

# Probing topological spin liquids in Rydberg atom arrays

Theoretical Condensed Matter Physics Principal Investigators' Meeting  
Office of Basic Energy Sciences, U.S. Department of Energy  
October 28, 2021

Subir Sachdev



INSTITUTE FOR  
ADVANCED STUDY

PHYSICS



HARVARD

Talk online: [sachdev.physics.harvard.edu](https://sachdev.physics.harvard.edu)

# Probing topological spin liquids in Rydberg atom arrays

(Distinct from the ten-fold “*band topology*” classification of “*topological insulators*” and “*topological superconductors*”)

Theoretical Condensed Matter Physics Principal Investigators’ Meeting  
Office of Basic Energy Sciences, U.S. Department of Energy  
October 28, 2021

Subir Sachdev



Talk online: [sachdev.physics.harvard.edu](https://sachdev.physics.harvard.edu)



**Rhine Samajdar**



**Seth Whitsitt**



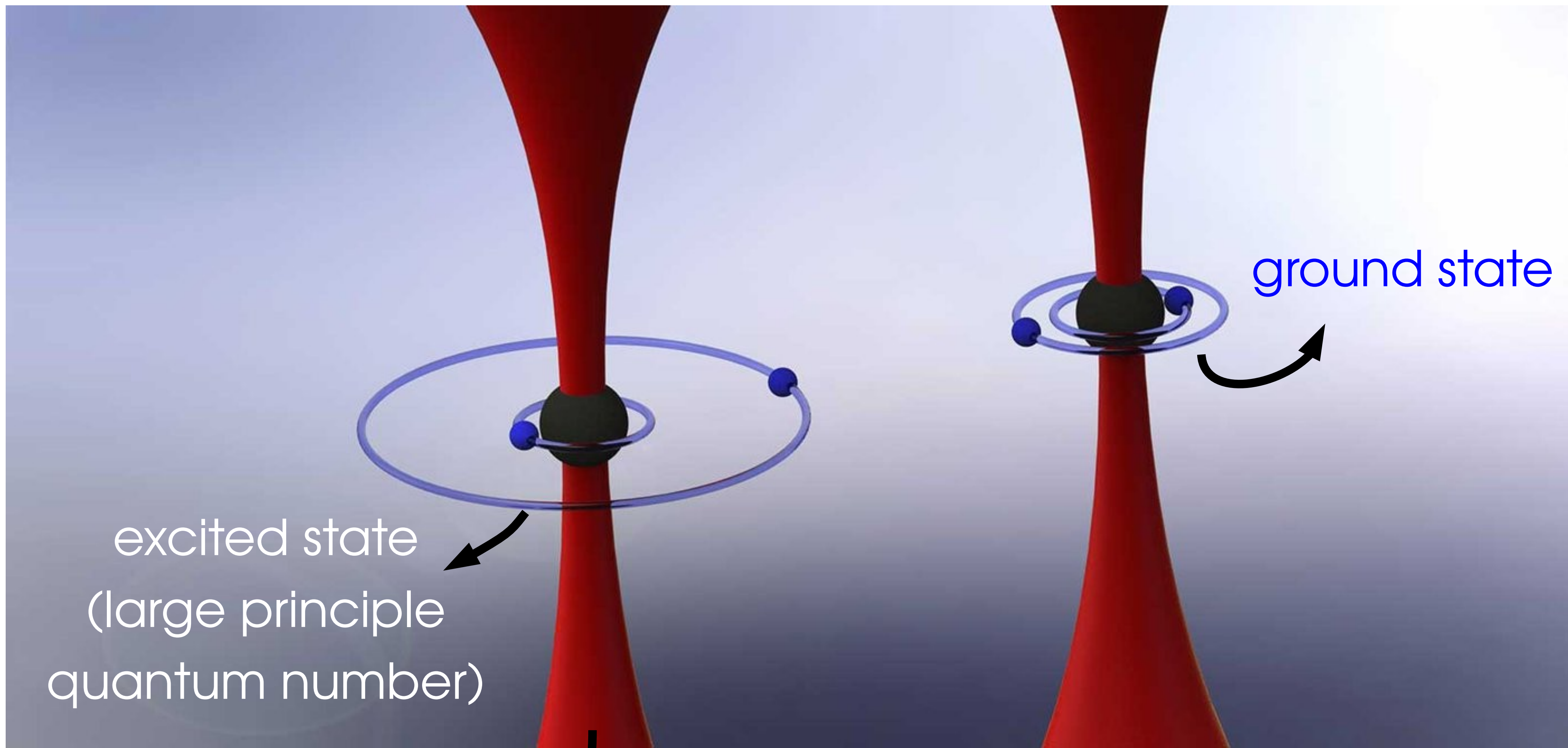
**Wen Wei Ho**



**Hannes Pichler**



**Mikhail Lukin**



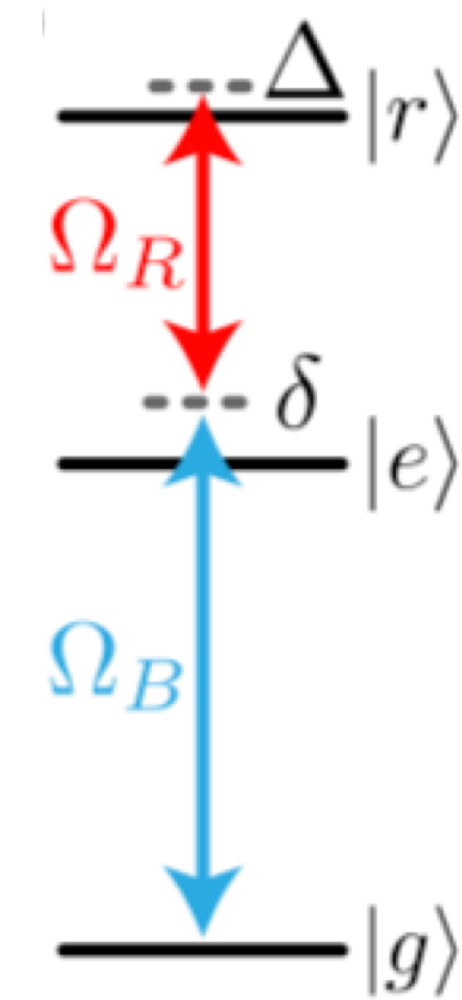
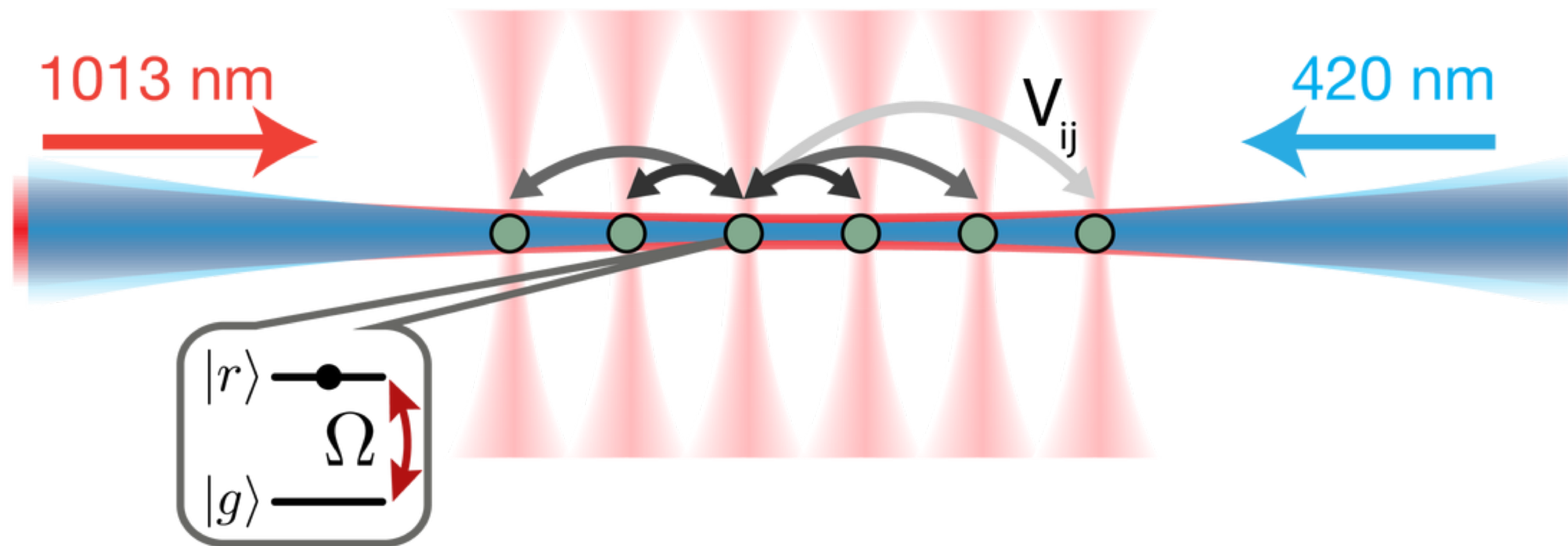
$$V_{|l-l'|} \sim \frac{1}{|l-l'|^6}$$

optical tweezer (traps atom)

Fig: <https://www.caltech.edu/about/news/quantum-innovations-achieved-using-alkaline-earth-atoms>

$$H_{\text{Ryd}} = \sum_{\ell} \left[ \frac{\Omega}{2} (|g\rangle\langle r| + |r\rangle\langle g|)_{\ell} - \Delta |r\rangle\langle r| \right] + \sum_{(\ell, \ell')} V_{|\ell-\ell'|} \left( |r\rangle\langle r|_{\ell} \otimes |r\rangle\langle r|_{\ell'} \right)$$

# QPTs in a Rydberg quantum simulator



$$|g\rangle \equiv |0\rangle$$

$$|r\rangle \equiv b^\dagger |0\rangle$$

$$\mathcal{H} = \sum_{\ell} \left[ \frac{\Omega}{2} (b_{\ell} + b_{\ell}^{\dagger}) - \Delta n_{\ell} \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_{\ell} n_{\ell'}$$

