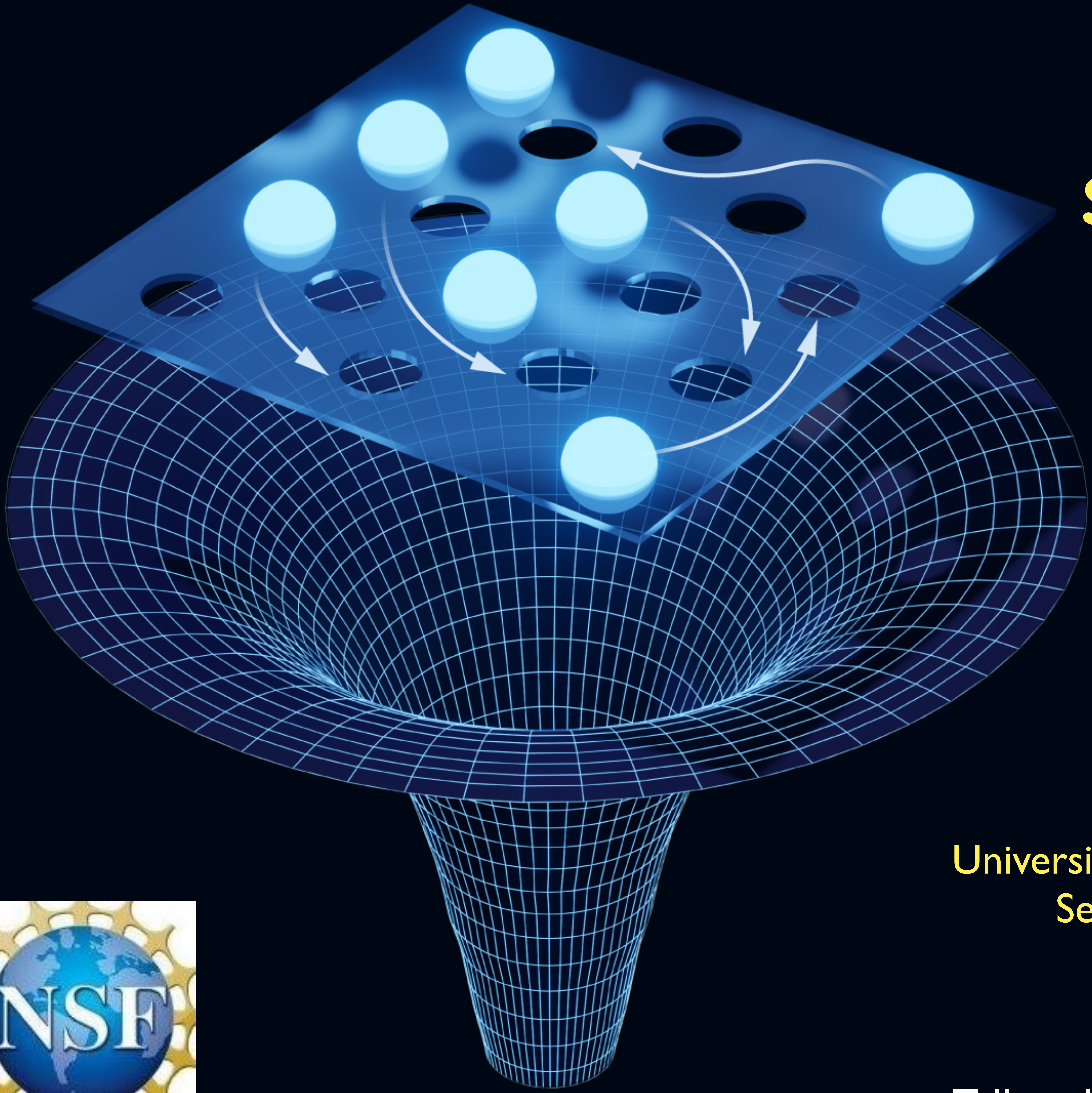


Quantum statistical mechanics of charged black holes and strange metals

University of Colorado, Boulder
September 30, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
VE RI TAS
HARVARD

SYK model

A simple model of a metal with quasiparticles

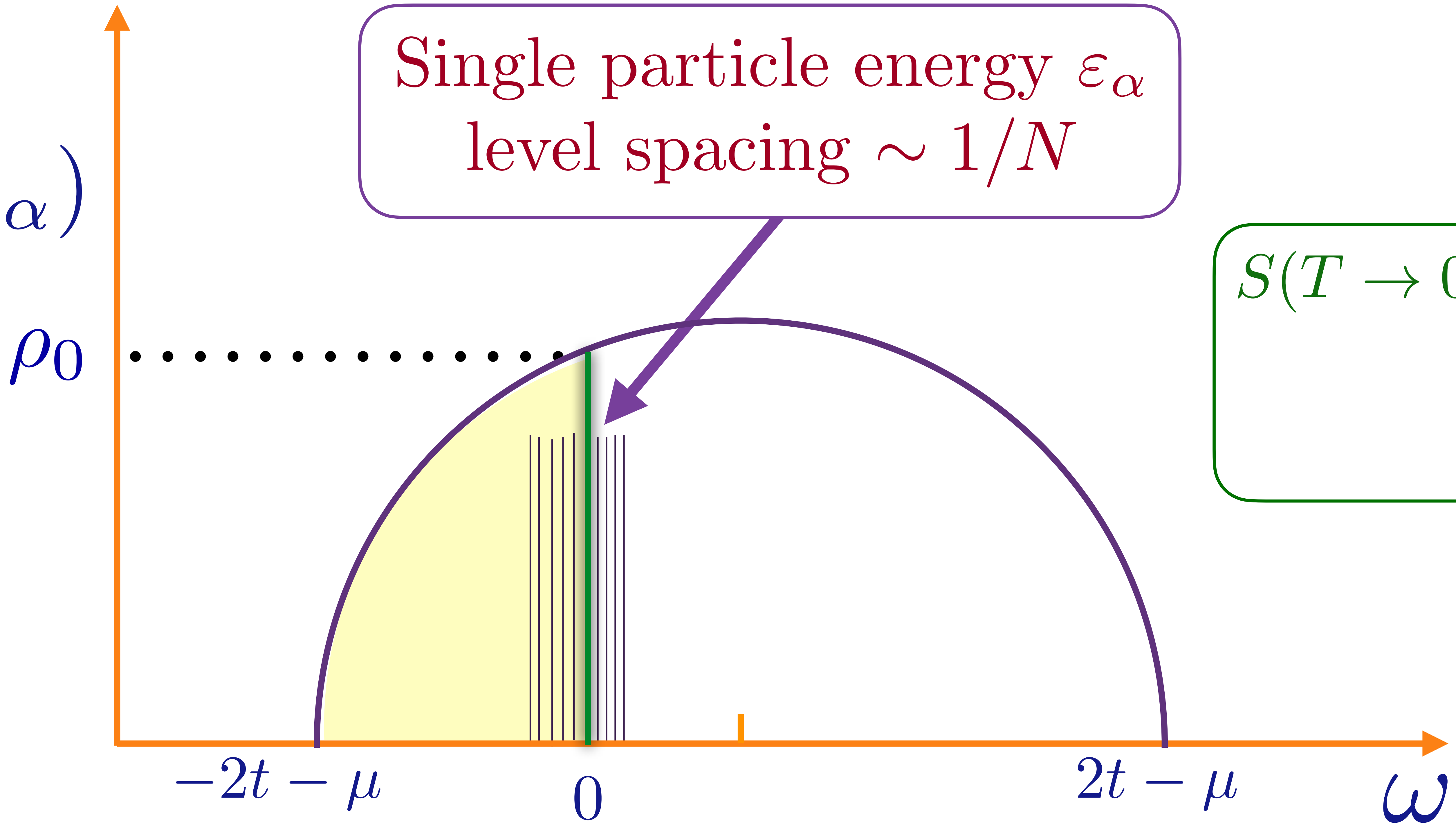
$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

$\rho(\omega) = \frac{1}{N} \sum_\alpha \delta(\omega - \varepsilon_\alpha)$



Single particle energy ε_α
level spacing $\sim 1/N$

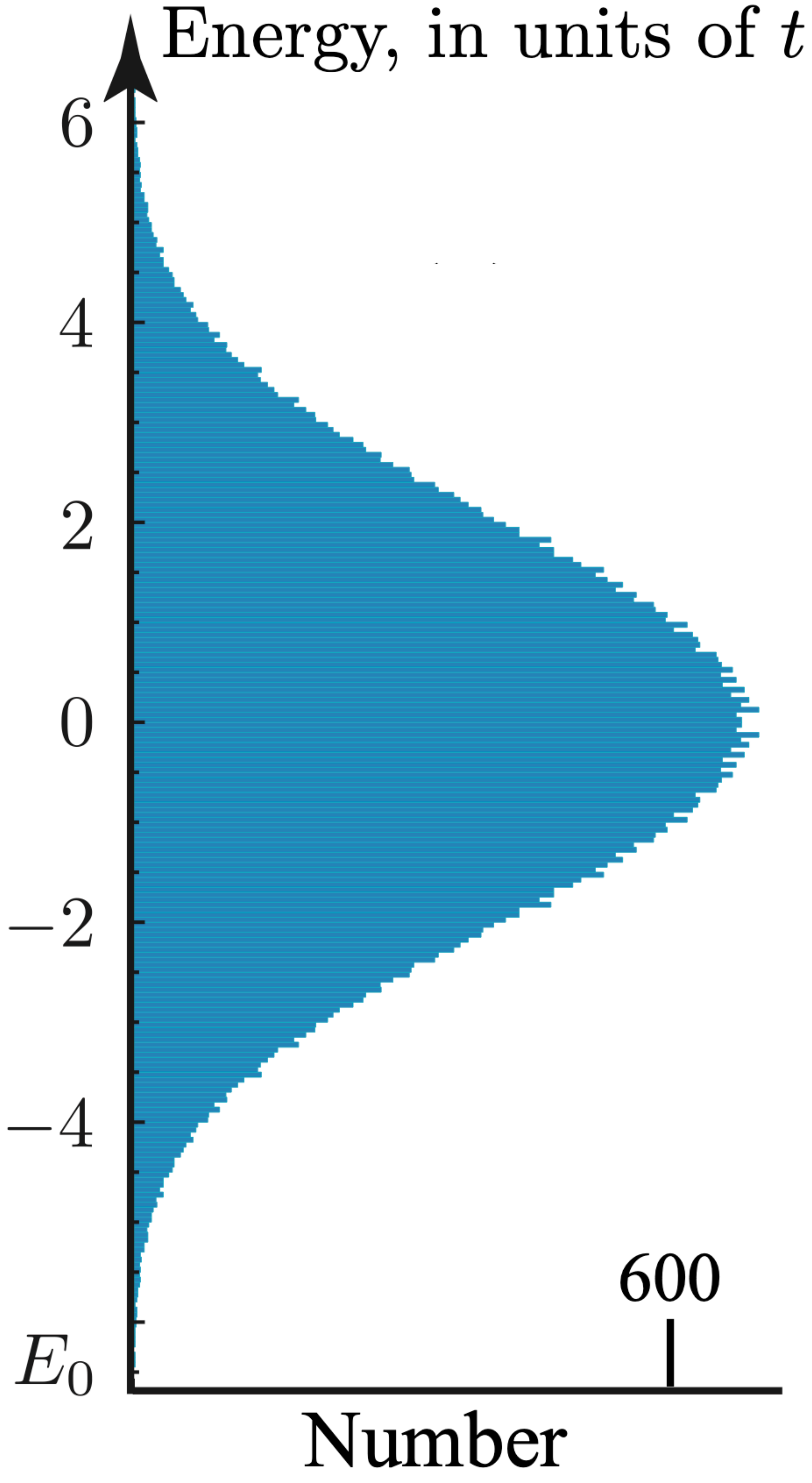
$$S(T \to 0) = N\gamma T$$

$$\gamma = \frac{\pi^2}{3} \rho_0$$

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

For random matrix model: $E_0 + E_i = \sum_\alpha n_\alpha \varepsilon_\alpha$ $n_\alpha = 0, 1,$ occupation number



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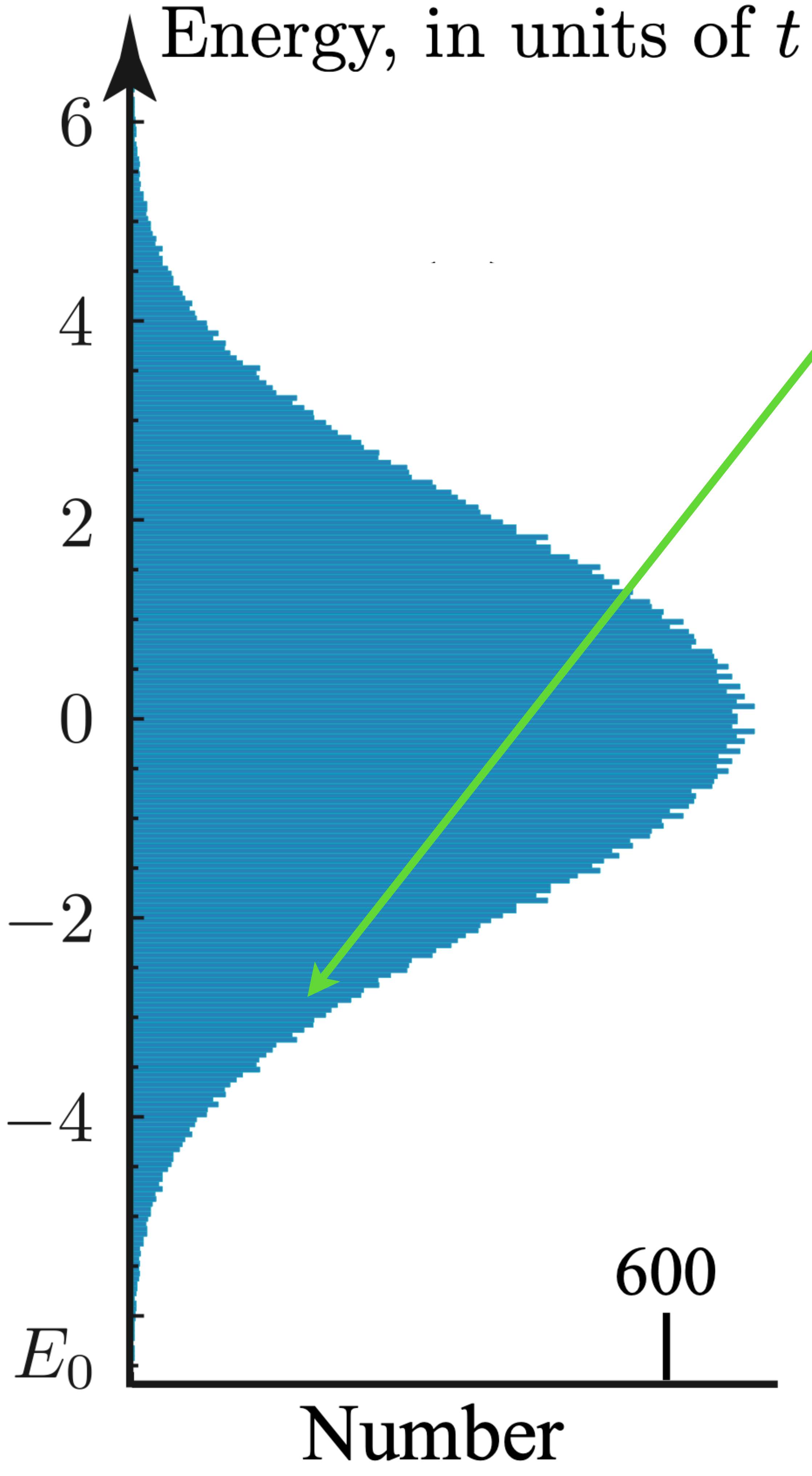

$$D(E) \sim e^{S(E)}$$

$$= e^{\sqrt{2N\gamma E}}$$

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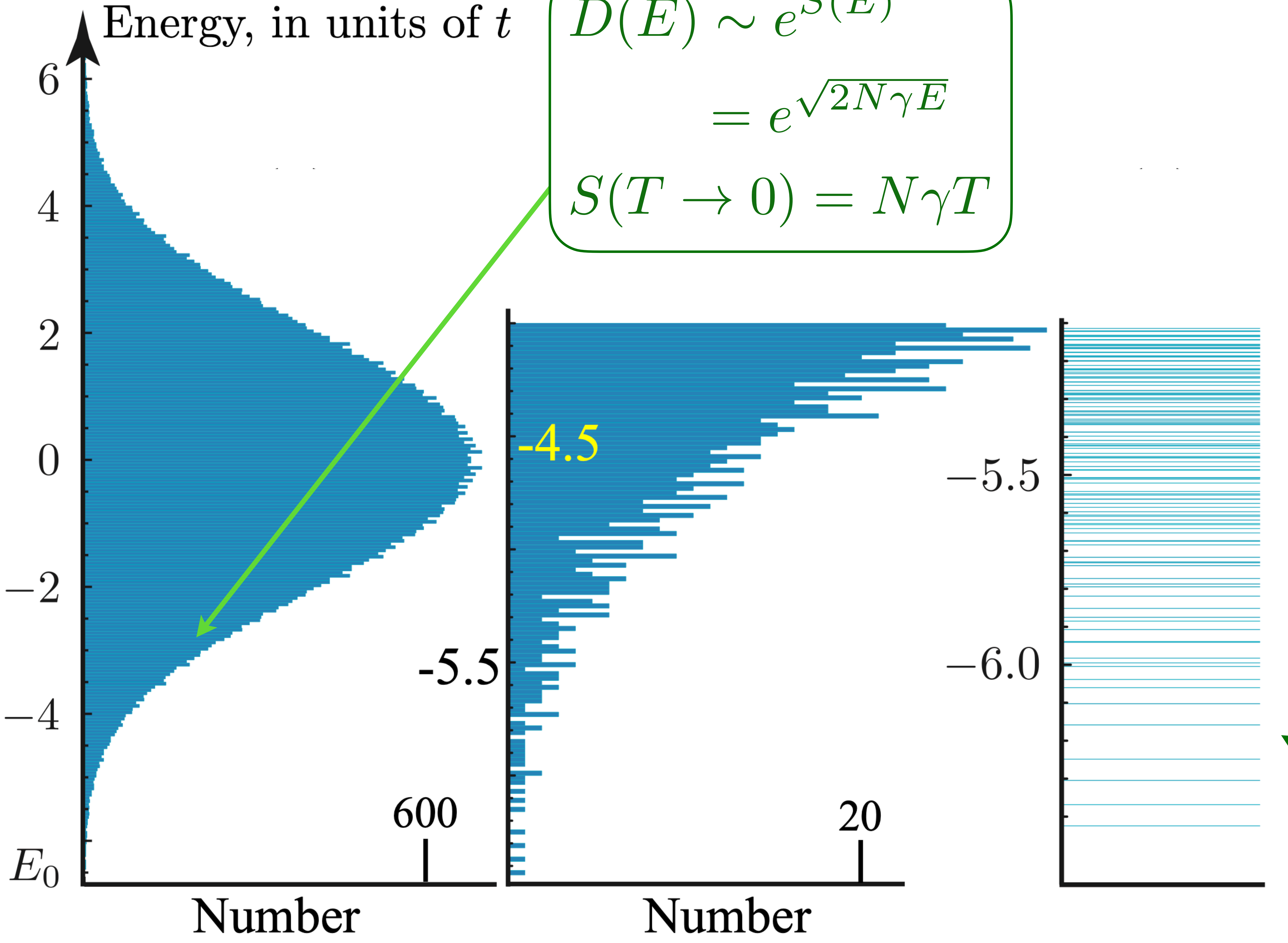

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$$= e^{\sqrt{2N\gamma E}}$$
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$$D(E) \sim N$$

Random matrix model

The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_\alpha c_\alpha^\dagger c_\alpha$$

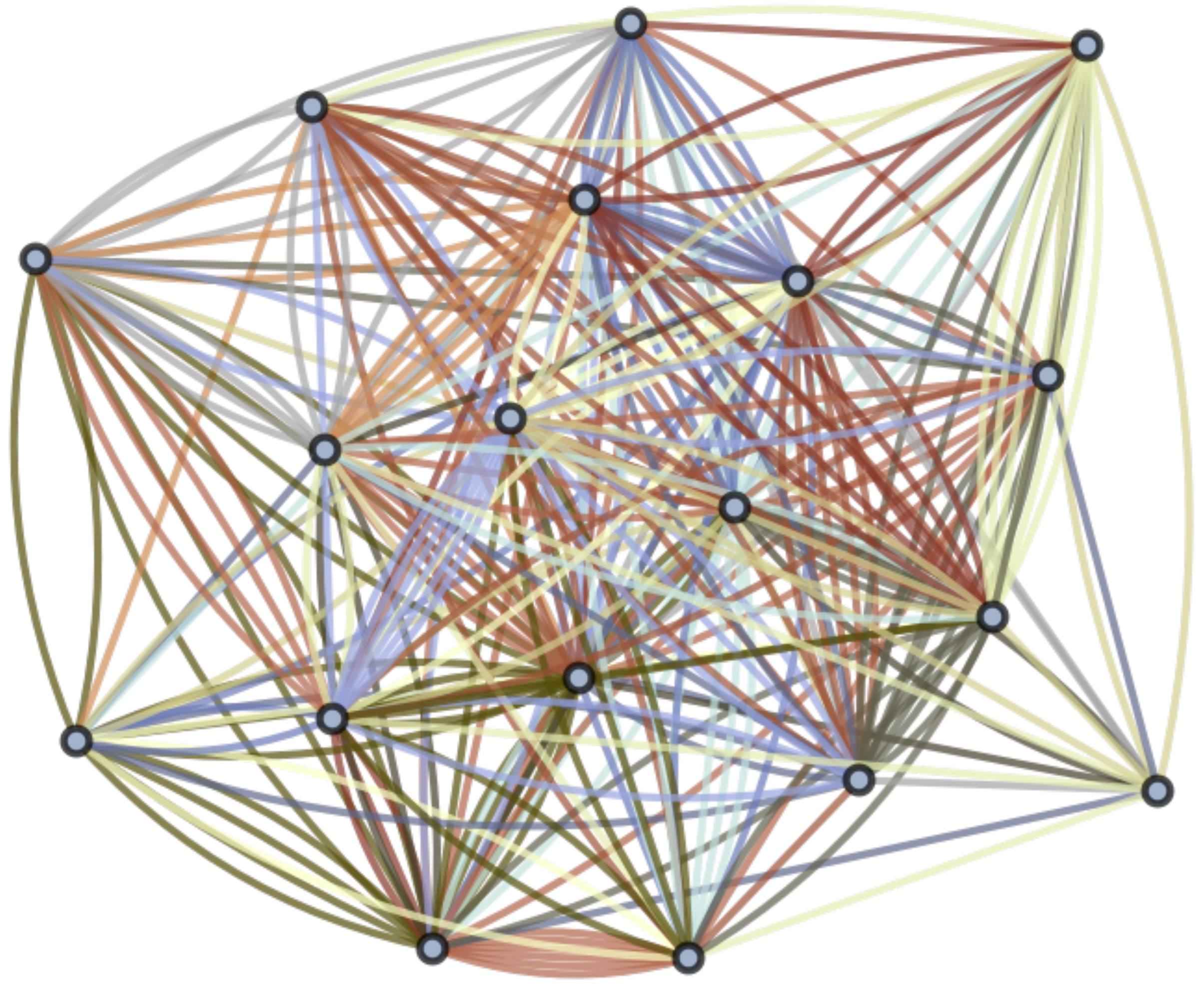
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

