

# Unveiling the order of the high temperature superconductors

Cornell University  
September 15, 2014

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PERIMETER INSTITUTE  
FOR THEORETICAL PHYSICS



PHYSICS



HARVARD

Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)

# Theorists at Harvard



Max Metlitski  
(KITP, UCSB)



Rolando La Placa  
(MIT)



Andrea Allais



Johannes  
Bauer



Debanjan  
Chowdhury



Jay Deep Sau  
(Maryland)



Alexandra  
Thomson

# Cornell



Kazuhiro Fujita  
Cornell/ BNL



Mohammad Hamidian  
Cornell / BNL



Stephen Edkins  
Cornell / St Andrews



Michael Lawler

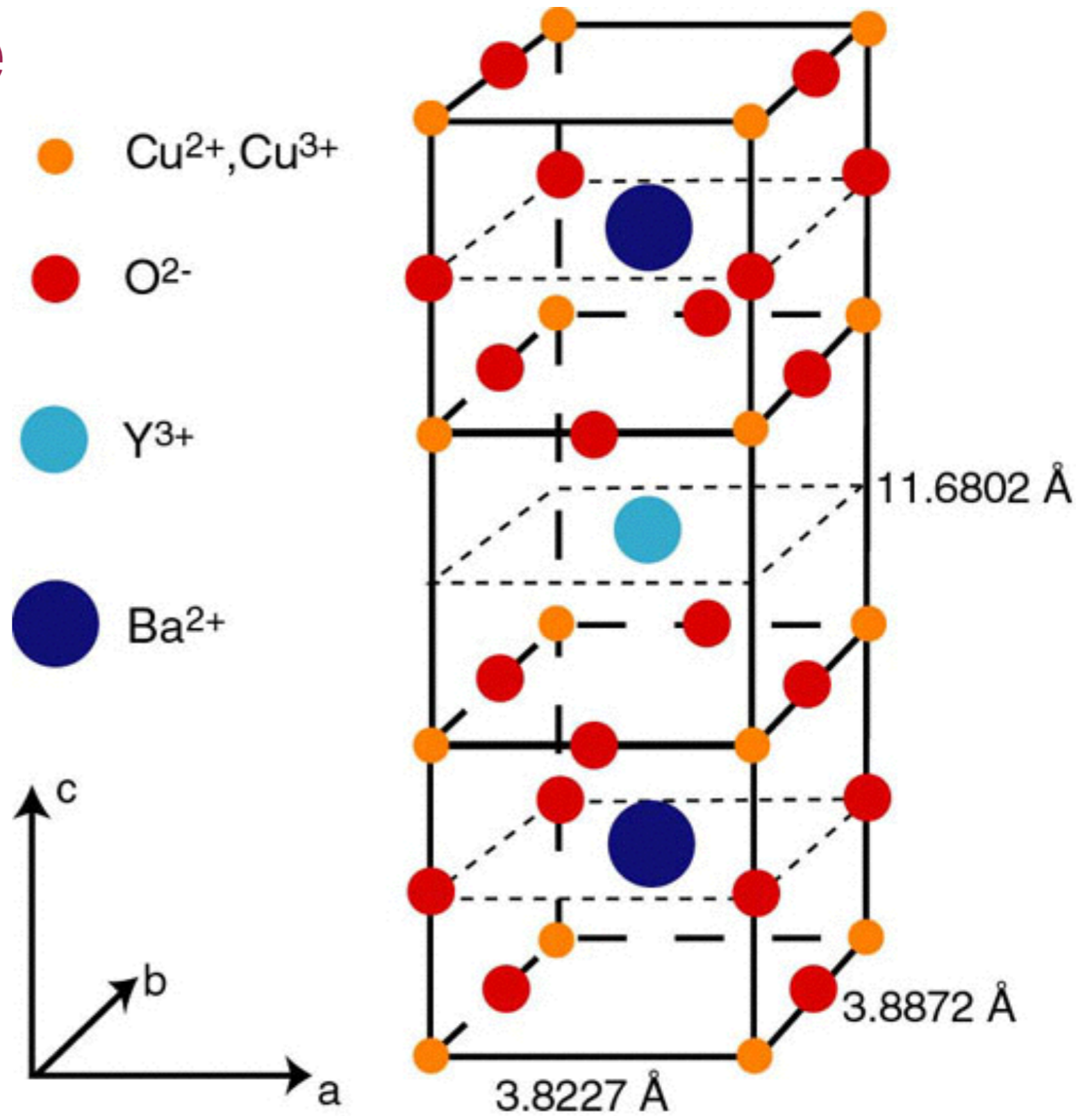


J. C. Seamus Davis

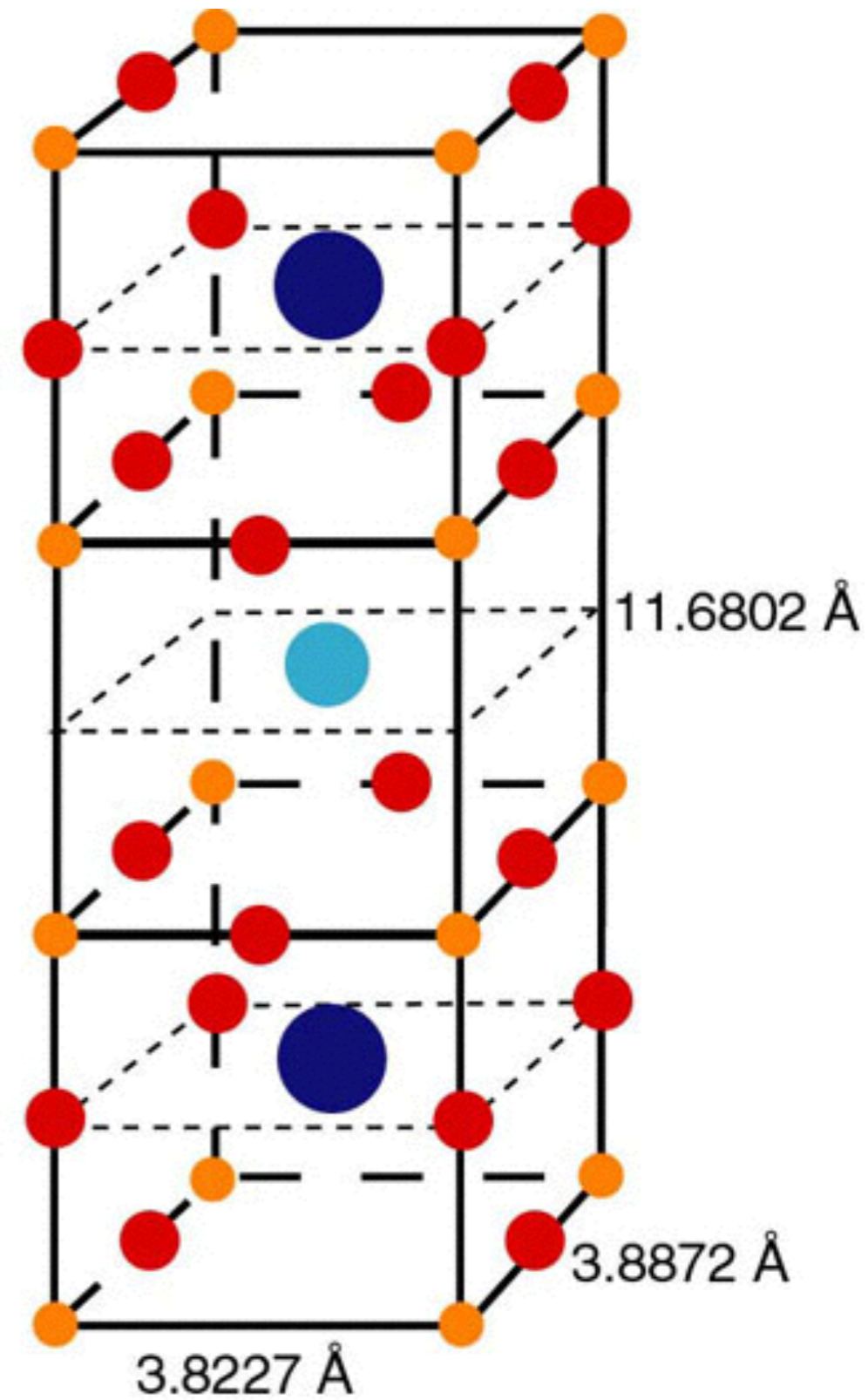
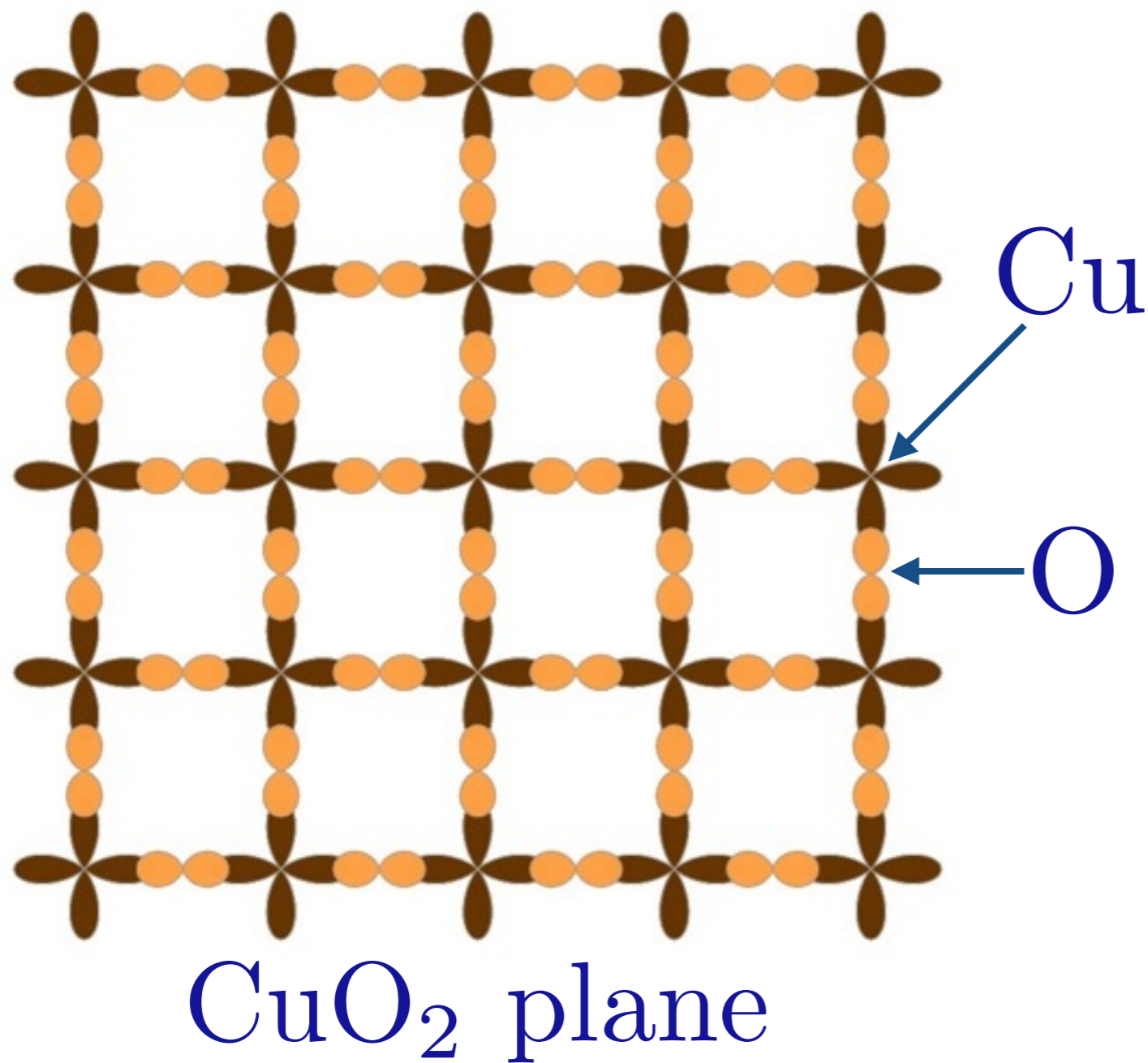


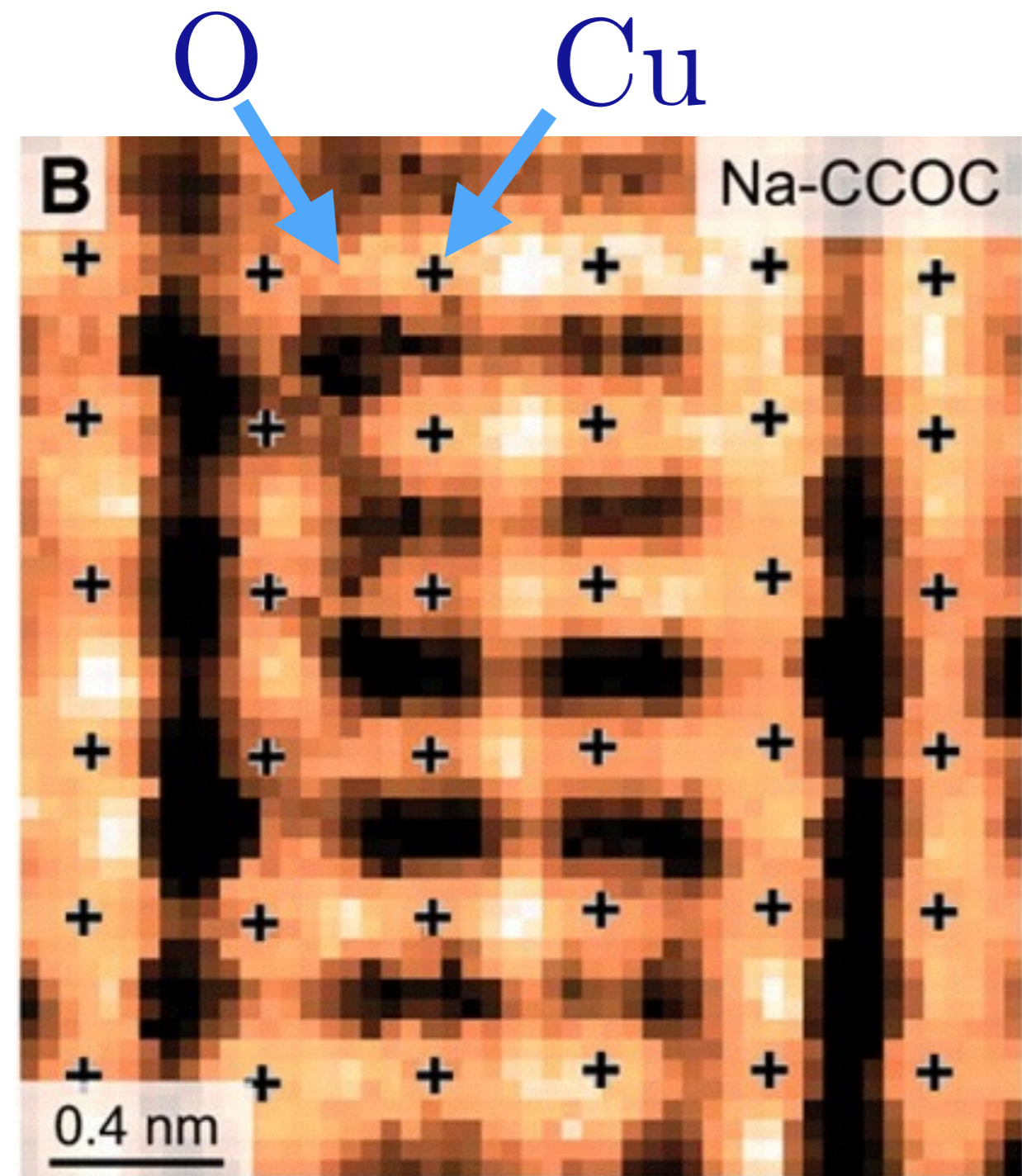
Eun-Ah Kim

# High temperature superconductors

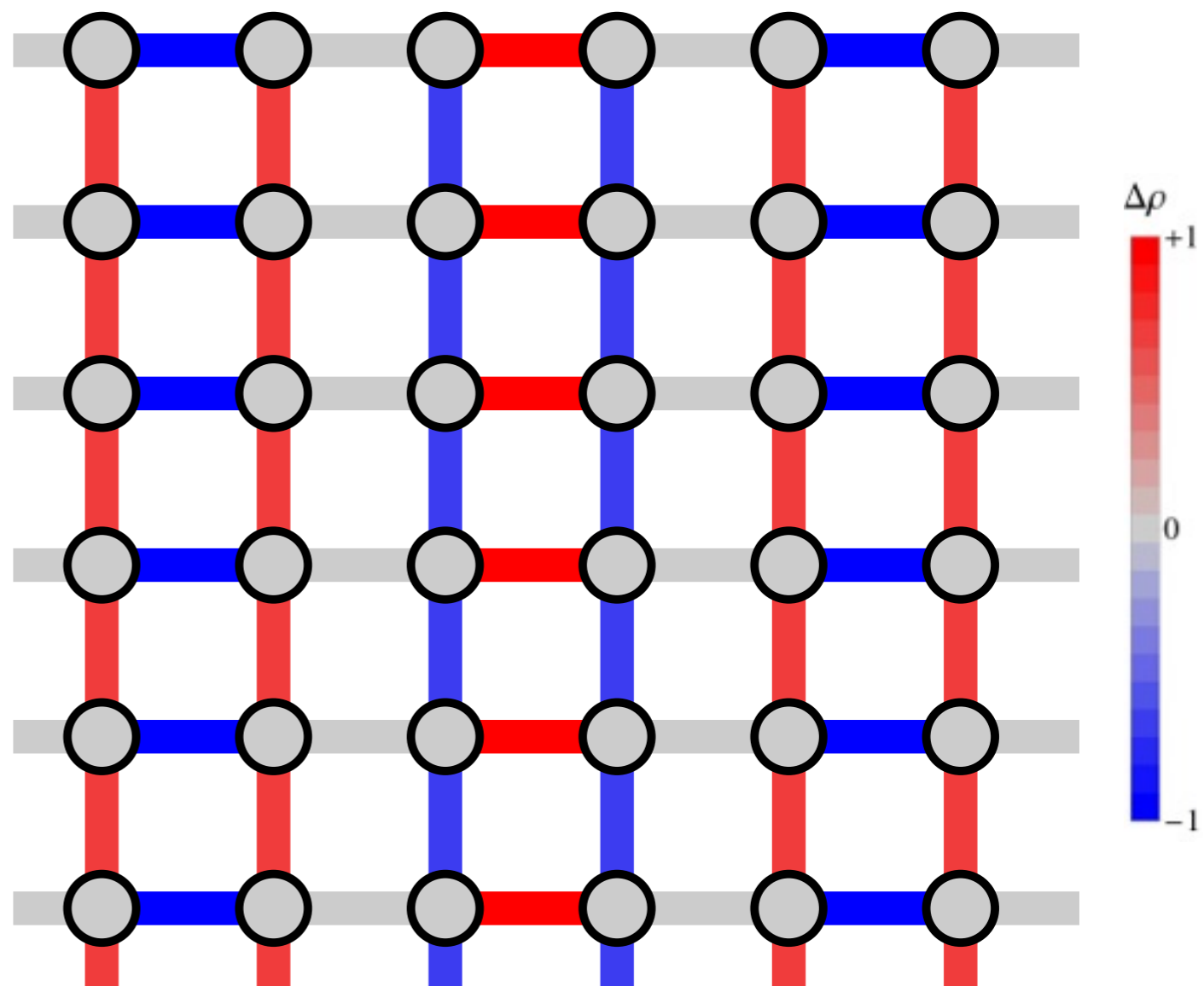


# High temperature superconductors

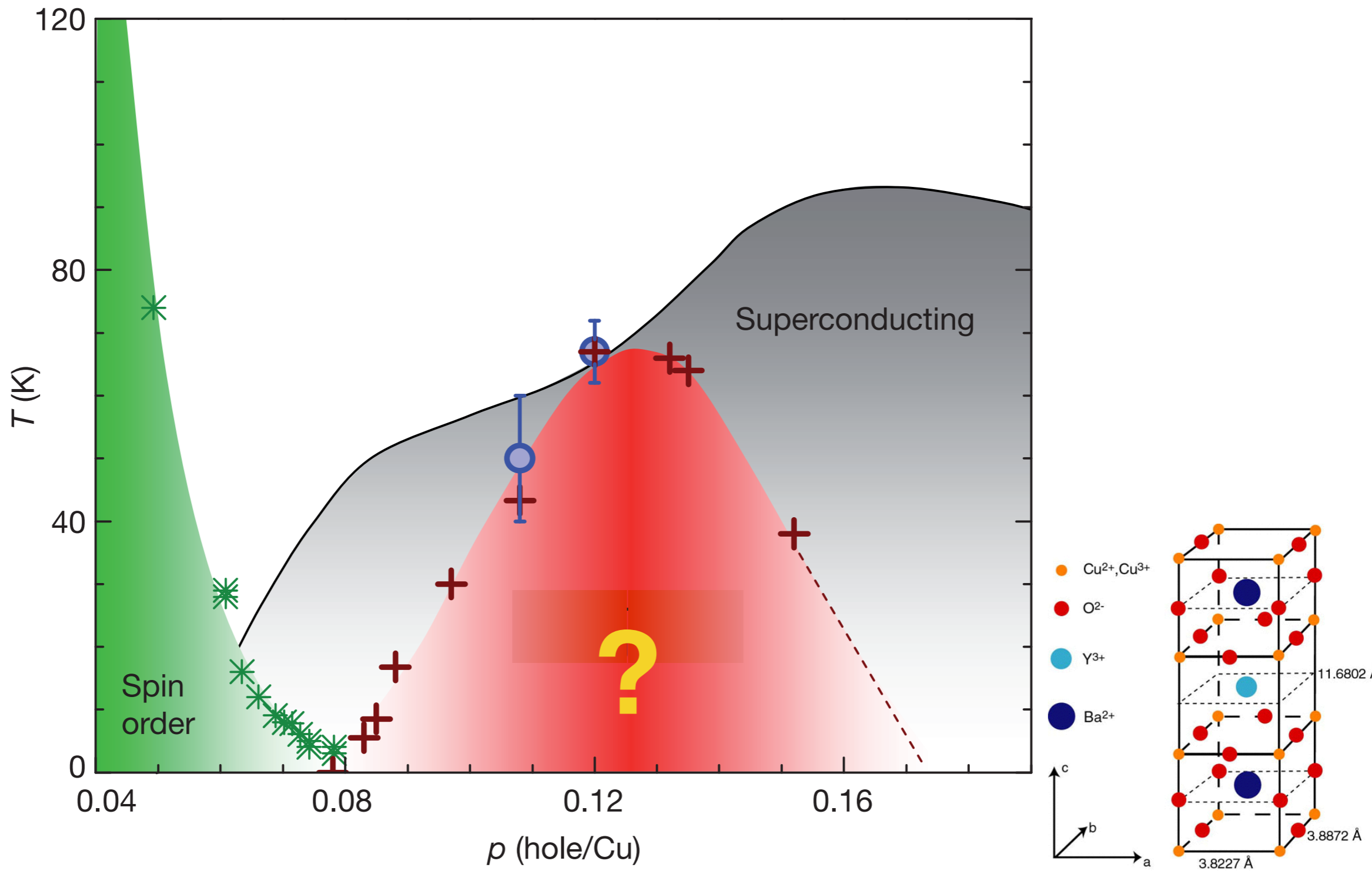




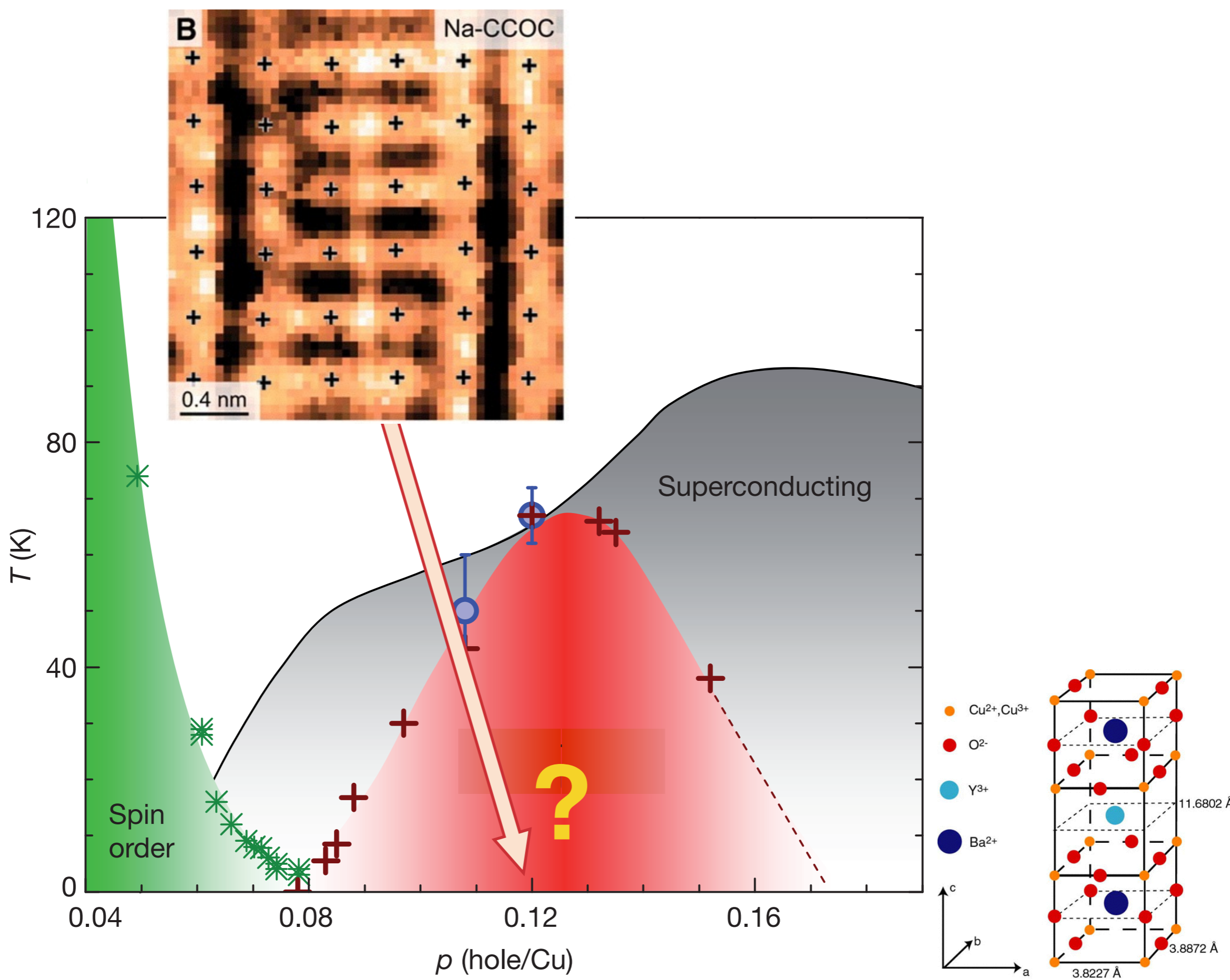
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



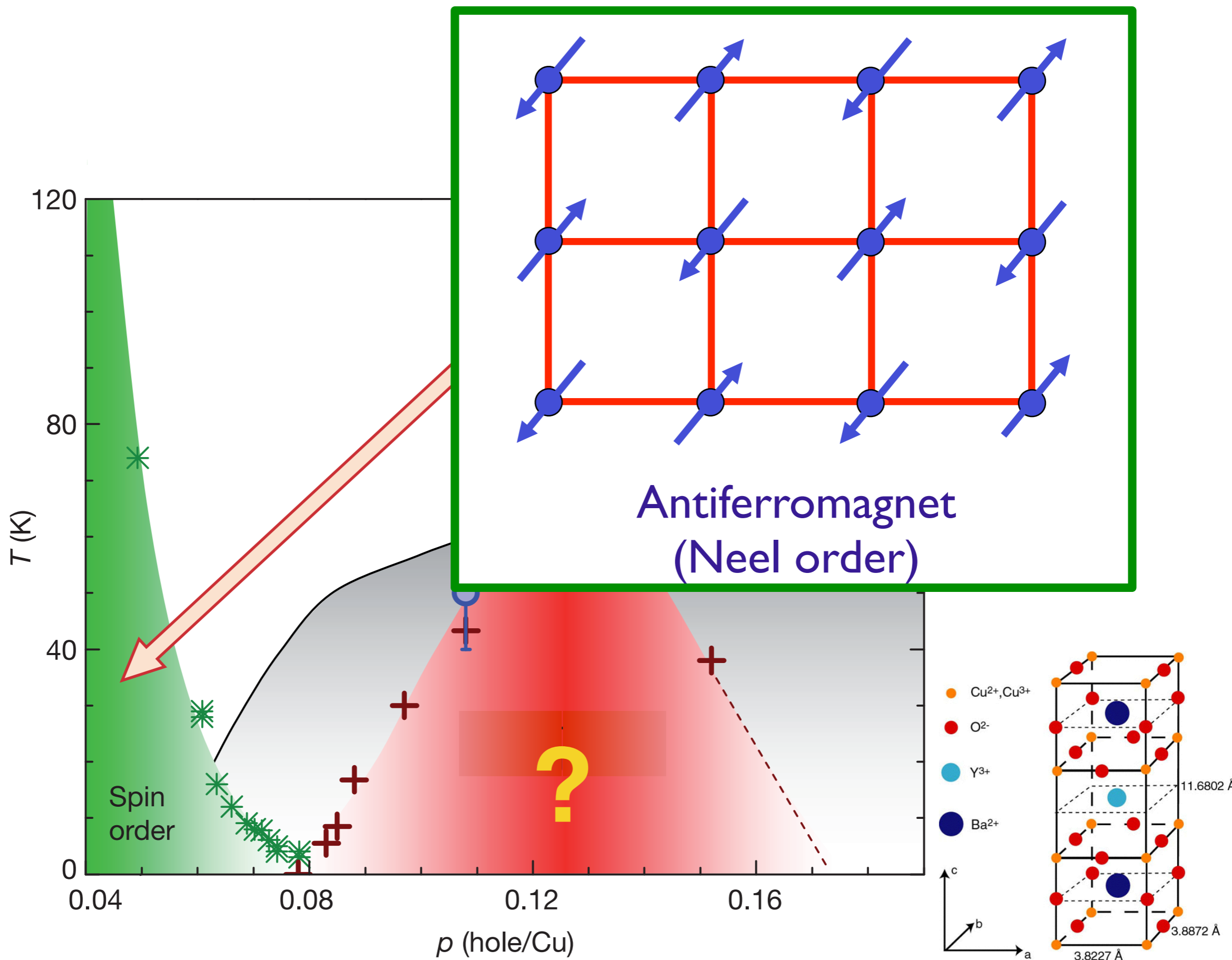
$d$ -form factor density wave order



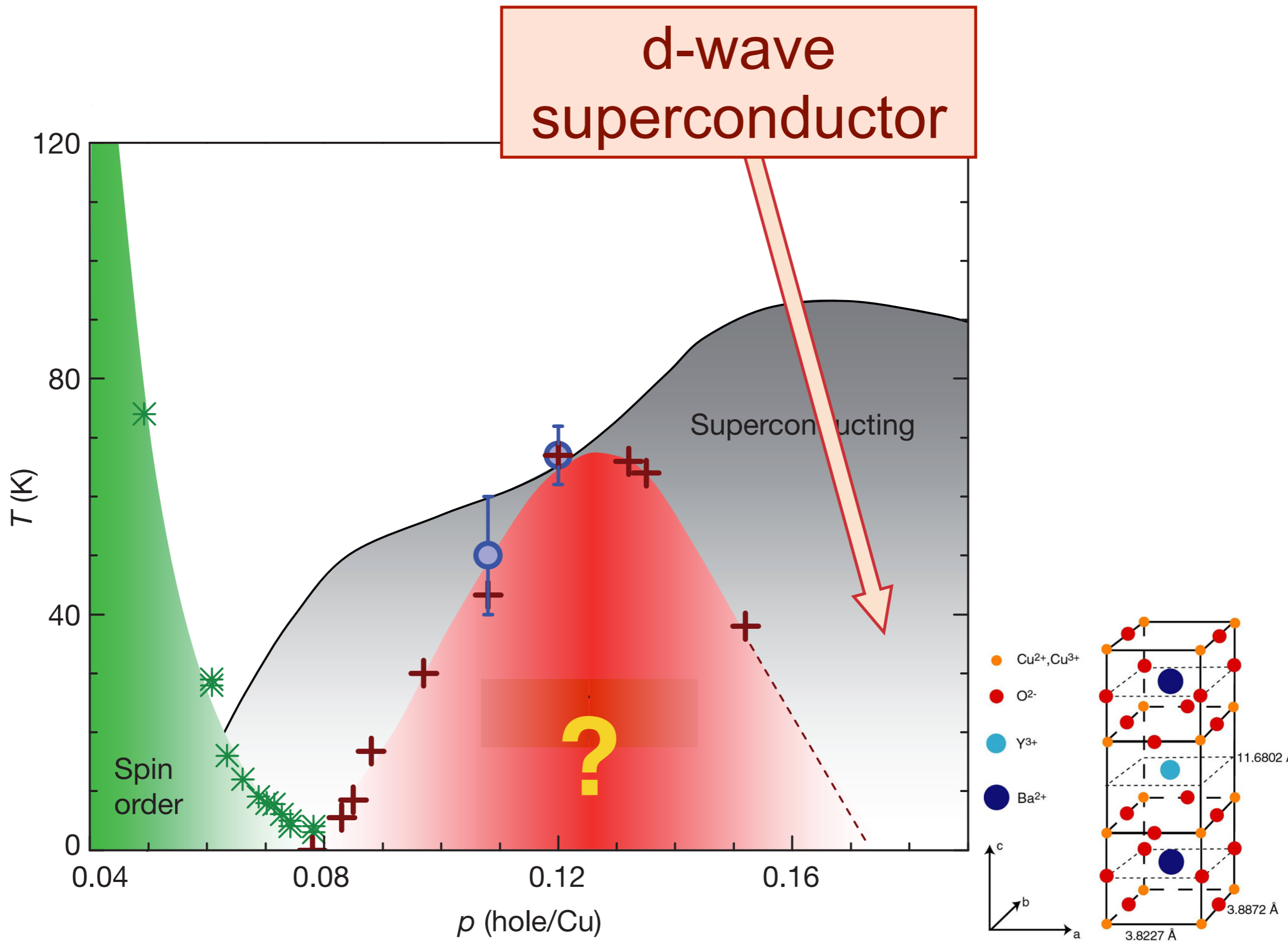
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# Superconductivity: Bose condensation of Cooper pairs of electrons

$$\varepsilon^{\alpha\beta} \left\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\beta}^{\dagger}(\mathbf{r}_2) \right\rangle = \left[ P(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{SC} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right)$$

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow} = -\varepsilon^{\downarrow\uparrow} = 1; \quad \varepsilon^{\uparrow\uparrow} = \varepsilon^{\downarrow\downarrow} = 0$$

# Superconductivity: Bose condensation of Cooper pairs of electrons

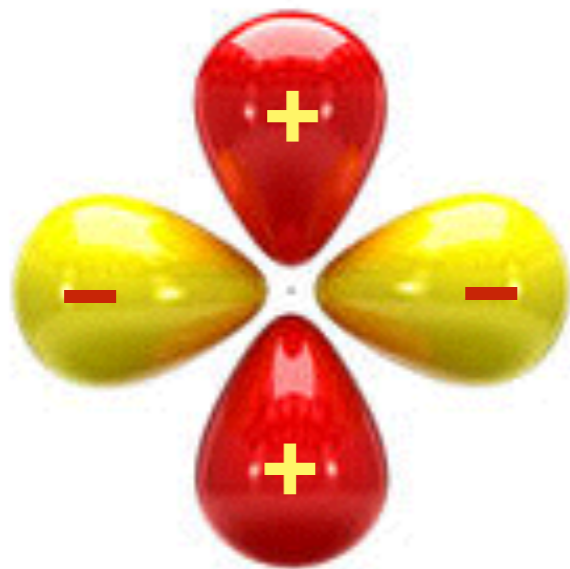
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Nearly constant condensate wavefunction  
(superconducting order parameter)

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \epsilon^{\uparrow\downarrow} = -\epsilon^{\downarrow\uparrow} = 1; \quad \epsilon^{\uparrow\uparrow} = \epsilon^{\downarrow\downarrow} = 0$$

# Superconductivity: Bose condensation of Cooper pairs of electrons

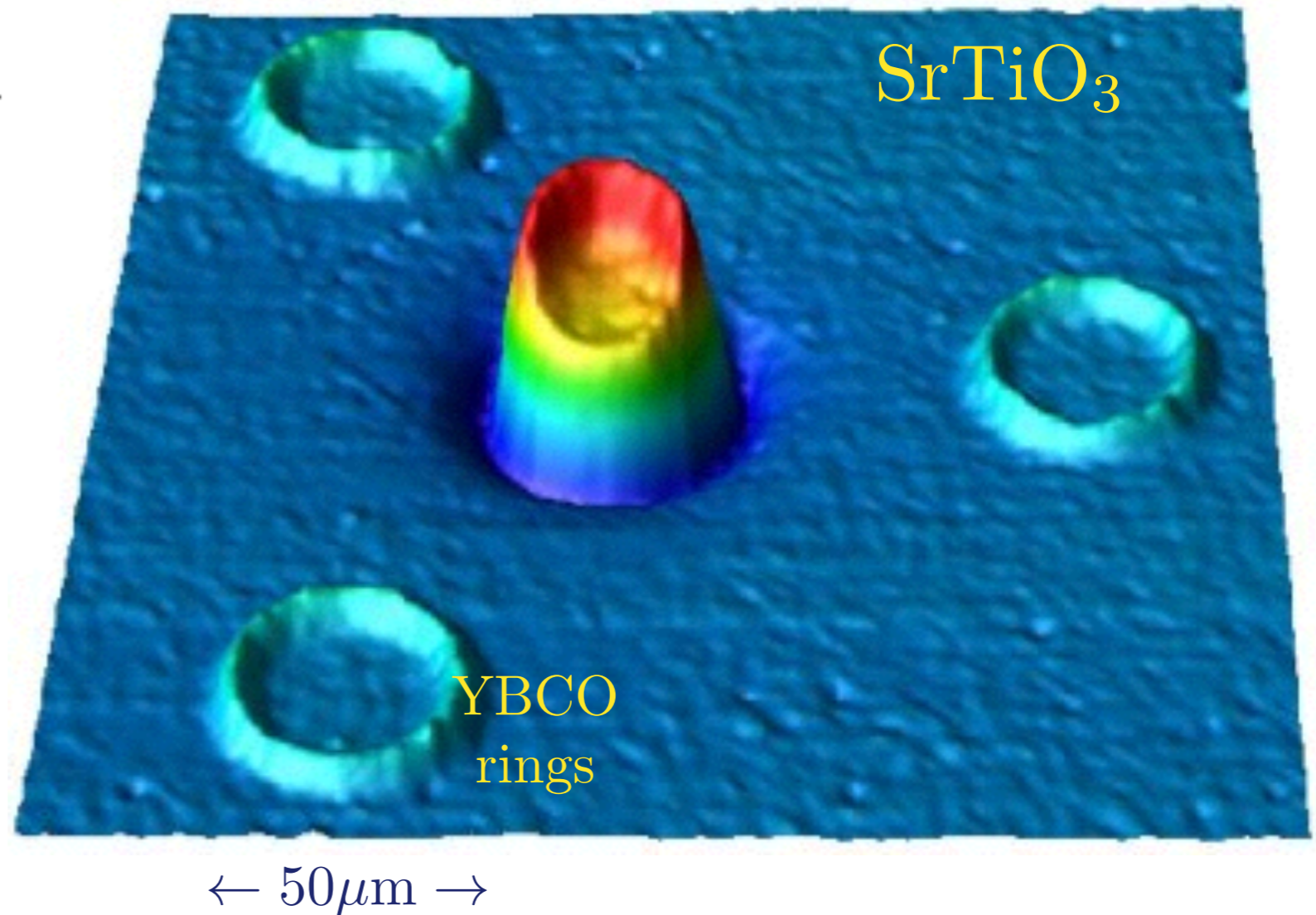
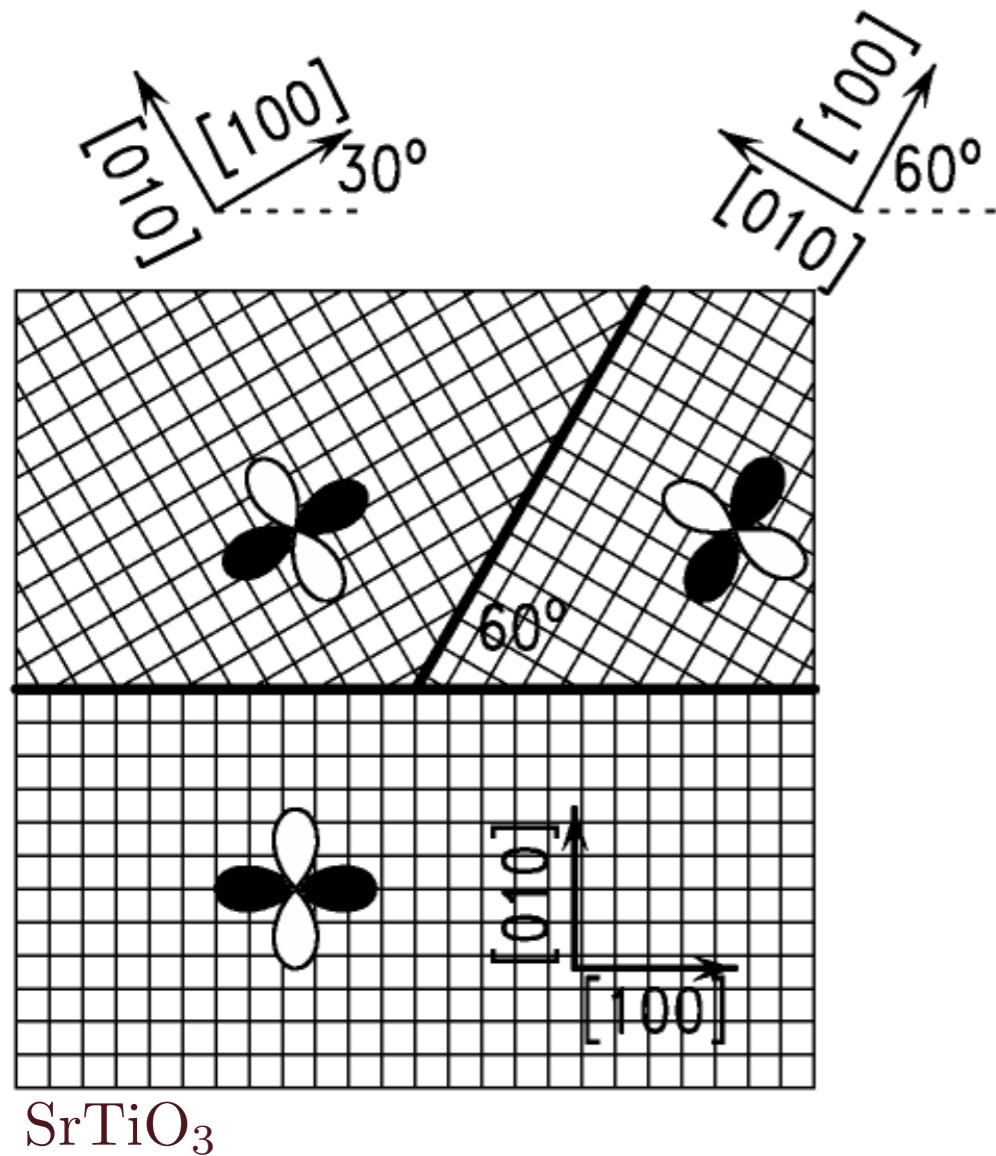
$$\varepsilon^{\alpha\beta} \left\langle c_{\alpha}^{\dagger}(\mathbf{r}_1) c_{\beta}^{\dagger}(\mathbf{r}_2) \right\rangle = \left[ P(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{SC} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right)$$



Internal Cooper-pair wavefunction.  
Has *d*-wave form in cuprates

$$\alpha, \beta = \uparrow, \downarrow \quad ; \quad \varepsilon^{\uparrow\downarrow} = -\varepsilon^{\downarrow\uparrow} = 1; \quad \varepsilon^{\uparrow\uparrow} = \varepsilon^{\downarrow\downarrow} = 0$$

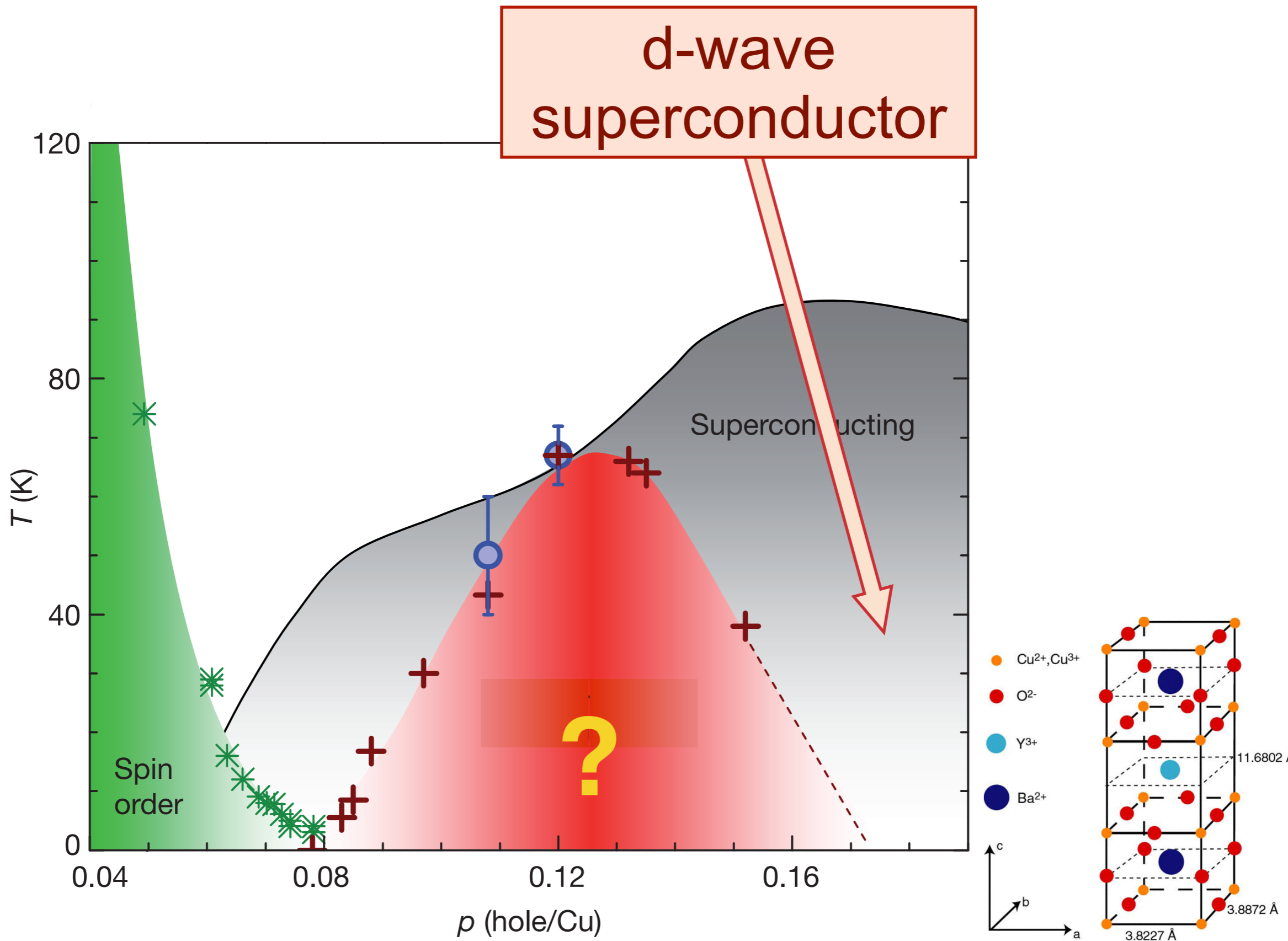
# Phase-sensitive measurement of the $d$ -wave symmetry of Cooper pairs



