SYK models and charged black holes: transport, chaos, and density of states

Many Body Quantum Dynamics:
Perspectives from Field Theory and Gravity,
The Graduate Center
City University of New York
May 9, 2019

Talk online: sachdev.physics.harvard.edu

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Ordinary metals and quasiparticles

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\rm eq} \sim \frac{\hbar E_F^3}{U^2 (k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where U is the strength of interactions, and E_F is the Fermi energy.

- Similarly, a quasiparticle model implies a resistivity

$$\rho = \frac{m^*}{ne^2}\frac{1}{\tau} \sim U^2 T^2 \quad \text{with} \quad \tau \sim \tau_{\rm eq}$$

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$$\rho = \frac{m^*}{ne^2} \frac{1}{\tau} \sim U^2 T^2 \quad \text{with} \quad \tau \sim \tau_{\rm eq}$$

- These times are much longer than the '<u>Planckian time</u>' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

$$\tau \sim \tau_{\rm eq} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, ρ, is

$$\rho = \frac{m^*}{ne^2}\,\frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$

independent of the strength of interactions!

Remarkable recent observation of 'Planckian' strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity is associated with a universal scattering time $\approx \hbar/(k_B T)$.

Universal *T*-linear resistivity and Planckian dissipation in overdoped cuprates

NATURE PHYSICS | VOL 15 | FEBRUARY 2019 | 142–147

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1], B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5], N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6]* and C. Proust[3,6]*

arXiv:1902.01034

Planckian dissipation and scale invariance in a quantum-critical disordered pnictide

Yasuyuki Nakajima,[1,2] Tristin Metz,[2] Christopher Eckberg,[2] Kevin Kirshenbaum,[2] Alex Hughes,[2] Renxiong Wang,[2] Limin Wang,[2] Shanta R. Saha,[2] I-Lin Liu,[2,3,4] Nicholas P. Butch,[2,4] Zhonghao Liu,[5,6] Sergey V. Borisenko,[5] Peter Y. Zavalij,[7] and Johnpierre Paglione[2,8]

Strange metal in magic-angle graphene with near Planckian dissipation

Yuan Cao,[1,*] Debanjan Chowdhury,[1,*] Daniel Rodan-Legrain,[1] Oriol Rubies-Bigordà,[1] Kenji Watanabe,[2] Takashi Taniguchi,[2] T. Senthil,[1,†] and Pablo Jarillo-Herrero[1,†]

arXiv:1901.03710

Bad metallic transport in a cold atom Fermi-Hubbard system

Science **363**, 379–382 (2019)

Peter T. Brown[1], Debayan Mitra[1], Elmer Guardado-Sanchez[1], Reza Nourafkan[2], Alexis Reymbaut[2], Charles-David Hébert[2], Simon Bergeron[2], A.-M. S. Tremblay[2,3], Jure Kokalj[4,5], David A. Huse[1], Peter Schauß[1*], Waseem S. Bakr[1]†

Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	P = 11 kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

A. Legros, S. Benhabib, W. Tabis, F. Laliberté, M. Dion, M. Lizaire, B. Vignolle, D. Vignolles, H. Raffy, Z. Z. Li, P. Auban-Senzier, N. Doiron-Leyraud, P. Fournier, D. Colson, L. Taillefer, and C. Proust, Nature Physics **15**, 142 (2019)

1. ‘Resonant’ SYK model for Planckian transport

   Linear-in-T resistivity and a remnant Fermi surface

2. SYK + phonons

   Transport and many-body chaos

3. Low temperature theory of charged black holes and complex SYK

   Density of states

1. ‘Resonant’ SYK model for Planckian transport

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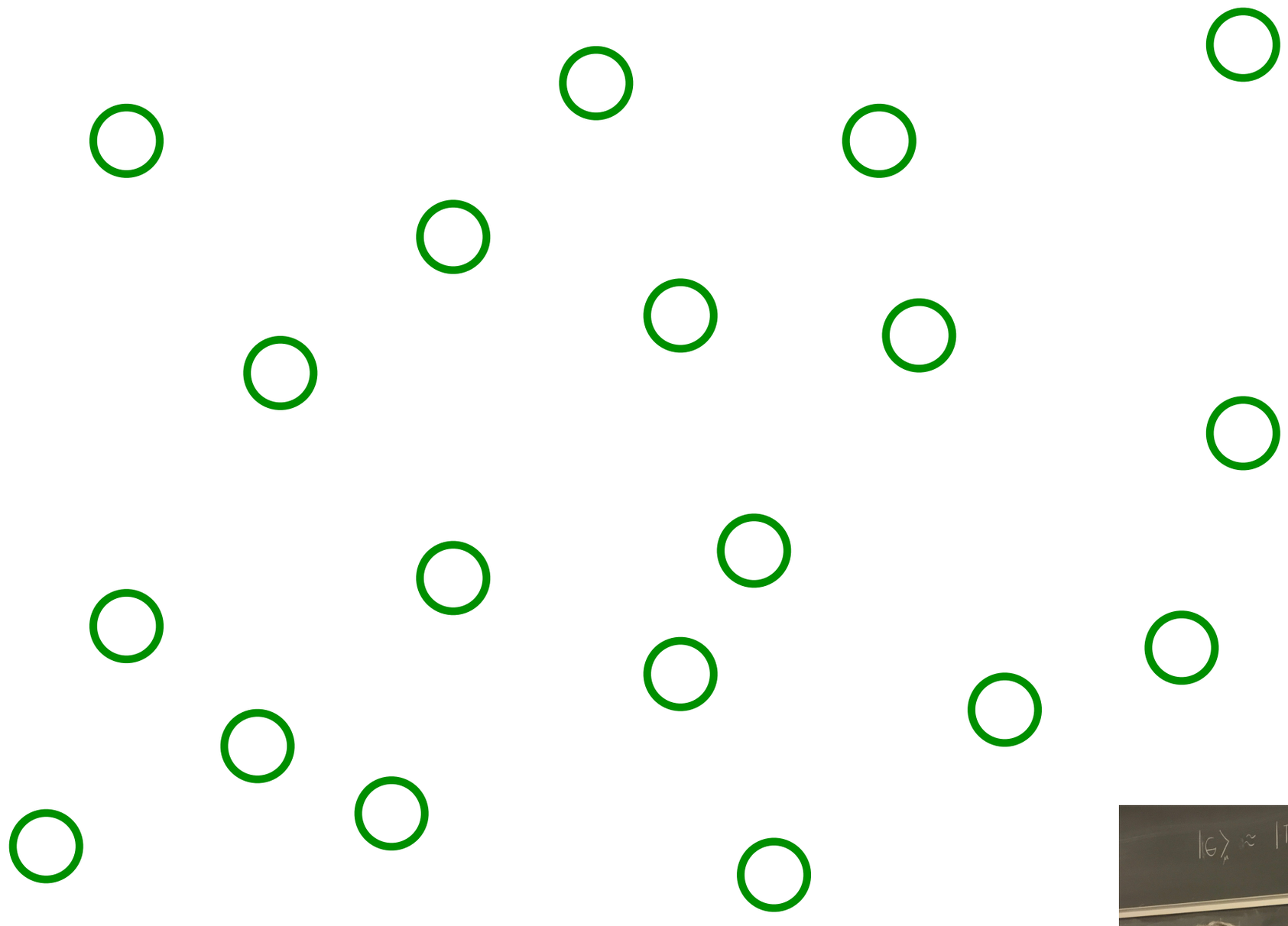
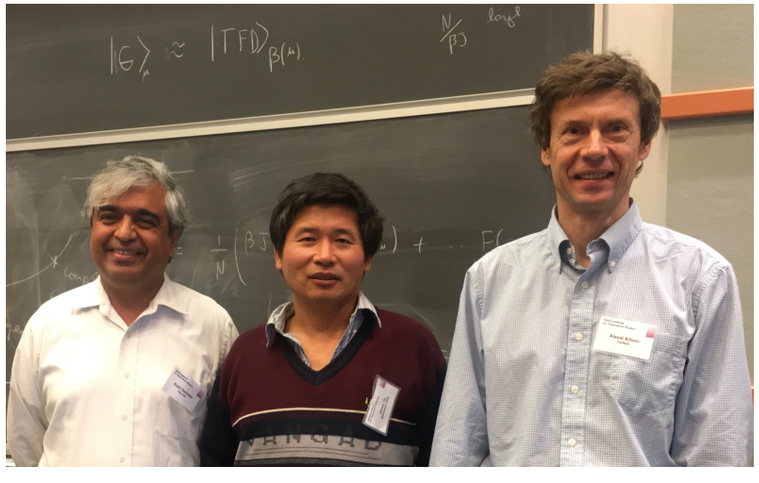
Density of states

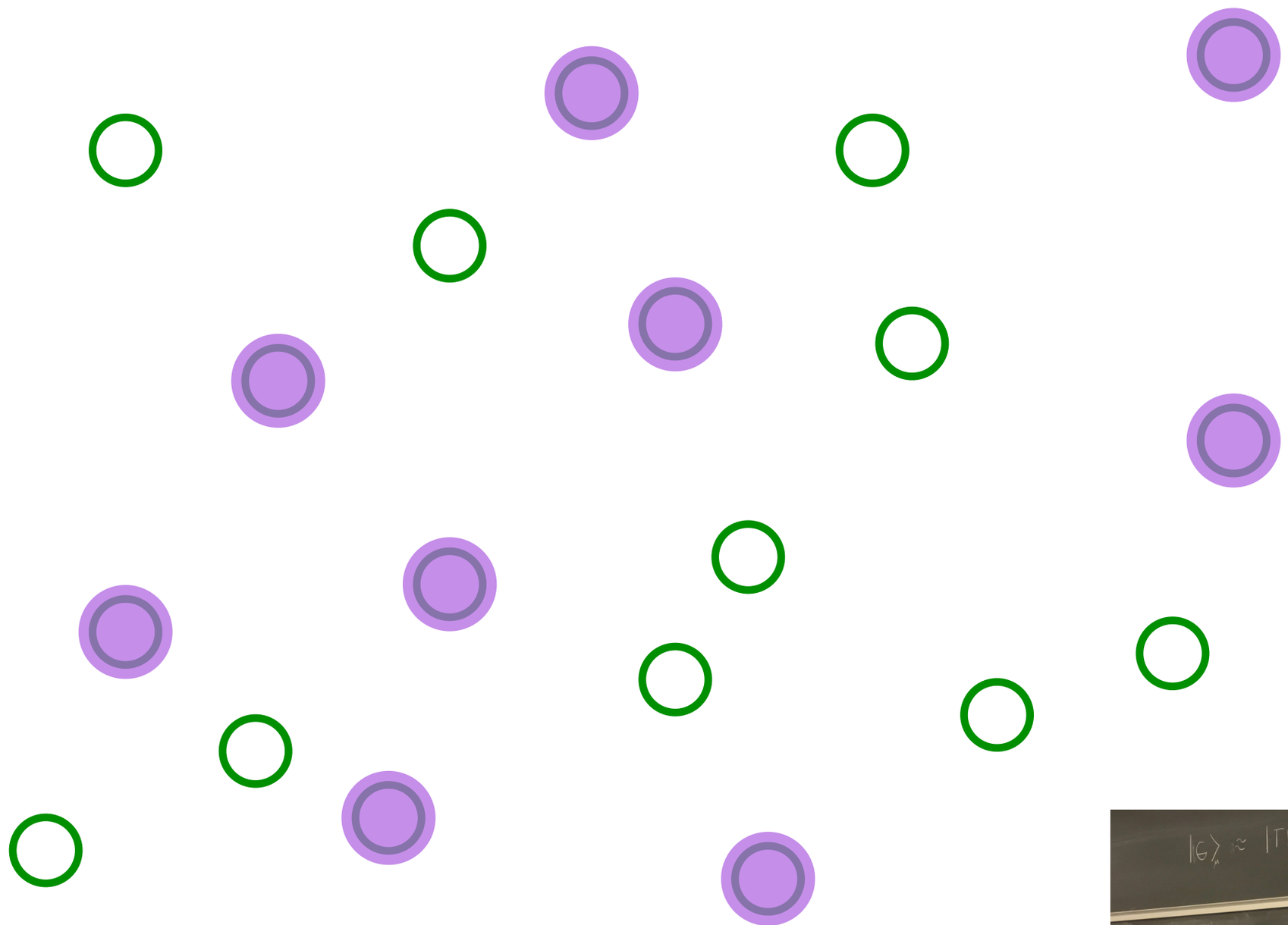
The Sachdev-Ye-Kitaev (SYK) model



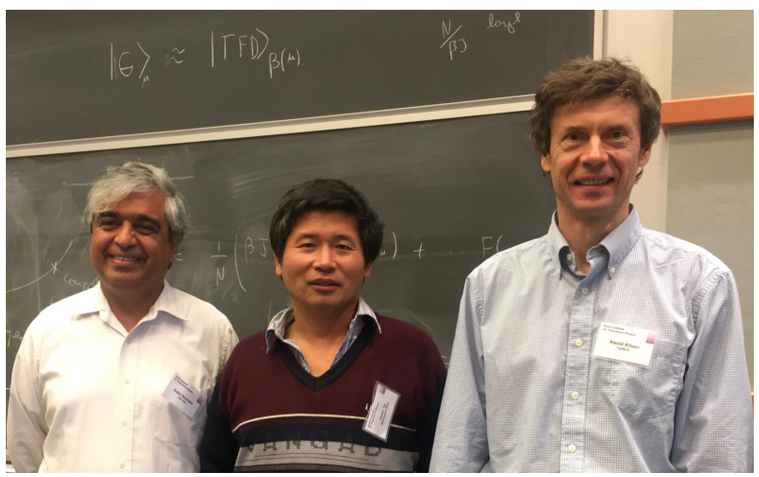
Pick a set of random positions



The SYK model



Place electrons randomly on some sites



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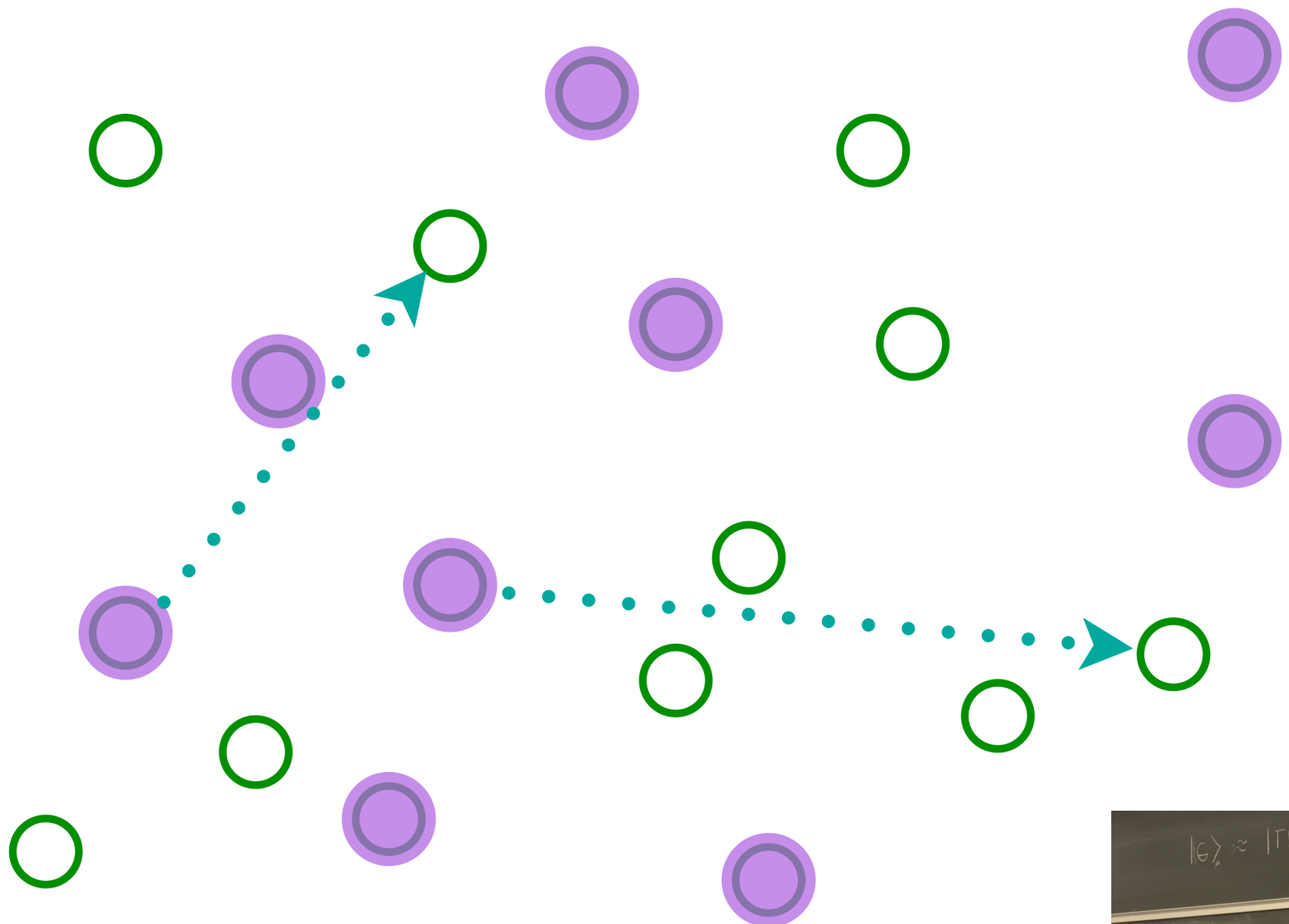

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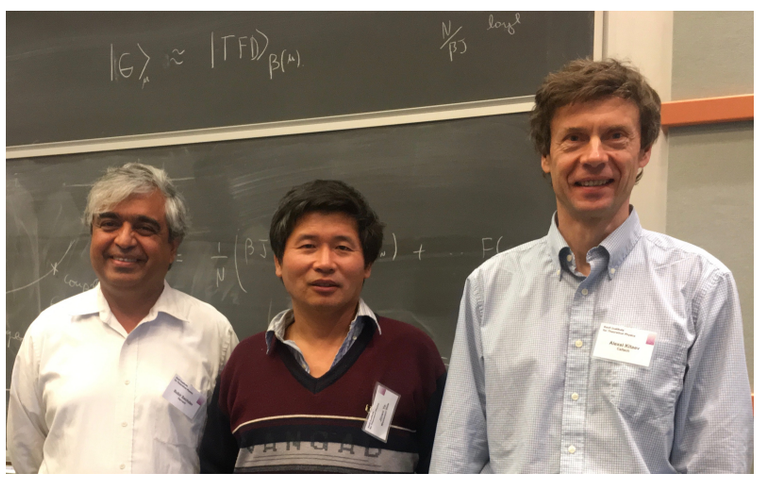

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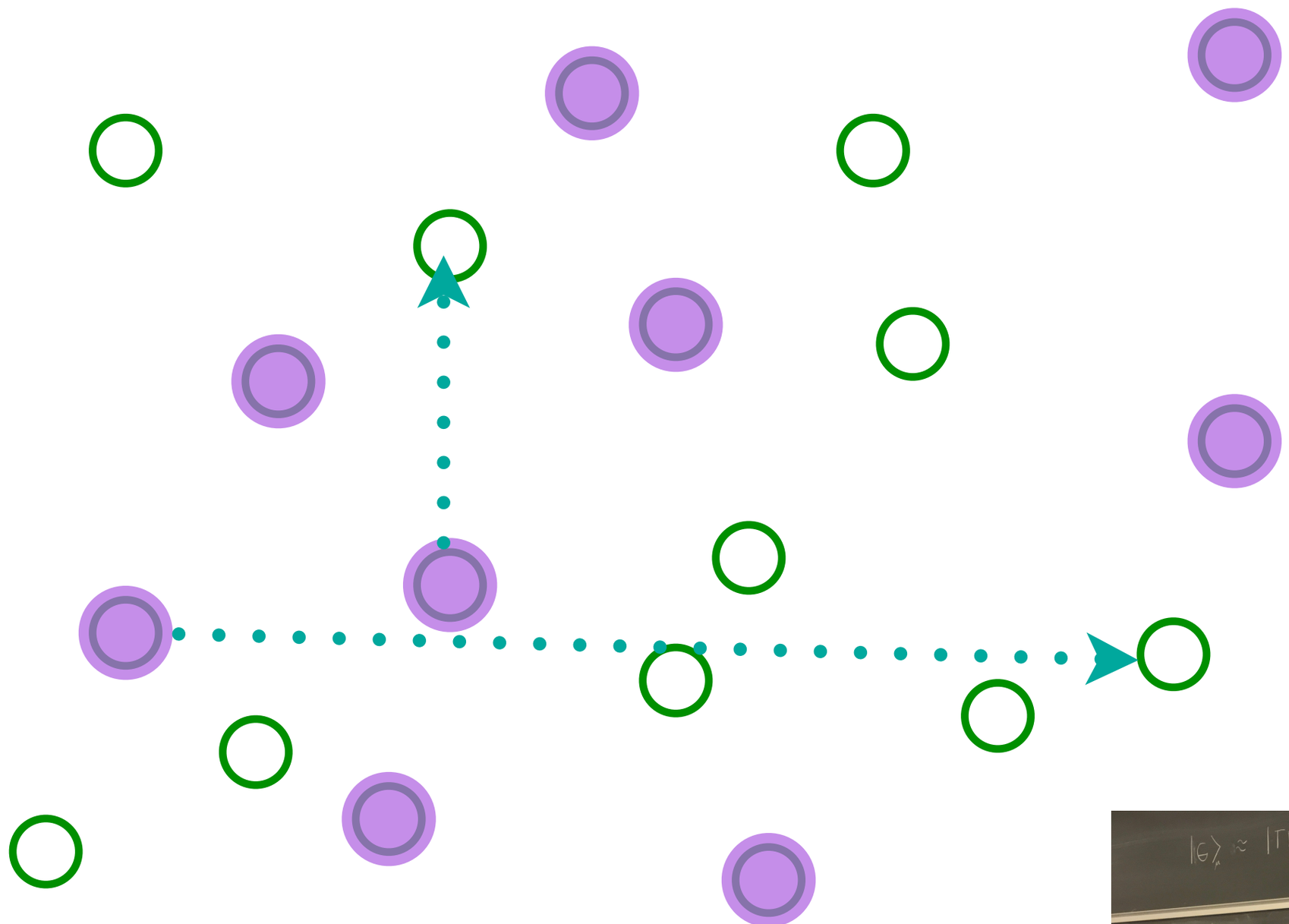

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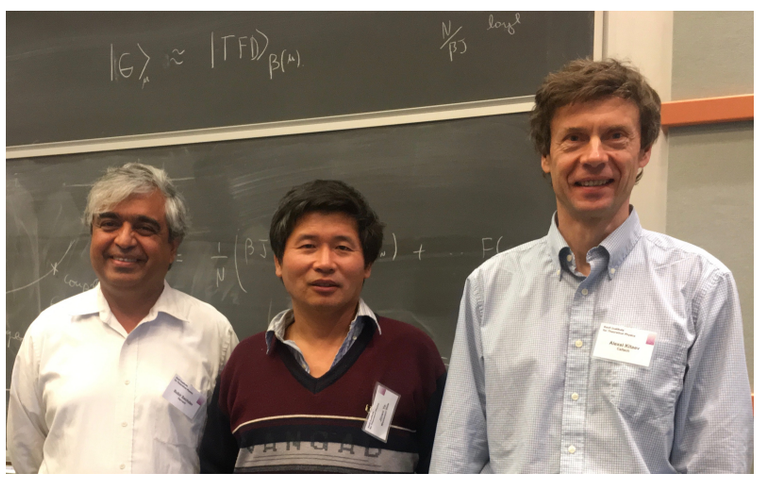

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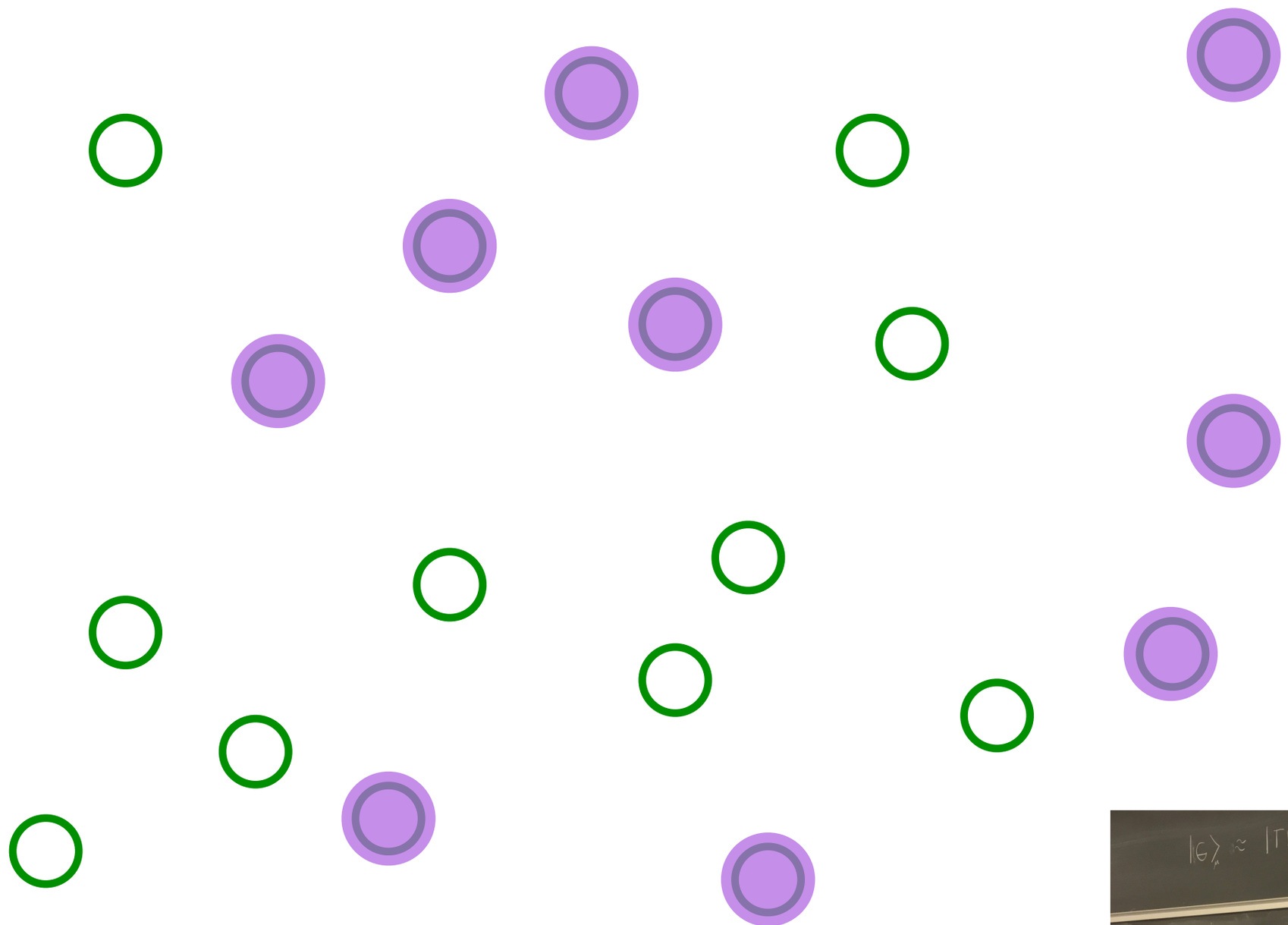

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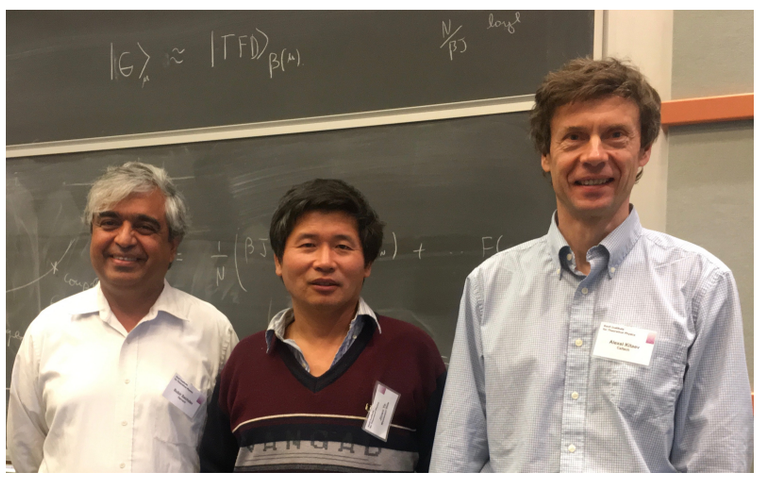

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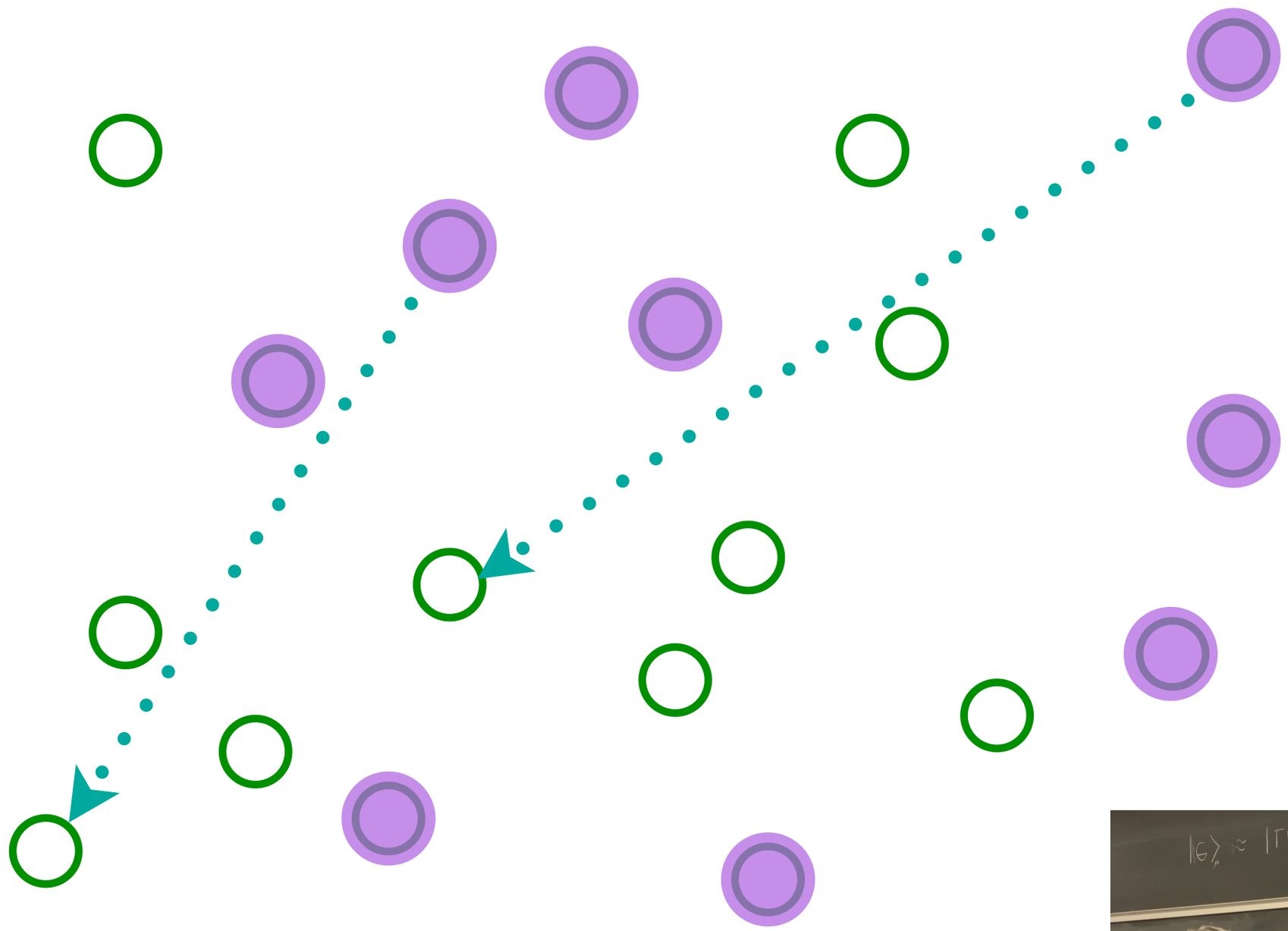

