De Sitter Lecture Series in Theoretical Physics 2009
University of Groningen

Quantum phase transitions: from antiferromagnets and superconductors to black holes

Talk online: sachdev.physics.harvard.edu

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Outline

1. Introduction to quantum phase transitions:
   quantum spin systems and relativistic field theories

2. Quantum phase transitions in metals

3. The AdS/CFT correspondence:
   quantum criticality at strong coupling

4. The cuprate high temperature superconductors:
   competing orders and quantum criticality

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A. Coupled dimer antiferromagnets
and Landau-Ginzburg-Wilson theory

B. Triangular lattice antiferromagnets :
Spinons, visons, and doubled
Chern-Simons theory

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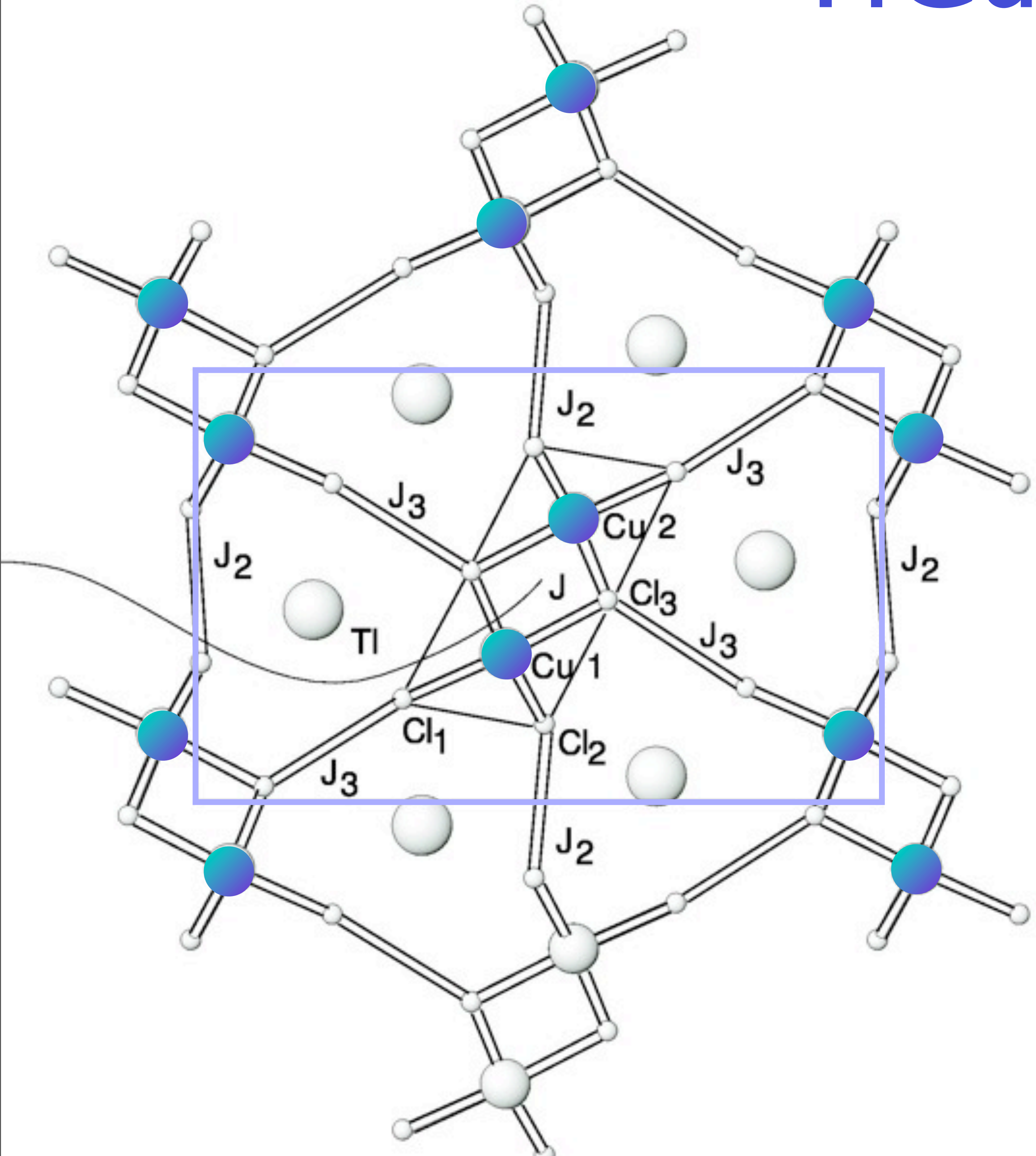
$TlCuCl_3$



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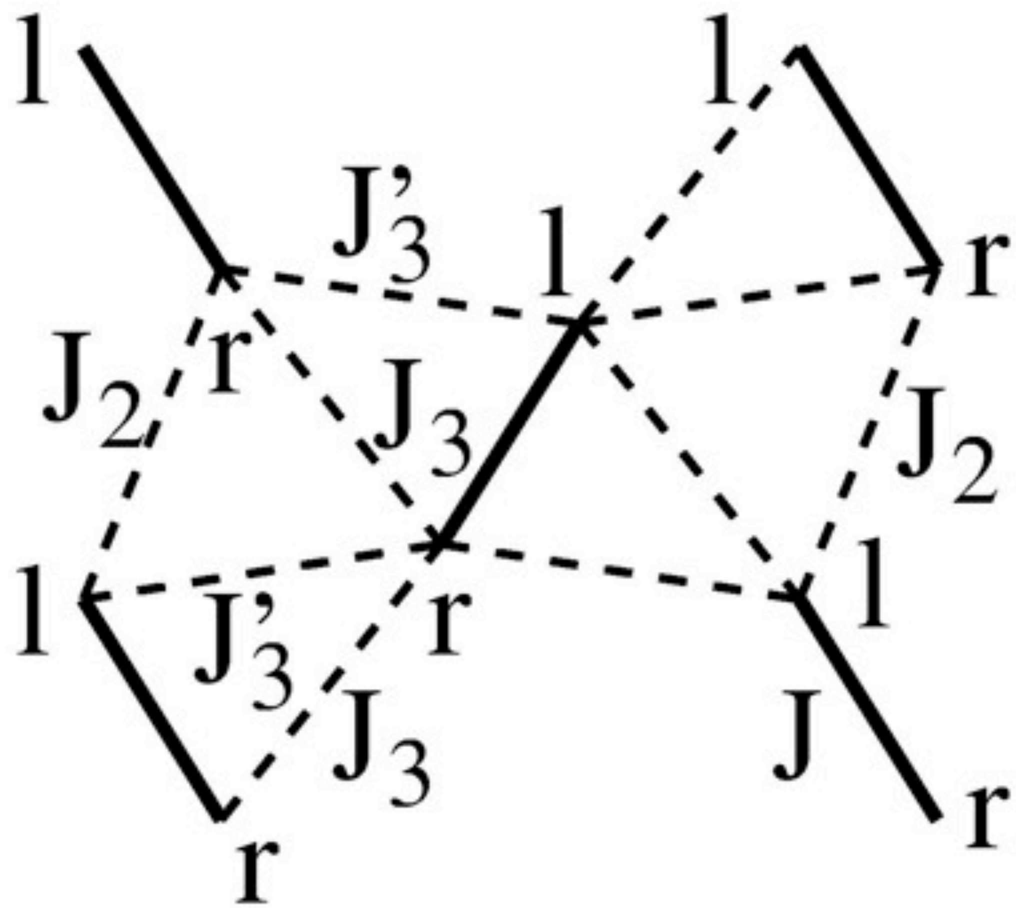

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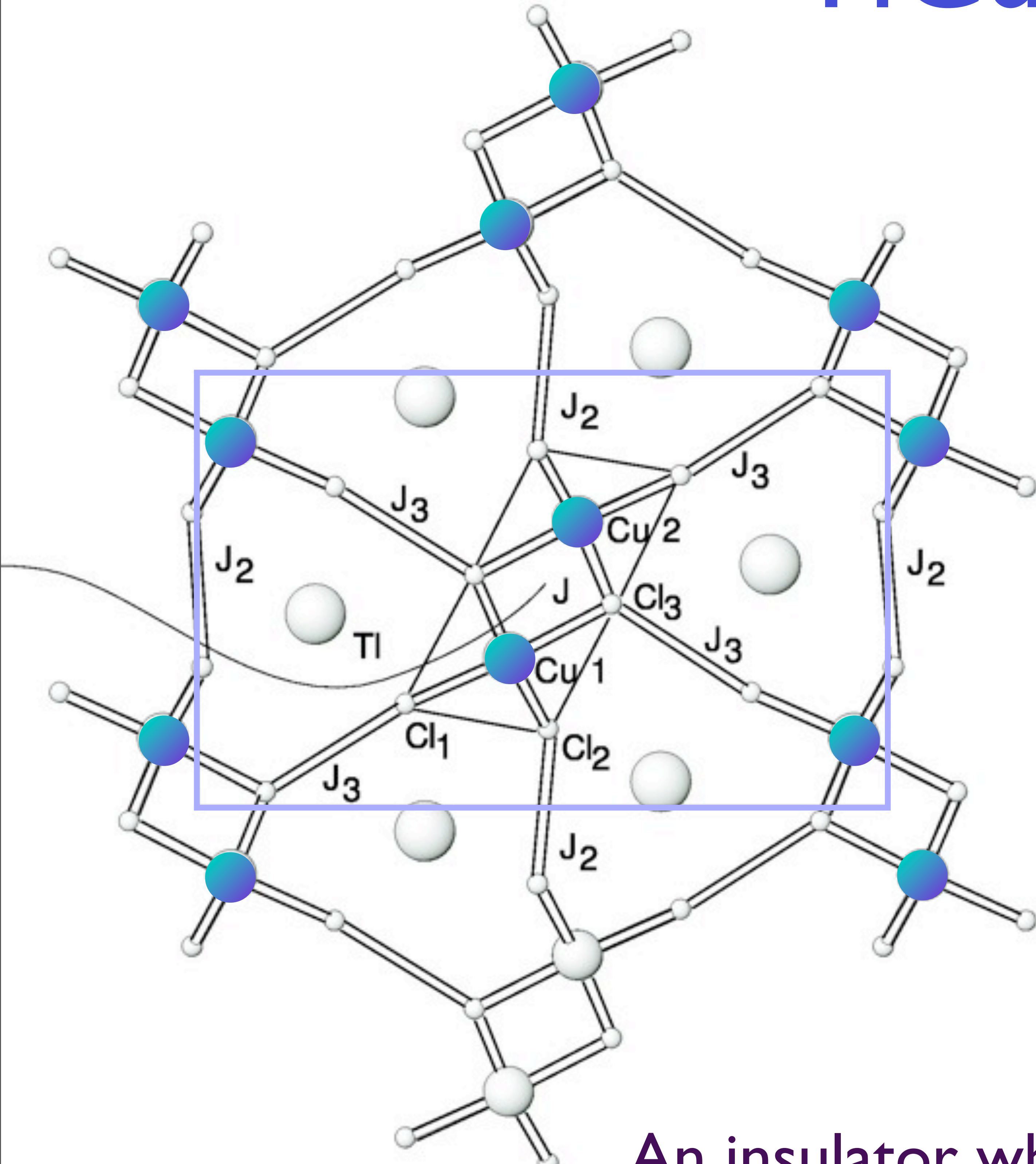
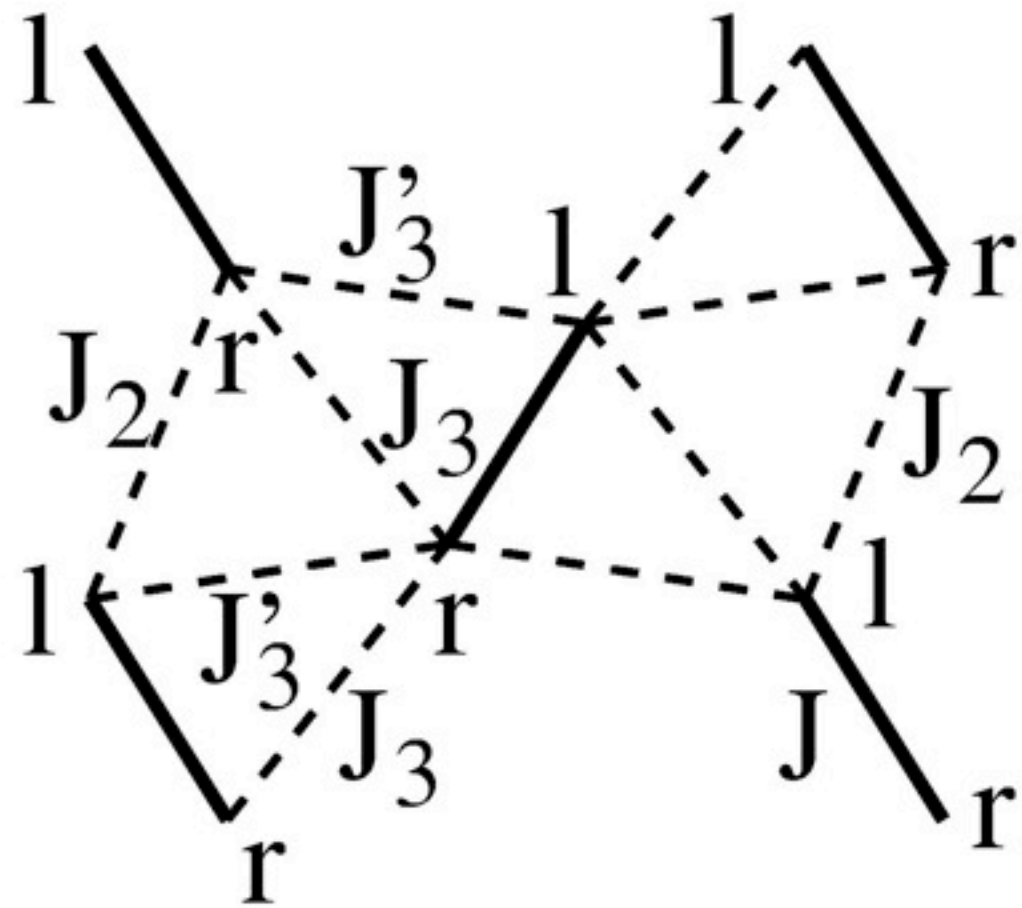
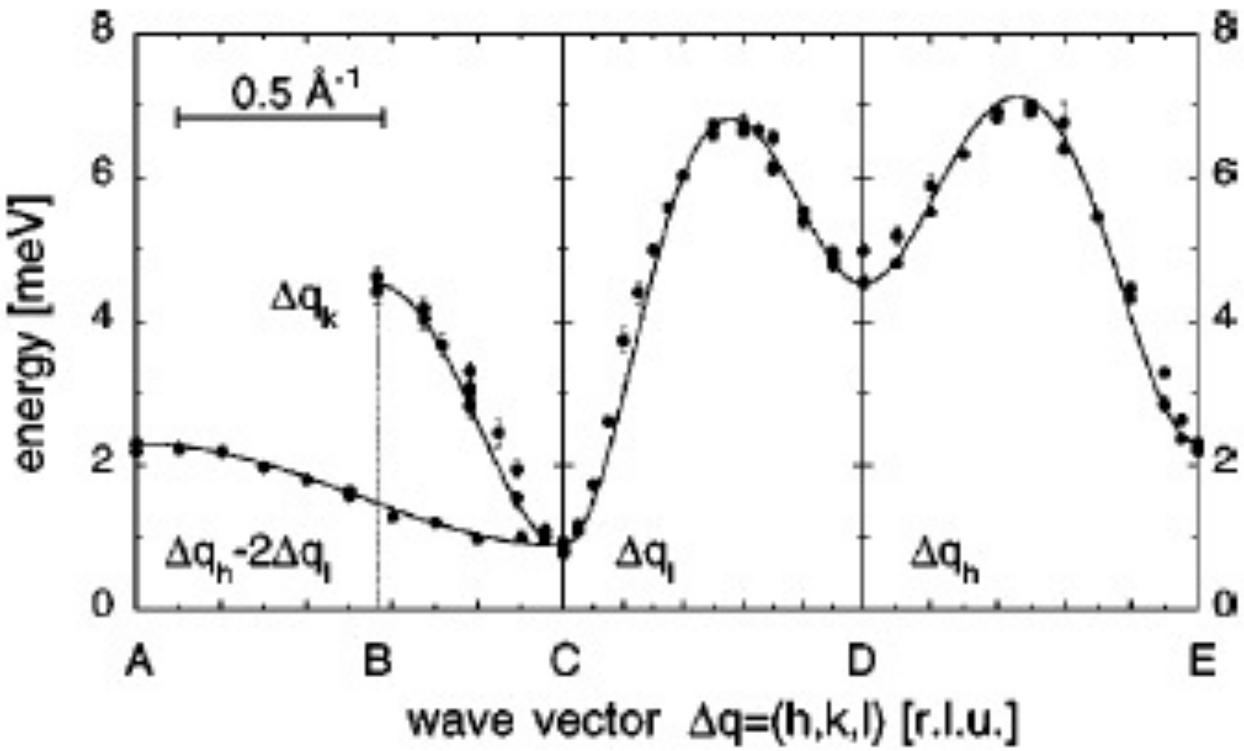
$TlCuCl_3$





An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

$TlCuCl_3$ at ambient pressure



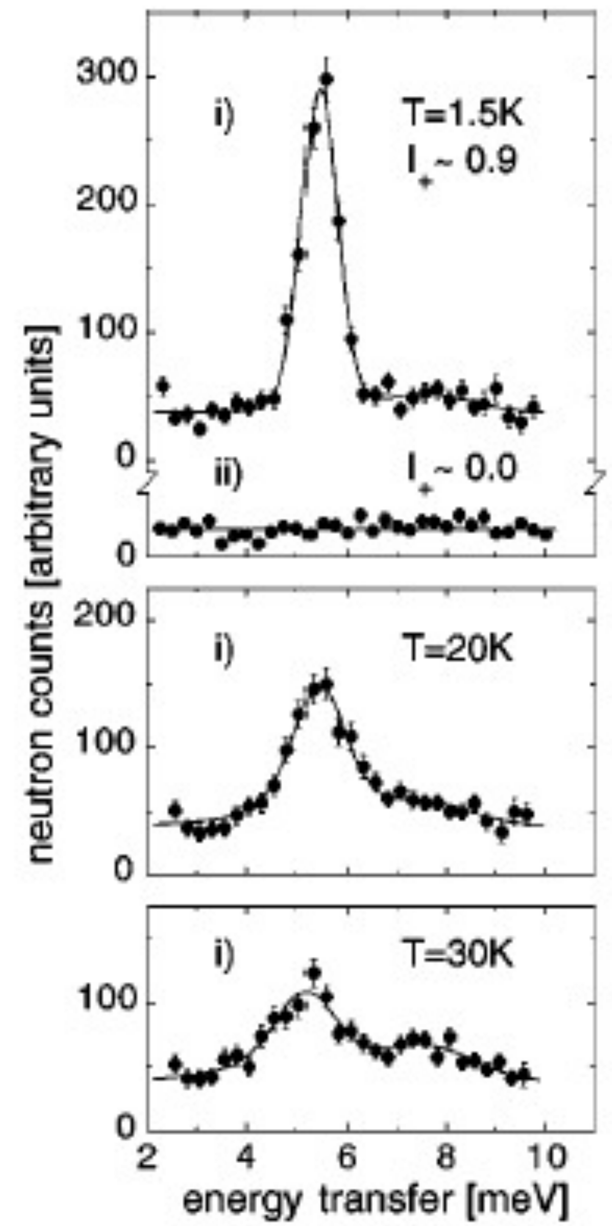
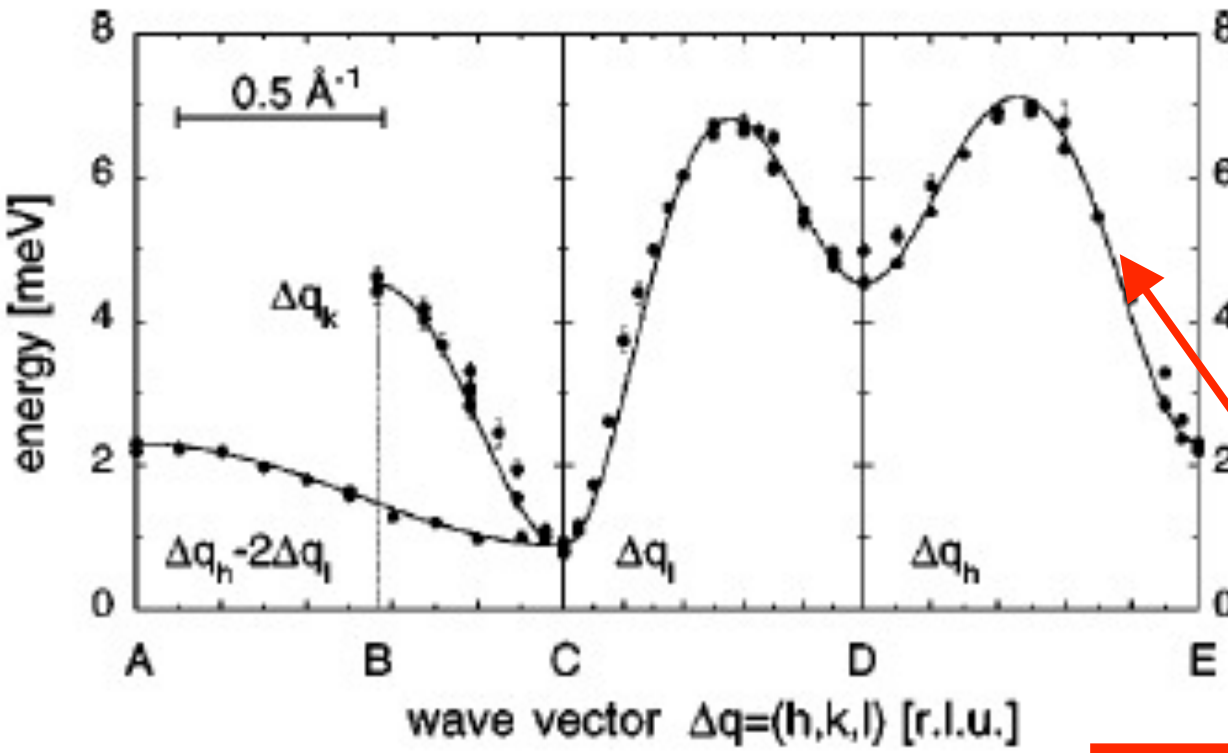
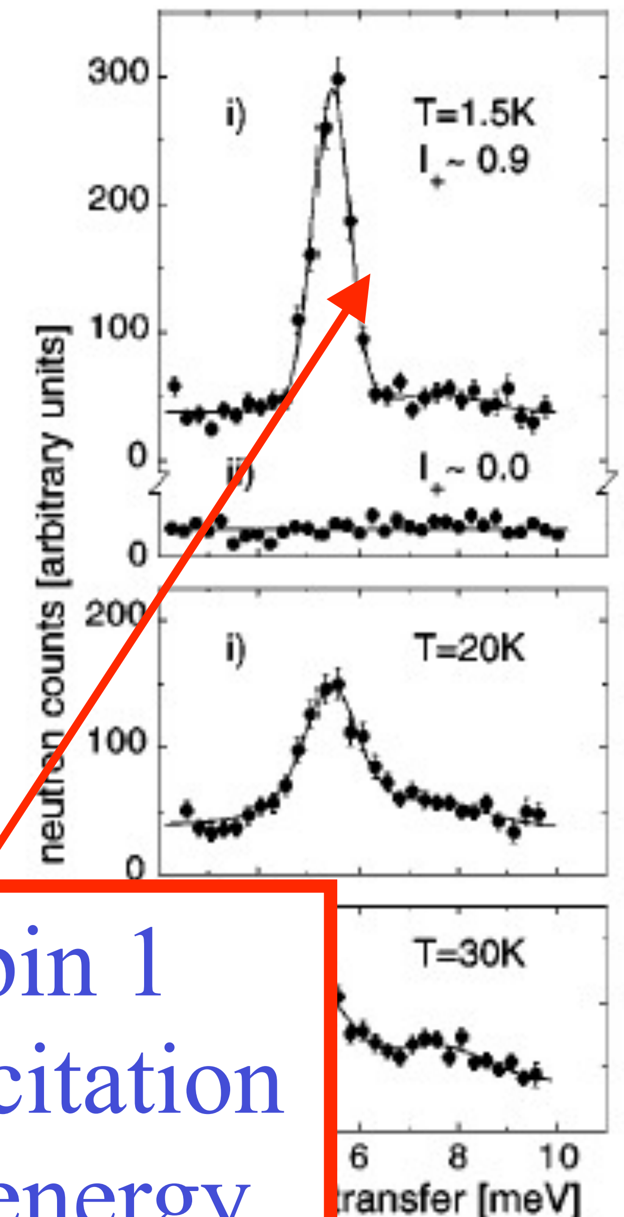


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

$TlCuCl_3$ at ambient pressure





Sharp spin 1 particle excitation above an energy gap (spin gap)

...les in the a^*c^* plane of $TlCuCl_3$
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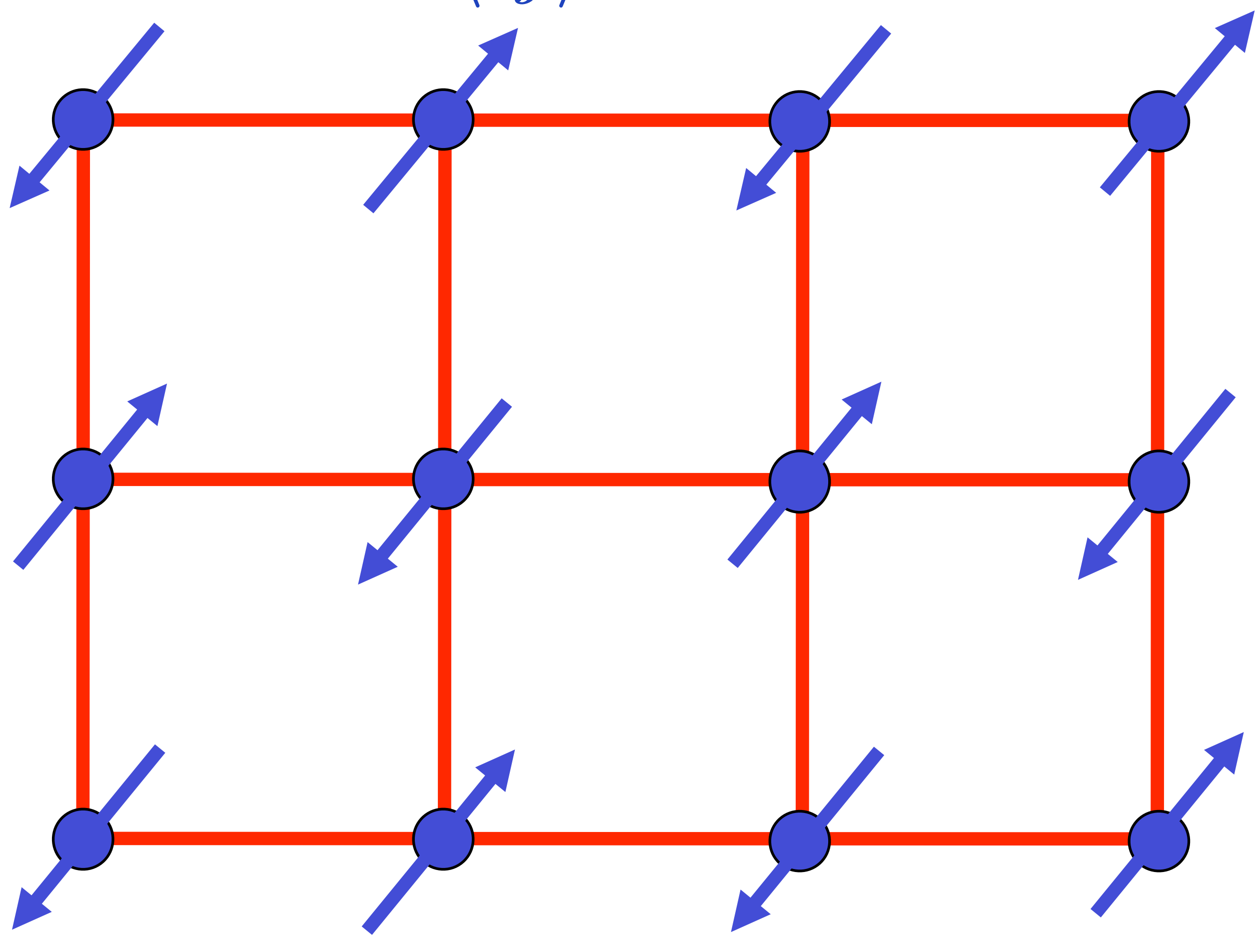
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

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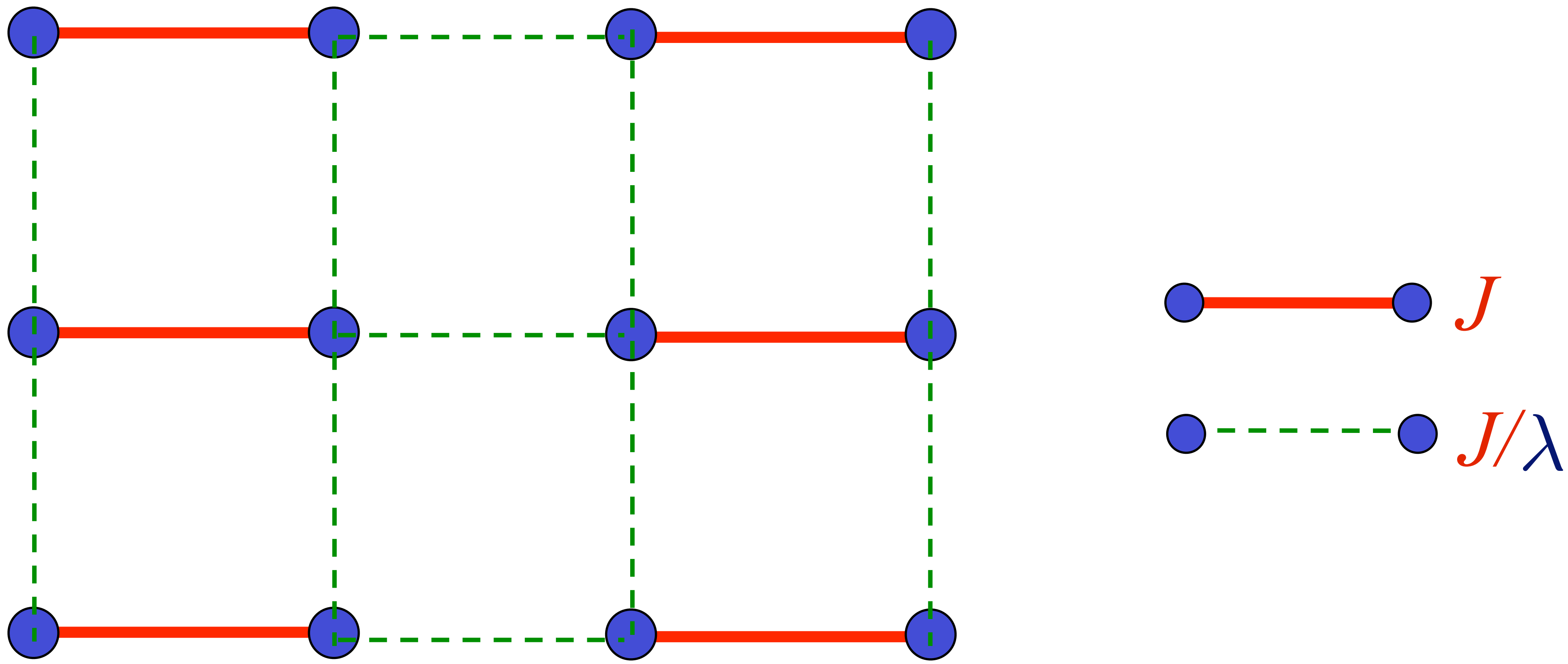
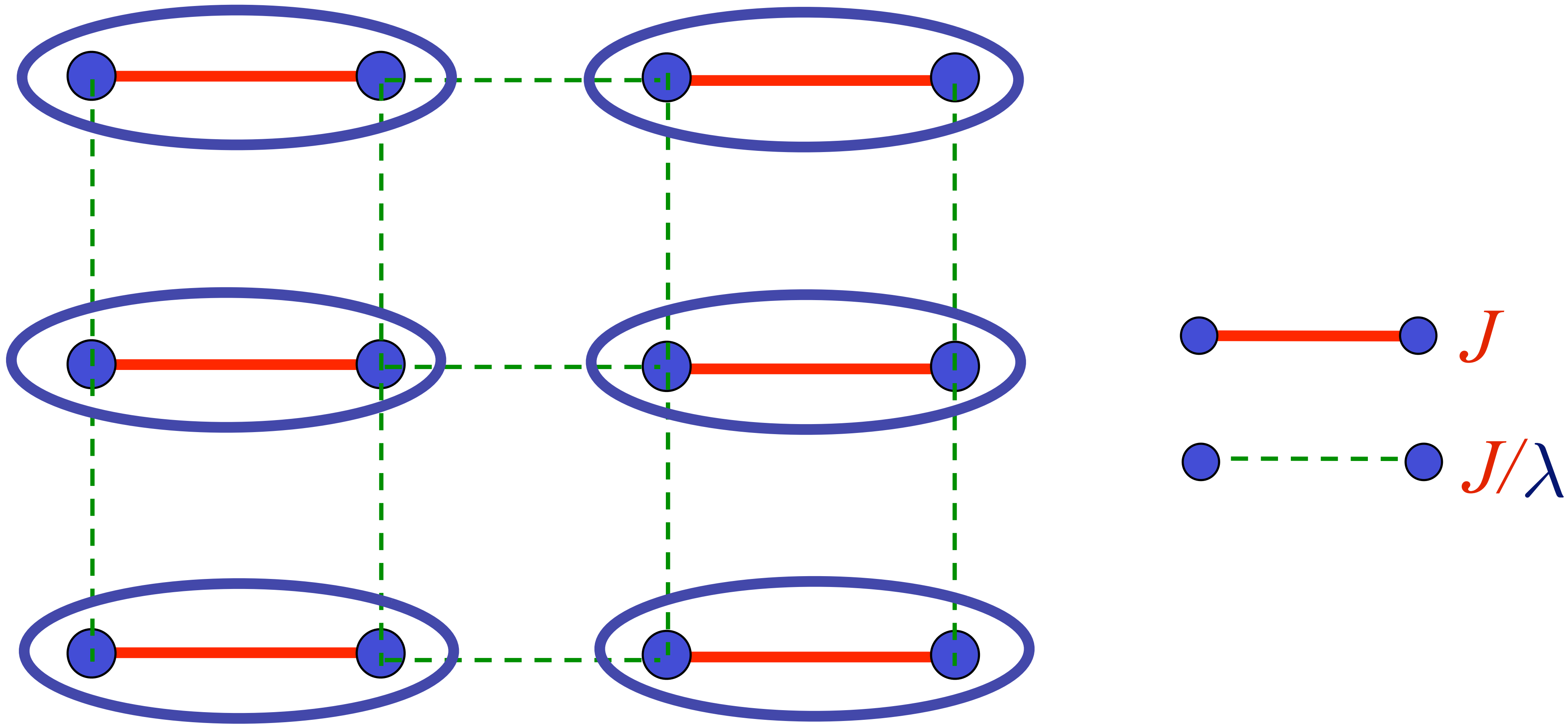


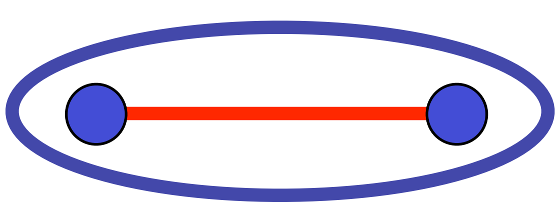
Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

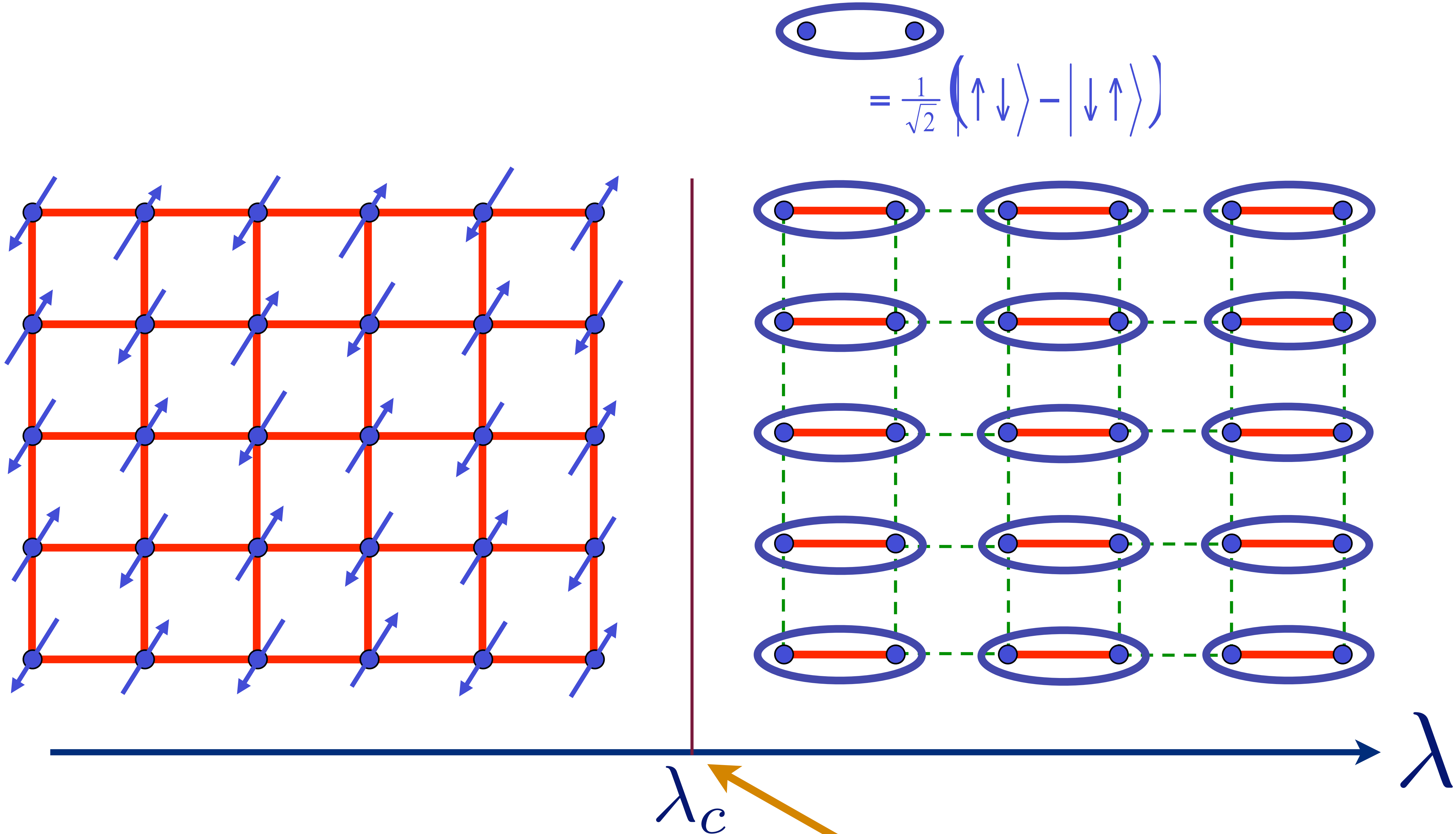
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state is a "quantum paramagnet"
with spins locked in valence bond singlets

