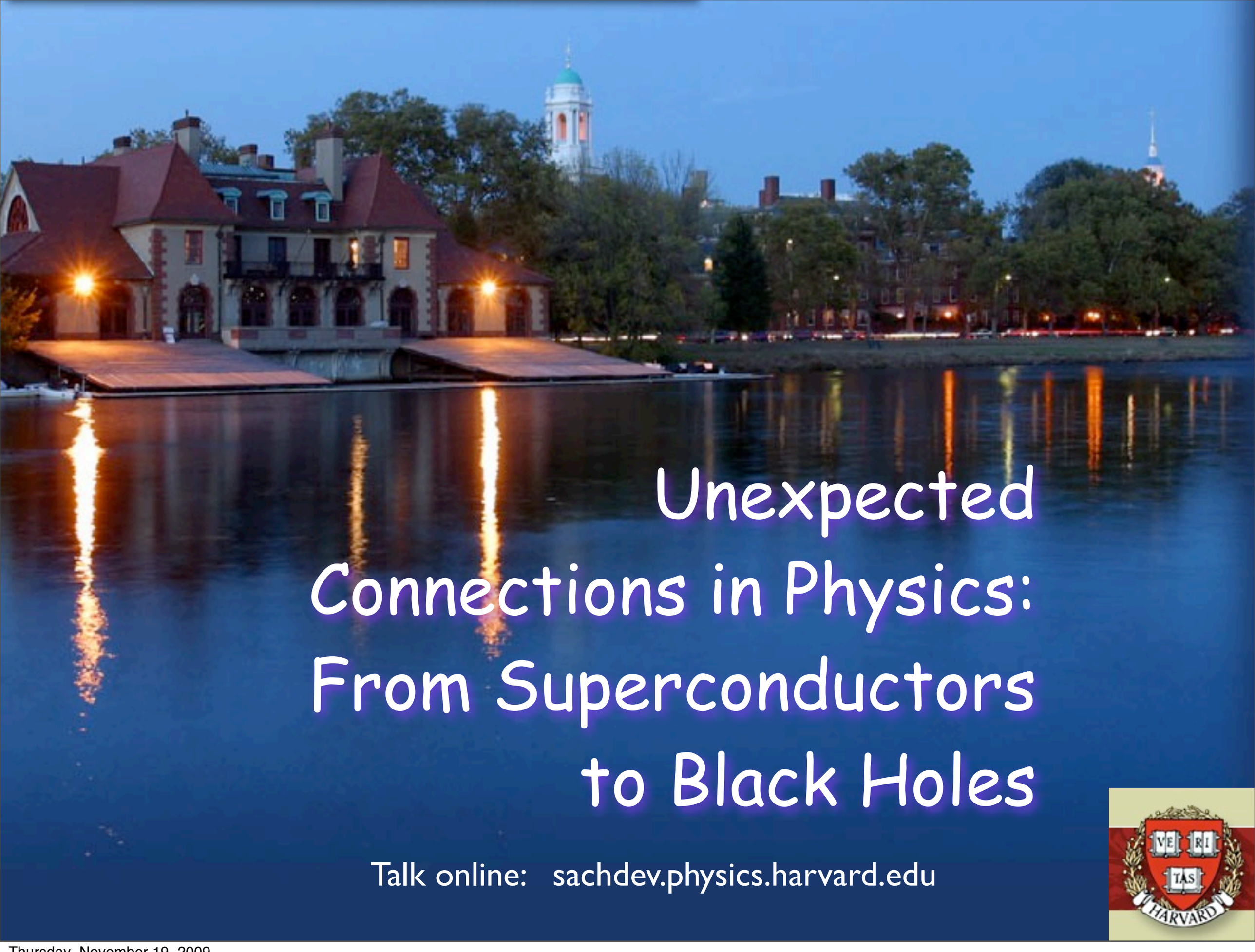

Unexpected Connections in Physics: From Superconductors to Black Holes

Talk online: sachdev.physics.harvard.edu

The main unsolved problem in theoretical physics today:

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Quantum theory of electrons, protons, neutron, atoms, molecules..

The main unsolved problem in theoretical physics today: Unification of

Quantum theory of electrons, protons, neutron, atoms, molecules..

and

Einstein's theory of General Relativity (gravitation) applied to stars, black holes, galaxies, the universe....

Work over the past 20+ years has finally led to a candidate theory which unifies gravitation and the quantum theory, and which is internally consistent.

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This theory is popularly known as "string theory"

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We do not know yet if string theory is the correct description of our universe.

The deepest new principle of physics to emerge from string theory is

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the quantum theory of gravity in 3 space dimensions

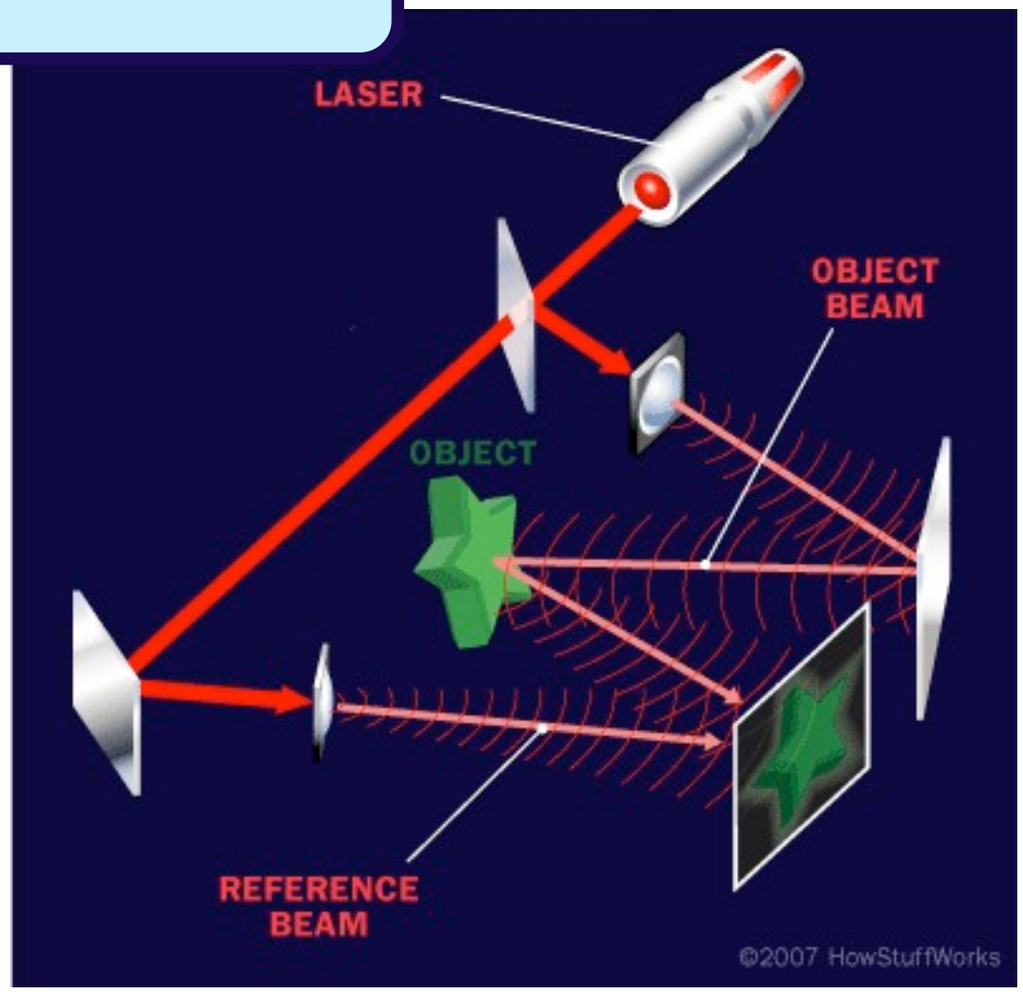
The deepest new principle of physics to emerge from string theory is the equivalence between

the quantum theory of gravity in 3 space dimensions

and

a theory of special quantum states of matter (electron, photons, quarks..) without gravity in 2 space dimensions

3-dimensional quantum gravity can be represented as a "hologram" in 2 dimensions



This equivalence has led to:

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- Complete formulations of consistent theories of quantum gravity

This equivalence has led to:

- Complete formulations of consistent theories of quantum gravity

- Solutions of previously unsolved problems in strongly interacting theories of quantum matter in 2 dimensions (without gravity)

Quantum superposition and entanglement

Superconductivity

Quantum phase transitions and Quantum critical points

Black Holes

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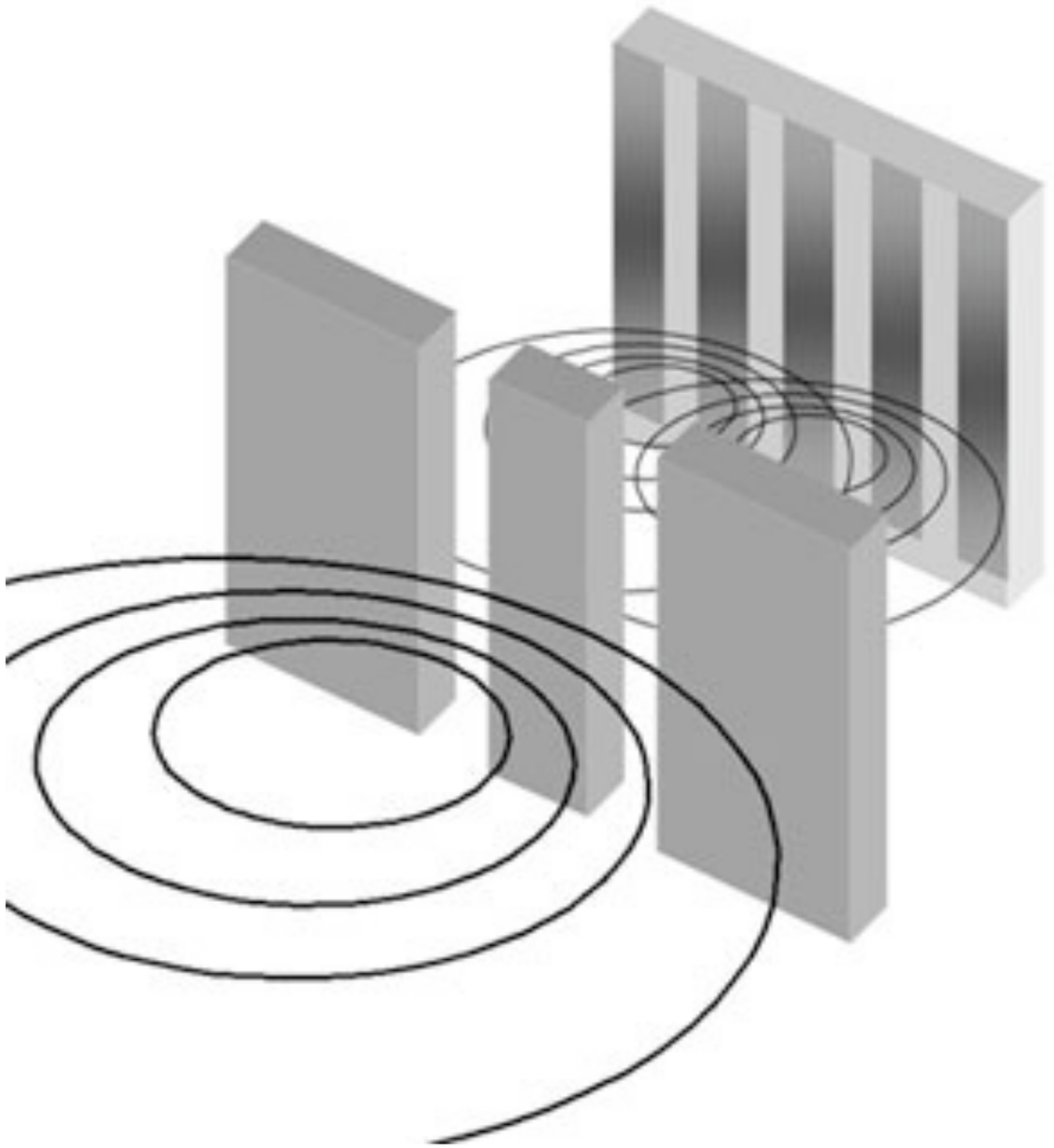
Quantum Superposition

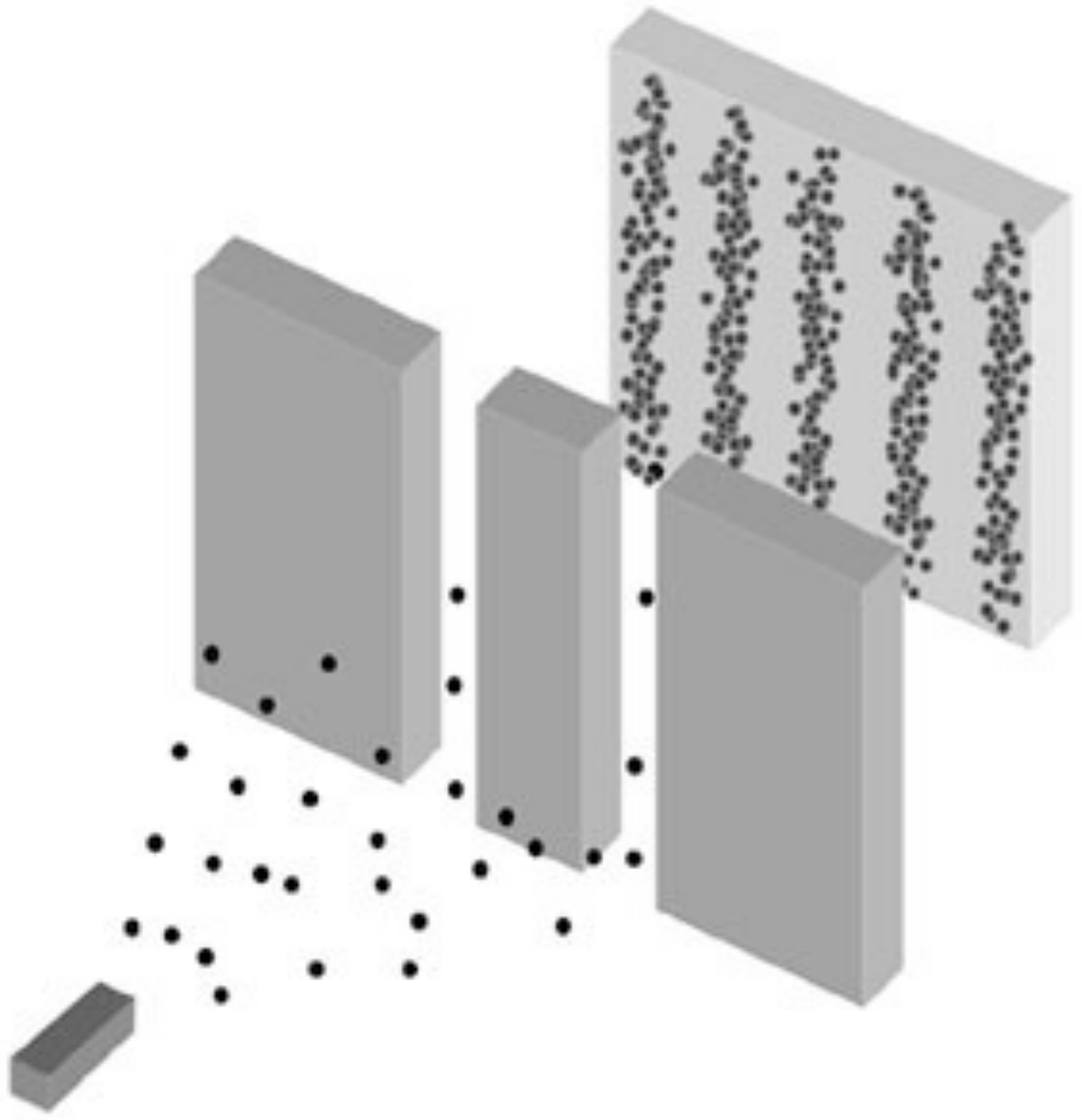
The double slit experiment



Interference of water waves

Quantum Superposition

The double slit experiment

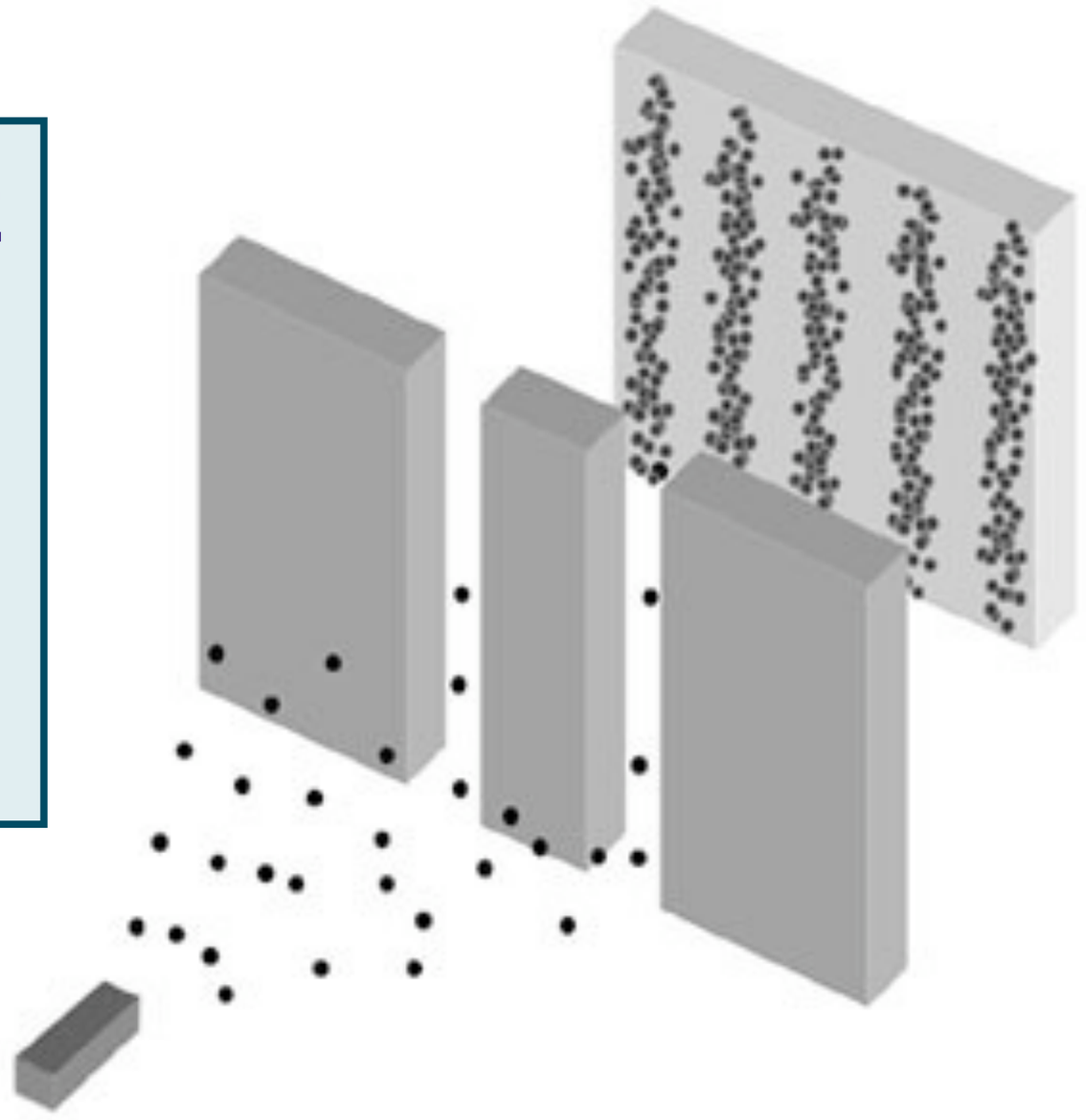


Interference of electrons

Quantum Superposition

The double slit experiment

Which slit does an electron pass through ?



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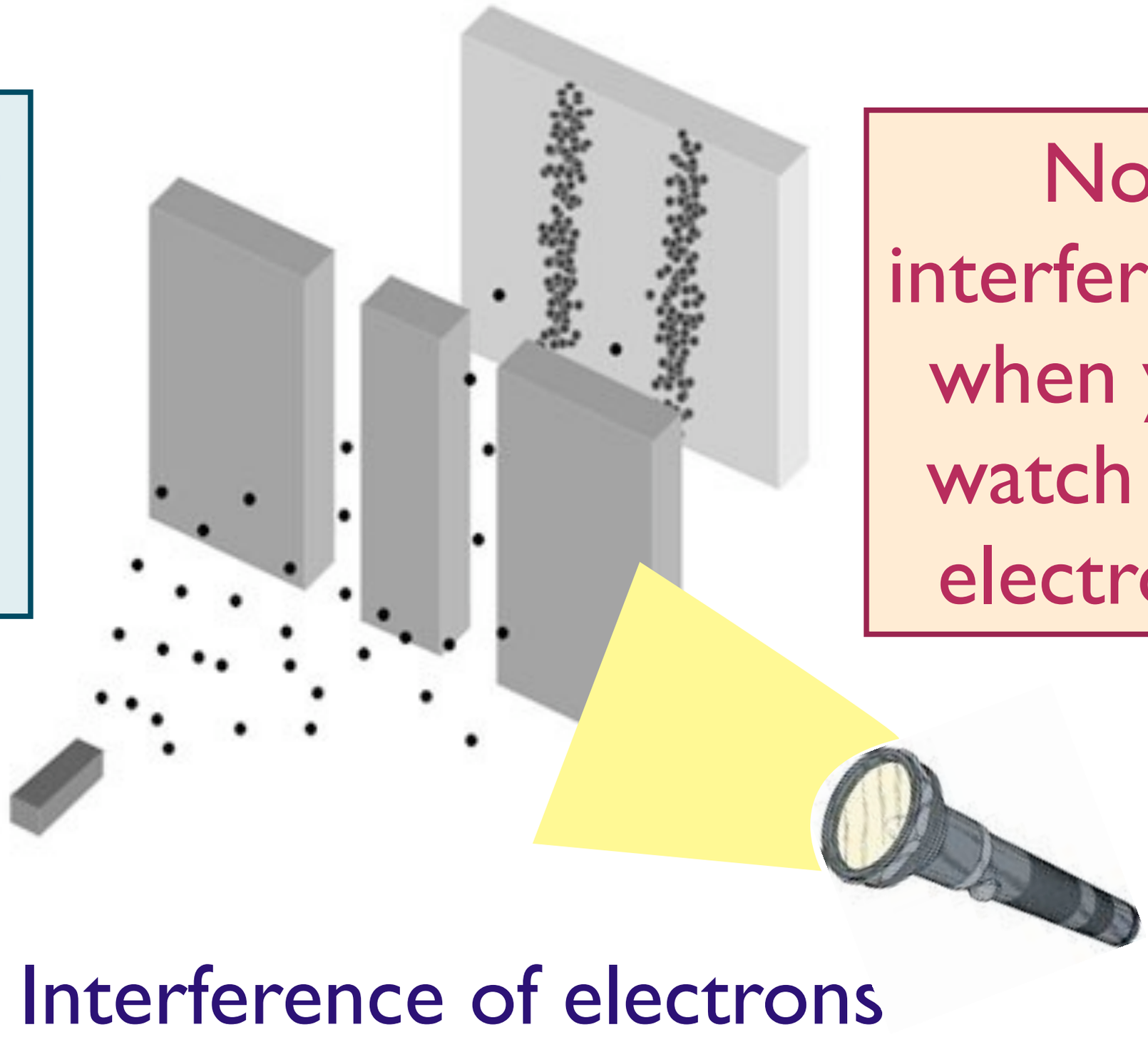


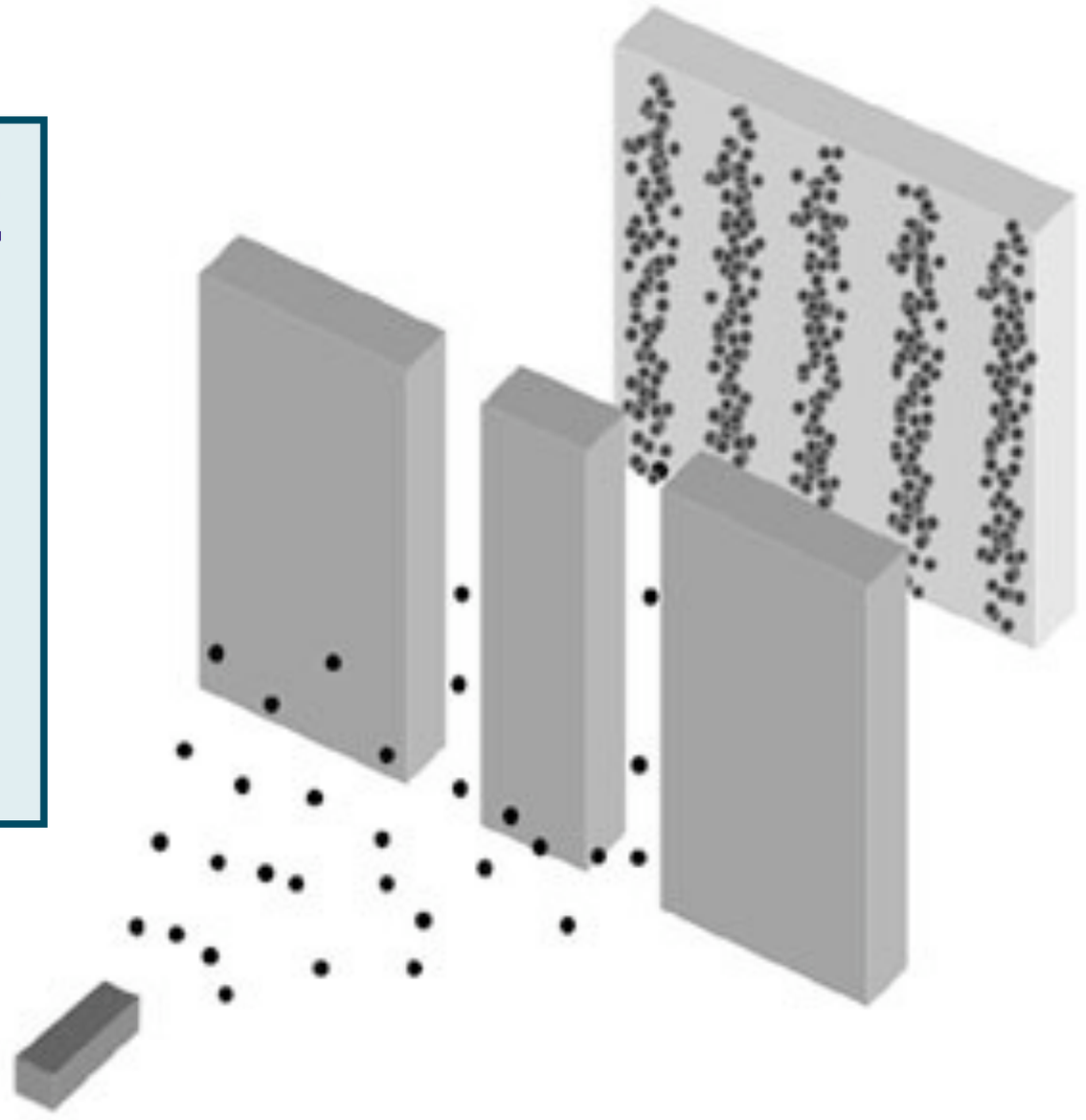
No interference when you watch the electrons

Interference of electrons

Quantum Superposition

The double slit experiment

Which slit does an electron pass through ?

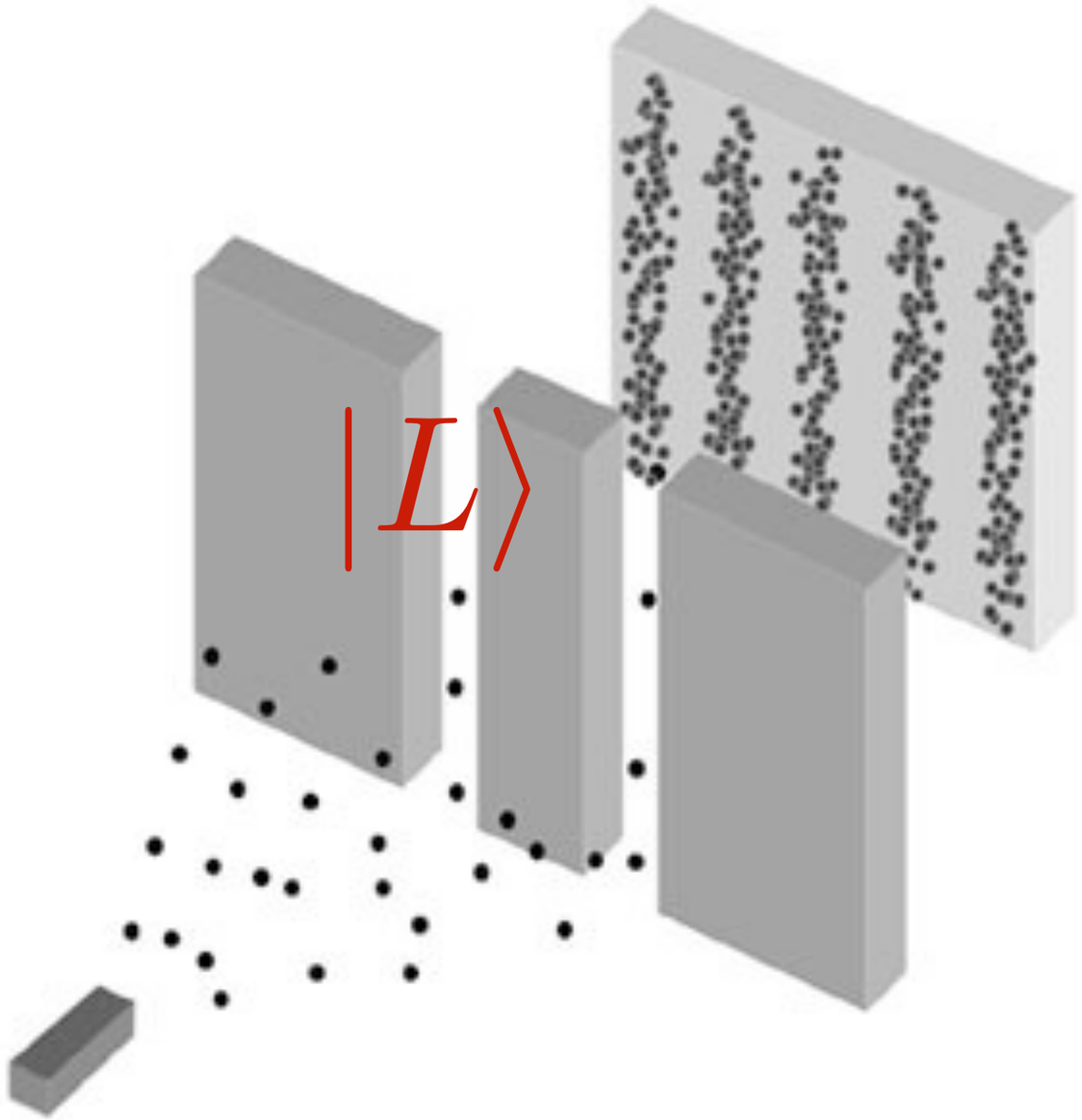


Each electron passes through both slits !

