Entanglement, holography, and strange metals

University of Cologne, June 8, 2012

Subir Sachdev

Lecture at the 100th anniversary Solvay conference,
Theory of the Quantum World, chair D.J. Gross.
arXiv:1203.4565

sachdev.physics.harvard.edu

PHYSICS
VE RI TAS
HARVARD

Liza Huijse

Max Metlitski

Brian Swingle

Modern phases of quantum matter

Not adiabatically connected

to independent electron states:

many-particle

quantum entanglement

“Complex entangled” states of quantum matter in *d* spatial dimensions

Useful classification is provided by nature of excitations with vanishing energy:

1. Gapped systems without zero energy excitations

2. “Relativistic” systems with zero energy excitations at isolated points in momentum space

3. “Compressible” systems with zero energy excitations on *d-1* dimensional surfaces in momentum space.

“Complex entangled” states of quantum matter in *d* spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

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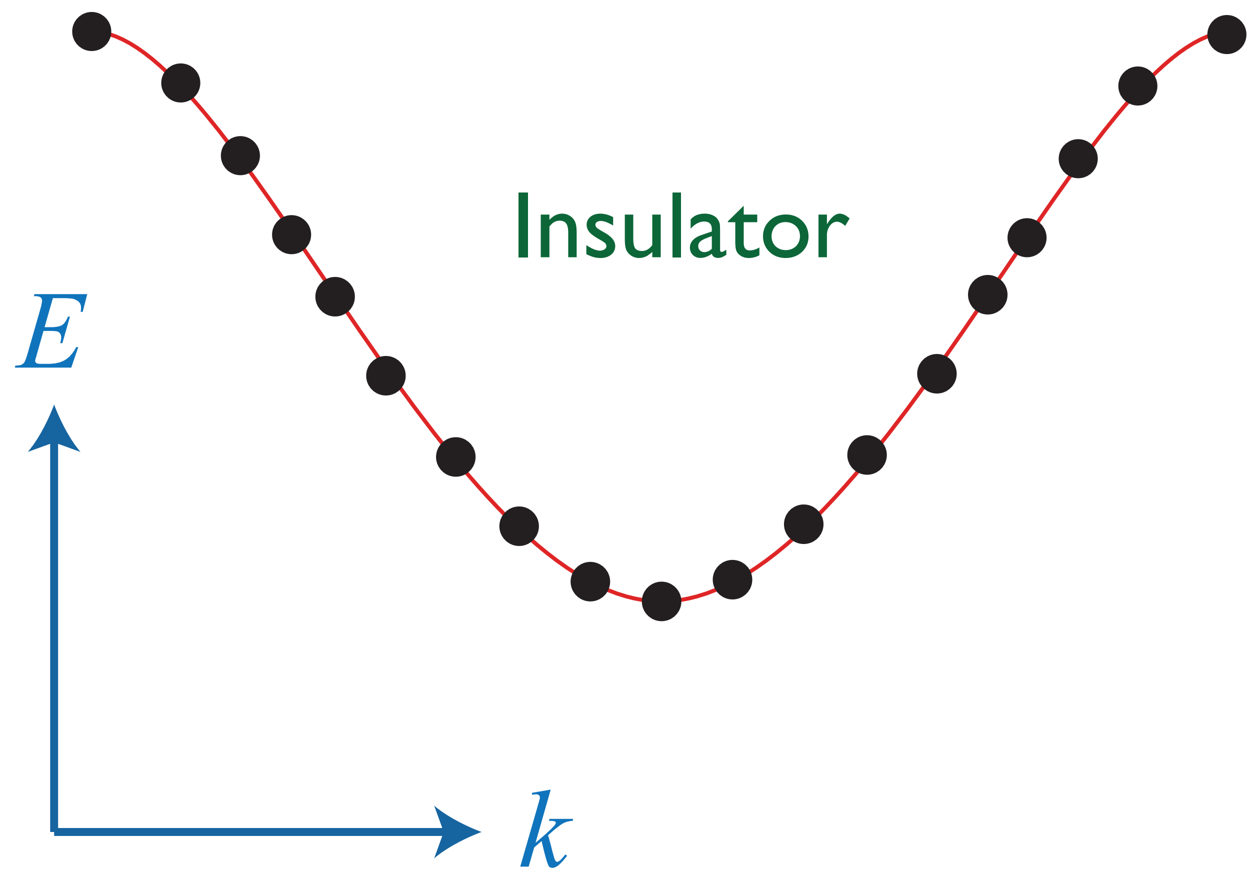
Band insulators



An even number of electrons per unit cell

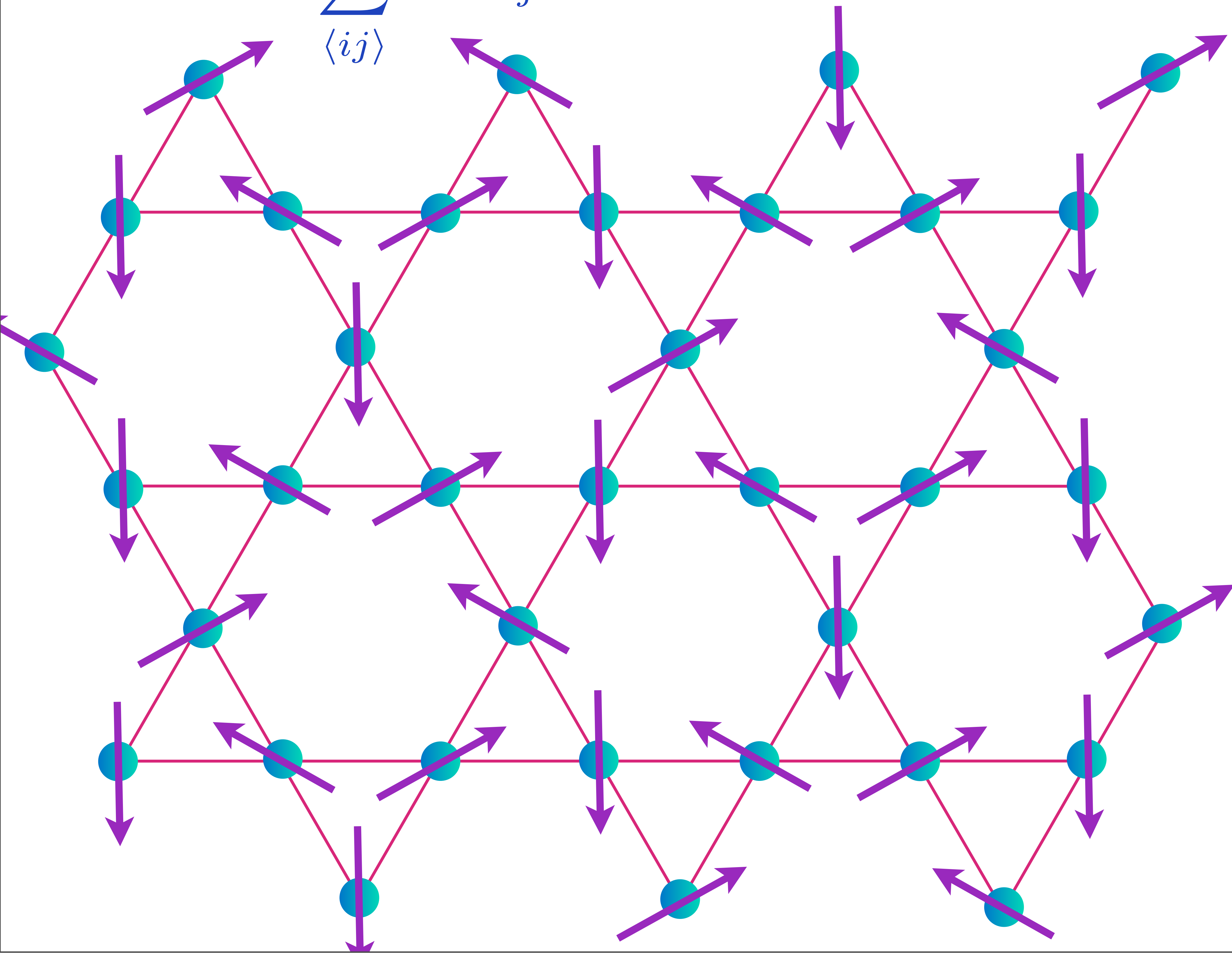
Mott insulator

Emergent excitations

An odd number of electrons per unit cell
but electrons are localized by Coulomb repulsion;
state has long-range entanglement

Mott insulator: Kagome antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



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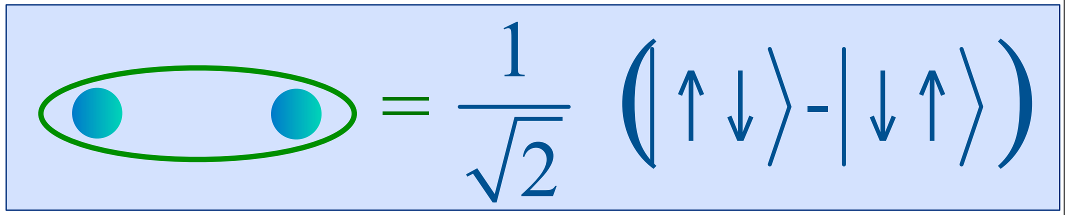

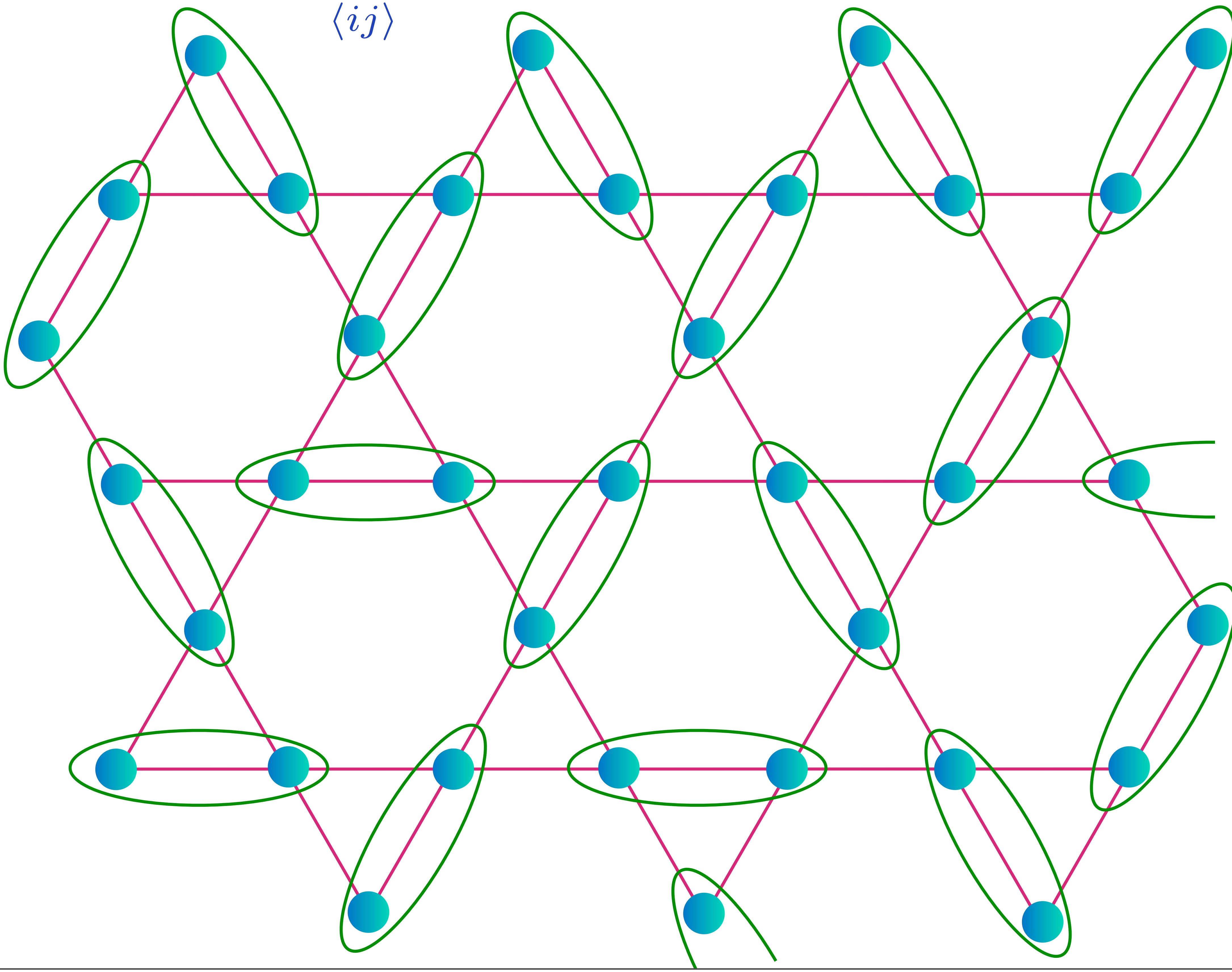

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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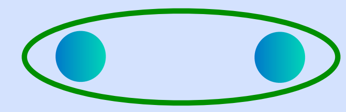
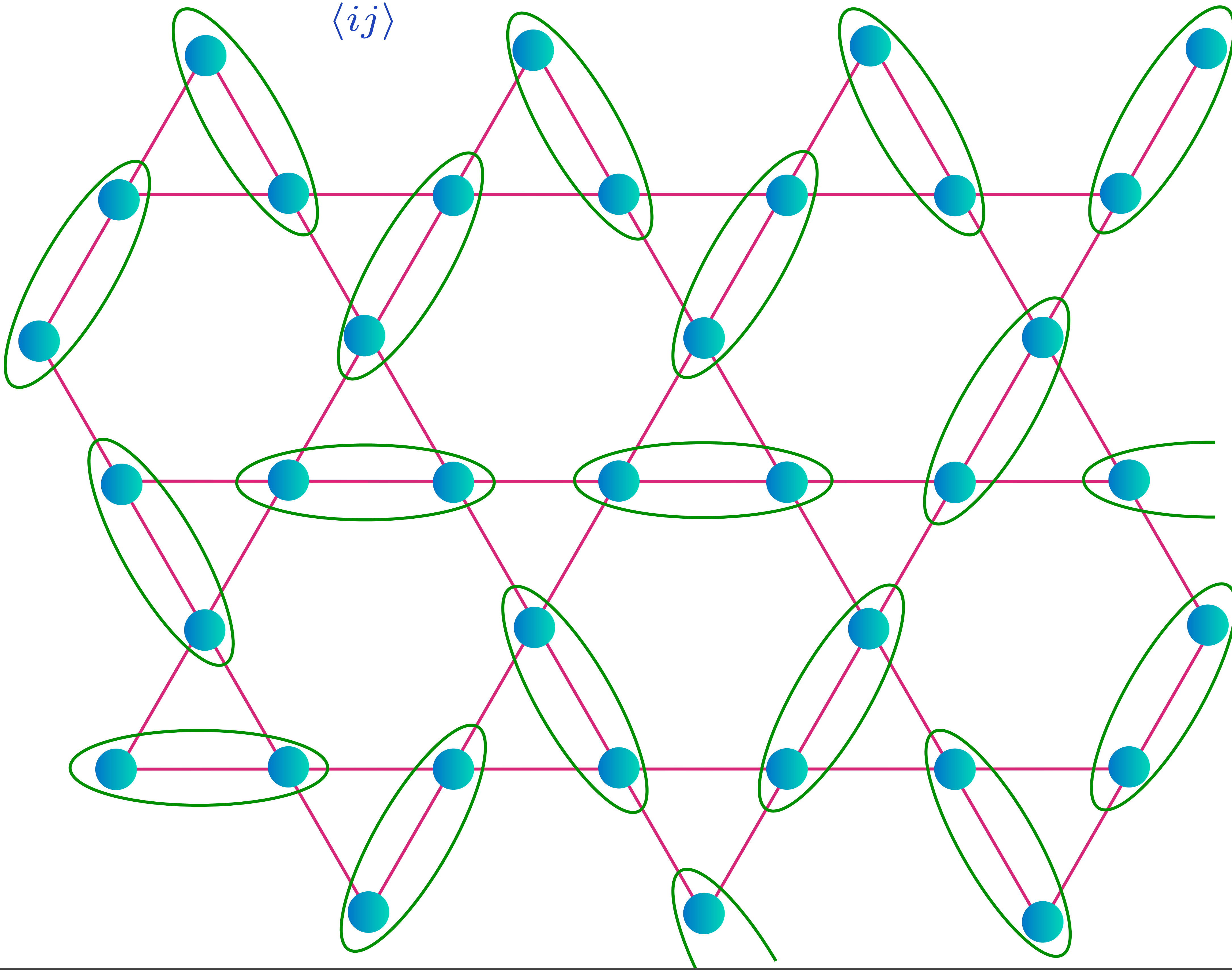
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$



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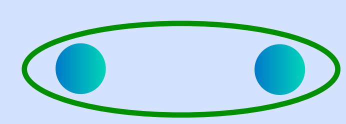

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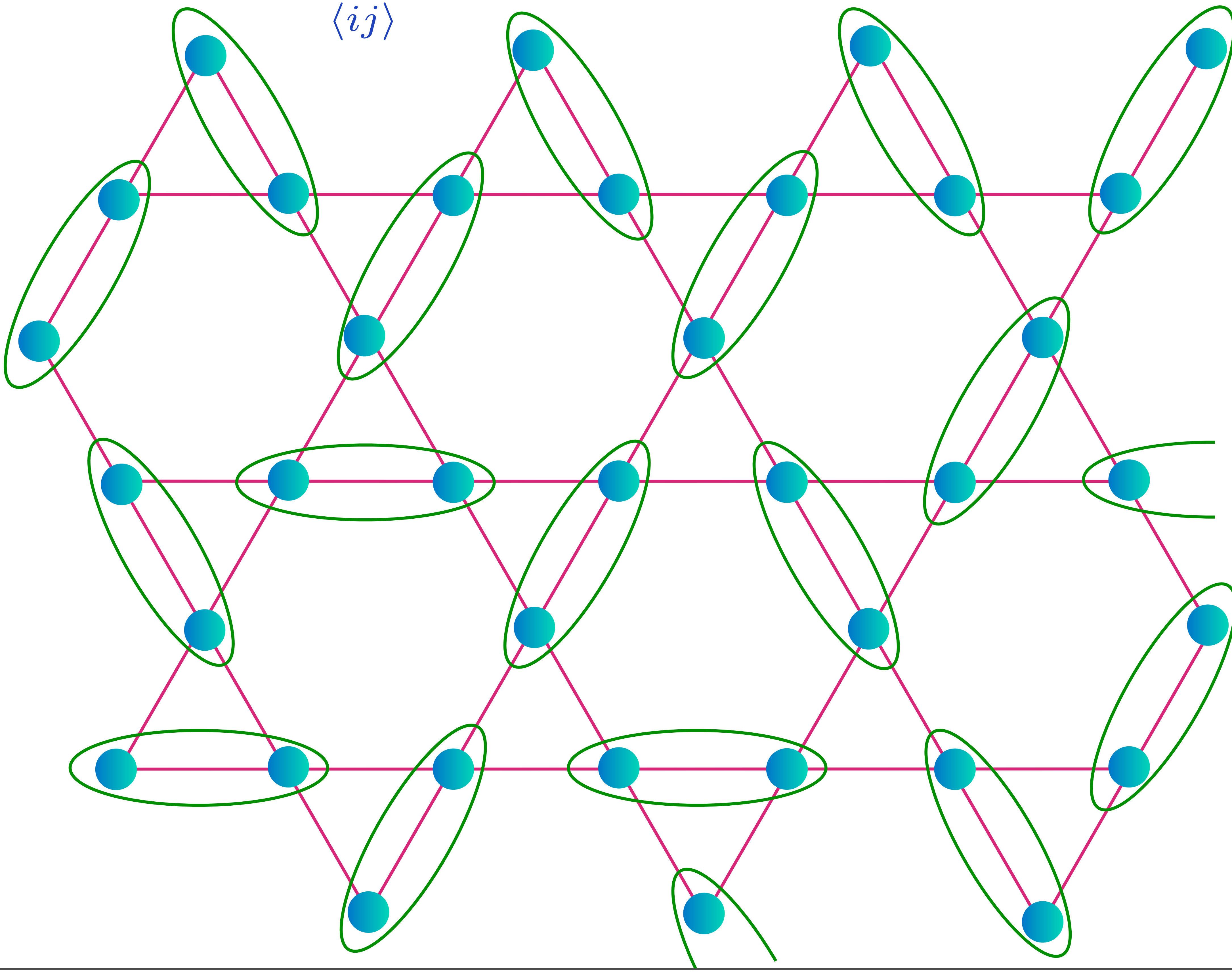

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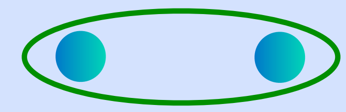

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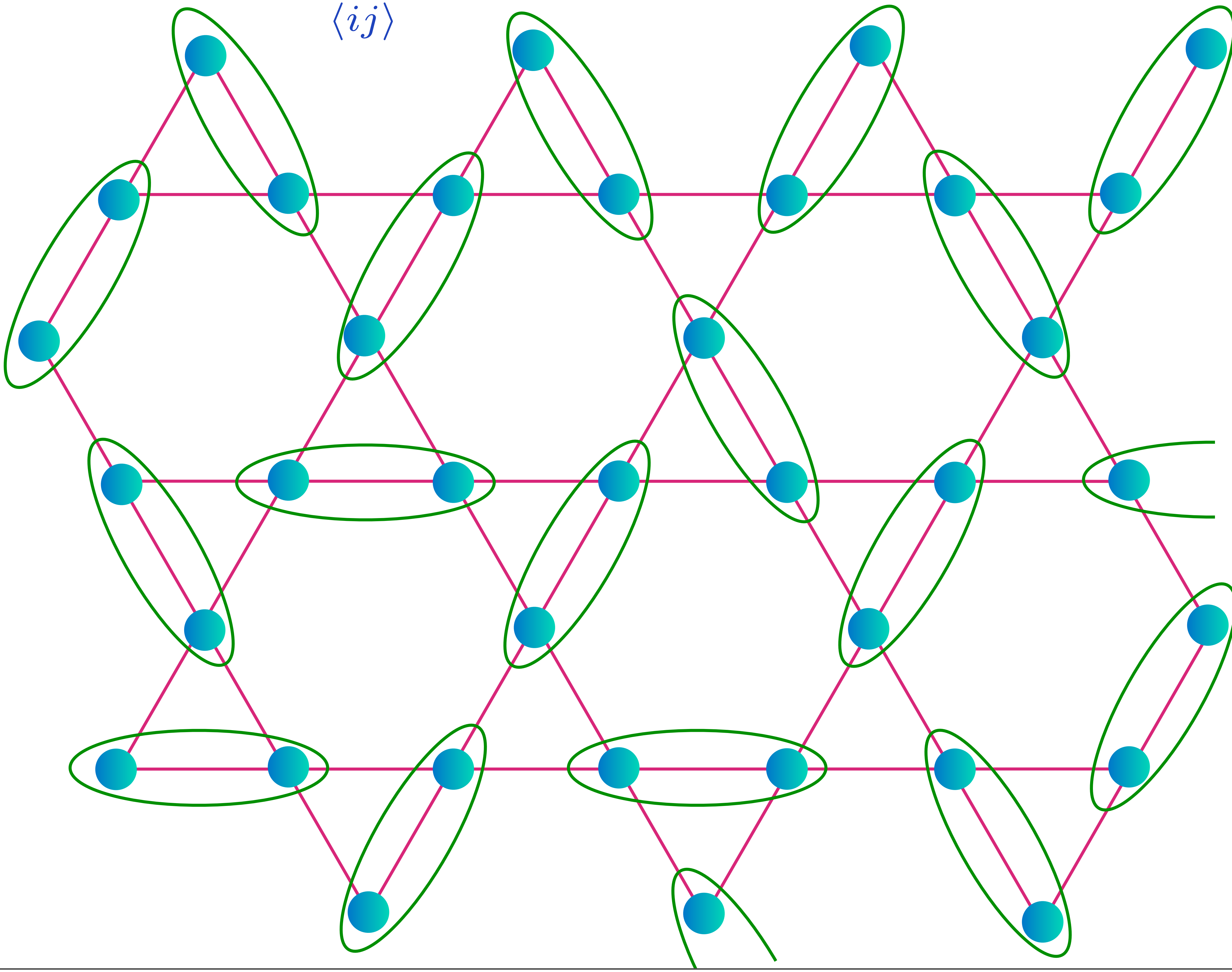