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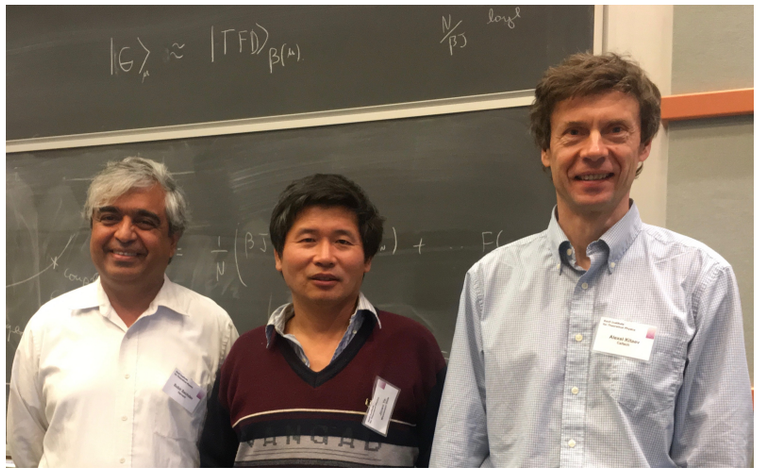

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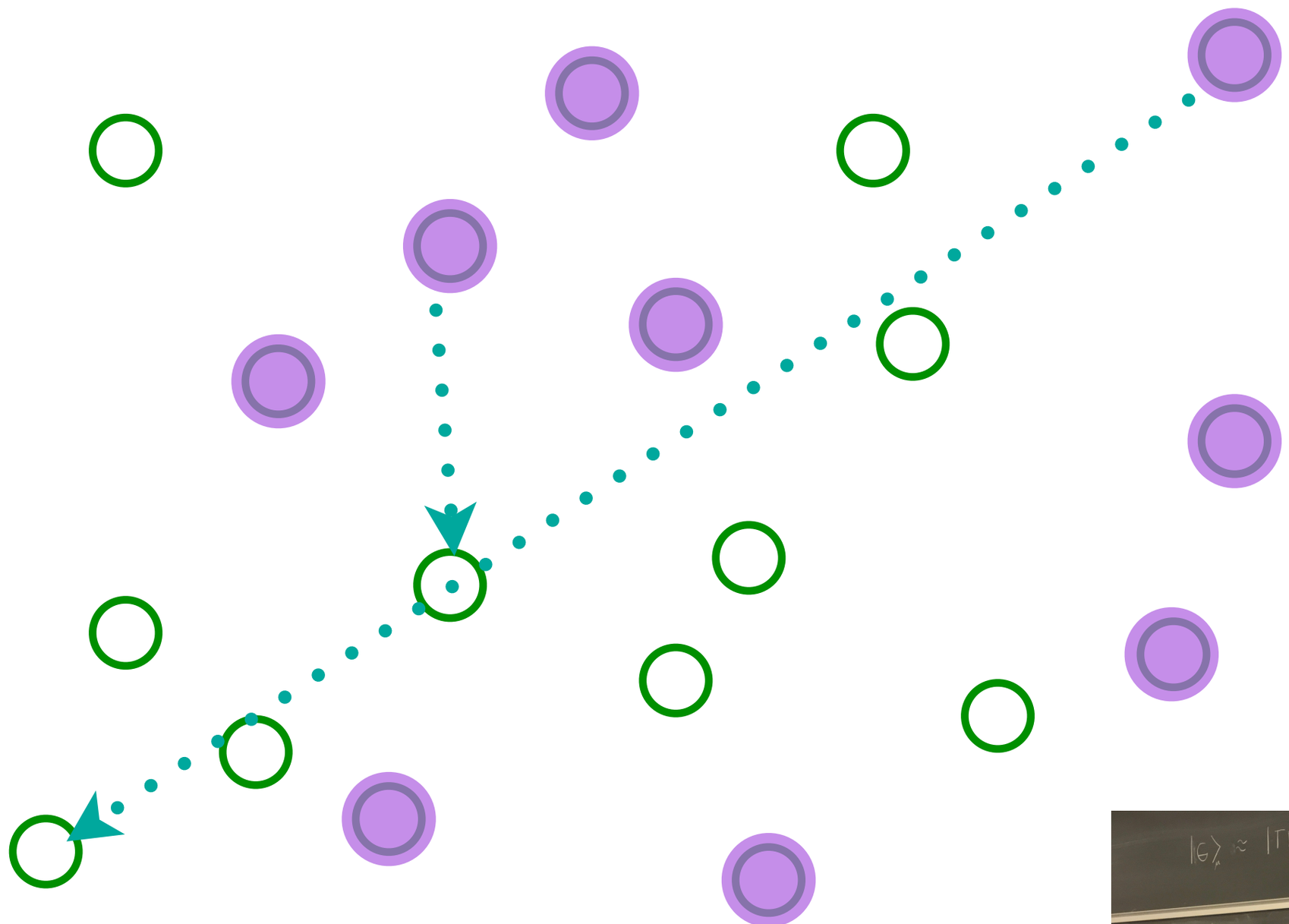

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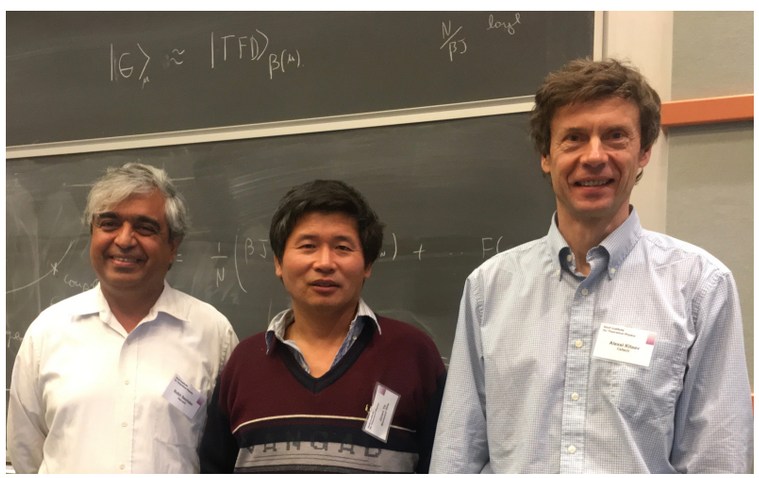

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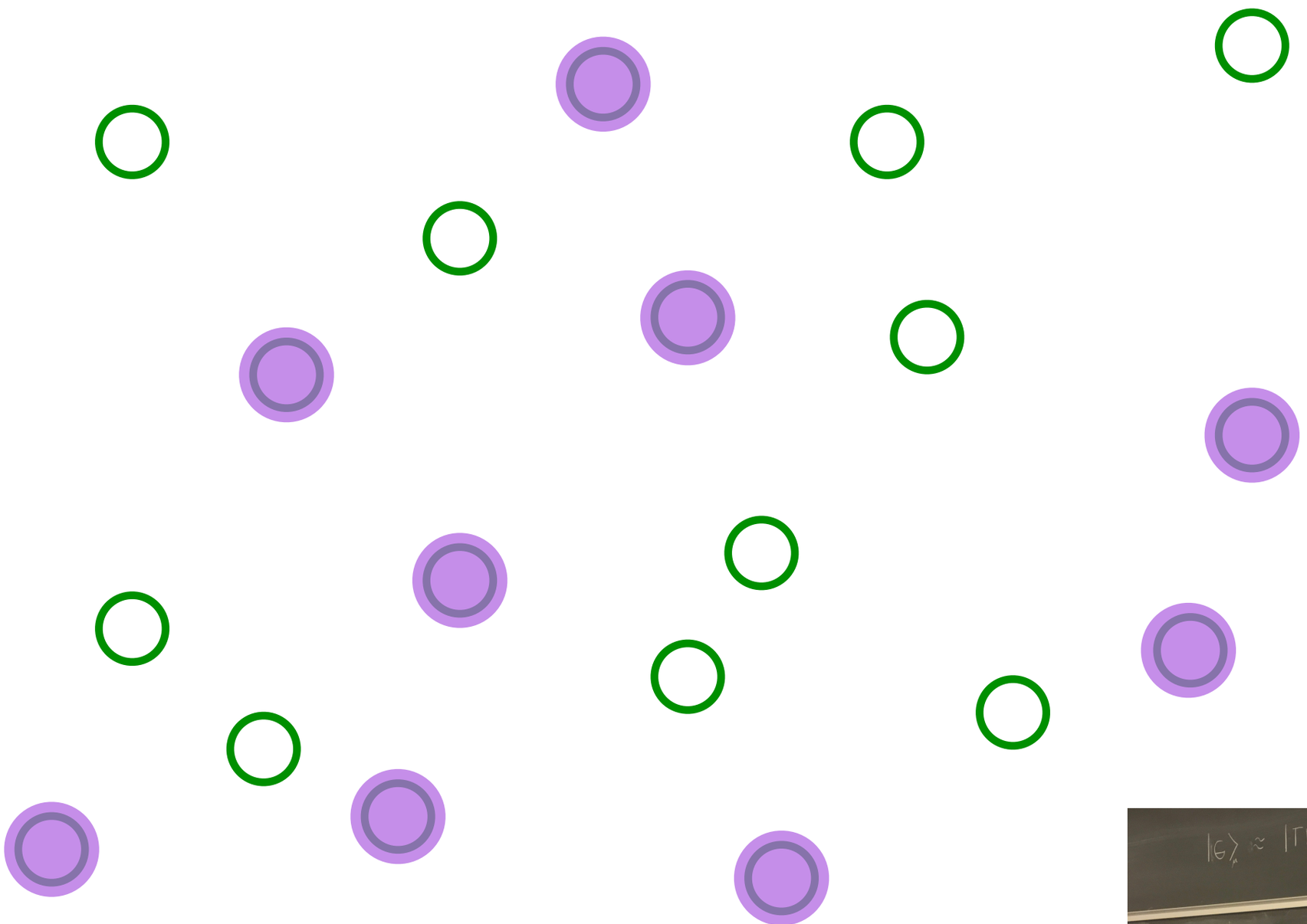

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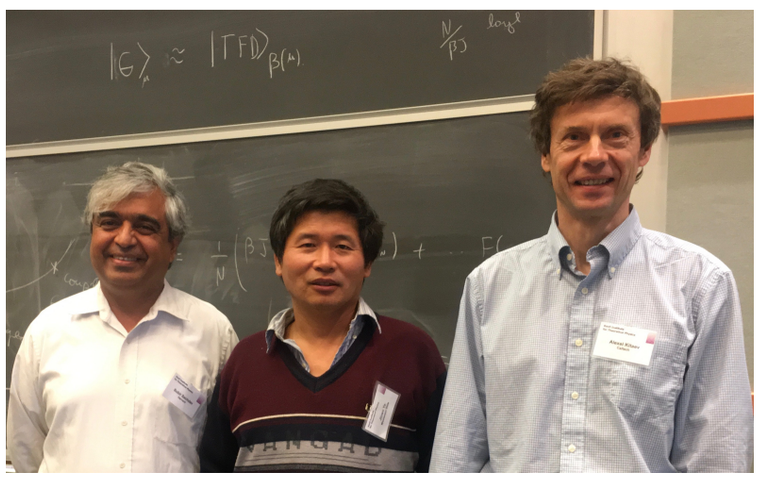

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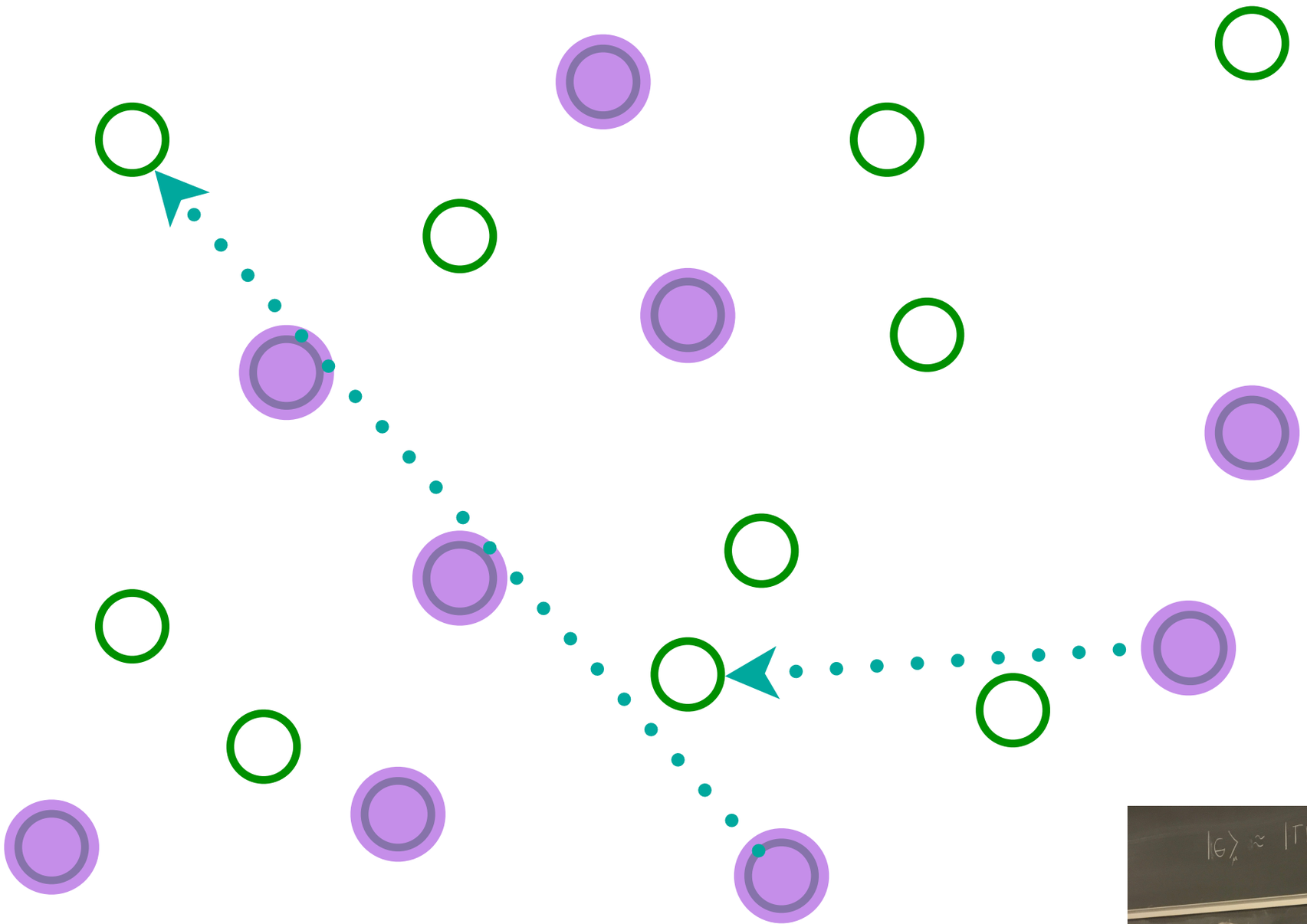

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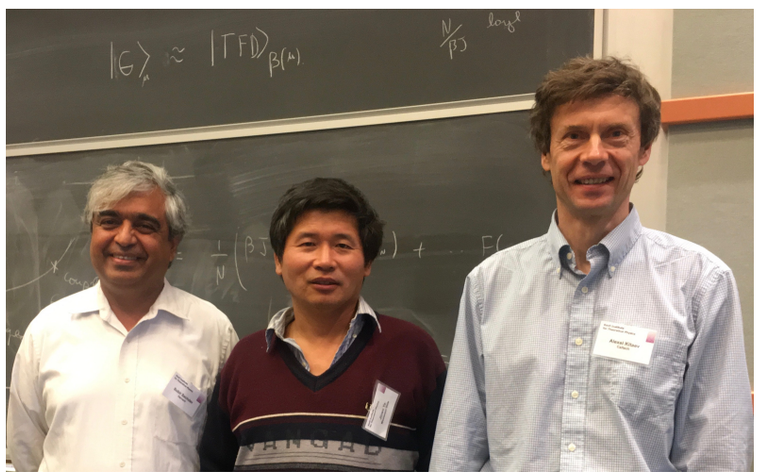

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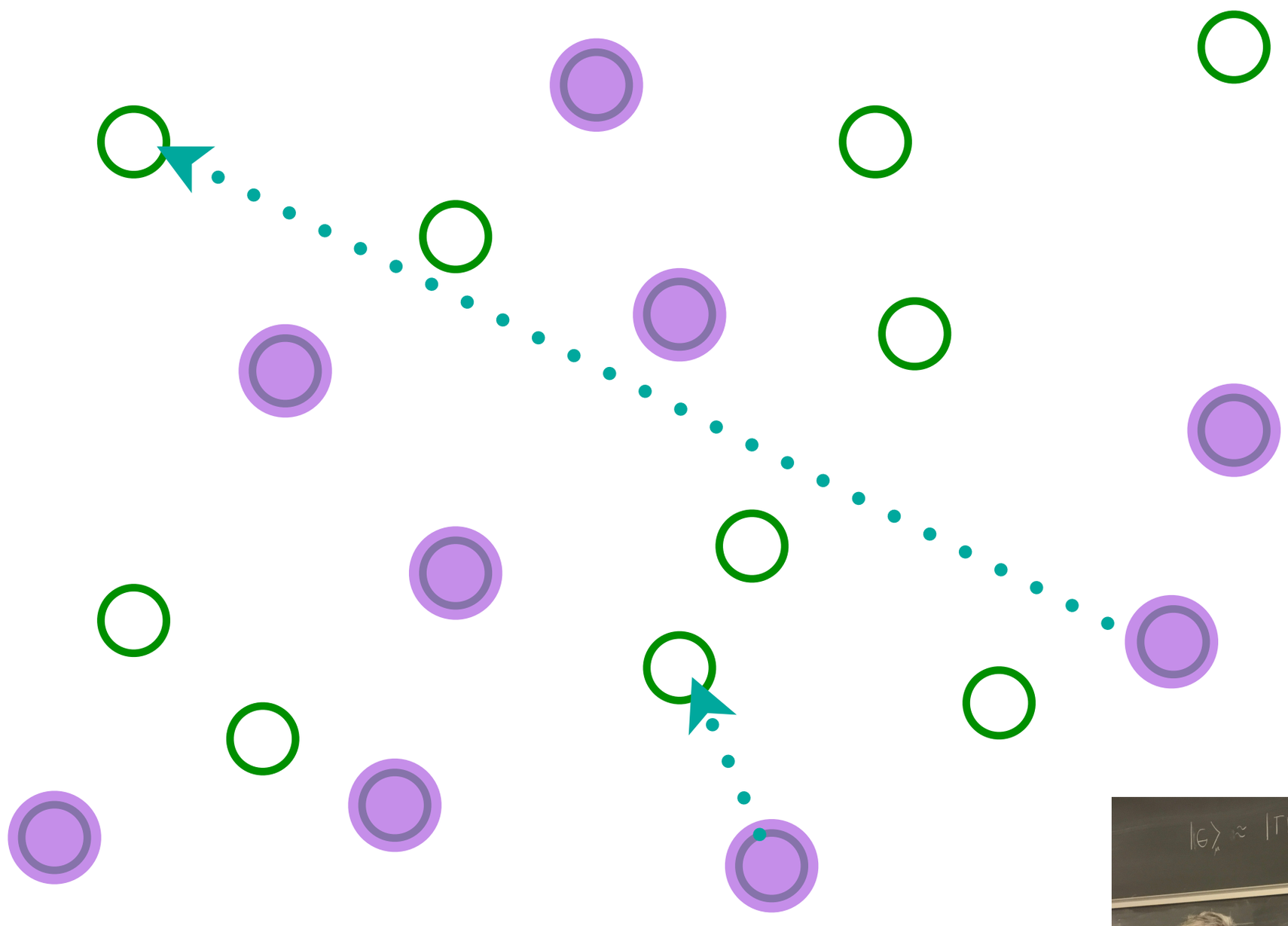

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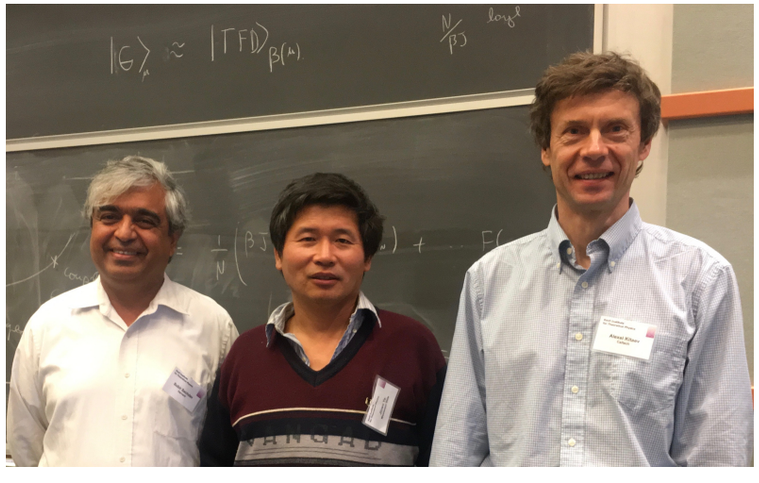

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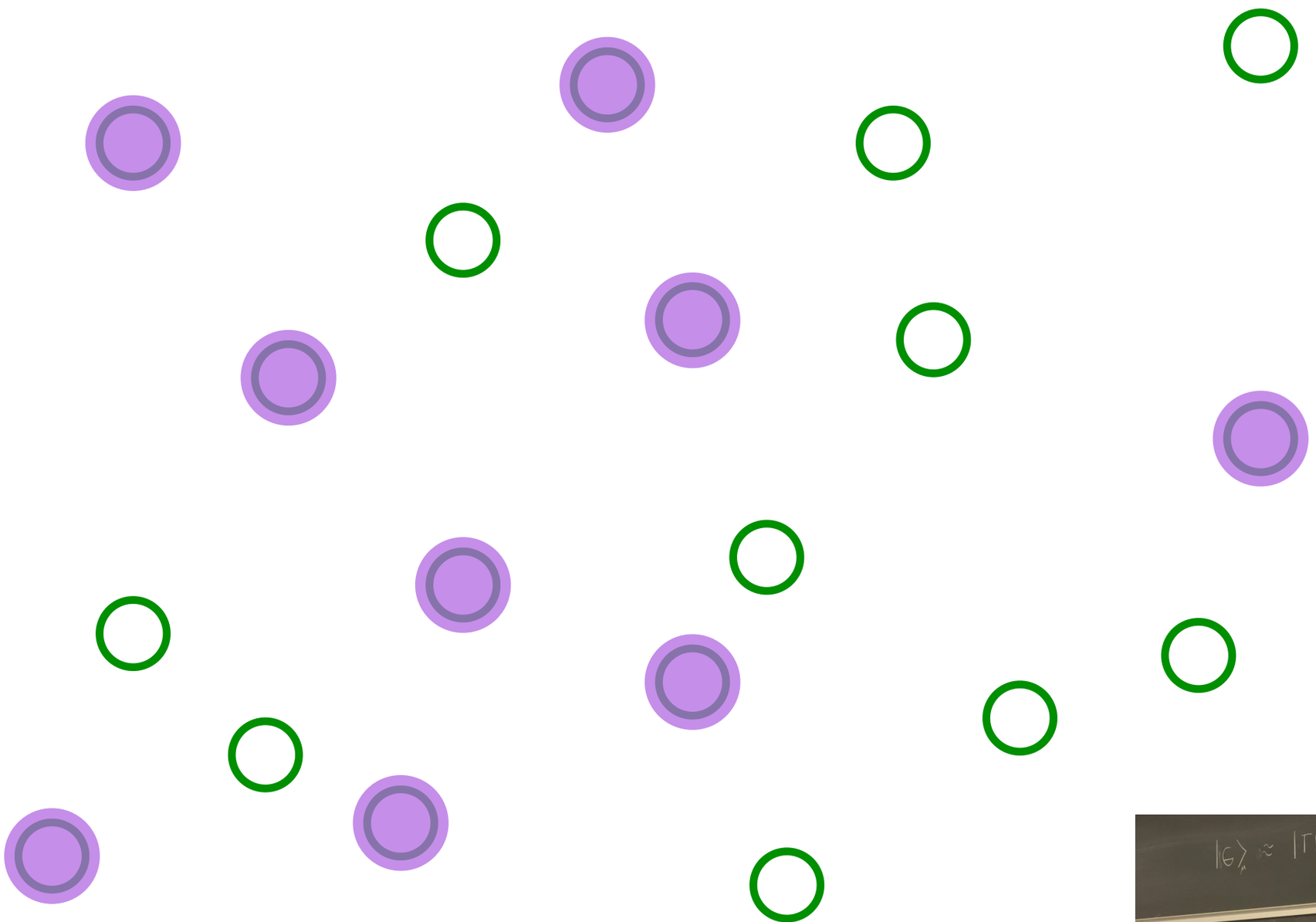

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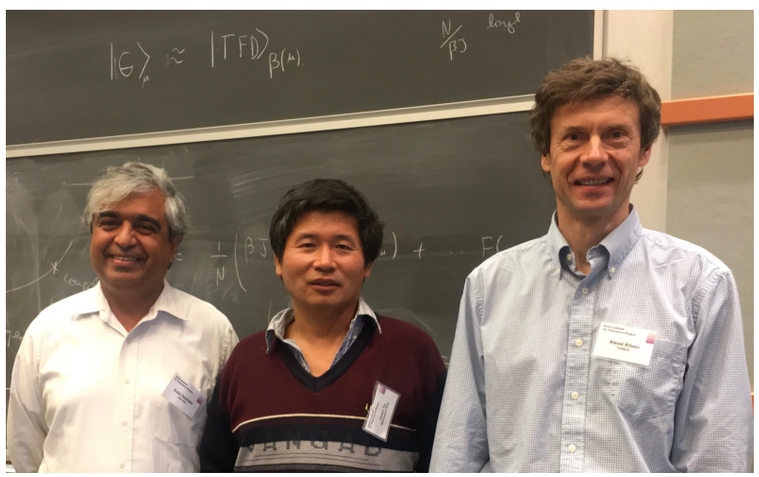

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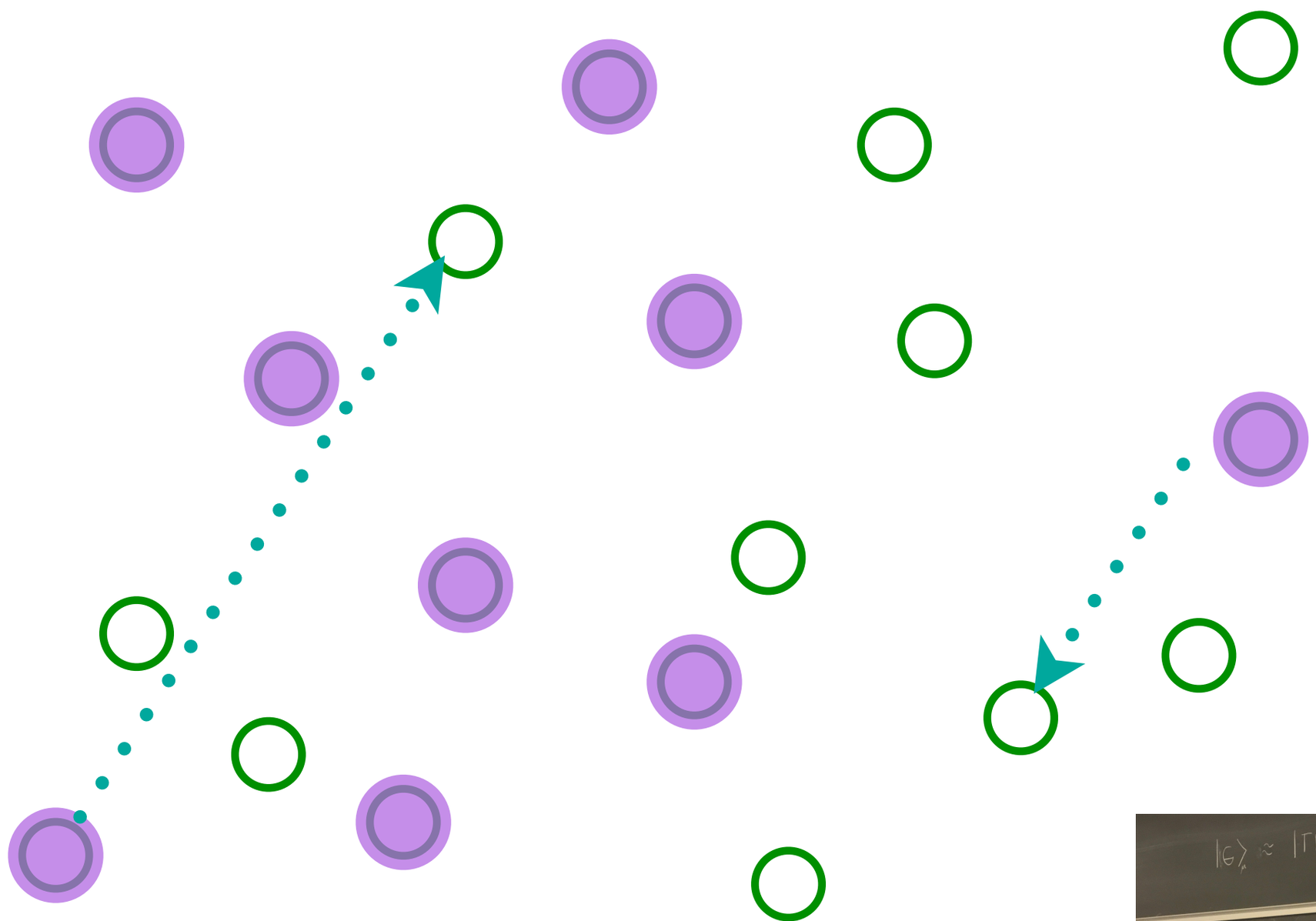