$$V_{|\ell - \ell'|} \sim \frac{1}{|\ell - \ell'|^6}$$

$$n_{\ell} \equiv b_{\ell}^{\dagger} b_{\ell}$$

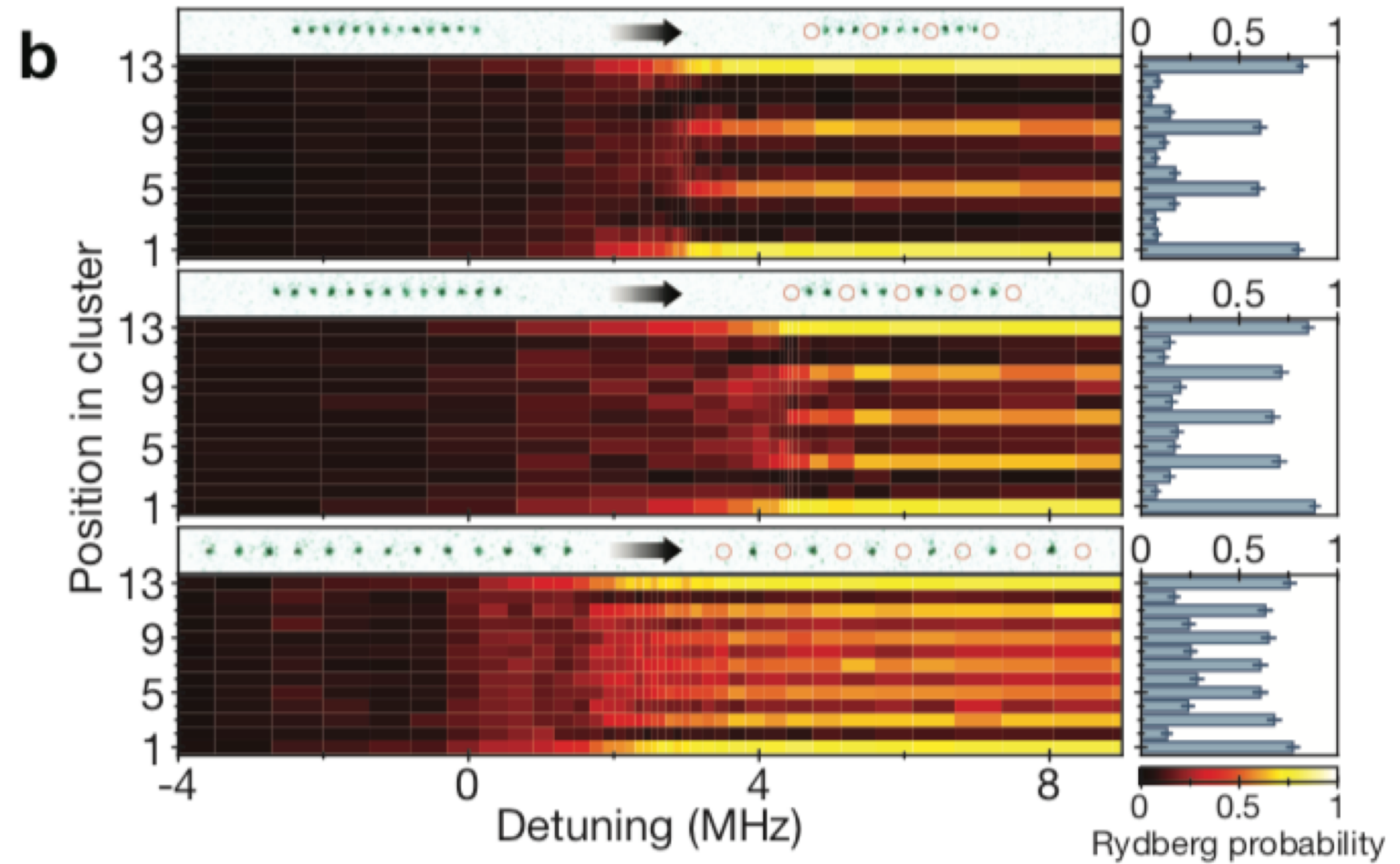
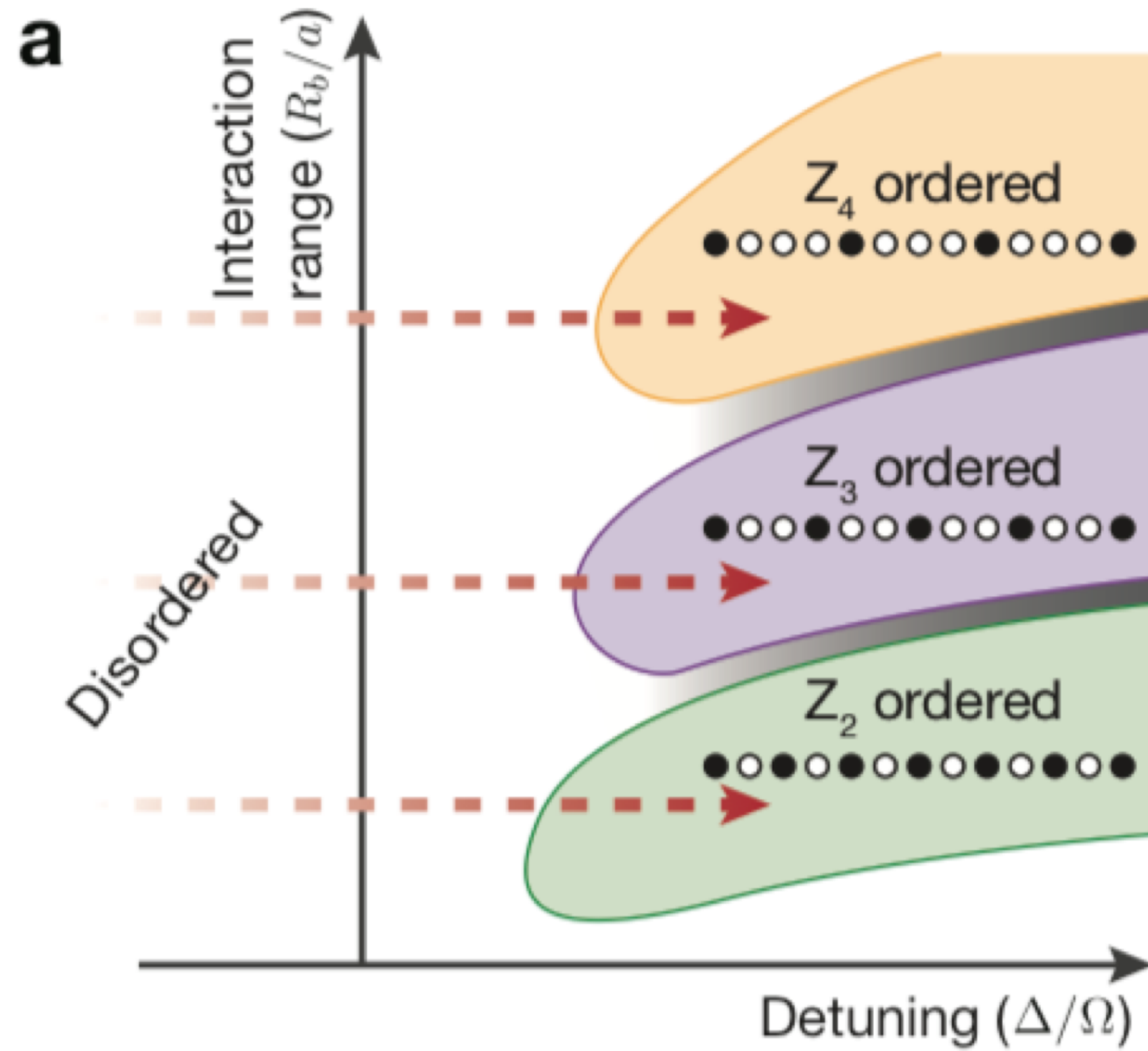
$n_{\ell} = 0, 1$  'hard core' bosons

**FSS model**

S. Sachdev, K. Sengupta, and S.M. Girvin, PRB **66**, 075128 (2002)

P. Fendley, K. Sengupta, S. Sachdev, PRB **69**, 075106 (2004)

# Density wave order and quantum phase transitions in one dimension



$$\mathcal{H} = \sum_{\ell} \left[ \frac{\Omega}{2} (b_{\ell} + b_{\ell}^{\dagger}) - \Delta n_{\ell} \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_{\ell} n_{\ell'}$$

$$n_{\ell} \equiv b_{\ell}^{\dagger} b_{\ell}$$

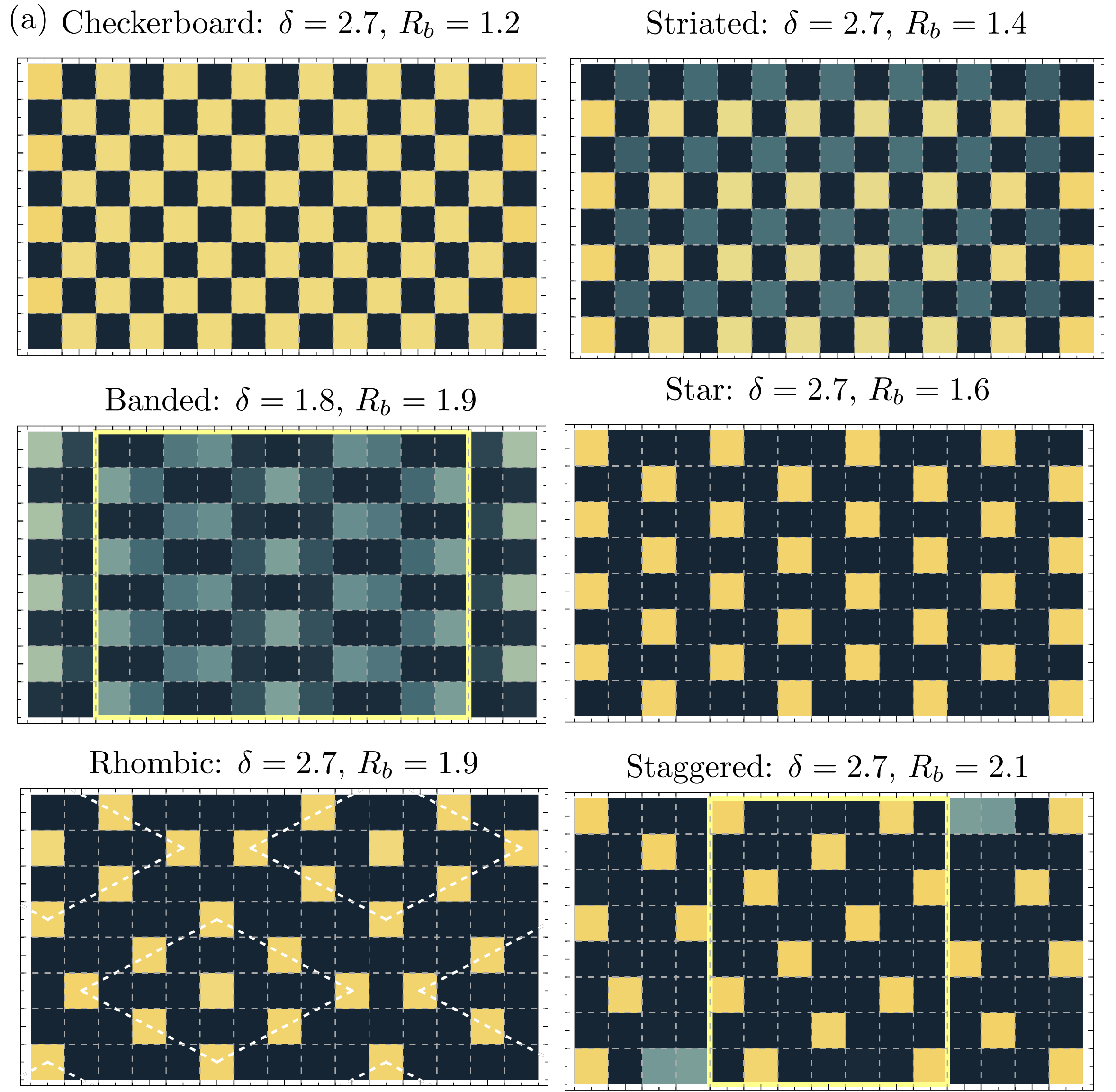
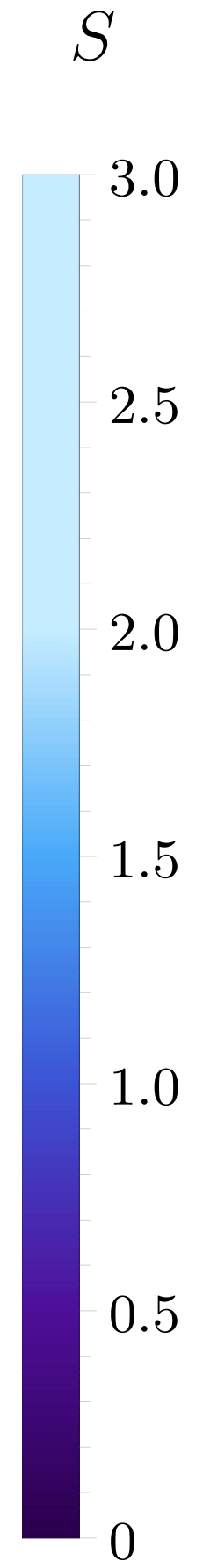
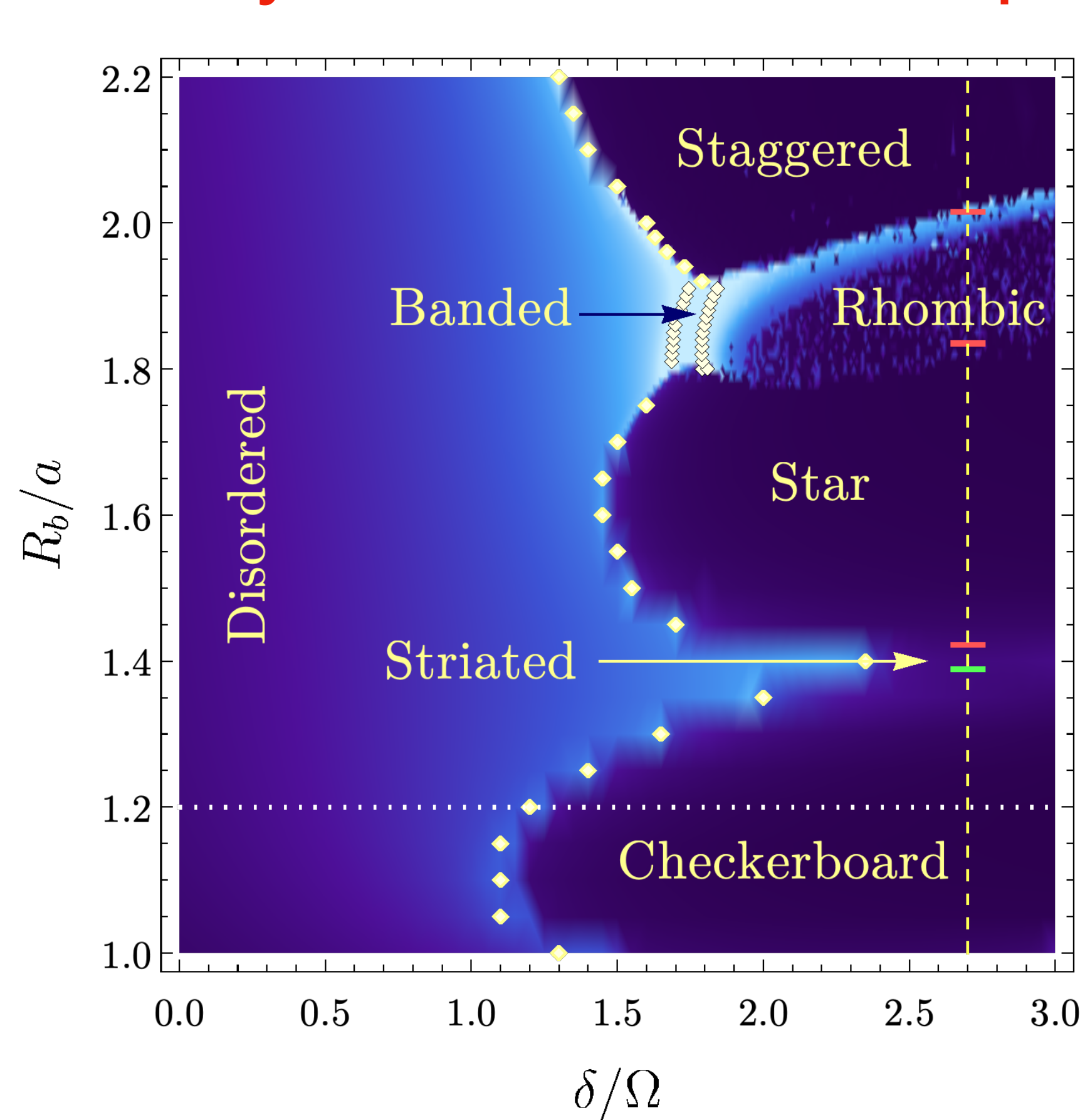
$n_{\ell} = 0, 1$  'hard core' bosons

FSS model

Bernien et. al., Nature **551**, 579 (2017)

Keesling et. al. Nature **568**, 207 (2019)

