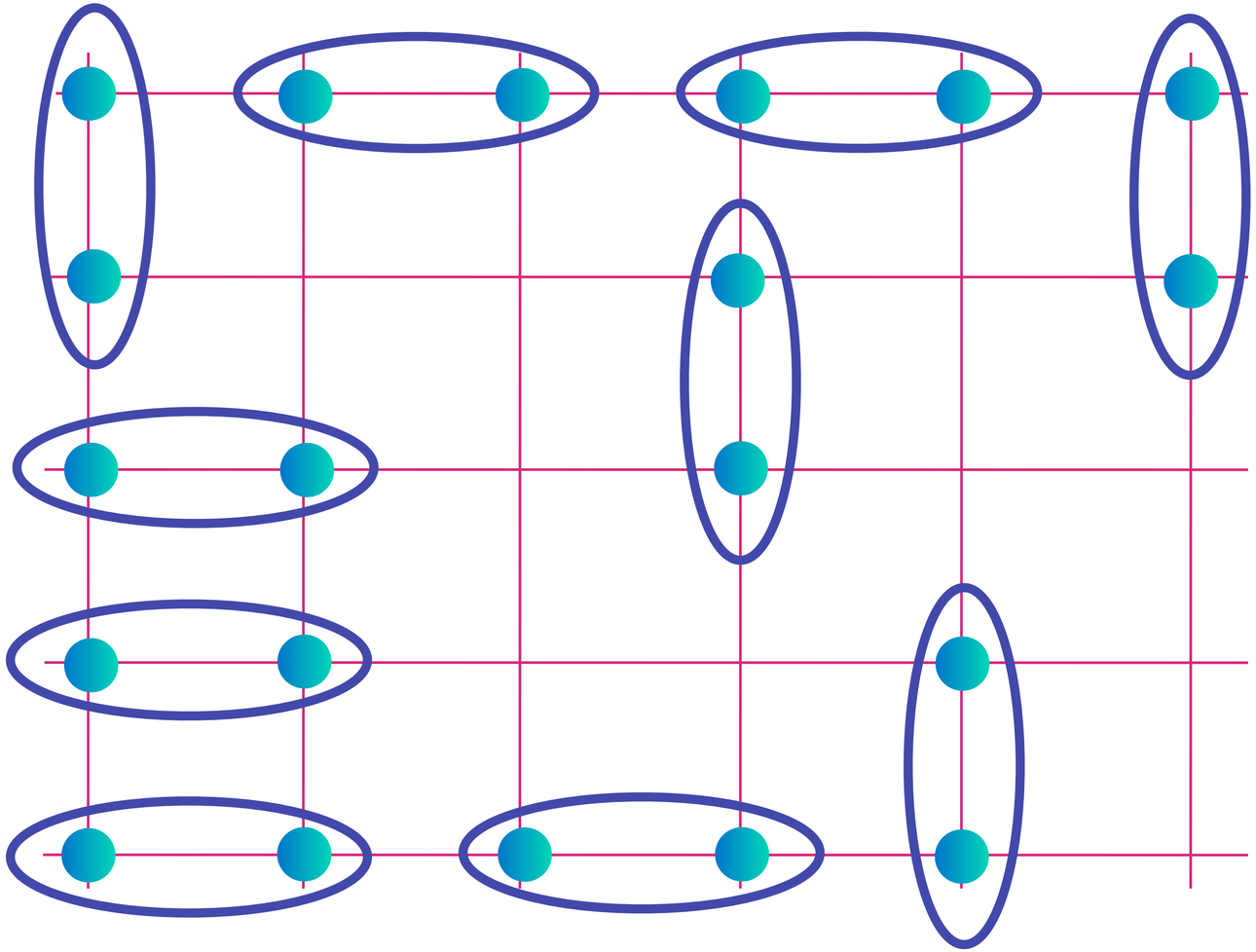

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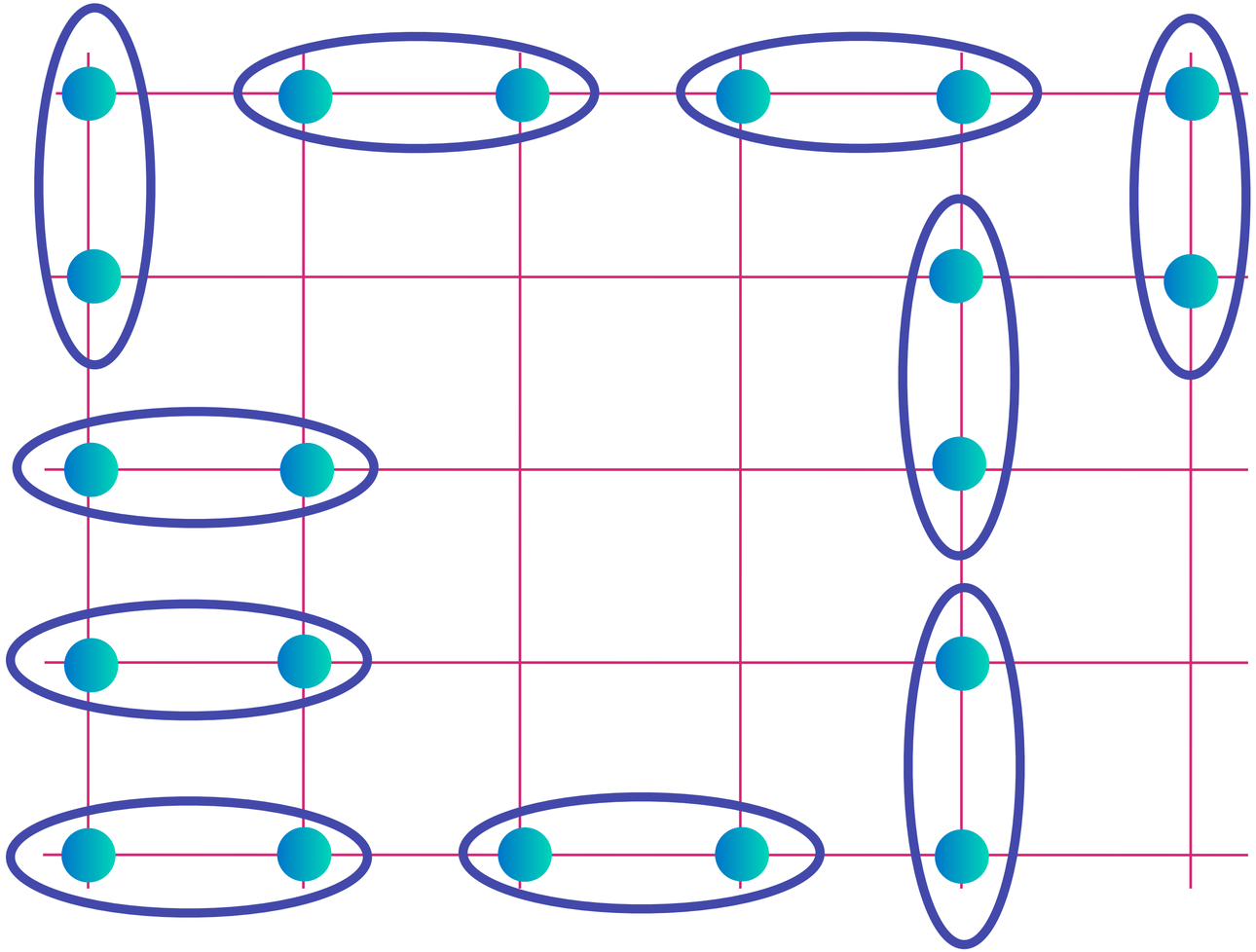

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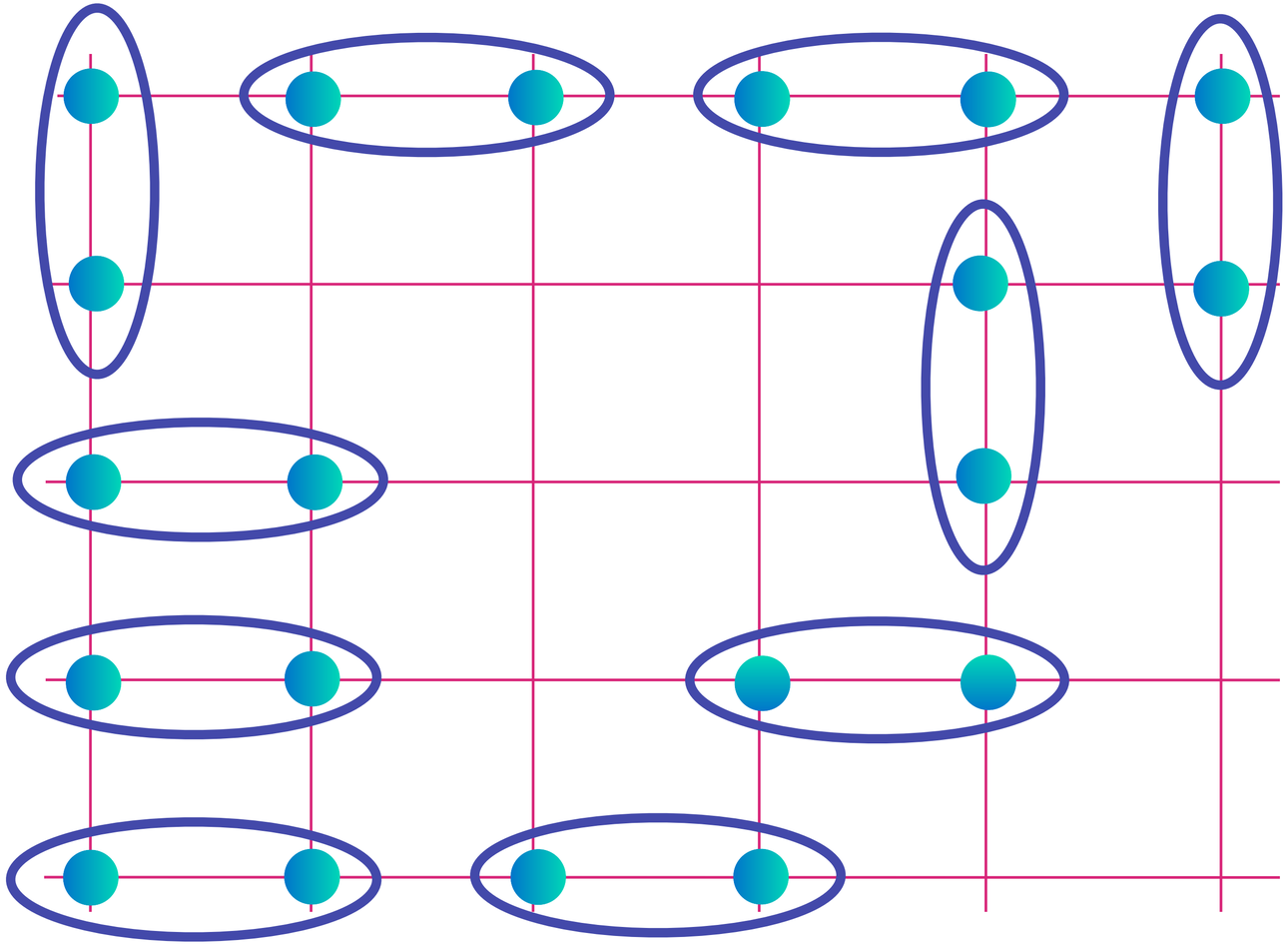

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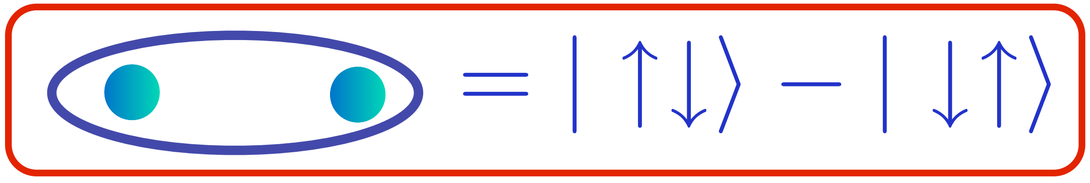
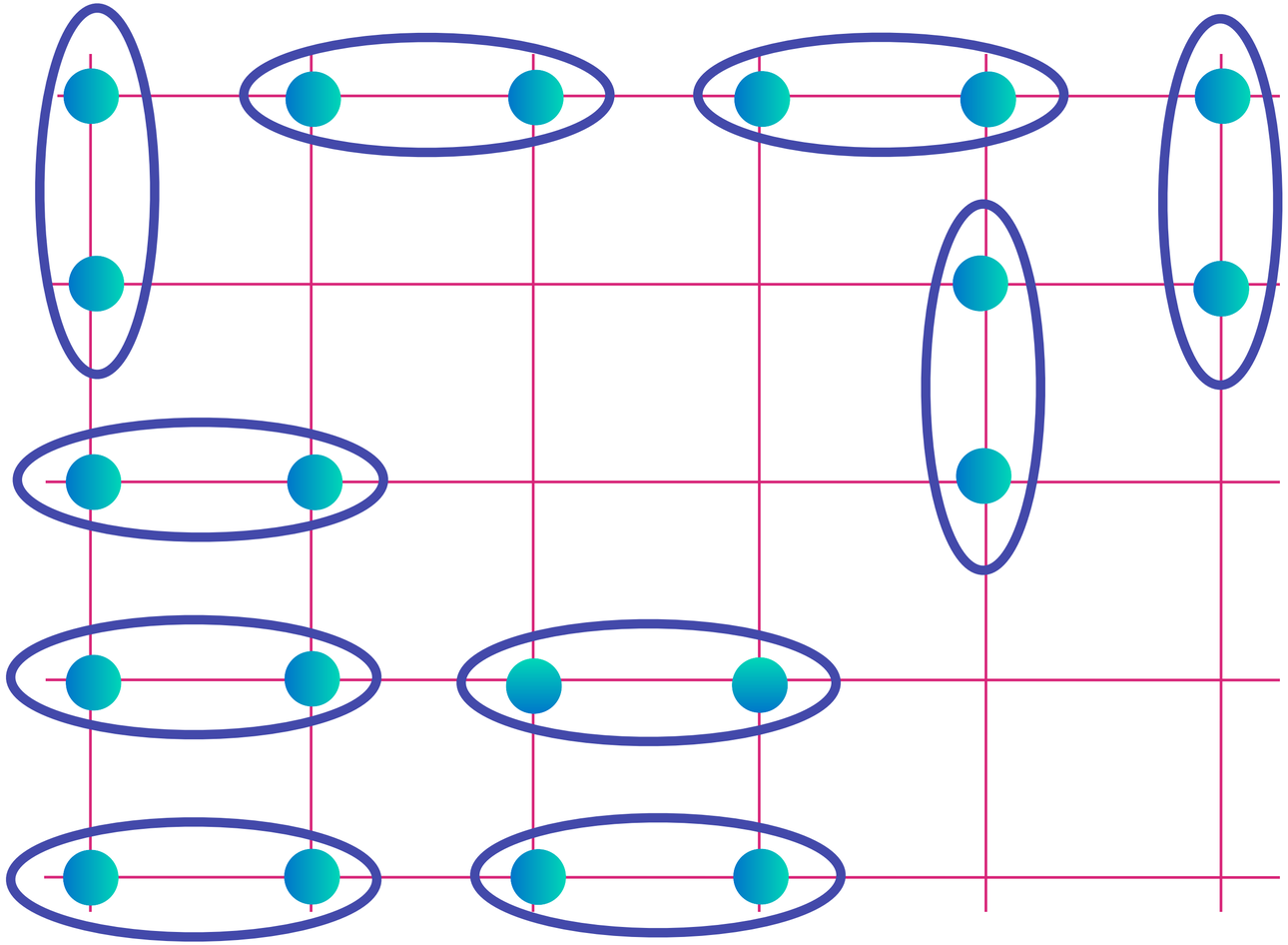


Antiferromagnet doped with hole density p



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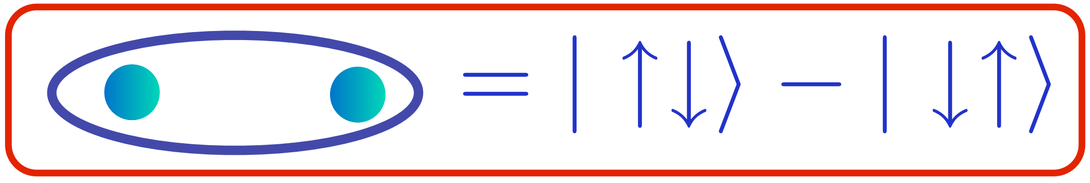

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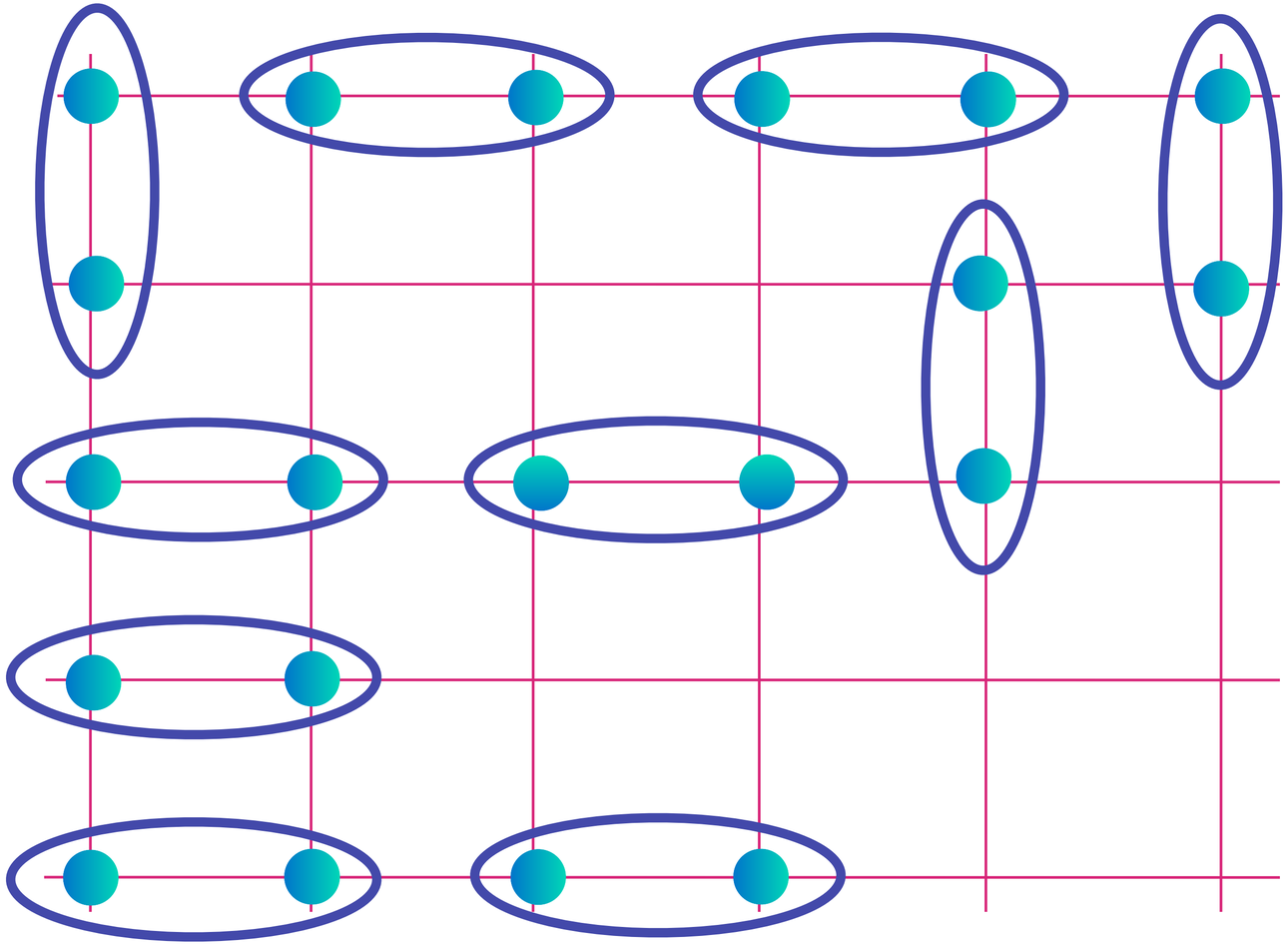

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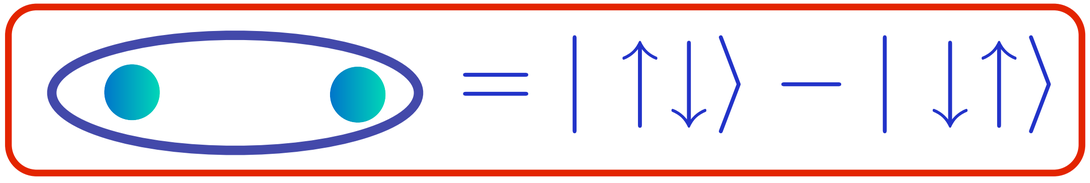