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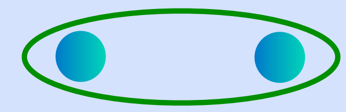

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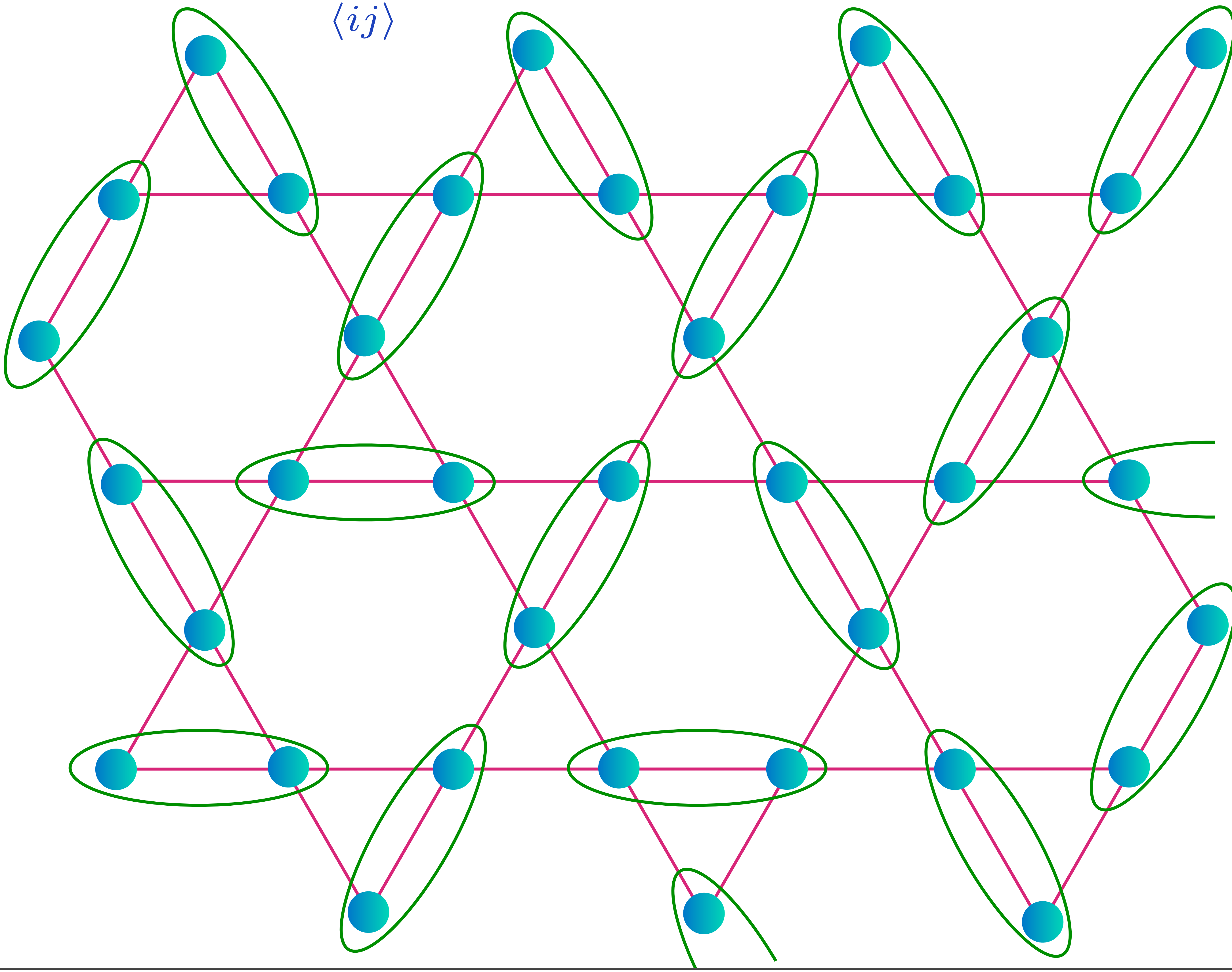

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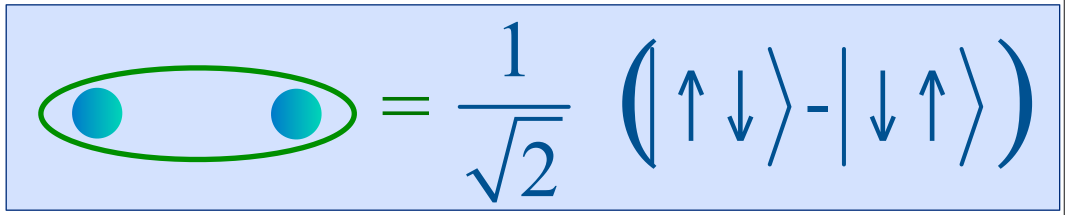

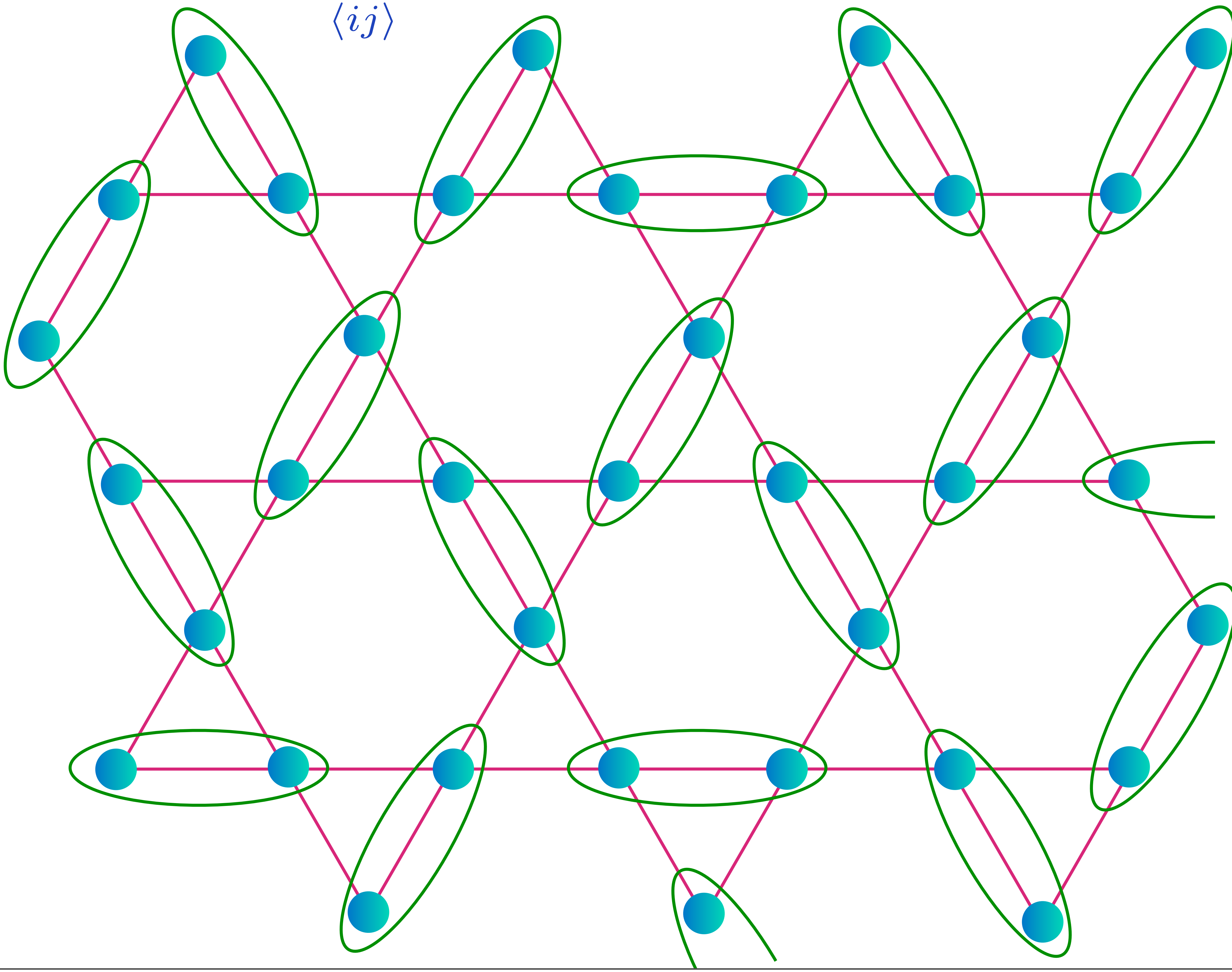

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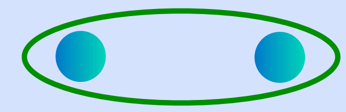

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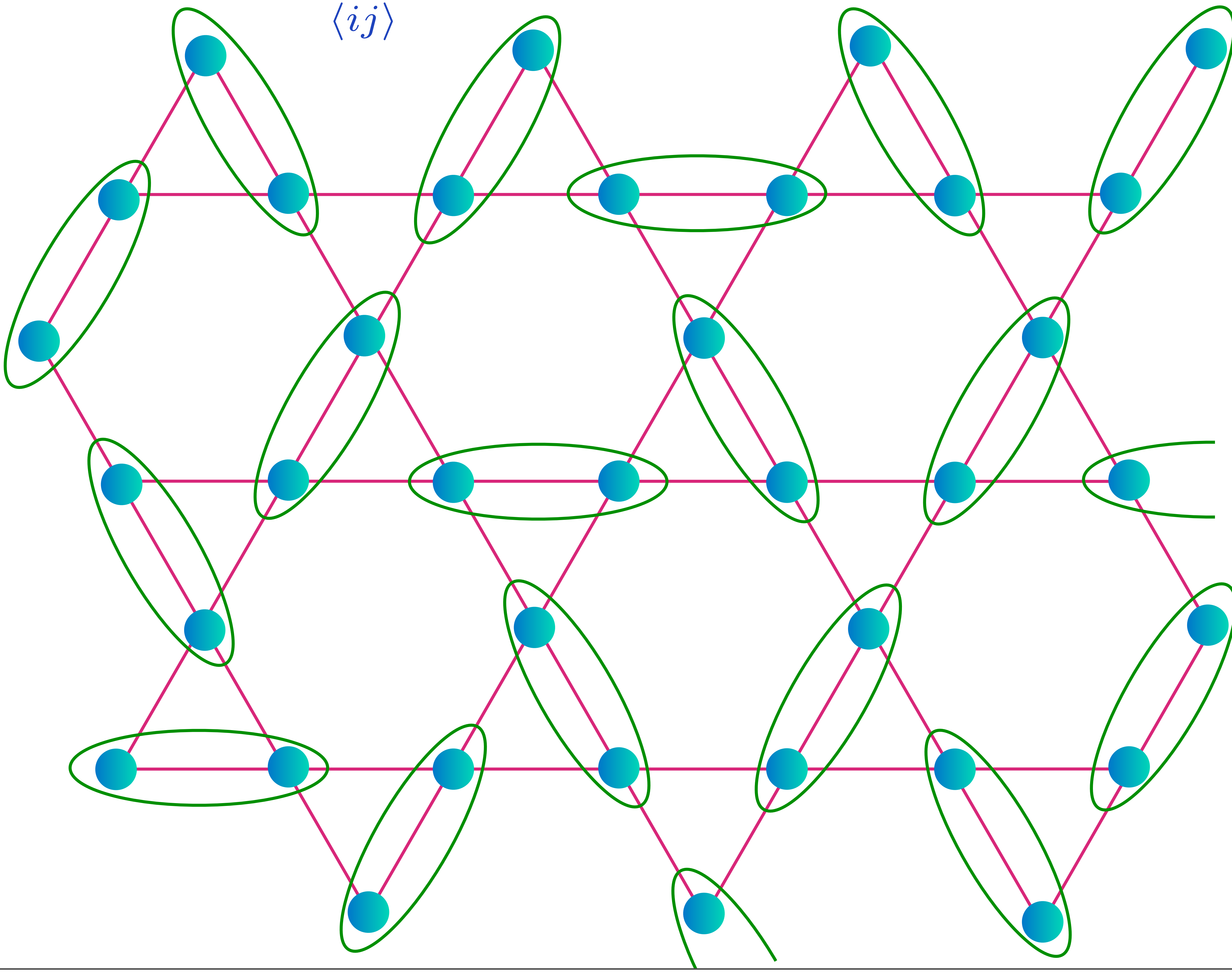

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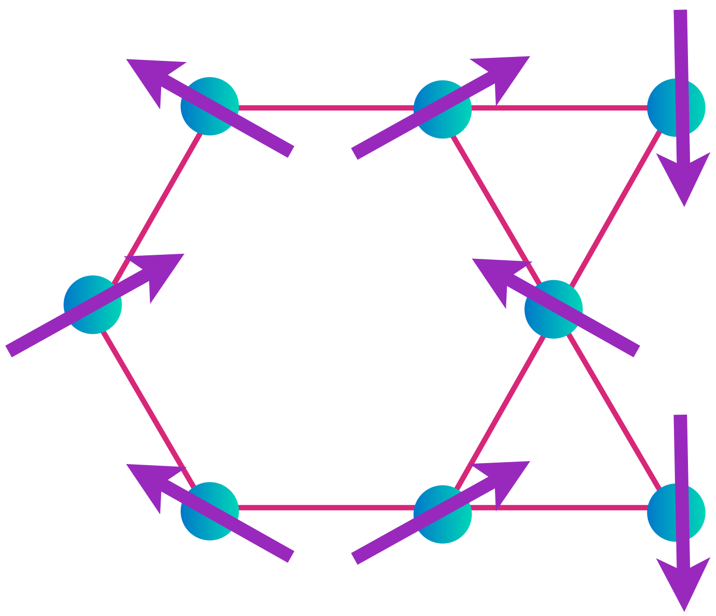


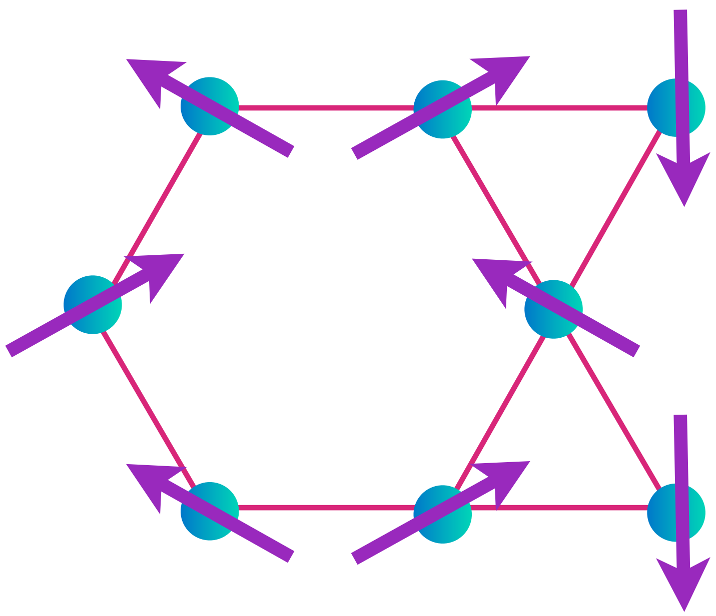
non-collinear Néel state

Quantum "disordered" state with exponentially decaying spin correlations.

s_c s

S. Sachdev, *Phys. Rev. B* **45**, 12377 (1992)

Mott insulator: kagome antiferromagnet



non-collinear Néel state

Quantum "disordered" state with exponentially decaying spin correlations.

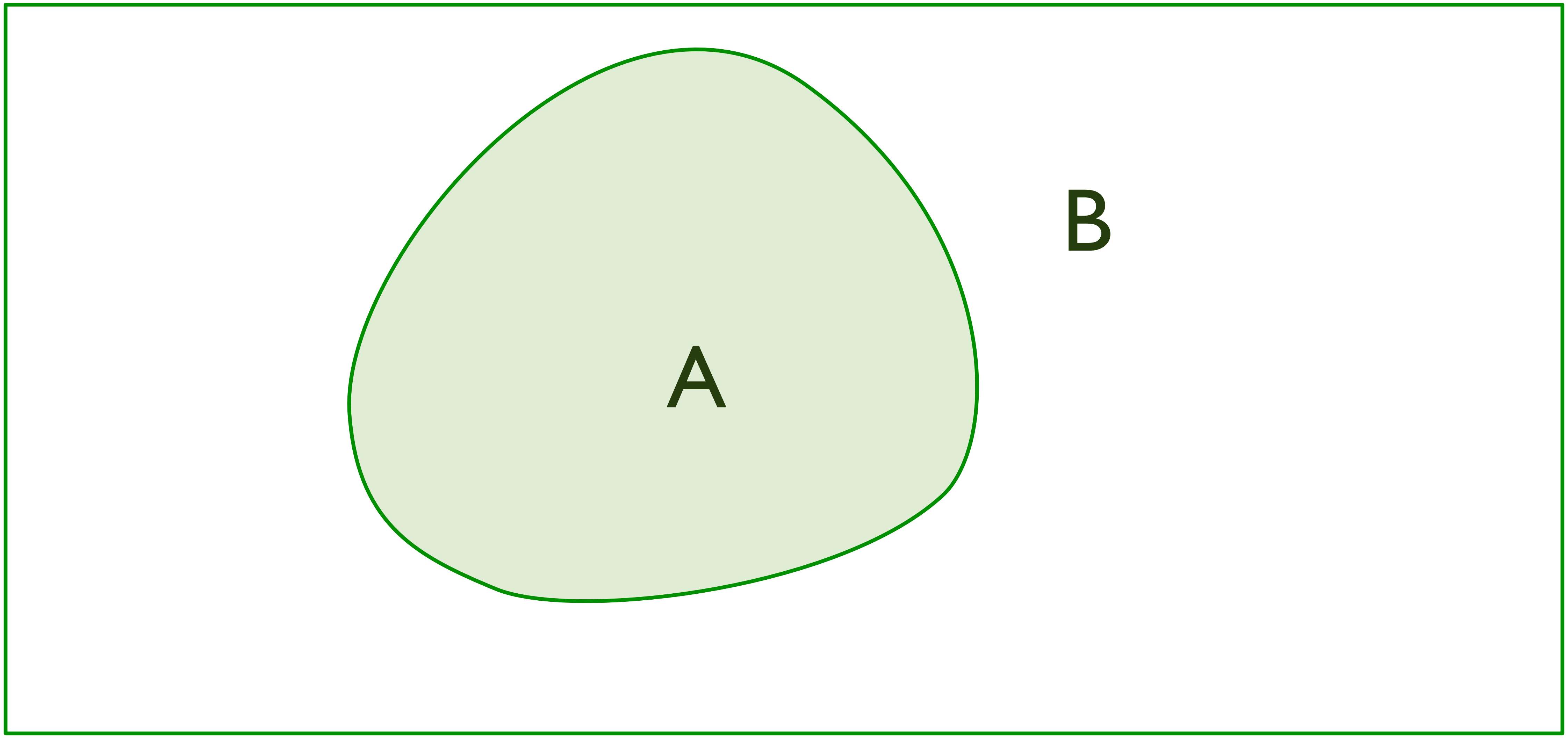
Spin liquid with topological features described by a $\mathbb{Z}_2$ gauge theory, or (equivalently) a doubled Chern-Simons field theory.

s_c $\qquad$ s

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

M. Freedman, C. Nayak, K. Shtengel, K. Walker, and Z. Wang, *Annals of Physics* **310**, 428 (2004).

Entanglement in the Z_2 spin liquid ground state

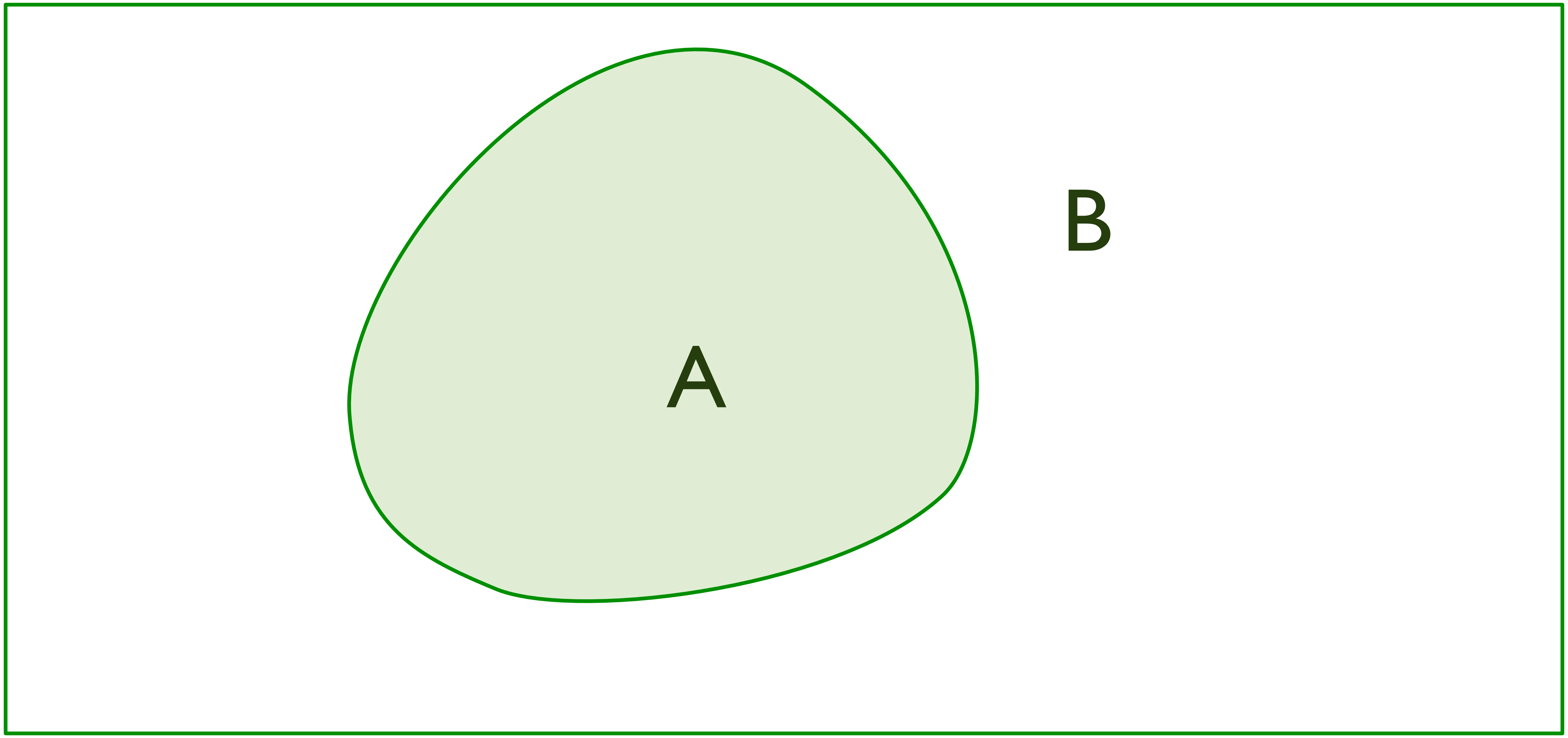


$$|\Psi\rangle \quad \Rightarrow \quad \text{Ground state of entire system,}$$
$$\rho = |\Psi\rangle\langle\Psi|$$

$$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_E = -\text{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement in the Z_2 spin liquid ground state

