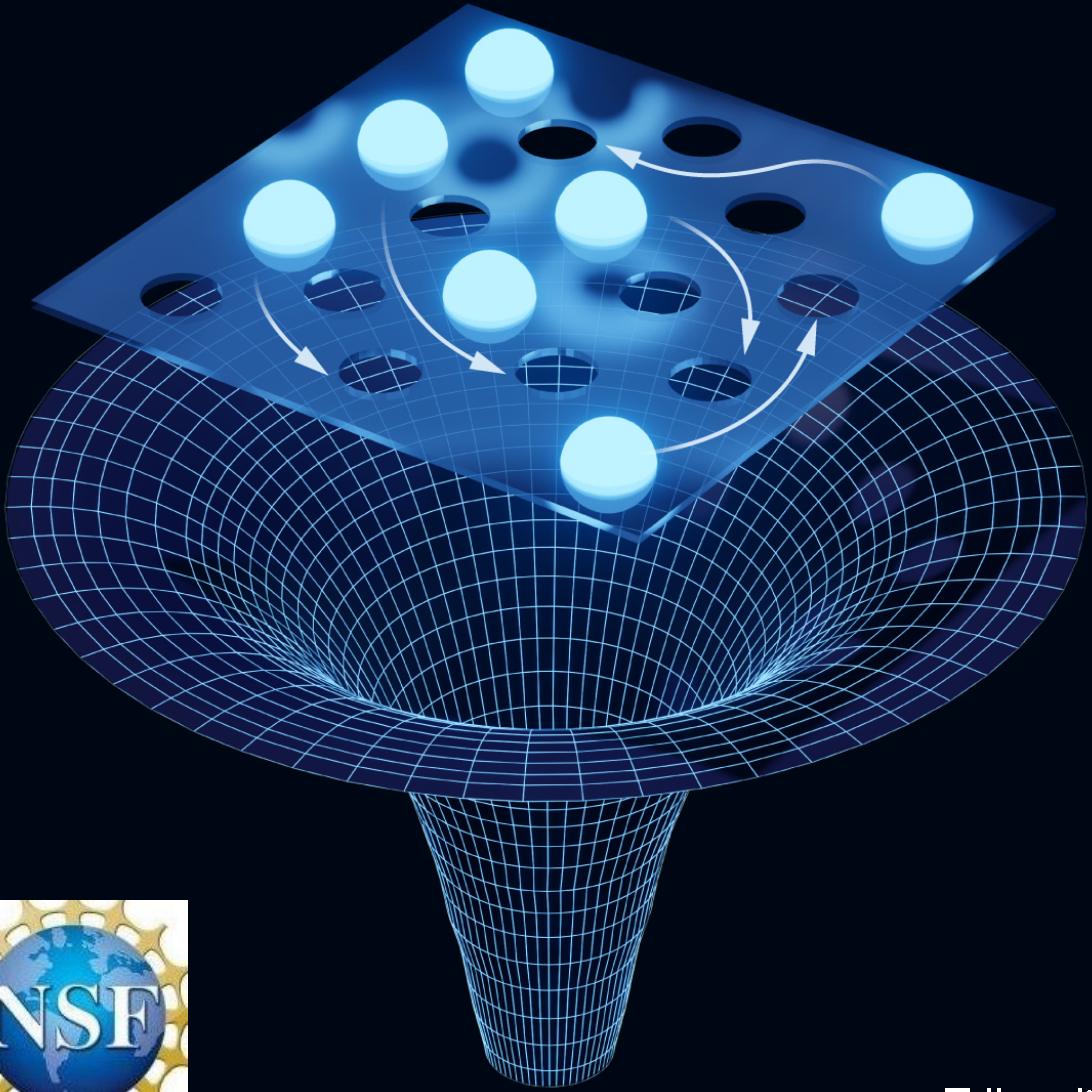


Quantum statistical mechanics of strange metals and black holes

12th Crete Regional Meeting
in String Theory
July 7, 2022

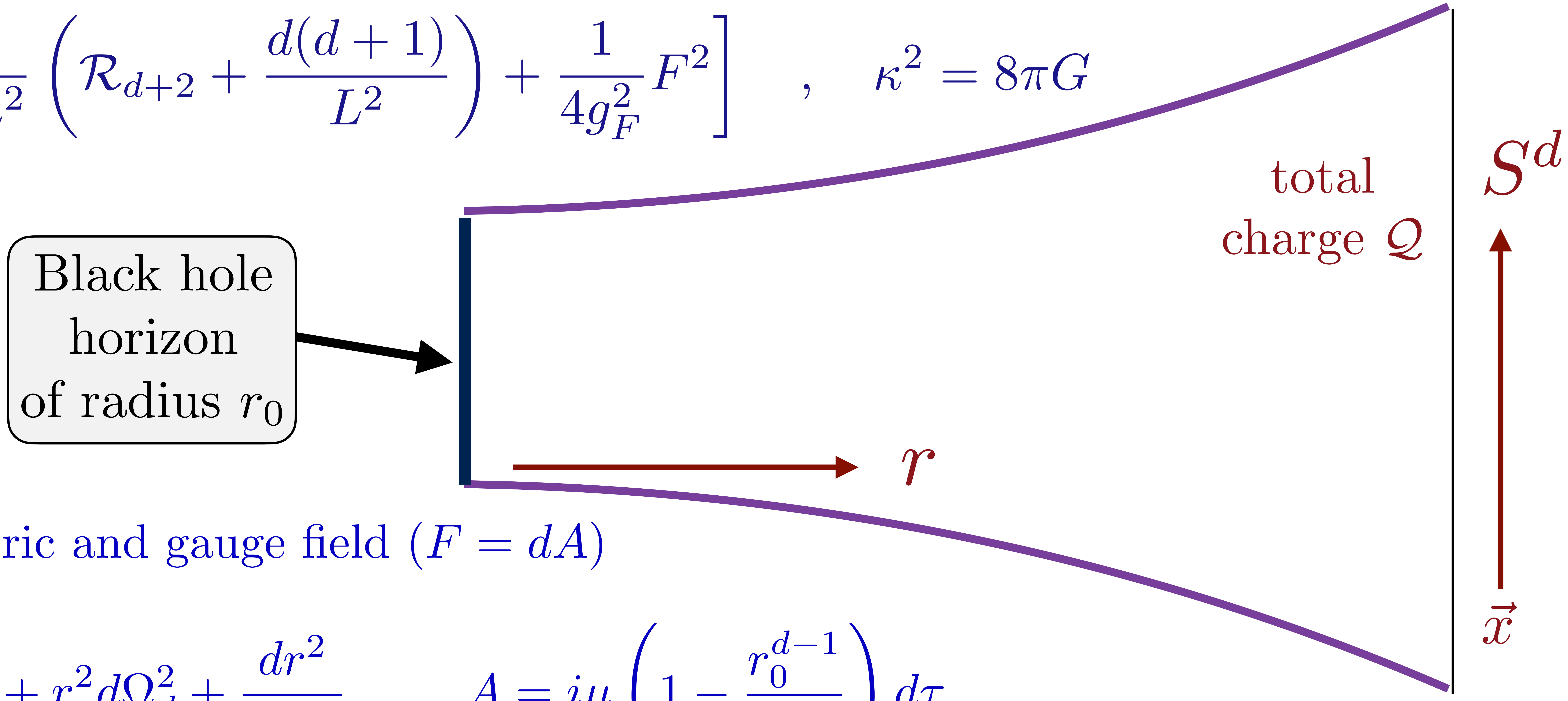
Subir Sachdev

Talk online: sachdev.physics.harvard.edu

Charged black holes

Charged black holes

$$I_{EM} = \int d^{d+2}x\sqrt{g}\left[-\frac{1}{2\kappa^2}\left(\mathcal{R}_{d+2} + \frac{d(d+1)}{L^2}\right) + \frac{1}{4g_F^2}F^2\right] \quad , \quad \kappa^2 = 8\pi G$$



Solutions of I_{EM} have metric and gauge field ($F = dA$)

$$ds^2 = V(r)d\tau^2 + r^2 d\Omega_d^2 + \frac{dr^2}{V(r)} \quad , \quad A = i\mu\left(1 - \frac{r_0^{d-1}}{r^{d-1}}\right)d\tau$$

$$V(r) = 1 + \frac{r^2}{L^2} + \frac{\Theta^2}{r^{2d-2}} - \frac{M}{r^{d-1}}.$$

where $d\Omega_d^2$ is the metric of the d-sphere. All parameters of the solution are determined in terms of the chemical potential μ (related to the charge Q), and the Hawking temperature of horizon, T_H (related to the mass M).

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD **60**, 064018 (1999)

SYK model

Charged black holes

$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{\mathcal{A}_0 c^3}{4} + \frac{\sqrt{\pi}\mathcal{A}_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right) + \ldots$$

$\mathcal{A}_0$ is the area of the horizon at $T = 0$.

SYK model

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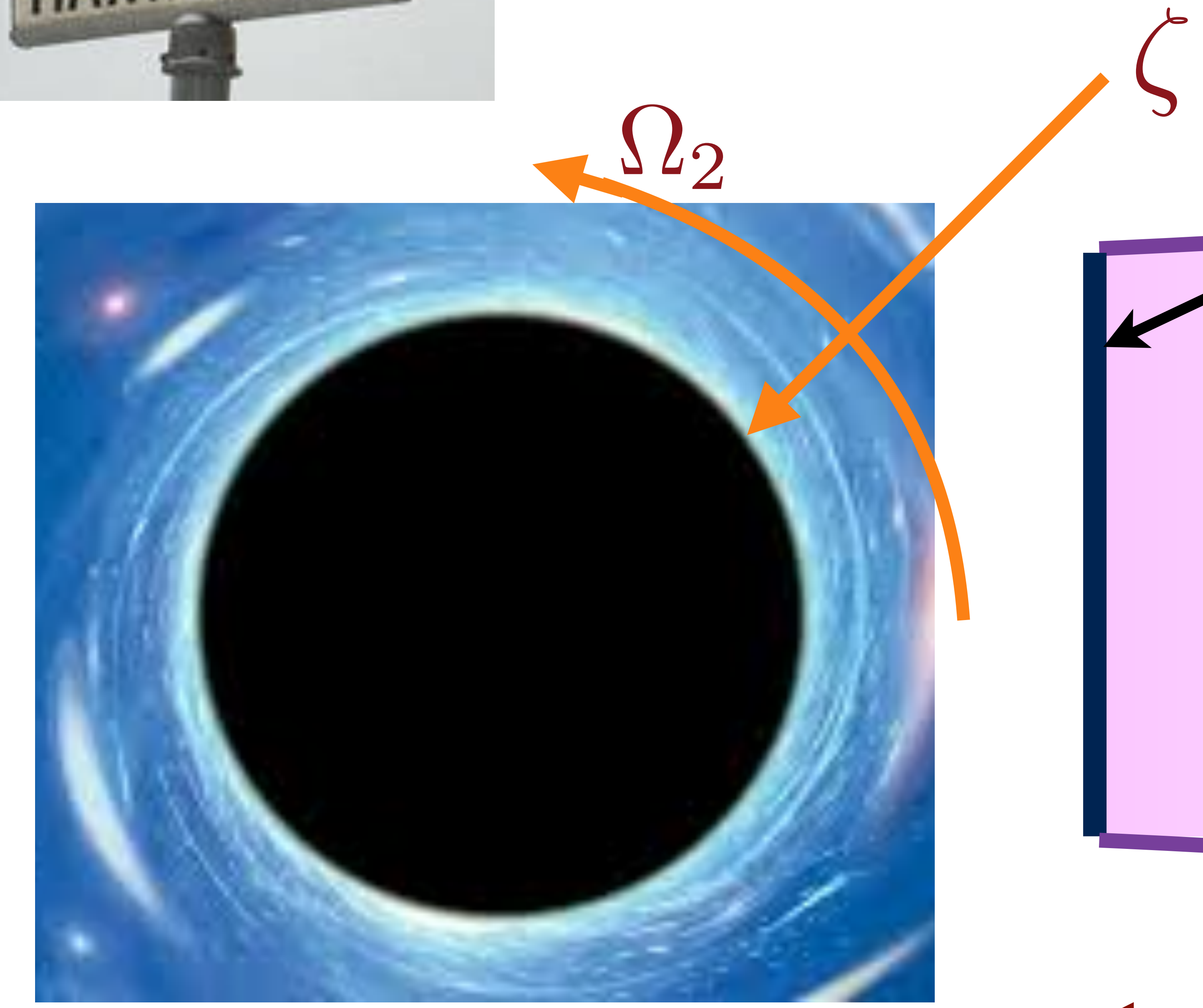
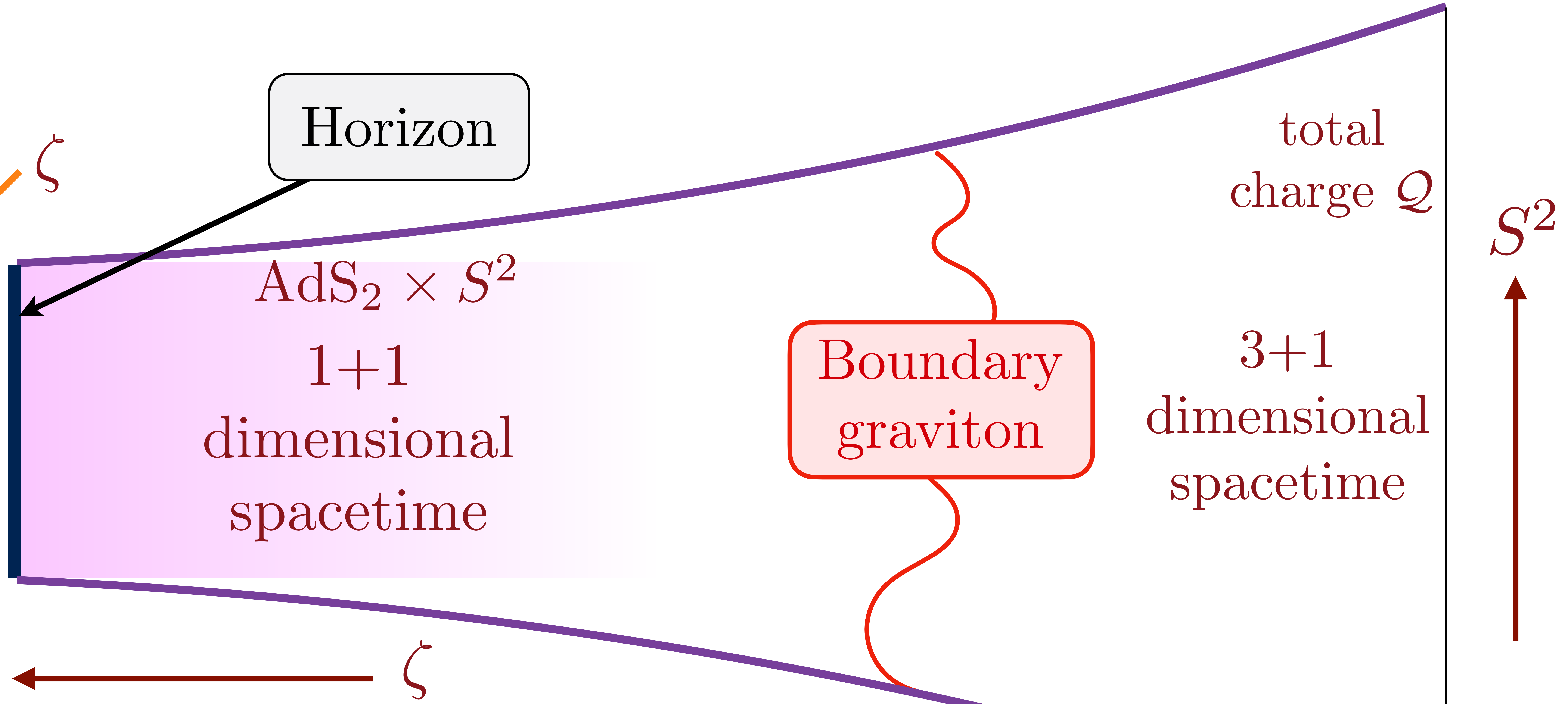
What is the origin of the low-T behavior?

Reissner-Nordstrom black hole of Einstein-Maxwell theory





Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Charged black holes

In the $T \to 0$ limit, at fixed μ, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of R_h

- In the near-horizon region, we change co-ordinates from r to ζ so that

$$r - R_h = \frac{R_2^2}{\zeta} \quad , \quad R_2 = \frac{L R_h}{\sqrt{d(d+1)R_h^2 + (d-1)^2 L^2}}.$$

Then the near-horizon metric becomes AdS$_2\times$ S$_d$, with

$$ds^2 = R_2^2 \left[\frac{-dt^2 + d\zeta^2}{\zeta^2} \right] + R_h^2 d\Omega_d^2 \quad , \quad A = \frac{\mathcal{E}}{\zeta} dt \,.$$

where the dimensionless electric field $\mathcal{E}$ is

$$\mathcal{E} = \frac{g_F R_h \sqrt{2d\left[(d+1)R_h^2 + (d-1)L^2\right]}}{2\left[d(d+1)R_h^2 + (d-1)^2 L^2\right]}.$$

S. Sachdev,
Journal of Mathematical Physics
60, 052303 (2019)

Quantum path integral for charged black holes

$$ds^2 = \frac{ds_2^2}{\Phi(\zeta,\tau)} + [\Phi(\zeta,\tau)]^2 \, d\Omega_2^2$$

is the metric ansatz, where ds_2^2 is an arbitrary metric in the (ζ,τ) spacetime, and Φ is a scalar field in the (ζ,τ) spacetime.
The low energy theory on the (ζ,τ) spacetime involves a metric h, and a scalar field Φ_1 given by $\lim_{\zeta\to\infty}[\Phi(\zeta,\tau)]^2 = R_h^2 + \Phi_1(\zeta,\tau)$, obeying the action

$$I_{JT} = -\frac{2\pi\mathcal{A}_0}{\kappa^2} + \int d^2x\sqrt{h}\left[-\frac{2\pi}{\kappa^2}\,\Phi_1\left(\mathcal{R}_2 + \frac{2}{R_h^3}\right)\right] - \frac{4\pi}{\kappa^2}\int_\partial dx\sqrt{h_b}\Phi_1\,\mathcal{K}_1$$

where $\mathcal{A}_0 = 4\pi R_h^2$ is the area of the horizon at $T=0$, $\mathcal{R}_2$ is the intrinsic curvature of two-dimensional spacetime with metric h, and $\mathcal{K}_1$ is the extrinsic curvature of the one-dimensional boundary $\zeta \to 0$ where

$$h_{\tau\tau}(\zeta\to 0) = \frac{R_h^3}{\zeta^2} \quad , \quad \Phi_1(\zeta\to 0) = \frac{2R_h^3}{\zeta}$$

S. Sachdev,
Journal of Mathematical Physics
60, 052303 (2019)

Saddle point of I_{JT} yields the low T entropy, including the co-efficient of the linear-T term.

SYK model

Charged black holes

$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{\mathcal{A}_0 c^3}{4} + \frac{\sqrt{\pi}\mathcal{A}_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right) + \ldots$$

$\mathcal{A}_0$ is the area of the horizon at $T = 0$.

SYK model

Charged black holes

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$\mathcal{A}_0$ is the area of the horizon at $T = 0$.