Interference of electrons

Quantum Superposition

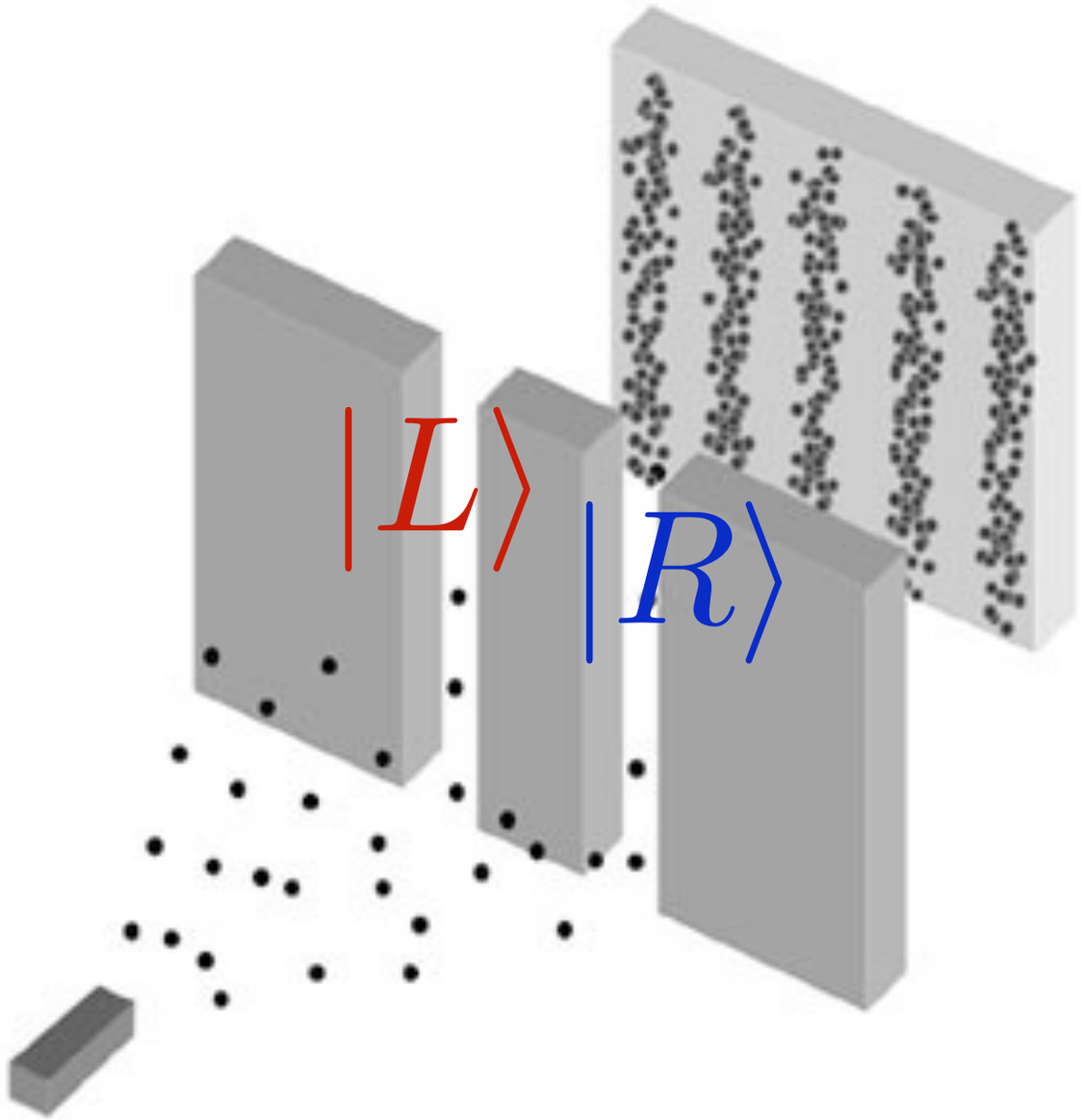
The double slit experiment



Let $|L\rangle$ represent the state
with the electron in the left slit

Quantum Superposition

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Quantum Superposition

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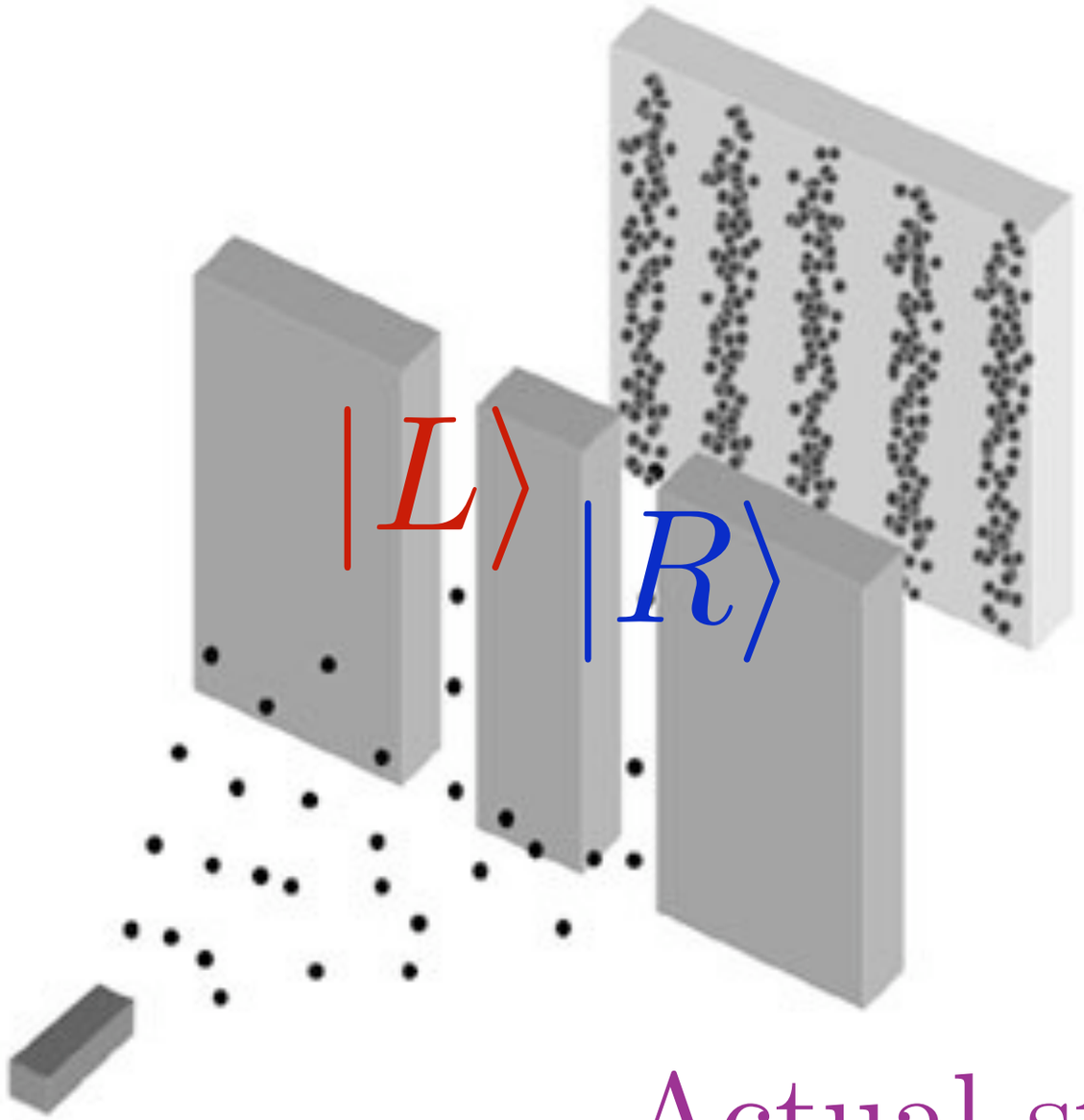

Let $|L\rangle$ represent the state with the electron in the left slit

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Actual state of the electron is

$$|L\rangle + |R\rangle$$

Quantum Entanglement: quantum superposition with more than one particle

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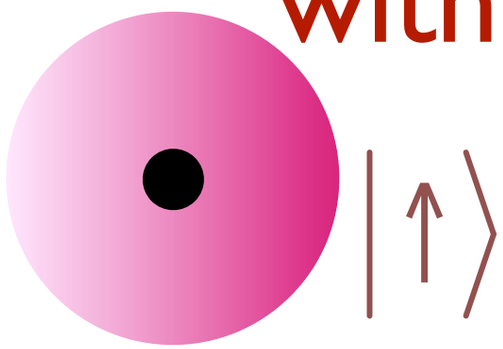
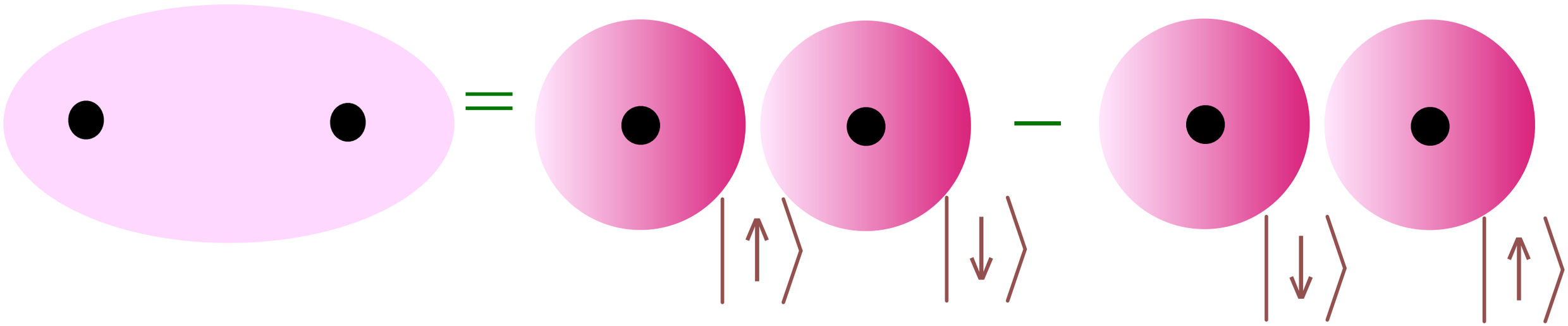
Hydrogen atom:

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:



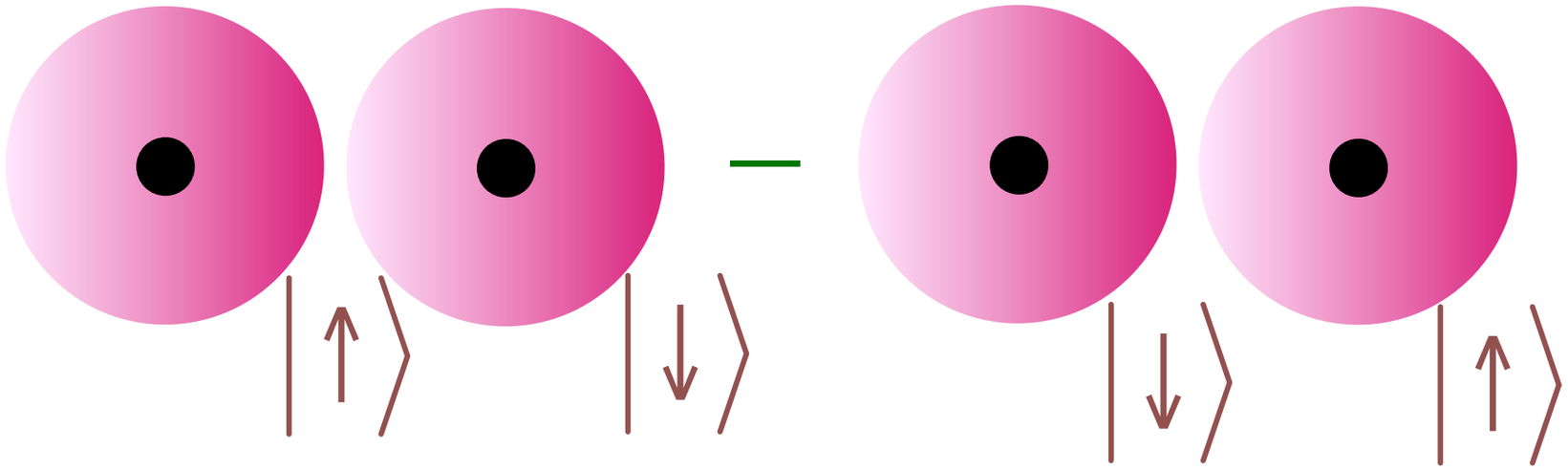
Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Superposition of two electron states leads to non-local correlations between spins

Quantum Entanglement: quantum superposition with more than one particle



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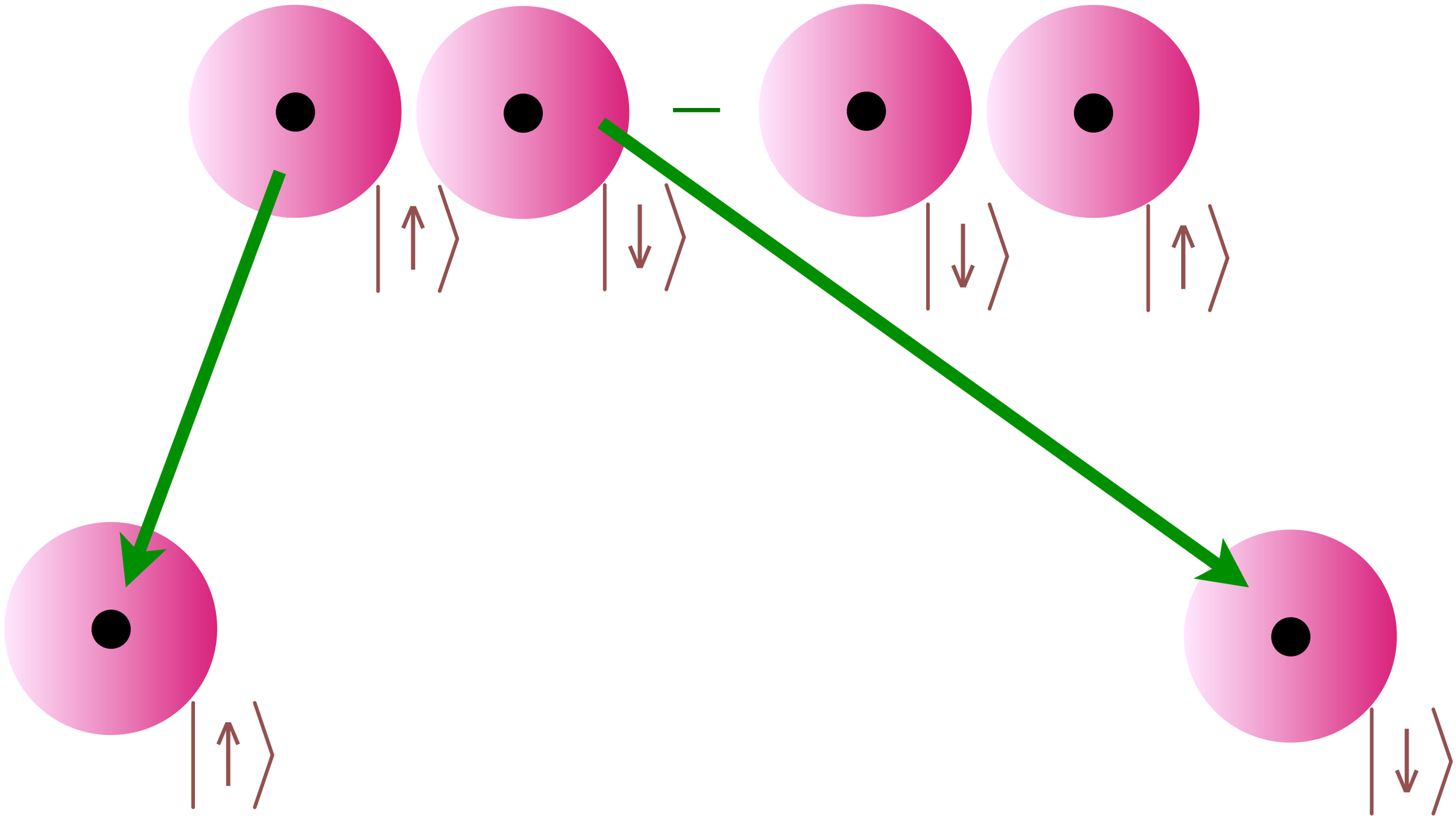

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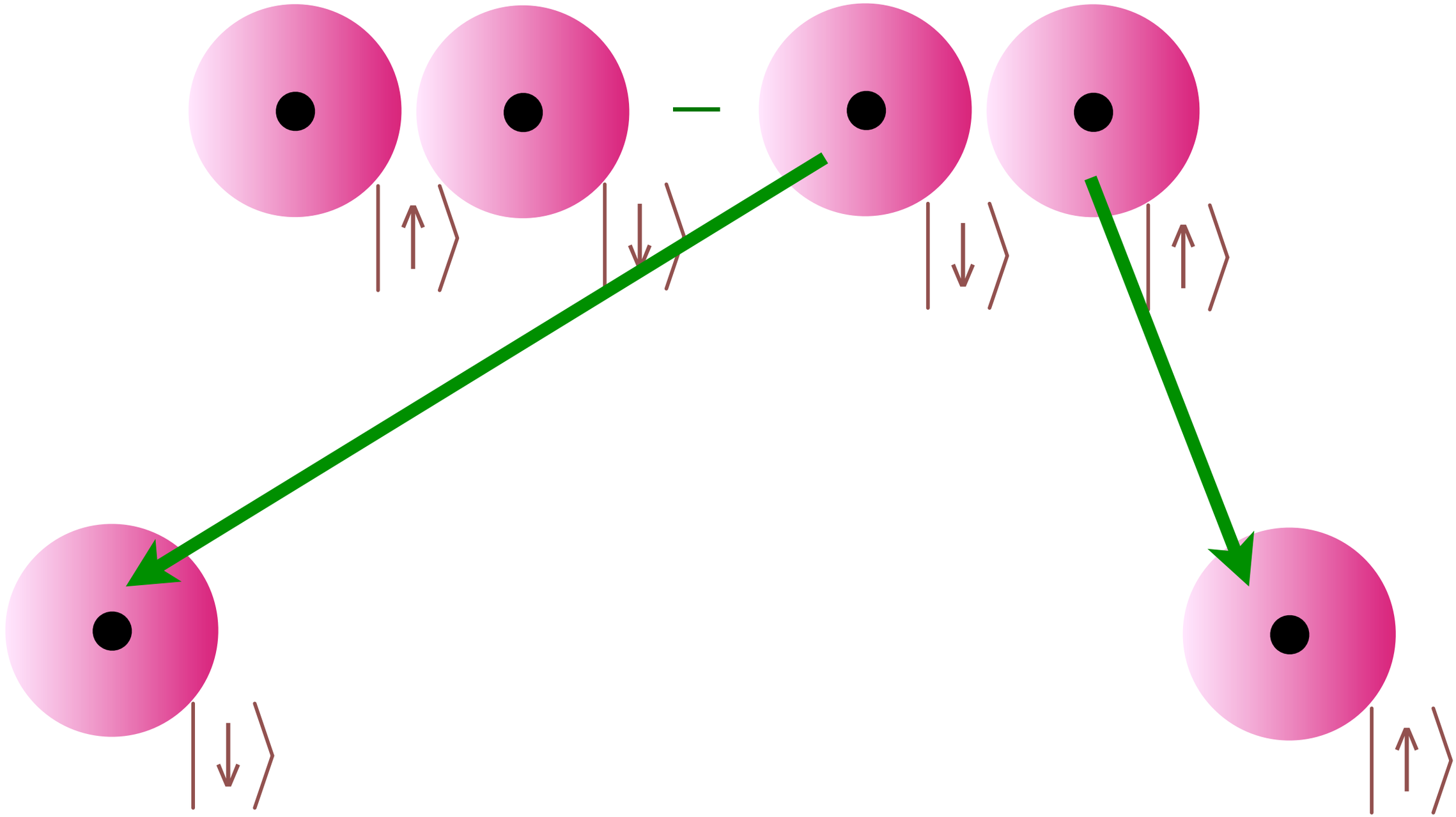

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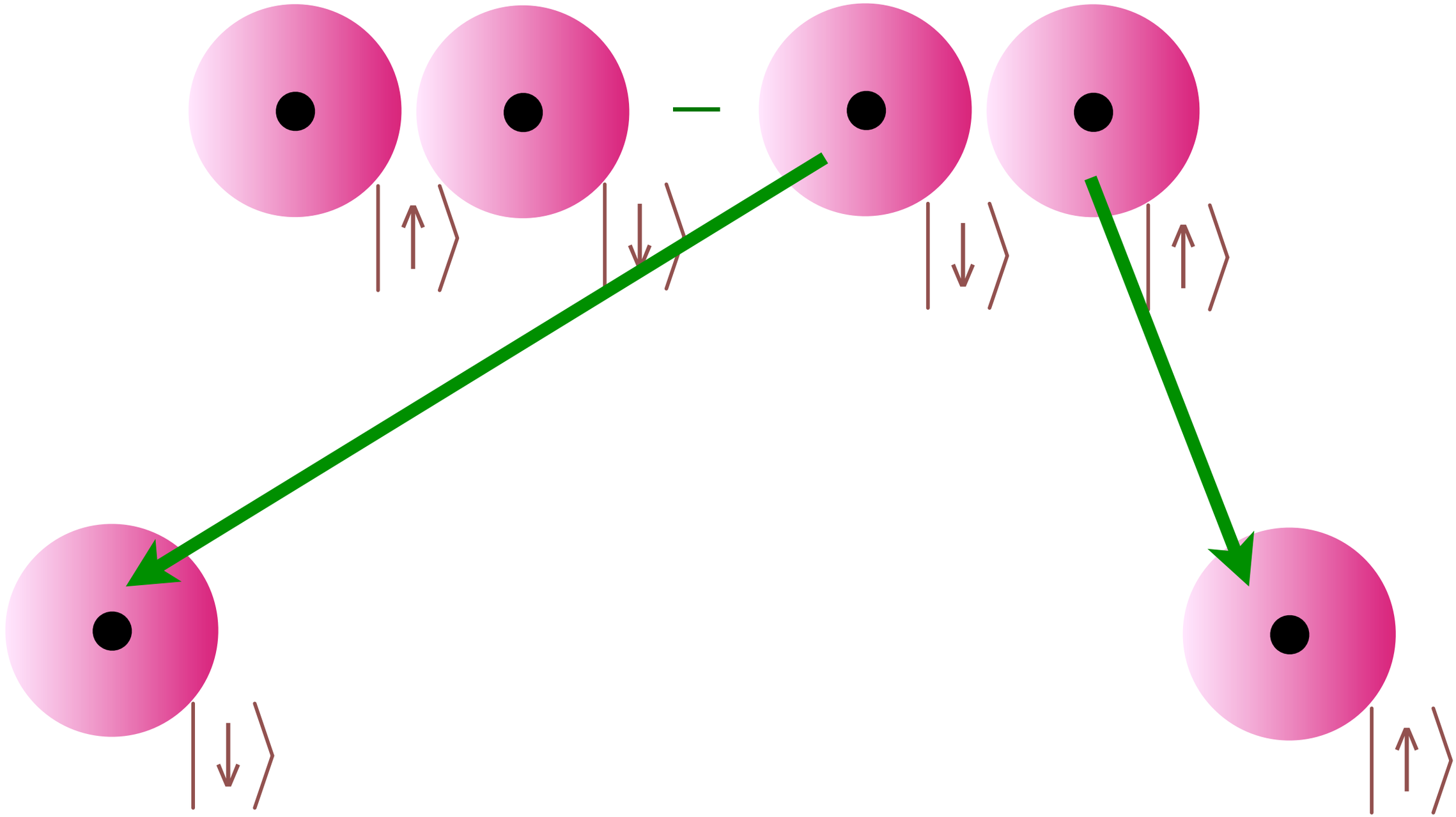
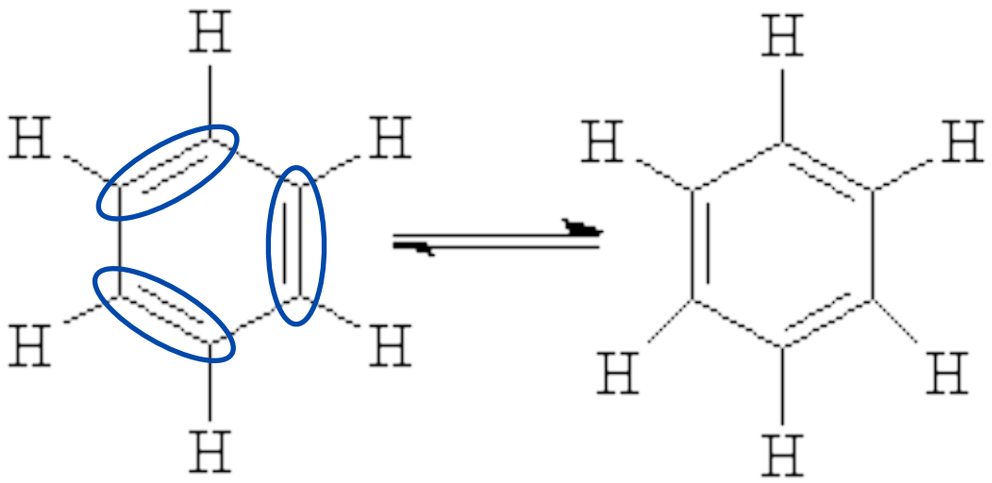
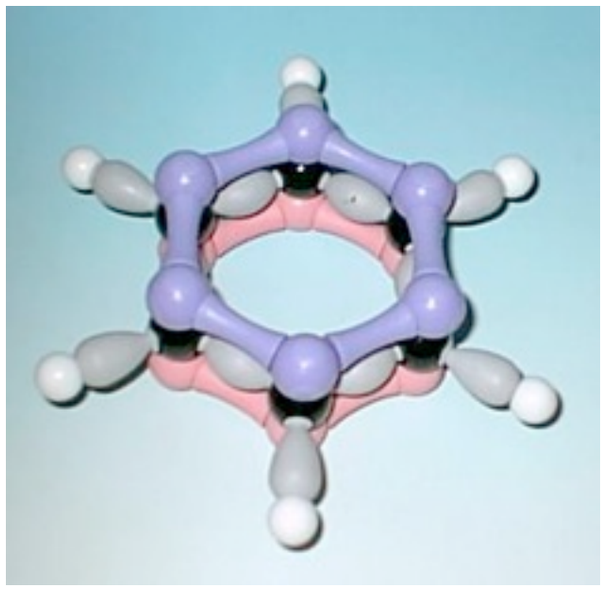


Einstein-Podolsky-Rosen "paradox": Non-local correlations between observations arbitrarily far apart

Entanglement of chemical bonds

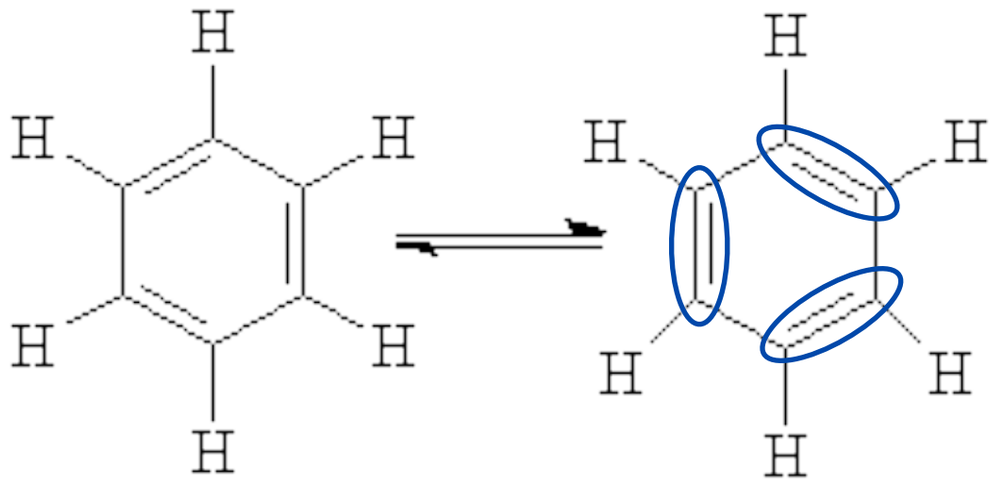


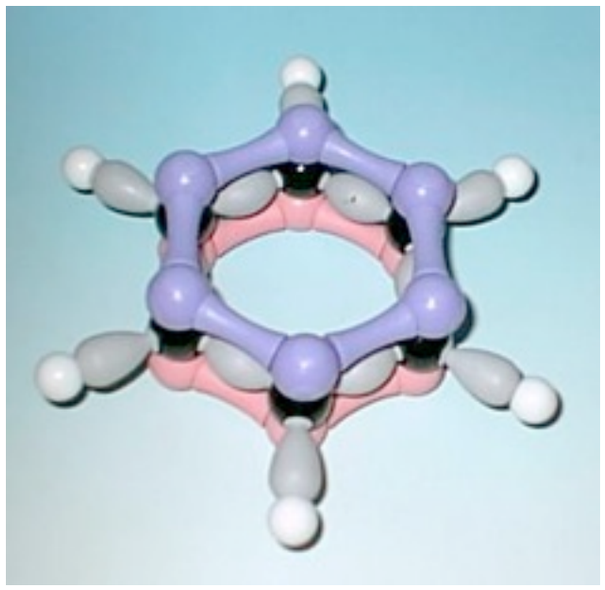


Resonance in benzene leads to a symmetric configuration of chemical bonds

(*F. Kekulé, L. Pauling*)

Entanglement of chemical bonds





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Quantum superposition and entanglement

Superconductivity

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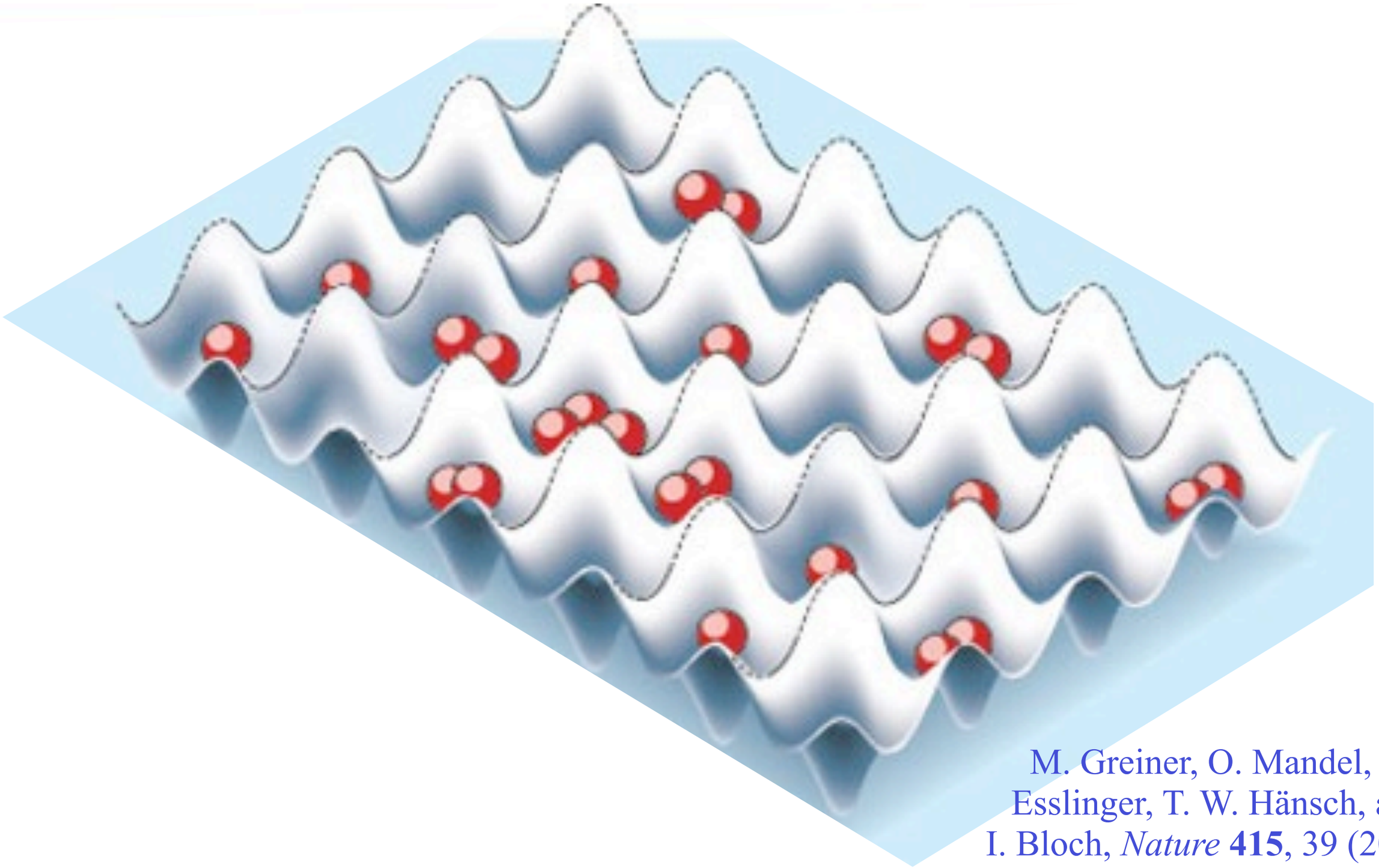
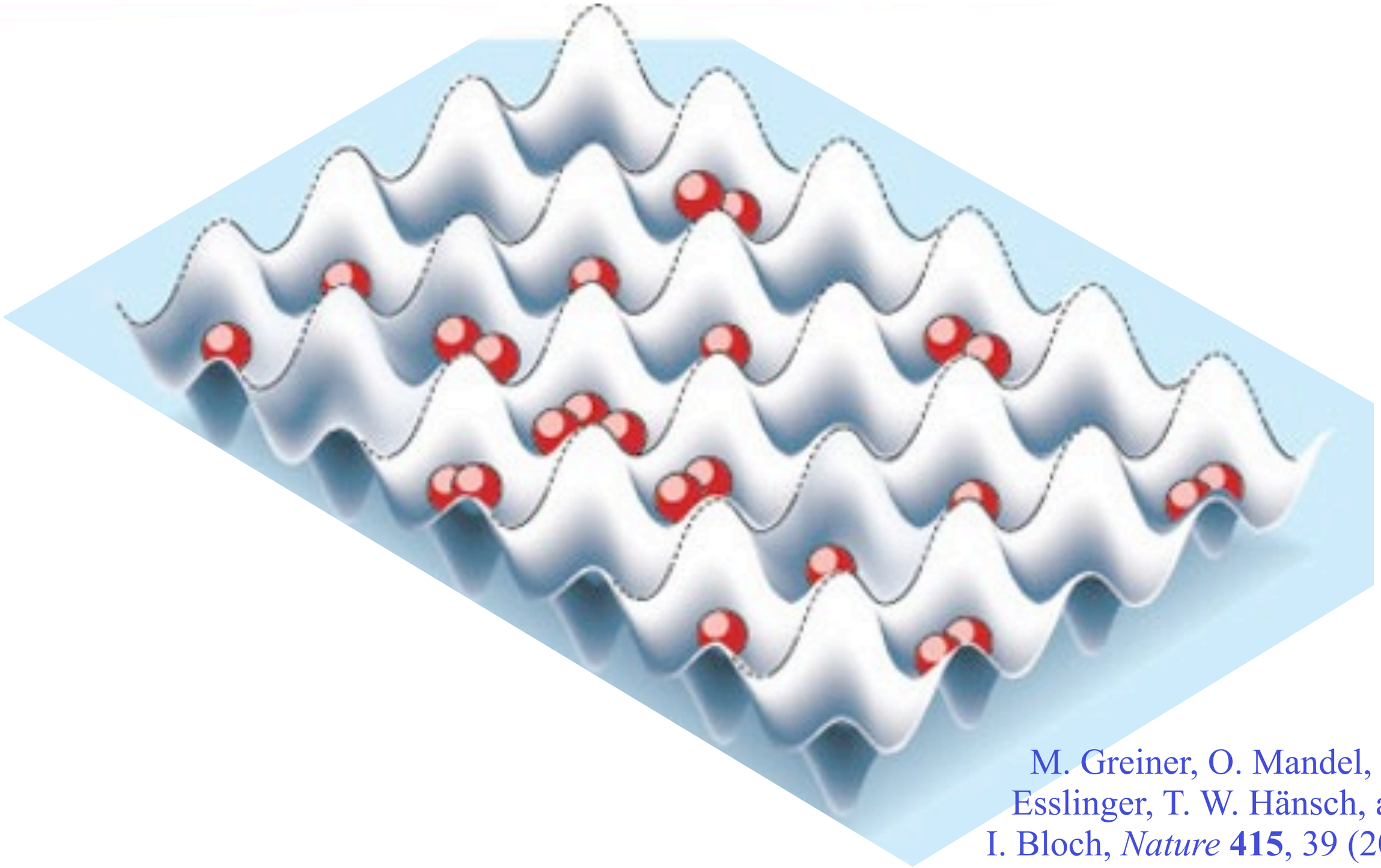
Rubidium atoms in a magnetic trap and standing waves of laser light



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

At very low temperatures and for a weak laser light, the Rubidium atoms form a Bose-Einstein condensate