# Density wave order and quantum phase transitions on the square lattice



$$\mathcal{H} = \sum_{\ell} \left[ \frac{\Omega}{2} (b_{\ell} + b_{\ell}^{\dagger}) - \Delta n_{\ell} \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_{\ell} n_{\ell'}$$

$$n_{\ell} \equiv b_{\ell}^{\dagger} b_{\ell}$$

$n_{\ell} = 0, 1$  'hard core' bosons

FSS model

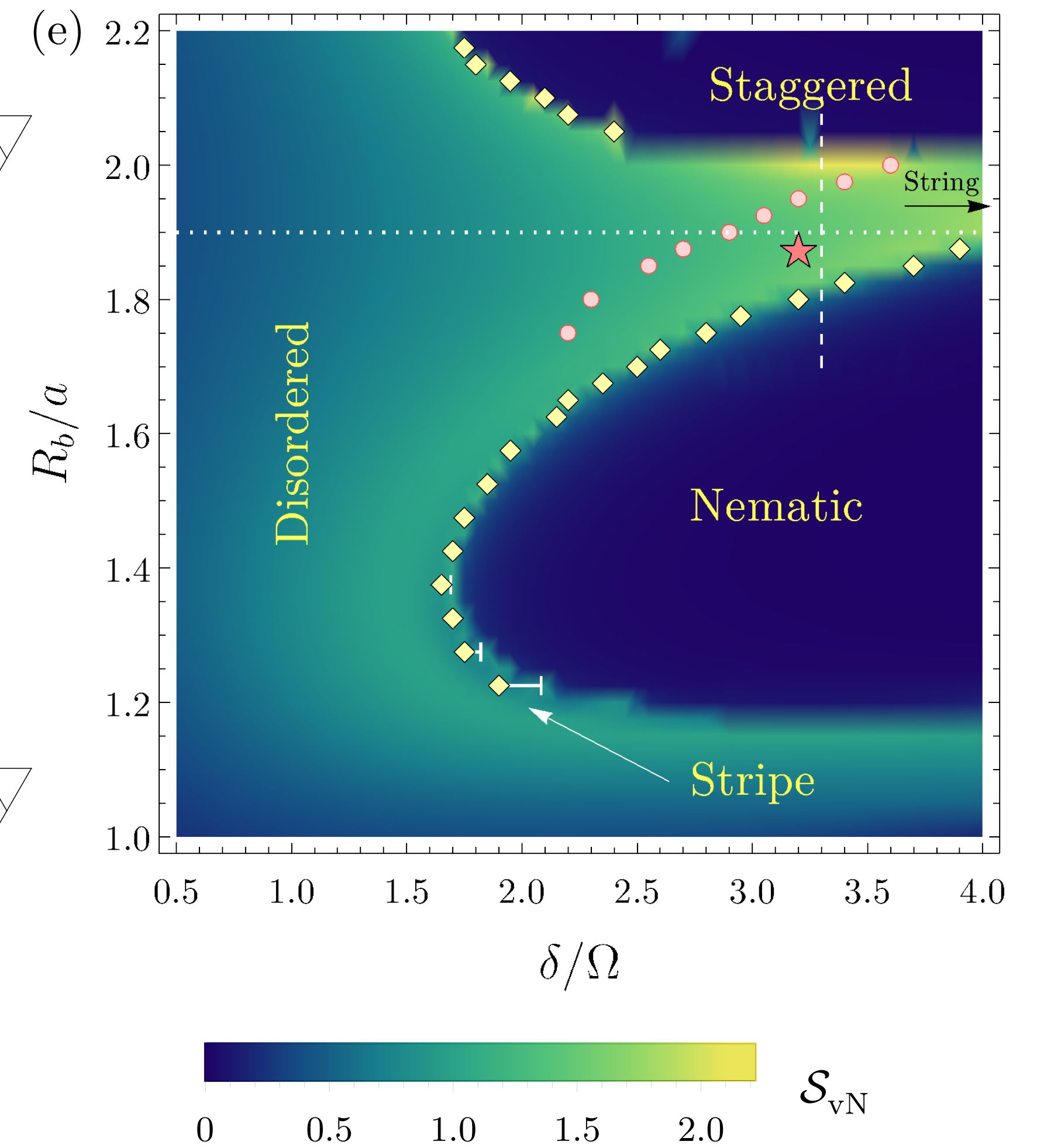
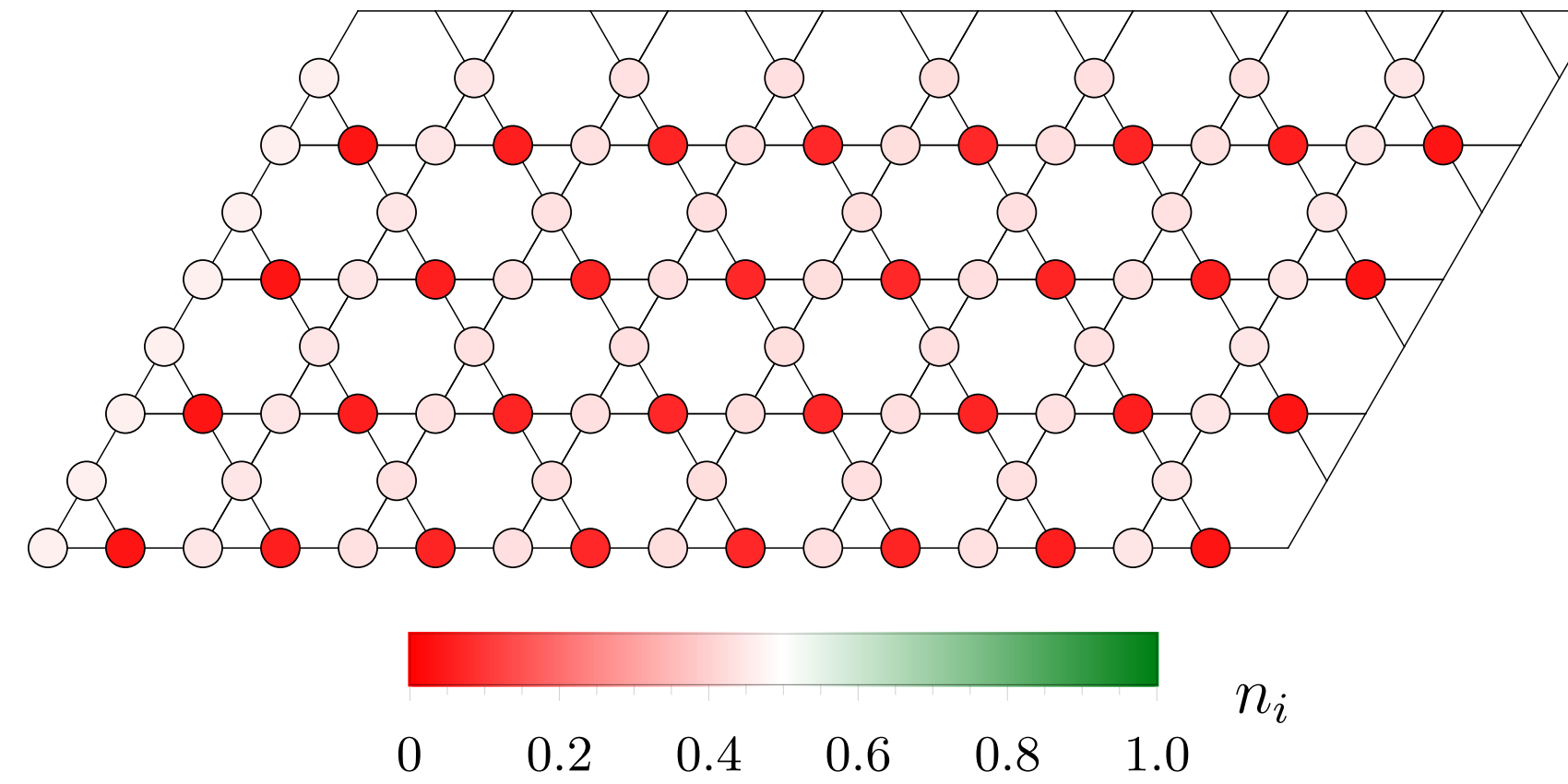


R. Samajdar et al., Physical Review Letters **124**, 103601 (2020).  
S. Ebadi et al., Nature **595**, 227 (2021).

# Rydberg atoms on site-kagome lattice: theory

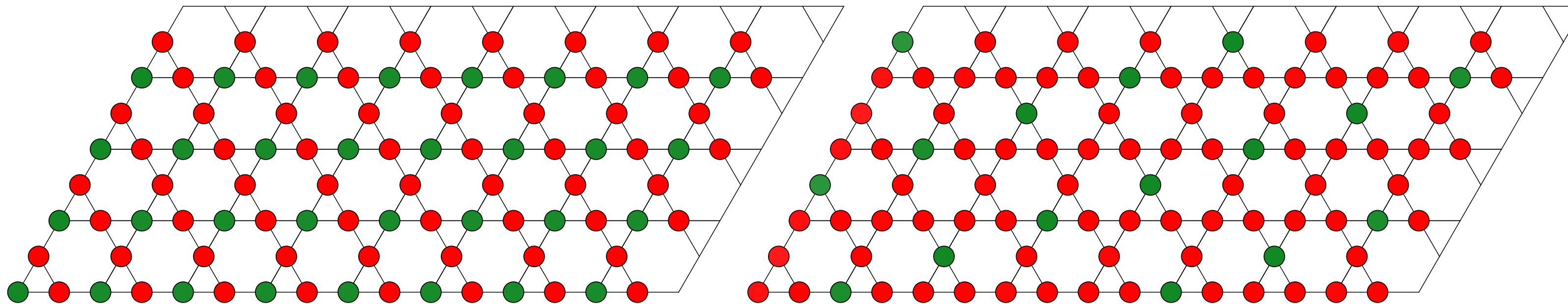


(b) Stripe:  $\delta = 2.2$ ,  $R_b = 1.2$



(c) Nematic:  $\delta = 3.3$ ,  $R_b = 1.7$

(d) Staggered:  $\delta = 3.3$ ,  $R_b = 2.1$

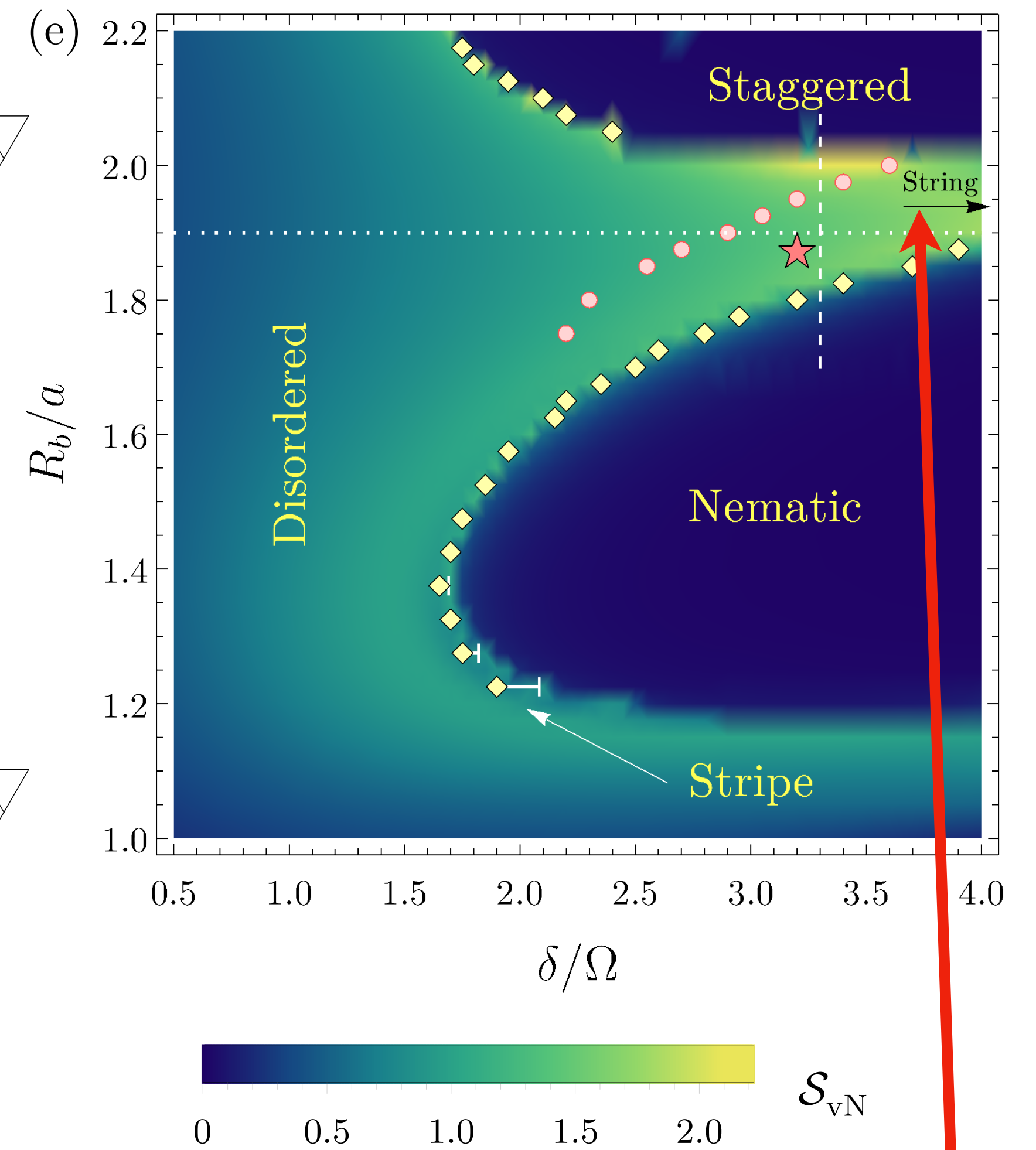
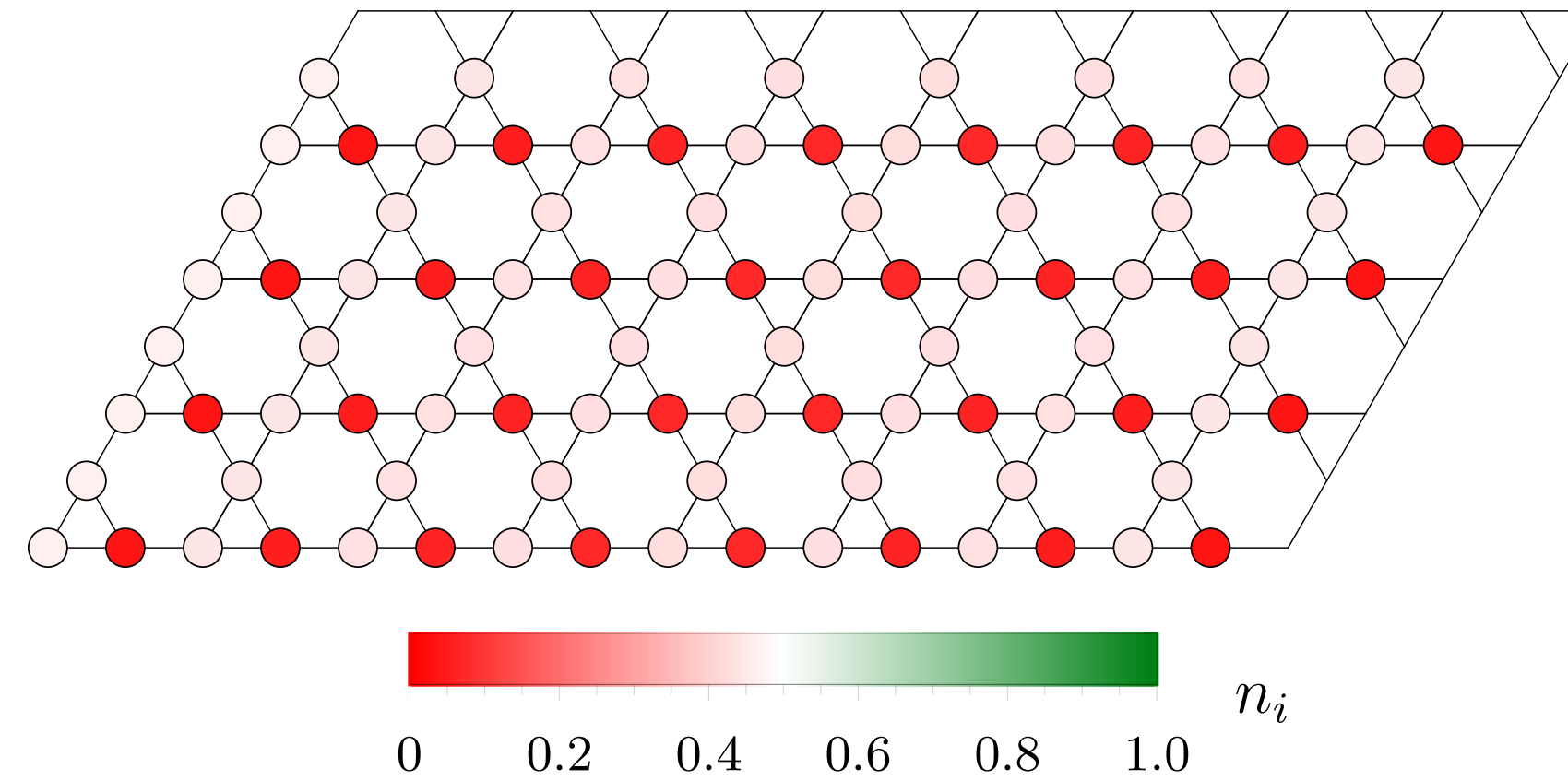