$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$

Probe fermion

Tom Faulkner, Hong Liu, John McGreevy, David Vegh (2009)

SYK model	Charged black holes
	$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{\mathcal{A}_0 c^3}{4} + \frac{\sqrt{\pi}\mathcal{A}_0^{3/2}c^2}{2}\frac{k_B T}{\hbar}\right) + \ldots$$ $\mathcal{A}_0$ is the area of the horizon at $T = 0$.
	$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$ Probe fermion
	$$\frac{\partial}{\partial \mathcal{Q}}\left(\frac{\mathcal{A}_0 c^3}{4\hbar G}\right) = 2\pi\mathcal{E}$$

A. Sen (2008)

Quantum path integral for charged black holes

Remarkably, the partition function of the $1+1$ dimensional JT gravity theory can be evaluated exactly (here we are ignoring the gauge field path integral, which is subdominant at fixed $\mathcal{Q}$)

$$\mathcal{Z}_{\mathcal{Q}} = \int \mathcal{D}h \mathcal{D}\Phi_1 \exp\left(-I_{JT}\right)$$

The action is linear in Φ_1, and the integral over Φ_1 yields a constraint $\mathcal{R}_2 = -2/R_h^3$ *i.e.* the metric h is rigidly AdS$_2$. The only dynamical degree of freedom in JT gravity is the shape of the boundary, determined by a time reparameterization $\tau \to f(\tau)$. To ensure that the bulk metric obeys its boundary condition, we make the spatial co-ordinate ζ a function of τ, so we map $(\tau, \zeta) \to (f(\tau), \zeta(\tau))$. Then the metric obeys its boundary condition provided $\zeta(\tau)$ is related to $f(\tau)$ by (here ζ_b is a small constant whose value cancels in the final result)

$$\zeta(\tau) = \zeta_b f'(\tau) + \zeta_b^3 \frac{\left[f''(\tau)\right]^2}{2f'(\tau)} + \mathcal{O}(\zeta_b^4)$$

Finally, we evaluate I_{GH} along this boundary curve. In this manner we obtain the action

$$I_{1,\text{eff}}[f] = -\frac{2\pi \mathcal{A}_0}{\kappa^2} - \frac{\gamma}{4\pi^2}\int d\tau \left\{f(\tau), \tau\right\} \quad , \quad \{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2$$

Maldacena, Stanford, Yang

where $\gamma = 32\pi^3 R_h^3/\kappa^2$ is precisely the linear-T co-efficient in the black hole entropy.

Quantum path integral for charged black holes

After a conformal map to finite temperature (and ignoring the contribution of the gauge field fluctuation), we can write the low energy partition function of a 3+1-dimensional black hole with charge $\mathcal{Q} = 4\pi R_h/(\kappa g_F)$, as a path integral over a single field $f(\tau)$ in one time dimension:

$$\mathcal{Z}_{\mathcal{Q}} = \exp\left(\frac{2\pi \mathcal{A}_0}{\kappa^2}\right) \int \frac{\mathcal{D}f}{||\text{SL(2,R)}||} \exp\left(\frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \left\{\tan(\pi T f(\tau)), \tau\right\}\right)$$

where $\gamma = 32\pi^3 R_h^3/\kappa^2$, $\mathcal{A}_0 = 4\pi R_h^2$, and $f(\tau)$ is a monotonic function of τ obeying

$$f(\tau + 1/T) = f(\tau) + 1/T\,.$$

We divide by the (infinite) volume of the SL(2,R) group because

$$\{f, \tau\} = \{\frac{af + b}{cf + d}, \tau\}$$

where a, b, c, d are constants with $ad - bc = 1$.

Maldacena, Stanford, Yang

At the saddle point, $f(\tau) = \tau$, and this yields the linear-T term in the entropy.

SYK model

Charged black holes

$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{\mathcal{A}_0 c^3}{4} + \frac{\sqrt{\pi}\mathcal{A}_0^{3/2}c^2}{2}\frac{k_B T}{\hbar}\right) + \ldots$$

$\mathcal{A}_0$ is the area of the horizon at $T = 0$.

$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$

Probe fermion

$$\frac{\partial}{\partial\mathcal{Q}}\left(\frac{\mathcal{A}_0 c^3}{4\hbar G}\right) = 2\pi\mathcal{E}$$

SYK model	Charged black holes
	$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{\mathcal{A}_0 c^3}{4} + \frac{\sqrt{\pi}\mathcal{A}_0^{3/2}c^2}{2}\frac{k_B T}{\hbar}\right) + \ldots - \frac{3}{2}\ln\left(\frac{\Lambda}{T}\right) + \ldots$$
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SYK
model

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

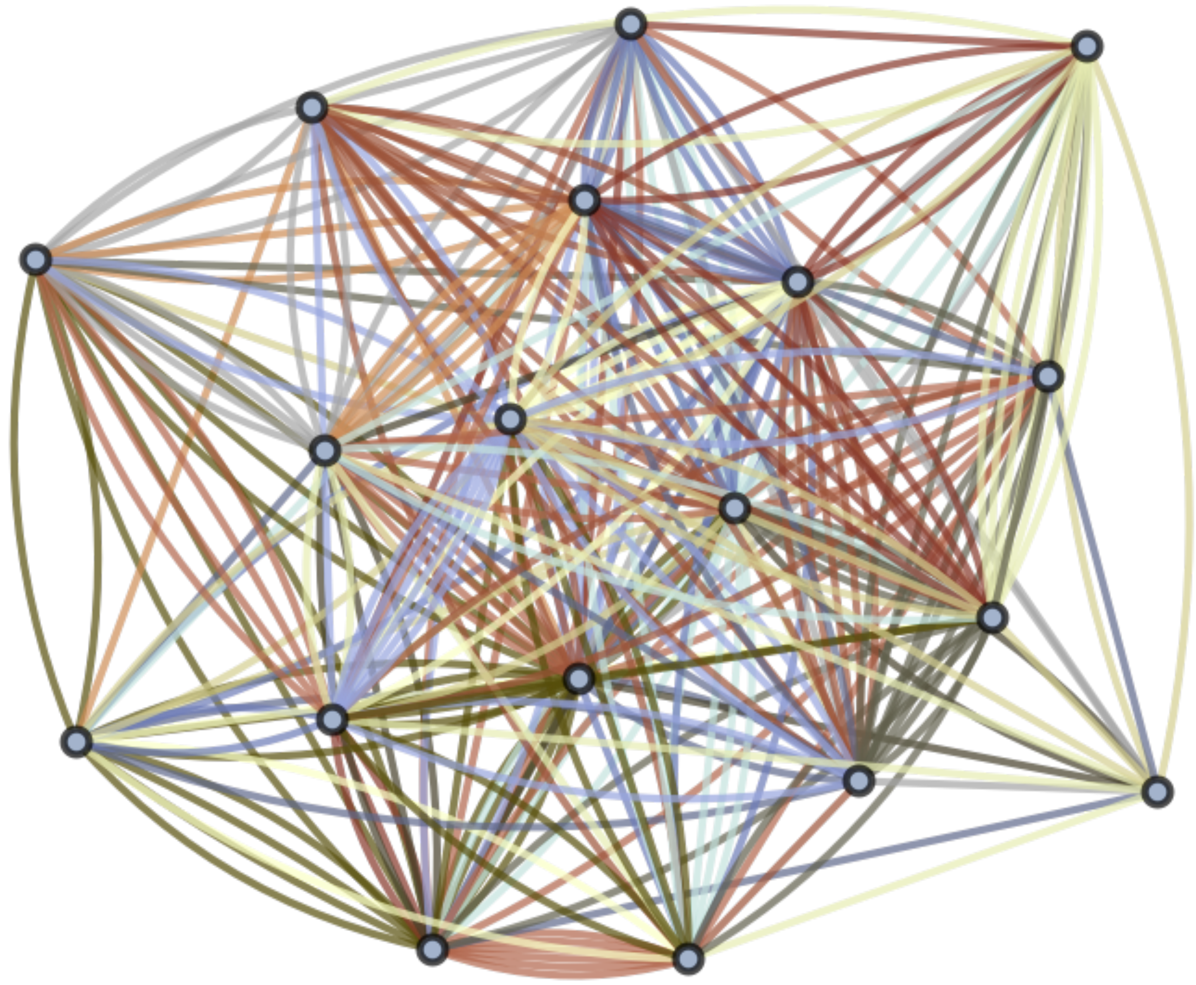
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

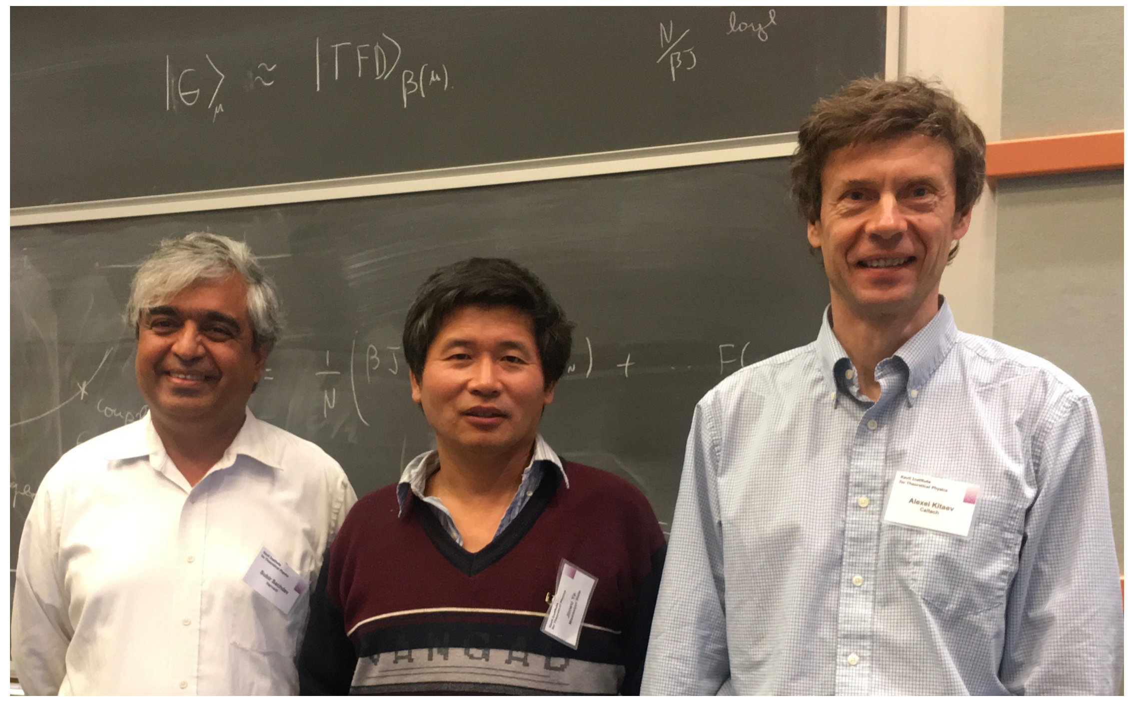
$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





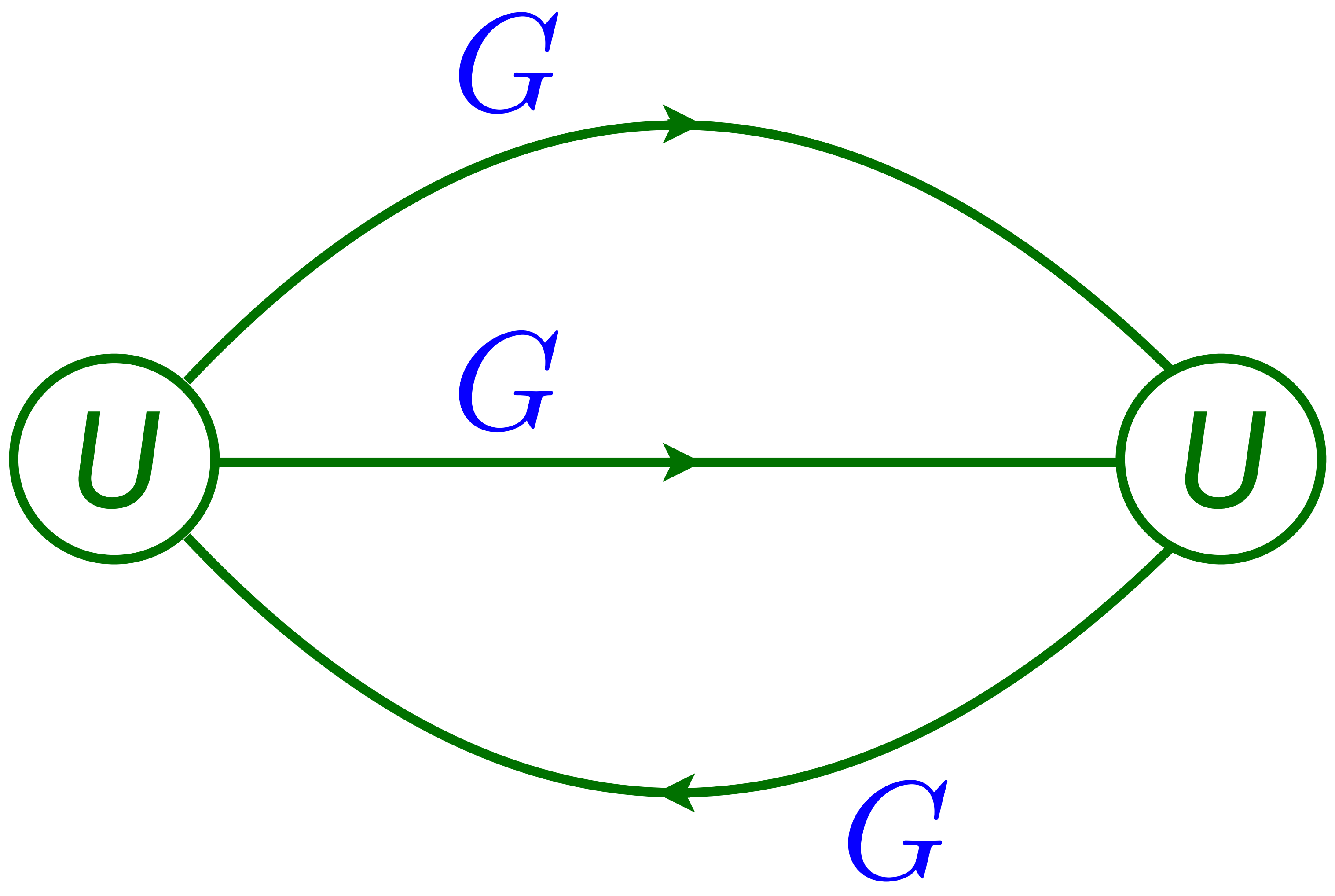
Complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$



Conformal solution at $\mu = 0$, $G(\tau) \sim \dfrac{\text{sgn}(\tau)}{\sqrt{|\tau|}}$.

S. Sachdev and J. Ye,
PRL **70**, 3339 (1993)

SYK model	Charged black holes
$$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T)$$	$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{\mathcal{A}_0 c^3}{4} + \frac{\sqrt{\pi}\mathcal{A}_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right) + \ldots$$ $\mathcal{A}_0$ is the area of the horizon at $T = 0$.
	$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$ Probe fermion
	$$\frac{\partial}{\partial \mathcal{Q}}\left(\frac{\mathcal{A}_0 c^3}{4\hbar G}\right) = 2\pi\mathcal{E}$$

S.S. 2010, 2015

SYK model

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S.S. 2010, 2015

G-Σ path integral

Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, G_s, Σ_s, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for Σ). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) e^{-E_0/T + Ns_0 - NS_{\rm eff}[f,\phi]},$$

where $E_0 \propto N$ is the ground state energy.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

G-Σ path integral

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\text{eff}}[f,\phi] = \frac{NK}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi\mathcal{E}T)\partial_\tau f)^2 - \frac{N\gamma}{4\pi^2}\int_0^{1/T} d\tau \left\{\tan(\pi T f(\tau)), \tau\right\},$$

where $f(\tau)$ is a monotonic map from $[0,1/T]$ to $[0,1/T]$, the couplings K, γ, and $\mathcal{E}$ can be related to thermodynamic derivatives and we have used the Schwarzian:

$$\{g,\tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2 .$$

Specifically, an argument constraining the effective at $T=0$ is

$$S_{\text{eff}}\left[f(\tau) = \frac{a\tau+b}{c\tau+d}, \phi(\tau) = 0\right] = 0,$$

and this is origin of the Schwarzian.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, PRB **95**, 155131 (2017);
A. Gaikwad, L.K. Joshi, G. Mandal, and S.R. Wadia, arXiv:1802.07746

SYK model	Charged black holes
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SYK model

$$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T) - \frac{3}{2}\ln\left(\frac{N^{1/3}U}{k_B T}\right) + \ldots$$

$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$

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Charged black holes

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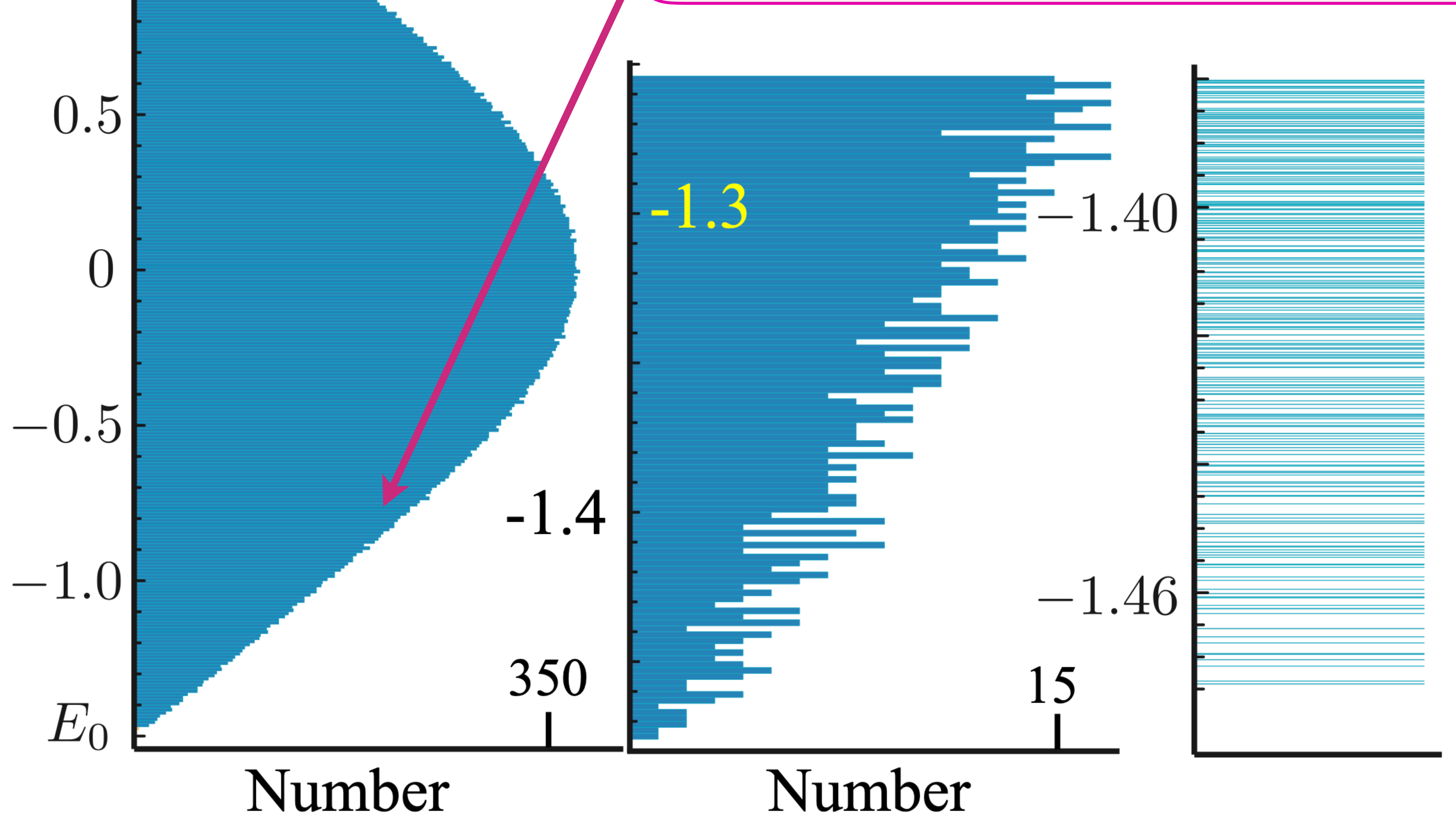
Probe fermion

$$\frac{\partial}{\partial \mathcal{Q}}\left(\frac{\mathcal{A}_0 c^3}{4\hbar G}\right) = 2\pi\mathcal{E}$$

$$D(E_0 + E) \sim 2\, e^{N s_0} \sinh(\sqrt{2N\gamma E})$$

$$\frac{S(T)}{k_B} = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{U}{T}\right)$$