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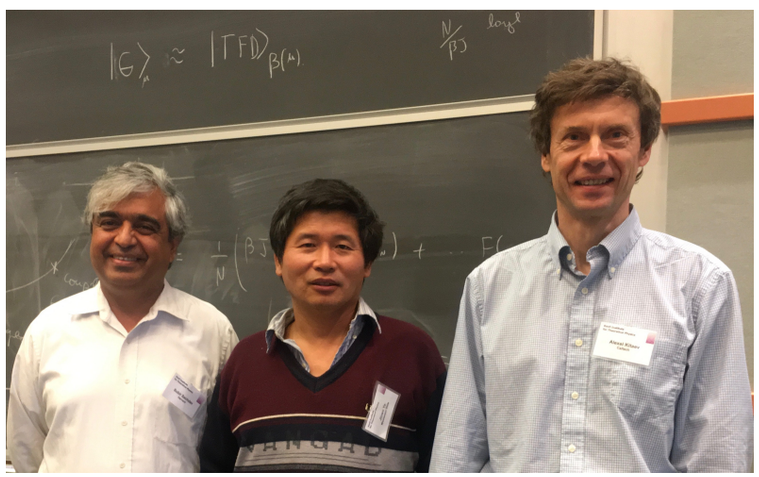

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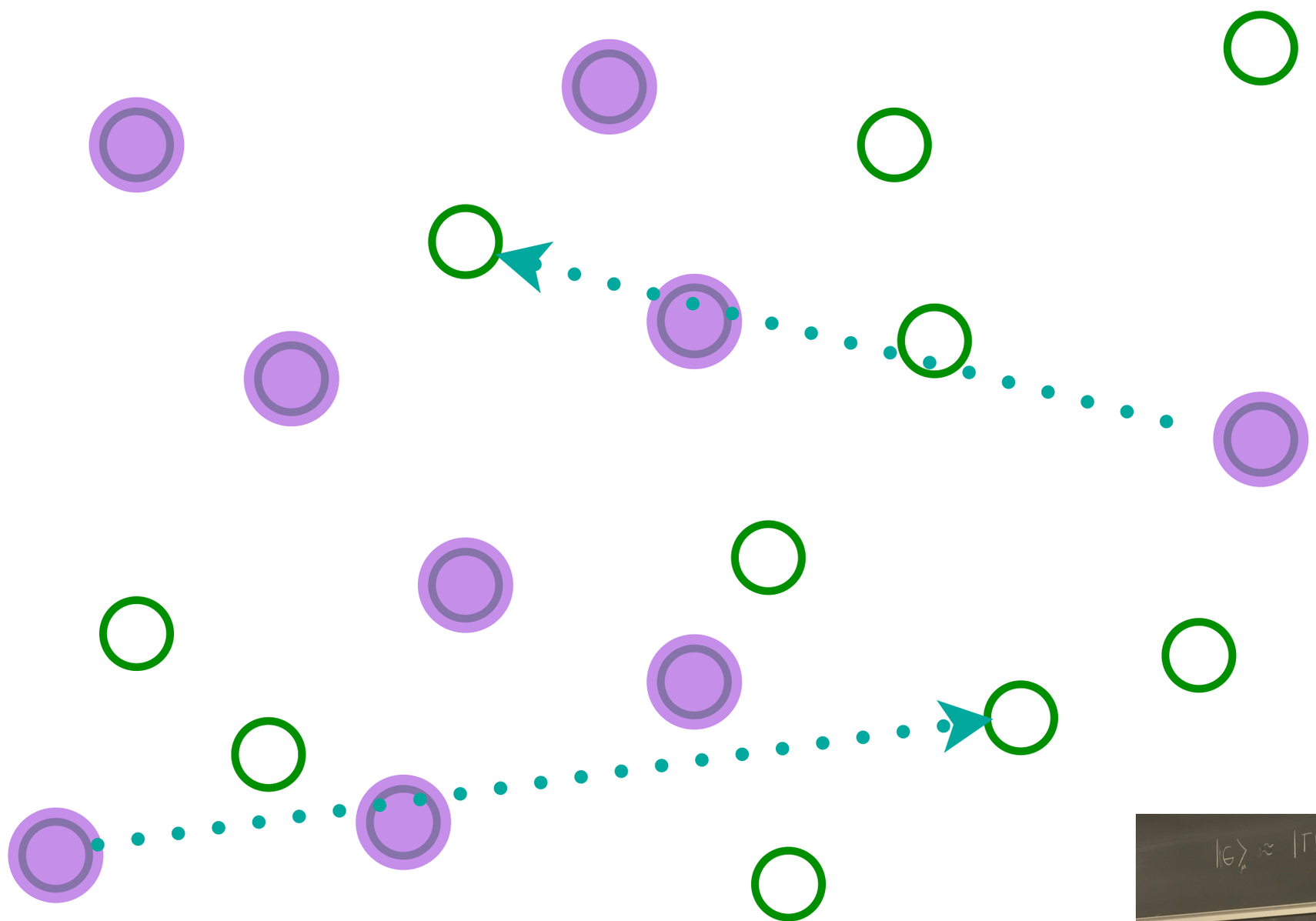

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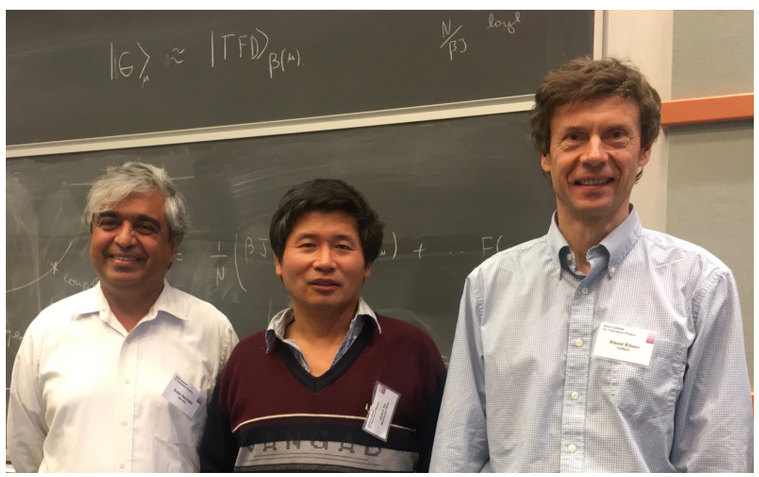

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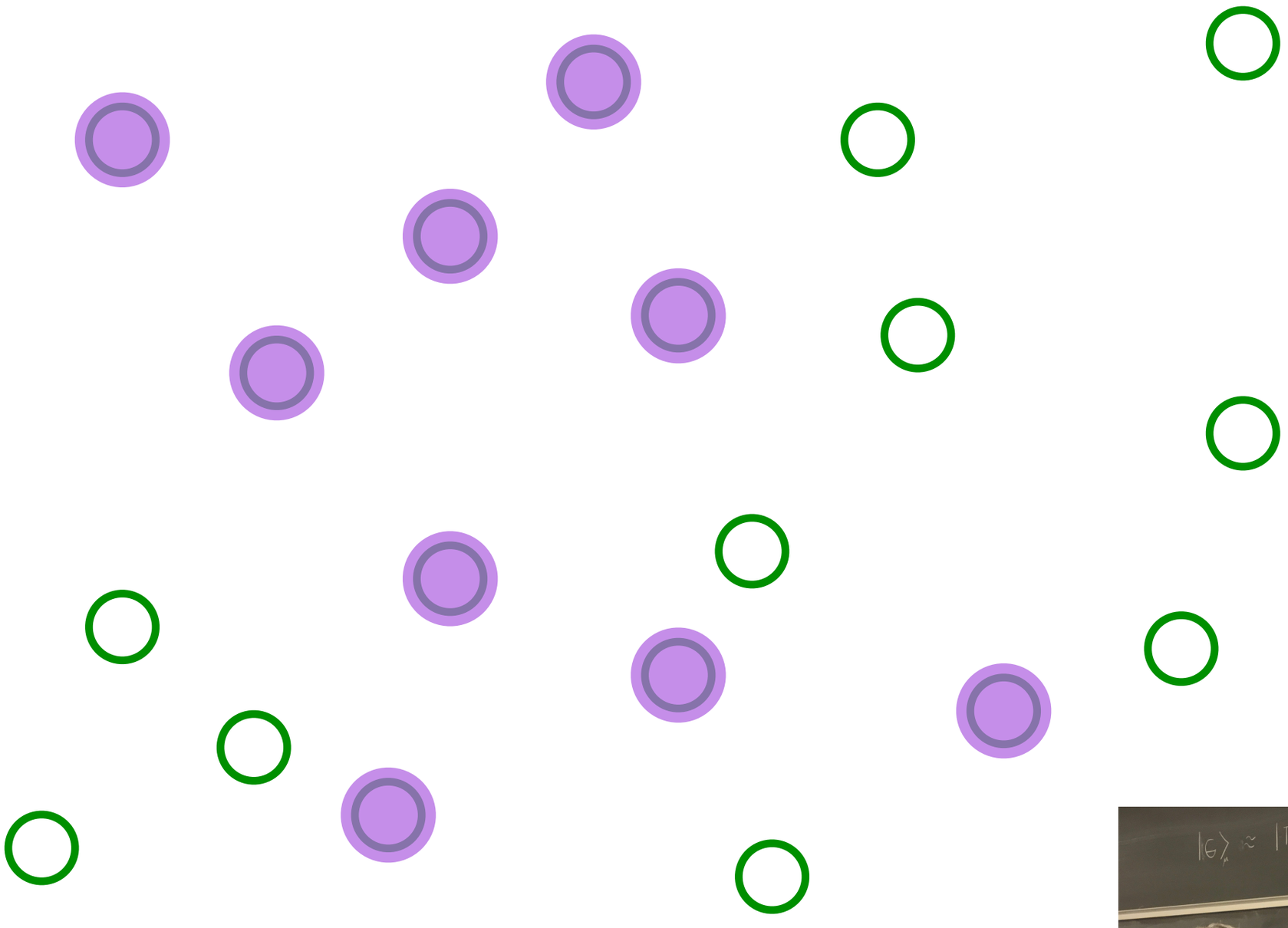

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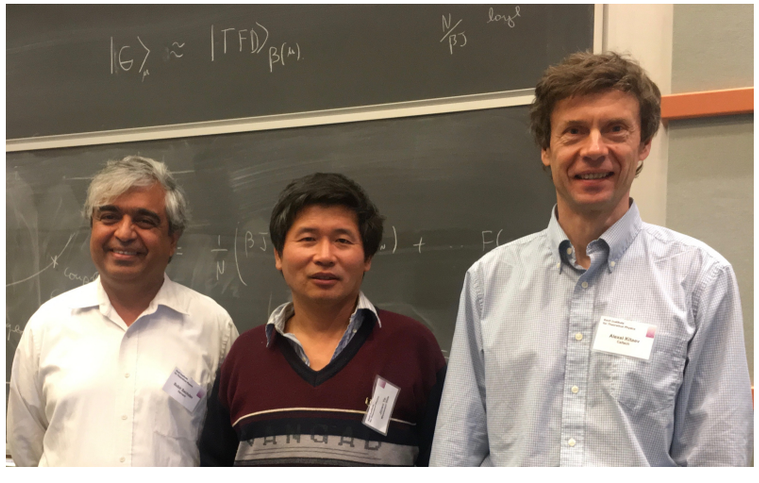

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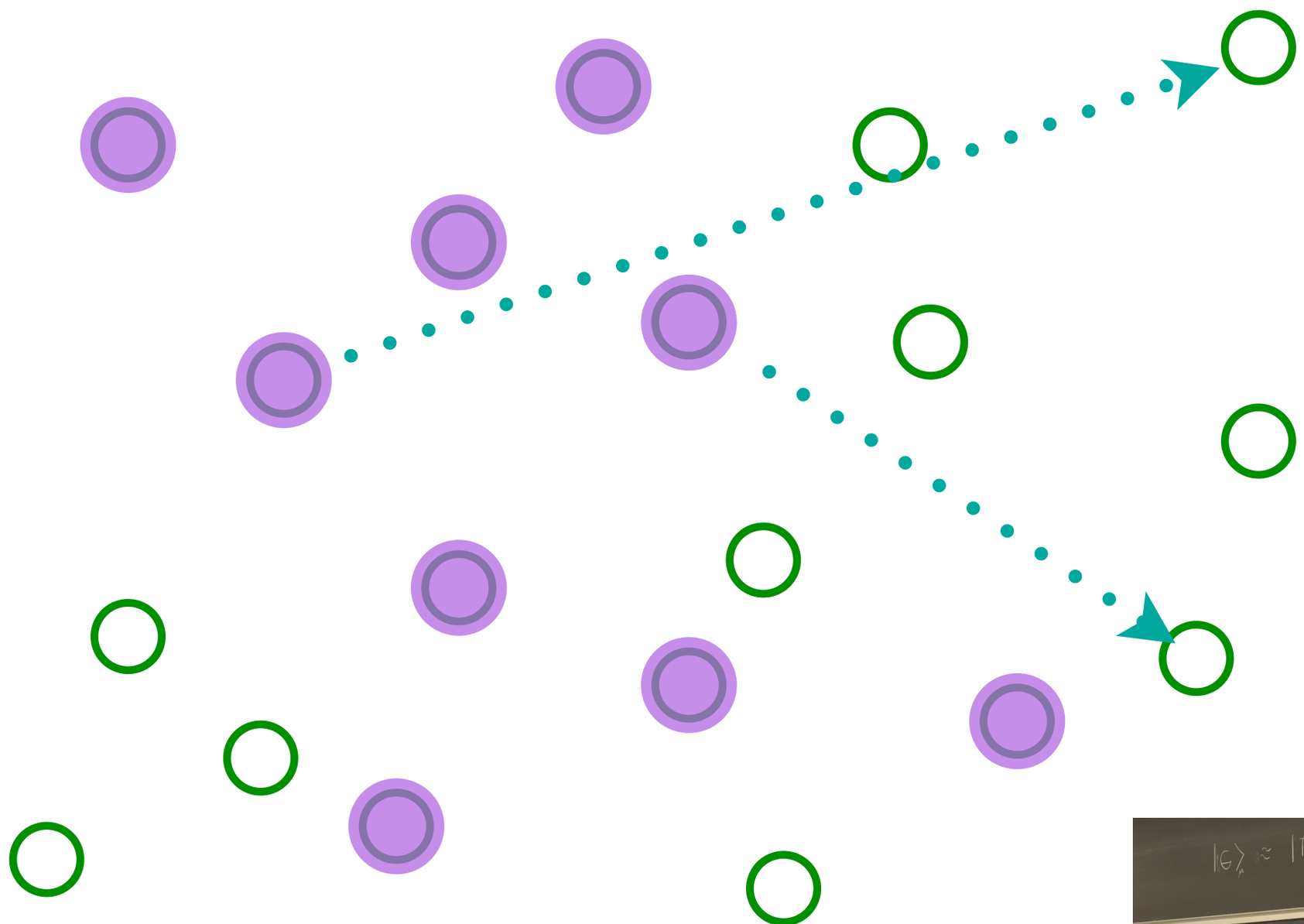

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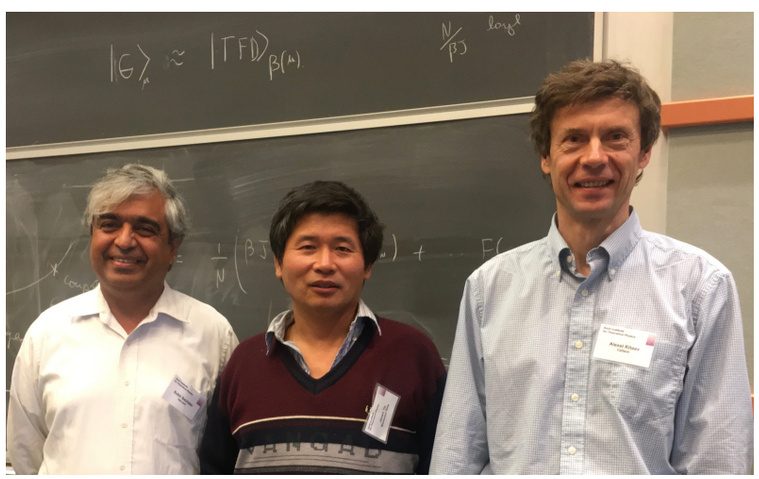

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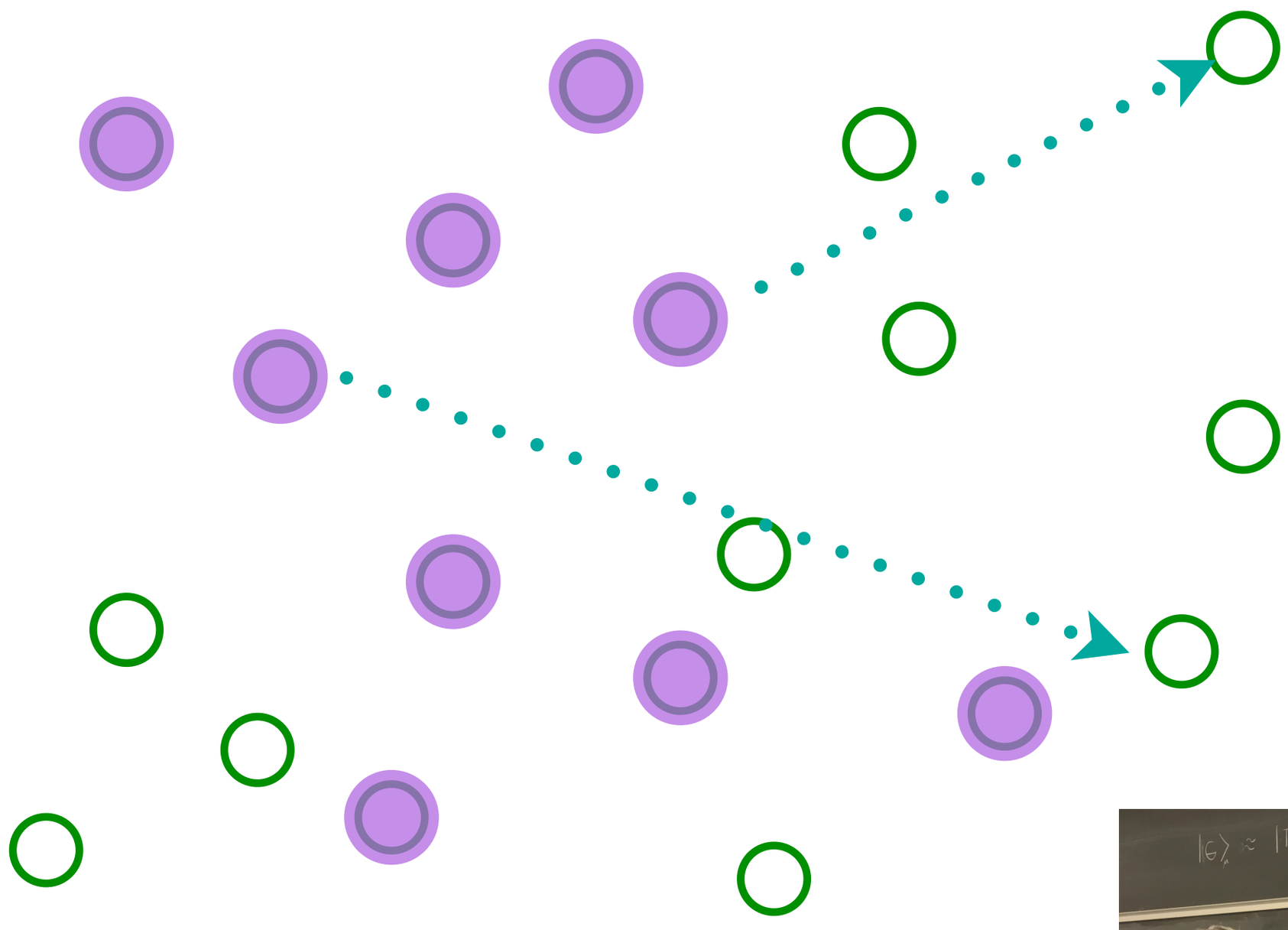

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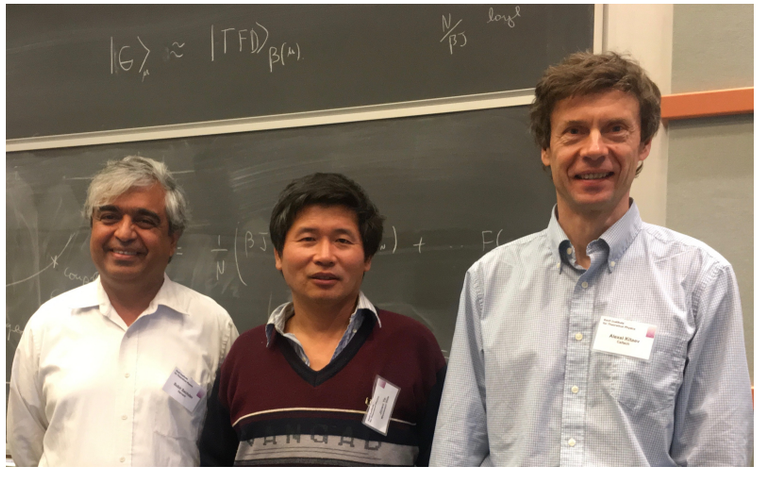

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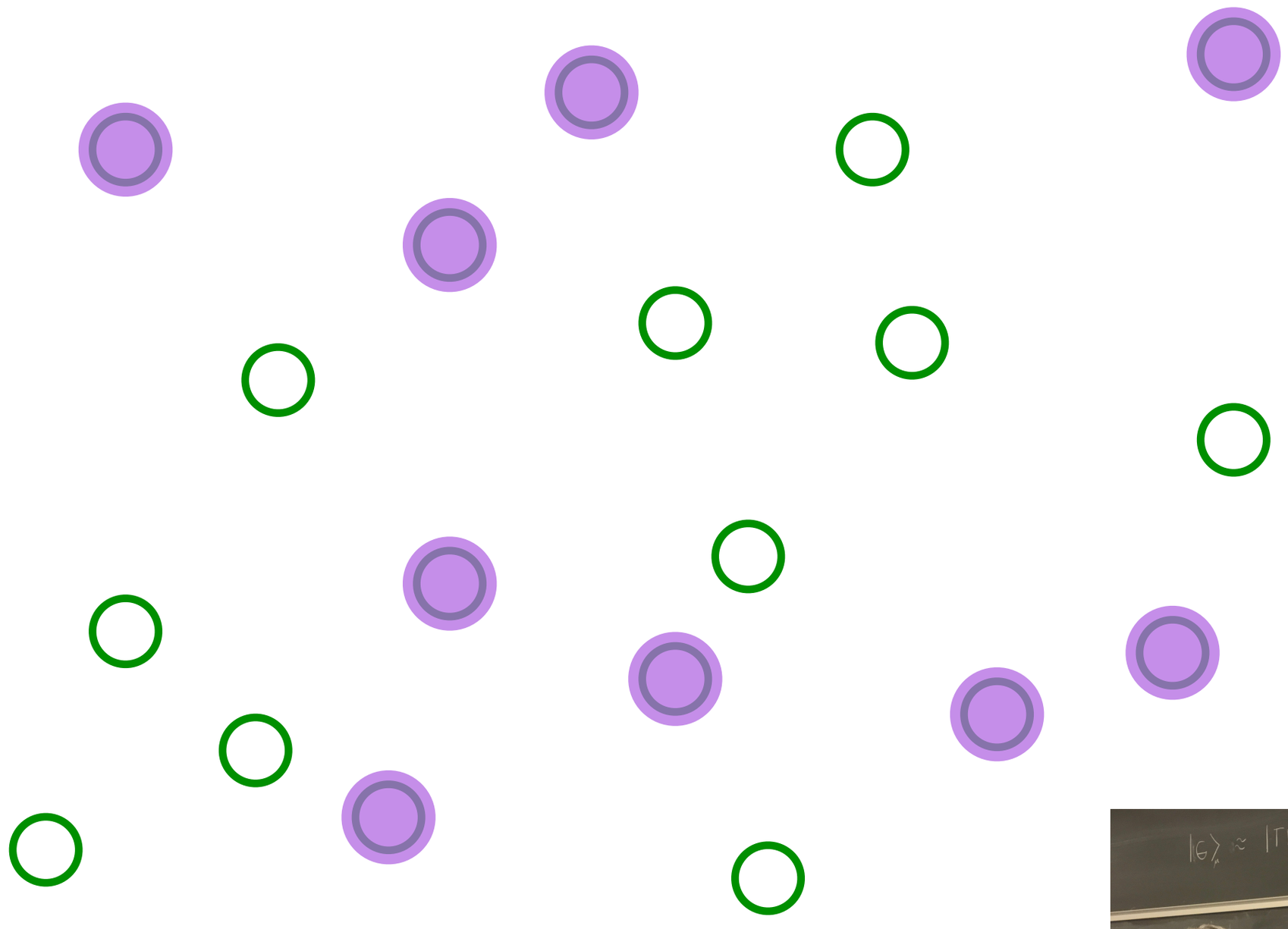

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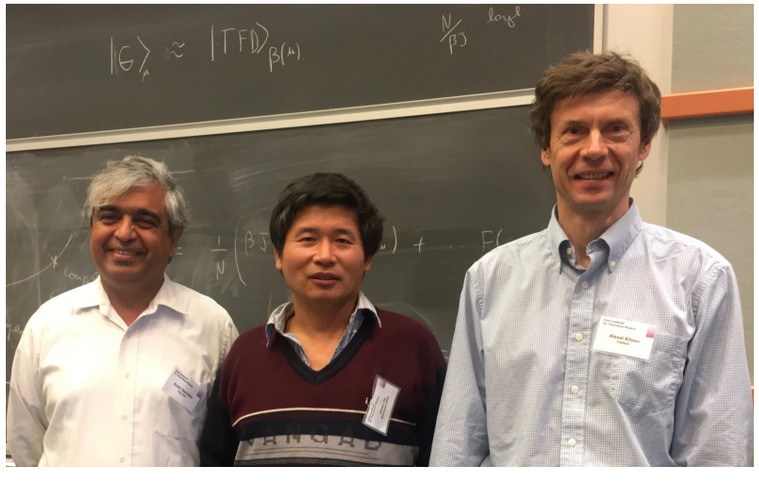

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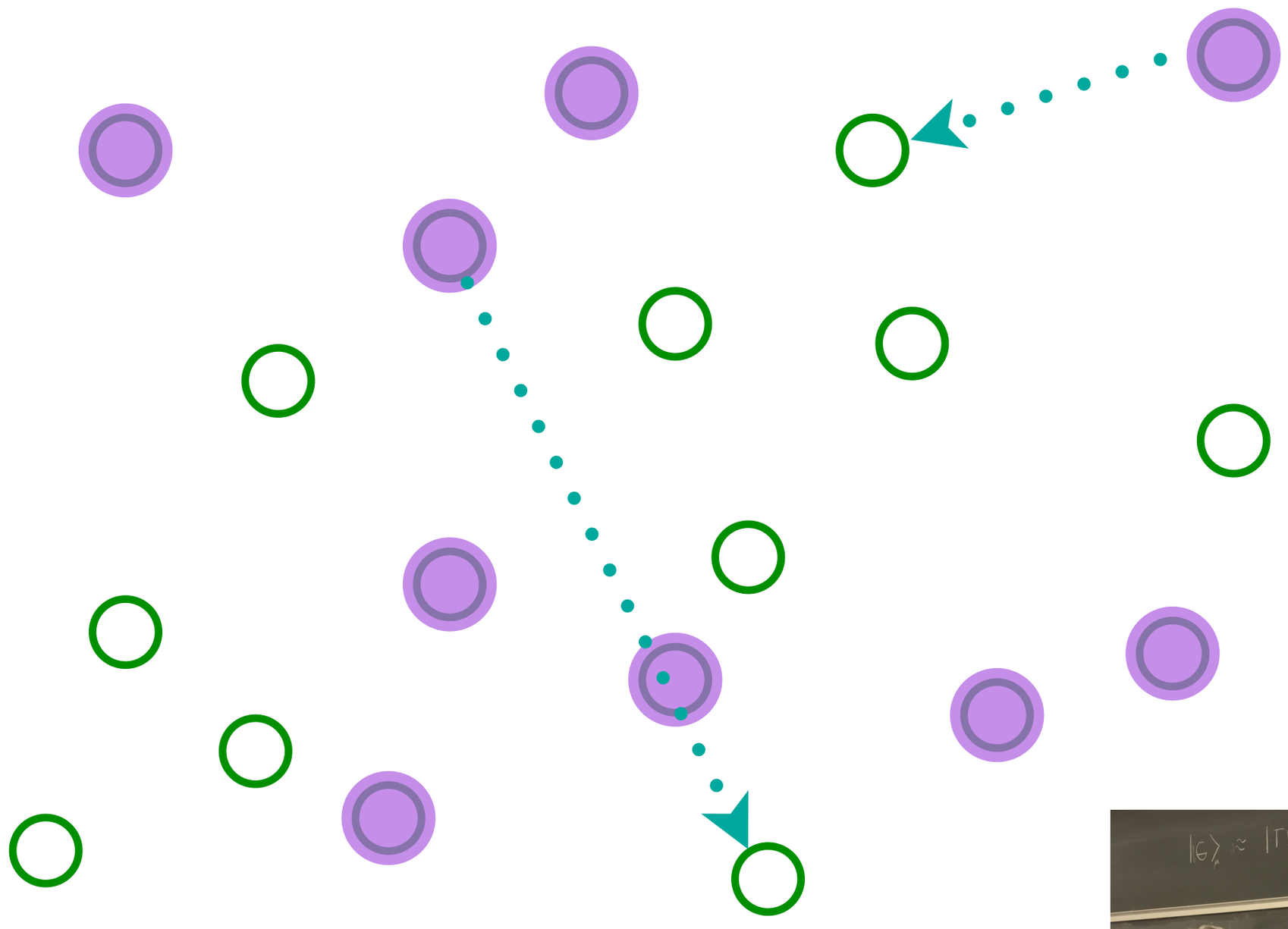