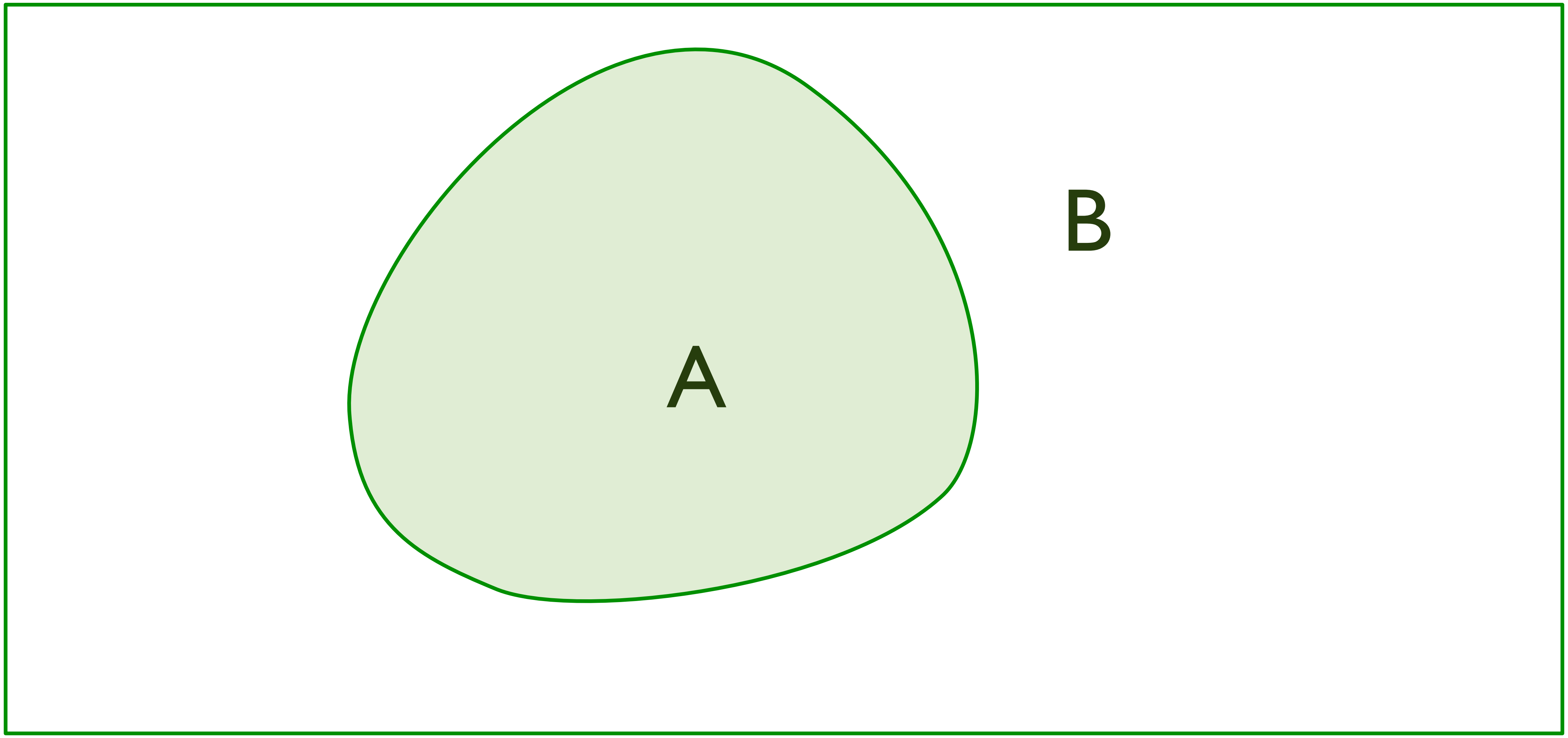


Entanglement entropy of a band insulator:

$$S_E = aP - b\exp(-cP)$$

where P is the surface area (perimeter) of the boundary between A and B.

Entanglement in the Z_2 spin liquid ground state



Entanglement entropy of a $\mathbb{Z}_2$ spin liquid:

$$S_E = aP - \ln(2)$$

where P is the surface area (perimeter) of the boundary between A and B.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
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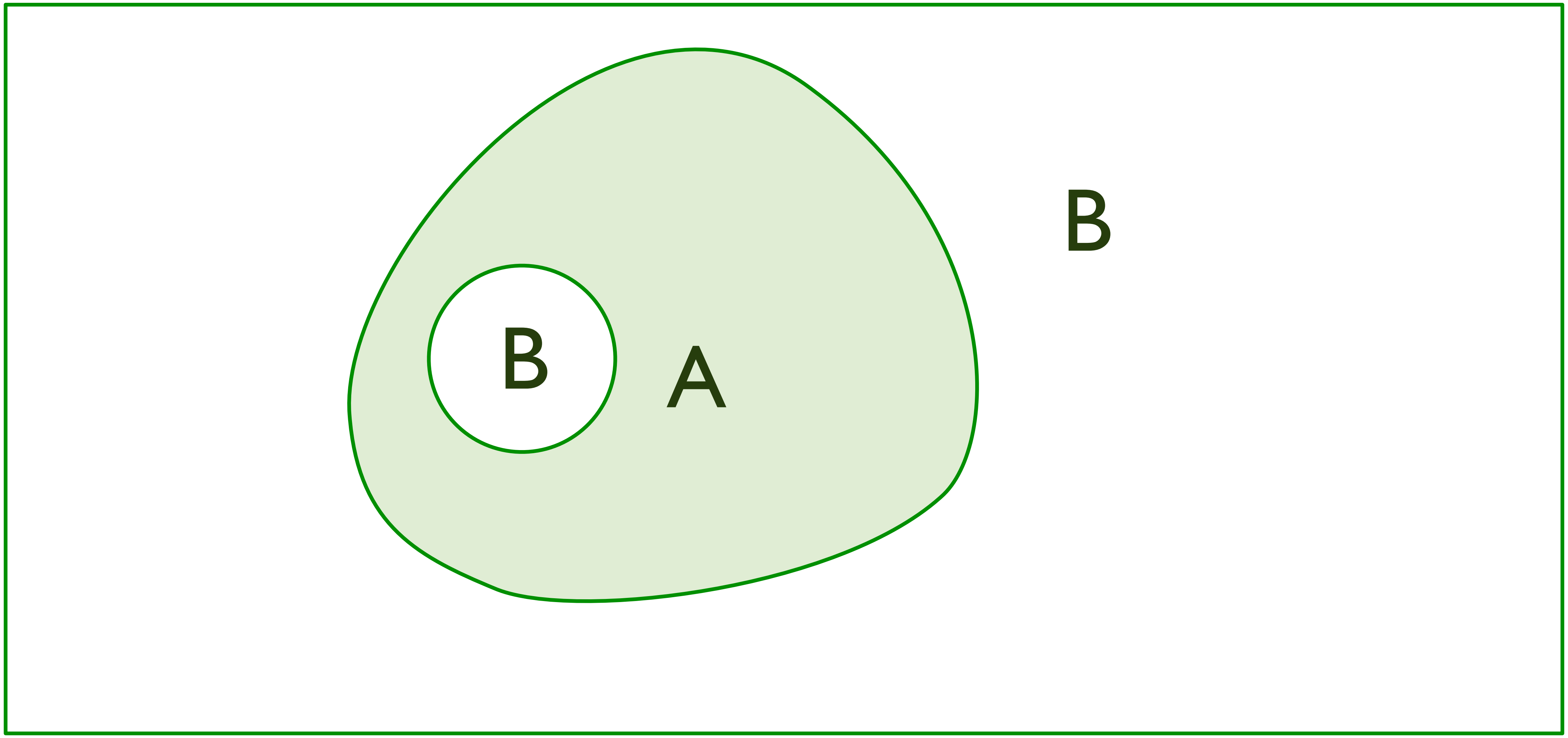
Entanglement in the Z_2 spin liquid ground state



Entanglement entropy of a $\mathbb{Z}_2$ spin liquid:

$$S_E = aP - \ln(4)$$

where P is the surface area (perimeter) of the boundary between A and B.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
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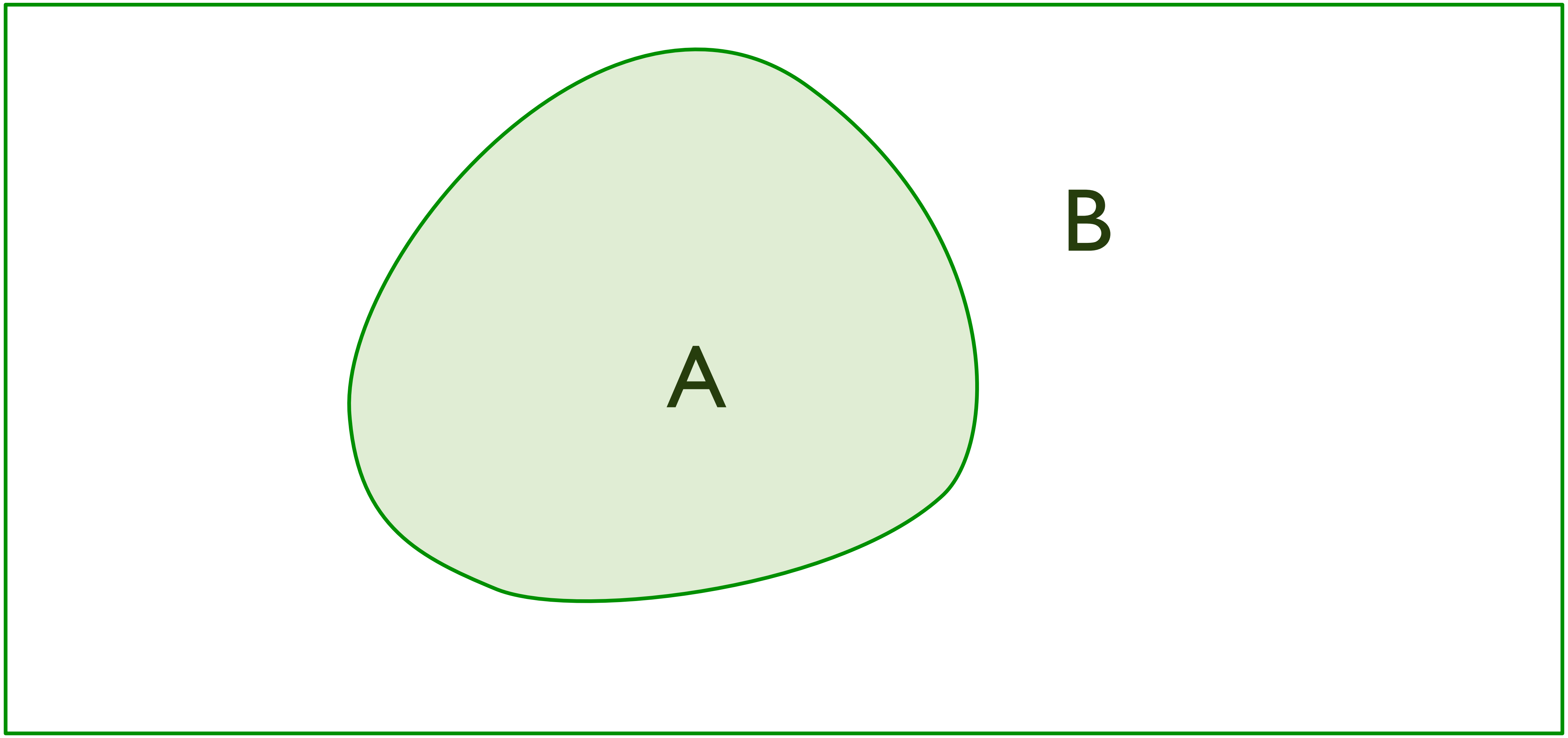

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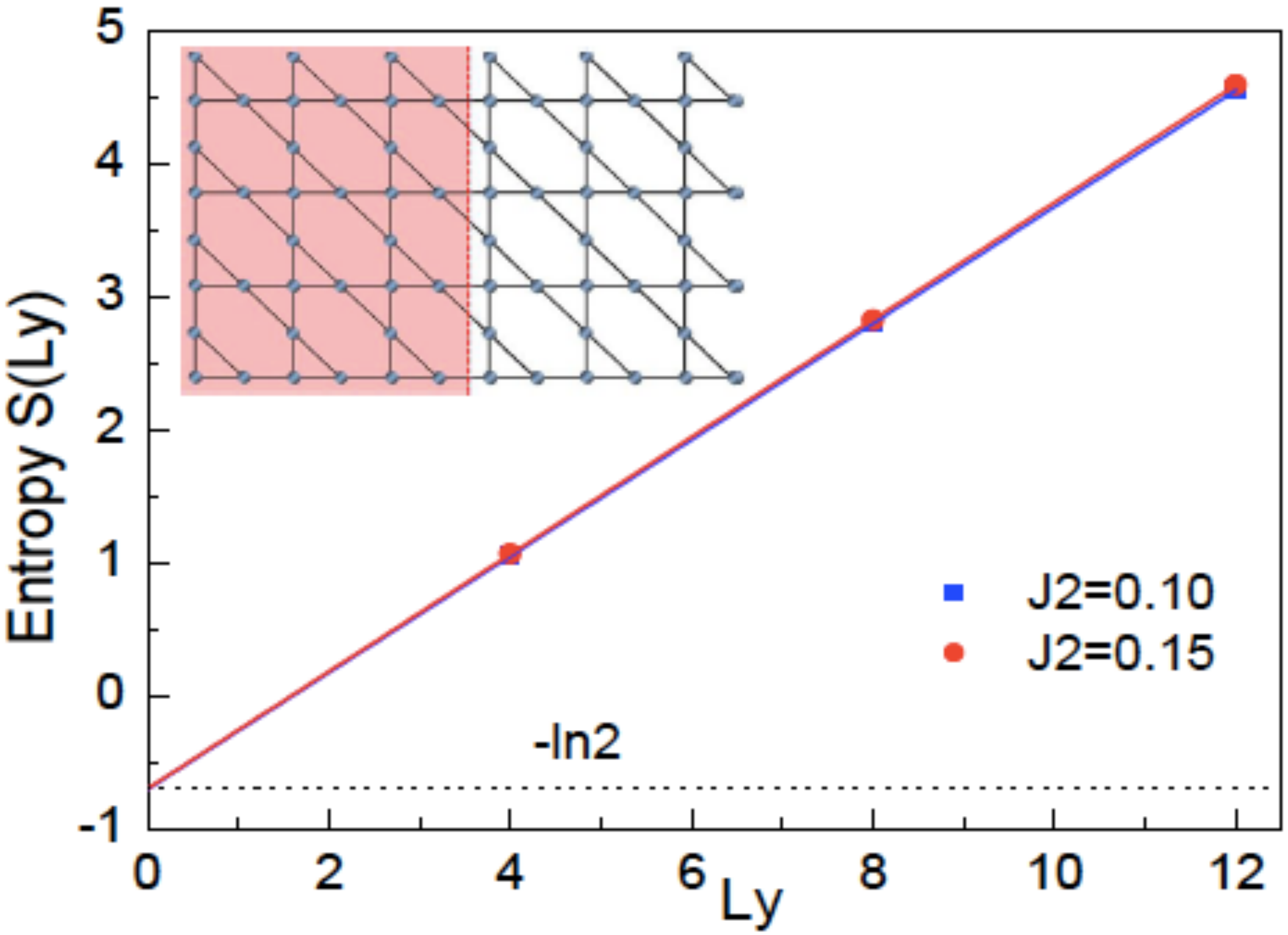
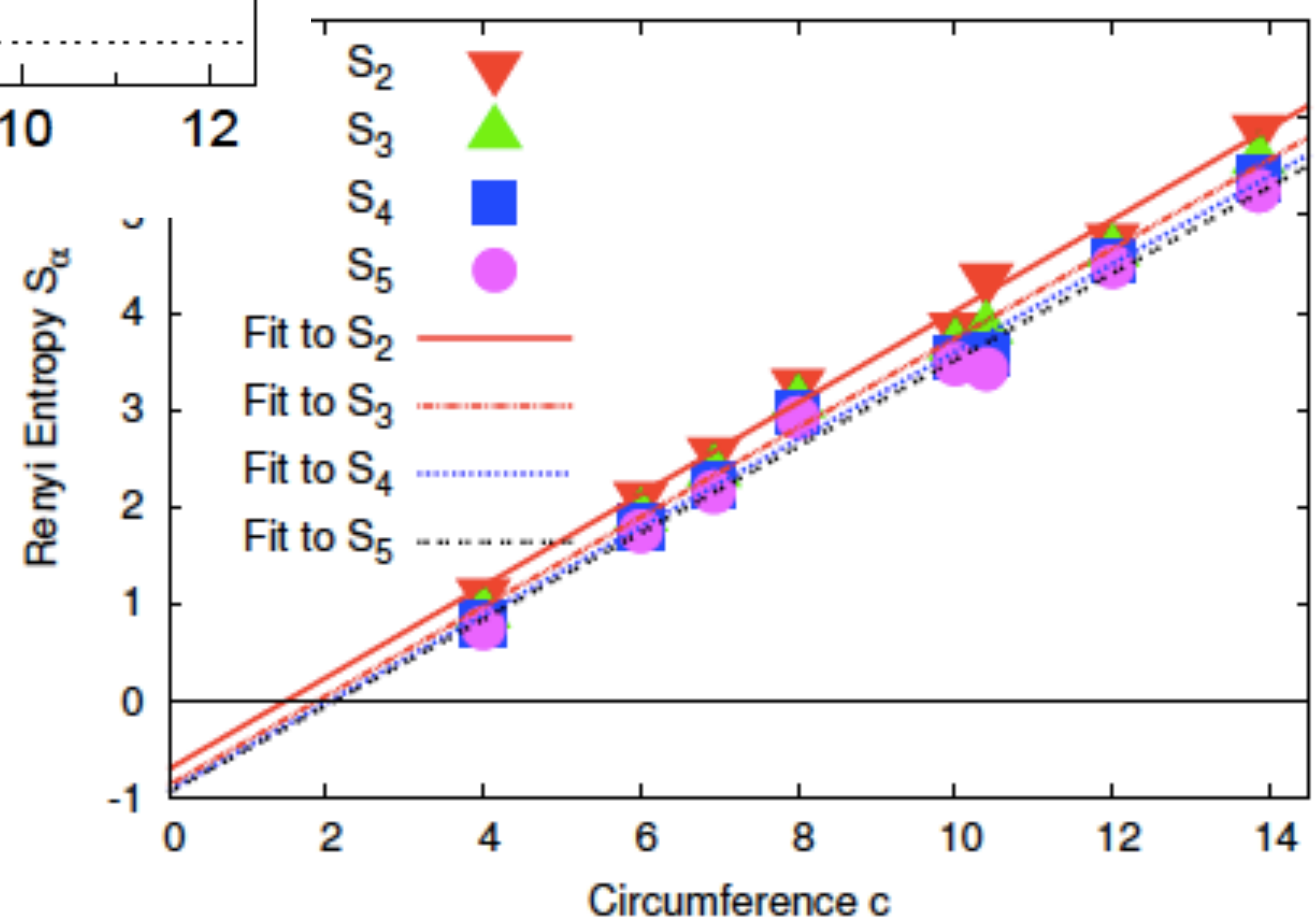
Kagome antiferromagnet



S. Depenbrock, I. P. McCulloch, and U. Schollwoeck, arXiv:1205.4858

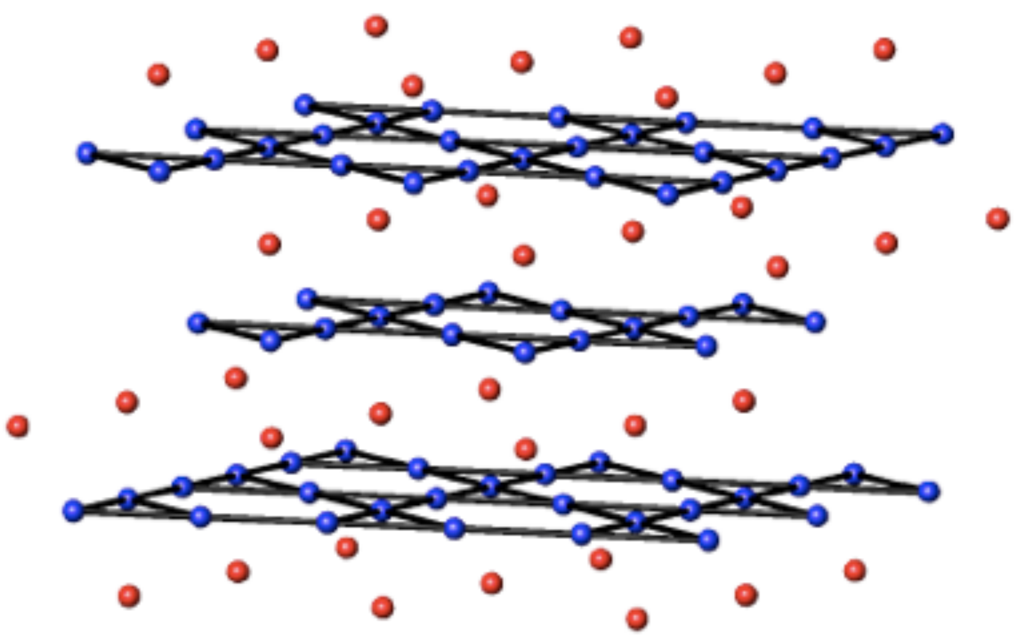
Hong-Chen Jiang, Z. Wang, and L. Balents, arXiv:1205.4289