SYK model

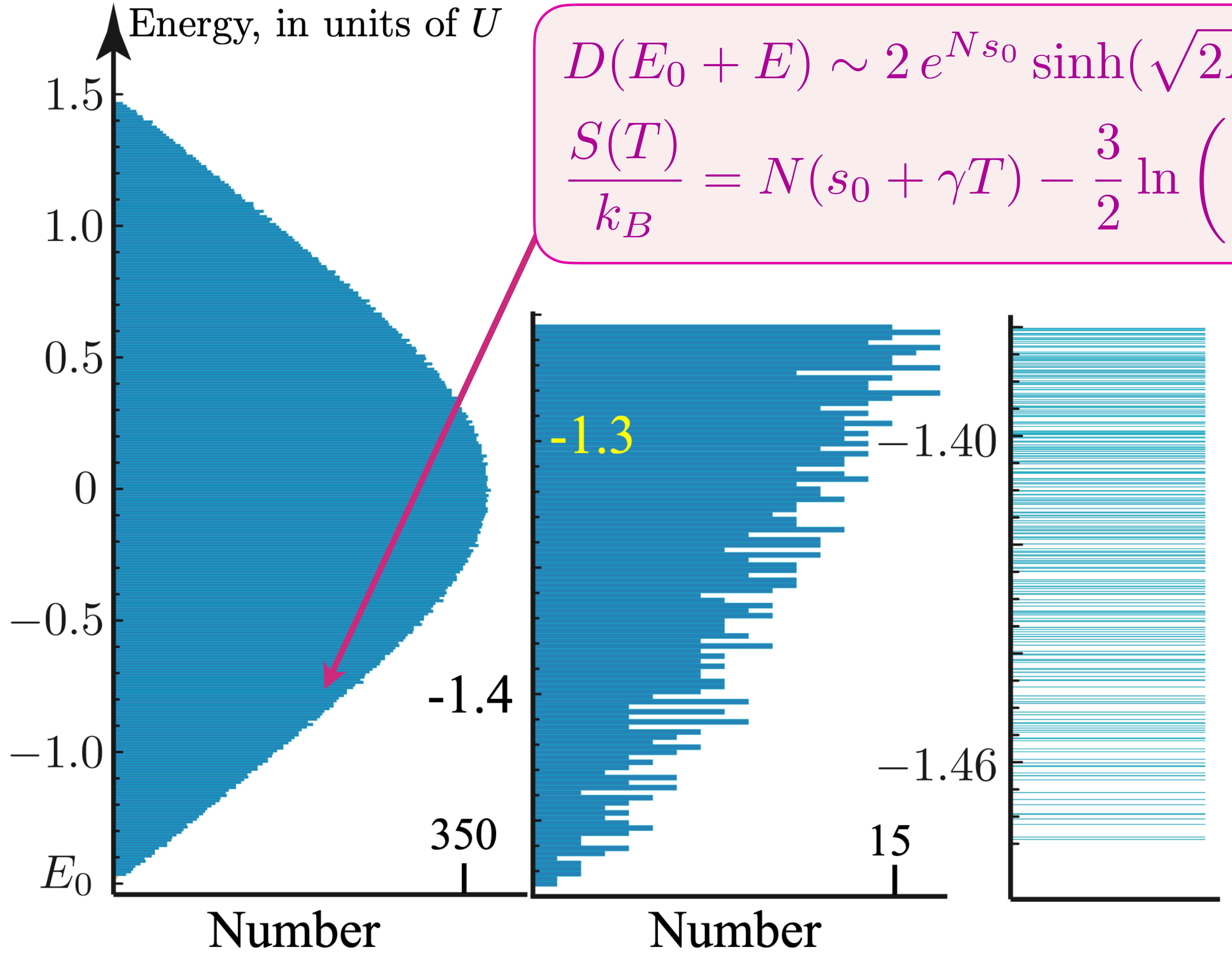
$$\mathcal{Z}(T) = e^{-F/T} = \int dE D(E) e^{-E/T}$$

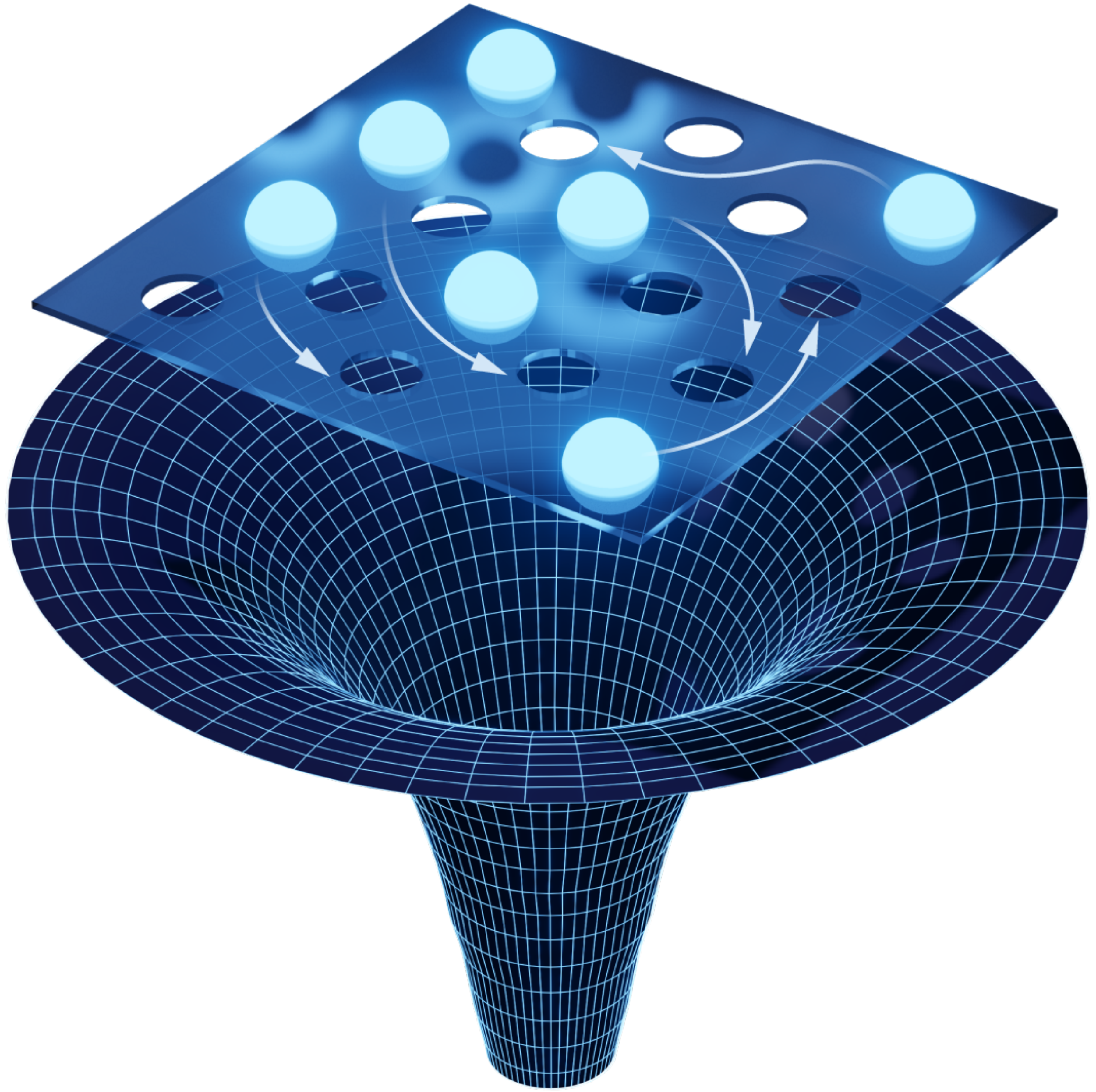
$$S(T) = -\frac{\partial F}{\partial T}$$

There is an extensive entropy as $T \to 0$ ($\lim_{T\to 0} \lim_{N\to\infty} S/N \neq 0$); however, the ground state is *not* extensively degenerate. Instead, the energy level spacing is exponentially small in N down to the ground state, implying the absence of quasiparticles.

$$D(E_0 + E) \sim 2\, e^{N s_0} \sinh(\sqrt{2N\gamma E})$$

$$\frac{S(T)}{k_B} = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{U}{T}\right)$$





SYK model
or
charged black hole

SYK model	Charged black holes
$$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T) - \frac{3}{2}\ln\left(\frac{N^{1/3}U}{k_B T}\right) + \ldots$$	$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{\mathcal{A}_0 c^3}{4} + \frac{\sqrt{\pi}\mathcal{A}_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right) + \ldots - \frac{3}{2}\ln\left(\frac{\Lambda}{T}\right) + \ldots$$
$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$	$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$ Probe fermion
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SYK model	Charged black holes
$$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T) - \frac{3}{2}\ln\left(\frac{N^{1/3}U}{k_B T}\right) + \ldots$$	$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{\mathcal{A}_0 c^3}{4} + \frac{\sqrt{\pi}\mathcal{A}_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right) + \ldots - \frac{3}{2}\ln\left(\frac{\Lambda}{T}\right) + \ldots$$
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$$D(E) \sim \exp(N s_0)\sinh\left(\sqrt{2N\gamma E}\right)$$	$$D(E) \sim \exp\left(\frac{\mathcal{A}_0 c^3}{4\hbar G}\right)\sinh\left(\left[\sqrt{\pi}\mathcal{A}_0^{3/2}\frac{c^3}{\hbar G}\frac{E}{\hbar c}\right]^{1/2}\right)$$

S. Sachdev,
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60, 052303 (2019)

SYK model	Charged black holes
$$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T) - \frac{3}{2}\ln\left(\frac{N^{1/3}U}{k_B T}\right) + \ldots$$	$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{\mathcal{A}_0 c^3}{4} + \frac{\sqrt{\pi}\mathcal{A}_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right) + \ldots - \frac{3}{2}\ln\left(\frac{\Lambda}{T}\right) + \ldots$$
$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$	$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$ Probe fermion
$$\frac{\partial s_0}{\partial \mathcal{Q}} = 2\pi\mathcal{E}$$	$$\frac{\partial}{\partial \mathcal{Q}}\left(\frac{\mathcal{A}_0 c^3}{4\hbar G}\right) = 2\pi\mathcal{E}$$
$$D(E) \sim \exp(N s_0)\sinh\left(\sqrt{2N\gamma E}\right)$$	$$D(E) \sim \exp\left(\frac{\mathcal{A}_0 c^3}{4\hbar G}\right)\sinh\left(\left[\sqrt{\pi}\mathcal{A}_0^{3/2}\frac{c^3}{\hbar G}\frac{E}{\hbar c}\right]^{1/2}\right)$$

S. Sachdev, Journal of Mathematical Physics **60**, 052303 (2019)

Questions and Answers

Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy? Yes, for charged black holes:

Questions and Answers

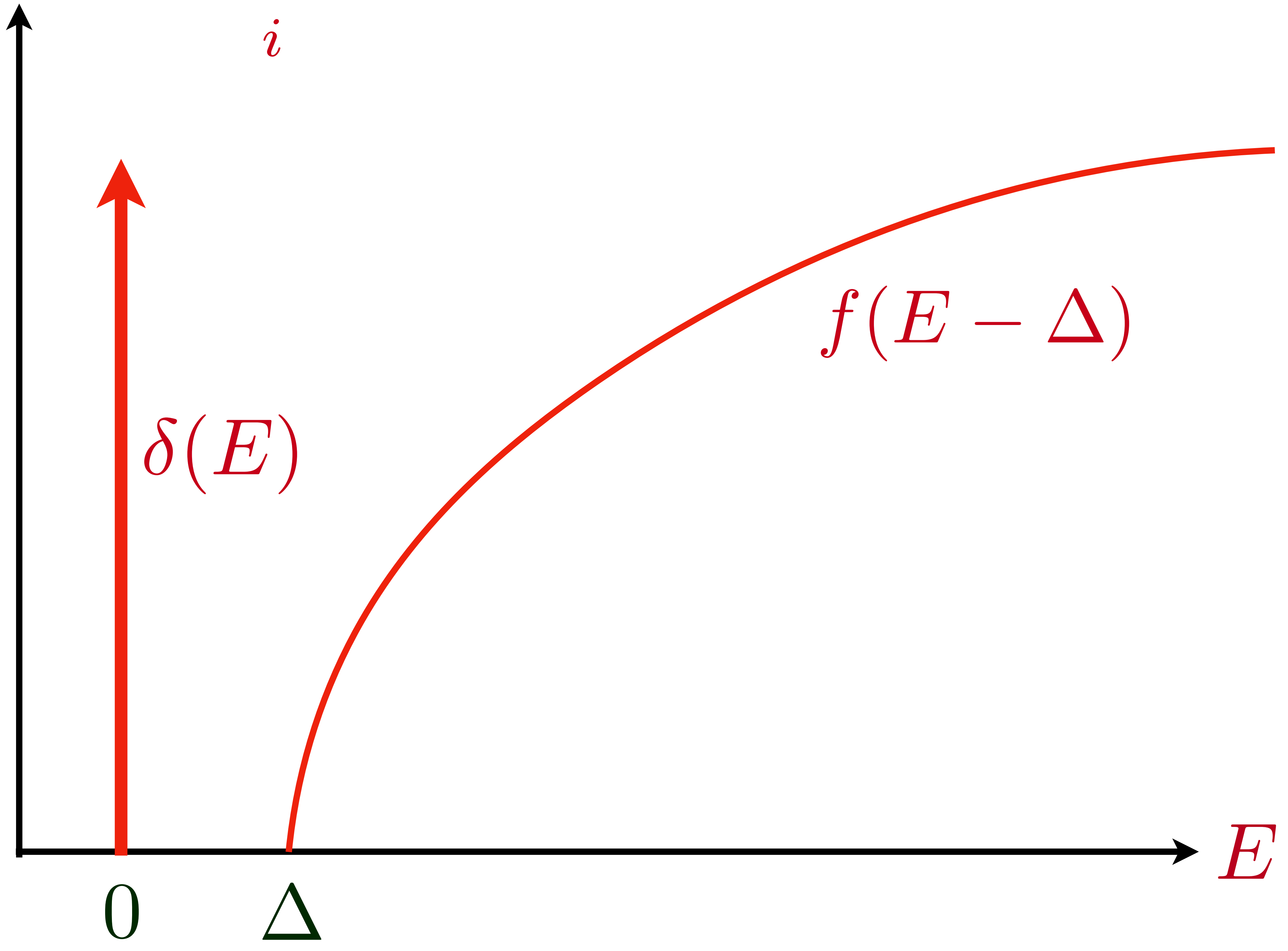
Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy? Yes, for charged black holes:

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{Ac^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta)f(E-\Delta) + \dots$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.

