Magnetic phases and critical points of insulators and superconductors

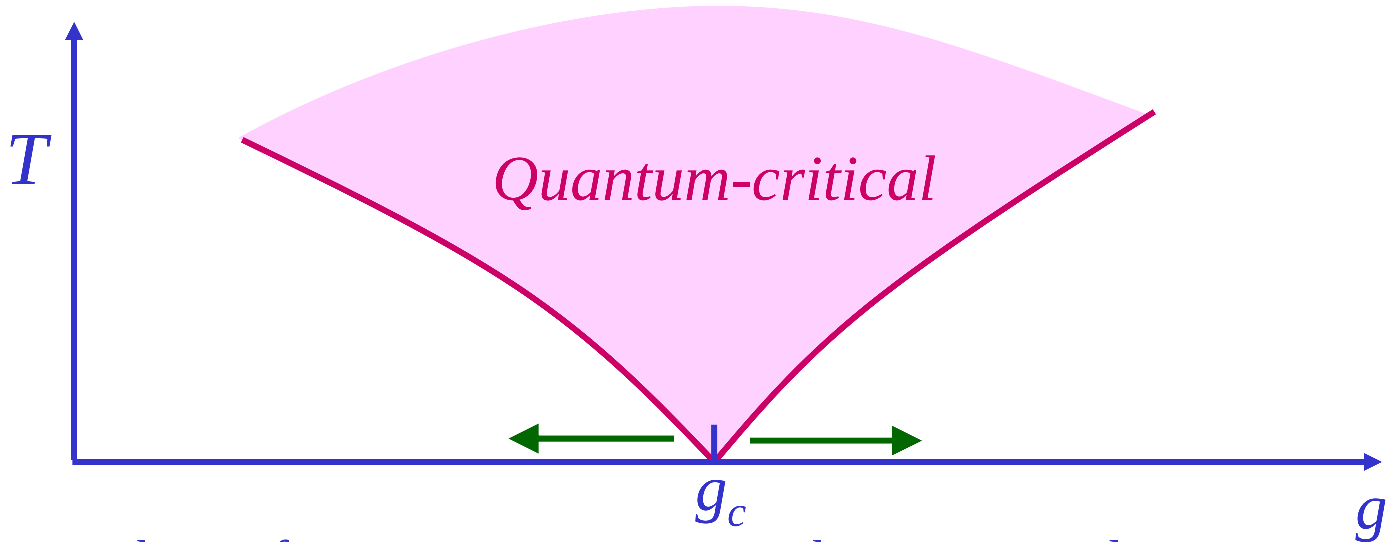
Colloquium article:
Reviews of Modern Physics, **75**, 913 (2003).

Talks online:
Google Sachdev

What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter g

Why study quantum phase transitions ?



- Theory for a quantum system with strong correlations: describe phases on either side of g_c by expanding in deviation from the quantum critical point.

- Critical point is a novel state of matter without quasiparticle excitations

- Critical excitations control dynamics in the wide *quantum-critical* region at non-zero temperatures.

Outline

A. Coupled dimer antiferromagnet

Effect of a magnetic field

B. Magnetic transitions in a superconductor

Effect of a magnetic field

C. Spin gap state on the square lattice

Spontaneous bond order

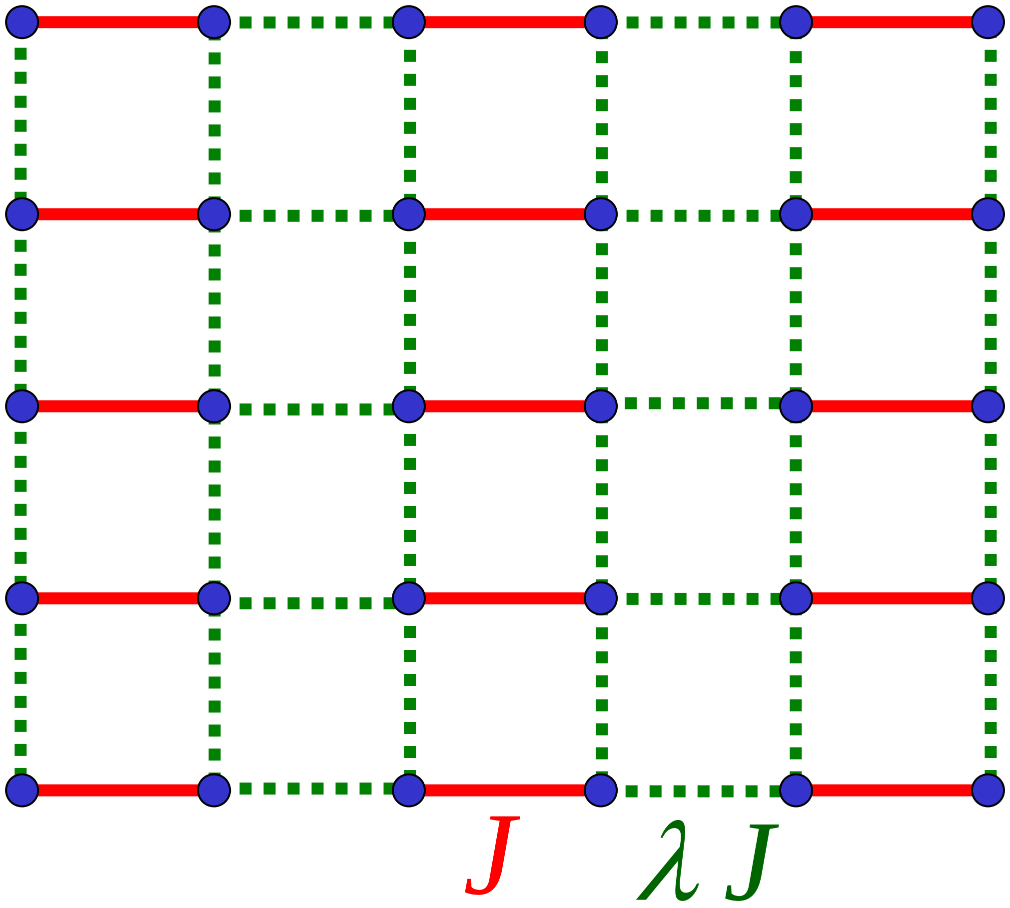
(A) Insulators

Coupled dimer antiferromagnet

Coupled Dimer Antiferromagnet

M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

S=1/2 spins on coupled dimers



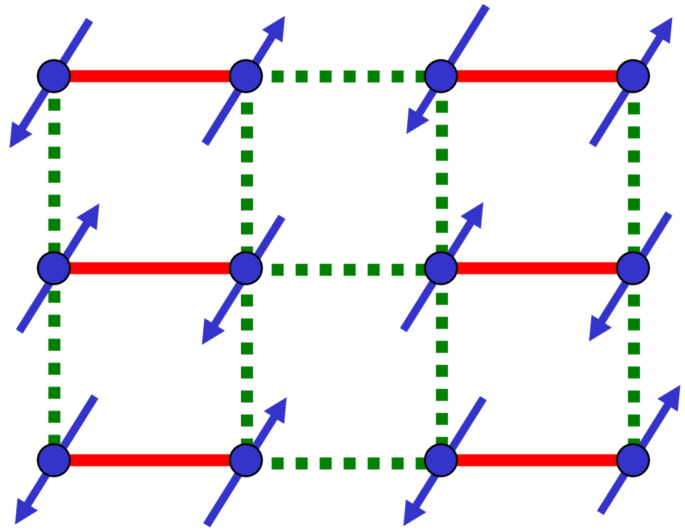
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



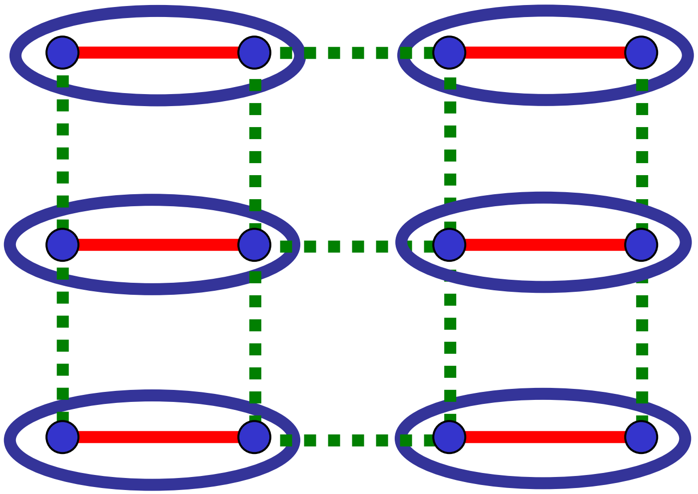
Ground state has long-range magnetic (Neel or spin density wave) order

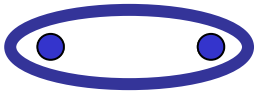
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x+i_y} N_0 \neq 0$$

Excitations: 2 spin waves (***magnons***) $\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$

$\boxed{\lambda \text{ close to } 0}$

Weakly coupled dimers

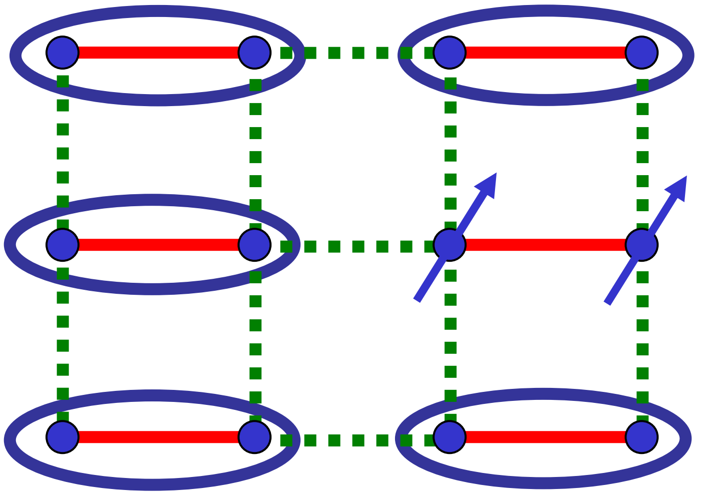


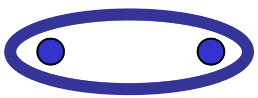
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Paramagnetic ground state $\left\langle \vec{S}_i \right\rangle = 0$

λ close to 0

Weakly coupled dimers



 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$

Excitation: S=1 ***triplon*** (***exciton***, spin collective mode)

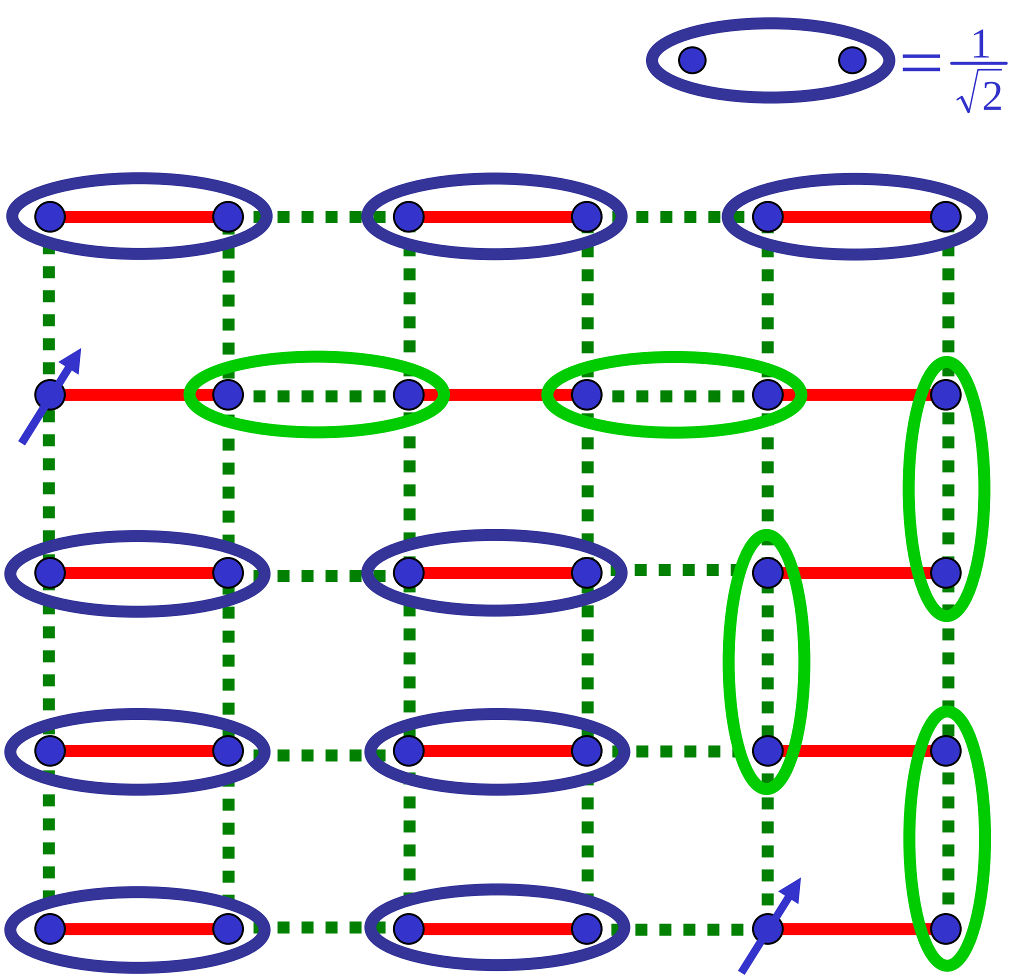
Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

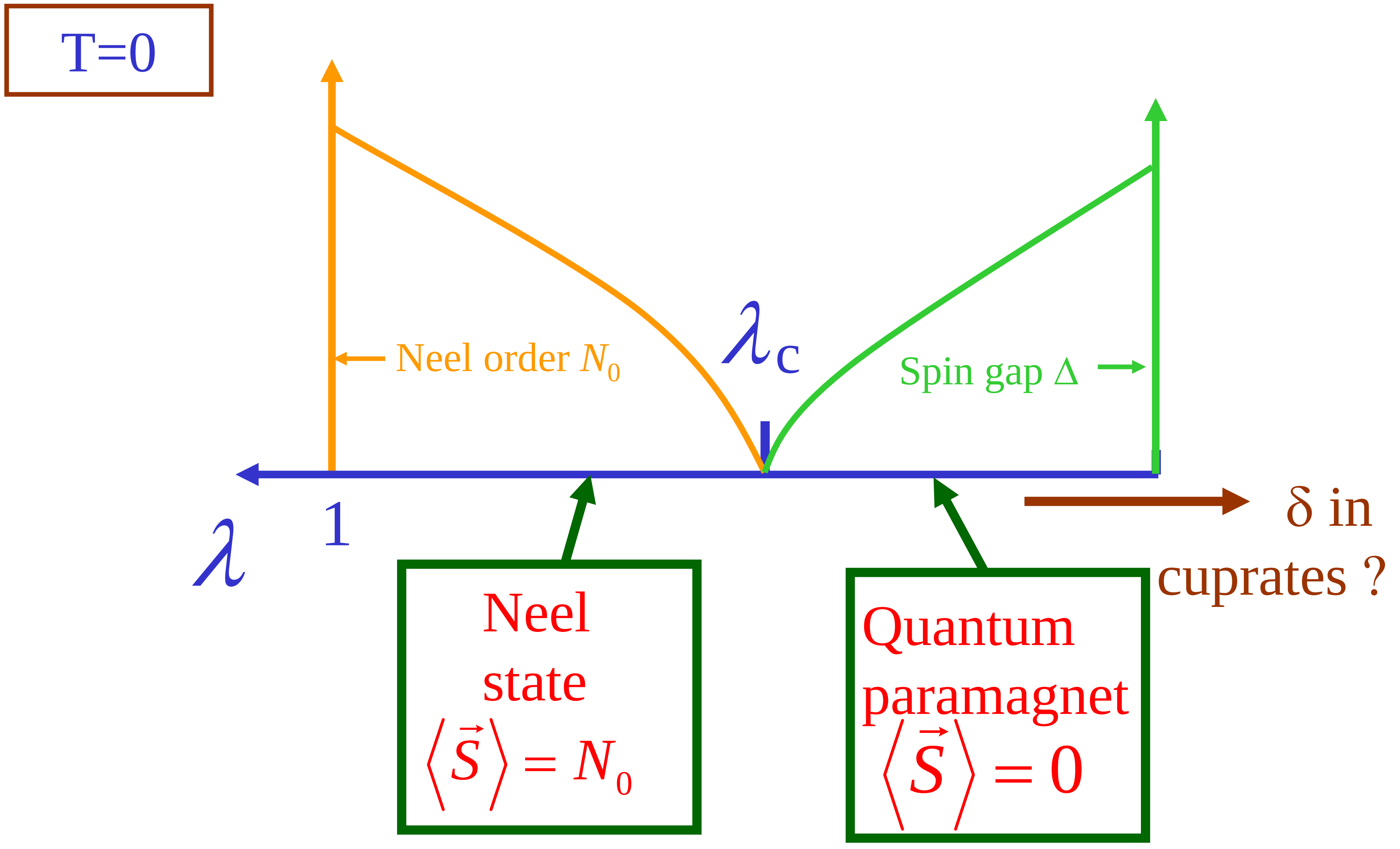
λ close to 0

Weakly coupled dimers



$$= \frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big)$$

S=1/2 ***spinons*** are <u>*confined*</u> by a linear potential into a *S*=1 ***triplon***