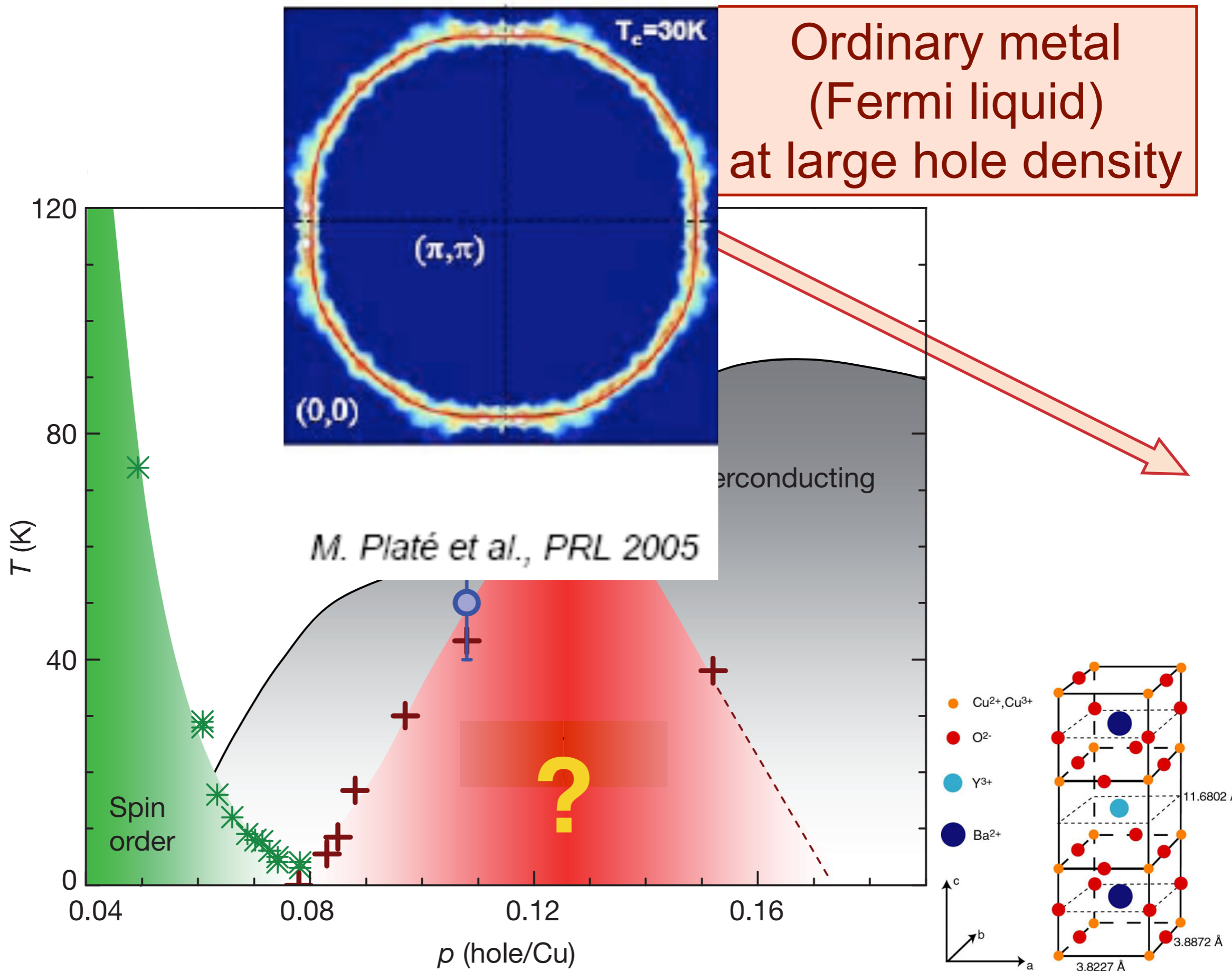
## Pairing Symmetry and Flux Quantization in a Tricrystal Superconducting Ring of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$

C. C. Tsuei, J. R. Kirtley, C. C. Chi,\* Lock See Yu-Jahnes, A. Gupta, T. Shaw, J. Z. Sun, and M. B. Ketchen  
*IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598*

Phys. Rev. Lett. **73**, 593 (1994)

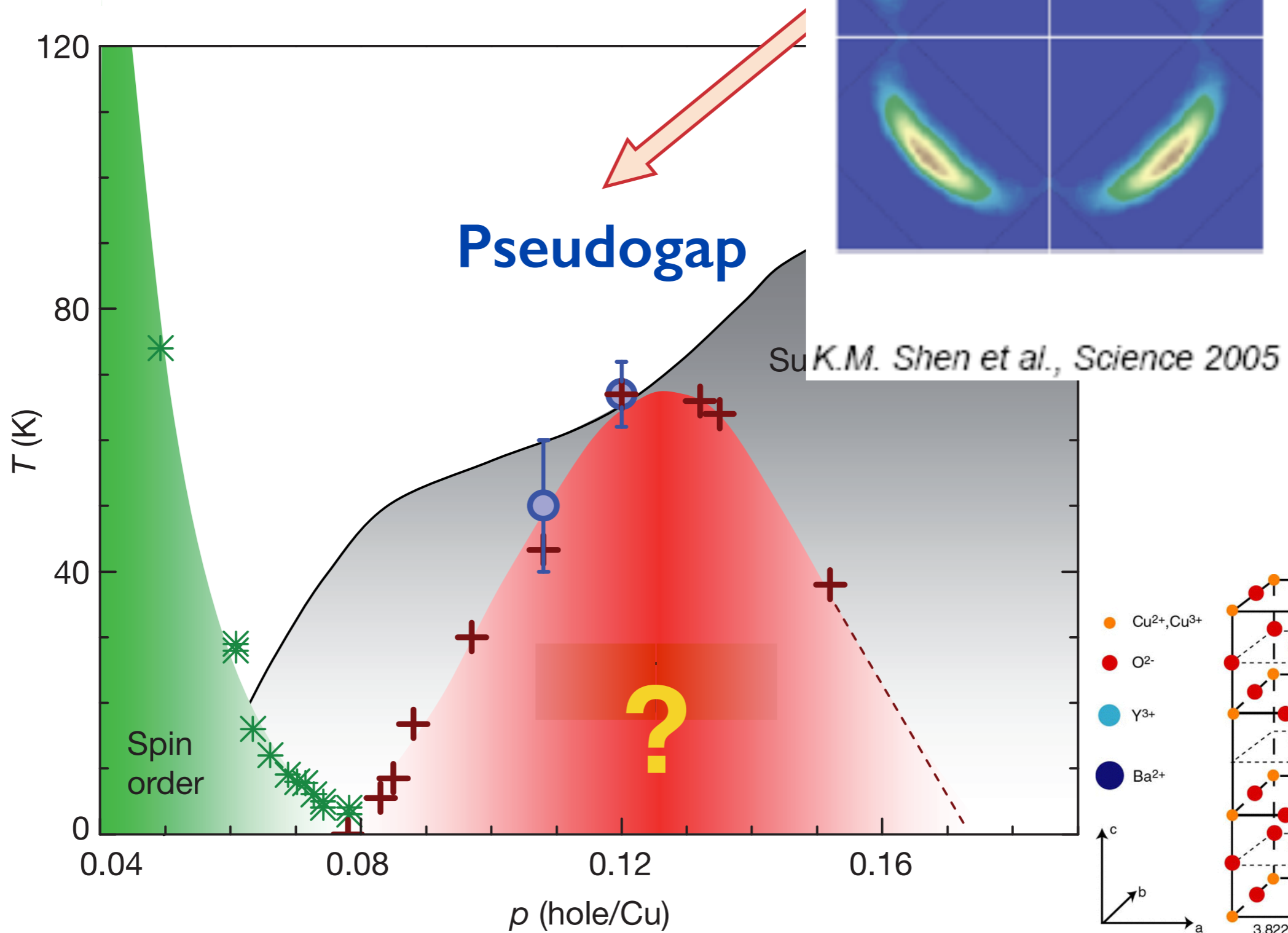
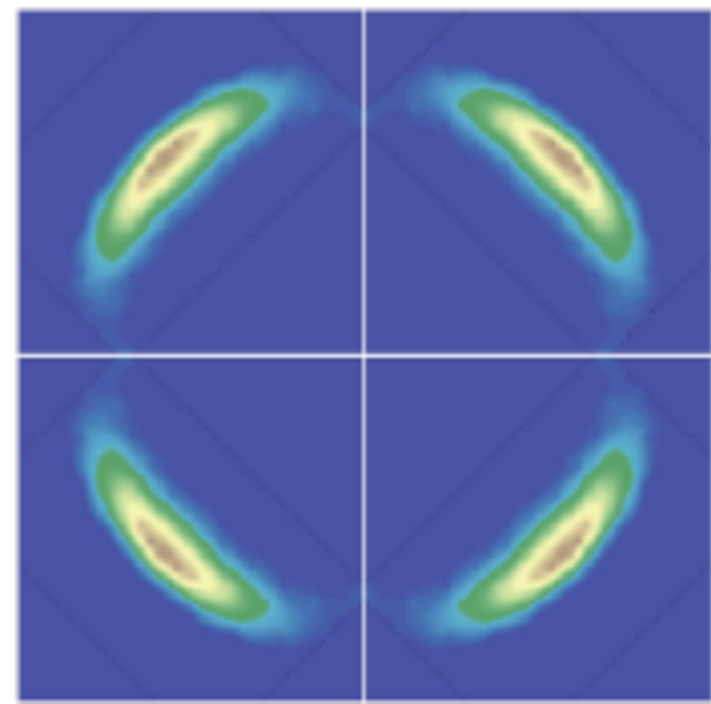


T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).

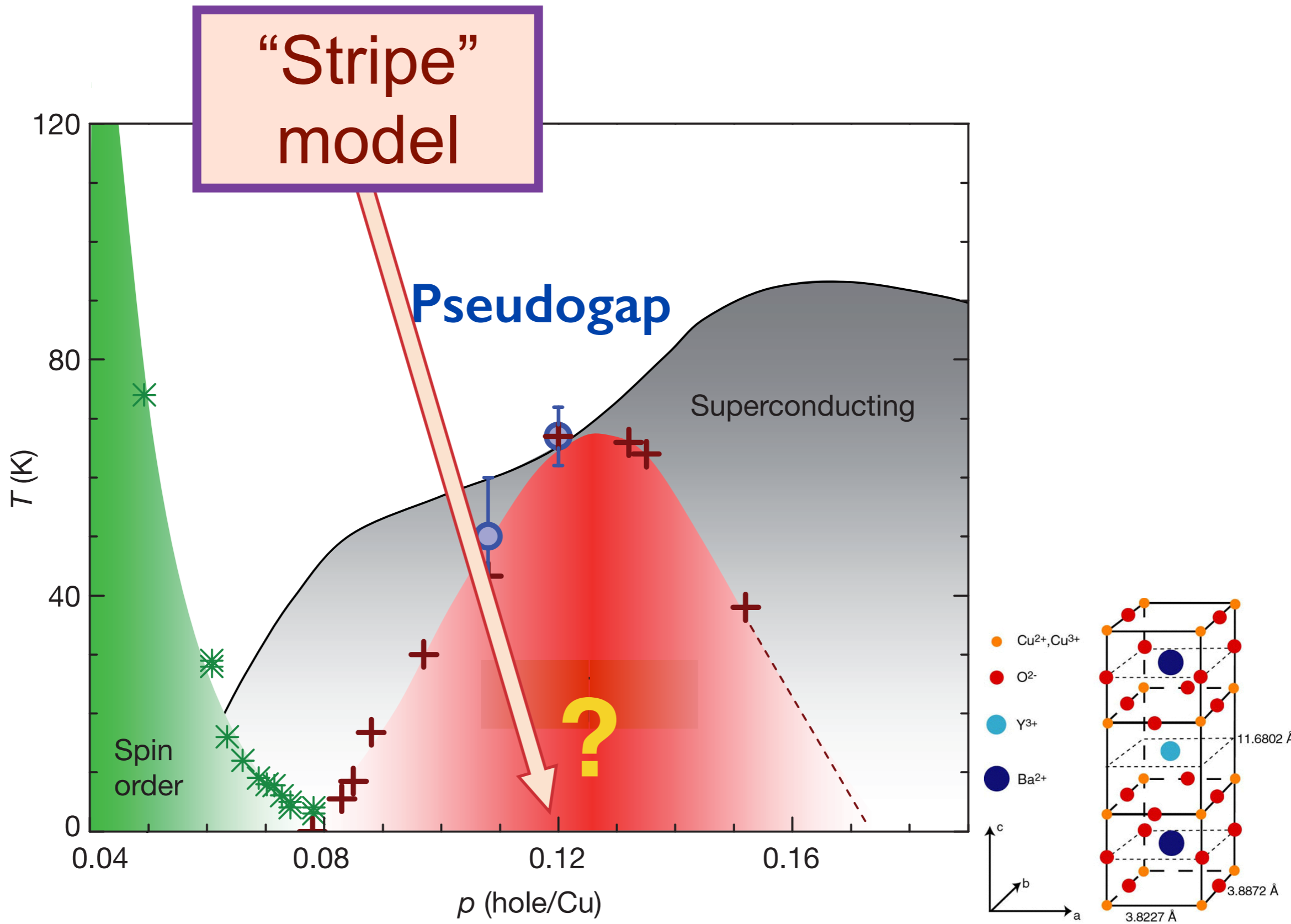


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“Fermi arcs” at low doping

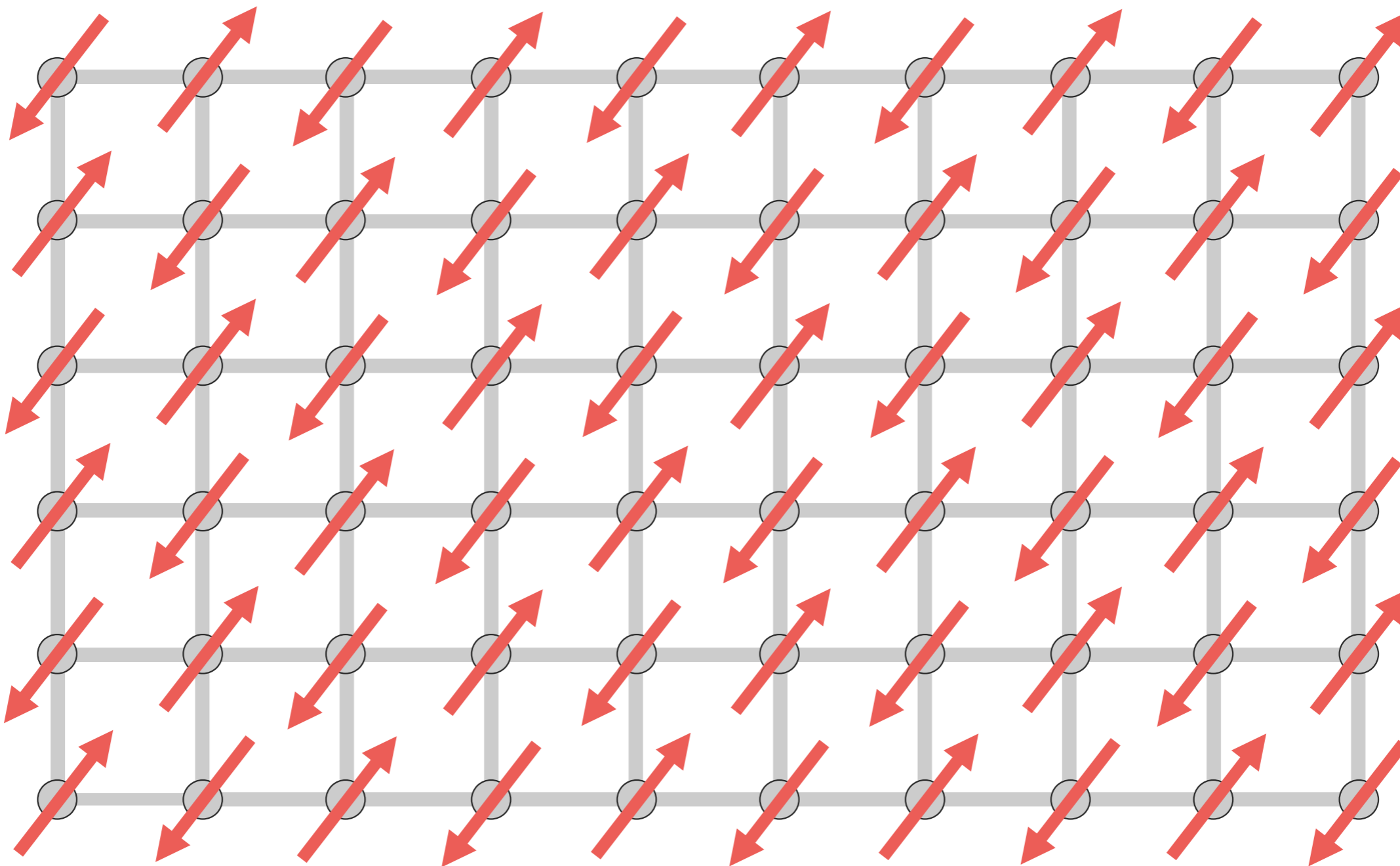


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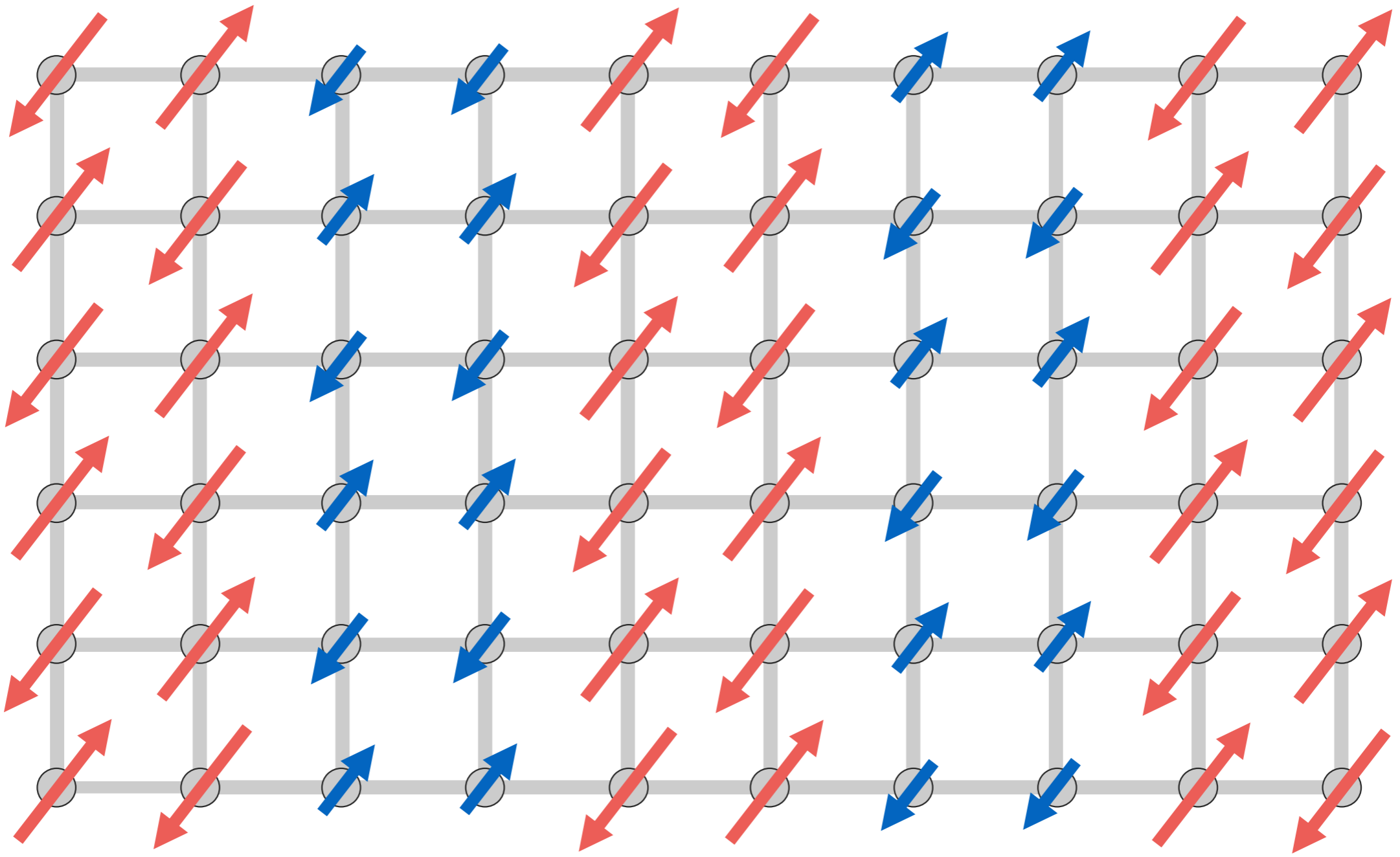
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# “Stripe” model



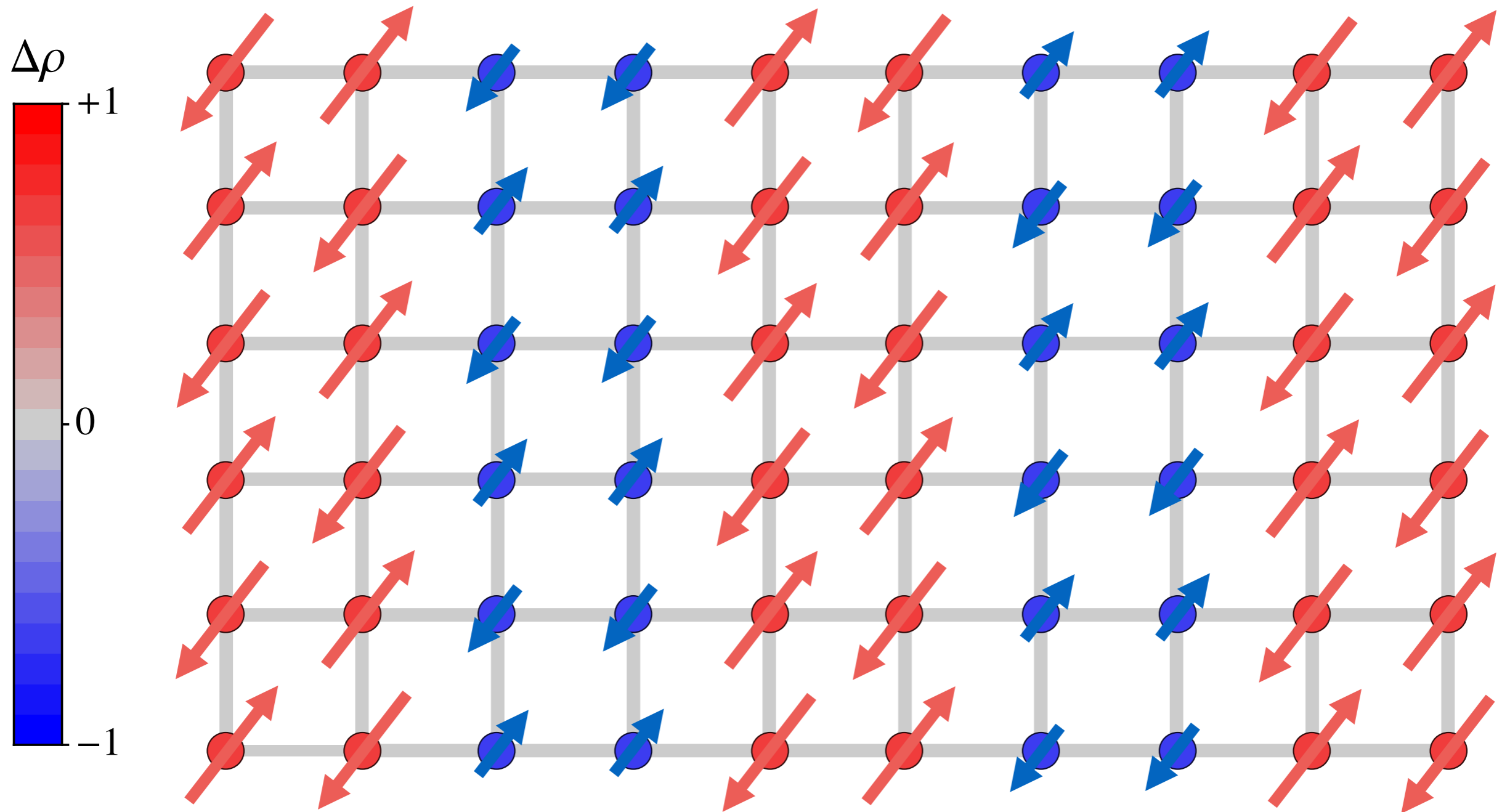
Start with an antiferromagnet

# “Stripe” model



Domain walls 4 lattice spacings apart

# “Stripe” model



Put the holes in the domain walls

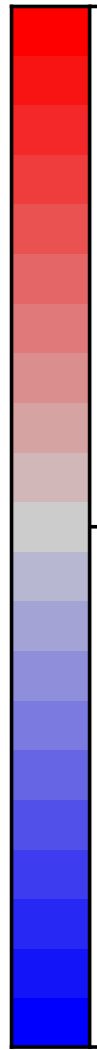
# “Stripe” model

Observed in La-based  
compounds (Tranquada..)

Theory: Zaanen, Kivelson, Fradkin....

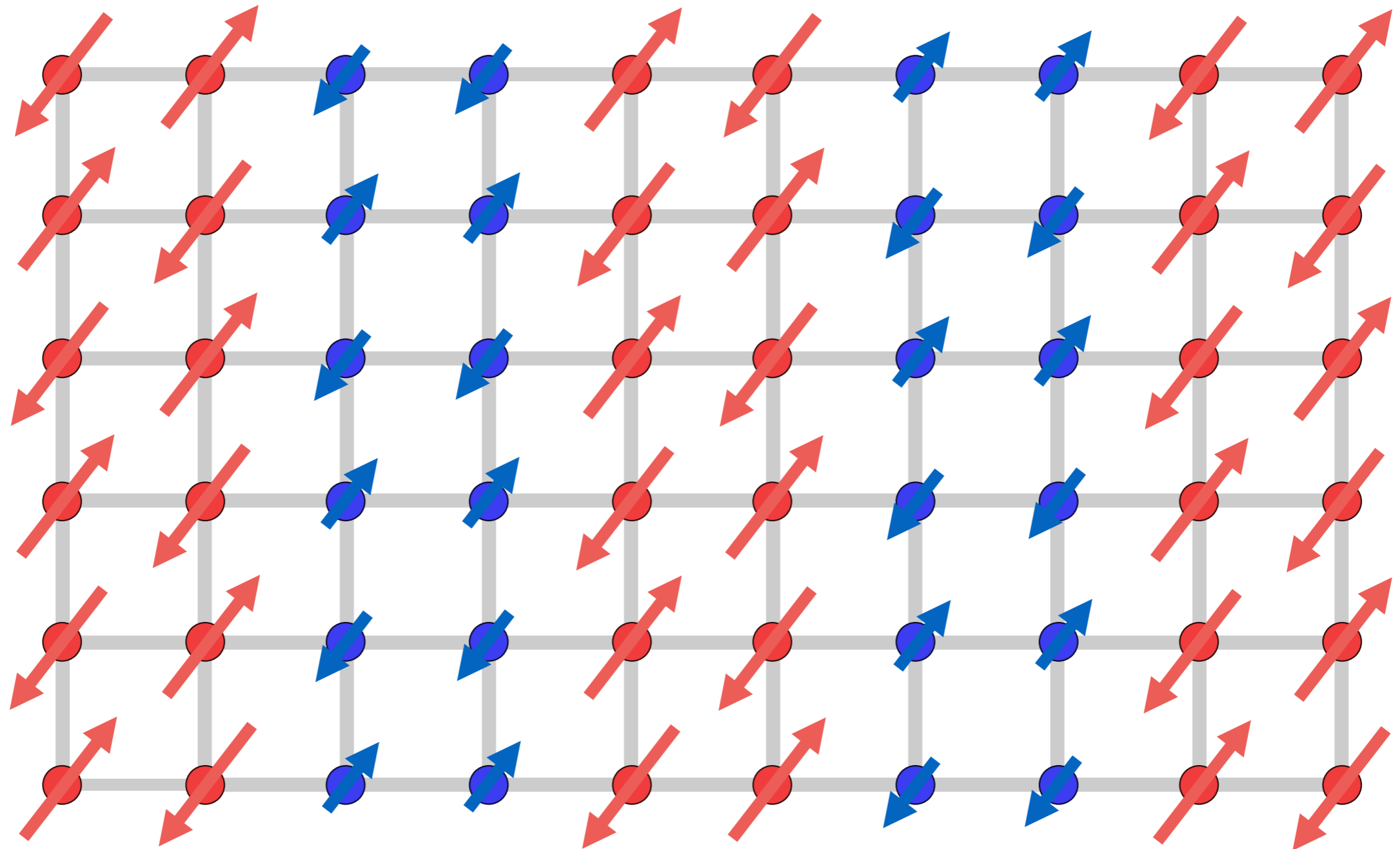
$\Delta\rho$

+1



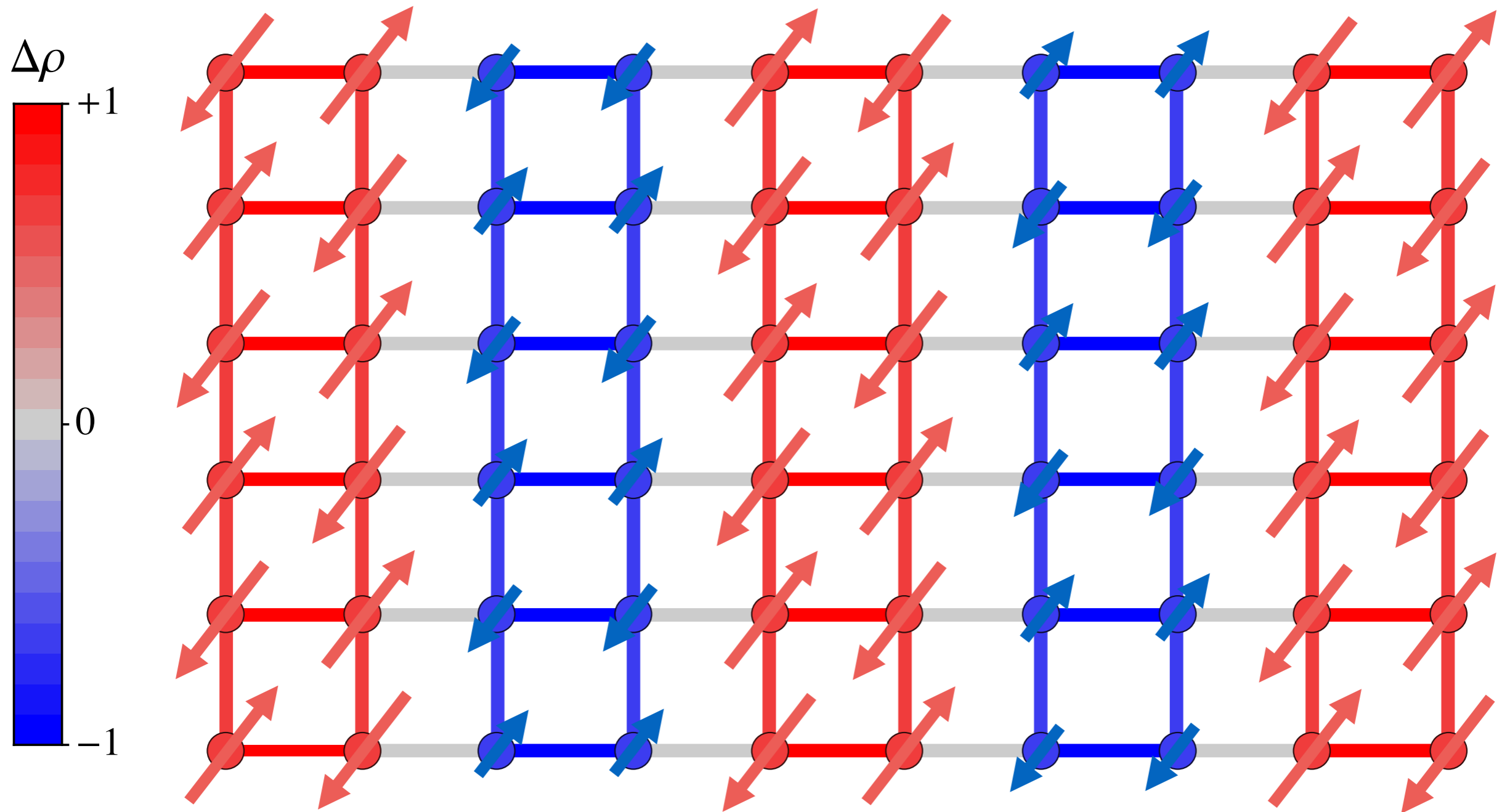
0

-1

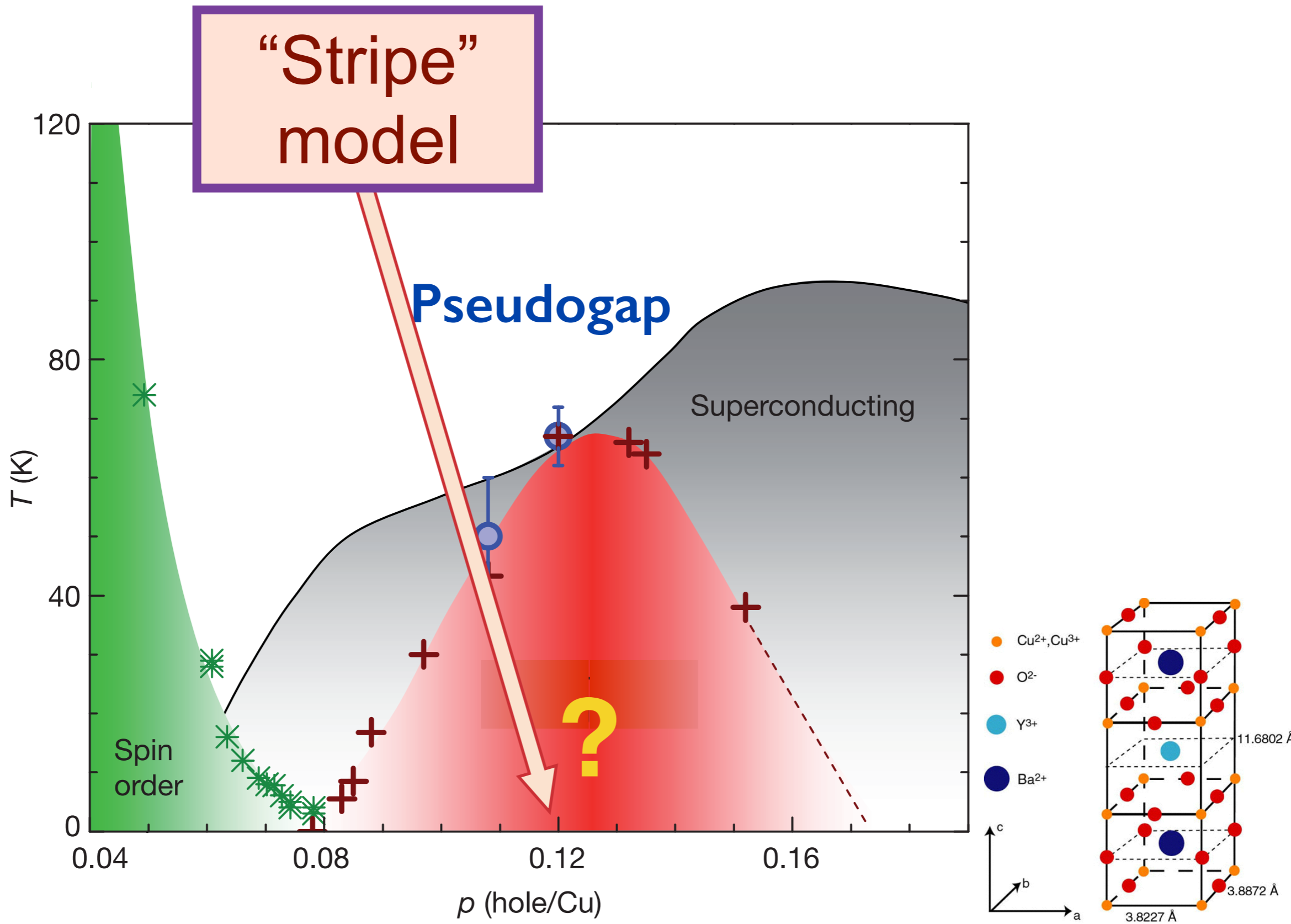


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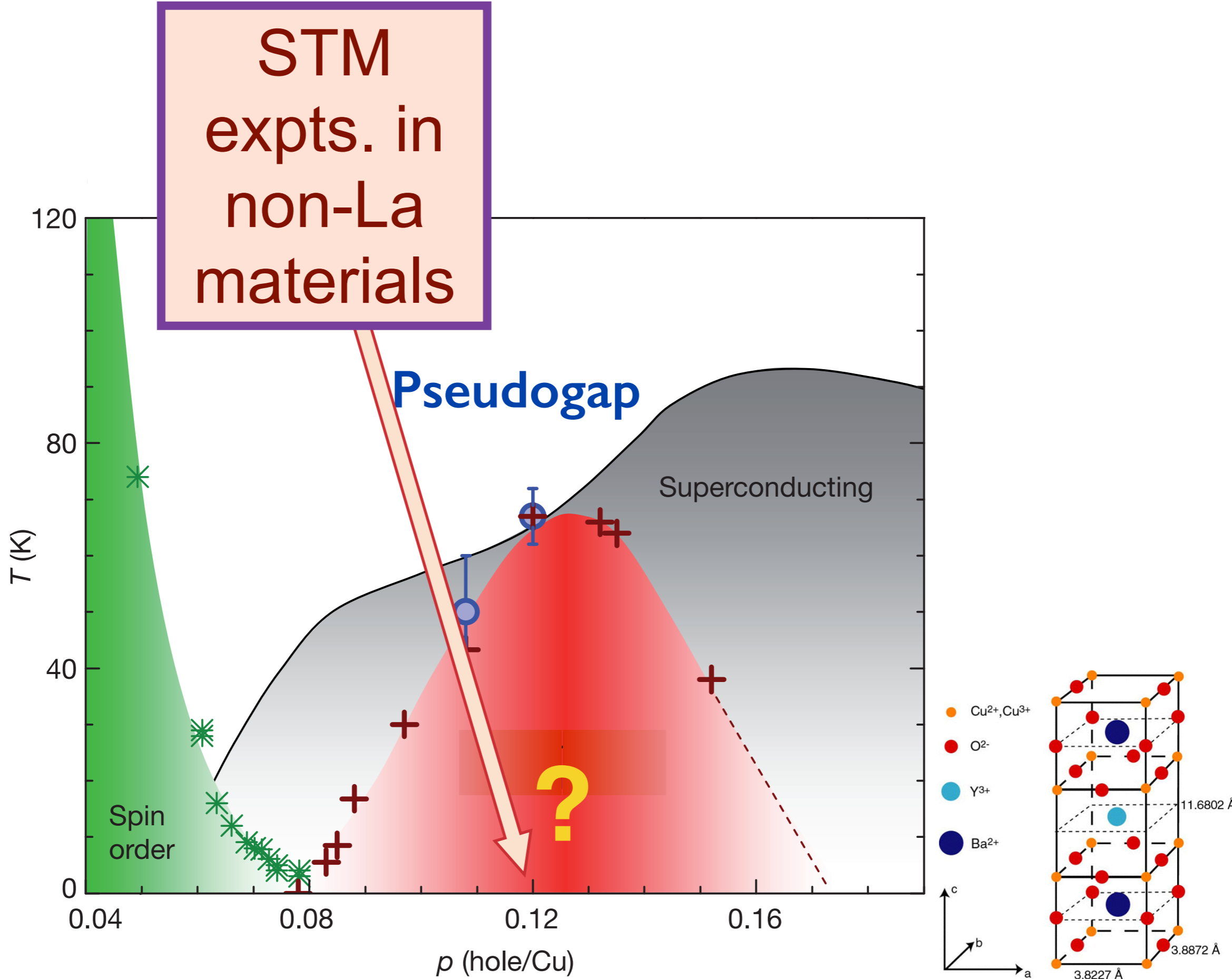
# “Stripe” model



Colors on the bonds map the local exchange energy



T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



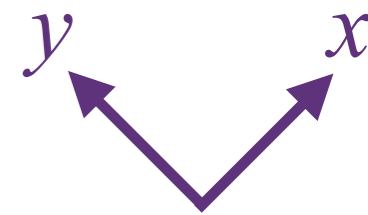
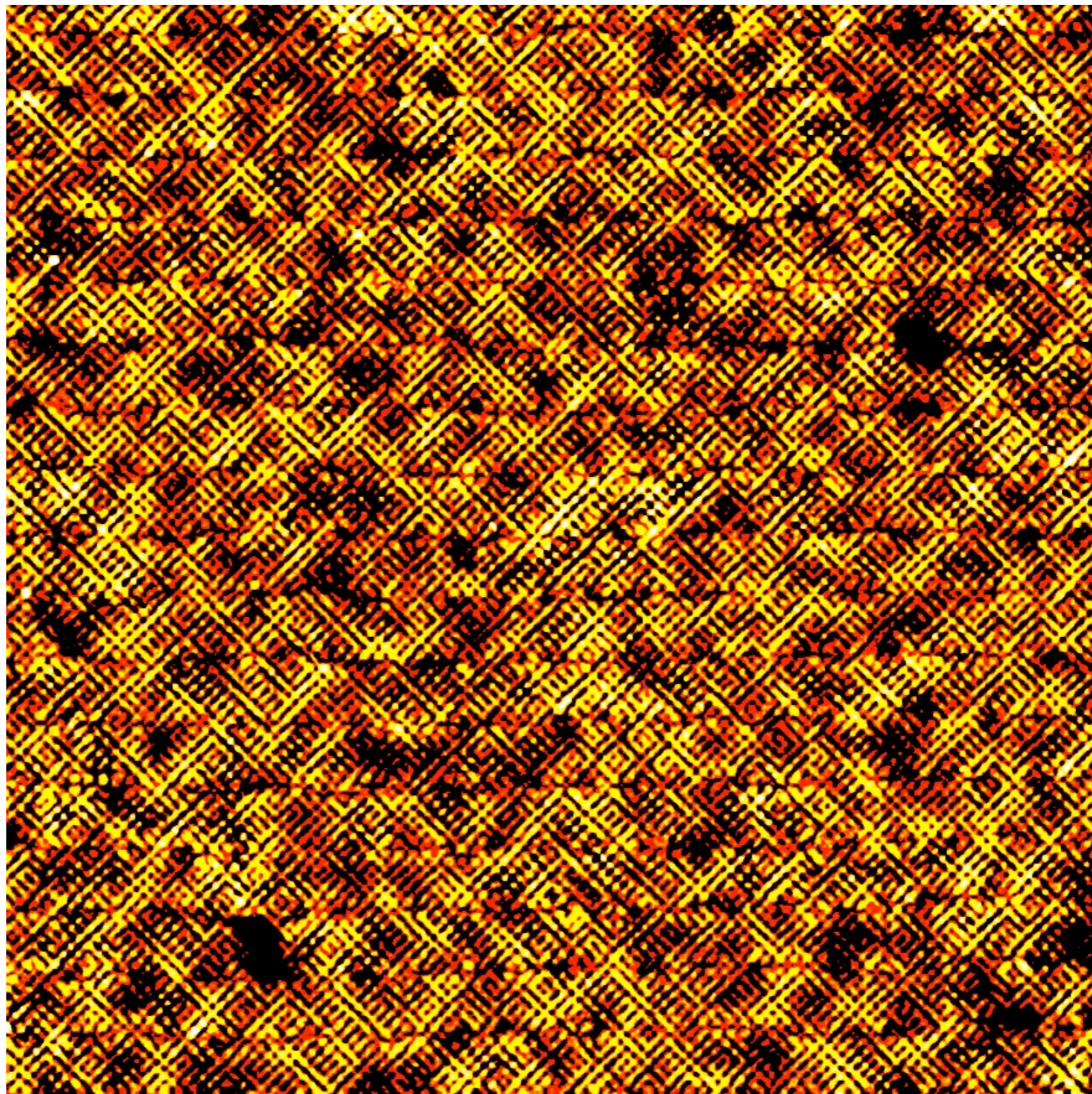
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See also