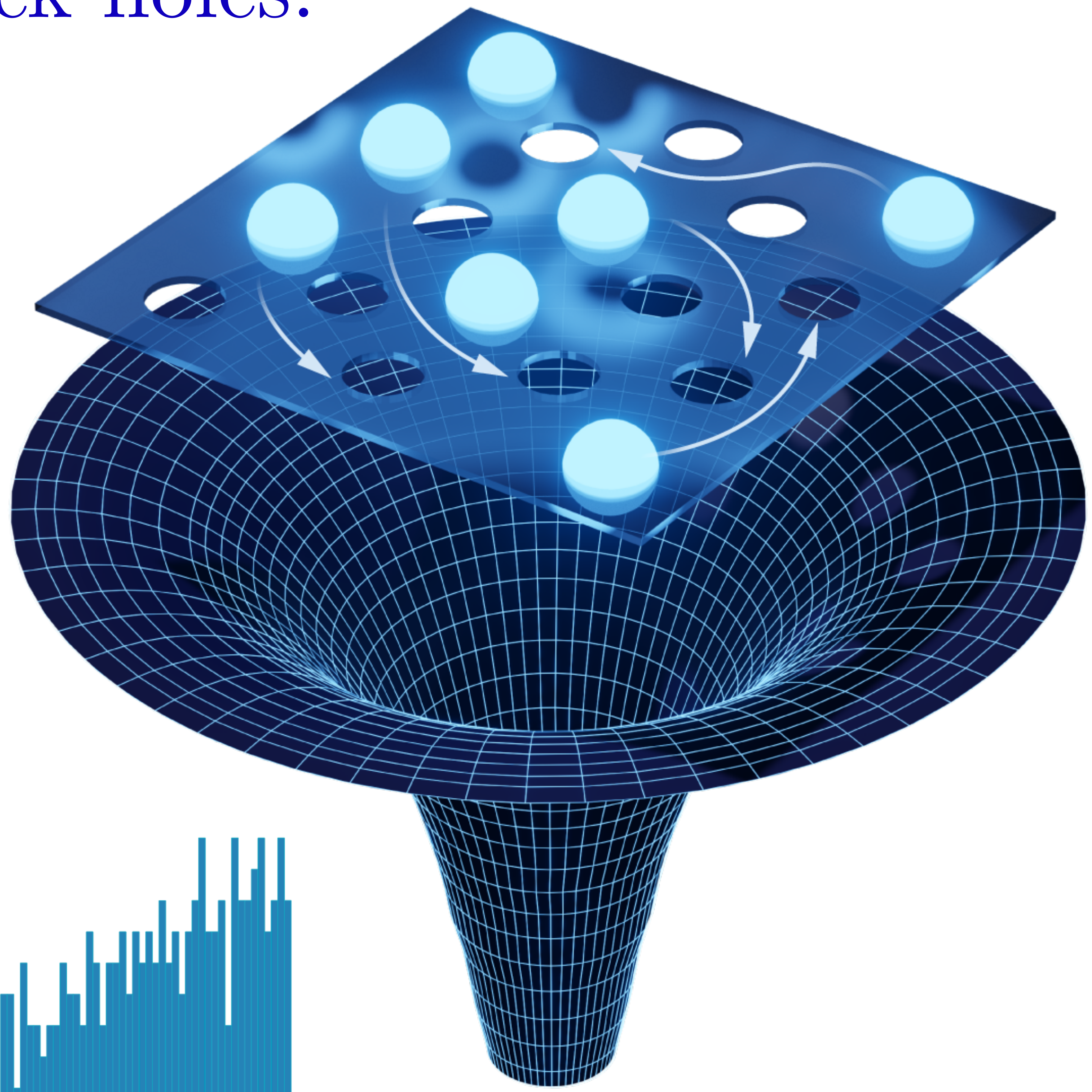
$$D(E) = \sum_i \delta(E - E_i)$$



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2022

Questions and Answers

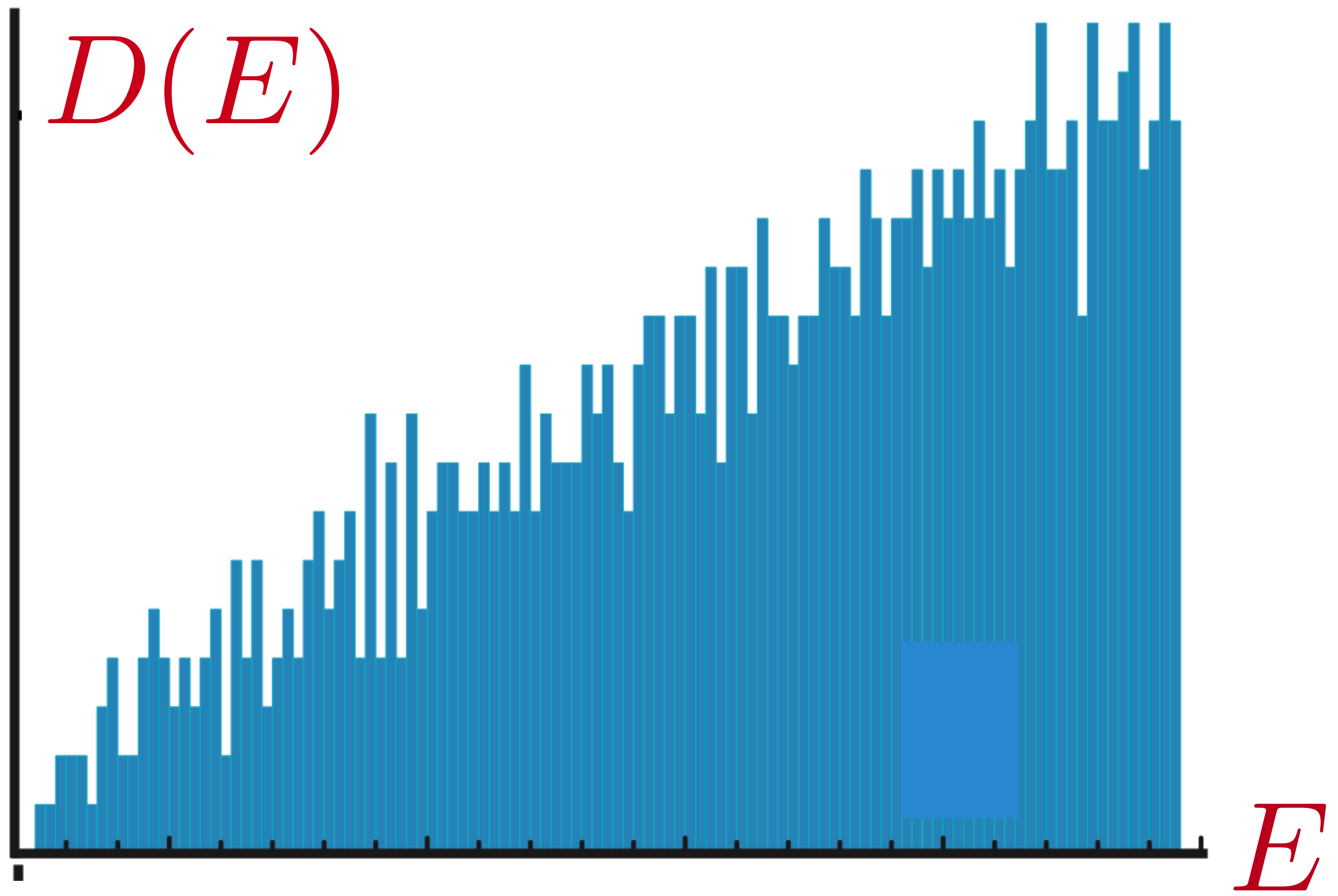
Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy? Yes, for charged black holes:



- For generic black holes in 3+1 dimensions, the SYK model yields:

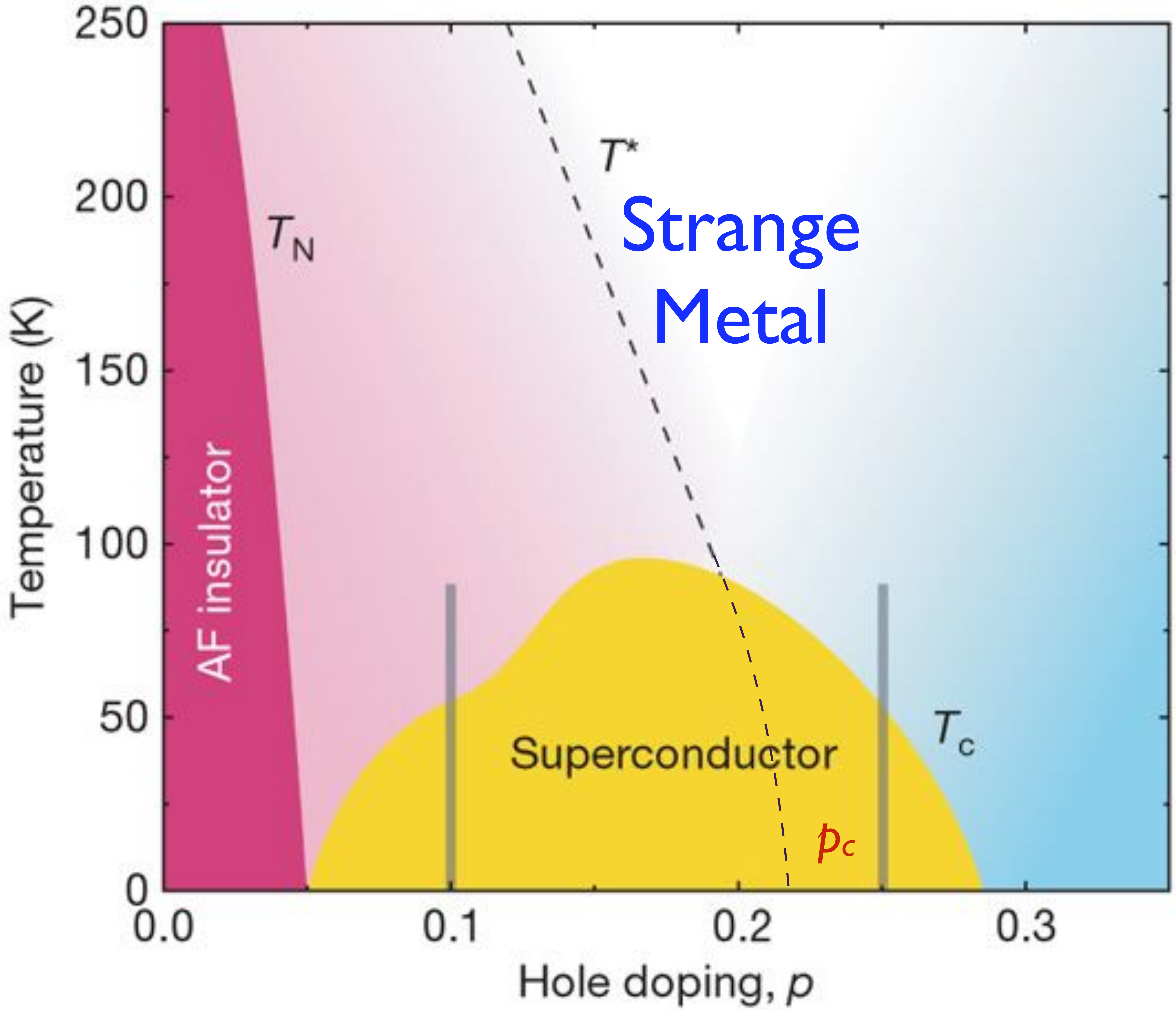
$$D(E) \sim \exp\left(\frac{\mathcal{A}_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}\mathcal{A}_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

where $\mathcal{A}_0$ is the horizon area at $T = 0$. There is no degeneracy, but an exponentially small level spacing down to the ground state.

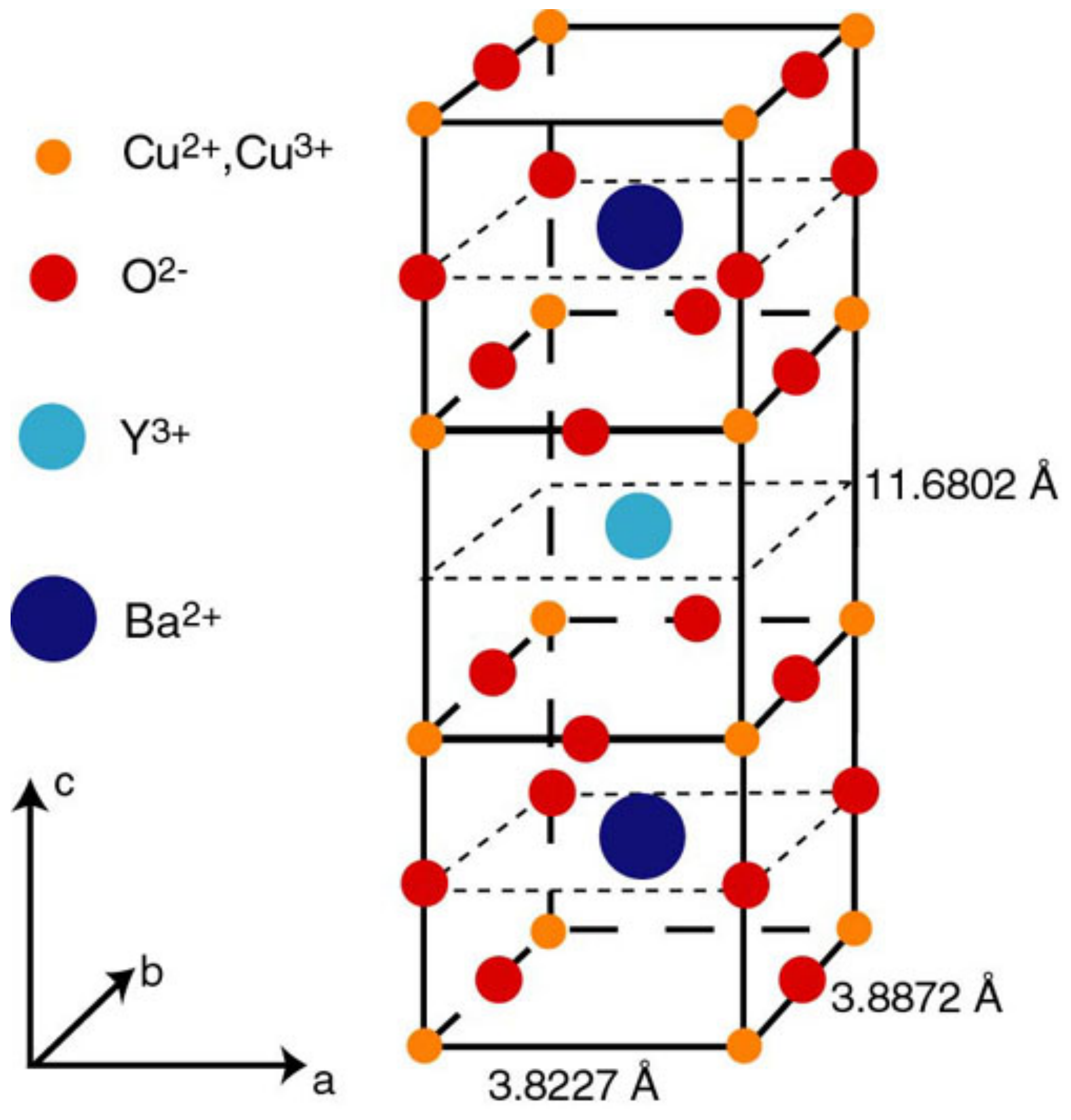


S. Sachdev,
Journal of Mathematical Physics
60, 052303 (2019)

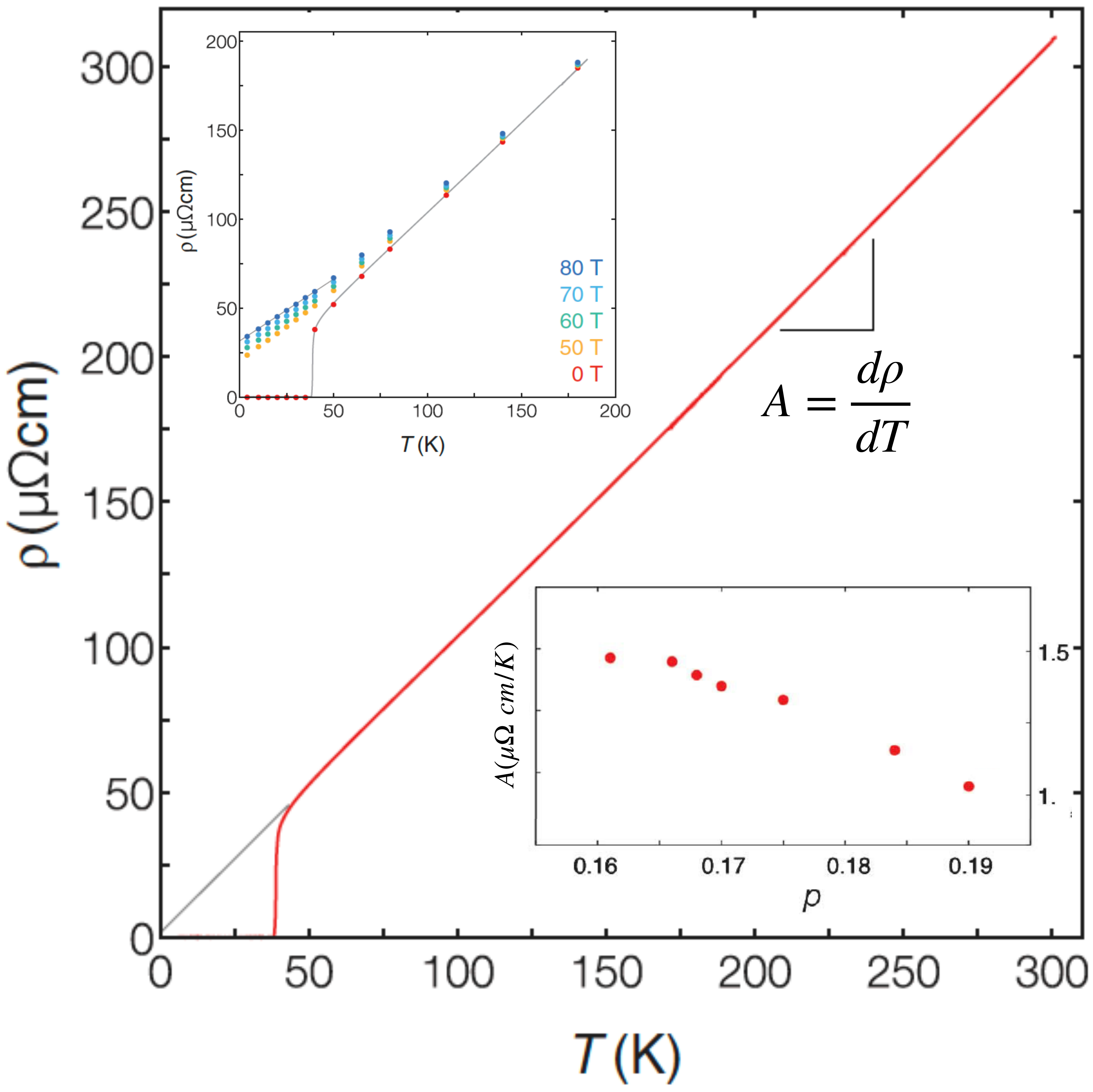
Strange metals



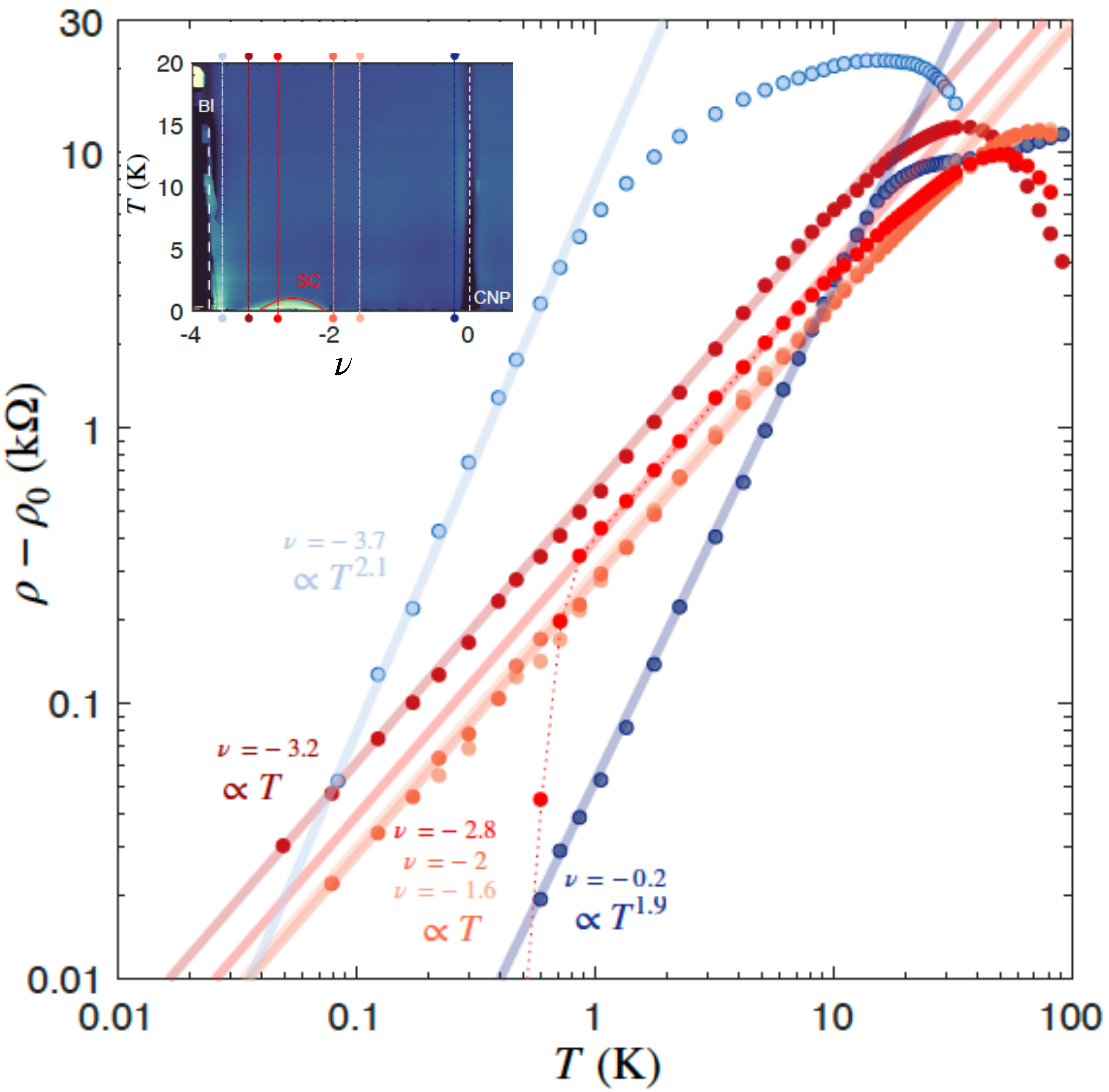
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å