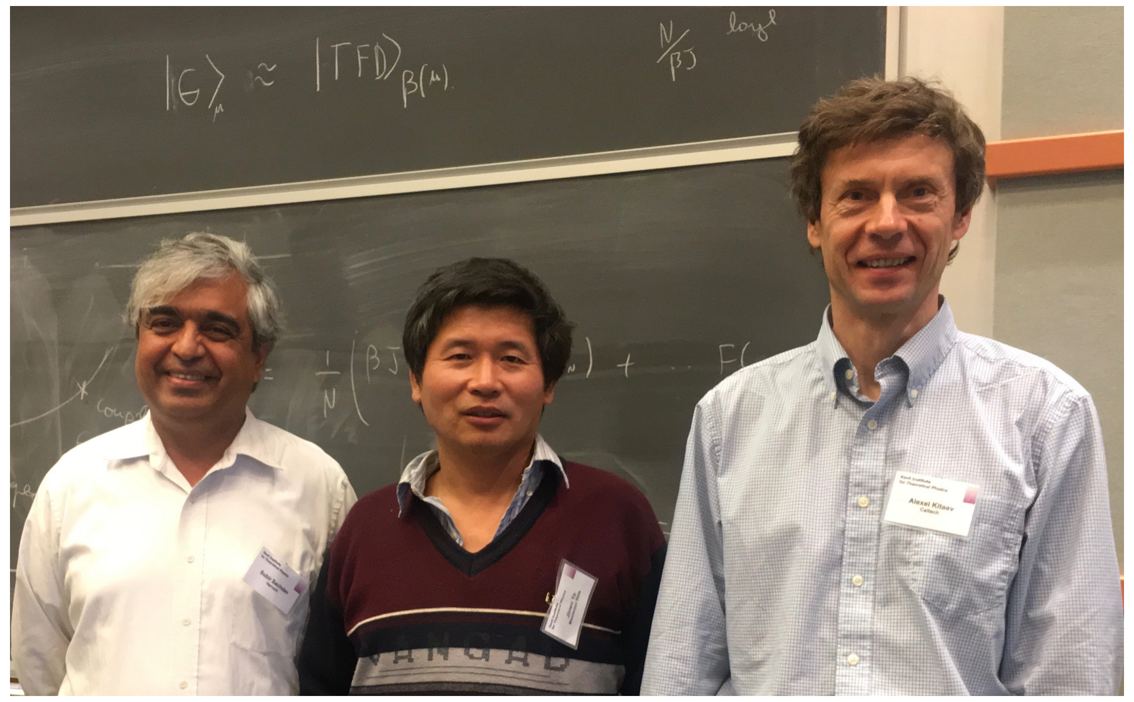
$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)





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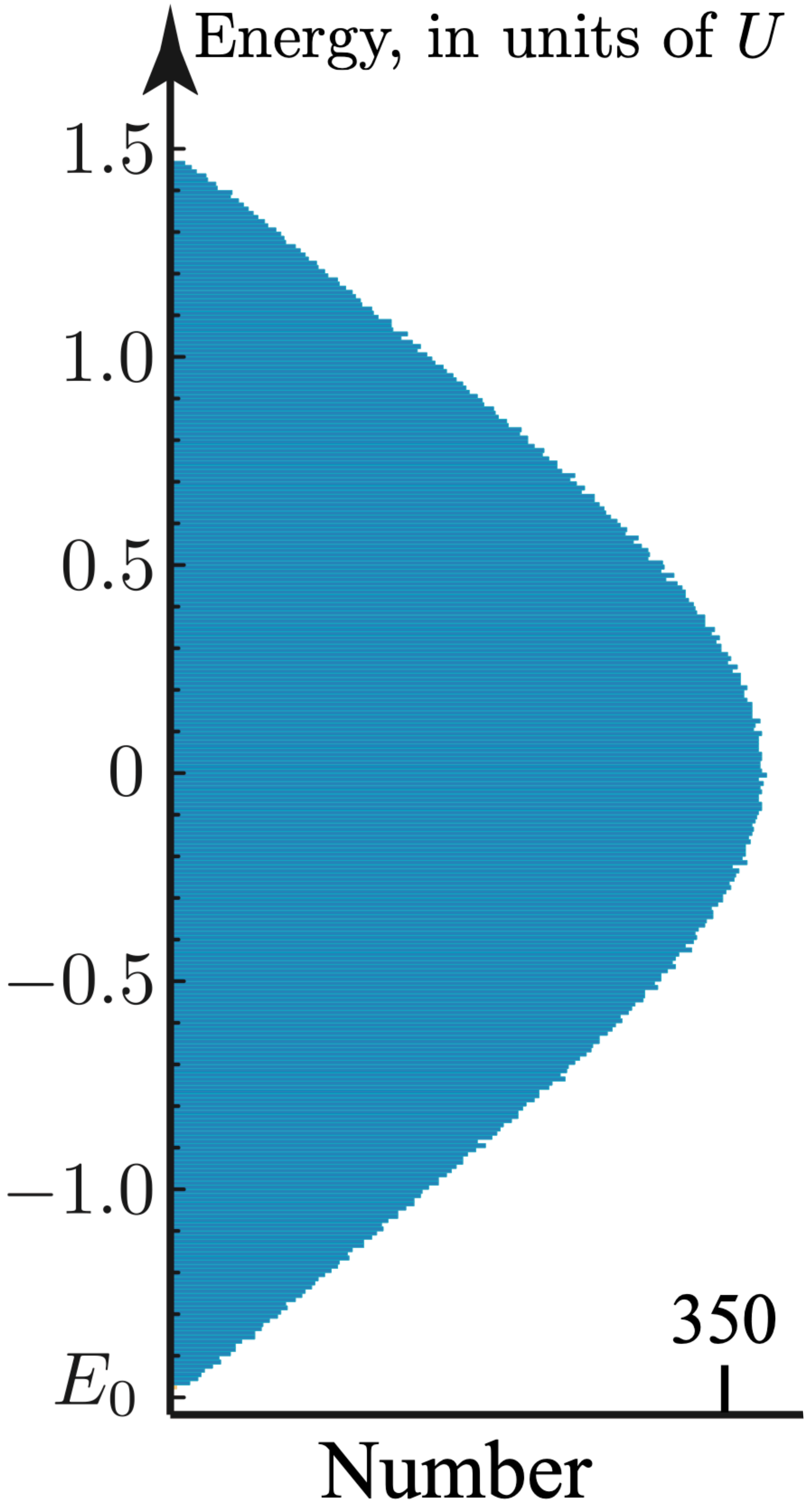

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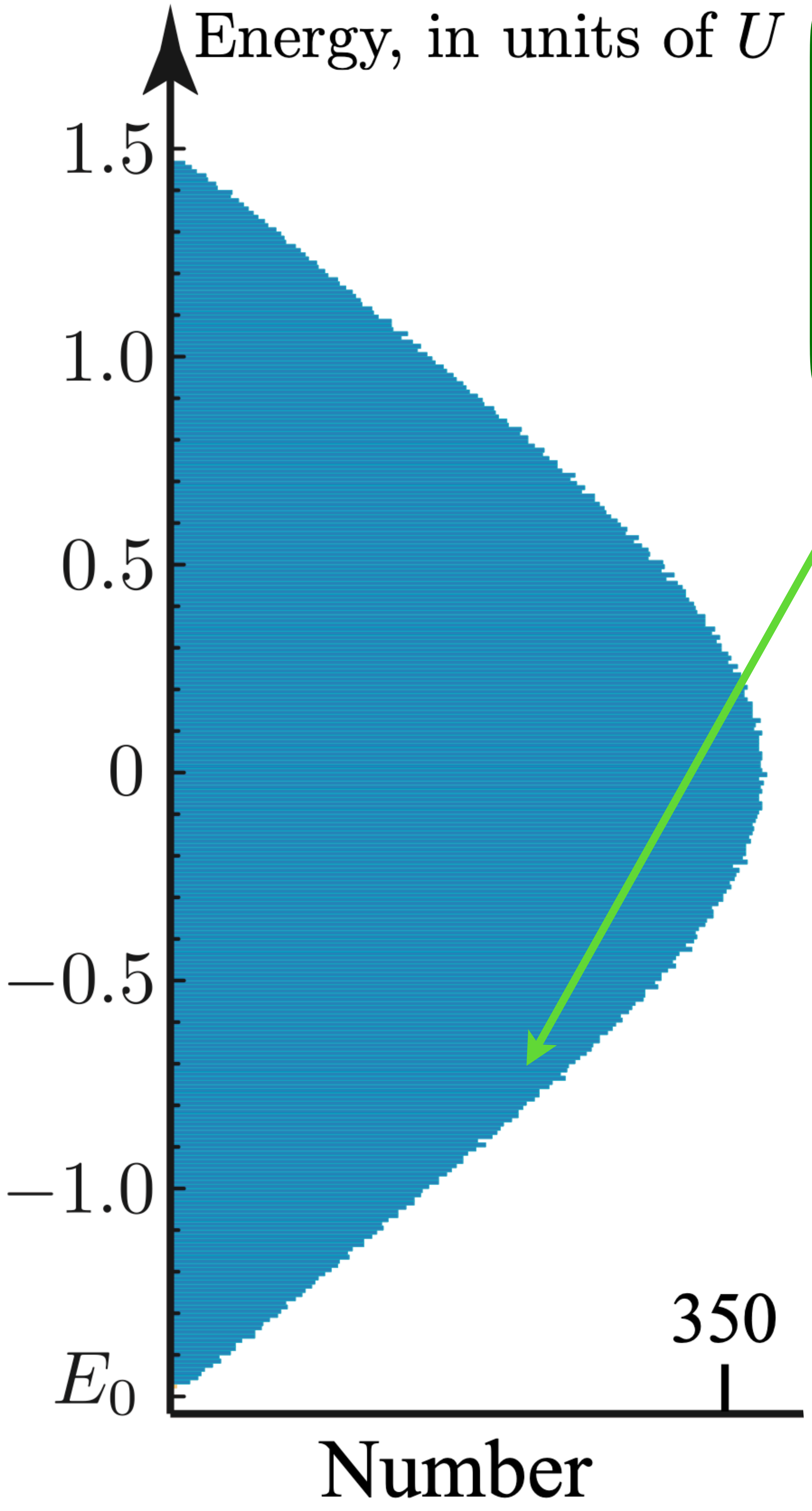

$$D(E) \sim e^{S(E)}$$
$$= e^{Ns_0 + \sqrt{2N\gamma E}}$$
$$S(T \to 0) = N(s_0 + \gamma T)$$

$$s_0 = 0.464848\ldots$$

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

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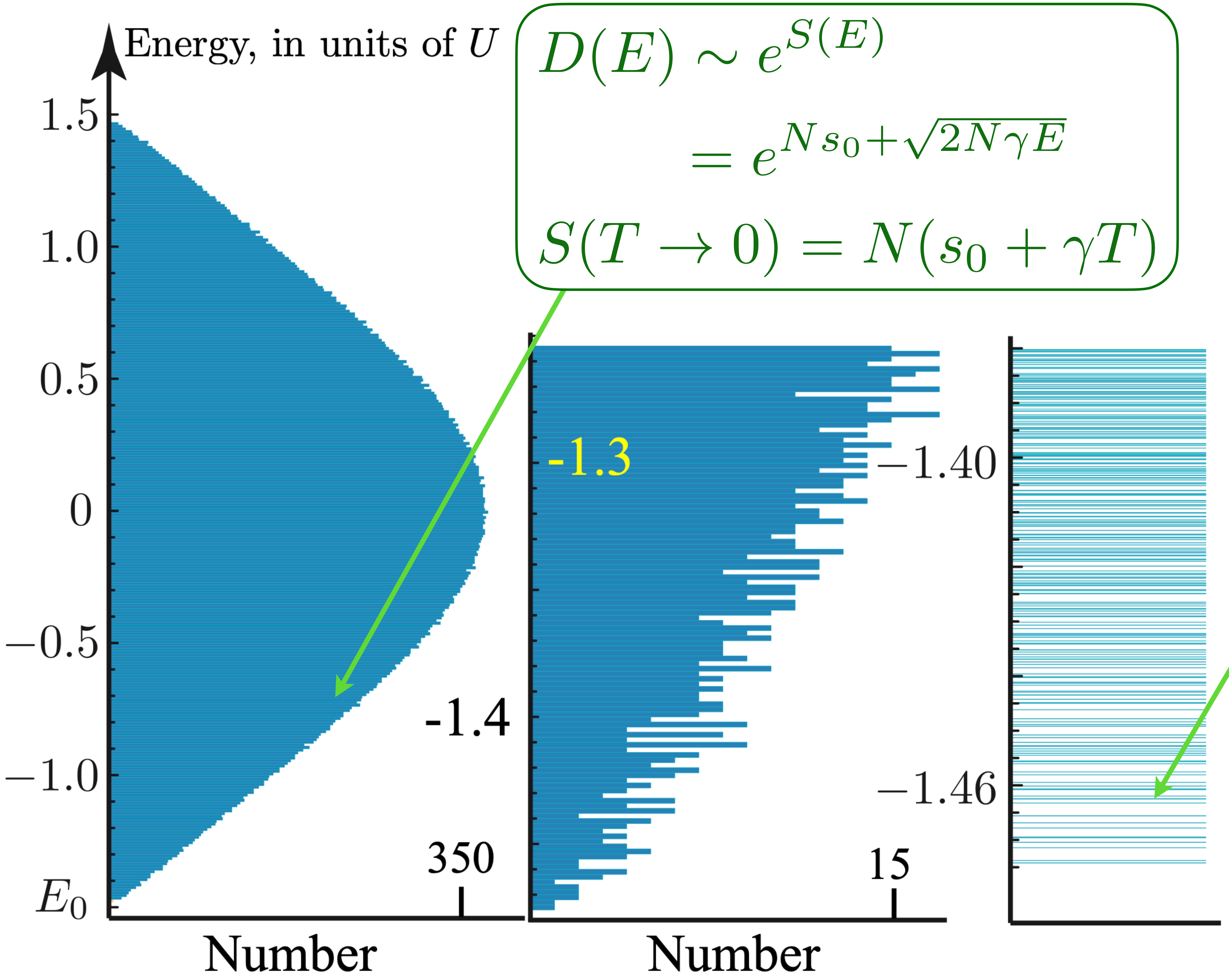

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$$D(E) \sim 2\, e^{N s_0} \sqrt{2N\gamma E}$$

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

$$s_0 = 0.464848\ldots$$

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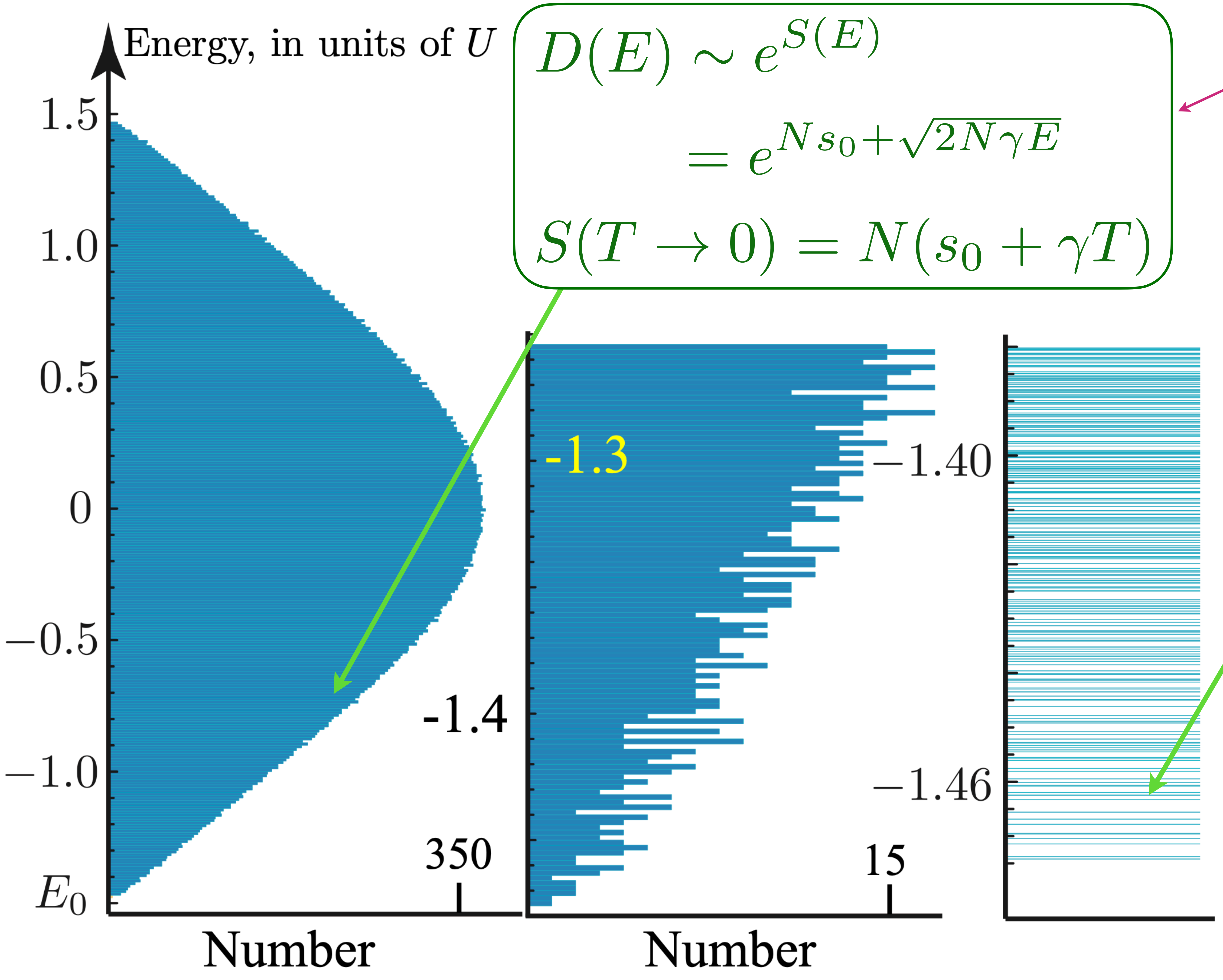

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$$D(E) \sim 2\, e^{N s_0} \sinh(\sqrt{2N\gamma E})$$

$$e^{-F(T)/T} = \int_0^\infty dE D(E) e^{-E/T}$$
$$S(T) = -\partial F/\partial T$$

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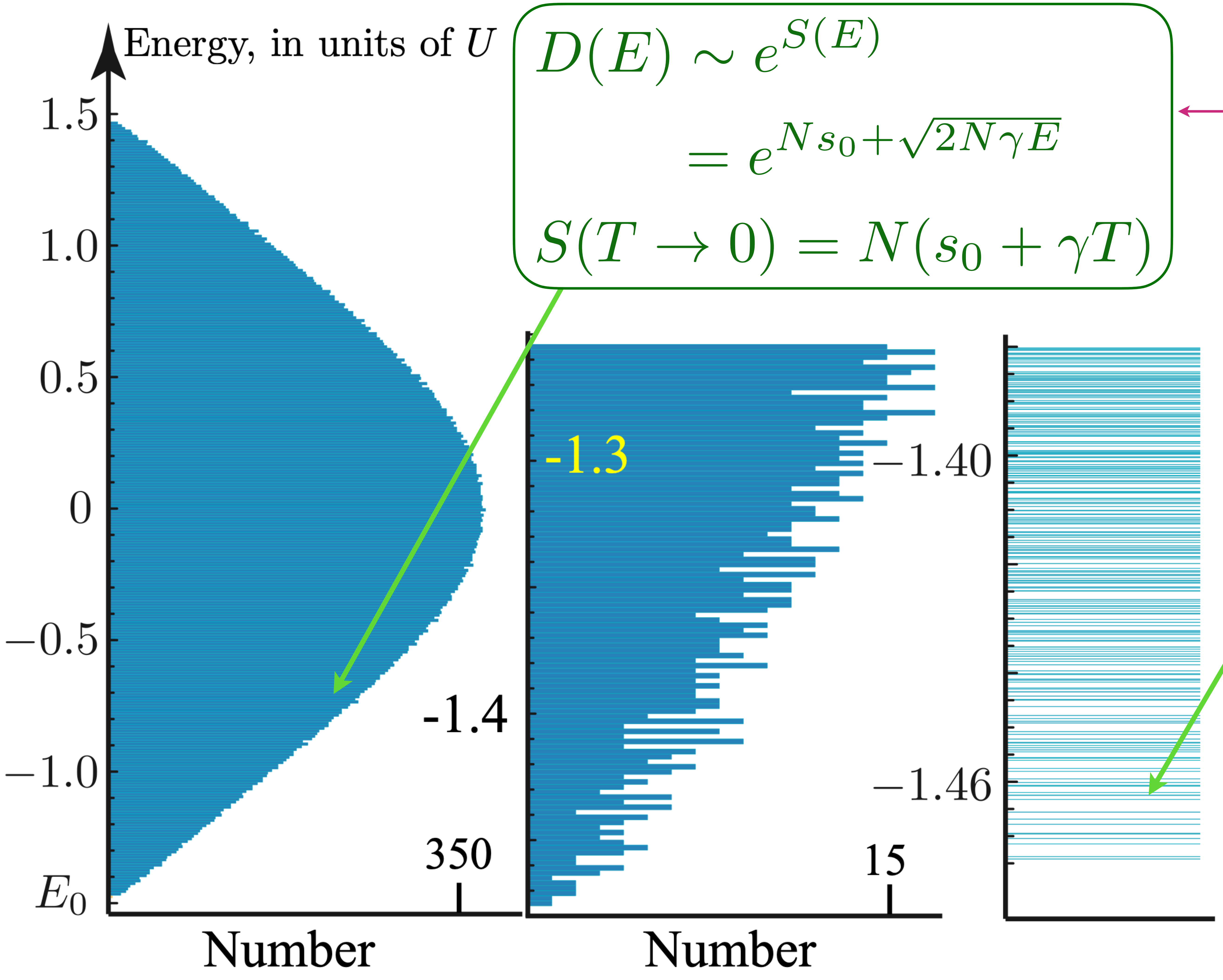

