VOLUME 78, NUMBER 12 PHYSICAL REVIEW LETTERS 24 MARCH 1997

Spin-Excitation-Instability-Induced Quantum Phase Transitions in Double-Layer Quantum Hall Systems

Lian Zheng, R. J. Radtke, and S. Das Sarma

Department of Physics, University of Maryland, College Park, Maryland 20742-4111

(Received 17 May 1996)

We study intersubband spin density collective modes in double-layer quantum Hall systems at $\nu = 2$ within the time-dependent Hartree-Fock approximation. We find that these intersubband spin density excitations may soften under experimentally accessible conditions, signaling a phase transition to a new quantum Hall state with interlayer inplane antiferromagnetic spin correlations. We show that this novel canted antiferromagnetic phase is energetically stable and that the phase transition is continuous.

VOLUME 79, NUMBER 5 PHYSICAL REVIEW LETTERS 4 AUGUST 1997

Double-Layer Quantum Hall Antiferromagnetism at Filling Fraction $\nu = 2/m$ where m is an Odd Integer

S. Das Sarma,[1] Subir Sachdev,[2] and Lian Zheng[1]

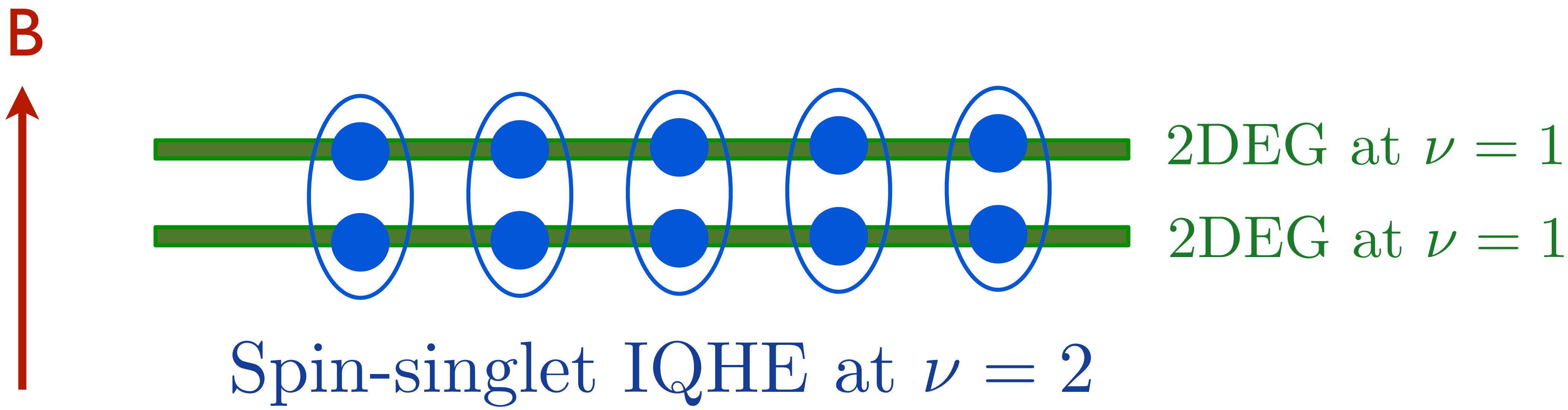
[1]*Department of Physics, University of Maryland, College Park, Maryland*

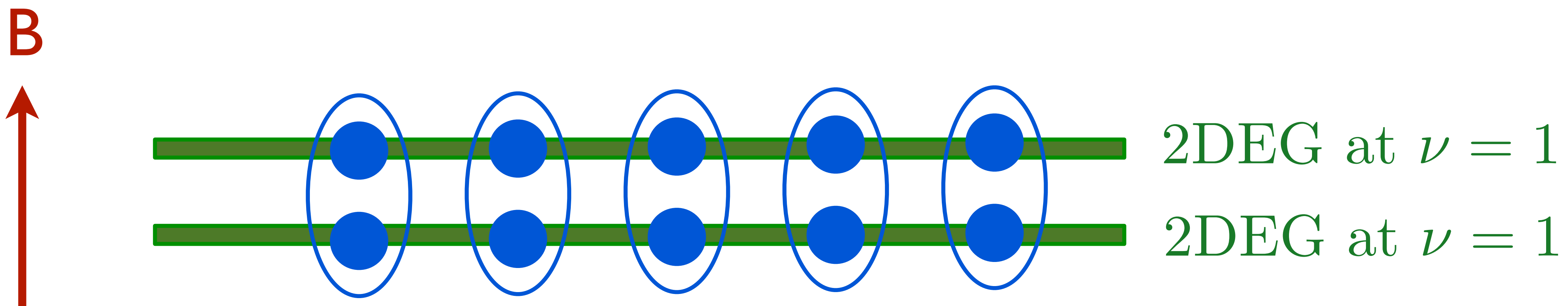
[2]*Department of Physics, Yale University, P.O. Box 208120, New Haven, Conne*

(Received 16 January 1997)

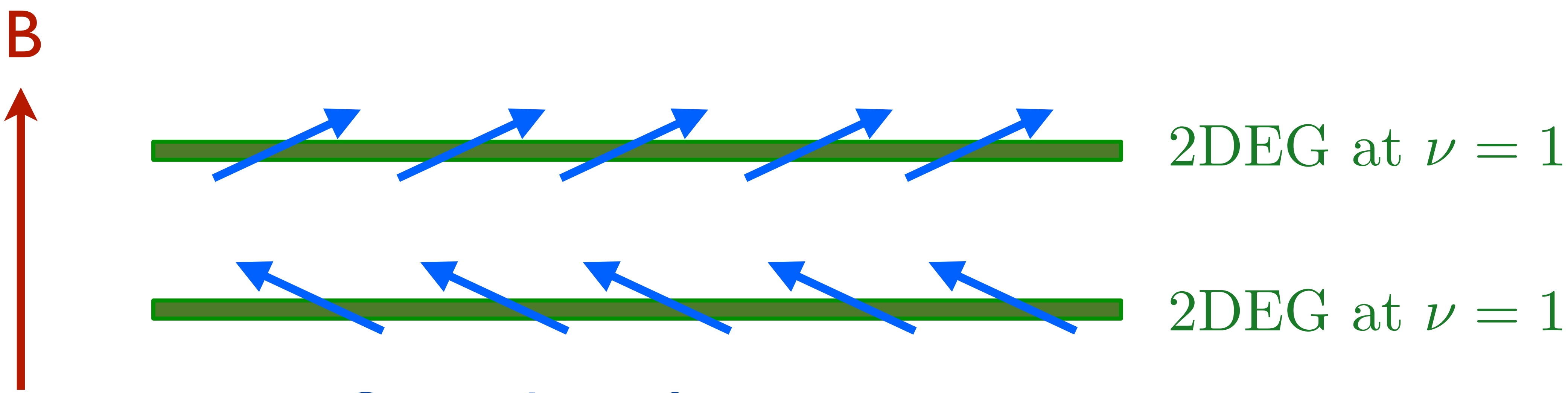
A low energy action for double-layer quantum Hall systems at filling fractio
m is an odd integer) is introduced. Interlayer antiferromagnetic exchange in
canted spin order, as well as a spin-singlet phase. Universal properties of zero a
transitions are obtained. We compute the critical temperature at which the cantec
Kosterlitz-Thouless transition. Implications for recent light scattering experiments

Sunday, March 17, 13

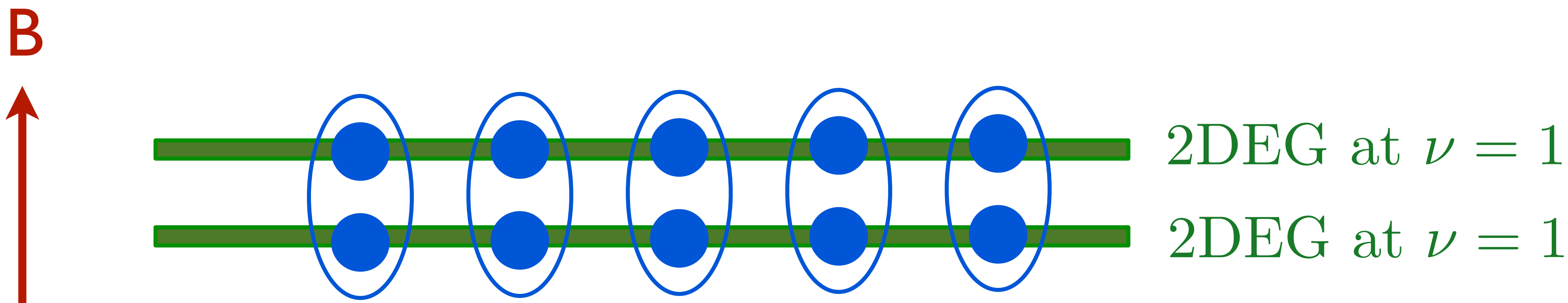

B
2DEG at $\nu = 1$
2DEG at $\nu = 1$
Spin-singlet IQHE at $\nu = 2$



Spin-singlet IQHE at $\nu = 2$

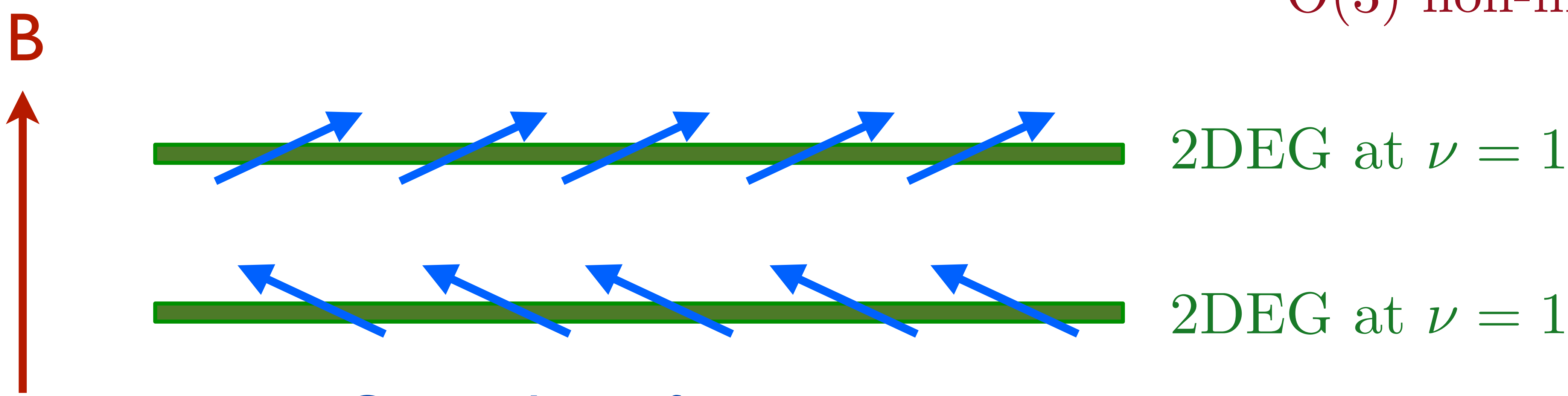


Canted antiferromagnet

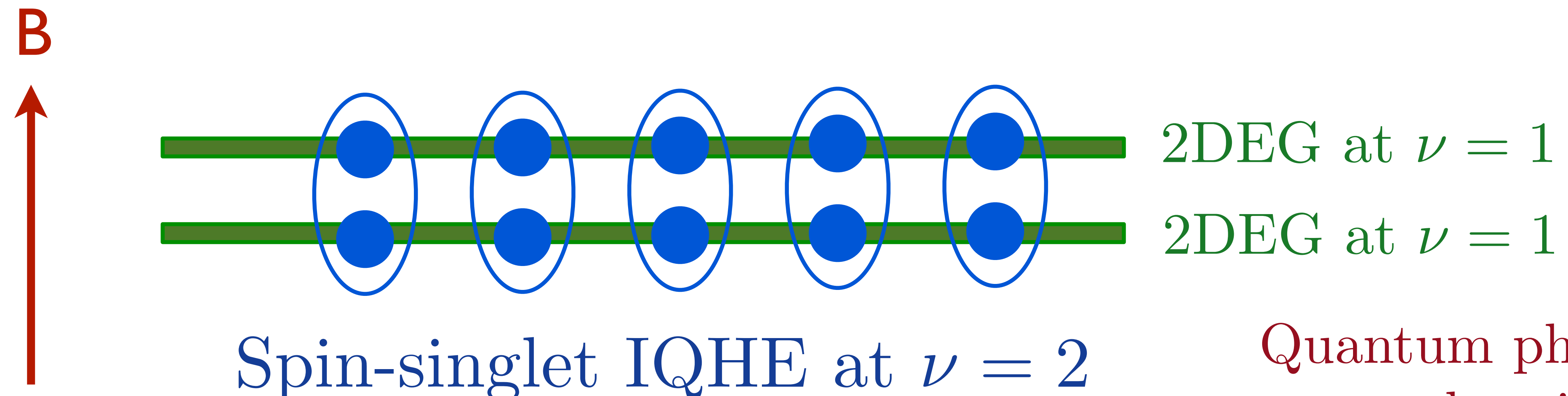


Spin-singlet IQHE at $\nu = 2$

Quantum phase transition
described by
O(3) non-linear σ model

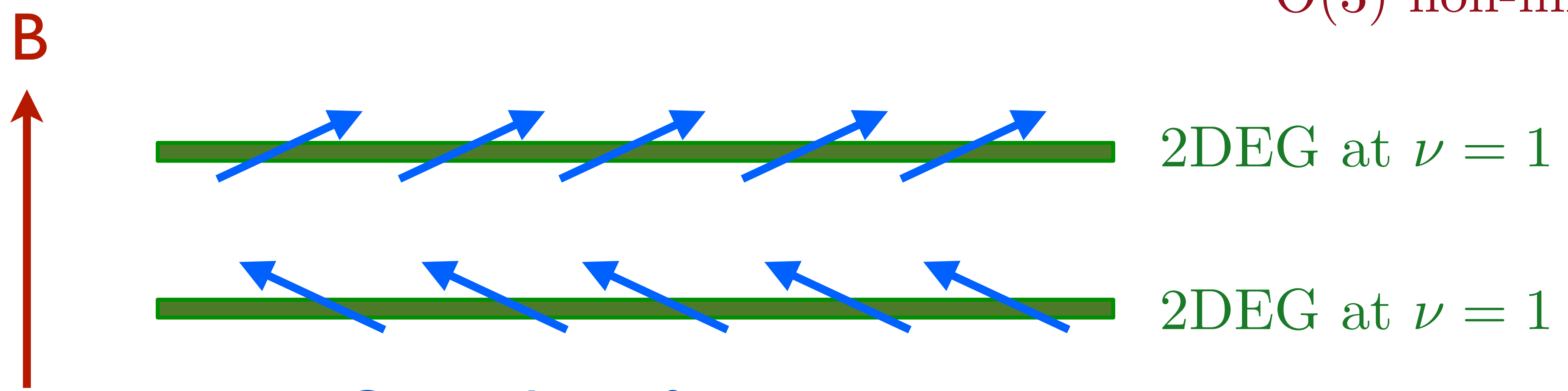


Canted antiferromagnet



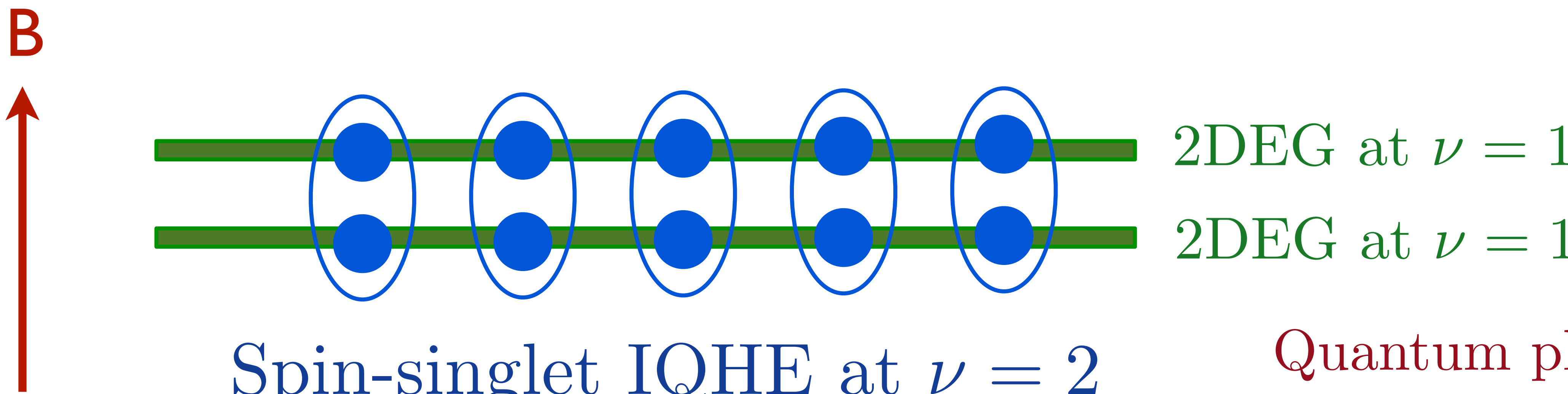
Spin-singlet IQHE at $\nu = 2$

Quantum phase transition
described by
O(3) non-linear σ model



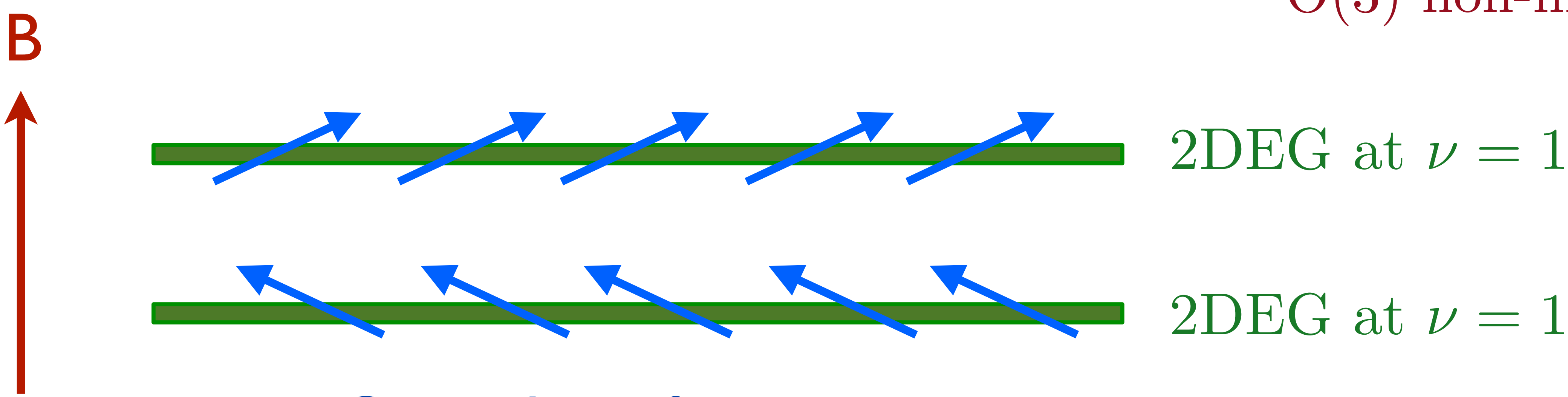
Canted antiferromagnet

Original paper: 4 pages



Spin-singlet IQHE at $\nu = 2$

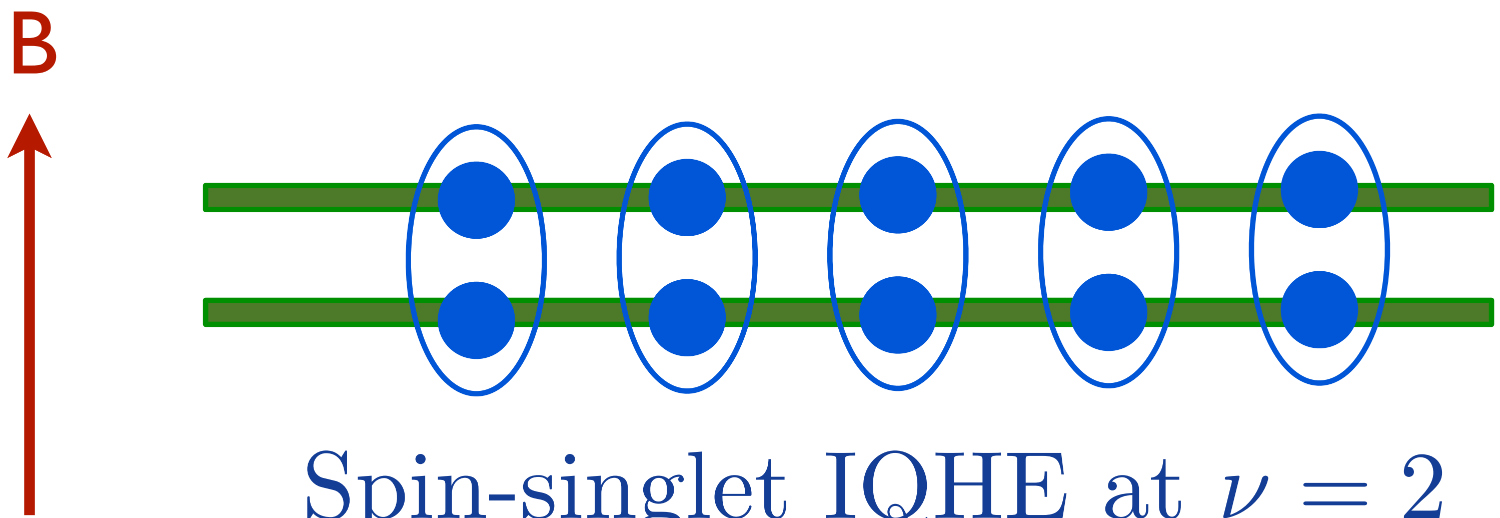
Quantum phase transition
described by
O(3) non-linear σ model



Canted antiferromagnet

Original paper: 4 pages

Referee report: 8 pages (in small print)



2DEG at $\nu = 1$

2DEG at $\nu = 1$

Spin-singlet IQHE at $\nu = 2$

Quantum phase transition
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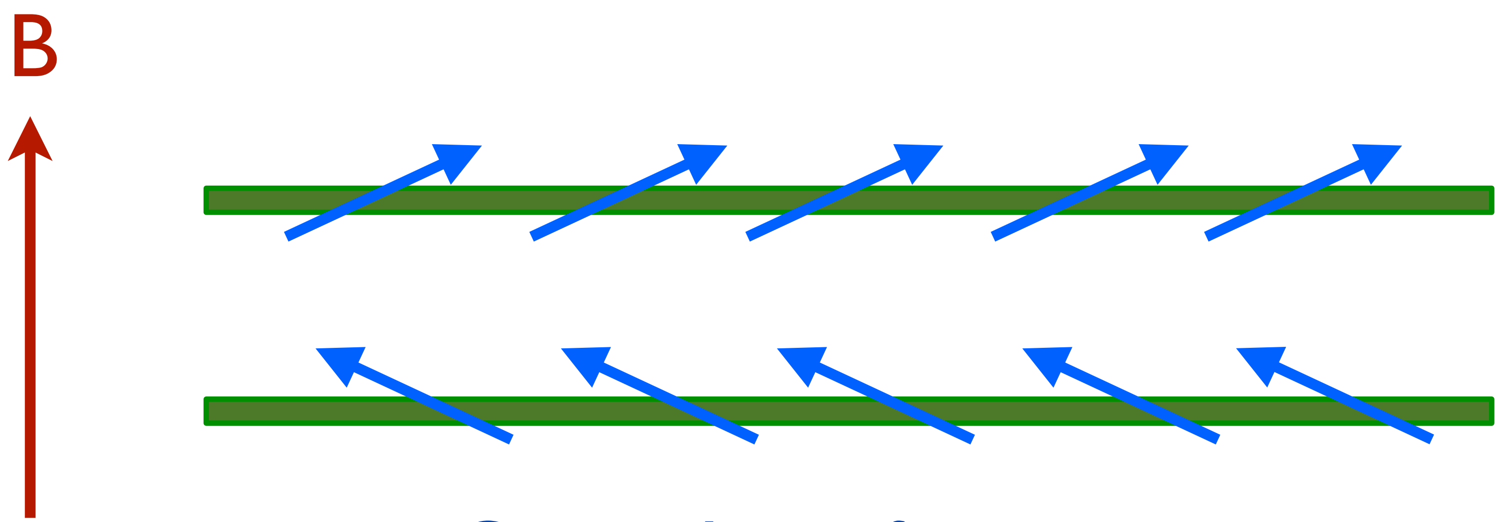
O(3) non-linear σ model



2DEG at $\nu = 1$

2DEG at $\nu = 1$

Canted antiferromagnet

Original paper: 4 pages

Referee report: 8 pages (in small print)

Sankar's *response*: **16** *pages* (in **1 day** with ***extensive punctuation*** !!!)

PERGAMON

Solid State Communications 119 (2001) 301–308

solid state communications

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Excitonic spin instabilities and continuous phase transitions in quantum Hall magnets

Vittorio Pellegrini[a,*], Aron Pinczuk[b,c]

[a]*Scuola Normale Superiore, INFM, Piazza dei Cavalieri 7, I-56126 Pisa, Italy*
[b]*Departments. of Physics, Appl. & Appl. Mathematics, Columbia University, New York, NY 10027 USA*
[c]*Bell Laboratories, Lucent Technologies, Murray Hill, NJ 07974, USA*

Low-Frequency Spin Dynamics in a Canted Antiferromagnet

Norio Kumada,[1*] Koji Muraki,[1] Yoshiro Hirayama[1,2,3]

Resistively detected nuclear spin relaxation measurements in closely separated two-dimensional electron systems reveal strong low-frequency electron-spin fluctuations in the quantum Hall regime. As the temperature is decreased, the spin fluctuations, manifested by a sharp enhancement of the nuclear spin-lattice relaxation rate $1/T_1$, continue to grow down to the lowest temperature of 66 millikelvin. The observed divergent behavior of $1/T_1$ signals a gapless spin excitation mode and is a hallmark of canted antiferromagnetic order. Our data demonstrate the realization of a two-dimensional system with planar broken symmetry, in which fluctuations do not freeze out when approaching the zero temperature limit.

SCIENCE VOL 313 21 JULY 2006

PHYSICAL REVIEW B **66**, 045318 (2002)

Tunneling gap collapse and $\nu = 2$ quantum Hall state in a bilayer electron system

S. J. Geer, A. G. Davies, C. H. W. Barnes, K. R. Zolleis, M. Y. Simmons, and D. A. Ritchie
Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge, CB3 OHE, United Kingdom

(Received 22 February 2002; published 24 July 2002)

PHYSICAL REVIEW B **73**, 165304 (2006)

Magnetotransport study of the canted antiferromagnetic phase in bilayer $\nu=2$ quantum Hall state

A. Fukuda* and A. Sawada
Research Center for Low Temperature and Materials Sciences, Kyoto University, Kyoto 606-8502, Japan

S. Kozumi, D. Terasawa, Y. Shimoda, and Z. F. Ezawa
Graduate School of Science, Department of Physics, Tohoku University, Sendai 980-8578, Japan

N. Kumada and Y. Hirayama
NTT Basic Research Laboratories, NTT Corporations, 3-1 Morinosato-Wakamiya, Atsugi 243-0198, Japan
(Received 28 December 2005; published 3 April 2006)

PHYSICAL REVIEW B **78**, 233305 (2008)

Observation of an in-plane magnetic-field-driven phase transition in a quantum Hall system with SU(4) symmetry

G. P. Guo,[1] Y. J. Zhao,[1] T. Tu,[1,*] X. J. Hao,[1] X. C. Zhang,[2] G. C. Guo,[1] and H. W. Jiang[2,†]

[1]*Key Laboratory of Quantum Information, Chinese Academy of Sciences, University of Science and Technology of China, Hefei 230026, People's Republic of China*

[2]*Department of Physics and Astronomy, University of California–Los Angeles, 405 Hilgard Avenue, Los Angeles, California 90095, USA*

(Received 22 July 2008; revised manuscript received 20 October 2008; published 9 December 2008)

Canted Antiferromagnetic Phase in a Double Quantum Well in a Tilted Quantizing Magnetic Field

V. S. Khrapai, E. V. Deviatov, A. A. Shashkin, and V. T. Dolgopolov
Institute of Solid State Physics, Chernogolovka, Moscow District 142432, Russia

F. Hastreiter and A. Wixforth
Ludwig-Maximilians-Universität, Geschwister-Scholl-Platz 1, D-80539 München, Germany

K. L. Campman and A. C. Gossard
Materials Department and Center for Quantized Electronic Structures, University of California, Santa Barbara, California 93106
(Received 29 March 1999)

PRL **99,** 076805 (2007) PHYSICAL REVIEW LETTERS week ending 17 AUGUST 2007

NMR Evidence for Spin Canting in a Bilayer $\nu = 2$ Quantum Hall System

N. Kumada,[1] K. Muraki,[1] and Y. Hirayama[1,2,3]

[1]*NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi 243-0198, Japan*
[2]*SORST-JST, 4-1-8 Honmachi, Kawaguchi, Saitama 331-0012, Japan*
[3]*Department of Physics, Tohoku University, Sendai 980-8578, Japan*

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Physica E 34 (2006) 77–80
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Quantum transport study of canted antiferromagnetic phase in the $\nu = 2$ bilayer quantum Hall state

Akira Fukuda[a,*], Shinsuke Kozumi[b], Daiju Terasawa[b], Yuichi Shimoda[b], Norio Kumada[c], Yoshiro Hirayama[c], Zyun F. Ezawa[b], Anju Sawada[a]

[a]*Research Center for Low Temperature and Materials Sciences, Kyoto University, Sakyo, 606-8502 Kyoto, Japan*
[b]*Graduate School of Science, Tohoku University, Aramaki-Aoba, Aoba, 980-8578 Sendai, Japan*
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Sunday, March 17, 13

PHYSICAL REVIEW B, VOLUME 63, 041305(R)

Spin effects in the magnetodrag between double quantum wells

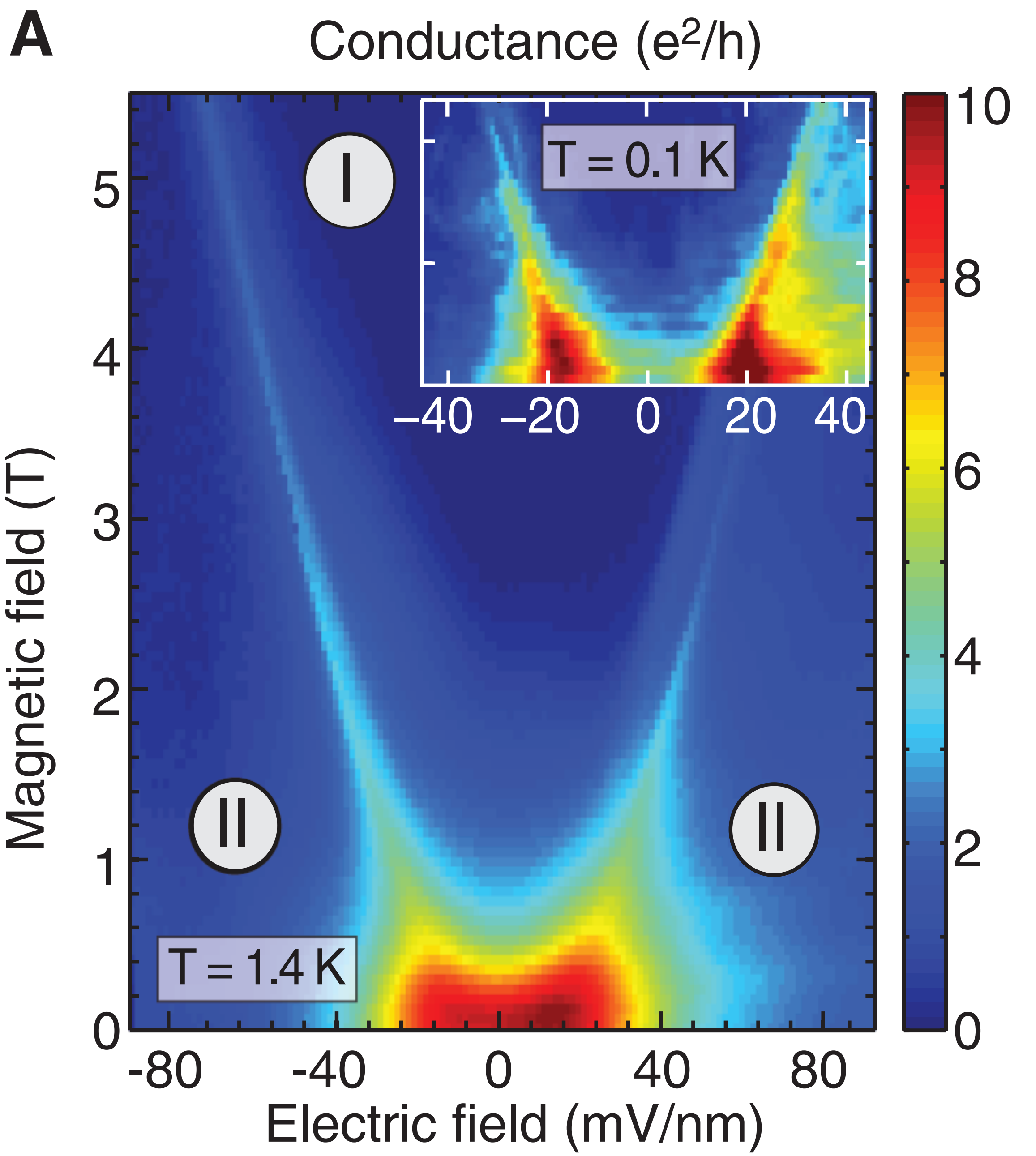
J. G. S. Lok,[1] S. Kraus,[1] M. Pohlt,[1] W. Dietsche,[1] K. von Klitzing,[1] W. Wegscheider,[2,3] and M. Bichler[2]
[1]*Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, 70569 Stuttgart, Germany*
[2]*Walter Schottky Institut, Technische Universität München, 85748 Garching, Germany*
[3]*Institut für angewandte und experimentelle Physik, Universität Regensburg, 93040 Regensburg, Germany*
(Received 2 November 2000; published 9 January 2001)

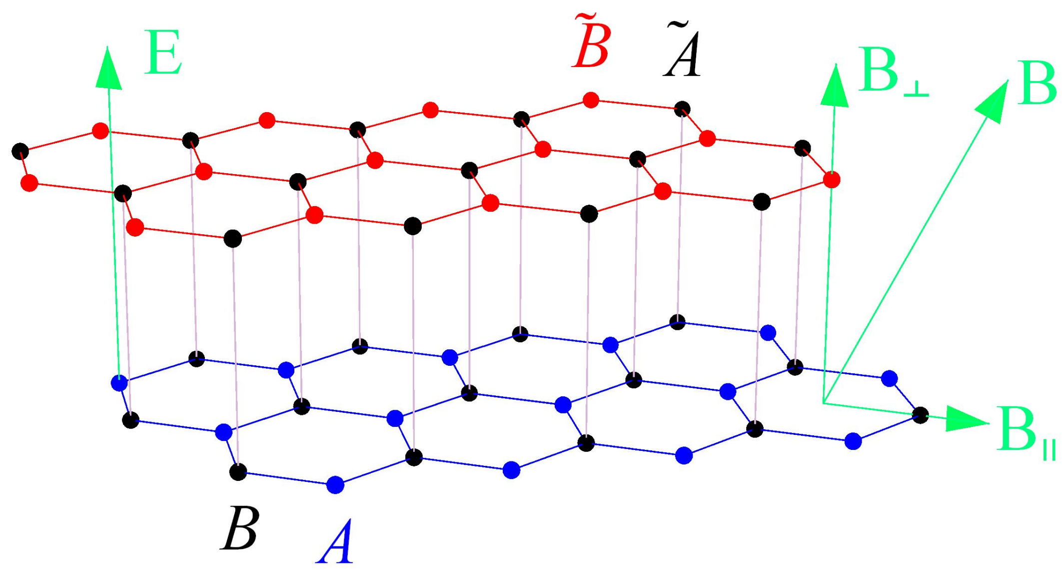
Sunday, March 17, 13

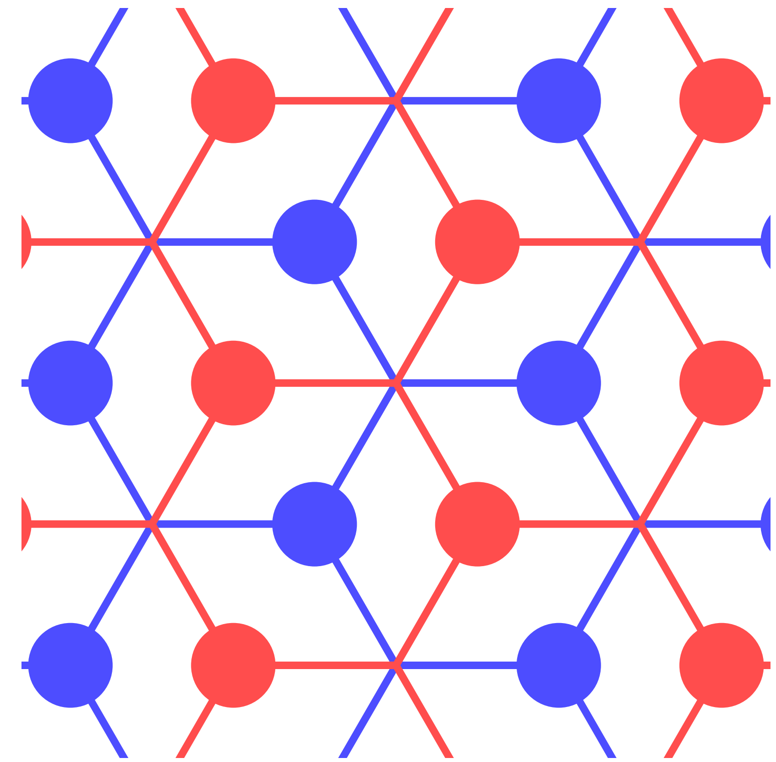
Broken-Symmetry States in Doubly Gated Suspended Bilayer Graphene