LSCO: Giraldo-Gallo et al. 2018

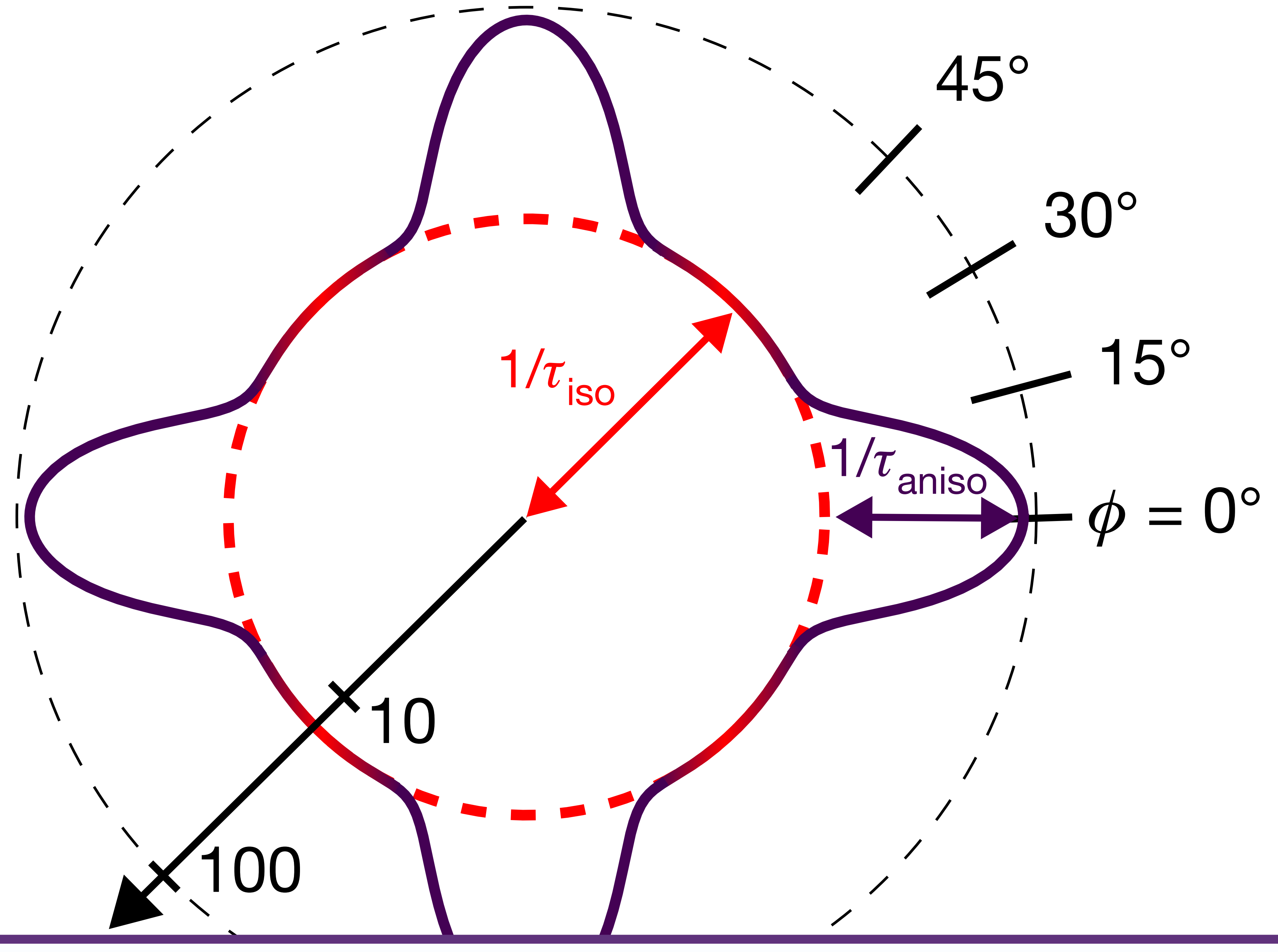


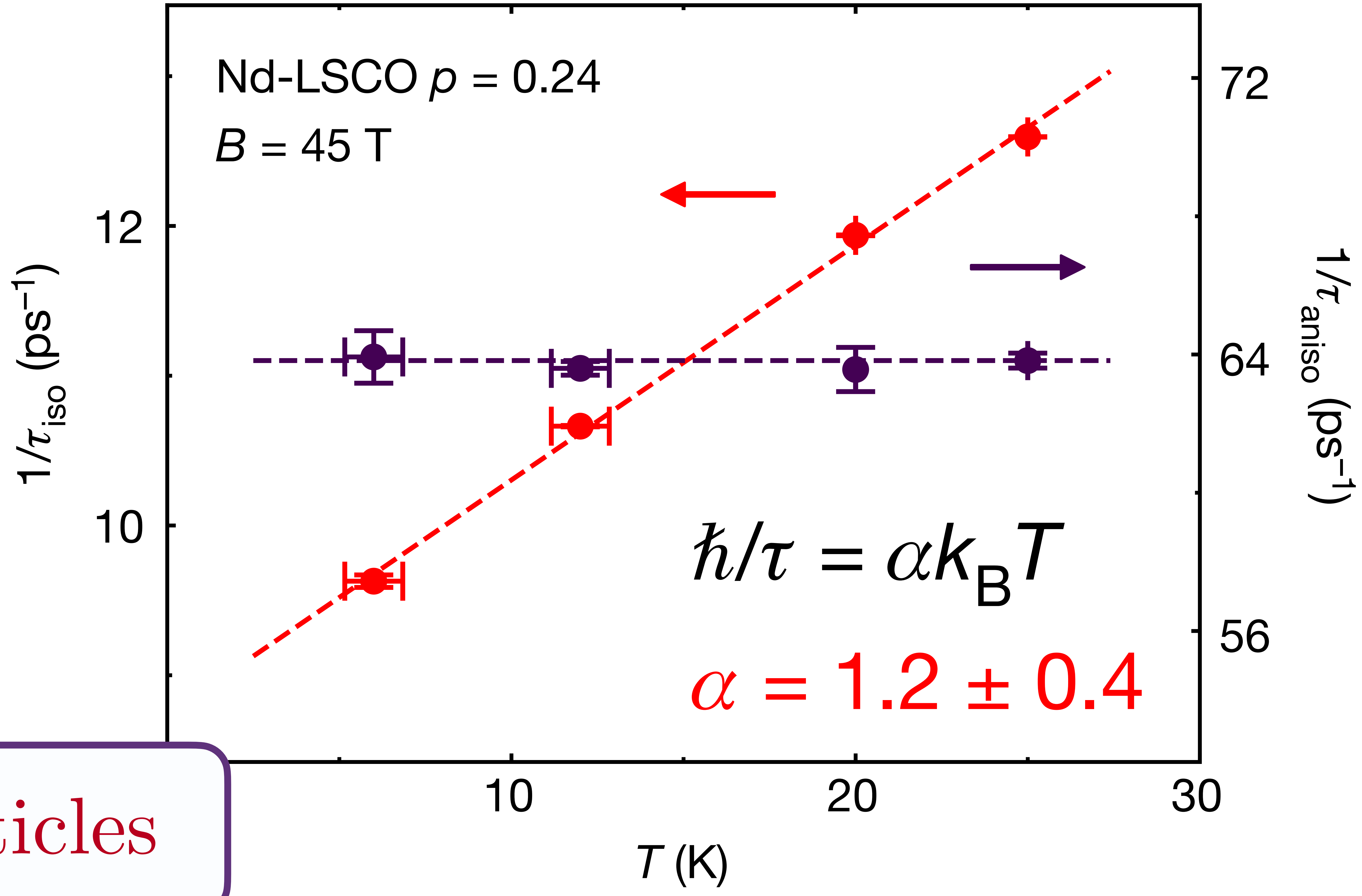
MATBG: Jaoui et al. 2021

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw





Current flow without quasiparticles

Properties of a strange metal:

- Resitivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
  and $\rho(T) < h/e^2$ (in $d = 2$).
  Metals with $\rho(T) > h/e^2$ are bad metals.

- Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

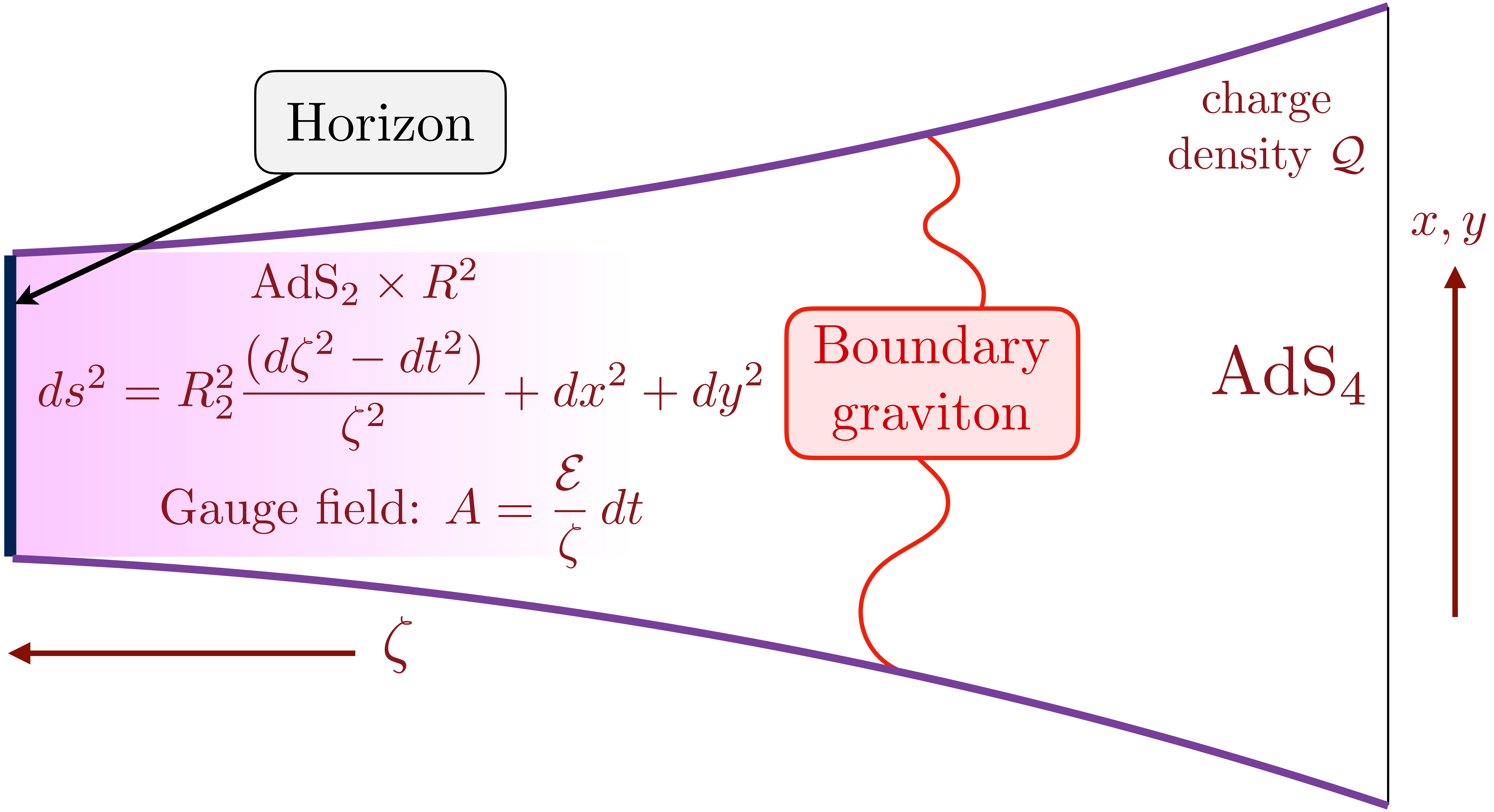
- Optical conductivity

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau(\omega)} - i\omega \dfrac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau(\omega)} = \frac{k_B T}{\hbar} G\left(\frac{\hbar\omega}{k_B T}\right)$$

B. Michon……A. Georges, arXiv:2205.04030

Non-Fermi liquid from a black brane



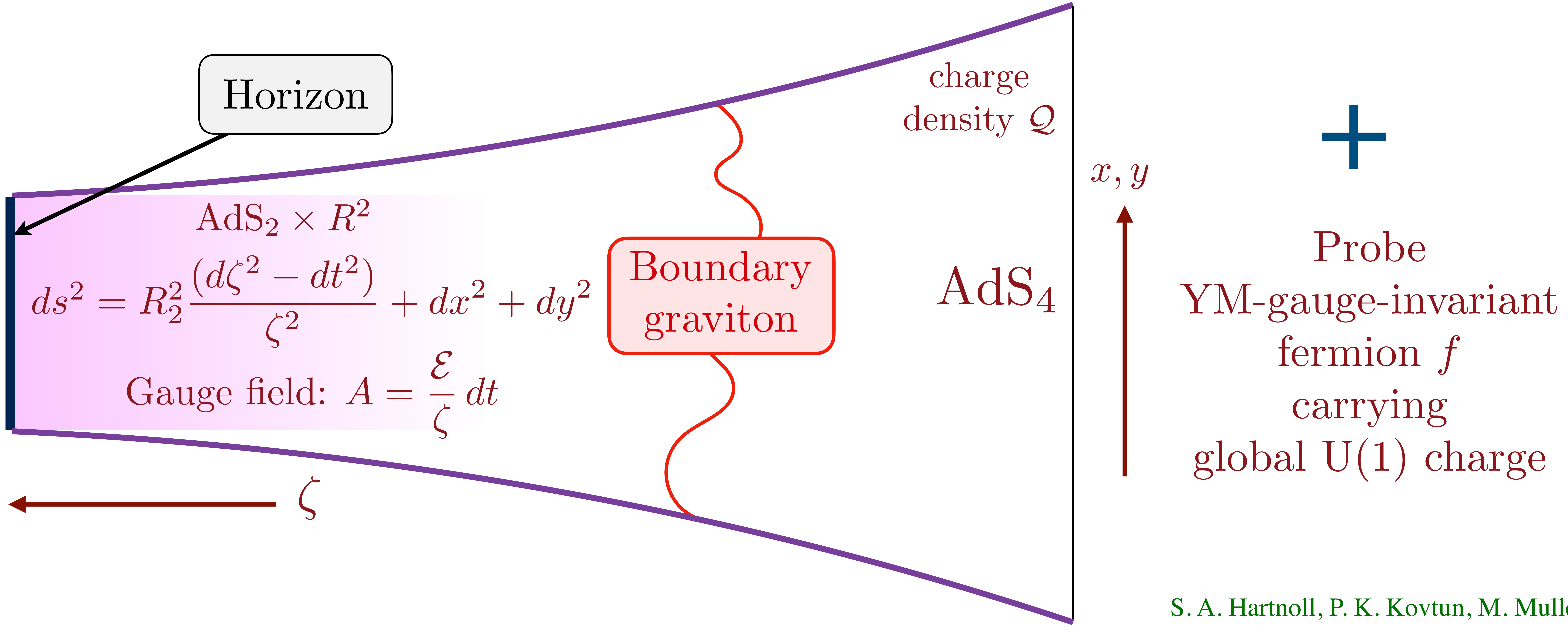
+

Probe
YM-gauge-invariant
fermion f
carrying
global U(1) charge

$$G_f = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) + iT^{\sigma(k)} F\left(\frac{\hbar\omega}{k_B T}\right)}$$

Tom Faulkner, Hong Liu, John McGreevy, David Vegh (2009)
Mihailo Cubrovic, Jan Zaanen, Koenraad Schalm (2009)

Non-Fermi liquid from a black brane



S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)

However, this model has resistivity $\rho(T) = 0$.
This is because of momentum conservation: the black brane also 'slides' in response to an external electric field. This non-Fermi liquid is *not* a strange metal.

Yukawa-SYK models and strange metals

Yukawa-SYK models

$$H = \sum_{ij} t_{ij} \psi_i^\dagger \psi_j + \sum_\ell \frac{1}{2} \left(\pi_\ell^2 + \omega_\ell^2 \phi_\ell^2 \right) + \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

Dionysios Anninos, Tarek Anous, Paul de Lange, George Konstantinidis, JHEP 03, 066 (2015)
W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_+ \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$
$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

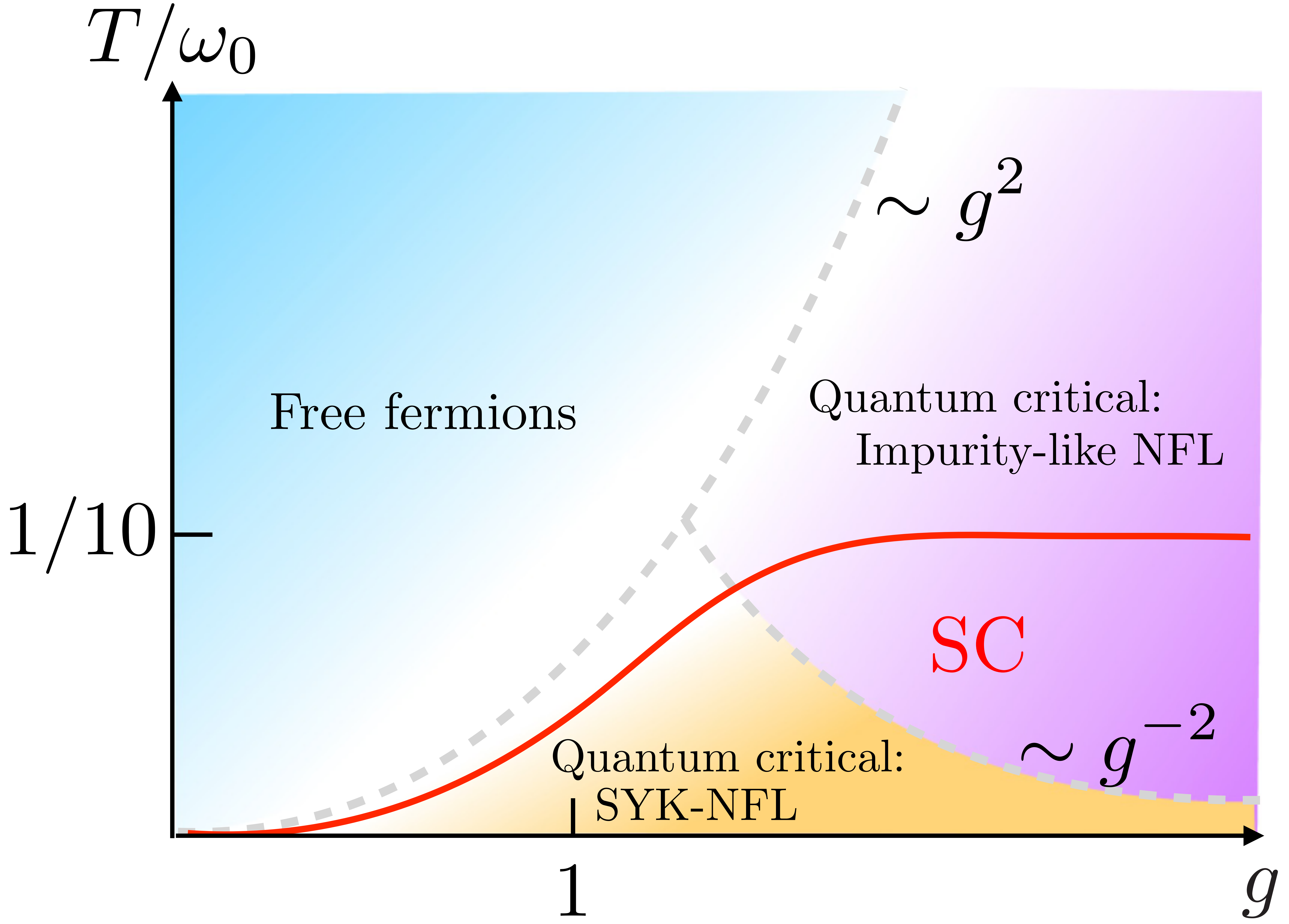
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