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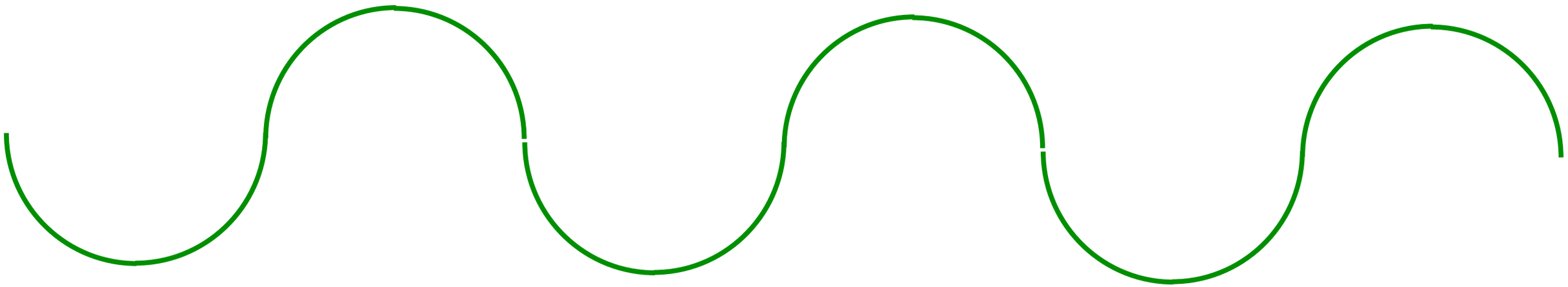
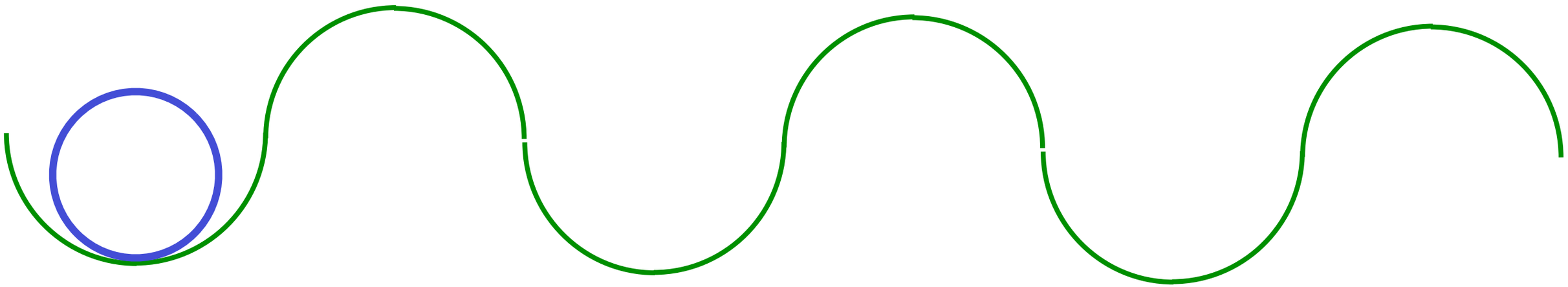
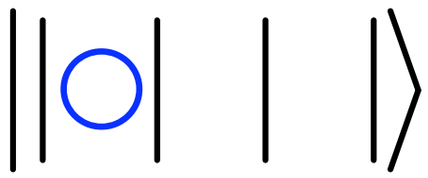
A Bose-Einstein condensate: An quantum superposition of all the atoms in all positions

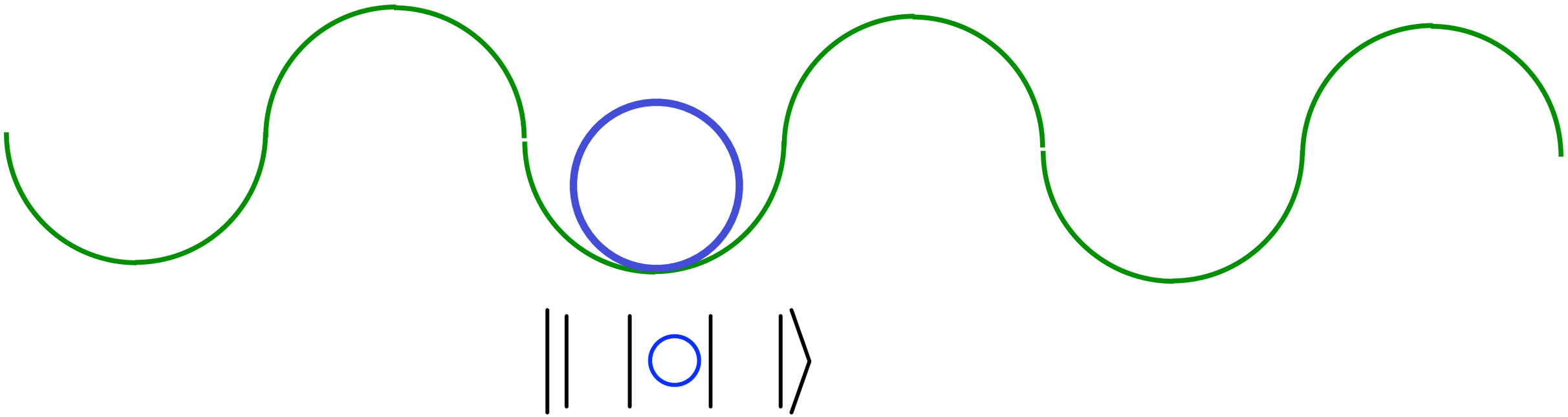
A liquid which flows without resistance (a superfluid)







A single atom is superposed between all positions



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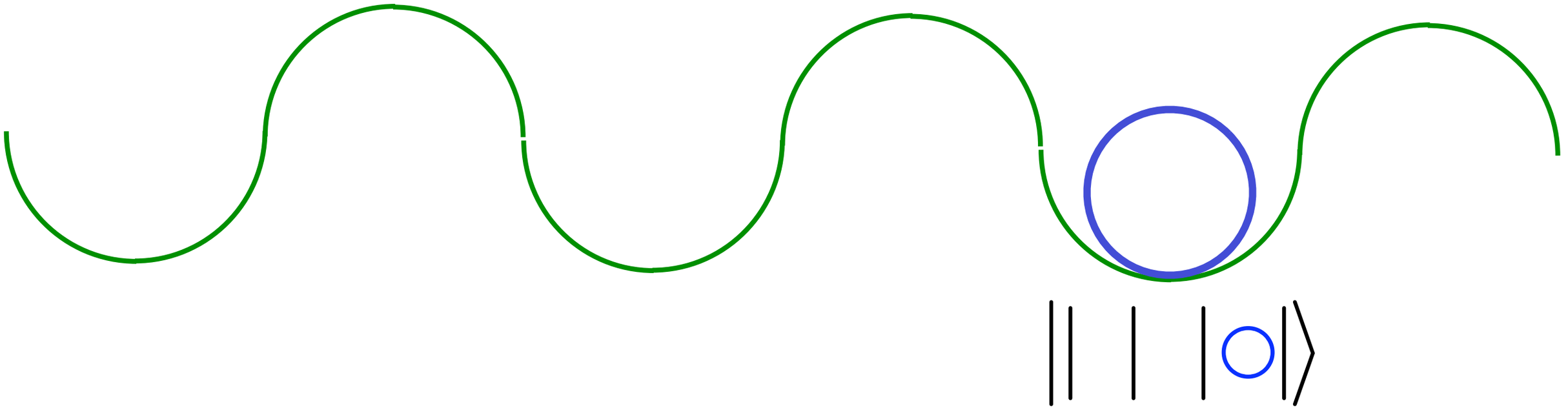

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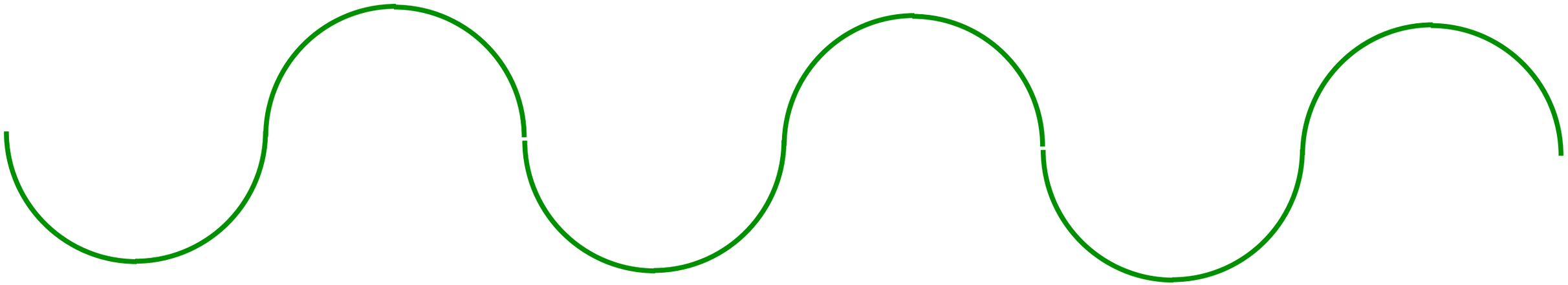

$$|\mathrm{G}\rangle = \left(|\bigcirc|\ |\ |\rangle + ||\ |\bigcirc|\ |\rangle + ||\ |\ |\bigcirc|\rangle\right)$$

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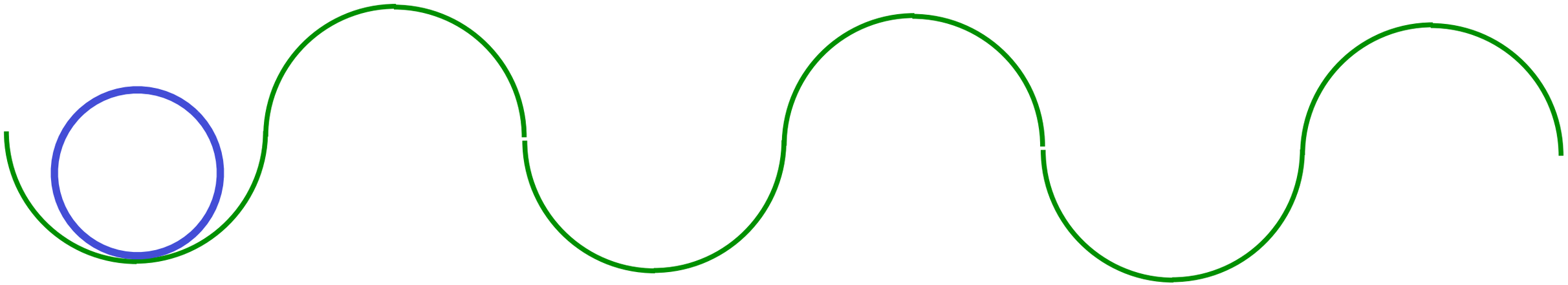

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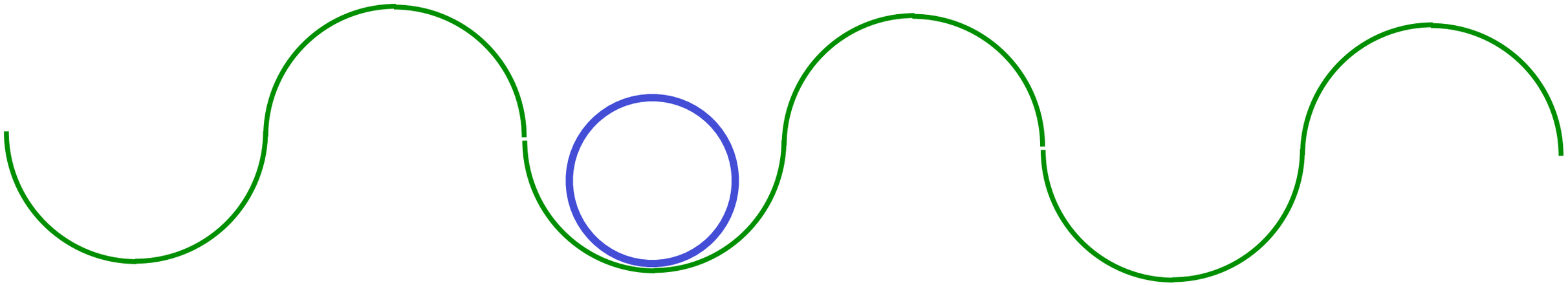

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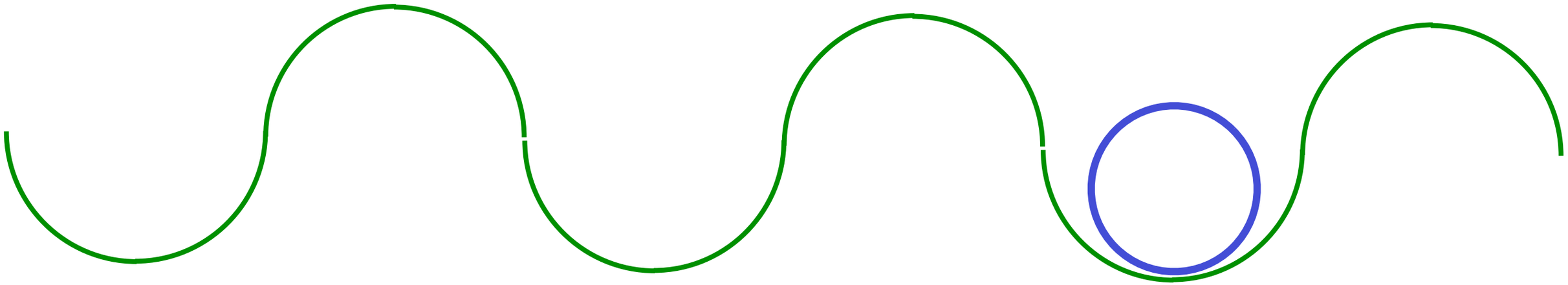

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Bose-Einstein condensate: superposition between all atoms

$$|\mathrm{BEC}\rangle = |\mathrm{G}\rangle|\mathrm{G}\rangle|\mathrm{G}\rangle$$

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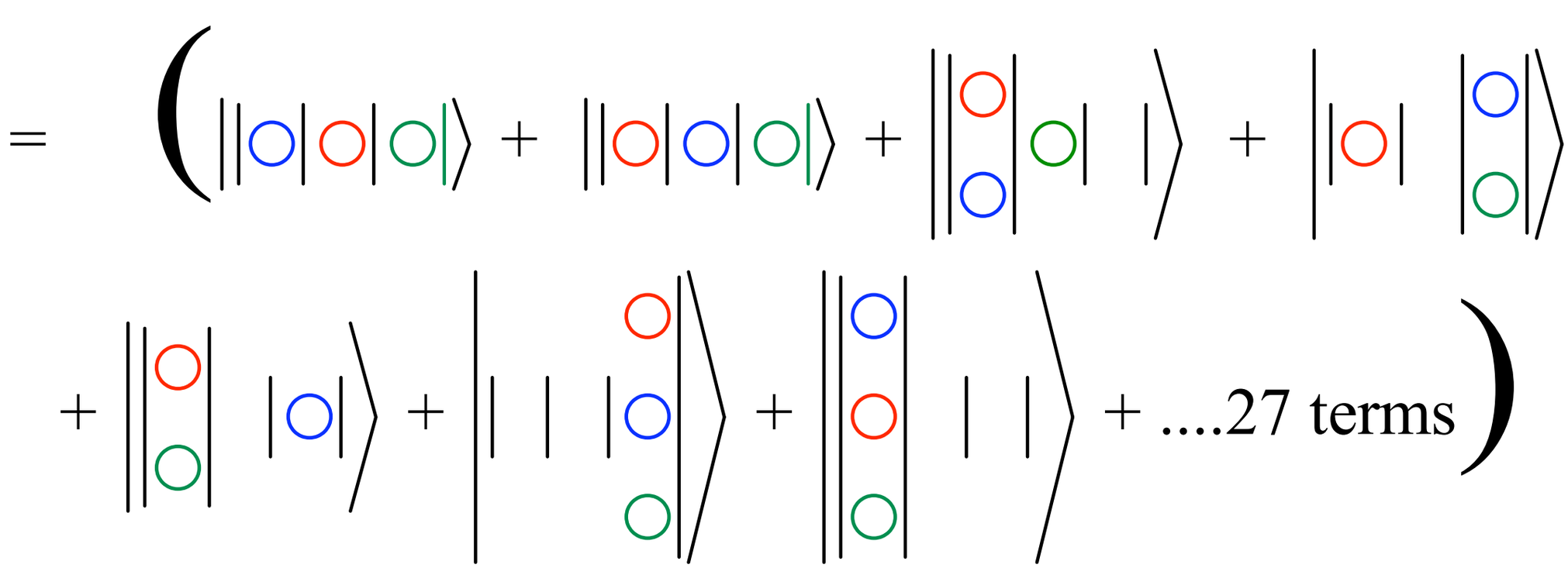
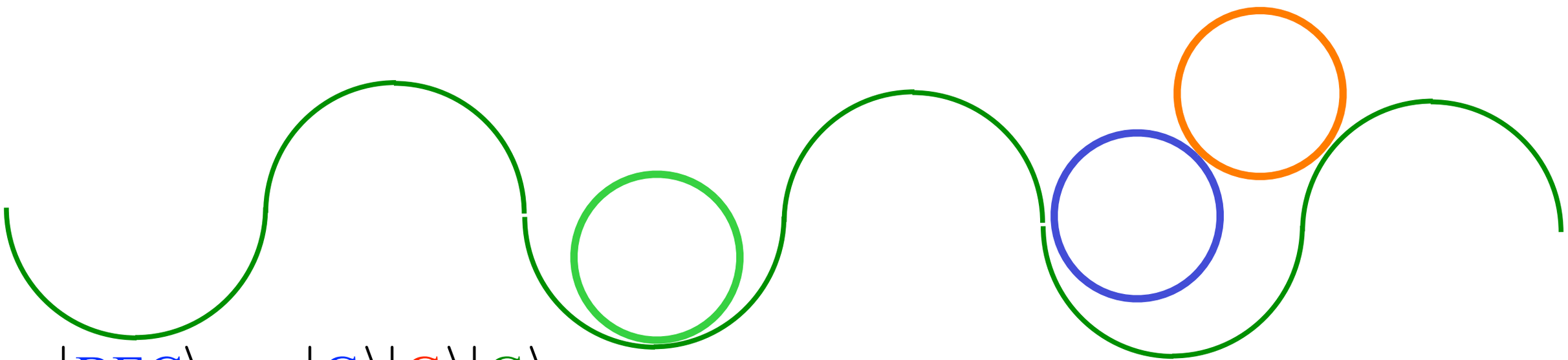


Large fluctuations in number of atoms in each site – *superfluidity* (atoms can "flow" without dissipation)

Bose-Einstein condensate: superposition between all atoms



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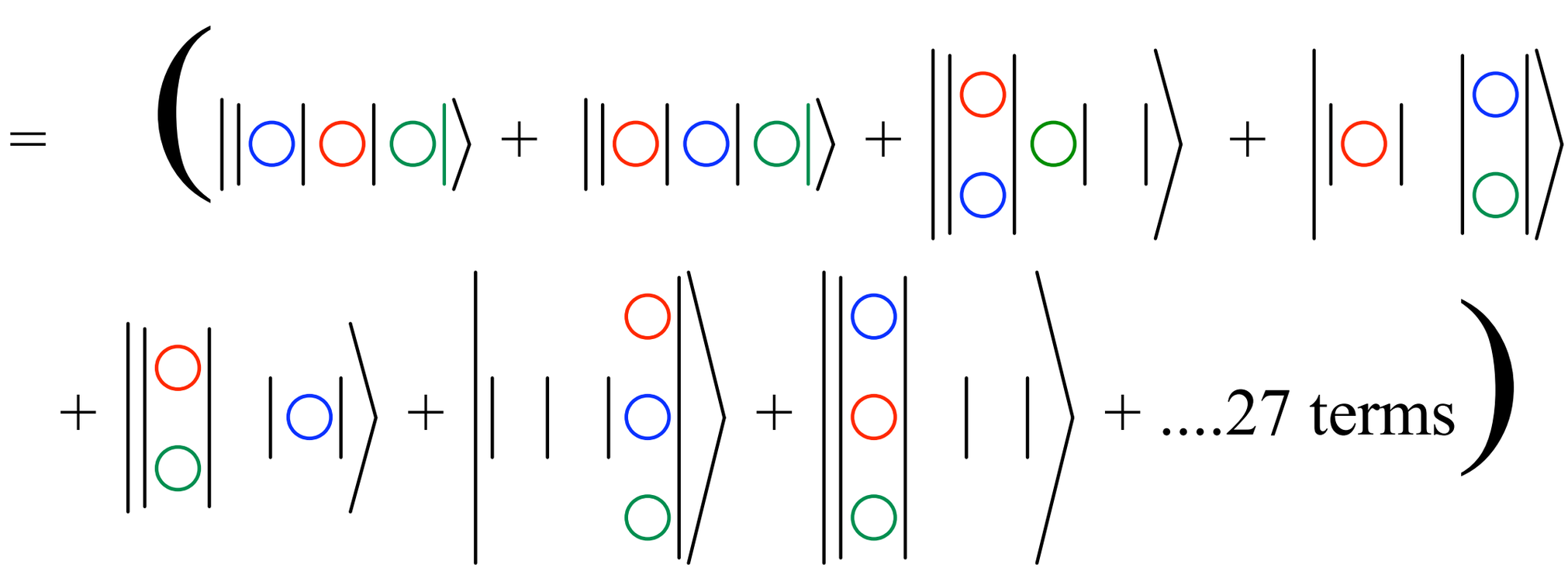

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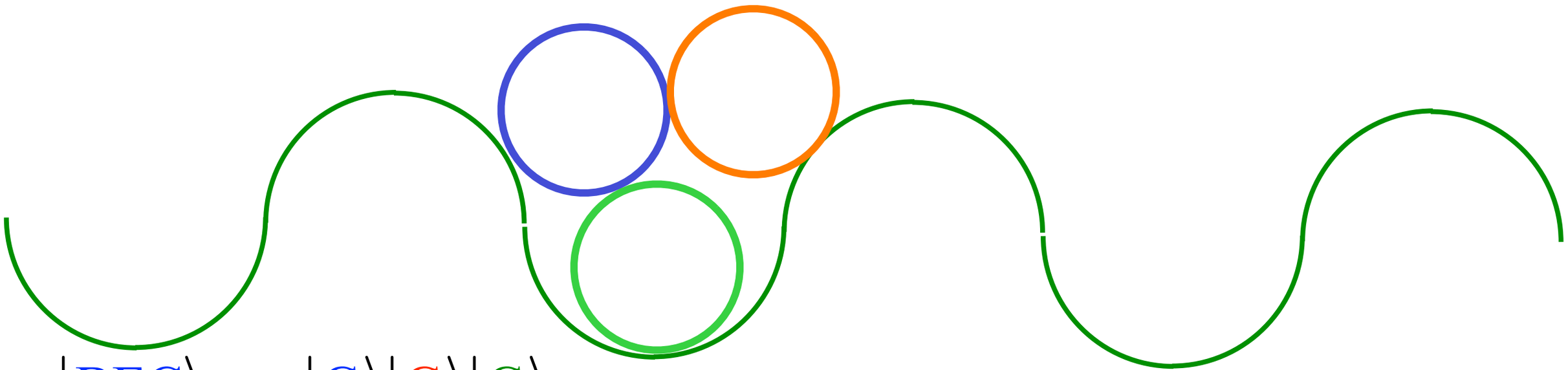

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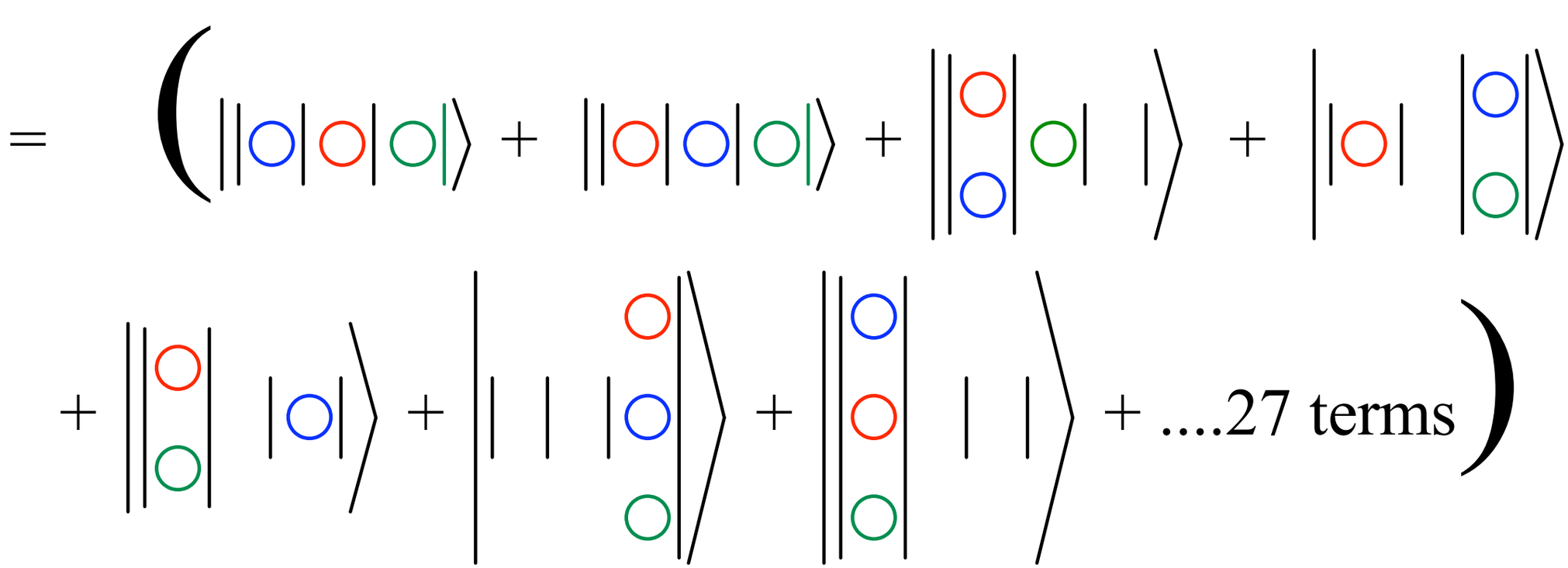

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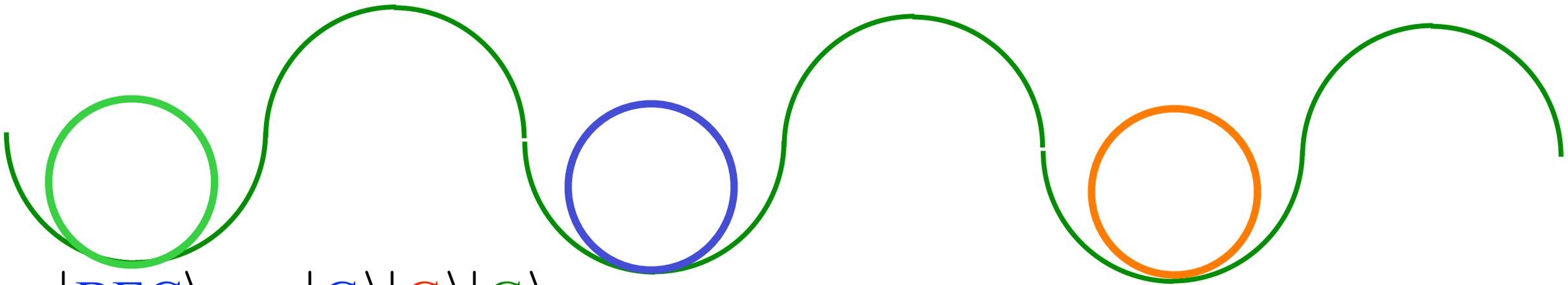

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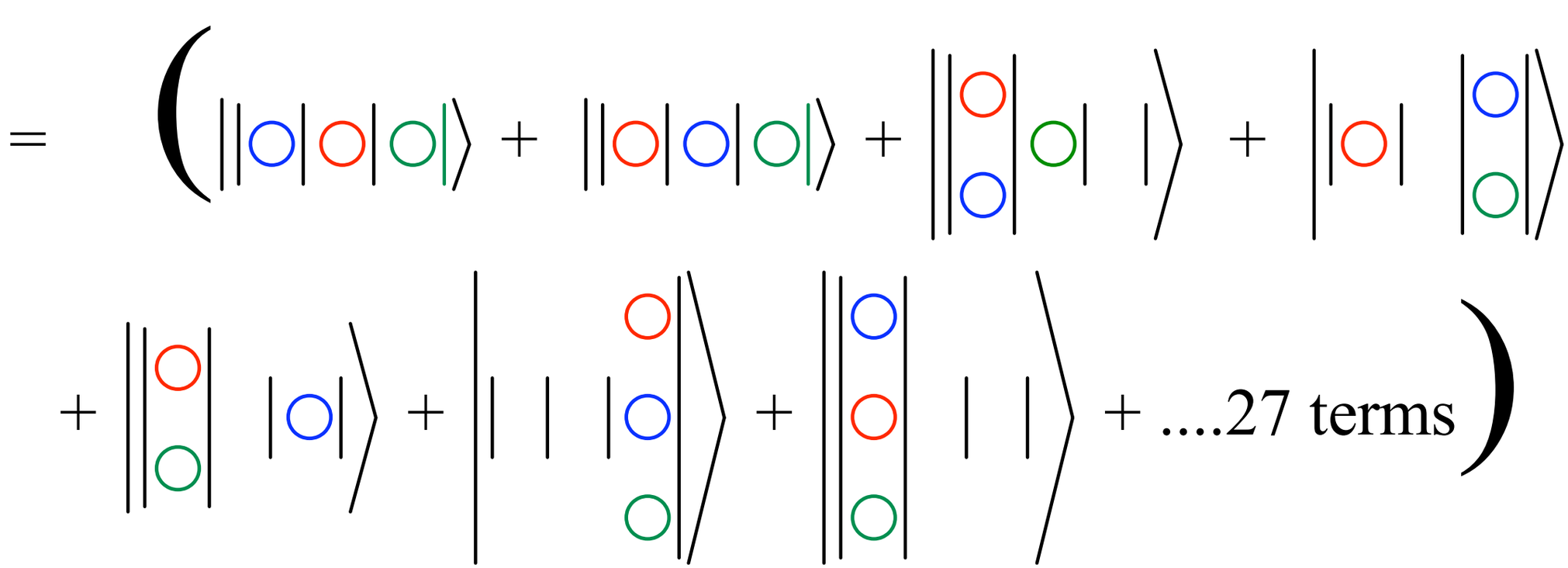

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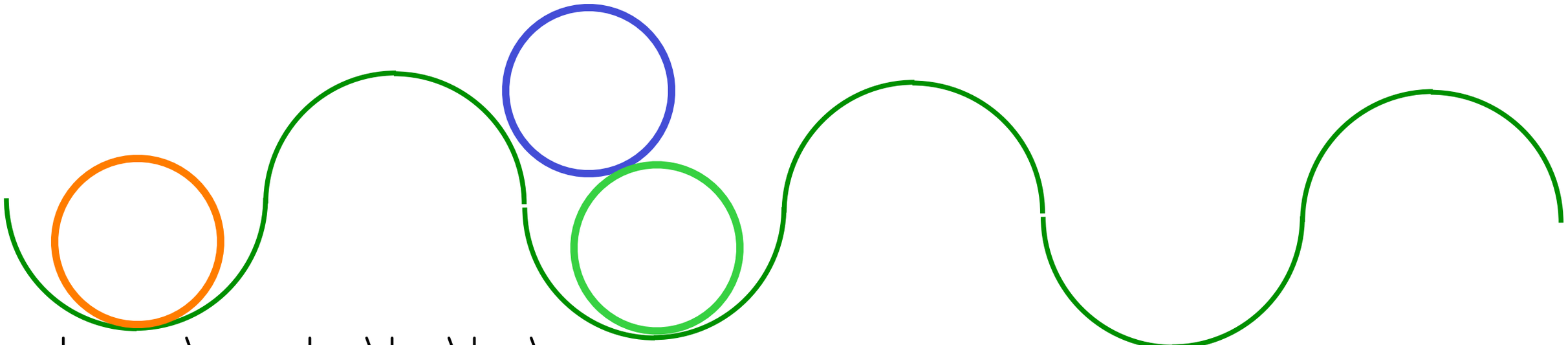

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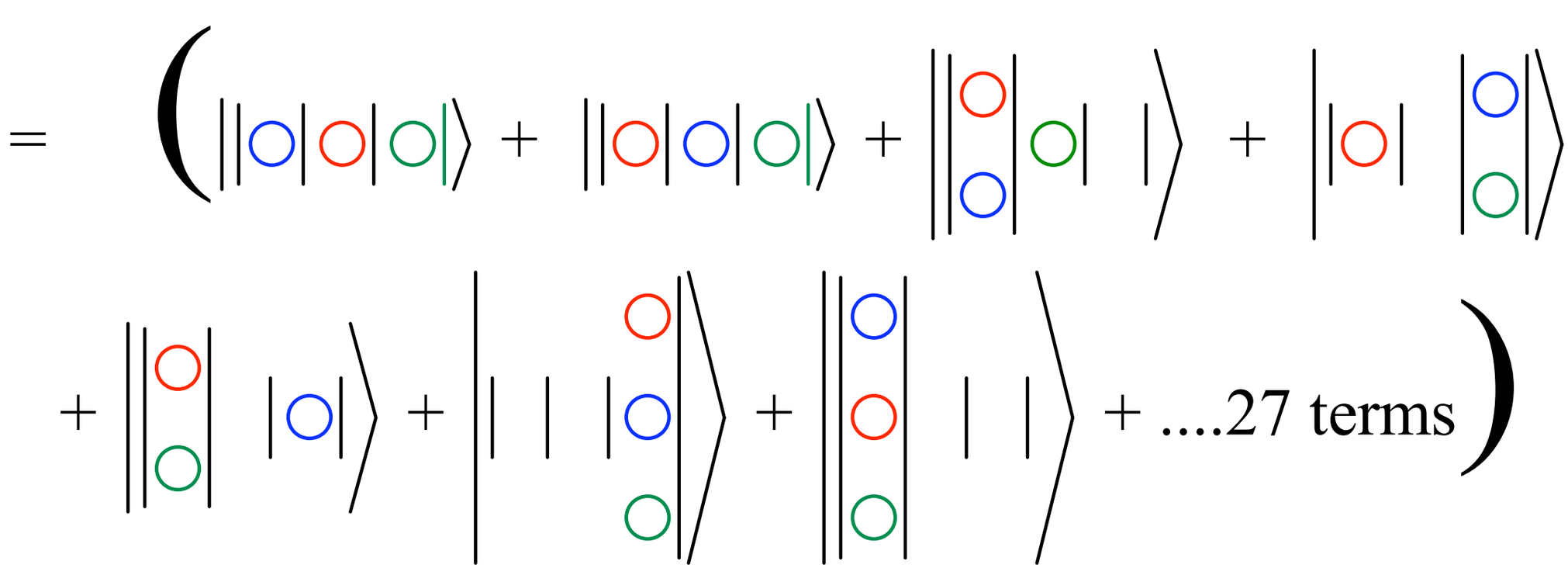