R. T. Weitz, M. T. Allen, B. E. Feldman, J. Martin, A. Yacoby*

Science **330**, 812 (2010)







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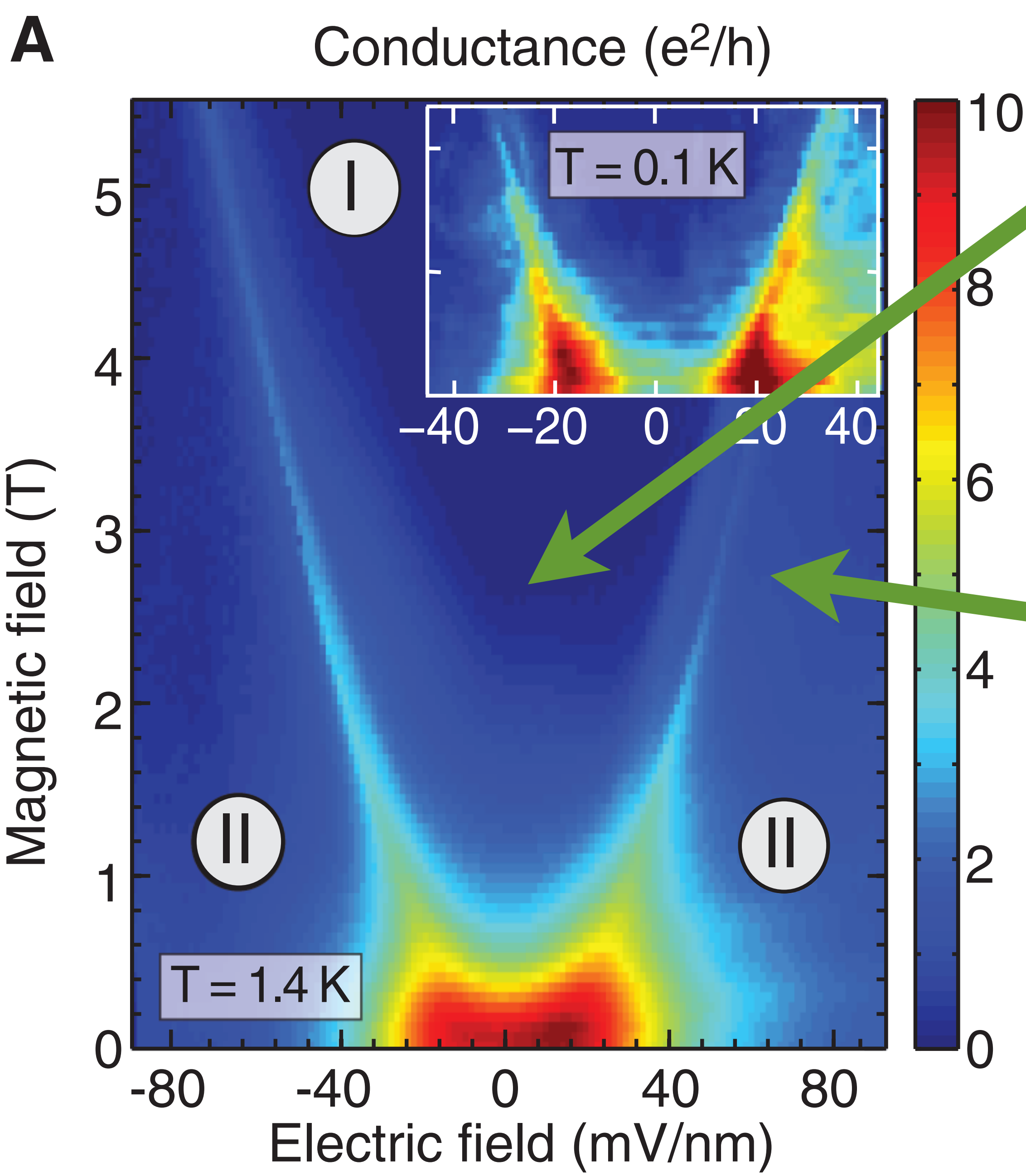
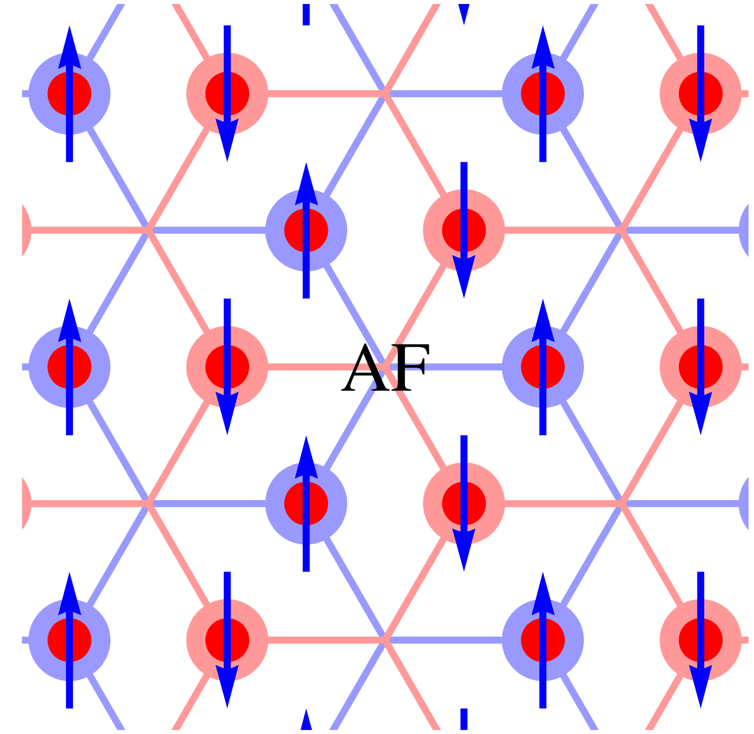
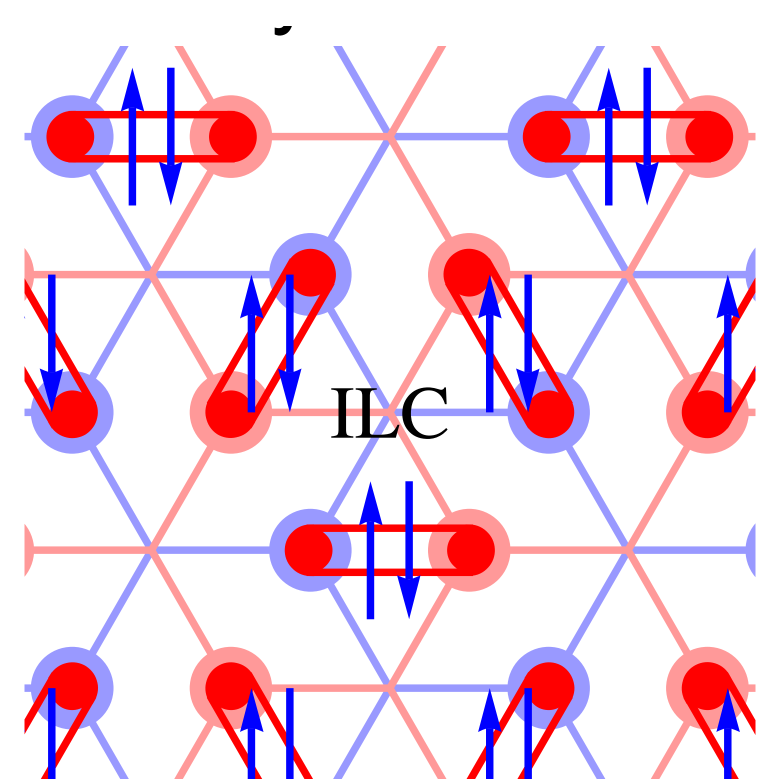


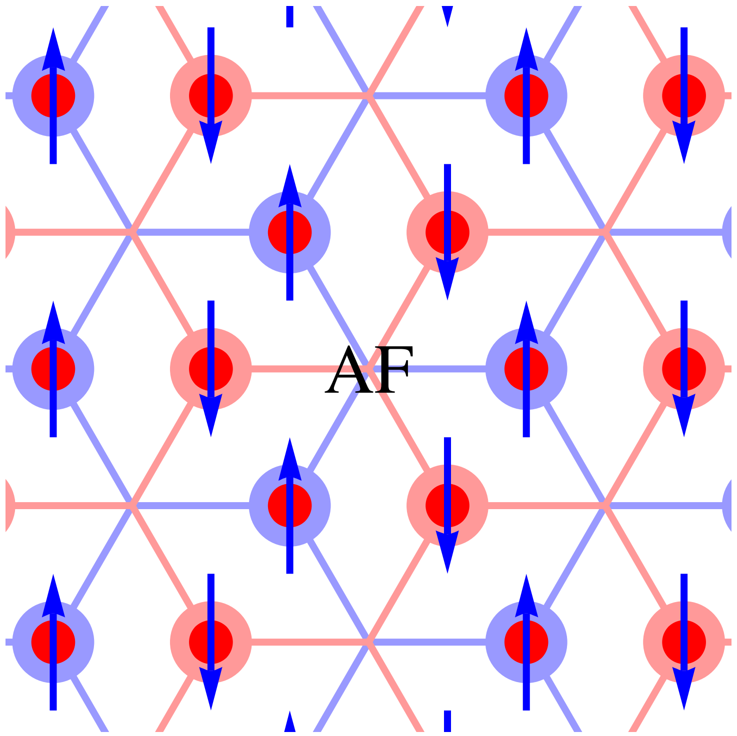


Canted antiferromagnet

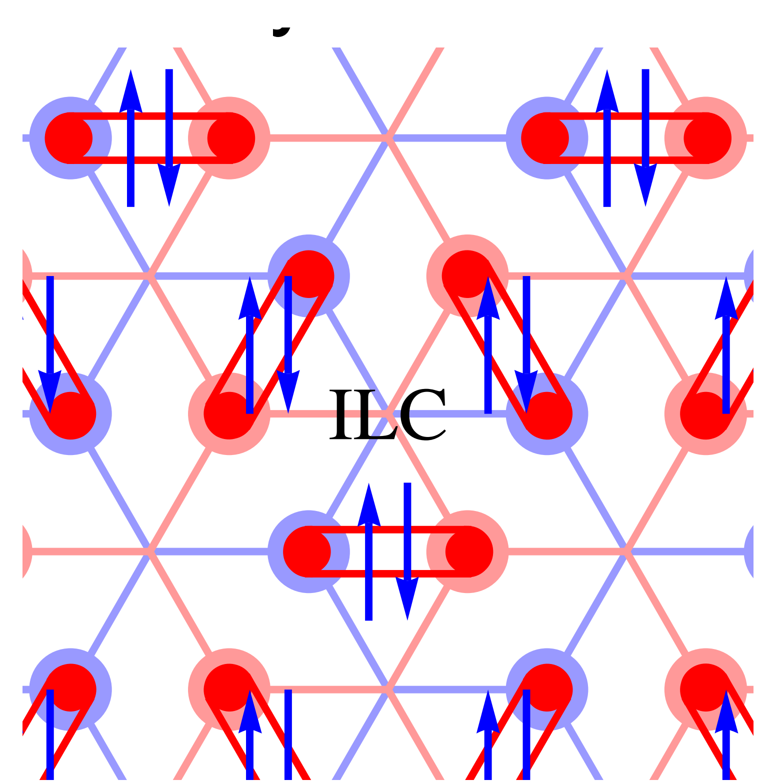


Kekule valence bond solid

Bilayer graphene



Canted antiferromagnet



Kekule valence bond solid

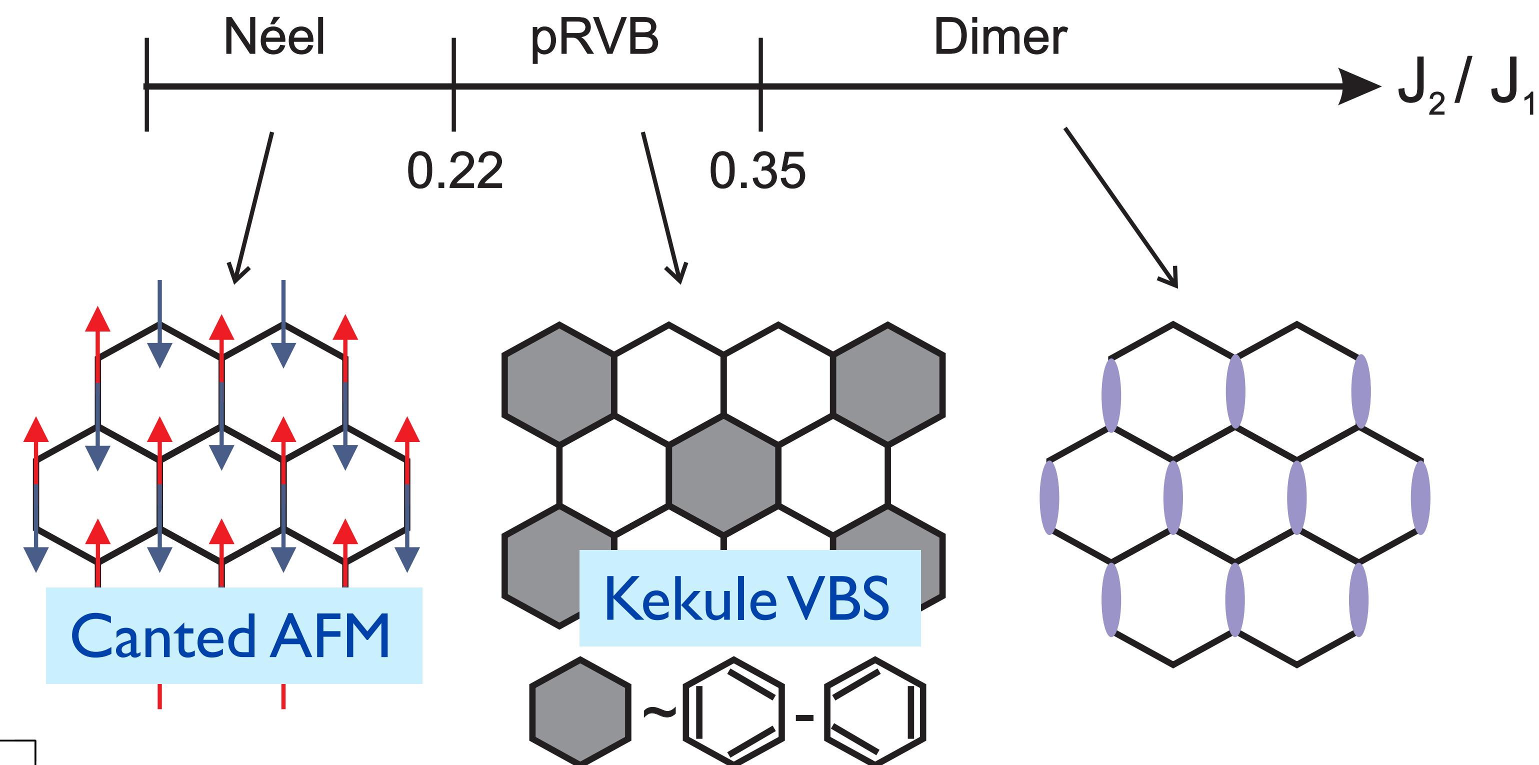
Quantum phase transition described by *deconfined criticality*: Theory of bosonic $S = 1/2$ spinons and U(1) gauge field.

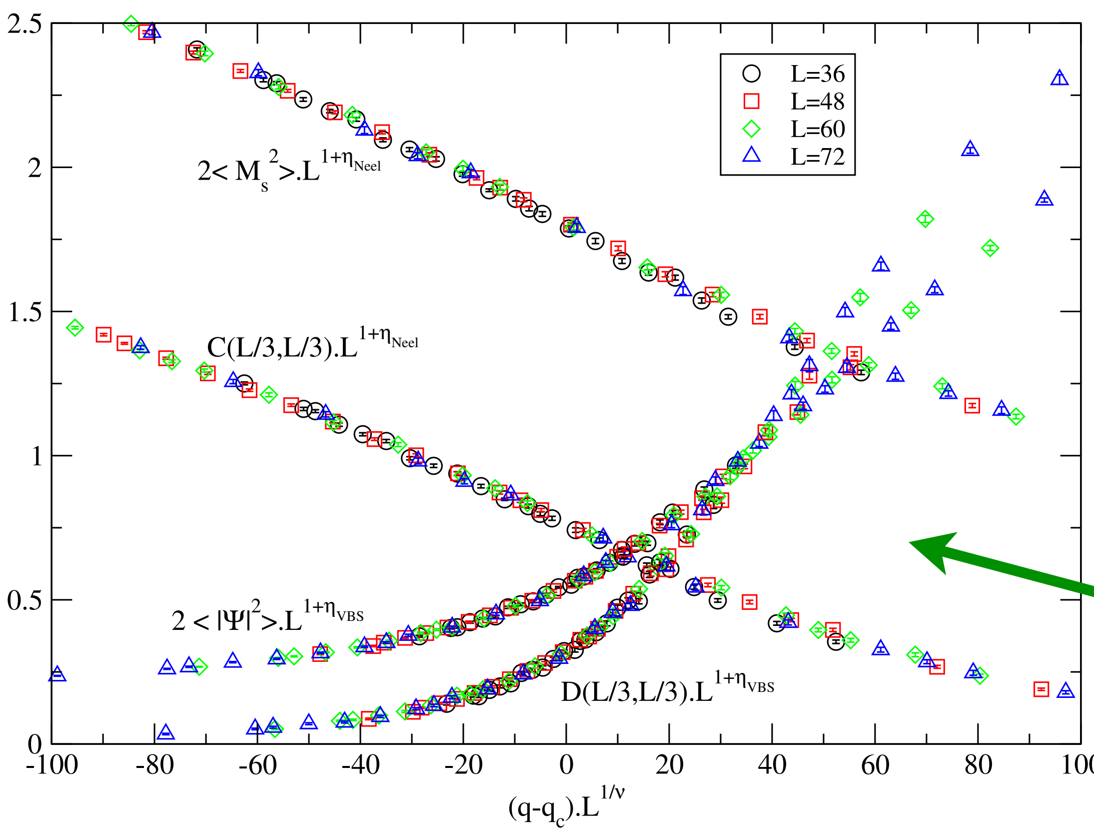
N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990)
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science **303**, 1490 (2004)
Junhyun Lee and S. Sachdev, to appear

Numerical studies of the *S*=1/2 antiferromagnet on the honeycomb lattice

evidence for the antiferromagnet, kekule order, and deconfined criticality

$$H = J_1 \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j$$





Z. Zhu, D.A. Huse, and S. R. White, arXiv:1212.6322

R. Ganesh, J. van den Brink, S. Nishimoto, arXiv:1301.0853

K. Damle, F. Alet, and S. Pujari, (J-Q model) arXiv:1302.1408

Progress on the physics of the underdoped cuprates

An open world of physics: A celebration of Sankar Das Sarma's research career on his 60th birthday

University of Maryland, College Park,
March 16, 2013

Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

Max Metlitski

Erez Berg

PHYSICS

VE RI TAS

HARVARD

Outline

1. The "modern era" of cuprate experiments

2. Antiferromagnetism in metals:
   d-wave superconductivity

3. Low energy theory, emergent pseudospin
   symmetry, and bond order

4. Unrestricted Hartree-Fock-BCS

5. Quantum Monte Carlo
   without the sign problem

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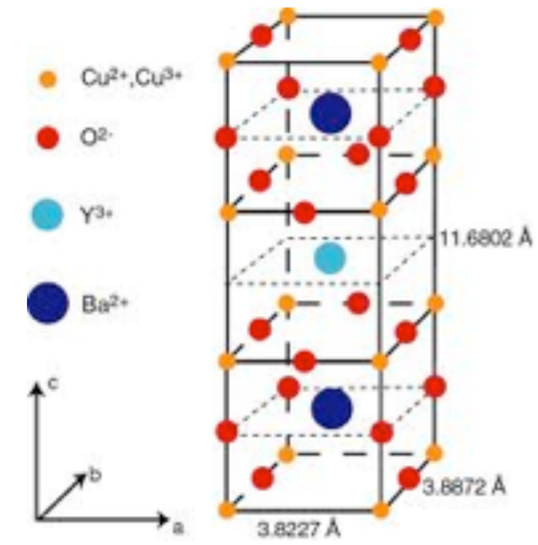
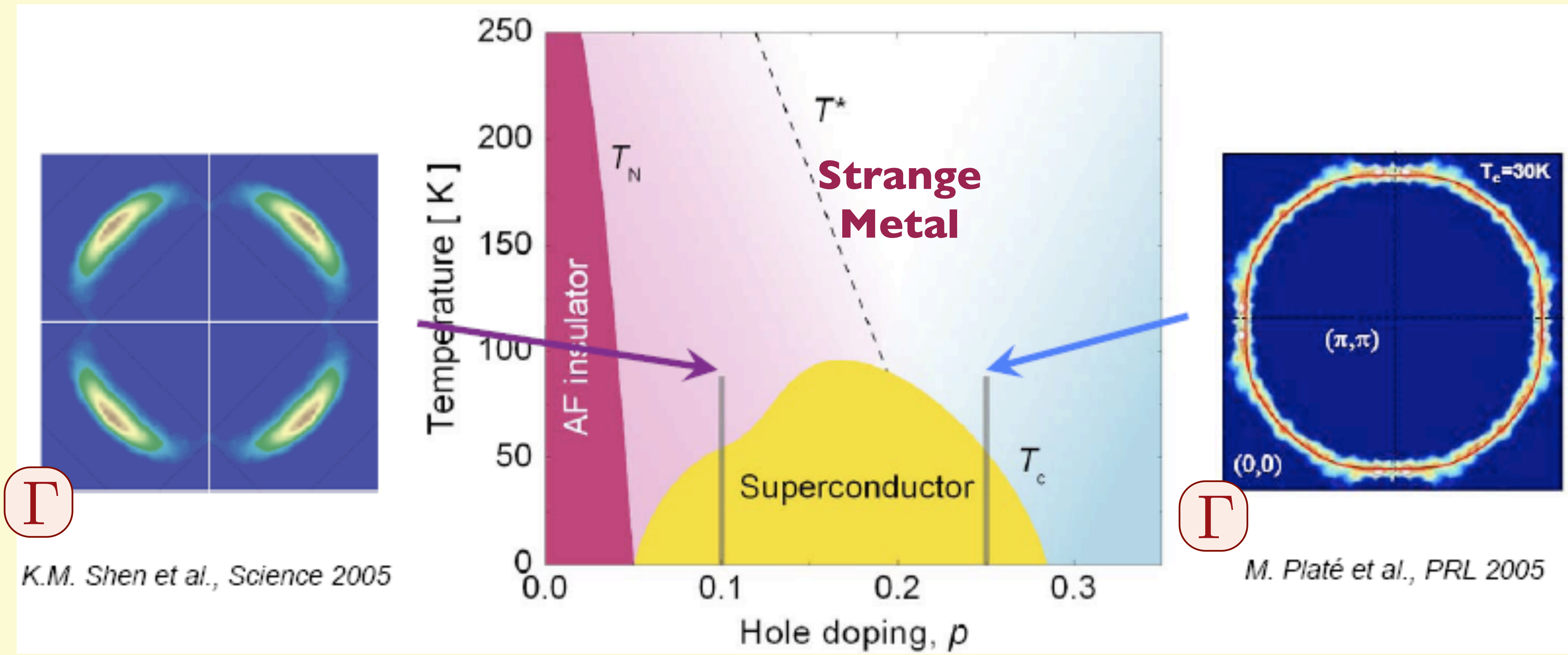




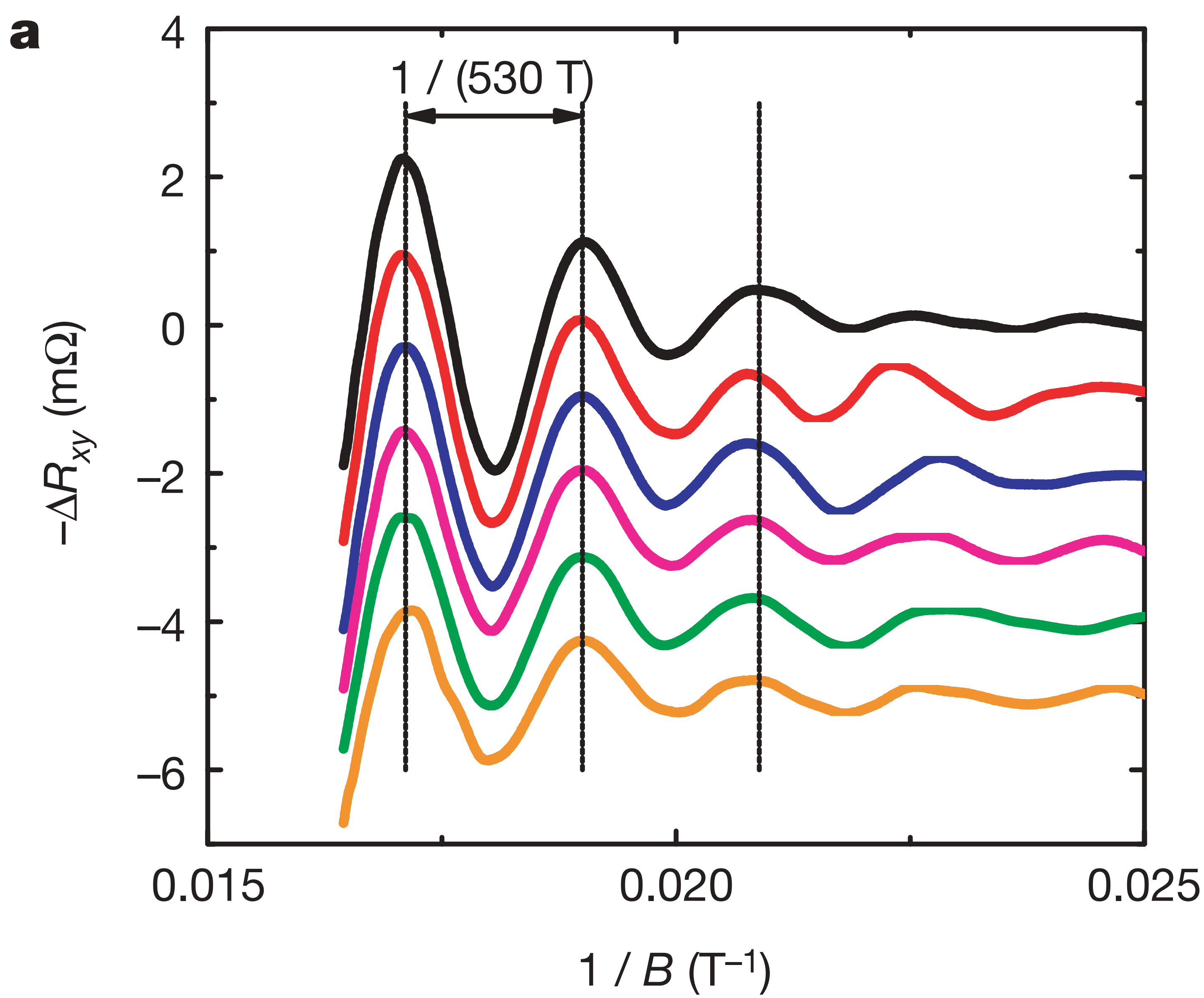
Smaller hole Fermi-pockets

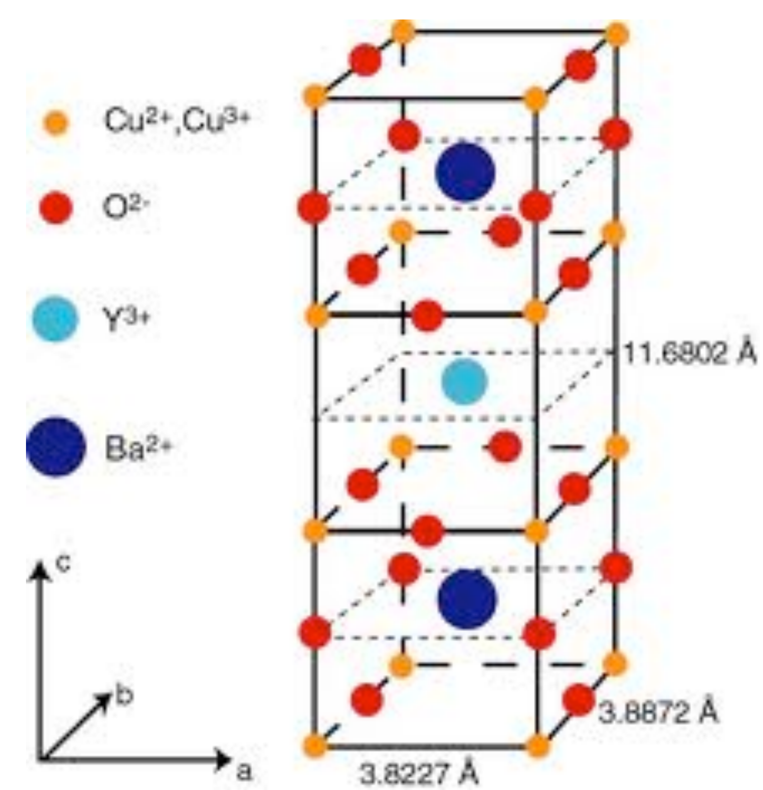
Large hole Fermi surface

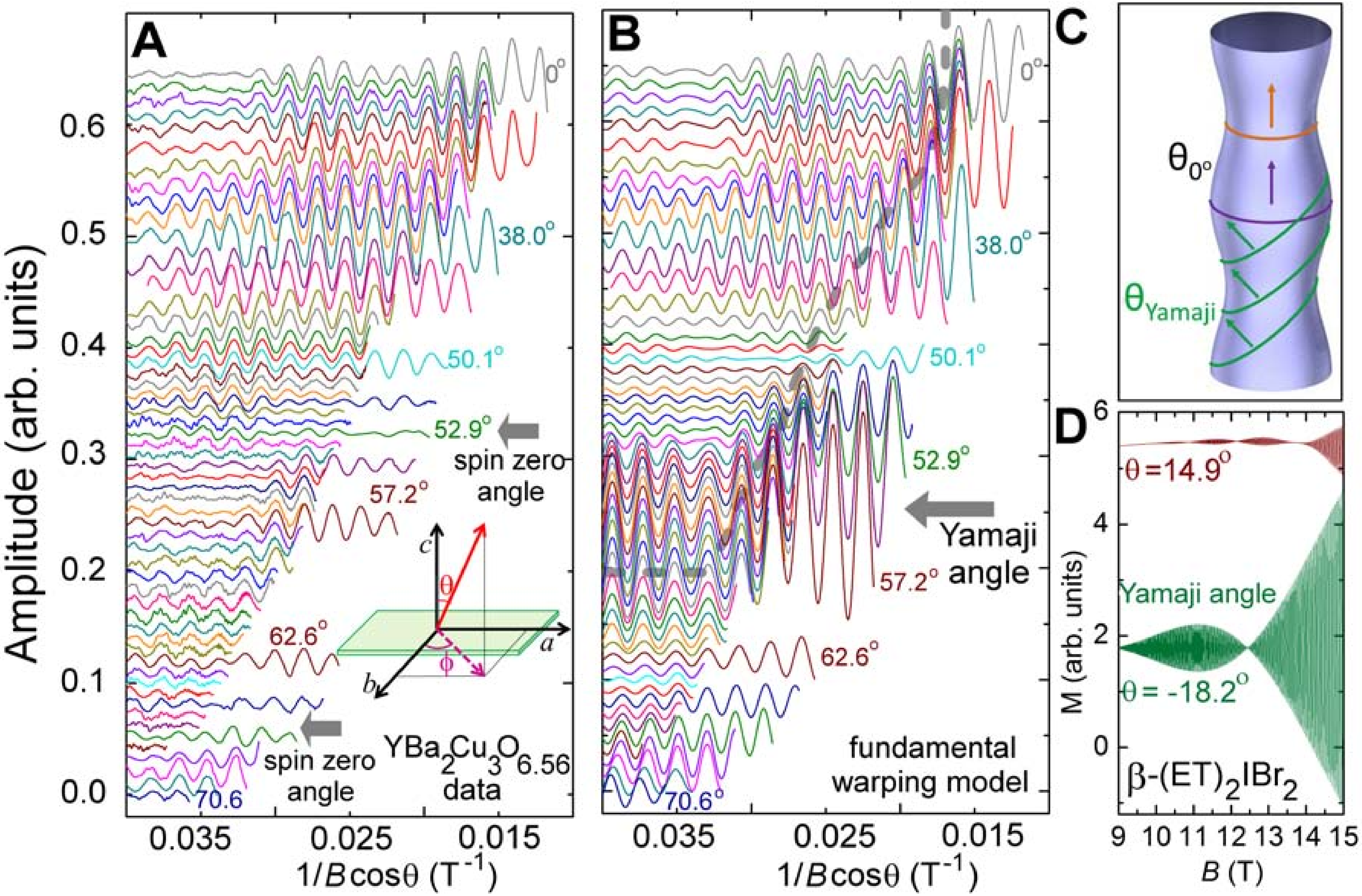
Quantum oscillations and the Fermi surface in an underdoped high-T_c superconductor

Nicolas Doiron-Leyraud[1], Cyril Proust[2], David LeBoeuf[1], Julien Levallois[2], Jean-Baptiste Bonnemaison[1], Ruixing Liang[3,4], D. A. Bonn[3,4], W. N. Hardy[3,4] & Louis Taillefer[1,4]

Nature **447**, 565 (2007)







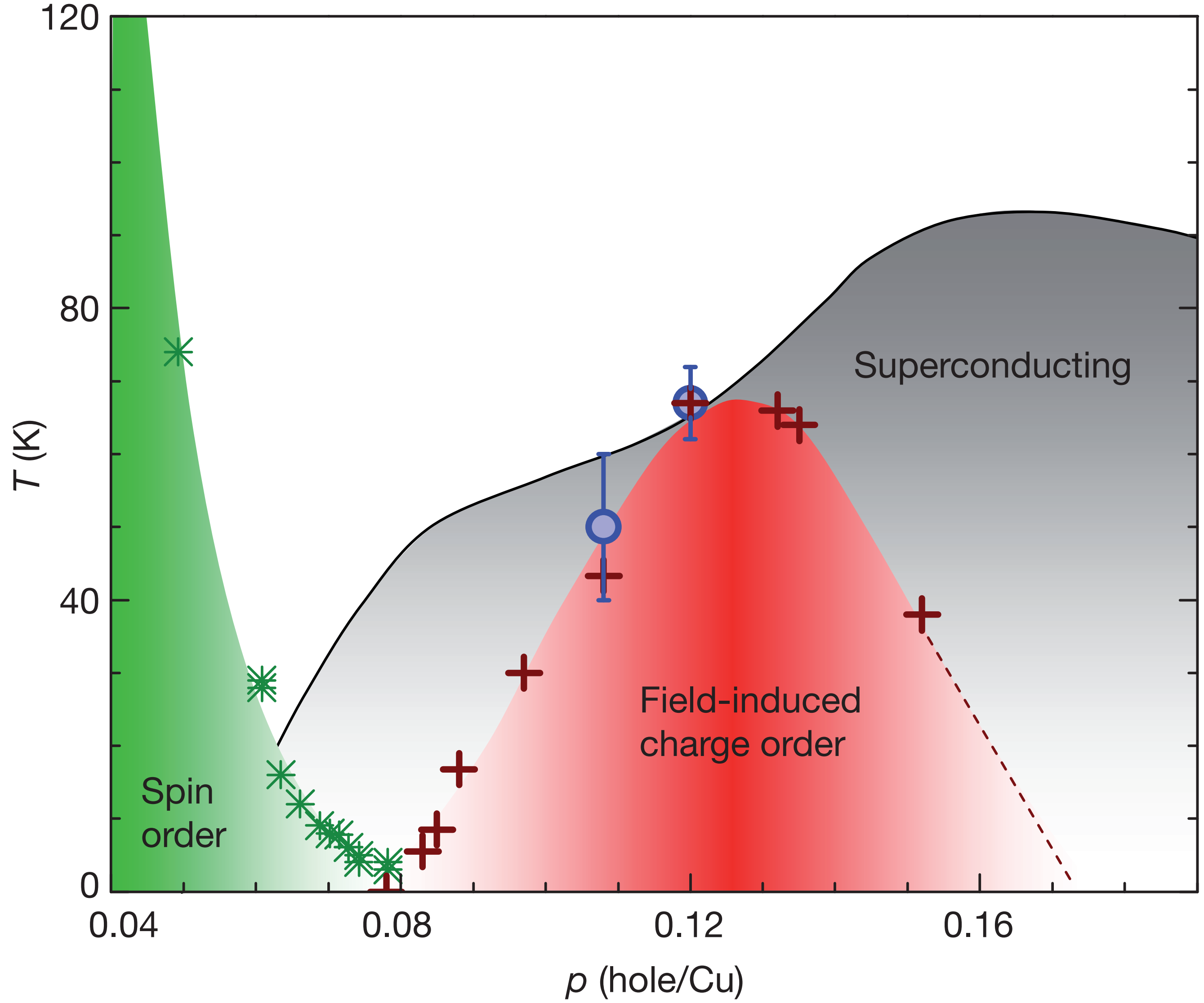
Twofold twisted Fermi surface from staggered order in an underdoped high T_c superconductor