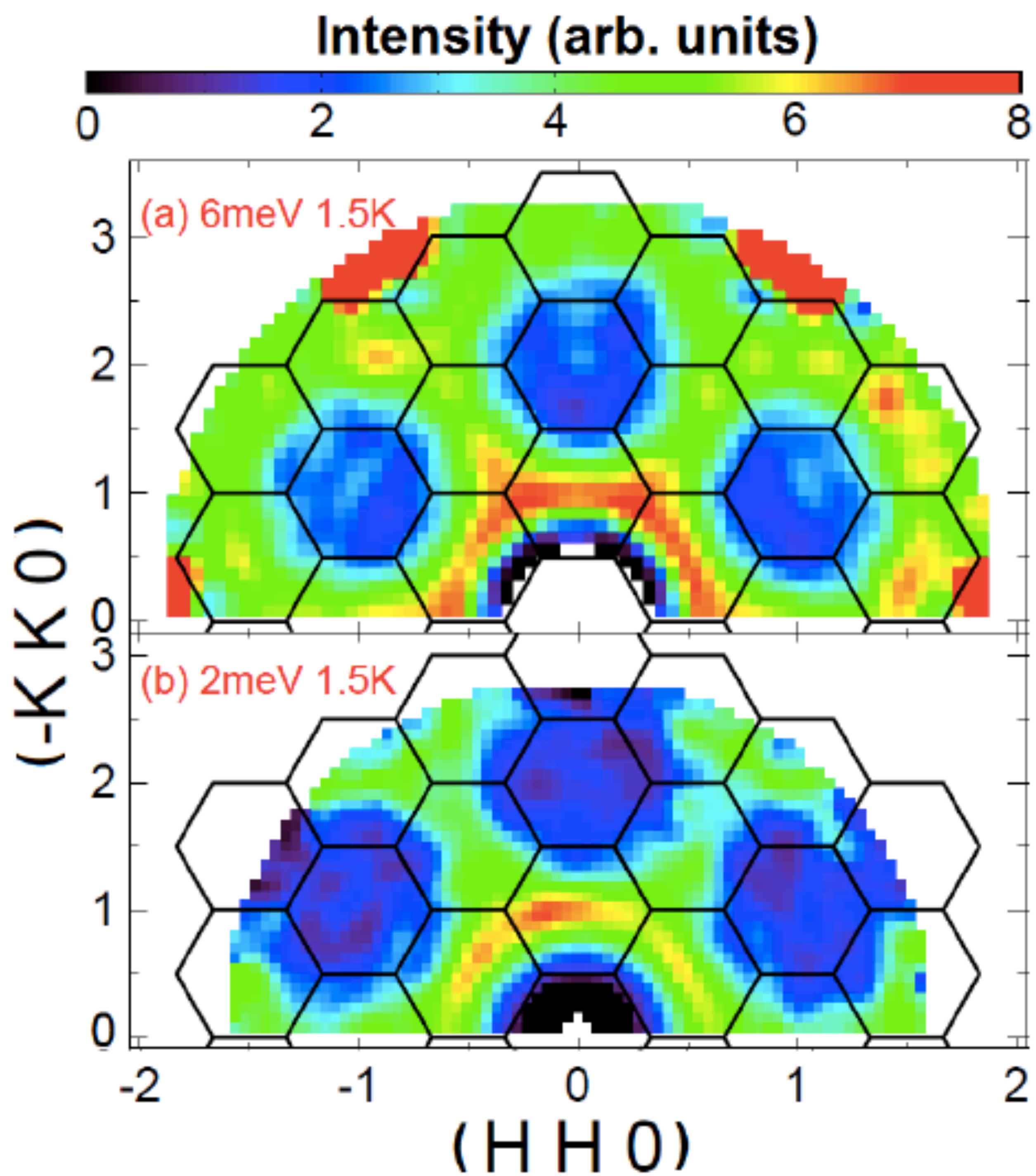


Kagome antiferromagnet: evidence for spinons

Young Lee,
APS meeting, March 2012

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





"Complex entangled" states of quantum matter in *d* spatial dimensions

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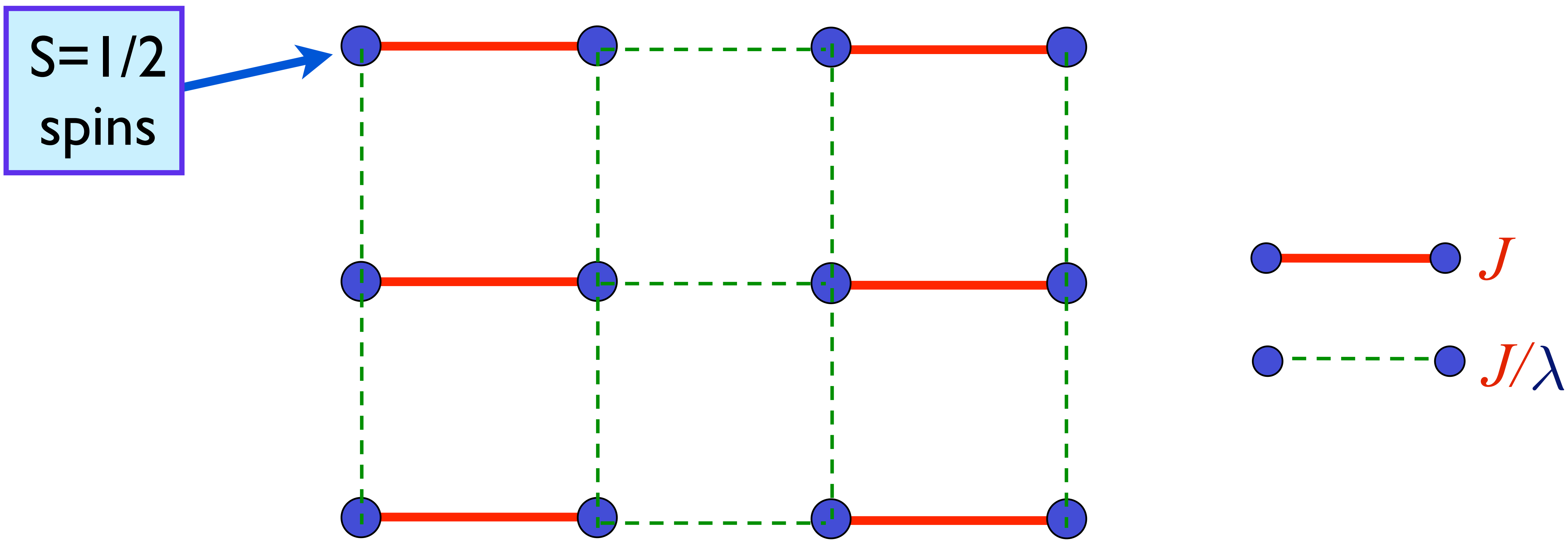
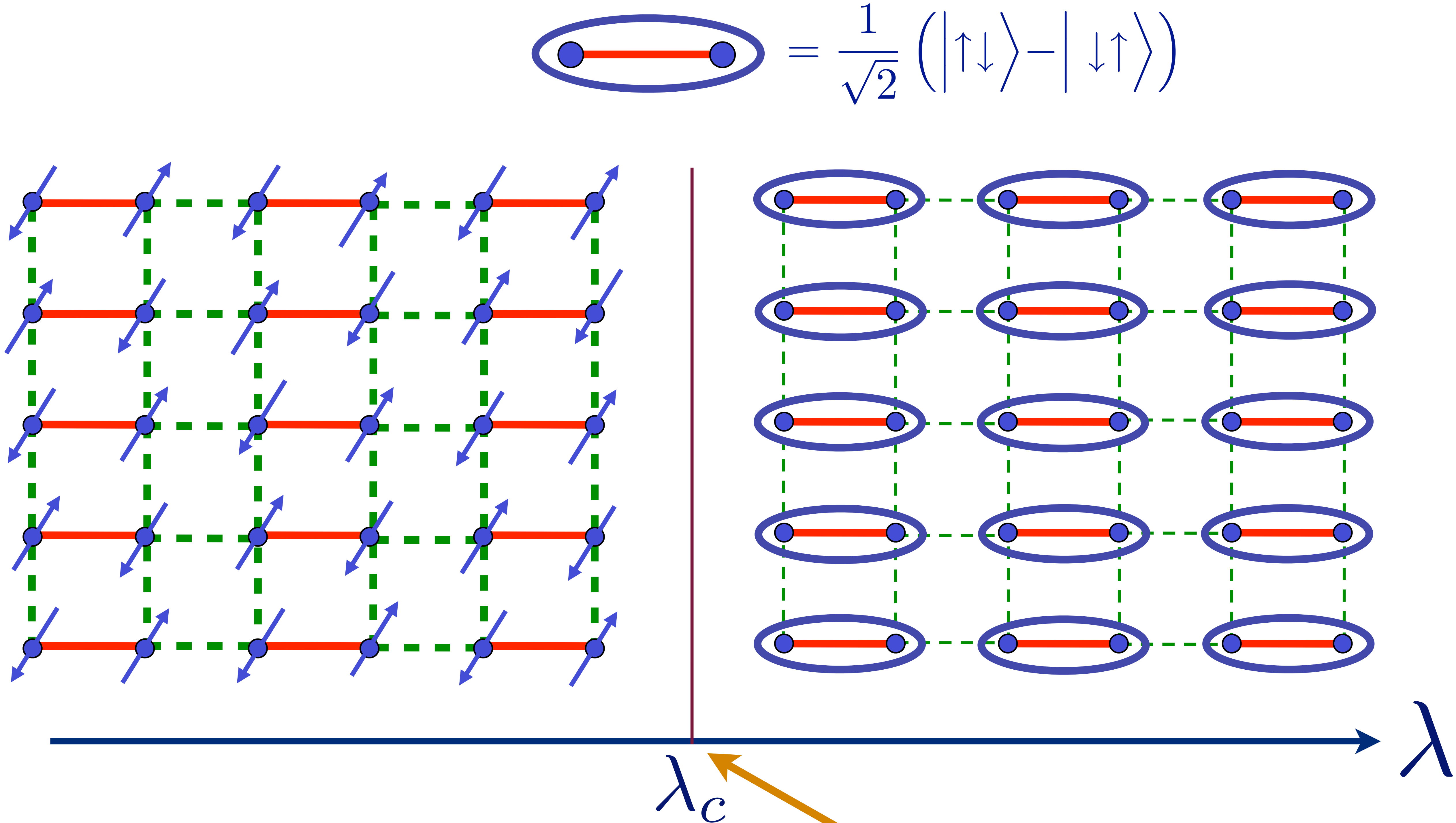
Spinning electrons localized on a square lattice

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

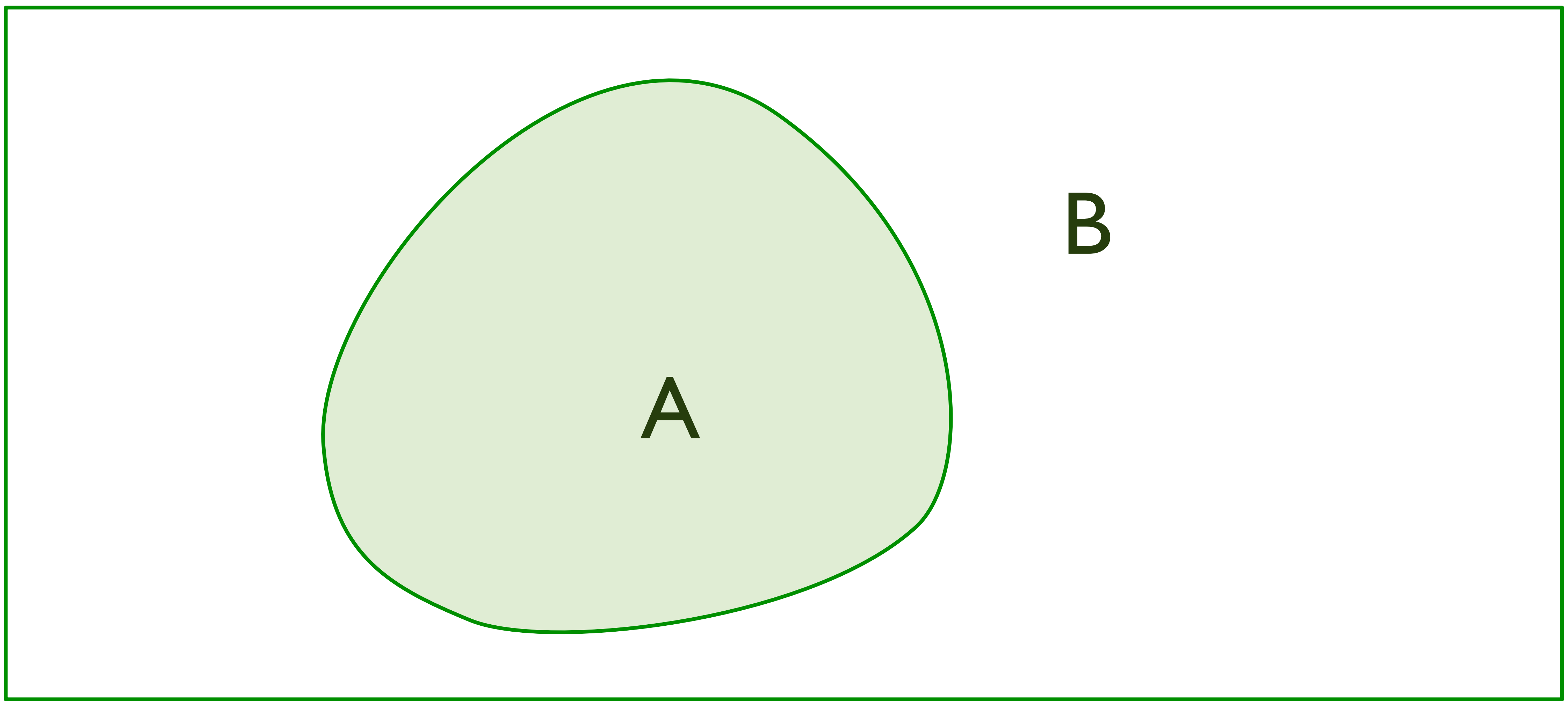


Quantum critical point described by a CFT3 (O(3) Wilson-Fisher)

A. W. Sandvik and D. J. Scalapino, Phys. Rev. Lett. **72**, 2777 (1994).

Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.

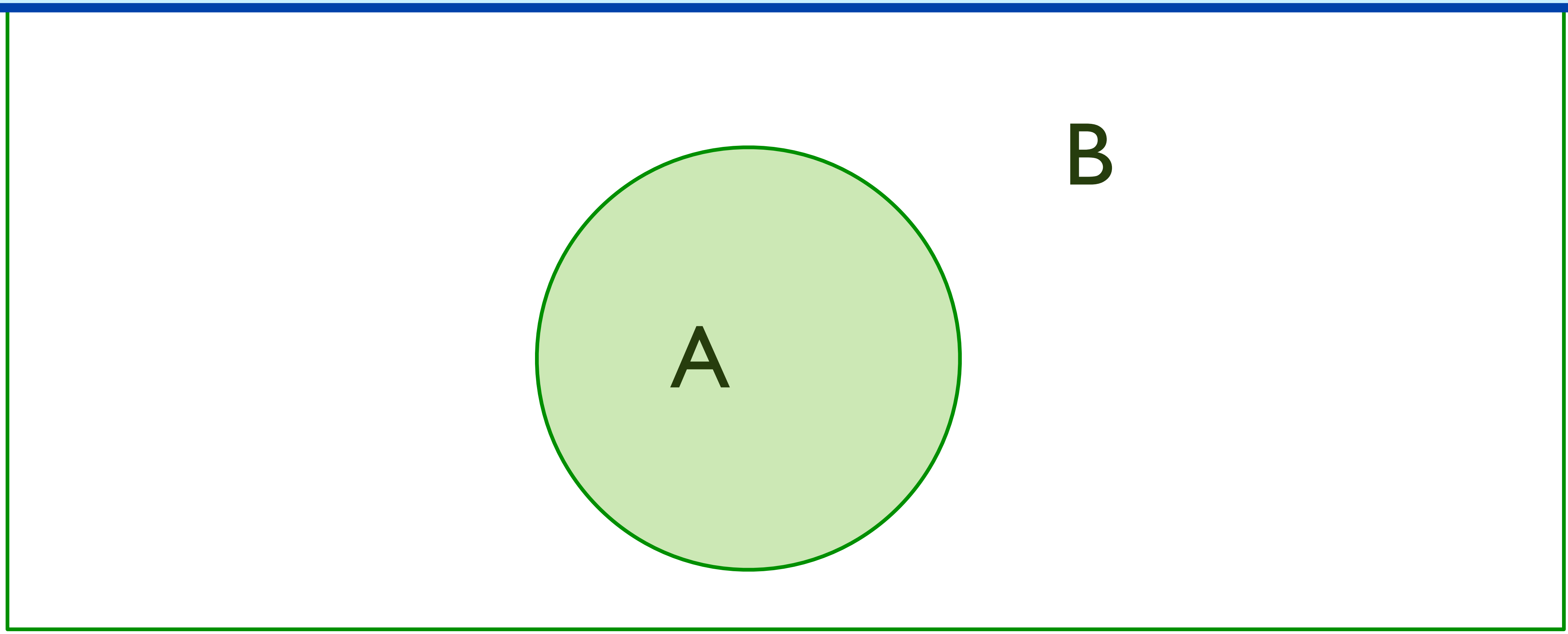


M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aP - \gamma$, where γ is a shape-dependent universal number associated with the CFT3.

- When A is a circle, $e^{-\gamma}$ = partition function of CFT3 on S^3.



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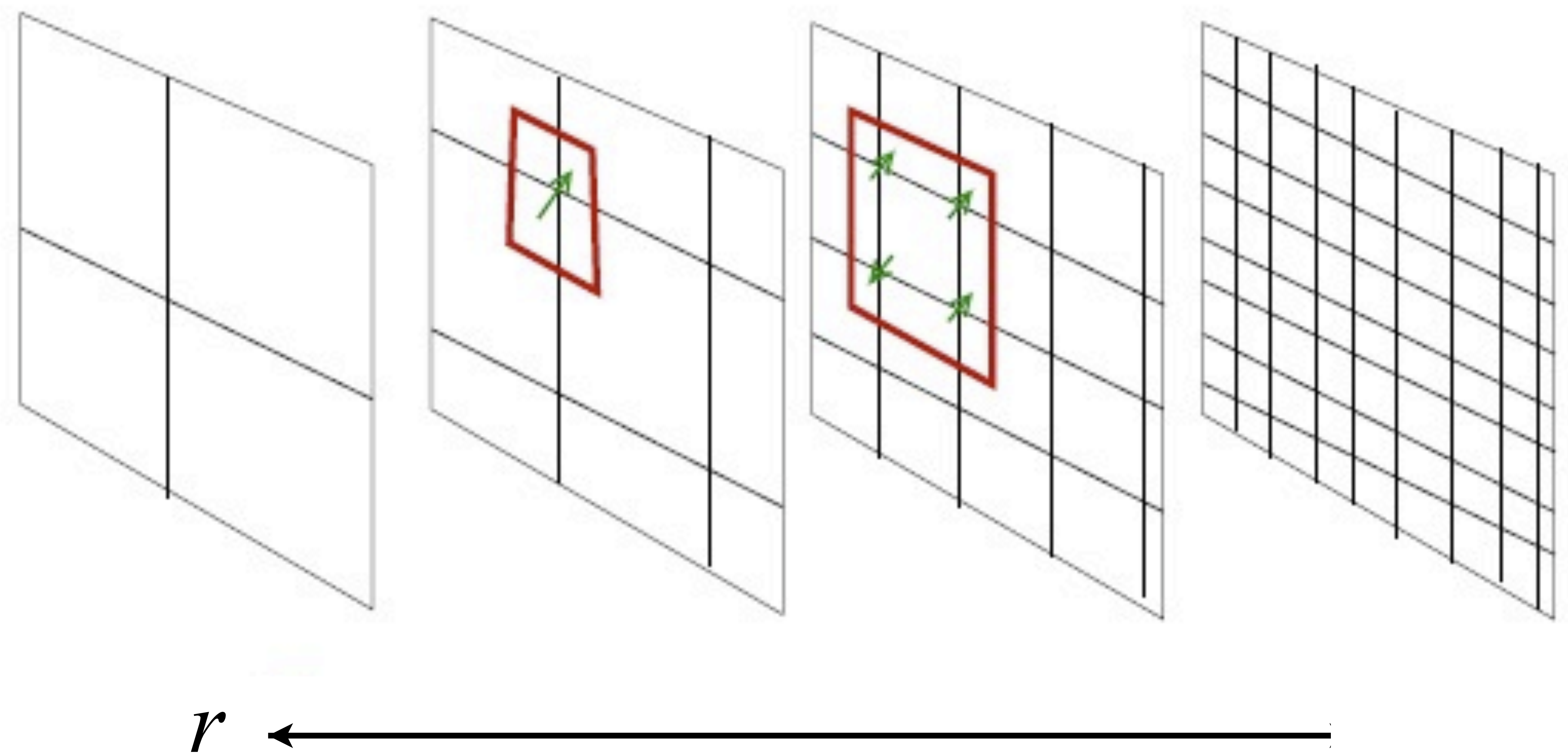

Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d + 2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

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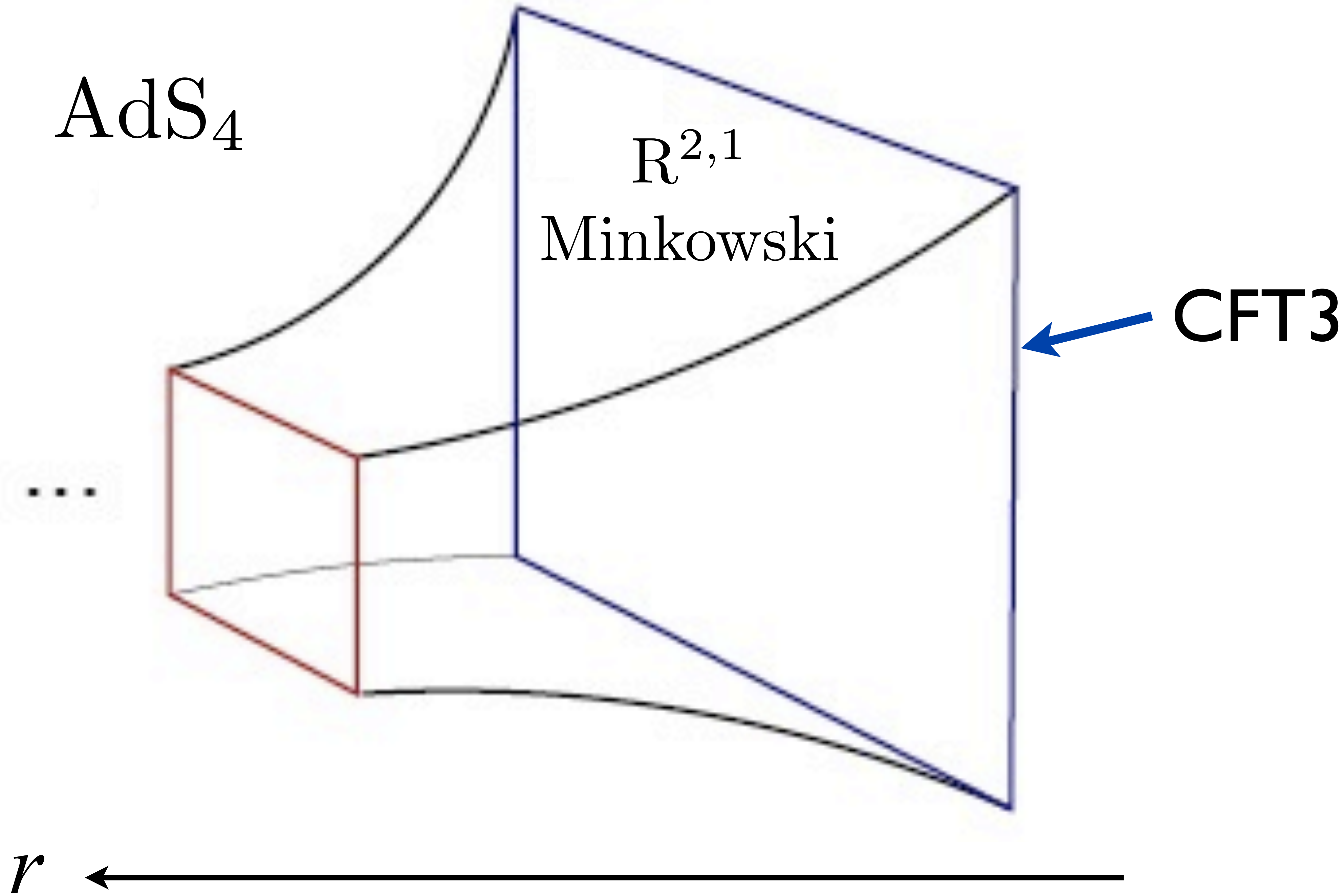
$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