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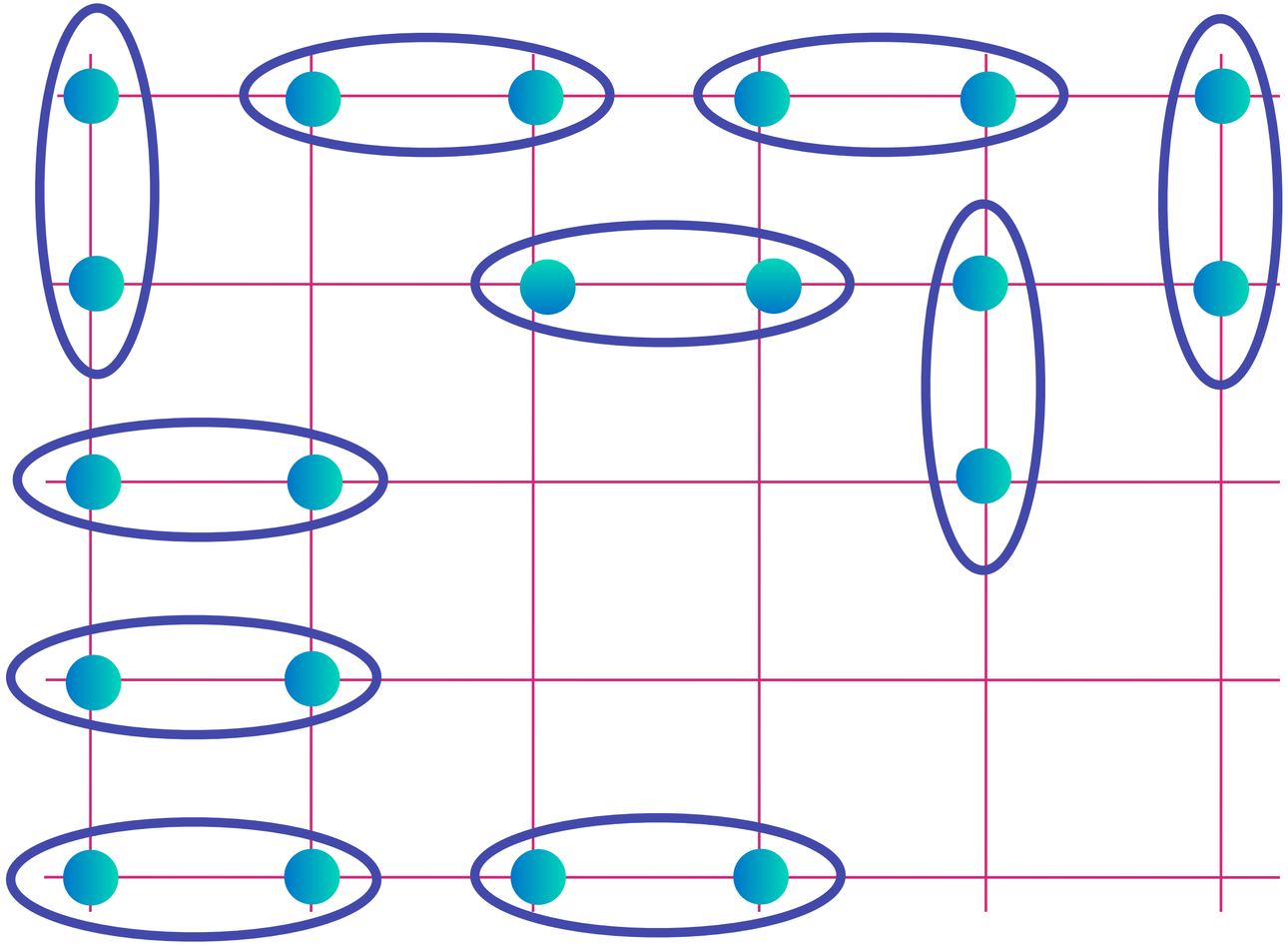

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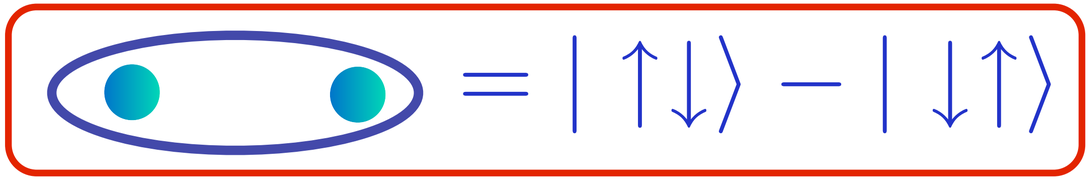

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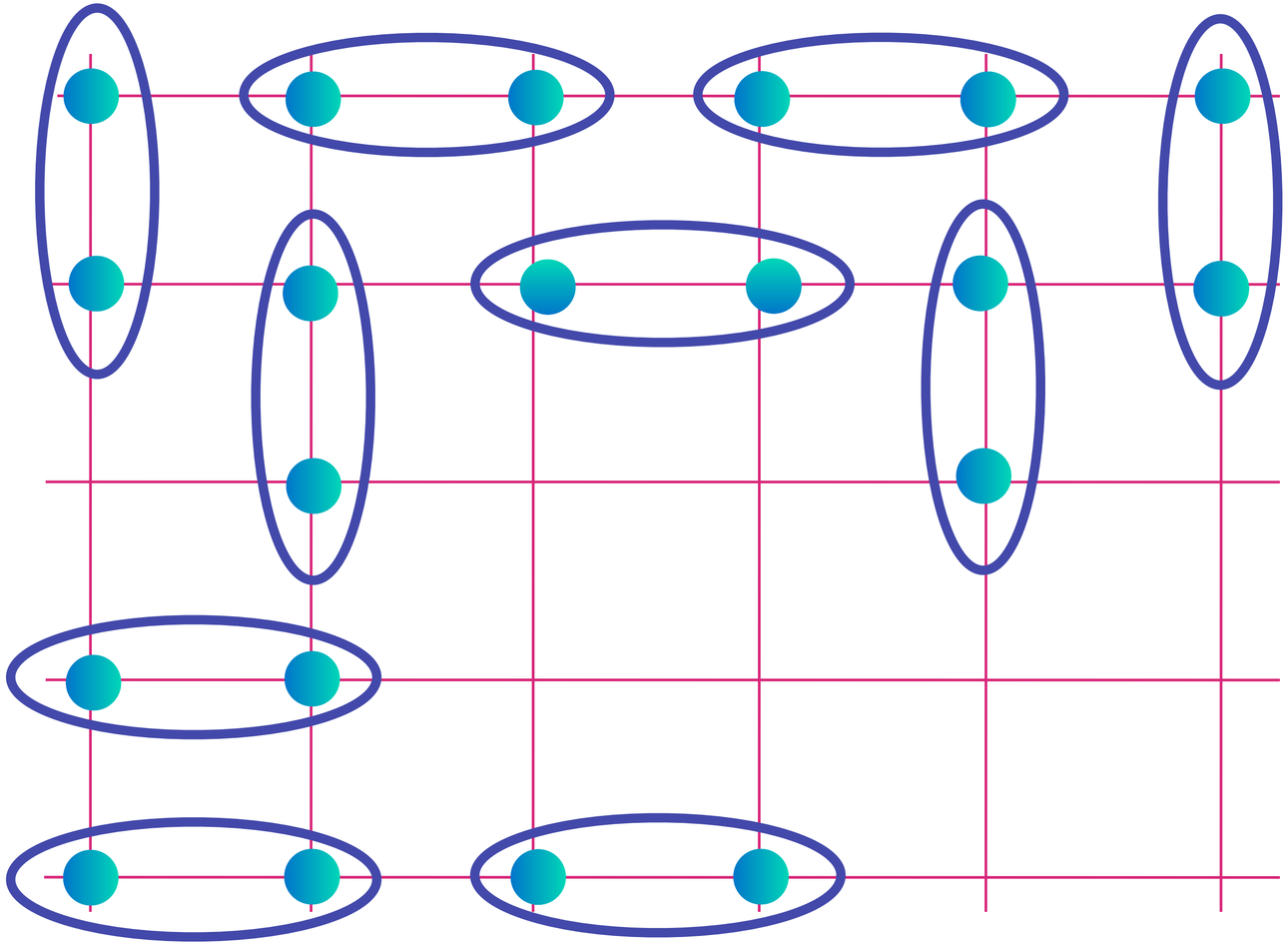

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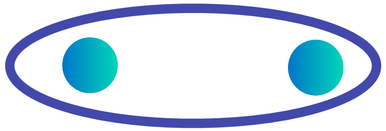

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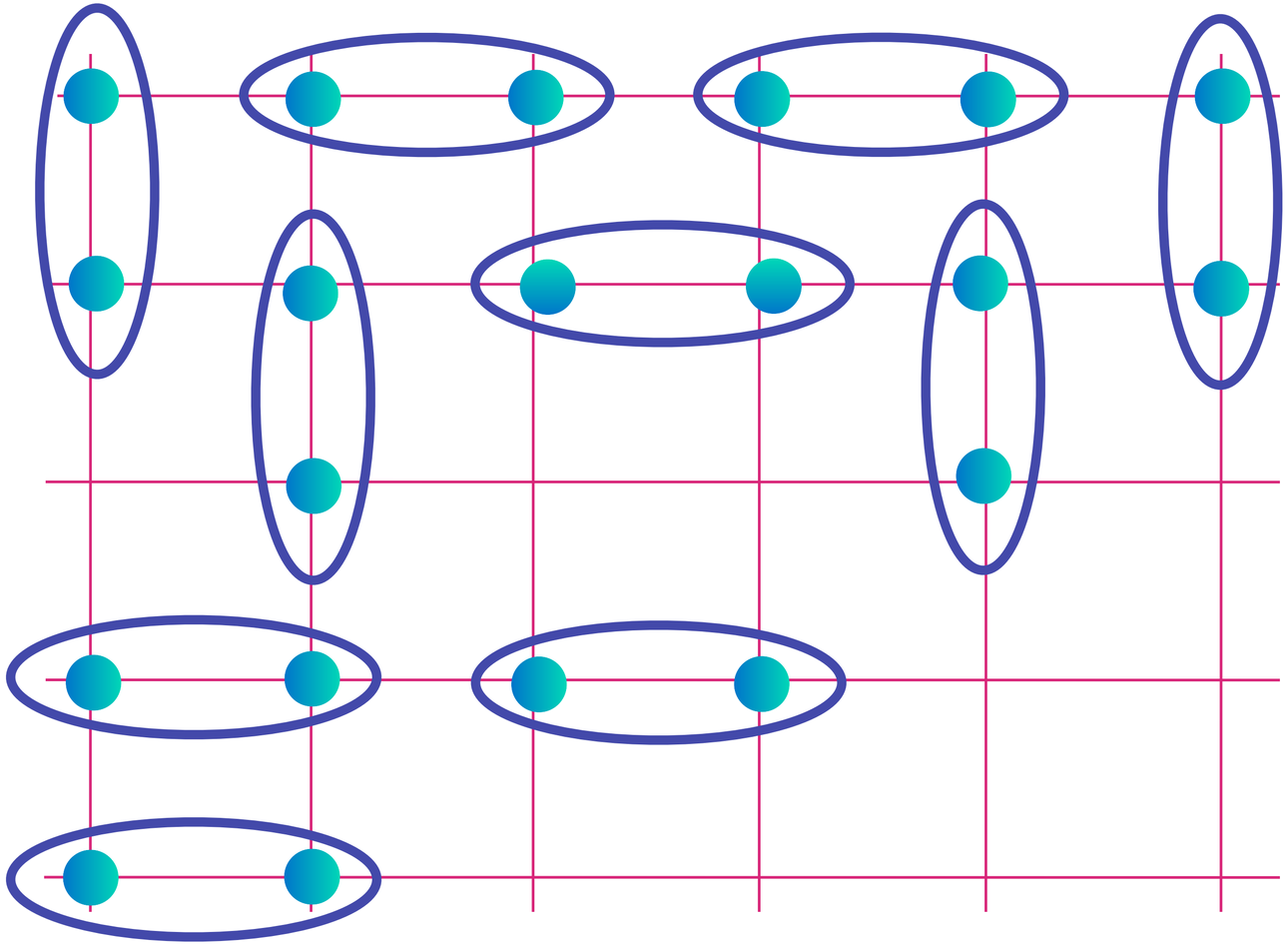

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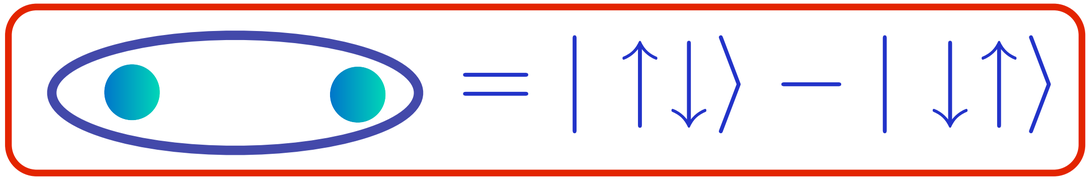

Quantum entanglement of 2,3,4,……∞ electrons: strange metals

The most remarkable new idea in the quantum theory is the

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a physical system can be in a
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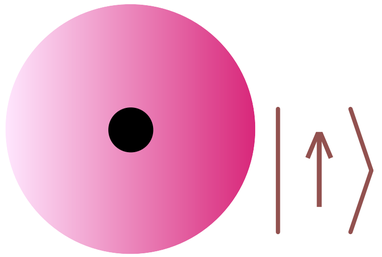
Can we make superpositions of objects
larger than a single electron ?

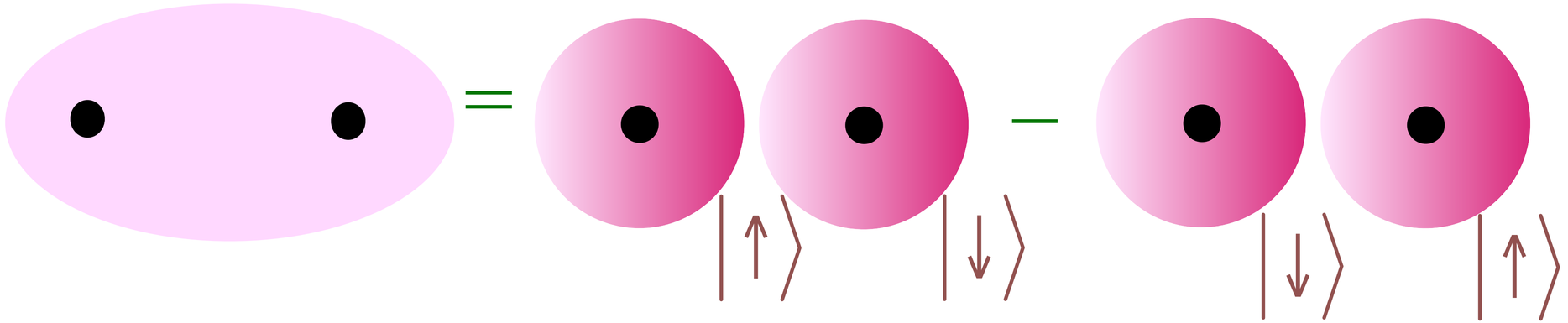
Molecules

Hydrogen atom:



Covalent bond

Hydrogen molecule:



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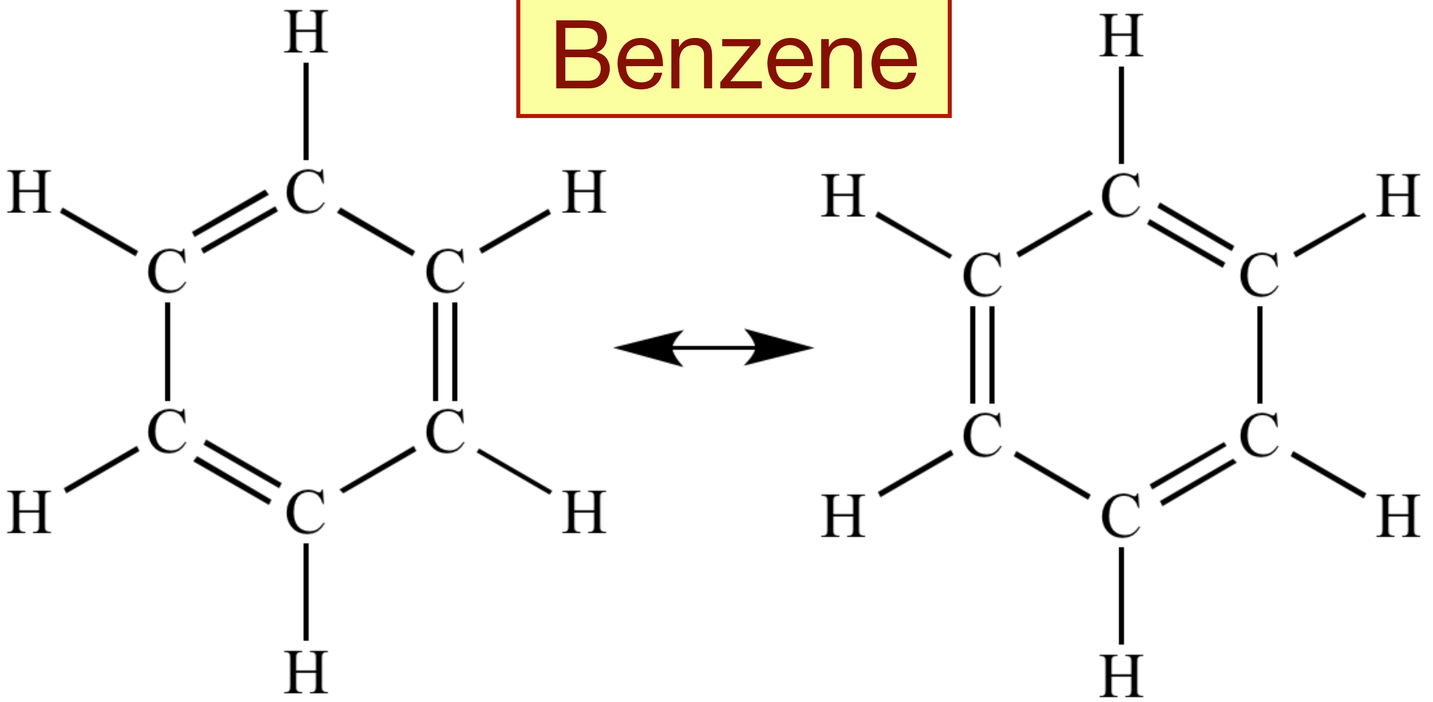
Benzene

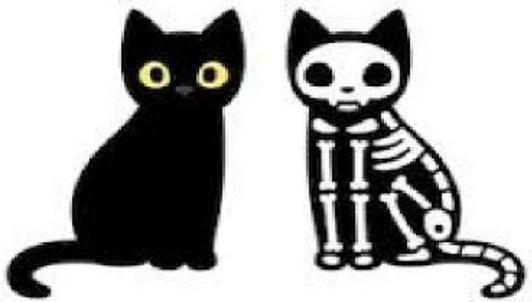


"Resonating" covalent bonds

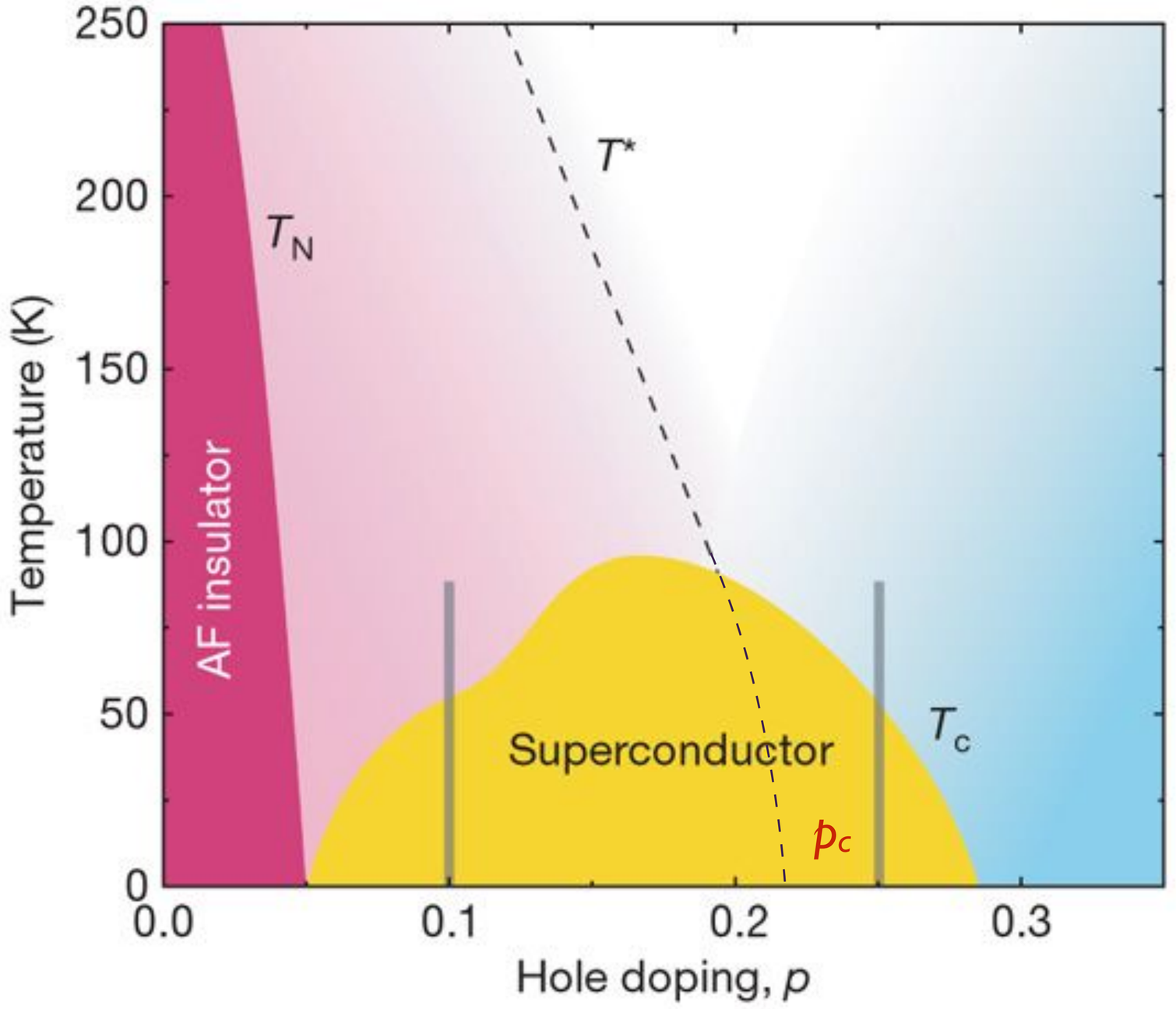
Benzene has a superposition of *covalent bonds*,
each of which is a superposition of a pair of electrons!

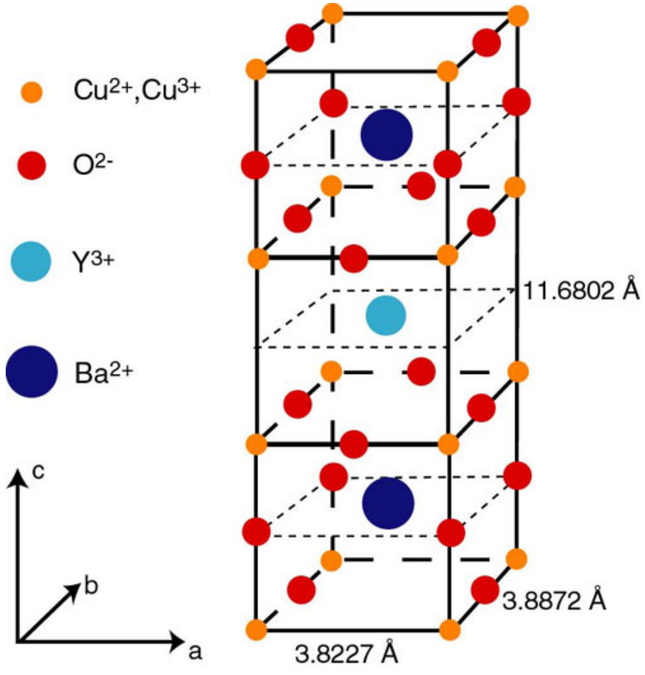
The most remarkable new idea in the quantum theory is the *principle of superposition*:
a physical system can be in a
superposition of two (or more) distinct states.

