Theory of the competition between spin density waves and d-wave superconductivity in the underdoped cuprates

Talk online: sachdev.physics.harvard.edu

PHYSICS

VE RI TAS

HARVARD

Where is the quantum critical point in the cuprate superconductors ?

arXiv:0907.0008

Talk online: sachdev.physics.harvard.edu

PHYSICS

VE RI TAS

HARVARD

Hole dynamics in an antiferromagnet across a deconfined quantum critical point,
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil,
Phys. Rev. B **75**, 235122 (2007).

Algebraic charge liquids
R. K. Kaul, Yong-Baek Kim, S. Sachdev, and T. Senthil,
Nature Physics **4**, 28 (2008).

Destruction of Neel order in the cuprates by electron doping,
R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu,
Physical Review B **78**, 045110 (2008).

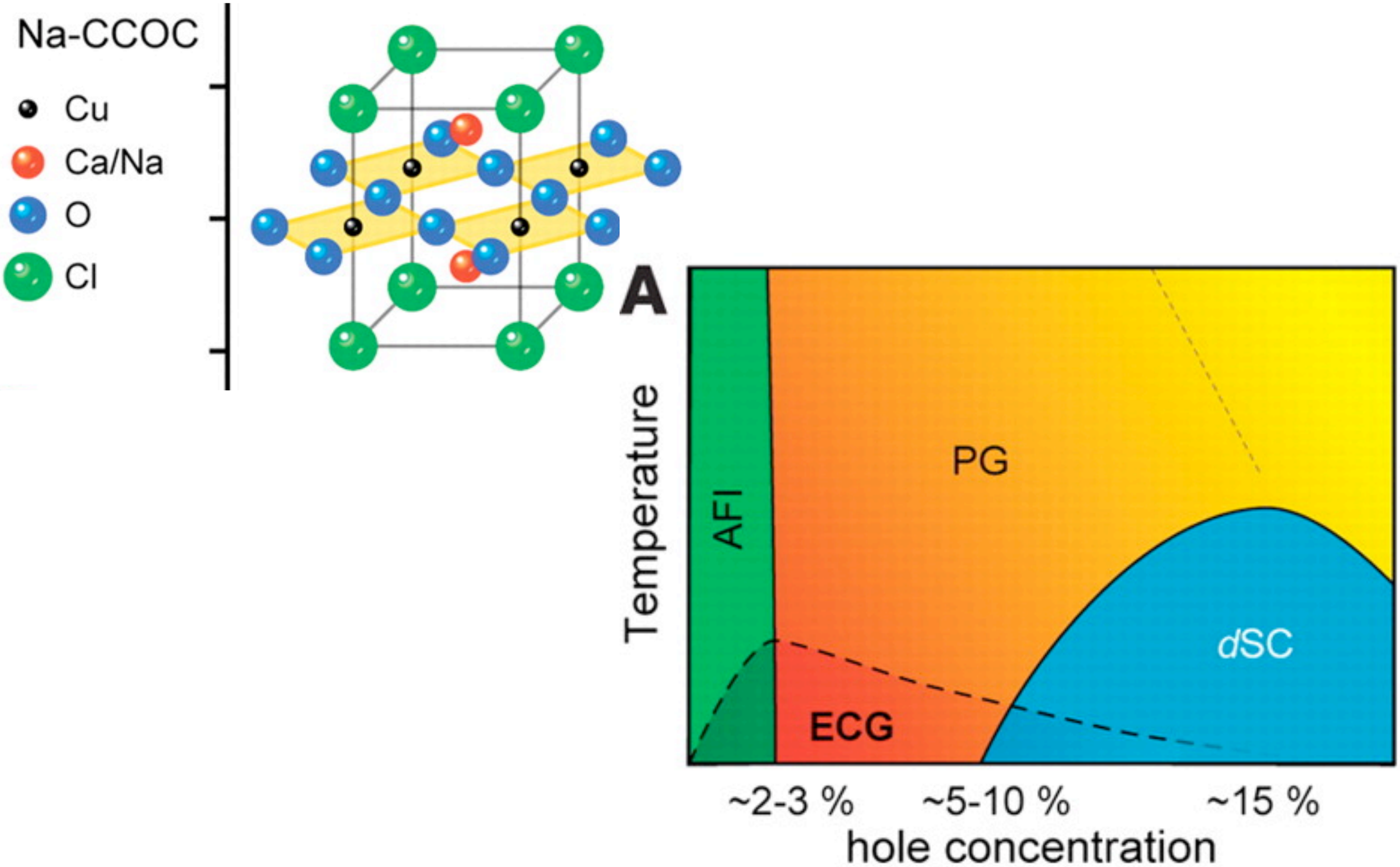
Paired electron pockets in the underdoped cuprates,
V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

Competition between spin density wave order and superconductivity in the underdoped cuprates,
Eun Gook Moon and S. Sachdev,
Physical Review B **80**, 035117 (2009).

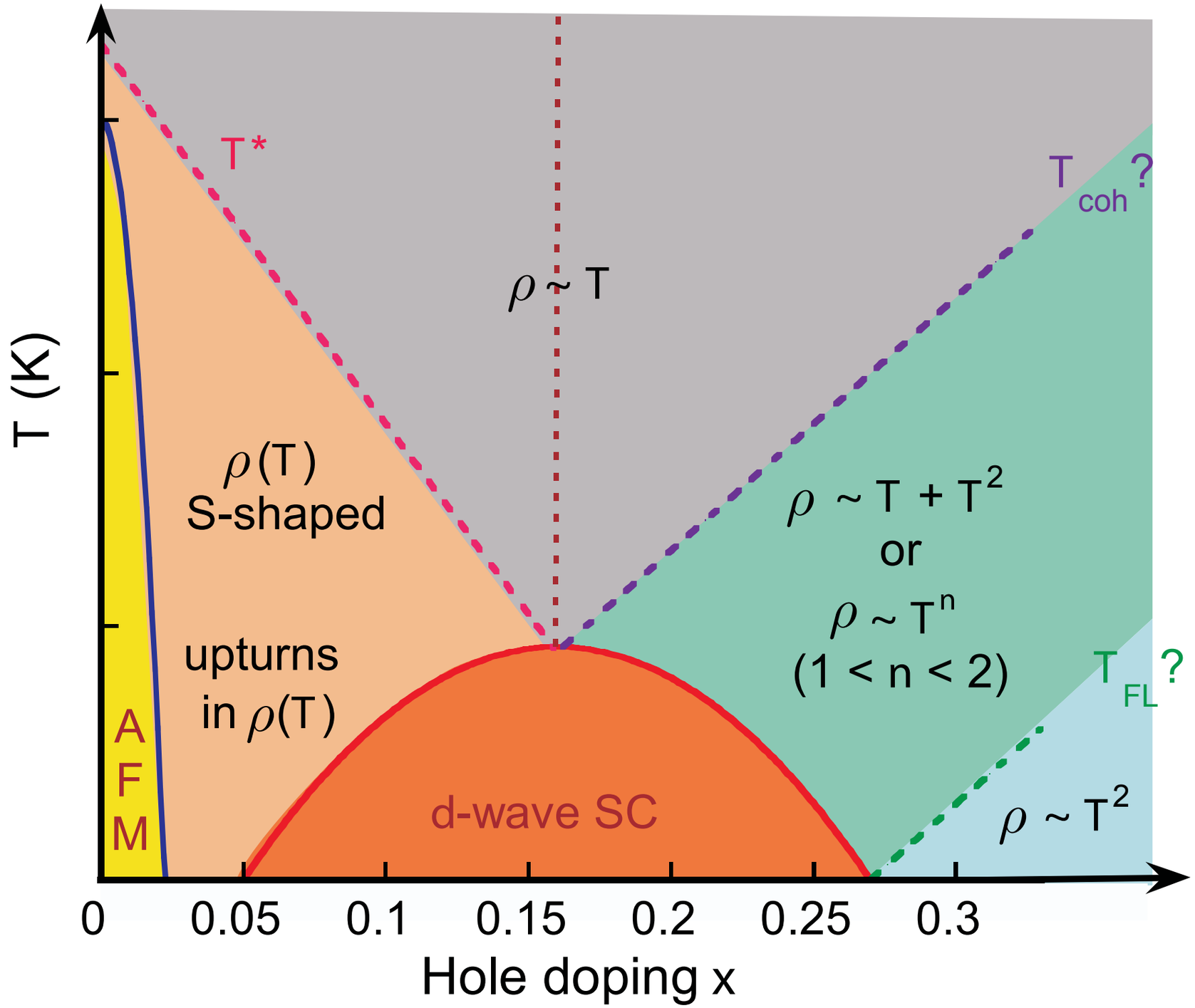
Fluctuating spin density waves in metals
S. Sachdev, M. Metlitski, Yang Qi, and Cenke Xu
arXiv:0907.3732

PHYSICS
VE RI TAS
HARVARD

The cuprate superconductors

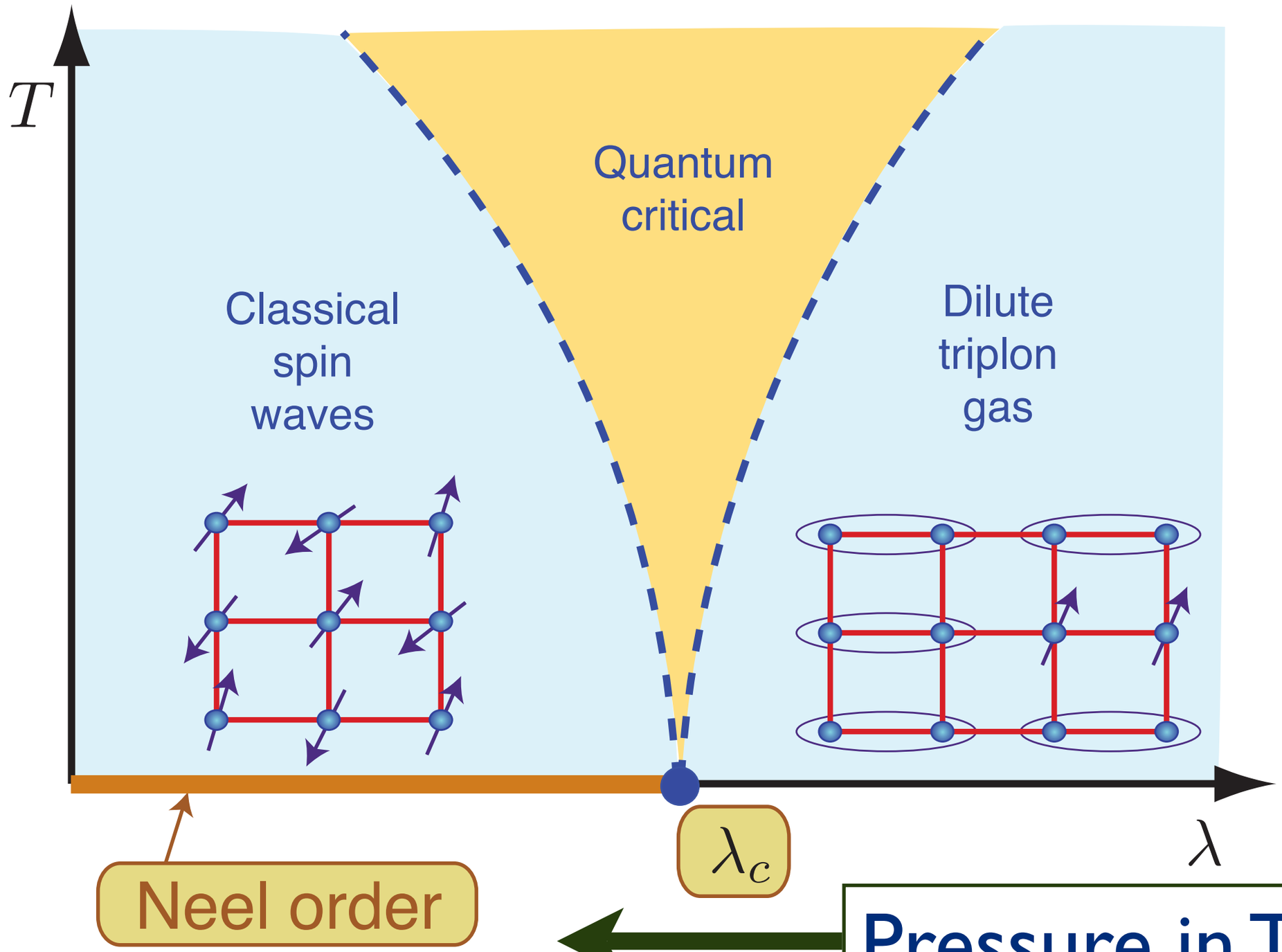


Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Canonical quantum critical phase diagram of coupled-dimer antiferromagnet

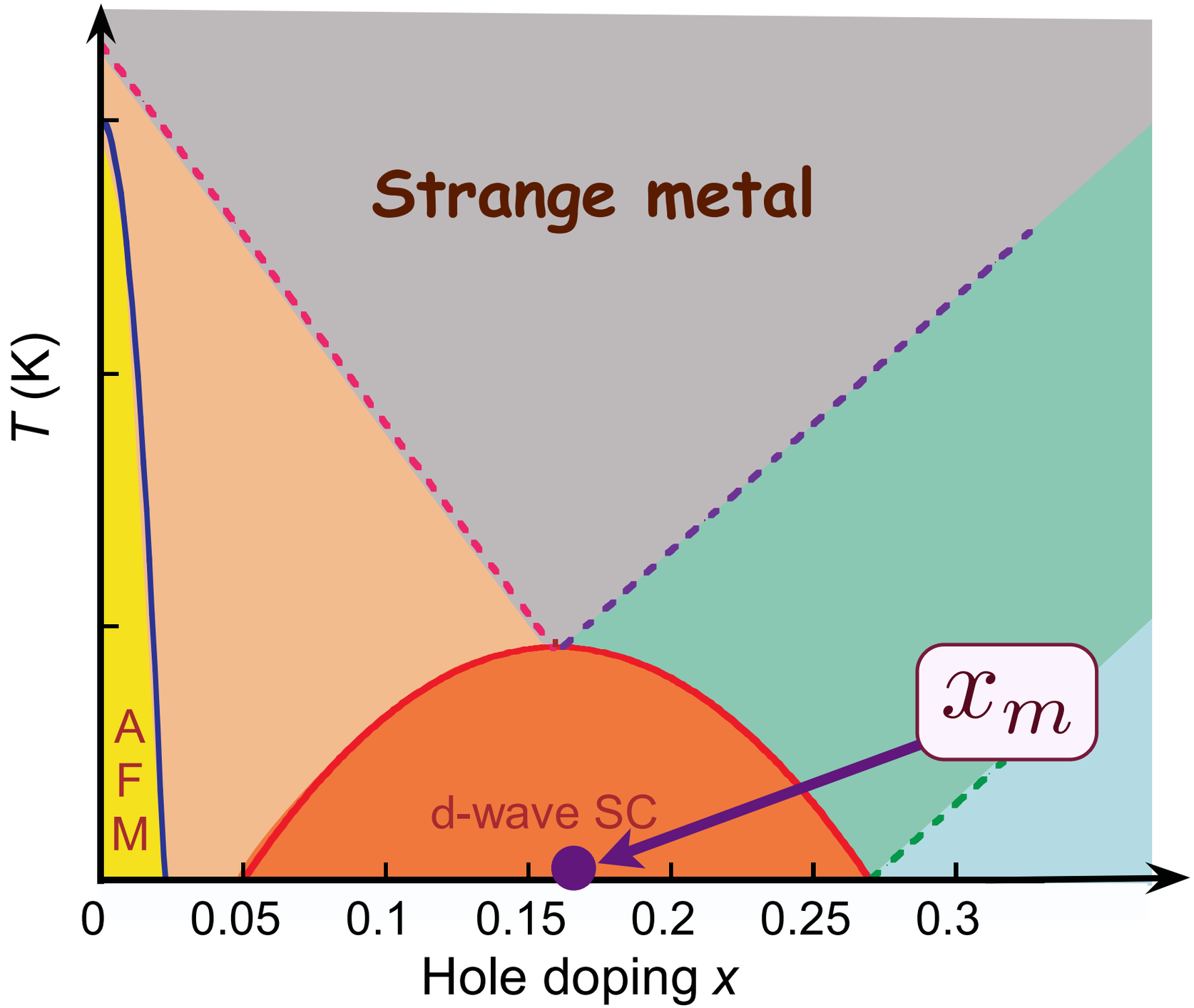


S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

Pressure in $TlCuCl_3$

Christian Ruegg et al. , *Phys. Rev. Lett.* **100**, 205701 (2008)

Crossovers in transport properties of hole-doped cuprates

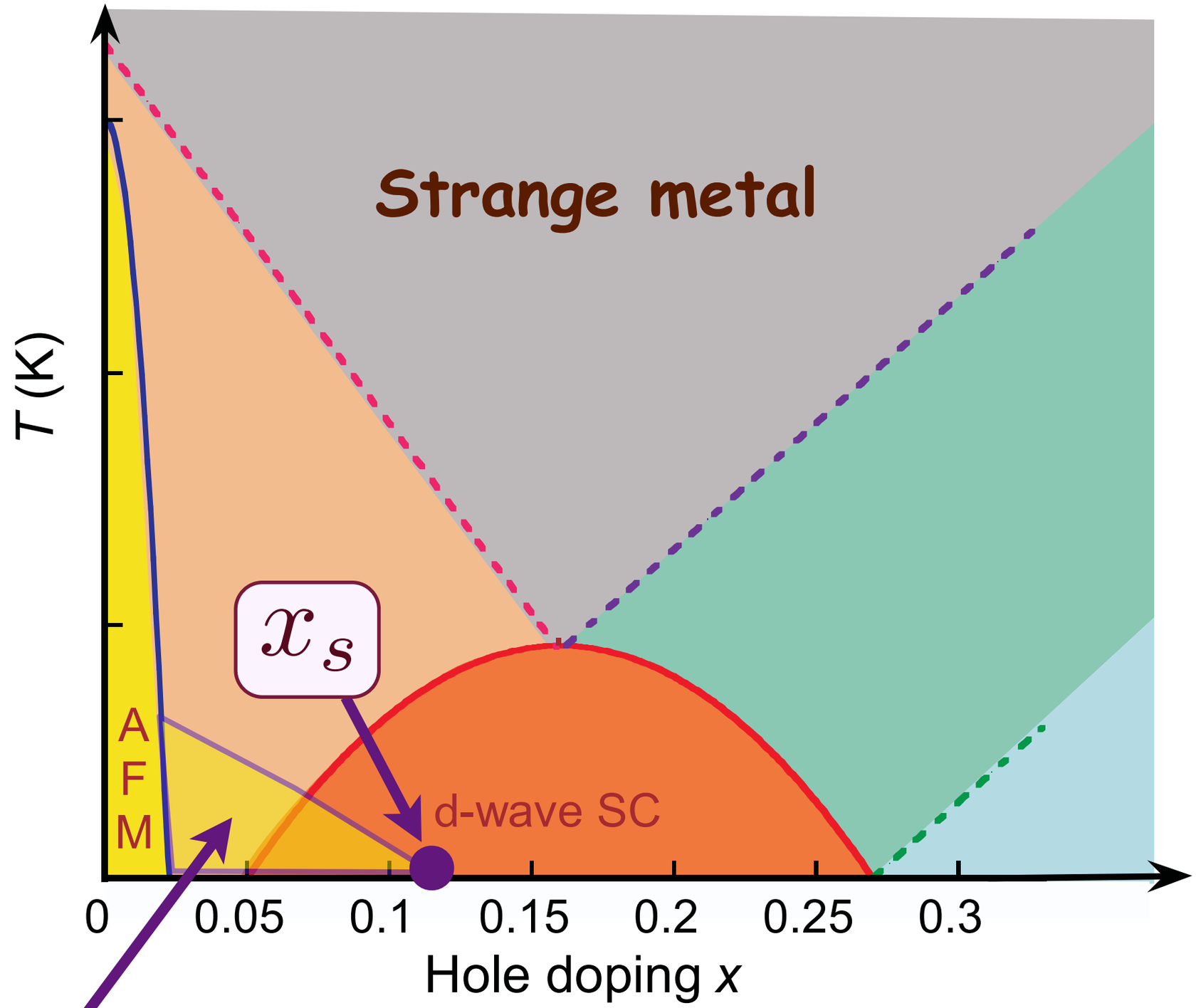


S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

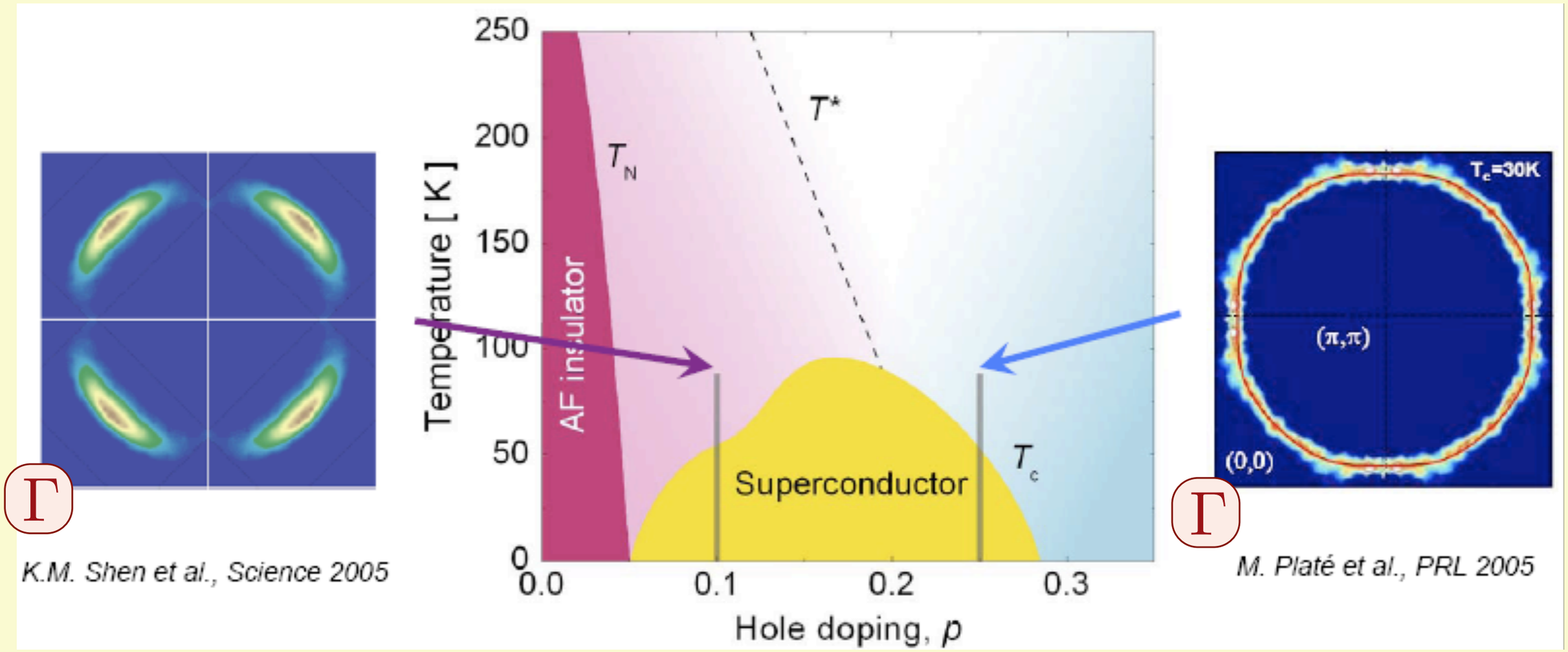
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T



Spin density wave order present
below a quantum critical point at $x = x_s$
with $x_s \approx 0.12$ in the La series of cuprates

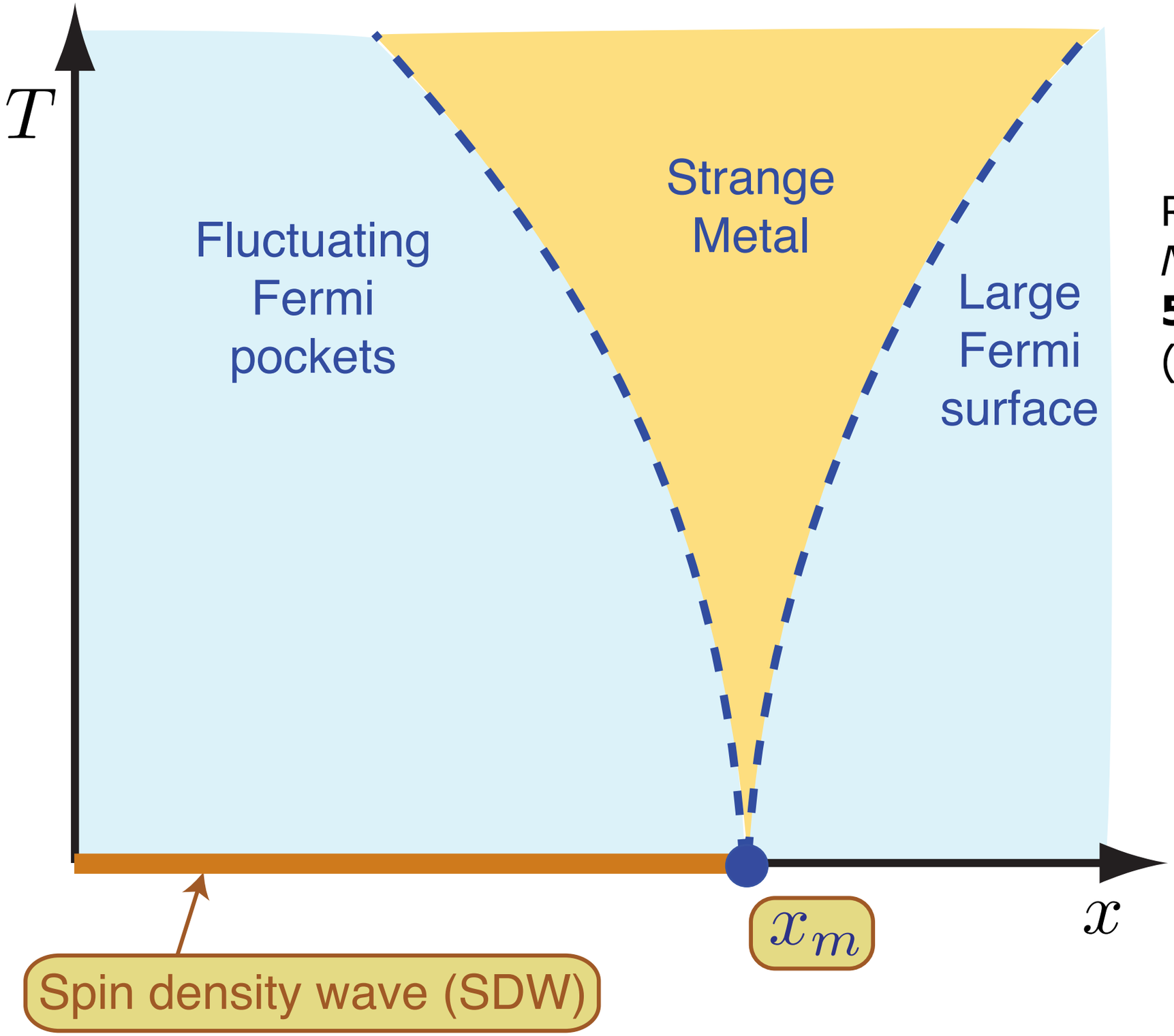
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole Fermi-pockets

Large hole Fermi surface

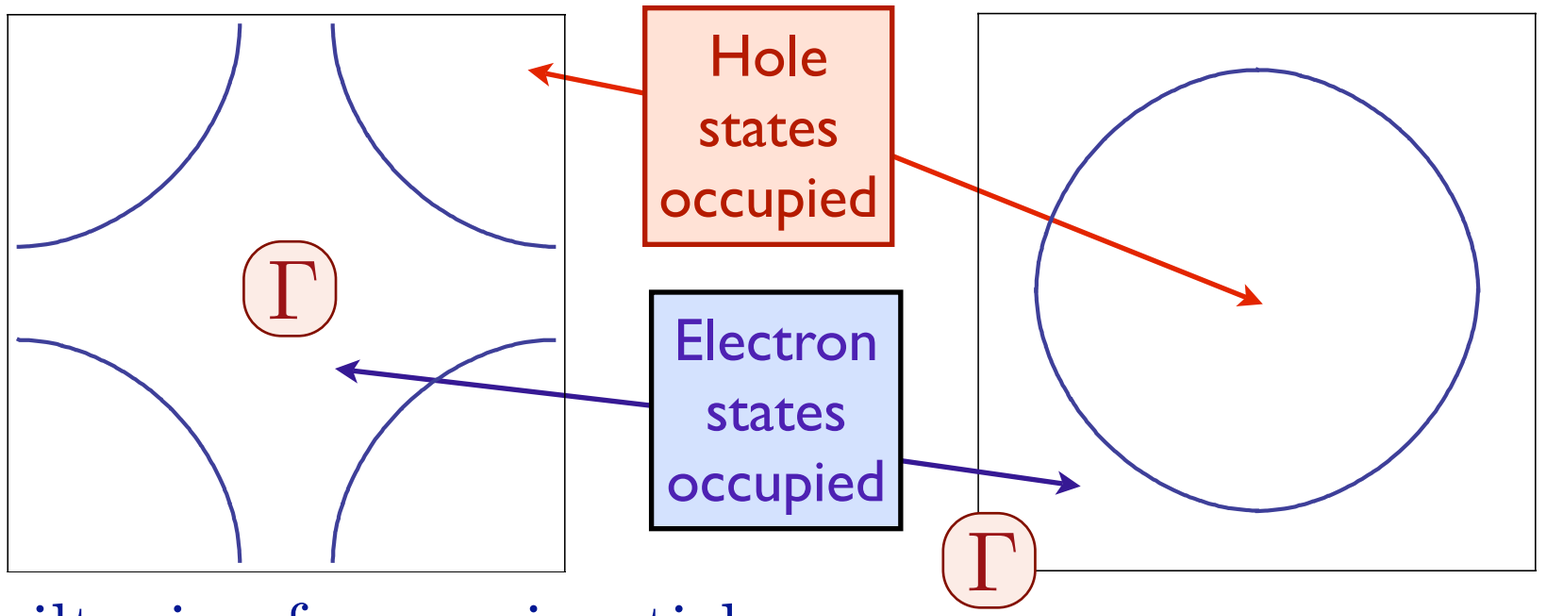
Theory of quantum criticality in the cuprates



R. Daou et al., *Nature Physics* **5**, 31 - 34 (2009)

Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

$$H_0 = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-p) & \text{for hole-doping } p \\ 2\pi^2(1+x) & \text{for electron-doping } x \end{cases}$$

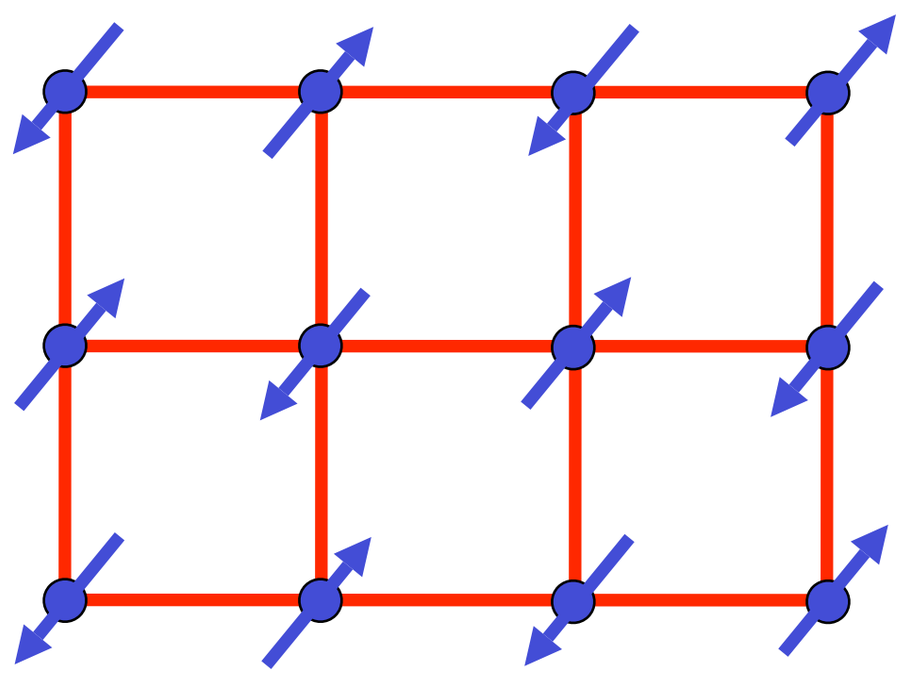
The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Spin density wave theory

A spin density wave (SDW) is the spontaneous appearance of an oscillatory spin polarization. The electron spin polarization is written as

$$\vec{S}(\mathbf{r},\tau) = \vec{\varphi}(\mathbf{r},\tau)e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\vec{\varphi}$ is the SDW order parameter, and $\mathbf{K}$ is a fixed ordering wavevector. For simplicity we will consider the case of $\mathbf{K} = (\pi,\pi)$, but our treatment applies to general $\mathbf{K}$.



Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

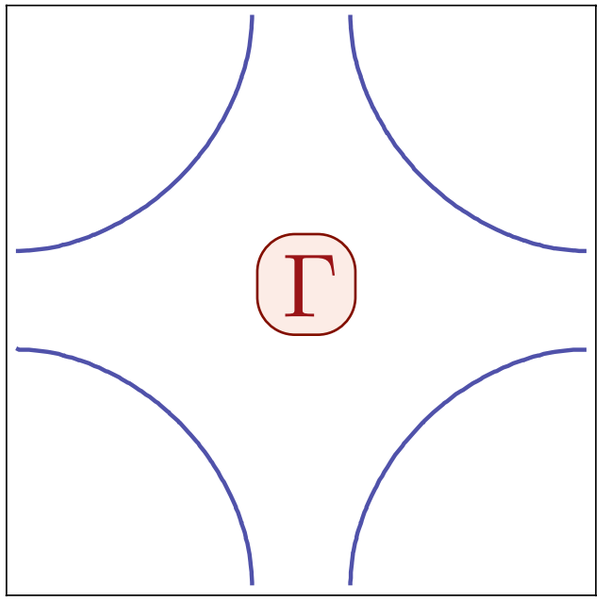
$$H_{\mathrm{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^{\dagger}_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\mathrm{sdw}}$ for $\vec{\varphi} \propto (0, 0, 1)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

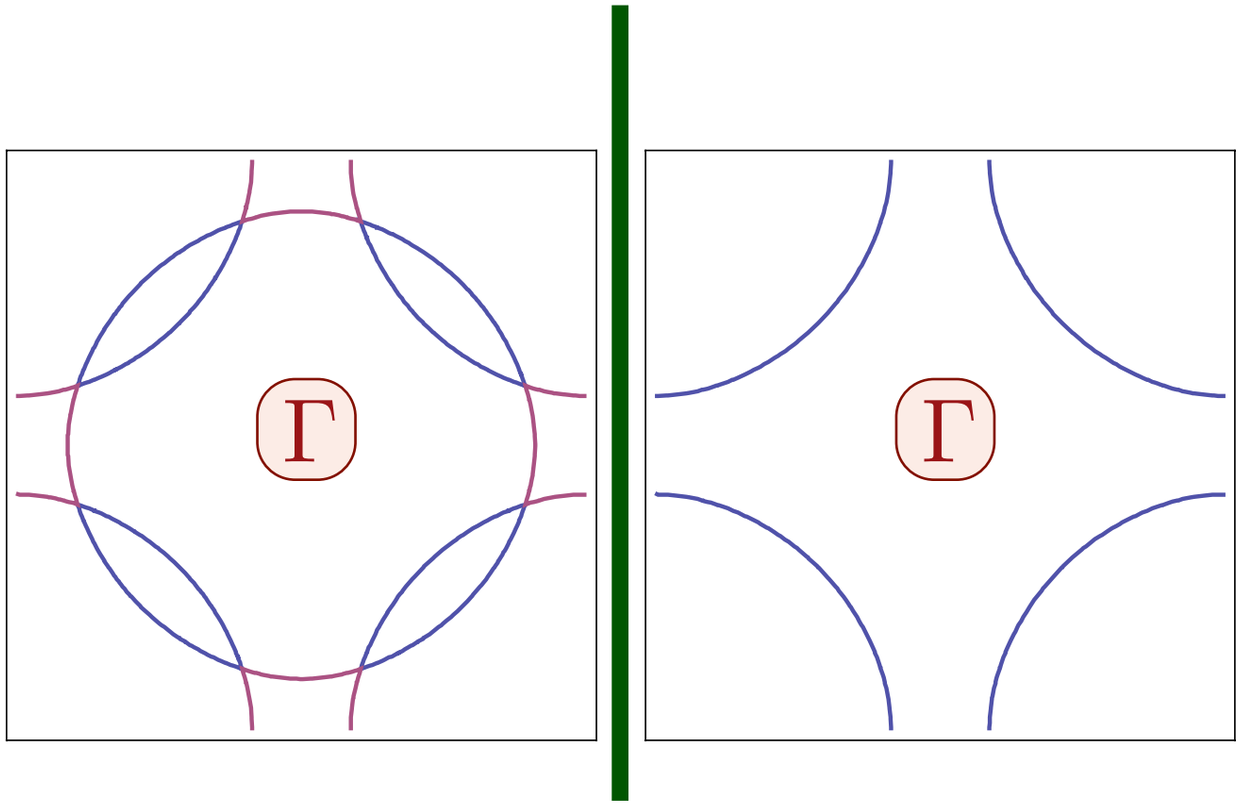
This leads to the Fermi surfaces shown in the following slides for electron and hole doping.

Spin density wave theory in hole-doped cuprates



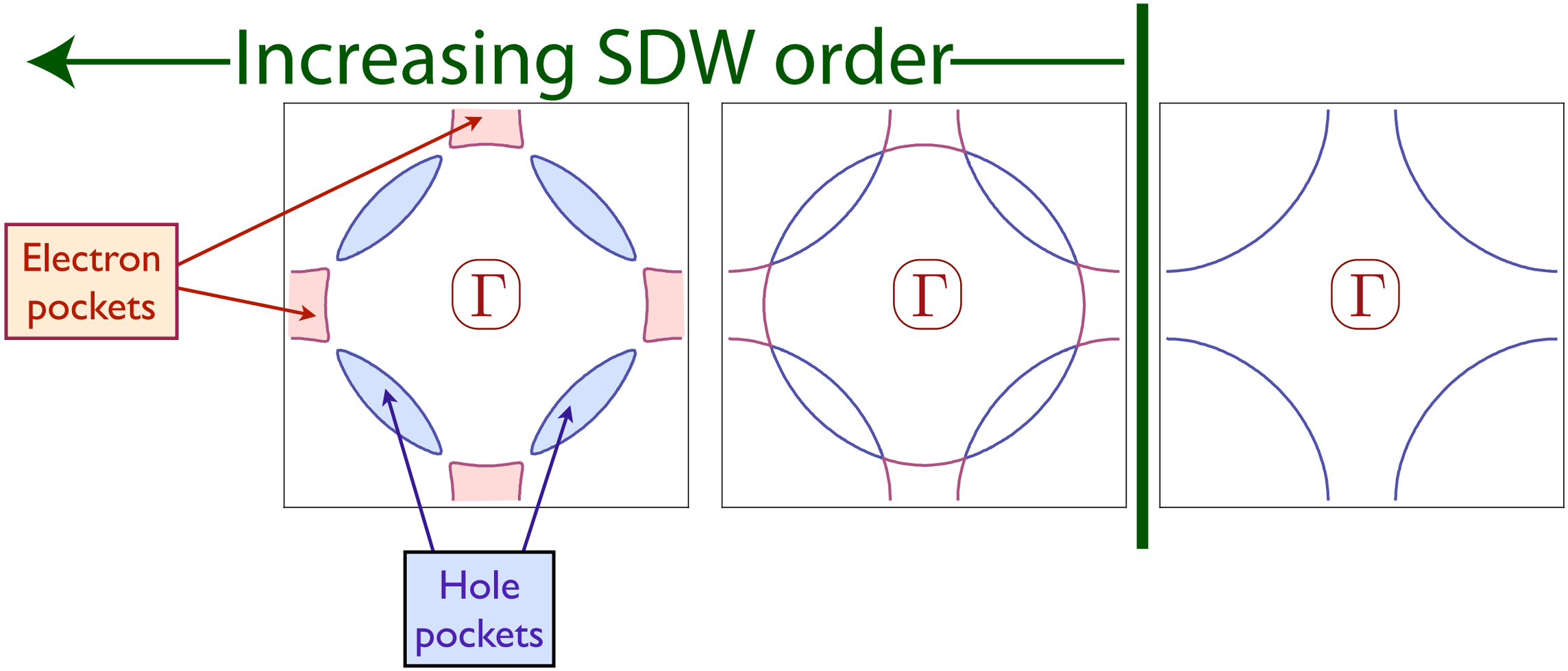
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates



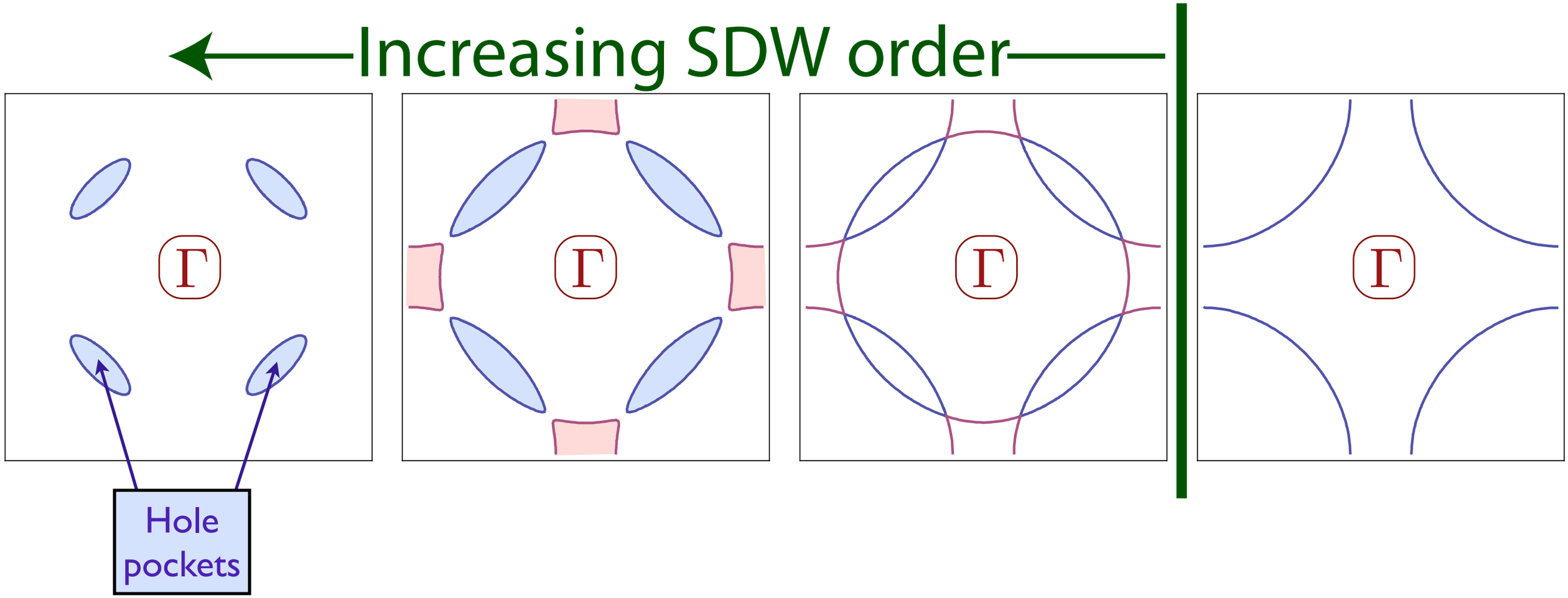
S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Spin density wave theory in hole-doped cuprates



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

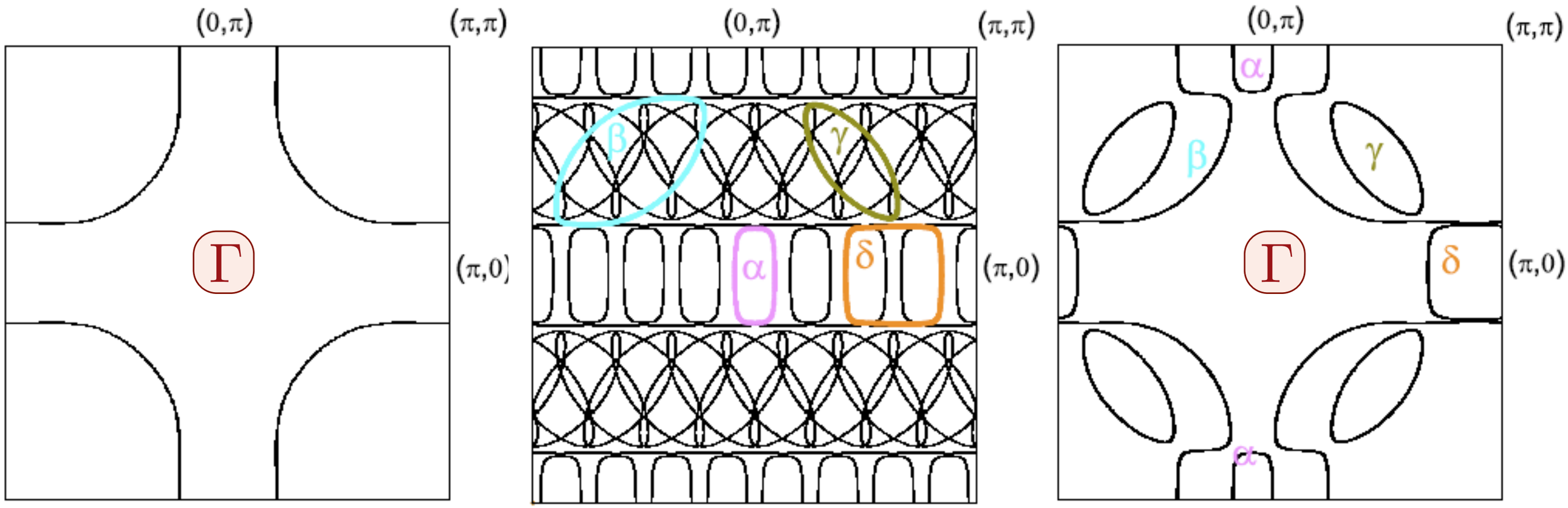
Spin density wave theory in hole-doped cuprates



SDW order parameter is a vector, $\vec{\varphi}$, whose amplitude vanishes at the transition to the Fermi liquid.

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

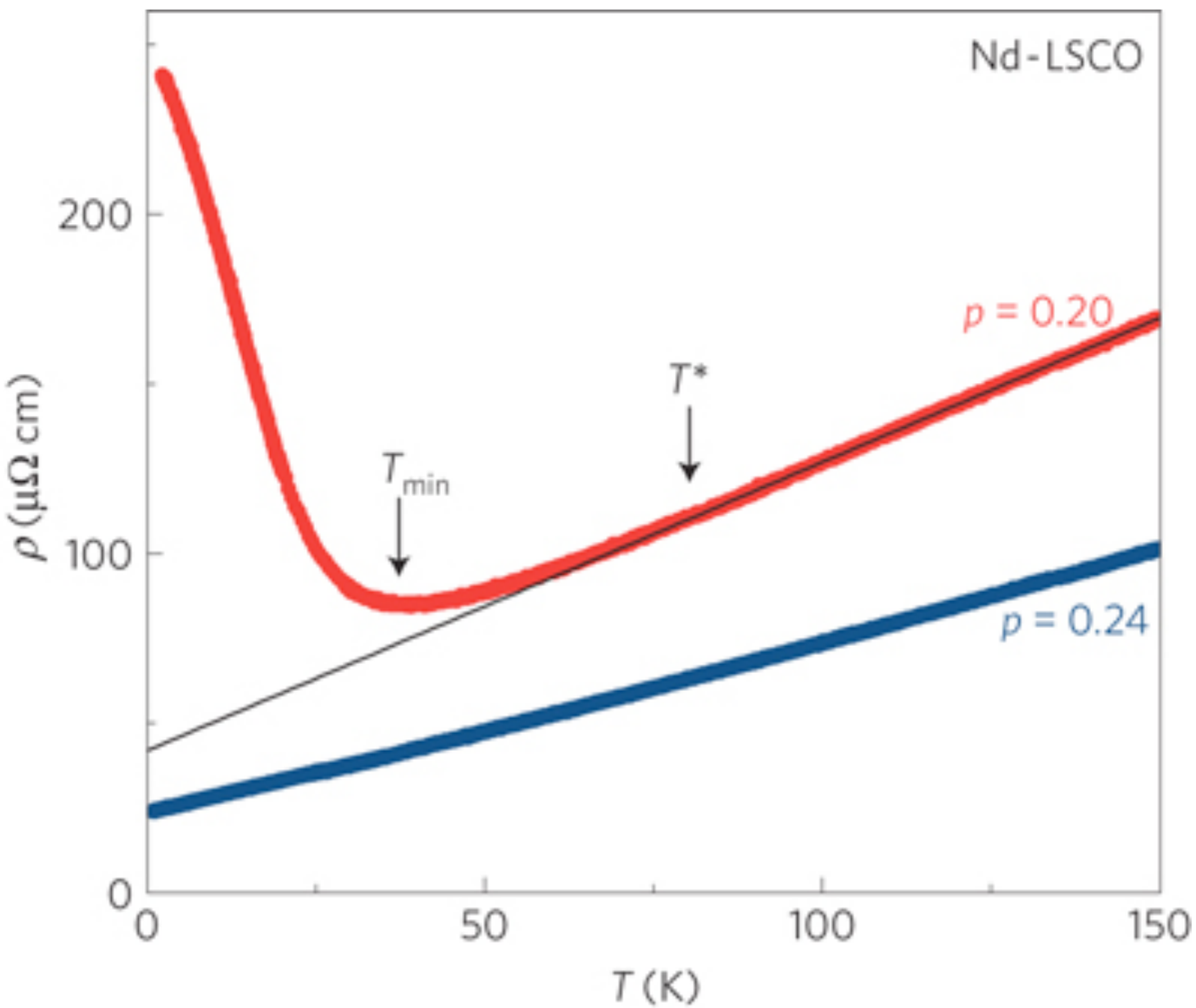
Spin density wave theory in hole-doped cuprates



Incommensurate order in $YBa_2Cu_3O_{6+x}$

A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).
N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c

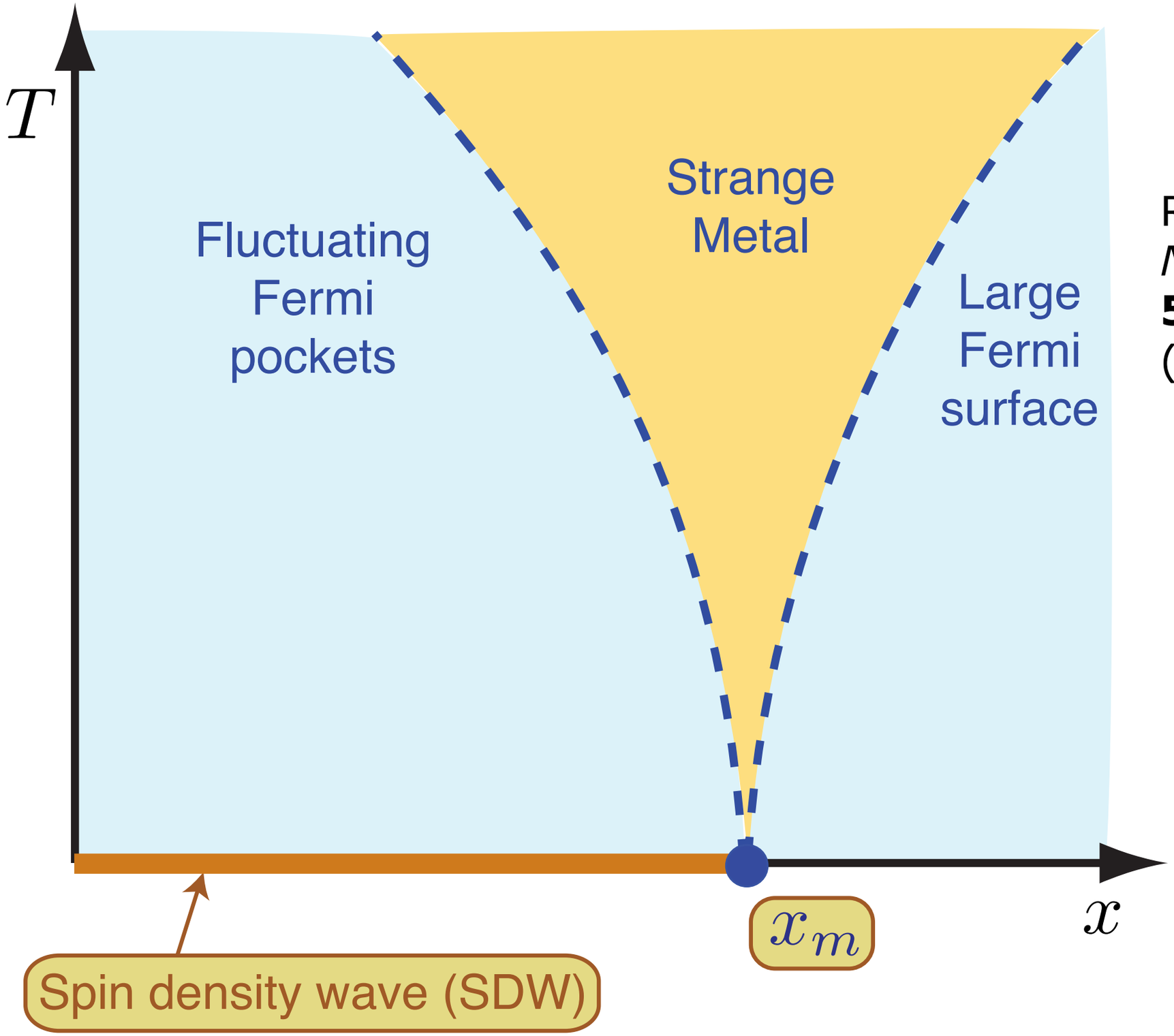


Magnetic field of upto 35 T used to suppress superconductivity

Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high-T_c superconductor

R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)

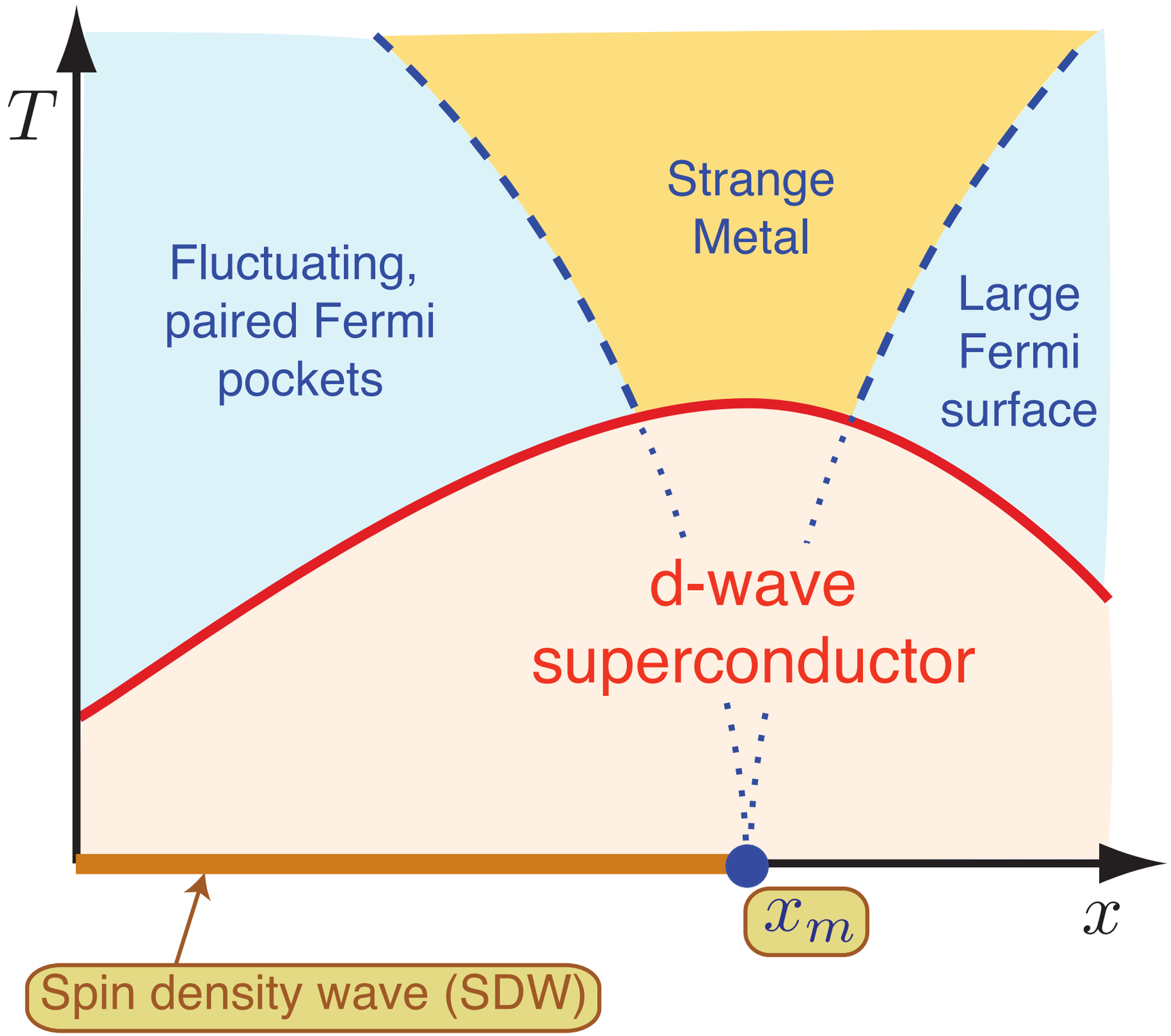
Theory of quantum criticality in the cuprates



R. Daou et al., *Nature Physics* **5**, 31 - 34 (2009)

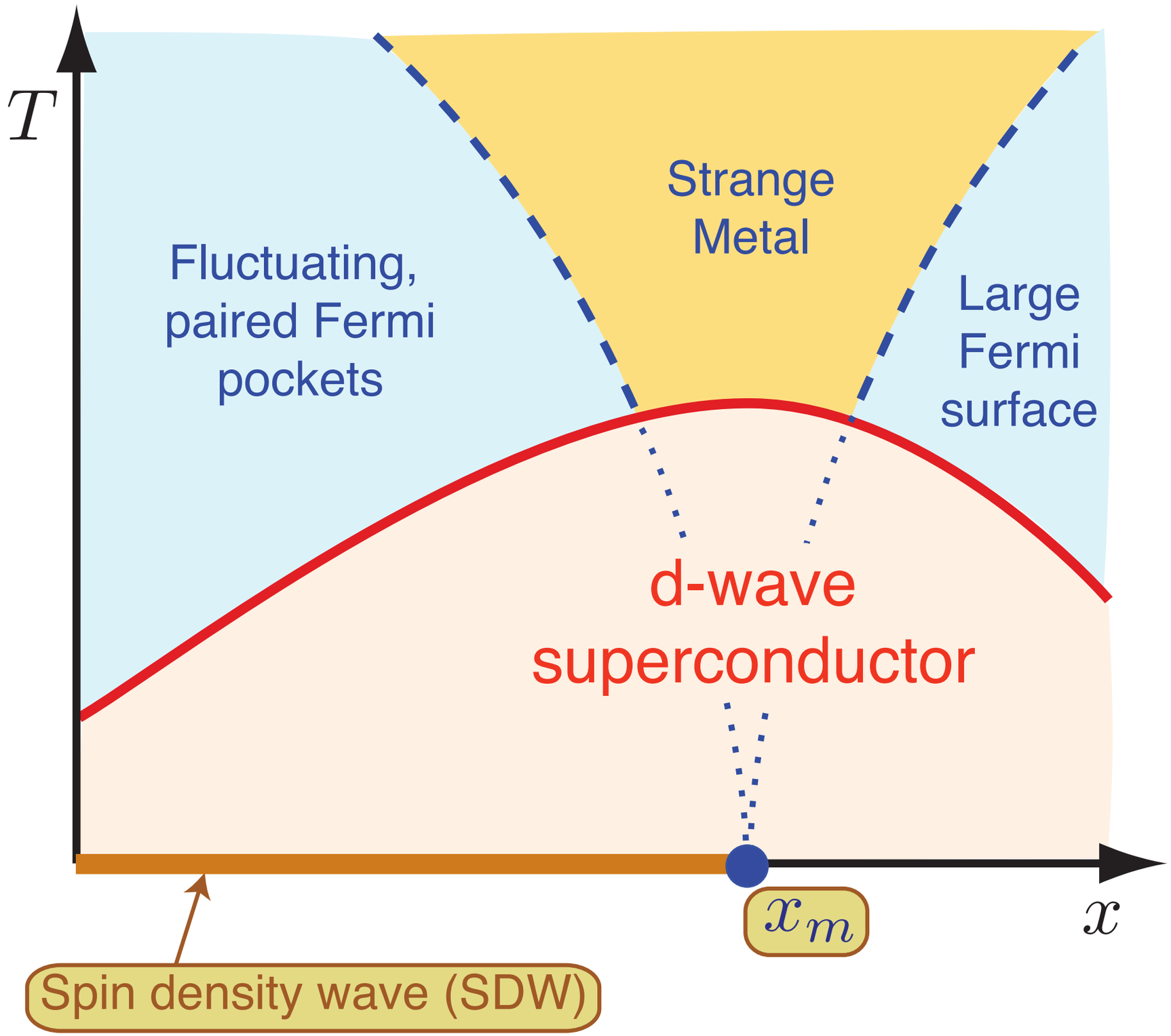
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



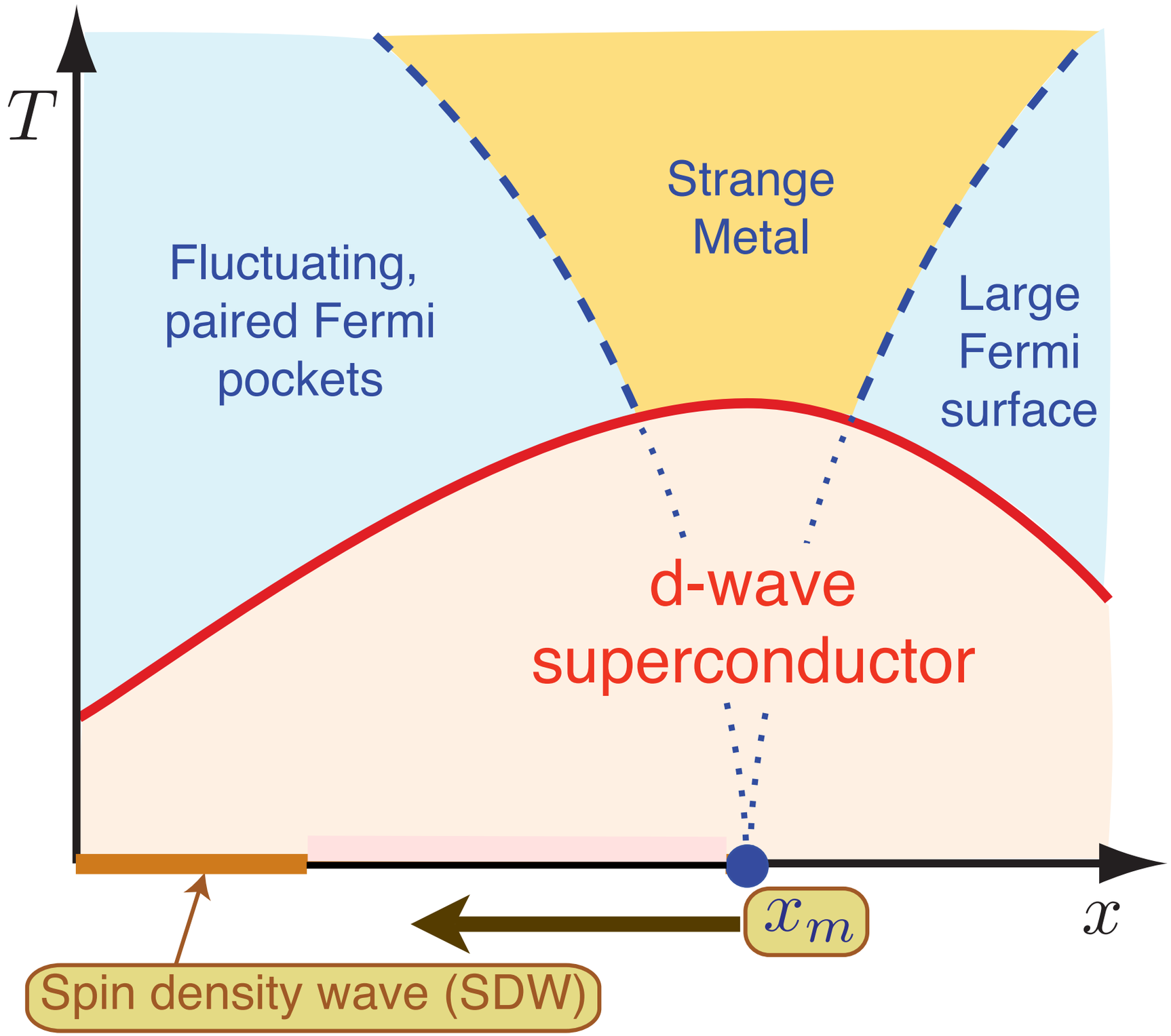
Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