AdS/CFT correspondence



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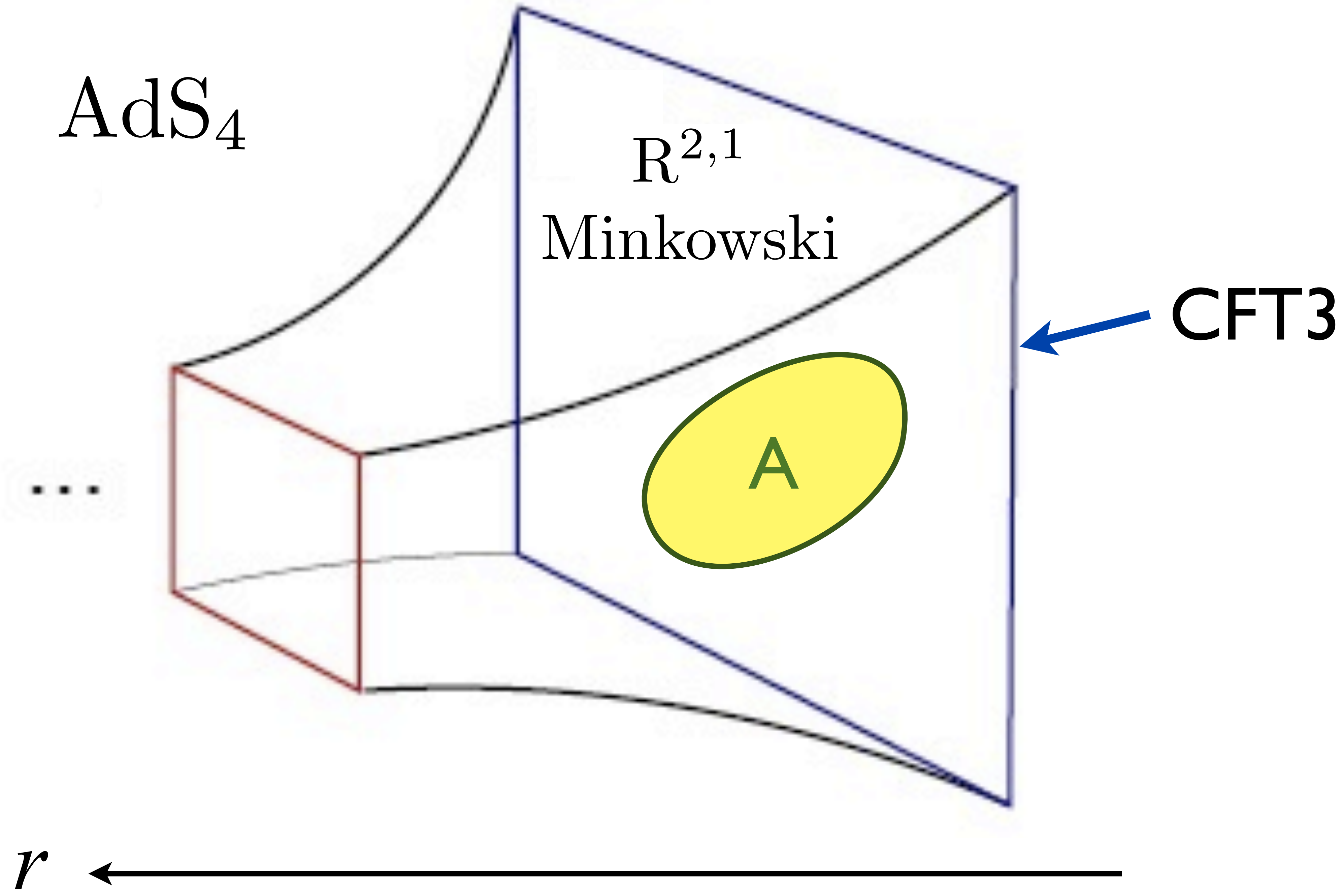

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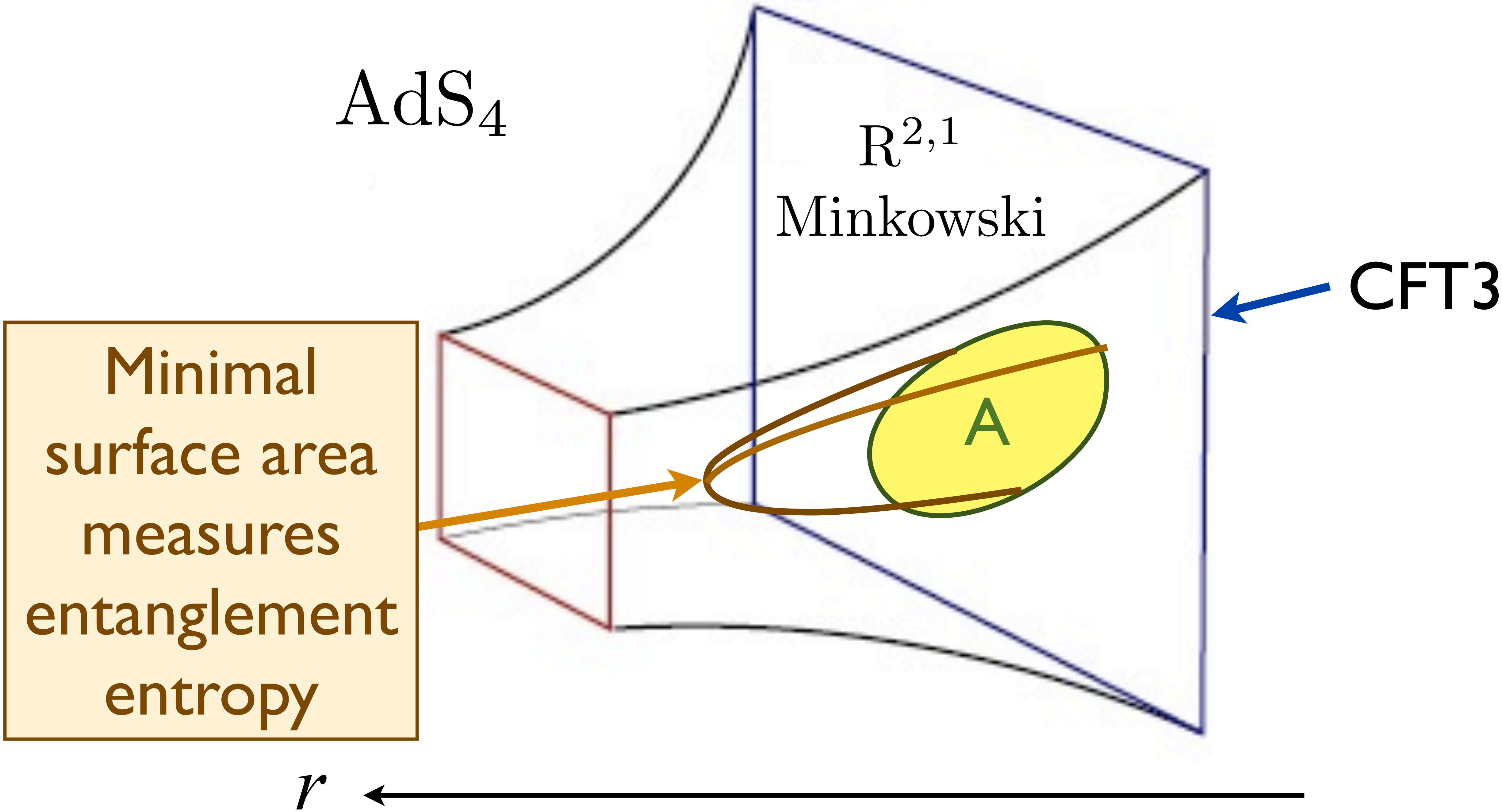

- Minimal surface area yields $S_E = aP - \gamma$, where γ is a shape-dependent universal number.

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

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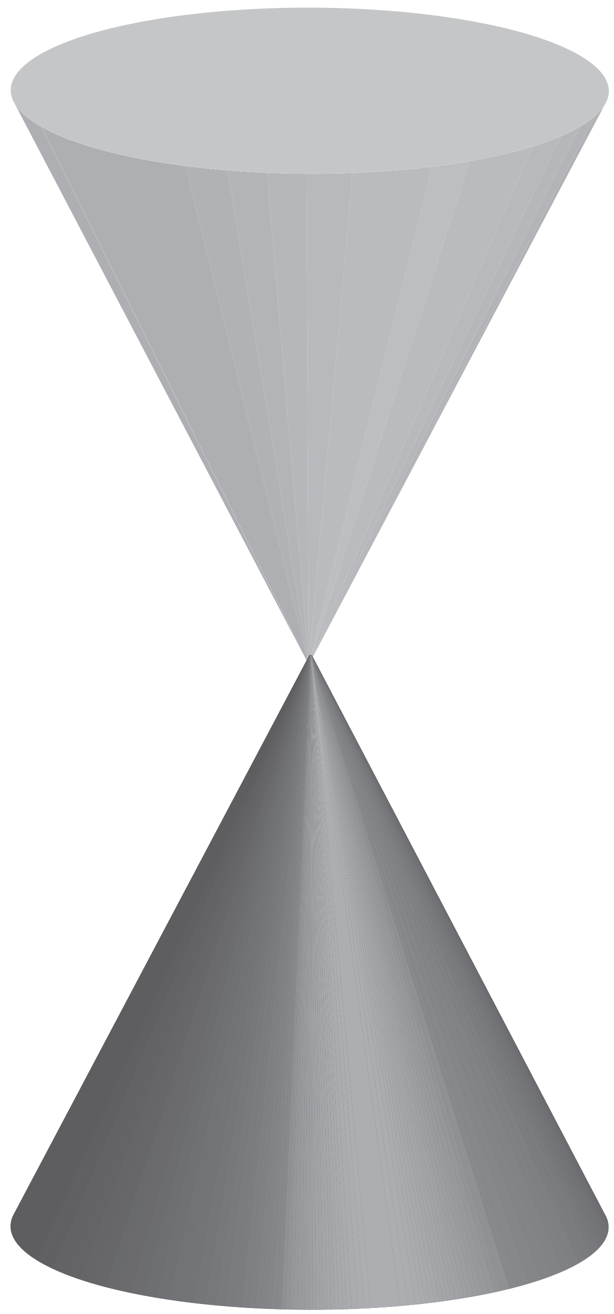
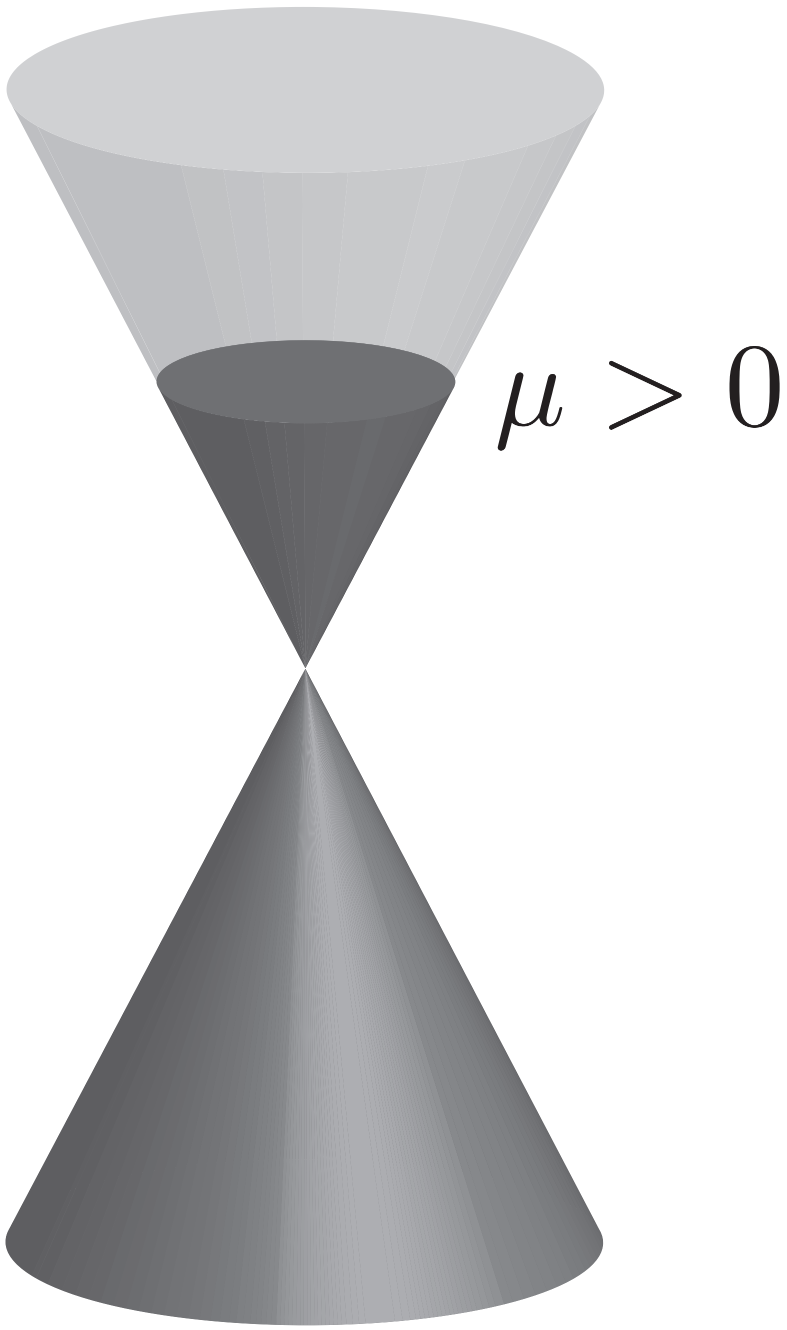
Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

- Describe *zero temperature* phases where $d\langle \mathcal{Q} \rangle / d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu \mathcal{Q}$.



Conformal quantum matter



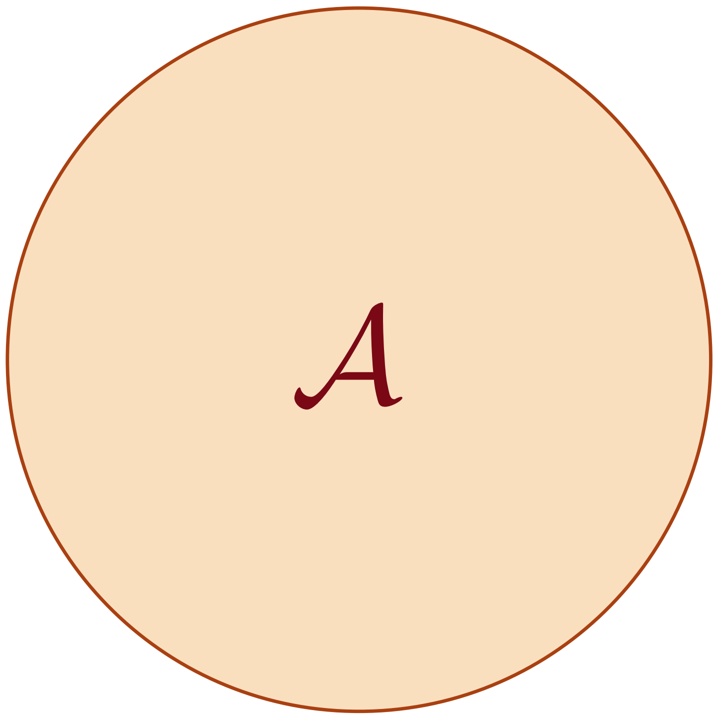
Compressible quantum matter

Compressible quantum matter

A. Field theory

B. Holography

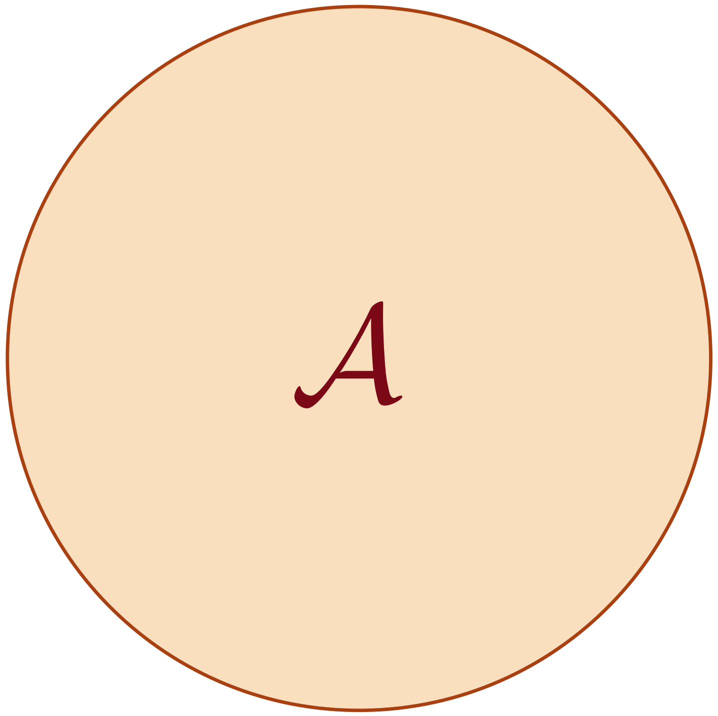
The Fermi liquid



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma$$

$$+ \text{ short-range 4-Fermi terms}$$

The Fermi liquid

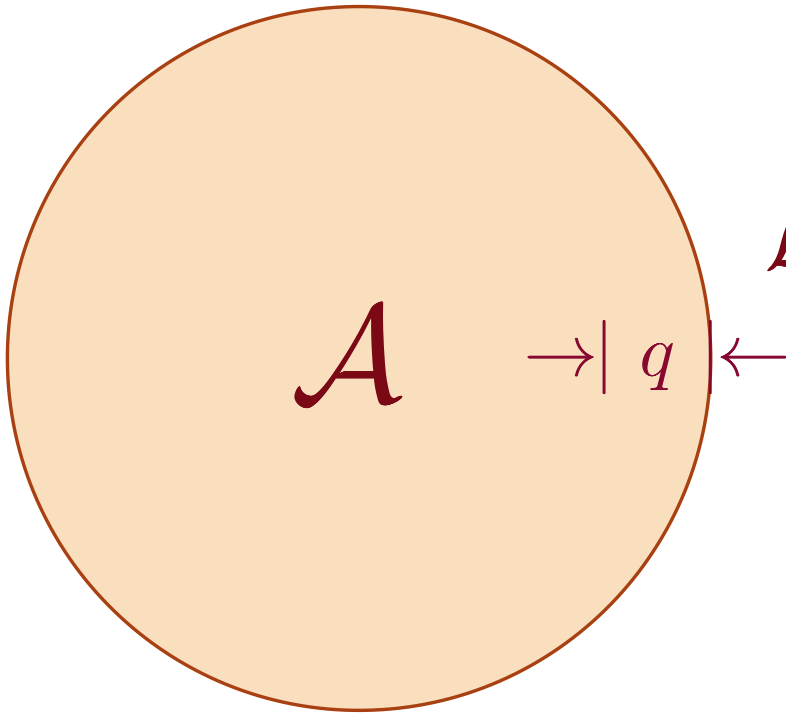


$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma$$

$$+ \text{short-range 4-Fermi terms}$$

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

The Fermi liquid

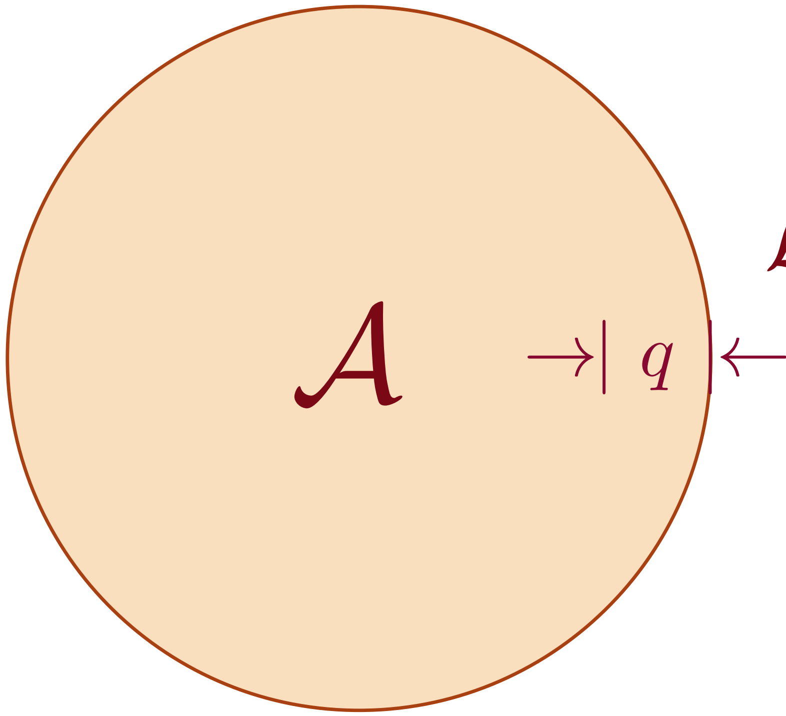


$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma$$

$+$ short-range 4-Fermi terms

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density
- Particle and hole of excitations near the Fermi surface with energy $\omega \sim |q|$.

The Fermi liquid

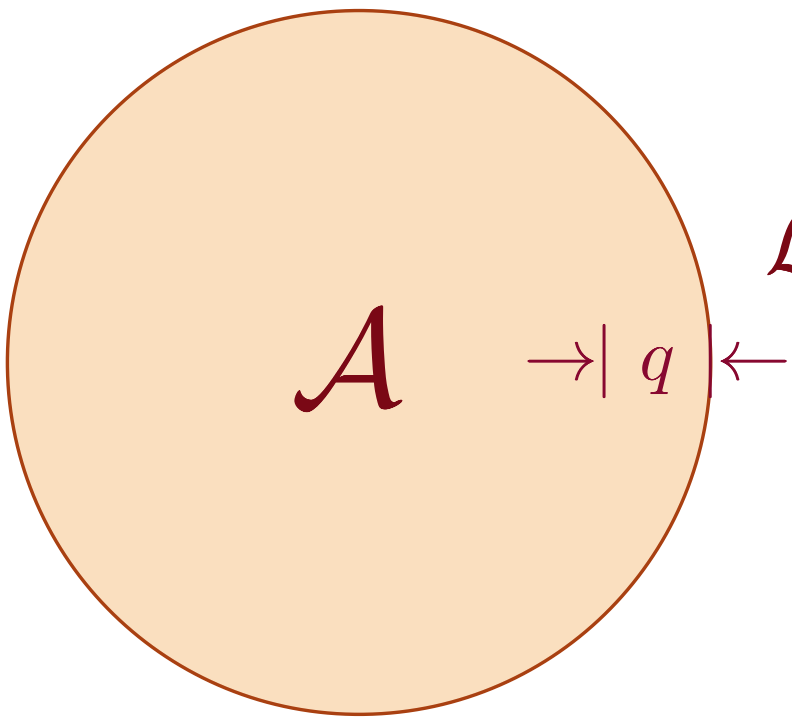


$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma$$

$$+ \text{ short-range 4-Fermi terms}$$

- Fermion Green's function $G_f^{-1} = \omega - v_F q + i\mathcal{O}(\omega^2, q^2)$.