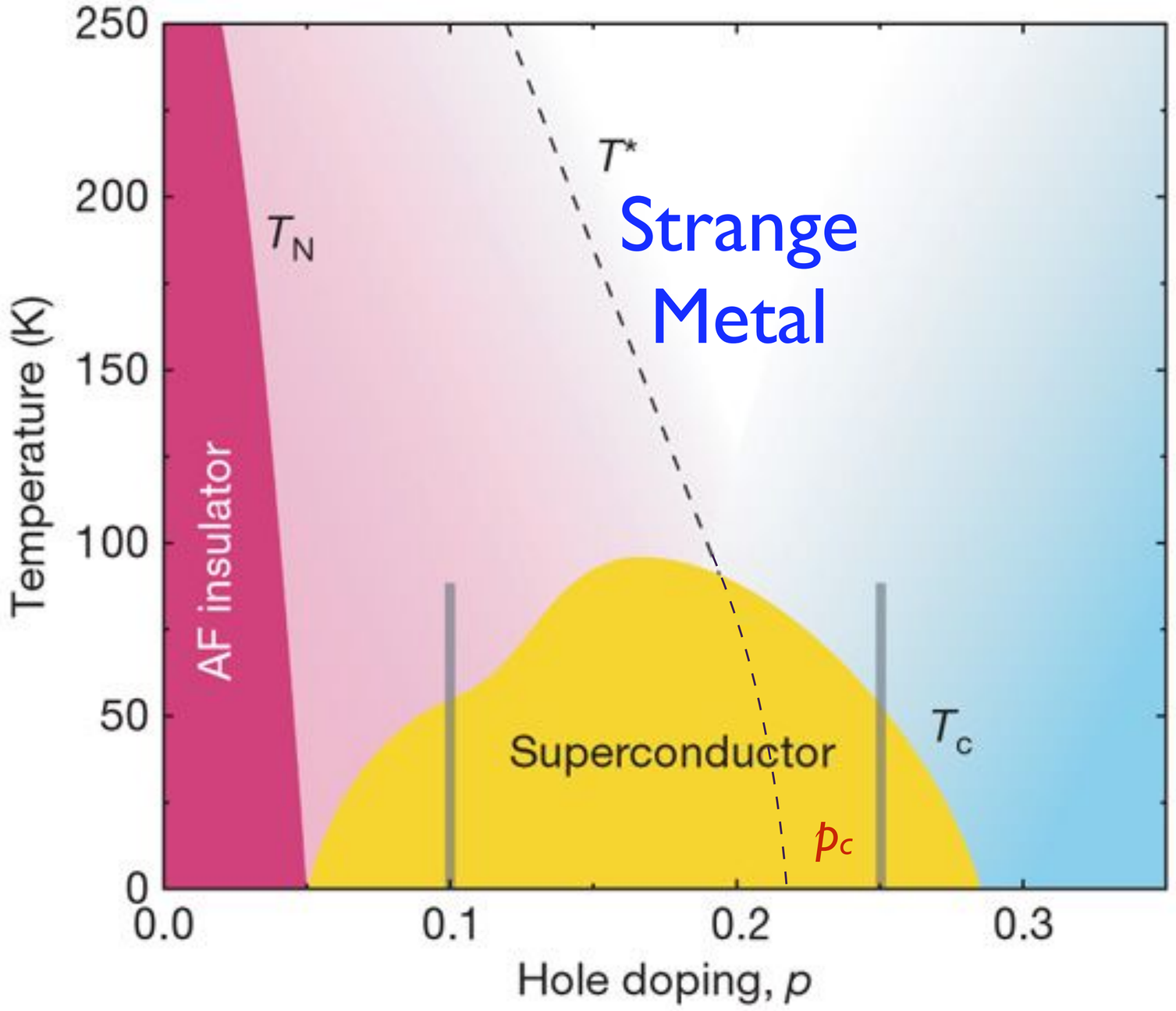
Schrodinger's Cat

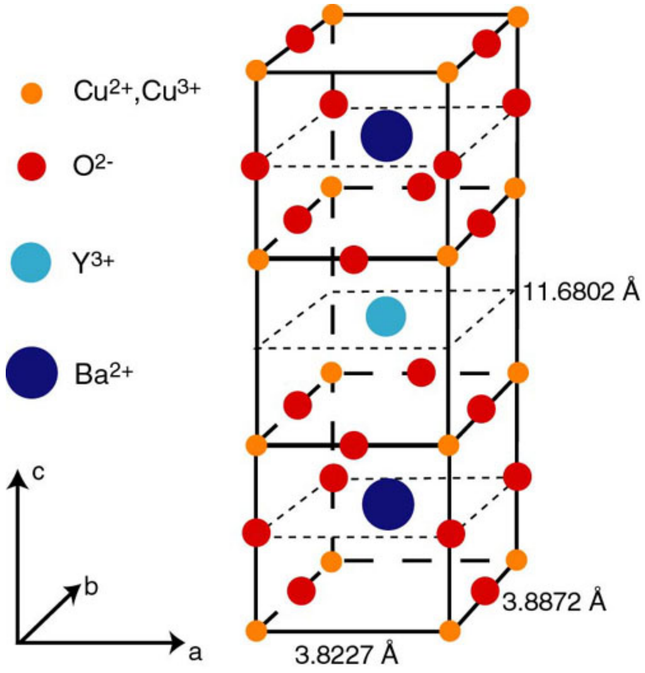


$$\frac{1}{\sqrt{2}}|\text{alive}\rangle + \frac{1}{\sqrt{2}}|\text{dead}\rangle$$

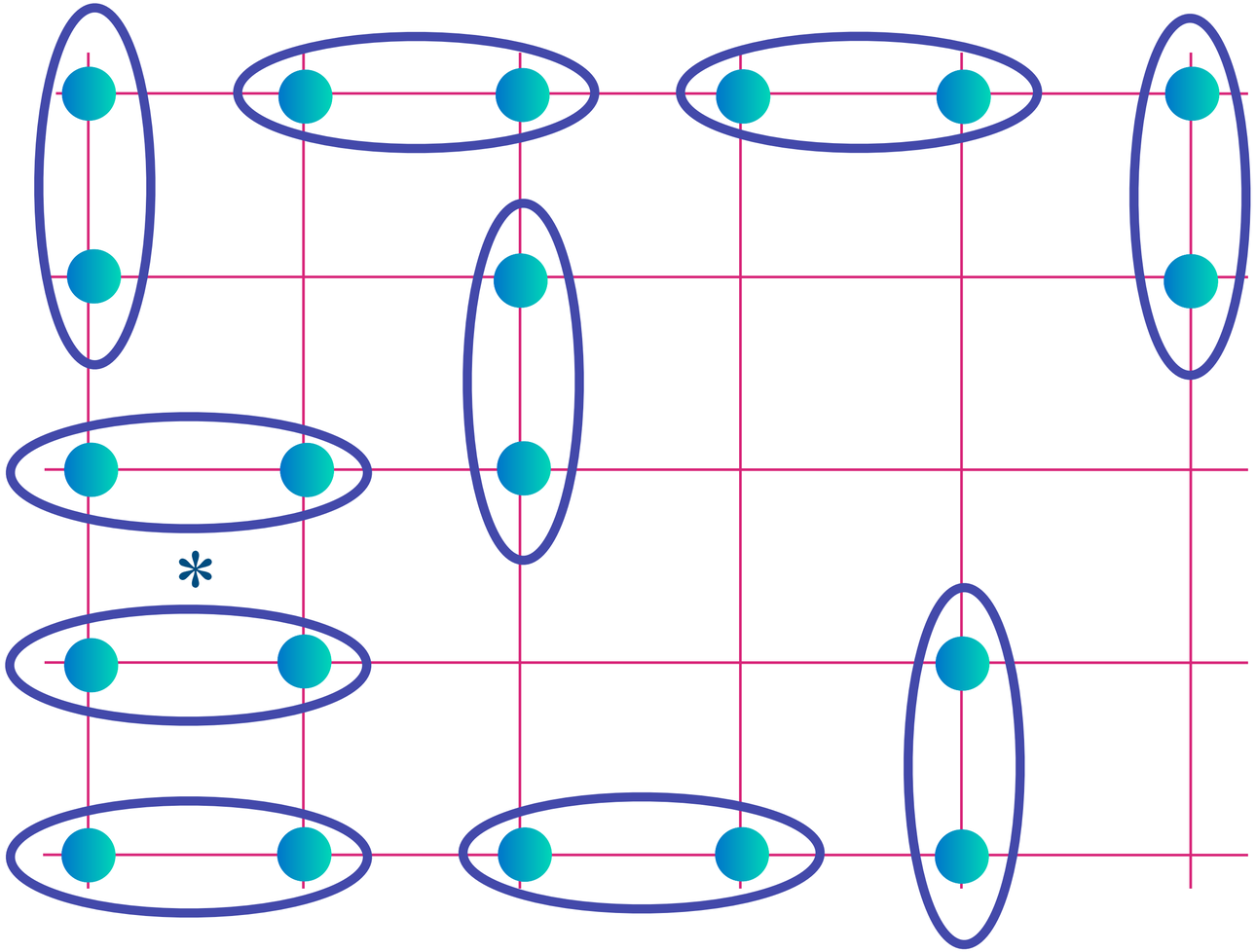

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
T_N
T^*
T_c
AF insulator
Superconductor
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
T_N
T^*
T_c
p_c
AF insulator
Superconductor
Strange
Metal


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

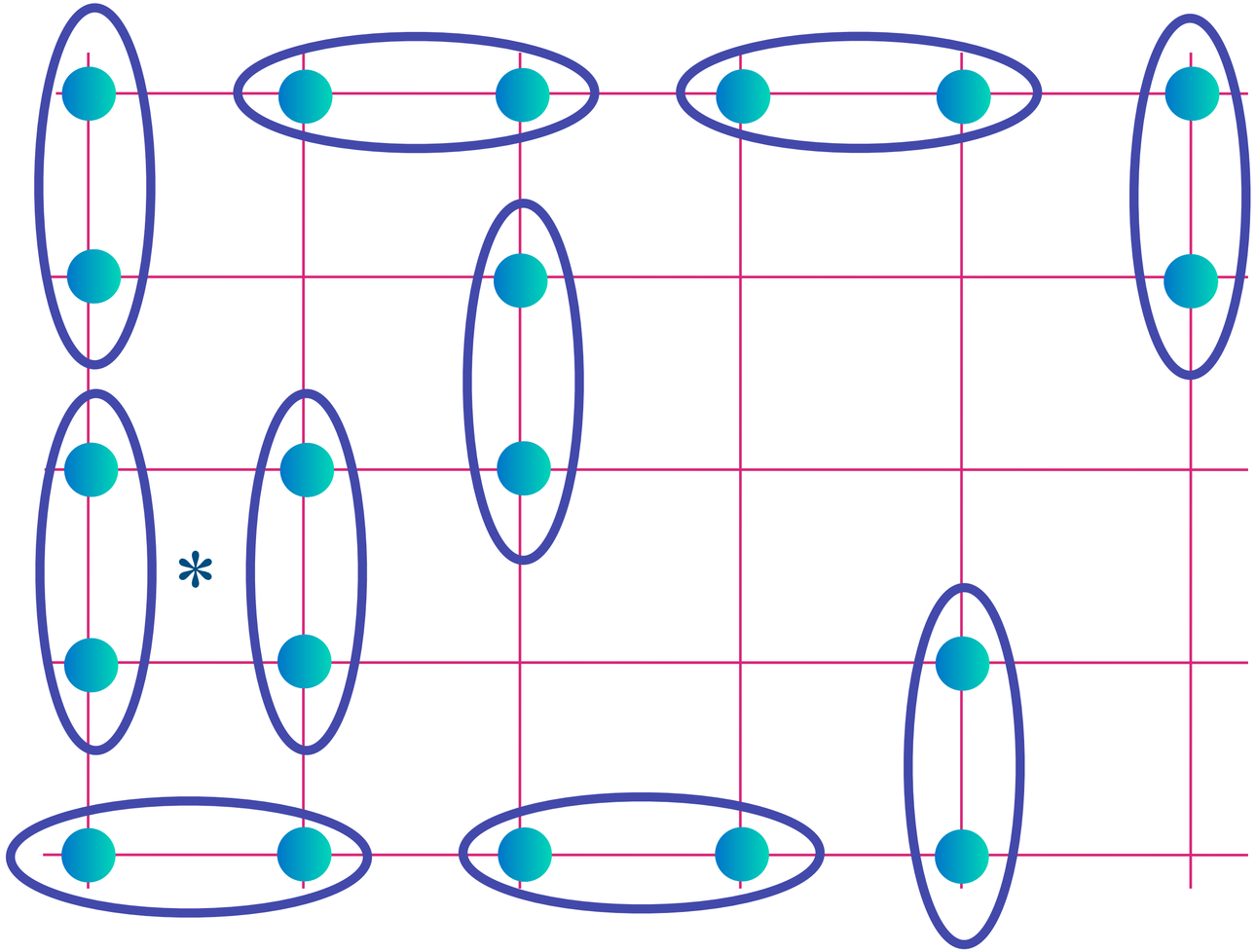
Antiferromagnet doped with hole density p



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Electrons
entangle
"en masse"
by
exchanging
partners,
and there is
long-range
quantum
entanglement

Antiferromagnet doped with hole density p



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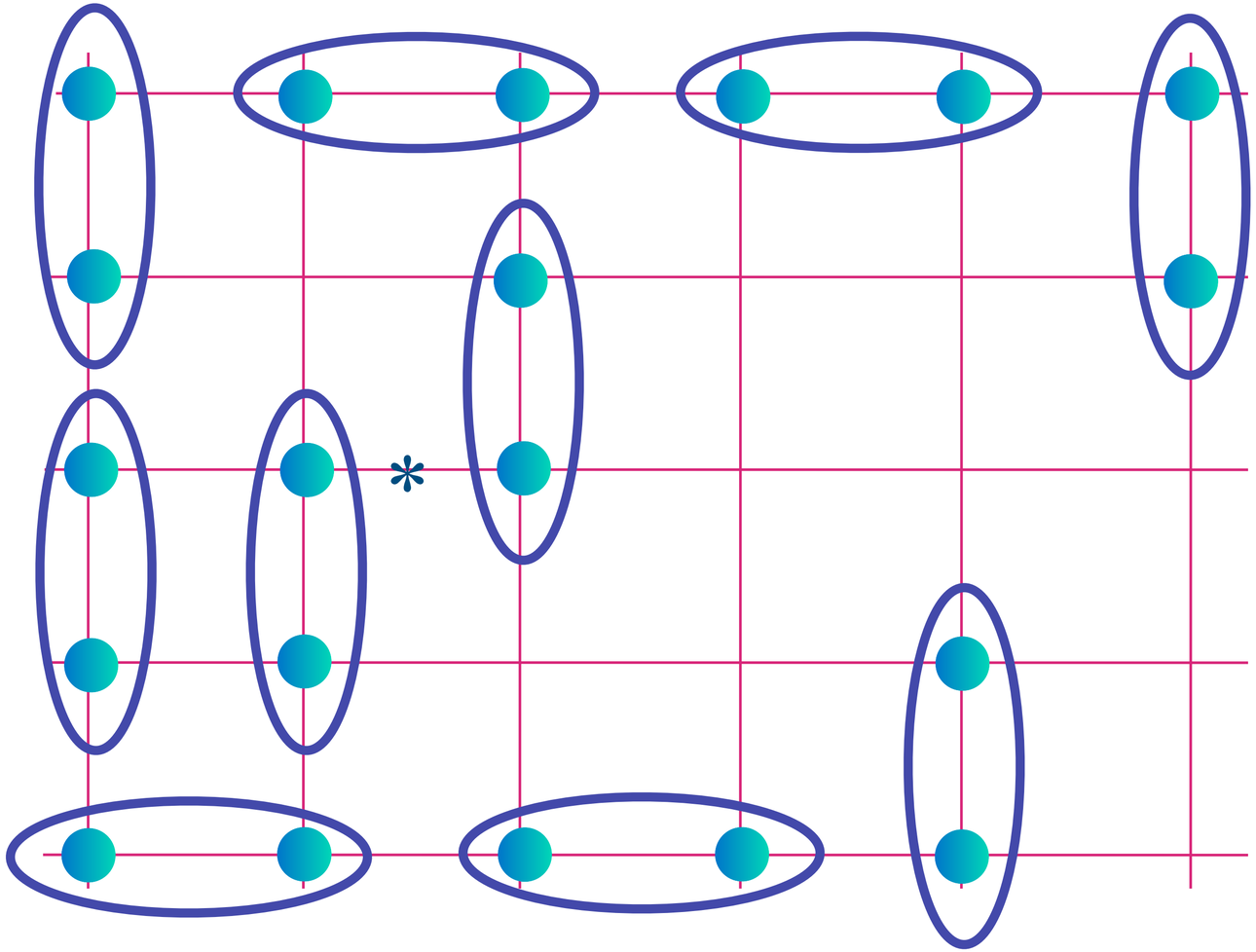
Electrons entangle "en masse" by exchanging partners, and there is long-range quantum entanglement

Antiferromagnet doped with hole density p



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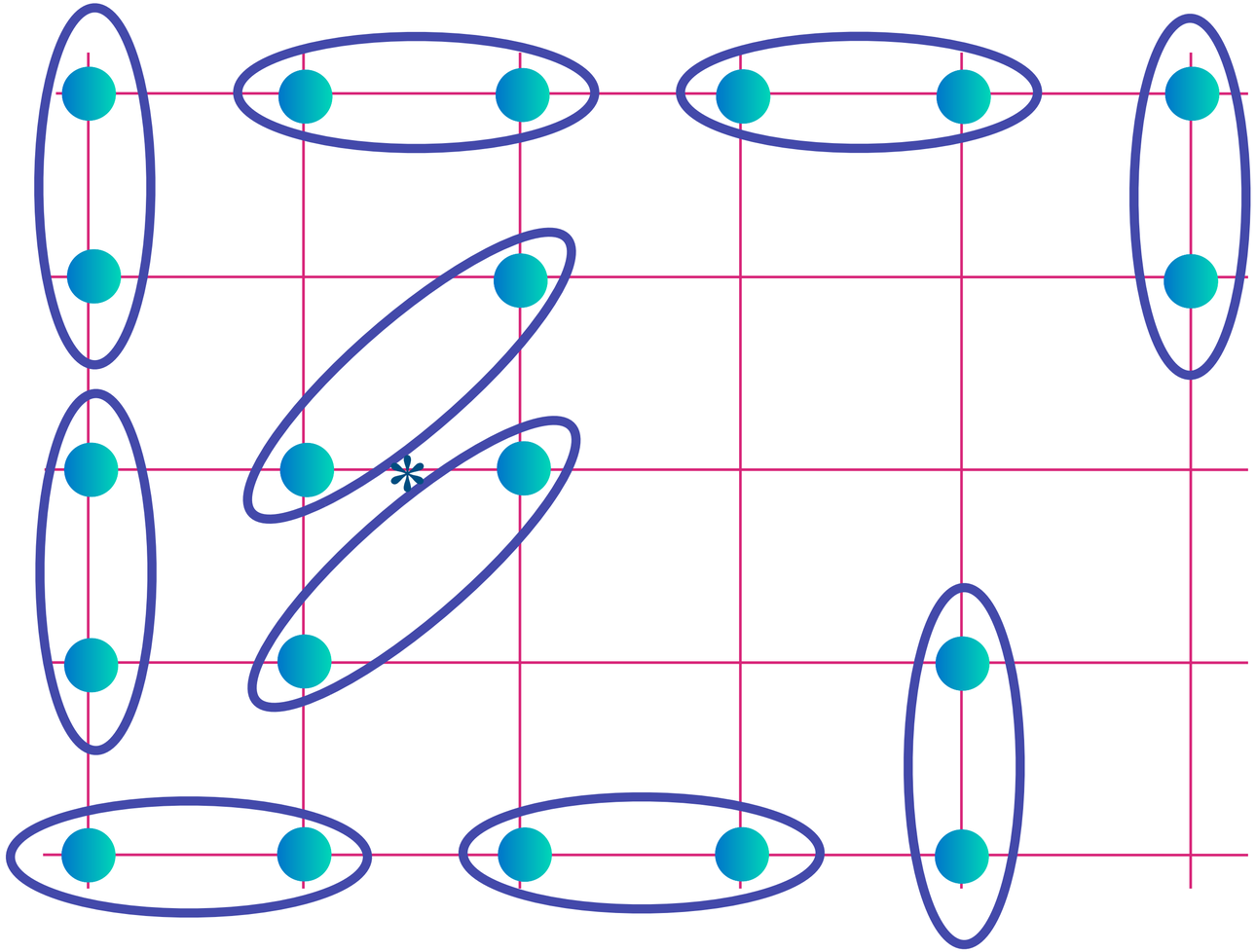
Antiferromagnet doped with hole density p