C. Howald, H. Eisaki,  
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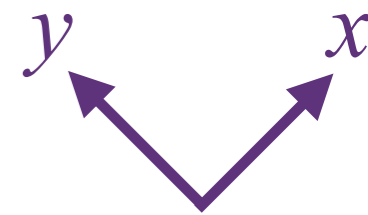
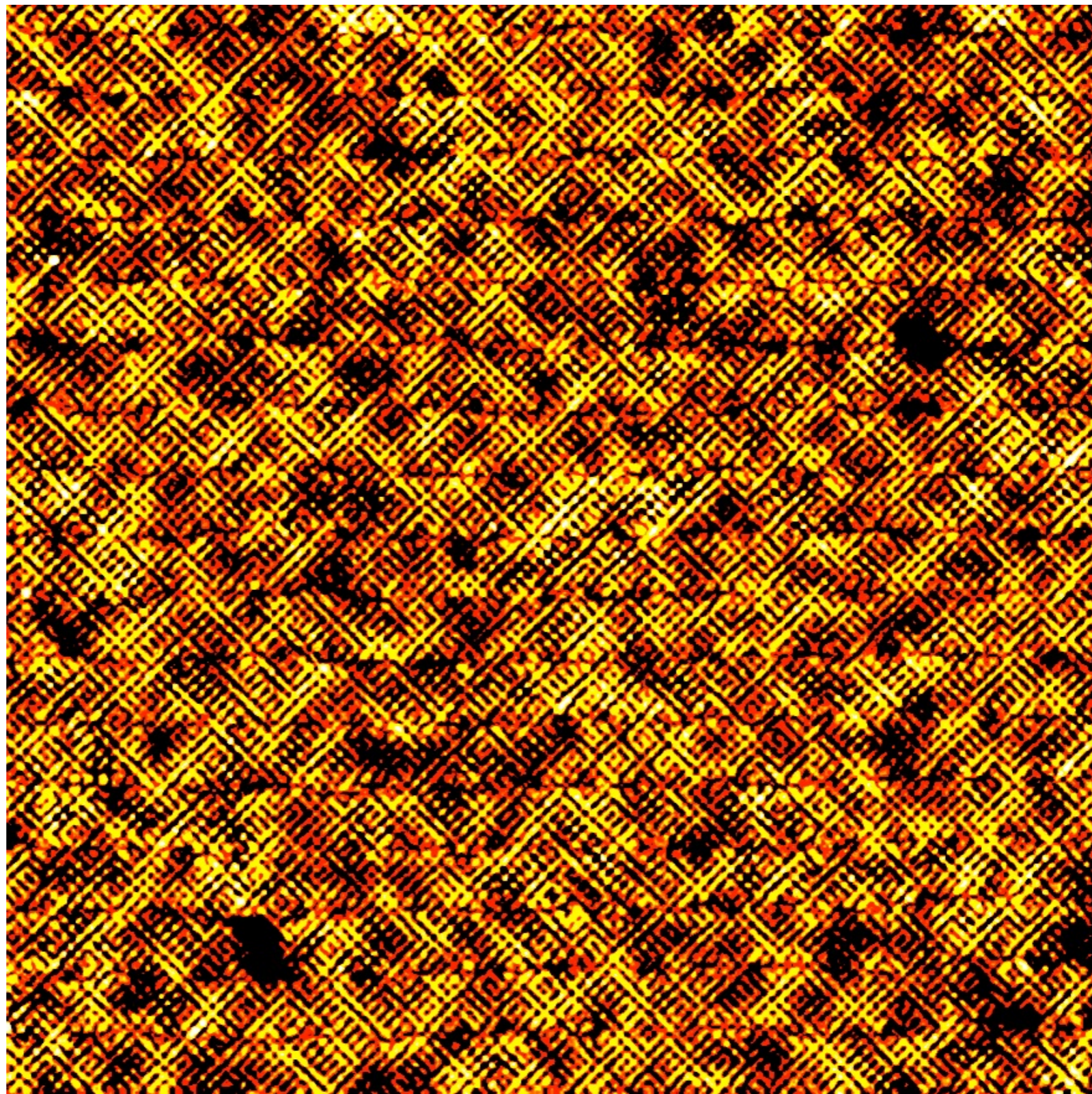
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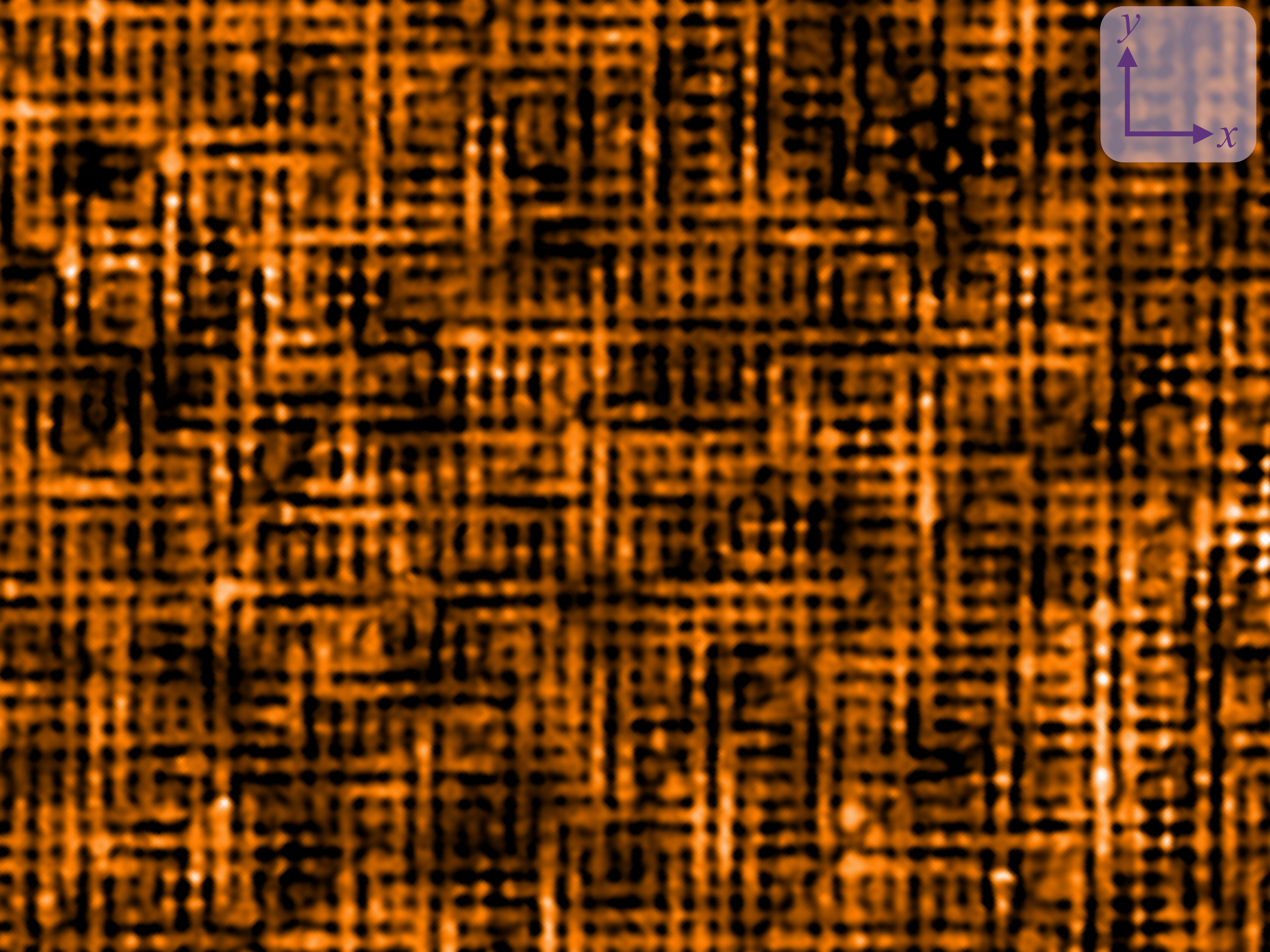
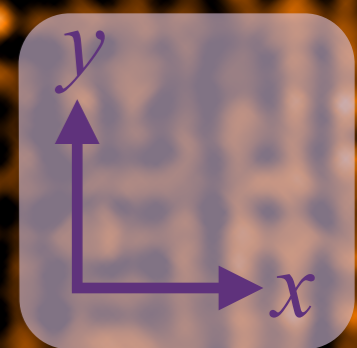
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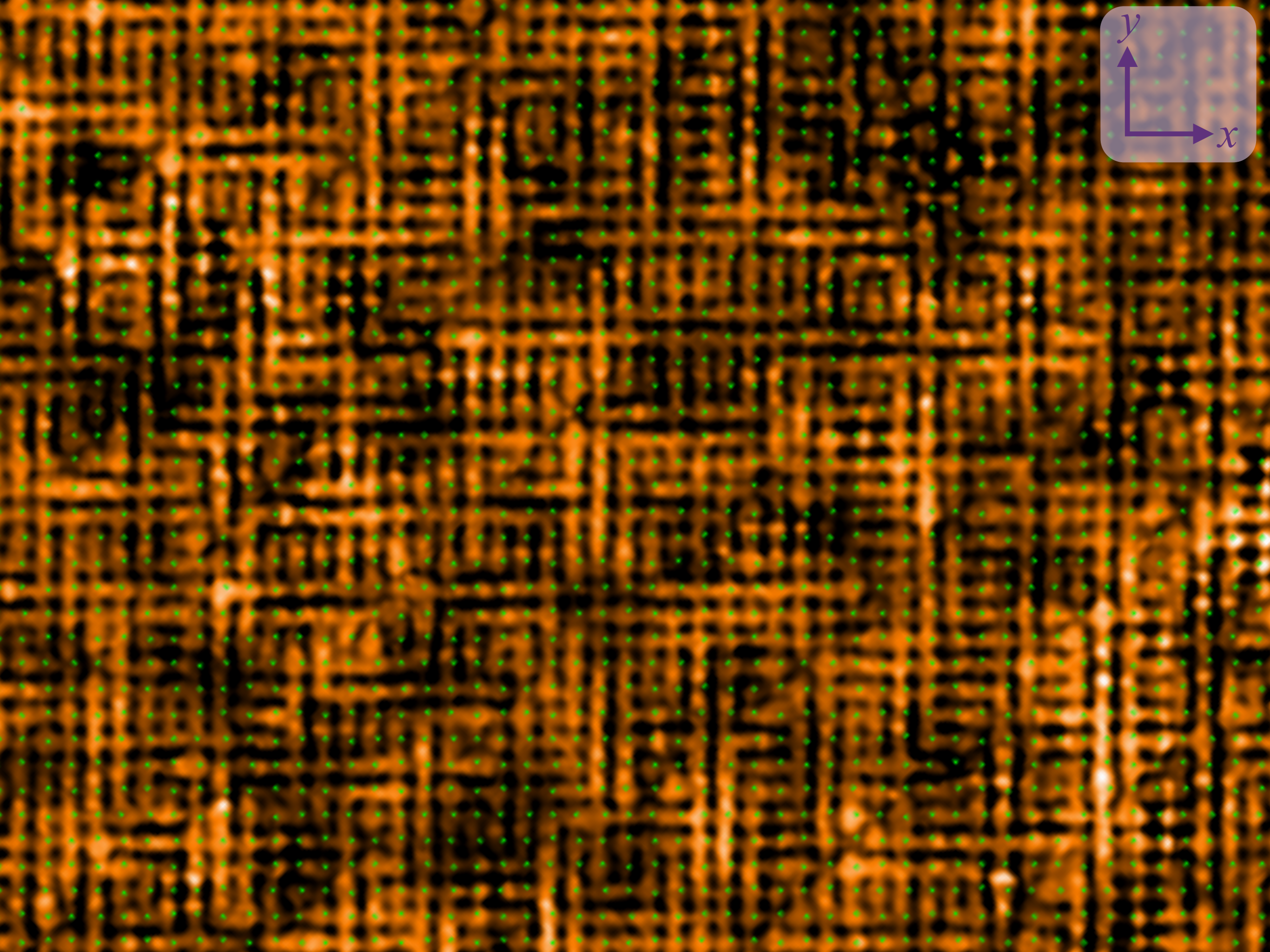
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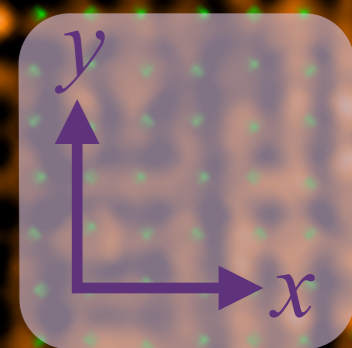


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A density wave with  
wavelength  $\approx 4$  lattice sites ?

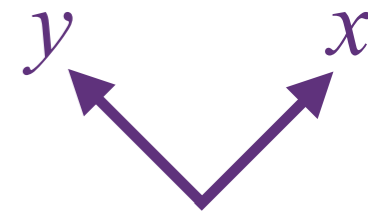
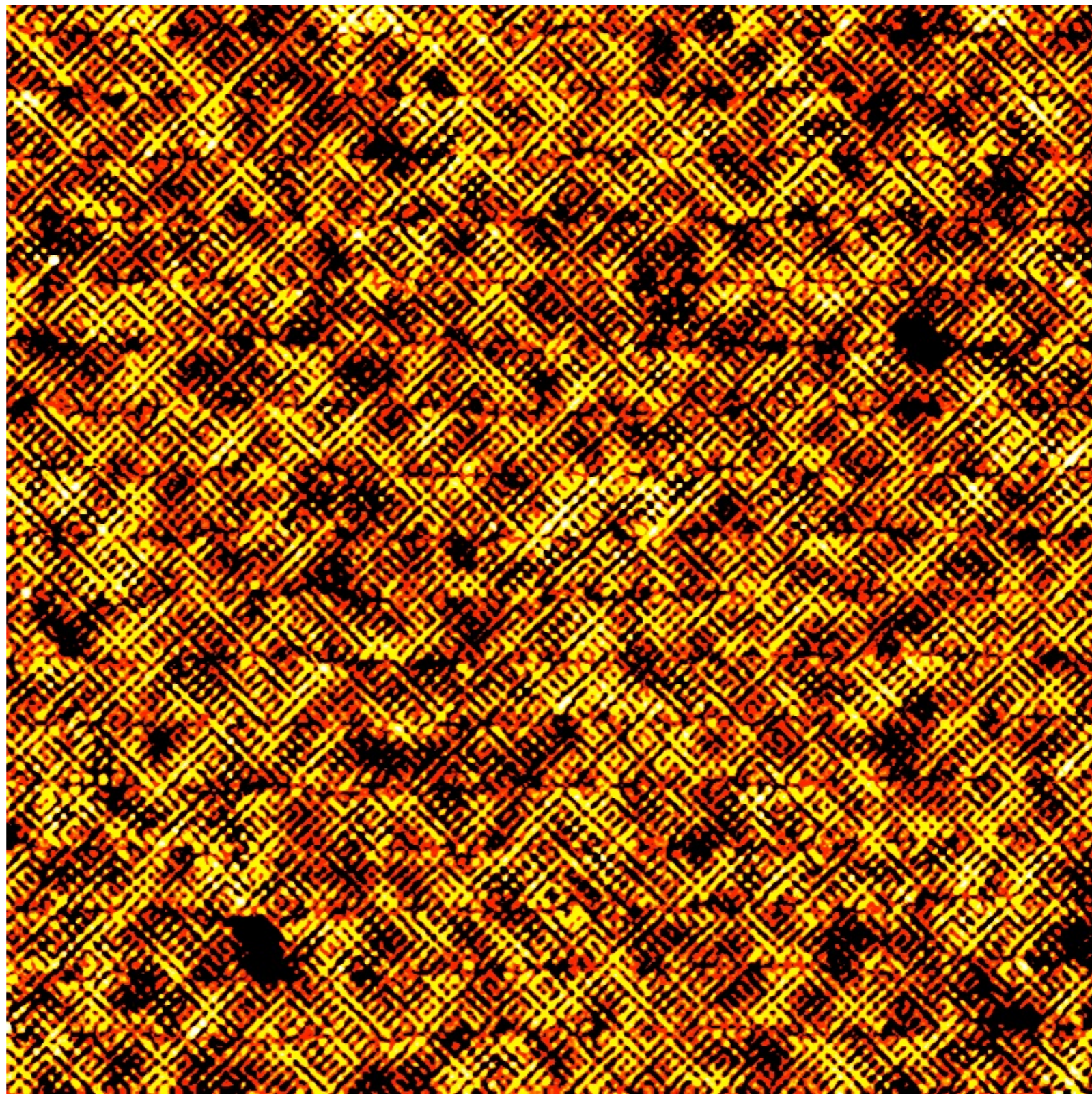


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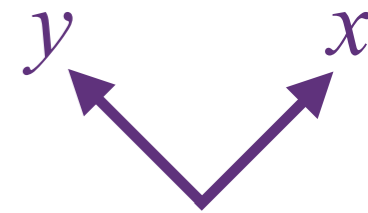
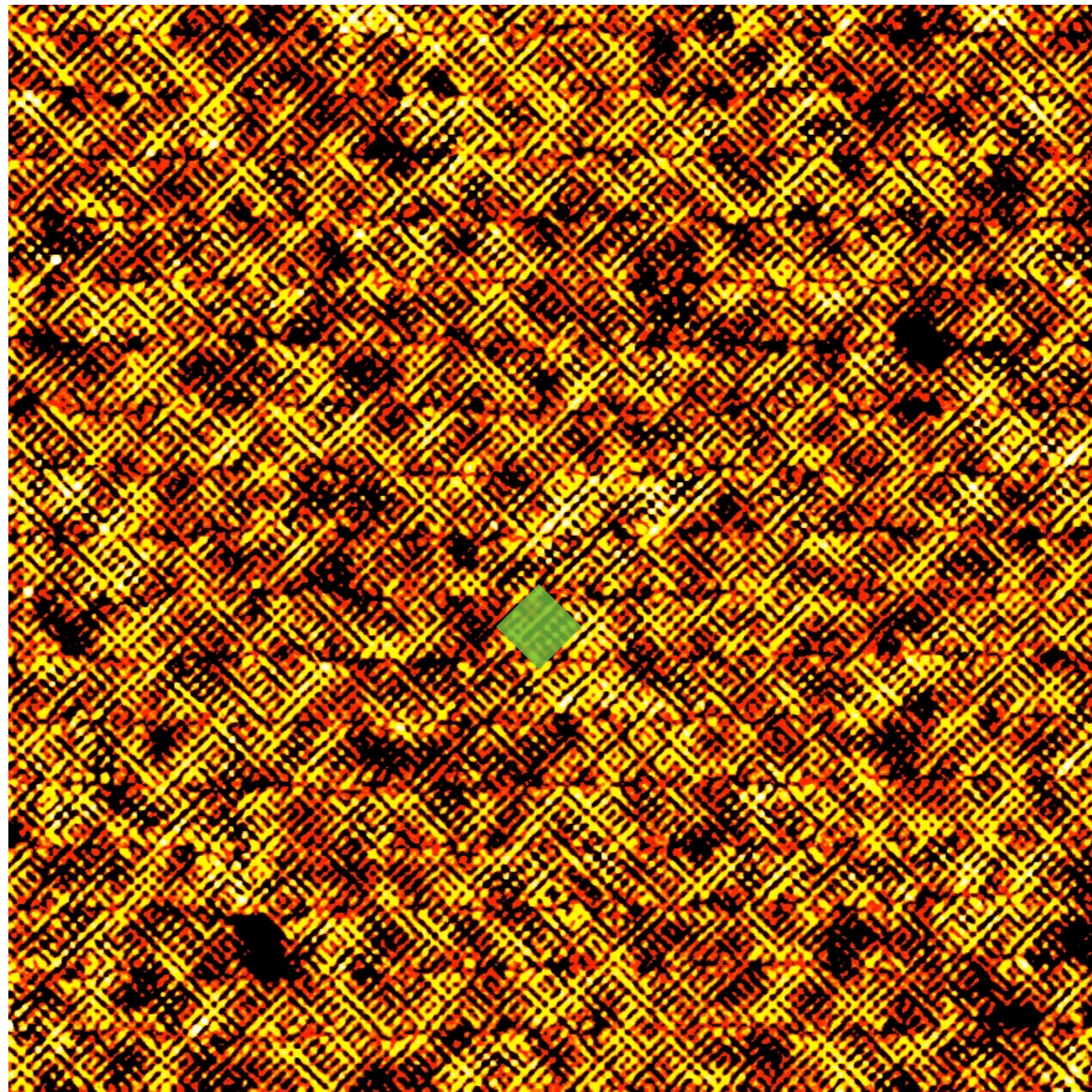
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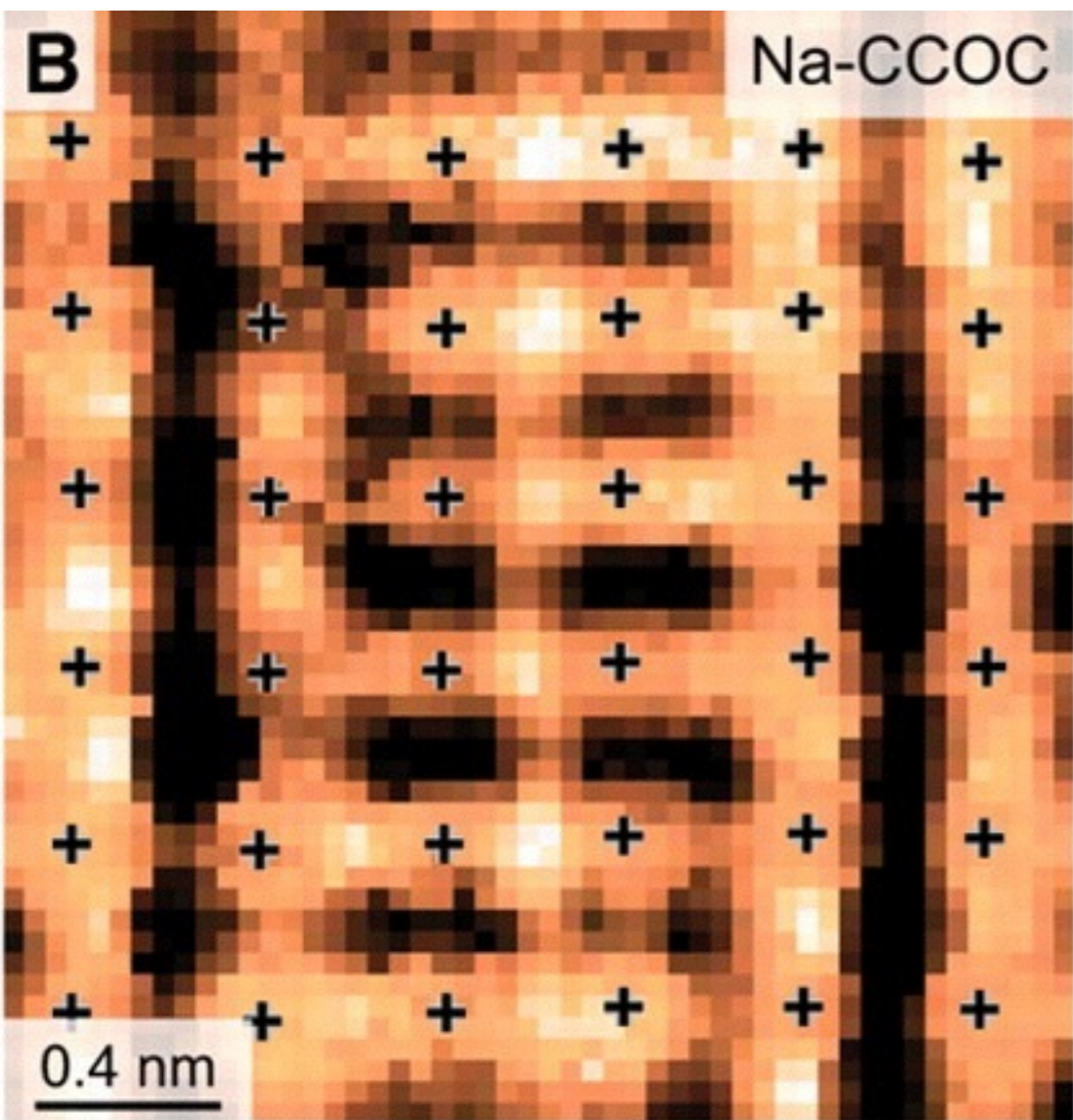
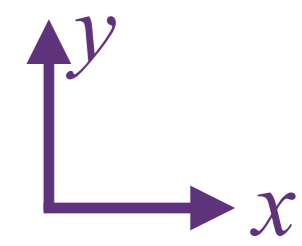
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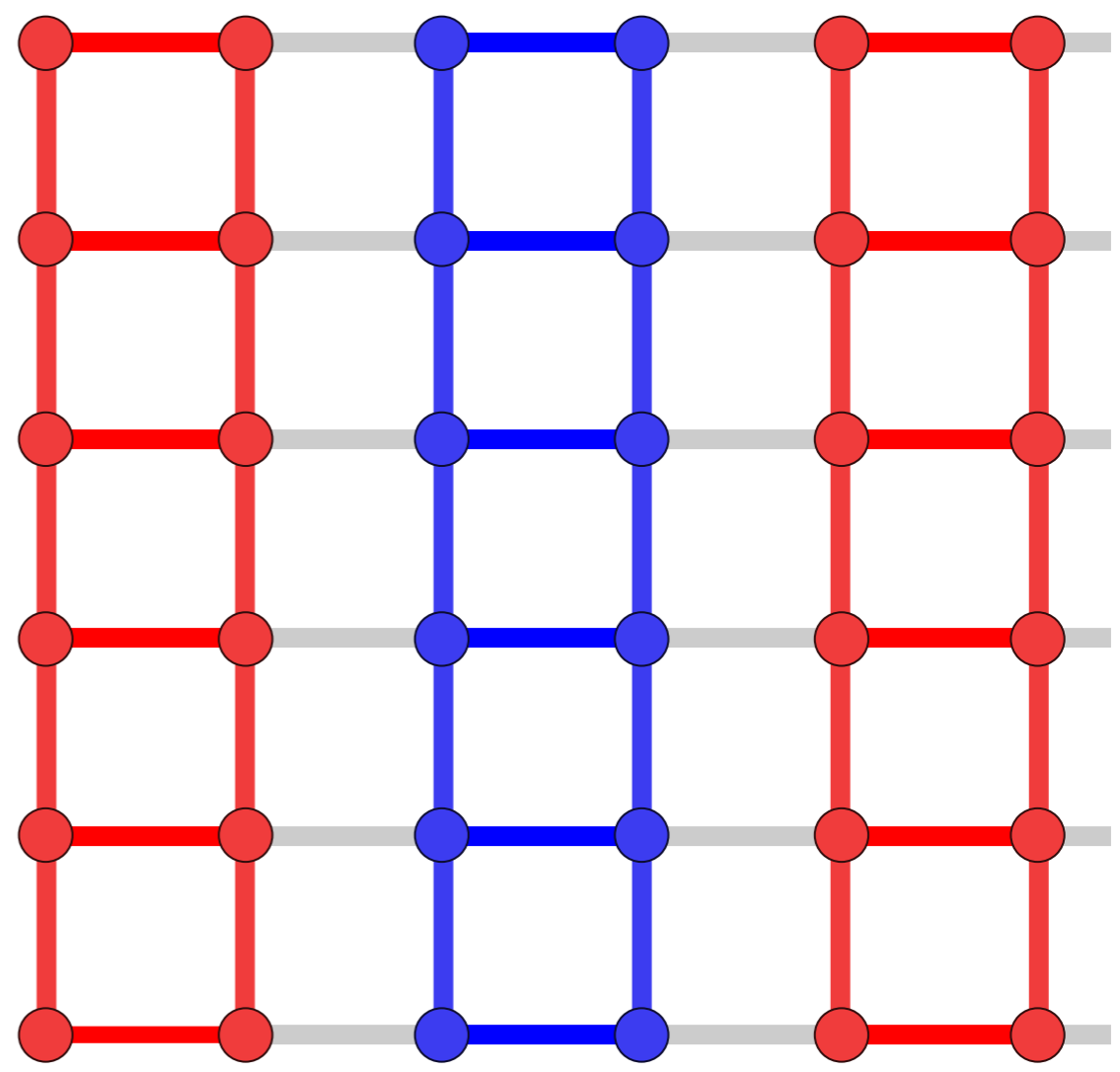
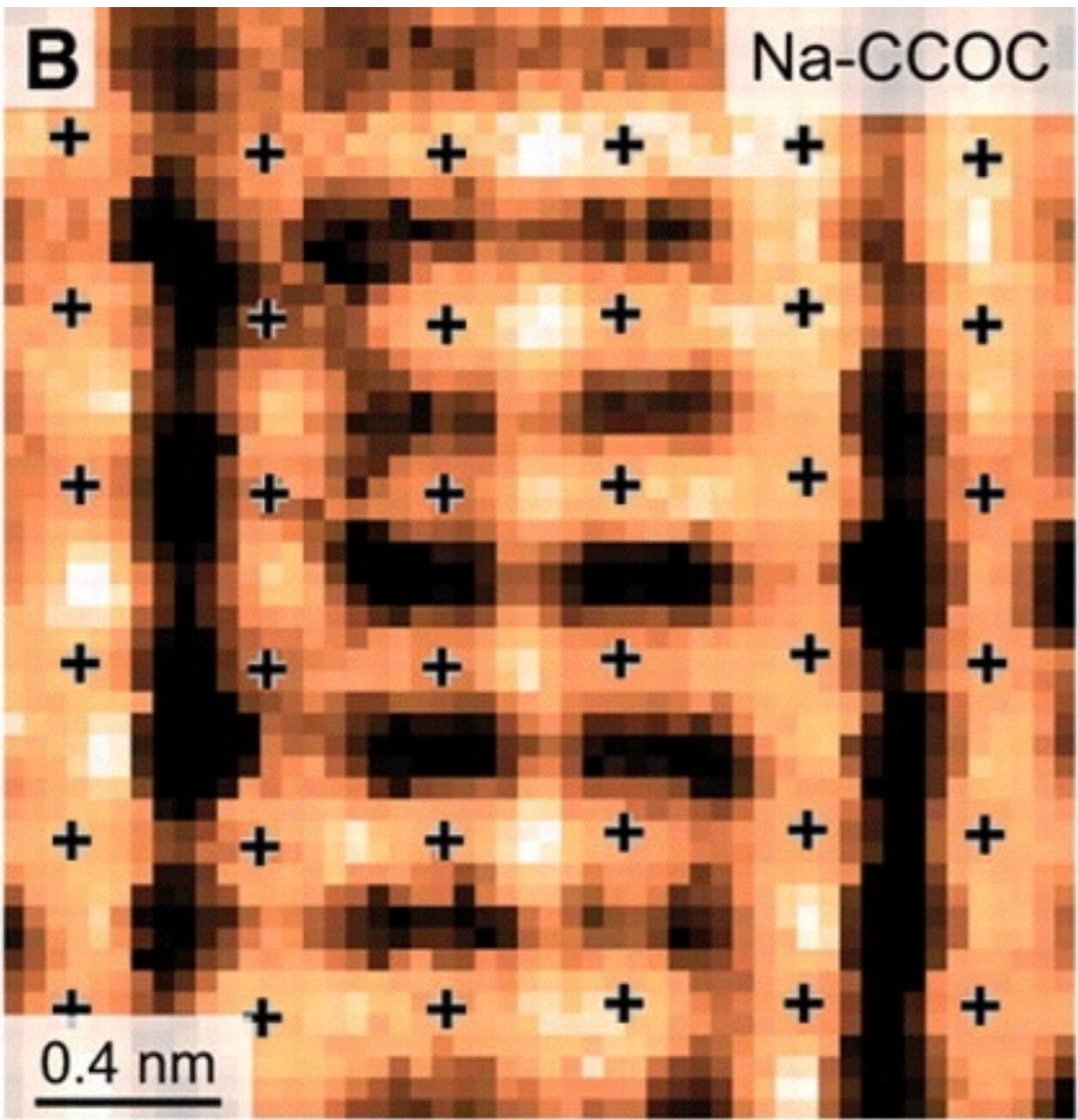
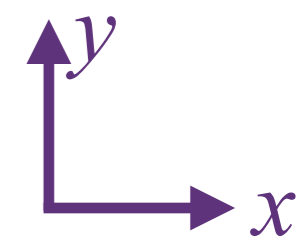
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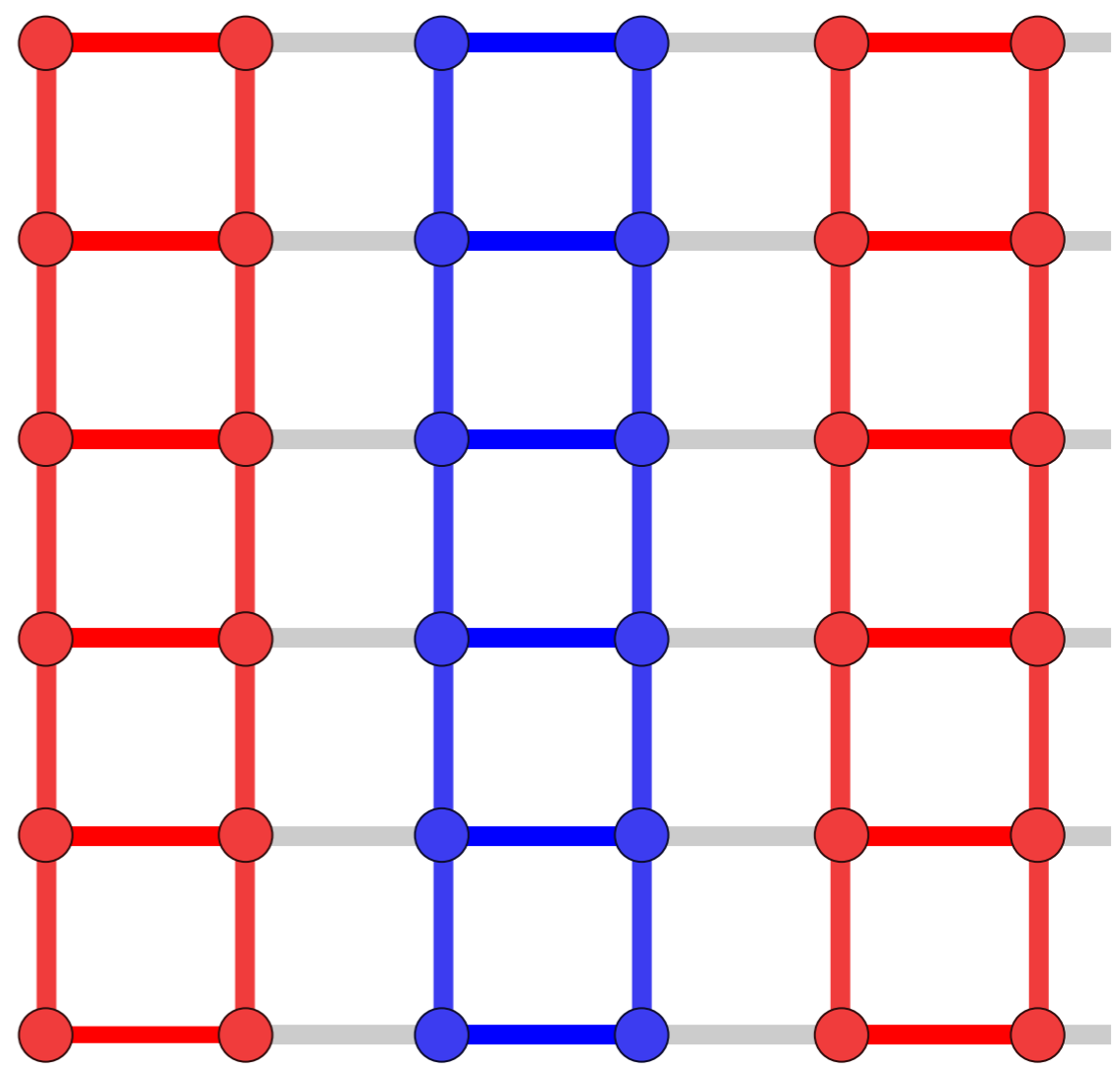
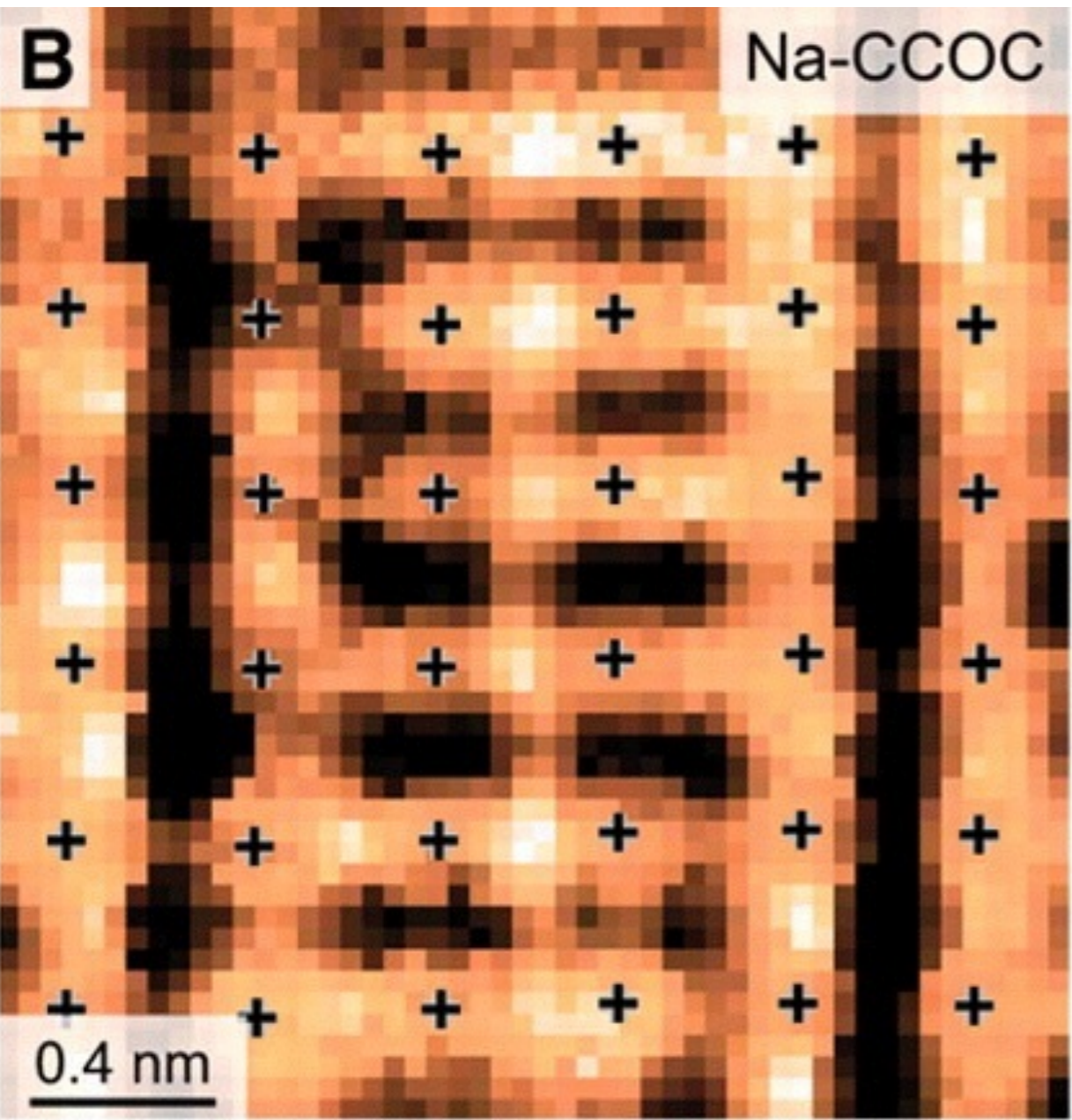
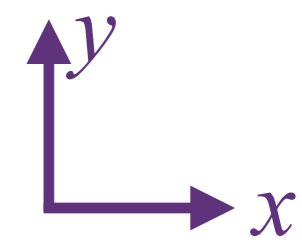


Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



“Stripe” model

Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



“Stripe” model

Microstructure of STM picture does not match “stripe” model

Y. Kohsaka *et al.*, SCIENCE 315, 1380 (2007)

# Charge density wave (CDW) order

$$\langle c_{\alpha}^{\dagger}(\mathbf{r})c_{\alpha}(\mathbf{r}) \rangle = \Psi_{CDW}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$

## Charge density wave (CDW) order

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Nearly constant CDW order parameter

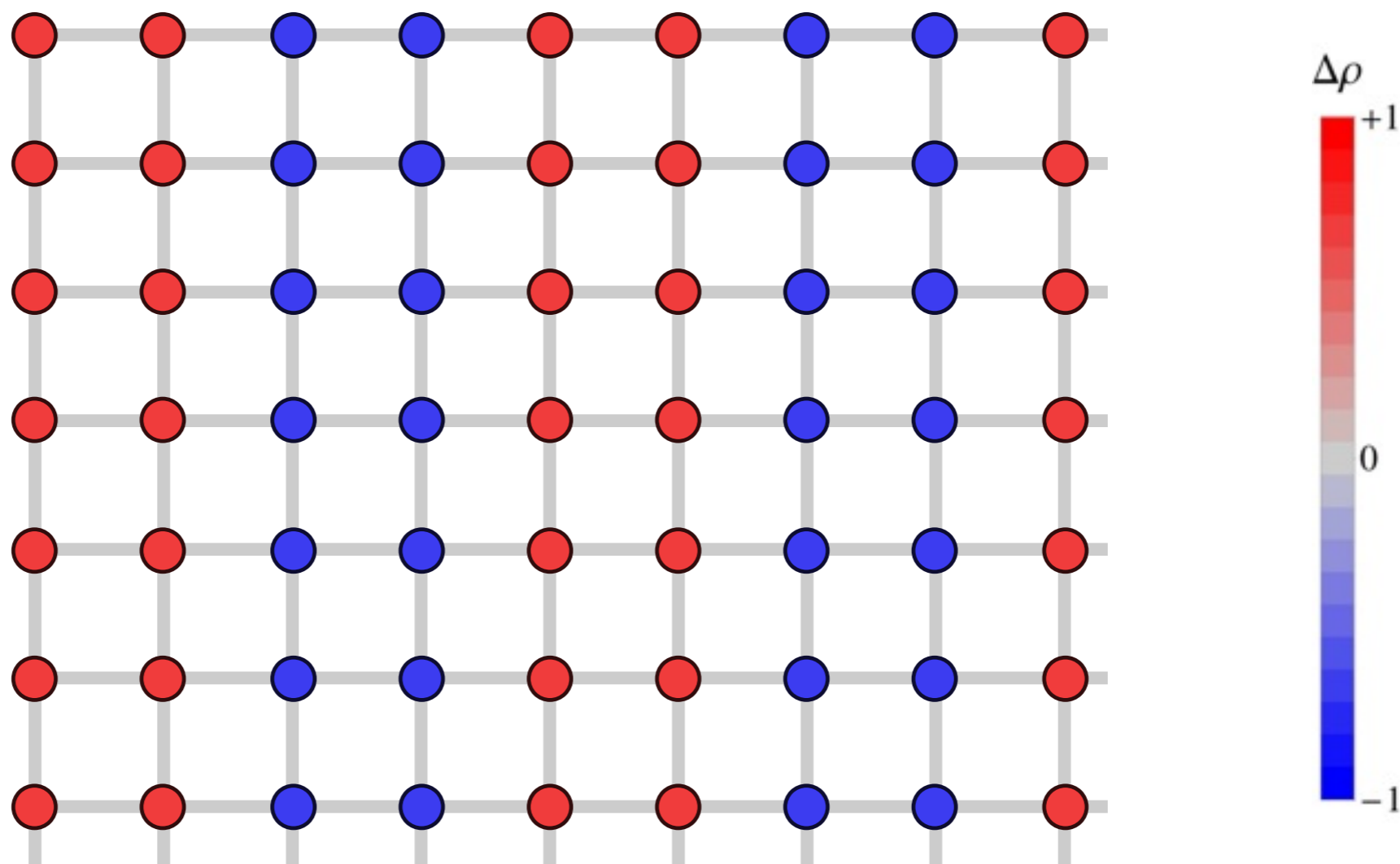
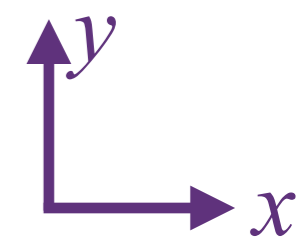


# CDW order.

Plot of  $P_{ii} = \langle c_{i\alpha}^\dagger c_{i\alpha} \rangle$  with

$$P_{ii} = e^{i\mathbf{Q}\cdot\mathbf{r}_i} + \text{c.c.}$$

$$\text{with } \mathbf{Q} = 2\pi(1/4, 0)$$



# Charge density wave (CDW) order

$$\langle c_{\alpha}^{\dagger}(\mathbf{r})c_{\alpha}(\mathbf{r}) \rangle = \Psi_{CDW}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} + \text{c.c.}$$



Nearly constant CDW order parameter

Unconventional density wave (DW) :  
Bose condensation of particle-hole pairs

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle$$
$$= \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}$$

Nearly constant CDW order parameter

**Unconventional density wave (DW) :**  
**Bose condensation of particle-hole pairs**

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle = \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}$$

Crucial “center-of-mass” co-ordinate.  
(Not used in previous work)  
Simplifies action of time-reversal

Unconventional density wave (DW) :  
Bose condensation of particle-hole pairs

$$\langle c_{\alpha}^{\dagger}(\mathbf{r}_1)c_{\alpha}(\mathbf{r}_2) \rangle = \left[ \mathcal{P}(\mathbf{r}_1 - \mathbf{r}_2) \right] \times \Psi_{DW} \left( \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} \right) e^{i\mathbf{Q} \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} + \text{c.c.}$$

Density wave form factor (internal particle-hole pair wavefunction)

$$\mathcal{P}(\mathbf{r}) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$$

Time-reversal symmetry requires  $\mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k})$ .