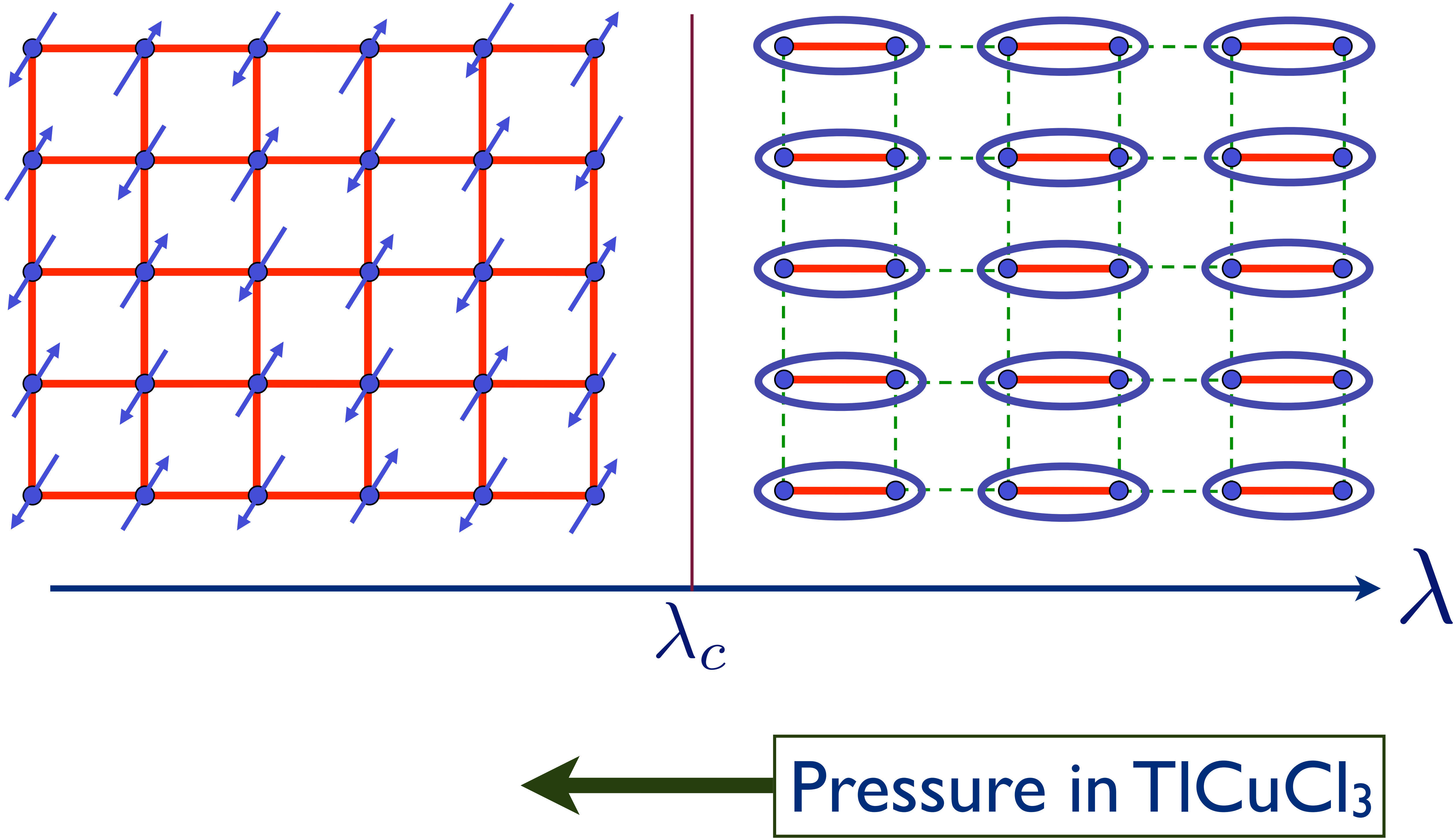


$$= \frac{1}{\sqrt{2}} \Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle \Big)$$

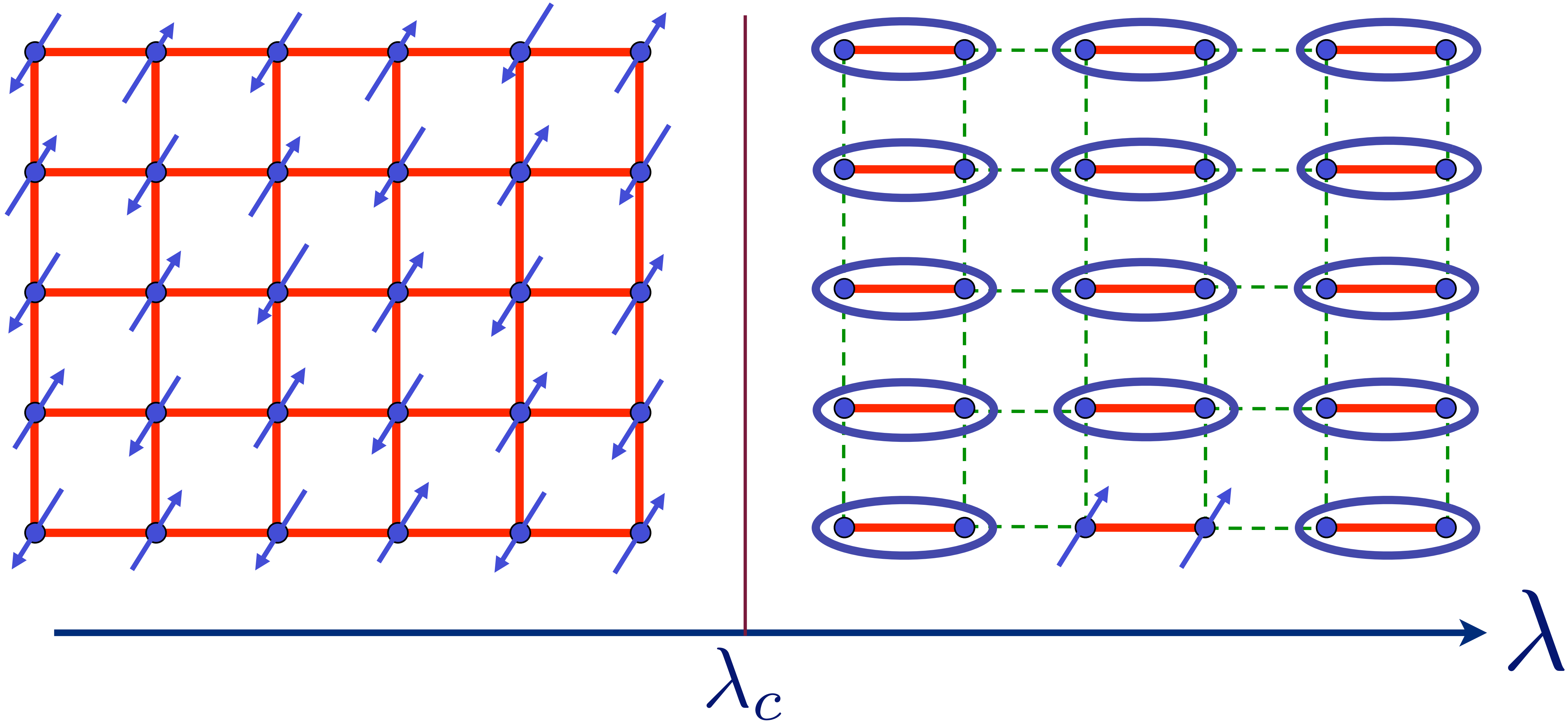


Quantum critical point with non-local entanglement in spin wavefunction

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.*B **65**, 014407 (2002).


λ
λ_c
Pressure in $TlCuCl_3$

Excitation spectrum in the paramagnetic phase



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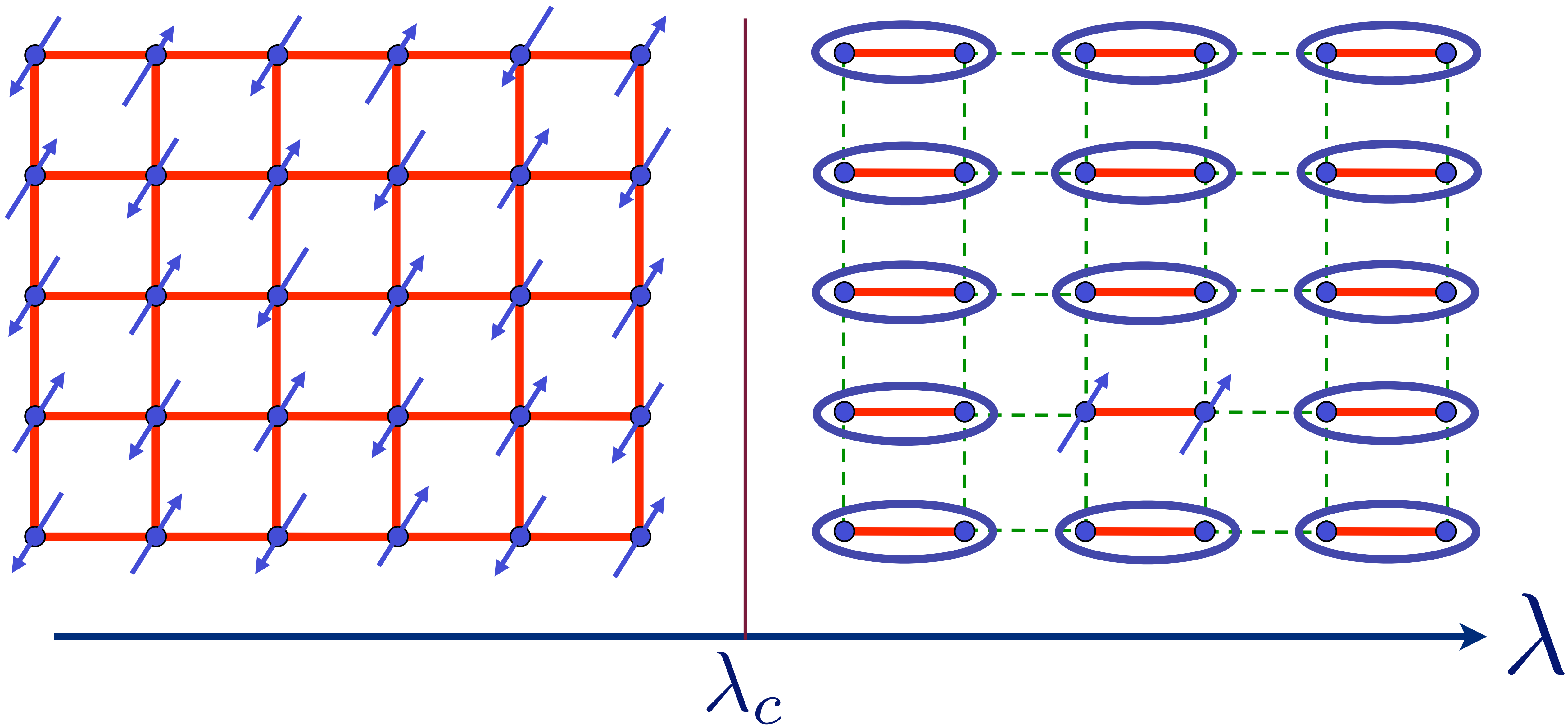

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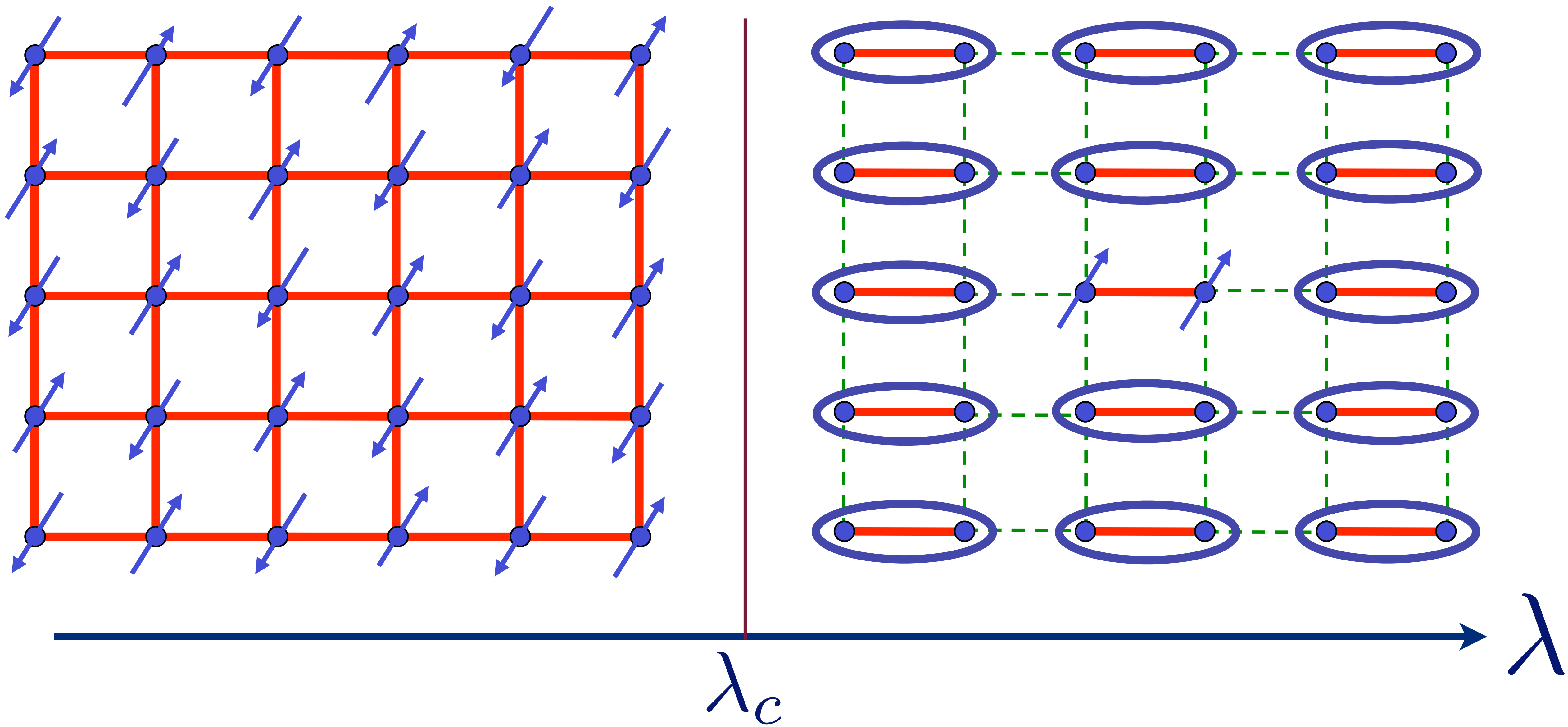

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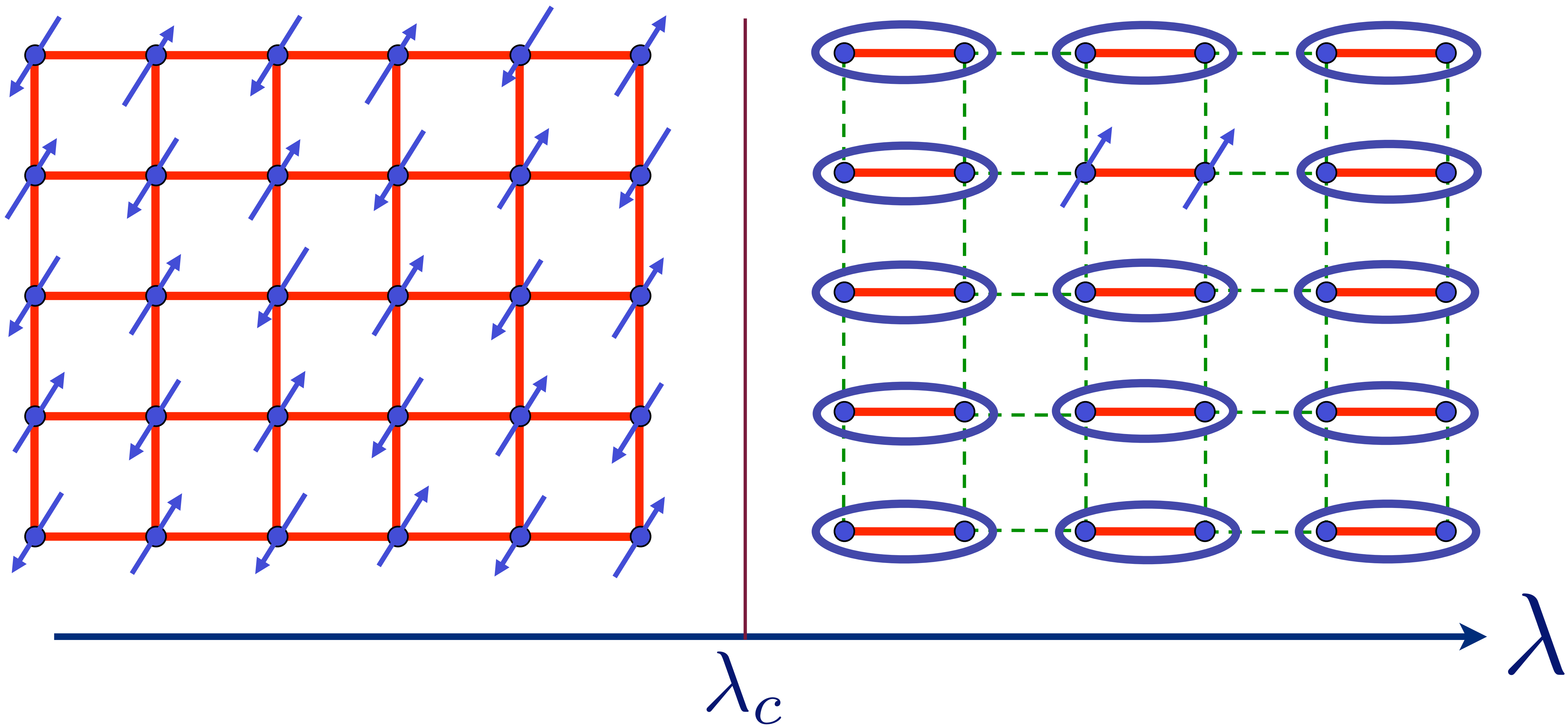
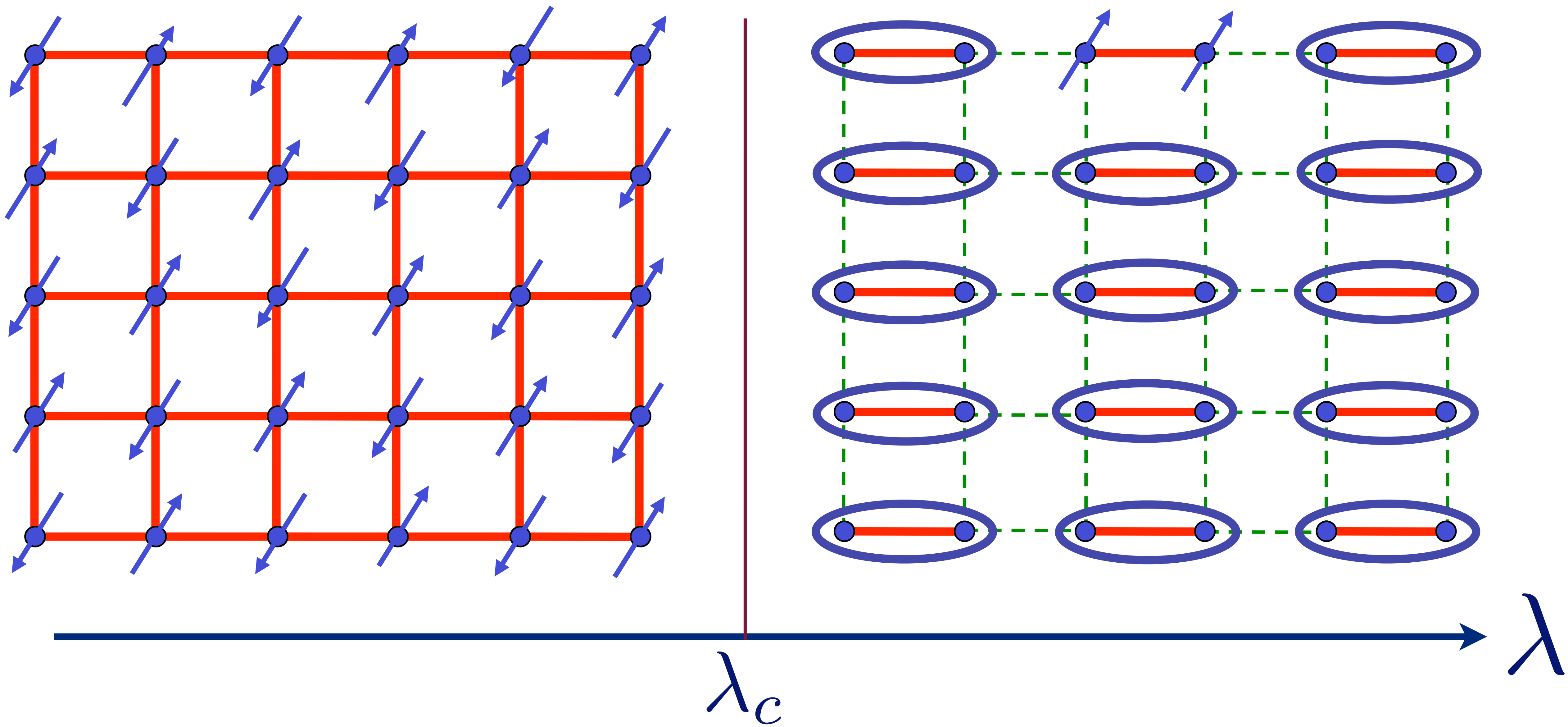
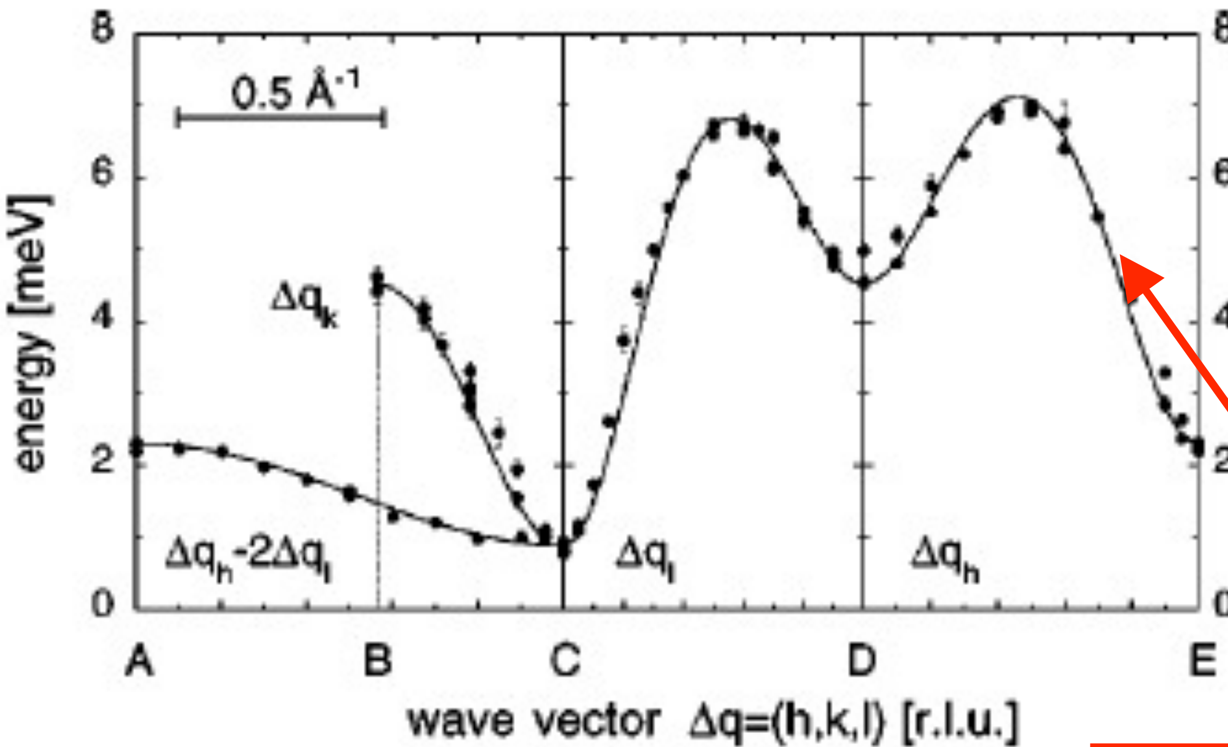


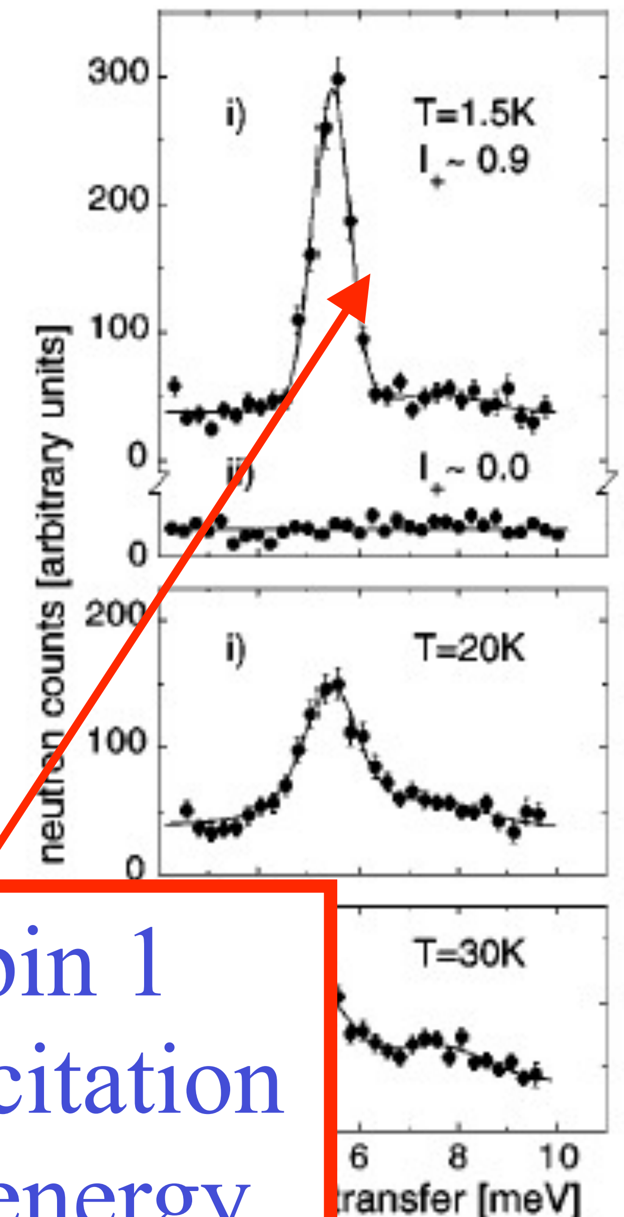
Excitation spectrum in the paramagnetic phase



Sharp spin 1
particle excitation
above an energy
gap (spin gap)

$TlCuCl_3$ at ambient pressure





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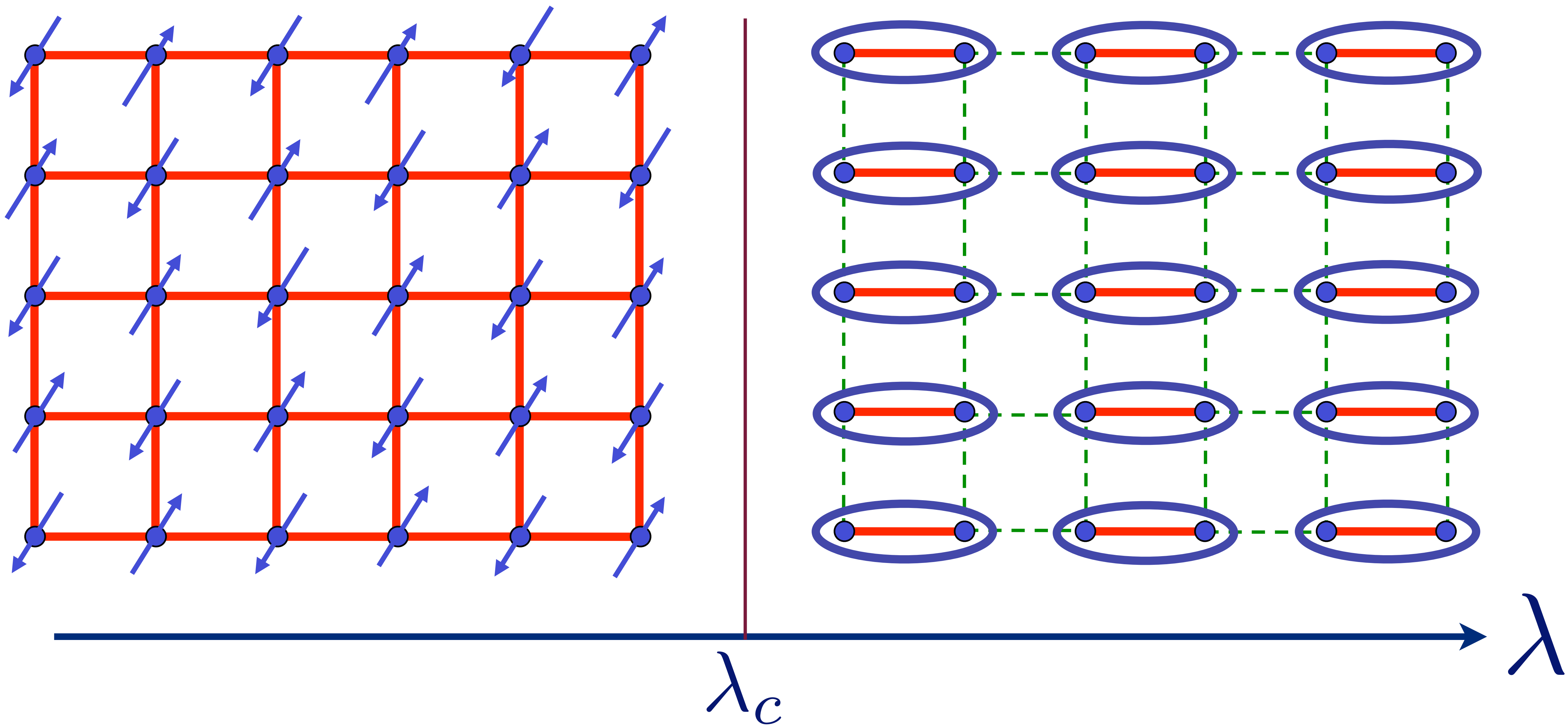
Excitation spectrum in the Néel phase



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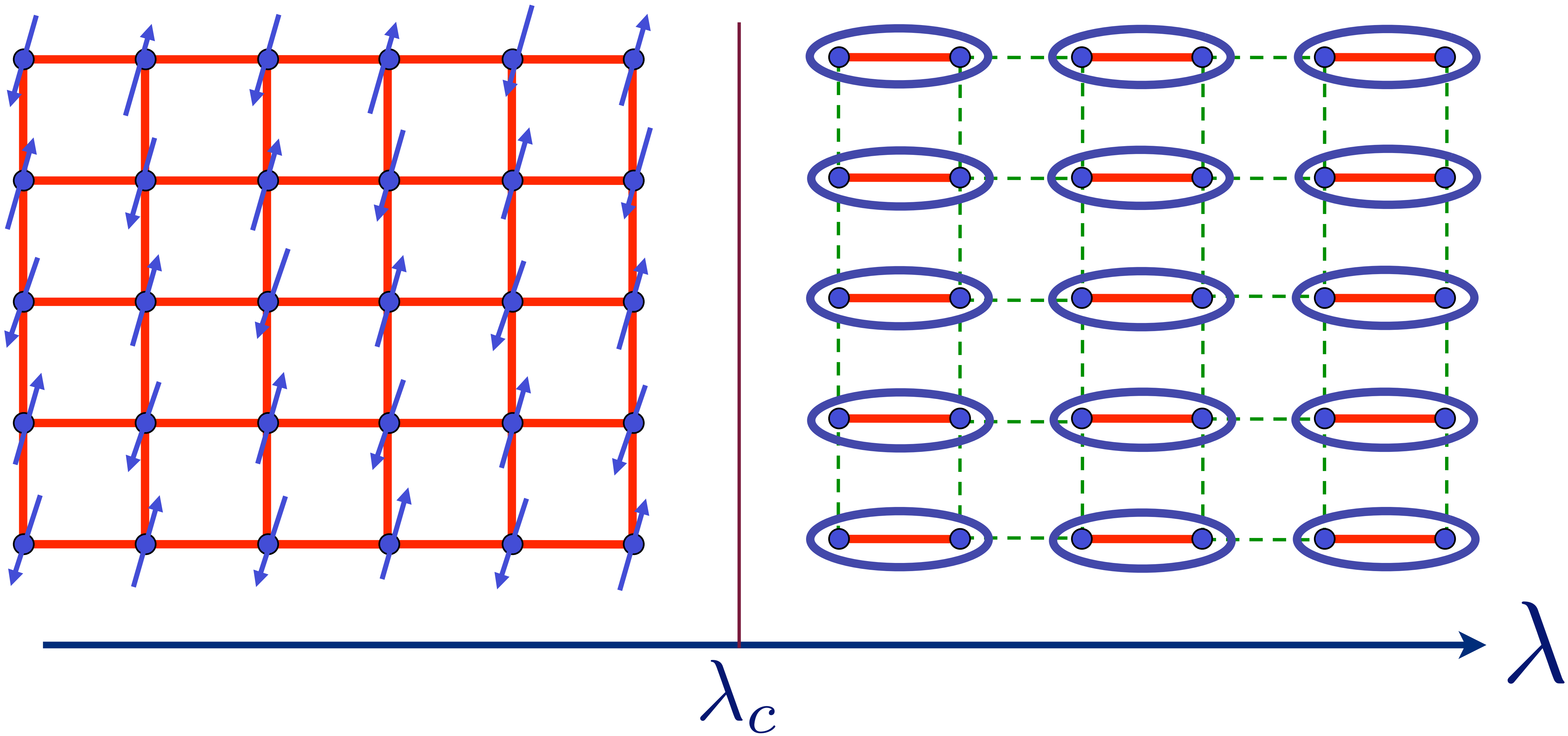
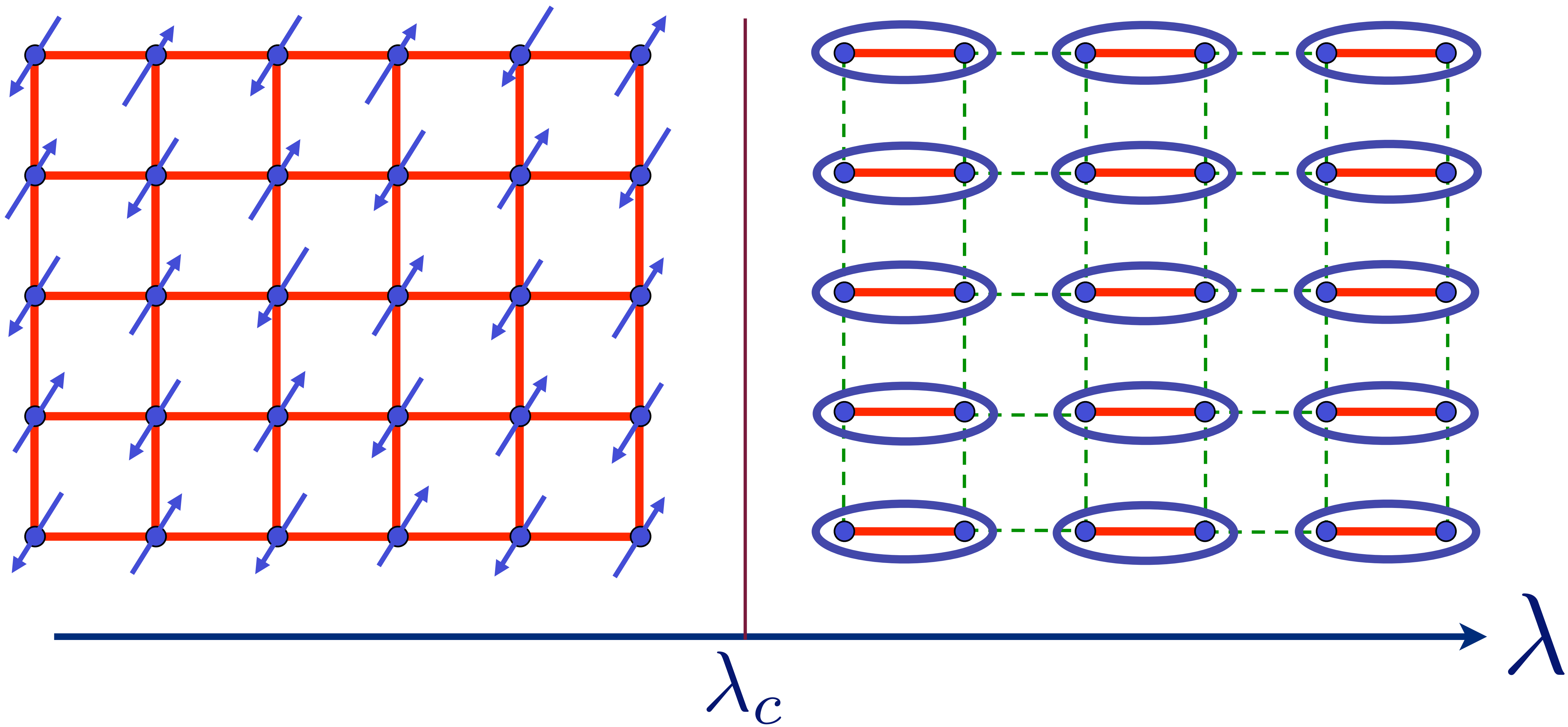