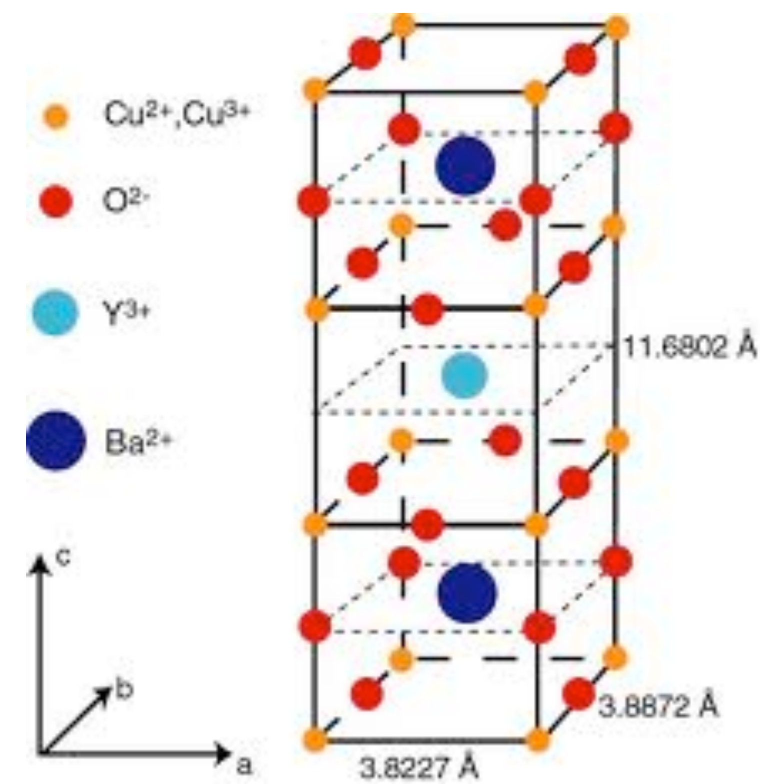
Suchitra E. Sebastian,[1*] N. Harrison,[2] F. F. Balakirev,[2] M. M. Altarawneh,[2,3]
Ruixing Liang,[4,5] D. A. Bonn,[4,5] W. N. Hardy,[4,5] G. G. Lonzarich,[1]

Magnetic-field-induced charge-stripe order in the high-temperature superconductor $YBa_2Cu_3O_y$

Tao Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Mladen Horvatić[1], Claude Berthier[1], W. N. Hardy[2,3], Ruixing Liang[2,3], D. A. Bonn[2,3] & Marc-Henri Julien[1]

8 SEPTEMBER 2011 | VOL 477 | NATURE | 191



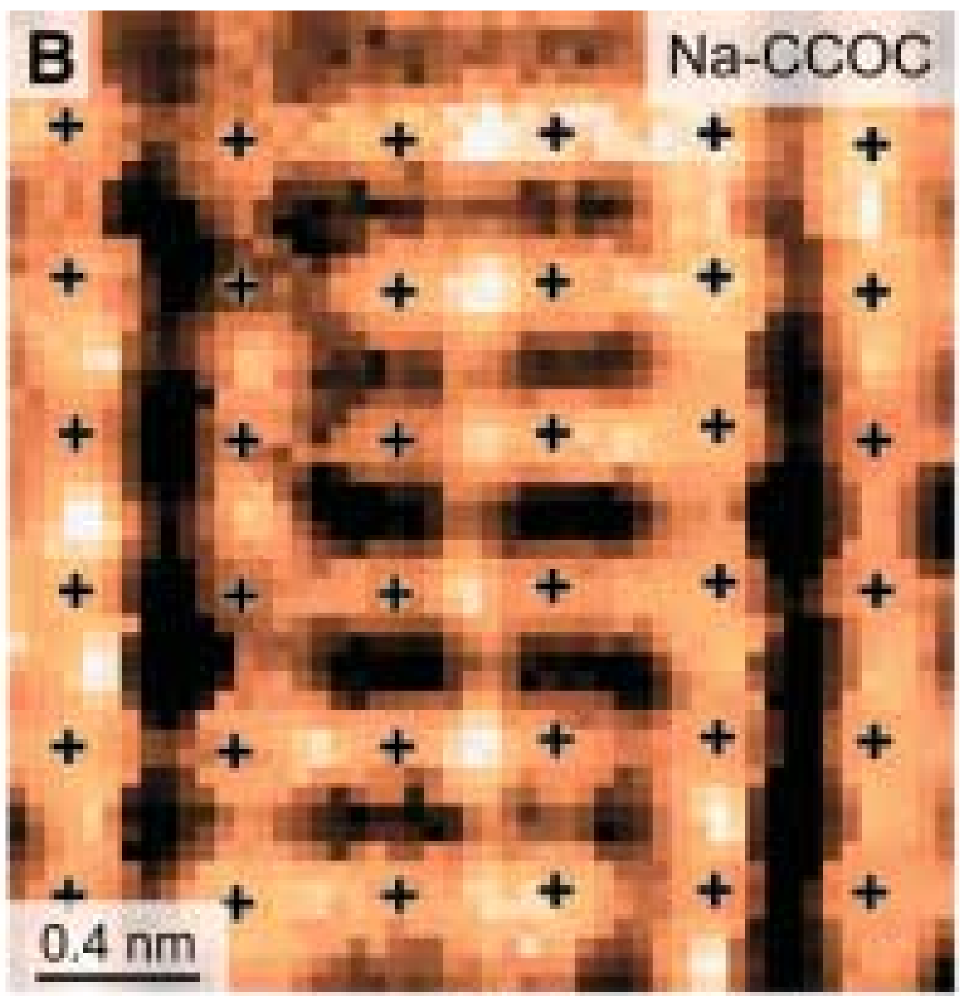


Sunday, March 17, 13

An Intrinsic Bond-Centered Electronic Glass with Unidirectional Domains in Underdoped Cuprates

Y. Kohsaka,[1] C. Taylor,[1] K. Fujita,[1,2] A. Schmidt,[1] C. Lupien,[3] T. Hanaguri,[4] M. Azuma,[5] M. Takano,[5] H. Eisaki,[6] H. Takagi,[2,4] S. Uchida,[2,7] J. C. Davis[1,8]*

9 MARCH 2007 VOL 315 SCIENCE

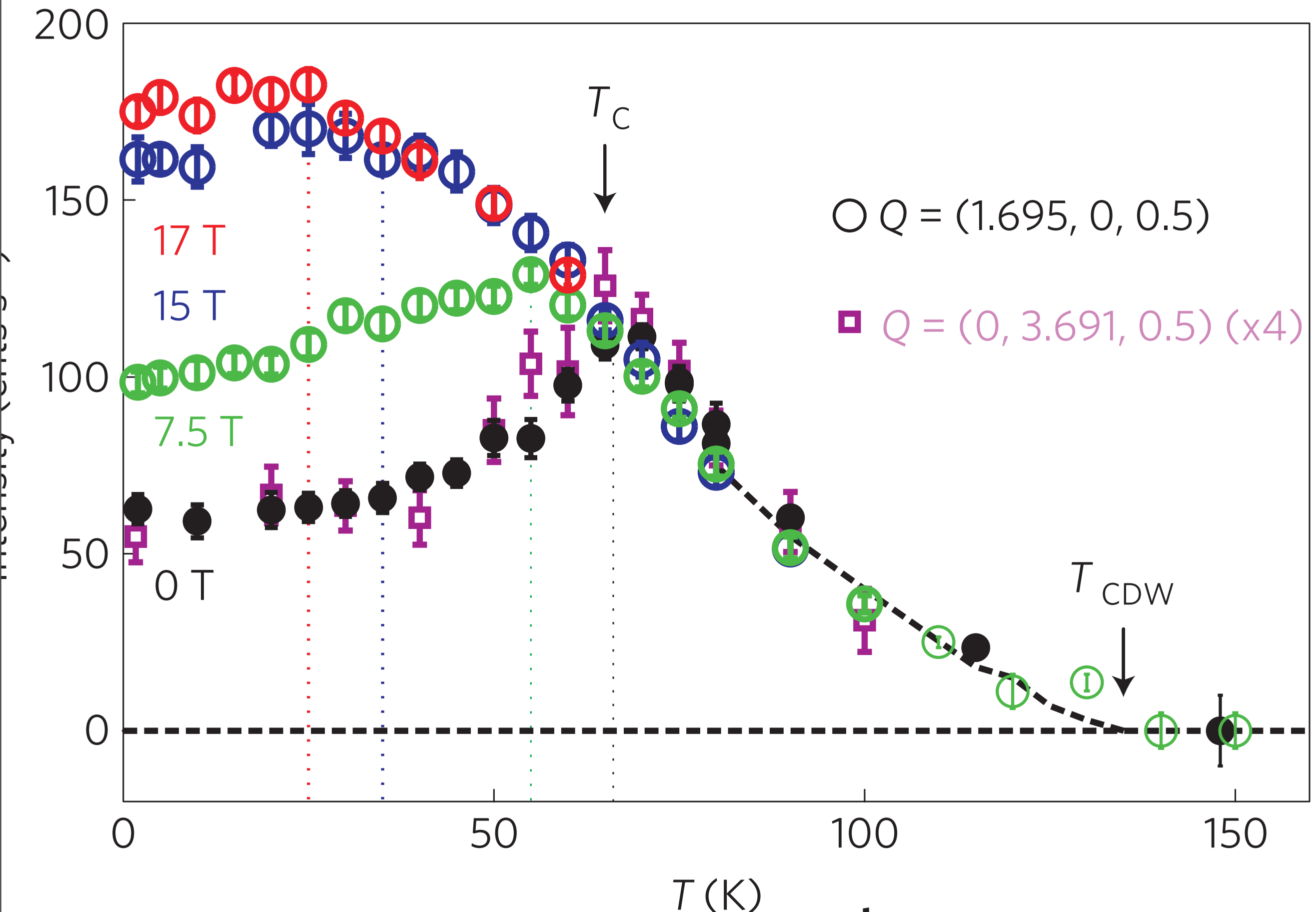


Sunday, March 17, 13

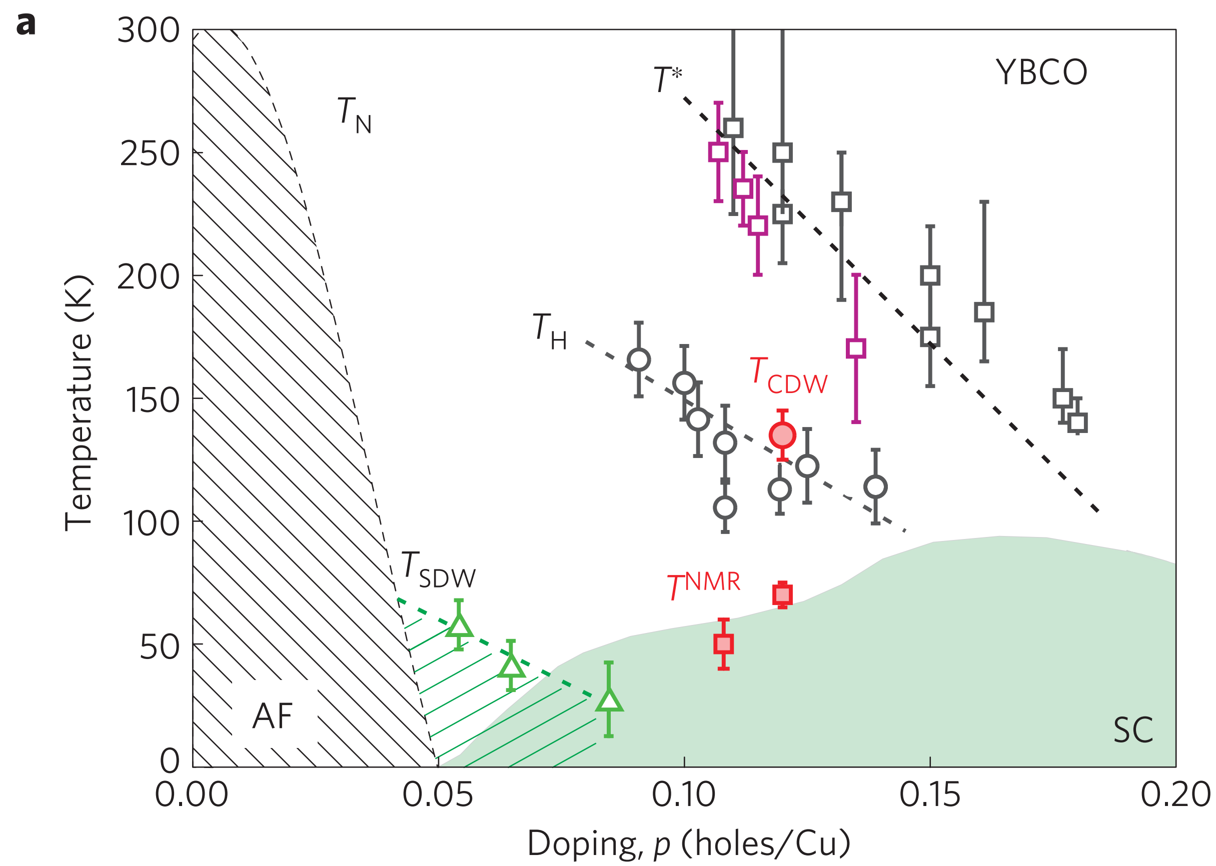
NATURE PHYSICS | VOL 8 | DECEMBER 2012 |

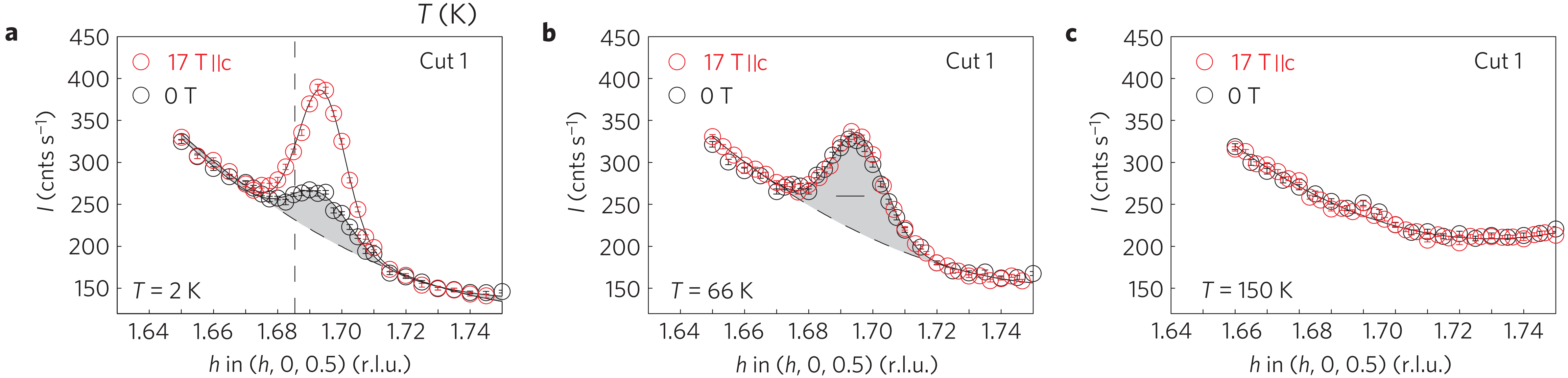
Direct observation of competition between superconductivity and charge density wave order in $YBa_2Cu_3O_{6.67}$

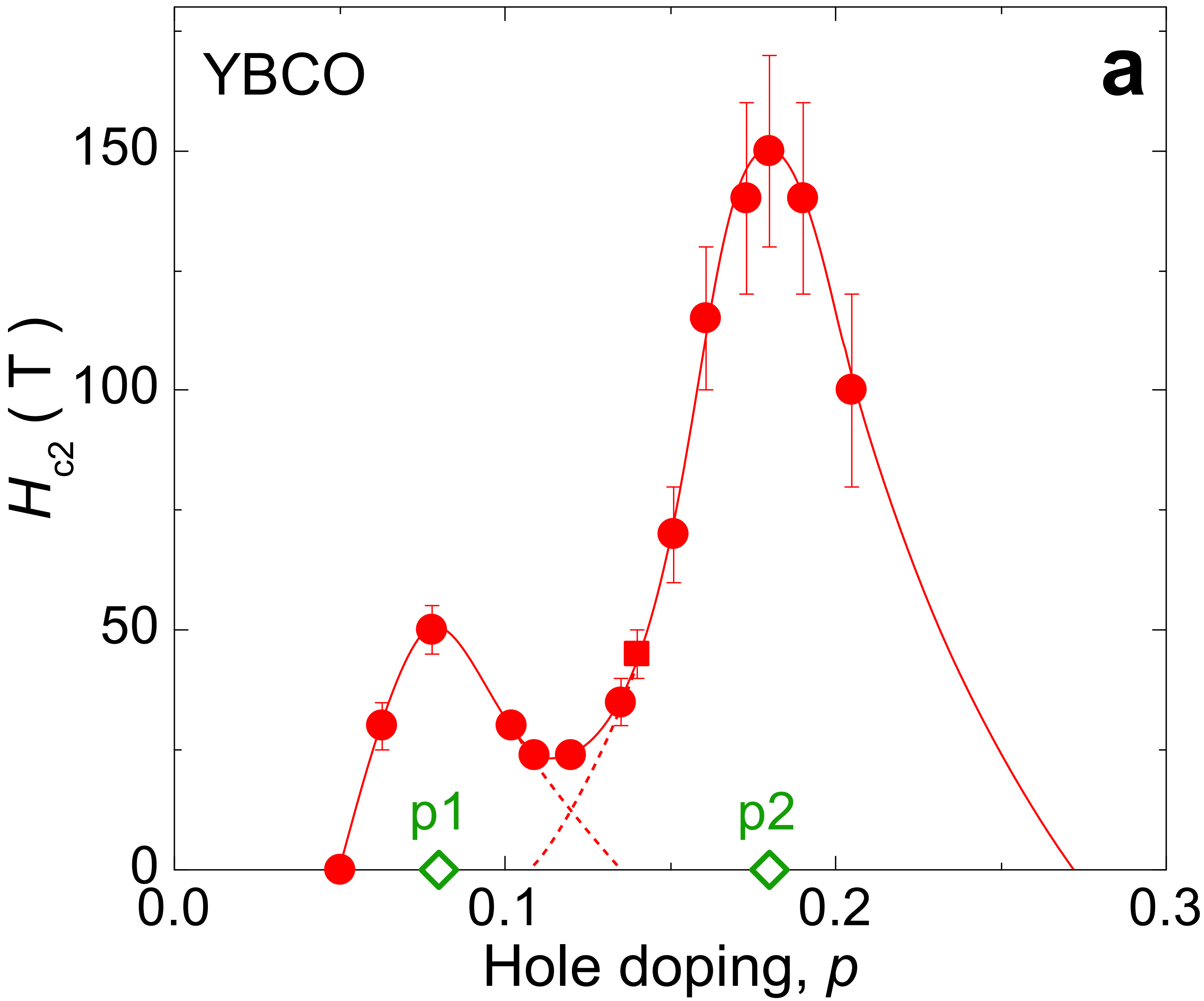
J. Chang[1,2]★, E. Blackburn[3], A. T. Holmes[3], N. B. Christensen[4], J. Larsen[4,5], J. Mesot[1,2], Ruixing Liang[6,7], D. A. Bonn[6,7], W. N. Hardy[6,7], A. Watenphul[8], M. v. Zimmermann[8], E. M. Forgan[3] and S. M. Hayden[9]

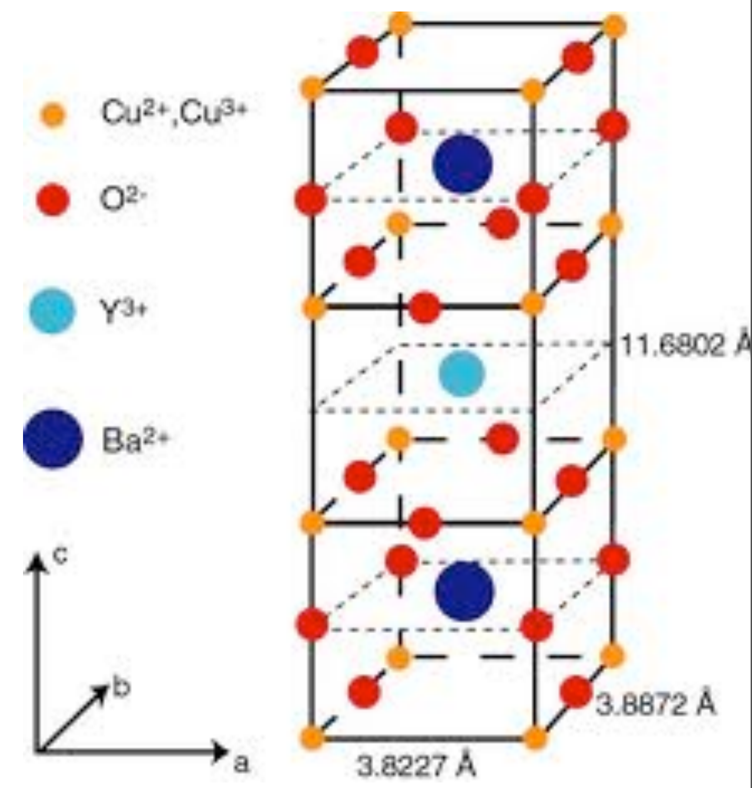


a

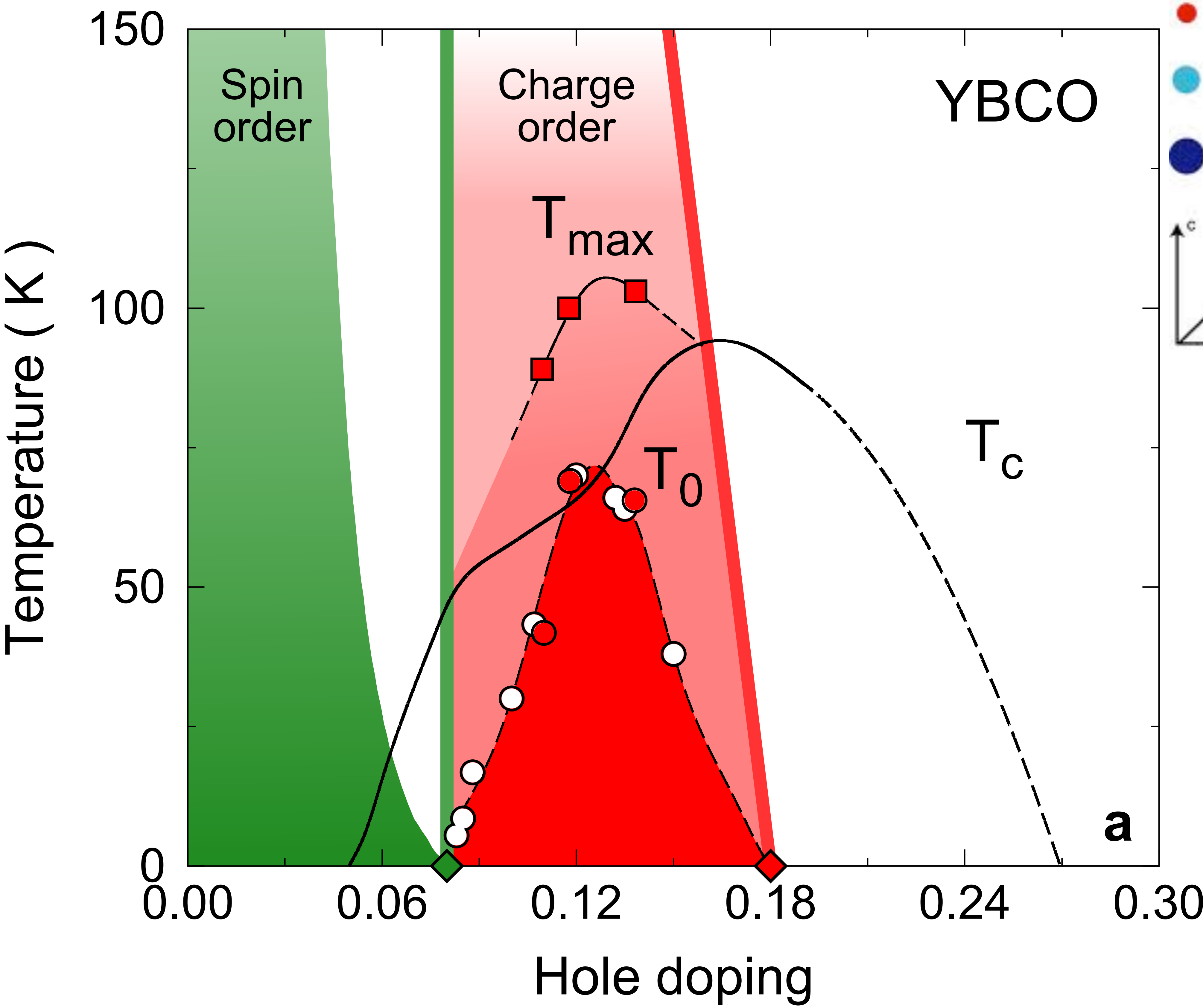


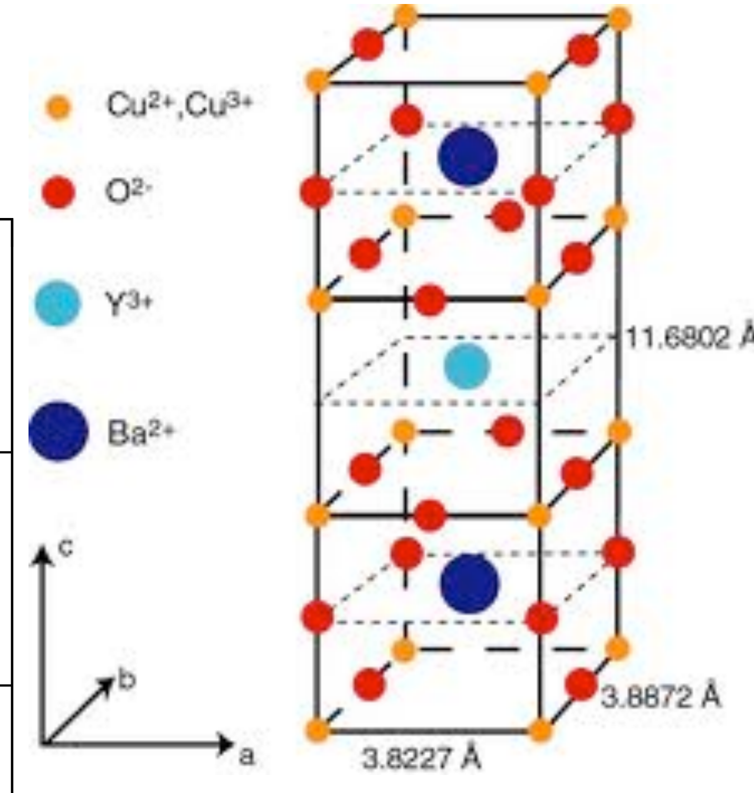






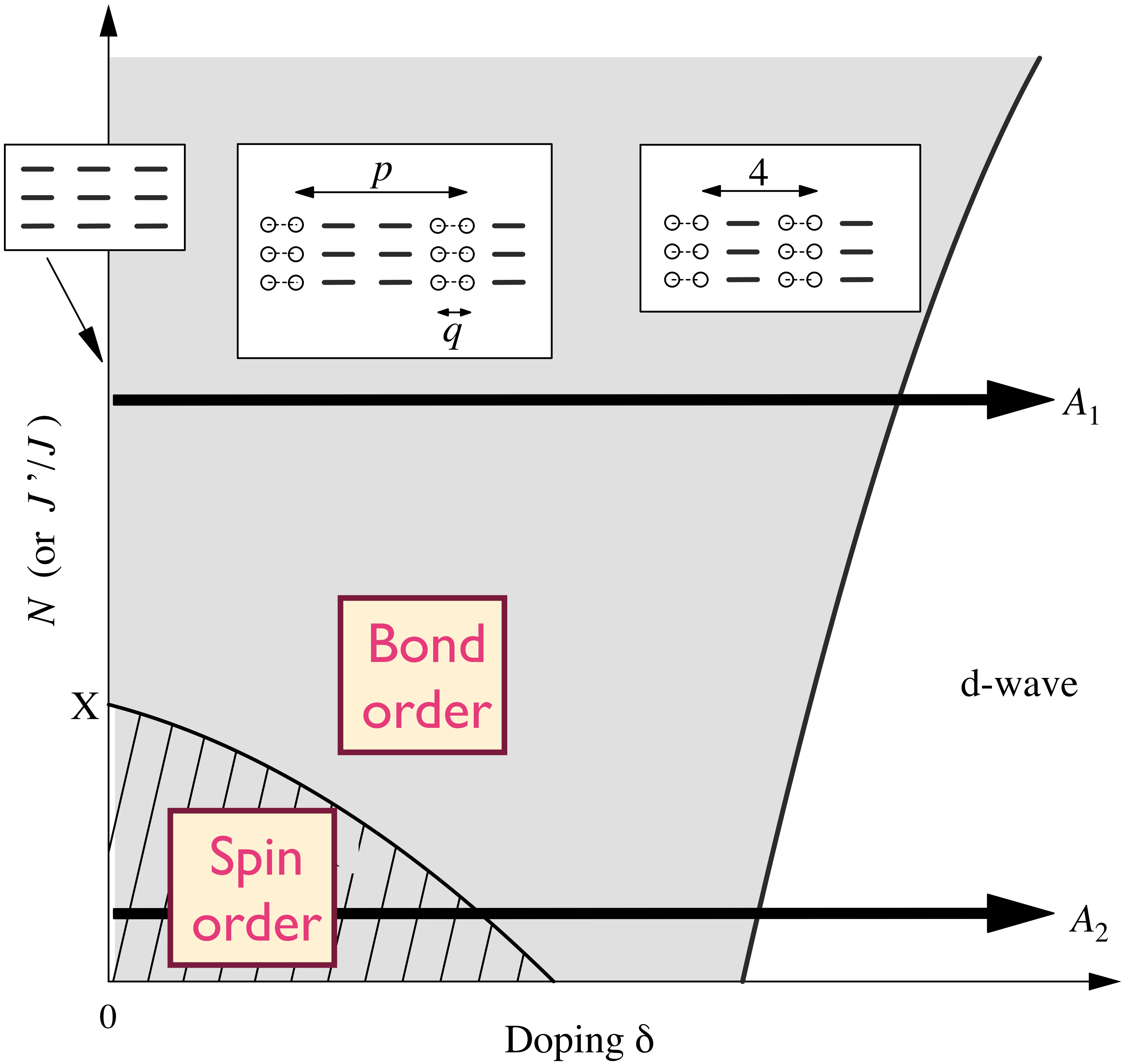
G. Grissonnanche et al., preprint





G. Grissonnanche et al., preprint

Predicted low T phase diagram



M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991)

Outline

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The Hubbard Model

$$H = -\sum_{i<j} t_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c^{\dagger}_{i\alpha} c_{i\alpha}$$

$t_{ij} \rightarrow$ "hopping". $U \rightarrow$ local repulsion, $\mu \rightarrow$ chemical potential

Spin index $\alpha = \uparrow, \downarrow$

$$n_{i\alpha} = c^{\dagger}_{i\alpha} c_{i\alpha}$$

$$c^{\dagger}_{i\alpha} c_{j\beta} + c_{j\beta} c^{\dagger}_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$
$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0$$

The Hubbard Model

Decouple *U* term by a Hubbard-Stratanovich transformation

$$\mathcal{S} = \int d^2 r d\tau \left[\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi} \right]$$

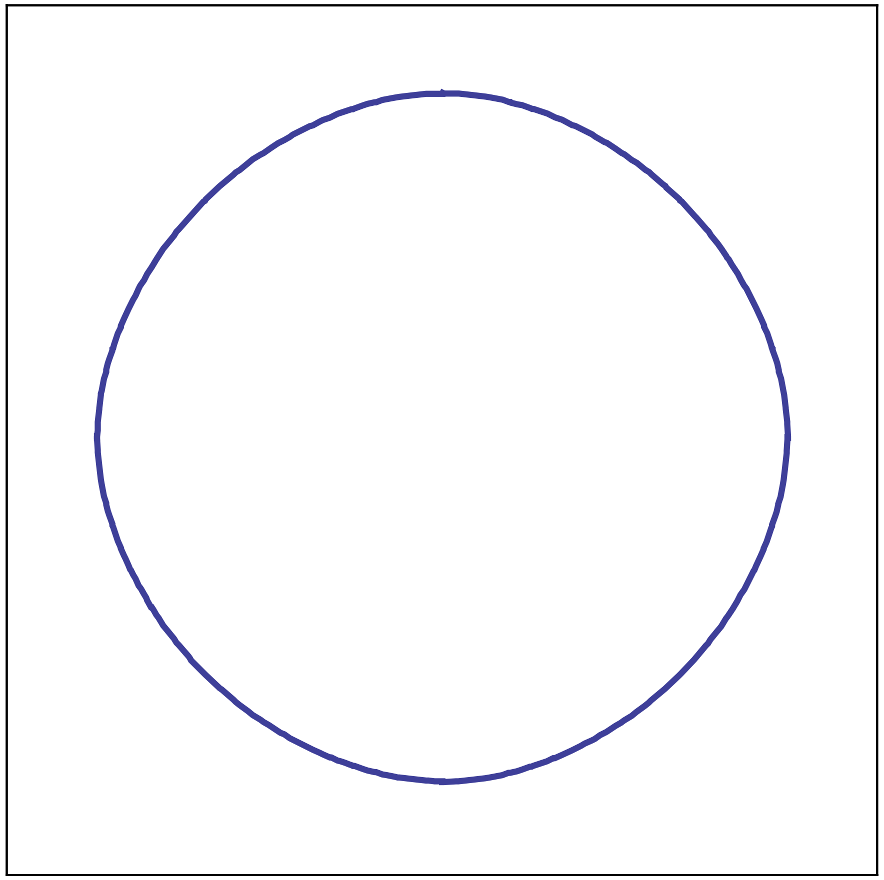
$$\mathcal{L}_c = c_a^\dagger \varepsilon(-i\boldsymbol{\nabla}) c_a$$

$$\mathcal{L}_\varphi = \frac{1}{2}(\boldsymbol{\nabla}\varphi_\alpha)^2 + \frac{r}{2}\varphi_\alpha^2 + \frac{u}{4}\left(\varphi_\alpha^2\right)^2$$

$$\mathcal{L}_{c\varphi} = \lambda\, \varphi_\alpha\, e^{i\mathbf{K}\cdot\mathbf{r}}\, c_a^\dagger\, \sigma_{ab}^\alpha\, c_b .$$

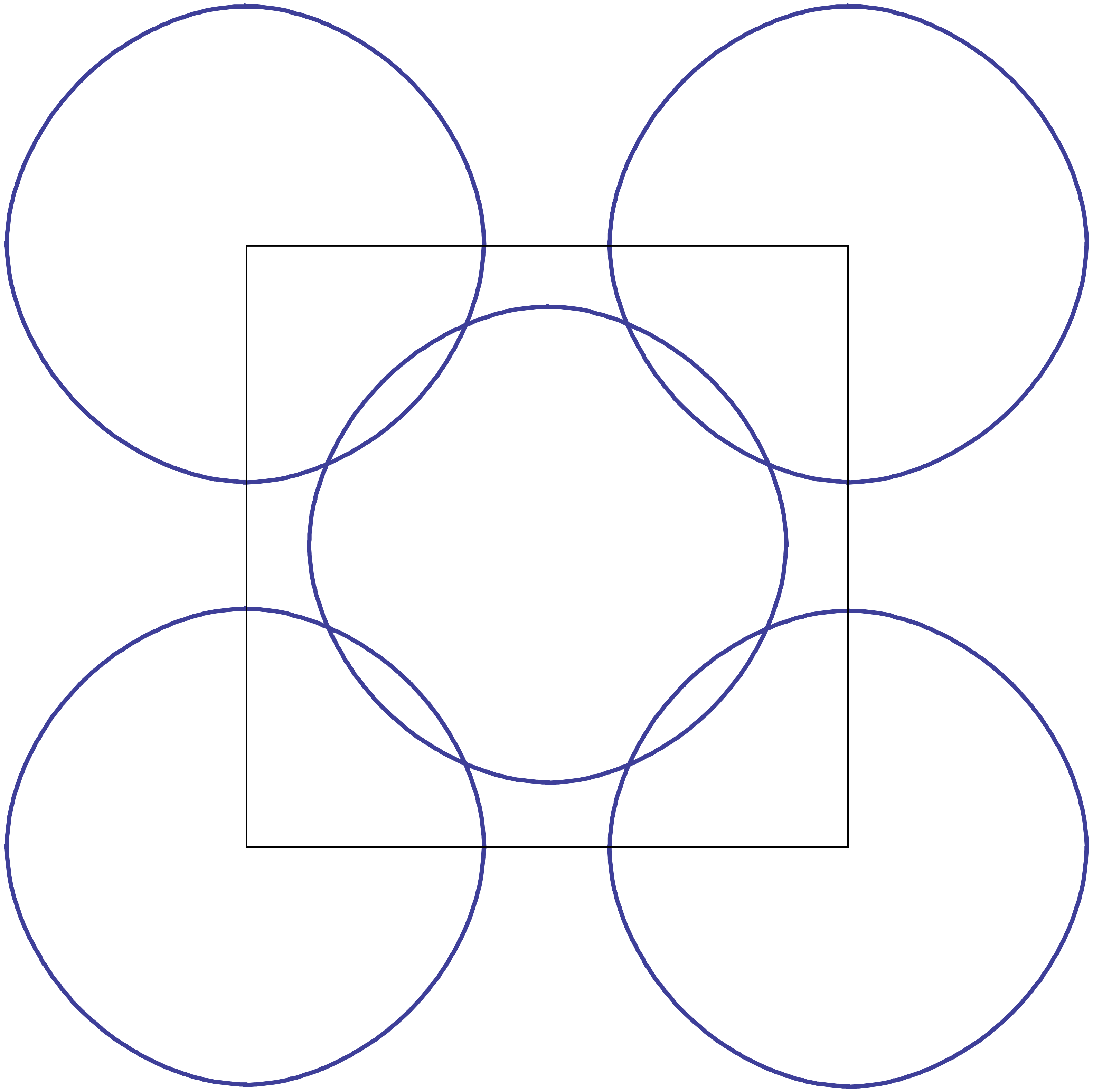
"Yukawa" coupling between fermions and antiferromagnetic order:
$\lambda^2 \sim U$, the Hubbard repulsion