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PRB **63**, 134406 (2001)

Complex SYK model

Charged black holes

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$

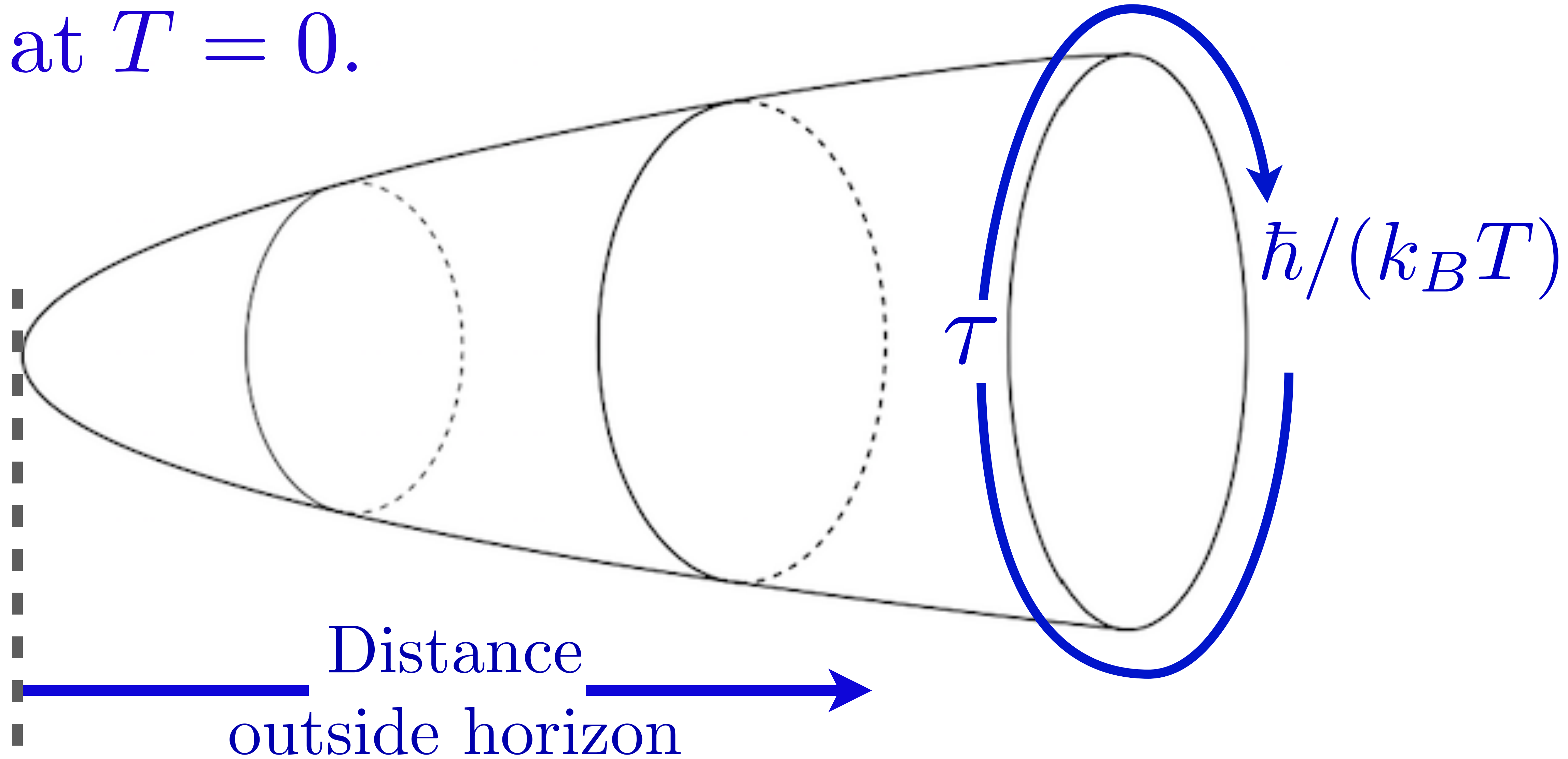
$$= \exp\left(S_{BH}\right) \times \left(\quad ....????... \quad \right)$$

Gibbons, Hawking (1977)
Chambin, Emparan, Johnson, Myers (1999)

$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T = 0$.
$\mathcal{Q}$ is the black hole charge.
A_0 is a function of $\mathcal{Q}$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



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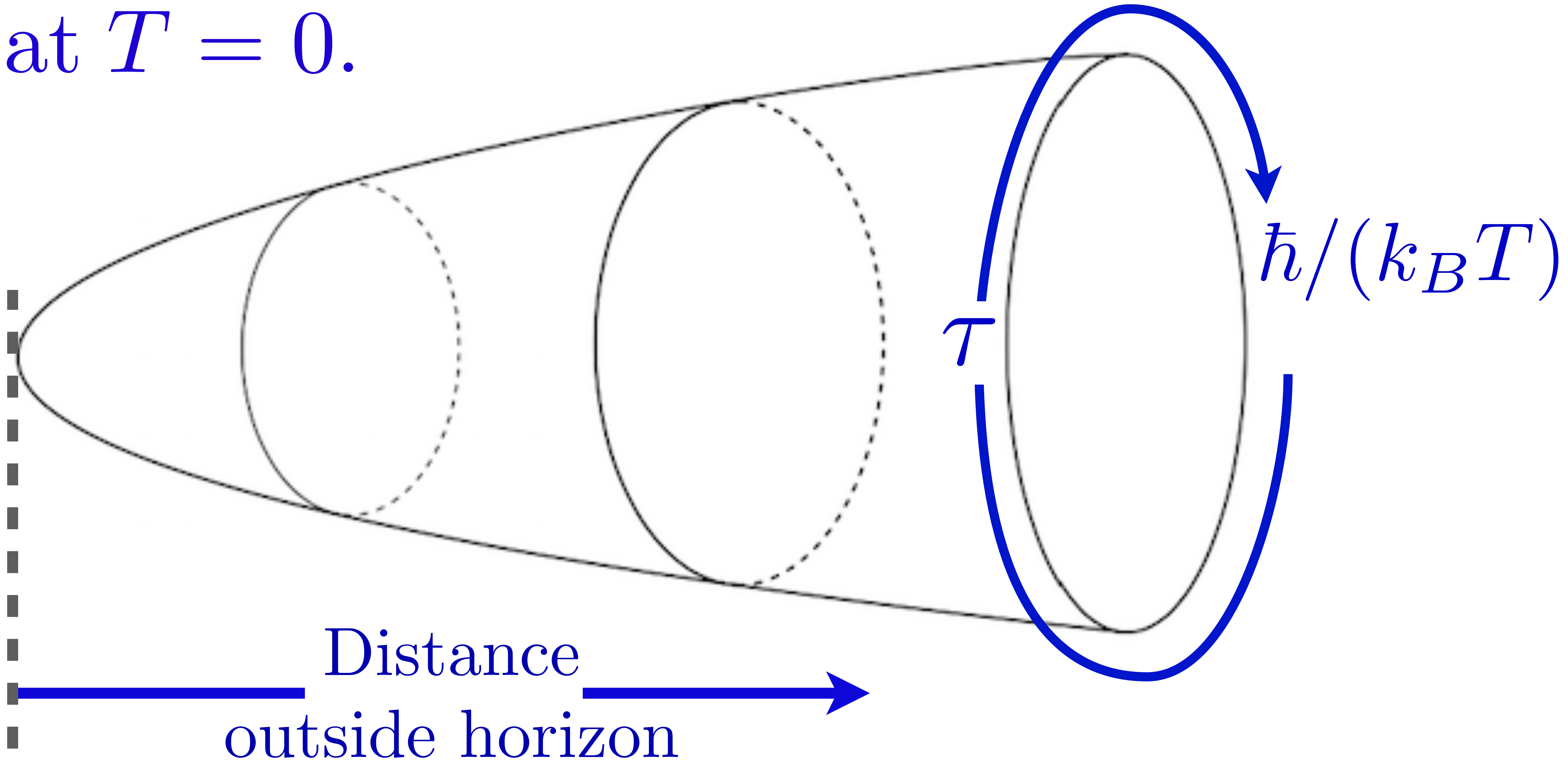
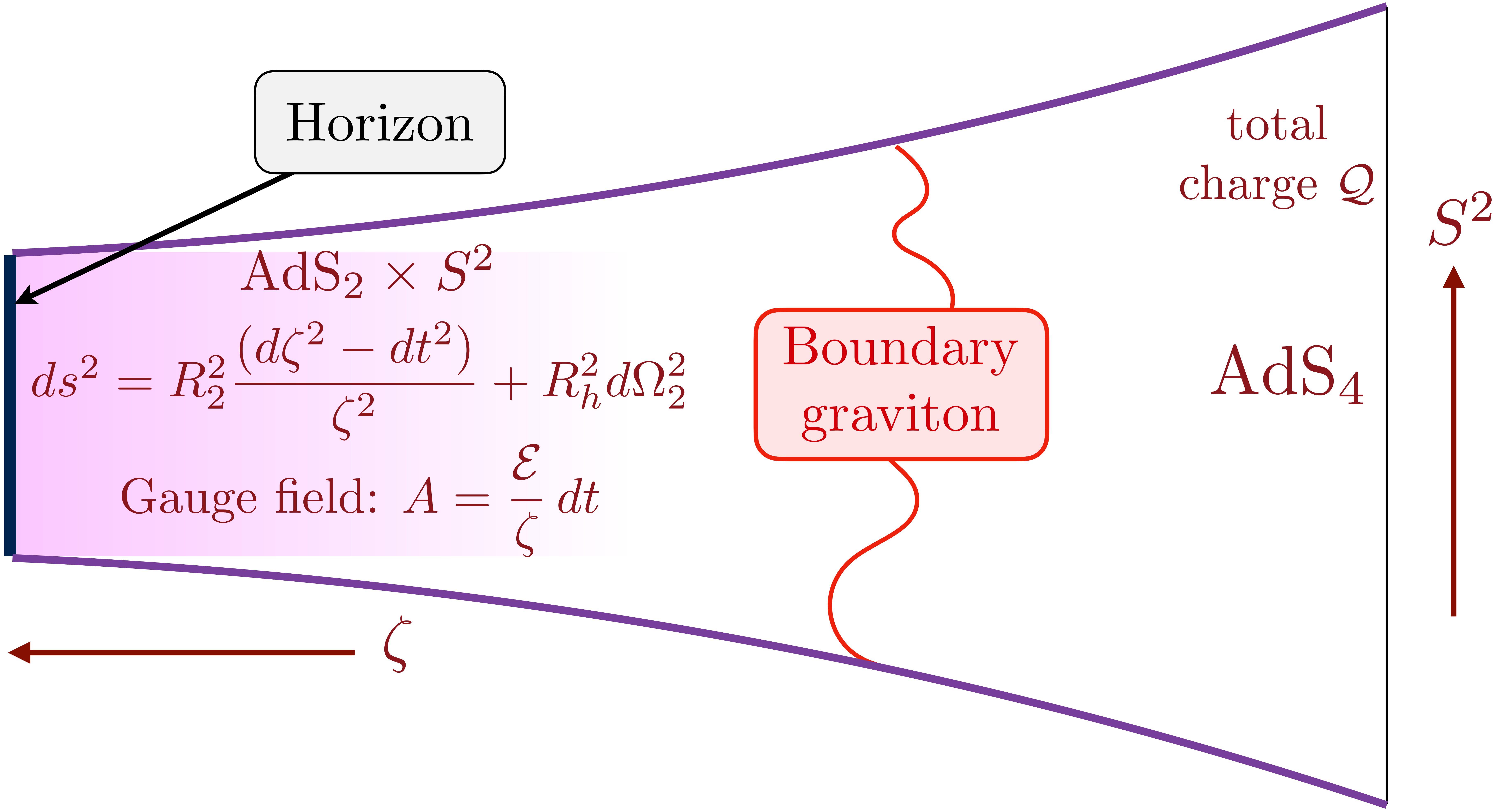


Note the similarity to the large *N* entropy of the SYK model !
(along with other similarities)

Sachdev PRL 2010

Reissner-Nordstrom black hole of Einstein-Maxwell theory



Horizon

$\mathrm{AdS}_2 \times S^2$

$$ds^2 = R_2^2 \frac{(d\zeta^2 - dt^2)}{\zeta^2} + R_h^2 d\Omega_2^2$$

Gauge field: $A = \dfrac{\mathcal{E}}{\zeta}\, dt$

ζ

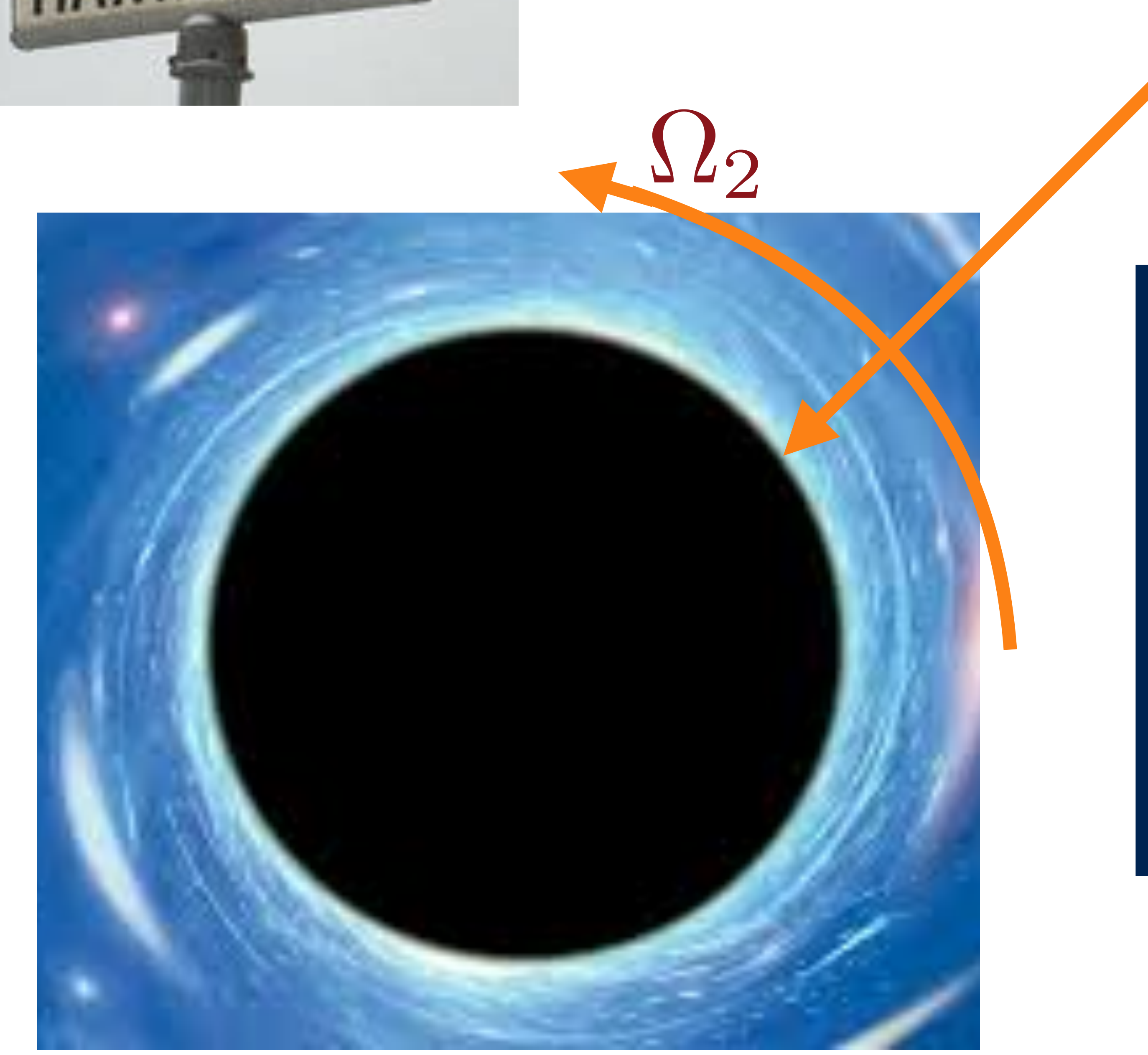
Boundary graviton

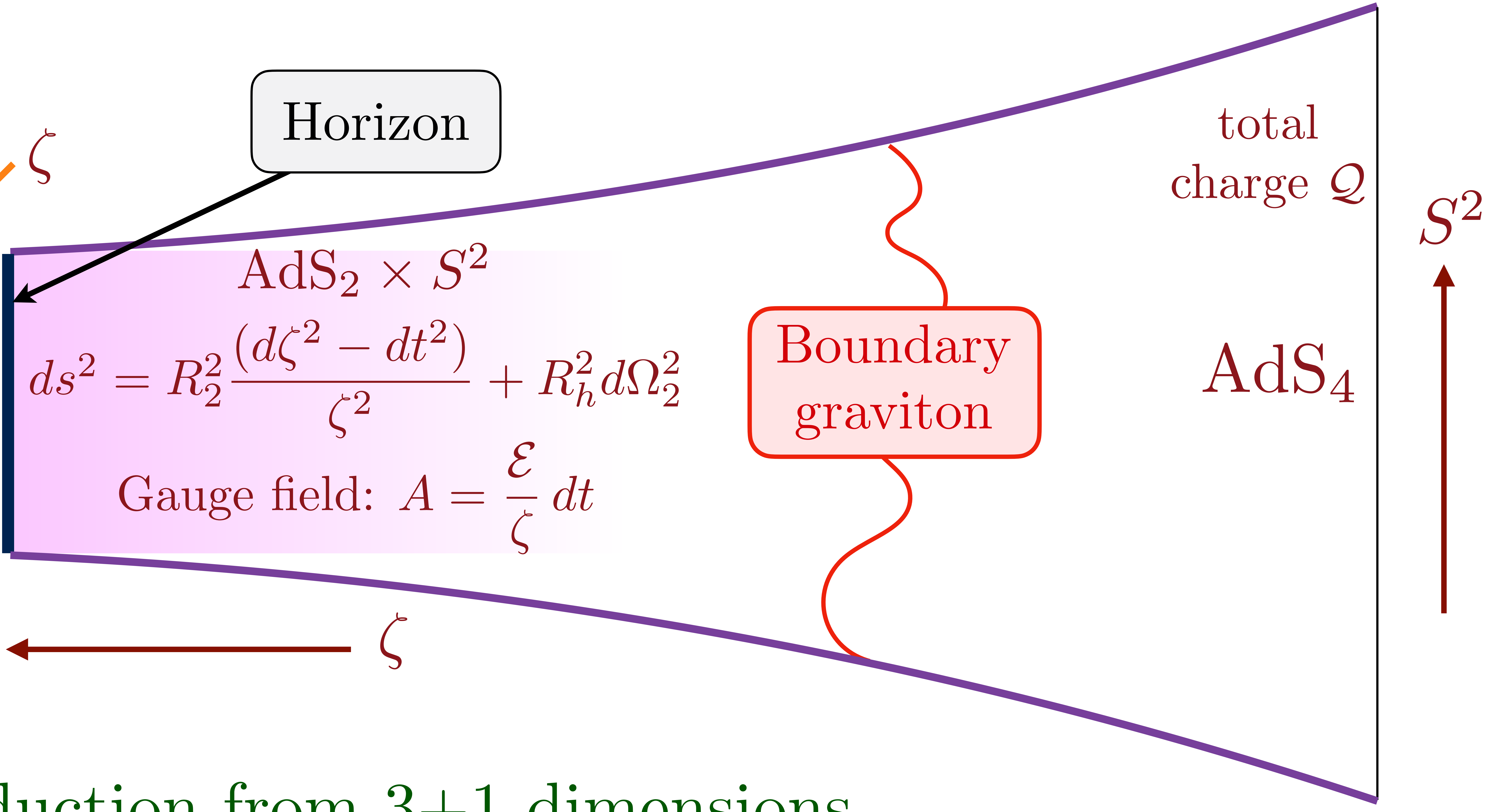
total charge Q

AdS_4

S^2

Reissner-Nordstrom black hole of Einstein-Maxwell theory





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Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

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$$\approx \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\,\mathcal{S}^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu},A_\mu]\right)$$

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Maldacena, Stanford, Yang (2016)

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau) \exp\left(-\frac{1}{\hbar}\mathcal{S}_{\text{SYK}}[\text{time reparameterizations } f(\tau), \text{ phase rotations } \phi(\tau)]\right)$$

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Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

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$$S(T\to 0,\mathcal{Q}) = S_{BH} - \frac{3}{4}\ln\left(\frac{\hbar c^5}{GT^2}\right)$$

$$S_{BH} = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

A_0 is the area of the charged black hole horizon at $T=0$, $\mathcal{Q}$ is the black hole charge. The $\ln T$ term is the contribution of the boundary graviton.