R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and  
S. Sachdev, PNAS **118**, e2015785118 (2021)

# Rydberg atoms on site-kagome lattice: theory

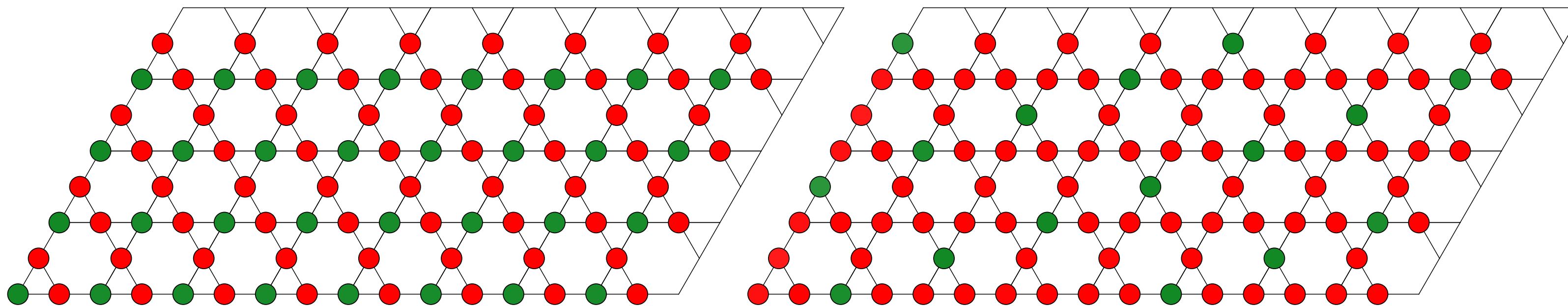


(b) Stripe:  $\delta = 2.2$ ,  $R_b = 1.2$



(c) Nematic:  $\delta = 3.3$ ,  $R_b = 1.7$

(d) Staggered:  $\delta = 3.3$ ,  $R_b = 2.1$



R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **118**, e2015785118 (2021)

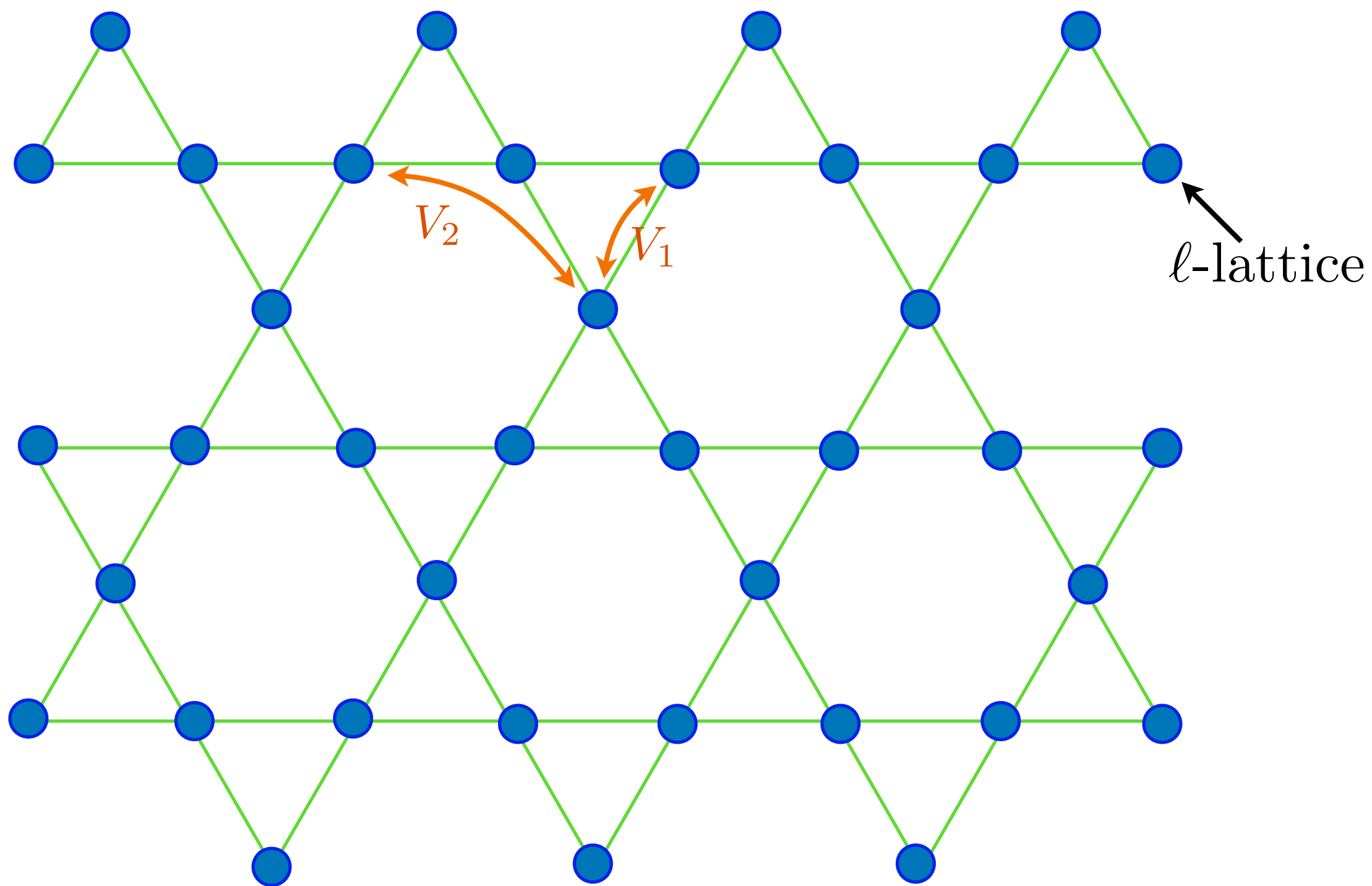
Topological spin liquid described by emergent  $\mathbb{Z}_2$  gauge theory?

# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell} \left[ \frac{\Omega}{2} (b_{\ell} + b_{\ell}^{\dagger}) - \Delta n_{\ell} \right] + \sum_{\ell < \ell'} V_{|\ell - \ell'|} n_{\ell} n_{\ell'}$$

$$n_{\ell} \equiv b_{\ell}^{\dagger} b_{\ell}$$

$n_{\ell} = 0, 1$  'hard core' bosons



Identify hard core bosons with a qubit  $\sigma^{x,y,z}$

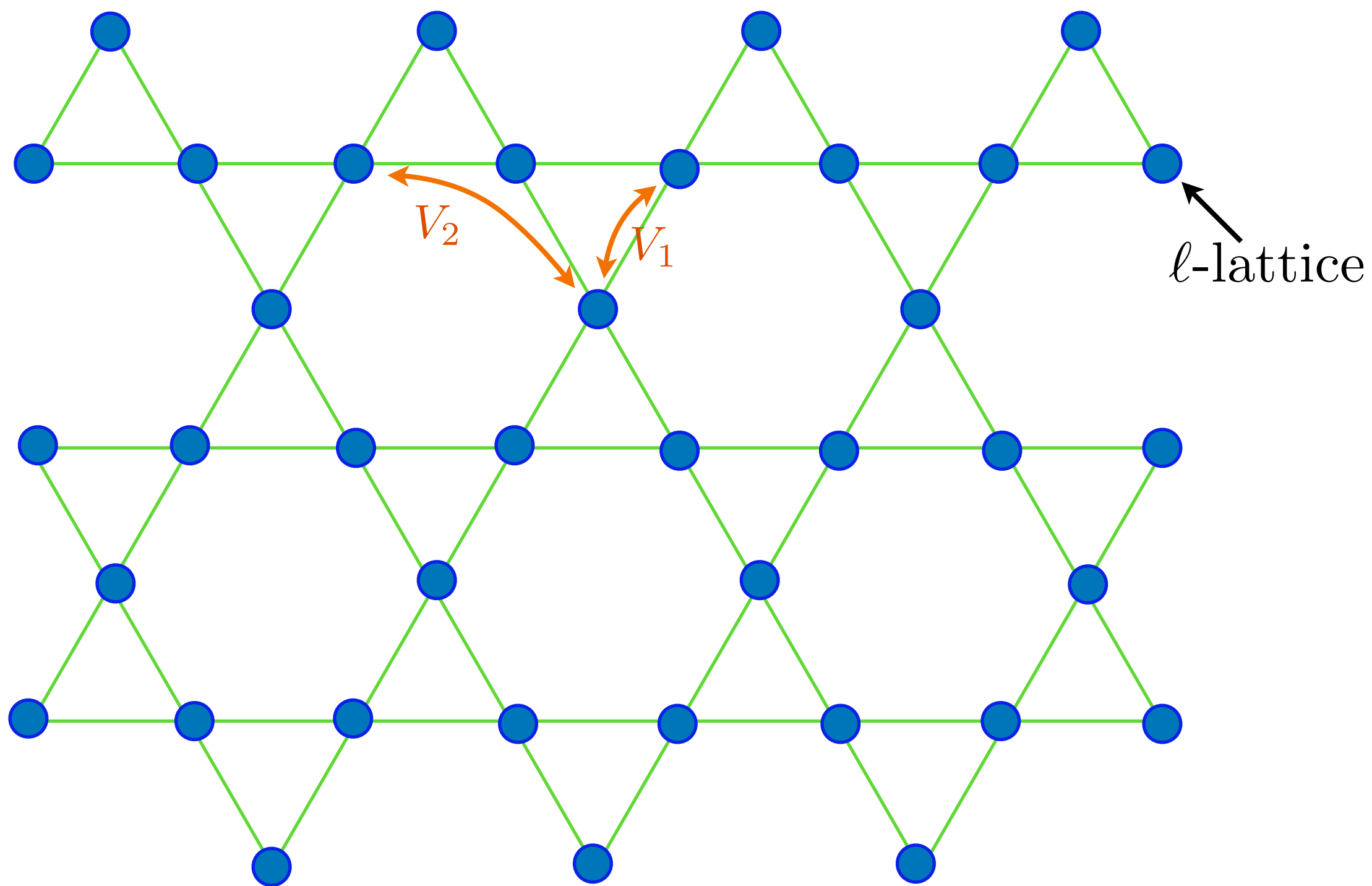
$$b_{\ell} + b_{\ell}^{\dagger} \Leftrightarrow \sigma_{\ell}^z$$

$$n_{\ell} \Leftrightarrow (1 - \sigma_{\ell}^x)/2$$

$\sigma^z$  will become the  $\mathbb{Z}_2$  gauge field

# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell} \left[ \frac{\Omega}{2} \sigma_{\ell}^z + \frac{\Delta}{2} \sigma_{\ell}^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell - \ell'|}}{4} (1 - \sigma_{\ell}^x)(1 - \sigma_{\ell'}^x)$$