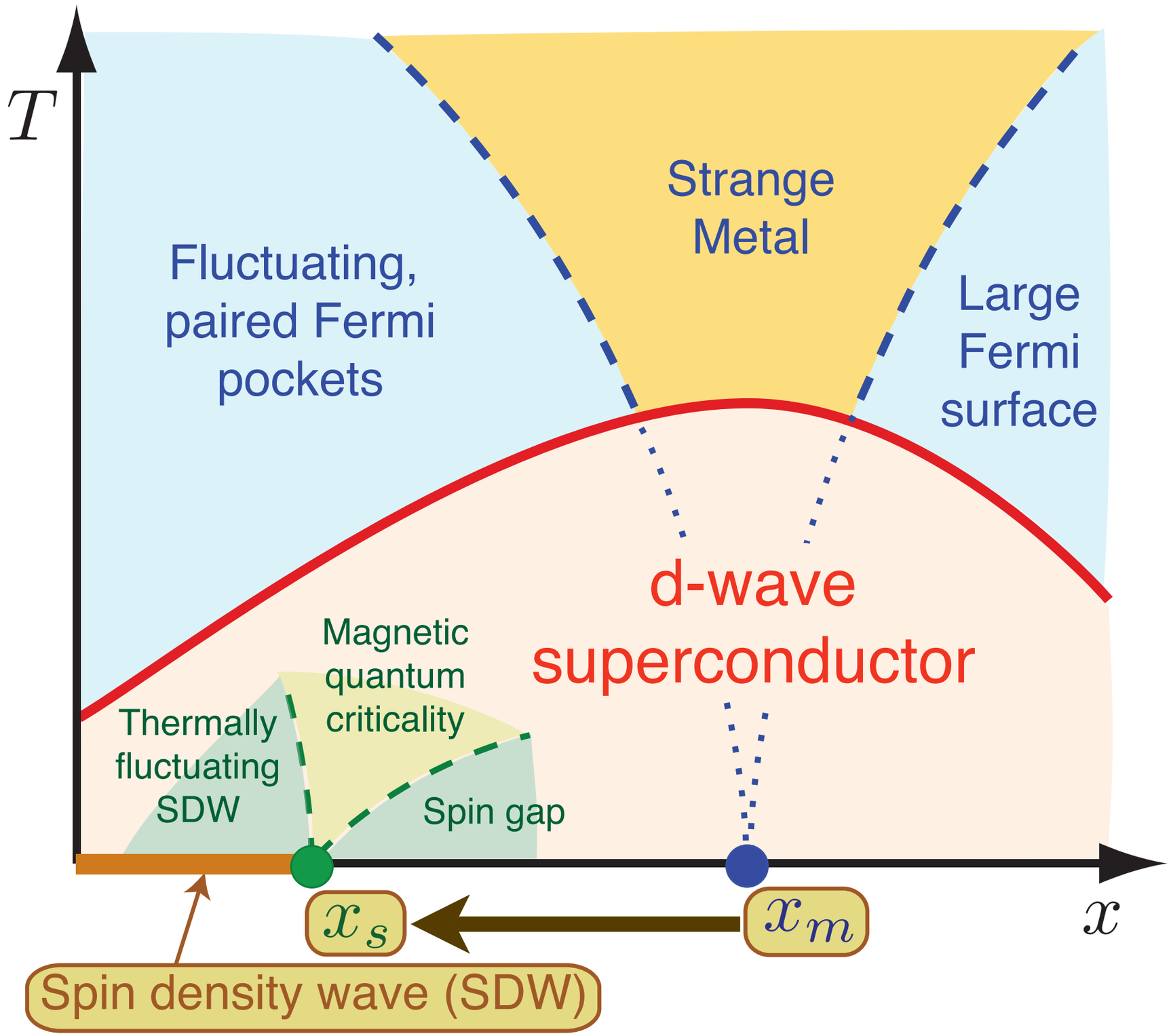
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order
   Survey of recent experiments

2. Superconductivity in the overdoped regime
   BCS pairing by spin fluctuation exchange

3. Superconductivity in the underdoped regime
   U(1) gauge theory of fluctuating SDW order

Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order
   Survey of recent experiments

2. Superconductivity in the overdoped regime
   BCS pairing by spin fluctuation exchange

3. Superconductivity in the underdoped regime
   U(1) gauge theory of fluctuating SDW order

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2 r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{r}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 + \kappa\, \vec{\varphi}^2\, |\psi|^2 \right]$$

$$+ \int d^2 r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
See also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004);
S. A. Kivelson, D.-H. Lee, E. Fradkin, and V. Oganesyan, *Phys. Rev. B* **66**, 144516 (2002).

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

$$\mathcal{S} = \int d^2 r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{r}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 + \boxed{\kappa\, \vec{\varphi}^2\, |\psi|^2} \right]$$

$$+ \int d^2 r \left[|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2 - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\nabla \times \mathcal{A} = H$ is the applied magnetic field.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
See also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004);
S. A. Kivelson, D.-H. Lee, E. Fradkin, and V. Oganesyan, *Phys. Rev. B* **66**, 144516 (2002).

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

Write down a Landau-Ginzburg action for the quantum fluctuations of the SDW order ($\vec{\varphi}$) and superconductivity (ψ):

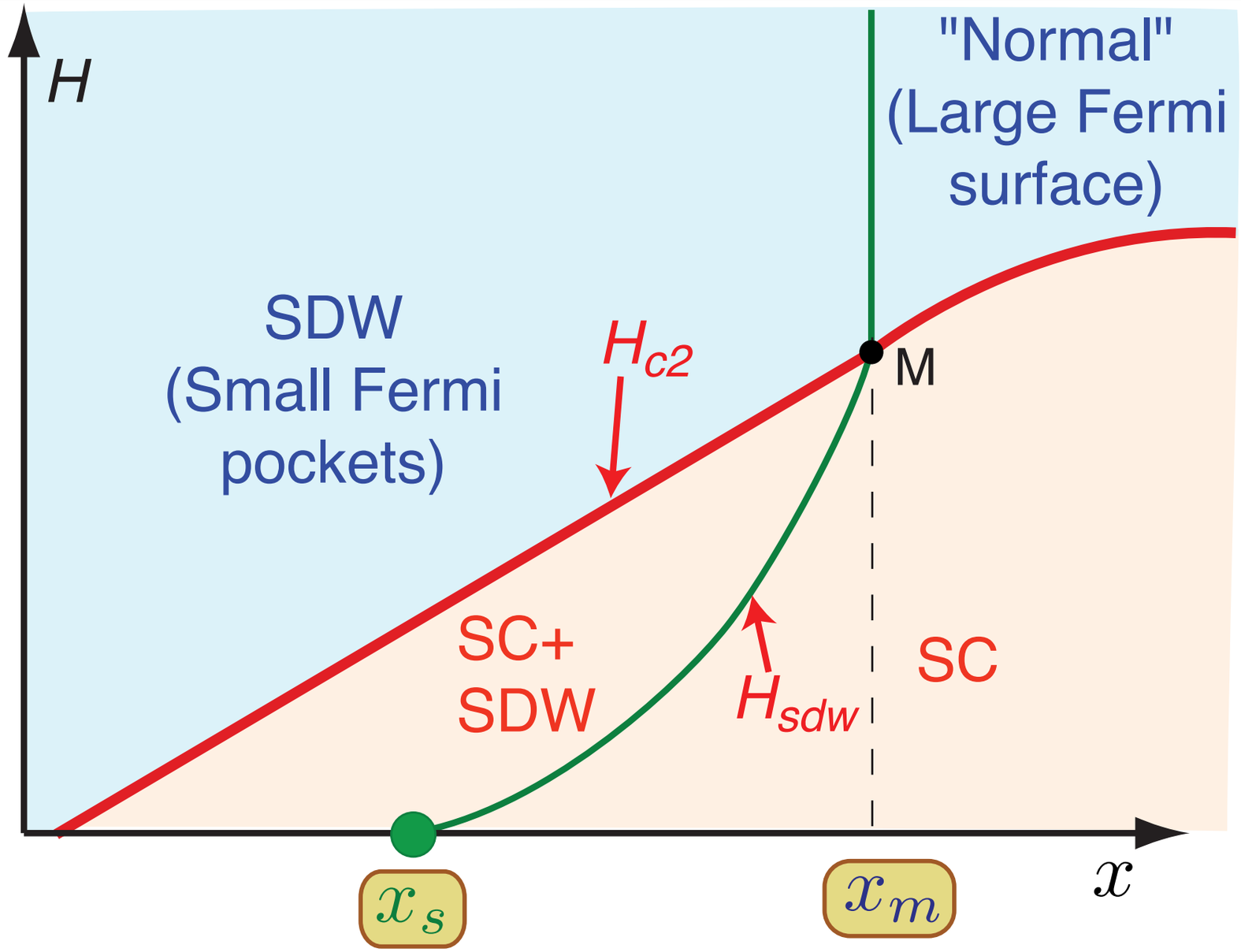
$$\mathcal{S} = \int d^2 r d\tau \left[\frac{1}{2}(\partial_\tau \vec{\varphi})^2 + \frac{c^2}{2}(\nabla_x \vec{\varphi})^2 + \frac{r}{2}\vec{\varphi}^2 + \frac{u}{4}\left(\vec{\varphi}^2\right)^2 + \kappa\, \vec{\varphi}^2\, |\psi|^2 \right]$$

$$+ \int d^2 r \left[\boxed{|(\nabla_x - i(2e/\hbar c)\mathcal{A})\psi|^2} - |\psi|^2 + \frac{|\psi|^4}{2} \right]$$

where $\kappa > 0$ is the repulsion between the two order parameters, and $\boxed{\nabla \times \mathcal{A} = H \text{ is the applied magnetic field.}}$

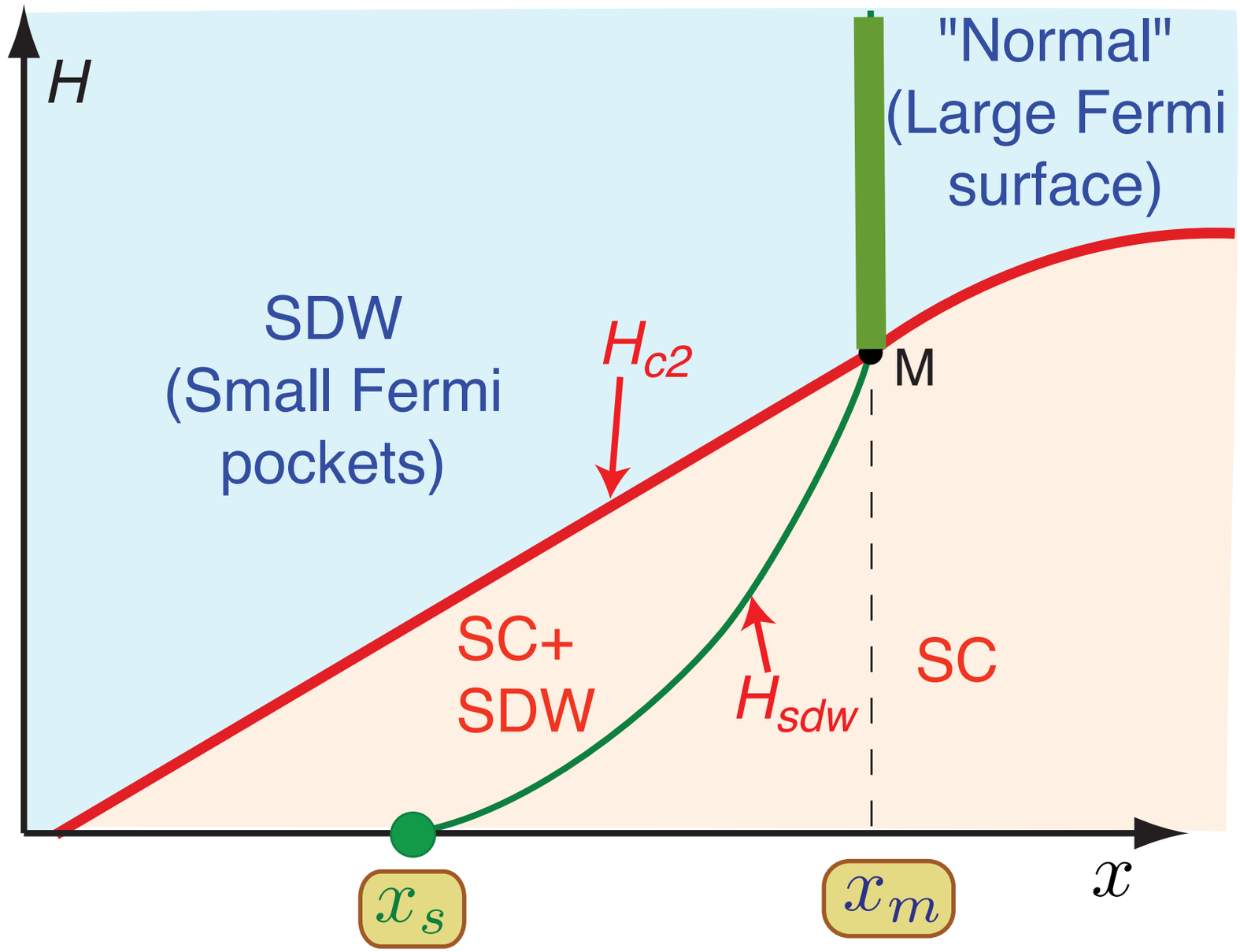
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).
See also E. Demler, W. Hanke, and S.-C. Zhang, *Rev. Mod. Phys.* **76**, 909 (2004);
S. A. Kivelson, D.-H. Lee, E. Fradkin, and V. Oganesyan, *Phys. Rev. B* **66**, 144516 (2002).

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

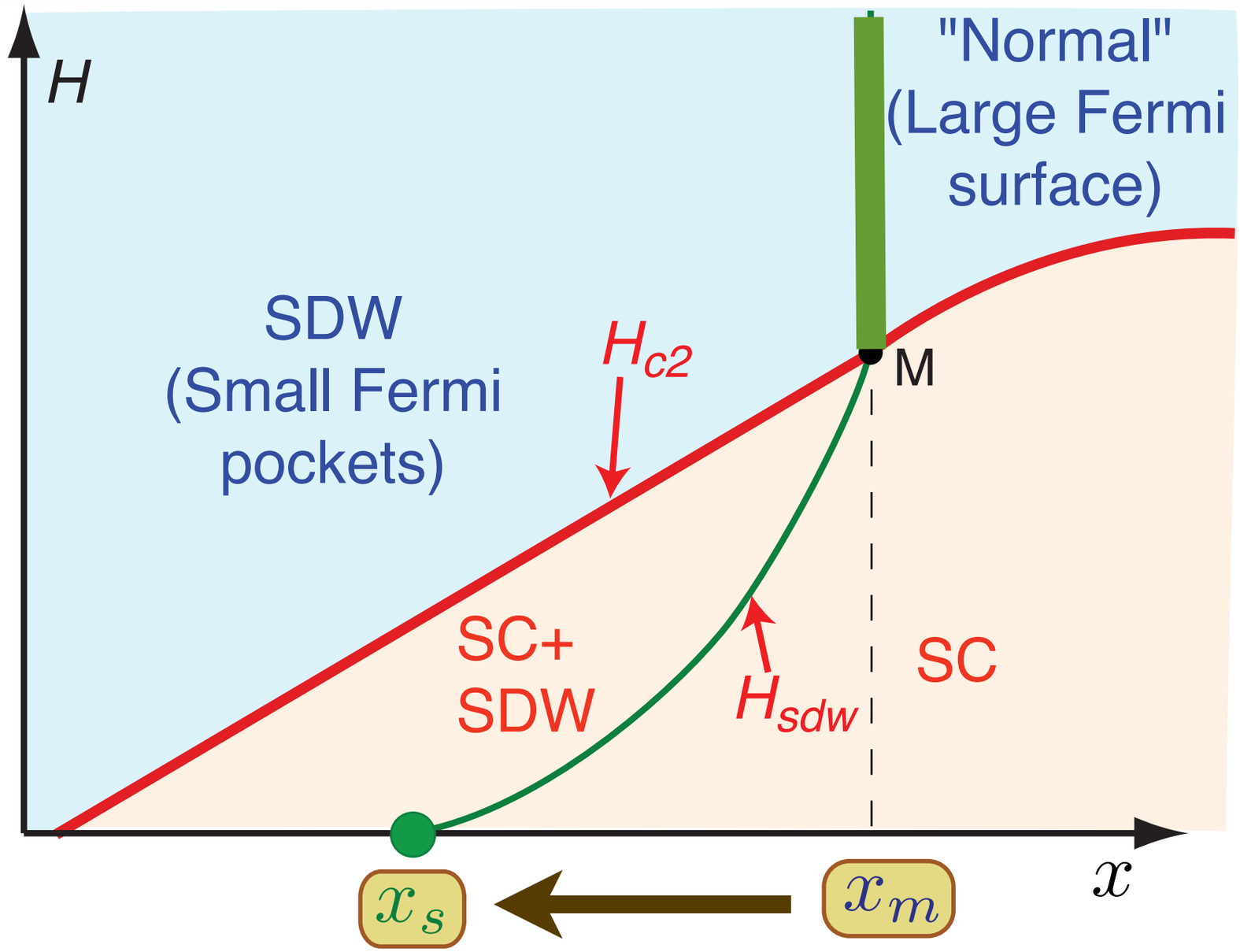
Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



- SDW order is more stable in the metal than in the superconductor: $x_m > x_s$.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

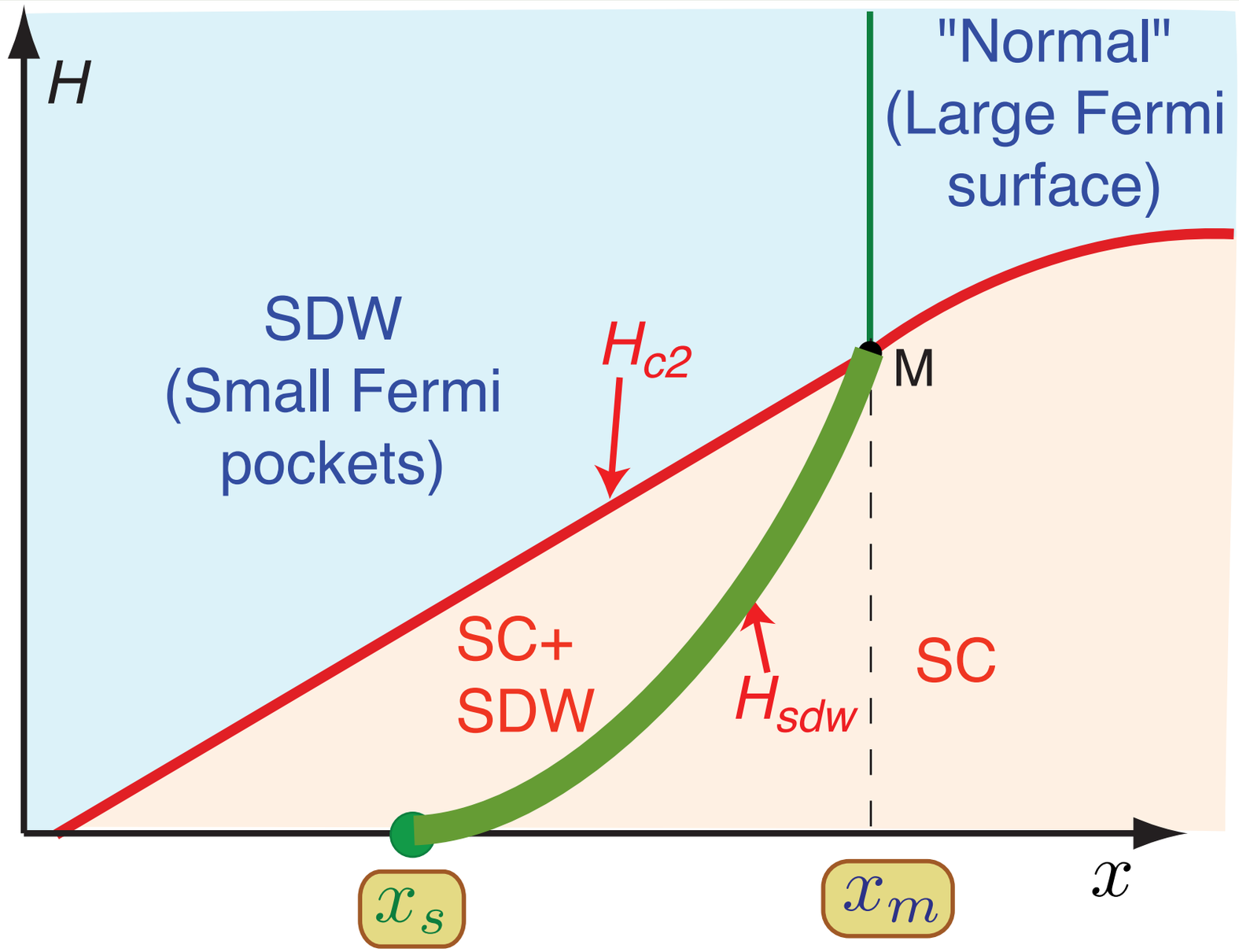
Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



- SDW order is more stable in the metal than in the superconductor: $x_m > x_s$.

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

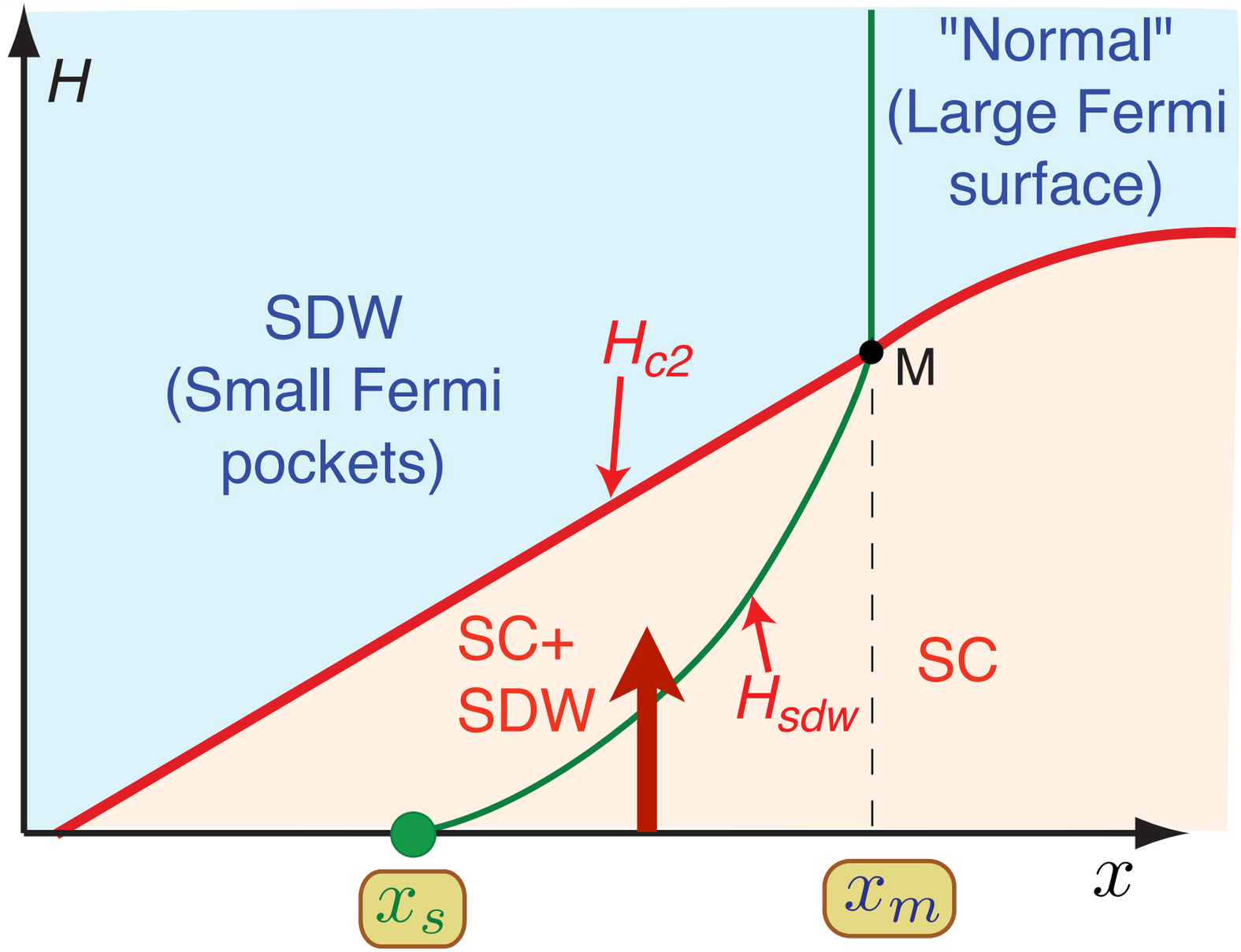
Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



- For doping with $x_s < x < x_m$, SDW order appears at a quantum phase transition at $H = H_{sdw} > 0$.

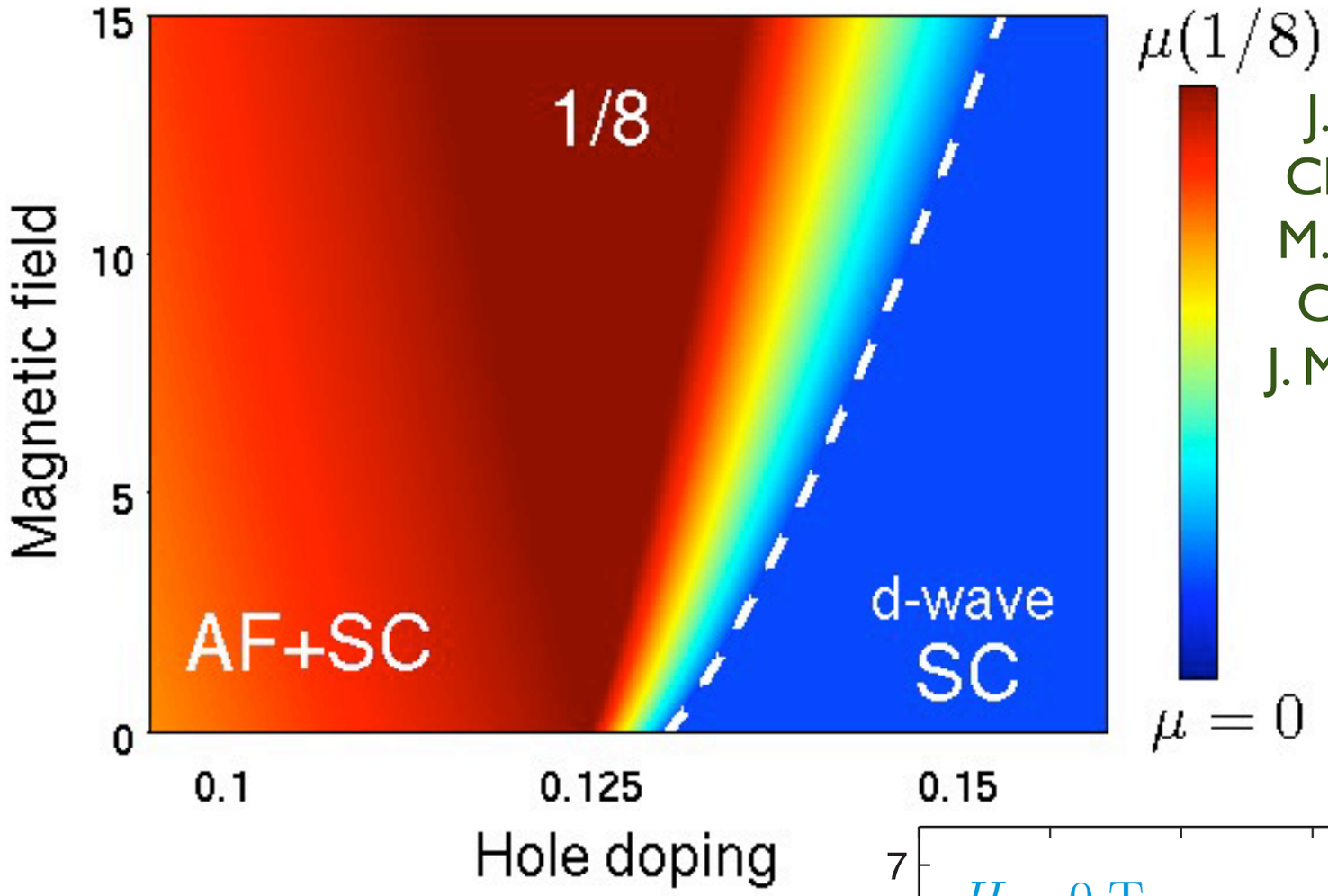
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

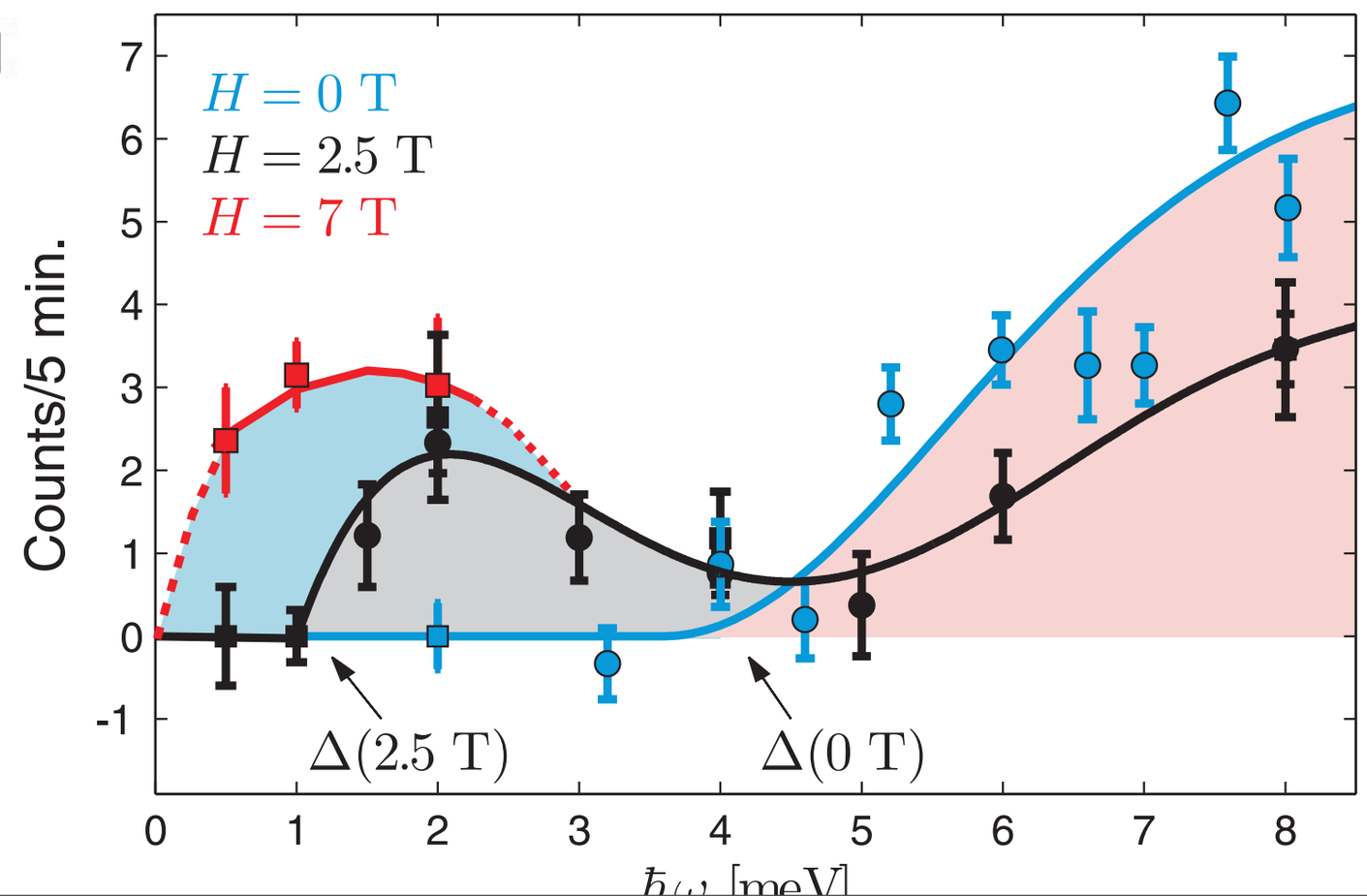


Neutron scattering on $La_{1.855}Sr_{0.145}CuO_4$
J. Chang *et al.*, *Phys. Rev. Lett.* **102**, 177006 (2009).