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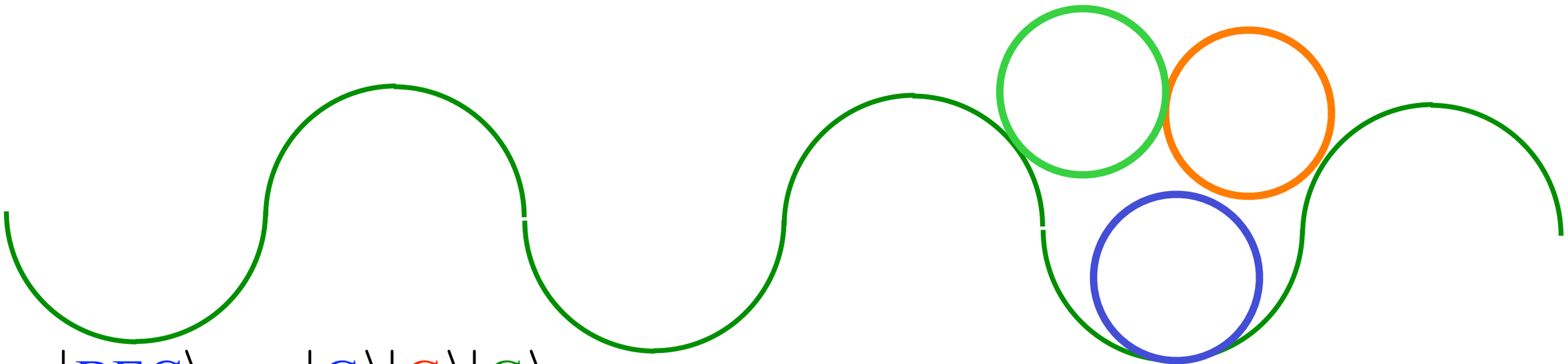

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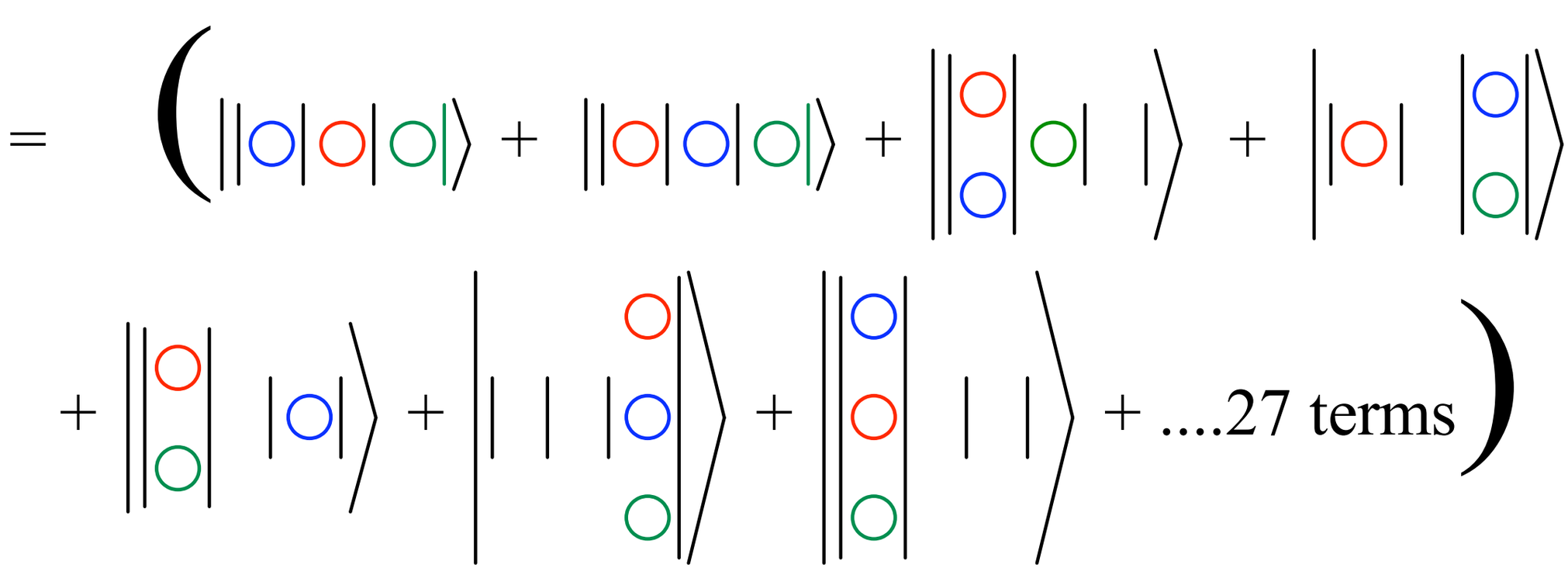

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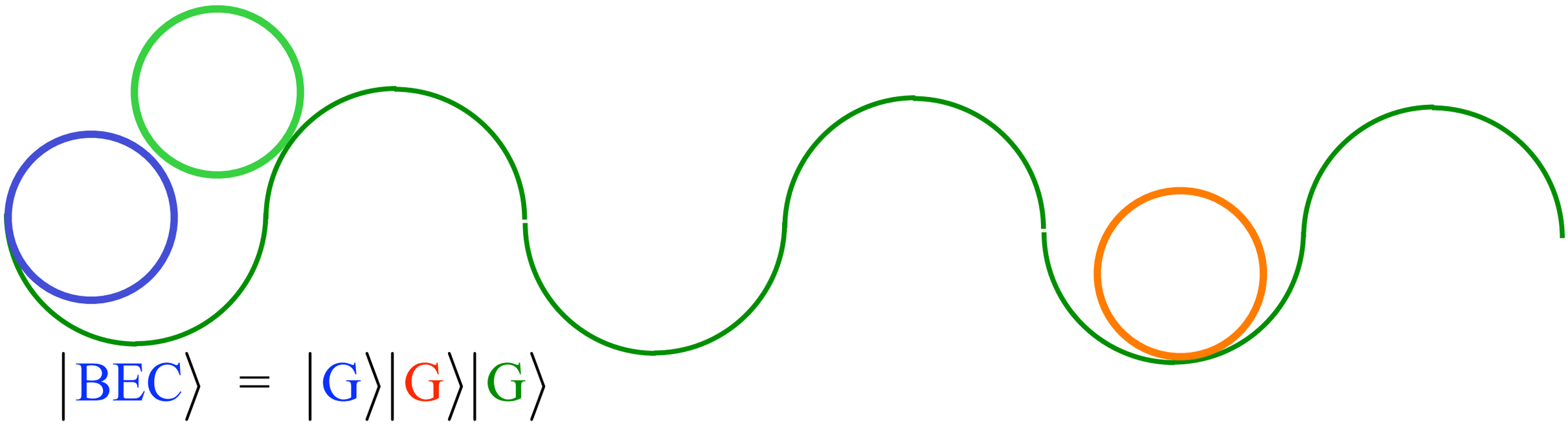

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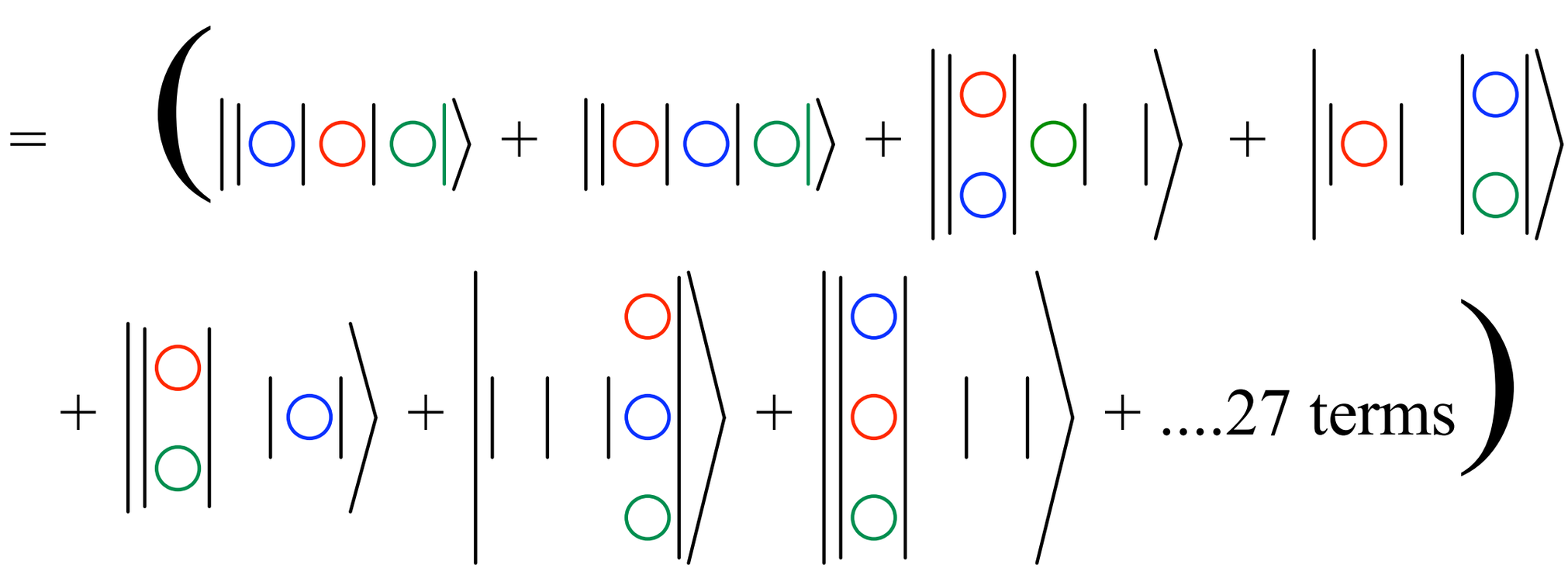

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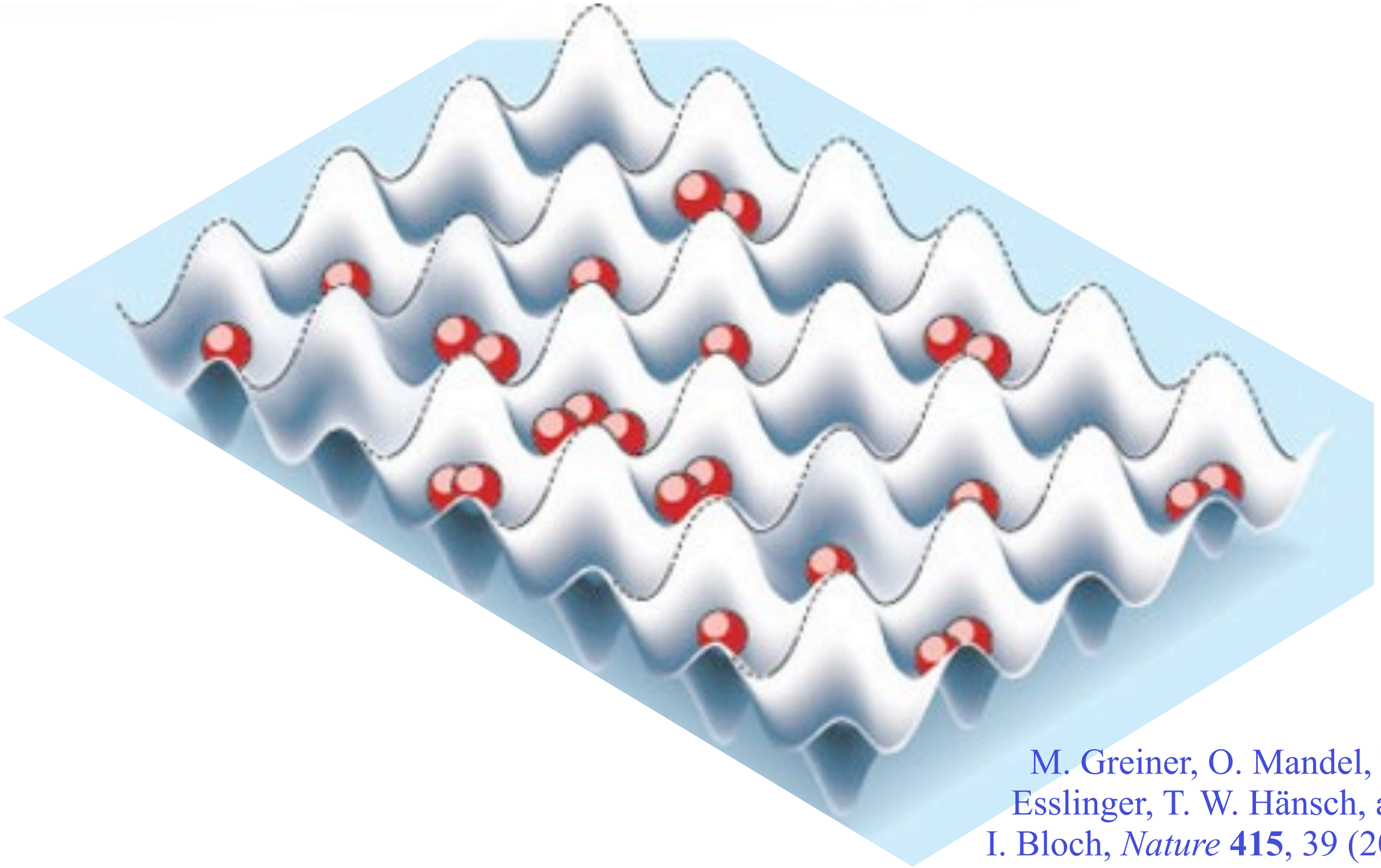
At very low temperatures and for a weak laser light, the Rubidium atoms form a Bose-Einstein condensate



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Bose-Einstein condensate: superposition between all atoms

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(Strictly speaking: this is not entanglement between the atoms because the BEC is a product of simple "wave" states of the atoms)

A superconductor: a Bose condensate of pairs of electrons in a "chemical bond" in a metal

$$|\mathrm{BEC}\rangle = |\mathrm{G}\rangle|\mathrm{G}\rangle|\mathrm{G}\rangle$$

$$|G\rangle \equiv |\uparrow\downarrow - \downarrow\uparrow\rangle$$

$$|G\rangle \equiv |\uparrow\downarrow - \downarrow\uparrow\rangle$$

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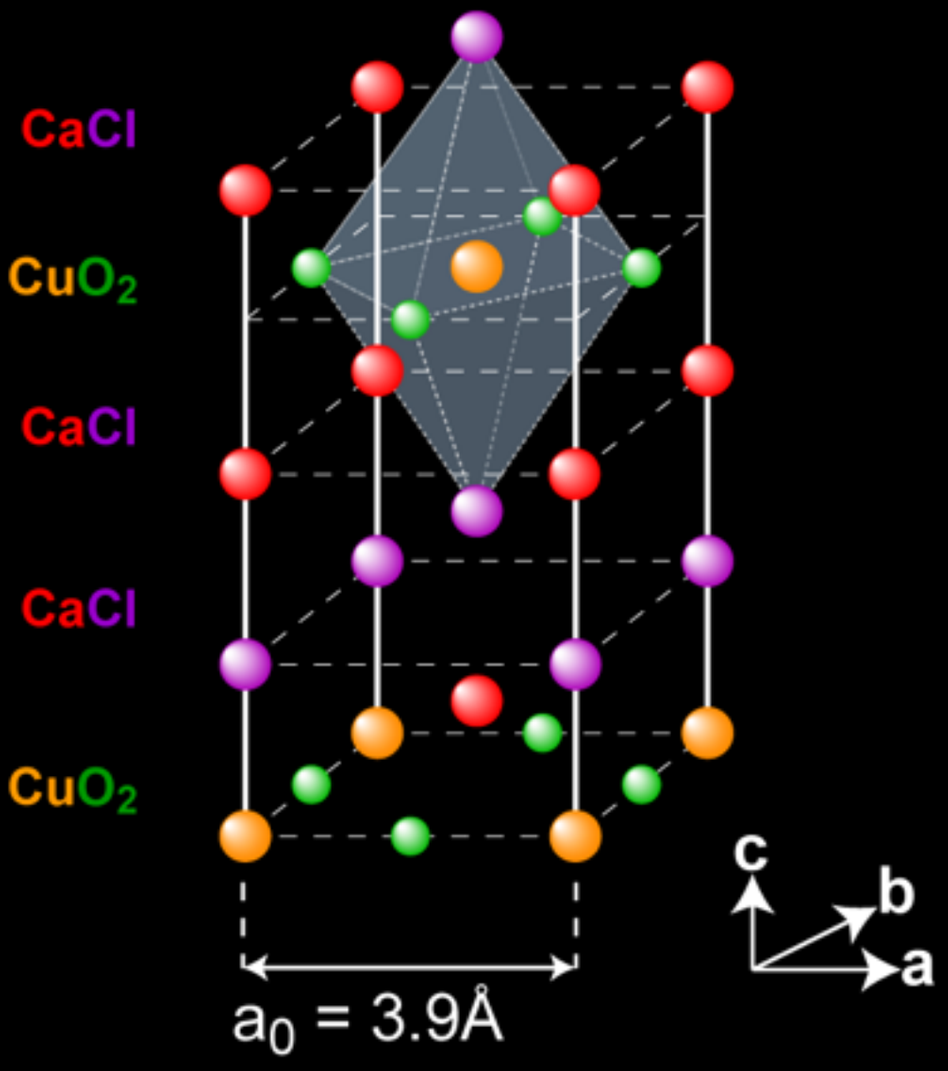
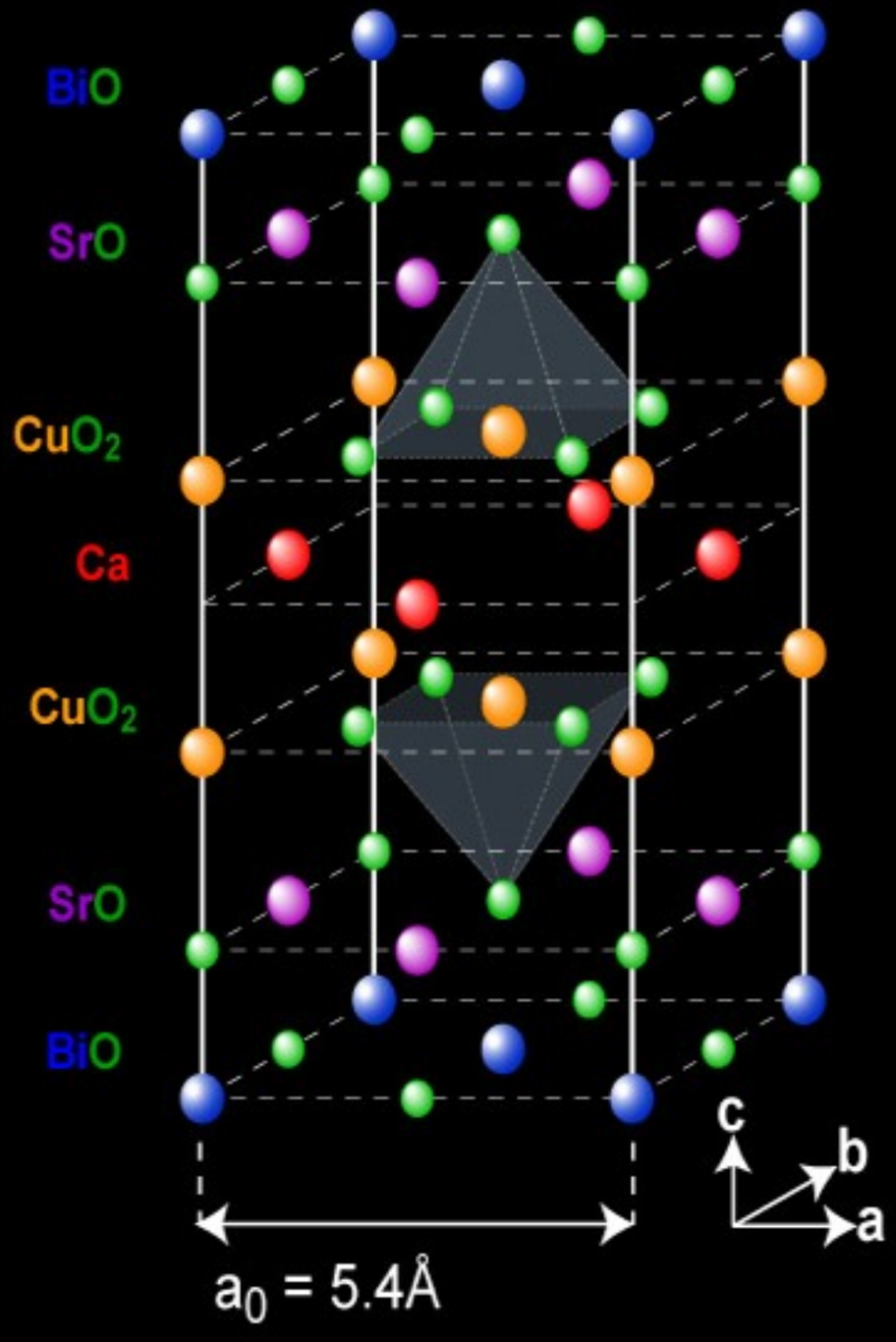
High temperature superconductors

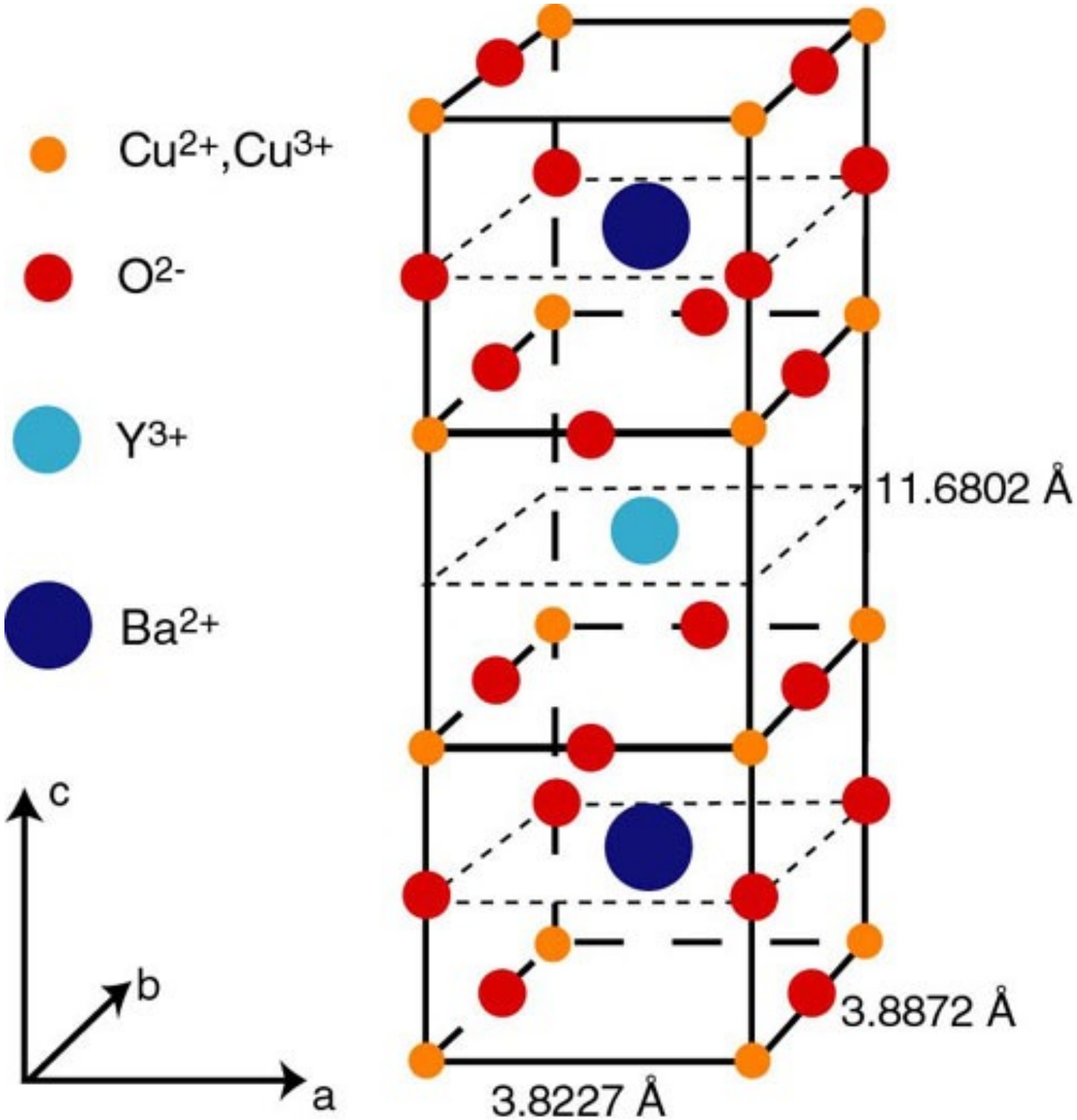
$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