(There is also a
$-(241/45)\ln(A_0/G)$
correction at $T=0$
A. Sen 2011)

Sachdev (2010); Kitaev (2015); Sachdev (2015); Bagrets, Altland, Kamenev (2016); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018) ; Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019); Iliesiu, Turaci (2020)

Black hole questions and answers

Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy? Yes, for charged black holes:

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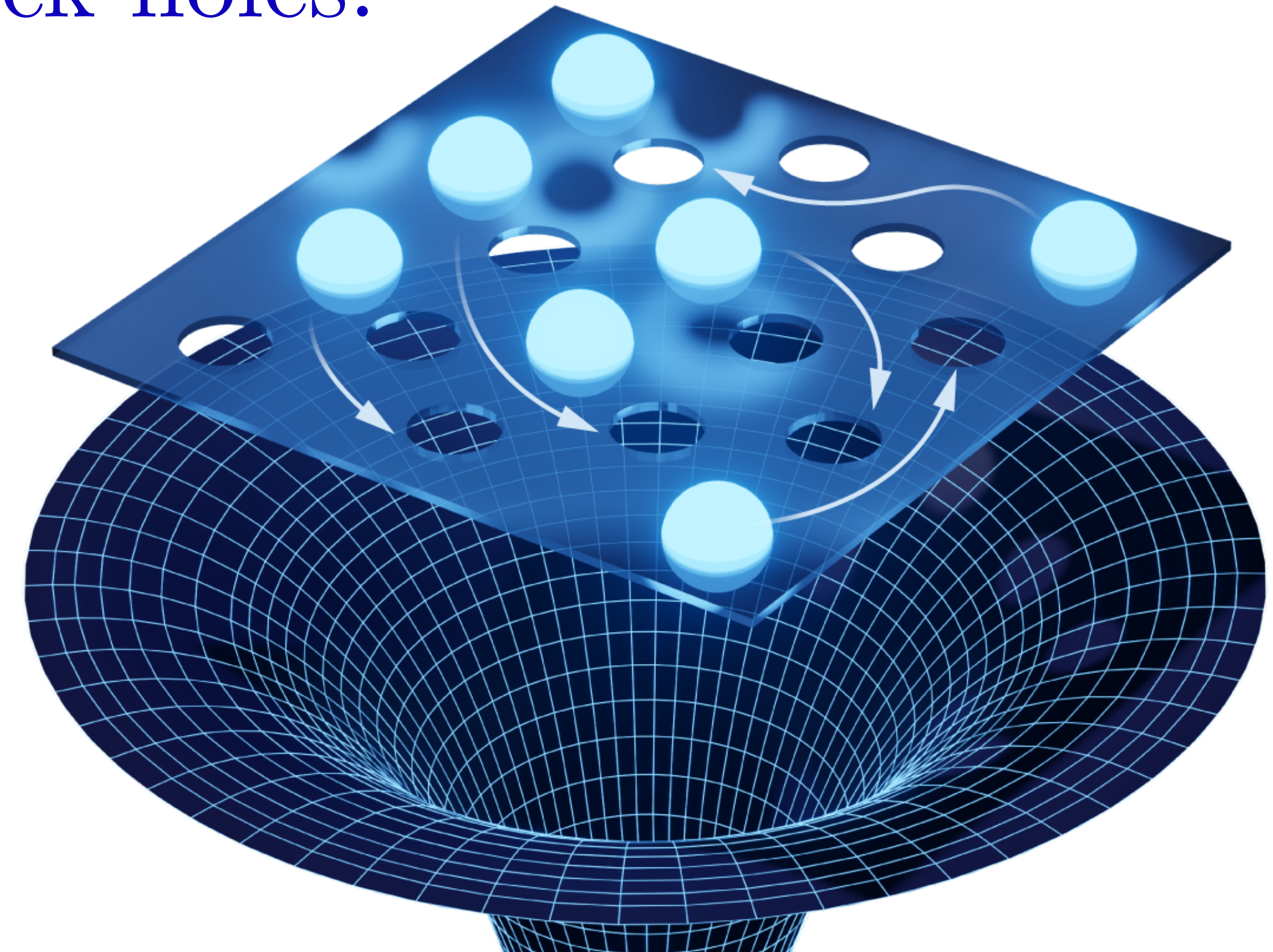
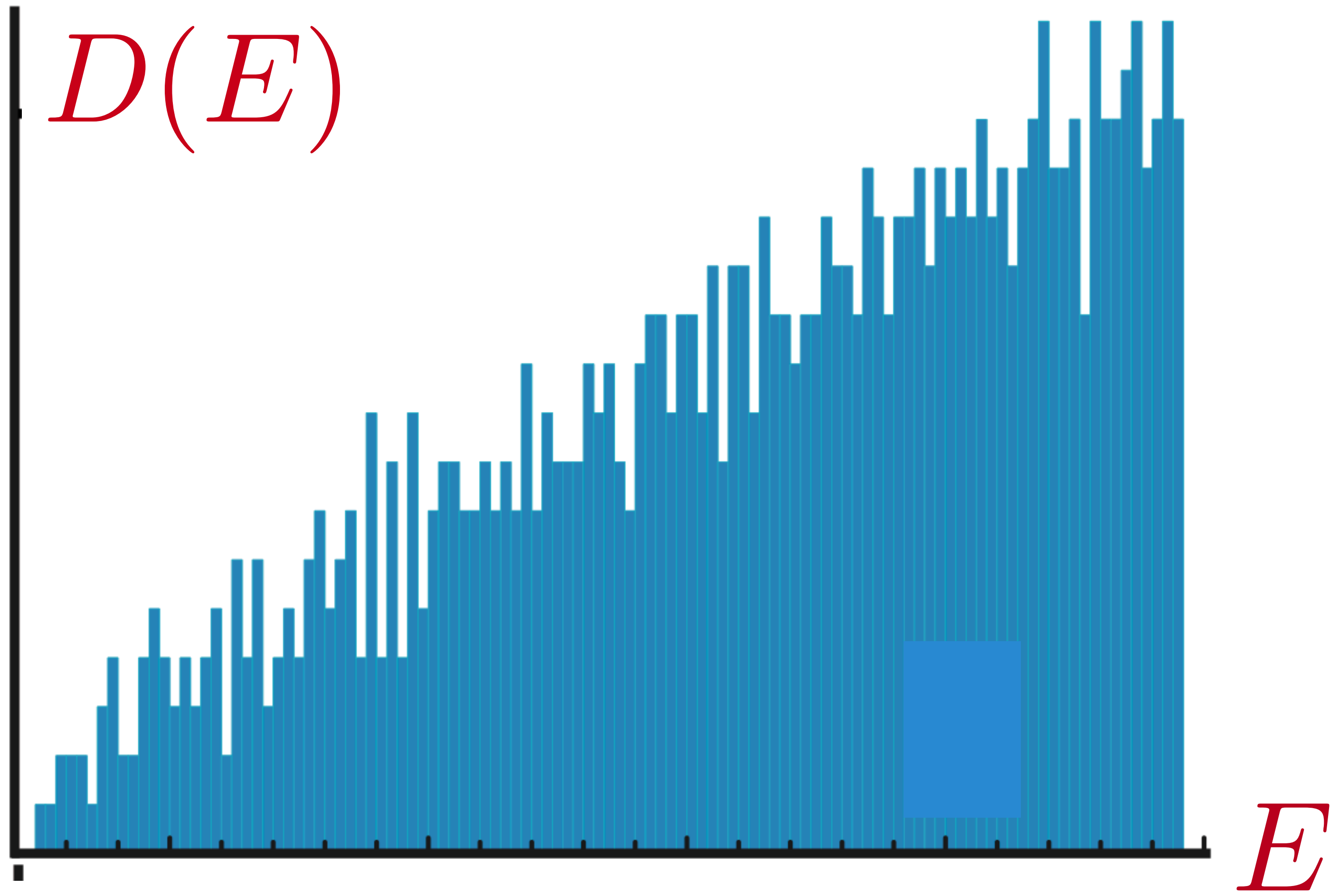


- For generic black holes in 3+1 dimensions, the SYK model yields:

$$D(E) \sim \exp\left(\frac{\mathcal{A}_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi}\mathcal{A}_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

where $\mathcal{A}_0$ is the horizon area at $T = 0$. There is no degeneracy, but an exponentially small level spacing down to the ground state.



S. Sachdev,
Journal of Mathematical Physics
60, 052303 (2019)

Black hole questions and answers

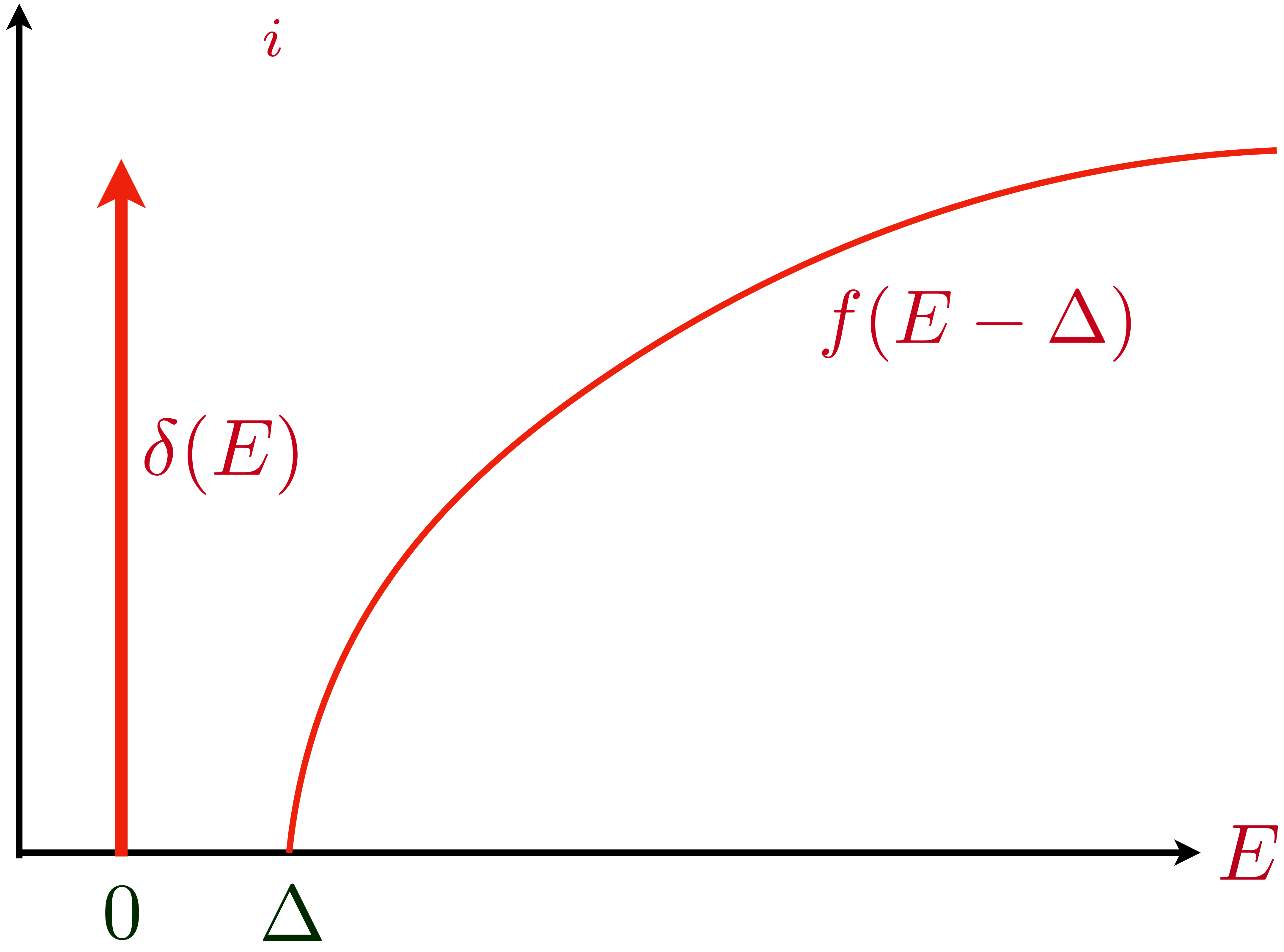
Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy? Yes, for charged black holes:

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{Ac^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta)f(E-\Delta) + \ldots$$

There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.

$$D(E) = \sum_i \delta(E - E_i)$$



M. Heydeman, L.V. Iliesiu, G. J. Turiaci, and W. Zhao, 2022

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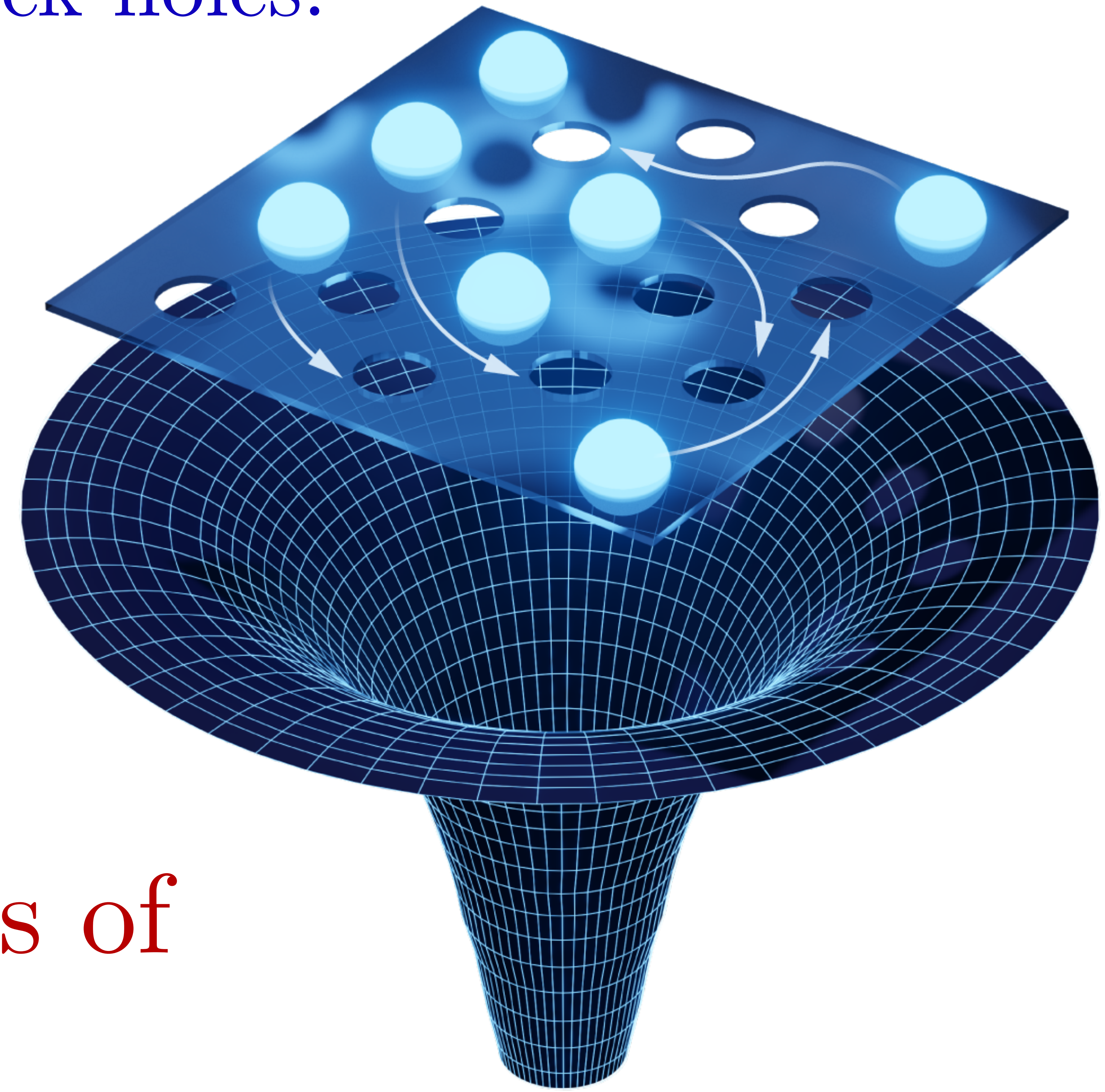

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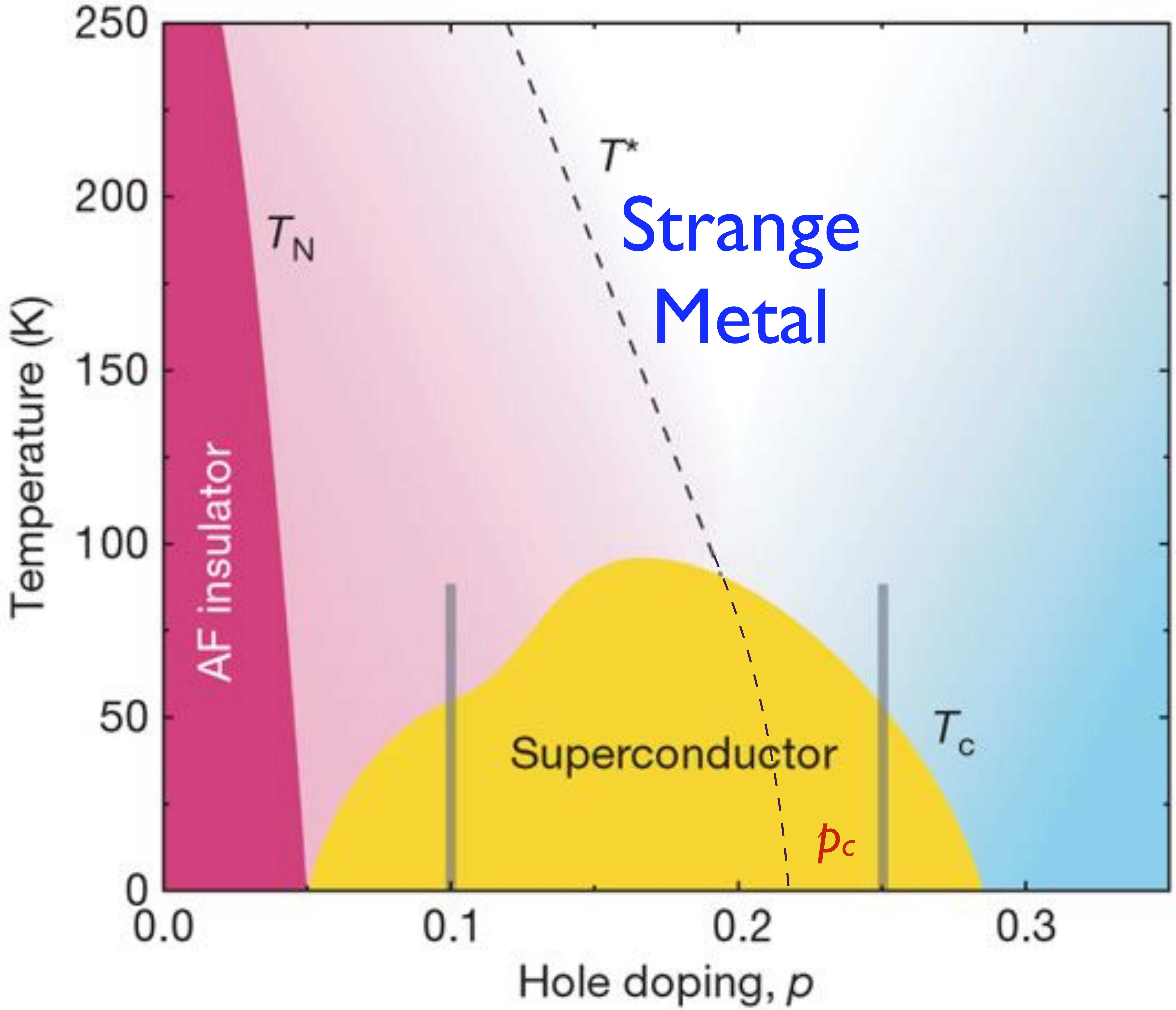
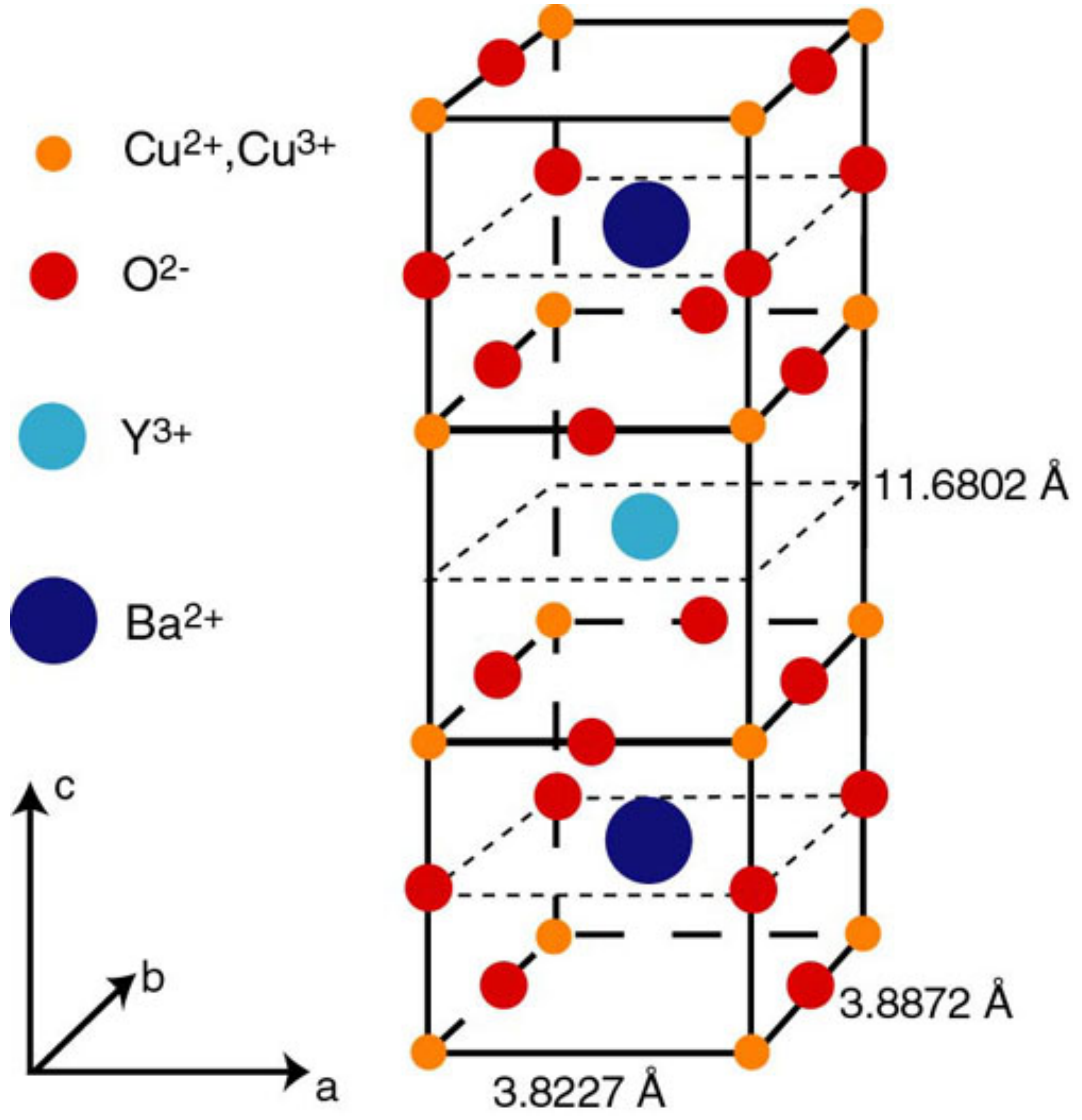
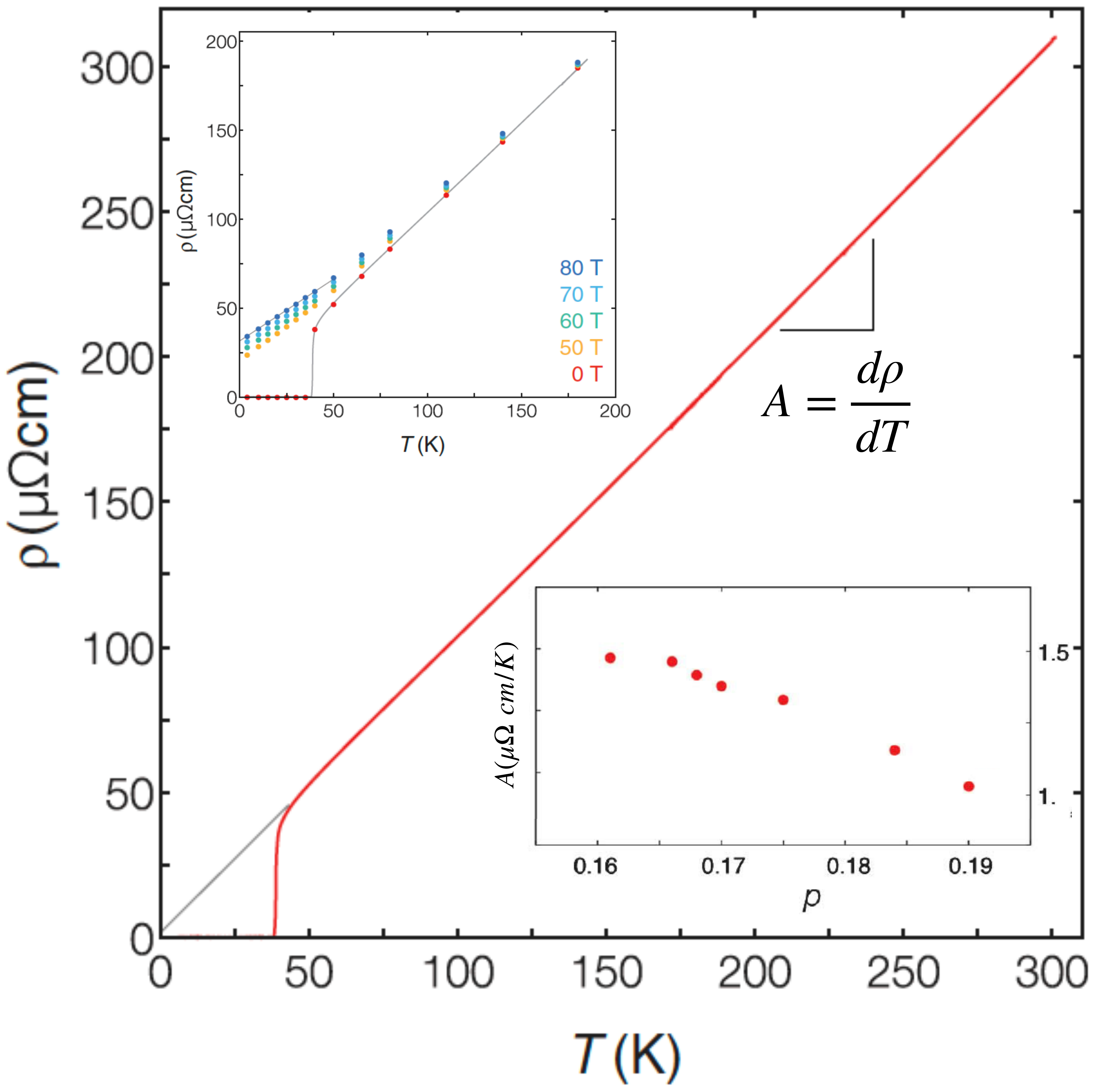
- This connection has led to resolutions of some of the “paradoxes” in applying quantum mechanics to black holes.