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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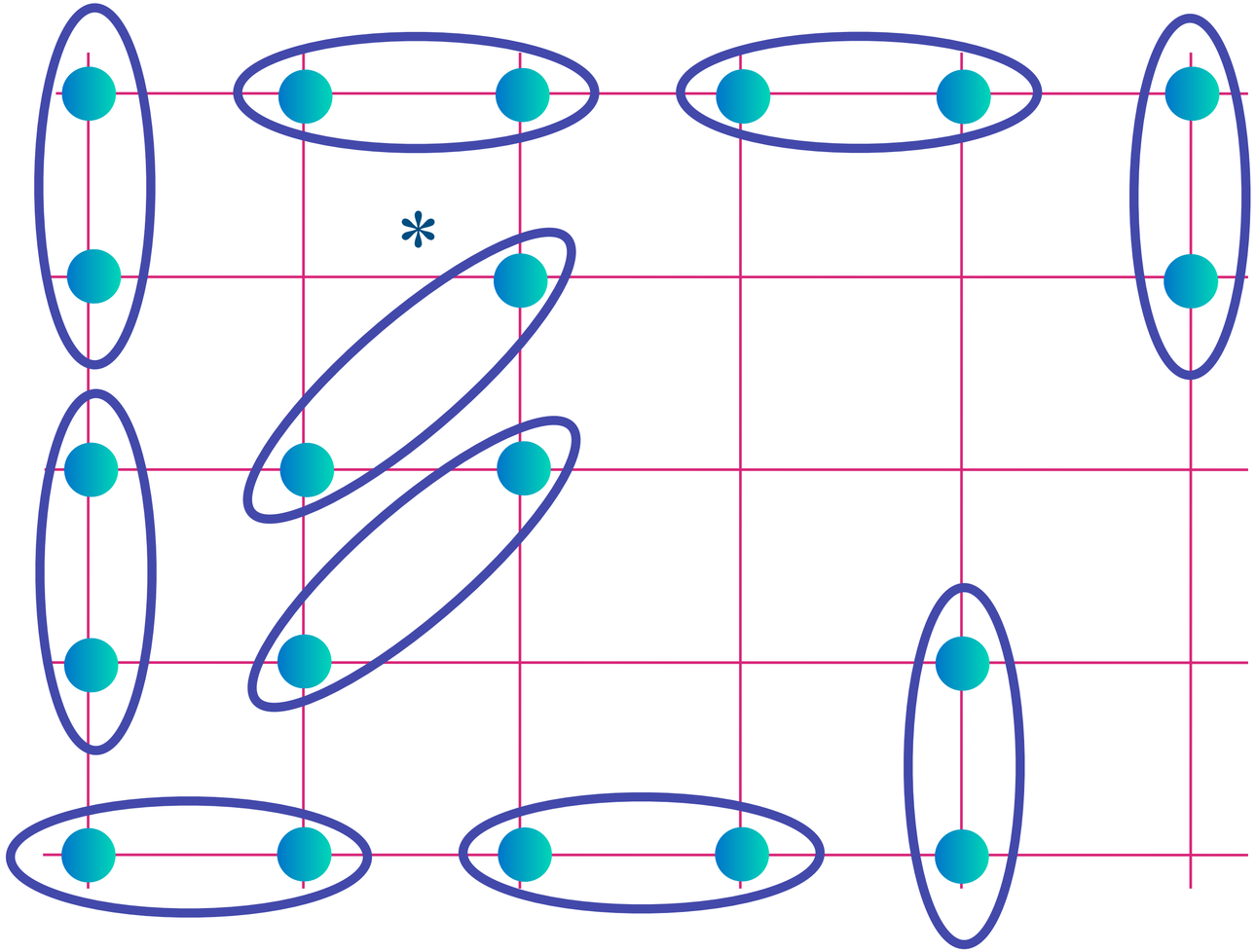
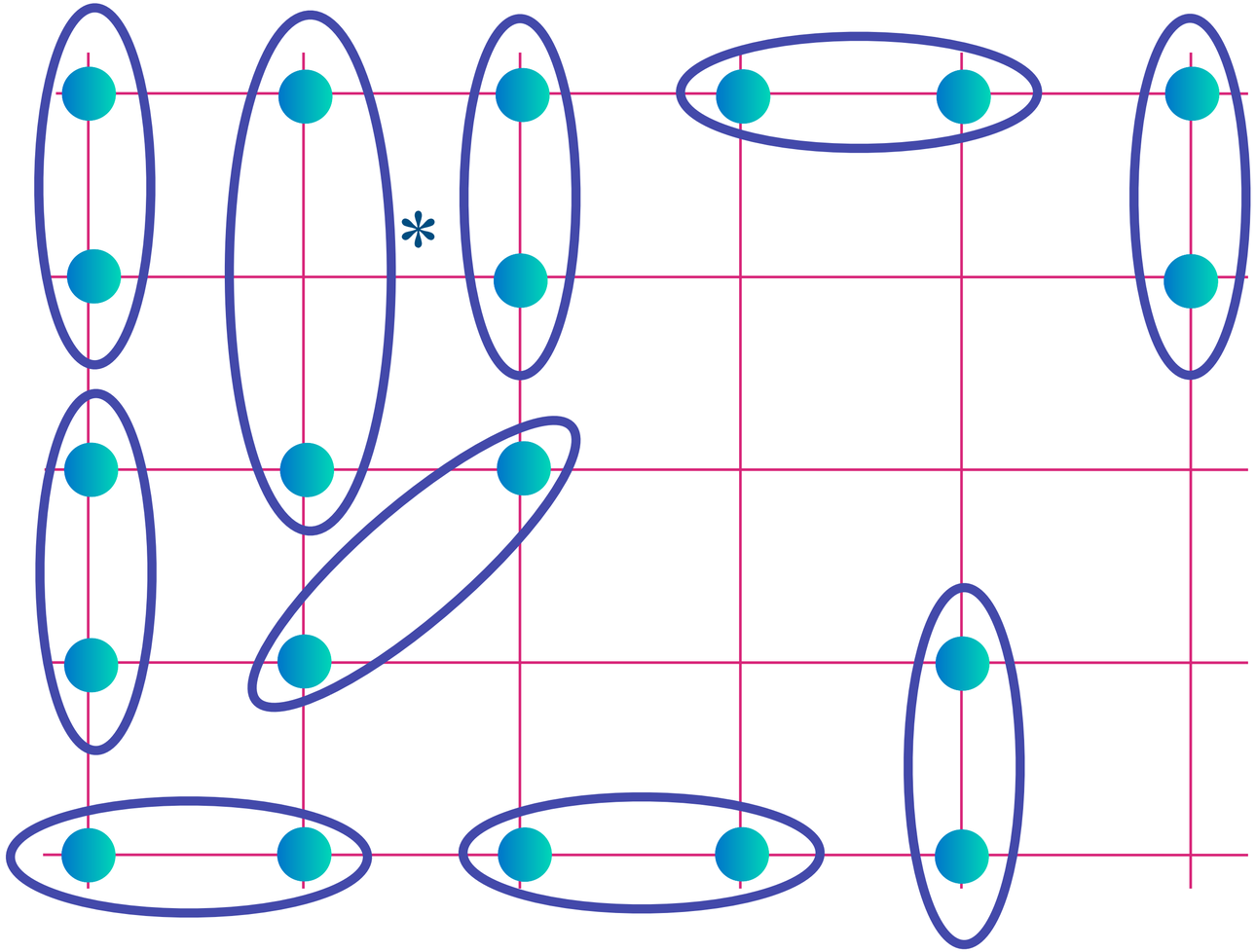
Antiferromagnet doped with hole density p



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Electrons entangle "en masse" by exchanging partners, and there is long-range quantum entanglement

Antiferromagnet doped with hole density p



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Electrons entangle "en masse" by exchanging partners, and there is long-range quantum entanglement

The Sachdev-Ye-Kitaev (SYK) model

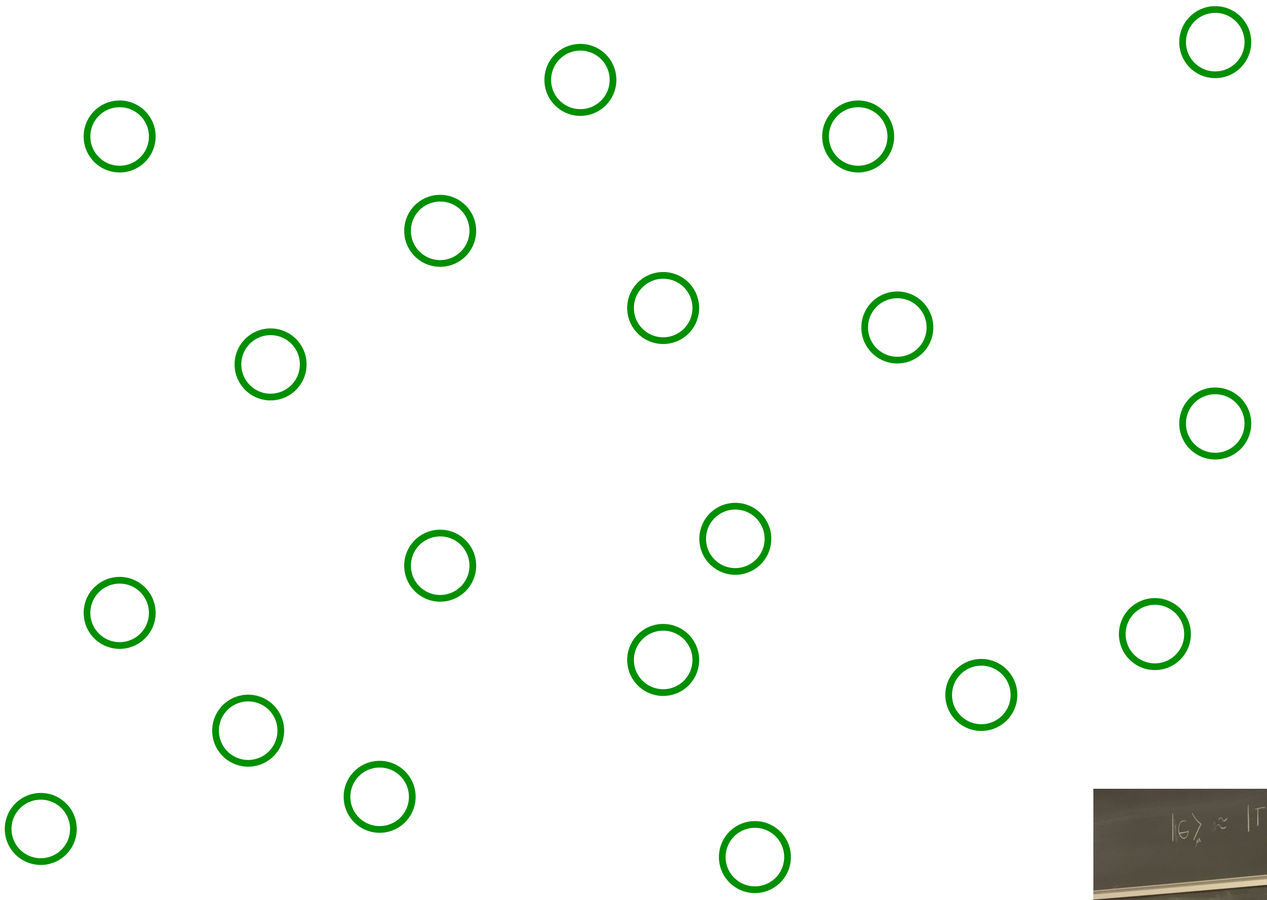
The SYK model has a scale-invariant entanglement structure:
i.e. electrons are entangled at all distance and time scales

It describes
certain ***strange metals***

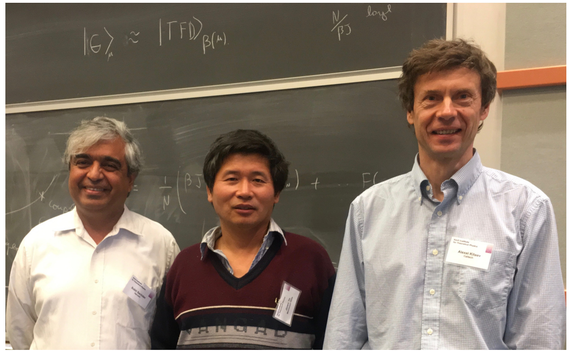
Sachdev, Ye (1993)

The Sachdev-Ye-Kitaev (SYK) model

Sachdev, Ye (1993); Kitaev (2015)

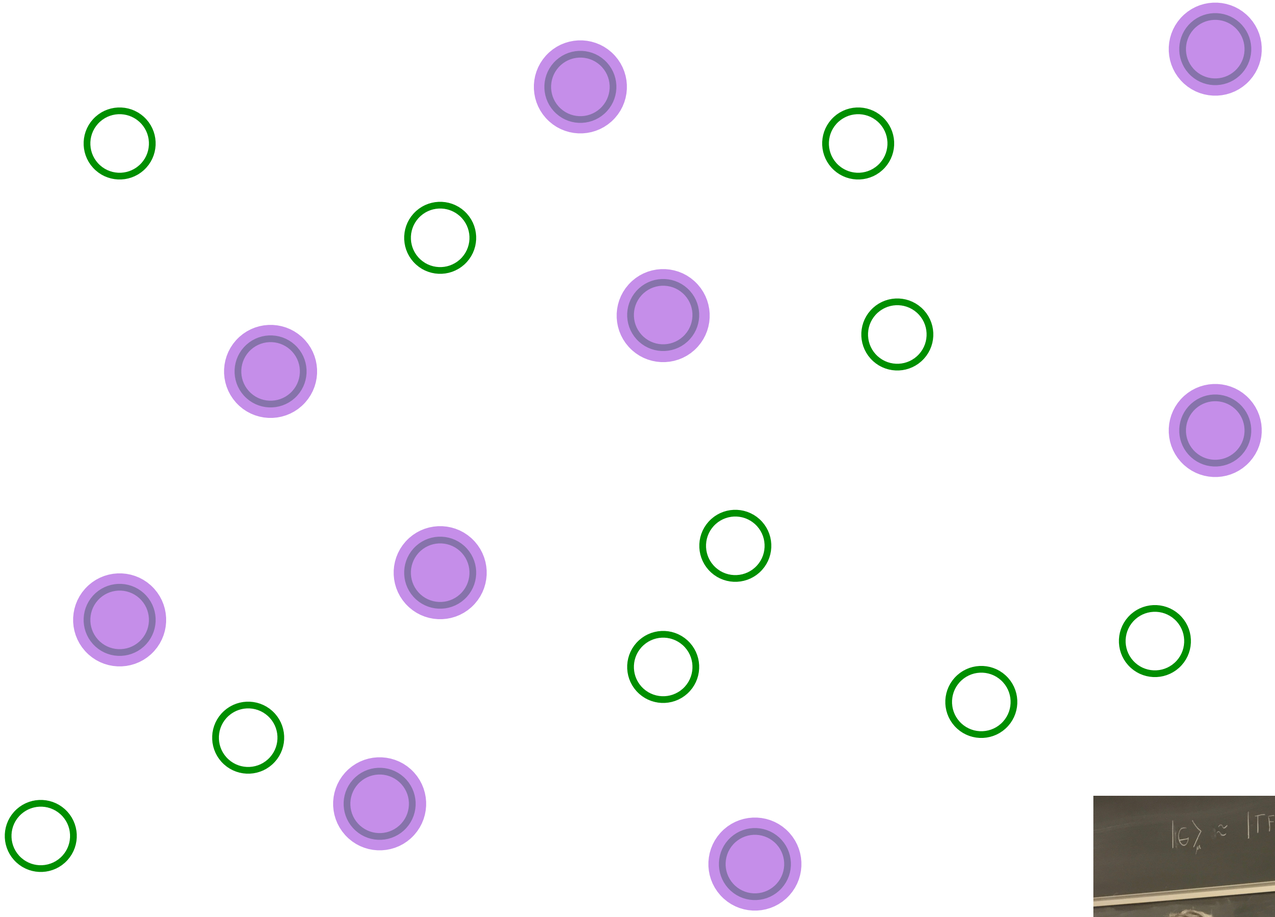


Pick a set of random positions

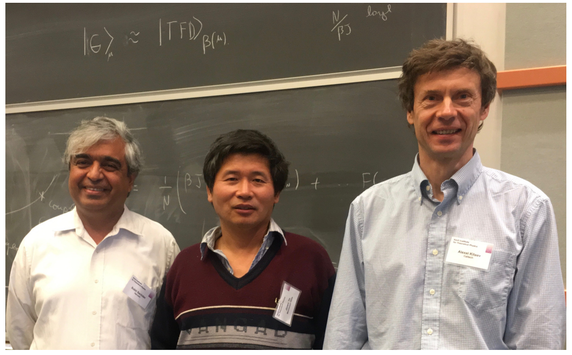


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

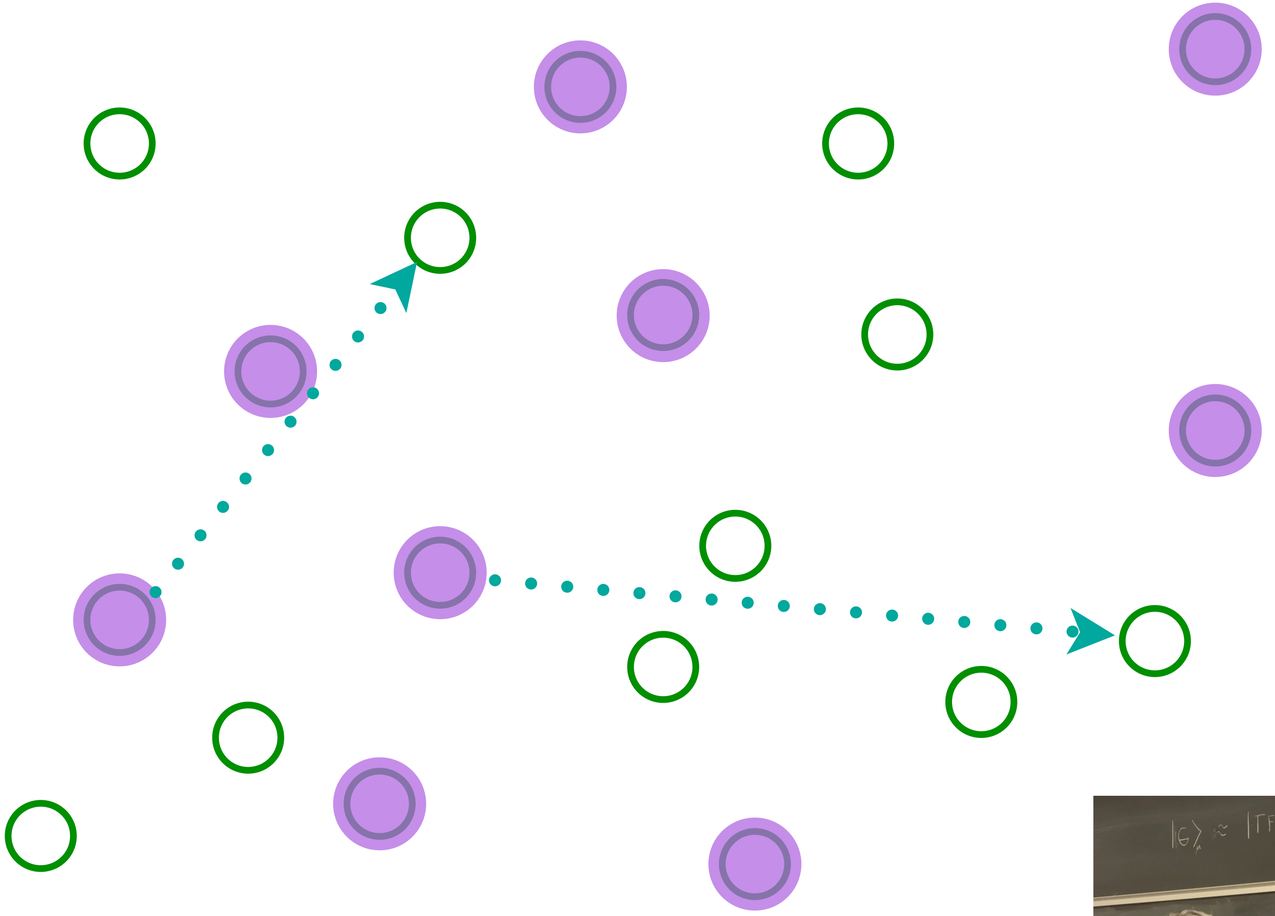


Place electrons randomly on some sites



The SYK model

Sachdev, Ye (1993); Kitaev (2015)



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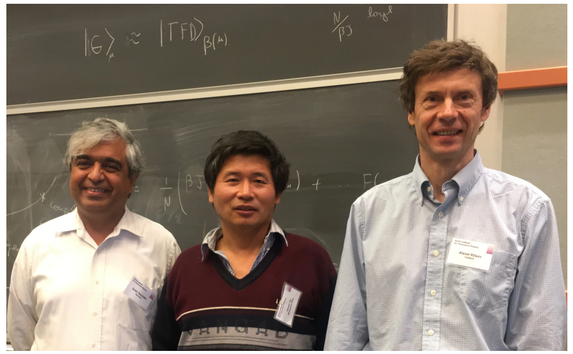
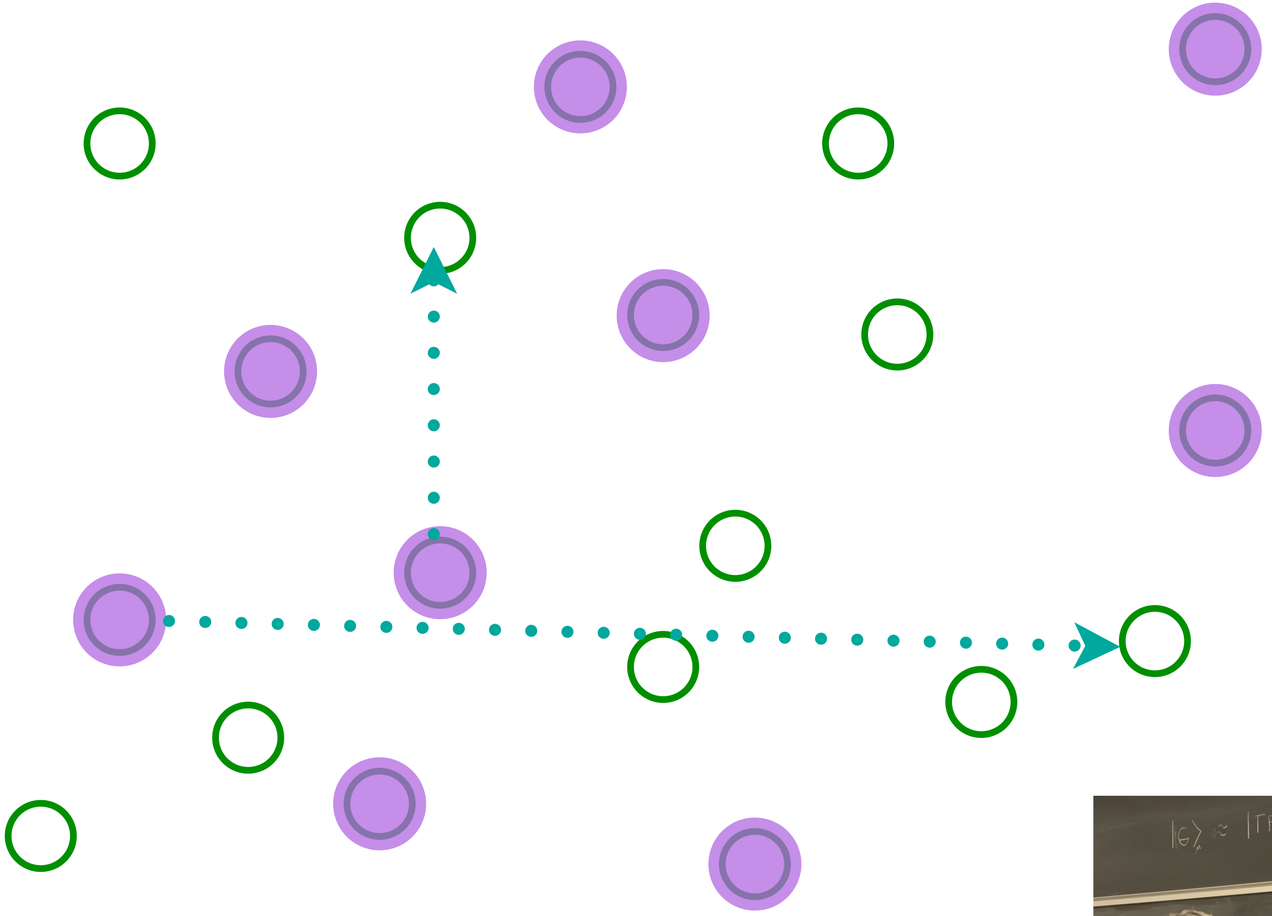
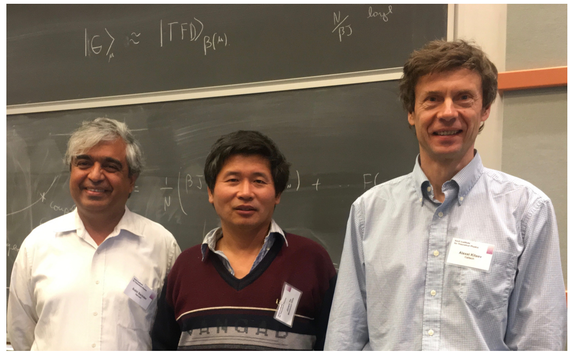


