Quantum criticality: from antiferromagnets and superconductors to black holes

Reviews:
arXiv:0907.0008
arXiv:0810.3005 (with Markus Mueller)

Talk online: sachdev.physics.harvard.edu

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Markus Mueller, Trieste
Joerg Schmalian, Iowa

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Sean Hartnoll, Harvard
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Dam Son, Washington

Outline

1. Coupled dimer antiferromagnets

   Order parameters and Landau-Ginzburg criticality

2. Graphene

   `Topological' Fermi surface transitions

3. Quantum criticality and black holes

   AdS_4 theory of compressible quantum liquids

4. Quantum criticality in the cuprates

   Global phase diagram and the spin
   density wave transition in metals

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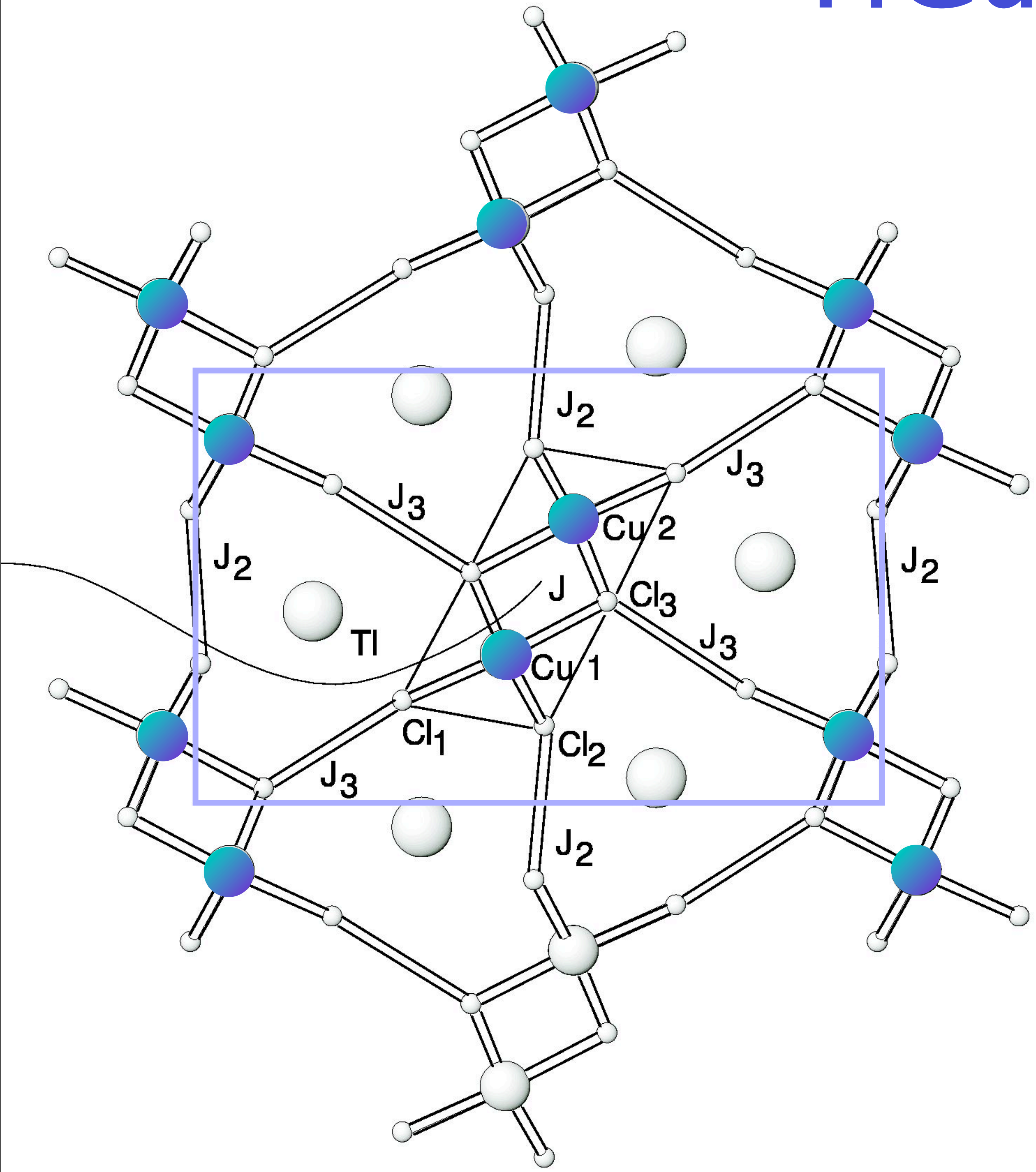
TlCuCl$_3$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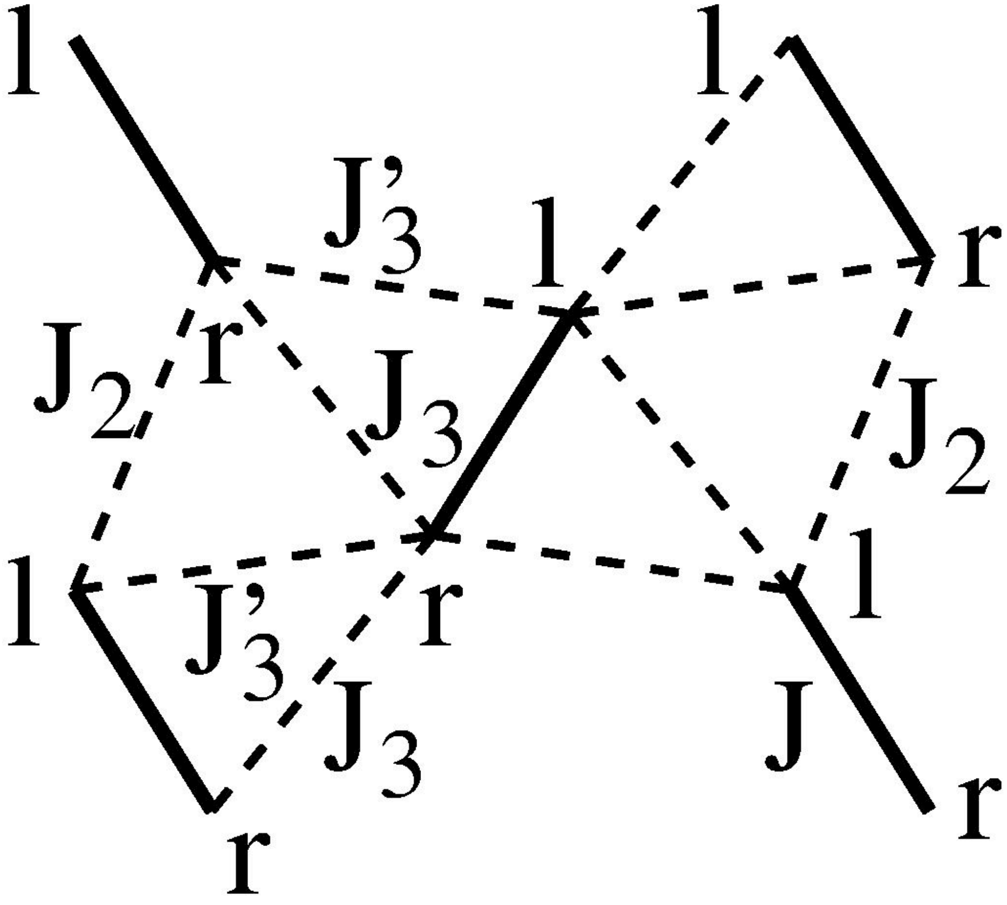
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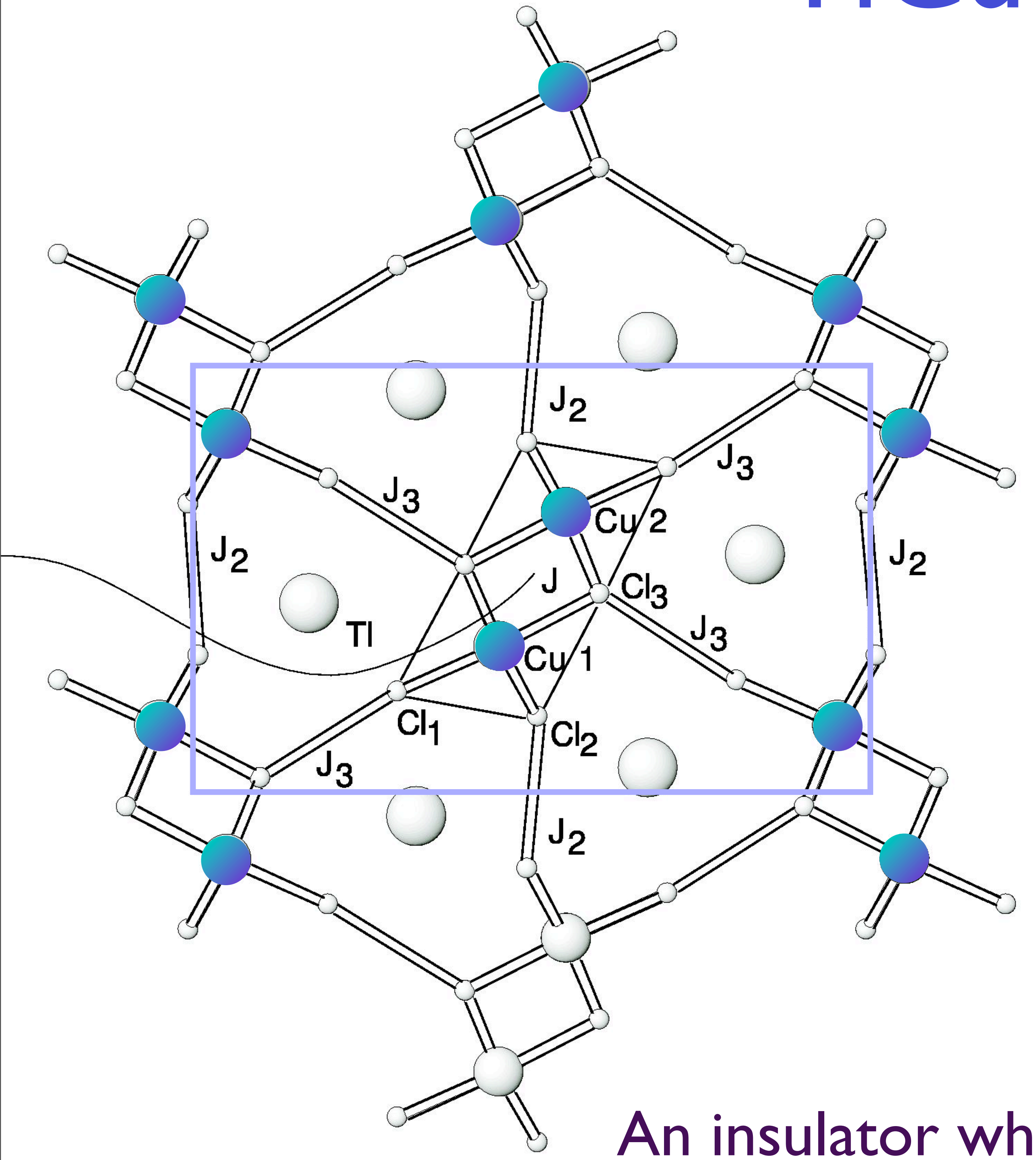
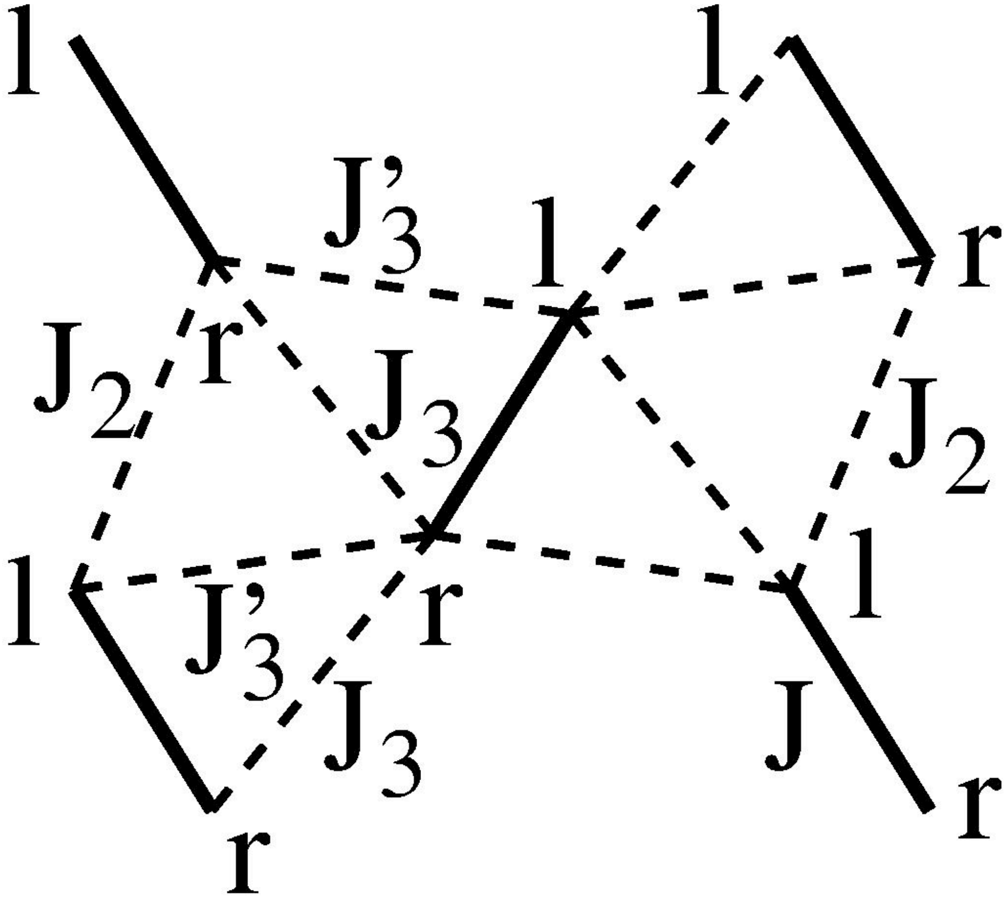
$TlCuCl_3$

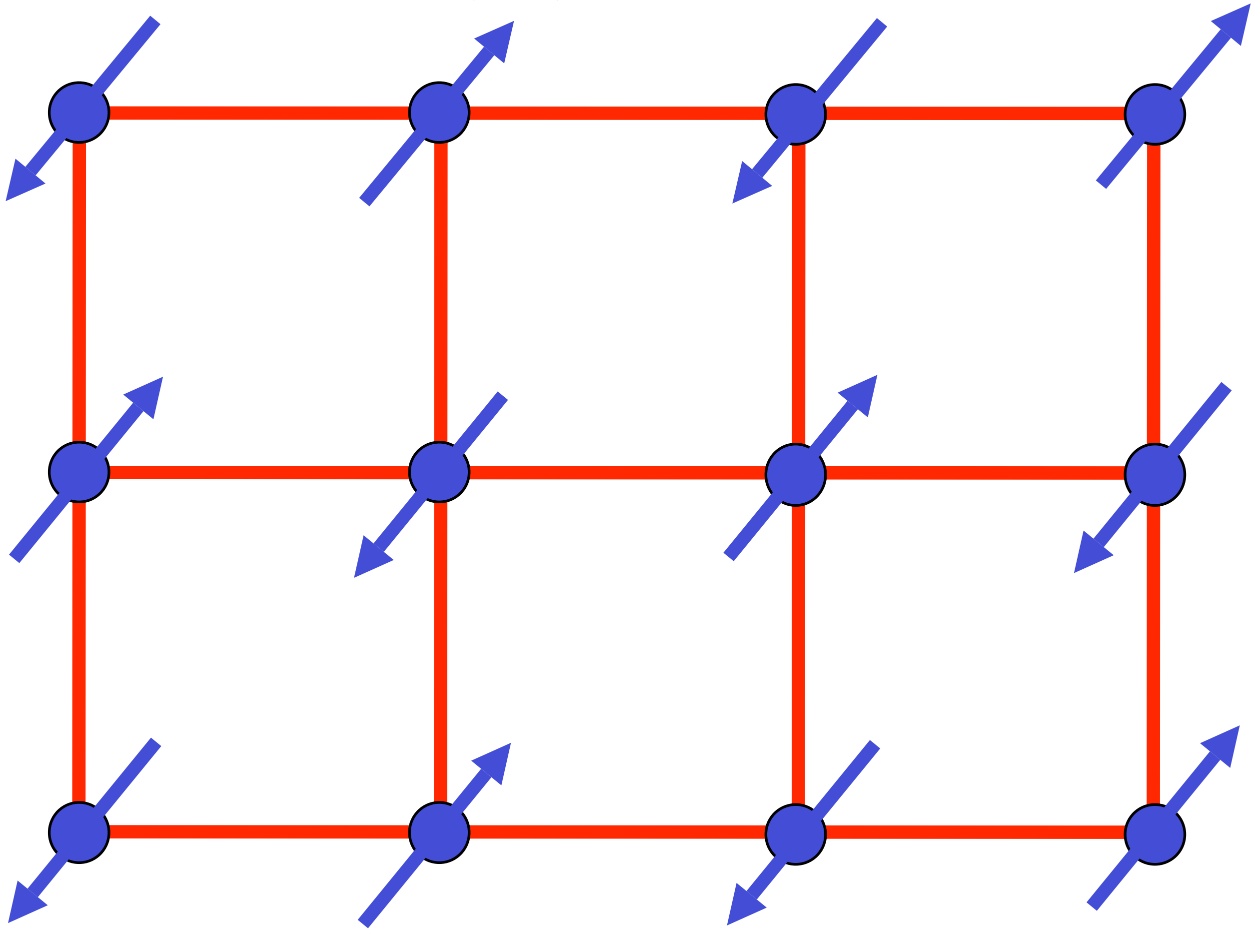




An insulator whose spin susceptibility vanishes exponentially as the temperature T tends to zero.

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

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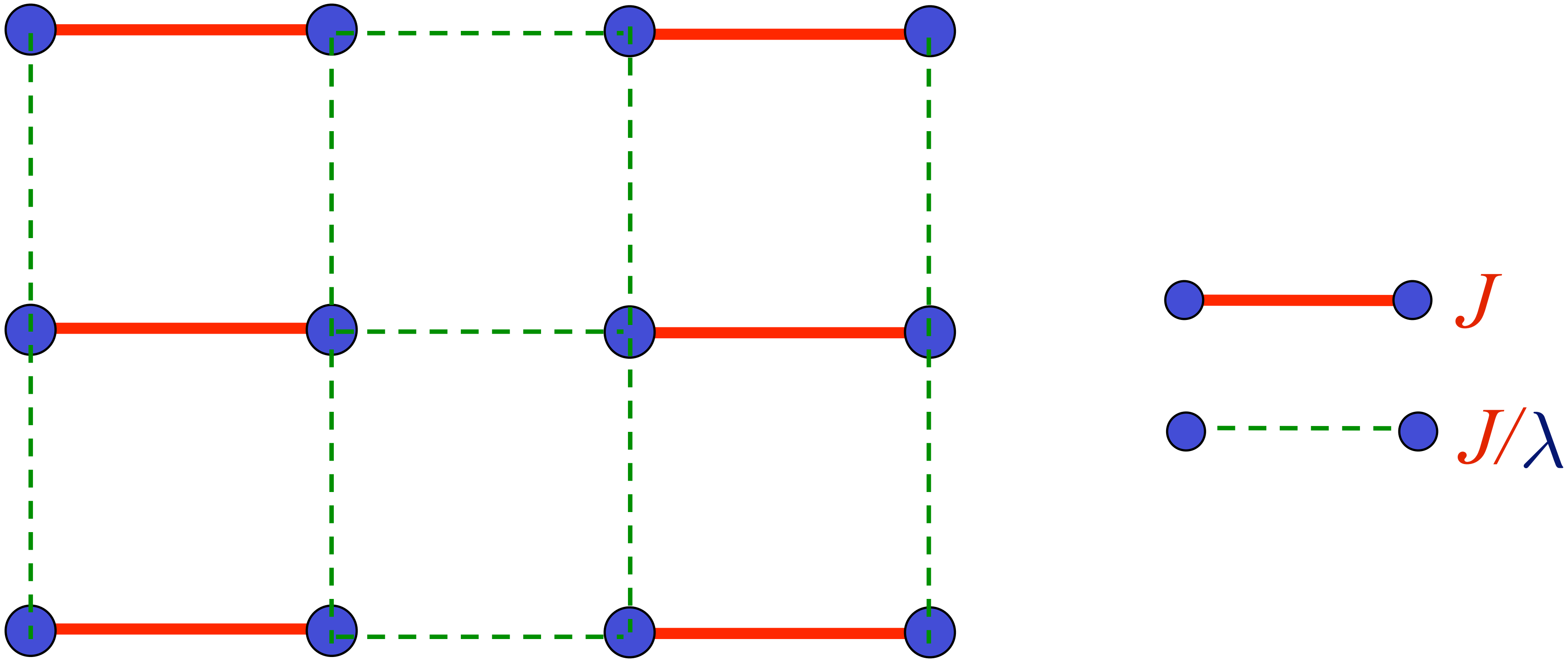
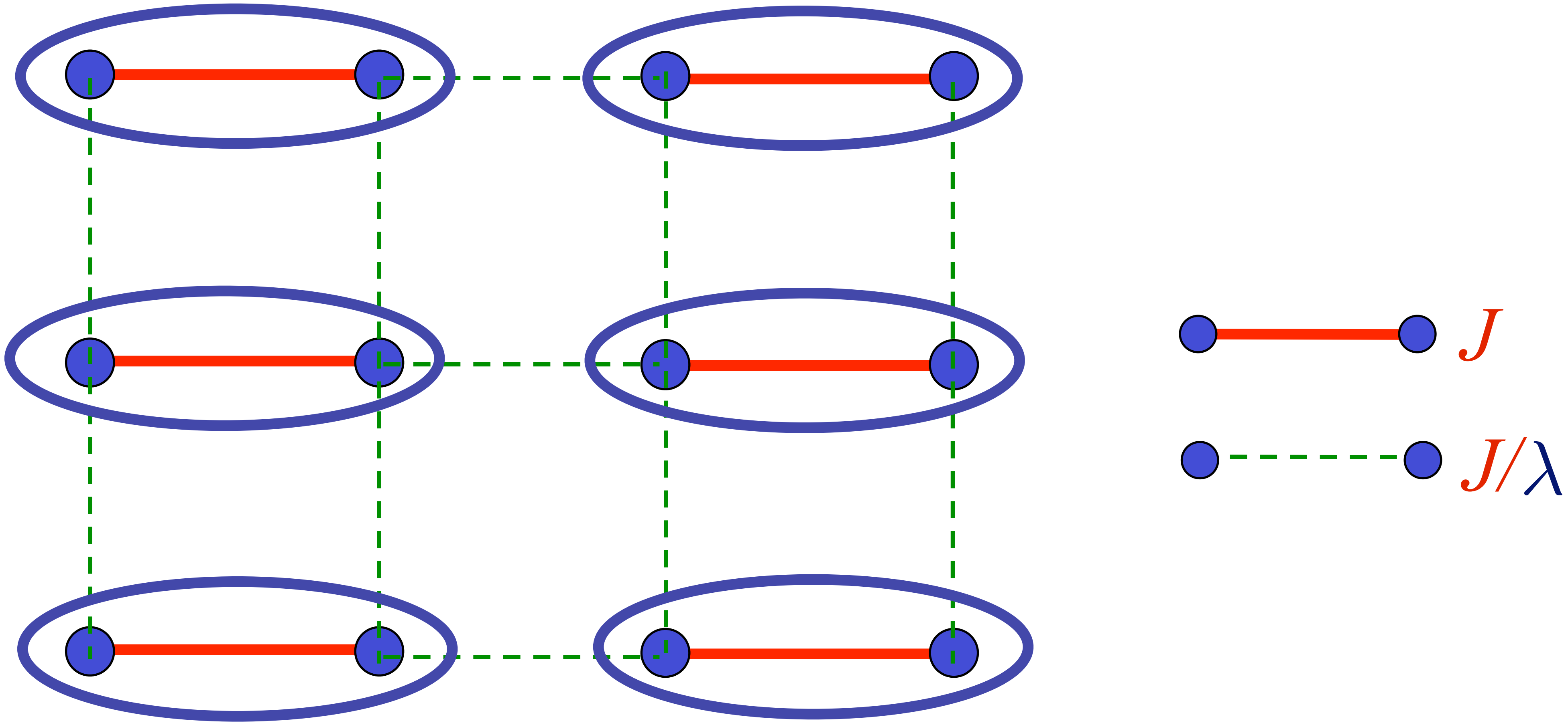


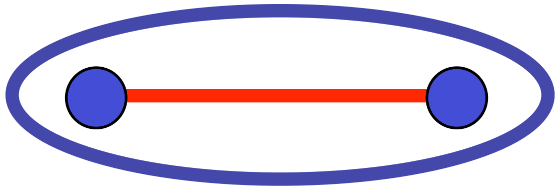
Weaken some bonds to induce spin entanglement in a new quantum phase

Square lattice antiferromagnet

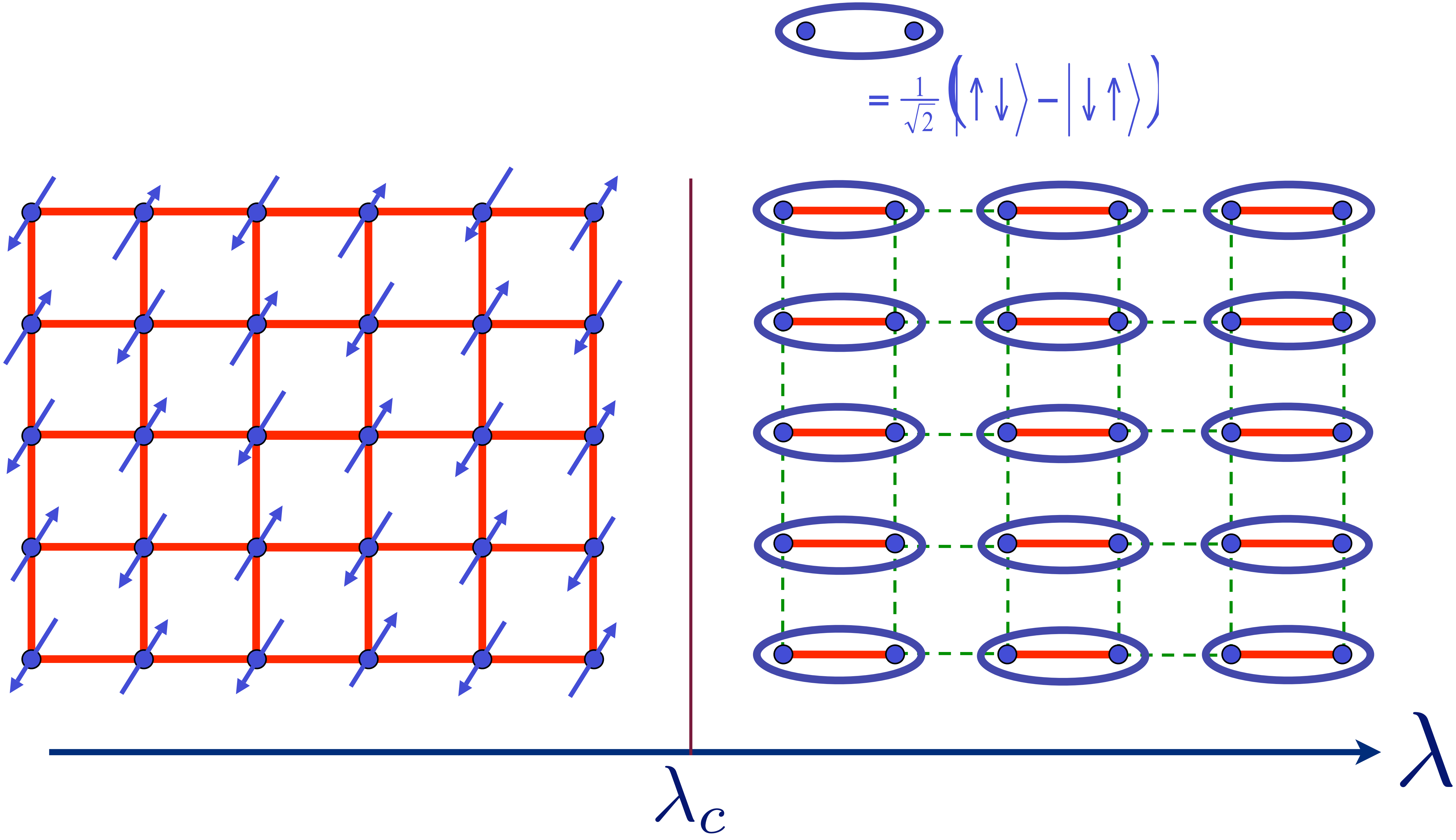
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state is a "quantum paramagnet" with spins locked in valence bond singlets

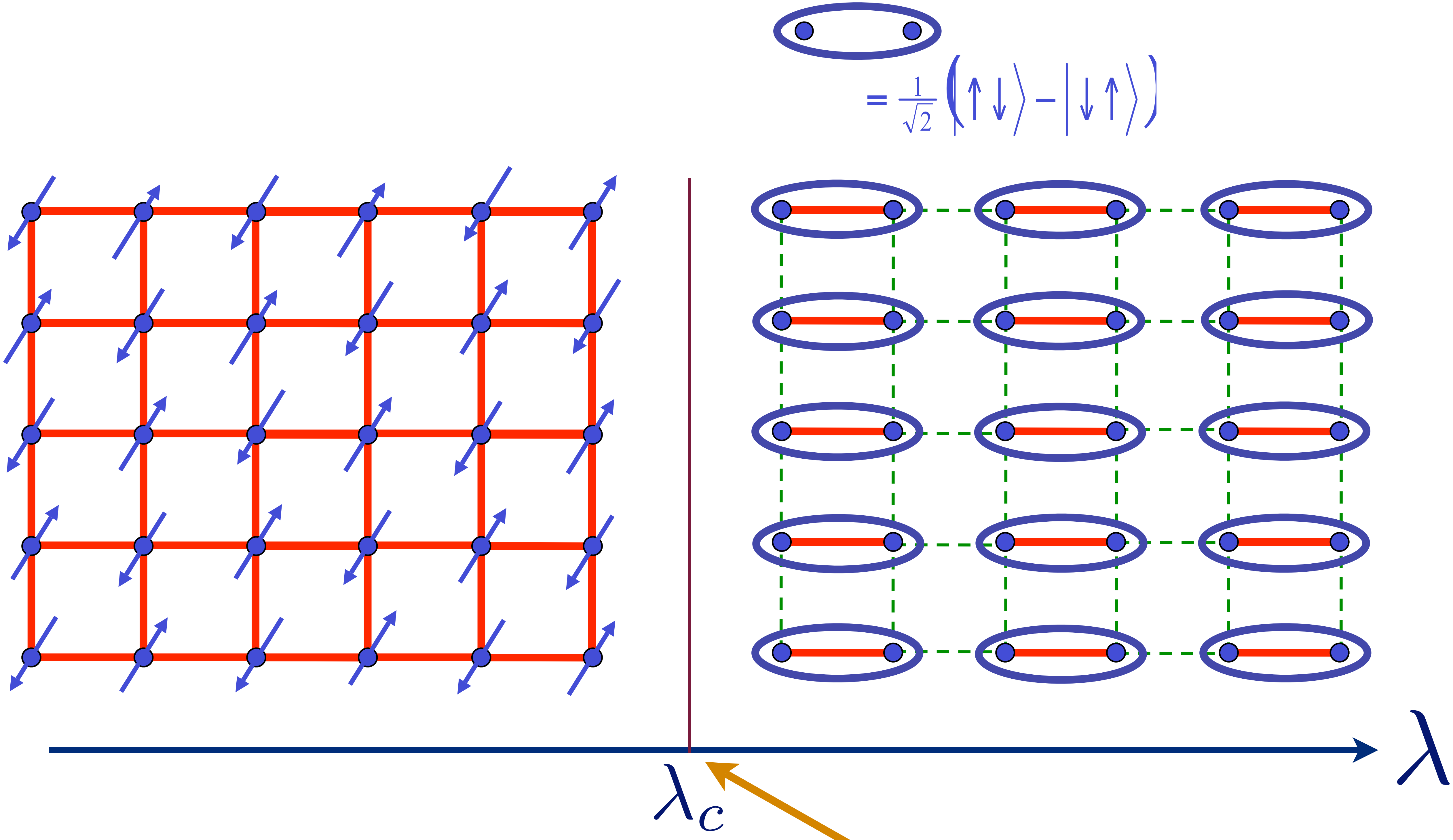


$$= \frac{1}{\sqrt{2}}\Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle\Big)$$



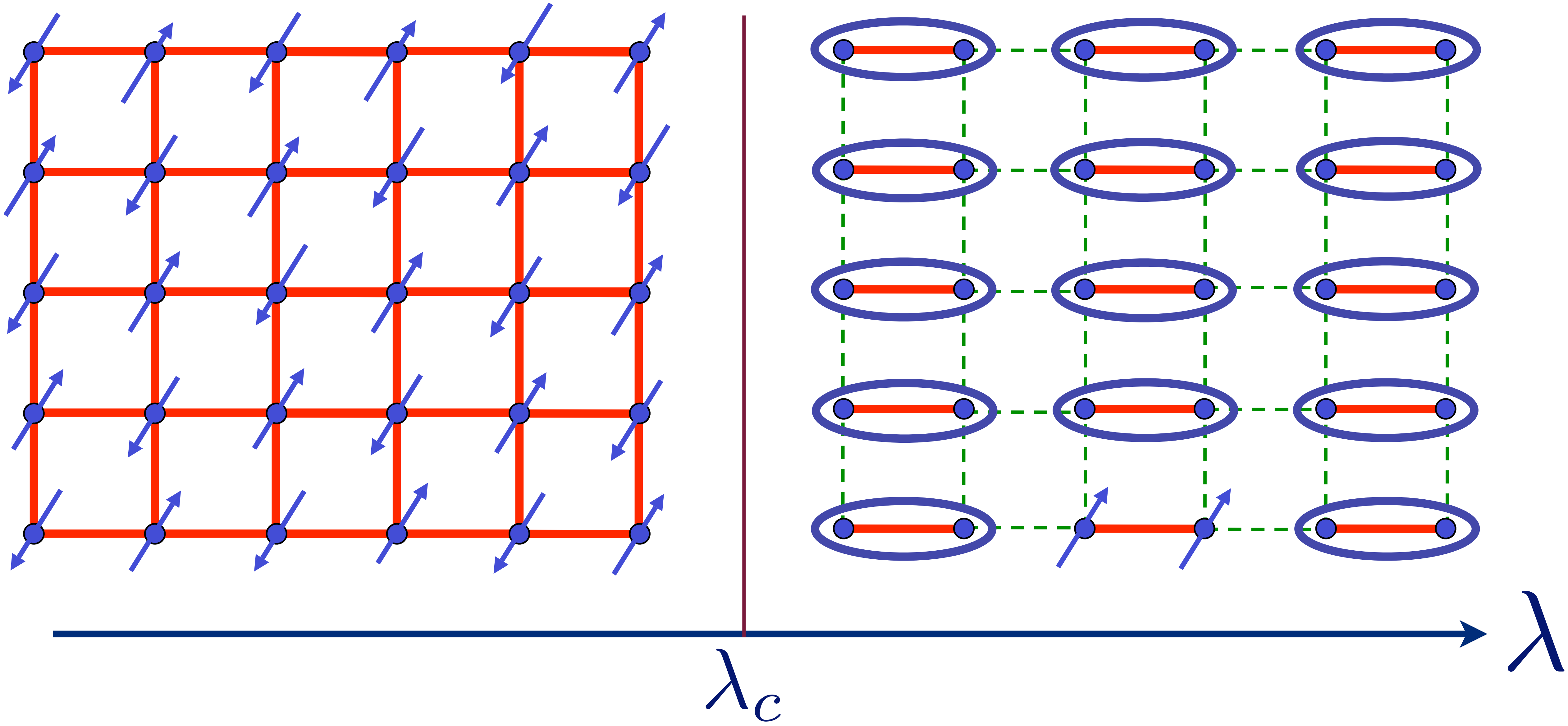
$$= \frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big)$$

Pressure in $TlCuCl_3$

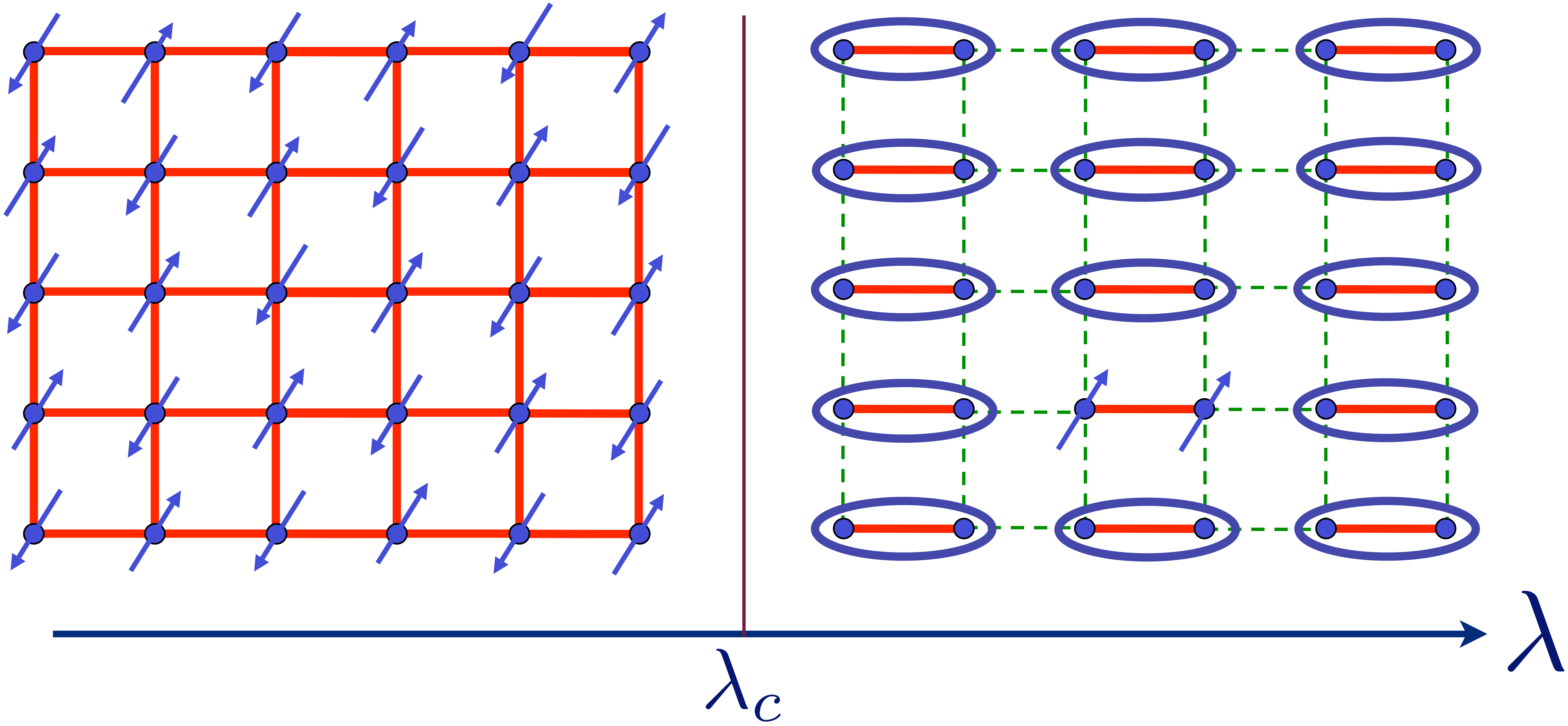

$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

Quantum critical point with non-local entanglement in spin wavefunction

Excitation spectrum in the paramagnetic phase



Excitation spectrum in the paramagnetic phase



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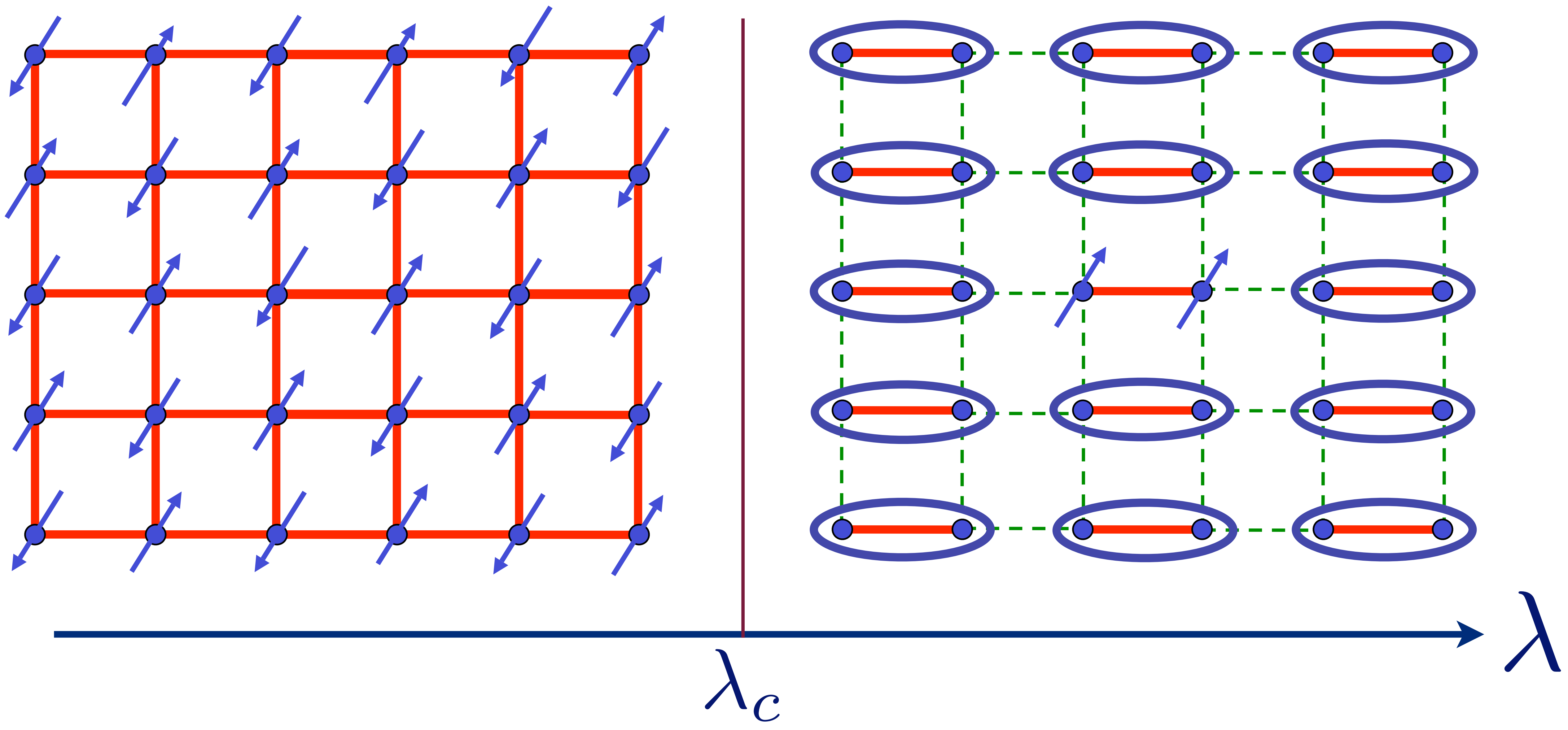
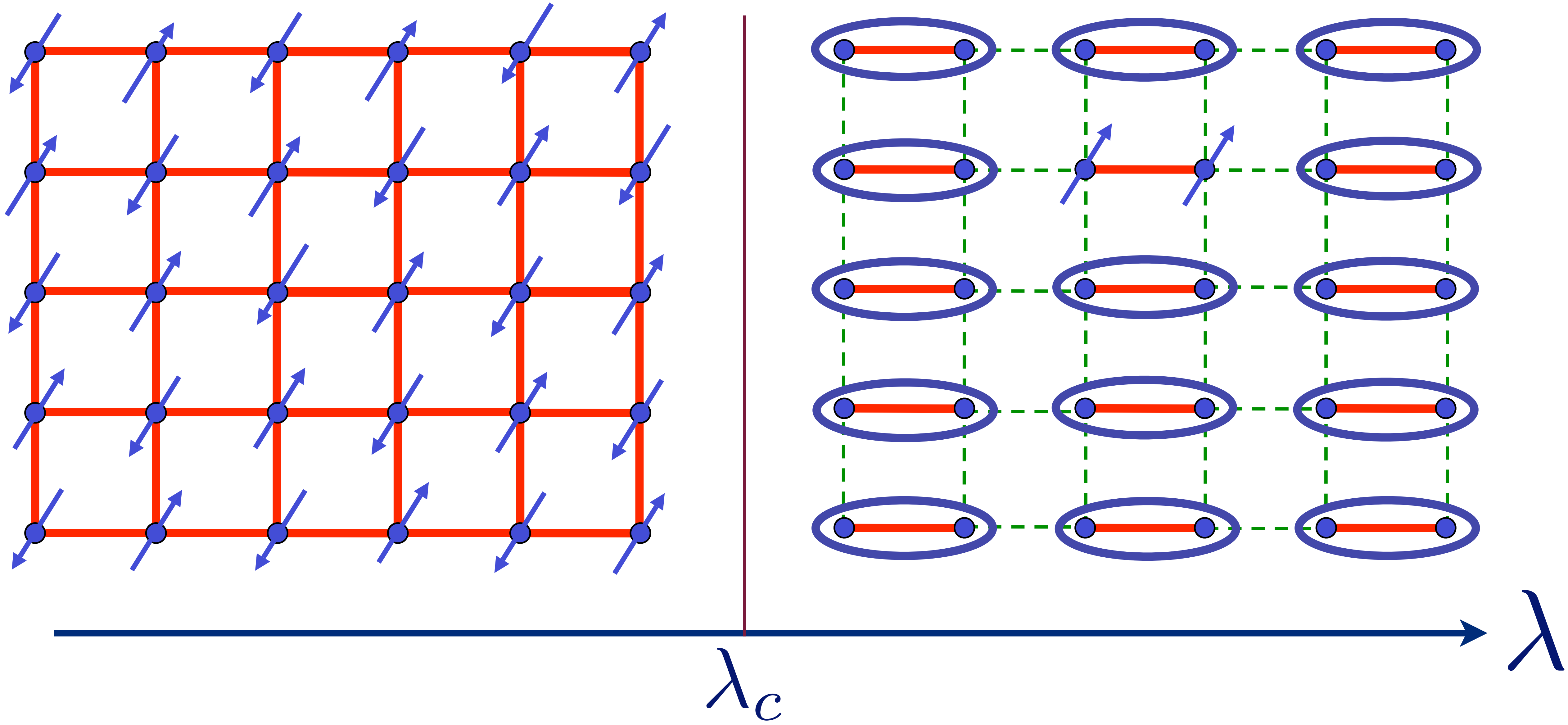