Spin waves

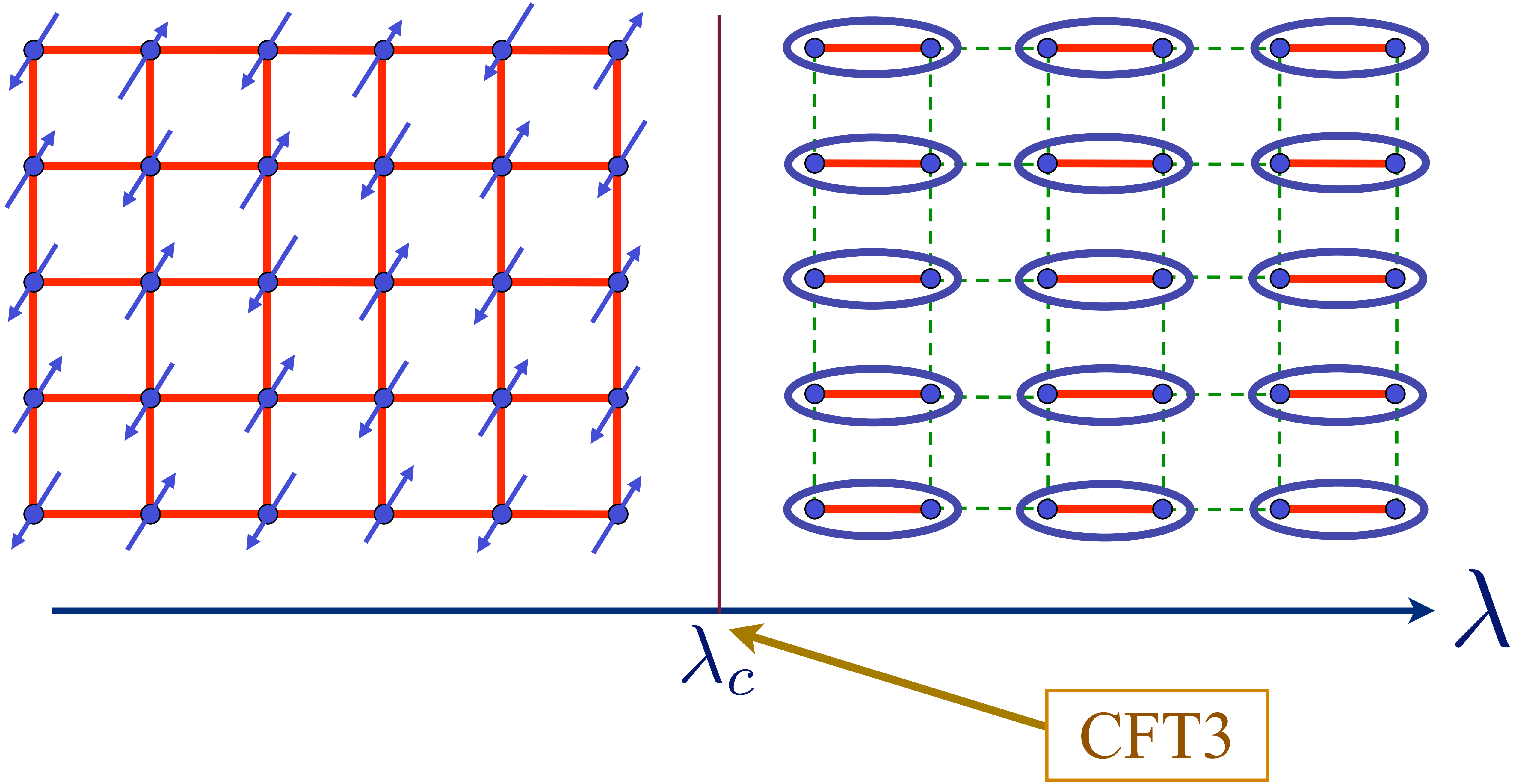
Excitation spectrum in the Néel phase



Spin waves

Derivation of field theory of critical point

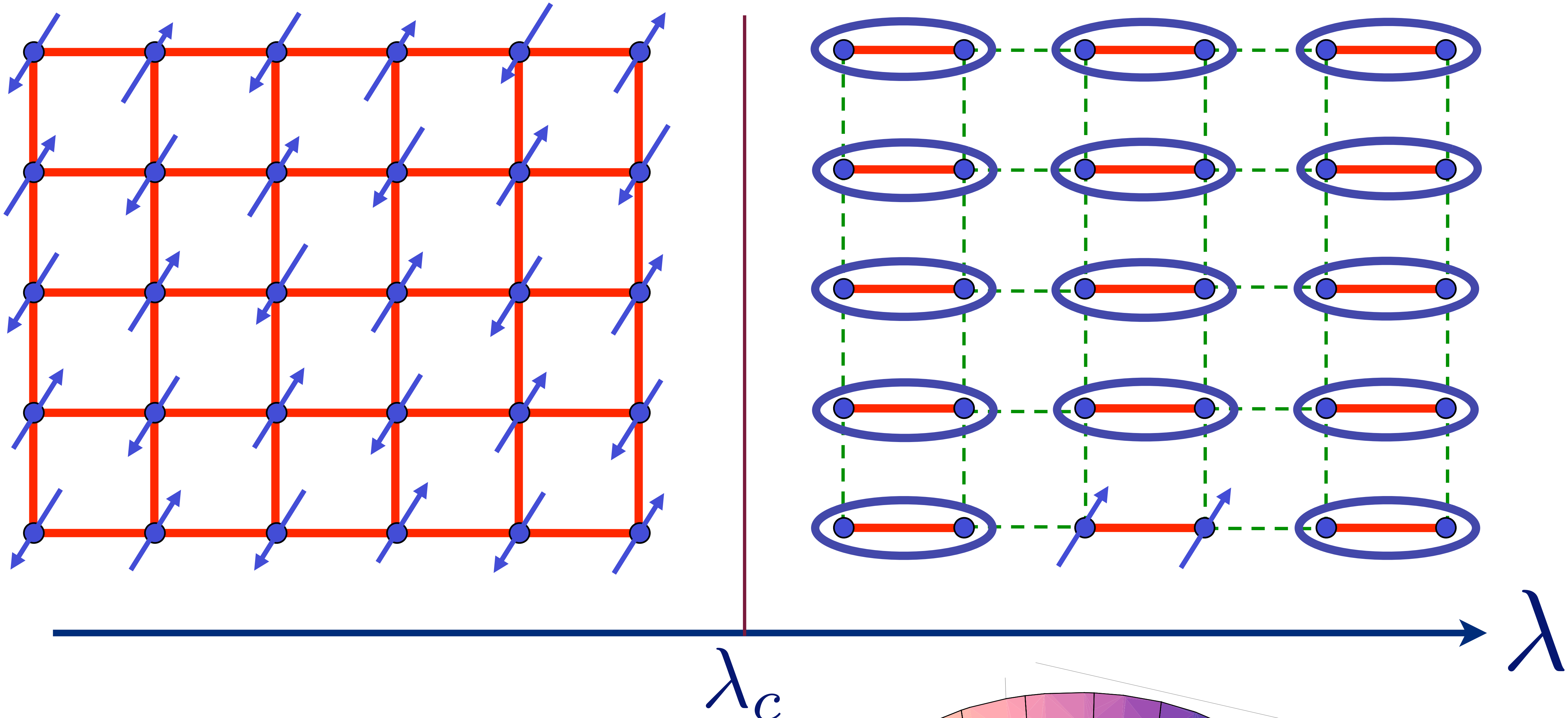
Description using Landau-Ginzburg field theory



O(3) order parameter $\vec{\varphi}$

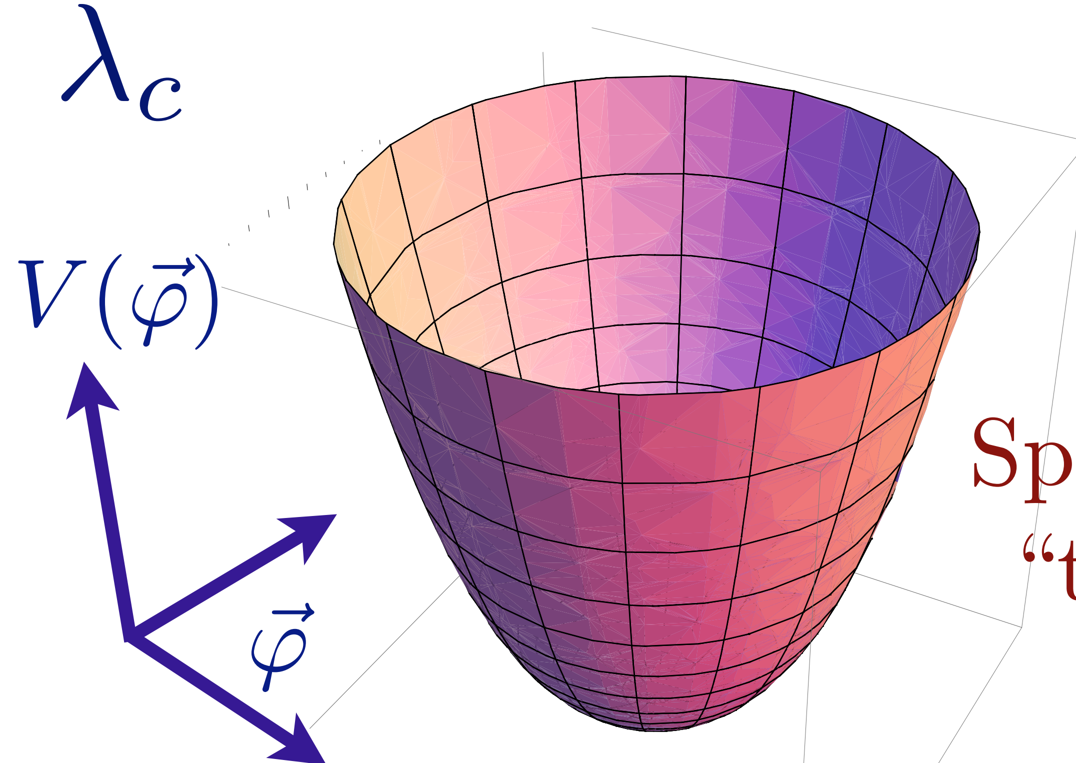
$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$

Excitation spectrum in the paramagnetic phase



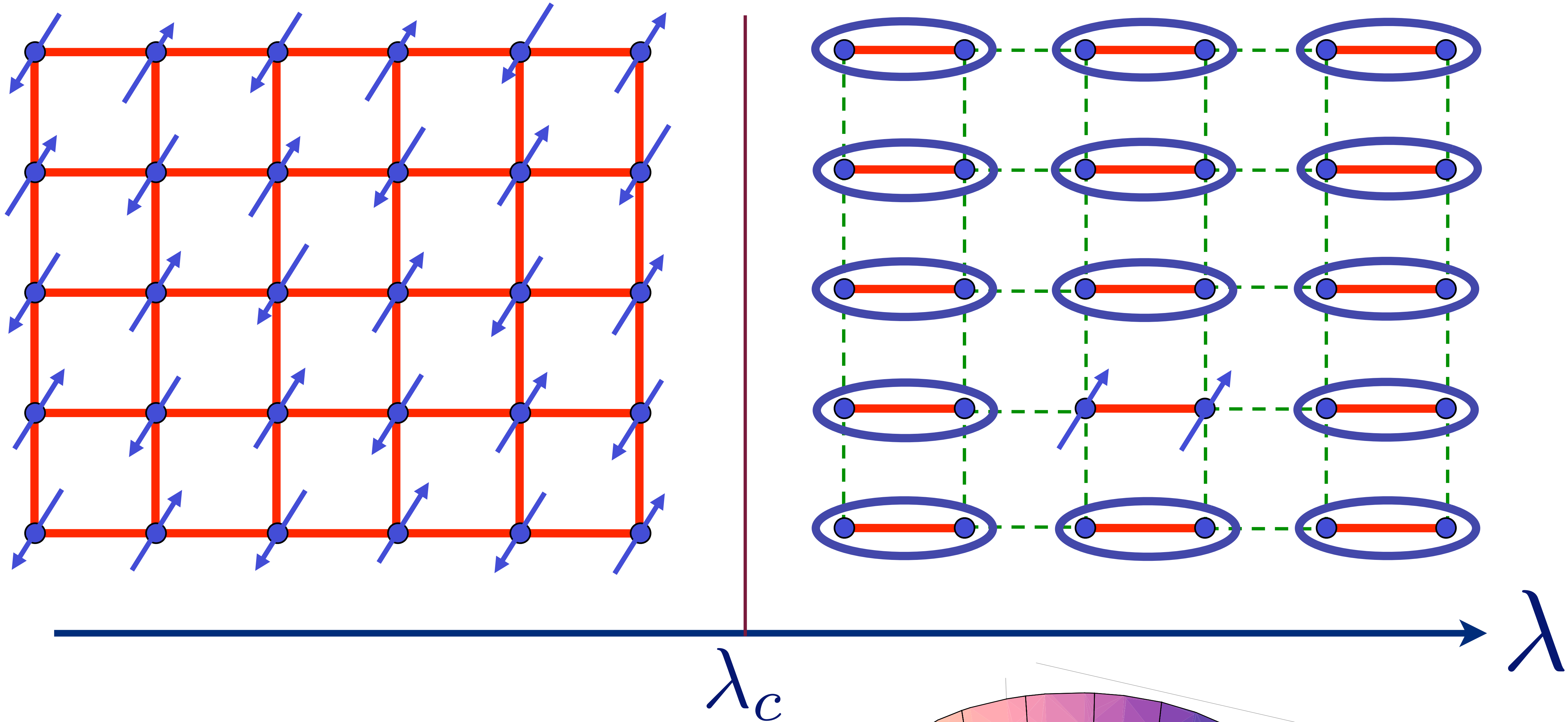
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda > \lambda_c$$



Spin $S = 1$
"triplon"

Excitation spectrum in the paramagnetic phase



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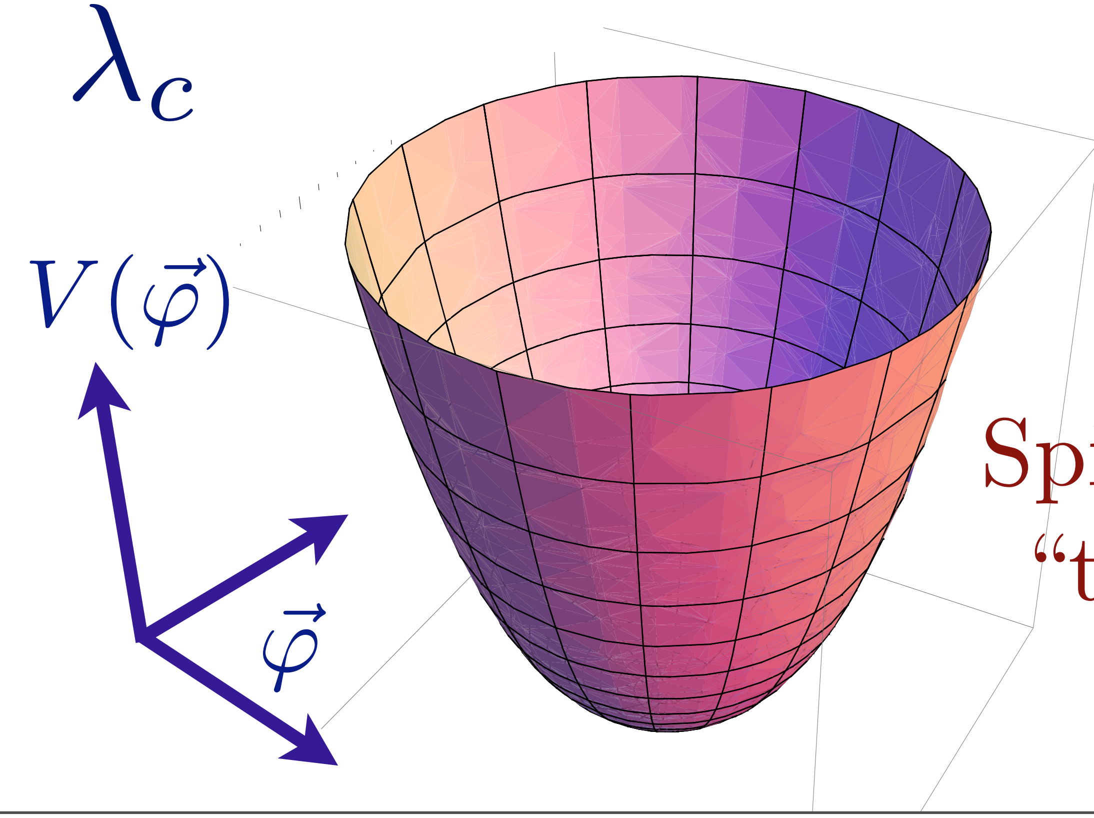

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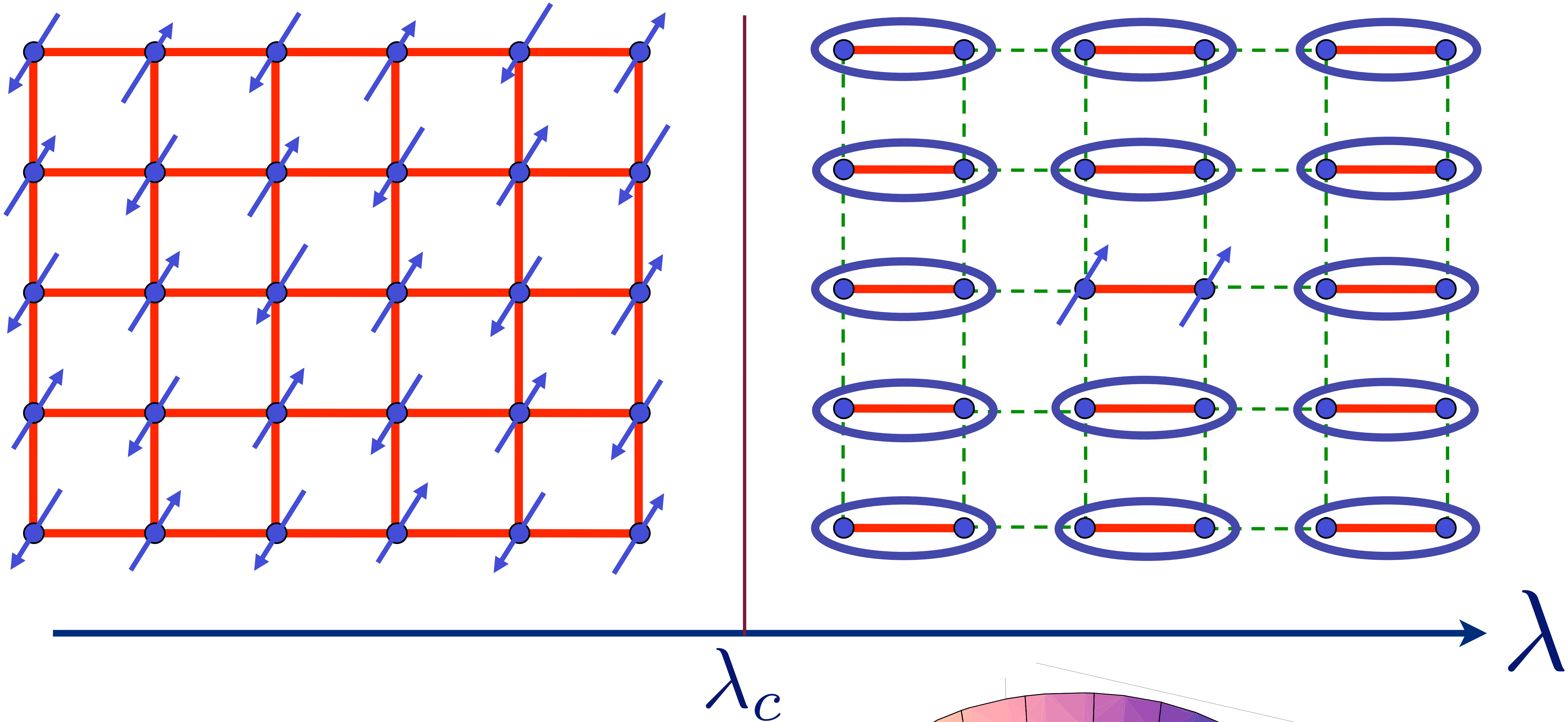

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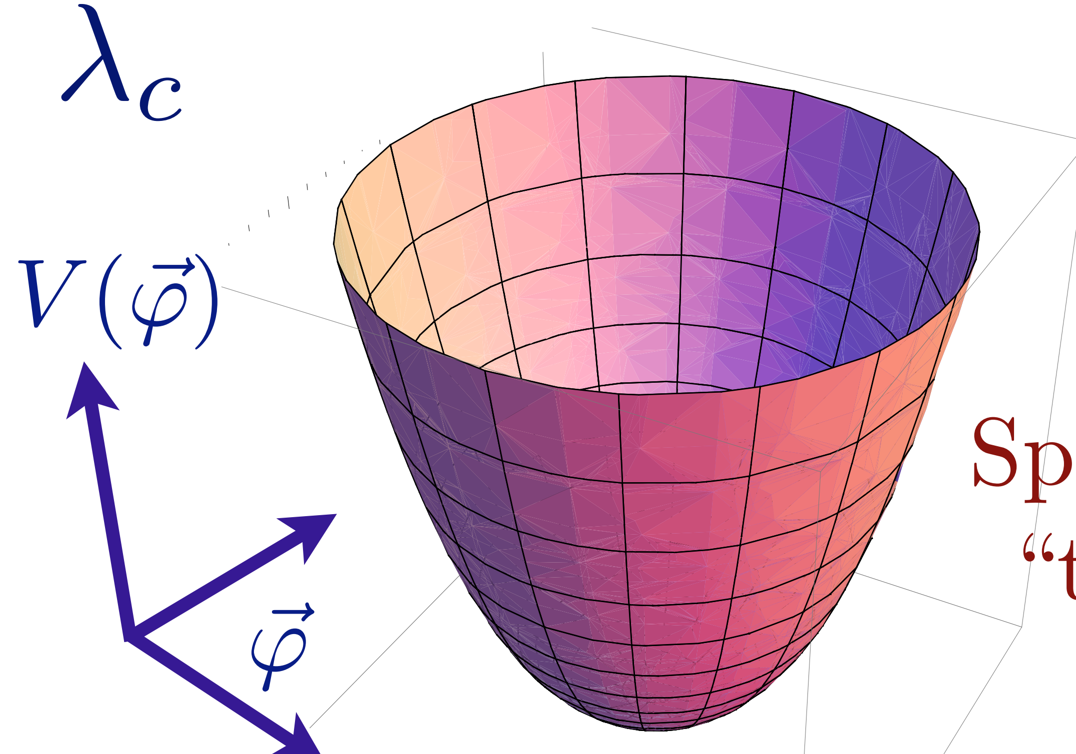

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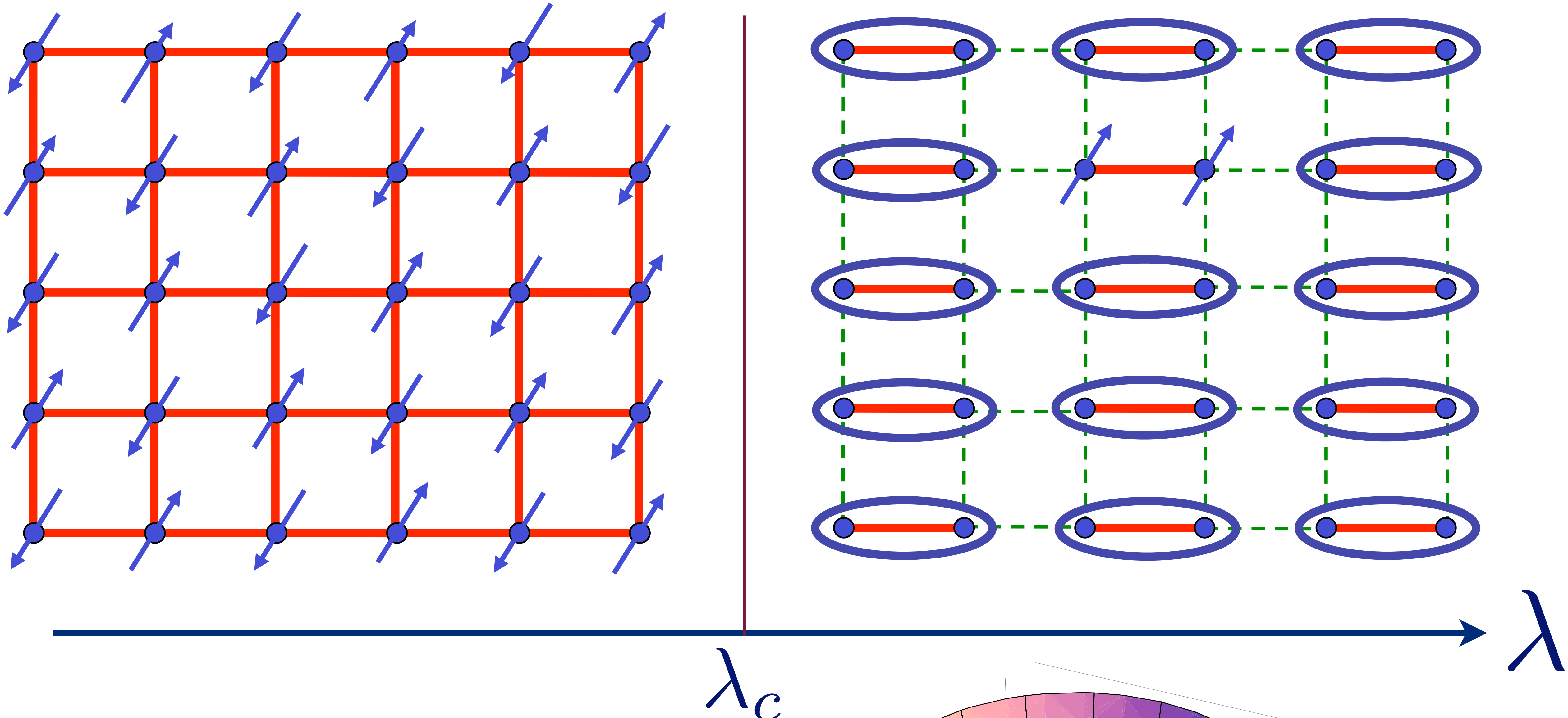

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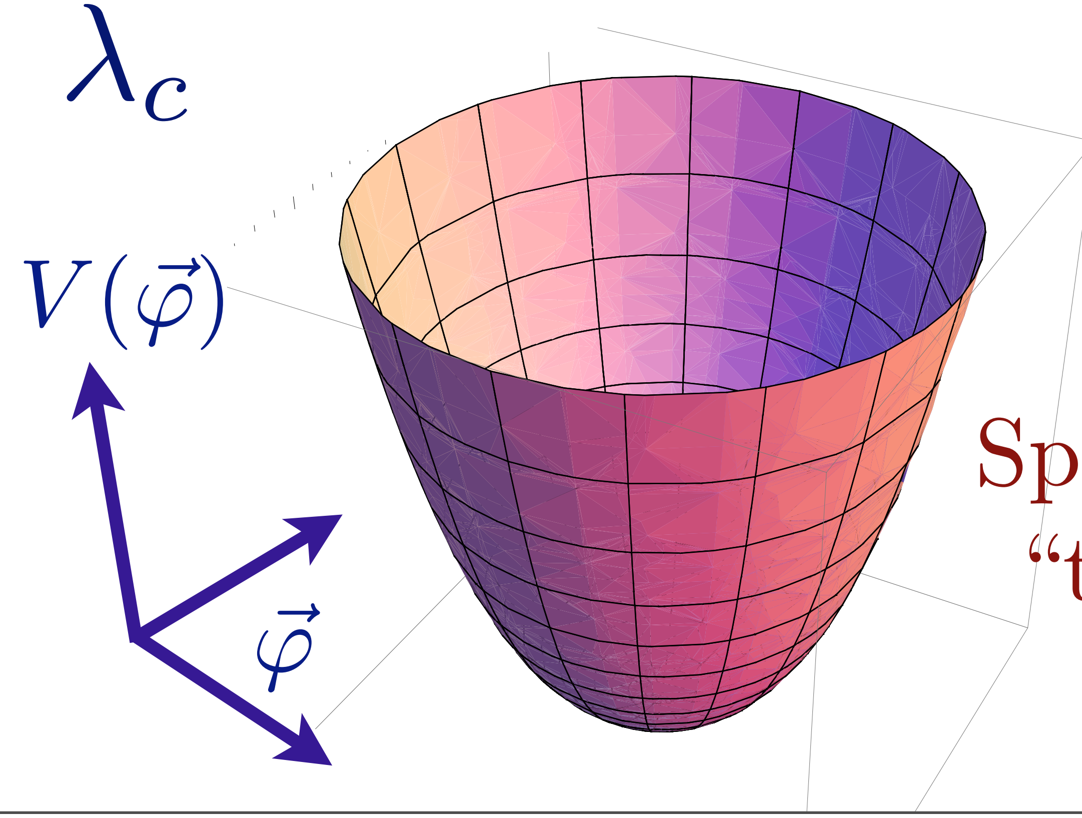

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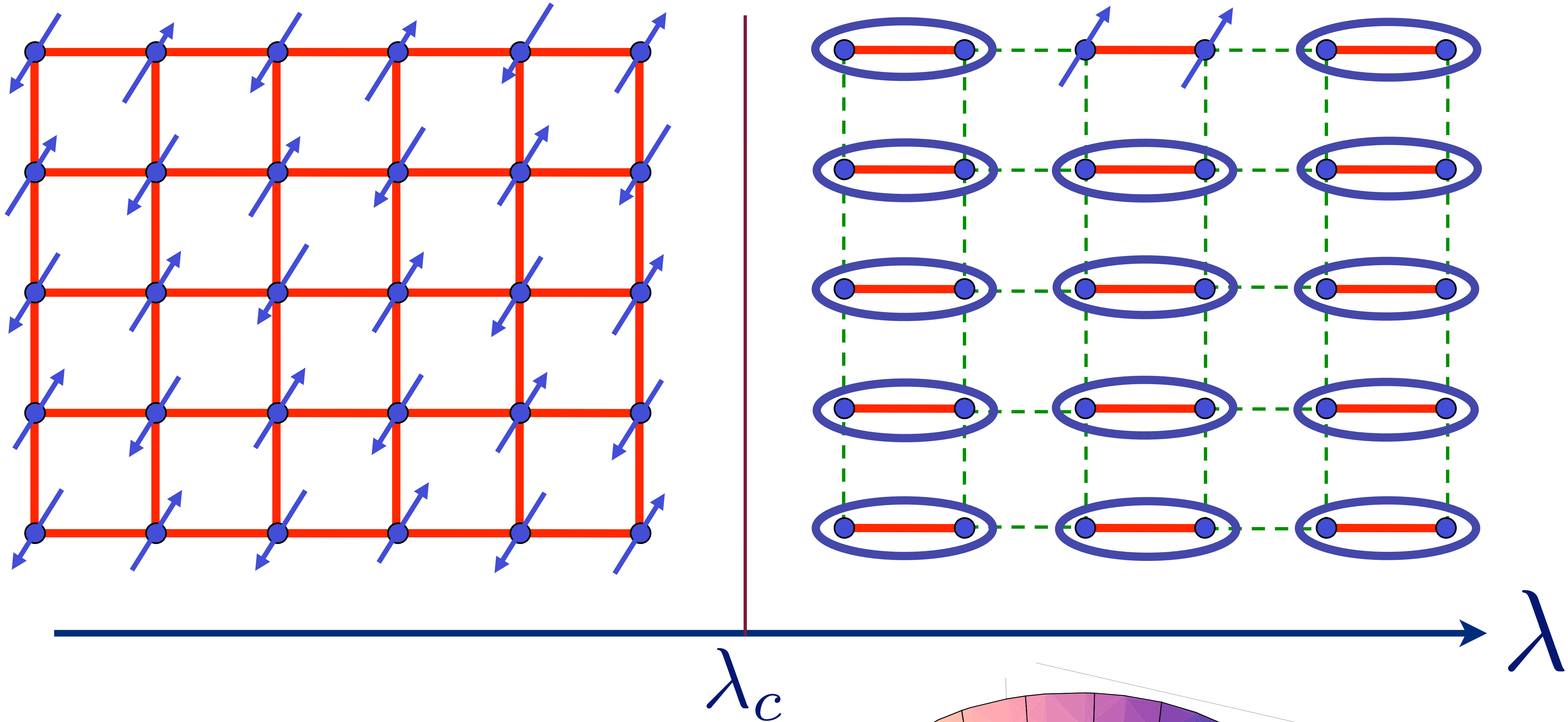

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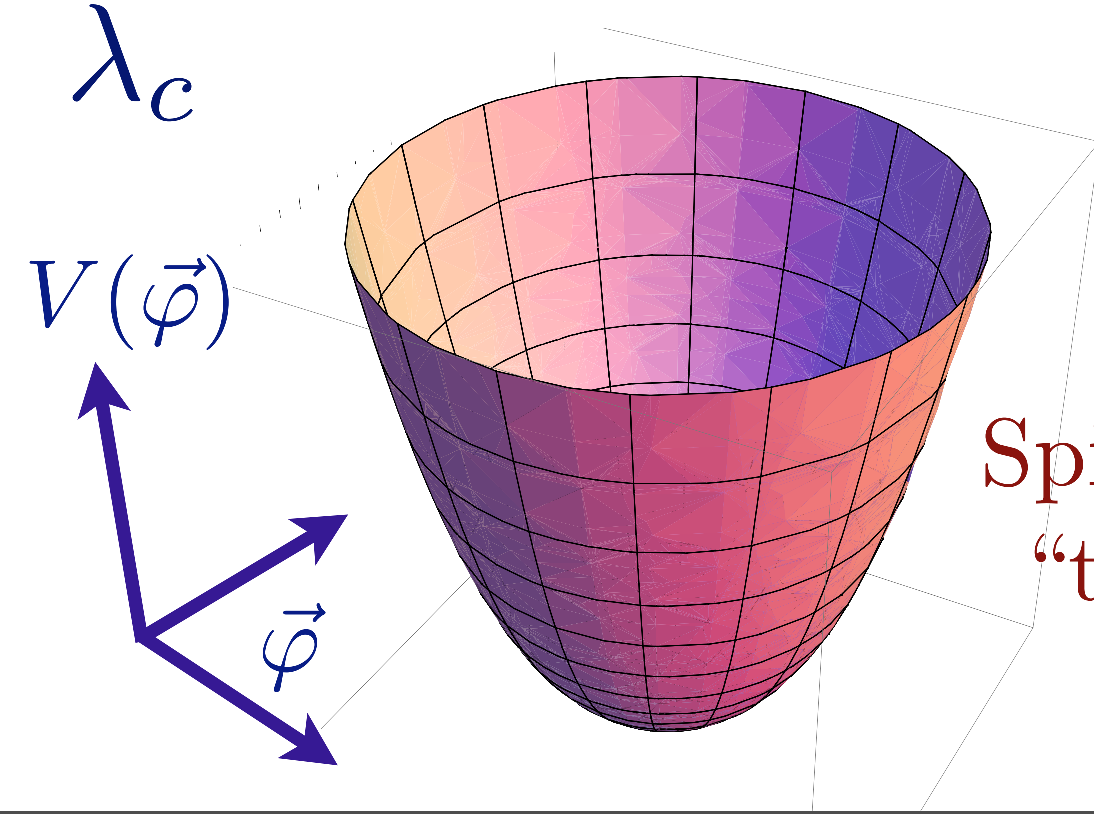