The Fermi liquid

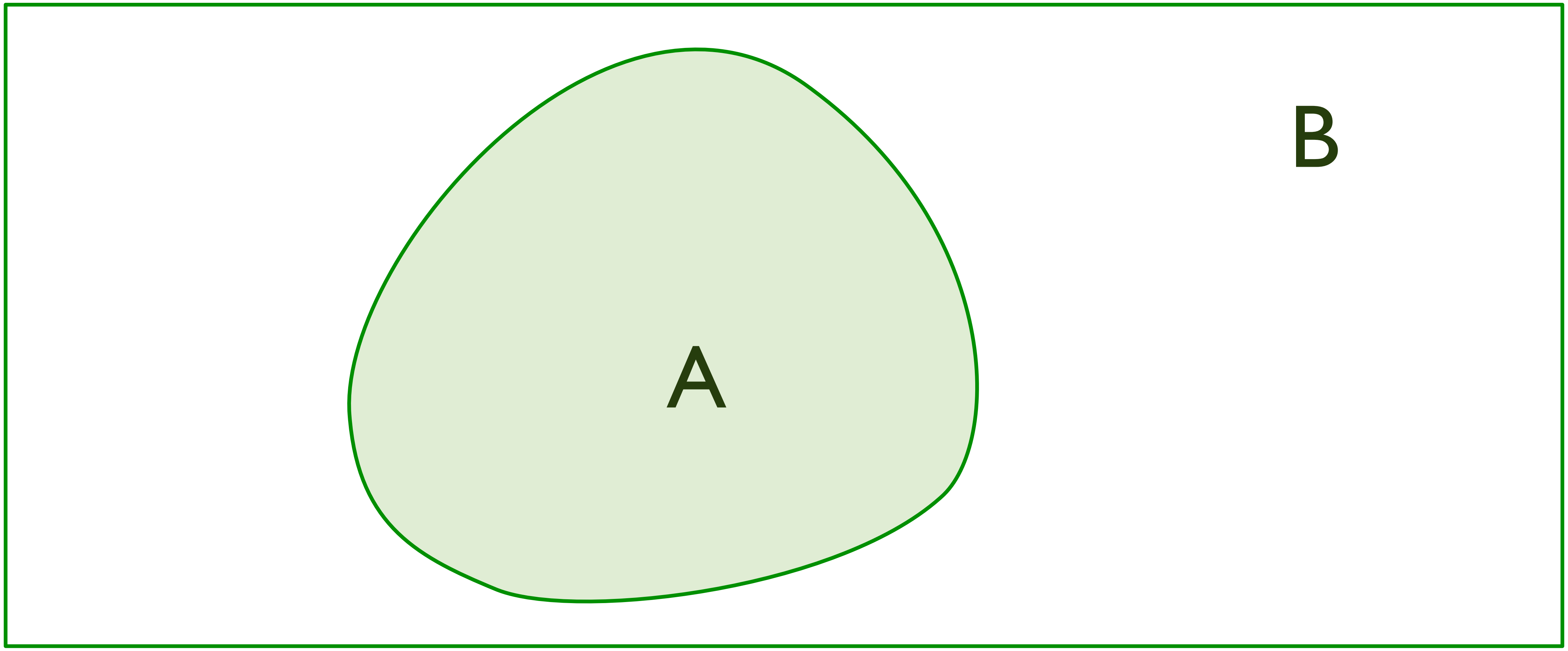


$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma$$

$$+ \text{ short-range 4-Fermi terms}$$

- Fermion Green's function $G_f^{-1} = \omega - v_F q + i\mathcal{O}(\omega^2, q^2)$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\text{eff}}}$ with $d_{\text{eff}} = 1$.

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F, where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

Non-Fermi liquids

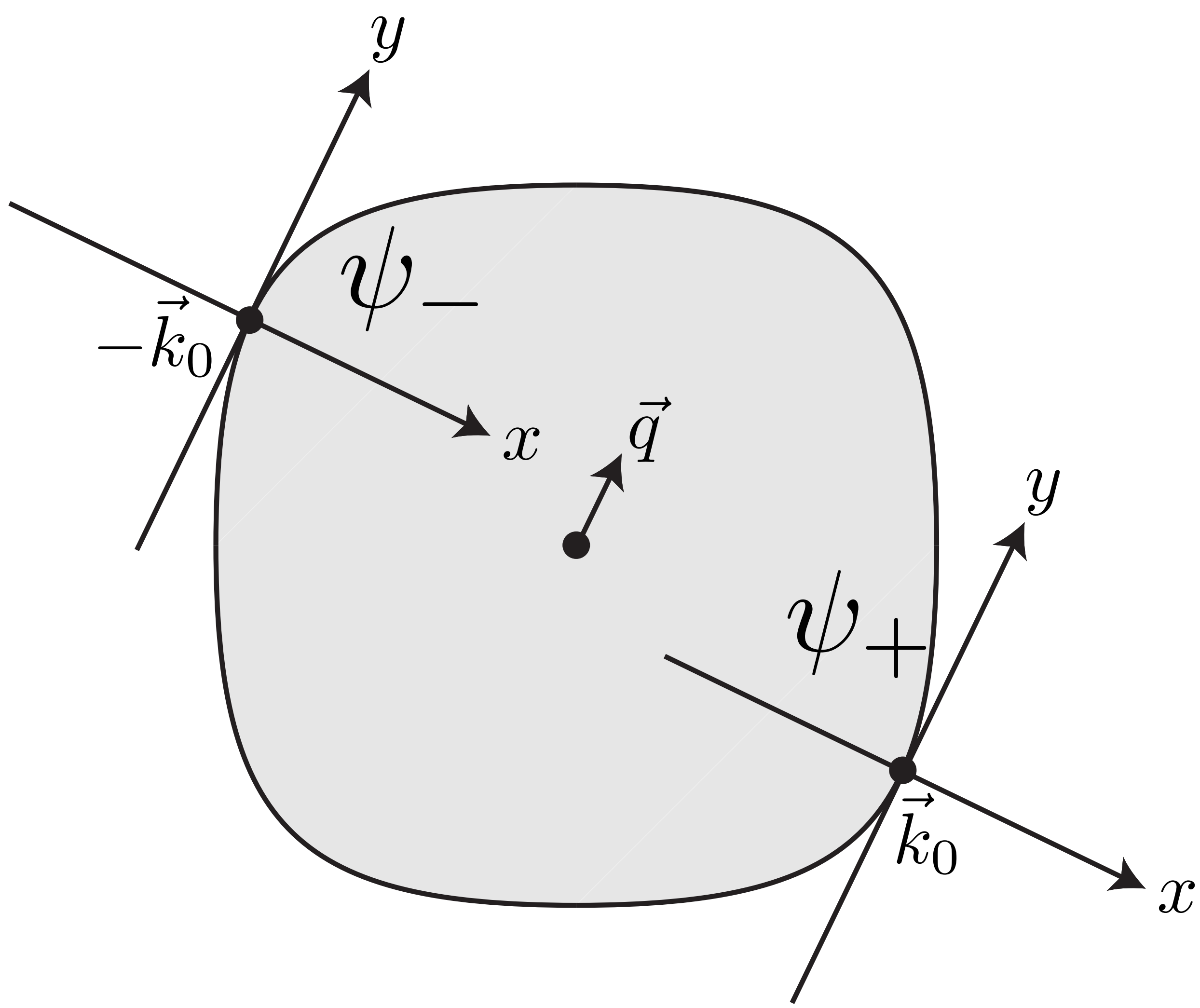
To obtain a compressible state which is not a Fermi liquid, take a Fermi surface in $d = 2$, and couple it to *any* gapless scalar field, ϕ, which has low energy excitations near $\mathbf{q} = 0$.

Non-Fermi liquids

To obtain a compressible state which is not a Fermi liquid, take a Fermi surface in $d = 2$, and couple it to *any* gapless scalar field, ϕ, which has low energy excitations near $\mathbf{q} = 0$. The field ϕ could represent

- ferromagnetic order
- breaking of point-group symmetry (Ising-nematic order)
- breaking of time-reversal symmetry
- circulating currents
- transverse component of an Abelian or non-Abelian gauge field.
- ...

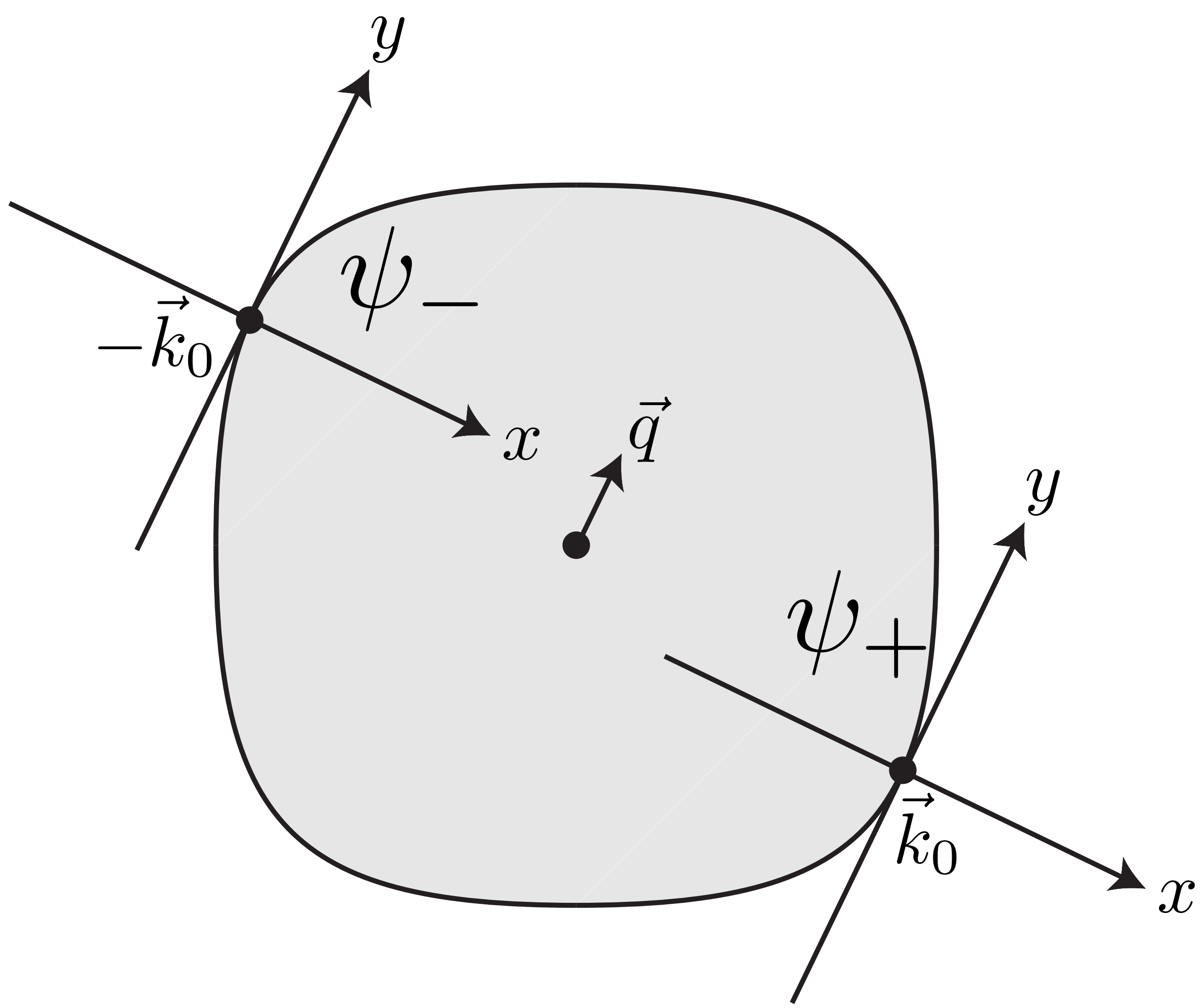
Non-Fermi liquids



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Non-Fermi liquids



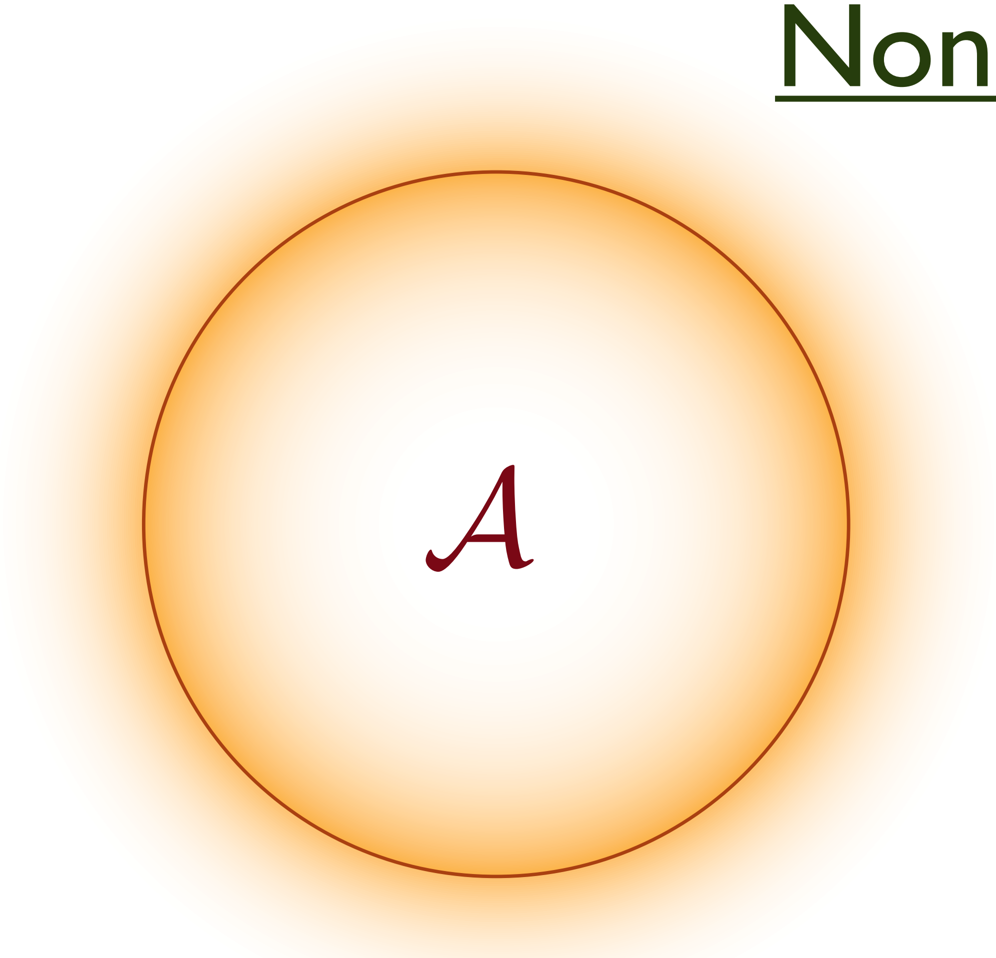
$$\mathcal{L}[\psi_\pm, \phi] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-g\phi \left(\psi_+^\dagger \psi_+ \pm \psi_-^\dagger \psi_-\right) + \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Non-Fermi liquids



- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density. Position of the Fermi surface defined by $G_f^{-1}(k = k_F, \omega = 0) = 0$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

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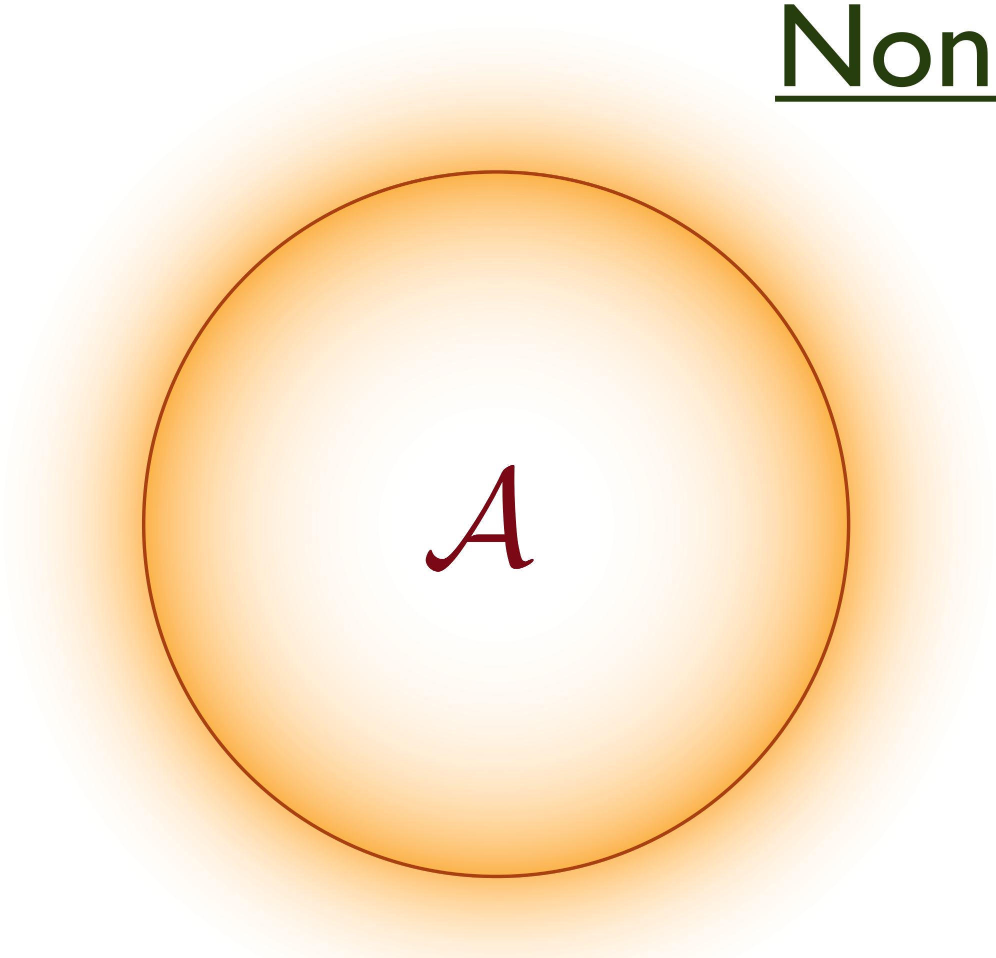

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- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
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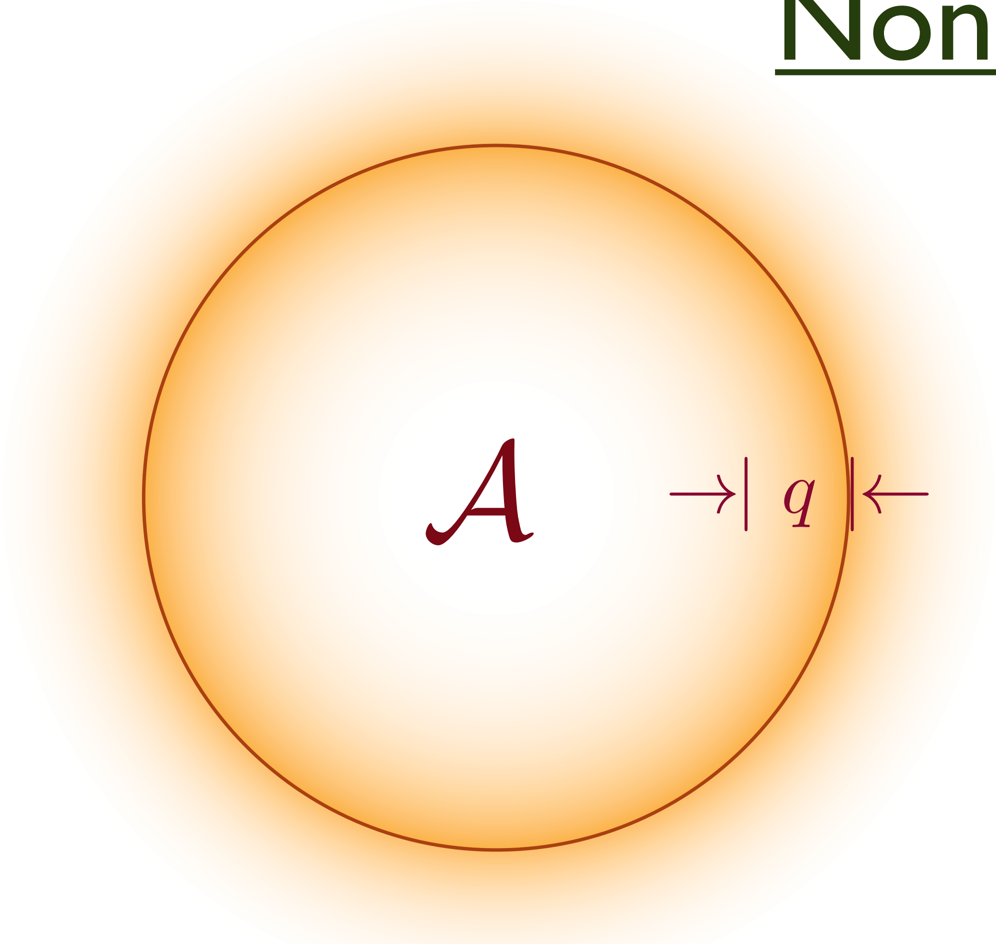

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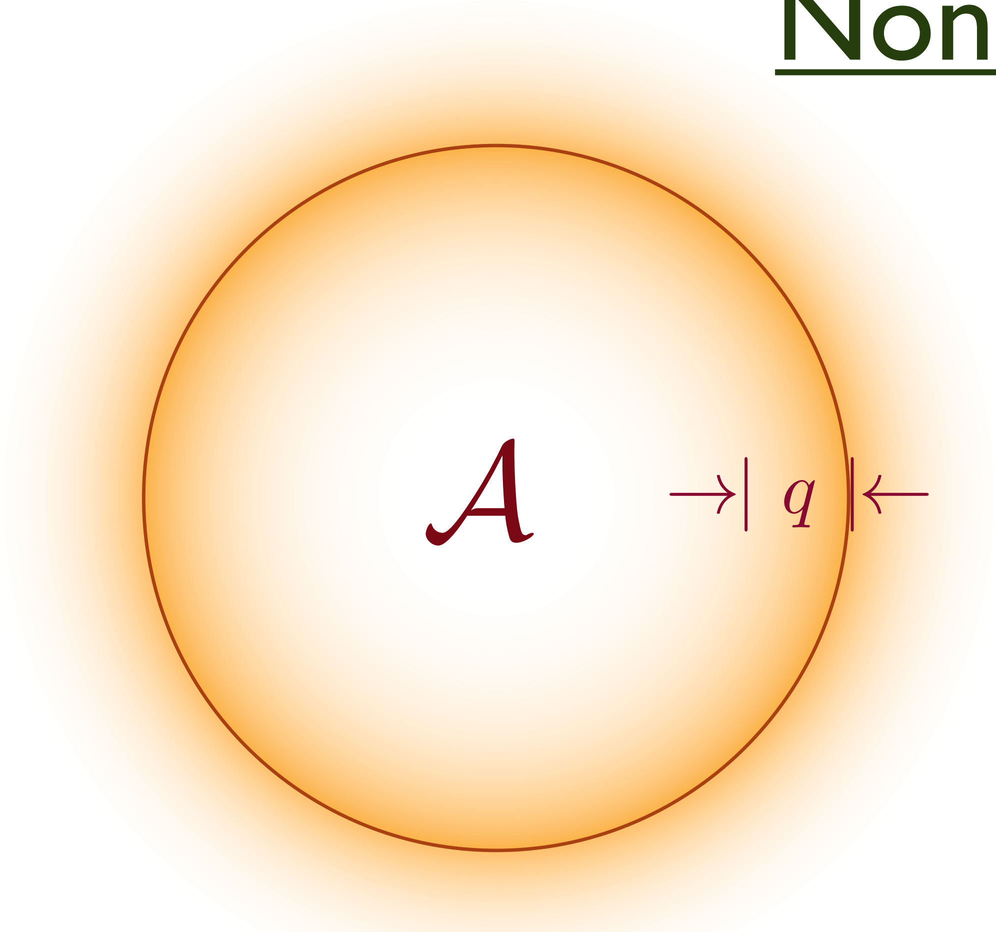

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Non-Fermi liquids

Simple scaling argument for $z = 3/2$.