Fermi surface+antiferromagnetism



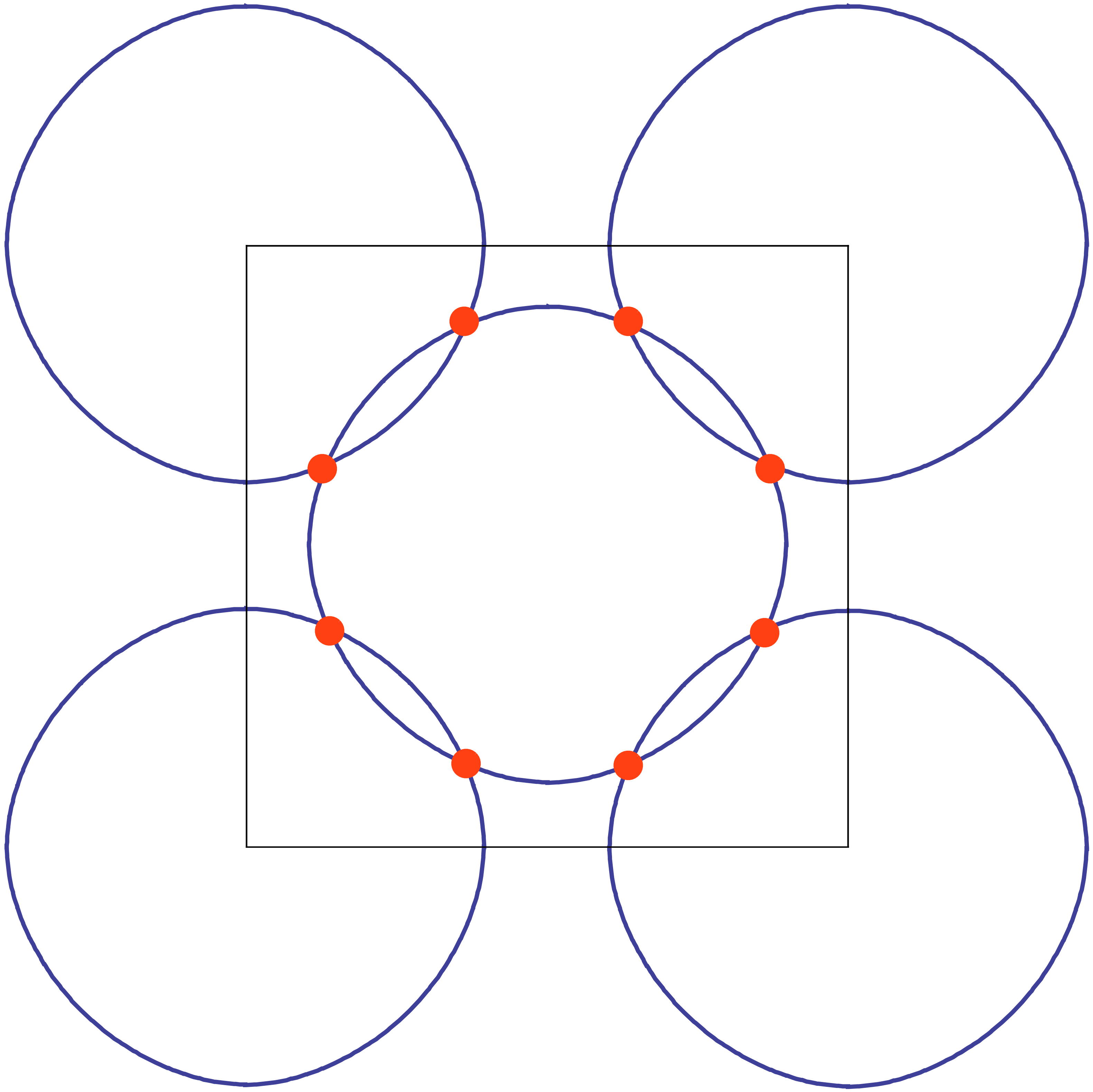
Metal with "large" Fermi surface

Fermi surface+antiferromagnetism



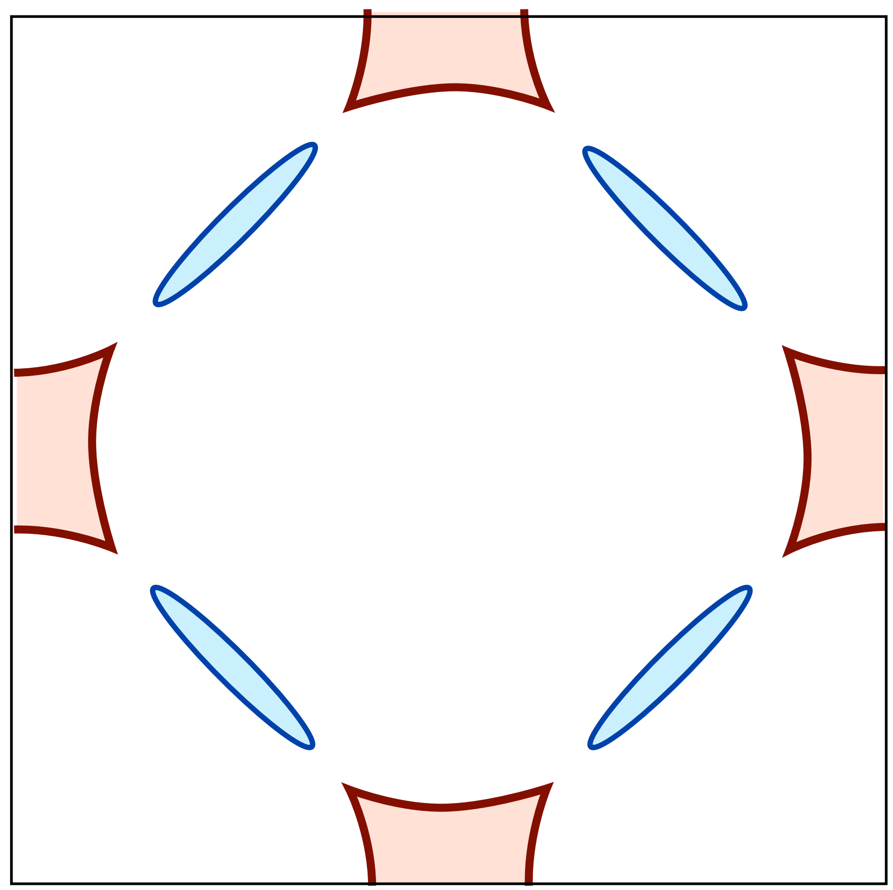
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



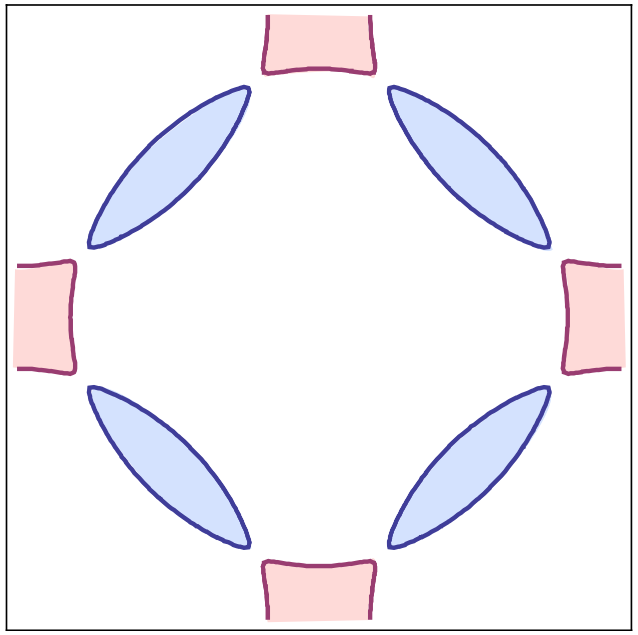
"Hot" spots

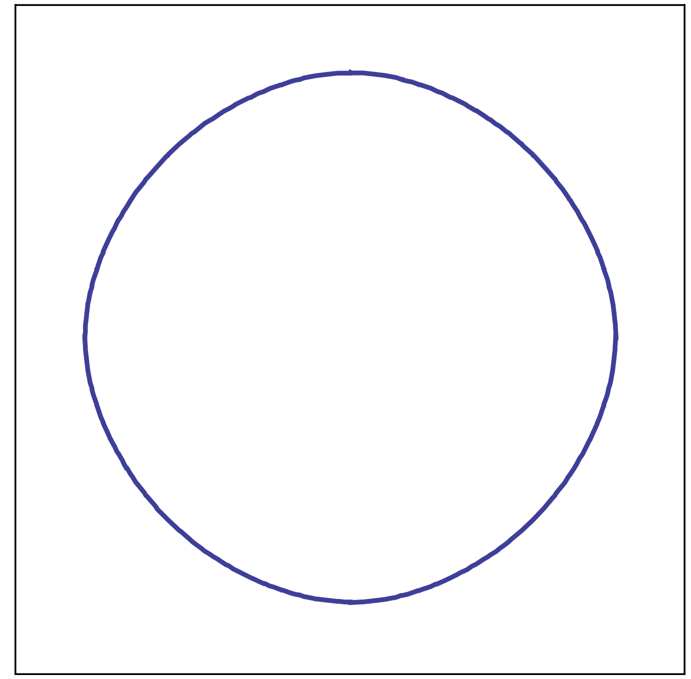
Fermi surface+antiferromagnetism



Electron and hole pockets in antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Fermi surface+antiferromagnetism





$\langle \vec{\varphi} \rangle \neq 0$

$\langle \vec{\varphi} \rangle = 0$

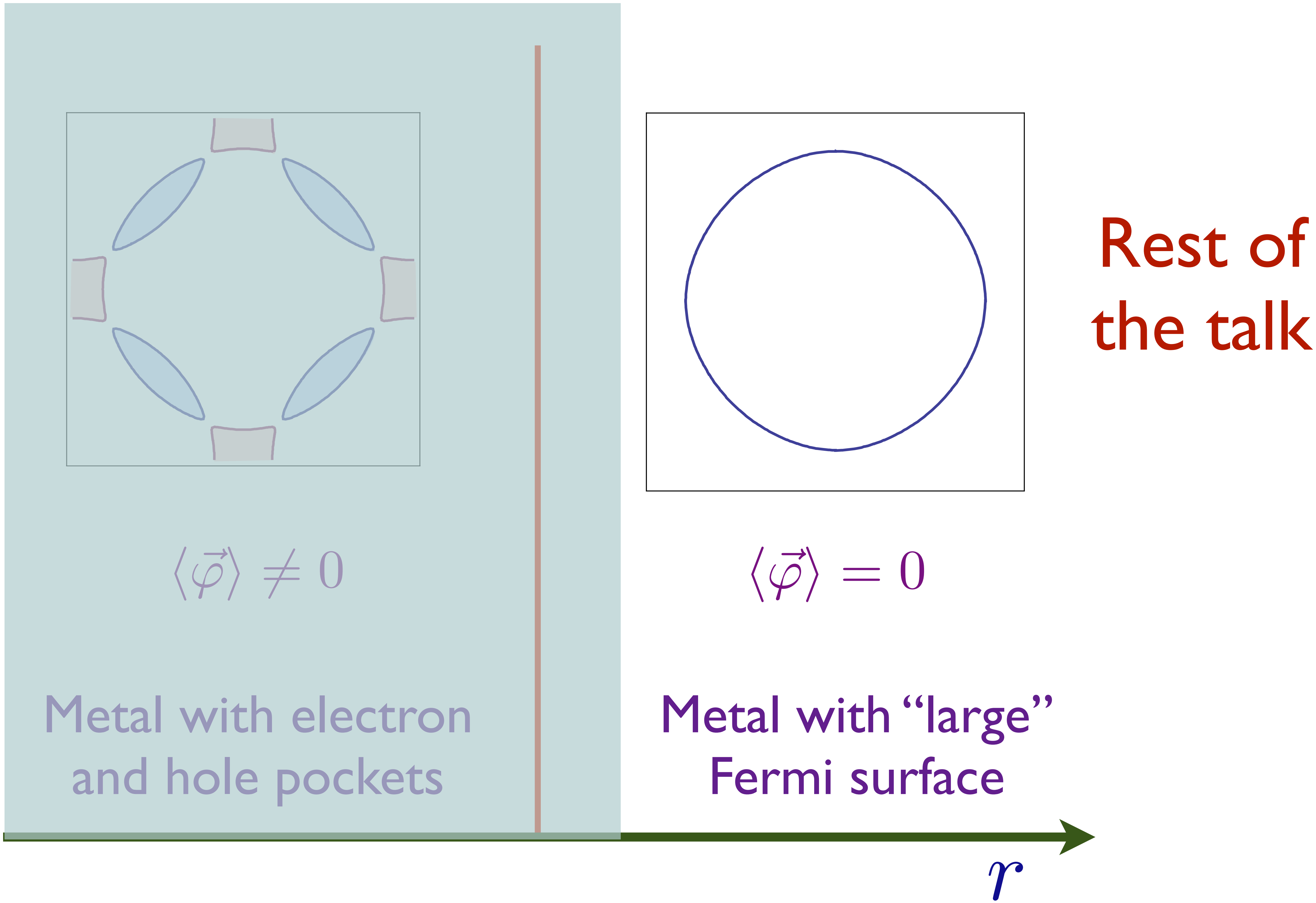
Metal with electron and hole pockets

Metal with "large" Fermi surface

r

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Fermi surface+antiferromagnetism



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A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Pairing by SDW fluctuation exchange

We now allow the SDW field $\vec{\varphi}$ to be dynamical, coupling to electrons as

$$H_{\rm sdw} = -\sum_{\mathbf{k},\mathbf{q},\alpha,\beta} \vec{\varphi}_{\mathbf{q}} \cdot c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K}+\mathbf{q},\beta}.$$

Exchange of a $\vec{\varphi}$ quantum leads to the effective interaction

$$H_{ee} = -\frac{1}{2} \sum_{\mathbf{q}} \sum_{\mathbf{p},\gamma,\delta} \sum_{\mathbf{k},\alpha,\beta} V_{\alpha\beta,\gamma\delta}(\mathbf{q}) c^{\dagger}_{\mathbf{k},\alpha} c_{\mathbf{k}+\mathbf{q},\beta} c^{\dagger}_{\mathbf{p},\gamma} c_{\mathbf{p}-\mathbf{q},\delta},$$

where the pairing interaction is

$$V_{\alpha\beta,\gamma\delta}(\mathbf{q}) = \vec{\sigma}_{\alpha\beta} \cdot \vec{\sigma}_{\gamma\delta} \frac{\chi_0}{\xi^{-2} + (\mathbf{q} - \mathbf{K})^2},$$

with $\chi_0 \xi^2$ the SDW susceptibility and ξ the SDW correlation length.

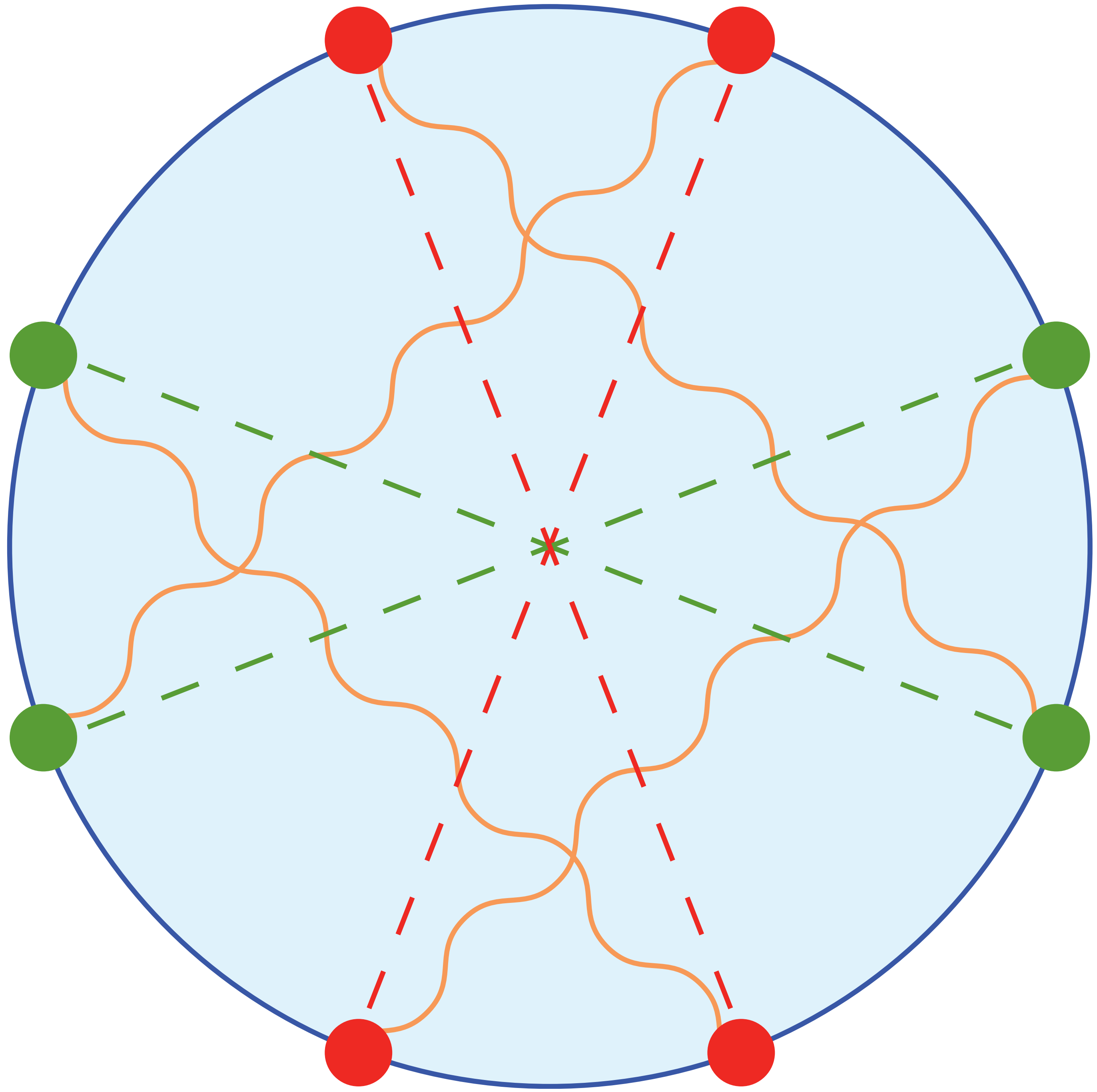
BCS Gap equation

In BCS theory, this interaction leads to the 'gap equation' for the pairing gap $\Delta_{\mathbf{k}} \propto \langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle$.

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{p}} \left(\frac{3\chi_0}{\xi^{-2} + (\mathbf{p} - \mathbf{k} - \mathbf{K})^2} \right) \frac{\Delta_{\mathbf{p}}}{2\sqrt{\varepsilon_{\mathbf{p}}^2 + \Delta_{\mathbf{p}}^2}}$$

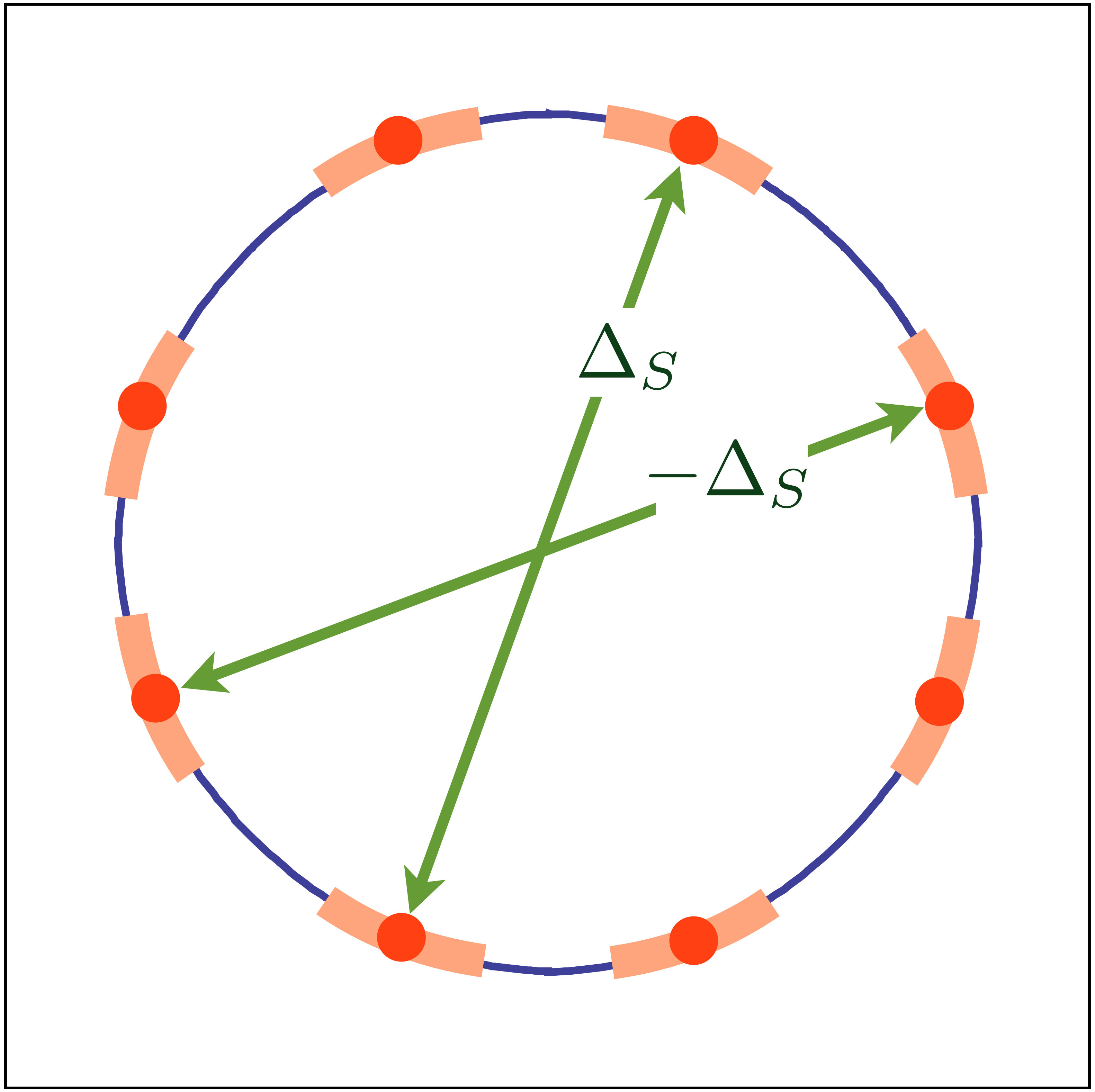
Non-zero solutions of this equation require that $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{p}}$ have opposite signs when $\mathbf{p} - \mathbf{k} \approx \mathbf{K}$.

Pairing "glue" from antiferromagnetic fluctuations



V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M.Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S.A. Kivelson, and D.J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

$$\left\langle c^\dagger_{\mathbf{k}\alpha} c^\dagger_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

Outline

1. The "modern era" of cuprate experiments

2. Antiferromagnetism in metals:
   d-wave superconductivity

3. Low energy theory, emergent pseudospin
   symmetry, and bond order

4. Unrestricted Hartree-Fock-BCS

5. Quantum Monte Carlo
   without the sign problem

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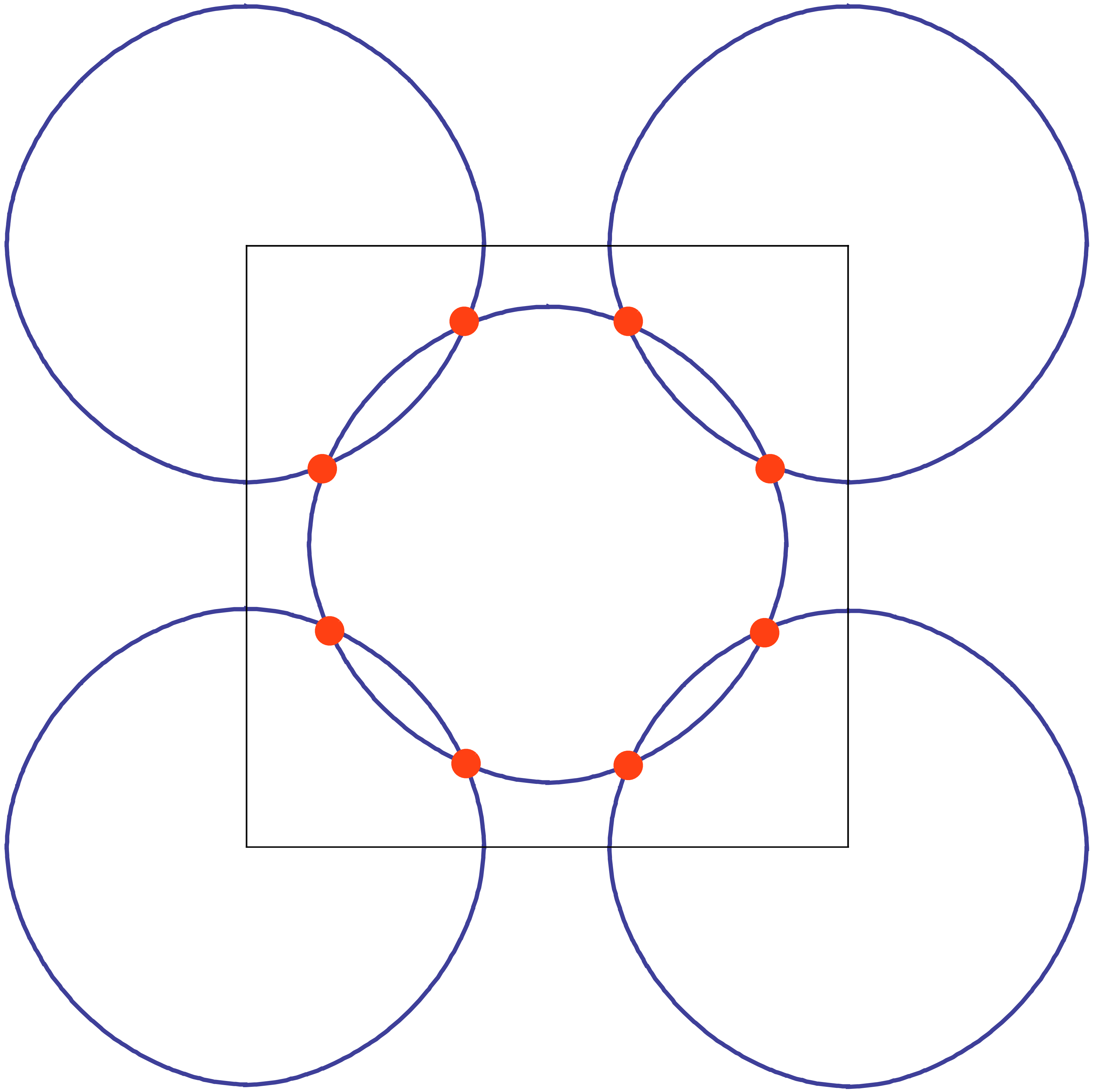
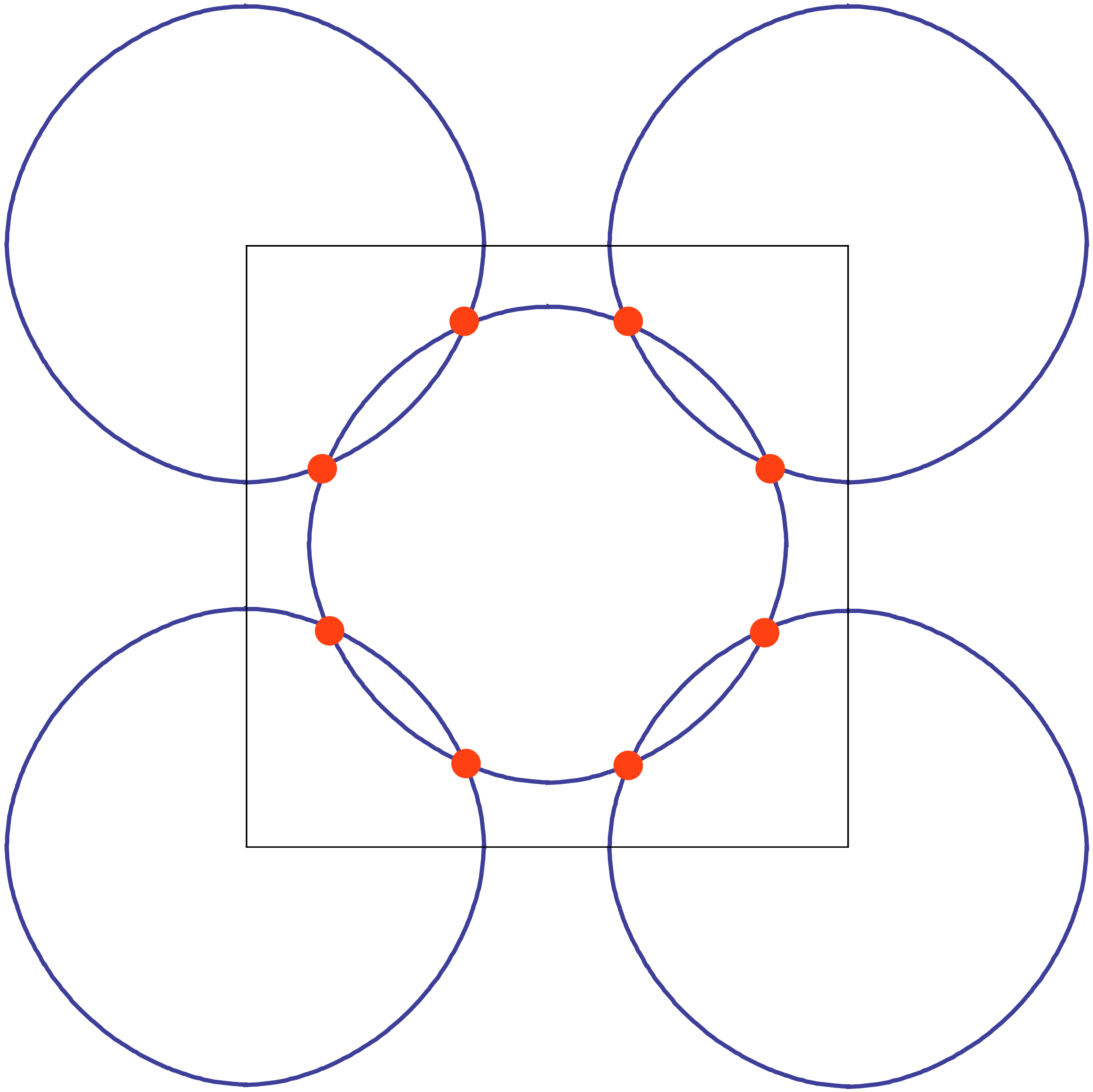
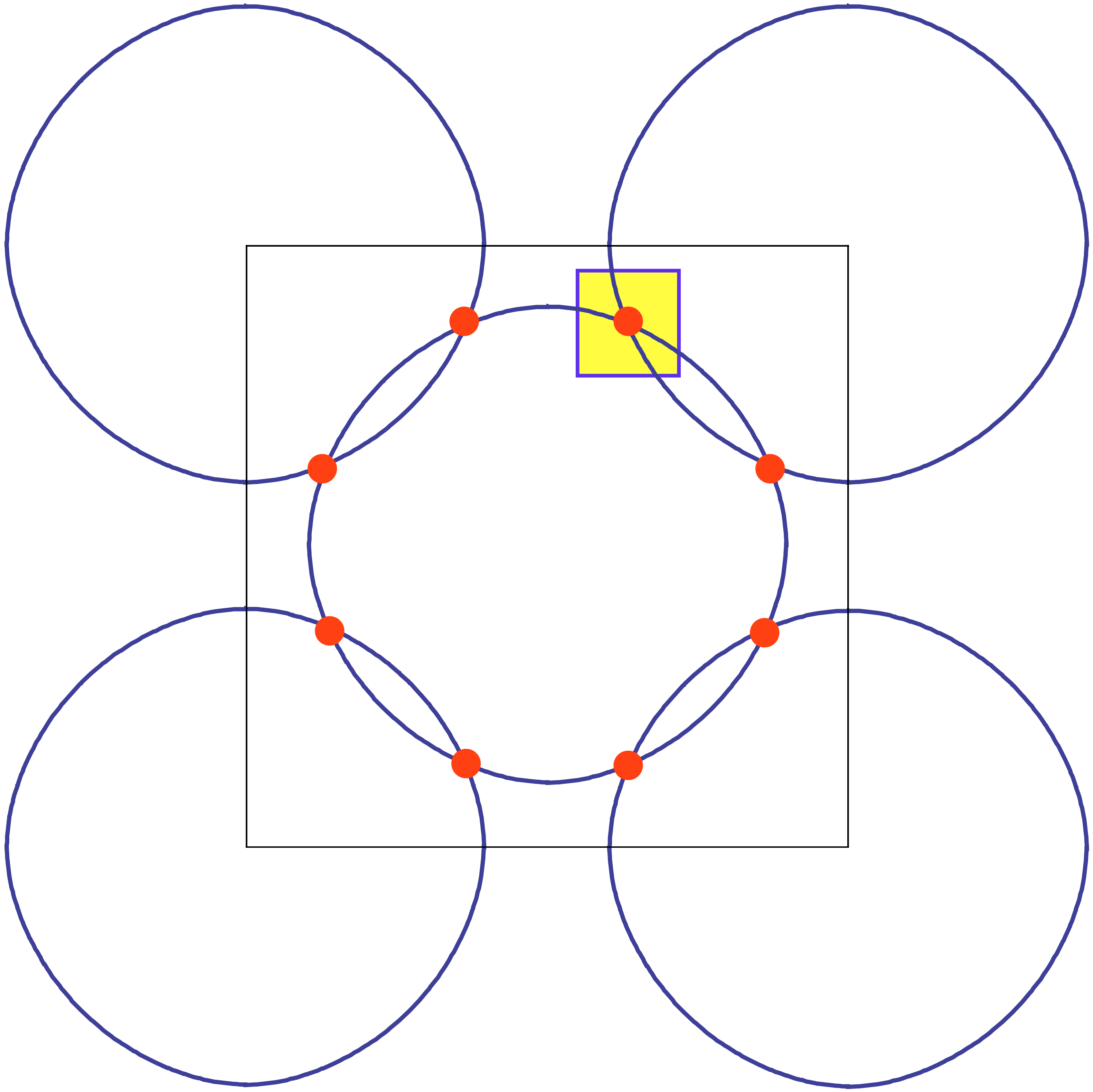


"Hot" spots

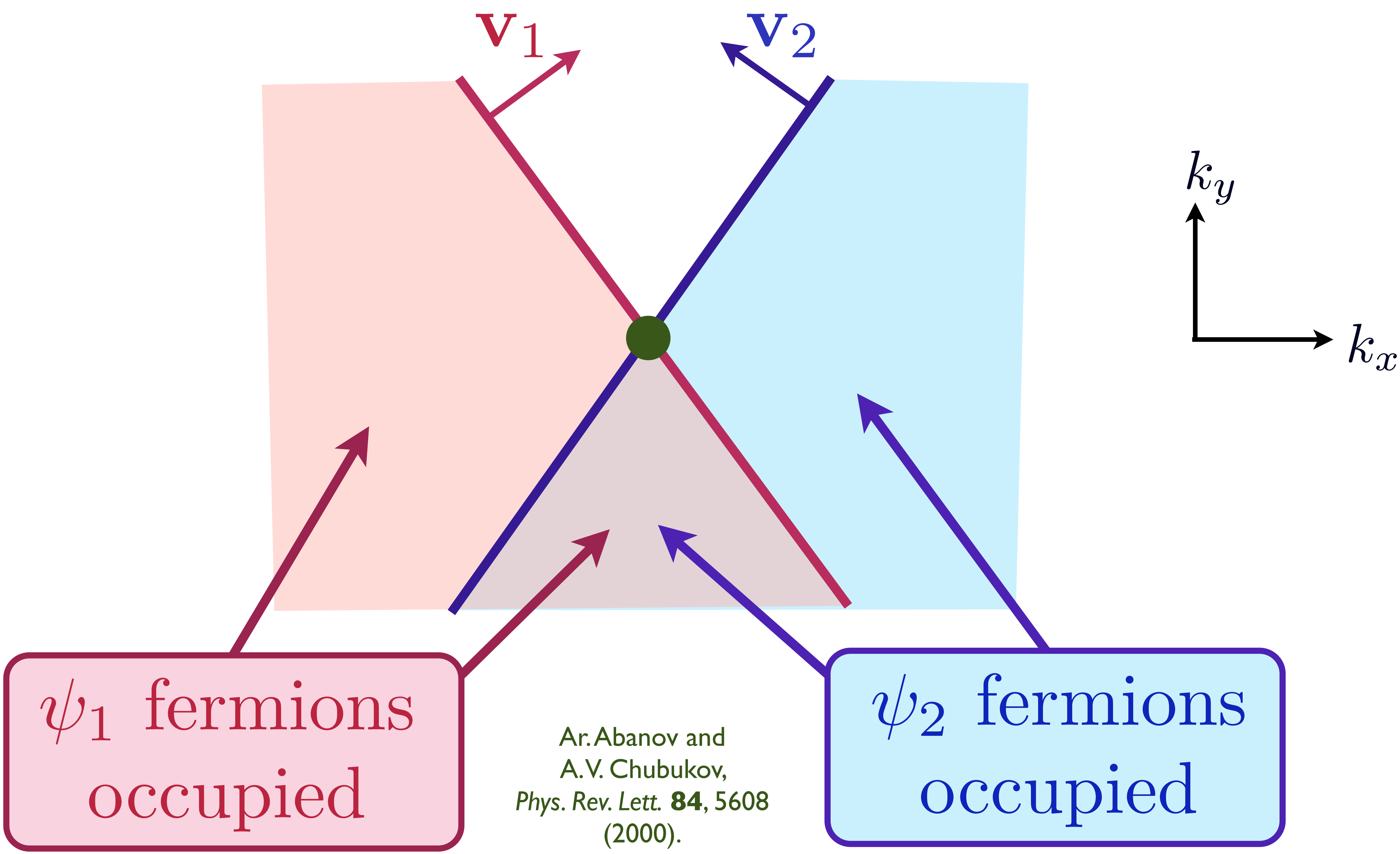


Low energy theory for critical point near hot spots



Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **84**, 5608 (2000).