Spin $S = 1$
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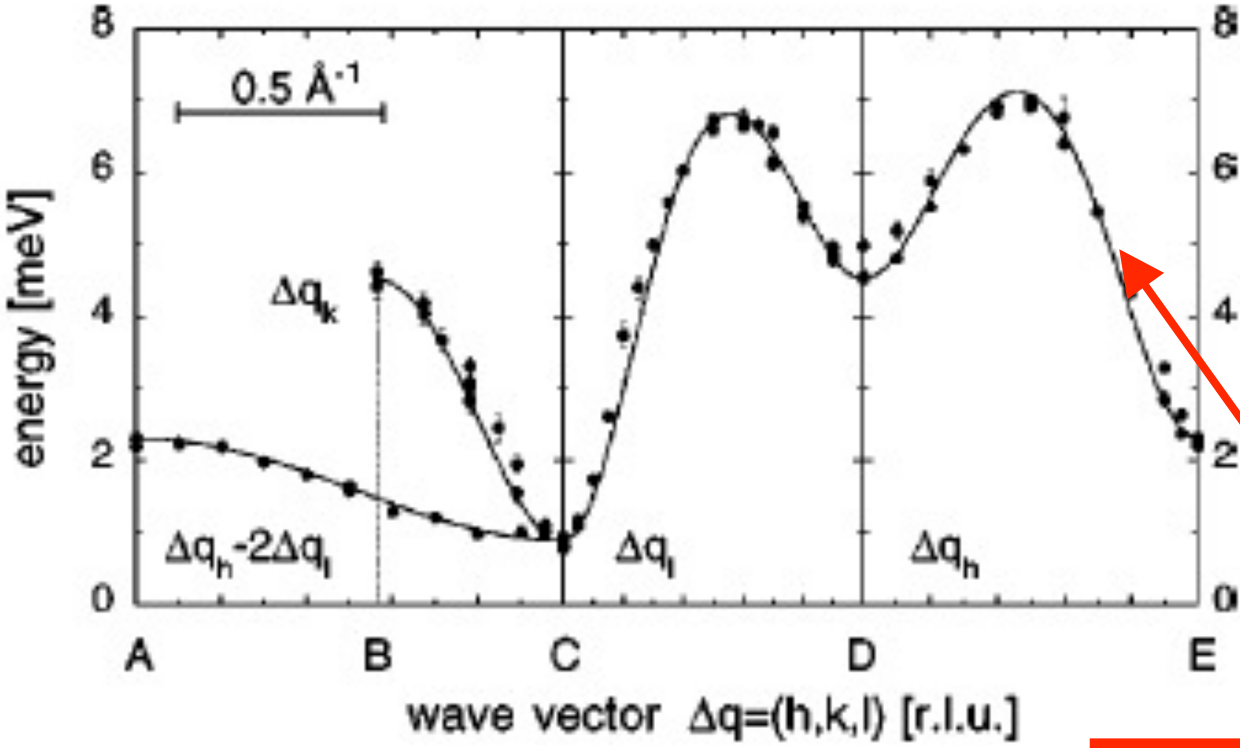
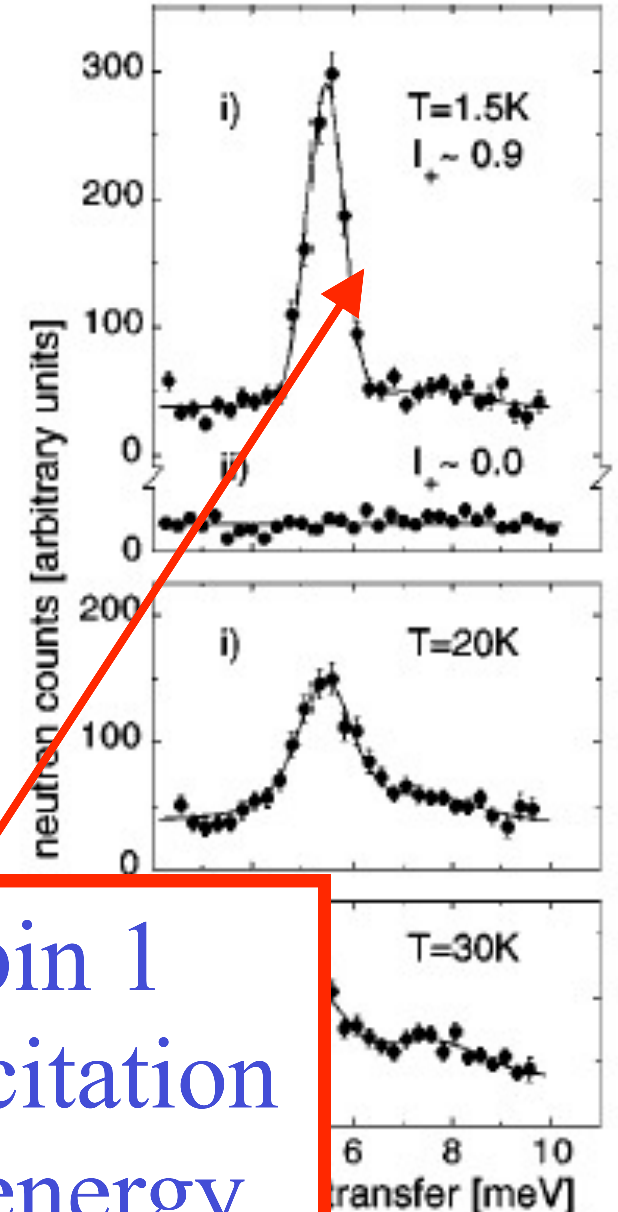
$TlCuCl_3$ at ambient pressure





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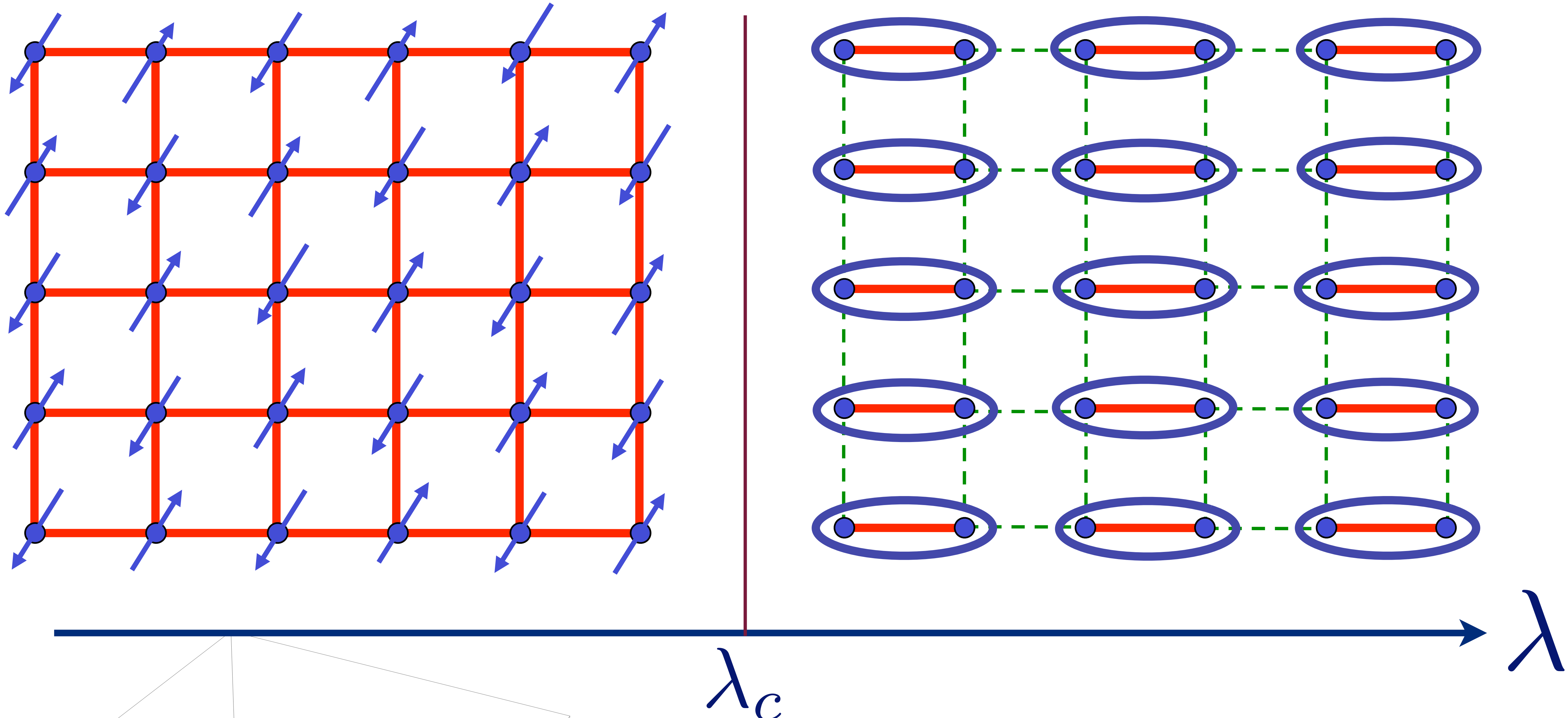

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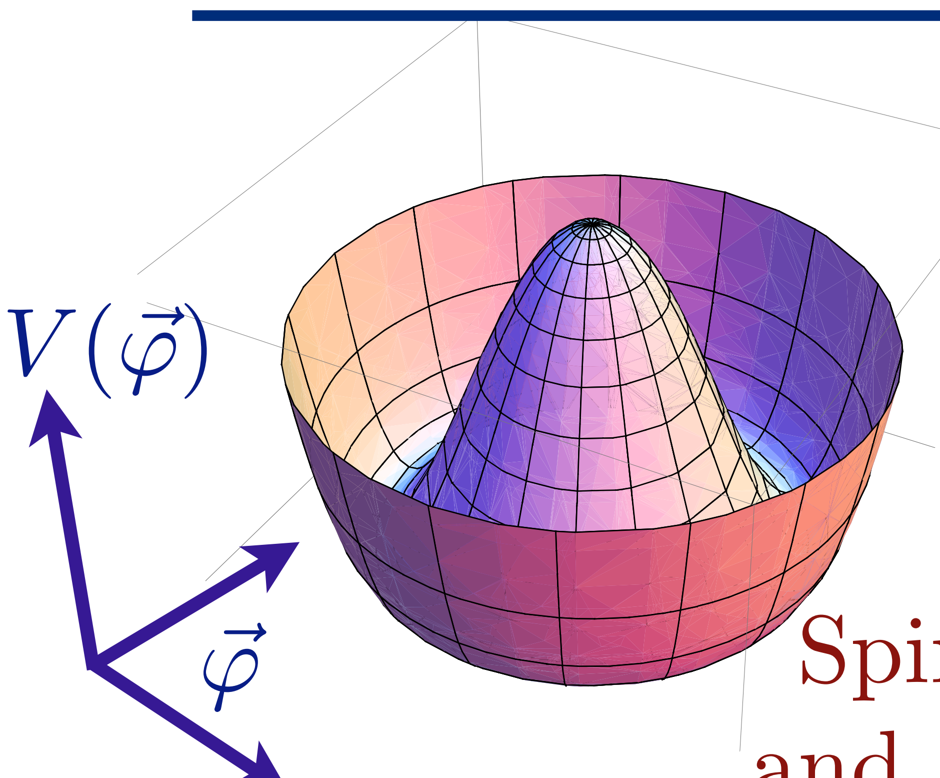
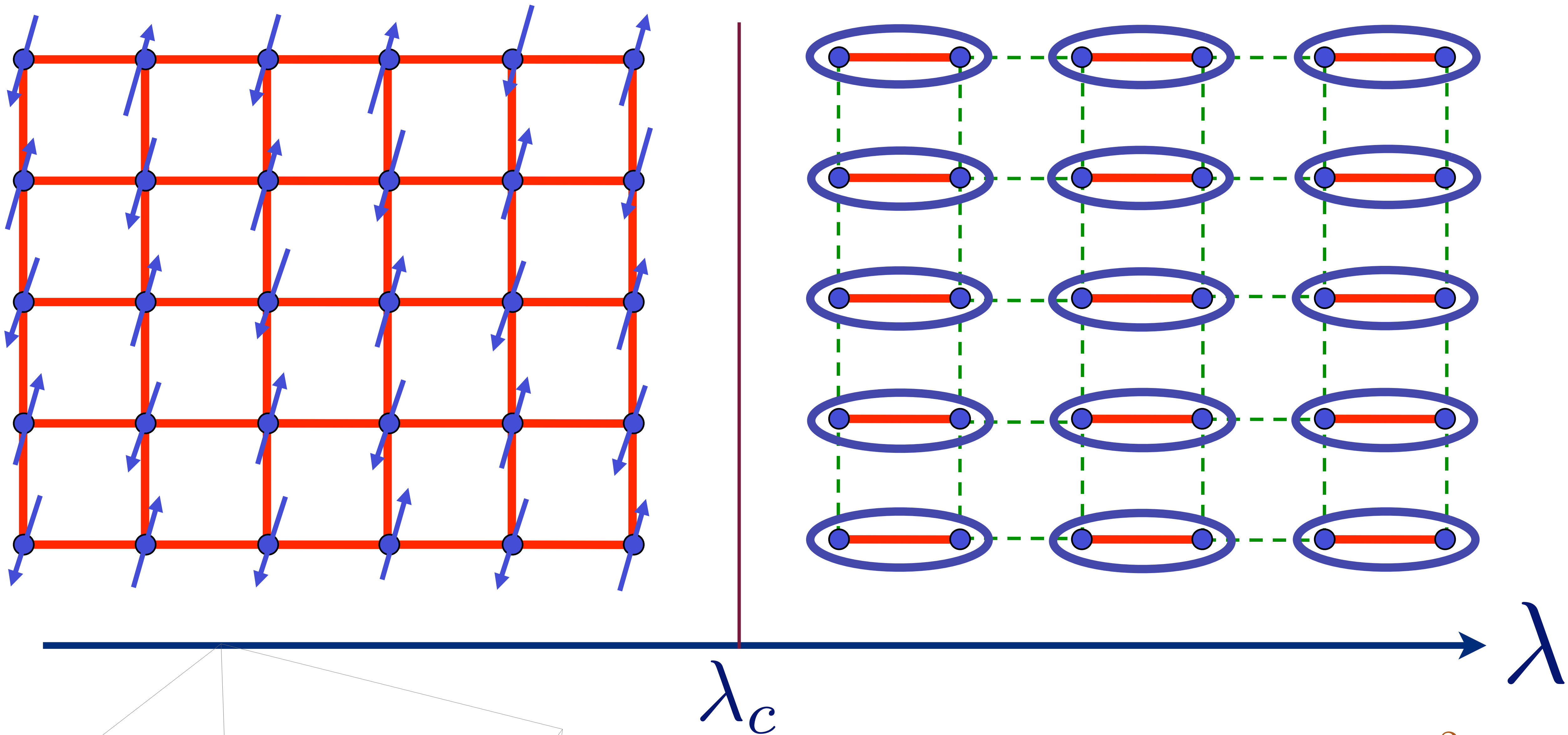
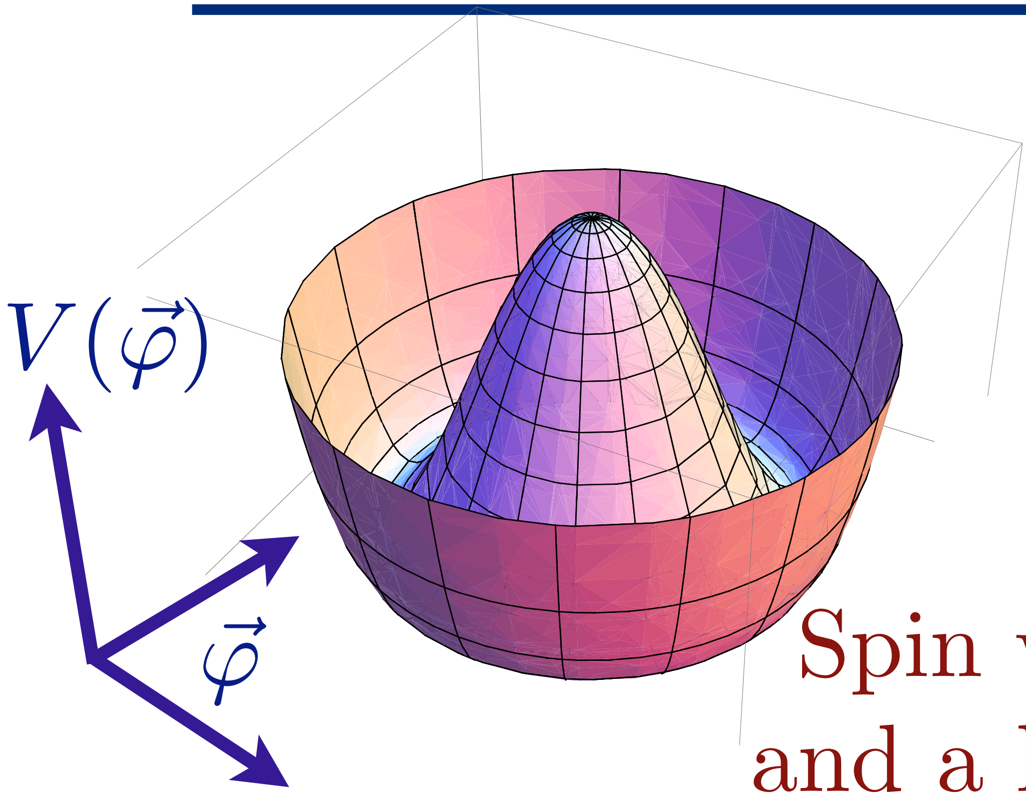


Spin waves ("Goldstone" modes)
and a longitudinal "Higgs" particle

Excitation spectrum in the Néel phase





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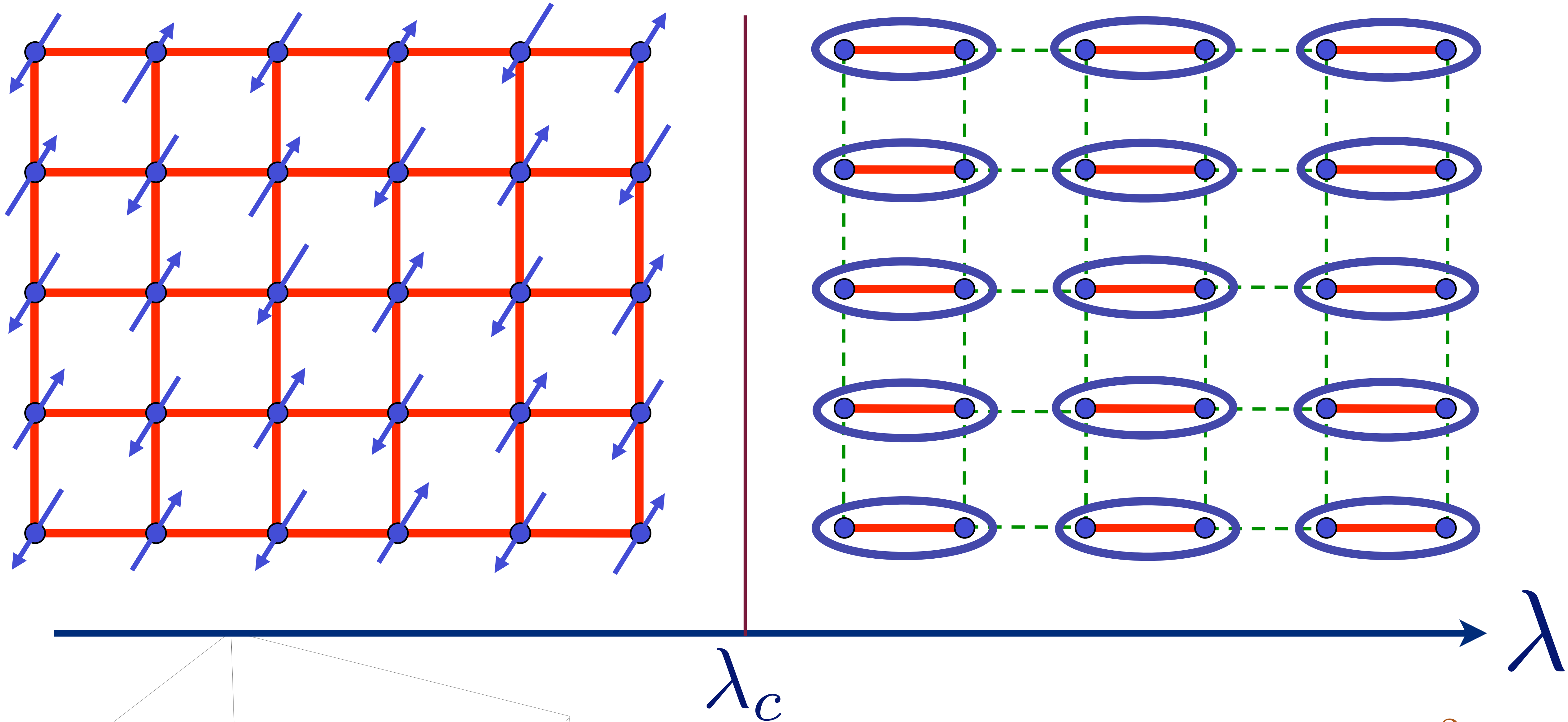

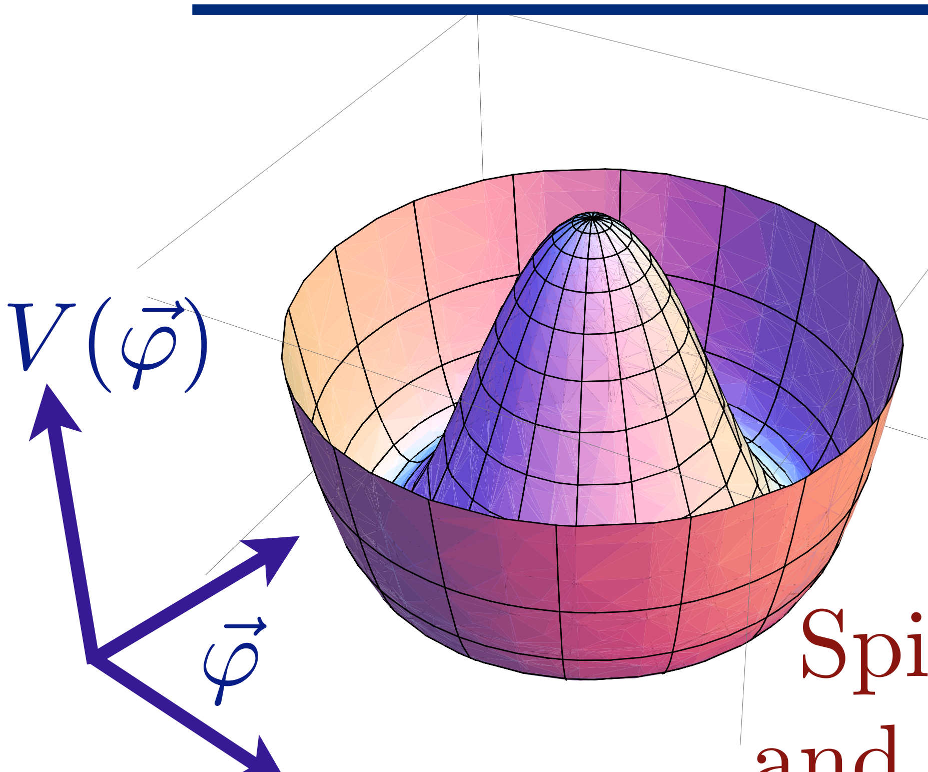

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Spin waves (“Goldstone” modes)
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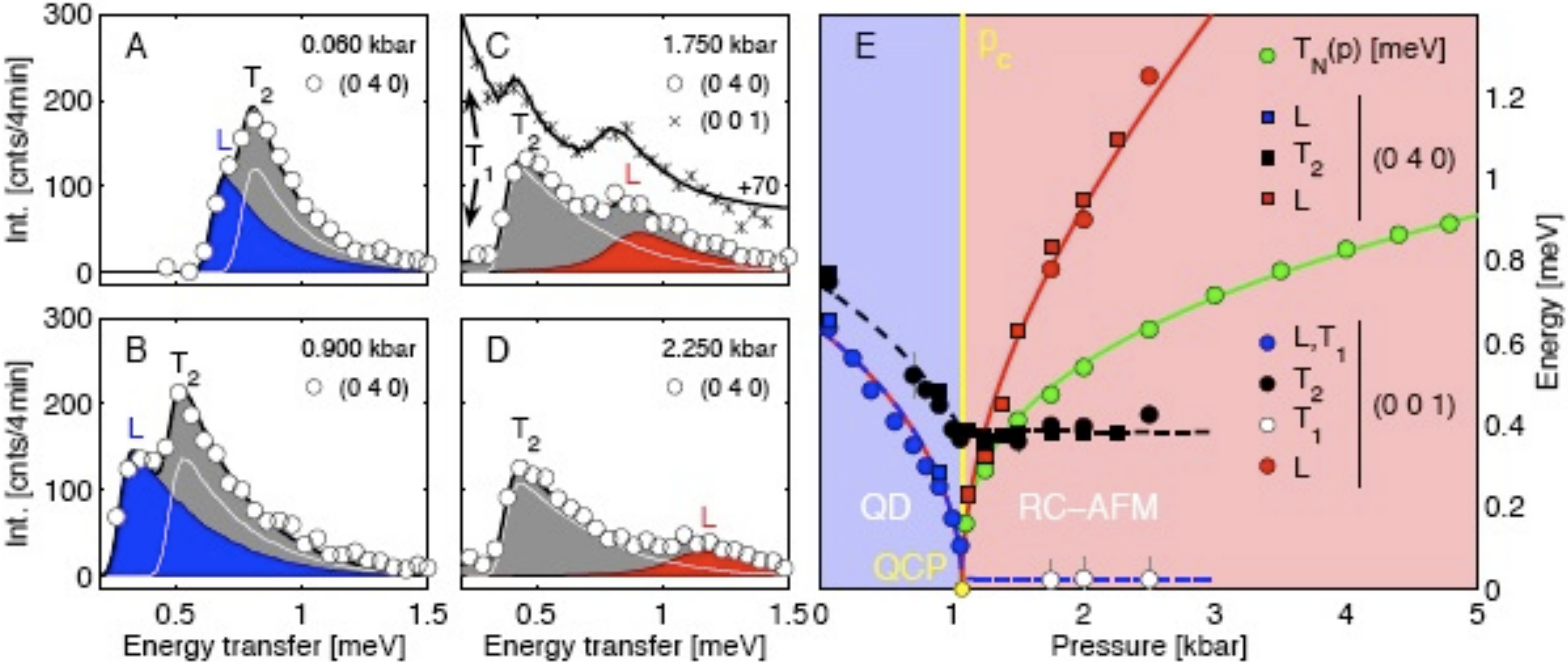
$TlCuCl_3$ with varying pressure



Observation of 3 → 2 low energy modes, emergence of new longitudinal mode (the "Higgs boson") in Néel phase, and vanishing of Néel temperature at quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\vec{\varphi} = 0$:

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

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$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

Néel phase, $\lambda < \lambda_c$

Expand $\vec{\varphi} = \left(0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}\right) + \vec{\varphi}_1$:

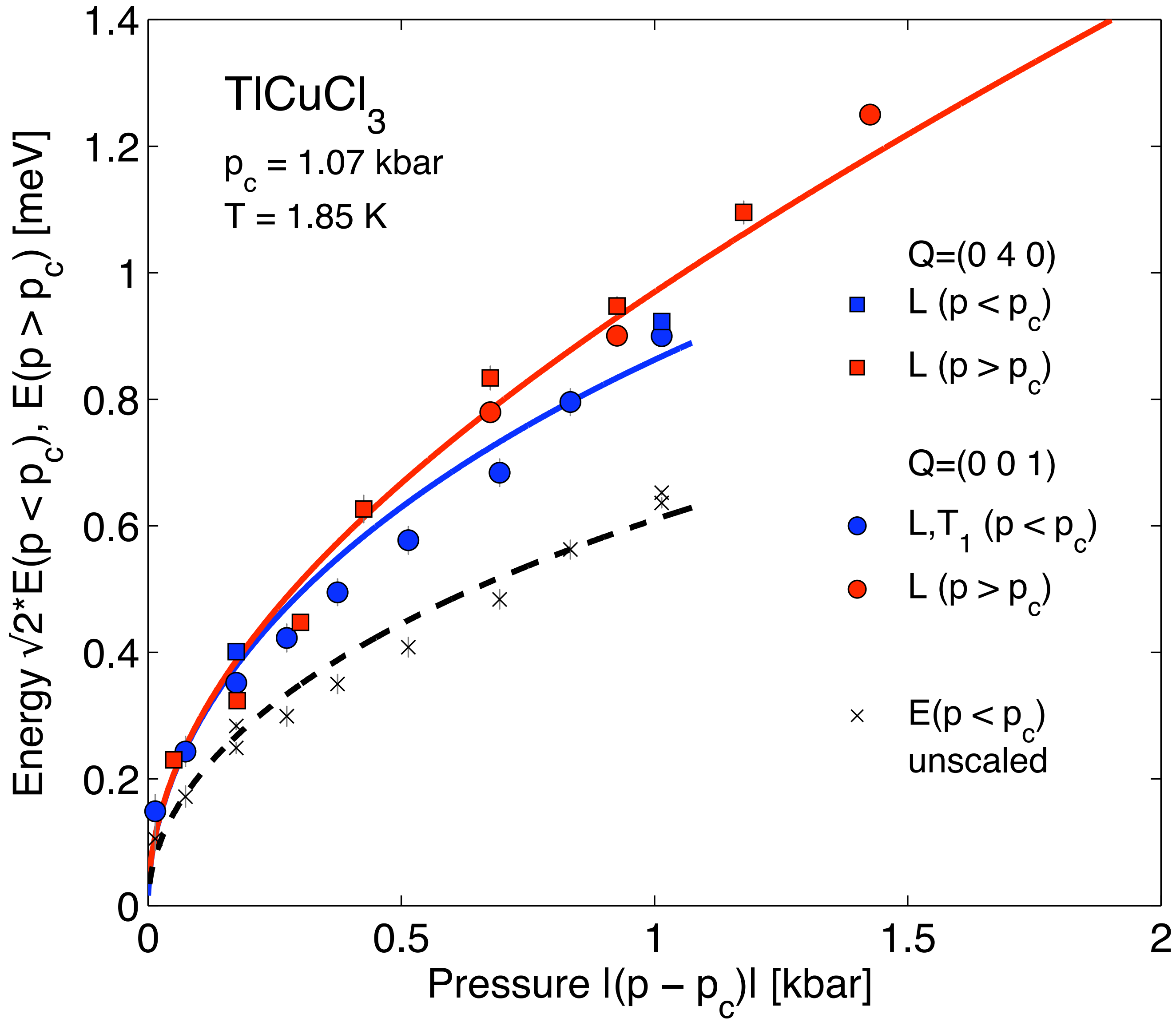
$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

Yields 2 gapless spin waves and one Higgs-Englert-Brout particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$

Prediction of quantum field theory

$$\frac{\text{Energy of Higgs-Englert-Brout particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$



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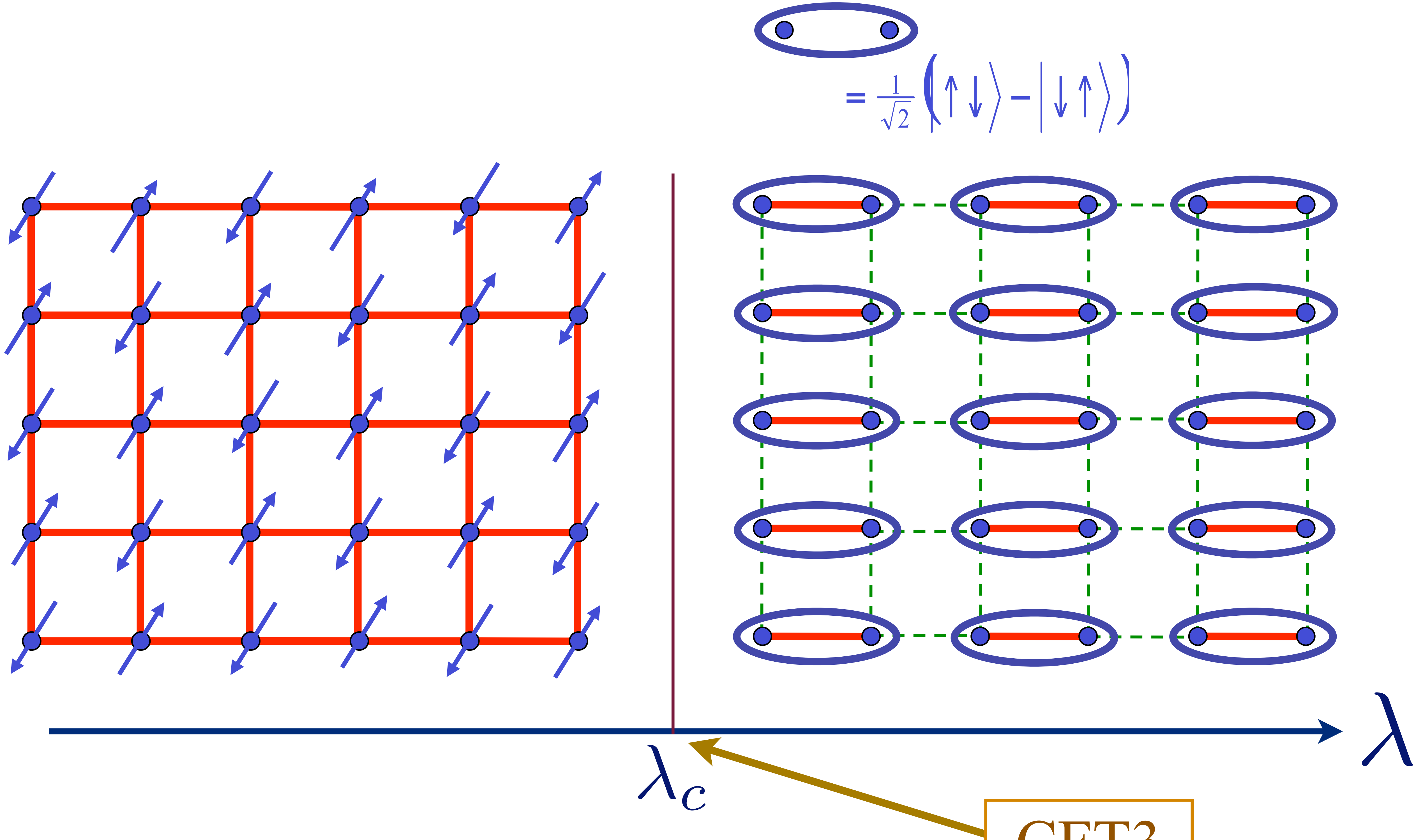

O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2 \right]$$

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν, β/ν, and η. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c. The bottom group are results for the plaquette model. Numbers in [...] brackets denote the χ^2/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
$1.9096 - \sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
$1.9096 + \sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107^d	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
$1.8230 - \sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
$1.8230 + \sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

[a] $L > 12$.
[b] $L > 16$.
[c] $L > 20$.
[d] Previous best estimate of Ref. 19.