Excitation spectrum in the paramagnetic phase



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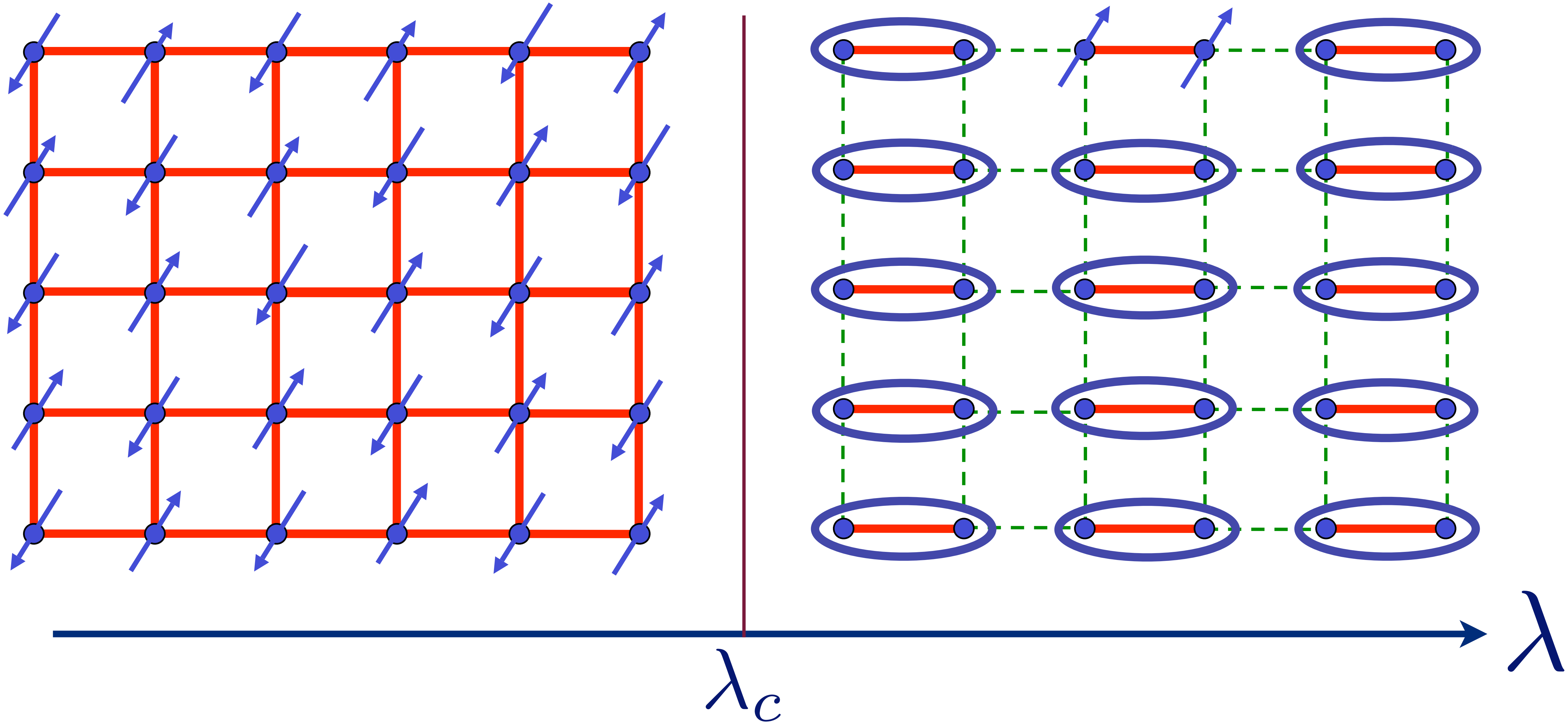
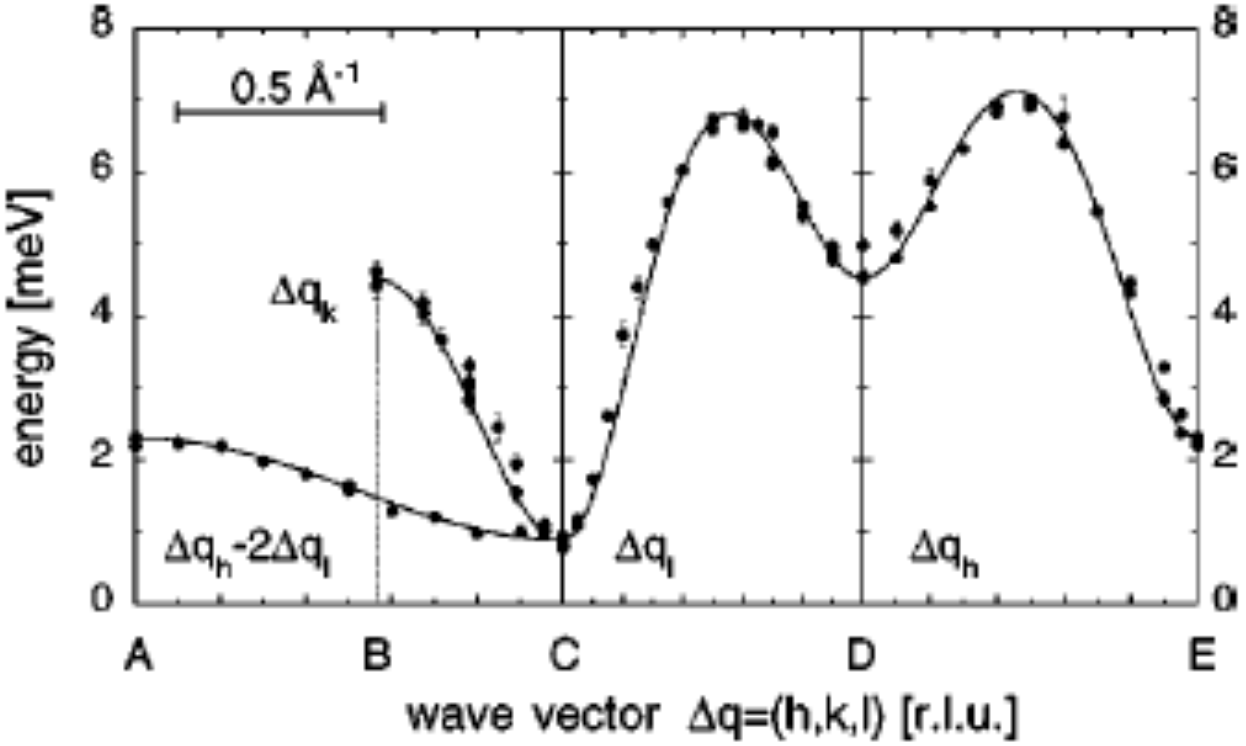


$TlCuCl_3$ at ambient pressure



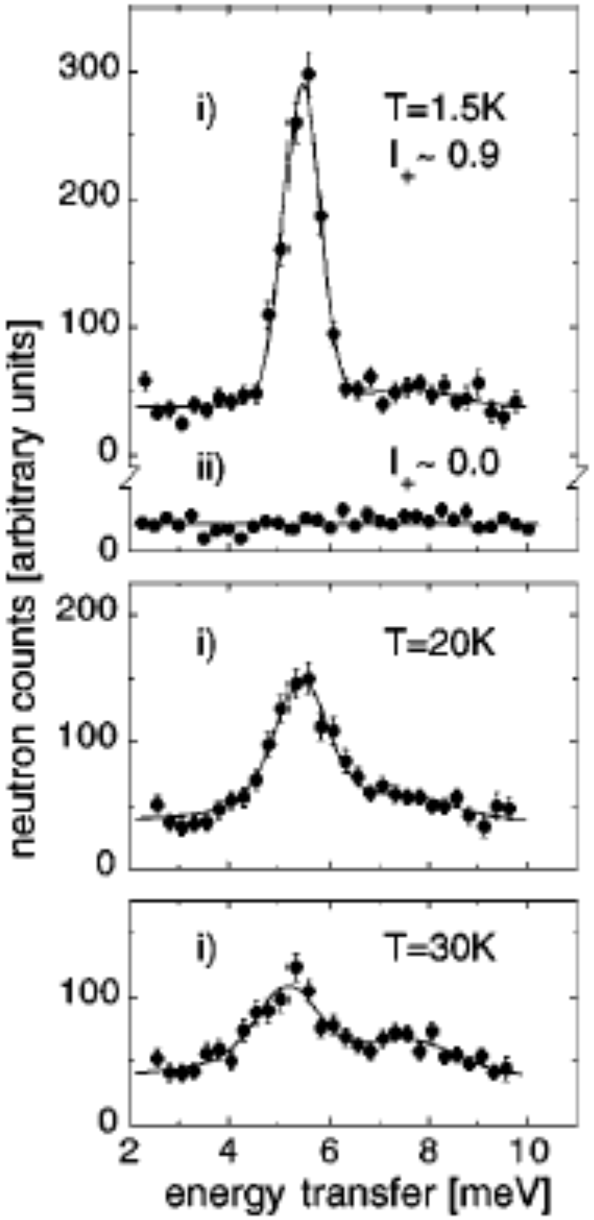
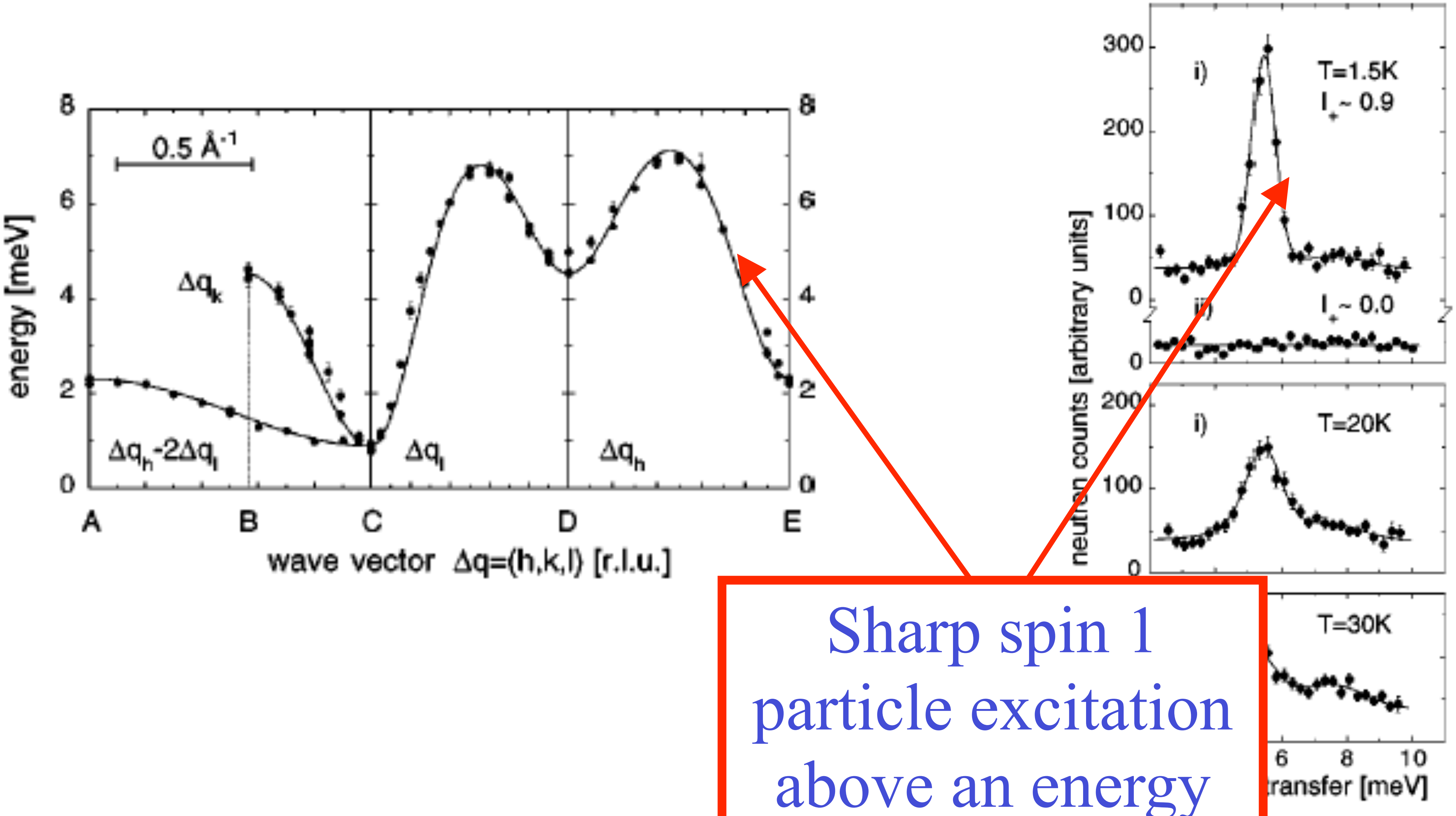


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev*. B 63 172414 (2001).

$TlCuCl_3$ at ambient pressure



Sharp spin 1 particle excitation above an energy gap (spin gap)

les in the a^*c^* plane of $TlCuCl_3$
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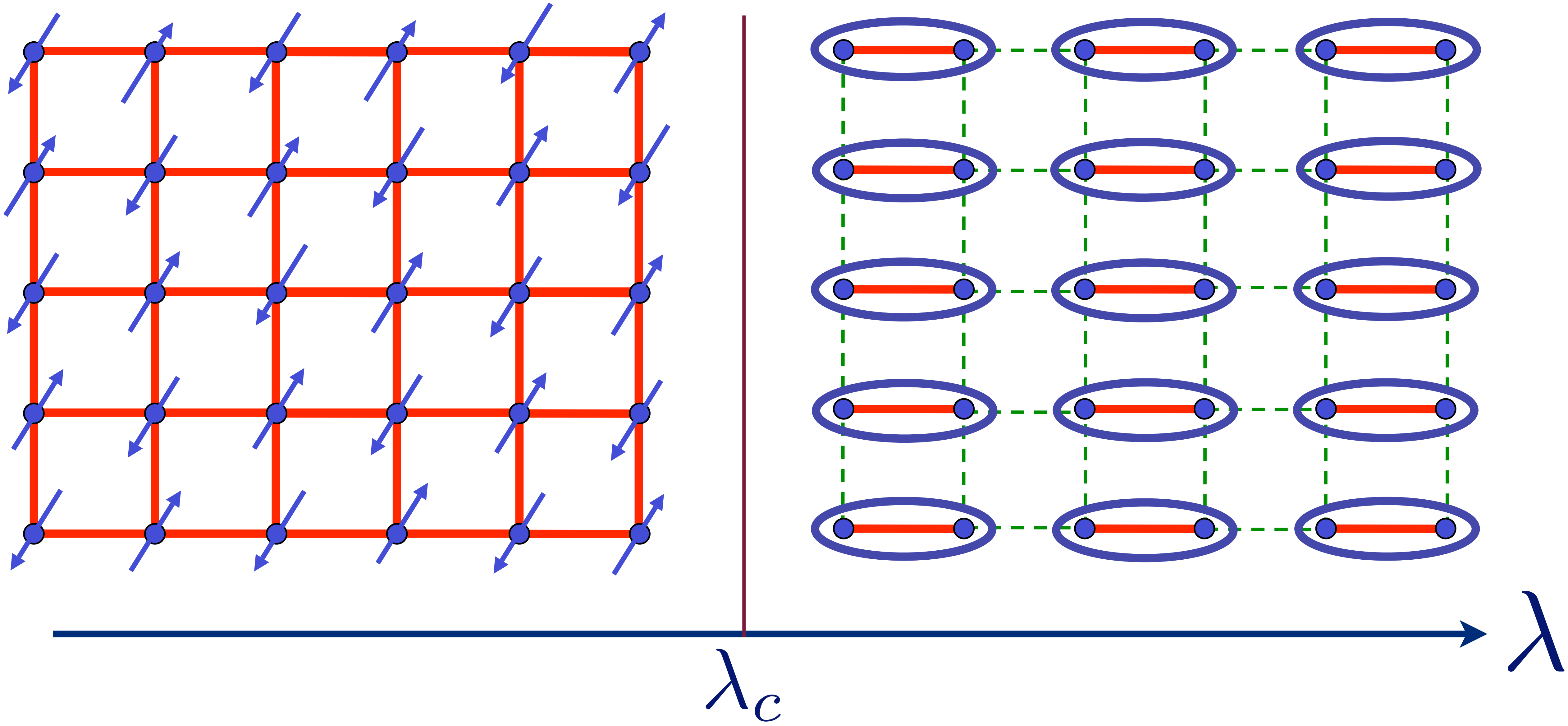
N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev*. B 63 172414 (2001).

Excitation spectrum in the Néel phase



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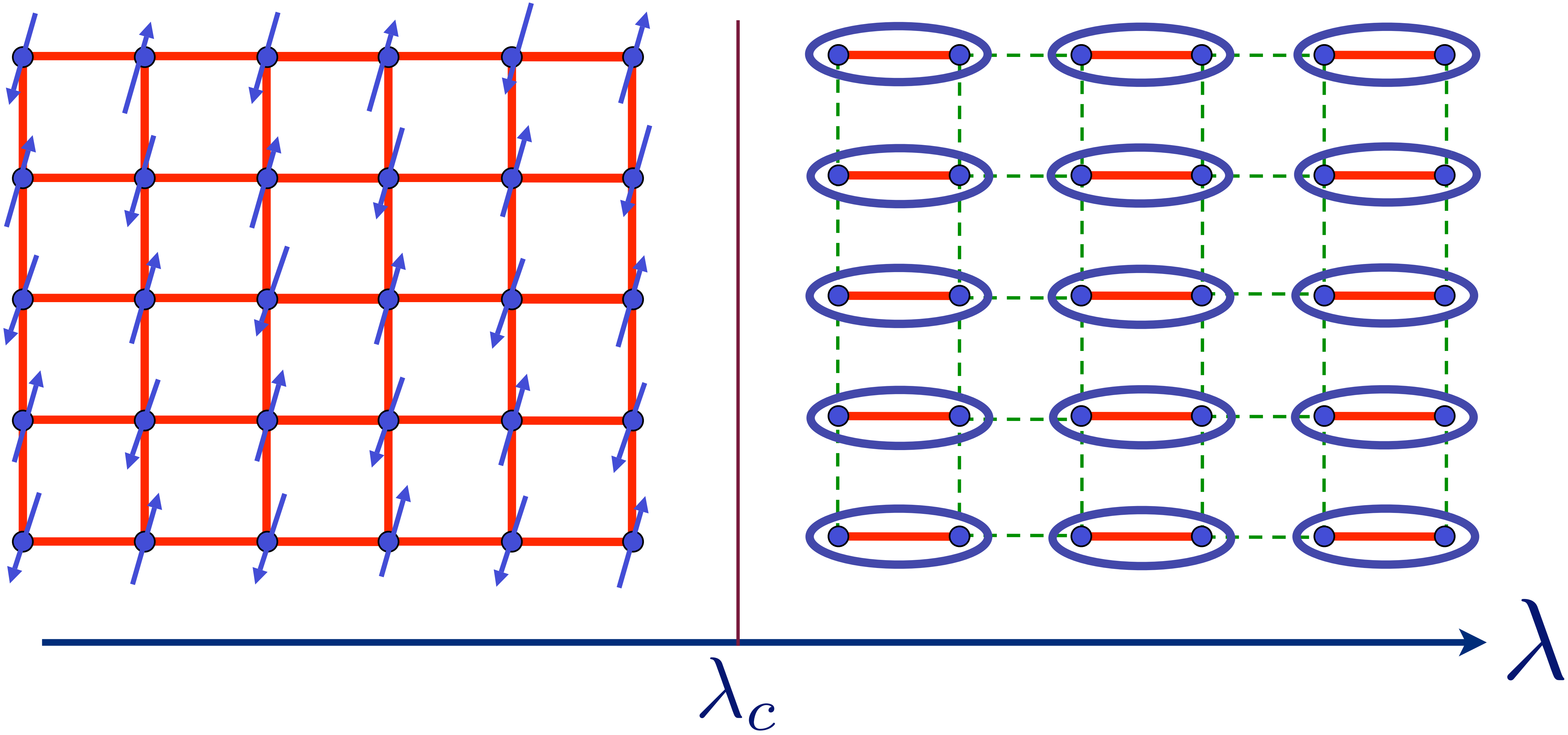
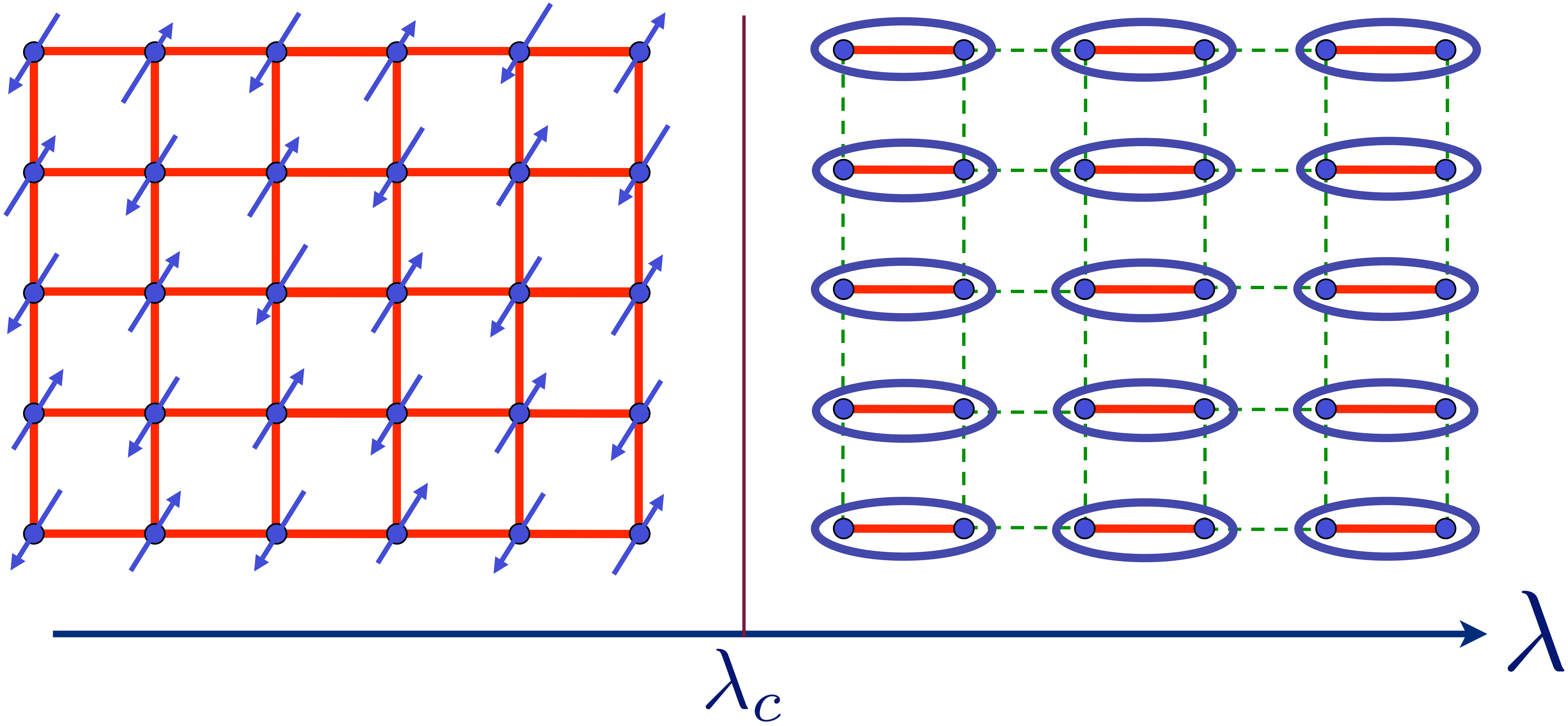


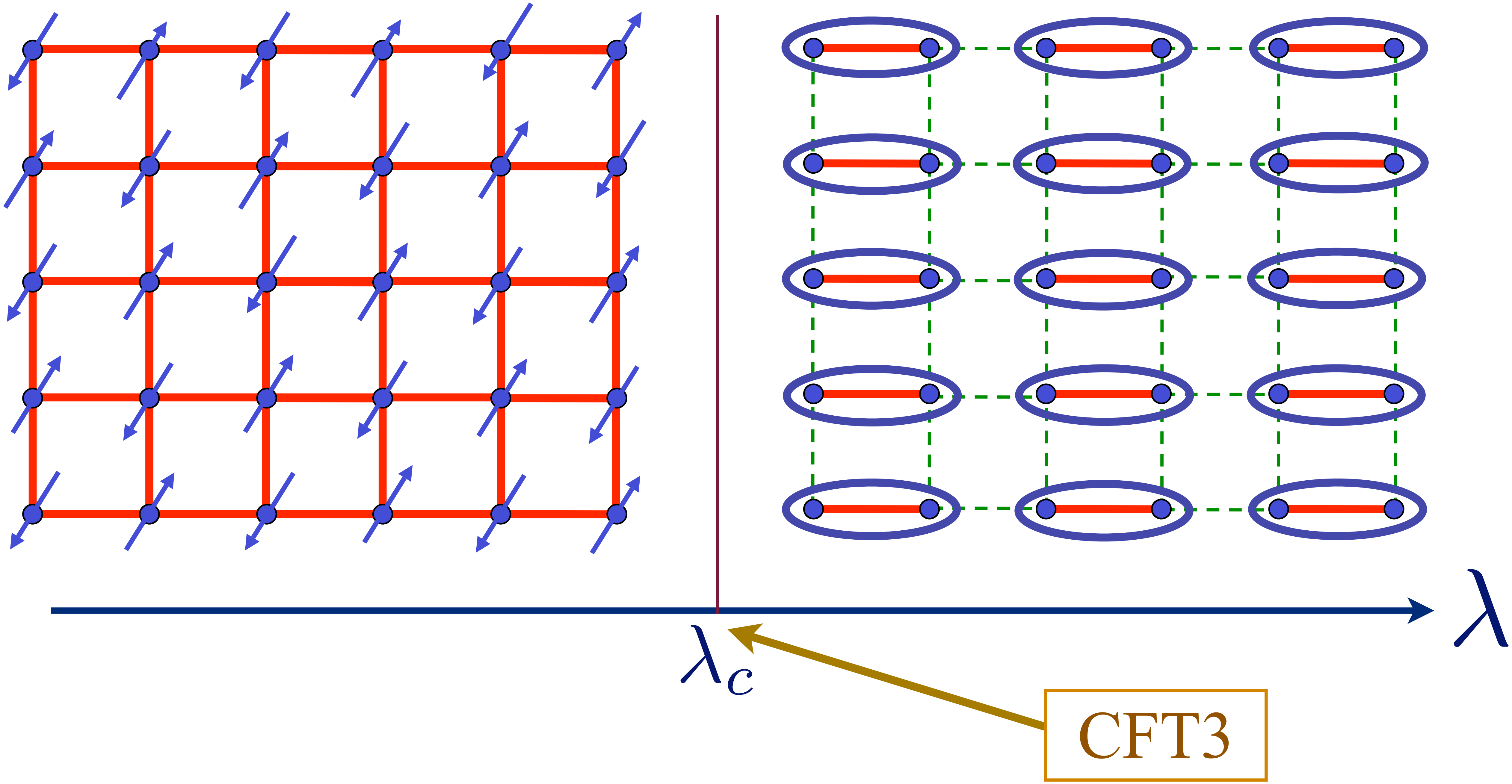
Spin waves

Excitation spectrum in the Néel phase



Spin waves

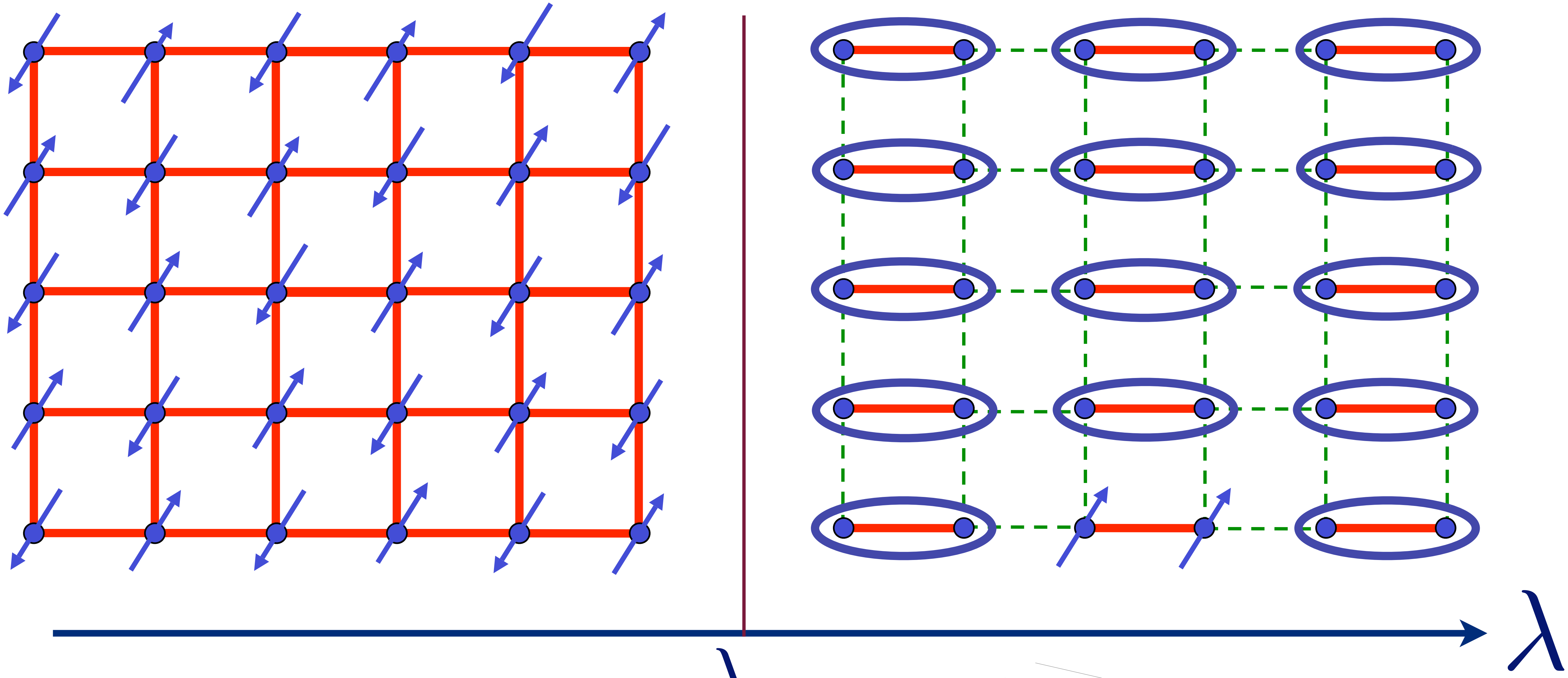
Description using Landau-Ginzburg field theory



O(3) order parameter $\vec{\varphi}$

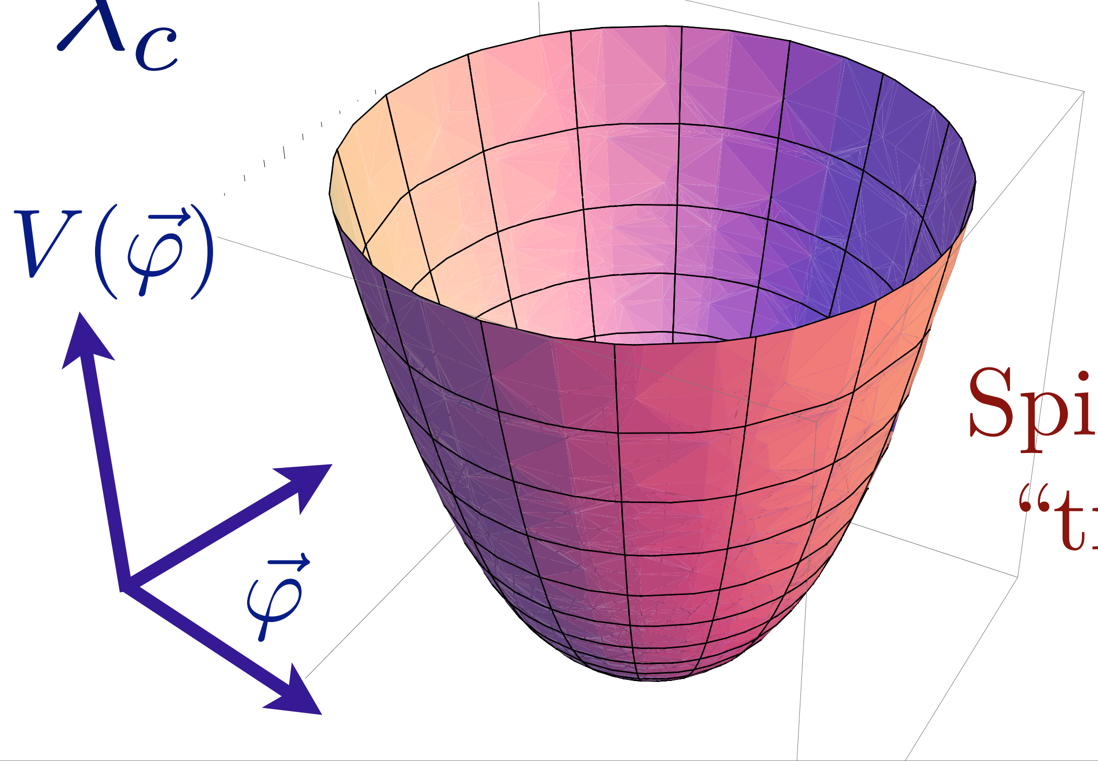
$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u \left(\vec{\varphi}^2 \right)^2 \right]$$

Excitation spectrum in the paramagnetic phase



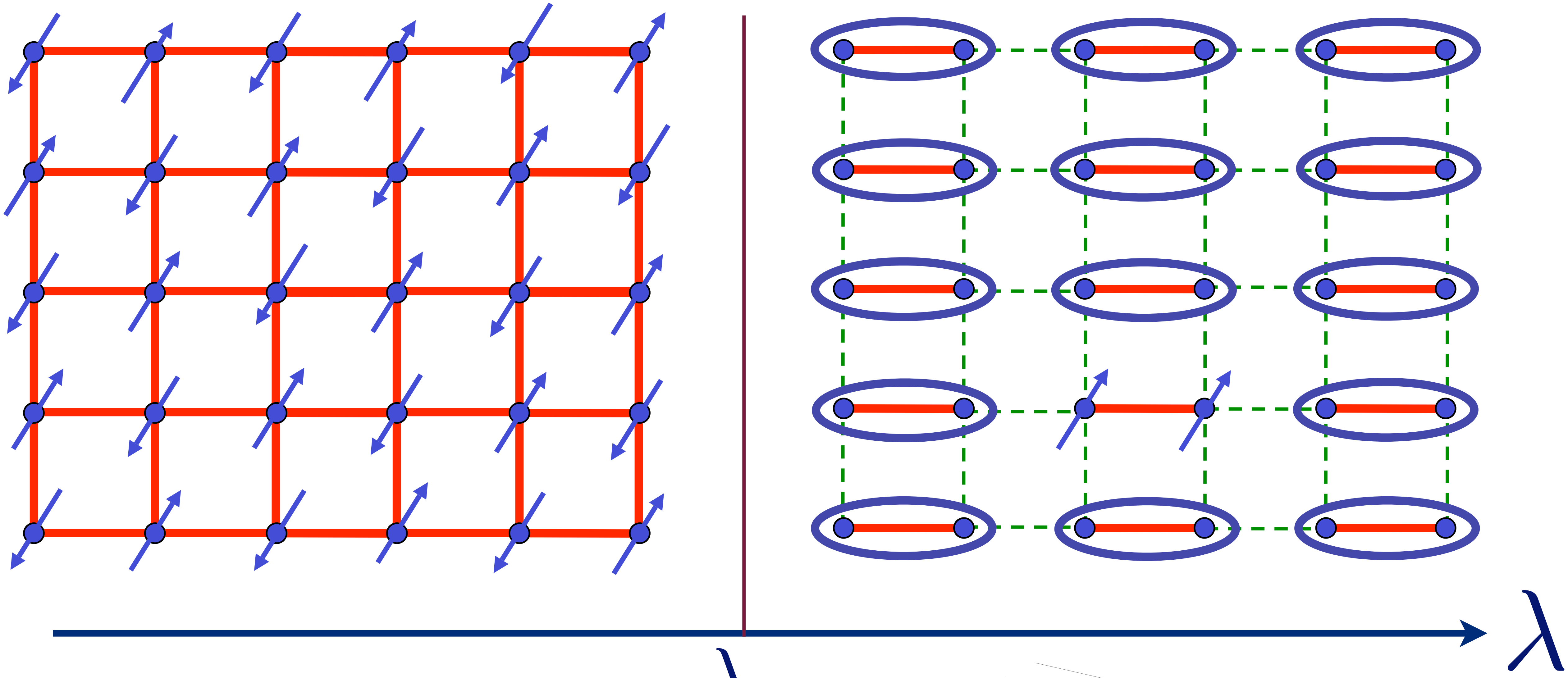
$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

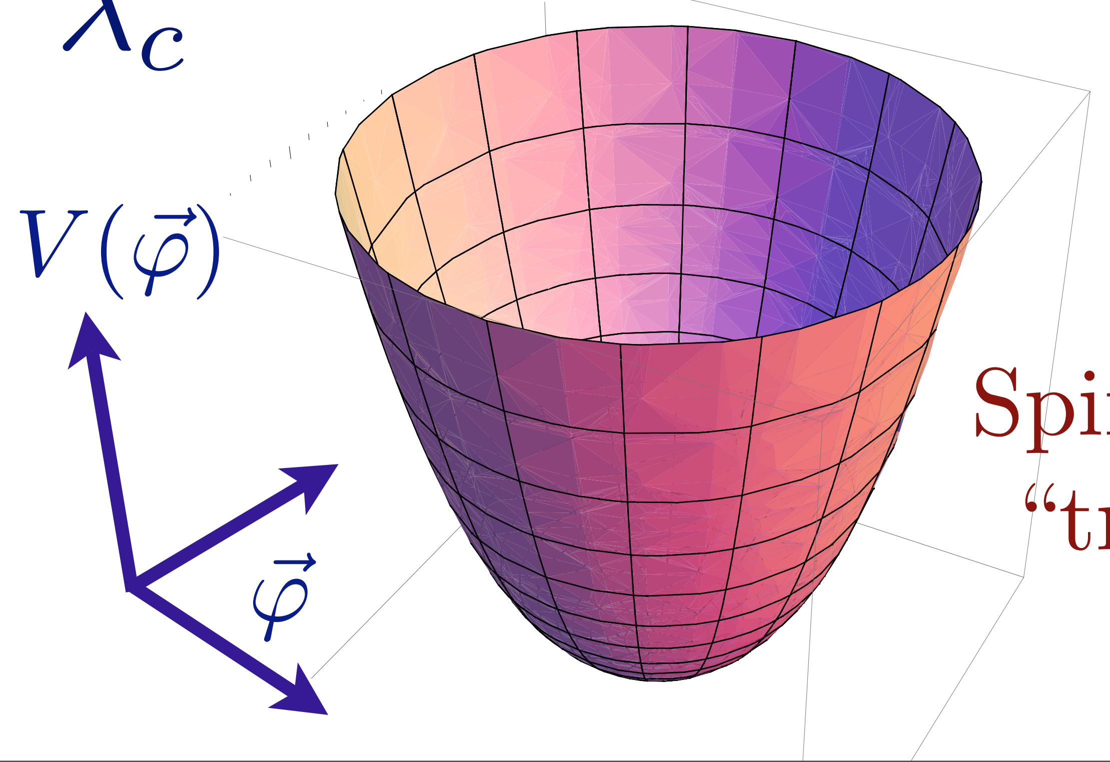
$$\lambda > \lambda_c$$



Spin $S = 1$
“triplon”