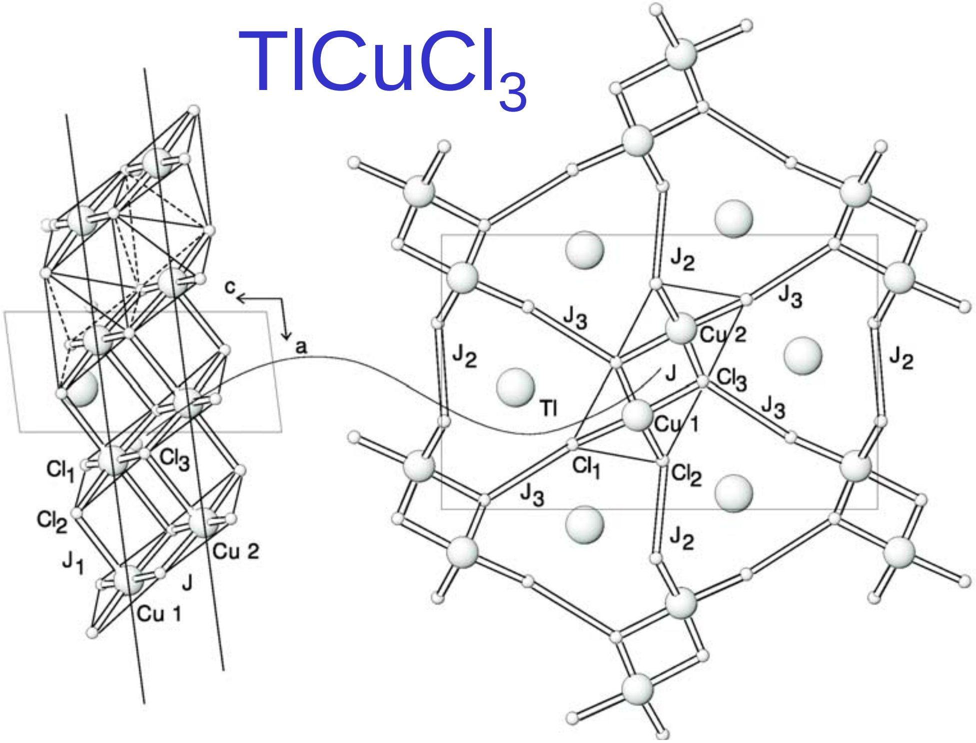


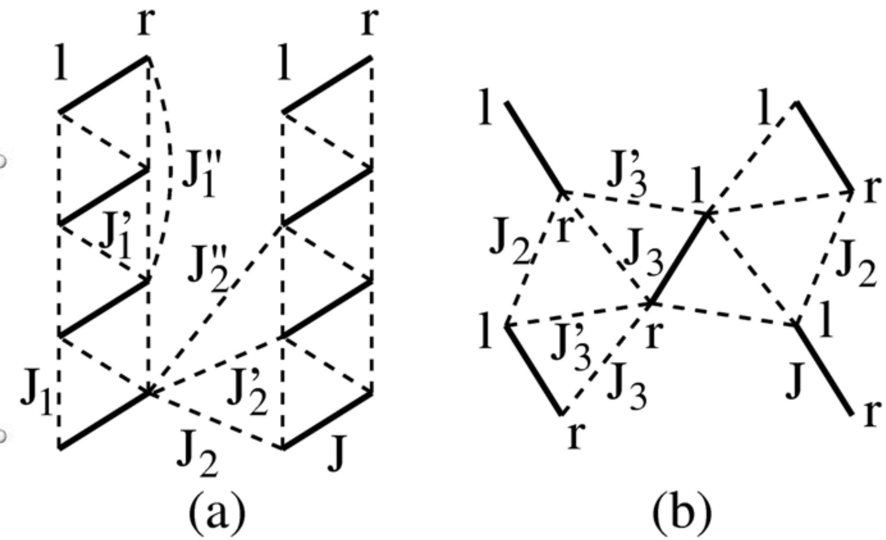
$$\lambda_c = 0.52337(3)$$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)

$TlCuCl_3$

M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.





Neutron Diffraction Study of the Pressure-Induced Magnetic Ordering in the Spin Gap System $TlCuCl_3$

Akira OOSAWA*, Masashi FUJISAWA[1], Toyotaka OSAKABE, Kazuhisa KAKURAI and Hidekazu TANAKA[2]

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[1]*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*
[2]*Research Center for Low Temperature Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*

(Received February 3, 2003)

Neutron elastic scattering measurements have been performed under a hydrostatic pressure in order to investigate the spin structure of the pressure-induced magnetic ordering in the spin gap system $TlCuCl_3$. Below the ordering temperature $T_N = 16.9$ K for the hydrostatic pressure $P = 1.48$ GPa, magnetic Bragg reflections were observed at reciprocal lattice points $\boldsymbol{Q} = (h, 0, l)$ with integer h and odd l, which are equivalent to those points with the lowest magnetic excitation energy at ambient pressure. This indicates that the spin gap closes due to the applied pressure. The spin structure of the pressure-induced magnetic ordered state for $P = 1.48$ GPa was determined.

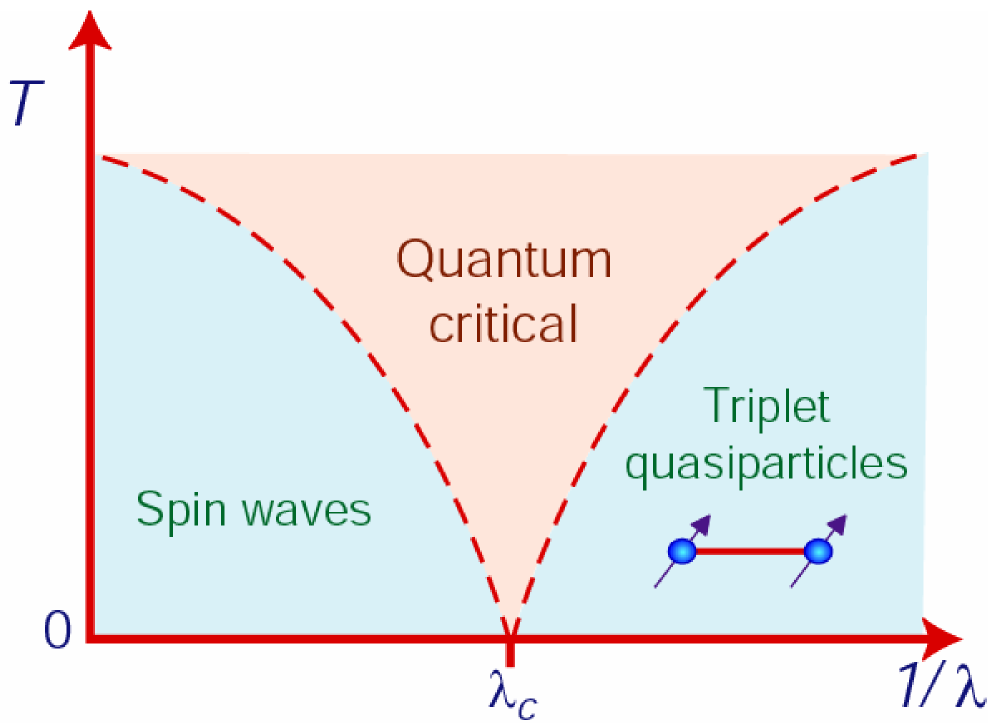
J. Phys. Soc. Jpn **72**, 1026 (2003)

Field theory for quantum criticality

λ close to λ_c : use "soft spin" field

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + (\lambda_c - \lambda)\phi_\alpha^2 \right) + \frac{u}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

$\phi_\alpha \longrightarrow$ 3-component antiferromagnetic order parameter



Quantum criticality described by strongly-coupled critical theory with universal dynamic response functions dependent on $\hbar\omega/k_B T$

$$\chi(\omega, T) = T^\eta g\left(\hbar\omega/k_B T\right)$$

Triplon scattering amplitude is determined by $k_B T$ alone, and not by the value of microscopic coupling u

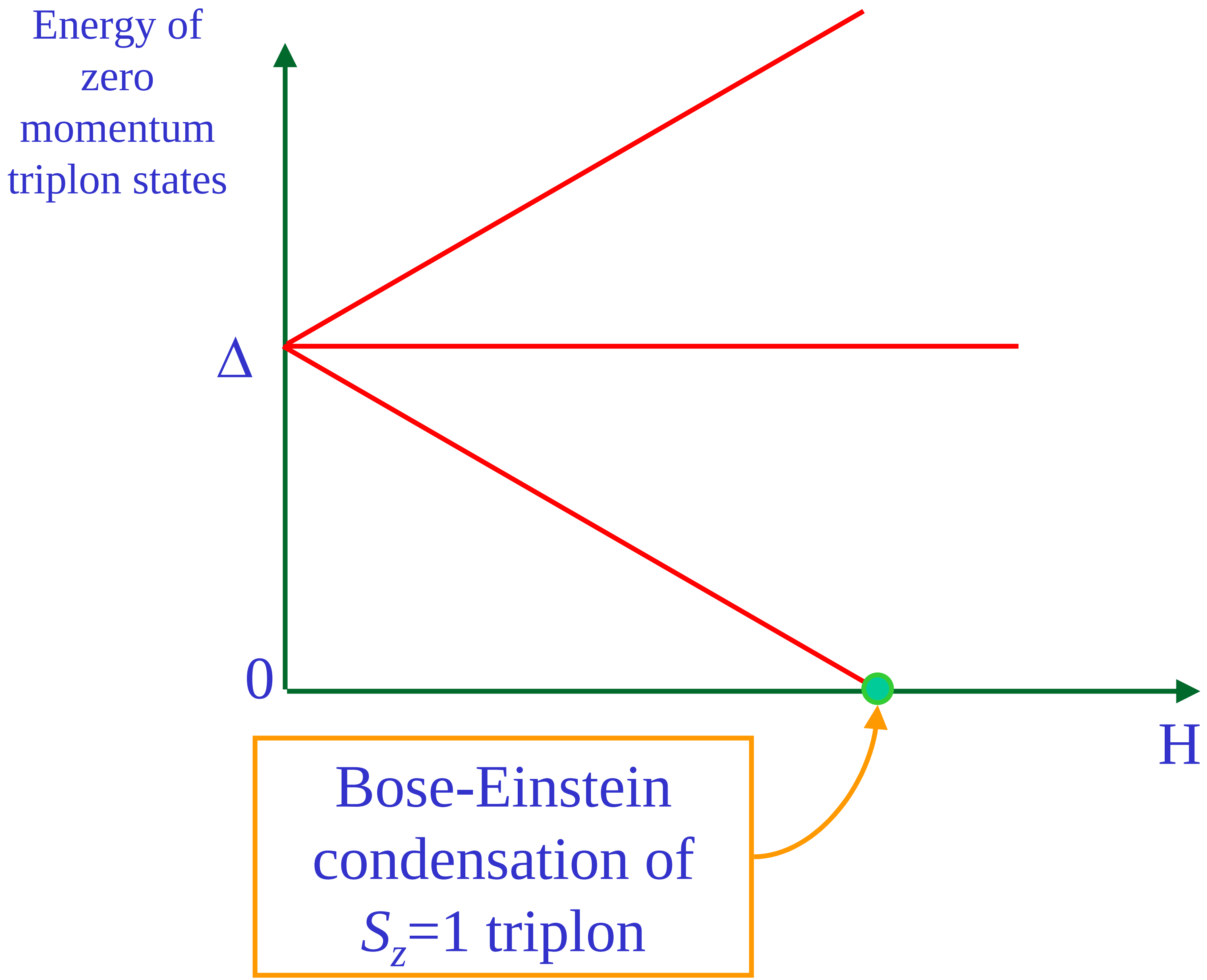
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

(A) Insulators

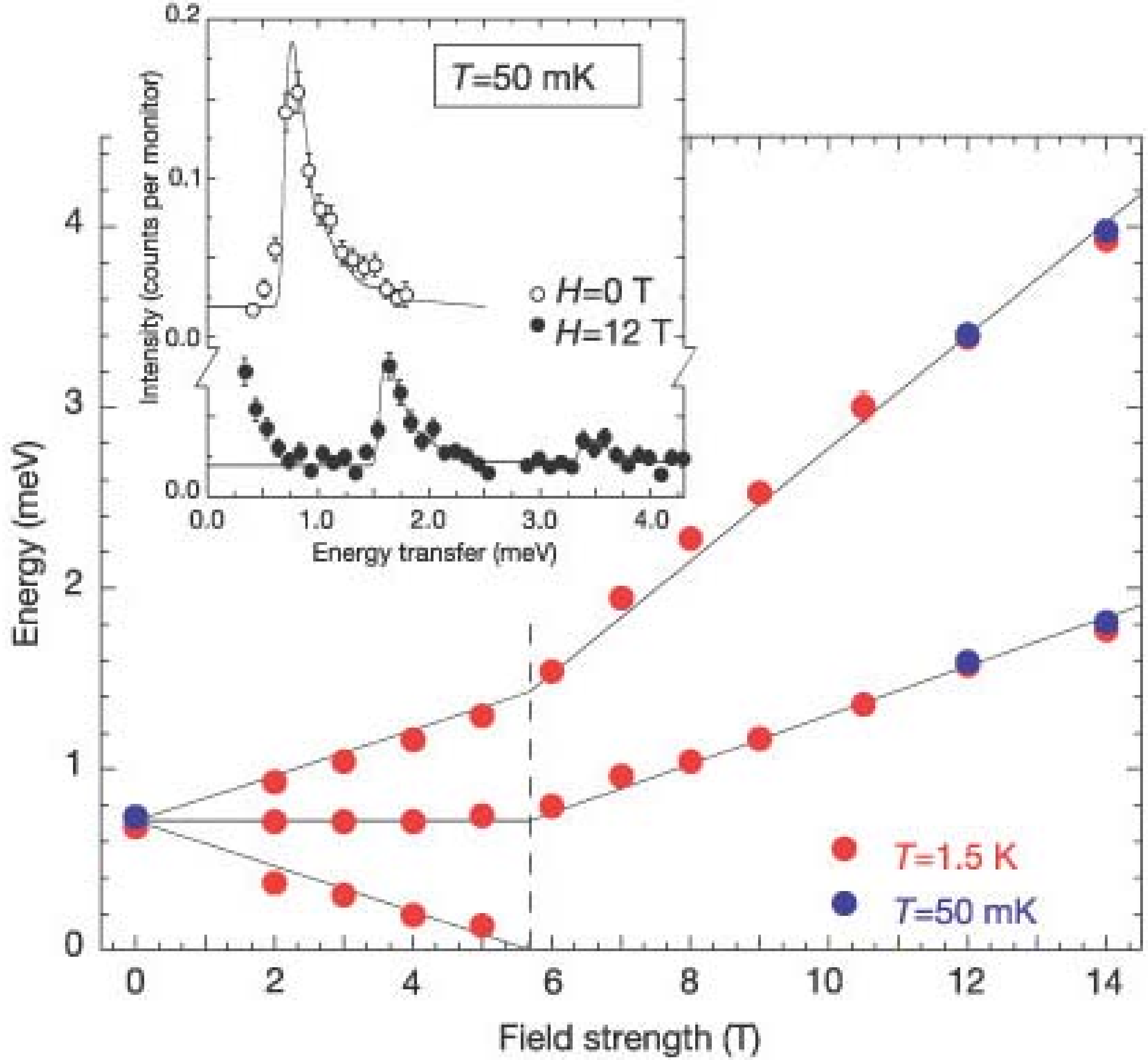
Coupled dimer antiferromagnet:

effect of a magnetic field.