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$
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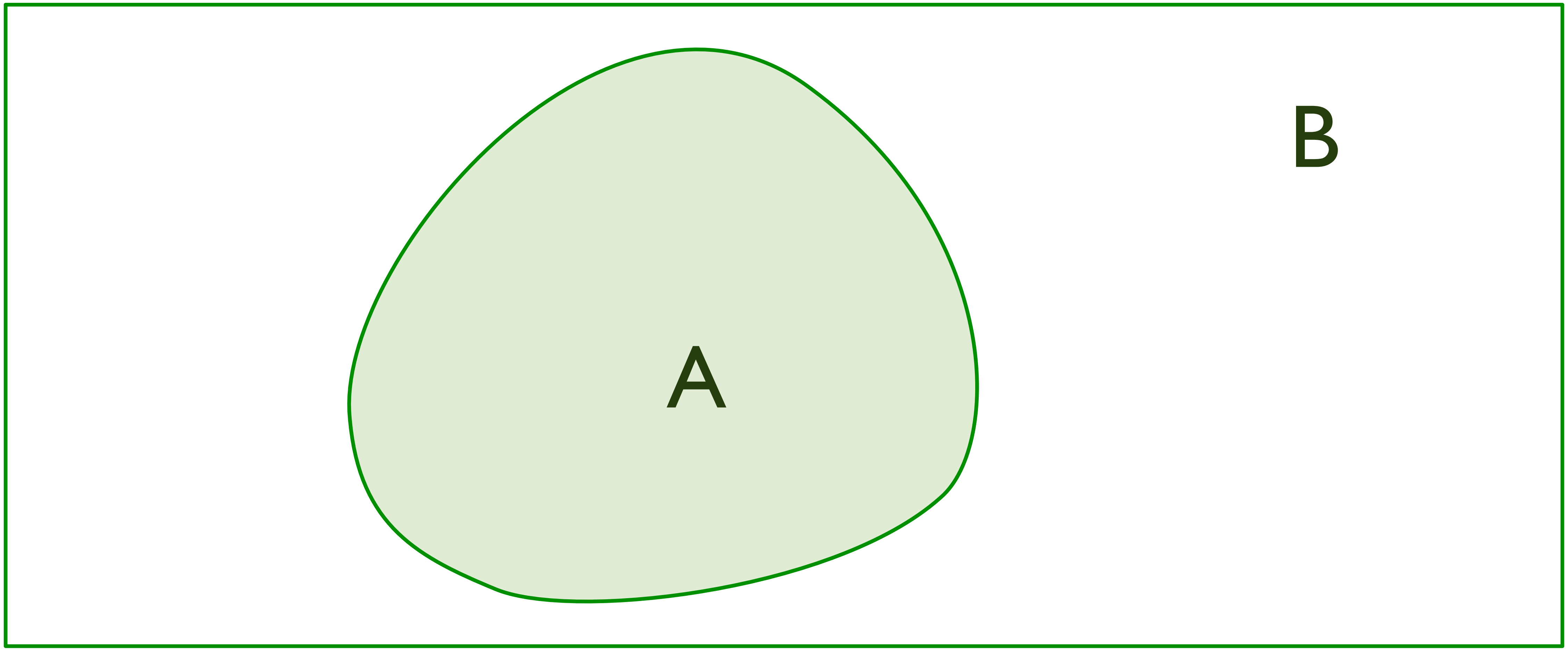
$$-g\phi \left(\psi_+^\dagger \psi_+ \pm \psi_-^\dagger \psi_- \right) + \left(\partial_y \phi \right)^2$$

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} \phi &\to \phi\, s^{(2z+1)/4} \\ \psi &\to \psi\, s^{(2z+1)/4} \\ g &\to g\, s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_E = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
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Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

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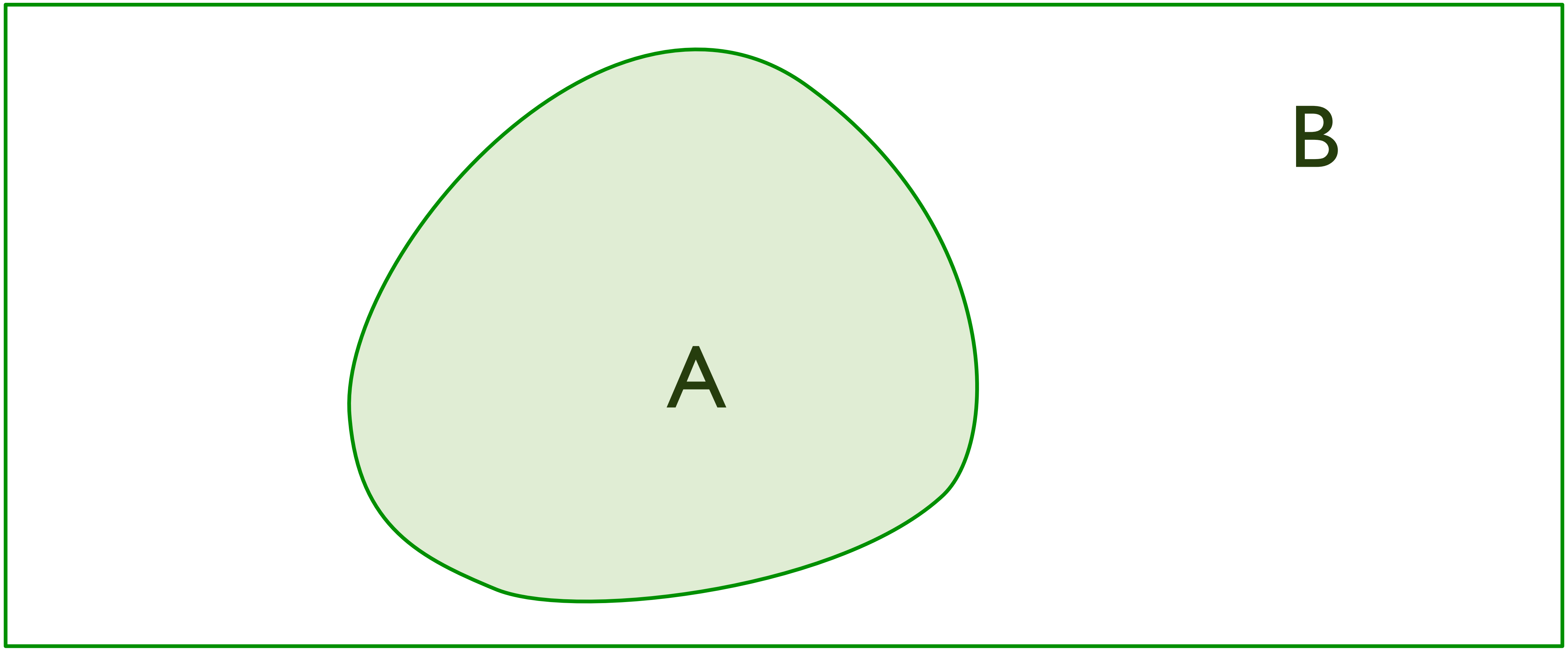

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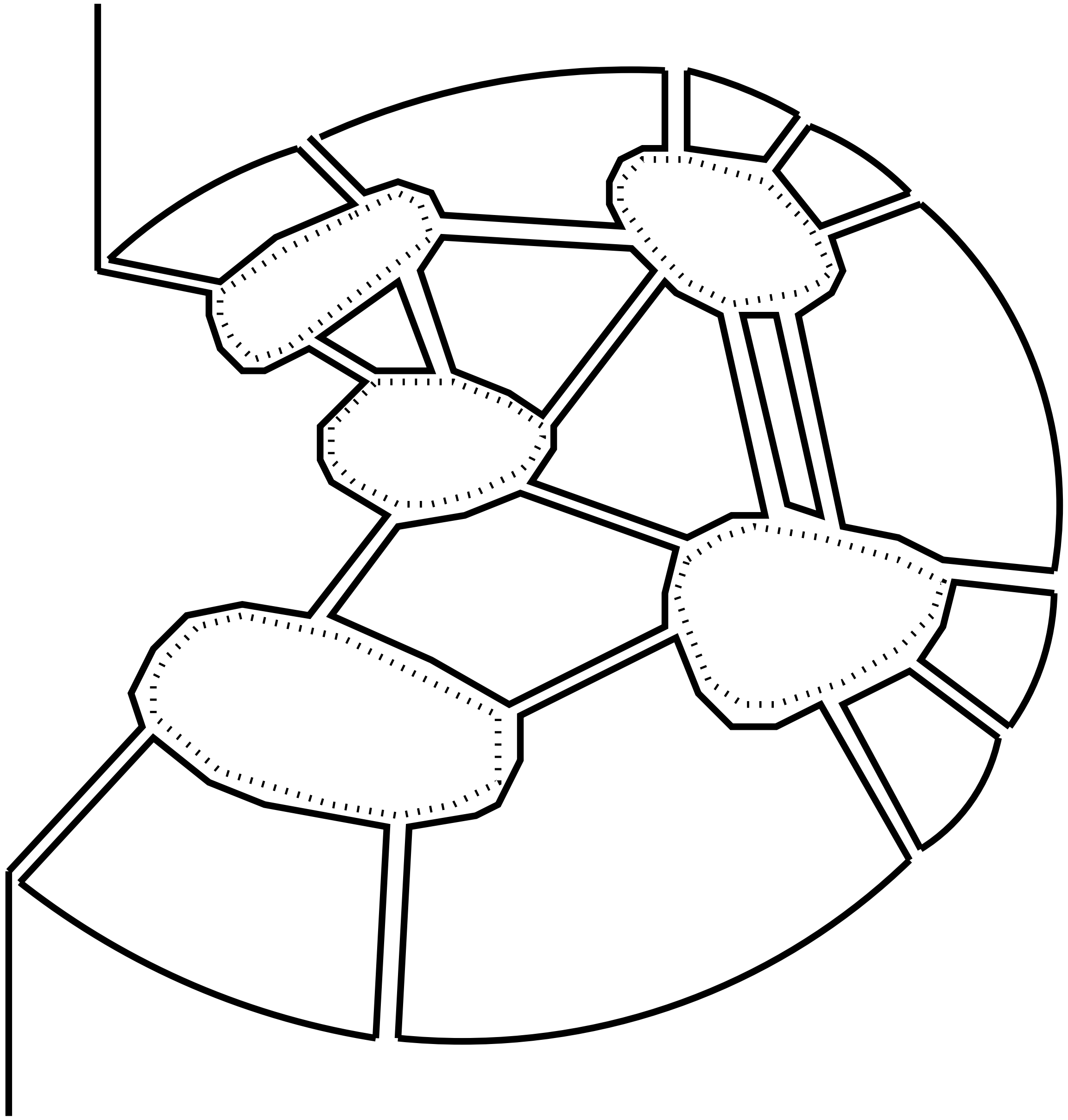
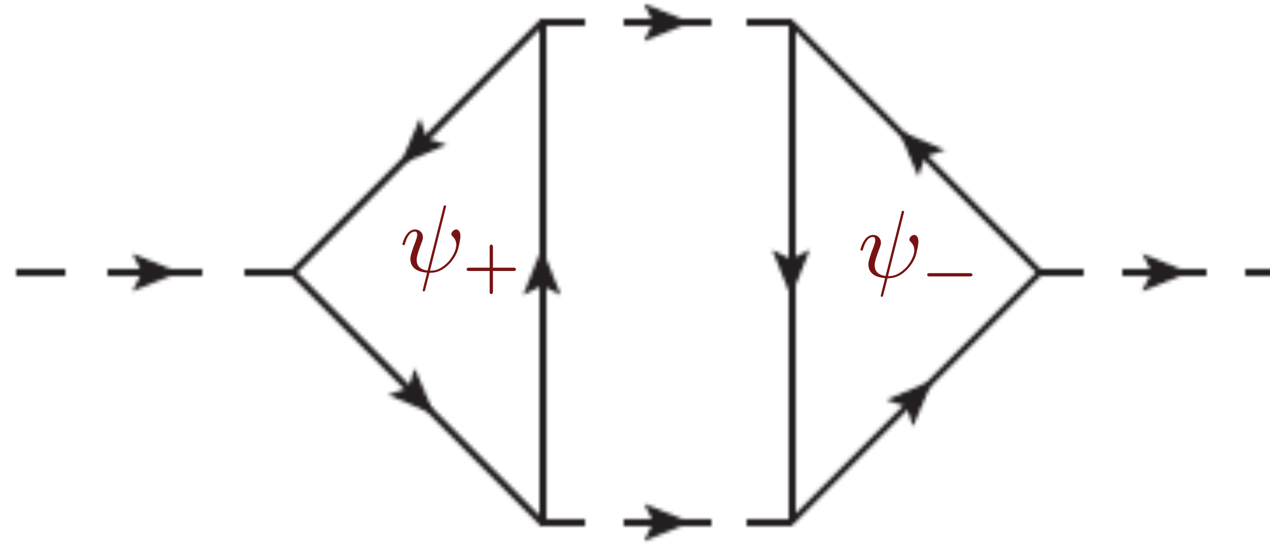
Computations in the 1/N expansion



All planar graphs of ψ_+ alone are as important as the leading term



Graph mixing ψ_+ and ψ_- is $\mathcal{O}\left(N^{3/2}\right)$ (instead of $\mathcal{O}\left(N\right)$), violating genus expansion

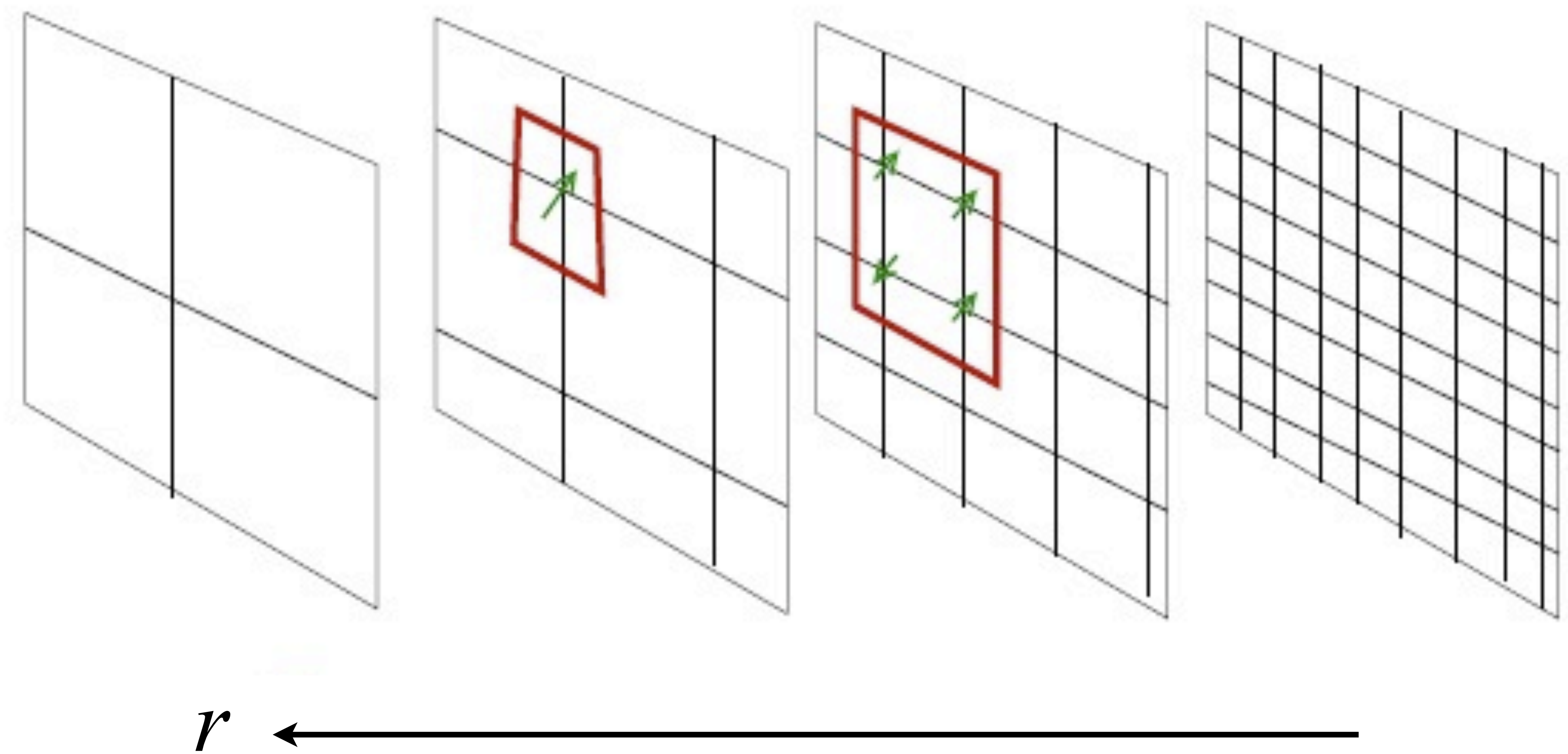
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Compressible quantum matter