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

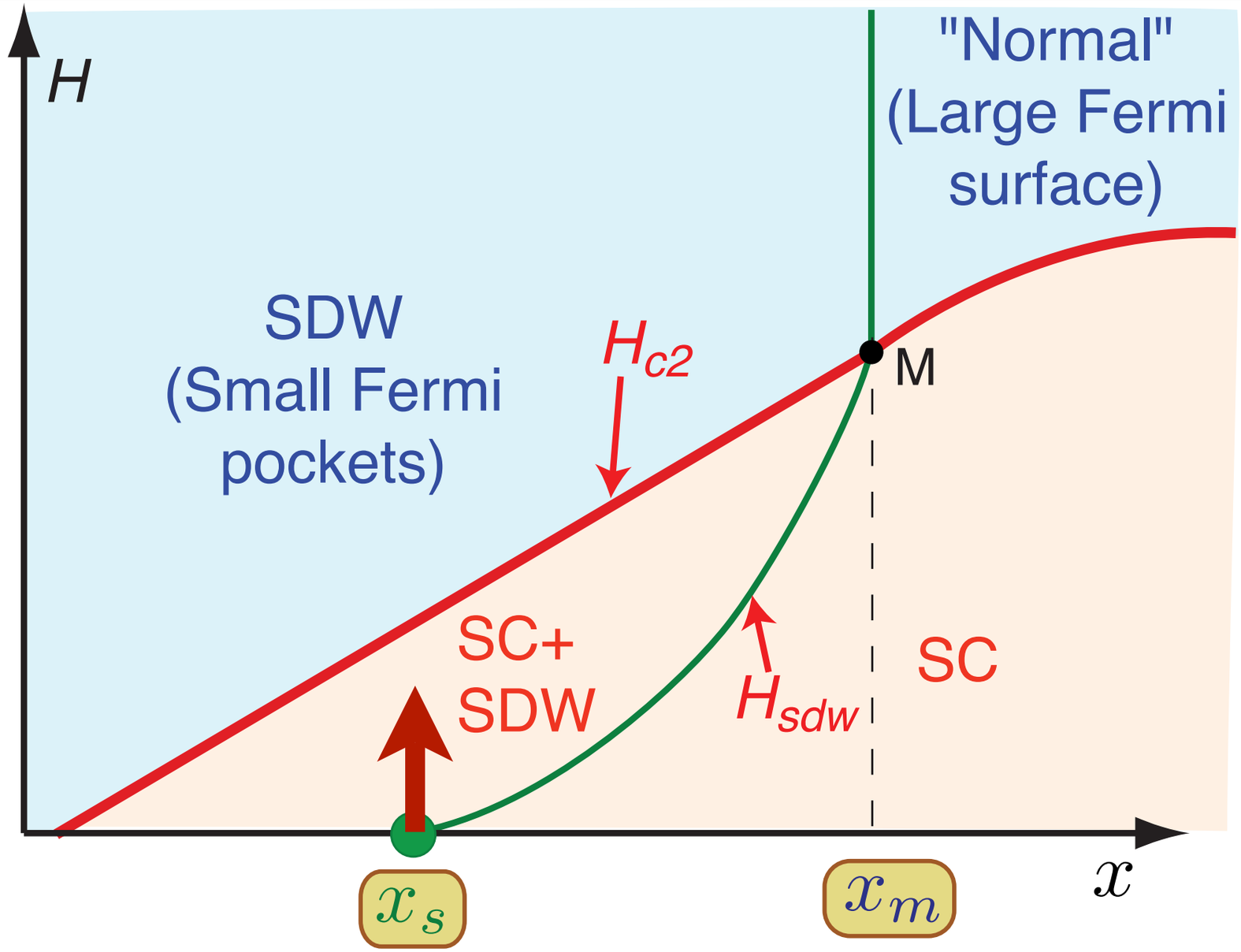


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).



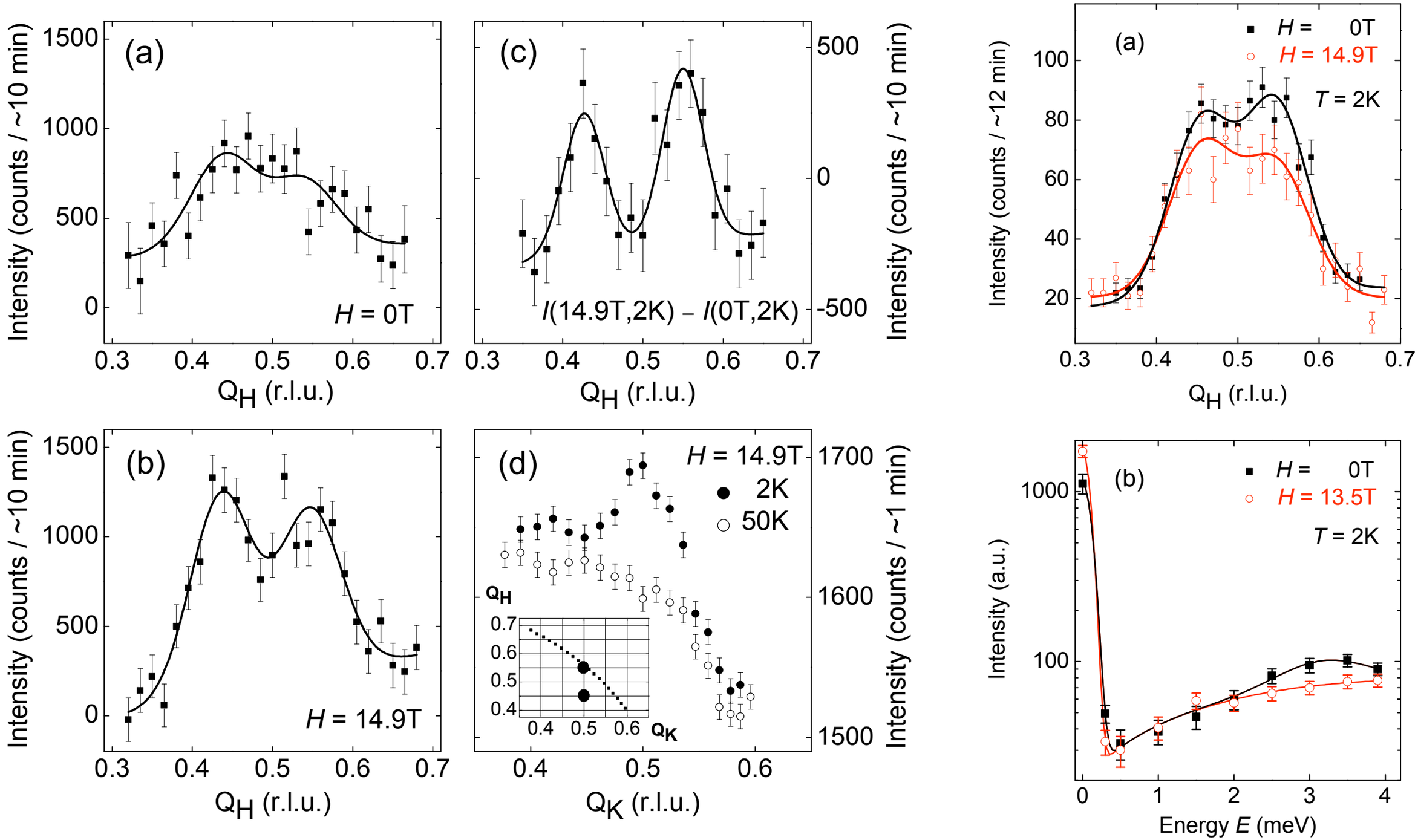
J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



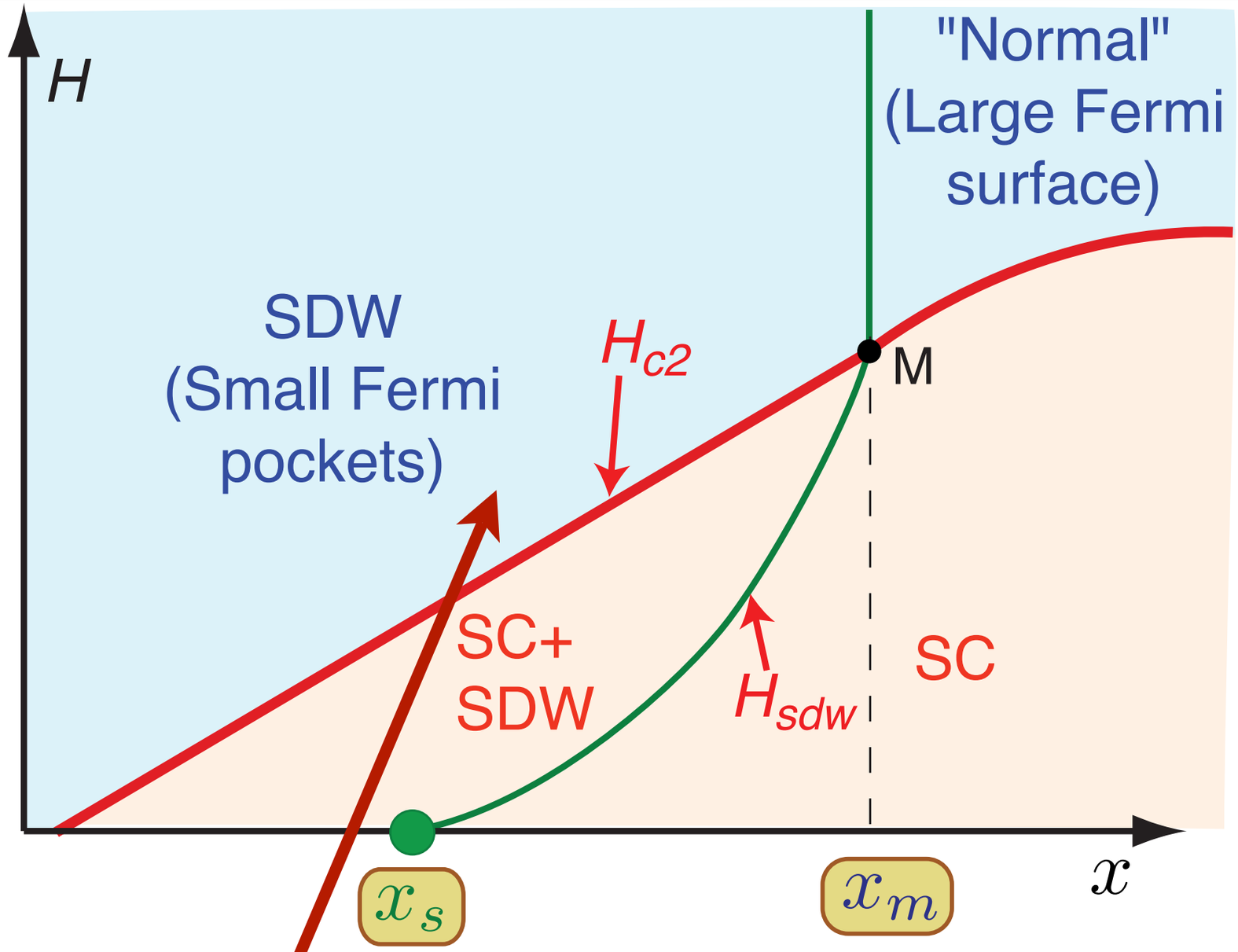
Neutron scattering on $YBa_2Cu_3O_{6.45}$
D. Haug *et al.*, *Phys. Rev. Lett.* **103**, 017001 (2009).

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



Quantum oscillations without Zeeman splitting

N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007).

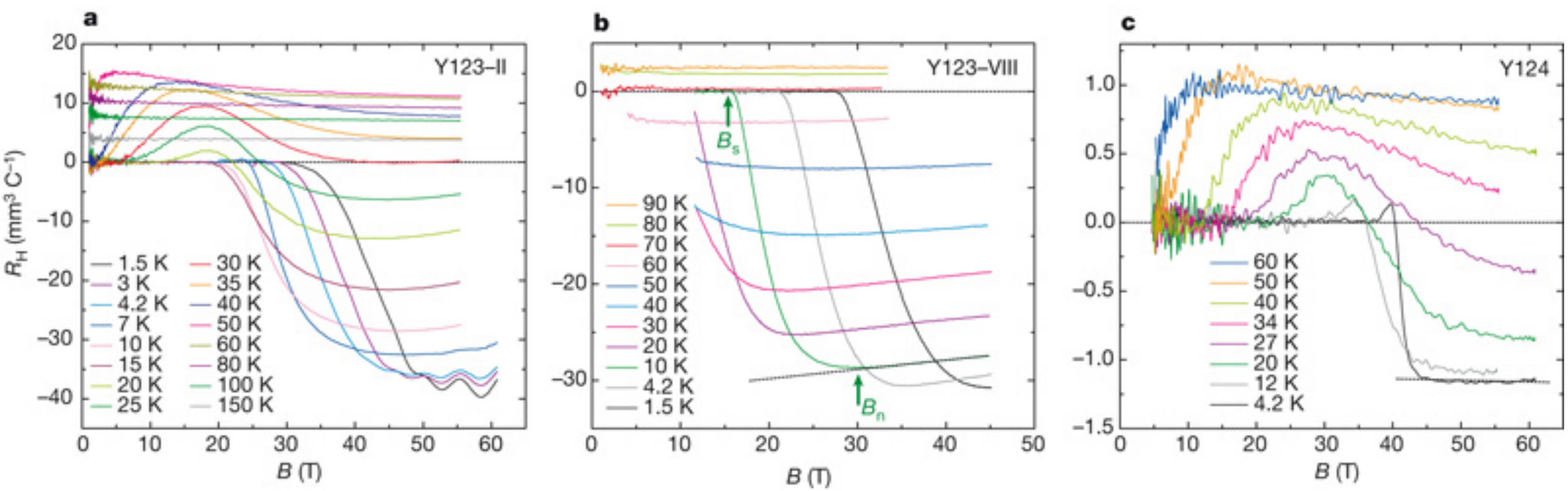
S. E. Sebastian, N. Harrison, C. H. Mielke, Ruixing Liang, D. A. Bonn, W.~N.~Hardy, and G. G. Lonzarich, arXiv:0907.2958

Quantum oscillations

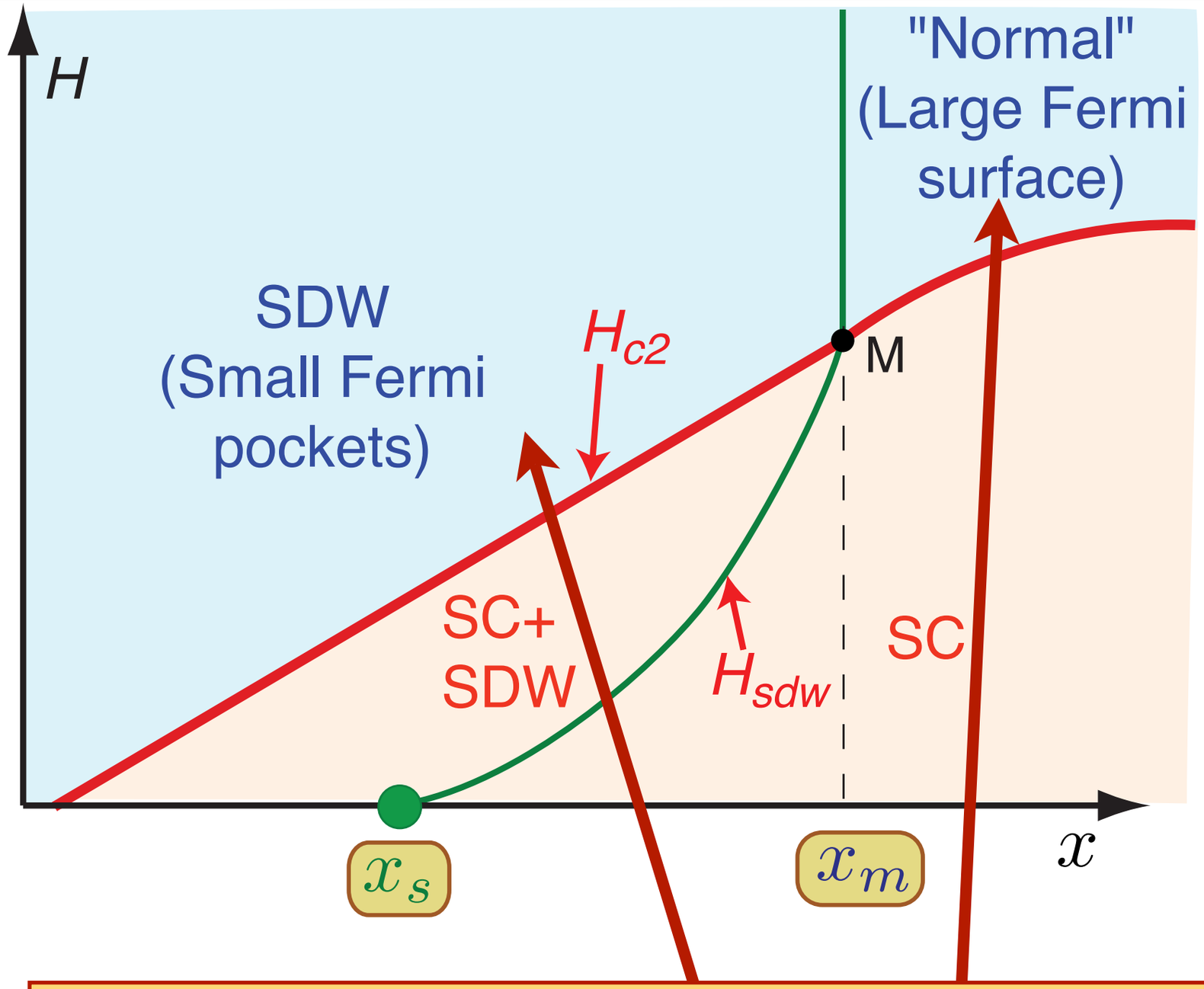
Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

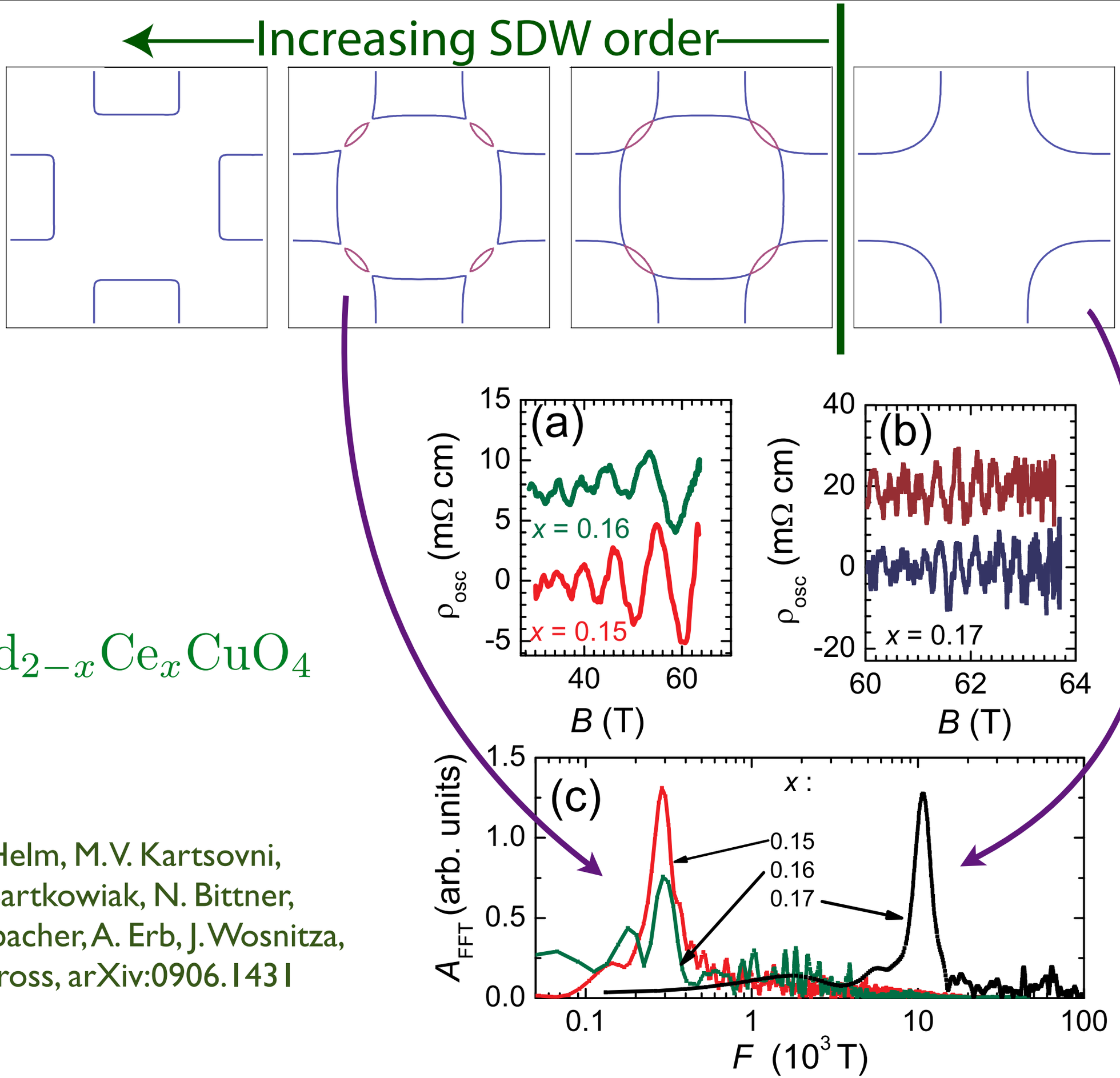
Nature **450**, 533 (2007)



Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



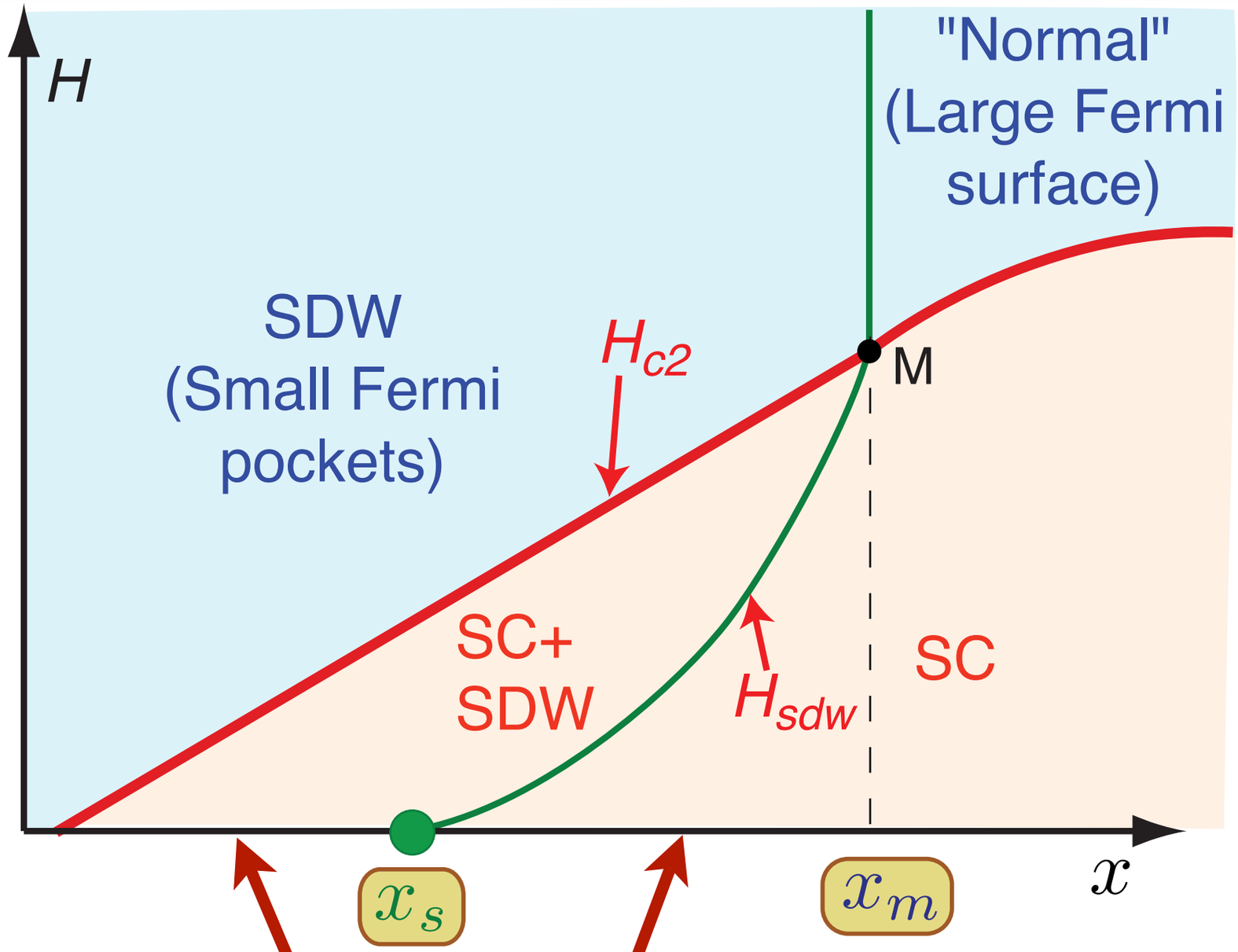
Change in frequency of quantum oscillations in electron-doped materials identifies $x_m = 0.165$



$Nd_{2-x}Ce_xCuO_4$

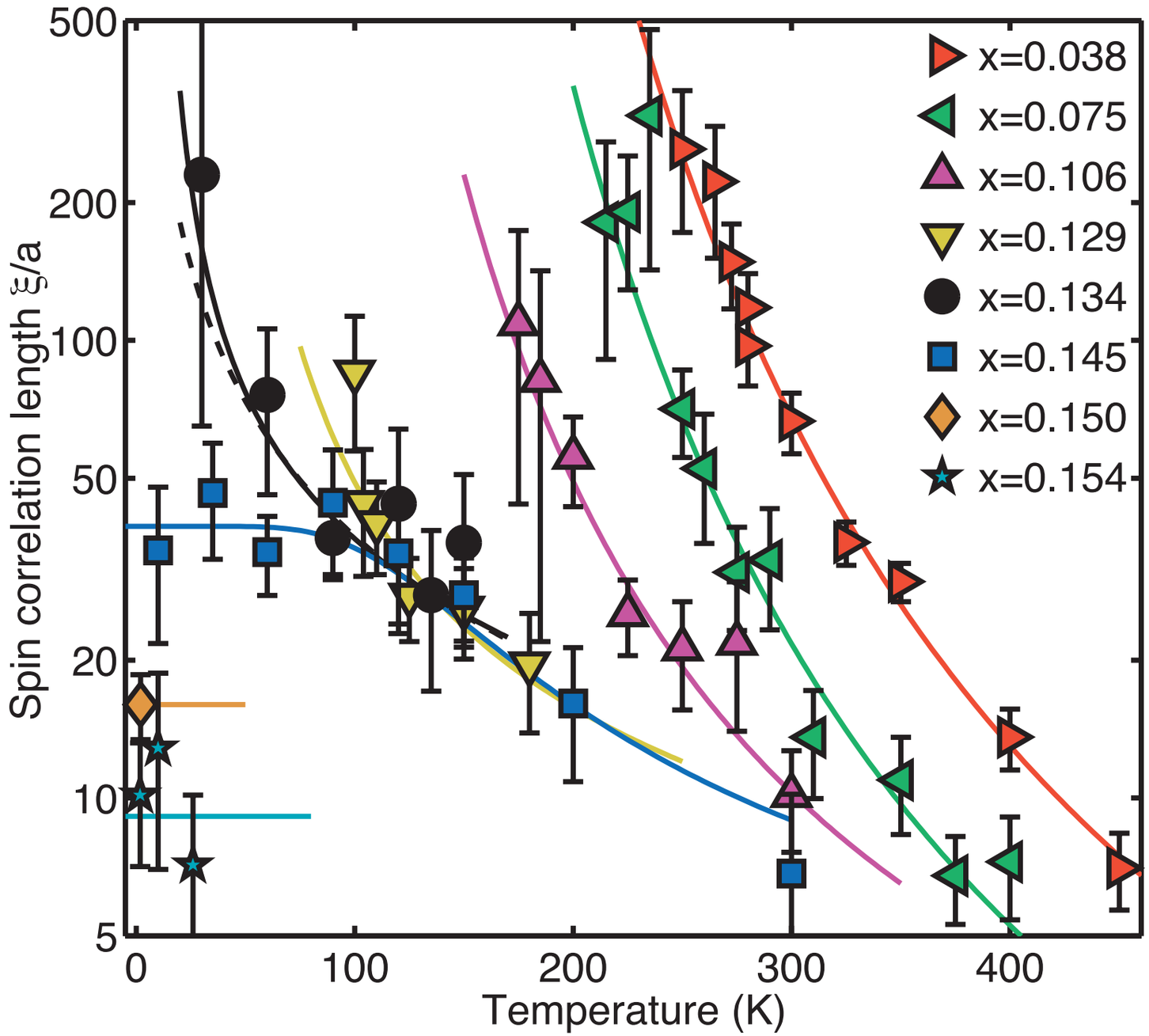
T. Helm, M.V. Kartsovni, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, R. Gross, arXiv:0906.1431