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

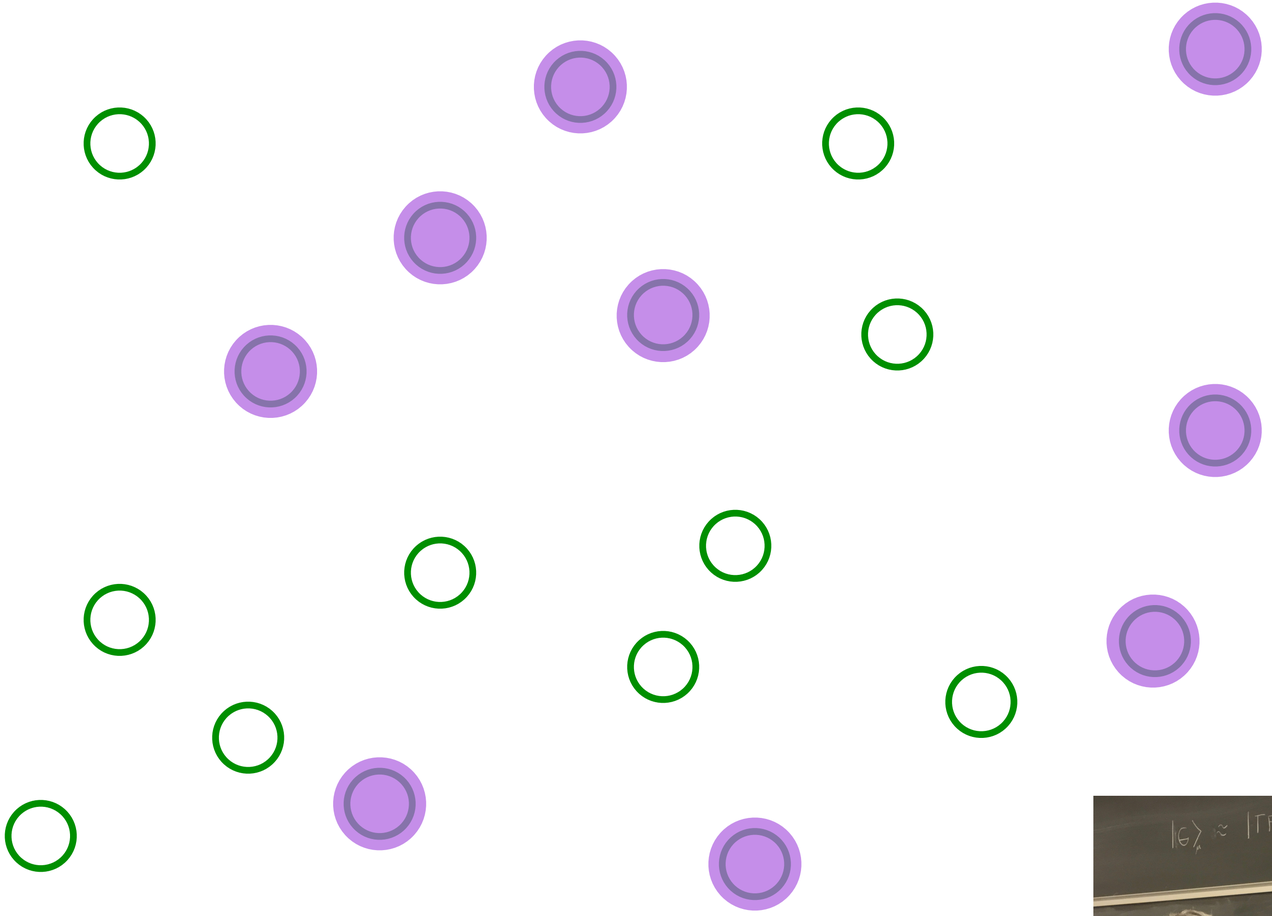


Place electrons randomly on some sites

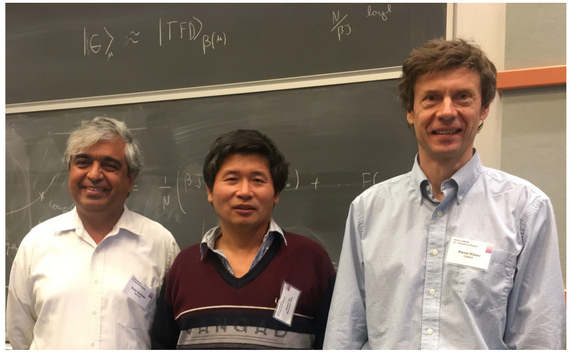


The SYK model

Sachdev, Ye (1993); Kitaev (2015)

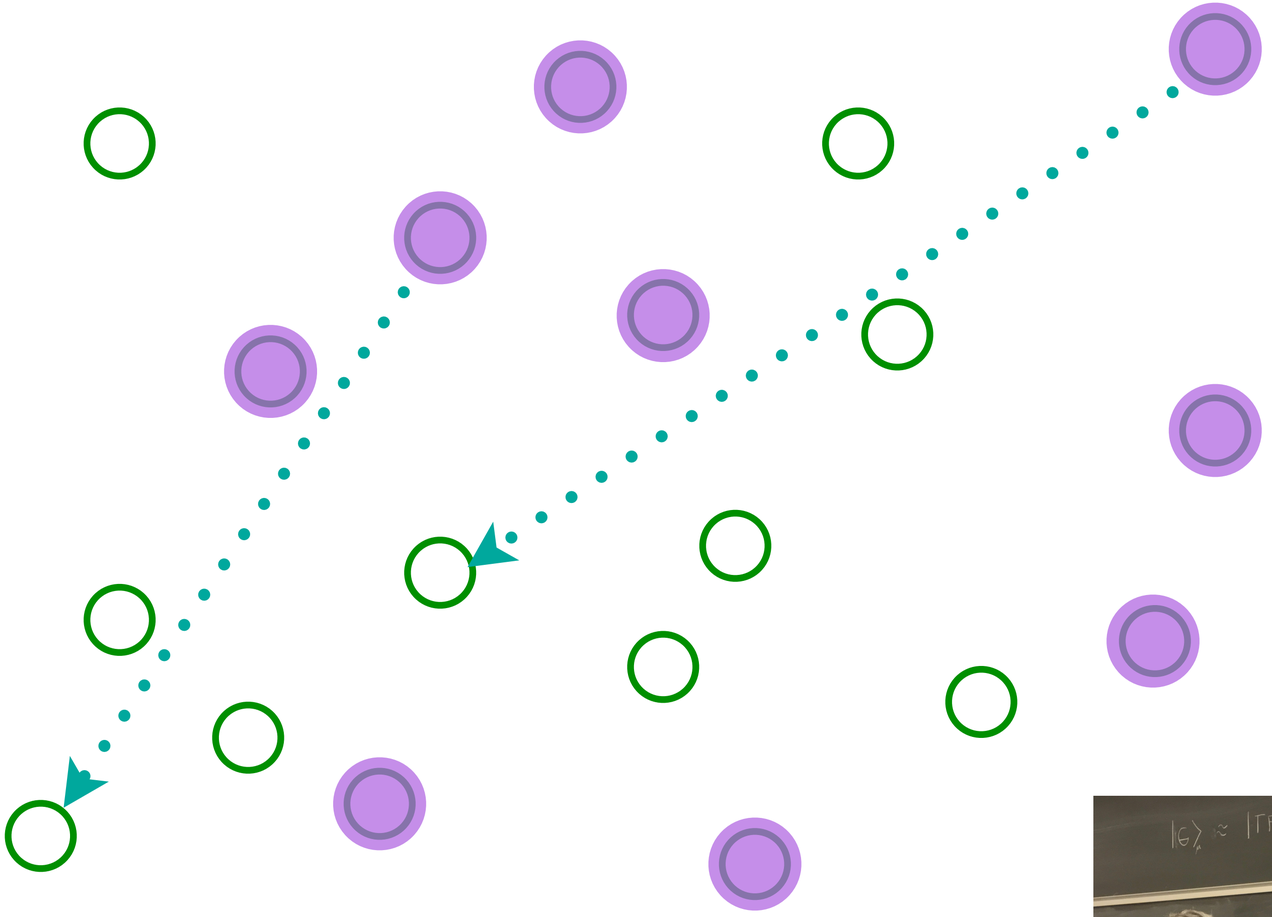


Entangle electrons pairwise randomly



The SYK model

Sachdev, Ye (1993); Kitaev (2015)



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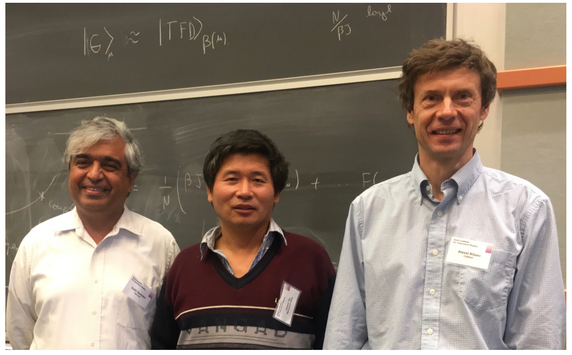
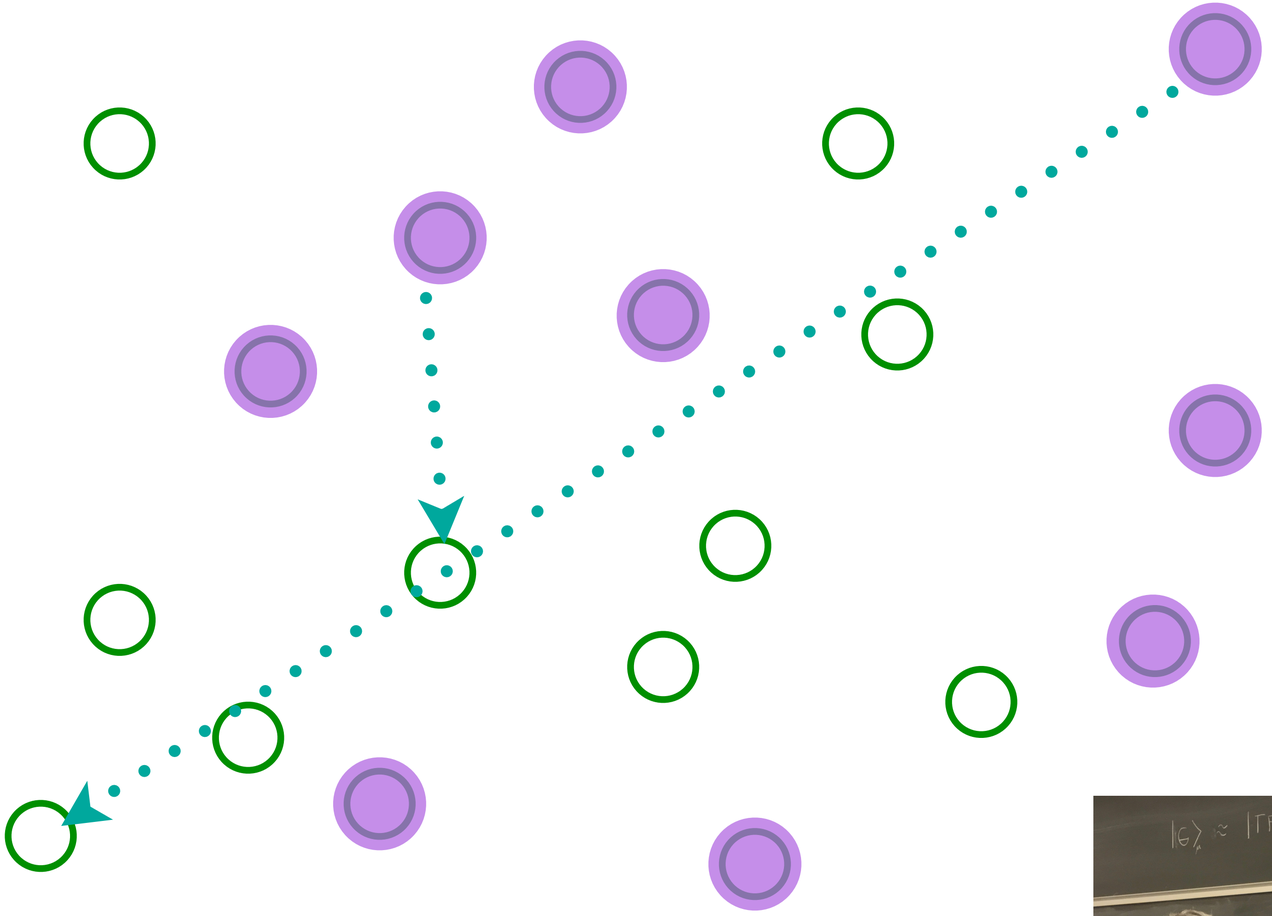


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



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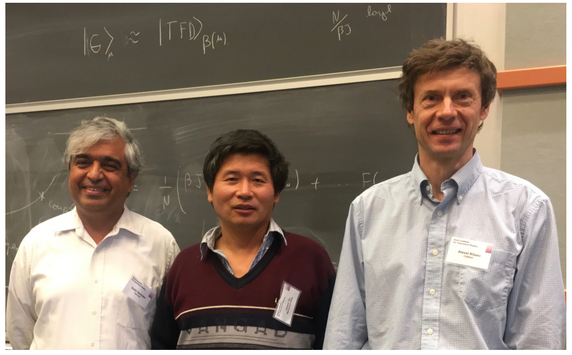

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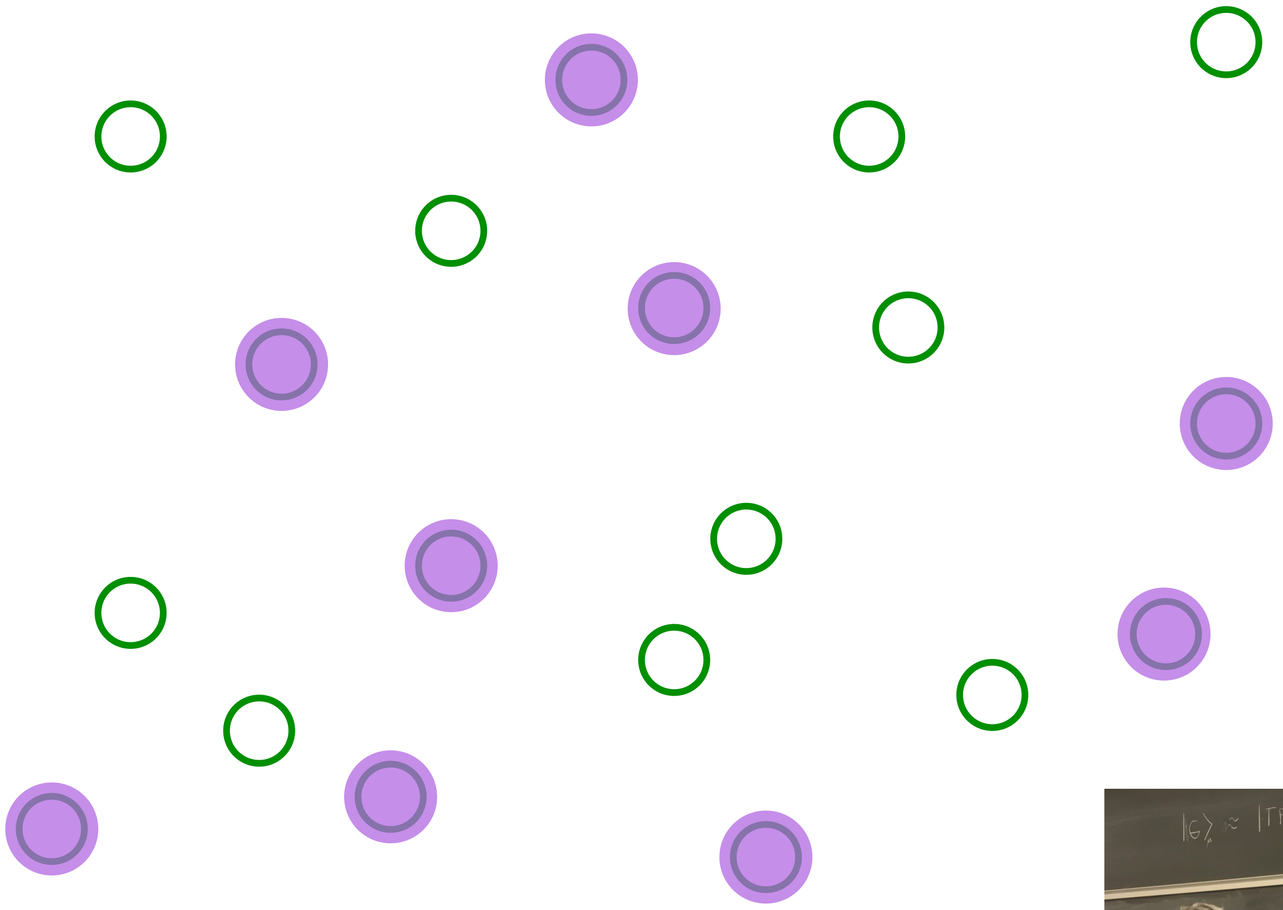

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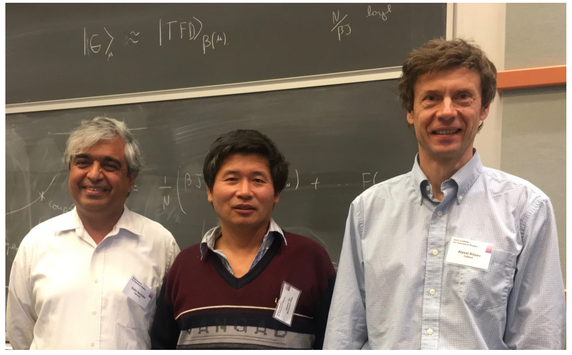
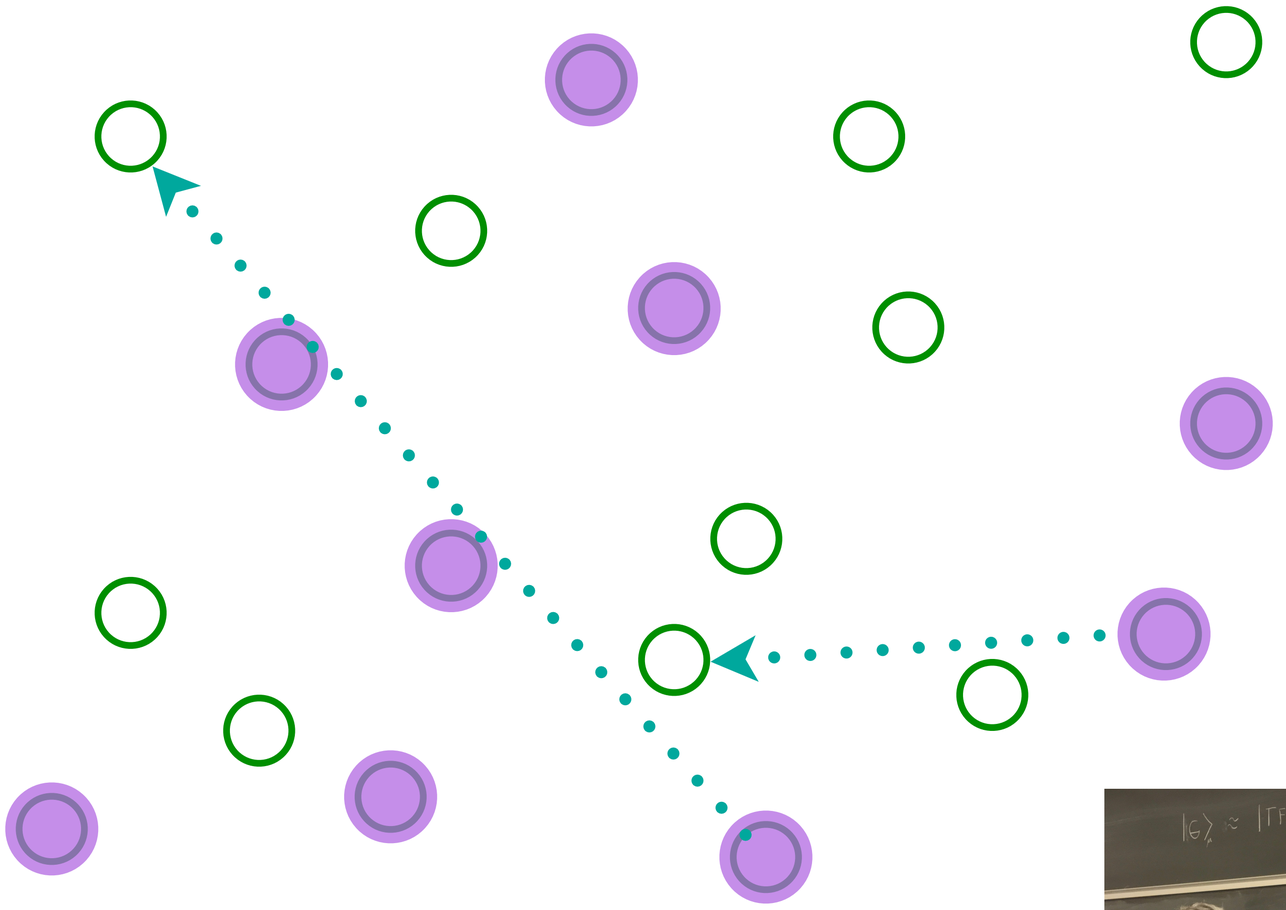