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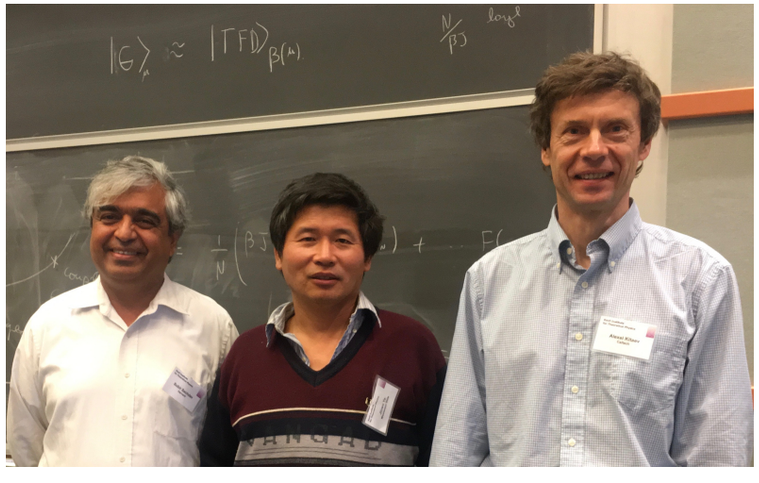

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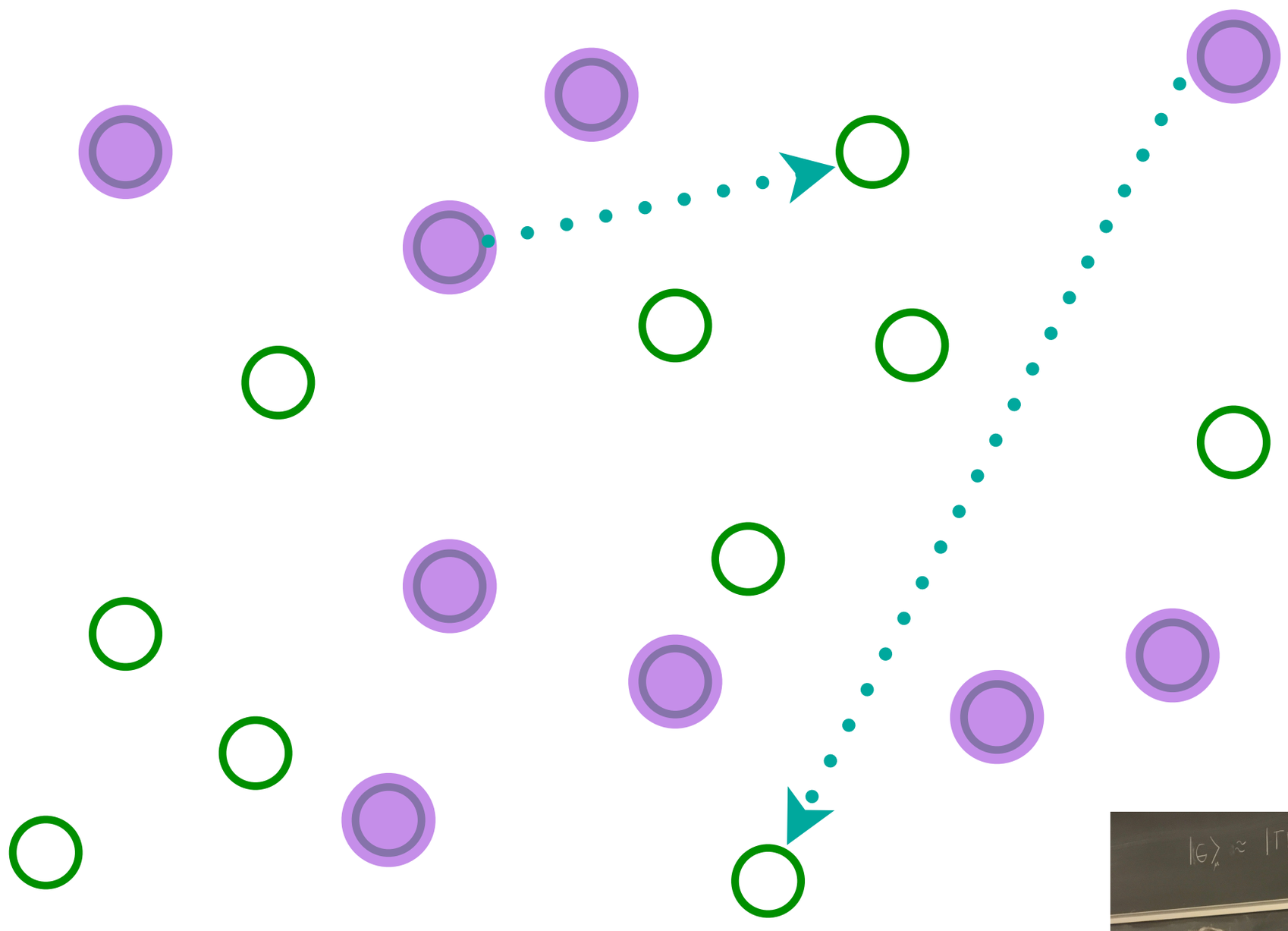

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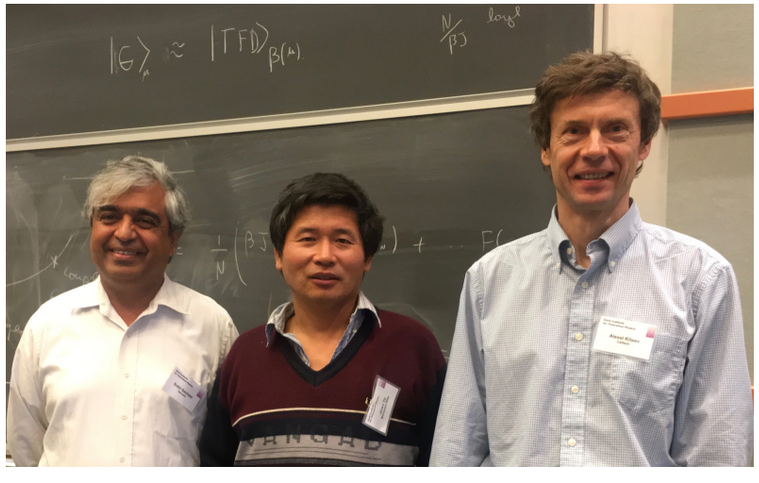

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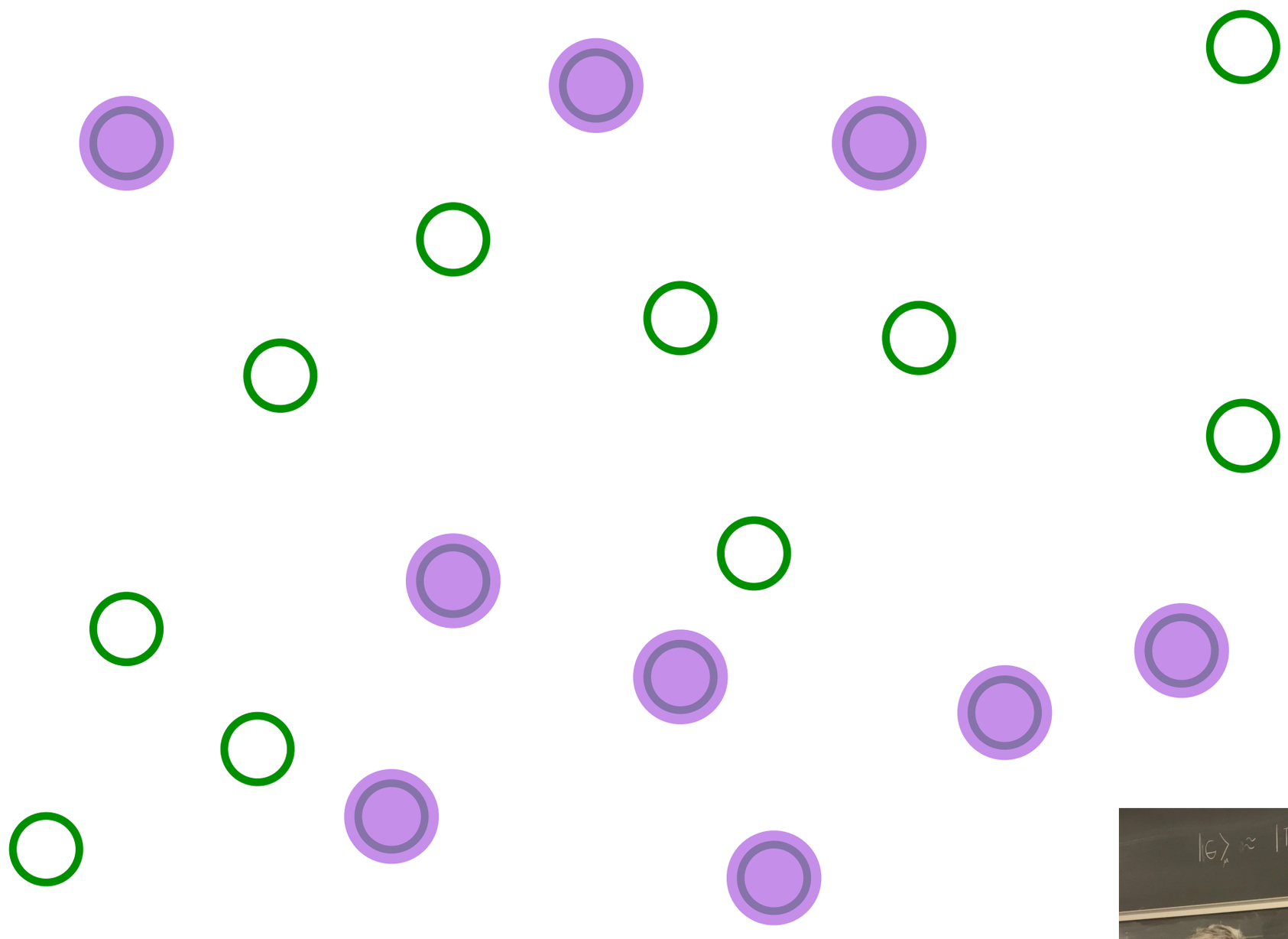

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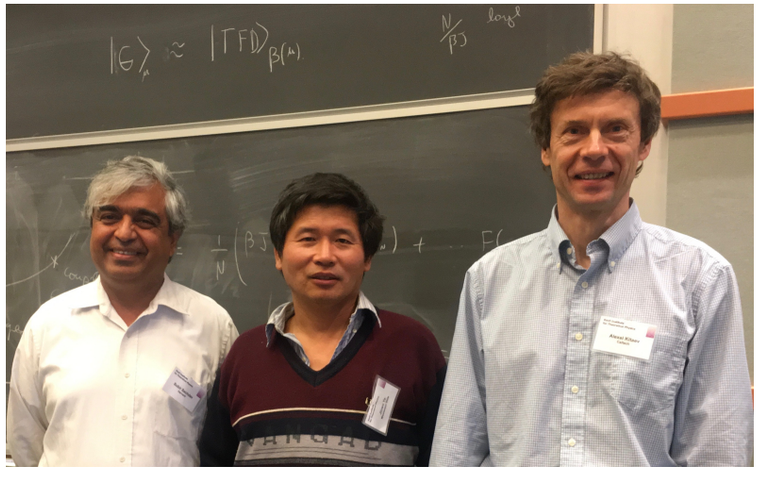

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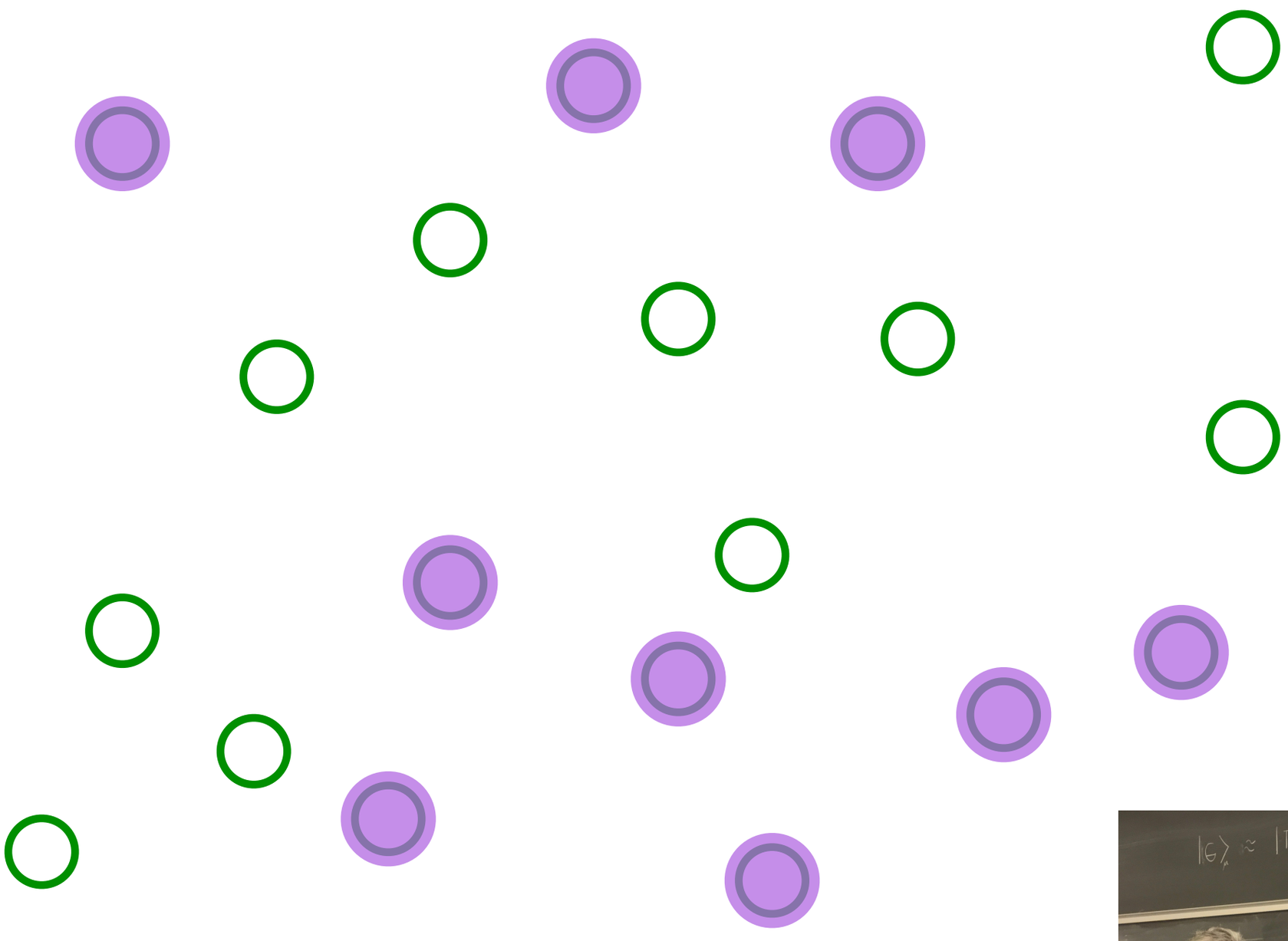
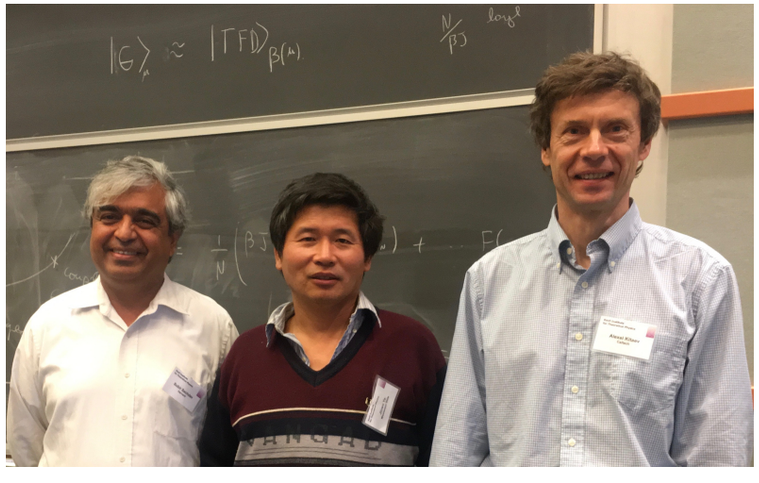


This describes both a strange metal and a black hole!



The complex SYK model

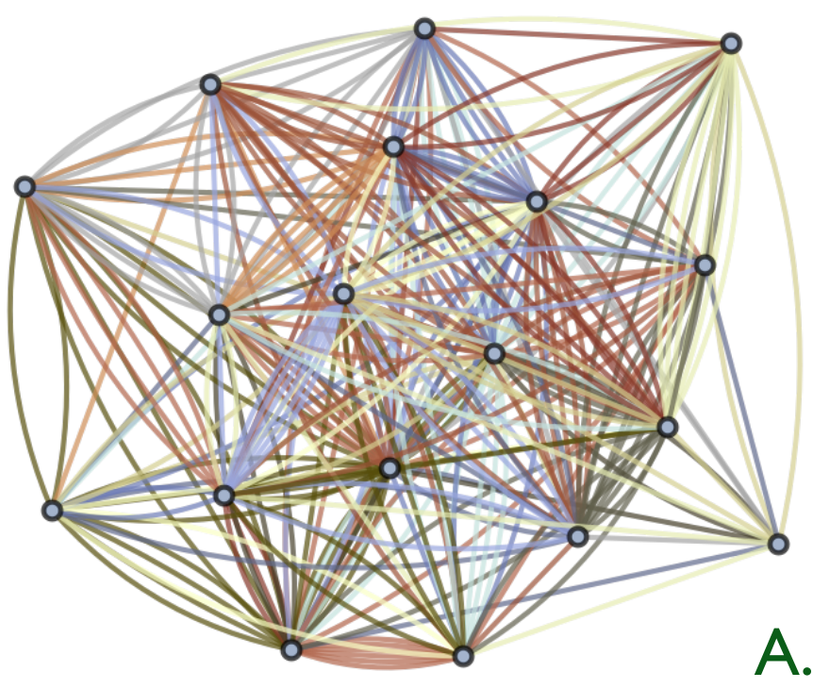
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

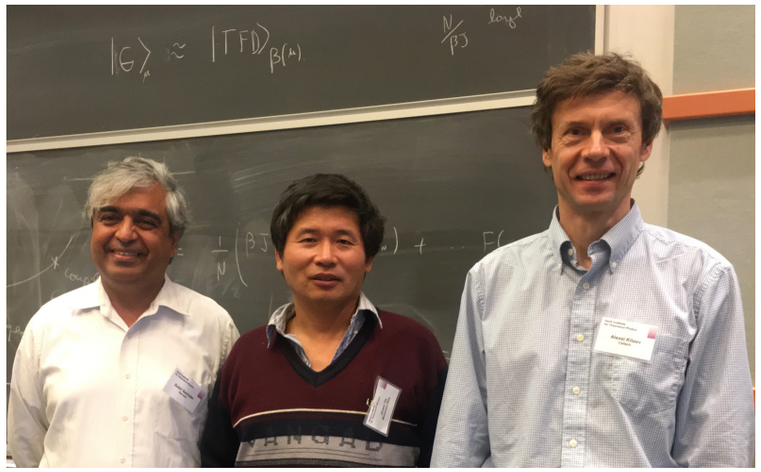
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta + \epsilon \sum_\alpha c_\alpha^\dagger c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.





S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

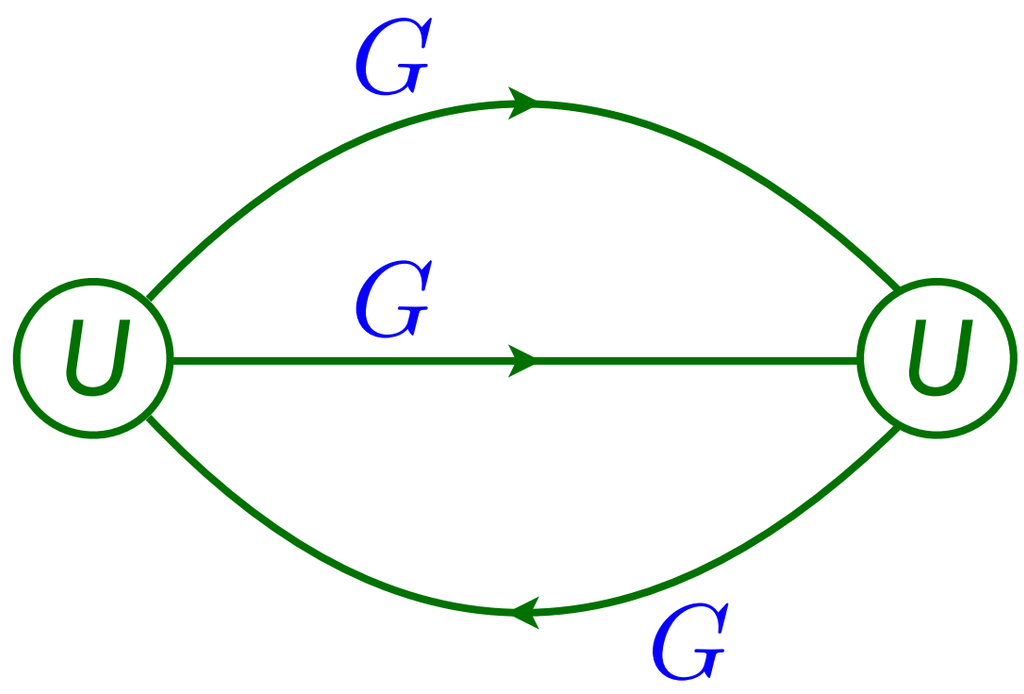
The complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega - \epsilon - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$



S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

The complex SYK model

The large N limit is given by the sum of "melon" Feynman graphs

For long times $\tau > 0$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = e^{\pi \mathcal{E}} \frac{A}{\sqrt{\tau}}$$

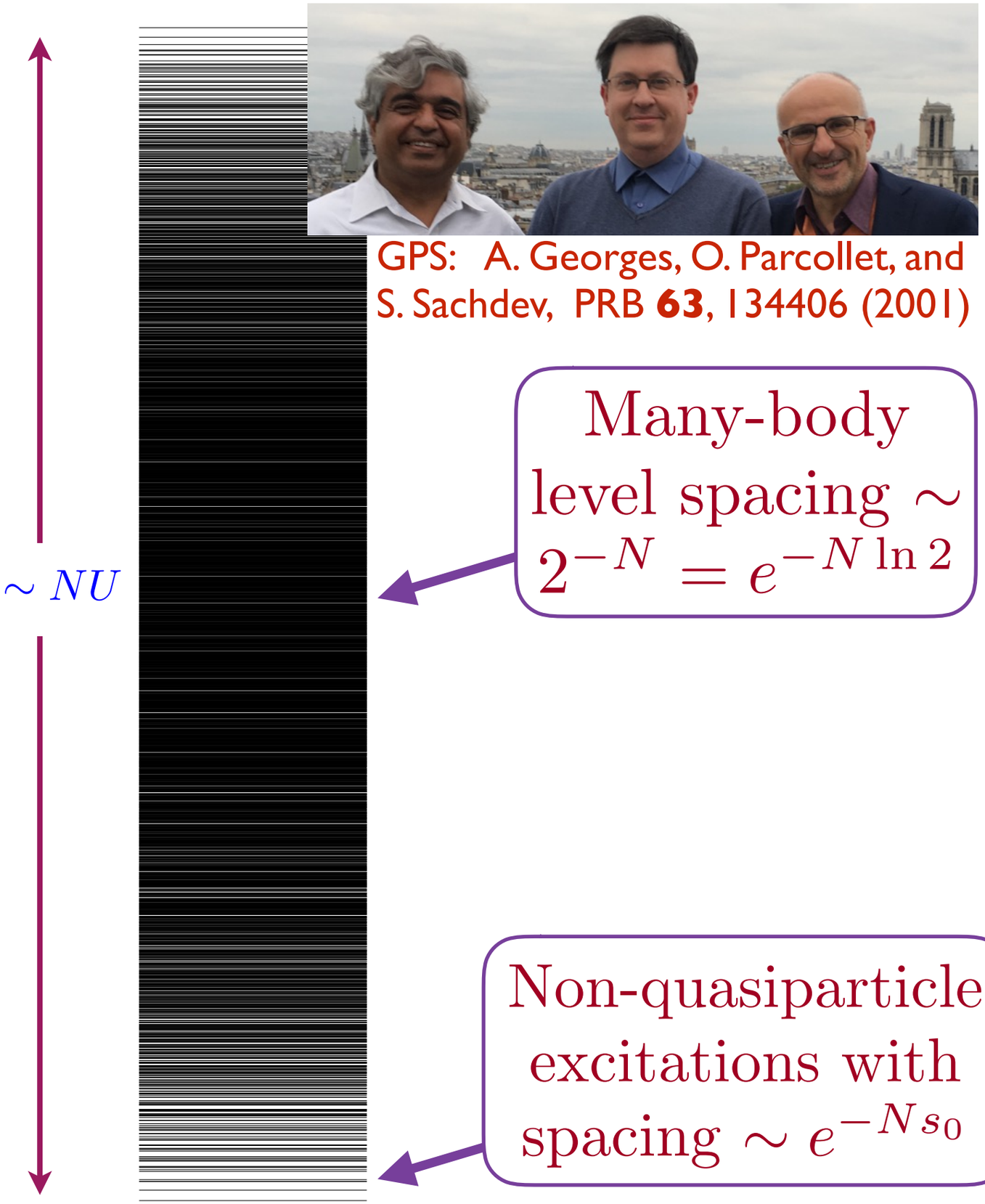
$$\left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = e^{-\pi \mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The parameter $\mathcal{E} = \mathbb{C}\left(\epsilon/U\right)$ determines the particle-hole asymmetry, and has a universal "Luttinger" relation to $\mathcal{Q}$.

In a Fermi liquid,

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = \left\langle c_\alpha^\dagger(\tau) c_\alpha(0) \right\rangle = \widetilde{A}/\tau$$

The SYK model



GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

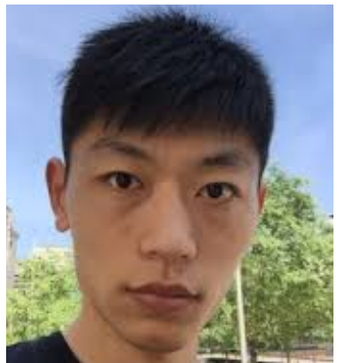
There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{dQ} = 2\pi \mathcal{E} \, .$$

At $\mathcal{Q} = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.



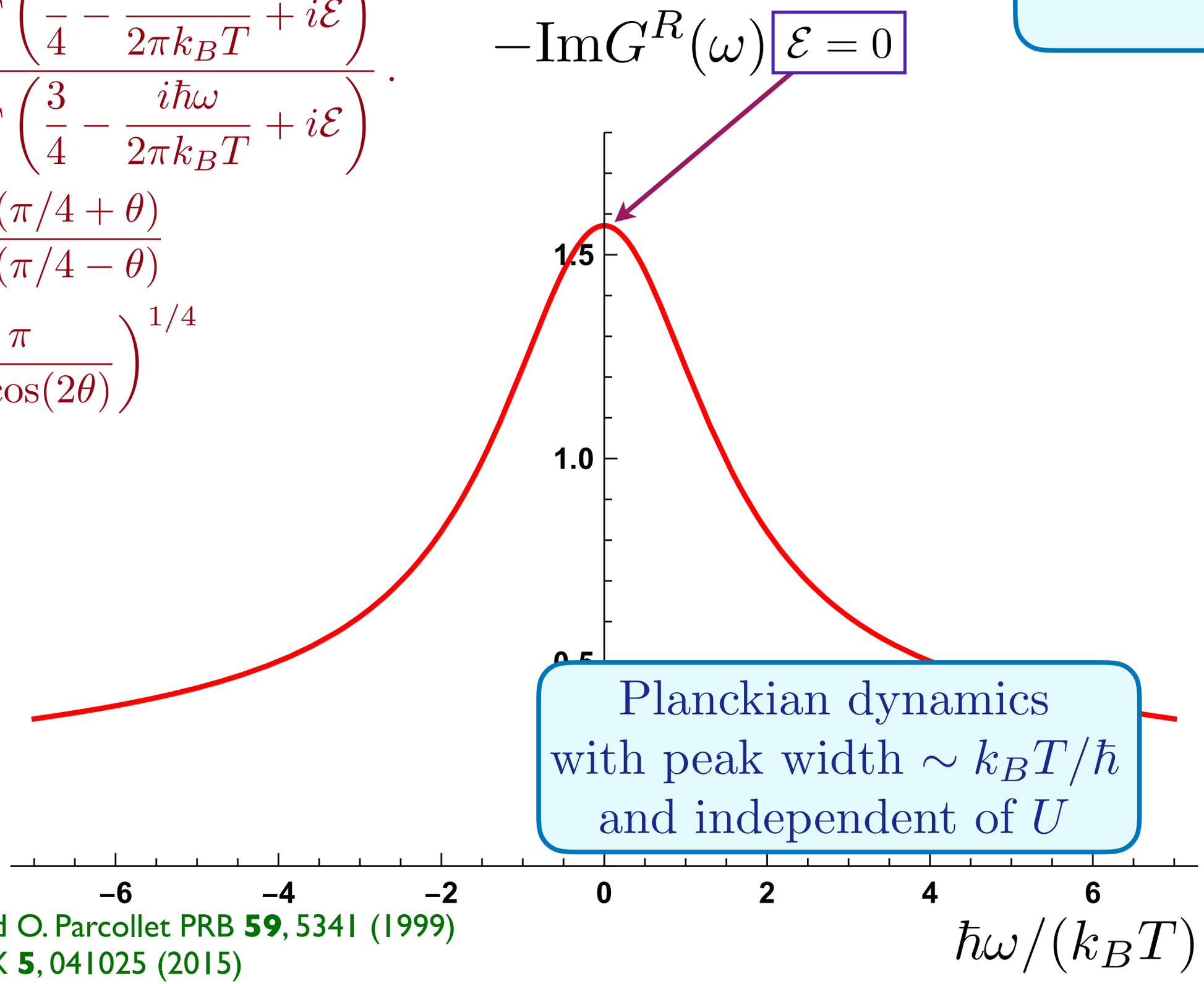
The complex SYK model

$$\mathcal{E} = \mathbb{C}\frac{\epsilon}{U}$$

$$G^R_{\rm SYK}(\epsilon, \hbar\omega/(k_B T)) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

$$C = \left(\frac{\pi}{U^2\cos(2\theta)}\right)^{1/4}$$



Planckian dynamics
with peak width $\sim k_B T/\hbar$
and independent of U

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

The complex SYK model

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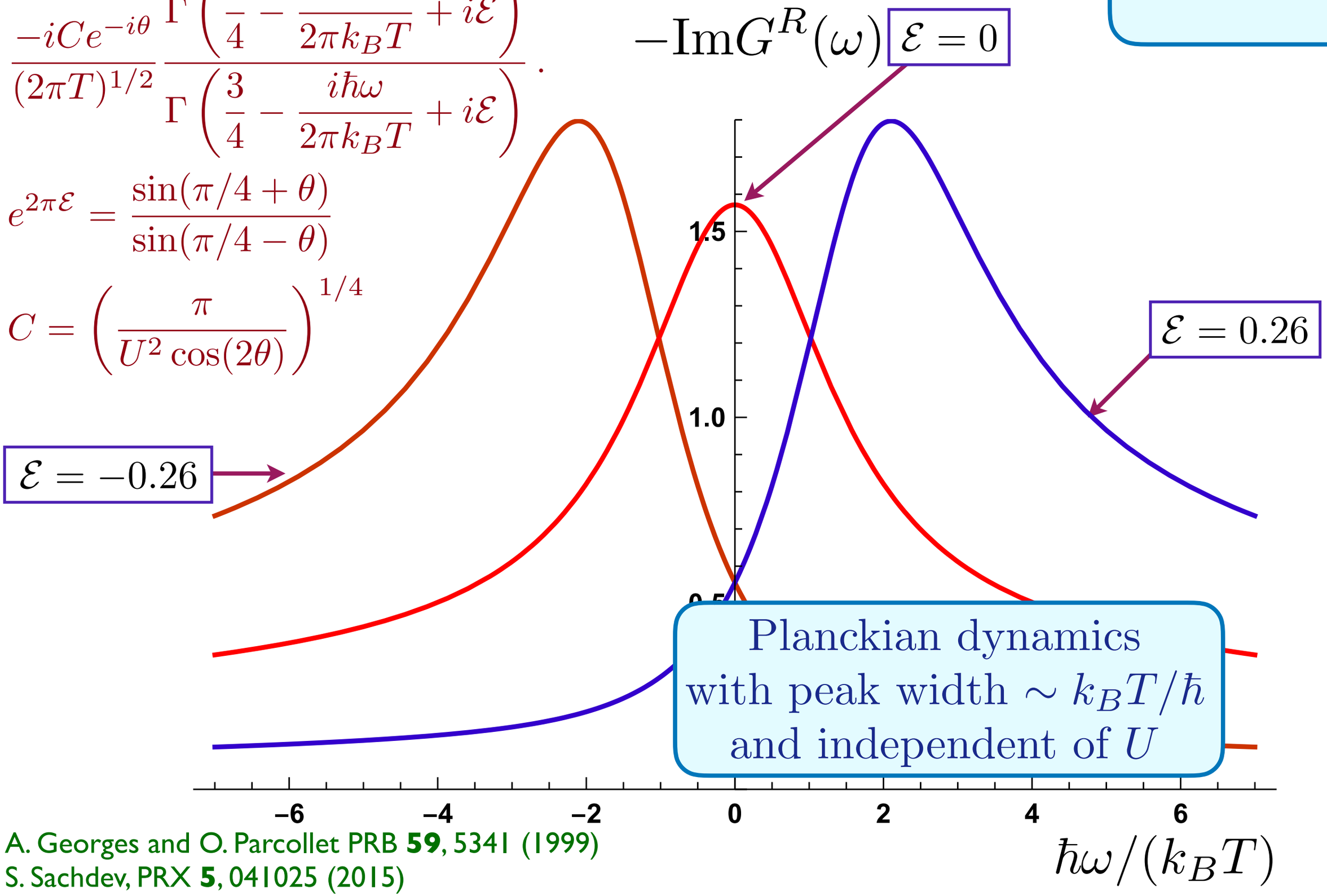

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$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta + \epsilon \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$

Generalized SYK models

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$
$$+ \sum_{k\alpha} \epsilon_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)
ϵ_k has a range of values of width W.