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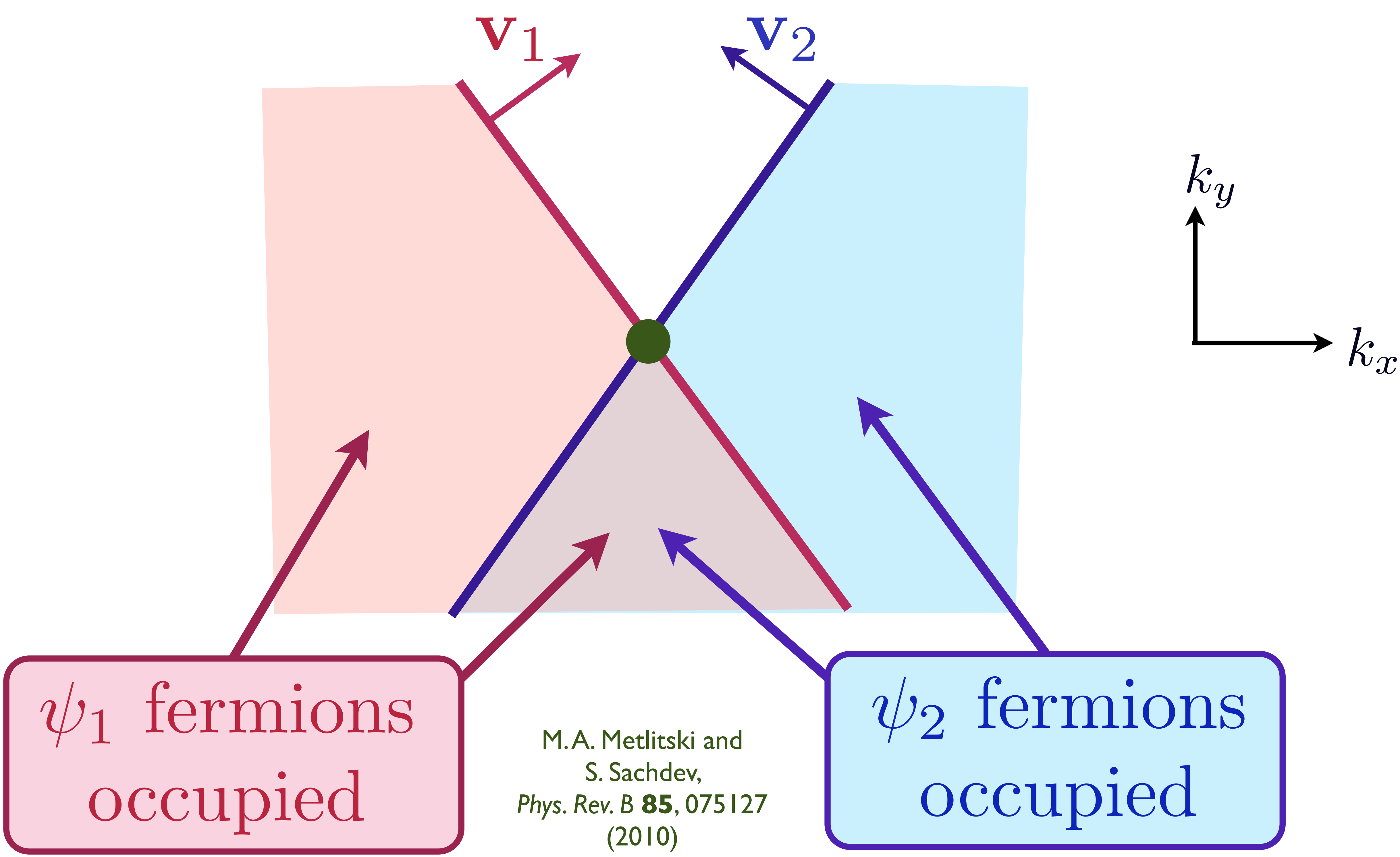

M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

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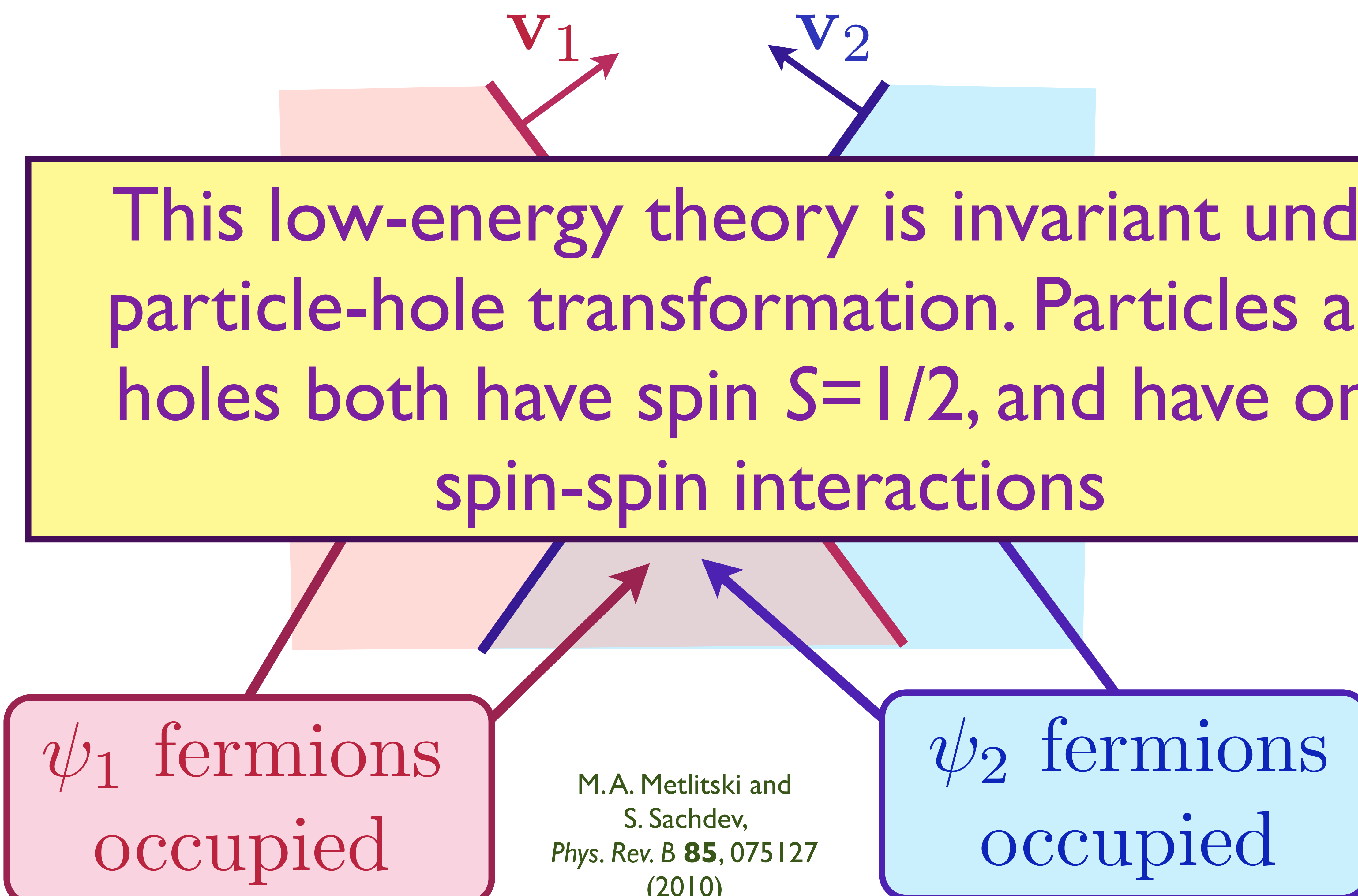
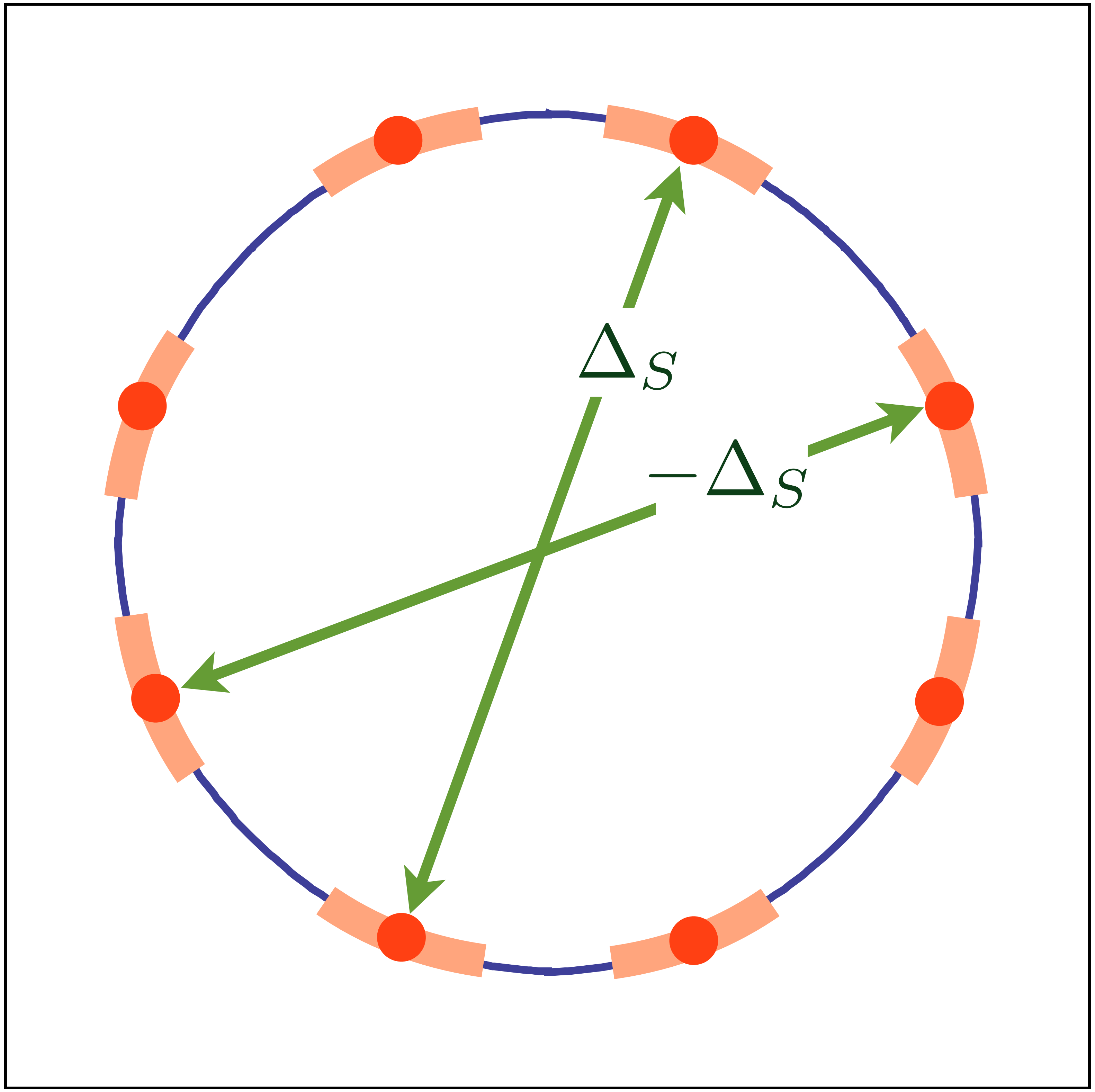


This low-energy theory is invariant under particle-hole transformation. Particles and holes both have spin *S*=1/2, and have only spin-spin interactions

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

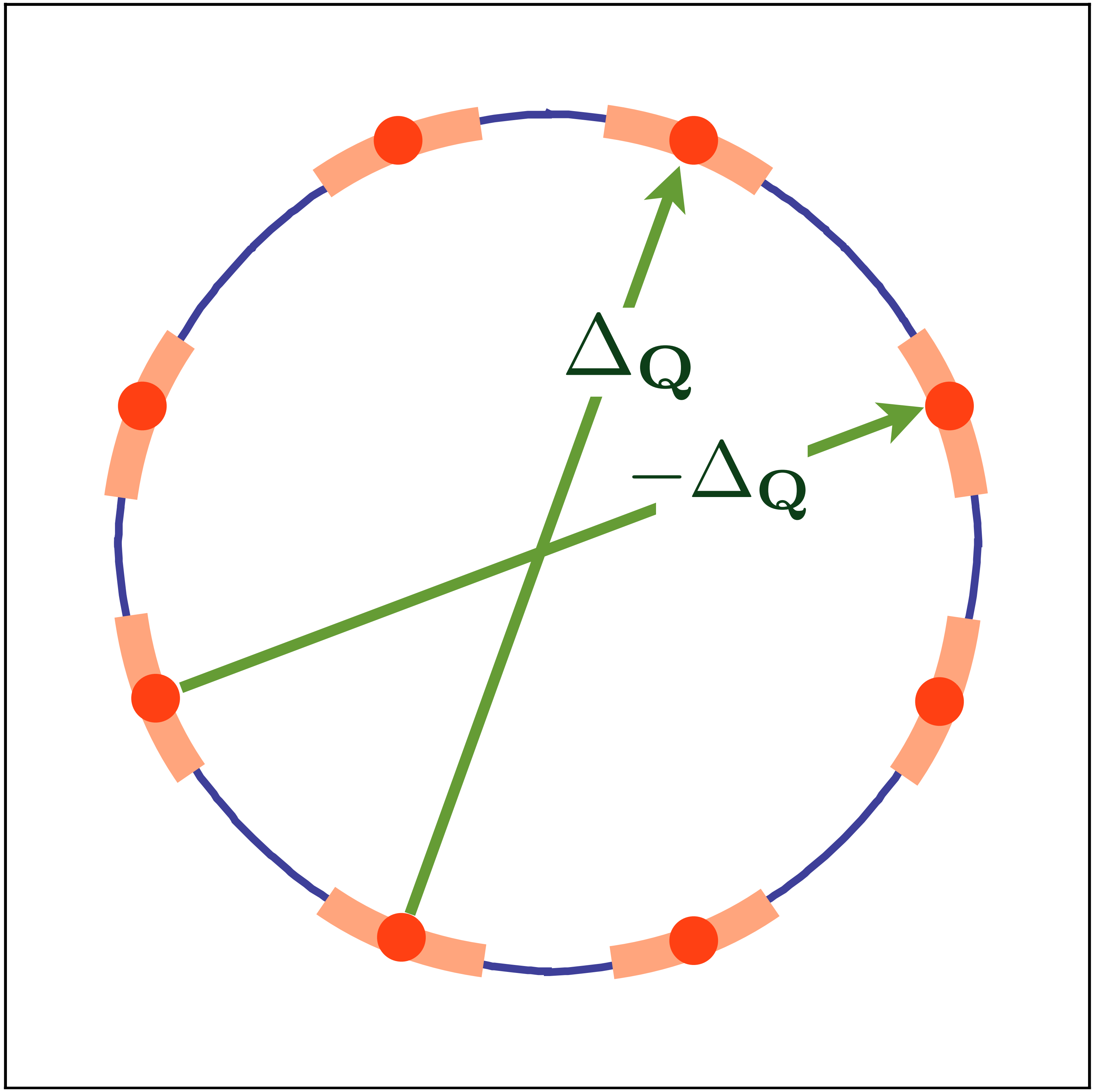
$$\left\langle c^{\dagger}_{\mathbf{k}\alpha} c^{\dagger}_{-\mathbf{k}\beta} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y)$$



Unconventional pairing at *and near* hot spots

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

After pseudospin rotation



$\mathbf{Q}$ is '$2k_F$' wavevector

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

Unconventional particle-hole pairing at *and near* hot spots

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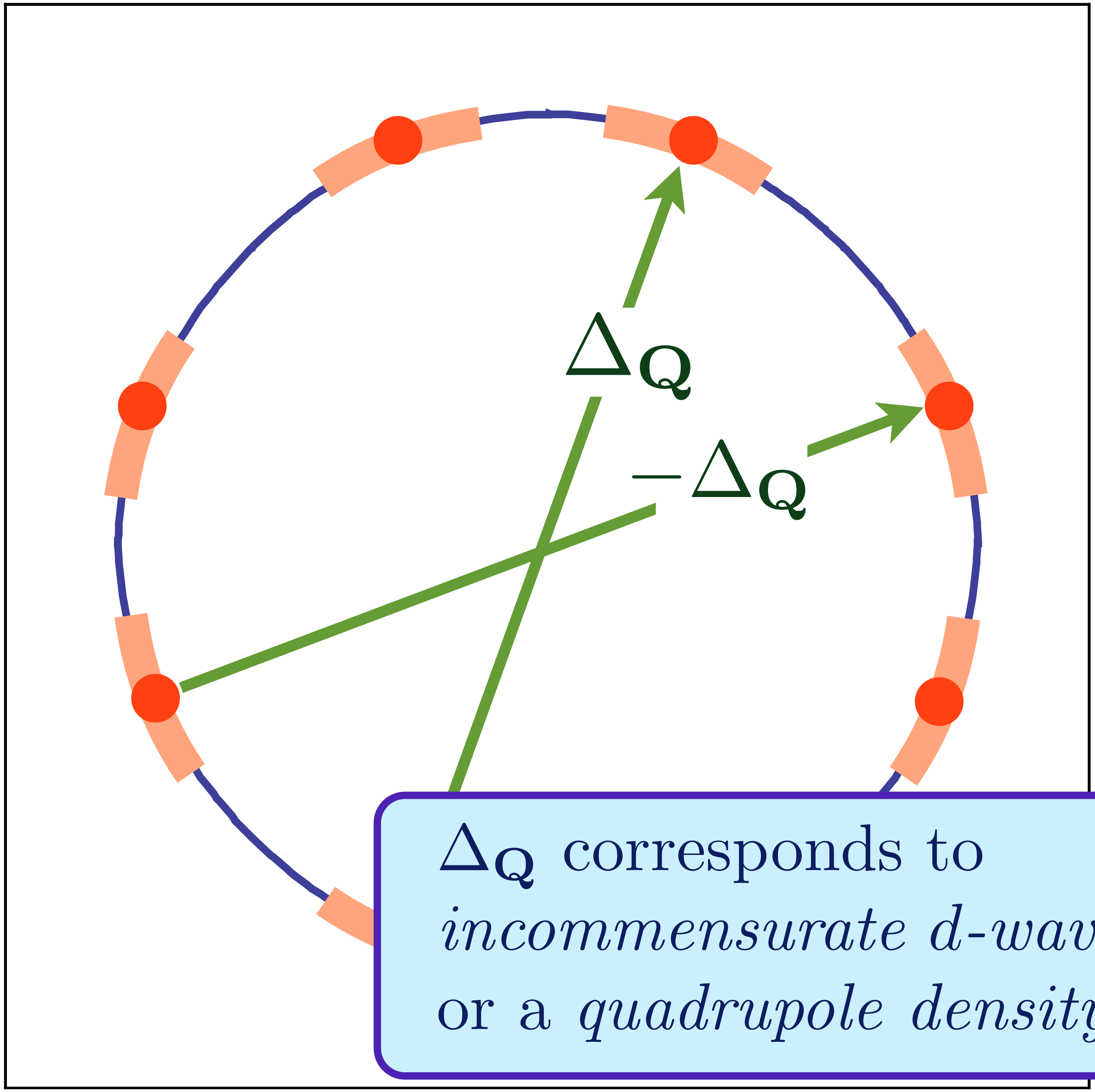


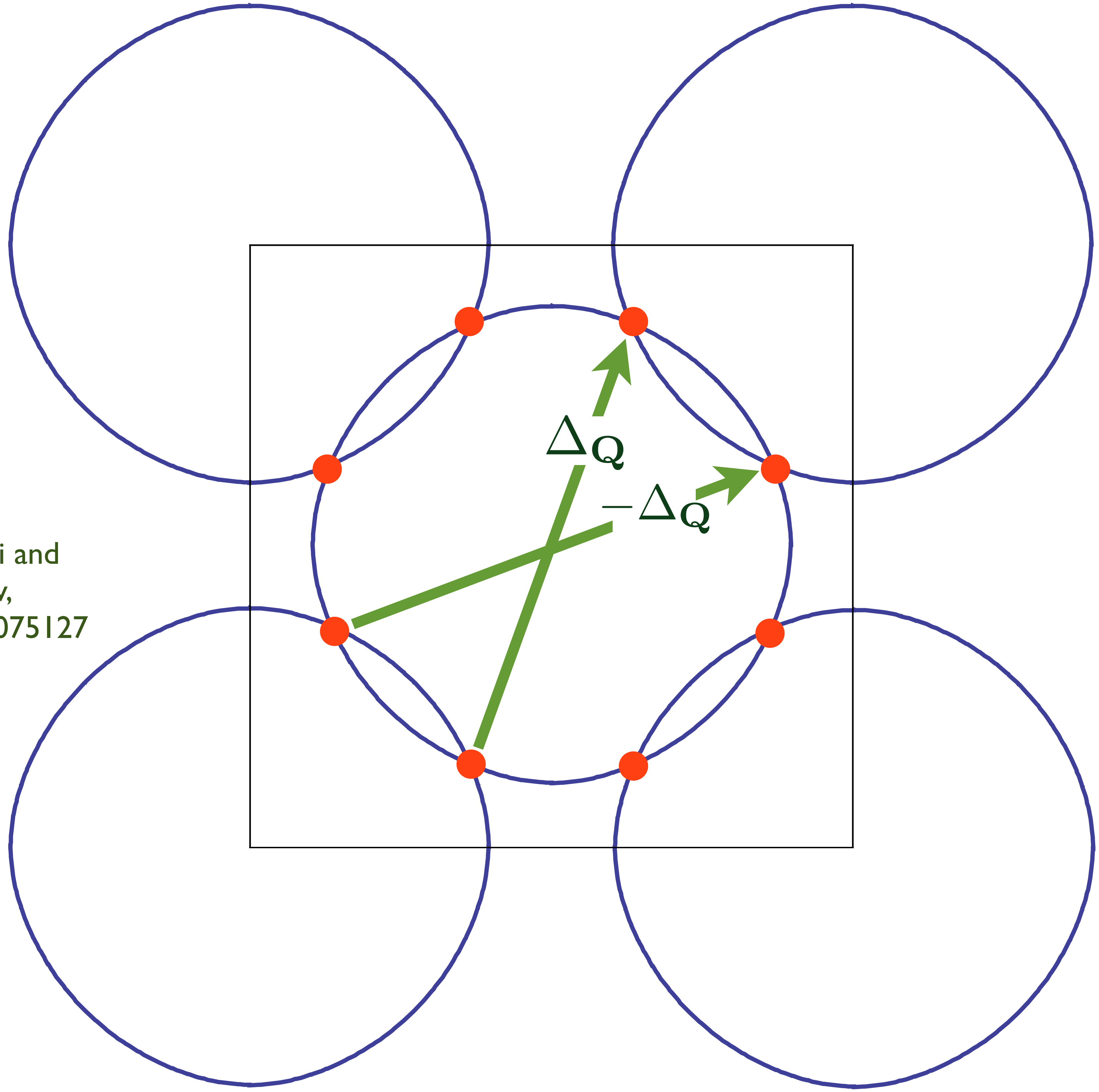
$\mathbf{Q}$ is '$2k_F$' wavevector

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$\Delta_{\mathbf{Q}}$ corresponds to *incommensurate d-wave bond order* or a *quadrupole density wave*

Unconventional particle-hole pairing at *and near* hot spots

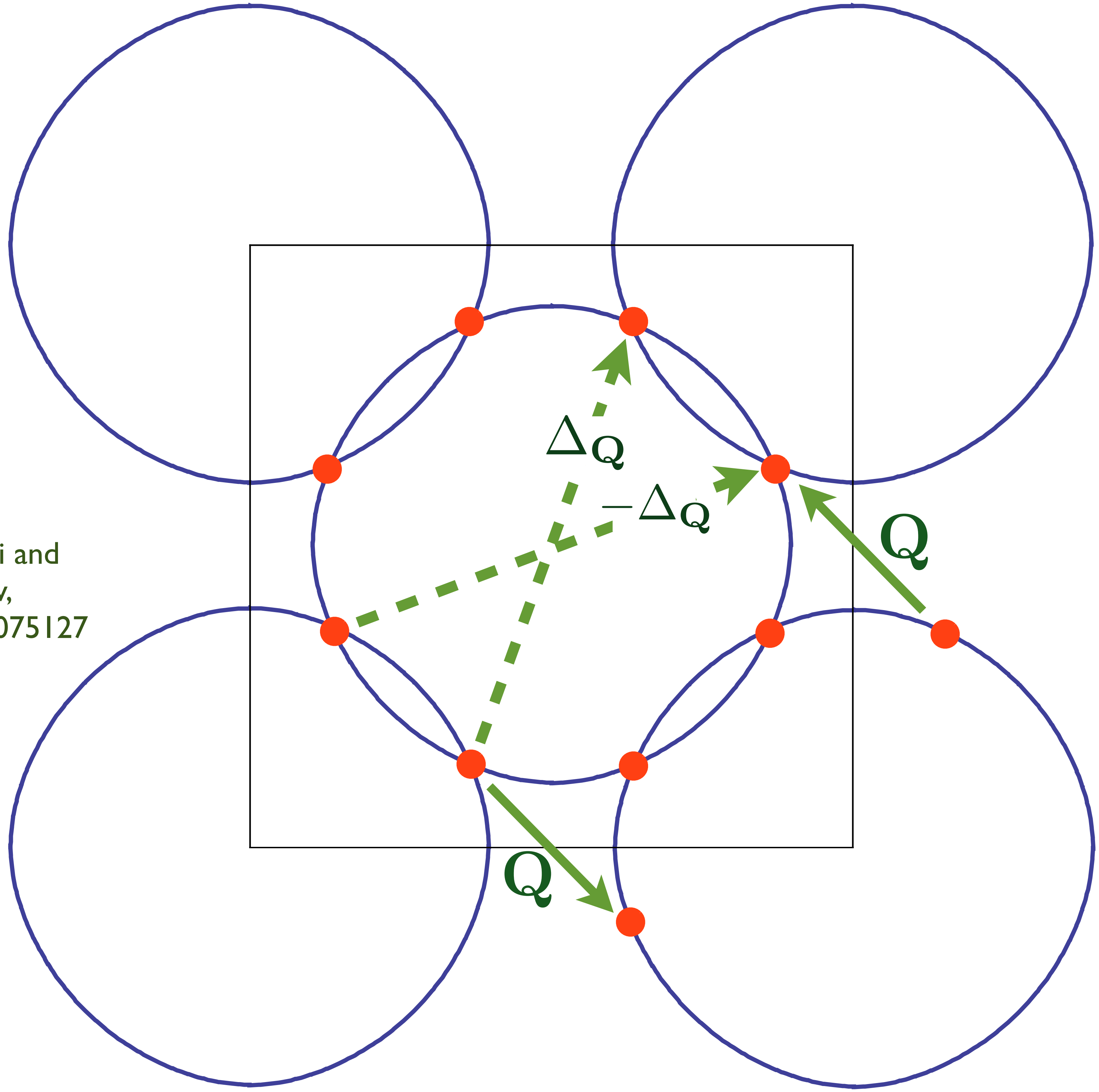
Incommensurate *d*-wave bond order



M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

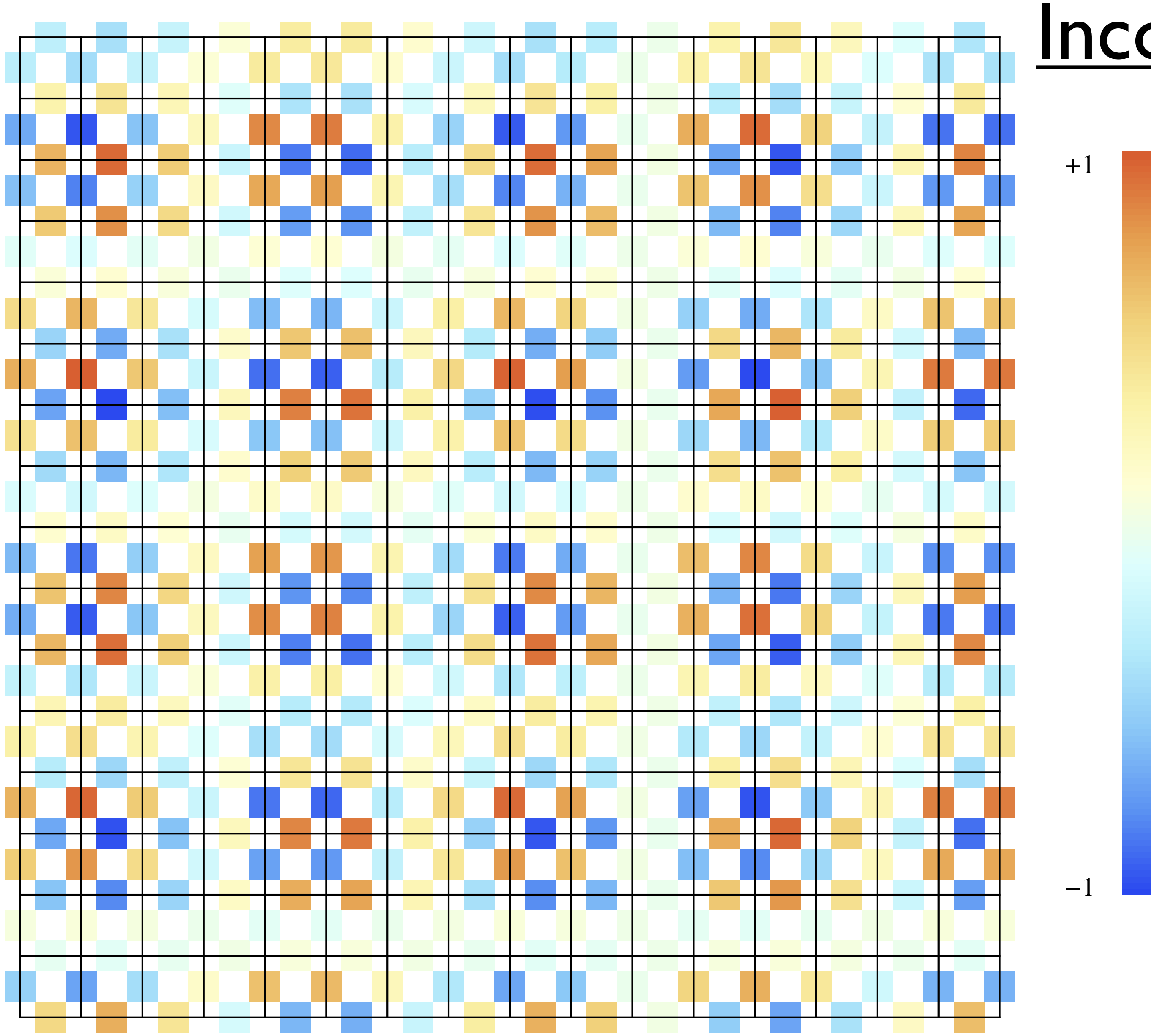
Incommensurate *d*-wave bond order



M. A. Metlitski and
S. Sachdev,
Phys. Rev. B **85**, 075127
(2010)

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Incommensurate *d*-wave bond order



"Bond density" measures amplitude for electrons to be in spin-singlet valence bond.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **85**, 075127 (2010)

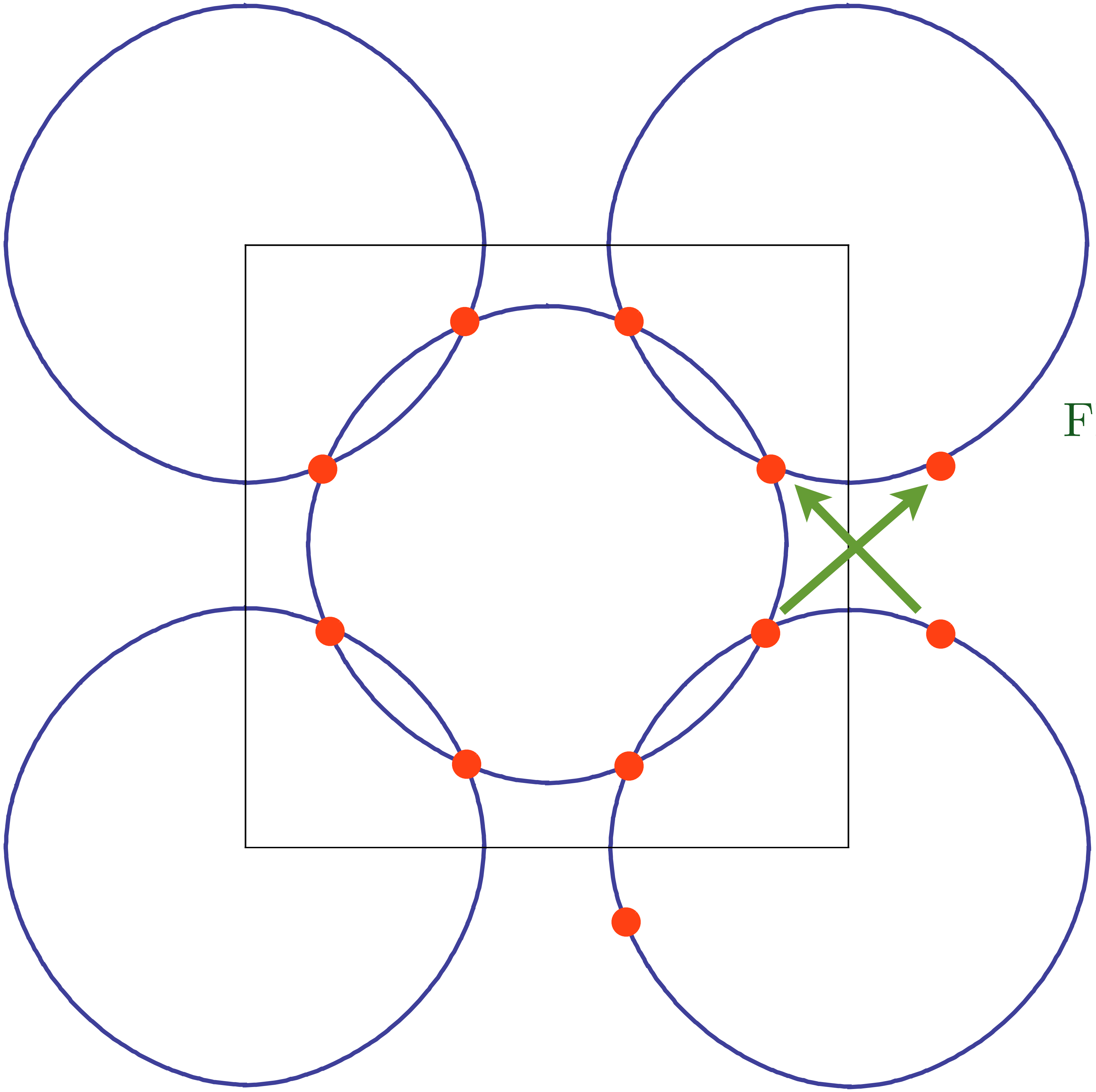
$$\left\langle c^\dagger_{\mathbf{r}\alpha} c_{\mathbf{s}\alpha} \right\rangle = \sum_{\mathbf{Q}} \sum_{\mathbf{k}} e^{i\mathbf{Q}\cdot(\mathbf{r}+\mathbf{s})/2} e^{-i\mathbf{k}\cdot(\mathbf{r}-\mathbf{s})} \left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle$$

where $\mathbf{Q}$ extends over $\mathbf{Q} = (\pm Q_0, \pm Q_0)$ with $Q_0 = 2\pi/(7.3)$ and

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Note $\left\langle c^\dagger_{\mathbf{r}\alpha} c_{\mathbf{s}\alpha} \right\rangle$ is non-zero *only* when $\mathbf{r}, \mathbf{s}$ are nearest neighbors.