We expand (using reflection symmetry for  $\mathbf{Q}$  along axes or diagonals)

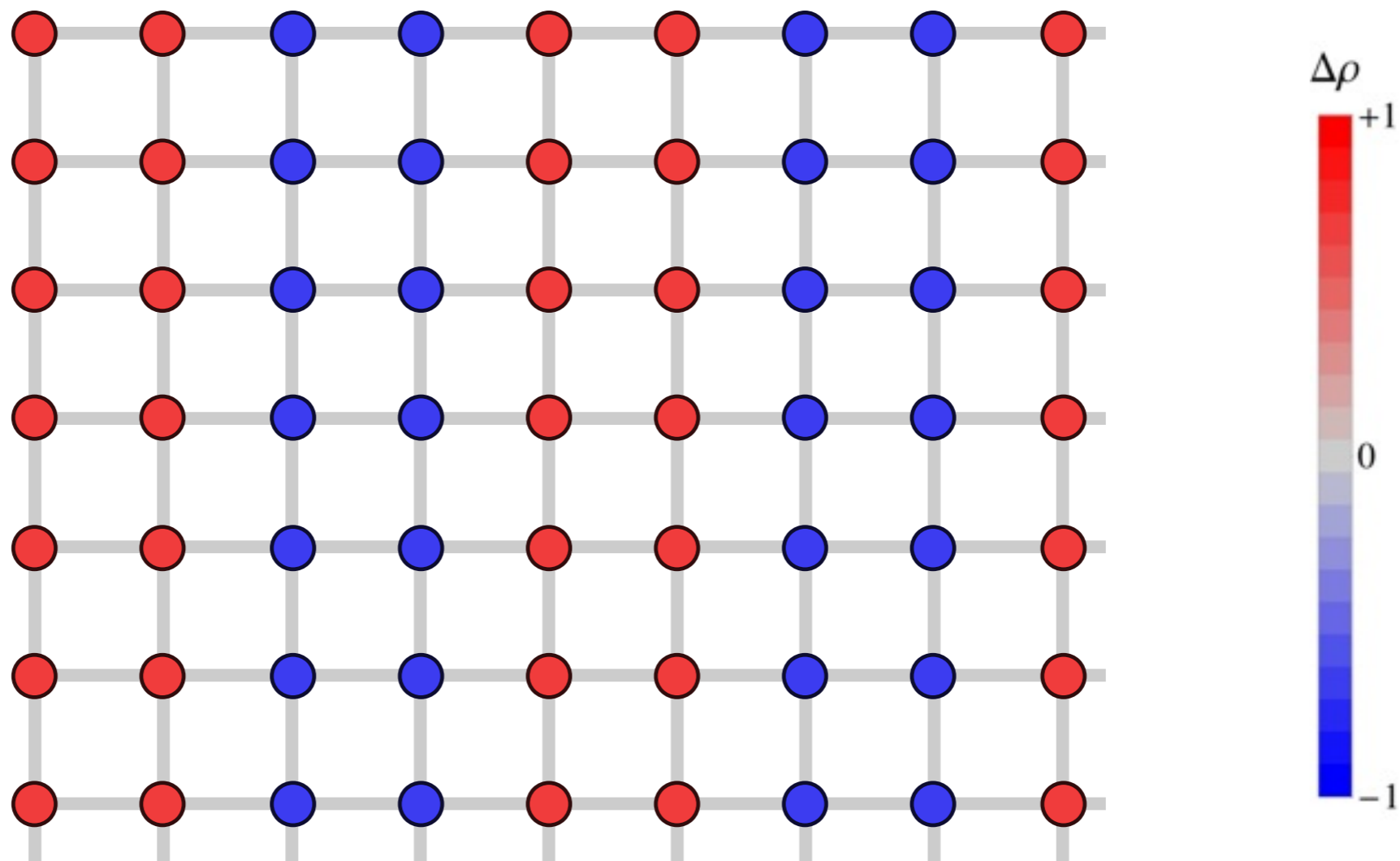
$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_{s'}(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)$$

# Conventional CDW order: $s$ -form factor

Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = 1 \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

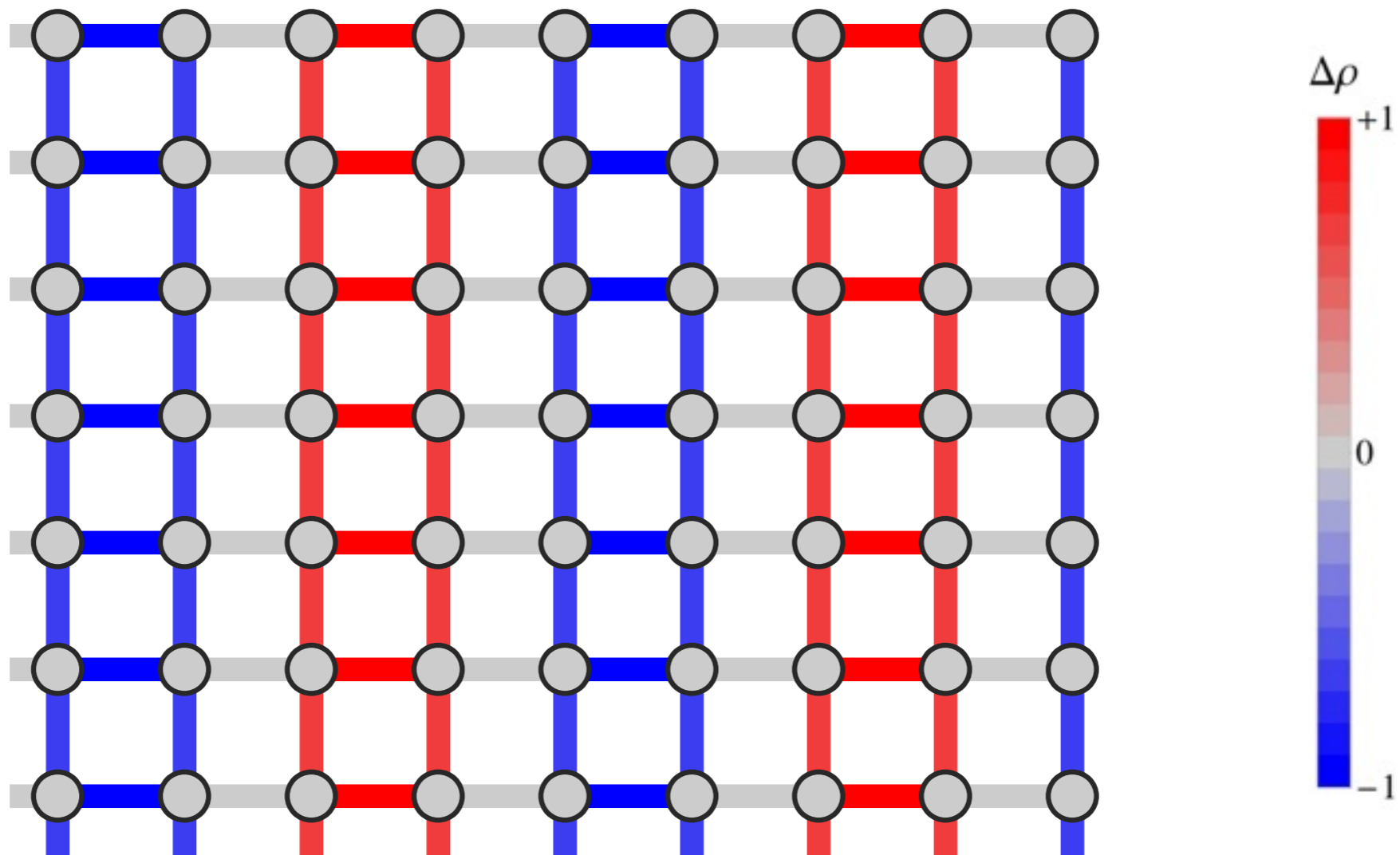


# Unconventional DW order: $s'$ -form factor

Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

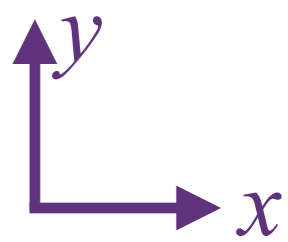


# Unconventional DW order: $s'$ -form factor

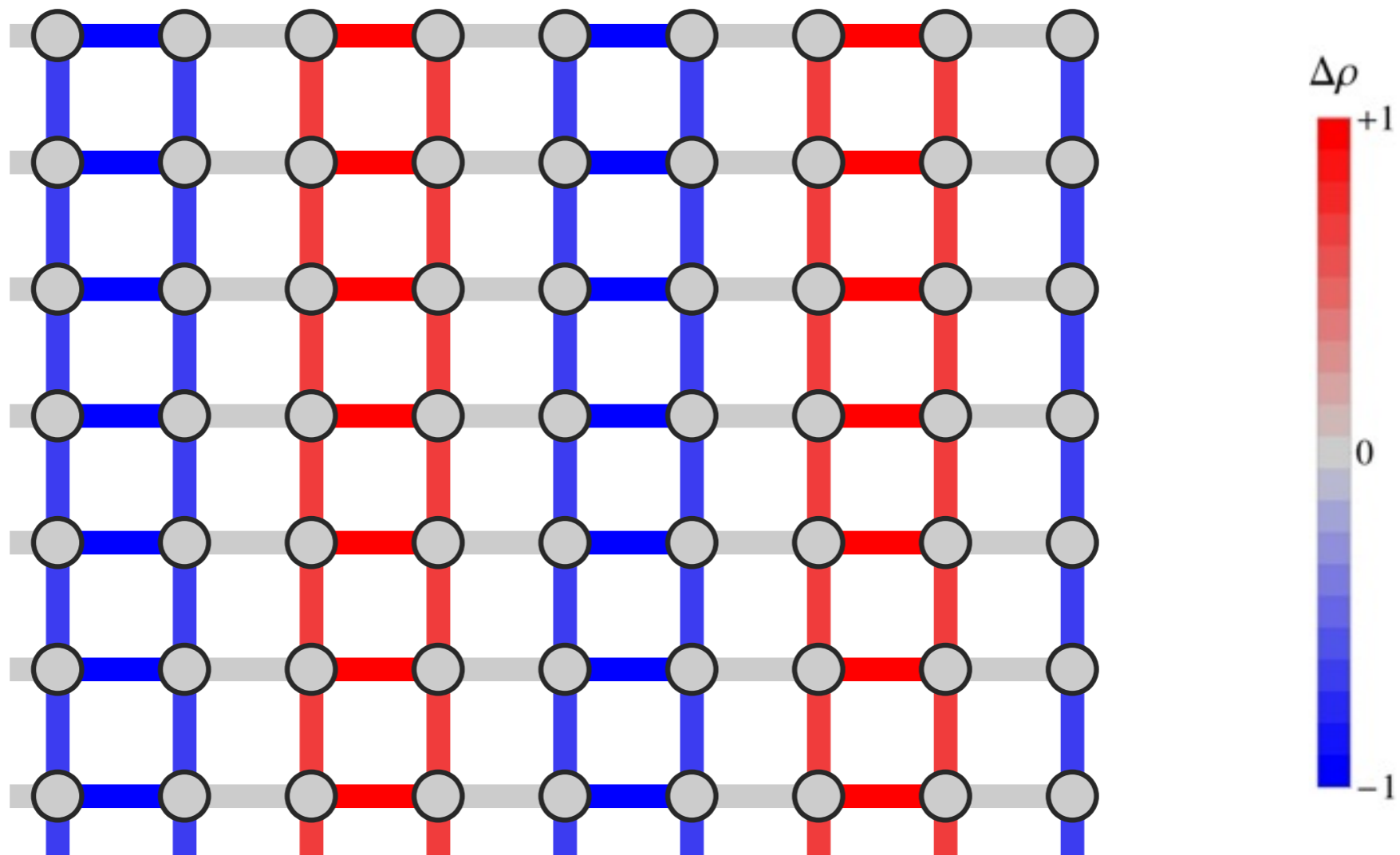
Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

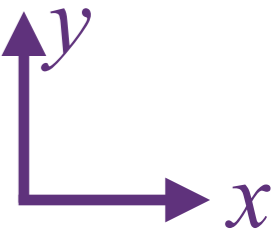


“Stripe”  
model !



# Unconventional DW order: $s'$ -form factor

Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

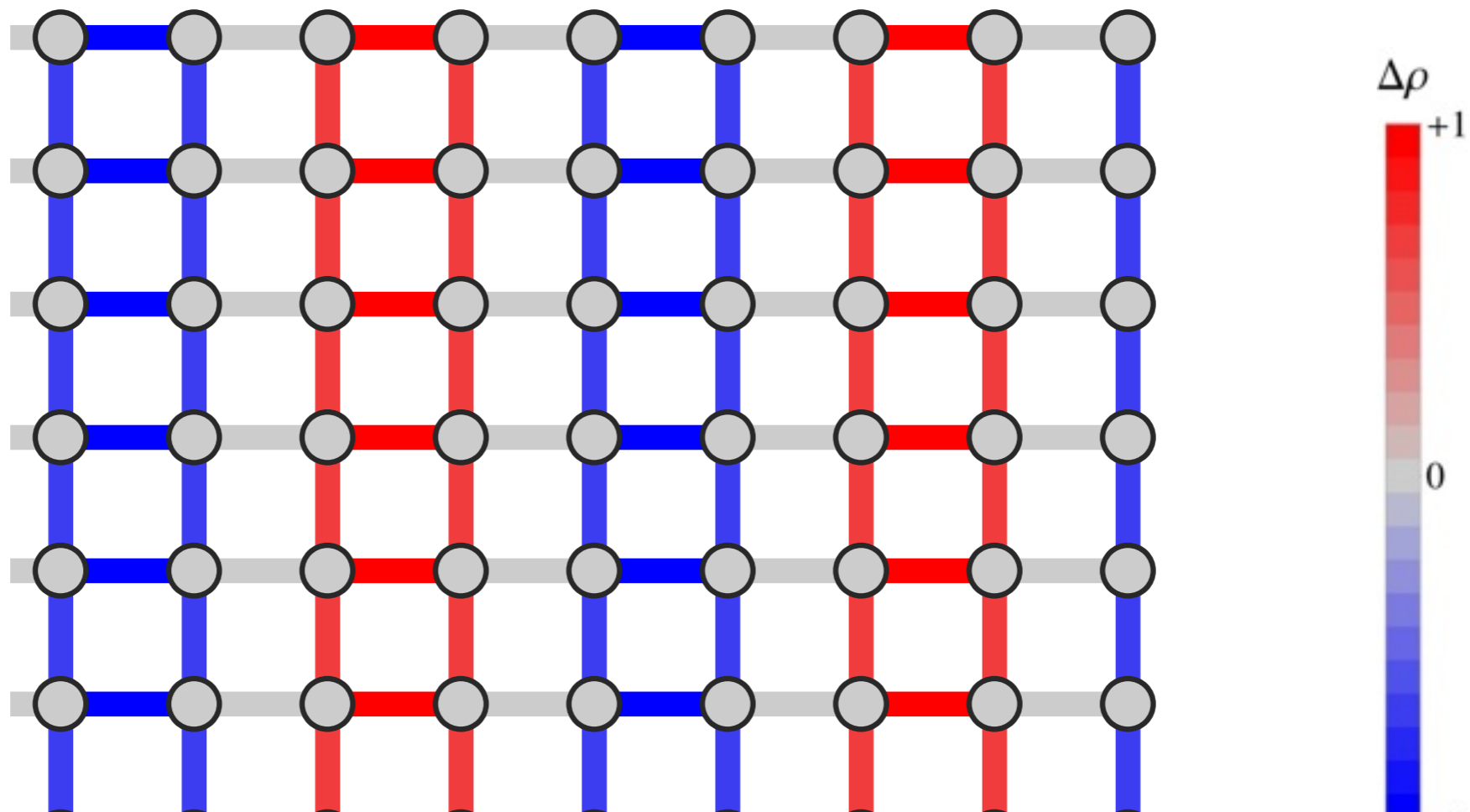


$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

“Stripe”  
model !

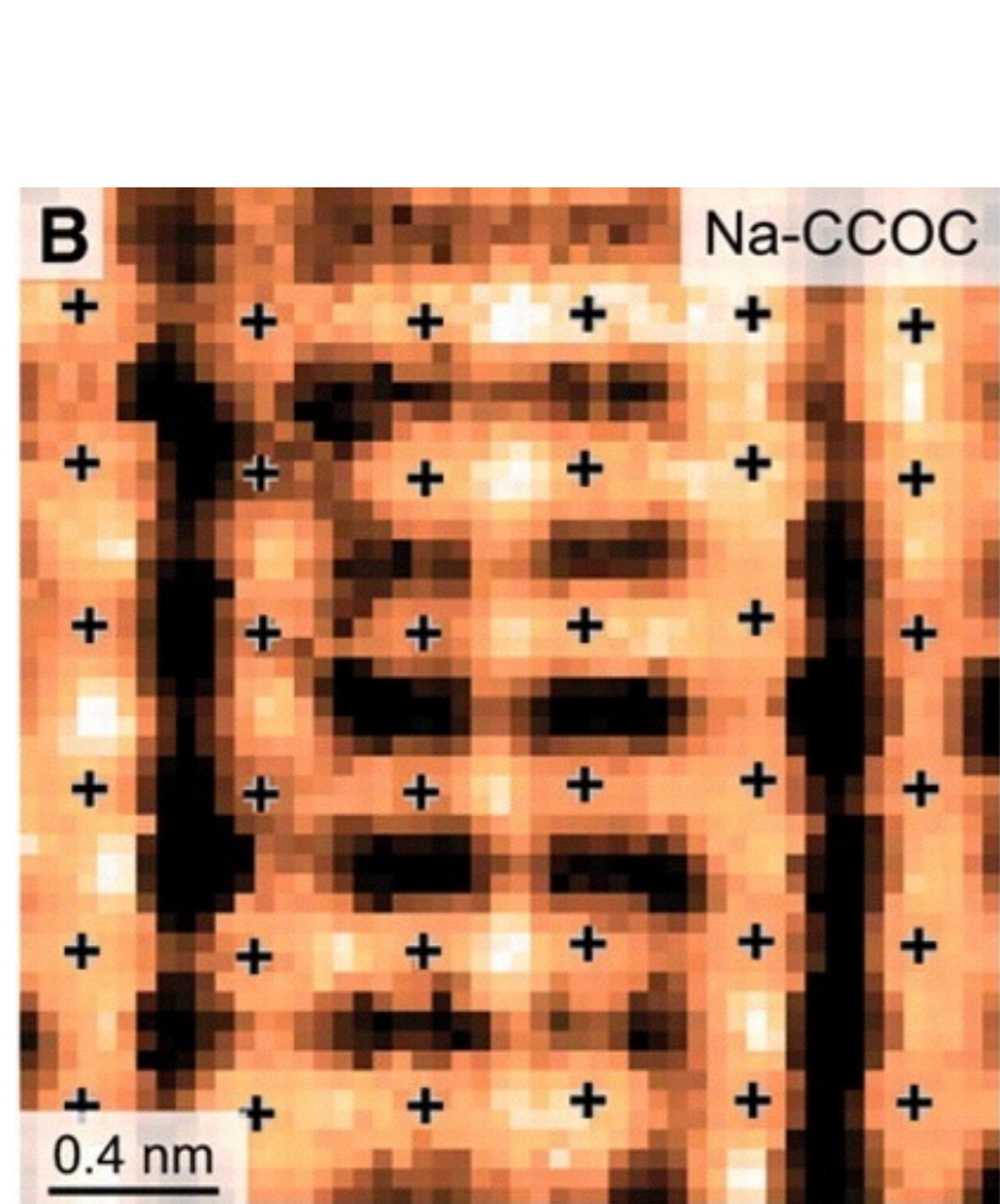
X-ray  
observations  
indicate  
strong  $s'$   
component in  
LBCO



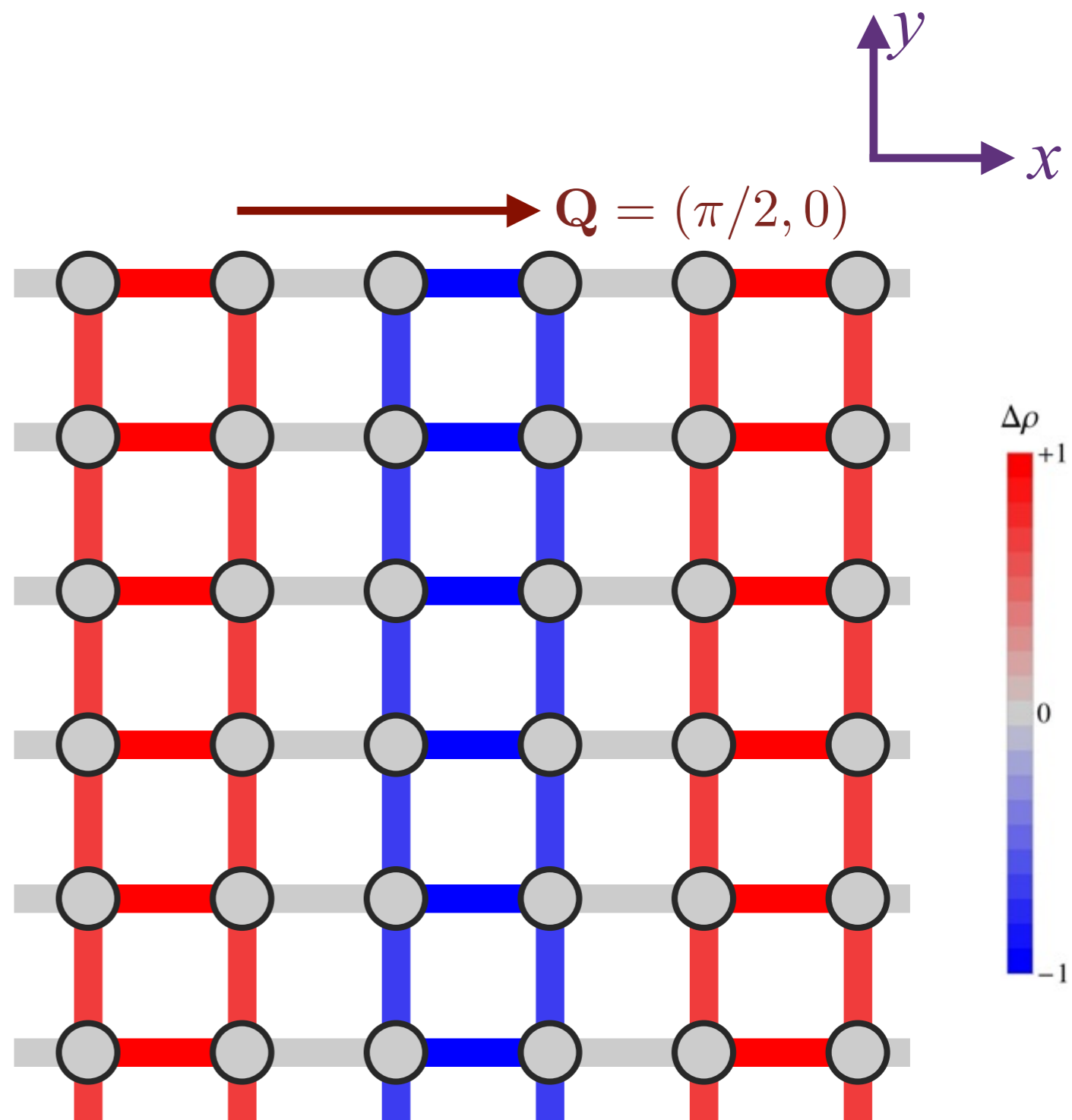
Orbital symmetry of charge density wave order in  $\text{La}_{1.88}\text{Ba}_{0.12}\text{CuO}_4$  and  $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$

A. J. Achkar,<sup>1</sup> F. He,<sup>2</sup> R. Sutarto,<sup>2</sup> Christopher McMahon,<sup>1</sup> M. Zwiebler,<sup>3</sup> M. Hücker,<sup>4</sup>  
G. Gu,<sup>4</sup> Ruixing Liang,<sup>5</sup> D. A. Bonn,<sup>5</sup> W. N. Hardy,<sup>5</sup> J. Geck,<sup>3</sup> and D. G. Hawthorn<sup>1</sup>

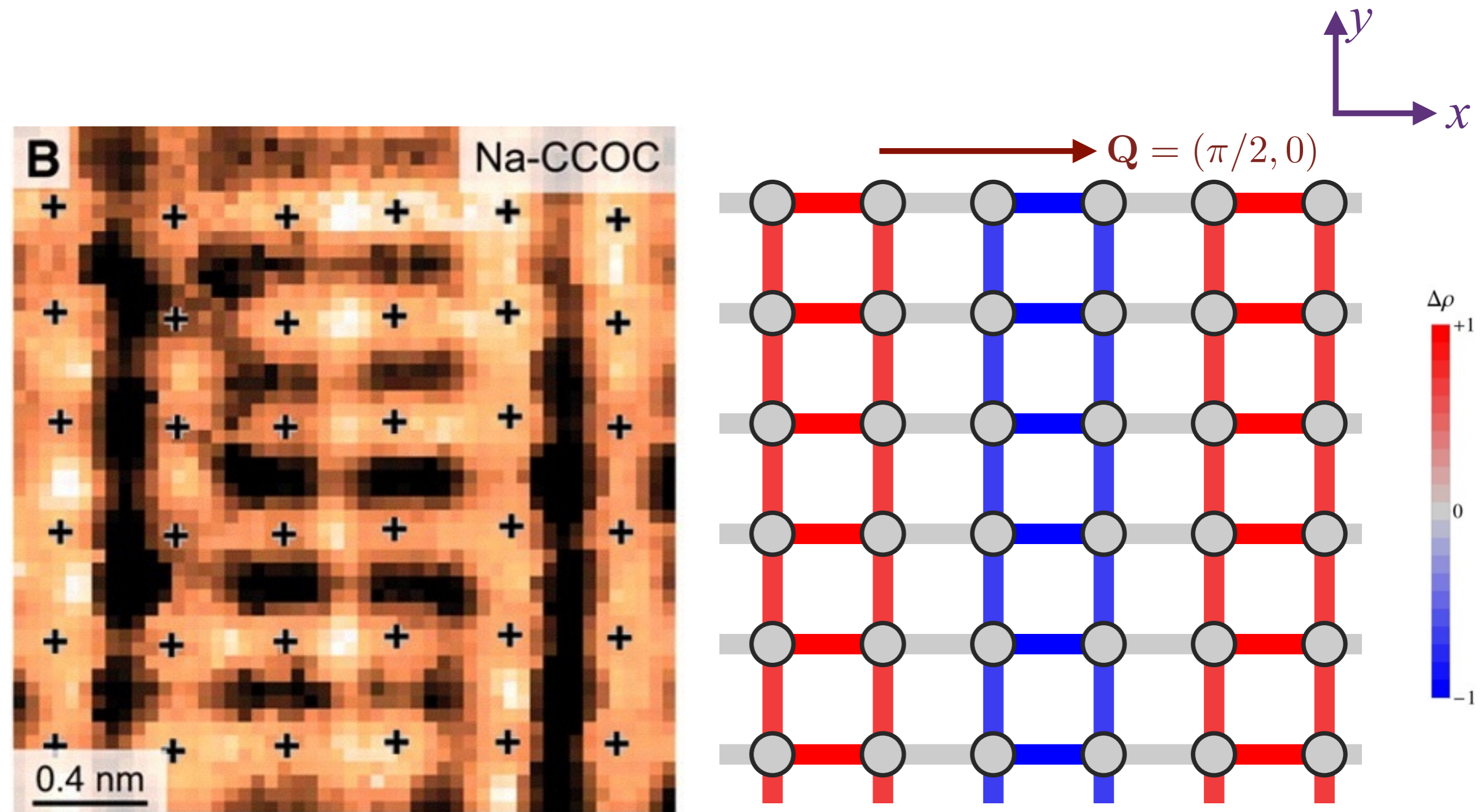
M. H. Fischer, Si Wu, M. Lawler, A. Paramakanti, and Eun-Ah Kim, arXiv:1406.2711



Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



$s'$ -form factor density wave order



Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

$s'$ -form factor density wave order

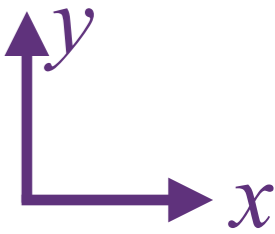
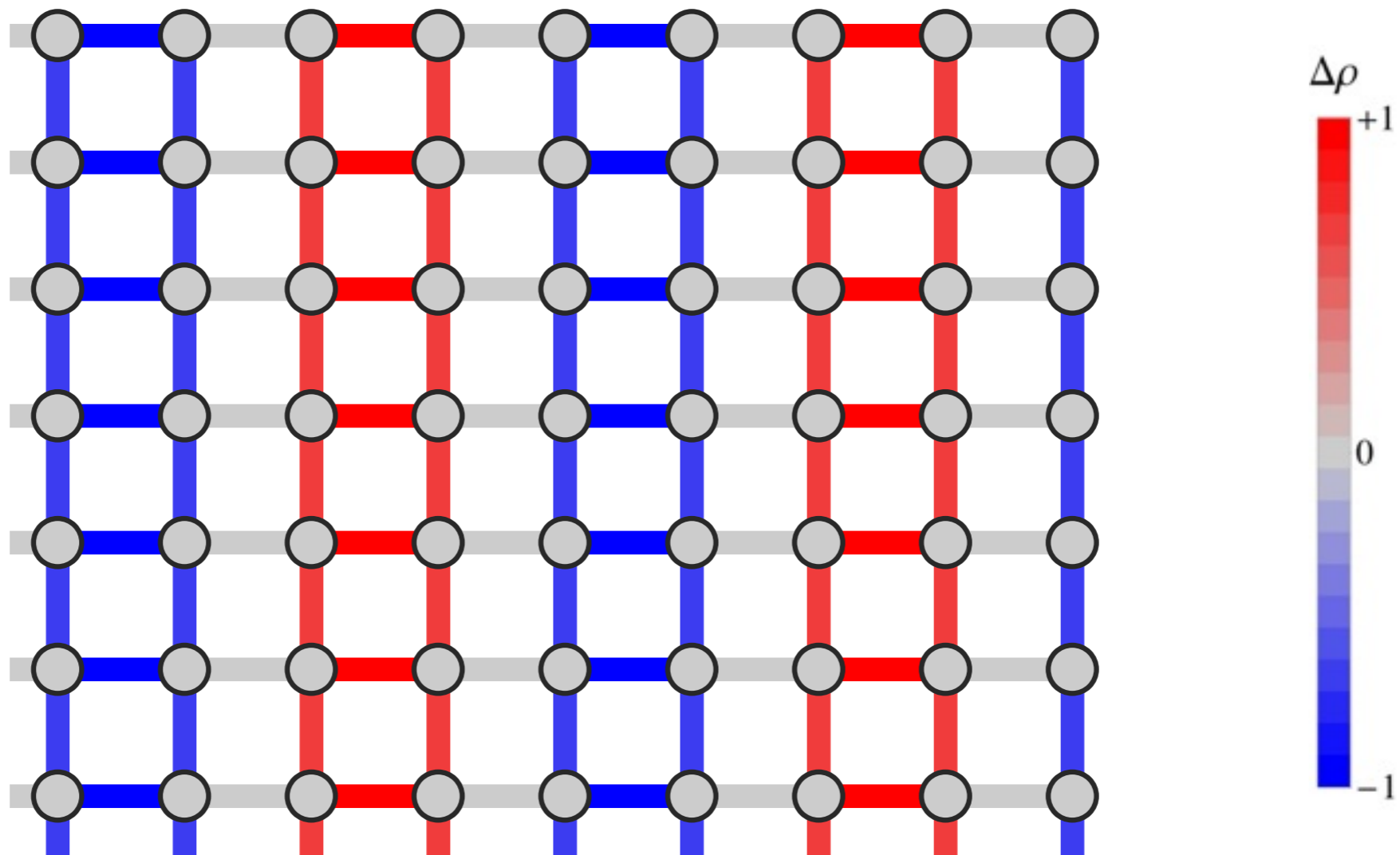
$s'$  form factor is incompatible with STM measurements on BSCCO, Na-CCOC.

# Unconventional DW order: $s'$ -form factor

Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) + \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$

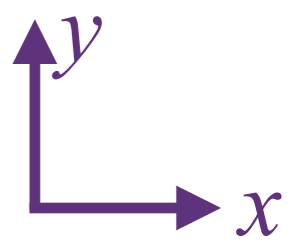


# Unconventional DW order: $d$ -form factor

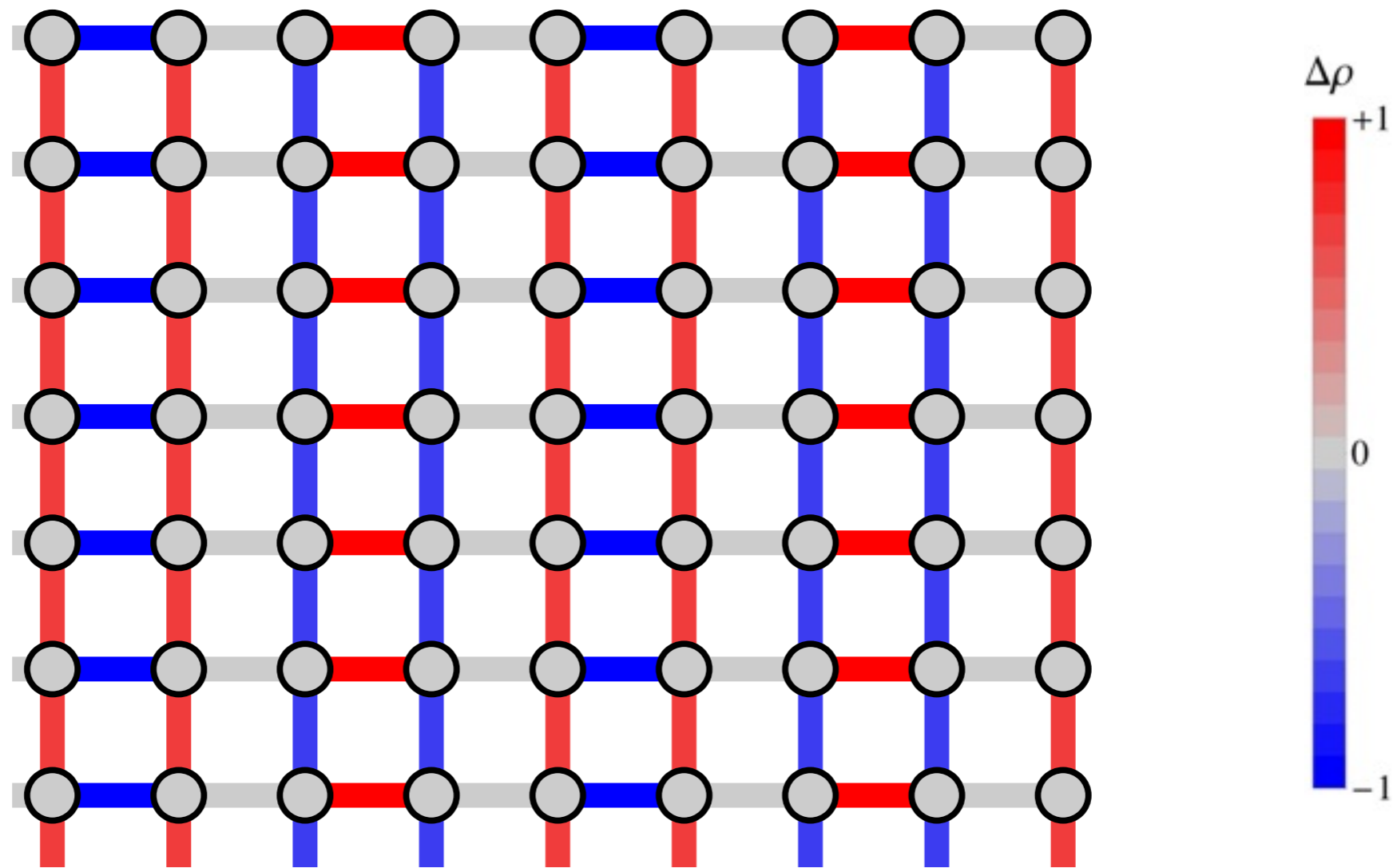
Plot of  $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$  for  $i = j$ , and  $i, j$  nearest neighbors.

$$P_{ij} = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

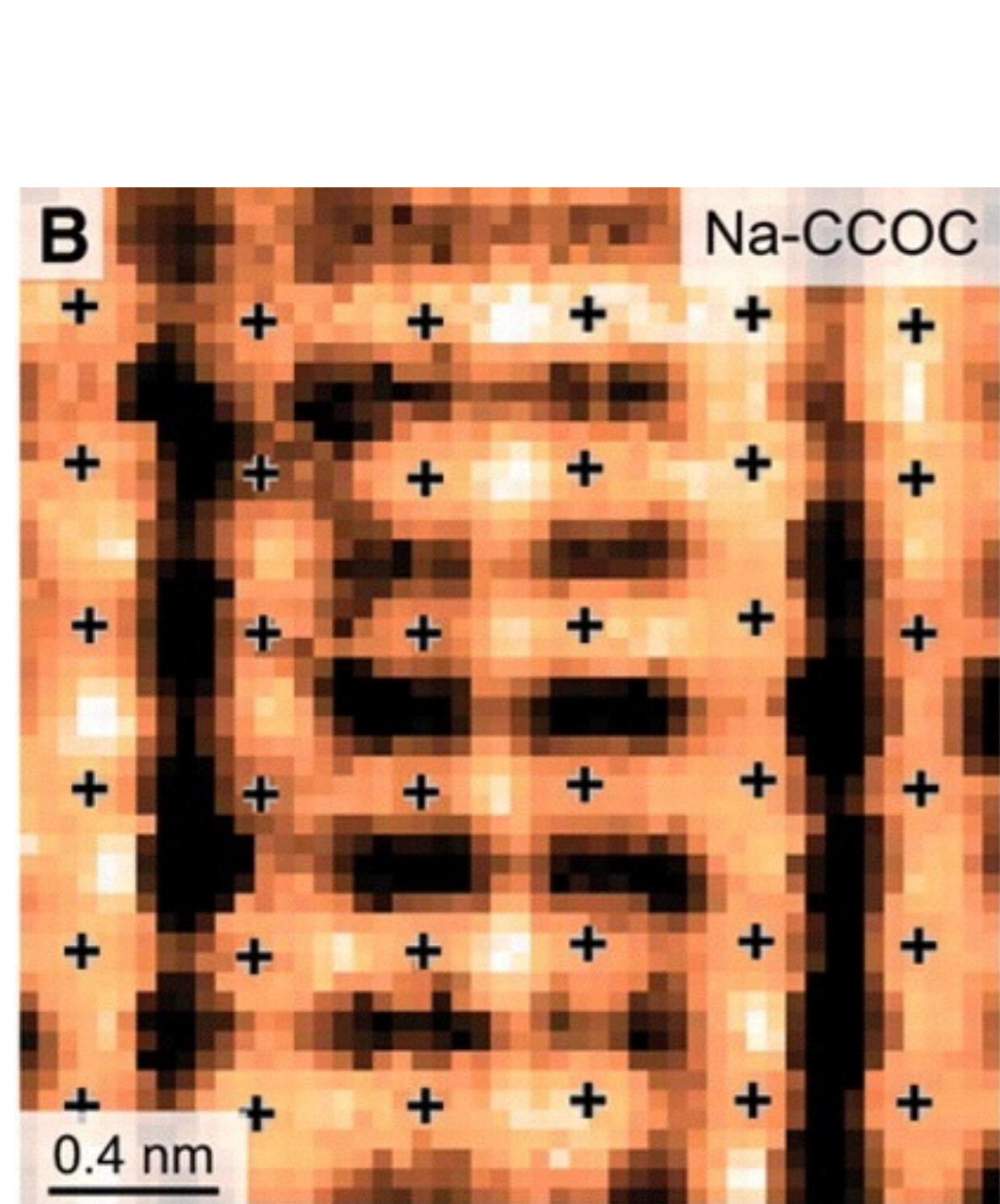
$$\mathcal{P}(\mathbf{k}) = e^{i\phi} [\cos(k_x) - \cos(k_y)] \quad \text{and} \quad \mathbf{Q} = 2\pi(1/4, 0)$$



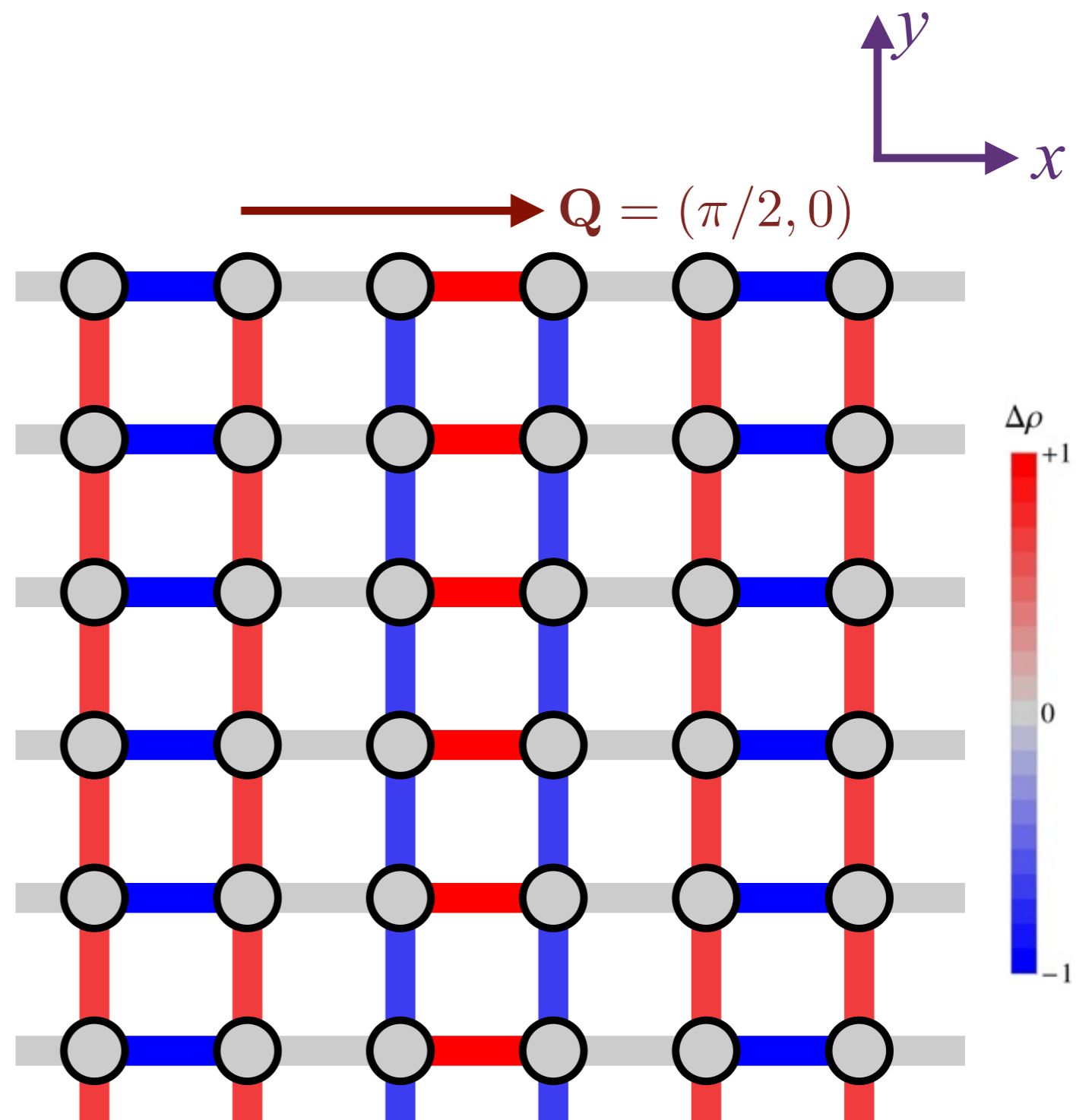
Density wave on horizontal bonds has a phase-shift of  $\pi$  relative to the wave on vertical bonds



M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).  
S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).

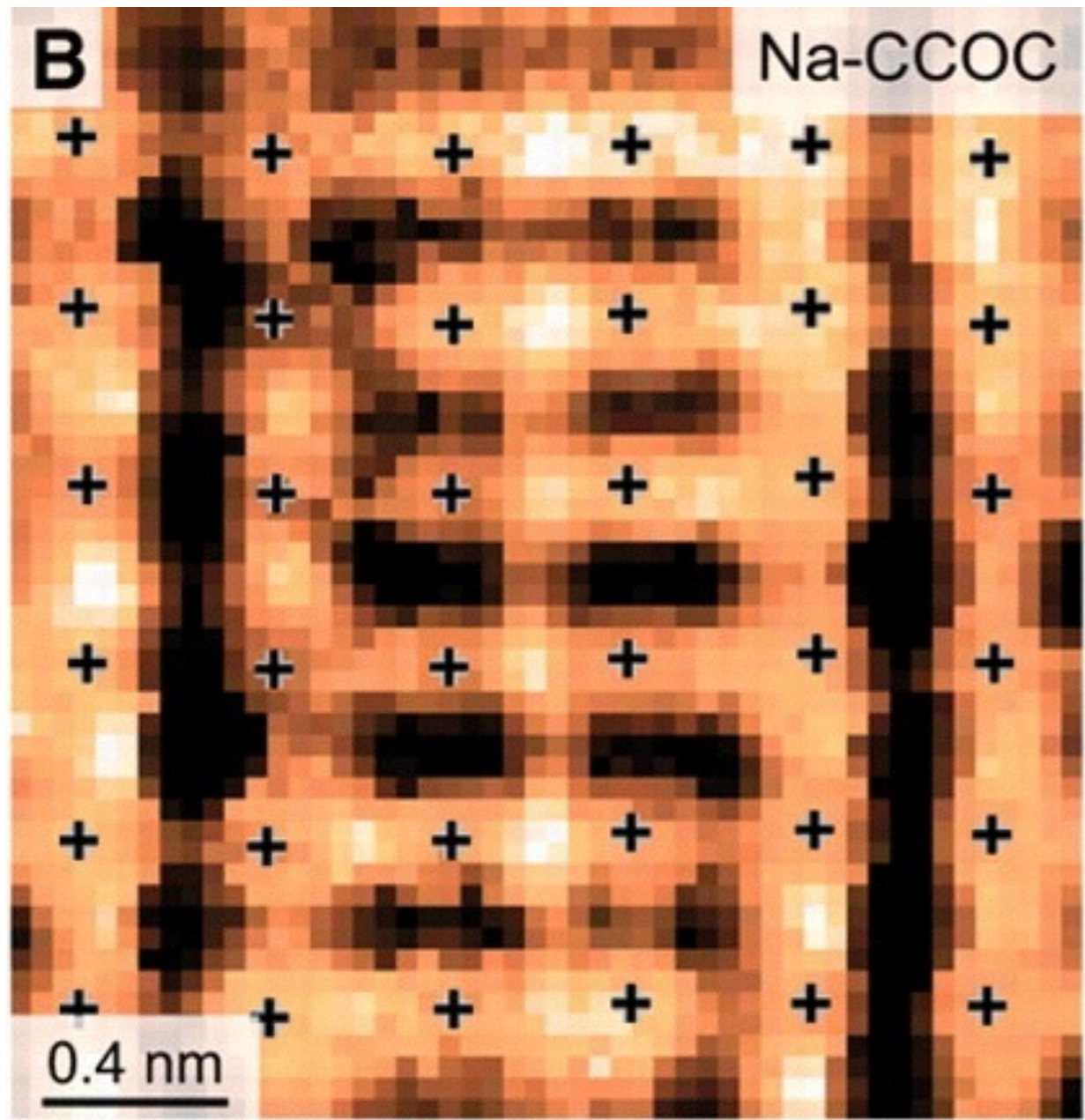
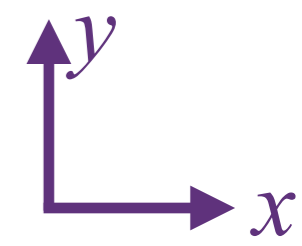


Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)

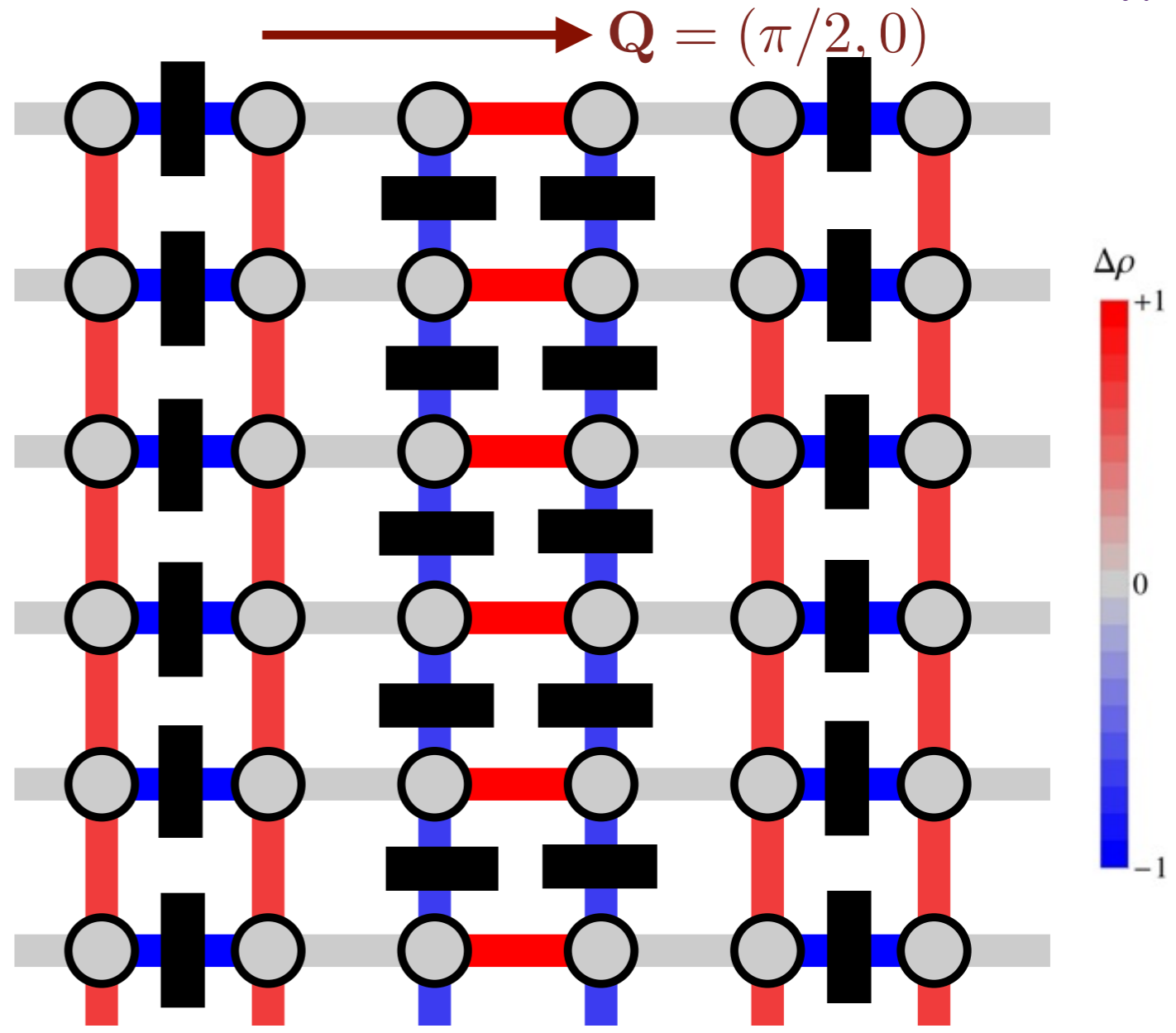


*d*-form factor density wave order

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075128 (2010).  
 S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).



Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)



*d*-form factor density wave order

*d* form factor is compatible with STM measurements on BSCCO, Na-CCOC !

# Direct phase-sensitive identification of a $d$ -form factor density wave in underdoped cuprates

Kazuhiro Fujita<sup>a,b,c,1</sup>, Mohammad H. Hamidian<sup>a,b,1</sup>, Stephen D. Edkins<sup>b,d</sup>, Chung Koo Kim<sup>a</sup>, Yuhki Kohsaka<sup>e</sup>, Masaki Azuma<sup>f</sup>, Mikio Takano<sup>g</sup>, Hidenori Takagi<sup>c,h,i</sup>, Hiroshi Eisaki<sup>j</sup>, Shin-ichi Uchida<sup>c</sup>, Andrea Allais<sup>k</sup>, Michael J. Lawler<sup>b,l</sup>, Eun-Ah Kim<sup>b</sup>, Subir Sachdev<sup>k,m</sup>, and J. C. Séamus Davis<sup>a,b,d,2</sup>

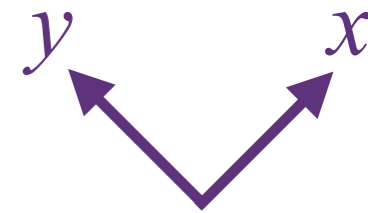
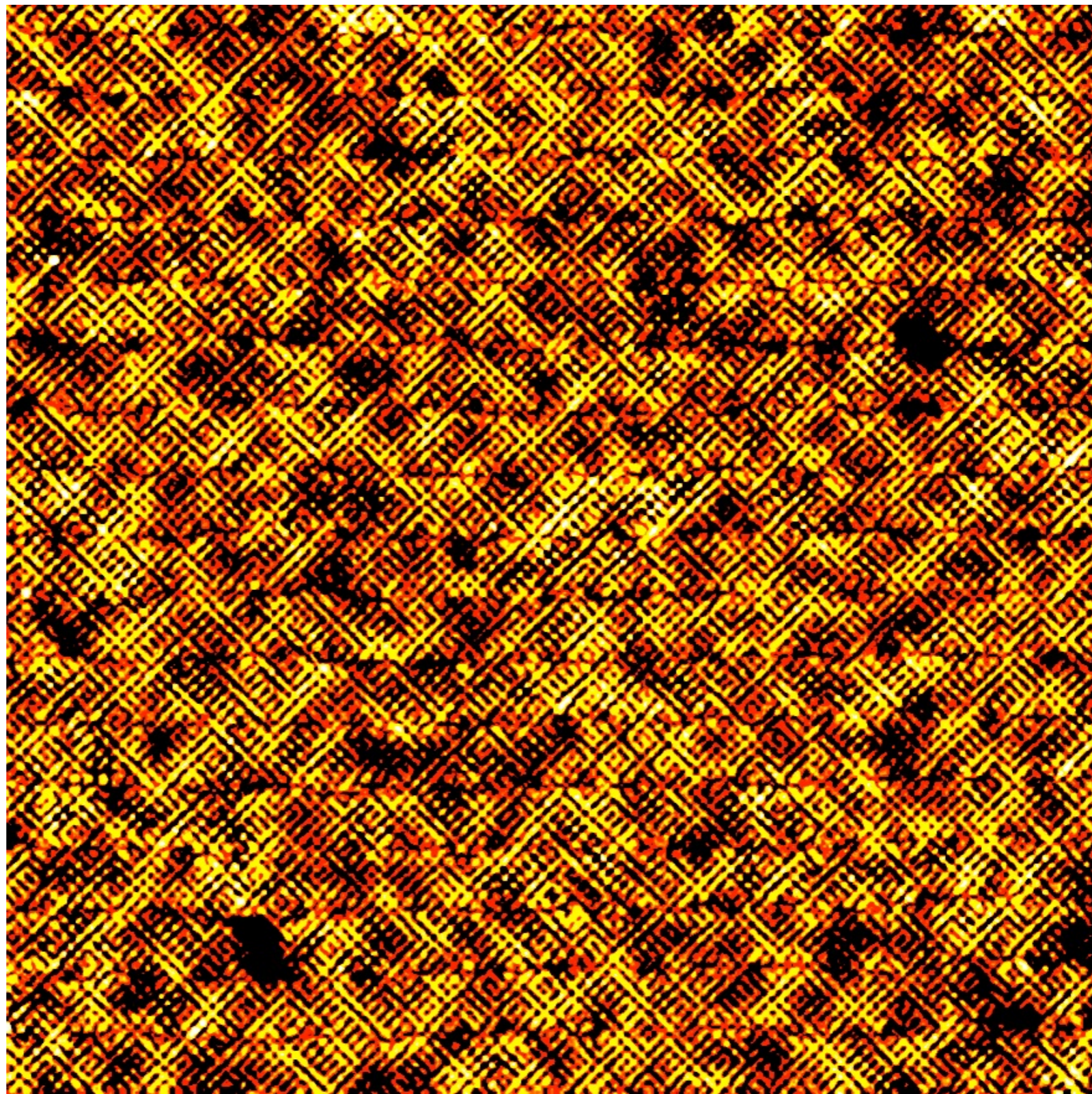
The identity of the fundamental broken symmetry (if any) in the underdoped cuprates is unresolved. However, evidence has been accumulating that this state may be an unconventional density wave. Here we carry out site-specific measurements within each  $\text{CuO}_2$  unit cell, segregating the results into three separate electronic structure images containing only the Cu sites [ $\text{Cu}(r)$ ] and only the  $x/y$  axis O sites [ $\text{O}_x(r)$  and  $\text{O}_y(r)$ ]. Phase-resolved Fourier analysis reveals directly that the modulations in the  $\text{O}_x(r)$  and  $\text{O}_y(r)$  sublattice images consistently exhibit a relative phase of  $\pi$ . We confirm this discovery on two highly distinct cuprate compounds, ruling out tunnel matrix-element and materials-specific systematics. These observations demonstrate by direct sublattice phase-resolved visualization that the density wave found in underdoped cuprates consists of modulations of the intraunit-cell states that exhibit a predominantly  $d$ -symmetry form factor.

See also

C. Howald, H. Eisaki,  
N. Kaneko, M. Greven,  
and A. Kapitulnik,  
*Phys. Rev. B* **67**,  
014533 (2003);

M. Vershinin, S. Misra,  
S. Ono, Y. Abe, Yoichi  
Ando, and  
A. Yazdani, *Science*  
**303**, 1995 (2004).

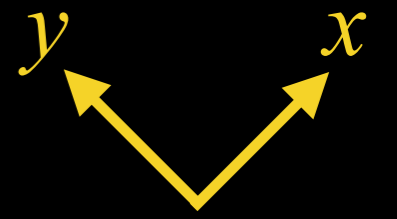
W. D. Wise, M. C. Boyer,  
K. Chatterjee, T. Kondo,  
T. Takeuchi, H. Ikuta,  
Y. Wang, and  
E. W. Hudson,  
*Nature Phys.* **4**, 696  
(2008).



“R-map” of BSCCO in zero magnetic field, similar to those published in Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007). Davis group has sub-angstrom resolution capabilities, with lattice drift corrections, which make sublattice phase-resolved STM possible.

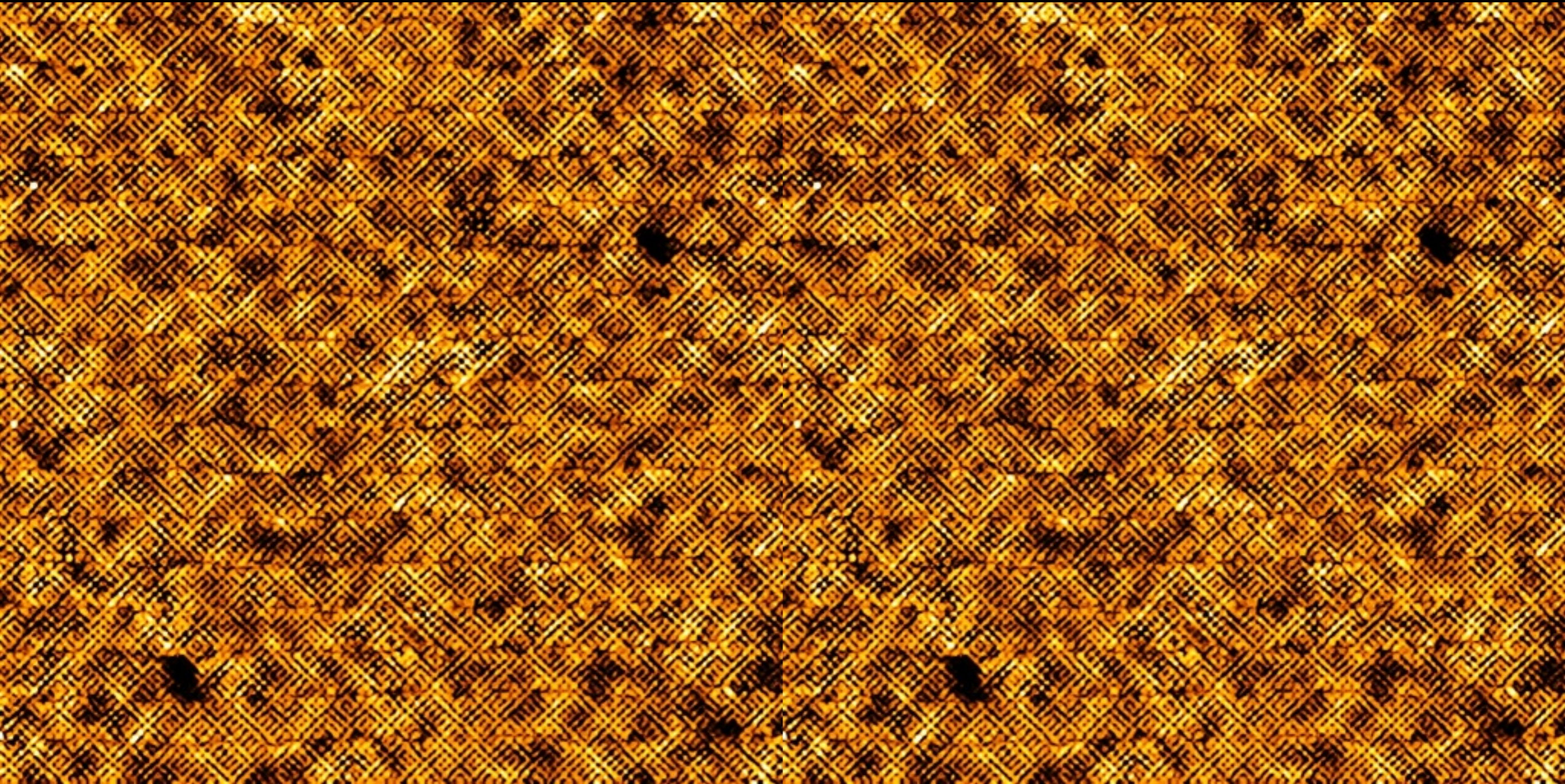
UD45K  
BSCCO

$R(r, 150\text{mV})$



$R(r, 150\text{mV})$

$R(r, 150\text{mV})$



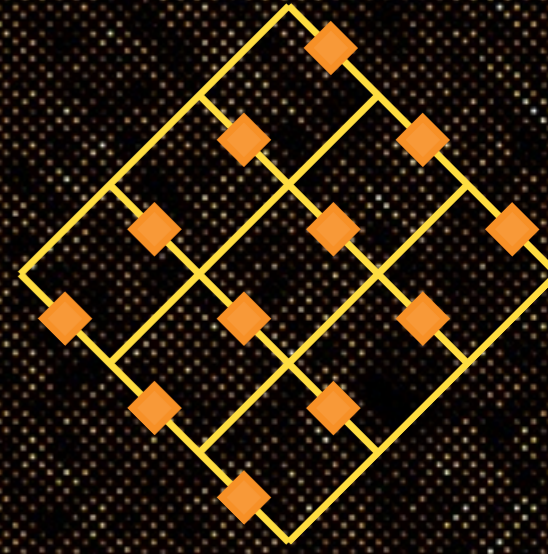
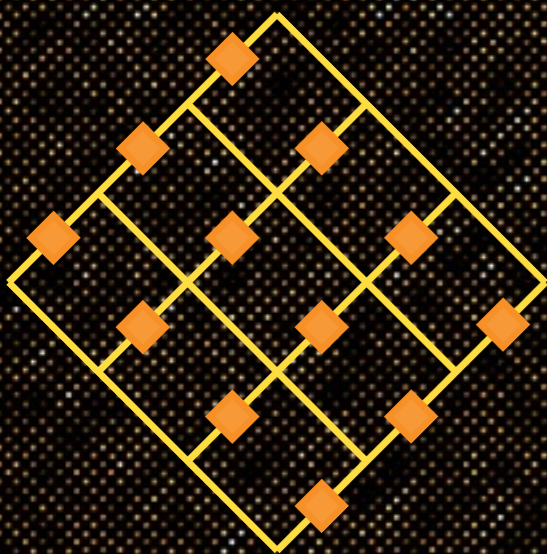
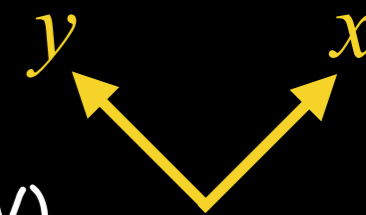
Note that these are identical images.

UD45K

$R(r=0, 150\text{mV})$

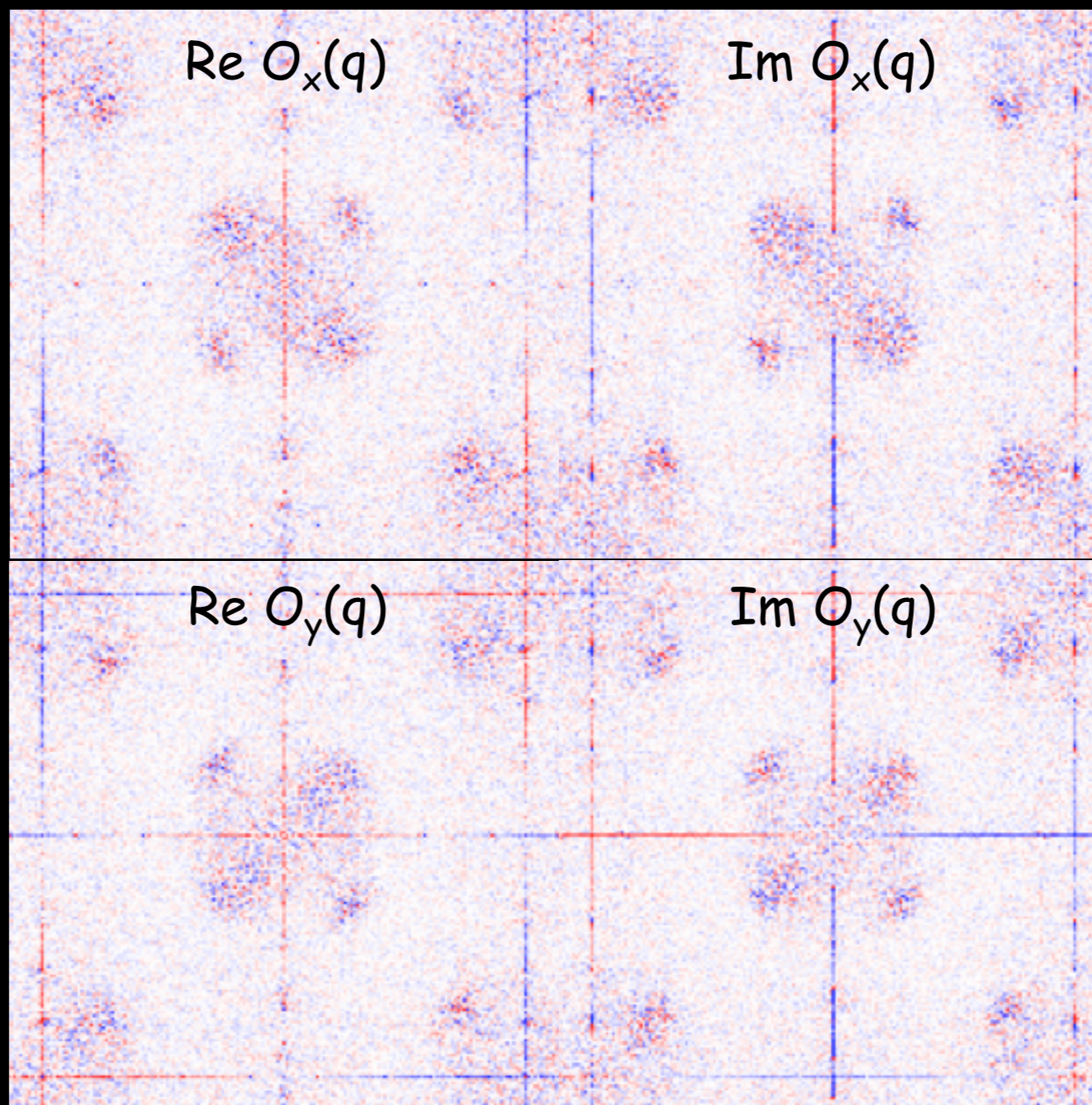
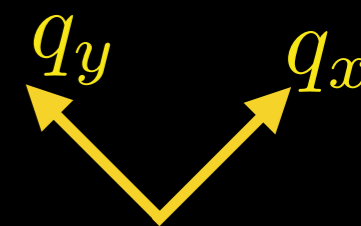
$R(r=O_x, 150\text{mV})$

$R(r=O_y, 150\text{mV})$

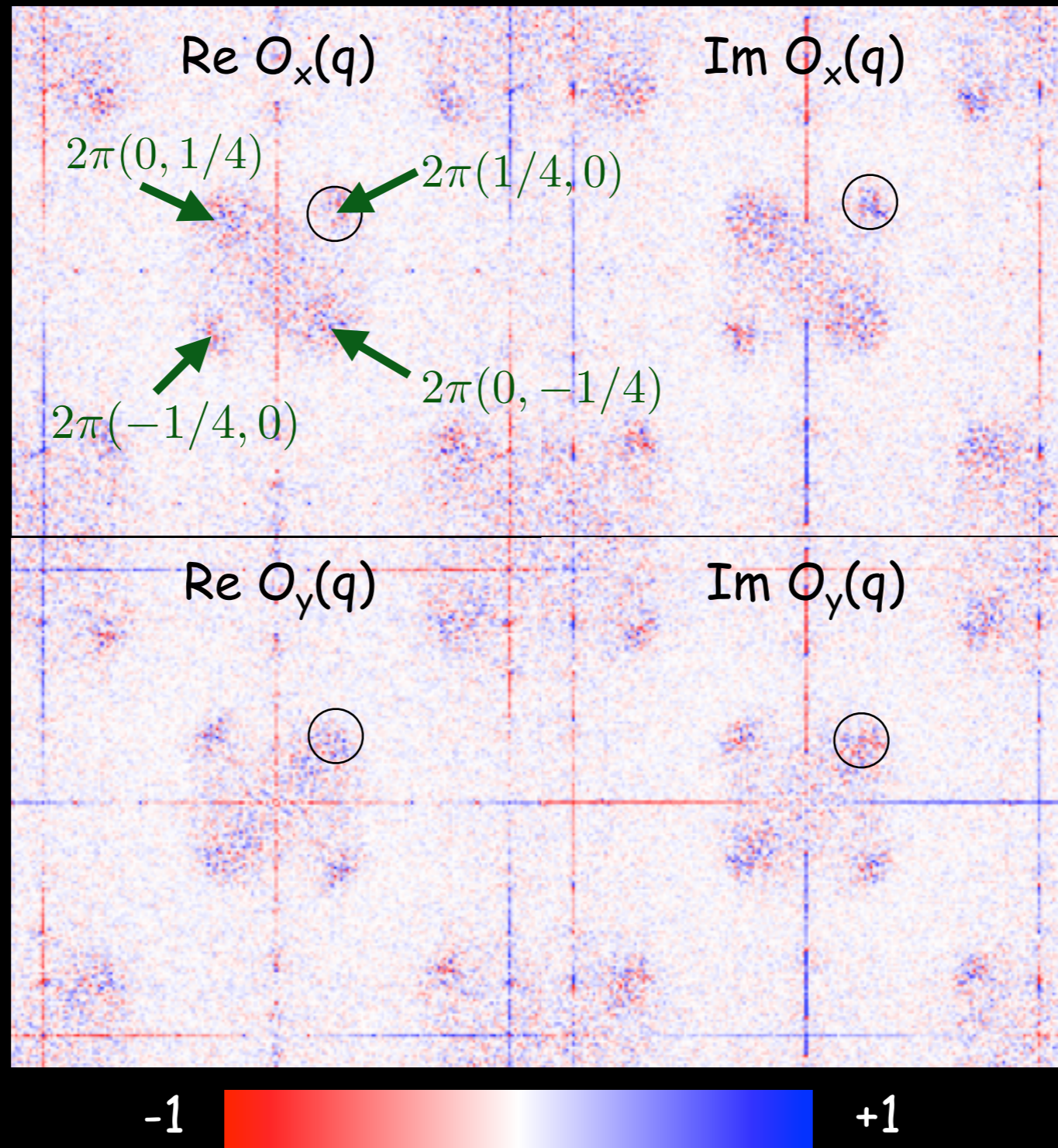
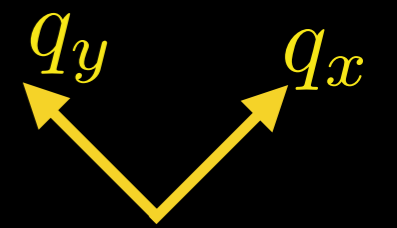


UD45K

# Broad (0,Q) and (Q,0) DW Features

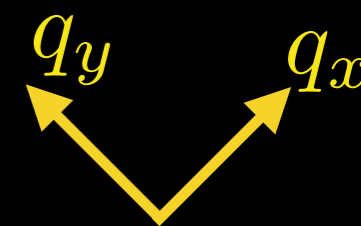


## Broad (0,Q) and (Q,0) DW Features

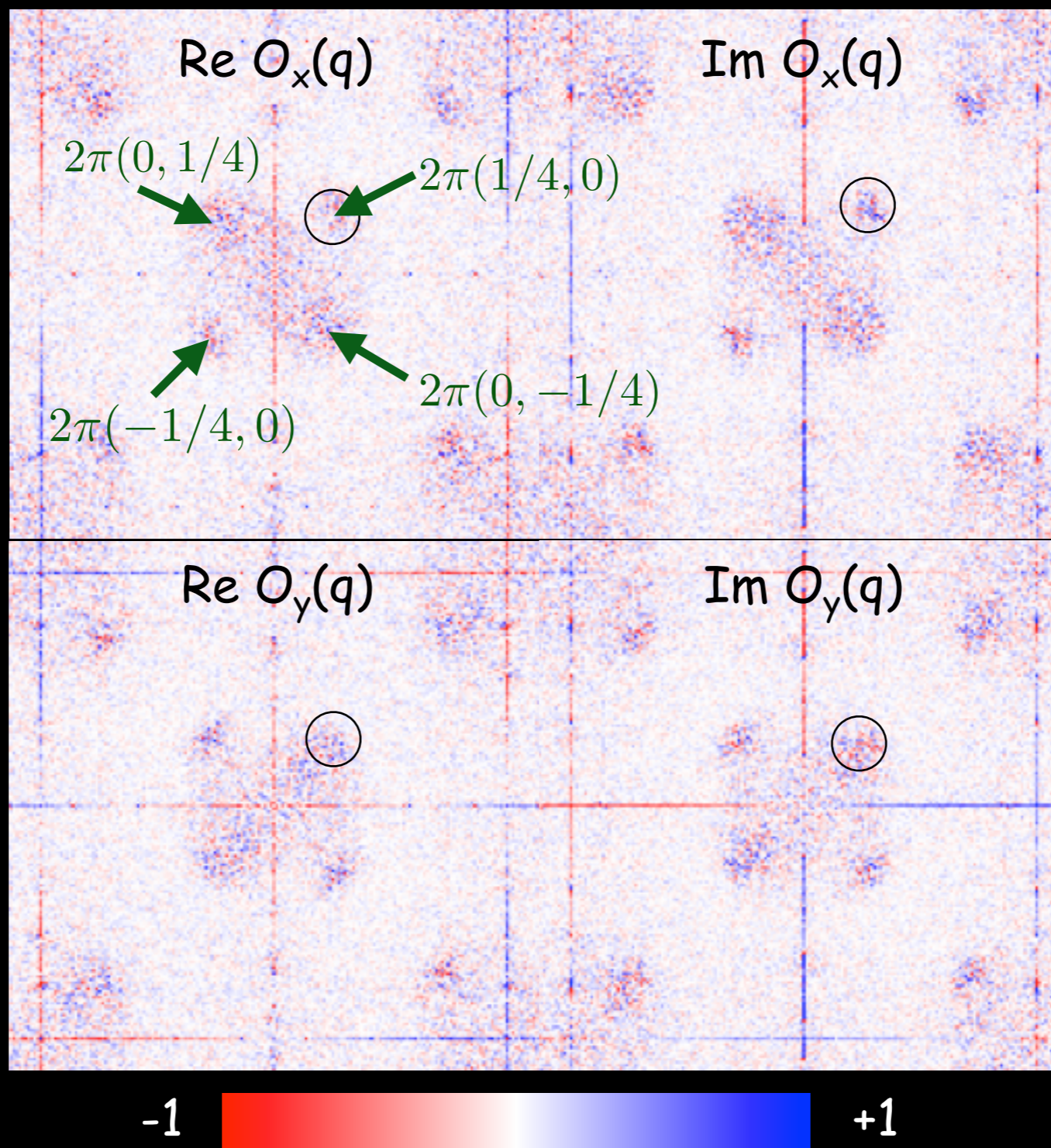


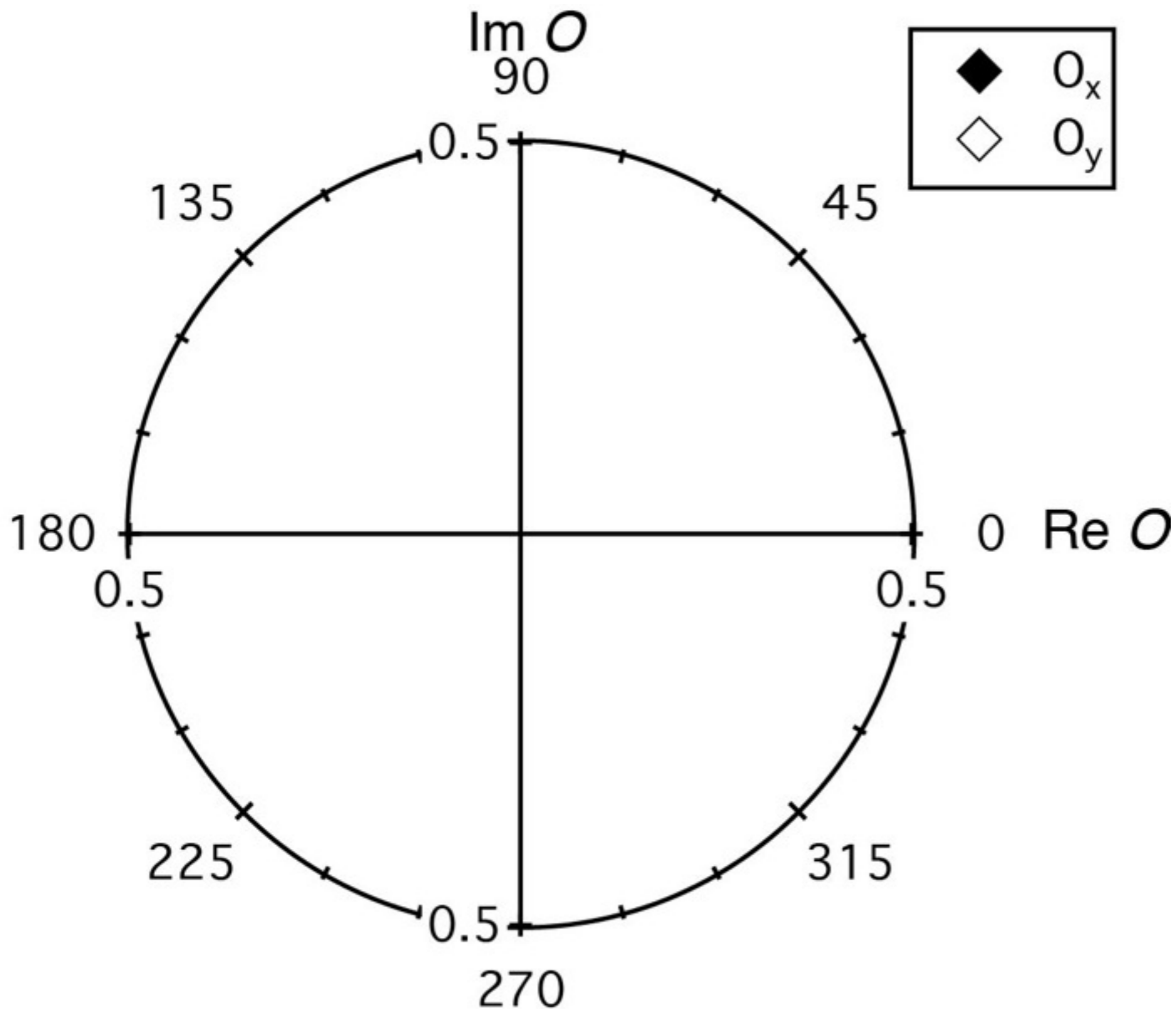
UD45K

## Broad (0,Q) and (Q,0) DW Features

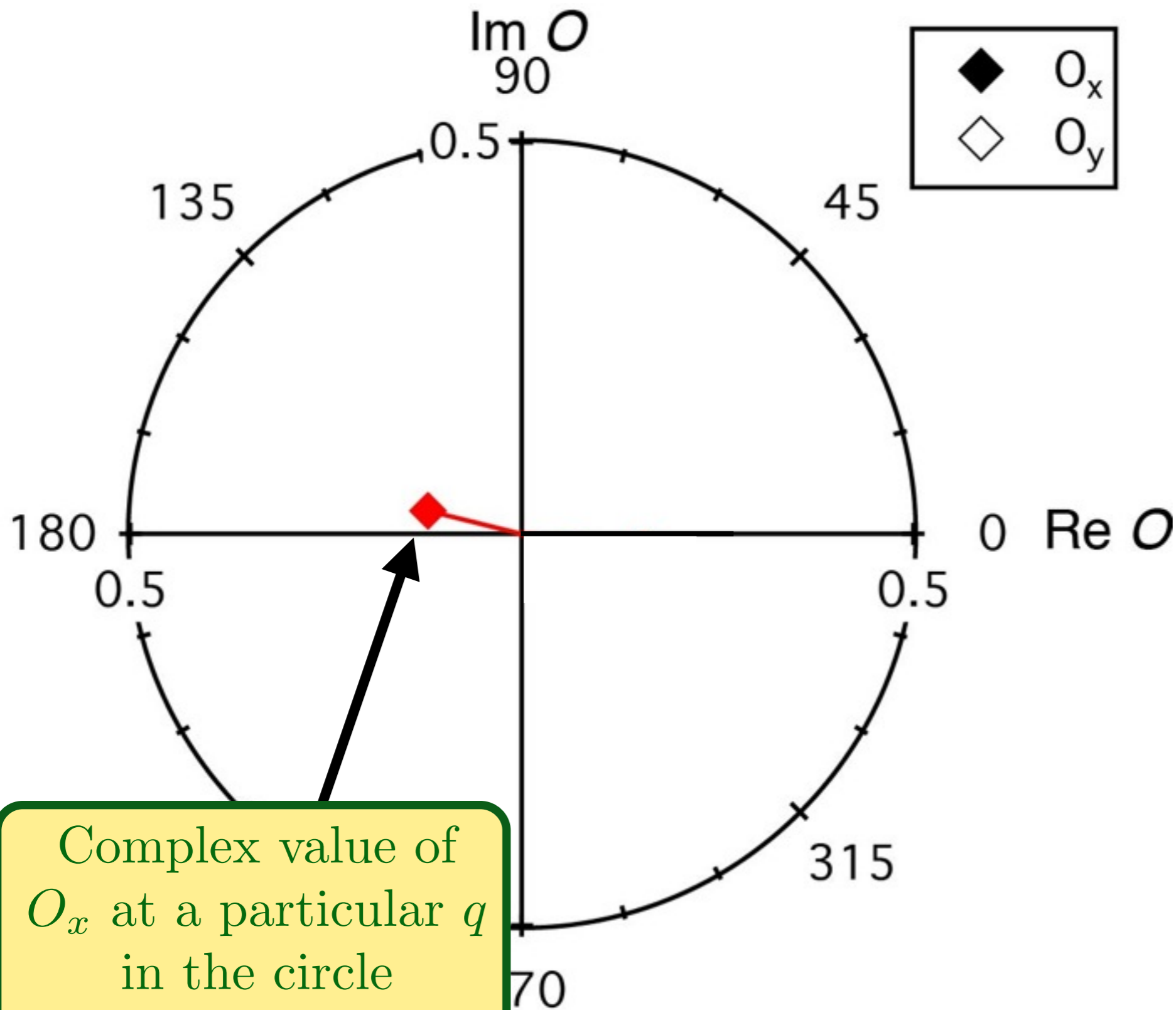


For each pixel in the circles, we obtain 2 complex numbers,  $O_x(q)$  and  $O_y(q)$ .



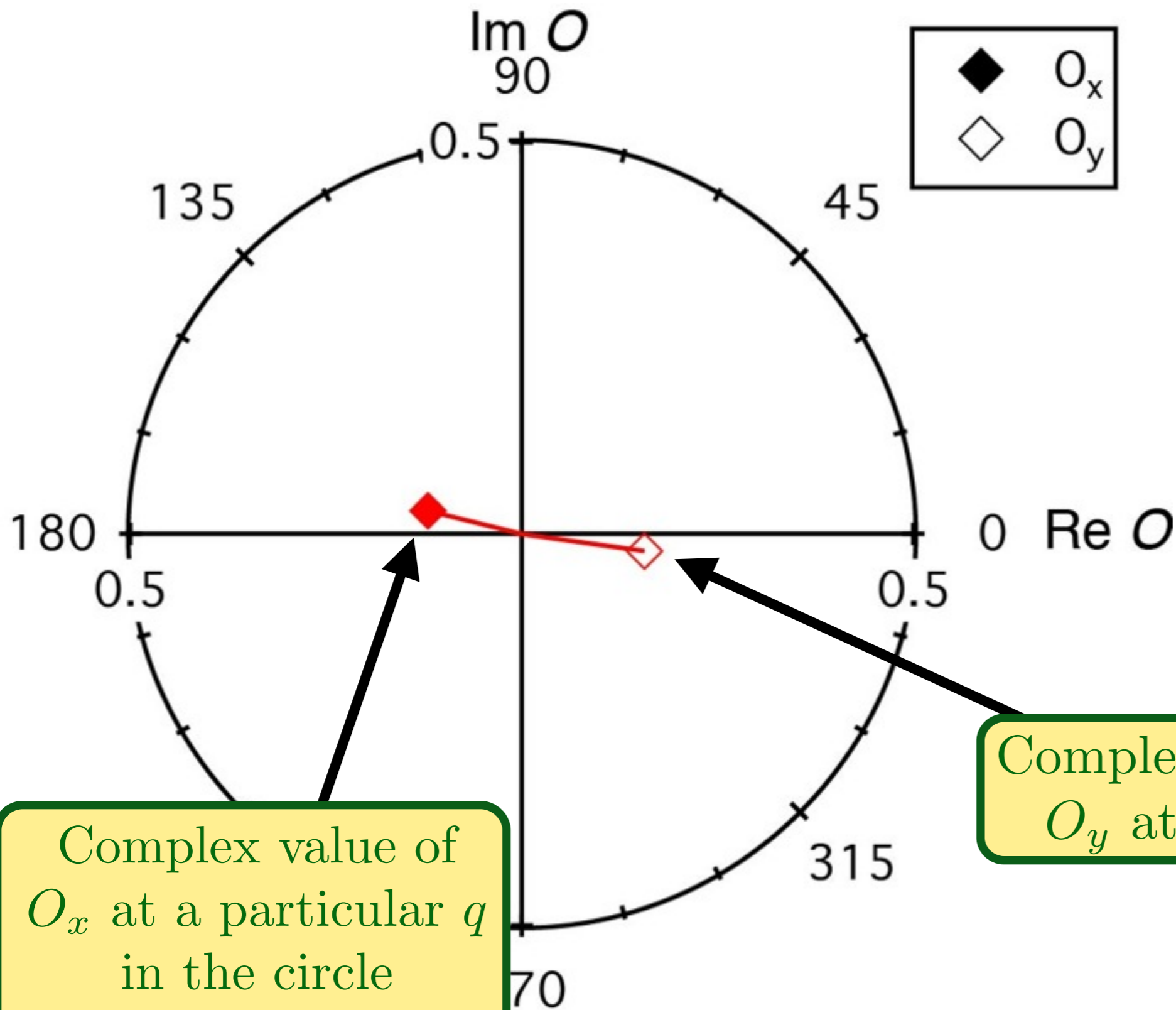


**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



**Phase-sensitive measurement of the  $d$ -form factor of density wave order**

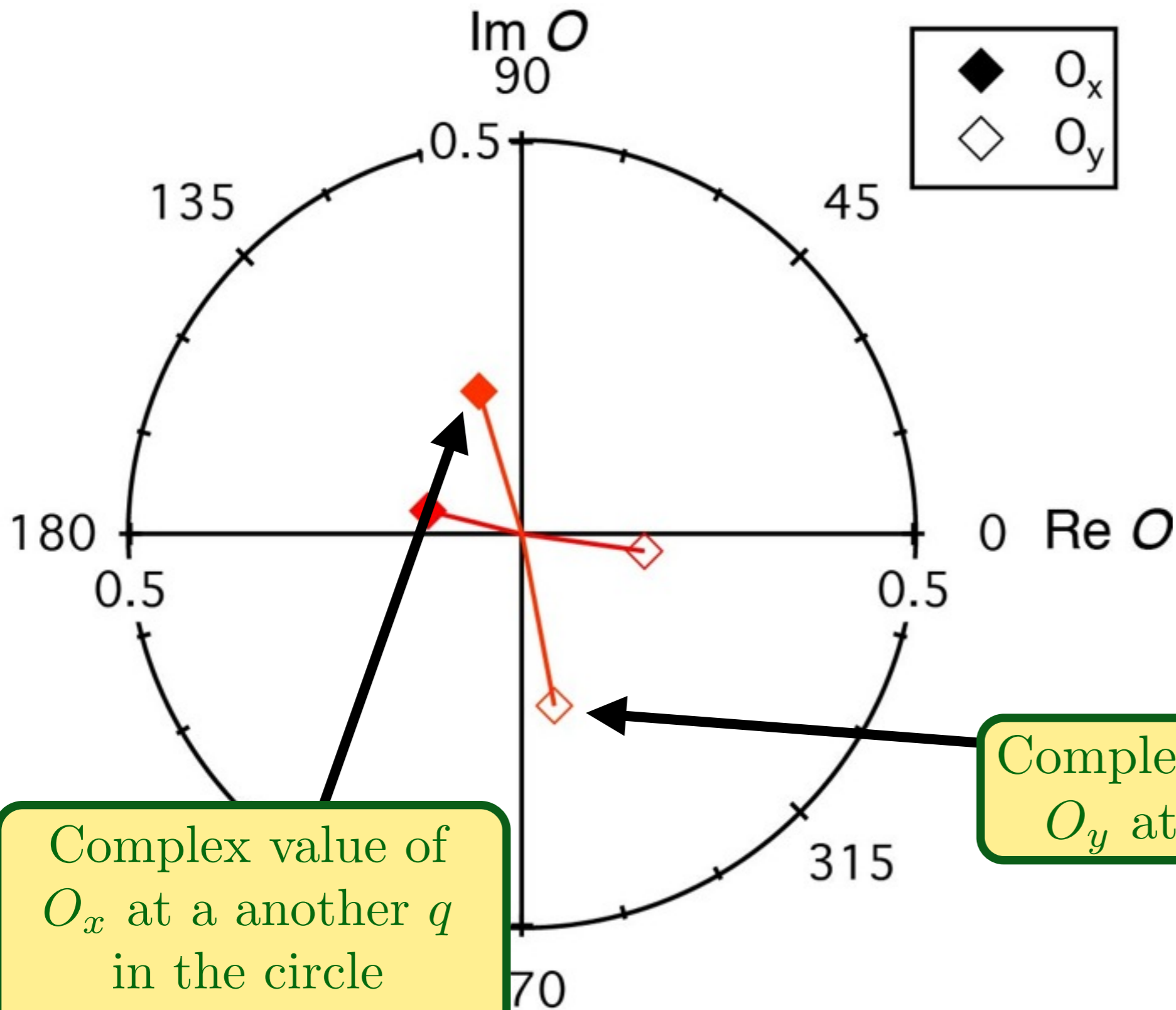
Complex value of  $O_x$  at a particular  $q$  in the circle around  $2\pi(1/4, 0)$ .



**Phase-sensitive measurement of the  $d$ -form factor of density wave order**

Complex value of  $O_x$  at a particular  $q$  in the circle around  $2\pi(1/4, 0)$ .