Effect of a field on paramagnet

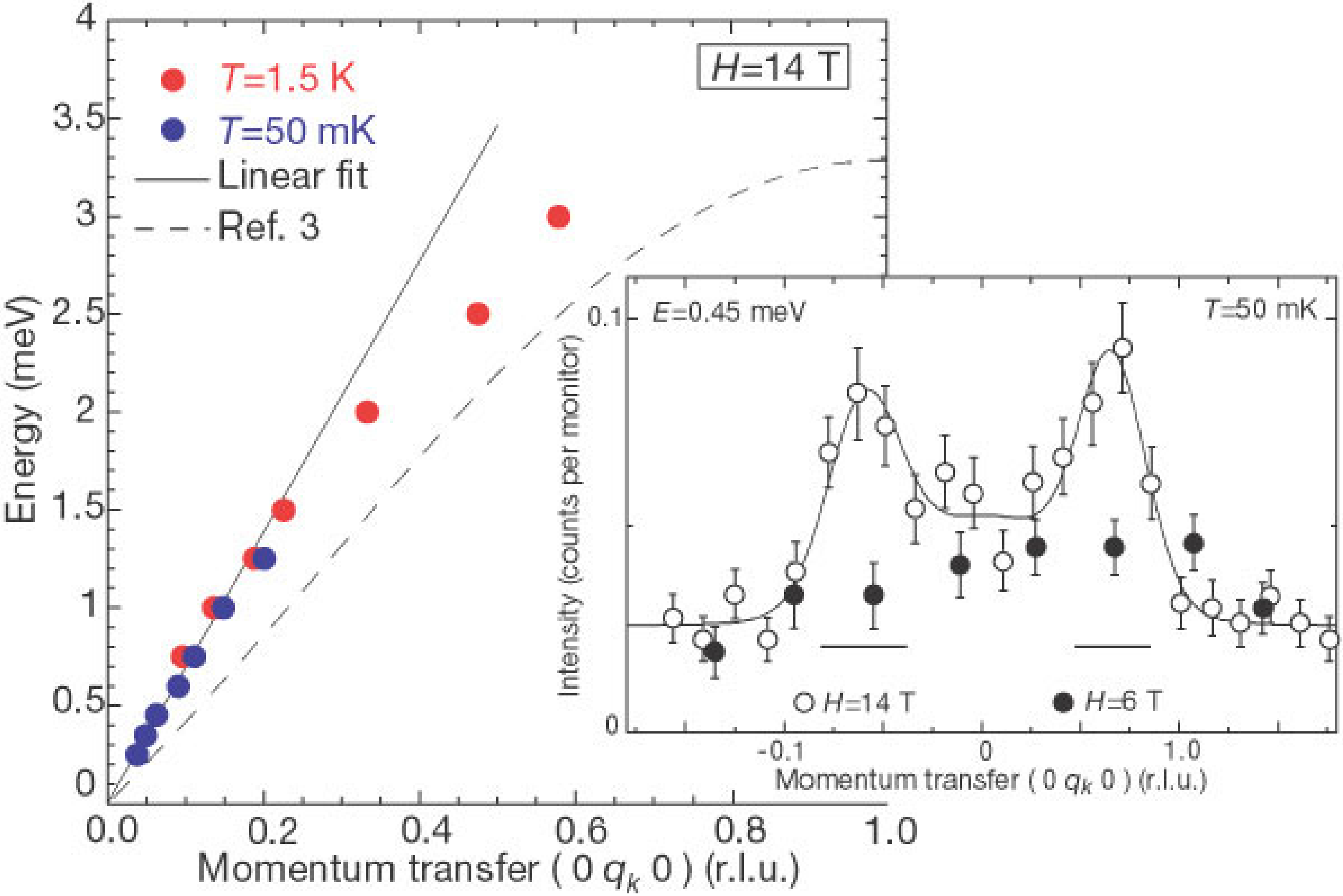


$TlCuCl_3$



Ch. Rüegg, N. Cavadini, A. Furrer, H.-U. Güdel, K. Krämer, H. Mutka, A. Wildes, K. Habicht, and P. Vorderwisch, *Nature* **423**, 62 (2003).

$TlCuCl_3$



“Spin wave (phonon) above critical field

Ch. Rüegg, N. Cavadini, A. Furrer, H.-U. Güdel, K. Krämer, H. Mutka, A. Wildes, K. Habicht, and P. Vorderwisch, *Nature* **423**, 62 (2003).

Phase diagram in a magnetic field.

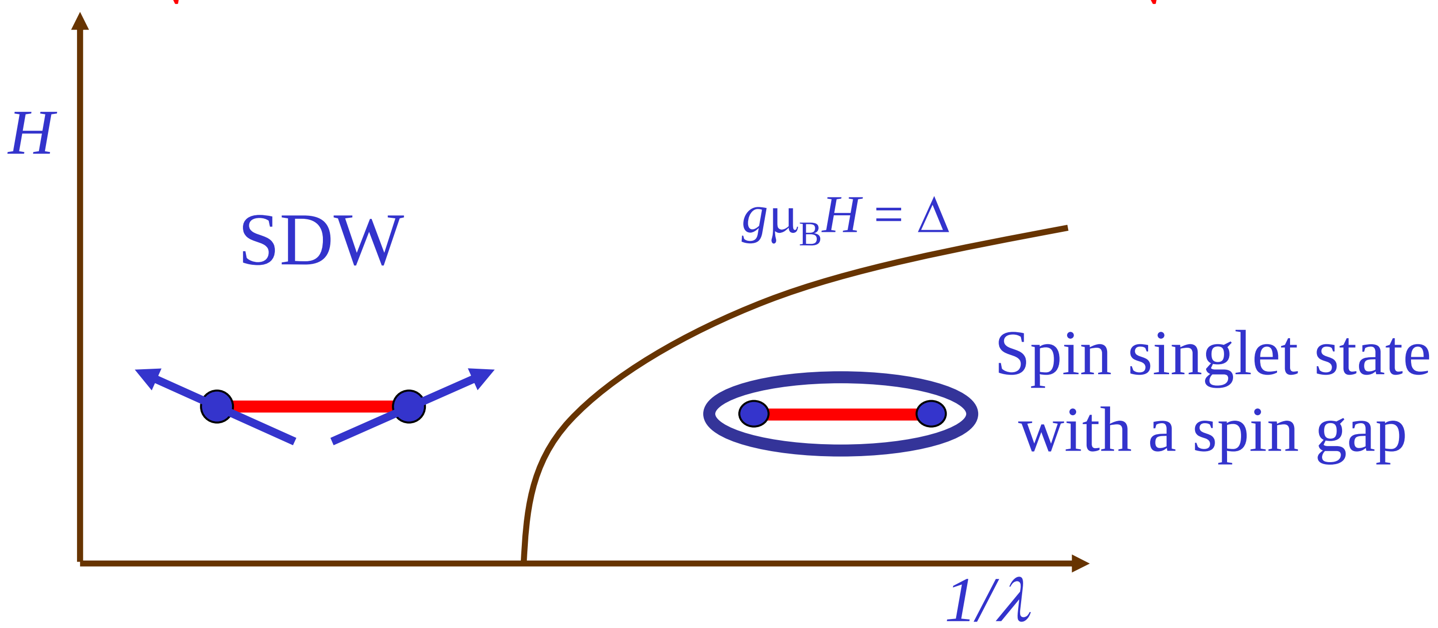
Zeeman term leads to a uniform precession of spins

$$\left|\partial_\tau \phi_\alpha\right|^2 \Rightarrow \left(\partial_\tau \phi_\alpha^* - i\varepsilon_{\alpha\sigma\rho} H_\sigma \phi_\rho\right)\left(\partial_\tau \phi_\alpha - i\varepsilon_{\alpha\beta\gamma} H_\beta \phi_\gamma\right)$$

Take H oriented along the z direction. Then

$$\left(\lambda_c - \lambda\right)\left(\phi_x^2 + \phi_y^2\right) \Rightarrow \left(\lambda_c - \lambda - H^2\right)\left(\phi_x^2 + \phi_y^2\right).$$

For $\lambda > \lambda_c$, $\phi_x \sim \sqrt{\lambda - \lambda_c + H^2}$, while for $\lambda < \lambda_c$, $H_c = \Delta \sim \sqrt{\lambda_c - \lambda}$



1 Tesla = 0.116 meV

Related theory applies to double layer quantum Hall systems at ν=2

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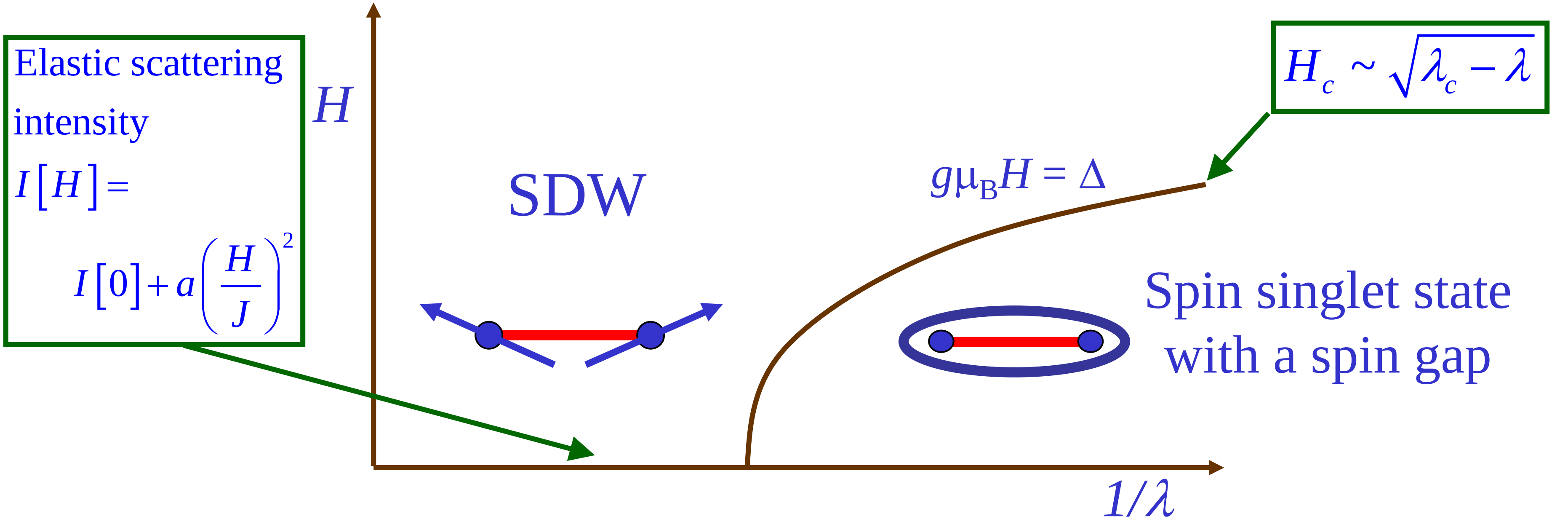

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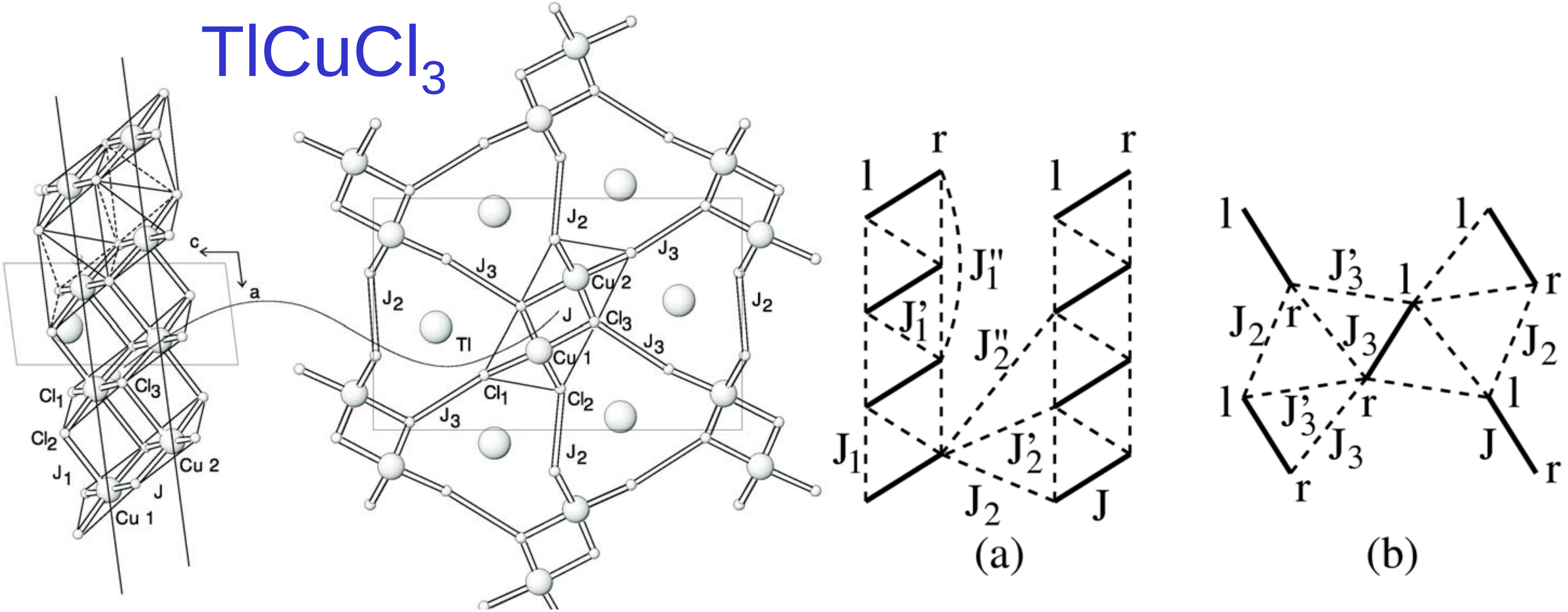
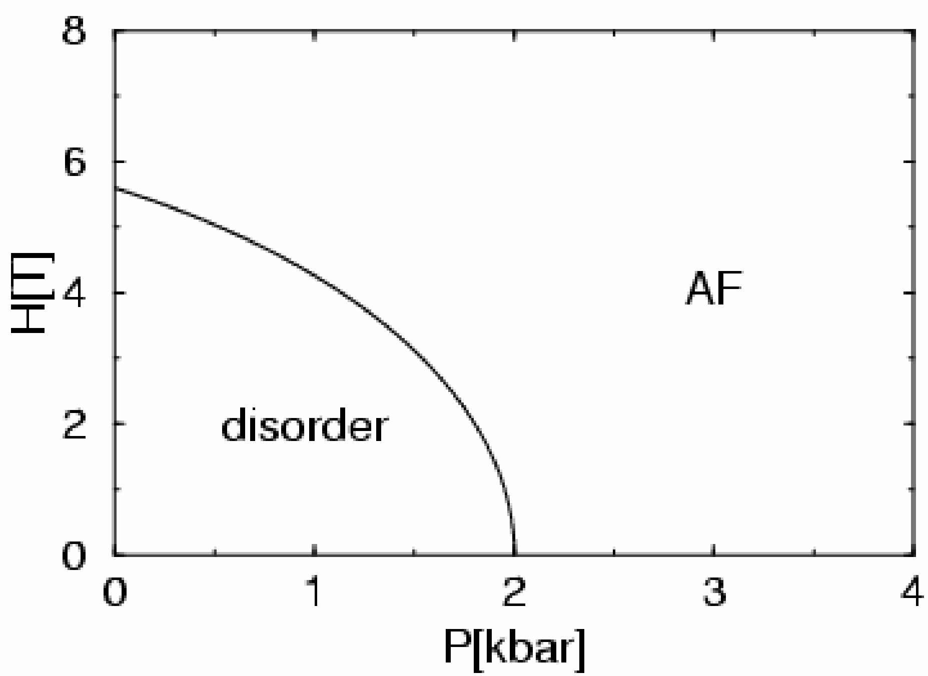
$TlCuCl_3$