S. Wenzel and W. Janke, *Phys. Rev. B* **79**, 014410 (2009)
M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν, β/ν, and η. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c. The bottom group are results for the plaquette model. Numbers in [...] brackets denote the χ^2/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
$1.9096 - \sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
$1.9096 + \sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107^d	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
$1.8230 - \sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
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Field-theoretic RG of CFT3 E. Vicari *et al.*

S. Wenzel and W. Janke, *Phys. Rev. B* **79**, 014410 (2009)
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Outline

1. Introduction to quantum phase transitions:
 quantum spin systems and relativistic field theories

A. Coupled dimer antiferromagnets and Landau-Ginzburg-Wilson theory

B. Triangular lattice antiferromagnets : Spinons, visons, and doubled Chern-Simons theory

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1. Introduction to quantum phase transitions:
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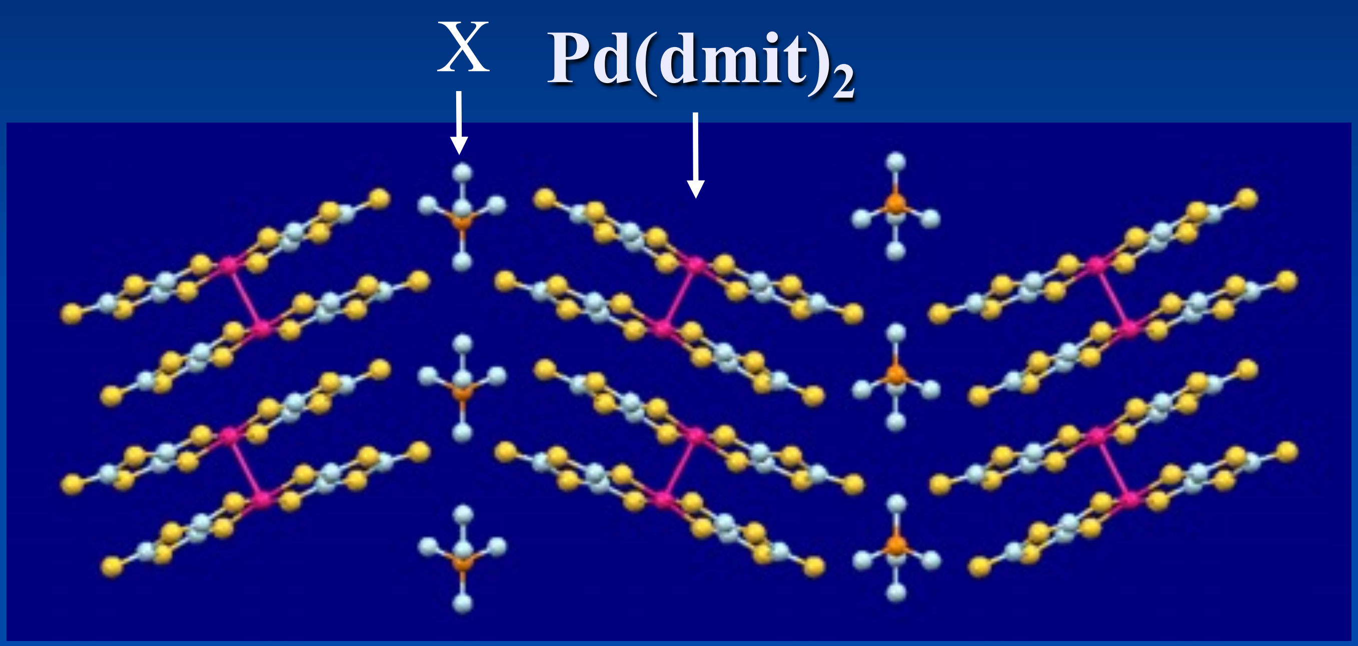
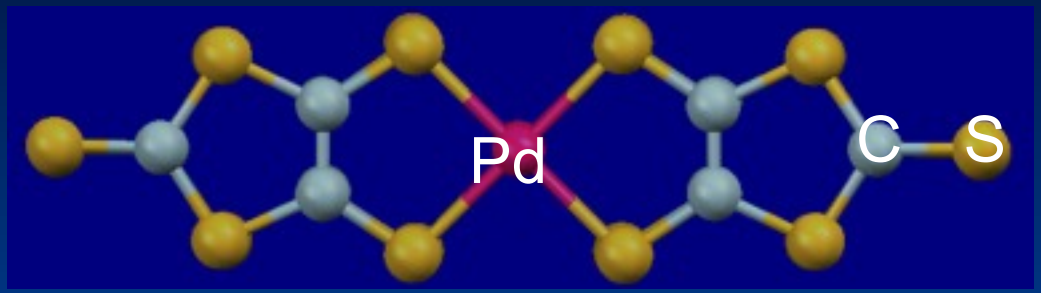
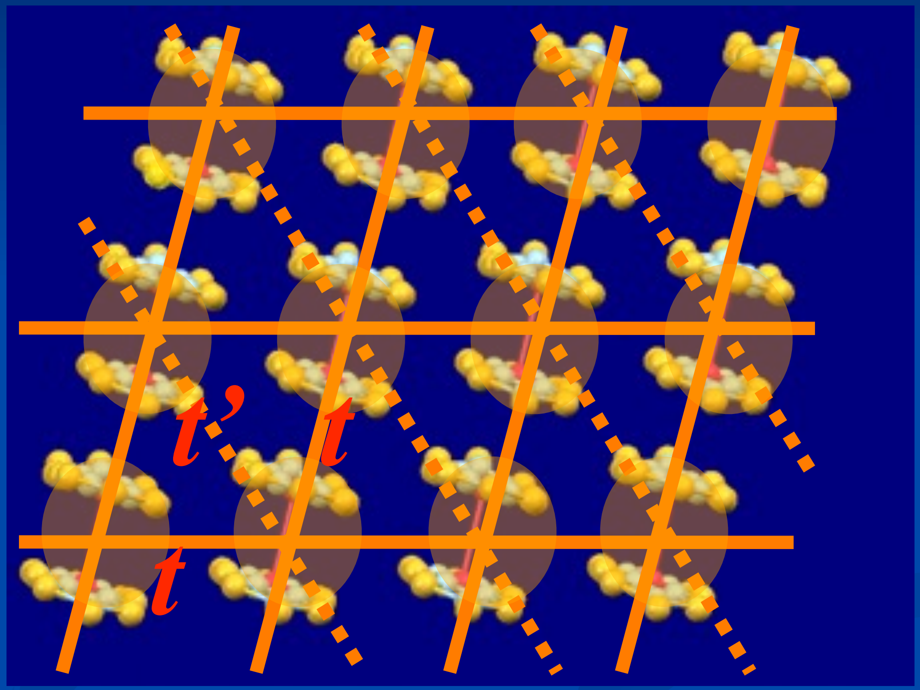
X[Pd(dmit)$_2$]$_2$







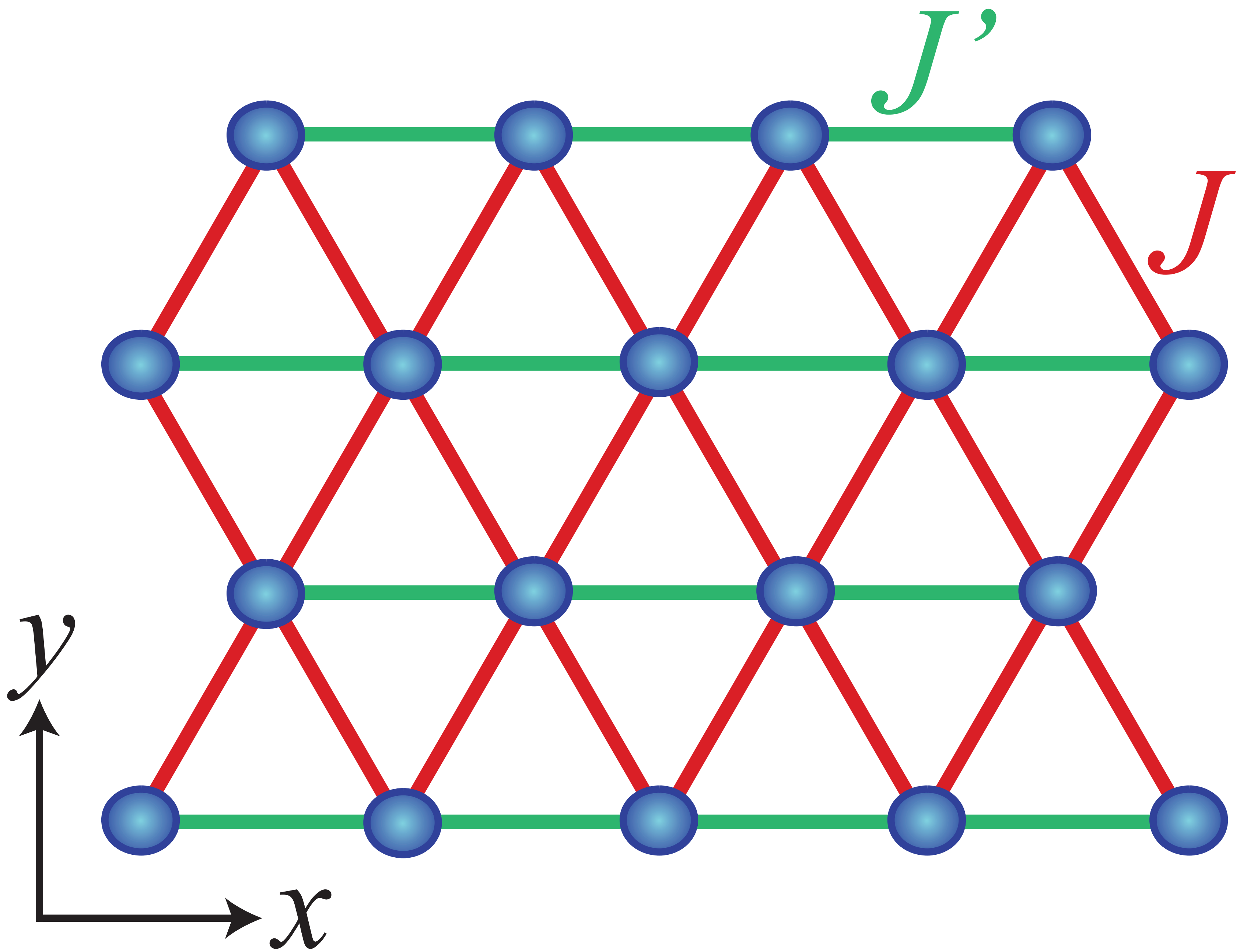
Half-filled band → Mott insulator with spin S = 1/2

Triangular lattice of [Pd(dmit)$_2$]$_2$
→ frustrated quantum spin system

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$



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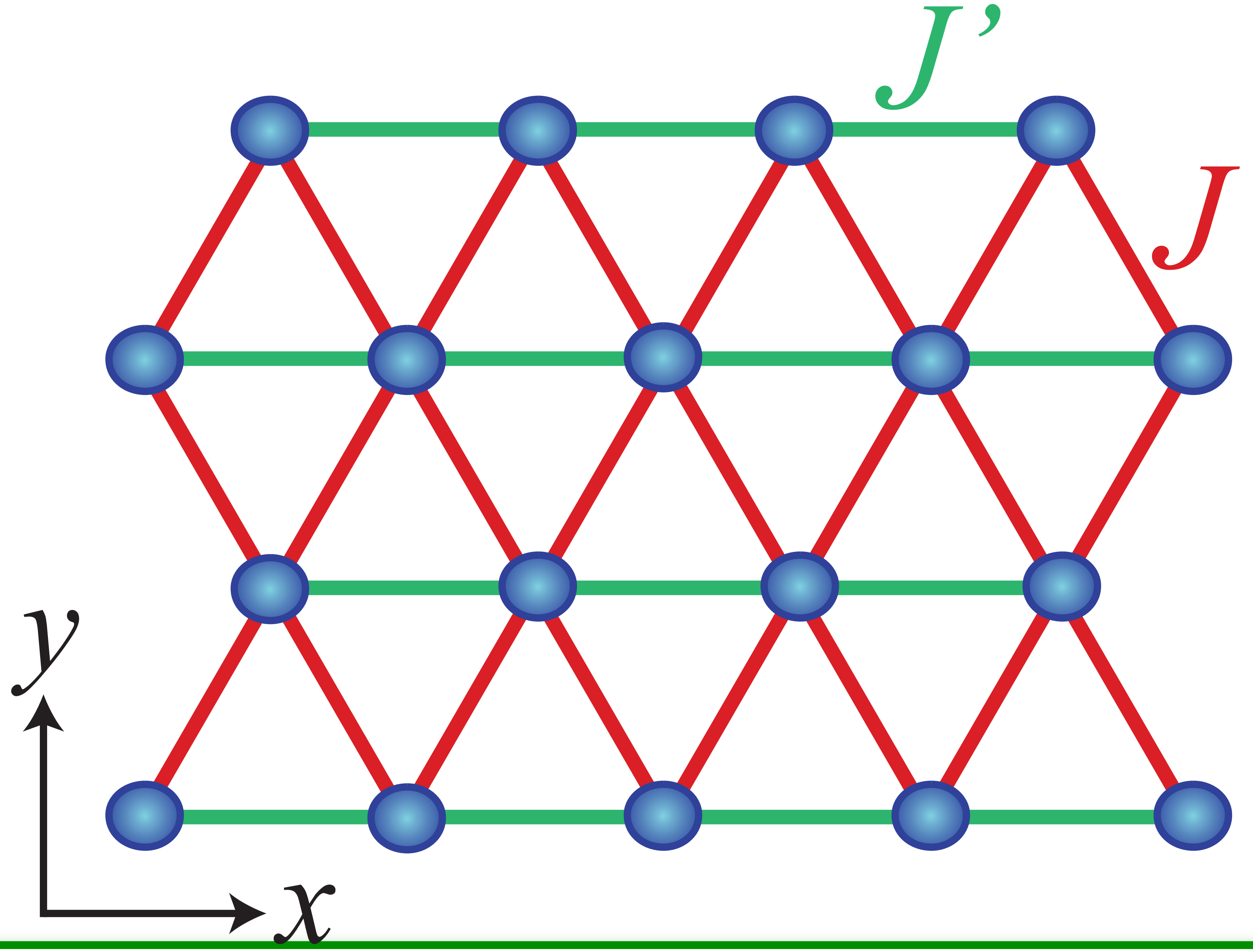
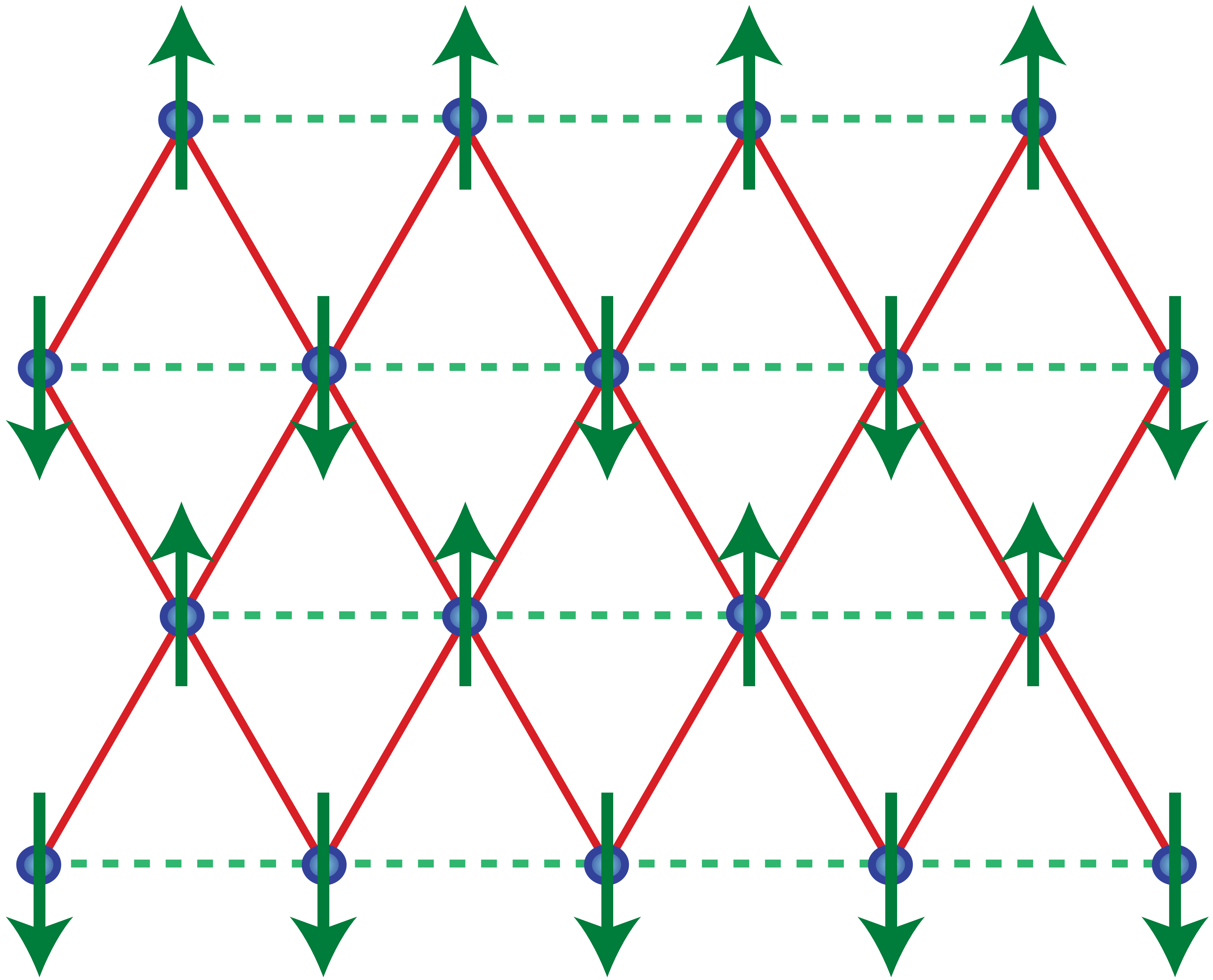


What is the ground state as a function of J'/J ?

Anisotropic triangular lattice antiferromagnet

Broken spin rotation symmetry



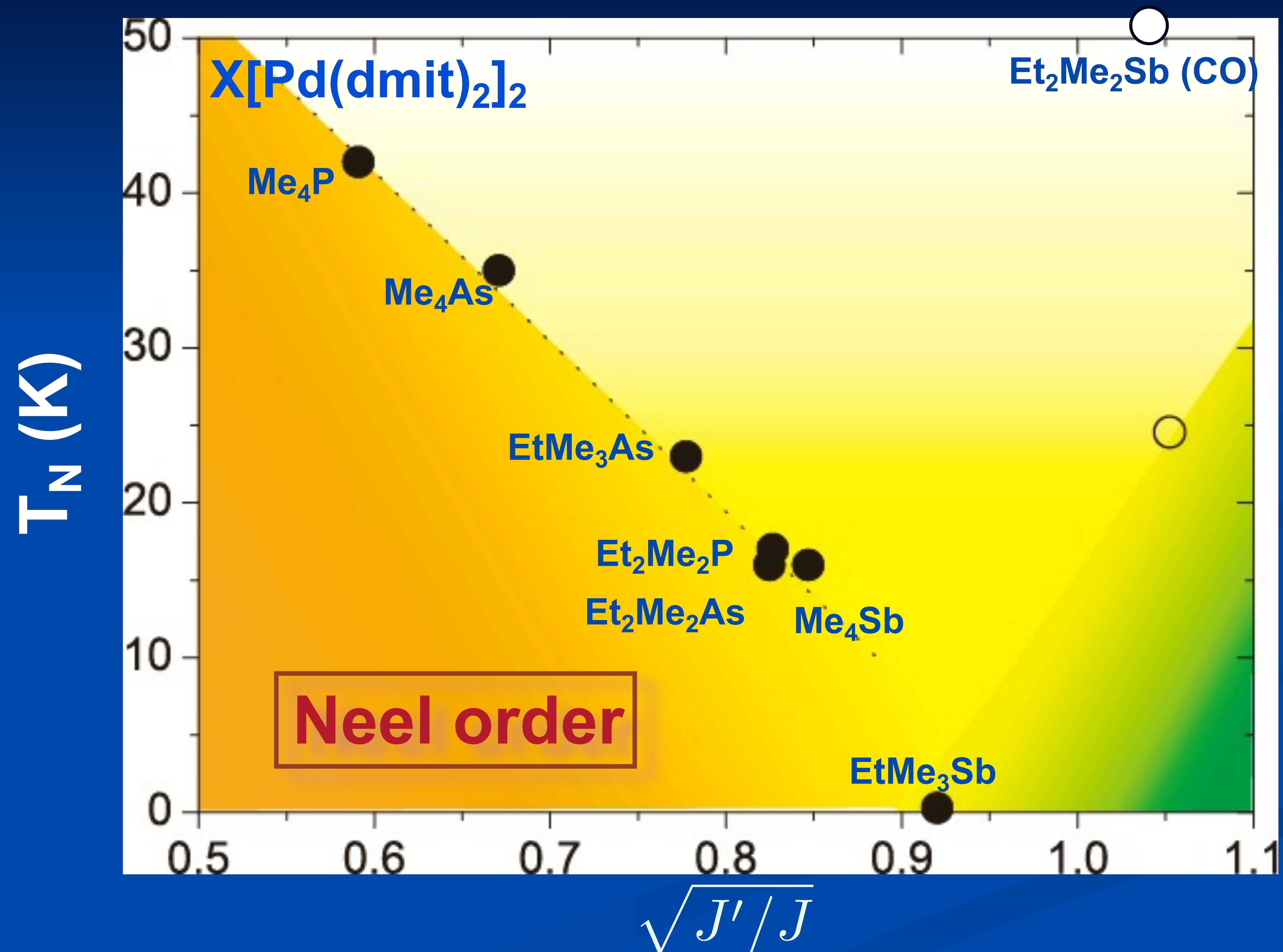
Neel ground state for small J'/J

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

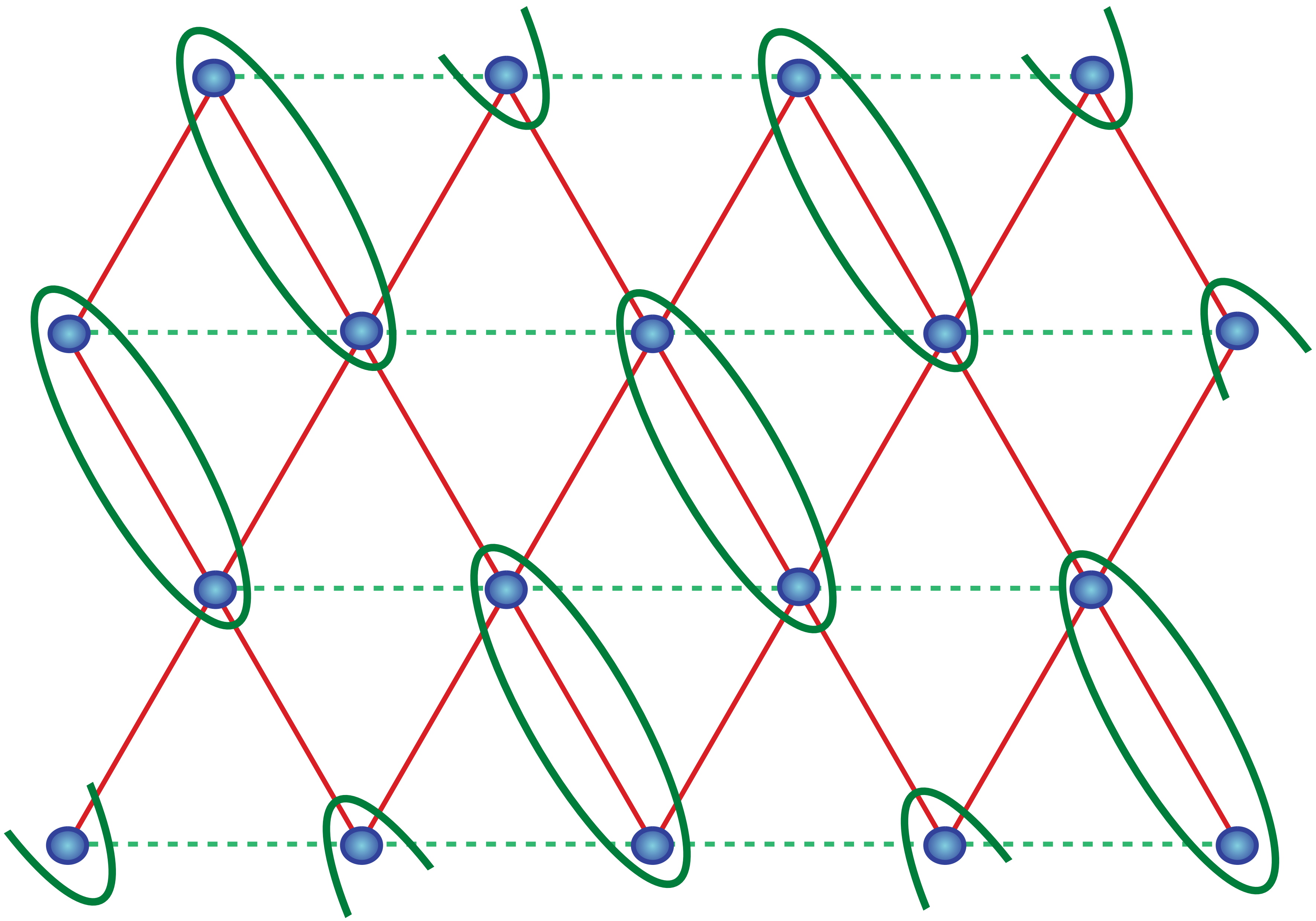
- Néel antiferromagnetic LRO

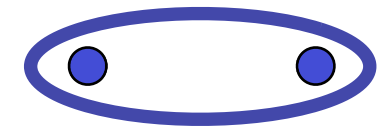
Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Anisotropic triangular lattice antiferromagnet





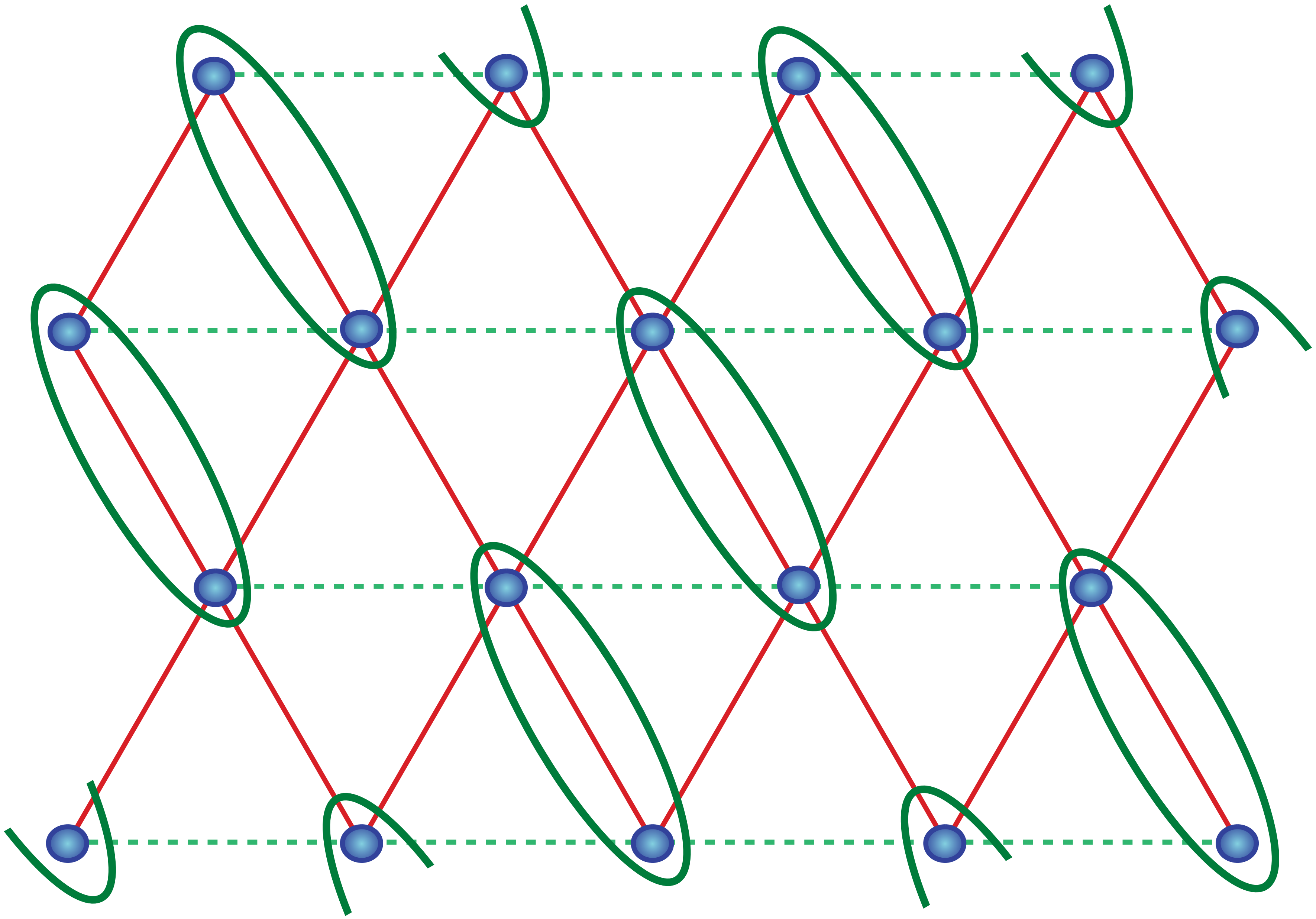
$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

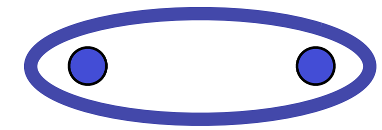
Possible ground state for intermediate *J'/J*

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989)

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry





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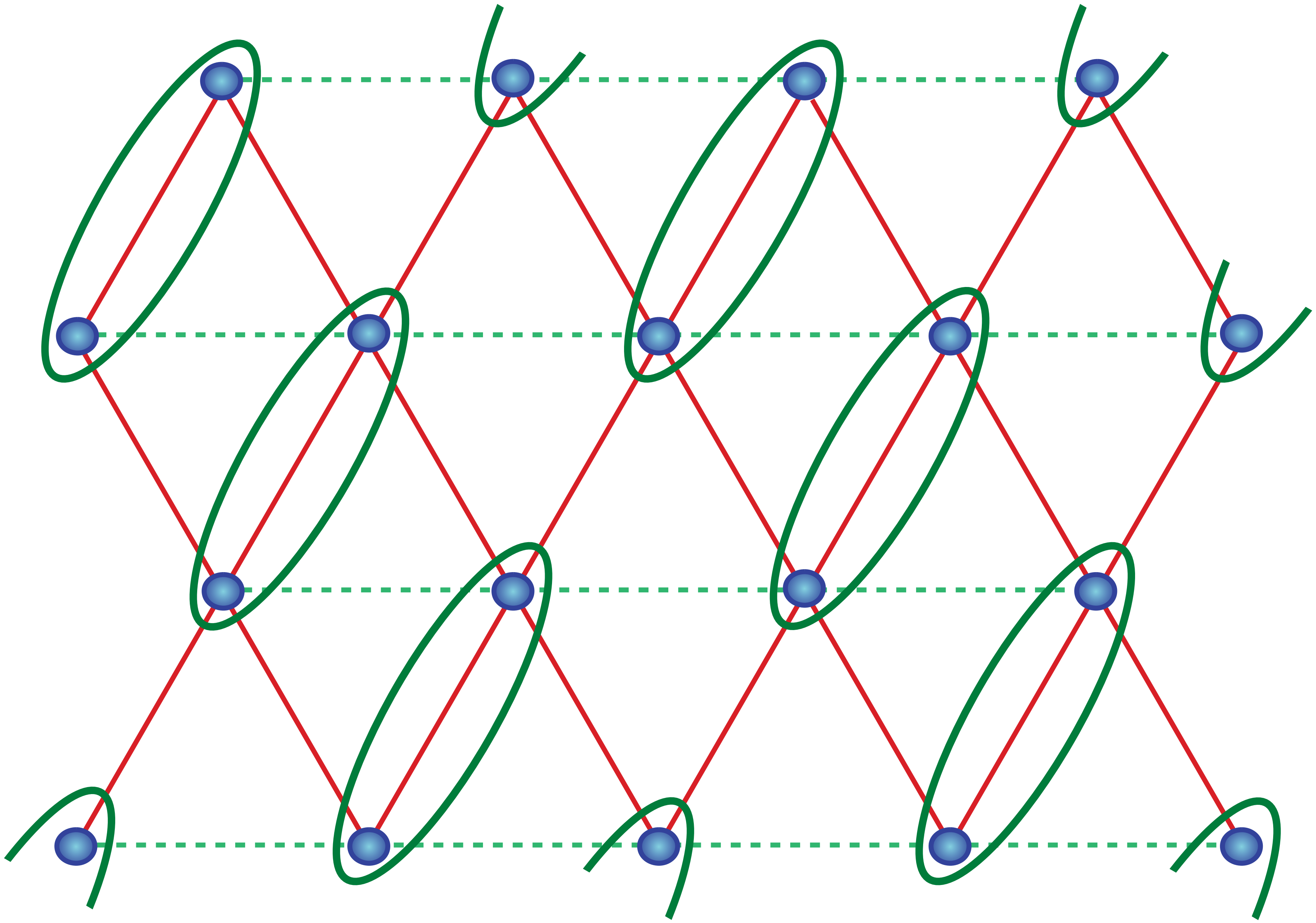

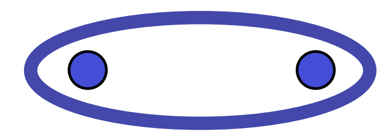

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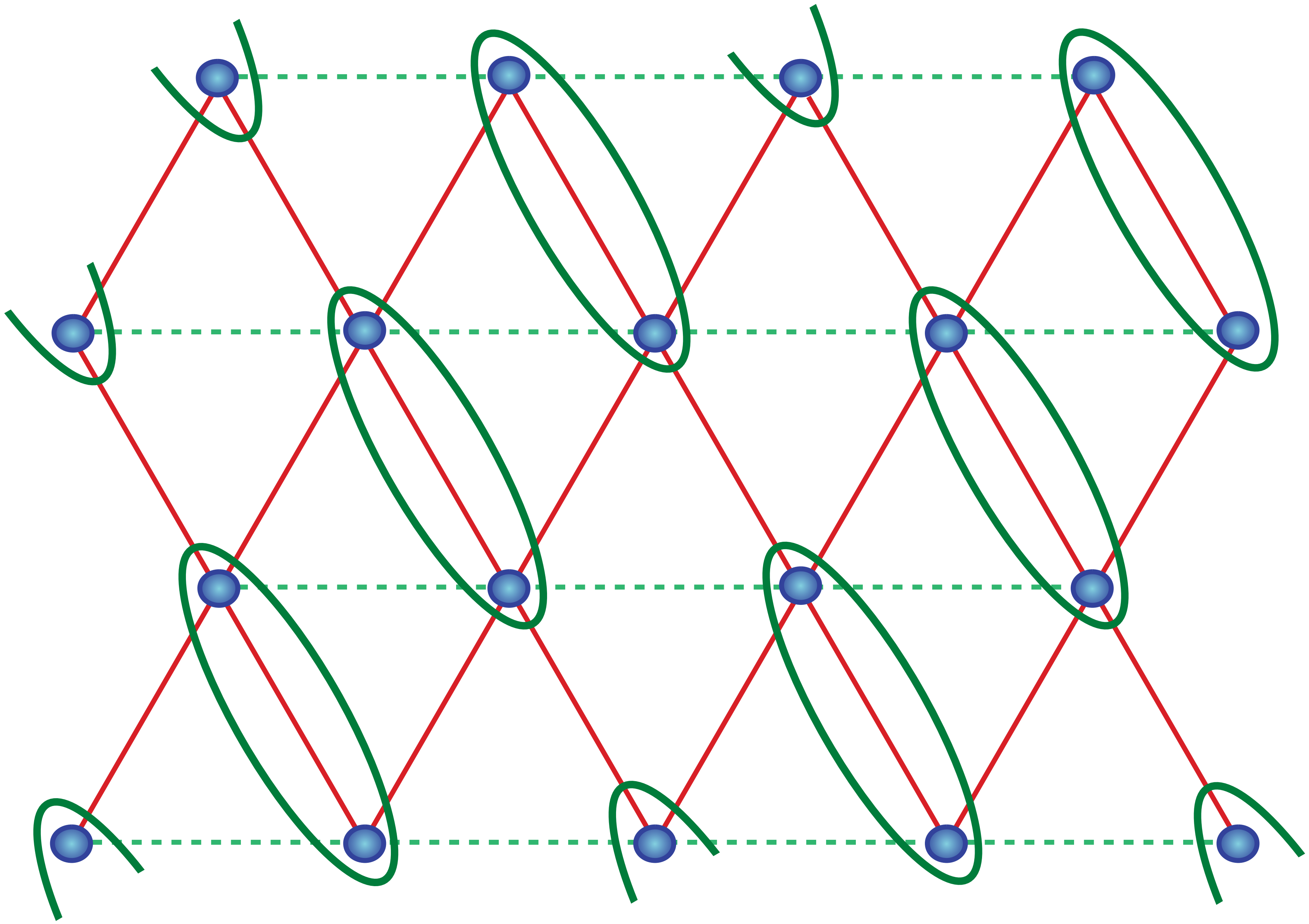

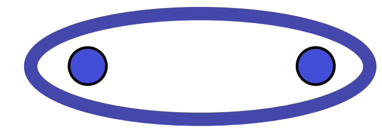

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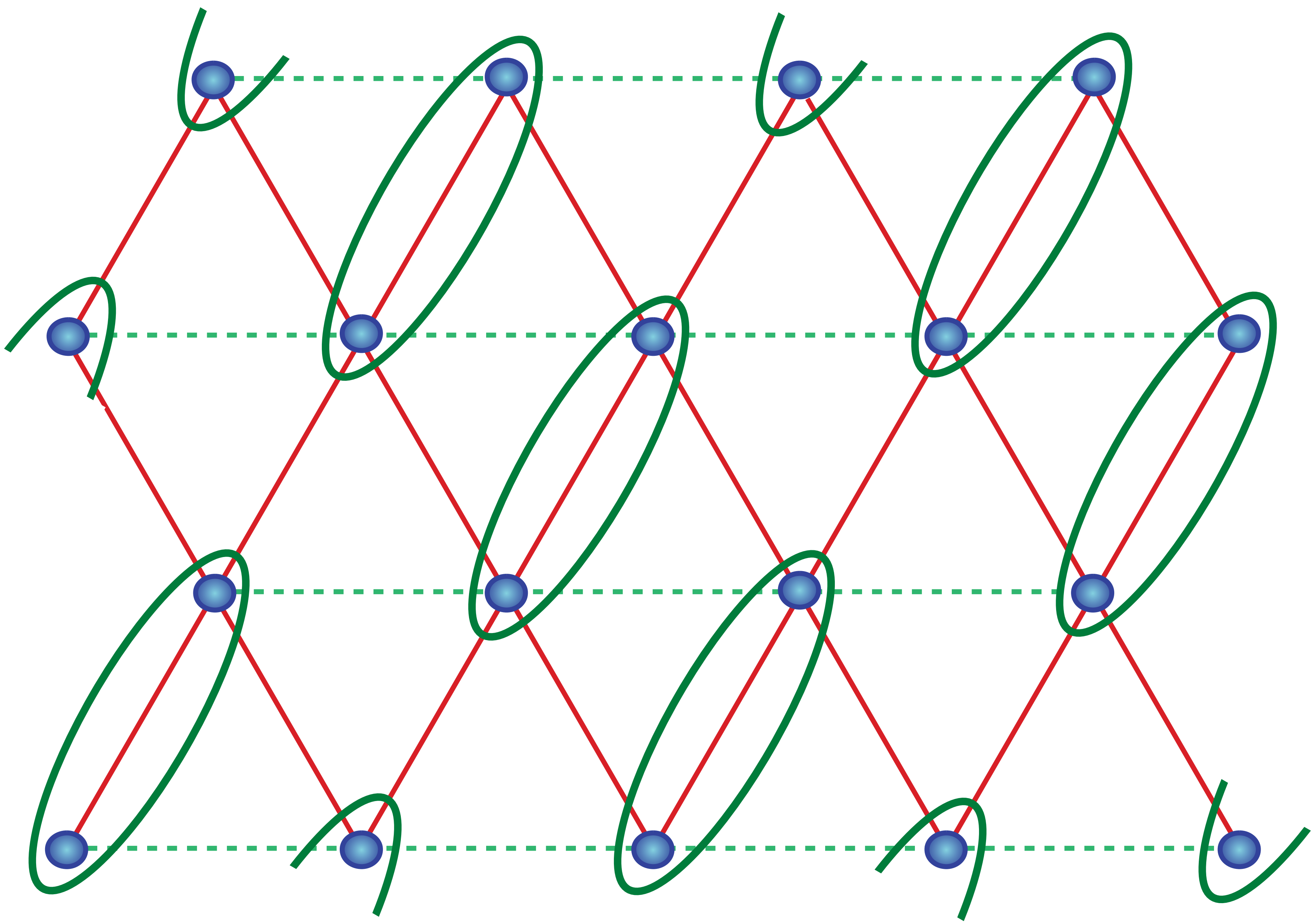