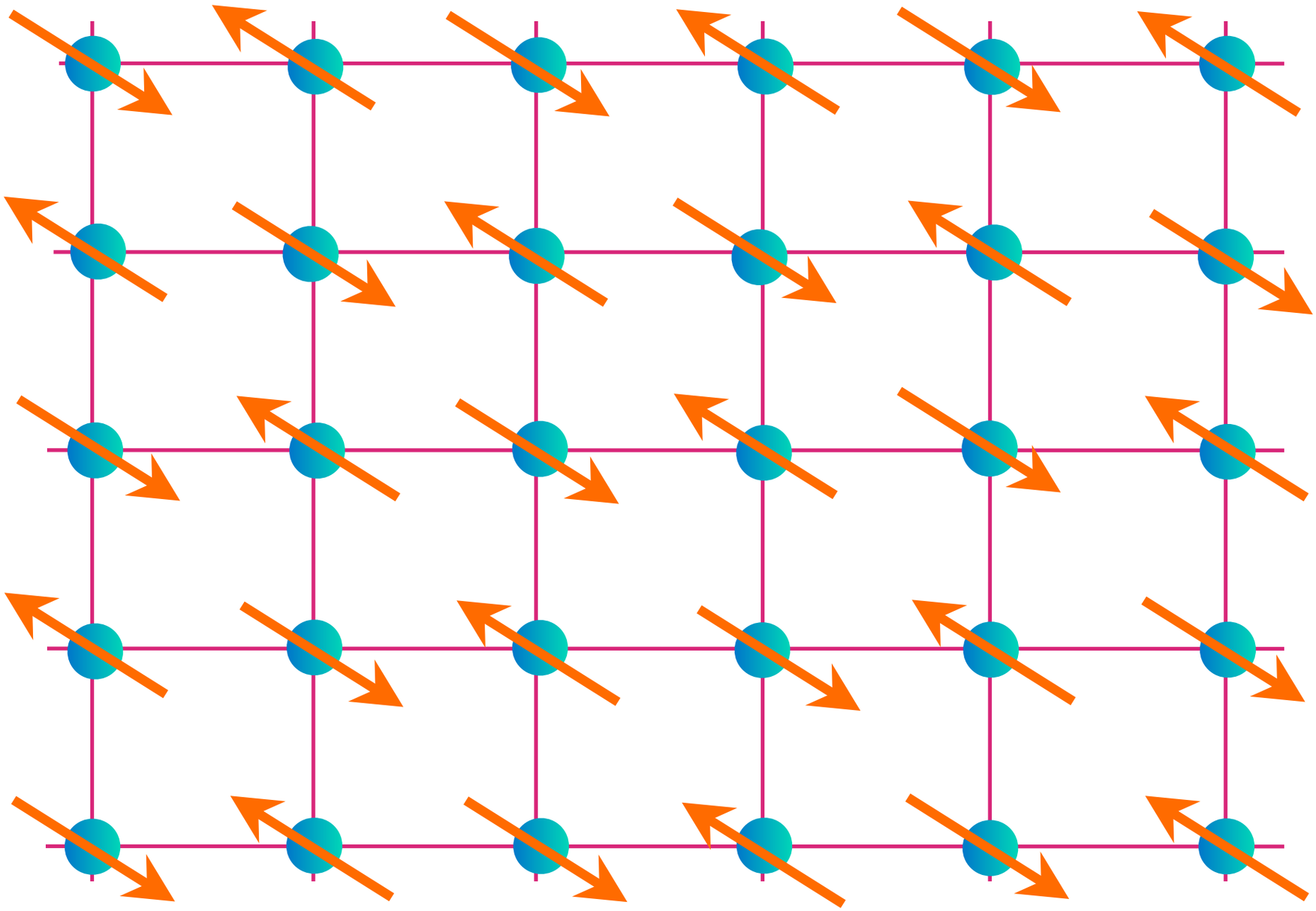


High temperature superconductors

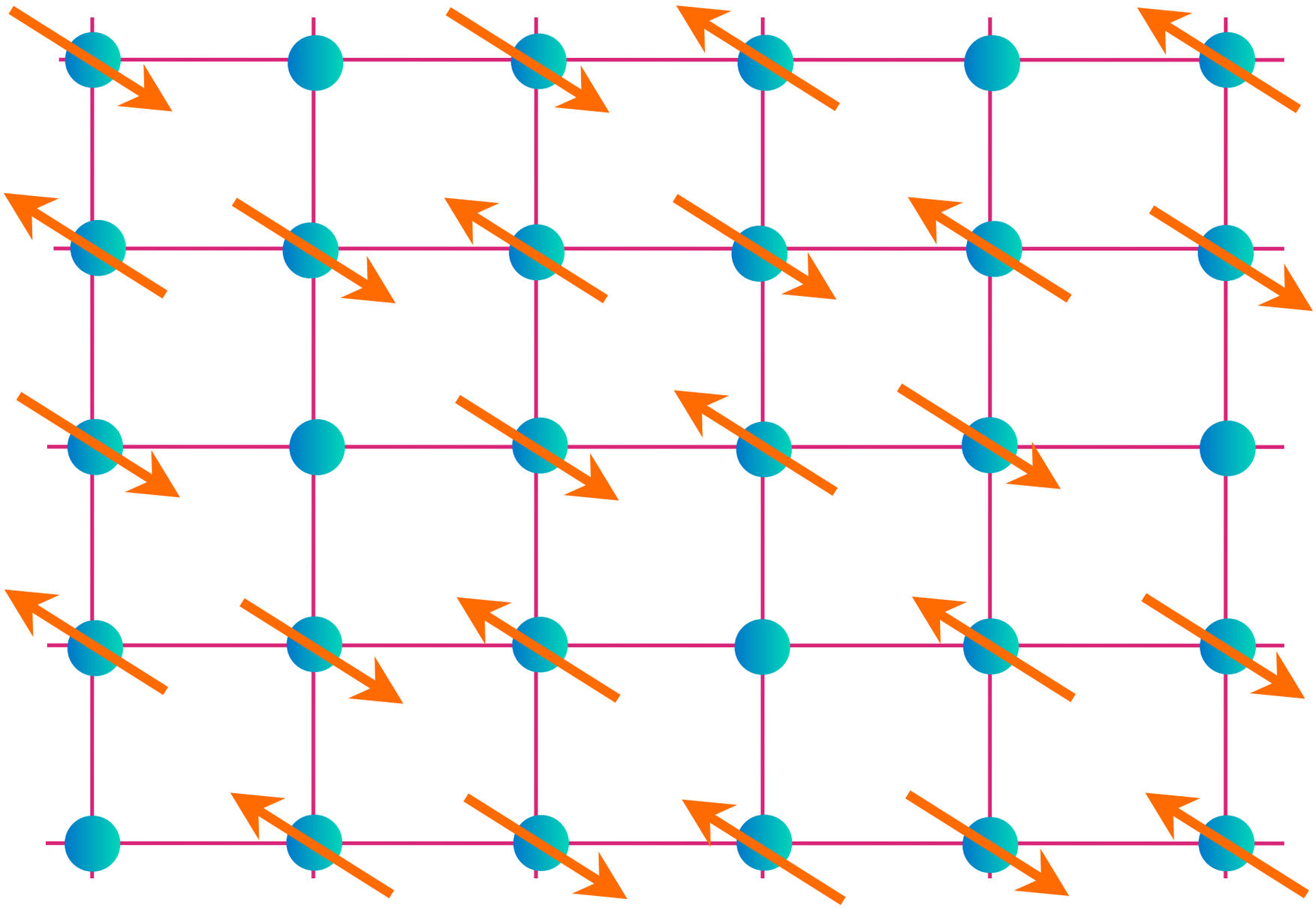


$$YBa_2Cu_3O_{6+x}$$

Square lattice of Cu sites

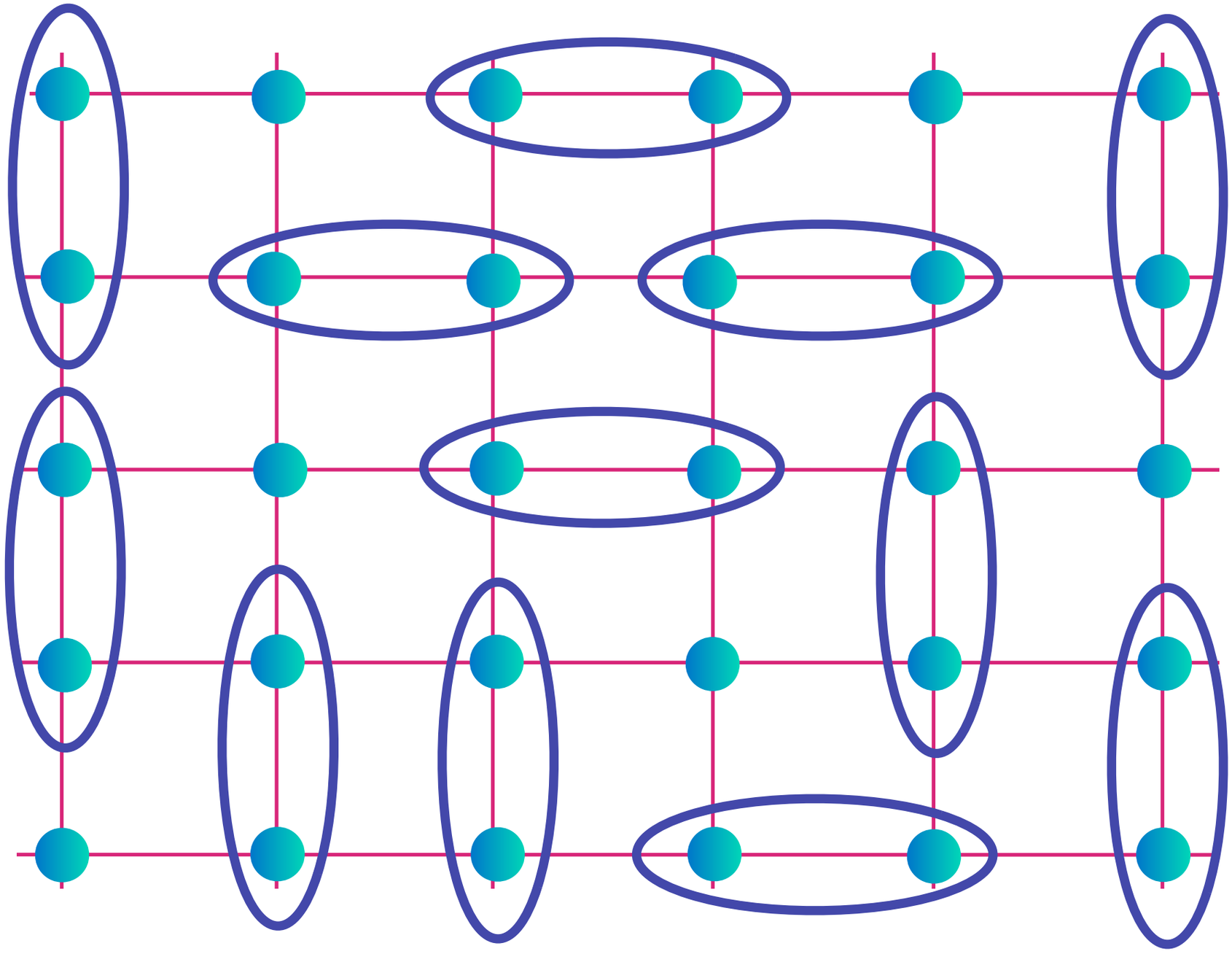


Square lattice of Cu sites



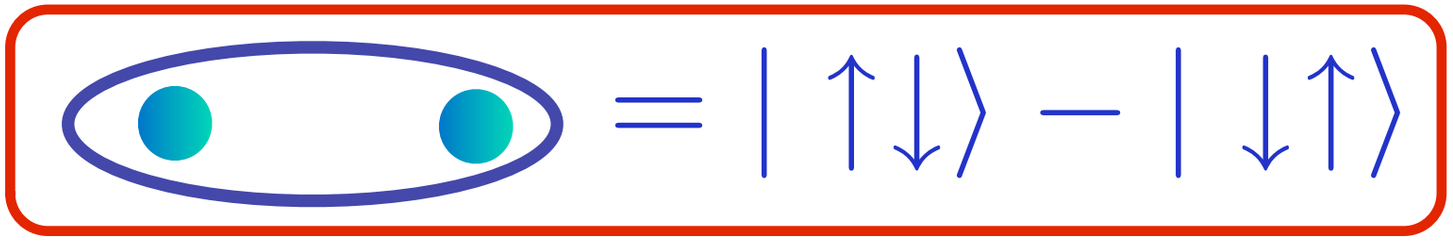
1. Remove some electrons

Square lattice of Cu sites



1. Remove some electrons

2. Electrons entangle into chemical bonds



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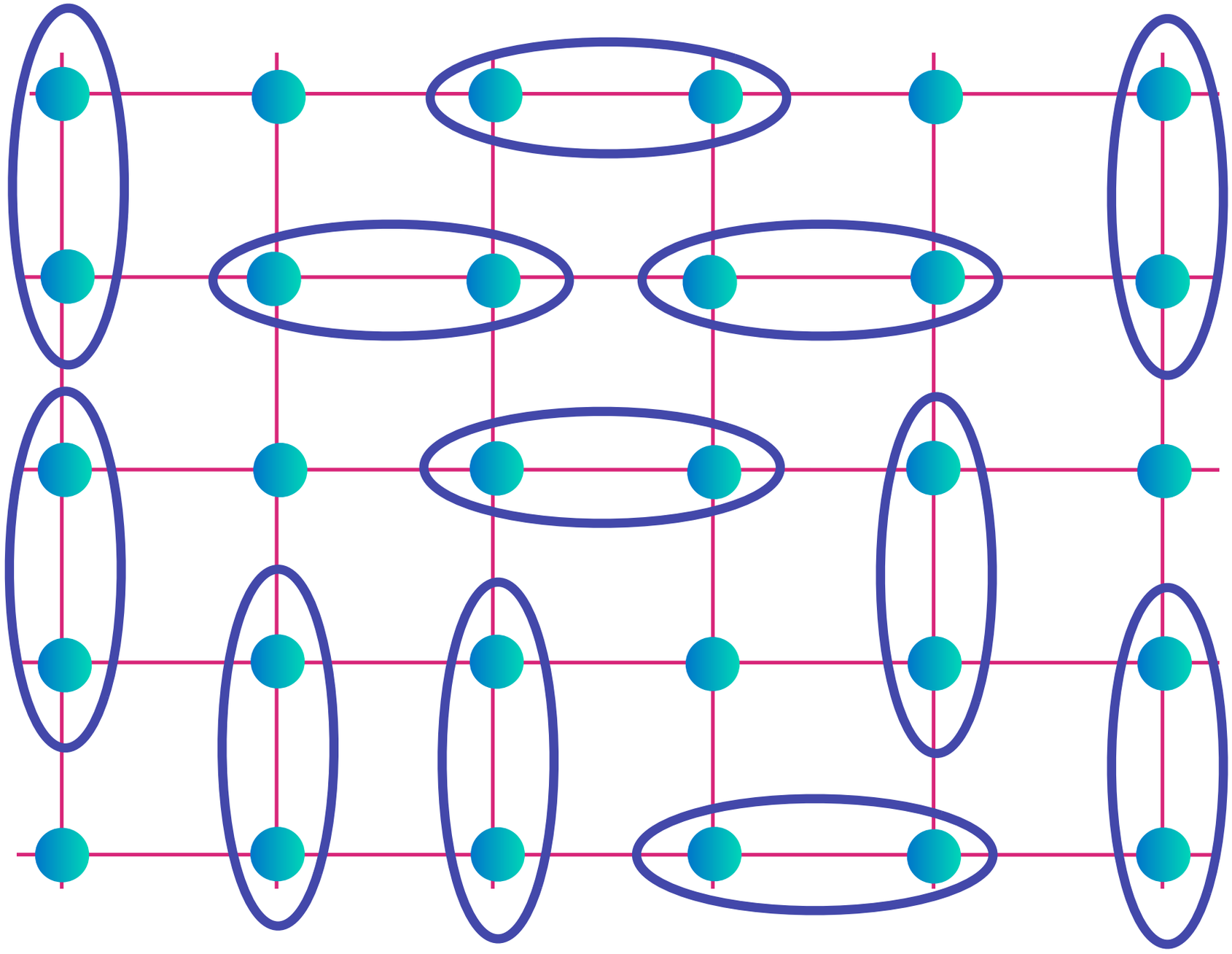
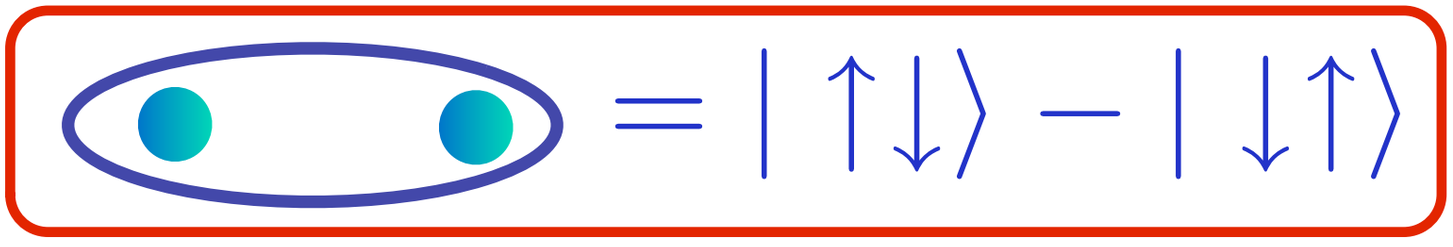


1. Remove some electrons

2. Electrons entangle into chemical bonds

3. Chemical bonds undergo Bose-Einstein condensation



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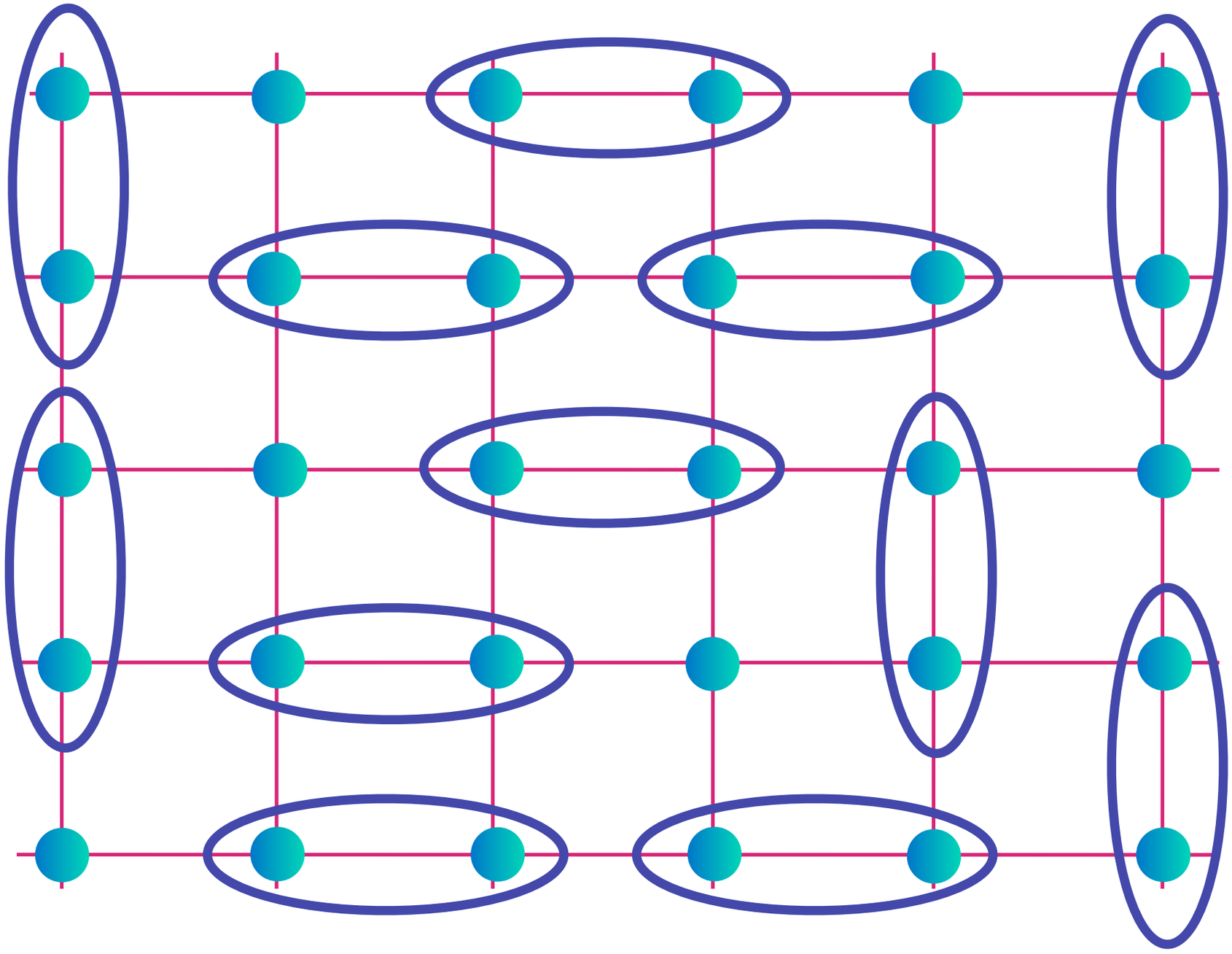

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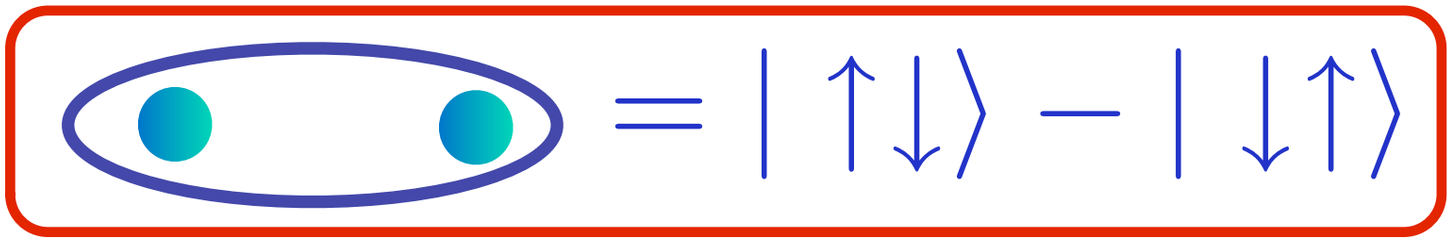

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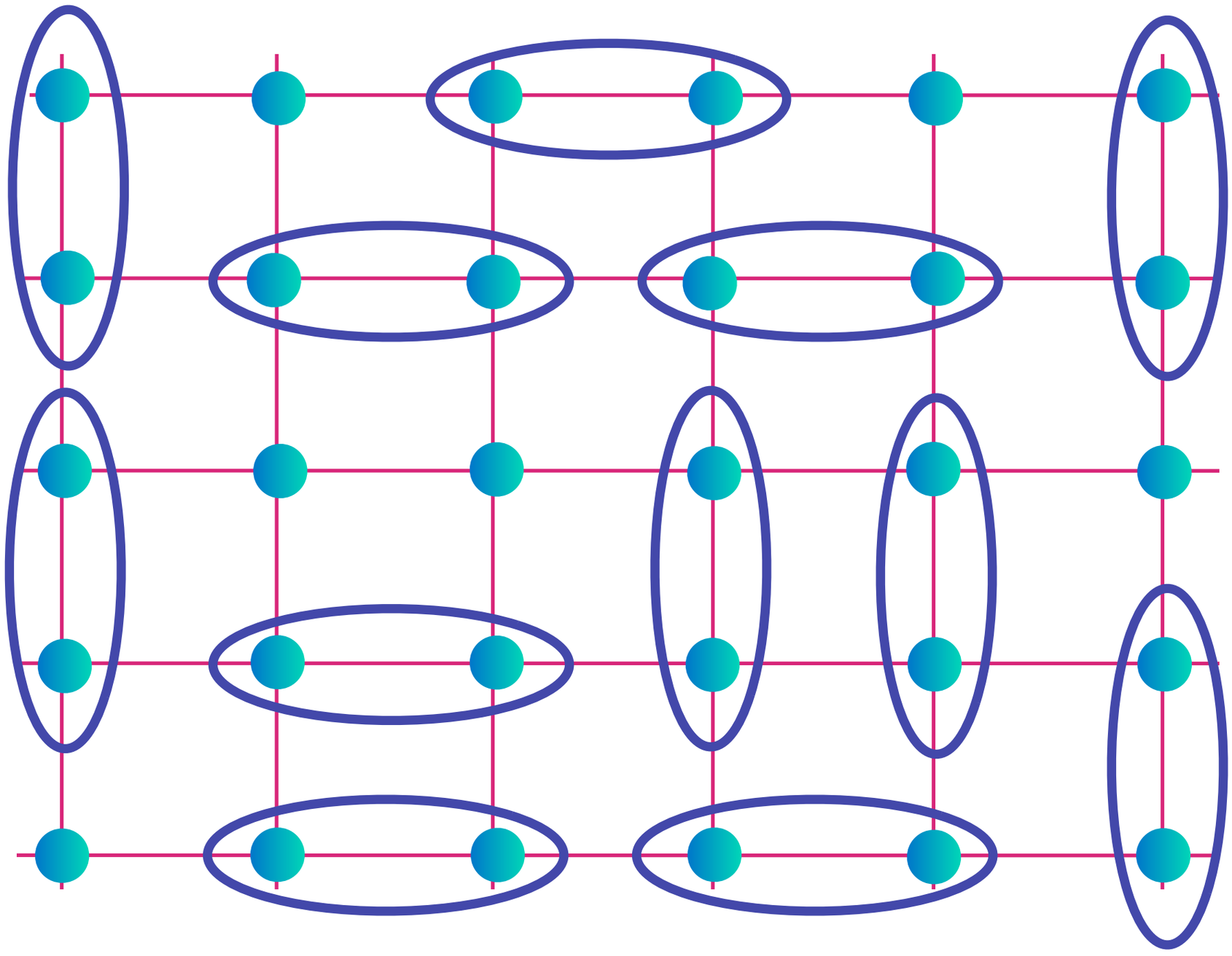

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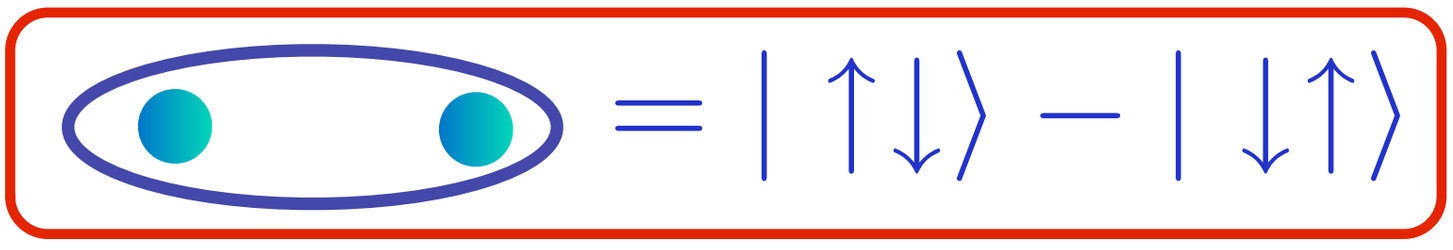

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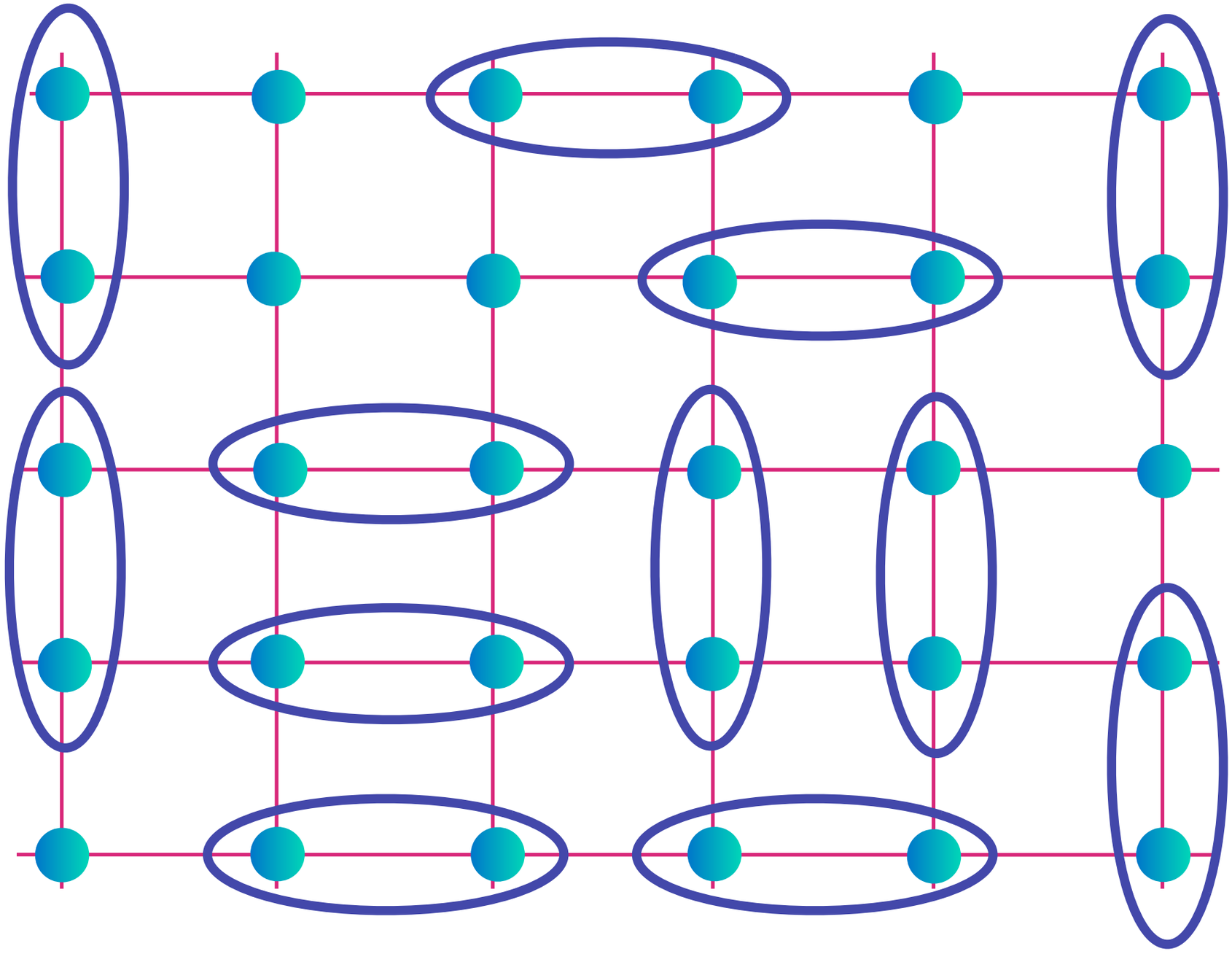

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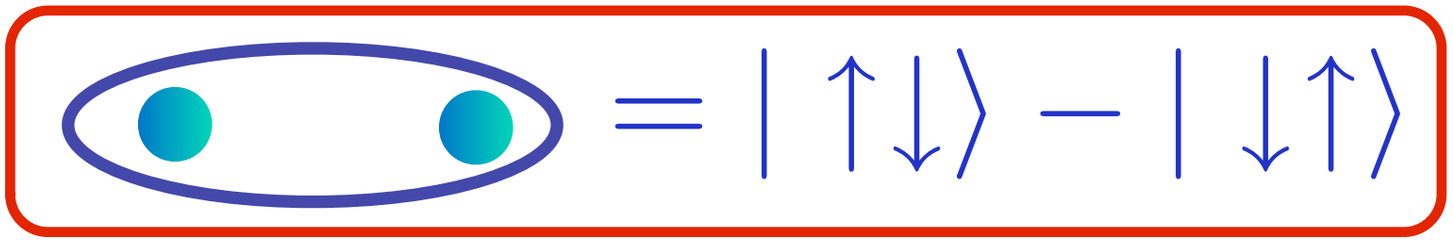
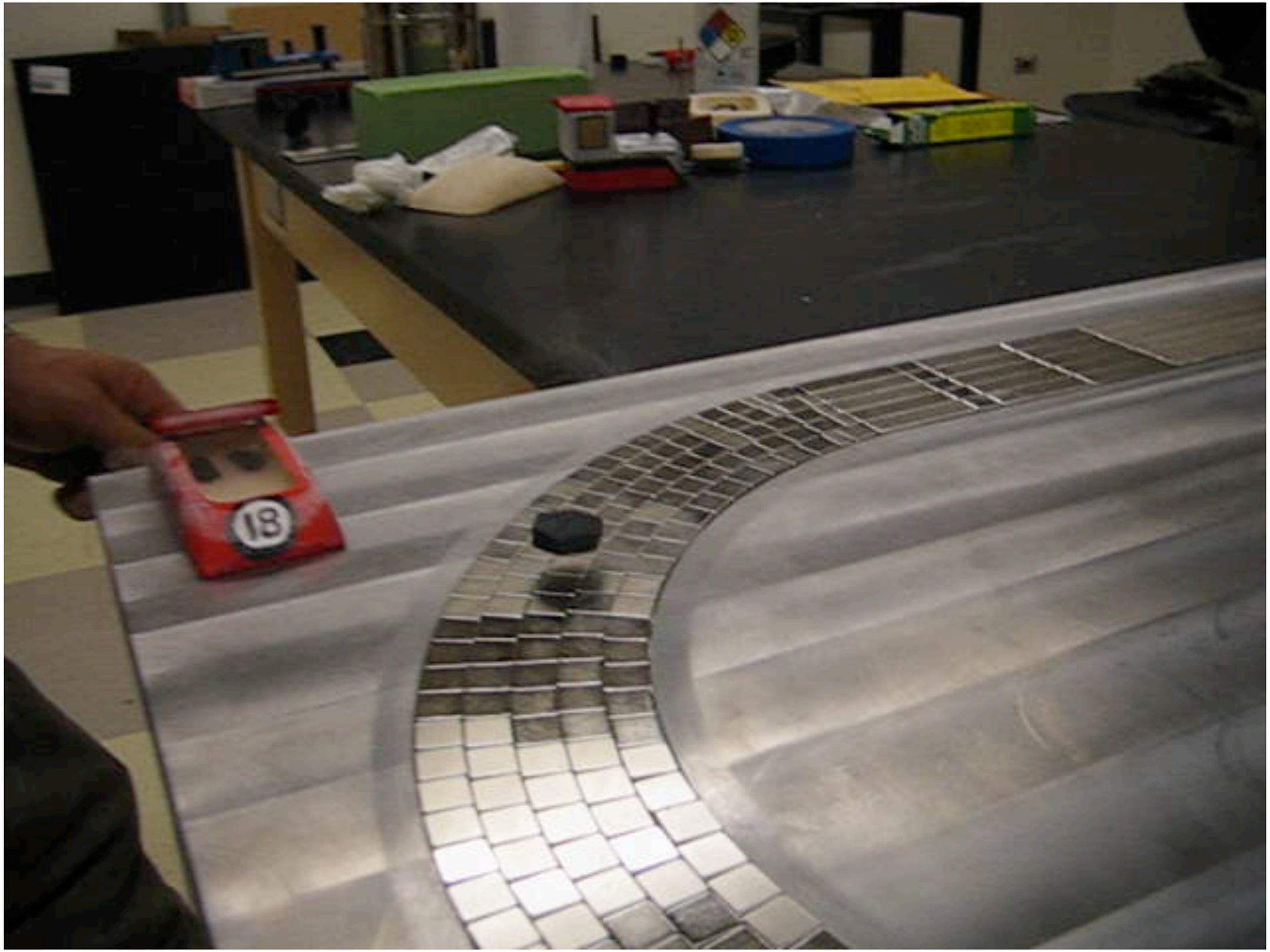


Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University



Nd-Fe-B magnets, YBaCuO superconductor

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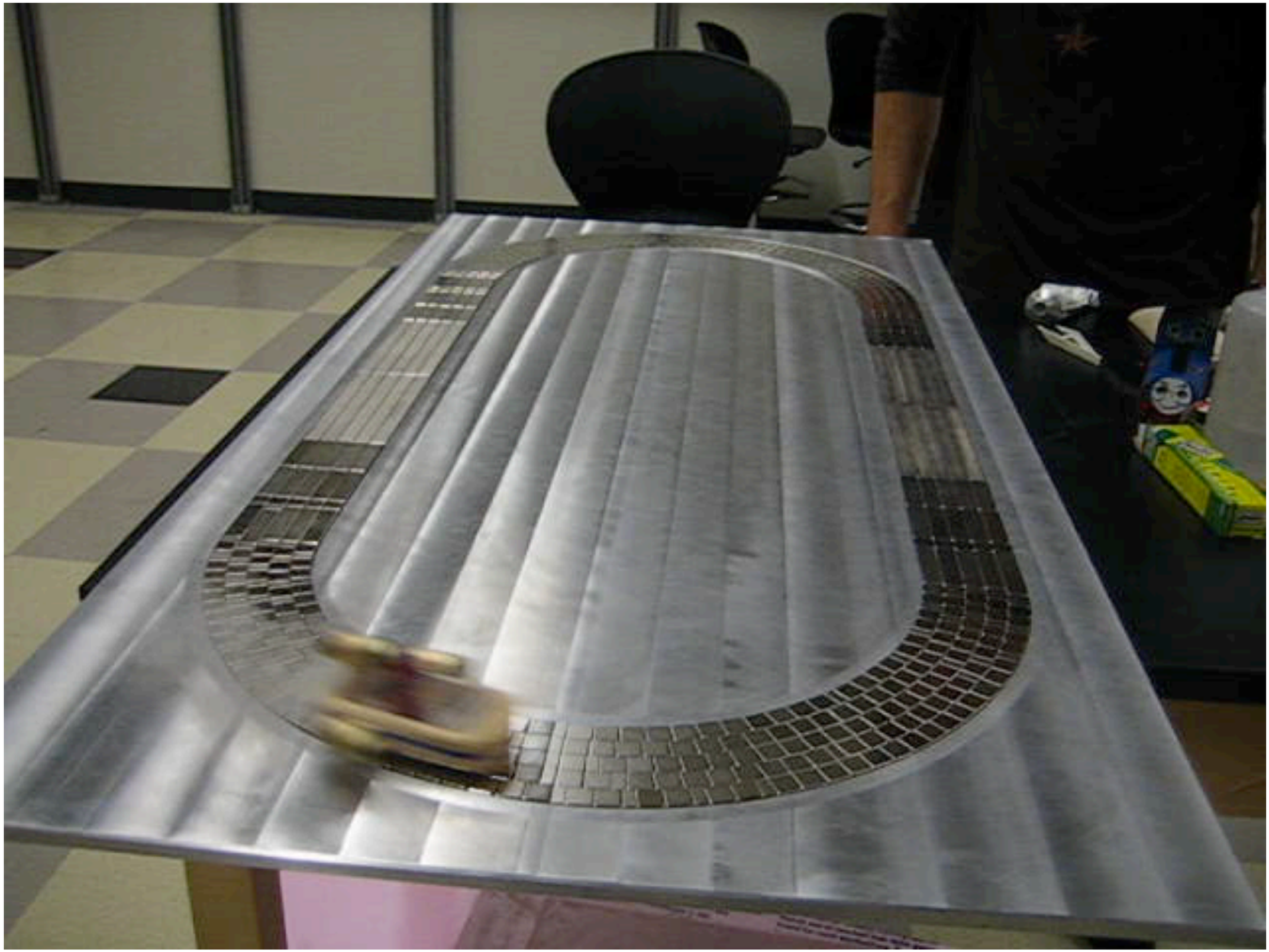

Nd-Fe-B magnets, YBaCuO superconductor

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Quantum entanglement

Superconductivity

Quantum phase transitions
and
Quantum critical points

Black Holes

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Black Holes

What is a “phase transition” ?

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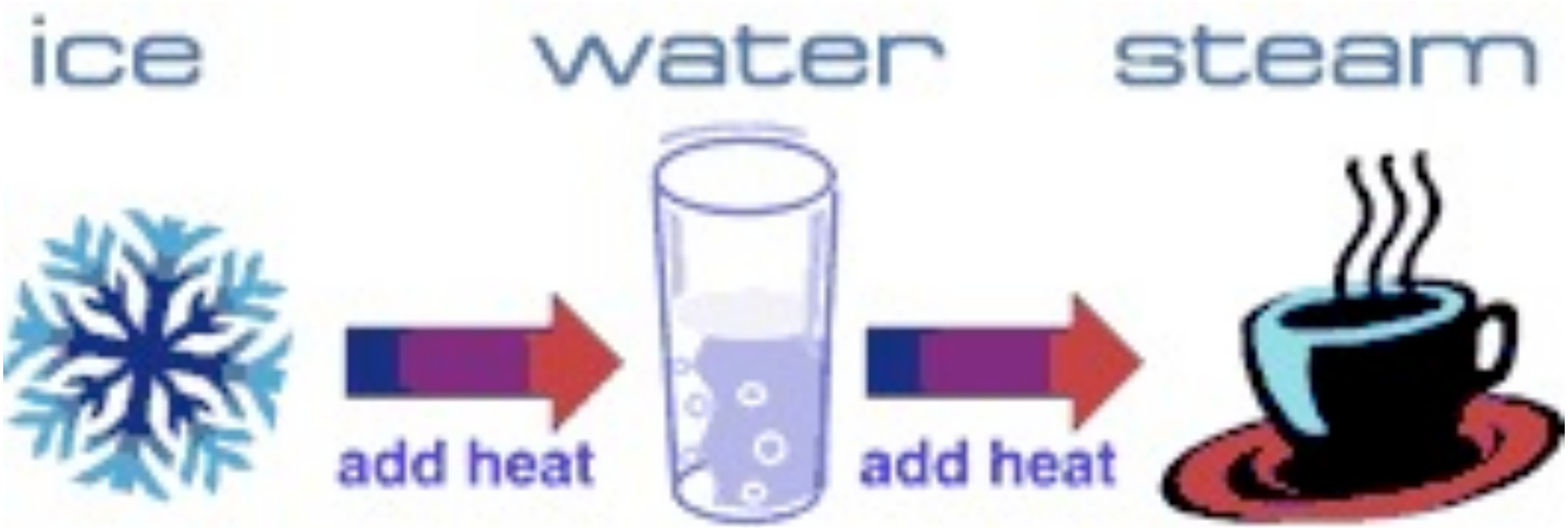
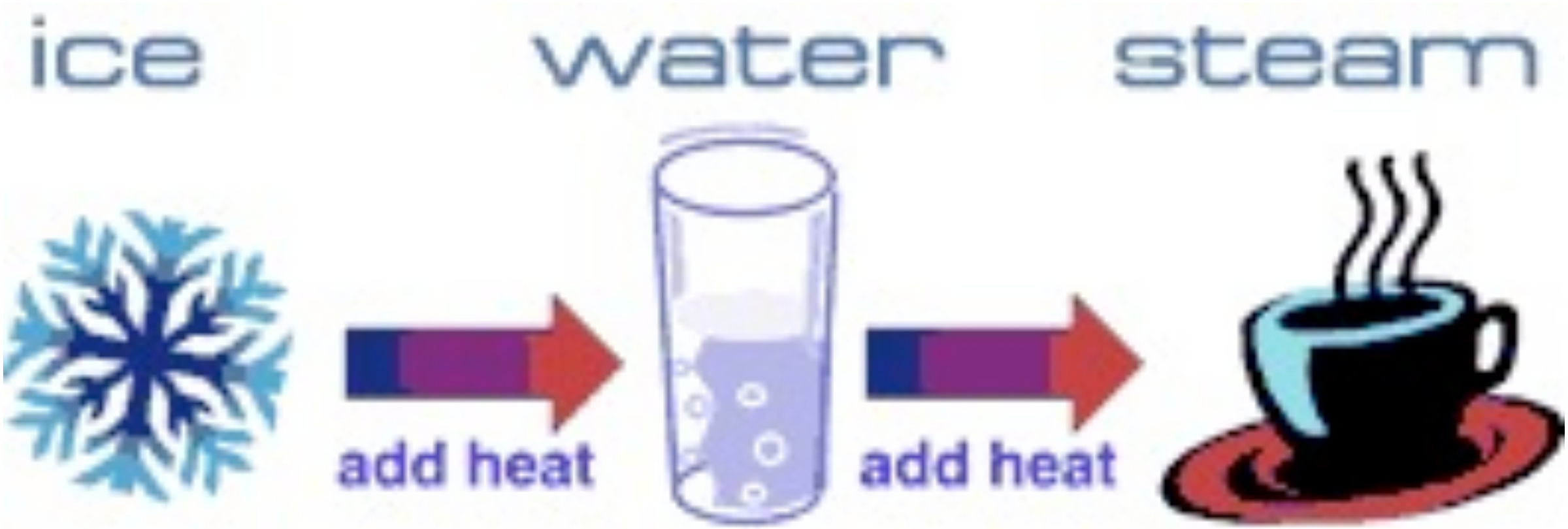


What is a "phase transition" ?