M. Matsumoto,
B. Normand, T.M. Rice,
and M. Sigrist,
cond-mat/0309440.

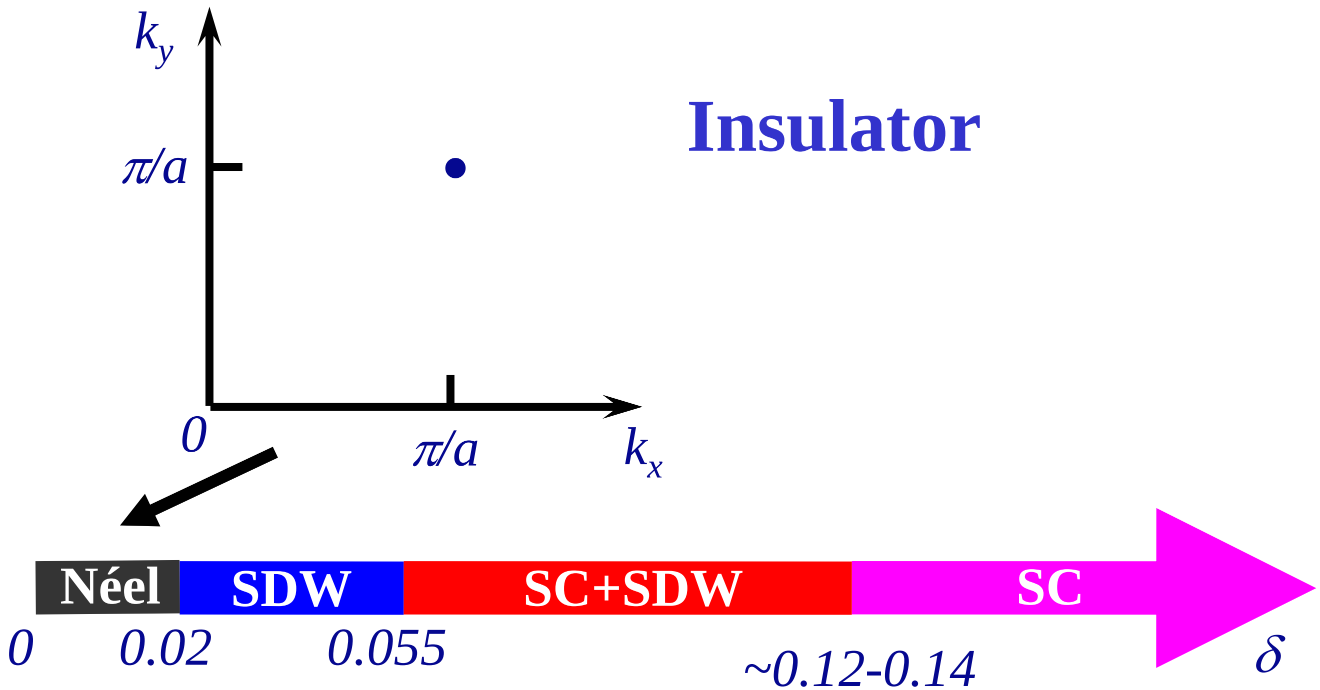
(B) Superconductors

Magnetic transitions in a superconductor:

effect of a magnetic field.

Interplay of SDW and SC order in the cuprates

T=0 phases of LSCO



(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

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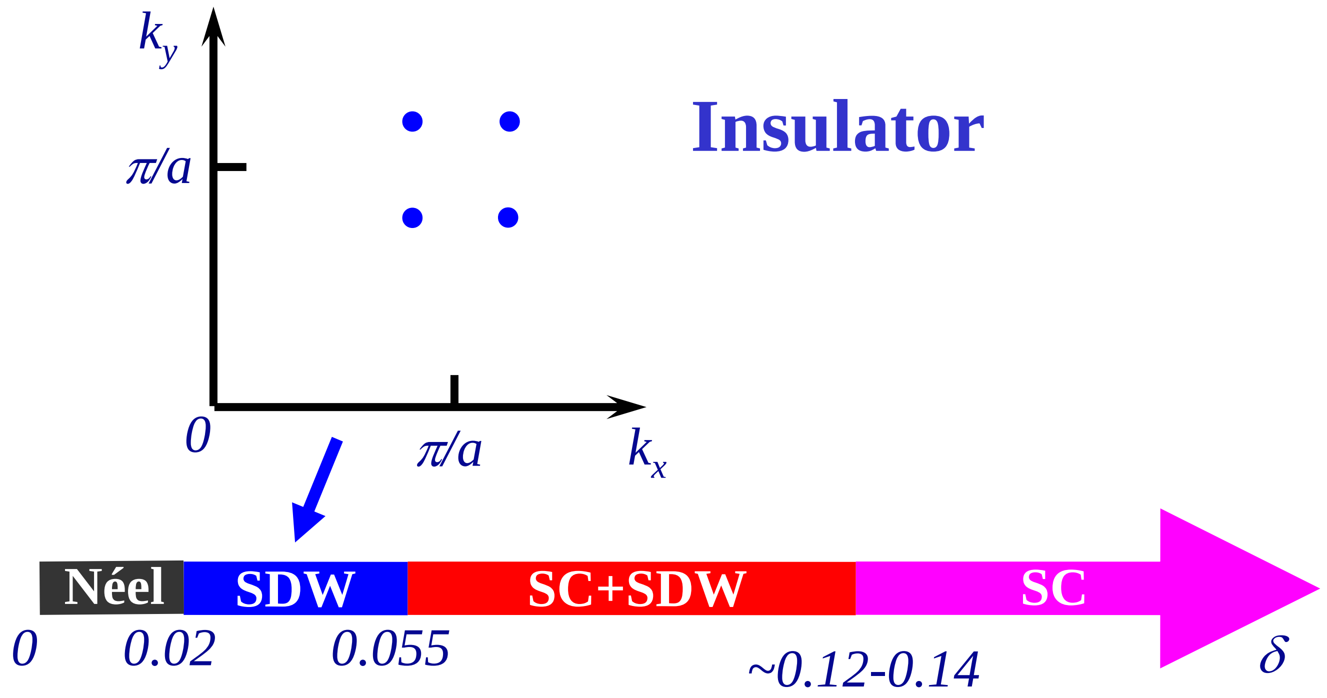

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J. M. Tranquada *et al*., *Phys*. *Rev*. B **54**, 7489 (1996).
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Interplay of SDW and SC order in the cuprates

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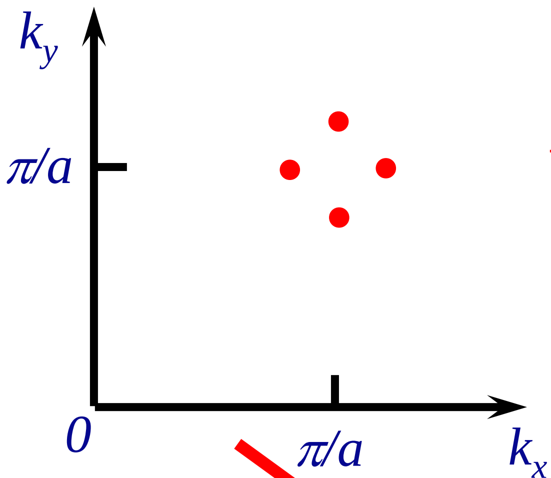
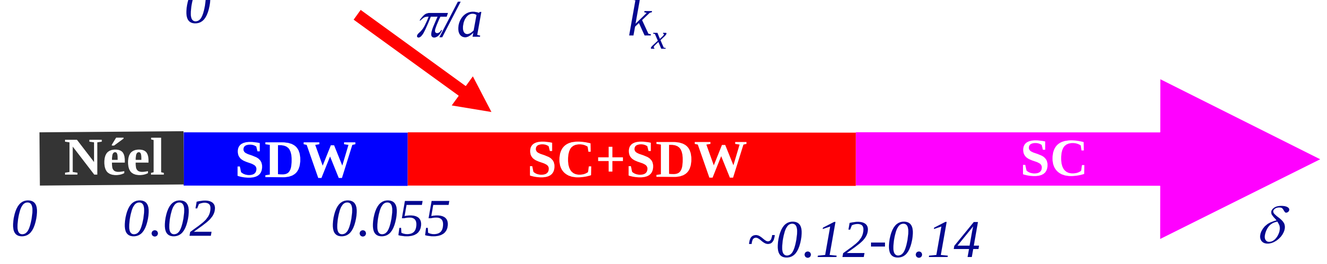


Superconductor with $T_{c,min}$ =10 K



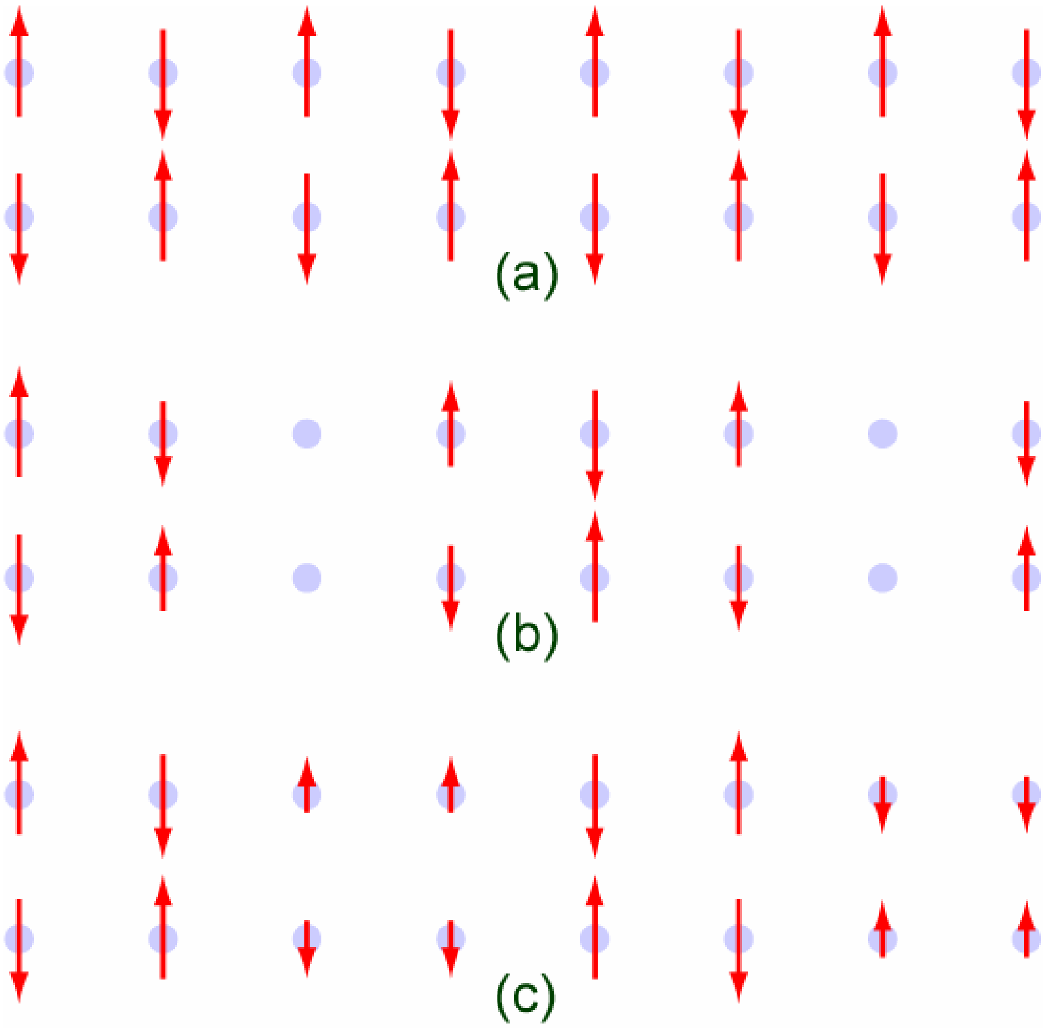
(additional commensurability effects near δ=0.125)

J. M. Tranquada *et al*., *Phys. Rev*. B **54**, 7489 (1996).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).
S. Wakimoto, G. Shirane *et al*., *Phys. Rev*. B **60**, R769 (1999).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., *Phys. Rev*. B **60**, 3643 (1999)
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev*. B **63**, 172501 (2001).