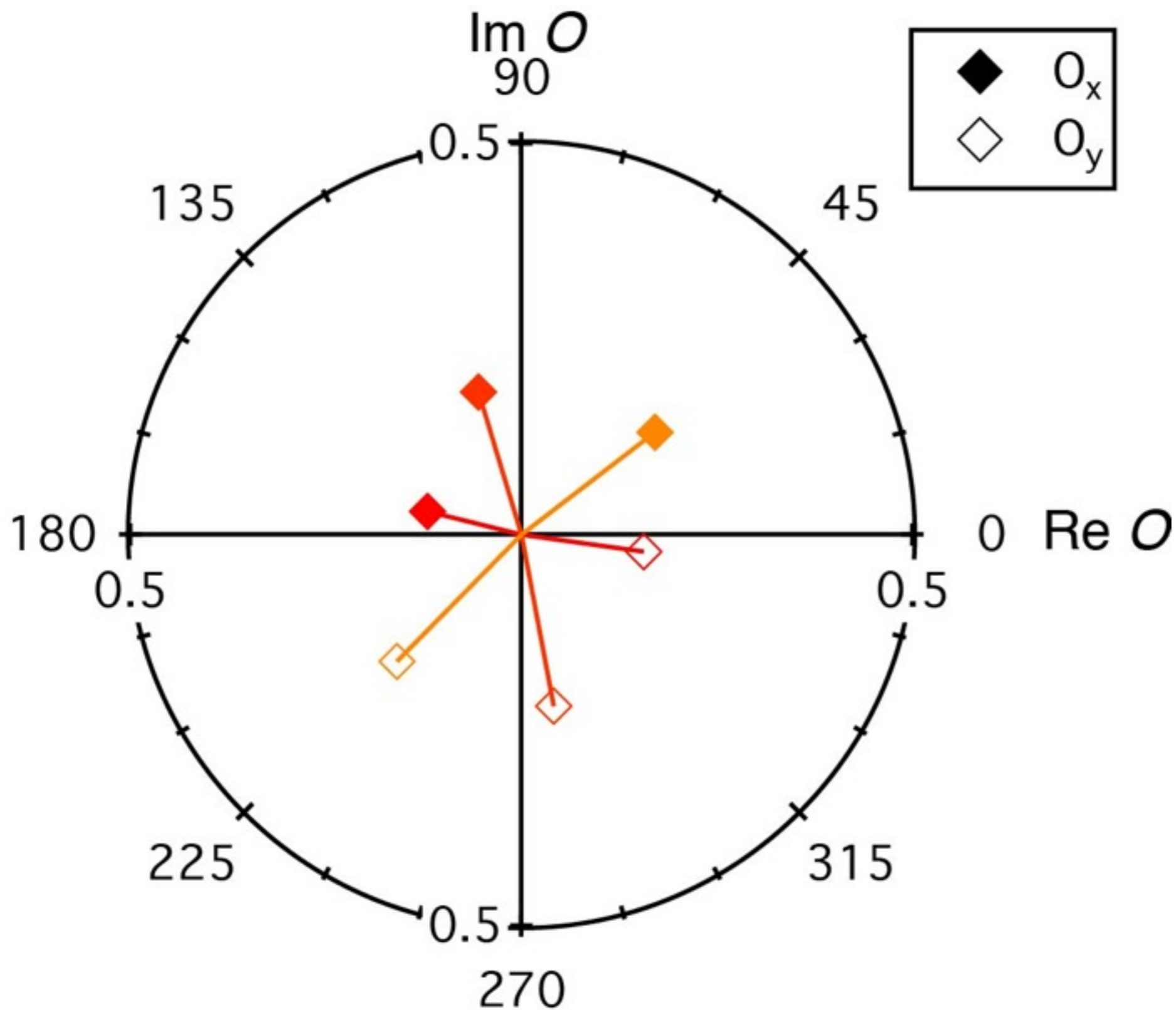
Complex value of  $O_y$  at same  $q$



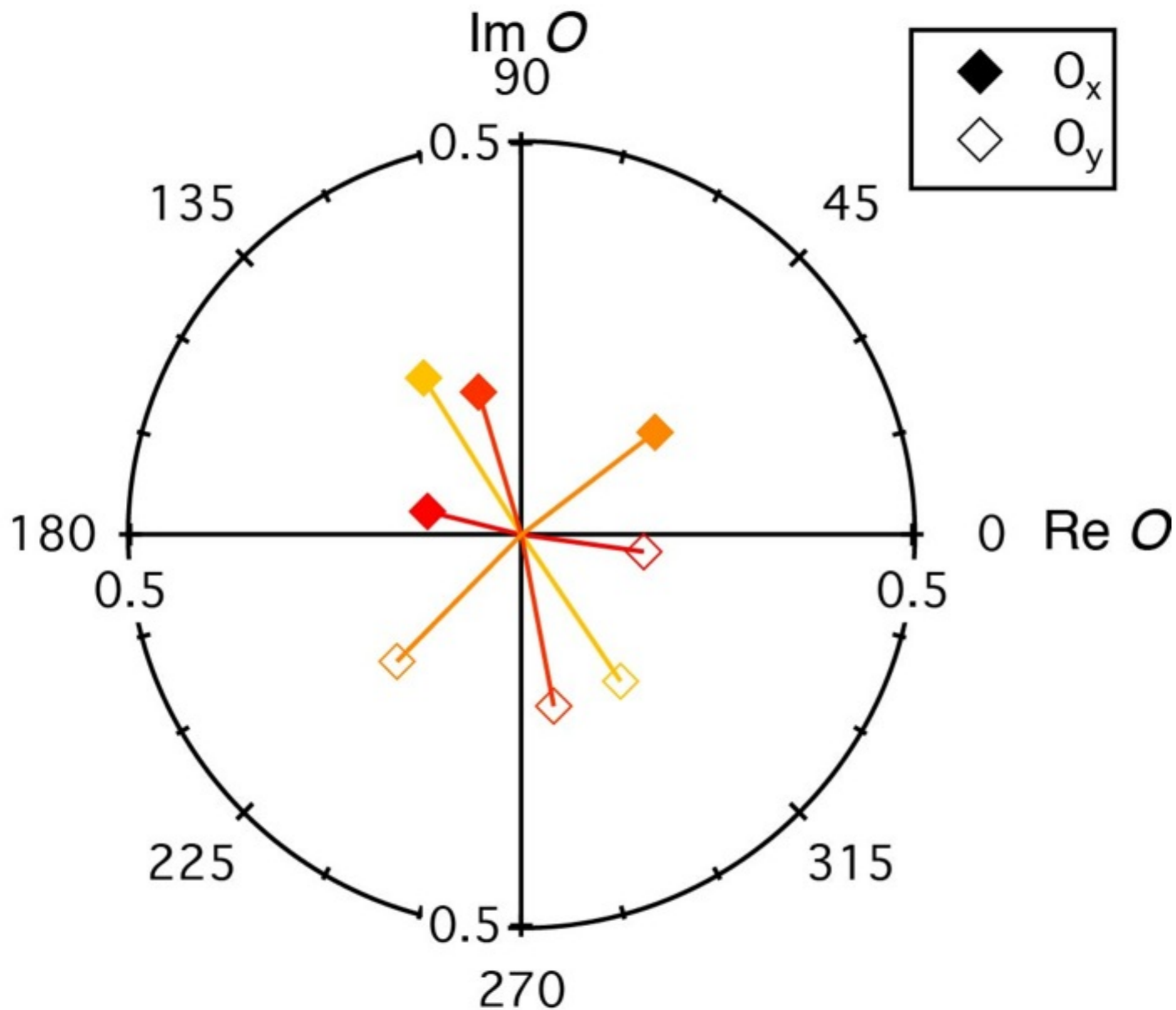
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**

Complex value of  $O_x$  at a another  $q$  in the circle around  $2\pi(1/4, 0)$ .

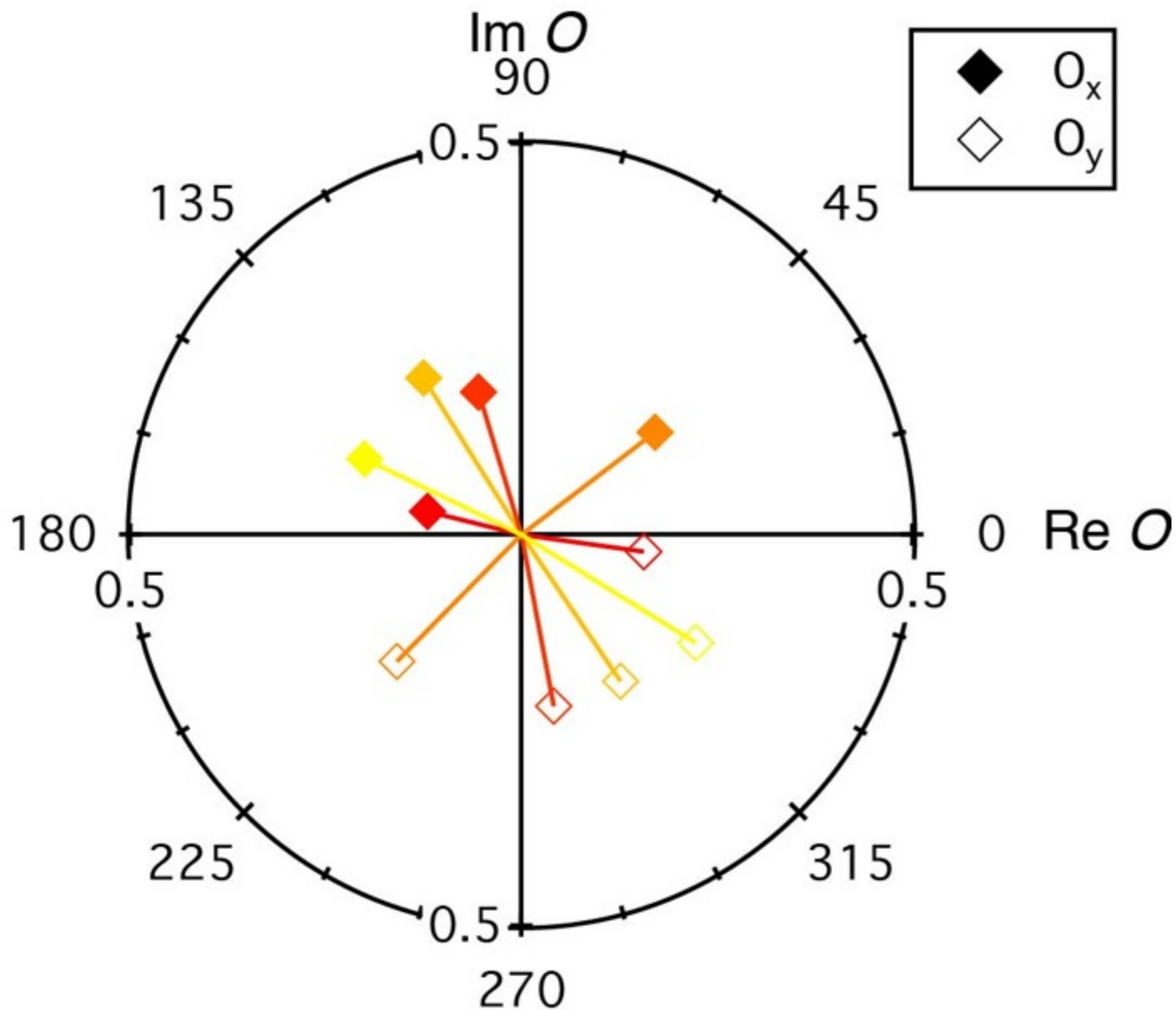
Complex value of  $O_y$  at same  $q$



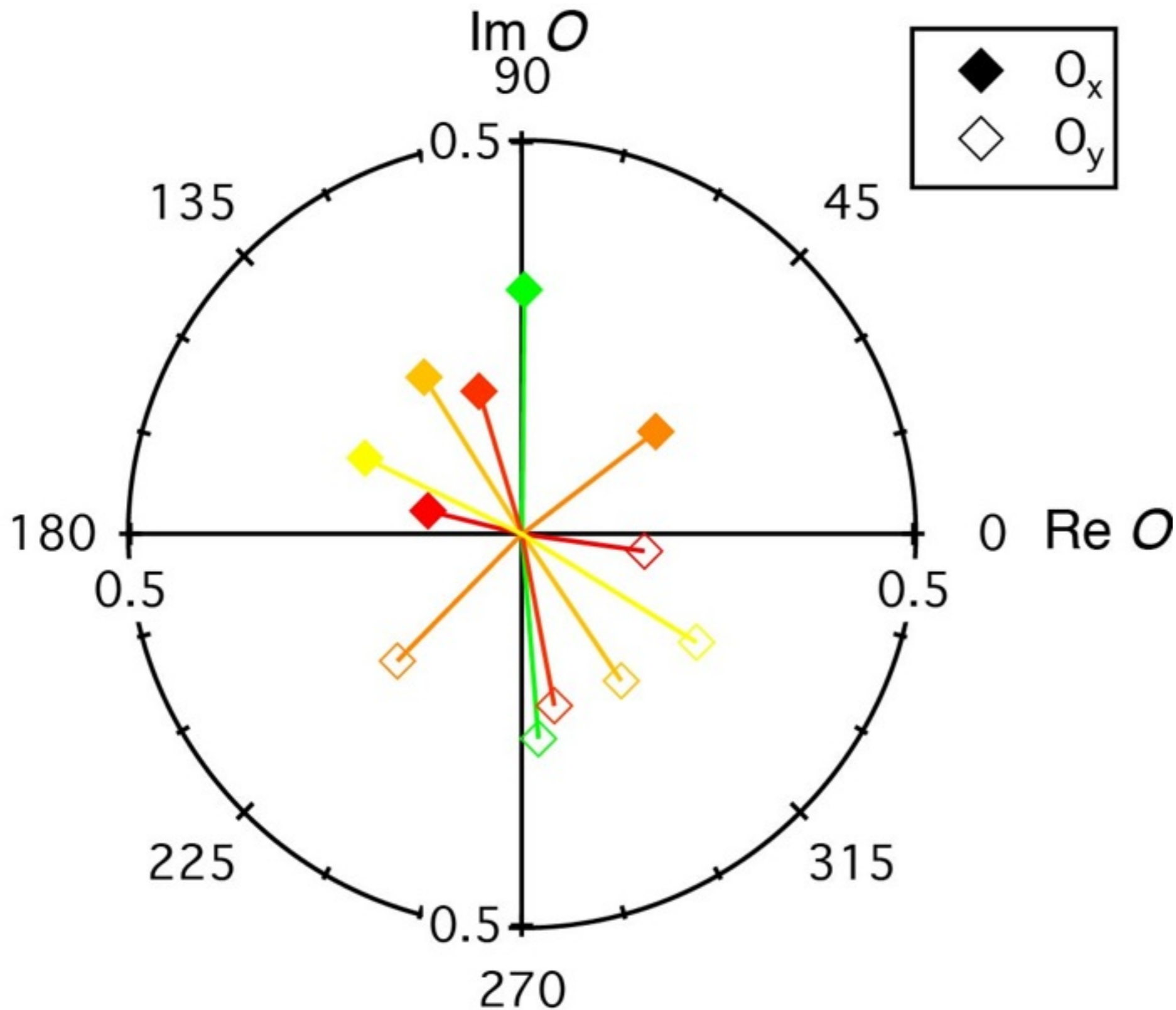
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



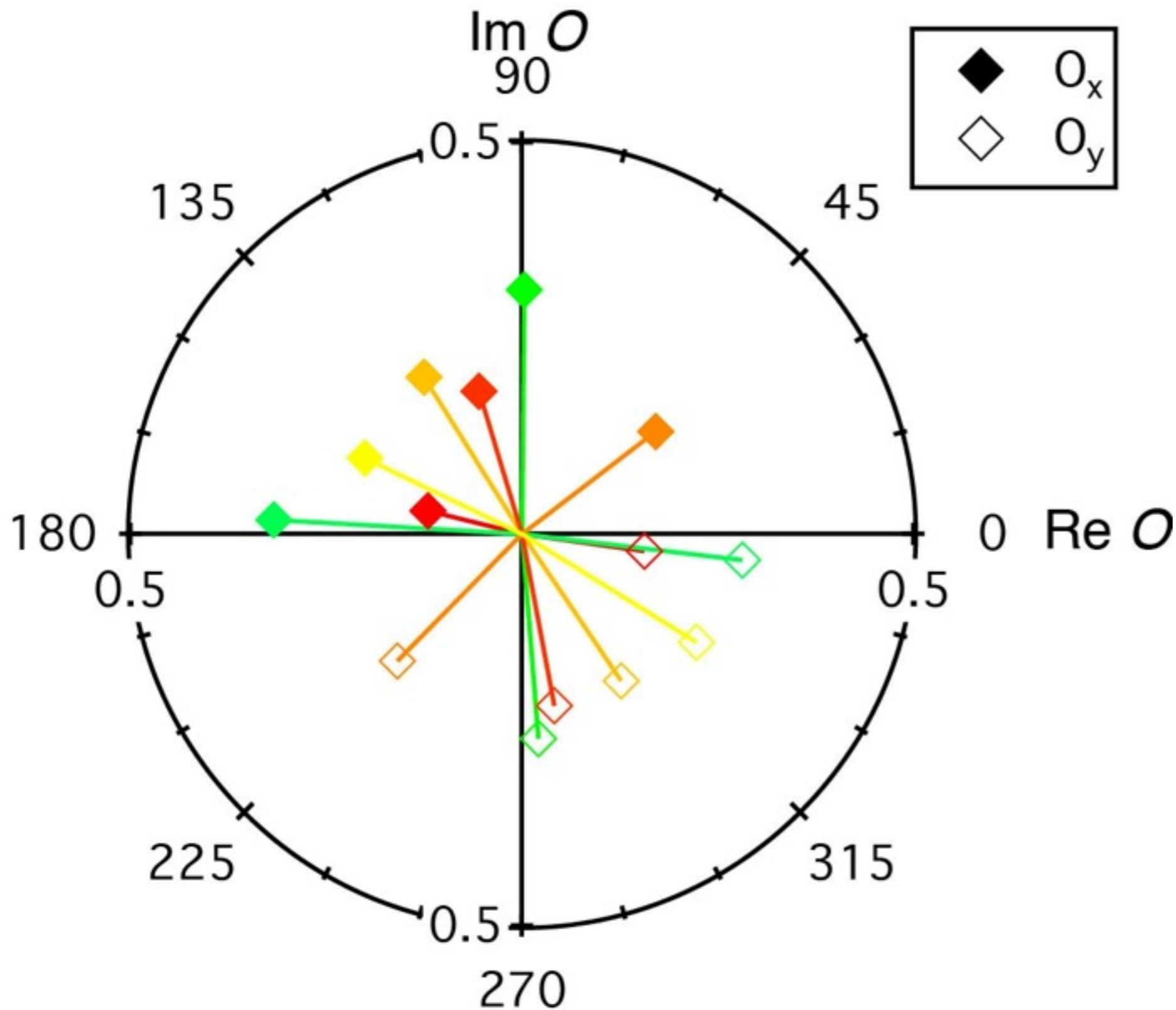
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



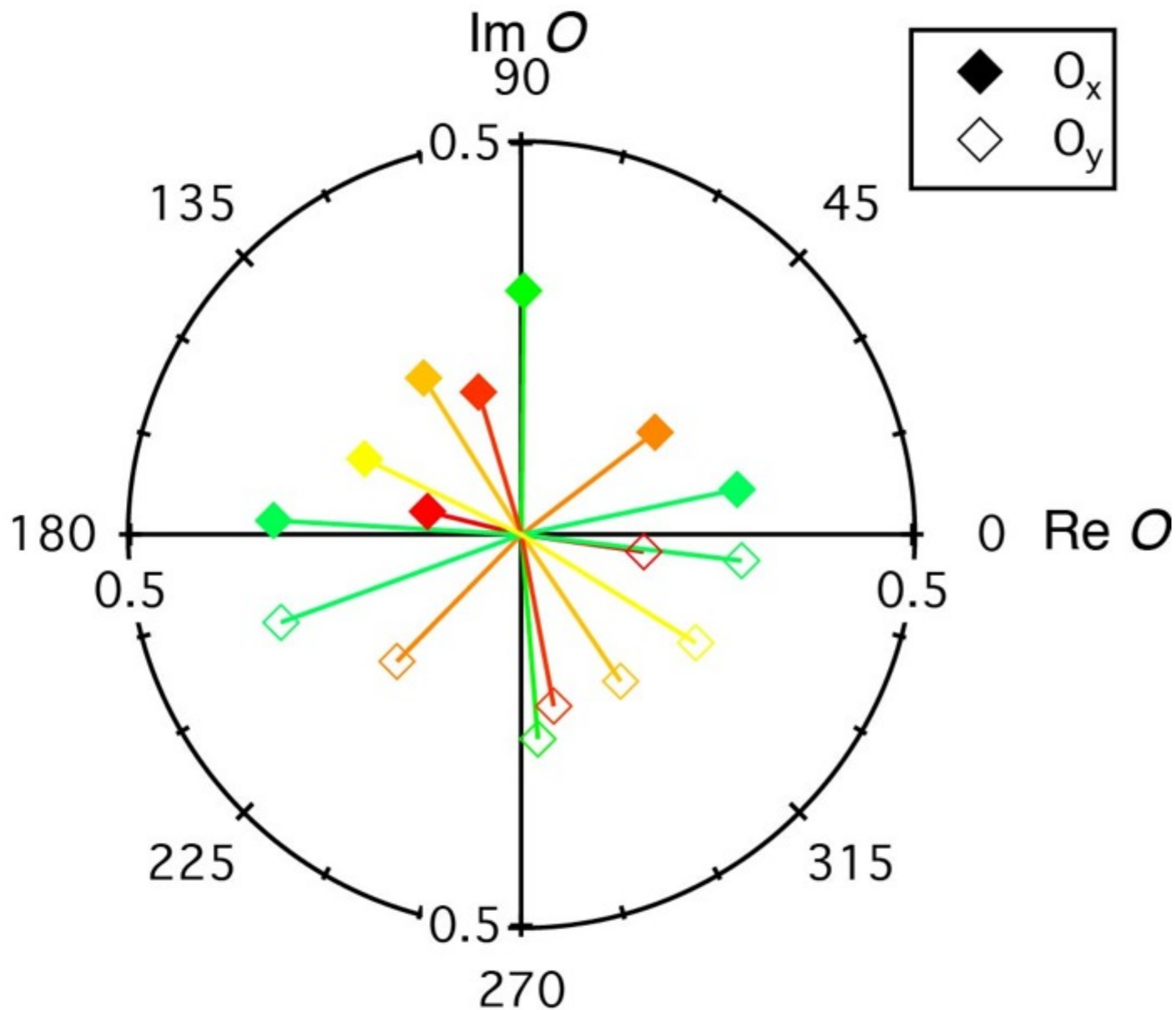
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



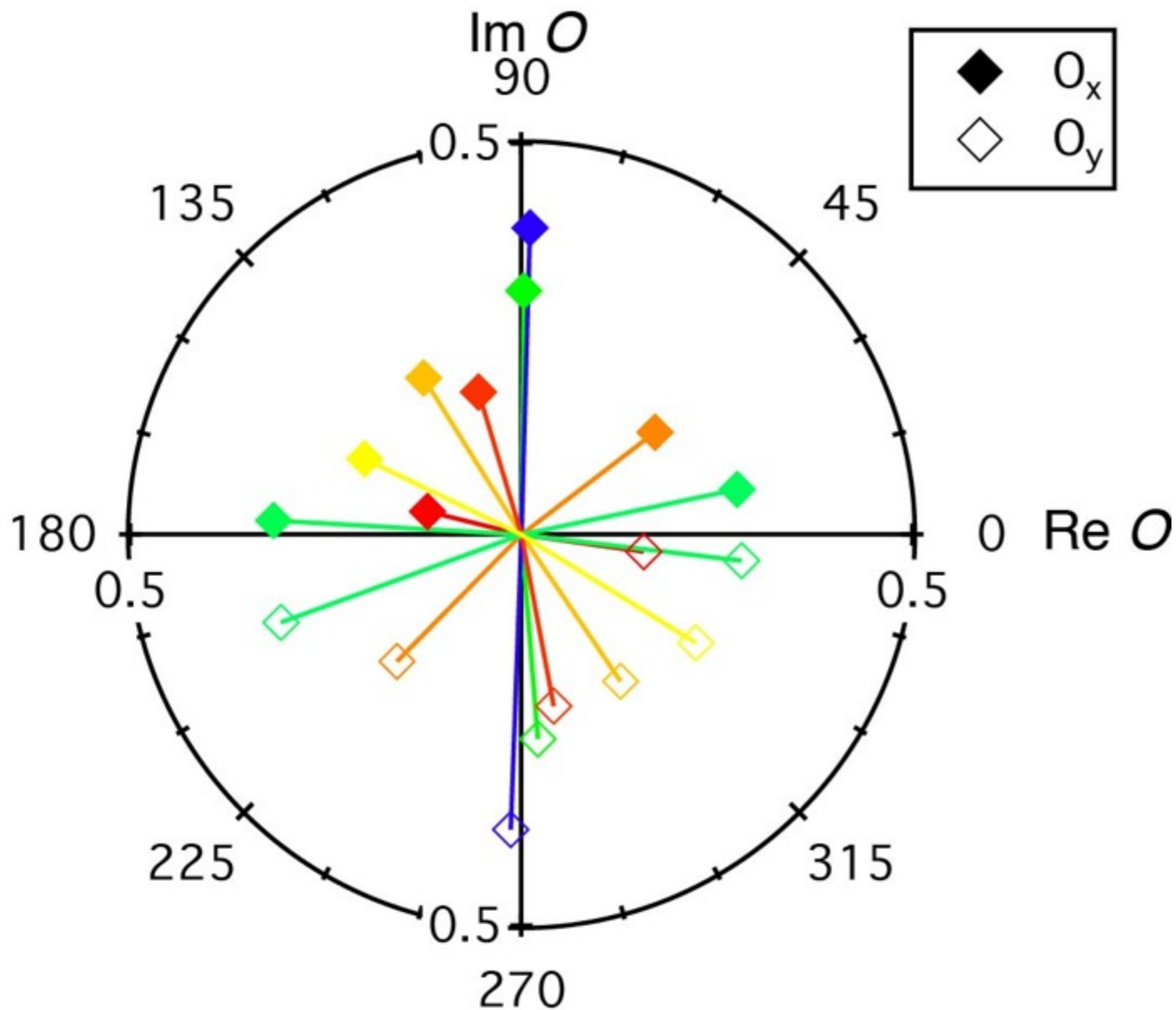
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



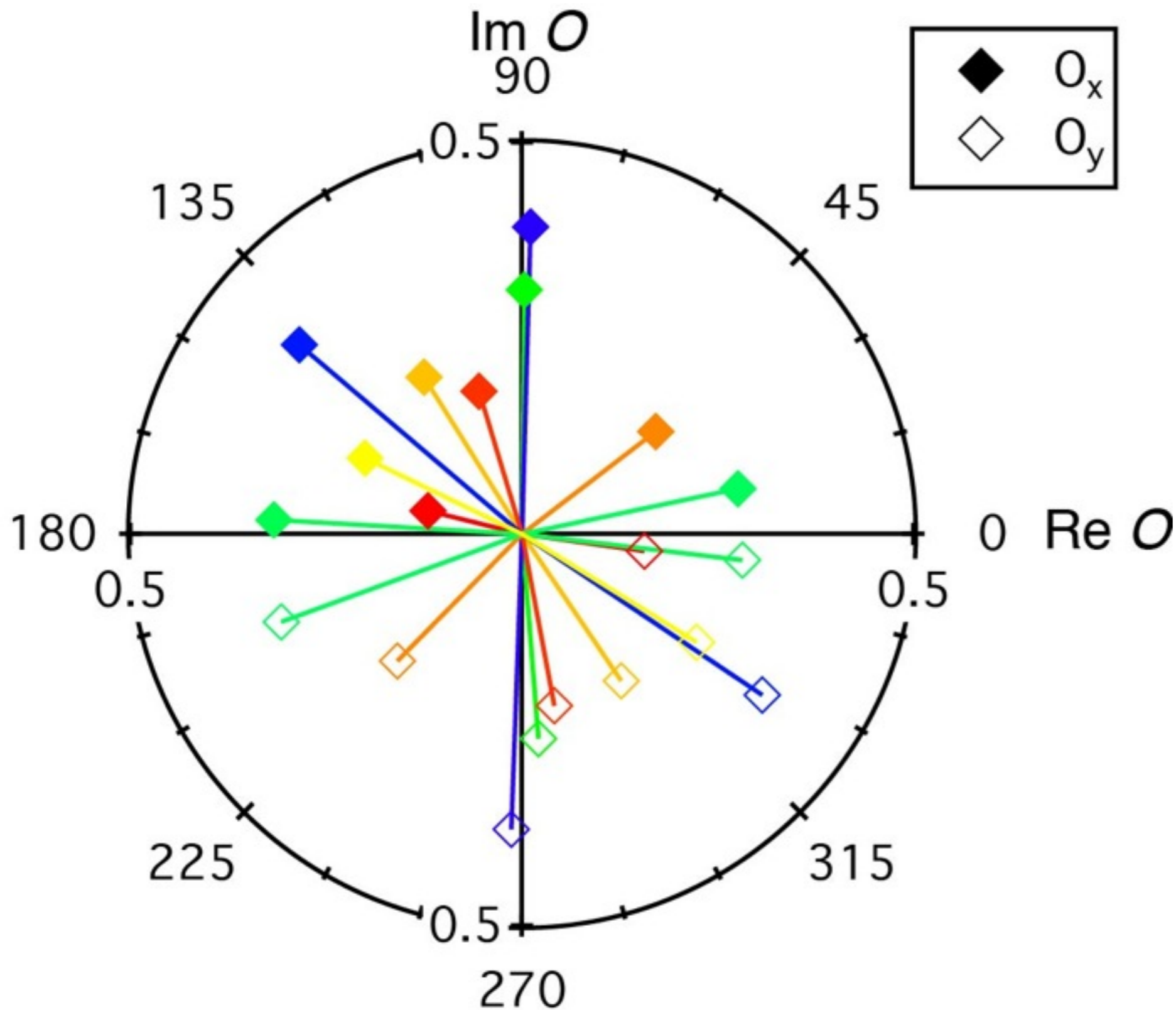
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



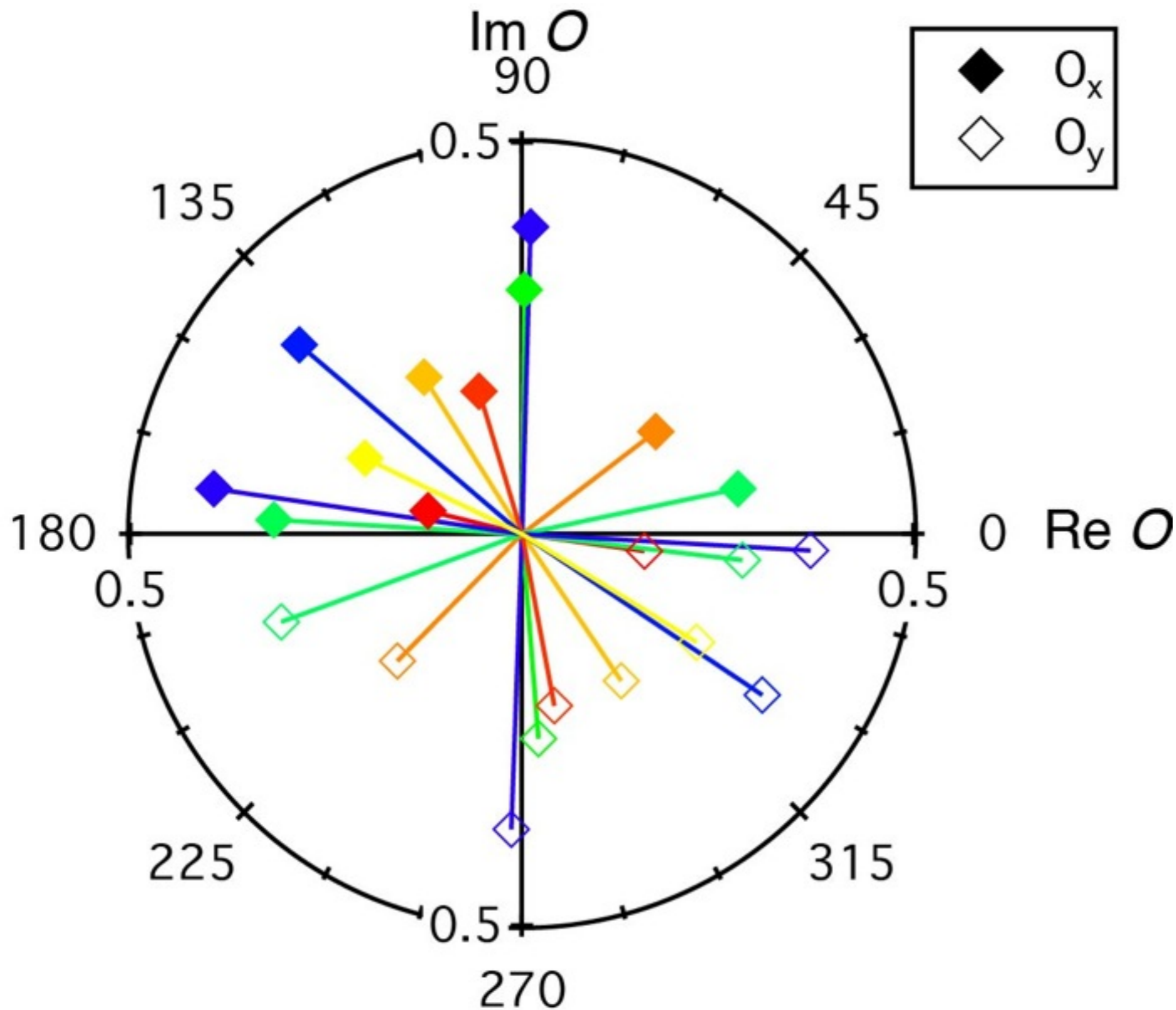
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



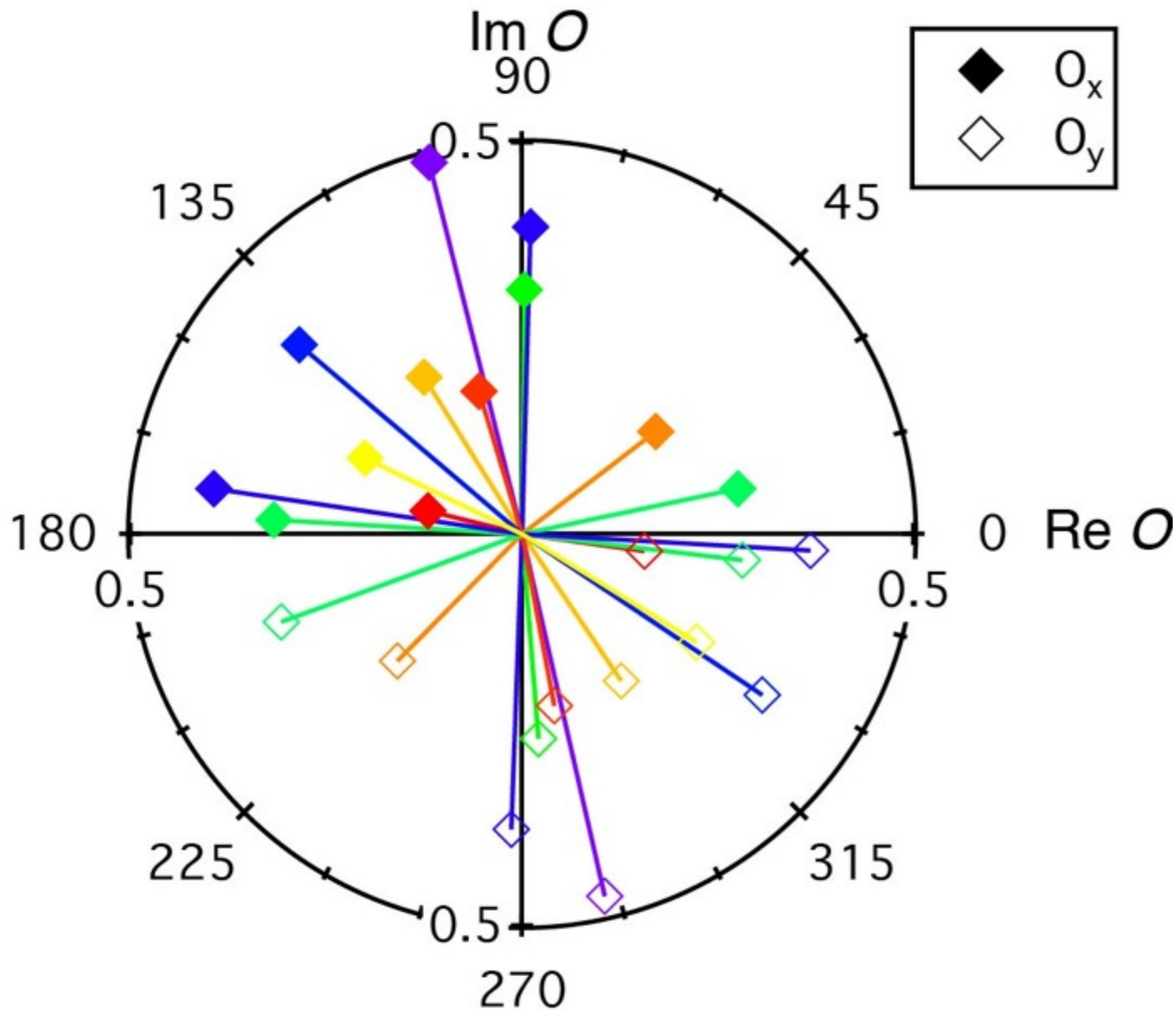
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



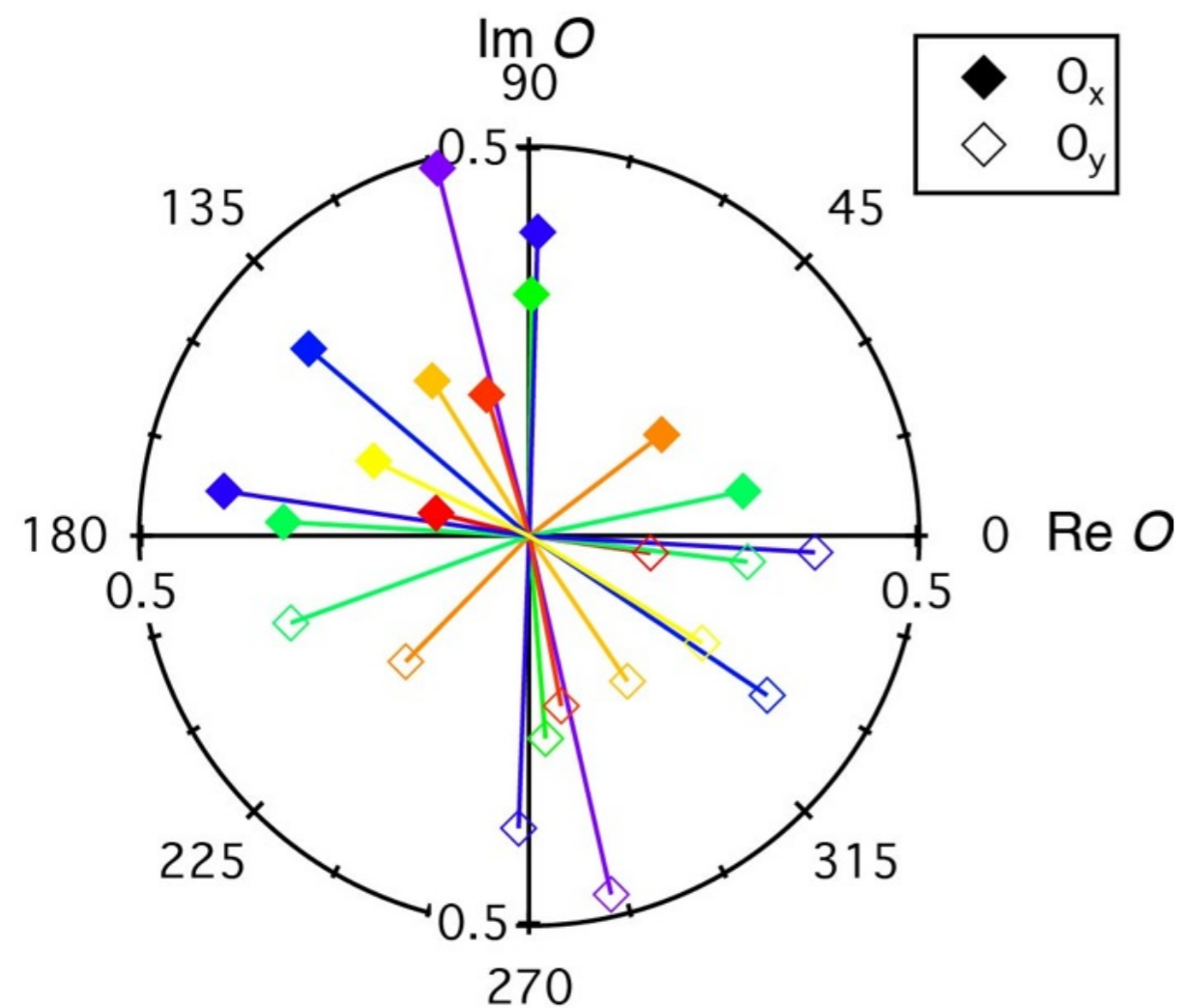
**Phase-sensitive measurement of the  $d$ -form factor of density wave order**



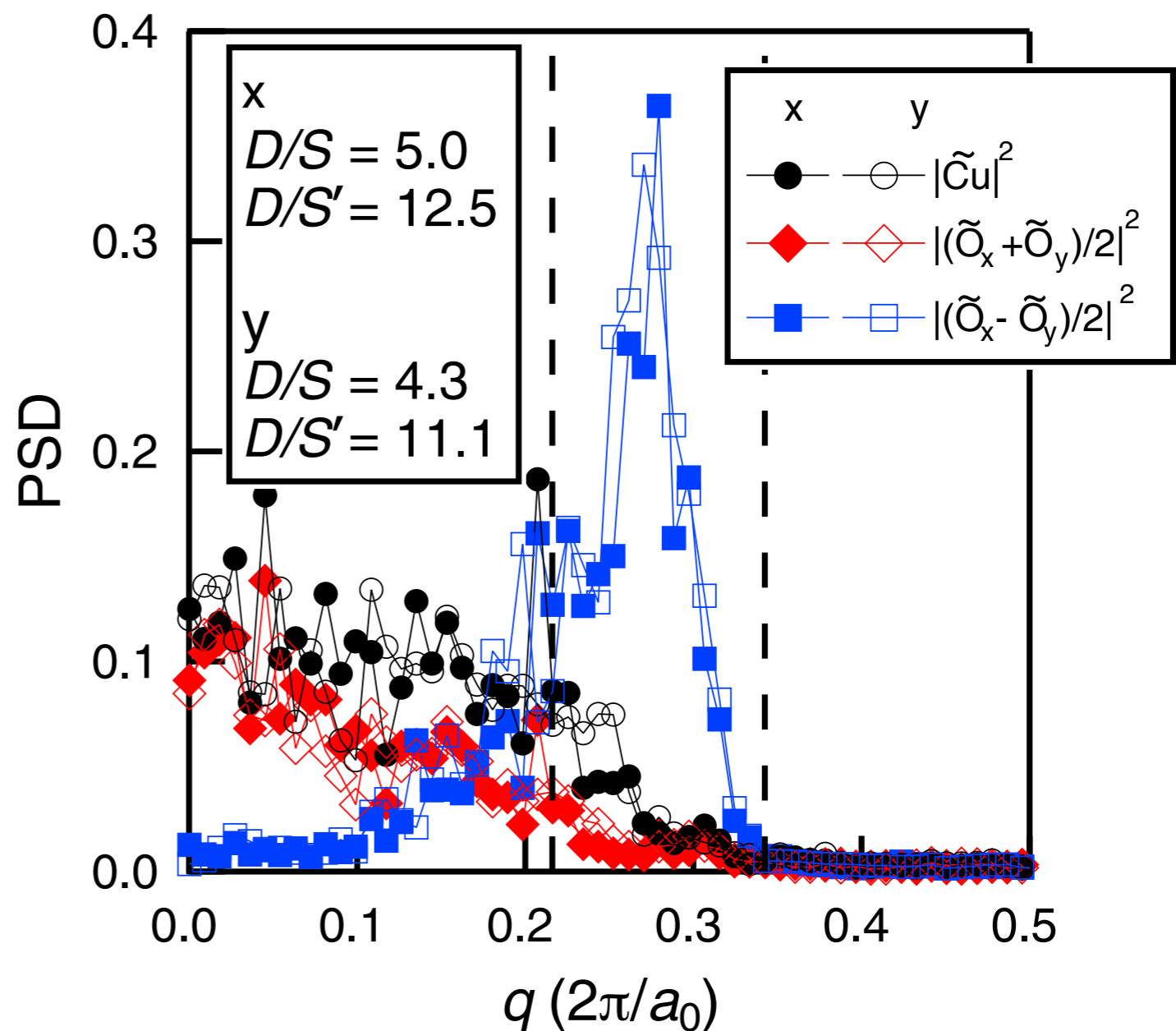
**Phase-sensitive measurement of the *d*-form factor of density wave order**

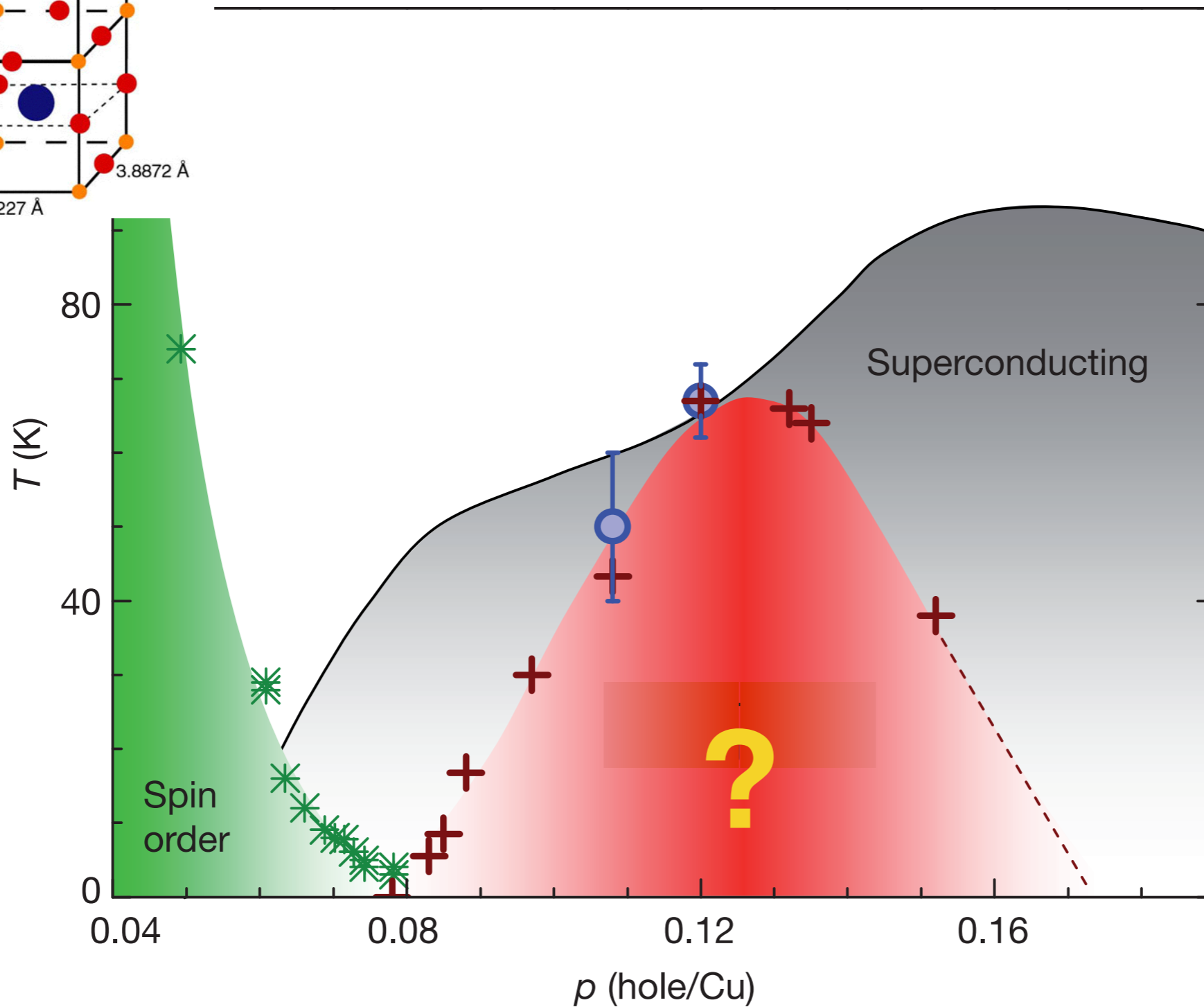
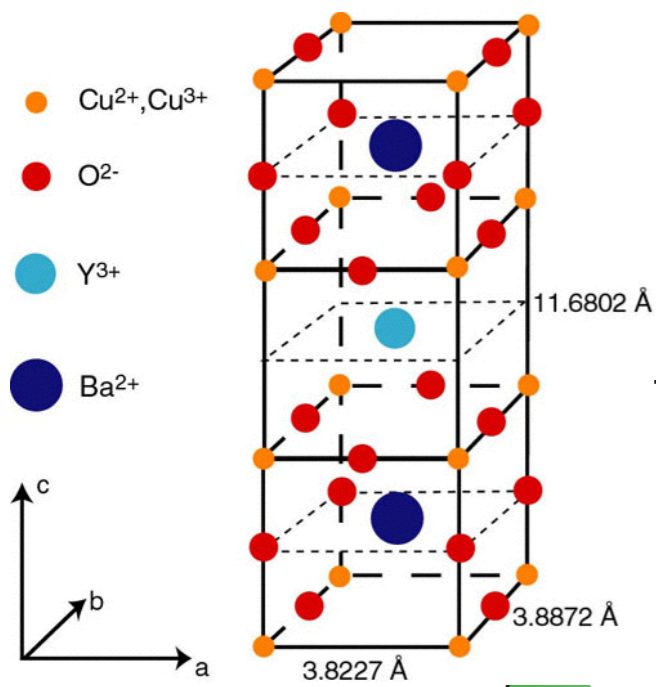


**Phase-sensitive measurement of the *d*-form factor of density wave order**

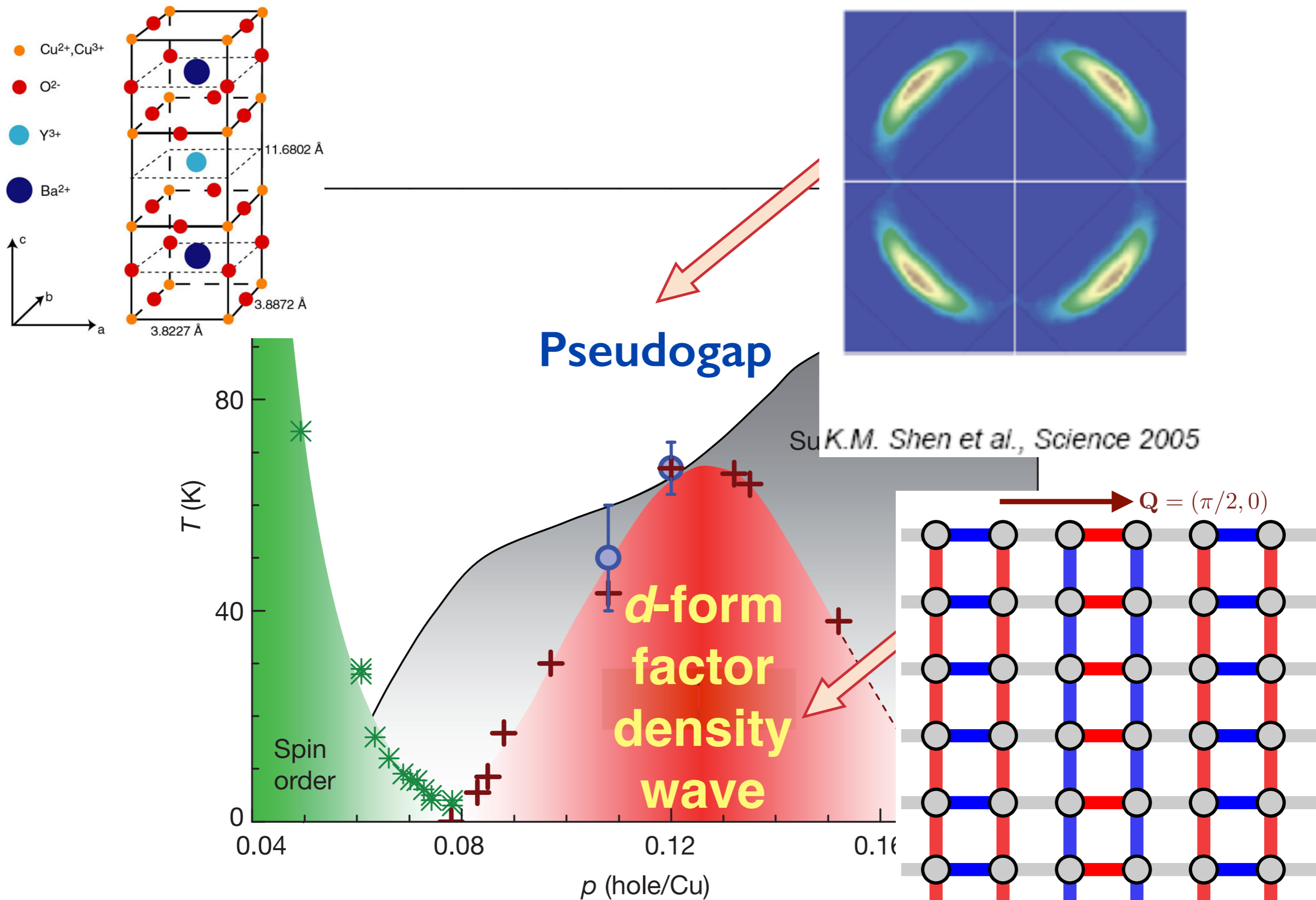


**Phase-sensitive measurement of the  $d$ -form factor of density wave order**





T. Wu, H. Mayaffre, S. Kramer, M. Horvatic, C. Berthier, W.N. Hardy, R. Liang, D.A. Bonn, and M.-H. Julien, *Nature* **477**, 191 (2011).