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order



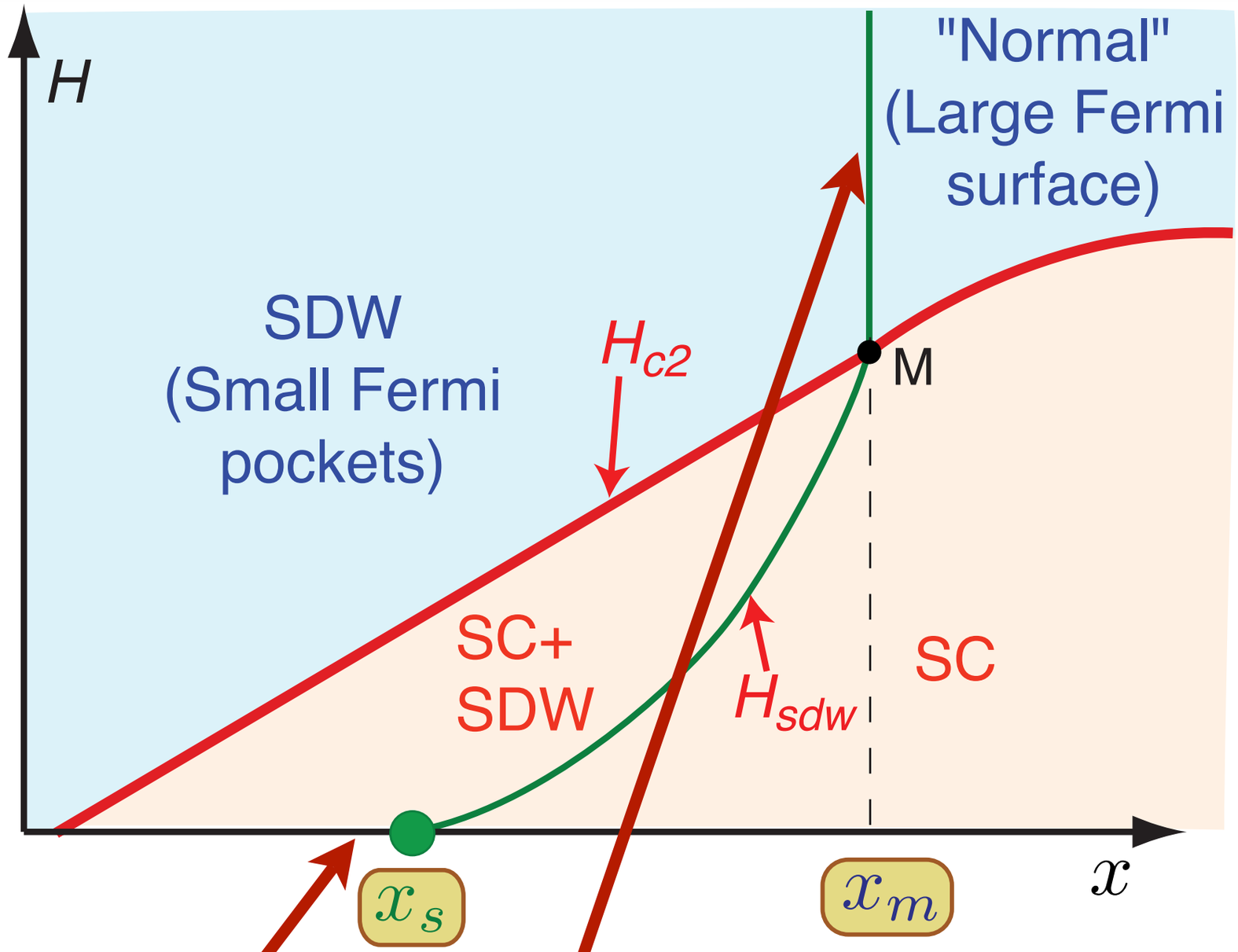
Neutron scattering at $H=0$ in **same** material identifies $x_s = 0.14 < x_m$

$Nd_{2-x}Ce_xCuO_4$

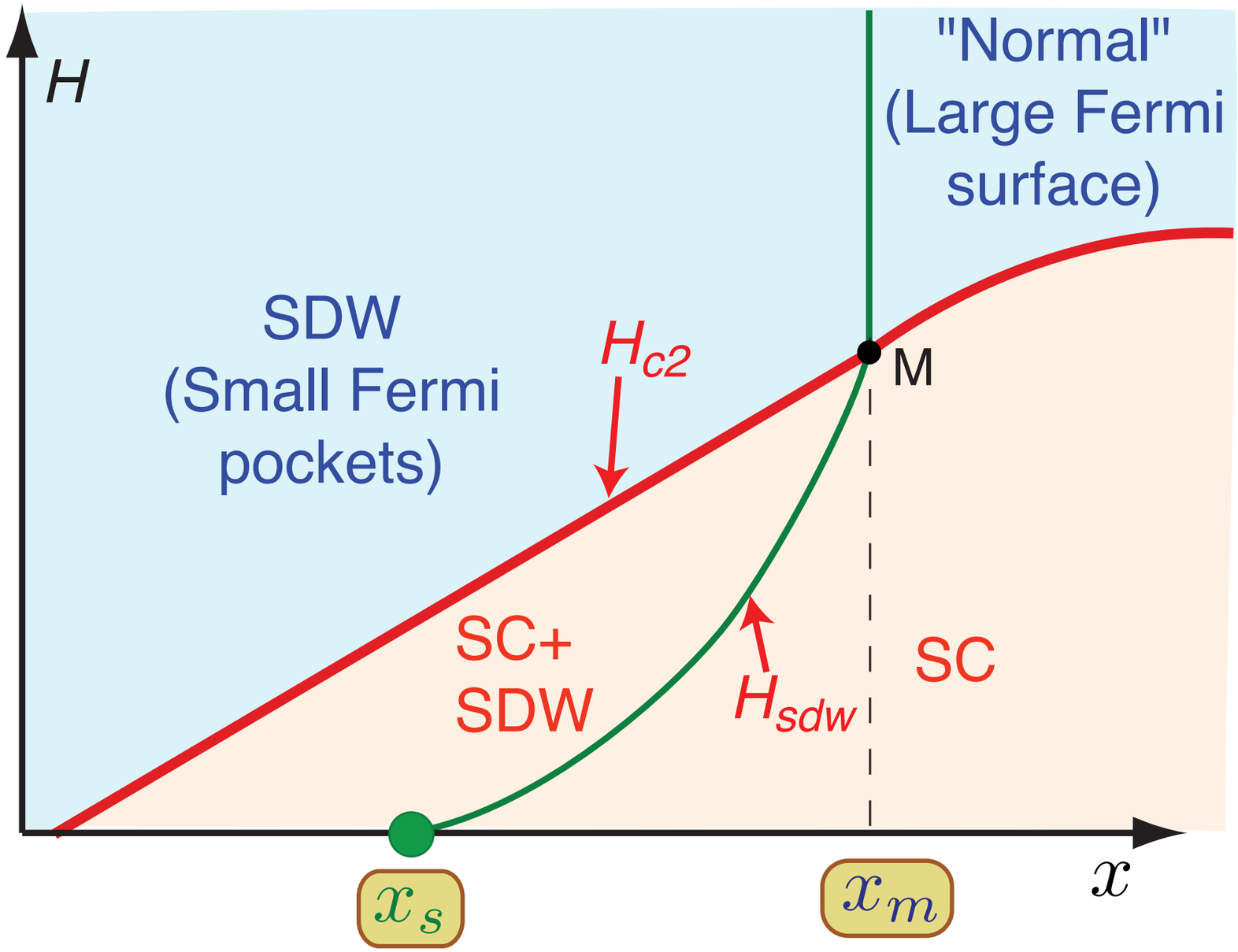


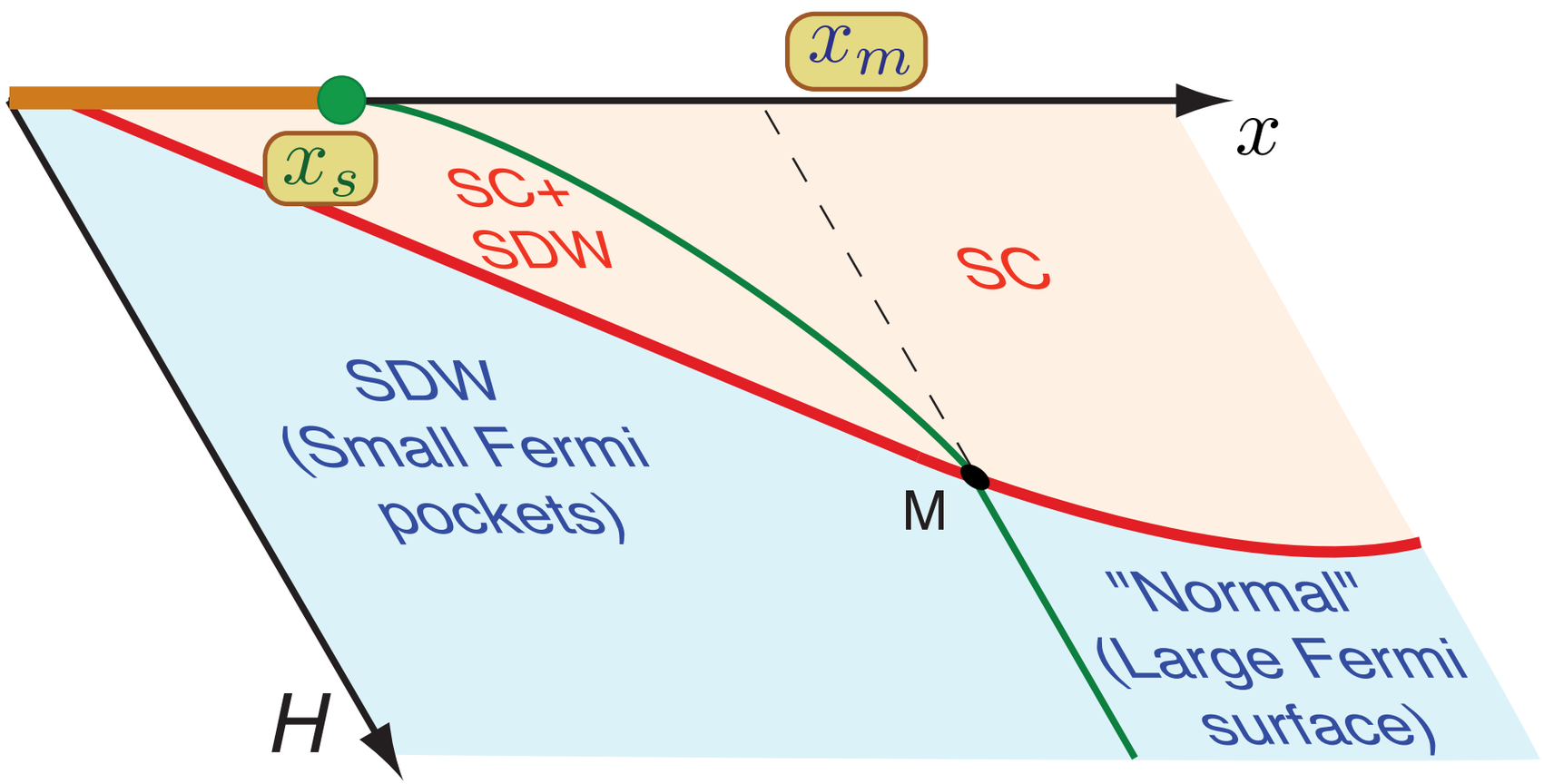
E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven,
Nature **445**, 186 (2007).

Phenomenological quantum theory of competition between superconductivity (SC) and spin-density wave (SDW) order

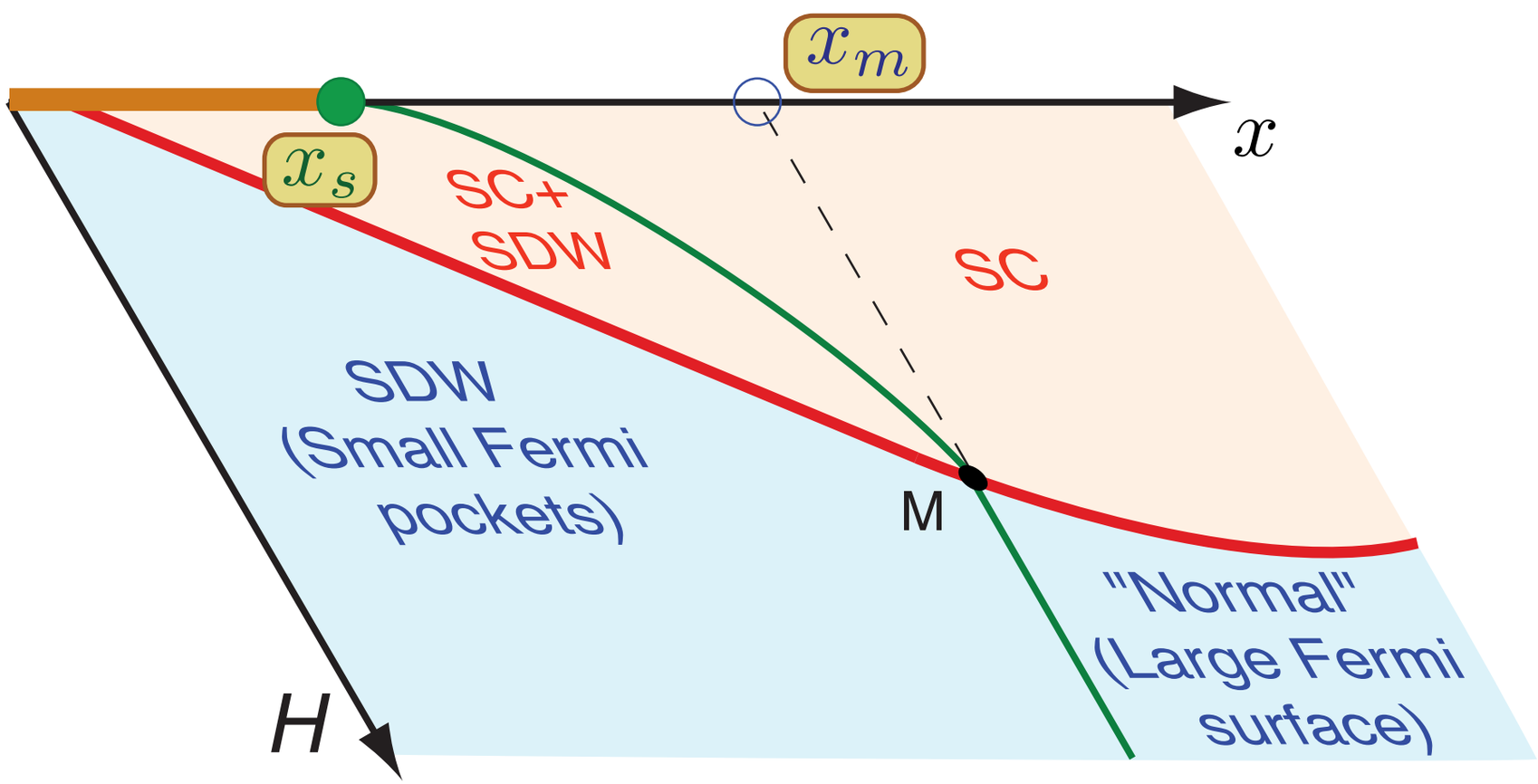


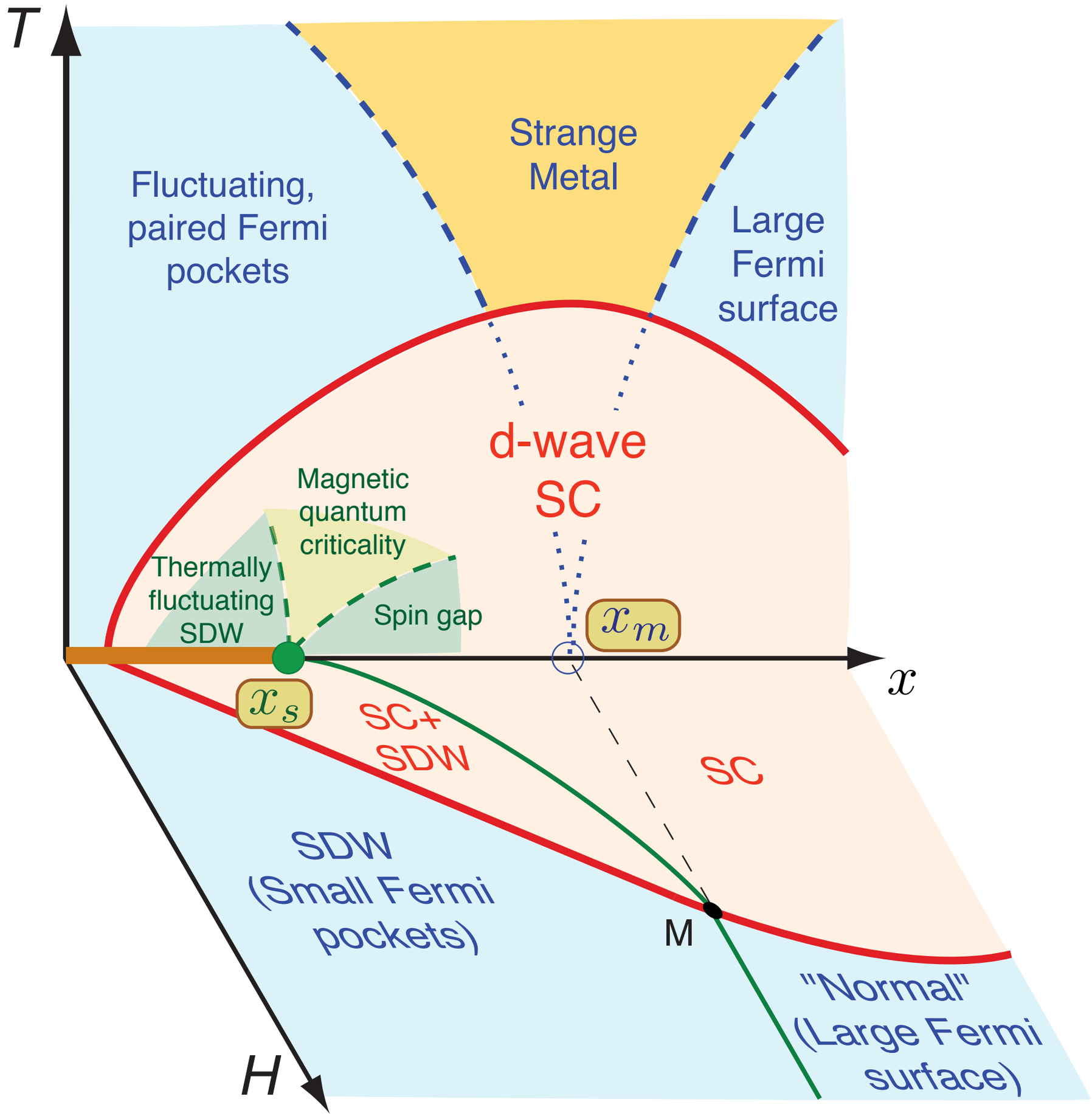
Experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while at high fields $x_m = 0.165$.


H
"Normal"
(Large Fermi
surface)
SDW
(Small Fermi
pockets)
H_{c2}
M
SC+
SDW
H_{sdw}
SC
x_s
x_m
x


x_m
x
x_s
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H

Fluctuating, paired Fermi pockets




T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order
   Survey of recent experiments

2. Superconductivity in the overdoped regime
   BCS pairing by spin fluctuation exchange

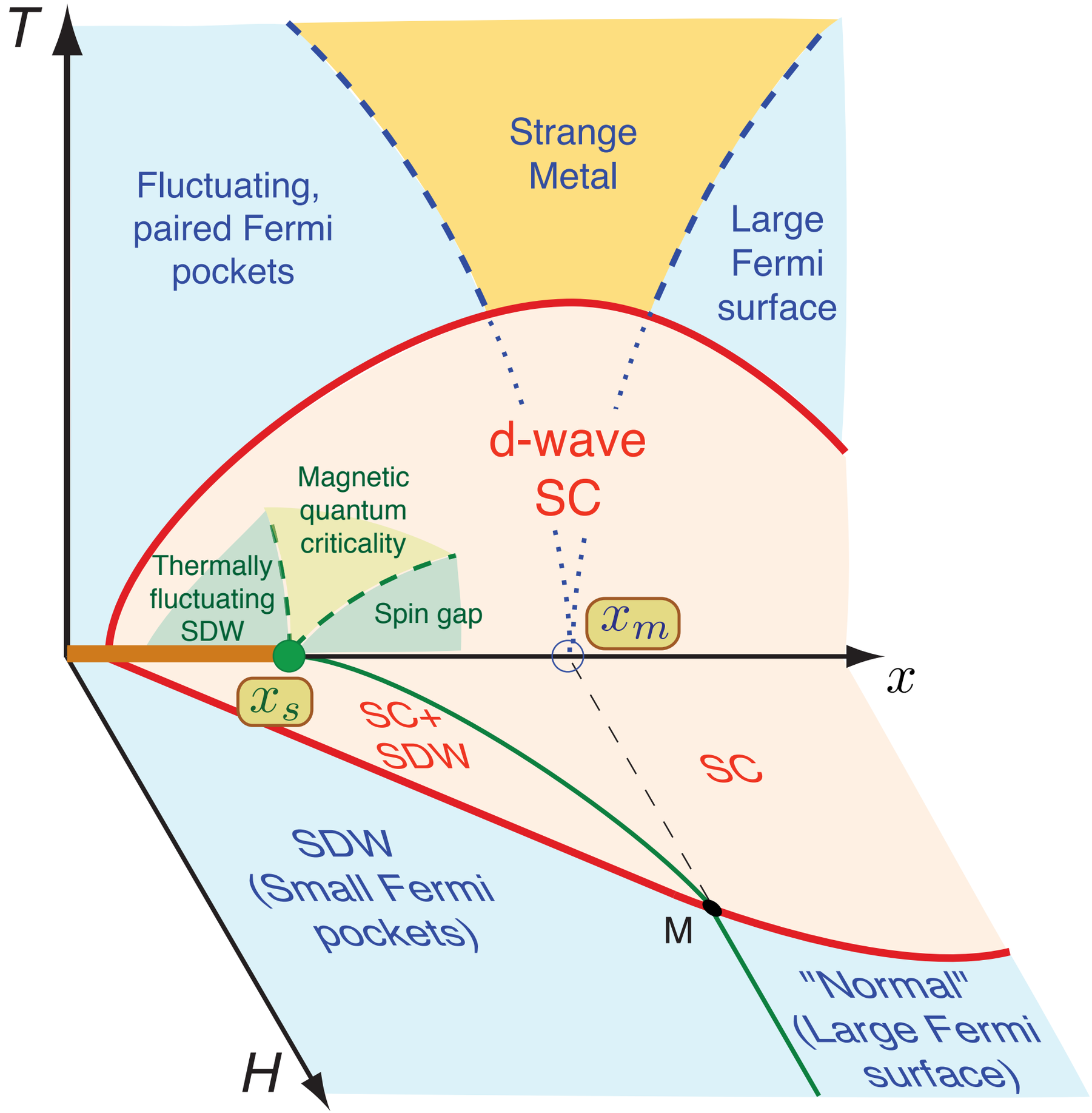
3. Superconductivity in the underdoped regime
   U(1) gauge theory of fluctuating SDW order

Outline

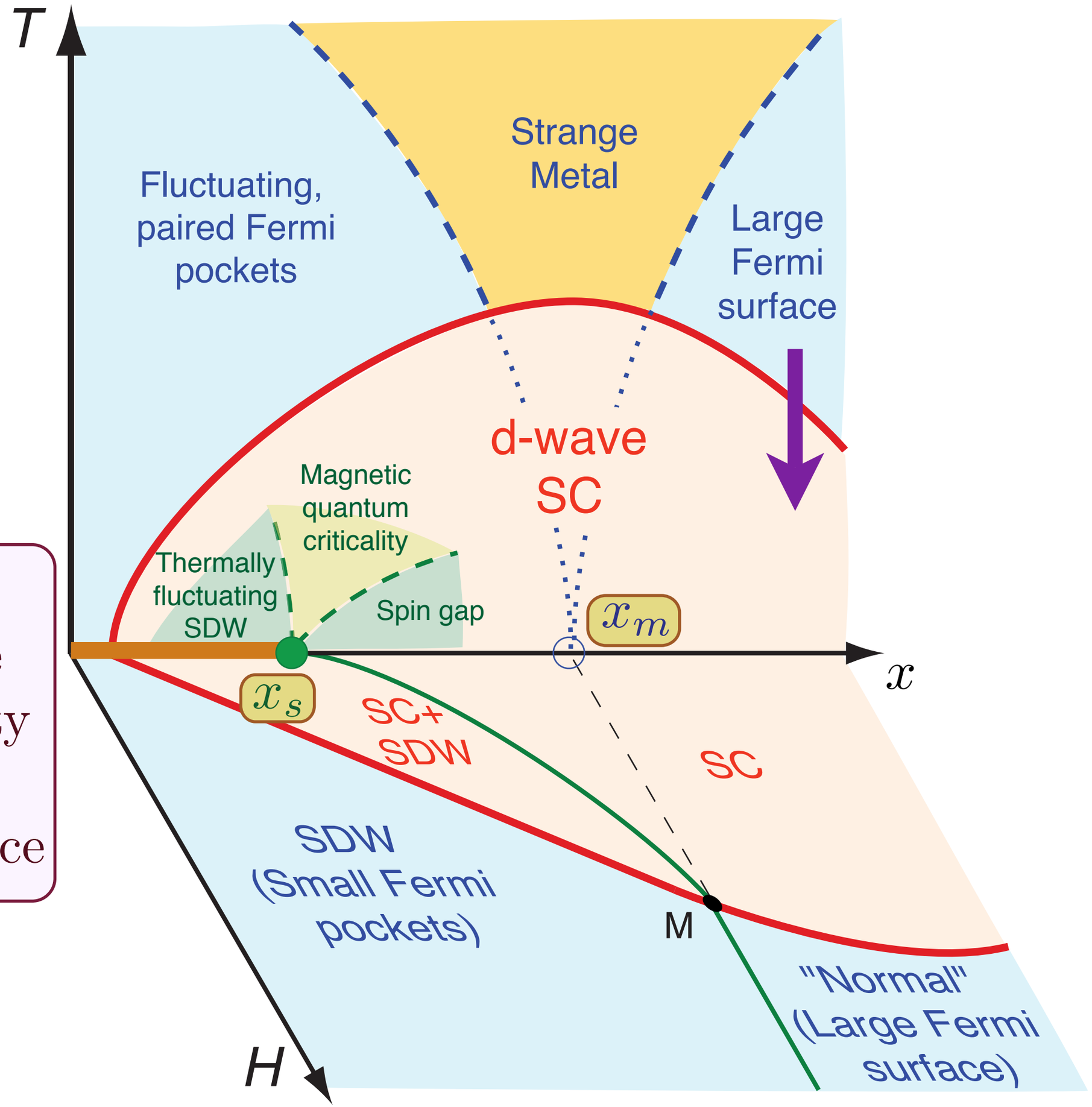
1. Phenomenological quantum theory of competition between superconductivity and SDW order
   Survey of recent experiments

2. Superconductivity in the overdoped regime
   BCS pairing by spin fluctuation exchange

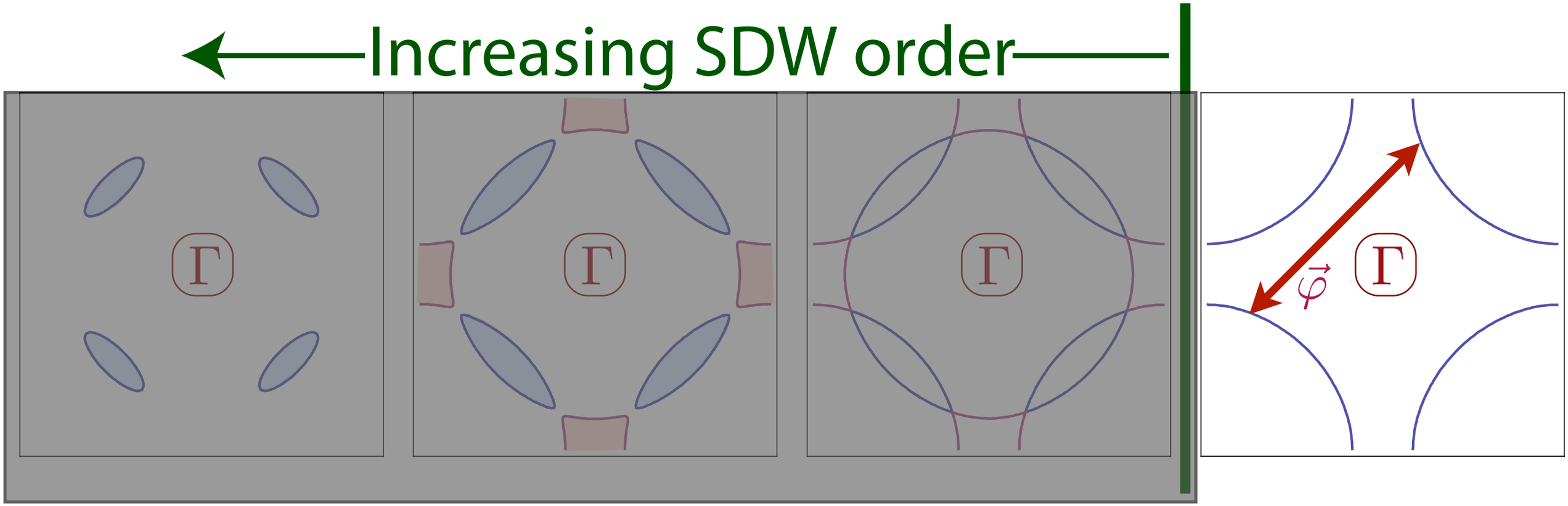
3. Superconductivity in the underdoped regime
   U(1) gauge theory of fluctuating SDW order


T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x
x_s
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Theory of the onset of d-wave superconductivity from a large Fermi surface



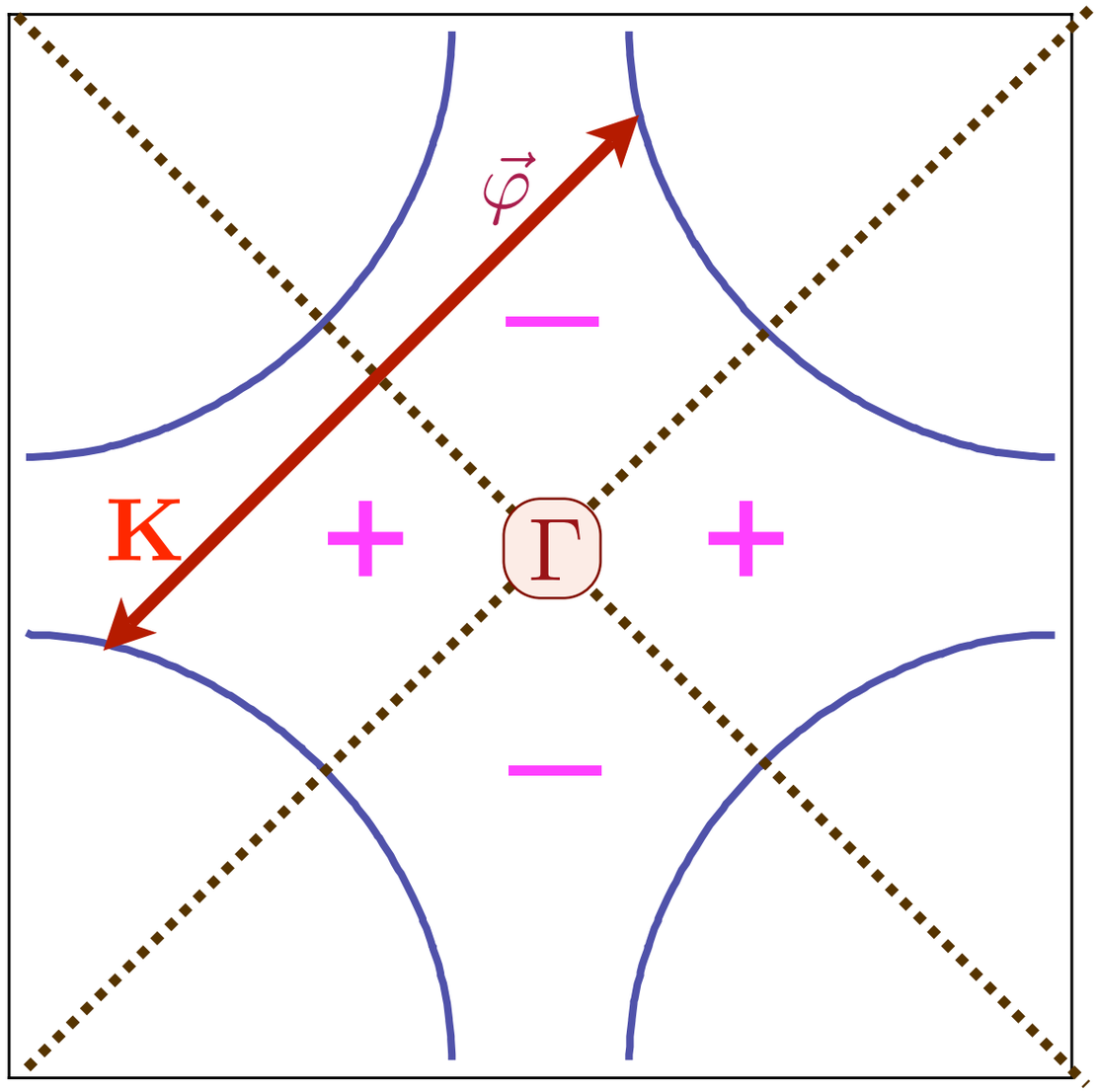
Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