Collinear magnetic (spin density wave) order

$$\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N_1} \cos\left(\vec{K}.\vec{r}_j\right) + \boldsymbol{N_2} \sin\left(\vec{K}.\vec{r}_j\right)$$

Collinear spins



$$\vec{K} = (\pi, \pi) \;\; ; \;\; \boldsymbol{N_2} = 0$$

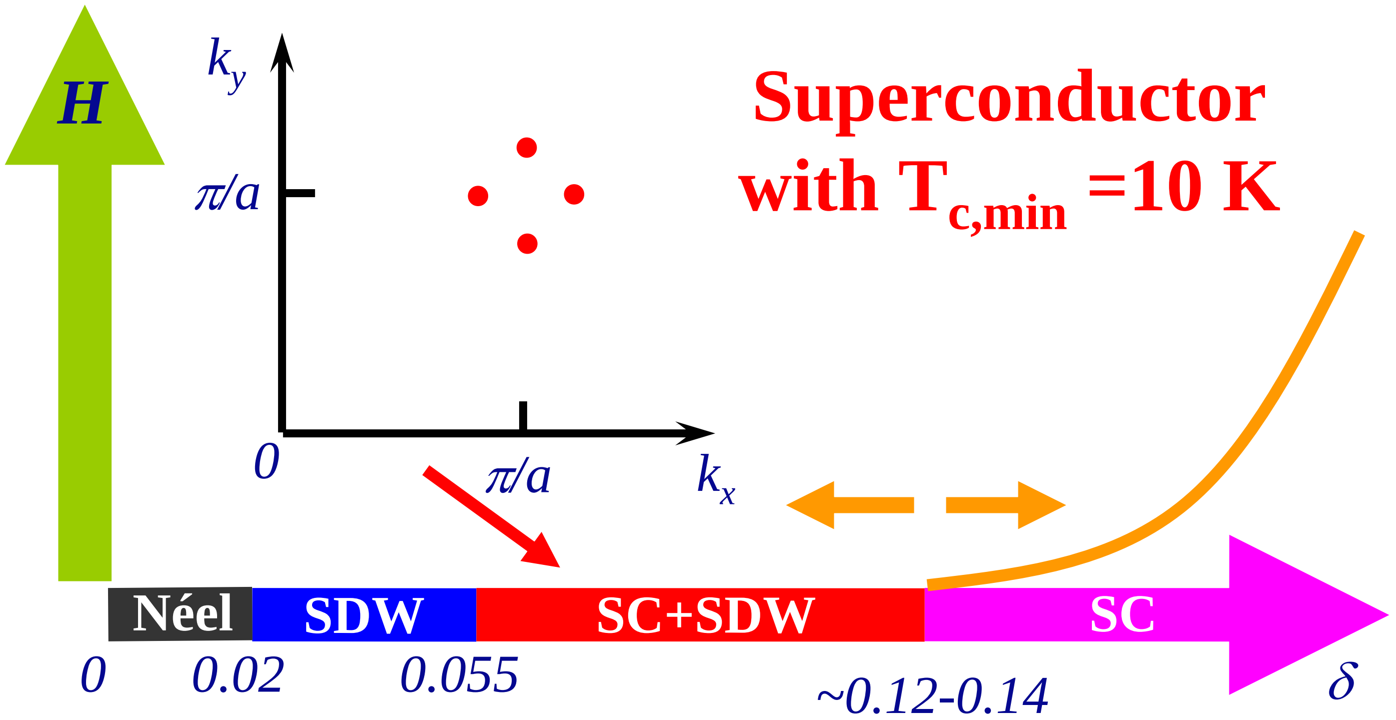
$$\vec{K} = (3\pi/4, \pi) \;\; ; \;\; \boldsymbol{N_2} = 0$$

$$\vec{K} = (3\pi/4, \pi) \;\; ;$$

$$\boldsymbol{N_2} = \left(\sqrt{2} - 1\right) \boldsymbol{N_1}$$

Interplay of SDW and SC order in the cuprates

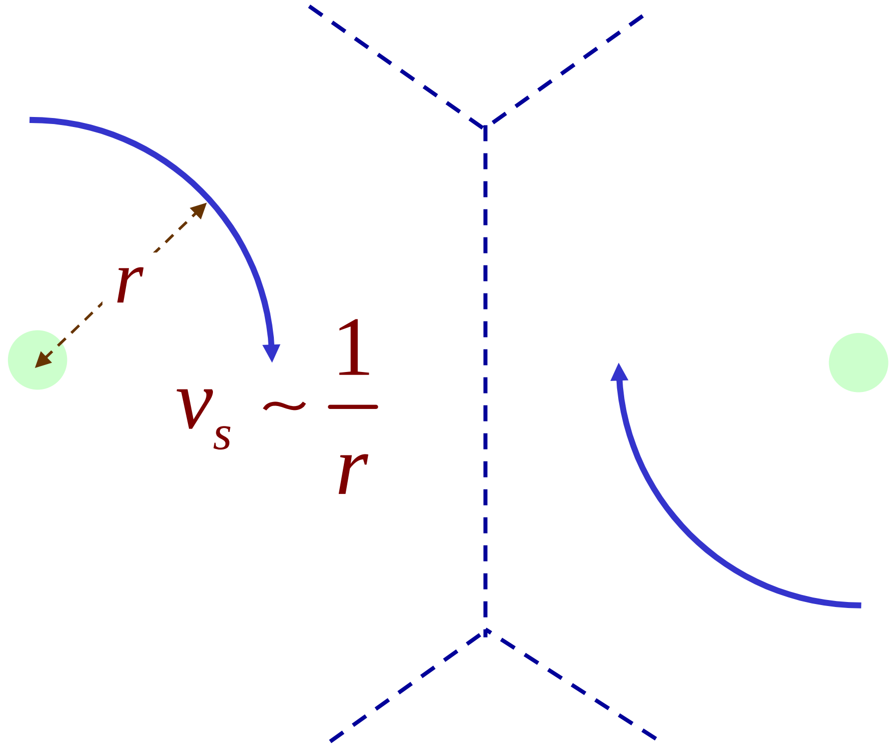
T=0 phases of LSCO



Use simplest assumption of a direct second-order quantum phase transition between SC and SC+SDW phases

Follow intensity of elastic Bragg spots in a magnetic field

Dominant effect of magnetic field: Abrikosov flux lattice



Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

$$\Phi_\alpha = N_{1\alpha} - iN_{2\alpha}$$

Quantum theory for dynamic and critical spin fluctuations

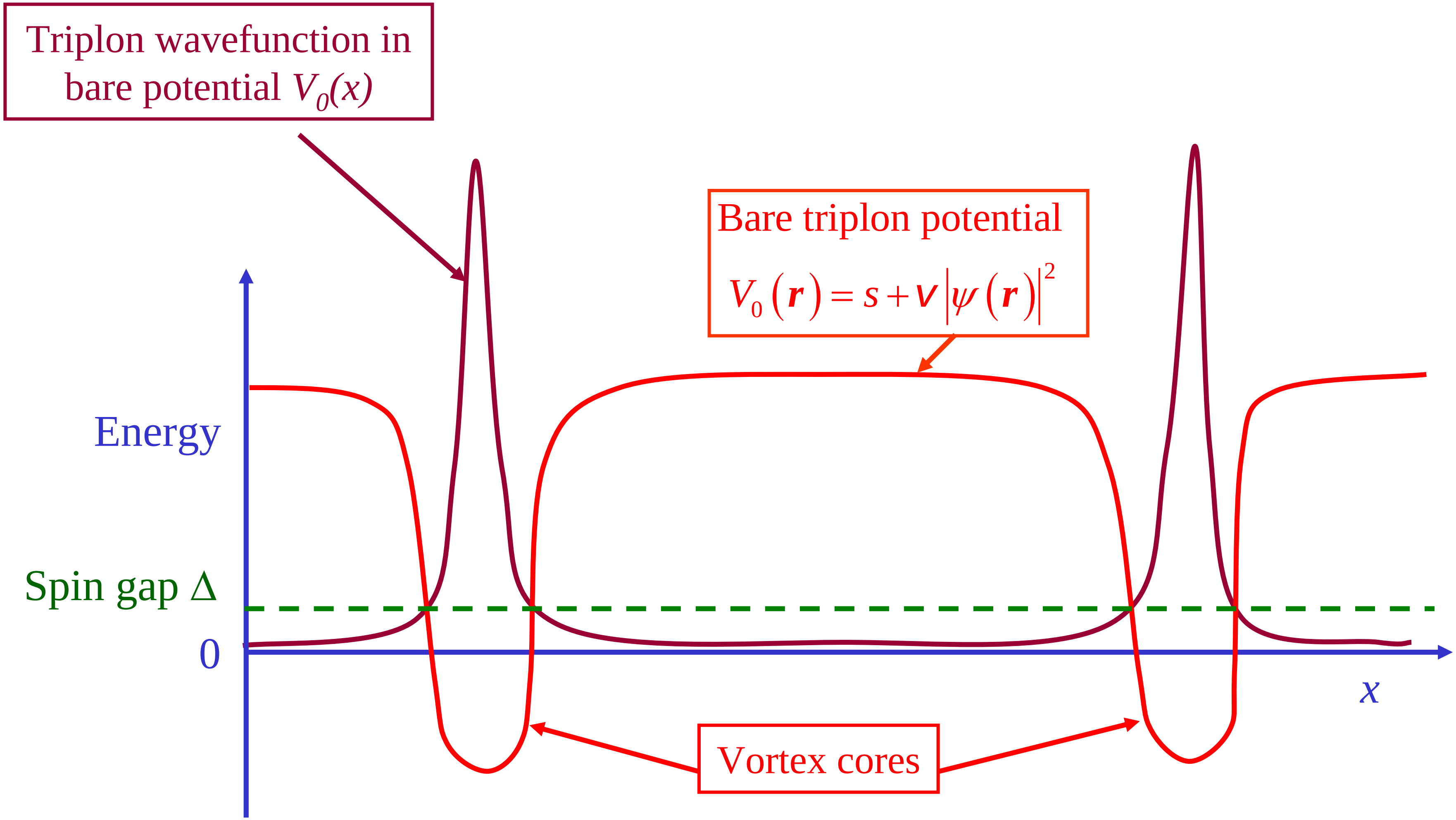
$$\boxed{S_b = \int d^2 r \int_0^{1/T} d\tau \left[\left|\nabla_r \Phi_\alpha\right|^2 + c^2 \left|\partial_\tau \Phi_\alpha\right|^2 + s\left|\Phi_\alpha\right|^2 + \frac{g_1}{2}\left(\left|\Phi_\alpha\right|^2\right)^2 + \frac{g_2}{2}\left|\Phi_\alpha^2\right|^2 \right]}$$

$$\boxed{S_c = \int d^2 r d\tau \left[\frac{v}{2} \left|\Phi_\alpha\right|^2 \left|\psi\right|^2 \right]}$$

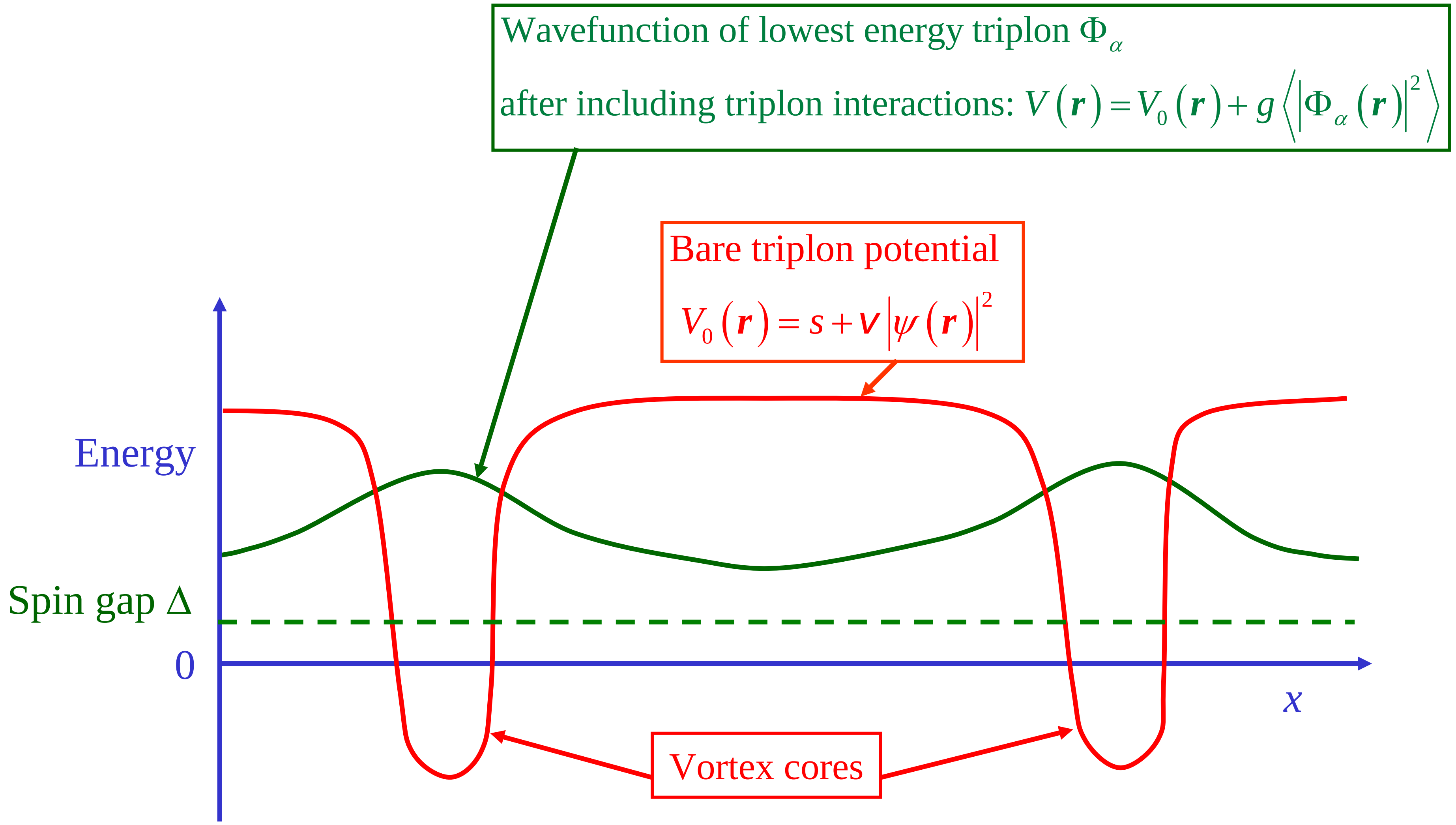
$$\boxed{\begin{gathered} Z\left[\psi(r)\right] = \int D\Phi(r,\tau) e^{-F_{GL} - S_b - S_c} \\ \frac{\delta \ln Z\left[\psi(r)\right]}{\delta \psi(r)} = 0 \end{gathered}}$$

$$\boxed{F_{GL} = \int d^2 r \left[-\left|\psi\right|^2 + \frac{\left|\psi\right|^4}{2} + \left|\left(\nabla_r - iA\right)\psi\right|^2 \right]}$$

Static Ginzburg-Landau theory for non-critical superconductivity



D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) proposed static magnetism (with Δ=0) localized within vortex cores



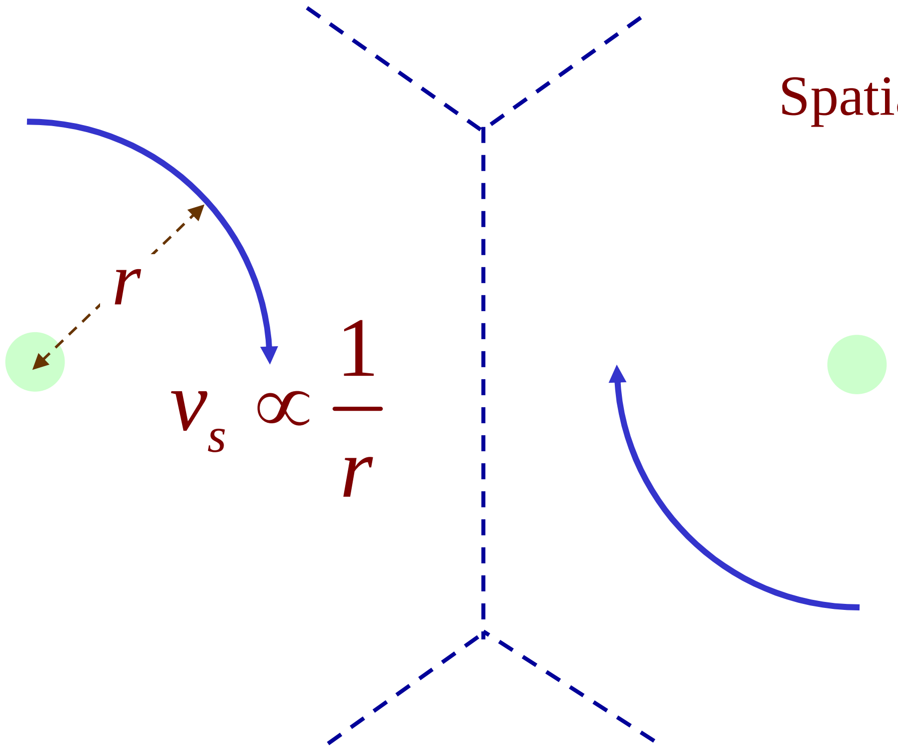
Strongly relevant repulsive interactions between excitons imply that triplons must be extended as $\Delta \to 0$.