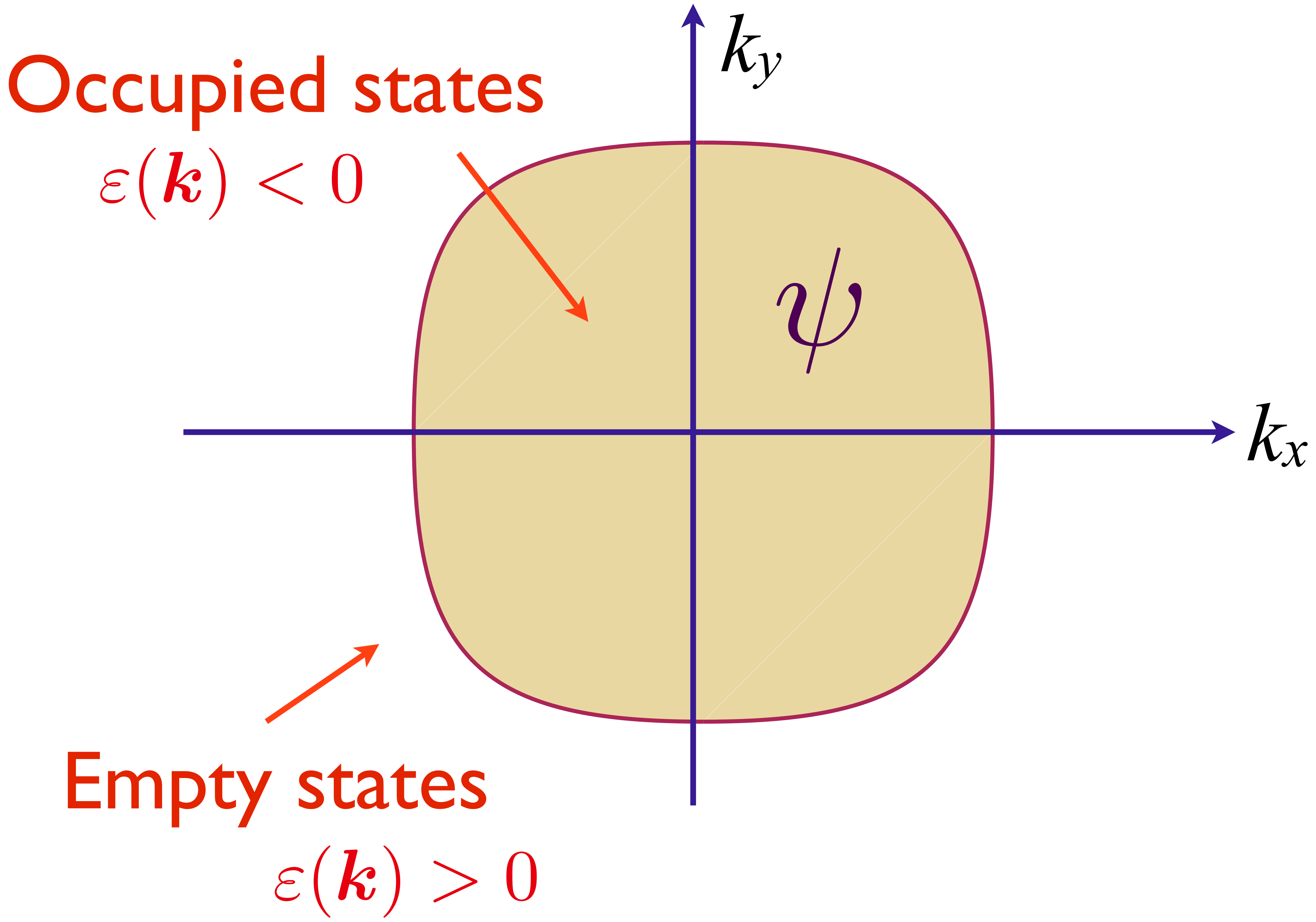
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Yukawa-SYK models



I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

Fermi surface coupled to a critical boson



$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

+

a critical boson

ϕ

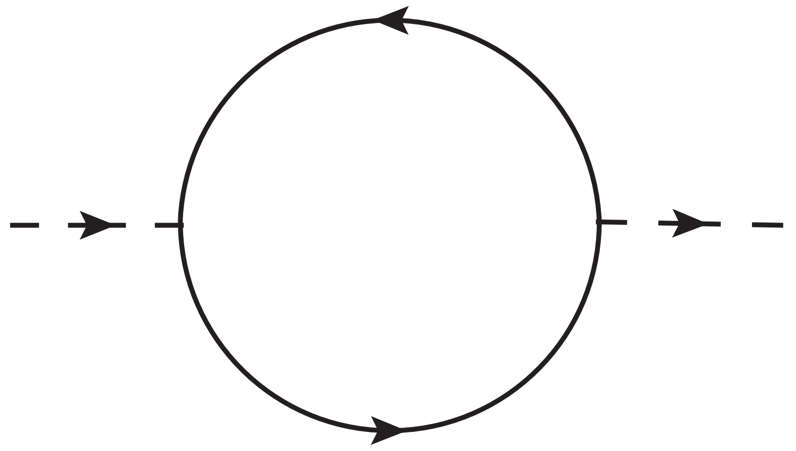
- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field
- Antiferromagnetic order...

$$\mathcal{L}_\phi = \frac{1}{2} \left[(\partial_\tau \phi)^2 + (\boldsymbol{\nabla} \phi)^2 + s\phi^2 \right]$$

Fermi surface coupled to a critical boson

"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

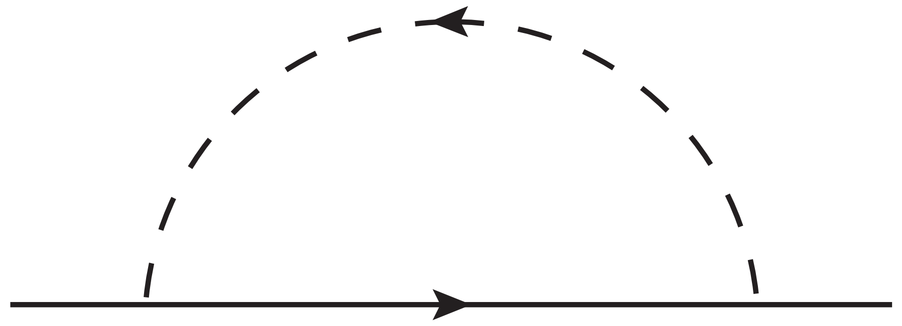
Boson self energy $\Pi(q, i\Omega) \sim -g^2 \frac{|\Omega|}{q}$ (Landau damping)



Boson Green's function $D(q, i\Omega) = \frac{1}{q^2 + \gamma|\Omega|/q}$

Fermion self energy $\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\text{sgn}(\omega)|\omega|^{2/3}$

Fermion Green's function $G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega \mp k_x - k_y^2 - \Sigma(\hat{\boldsymbol{k}}, i\omega)}$



P.A. Lee (1989)

Yields a state without quasiparticle excitations, but the theory is not systematic at large N

Sung-Sik Lee (2009)

Fermi surface coupled to a critical boson

"Yukawa" coupling: $\dfrac{g_{ij\ell}}{N}\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

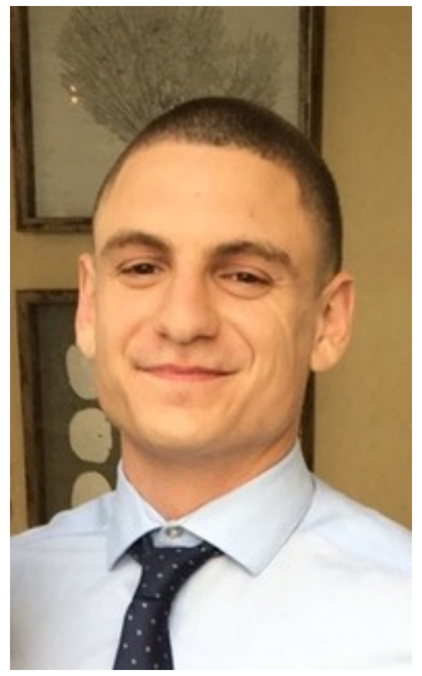
$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

Main idea:

Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.

Ilya Esterlis, J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A.V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

G-Σ-D-Π Theory



The theory self-averages, and the average partition function can be written exactly as a 'G-Σ' theory involving a path integral over *bilocal in spacetime*. We introduce the spacetime co-ordinate $X \equiv (\tau, x, y)$, and all Green's functions and self energies in the path integral are functions of two spacetime co-ordinates X_1 and X_2.

$$\overline{\mathcal{Z}} = \int \mathcal{D}G(X_1, X_2)\mathcal{D}\Sigma(X_1, X_2)\mathcal{D}D(X_1, X_2)\mathcal{D}\Pi(X_1, X_2) \exp\left[-N I(G, \Sigma, D, \Pi)\right] .$$

The G-Σ-D-Π action is now

$$\begin{aligned} I(G, \Sigma, D, \Pi) = \frac{g^2}{2}\mathrm{Tr}\left(G \cdot [GD]\right) - \mathrm{Tr}(G \cdot \Sigma) + \frac{1}{2}\mathrm{Tr}(D \cdot \Pi) \\ - \ln\det\left[(\partial_{\tau_1} + \varepsilon(-i\boldsymbol{\nabla}_1))\,\delta(X_1 - X_2) + \Sigma(X_1, X_2)\right] \\ + \frac{1}{2}\ln\det\left[\left(-\partial_{\tau_1}^2 - \boldsymbol{\nabla}_1^2 + s\right)\delta(X_1 - X_2) - \Pi(X_1, X_2)\right] . \end{aligned}$$

where we have introduced notation

$$\mathrm{Tr}\,(f \cdot g) \equiv \int dX_1 dX_2\, f(X_2, X_1) g(X_1, X_2) .$$

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

G-Σ-D-Π Theory

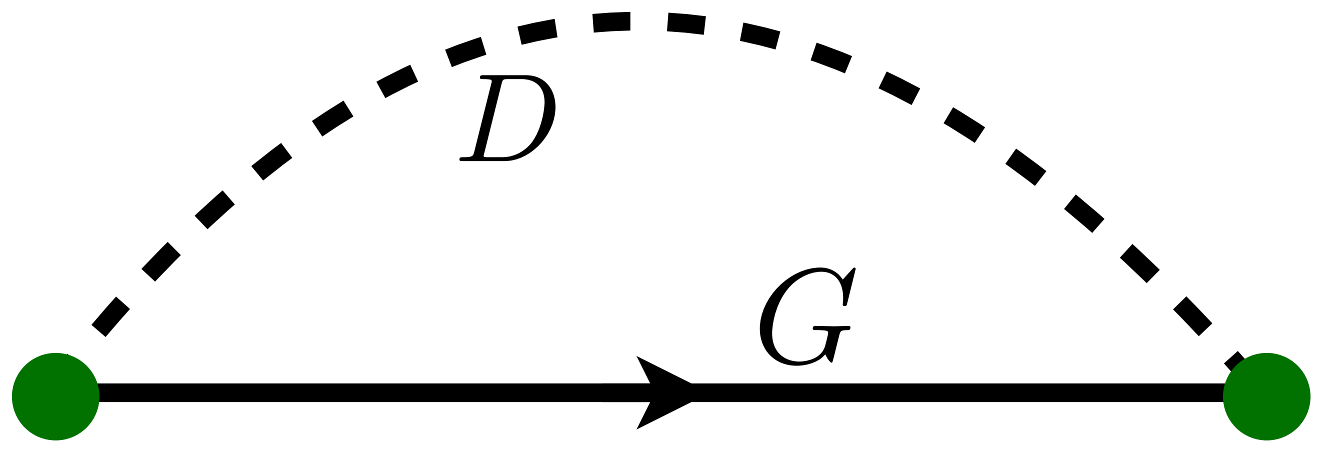
The saddle point equations are

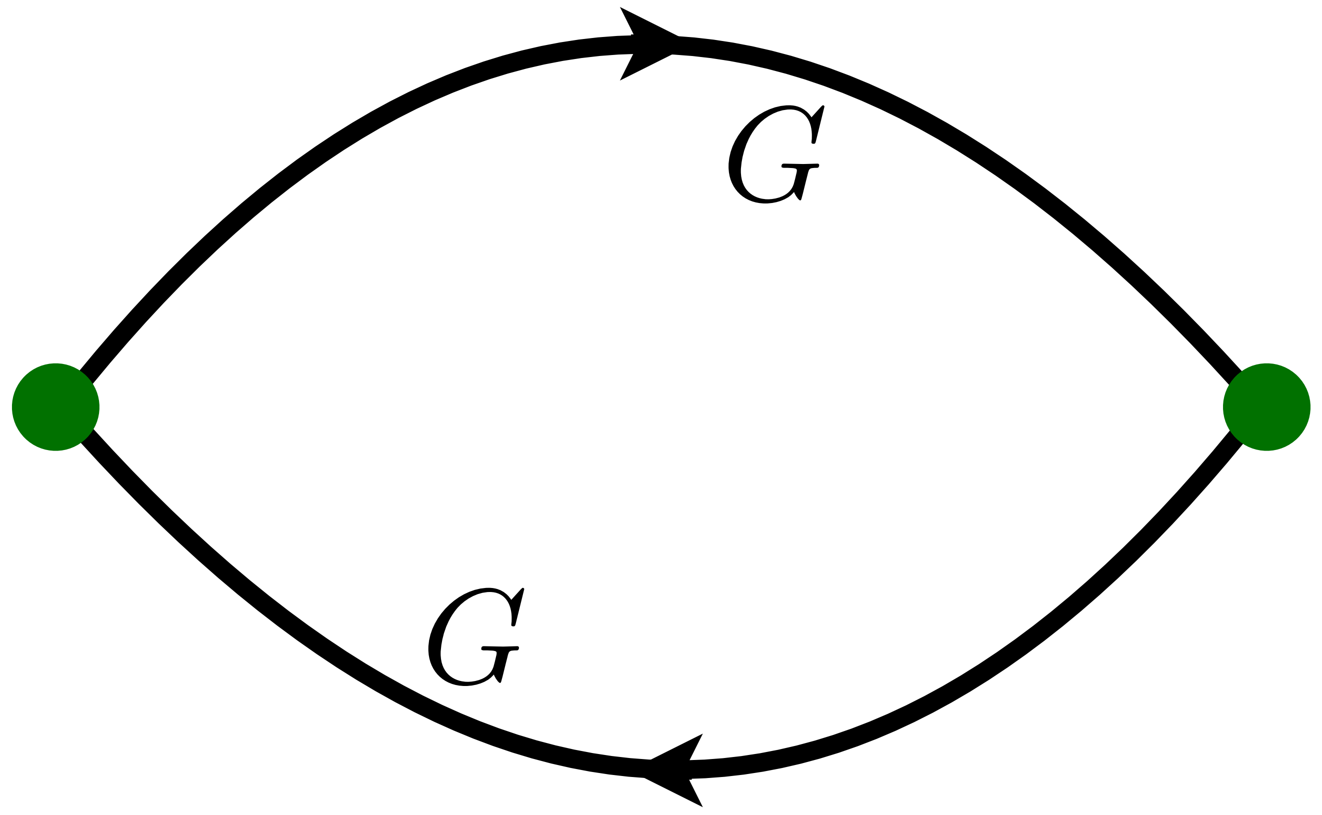
$$\Sigma(\boldsymbol{r},\tau) = g^2\lambda D(\boldsymbol{r},\tau)G(\boldsymbol{r},\tau),$$

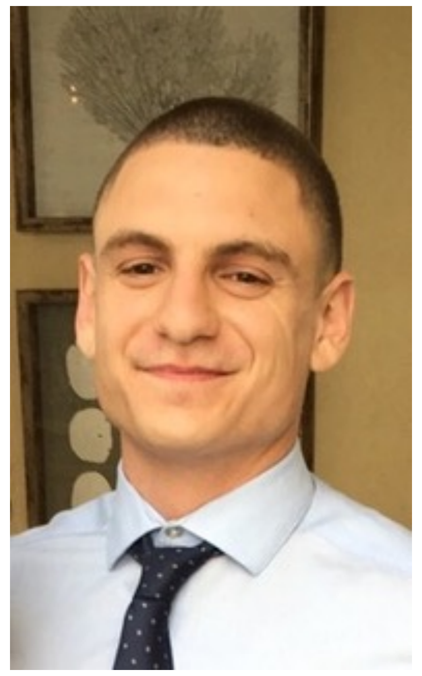
$$\Pi(\boldsymbol{r},\tau) = -g^2 G(-\boldsymbol{r},-\tau)G(\boldsymbol{r},\tau),$$

$$G(\boldsymbol{k}, i\omega_n) = \frac{1}{i\omega_n - \varepsilon(\boldsymbol{k}) - \Sigma(\boldsymbol{k}, i\omega_n)},$$

$$D(\boldsymbol{q}, i\Omega_m) = \frac{1}{\Omega_m^2 + q^2 + s - \Pi(\boldsymbol{q}, i\Omega_m)}.$$

$\Sigma =$ 

$\Pi =$ 



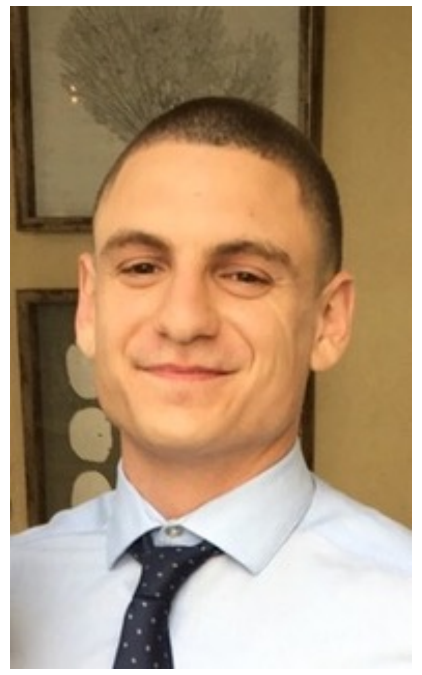
Exact Solution at small ω:

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{2/3}\,, \quad G(\boldsymbol{k}, i\omega) = \frac{-1}{\varepsilon(\boldsymbol{k}) + \Sigma(\hat{\boldsymbol{k}}, i\omega)}$$

where the co-efficient is known exactly in terms of the Fermi velocity and Fermi surface curvature at the Fermi surface point along the direction $\hat{\boldsymbol{k}}$.