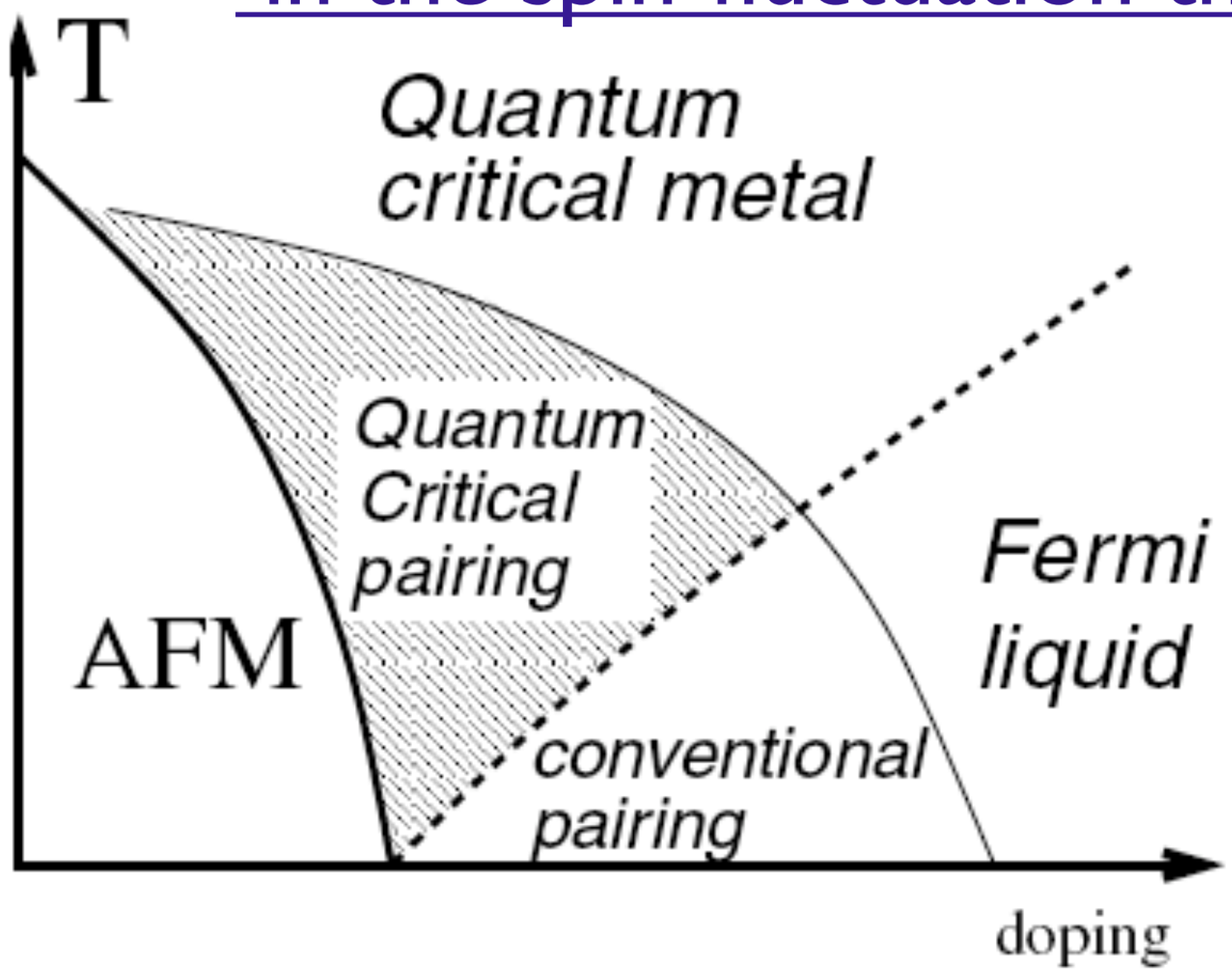
d-wave pairing of the large Fermi surface



$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0 (\cos(k_x) - \cos(k_y))$$

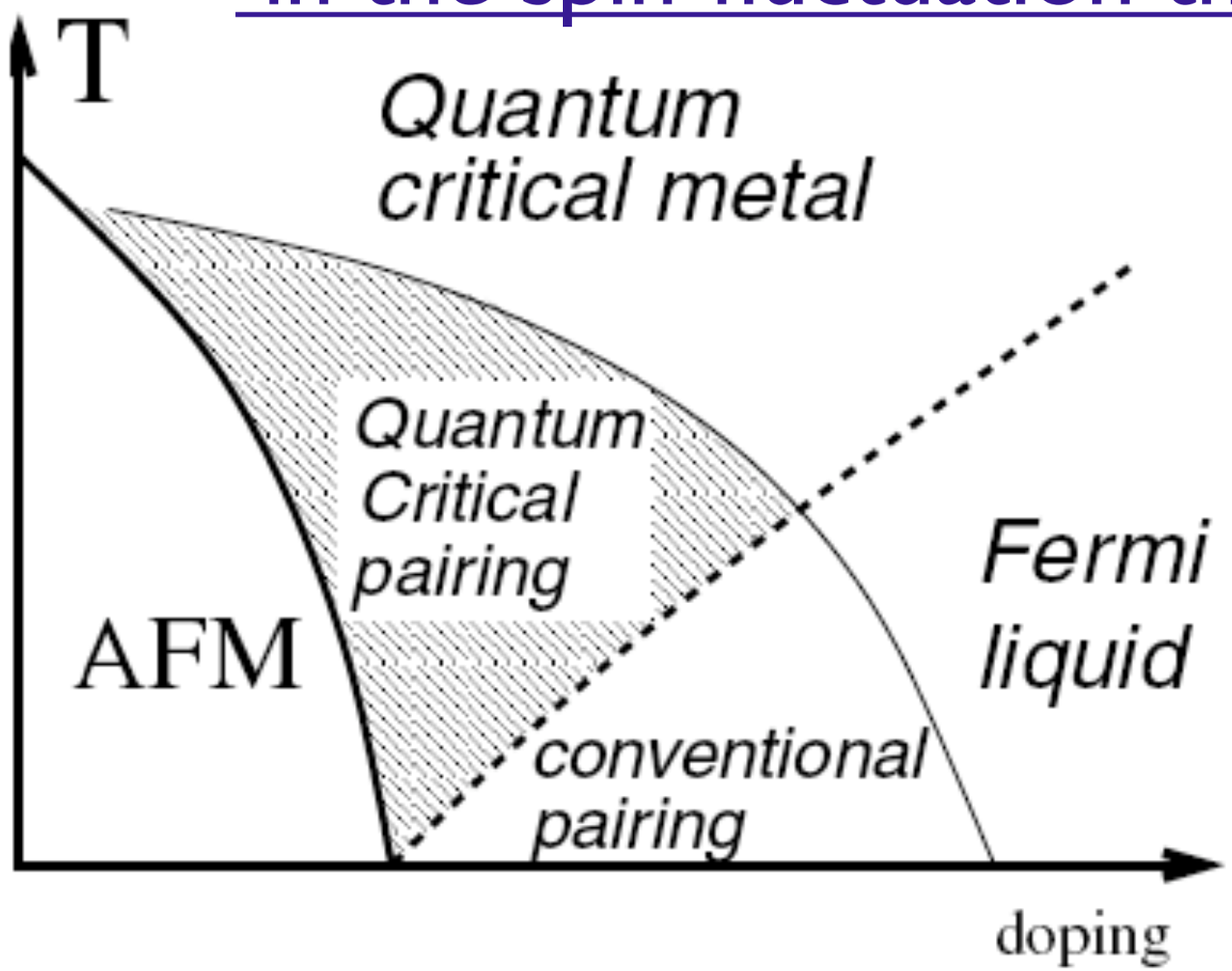
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

Approaching the onset of antiferromagnetism in the spin-fluctuation theory



Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

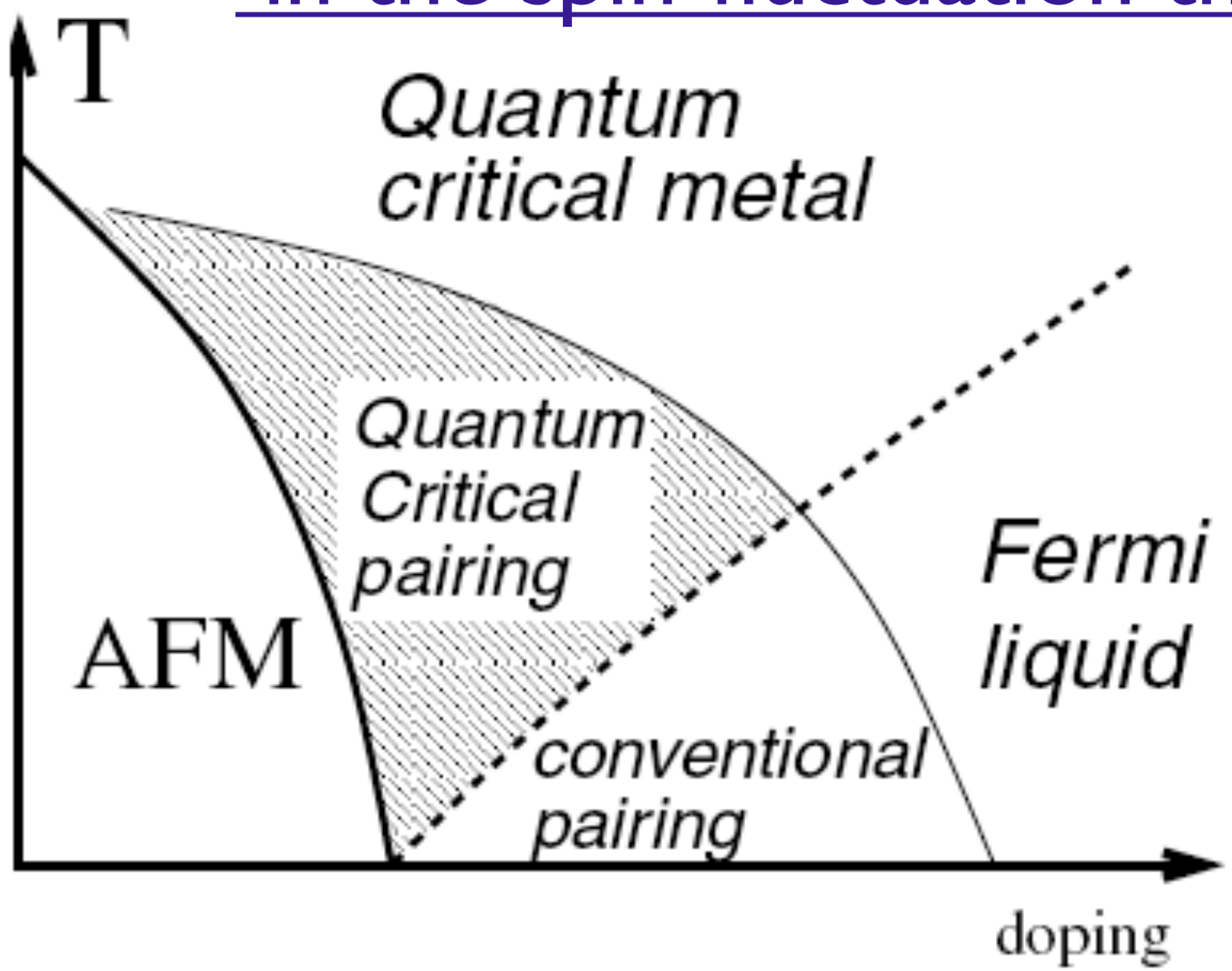
Approaching the onset of antiferromagnetism in the spin-fluctuation theory



- T_c *increases* upon approaching the SDW transition.
  SDW and SC orders do not compete, but attract each other.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

Approaching the onset of antiferromagnetism in the spin-fluctuation theory

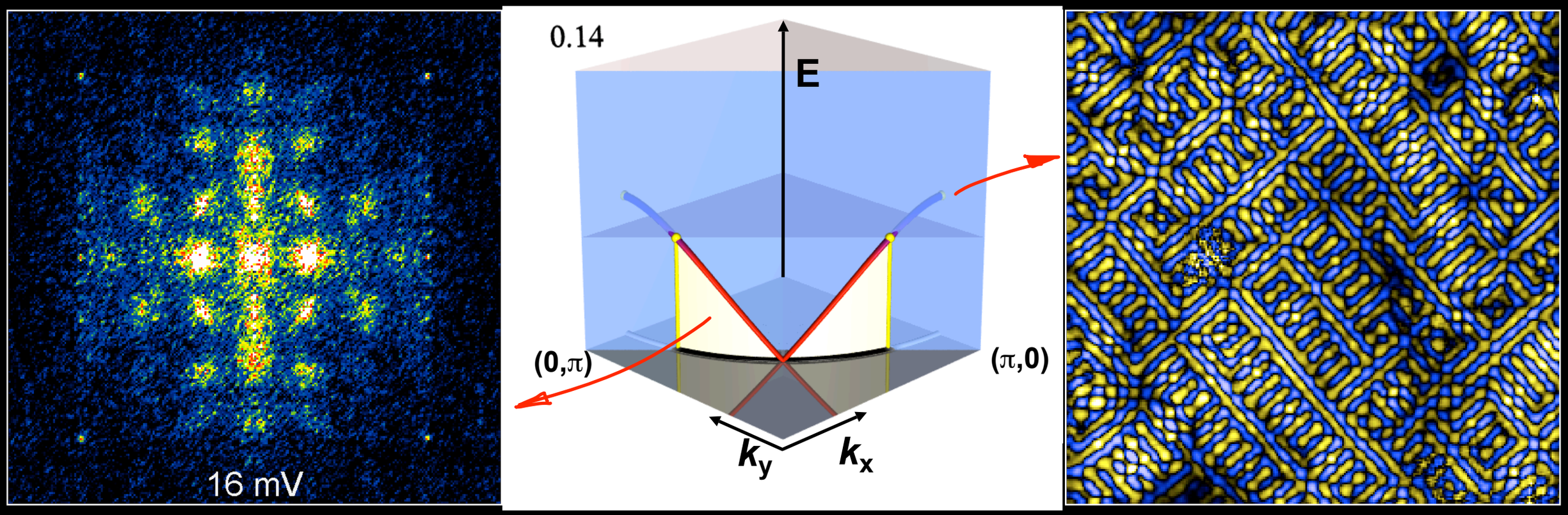


- T_c *increases* upon approaching the SDW transition.
  SDW and SC orders do not compete, but attract each other.

- No simple mechanism for nodal-anti-nodal dichotomy.

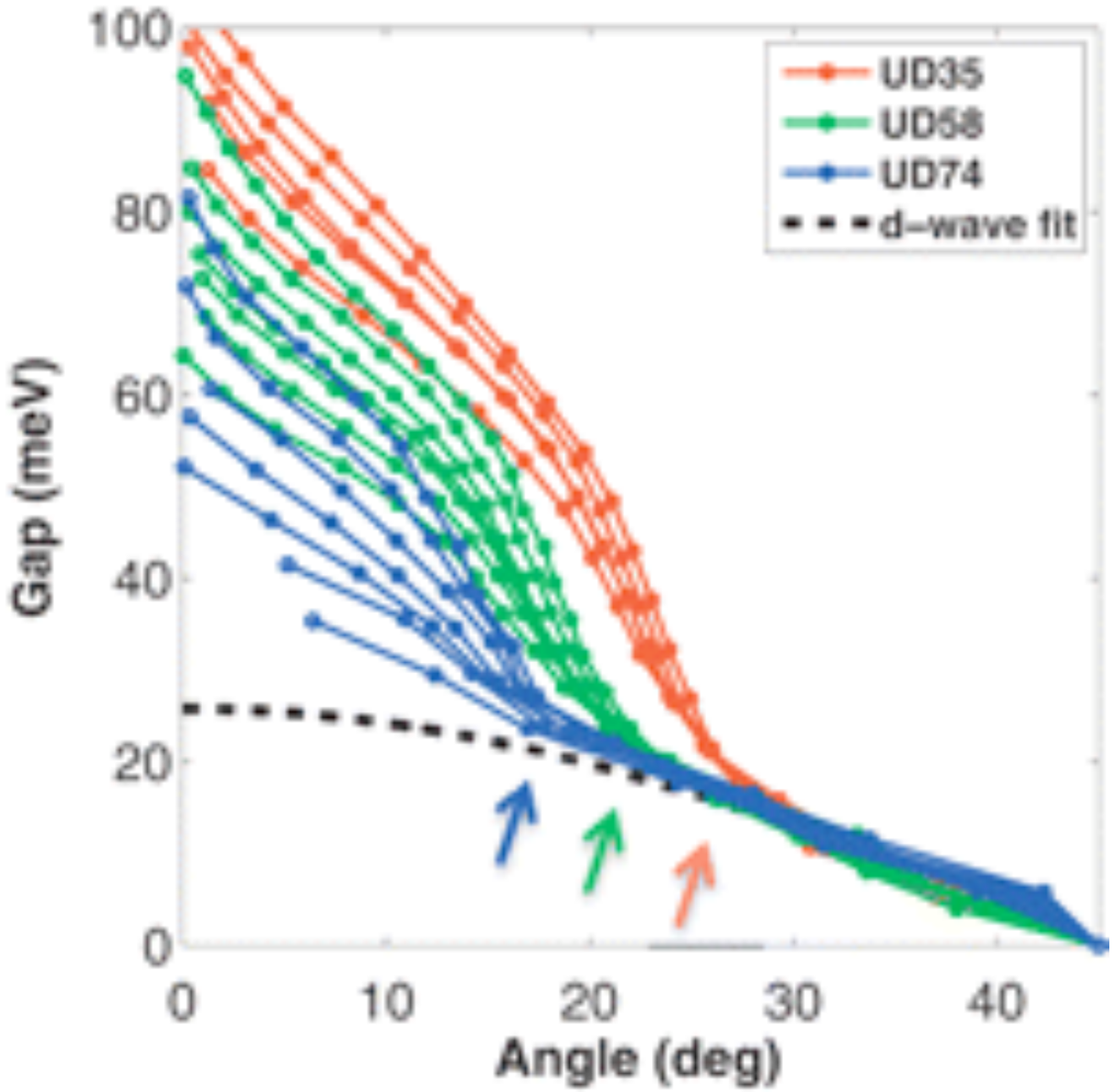
Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

"Nodal/anti-nodal dichotomy" in STM measurements



J.C. Davis and collaborators

STM in BSCCO



A. Pushp, C.V. Parker, A. N. Pasupathy, K. K. Gomes, S. Ono, J. Wen, Z. Xu, G. Gu, and A. Yazdani, *Science* **324**, 1689 (2009)

Outline

1. Phenomenological quantum theory of competition between superconductivity and SDW order
   Survey of recent experiments

2. Superconductivity in the overdoped regime
   BCS pairing by spin fluctuation exchange

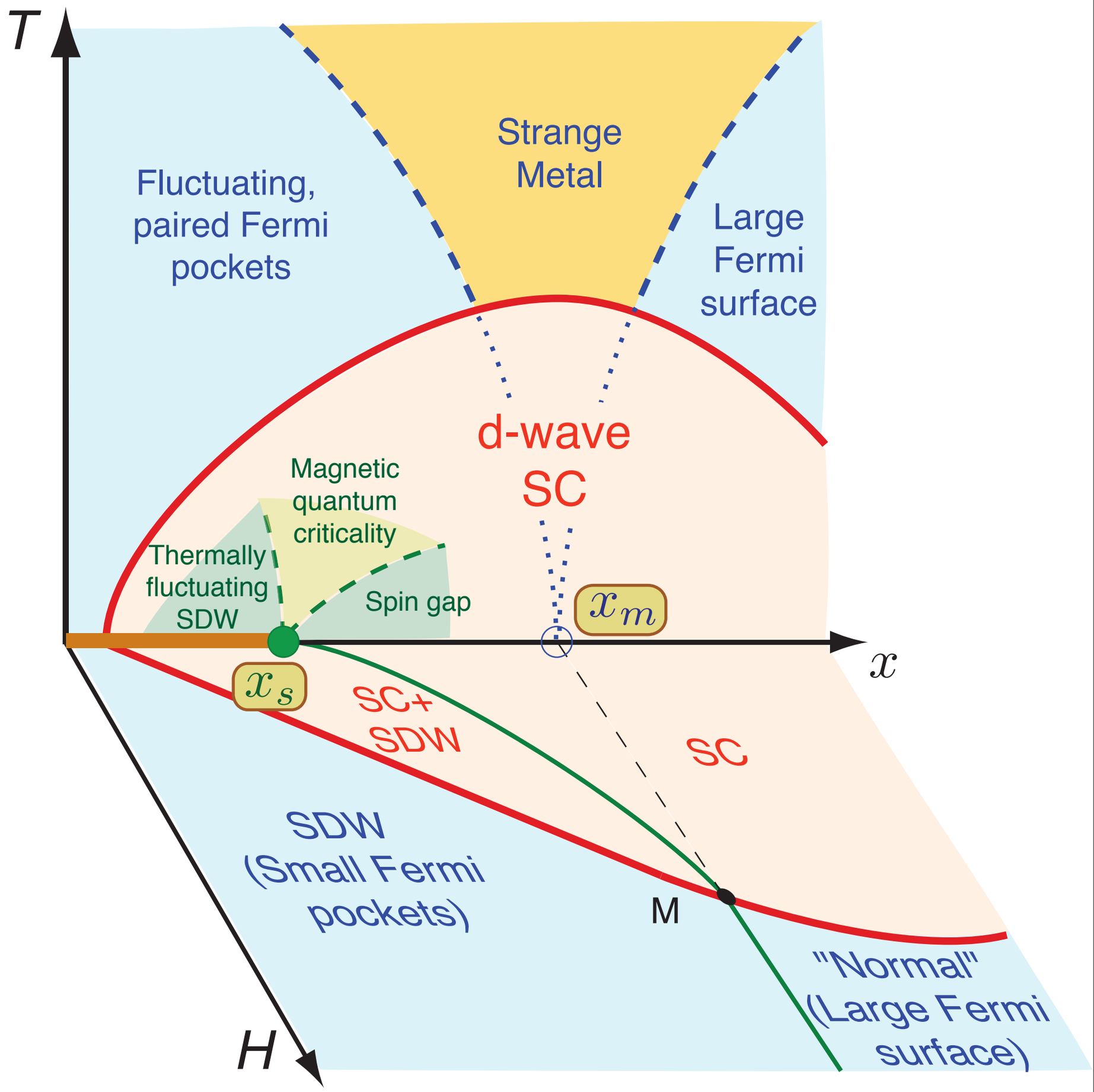
3. Superconductivity in the underdoped regime
   U(1) gauge theory of fluctuating SDW order

Outline

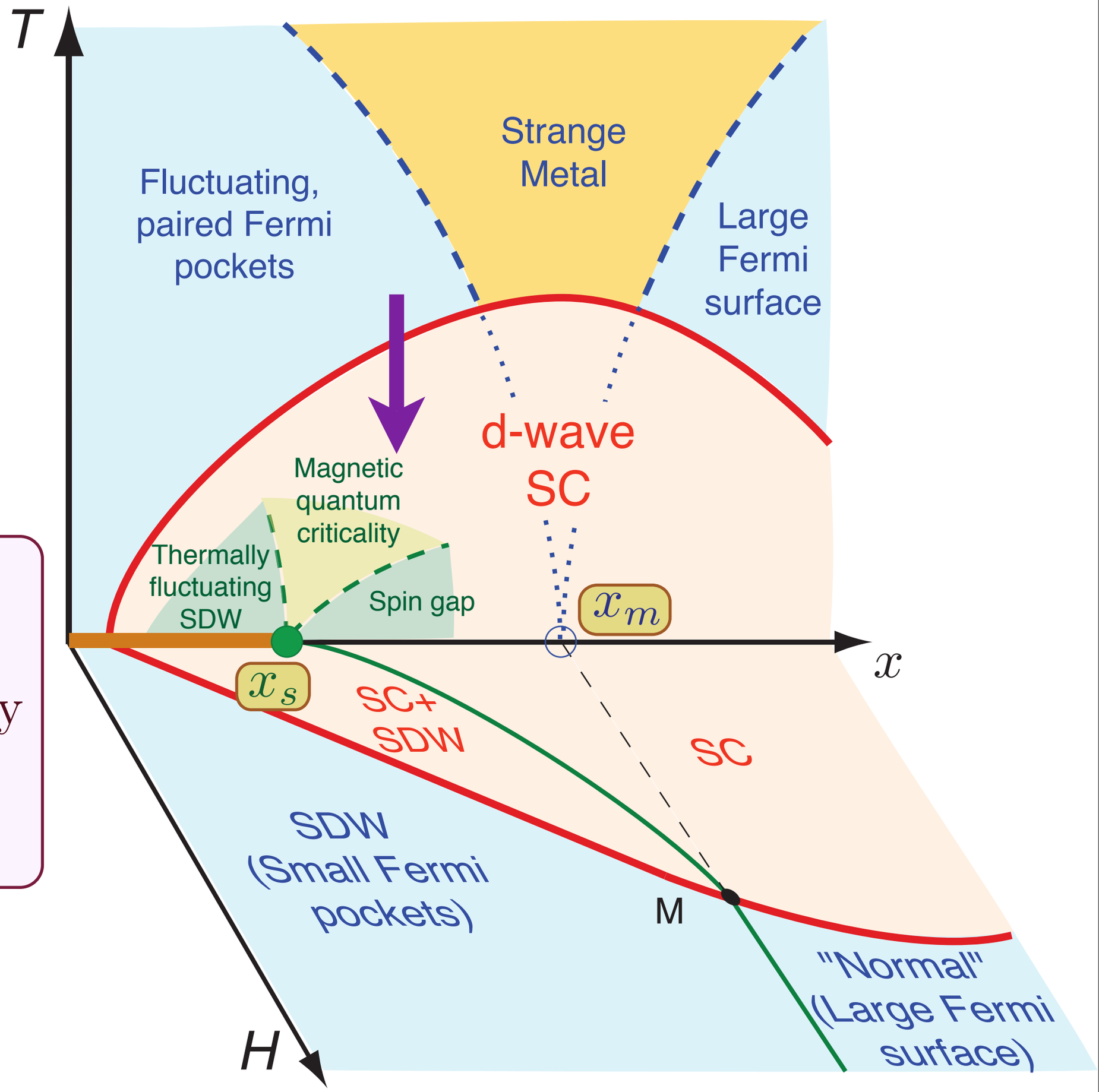
1. Phenomenological quantum theory of competition between superconductivity and SDW order
   Survey of recent experiments

2. Superconductivity in the overdoped regime
   BCS pairing by spin fluctuation exchange

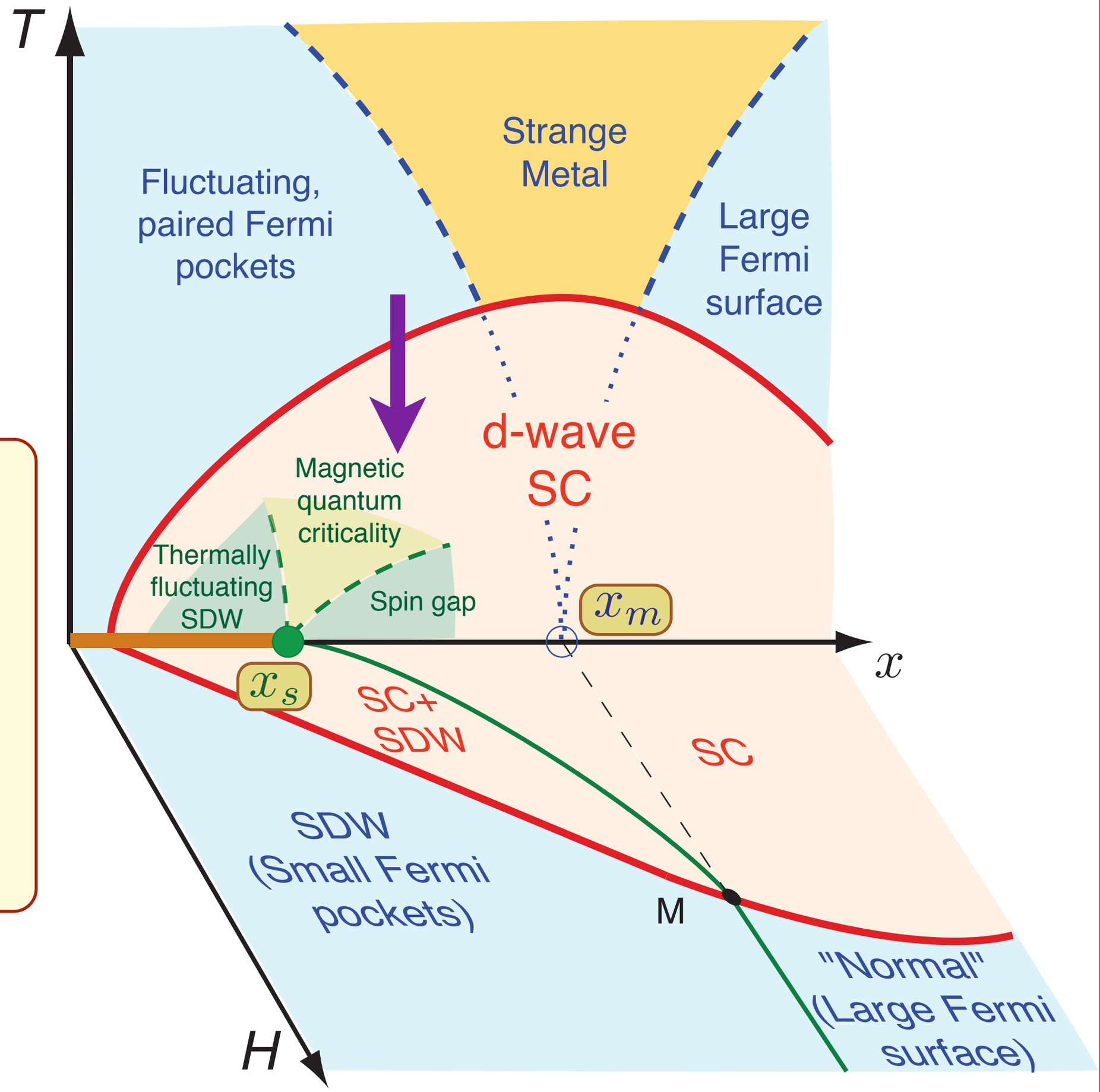
3. Superconductivity in the underdoped regime
   U(1) gauge theory of fluctuating SDW order


T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

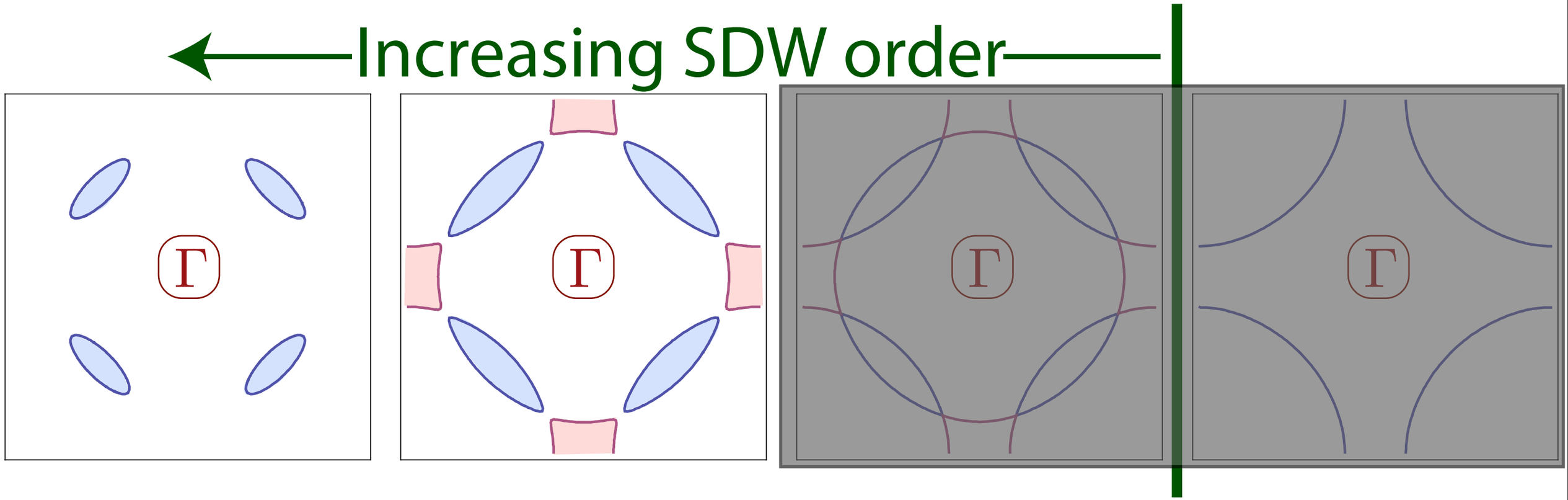
Theory of the onset of d-wave superconductivity from small Fermi pockets



Physics of competition: d-wave SC and SDW "eat up' same pieces of the large Fermi surface.