Identify hard core bosons with a qubit  $\sigma^{x,y,z}$

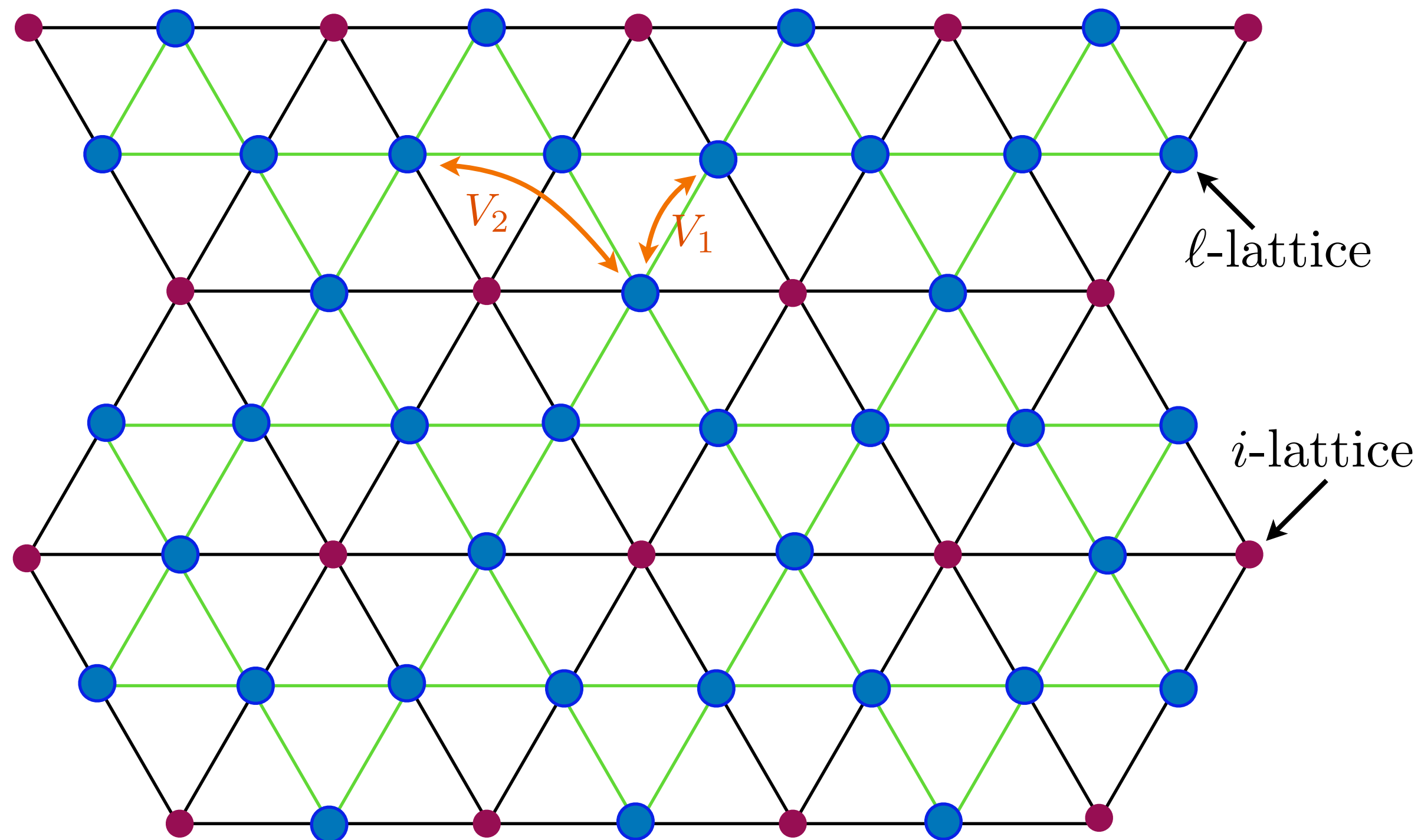
$$\begin{aligned} b_{\ell} + b_{\ell}^{\dagger} &\Leftrightarrow \sigma_{\ell}^z \\ n_{\ell} &\Leftrightarrow (1 - \sigma_{\ell}^x)/2 \end{aligned}$$

$\sigma^z$  will become the  $\mathbb{Z}_2$  gauge field

# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[ \frac{\Omega}{2} \tau_i^z \sigma_\ell^z \tau_j^z + \frac{\Delta}{2} \sigma_\ell^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - \sigma_\ell^x)(1 - \sigma_{\ell'}^x)$$

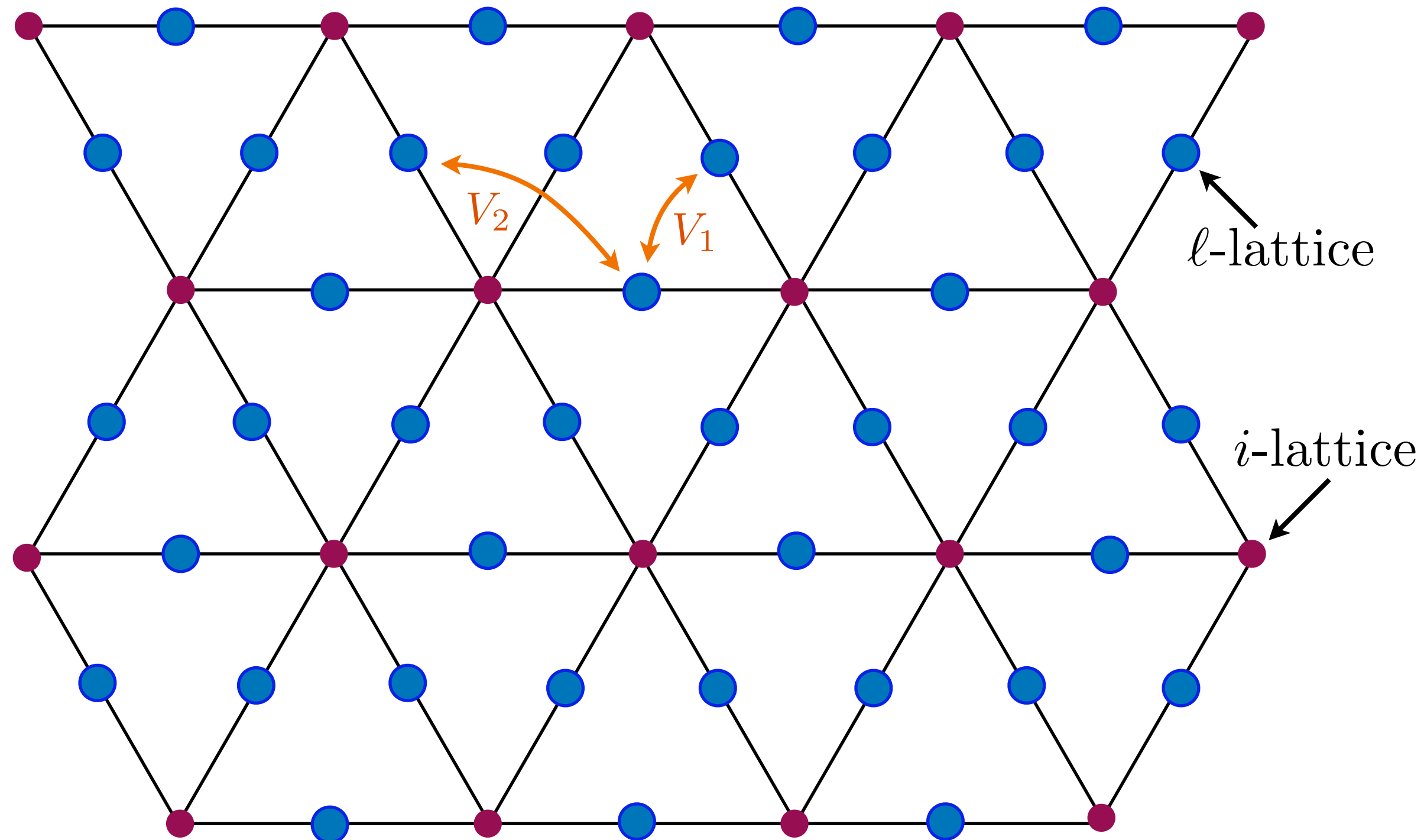
Introduce  $\mathbb{Z}_2$  matter fields on ' $i$  sites'. Gauge invariance:  $\tau_i^z \rightarrow \rho_i \tau_i^z$ ,  $\sigma_{ij}^z \rightarrow \rho_i \sigma_{ij}^z \rho_j$ ,  
 $\tau_i^x \rightarrow \tau_i^x$ ,  $\sigma_i^x \rightarrow \sigma_i^x$ ,  $\rho_i = \pm 1$ .



# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[ \frac{\Omega}{2} \tau_i^z \sigma_\ell^z \tau_j^z + \frac{\Delta}{2} \sigma_\ell^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - \sigma_\ell^x)(1 - \sigma_{\ell'}^x) - \sum_{\text{loops}} K_{\text{loop}} \prod_{\ell_1, \ell_2, \ell_3 \dots \in \text{loop}} \sigma_{\ell_1}^z \sigma_{\ell_2}^z \sigma_{\ell_3}^z \dots$$

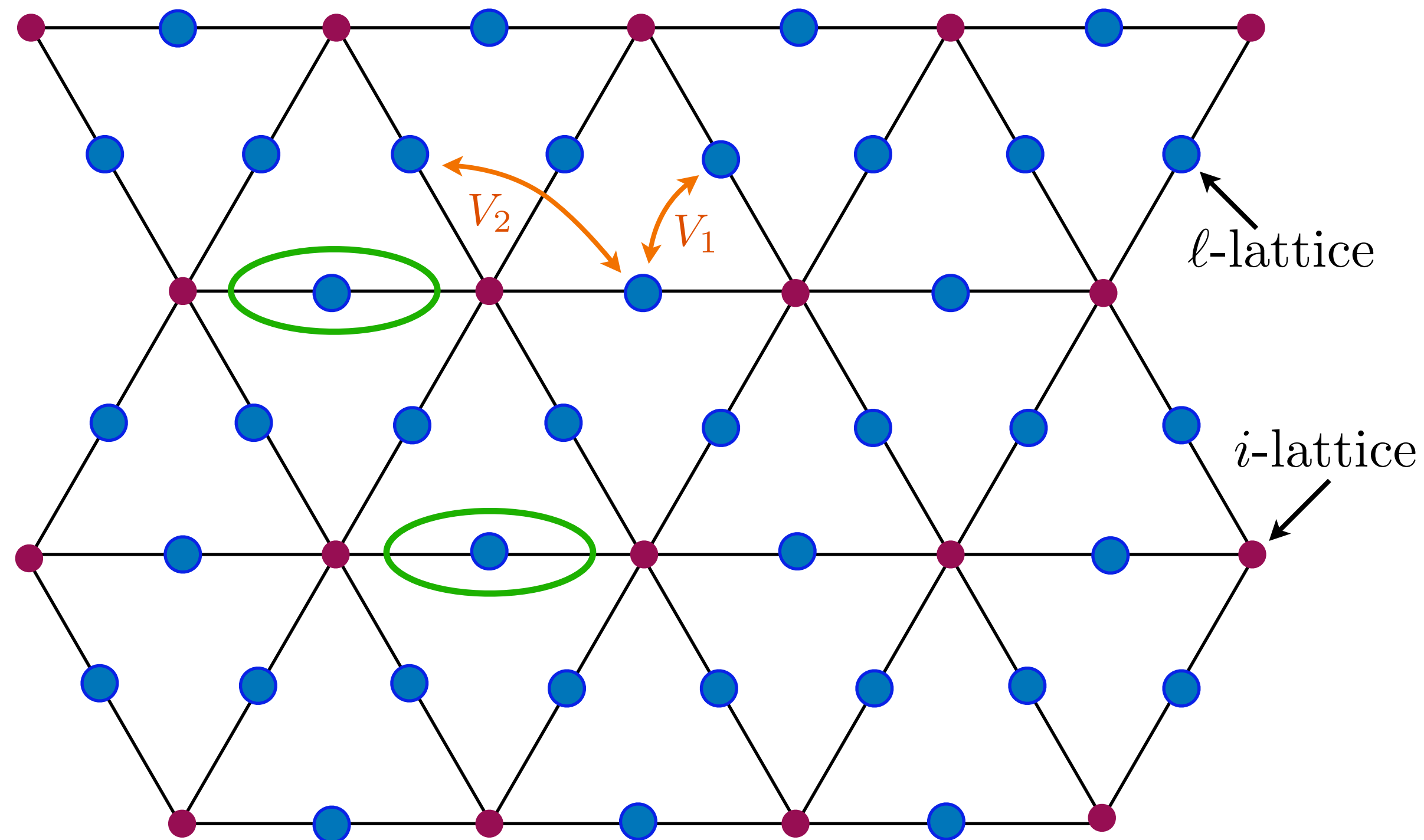
The  $K_{\text{loop}}$  terms are generated in a large  $V$  expansion: ‘resonance’ between Rydberg states stabilizes a phase with deconfined  $\mathbb{Z}_2$  gauge charges *i.e.* a  $\mathbb{Z}_2$  spin liquid



# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[ \frac{\Omega}{2} \tau_i^z \sigma_\ell^z \tau_j^z + \frac{\Delta}{2} \sigma_\ell^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - \sigma_\ell^x)(1 - \sigma_{\ell'}^x) - \sum_{\text{loops}} K_{\text{loop}} \prod_{\ell_1, \ell_2, \ell_3 \dots \in \text{loop}} \sigma_{\ell_1}^z \sigma_{\ell_2}^z \sigma_{\ell_3}^z \dots$$