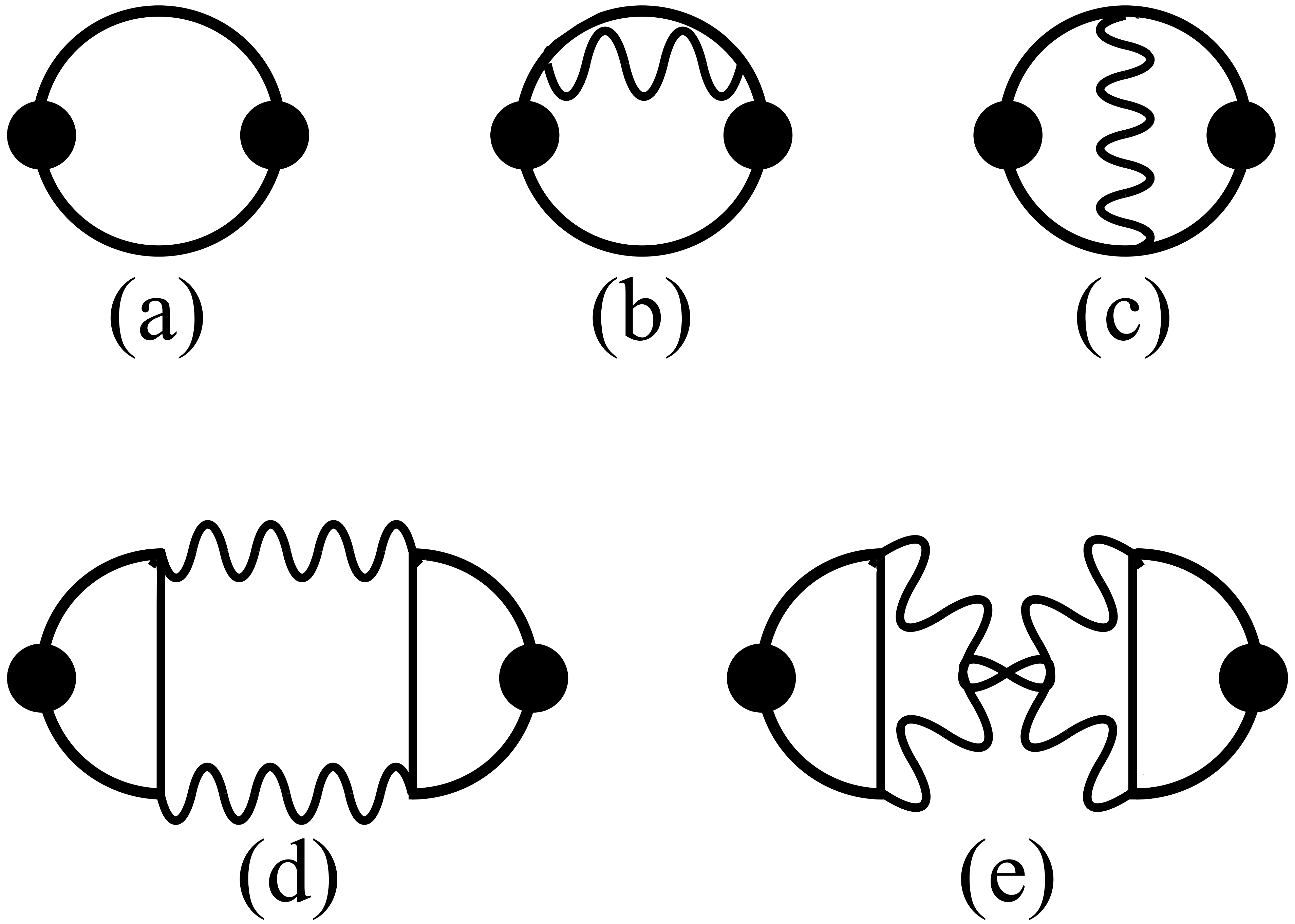
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

Fermi surface coupled to a critical boson



"Yukawa" coupling: $$\frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$



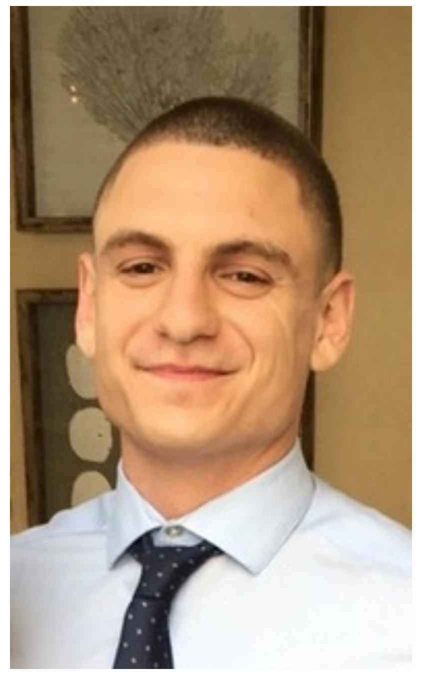
+ all ladders and bubbles…..

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994) examined these graphs and concluded that the d.c. resistivity $\rho(T) \sim T^{4/3}$ and $\sigma(\omega \gg T) \sim \omega^{-2/3}$.

These results do not account for conservation of total momentum *i.e.* 'boson drag'.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson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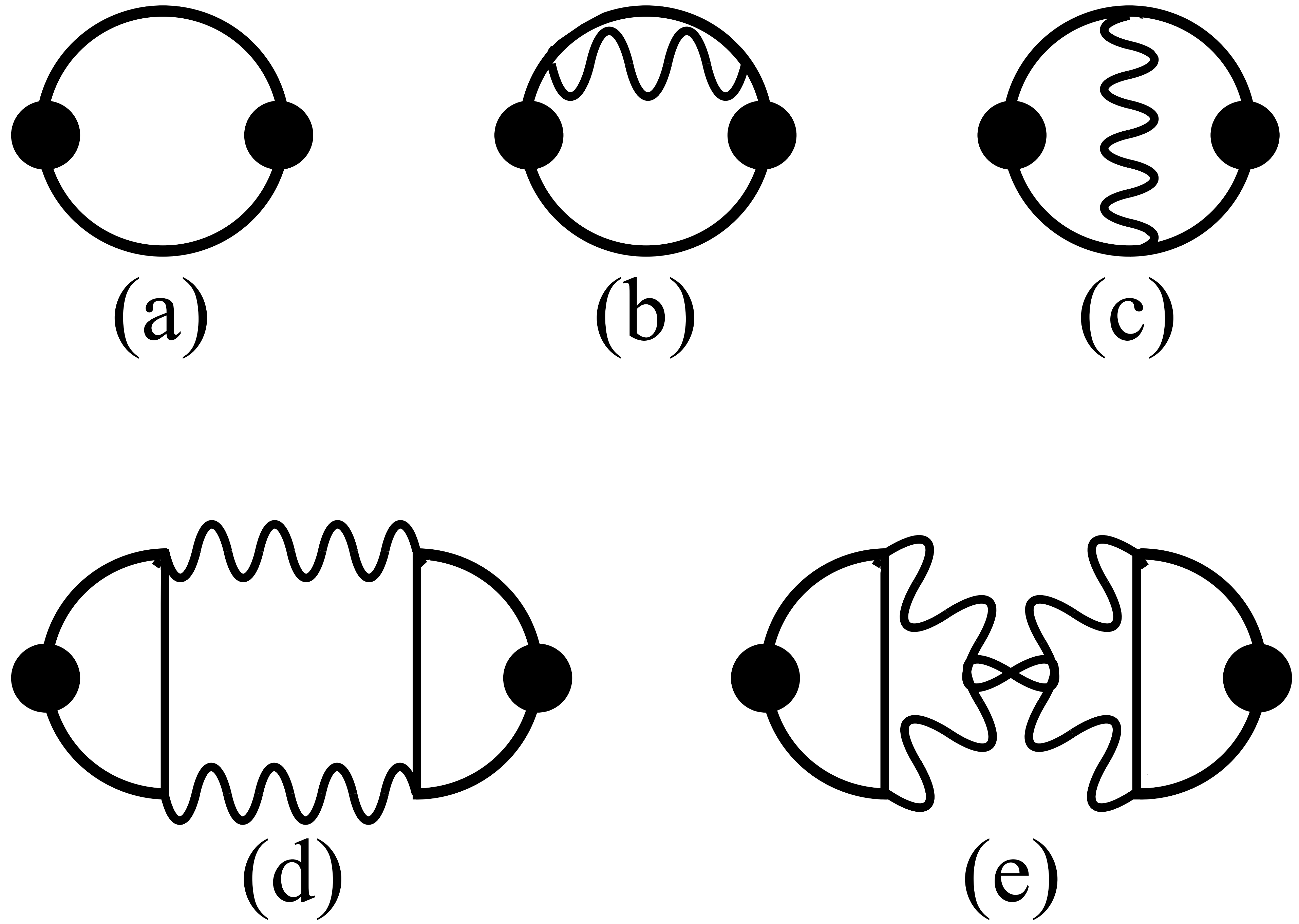


+ all ladders and bubbles…..

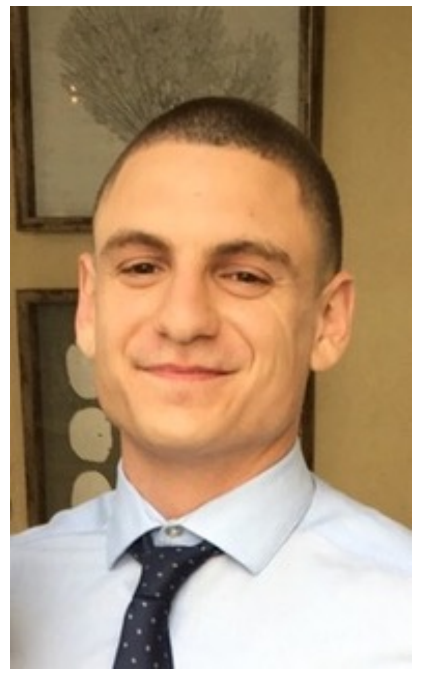
Conservation of momentum implies the d.c. conductivity is infinite

$$\mathrm{Re}\,\sigma(\omega) = D\delta(\omega) + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson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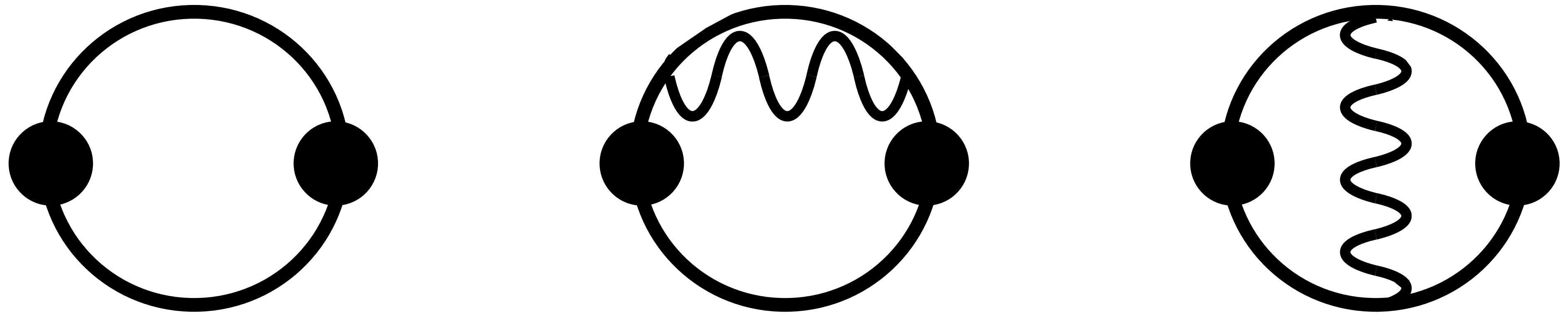
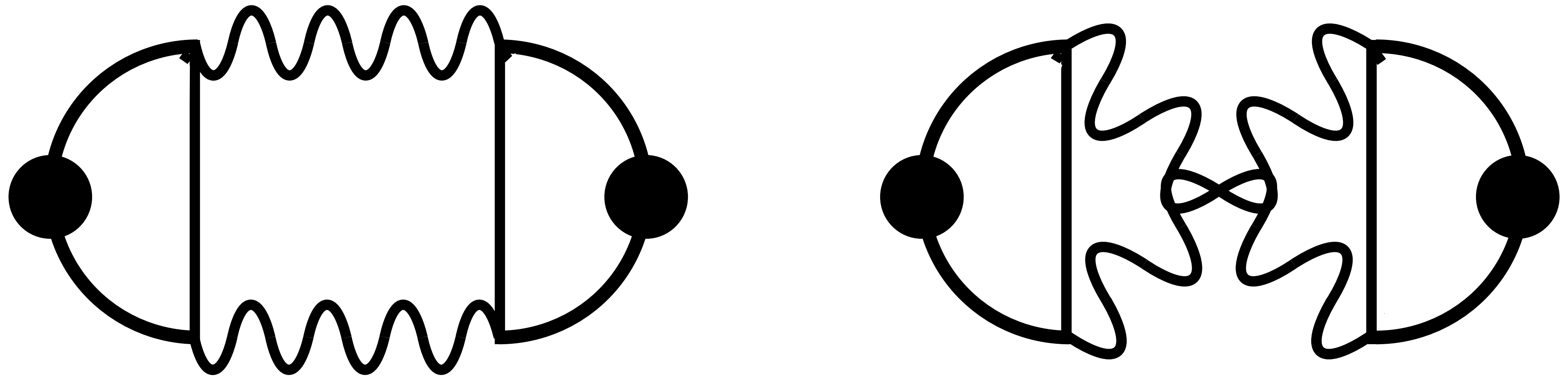


(a) (b) (c)



(d) (e)

+ all ladders and bubbles…..

Conservation of momentum implies the d.c. conductivity is infinite

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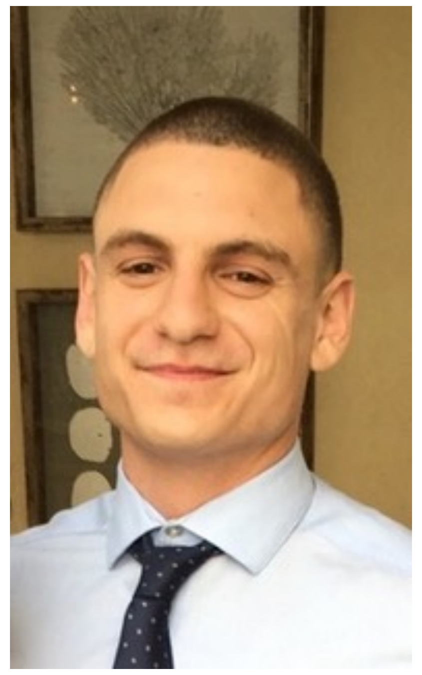
S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

$$\sigma(\omega) \sim \frac{1}{-i\omega} + |\omega|^0 + \ldots$$

Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder

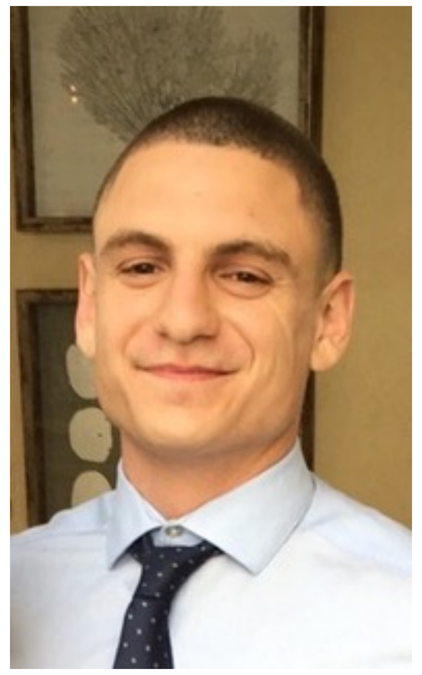

"Yukawa" coupling: $$\frac{g_{ij\ell}}{N} \int d^2r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

Random potential: $$+ \frac{1}{\sqrt{N}} \int d^2r d\tau \, v_{ij}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{g^*_{ijl} g_{abc}} = g^2 \, \delta_{ia}\delta_{jb}\delta_{lc} \quad , \quad \overline{v_{ij}(r)} = 0 \quad , \quad \overline{v^*_{ij}(r) v_{lm}(r')} = v^2 \, \delta(r - r')\delta_{il}\delta_{jm}$$

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder


"Yukawa" coupling: $\dfrac{g_{ij\ell}}{N}\displaystyle\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

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Marginal Fermi liquid self energy and *T log T* specific heat

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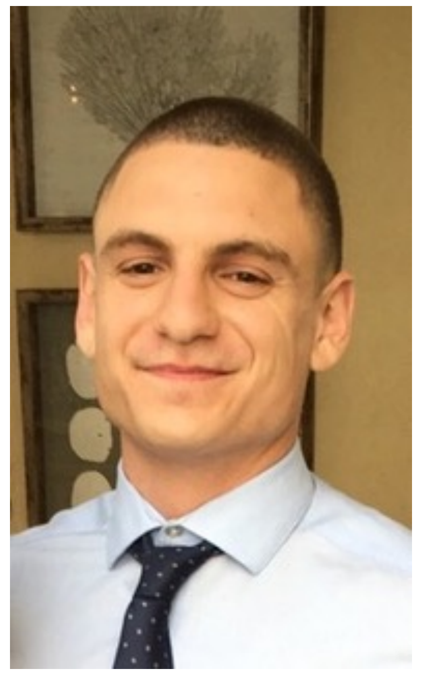

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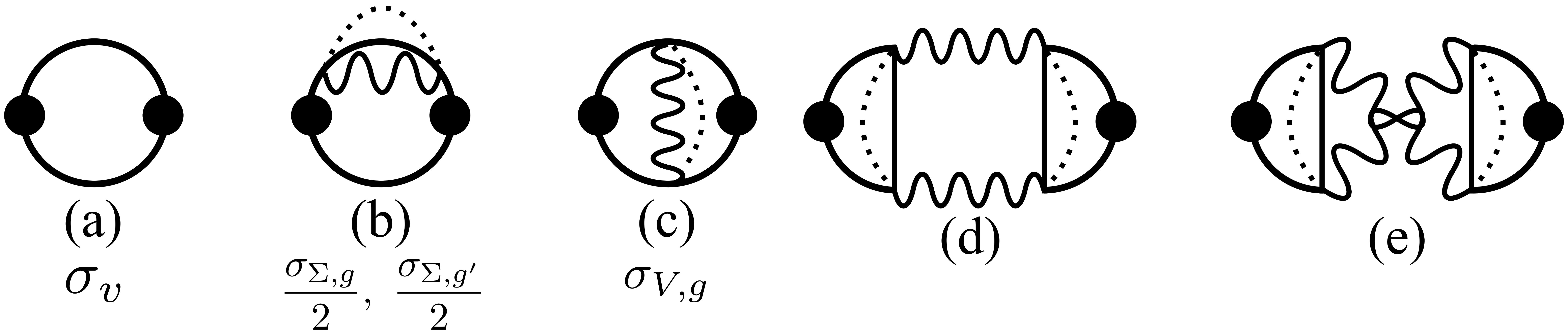

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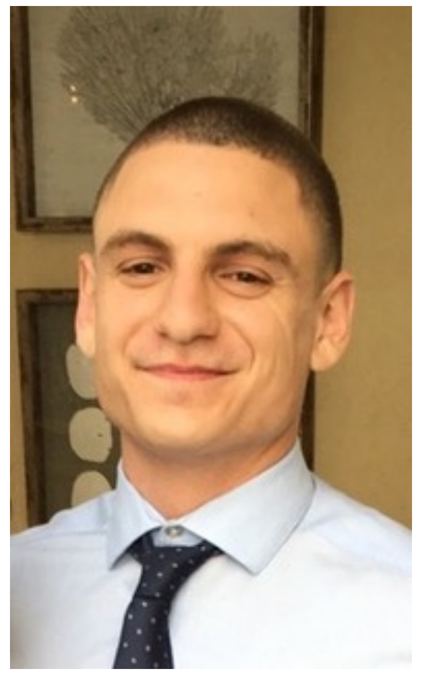


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The g^2 log term does not contribute to transport

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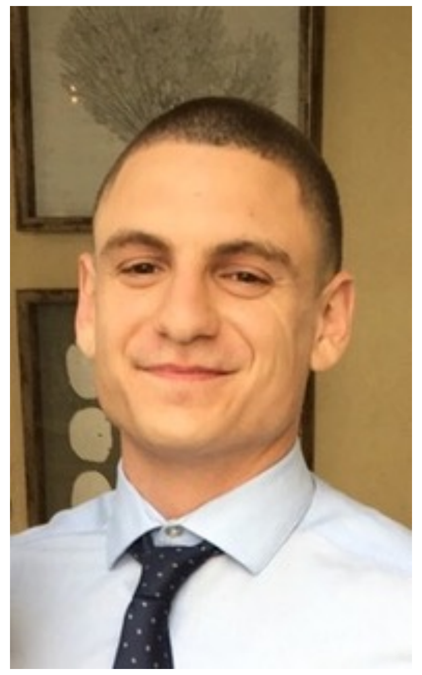

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With g and v non-zero, we obtain a non-zero residual resistivity and Fermi liquid like corrections
$$\rho(T) = \rho(0) + AT^2 + \ldots.$$
with $1/\rho(0) \sim 1/\tau_{\rm trans} \sim v^2$.