A change in the collective properties of a macroscopic number of atoms

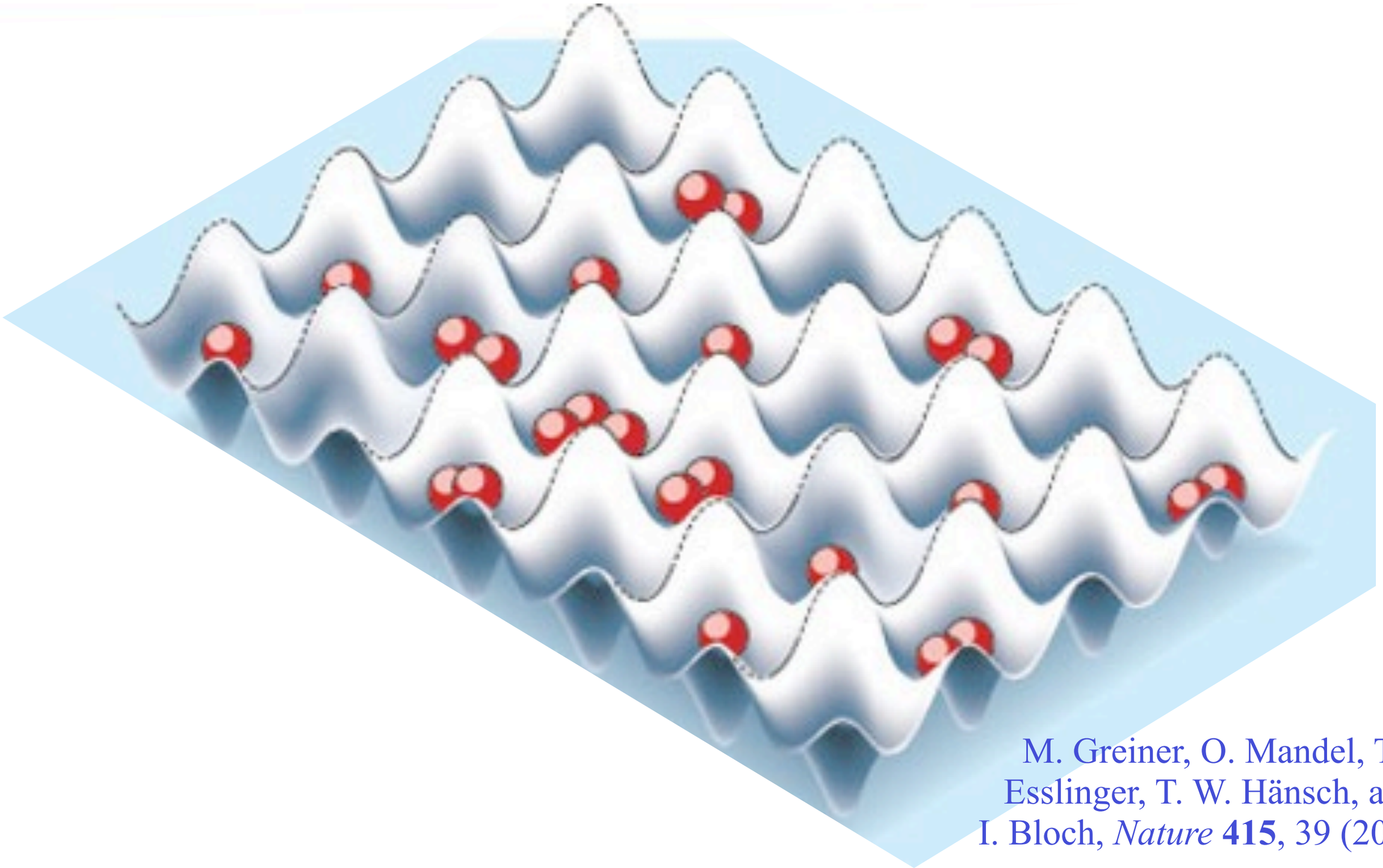
What is a "*quantum phase transition*" ?

Change in the nature of quantum superposition in a macroscopic quantum system.

Quantum Criticality

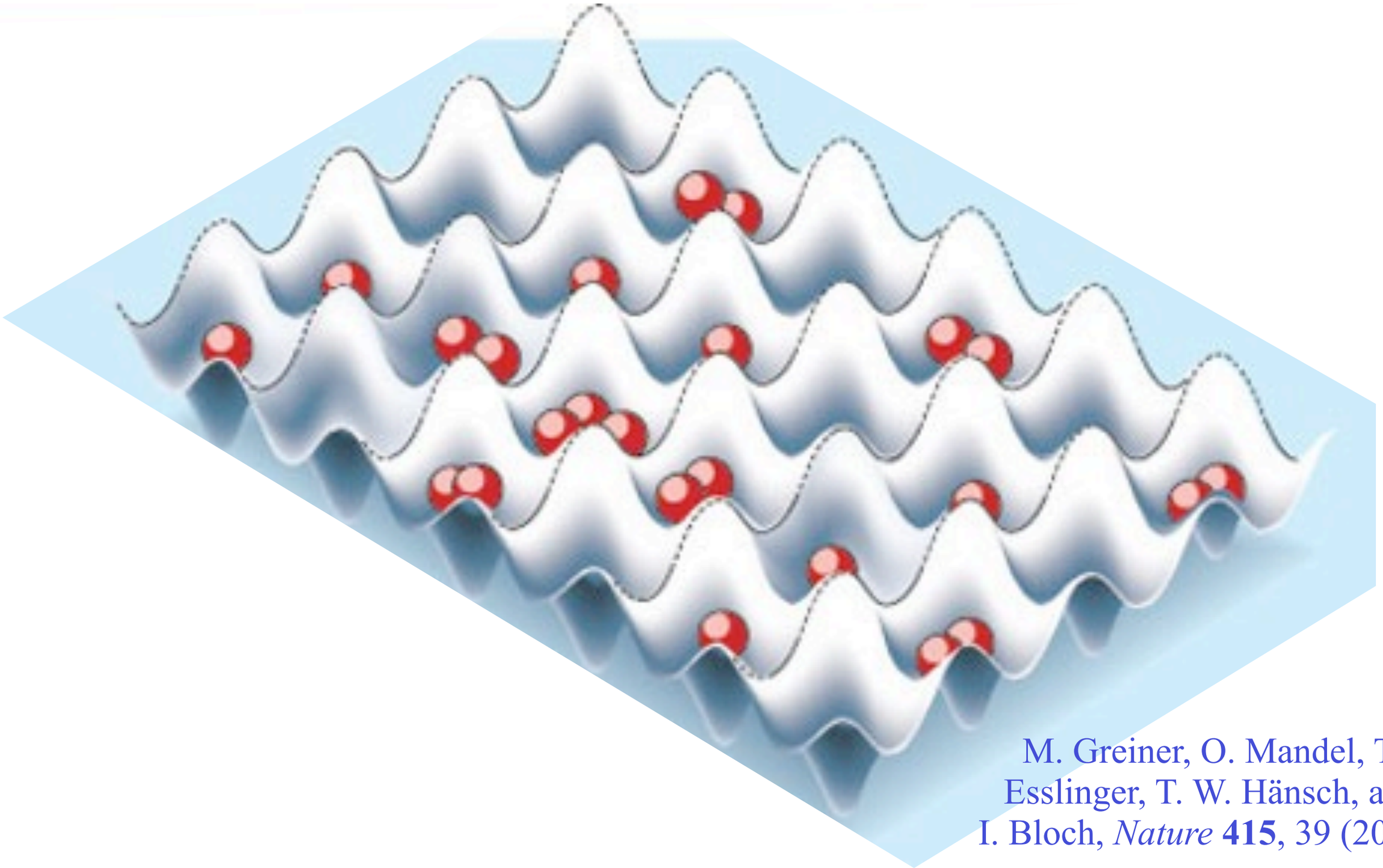
The complex and non-local entanglement at the critical point between two quantum phases. There is entanglement between electrons/atoms at all distance scales.

Rubidium atoms in a magnetic trap and standing waves of laser light



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Increase the intensity of laser light



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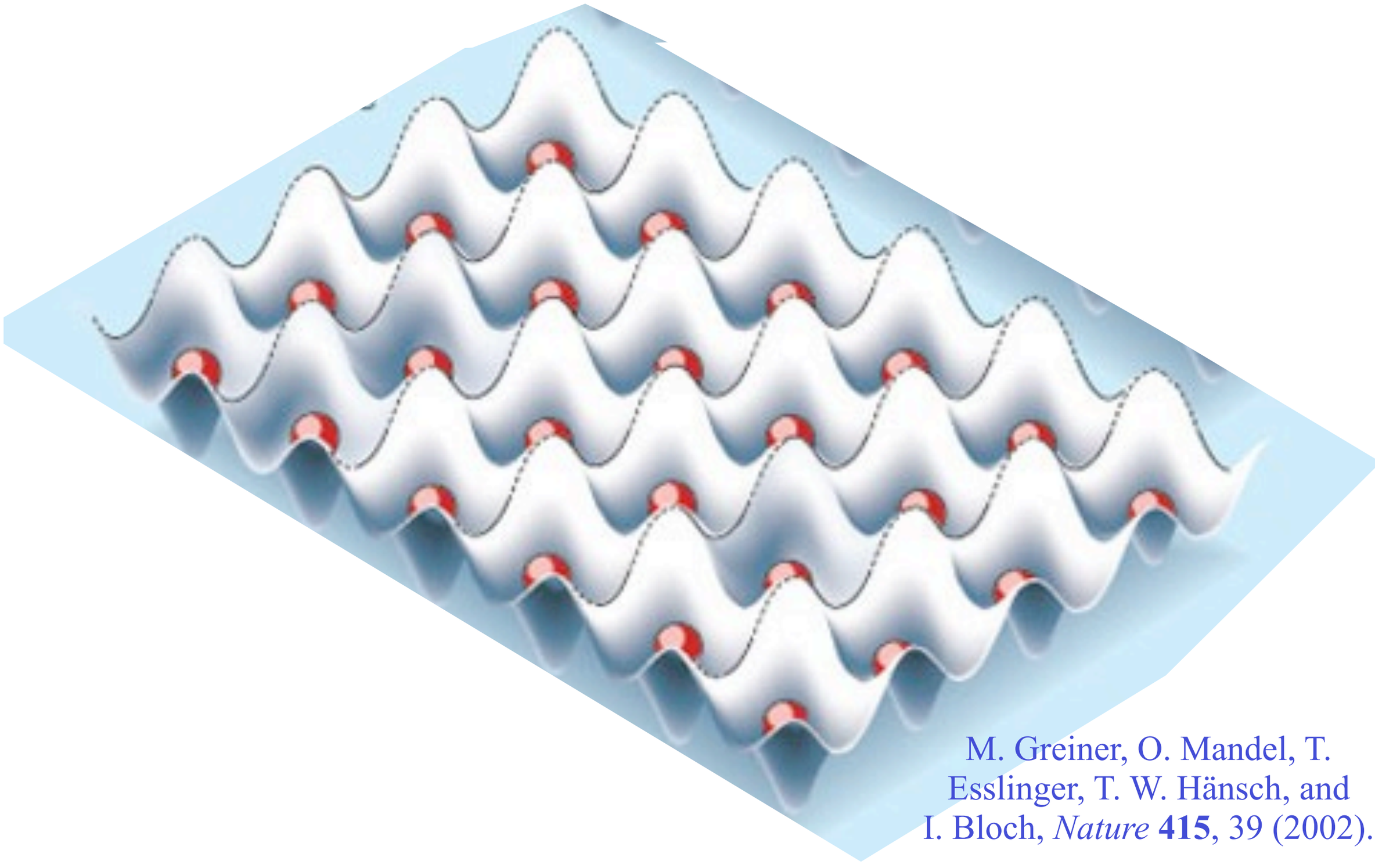

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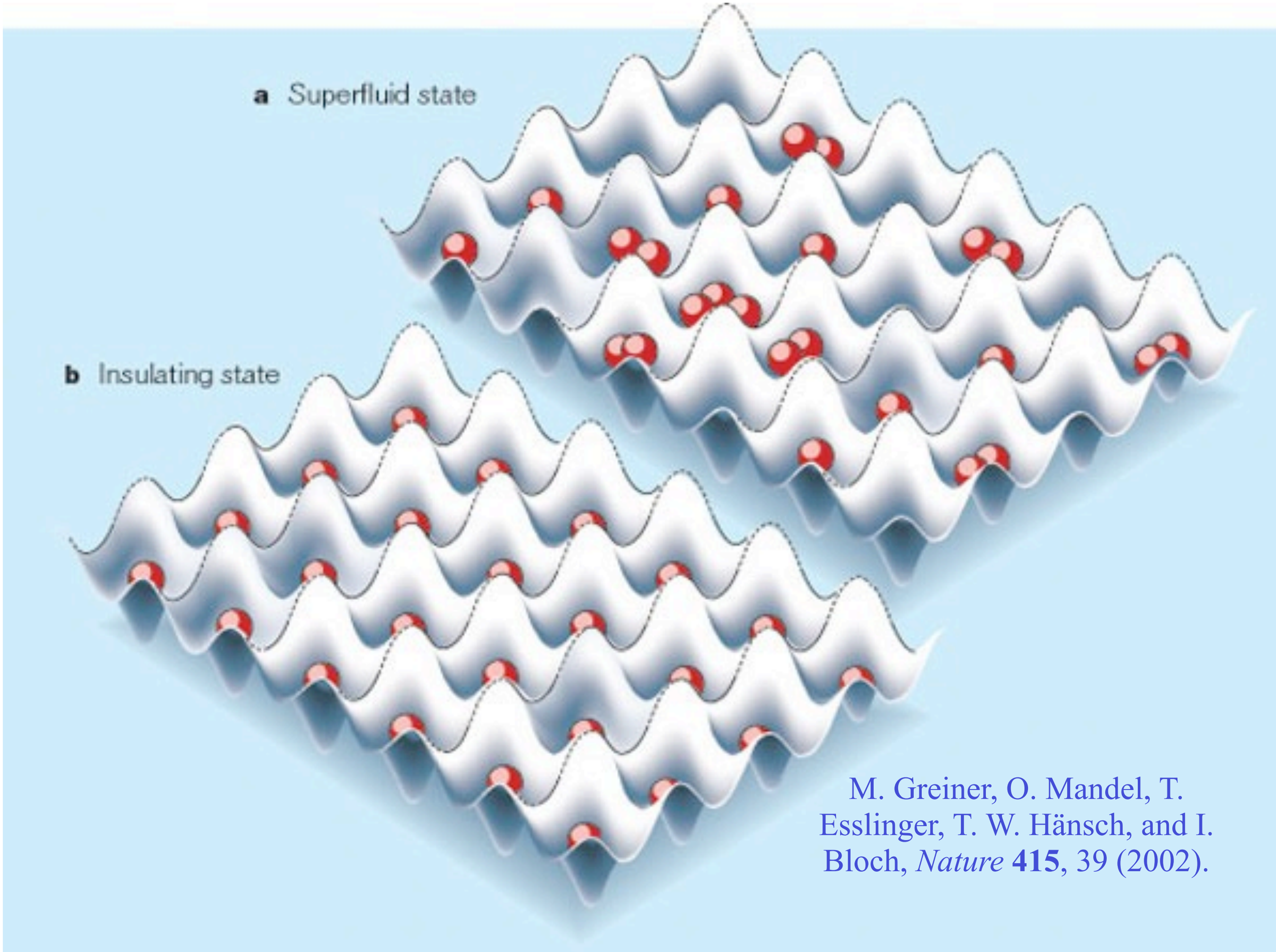

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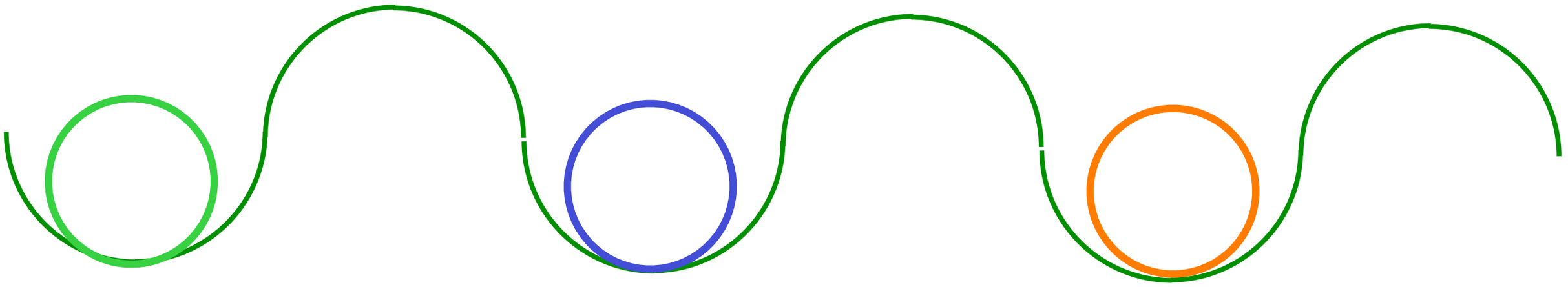
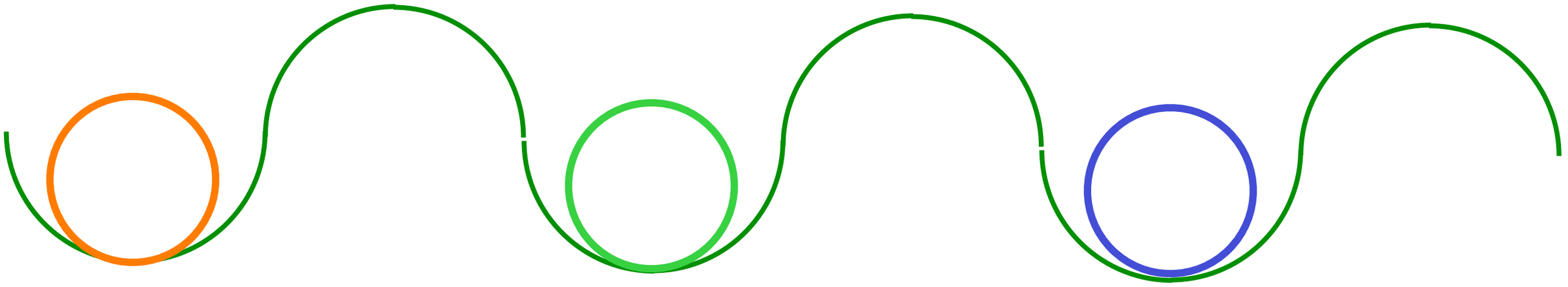
Strong laser light

"Eggs in an egg carton"



Strong laser light

"Eggs in an egg carton"



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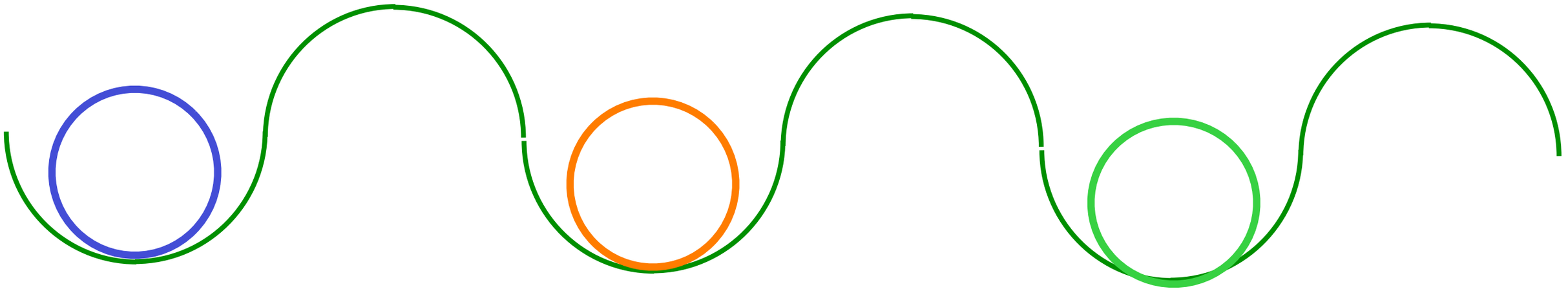

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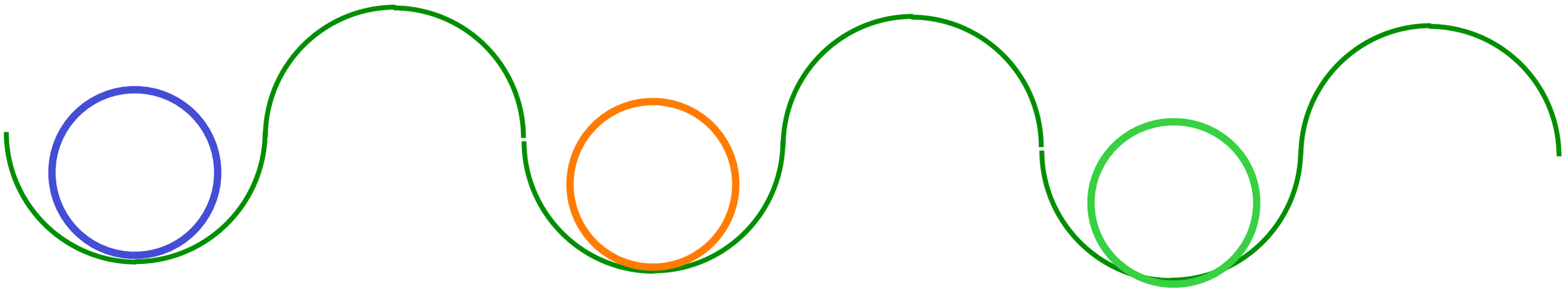
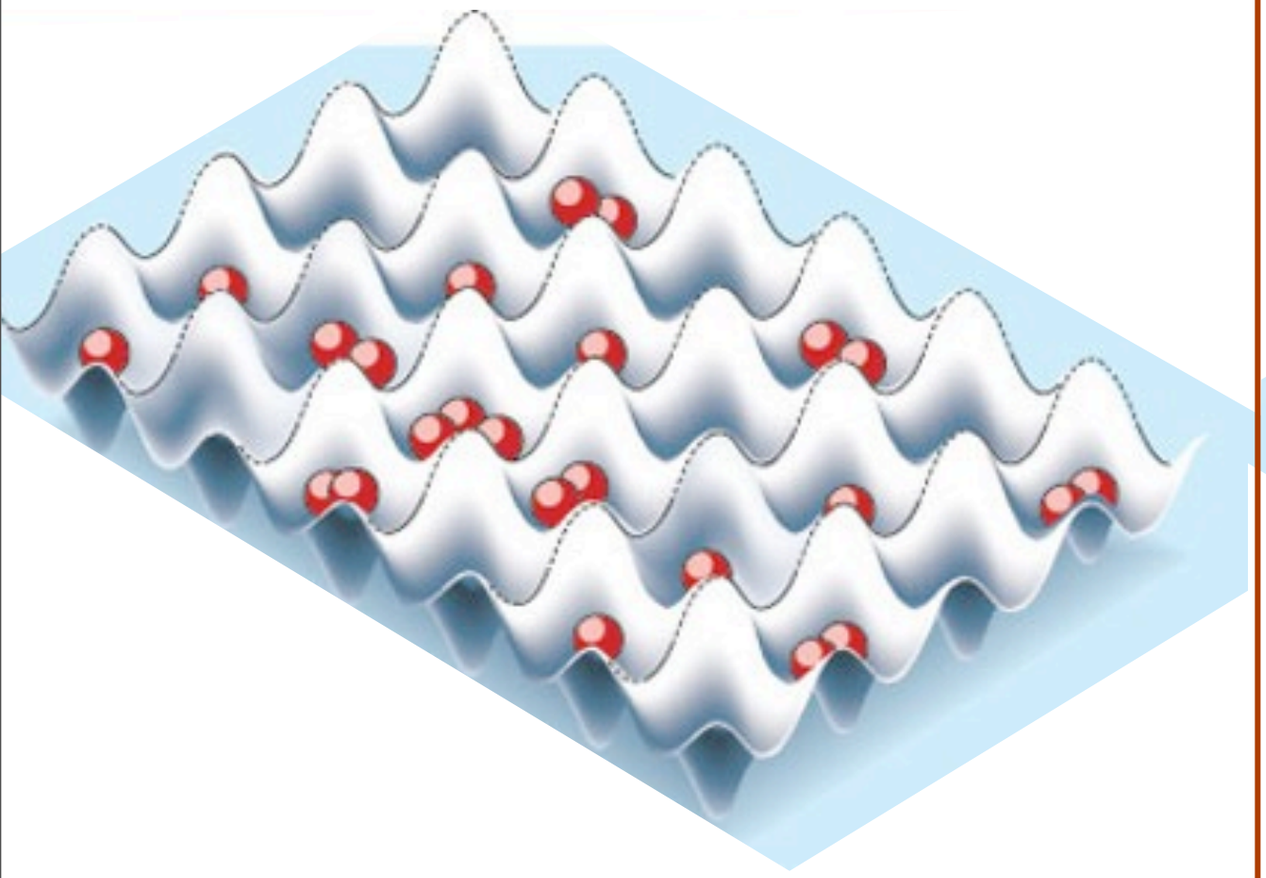
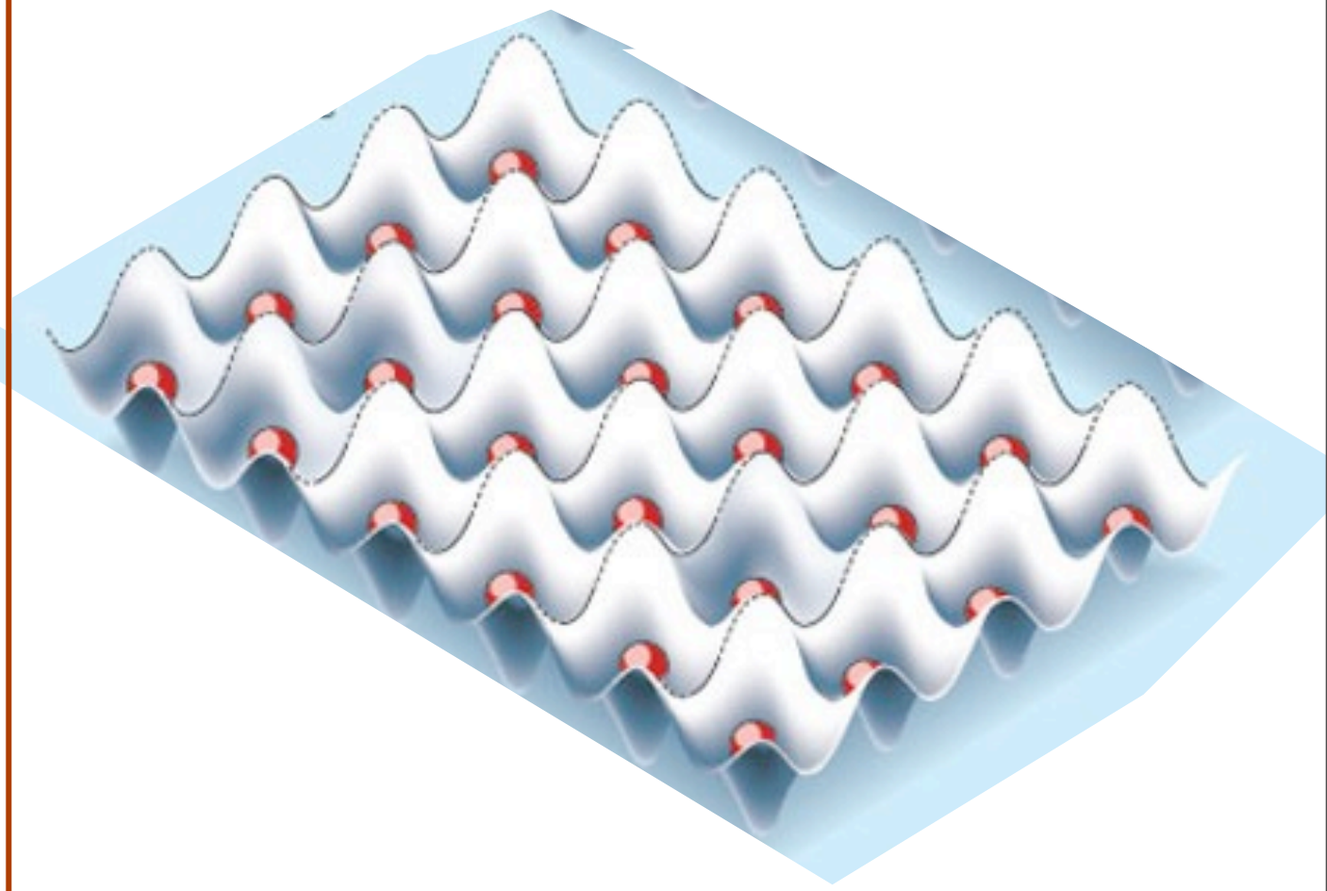


$$|\text{Insulator}\rangle = ||\bigcirc|\bigcirc|\bigcirc|\rangle + ||\bigcirc|\bigcirc|\bigcirc|\rangle + ||\bigcirc|\bigcirc|\bigcirc|\rangle$$
$$+ ||\bigcirc|\bigcirc|\bigcirc|\rangle + ||\bigcirc|\bigcirc|\bigcirc|\rangle + ||\bigcirc|\bigcirc|\bigcirc|\rangle$$