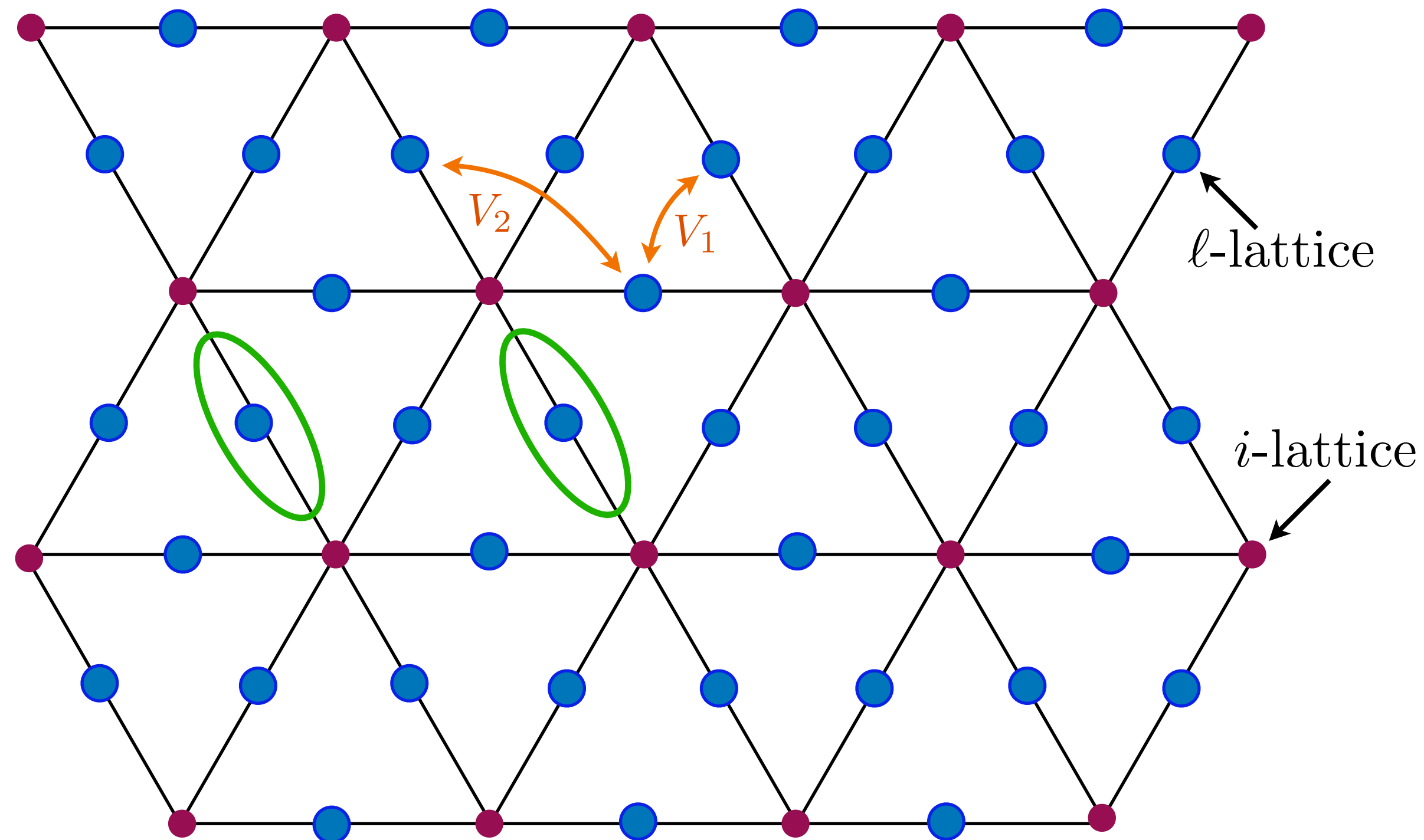
The  $K_{\text{loop}}$  terms are generated in a large  $V$  expansion: ‘resonance’ between Rydberg states stabilizes a phase with deconfined  $\mathbb{Z}_2$  gauge charges *i.e.* a  $\mathbb{Z}_2$  spin liquid



# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[ \frac{\Omega}{2} \tau_i^z \sigma_\ell^z \tau_j^z + \frac{\Delta}{2} \sigma_\ell^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - \sigma_\ell^x)(1 - \sigma_{\ell'}^x) - \sum_{\text{loops}} K_{\text{loop}} \prod_{\ell_1, \ell_2, \ell_3 \dots \in \text{loop}} \sigma_{\ell_1}^z \sigma_{\ell_2}^z \sigma_{\ell_3}^z \dots$$

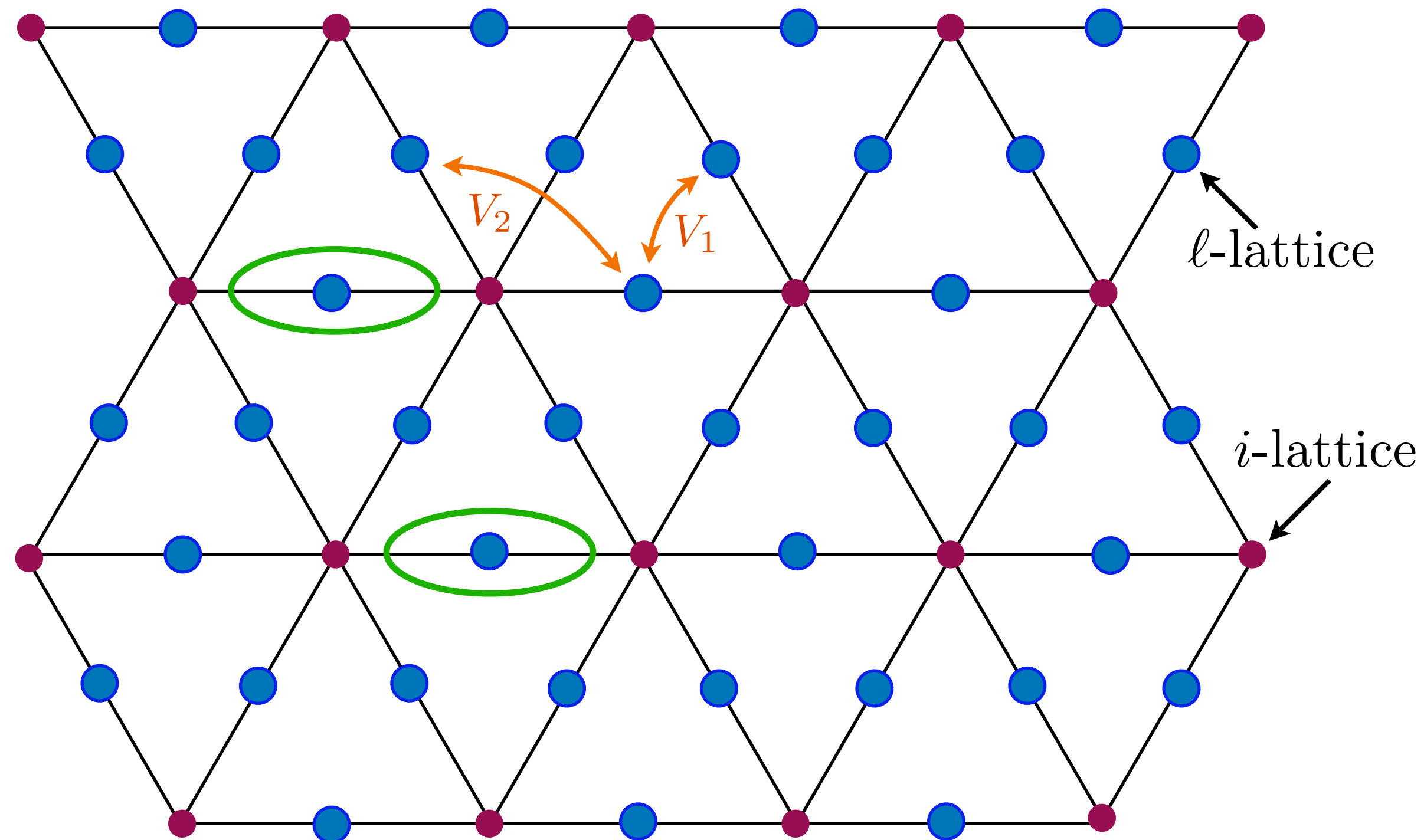
The  $K_{\text{loop}}$  terms are generated in a large  $V$  expansion: ‘resonance’ between Rydberg states stabilizes a phase with deconfined  $\mathbb{Z}_2$  gauge charges *i.e.* a  $\mathbb{Z}_2$  spin liquid



# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[ \frac{\Omega}{2} \tau_i^z \sigma_\ell^z \tau_j^z + \frac{\Delta}{2} \sigma_\ell^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - \sigma_\ell^x)(1 - \sigma_{\ell'}^x) - \sum_{\text{loops}} K_{\text{loop}} \prod_{\ell_1, \ell_2, \ell_3 \dots \in \text{loop}} \sigma_{\ell_1}^z \sigma_{\ell_2}^z \sigma_{\ell_3}^z \dots$$

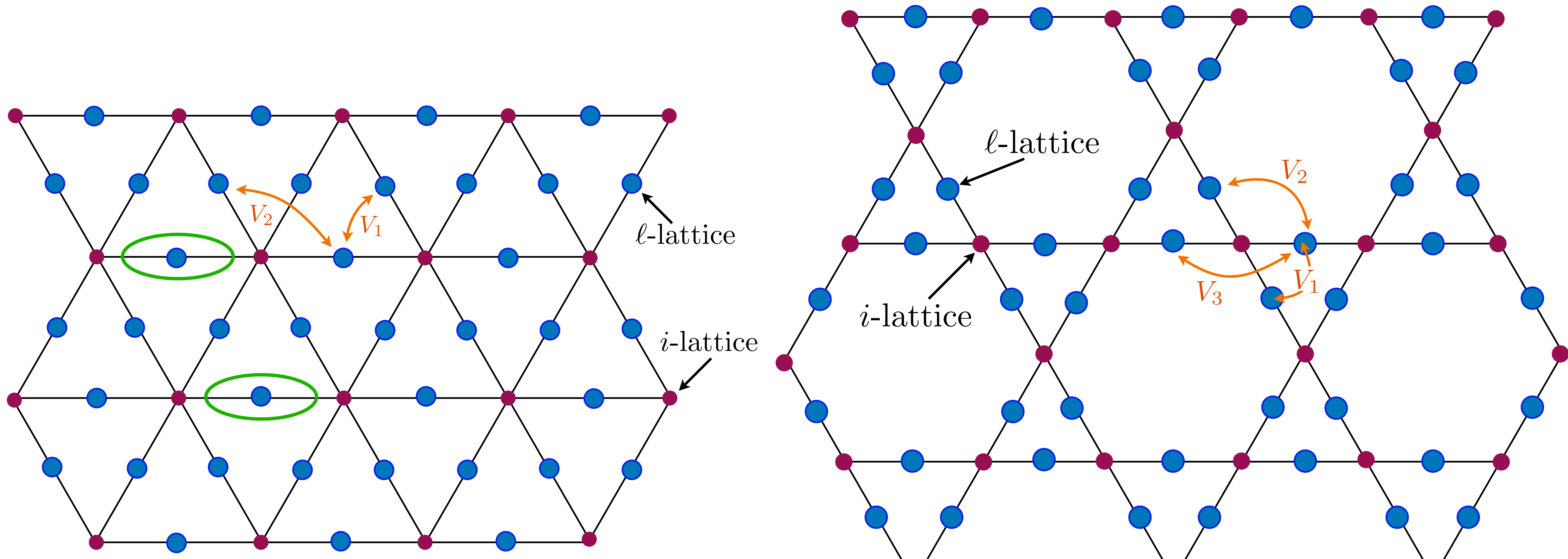
The  $K_{\text{loop}}$  terms are generated in a large  $V$  expansion: ‘resonance’ between Rydberg states stabilizes a phase with deconfined  $\mathbb{Z}_2$  gauge charges *i.e.* a  $\mathbb{Z}_2$  spin liquid



# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[ \frac{\Omega}{2} \tau_i^z \sigma_\ell^z \tau_j^z + \frac{\Delta}{2} \sigma_\ell^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - \sigma_\ell^x)(1 - \sigma_{\ell'}^x) - \sum_{\text{loops}} K_{\text{loop}} \prod_{\ell_1, \ell_2, \ell_3 \dots \in \text{loop}} \sigma_{\ell_1}^z \sigma_{\ell_2}^z \sigma_{\ell_3}^z \dots$$

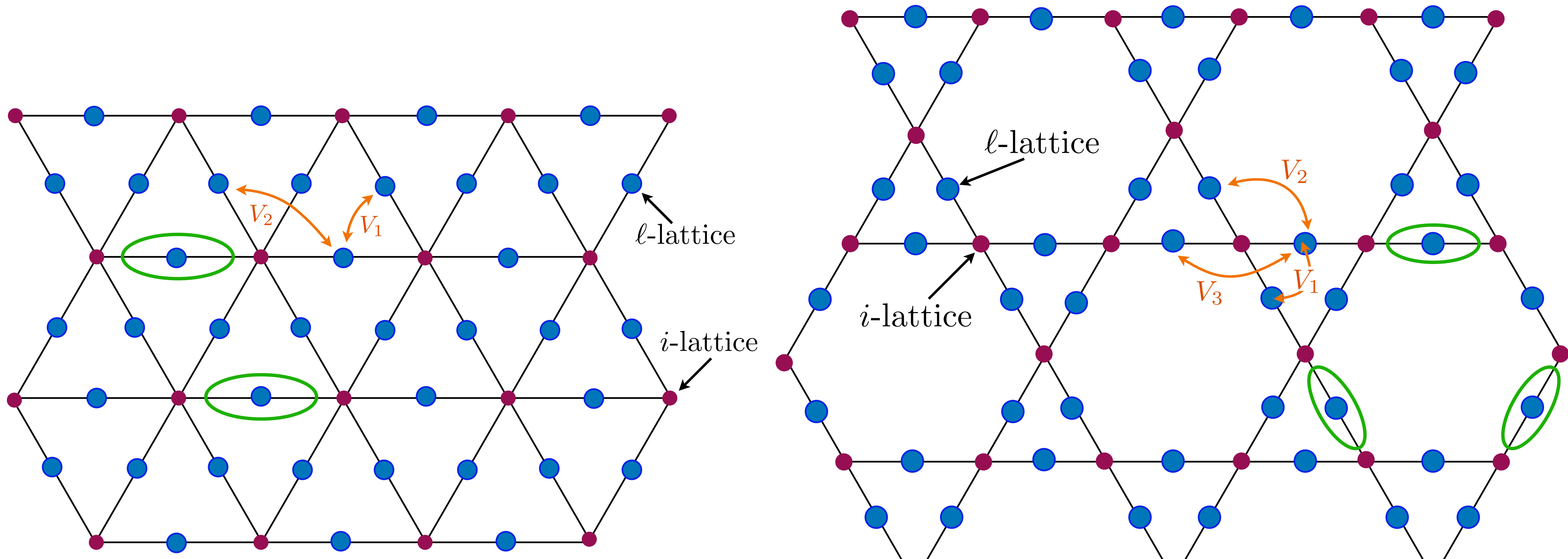
The  $K_{\text{loop}}$  terms are generated in a large  $V$  expansion: ‘resonance’ between Rydberg states stabilizes a phase with deconfined  $\mathbb{Z}_2$  gauge charges *i.e.* a  $\mathbb{Z}_2$  spin liquid



# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[ \frac{\Omega}{2} \tau_i^z \sigma_\ell^z \tau_j^z + \frac{\Delta}{2} \sigma_\ell^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - \sigma_\ell^x)(1 - \sigma_{\ell'}^x) - \sum_{\text{loops}} K_{\text{loop}} \prod_{\ell_1, \ell_2, \ell_3 \dots \in \text{loop}} \sigma_{\ell_1}^z \sigma_{\ell_2}^z \sigma_{\ell_3}^z \dots$$

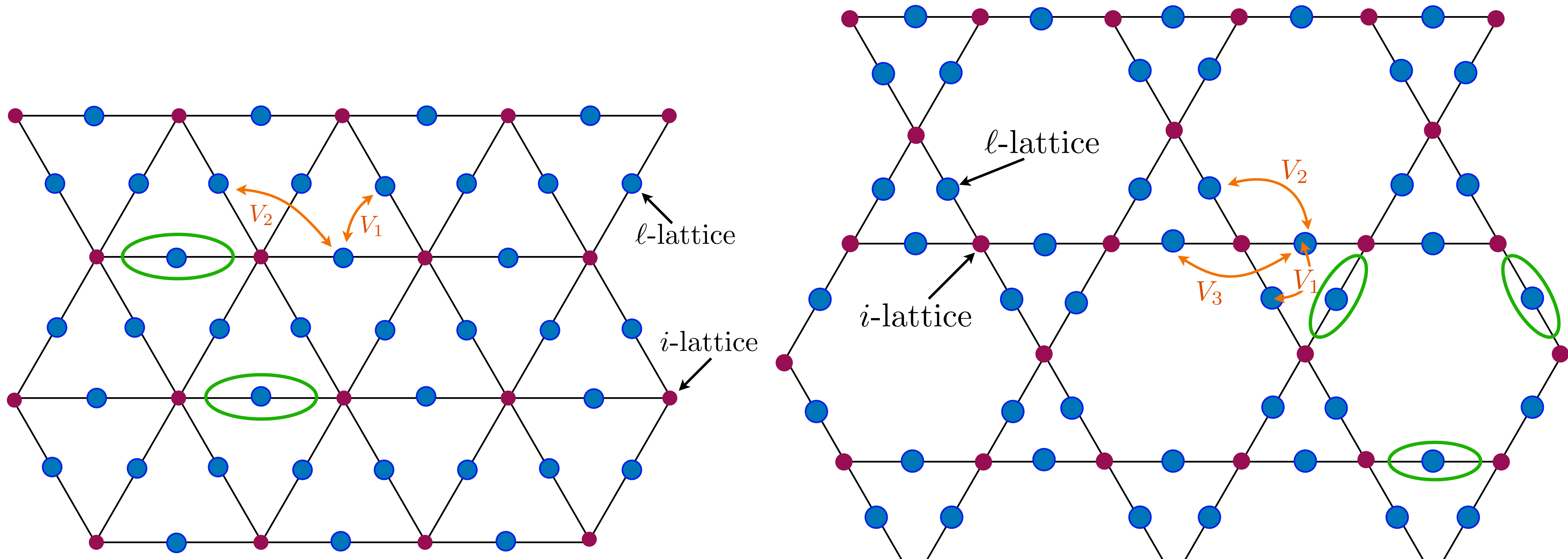
The  $K_{\text{loop}}$  terms are generated in a large  $V$  expansion: ‘resonance’ between Rydberg states stabilizes a phase with deconfined  $\mathbb{Z}_2$  gauge charges *i.e.* a  $\mathbb{Z}_2$  spin liquid



# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[ \frac{\Omega}{2} \tau_i^z \sigma_\ell^z \tau_j^z + \frac{\Delta}{2} \sigma_\ell^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - \sigma_\ell^x)(1 - \sigma_{\ell'}^x) - \sum_{\text{loops}} K_{\text{loop}} \prod_{\ell_1, \ell_2, \ell_3 \dots \in \text{loop}} \sigma_{\ell_1}^z \sigma_{\ell_2}^z \sigma_{\ell_3}^z \dots$$

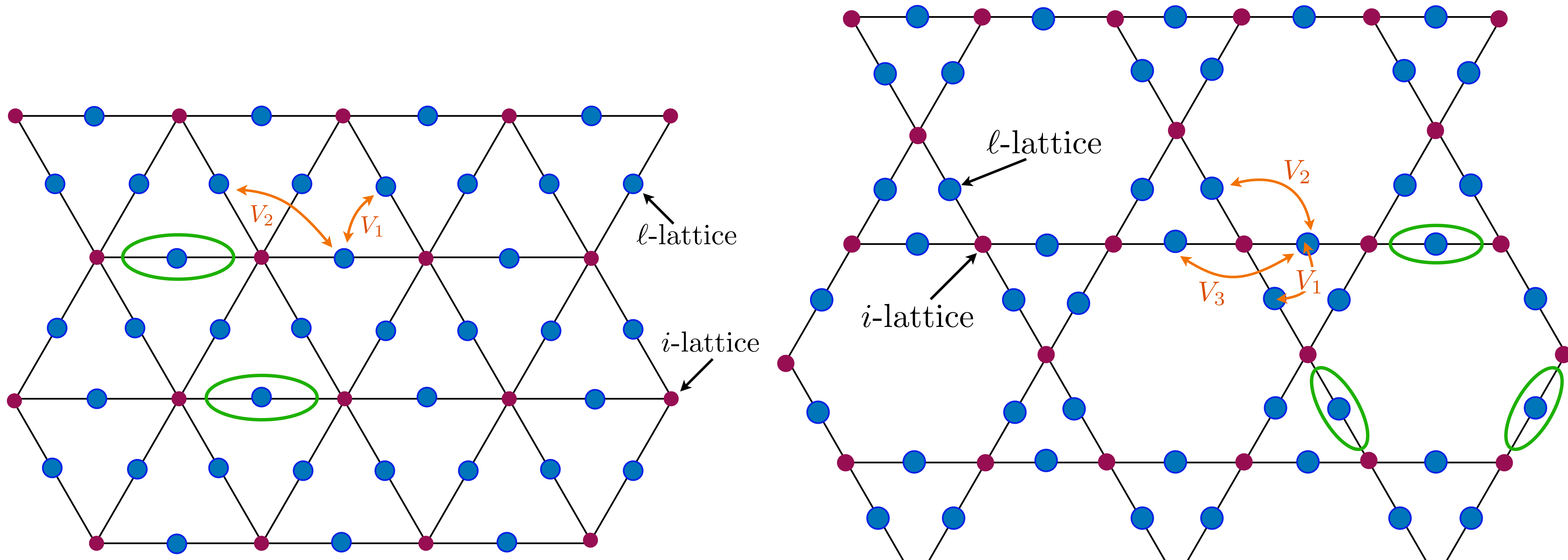
The  $K_{\text{loop}}$  terms are generated in a large  $V$  expansion: ‘resonance’ between Rydberg states stabilizes a phase with deconfined  $\mathbb{Z}_2$  gauge charges *i.e.* a  $\mathbb{Z}_2$  spin liquid



# From the FSS model to an emergent $\mathbb{Z}_2$ gauge theory

$$\mathcal{H} = \sum_{\ell \in (i,j)} \left[ \frac{\Omega}{2} \tau_i^z \sigma_\ell^z \tau_j^z + \frac{\Delta}{2} \sigma_\ell^x \right] + \sum_{\ell < \ell'} \frac{V_{|\ell-\ell'|}}{4} (1 - \sigma_\ell^x)(1 - \sigma_{\ell'}^x) - \sum_{\text{loops}} K_{\text{loop}} \prod_{\ell_1, \ell_2, \ell_3 \dots \in \text{loop}} \sigma_{\ell_1}^z \sigma_{\ell_2}^z \sigma_{\ell_3}^z \dots$$