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

A.J. Bray and M.A. Moore, *J. Phys.* C **15**, L7 65 (1982).

J.A. Hertz, A. Fleishman, and P.W. Anderson, *Phys. Rev. Lett.* **43**, 942 (1979).

Phase diagram of SC and SDW order in a magnetic field



Spatially averaged superflow kinetic energy

$$\left\langle v_s^2 \right\rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

The suppression of SC order appears to the SDW order as a ***uniform*** effective "doping" δ:

$$\delta_{\text{eff}}\left(H\right) = \delta - C\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

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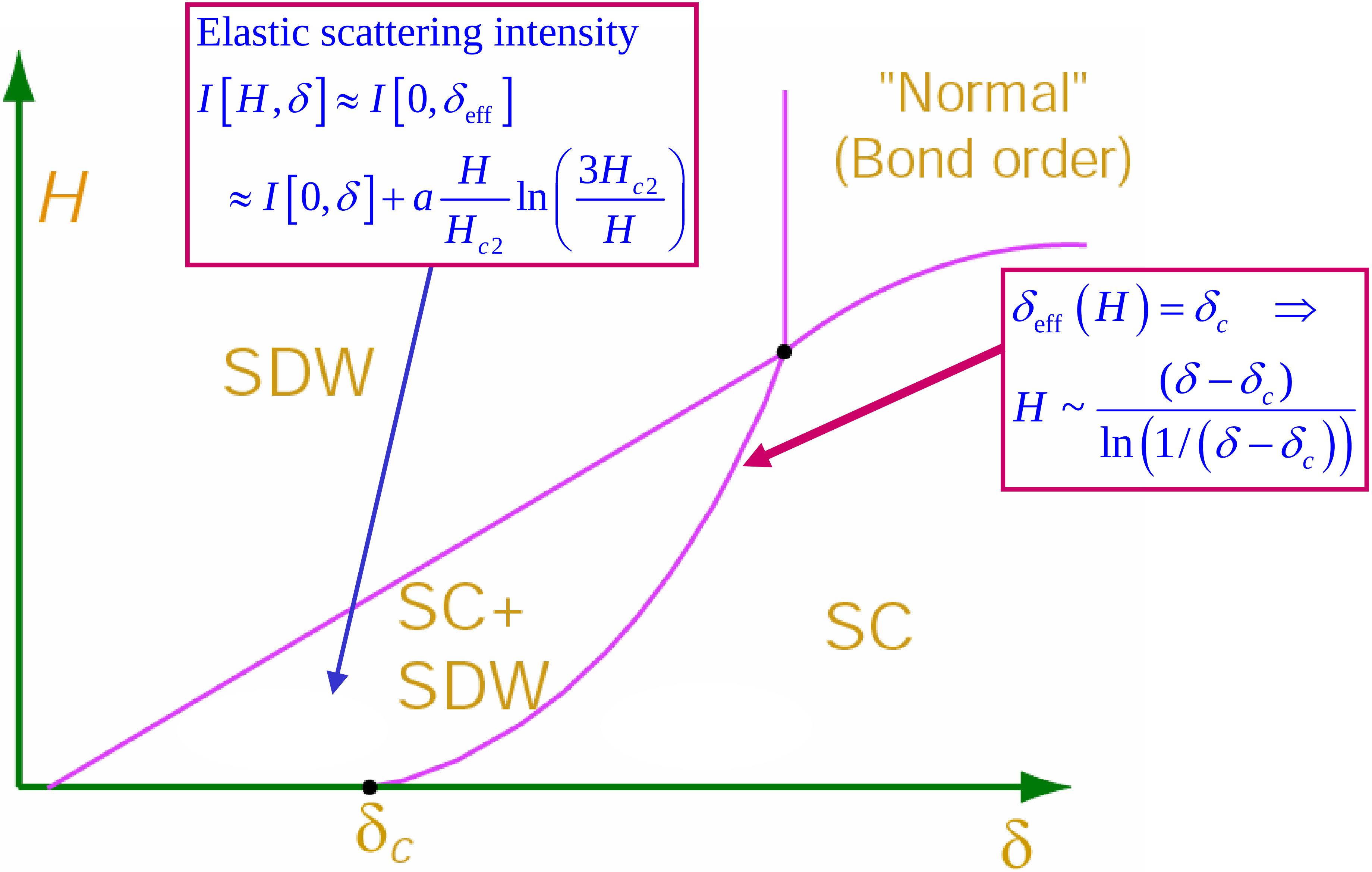
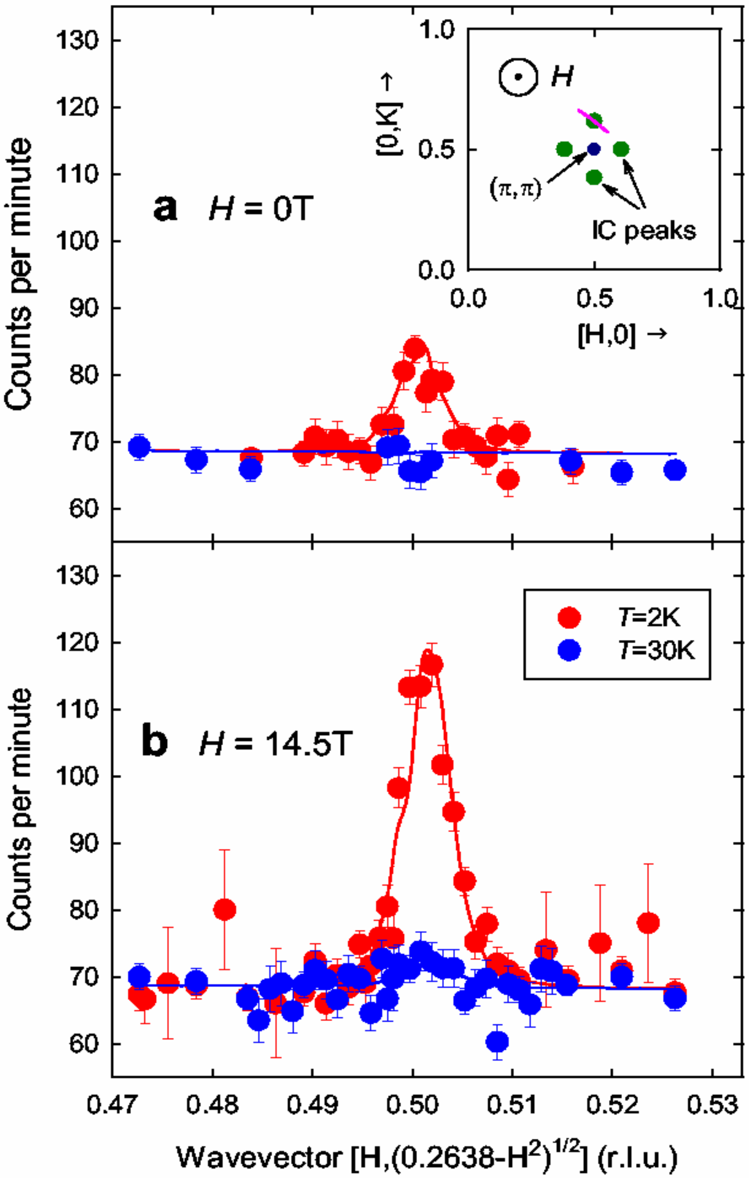
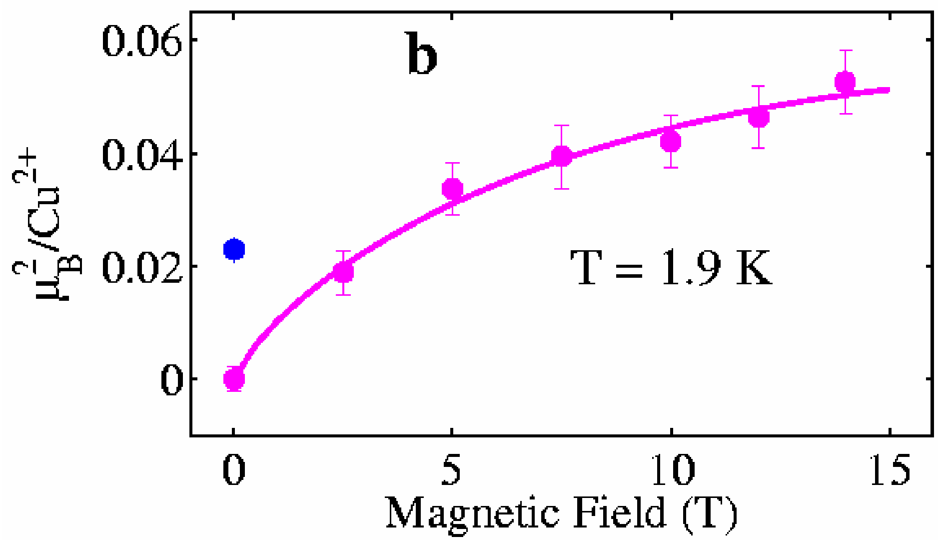


E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1



B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



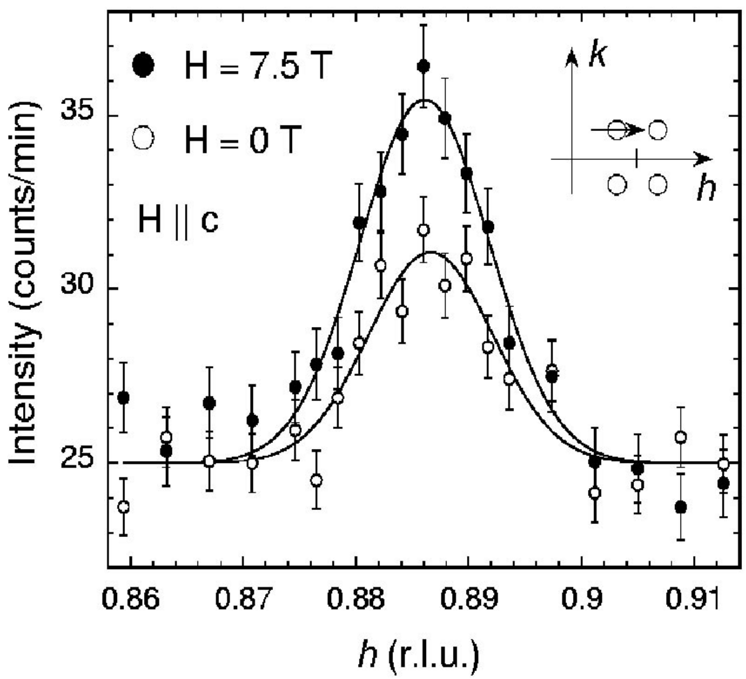
Solid line - fit to : $I(H) = a \frac{H}{H_{c2}} \ln\left(\frac{H_{c2}}{H}\right)$

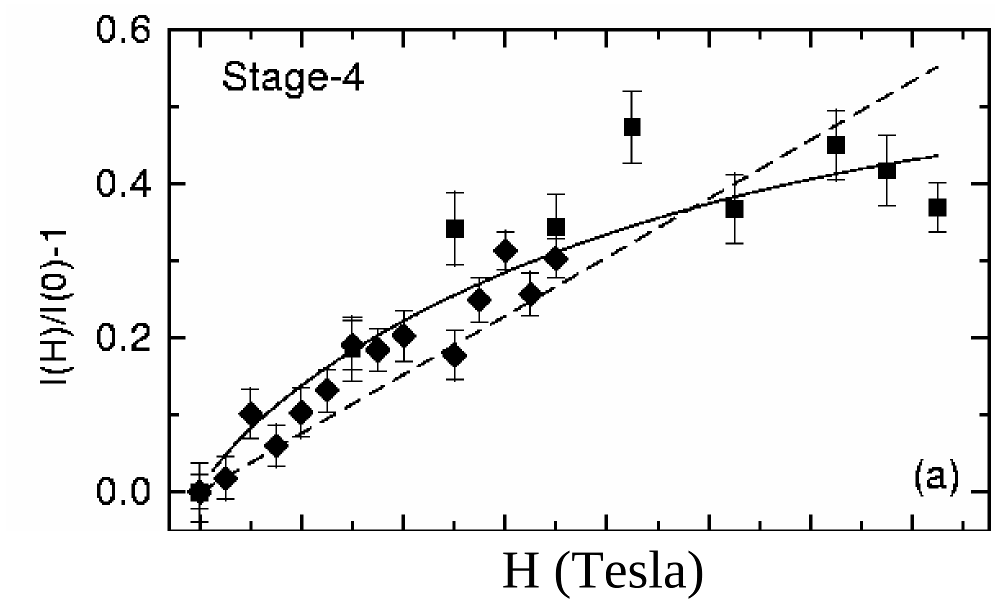
See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev.* B **62**, R14677 (2000).

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+CM) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, *Phys. Rev.* B **66**, 014528 (2002).