The large N limit is still given by the sum of "melon" diagrams.

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)
Pengfei Zhang, PRB **96**, 205138 (2017)
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018)
Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

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The large N limit is still given by the sum of "melon" diagrams.

For many generic models in this class, $\hbar\omega/(k_B T)$ scaling of SYK holds for $W^2/U \ll k_B T \ll U$, and Fermi liquid theory is recovered for $k_B T \ll W^2/U$.

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)
Pengfei Zhang, PRB **96**, 205138 (2017)
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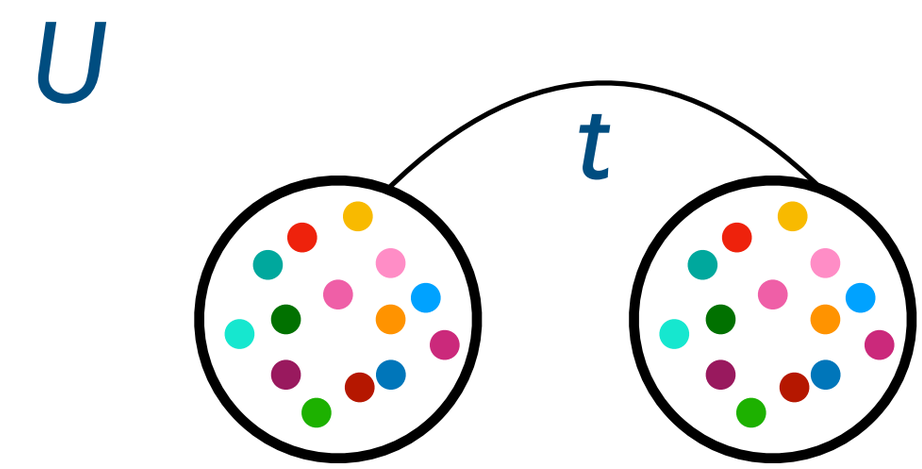
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_{i\alpha} c^\dagger_{i\beta} c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_\alpha c^\dagger_{i\alpha} c_{j\alpha}$$

Equivalent to an
"eternal traversable wormhole"
between two black holes with
AdS_2 horizons



J. Maldacena and Xiao-Liang Qi, arXiv:1804.00491

A lattice SYK model

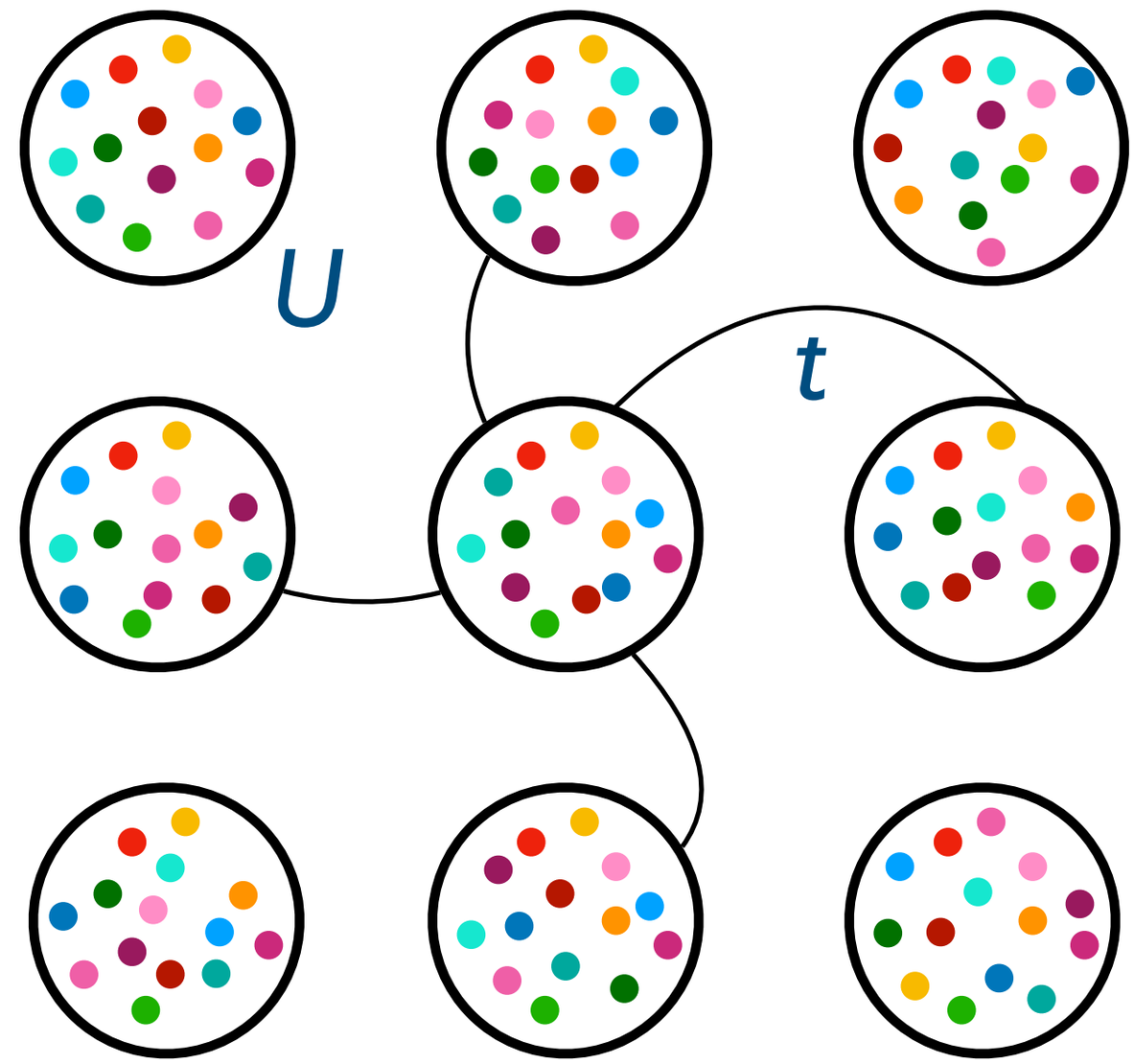
$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_\alpha c_{i\alpha}^\dagger c_{j\alpha}$$

Choose U on-site,
and uncorrelated between sites;
yields 'incoherent metal'
with no Fermi surface
for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k}, \omega) = G_{\rm SYK}(\epsilon, \hbar\omega/(k_B T))$$

independent of $\mathbf{k}$.
There is linear-in-T resistivity
but only with $\rho > h/e^2$.

$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)
Pengfei Zhang, PRB **96**, 205138 (2017)
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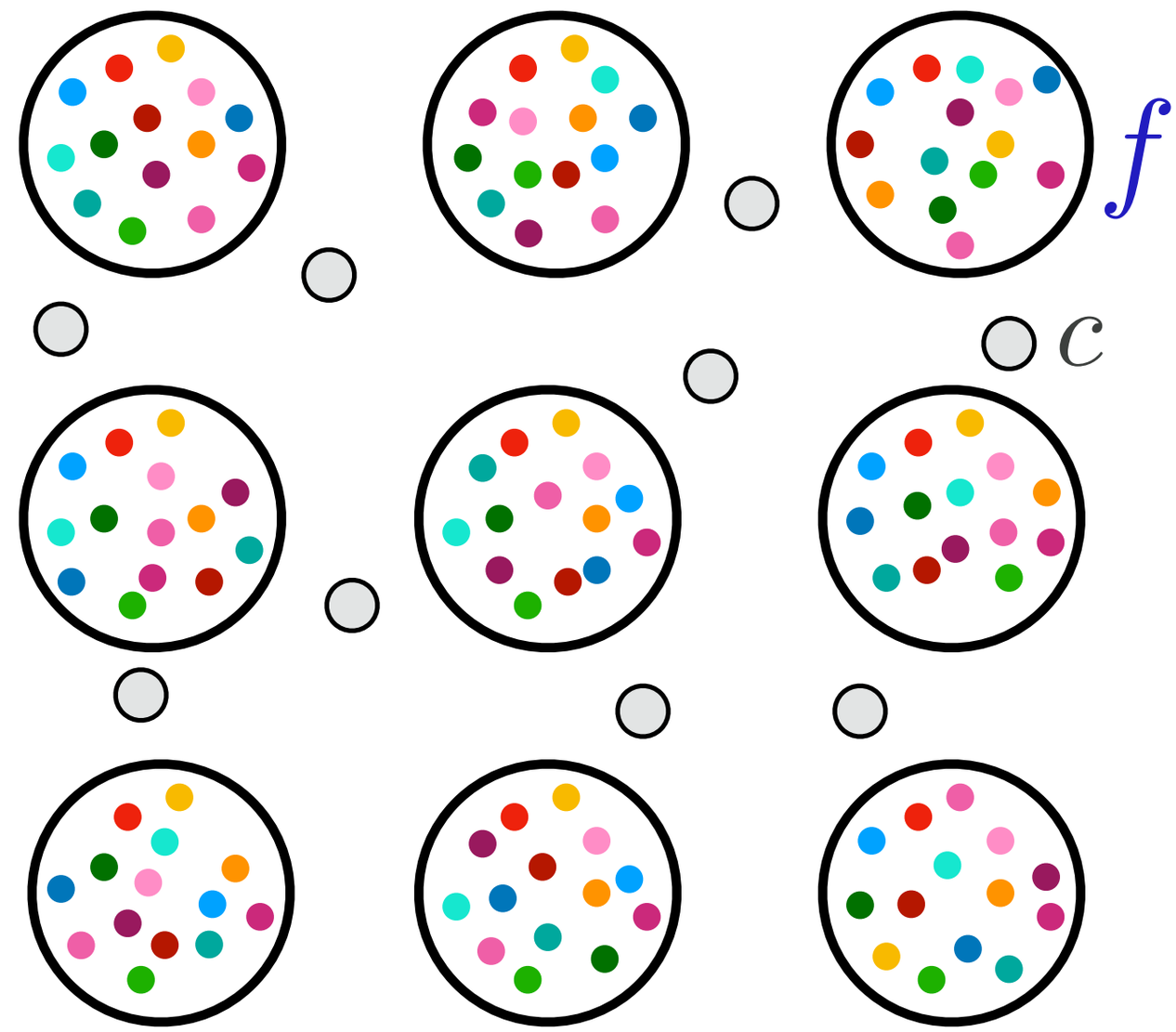
See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999)

A Kondo-SYK model

Mobile electrons (c) coupled to SYK quantum islands (f) with exchange interactions.

Has a regime where the c electrons form a marginal Fermi liquid with a linear-in-T resistivity dependent upon interaction strength, and a small Fermi surface which does not count the f electrons.



Similar results for many earlier `marginal Fermi liquid' and holographic models

Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018)
Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

A lattice SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_{i\alpha}^\dagger c_{i\beta}^\dagger c_{i\gamma} c_{i\delta} - t \sum_{\langle ij \rangle} \sum_\alpha c_{i\alpha}^\dagger c_{j\alpha}$$

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There is linear-in-T resistivity
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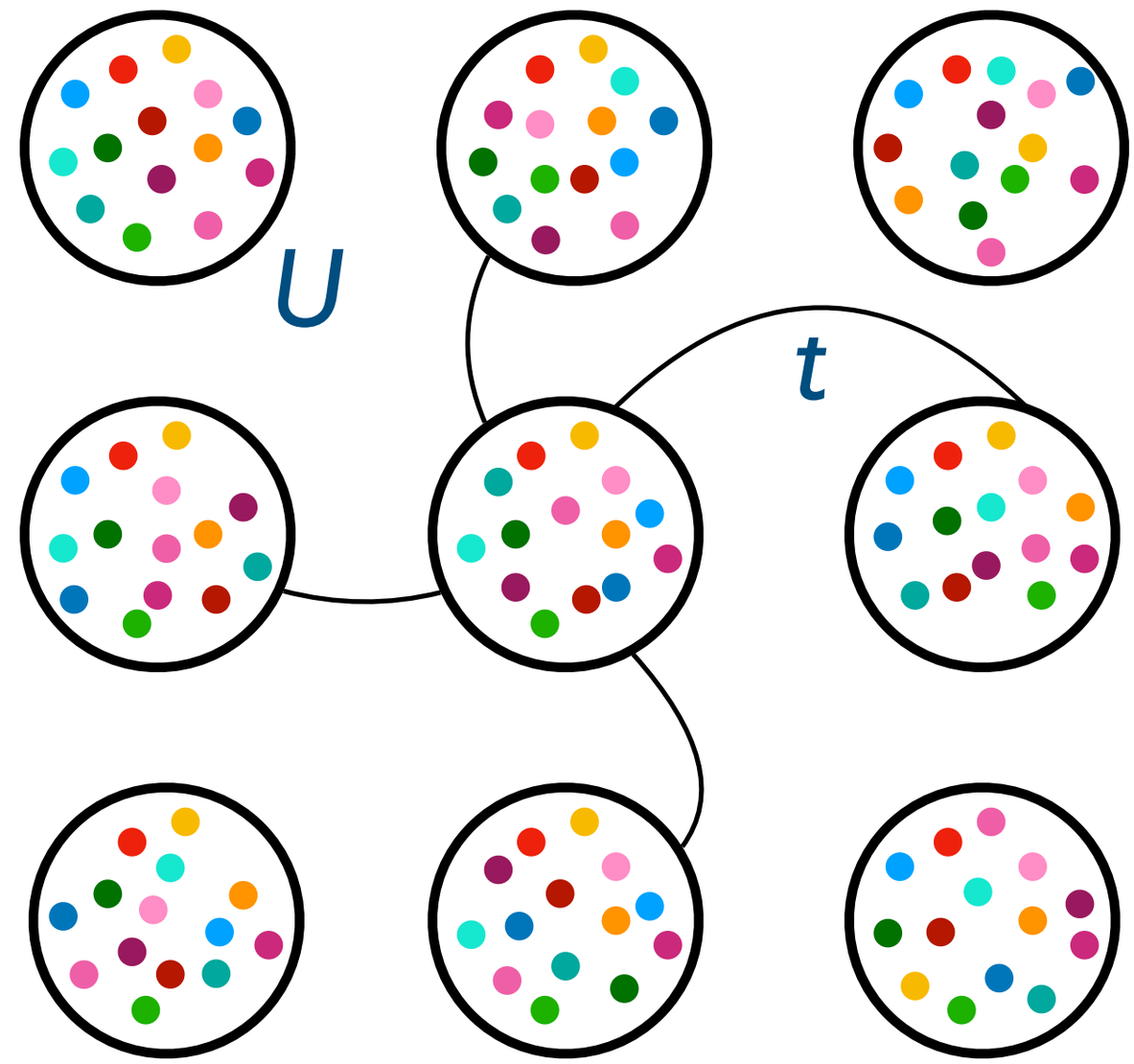
$$\rho \sim \frac{h}{e^2} \frac{k_B T}{t^2/U}$$



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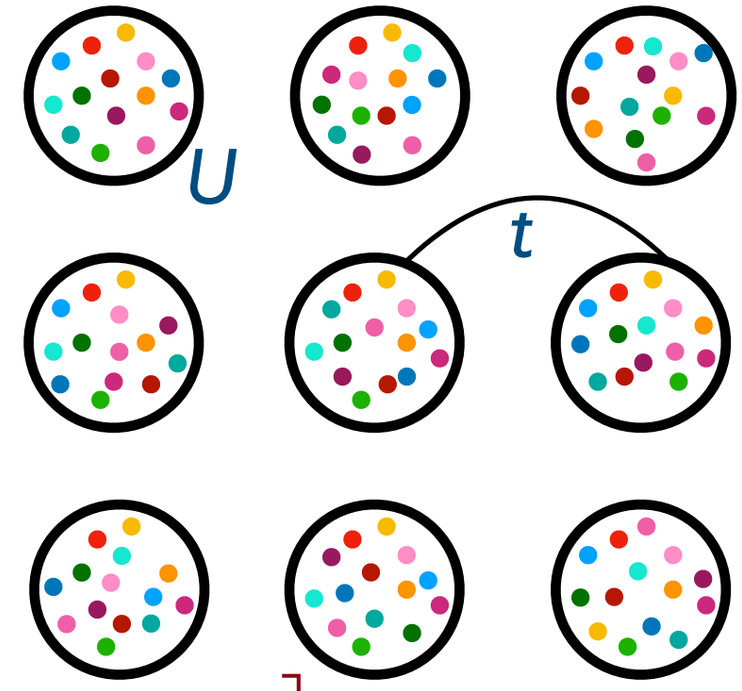
$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$

$$+ \sum_{k\alpha} \epsilon_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

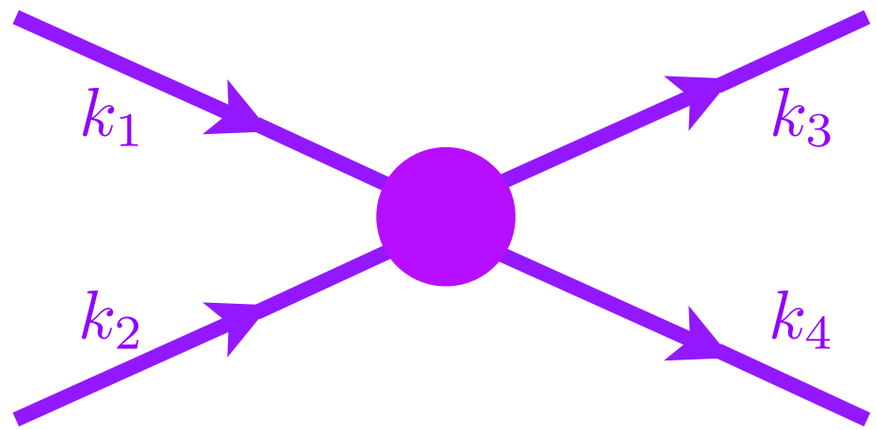
ϵ_k has a bandwidth W.



Rewriting of lattice model of incoherent and bad metal in momentum space

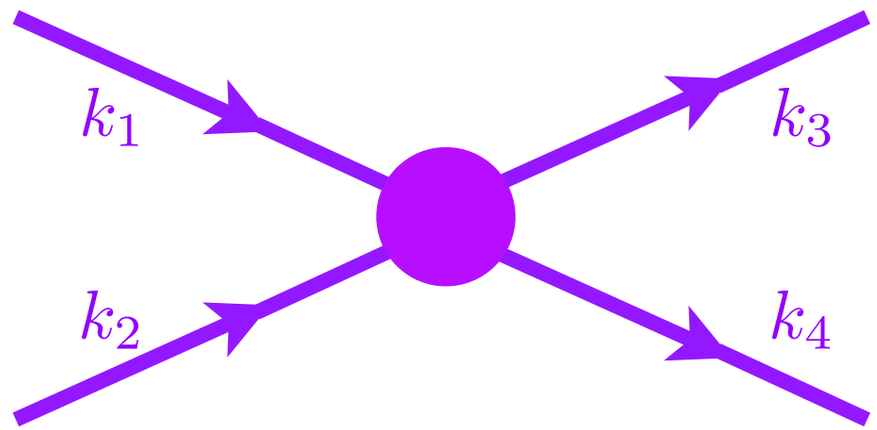
$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} =$$
$$U^2\Big[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\Big]$$

Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we "integrate these out" in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k, which we have already accounted for.

Resonant SYK model



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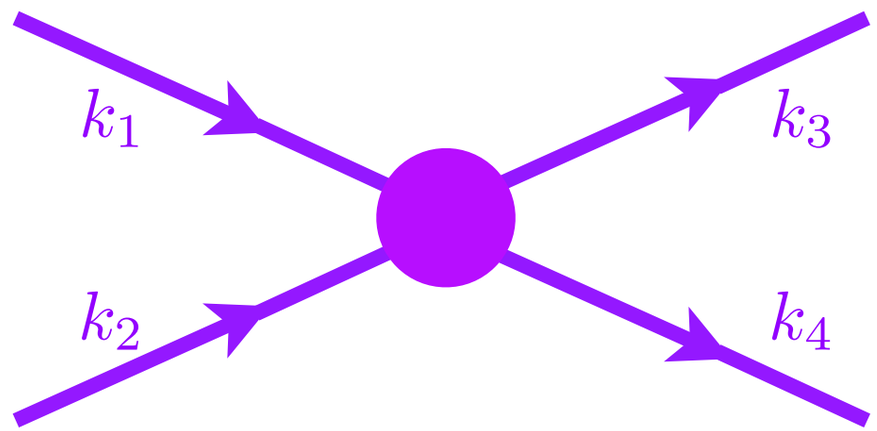
Keep only the
interactions resonant in the bare quasiparticle energy
with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

Resonant SYK model



Interactions with $\epsilon_{k_1} + \epsilon_{k_2} \neq \epsilon_{k_3} + \epsilon_{k_4}$ are non-resonant: we "integrate these out" in a RG procedure, and assume that their main effect is a renormalization of the quasiparticle dispersion ϵ_k, which we have already accounted for.

Keep only the
interactions resonant in the bare quasiparticle energy
with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$ and account for them with a self-consistent SYK-like analysis.

This is precisely the effective Hamiltonian method, when low energy states are separated from high energy states by a gap; we are assuming it can also apply in a gapless system.

Resonant SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$

$$+ \sum_{k\alpha} \epsilon_k c^\dagger_{k\alpha} c_{k\alpha}$$

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)

The random k_i dependence of U allows only

interactions resonant in the bare quasiparticle energies

with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$.

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} =$$

$$U^2 \Big[\delta(k_1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8)\Big]$$

$$\times \Big[\delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_3} - \epsilon_{k_4}) + \delta(\epsilon_{k_5} + \epsilon_{k_6} - \epsilon_{k_7} - \epsilon_{k_8})\Big]$$

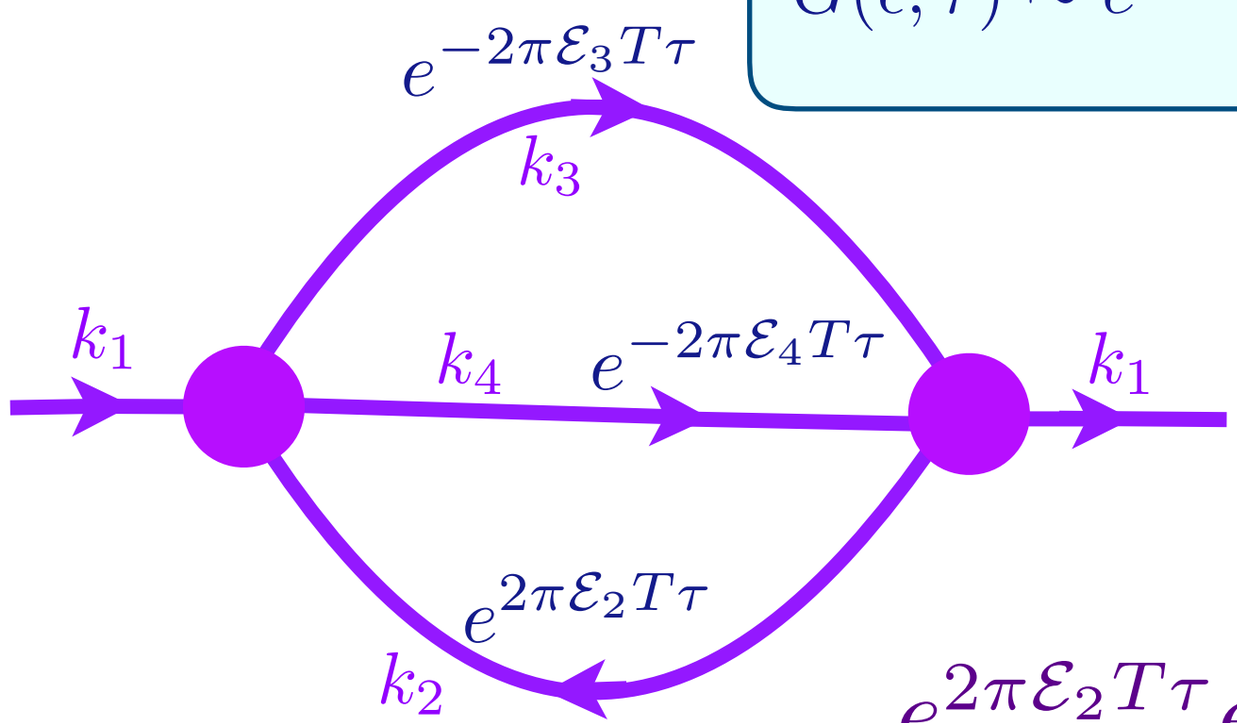
Aavishkar Patel

This implies off-site interactions with correlations which decay with a power-law in space.