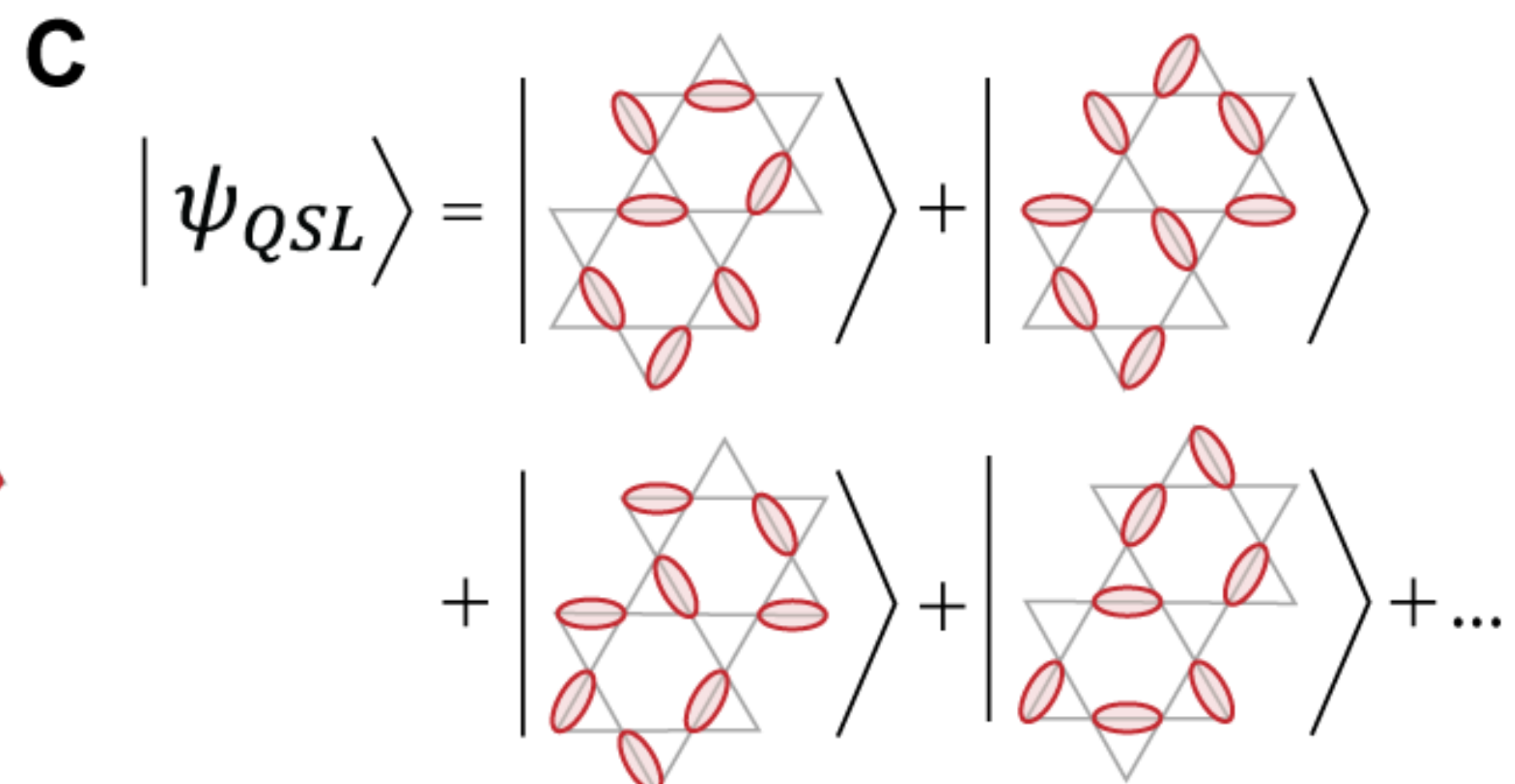
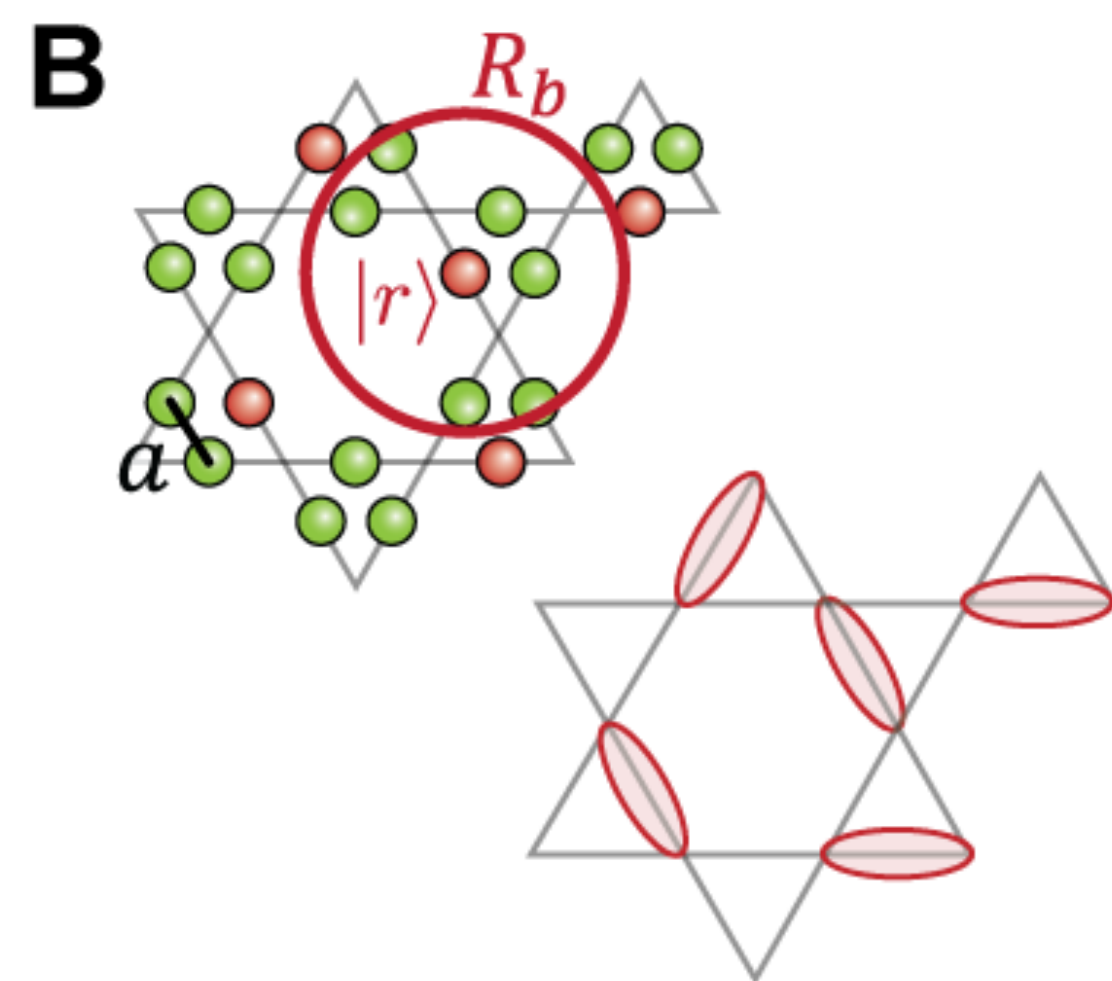
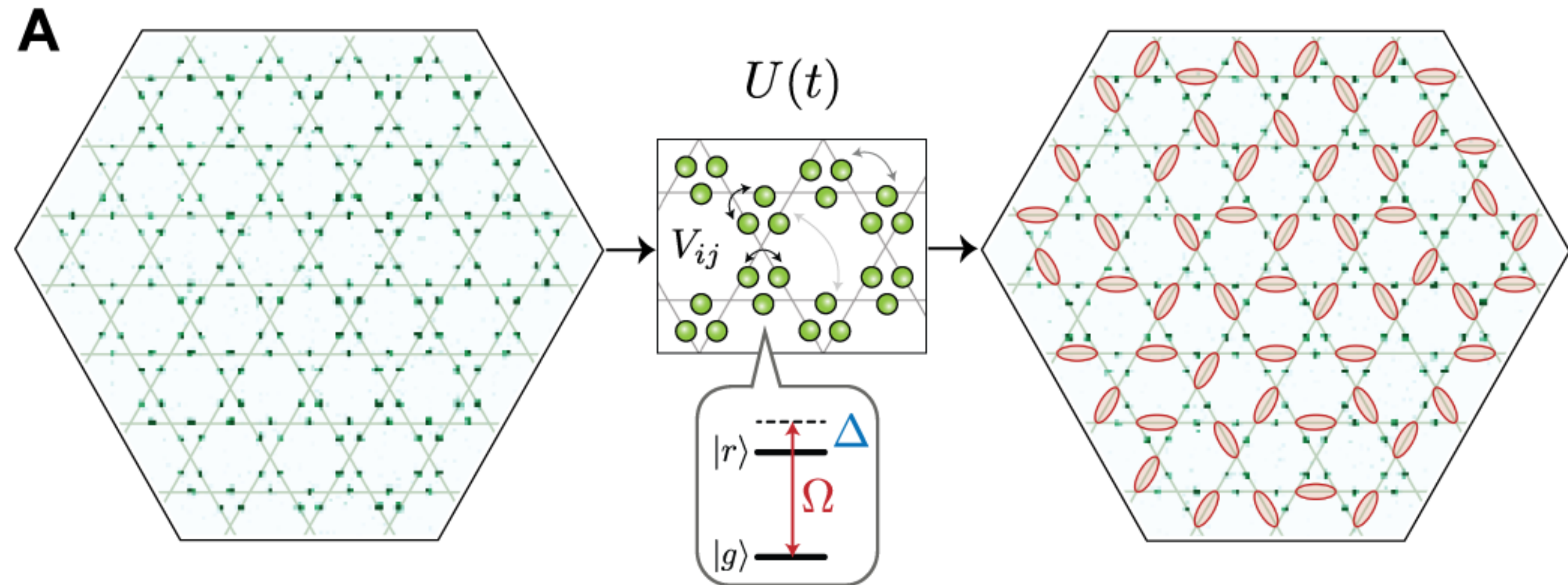
The  $K_{\text{loop}}$  terms are generated in a large  $V$  expansion: ‘resonance’ between Rydberg states stabilizes a phase with deconfined  $\mathbb{Z}_2$  gauge charges *i.e.* a  $\mathbb{Z}_2$  spin liquid



# Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T.T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, arXiv:2104.04119; Science, to appear.

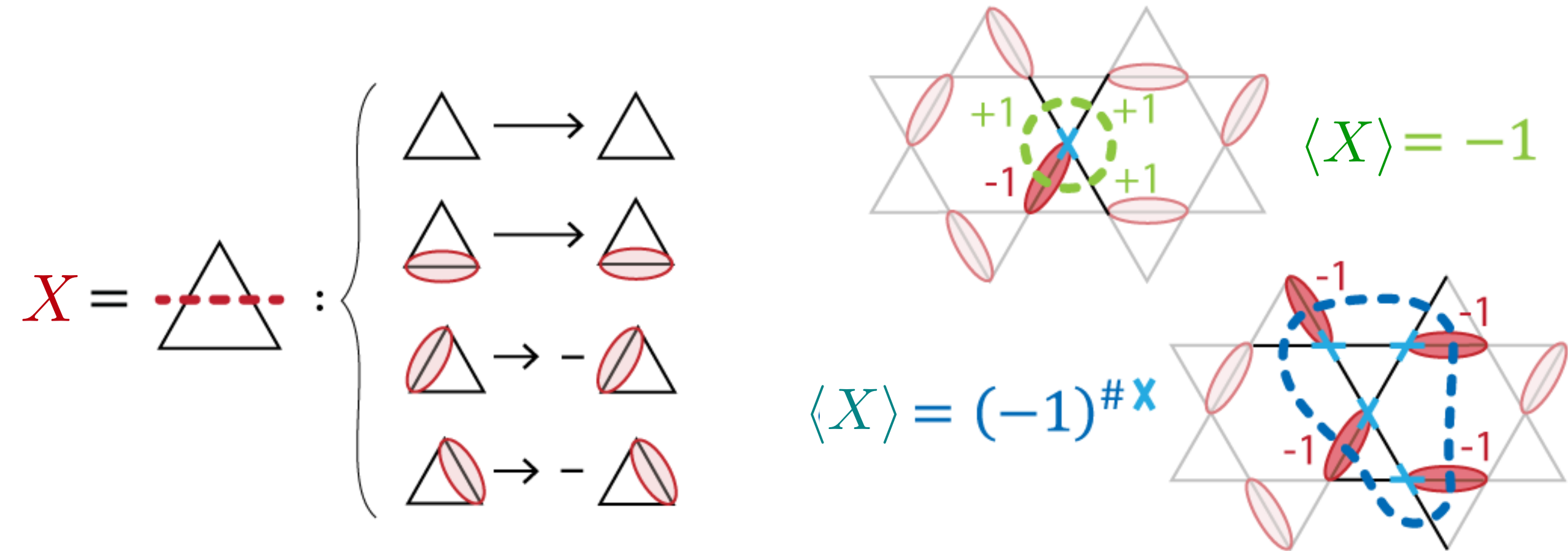
Rydberg atoms  
on the  
link-kagome lattice:  
experiment



# Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T.T.Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, arXiv:2104.04119; Science, to appear.

Rydberg atoms  
on the  
link-kagome lattice:  
experiment

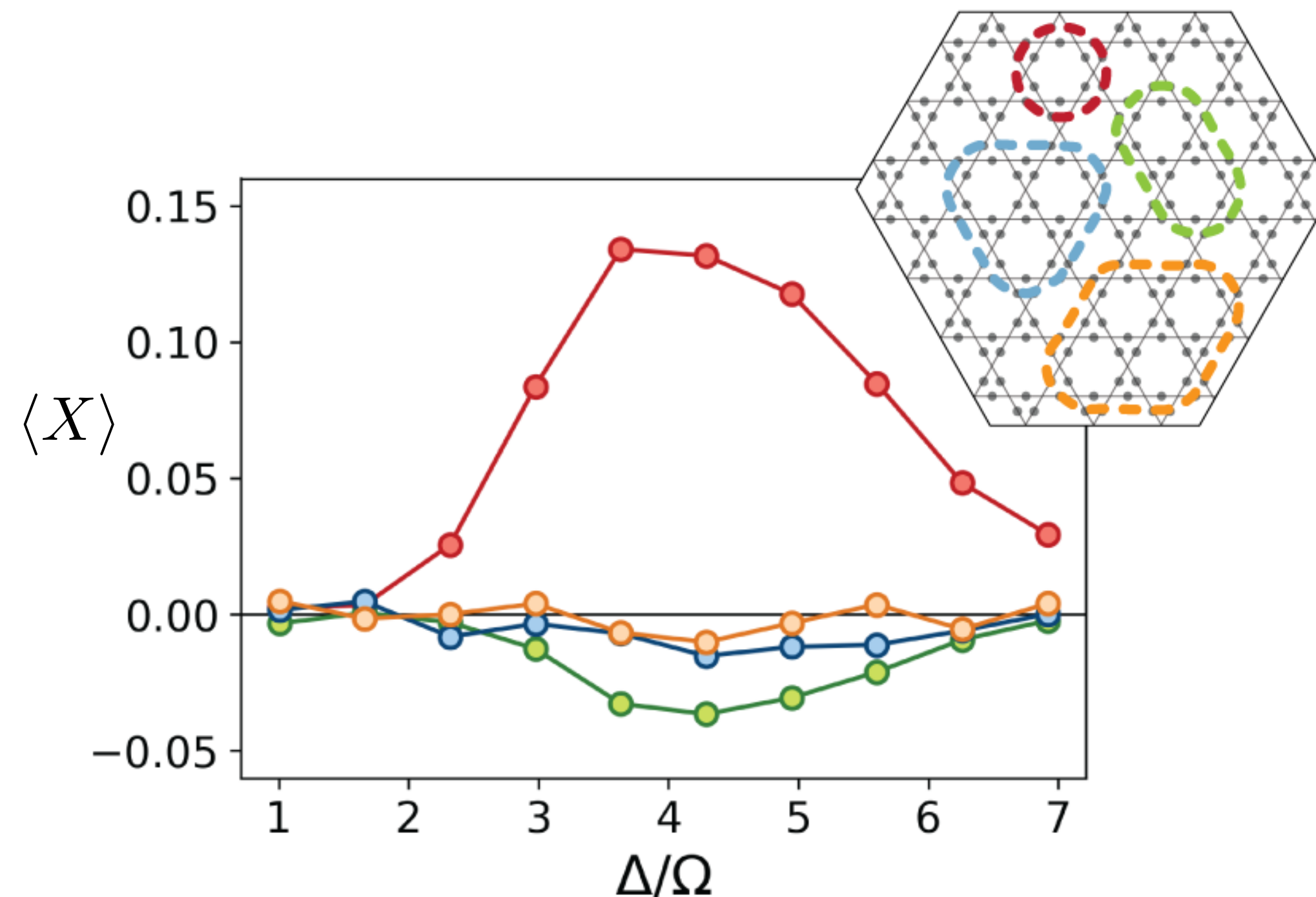


Measurement of  
the topological

$X$  operator

$$= \prod_{\text{loop}} \sigma_{\ell}^x.$$

Detects close-packed dimers.

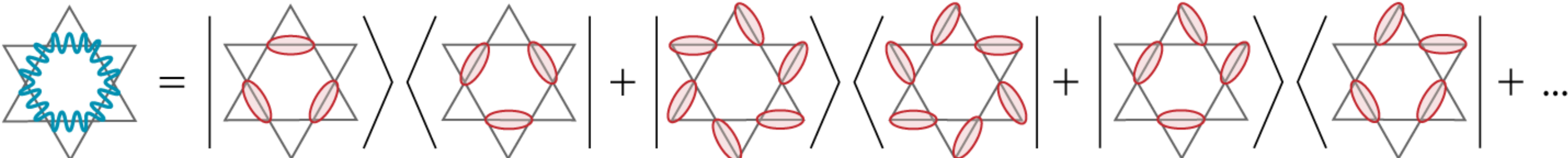


# Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T.T.Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, arXiv:2104.04119; Science, to appear.

Rydberg atoms  
on the  
link-kagome lattice:  
experiment

$$Z = \begin{array}{c} \triangle \\ \text{wavy line} \end{array} : \left\{ \begin{array}{l} \triangle \leftrightarrow (-1) \triangle \\ \triangle \leftrightarrow \triangle \end{array} \right.$$



Measurement of  
the topological  
 $Z$  operator.  
Detects resonance  
between dimer loops.