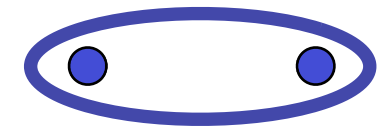

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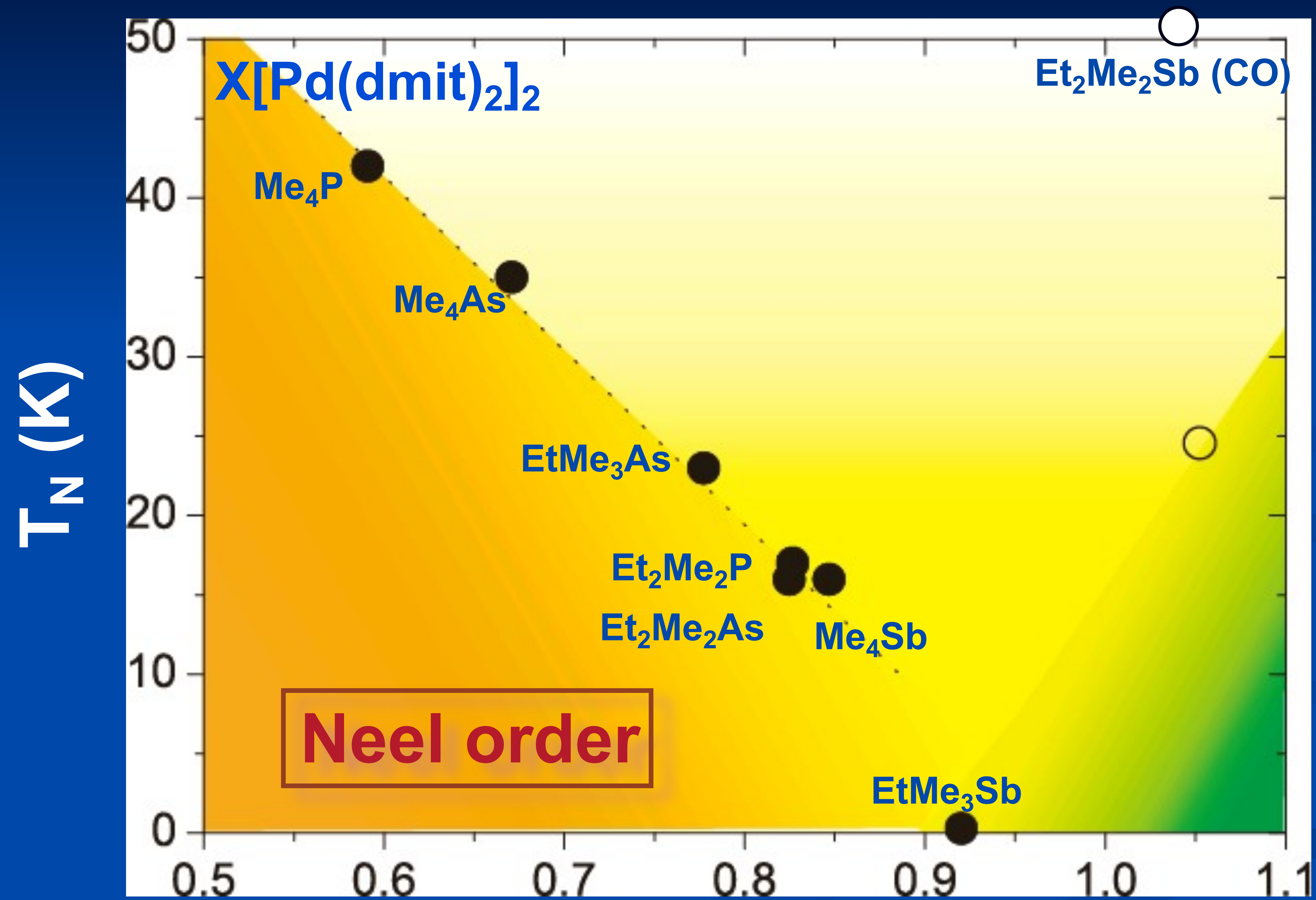
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989)

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid

Magnetic Criticality



$$\sqrt{J'/J}$$

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

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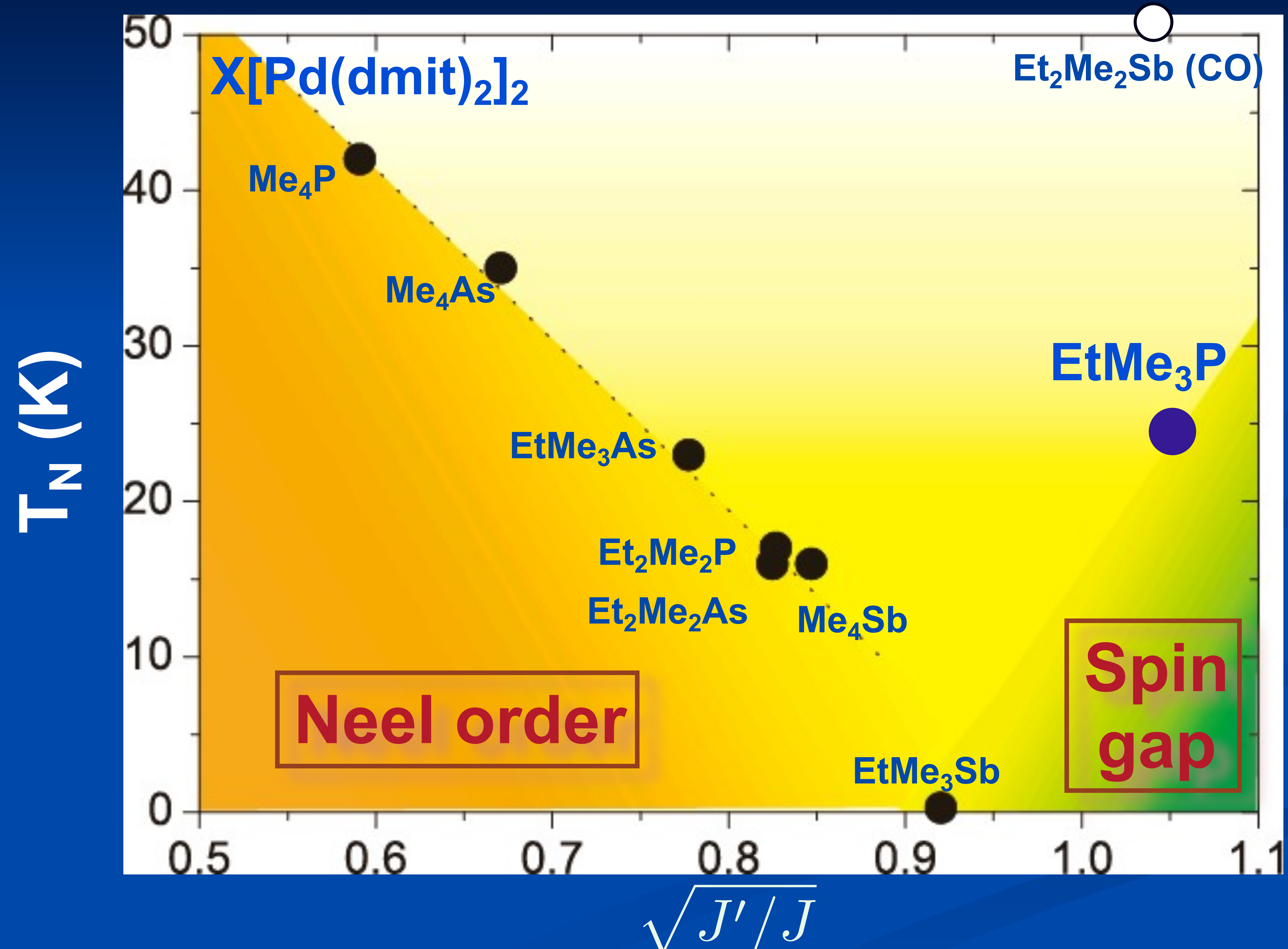

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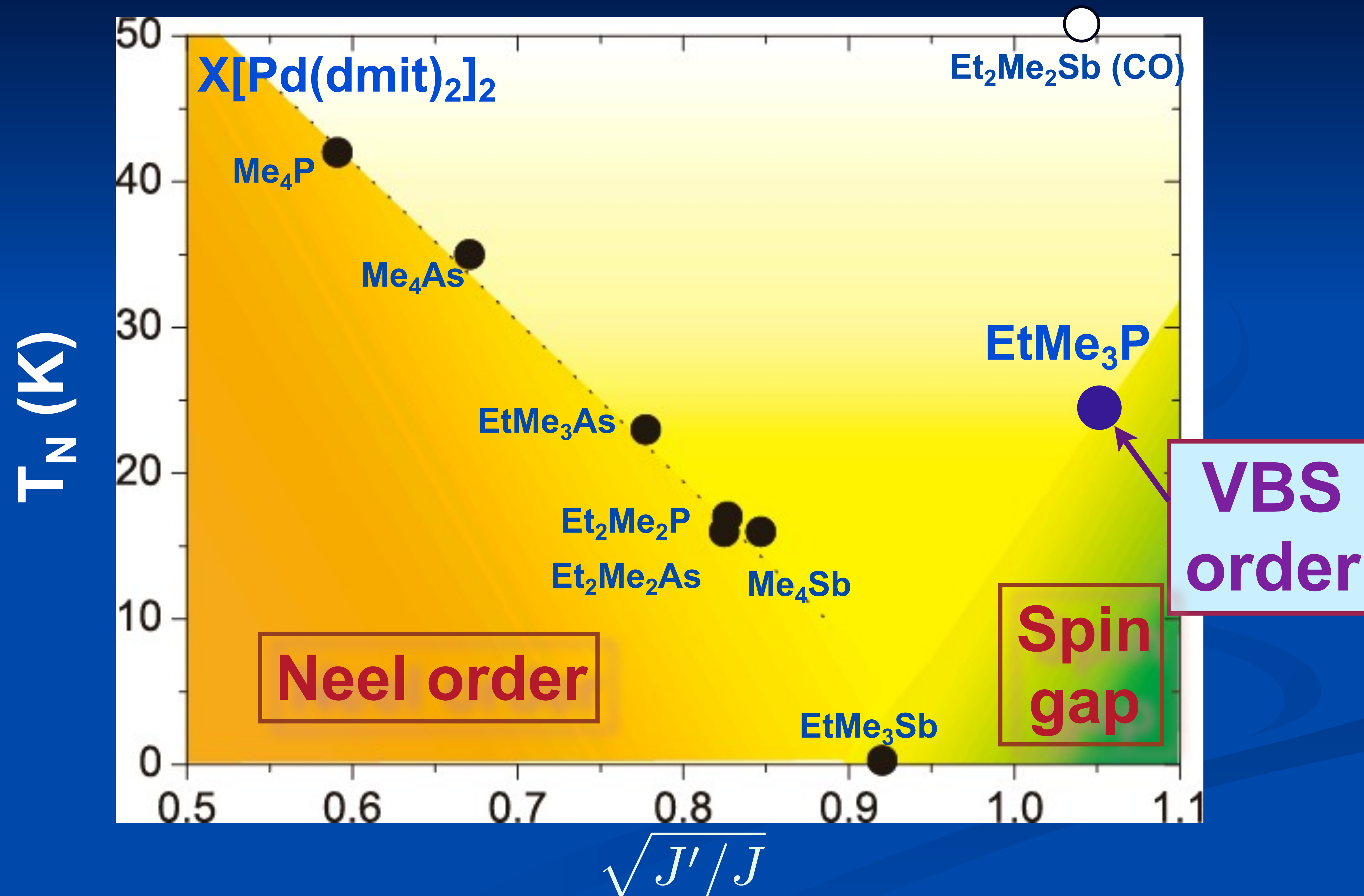
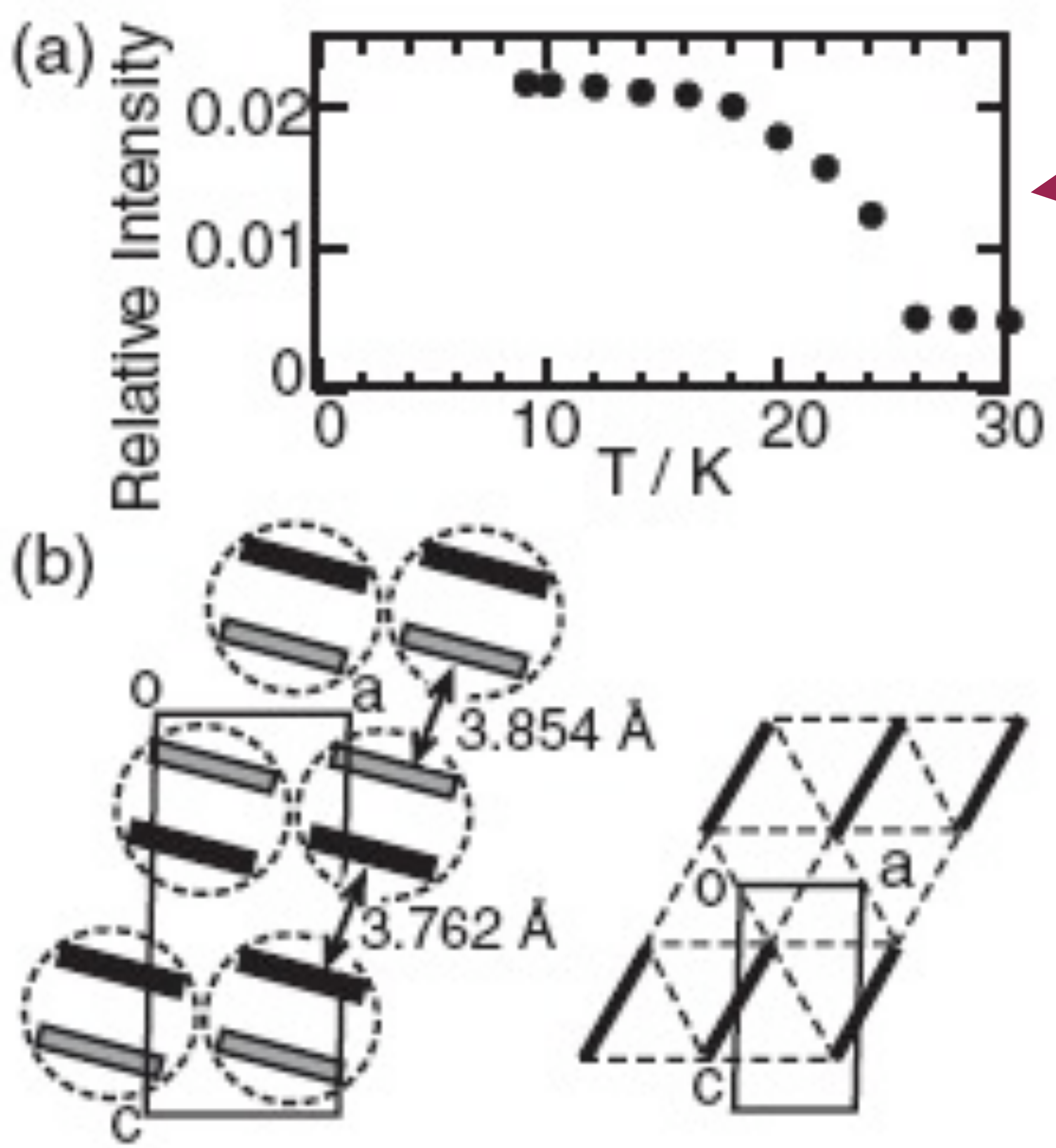


Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Observation of a valence bond solid (VBS) in $ETMe_3P[Pd(dmit)_2]_2$

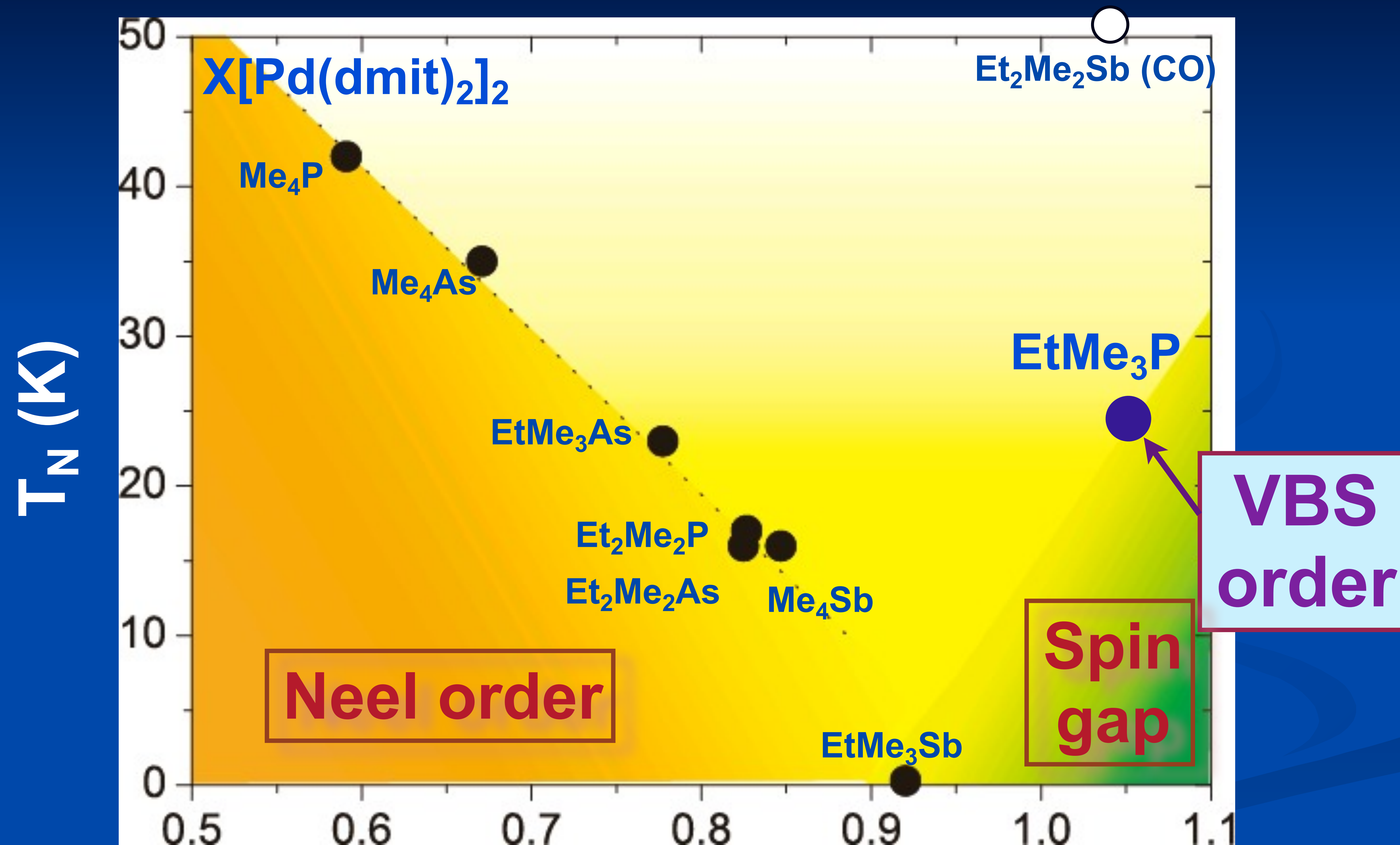


X-ray scattering

Spin gap ~ 40 K
J ~ 250 K

M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

Magnetic Criticality



$\sqrt{J'/J}$

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
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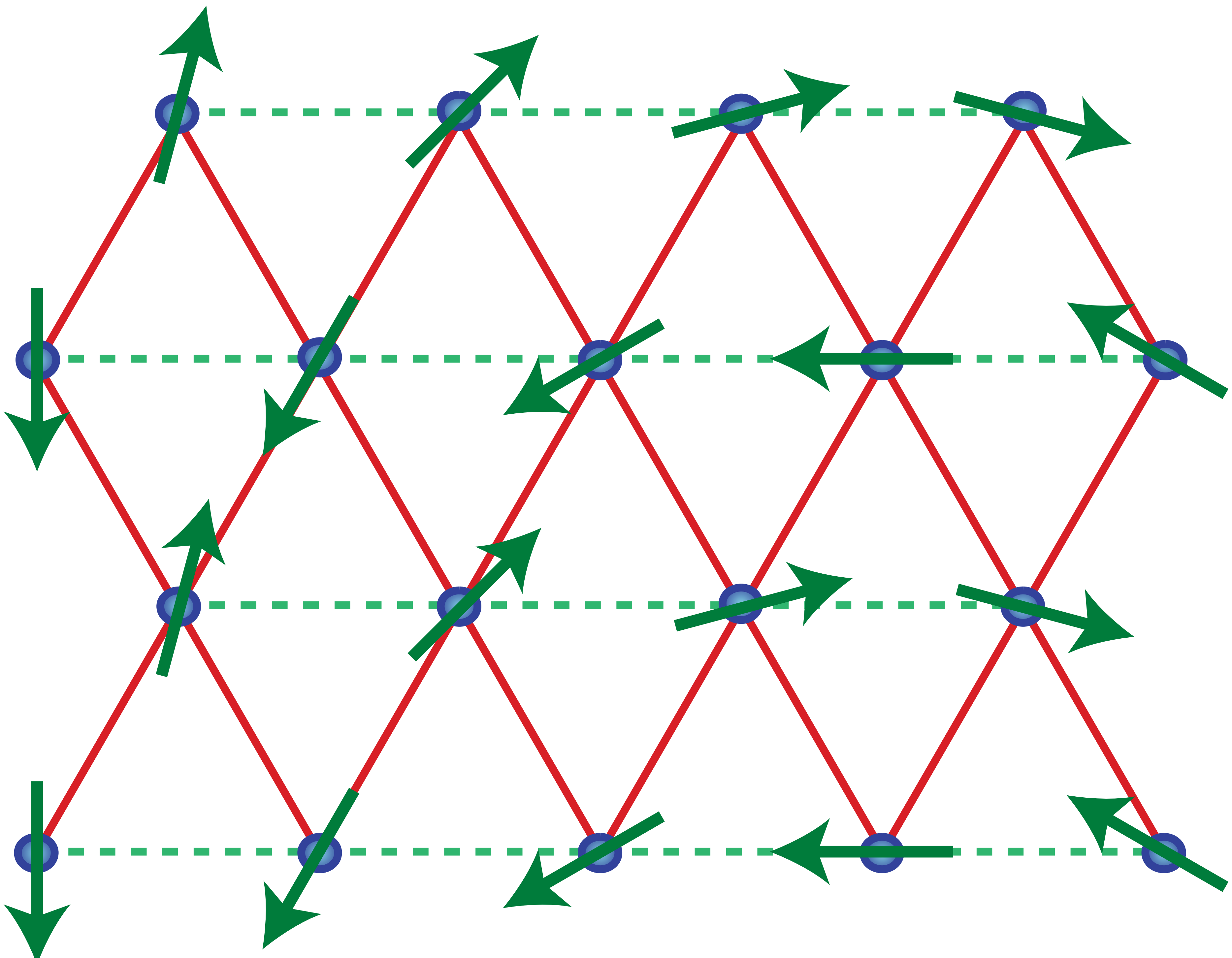
Anisotropic triangular lattice antiferromagnet



Classical ground state for large J'/J

Found in Cs_2CuCl_4

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO

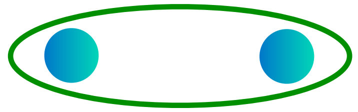
Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

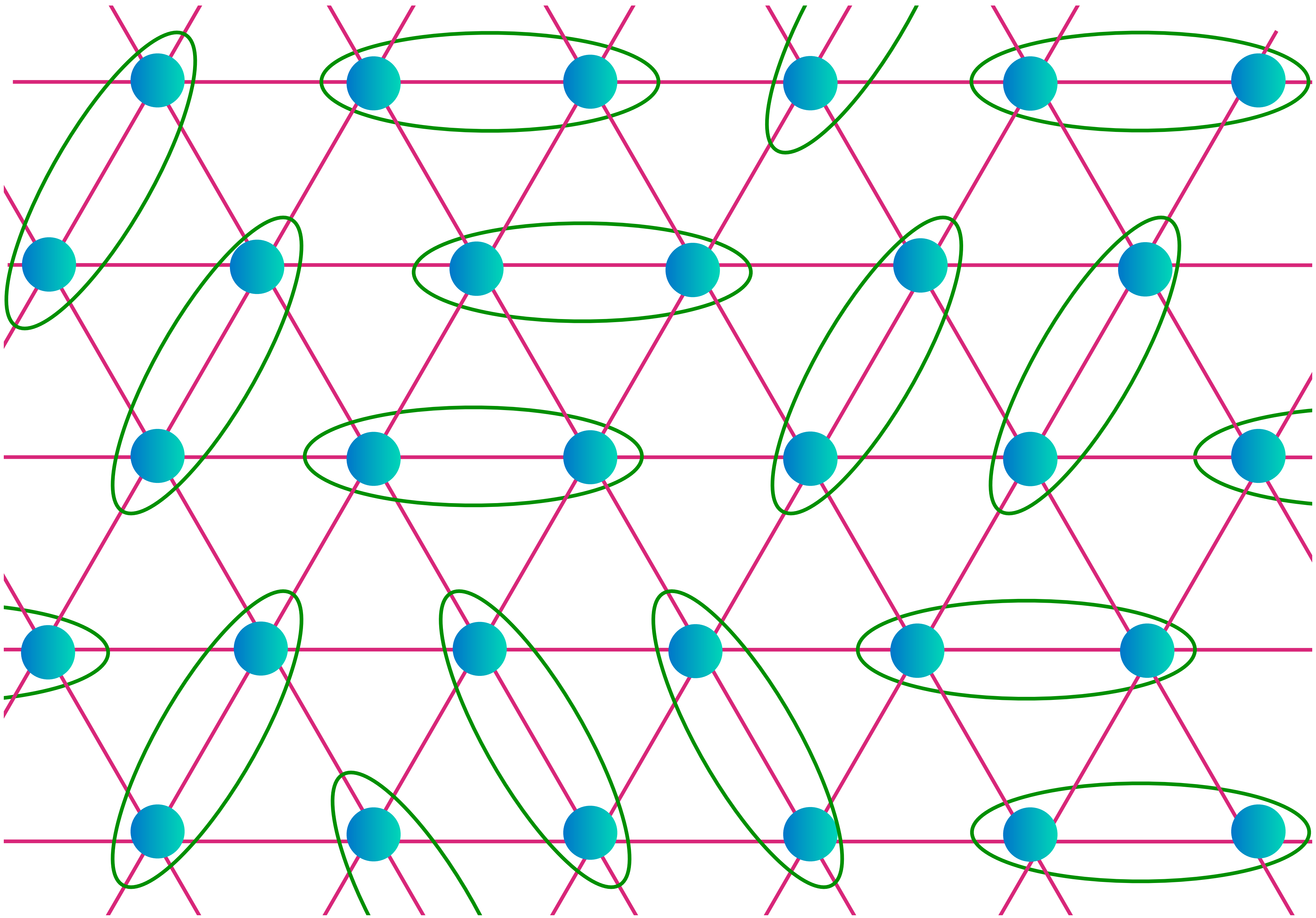
- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO
- Z_2 spin liquid: preserves all symmetries of Hamiltonian

Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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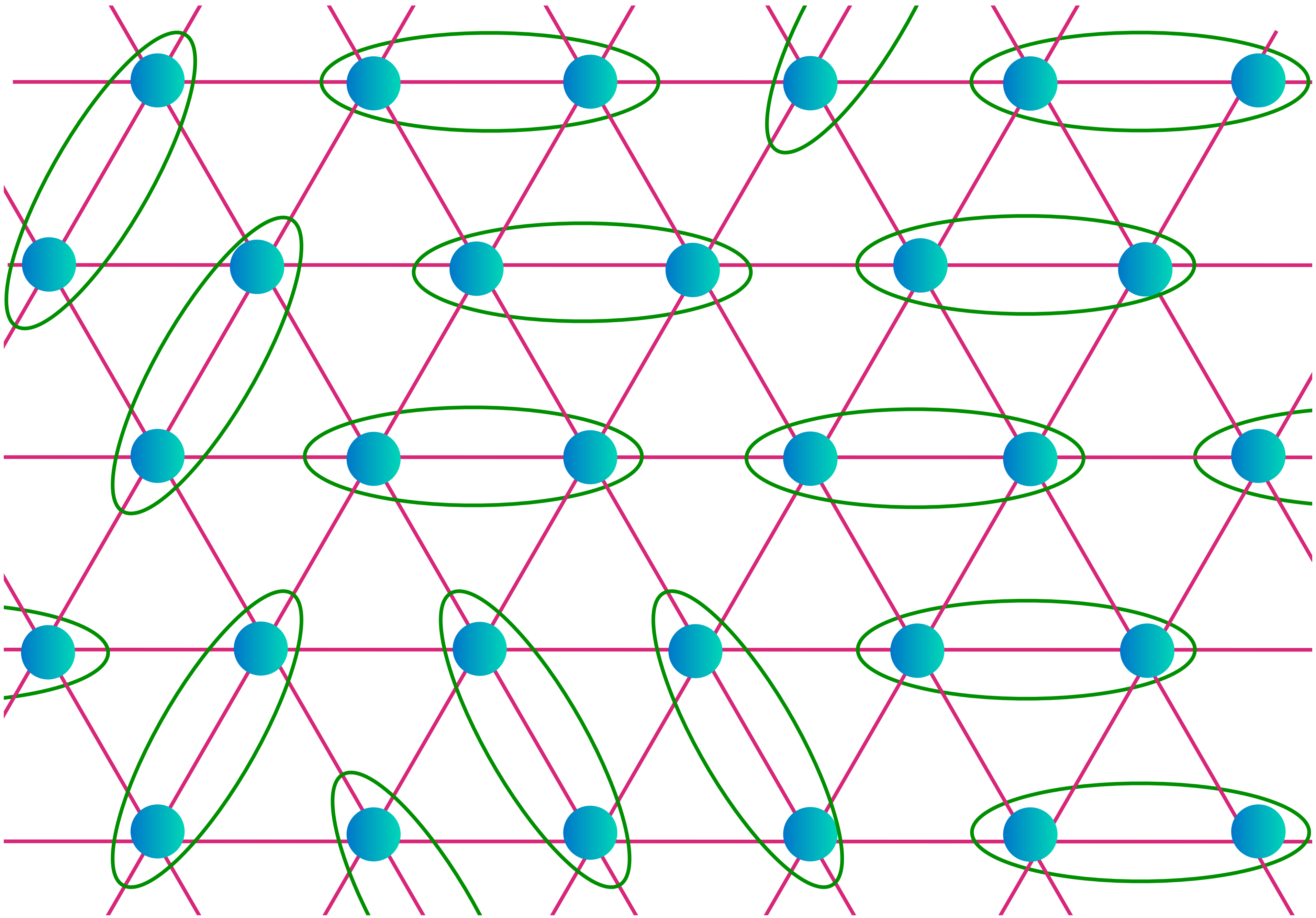

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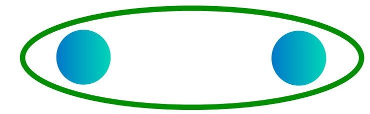

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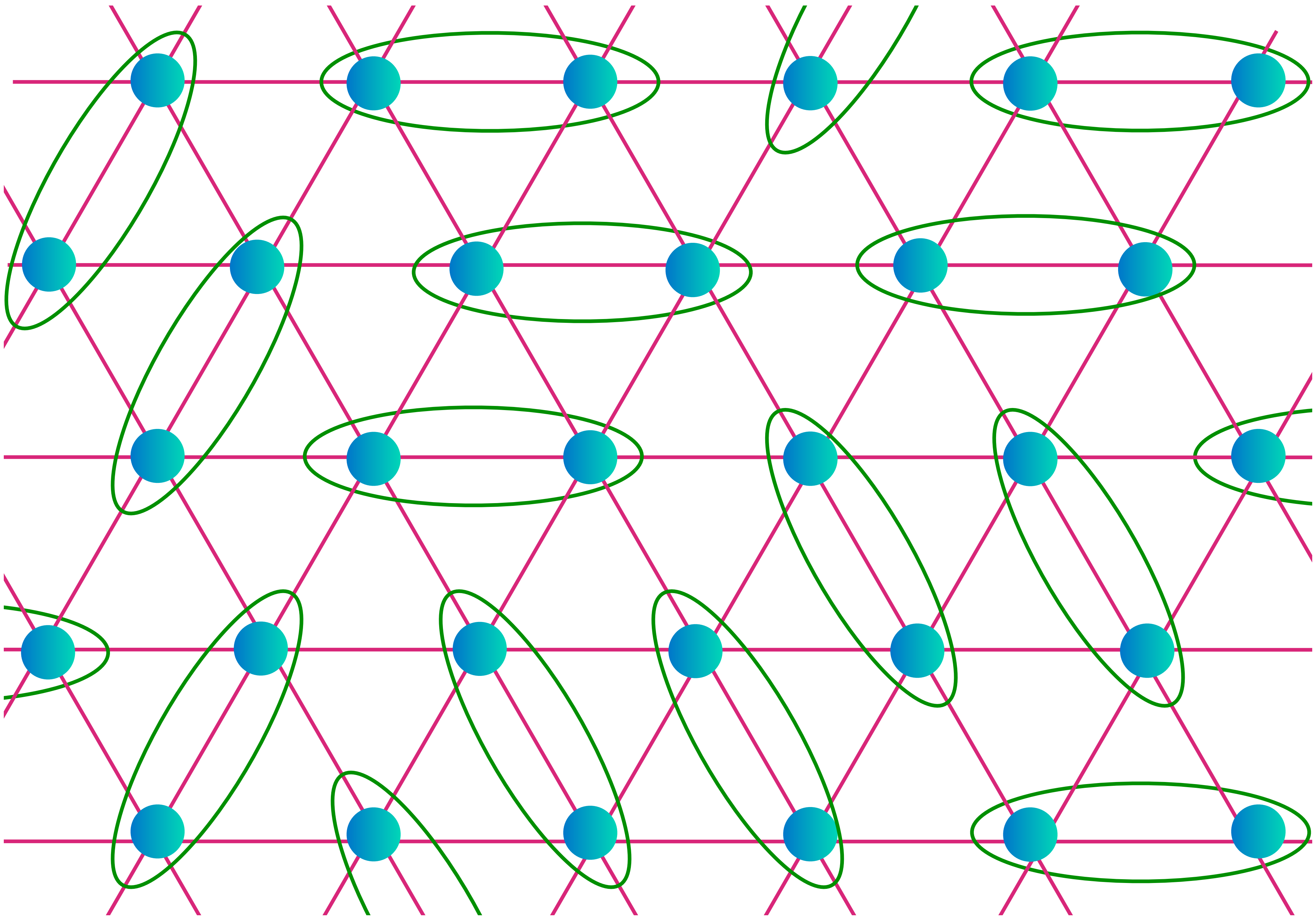

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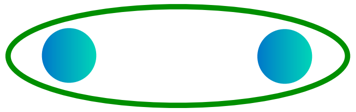

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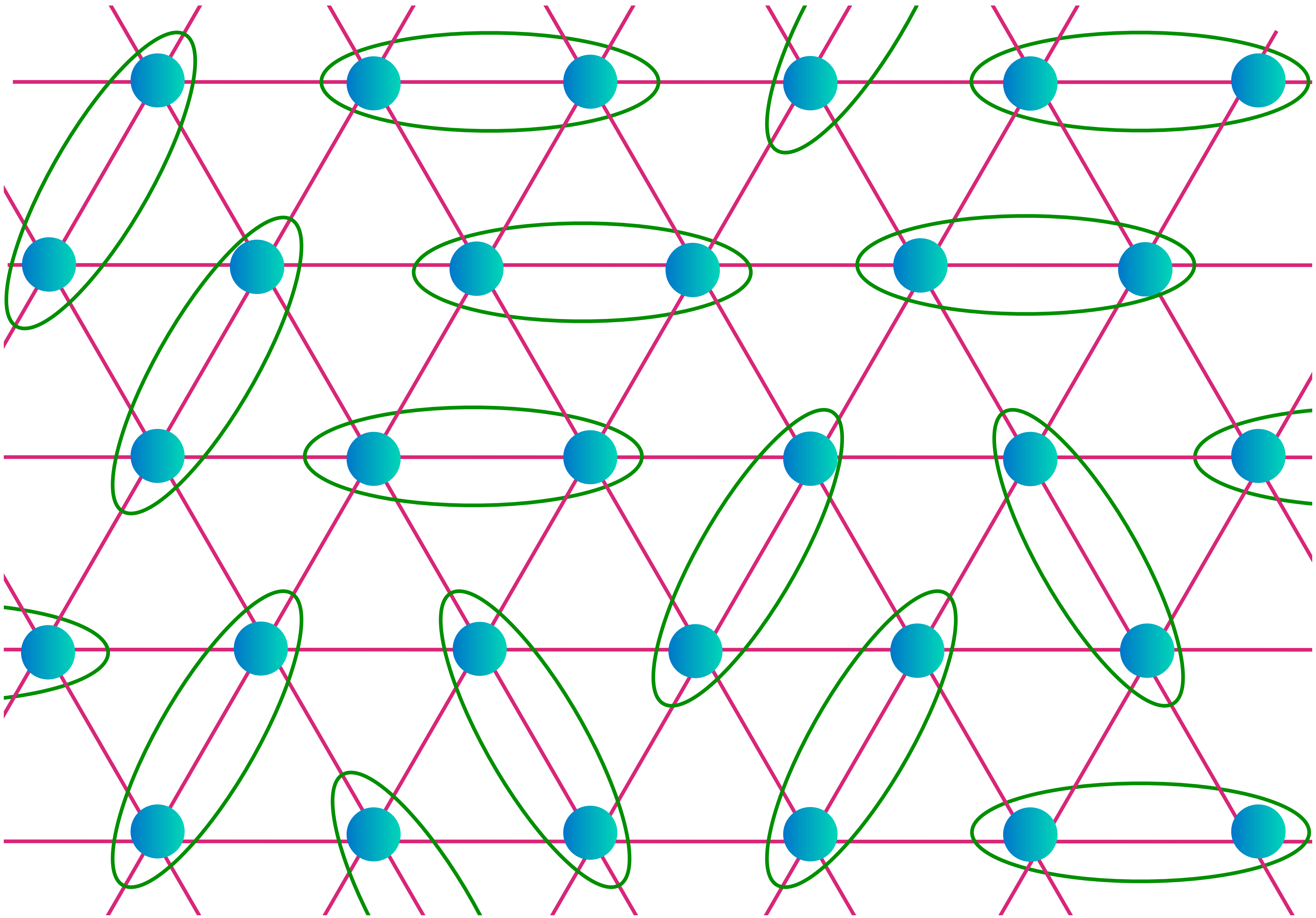

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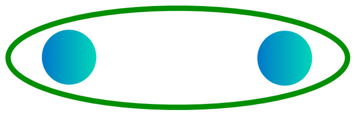