Resonant SYK model

Conformal Green's function at $T > 0$ must have the form

$$G(\epsilon, \tau) \sim e^{-2\pi \mathcal{E} T \tau} \left(\frac{T}{\sin(\pi T \tau)} \right)^{1/2} \quad , \quad 0 < \tau < 1/T \, .$$



$$e^{2\pi \mathcal{E}_2 T \tau} e^{-2\pi \mathcal{E}_3 T \tau} e^{-2\pi \mathcal{E}_4 T \tau} = e^{-2\pi \mathcal{E}_1 T \tau}$$

if

$$\mathcal{E}_a = \mathbb{C} \epsilon_a / U$$

and

$$\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4$$

SYK behavior in a
Planckian metal as $T \to 0$
with a remnant Fermi surface:
$G(k, \omega) = G_{\mathrm{SYK}}(\epsilon_k, \hbar\omega/(k_B T))$,
with $\mathcal{E}_k = \mathbb{C}\, \epsilon_k / U$

Incoherent metal

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathcal{E}} \frac{A}{\sqrt{\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The parameter $\mathcal{E}$ is independent of k,
and determined by the total density

Planckian metal with remnant Fermi surface

For long times $\tau > 0$

$$\left\langle c_k(\tau) c_k^\dagger(0) \right\rangle = e^{\pi \mathbb{C} \epsilon_k / U} \frac{A}{\sqrt{\tau}}$$

$$\left\langle c_k^\dagger(\tau) c_k(0) \right\rangle = e^{-\pi \mathbb{C} \epsilon_k / U} \frac{A}{\sqrt{\tau}}$$

The particle-hole asymmetry changes as we cross the Fermi surface

Aavishkar Patel

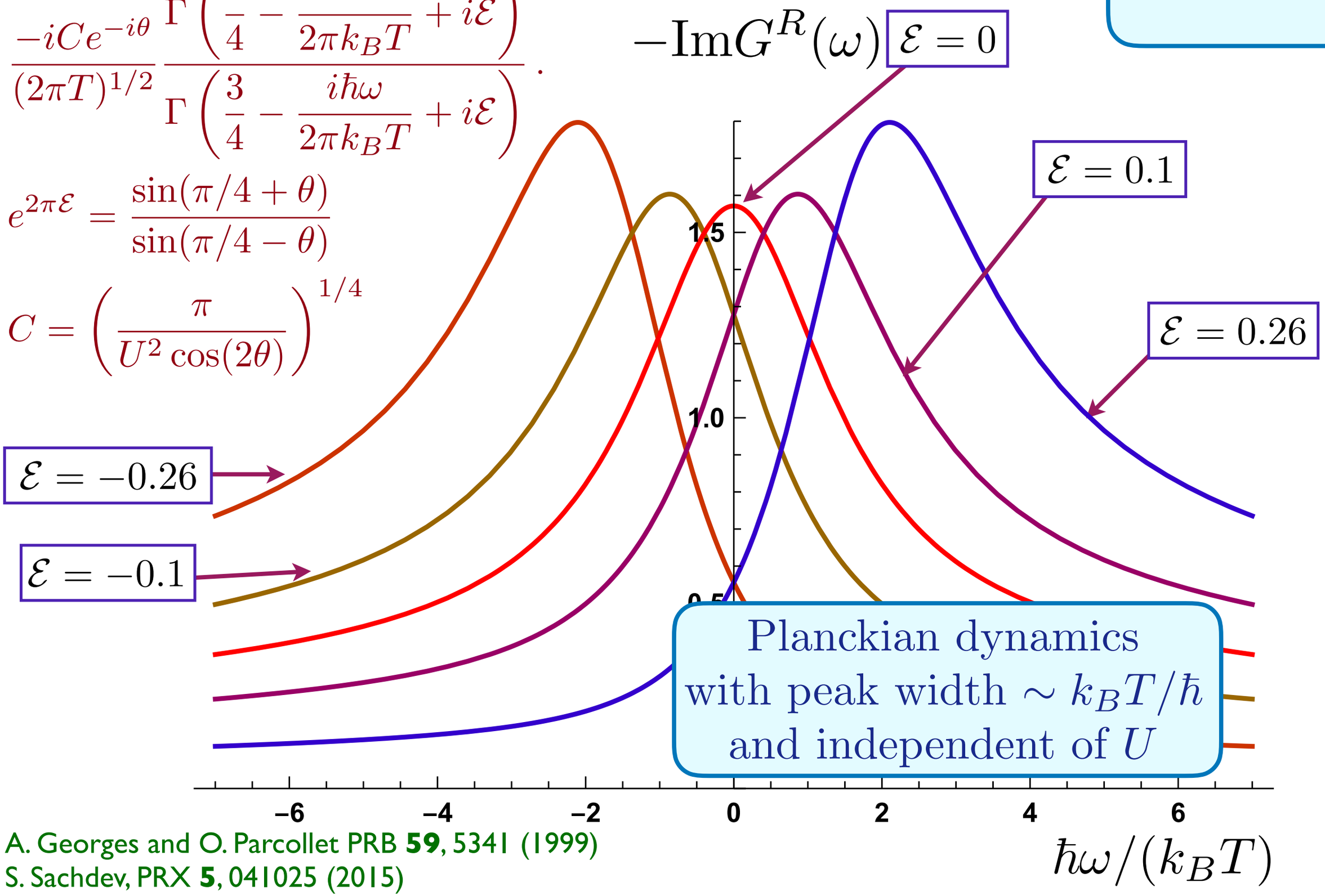
The complex SYK model

$$\mathcal{E} = \mathbb{C}\frac{\epsilon}{U}$$

$$G^R_{\rm SYK}(\epsilon, \hbar\omega/(k_B T)) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

$$C = \left(\frac{\pi}{U^2\cos(2\theta)}\right)^{1/4}$$



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A. Georges and O. Parcollet PRB **59**, 5341 (1999)
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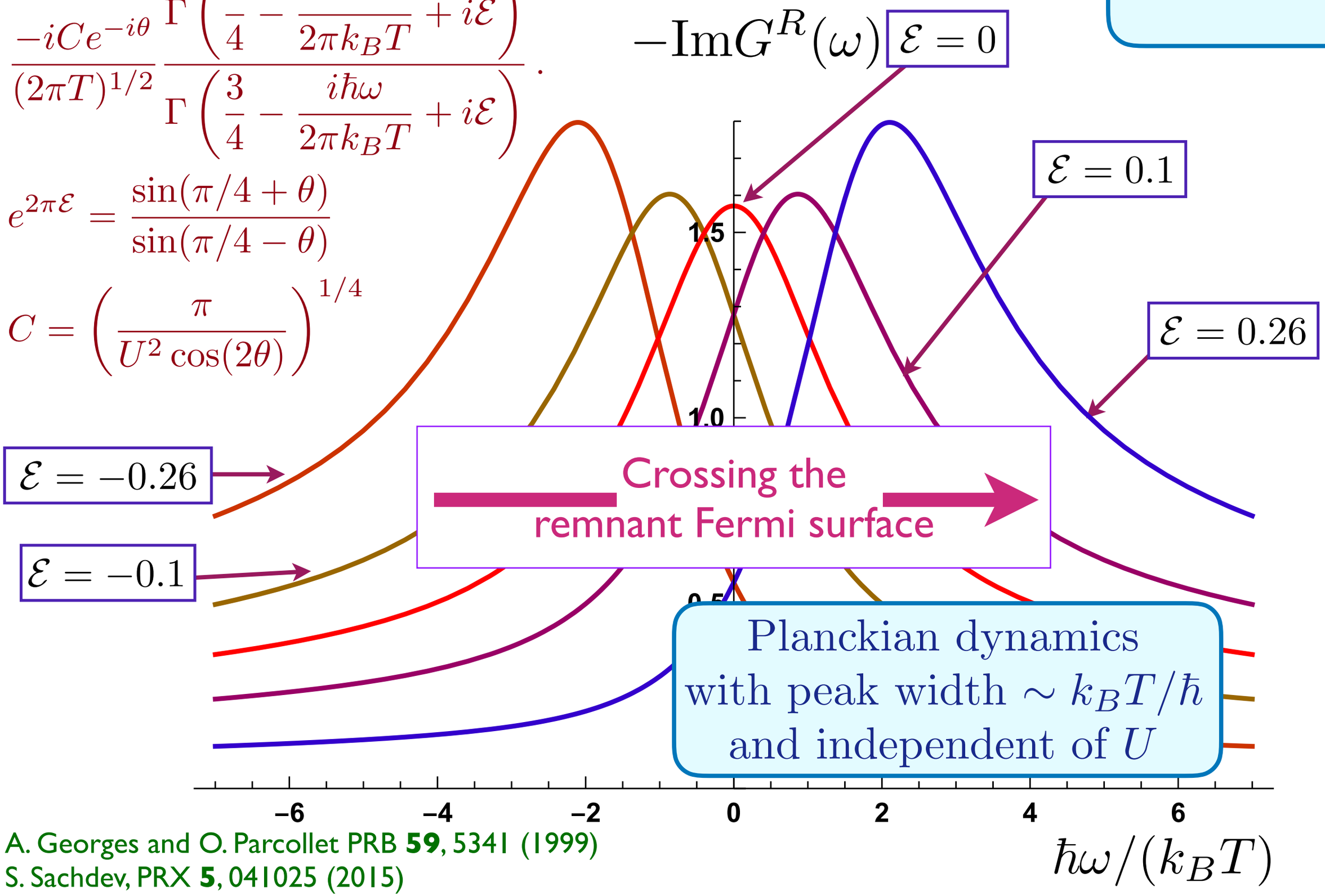
$$G^R_{\rm SYK}(\epsilon, \hbar\omega/(k_B T)) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1/2}} \frac{\Gamma\left(\frac{1}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}{\Gamma\left(\frac{3}{4} - \frac{i\hbar\omega}{2\pi k_B T} + i\mathcal{E}\right)}.$$

$$e^{2\pi\mathcal{E}} = \frac{\sin(\pi/4+\theta)}{\sin(\pi/4-\theta)}$$

$$C = \left(\frac{\pi}{U^2\cos(2\theta)}\right)^{1/4}$$



A. Georges and O. Parcollet PRB **59**, 5341 (1999)
S. Sachdev, PRX **5**, 041025 (2015)

Resonant SYK model

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$ (as before)

The random k_i dependence of U allows only

interactions resonant in the bare quasiparticle energies

with $\epsilon_{k_1} + \epsilon_{k_2} = \epsilon_{k_3} + \epsilon_{k_4}$.

Resistivity of a Planckian metal as $T \to 0$

From the Kubo formula, in the large N limit

$$\sigma = \frac{N e^2 m^* v_F^2}{2T} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega}{4\pi} \left[\mathrm{Im}\, G^R_{\mathrm{SYK}} \left(\epsilon, \frac{\omega}{T} \right) \right]^2 \mathrm{sech}^2 \left(\frac{\omega}{2T} \right)$$

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$$\boxed{\rho = \frac{m^*}{n e^2} \, 2.71 \mathbb{C} \, \frac{k_B T}{\hbar}} \quad , \quad \text{using } \mathcal{E} = \mathbb{C}\epsilon/U \, ,$$

where

$$m^* = \frac{d\, V_{FS}}{\oint_{FS} |\boldsymbol{v}_F|} \, ,$$

where d is spatial dimensionality and V_{FS} is the volume enclosed by the Fermi surface. For a circular Fermi surface, this is the usual m^*.

Resonant SYK model

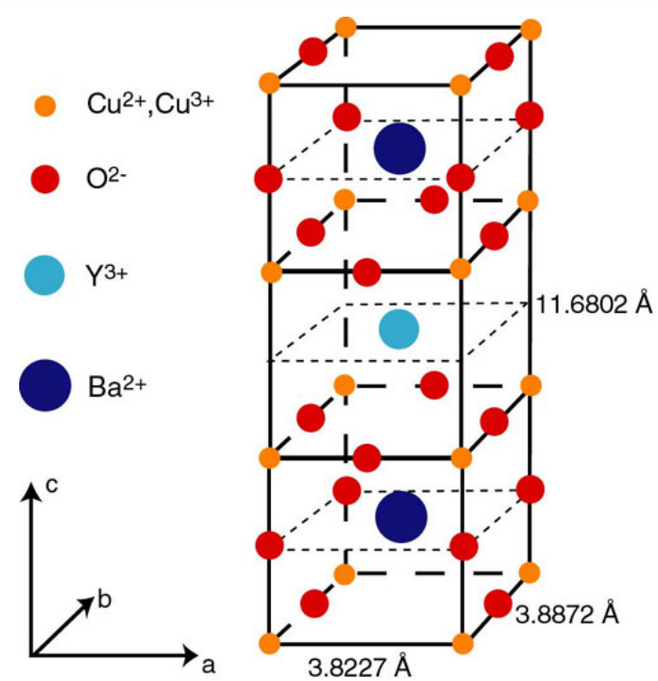
Resistivity of a Planckian metal as $T \to 0$

$$\rho = \frac{m^*}{ne^2}\, 2.71\mathbb{C}\, \frac{k_B T}{\hbar}$$

Note that all explicit dependence on U has cancelled out!

The number $\mathbb{C}$ is defined by $\mathcal{E}_k = \mathbb{C}\,\epsilon_k/U$ as $|\epsilon_k| \to 0$. This is determined by UV physics, and is very weakly dependent upon the ratio of the energy width of the interactions, W_U, to U.

Aavishkar Patel



Resonant SYK model

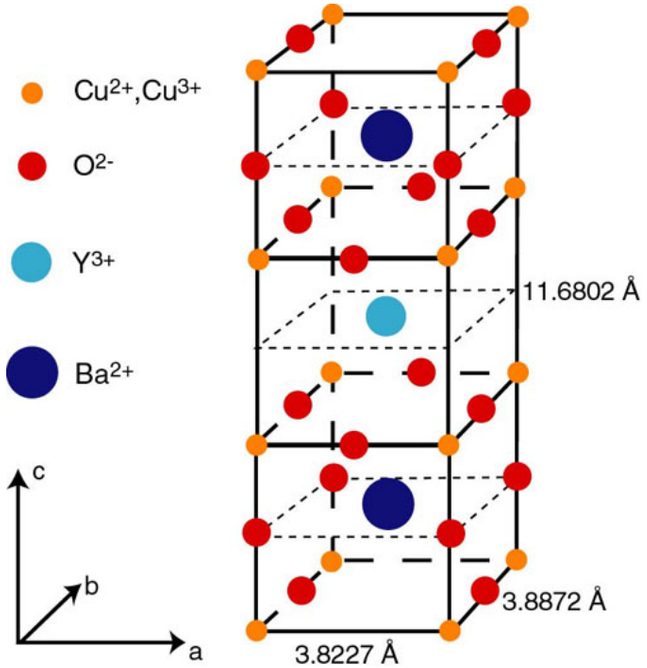
Take the independent momentum shell limit, $W_U/U \to 0$,

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2\Big[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\Big]$$
$$\times\Big[\delta(\epsilon_{k_1}-\epsilon_{k_2})\delta(\epsilon_{k_2}-\epsilon_{k_3})\delta(\epsilon_{k_3}-\epsilon_{k_4})+\delta(\epsilon_{k_5}-\epsilon_{k_6})\delta(\epsilon_{k_6}-\epsilon_{k_7})\delta(\epsilon_{k_7}-\epsilon_{k_8})\Big]$$

$\mathbb{C} = 0.41$ as in a single SYK model,
and we obtain a Planckian metal with

$$\rho = \frac{m^*}{ne^2}\,1.11\,\frac{k_B T}{\hbar}$$

Aavishkar Patel



Planckian metals with a remnant Fermi surface

- Planckian dynamics, *i.e.* $\hbar\omega/(k_B T)$ scaling of response functions and fastest possible local thermalization in a time $\sim \hbar/(k_B T)$), is realized in the 'solvable' SYK models.

- Resonant SYK models are compressible and dispersive quantum systems with Planckian dynamics as $T \to 0$.

- The resonance condition is supported by a RG argument: non-resonant interactions mainly renormalize the underlying quasiparticle dispersion ϵ_k, while resonant interactions have to be treated self-consistently.

- Resonant SYK models realize Planckian metals with remnant large Fermi surface at $\epsilon_k = 0$, and an effective mass m^* defined by the dispersion of ϵ_k.

- Planckian metals have a resistivity $\rho \sim (m^*/(ne^2))k_B T/\hbar$.

Aavishkar Patel (Harvard ⟶ Miller Fellow at Berkeley)

1. ‘Resonant’ SYK model for Planckian transport

Linear-in-T resistivity and a remnant Fermi surface

2. SYK + phonons

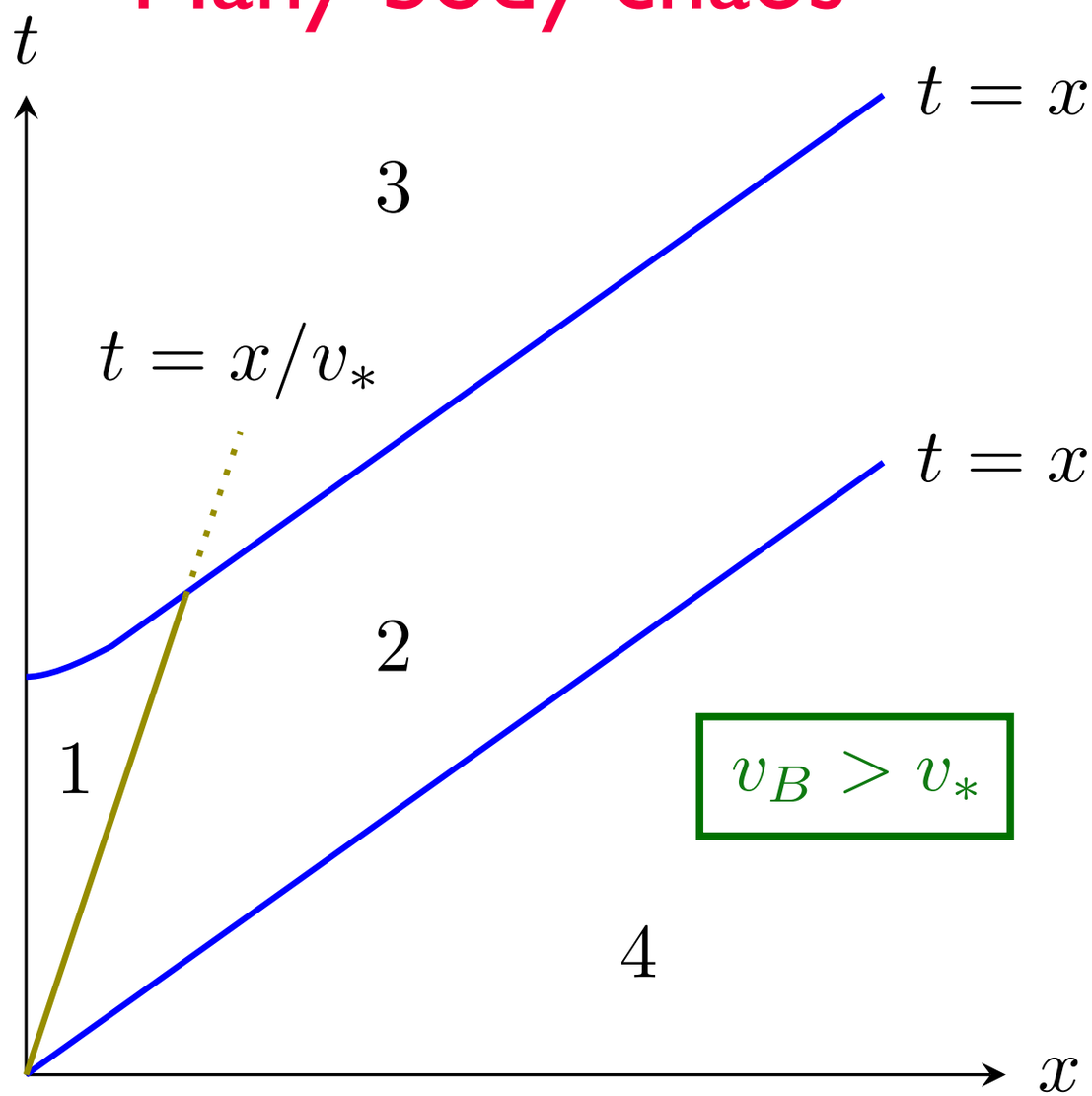
Transport and many-body chaos

3. Low temperature theory of charged black holes and complex SYK

Density of states

Many-body chaos

Yingfei Gu and A. Kitaev, arXiv:1812.00120



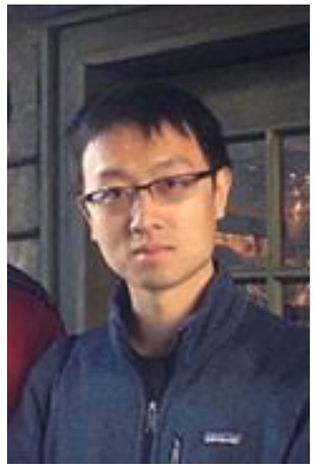
$$\text{OTOC} \sim \frac{1}{N}\frac{1}{\cos(\lambda_L(q)/(4T))}\frac{1}{[\omega - i\lambda_L(q)]}$$

This has a pole at imaginary q at $q_1 = i|q_1|$ where $\lambda_L(q_1) = 2\pi T$.

We can define two characteristic velocities

$$\begin{aligned} v_* &= |\lambda_L'(q_1)| \\ v_B &= 2\pi T/|q_1|\,, \end{aligned}$$

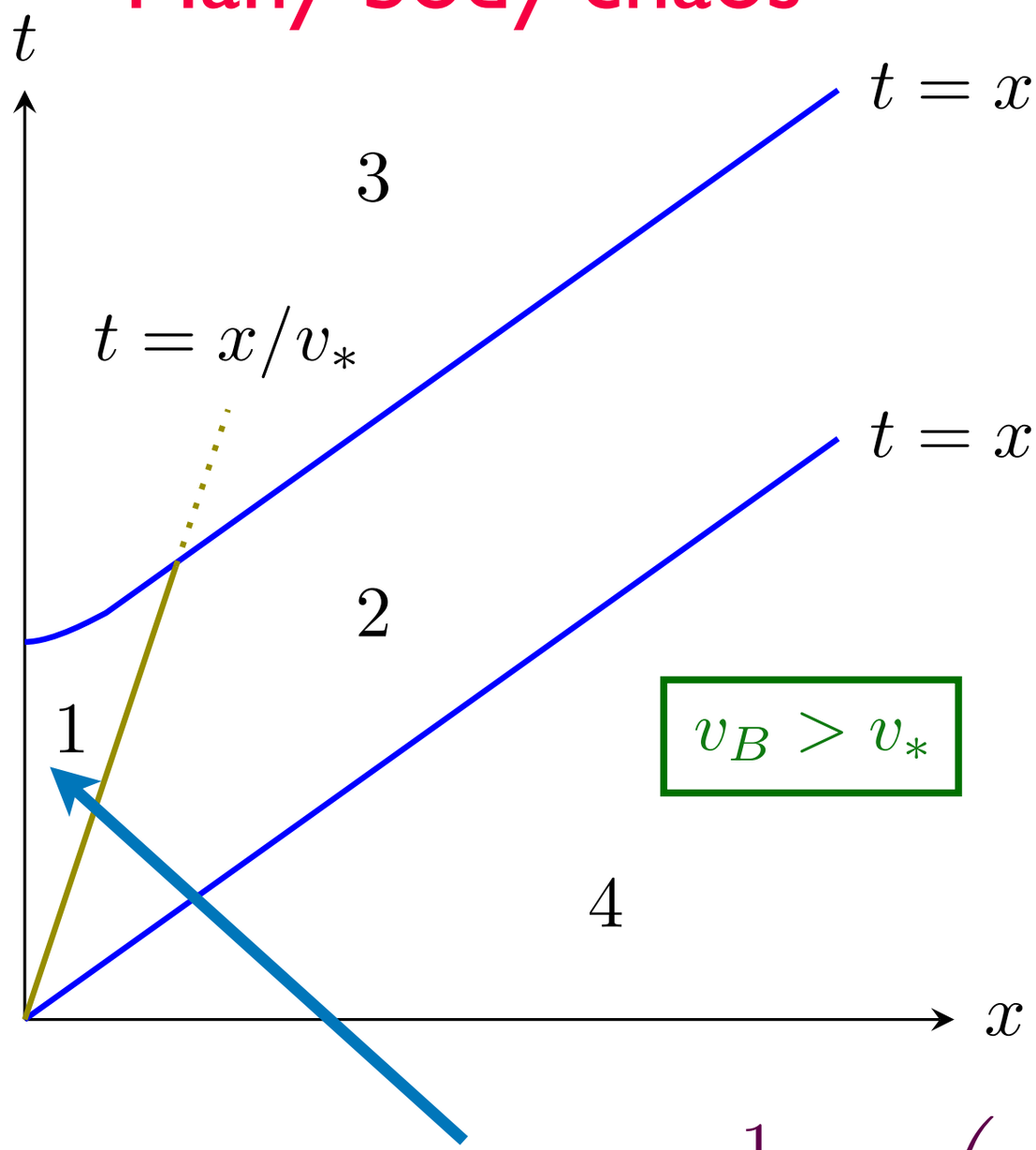
and the scrambling time $t_{\rm scr} \approx (\ln N)/\lambda_L$



Yingfei Gu

Many-body chaos

Yingfei Gu and A. Kitaev, arXiv:1812.00120



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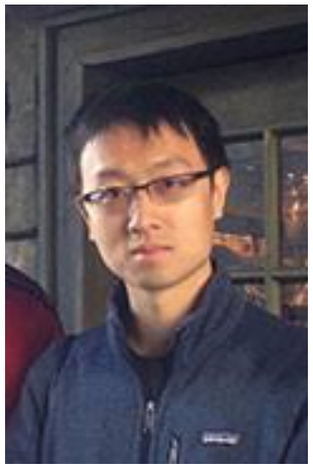
$$\text{OTOC} \sim \frac{1}{N}\exp\left(\lambda_L(0)t - \frac{x^2}{D_* t}\right) \quad \text{in region 1}$$

where the chaos diffusion co-efficient D_* is defined by

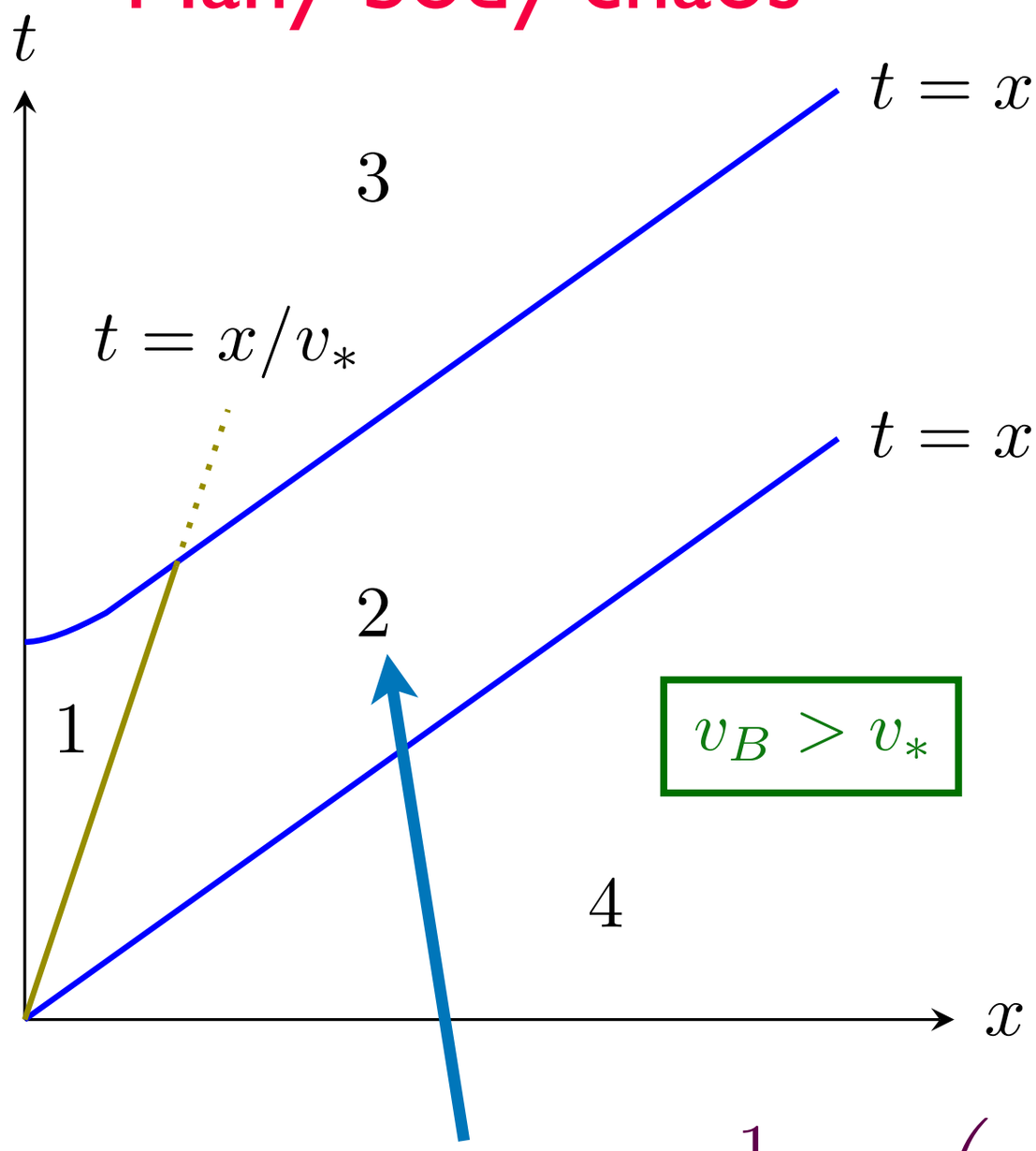
$$\lambda_L(q) = \lambda_L(0) - \frac{D_*}{4}q^2 + \ldots$$



Yingfei Gu

Many-body chaos

Yingfei Gu and A. Kitaev, arXiv:1812.00120



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We can define two characteristic velocities

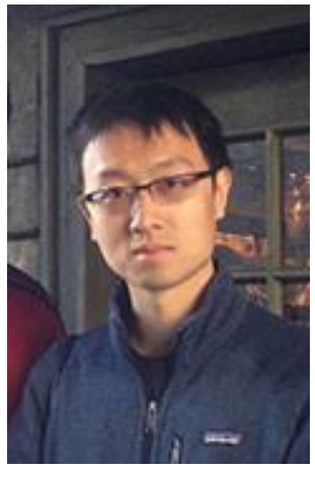
$$\begin{aligned} v_* &= |\lambda_L'(q_1)| \\ v_B &= 2\pi T/|q_1|\,, \end{aligned}$$

and the scrambling time $t_{\rm scr} \approx (\ln N)/\lambda_L$

$$\text{OTOC} \sim \frac{1}{N}\exp\left(2\pi T\left[t - \frac{|x|}{v_B}\right]\right) \quad \text{in region 2}$$

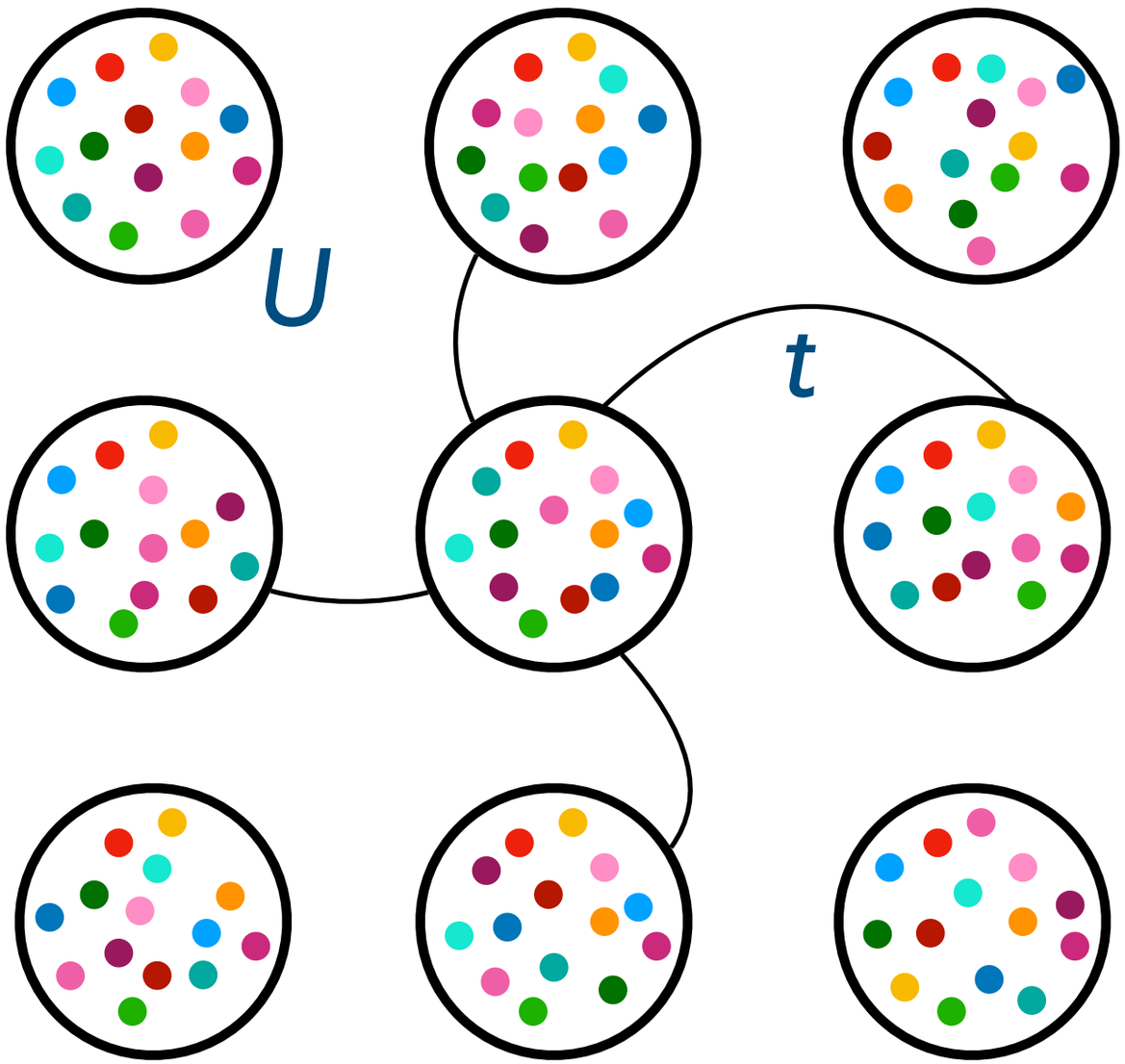
Now we define the chaos diffusion co-efficient D_1 by

$$D_1 = \frac{v_B^2}{2\pi T} = \frac{2\pi T}{|q_1|^2}$$



Yingfei Gu

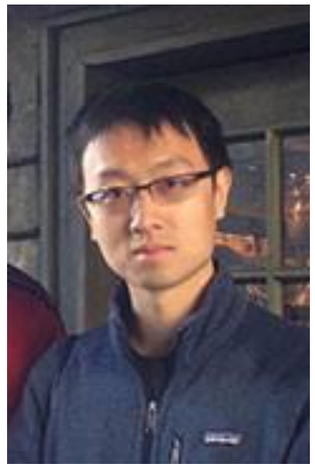
SYK+ phonons



coupled to N^2 local oscillators on each site.