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



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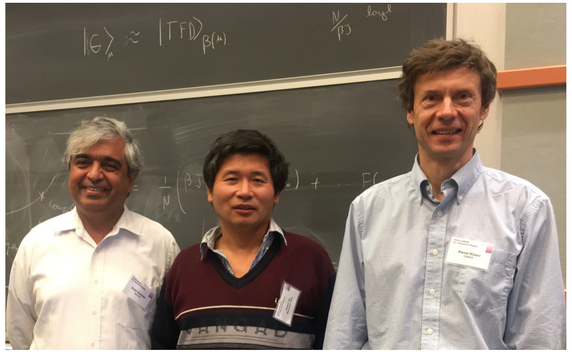

The SYK model

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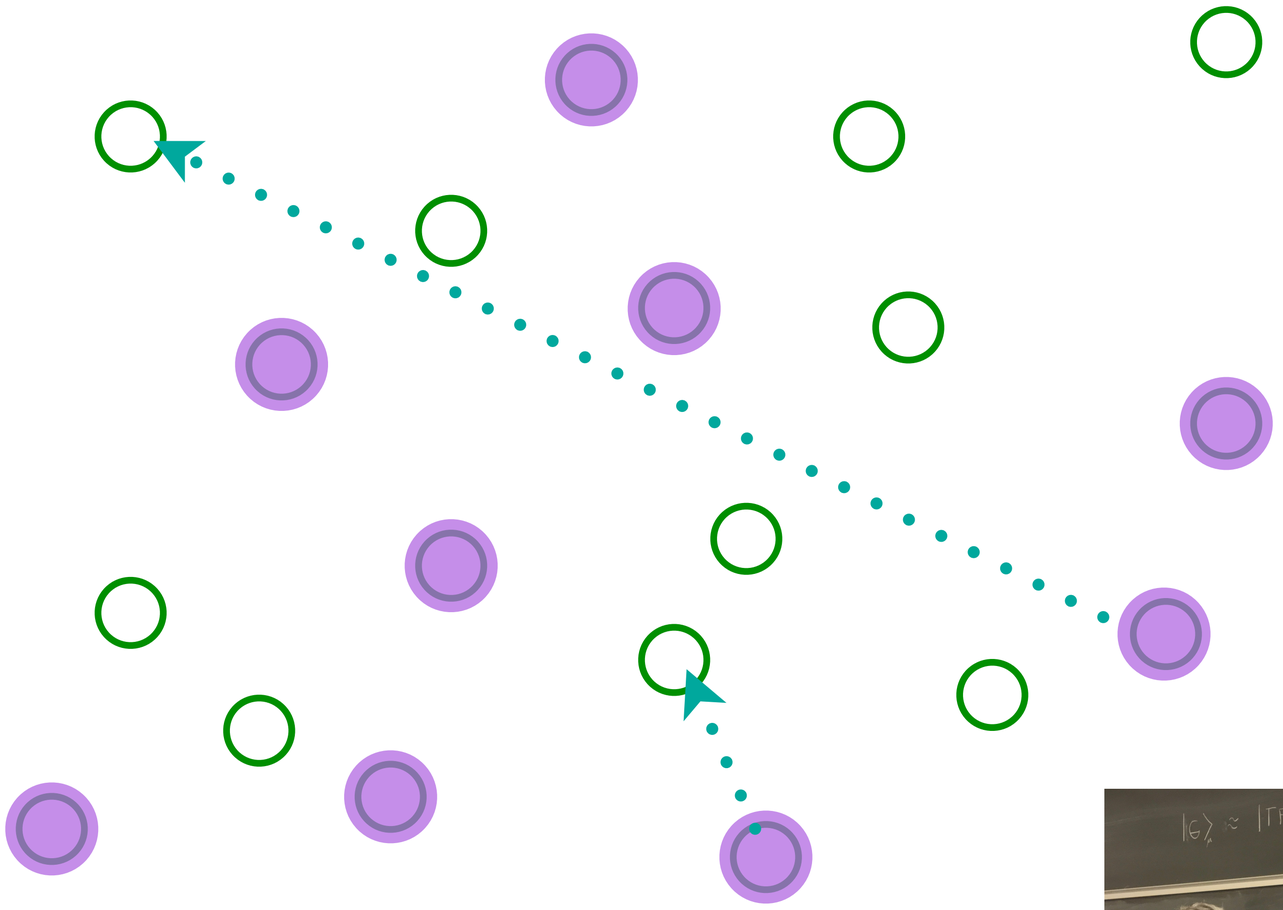

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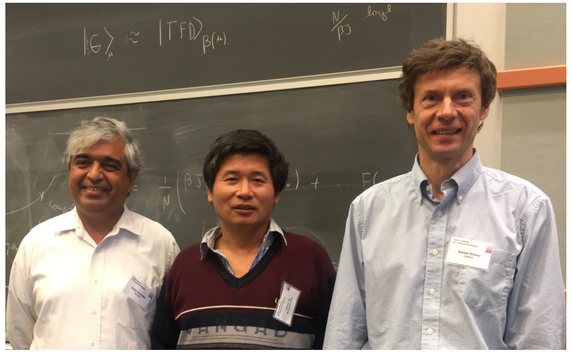

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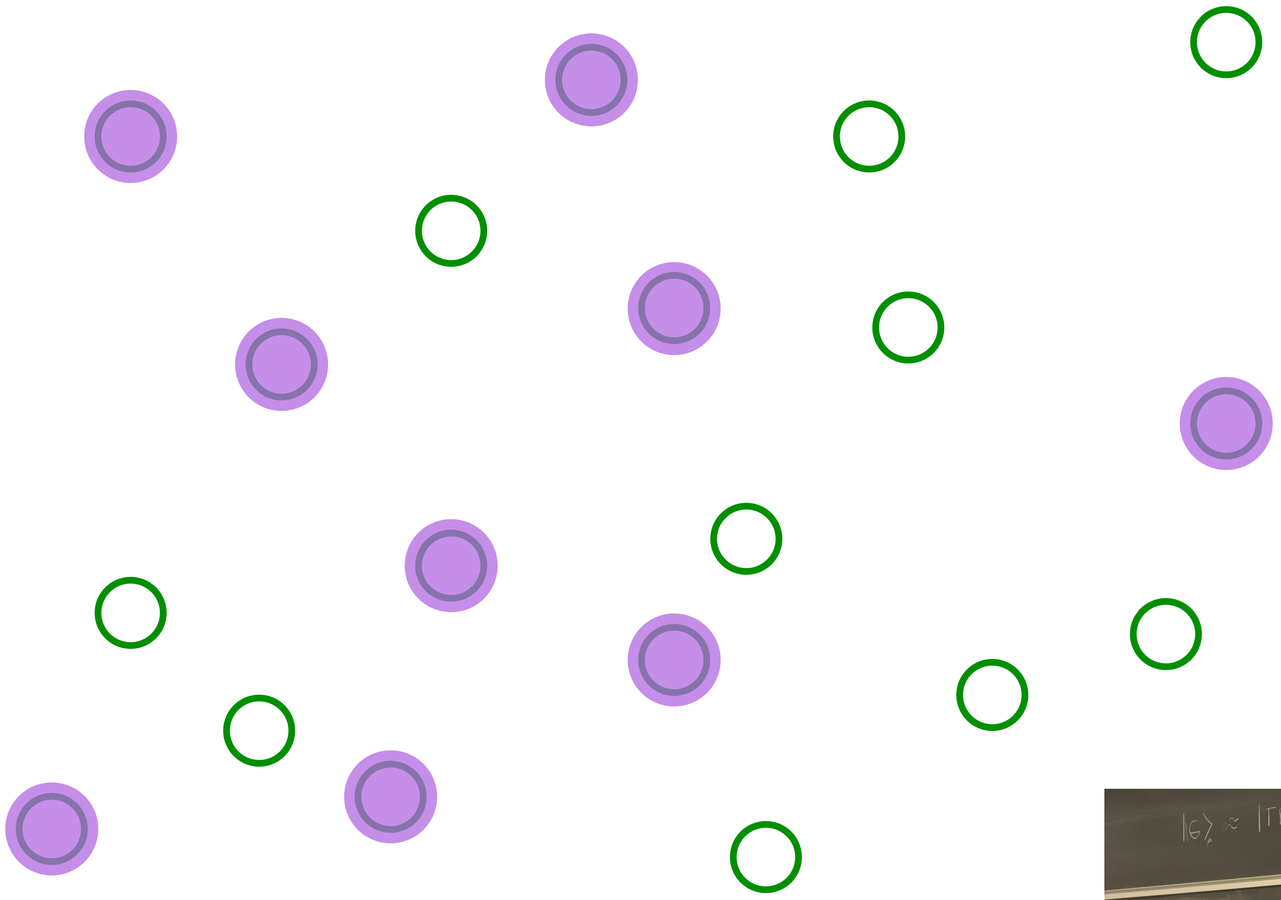
Sachdev, Ye (1993); Kitaev (2015)



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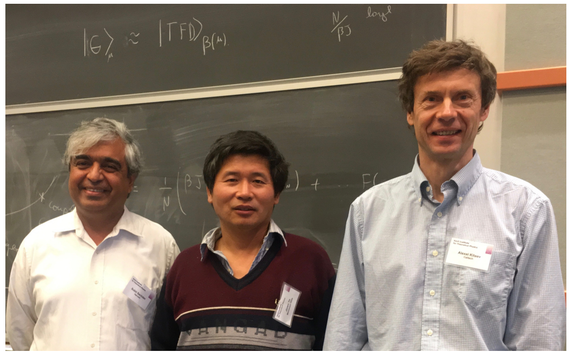
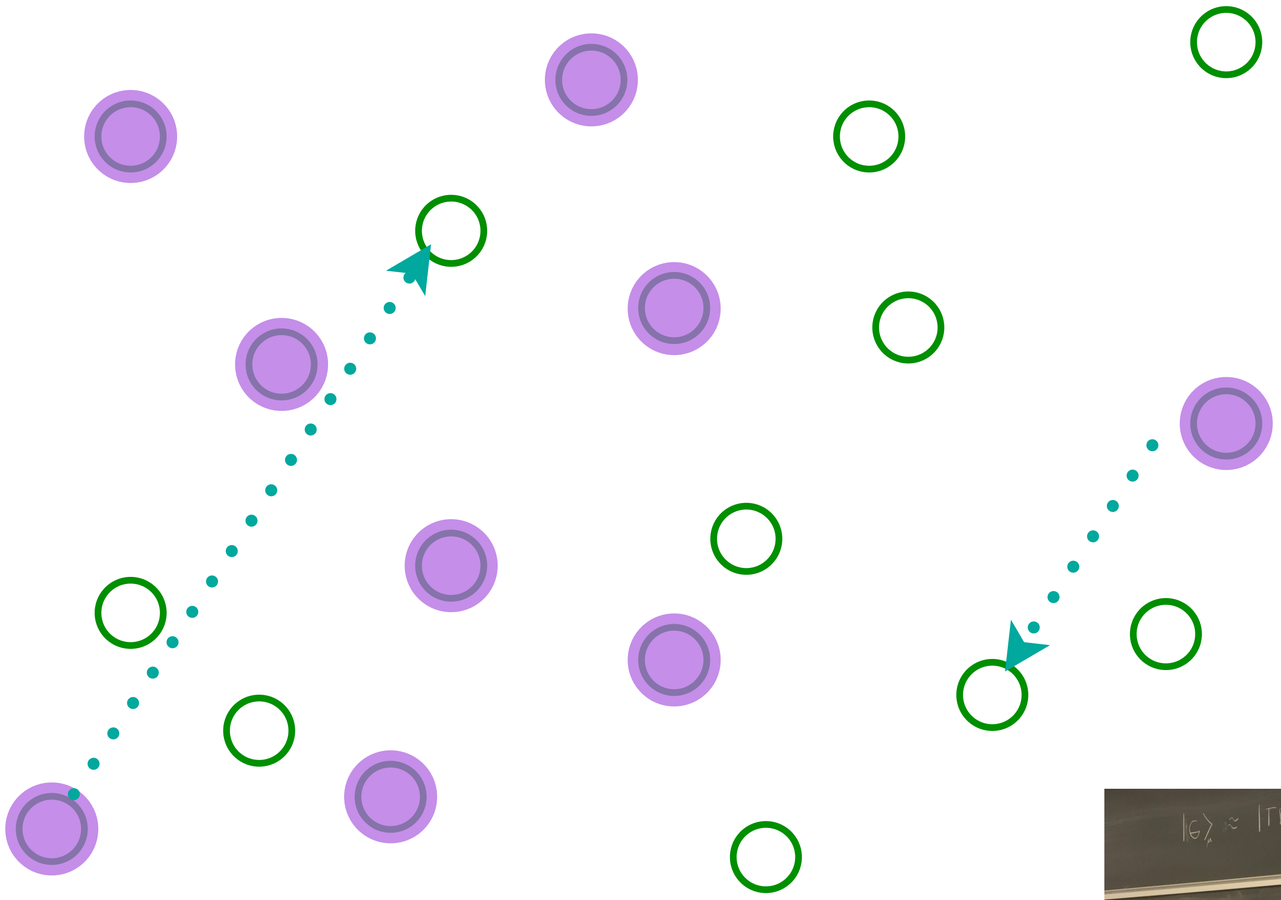
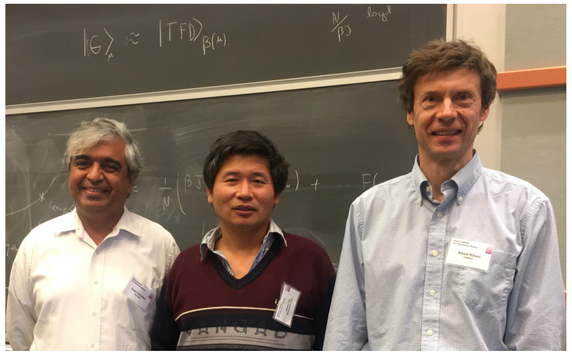


The SYK model

Sachdev, Ye (1993); Kitaev (2015)



Entangle electrons pairwise randomly



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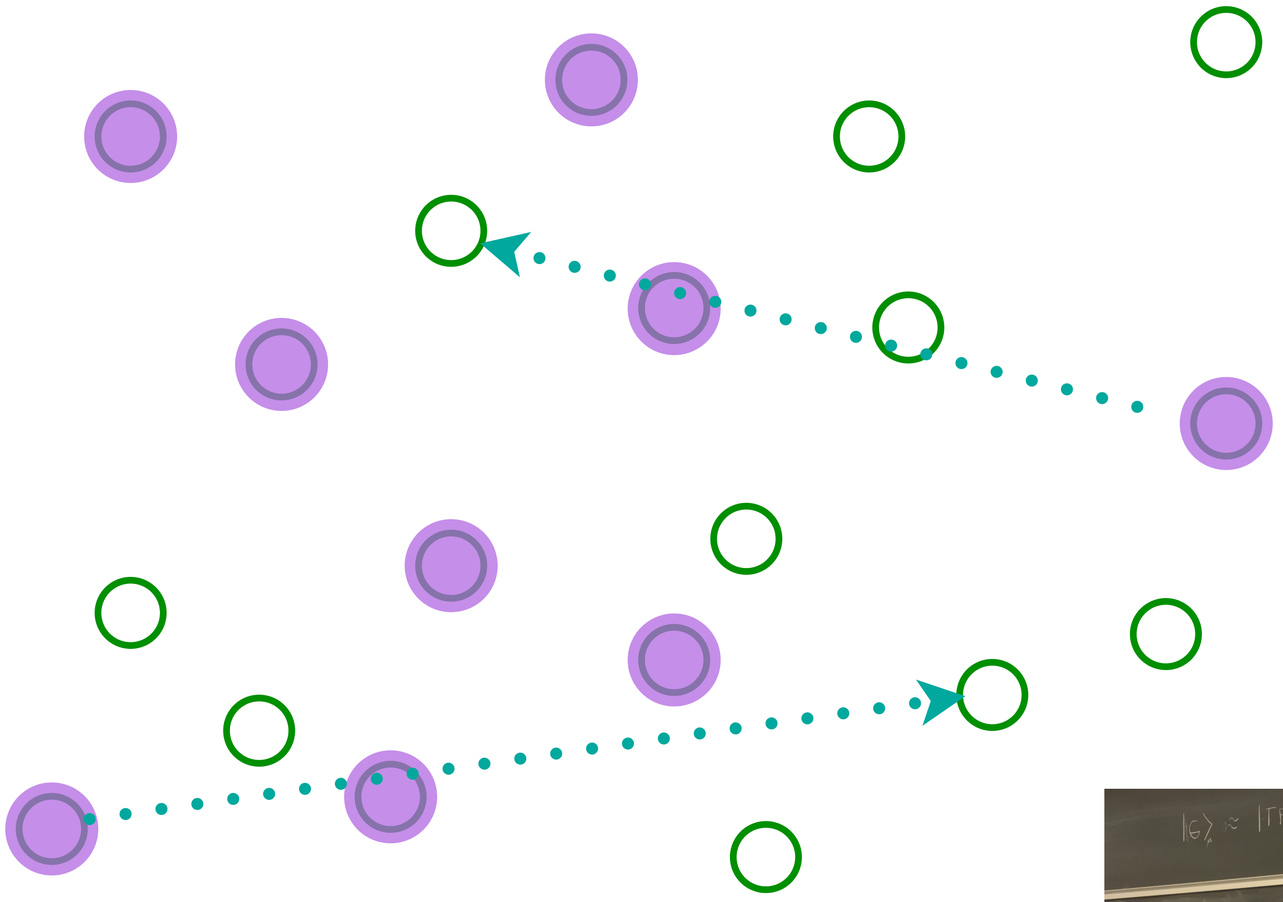
Sachdev, Ye (1993); Kitaev (2015)



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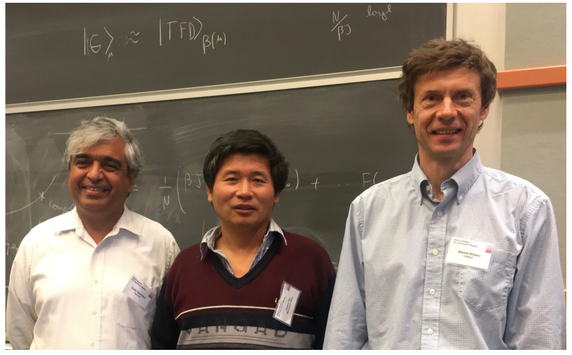

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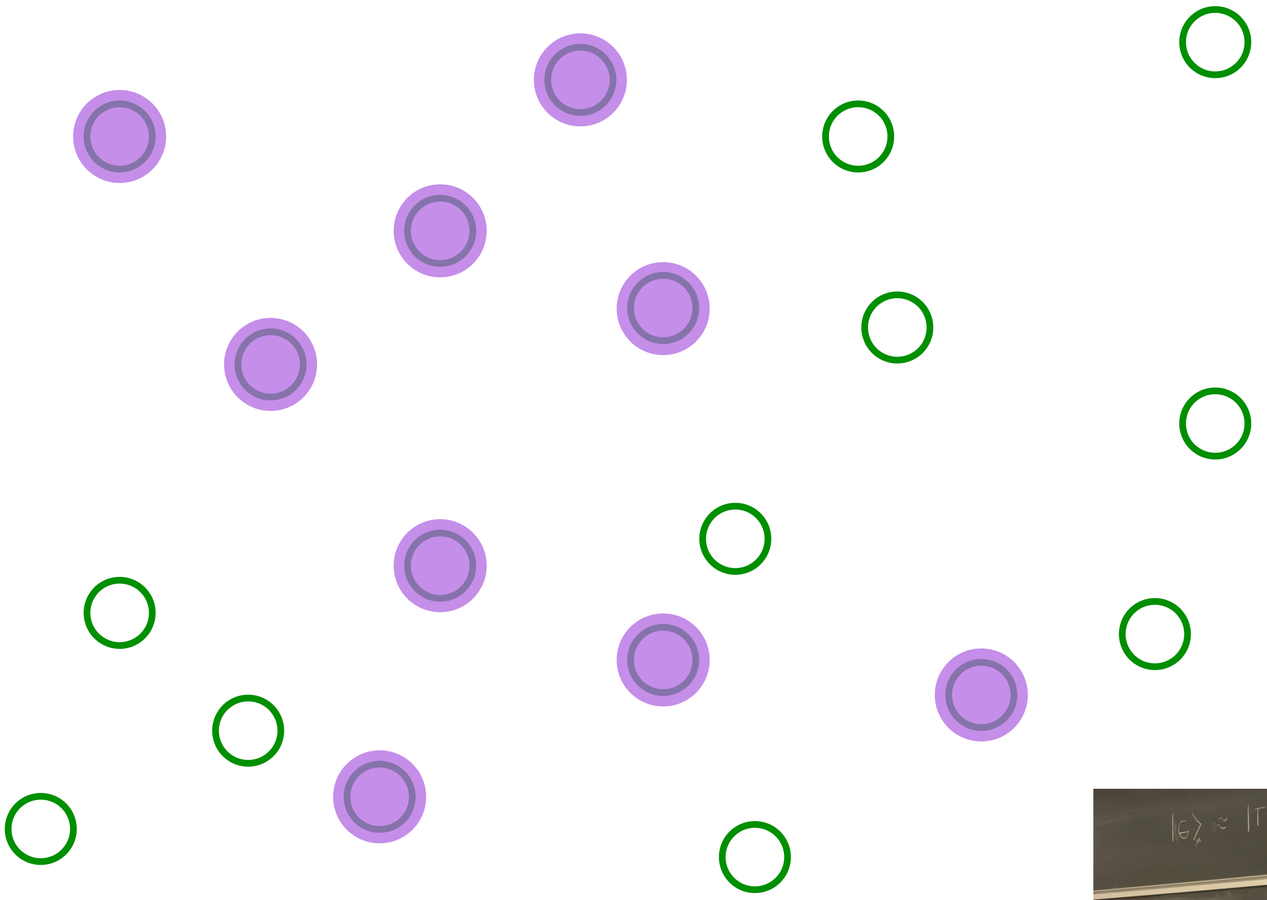
Sachdev, Ye (1993); Kitaev (2015)