Excitation spectrum in the paramagnetic phase





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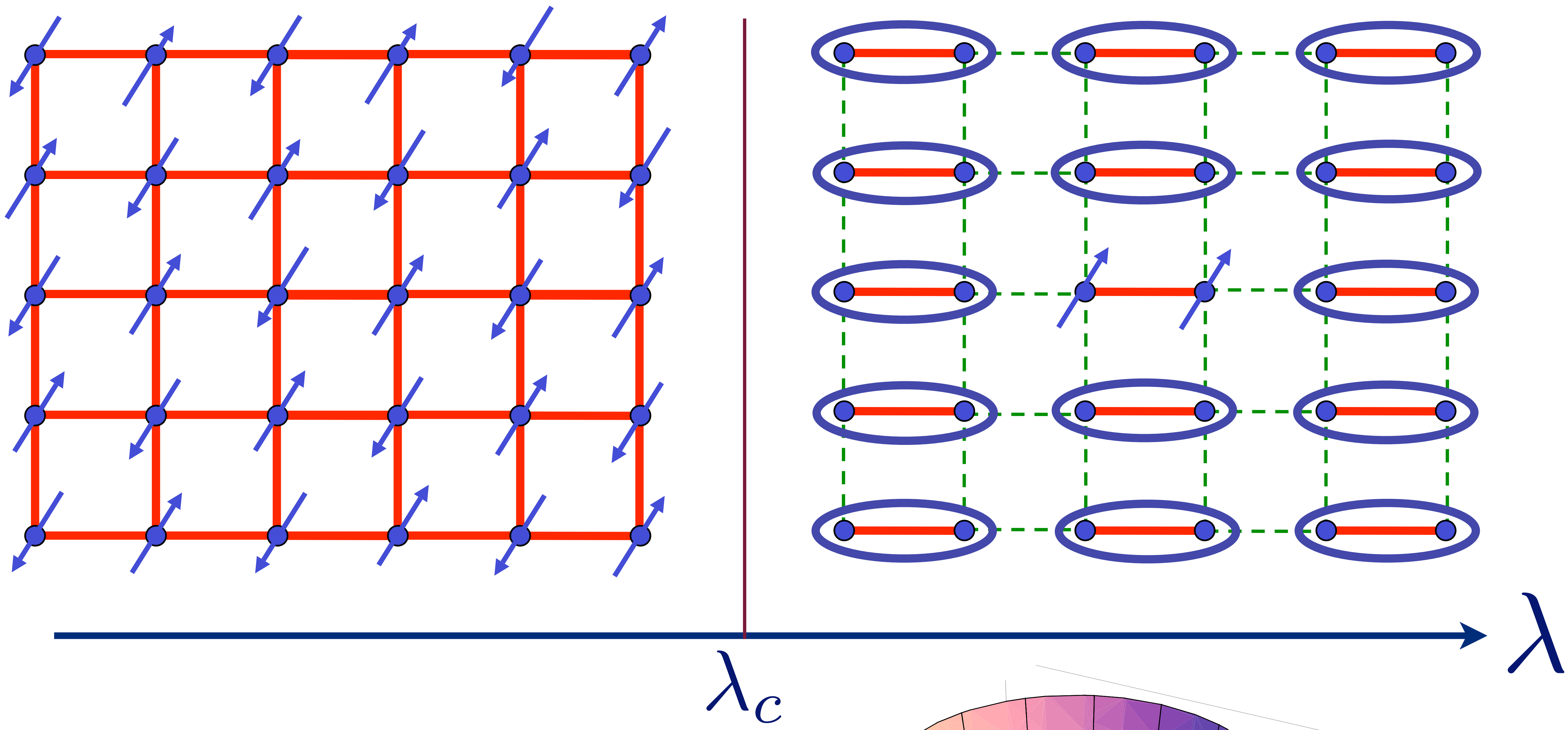

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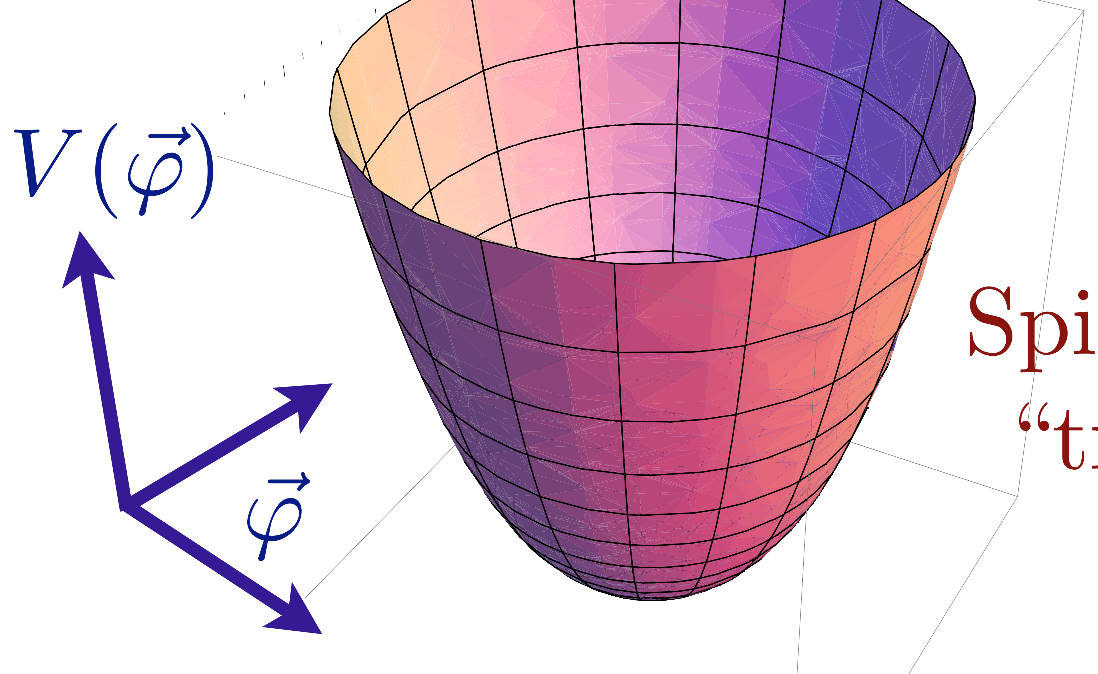

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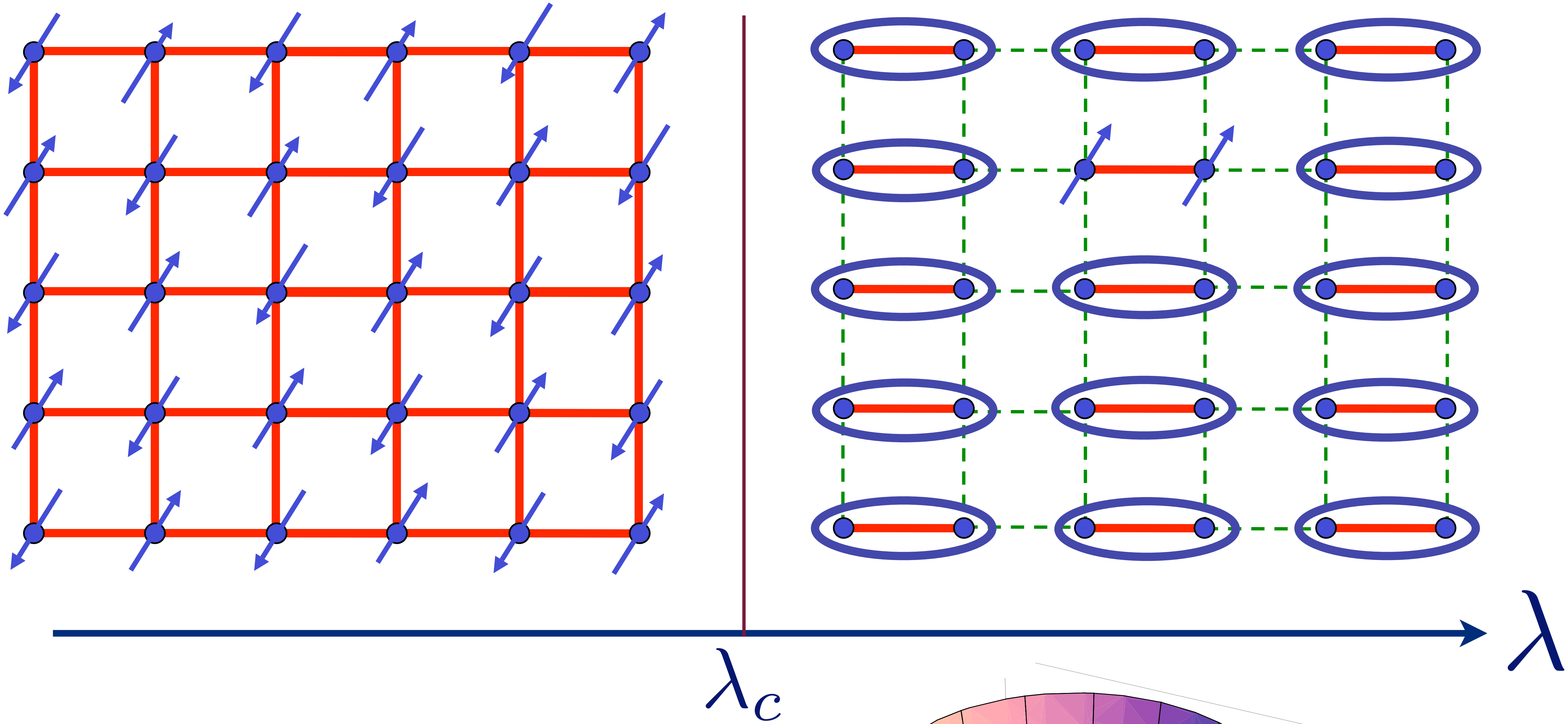

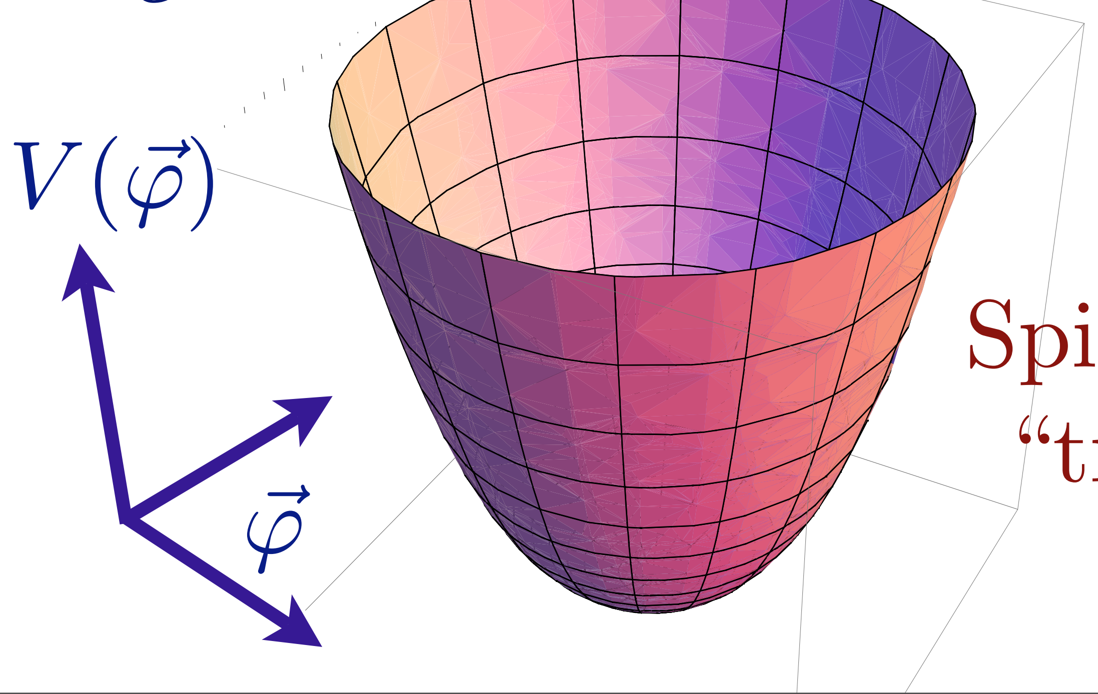

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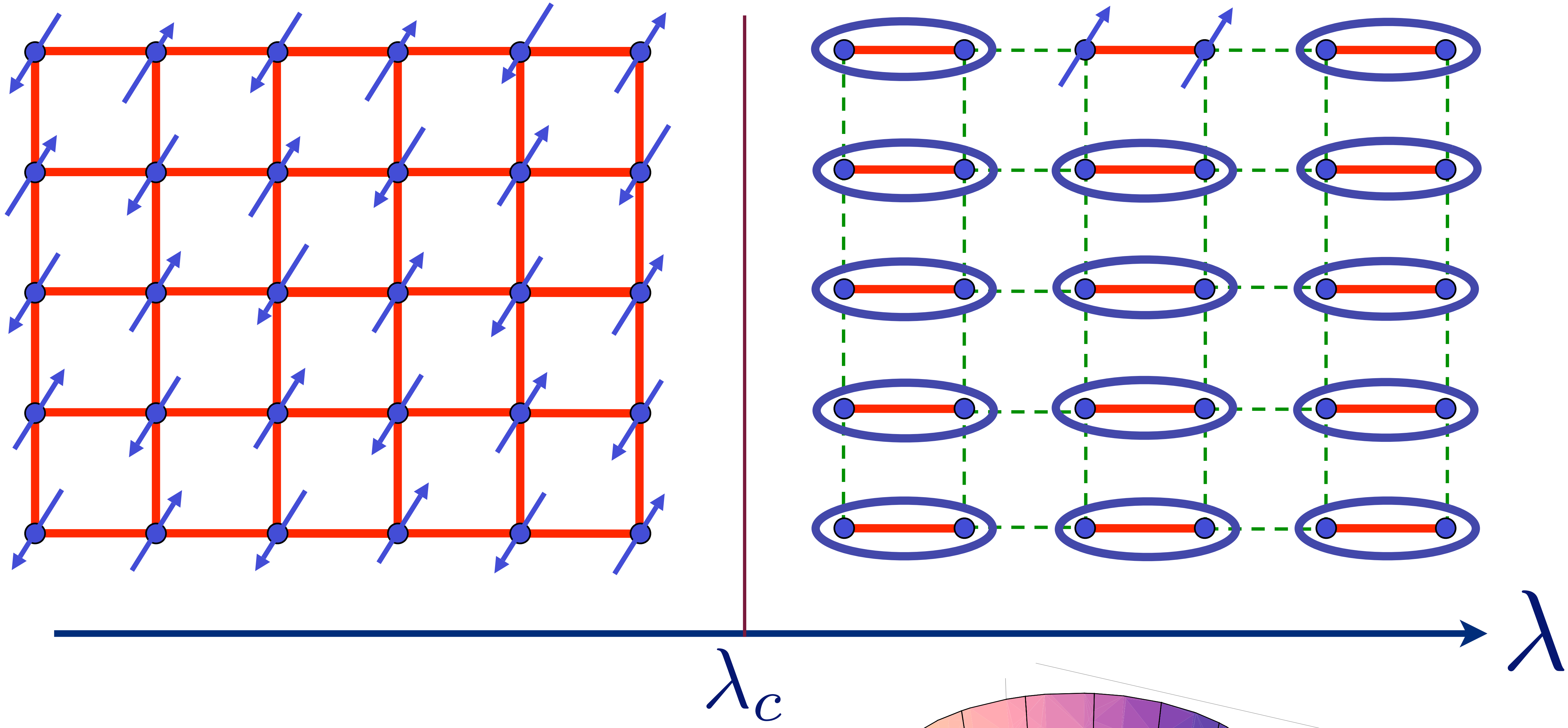

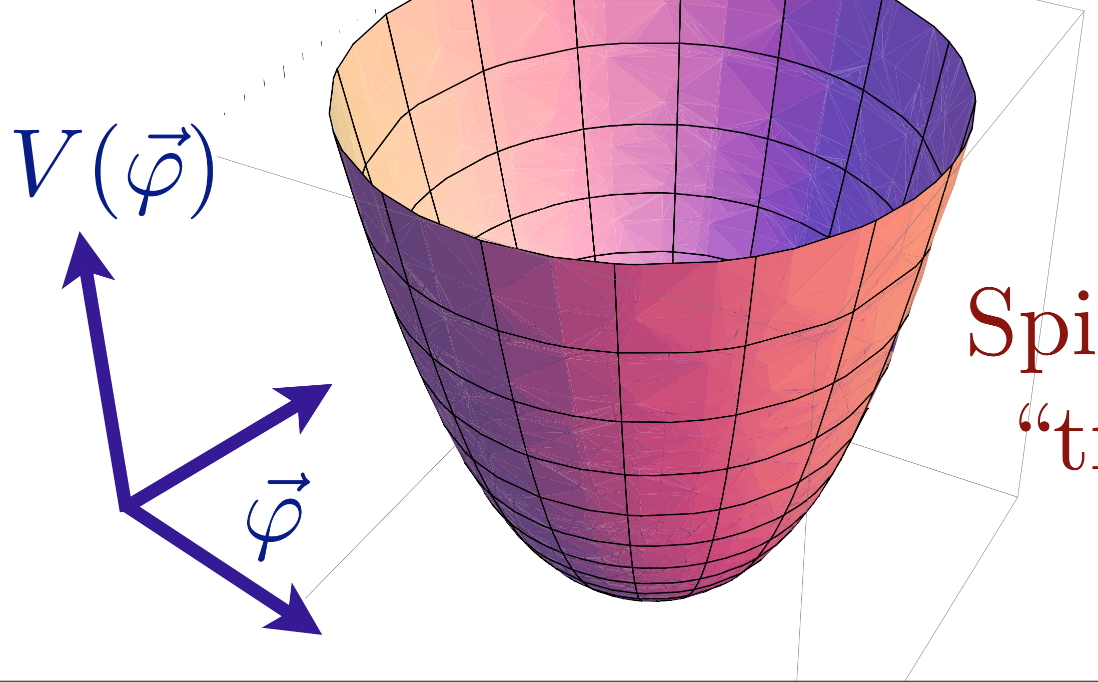

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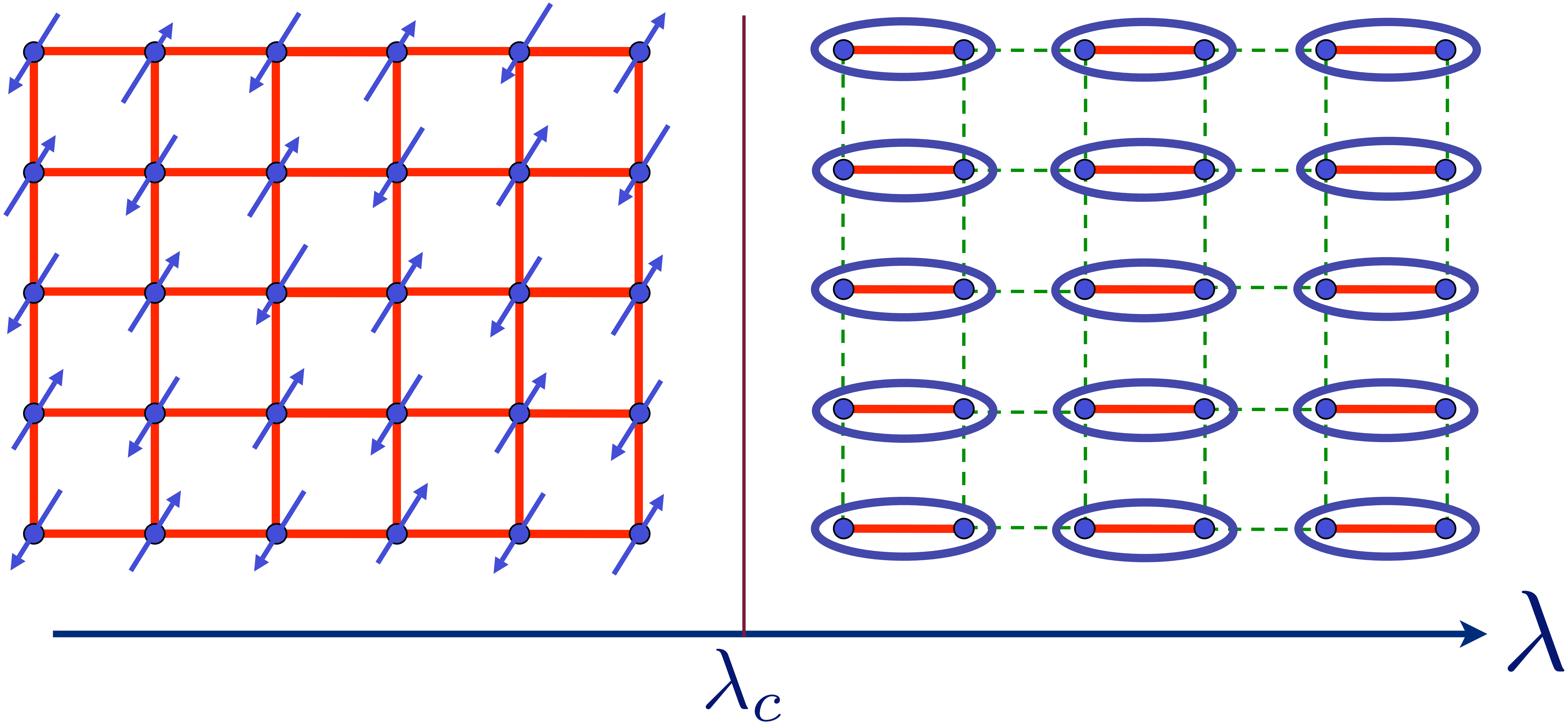
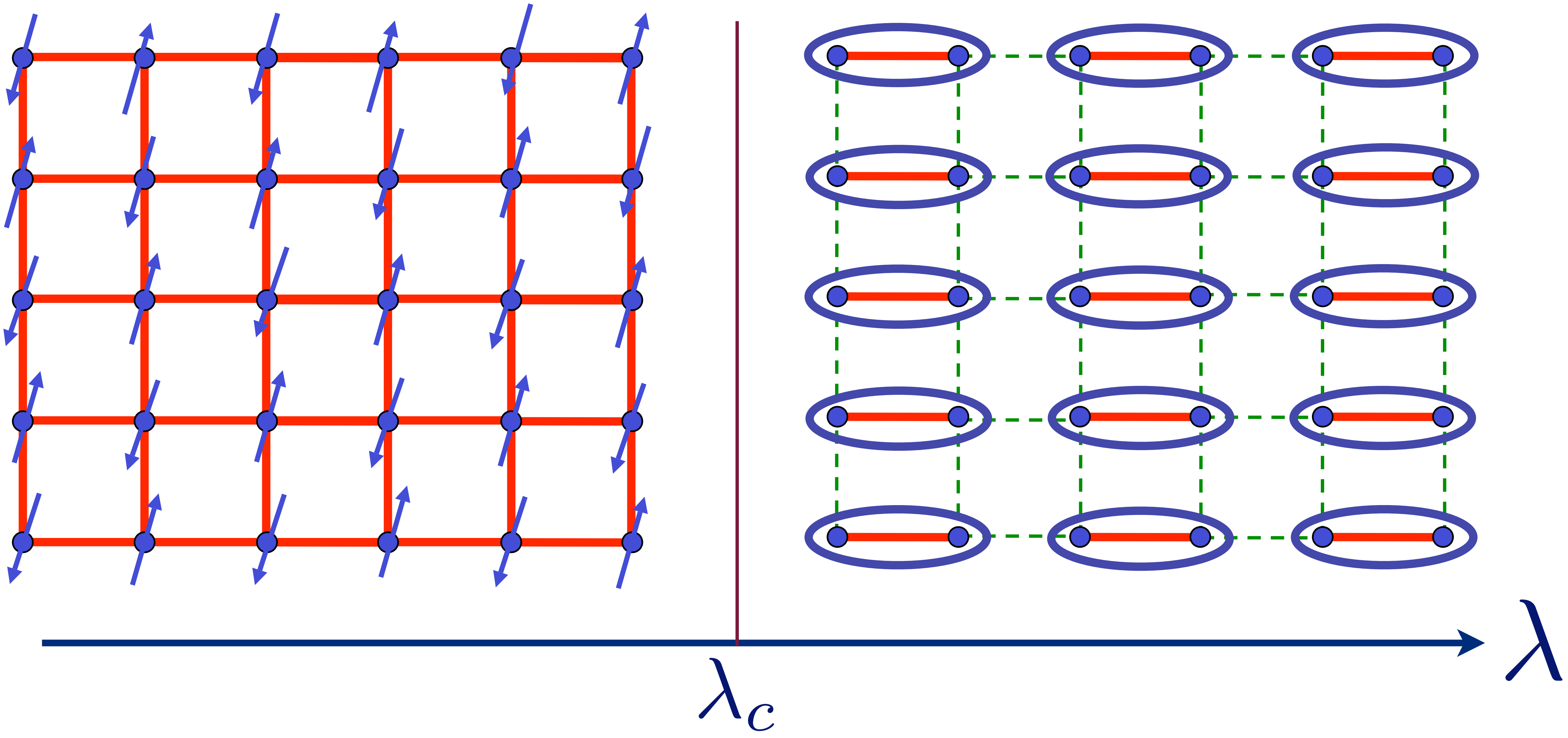
“triplon”

Excitation spectrum in the Néel phase

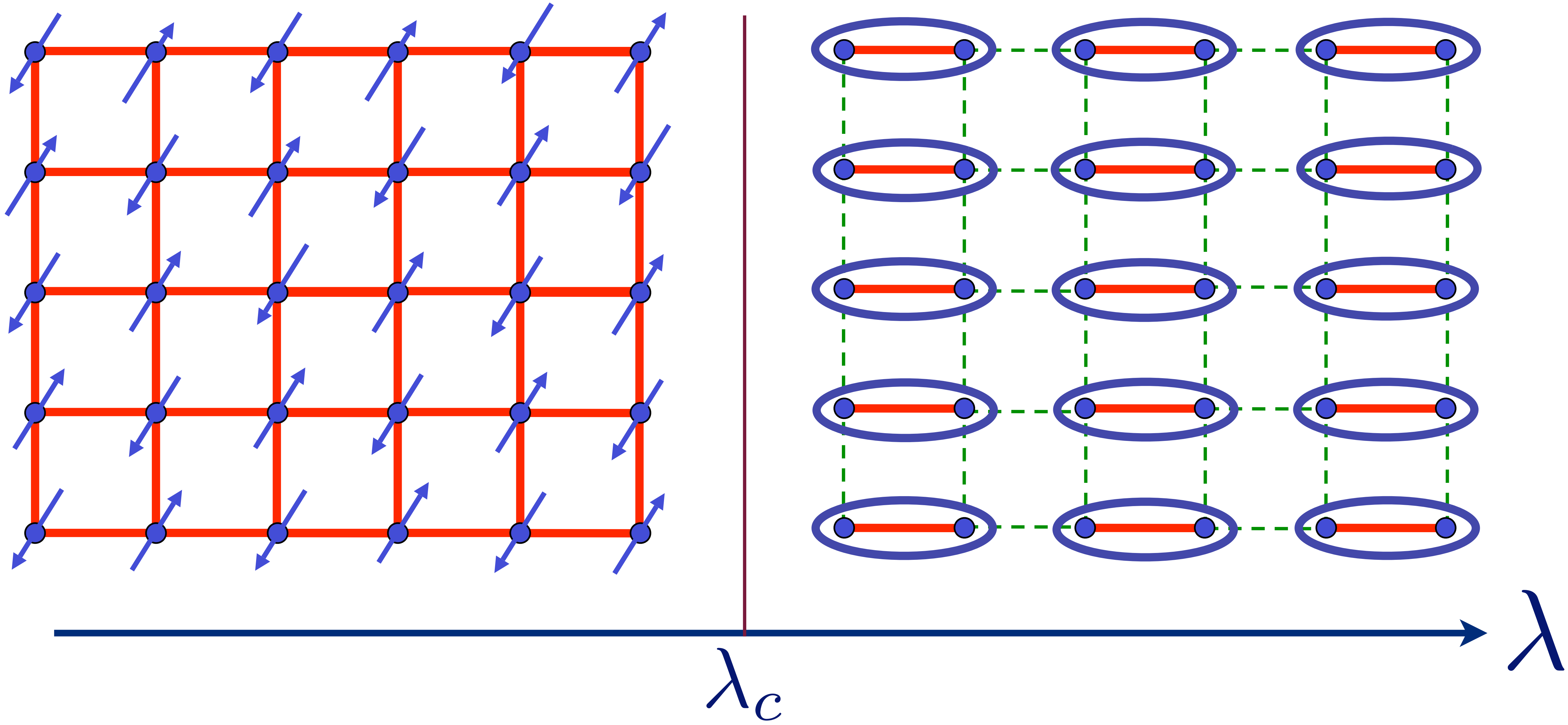


Excitation spectrum in the Néel phase



Spin waves

Excitation spectrum in the Néel phase



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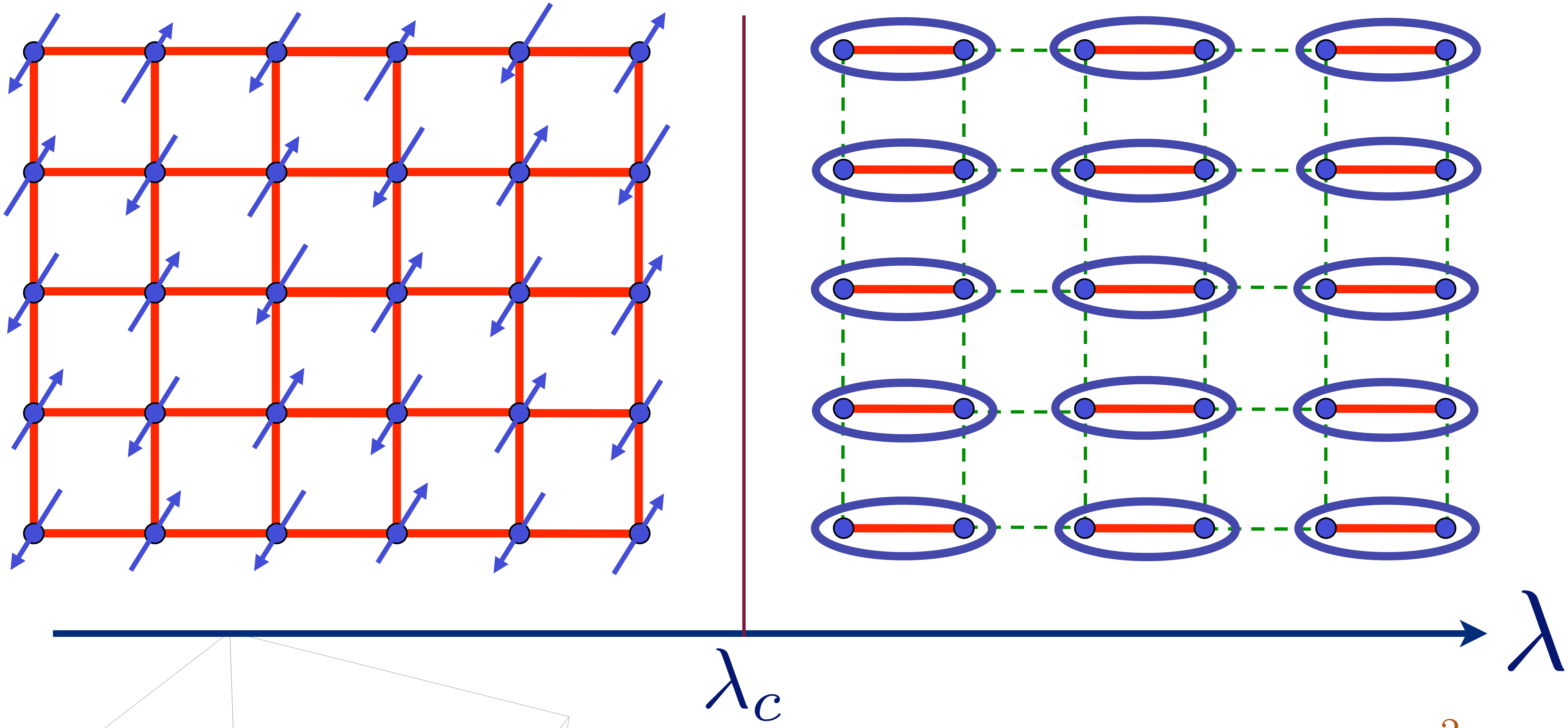

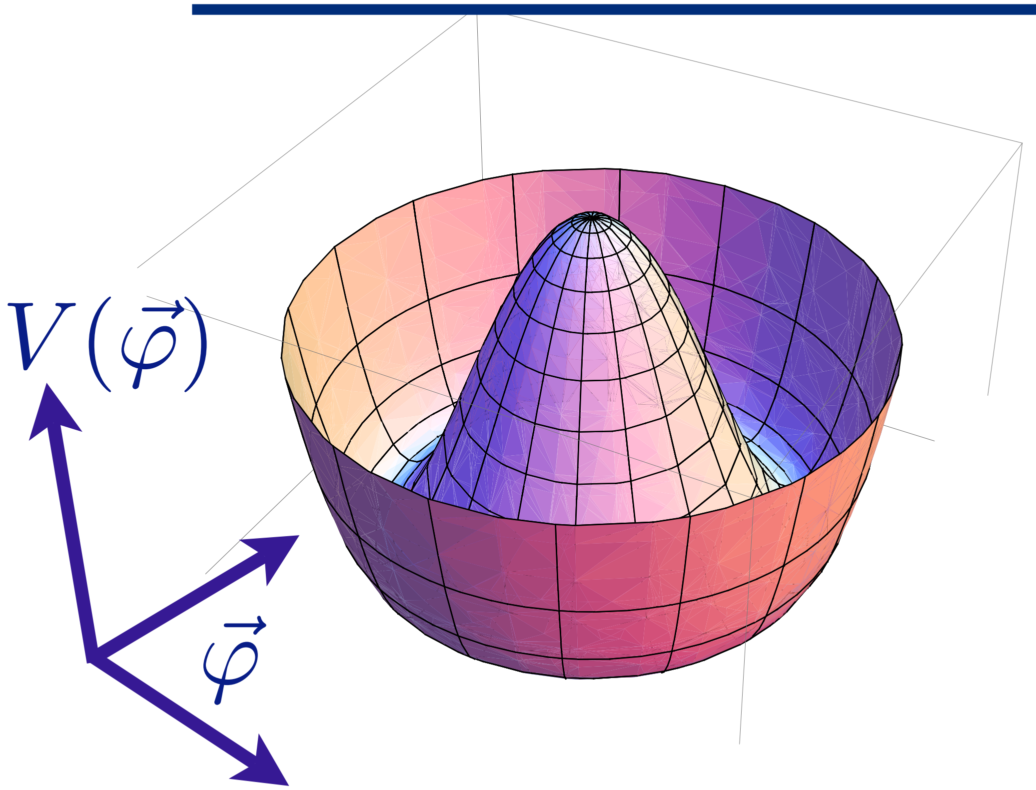

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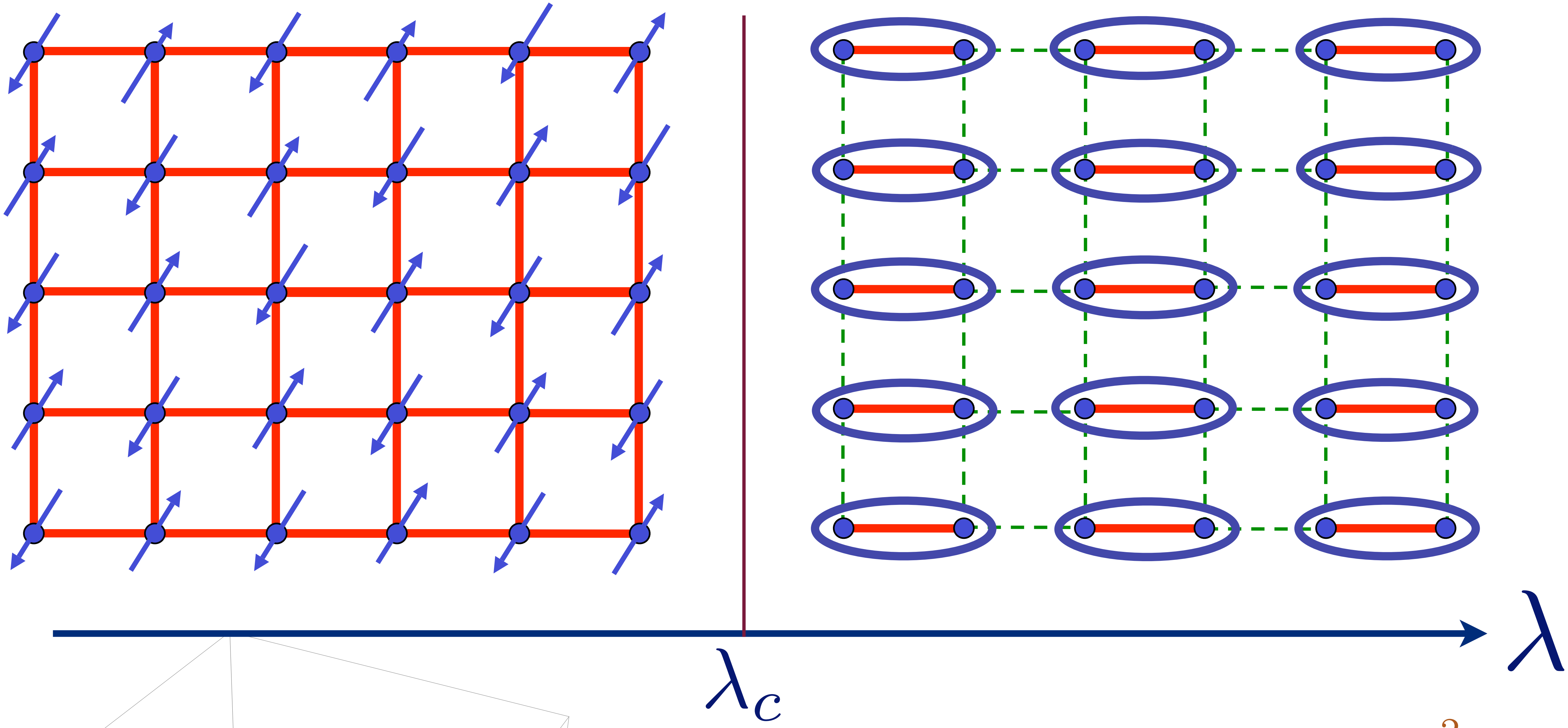
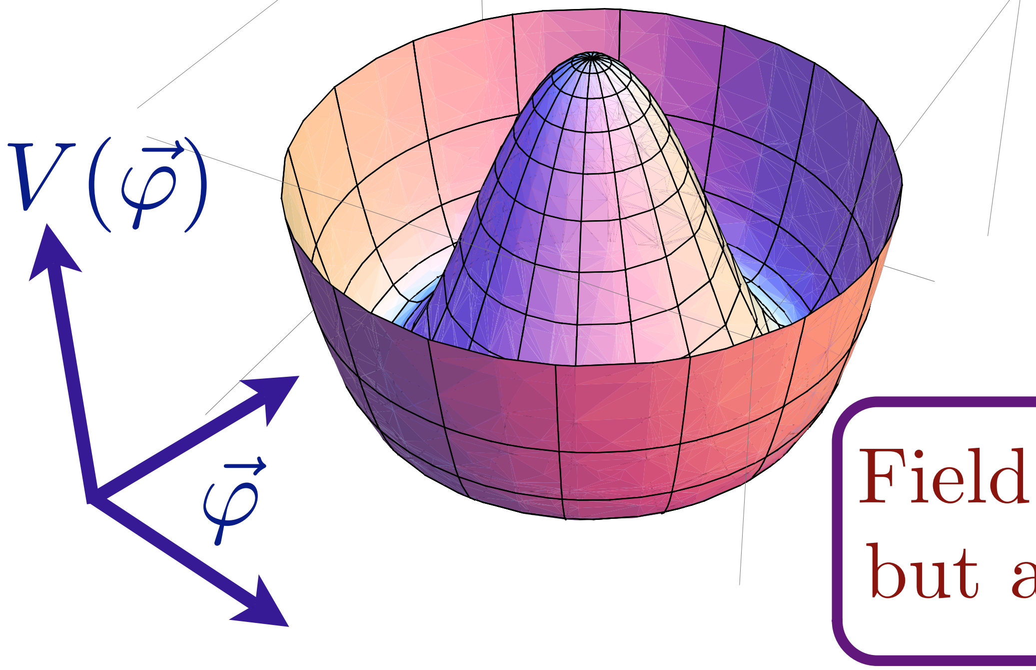
$$\lambda < \lambda_c$$

Excitation spectrum in the Néel phase



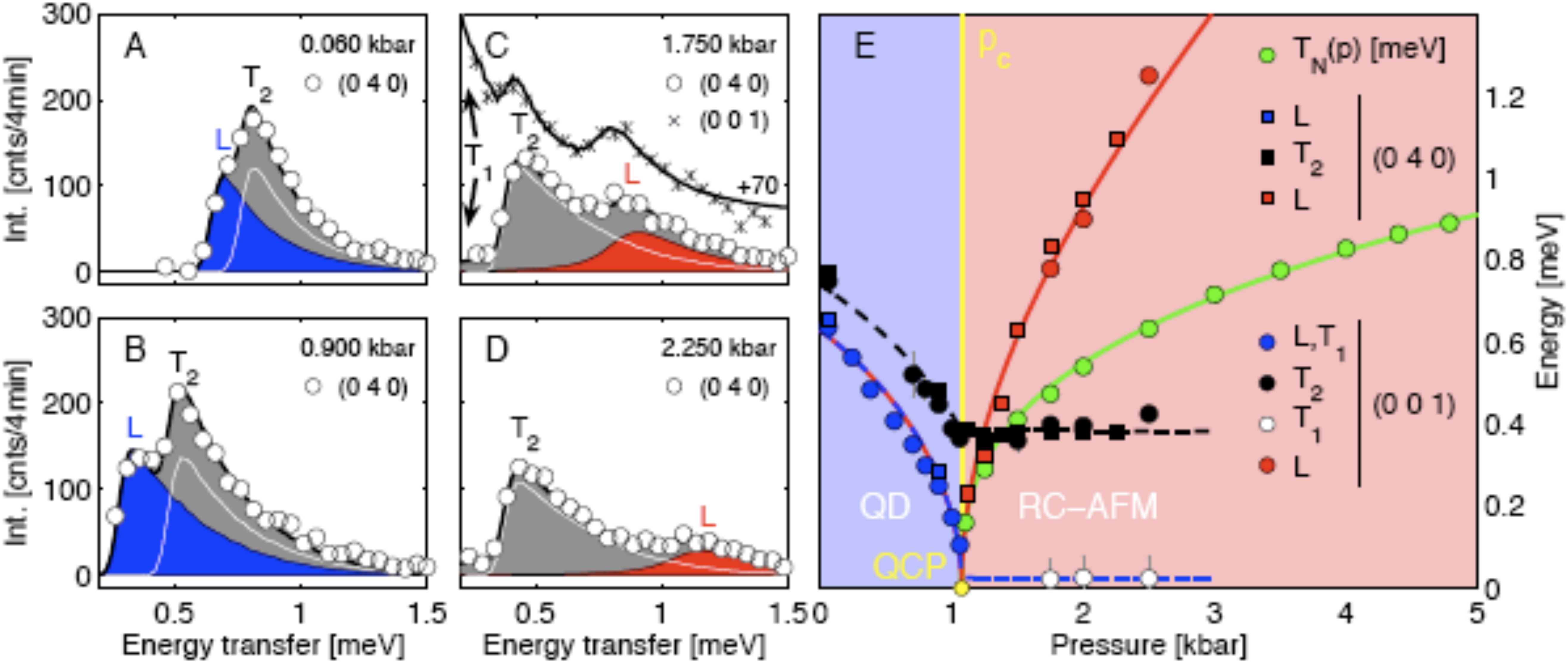


$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$

$$\lambda < \lambda_c$$

Field theory yields spin waves (“Goldstone” modes) but also an additional longitudinal “Higgs” particle