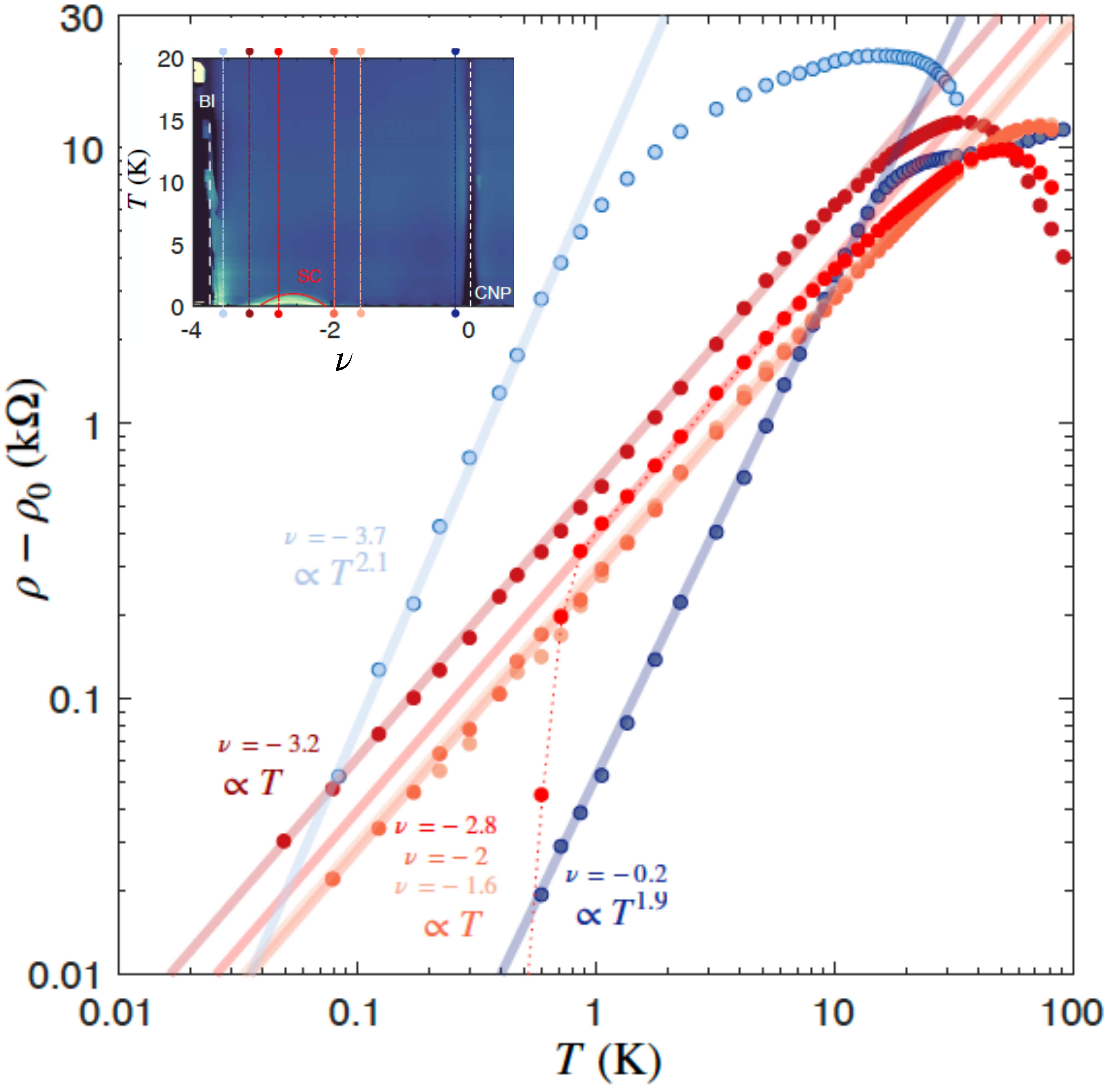
Strange metals


250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
0.3
Hole doping, p
AF insulator
T_N
T^*
Strange Metal
Superconductor
T_c
p_c


Cu^{2+}, Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å



LSCO: Giraldo-Gallo et al. 2018



MATBG: Jaoui et al. 2021

Properties of a strange metal:

- Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
  and $\rho(T) < h/e^2$ (in $d = 2$).
  Metals with $\rho(T) > h/e^2$ are bad metals.

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  Metals with $\rho(T) > h/e^2$ are bad metals.

- Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

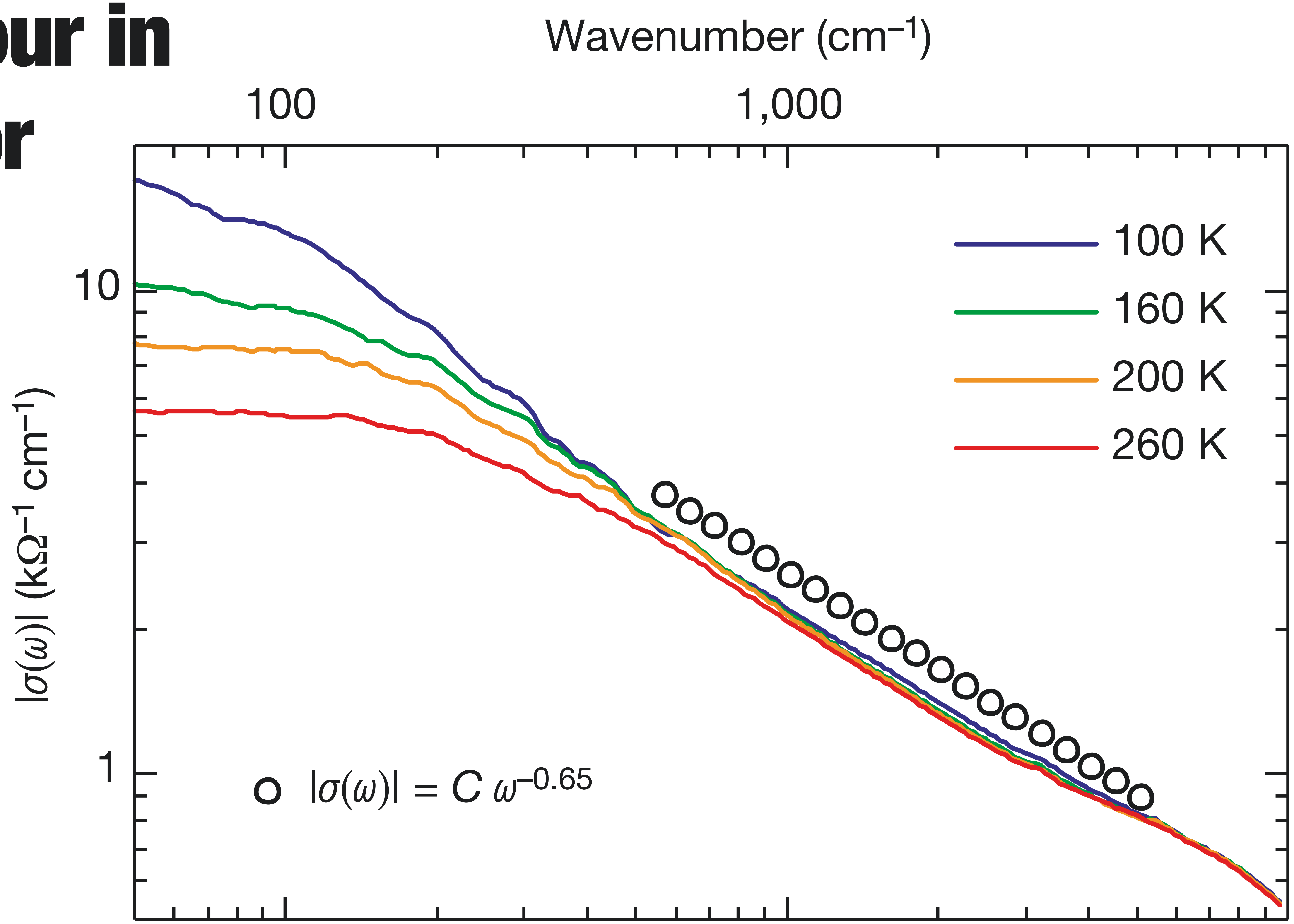
Properties of a strange metal:

- Optical conductivity

Nature **425**, 271 (2003)

D. van der Marel[1]*, H. J. A. Molegraaf[1]*, J. Zaanen[2], Z. Nussinov[2]*, F. Carbone[1]*, A. Damascelli[3]*, H. Eisaki[3]*, M. Greven[3], P. H. Kes[2] & M. Li[2]

Quantum critical behaviour in a high-T_c superconductor



But no $\hbar\omega/(k_B T)$ scaling.

Properties of a strange metal:

arXiv:2205.04030

- Optical conductivity

B. Michon,[1,2,3] C. Berthod,[3] C. W. Rischau,[3] A. Ataei,[4] L. Chen,[4]
S. Komiya,[5] S. Ono,[5] L. Taillefer,[4,6] D. van der Marel,[3] and A. Georges[7,8,3,

Planckian Behavior of Cuprate Superconductors: Reconciling the Scaling of Optical Conductivity with Resistivity and Specific Heat

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\mathrm{trans}}(\omega)} - i\omega \dfrac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\mathrm{trans}}(\omega)} \sim |\omega| \Phi_\sigma\left(\frac{\hbar\omega}{k_B T}\right)$$

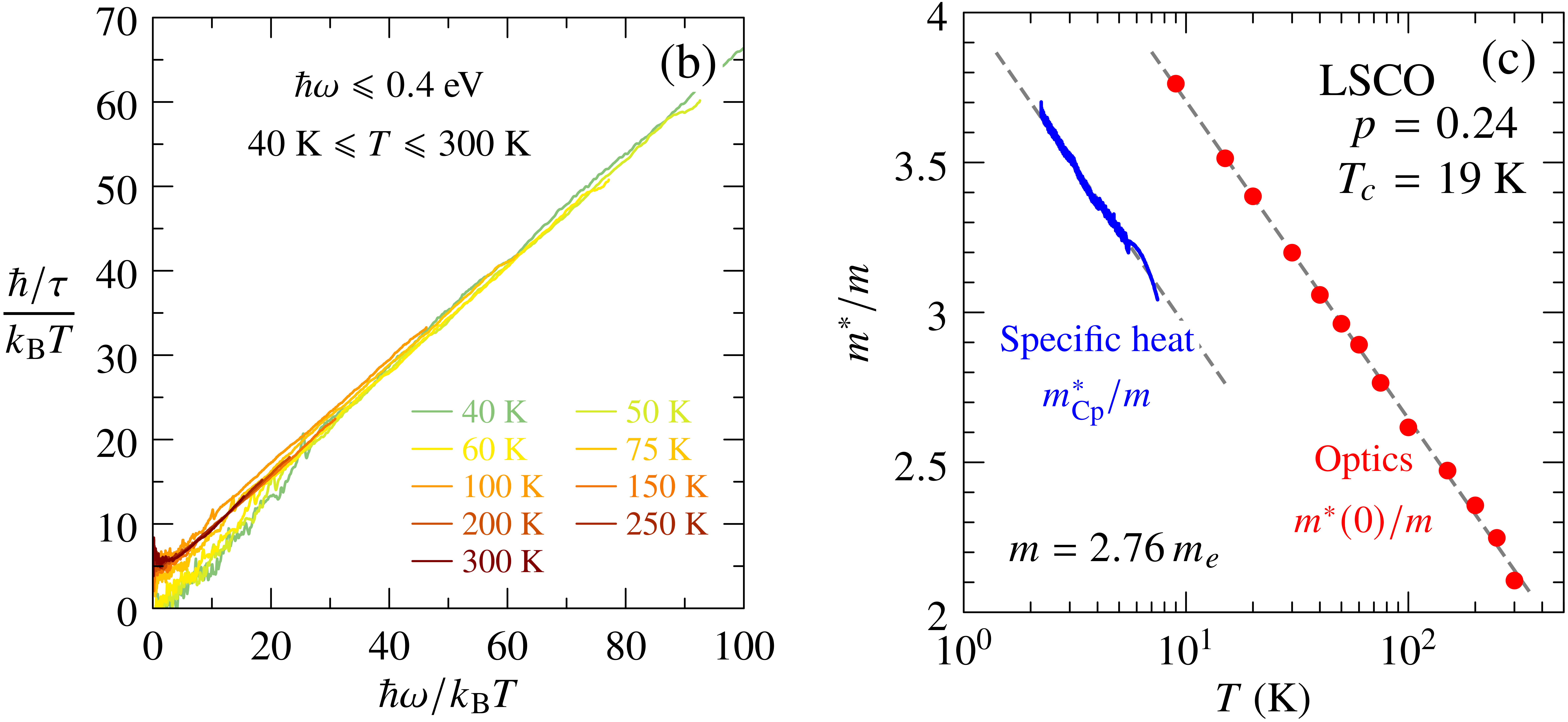
$$\text{Causality:} \quad \frac{m^*(\omega)}{m} \sim \ln\left(\frac{\Lambda}{\mathrm{Max}(\hbar\omega, k_B T)}\right)$$

Properties of a strange metal:

arXiv:2205.04030

- Optical conductivity

B. Michon,[1,2,3] C. Berthod,[3] C. W. Rischau,[3] A. Ataei,[4] L. Chen,[4]
S. Komiya,[5] S. Ono,[5] L. Taillefer,[4,6] D. van der Marel,[3] and A. Georges[7,8,3,



Properties of a strange metal:

- Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
  <u>and</u> $\rho(T) < h/e^2$ (in $d = 2$).
  Metals with $\rho(T) > h/e^2$ are <u>bad metals</u>.

- Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

- Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\text{trans}}(\omega)} - i\omega \dfrac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\text{trans}}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

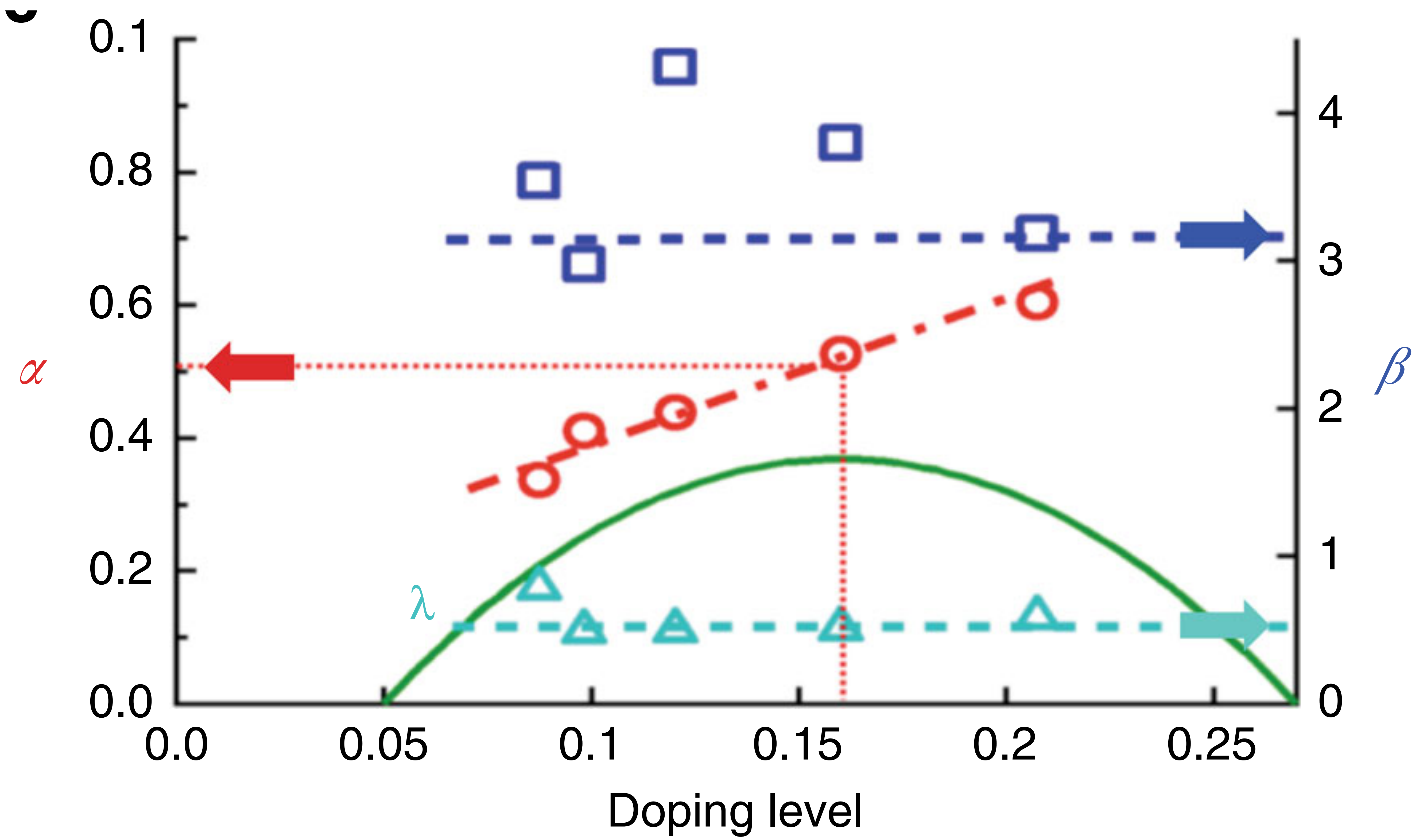
Properties of a strange metal:

Nature Communications **10**, 5737 (2019)

- Photoemission

A unified form of low-energy nodal electronic interactions in hole-doped cuprate superconductors

T.J. Reber[1,5*], X. Zhou[1*], N.C. Plumb[1,6], S. Parham[1], J.A. Waugh[1], Y. Cao[1], Z. Sun[1,7], H. Li[1], Q. Wang[1], J.S. Wen[2], Z.J. Xu[2], G. Gu[2], Y. Yoshida[3], H. Eisaki[3], G.B. Arnold[1] & D.S. Dessau[1,4*]



$$\Sigma''_{\mathrm{PLL}}(\omega) = \Gamma_0 + \lambda \frac{\left[(\hbar\omega)^2+(\beta k_{\mathrm{B}} T)^2\right]^{\alpha}}{(\hbar\omega_N)^{2\alpha-1}}$$

Properties of a strange metal:

- Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
  and $\rho(T) < h/e^2$ (in $d = 2$).
  Metals with $\rho(T) > h/e^2$ are bad metals.