Theory of underdoped cuprates



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\vec{\varphi}}$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad ; \quad R^\dagger \hat{\vec{\varphi}} \cdot \vec{\sigma} R = \sigma^z \quad ; \quad R^\dagger R = 1$$

Theory of underdoped cuprates

$$\text{With} \quad R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}$$

the theory is invariant under the U(1) gauge transformation

$$z_\alpha \to e^{i\theta} z_\alpha \quad ; \quad \psi_+ \to e^{-i\theta}\psi_+ \quad ; \quad \psi_- \to e^{i\theta}\psi_-$$

and the SDW order is given by

$$\hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

Theory of underdoped cuprates

Starting from the "SDW-fermion" model with Lagrangian

$$\begin{aligned}\mathcal{L} &= \sum_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha}\left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}\right) c_{\mathbf{k}\alpha} \\ &\quad - E_{sdw}\sum_{i} c^{\dagger}_{i\alpha}\hat{\vec{\varphi}}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} \\ &\quad + \frac{1}{2t}\left(\partial_\mu \hat{\vec{\varphi}}\right)^2\end{aligned}$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,

$$\mathcal{L} = \sum_{\mathbf{k},p=\pm} \left[\psi^\dagger_{\mathbf{k}p} \left(\frac{\partial}{\partial \tau} - ipA_\tau + \varepsilon_{\mathbf{k}-p\mathbf{A}} \right) \psi_{\mathbf{k}p} \right.$$

$$\left. - E_{sdw} \psi^\dagger_{\mathbf{k}p} p \psi_{\mathbf{k}+\mathbf{K},p} \right]$$

Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,

- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order

$$+ \frac{1}{t}\left[|(\partial_\tau - iA_\tau) z_\alpha|^2 + v^2 |\nabla - i\mathbf{A}) z_\alpha|^2 + i\lambda(|z_\alpha|^2 - 1) \right]$$

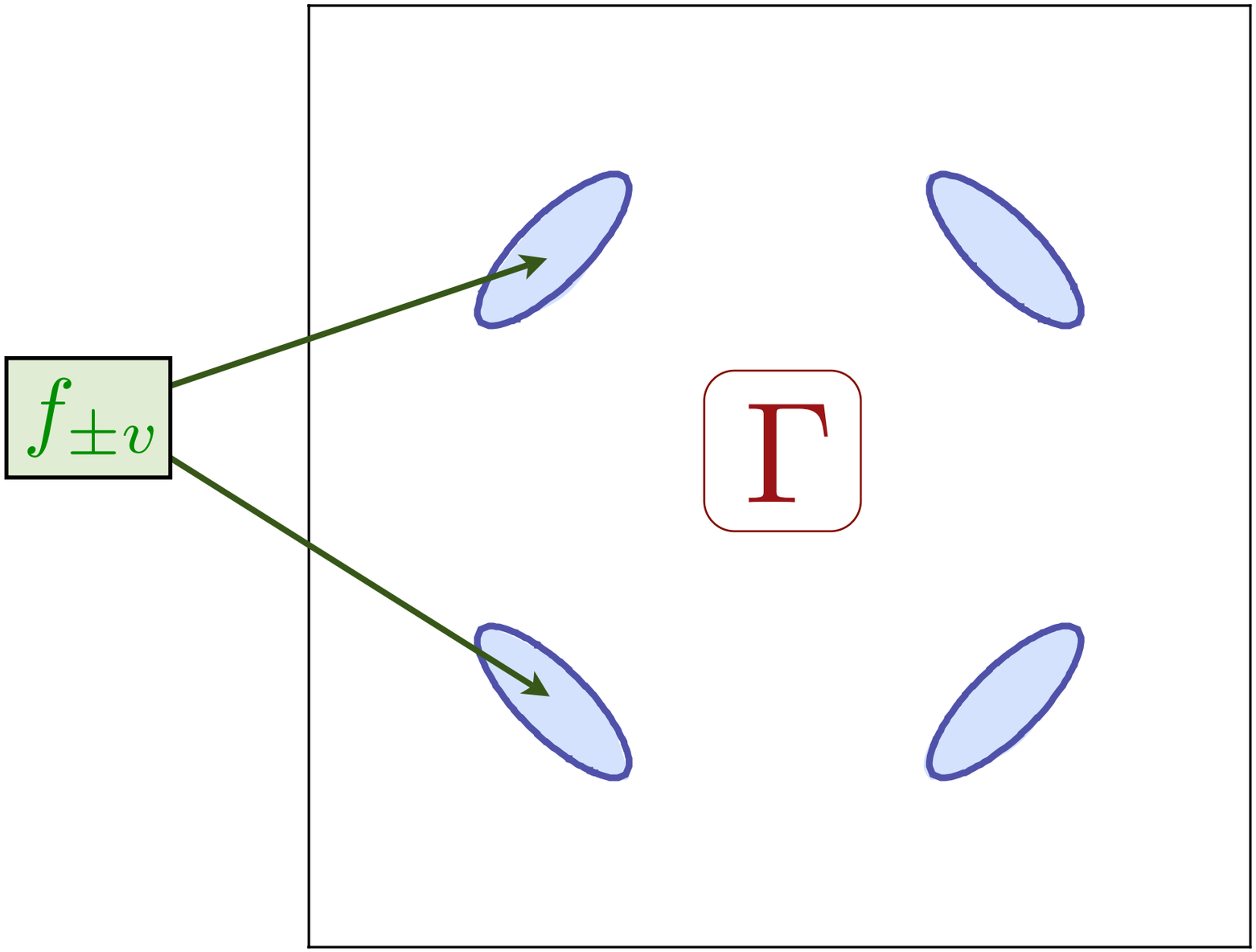
Theory of underdoped cuprates

we obtain a U(1) gauge theory of

- fermions ψ_p with U(1) charge $p = \pm 1$ and pocket Fermi surfaces,
- relativistic complex scalars z_α with charge 1, representing the orientational fluctuations of the SDW order
- Shraiman-Siggia term

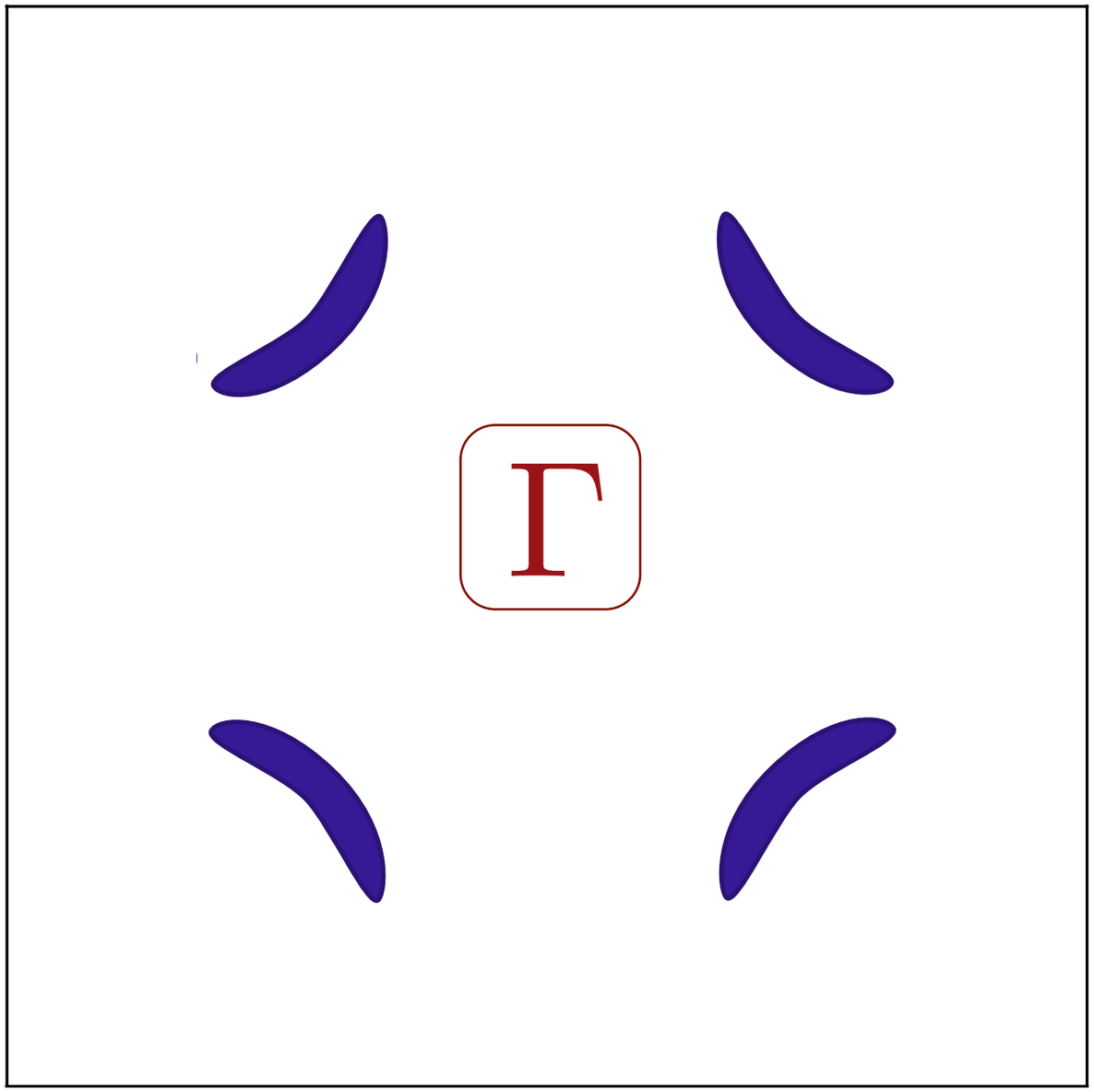
$$\sum_{\mathbf{k},\mathbf{p},\mathbf{q}} z_{\mathbf{q}-\mathbf{p}/2,\downarrow} z_{\mathbf{q}+\mathbf{p}/2,\uparrow} \left(\mathbf{p} \cdot \frac{\partial \varepsilon(\mathbf{k})}{\partial \mathbf{k}} \right) \psi^\dagger_{\mathbf{k}+\mathbf{q},-} \psi_{\mathbf{k}-\mathbf{q},+} + \text{c.c.}$$

Normal state above T_c



$\psi_\pm = f_{\pm v}$ near $(\pm\pi/2, \pm\pi/2)$. These are spinless fermions, and are invisible to photoemission.

Normal state above T_c



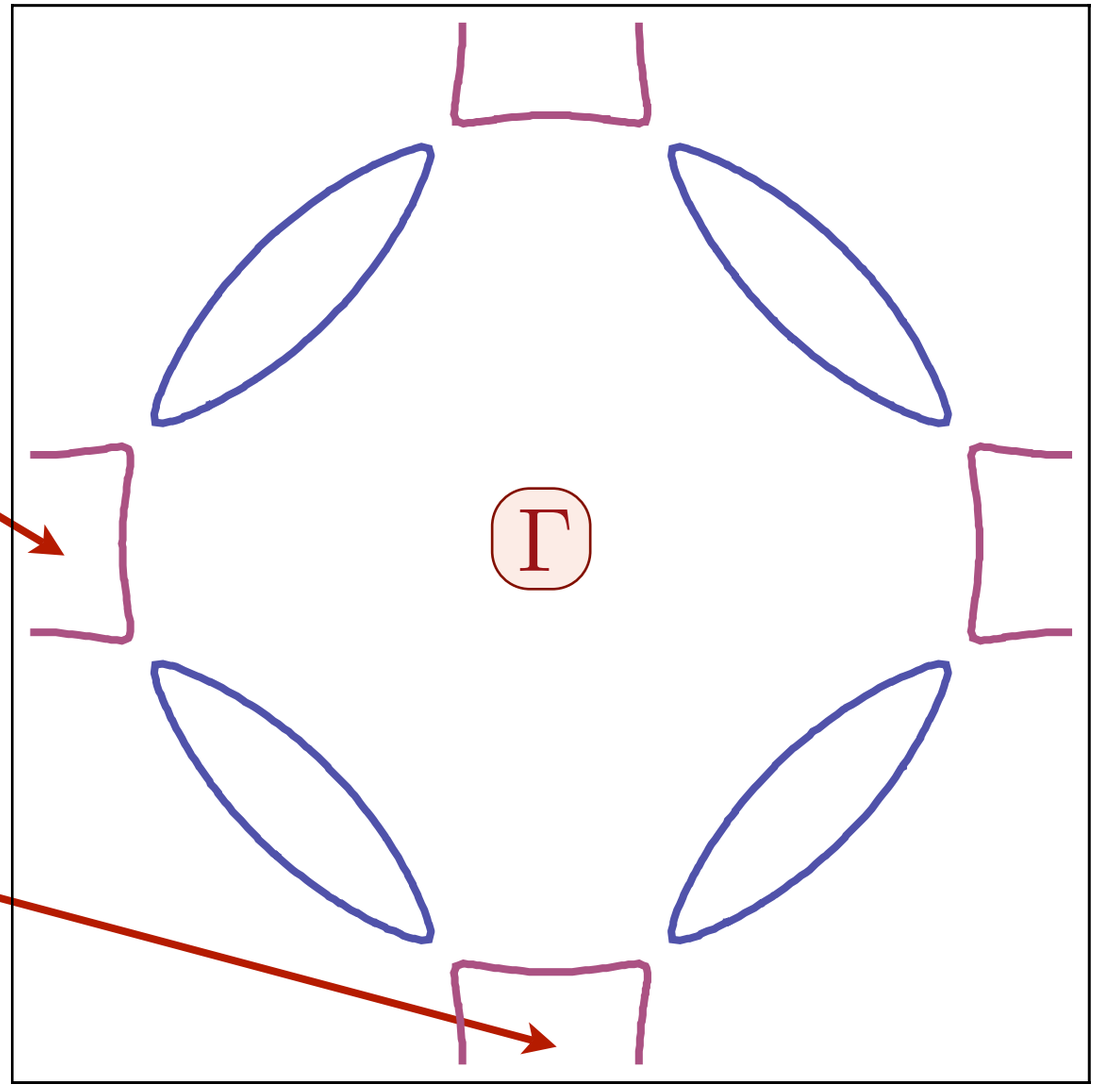
Gauge forces lead to bound states of $f_{\pm v}$ and z_α which form "banana Fermi surfaces" of spinful fermions visible to photoemission.

R. K. Kaul, Yong-Baek Kim, S. Sachdev, and T. Senthil,
Nature Physics **4**, 28 (2008).

Pairing across T_c

Electron $c_{2\alpha}$,
spinless fermion $g_\pm$

Electron $c_{1\alpha}$,
spinless fermion $g_\pm$



Focus on pairing near $(\pi, 0)$, $(0, \pi)$, where $\psi_\pm \equiv g_\pm$,
and the electron operators are

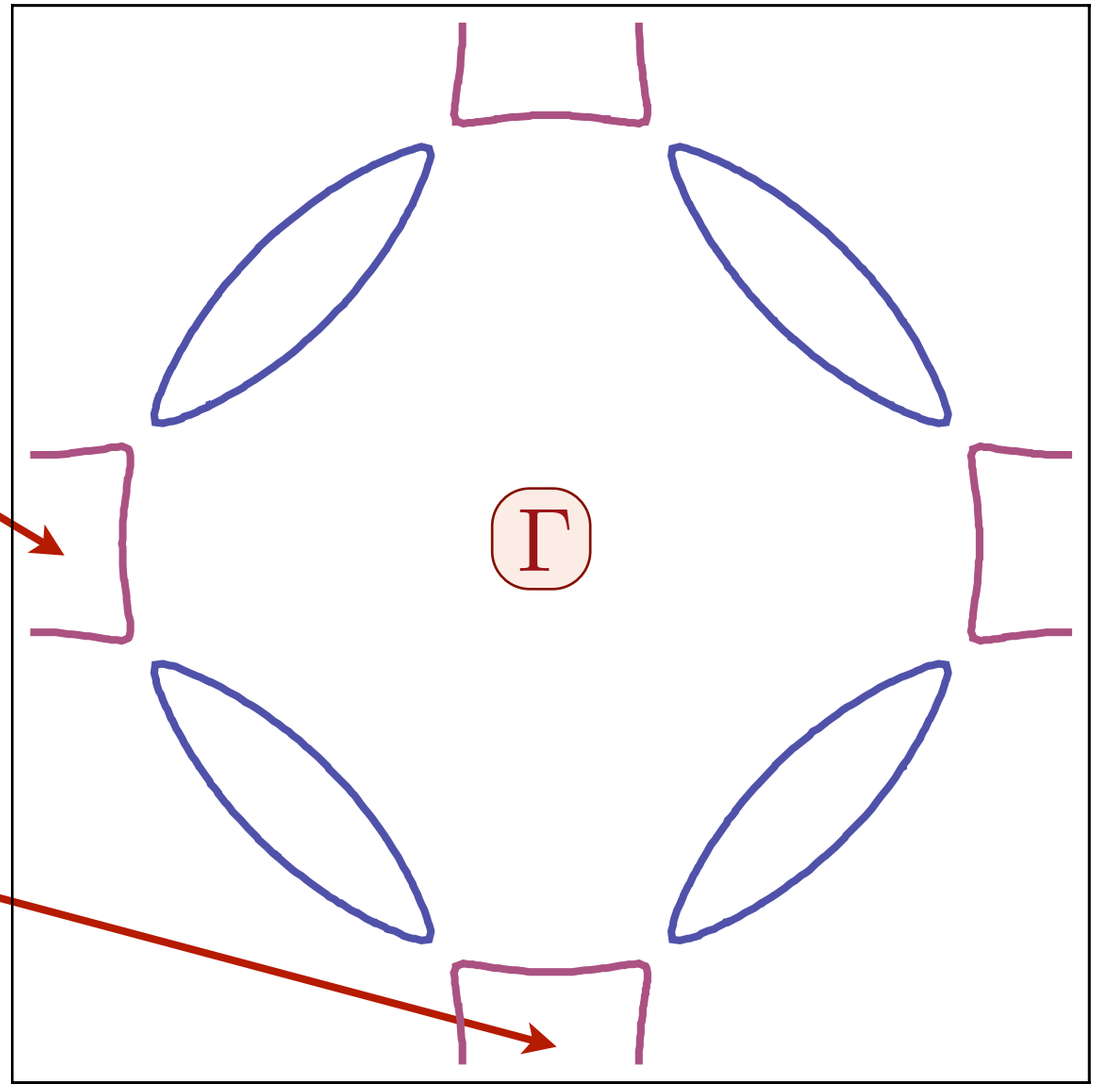
$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix} \quad ; \quad \begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ -g_- \end{pmatrix}$$

$$\mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.$$

Pairing across T_c

Electron $c_{2\alpha}$,
spinless fermion $g_\pm$

Electron $c_{1\alpha}$,
spinless fermion $g_\pm$



Focus on pairing near $(\pi, 0)$, $(0, \pi)$, where $\psi_\pm \equiv g_\pm$,
and the electron operators are

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix} \quad ; \quad \begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ -g_- \end{pmatrix}$$

$$\mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.$$

Fluctuating pocket theory for electrons near $(0, \pi)$ and $(\pi, 0)$

Attractive gauge forces lead to simple s-wave pairing of the $g_\pm$

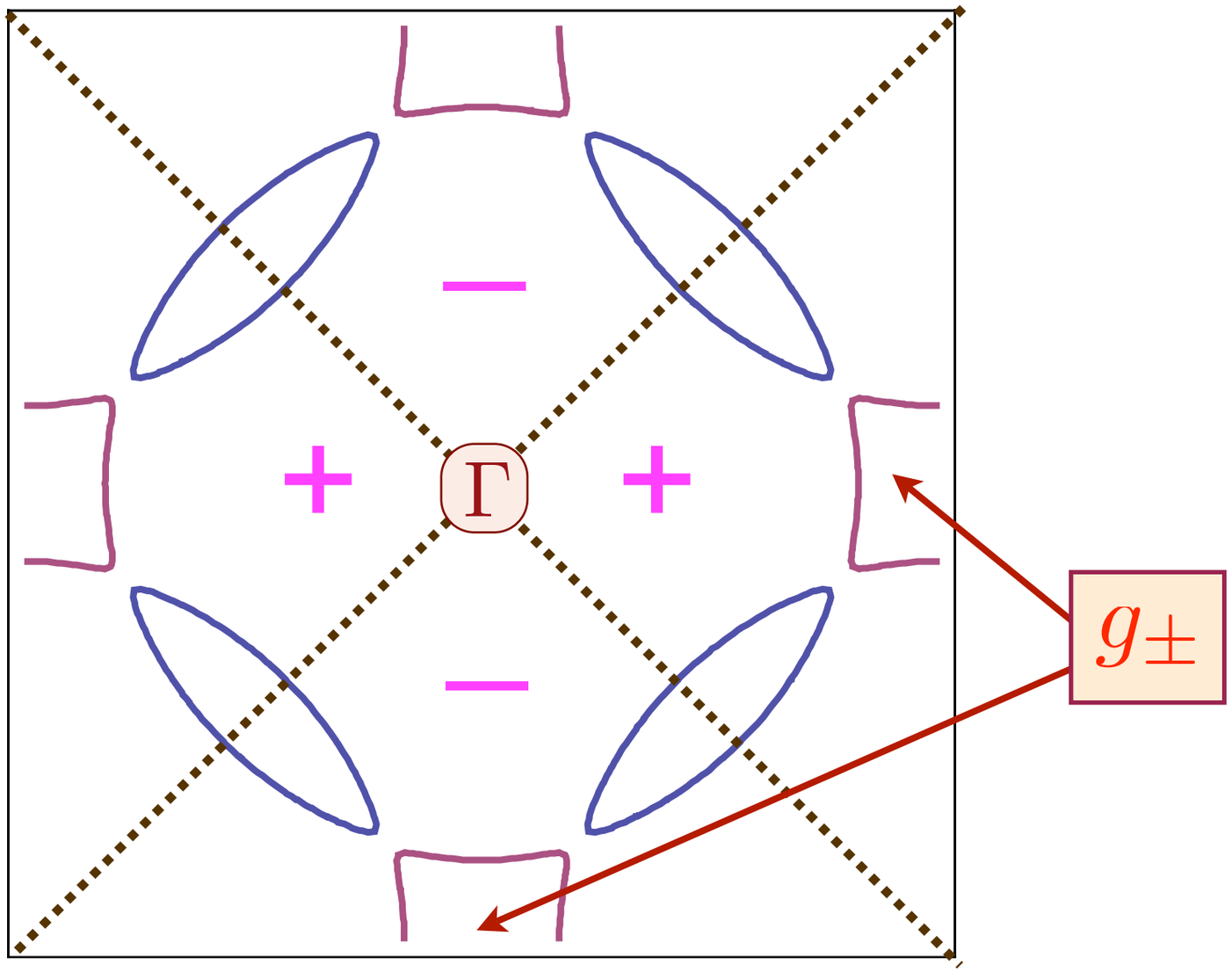
$$\langle g_+ g_- \rangle = \Delta$$

For the physical electron operators, this pairing implies

$$\begin{aligned} \langle c_{1\uparrow} c_{1\downarrow} \rangle &= \Delta \left\langle |z_\alpha|^2 \right\rangle \\ \langle c_{2\uparrow} c_{2\downarrow} \rangle &= -\Delta \left\langle |z_\alpha|^2 \right\rangle \end{aligned}$$

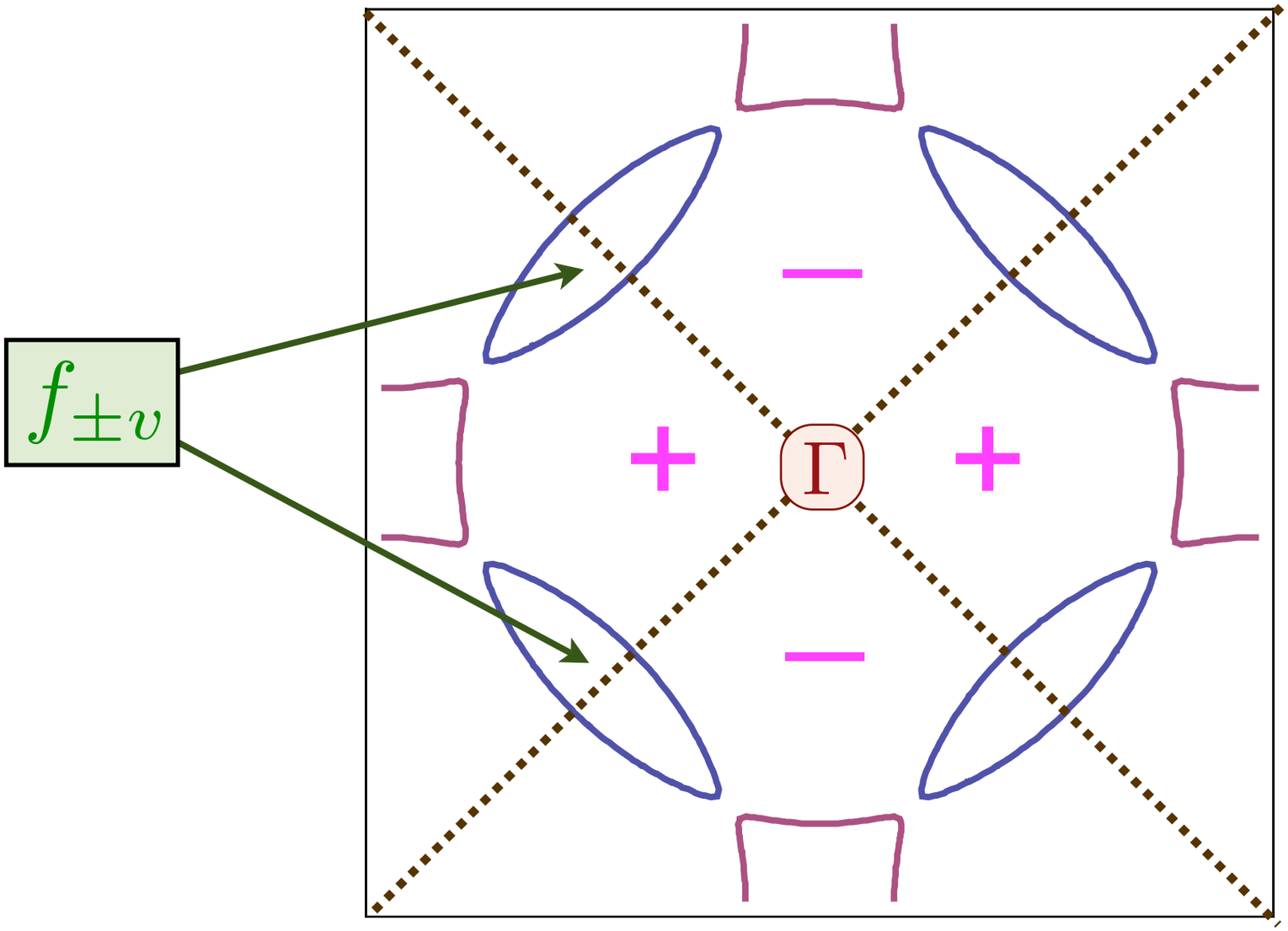
i.e. d-wave pairing !