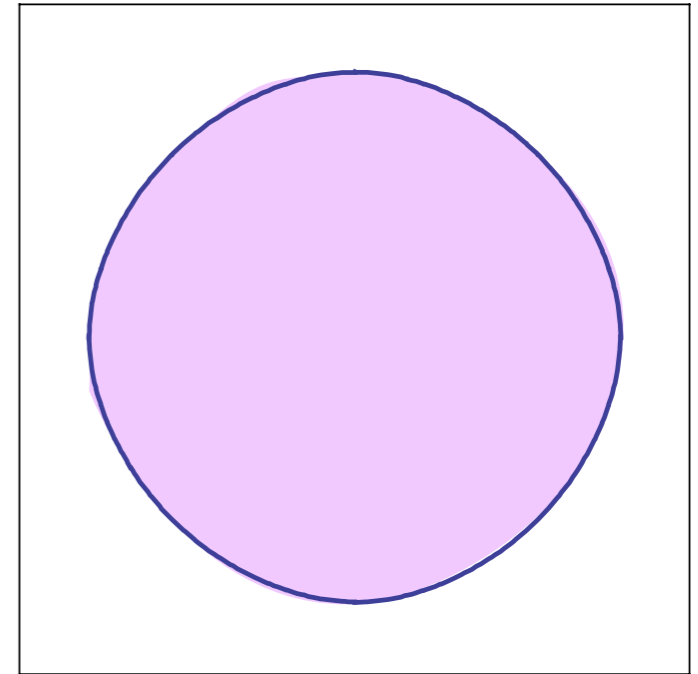
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS 111, E3026 (2014)



Theory

# Fermi surface+antiferromagnetism

Metal with “large”  
Fermi surface

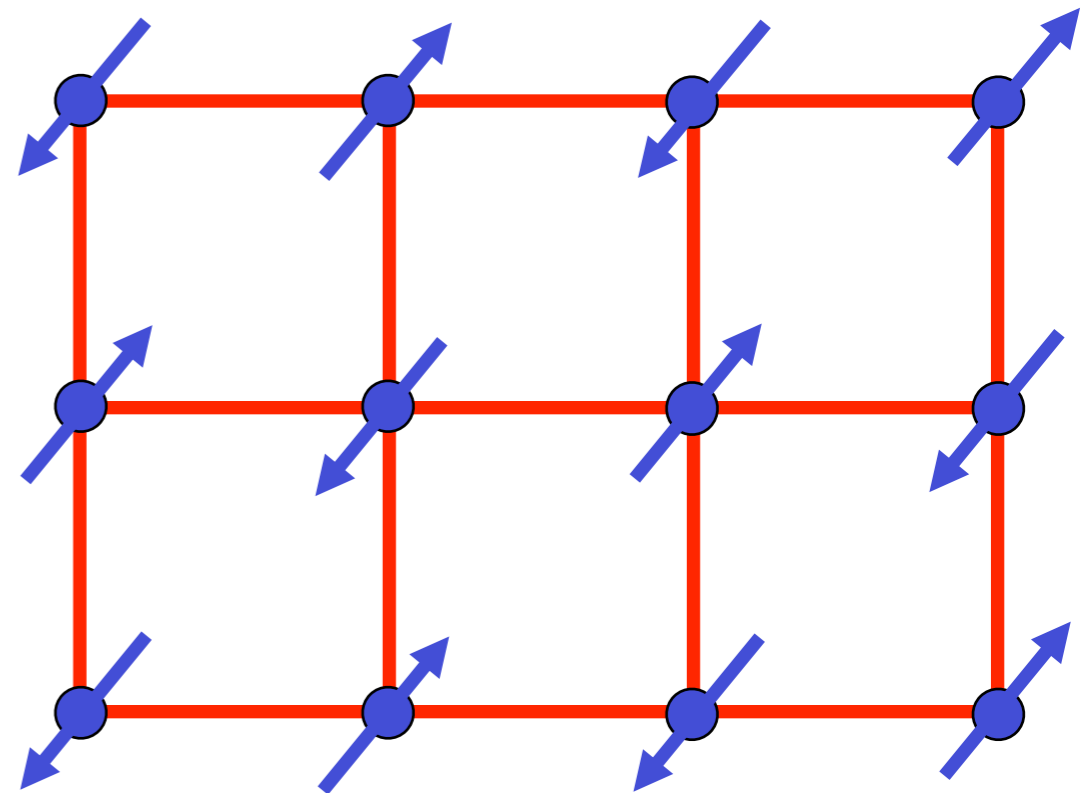


+

The electron spin polarization obeys

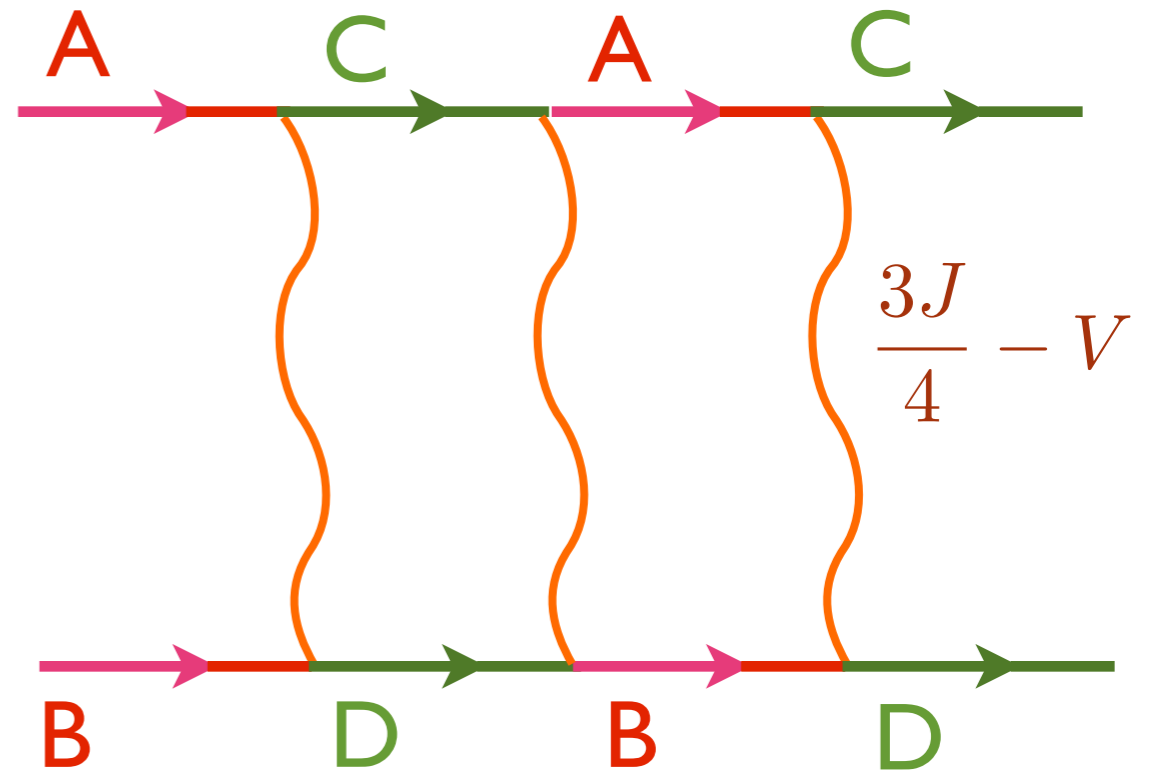
$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

where  $\mathbf{K} = (\pi, \pi)$  is the ordering  
wavevector.



# Pairing “glue” from antiferromagnetic fluctuations

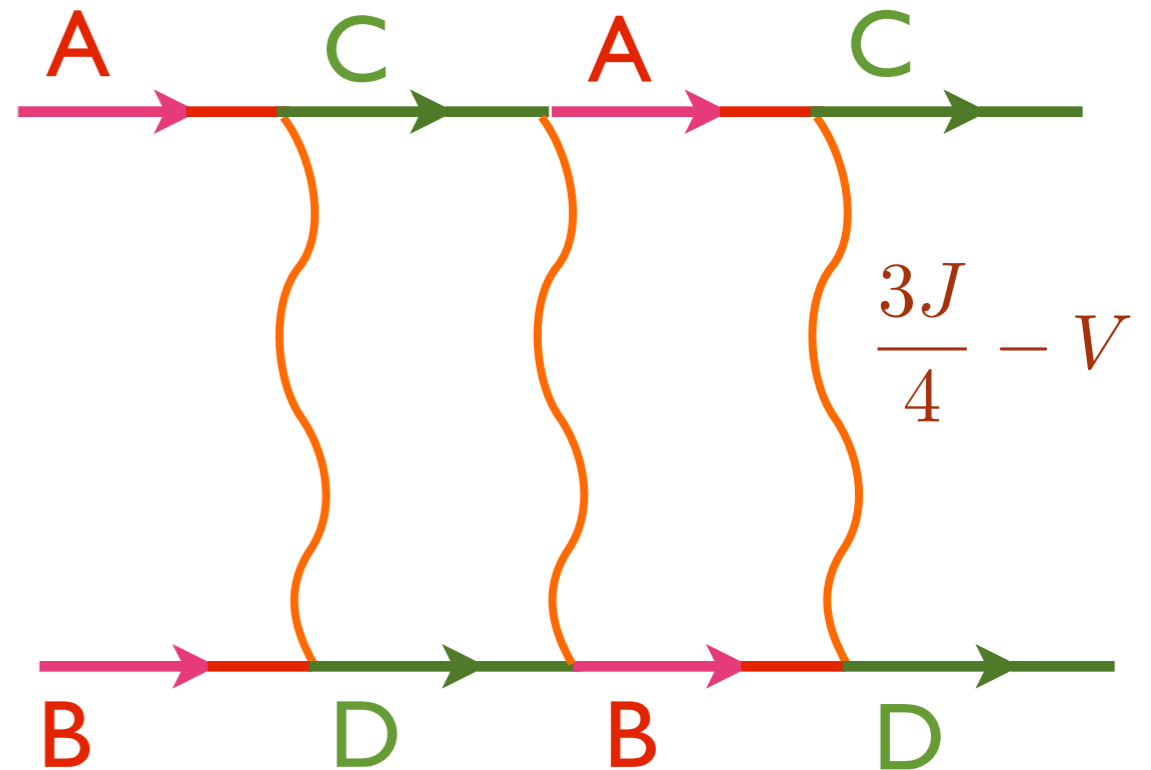
$$\begin{aligned}
 H = & - \sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} \\
 & + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \\
 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$



# Pairing “glue” from antiferromagnetic fluctuations

Electron hopping

$$\begin{aligned}
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 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$



V. J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)

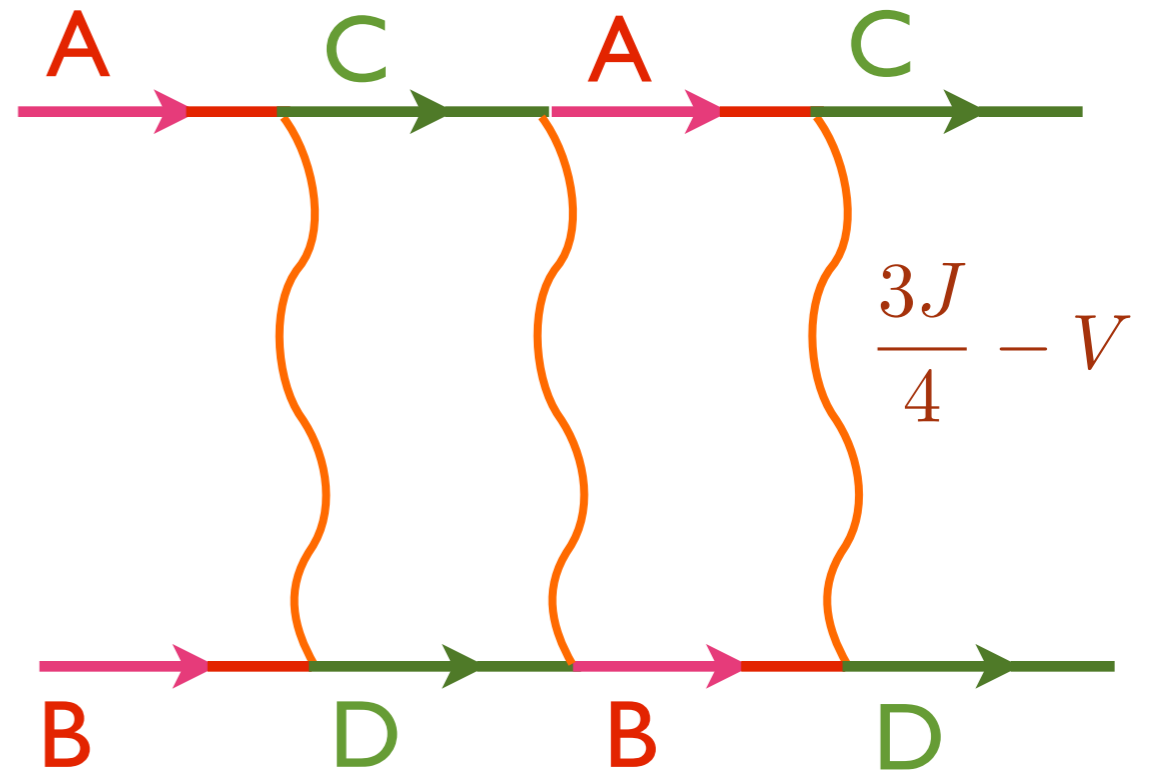
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)

K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

# Pairing “glue” from antiferromagnetic fluctuations

$$\begin{aligned}
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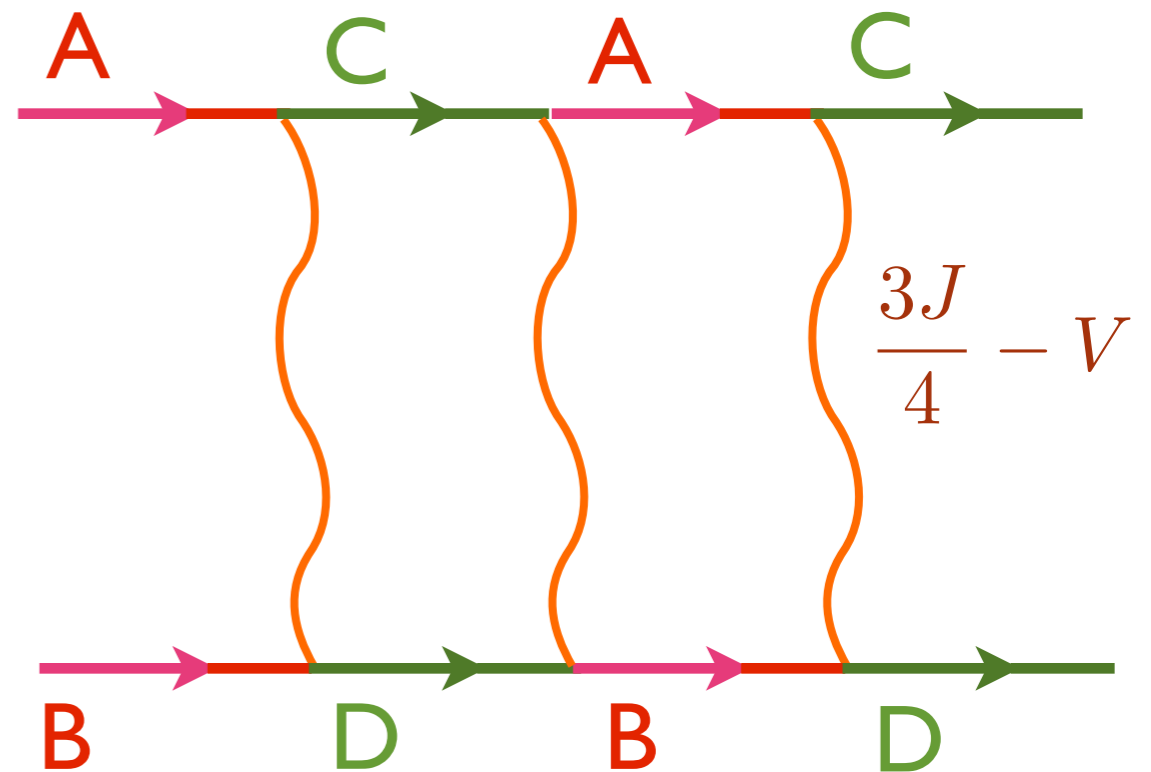
Antiferromagnetic  
exchange  
interaction



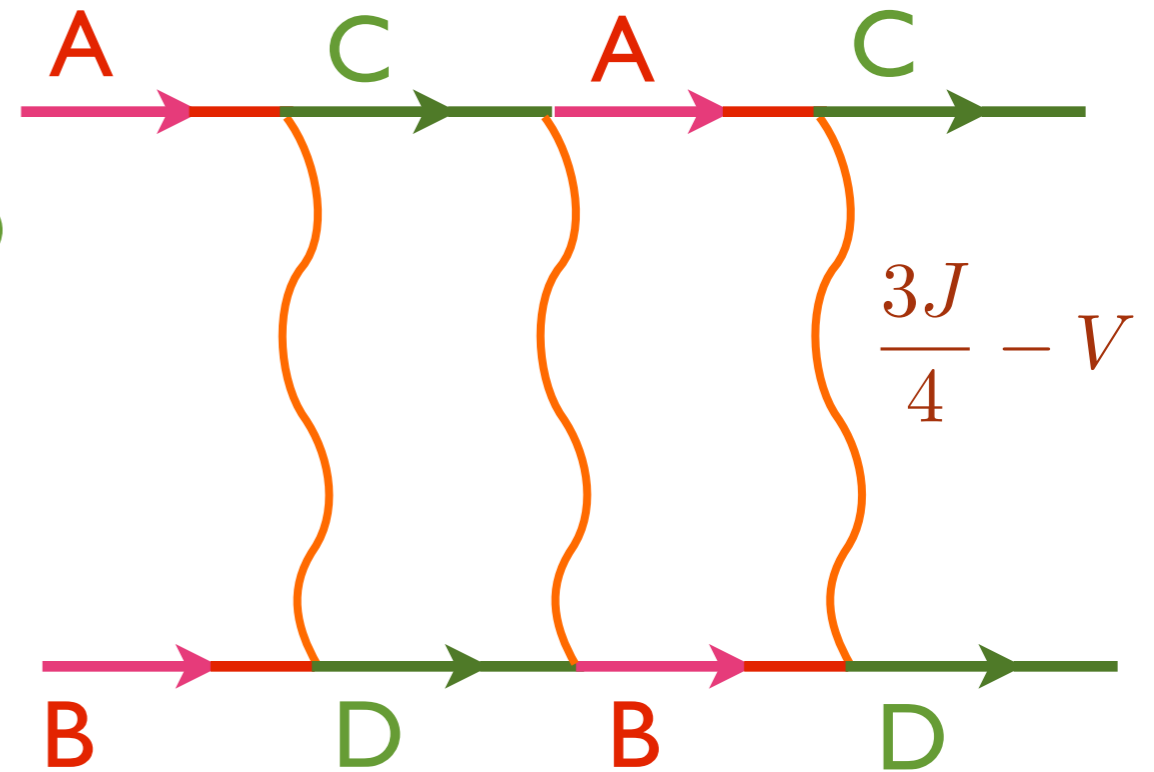
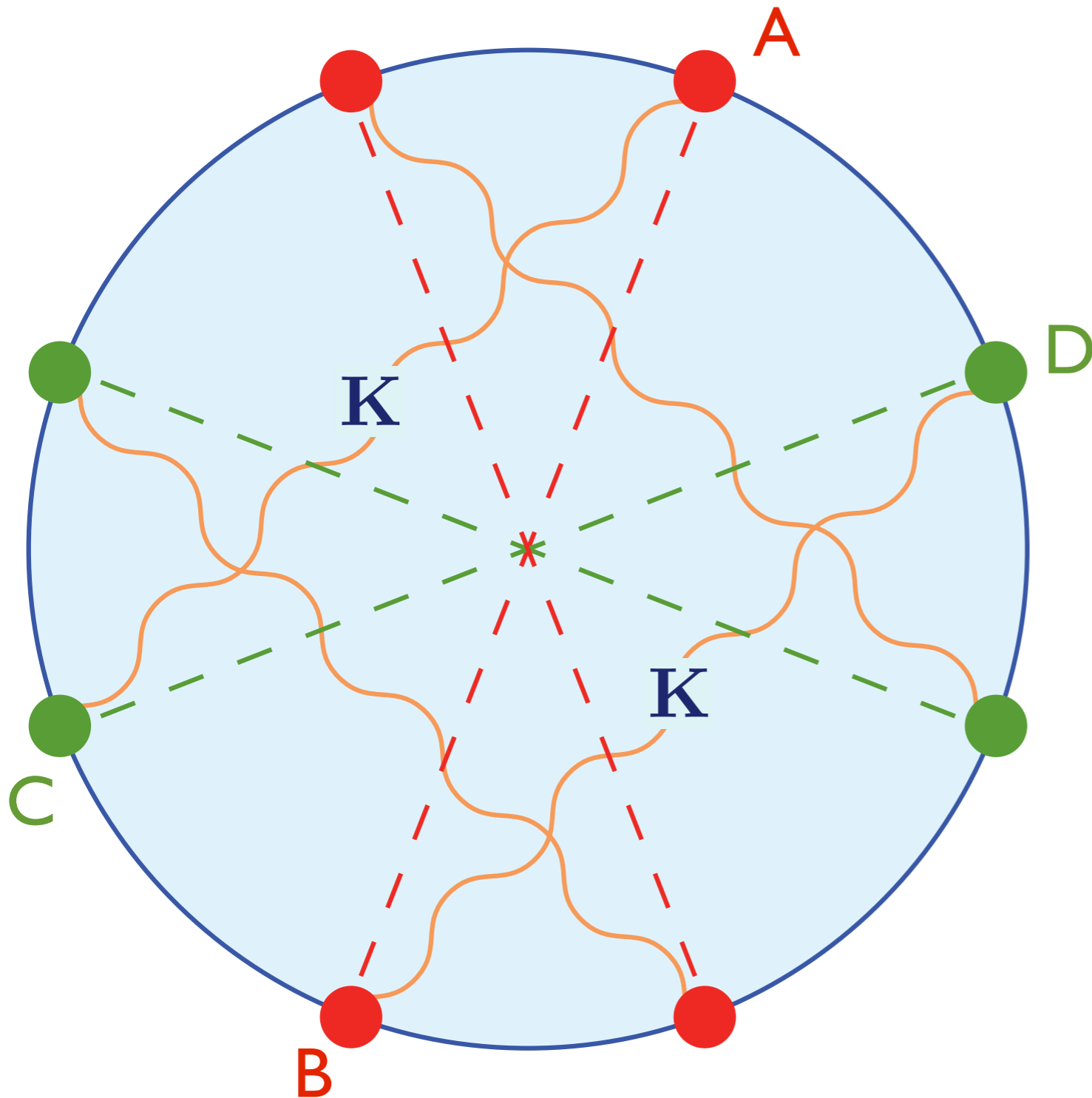
# Pairing “glue” from antiferromagnetic fluctuations

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 & + V \sum_{\langle ij \rangle} n_i n_j + \dots
 \end{aligned}$$

Coulomb repulsion

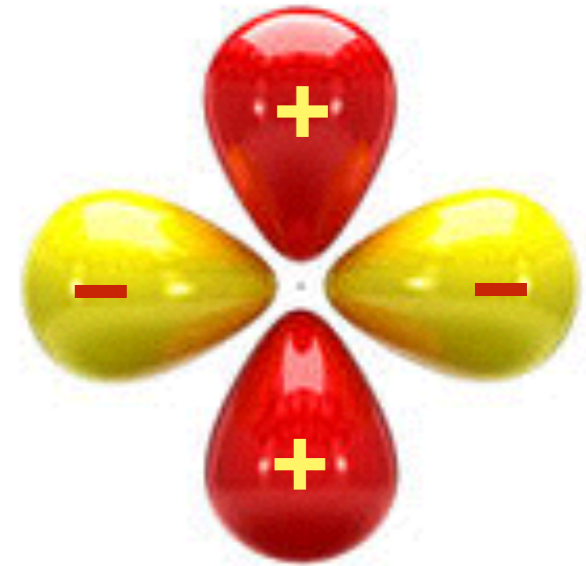
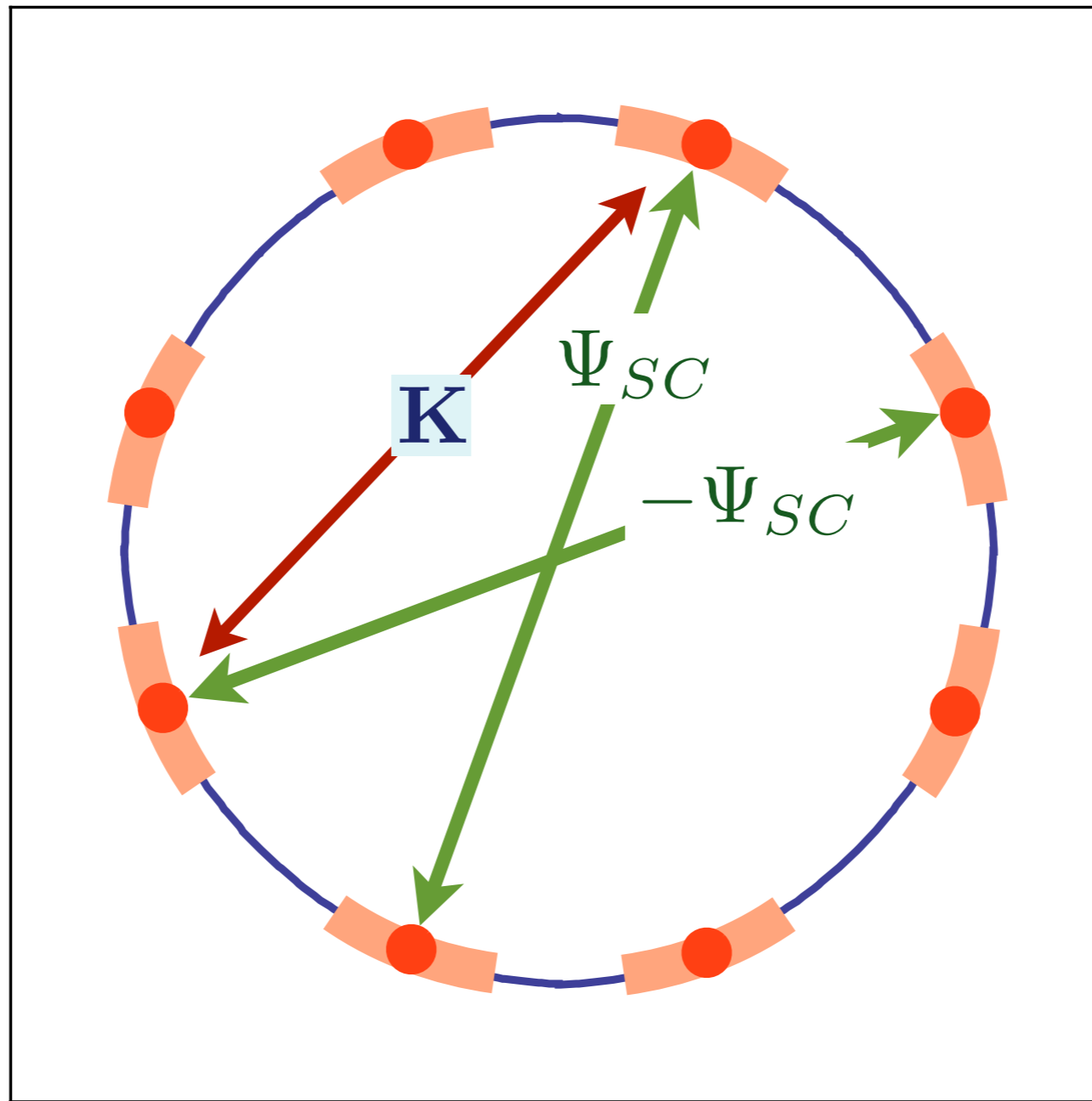


# Pairing “glue” from antiferromagnetic fluctuations



V.J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)  
D.J. Scalapino, E. Loh, and J.E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)  
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} (\cos k_x - \cos k_y) \Psi_{SC}$$



**d-wave superconductor:  
sign-changing pairing amplitude**

V. J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)  
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)  
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)

## Pseudospin symmetry of the exchange interaction

$$H_J = \sum_{i < j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

with  $\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}$  is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

$$\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow}^\dagger \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}$$

Then we can write

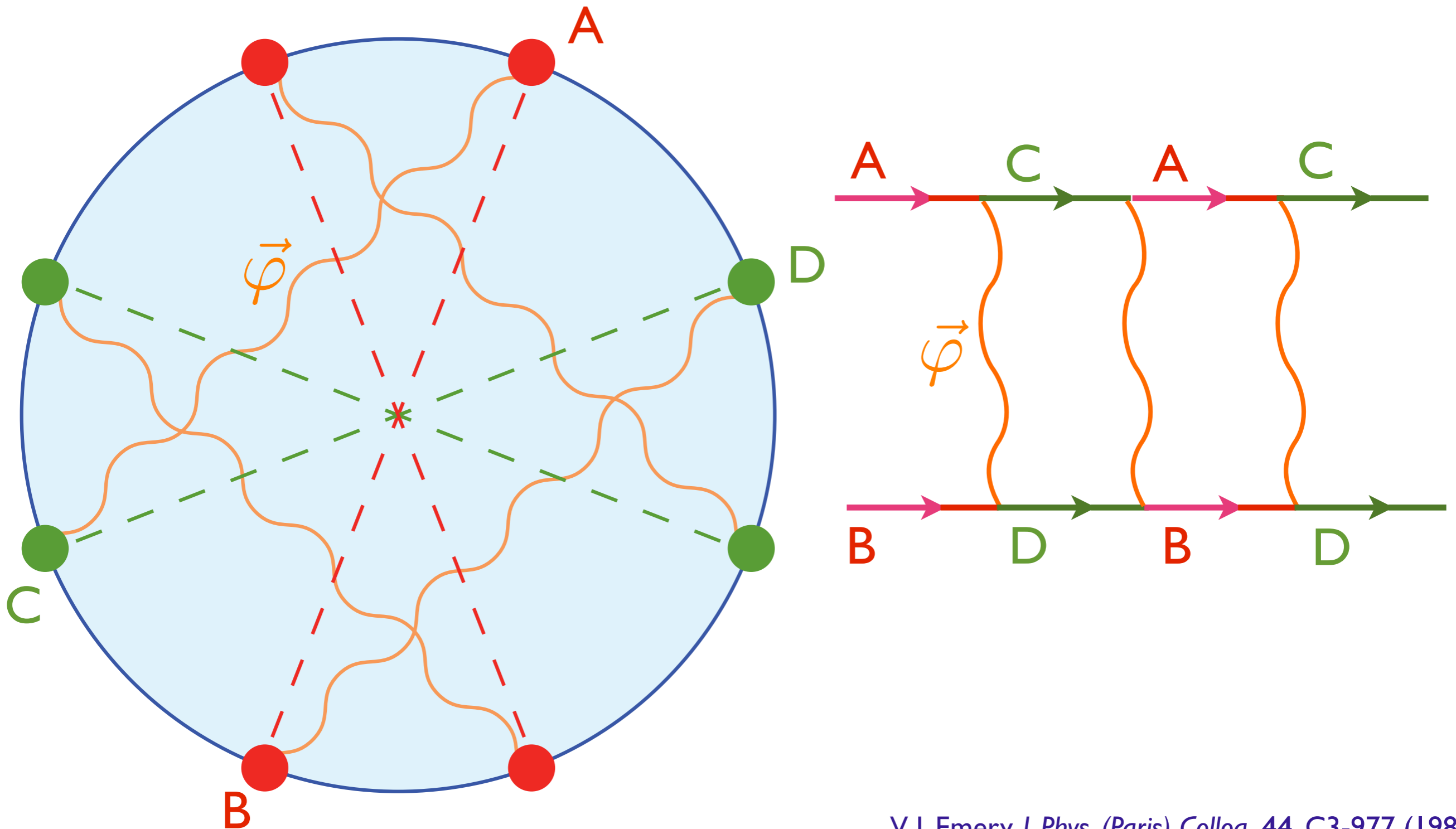
$$H_J = \frac{1}{8} \sum_{i < j} J_{ij} \left( \Psi_{i\alpha a}^\dagger \vec{\sigma}_{\alpha\beta} \Psi_{i\beta a} \right) \cdot \left( \Psi_{j\gamma b}^\dagger \vec{\sigma}_{\gamma\delta} \Psi_{j\delta b} \right)$$

where  $a, b$  are the Nambu indices. This form makes explicit the symmetry under *independent* SU(2) pseudospin transformations on each site

$$\Psi_{i\alpha a} \rightarrow U_{i,ab} \Psi_{i\alpha b}$$

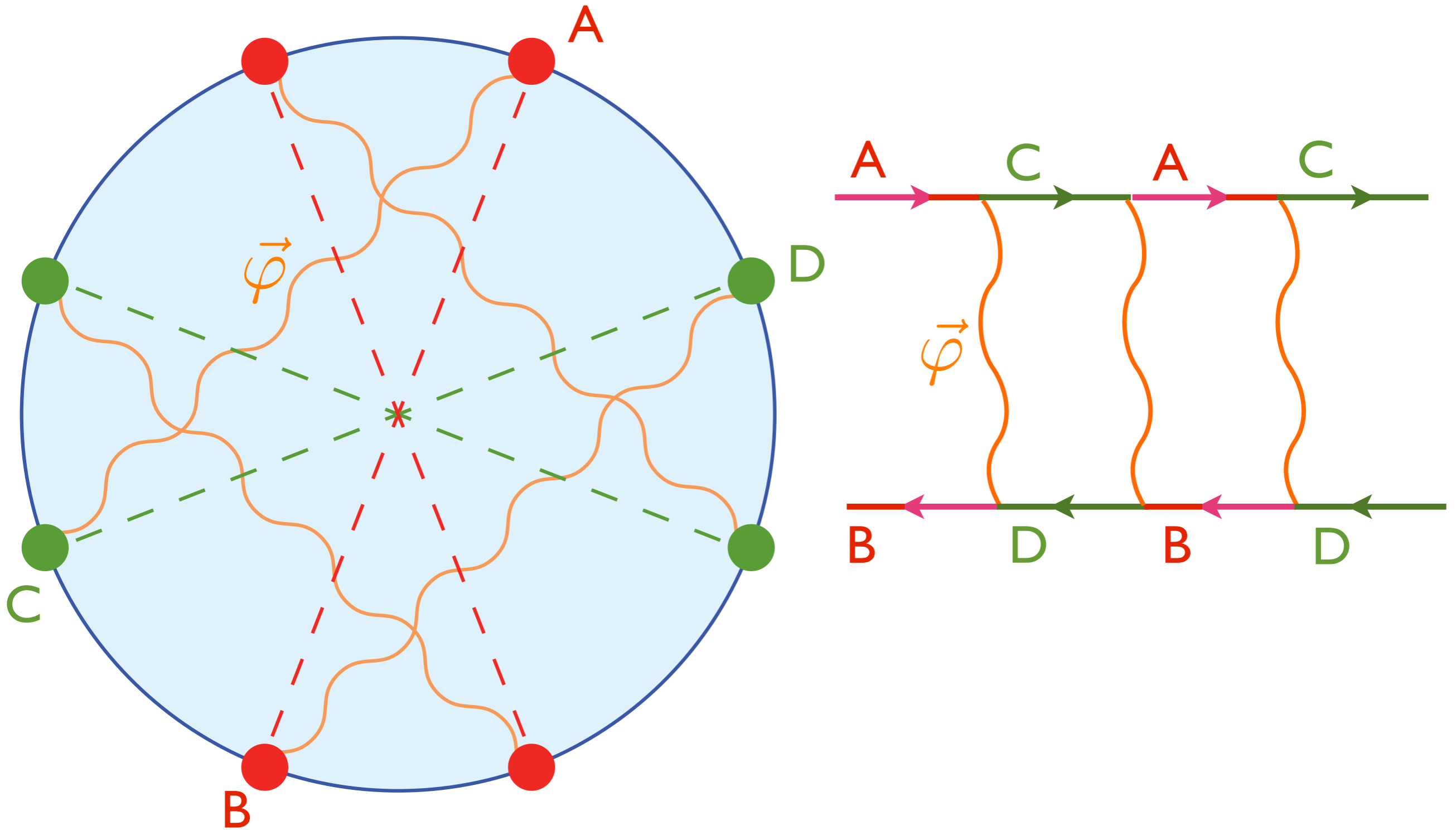
- I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988)
- E. Dagotto, E. Fradkin, and A. Moreo, Phys. Rev. B **38**, 2926 (1988)
- P. A. Lee, N. Nagaosa, and X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)

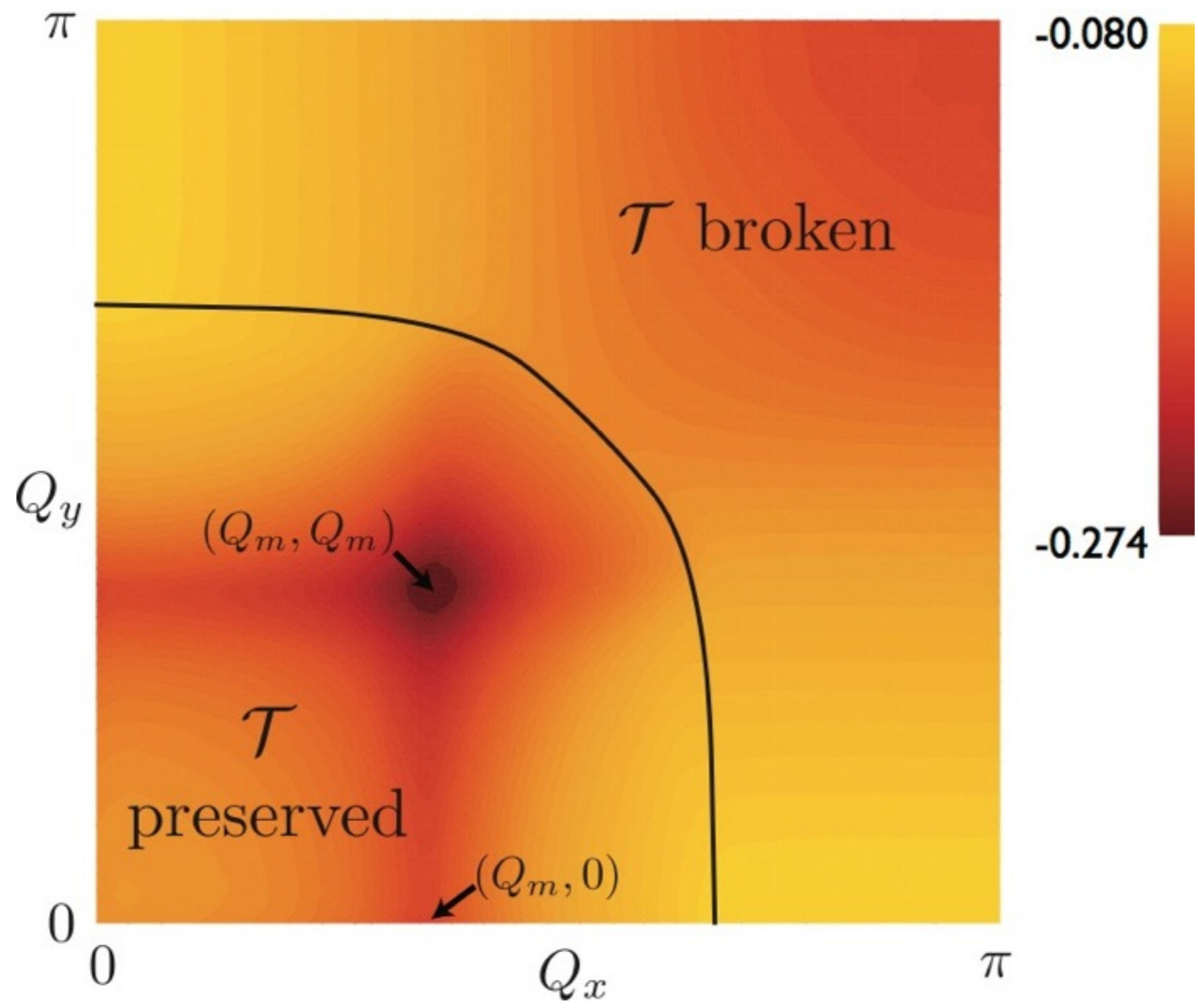
# Pairing “glue” from antiferromagnetic fluctuations