TlCuCl$_3$ with varying pressure



Observation of 3 → 2 low energy modes,
emergence of new Higgs particle in the Néel phase,
and vanishing of Néel temperature at the quantum critical point

Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer,
Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya,
Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

Prediction of quantum field theory

Potential for $\vec{\varphi}$ fluctuations: $V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\vec{\varphi} = 0$:

$$V(\vec{\varphi}) \approx (\lambda - \lambda_c)\vec{\varphi}^2$$

Yields 3 particles with energy gap $\sim \sqrt{(\lambda - \lambda_c)}$

Néel phase, $\lambda < \lambda_c$

Expand $\vec{\varphi} = \left(0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}\right) + \vec{\varphi}_1$:

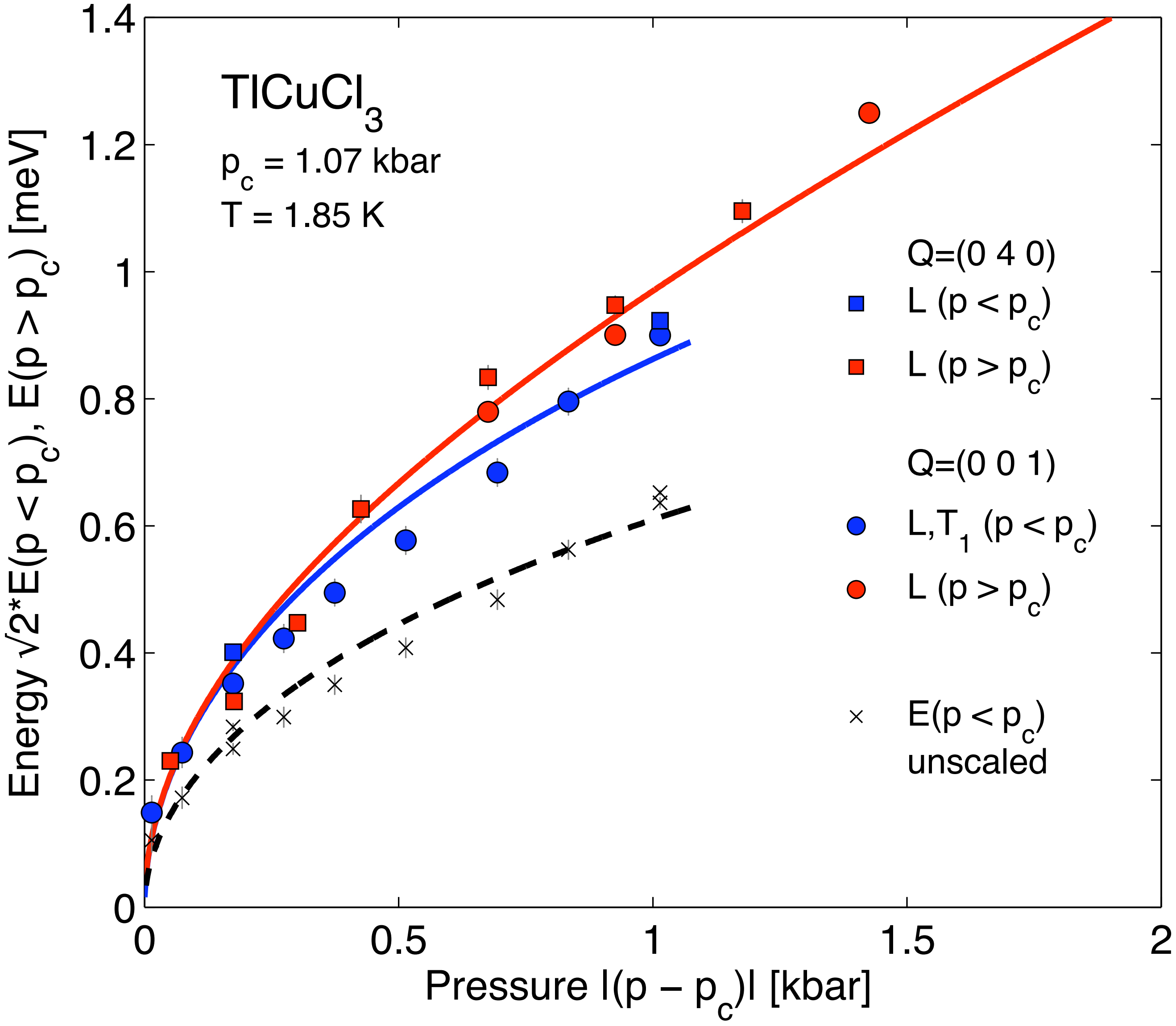
$$V(\vec{\varphi}) \approx 2(\lambda_c - \lambda)\varphi_{1z}^2$$

Yields 2 gapless spin waves and one Higgs particle
with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$

Prediction of quantum field theory

$$\frac{\text{Energy of “Higgs” particle}}{\text{Energy of triplon}} = \sqrt{2}$$

$$V(\vec{\varphi}) = (\lambda - \lambda_c)\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2$$



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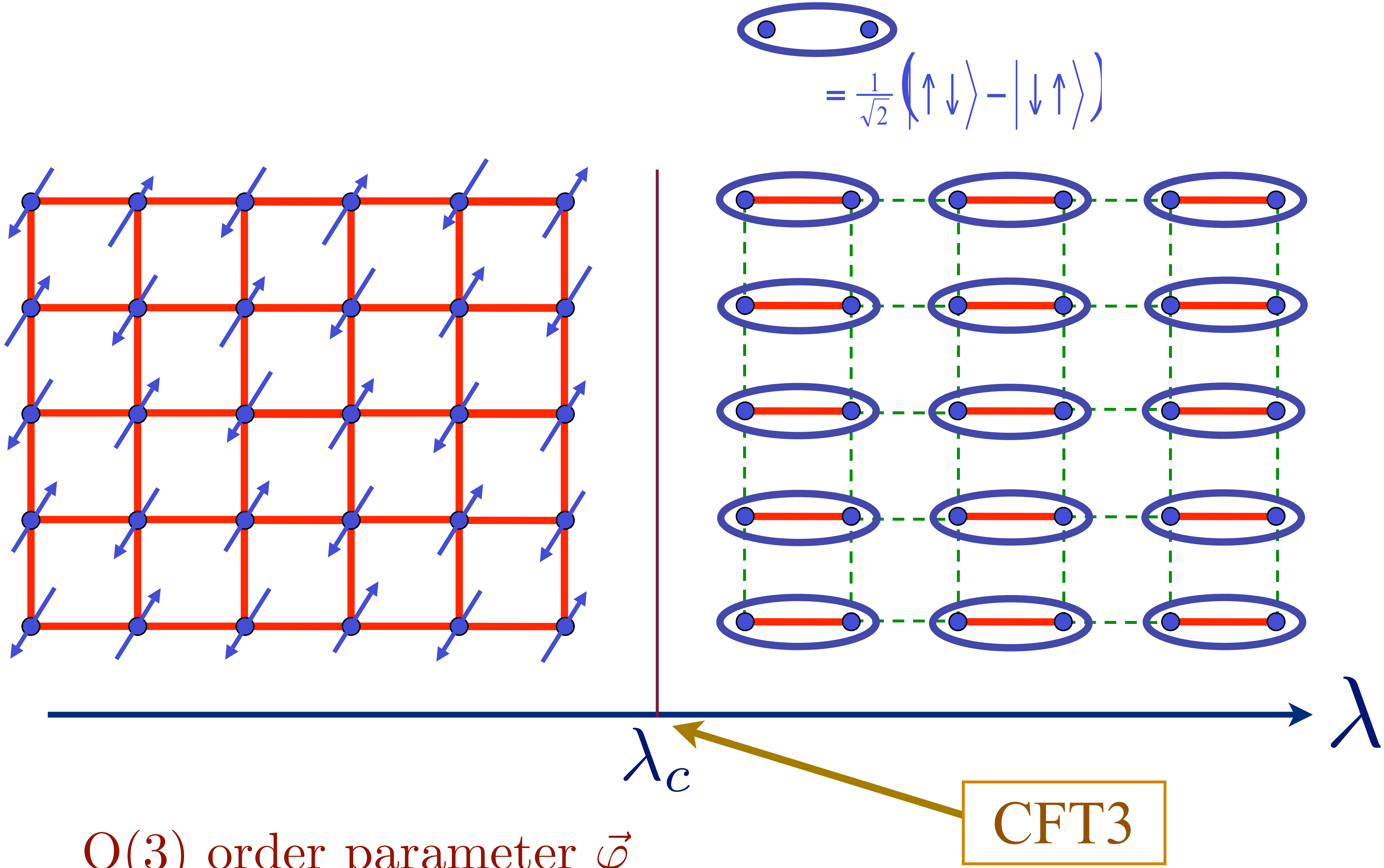

O(3) order parameter $\vec{\varphi}$

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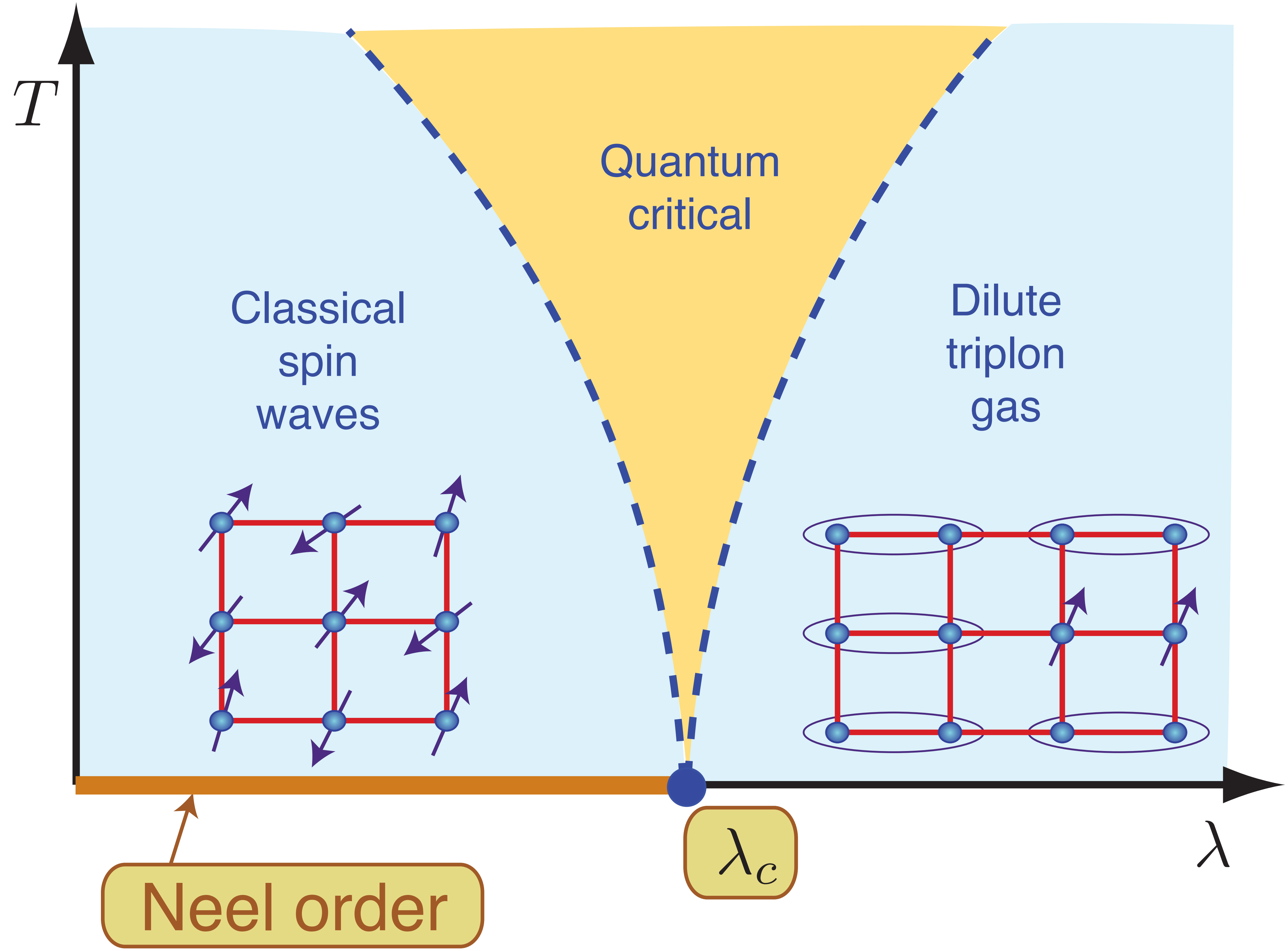
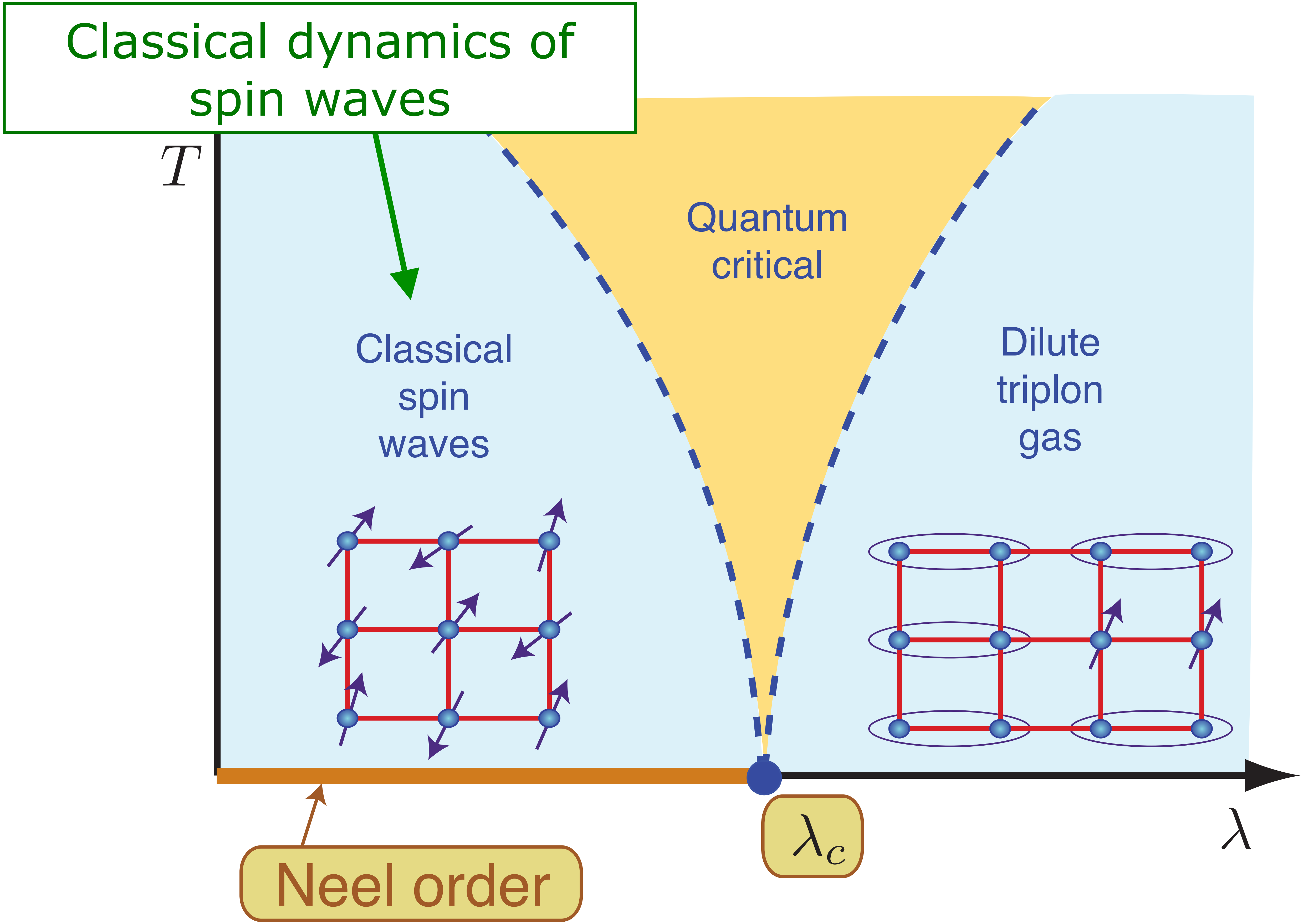
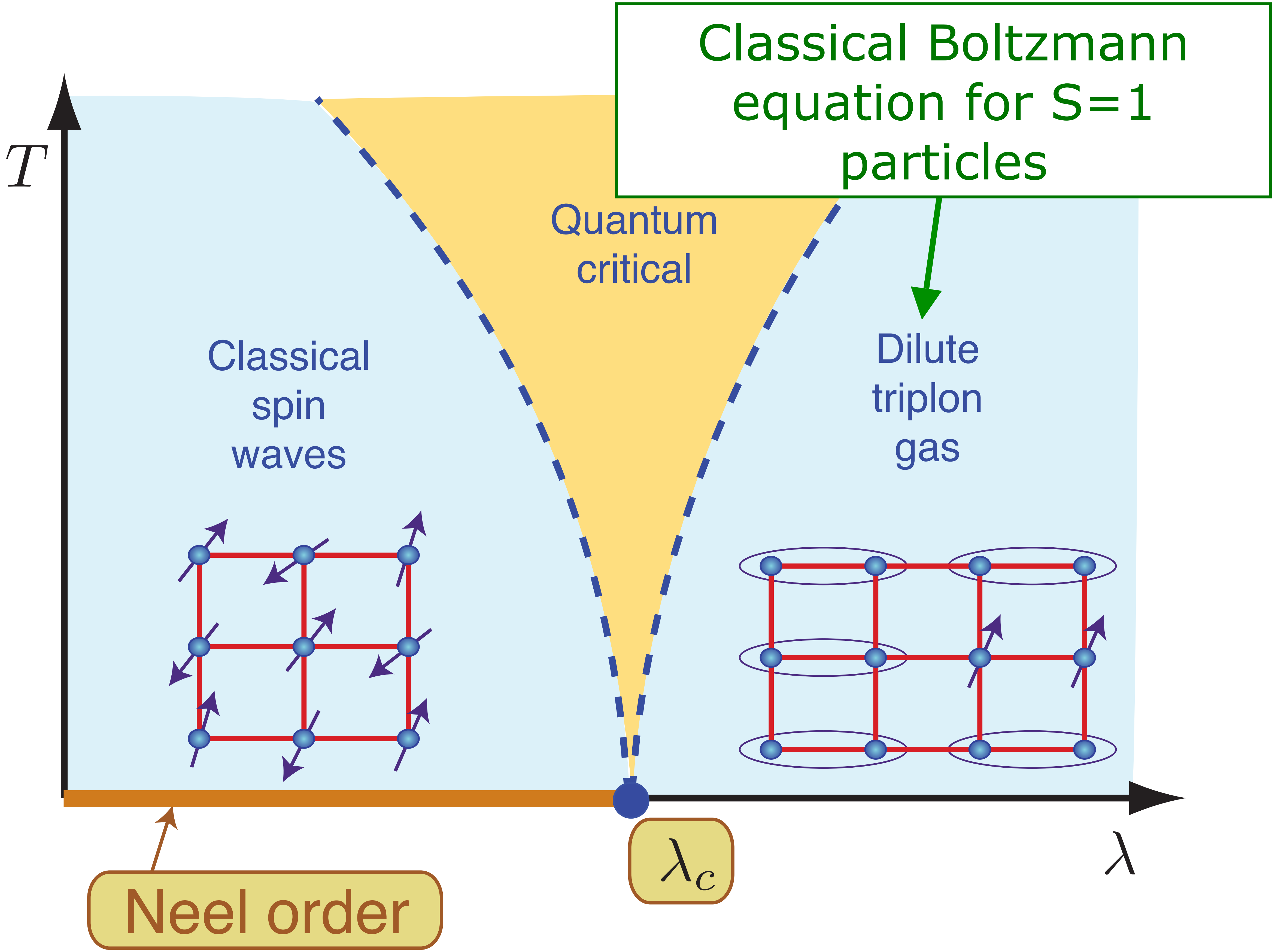

T
Quantum critical
Classical spin waves
Dilute triplon gas
λ
λ_c
Neel order



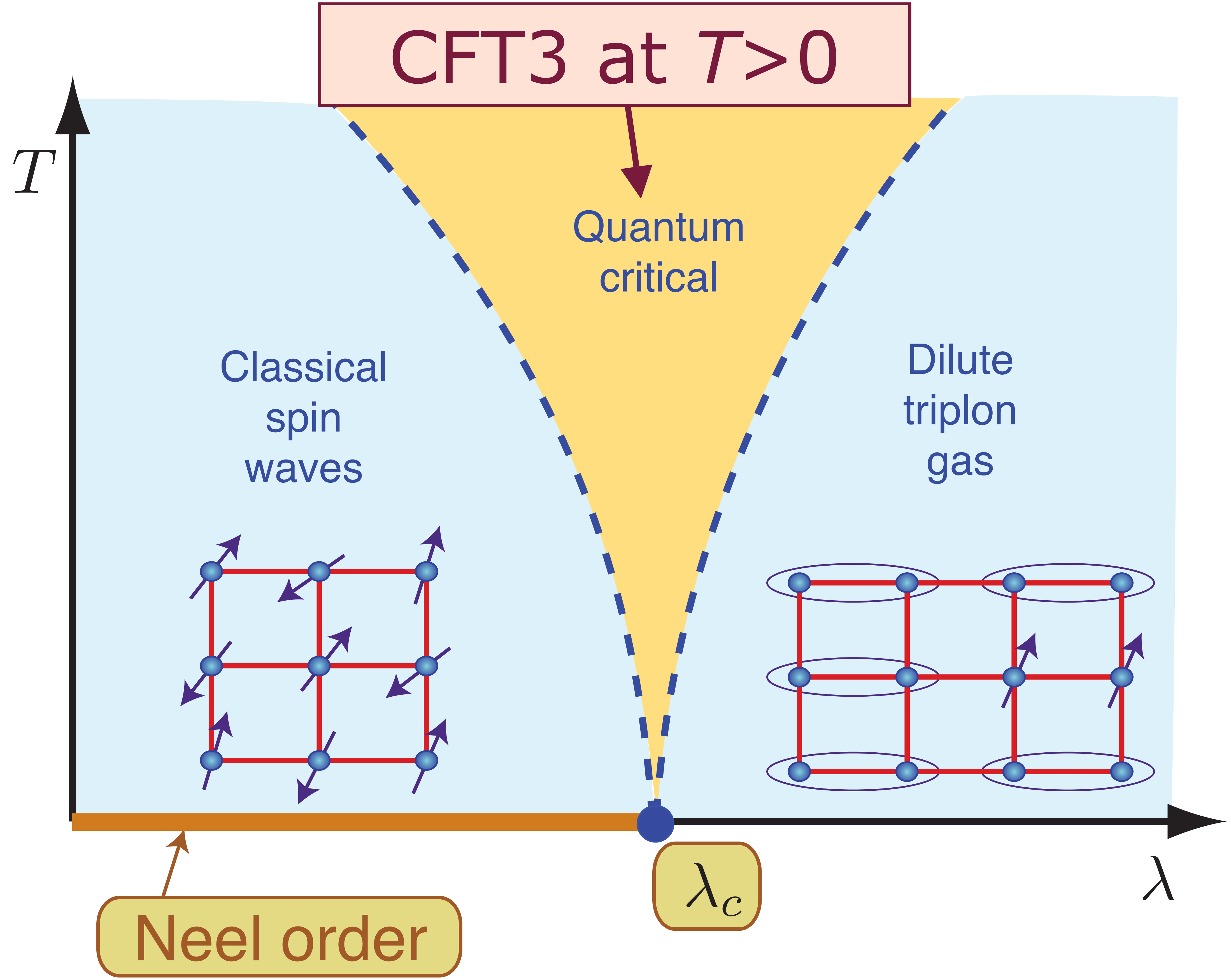
Classical dynamics of spin waves

Neel order



Classical Boltzmann equation for S=1 particles

Neel order


CFT3 at $T>0$
T
Quantum critical
Classical spin waves
Dilute triplon gas
λ
λ_c
Neel order

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   `Topological' Fermi surface transitions

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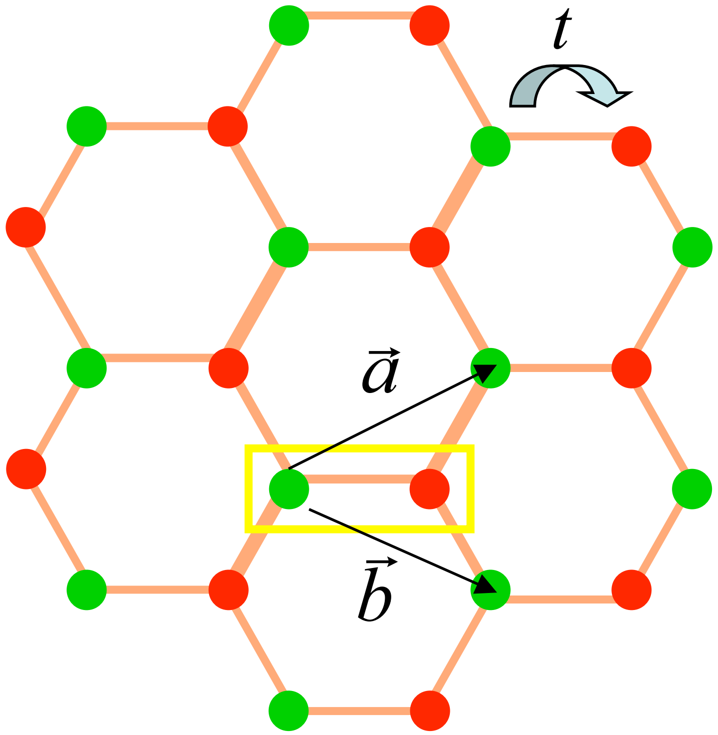
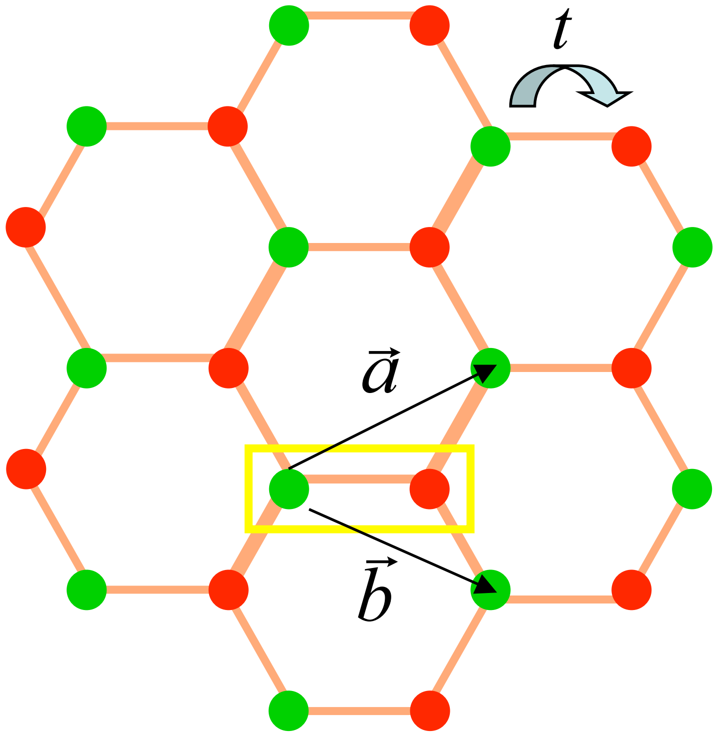
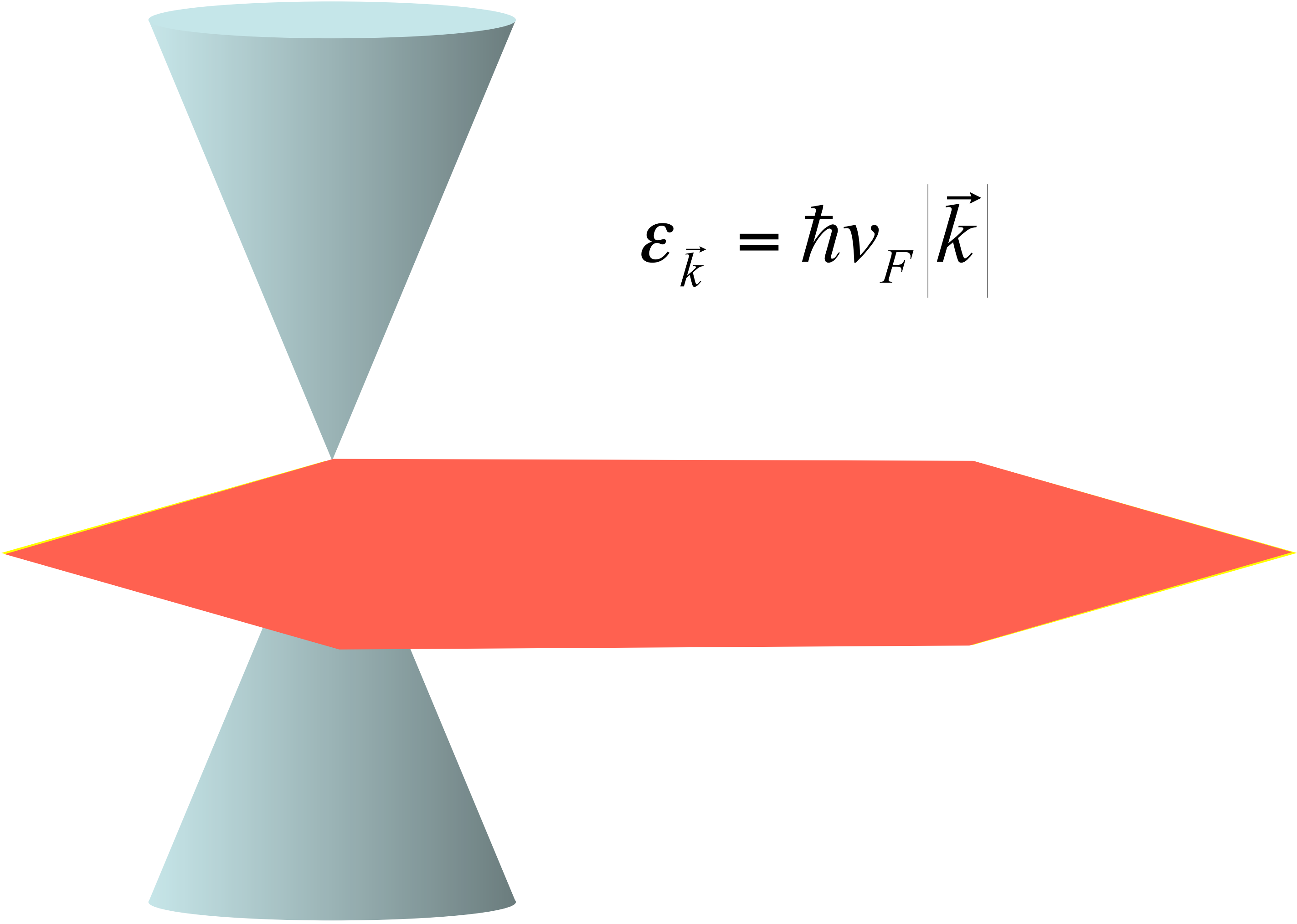
Graphene

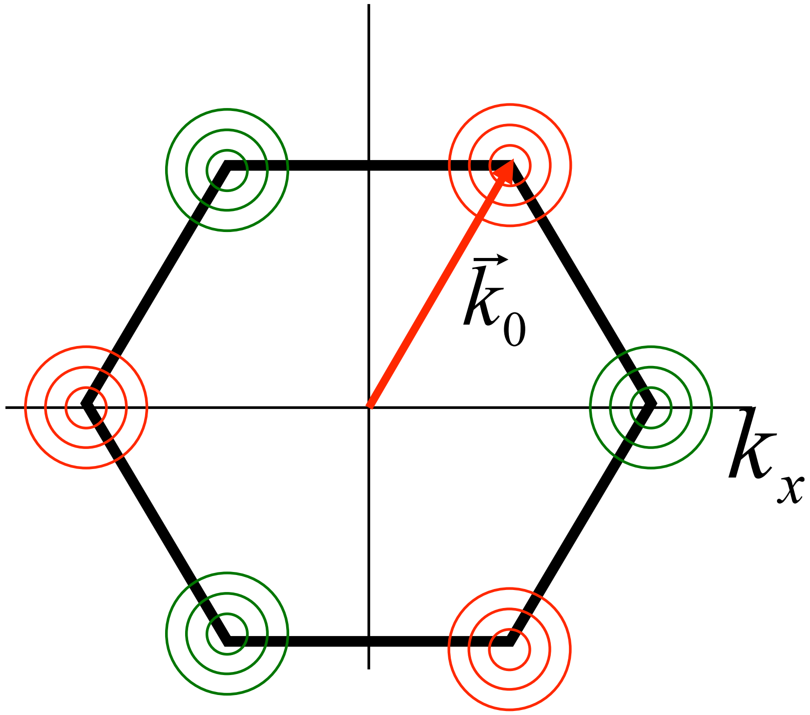


Graphene



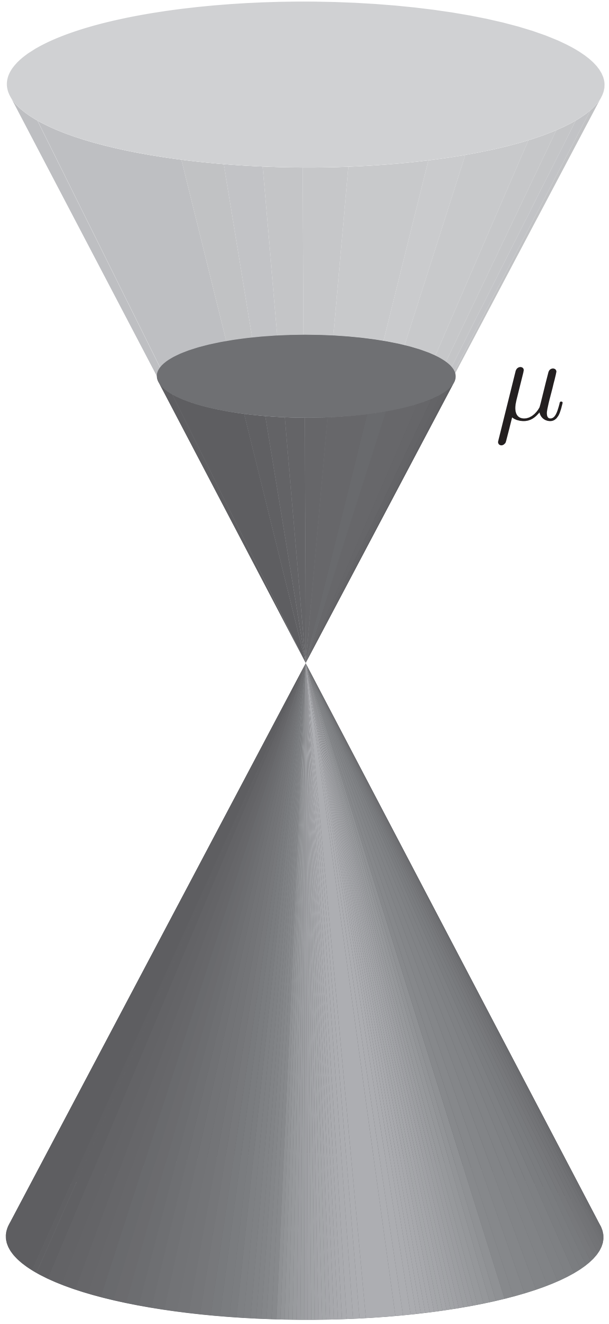


$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$



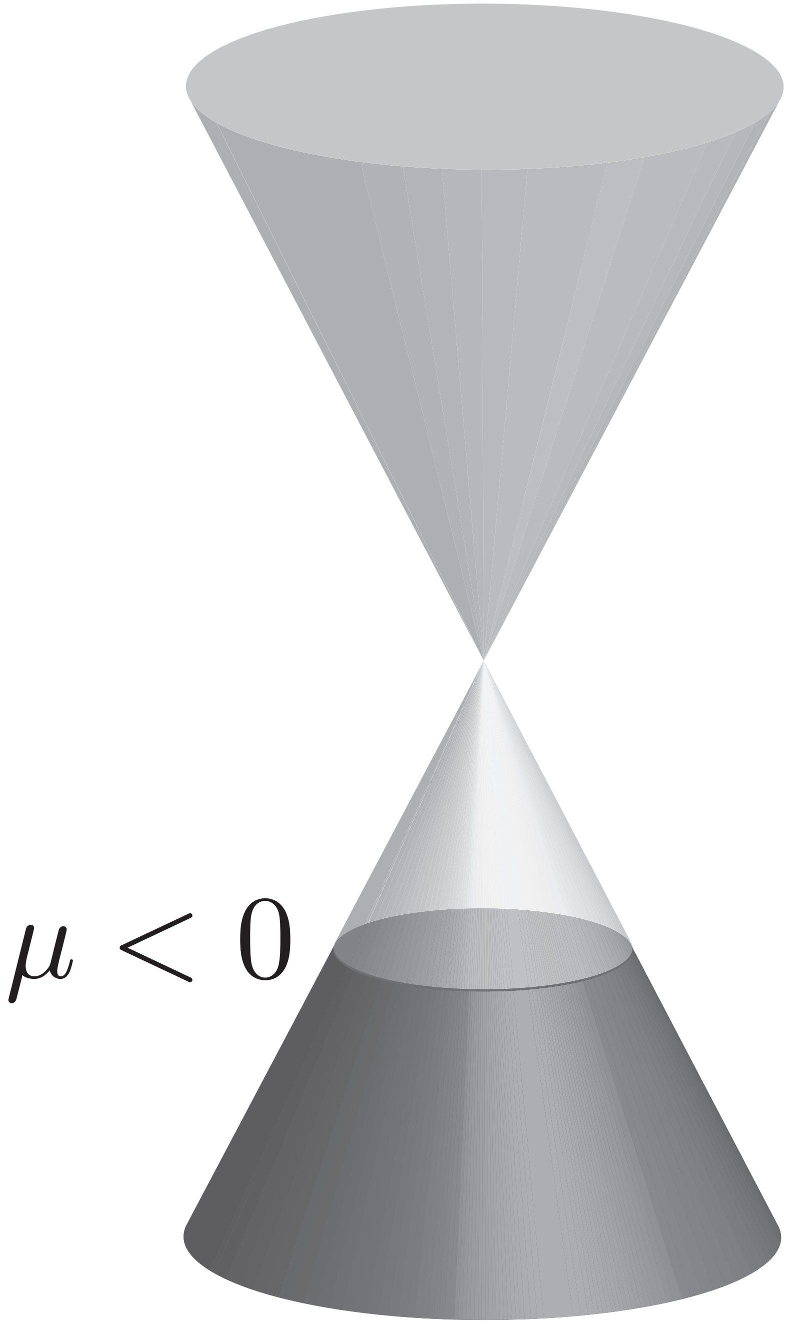
Conical Dirac dispersion

Quantum phase transition in graphene tuned by a bias voltage

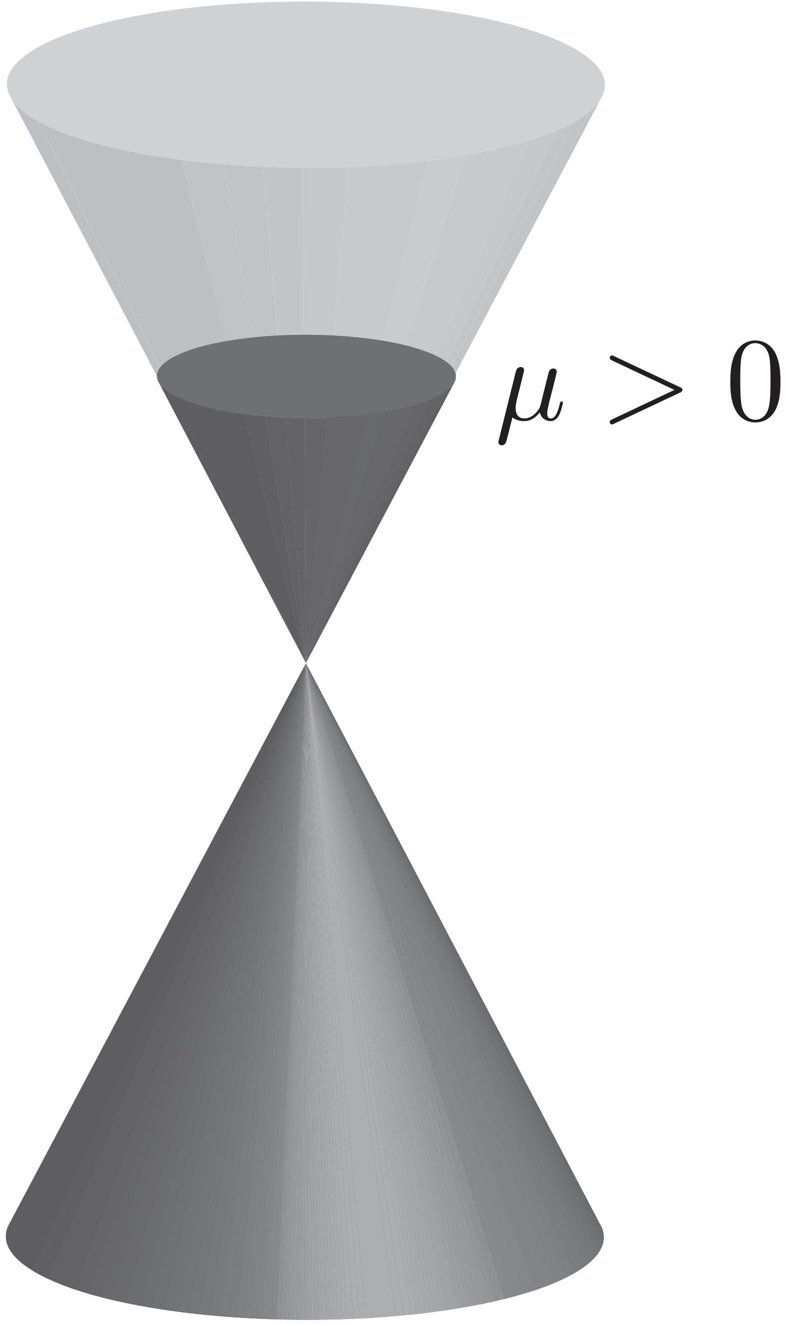


Electron
Fermi surface

Quantum phase transition in graphene tuned by a bias voltage

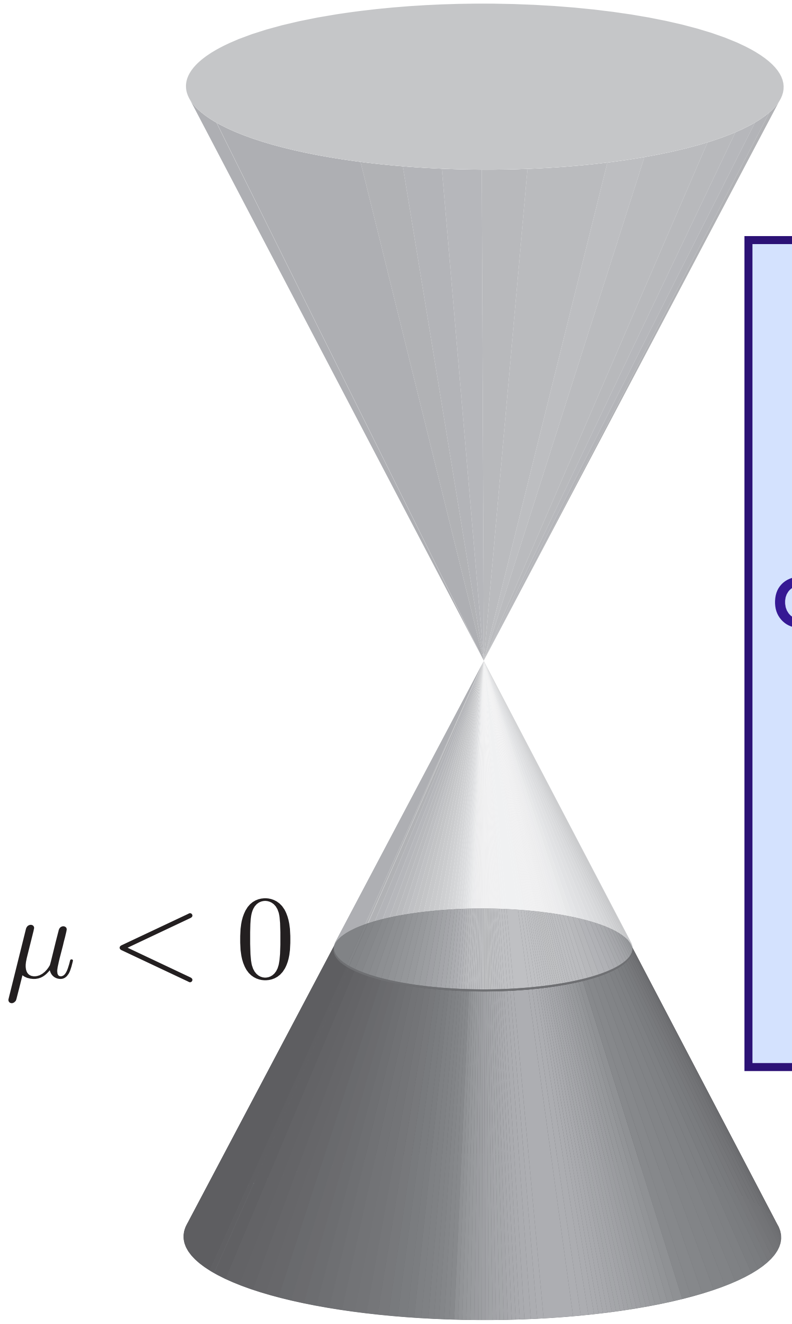


Hole
Fermi surface



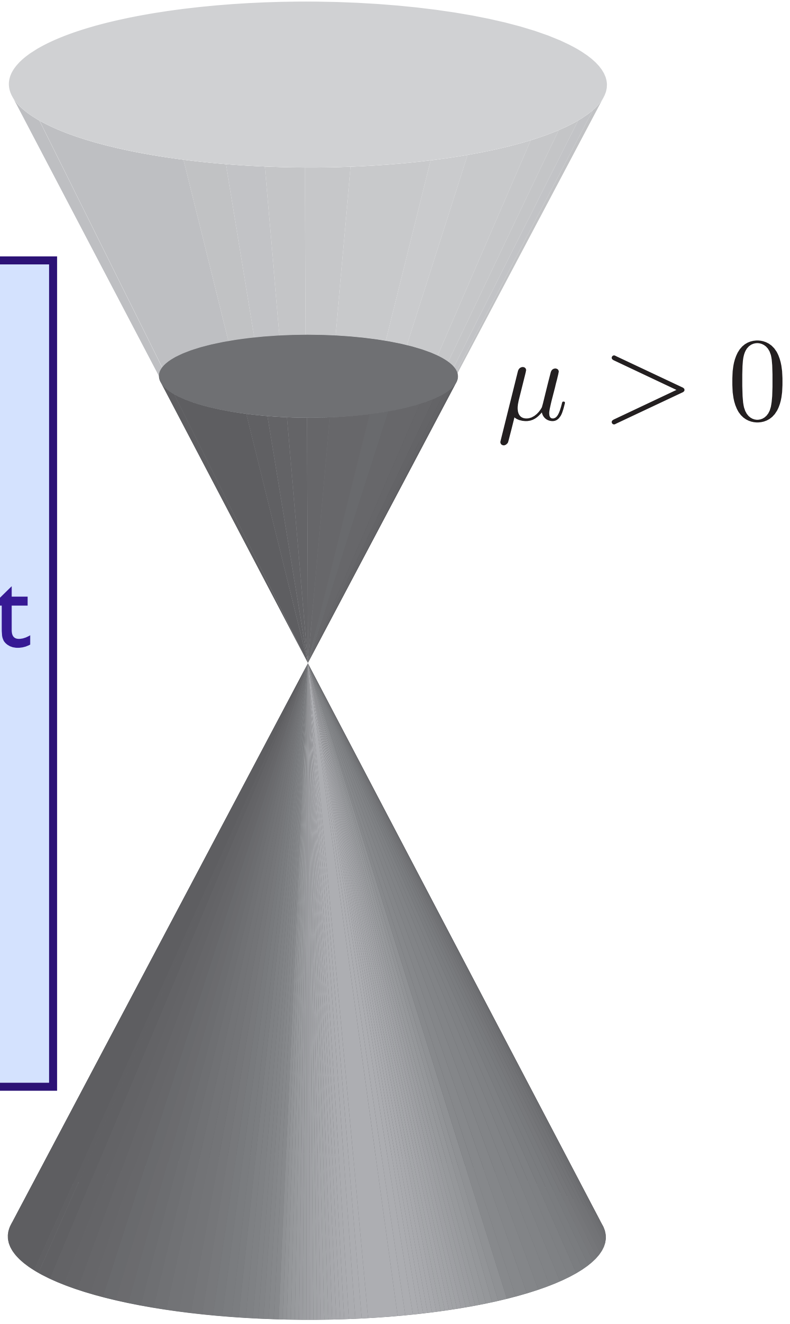
Electron
Fermi surface

Quantum phase transition in graphene tuned by a bias voltage



Hole
Fermi surface

There must be an intermediate quantum critical point where the Fermi surfaces reduce to a Dirac point