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S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

- Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

B. Michon……A. Georges, arXiv:2205.04030

- Photoemission: nearly "marginal Fermi liquid" electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2$$

T.J. Reber….D. Dessau,
Nature Communications **10**, 5737 (2019)

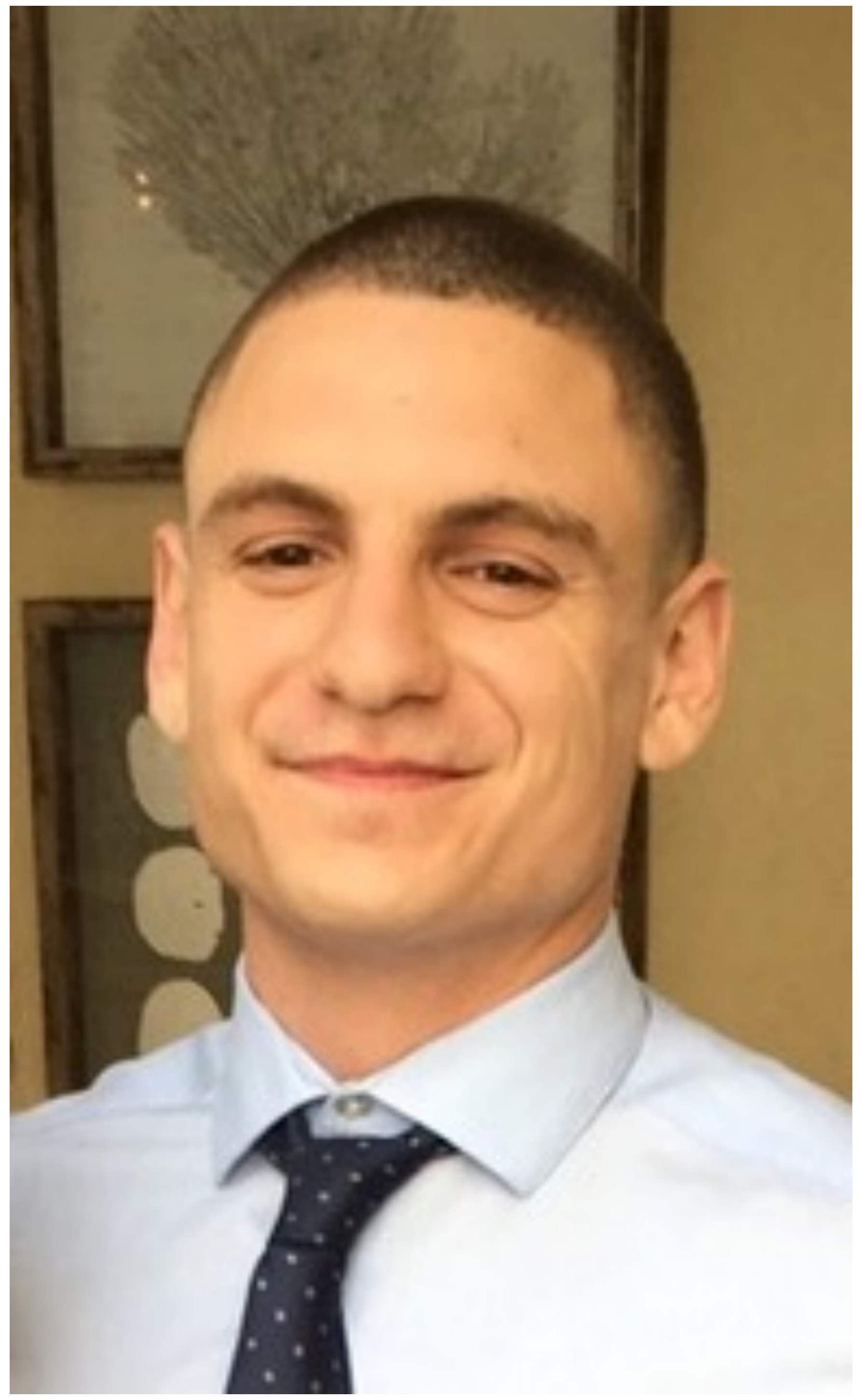
From
Yukawa-SYK models
to a universal theory of
strange metals

Aavishkar Patel

Flatiron Institute, NYC

Haoyu Guo

Harvard



Ilya Esterlis

Harvard → Wisconsin

arXiv: 2103.08615, 2203.04990, 2207.08841

E. E. Aldape, T. Cookmeyer, Aavishkar A. Patel, and Ehud Altman, arXiv:2012.00763

Yukawa-SYK models

$$H = \sum_{ij} t_{ij} \psi_i^\dagger \psi_j + \sum_\ell \frac{1}{2} \left(\pi_\ell^2 + \omega_\ell^2 \phi_\ell^2 \right) + \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Yukawa-SYK models

$$\mathcal{H} = -\mu \sum_{i} \psi_i^\dagger \psi_+ \sum_{\ell} \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N saddle point equations are

$$\boxed{\begin{aligned} G(i\omega_n) &= \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)} \\ \Sigma(\tau) &= g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau) \end{aligned}}$$

Make the low frequency ansatz

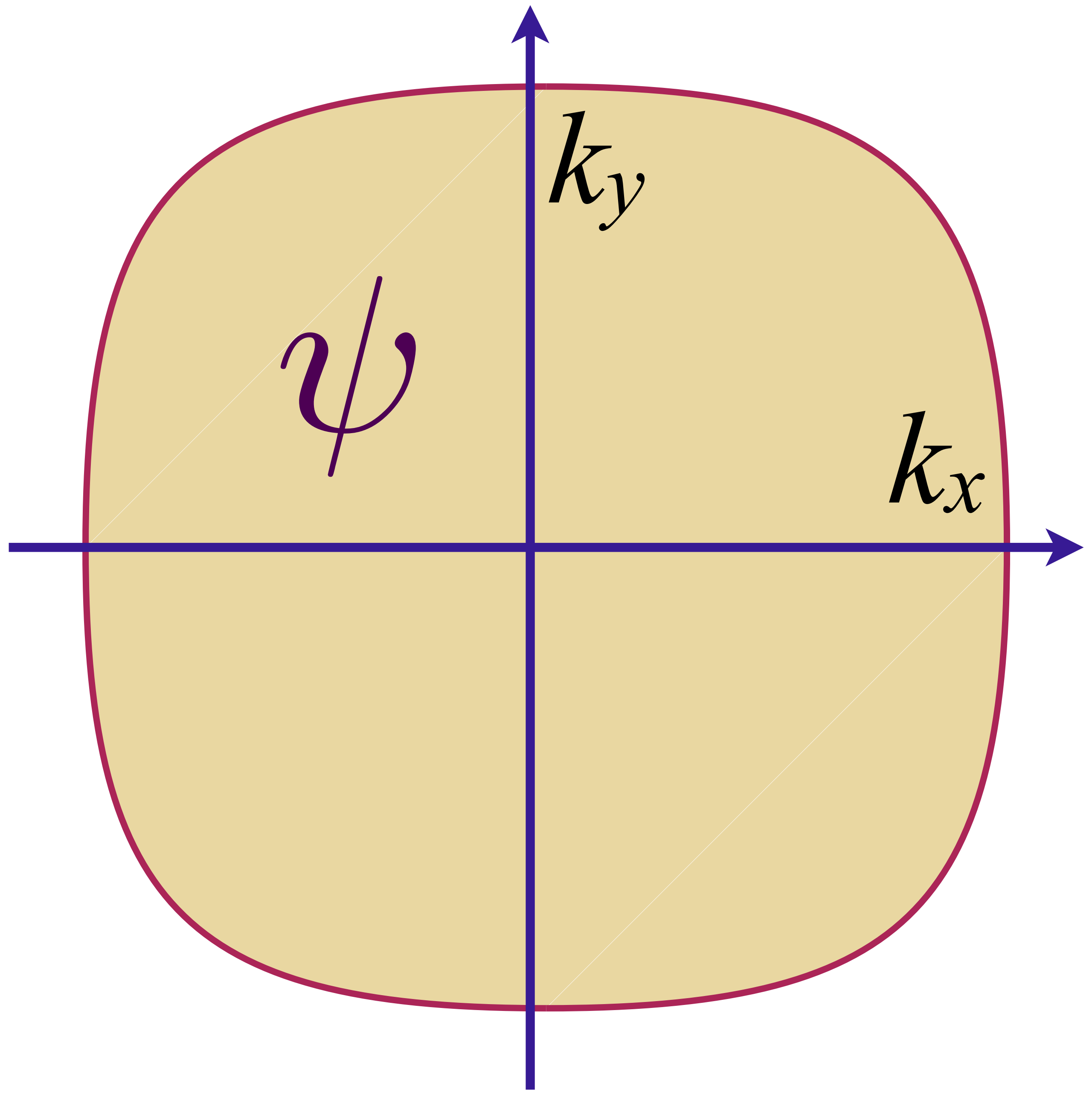
$$G(i\omega) \sim -i \operatorname{sgn}(\omega) |\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
See also Yuxuan Wang, PRL **124**, 017002 (2020)

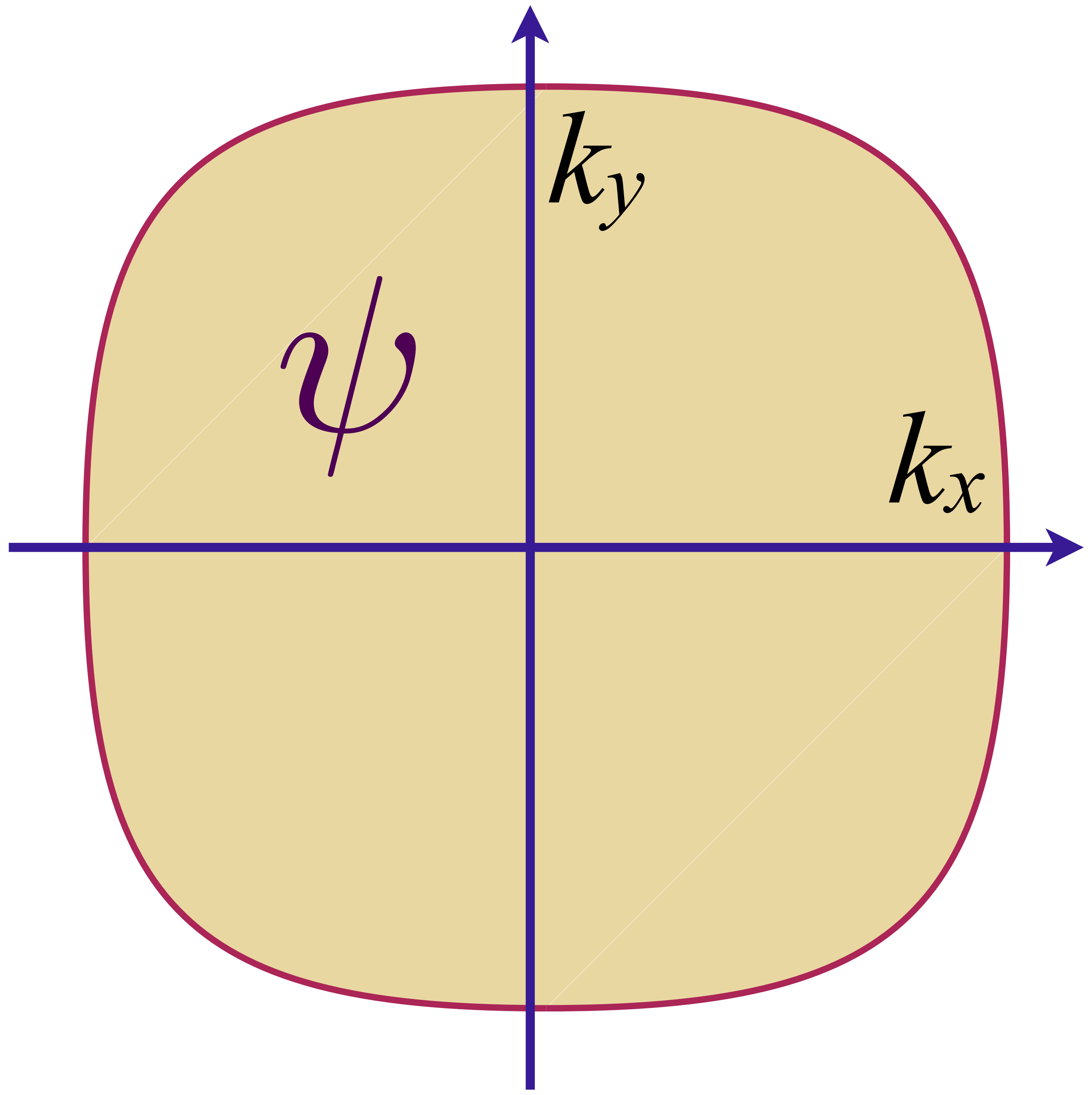
$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



$$-\boldsymbol{J}\,\psi^\dagger \psi^\dagger \psi \psi$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

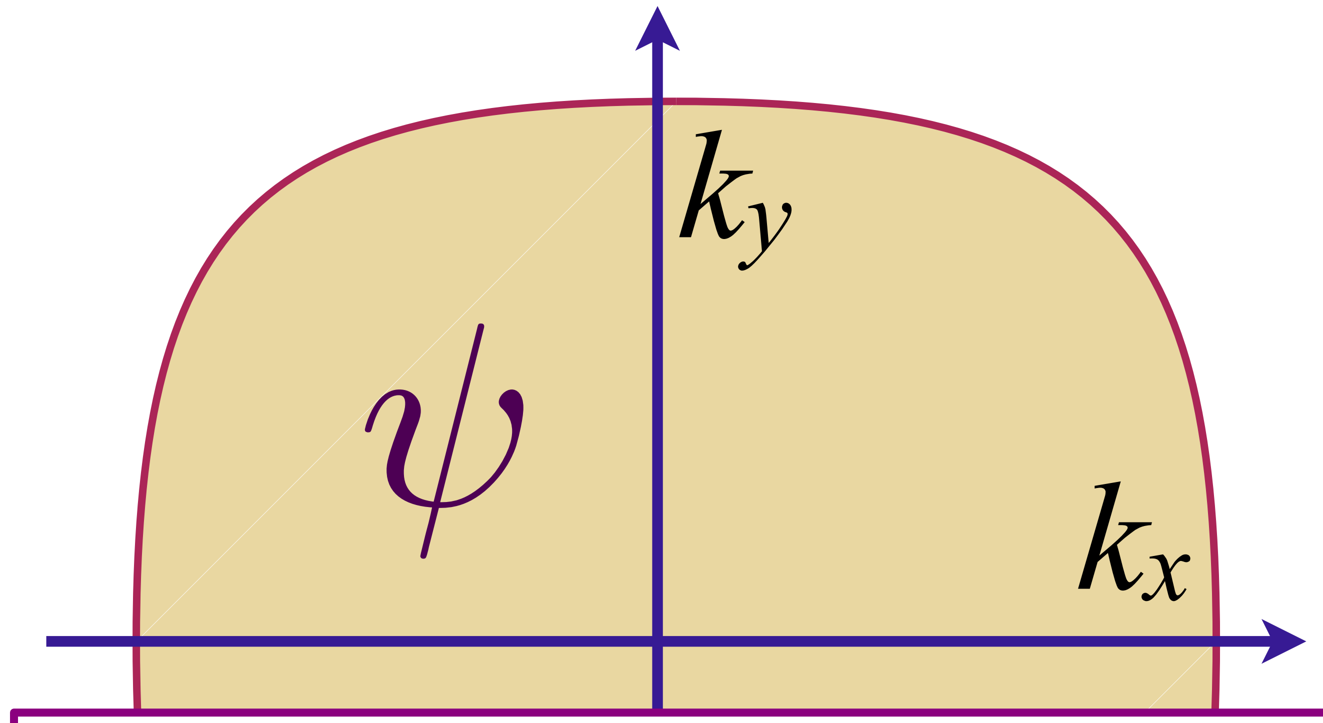


a critical boson ϕ
e.g. Ising-nematic order

$$\frac{\phi^2}{J} + \psi^\dagger \psi \, \phi$$

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{\phi^2}{J} + \psi^\dagger \psi \, \phi$$

Solve in a large N limit with Yukawa coupling

$$\frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi^\dagger_i(r,\tau) \psi_j(r,\tau) \phi_l(r,\tau) \quad , \quad \overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

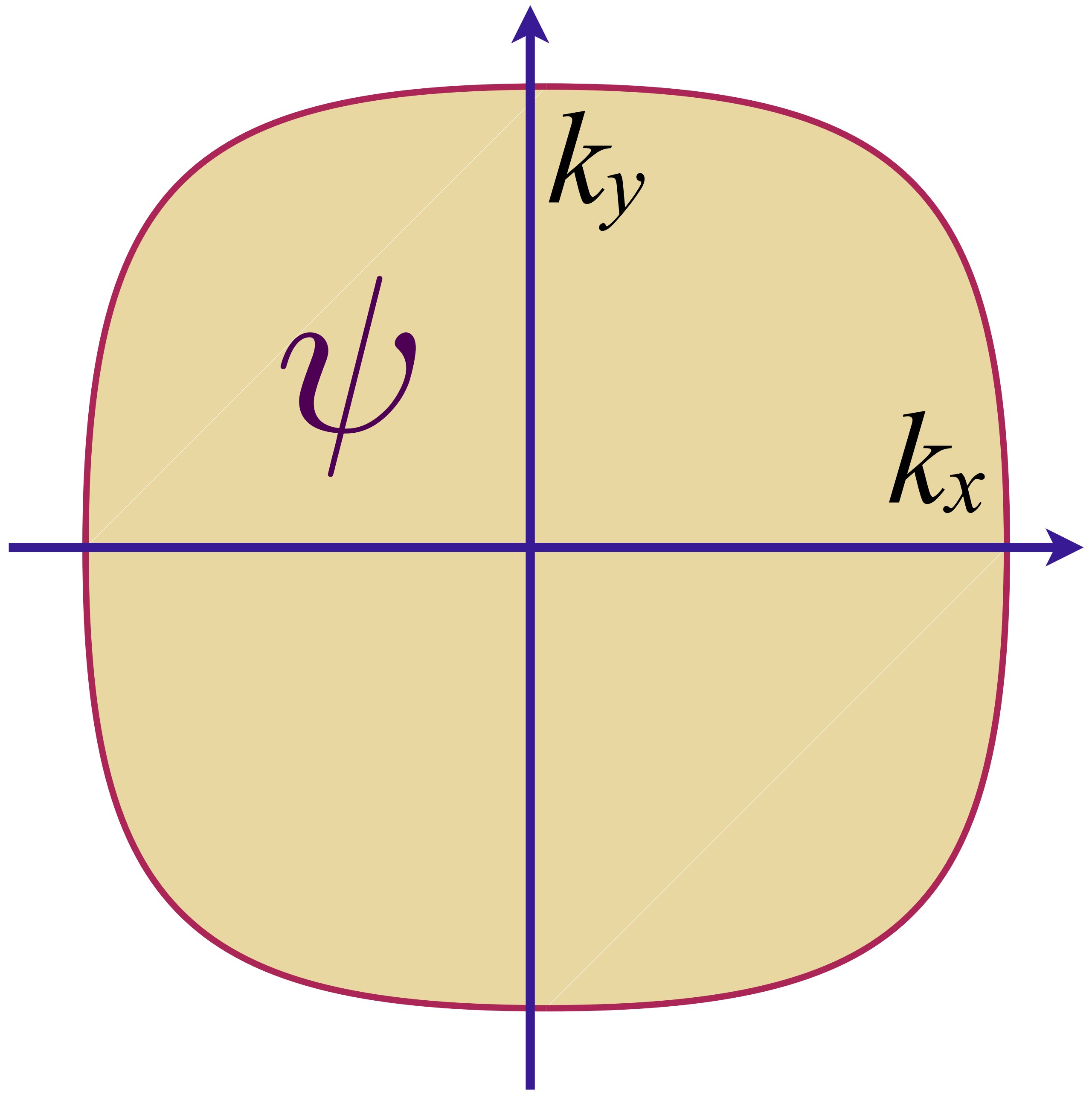
to obtain Eliashberg solution for electron (G) and boson (D) Green's functions at small ω:

$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i \text{sgn}(\omega) |\omega|^{2/3} , \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)} , \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

P.A. Lee (1989)

Fermi surface coupled to a critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{\phi^2}{J} + \psi^\dagger \psi \, \phi$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov PRL **106**, 106403 (2011)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S.S. PRB **94**, 045133 (2016)
Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990
Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585

Transport—a perfect metal!