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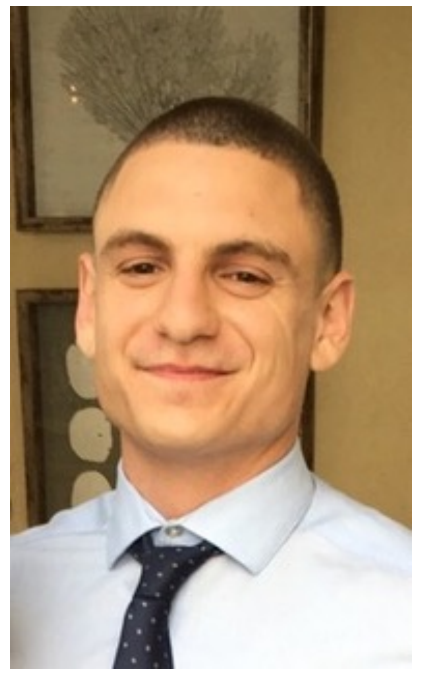


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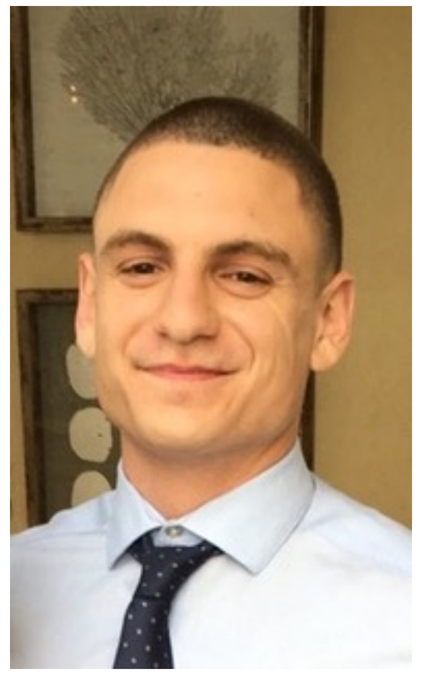

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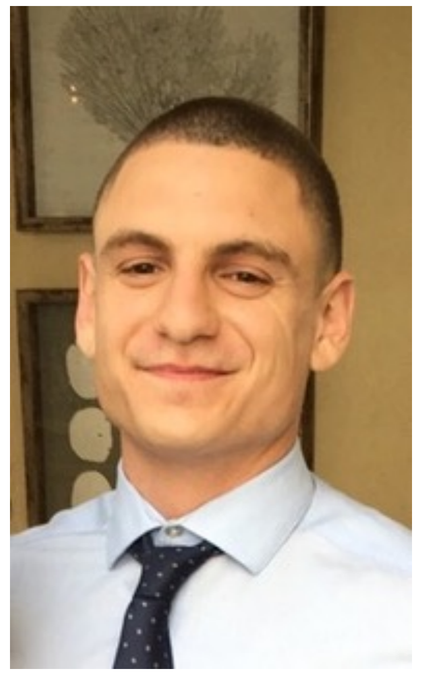


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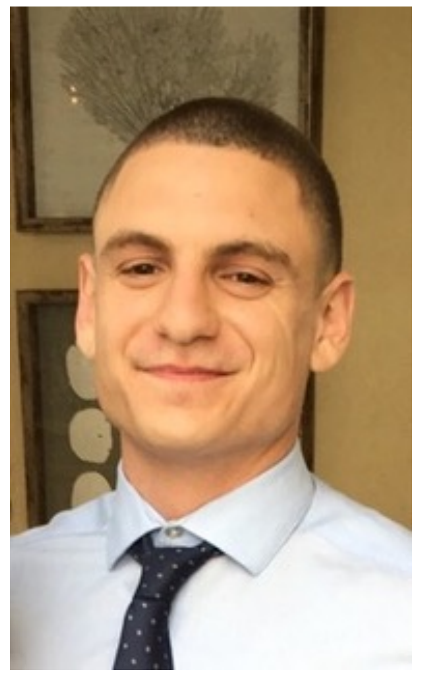


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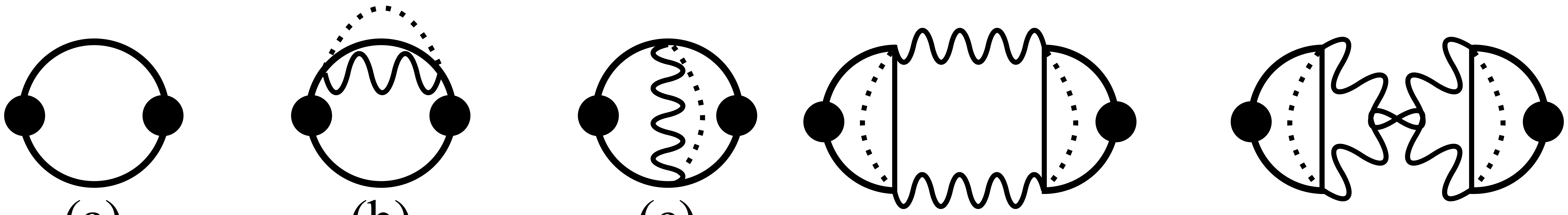

(a) σ_v

(b) $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$

(c) $\sigma_{V,g}$

(d)

(e)

+ all ladders and bubbles.....

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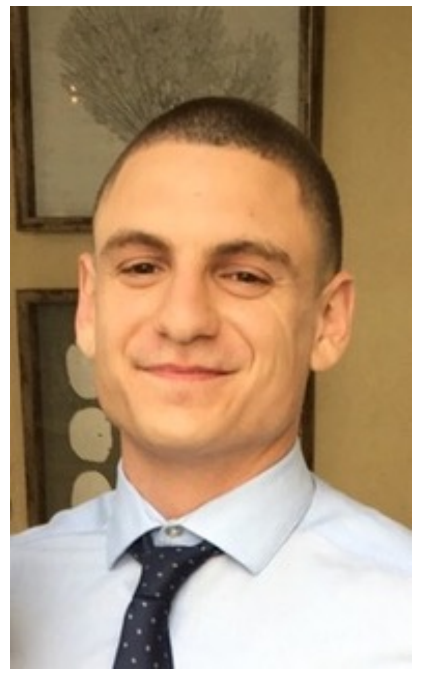


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The g^2 log term does not contribute to transport but the g'^2 log term does!

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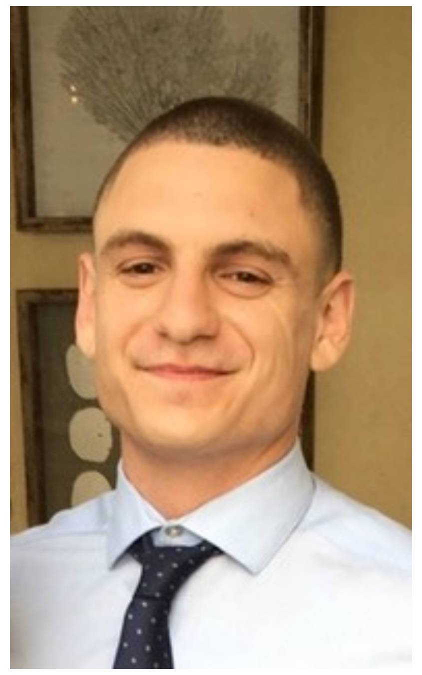

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Conductivity: $\sigma(\omega) \sim [1/\tau_{\text{trans}}(\omega) - i\omega \, m^*(\omega)/m]^{-1}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2.

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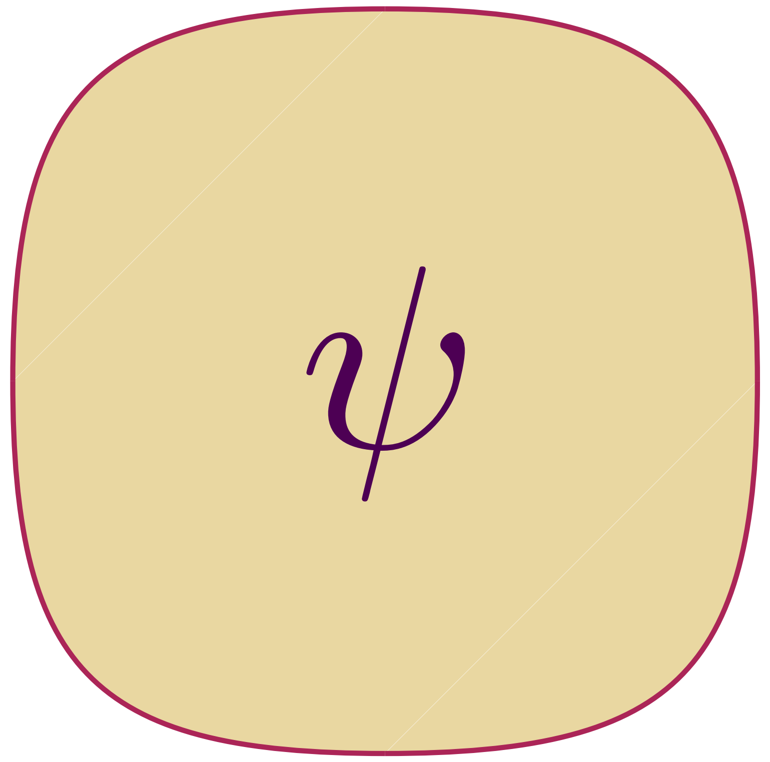
Strange metal from a Yukawa-SYK model



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Strange metal from a Yukawa-SYK model

ψ + a critical boson ϕ

Non-Fermi liquid with $T^{2/3}$ specific heat,
but conductivity $\sigma(\omega) \sim \delta(\omega)$

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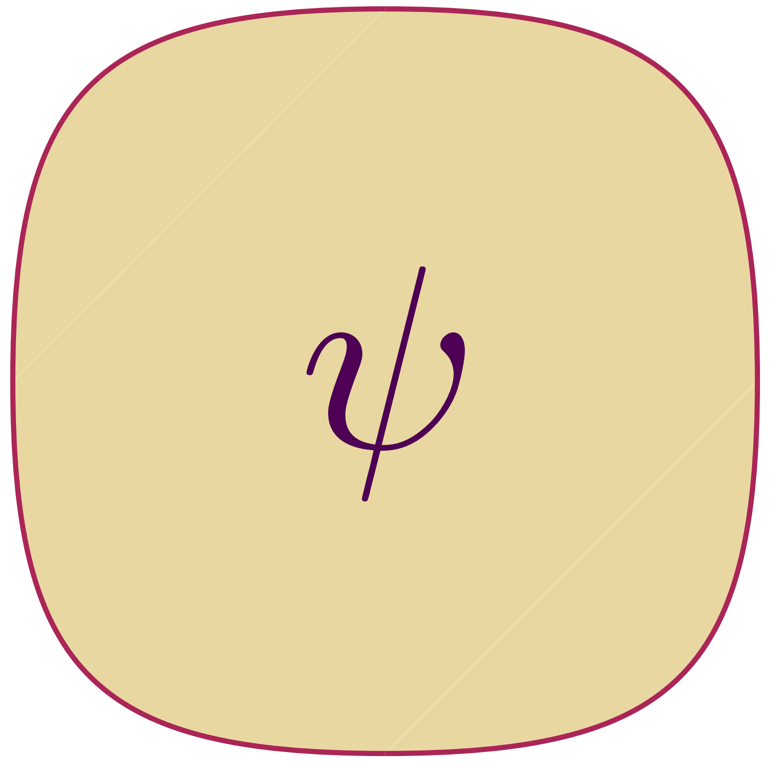

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MFL self-energy, $T \ln(1/T)$ specific heat, but T-independent 'residual' resistivity, and negligible optical conductivity

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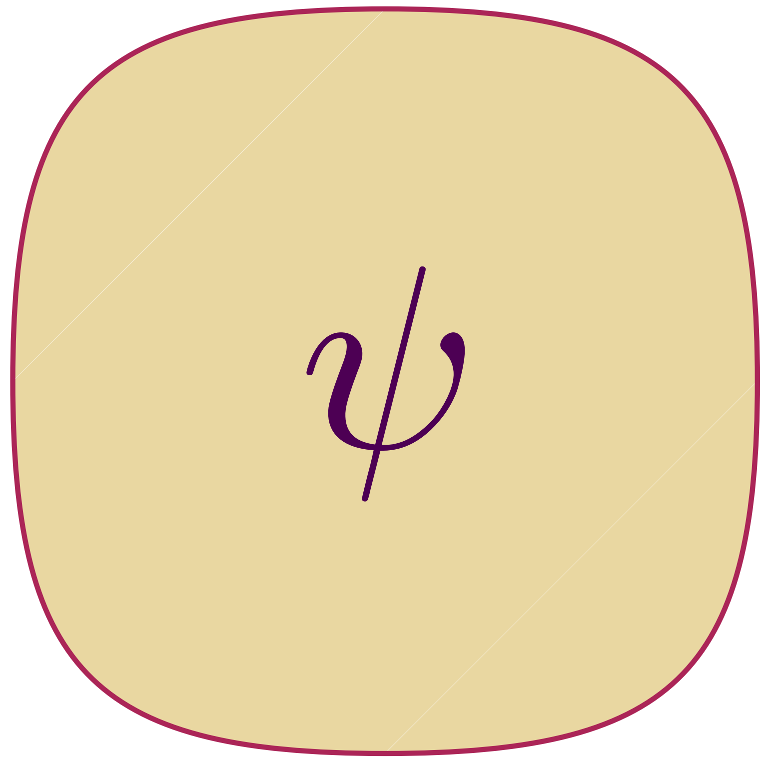

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MFL self-energy, $T\ln(1/T)$ specific heat, linear-T resistivity and $1/[\omega - i(2\omega/\pi)\ln(\Lambda/\omega)]$ optical conductivity

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Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

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- Low energy theory of time reparameterizations is the theory of the boundary graviton in 1+1 dimensional quantum gravity on AdS_2.

Summary

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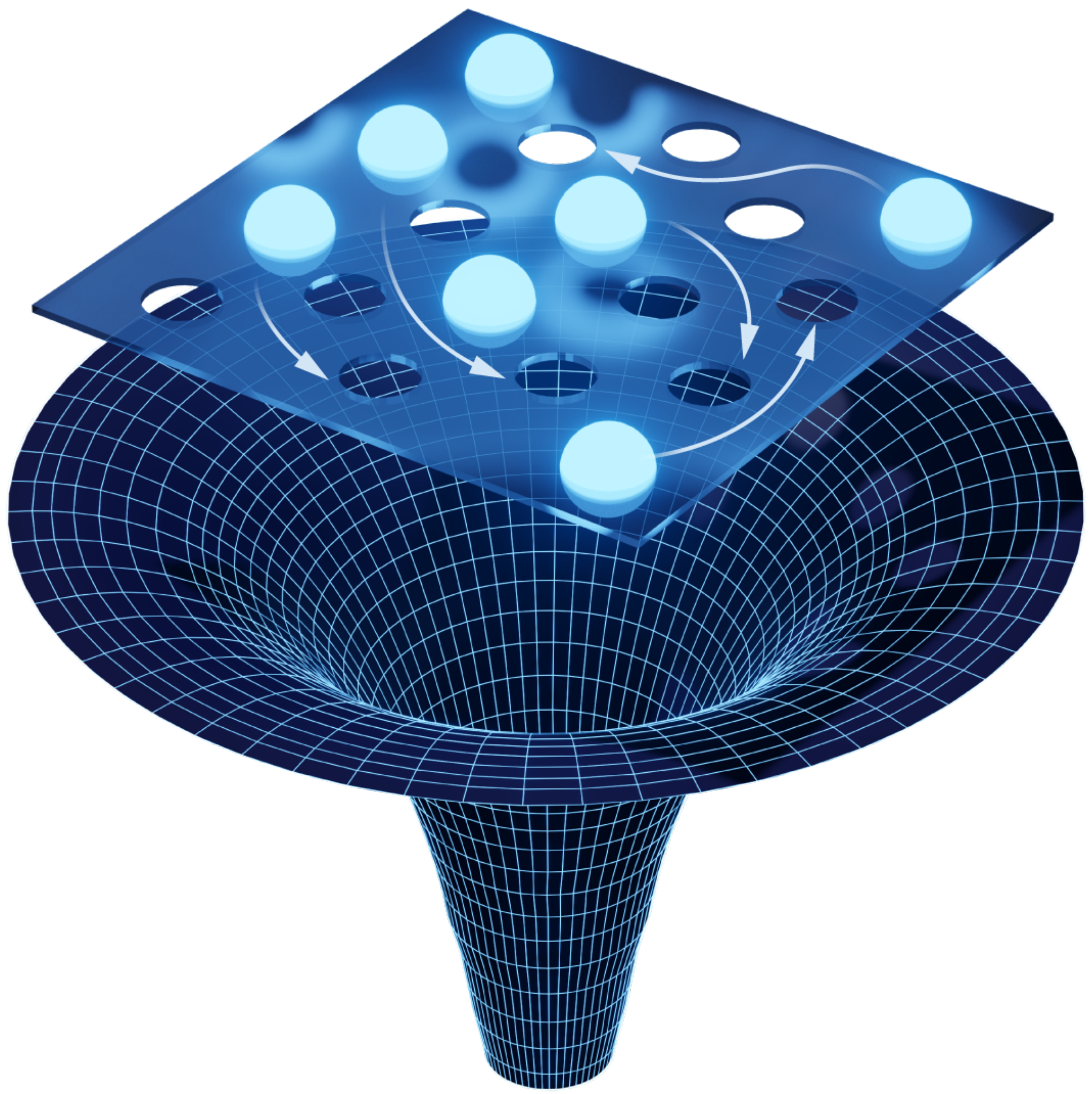

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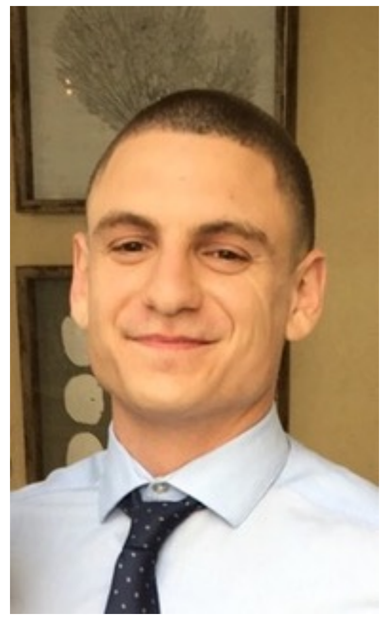
- Linear-T resistivity arises from spatially random interactions in a two-dimensional quantum-critical metal.





Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990