Superfluid

Insulator



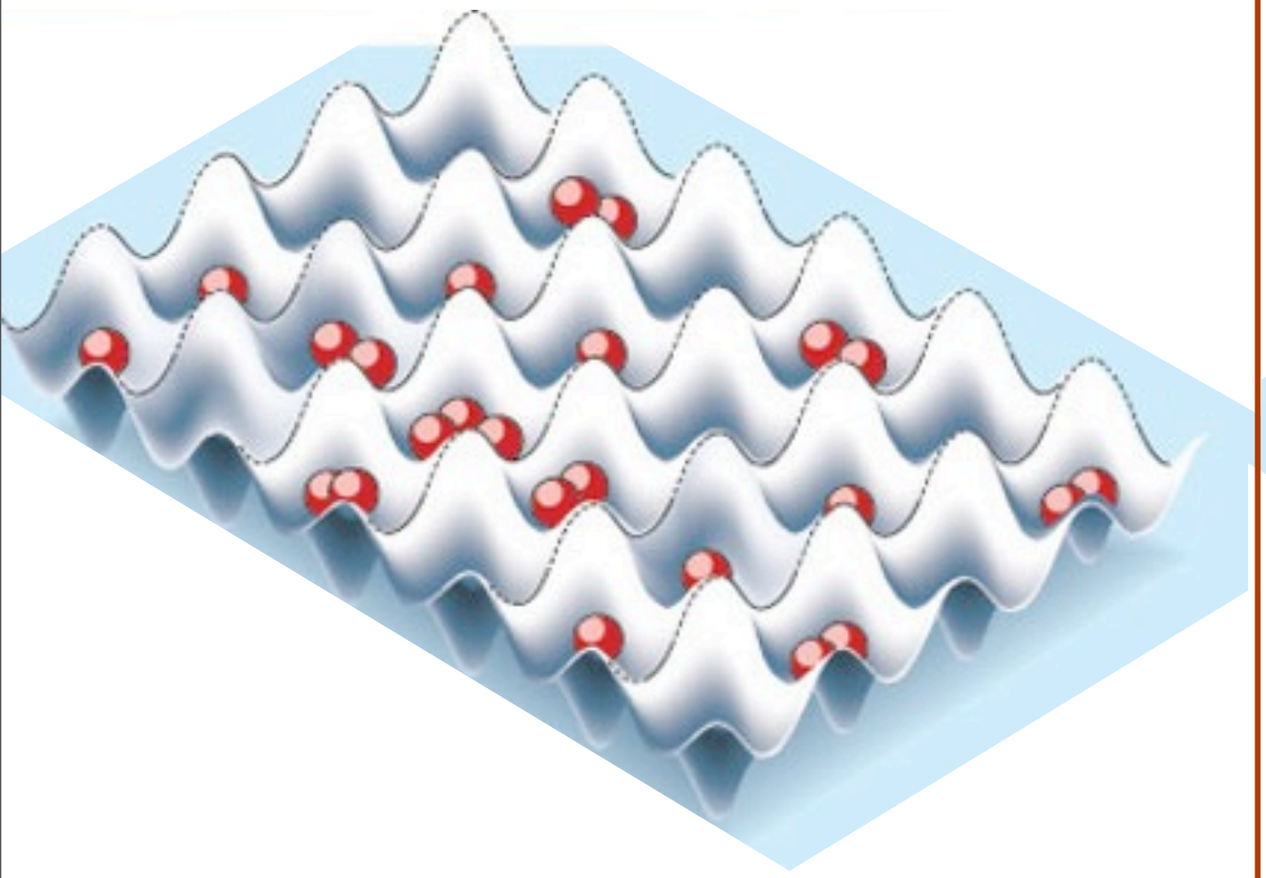


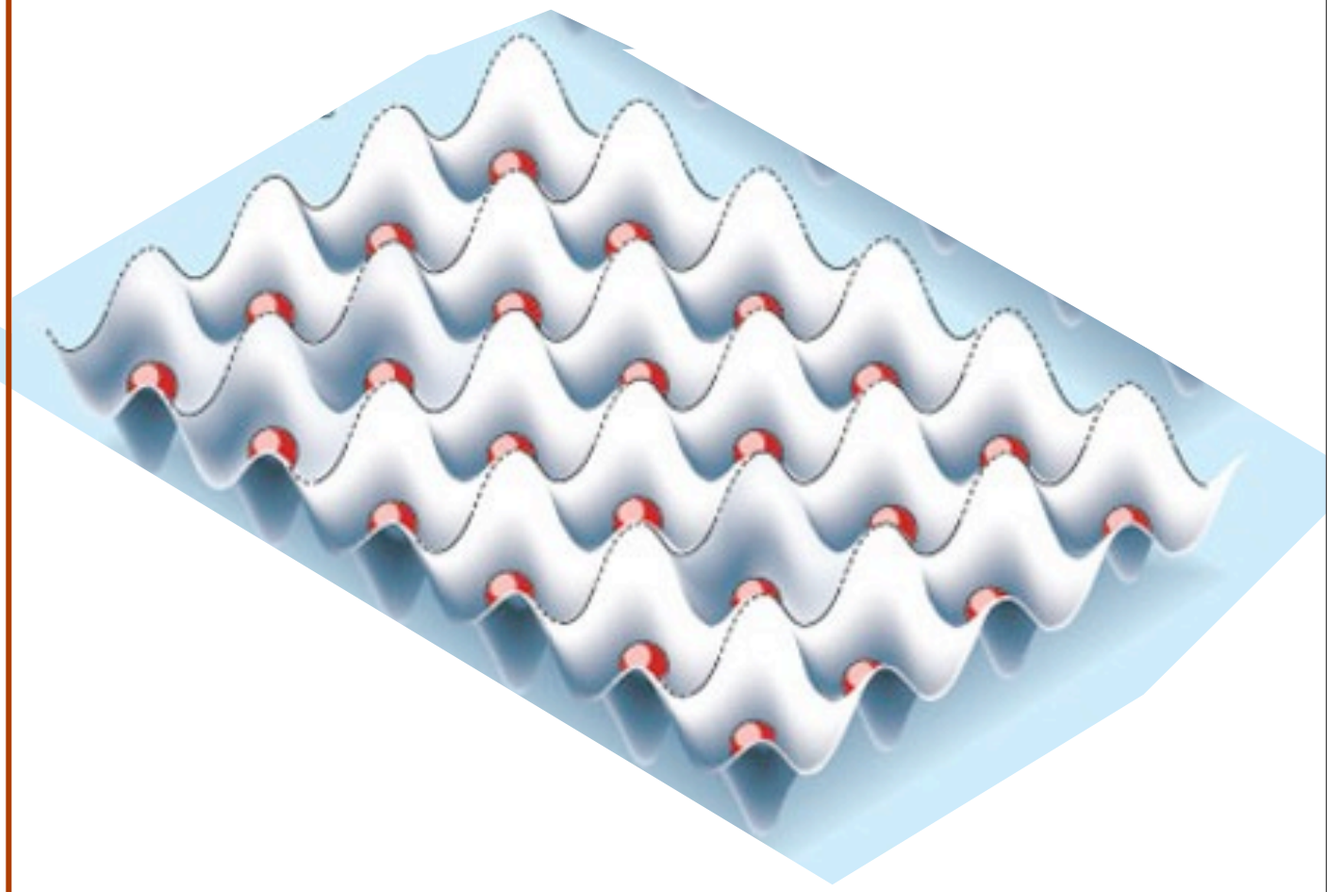
Intensity of laser light

Quantum critical point

Superfluid

Insulator

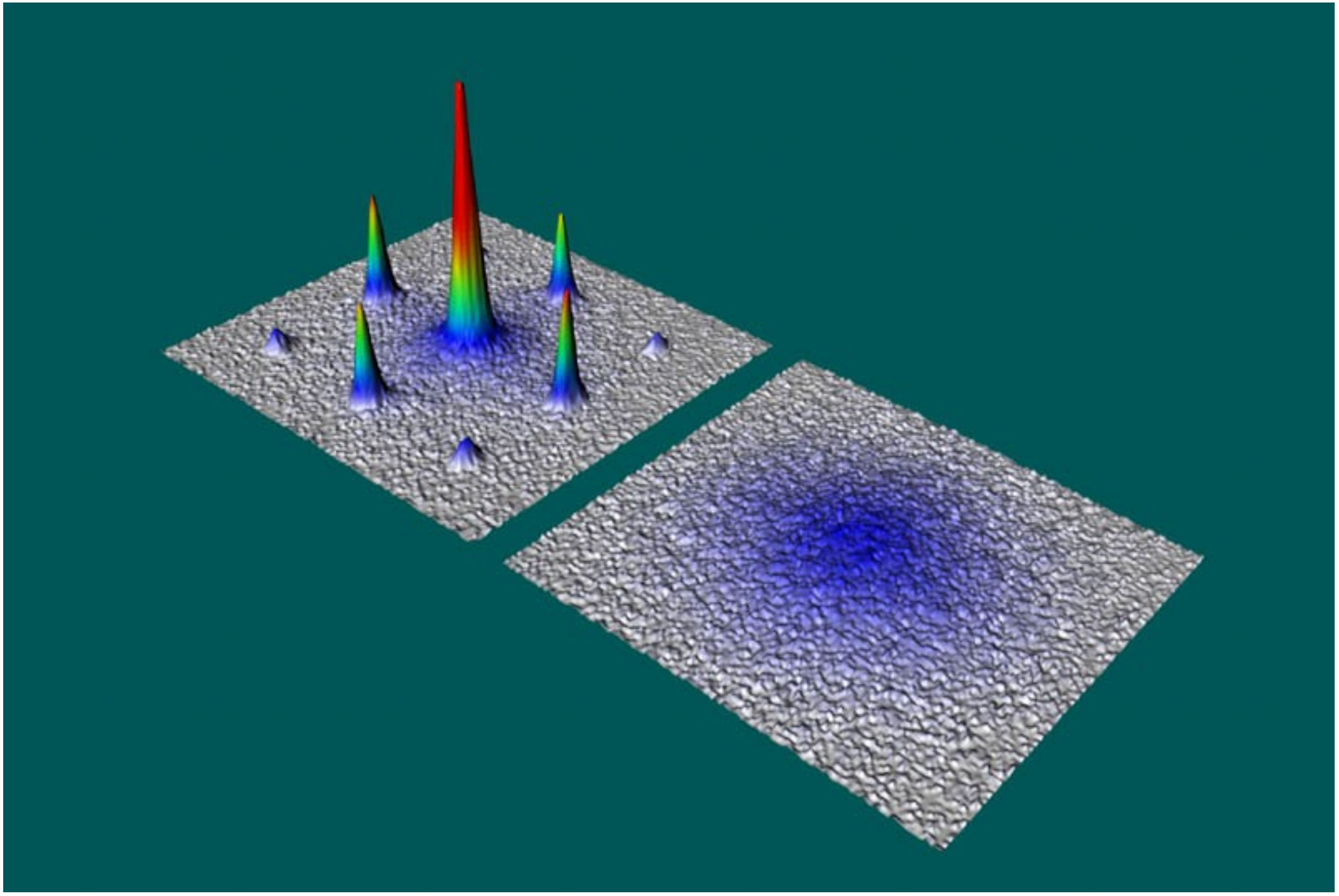




Intensity of laser light

Entanglement between
atoms at all separations

Superfluid-insulator quantum phase transition



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Quantum Criticality

The complex and non-local entanglement at the critical point between two quantum phases. There is entanglement between electrons/atoms at all distance scales.

Quantum entanglement

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Black Holes

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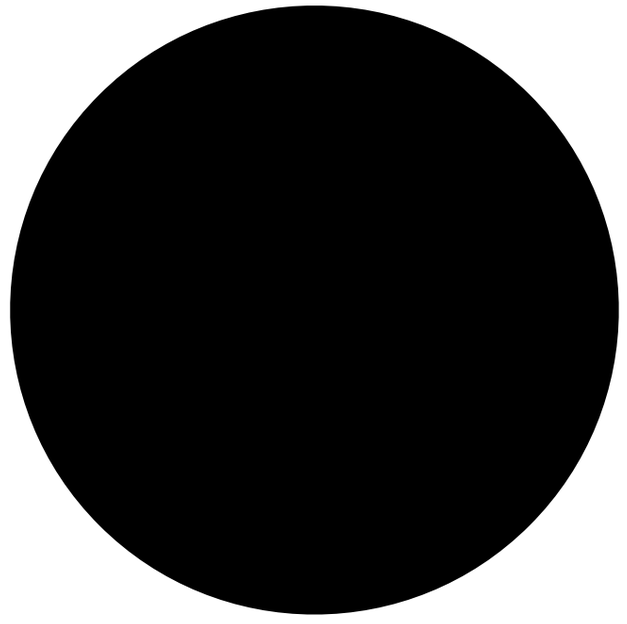
Black Holes

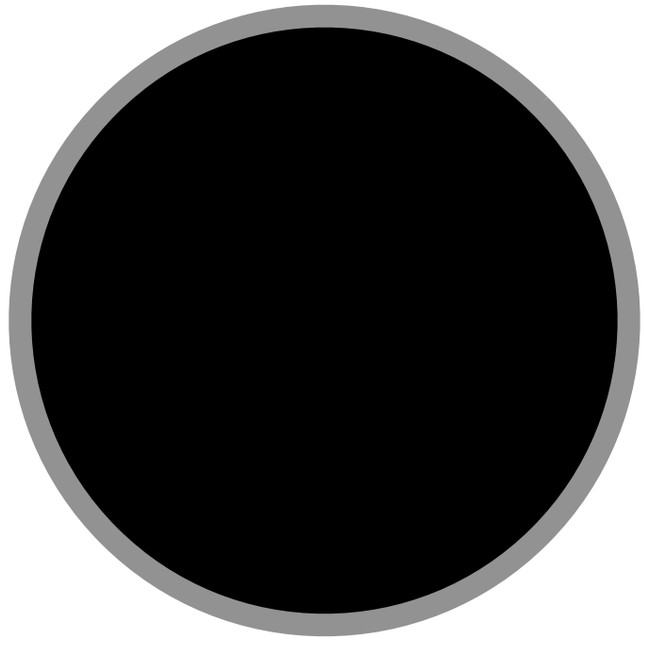
Objects so massive that light is gravitationally bound to them.



Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole horizon is disconnected from the rest of the universe.

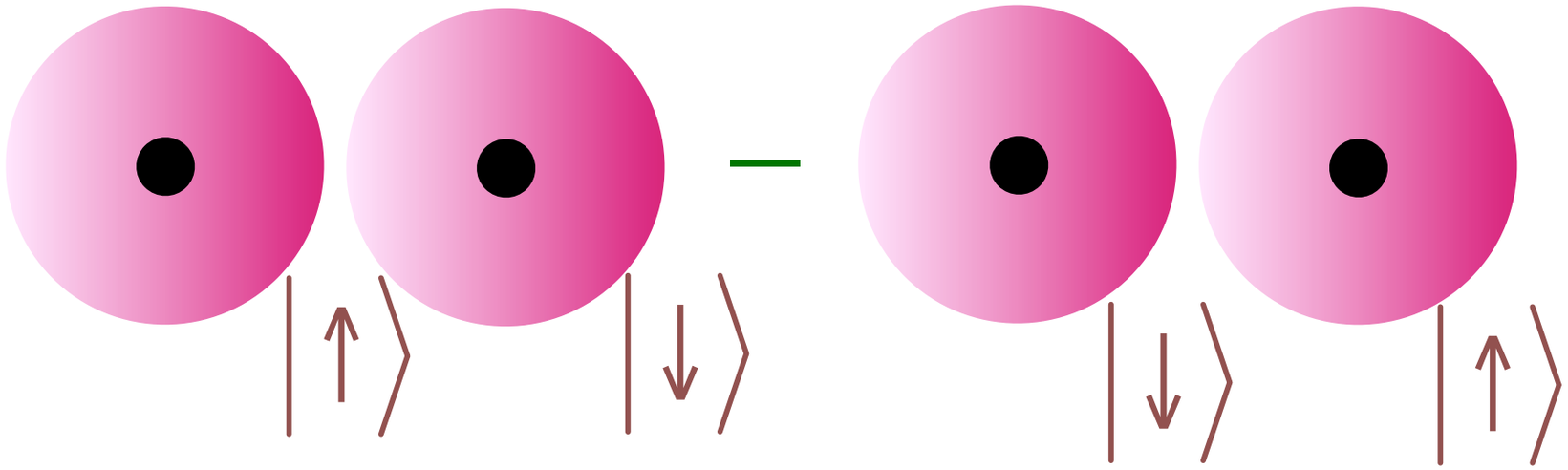


$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

Quantum Entanglement across a black hole horizon



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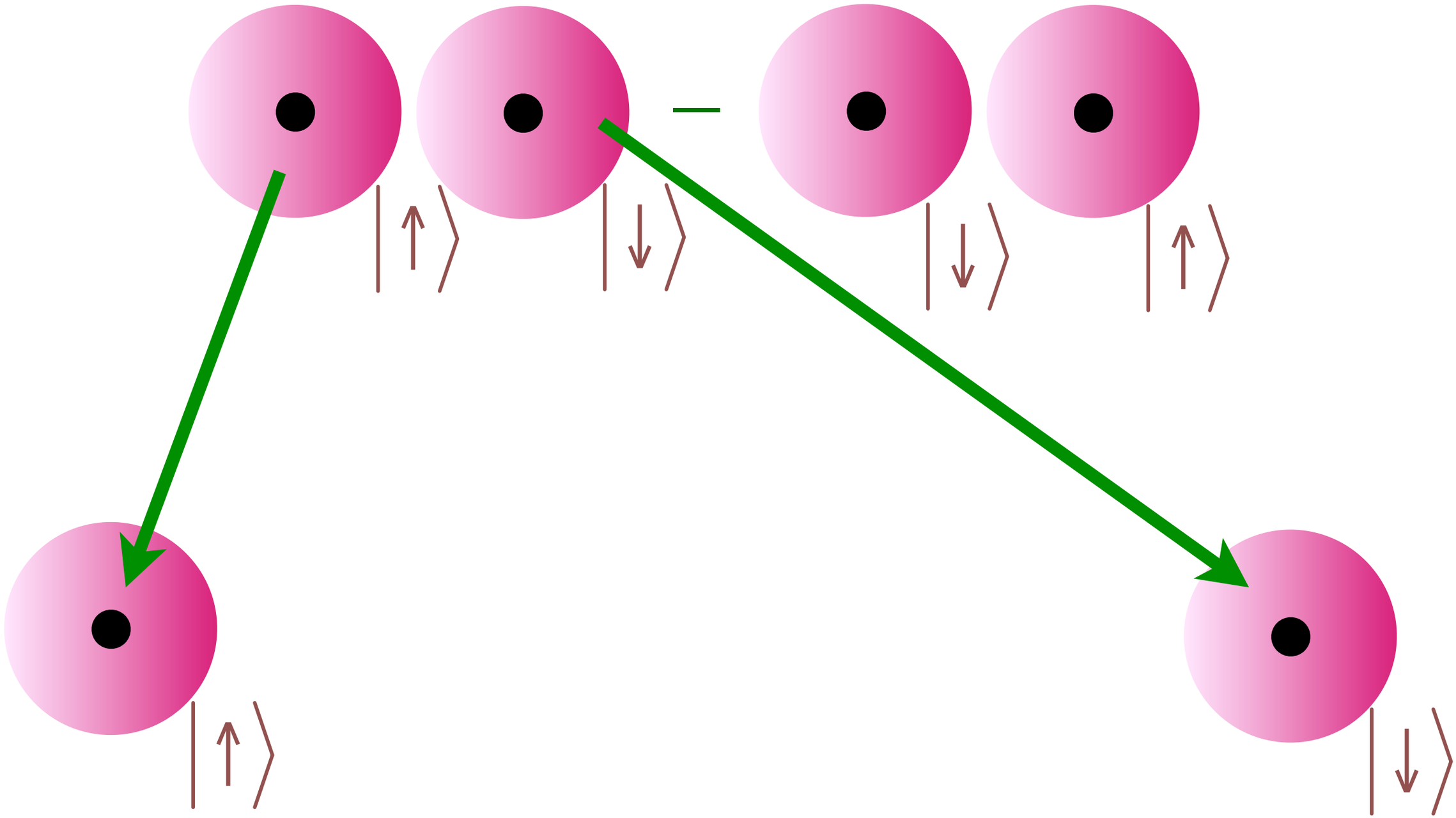

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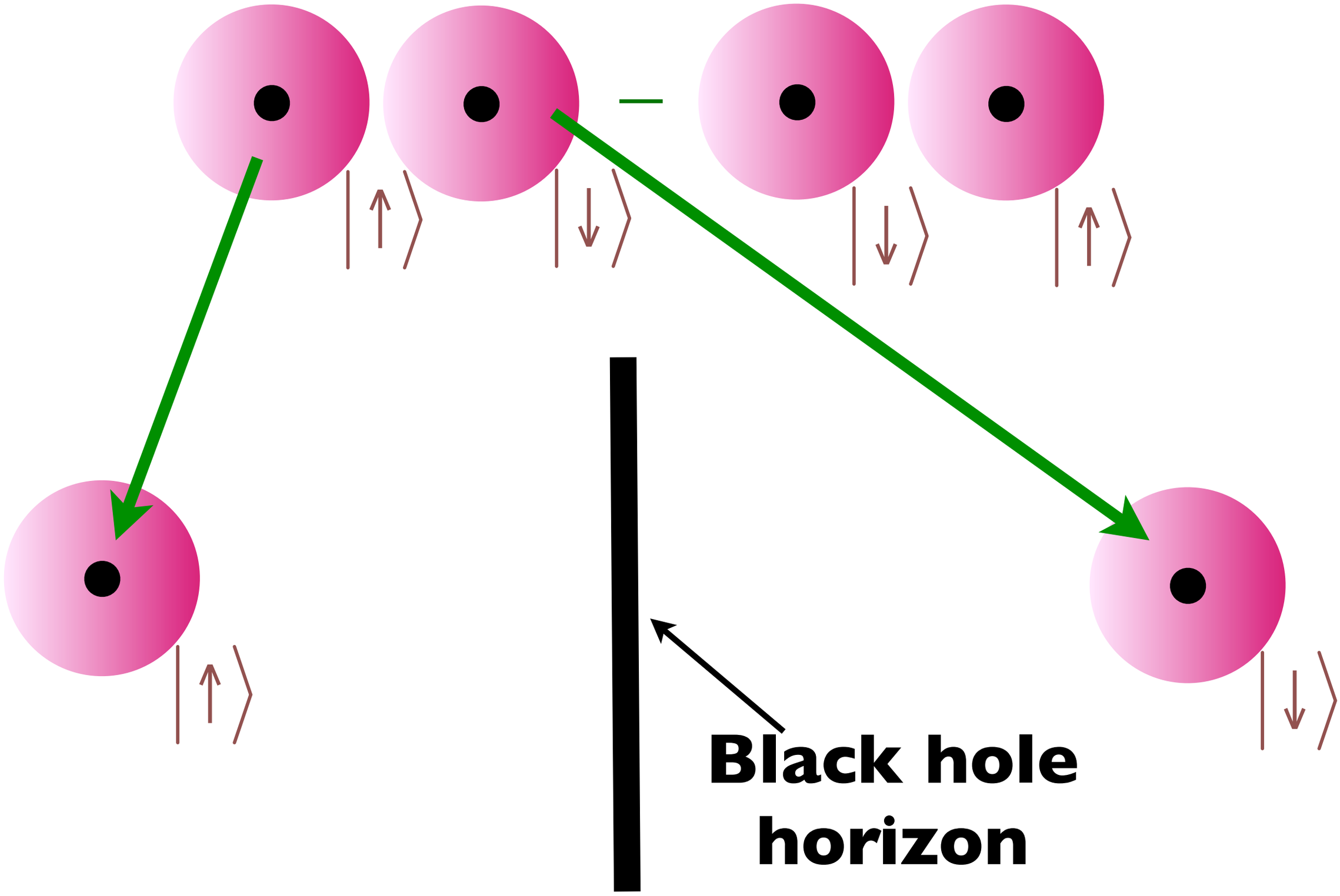

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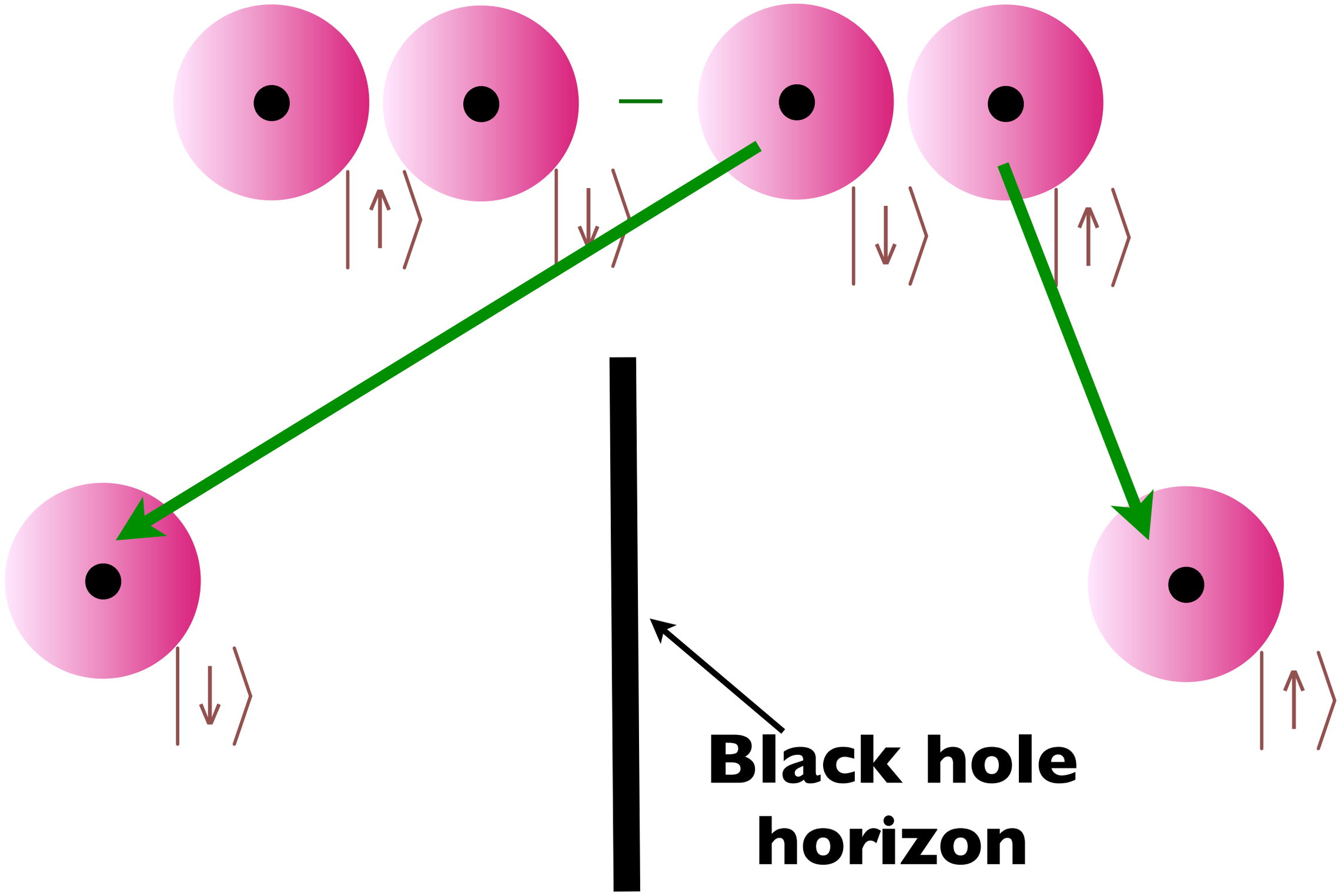
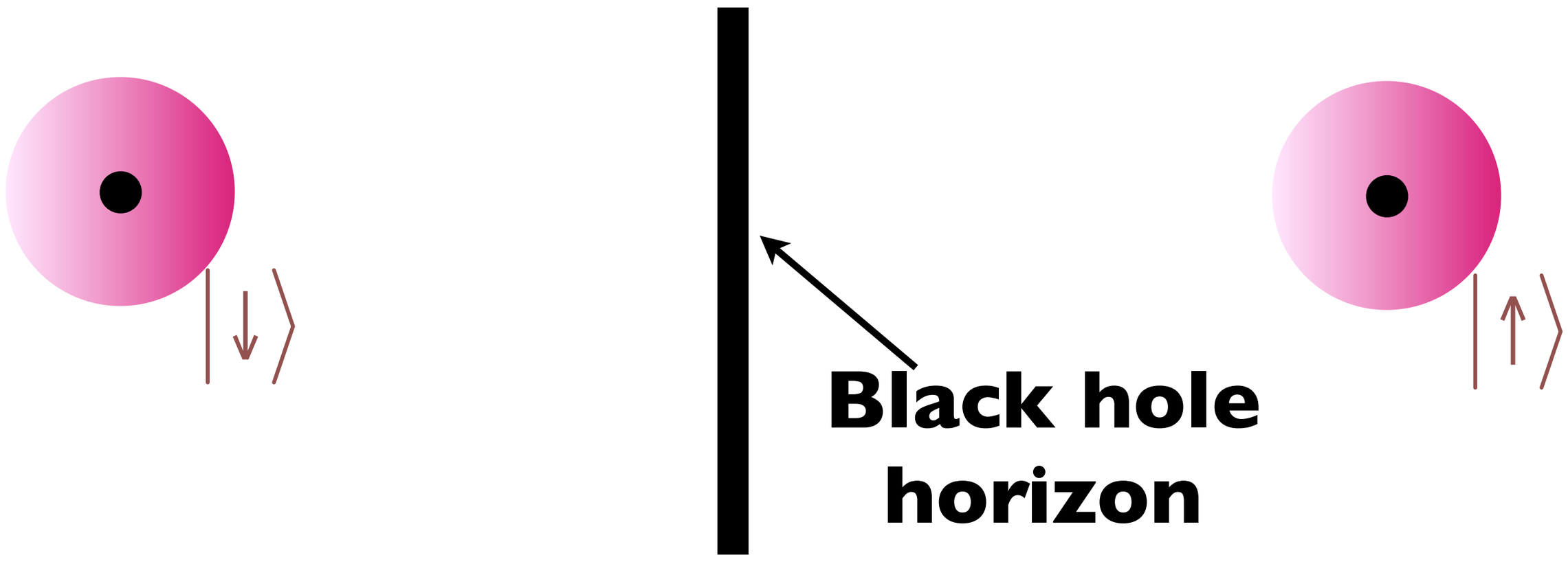


Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole



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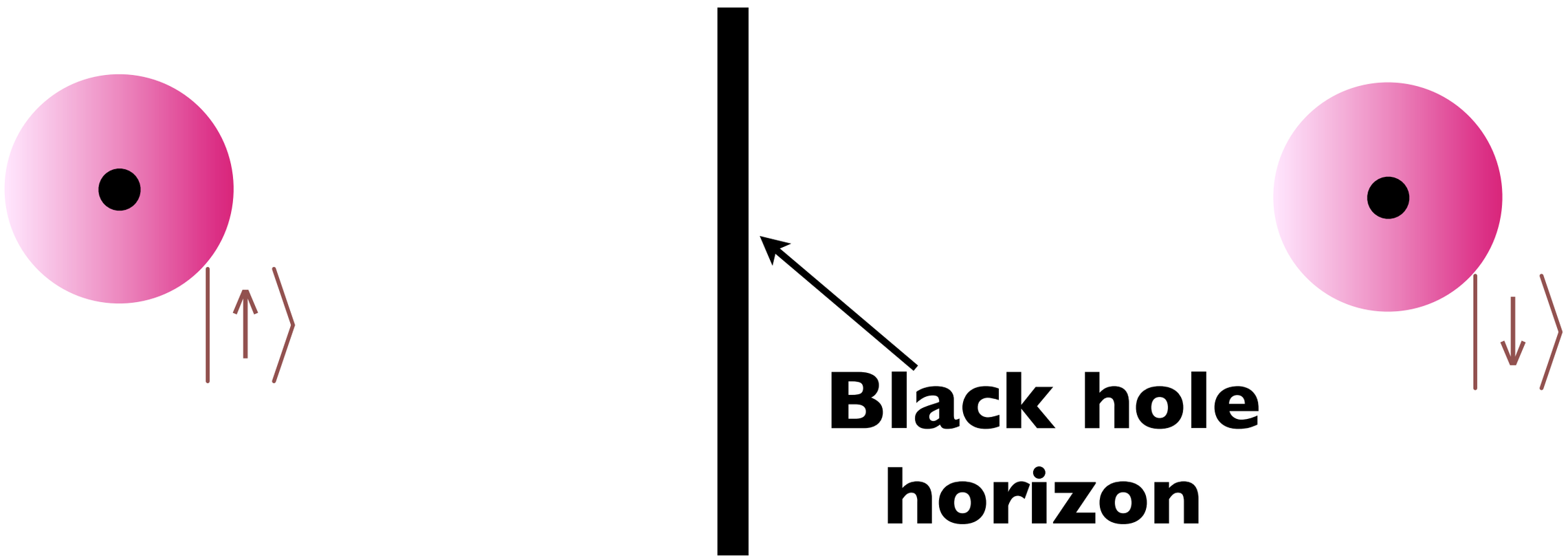

Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

Quantum entanglement

Superconductivity

Quantum phase transitions
and
Quantum critical points

Black Holes

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Quantum Entanglement across a black hole horizon

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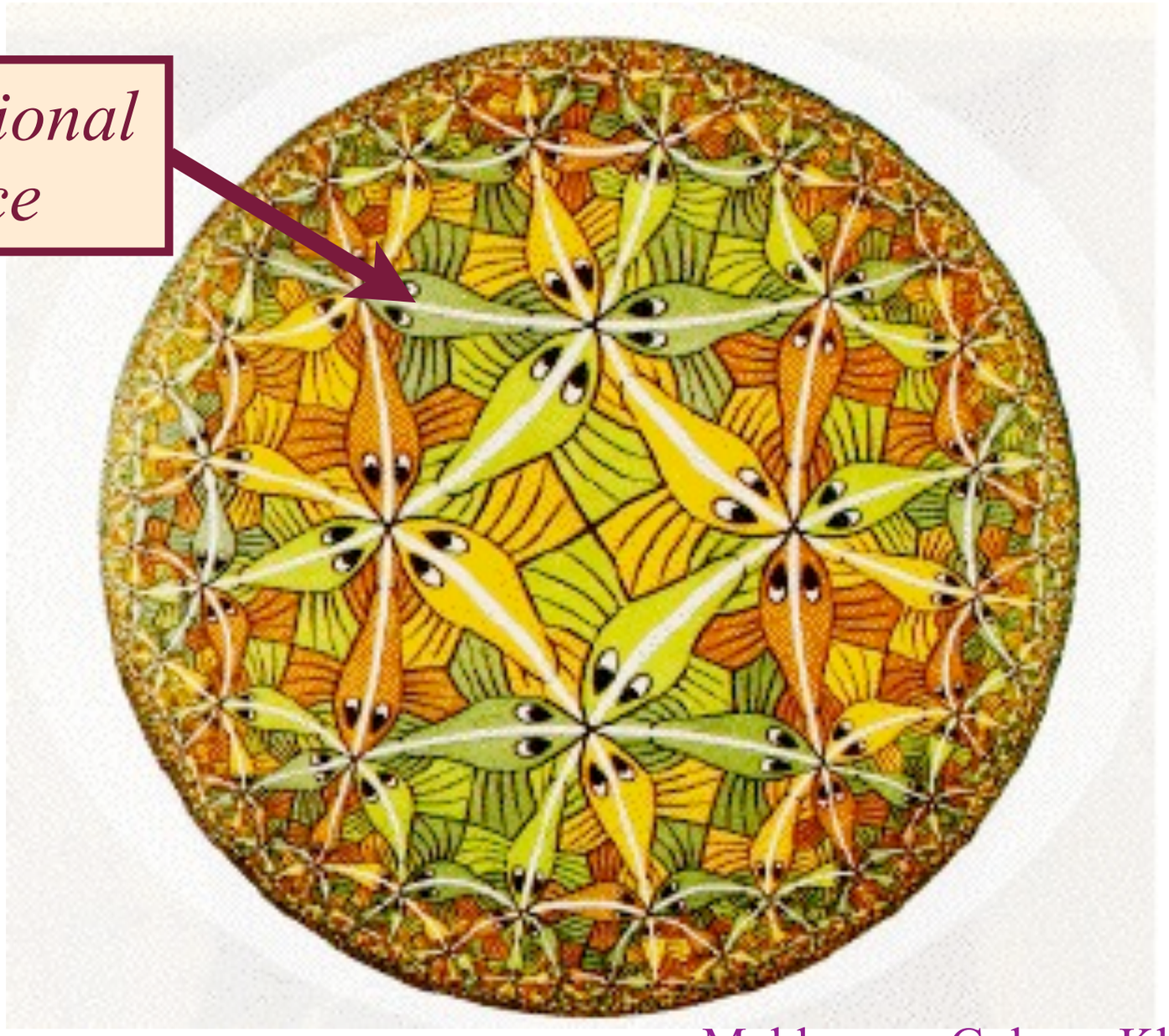
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole

In anti-de Sitter (AdS) space, this entanglement is equivalent to that of a quantum critical point of quantum matter in 2 dimensions

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by the theory of a quantum critical point in 2+1 dimensions