A. Field theory

B. Holography


r

Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

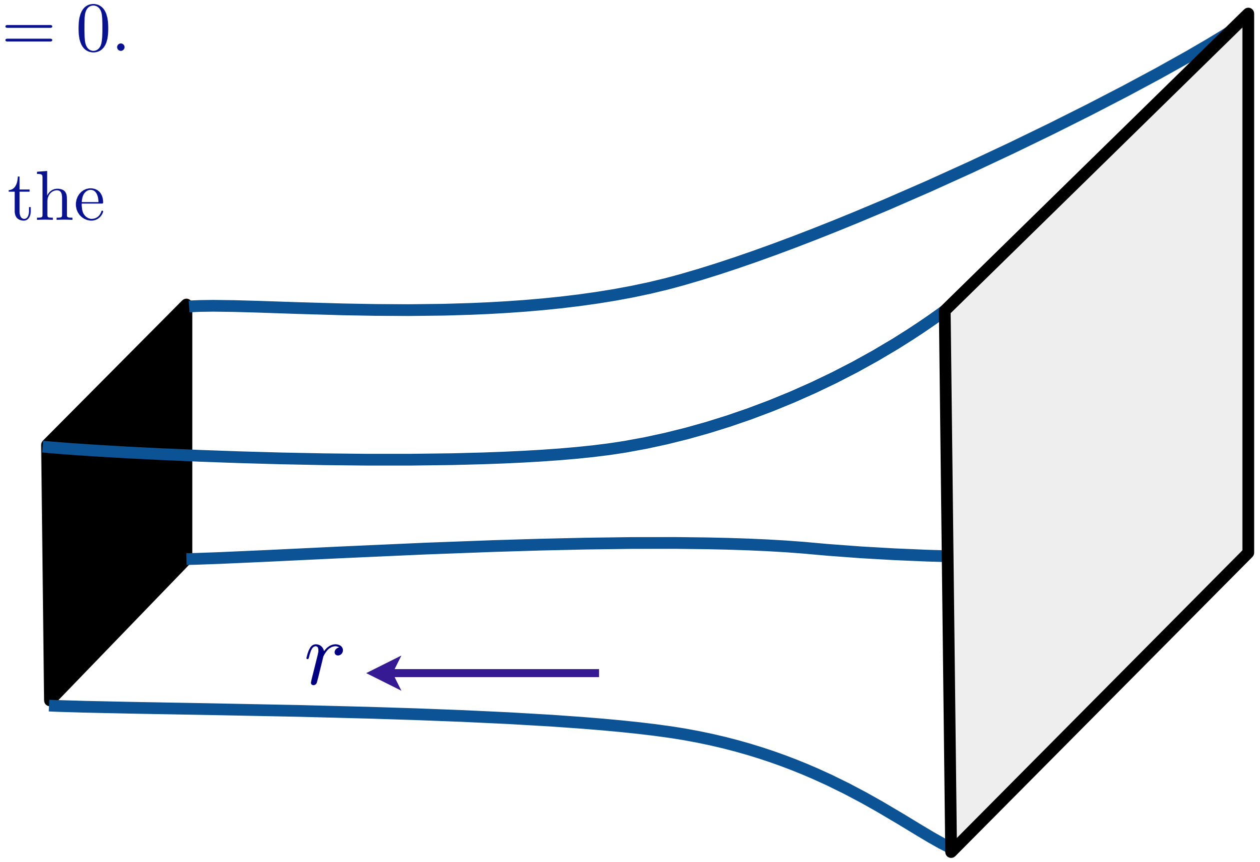
We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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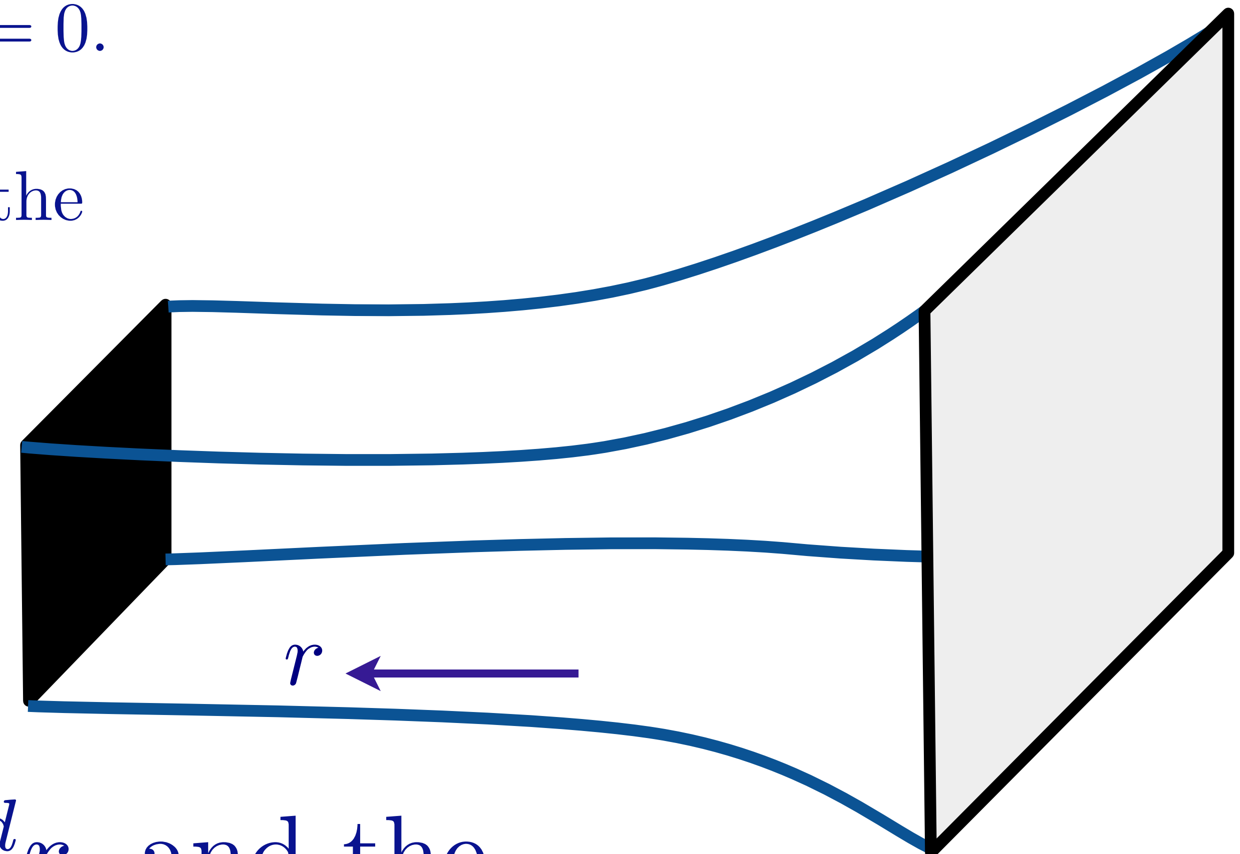
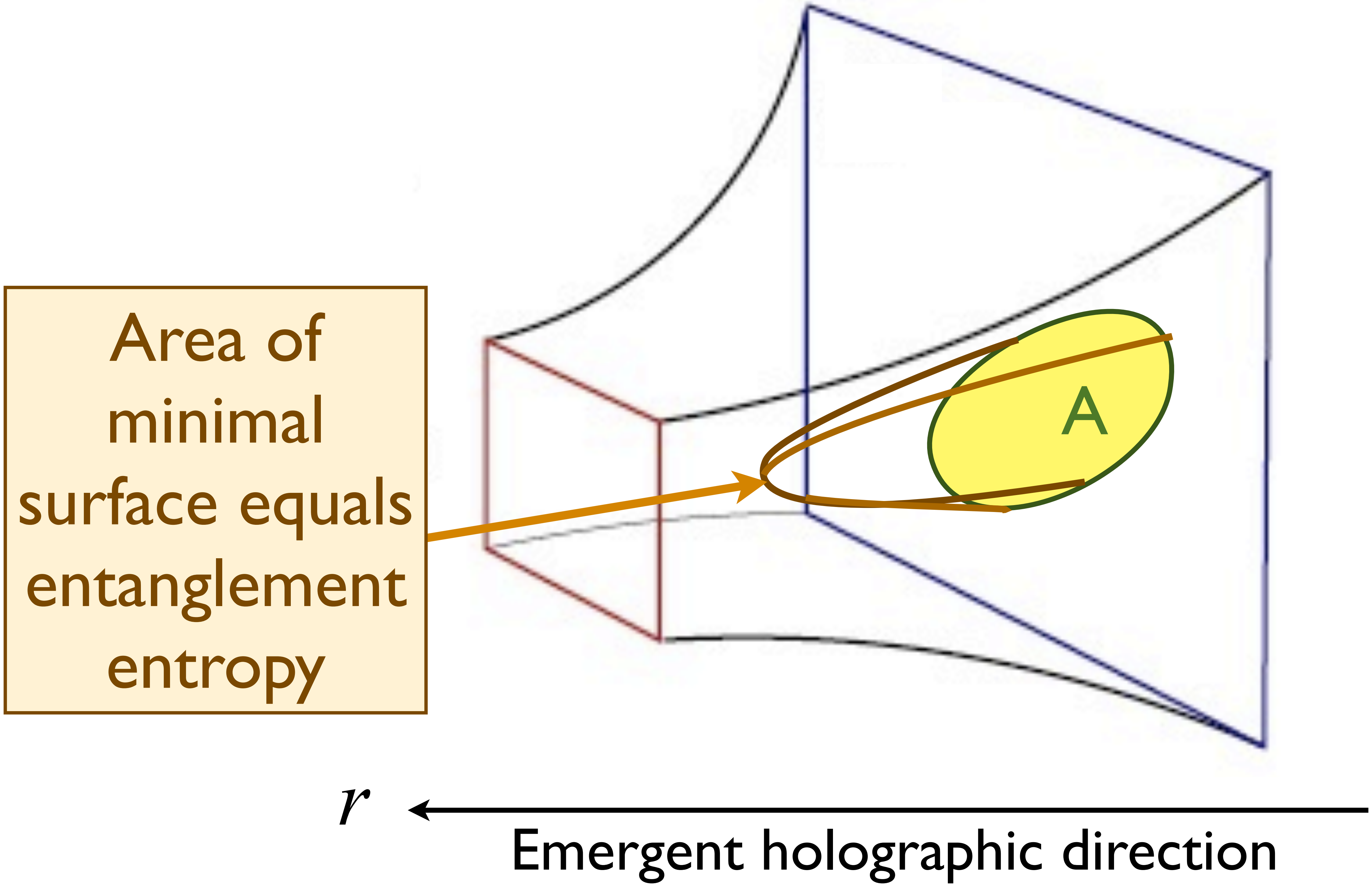


Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\rm eff}/z}$$

where $\theta = d - d_{\rm eff}$ measures "dimension deficit" in the phase space of low energy degrees of a freedom.

Holographic entanglement entropy



S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

- The thermal entropy density scales as

$$S \sim T^{(d-\theta)/z}.$$

  The third law of thermodynamics requires $\theta < d$.

- The entanglement entropy, S_E, of an entangling region with boundary surface 'area' P scales as

$$S_E \sim \begin{cases} P & , \quad \text{for } \theta < d-1 \\ P \ln P & , \quad \text{for } \theta = d-1 \\ P^{\theta/(d-1)} & , \quad \text{for } \theta > d-1 \end{cases}$$

  All local quantum field theories obey the "area law" (upto log violations) and so $\theta \leq d-1$.

- The null energy condition implies $z \geq 1 + \dfrac{\theta}{d}$.

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

- The value of θ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general d.
  Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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- The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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$$\theta = d - 1$$

- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!

- The logarithmic violation is of the form $P \ln P$, where P is the perimeter of the entangling region. This form is *independent* of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

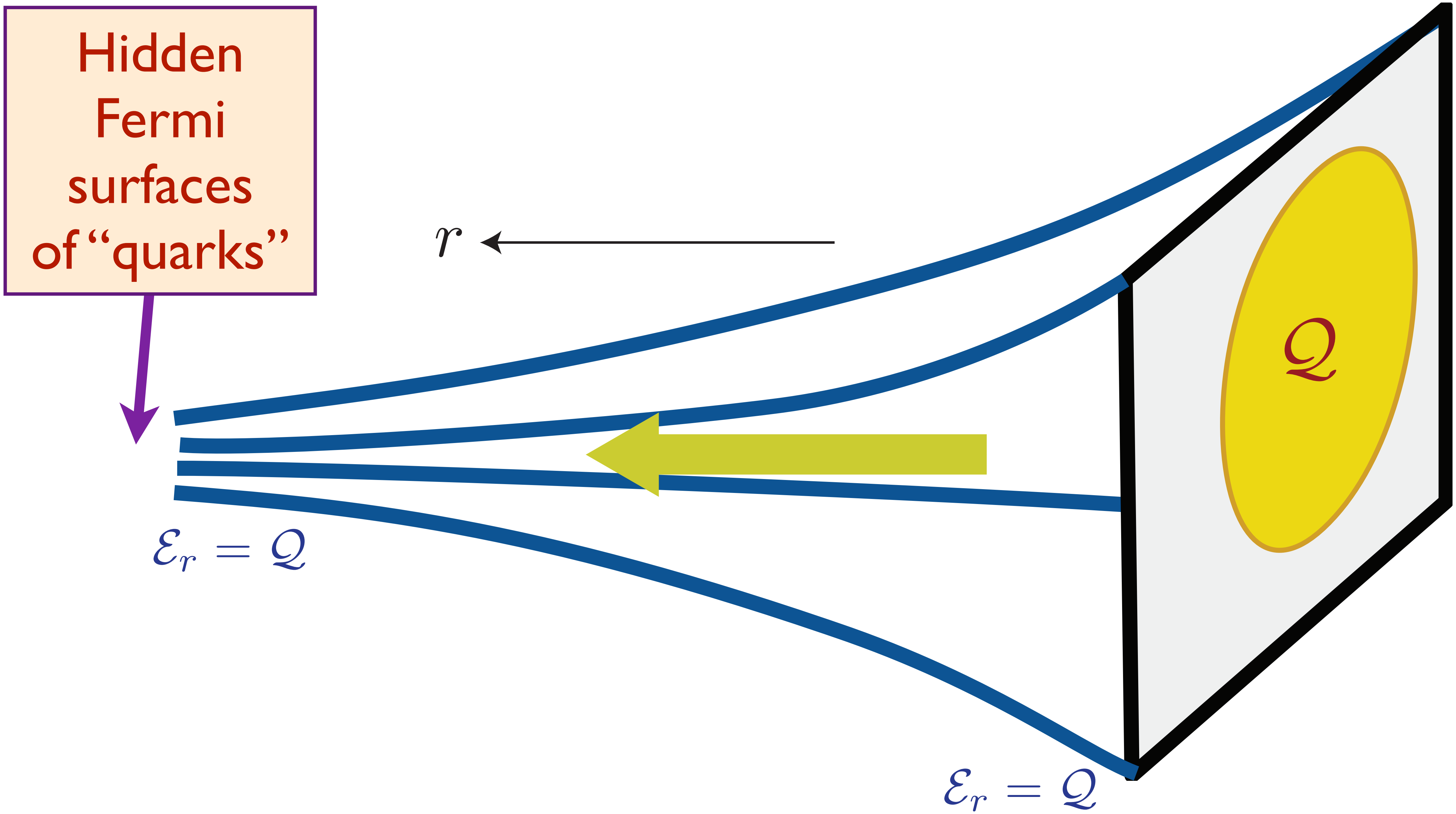
- This metric can be realized in a Maxwell-Einstein-dilaton theory, which may be viewed as a "bosonization" of the non-Fermi liquid state. The entanglement entropy of this theory has log-violation of the area law with

$$S_E = \Xi\, \mathcal{Q}^{(d-1)/d} P \ln P.$$

where P is surface area of the entangling region, and Ξ is a dimensionless constant which is independent of all UV details, of $\mathcal{Q}$, and of any property of the entangling region.
Note $\mathcal{Q}^{(d-1)/d} \sim k_F^{d-1}$ via the Luttinger relation, and then S_E is just as expected for a Fermi surface !!!!

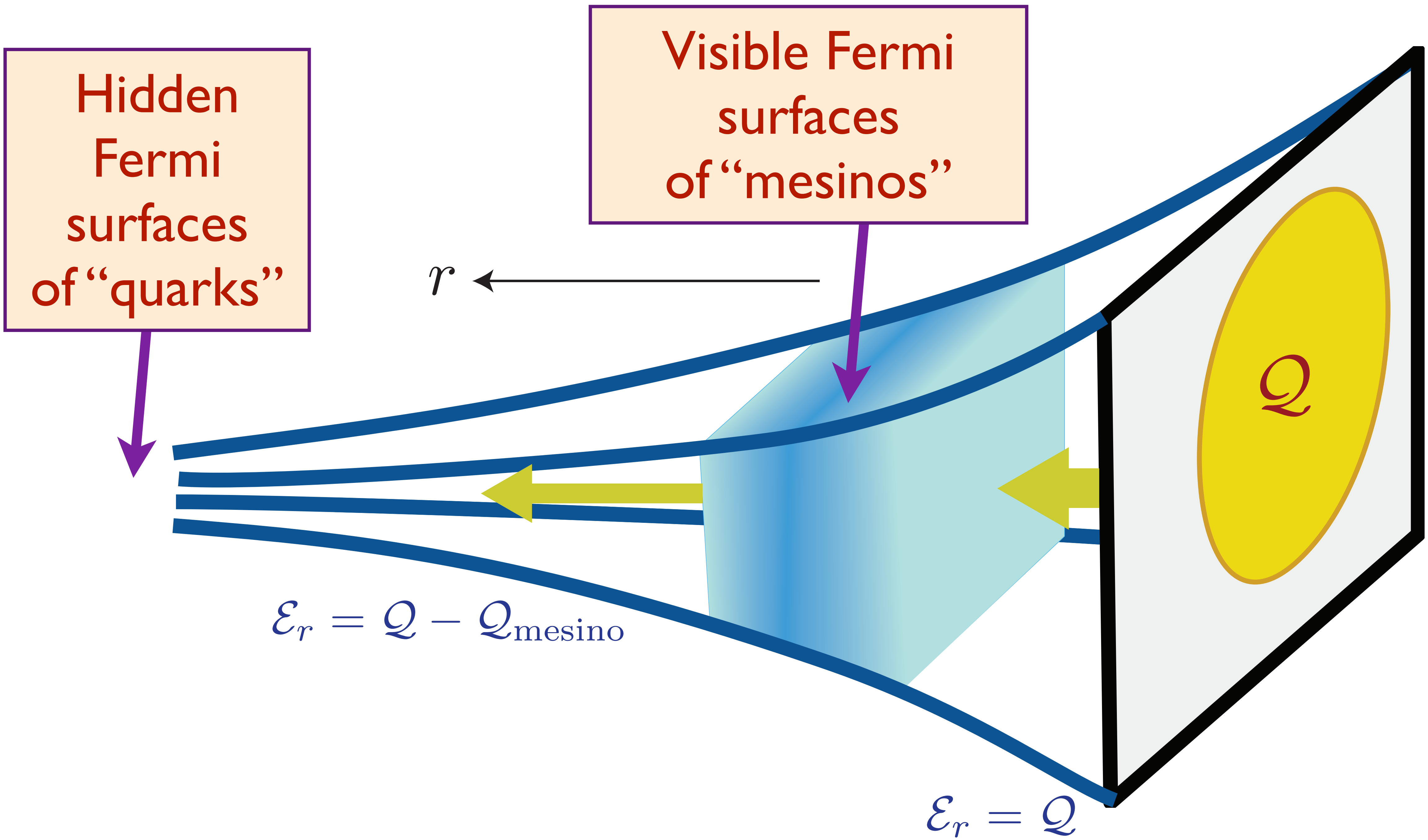
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holographic theory of a non-Fermi liquid (NFL)



Gauss Law and the "attractor" mechanism
$\Leftrightarrow$ Luttinger theorem on the boundary

Holographic theory of a fractionalized-Fermi liquid (FL*)



A state with *partial* confinement

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)
S. Sachdev, *Physical Review D* **84**, 066009 (2011)

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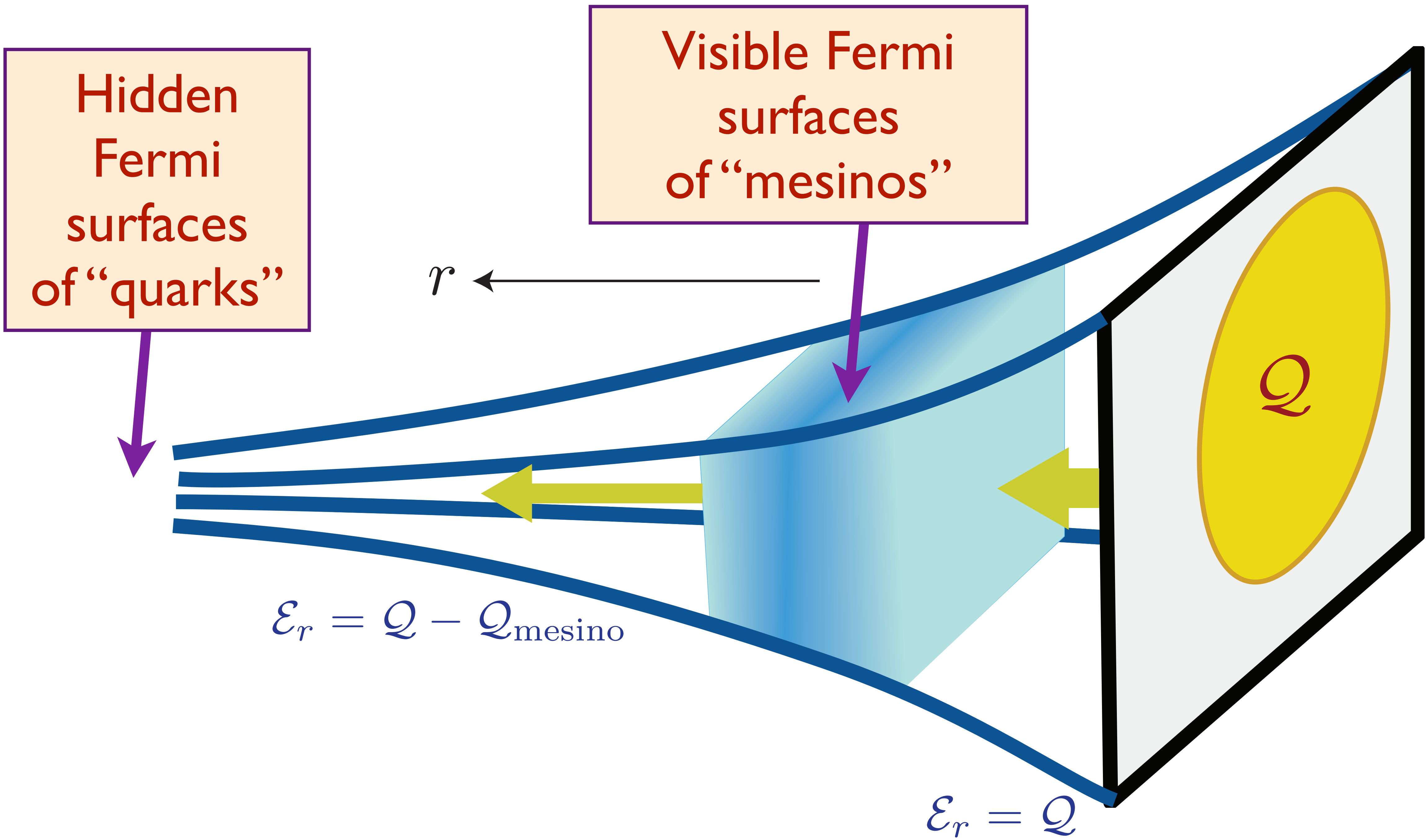
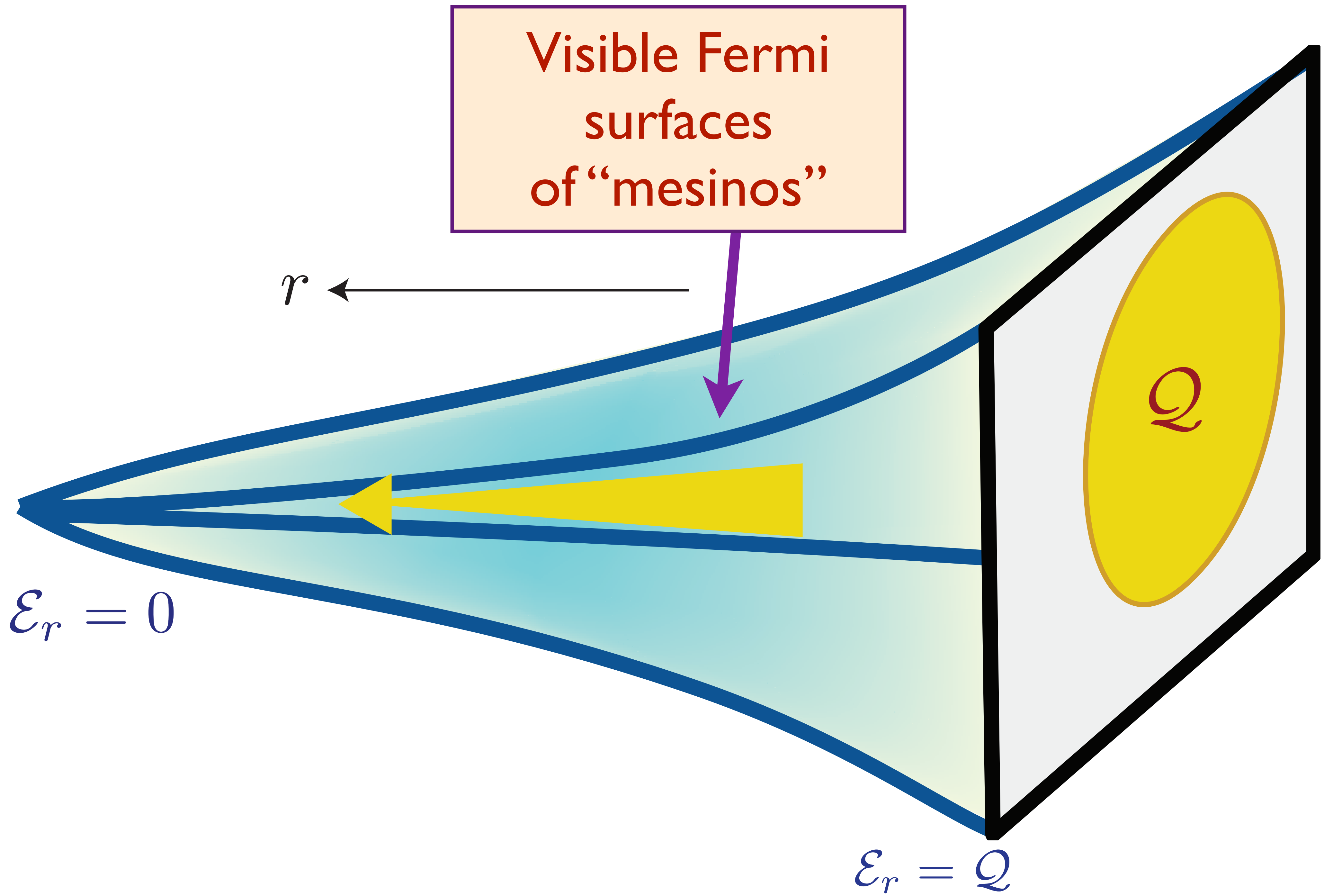


The "mesinos" corresponds to the Fermi surfaces obtained in the early probe fermion computation (S.-S. Lee, Phys. Rev. D **79**, 086006 (2009); H. Liu, J. McGreevy, and D. Vegh, arXiv:0903.2477; M. Čubrović, J. Zaanen, and K. Schalm, Science **325**, 439 (2009)).

These are spectators, and are expected to have well-defined quasiparticle excitations.

Holographic theory of a Fermi liquid (FL)



- Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D **84**, 066009 (2011)

Conclusions

Compressible quantum matter

- Field theory of a non-Fermi liquid obtained by coupling a Fermi surface to a gapless scalar field with low energy excitations near zero wavevector

Conclusions

Compressible quantum matter

- Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.

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- Evidence for Luttinger theorem in prefactor of S_E.