Incommensurate *d*-wave bond order

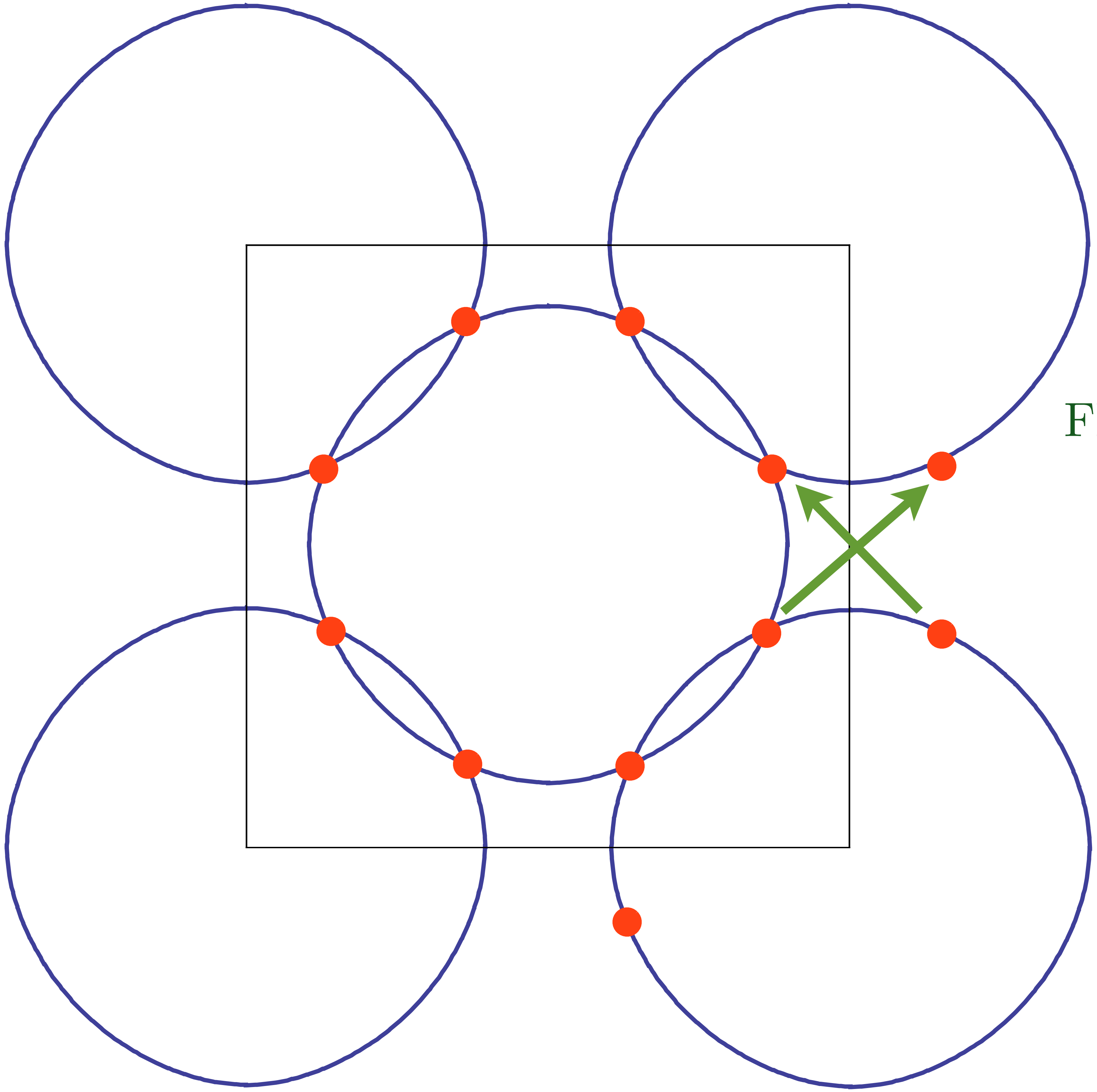


High T pseudogap: Fluctuating composite order parameter of nearly degenerate d-wave pairing and incommensurate d-wave bond order

K. B. Efetov, H. Meier, and C. Pepin, arXiv:1210.3276

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$

Incommensurate *d*-wave bond order



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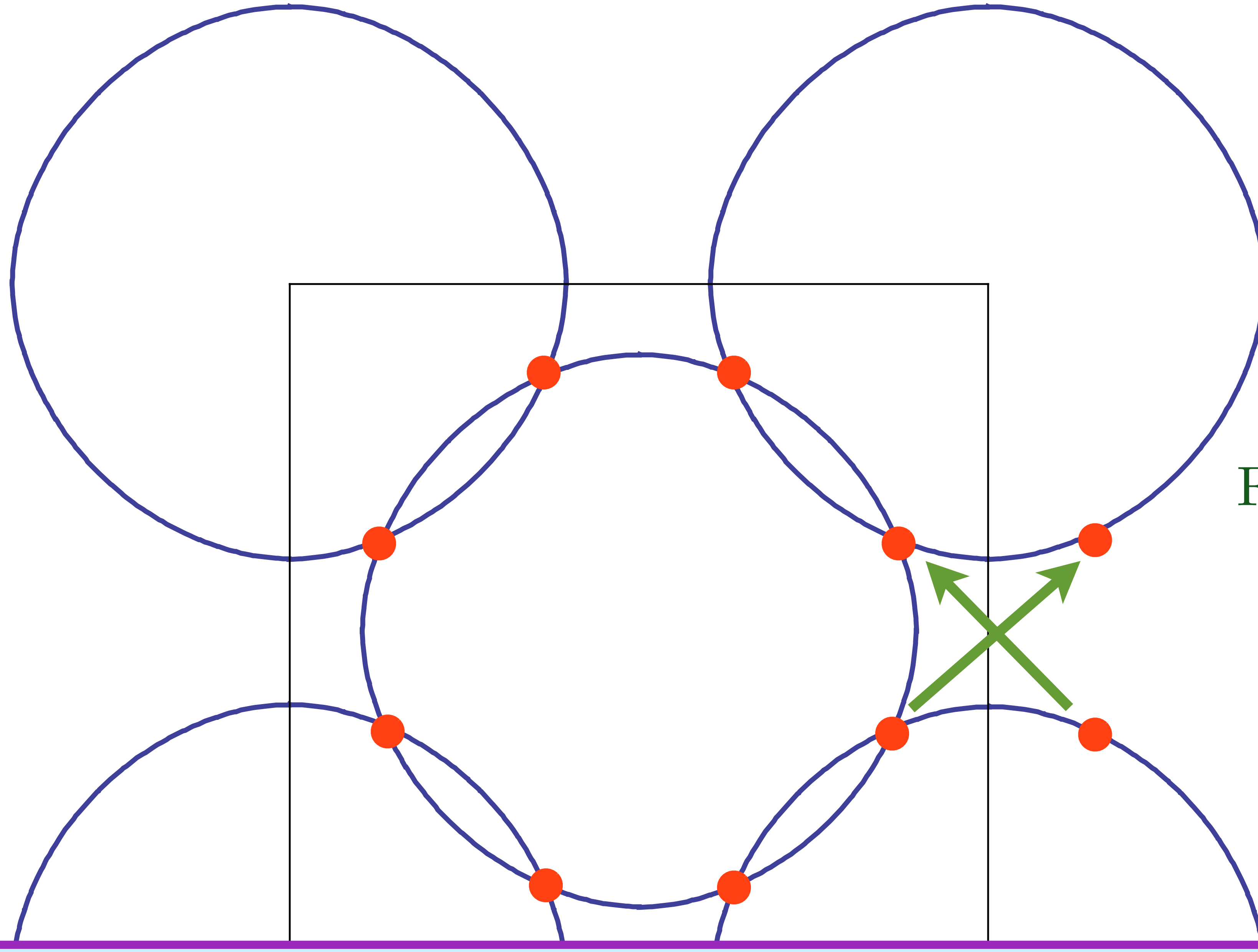
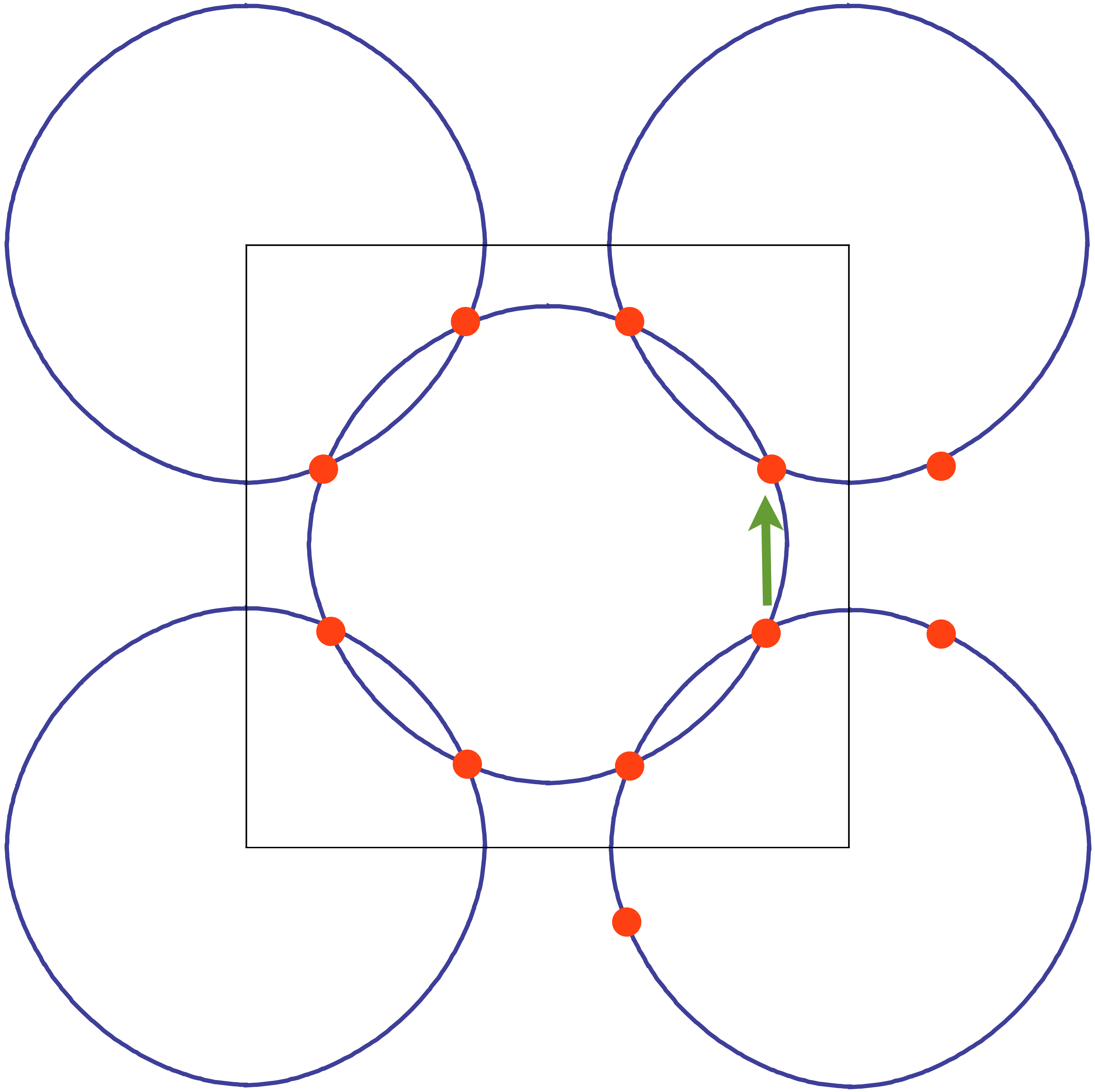
Incommensurate *d*-wave bond order



High T pseudogap: Fluctuating composite order parameter of nearly degenerate d-wave pairing and incommensurate d-wave bond order

The theory of the composite order parameter of *d*-wave pairing and *d*-wave bond order has similarities to the theory of the composite order parameter of antiferromagnetism and kekule bond order on the honeycomb lattice



Observed
low T ordering

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Hartree-Fock computation on lattice model

$$H = \sum_{\boldsymbol{k}} \varepsilon(\boldsymbol{k})\, c^{\dagger}_{\boldsymbol{k},\alpha} c_{\boldsymbol{k},\alpha} - \frac{1}{2V} \sum_{\boldsymbol{q}} \chi(\boldsymbol{q})\, \vec{S}(-\boldsymbol{q}) \cdot \vec{S}(\boldsymbol{q}).$$

$$\vec{S}(\boldsymbol{q}) = \sum_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}+\boldsymbol{q},\alpha}\, \vec{\sigma}_{\alpha\beta}\, c_{\boldsymbol{k},\beta}$$

S. Sachdev and R. La Placa, arXiv:1303.2114

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$$H_{MF} = \sum_{\boldsymbol{k}} \Big[\varepsilon(\boldsymbol{k})\, c^\dagger_{\boldsymbol{k},\alpha} c_{\boldsymbol{k},\alpha} + \Delta_S(\boldsymbol{k})\, \epsilon_{\alpha\beta}\, c_{\boldsymbol{k},\alpha} c_{-\boldsymbol{k}\beta} + \text{H.c.}$$

$$+ \sum_{\boldsymbol{Q}} \Delta_{\boldsymbol{Q}}(\boldsymbol{k})\, c^\dagger_{\boldsymbol{k}+\boldsymbol{Q}/2,\alpha} c_{\boldsymbol{k}-\boldsymbol{Q}/2,\alpha} \Big],$$

$$F \leq F_{MF} + \langle H - H_{MF} \rangle_{MF}$$

S. Sachdev and R. La Placa, arXiv:1303.2114

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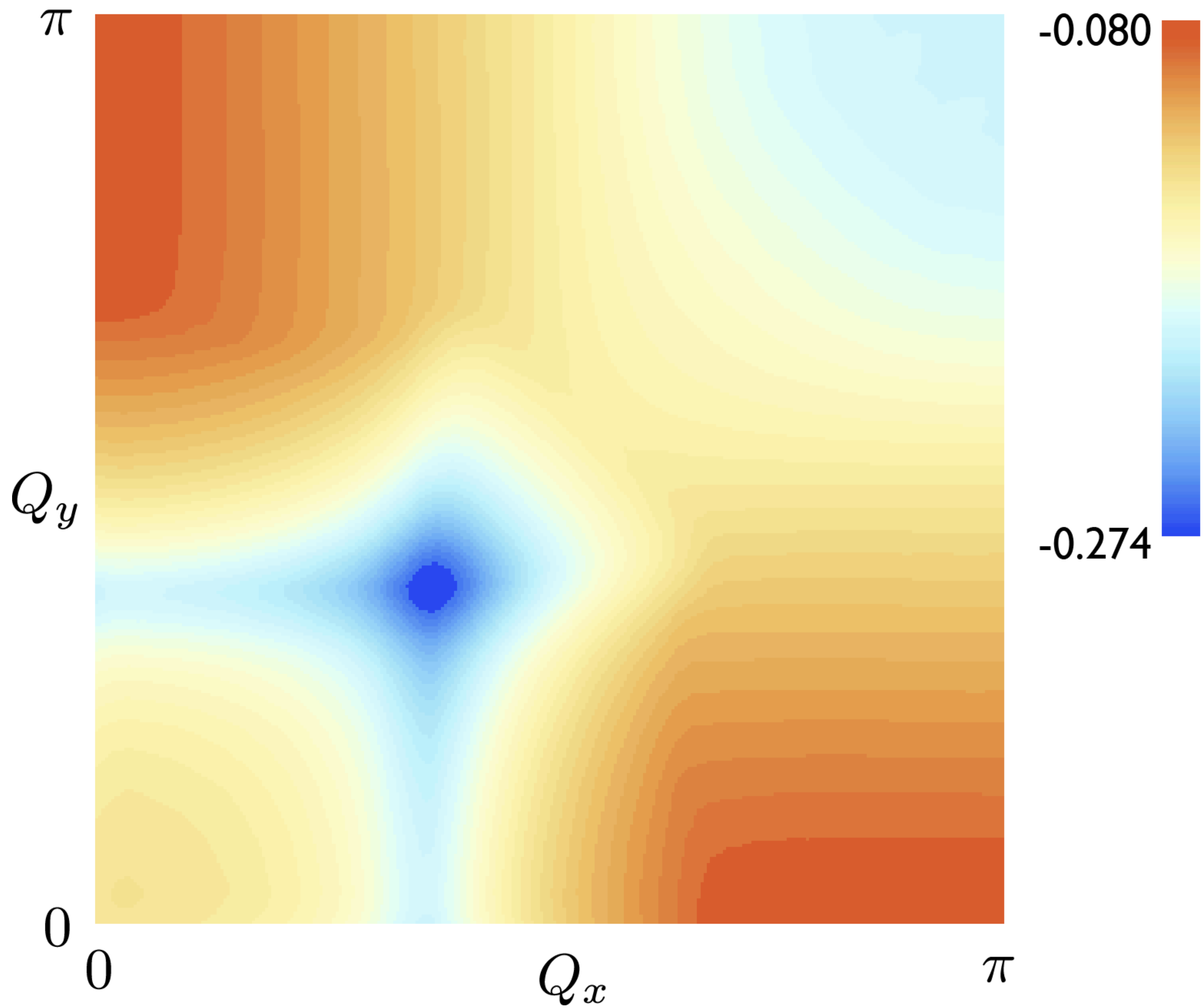
$$H_{MF} = \sum_{\boldsymbol{k}} \Big[\varepsilon(\boldsymbol{k})\, c^{\dagger}_{\boldsymbol{k},\alpha} c_{\boldsymbol{k},\alpha} + \Delta_S(\boldsymbol{k})\, \epsilon_{\alpha\beta}\, c_{\boldsymbol{k},\alpha} c_{-\boldsymbol{k}\beta} + \text{H.c.} + \sum_{\boldsymbol{Q}} \Delta_{\boldsymbol{Q}}(\boldsymbol{k})\, c^{\dagger}_{\boldsymbol{k}+\boldsymbol{Q}/2,\alpha} c_{\boldsymbol{k}-\boldsymbol{Q}/2,\alpha} \Big],$$

Expand F to second order in $\Delta_S(\mathbf{k})$ and $\Delta_{\mathbf{Q}}(\mathbf{k})$, and obtain lowest eigenvalues λ_S and $\lambda_{\mathbf{Q}}$ and corresponding eigenvectors $\Delta_S(\mathbf{k})$ and $\Delta_{\mathbf{Q}}(\mathbf{k})$.

S. Sachdev and R. La Placa, arXiv:1303.2114

Hartree-Fock computation on lattice model



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}$.

S. Sachdev and R. La Placa, arXiv:1303.2114

Hartree-Fock computation on lattice model

$$\Delta_{\boldsymbol{Q}}(\boldsymbol{k}) = \sum_{\gamma} c_{\boldsymbol{Q},\gamma}\, \psi_\gamma(\boldsymbol{k})$$

γ	$\psi_\gamma(\boldsymbol{k})$	$\boldsymbol{Q}$ = (1.15,1.15)	$\boldsymbol{Q}$ = (1.15, 0)	$\boldsymbol{Q}$ = (0,0)	$\boldsymbol{Q}$ = (π,π)	$\Delta_S(\boldsymbol{k})$
s	1	0	-0.231	0	0	0
s'	$\cos k_x + \cos k_y$	0	0.044	0	0	0
s''	$\cos(2k_x) + \cos(2k_y)$	0	-0.046	0	0	0
d	$\cos k_x - \cos k_y$	0.993	0.963	0.997	0	0.997
d'	$\cos(2k_x) - \cos(2k_y)$	- 0.069	-0.067	-0.057	0	-0.056
d''	$2 \sin k_x \sin k_y$	0	0	0	0	0
p_x	$\sqrt{2} \sin k_x$	0	0	0	0.706	0
p_y	$\sqrt{2} \sin k_y$	0	0	0	-0.706	0
g	$(\cos k_x - \cos k_y) \times \sqrt{8} \sin k_x \sin k_y$	-0.009	0	0	0	0

Charge-ordering eigenvector

S. Sachdev and R. La Placa, arXiv:1303.2114

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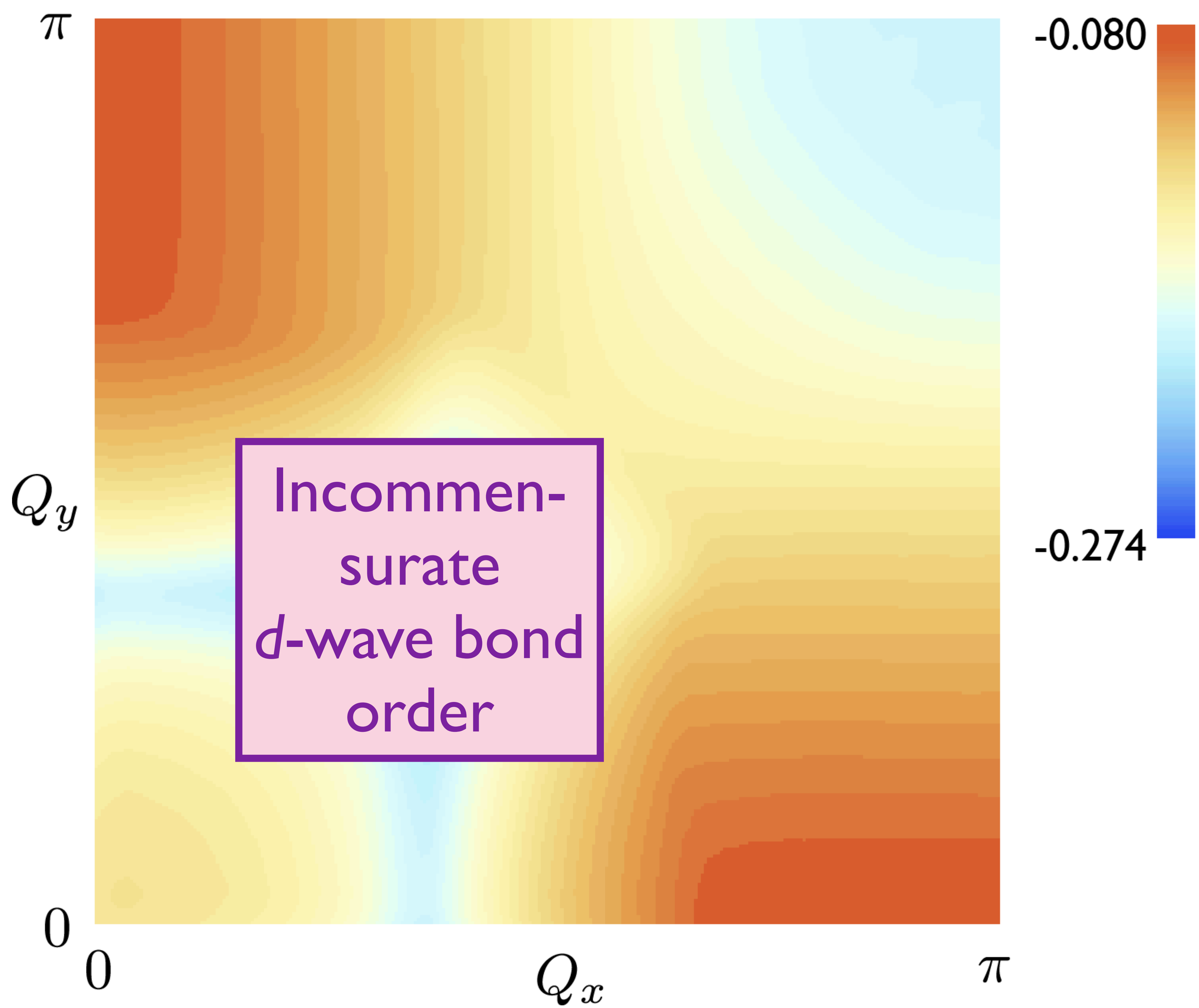
S. Sachdev and R. La Placa, arXiv:1303.2114

Hartree-Fock computation on lattice model



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}$.

S. Sachdev and R. La Placa, arXiv:1303.2114

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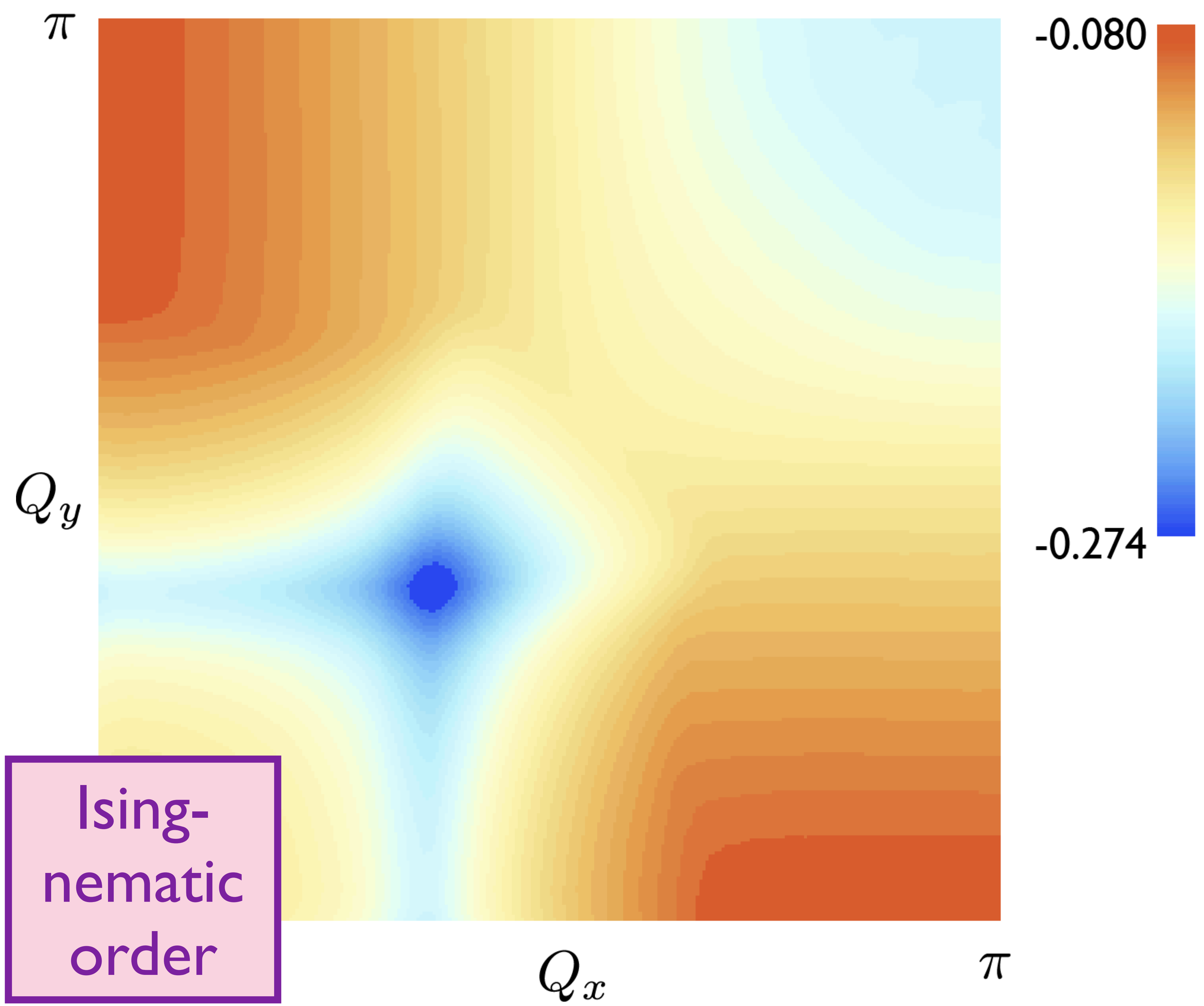
s''	$\cos(2k_x) + \cos(2k_y)$	0	-0.046	0	0	0
d	$\cos k_x - \cos k_y$	0.993	0.963	0.997	0	0.997
d'	$\cos(2k_x) - \cos(2k_y)$	- 0.069	-0.067	-0.057	0	-0.056
d''	$2 \sin k_x \sin k_y$	0	0	0	0	0
p_x	$\sqrt{2} \sin k_x$	0	0	0	0.706	0
p_y	$\sqrt{2} \sin k_y$	0	0	0	-0.706	0
g	$(\cos k_x - \cos k_y) \times \sqrt{8} \sin k_x \sin k_y$	-0.009	0	0	0	0

Charge-ordering eigenvector

S. Sachdev and R. La Placa, arXiv:1303.2114

Hartree-Fock computation on lattice model



Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}$.

S. Sachdev and R. La Placa, arXiv:1303.2114

Hartree-Fock computation on lattice model

$$\Delta_{\boldsymbol{Q}}(\boldsymbol{k}) = \sum_{\gamma} c_{\boldsymbol{Q},\gamma}\, \psi_\gamma(\boldsymbol{k})$$

γ	$\psi_\gamma(\boldsymbol{k})$	$\boldsymbol{Q} =$ (1.15,1.15)	$\boldsymbol{Q} =$ (1.15, 0)	$\boldsymbol{Q} =$ (0,0)	$\boldsymbol{Q} =$ (π,π)	$\Delta_S(\boldsymbol{k})$
s	1	0	-0.231	0	0	0
s'	$\cos k_x + \cos k_y$	0	0.044	0	0	0
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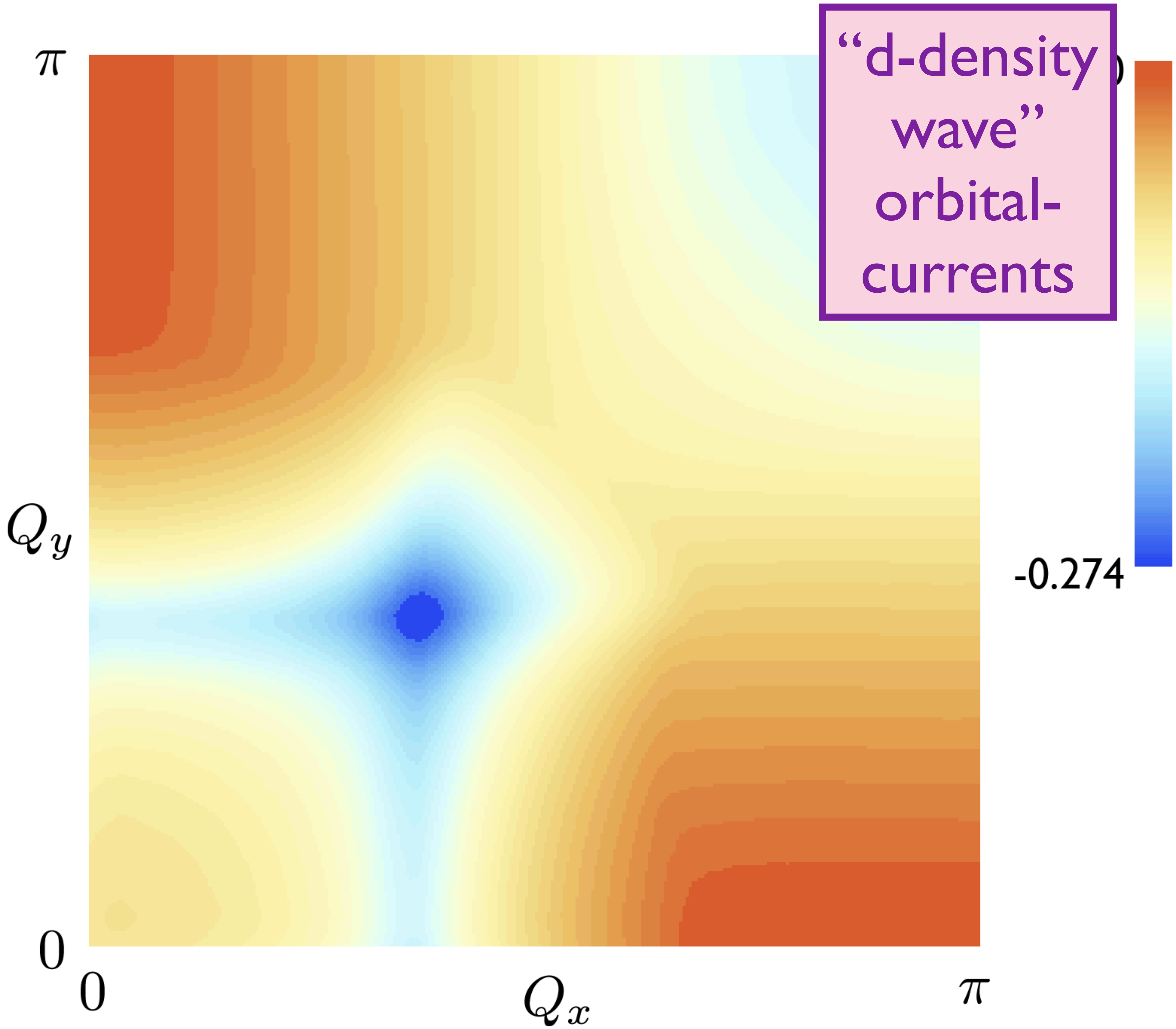

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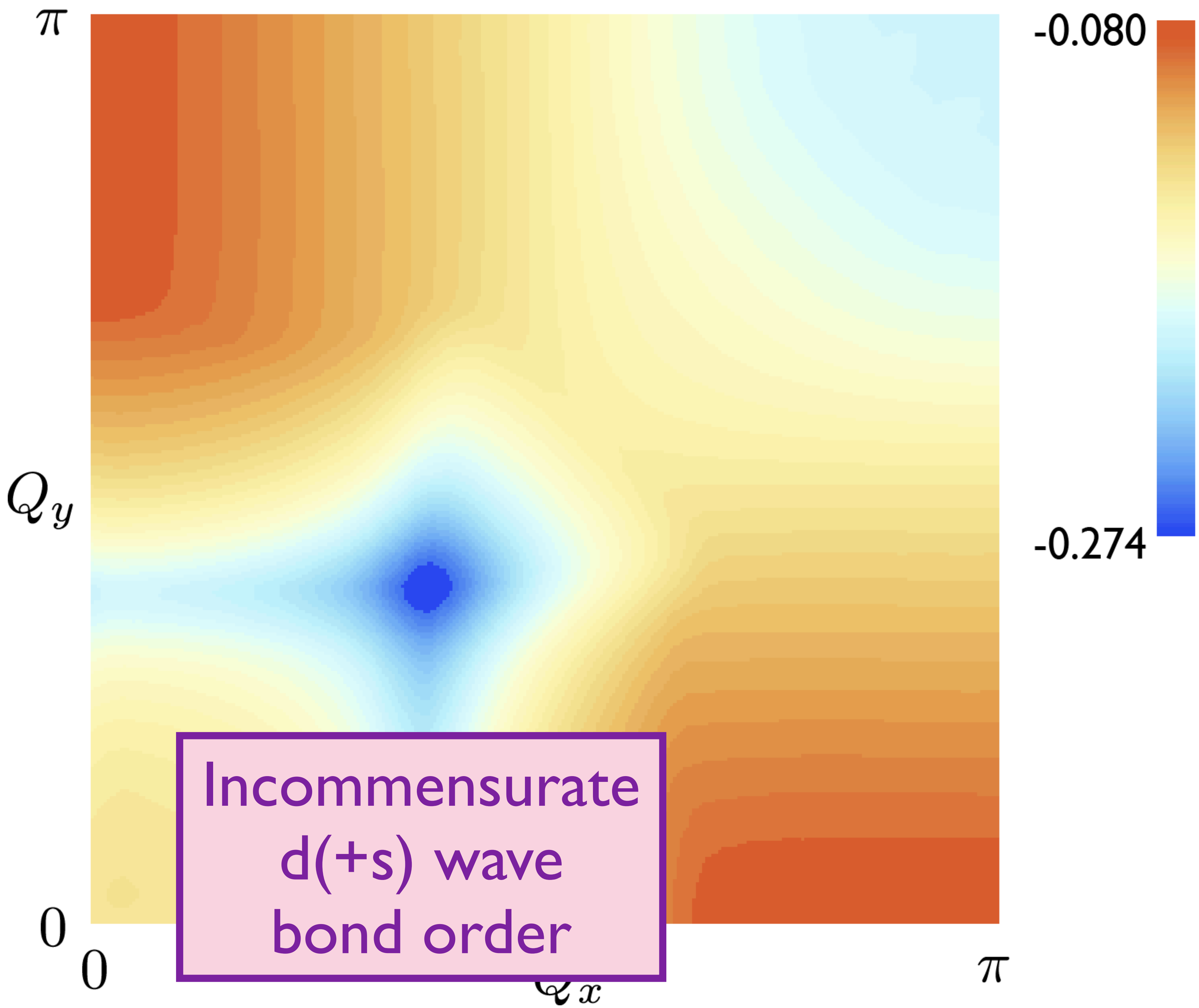
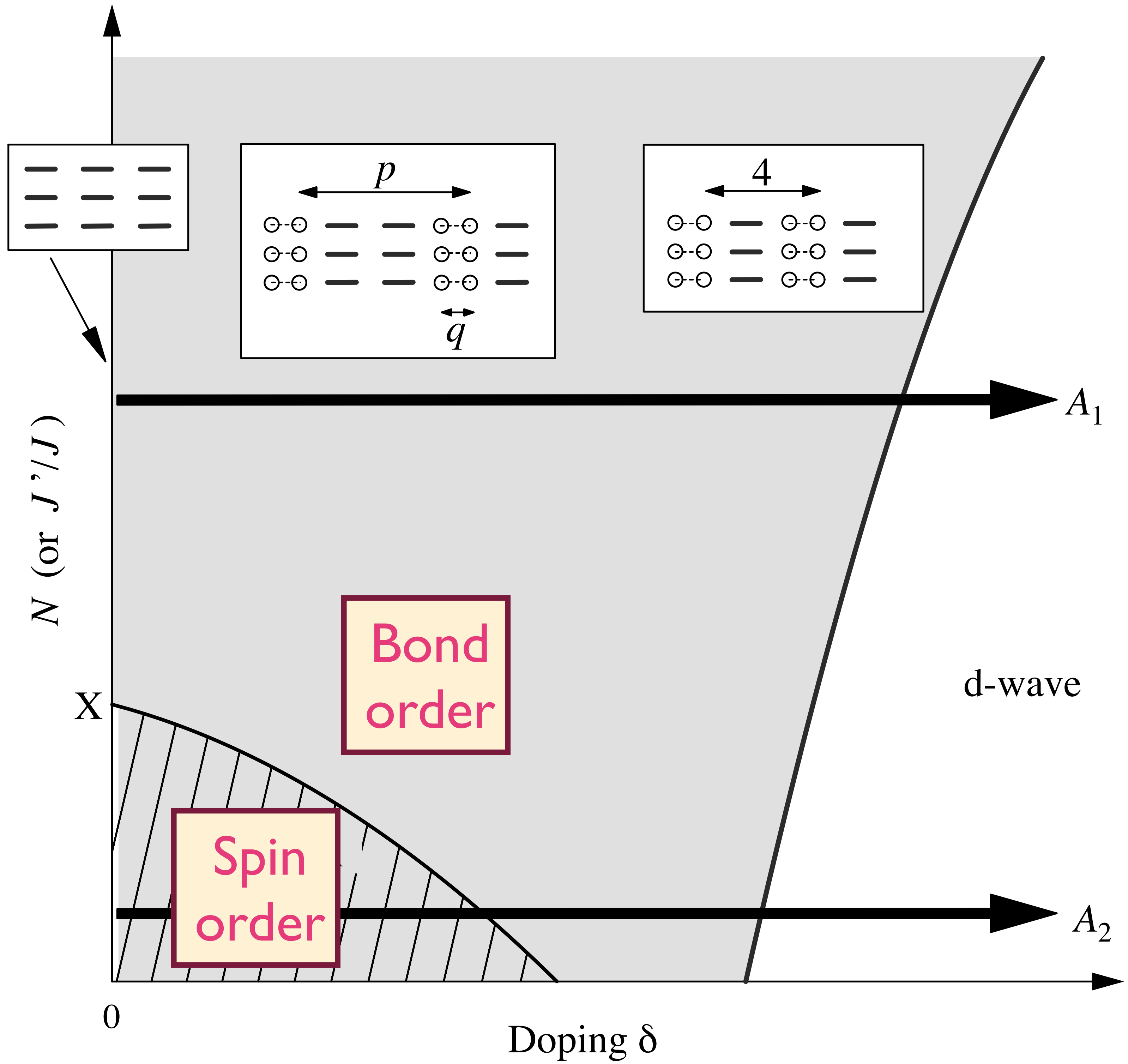


Charge-ordering eigenvalue $\lambda_{\mathbf{Q}}$.