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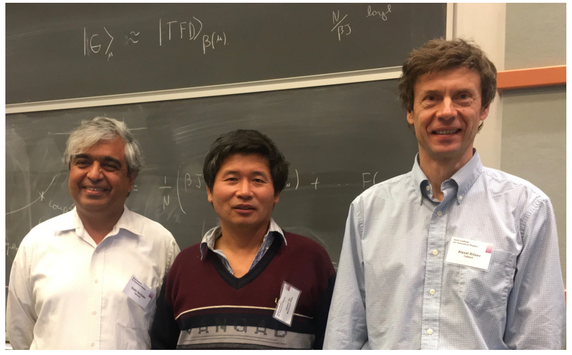

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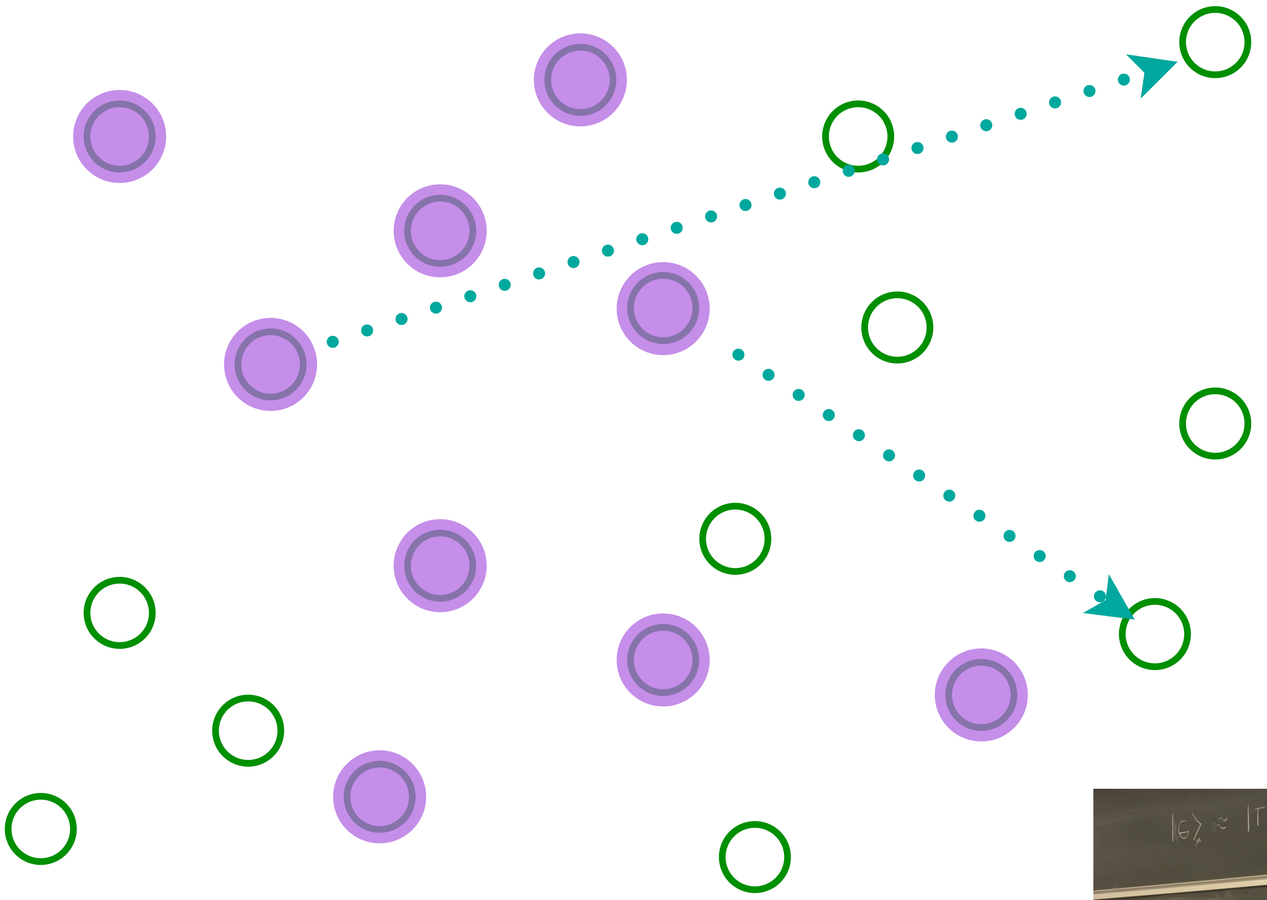

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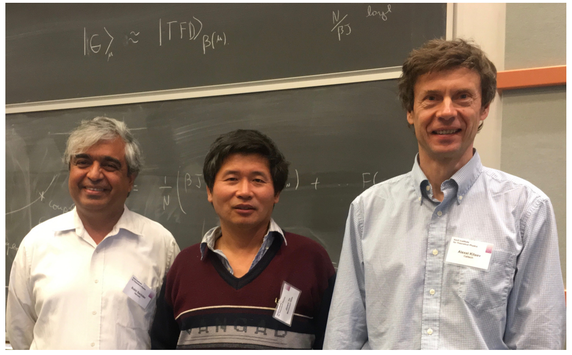

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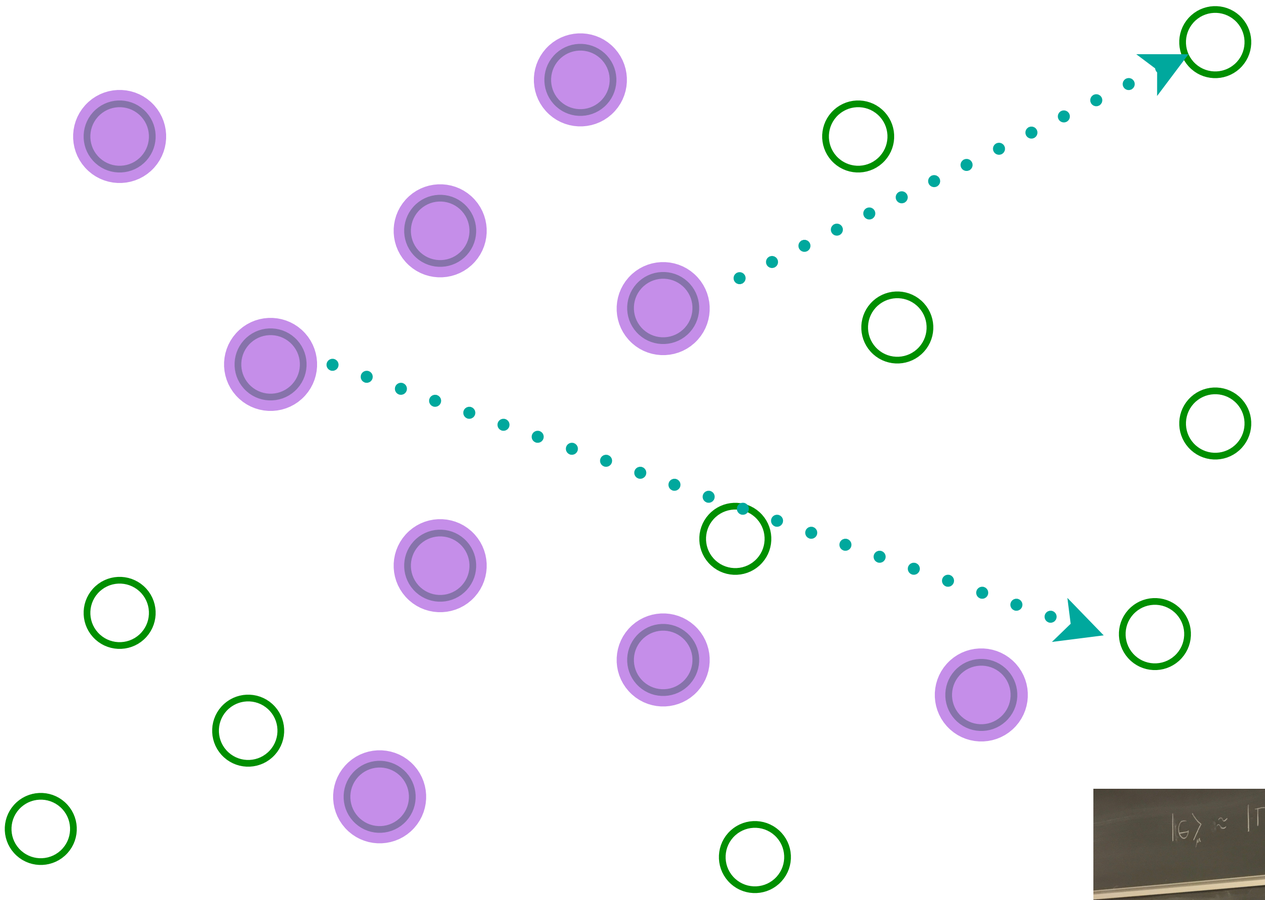

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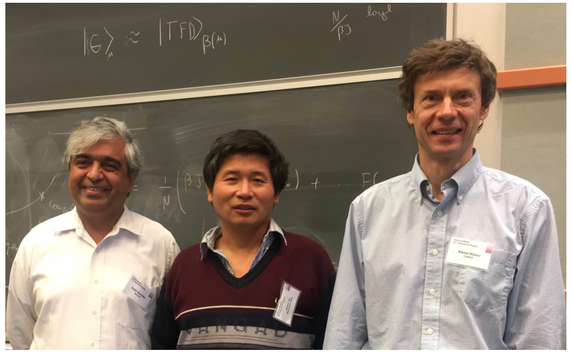

The SYK model

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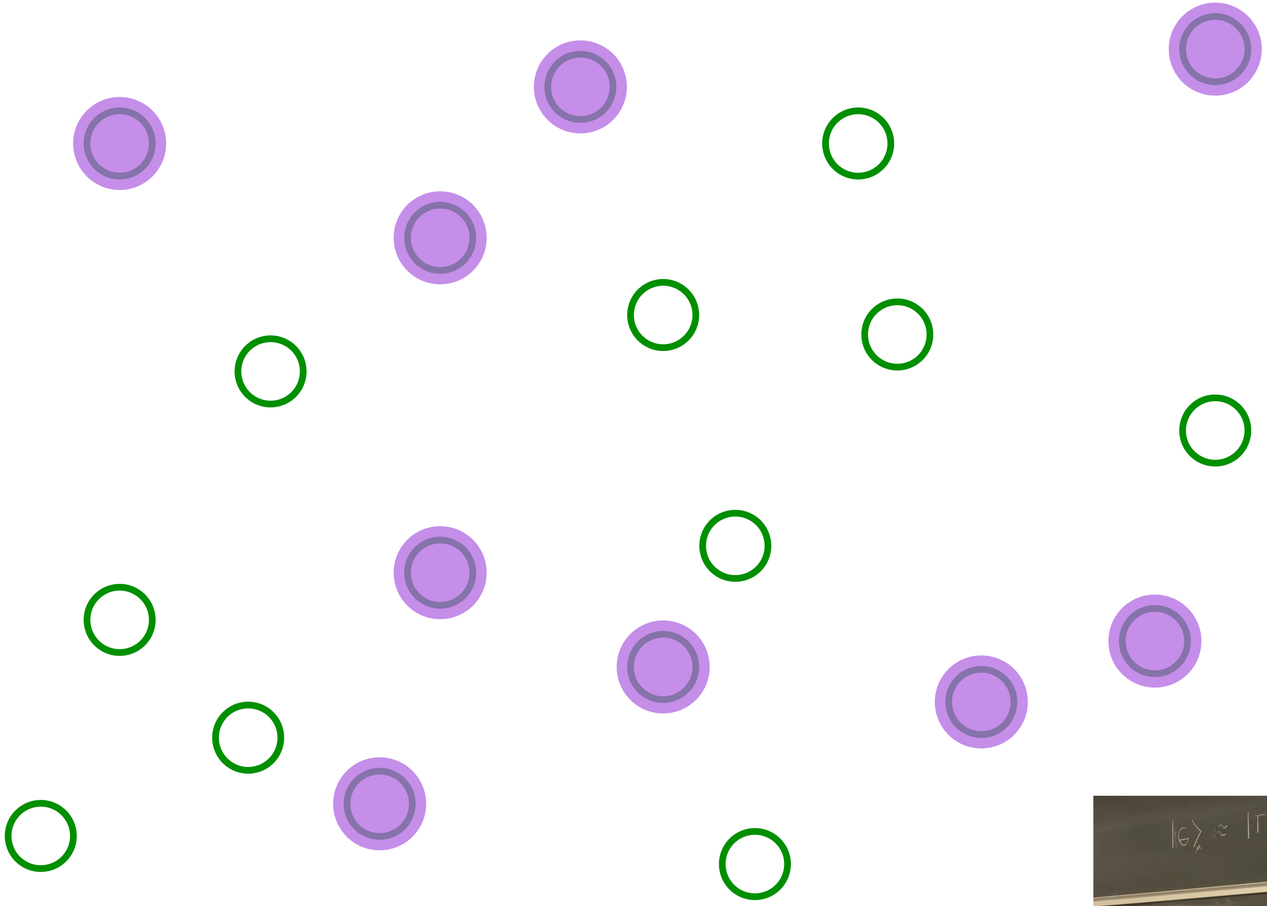

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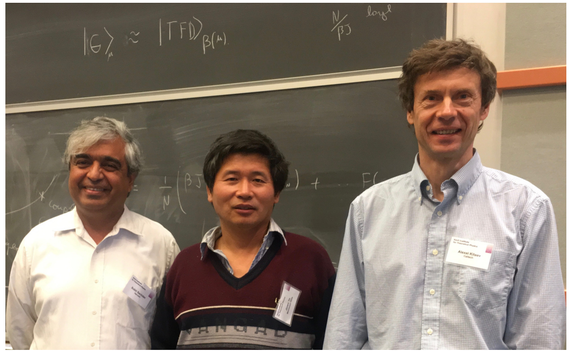

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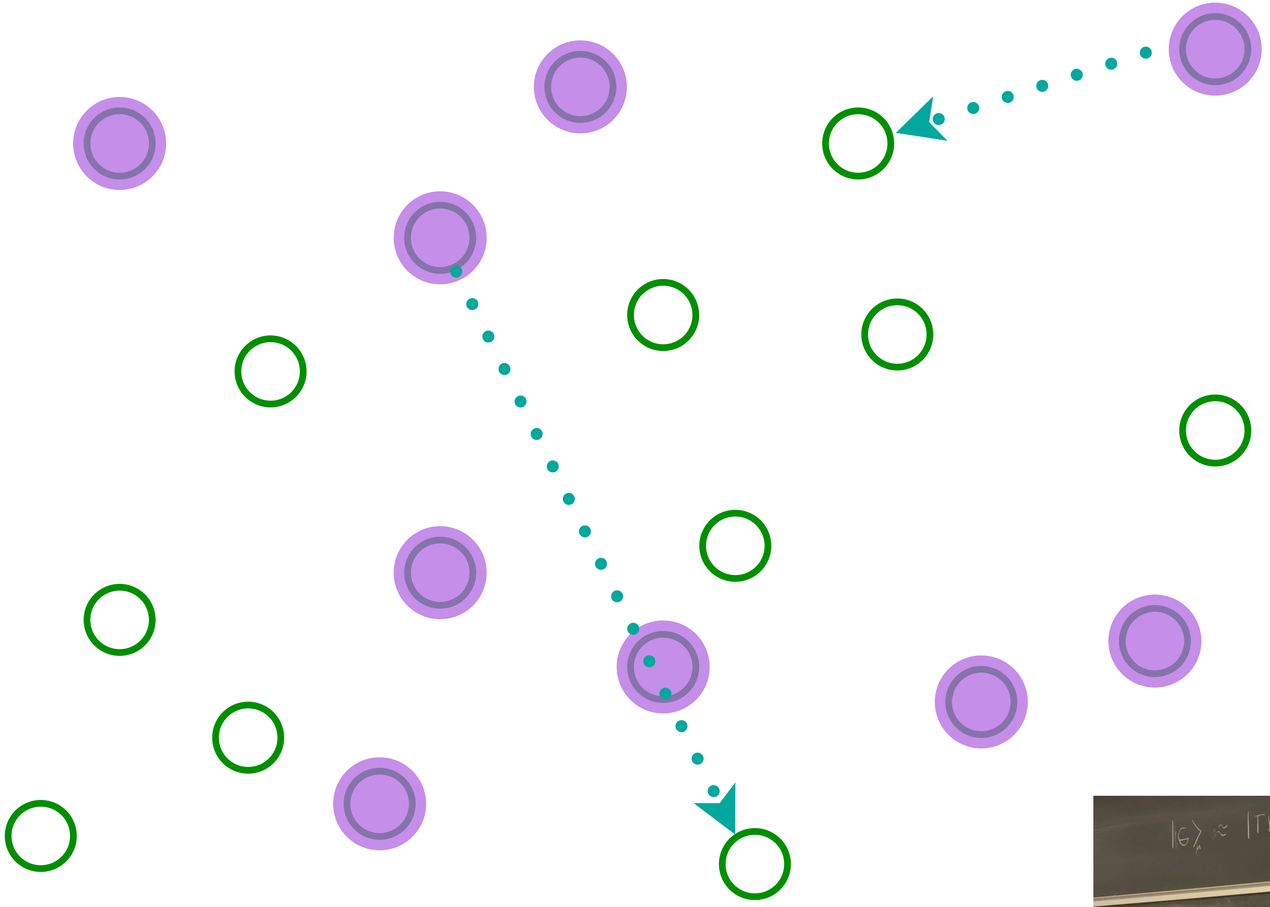

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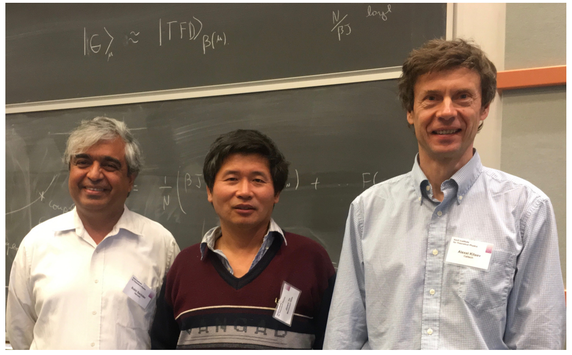

The SYK model

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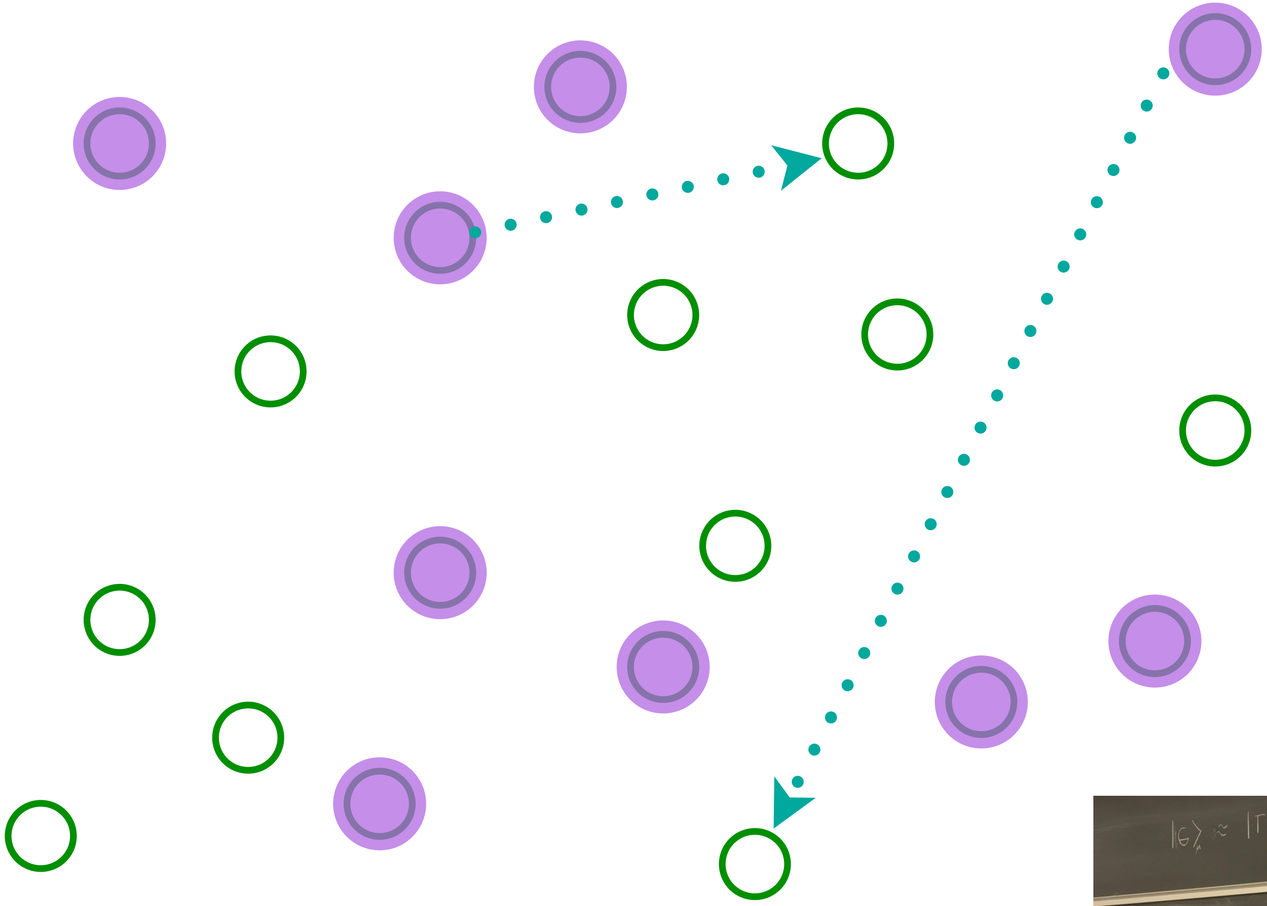