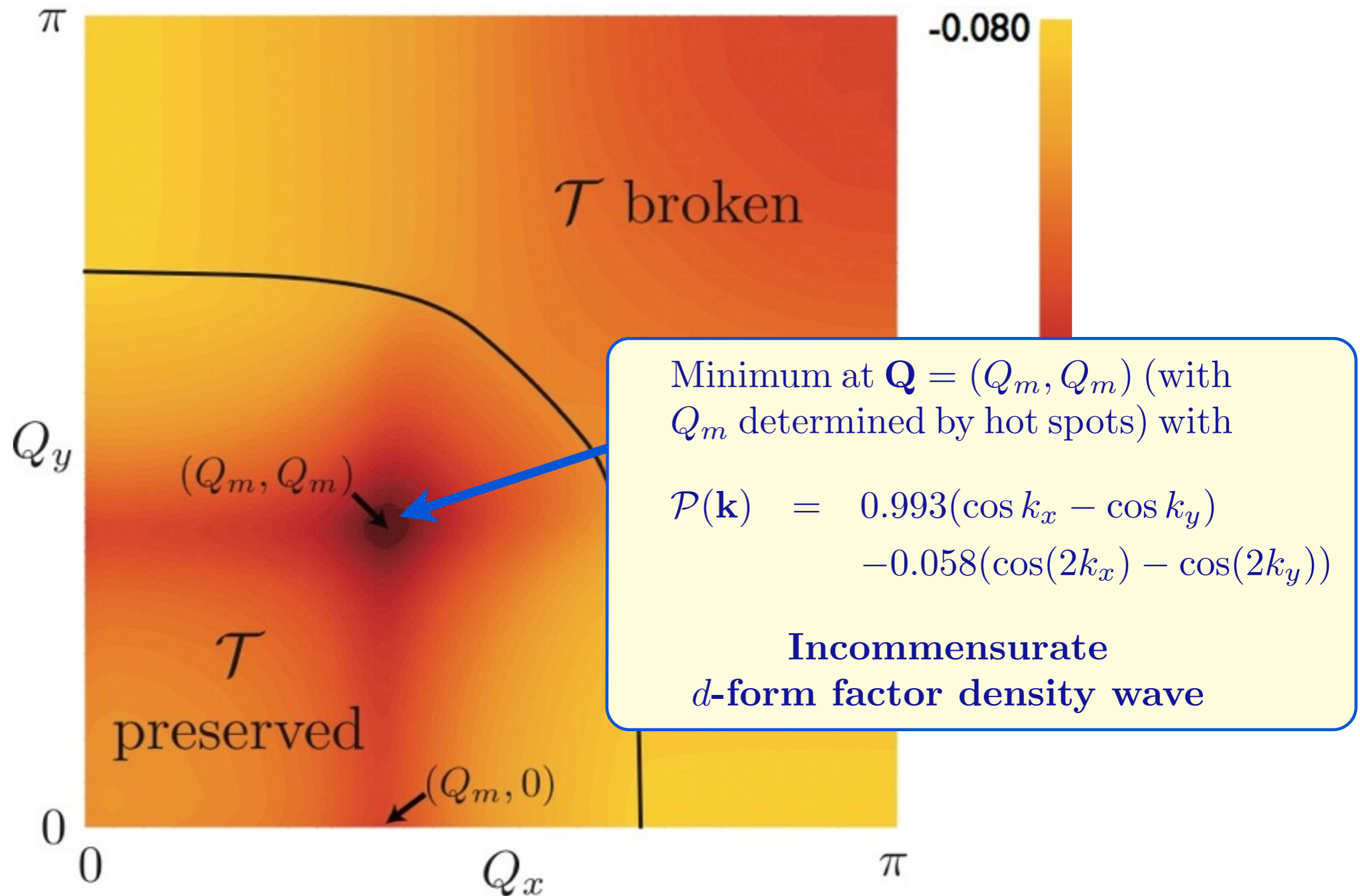
V. J. Emery, *J. Phys. (Paris) Colloq.* 44, C3-977 (1983)  
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* 34, 8190 (1986)  
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* 34, 6554 (1986)  
S. Raghu, S. A. Kivelson, and D. J. Scalapino, *Phys. Rev. B* 81, 224505 (2010)

Perform pseudospin rotation on **B** and **D** electrons, but not on **A** and **C** electrons: Same “glue” leads to particle-hole pairing



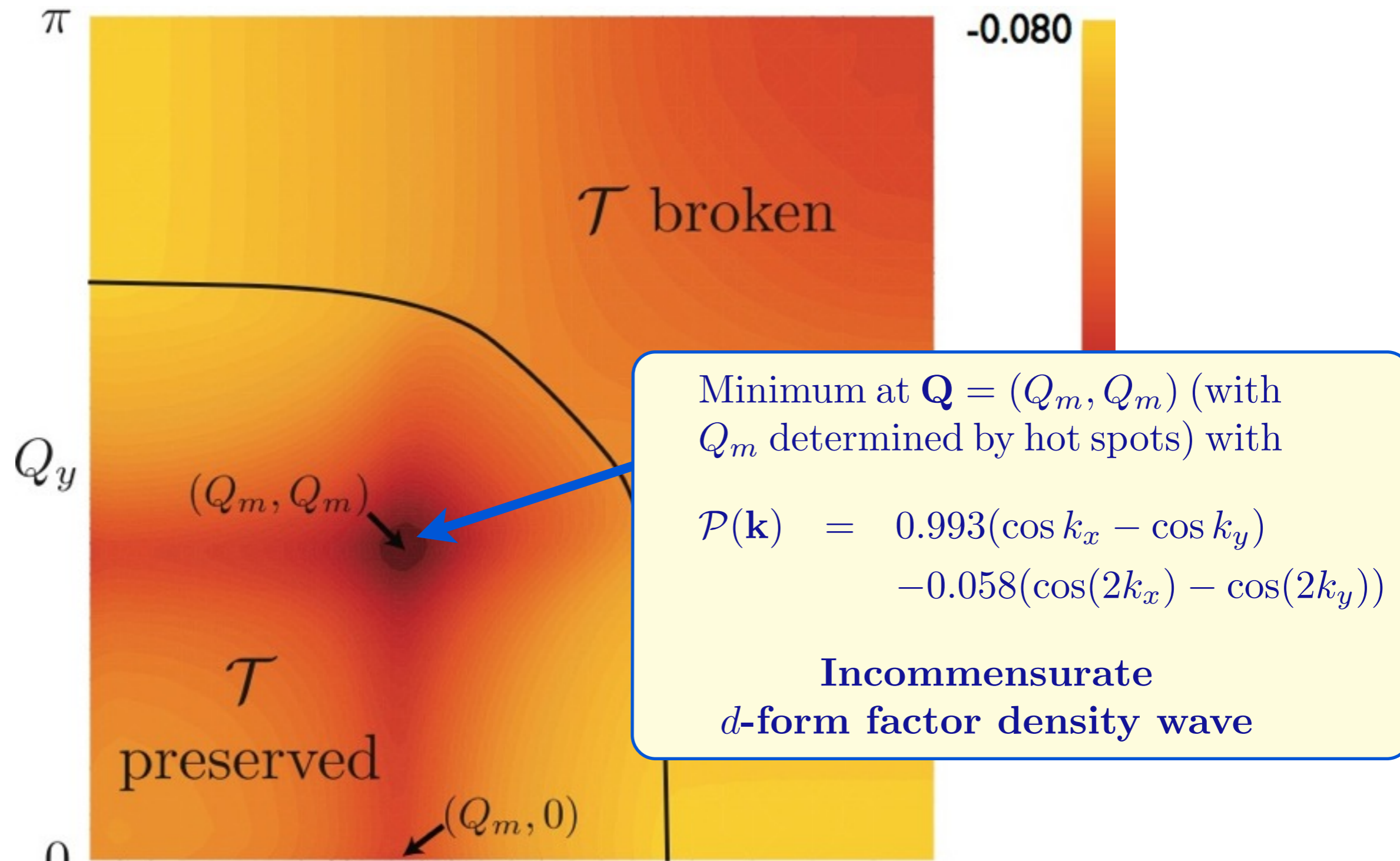


Eigenvalues,  $\lambda(\mathbf{Q})$ , of the spin-singlet, particle-hole propagator. The corresponding eigenvector is  $\mathcal{P}(\mathbf{k})$  and this leads to the order  $\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = [\int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}] e^{i\mathbf{Q}\cdot(\mathbf{r}_i + \mathbf{r}_j)/2}$



Eigenvalues,  $\lambda(\mathbf{Q})$ , of the spin-singlet, particle-hole propagator. The corresponding eigenvector is  $\mathcal{P}(\mathbf{k})$  and this leads to the order

$$\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle = \left[ \int_{\mathbf{k}} \mathcal{P}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{i\mathbf{Q} \cdot (\mathbf{r}_i + \mathbf{r}_j)/2}$$

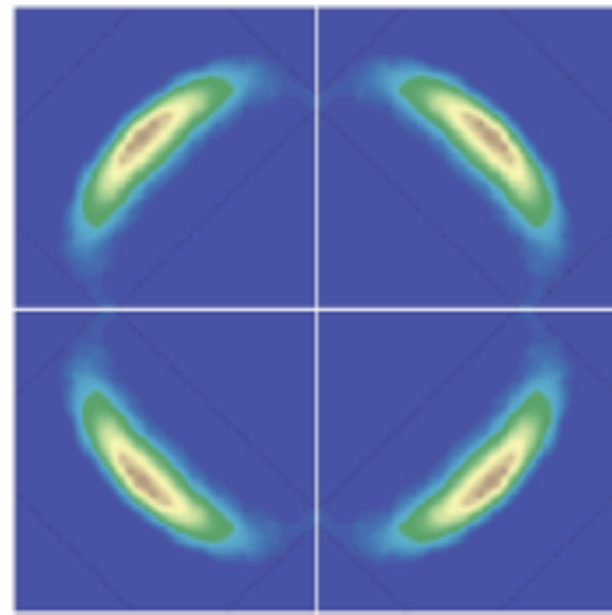
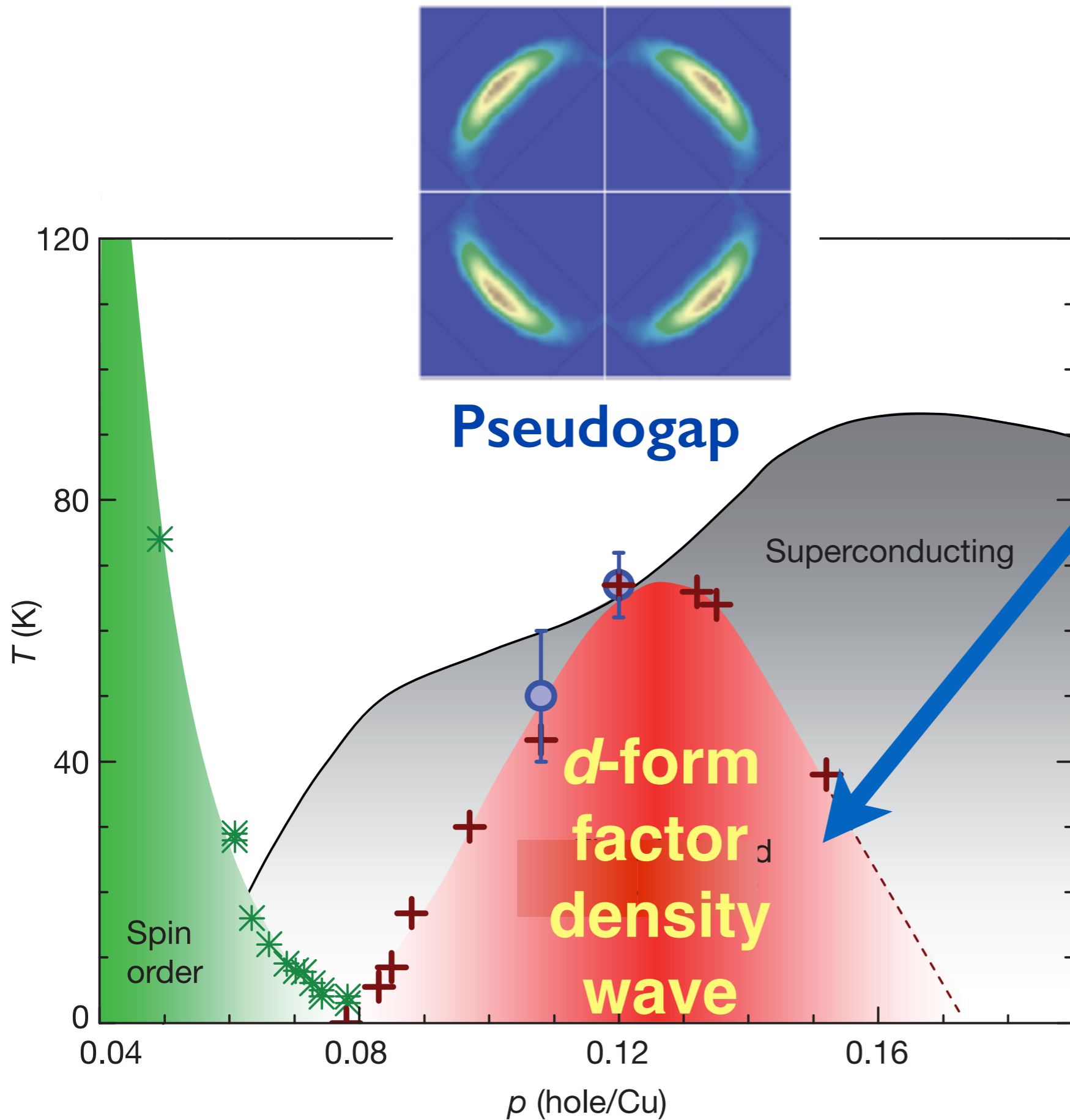


Minimum at  $\mathbf{Q} = (Q_m, Q_m)$  (with  $Q_m$  determined by hot spots) with

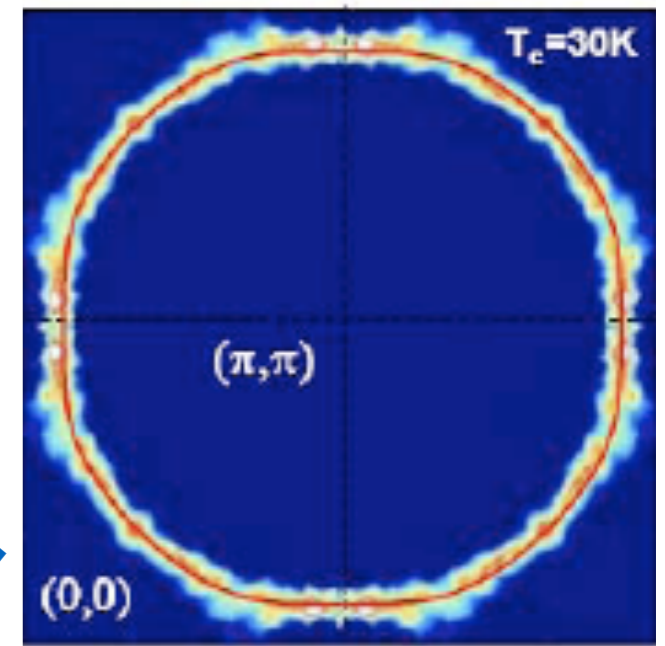
$$\mathcal{P}(\mathbf{k}) = 0.993(\cos k_x - \cos k_y) - 0.058(\cos(2k_x) - \cos(2k_y))$$

**Incommensurate  
*d*-form factor density wave**

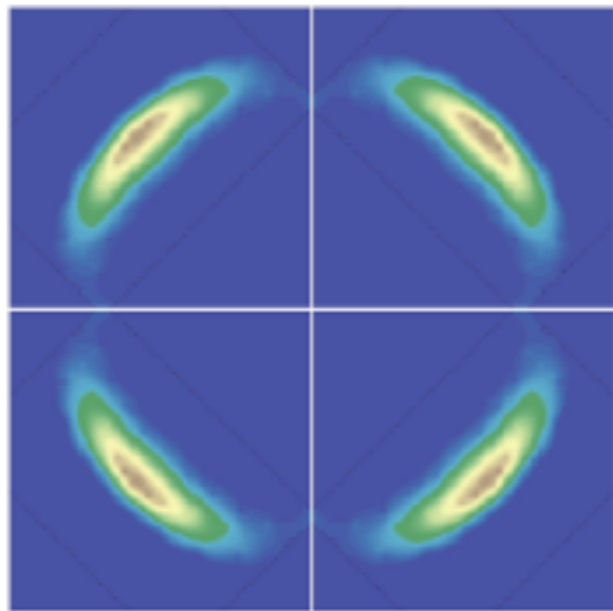
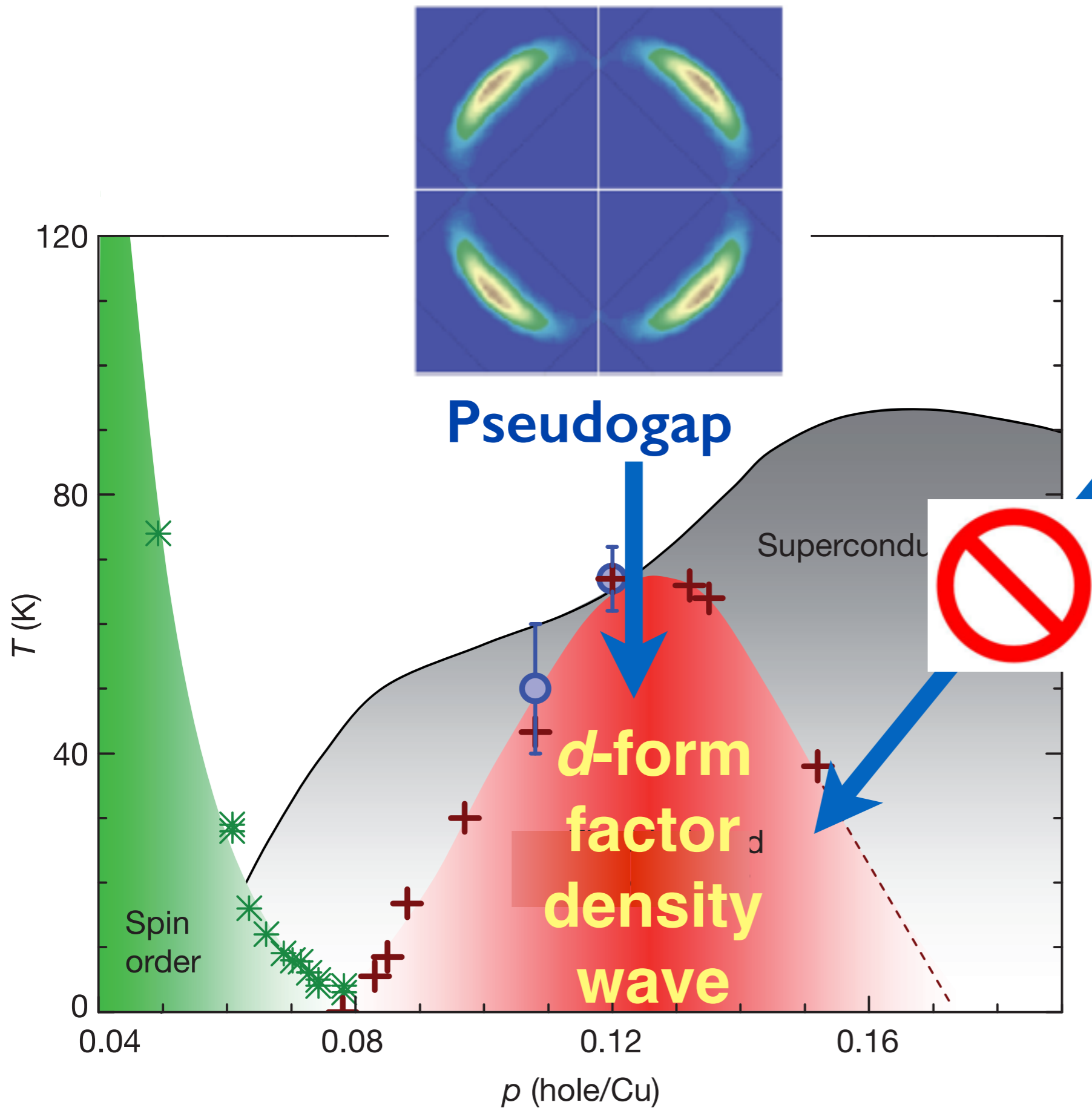
This theory yields the correct form factor, but the incorrect  $\mathbf{Q}$



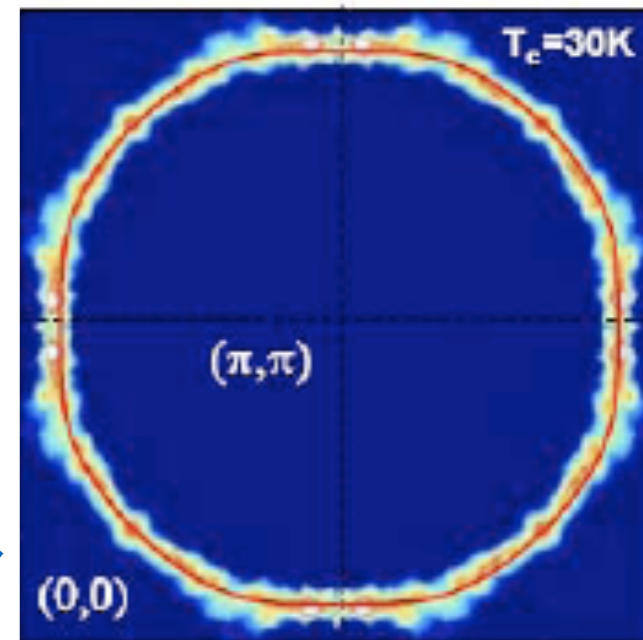
**Pseudogap**



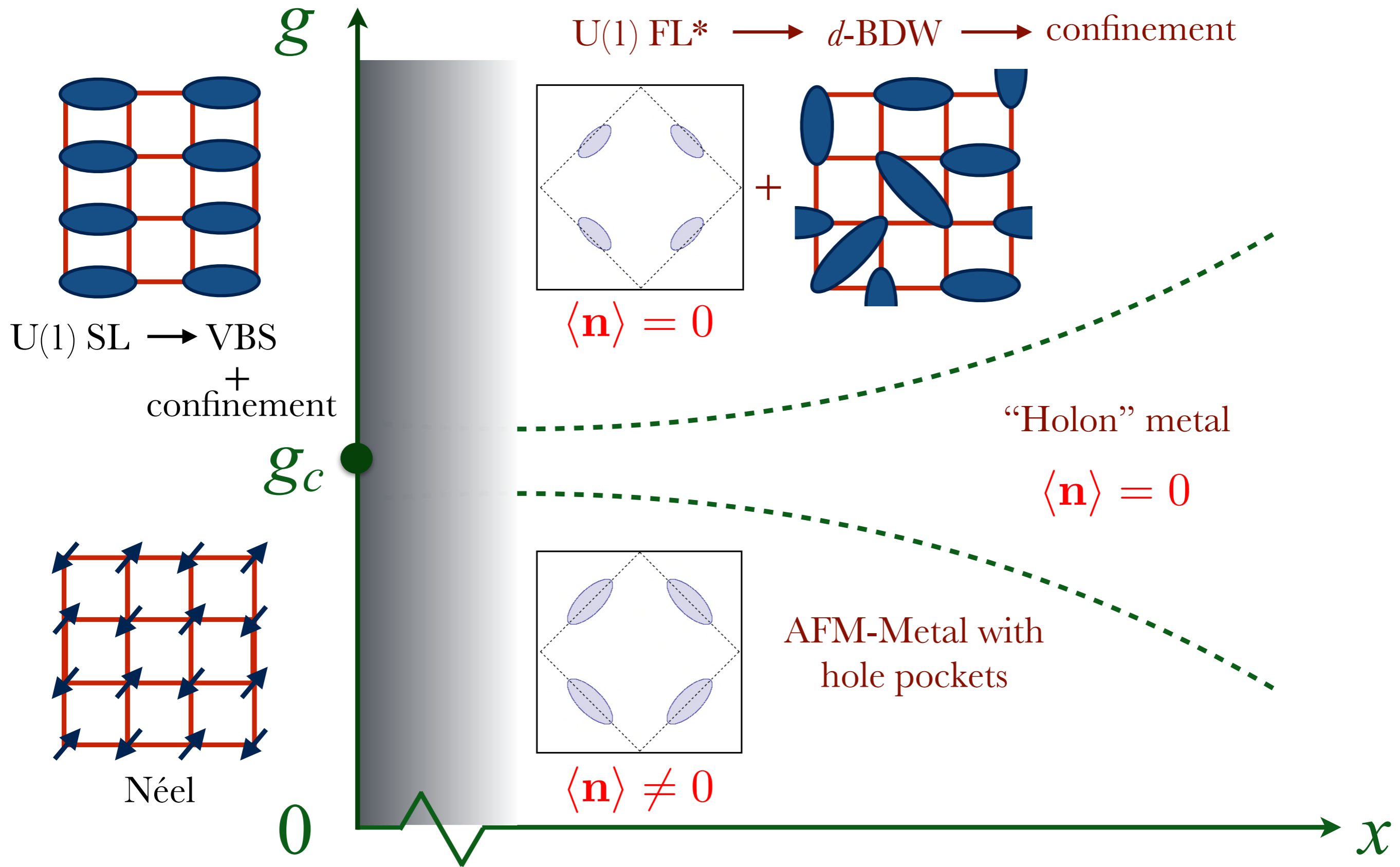
Have examined density wave instabilities of large Fermi surface, not the “Fermi arc” state found at low doping.

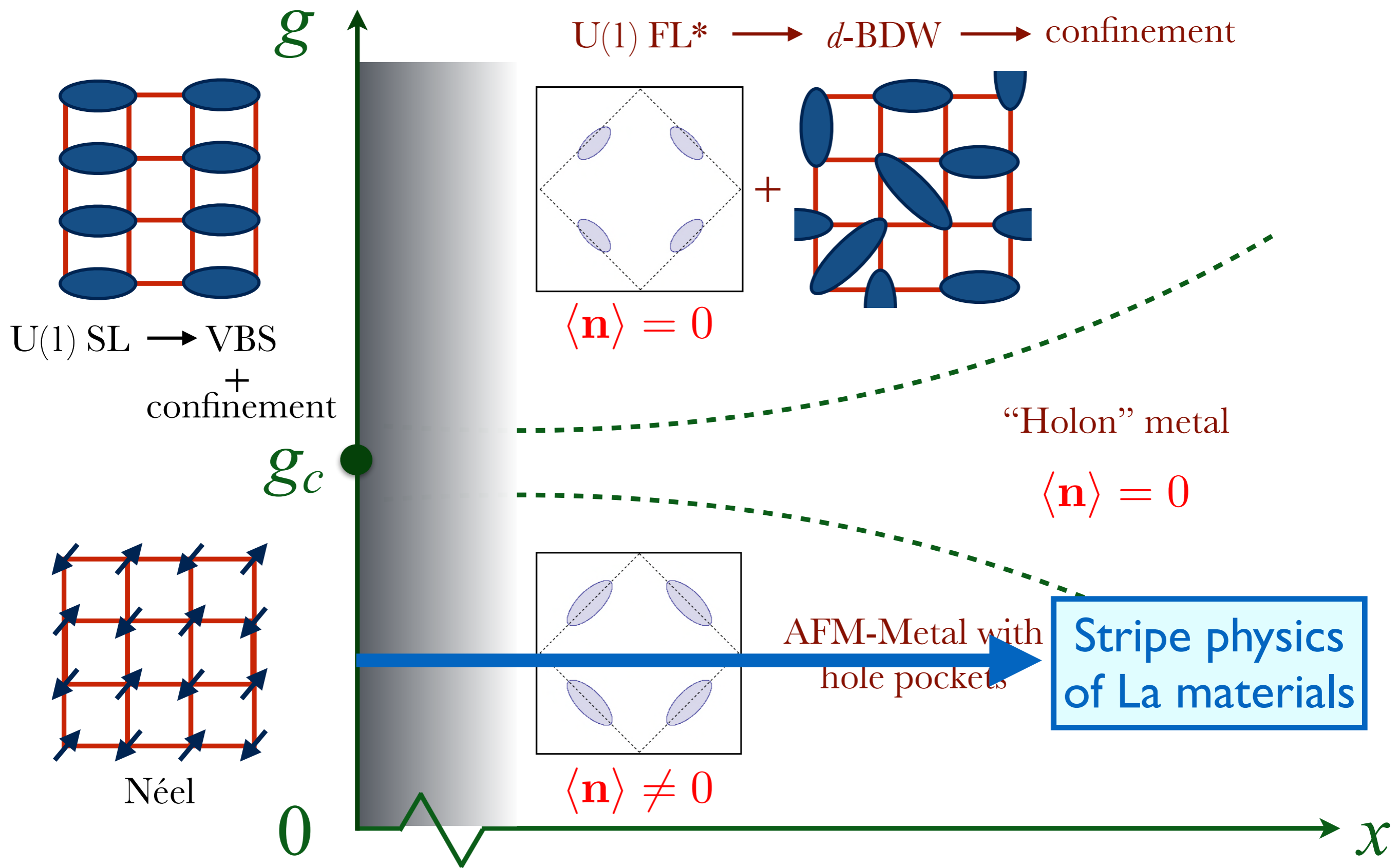


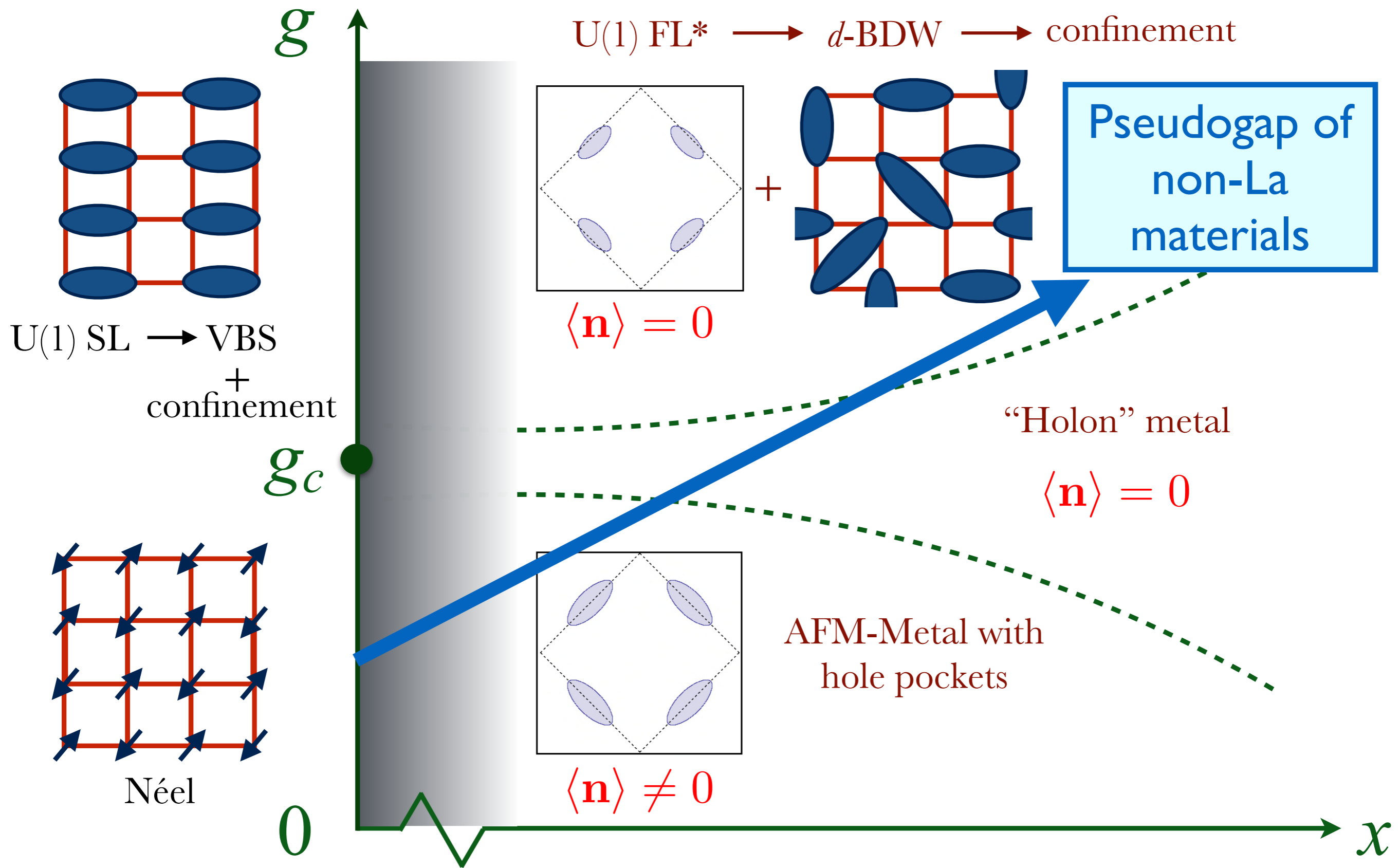
**Pseudogap**

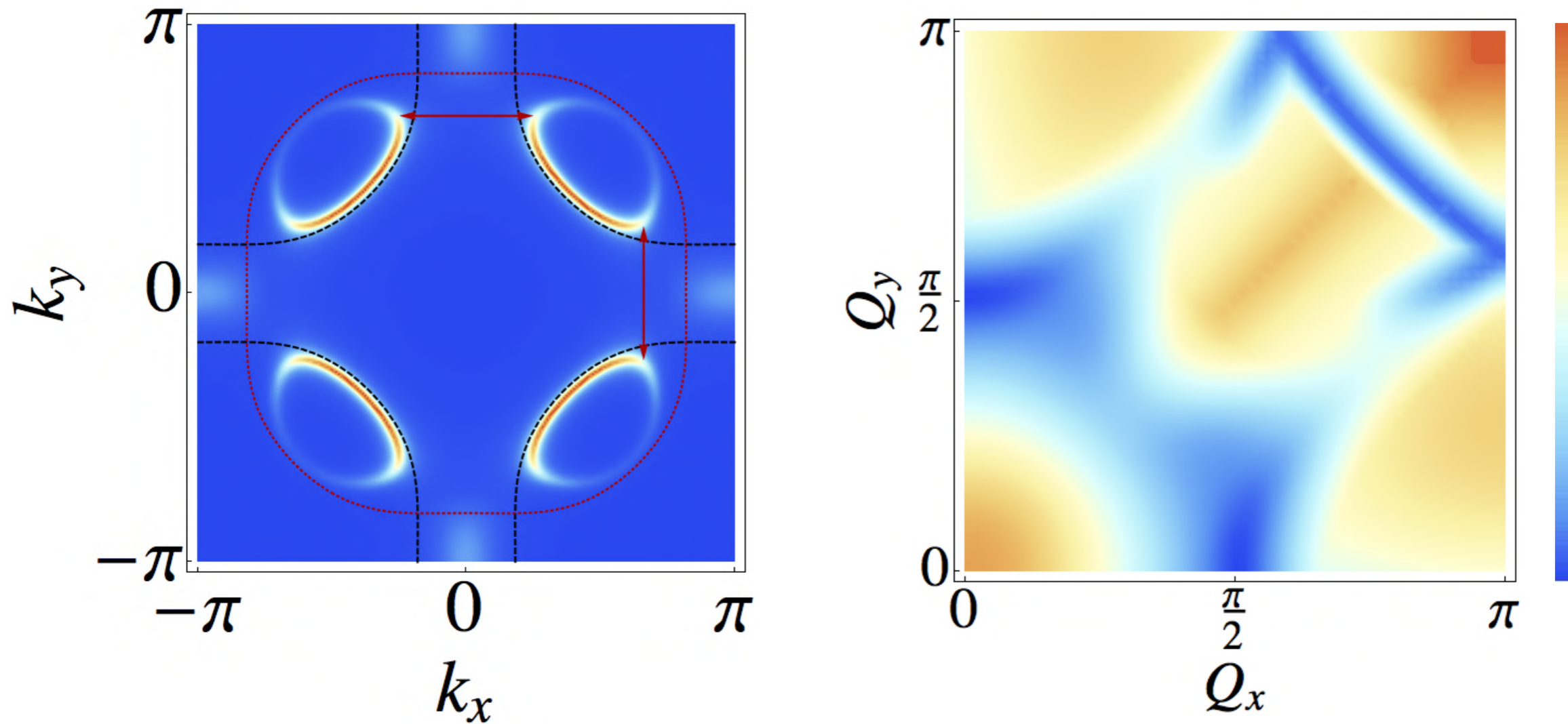


Have examined density wave instabilities of large Fermi surface, not the “Fermi arc” state found at low doping.









The pseudogap is described by the U(1)-FL<sup>\*</sup>: a state with hole pockets on a background of a spin-liquid described by a U(1) gauge theory. Its dominant density wave instability is a  $d$ -form factor density wave with a wavevector  $\mathbf{Q}$  along the  $(1, 0)$  and  $(0, 1)$  square lattice directions, in agreement with observations on the non-La-based cuprates.

# Conclusions

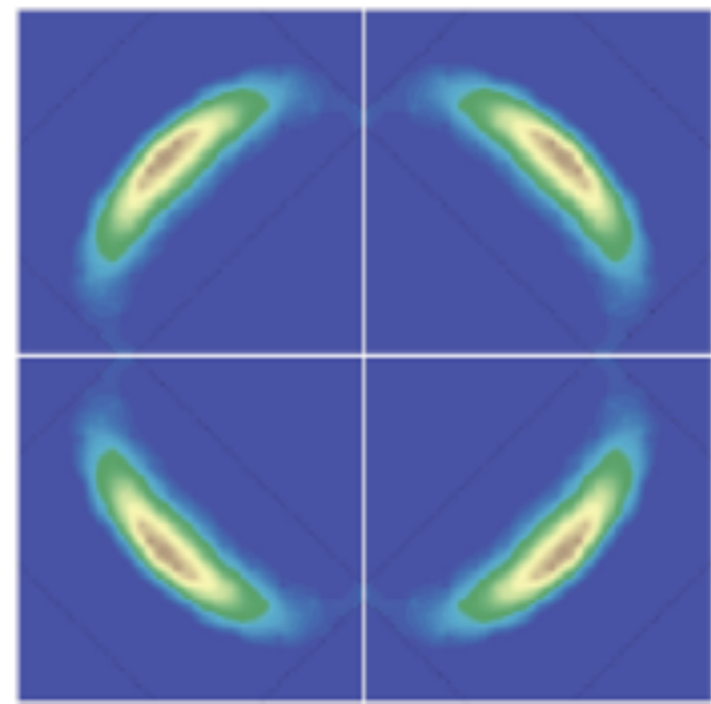
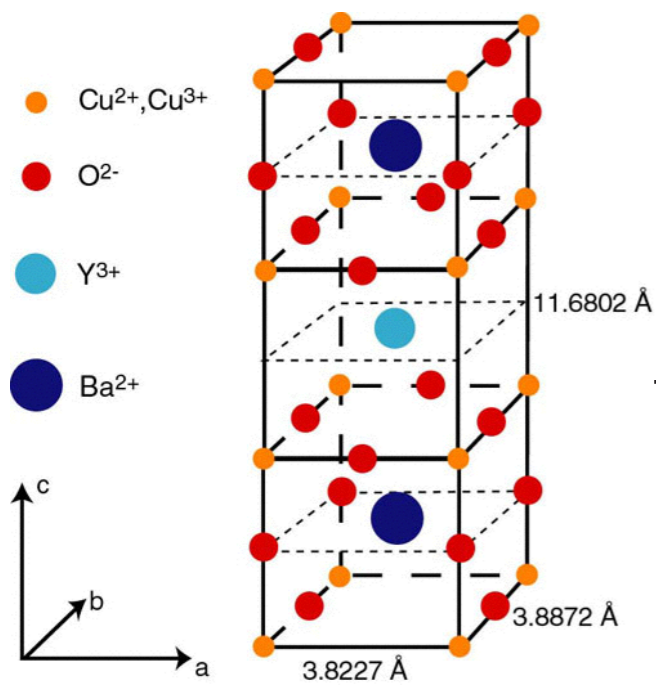
1.  $d$ -form factor density wave order established in the non-La hole-doped cuprate superconductors.

# Conclusions

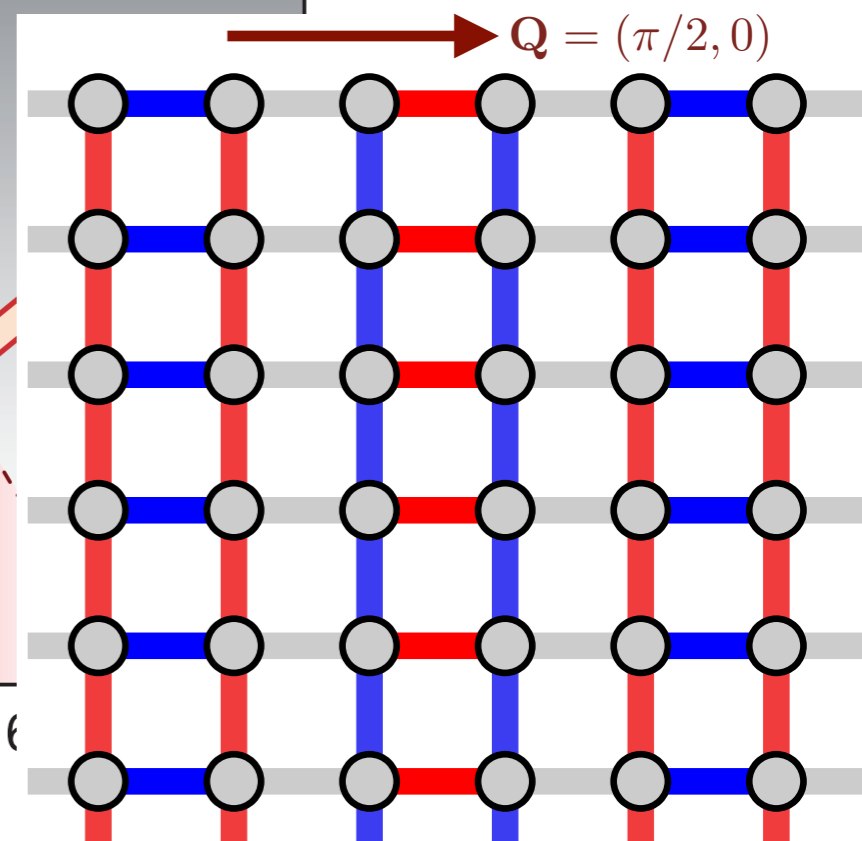
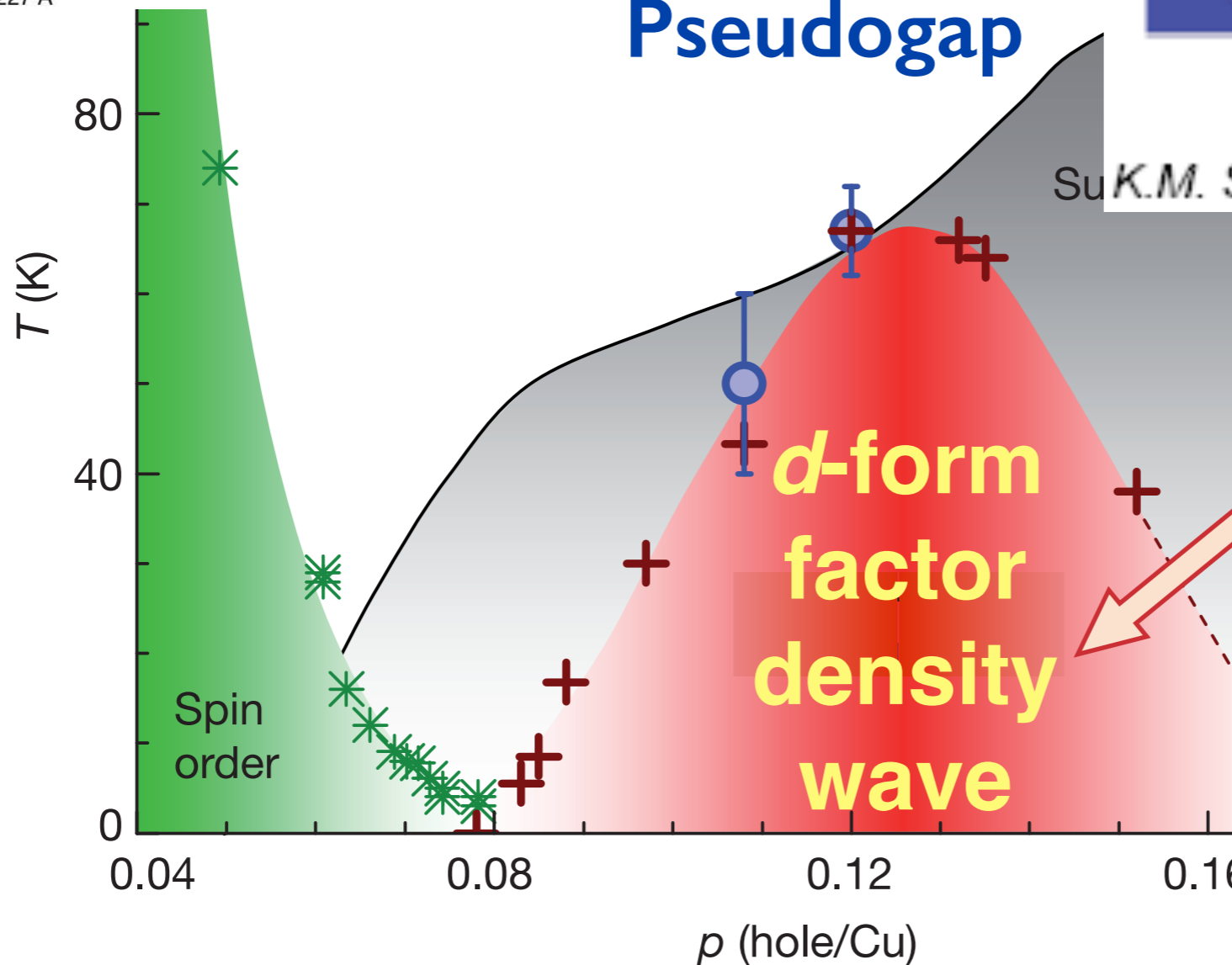
1.  $d$ -form factor density wave order established in the non-La hole-doped cuprate superconductors.
2. The “stripe” model corresponds to a  $s'$ -form factor, and this describes the La-based, lower  $T_c$ , hole-doped cuprate superconductors.

# Conclusions

1.  $d$ -form factor density wave order established in the non-La hole-doped cuprate superconductors.
2. The “stripe” model corresponds to a  $s'$ -form factor, and this describes the La-based, lower  $T_c$ , hole-doped cuprate superconductors.
3. The  $d$ -form factor may be an unexpected window into the spin-liquid physics of the pseudogap.



**Pseudogap**



K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS 111, E3026 (2014)