S. Sachdev and R. La Placa, arXiv:1303.2114

Evidence bond order is along (1,0), (0,1) directions in low T superconducting phase



M. Vojta and S. Sachdev, Physical Review Letters **83**, 3916 (1999)
S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991)

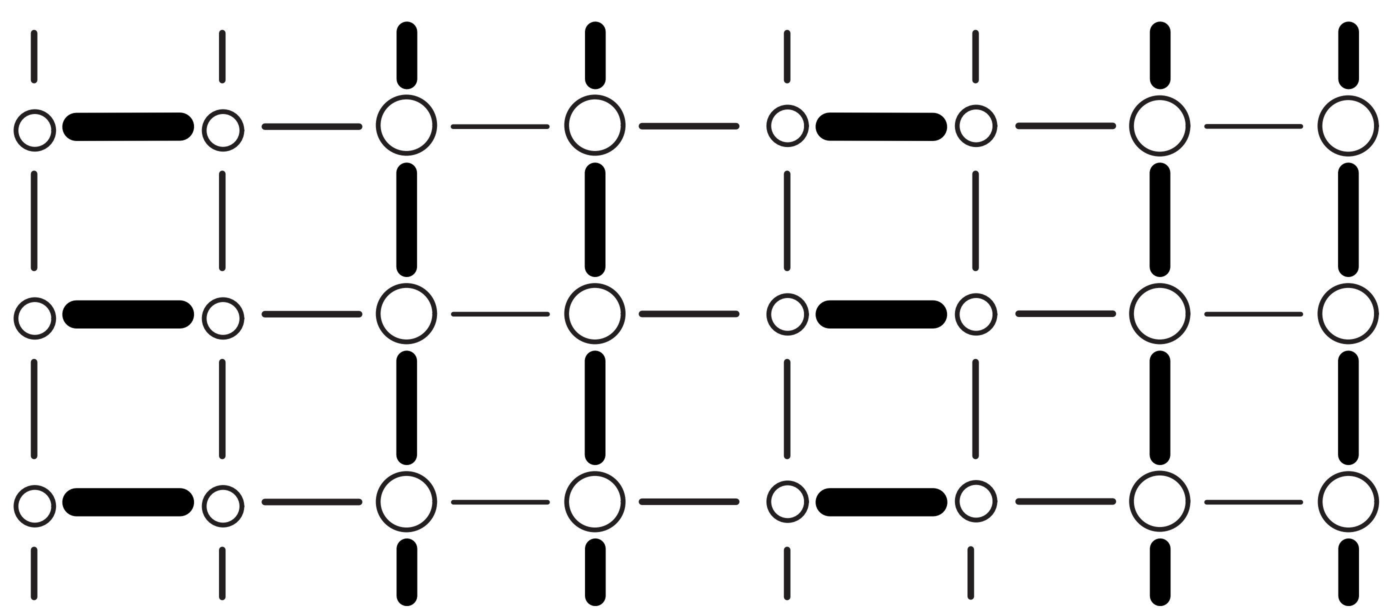
Evidence bond order is along (1,0), (0,1) directions in low T superconducting phase

PHYSICAL REVIEW B **77**, 094504 (2008)

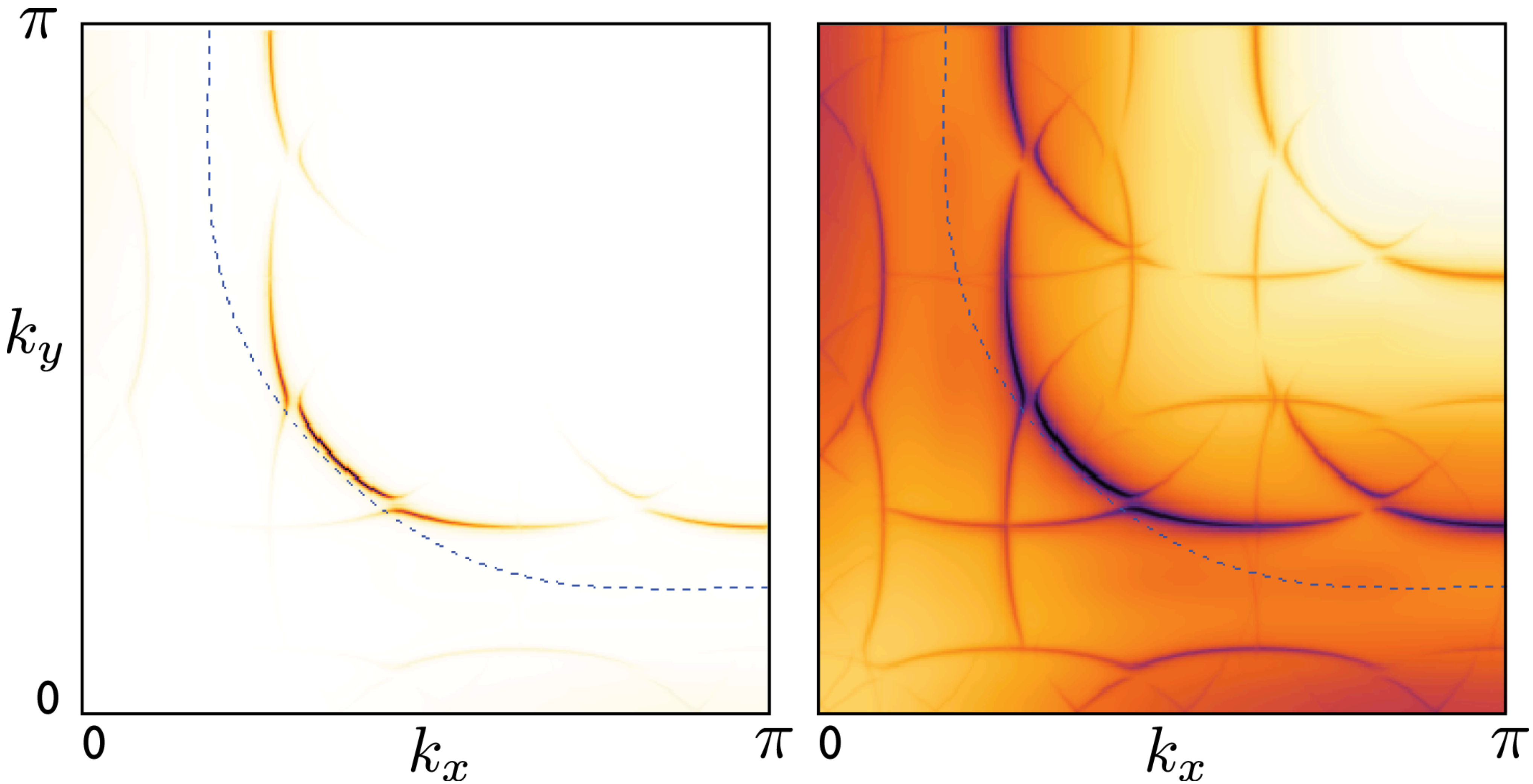
Superconducting d-wave stripes in cuprates: Valence bond order coexisting with nodal quasiparticles

Matthias Vojta and Oliver Rösch
Institut für Theoretische Physik, Universität zu Köln, Zülpicher Straße 77, 50937 Köln, Germany
(Received 8 January 2008; revised manuscript received 10 January 2008; published 6 March 2008)

We point out that unidirectional bond-centered charge-density-wave states in cuprates involve electronic order in both s- and d-wave channels, with nonlocal Coulomb repulsion suppressing the s-wave component. The resulting bond-charge-density wave, coexisting with superconductivity, is compatible with recent photoemission and tunneling data and as well as neutron-scattering measurements, once long-range order is destroyed by slow fluctuations or glassy disorder. In particular, the real-space structure of d-wave stripes is consistent with the scanning-tunneling-microscopy measurements on both underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ and $Ca_{2-x}Na_xCuO_2Cl_2$ of Kohsaka *et al.* [Science **315**, 1380 (2007)].



Electron spectral function



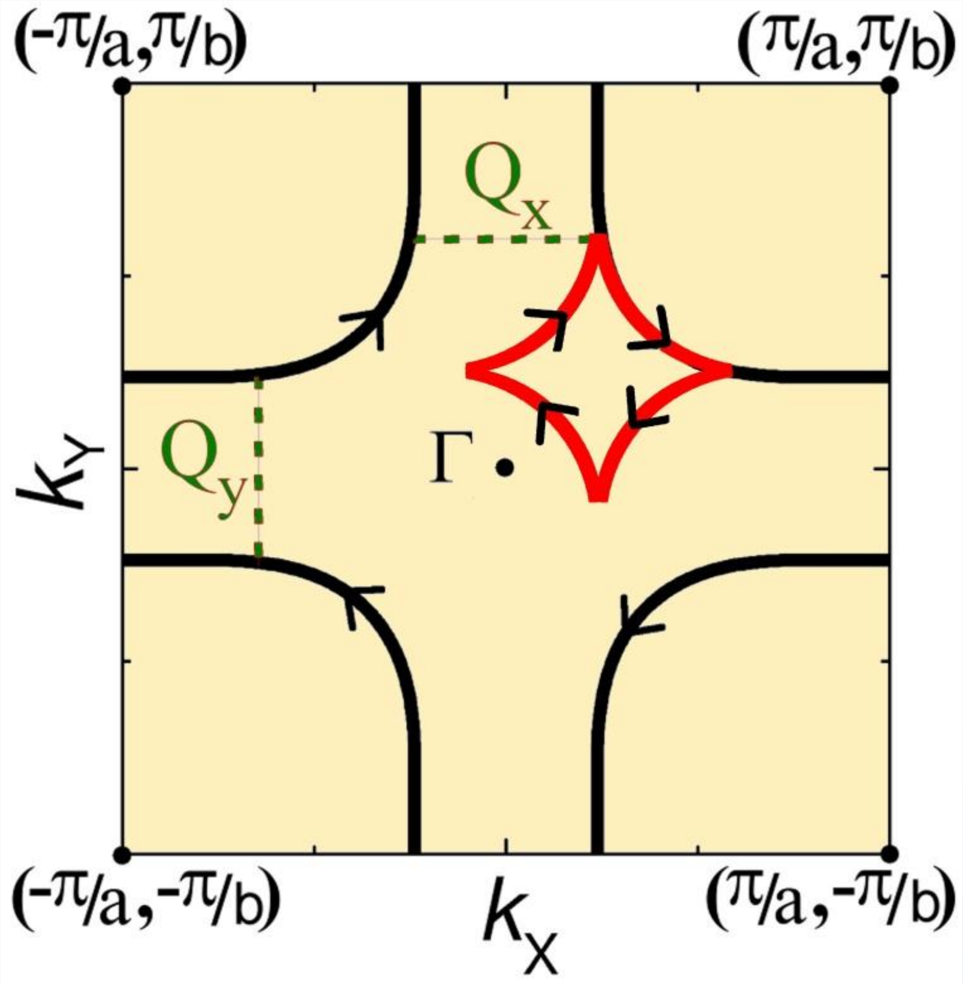
$\mathrm{Im}G(\mathbf{k}, \omega + i\eta)$ $\quad\quad$ $\log \mathrm{Im}G(\mathbf{k}, \omega + i\eta)$

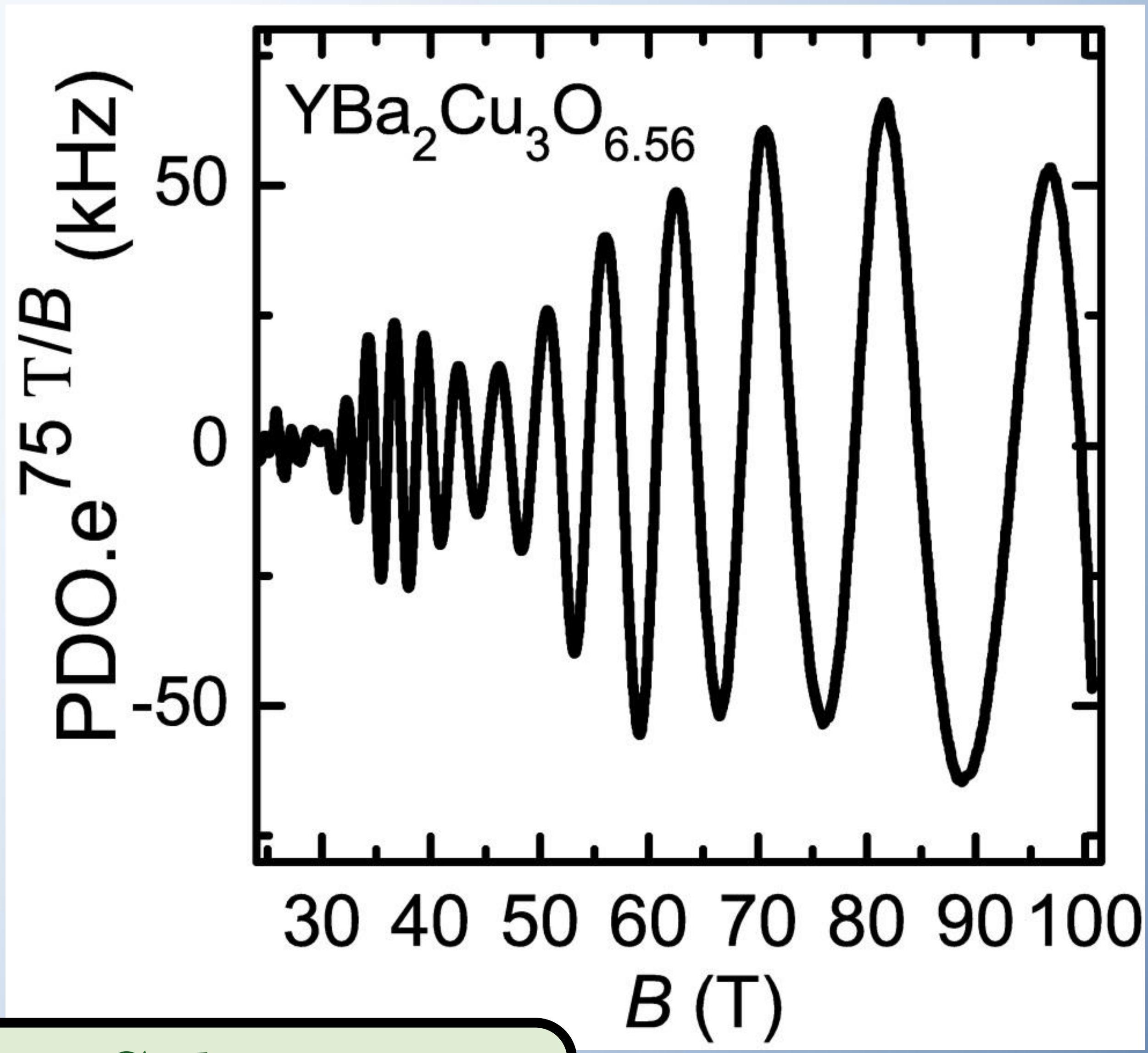
$$\left\langle c^{\dagger}_{\boldsymbol{k}-\boldsymbol{Q}/2,\alpha} c_{\boldsymbol{k}+\boldsymbol{Q}/2,\alpha} \right\rangle \propto \Delta_{\boldsymbol{Q}}(\boldsymbol{k}) = \begin{cases} \Delta_s + \Delta_d(\cos k_x - \cos k_y) & , \quad \boldsymbol{Q} = (\pm Q_0, 0) \\ \Delta_s - \Delta_d(\cos k_x - \cos k_y) & , \quad \boldsymbol{Q} = (0, \pm Q_0) \end{cases}$$

with $\Delta_s/\Delta_d = -0.234$.

S. Sachdev and R. La Placa, arXiv:1303.2114

Do we finally have a resolution to the low energy electronic structure of underdoped YBCO?





N. Harrison and S. E. Sebastian
Phys. Rev. Lett. **106**, 226402 (2011).

Outline

1. The "modern era" of cuprate experiments

2. Antiferromagnetism in metals:
   d-wave superconductivity

3. Low energy theory, emergent pseudospin
   symmetry, and bond order

4. Unrestricted Hartree-Fock-BCS

5. Quantum Monte Carlo
   without the sign problem

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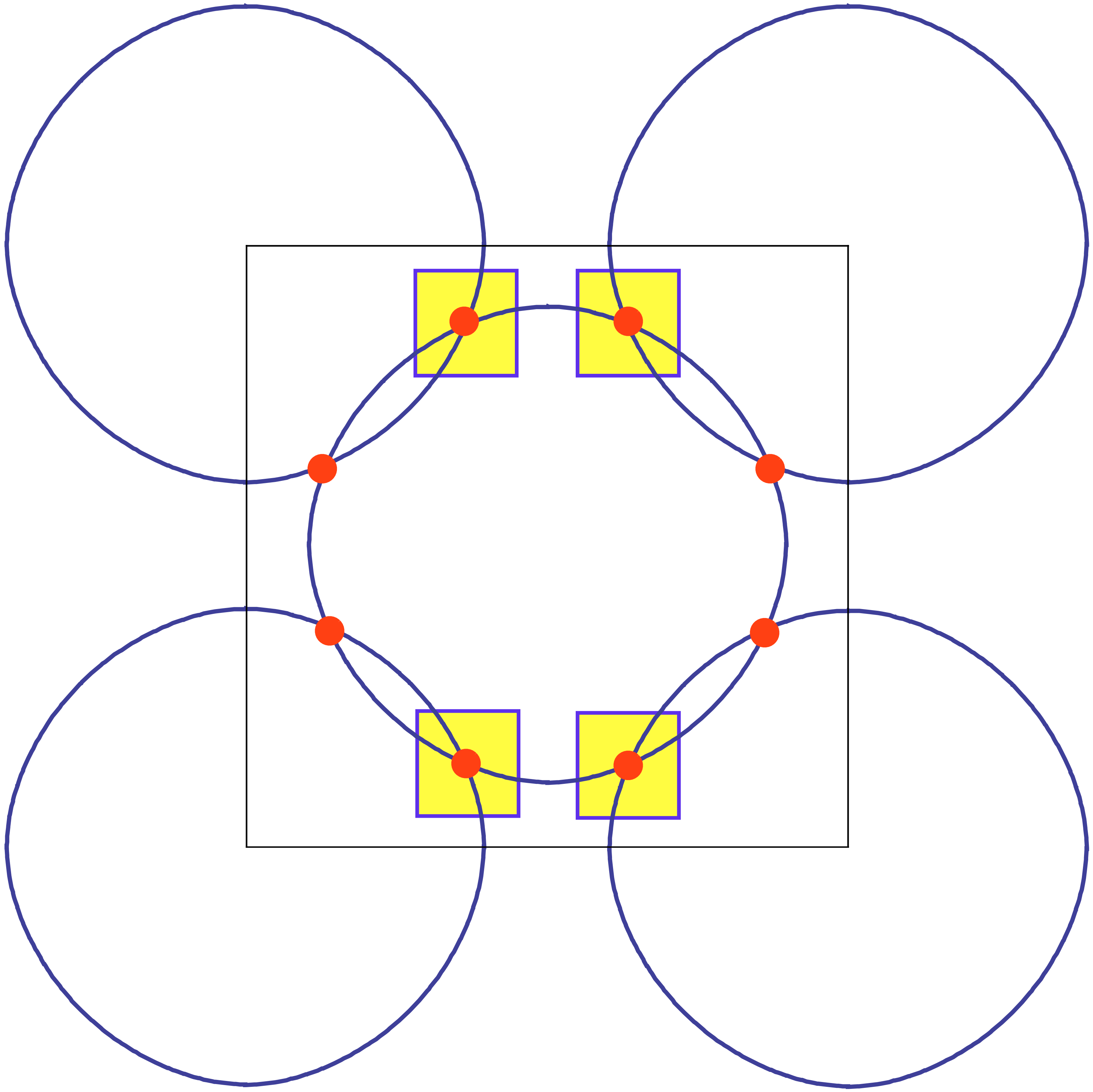
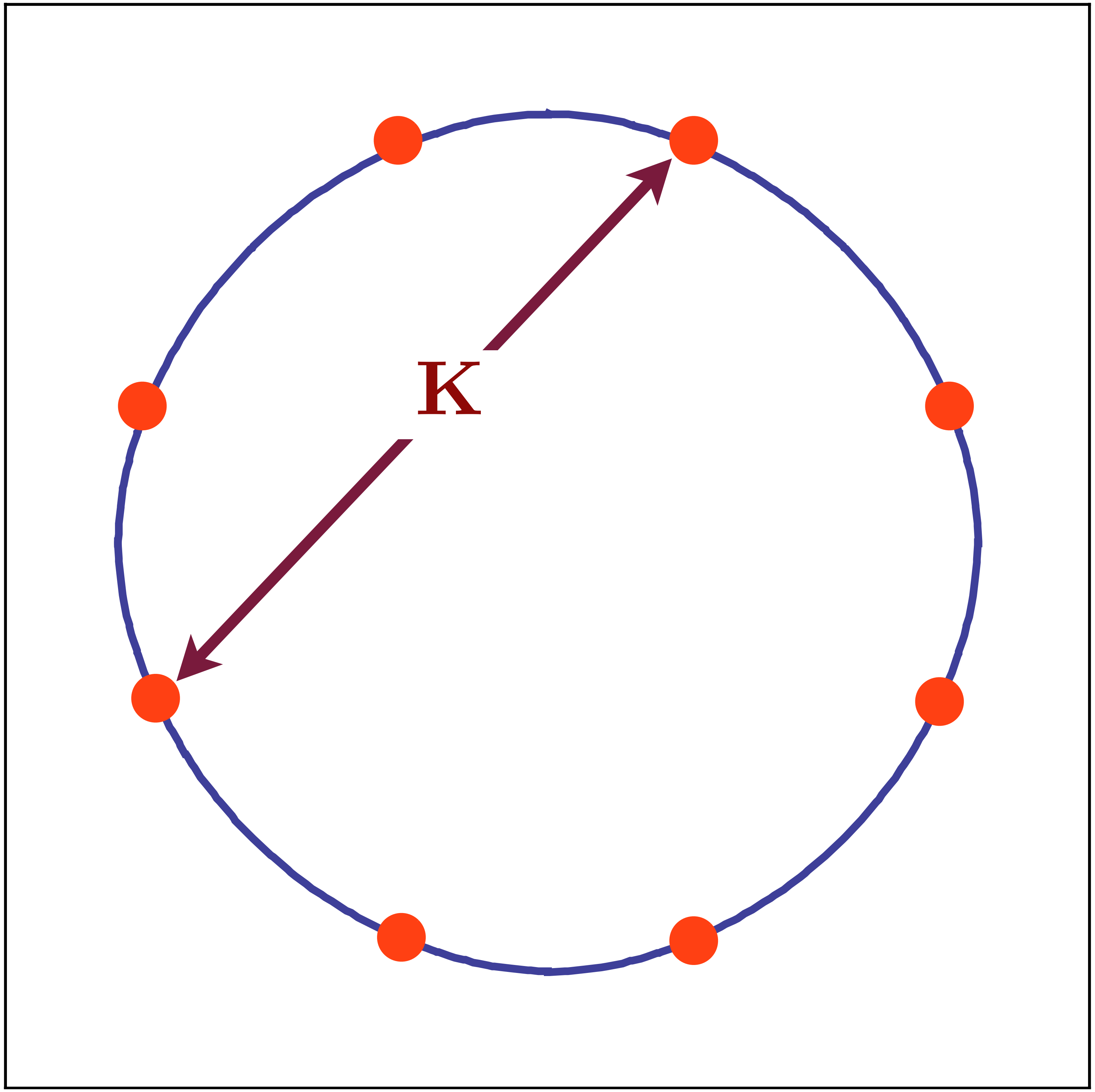


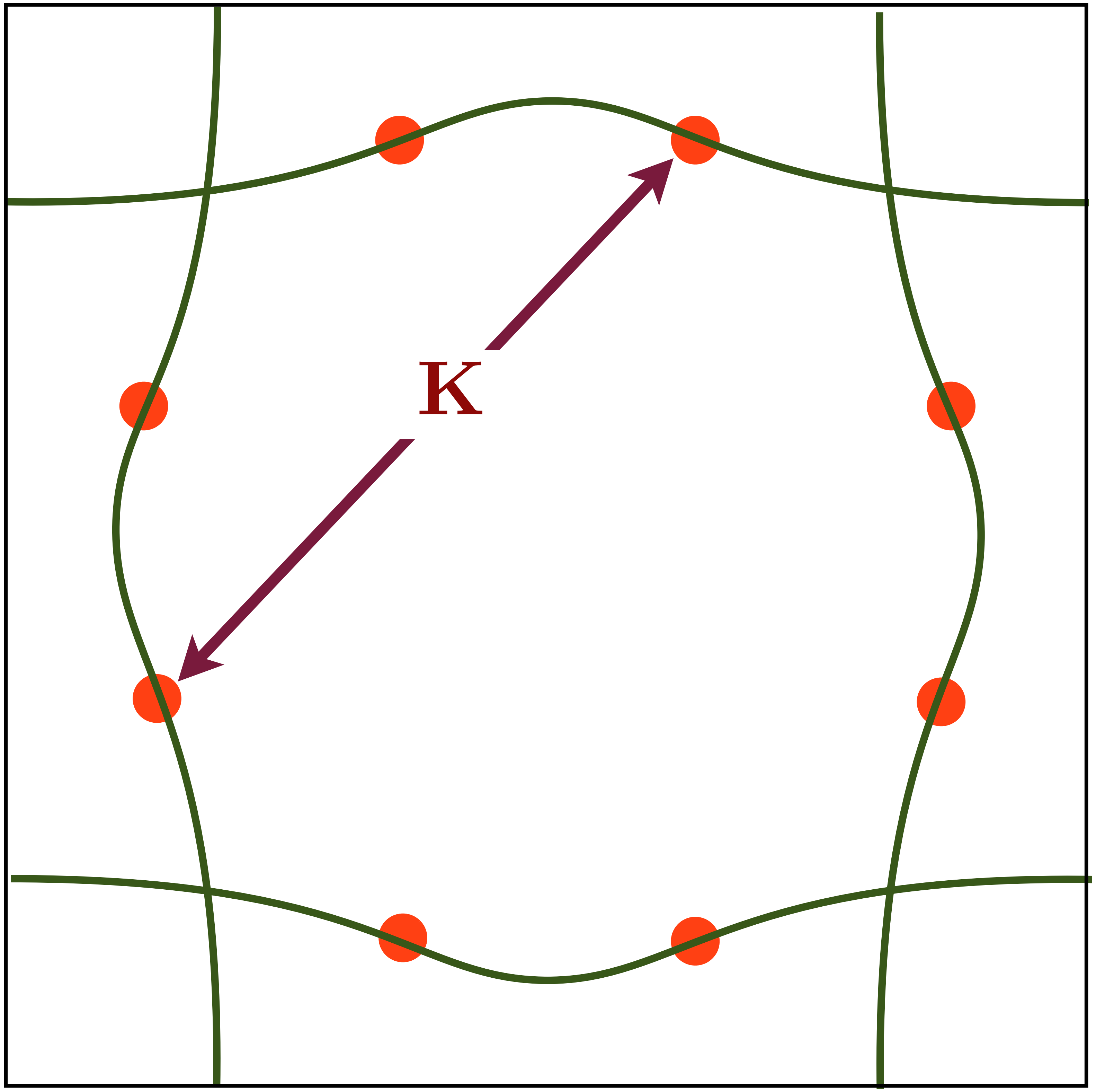
Low energy theory for critical point near hot spots

QMC for the onset of antiferromagnetism



Hot spots in a single band model

QMC for the onset of antiferromagnetism

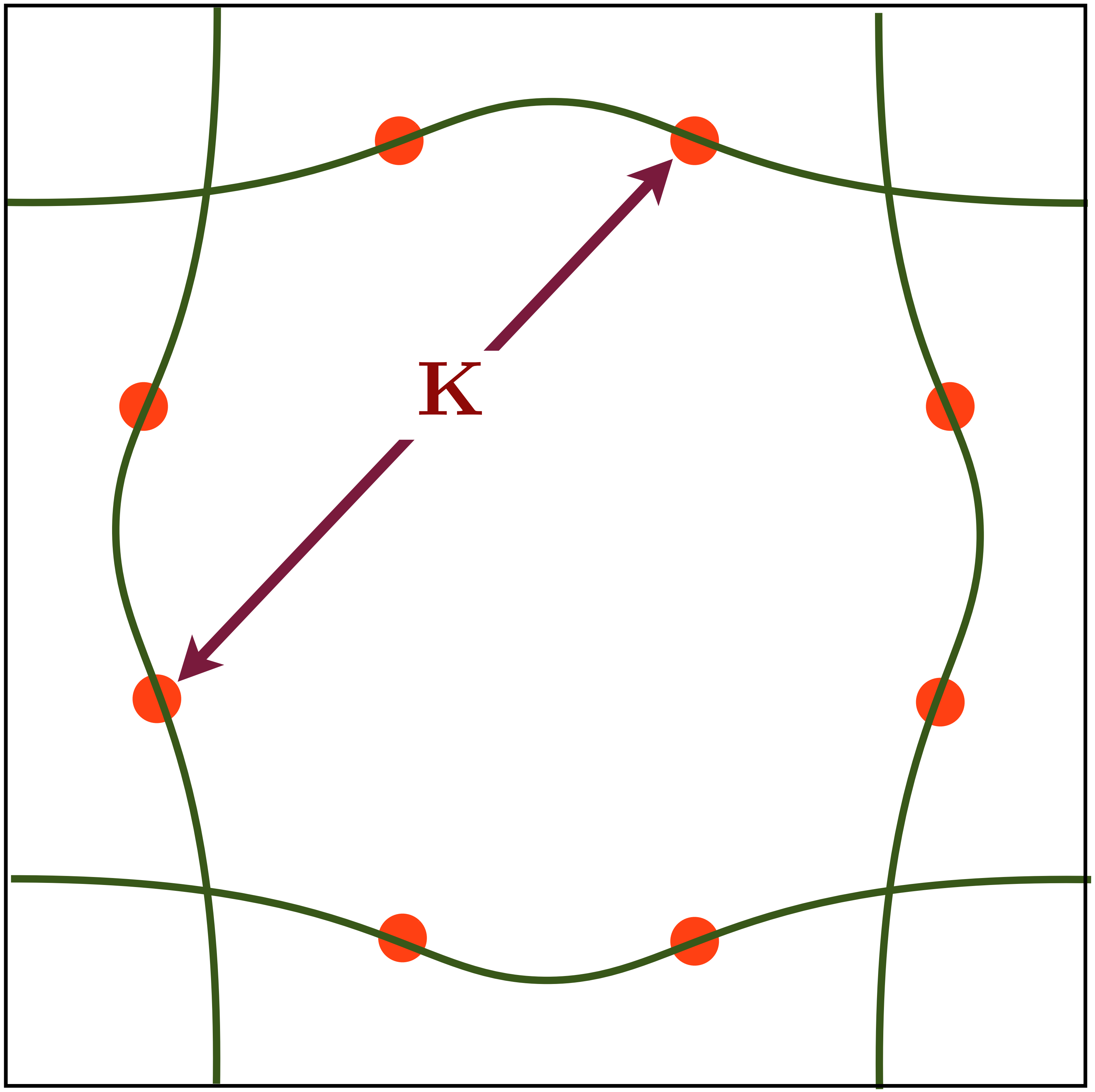


E. Berg,
M. Metlitski, and
S. Sachdev,
Science **338**, 1606
(2012).

Hot spots in a two band model

QMC for the onset of antiferromagnetism

Faithful realization of the *generic* universal low energy theory for the onset of antiferro-magnetism.



Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

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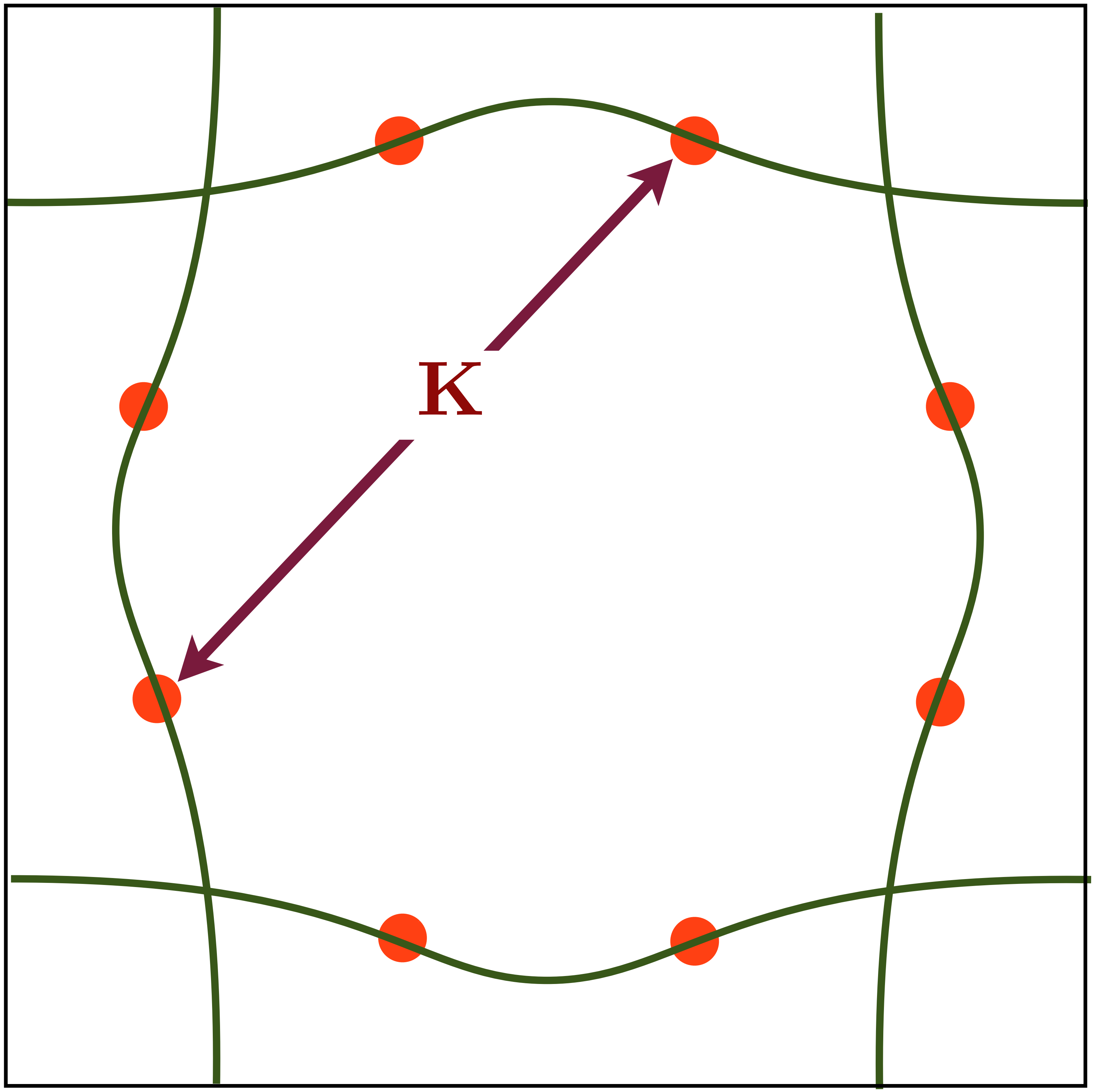

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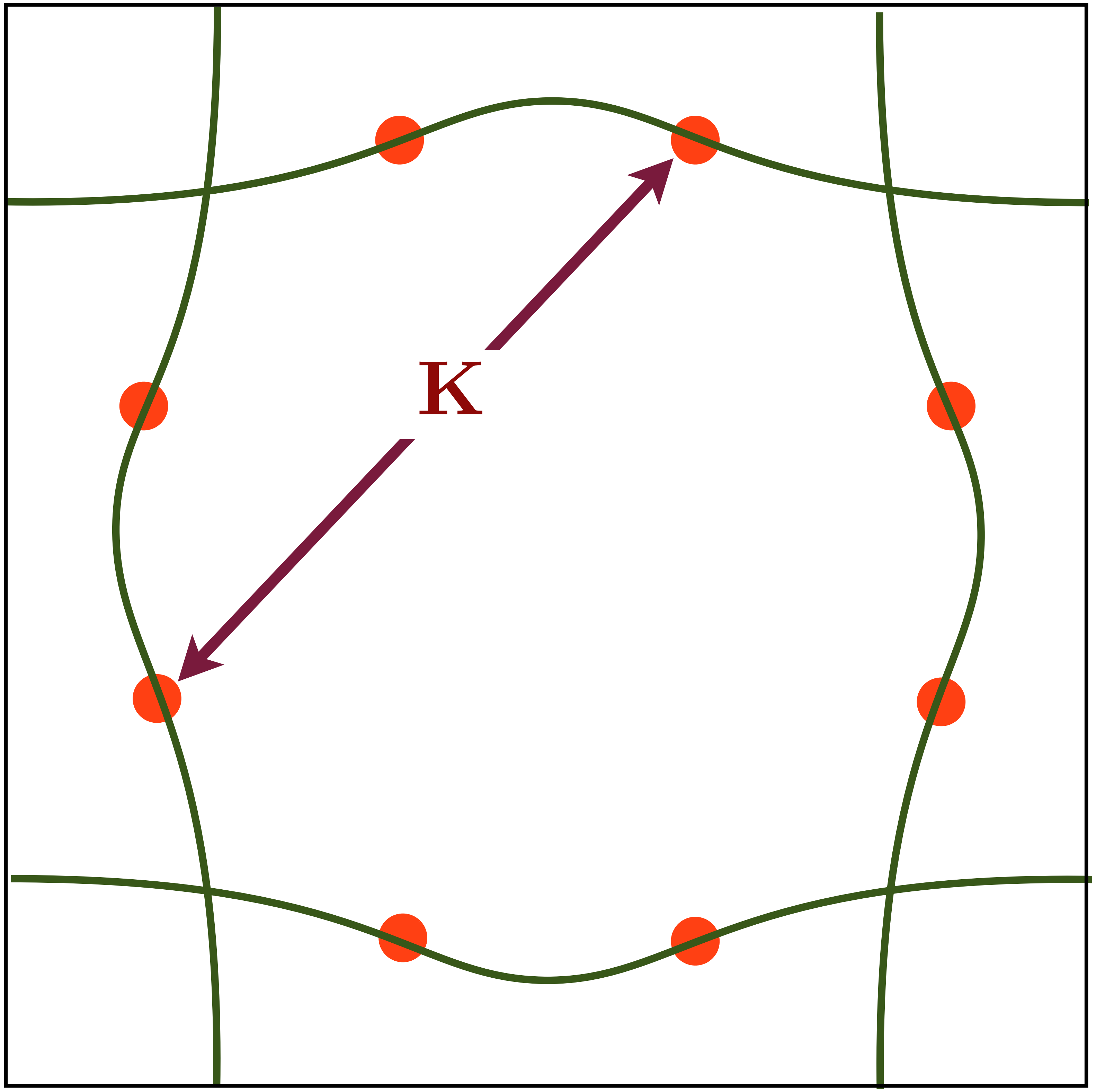
Sign problem is absent as long as ***K*** connects hotspots in distinct bands



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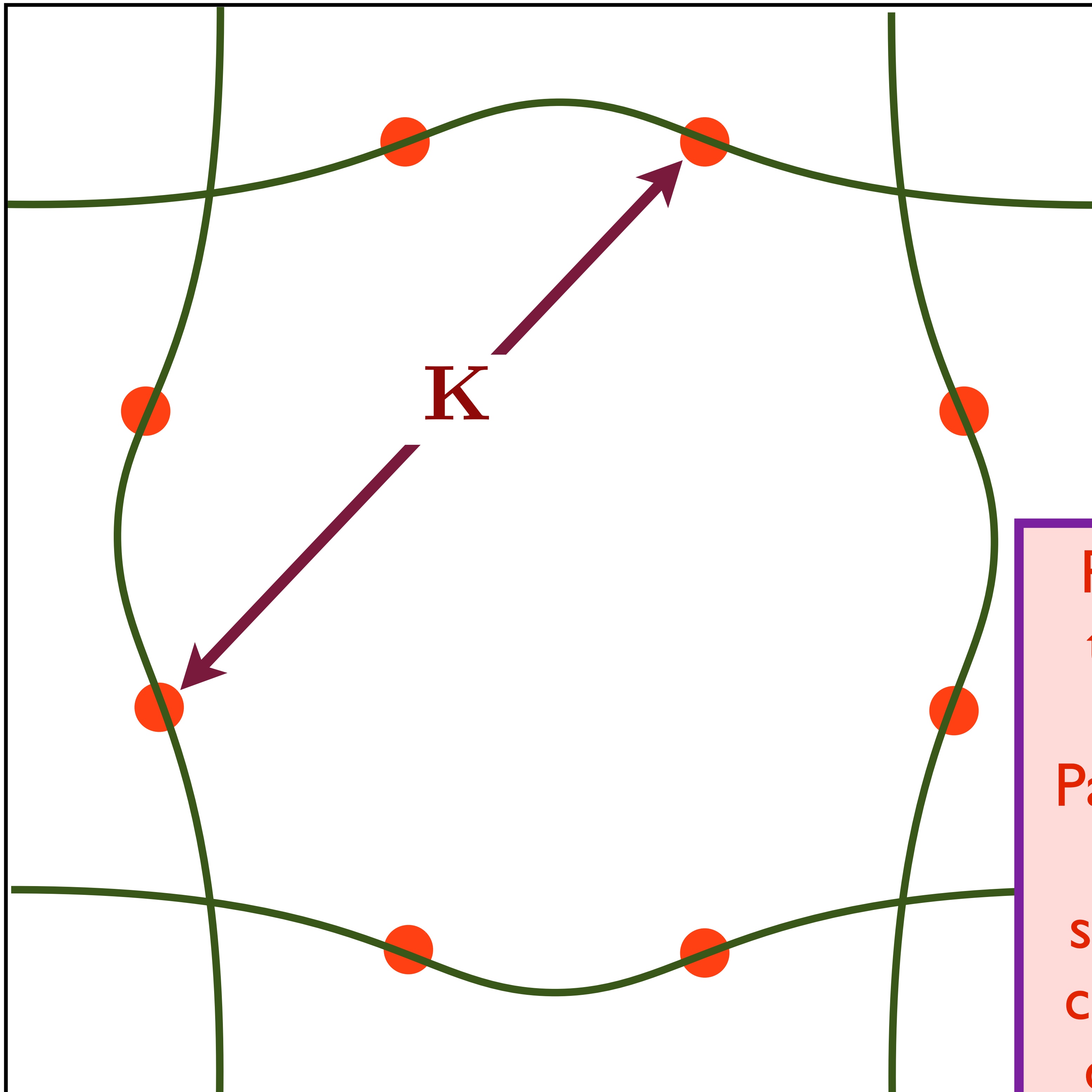

Hot spots in a two band mod

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Requires only time-reversal symmetry. Particle-hole or point-group symmetries or commensurate densities ***not*** required !

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$+ \int d\tau d^2 x \left[\frac{1}{2} \left(\boldsymbol{\nabla}_x \vec{\varphi} \right)^2 + \frac{r}{2} \vec{\varphi}^2 + \ldots \right]$$

$$- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

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E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

No sign problem !

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E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Applies without changes to the microscopic band structure in the iron-based superconductors

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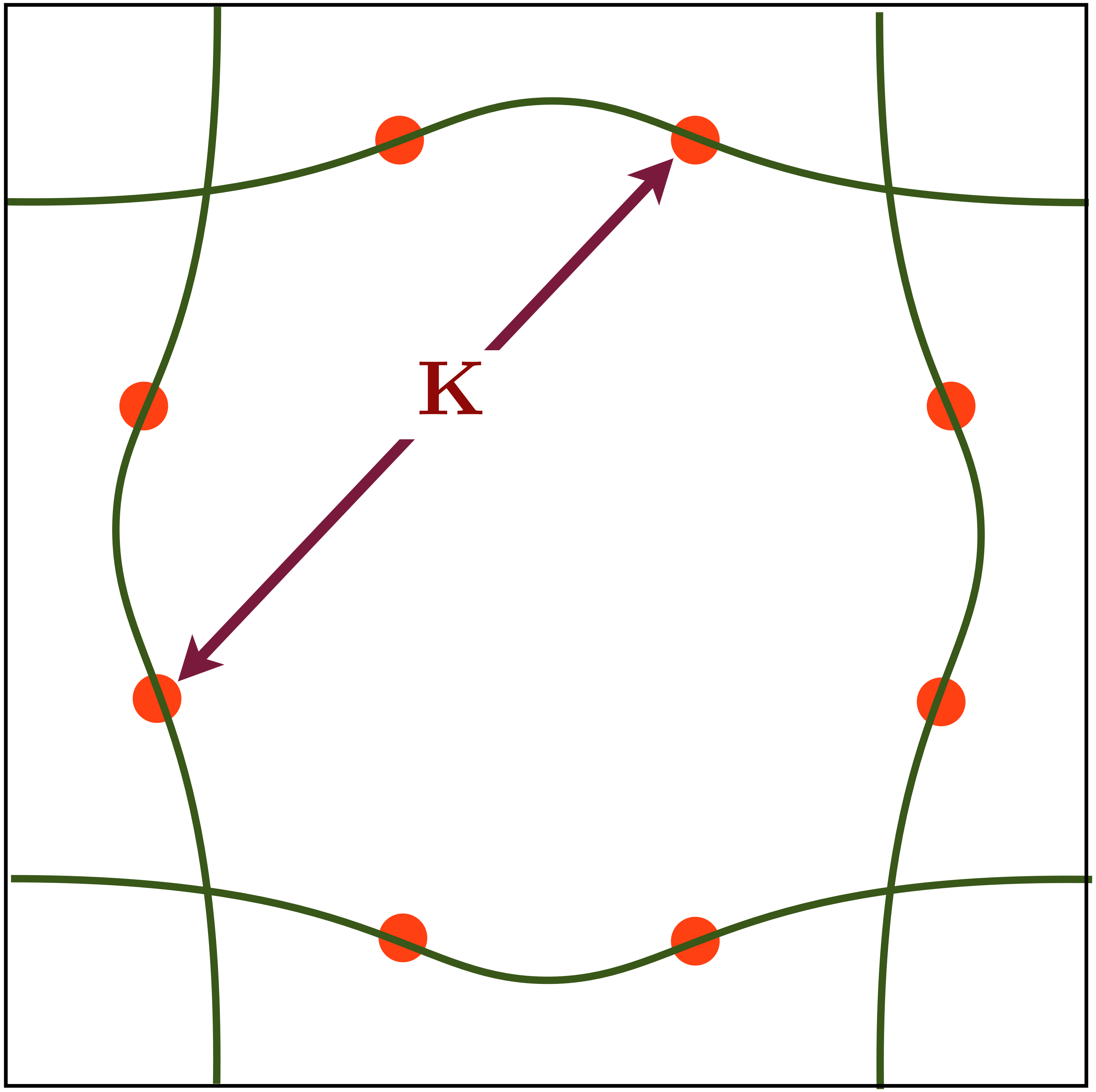
E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Can integrate out $\vec{\varphi}$ to obtain an extended Hubbard model. The interactions in this model only couple electrons in separate bands.

QMC for the onset of antiferromagnetism



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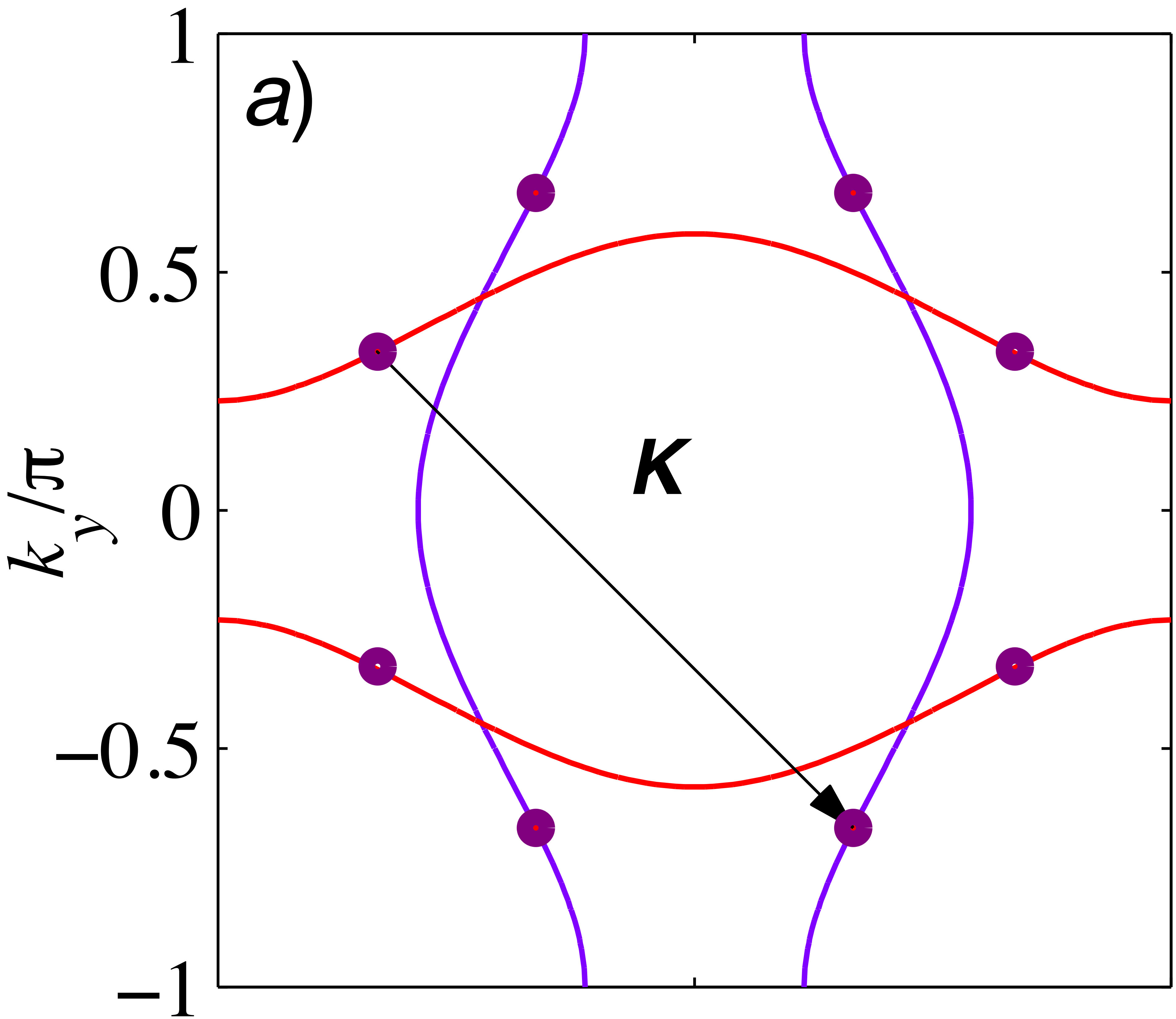

Center Brillouin zone at (π,π,)

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Science **338**, 1606
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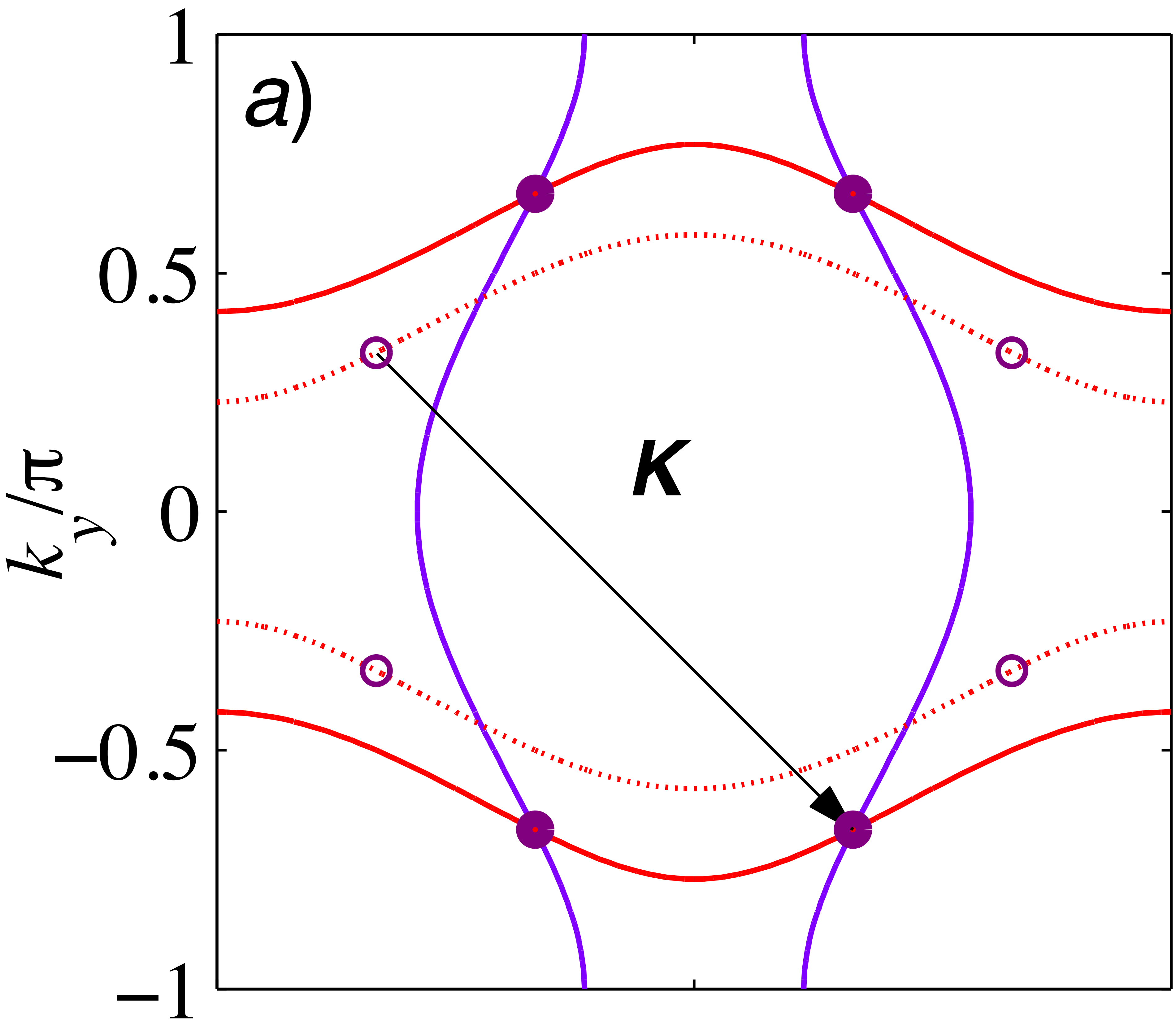
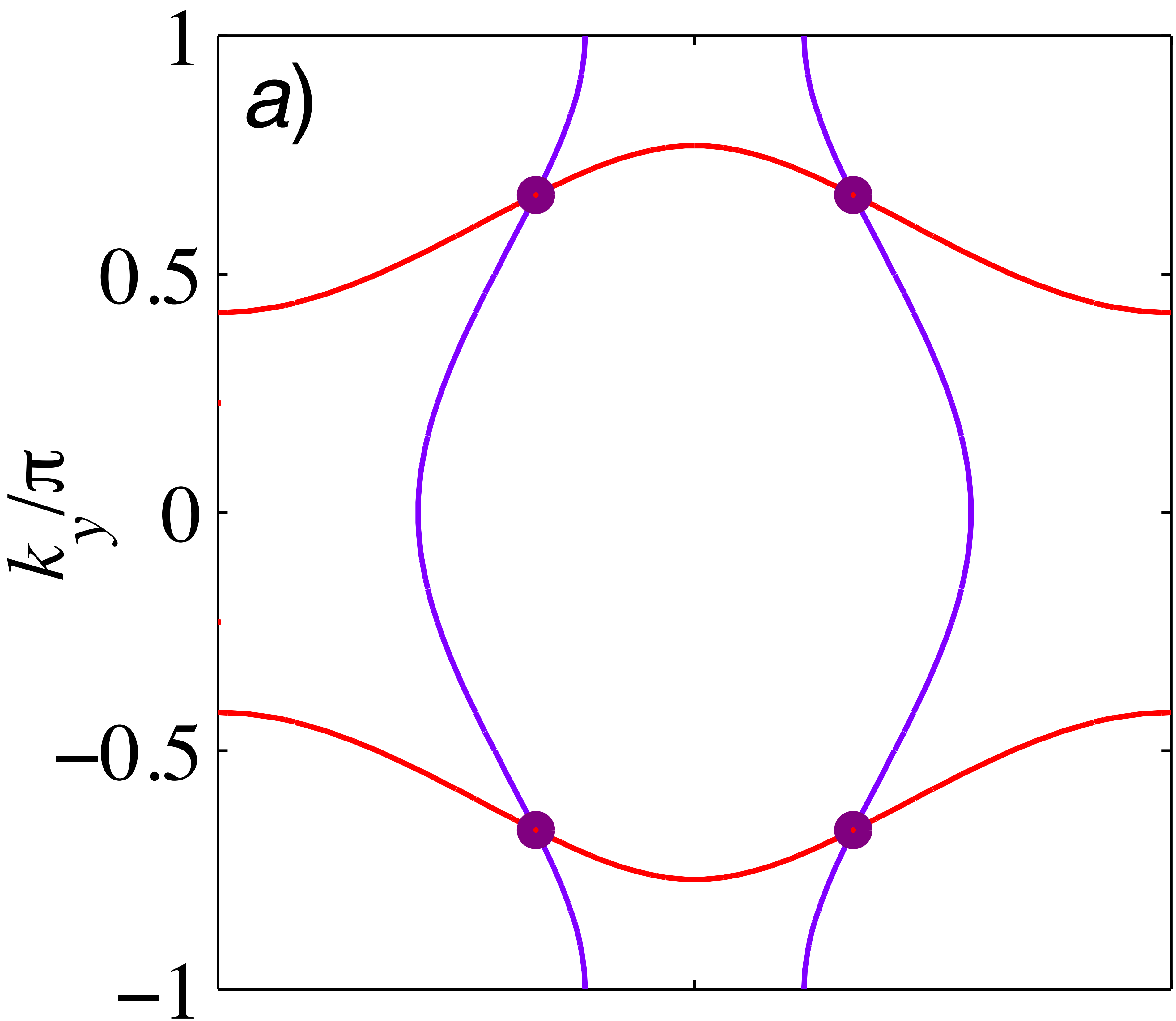
QMC for the onset of antiferromagnetism



E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Move one of the Fermi surface by $(\pi,\pi,)$

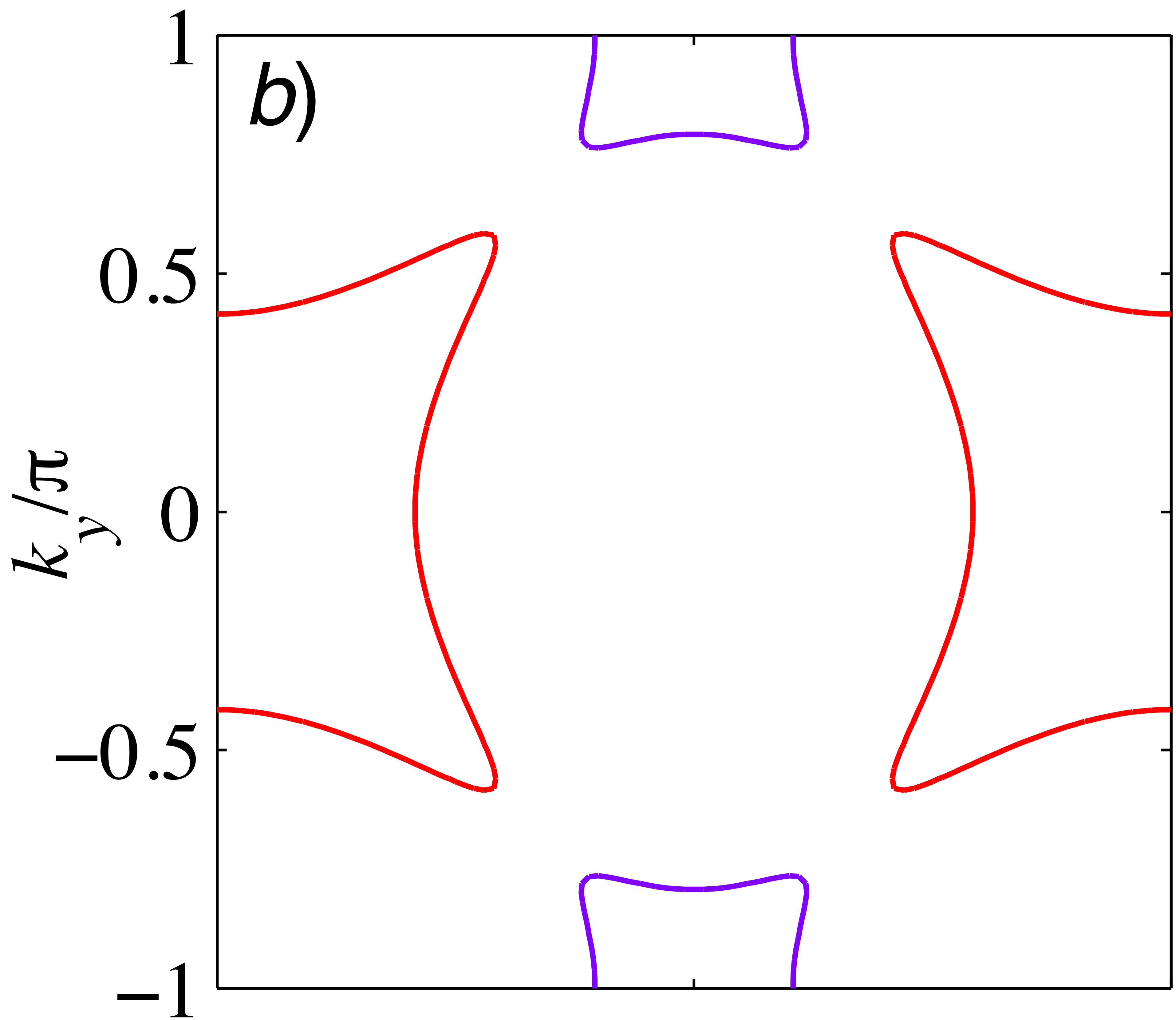
QMC for the onset of antiferromagnetism



E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Now hot spots are at Fermi surface intersections

QMC for the onset of antiferromagnetism



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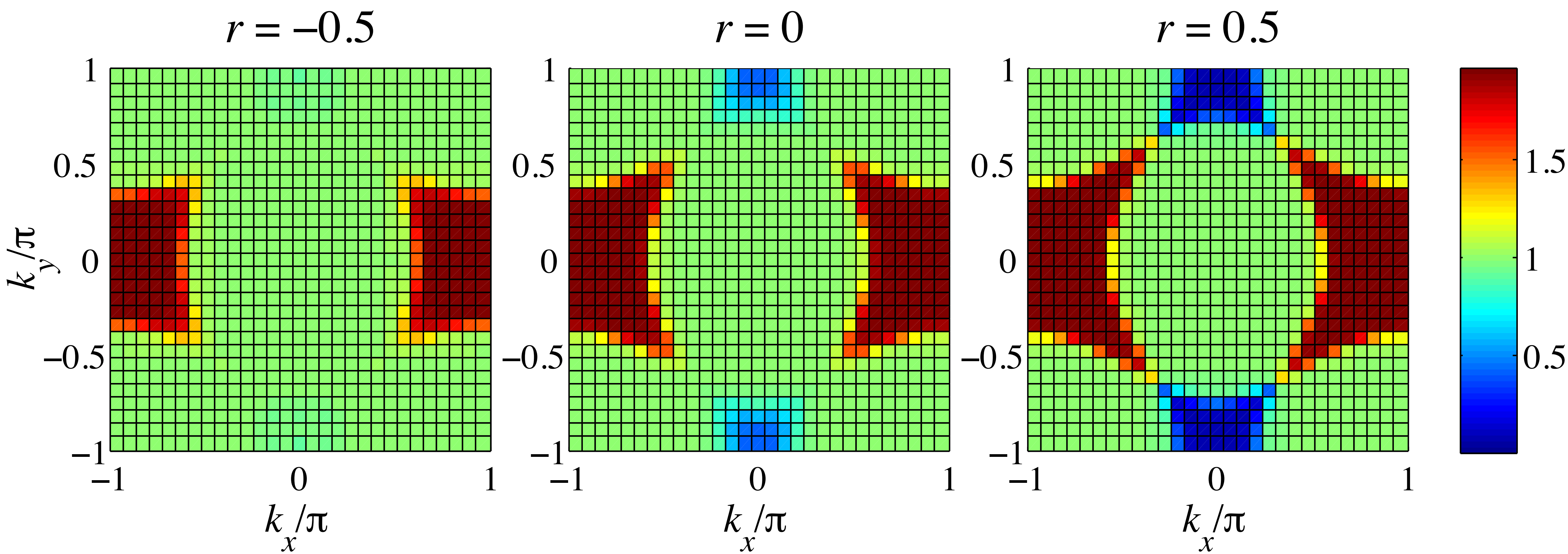
Expected Fermi surfaces in the AFM ordered phase

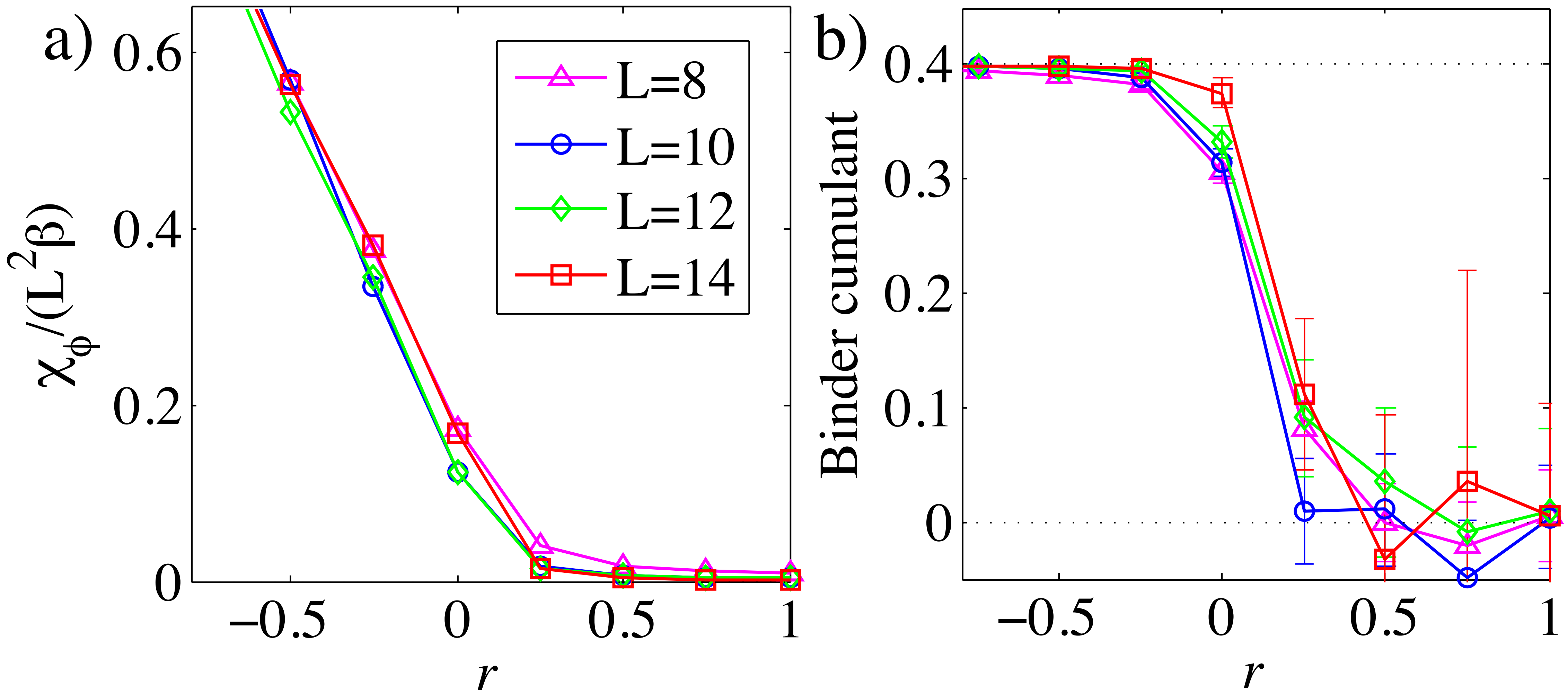
QMC for the onset of antiferromagnetism



Electron occupation number $n_{\mathbf{k}}$
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

QMC for the onset of antiferromagnetism



AF susceptibility, χ_φ, and Binder cumulant as a function of the tuning parameter r

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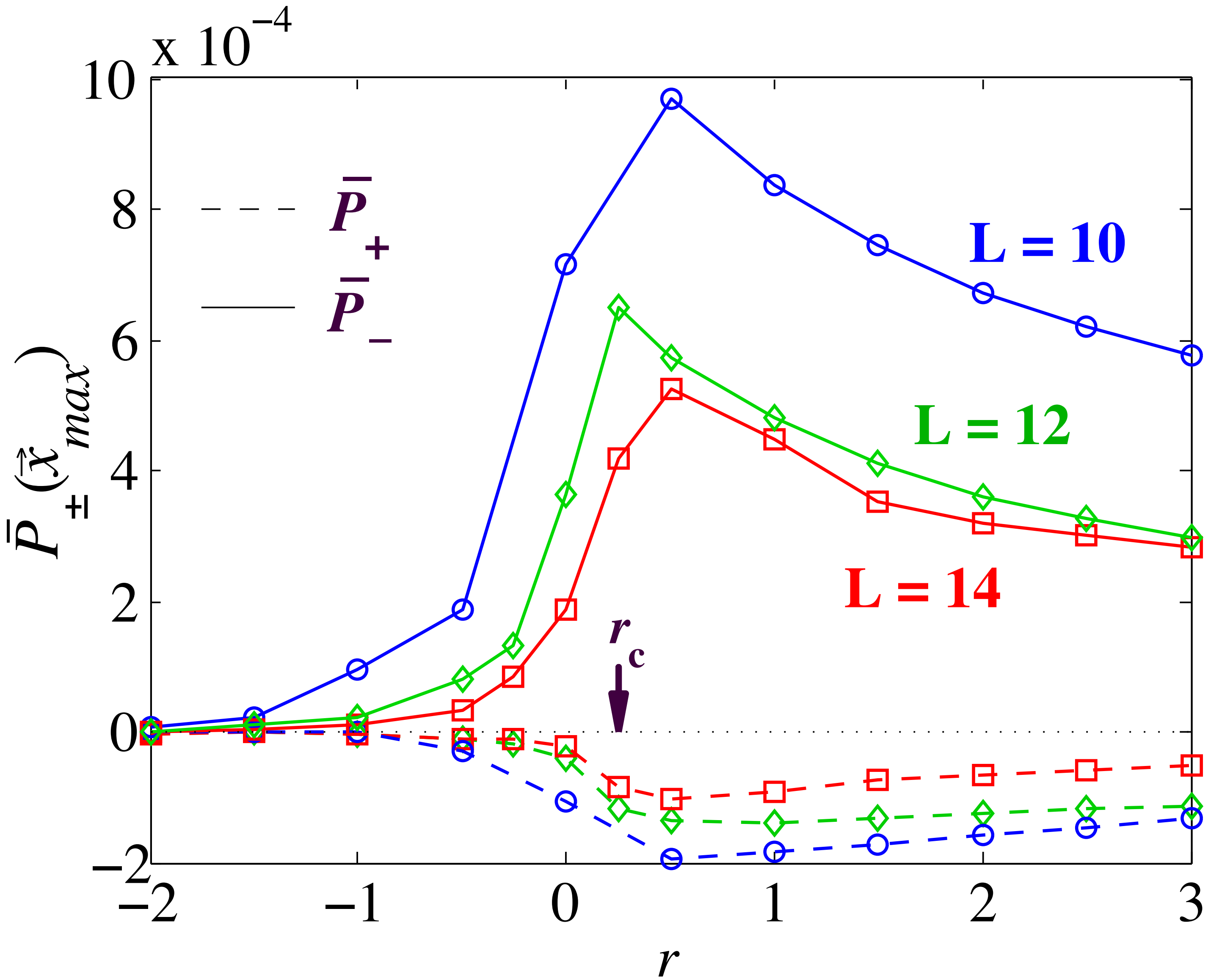
QMC for the onset of antiferromagnetism



s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r

E. Berg, M. Metlitski, and S. Sachdev, *Science* **338**, 1606 (2012).

Conclusions

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- Good prospects for studying competing charge orders, and non-Fermi liquid physics at non-zero temperature.