Haoyu Guo, Yingfei Gu, S. Sachdev, arXiv:1904.02174

Haoyu Guo



Yingfei Gu

$$
\begin{aligned}
H &= H_{\text{hopping}} + H_{\text{SYK}} + H_{\text{ph}} + H_{\text{e-ph}}, \\
H_{\text{hopping}} &= \frac{1}{\sqrt{zN}} \sum_{<xx'>} \sum_{ab} t^{ab}_{xx'} c^\dagger_{ax} c_{bx'}, \\
H_{\text{SYK}} &= \frac{1}{(2N)^{3/2}} \sum_x U_{abcd,x} c^\dagger_{ax} c^\dagger_{bx} c_{cx} c_{dx}, \\
H_{\text{ph}} &= \frac{M}{2} \sum_x \sum_{ab} \left(|\partial_t X_{abx}|^2 + \omega_0^2 |X_{abx}|^2 \right), \\
H_{\text{e-ph}} &= -\frac{\alpha}{\sqrt{N}} \sum_x \sum_{ab} X_{abx} c^\dagger_{ax} c_{bx}.
\end{aligned}
$$

Yochai Werman, Steven A. Kivelson, and Erez Berg, arXiv:1705.07895

Lattice of SYK islands coupled to N^2 local Einstein phonons.

We study the limit $\omega_0 \to 0$ with fixed electron-phonon coupling $g = \alpha^2/(M\omega_0^2 t_0)$. Then all properties are characterized by the energy scale $E_c = t_0^2/U$ and the dimensionless parameter $g t_0/U$

Haoyu Guo, Yingfei Gu, S. Sachdev, arXiv:1904.02174

$E_c = t_0^2/U$

Heavy Fermi liquid

$$\rho = \rho_0 + \rho_1 \frac{T^2}{E_c^2} + \frac{\pi}{2}\frac{g t_0}{U}\frac{T}{E_c}$$

$$\frac{T}{\kappa} = a_0 + a_1 \frac{T^2}{E_c^2} + \frac{3}{2\pi}\frac{g t_0}{U}\frac{T}{E_c}$$

$$\lambda_L \sim \frac{T^2}{E_c}$$

$$v_* > v_B,\ D_E \sim D_* \approx D_{\text{chaos}}$$

Incoherent metal

$$\rho = \frac{2}{\sqrt{\pi}}\frac{T}{E_c} + \text{const}$$

$$\frac{T}{\kappa} = \frac{16}{\pi^{5/2}}\frac{T}{E_c} + \text{const}$$

$$\frac{\lambda_L}{T} = 2\pi - \mathcal{O}(g t_0/U)$$

$$v_* < v_B,\ D_E \approx D_{\text{chaos}}$$

$$g t_0/U \ll 1$$

T

t_0^2/U $\quad$ t_0/g $\quad$ U

Nearly-free electrons

$$\rho = \frac{\pi}{2}\frac{g t_0}{U}\frac{T}{E_c} + \text{const}$$

$$\kappa = \kappa_{\text{electron}} + \kappa_{\text{phonon}}$$

$$\frac{T}{\kappa_{\text{electron}}} = \frac{3}{2\pi}\frac{g t_0}{U}\frac{T}{E_c} + \text{constant}$$

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$$\kappa_{\text{phonon}} \gg \kappa_{\text{electron}}$$

Strong electron-phonon scattering

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T

t_0/g $\quad$ t_0^2/U $\quad$ U

Haoyu Guo, Yingfei Gu, S. Sachdev, arXiv:1904.02174

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$$g t_0/U \ll 1$$

t_0^2/U

Weak electron-phonon scattering
Well-defined quasipartices

U

T

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$$g t_0/U \gg 1$$

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t_0/g $\quad$ t_0^2/U $\quad$ U $\quad$ T

Haoyu Guo, Yingfei Gu, S. Sachdev, arXiv:1904.02174

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v_*

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$$\rho = \frac{2}{\sqrt{\pi}}\frac{T}{E_c} + \text{const}$$

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$$v_* < v_B,\ D_E \approx D_{\text{chaos}}$$

$g t_0/U \ll 1$

Weak electron-phonon scattering; Bad metal, no quasiparticles, and maximal chaos

T

$/U$ $\quad t_0/g \quad U$

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$t_0/g \quad t_0^2/U \quad U$

Haoyu Guo, Yingfei Gu, S. Sachdev, arXiv:1904.02174

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$$\lambda_L$$

$$g t_0/U \ll 1$$

Strong electron-phonon scattering; quasiparticles well defined

t_0^2/U $\quad U$ $\quad T$

Nearly-free electrons

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t_0/g $\quad t_0^2/U$ $\quad U$ $\quad T$

Haoyu Guo, Yingfei Gu, S. Sachdev, arXiv:1904.02174

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$$v_* < v_B,\ D_E \approx D_{\text{chaos}}$$

$gt_0/U \ll 1$

Strong electron-phonon scattering; short quasiparticle lifetime from near-elastic scattering; little chaos

$$\rho = \frac{\pi}{2}\frac{g t_0}{U}\frac{T}{E_c} + \text{const}$$

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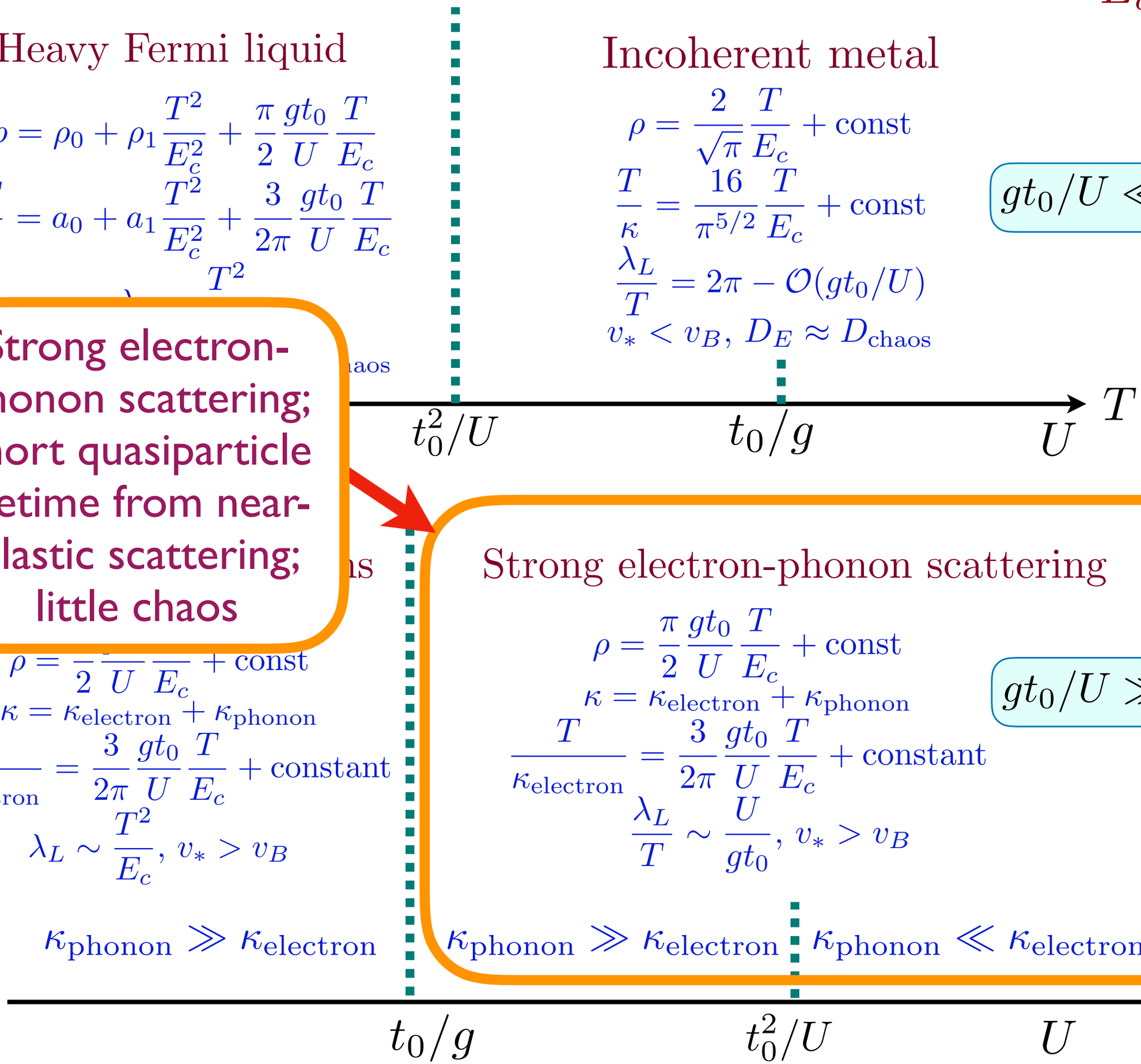

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$$\frac{T}{\kappa_{\text{electron}}} = \frac{3}{2\pi}\frac{g t_0}{U}\frac{T}{E_c} + \text{constant}$$

$$\frac{\lambda_L}{T} \sim \frac{U}{g t_0},\ v_* > v_B$$

$gt_0/U \gg 1$

$\kappa_{\text{phonon}} \gg \kappa_{\text{electron}}$ | $\kappa_{\text{phonon}} \ll \kappa_{\text{electron}}$

Haoyu Guo, Yingfei Gu, S. Sachdev, arXiv:1904.02174

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Incoherent metal

$$\rho = \frac{2}{\sqrt{\pi}}\frac{T}{E_c} + \text{const}$$

$$\frac{T}{\kappa} = \frac{16}{\pi^{5/2}}\frac{T}{E_c} + \text{const}$$

$$\frac{\lambda_L}{T} = 2\pi - \mathcal{O}(g t_0/U)$$

$$v_* < v_B,\ D_E \approx D_{\text{chaos}}$$

$$g t_0/U \ll 1$$

T

t_0^2/U $\quad$ t_0/g $\quad$ U

Nearly-free electrons

$$\rho = \frac{\pi}{2}\frac{g t_0}{U}\frac{T}{E_c} + \text{const}$$

$$\kappa = \kappa_{\text{electron}} + \kappa_{\text{phonon}}$$

$$\frac{T}{\kappa_{\text{electron}}} = \frac{3}{2\pi}\frac{g t_0}{U}\frac{T}{E_c} + \text{constant}$$

$$\lambda_L \sim \frac{T^2}{E_c},\ v_* > v_B$$

$$\kappa_{\text{phonon}} \gg \kappa_{\text{electron}}$$

Strong electron-phonon scattering

$$\rho = \frac{\pi}{2}\frac{g t_0}{U}\frac{T}{E_c} + \text{const}$$

$$\kappa = \kappa_{\text{electron}} + \kappa_{\text{phonon}}$$

$$\frac{T}{\kappa_{\text{electron}}} = \frac{3}{2\pi}\frac{g t_0}{U}\frac{T}{E_c} + \text{constant}$$

$$\frac{\lambda_L}{T} \sim \frac{U}{g t_0},\ v_* > v_B$$

$$g t_0/U \gg 1$$

$\kappa_{\text{phonon}} \gg \kappa_{\text{electron}}$ $\quad$ $\kappa_{\text{phonon}} \ll \kappa_{\text{electron}}$

T

t_0/g $\quad$ t_0^2/U $\quad$ U

Haoyu Guo, Yingfei Gu, S. Sachdev, arXiv:1904.02174

Phonons can act as a heat bath which leads to large near-elastic scattering of electrons at higher temperatures (some similarity to "random unitary" models"). Such scattering leads to large linear-in-T resistivity. However it does not lead to strong chaos, and the Lyapunov rate is well below the chaos bound.

On the other hand, strong electron-electron scattering does lead to near-maximal chaos, propagating with a well-defined butterfly velocity, along with a linear-in-T resistivity

$$\frac{T}{\kappa_{\text{electron}}} = \frac{3}{2\pi}\frac{g t_0}{U}\frac{T}{E_c} + \text{constant}$$

$$\lambda_L \sim \frac{T^2}{E_c}, \; v_* > v_B$$

$$\kappa_{\text{phonon}} \gg \kappa_{\text{electron}}$$

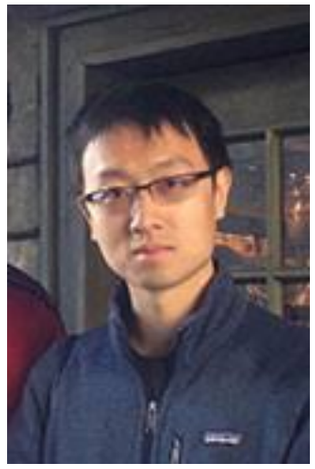
$$\frac{T}{\kappa_{\text{electron}}} = \frac{3}{2\pi}\frac{g}{l}$$

$$\frac{\lambda_L}{T} \sim \frac{l}{g}$$

$$\kappa_{\text{phonon}} \gg \kappa_{\text{electr}}$$

t_0/g

Haoyu Guo



Yingfei Gu

1. ‘Resonant’ SYK model for Planckian transport

Linear-in-T resistivity and a remnant Fermi surface

2. SYK + phonons

Transport and many-body chaos

3. Low temperature theory of charged black holes and complex SYK

Density of states

Quantum Black holes

- Black holes have an entropy and a temperature, T_H
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)

C. V. Vishveshwara, Nature **227**, 936 (1970)

Quantum Black holes

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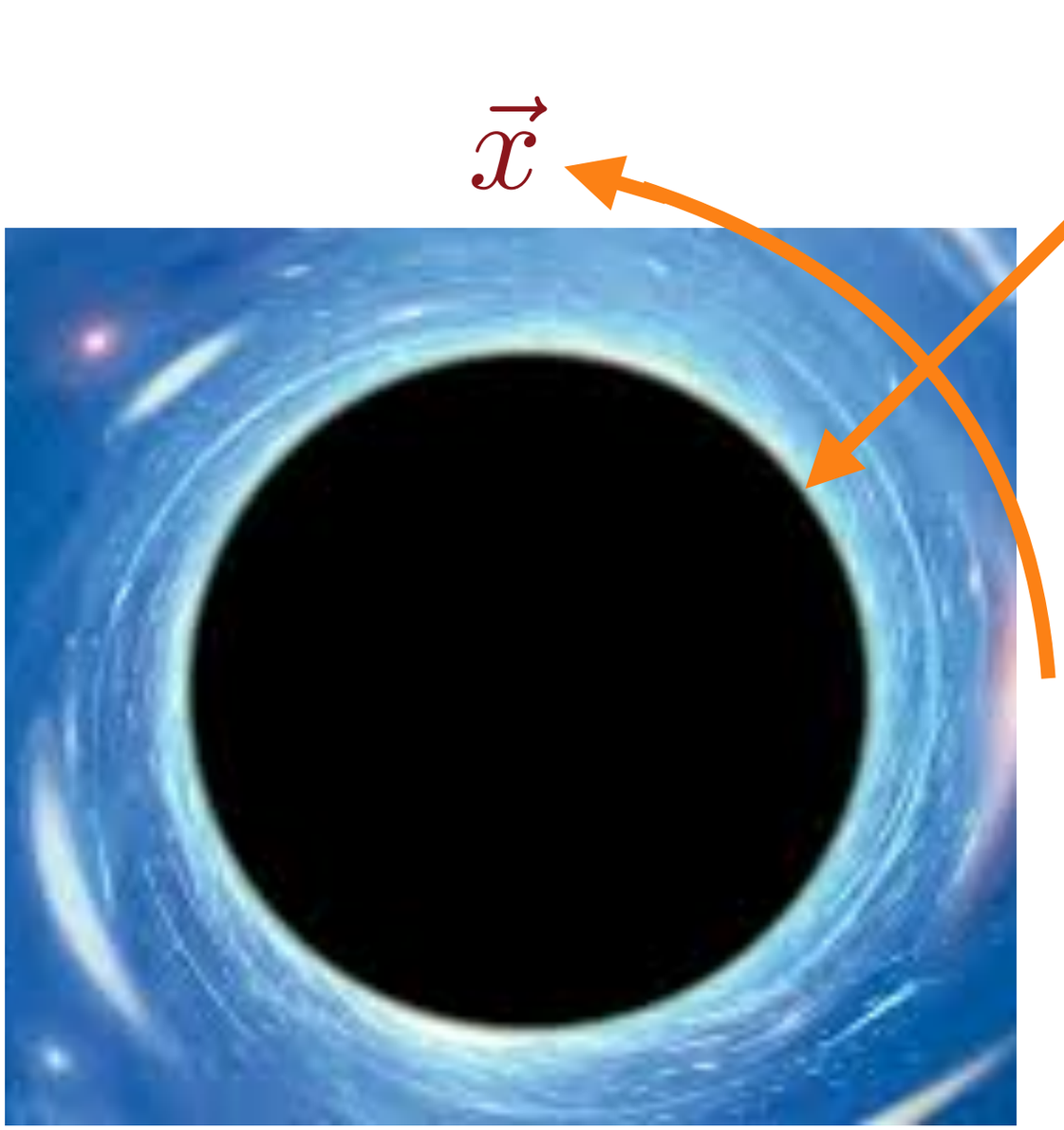
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

<u>Holography</u>:
Quantum black holes "look like"
quantum many-particle systems
without quasiparticle excitations,
residing "on" the surface of the black hole

J. Maldacena, IJTP **38**, 1113 (1999); S.S. Gubser, I.R. Klebanov, and A.M. Polyakov Phys. Lett. B **428**, 105 (1998);
E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998)

Work with a theory of Maxwell's electromagnetism and Einstein's general relativity. Include a negative cosmological constant, and examine black hole solutions with a net charge

Work with a theory of Maxwell's electromagnetism and Einstein's general relativity. Include a negative cosmological constant, and examine black hole solutions with a net charge



Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space (ζ) and one time dimension

SYK model and charged black holes

Black hole horizon

$$\mathrm{AdS}_2 \times S^2$$

$$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

charge density $\mathcal{Q}$

S^2

$\zeta = \infty$

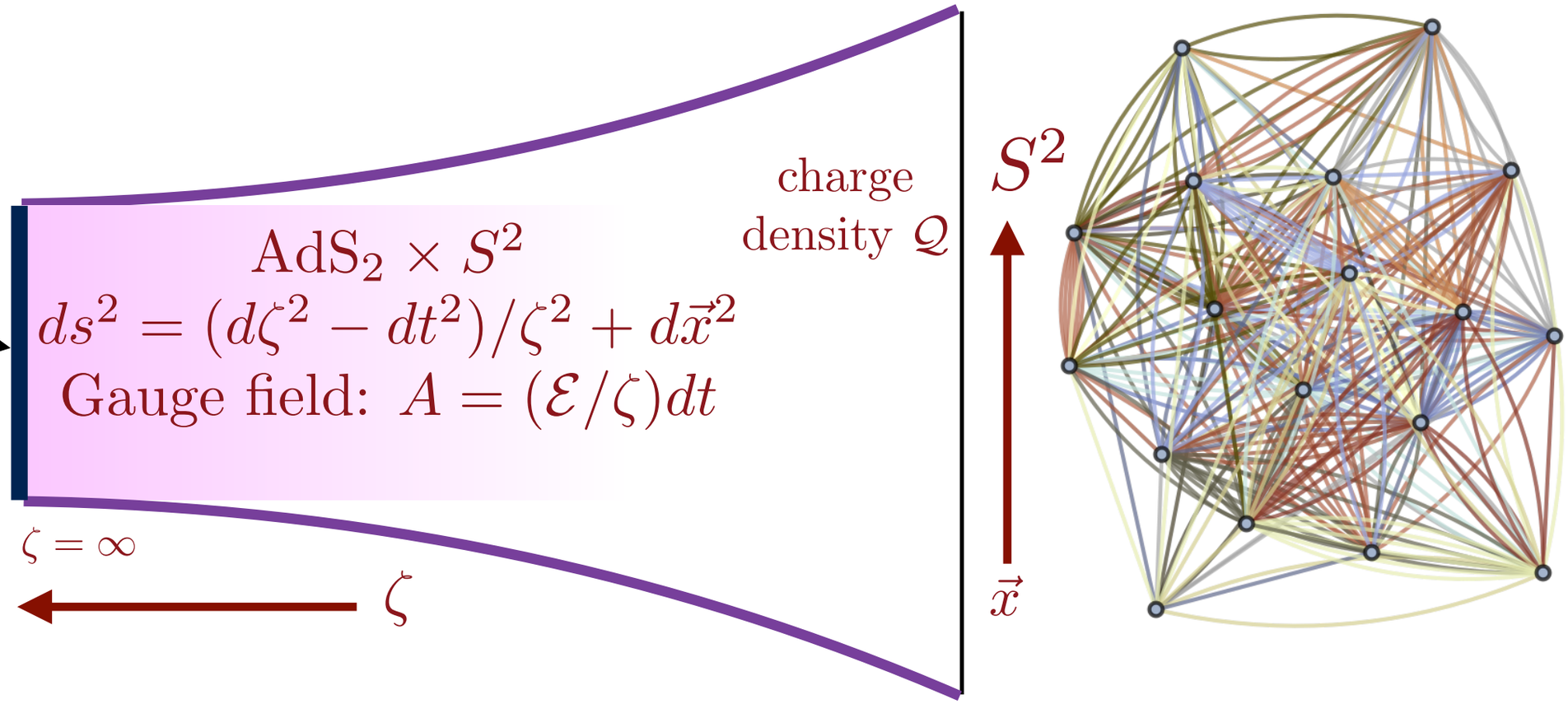
ζ

$\vec{x}$

The near-horizon region of a charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model

SYK model and charged black holes

Black hole horizon



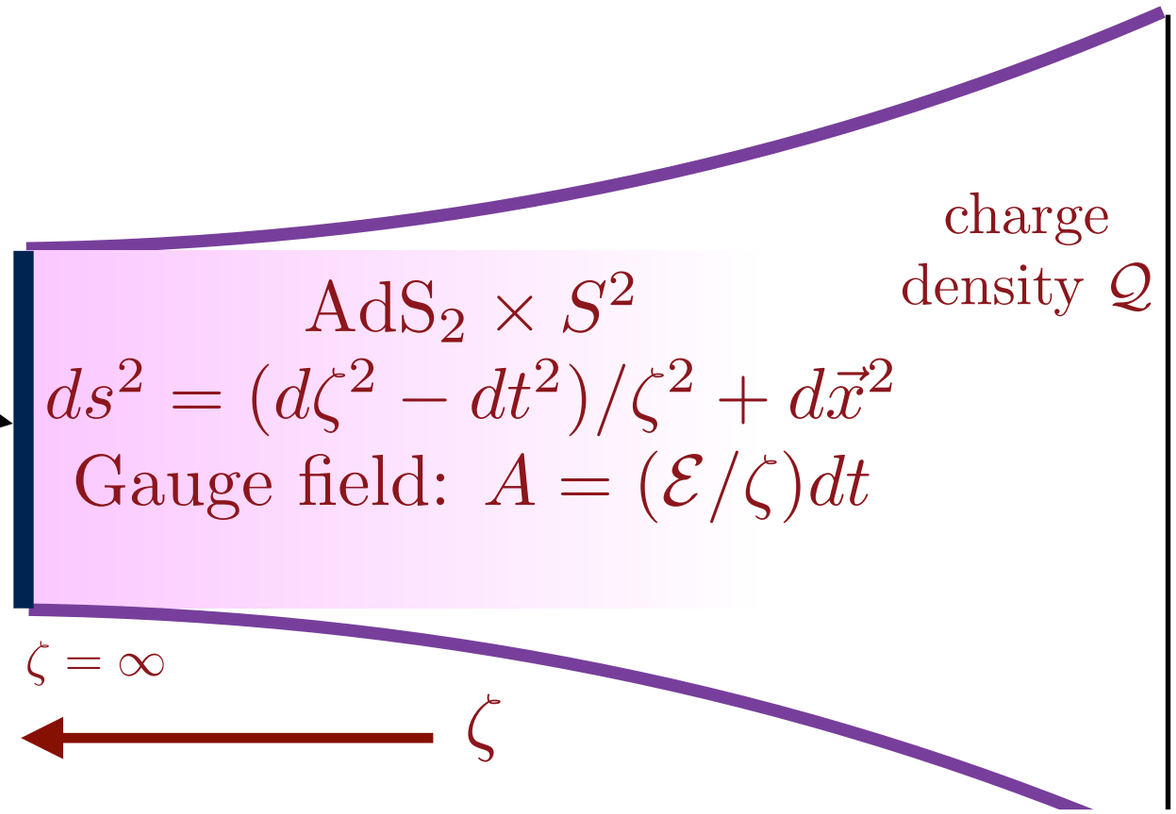
Bekenstein-Hawking entropy of AdS_2 horizon at $T = 0 \Leftrightarrow N s_0$ entropy of SYK model.

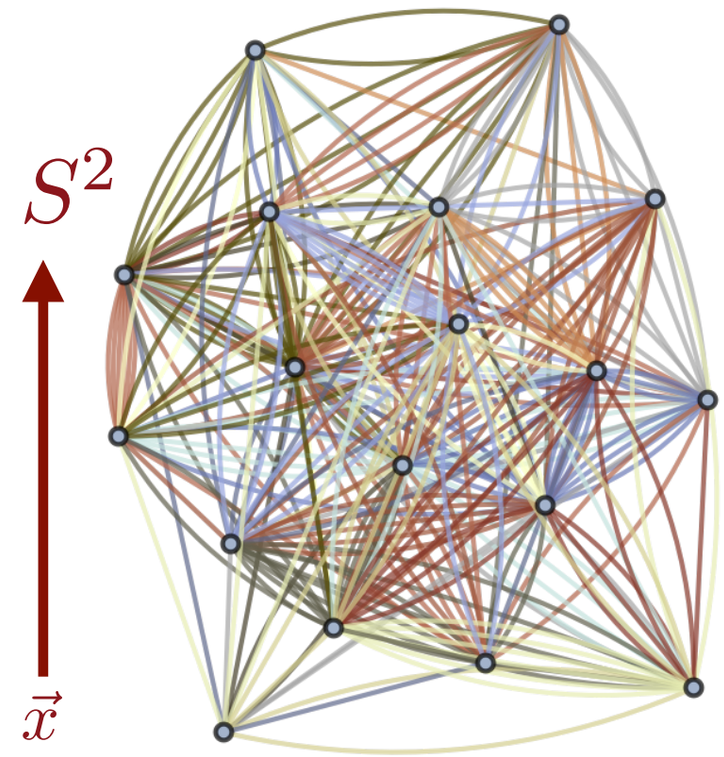
$\dfrac{\partial s_0}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$ holds for both the black hole and the SYK model, where $\mathcal{E}$ determines identical fermion spectral functions.

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010); PRX **5,** 041025 (2015)

SYK model and charged black holes

Black hole horizon





PHYSICAL REVIEW LETTERS **105**, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

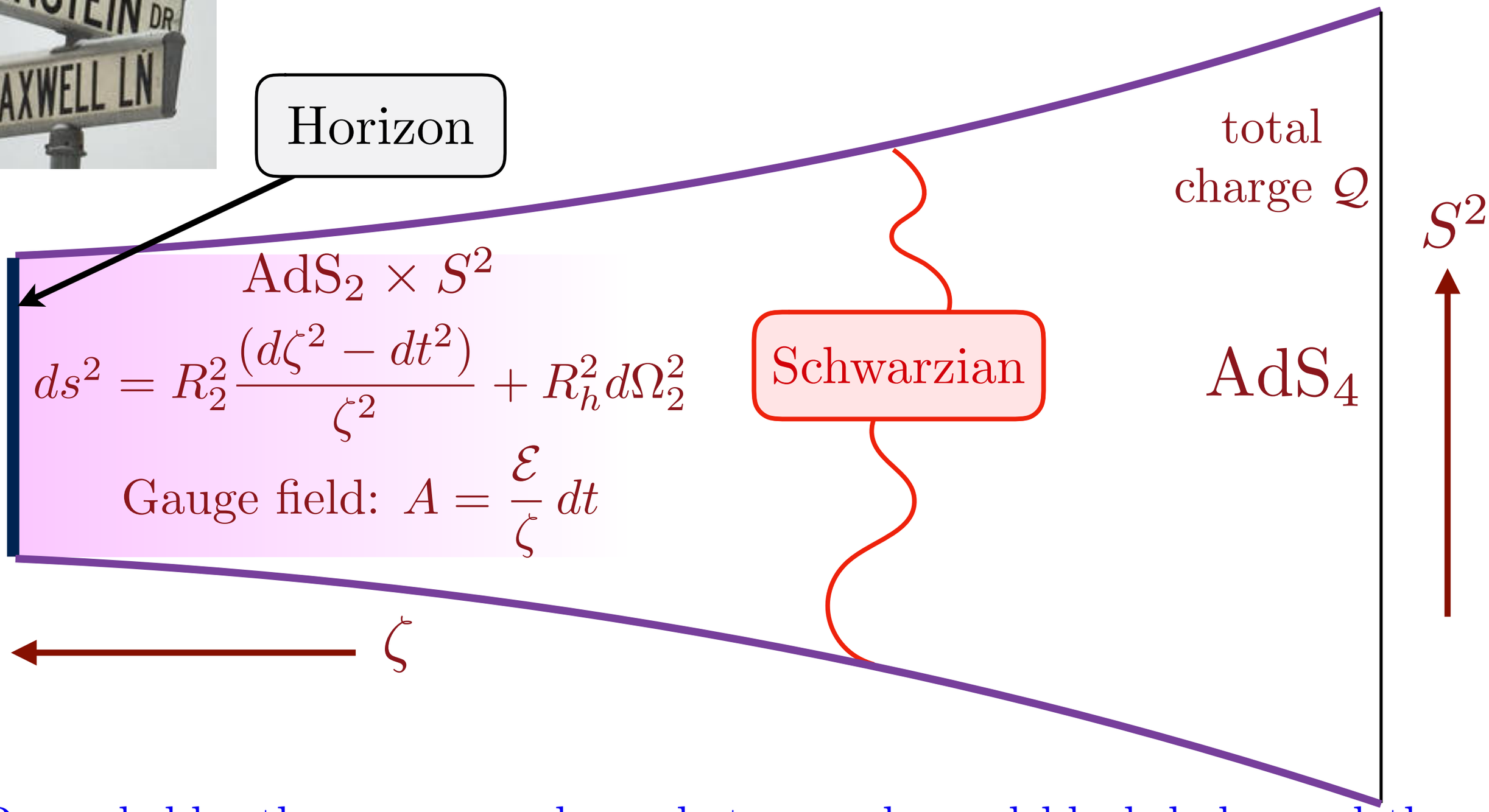
Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $\mathrm{AdS}_2 \times \mathrm{R}^2$ physics of Reissner-Nordström black holes.