Electron
Fermi surface

Quantum critical graphene

Low energy theory has 4 two-component Dirac fermions, ψ_σ, $\sigma = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$\begin{aligned} \mathcal{S} &= \int d^2 r d\tau \psi_\sigma^\dagger \left(\partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma \\ &\quad + \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\sigma^\dagger \psi_\sigma (r) \frac{1}{|r - r'|} \psi_{\sigma'}^\dagger \psi_{\sigma'} (r') \end{aligned}$$

Quantum critical graphene

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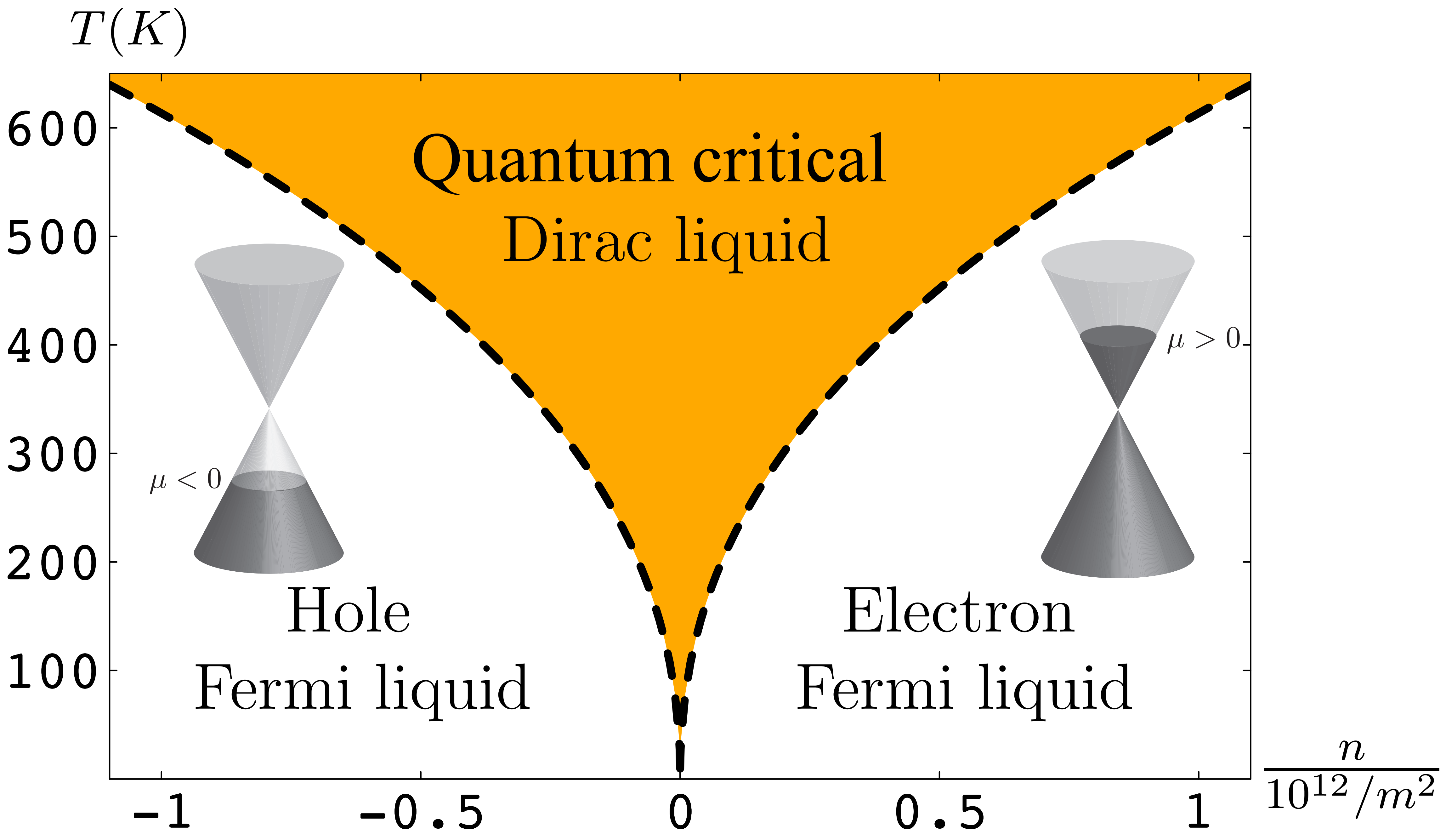
$$\begin{aligned} \mathcal{S} &= \int d^2 r d\tau \psi_\sigma^\dagger \left(\partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\sigma \\ &\quad + \frac{e^2}{2} \int d^2 r d^2 r' d\tau \psi_\sigma^\dagger \psi_\sigma(r) \frac{1}{|r - r'|} \psi_{\sigma'}^\dagger \psi_{\sigma'}(r') \end{aligned}$$

Dimensionless "fine-structure" constant $\alpha = e^2/(\hbar v_F)$.
RG flow of α:

$$\frac{d\alpha}{d\ell} = -\alpha^2 + \ldots$$

Behavior is similar to a conformal field theory (CFT) in 2+1 dimensions with $\alpha \sim 1/\ln(\text{scale})$

Quantum phase transition in graphene



Quantum critical transport

Quantum *"perfect fluid"*
with shortest possible
relaxation time, τ_R

$$\boxed{\tau_R \gtrsim \frac{\hbar}{k_B T}}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\boxed{\sigma = \frac{e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]}$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport in graphene

$$\sigma(\omega) = \begin{cases} \dfrac{e^2}{h}\left[\dfrac{\pi}{2} + \mathcal{O}\left(\dfrac{1}{\ln(\Lambda/\omega)}\right)\right] & , \quad \hbar\omega \gg k_B T \\ \dfrac{e^2}{h\alpha^2(T)}\left[0.760 + \mathcal{O}\left(\dfrac{1}{|\ln(\alpha(T))|}\right)\right] & , \quad \hbar\omega \ll k_B T \alpha^2(T) \end{cases}$$

$$\frac{\eta}{s} = \frac{\hbar}{k_B \alpha^2(T)} \times 0.130$$

where the "fine structure constant" is

$$\alpha(T) = \frac{\alpha}{1 + (\alpha/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)}$$

L. Fritz, J. Schmalian, M. Müller and S. Sachdev, *Physical Review B* **78**, 085416 (2008)
M. Müller, J. Schmalian, and L. Fritz, *Physical Review Letters* **103**, 025301 (2009)

Outline

1. Coupled dimer antiferromagnets

Order parameters and Landau-Ginzburg criticality

2. Graphene

`Topological' Fermi surface transitions

3. Quantum criticality and black holes

AdS_4 *theory of compressible quantum liquids*

4. Quantum criticality in the cuprates

Global phase diagram and the spin density wave transition in metals

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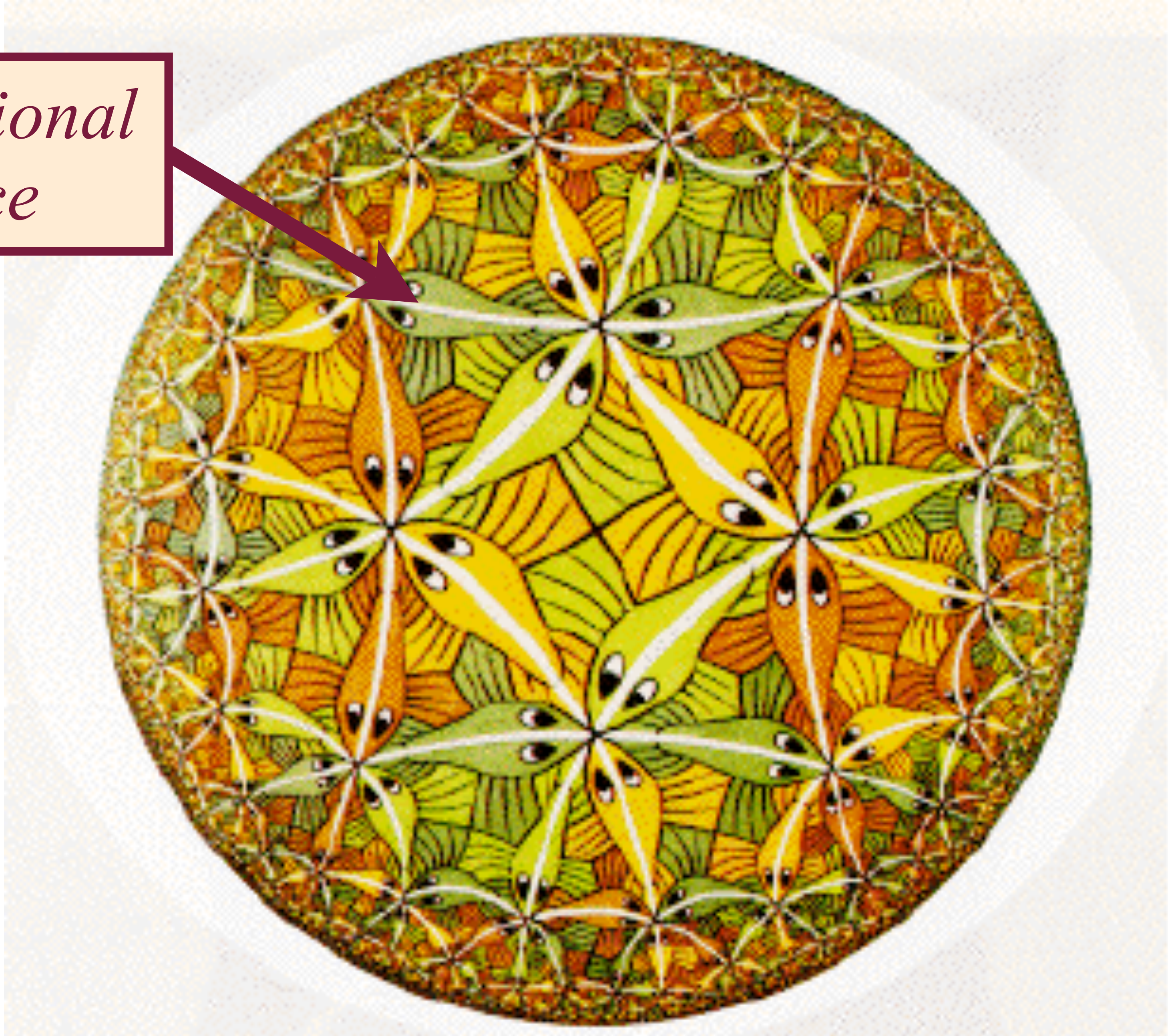
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



3+1 dimensional AdS space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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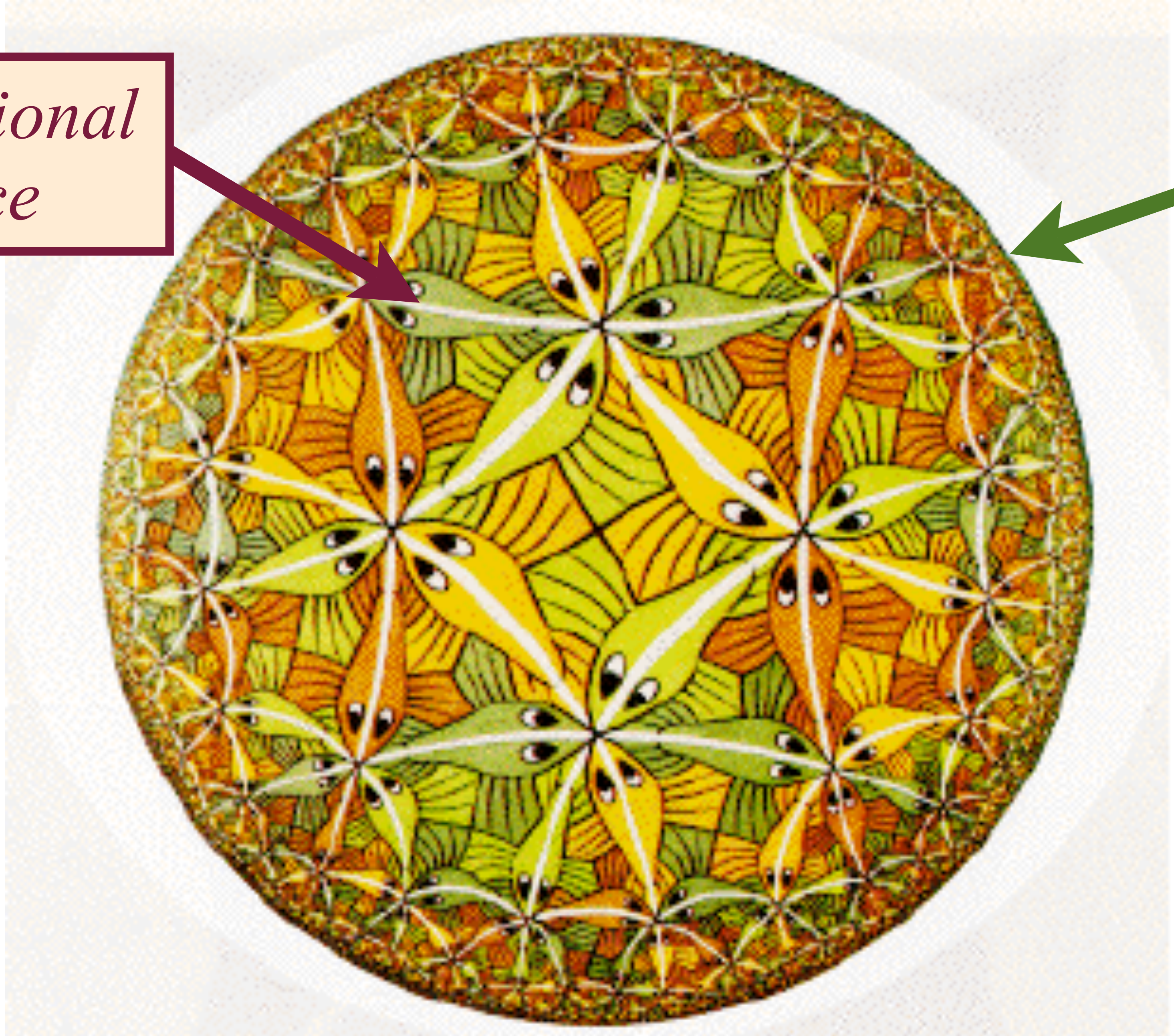

3+1 dimensional AdS space

A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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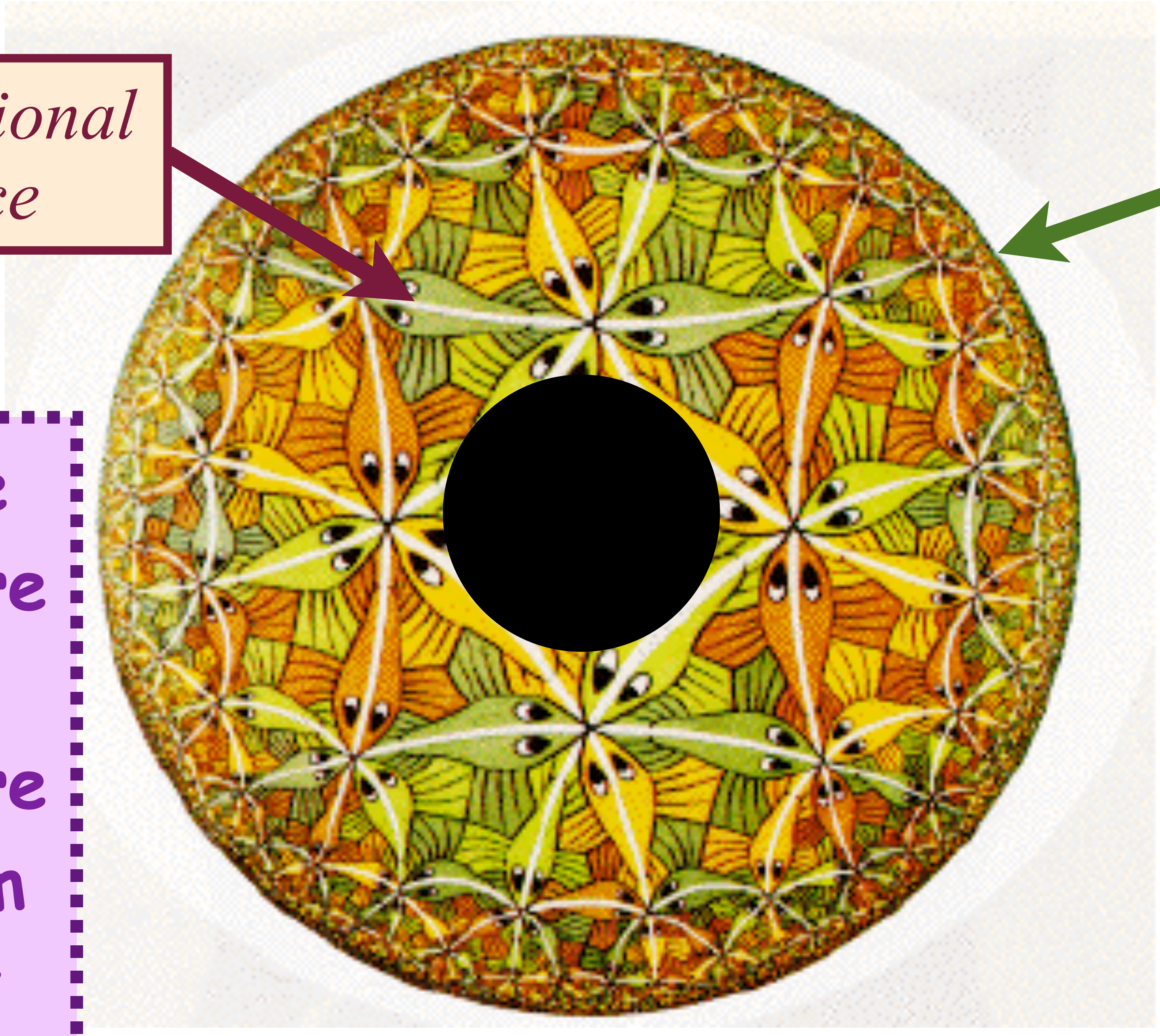

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Black hole temperature = temperature of quantum criticality

Maldacena, Gubser, Klebanov, Polyakov, Witten

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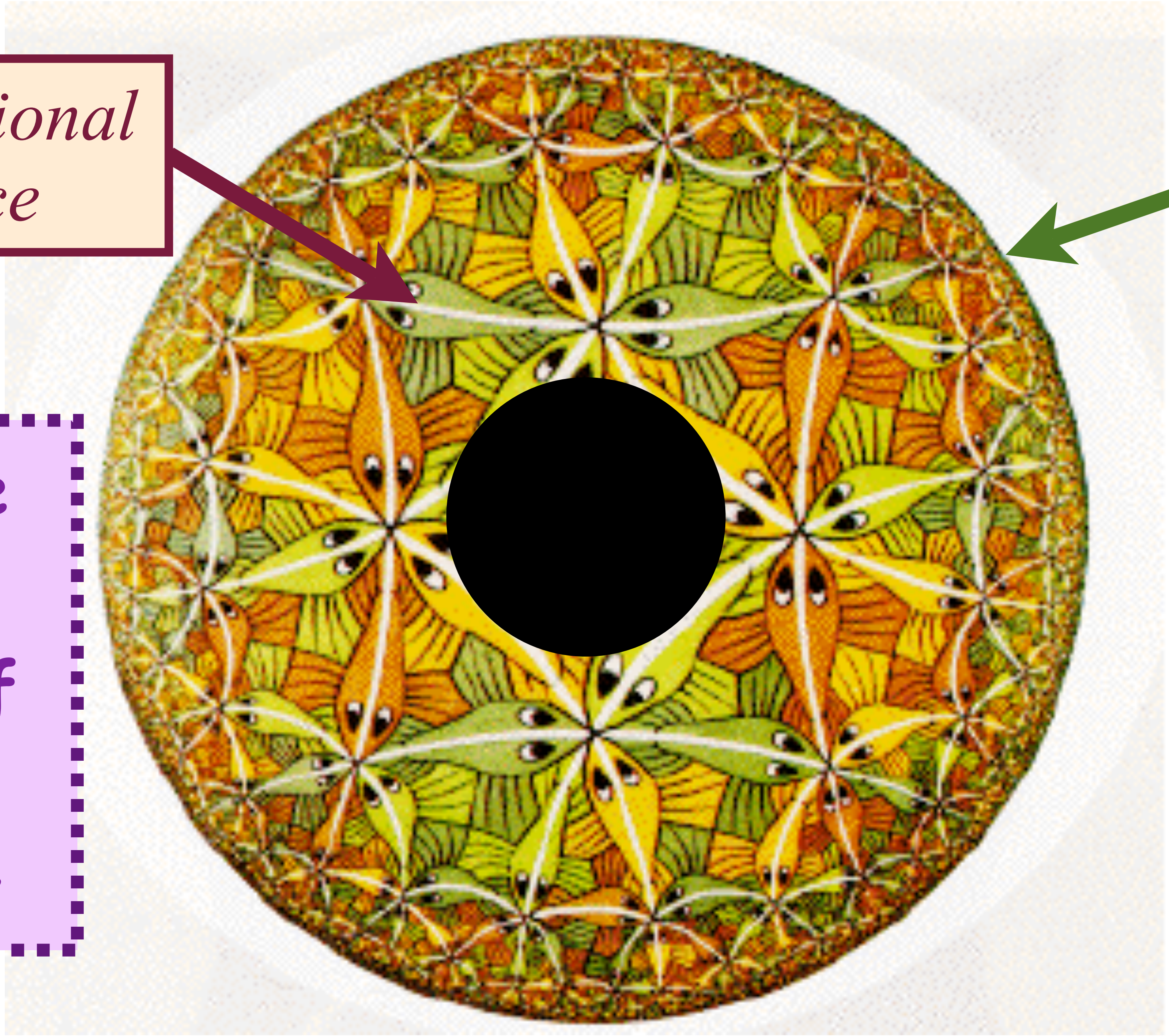


3+1 dimensional AdS space

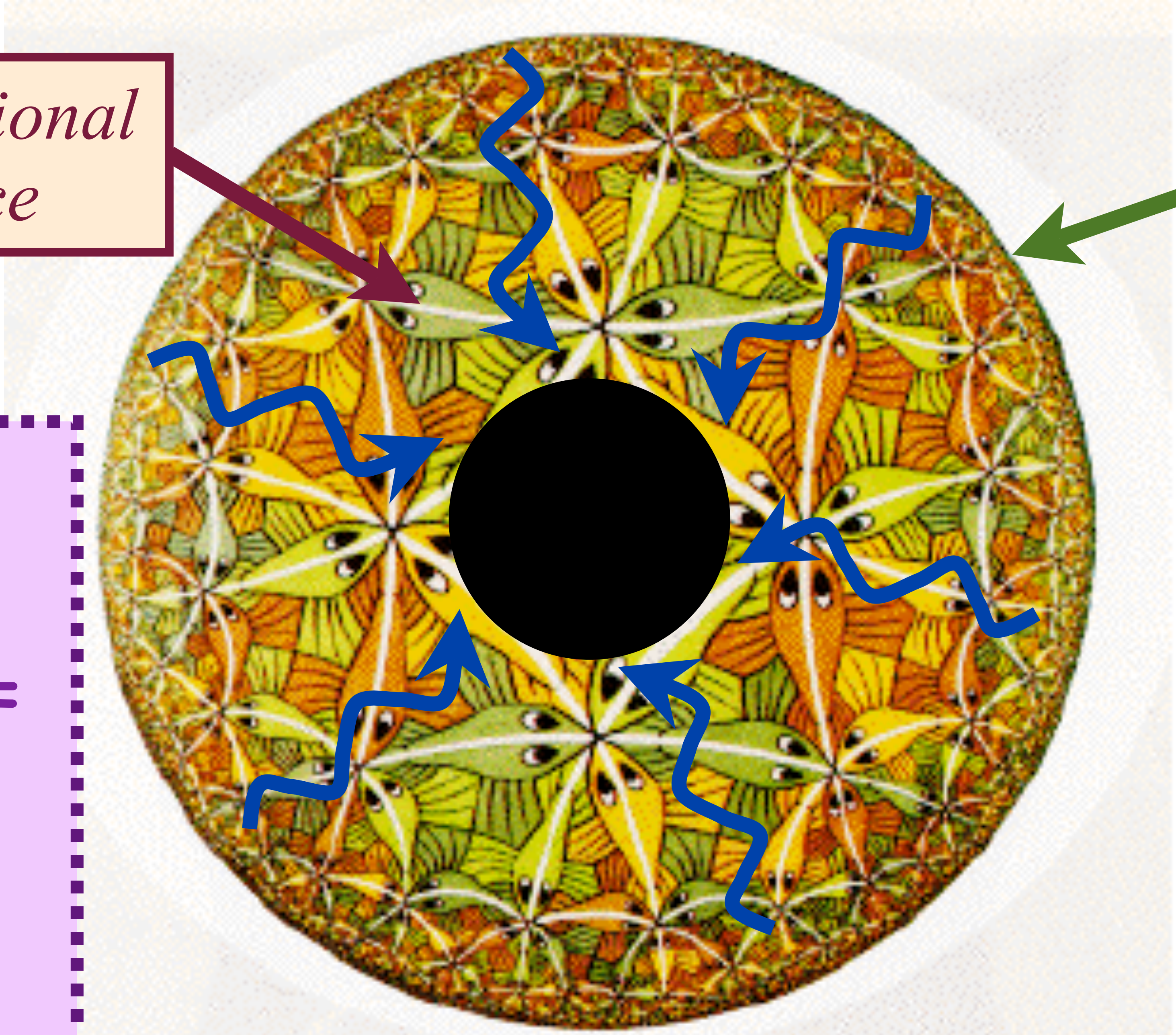
Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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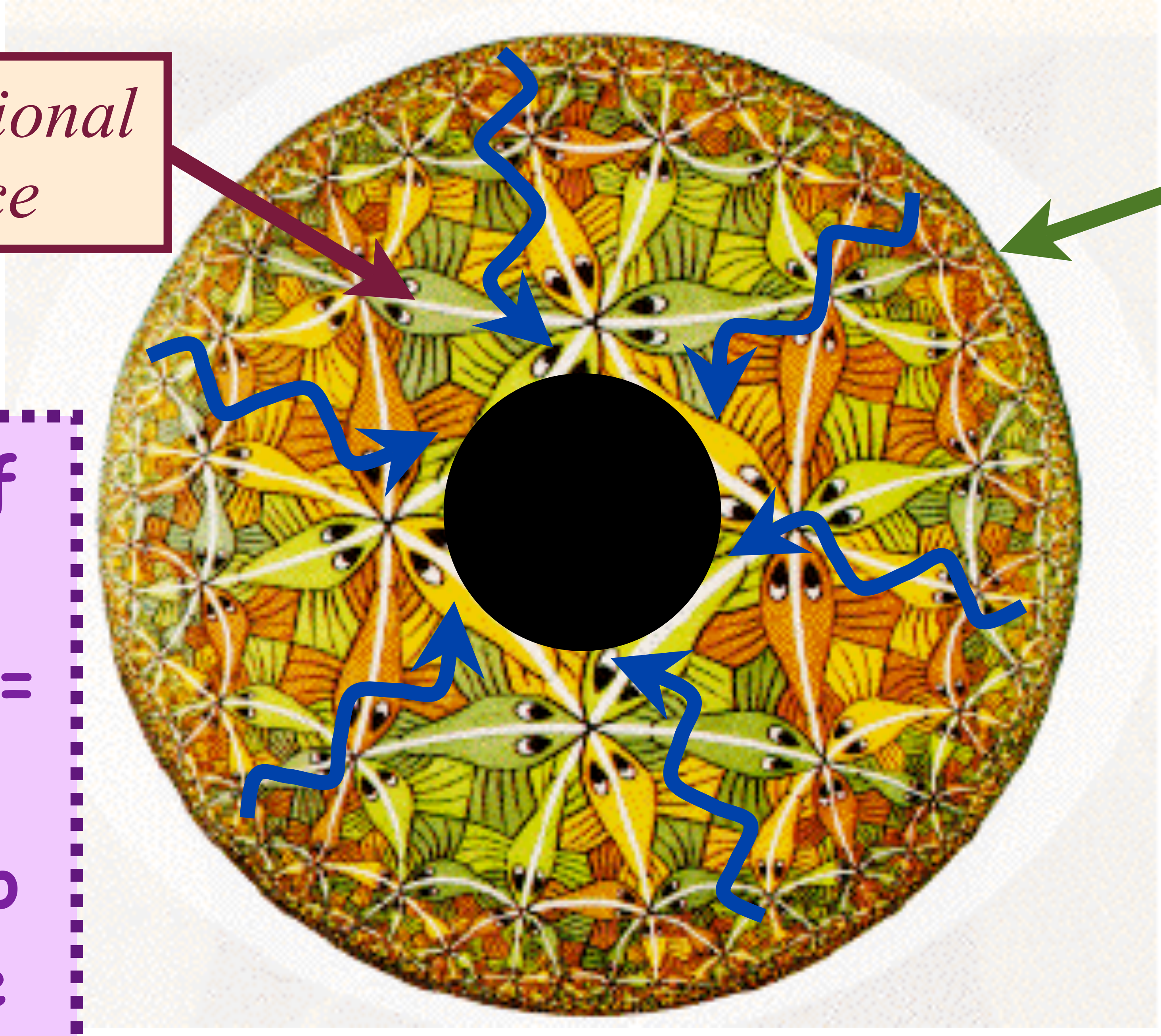


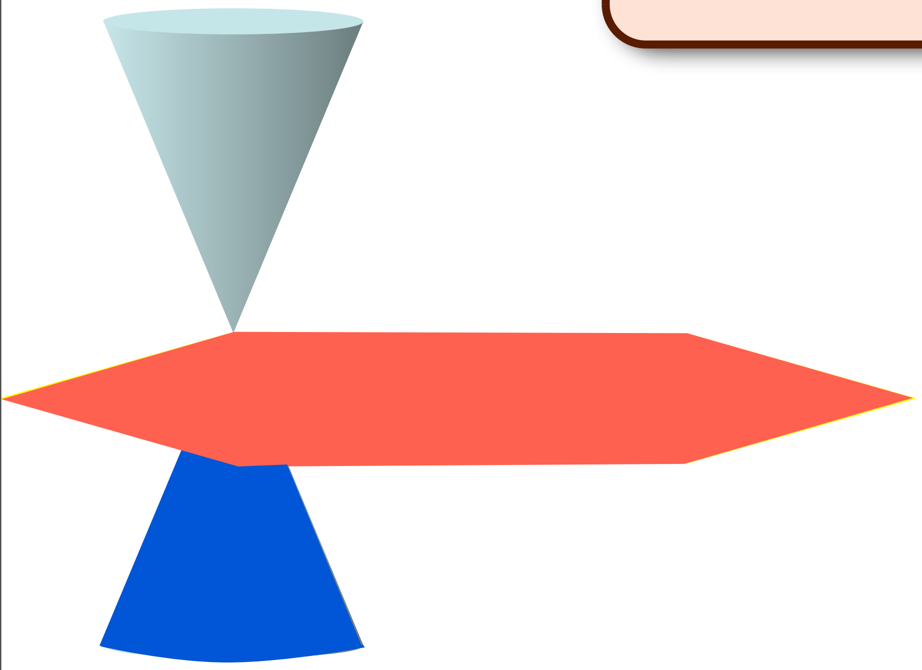
3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Friction of quantum criticality = waves falling into black hole

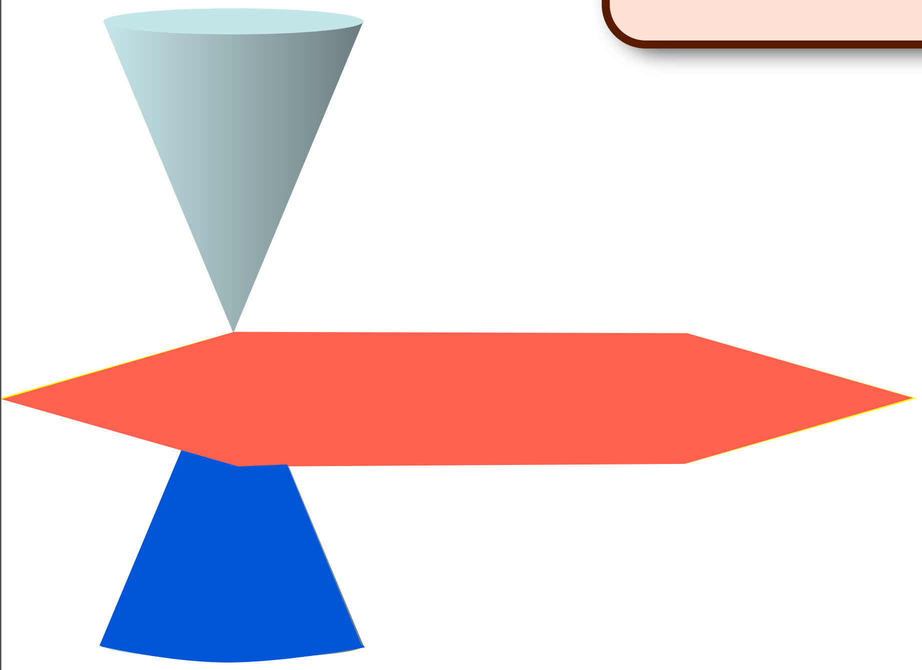
Kovtun, Policastro, Son

Conformal field theory in 2+1 dimensions at $T = 0$

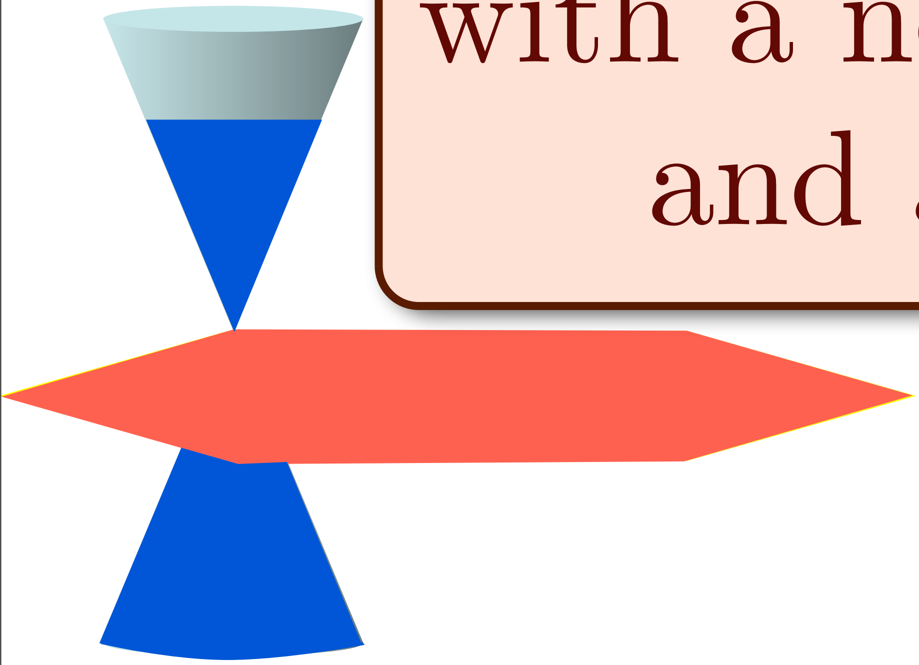


Einstein gravity on AdS_4

Conformal field theory
in 2+1 dimensions at $T > 0$

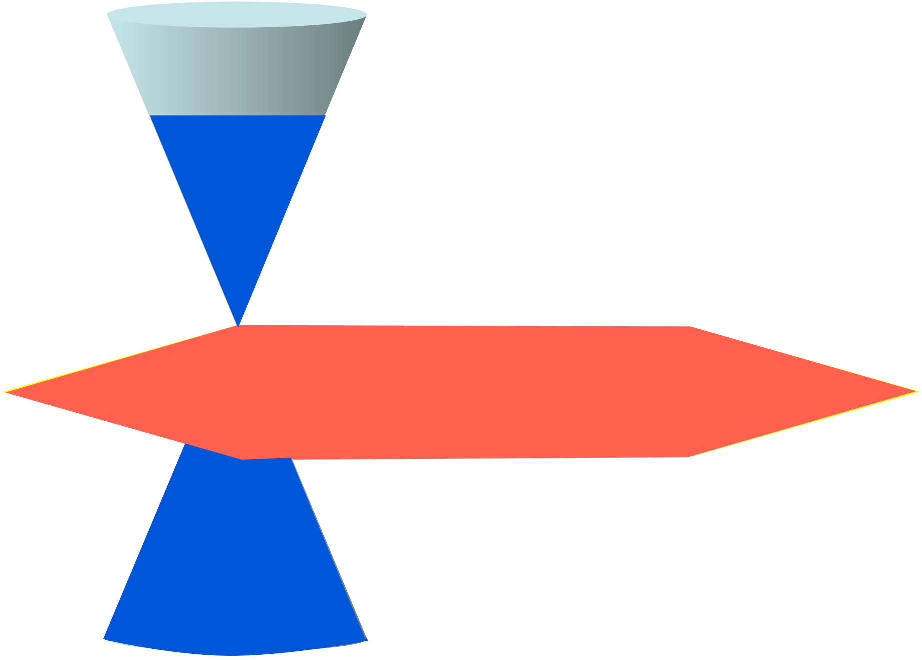


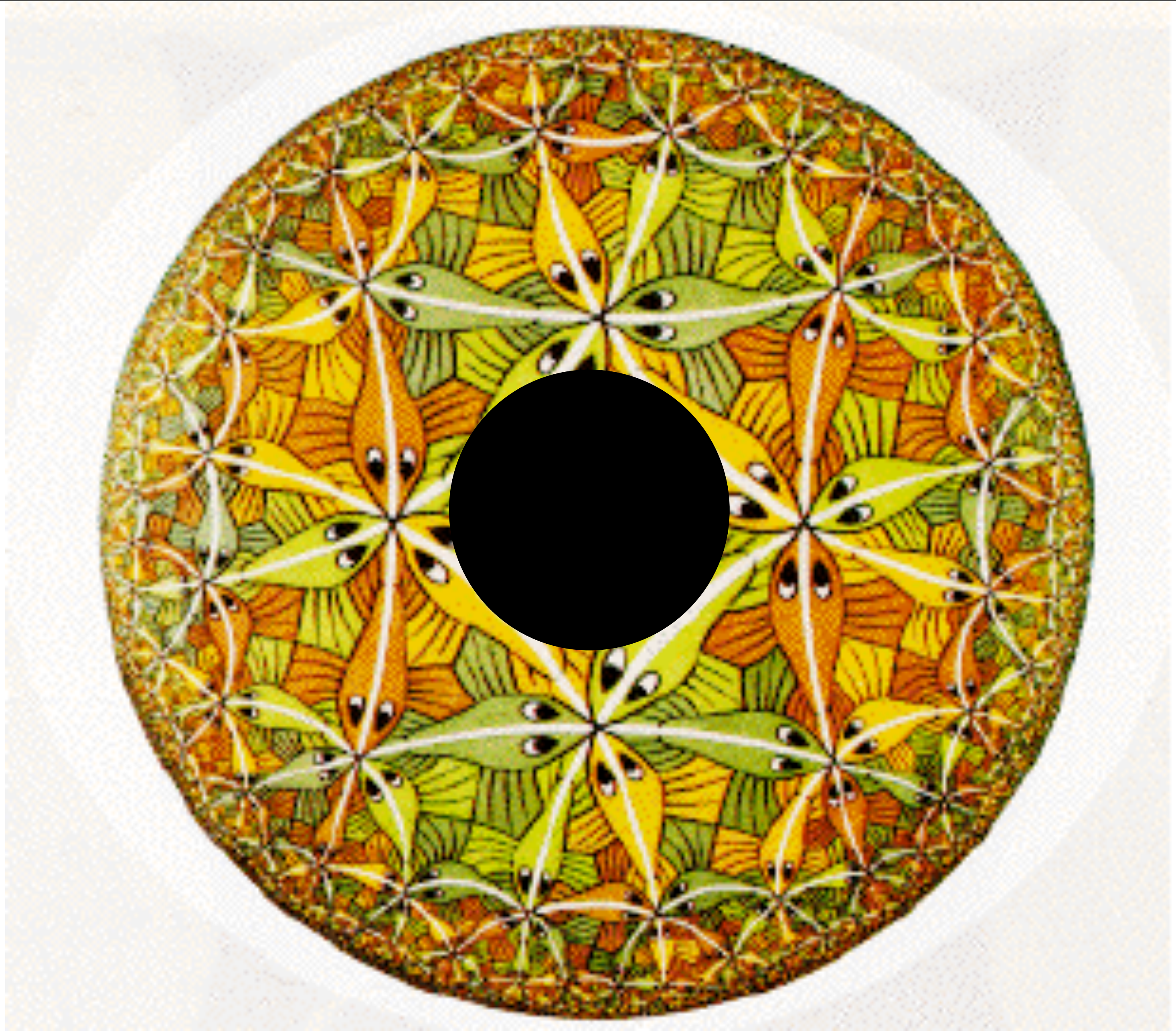
Einstein gravity on AdS_4
with a Schwarzschild
black hole