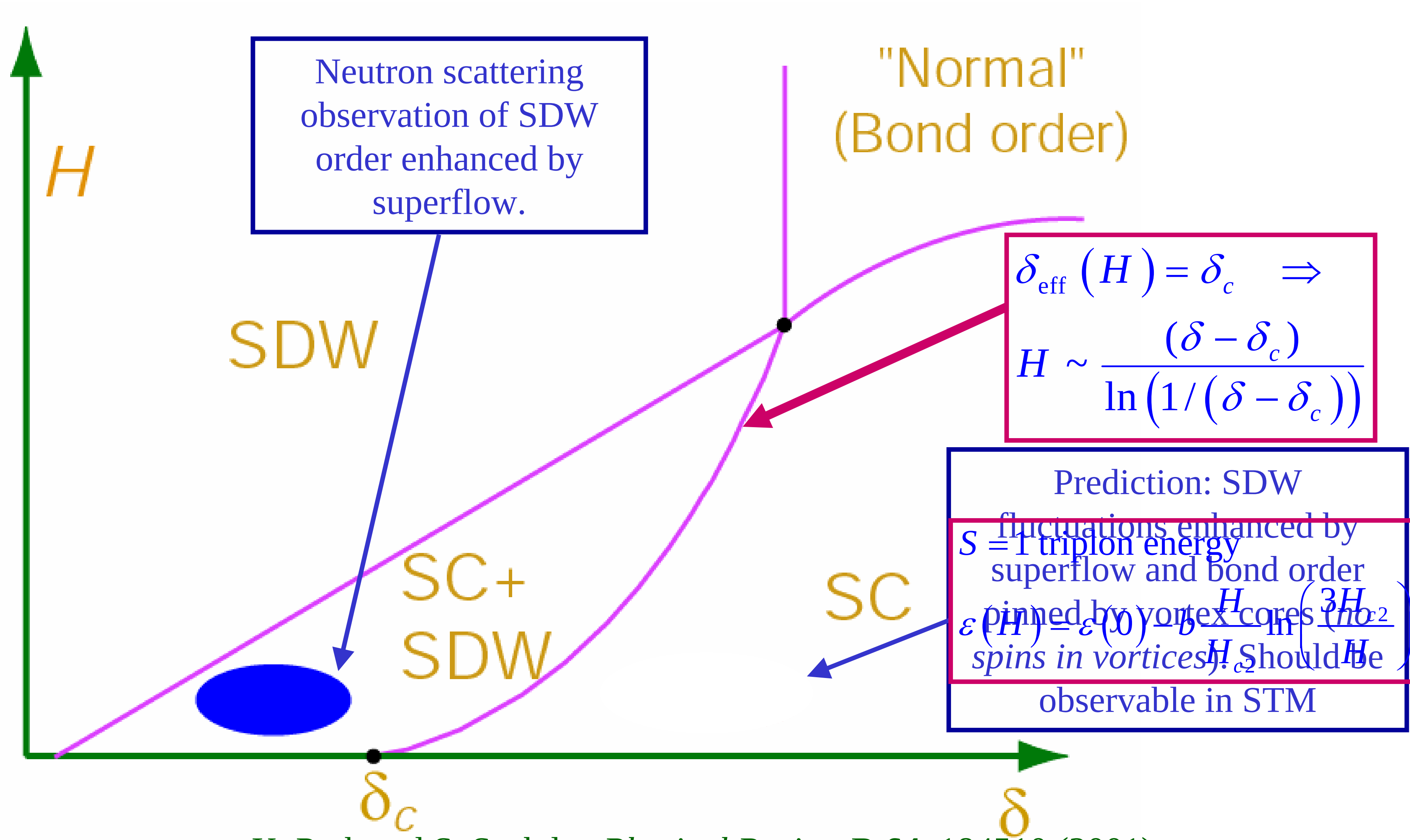


Solid line --- fit to : $\dfrac{I(H)}{I(0)} = 1 + a\dfrac{H}{H_{c2}}\ln\left(\dfrac{3.0H_{c2}}{H}\right)$

a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Phase diagram of a superconductor in a magnetic field



K. Park and S. Sachdev *Physical Review* B **64**, 184510 (2001);

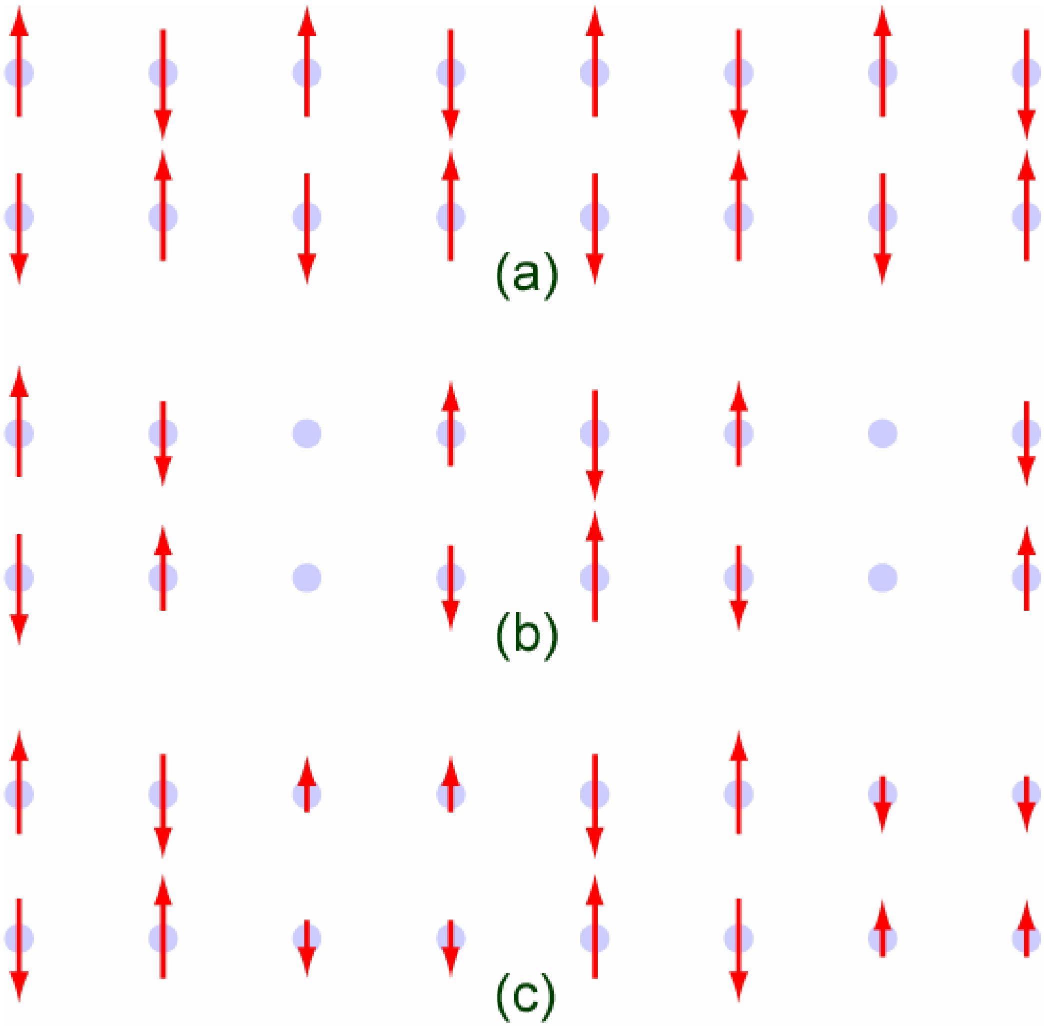
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Y. Zhang, E. Demler and S. Sachdev, *Physical Review* B **66**, 094501 (2002).

Collinear magnetic (spin density wave) order

$$\langle \boldsymbol{S}_j \rangle = \boldsymbol{N_1} \cos\left(\vec{K}.\vec{r}_j\right) + \boldsymbol{N_2} \sin\left(\vec{K}.\vec{r}_j\right)$$

Collinear spins



$$\vec{K} = (\pi, \pi) \;\; ; \;\; \boldsymbol{N_2} = 0$$

$$\vec{K} = (3\pi/4, \pi) \;\; ; \;\; \boldsymbol{N_2} = 0$$

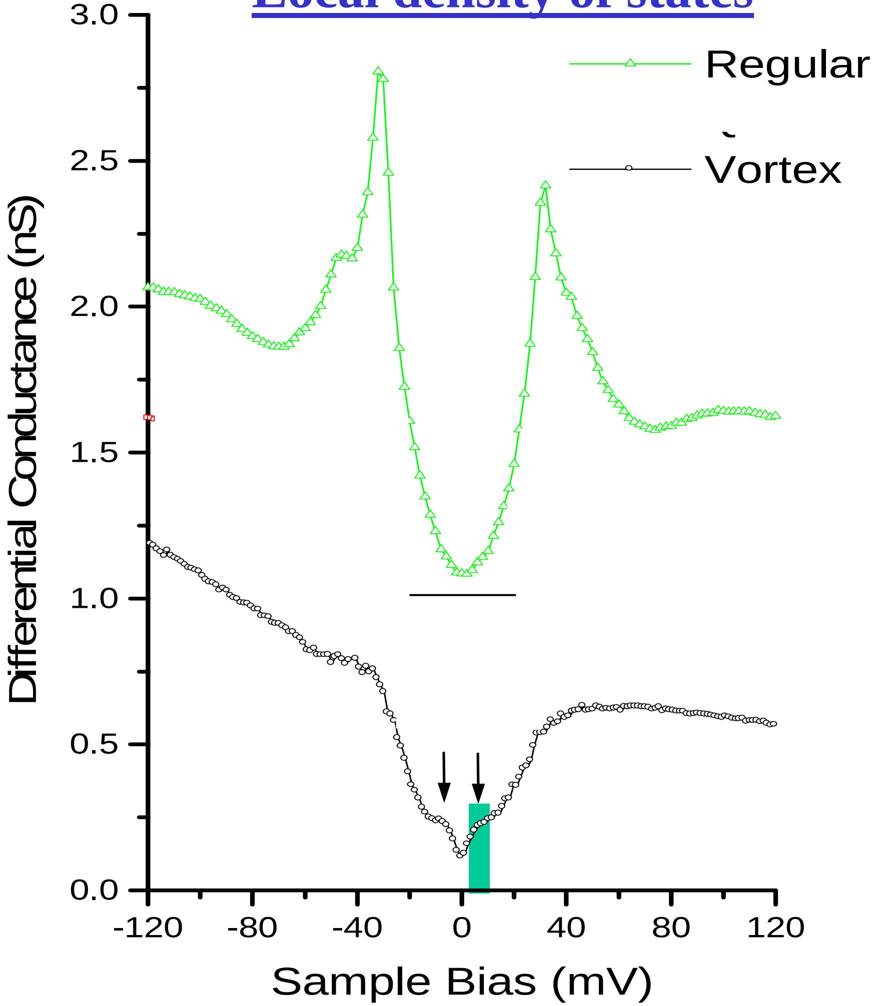
$$\vec{K} = (3\pi/4, \pi) \;\; ;$$

$$\boldsymbol{N_2} = \left(\sqrt{2} - 1\right) \boldsymbol{N_1}$$

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states

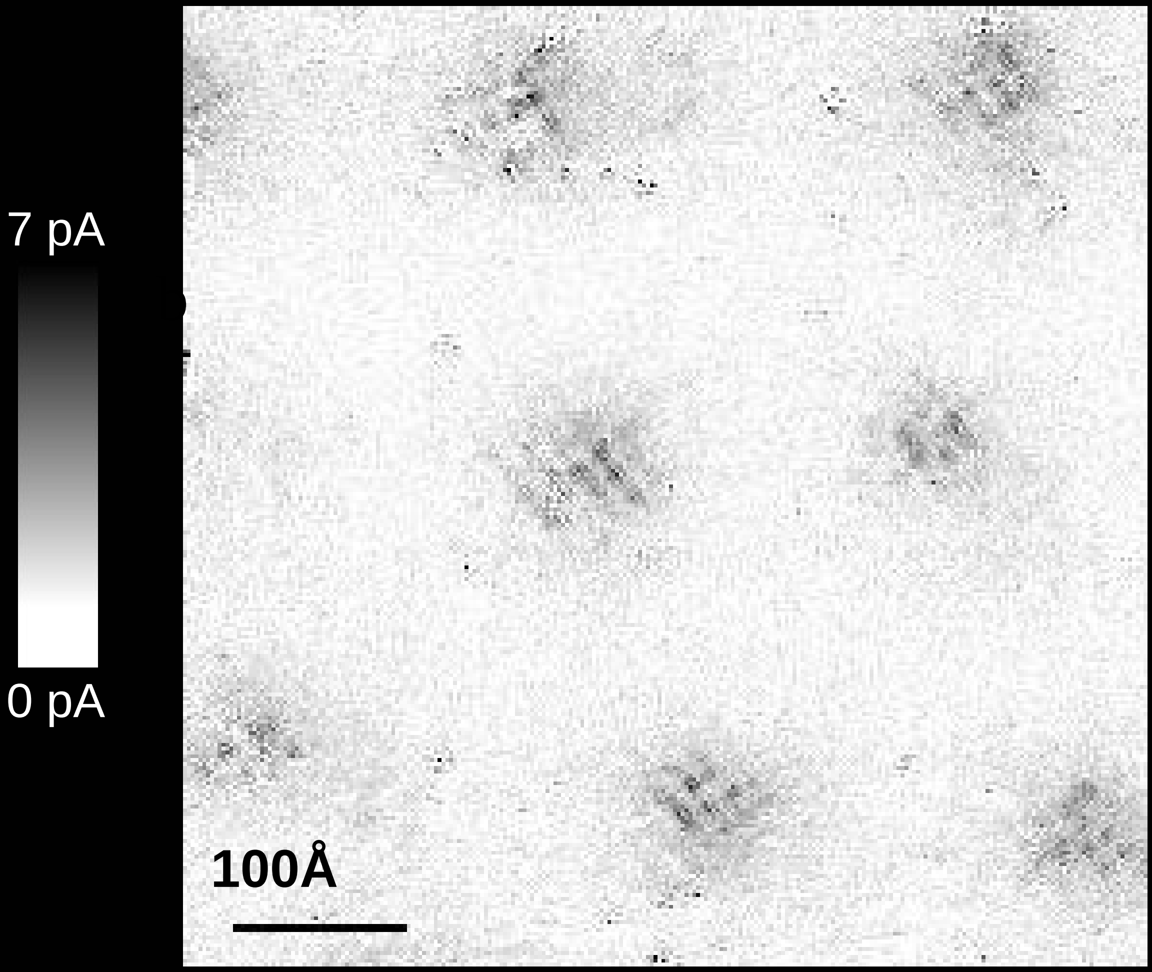


S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



Our interpretation:
LDOS modulations are signals of bond order of period 4 revealed in vortex halo

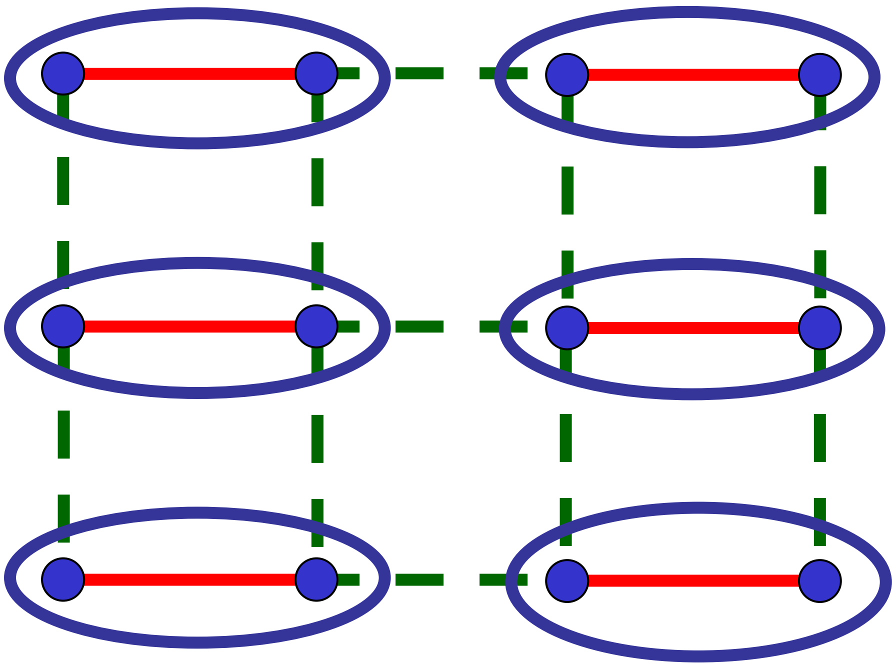
See also:
S. A. Kivelson, E. Fradkin, V. Oganesyan, I. P. Bindloss, J. M. Tranquada, A. Kapitulnik, and C. Howald, cond-mat/0210683.