Conservation of momentum and fermion-boson drag imply:

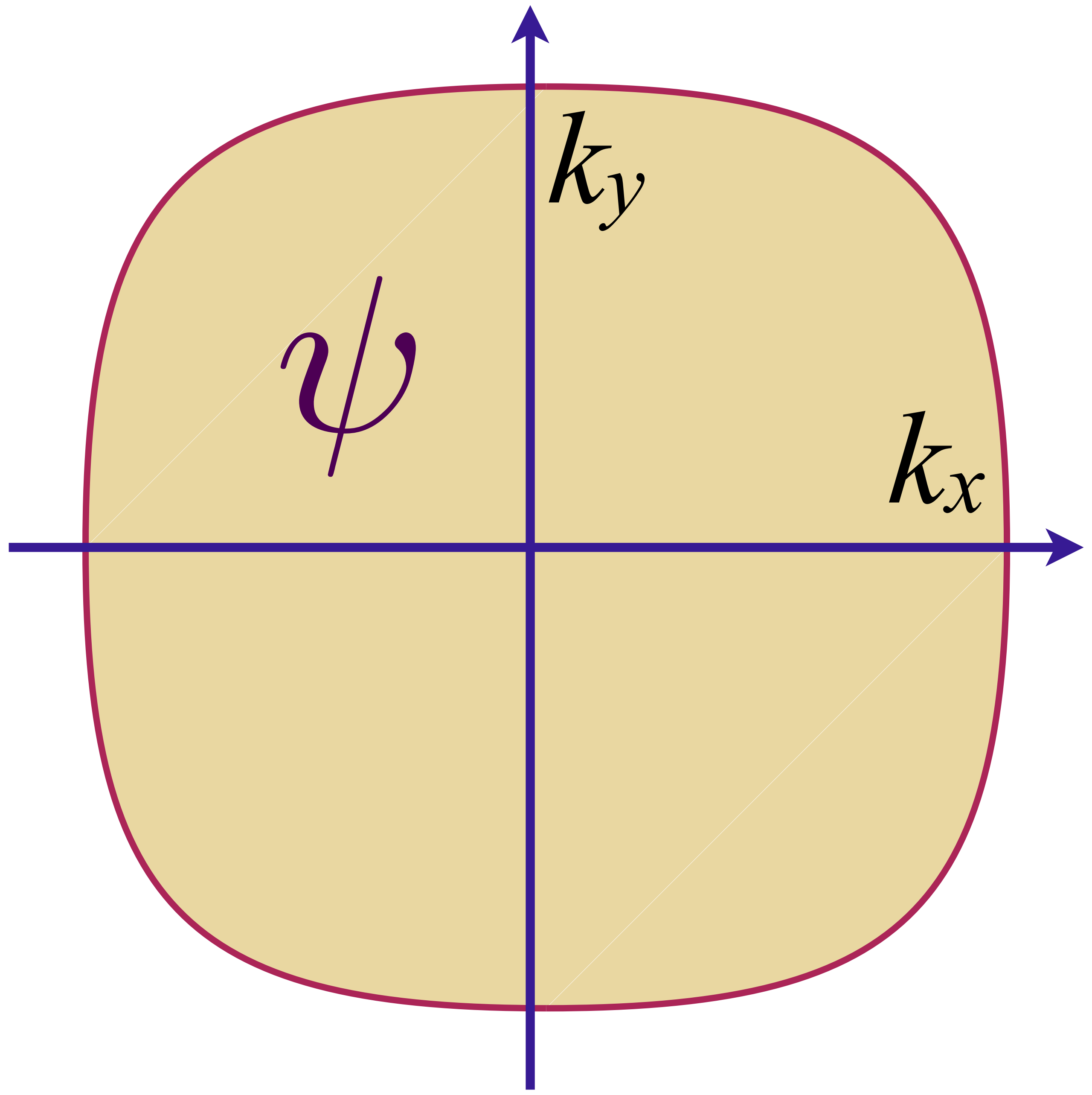
$$\mathrm{Re}\,[\sigma(\omega)] = D\delta(\omega) + \ldots$$

Fermi surface coupled to a critical boson:
No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ

e.g. Ising-nematic order

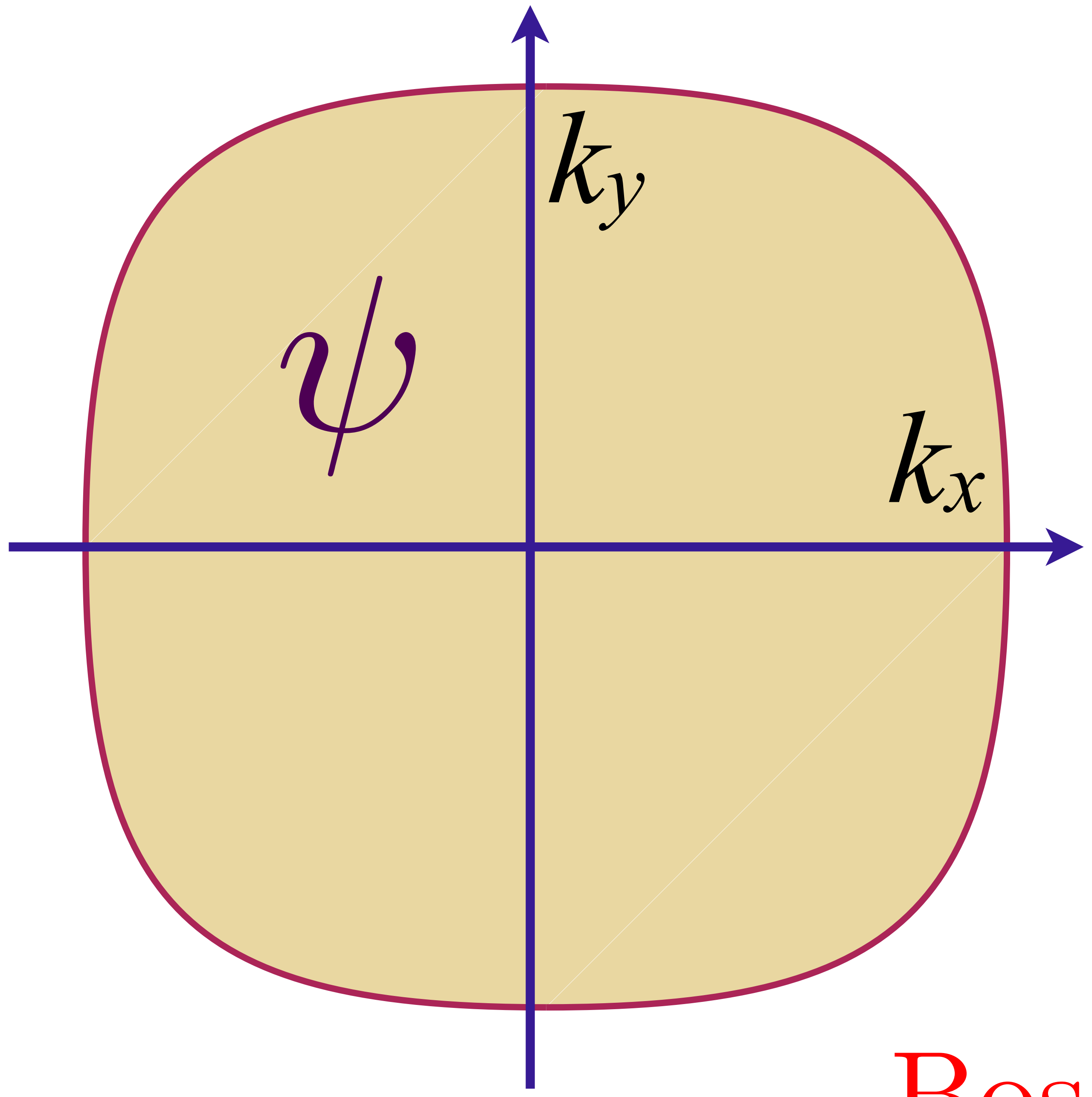
$$\frac{\phi^2}{J} + \psi^\dagger \psi \, \phi$$

$$+v(\boldsymbol{r})\psi^\dagger\psi$$

Spatially random potential $v(r)$ with $\overline{v(r)} = 0$, $\overline{v(r)v(r')} = v^2\delta(r - r')$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{\phi^2}{J} + \psi^\dagger \psi \, \phi$$

$$+ v(\boldsymbol{r}) \psi^\dagger \psi$$

Boson Green's function: $D(q, i\Omega) \sim 1/(q^2 + \gamma|\Omega|)$

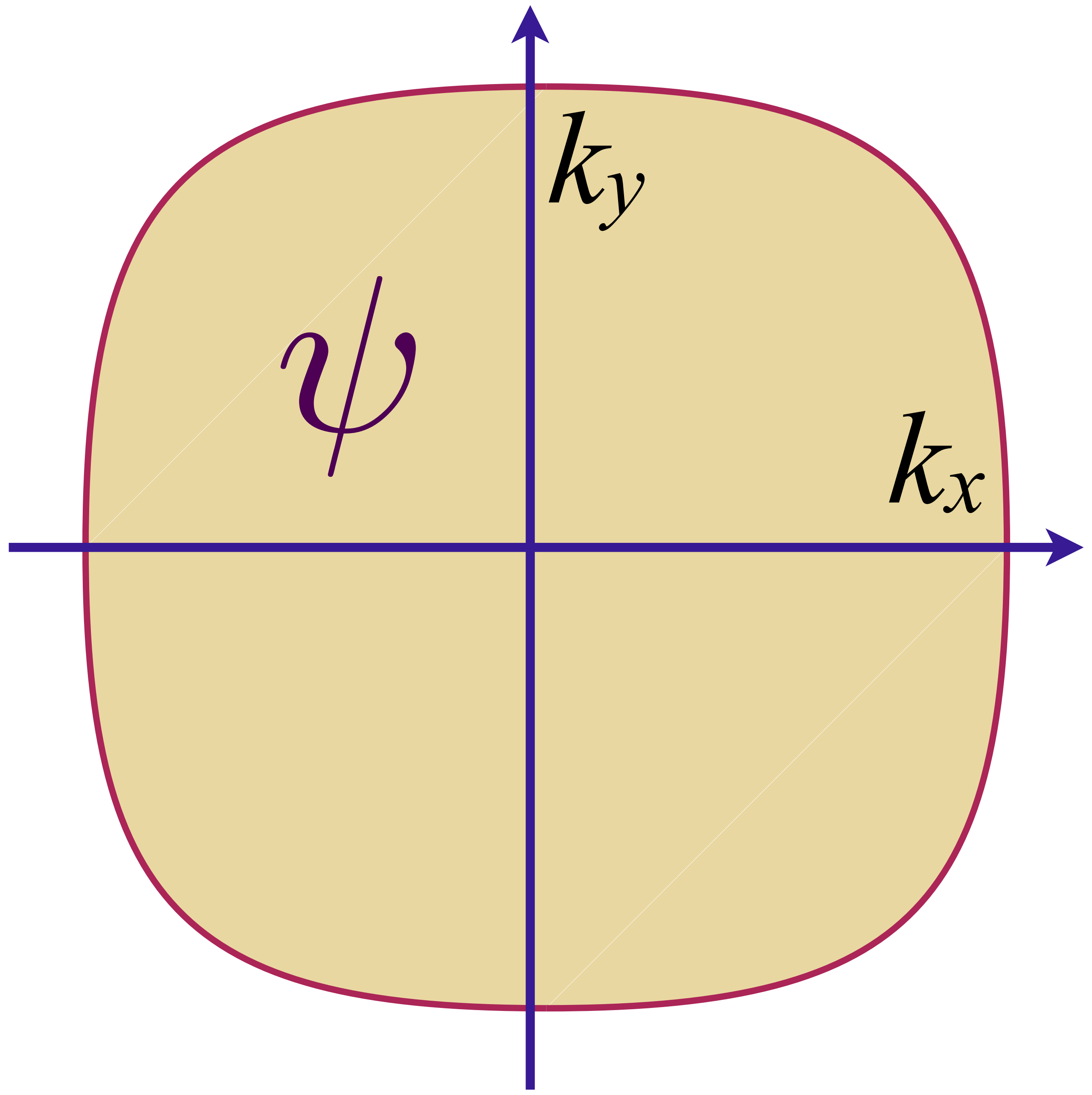
Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \text{sgn}(\omega) - i\dfrac{g^2}{v^2}\omega \ln(1/|\omega|); \quad \dfrac{1}{\tau(\varepsilon)} \sim |\varepsilon|$

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat

HLR (1993)

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{\phi^2}{J} + \psi^\dagger \psi \, \phi$$

$$+v(\boldsymbol{r})\psi^\dagger\psi$$

But resistivity and optical conductivity are like a Fermi liquid, with residual resistivity $\sim 1/v^2$.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990; T. C. Wu, Y. Liao, and M. S. Foster, arXiv:2206.01762

Fermi surface coupled to a critical boson:
No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:
Potential disorder v

A marginal Fermi liquid but NOT a strange metal

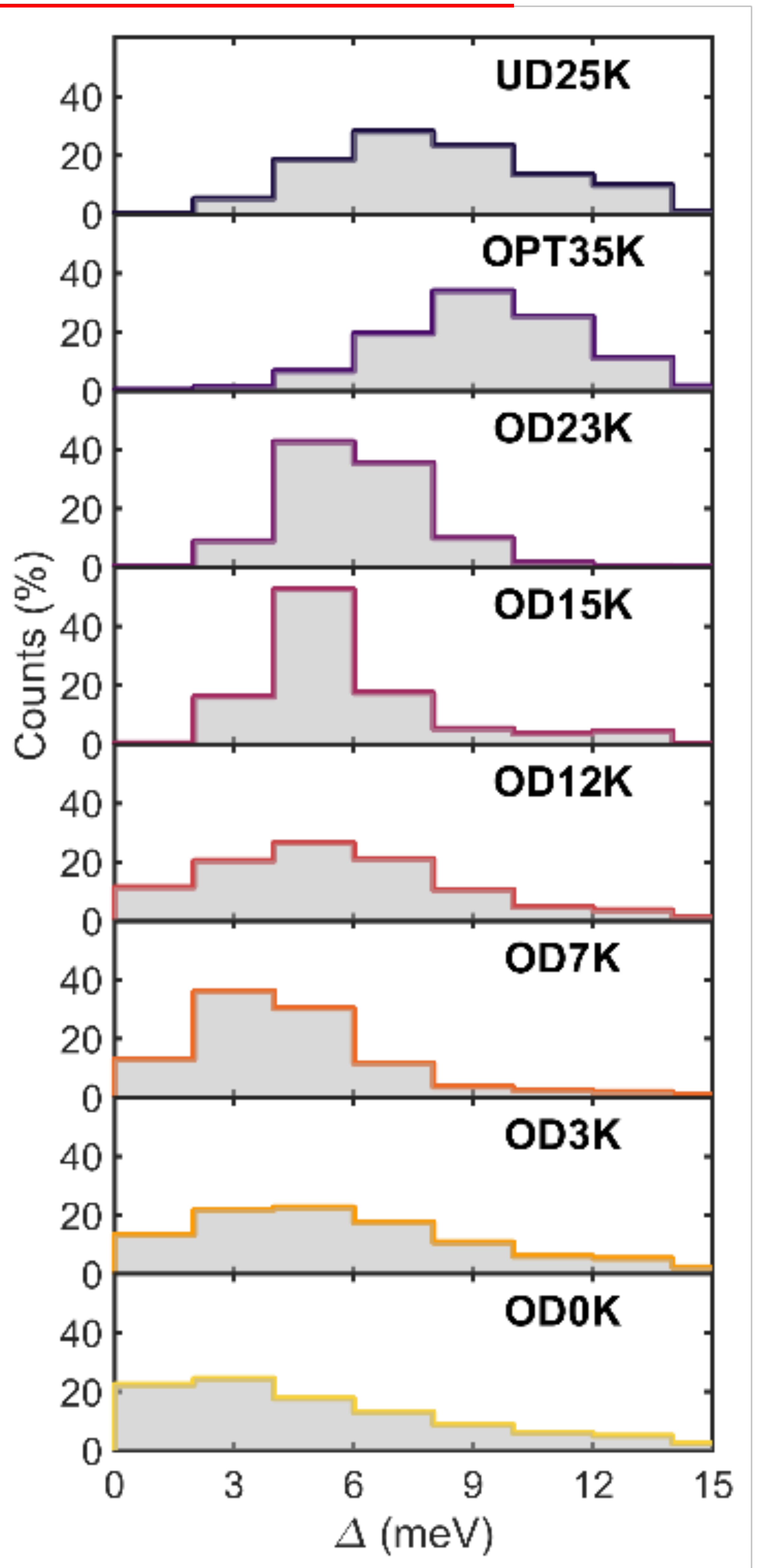
Spatially random interactions!

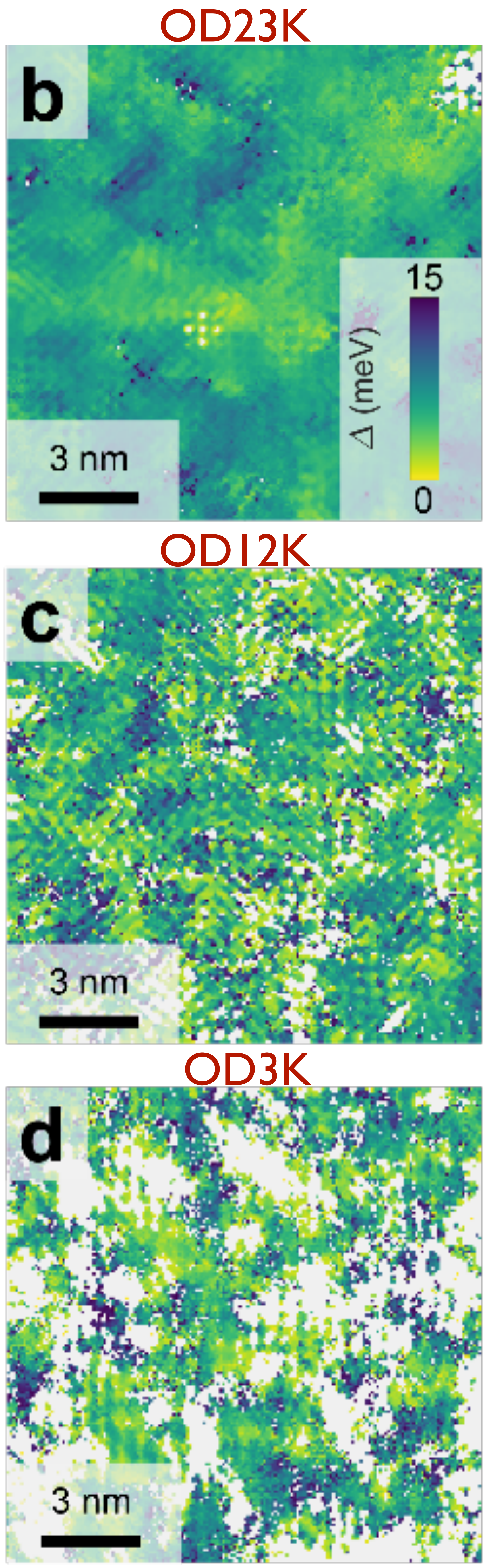
Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped (Pb,Bi)$_2$Sr$_2$CuO$_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge,
Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos,
Amber Vervloet, Steef Smit, Erik van Heumen,
Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin,
Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen,
Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the (Pb,Bi)$_2$Sr$_2$CuO$_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