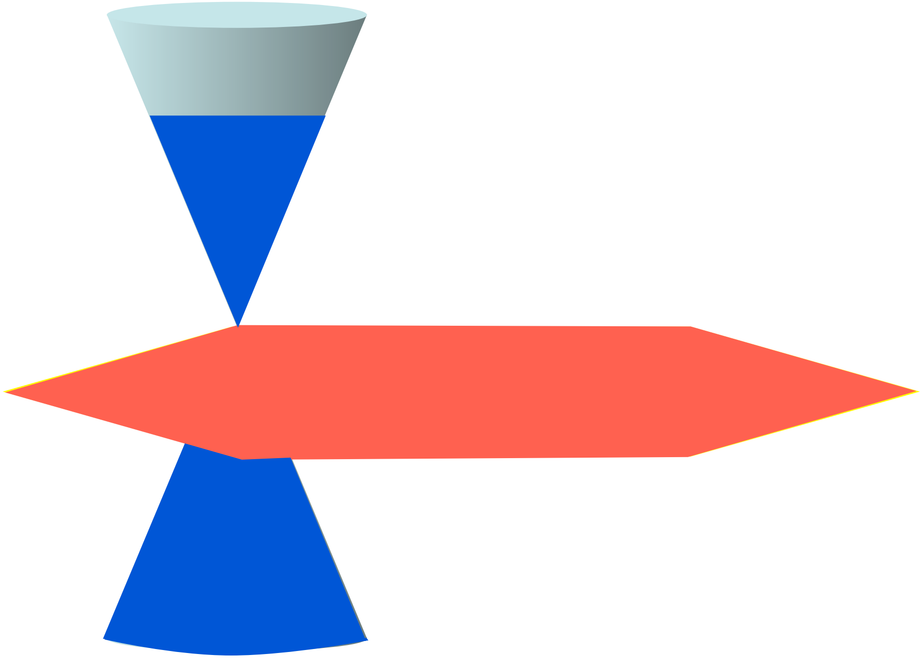


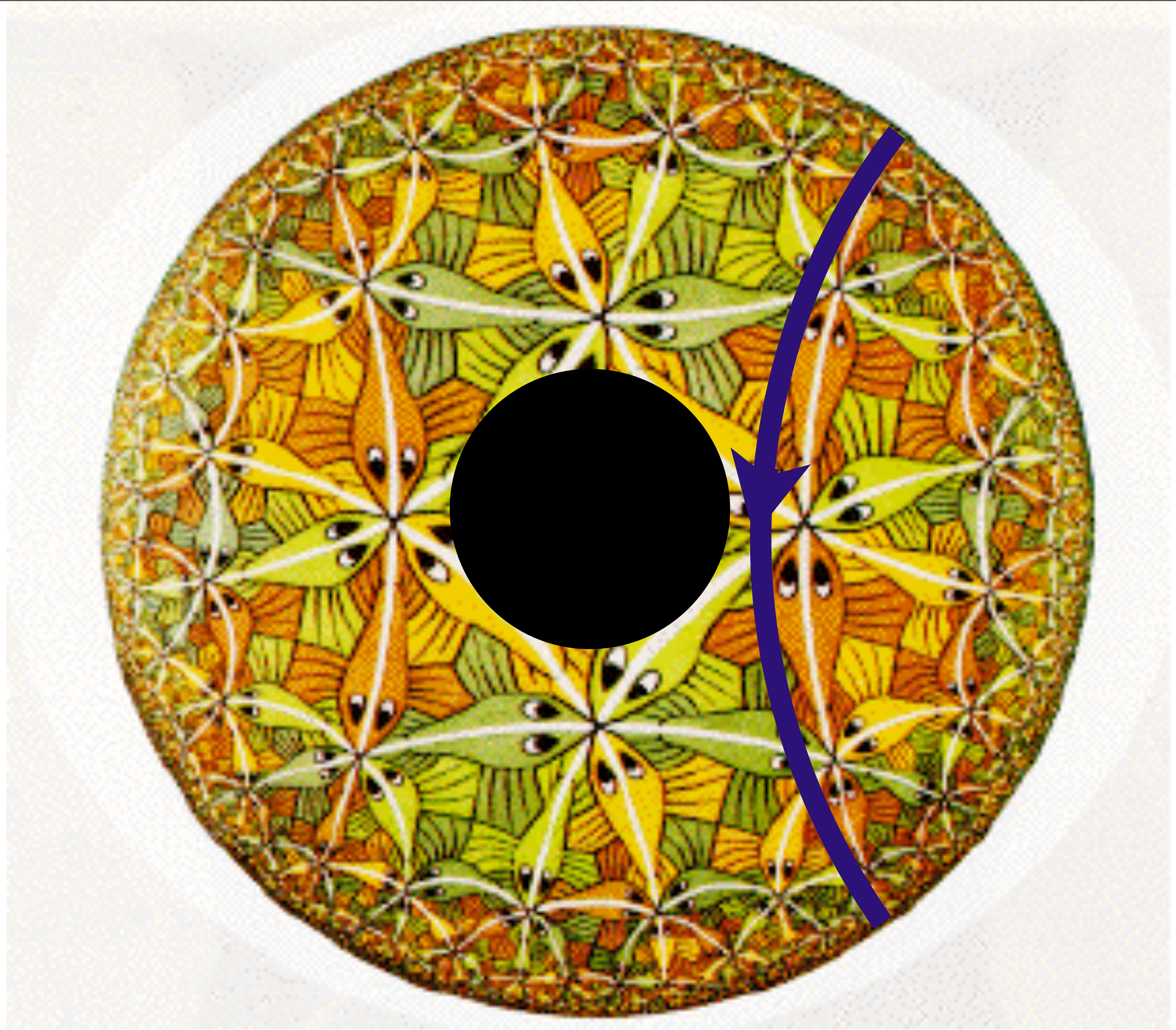
Conformal field theory in 2+1 dimensions at $T > 0$, with a non-zero chemical potential, μ and applied magnetic field, B

Einstein gravity on AdS$_4$ with a Reissner-Nordstrom black hole carrying electric and magnetic charges

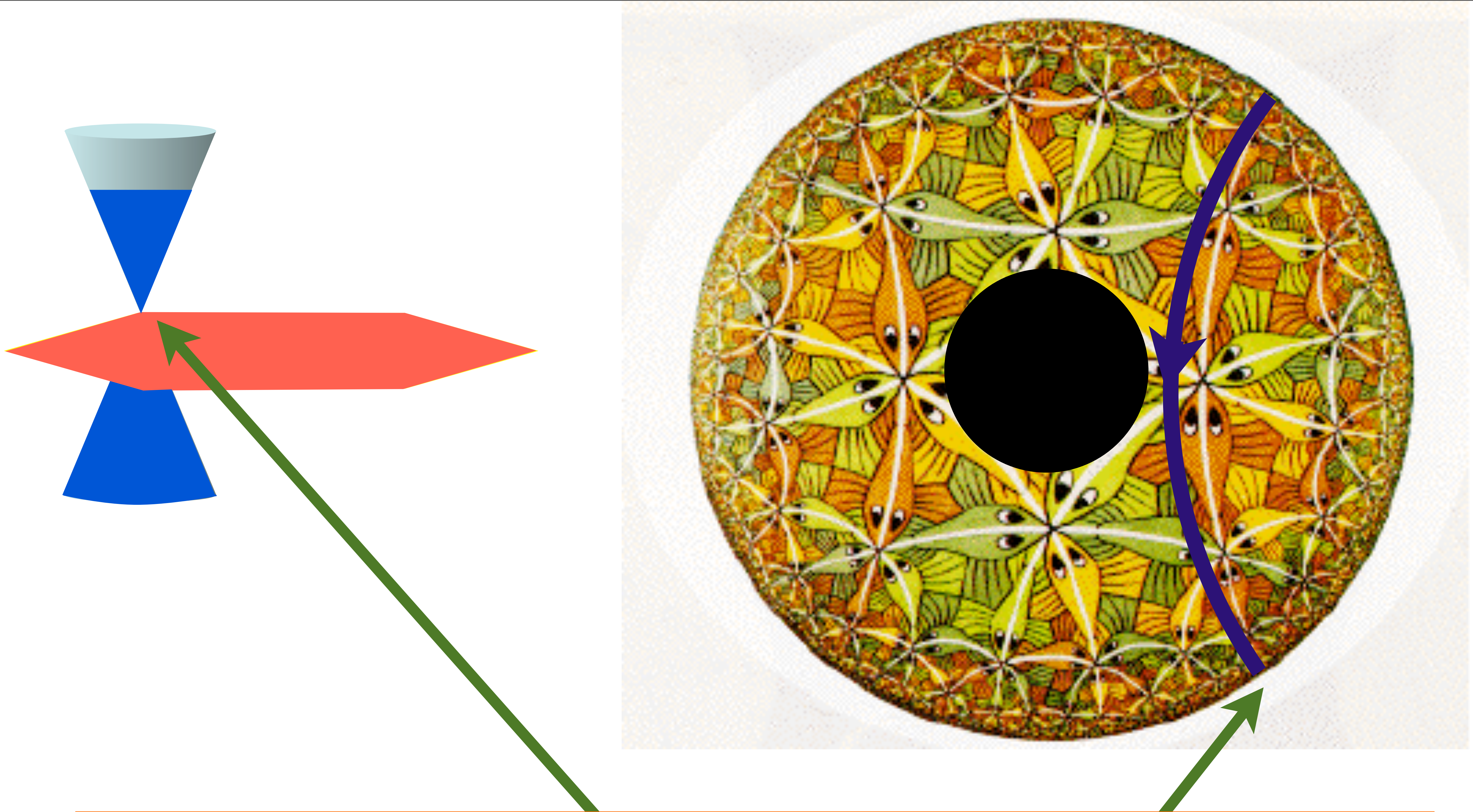




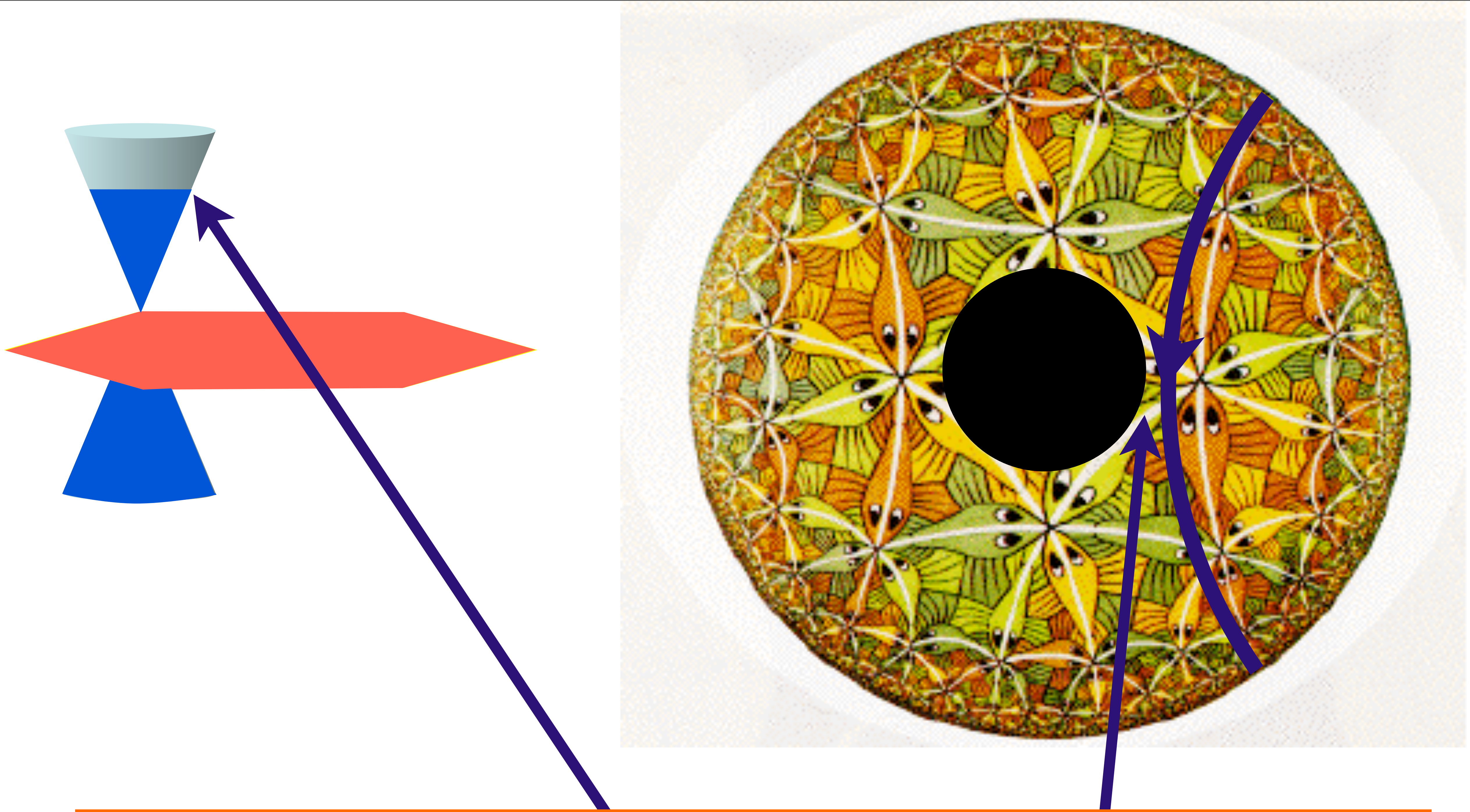




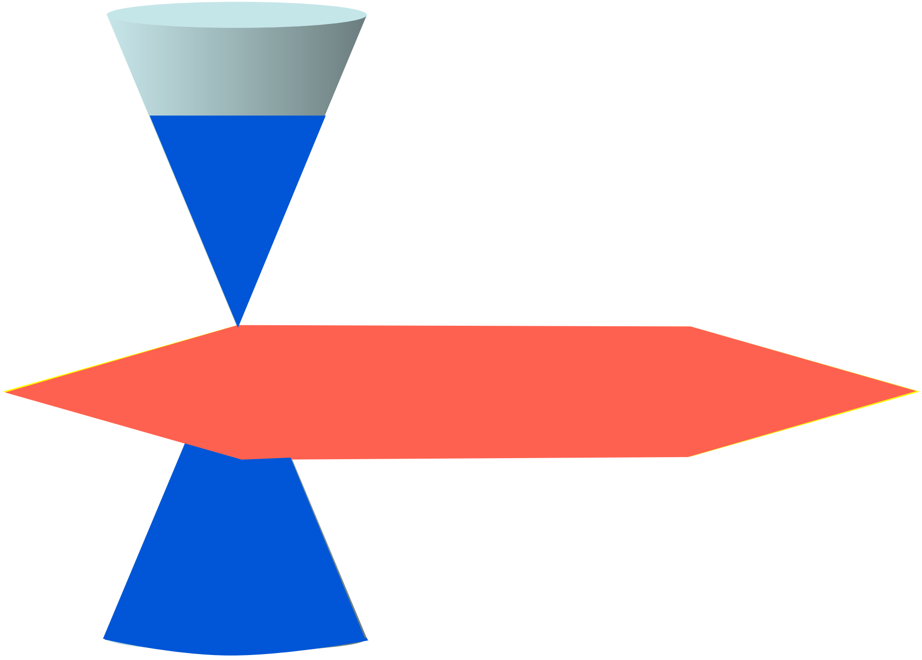
Examine free energy and Green's function
of a probe particle

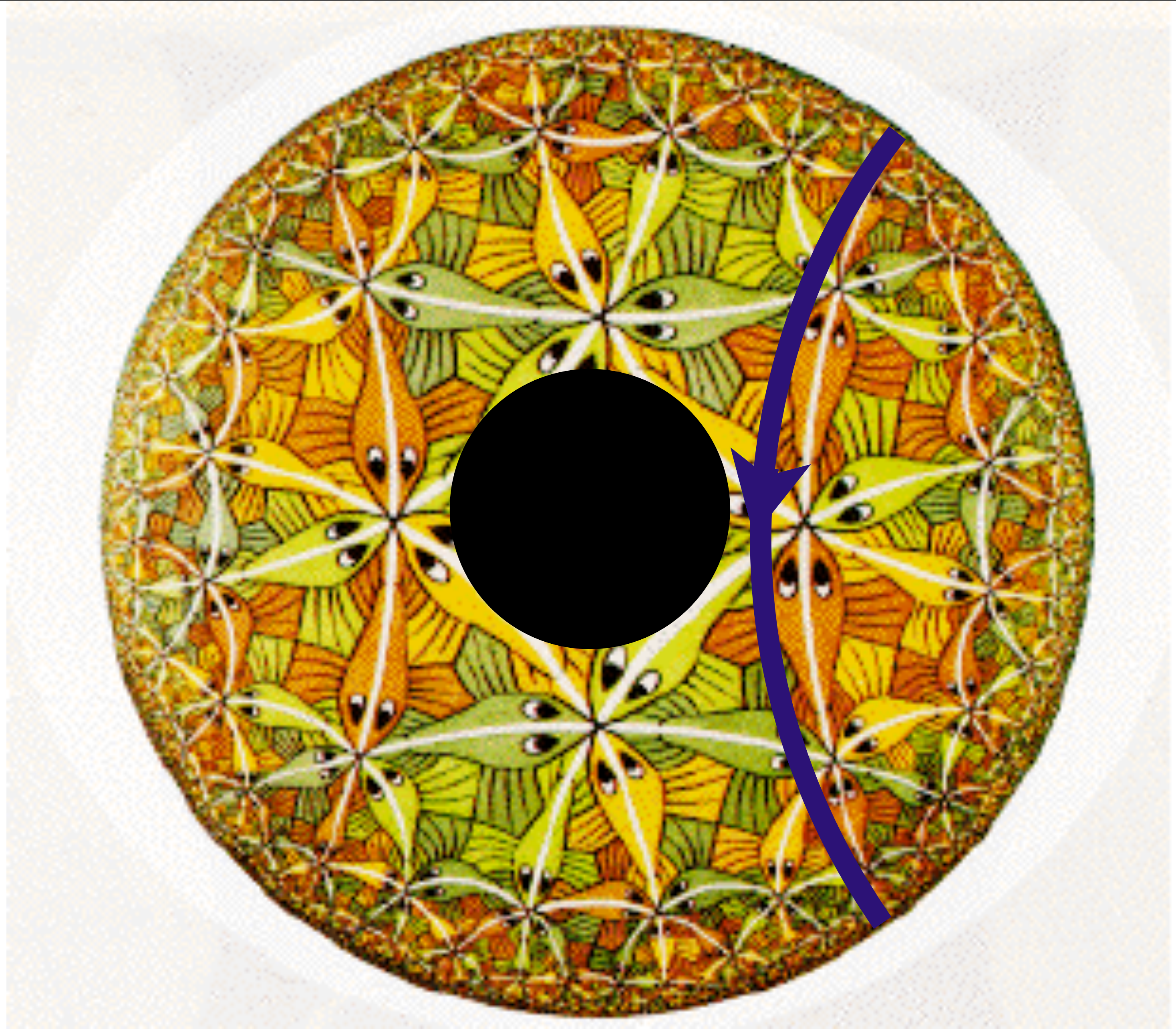


Short time behavior depends upon conformal AdS$_4$ geometry near boundary

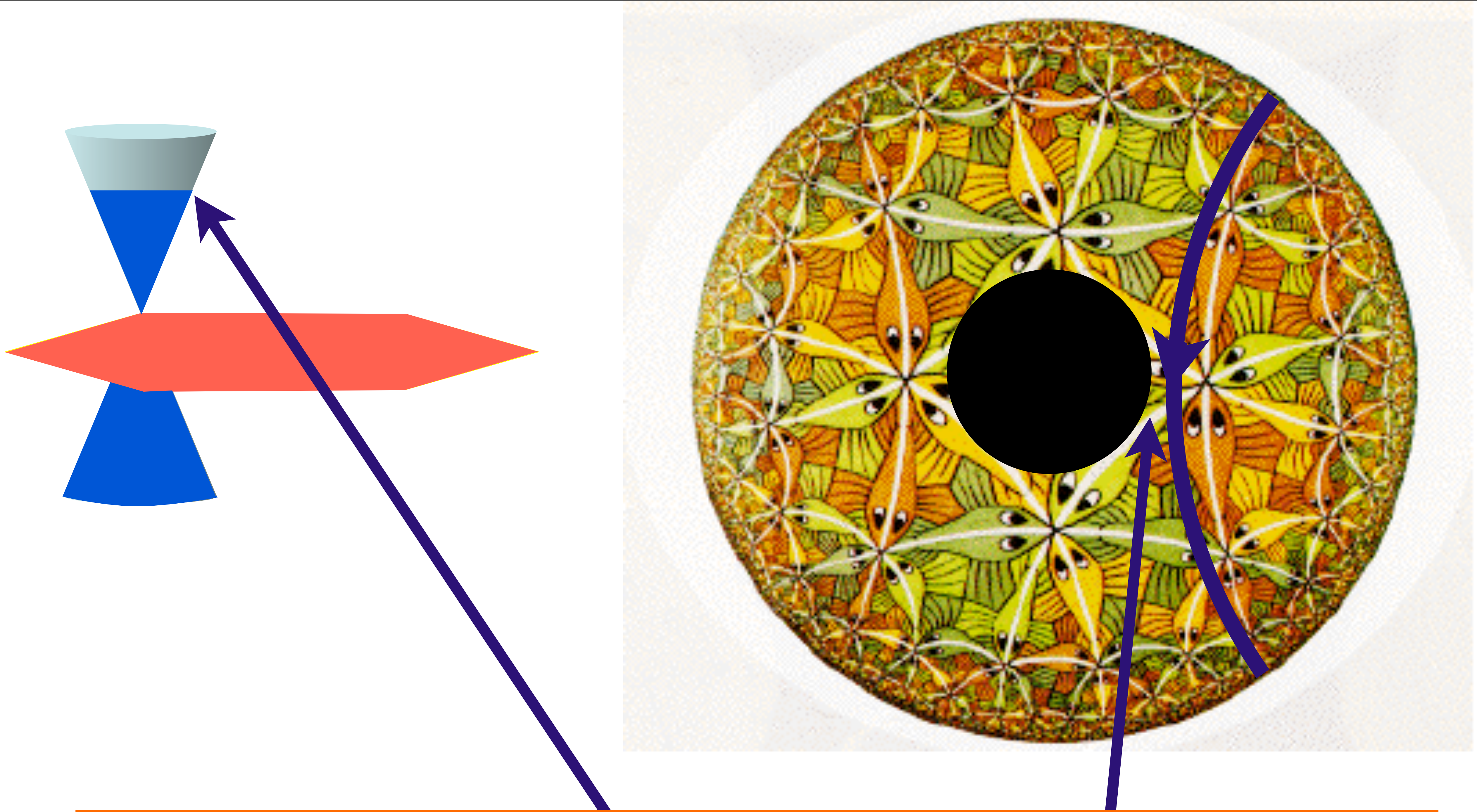


Long time behavior depends upon near-horizon geometry of black hole



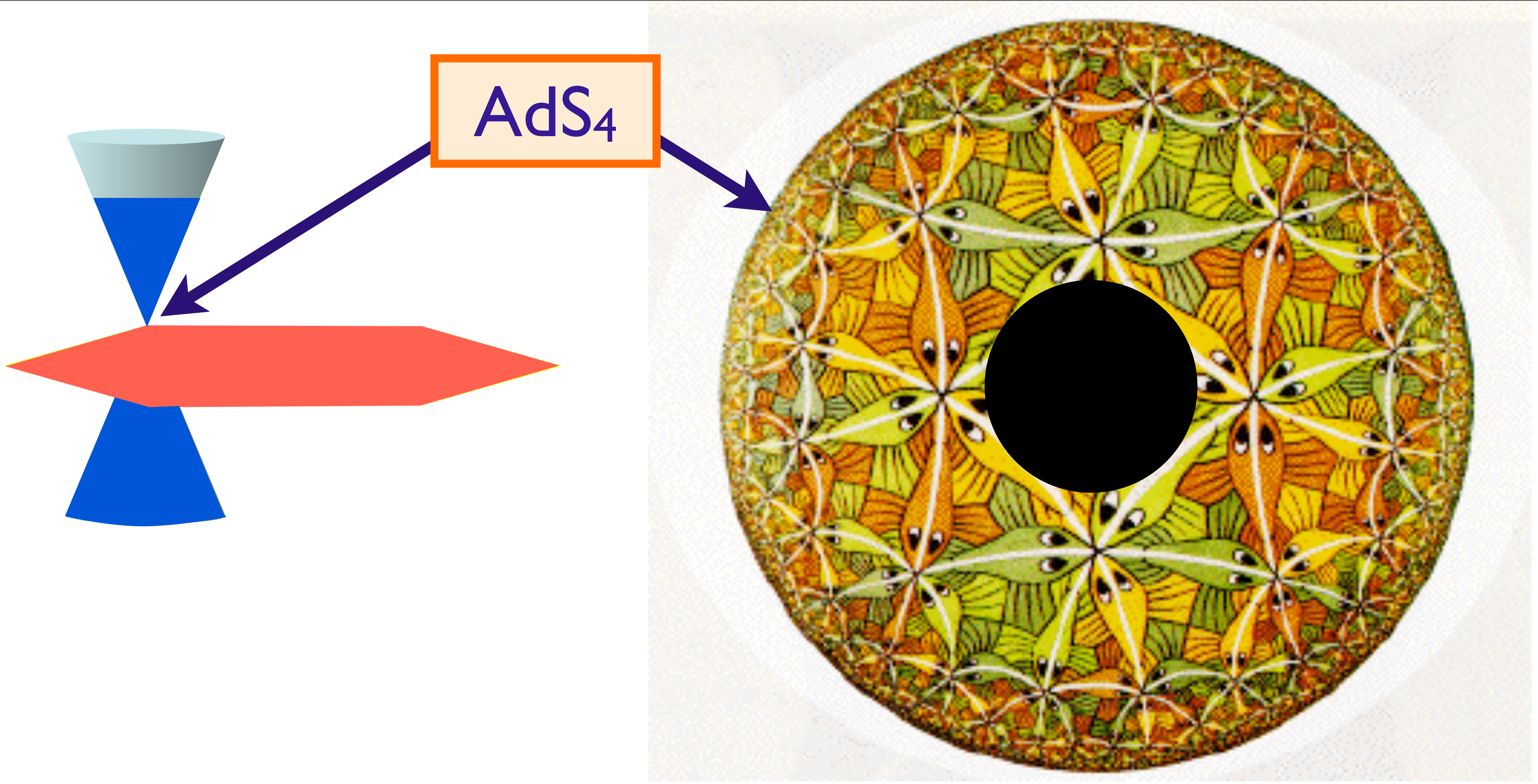


Radial direction of gravity theory is measure of energy scale in CFT



Infrared physics of Fermi surface is linked to the near horizon AdS$_2$ geometry of Reissner-Nordstrom black hole

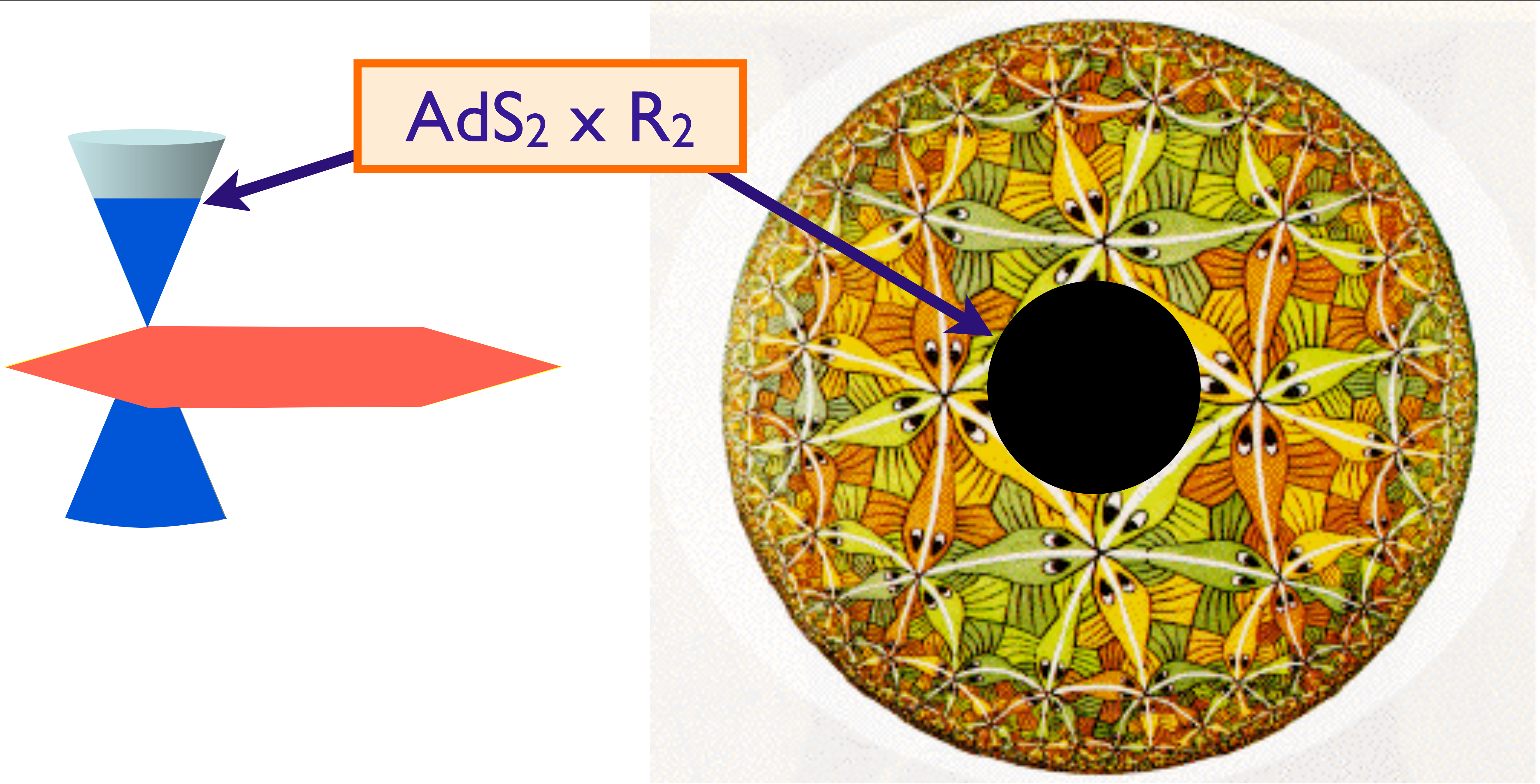
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694



AdS$_4$

Geometric interpretation of RG flow

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694



AdS$_2$ x R$_2$

Geometric interpretation of RG flow

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

The **same** results were later obtained from the equations of generalized relativistic magnetohydrodynamics.

So the results apply to experiments on graphene, *and* to the dynamics of black holes.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

As a simple example, in zero magnetic field, we can write the electrical conductivity as

$$\sigma = \sigma_Q + \frac{e^{*2}\rho^2 v^2}{\varepsilon + P} \pi \delta(\omega)$$

where σ_Q is the universal conductivity of the CFT, ρ is the charge density, ε is the energy density and P is the pressure.

The same quantities also determine the thermal conductivity, κ:

$$\kappa = \sigma_Q \left(\frac{k_B^2 T}{e^{*2}} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right)^2$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

A second example: In an applied magnetic field B, the dynamic transport co-efficients exhibit a **hydrodynamic cyclotron resonance** at a frequency ω_c

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

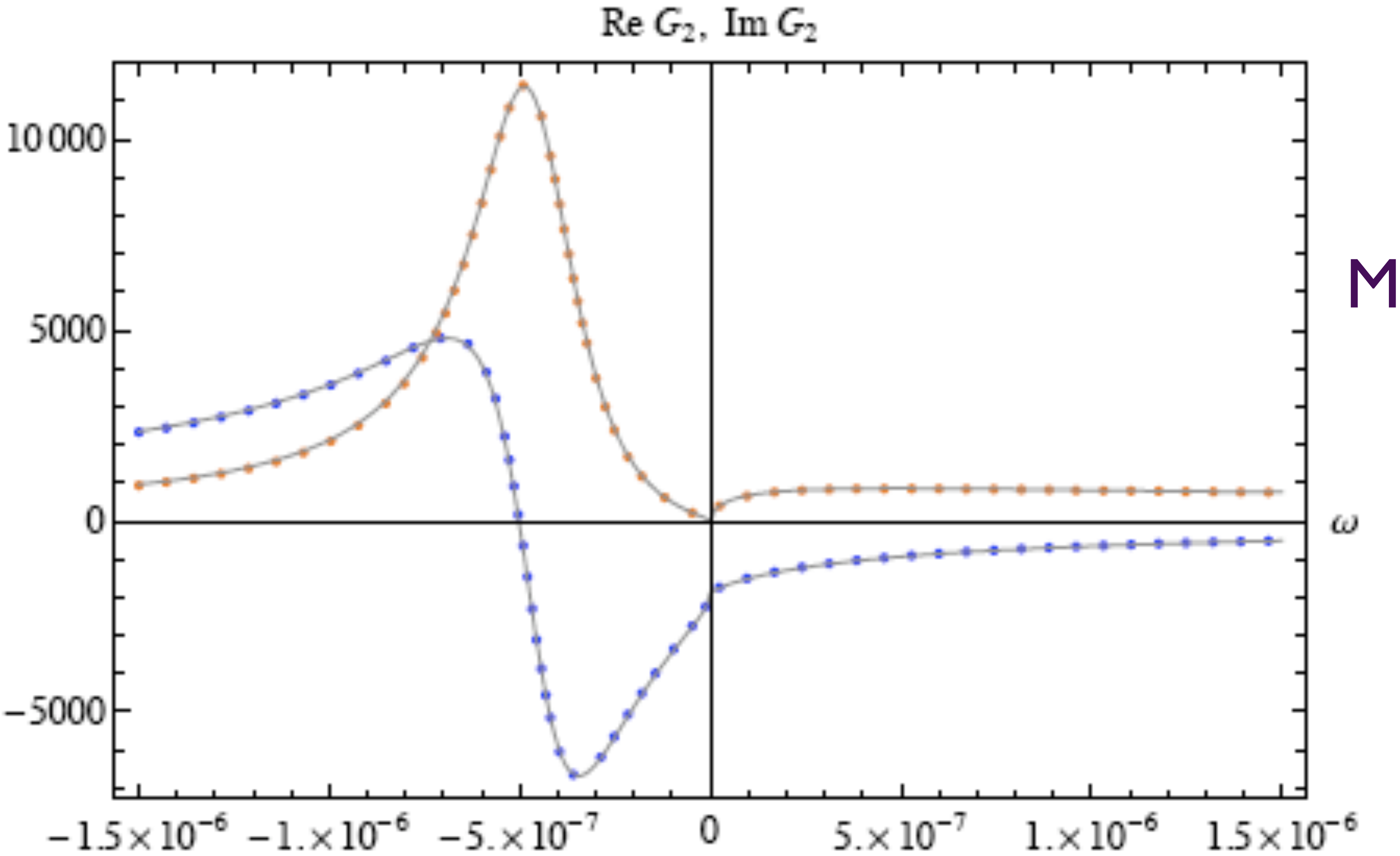
and damping constant γ

$$\gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}.$$

The same constants determine the **quasinormal frequency** of the Reissner-Nordstrom black hole.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Green's function of a fermion

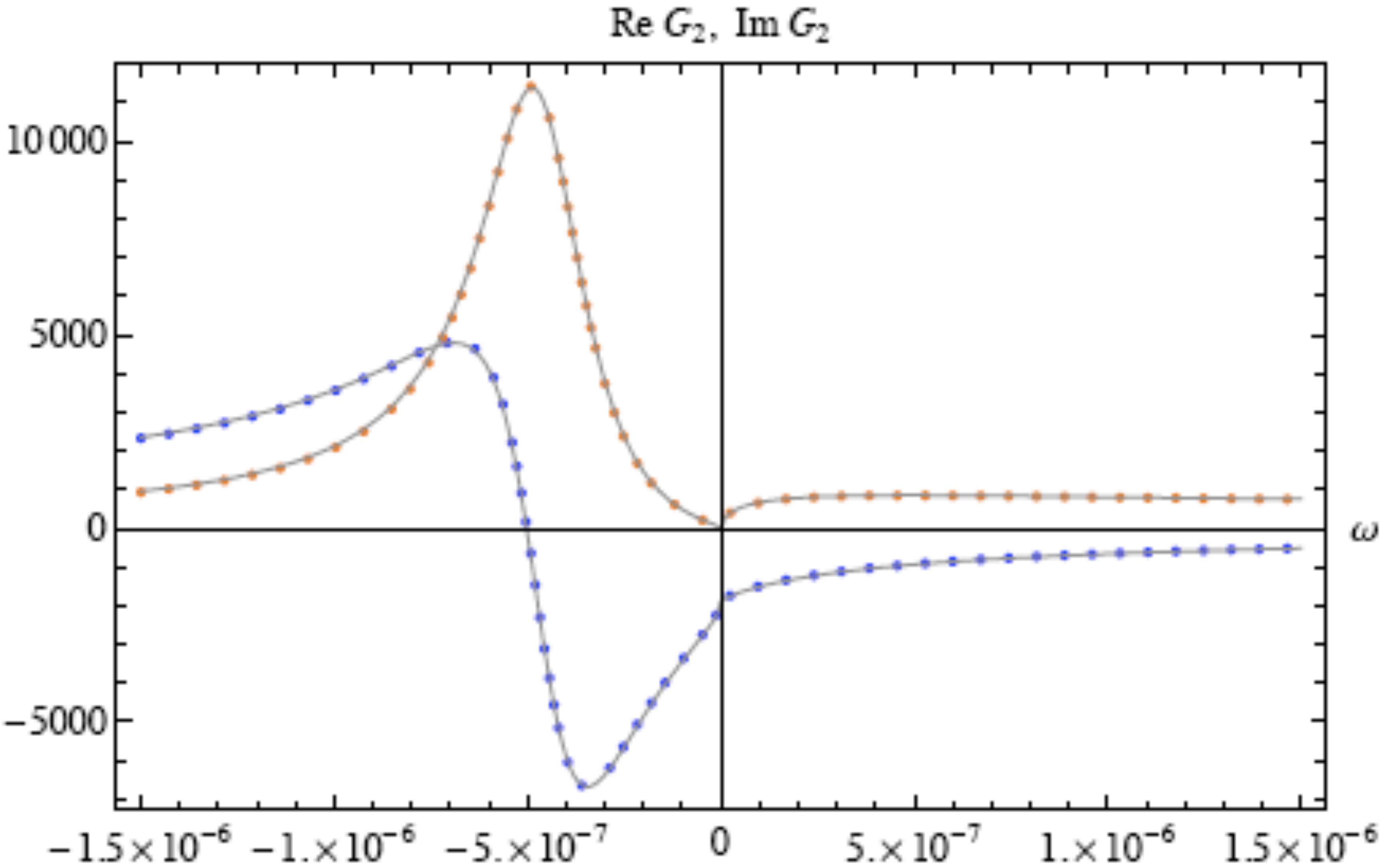


Sung-Sik Lee,
arXiv:0809.3402;
M. Cubrovic, J. Zaanen,
and K. Schalm,
arXiv:0904.1993

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

T. Faulkner, H. Liu, J. McGreevy, and D.Vegh, arXiv:0907.2694

Green's function of a fermion



T. Faulkner, H. Liu,
J. McGreevy, and
D. Vegh,
arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

Similar to non-Fermi liquid theories of Fermi surfaces coupled to gauge fields, and at quantum critical points

Free energy from gravity theory

The free energy is expressed as a sum over the "quasinormal frequencies", z_ℓ, of the black hole. Here ℓ represents any set of quantum numbers:

$$\begin{aligned} \mathcal{F}_{\text{boson}} &= -T\sum_\ell \ln\left(\frac{|z_\ell|}{2\pi T}\left|\Gamma\left(\frac{iz_\ell}{2\pi T}\right)\right|^2\right) \\ \mathcal{F}_{\text{fermion}} &= T\sum_\ell \ln\left(\left|\Gamma\left(\frac{iz_\ell}{2\pi T}+\frac{1}{2}\right)\right|^2\right) \end{aligned}$$

Application of this formula shows that the fermions exhibit the dHvA quantum oscillations with expected period (2π/(Fermi surface ares)) in $1/B$, but with an amplitude corrected from the Fermi liquid formula of Lifshitz-Kosevich.