3+1 dimensional AdS space



Maldacena, Gubser, Klebanov, Polyakov, Witten

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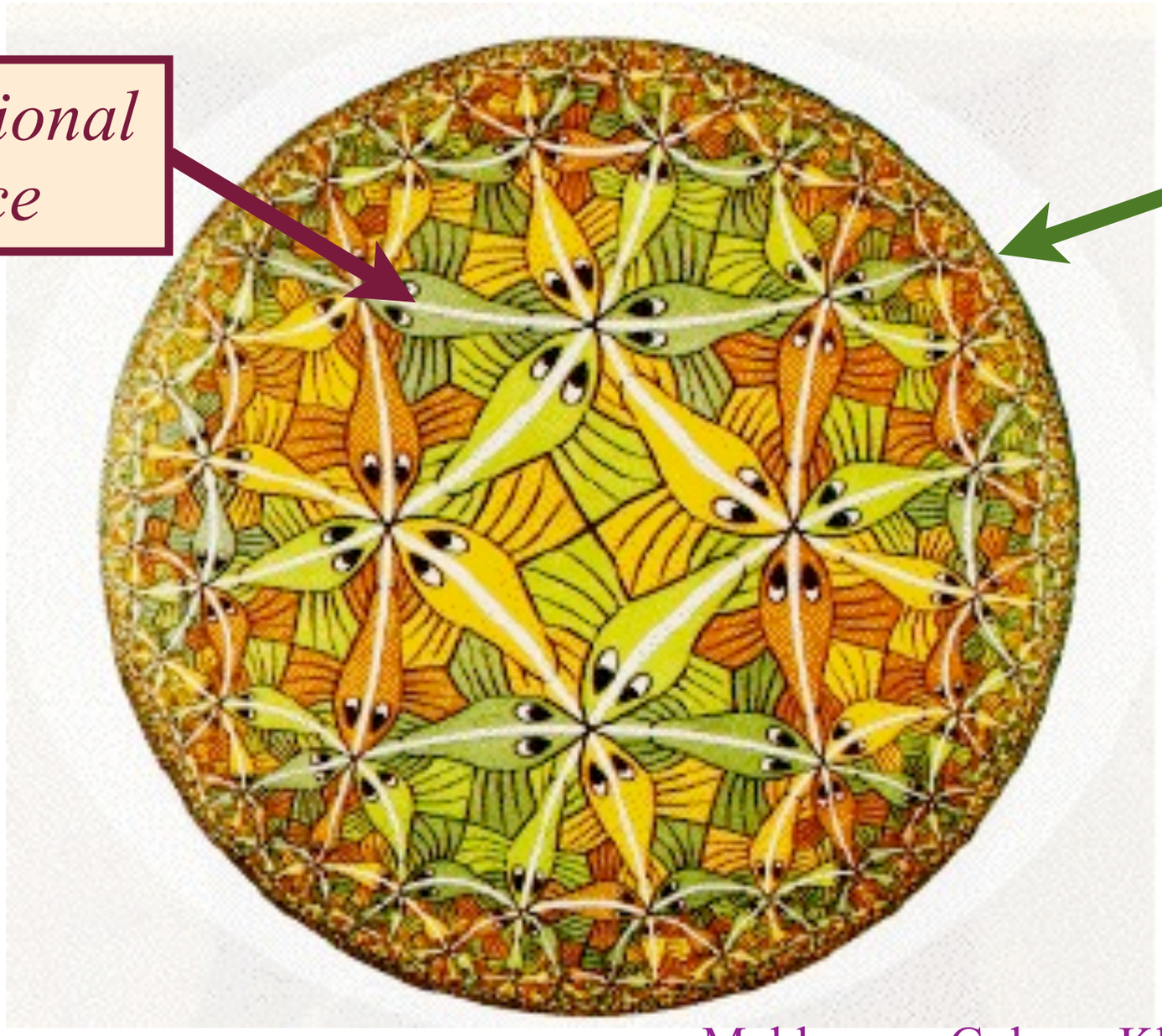

A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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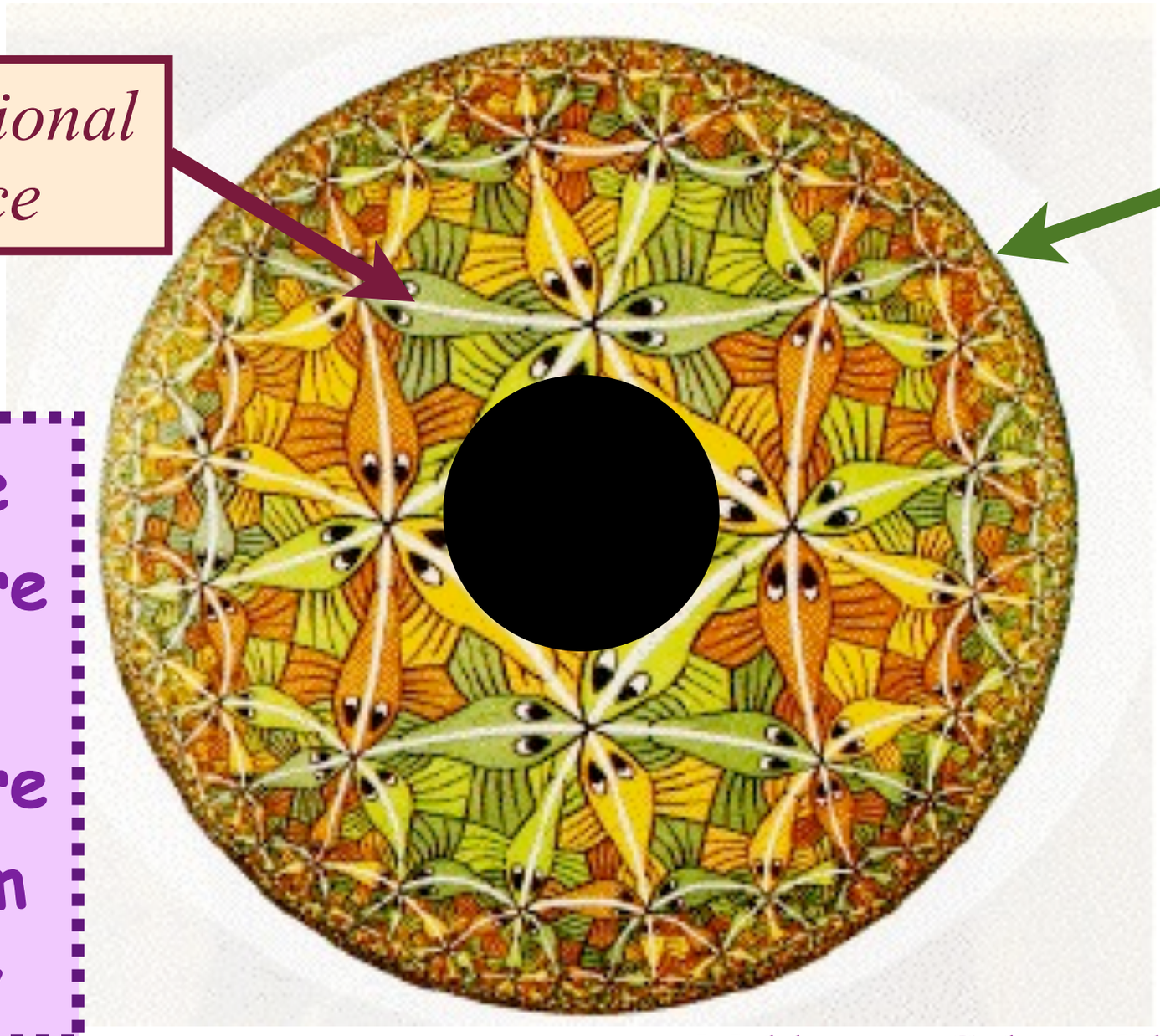
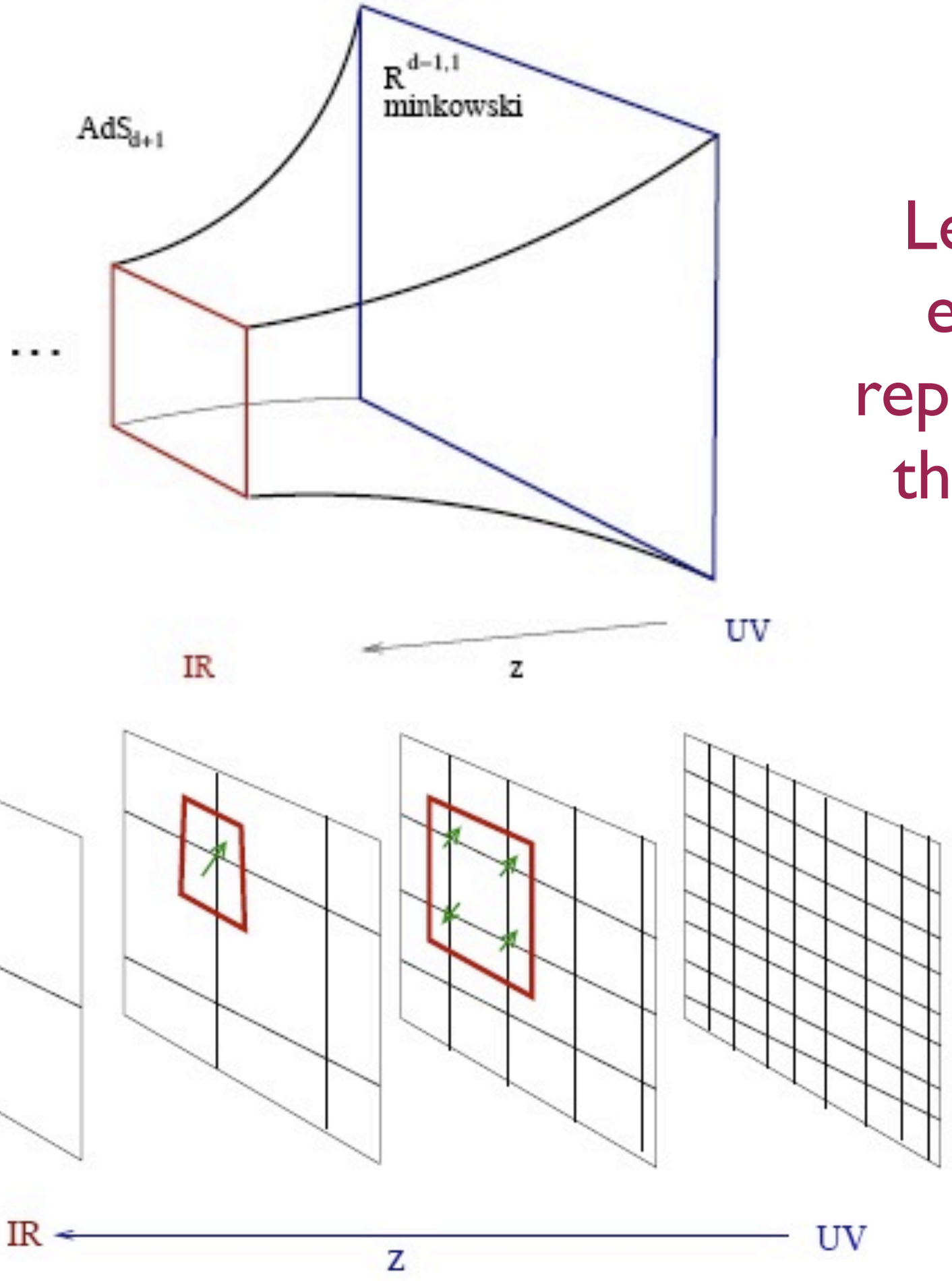
3+1 dimensional AdS space



Quantum criticality in 2+1 dimensions

Black hole temperature = temperature of quantum criticality

Maldacena, Gubser, Klebanov, Polyakov, Witten



Length-scale of entanglement represents "extra" third dimension

J. McGreevy

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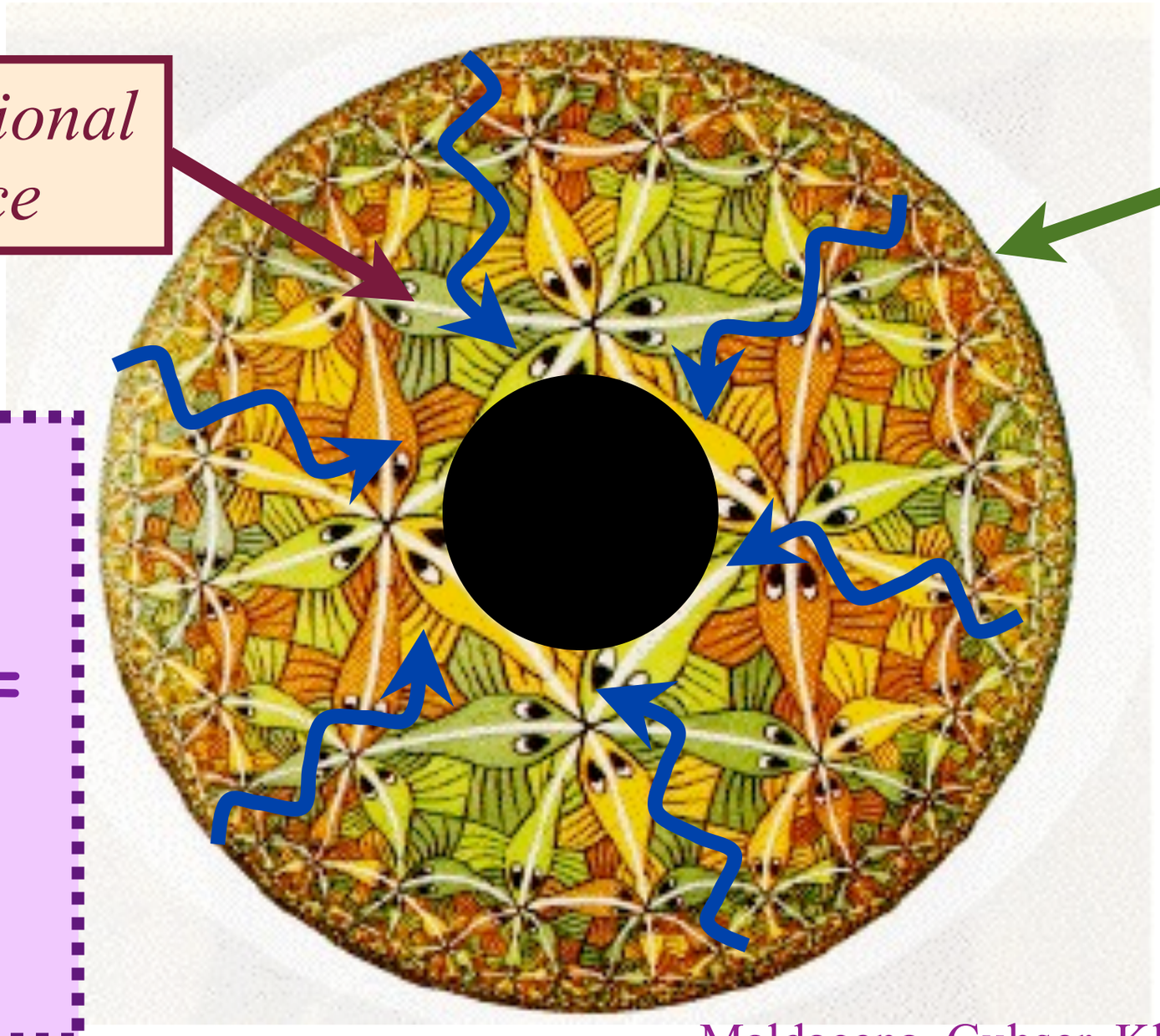
3+1 dimensional AdS space



Quantum criticality in 2+1 dimensions

Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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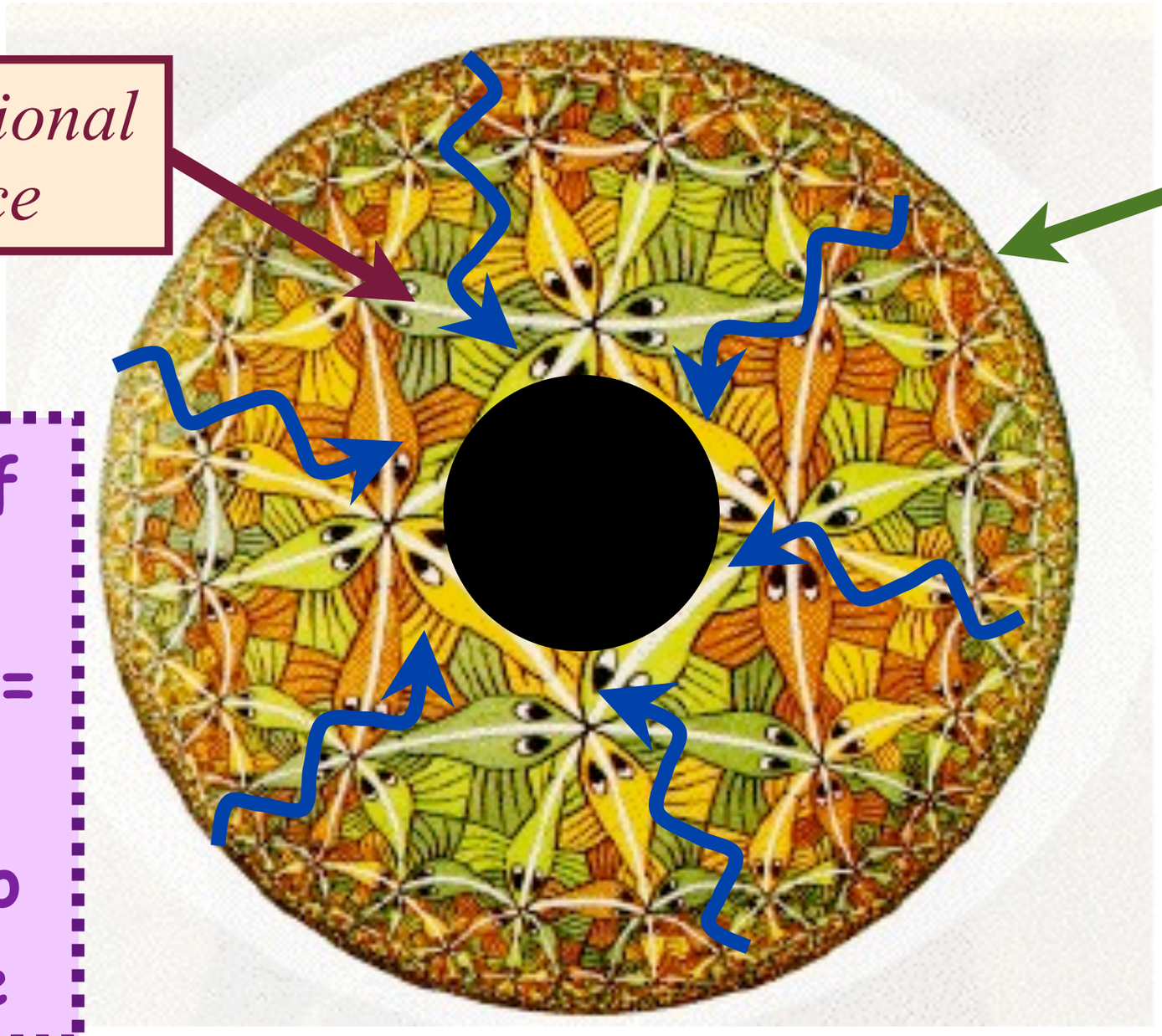
3+1 dimensional AdS space



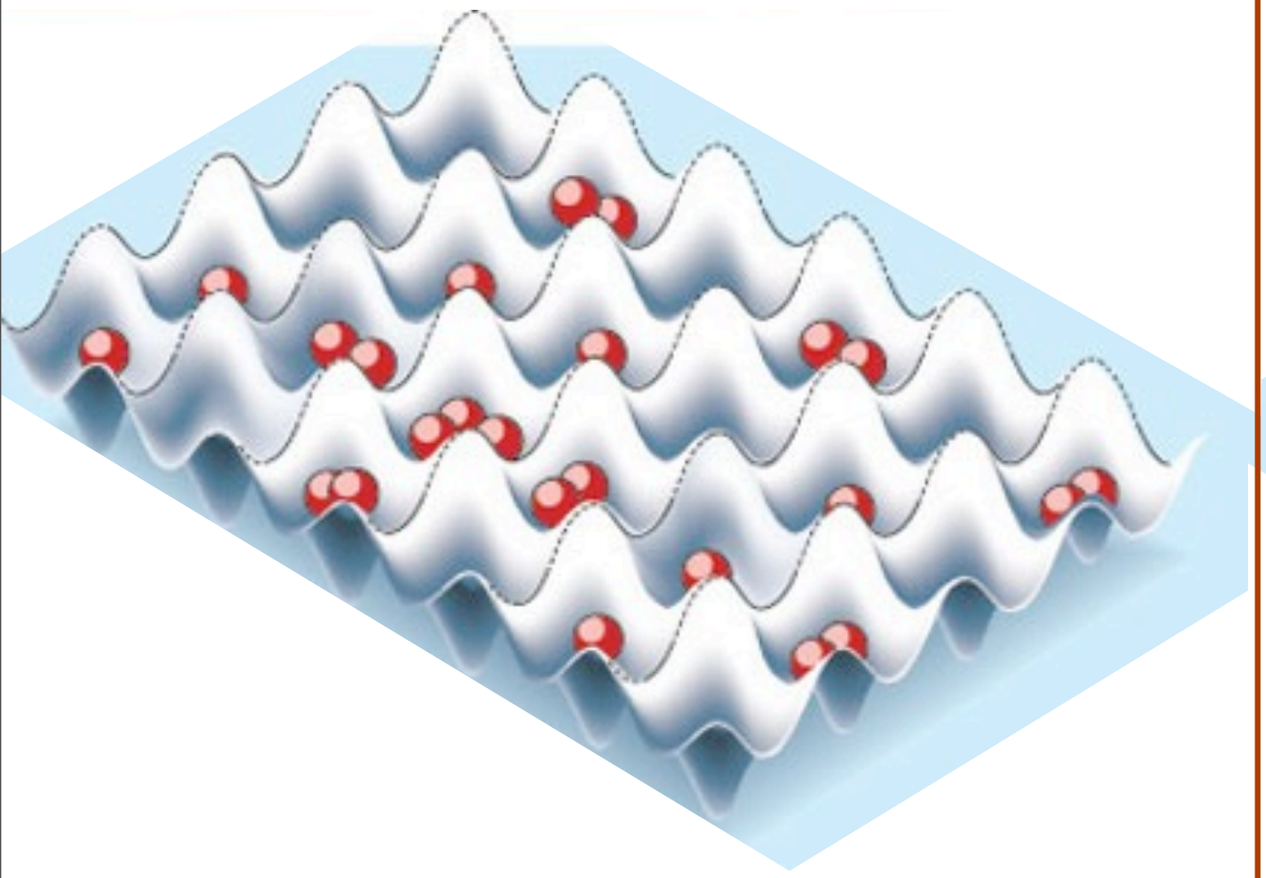
Quantum criticality in 2+1 dimensions

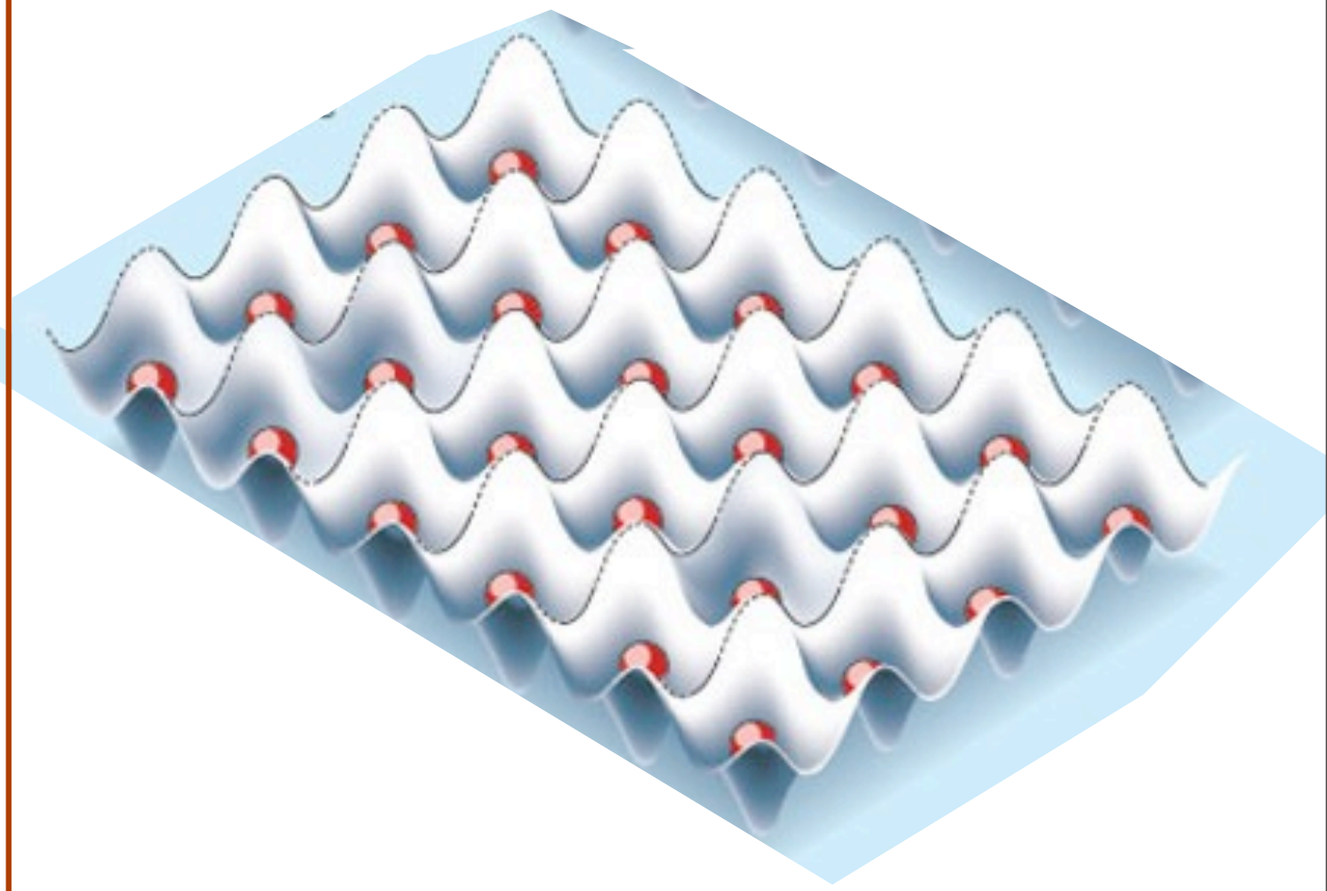
Friction of quantum criticality = waves falling into black hole

Kovtun, Policastro, Son

Superfluid

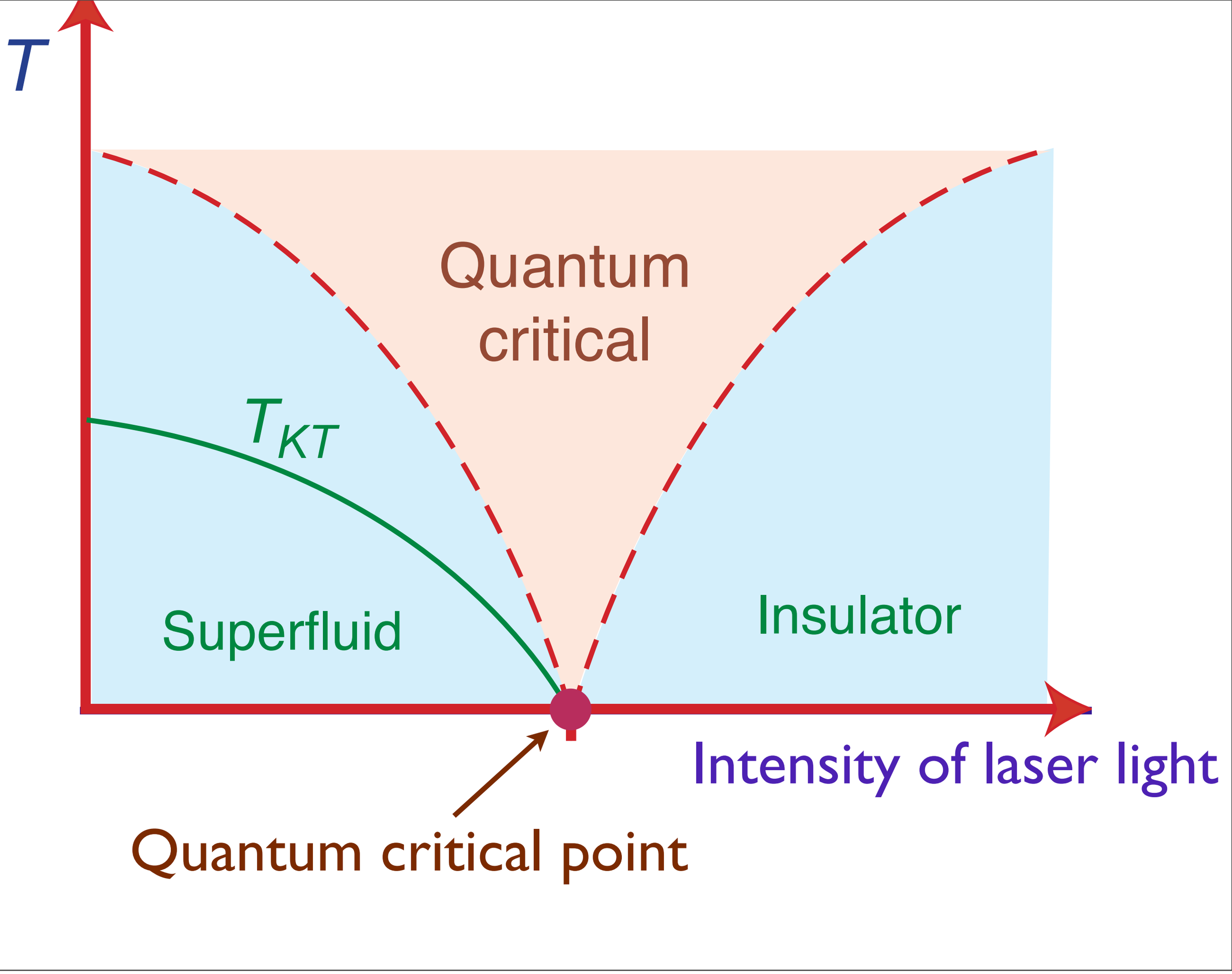
Insulator





Intensity of laser light

Quantum critical point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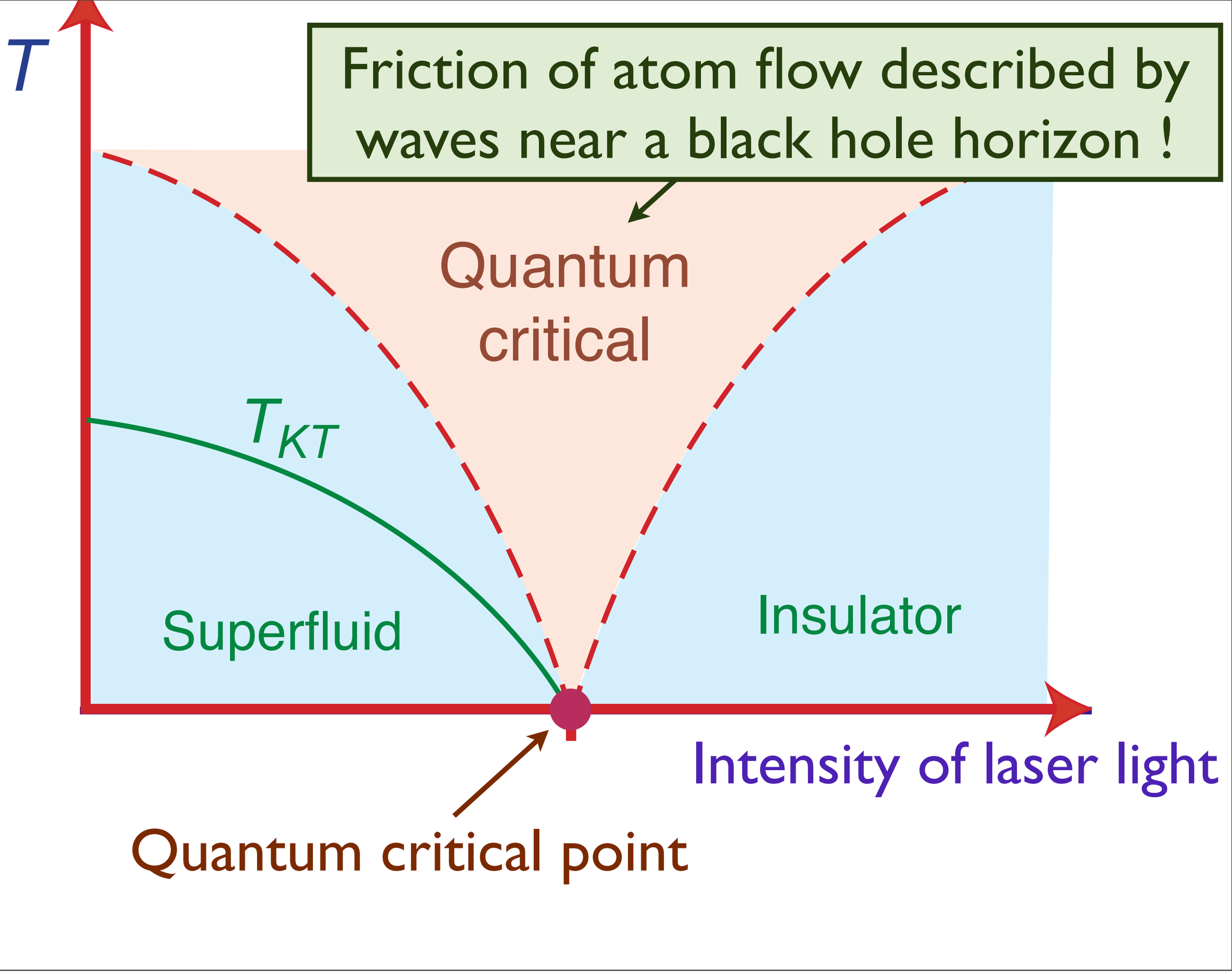
Quantum
critical
T_{KT}
Superfluid
Insulator
Intensity of laser light
Quantum critical point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Friction of atom flow described by waves near a black hole horizon !
Quantum critical
T_{KT}
Superfluid
Insulator
Intensity of laser light
Quantum critical point

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by the theory of a quantum critical point in 2+1 dimensions

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Question: if you lived in this world, would you think you lived in 2 or 3 spatial dimensions ?

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Question: if you lived in this world, would you think you lived in 2 or 3 spatial dimensions ?

Answer: it depends.....

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by the theory of a quantum critical point in 2+1 dimensions

Question: if you lived in this world, would you think you lived in 2 or 3 spatial dimensions ?

Answer: for some values of the parameters in the theory, the world would appear 3 dimensional, while for other parameters, it would appear 2 dimensional, and there many theories which would appear "in-between"

The deepest new principle of physics to emerge from string theory is the equivalence between

the quantum theory of gravity in 3 space dimensions

and

a theory of special quantum states of matter (electron, photons, quarks..) without gravity in 2 space dimensions

Quantum superposition and entanglement

Superconductivity

Quantum phase transitions and Quantum critical points

Black Holes

Quantum superposition and entanglement

Superconductivity

Quantum phase transitions and Quantum critical points

Black Holes