SYK model and charged black holes



Remarkably, the correspondence between charged black holes and the SYK model also holds for the leading fluctuations at higher temperatures: both are described by a 'Schwarzian' theory with emergent SL(2,R) and U(1) gauge symmetries. For the black hole, the Schwarzian describes the fluctuations of the boundary between AdS_2 and AdS_4.

Main result

S. Sachdev, Phys. Rev. Lett. **105**, 151602 (2010)

A. Kitaev (2015)

S. Sachdev, Phys. Rev. X **5**, 041025 (2015)

J. Maldacena and D. Stanford, Phys. Rev. D **94,** 106002 (2016)

J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)

K. Jensen, Phys. Rev. Lett. **117**, 111601 (2016)

J. Engelsoy, T.G. Mertens, and H. Verlinde, JHEP 1607 (2016) 139

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev,
Phys. Rev. B **95**, 155131 (2017)

A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746 P. Nayak, A. Shukla,

R.M. Soni, S.P. Trivedi, and V. Vishal, arXiv:1802.09547

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062

U. Moitra, S. P. Trivedi, and V. Vishal, arXiv:1808.08239

S. Sachdev, arXiv:1902.04078

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number $\mathcal{Q}$, at temperatures $T \ll U$

Main result

SYK model of fermions with random interactions of mean-square-value U, with total fermion number $\mathcal{Q}$, at temperatures $T \ll U$

and

Charged black holes in 3+1 dimensions of radius R_h, with total charge $\mathcal{Q}$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in $0+1$ dimensions

Main result

The common low T path integral is $\mathcal{Z} = \int \mathcal{D}f\, \mathcal{D}\phi\, e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left(\frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E} T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \mathrm{Sch}[\tan(\pi T f(\tau)), \tau] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with

$$f(\tau + 1/T) = f(\tau) + 1/T,$$

ϕ is a phase conjugate to the charge density with

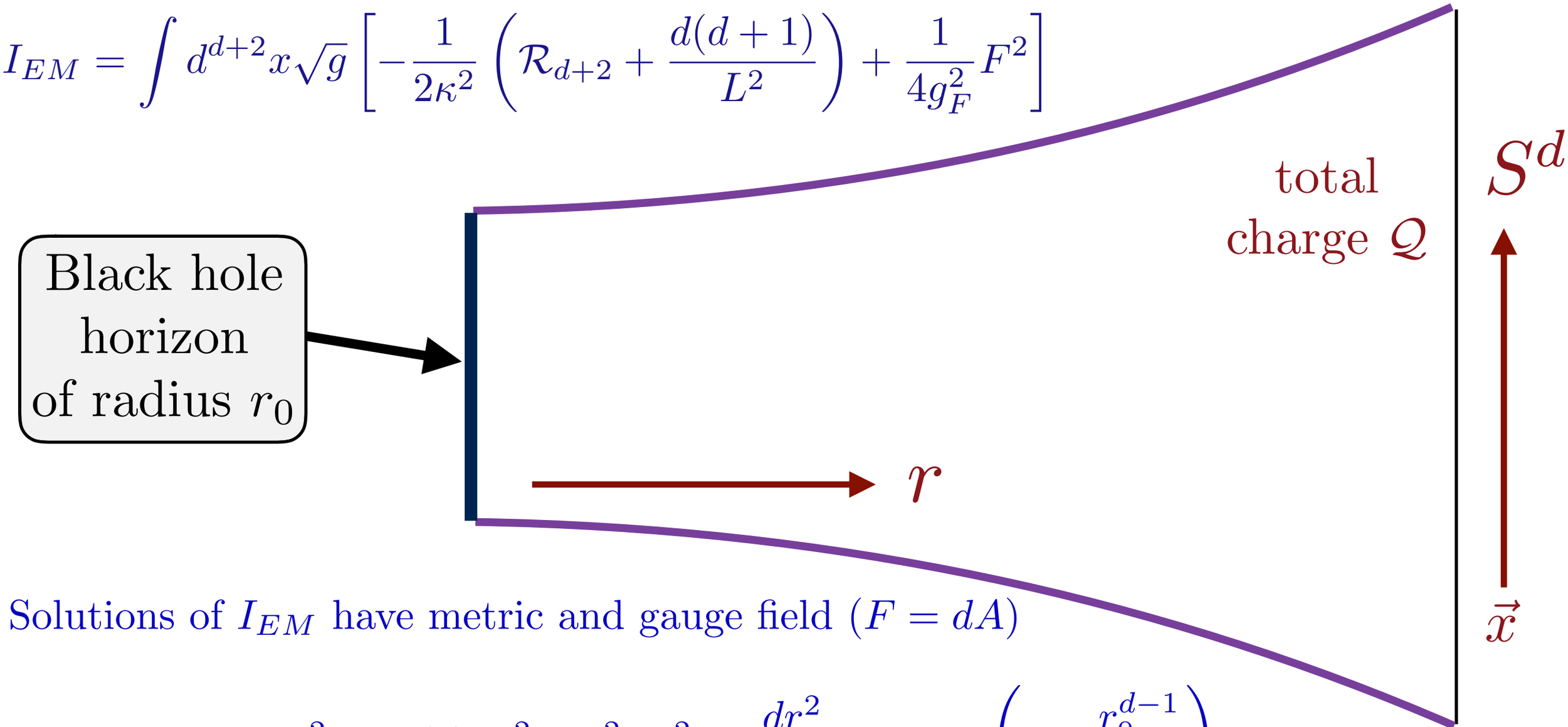
$$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n,\ n \text{ integer},$$

$\mathrm{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$.
The couplings are related to the entropy $S(T, \mathcal{Q})$ and the chemical potential μ via

$$S(T \to 0, \mathcal{Q}) = s_0 + \gamma T, \quad K = \left(\frac{d\mathcal{Q}}{d\mu} \right)_{T \to 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{d\mathcal{Q}}$$

Charged black holes

$$I_{EM} = \int d^{d+2}x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2}\right) + \frac{1}{4g_F^2}F^2\right]$$



Solutions of I_{EM} have metric and gauge field ($F = dA$)

$$ds^2 = V(r)d\tau^2 + r^2 d\Omega_d^2 + \frac{dr^2}{V(r)} \quad , \quad i\mu\left(1 - \frac{r_0^{d-1}}{r^{d-1}}\right)d\tau$$

$$V(r) = 1 + \frac{r^2}{L^2} + \frac{\Theta^2}{r^{2d-2}} - \frac{M}{r^{d-1}}.$$

where $d\Omega_d^2$ is the metric of the d-sphere. All parameters of the solution are determined in terms of the chemical potential μ, and the Hawking temperature of horizon, T.

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

Charged black holes

We write the $(d+2)$-dimensional metric g of I_{EM} in terms of a two-dimensional metric h and a scalar field Φ:

$$ds^2 = \frac{ds_2^2}{\Phi^{d-1}} + \Phi^2 \, d\Omega_d^2 \, .$$

The Einstein-Maxwell and Gibbons-Hawking actions reduce to and extension of Jackiw-Tietelbaum gravity $(x \equiv (\tau, \zeta))$

$$\begin{aligned} I_{EM} &= \int d^2x \sqrt{h} \left[-\frac{s_d}{2\kappa^2} \, \Phi^d \, \mathcal{R}_2 + U(\Phi) + \frac{Z(\Phi)}{4g_F^2} F^2 \right] \\ I_{GH} &= -\frac{s_d}{\kappa^2} \int_\partial dx \sqrt{h_b} \Phi^d \, \mathcal{K}_1 \end{aligned}$$

The explicit forms of the potentials $U(\Phi)$ and $Z(\Phi)$ are,

$$U(\Phi) = -\frac{s_d}{2\kappa^2} \left(\frac{d(d-1)}{\Phi} + \frac{d(d+1)\Phi}{L^2} \right) \quad , \quad Z(\Phi) = s_d \Phi^{2d-1} \, .$$

S. Sachdev, arXiv:1902.04078

Charged black holes

The exact saddle point of Φ relates to R_h the horizon radius at $T = 0$

$$\Phi(\zeta) = R_h + \frac{R_2^2}{\zeta} \quad , \quad R_h \equiv \frac{L}{g_F} \left[\frac{(d-1)(\mu_0^2 \kappa^2 (d-1) - d g_F^2)}{d(d+1)} \right]^{1/2} ,$$

while the near-horizon, low $T \ll 1/R_h$ metric is AdS_2

$$ds_2^2 = \frac{R_2^2 R_h^{d-1}}{\zeta^2} \left[(1 - 4\pi^2 T^2 \zeta^2) d\tau^2 + \frac{d\zeta^2}{1 - 4\pi^2 T^2 \zeta^2} \right] ,$$

where

$$R_2 = \frac{L R_h}{\sqrt{d(d+1) R_h^2 + (d-1)^2 L^2}}$$

The field coupling to $\mathcal{R}_2$ is Φ^d

$$[\Phi(\zeta)]^d = R_h^d + \frac{\Phi_1}{\zeta} + \ldots \quad , \quad \Phi_1 = d R_h^{d-1} R_2^2,$$

S. Sachdev, arXiv:1902.04078

Charged black holes

The field coupling to $\mathcal{R}_2$ is Φ^d

$$[\Phi(\zeta)]^d = R_h^d + \frac{\Phi_1}{\zeta} + \ldots \quad , \quad \Phi_1 = d R_h^{d-1} R_2^2,$$

We choose the boundary of the AdS_2 region at bulk co-ordinates $(f(\tau), \zeta(\tau))$ with the induced boundary metric fixed at $(R_2^2 R_h^{d-1}/\zeta_b^2) d\tau^2$ by choosing

$$\zeta(\tau) = \zeta_b f'(\tau) + \zeta_b^3 \left(\frac{[f''(\tau)]^2}{2 f'(\tau)} - 2\pi^2 T^2 [f'(\tau)]^3 \right) + \ldots .$$

Finally, we evaluate I_{GH} along this boundary curve

$$I_1[f] = -\frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \, \{\tan(\pi T f(\tau)), \tau\} ,$$

where

$$\gamma = \frac{4\pi^2 s_d \Phi_1}{\kappa^2} ,$$

matches the linear-in-T co-efficient of the specific heat of the full Reissner-Nördstorm solution in $d+2$ dimensions.

S. Sachdev, arXiv:1902.04078

Fluctuations

An *exact* path integral over the effective action leads to the following physical consequences

- The ground state energy with fermion number $N\mathcal{Q} + p$ (p integer) varies as

$$E_p = E_0 + \frac{p^2}{2NK}$$

  This identifies K with the compressibility $K = d\mathcal{Q}/d\mu$ at $T = 0$.

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, to appear

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- The low temperature corrections to the entropy at *fixed* $\mathcal{Q}$ are

$$S(T \to 0, \mathcal{Q}) = N\Big[s_0 + \gamma T + \ldots\Big] + 2\ln(U/T)\ldots$$

  This defines γ as the co-efficient of the linear-in-T specific heat (at fixed $\mathcal{Q}$)

D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S. Josephine Suh, arXiv:1711.08467

Fluctuations

An *exact* path integral over the effective action leads to the following physical consequences

- The *many*-body density of states, $D(E)$, is related to the grand potential, $\Omega(T)$ by

$$Z = e^{-\Omega(T)/T} = \int_{-\infty}^{\infty} dE D(E) e^{-E/T}$$

We obtain

$$D(E) = \sum_{p=-\infty}^{\infty} e^{2\pi p \mathcal{E}} \, d\left(E - E_p\right)$$

where $N\mathcal{Q} + p$ is the integer fermion number,

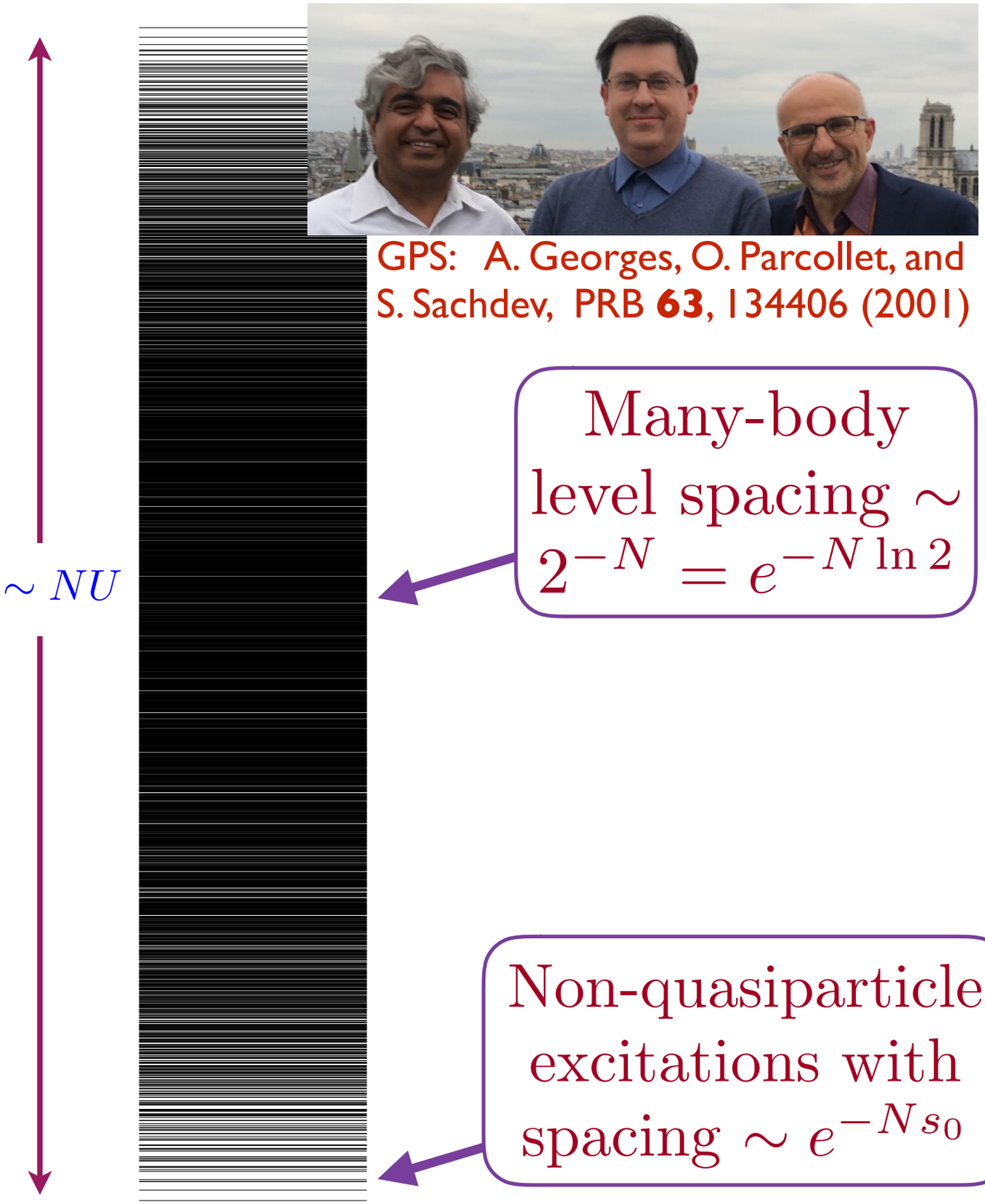
$$d(E) \sim \exp\left(N s_0\right) \sinh\left(\sqrt{2N\gamma E}\right) \quad , \quad E > 0 \ , \ e^{-cN} \ll \gamma E \ll N$$

There are exponentially more low energy states than for the quasiparticle case, and $D(E)$ self-averages down to energies exponentially small in N.

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, to appear

J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, arXiv:1611.04650; A.M. Garcia-Garcia and J.J.M. Verbaarschot, arXiv:1701.06593; D. Bagrets, A. Altland, and A. Kamenev, arXiv:1702.08902; D. Stanford and E. Witten, arXiv:1703.04612; A. Kitaev and S.J. Suh, arXiv:1711.08467

The SYK model



GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

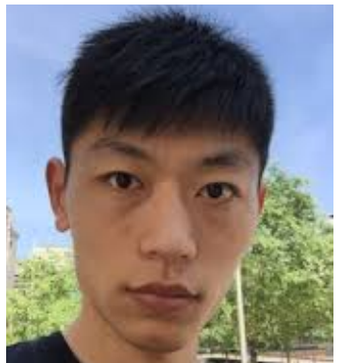
There are 2^N many body levels with energy E. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{dQ} = 2\pi \mathcal{E} \, .$$

At $\mathcal{Q} = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where G is Catalan's constant.



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Fluctuations

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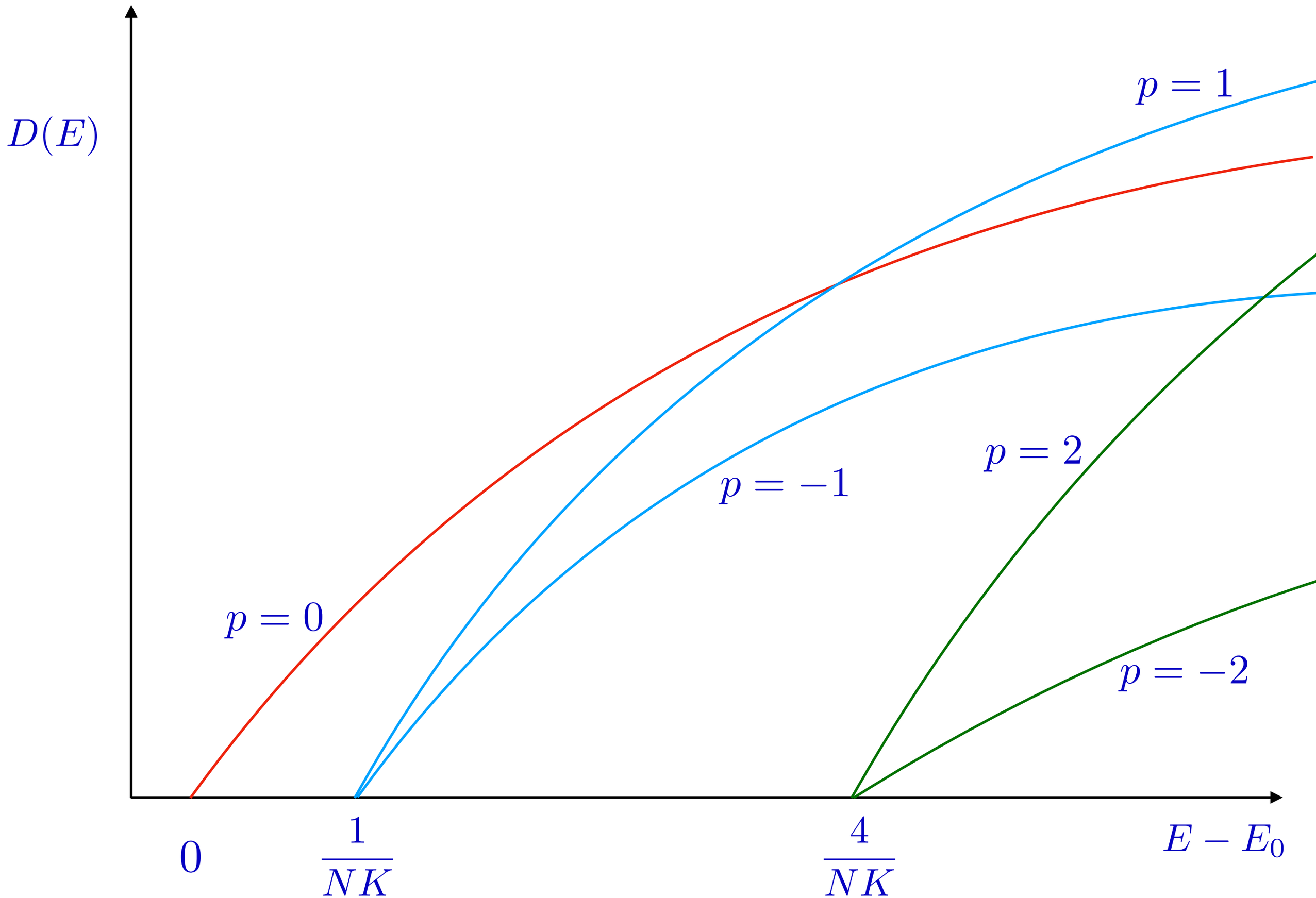
- At charge $N\mathcal{Q}+p$, the prefactor of the $\sinh(\sqrt{2N\gamma(E-E_p)})$ term is

$$\exp\left[Ns_0(\mathcal{Q})+2\pi p\mathcal{E}\right]\approx\exp\left[Ns_0(\mathcal{Q}+p/N)\right]$$

using

$$\boxed{\frac{ds_0}{d\mathcal{Q}}=2\pi\mathcal{E}}$$

Yingfei Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, to appear


$D(E)$
$p = 1$
$p = 0$
$p = -1$
$p = 2$
$p = -2$
0
$\frac{1}{NK}$
$\frac{4}{NK}$
$E - E_0$

SYK with $\mathcal{N} = 2$ SUSY

$$Q = i \sum_{1 \le i < j < k \le N} C_{ijk} c_i c_j c_k$$

$$H = QQ^\dagger + Q^\dagger Q = \sum_{i,j,k,\ell}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell$$

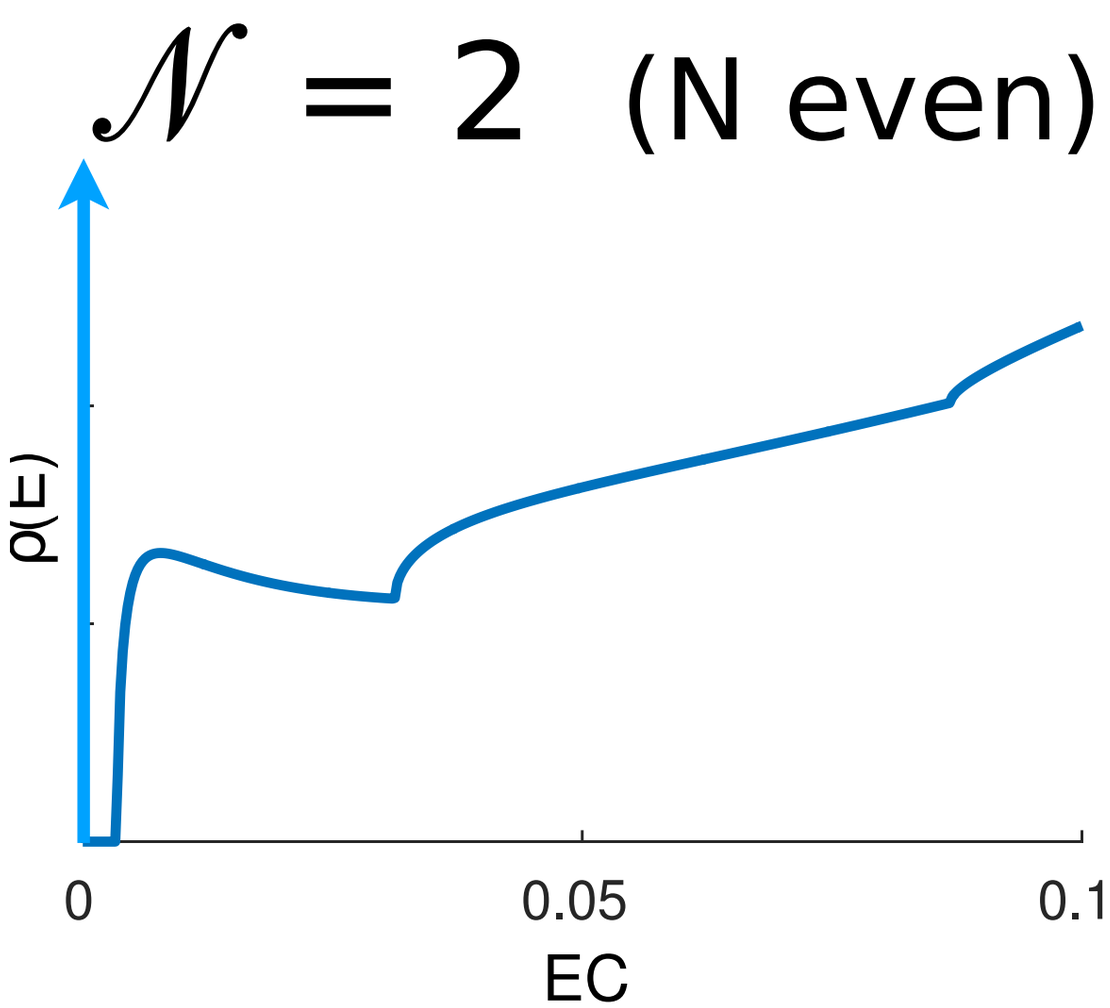
C_{ijk} are independent random variables with $\overline{C_{ijk}} = 0$ and $\overline{|C_{ijk}|^2} = 2U/N^2$.

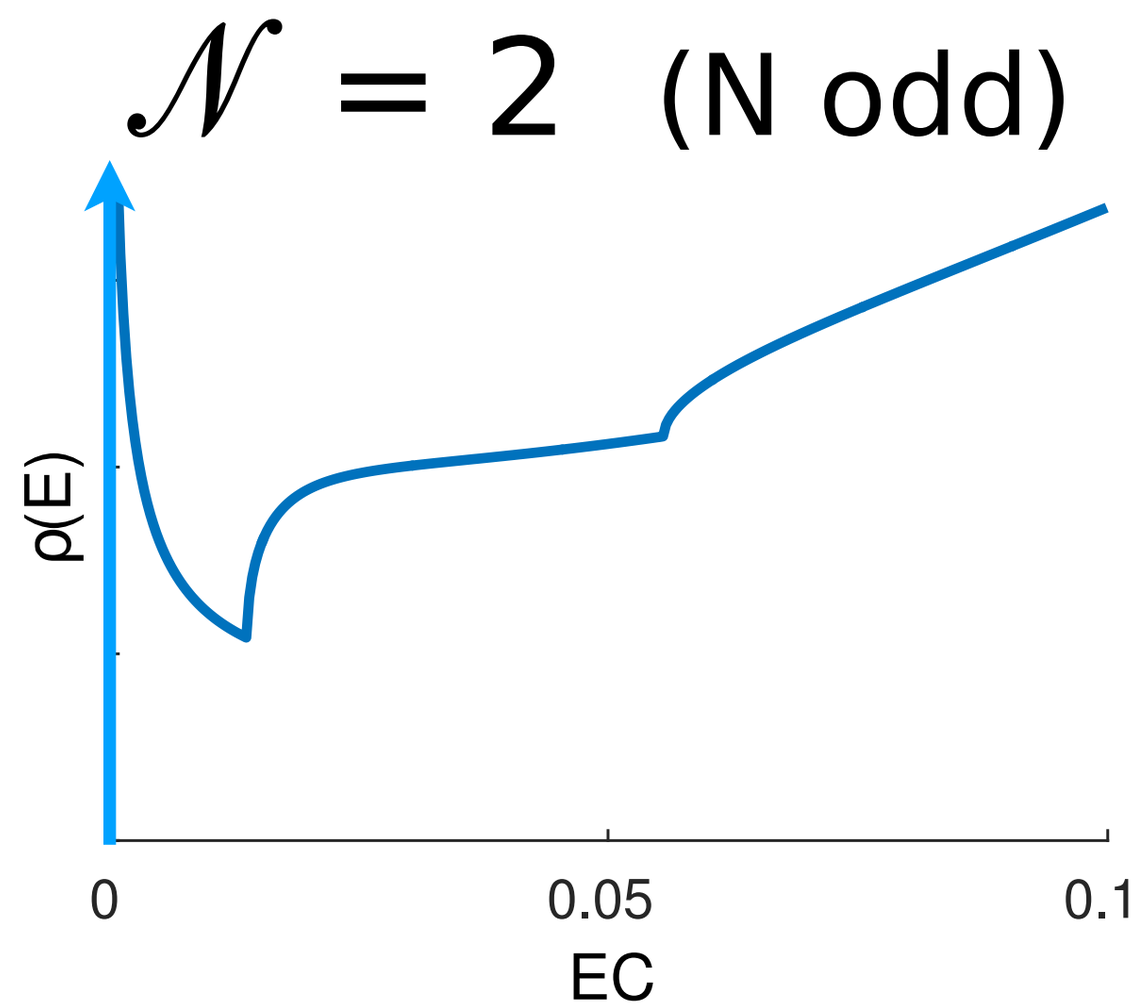
Wenbo Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)

SYK with $\mathcal{N} = 2$ SUSY

- Fluctuations are described by a super-Schwarzian theory for $\lim_{N\to\infty} \mathcal{Q}_R/N = 0$. The density of states has delta function at zero energy, and a continuum contribution at non-zero energies, both of order $e^{s_0 N}$. There is a gap between the ground states and the excited states $\sim 1/N$ for even N.

D. Stanford and E. Witten, arXiv:1703.04612





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D. Stanford and E. Witten, arXiv:1703.04612

- Similar results are expected to $\mathcal{N} = 2$ black holes with AdS_2 horizons obtained from string theory (Sen, Dabholkar, Gomes, Murthy ...).

- For $\lim_{N\to\infty} \mathcal{Q}_R/N \neq 0$, we expect that the density of states of the generic complex SYK model will apply.

Charged black holes and complex SYK

- A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, describes the quantum dynamics of
  - the SYK models
  - black holes with near-extremal AdS_2 horizons
- The path-integral over this action yields a low energy density of states at integer charge $\mathcal{Q}$ of the form $\sim \exp(NS_0(\mathcal{Q}))f(N(E-E_Q))$, where f is some smooth function.
- The $T \to 0$ black hole entropy is *not* realized by a ground state degeneracy. This is in contrast to all supersymmetric and string-theoretic computations of black hole entropy.