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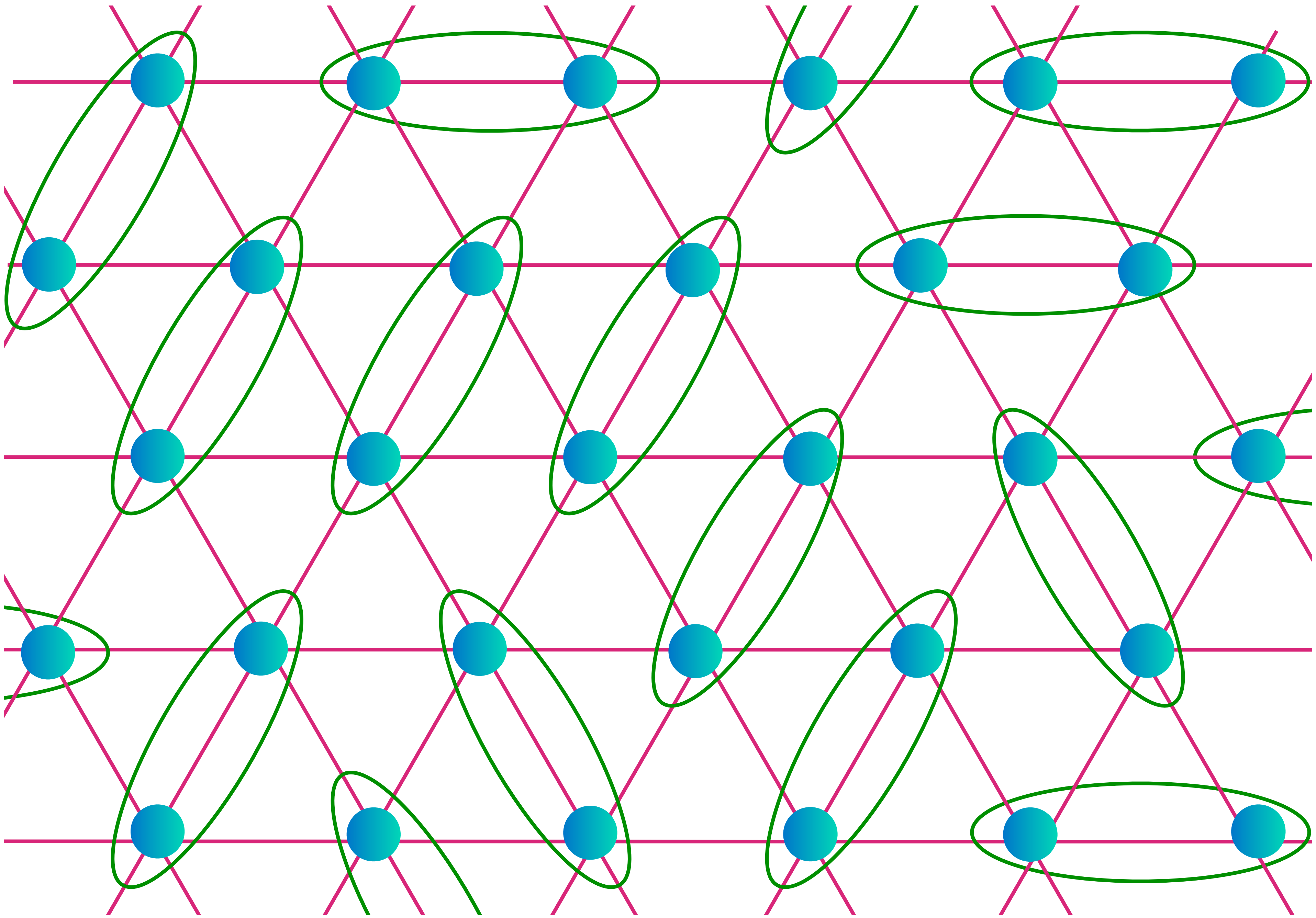

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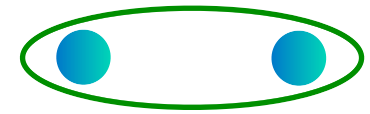

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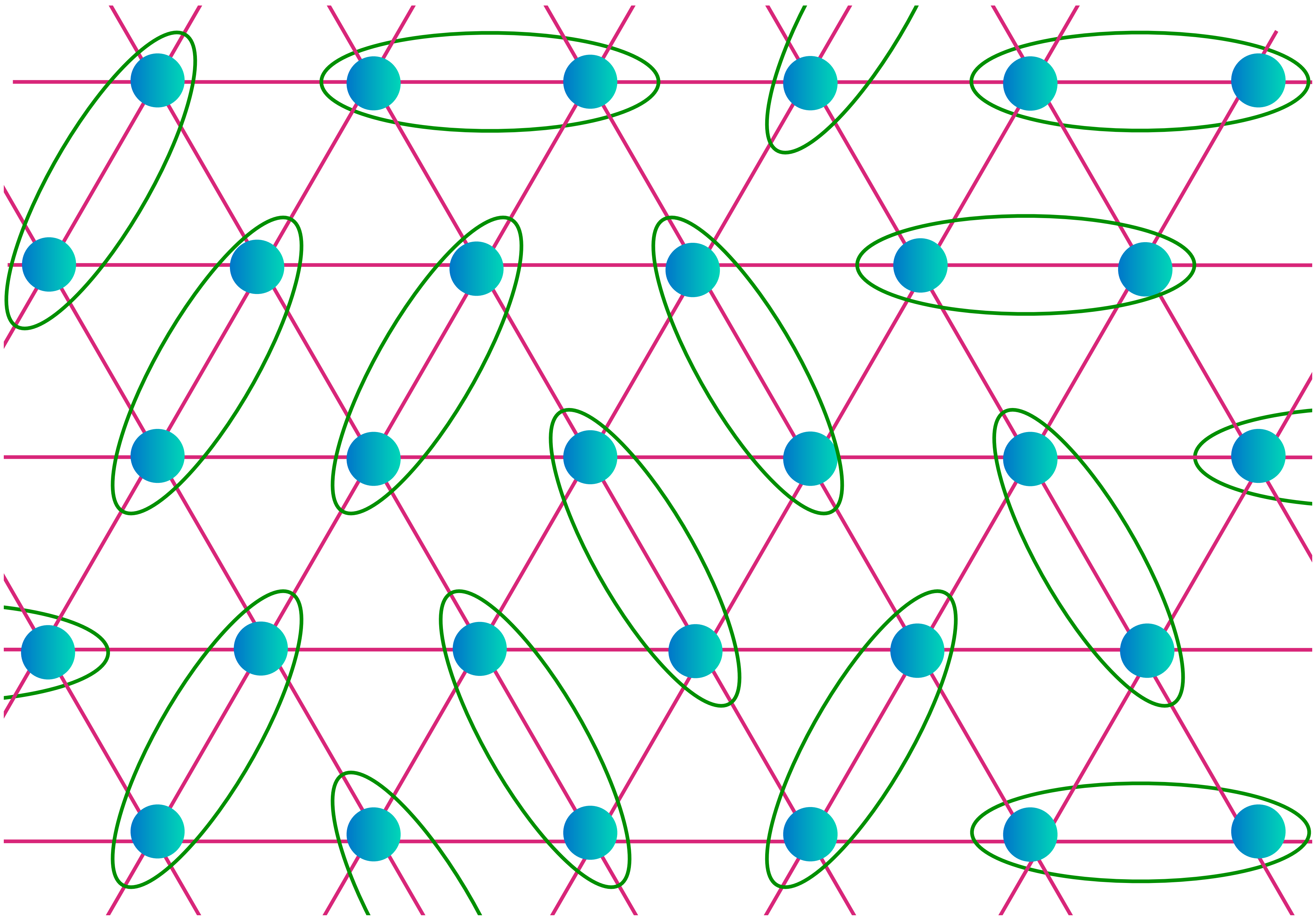

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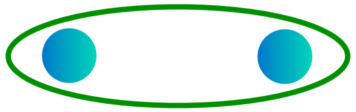
Excitations of the Z_2 Spin liquid

A spinon



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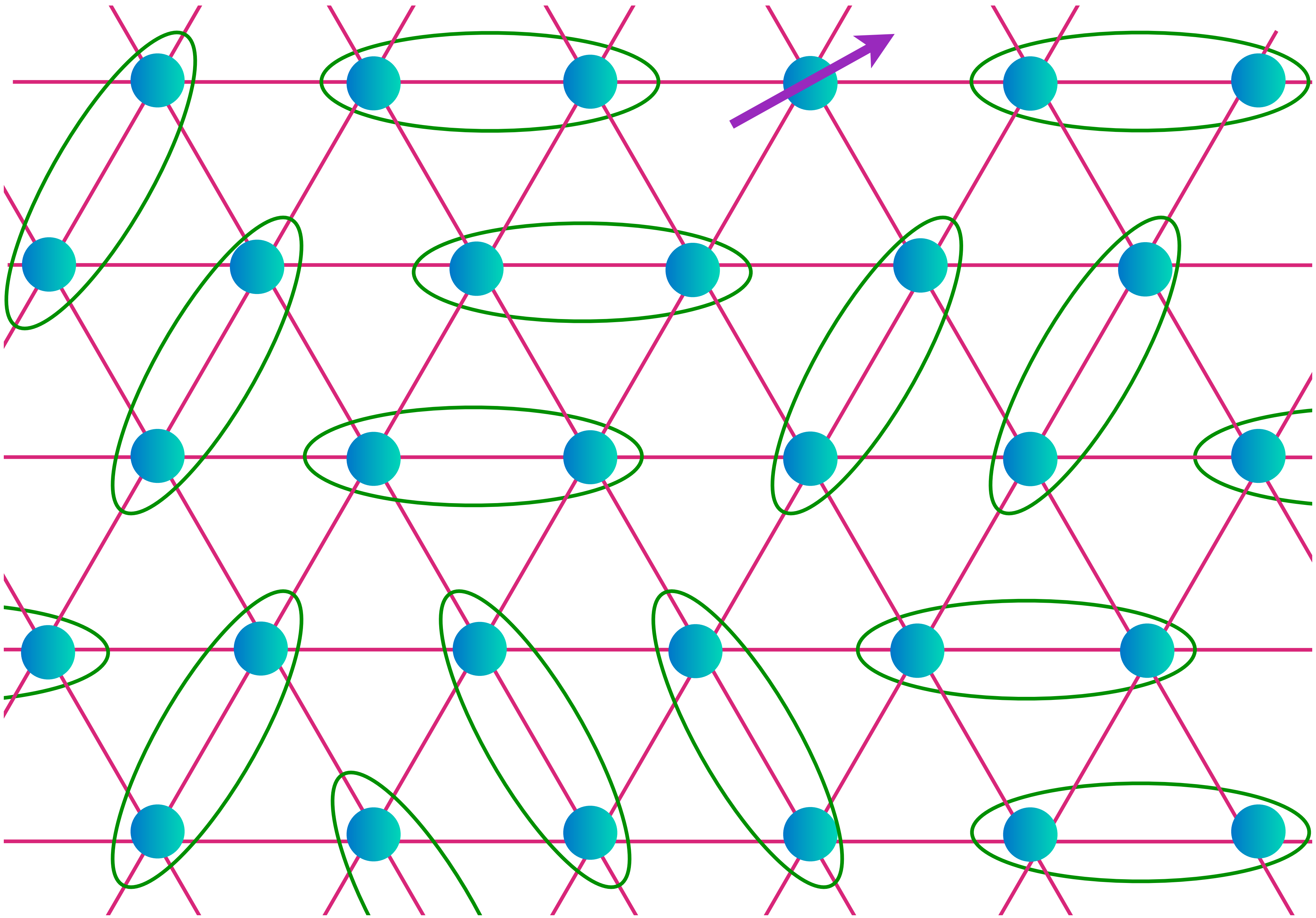

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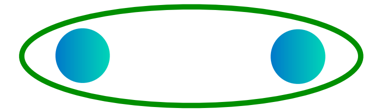

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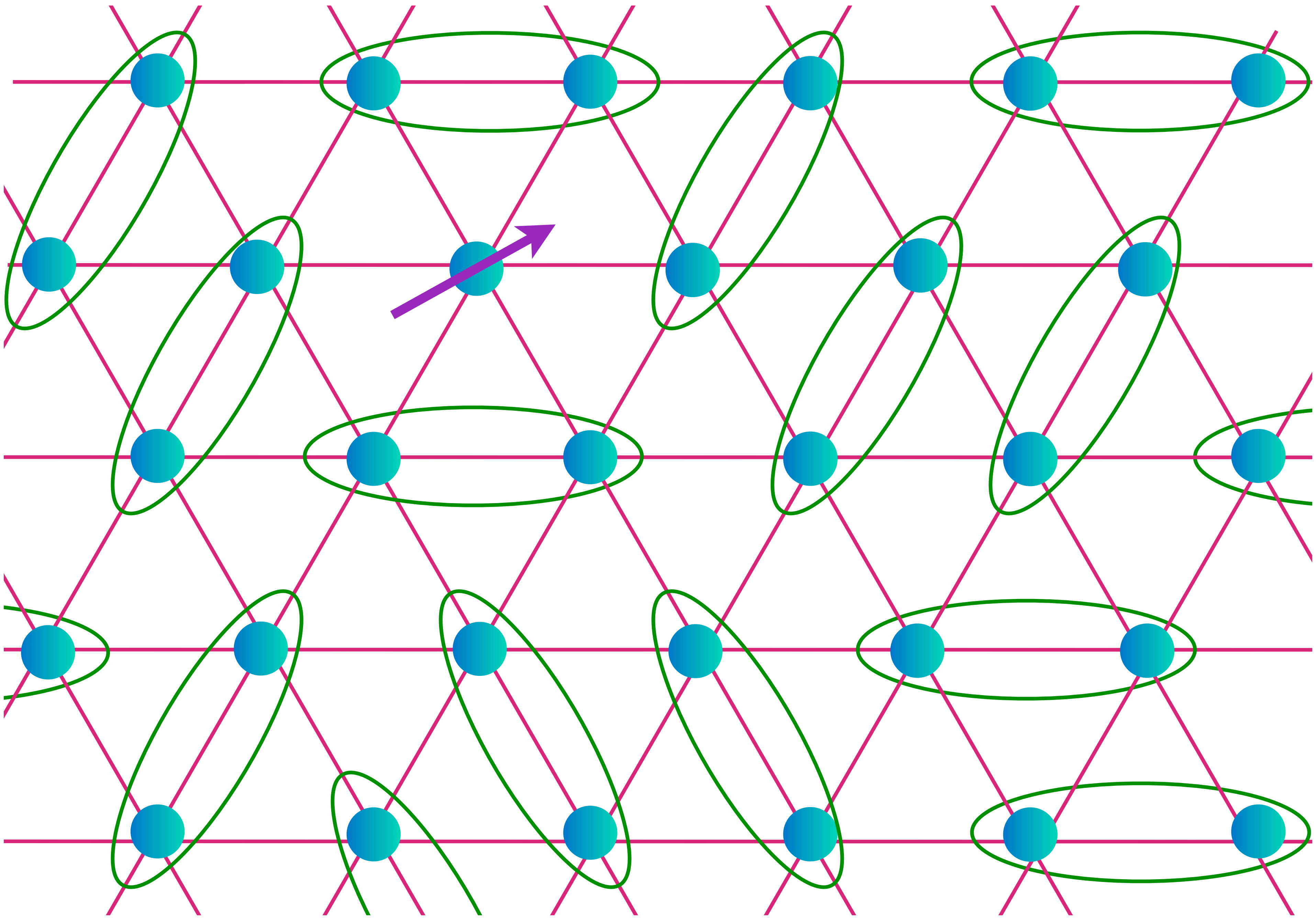

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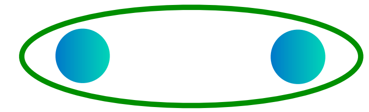

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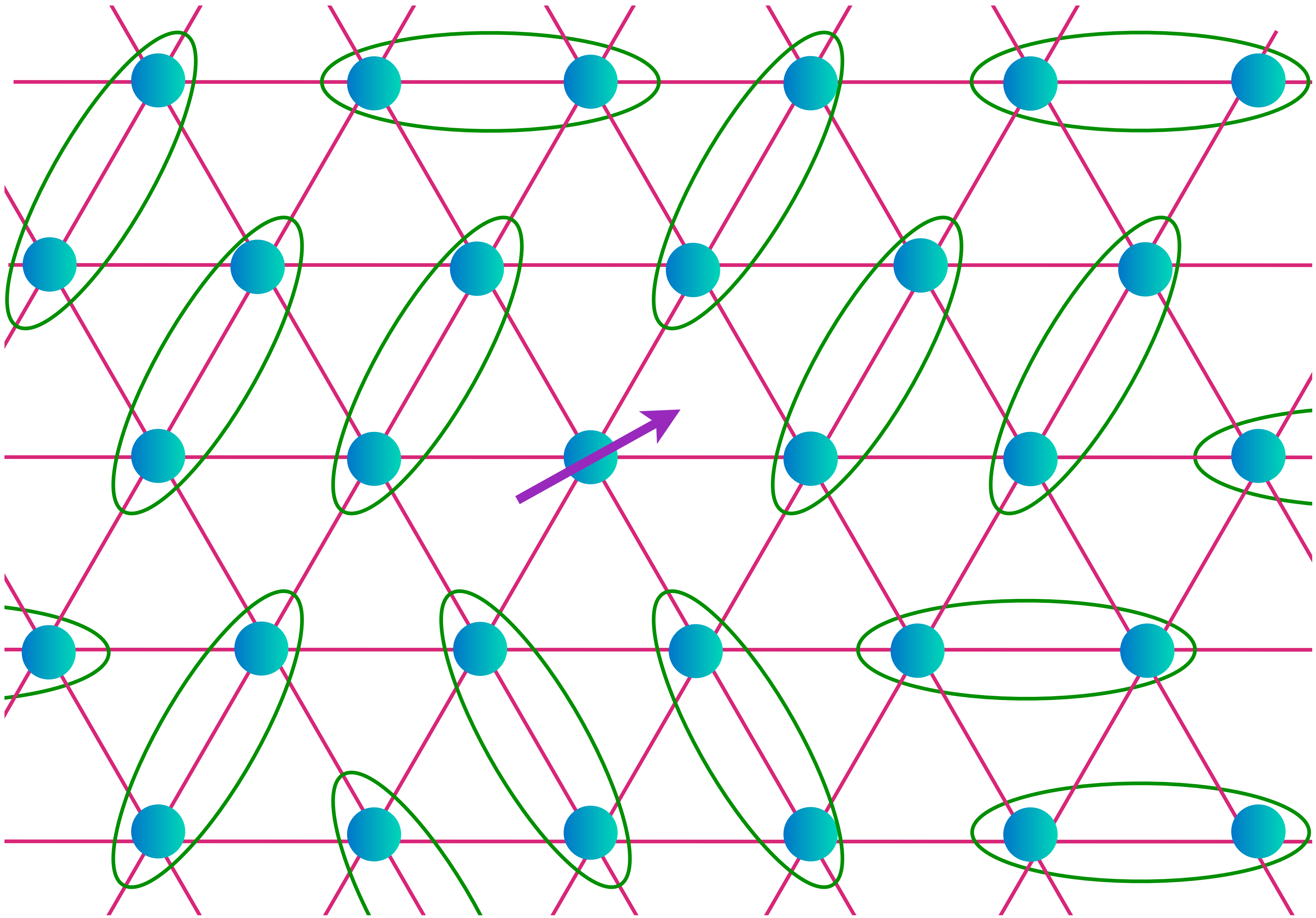

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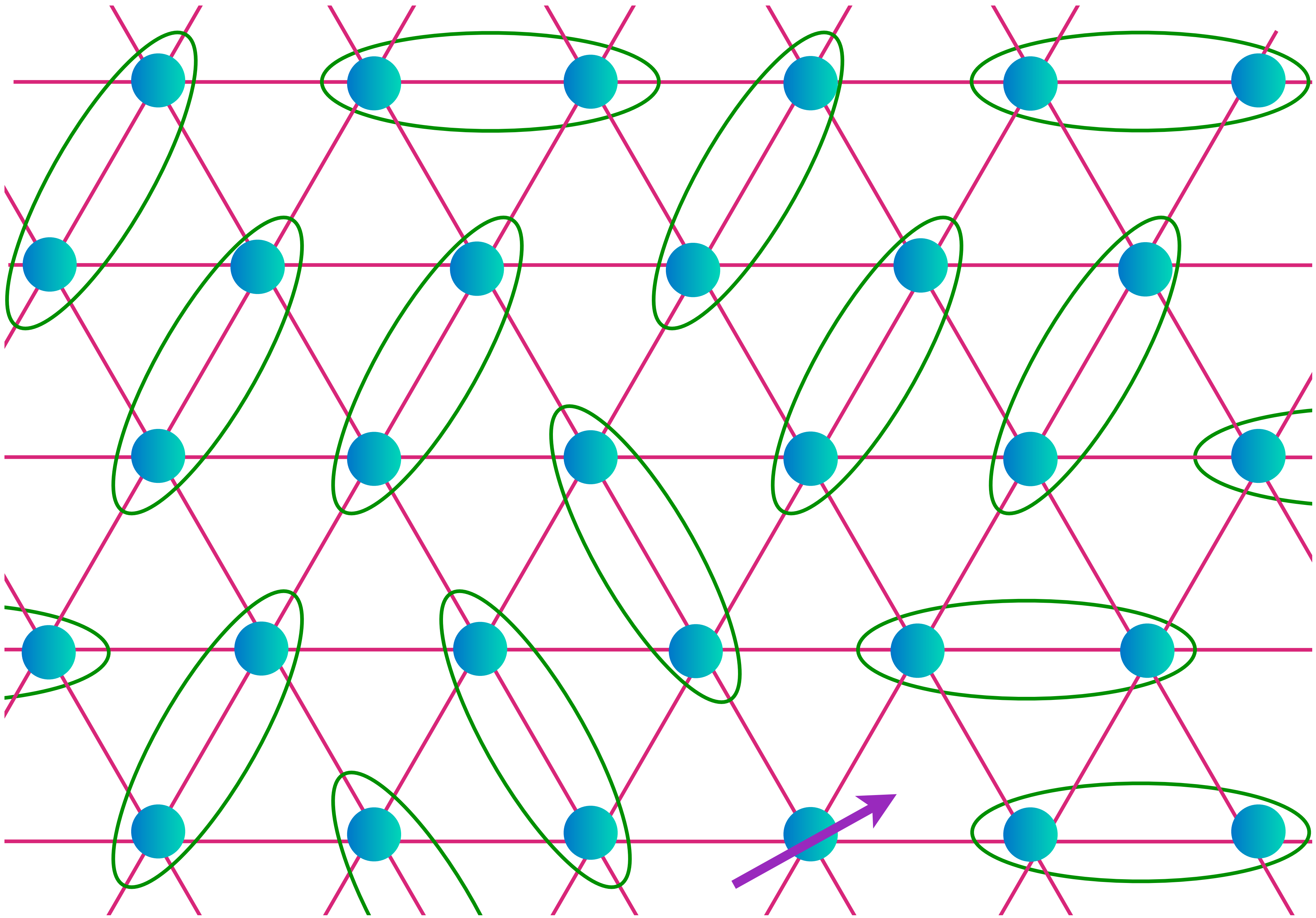

Excitations of the Z_2 Spin liquid

A spinon

The spinon annihilation operator is a spinor z_α, where $\alpha = \uparrow, \downarrow$.

The Néel order parameter, $\vec{\varphi}$ is a composite of the spinons:

$$\vec{\varphi} = z_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} z_{i\beta}$$

where $\vec{\sigma}$ are Pauli matrices

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The theory for quantum phase transitions is expressed in terms of fluctuations of z_α, and *not* the order parameter $\vec{\varphi}$.

Effective theory for z_α must be invariant under the U(1) gauge transformation

$$z_{i\alpha} \to e^{i\theta} z_{i\alpha}$$

Excitations of the Z_2 Spin liquid

A vison

- A characteristic property of a Z_2 spin liquid is the presence of a spinon pair condensate $\Phi \sim \varepsilon^{\alpha\beta} z_{i\alpha} z_{j\beta}$ which breaks the U(1) "symmetry" down to Z_2.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

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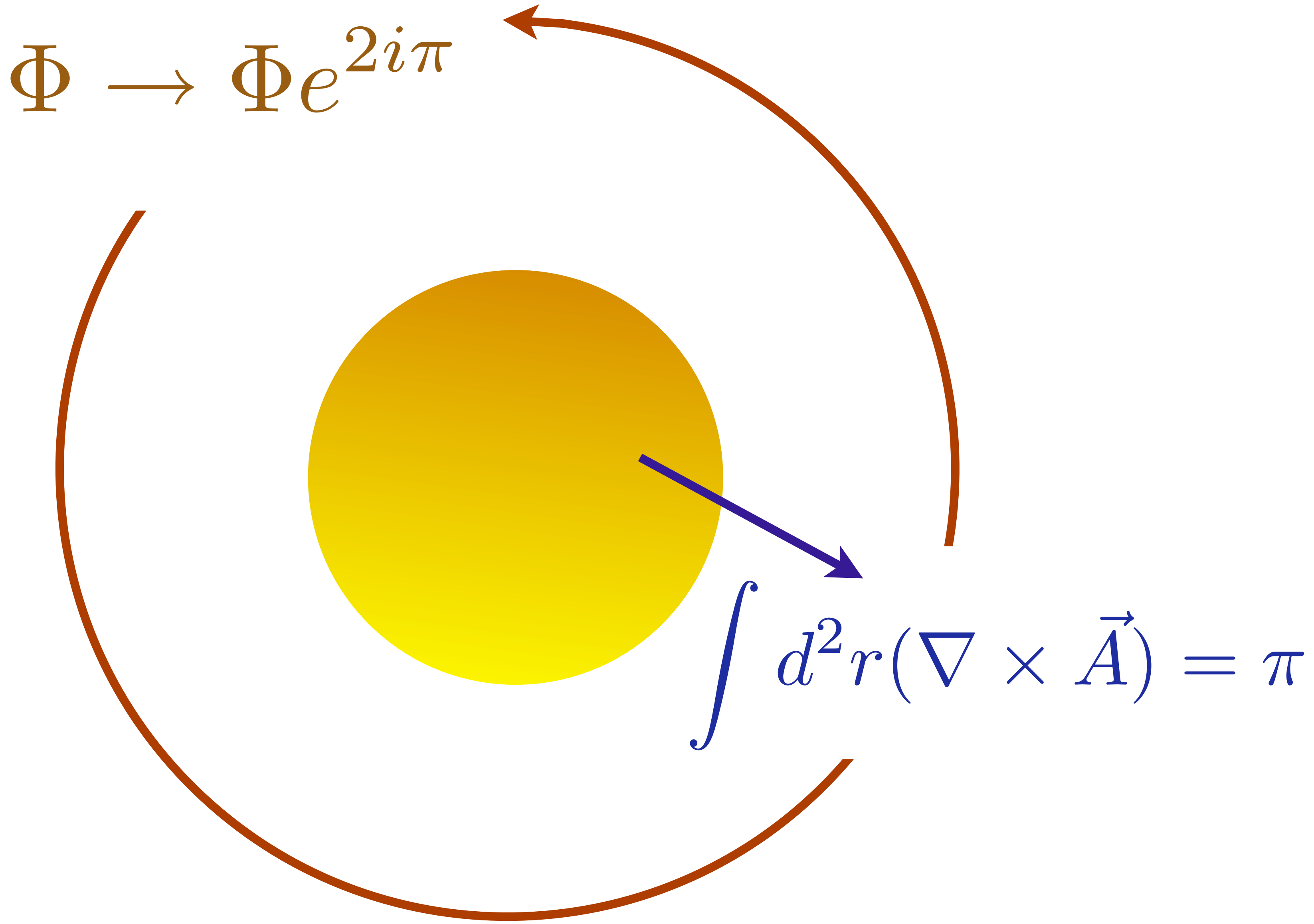
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Topological excitation of a Z_2 spin liquid (vison)

An Abrikosov vortex in the Φ condensate



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Excitations of the Z_2 Spin liquid

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Excitations of the Z_2 Spin liquid

A vison

- A characteristic property of a Z_2 spin liquid is the presence of a spinon pair condensate $\Phi \sim \varepsilon^{\alpha\beta} z_{i\alpha} z_{j\beta}$ which breaks the U(1) "symmetry" down to Z_2.

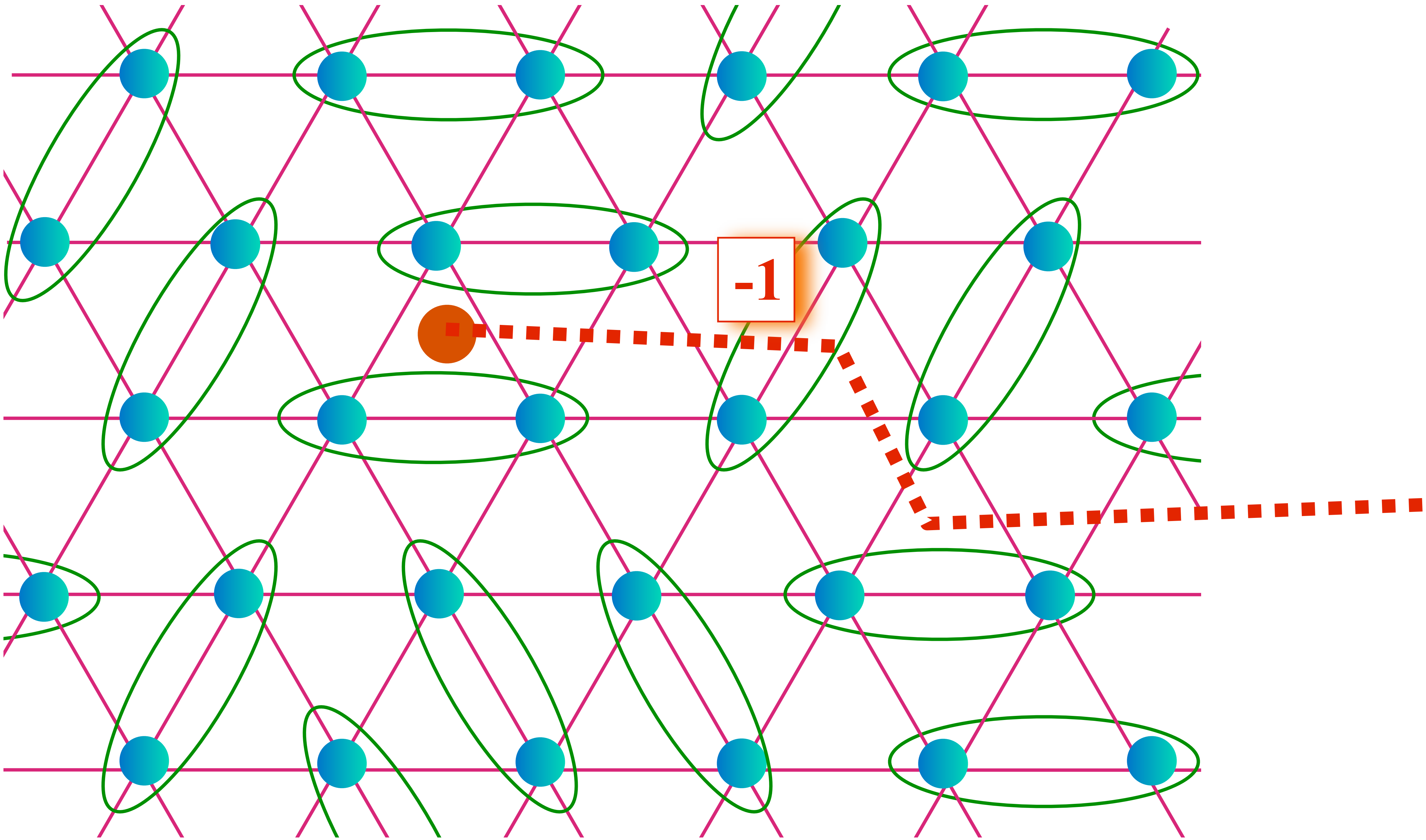
- A vison is an Abrikosov vortex in the pair condensate of spinons

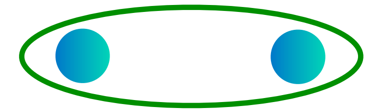
- Visons are are the **dark matter** of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Excitations of the Z_2 Spin liquid

A vison





$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

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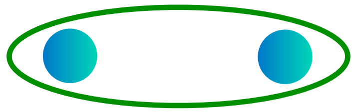
Excitations of the Z_2 Spin liquid

A vison



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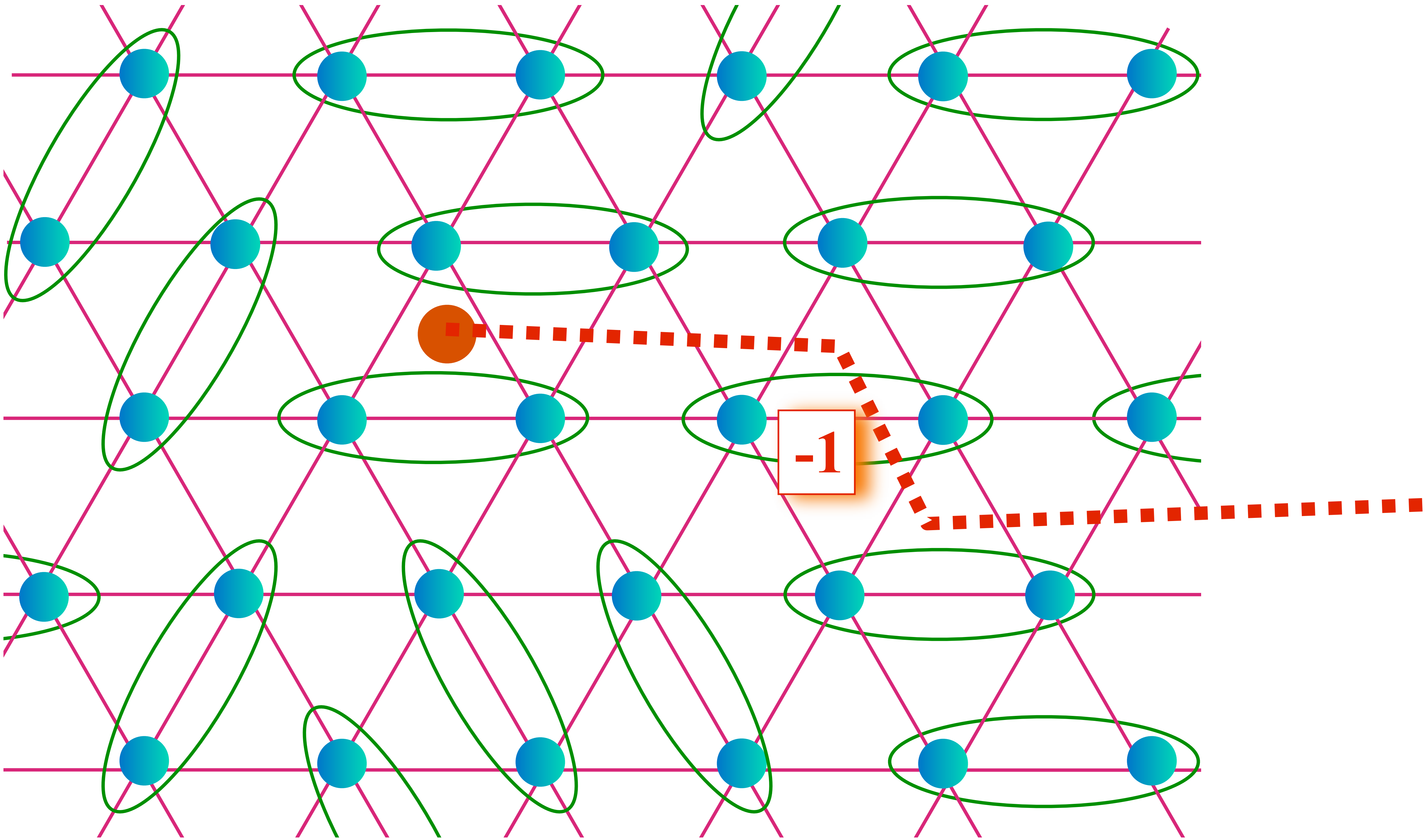

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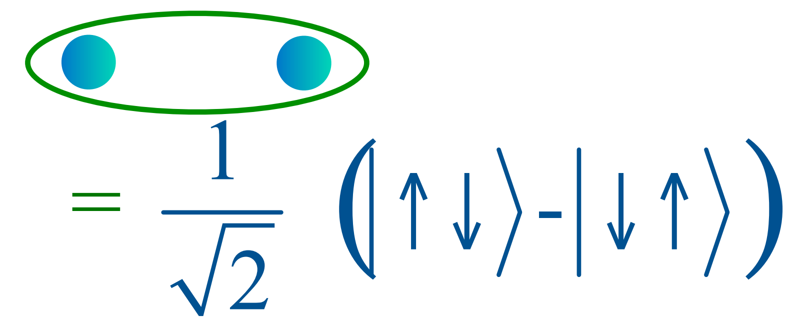
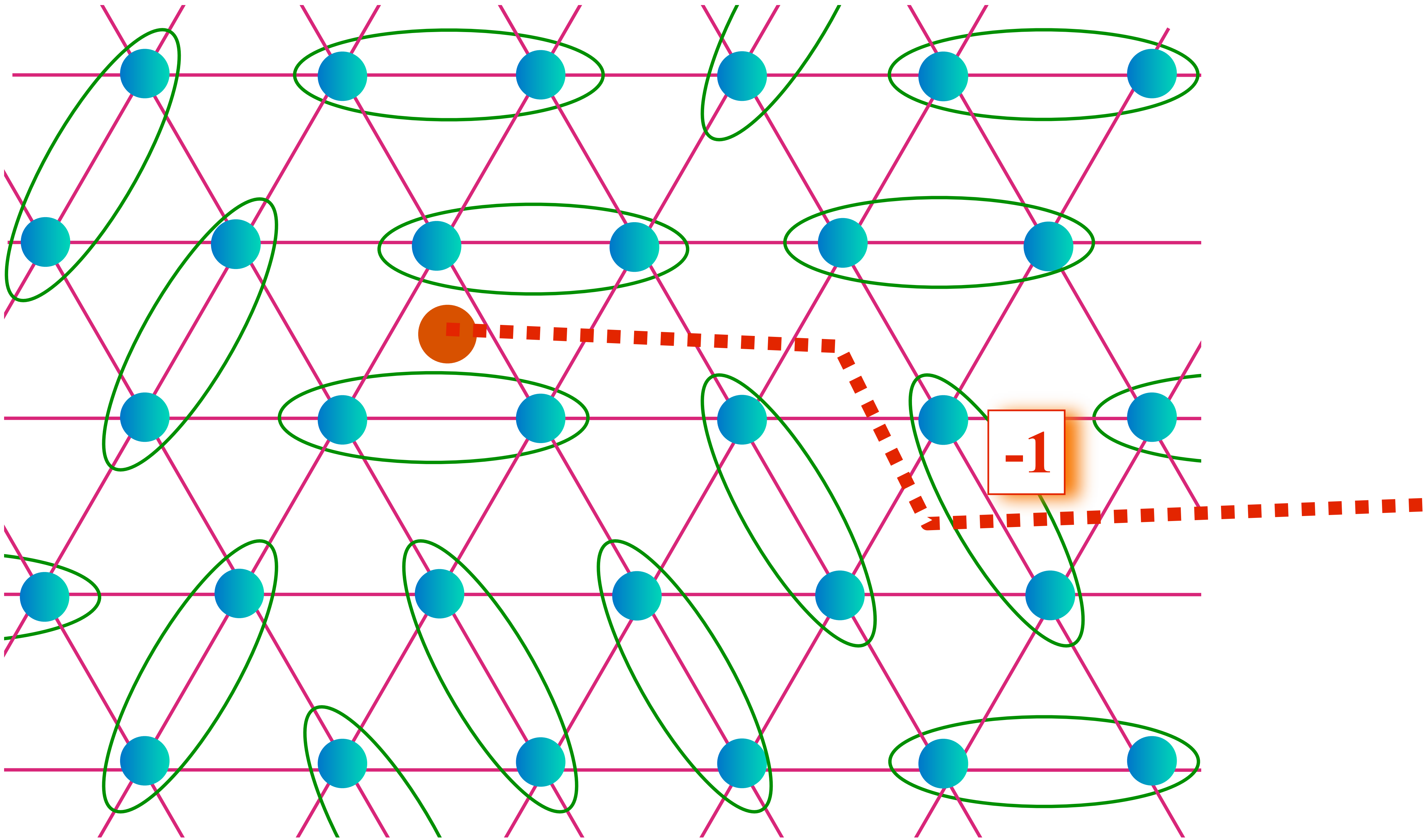
Excitations of the Z_2 Spin liquid