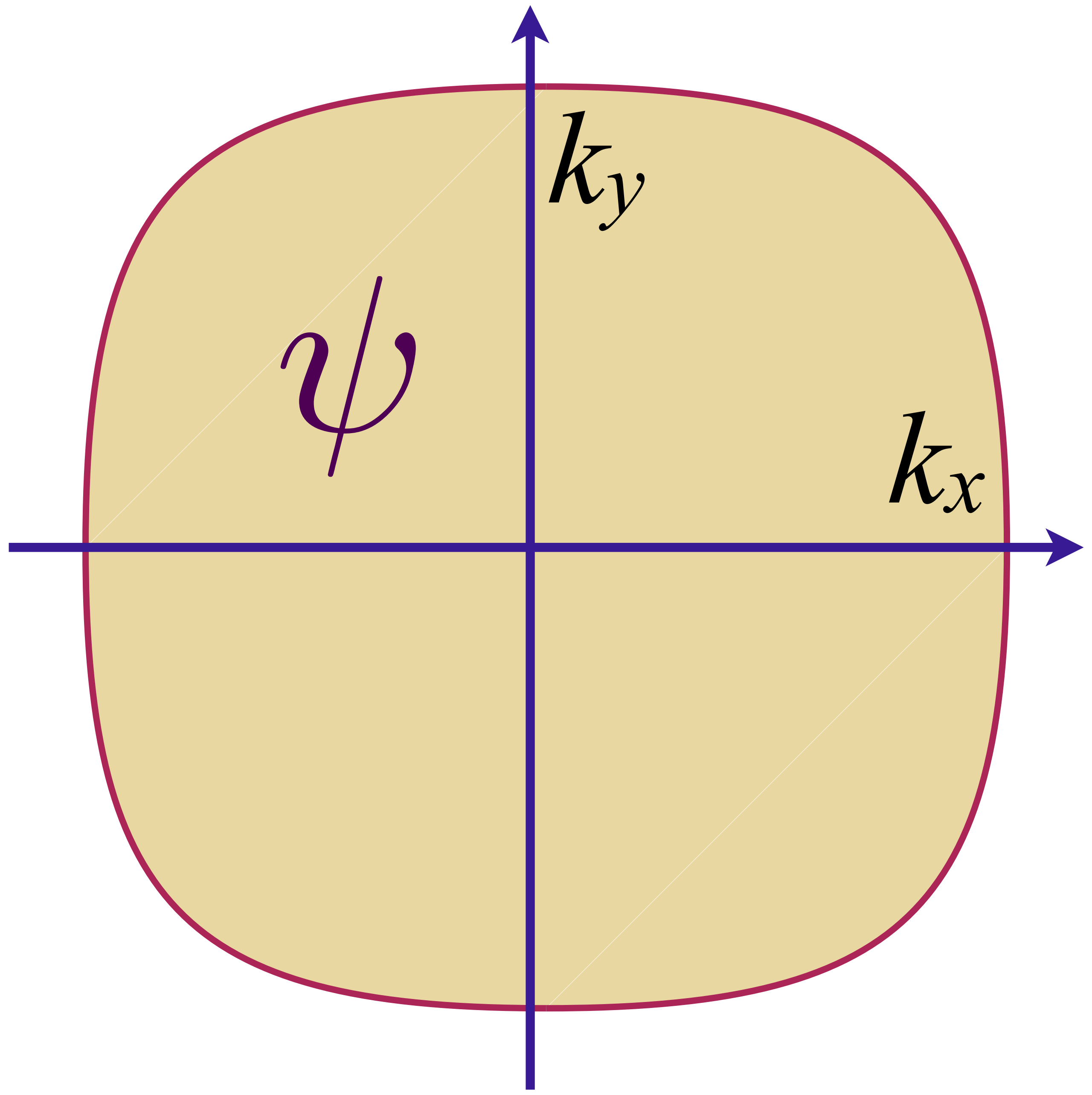
arXiv:2205.09740





Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



a critical boson ϕ
e.g. Ising-nematic order

$$\frac{\phi^2}{J + J'(\boldsymbol{r})} + \psi^\dagger \psi \, \phi$$

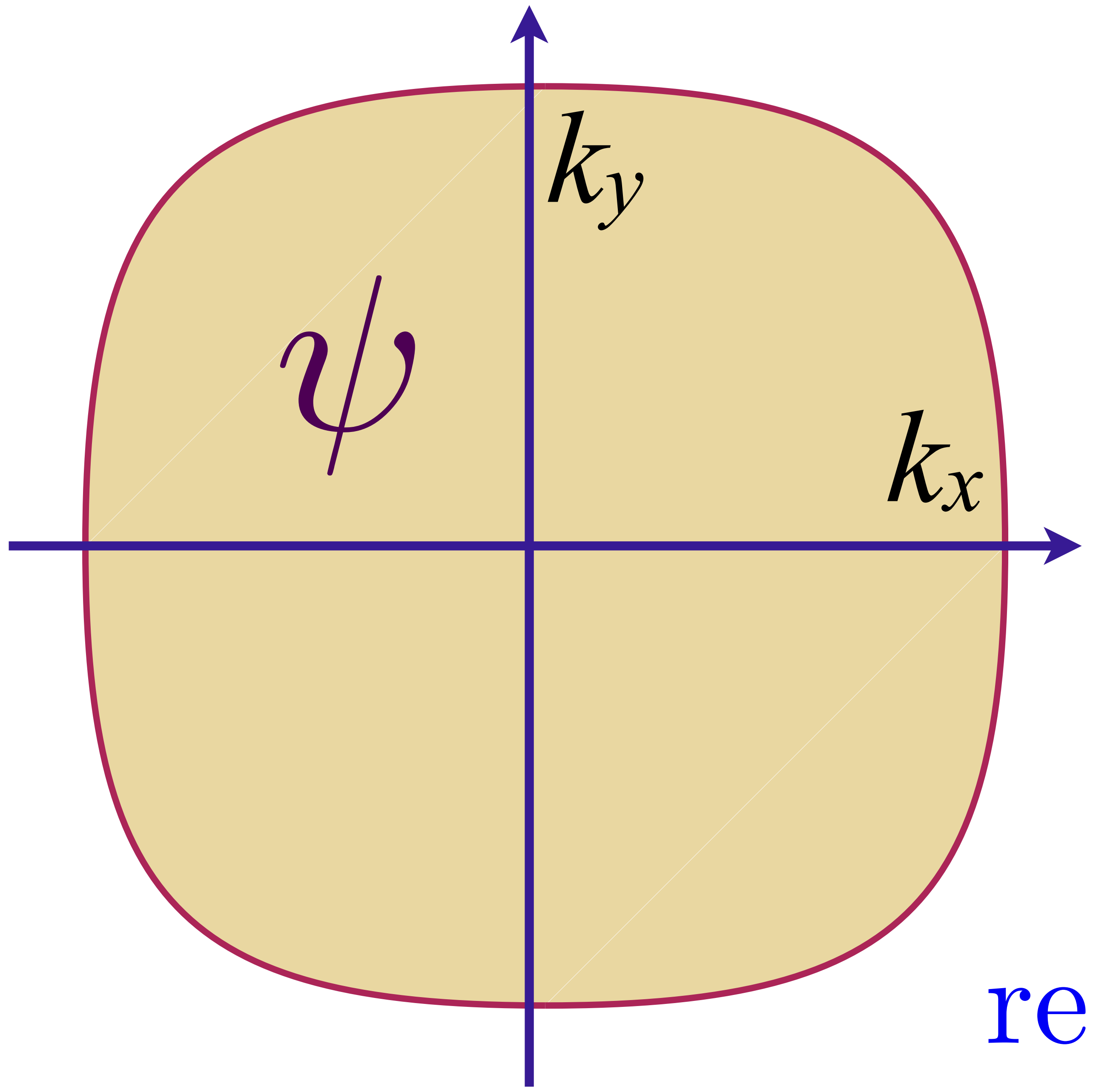
$$+ v(\boldsymbol{r}) \psi^\dagger \psi$$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ

e.g. Ising-nematic order



$$\phi^2 + \; [g + g'(\boldsymbol{r})]\psi^\dagger \psi \, \phi$$

$$+v(\boldsymbol{r})\psi^\dagger \psi$$

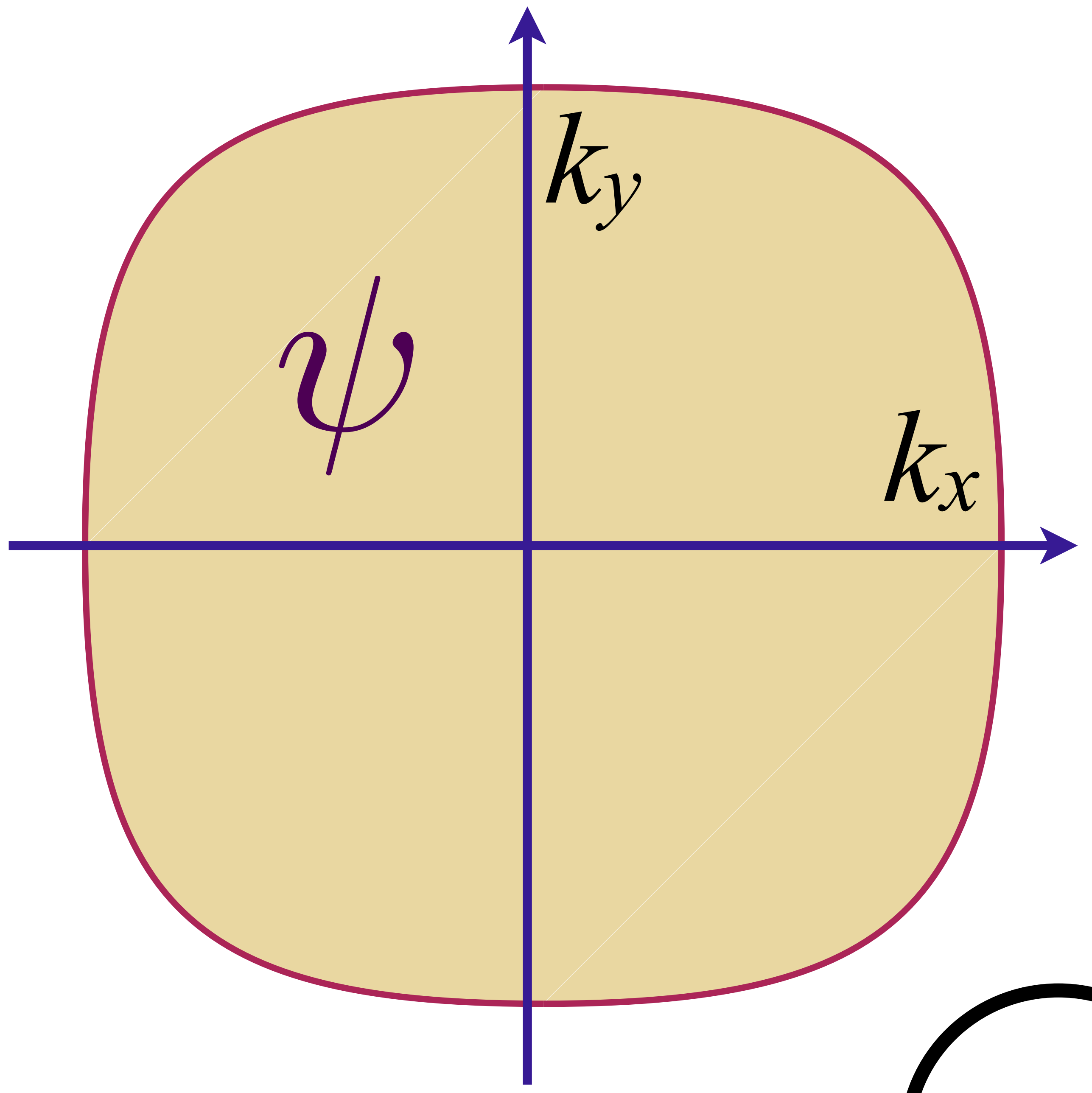
ϕ^2 "mass" disorder $J'(\boldsymbol{r})$ is strongly relevant;
rescale ϕ to move disorder to the Yukawa coupling;
can then be controlled in a SYK-like large N limit of 'flavor' indices,
leading to a G-Σ-D-Π bi-local field theory.

Spatially random Yukawa coupling $g'(r)$ with $\overline{g'(r)} = 0$, $\overline{g'(r)g'(r')} = g'^2 \delta(r - r')$

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

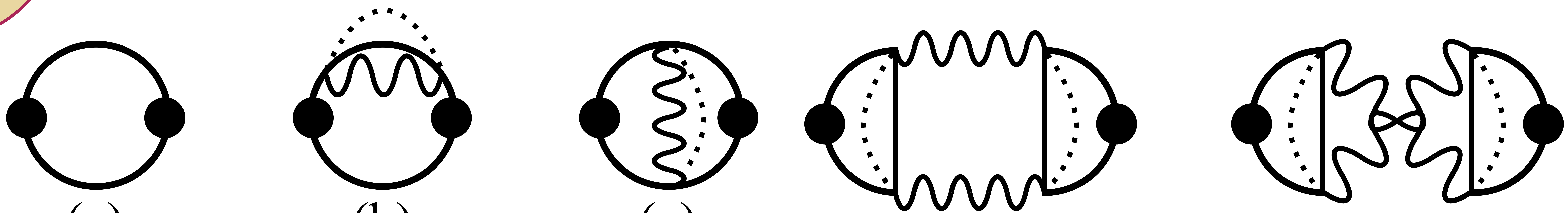
a critical boson ϕ
e.g. Ising-nematic order



$$\phi^2 + \left[g + g'(\boldsymbol{r}) \right] \psi^\dagger \psi \, \phi$$

$$+ v(\boldsymbol{r}) \psi^\dagger \psi$$

Conductivity:



(a) σ_v

(b) $\frac{\sigma_{\Sigma,g}}{2}, \ \frac{\sigma_{\Sigma,g'}}{2}$

(c) $\sigma_{V,g}$

(d)

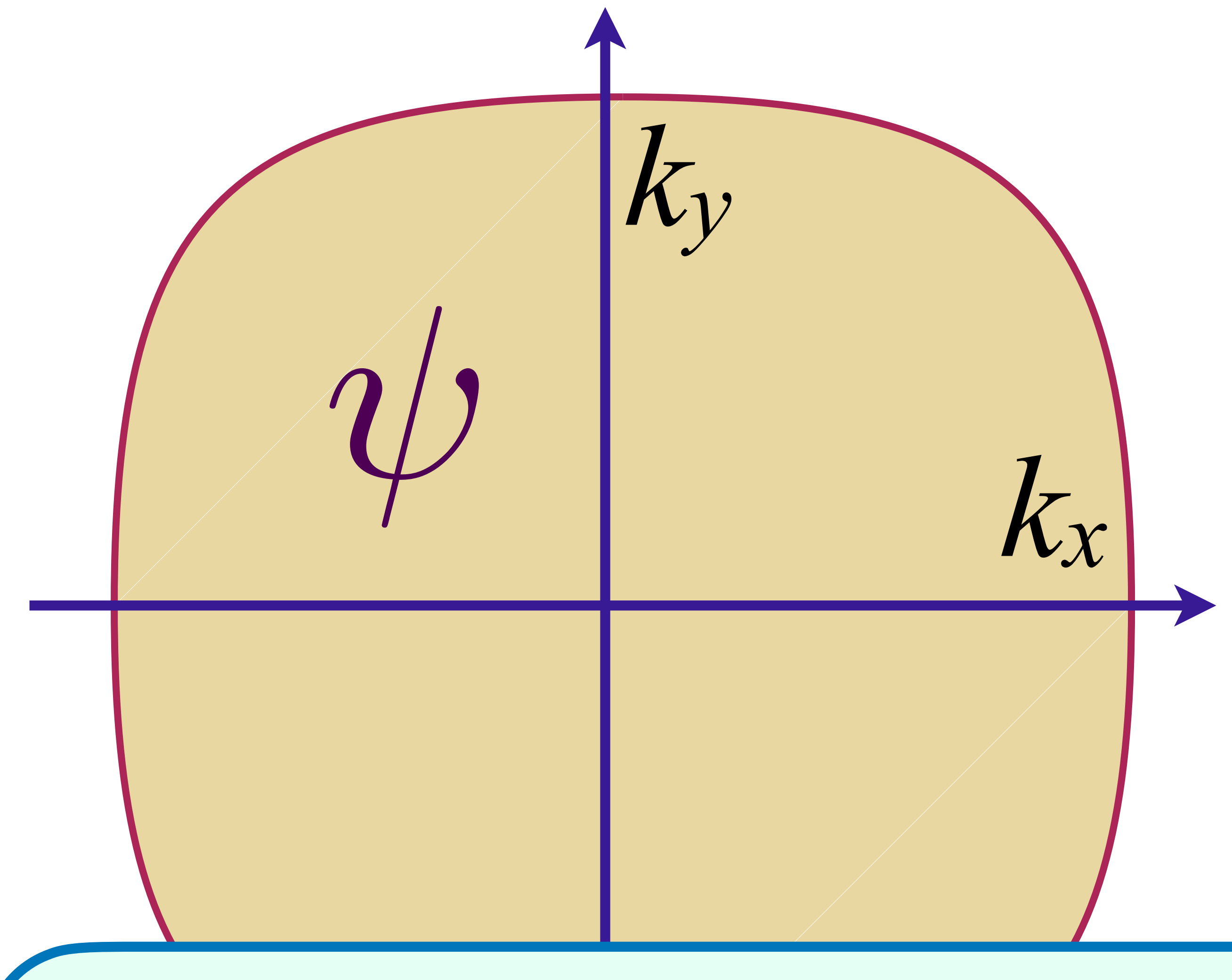
(e)

+ all ladders and bubbles…..

Fermi surface coupled to a critical boson with disorder

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

a critical boson ϕ
e.g. Ising-nematic order



$$\phi^2 + \; [g + g'(\boldsymbol{r})]\psi^\dagger \psi \, \phi$$

$$+v(\boldsymbol{r})\psi^\dagger \psi$$

Conductivity: $\sigma(\omega) \sim [1/\tau_{\text{trans}}(\omega) - i\omega \, m^*(\omega)/m]^{-1}$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2; Transport insensitive to g; Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Fermi surface coupled to a critical boson:

No spatial disorder

A non-Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Potential disorder v

A marginal Fermi liquid but NOT a strange metal

Fermi surface coupled to a critical boson:

Interaction disorder g'

A marginal Fermi liquid AND a strange metal

Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

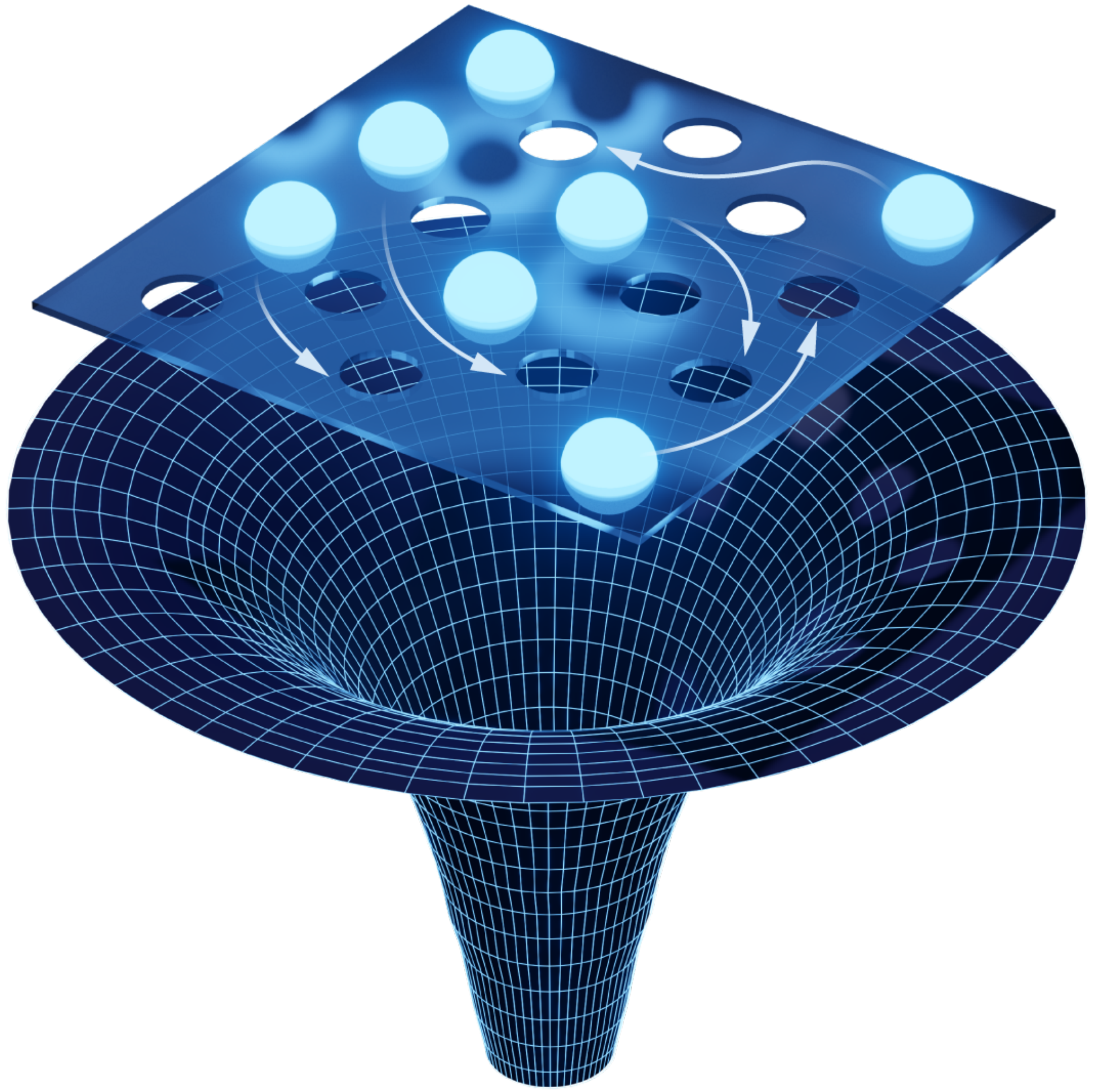
Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Low energy theory of time reparameterizations is the theory of the boundary graviton in 1+1 dimensional quantum gravity on AdS$_2$.

Summary

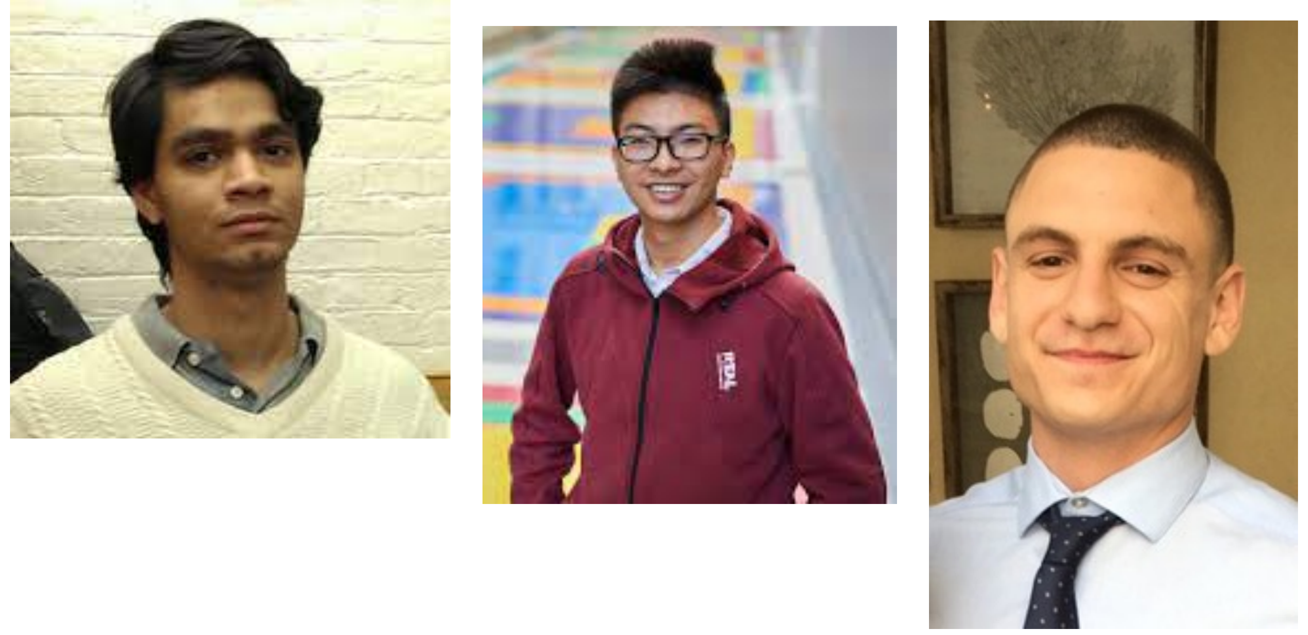
- The density of states of a charged black holes in Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.



Summary

- The density of states of a charged black holes in Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.

- Linear-T resistivity arises from spatially random interactions in a two-dimensional quantum-critical metal.



Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990