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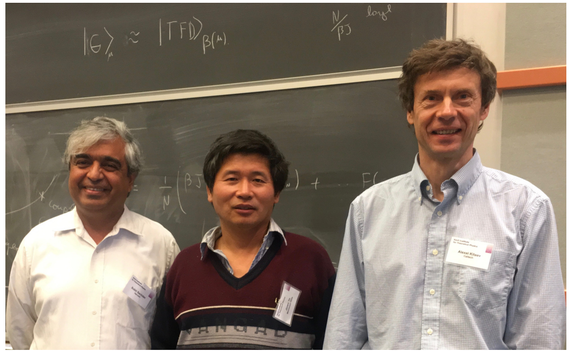

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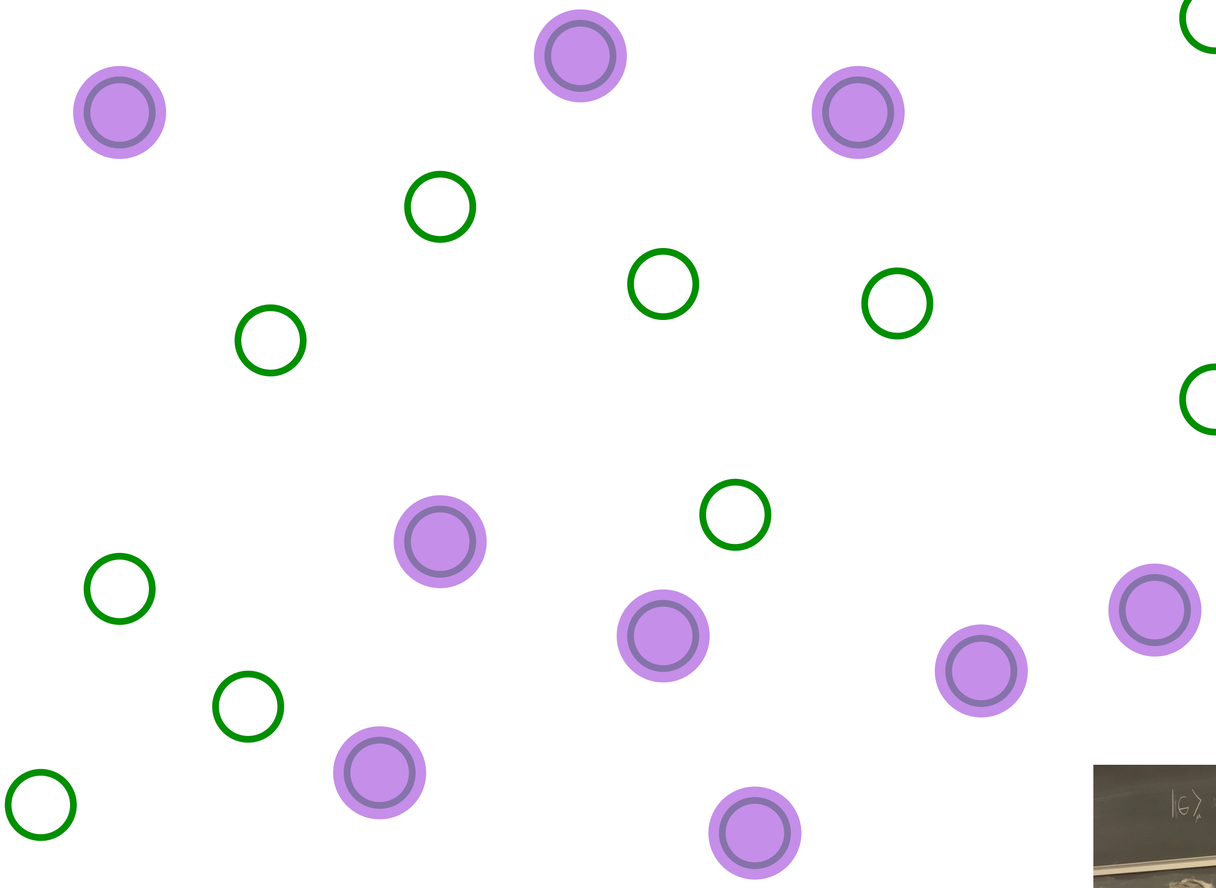
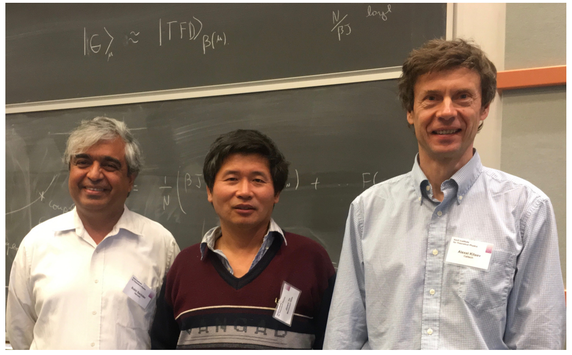


Electron scattering time τ in the SYK model

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

Entangle electrons pairwise randomly

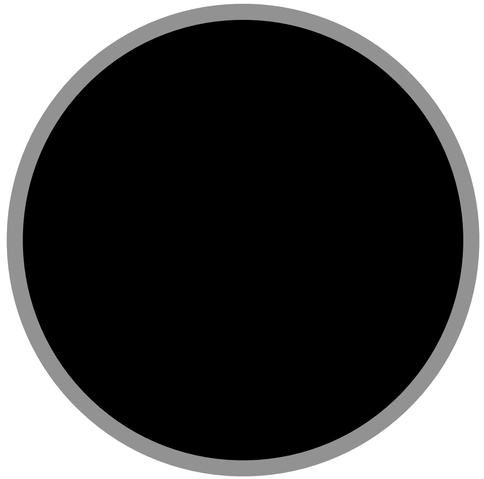


Complex quantum entanglement in black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

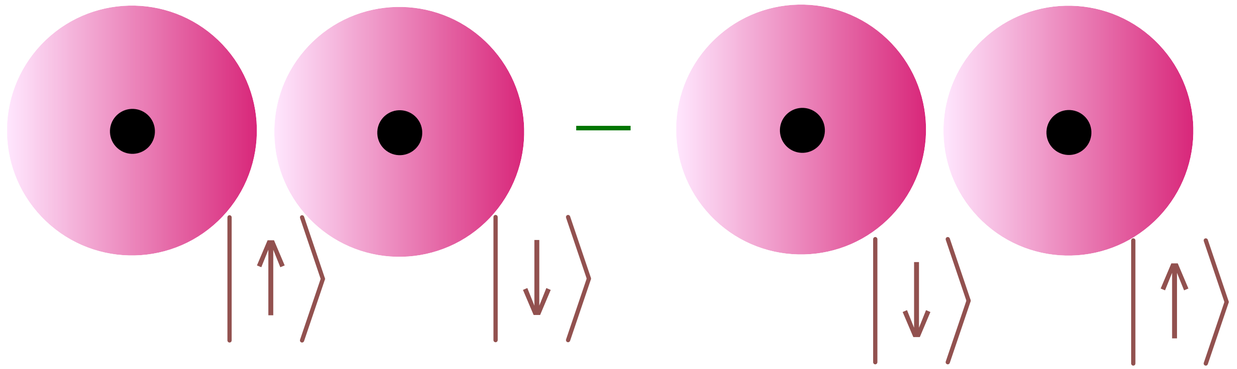
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



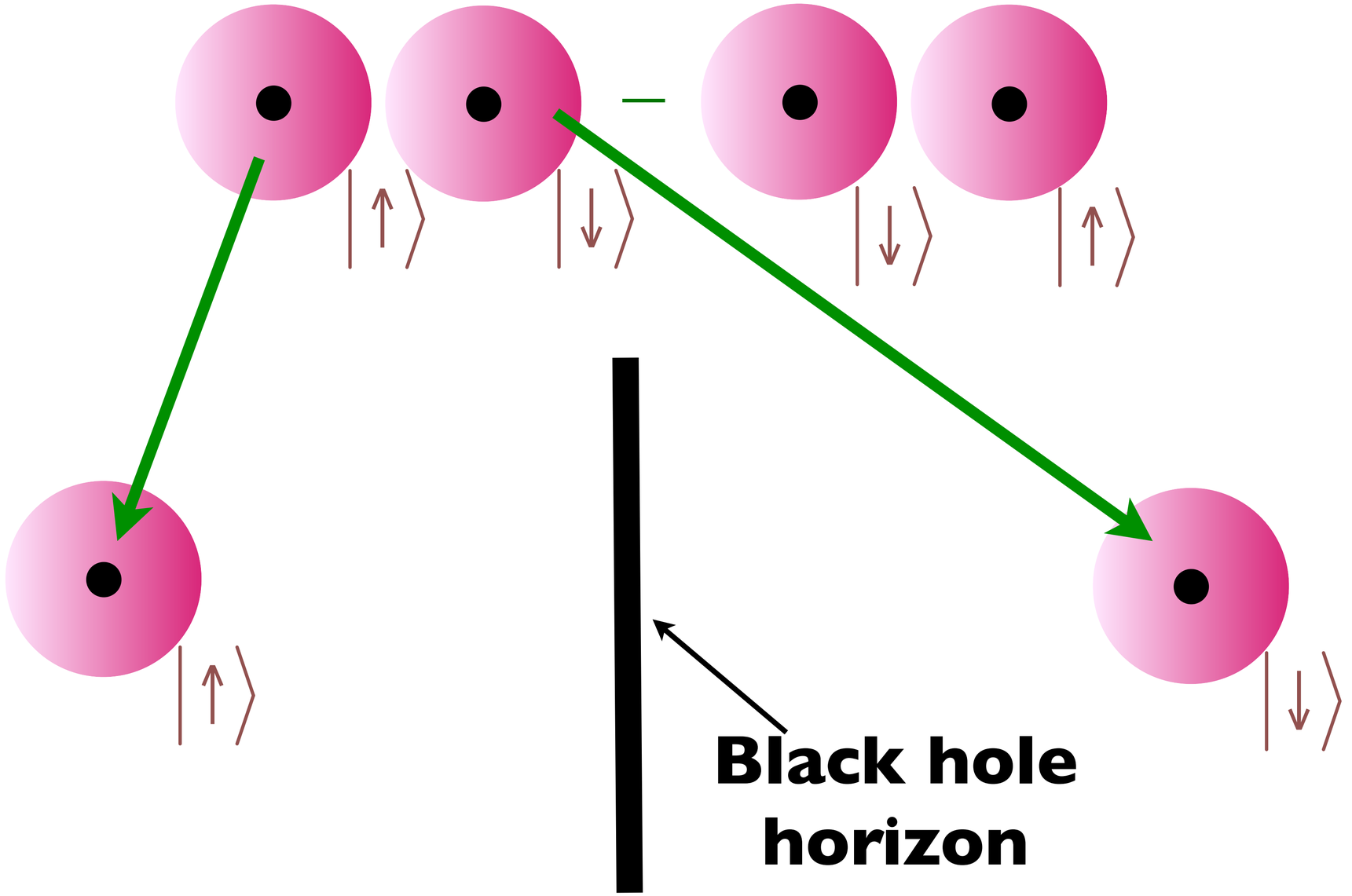
G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

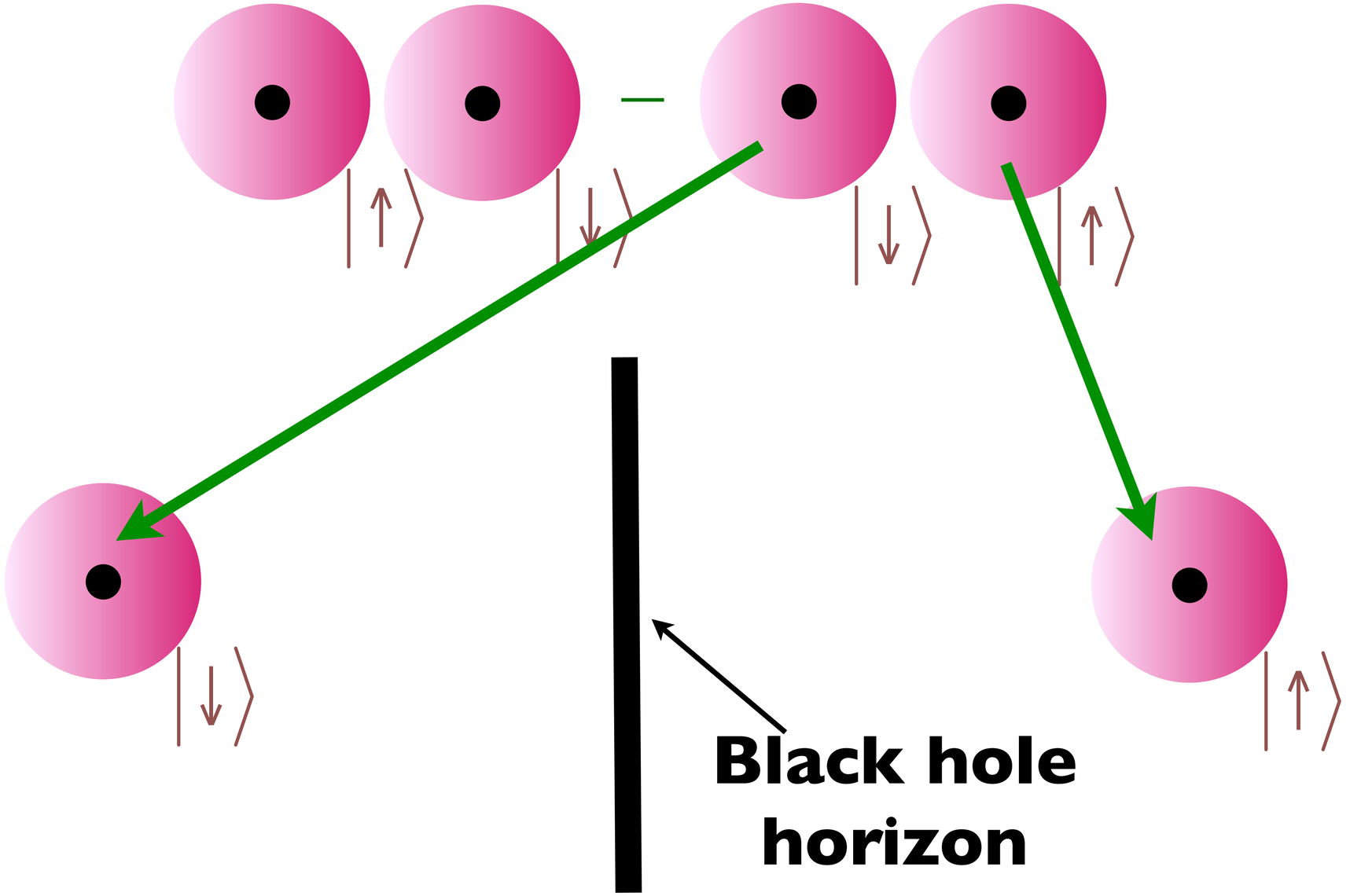
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

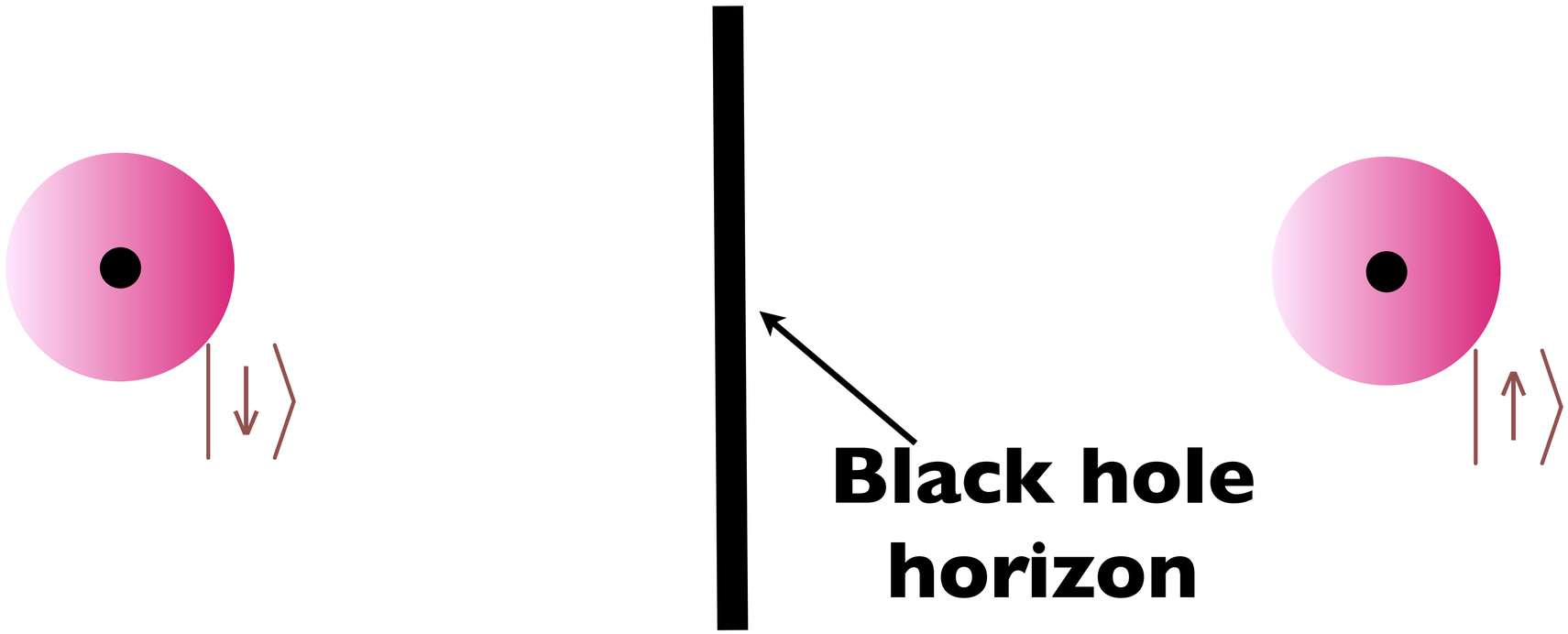


Quantum Entanglement across a black hole horizon



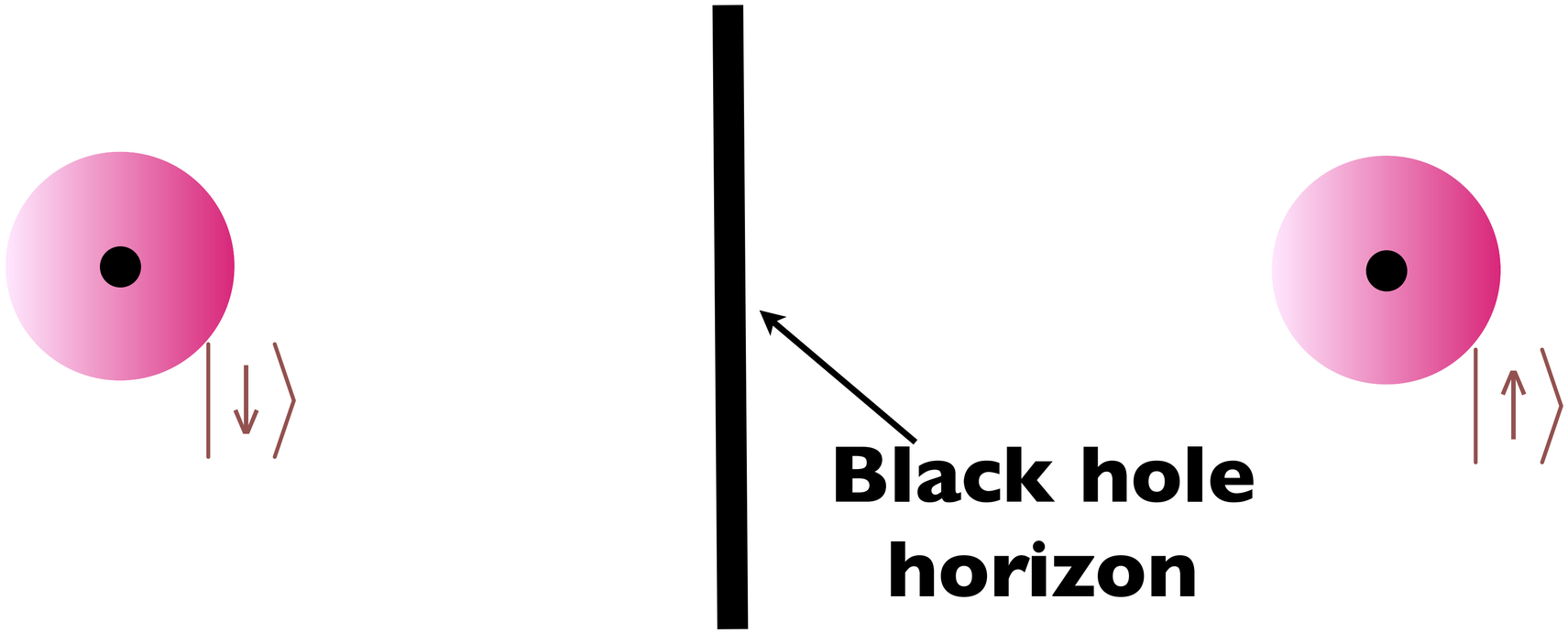
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature
(The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)



The Sachdev-Ye-Kitaev (SYK) model

The SYK model has a scale-invariant entanglement structure:
i.e. electrons are entangled at all distance and time scales

It describes
certain ***strange metals***

Sachdev, Ye (1993)

The Sachdev-Ye-Kitaev (SYK) model

The SYK model has a scale-invariant entanglement structure:
i.e. electrons are entangled at all distance and time scales

In one set of variables, it describes certain ***strange metals***

Sachdev, Ye (1993)

In a *dual* set of variables it describes certain

black holes

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

Quantum theory of electrons,
one at a time:
metals and insulators

Quantum entanglement of
electron pairs:
superconductivity

Quantum entanglement of 2,3,4,……∞ electrons: strange metals

Complex quantum entanglement in black holes