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

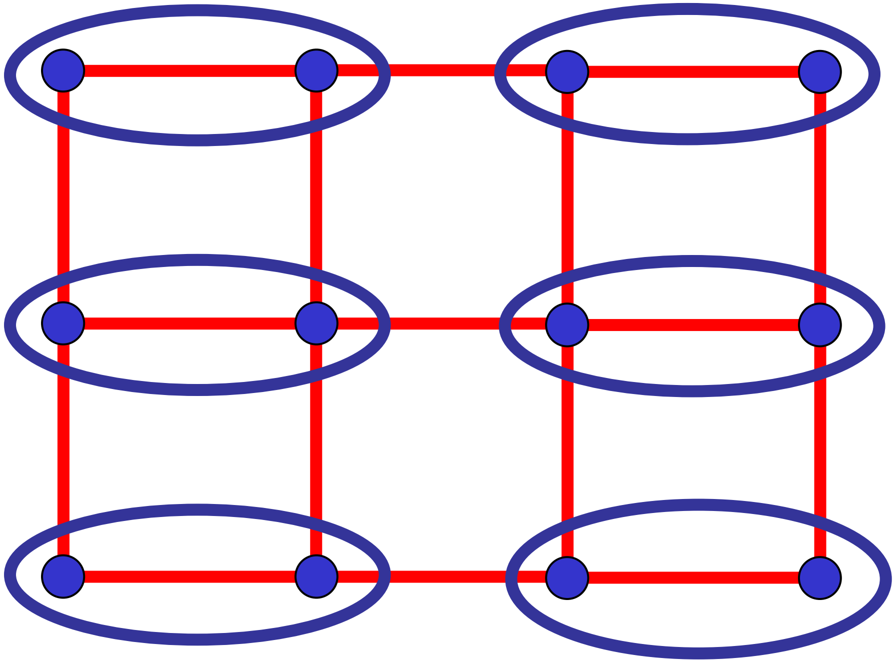
(C) Spin gap state on the square lattice:

Spontaneous bond order

Paramagnetic ground state of coupled ladder model



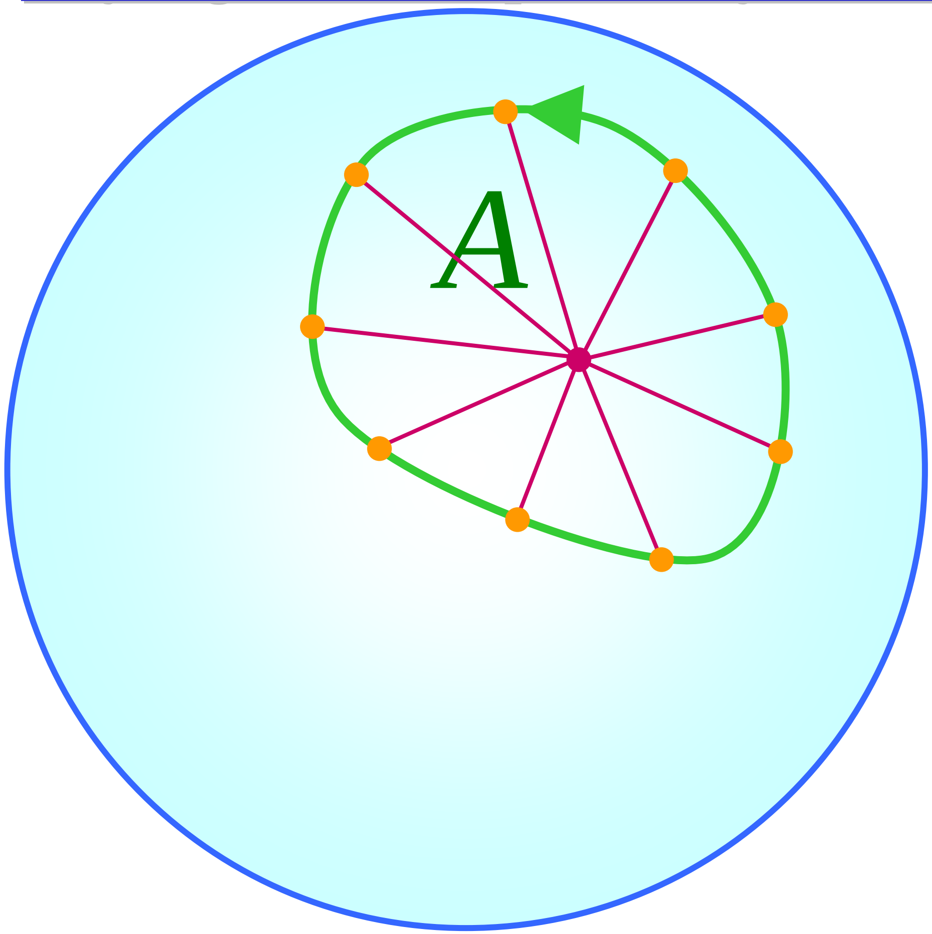
Can such a state with ***bond order*** be the ground state of a system with full square lattice symmetry ?



Collinear spins and compact U(1) gauge theory

Write down path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases

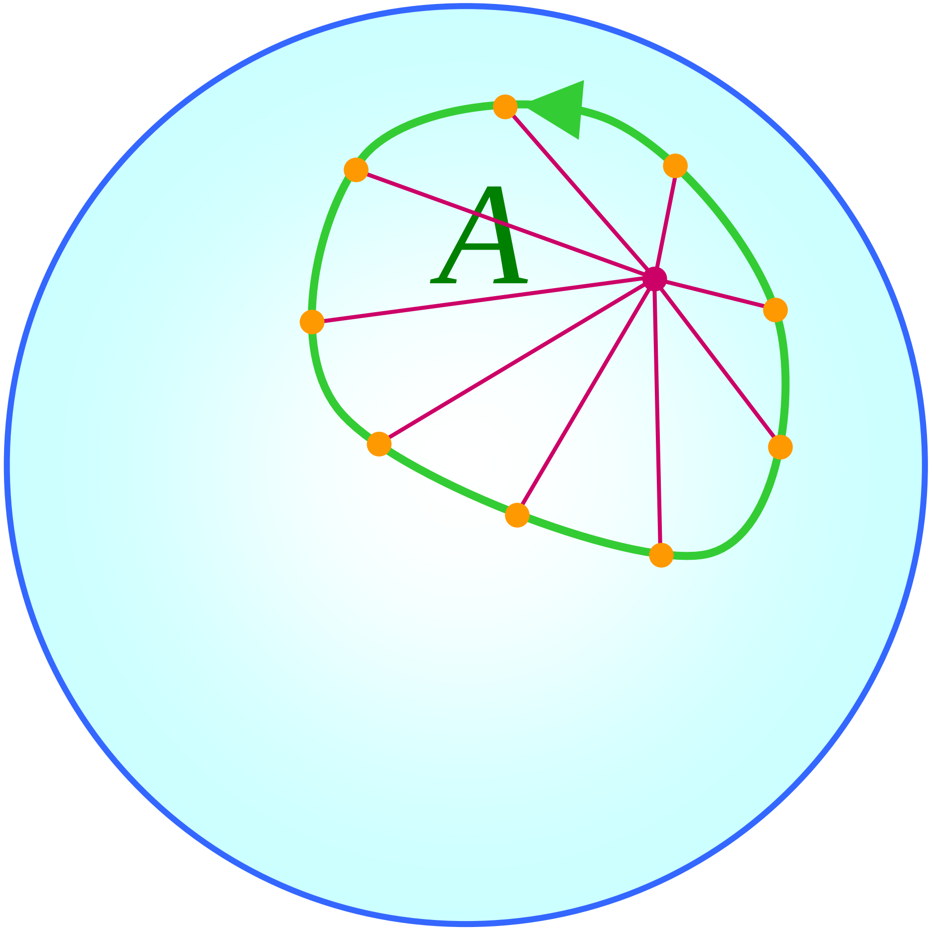


$$e^{iSA}$$

Collinear spins and compact U(1) gauge theory

Write down path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases



$$e^{iSA}$$

Class A: Collinear spins and compact U(1) gauge theory

$S=1/2$ square lattice antiferromagnet with non-nearest neighbor exchange

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

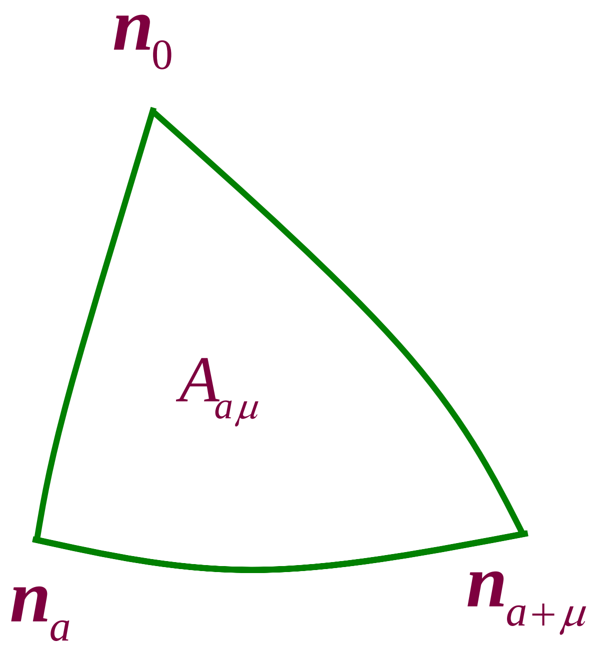
$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_\mathrm{a} \rightarrow \pm 1$ on two square sublattices ;

$\boldsymbol{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$

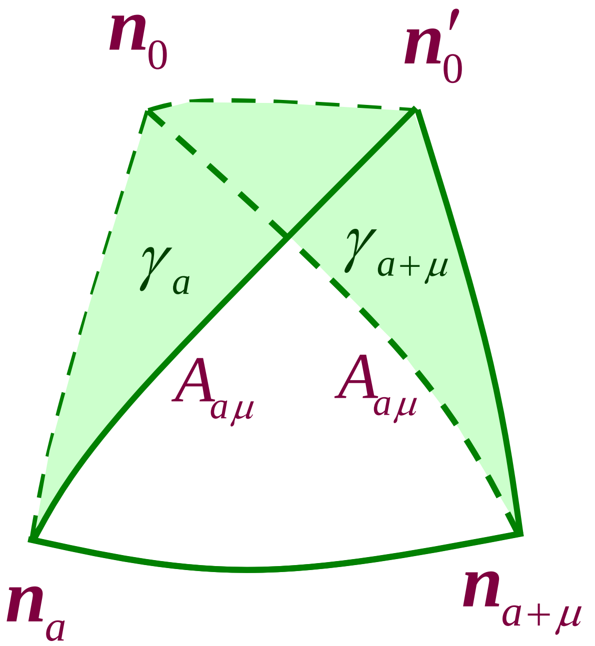


$\boldsymbol{n}_0$
$A_{a\mu}$
$\boldsymbol{n}_a$
$\boldsymbol{n}_{a+\mu}$

Change in choice of $\boldsymbol{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \to A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{n}_a$ and the two choices for $\boldsymbol{n}_0$).



The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \to A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$ which provides description of the large g phase

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(-\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2}\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right)\right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in $d+1$ dimensions with static charges ± 1 on two sublattices.

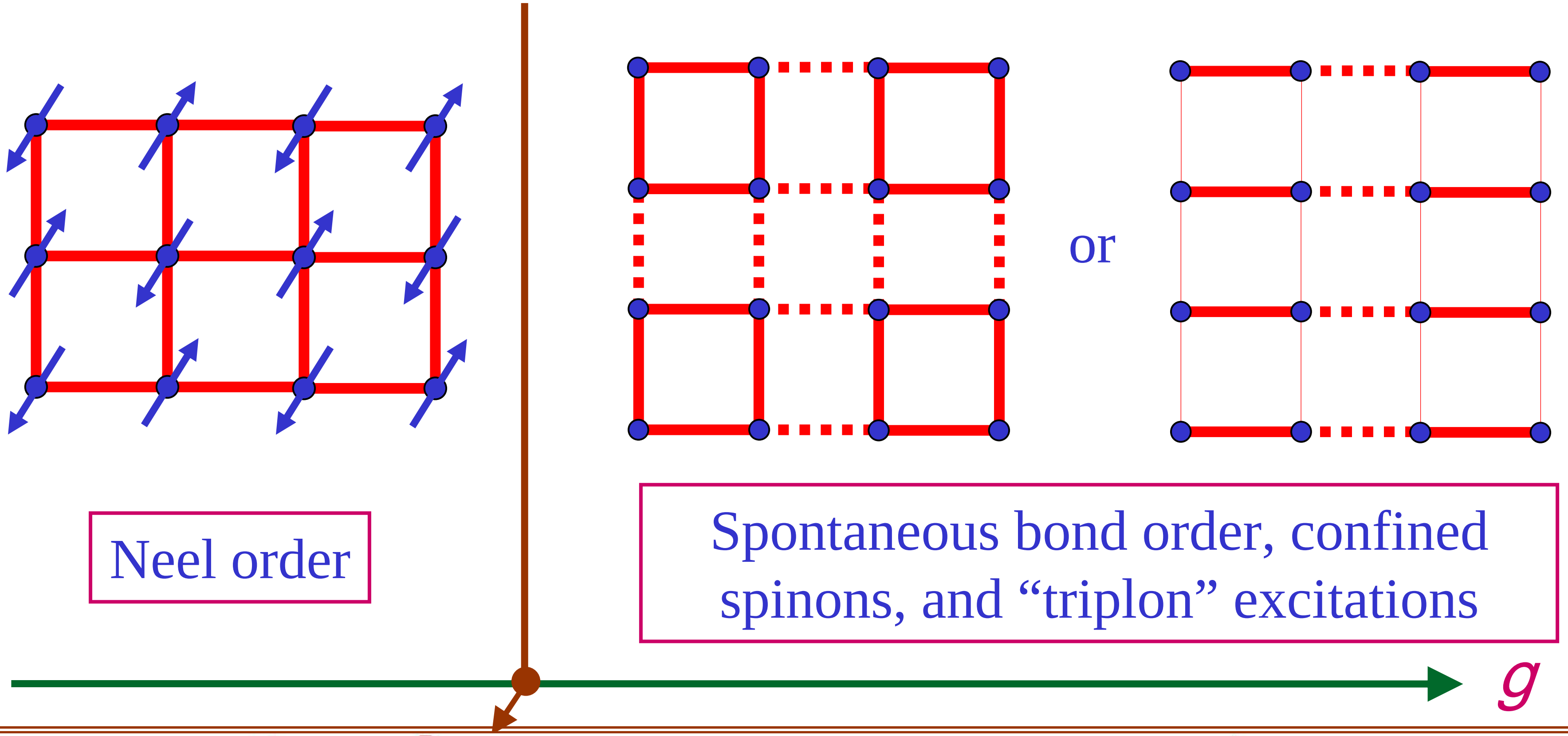
This theory can be reliably analyzed by a duality mapping.

***d*=2**: The gauge theory is *always* in a *confining* phase and there is bond order in the ground state.

***d*=3**: A deconfined phase with a gapless "photon" is possible.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev.* B **65**, 220405 (2002).

Phase diagram of S=1/2 square lattice antiferromagnet



$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + r |z_\alpha|^2 + \frac{u}{2} \left(|z_\alpha|^2 \right)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

where $\mathbf{n} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$ and z_α are bosonic spinors

Critical theory is not expressed in terms of order parameter of either phase, but instead contains spinons interacting the a non-compact U(1) gauge force

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, submitted to *Science*

Conclusions

I. Introduction to magnetic quantum criticality in coupled dimer antiferromagnet.

II. Theory of quantum phase transitions provides semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also proposes a connection to STM experiments.

III. Spontaneous bond order in spin gap state on the square lattice: possible connection to modulations observed in vortex halo.