Strong pairing of the $g_\pm$ electron pockets



$$\langle g_+ g_- \rangle = \Delta$$

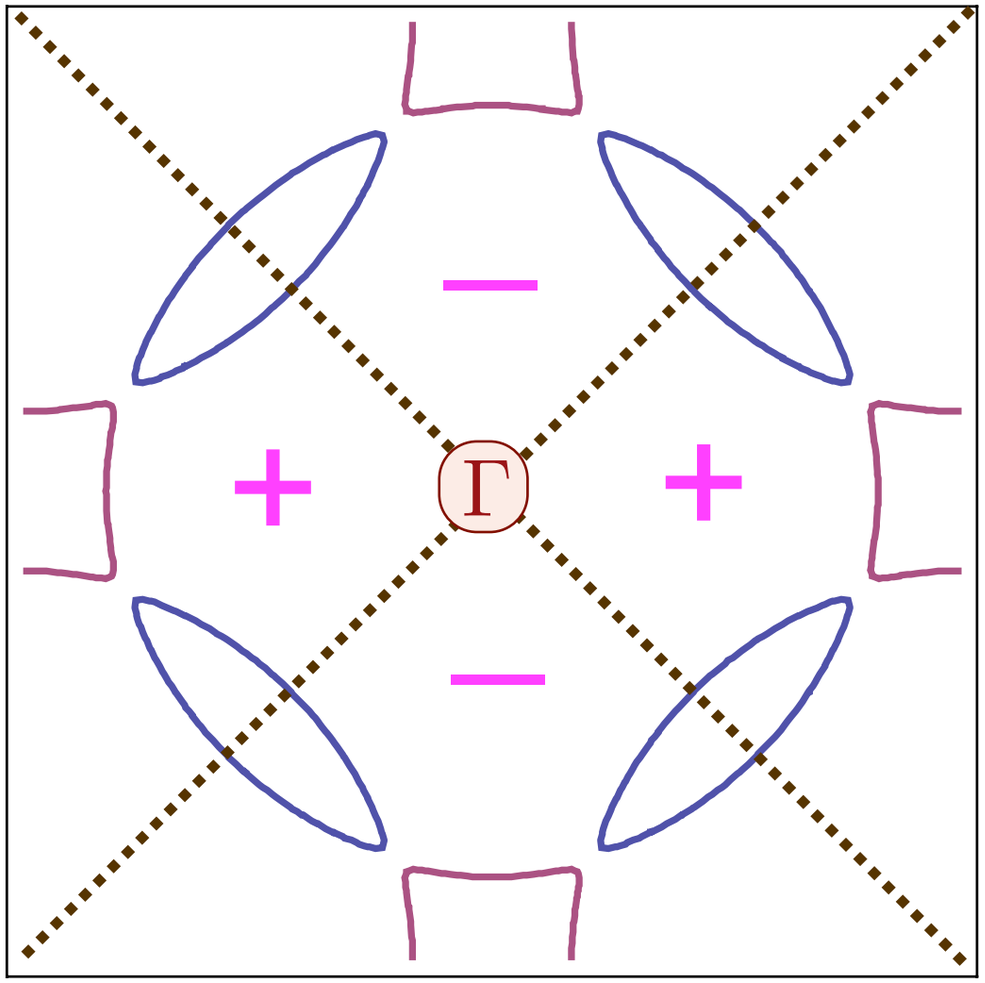
Weak pairing of the $f_\pm$ hole pockets



$$
\begin{aligned}
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k})\rangle &\sim (k_x - k_y) J \langle g_+ g_-\rangle; \\
\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &\sim (k_x + k_y) J \langle g_+ g_-\rangle; \\
\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k})\rangle &= 0,
\end{aligned}
$$

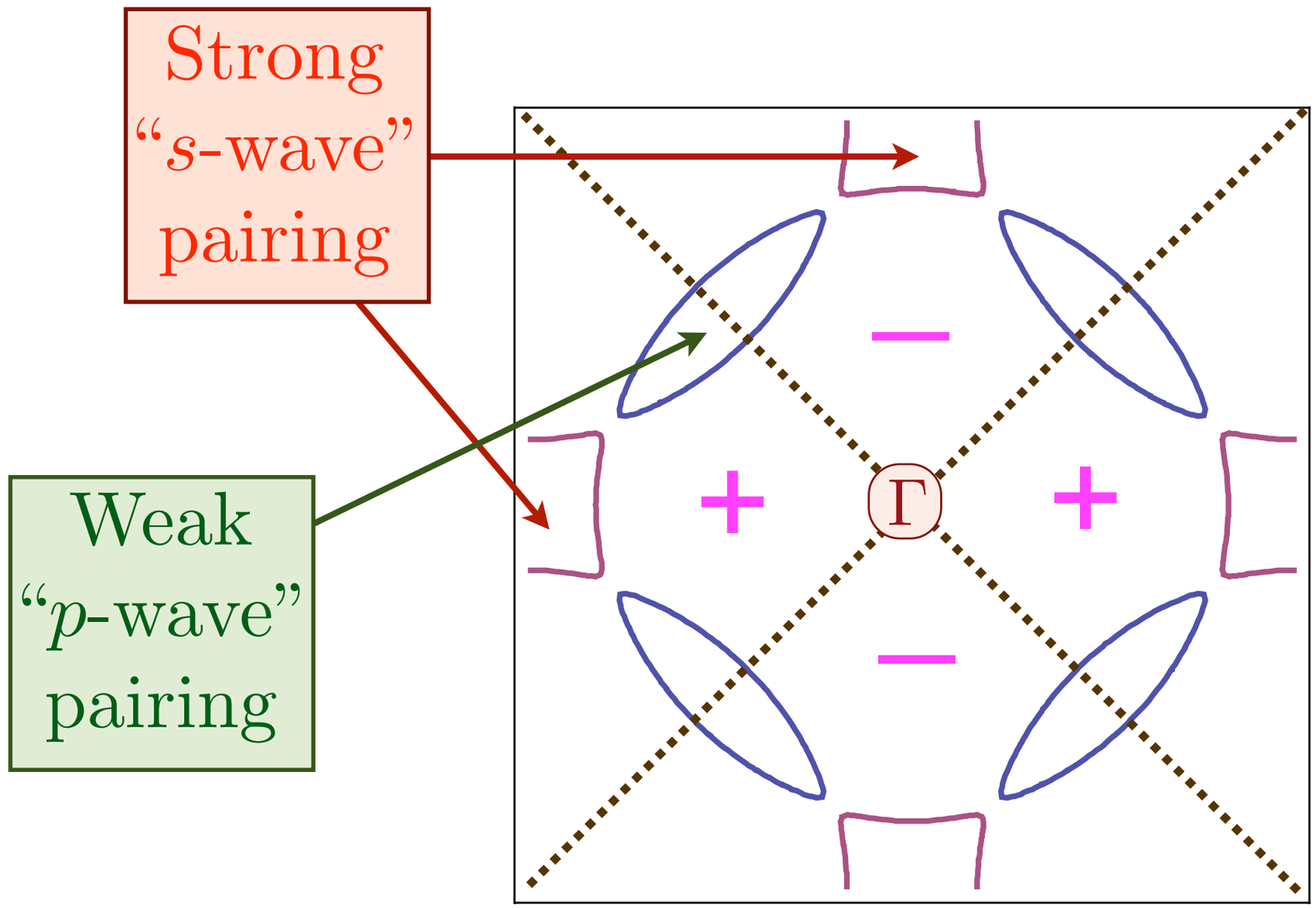
Features of superconductivity

- d-wave superconductivity.



Features of superconductivity

- d-wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.

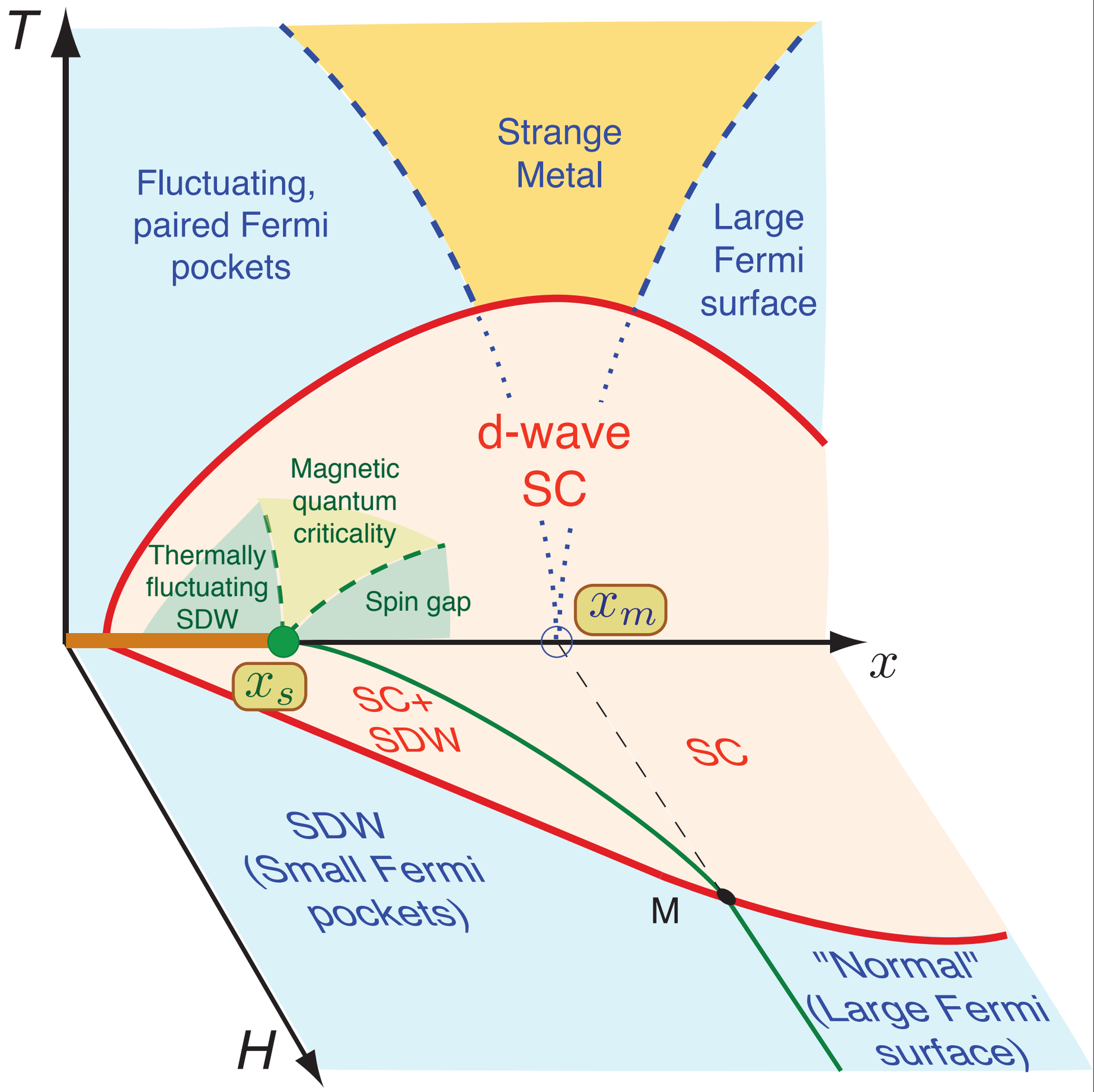


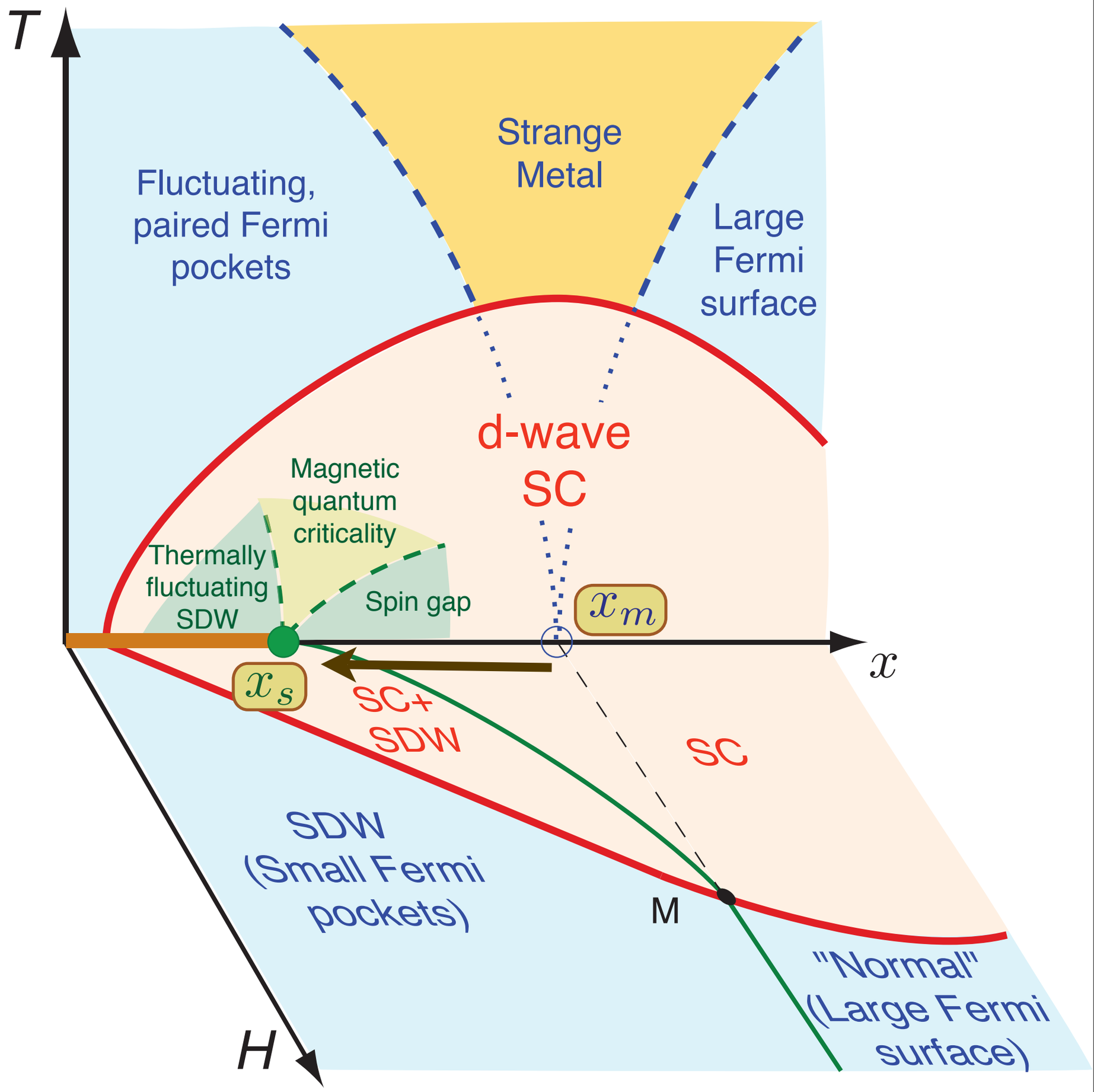
Features of superconductivity

- d-wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.
- T_c decreases as spin correlation increases (competing order effect).

Features of superconductivity

- d-wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.
- T_c decreases as spin correlation increases (competing order effect).
- Shift in quantum critical point of SDW ordering: gauge fluctuations are stronger in the superconductor.


T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H


T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

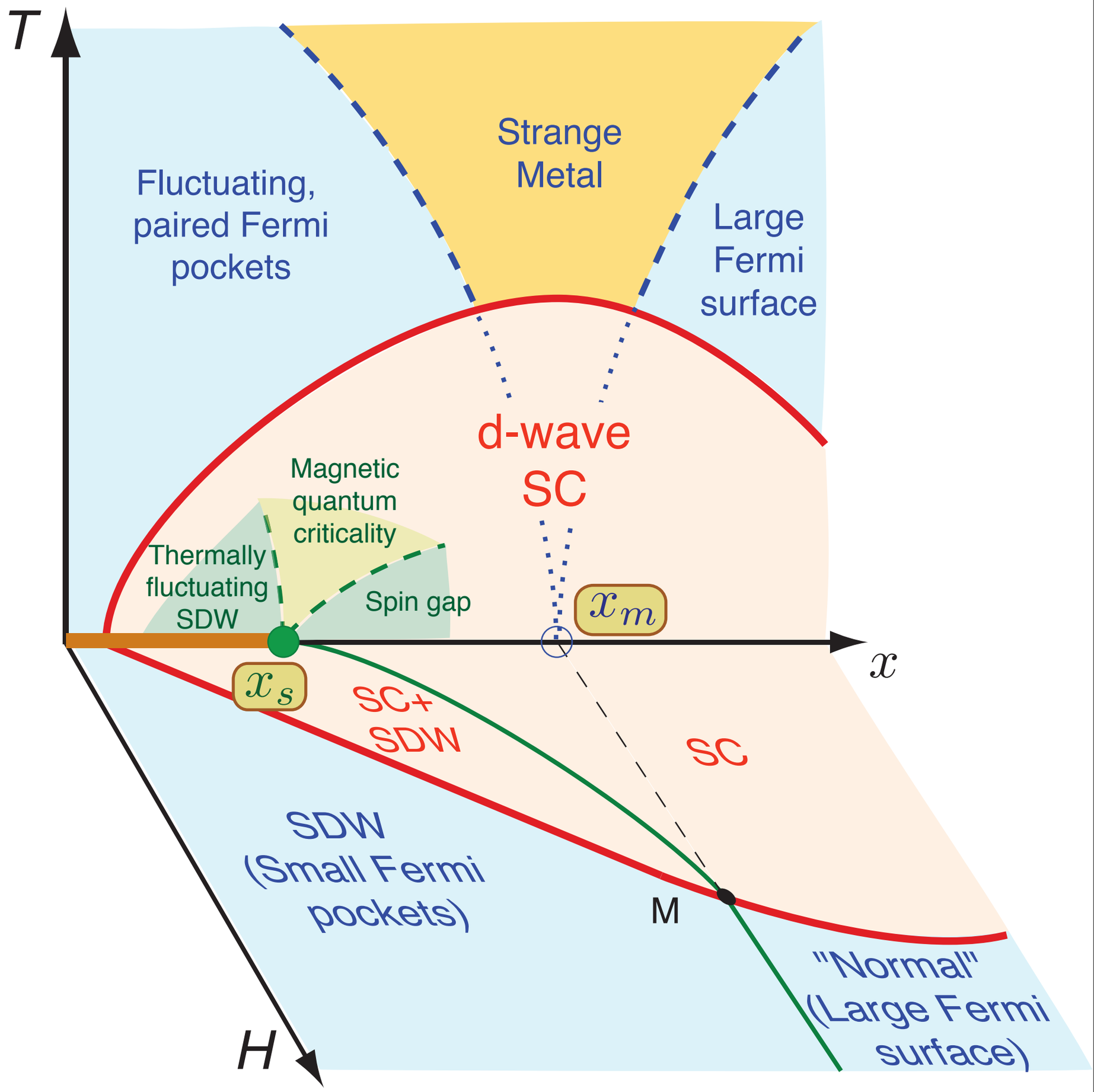
Features of superconductivity

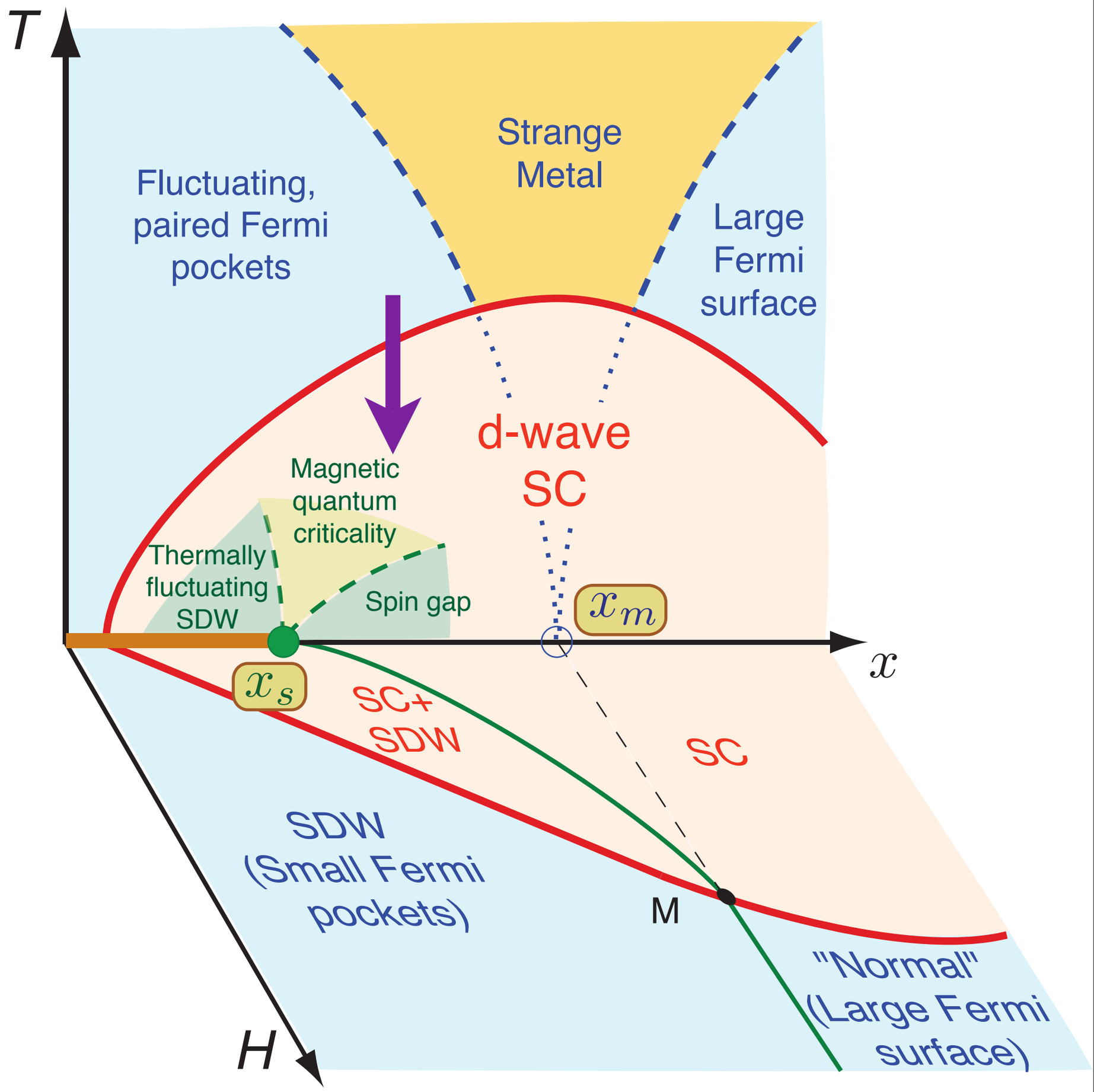
- d-wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.
- T_c decreases as spin correlation increases (competing order effect).
- Shift in quantum critical point of SDW ordering: gauge fluctuations are stronger in the superconductor.

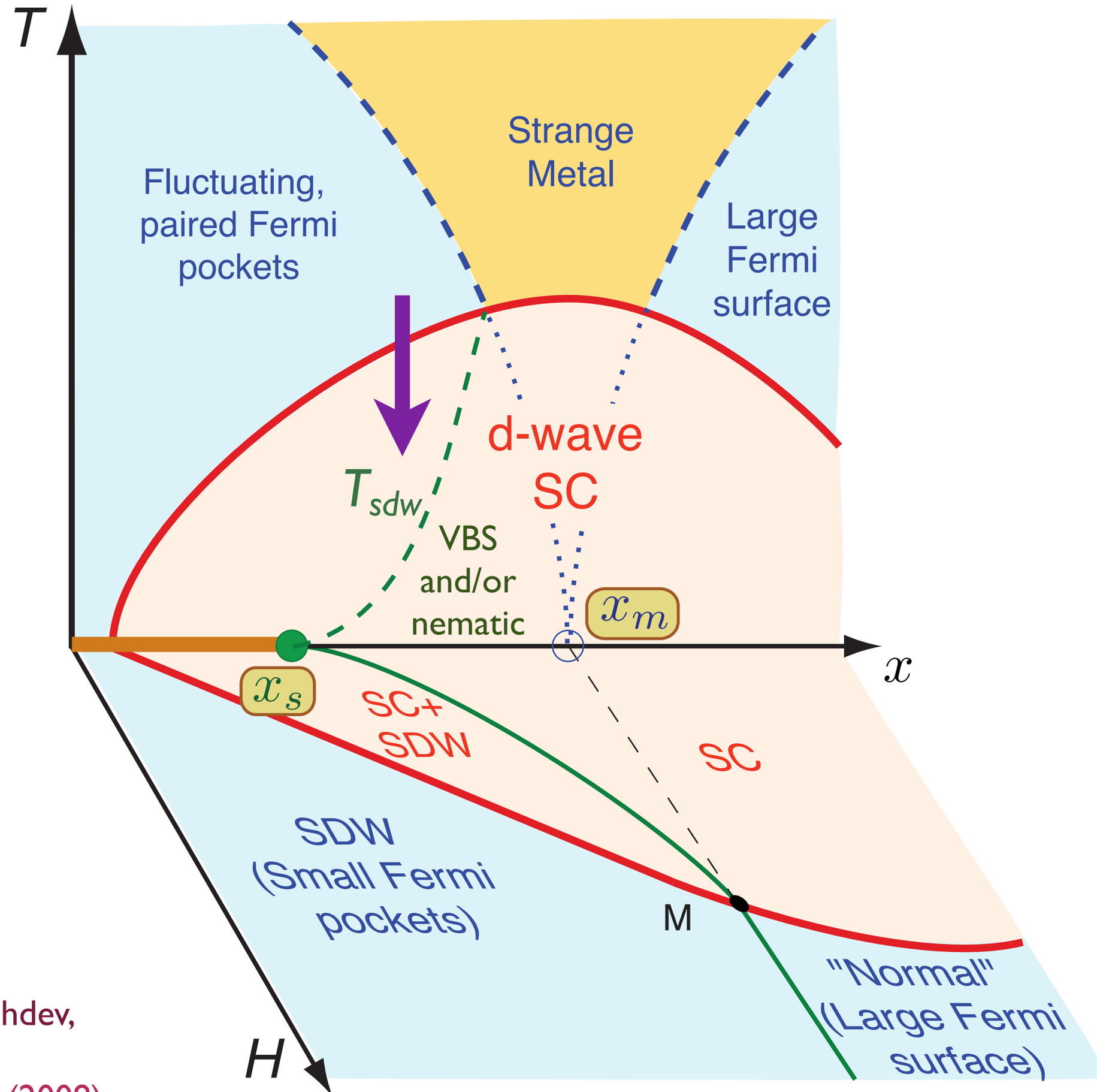
Features of superconductivity

- d-wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.
- T_c decreases as spin correlation increases (competing order effect).
- Shift in quantum critical point of SDW ordering: gauge fluctuations are stronger in the superconductor.
- After onset of superconductivity, monopoles condense and lead to confinement and **nematic** and/or **valence bond solid** (VBS) order.

R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).

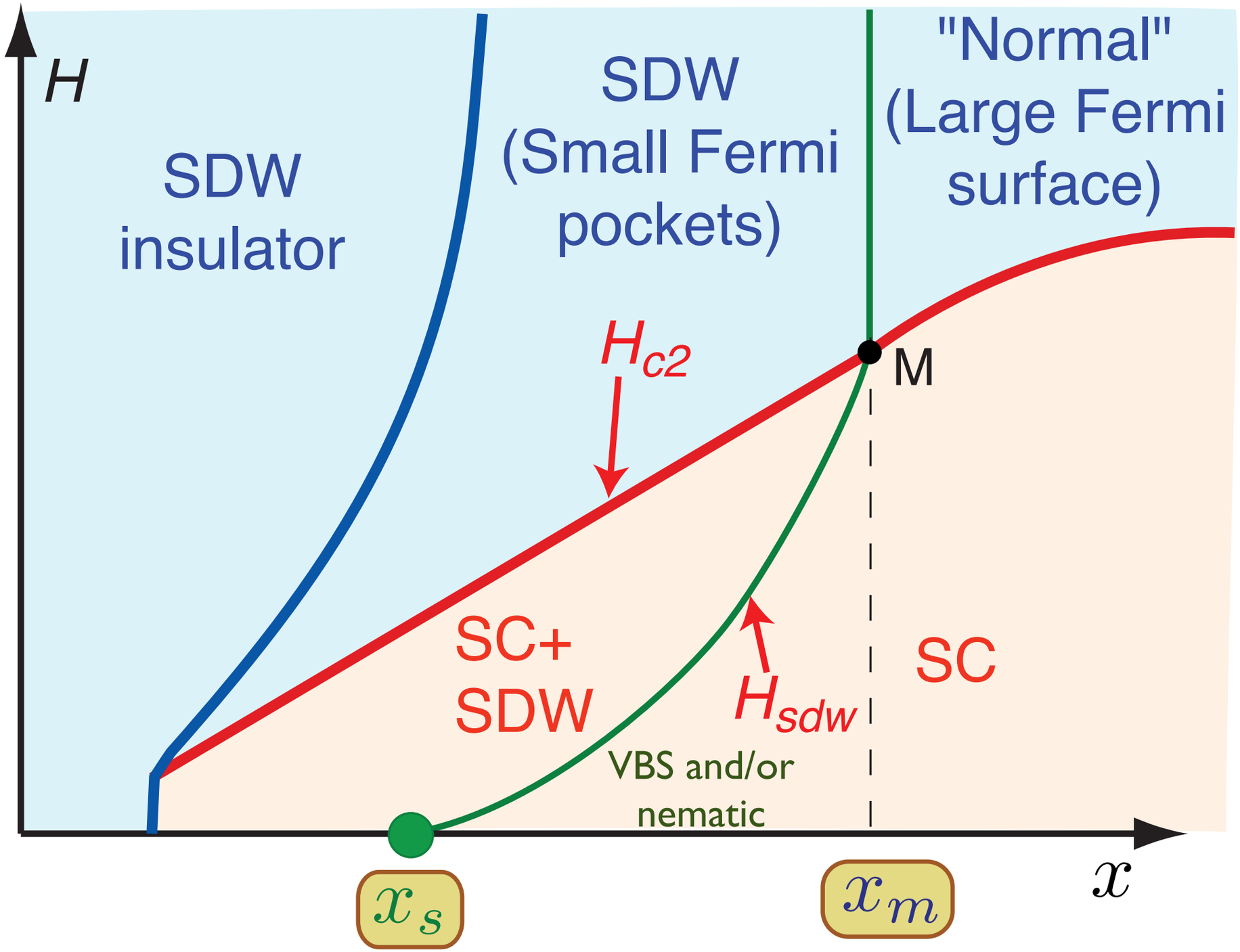

T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H


T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H



R. K. Kaul, M. Metlitksi, S. Sachdev,
and Cenke Xu,
Physical Review B **78**, 045110 (2008).

Include metal-insulator transition



Conclusions

Theory of competition between spin density wave order and superconductivity:

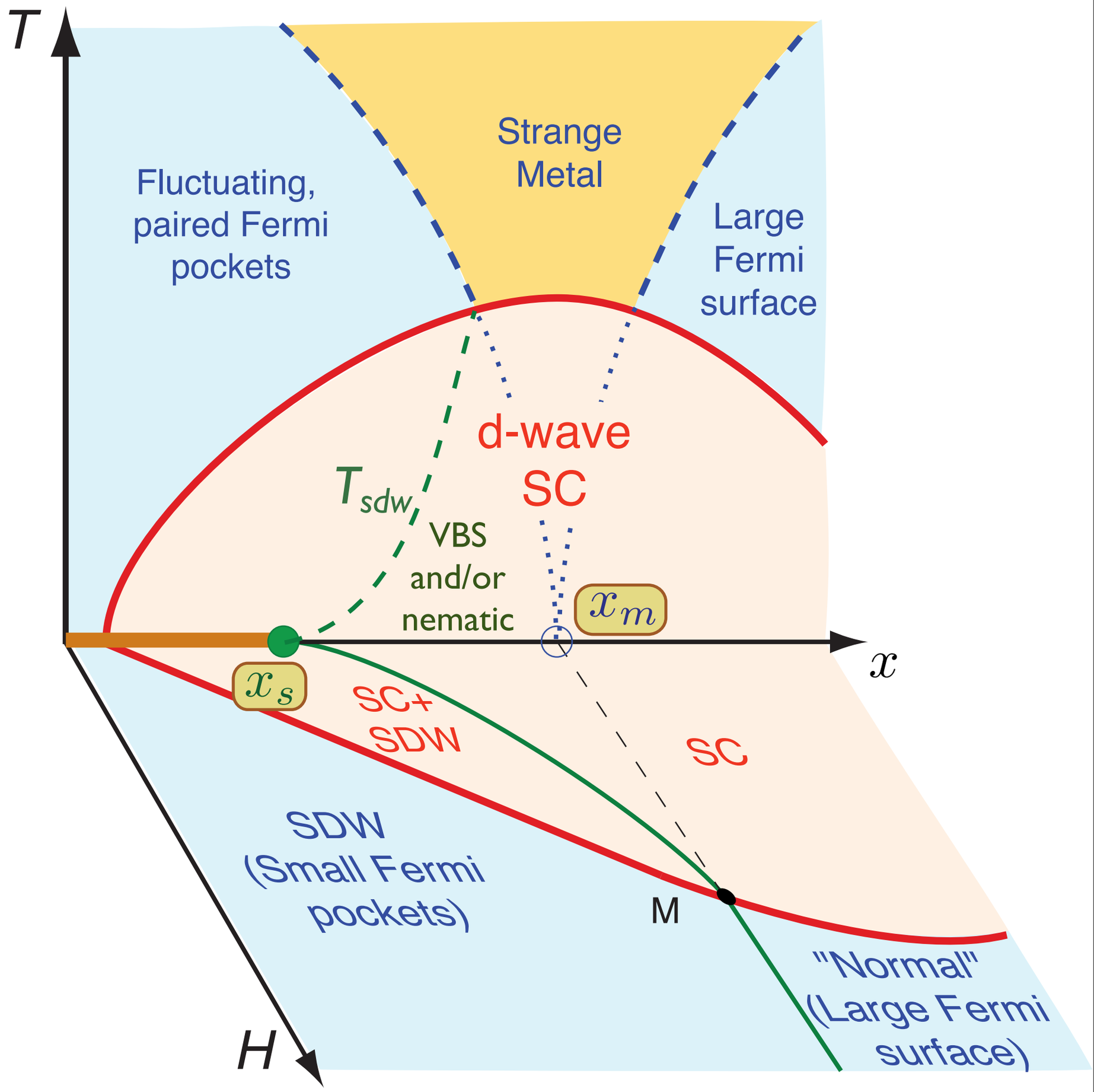
Spin density waves and *d*-wave superconductivity compete for the same portion of the Fermi surface: expressed by U(1) gauge theory of "pocket" fermions and CP^1 model

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity


T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
T_{sdw}
VBS and/or nematic
x_m
x
x_s
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H