F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

Outline

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2. Graphene

   `Topological' Fermi surface transitions

3. Quantum criticality and black holes

   AdS_4 theory of compressible quantum liquids

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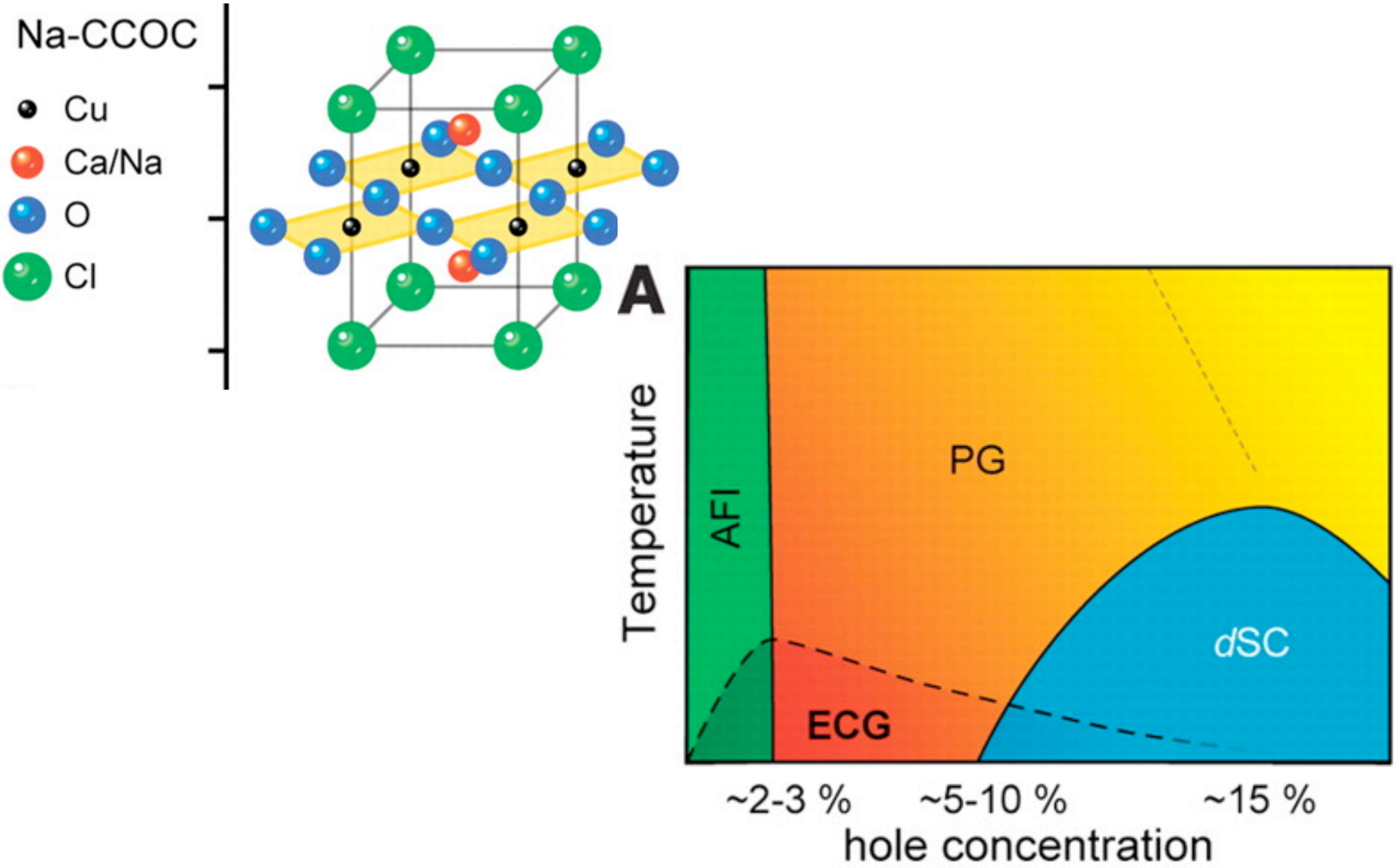
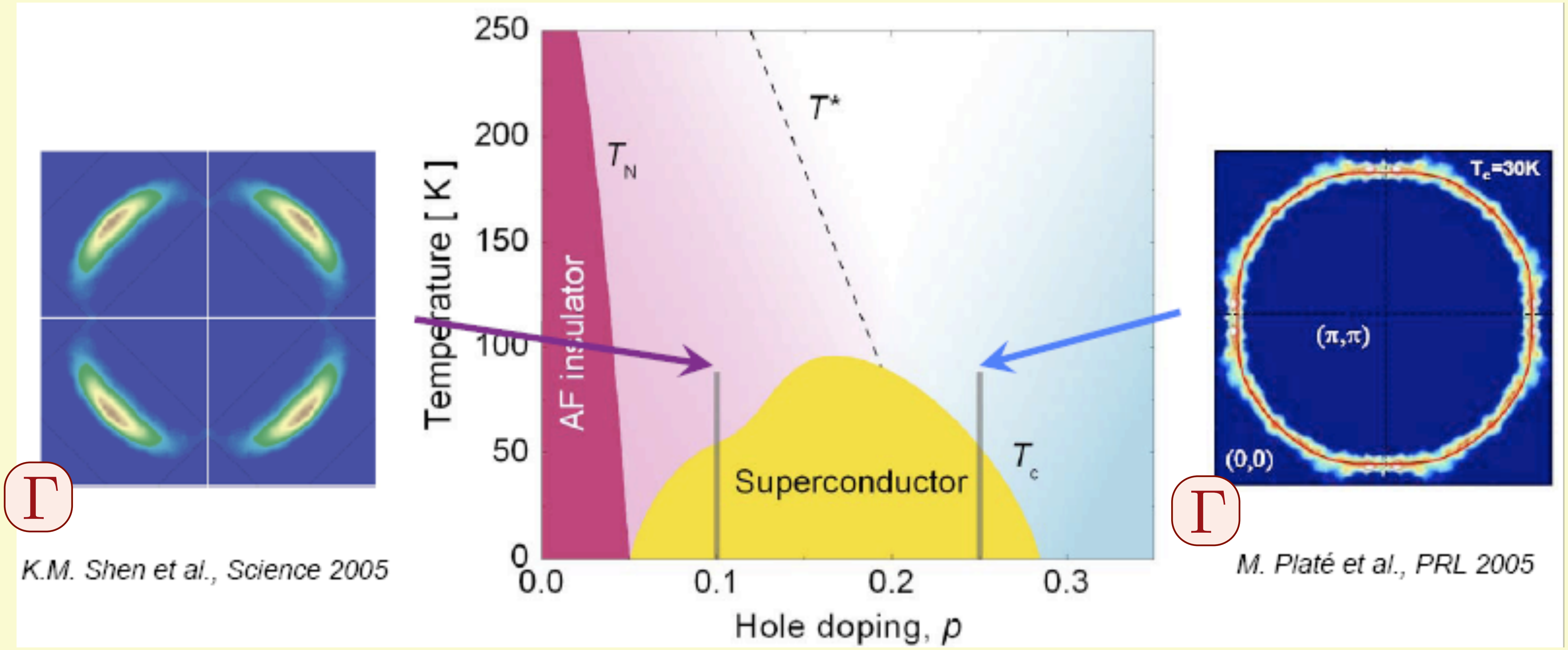
The cuprate superconductors



Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005

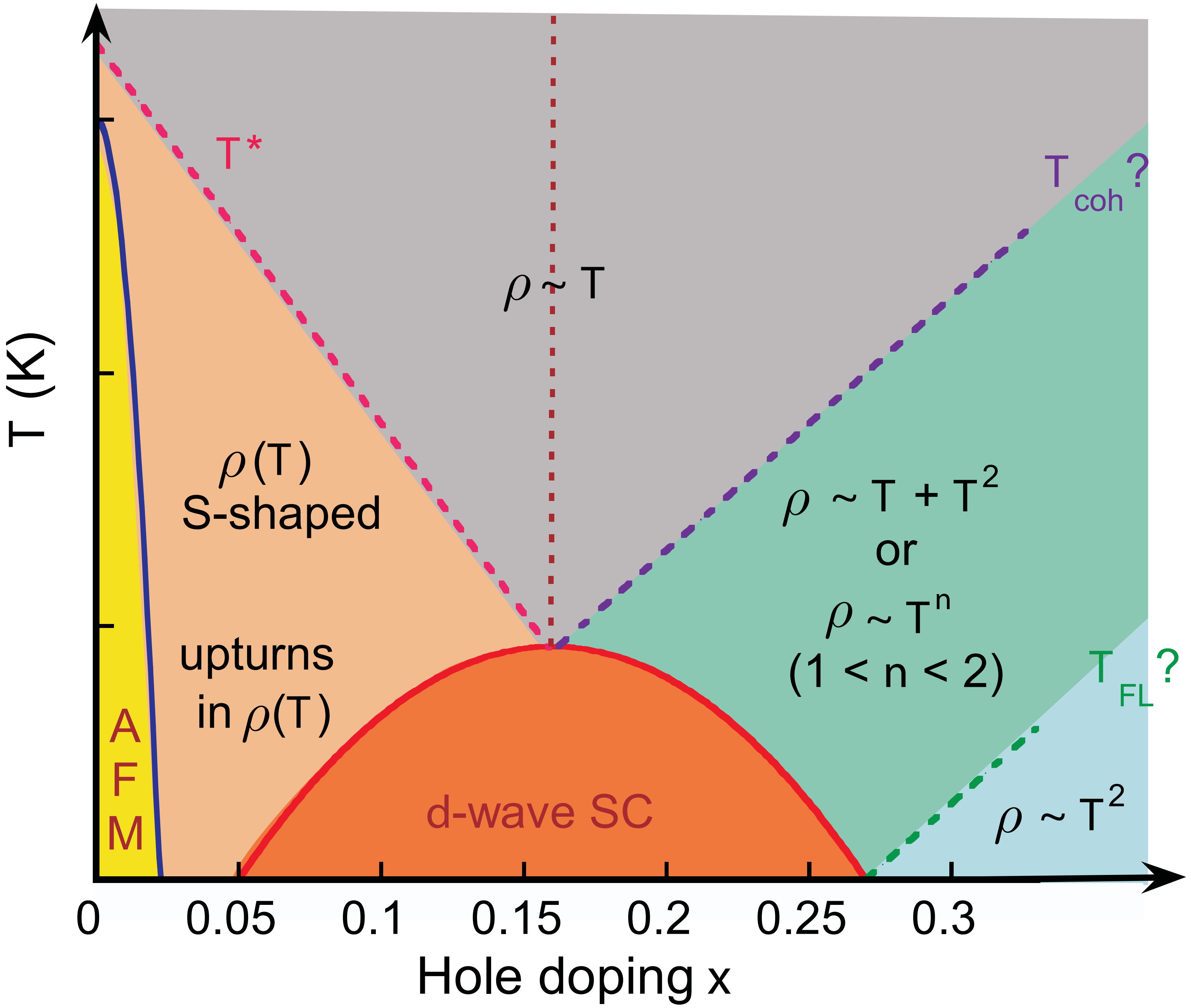
Smaller hole
Fermi-pockets

Large hole
Fermi surface

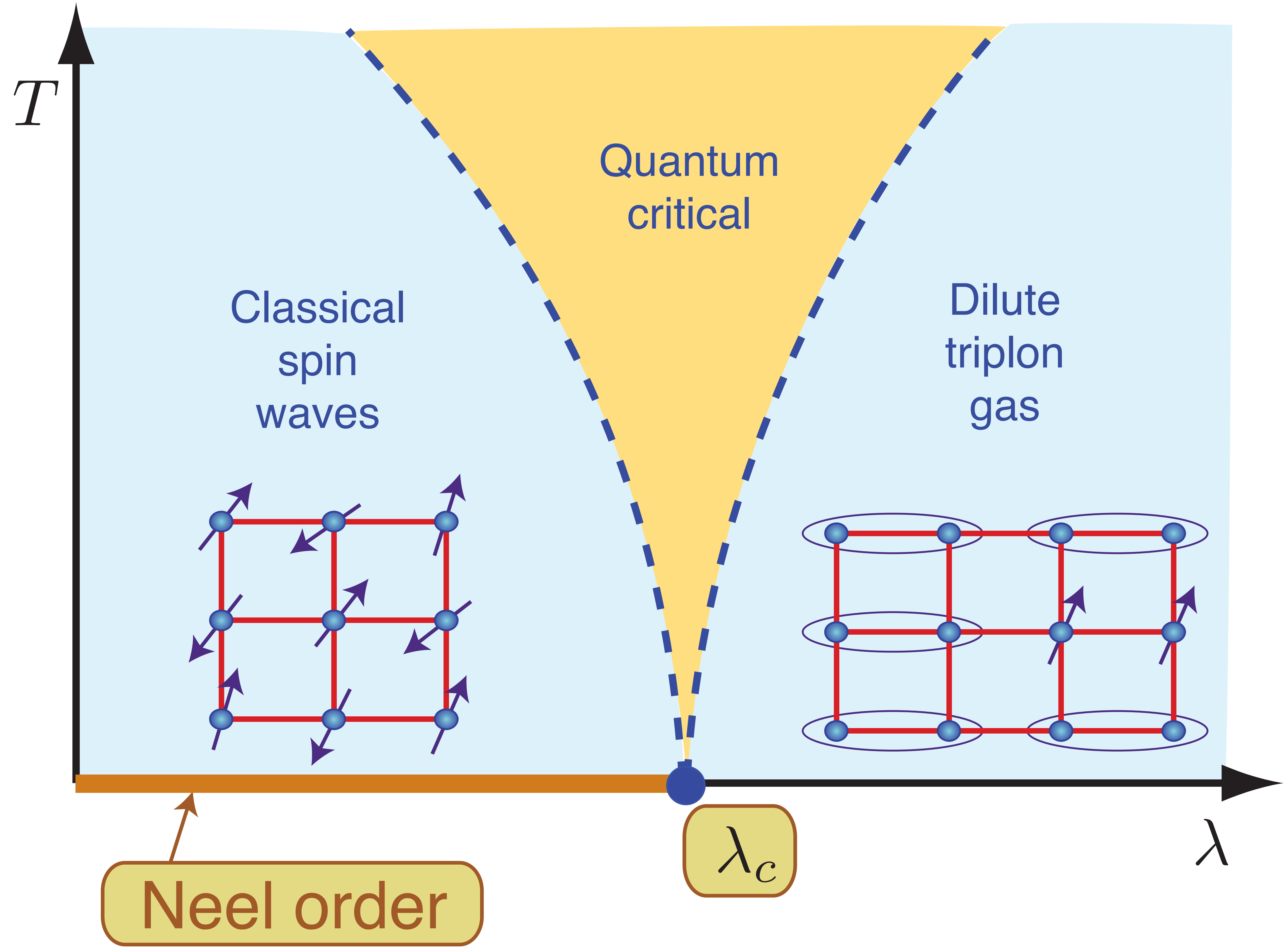
The cuprate superconductors

Multiple quantum phase transitions involving at least two order parameters (antiferromagnetism and superconductivity) and a topological change in the Fermi surface

Crossovers in transport properties of hole-doped cuprates

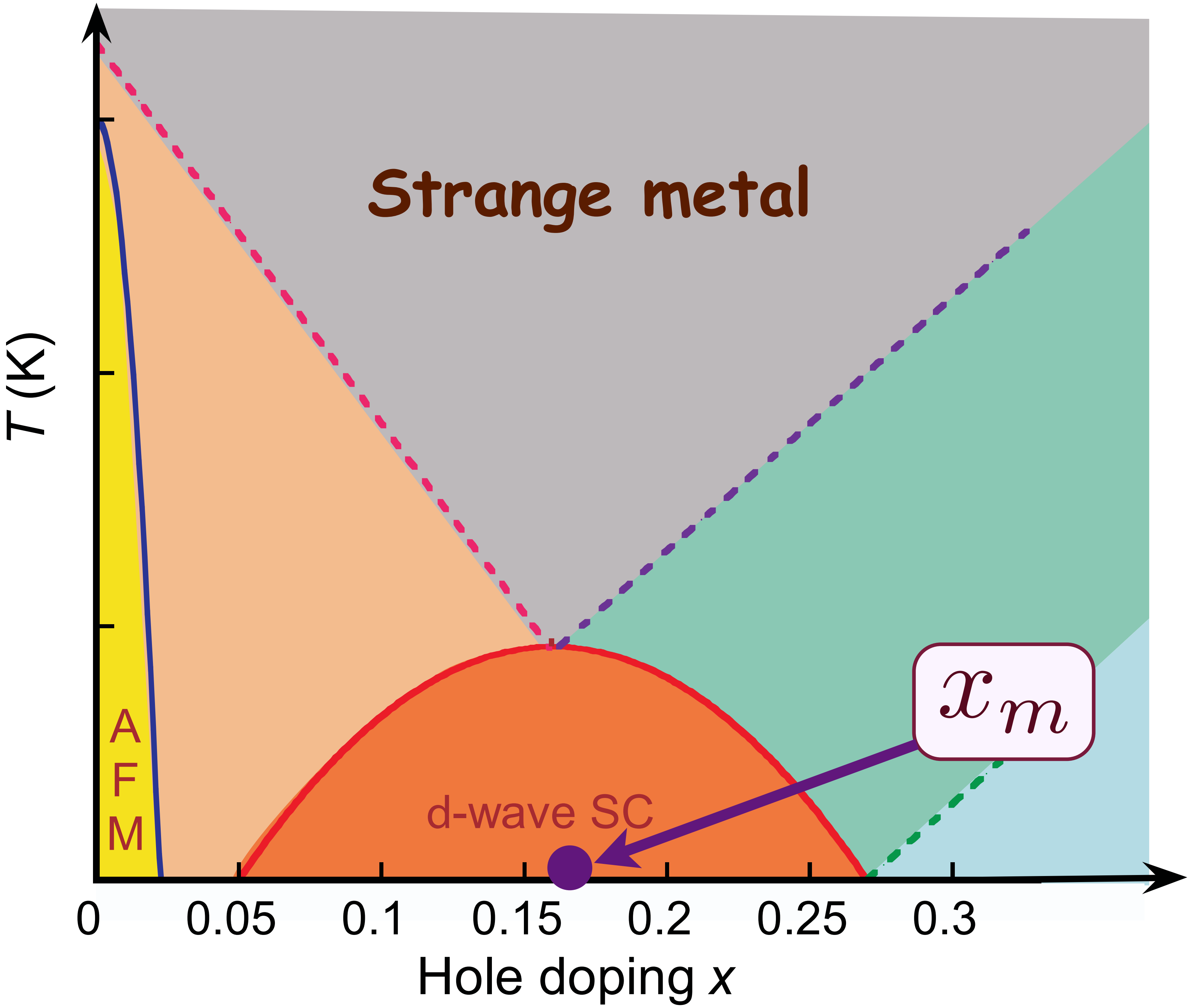


N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)



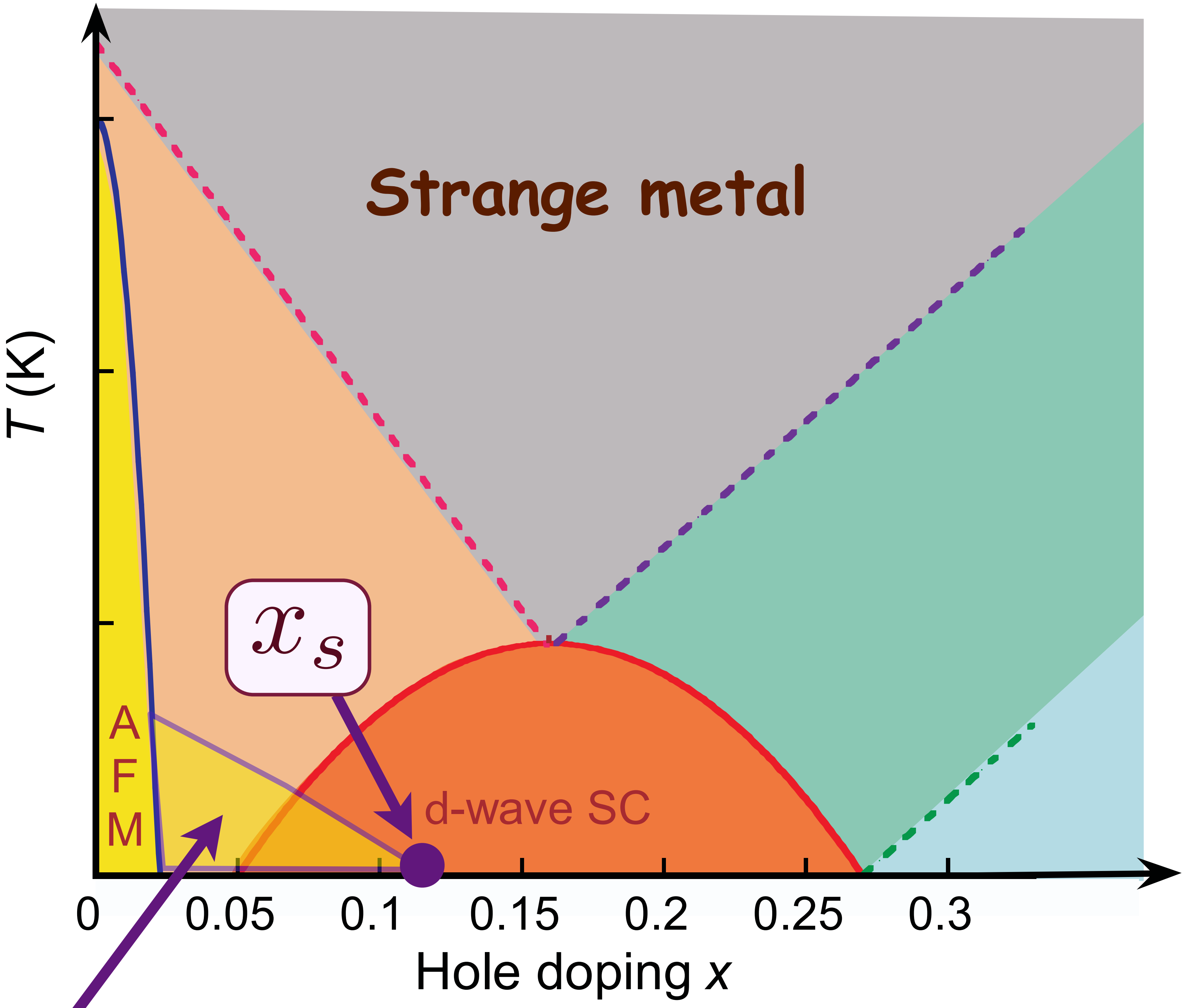
T
Quantum critical
Classical spin waves
Dilute triplon gas
λ
λ_c
Neel order

Crossovers in transport properties of hole-doped cuprates



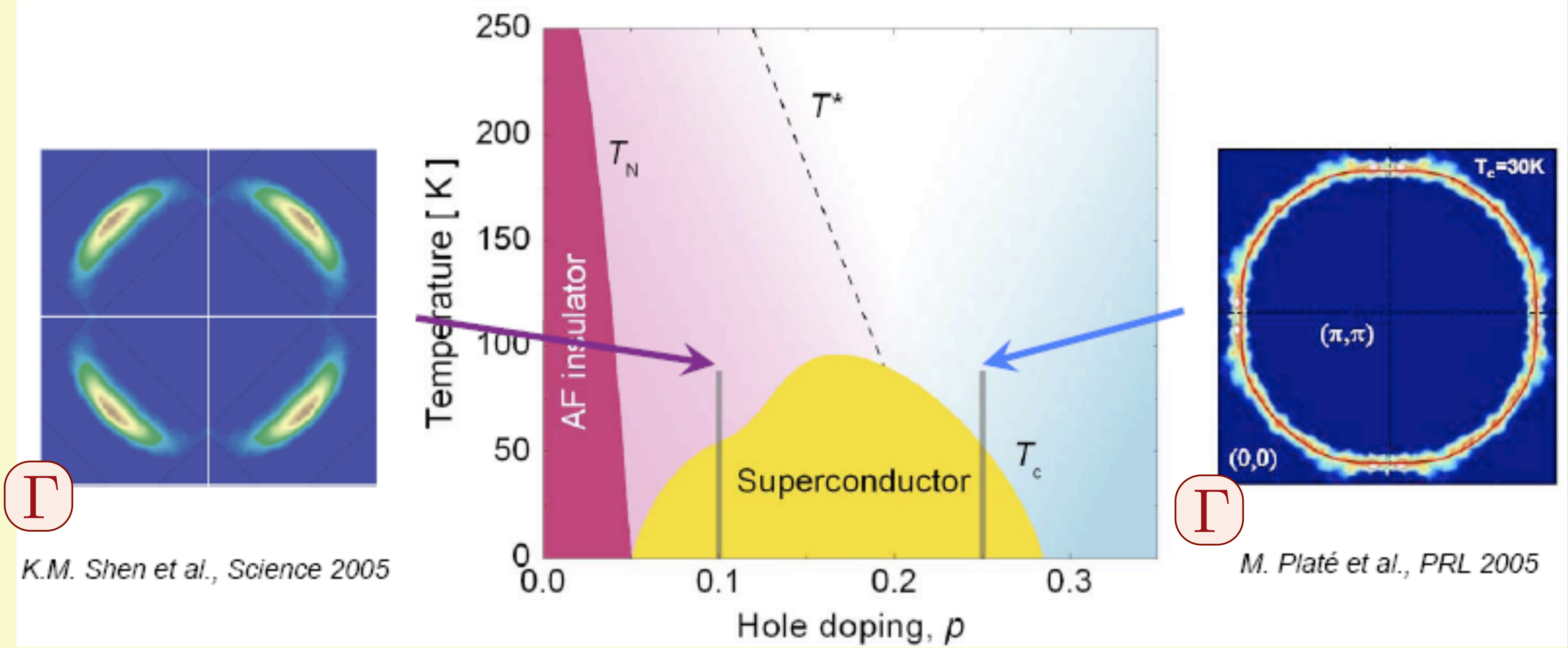
Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low *T*



Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



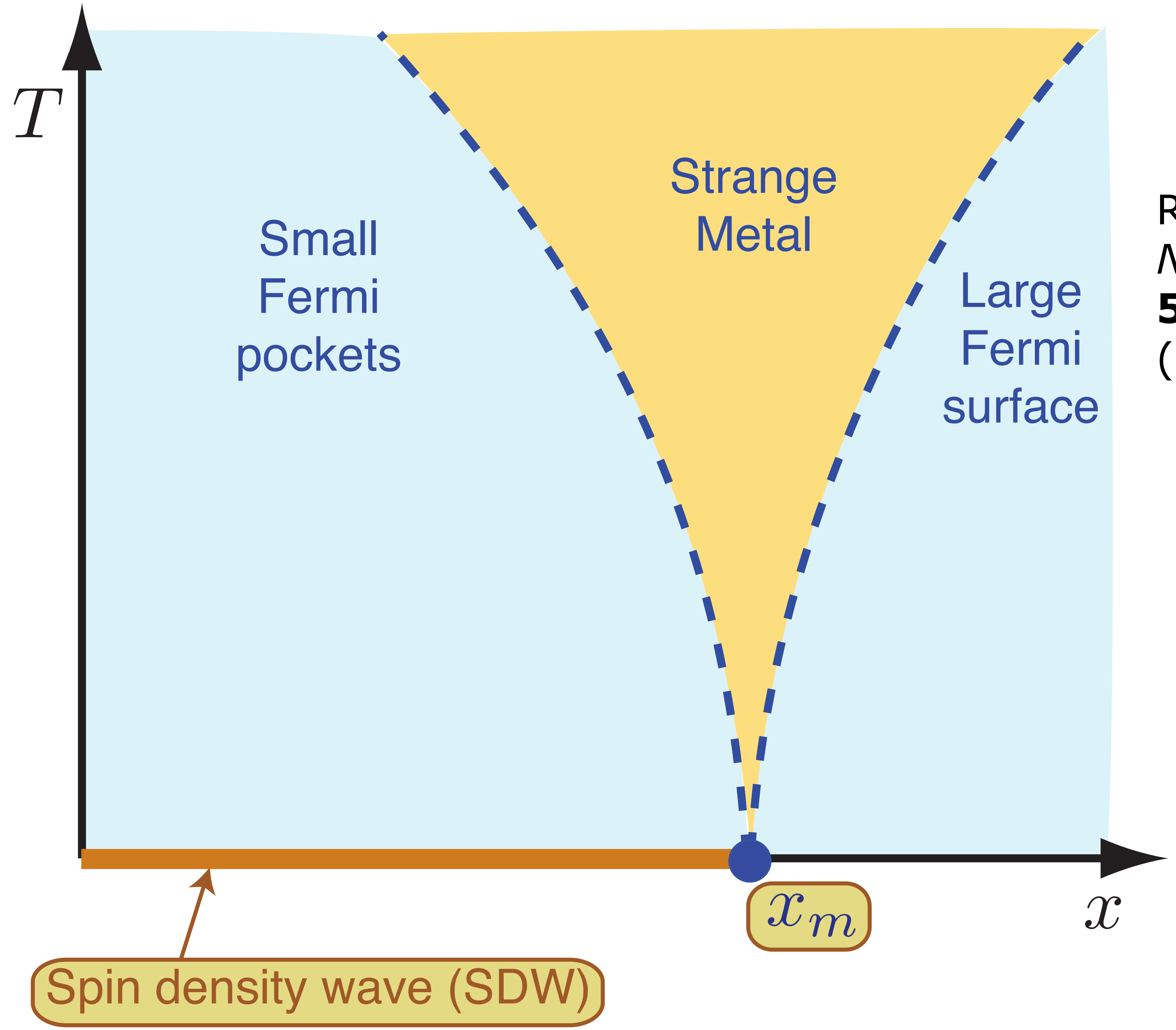
K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005

Smaller hole Fermi-pockets

Large hole Fermi surface

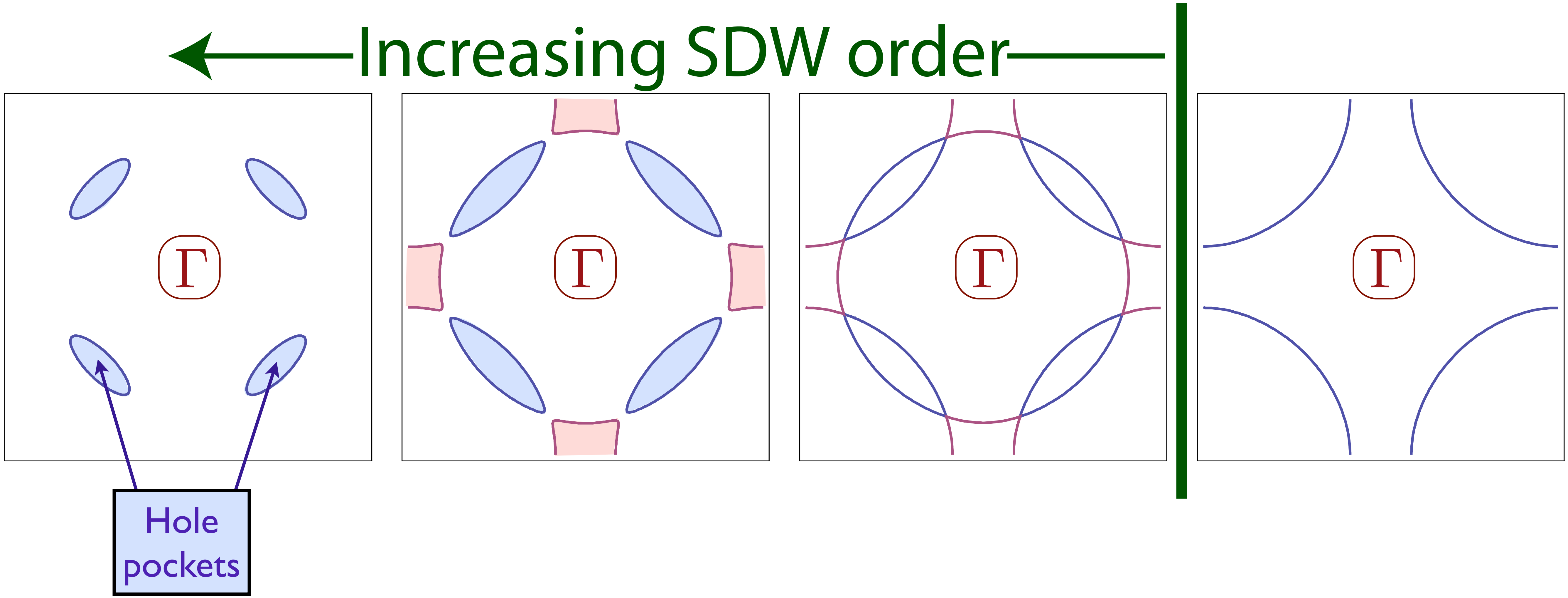
Theory of quantum criticality in the cuprates



R. Daou et al., *Nature Physics* **5**, 31 - 34 (2009)

Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Spin density wave theory in hole-doped cuprates



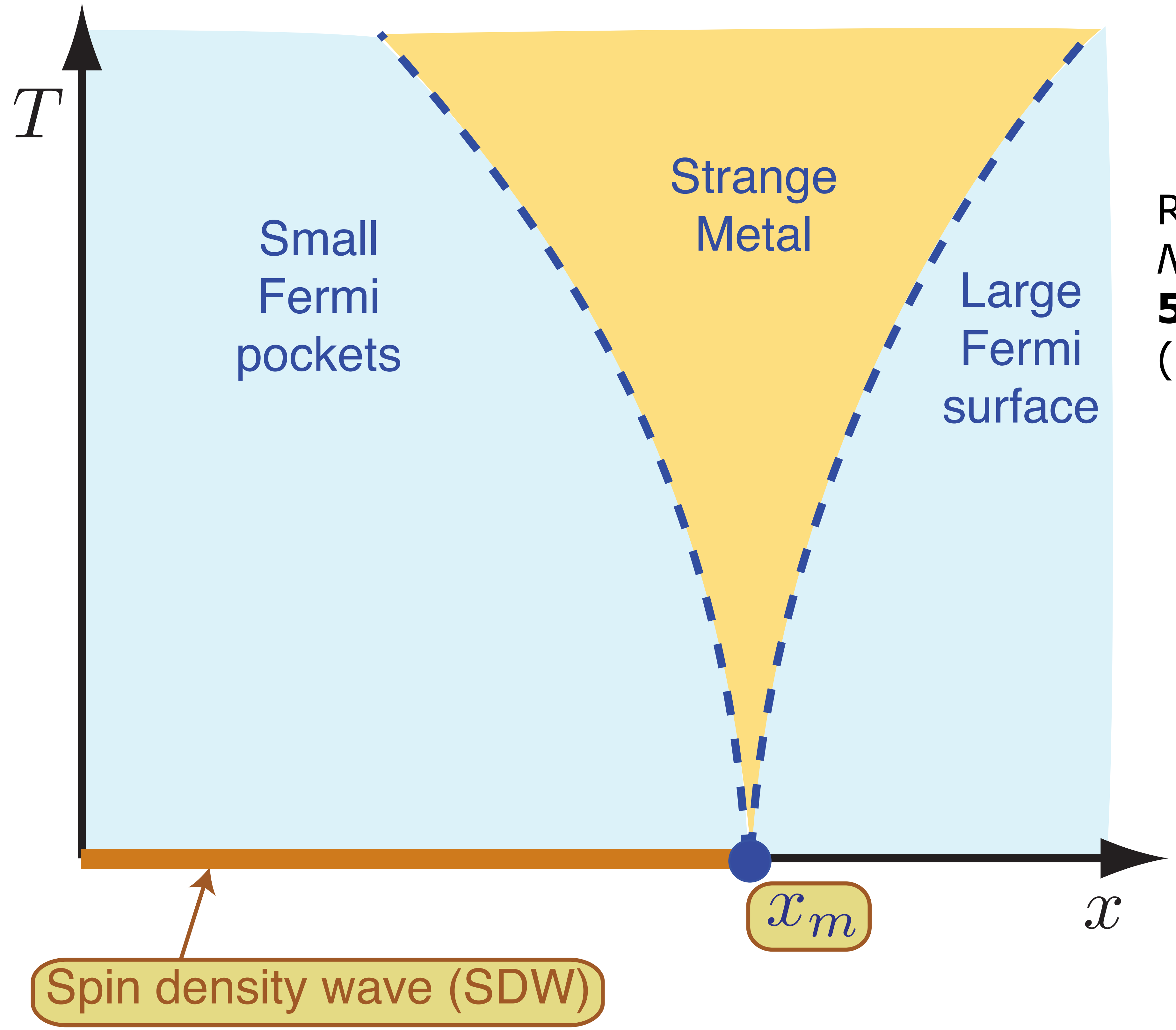
Increasing SDW order

Hole pockets

Quantum phase transition involves *both*
a SDW order parameter $\vec{\varphi}$,
and a topological change in the Fermi surface

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Theory of quantum criticality in the cuprates



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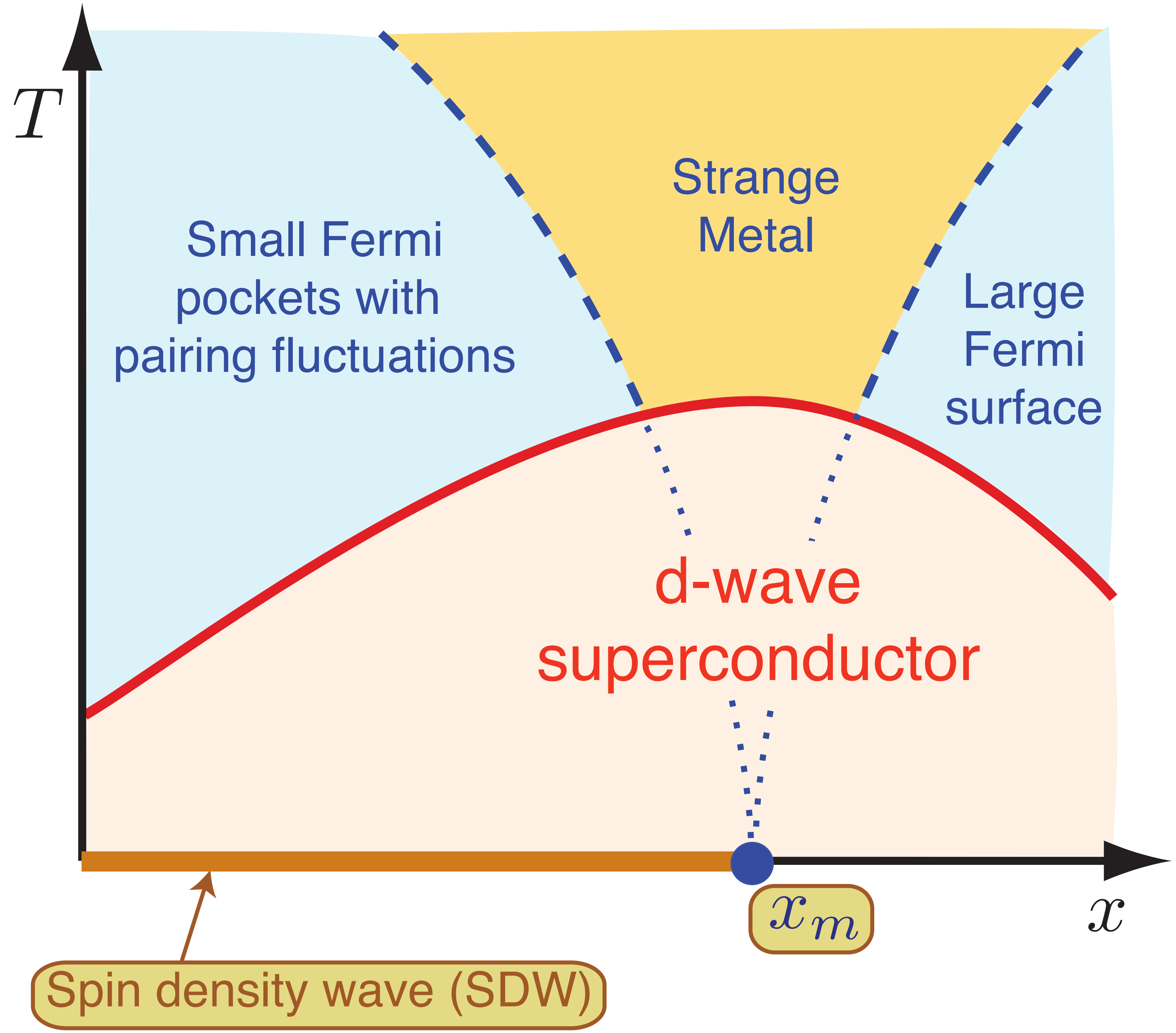
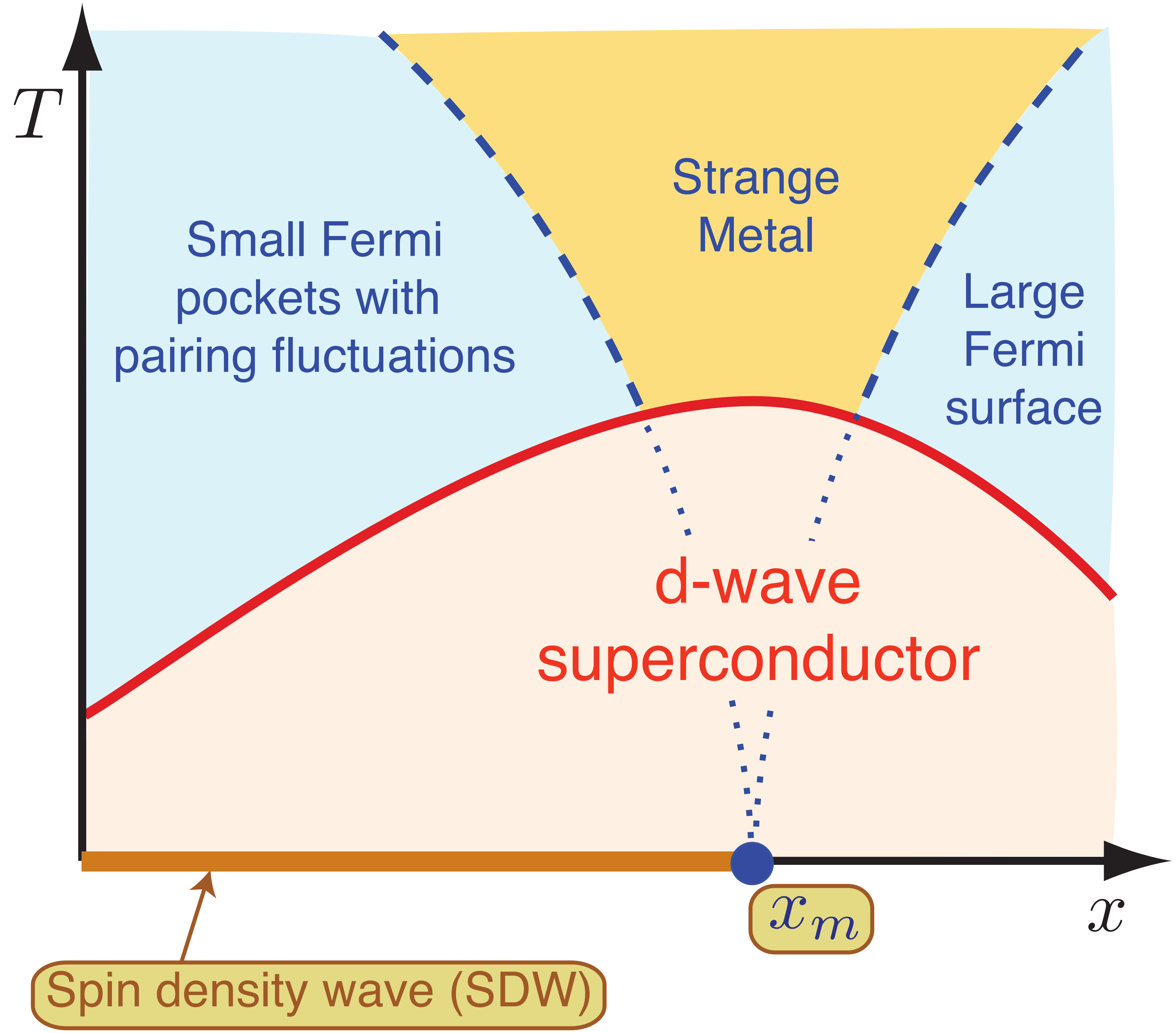
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