A vison



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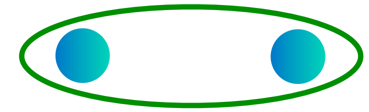
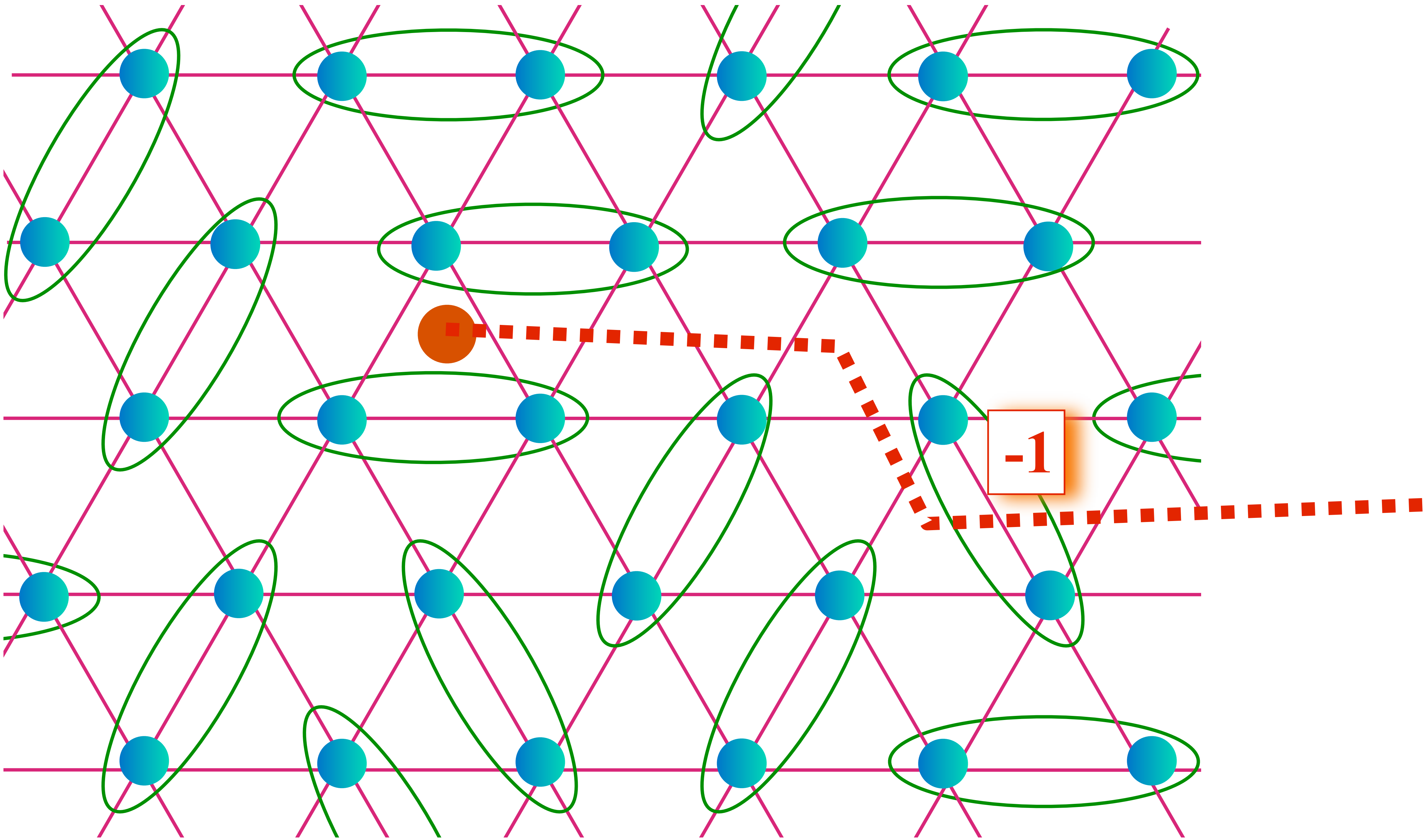
Excitations of the Z_2 Spin liquid



$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

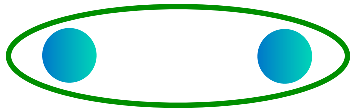
A vison



N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Excitations of the Z_2 Spin liquid

A vison



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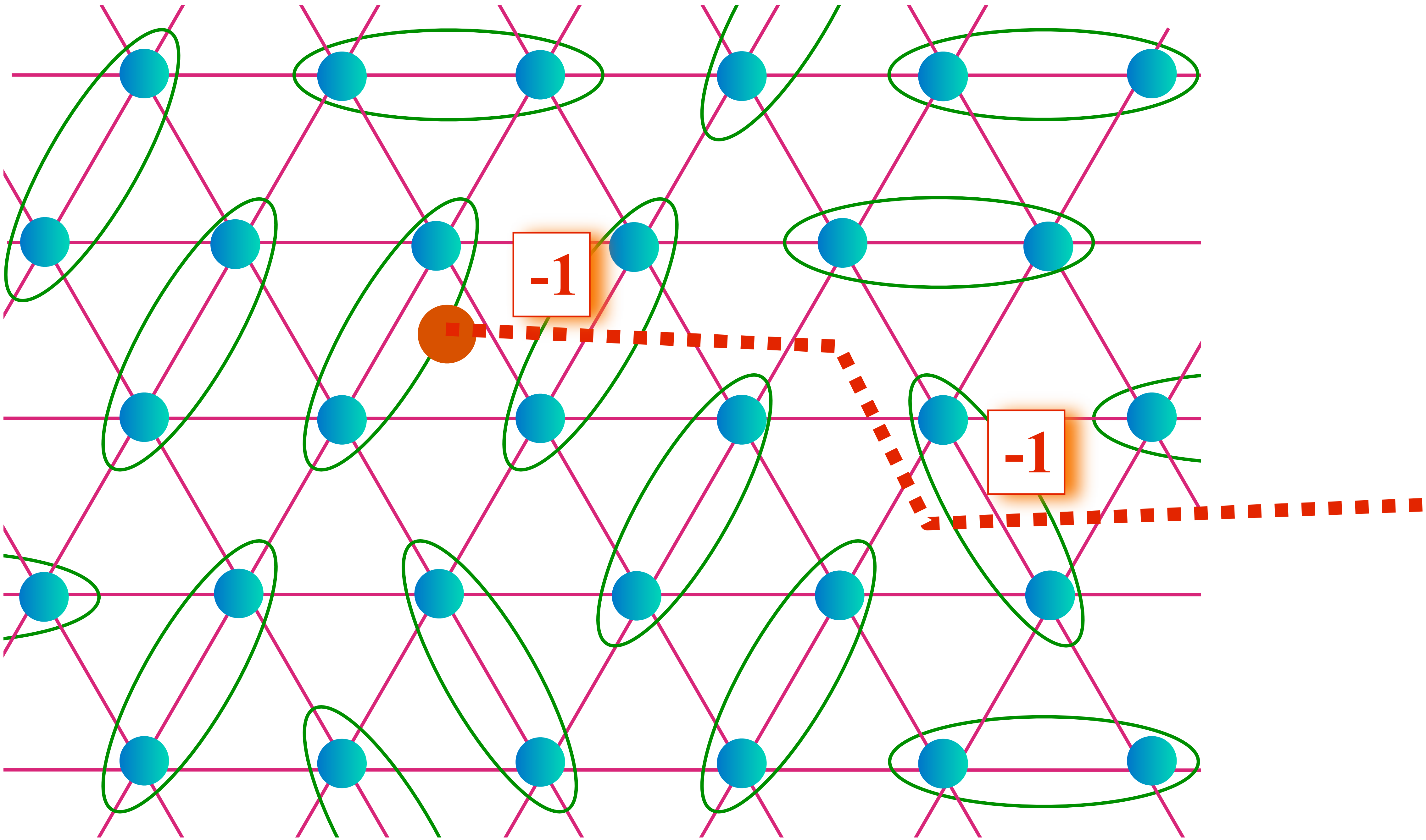
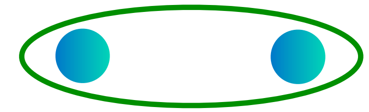


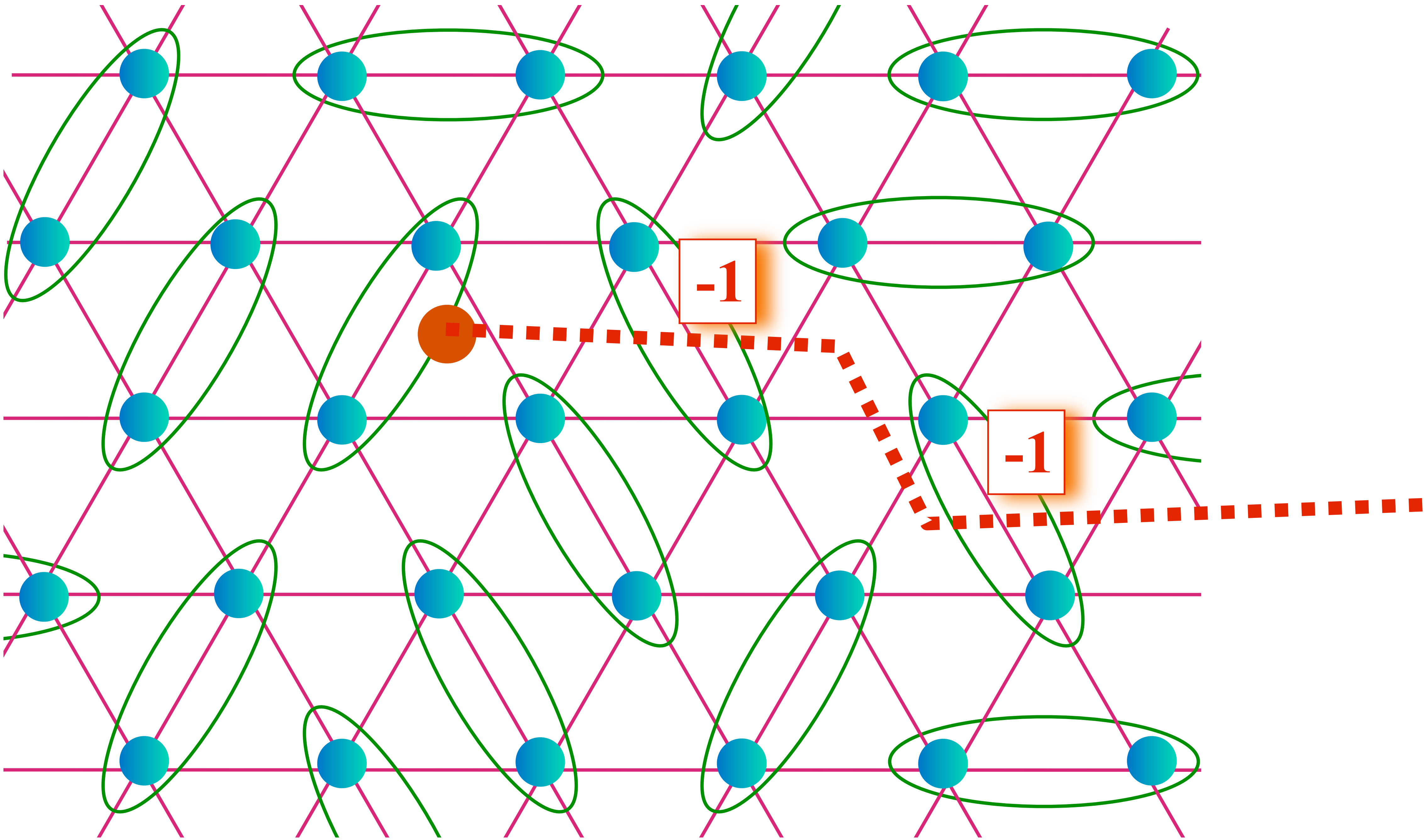
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Excitations of the Z_2 Spin liquid



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

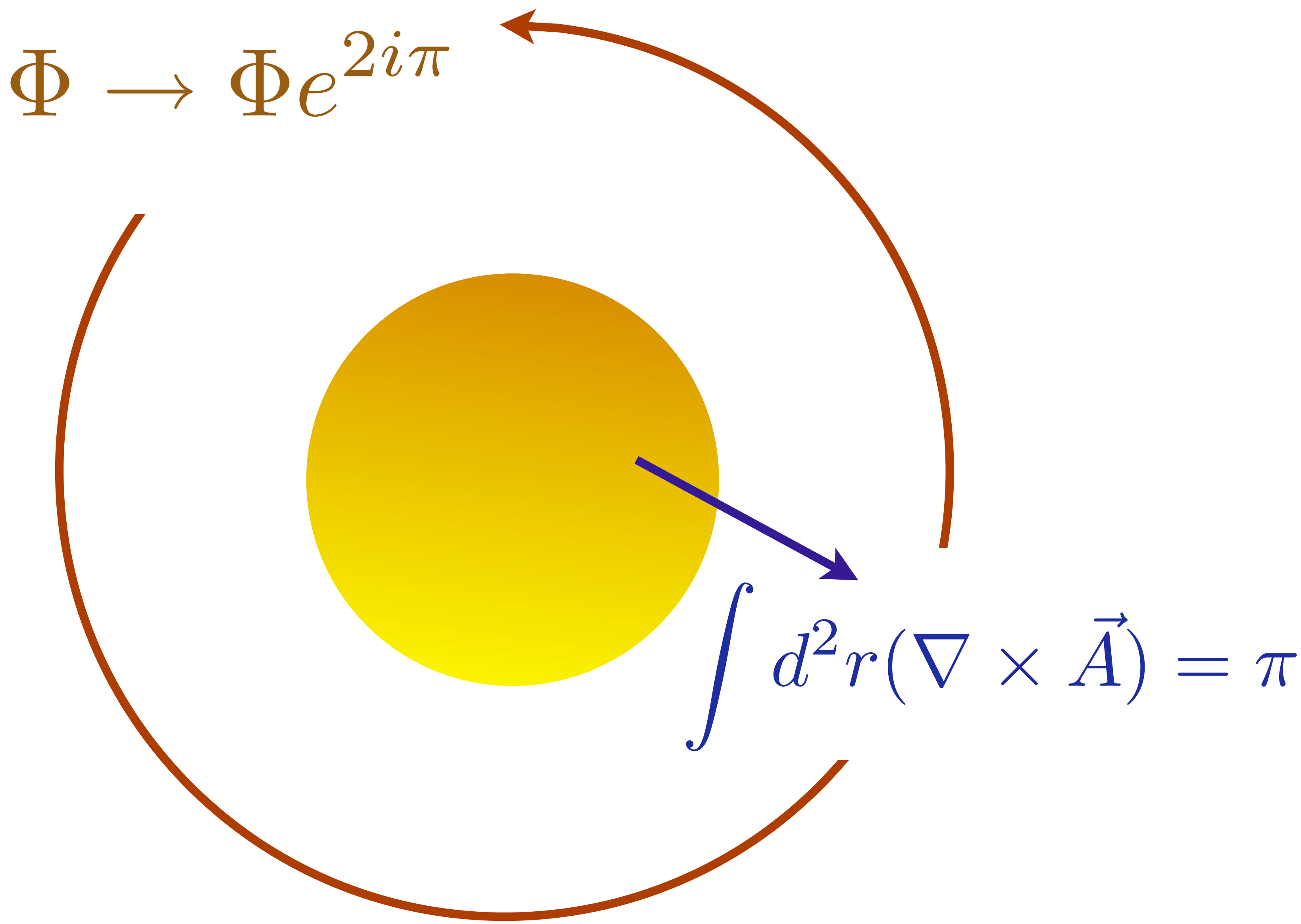
A vison



N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

Topological excitation of a Z_2 spin liquid (vison)

An Abrikosov vortex in the Φ condensate



Parallel transport of a gapped spinon z_α around a vison yields $z_\alpha \rightarrow -z_\alpha$.

Spinons and visons are **mutual semions**.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

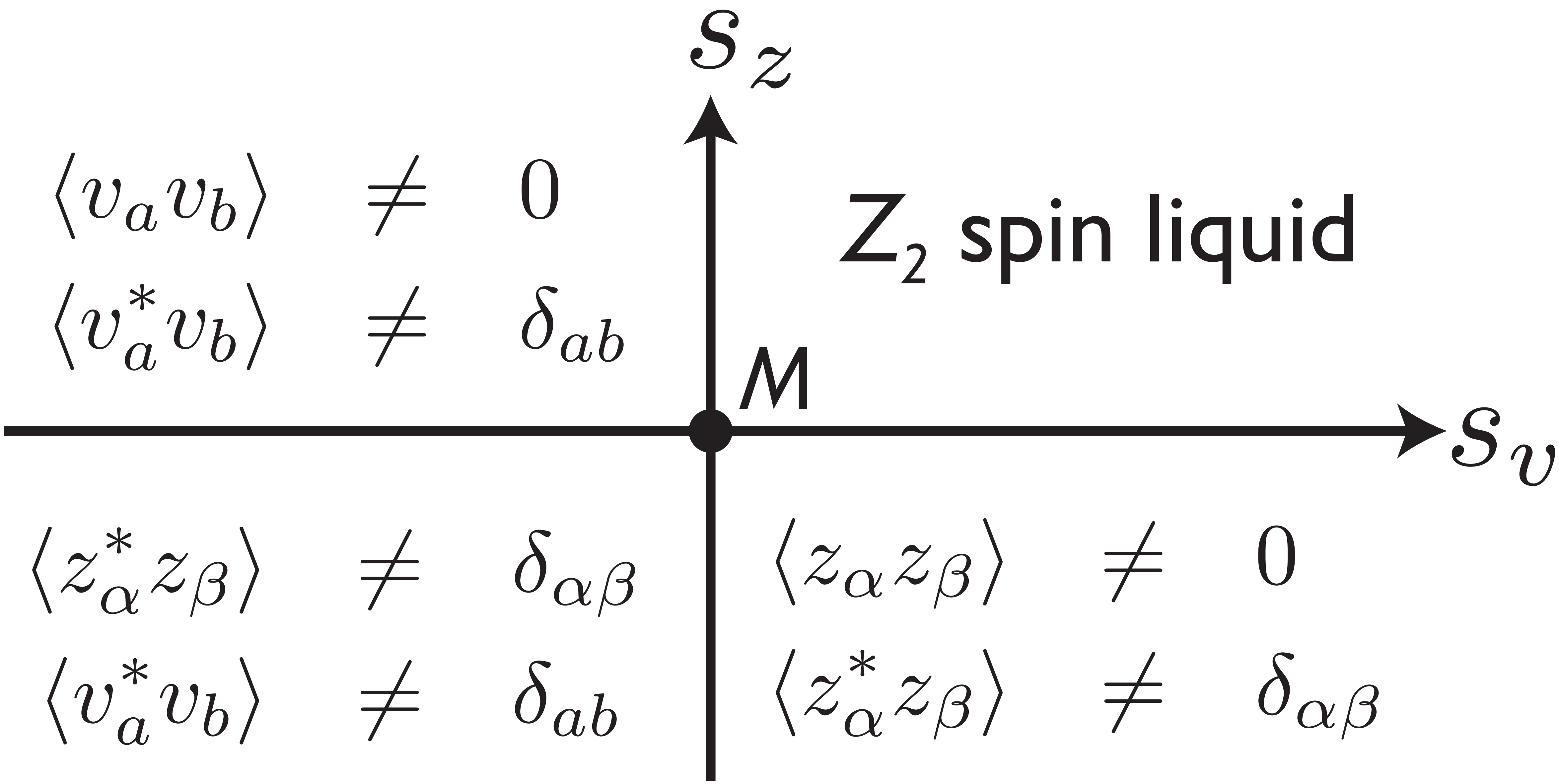
Mutual Chern-Simons Theory

Express theory in terms of the physical excitations of the Z_2 spin liquid: the spinons, z_α, and the visons. After accounting for Berry phase effects, the visons can be described by complex fields v_a, which transforms non-trivially under the square lattice space group operations.

A related Berry phase is the phase of -1 acquired by a spinon encircling a vortex. This is implemented in the following "mutual Chern-Simons" theory at $k = 2$:

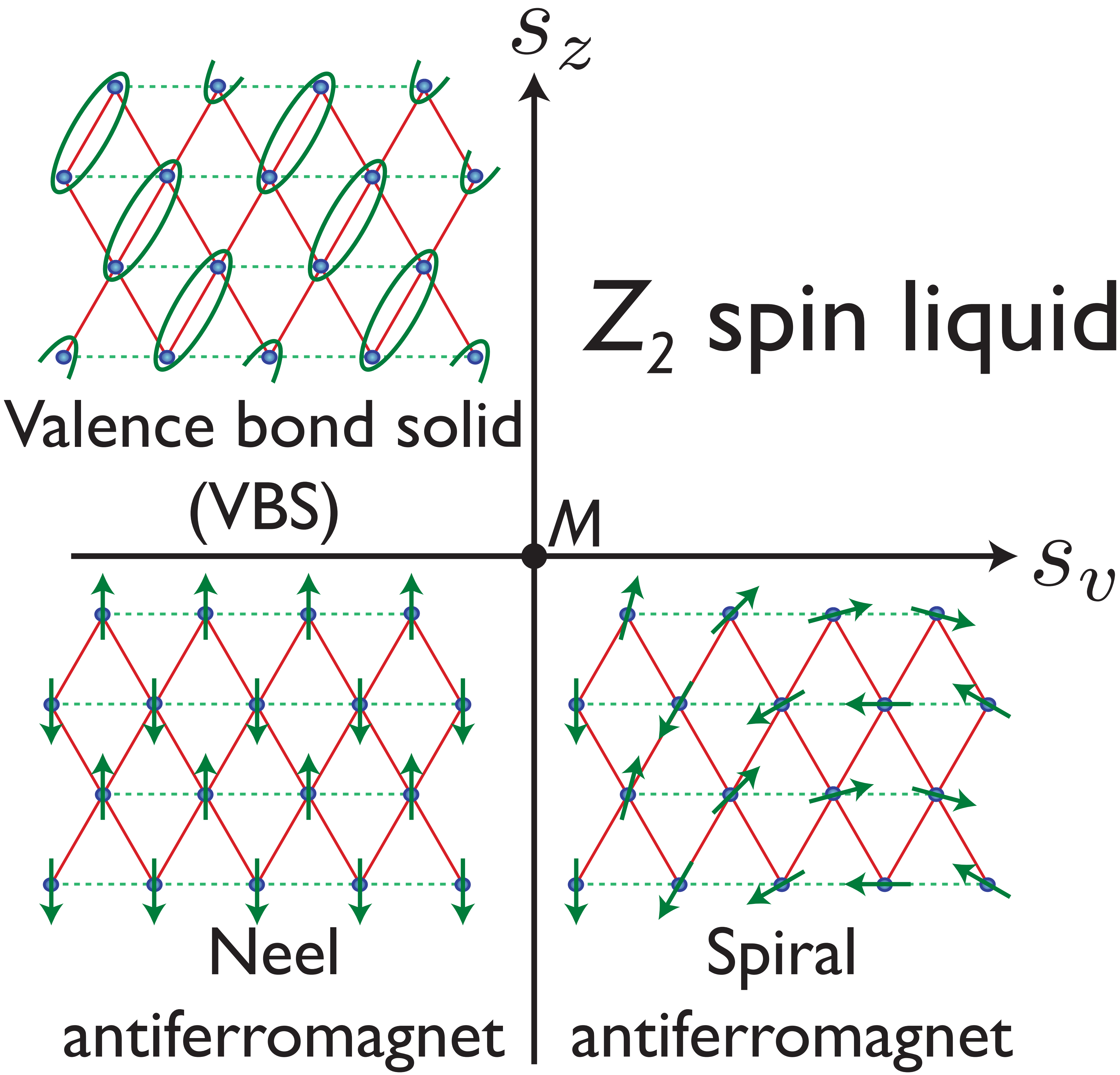
$$
\begin{aligned}
\mathcal{L} &= \sum_{\alpha=1}^{2} \left\{ |(\partial_\mu - i a_\mu) z_\alpha|^2 + s_z |z_\alpha|^2 + u_z (|z_\alpha|^2)^2 \right\} \\
&+ \sum_{a=1}^{N_v} \left\{ |(\partial_\mu - i b_\mu) v_a|^2 + s_v |v_a|^2 + u_v (|v_a|^2)^2 \right\} \\
&+ \frac{ik}{2\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda + \cdots
\end{aligned}
$$

Cenke Xu and S. Sachdev, *Phys. Rev. B* 79 064405 (2009)



Cenke Xu and S. Sachdev, *Phys. Rev. B* 79 064405 (2009)

Theoretical global phase diagram



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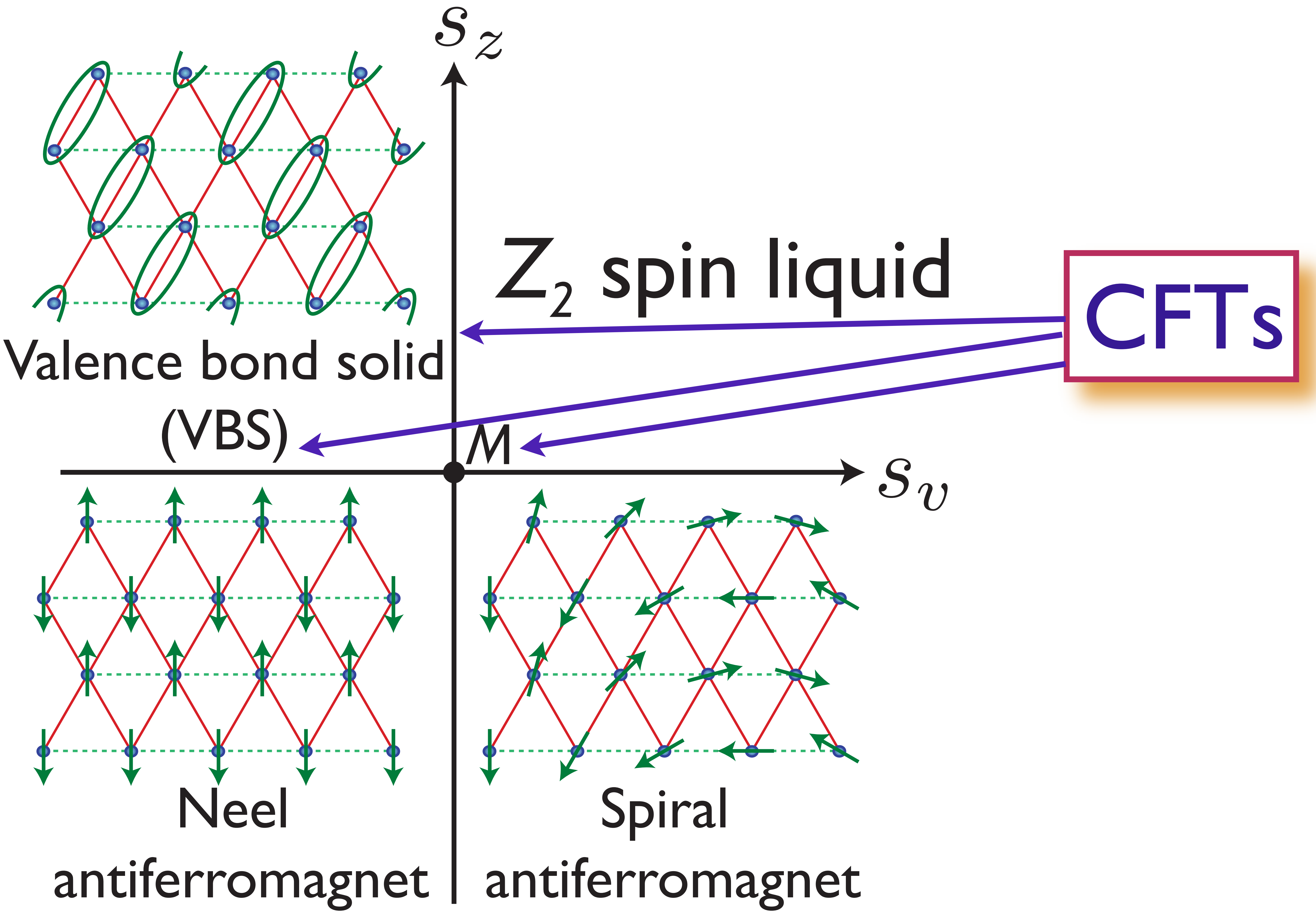
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).
Cenke Xu and S. Sachdev, *Phys. Rev. B* 79 064405 (2009)

Monday, November 16, 2009

Theoretical global phase diagram



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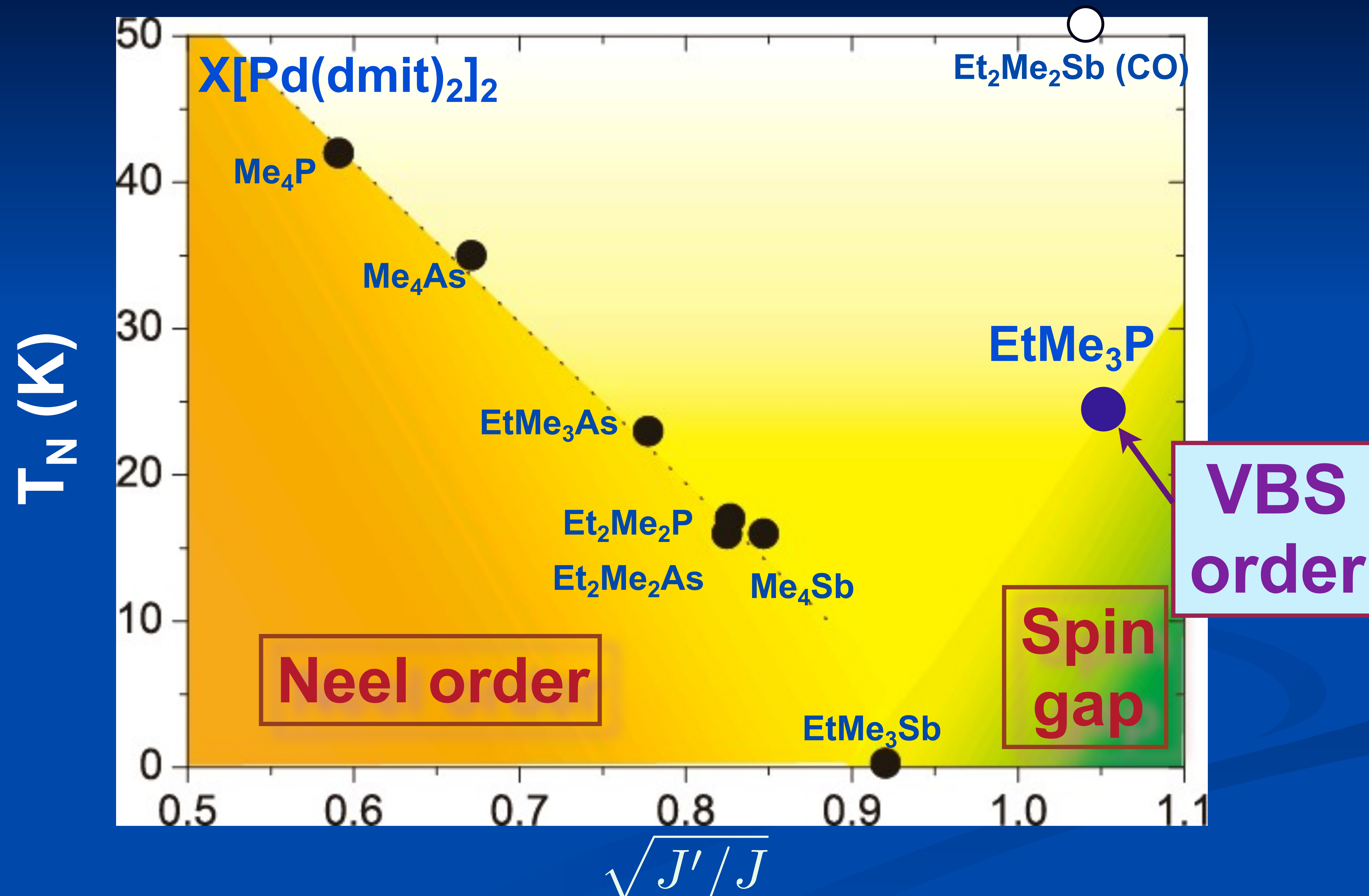
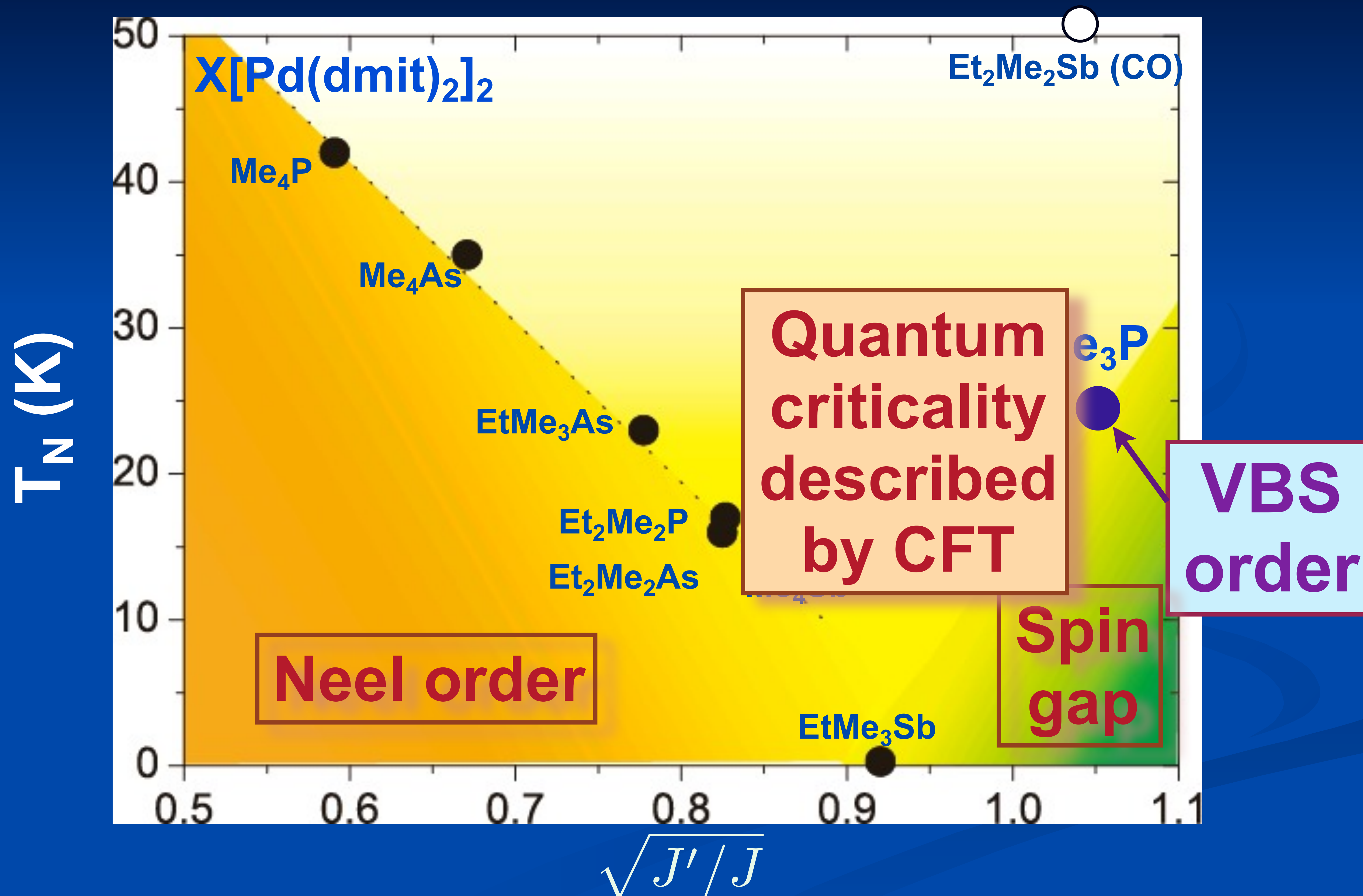
Cenke Xu and S. Sachdev, *Phys. Rev. B* 79 064405 (2009)

Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

From quantum antiferromagnets to string theory

A direct generalization of the CFT of the multicritical point M ($s_z = s_v = 0$) to $\mathcal{N} = 4$ supersymmetry and the U(N) gauge group was shown by O. Aharony, O. Bergman, D. L. Jafferis, J. Maldacena, JHEP **0810**, 091 (2008) to be dual to a theory of quantum gravity (M theory) on $AdS_4 \times S_7/Z_k$.

$$\begin{aligned}
\mathcal{L} \;&=\; \sum_{\alpha=1}^{2} \left\{ |(\partial_\mu - i a_\mu) z_\alpha|^2 + s_z |z_\alpha|^2 + u_z (|z_\alpha|^2)^2 \right\} \\
&+\; \sum_{a=1}^{N_v} \left\{ |(\partial_\mu - i b_\mu) v_a|^2 + s_v |v_a|^2 + u_v (|v_a|^2)^2 \right\} \\
&+\; \frac{ik}{2\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda + \cdots
\end{aligned}$$

Cenke Xu and S. Sachdev, *Phys. Rev. B* 79 064405 (2009)