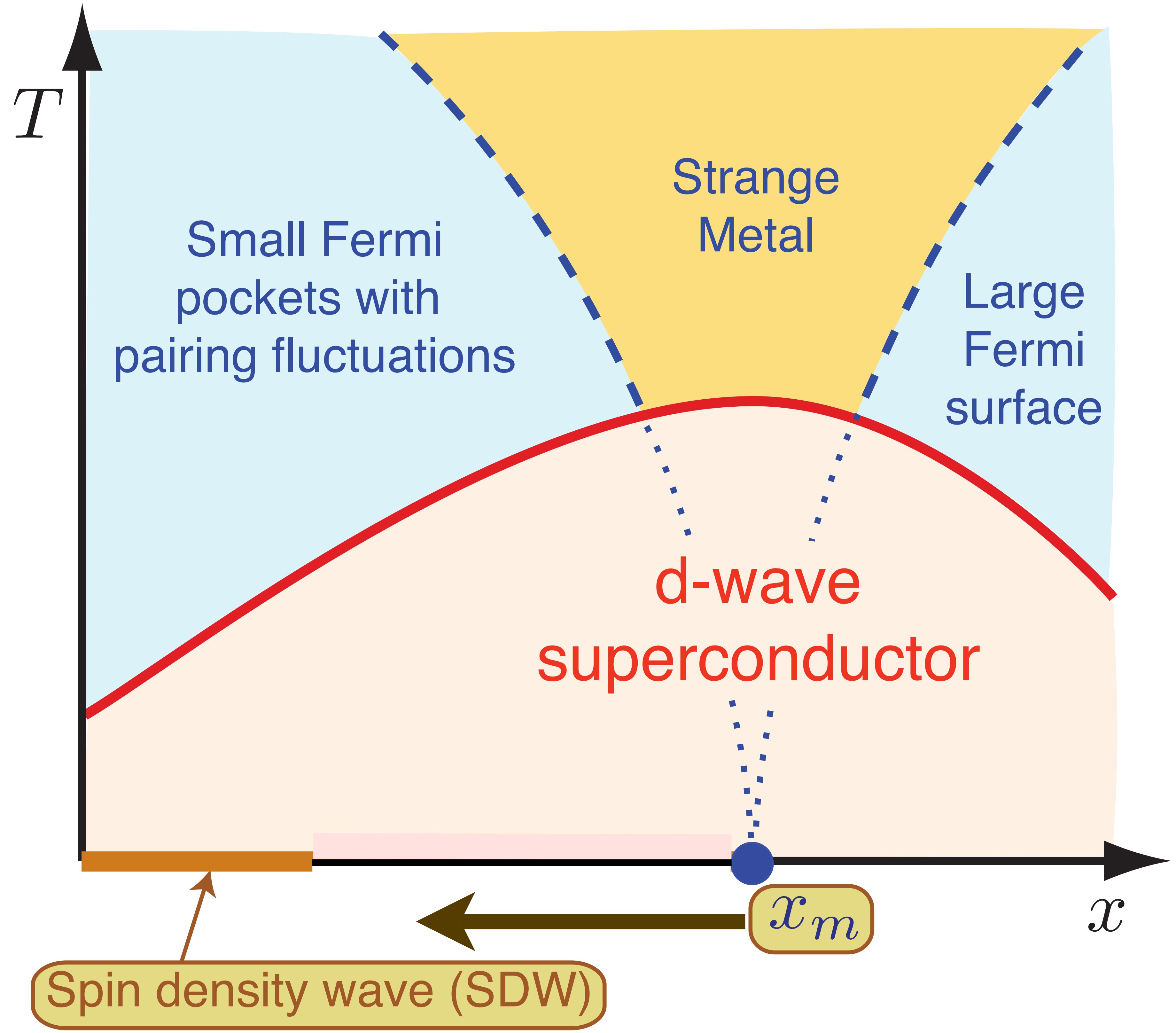
Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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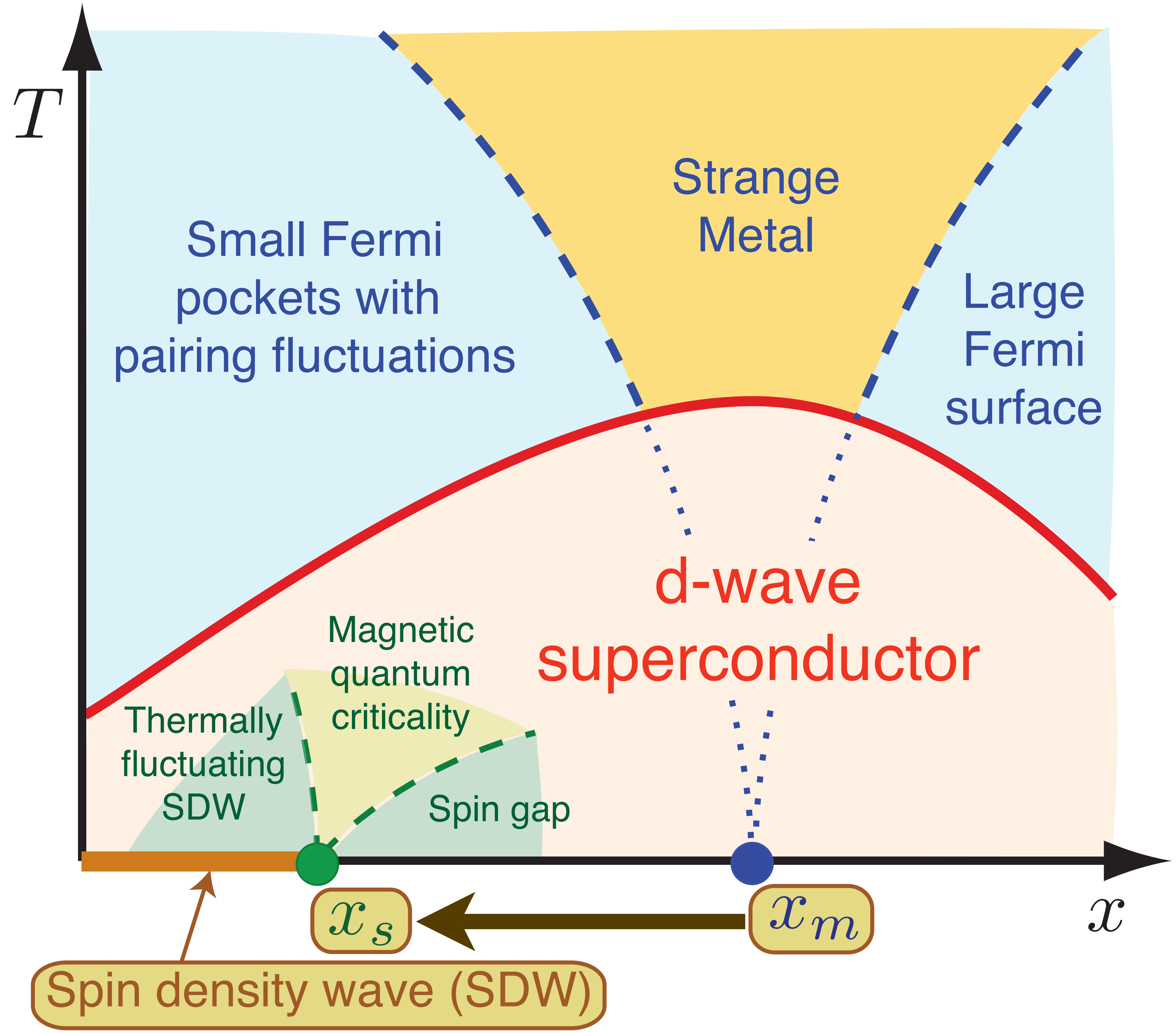
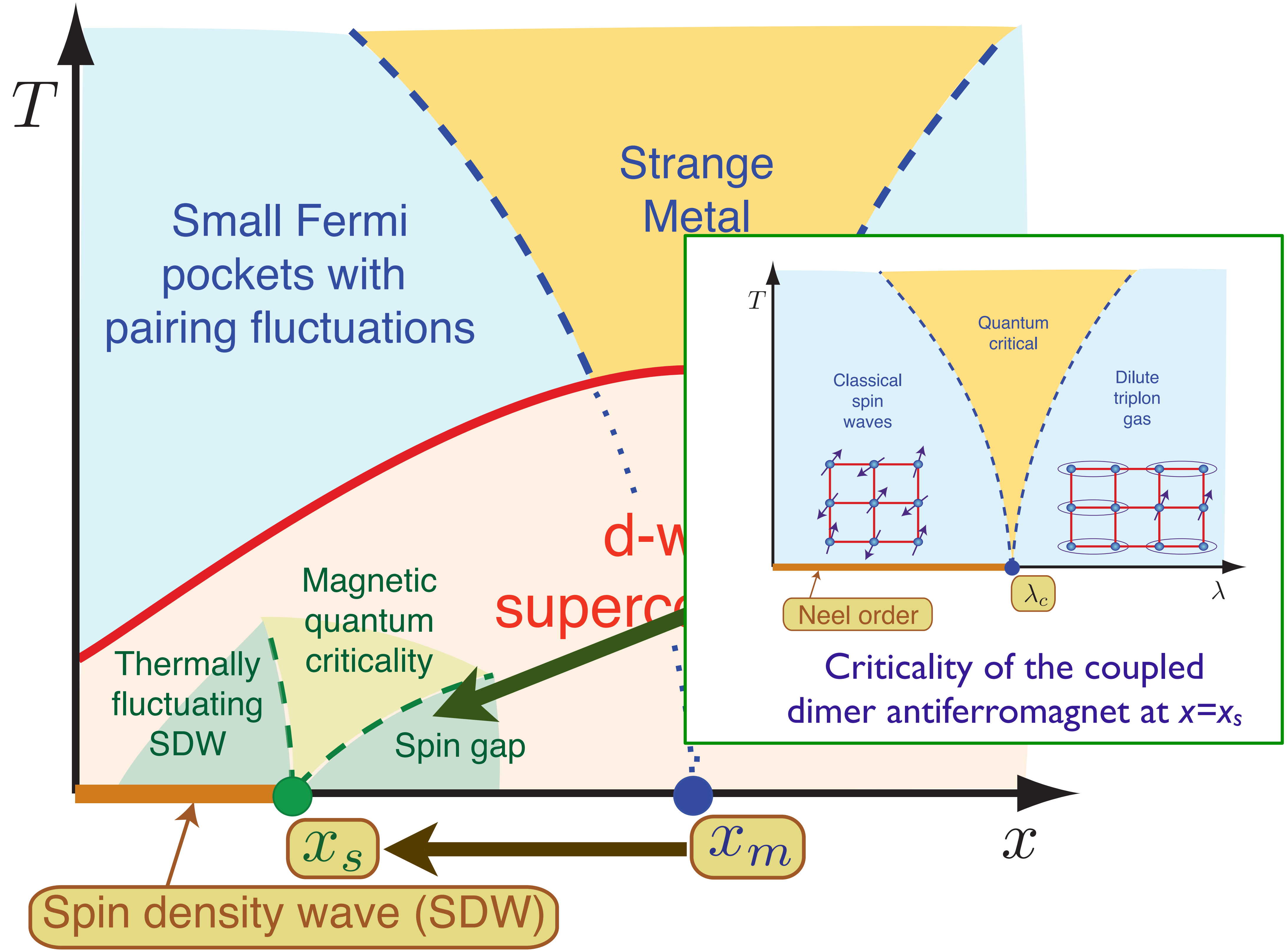
Theory of quantum criticality in the cuprates



Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

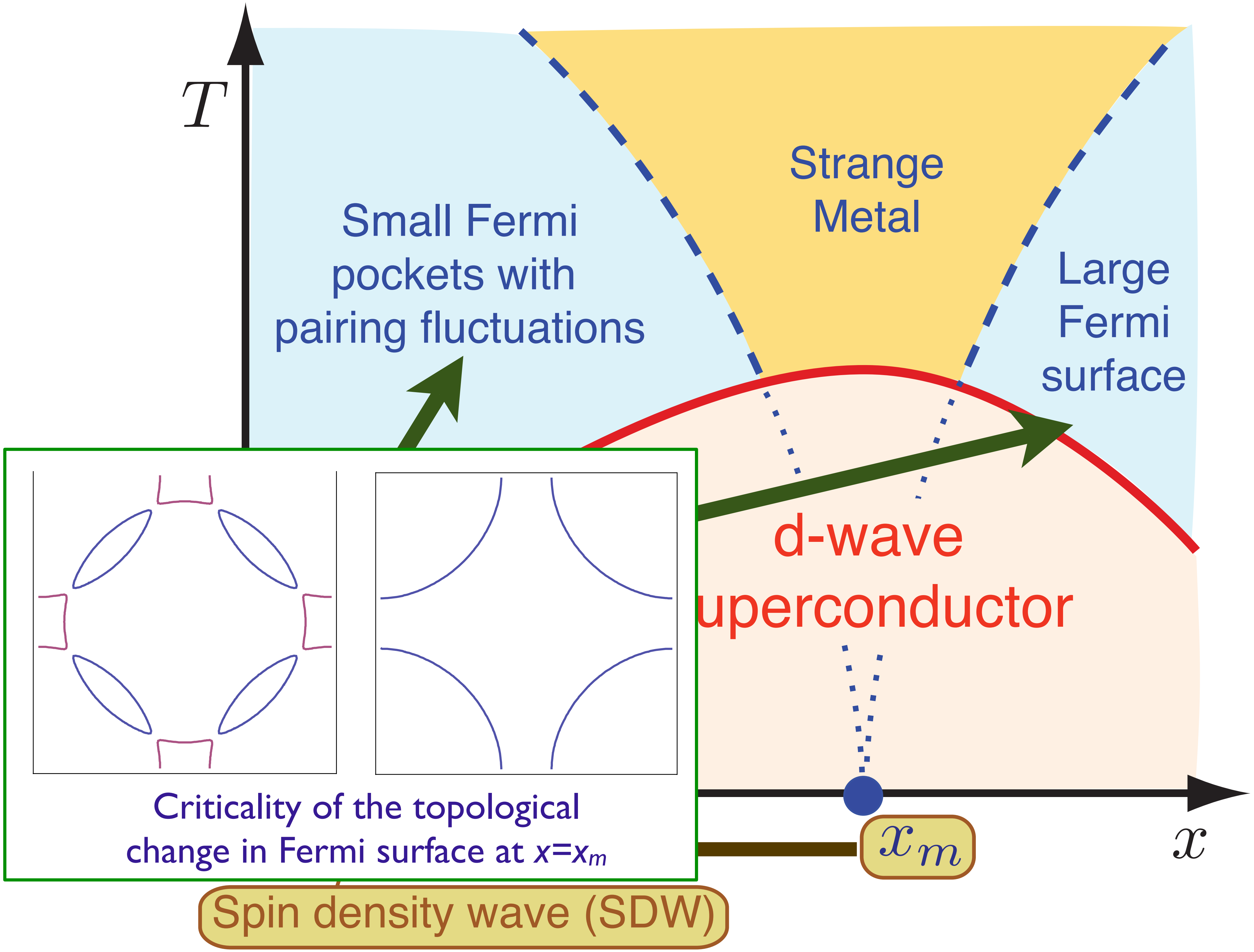
Theory of quantum criticality in the cuprates



Criticality of the coupled dimer antiferromagnet at $x=x_s$

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



T

Small Fermi pockets with pairing fluctuations

Strange Metal

Large Fermi surface

d-wave uperconductor

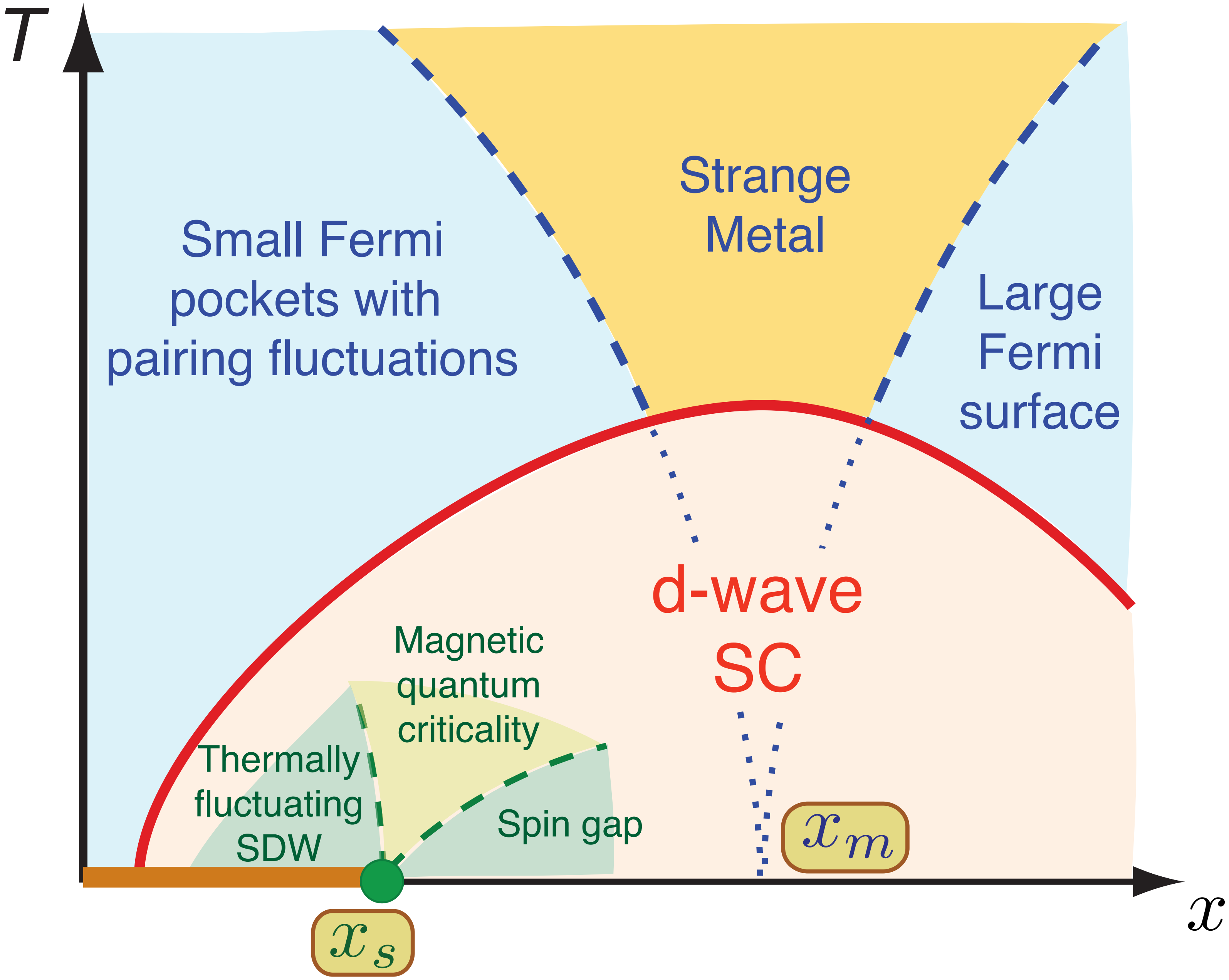
x

x_m

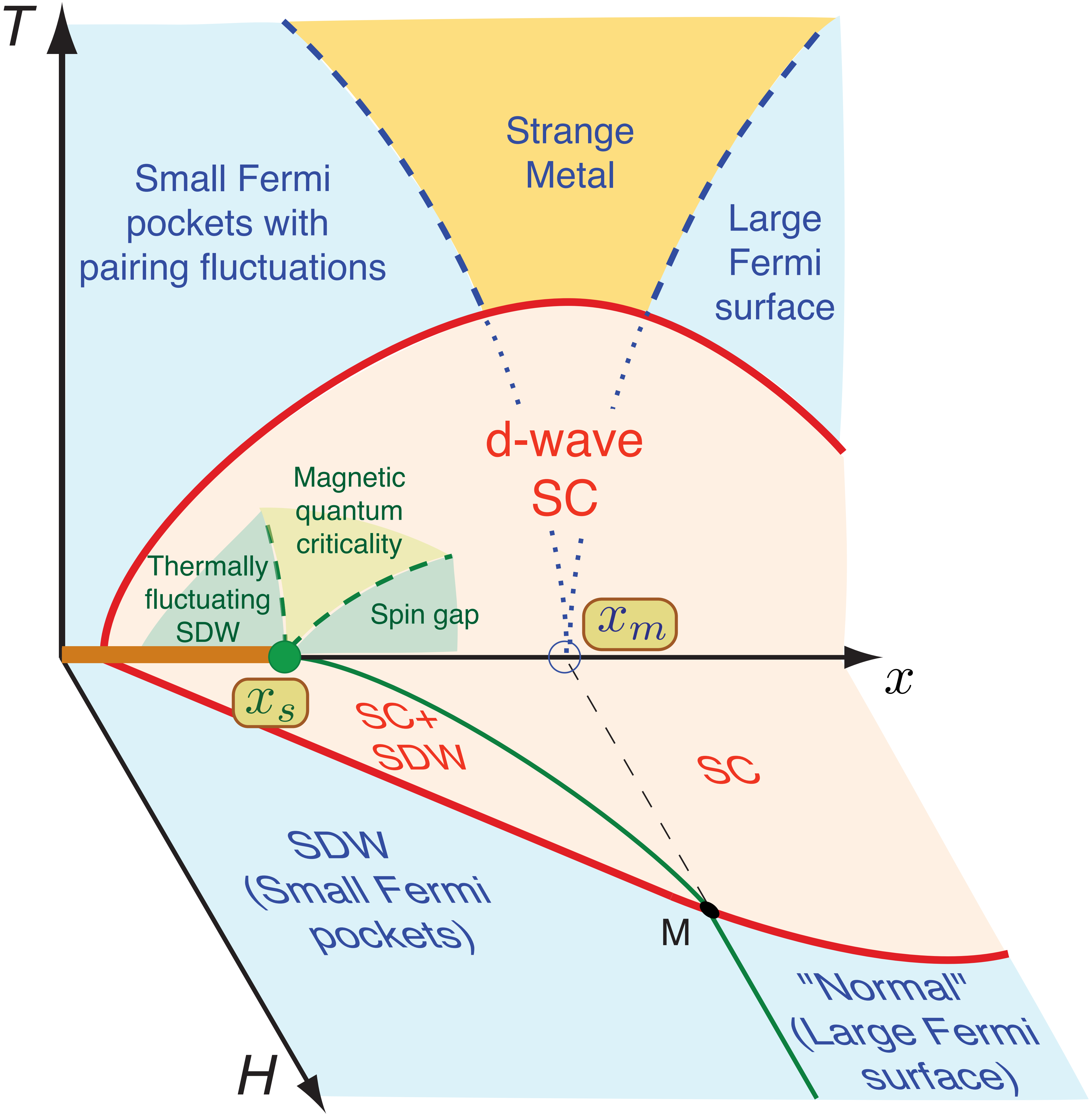
Criticality of the topological change in Fermi surface at $x=x_m$

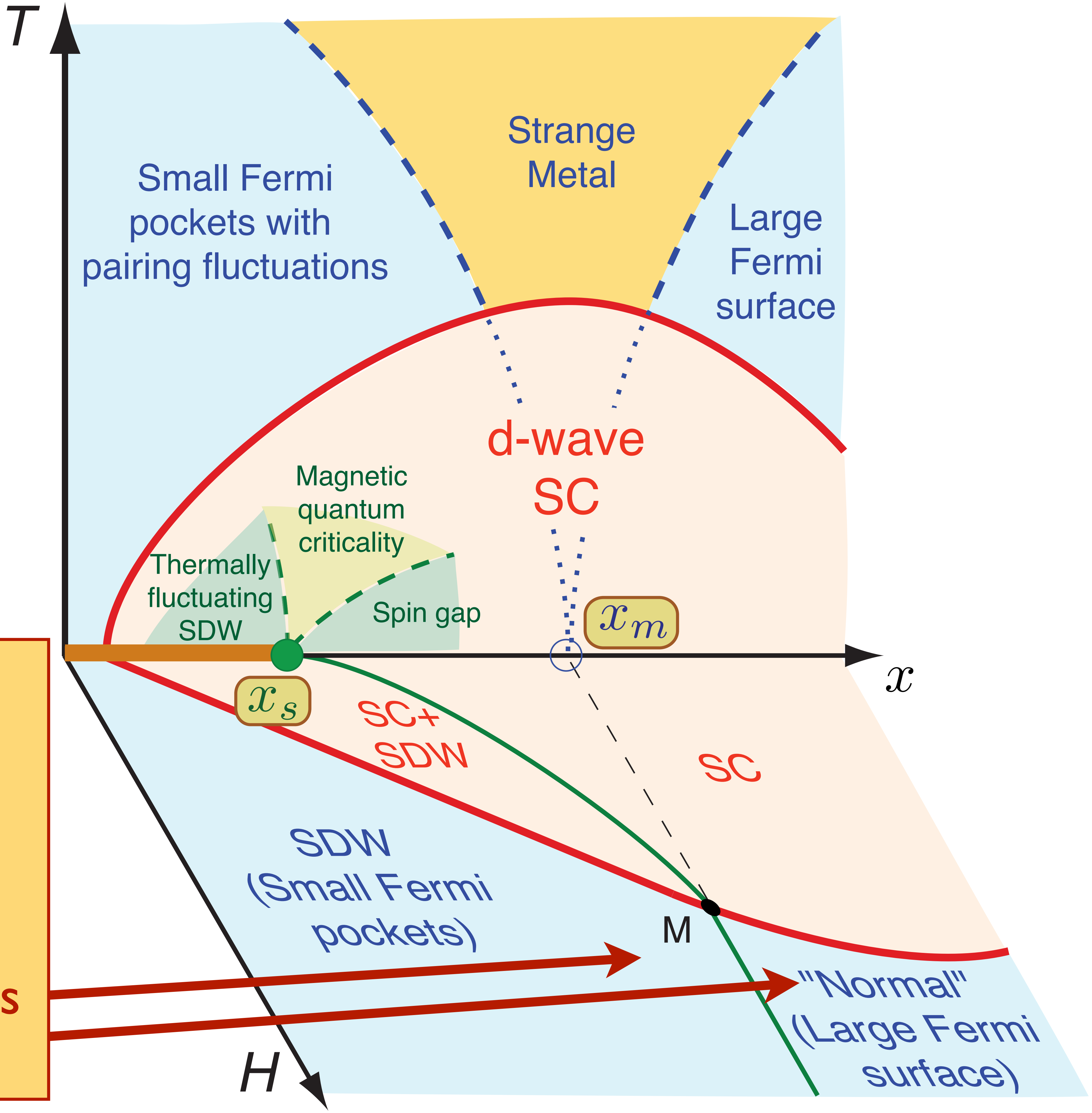
Spin density wave (SDW)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

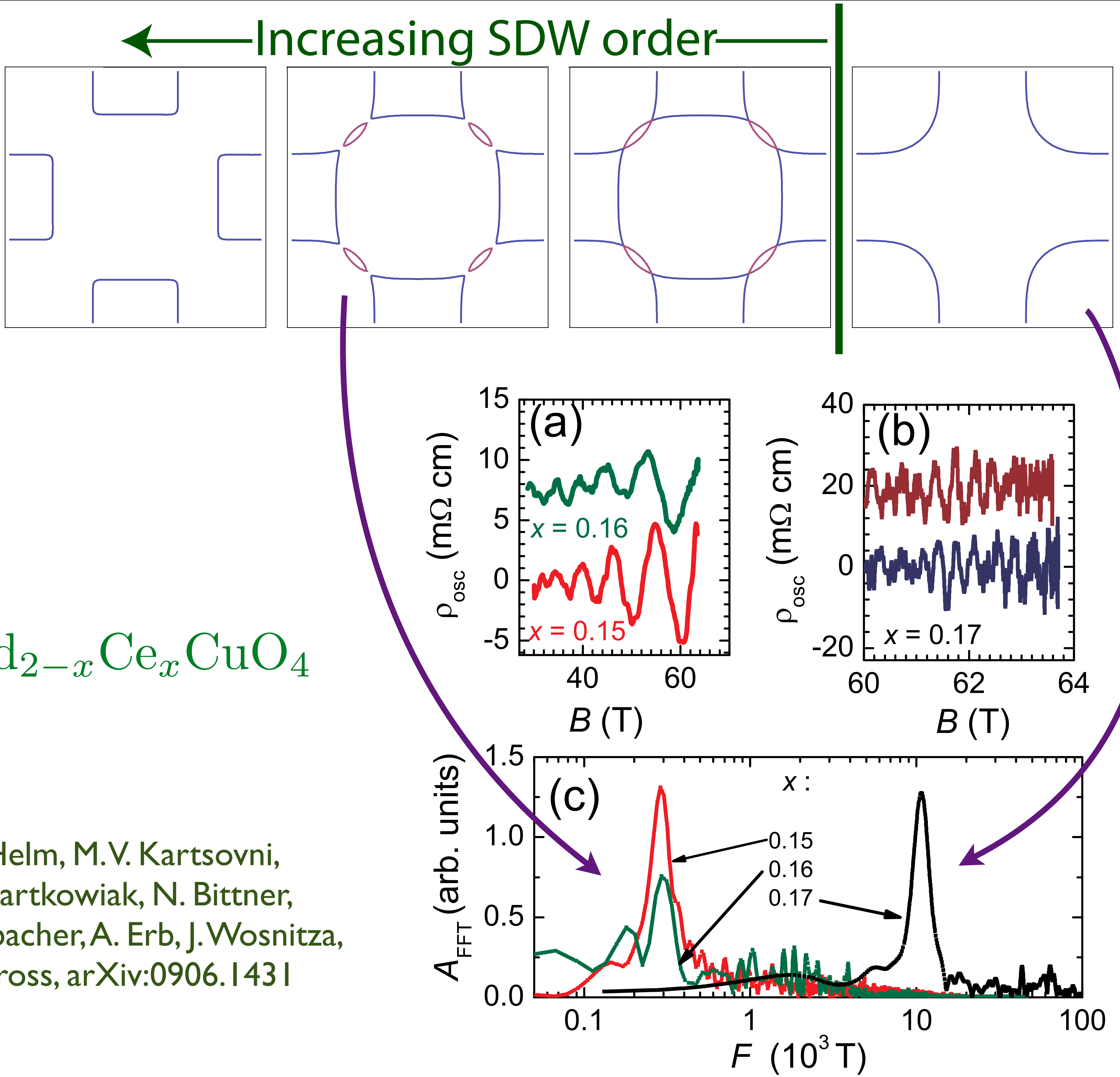


T
Small Fermi pockets with pairing fluctuations
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x


T
Small Fermi
pockets with
pairing fluctuations
Strange
Metal
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x
x_s
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H



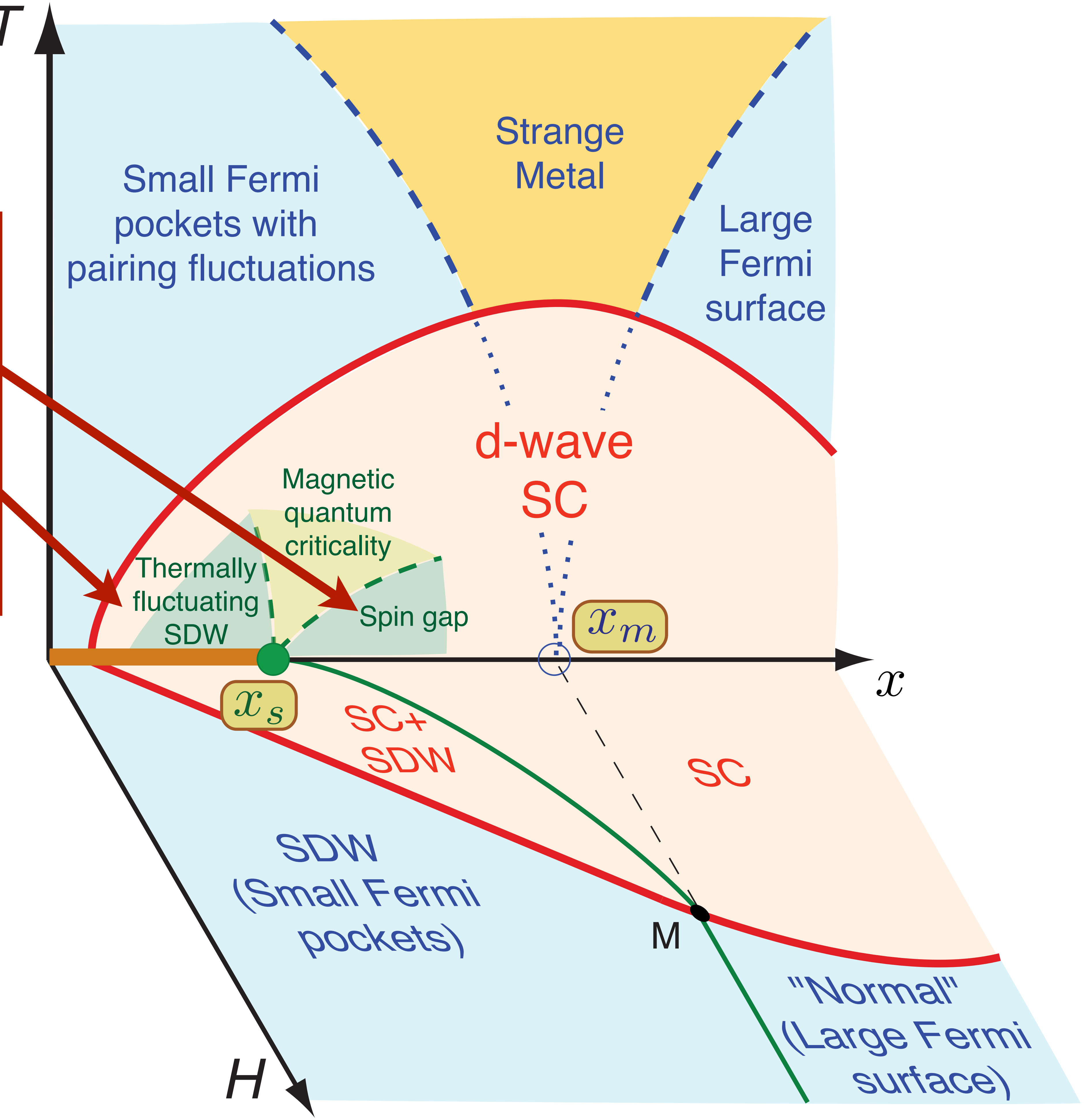
Change in frequency of quantum oscillations in electron-doped materials identifies $x_m = 0.165$



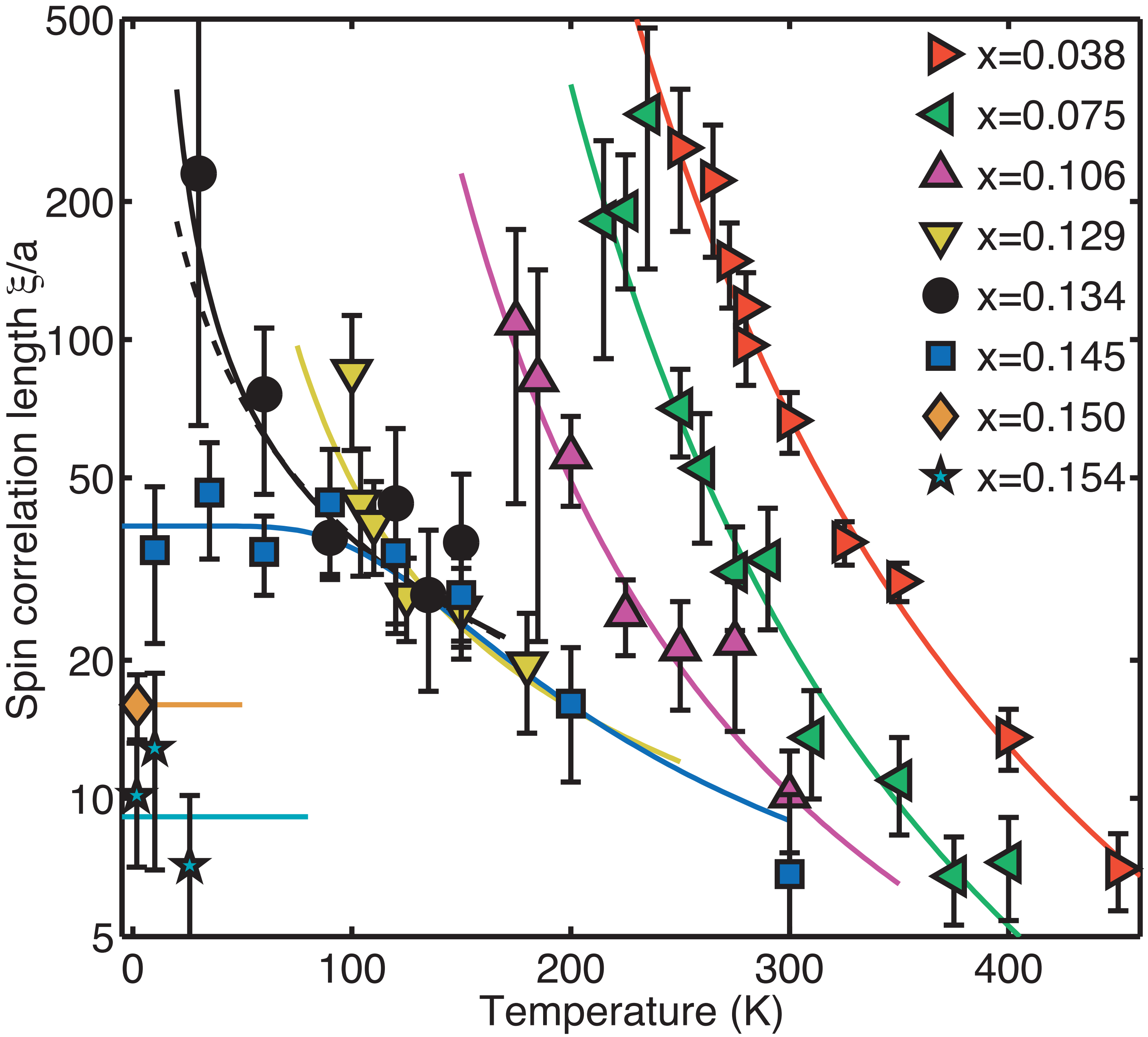
$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovni,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
R. Gross, arXiv:0906.1431

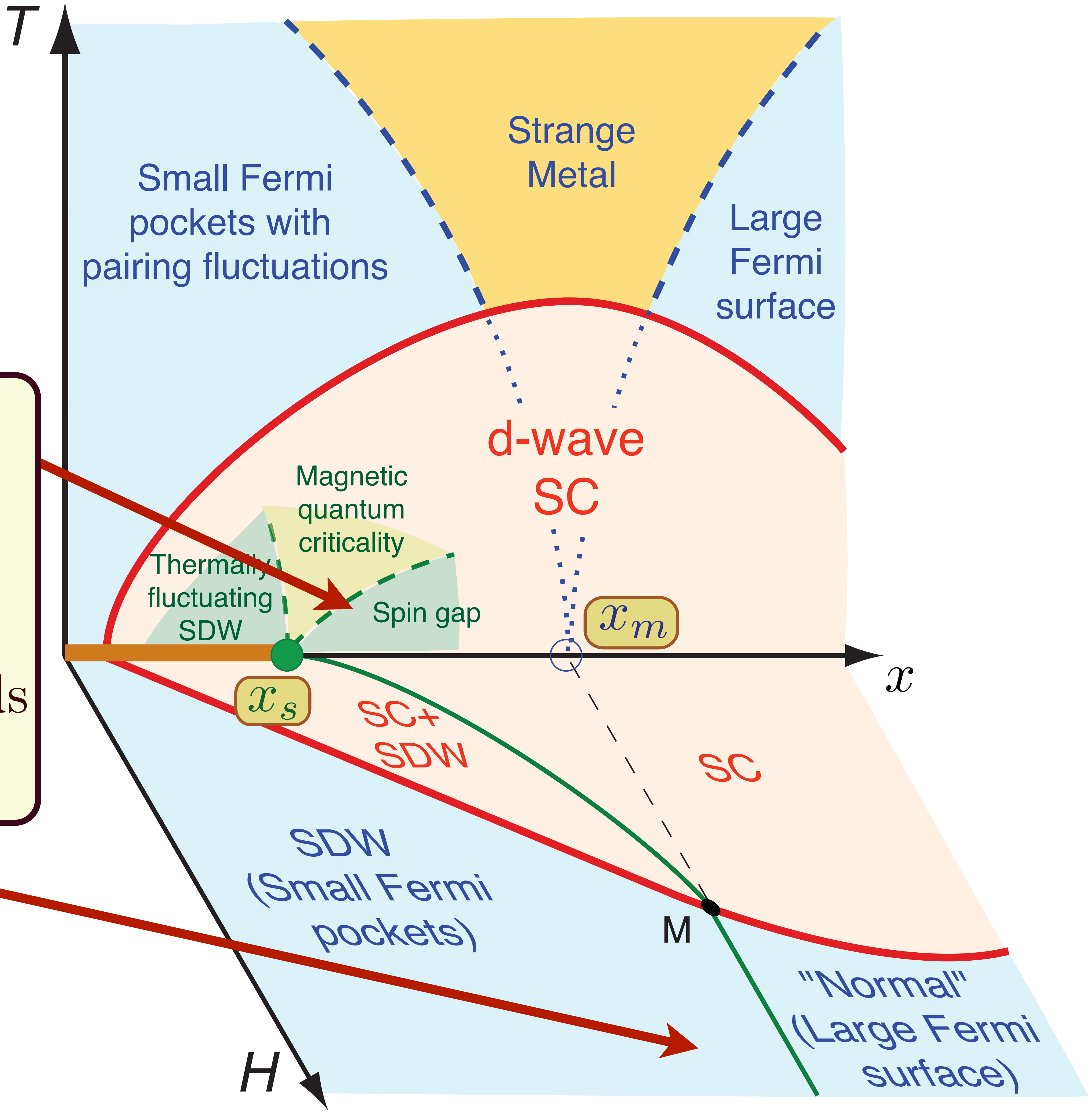
Neutron scattering at $H=0$ in **same** material identifies $x_s = 0.14 < x_m$



$Nd_{2-x}Ce_xCuO_4$



E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).



Experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while at high fields $x_m = 0.165$.

Conclusions

General theory of finite temperature dynamics and transport near quantum critical points, with applications to antiferromagnets, graphene, and superconductors

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been “hiding in plain sight”.

It is shifted to lower doping by the onset of superconductivity